

JEE Main 2024 Paper 1 (B.E./B.Tech) Apr 6 Shift 2

Question Paper with Solutions PDF



Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours

Maximum Marks: 300

General Instructions

- (i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 6 April 2024, Shift 2. The total duration is 3 hours.
- (ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.
- (iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.
- (iv) Marking scheme (multiple choice questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (v) Marking scheme (numerical value questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If the function $f(x) = \left(\frac{1}{x}\right)^{2x}$; $x > 0$ attains the maximum value at $x = \frac{1}{e}$ then :

[4 marks]

- (A) $e^\pi < \pi^e$
- (B) $e^{2\pi} < (2\pi)^e$
- (C) $(2e)^\pi > \pi^{(2e)}$
- (D) $e^\pi > \pi^e$

Correct Answer: (D) $e^\pi > \pi^e$

Solution:

Concept:

- For positive bases, compare powers by taking logarithms: $a^b > b^a$ is equivalent to $\log a/a > \log b/b$. The logarithm is strictly increasing, so this preserves the original comparison.
- For $h(t) = \log t/t$, the derivative is $h'(t) = (1 - \log t)/t^2$. Thus h increases up to $t = e$ and decreases strictly afterwards.

Step 1: Relate the given maximum to a logarithmic function The function is positive on its domain, and $\log f(x) = -2x \log x$. Its derivative is $-2(1 + \log x)$, which changes from positive to negative at $x = 1/e$. Equivalently, putting $t = 1/x$ gives $\log f(1/t) = 2 \log t/t = 2h(t)$. Hence the maximum mentioned in the question corresponds to the maximum of h at $t = e$.

Step 2: Compare the bases and eliminate the other inequalities Since $\pi > e$, strict decrease to the right of e gives

$$\frac{1}{e} > \frac{\log \pi}{\pi} \implies \pi > e \log \pi.$$

Exponentiating yields $e^\pi > \pi^e$, so (D) holds and (A) reverses the correct inequality. The same comparison with $2\pi > e$ gives $e^{2\pi} > (2\pi)^e$, excluding (B). Finally $e < \pi < 2e$ implies $h(\pi) > h(2e)$, or $\pi^{2e} > (2e)^\pi$, excluding (C).

Step 3: Conclusion The required comparison is $e^\pi > \pi^e$. Hence, the correct option is (D).

Quick Tip to Solve: To compare a^b with b^a , compare $\log a/a$ with $\log b/b$; remember that this function decreases for bases greater than e .

2. If z_1, z_2 are two distinct complex number such that $\left| \frac{z_1 - 2z_2}{\frac{1}{2} - z_1 \bar{z}_2} \right| = 2$, then

[4 marks]

- (A) either z_1 lies on a circle of radius $\frac{1}{2}$ or z_2 lies on a circle of radius 1.
(B) z_1 lies on a circle of radius $\frac{1}{2}$ and z_2 lies on a circle of radius 1.
(C) either z_1 lies on a circle of radius 1 or z_2 lies on a circle of radius $\frac{1}{2}$.
(D) both z_1 and z_2 lie on the same circle.

Correct Answer: (C) either z_1 lies on a circle of radius 1 or z_2 lies on a circle of radius $\frac{1}{2}$.

Solution:

Concept:

- For any complex number w , $|w|^2 = w\bar{w}$. A quotient is defined only when its denominator is nonzero; this restriction must be retained when squaring the given modulus equation.
- The locus $|z| = r$ is the circle of radius r centred at the origin. A product of real factors equals zero precisely when at least one factor vanishes.

Step 1: Square the modulus and expand both sides Write $a = |z_1|^2$, $b = |z_2|^2$ and $c = \operatorname{Re}(z_1 \bar{z}_2)$. The given quotient has modulus 2, so

$$|z_1 - 2z_2|^2 = 4 \left| \frac{1}{2} - z_1 \bar{z}_2 \right|^2.$$

The left side is $a + 4b - 4c$. The square on the right is $1/4 + ab - c$, before multiplication by 4. Thus the mixed real-part terms cancel exactly, leaving $a + 4b = 1 + 4ab$.

Step 2: Factor and interpret the alternatives Rearrange the remaining equation as

$$a - 1 + 4b - 4ab = (a - 1)(1 - 4b) = 0.$$

Therefore $a = 1$ or $b = 1/4$, giving $|z_1| = 1$ or $|z_2| = 1/2$. These are alternatives, not simultaneous requirements. They are subject to the original nonzero-denominator condition. The radii in (A) and (B) are interchanged. To see why the same circle is not forced in the intended origin-centred sense, take $z_1 = 1, z_2 = 0$: the quotient equals 2 and the moduli differ. Taking $z_1 = 0, z_2 = 1/2$ also satisfies the equation, demonstrating that the second branch is possible independently.

Step 3: Conclusion Either z_1 lies on the origin-centred circle of radius 1, or z_2 lies on the origin-centred circle of radius $1/2$. Hence, the correct option is (C).

Quick Tip to Solve: Expand squared moduli before attempting geometric reasoning: the common real-part term cancels, exposing the two possible radii.

3. Let $A = \{1, 2, 3, 4, 5\}$. Let R be a relation on A defined by xRy if and only if $4x \leq 5y$. Let m be the number of elements in R and n be the minimum number of elements from $A \times A$ that are required to be added to R to make it a symmetric relation. Then $m + n$ is equal to : [4 marks]

- (A) 23
(B) 26
(C) 25
(D) 24

Correct Answer: (C) 25

Solution:

Concept:

- A relation on A is a subset of $A \times A$. It is symmetric when $(x, y) \in R$ implies $(y, x) \in R$ for every ordered pair.
- The smallest symmetric relation containing R is $R \cup R^{-1}$. Only missing reverse pairs must be added; pairs already present, including diagonal pairs, need no additions.

Step 1: Count the ordered pairs already in the relation For a fixed x , the allowed values of y satisfy $y \geq 4x/5$, with $y \in \{1, 2, 3, 4, 5\}$. For $x = 1, 2, 3, 4, 5$, respectively, the allowed sets are $\{1, 2, 3, 4, 5\}$, $\{2, 3, 4, 5\}$, $\{3, 4, 5\}$, $\{4, 5\}$, and $\{4, 5\}$. Notice particularly that $(5, 4)$ belongs to the relation, since $20 \leq 20$. Consequently,

$$m = 5 + 4 + 3 + 2 + 2 = 16.$$

Step 2: Add exactly the missing reverse pairs Whenever $x \leq y$, the inequality $4x \leq 5y$ holds. Thus all upper-triangular pairs, and all five diagonal pairs, are present. Among the ten pairs with $x > y$, only $(5, 4)$ is already present. Each of the other nine lower-triangular pairs is the reverse of an existing upper-triangular pair and must be added. Adding them makes the relation all of $A \times A$, which is symmetric. No smaller addition can suffice, because each missing reverse is individually required. Hence $n = 9$ and $m + n = 16 + 9 = 25$. Values 23, 24 and 26 fail this explicit pair count.

Step 3: Conclusion $m + n = 25$. Hence, the correct option is (C).

Quick Tip to Solve: Count missing reverse pairs, not all off-diagonal pairs. Here every upper-triangular pair exists, so the symmetric closure is the entire Cartesian product.

4. Let $f(x) = \frac{1}{7 - \sin 5x}$ be a function defined on $\mathbb{R}$. Then the range of the function $f(x)$ is equal to : [4 marks]

- (A) $[\frac{1}{8}, \frac{1}{5}]$
(B) $[\frac{1}{7}, \frac{1}{6}]$
(C) $[\frac{1}{7}, \frac{1}{5}]$
(D) $[\frac{1}{8}, \frac{1}{6}]$

Correct Answer: (D) $[\frac{1}{8}, \frac{1}{6}]$

Solution:

Concept:

- For real x , the sine function assumes every value in $[-1, 1]$. Multiplying its argument by 5 changes its period but does not change this range.
- The reciprocal function $1/t$ is continuous and strictly decreasing for $t > 0$. Taking reciprocals of positive bounds reverses their order.

Step 1: Bound the denominator and check the domain Begin with $-1 \leq \sin 5x \leq 1$. Subtracting this quantity from 7 gives

$$6 \leq 7 - \sin 5x \leq 8.$$

In particular, the denominator never vanishes and remains positive for every real x . Thus there is no excluded point that could remove an endpoint or divide the range into separate intervals. The smallest denominator is 6, obtained when $\sin 5x = 1$; the largest denominator is 8, obtained when $\sin 5x = -1$.

Step 2: Take reciprocals and establish attainment Using positivity of the denominator, reverse the two bounds when taking reciprocals:

$$\frac{1}{8} \leq \frac{1}{7 - \sin 5x} \leq \frac{1}{6}.$$

Both endpoints occur: $x = \pi/10$ gives $1/6$, while $x = -\pi/10$ gives $1/8$. As sine takes every intermediate value and the reciprocal is continuous on $[6, 8]$, every value between these endpoints is attained. Options (A) and (C) incorrectly include $1/5$, which would require denominator 5. Option (B) omits attainable values below $1/7$, including the minimum $1/8$. Hence only (D) gives the full range.

Step 3: Conclusion The range is $[\frac{1}{8}, \frac{1}{6}]$. Hence, the correct option is (D).

Quick Tip to Solve: Find the denominator range first, verify that it stays positive, and then reverse the endpoint order when taking reciprocals.

5. $\lim_{n \rightarrow \infty} \frac{(1^2-1)(n-1) + (2^2-2)(n-2) + \dots + ((n-1)^2-(n-1)) \cdot 1}{(1^3+2^3+\dots+n^3)-(1^2+2^2+\dots+n^2)}$ is equal to :

[4 marks]

- (A) $\frac{2}{3}$
 (B) $\frac{1}{2}$
 (C) $\frac{3}{4}$
 (D) $\frac{1}{3}$

Correct Answer: (D) $\frac{1}{3}$

Solution:

Concept:

- The numerator is a finite sum $\sum_{k=1}^{n-1} (k^2 - k)(n - k)$. Expanding its summand reduces it to standard sums of the first three powers.
- The identities are $\sum_{k=1}^r k = r(r+1)/2$, $\sum_{k=1}^r k^2 = r(r+1)(2r+1)/6$, and $\sum_{k=1}^r k^3 = r^2(r+1)^2/4$.

Step 1: Evaluate and factor the numerator Let N_n denote the numerator. Expanding before summing gives

$$N_n = (n+1) \sum_{k=1}^{n-1} k^2 - n \sum_{k=1}^{n-1} k - \sum_{k=1}^{n-1} k^3.$$

Insert the standard formulae with upper limit $n-1$, obtaining

$$\begin{aligned} N_n &= \frac{(n+1)n(n-1)(2n-1)}{6} - \frac{n^2(n-1)}{2} - \frac{n^2(n-1)^2}{4} \\ &= \frac{n(n-1)(n+1)(n-2)}{12}. \end{aligned}$$

Step 2: Simplify the denominator and take the limit The denominator has upper limit n , rather than $n-1$. Therefore

$$\begin{aligned} D_n &= \frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6} \\ &= \frac{n(n+1)(n-1)(3n+2)}{12}. \end{aligned}$$

For $n > 1$, cancel the common nonzero factors to obtain $N_n/D_n = (n-2)/(3n+2)$. Dividing numerator and denominator by n now gives the limit $1/3$. As a check, the leading fourth-degree coefficients are $1/12$ and $1/4$; their ratio is also $1/3$, excluding the three larger fractions in (A), (B), and (C).

Step 3: Conclusion The limit is $\boxed{1/3}$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the different upper limits straight: the numerator stops at $n-1$, while the denominator stops at n . Expand the product before applying power-sum formulae.

6. If A is a square matrix of order 3 such that $\det(A) = 3$ and $\det(\operatorname{adj}(-4 \operatorname{adj}(-3 \operatorname{adj}(3 \operatorname{adj}((2A)^{-1})))) = 2^m 3^n$, then $m + 2n$ is equal to : [4 marks]

- (A) 2
(B) 4
(C) 3
(D) 6

Correct Answer: (B) 4

Solution:

Concept:

- For a square matrix M of order 3, $\det(\operatorname{adj} M) = (\det M)^2$. Also $\det(cM) = c^3 \det M$, so a scalar multiplier must be cubed before applying an adjugate.
- Since $\det A = 3 \neq 0$, the inverse exists and $\det((2A)^{-1}) = 1/(2^3 \det A) = 2^{-3} 3^{-1}$. Track powers of 2 and 3 separately through the nested operations.

Step 1: Work through the first two adjugates Let $B = (2A)^{-1}$, and denote $C = \operatorname{adj} B$. The determinant of C is

$$\det C = (2^{-3} 3^{-1})^2 = 2^{-6} 3^{-2}.$$

The next inner matrix is $D = \operatorname{adj}(3C)$. Apply scalar multiplication first and the adjugate second:

$$\det D = (3^3 \det C)^2 = (2^{-6} 3)^2 = 2^{-12} 3^2.$$

Step 2: Complete the nesting and read off the exponents Write $E = \operatorname{adj}(-3D)$ and $F = \operatorname{adj}(-4E)$, where $\det F$ is the required expression. Then

$$\det E = ((-3)^3 \det D)^2 = (-2^{-12} 3^5)^2 = 2^{-24} 3^{10},$$

followed by

$$\det F = ((-4)^3 \det E)^2 = (-2^{-18} 3^{10})^2 = 2^{-36} 3^{20}.$$

The negative scalar signs disappear when their determinants are squared, but their magnitudes still contribute. Comparing with $2^m 3^n$ gives $m = -36$ and $n = 20$. Thus $m + 2n = -36 + 40 = 4$, not 2, 3 or 6. In particular, the exponents need not both be positive: the inverse at the start contributes negative exponents.

Step 3: Conclusion $m = -36$, $n = 20$, so $m + 2n = 4$. Hence, the correct option is (B).

Quick Tip to Solve: For each layer $\operatorname{adj}(cM)$, update the determinant by $d \mapsto (c^3 d)^2$; this avoids expanding any matrix entries.

7. If three letters can be posted to any one of the 5 different addresses, then the probability that the three letters are posted to exactly two addresses is : [4 marks]

- (A) $\frac{18}{25}$
(B) $\frac{12}{25}$
(C) $\frac{6}{25}$
(D) $\frac{4}{25}$

Correct Answer: (B) $\frac{12}{25}$

Solution:

Concept:

- Each of the three distinct letters has five independent address choices. Under equal likelihood, the sample space consists of 5^3 assignments, and probability is favourable assignments divided by total assignments.
- Exactly two occupied addresses means the letters are distributed as two letters at one address and one letter at the other. The addresses are distinct, so choosing which address gets the pair is necessary.

Step 1: Count all assignments and choose the used addresses The total number of ways to address the three letters is $5 \times 5 \times 5 = 125$. Choose the unordered pair of addresses that receive letters in $\binom{5}{2} = 10$ ways. For any selected pair, there are $2^3 = 8$ assignments of the three letters to those addresses. Two of these put all letters at one address, so exactly $8 - 2 = 6$ assignments use both selected addresses. This subtraction enforces the word exactly in the question.

Step 2: Calculate the probability and cross-check the count The favourable count is therefore $10 \times 6 = 60$. An equivalent direct count chooses the address receiving two letters in 5 ways, chooses those two letters in $\binom{3}{2} = 3$ ways, and chooses the different address for the remaining letter in 4 ways: $5 \times 3 \times 4 = 60$. There is no overcount because the address receiving the pair is uniquely determined by each favourable assignment. Thus

$$P = \frac{60}{125} = \frac{12}{25}.$$

For a completeness check, exactly one occupied address has 5 assignments and three distinct addresses have $5 \times 4 \times 3 = 60$; together with our 60 they total 125. The other proposed probabilities do not match the favourable count.

Step 3: Conclusion The required probability is $\boxed{12/25}$. Hence, the correct option is (B).

Quick Tip to Solve: With three letters and exactly two used addresses, identify the unique address receiving two letters; counting the pair first prevents double counting.

8. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = ((\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i}) \times \hat{i}$. Then the square of the projection of $\vec{a}$ on $\vec{b}$ is: [4 marks]

- (A) $\frac{1}{3}$
(B) $\frac{1}{5}$
(C) 2
(D) $\frac{2}{3}$

Correct Answer: (C) 2

Solution:

Concept:

- The scalar projection of a vector $\vec{a}$ on a nonzero vector $\vec{b}$ is $(\vec{a} \cdot \vec{b})/|\vec{b}|$. Its square is therefore $(\vec{a} \cdot \vec{b})^2/|\vec{b}|^2$.
- Cross products must be evaluated in the parenthesised order: they are not associative. In components, $(u, v, w) \times (1, 0, 0) = (0, w, -v)$.

Step 1: Evaluate the innermost cross product Write the given vectors in Cartesian component form. The first cross product is

$$\vec{a} \times (\hat{i} + \hat{j}) = (2, 1, -1) \times (1, 1, 0) = (1, -1, 1).$$

For example, its first component is $1 \cdot 0 - (-1) \cdot 1 = 1$, while the third is $2 \cdot 1 - 1 \cdot 1 = 1$. Keeping these signs matters because two further cross products follow.

Step 2: Apply the remaining two cross products Cross the intermediate vector with $\hat{i}$, then cross that result with $\hat{i}$ once again:

$$\begin{aligned}(1, -1, 1) \times (1, 0, 0) &= (0, 1, 1), \\ \vec{b} &= (0, 1, 1) \times (1, 0, 0) = (0, 1, -1).\end{aligned}$$

Thus $\vec{b} = \hat{j} - \hat{k}$. Its length is $\sqrt{2}$, so this direction is well defined.

Step 3: Square the scalar projection Use the dot product only after determining the final vector $\vec{b}$:

$$\vec{a} \cdot \vec{b} = 2 \cdot 0 + 1 \cdot 1 + (-1)(-1) = 2, \quad |\vec{b}|^2 = 0^2 + 1^2 + (-1)^2 = 2.$$

Consequently,

$$\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \right)^2 = \frac{2^2}{2} = 2.$$

The fractions in (A), (B) and (D) do not equal this value; in particular, dividing by $|\vec{a}|^2$ would project onto the wrong vector.

Step 4: Conclusion The square of the projection is $\boxed{2}$. Hence, the correct option is (C).

Quick Tip to Solve: Evaluate nested cross products from the innermost brackets outward, then use the squared-projection formula to avoid unnecessary square roots.

9. Let $\vec{a} = 6\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $|\vec{c}| \geq 6$, $\vec{a} \cdot \vec{c} = 6|\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is 60° , then $|(\vec{a} \times \vec{b}) \times \vec{c}|$ is equal to: [4 marks]

- (A) $\frac{3}{2}\sqrt{6}$
 (B) $\frac{9}{2}(6 - \sqrt{6})$
 (C) $\frac{9}{2}(6 + \sqrt{6})$
 (D) $\frac{3}{2}\sqrt{3}$

Correct Answer: No vector satisfies all the stated conditions. Formal substitution yields option (C), the key answer, but the angle and dot-product premises are incompatible.

Solution:

Concept:

- The squared-distance identity is

$$|\vec{c} - \vec{a}|^2 = |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a} \cdot \vec{c}.$$

It converts the given distance into a quadratic in $|\vec{c}|$.

- The magnitude of a cross product is $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|\sin\theta$, where θ is their included angle.

Step 1: Determine the magnitude of the unknown vector Let $r = |\vec{c}|$. From $\vec{a} = (6, 1, -1)$, we have $|\vec{a}|^2 = 36 + 1 + 1 = 38$. Substitute the supplied dot product and distance:

$$8 = r^2 + 38 - 12r.$$

Hence

$$r^2 - 12r + 30 = 0 \implies (r - 6)^2 = 6.$$

The two algebraic possibilities are $6 + \sqrt{6}$ and $6 - \sqrt{6}$. The explicit condition $r \geq 6$ rejects the smaller one, leaving $r = 6 + \sqrt{6}$.

Step 2: Find the first cross-product magnitude Compute the vector from the given $\vec{a}$ and $\vec{b}$:

$$\vec{a} \times \vec{b} = (6, 1, -1) \times (1, 1, 0) = (1, -1, 5).$$

Its magnitude is

$$|\vec{a} \times \vec{b}| = \sqrt{1 + 1 + 25} = \sqrt{27} = 3\sqrt{3}.$$

This is the magnitude of the first factor in the final cross product; it is not the magnitude of $\vec{a}$ itself.

Step 3: Use the specified included angle The angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is 60° . Therefore

$$\begin{aligned} |(\vec{a} \times \vec{b}) \times \vec{c}| &= 3\sqrt{3}(6 + \sqrt{6}) \sin 60^\circ \\ &= 3\sqrt{3}(6 + \sqrt{6}) \frac{\sqrt{3}}{2} \\ &= \frac{9}{2}(6 + \sqrt{6}). \end{aligned}$$

Option (B) uses the inadmissible smaller root. Options (A) and (D) omit the required dependence on the admissible value of $|\vec{c}|$, and neither equals the computed magnitude.

Step 4: Check that the stated vector can exist Let $\vec{u} = \vec{c}/|\vec{c}|$, $\vec{e}_1 = \vec{a}/\sqrt{38}$, and $\vec{e}_2 = (\vec{a} \times \vec{b})/(3\sqrt{3})$. The vectors $\vec{e}_1, \vec{e}_2$ are orthogonal unit vectors. The dot-product condition gives $\vec{u} \cdot \vec{e}_1 = 6/\sqrt{38}$, while the stated angle gives $\vec{u} \cdot \vec{e}_2 = \cos 60^\circ = 1/2$. For any unit vector, the sum of squared components in two orthogonal unit directions cannot exceed 1. Here, however,

$$\left(\frac{6}{\sqrt{38}}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{18}{19} + \frac{1}{4} = \frac{91}{76} > 1.$$

Thus no real vector $\vec{c}$ can satisfy all the printed premises. The preceding calculation identifies the intended conditional option but does not establish that such a configuration exists.

Step 5: Conclusion Formal use of the given magnitude and angle yields $\frac{9}{2}(6 + \sqrt{6})$, corresponding to option (C), the answer key. Literally, the premises are incompatible, so there is no existing vector and no well-defined required magnitude.

Quick Tip to Solve: After calculating a vector length, check existence: the sum of squared projections of a unit vector onto orthogonal unit directions cannot exceed one.

10. If the area of the region $\{(x, y) : \frac{a}{x^2} \leq y \leq \frac{1}{x}, 1 \leq x \leq 2, 0 < a < 1\}$ is $(\log_e 2) - \frac{1}{7}$ then the value of $7a - 3$ is equal to : [4 marks]

- (A) 1
- (B) 0
- (C) 2
- (D) -1

Correct Answer: (D) -1

Solution:

Concept:

- For upper curve $y = u(x)$ and lower curve $y = l(x)$, the area of a vertical strip is $[u(x) - l(x)] dx$, so the total area is their difference integrated over the stated interval.
- The antiderivatives needed here are $\int x^{-1} dx = \log_e x$ for $x > 0$, and $\int x^{-2} dx = -x^{-1}$. The parameter a is constant during integration.

Step 1: Identify the upper and lower boundaries The region specifies $a/x^2 \leq y \leq 1/x$ and $1 \leq x \leq 2$. The assumption $0 < a < 1$ ensures that the ordering holds throughout: $a < x$, so $a/x^2 < 1/x$. There is no crossing within the integration interval and no need to split the integral. Thus its area is

$$A = \int_1^2 \left(\frac{1}{x} - \frac{a}{x^2} \right) dx.$$

Step 2: Evaluate the definite integral An antiderivative of the integrand is $\log_e x + a/x$, since

differentiating a/x gives $-a/x^2$. Evaluating at both endpoints gives

$$\begin{aligned} A &= \left[\log_e x + \frac{a}{x} \right]_1^2 \\ &= \left(\log_e 2 + \frac{a}{2} \right) - (0 + a) \\ &= \log_e 2 - \frac{a}{2}. \end{aligned}$$

The negative contribution of the lower curve is essential; adding the two curves would calculate a different quantity.

Step 3: Match the specified area and evaluate the expression The given area is $\log_e 2 - 1/7$. Equating it with the result cancels the logarithmic terms:

$$\frac{a}{2} = \frac{1}{7} \implies a = \frac{2}{7}.$$

This satisfies $0 < a < 1$, as required. Now

$$7a - 3 = 7 \left(\frac{2}{7} \right) - 3 = 2 - 3 = -1.$$

The values 1, 0 and 2 in the other options are different from the requested expression; 2 is only the intermediate value of $7a$.

Step 4: Conclusion The required value is $\boxed{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: Check the upper curve first, then cancel any common logarithmic term when comparing the calculated area with the given area.

11. Let $P(\alpha, \beta, \gamma)$ be the image of the point $Q(3, -3, 1)$ in the line $\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-1}{-1}$ and R be the point $(2, 5, -1)$. If the area of the triangle PQR is λ and $\lambda^2 = 14$ K, then K is equal to : [4 marks]

- (A) 18
- (B) 81
- (C) 72
- (D) 36

Correct Answer: (B) 81

Solution:

Concept:

- For reflection of a point Q in a line, the perpendicular foot H on the line is the midpoint of PQ , where P is the image. Thus $P = 2H - Q$.
- For a line $A + t\vec{d}$, the perpendicular-foot parameter is $t = ((Q - A) \cdot \vec{d}) / |\vec{d}|^2$. A triangle with edge vectors $\vec{u}, \vec{v}$ has area $\frac{1}{2} |\vec{u} \times \vec{v}|$.

Step 1: Locate the perpendicular foot on the line The given line passes through $A = (0, 3, 1)$ with

direction $\vec{d} = (1, 1, -1)$. For $Q = (3, -3, 1)$,

$$Q - A = (3, -6, 0), \quad |\vec{d}|^2 = 3.$$

The required parameter is

$$t = \frac{(3, -6, 0) \cdot (1, 1, -1)}{3} = \frac{-3}{3} = -1.$$

Therefore $H = A - \vec{d} = (-1, 2, 2)$. The perpendicularity check is $(Q - H) \cdot \vec{d} = (4, -5, -1) \cdot (1, 1, -1) = 0$.

Step 2: Reflect the point and form triangle edges Since H bisects PQ , the image is

$$P = 2(-1, 2, 2) - (3, -3, 1) = (-5, 7, 3).$$

Using $R = (2, 5, -1)$, choose both edges from the same vertex:

$$\overrightarrow{PQ} = (8, -10, -2), \quad \overrightarrow{PR} = (7, -2, -4).$$

Using the foot H as the reflected point would halve the displacement and give an incorrect triangle.

Step 3: Calculate the squared area and determine K The cross product is

$$\overrightarrow{PQ} \times \overrightarrow{PR} = (36, 18, 54).$$

Thus

$$\lambda^2 = \frac{1}{4}(36^2 + 18^2 + 54^2) = \frac{4536}{4} = 1134.$$

As $\lambda^2 = 14K$,

$$K = \frac{1134}{14} = 81.$$

The other candidate values 18, 72 and 36 do not reproduce the squared area. Working directly with λ^2 avoids introducing a square root and then squaring it again.

Step 4: Conclusion The value is $K = 81$. Hence, the correct option is (B).

Quick Tip to Solve: For reflection in a line, find the perpendicular foot first and double its position before subtracting the original point. Compute area squared directly when asked.

12. Let ABC be an equilateral triangle. A new triangle is formed by joining the middle points of all sides of the triangle ABC and the same process is repeated infinitely many times. If P is the sum of perimeters and Q is the sum of areas of all the triangles formed in this process, then : [4 marks]

(A) $P^2 = 72\sqrt{3}Q$

(B) $P^2 = 36\sqrt{3}Q$

(C) $P = 36\sqrt{3}Q^2$

(D) $P^2 = 6\sqrt{3}Q$

Correct Answer: (B) $P^2 = 36\sqrt{3}Q$

Solution:

Concept:

- The triangle joining the three side-midpoints is similar to its parent with side ratio $1/2$. Its perimeter is halved and its area is multiplied by $1/4$.
- An infinite geometric series with first term u and ratio r , where $|r| < 1$, has sum $u/(1-r)$. Apply this separately to lengths and areas.

Step 1: Identify the first triangle formed in the process Let the original equilateral triangle have side s , perimeter $3s$, and area $\sqrt{3}s^2/4$. The first new triangle is the medial triangle; its side is $s/2$. Therefore its perimeter and area are

$$p_1 = \frac{3s}{2}, \quad a_1 = \frac{\sqrt{3}s^2}{16}.$$

The sums in the question concern the triangles formed by repeating the construction, starting with this new triangle.

Step 2: Sum the two geometric progressions Each following perimeter is half the preceding perimeter. Each following area is one quarter of the preceding area. Both ratios have absolute value less than one, so

$$P = \frac{3s/2}{1 - 1/2} = 3s,$$

and

$$Q = \frac{\sqrt{3}s^2/16}{1 - 1/4} = \frac{\sqrt{3}s^2}{12}.$$

Using the perimeter ratio for the area series would incorrectly treat area as scaling linearly with side length.

Step 3: Eliminate the original side length Squaring the perimeter sum gives $P^2 = 9s^2$. From the area sum,

$$36\sqrt{3}Q = 36\sqrt{3} \left(\frac{\sqrt{3}s^2}{12} \right) = 9s^2.$$

Hence $P^2 = 36\sqrt{3}Q$. The coefficients in (A) and (D) disagree with this ratio. Option (C) also has inconsistent length dimensions: its left side is a length, whereas Q^2 has the fourth power of length. As a numerical check, setting $s = 2$ gives $P = 6$ and $Q = \sqrt{3}/3$, again yielding $P^2 = 36$.

Step 4: Conclusion The relation is $P^2 = 36\sqrt{3}Q$. Hence, the correct option is (B).

Quick Tip to Solve: In repeated midpoint constructions, lengths have ratio one half but areas have ratio one quarter. Start each series with the first triangle actually formed.

13. Suppose the solution of the differential equation $\frac{dy}{dx} = \frac{(2+\alpha)x - \beta y + 2}{\beta x - 2\alpha y - (\beta\gamma - 4\alpha)}$ represents a circle passing through origin. Then the radius of this circle is : [4 marks]

- (A) $\sqrt{17}$
 (B) 2
 (C) $\frac{\sqrt{17}}{2}$
 (D) $\frac{1}{2}$

Correct Answer: (C) $\frac{\sqrt{17}}{2}$

Solution:

Concept:

- A circle with centre (h, k) satisfies $(x - h)^2 + (y - k)^2 = r^2$. Implicit differentiation gives $dy/dx = -(x - h)/(y - k)$, wherever the denominator is nonzero.
- The numerator of this circle slope depends only on x , while its denominator depends only on y , and the leading coefficients have opposite signs and equal magnitudes after a common scaling.

Step 1: Compare the given slope with a circle slope The given numerator is $(2 + \alpha)x - \beta y + 2$, and the denominator is $\beta x - 2\alpha y - (\beta\gamma - 4\alpha)$. For the nondegenerate circle slope, the unwanted y -term in the numerator and x -term in the denominator must vanish, giving $\beta = 0$. The coefficients of x and y must then be negatives of each other:

$$2 + \alpha = 2\alpha \implies \alpha = 2.$$

A constant slope, which could arise after cancellation of proportional linear factors, cannot represent a circle.

Step 2: Identify the centre and integrate Substituting these parameter values, the differential equation becomes

$$\frac{dy}{dx} = \frac{4x + 2}{-4y + 8} = -\frac{x + 1/2}{y - 2}.$$

The parameter γ disappears because $\beta = 0$. Rearrangement and integration give

$$(y - 2) dy = -(x + 1/2) dx,$$

$$(x + 1/2)^2 + (y - 2)^2 = C.$$

Thus the centre is $(-1/2, 2)$ and C is the squared radius.

Step 3: Use the point through which the circle passes The origin lies on the required circle. Substituting $x = y = 0$ gives

$$C = \left(\frac{1}{2}\right)^2 + (-2)^2 = \frac{1}{4} + 4 = \frac{17}{4}.$$

Therefore the positive radius is $r = \sqrt{17}/2$. The value $\sqrt{17}$ in (A) omits division by two, while 2 and $1/2$ account for only one coordinate displacement of the centre from the origin. Both displacements must enter the distance formula.

Step 4: Conclusion The radius is $\frac{\sqrt{17}}{2}$. Hence, the correct option is (C).

Quick Tip to Solve: Compare a circle slope with $-(x-h)/(y-k)$, read off the centre, and use the given point to obtain the radius as a distance.

14. A software company sets up m number of computer systems to finish an assignment in 17 days. If 4 computer systems crashed on the start of the second day, 4 more computer systems crashed on the start of the third day and so on, then it took 8 more days to finish the assignment. The value of m is equal to: [4 marks]

- (A) 125
(B) 160
(C) 150
(D) 180

Correct Answer: (C) 150

Solution:

Concept:

- With identical computer systems working at a constant rate, total work is proportional to the sum of the numbers of functioning systems over the working days. Use one system-day as the work unit.
- An arithmetic progression with n terms, first term a , and common difference d has sum $S_n = n[2a + (n-1)d]/2$. Here the daily system count drops by four.

Step 1: Express the planned and actual work Without failures, m systems finish in 17 days, so the assignment requires $17m$ system-days. The actual completion time is $17 + 8 = 25$ days. On day 1 all m systems work; four crash at the start of day 2, so $m - 4$ work that day. The successive daily counts are

$$m, m - 4, m - 8, \dots, m - 96.$$

There are 24 reductions over 25 days; no systems crash before the first day.

Step 2: Sum the actual daily capacities The counts form an arithmetic progression with $a = m$, $d = -4$ and $n = 25$. The actual completed work is therefore

$$\begin{aligned} S_{25} &= \frac{25}{2} [2m + 24(-4)] \\ &= 25m - 1200. \end{aligned}$$

Equivalently, the lost system-days are $4(1 + 2 + \dots + 24) = 1200$. The later failures affect fewer working days, which is why using the final system count for all 25 days would be incorrect.

Step 3: Equate work and check the result The assignment is the same under the planned and actual

schedules, giving

$$25m - 1200 = 17m \implies 8m = 1200 \implies m = 150.$$

This is physically consistent: the last day still has $150 - 96 = 54$ functioning systems. The actual work is $25(150) - 1200 = 2550$ system-days, equal to the planned $17(150) = 2550$. Values 125, 160 and 180 do not satisfy the work equation and hence cannot be the original system count.

Step 4: Conclusion The company initially set up 150 computer systems. Hence, the correct option is (C).

Quick Tip to Solve: Convert a changing workforce into system-days. Count the first day separately in the progression: failures beginning on day 2 give one fewer reduction than working days.

15. If $P(6, 1)$ be the orthocentre of the triangle whose vertices are $A(5, -2)$, $B(8, 3)$ and $C(h, k)$, then the point C lies on the circle : [4 marks]

(A) $x^2 + y^2 - 74 = 0$

(B) $x^2 + y^2 - 65 = 0$

(C) $x^2 + y^2 - 61 = 0$

(D) $x^2 + y^2 - 52 = 0$

Correct Answer: (B) $x^2 + y^2 - 65 = 0$

Solution:

Concept:

- The orthocentre is the common point of the three altitudes. Thus the line joining a vertex to the orthocentre is perpendicular to the opposite side.
- For perpendicular vectors, the scalar product is zero: $(u_1, u_2) \cdot (v_1, v_2) = u_1v_1 + u_2v_2 = 0$. This avoids special cases involving vertical slopes.

Step 1: Use the altitude through A The given orthocentre is $P(6, 1)$, so $\overrightarrow{AP} = (1, 3)$. The opposite side has direction $\overrightarrow{BC} = (h - 8, k - 3)$. Since these directions are perpendicular,

$$1(h - 8) + 3(k - 3) = 0 \implies h + 3k = 17.$$

This equation describes every point C for which AP is the altitude from A .

Step 2: Use the altitude through B Also $\overrightarrow{BP} = (-2, -2)$ and $\overrightarrow{AC} = (h - 5, k + 2)$. The second altitude therefore gives

$$-2(h - 5) - 2(k + 2) = 0 \implies h + k = 3.$$

Subtracting this equation from the first gives $2k = 14$, so $k = 7$ and $h = -4$. The third vertex is consequently $C(-4, 7)$.

Step 3: Test the circle equations Every offered circle is centred at the origin. Substitute the coordinates of C into $x^2 + y^2$:

$$(-4)^2 + 7^2 = 16 + 49 = 65.$$

Thus $x^2 + y^2 - 65 = 0$ contains C. The constants 74, 61 and 52 in the other equations do not equal this squared distance, so those circles do not contain C.

Step 4: Conclusion The point is $C(-4, 7)$, and it lies on $x^2 + y^2 - 65 = 0$. Hence, the correct option is (B).

Quick Tip to Solve: Write two altitude conditions as dot products, solve for the unknown vertex, and only then test its squared distance from the circle centre.

16. If all the words with or without meaning made using all the letters of the word “NAGPUR” are arranged as in a dictionary, then the word at 315th position in this arrangement is : [4 marks]

- (A) NRAPUG
- (B) NRAGUP
- (C) NRAPGU
- (D) NRAGPU

Correct Answer: (C) NRAPGU

Solution:

Concept:

- All six letters of NAGPUR are distinct. Their alphabetical order is A, G, N, P, R, U , and each fixed first letter occupies a block of $5! = 120$ words.
- After fixing a prefix, its possible next letters occur in equal factorial-sized blocks. Keep a one-based rank within the current block when subtracting earlier blocks.

Step 1: Locate the first letter Words beginning with A occupy positions 1–120, while those beginning with G occupy positions 121–240. The requested position 315 therefore lies among words beginning with N, in positions 241–360. Its rank within the N block is

$$315 - 2(120) = 75.$$

The remaining letters are A, G, P, R and U.

Step 2: Locate the next two letters For each second letter there are $4! = 24$ completions. The second-letter blocks A, G and P account for the first 72 words in the N block. The 75th word therefore begins NR, with local rank $75 - 72 = 3$. The remaining letters are A, G, P and U. A fixed third letter has $3! = 6$ completions, so local rank 3 belongs to the first third-letter block, A. We now have prefix NRA and rank 3 among arrangements of G, P and U.

Step 3: Complete the word and verify its rank For the fourth letter, G occupies local ranks 1–2, P occupies ranks 3–4, and U occupies ranks 5–6. Rank 3 gives P, leaving G and U in their first order, GU. Thus the word is NRAPGU. Its global rank is

$$2(5!) + 3(4!) + 1(2!) + 1 = 240 + 72 + 2 + 1 = 315.$$

The other options NRAGPU and NRAGUP precede it, while NRAPUG follows it.

Step 4: Conclusion The word in position 315 is NRAPGU. Hence, the correct option is (C).

Quick Tip to Solve: At each position subtract complete factorial blocks, keeping the leftover rank one-based; verify the result by counting all preceding prefixes.

17. Suppose for a differentiable function h , $h(0) = 0$, $h(1) = 1$ and $h'(0) = h'(1) = 2$. If $g(x) = h(e^x)e^{h(x)}$, then $g'(0)$ is equal to: [4 marks]

- (A) 4
- (B) 5
- (C) 3
- (D) 8

Correct Answer: (A) 4

Solution:

Concept:

- For a product $u(x)v(x)$, the derivative is $u'(x)v(x) + u(x)v'(x)$. Both contributions must be retained.
- The chain rule gives $\frac{d}{dx}h(e^x) = h'(e^x)e^x$ and $\frac{d}{dx}e^{h(x)} = e^{h(x)}h'(x)$. The inner argument determines which supplied value of h or its derivative is used.

Step 1: Differentiate the two factors Set $u(x) = h(e^x)$ and $v(x) = e^{h(x)}$. Applying the chain rule separately gives

$$u'(x) = h'(e^x)e^x, \quad v'(x) = e^{h(x)}h'(x).$$

Notice that the first derivative is evaluated at e^x , whereas the second includes the derivative of h at x . These arguments are different even though the same function h occurs in both factors.

Step 2: Apply the product rule before substitution Combine the two derivatives to obtain

$$g'(x) = e^{h(x)} [h'(e^x)e^x + h(e^x)h'(x)].$$

At $x = 0$, the inner exponential equals $e^0 = 1$, so the needed values are $h'(1) = 2$, $h(1) = 1$, $h'(0) = 2$ and $e^{h(0)} = e^0 = 1$.

Step 3: Evaluate and check both contributions Substitution now yields

$$g'(0) = 1 [2(1) + 1(2)] = 2 + 2 = 4.$$

Each term in the product rule contributes 2. Replacing h evaluated at 1 by h evaluated at 0 would incorrectly erase the second contribution. The alternatives 5, 3 and 8 do not equal the result after the given values and both chain-rule factors are included.

Step 4: Conclusion $g'(0) = 4$. Hence, the correct option is (A).

Quick Tip to Solve: Differentiate the whole product first. At $x = 0$, write $e^0 = 1$ explicitly so that $h(1)$ and $h(0)$ are not confused.

18. Let $0 \leq r \leq n$. If ${}^{n+1}C_{r+1} : {}^nC_r : {}^{n-1}C_{r-1} = 55 : 35 : 21$, then $2n + 5r$ is equal to :

[4 marks]

- (A) 62
- (B) 60
- (C) 55
- (D) 50

Correct Answer: (D) 50

Solution:

Concept:

- For admissible integer n and r , factorial cancellation gives $\frac{{}^{n+1}C_{r+1}}{{}^nC_r} = \frac{(n+1)}{(r+1)}$.
- Similarly, $\frac{{}^nC_r}{{}^{n-1}C_{r-1}} = \frac{n}{r}$. The nonzero ratio in the question ensures that the lower coefficient is defined and r is nonzero.

Step 1: Convert the coefficient ratios into equations Take the first coefficient divided by the second, and then the second divided by the third. Using the supplied ratio $55 : 35 : 21$,

$$\frac{n+1}{r+1} = \frac{55}{35} = \frac{11}{7}, \quad \frac{n}{r} = \frac{35}{21} = \frac{5}{3}.$$

Cross-multiplication gives

$$7n - 11r = 4, \quad 3n = 5r.$$

Taking adjacent ratios eliminates the common proportionality factor without evaluating large factorials.

Step 2: Solve for n and r From $3n = 5r$, write $r = 3n/5$ and substitute into the first equation:

$$7n - 11\left(\frac{3n}{5}\right) = 4.$$

Therefore $35n - 33n = 20$, so $n = 10$, and hence $r = 6$. Both are integers and satisfy $0 \leq r \leq n$, as required.

Step 3: Check the ratio and evaluate the expression The actual coefficients are

$$\binom{11}{7} = 330, \quad \binom{10}{6} = 210, \quad \binom{9}{5} = 126.$$

Dividing all three by 6 reproduces $55 : 35 : 21$, which checks the solution. Finally,

$$2n + 5r = 2(10) + 5(6) = 20 + 30 = 50.$$

The alternatives 62, 60 and 55 are excluded because the ratios uniquely give $n=10$ and $r=6$.

Step 4: Conclusion The required value is 50. Hence, the correct option is (D).

Quick Tip to Solve: Use ratios of adjacent binomial coefficients; factorial cancellation turns a three-term coefficient ratio into two linear equations.

19. If $\int \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx = \frac{1}{12} \tan^{-1}(3 \tan x) + \text{constant}$, then the maximum value of $a \sin x + b \cos x$, is : [4 marks]

(A) $\sqrt{41}$

(B) $\sqrt{39}$

(C) $\sqrt{40}$

(D) $\sqrt{42}$

Correct Answer: (C) $\sqrt{40}$

Solution:

Concept:

- If two expressions are equal up to an additive constant, their derivatives are equal wherever the expressions are differentiable.
- The maximum of $a \sin x + b \cos x$ over real x is $\sqrt{a^2 + b^2}$. Cauchy–Schwarz gives this bound, and it is attained by aligning $(\sin x, \cos x)$ with (a, b) .

Step 1: Differentiate the given antiderivative Use the chain rule for the inverse tangent:

$$\frac{d}{dx} \left[\frac{1}{12} \tan^{-1}(3 \tan x) \right] = \frac{1}{12} \frac{3 \sec^2 x}{1 + 9 \tan^2 x}.$$

Multiplying numerator and denominator by $\cos^2 x$ simplifies this to

$$\frac{1}{4(\cos^2 x + 9 \sin^2 x)} = \frac{1}{36 \sin^2 x + 4 \cos^2 x}.$$

This equality is valid on intervals on which the displayed antiderivative is differentiable.

Step 2: Identify the squared coefficients The derivative must be the original integrand. Comparing the denominators,

$$a^2 \sin^2 x + b^2 \cos^2 x = 36 \sin^2 x + 4 \cos^2 x.$$

The identity for all x gives $a^2 = 36$ and $b^2 = 4$. Only these squares are required; the signs of a and b do not affect the largest attainable value of the linear combination.

Step 3: Find and attain the maximum By Cauchy–Schwarz,

$$(a \sin x + b \cos x)^2 \leq (a^2 + b^2)(\sin^2 x + \cos^2 x) = 40.$$

The positive bound is attained when $(\sin x, \cos x) = (a, b)/\sqrt{40}$, a unit vector. Thus the maximum is $\sqrt{40}$, regardless of the possible signs of a and b . The other square roots correspond to squared amplitudes 41, 39 and 42, inconsistent with the coefficients just found.

Step 4: Conclusion The maximum is $\sqrt{40}$. Hence, the correct option is (C).

Quick Tip to Solve: Differentiate a stated antiderivative to identify the denominator, then use the amplitude $\sqrt{a^2 + b^2}$ for a sine-cosine combination.

20. If the locus of the point, whose distances from the point (2, 1) and (1, 3) are in the ratio 5 : 4, is $ax^2 + by^2 + cxy + dx + ey + 170 = 0$, then the value of $a^2 + 2b + 3c + 4d + e$ is equal to : [4 marks]

- (A) 37
(B) -27
(C) 437
(D) 5

Correct Answer: (A) 37

Solution:

Concept:

- For a moving point $P(x, y)$, squared distances to (u, v) equal $(x - u)^2 + (y - v)^2$. Squaring a positive distance ratio preserves its meaning.
- An equation of a locus may be multiplied by a nonzero constant. Here the specified constant term 170 fixes the scale when identifying a, b, c, d and e.

Step 1: Translate the distance ratio The distances to $A(2, 1)$ and $B(1, 3)$, in that order, are in the ratio 5 : 4. Hence

$$\frac{PA^2}{PB^2} = \frac{25}{16}.$$

Cross-multiplication gives

$$16[(x - 2)^2 + (y - 1)^2] = 25[(x - 1)^2 + (y - 3)^2].$$

The factors 16 and 25 multiply the opposite squared distances; interchanging them would describe the reciprocal ratio.

Step 2: Expand and match the specified constant The two squared-distance expressions are $x^2 + y^2 - 4x - 2y + 5$ and $x^2 + y^2 - 2x - 6y + 10$. Moving the left side to the right gives

$$9x^2 + 9y^2 + 14x - 118y + 170 = 0.$$

Its constant already matches the required 170. There is no xy term because expanding each squared Euclidean distance produces only x squared, y squared and linear terms. Therefore

$$a = 9, \quad b = 9, \quad c = 0, \quad d = 14, \quad e = -118.$$

Step 3: Substitute the coefficients carefully The requested expression contains a^2 , but only $2b$, not $2b^2$. Thus

$$a^2 + 2b + 3c + 4d + e = 81 + 18 + 0 + 56 - 118 = 37.$$

The alternatives -27, 437 and 5 fail this direct substitution. The positive constant in the required equation also prevents using the negatives of all five coefficients without rescaling the equation.

Step 4: Conclusion The required value is $\boxed{37}$. Hence, the correct option is (A).

Quick Tip to Solve: After expanding a distance-ratio locus, match the stated constant term before reading off coefficients; this fixes the overall sign and scale.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $[t]$ denote the greatest integer less than or equal to t . Let $f : [0, \infty) \rightarrow \mathbb{R}$ be a function defined by $f(x) = \left\lfloor \frac{x}{2} + 3 \right\rfloor - \lfloor \sqrt{x} \rfloor$. Let S be the set of all points in the interval $[0, 8]$ at which f is not continuous. Then $\sum_{a \in S} a$ is equal to _____.

[4 marks]

Solution:

Concept:

- A floor function is locally constant except where its argument passes through an integer. Since both inner functions are increasing here, each such crossing creates an upward jump of size one.
- For the difference of two floor functions, simultaneous equal jumps cancel. Continuity at the domain endpoint 0 is tested from the right, while 8 is an interior point of the full domain and needs both sides.

Step 1: List the candidate jumps of each term For $A(x) = \lfloor x/2 + 3 \rfloor$, the positive integer crossings in $[0, 8]$ occur at $x = 2, 4, 6, 8$. The argument is 3 at $x=0$, but there is no left-hand part of the domain there. For $B(x) = \lfloor \sqrt{x} \rfloor$, the positive crossings occur at $x = 1, 4$, since the next square 9 lies outside the interval. Away from these points both terms, and hence their difference, are locally constant.

Step 2: Check which jumps survive subtraction At $x=1$, only B jumps, so f has a downward jump. At $x=2$ and $x=6$, only A jumps, so f has an upward jump. At $x=4$ both jump together: immediately to its left, $A = 4$ and $B = 1$, giving $f=3$; at $x=4$ and immediately to its right, $A = 5$ and $B = 2$, again giving $f=3$. Thus $x=4$ is continuous, not a discontinuity.

Step 3: Check the endpoints and sum the set At $x=0$ the value is $3 - 0 = 3$, and f remains 3 for sufficiently small nonnegative x , so it is continuous relative to its domain. At $x=8$ the left value is $6 - 2 = 4$, whereas $f(8) = 7 - 2 = 5$; the jump persists. Therefore

$$S = \{1, 2, 6, 8\}, \quad \sum_{a \in S} a = 1 + 2 + 6 + 8 = 17.$$

Including $x=4$ would incorrectly count a jump that cancels between the two terms.

Step 4: Conclusion The sum of the discontinuity points is $\boxed{17}$.

Quick Tip to Solve: List the integer crossings of each floor separately, then compare their jump sizes at common crossings before deciding the discontinuity set.

22. In a triangle ABC, $BC = 7$, $AC = 8$, $AB = \alpha \in \mathbb{N}$ and $\cos A = \frac{2}{3}$. If $49 \cos(3C) + 42 = \frac{m}{n}$, where $\gcd(m, n) = 1$, then $m + n$ is equal to _____. [4 marks]

Solution:

Concept:

- The cosine rule relates the side opposite an angle to the other two sides. At vertex A, it gives $BC^2 = AC^2 + AB^2 - 2(AC)(AB) \cos A$.
- The triple-angle identity is $\cos(3C) = 4 \cos^3 C - 3 \cos C$. First determine the permitted integer side, then evaluate the required rational number in lowest terms.

Step 1: Determine the natural-number side Put $AB = \alpha$, using the sides adjacent to angle A as 8 and α :

$$49 = 64 + \alpha^2 - \frac{32\alpha}{3}.$$

Multiplication by 3 and factorisation give

$$3\alpha^2 - 32\alpha + 45 = (\alpha - 9)(3\alpha - 5) = 0.$$

The candidates are 9 and $5/3$. Only $\alpha = 9$ is a natural number. It also satisfies the strict triangle inequalities, since $7 + 8 > 9$. Thus no degenerate triangle has been introduced.

Step 2: Evaluate the angle at C Angle C is opposite AB, so apply the cosine rule again with the correct opposite side:

$$\cos C = \frac{7^2 + 8^2 - 9^2}{2 \cdot 7 \cdot 8} = \frac{32}{112} = \frac{2}{7}.$$

Consequently,

$$\begin{aligned} \cos(3C) &= 4 \left(\frac{2}{7}\right)^3 - 3 \left(\frac{2}{7}\right) = \frac{32 - 294}{343} = -\frac{262}{343}, \\ 49 \cos(3C) + 42 &= -\frac{262}{7} + 42 = \frac{32}{7}. \end{aligned}$$

The numerator 32 and denominator 7 have no common factor, so they already satisfy the required gcd condition.

Step 3: Conclusion Thus $m = 32$, $n = 7$, and the required integer is $m + n = 39$.

Quick Tip to Solve: Label the side opposite each angle before applying the cosine rule a second time; here AB is opposite C, not A.

23. Let $[t]$ denote the largest integer less than or equal to t . If $\int_0^3 \left([x^2] + \left[\frac{x^2}{2} \right] \right) dx = a + b\sqrt{2} - \sqrt{3} - \sqrt{5} + c\sqrt{6} - \sqrt{7}$, where $a, b, c \in \mathbb{Z}$, then $a + b + c$ is equal to _____. [4 marks]

Solution:

Concept:

- For a nonnegative increasing expression, its floor counts the integer thresholds it has crossed. Thus $[x^2] = \sum_{k=1}^9 \mathbf{1}_{x \geq \sqrt{k}}$ on $[0, 3]$.

- Integrating an indicator over this interval gives its interval length. Values at isolated jump points do not affect the definite integral, so each threshold contributes $3 - \sqrt{k}$.

Step 1: Integrate the first floor function The thresholds of x^2 occur at $x = \sqrt{k}$. Its possible positive integer thresholds on the interval are $k = 1, \dots, 9$; the last contributes zero length. Therefore

$$\int_0^3 \lfloor x^2 \rfloor dx = \sum_{k=1}^9 (3 - \sqrt{k}).$$

Collect the square roots of perfect squares and combine $\sqrt{8} = 2\sqrt{2}$:

$$\sum_{k=1}^9 \sqrt{k} = 6 + 3\sqrt{2} + \sqrt{3} + \sqrt{5} + \sqrt{6} + \sqrt{7}.$$

This gives the first integral as 21 minus the displayed irrational terms. Counting intervals through thresholds avoids having to integrate eight separate constant pieces.

Step 2: Integrate the second floor function and collect coefficients For $\lfloor x^2/2 \rfloor$, the thresholds are $\sqrt{2k}$. Only $k = 1, 2, 3, 4$ occur before 3, since $\sqrt{10} > 3$. Hence

$$\begin{aligned} \int_0^3 \left\lfloor \frac{x^2}{2} \right\rfloor dx &= \sum_{k=1}^4 (3 - \sqrt{2k}) \\ &= 10 - 3\sqrt{2} - \sqrt{6}. \end{aligned}$$

Adding the two integrals now yields

$$31 - 6\sqrt{2} - \sqrt{3} - \sqrt{5} - 2\sqrt{6} - \sqrt{7}.$$

Compare this with the expression in the question, keeping the signs attached to their respective radicals.

Step 3: Conclusion The coefficients are $a = 31$, $b = -6$ and $c = -2$. Thus $\boxed{a + b + c = 23}$.

Quick Tip to Solve: For an increasing floor function, sum the lengths beyond each integer threshold instead of listing every constant interval.

24. If the solution $y(x)$ of the given differential equation $(e^y + 1) \cos x \, dx + e^y \sin x \, dy = 0$ passes through the point $(\frac{\pi}{2}, 0)$, then the value of $e^{y(\frac{\pi}{6})}$ is equal to _____. [4 marks]

Solution:

Concept:

- The product rule gives $d[(e^y + 1) \sin x] = (e^y + 1) \cos x \, dx + e^y \sin x \, dy$. Thus the given equation is an exact differential.
- An equation $dF = 0$ has $F = C$ along its solution curve. The initial point determines this constant before the requested argument is substituted.

Step 1: Recognise and integrate the product differential The coefficient of dx is the derivative of $\sin x$ multiplied by $e^y + 1$; the coefficient of dy is the derivative of $e^y + 1$ multiplied by $\sin x$. Therefore the two terms combine without dividing by $\cos x$:

$$d[(e^y + 1) \sin x] = 0.$$

Integration gives the implicit solution

$$(e^y + 1) \sin x = C.$$

This method remains valid at the initial point $x = \pi/2$, where $\cos x = 0$. Dividing by that factor would be unnecessary and could obscure the use of the given point.

Step 2: Use the initial condition and evaluate At $(x, y) = (\pi/2, 0)$, the exponential and sine are both 1. Hence

$$C = (e^0 + 1) \sin \frac{\pi}{2} = 2.$$

The particular solution is consequently $(e^{y(x)} + 1) \sin x = 2$. At the required argument it gives

$$\left(e^{y(\pi/6)} + 1\right) \frac{1}{2} = 2, \quad e^{y(\pi/6)} = 3.$$

The resulting exponential is positive, as required for a real-valued y . Indeed the local explicit form is $y(x) = \ln(2 \csc x - 1)$, and both the initial argument and $\pi/6$ lie in an interval on which its logarithm is defined.

Step 3: Conclusion The requested exponential value is $e^{y(\pi/6)} = 3$.

Quick Tip to Solve: Before separating variables, look for a product differential; it can avoid awkward divisions at an initial point.

25. If the shortest distance between the lines $\frac{x-\lambda}{3} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $\frac{x+2}{-3} = \frac{y+5}{2} = \frac{z-4}{4}$ is $\frac{44}{\sqrt{30}}$, then the largest possible value of $|\lambda|$ is equal to _____. [4 marks]

Solution:

Concept:

- For lines through position vectors $\mathbf{a}_1, \mathbf{a}_2$ with nonparallel directions $\mathbf{b}_1, \mathbf{b}_2$, their shortest distance is $d = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)| / |\mathbf{b}_1 \times \mathbf{b}_2|$.
- The absolute value in the scalar triple product is essential: a prescribed positive distance generally permits two signed equations. Both must be solved before maximising $|\lambda|$.

Step 1: Read the points and compute a common normal The symmetric forms give

$$\begin{aligned} \mathbf{a}_1 &= (\lambda, 2, 1), & \mathbf{b}_1 &= (3, -1, 1), \\ \mathbf{a}_2 &= (-2, -5, 4), & \mathbf{b}_2 &= (-3, 2, 4). \end{aligned}$$

Their cross product is

$$\mathbf{b}_1 \times \mathbf{b}_2 = (-6, -15, 3) = 3(-2, -5, 1).$$

We may use the simpler normal $\mathbf{n} = (-2, -5, 1)$, because its scale cancels in the distance formula. Its magnitude is $\sqrt{4 + 25 + 1} = \sqrt{30}$, which is nonzero, confirming that the directions are not parallel.

Step 2: Impose the distance and compare both values The vector joining the chosen points is $(-2 - \lambda, -7, 3)$. Its projection numerator along the normal is

$$(-2 - \lambda, -7, 3) \cdot (-2, -5, 1) = 2\lambda + 42.$$

Thus the distance condition becomes

$$\frac{|2\lambda + 42|}{\sqrt{30}} = \frac{44}{\sqrt{30}}, \quad 2\lambda + 42 = \pm 44.$$

The positive sign gives $\lambda = 1$, while the negative sign gives $\lambda = -43$. Substitution returns a numerator of magnitude 44 in both cases. Their absolute values are 1 and 43, so selecting only the positive signed equation would miss the required maximum.

Step 3: Conclusion The largest possible absolute value is $|\lambda|_{\max} = 43$.

Quick Tip to Solve: Reduce a cross-product normal by any common factor, and retain both signs when undoing an absolute-value distance equation.

26. The length of the latus rectum and directrices of hyperbola with eccentricity e are 9 and $x = \pm \frac{4}{\sqrt{3}}$, respectively. Let the line $y - \sqrt{3}x + \sqrt{3} = 0$ touch this hyperbola at (x_0, y_0) . If m is the product of the focal distances of the point (x_0, y_0) , then $4e^2 + m$ is equal to _____. [4 marks]

Solution:

Concept:

- For the standard hyperbola $x^2/a^2 - y^2/b^2 = 1$, the eccentricity obeys $e^2 = 1 + b^2/a^2$, the latus rectum has length $2b^2/a$, and the directrices are $x = \pm a/e$.
- A line $y = sx + c$ is tangent to this hyperbola when $c^2 = a^2s^2 - b^2$. All the supplied conditions must describe the same positive parameters a and b .

Step 1: Check the latus rectum against the tangent The given line has slope $\sqrt{3}$ and intercept $-\sqrt{3}$. Its tangency condition is

$$3 = 3a^2 - b^2.$$

The stated latus rectum gives $b^2 = 9a/2$. Combining these equations yields

$$6a^2 - 9a - 6 = 0, \quad (2a + 1)(a - 2) = 0.$$

Since a is a positive semitransverse axis, $a = 2$ and $b^2 = 9$. These force $e^2 = 13/4$, and hence the directrices would be $x = \pm 4/\sqrt{13}$. This differs from the printed $x = \pm 4/\sqrt{3}$. Thus the three stated conditions are inconsistent for a standard hyperbola.

Step 2: Identify the value associated with the stated tangent and latus rectum Using the hyperbola forced by the tangent and latus rectum, the point of contact satisfies

$$\frac{x_0x}{4} - \frac{y_0y}{9} = 1.$$

Comparison with $y = \sqrt{3}x - \sqrt{3}$ gives $(x_0, y_0) = (4, 3\sqrt{3})$. Its squared focal distances are

$$(4 - \sqrt{13})^2 + 27 = 56 - 8\sqrt{13}, \quad (4 + \sqrt{13})^2 + 27 = 56 + 8\sqrt{13}.$$

Their product is therefore $m = \sqrt{56^2 - 64 \cdot 13} = 48$, and $4e^2 + m = 13 + 48 = 61$. This calculation explains the keyed value, but it requires the directrix denominator to be $\sqrt{13}$, rather than the printed $\sqrt{3}$.

Step 3: Conclusion As printed, the conditions are inconsistent, so no such hyperbola exists. The keyed value $\boxed{61}$ follows from the latus rectum and tangent, with directrices $x = \pm 4/\sqrt{13}$.

Quick Tip to Solve: Check the tangent relation, latus rectum and directrix together; two conditions can force parameters that contradict the third.

27. Let α, β be roots of $x^2 + \sqrt{2}x - 8 = 0$. If $U_n = \alpha^n + \beta^n$, then $\frac{U_{10} + \sqrt{2}U_9}{2U_8}$ is equal to _____. [4 marks]

Solution:

Concept:

- Every root of $t^2 + \sqrt{2}t - 8 = 0$ satisfies $t^2 + \sqrt{2}t = 8$. Multiplying this identity by a power of the root gives a recurrence without explicitly evaluating the roots.
- Adding the identities for α and β converts them into the power sums $U_n = \alpha^n + \beta^n$. Even powers also make it easy to verify that the required denominator is nonzero.

Step 1: Build the required recurrence directly Apply the quadratic identity separately to the two roots:

$$\alpha^2 + \sqrt{2}\alpha = 8, \quad \beta^2 + \sqrt{2}\beta = 8.$$

Multiply the first equality by α^8 and the second by β^8 . Adding gives

$$\alpha^{10} + \beta^{10} + \sqrt{2}(\alpha^9 + \beta^9) = 8(\alpha^8 + \beta^8).$$

By the definition of the power sums, this is exactly

$$U_{10} + \sqrt{2}U_9 = 8U_8.$$

No approximation to either irrational root is needed, so there is no loss of accuracy from cancellation between large powers.

Step 2: Verify division is valid and simplify The discriminant of the quadratic is $2 + 32 = 34 > 0$, so both roots are real. Their product is -8 , so neither root is zero. Consequently their eighth powers are

strictly positive, and

$$U_8 = \alpha^8 + \beta^8 > 0.$$

Division by the denominator in the question is therefore legitimate. Substitute the exact recurrence:

$$\frac{U_{10} + \sqrt{2}U_9}{2U_8} = \frac{8U_8}{2U_8} = 4.$$

The positive sign follows from moving the constant term of the original quadratic to the other side; using a recurrence with $-8U_8$ would reverse that sign incorrectly.

Step 3: Conclusion The required numerical value is $\boxed{4}$.

Quick Tip to Solve: When a quotient contains consecutive power sums of quadratic roots, multiply the root equation by the matching lower power.

28. If the system of equations

$$2x + 7y + \lambda z = 3$$

$$3x + 2y + 5z = 4$$

$$x + \mu y + 32z = -1$$

has infinitely many solutions, then $(\lambda - \mu)$ is equal to _____ :

[4 marks]

Solution:

Concept:

- A system with infinitely many solutions must have equal coefficient and augmented ranks, both smaller than the number of unknowns. A dependence among coefficient rows must also hold for the constants.
- The first two equations here are independent because the determinant of their x,y coefficients is $2 \cdot 2 - 3 \cdot 7 = -17 \neq 0$. Thus their common solutions form a line, and the third equation must be their linear combination.

Step 1: Find the necessary linear combination Write the third equation as p times the first plus q times the second. The x coefficients and constant terms give

$$2p + 3q = 1, \quad 3p + 4q = -1.$$

Eliminate p: multiplying these equalities by 3 and 2 respectively and subtracting gives $q = 5$. Substitution then gives $p = -7$. Therefore the required dependence, including the right-hand side, is

$$R_3 = -7R_1 + 5R_2.$$

Using just a zero coefficient determinant would not be enough: it could also describe an inconsistent system if the constants failed the same dependence.

Step 2: Determine the parameters and check the ranks Match the remaining coefficients in that row

relation. For the y coefficient,

$$\mu = -7(7) + 5(2) = -39.$$

For the z coefficient,

$$32 = -7\lambda + 5(5), \quad \lambda = -1.$$

With these values, the x coefficient is $-14 + 15 = 1$ and the constant is $-21 + 20 = -1$, so all four entries of the augmented third row agree. The first two coefficient rows retain rank 2 because their x, y minor remains -17 . Hence the full coefficient and augmented ranks are both 2, leaving one free variable and proving that the solution set is infinite.

Step 3: Conclusion The parameters are $\lambda = -1$ and $\mu = -39$, so $\boxed{\lambda - \mu = 38}$.

Quick Tip to Solve: For infinite solutions, match the constants in the same row dependence as the coefficients; a zero determinant alone is insufficient.

29. If $S(x) = (1+x) + 2(1+x)^2 + 3(1+x)^3 + \cdots + 60(1+x)^{60}$, $x \neq 0$, and $(60)^2 S(60) = a(b)^b + b$, where $a, b \in N$, then $(a+b)$ equal to _____. [4 marks]

Solution:

Concept:

- An arithmetic-geometric series has terms of the form kr^k . Obtain its finite sum by differentiating the finite geometric series with respect to r .
- For $r \neq 1$, $G(r) = 1 + r + \cdots + r^{60} = (r^{61} - 1)/(r - 1)$, and $rG'(r) = \sum_{k=1}^{60} kr^k$. Here the required ratio is $r = 1 + x$.

Step 1: Differentiate the finite geometric sum Differentiation is term by term because the sum is finite. The constant term differentiates to zero, and multiplication by r restores the power of each term:

$$\begin{aligned} S(x) &= r \frac{d}{dr} \left(\frac{r^{61} - 1}{r - 1} \right) \\ &= \frac{r \{ 61r^{60}(r - 1) - (r^{61} - 1) \}}{(r - 1)^2} \\ &= \frac{60r^{62} - 61r^{61} + r}{(r - 1)^2}. \end{aligned}$$

The condition $x \neq 0$ ensures that the denominator $(r - 1)^2 = x^2$ is nonzero. This also explains why the question multiplies the value by 60^2 : that factor will cancel the denominator at $x = 60$.

Step 2: Substitute the required ratio and collect the power At $x = 60$, the ratio is $r = 61$, rather than 60. Hence

$$\begin{aligned} 60^2 S(60) &= 60(61)^{62} - 61(61)^{61} + 61 \\ &= (61)^{61} \{ 60 \cdot 61 - 61 \} + 61 \\ &= 3599(61)^{61} + 61. \end{aligned}$$

Comparing this exact expression with $a(b)^b + b$ gives $a = 3599$ and $b = 61$. Both are natural numbers, as required. The base, exponent and final additive term are all 61, so the comparison is consistent

throughout the expression.

Step 3: Conclusion Therefore, $a + b = 3599 + 61 = \boxed{3660}$.

Quick Tip to Solve: For a sum of kr^k , differentiate a finite geometric sum; substitute $r = 1 + x$ only after simplifying.

30. From a lot of 12 items containing 3 defectives, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. Let items in the sample be drawn one by one without replacement. If variance of X is $\frac{m}{n}$, where $\gcd(m, n) = 1$, then $n - m$ is equal to _____. [4 marks]

Solution:

Concept:

- Sampling without replacement from a finite population gives a hypergeometric count. The success proportion here is $p = 3/12 = 1/4$, where a success denotes a defective item.
- For population size N , sample size s and defective proportion p , the variance is $\text{Var}(X) = sp(1-p)(N-s)/(N-1)$. The final factor accounts for dependence between draws.

Step 1: Identify the population and sample parameters There are $N = 12$ items, of which 3 are defective and 9 are good. We select $s = 5$ distinct items. Every defective item in the sample contributes one to X , so X can take the values 0, 1, 2 and 3. Its mean is $sp = 5/4$. Drawing one by one does not change this distribution, because all ordered samples without replacement are equally likely. However, successive defective indicators are not independent: after drawing a defective item, fewer defectives remain. Thus a binomial variance would omit an essential correction.

Step 2: Calculate and reduce the variance Apply the finite-population correction with the number remaining outside the sample equal to $12 - 5 = 7$:

$$\begin{aligned}\text{Var}(X) &= 5 \left(\frac{3}{12} \right) \left(\frac{9}{12} \right) \frac{12-5}{12-1} \\ &= 5 \cdot \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{7}{11} \\ &= \frac{105}{176}.\end{aligned}$$

The numerator factors as $105 = 3 \cdot 5 \cdot 7$, whereas $176 = 2^4 \cdot 11$, so the two integers have no common factor. Therefore this is already the fraction in lowest terms, giving $m = 105$ and $n = 176$. As a check, the variance is less than the corresponding binomial value $15/16$, consistent with the reduced variability caused by sampling without replacement.

Step 3: Conclusion The required difference is $n - m = 176 - 105 = \boxed{71}$.

Quick Tip to Solve: For sampling without replacement, multiply the binomial variance by $(N-s)/(N-1)$ before reducing the fraction.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A car of 800 kg is taking turn on a banked road of radius 300 m and angle of banking 30° . If coefficient of static friction is 0.2 then the maximum speed with which car can negotiate the turn safely: ($g = 10 \text{ m/s}^2$, $\sqrt{3} = 1.73$) [4 marks]

- (A) 51.4 m/s
- (B) 102.8 m/s
- (C) 70.4 m/s
- (D) 264 m/s

Correct Answer: (A) 51.4 m/s

Solution:

Concept:

- On a banked turn, the inward horizontal resultant provides the centripetal force mv^2/r , and the vertical forces balance because the car has no vertical acceleration.
- At the maximum safe speed the car tends to slip up the bank. Limiting static friction therefore acts down the slope with magnitude $f = \mu_s N$, where N is the normal reaction.

Step 1: Resolve the normal reaction and limiting friction Let the banking angle be $\theta = 30^\circ$. The inward component of the normal reaction is $N \sin \theta$, and its upward component is $N \cos \theta$. Down-slope friction contributes an inward component $f \cos \theta$ and a downward component $f \sin \theta$. Consequently,

$$\begin{aligned} N \cos \theta - \mu_s N \sin \theta &= mg, \\ N \sin \theta + \mu_s N \cos \theta &= \frac{mv_{\max}^2}{r}. \end{aligned}$$

Dividing the second equation by the first eliminates both the normal reaction and the car's mass. Friction increases the maximum speed above the frictionless banking speed.

Step 2: Substitute the angle and coefficient of friction The limiting-speed relation becomes

$$v_{\max}^2 = rg \frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta}.$$

Use $\sin 30^\circ = 1/2$, $\cos 30^\circ = 1.73/2$, $r = 300 \text{ m}$, $g = 10 \text{ m/s}^2$, and $\mu_s = 0.2$:

$$\begin{aligned} v_{\max}^2 &= 3000 \frac{1 + 0.2(1.73)}{1.73 - 0.2} \\ &= 3000 \frac{1.346}{1.53} \approx 2639.2 \text{ m}^2 \text{ s}^{-2}, \\ v_{\max} &\approx 51.37 \text{ m/s}. \end{aligned}$$

Rounding gives 51.4. The other proposed speeds, 102.8, 70.4 and 264, all exceed this friction-limited value and would demand more inward force than the road can provide.

Step 3: Conclusion The maximum safe speed is 51.4 m/s . Hence, the correct option is (A).

Quick Tip to Solve: For the maximum speed on a banked road, take friction down the slope; the numerator gains the friction term and the denominator loses it.

32. A body projected vertically upwards with a certain speed from the top of a tower reaches the ground in t_1 . If it is projected vertically downwards from the same point with the same speed, it reaches the ground in t_2 . Time required to reach the ground, if it is dropped from the top of the tower, is :

[4 marks]

- (A) $\sqrt{t_1 + t_2}$
 (B) $\sqrt{t_1 t_2}$
 (C) $\sqrt{\frac{t_1}{t_2}}$
 (D) $\sqrt{t_1 - t_2}$

Correct Answer: (B) $\sqrt{t_1 t_2}$

Solution:

Concept:

- For constant gravitational acceleration, use $s = ut + \frac{1}{2}at^2$ with a consistent sign convention. Choose downward as positive and write the tower height as h .
- The upward and downward projections have equal initial speed magnitudes u , but opposite initial velocities. Eliminating u relates their flight times to the height.

Step 1: Write the two displacement equations For the upward launch, the initial velocity is $-u$ and the displacement to the ground is $+h$. For the downward launch, the initial velocity is $+u$. Therefore,

$$h = -ut_1 + \frac{1}{2}gt_1^2,$$

$$h = ut_2 + \frac{1}{2}gt_2^2.$$

The first flight takes longer because the body first rises before falling to the ground. Both equations nevertheless refer to the same net downward displacement. Dividing by the respective positive flight times gives $h/t_1 = -u + gt_1/2$ and $h/t_2 = u + gt_2/2$.

Step 2: Eliminate the projection speed and compare with free fall Add the divided equations to remove u :

$$h \left(\frac{1}{t_1} + \frac{1}{t_2} \right) = \frac{g}{2}(t_1 + t_2),$$

$$h \frac{t_1 + t_2}{t_1 t_2} = \frac{g}{2}(t_1 + t_2),$$

$$\frac{2h}{g} = t_1 t_2.$$

The factor $t_1 + t_2$ is positive, so cancellation is valid. If the body is simply dropped, its initial speed is zero and $h = gT^2/2$. Thus $T^2 = 2h/g = t_1 t_2$. We take the positive square root since a flight time is positive. The product has dimensions of time squared; a sum or difference under a square root would instead have the wrong dimensions, while a ratio gives a dimensionless result.

Step 3: Conclusion The required time is $T = \sqrt{t_1 t_2}$. Hence, the correct option is (B).

Quick Tip to Solve: Eliminate the equal and opposite initial velocities by dividing each height equation by its flight time and then adding.

33. Given below are two statements:

Statement (I) : Dimensions of specific heat is $[L^2 T^{-2} K^{-1}]$.

Statement (II) : Dimensions of gas constant is $[ML^2 T^{-1} K^{-1}]$.

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]

(A) Statement (I) is incorrect but statement (II) is correct

(B) Both statement (I) and statement (II) are incorrect

(C) Both statement (I) and statement (II) are correct

(D) Statement (I) is correct but statement (II) is incorrect

Correct Answer: (D) Statement (I) is correct but statement (II) is incorrect

Solution:

Concept:

- Specific heat capacity is heat energy per unit mass per unit temperature rise: $c = Q/(m\Delta T)$. Energy has dimensions $[ML^2T^{-2}]$.
- The universal gas constant is defined by $pV = nR\Theta$, with n the amount of substance and Θ absolute temperature. Its dimensions must include the energy time exponent -2 .

Step 1: Test the dimensions of specific heat Temperature is represented by K in the question. From the heat relation $Q = mc\Delta T$,

$$[c] = \frac{[Q]}{[m][\Delta T]} = \frac{ML^2T^{-2}}{MK} = L^2T^{-2}K^{-1}.$$

The mass dimension cancels because specific heat is measured per unit mass. Its SI unit is $J kg^{-1} K^{-1}$, which independently reduces to $m^2 s^{-2} K^{-1}$. The expression in Statement (I) agrees with this result, so Statement (I) is correct.

Step 2: Test the dimensions of the gas constant Pressure has dimensions $ML^{-1}T^{-2}$, and volume has dimensions L^3 . Hence their product has the dimensions of energy:

$$[pV] = ML^2T^{-2}.$$

Writing the amount-of-substance dimension as N , the ideal gas equation gives

$$[R] = ML^2T^{-2}K^{-1}N^{-1}.$$

When amount is suppressed in the usual four-dimension notation, this is $[ML^2T^{-2}K^{-1}]$. In either notation the time exponent is -2 , whereas Statement (II) prints -1 ; it is therefore incorrect. The SI unit $J mol^{-1} K^{-1}$ confirms the same exponent. Options asserting that Statement (I) is false, or that Statement (II) is true, are consequently ruled out.

Step 3: Conclusion Statement (I) is correct and Statement (II) is incorrect. Hence, the correct option is (D).

Quick Tip to Solve: Derive thermal dimensions from energy first: specific heat divides by mass, whereas the universal gas constant divides by amount of substance.

34. A body of weight 200 N is suspended from a tree branch through a chain of mass 10 kg. The branch pulls the chain by a force equal to (if $g = 10 \text{ m/s}^2$) : [4 marks]

- (A) 300 N
- (B) 100 N
- (C) 150 N
- (D) 200 N

Correct Answer: (A) 300 N

Solution:

Concept:

- A stationary system has zero net force. The support force on the entire chain-plus-body system must balance its total downward weight.
- A massive chain has its own weight $m_c g$; the tension at its upper end supports both that weight and the load attached at its lower end.

Step 1: Calculate the weight of the chain The body is specified by its weight, $W_b = 200 \text{ N}$, so this number is already a force and must not be multiplied by g again. The chain, however, is specified by its mass $m_c = 10 \text{ kg}$. Its gravitational force is

$$W_c = m_c g = 10 \times 10 = 100 \text{ N}.$$

Both forces act vertically downward. The body pulls downward on the lower end of the chain with 200 N. Each higher part of the chain additionally carries the weight of the chain below it, so the upper-end tension is larger than the lower-end tension.

Step 2: Balance the full suspended system Choose the entire chain together with the attached body as one system. Forces between the body and chain are internal to this system and cancel in the total force balance. The external vertical forces are the branch's upward pull F , the body's weight, and the chain's weight. Static equilibrium therefore requires

$$\begin{aligned} F - W_b - W_c &= 0, \\ F &= 200 + 100 \\ &= 300 \text{ N}. \end{aligned}$$

The 100 N choice counts only the chain, while 200 N counts only the body and would be appropriate for a massless chain. The 150 N choice is also smaller than the body's weight alone and cannot support the system. By Newton's third law, the chain exerts an equal 300 N downward force on the branch, but the asked force on the chain is upward.

Step 3: Conclusion The branch pulls the chain upward with a force of 300 N. Hence, the correct option is (A).

Quick Tip to Solve: At the top of a heavy chain, include the weight of the chain itself as well as the attached load; treat the whole suspended system together.

35. The number of electrons flowing per second in the filament of a 110 W bulb operating at 220 V is :

(Given $e = 1.6 \times 10^{-19} \text{C}$)

[4 marks]

(A) 1.25×10^{19}

(B) 31.25×10^{17}

(C) 6.25×10^{18}

(D) 6.25×10^{17}

Correct Answer: (B) 31.25×10^{17}

Solution:

Concept:

- For a bulb operating at voltage V with electrical power P , the current magnitude follows from $P = VI$, so $I = P/V$.
- Current is charge crossing a filament section per unit time. If $\dot{N}$ electrons cross per second and each has charge magnitude e , then $I = \dot{N}e$.

Step 1: Determine the operating current Use the stated operating power and voltage, which already describe the bulb at its working resistance:

$$I = \frac{P}{V} = \frac{110 \text{ W}}{220 \text{ V}} = 0.5 \text{ A}.$$

A watt per volt is an ampere, so this is 0.5 C s^{-1} of charge passing through a cross-section of the filament. There is no need to calculate resistance separately. As a consistency check, $R = V/I = 440 \Omega$ would give $I^2 R = 110 \text{ W}$, the same power.

Step 2: Convert the charge flow into an electron count Divide the charge passing per second by the magnitude of one electron's charge:

$$\begin{aligned}\dot{N} &= \frac{I}{e} \\ &= \frac{0.5}{1.6 \times 10^{-19}} \\ &= 0.3125 \times 10^{19} \\ &= 3.125 \times 10^{18} \text{ s}^{-1} \\ &= 31.25 \times 10^{17} \text{ s}^{-1}.\end{aligned}$$

The units are $(\text{C s}^{-1})/\text{C} = \text{s}^{-1}$, as required for a number per second. The electron's negative sign sets its direction relative to conventional current; it does not make the count negative. Option (C) would correspond to a current of 1 A, twice the actual value. Options (A) and (D) also do not reproduce the charge flow of 0.5 C per second. The scientific-notation form in option (B) is exactly the computed value, even though its coefficient exceeds 10.

Step 3: Conclusion The number is 3.125×10^{18} electrons per second, or 31.25×10^{17} . Hence, the correct option is (B).

Quick Tip to Solve: First find $I = P/V$, then divide by e . Compare numerical options after writing all of them with the same power of ten.

36. The acceptor level of a p-type semiconductor is 6 eV. The maximum wavelength of light which can create a hole would be : Given $hc = 1242 \text{ eV nm}$. [4 marks]

- (A) 407 nm
- (B) 103.5 nm
- (C) 414 nm
- (D) 207 nm

Correct Answer: (D) 207 nm

Solution:

Concept:

- A photon has energy $E = hc/\lambda$, where h is Planck's constant, c is the speed of light and λ is its wavelength. Photon energy decreases as wavelength increases.
- An electron transferred from the valence band to an acceptor state leaves a mobile hole in the valence band. The threshold photon must provide the specified excitation energy; the longest useful wavelength occurs at equality.

Step 1: Identify the threshold energy The question supplies an acceptor excitation energy of 6 eV. Creating a hole requires transferring an electron into the acceptor state, so the photon energy must be at least this amount. At the limiting wavelength, use $E_{\min} = 6 \text{ eV}$. Larger wavelengths have smaller energies and cannot produce this excitation. It is the energy needed for the transition that enters this calculation, rather than twice or half that energy.

Step 2: Convert the threshold to wavelength Use the supplied product $hc = 1242 \text{ eV nm}$, which lets electronvolts cancel directly without converting to joules.

$$\lambda_{\max} = \frac{hc}{E_{\min}} = \frac{1242 \text{ eV nm}}{6 \text{ eV}} = 207 \text{ nm}.$$

The alternatives 407 nm and 414 nm have energies below the required threshold. The shorter wavelength 103.5 nm supplies 12 eV; although sufficient energetically, it is not the maximum wavelength.

Step 3: Conclusion The maximum wavelength that can create a hole is 207 nm. Hence, the correct option is (D).

Quick Tip to Solve: For a threshold wavelength with energy in eV, divide the supplied hc in eV nm by the transition energy; the largest wavelength corresponds to the smallest permitted photon energy.

37. In a vernier calliper, when both jaws touch each other, zero of the vernier scale shifts towards left and its 4th division coincides exactly with a certain division on main scale. If 50 vernier scale divisions equal to 49 main scale divisions and zero error in the instrument is 0.04 mm then how many main scale divisions are there in 1 cm ? [4 marks]

- (A) 5
(B) 40
(C) 10
(D) 20

Correct Answer: No listed option agrees with the printed negative zero error: the literal result is 230 main-scale divisions per centimetre; option (D), 20, follows from a positive-error interpretation.

Solution:

Concept:

- Let one main-scale division be d . From 50 vernier divisions equal to 49 main-scale divisions, one vernier division is $49d/50$, so the least count is $LC = d/50$.
- If the vernier zero is to the left of the main-scale zero, the zero error is negative. When its n -th division coincides, the magnitude of the usual negative zero error is $(N - n)LC$, with N vernier divisions.

Step 1: Use the stated leftward displacement Here $N = 50$ and the fourth vernier division coincides. The negative-error magnitude is therefore $(50 - 4)LC = 46LC$, rather than $4LC$. The latter expression applies to a positive zero error. Using the given magnitude,

$$46 \frac{d}{50} = 0.04 \text{ mm} \implies d = \frac{0.04 \times 50}{46} \text{ mm} = \frac{1}{23} \text{ mm}.$$

This also follows geometrically: the fourth vernier mark lies $4(49d/50) = 3.92d$ to the right of its own zero. For a small leftward zero shift it meets the third main-scale mark, requiring a shift of $0.92d$.

Step 2: Count the divisions and test the options One centimetre is 10 mm, so the number of main-scale divisions is

$$\frac{10 \text{ mm}}{(1/23) \text{ mm}} = 230.$$

None of the listed numbers 5, 40, 10, 20 equals this result. In particular, choosing 20 would give $d = 0.5 \text{ mm}$ and $LC = 0.01 \text{ mm}$. The fourth-division negative error would then have magnitude $46 \times 0.01 = 0.46 \text{ mm}$, contradicting the stated 0.04 mm. The value 20 arises only by treating the displacement as a positive zero error, contrary to the word "left".

Step 3: Conclusion With the leftward zero shift as printed, the answer is 230 main-scale divisions per centimetre, so no listed option is correct. The key selects (D), 20, which corresponds to using the positive-error formula rather than the stated negative error.

Quick Tip to Solve: First decide the sign of the zero error: use nLC for a positive error and $(N - n)LC$ for a negative error before solving for the scale division.

38. In a coil, the current changes from -2 A to $+2\text{ A}$ in 0.2 s and induces an emf of 0.1 V . The self inductance of the coil is :

[4 marks]

- (A) 4 mH
- (B) 2.5 mH
- (C) 1 mH
- (D) 5 mH

Correct Answer: (D) 5 mH

Solution:

Concept:

- The self-induced emf obeys $\varepsilon = -L di/dt$, where L is the self-inductance. For a constant rate or the stated interval average, its magnitude is $|\varepsilon| = L|\Delta i|/\Delta t$.
- The change of current is its final algebraic value minus its initial algebraic value. A reversal from a negative current to a positive current includes the change up to zero and the change away from zero.

Step 1: Calculate the complete change in current The initial current is $i_i = -2\text{ A}$ and the final current is $i_f = +2\text{ A}$. Therefore,

$$\Delta i = i_f - i_i = 2 - (-2) = 4\text{ A}.$$

The interval is 0.2 s , giving

$$\left| \frac{\Delta i}{\Delta t} \right| = \frac{4}{0.2} = 20\text{ A s}^{-1}.$$

Subtracting the magnitudes would incorrectly give zero; the current directions are opposite, so the signed values must be used.

Step 2: Find the inductance and convert the unit Rearrange the magnitude form of the induction law and substitute the induced emf 0.1 V :

$$L = \frac{|\varepsilon|}{|\Delta i|/\Delta t} = \frac{0.1}{20}\text{ V s A}^{-1} = 0.005\text{ H} = 5\text{ mH}.$$

The minus sign in the induction law specifies opposition to the change of current; it does not make the inductance negative. As a check, an inductance of 5 mH multiplied by 20 A s^{-1} gives 0.1 V , as required. The alternatives $4, 2.5, 1\text{ mH}$ would instead give emfs $0.08, 0.05, 0.02\text{ V}$.

Step 3: Conclusion The self-inductance is 5 mH . Hence, the correct option is (D).

Quick Tip to Solve: For a current reversal, subtract signed currents before using $L = |\varepsilon|\Delta t/|\Delta i|$; then convert henries to millihenries at the end.

39. Energy of 10 non rigid diatomic molecules at temperature T is :

[4 marks]

- (A) $35 RT$
- (B) $\frac{7}{2} RT$
- (C) $70 K_B T$
- (D) $35 K_B T$

Correct Answer: (D) $35 K_B T$

Solution:

Concept:

- Each independent quadratic contribution to a molecule's energy contributes $\frac{1}{2}k_B T$ in classical equipartition, where k_B is Boltzmann's constant and T is absolute temperature.
- A non-rigid diatomic molecule has three translational and two rotational contributions. Its one bond-vibration mode supplies two further quadratic contributions, one kinetic and one potential, giving seven in all.

Step 1: Count all active energy contributions Because the molecule is specified as non-rigid, include the vibrational energy in addition to translation and rotation. Translation contributes $3k_B T/2$, rotation contributes $2k_B T/2$, and vibration contributes $2k_B T/2$. Thus the energy of one molecule is

$$u = \left(\frac{3}{2} + 1 + 1 \right) k_B T = \frac{7}{2} k_B T.$$

This is the classical non-rigid model; treating the bond as rigid would omit the vibrational contribution.

Step 2: Multiply by the number of molecules The question specifies ten molecules, not ten moles. The total is therefore

$$U = Nu = 10 \left(\frac{7}{2} k_B T \right) = 35 k_B T.$$

The gas constant $R = N_A k_B$ is appropriate when the quantity of gas is expressed in moles, so neither $35RT$ nor $7RT/2$ describes these ten molecules. The value $70k_B T$ doubles the equipartition result by assigning $k_B T$, instead of half of it, to each quadratic contribution. The factor of ten counts molecules only once.

Step 3: Conclusion The total energy is $35k_B T$. Hence, the correct option is (D).

Quick Tip to Solve: Use Nk_B for a stated number of molecules and nR for moles. A non-rigid diatomic molecule contributes seven halves of $k_B T$.

40. When UV light of wavelength 300 nm is incident on the metal surface having work function 2.13 eV, electron emission takes place. The stopping potential is :

(Given $hc = 1240 \text{ eV nm}$)

[4 marks]

- (A) 4 V
- (B) 2 V
- (C) 4.1 V
- (D) 1.5 V

Correct Answer: (B) 2 V

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = hc/\lambda - \phi$, where ϕ is the work function and $K_{\max}$ is the largest emitted-electron kinetic energy.
- The stopping potential magnitude satisfies $eV_s = K_{\max}$. Consequently a kinetic energy expressed numerically in electronvolts gives the same numerical stopping-potential magnitude in volts.

Step 1: Calculate the incident photon energy Use the given $hc = 1240 \text{ eV nm}$ and $\lambda = 300 \text{ nm}$. Keeping both quantities in compatible units avoids an unnecessary conversion to joules:

$$E_{\gamma} = \frac{1240}{300} \text{ eV} = 4.1333 \dots \text{ eV}.$$

This exceeds the work function 2.13 eV, consistently with the stated electron emission. The work function is the part of this photon energy spent in freeing the electron from the surface.

Step 2: Find the available kinetic energy and retarding voltage Subtract the work function before identifying the stopping voltage:

$$K_{\max} = 4.1333 \dots - 2.13 = 2.0033 \dots \text{ eV}.$$

Therefore $V_s = 2.0033 \dots \text{ V}$, which is 2 V at the precision of the options. A stopping potential of 4 or 4.1 V would nearly equate the kinetic energy to the full photon energy, neglecting the escape energy. A value 1.5 V is too small to stop the fastest electrons because they have approximately 2 eV of kinetic energy. The stopping potential is reported here as a positive magnitude; the collector is retarding relative to the emitter.

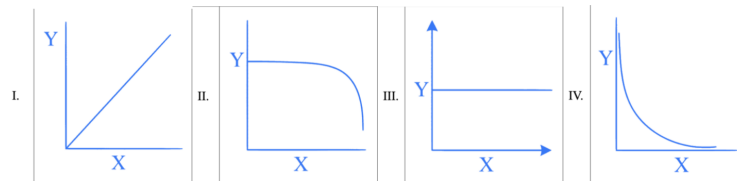
Step 3: Conclusion The stopping potential magnitude is approximately 2 V. Hence, the correct option is (B).

Quick Tip to Solve: When hc is supplied in eV nm, compute $hc/\lambda - \phi$ in eV; its numerical value is the stopping potential in volts.

41. Match List I with List II:

	List I (Y vs X)
A	Y : magnetic susceptibility; X : magnetising field
B	Y : magnetic field; X : distance from centre of a current-carrying wire for $x < a$, where a is the wire radius
C	Y : magnetic field; X : distance from centre of a current-carrying wire for $x > a$, where a is the wire radius
D	Y : magnetic field inside solenoid; X : distance from centre

List II (shape of graph): graphs I, II, III and IV are shown in that order from left to right. Choose the correct answer from the options given below: [4 marks]



- (A) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
 (B) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)
 (C) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)
 (D) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

Correct Answer: (B) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Solution:

Concept:

- For an ordinary linear magnetic material at fixed temperature, susceptibility $\chi = M/H$ is independent of the magnetising field H . A constant ordinate corresponds to a horizontal graph.
- Ampere's law gives $B = \mu_0 I x / (2\pi a^2)$ inside a long uniformly current-carrying wire and $B = \mu_0 I / (2\pi x)$ outside it. These are respectively proportional and inversely proportional dependences.

Step 1: Match susceptibility and the wire interior For A, susceptibility is constant with magnetising field in the linear model, so select the horizontal graph III. For B, the enclosed current at radius $x < a$ is $I x^2 / a^2$. Substituting it in $B(2\pi x) = \mu_0 I_{\text{enc}}$ gives $B \propto x$. The field starts from zero on the axis and increases linearly, giving graph I. These two matches already distinguish the correct mapping from those assigning a curved dependence to the wire interior.

Step 2: Match the wire exterior and solenoid For C, when $x > a$, all the wire current is enclosed and $B = \mu_0 I / (2\pi x)$. Thus the field decreases like a rectangular hyperbola: graph IV. For D, the field near the centre of a finite solenoid is almost uniform and decreases towards its ends, corresponding to graph II. Graph II is distinct from the exact $1/x$ decay outside the straight wire. Collecting the four results yields

$$A \rightarrow \text{III}, \quad B \rightarrow \text{I}, \quad C \rightarrow \text{IV}, \quad D \rightarrow \text{II}.$$

The other alternatives interchange a constant, a linear or an inverse-distance behaviour and therefore fail at least one of these laws.

Step 3: Conclusion The matching is A-III, B-I, C-IV, D-II. Hence, the correct option is (B).

Quick Tip to Solve: Identify the three basic shapes first: susceptibility is constant, the wire interior gives $B \propto x$, and the exterior gives $B \propto 1/x$. The remaining curve belongs to the finite solenoid.

42. For the thin convex lens, the radii of curvature are at 15 cm and 30 cm respectively. The focal length the lens is 20 cm. The refractive index of the material is : [4 marks]

- (A) 1.2
- (B) 1.5
- (C) 1.4
- (D) 1.8

Correct Answer: (B) 1.5

Solution:

Concept:

- For a thin lens in air, the lens maker relation is $1/f = (\mu - 1)(1/R_1 - 1/R_2)$, where μ is the refractive index relative to air and the radii carry Cartesian signs.
- For a biconvex lens with light travelling from left to right, the first surface has $R_1 > 0$ and the second has $R_2 < 0$. Consequently the two positive radius magnitudes contribute as a sum to its converging power.

Step 1: Assign consistent signs to the surface radii Take $R_1 = +15$ cm, $R_2 = -30$ cm, and $f = +20$ cm because the lens is converging. The curvature factor is

$$\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{15} - \frac{1}{-30} = \frac{1}{15} + \frac{1}{30} = \frac{1}{10} \text{ cm}^{-1}.$$

Reversing the lens would interchange the magnitudes and signs appropriately, leaving the power unchanged. Taking both signed radii positive would incorrectly subtract their curvature magnitudes.

Step 2: Solve for the refractive index and check the options Substitute the focal length and curvature factor in the lens maker formula:

$$\frac{1}{20} = (\mu - 1) \frac{1}{10} \implies \mu - 1 = \frac{10}{20} = 0.5.$$

Hence $\mu = 1.5$. All lengths were expressed in centimetres, so the ratio is dimensionless as a refractive index must be. A direct check gives $f = 10/(1.5 - 1) = 20$ cm. By contrast the indices 1.2, 1.4, 1.8 would give focal lengths 50, 25, 12.5 cm, respectively; none matches the specified lens. Thus the sign convention and numerical substitution both select the same option.

Step 3: Conclusion The refractive index of the lens material is 1.5. Hence, the correct option is (B).

Quick Tip to Solve: For a biconvex lens in air, add the reciprocal radius magnitudes in the lens maker formula; keep every length in the same unit before solving for μ .

43. Two identical conducting spheres P and S with charge Q on each, repel each other with a force 16 N. A third identical uncharged conducting sphere R is successively brought in contact with the two spheres. The new force of repulsion between P and S is :

[4 marks]

- (A) 1 N
- (B) 6 N
- (C) 12 N
- (D) 4 N

Correct Answer: (B) 6 N

Solution:

Concept:

- Identical conducting spheres share their total charge equally on contact because their potentials become equal. The charge of the isolated two-sphere system is conserved during each contact.
- At fixed separation r , Coulomb's force is $F = kq_Pq_S/r^2$, where $k = 1/(4\pi\epsilon_0)$. Thus the ratio of final to initial force equals the ratio of charge products.

Step 1: Track the first contact Take the initially uncharged sphere R into contact with P first. Their combined charge is $Q + 0 = Q$, so, after separation, each carries $Q/2$. Sphere S has not yet been touched and still carries Q . The charge on P remains $Q/2$ during the next contact because P is then isolated.

Step 2: Track the second contact Now R, carrying $Q/2$, touches S, carrying Q . Their total charge is $3Q/2$; identical spheres divide it equally. Hence

$$q_P = \frac{Q}{2}, \quad q_S = q_R = \frac{3Q}{4}.$$

Reversing the order of contacts only interchanges P and S, and leaves their final charge product unchanged.

Step 3: Compare the force products The initial force is $F_0 = kQ^2/r^2 = 16$ N. At the same separation,

$$\frac{F}{F_0} = \frac{(Q/2)(3Q/4)}{Q^2} = \frac{3}{8}, \quad F = 16 \times \frac{3}{8} = 6 \text{ N}.$$

The charges have the same sign, so the new force remains repulsive. The value 4 N would incorrectly halve both initial charges independently; the other values also do not use the successive sharing correctly.

Step 4: Conclusion The new repulsive force between P and S is 6 N. Hence, the correct option is (B).

Quick Tip to Solve: After each contact, update the travelling sphere's charge before the next contact; it is no longer uncharged.

44. The longest wavelength associated with Paschen series is : (Given $R_H = 1.097 \times 10^7 \text{ SI unit}$)

[4 marks]

(A) $2.973 \times 10^{-6} \text{ m}$

(B) $1.876 \times 10^{-6} \text{ m}$

(C) $1.094 \times 10^{-6} \text{ m}$

(D) $3.646 \times 10^{-6} \text{ m}$

Correct Answer: (B) $1.876 \times 10^{-6} \text{ m}$

Solution:

Concept:

- The Paschen series consists of hydrogen emission transitions ending at $n_f = 3$, with initial principal quantum number $n_i = 4, 5, \dots$
- The Rydberg formula is $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$. The longest wavelength corresponds to the smallest positive photon energy, since $E = hc/\lambda$.

Step 1: Identify the smallest energy transition All Paschen lines terminate at the same level $n_f = 3$. The nearest allowed upper level is $n_i = 4$, so $4 \rightarrow 3$ has the smallest energy difference and the longest wavelength. Increasing n_i increases $1/9 - 1/n_i^2$, which increases the photon energy and reduces its wavelength. The series limit therefore gives the shortest, rather than the longest, wavelength.

Step 2: Substitute into the Rydberg formula With the supplied $R_H = 1.097 \times 10^7 \text{ m}^{-1}$,

$$\frac{1}{\lambda_{\max}} = 1.097 \times 10^7 \left(\frac{1}{9} - \frac{1}{16} \right) = 1.097 \times 10^7 \frac{7}{144} \text{ m}^{-1}.$$

Inverting this expression gives

$$\lambda_{\max} = \frac{144}{7(1.097 \times 10^7)} \text{ m} = 1.875 \dots \times 10^{-6} \text{ m}.$$

Step 3: Compare with the listed values To the precision of the options, this is $1.876 \times 10^{-6} \text{ m}$. The value $1.094 \times 10^{-6} \text{ m}$ corresponds approximately to another Paschen transition, not its first line. The larger values $2.973 \times 10^{-6} \text{ m}$ and $3.646 \times 10^{-6} \text{ m}$ cannot result from the required $4 \rightarrow 3$ transition with the given constant.

Step 4: Conclusion The longest Paschen wavelength is $1.876 \times 10^{-6} \text{ m}$. Hence, the correct option is (B).

Quick Tip to Solve: For a series ending at level n , its longest wavelength comes from the transition from $n+1$ to n ; its shortest comes from infinity to n .

45. A total of 48 J heat is given to one mole of helium kept in a cylinder. The temperature of helium increases by 2°C. The work done by the gas is: Given, $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$. [4 marks]

(A) 23.1 J

(B) 48 J

(C) 24.9 J

(D) 72.9 J

Correct Answer: (A) 23.1 J

Solution:

Concept:

- Helium is a monatomic ideal gas. Its molar heat capacity at constant volume is $C_V = 3R/2$, and its internal-energy change is $\Delta U = nC_V\Delta T$, whatever the path between the two equilibrium states.
- Use the first law with work done by the gas positive: $Q = \Delta U + W$. Heat supplied to the gas is positive, and a temperature difference of 2°C equals 2 K.

Step 1: Evaluate the internal-energy increase The supplied quantities are $Q = 48 \text{ J}$, $n = 1 \text{ mol}$, $\Delta T = 2 \text{ K}$ and $R = 8.3 \text{ J mol}^{-1}\text{K}^{-1}$. For helium,

$$\Delta U = \frac{3}{2}nR\Delta T = \frac{3}{2} \times 1 \times 8.3 \times 2 = 24.9 \text{ J}.$$

This part of the supplied heat raises the random translational kinetic energy of the atoms.

Step 2: Apply energy conservation The rest of the supplied heat becomes work done by the gas on its surroundings:

$$W = Q - \Delta U = 48 - 24.9 = 23.1 \text{ J}.$$

The positive sign is consistent with net work output. No assumption of constant pressure or constant volume is necessary: the measured heat and temperature rise determine the work directly through the first law.

Step 3: Check the alternative energy assignments The value 48 J assigns all supplied heat to work and ignores the positive temperature rise. The value 24.9 J is the internal-energy increase, not the work. Adding that increase to the supplied heat gives 72.9 J, which uses the wrong sign in the first law and would violate the stated energy balance.

Step 4: Conclusion The work done by helium is 23.1 J. Hence, the correct option is (A).

Quick Tip to Solve: For monatomic helium, calculate the internal-energy rise using $3nR\Delta T/2$ first, then subtract it from the heat supplied.

46. When kinetic energy of a body becomes 36 times of its original value, the percentage increase in the momentum of the body will be : [4 marks]

- (A) 60%
- (B) 500%
- (C) 6%
- (D) 600%

Correct Answer: (B) 500%

Solution:

Concept:

- For a nonrelativistic body of fixed mass m , kinetic energy and momentum magnitude satisfy $K = p^2/(2m)$. Therefore momentum magnitude is proportional to the square root of kinetic energy.
- Percentage increase uses the change relative to the original value: $100(p_f - p_i)/p_i$. It differs from the final value expressed as a percentage of the original.

Step 1: Relate the two momentum magnitudes Let the initial kinetic energy be K_i and initial momentum magnitude be p_i . The final energy is $K_f = 36K_i$. Since the same body retains its mass,

$$\frac{K_f}{K_i} = \frac{p_f^2/(2m)}{p_i^2/(2m)} = \left(\frac{p_f}{p_i}\right)^2 = 36.$$

Taking the positive square root for momentum magnitudes gives $p_f = 6p_i$.

Step 2: Calculate the percentage increase The increase is $\Delta p = p_f - p_i = 6p_i - p_i = 5p_i$. Thus

$$\text{Percentage increase} = \frac{\Delta p}{p_i} \times 100\% = 5 \times 100\% = 500\%.$$

The kinetic-energy information specifies momentum magnitude; it does not determine a change of direction, and no directional information is required here.

Step 3: Distinguish increase from final percentage The final momentum is 600% of its original magnitude, but the question asks for the increase, so the original 100% must be subtracted. Hence 600% is the tempting incorrect option. The options 60% and 6% confuse the factor six with a percentage and do not satisfy the squared relation between kinetic energy and momentum.

Step 4: Conclusion The percentage increase in momentum magnitude is 500%. Hence, the correct option is (B).

Quick Tip to Solve: Take the square root of the kinetic-energy multiplier, subtract one, and only then multiply by 100 to obtain the momentum percentage increase.

47. Pressure inside a soap bubble is greater than the pressure outside by an amount : (given : R = Radius of bubble, S = Surface tension of bubble) [4 marks]

- (A) $\frac{S}{R}$
(B) $\frac{4S}{R}$
(C) $\frac{4R}{S}$
(D) $\frac{2S}{R}$

Correct Answer: (B) $\frac{4S}{R}$

Solution:

Concept:

- A thin soap bubble has two liquid-air surfaces, an inner surface and an outer surface. Surface tension S acts along each surface and tends to reduce its area.
- The excess pressure $\Delta P = P_{\text{in}} - P_{\text{out}}$ supports these surface-tension forces. For a spherical bubble of radius R , both surfaces contribute to the pressure balance.

Step 1: Balance forces on one hemisphere Imagine cutting the spherical soap bubble into two hemispheres through its centre. The resultant outward force caused by the excess pressure on one hemisphere equals the pressure difference times its circular projected area:

$$F_P = \Delta P \pi R^2.$$

The absolute inside and outside pressures are not added; their opposing contributions leave only their difference.

Step 2: Include both surfaces of the film At the circular rim, the inward force due to surface tension on each surface is its circumference multiplied by S , namely $2\pi RS$. A soap film has two such surfaces, so the total is $4\pi RS$. In equilibrium,

$$\Delta P \pi R^2 = 4\pi RS, \quad \Delta P = \frac{4S}{R}.$$

The expression has pressure units because $(\text{N m}^{-1})/\text{m} = \text{N m}^{-2}$.

Step 3: Check the listed expressions The expression $2S/R$ applies to a single spherical interface, such as a liquid drop, so it omits one surface of the soap film. The expression S/R also lacks the required geometric factor. The ratio $4R/S$ has inverse-pressure dimensions and cannot represent an excess pressure. The pressure difference is positive, as stated in the question.

Step 4: Conclusion The inside pressure exceeds the outside pressure by $4S/R$. Hence, the correct option is (B).

Quick Tip to Solve: Count interfaces before applying the pressure formula: a liquid drop has one surface, while a thin soap bubble has two.

48. Assuming the earth to be a sphere of uniform mass density, a body weighed 300 N on the surface of earth. How much it would weigh at $R/4$ depth under surface of earth ? [4 marks]

- (A) 75 N
- (B) 375 N
- (C) 300 N
- (D) 225 N

Correct Answer: (D) 225 N

Solution:

Concept:

- For a sphere of uniform mass density, only the mass enclosed within radius r contributes to gravitational force at that radius; the outer spherical shells produce zero net field there.
- The enclosed mass is proportional to r^3 , so inside the Earth $g(r) = g_s r/R$, where R is the Earth radius and g_s is surface gravity. The weight is $W = mg$.

Step 1: Convert depth to distance from the centre The stated depth is $d = R/4$ below the surface. Thus the radial distance from the centre is

$$r = R - d = R - \frac{R}{4} = \frac{3R}{4}.$$

Depth and distance from the centre are different quantities; using $R/4$ directly as r would describe a point much deeper inside the Earth.

Step 2: Find the gravity ratio Writing the total Earth mass as M , uniform density gives $M_r = M(r/R)^3$. Therefore

$$g(r) = \frac{GM_r}{r^2} = \frac{GM_r}{R^3}, \quad \frac{g(r)}{g_s} = \frac{GM_r/R^3}{GM/R^2} = \frac{r}{R} = \frac{3}{4}.$$

Inside a uniform Earth the field decreases linearly with depth, rather than obeying the external inverse-square dependence on the full Earth mass.

Step 3: Scale the original weight The body retains the same mass, so its weight scales by the same factor as gravity:

$$W(r) = 300 \times \frac{3}{4} = 225 \text{ N}.$$

The option 75 N is the decrease in weight, not the remaining weight. A weight of 300 N would imply no change in gravity, and 375 N would incorrectly make gravity increase at the specified interior point.

Step 4: Conclusion The body weighs 225 N at a depth $R/4$. Hence, the correct option is (D).

Quick Tip to Solve: For a uniform Earth use the remaining radius fraction, 1 minus depth/ R , to multiply the surface weight; do not use depth/ R itself.

49. In finding out refractive index of glass slab the following observations were made through travelling microscope 50 vernier scale division = 49 MSD; 20 divisions on main scale in each cm
For mark on paper

$$\text{MSR} = 8.45 \text{ cm}, \text{VC} = 26$$

For mark on paper seen through slab

$$\text{MSR} = 7.12 \text{ cm}, \text{VC} = 41$$

For powder particle on the top surface of the glass slab

$$\text{MSR} = 4.05 \text{ cm}, \text{VC} = 1$$

(MSR = Main Scale Reading, VC = Vernier Coincidence)

Refractive index of the glass slab is :

[4 marks]

(A) 1.52

(B) 1.35

(C) 1.24

(D) 1.42

Correct Answer: (D) 1.42

Solution:

Concept:

- A vernier microscope reading equals the main-scale reading plus vernier coincidence multiplied by the least count: $x = \text{MSR} + \text{VC} \times \text{LC}$.
- For viewing a plane glass slab nearly normally from air, refractive index is $\mu = t/t_{\text{app}}$, the ratio of real thickness to apparent thickness. Both thicknesses are measured relative to the top surface.

Step 1: Calculate the least count and corrected readings Twenty main-scale divisions occupy 1 cm, so 1 MSD = 0.05 cm. Since 50 VSD = 49 MSD,

$$\text{LC} = 1 \text{ MSD} - 1 \text{ VSD} = \frac{0.05}{50} = 0.001 \text{ cm}.$$

The readings for the uncovered mark, its image through the slab, and the top-surface powder are respectively

$$x_1 = 8.45 + 26(0.001) = 8.476 \text{ cm},$$

$$x_2 = 7.12 + 41(0.001) = 7.161 \text{ cm},$$

$$x_3 = 4.05 + 1(0.001) = 4.051 \text{ cm}.$$

Step 2: Determine real and apparent thickness The uncovered mark gives the true bottom position, while the powder gives the actual top surface. Their separation is

$$t = x_1 - x_3 = 8.476 - 4.051 = 4.425 \text{ cm}.$$

The image of the bottom mark through glass appears closer to the top, giving

$$t_{\text{app}} = x_2 - x_3 = 7.161 - 4.051 = 3.110 \text{ cm.}$$

The shift $x_1 - x_2$ is neither the real nor the apparent thickness.

Step 3: Form the refractive-index ratio Both distances use the same unit, so their ratio is dimensionless:

$$\mu = \frac{4.425}{3.110} = 1.4228 \dots \simeq 1.42.$$

The apparent thickness is smaller than the real thickness, consistent with an index exceeding one. The values 1.52, 1.35 and 1.24 do not equal the ratio obtained after applying the vernier corrections and subtracting the top-surface reading.

Step 4: Conclusion The refractive index of the glass slab is approximately 1.42. Hence, the correct option is (D).

Quick Tip to Solve: Correct every microscope reading with the least count before subtraction, and measure both real and apparent thickness from the top-surface reading.

50. In the given electromagnetic wave $E_y = 600 \sin(\omega t - kx) \text{ V m}^{-1}$, intensity of the associated light beam is (in W/m^2 : (Given $\epsilon_0 = 9 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$) [4 marks]

- (A) 486
- (B) 729
- (C) 243
- (D) 972

Correct Answer: (A) 486

Solution:

Concept:

- For a sinusoidal electromagnetic wave in free space, the electric amplitude is E_0 . The total instantaneous energy density is $u = \epsilon_0 E^2$, including equal electric and magnetic contributions.
- The intensity is the time-averaged energy crossing unit area per unit time: $I = c \langle u \rangle = \frac{1}{2} \epsilon_0 c E_0^2$, since the average of $\sin^2$ over a full cycle is $1/2$.

Step 1: Identify the peak electric field Compare the wave with $E_y = E_0 \sin(\omega t - kx)$. The coefficient of the sine is the peak field, not the root-mean-square field. Thus $E_0 = 600 \text{ V m}^{-1}$, and $E_{\text{rms}} = 600/\sqrt{2} \text{ V m}^{-1}$. Use the supplied permittivity $\epsilon_0 = 9 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ and the free-space wave speed $c = 3 \times 10^8 \text{ m s}^{-1}$.

Step 2: Average the energy flux The electric and magnetic fields both transport energy; their combined instantaneous flux is $\epsilon_0 c E^2$. Averaging this expression gives

$$I = \frac{1}{2} (9 \times 10^{-12}) (3 \times 10^8) (600)^2 = 486 \text{ W m}^{-2}.$$

The value 972 W m^{-2} in (D) omits the time-average factor. The value 243 in (C) halves the result again, effectively leaving out half the total field energy. The value 729 in (B) does not follow from the amplitude and constants given. The direction of propagation does not affect this positive scalar intensity.

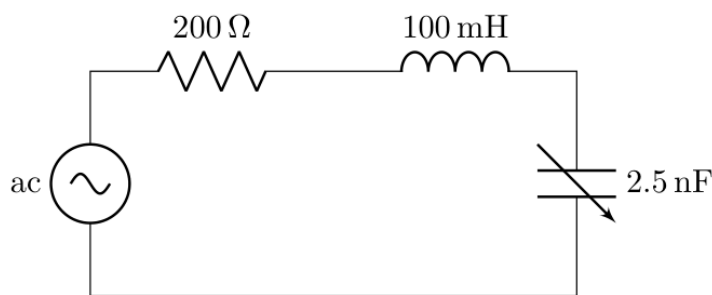
Step 3: Conclusion The intensity of the light beam is 486 W m^{-2} . Hence, the correct option is (A).

Quick Tip to Solve: When a wave is written as a sine, its coefficient is the peak field: use the factor $1/2$ in intensity, or first convert to the rms field.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. For a given series LCR circuit it is found that maximum current is drawn when value of variable capacitance is 2.5 nF . If resistance of 200Ω and 100 mH inductor is being used in the given circuit. The frequency of ac source is _____ $\times 10^3 \text{ Hz}$ (given $a^2 = 10$) [4 marks]



Solution:

Concept:

- The impedance of a series LCR circuit is $Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}$. For a fixed source voltage and frequency, the current is largest when the inductive and capacitive reactances cancel.
- At series resonance, $\omega_0^2 LC = 1$, so $f_0 = 1/(2\pi\sqrt{LC})$. Resistance determines the maximum current but does not change this resonance condition for the ideal components here.

Step 1: Apply the maximum-current condition The capacitance is varied until the circuit draws maximum current. This minimises the magnitude of the impedance. Since the resistance stays at 200Ω , the minimum occurs at $X_L = X_C$, giving

$$2\pi fL = \frac{1}{2\pi fC}, \quad f = \frac{1}{2\pi\sqrt{LC}}.$$

The value of the source voltage is not needed because we seek the resonant frequency rather than the current amplitude.

Step 2: Convert the component values and evaluate Use $L = 100 \text{ mH} = 0.100 \text{ H}$ and $C = 2.5 \text{ nF} =$

2.5×10^{-9} F. Hence

$$LC = 2.5 \times 10^{-10} \text{ s}^2, \quad f = \frac{1}{2\pi\sqrt{2.5 \times 10^{-10}}} \approx 1.0066 \times 10^4 \text{ Hz}.$$

Thus $f/(10^3 \text{ Hz}) \approx 10.066$, whose nearest-integer numerical entry is 10. With the customary approximation $\pi^2 \simeq 10$, the same expression gives $f = 10^4 \text{ Hz}$ directly. Keeping angular frequency and ordinary frequency distinct is essential; $1/\sqrt{LC}$ alone would give radians per second.

Step 3: Conclusion The source frequency is approximately $10 \times 10^3 \text{ Hz}$; the required numerical entry is 10.

Quick Tip to Solve: For maximum current in a series LCR circuit, set $X_L = X_C$ first; use resistance only if the current itself is requested.

52. Three balls of masses 2 kg, 4 kg and 6 kg respectively are arranged at centre of the edges of an equilateral triangle of side 2 m. The moment of inertia of the system about an axis through the centroid and perpendicular to the plane of triangle, will be _____ kg m². [4 marks]

Solution:

Concept:

- For particles at perpendicular distances r_i from a fixed axis, the moment of inertia is $I = \sum_i m_i r_i^2$. The balls are treated as particles at the stated positions because no radii or intrinsic moments of inertia are supplied.
- In an equilateral triangle of side a , the centroid lies two-thirds of the way along a median from a vertex. Its distance from the midpoint of any side is one-third of the altitude, $r = a\sqrt{3}/6$.

Step 1: Find the common distance from the axis For the triangle of side $a = 2 \text{ m}$, the altitude is

$$h = \frac{\sqrt{3}}{2}a = \sqrt{3} \text{ m}.$$

The three masses lie at the centres of the edges, meaning the side midpoints. The centroid divides every median in a 2 : 1 ratio measured from the vertex. Therefore the distance from the centroid to each side midpoint is

$$r = \frac{h}{3} = \frac{\sqrt{3}}{3} \text{ m}, \quad r^2 = \frac{1}{3} \text{ m}^2.$$

Because the axis is perpendicular to the plane at the centroid, these in-plane distances are precisely the perpendicular distances to the axis.

Step 2: Add the three contributions Each mass is at the same distance r , even though the masses differ. Thus

$$I = (2 + 4 + 6) \frac{1}{3} = 4 \text{ kg m}^2.$$

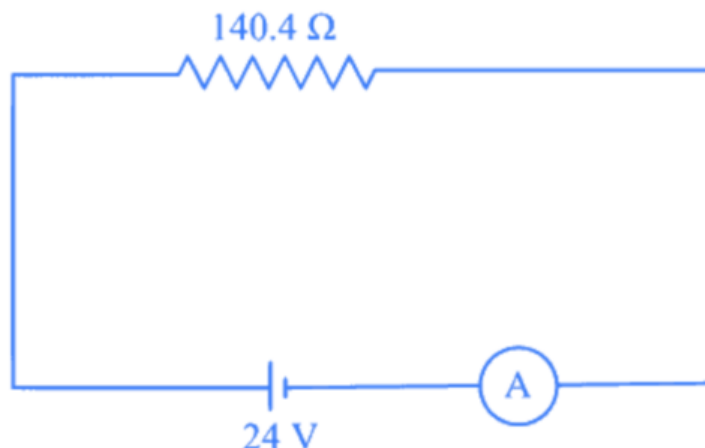
Individually the contributions are $2/3$, $4/3$ and 2 kg m^2 . Their sum is 4. Using the distance from the centroid to a vertex would be inappropriate because the balls occupy edge midpoints. The unequal masses do not move the prescribed rotation axis: the question specifies the geometric centroid of the

triangle, rather than the centre of mass of the system.

Step 3: Conclusion The moment of inertia about the stated axis is 4 kg m^2 . The numerical entry is 4.

Quick Tip to Solve: Distinguish side midpoints from vertices: their distances from an equilateral triangle's centroid are $h/3$ and $2h/3$, respectively.

53. In the given figure an ammeter A consists of a 240Ω coil connected in parallel to a 10Ω shunt. The reading of the ammeter is _____ mA. [4 marks]



Solution:

Concept:

- A practical ammeter can consist of a galvanometer coil resistance R_g in parallel with a small shunt resistance R_s . Its effective resistance is $R_A = R_g R_s / (R_g + R_s)$.
- The ammeter reading is the total current entering the parallel coil-and-shunt combination, rather than the smaller current through the coil alone. In the external series circuit, Ohm's law gives $I = V / (R + R_A)$.

Step 1: Replace the ammeter by its equivalent resistance The diagram shows a 24 V supply in series with an external resistor of 140.4Ω and ammeter A. Inside A, the 240Ω coil and 10Ω shunt have the same potential difference. Their parallel resistance is

$$R_A = \frac{240 \times 10}{240 + 10} = \frac{2400}{250} = 9.6\Omega.$$

The shunt reduces the meter resistance considerably below the coil resistance, allowing most of the current to bypass the sensitive coil.

Step 2: Find the total circuit current The meter is in series with the external resistor, so

$$R_{\text{total}} = 140.4 + 9.6 = 150\Omega, \quad I = \frac{24}{150} = 0.160 \text{ A} = 160 \text{ mA}.$$

As a check, the potential difference across the meter is $0.160 \times 9.6 = 1.536 \text{ V}$. The coil current

is $1.536/240 = 0.0064 \text{ A}$, while the shunt current is $1.536/10 = 0.1536 \text{ A}$. They add to 0.160 A , confirming the indicated total. Treating the ammeter as ideal would omit its stated resistance and give a different current.

Step 3: Conclusion The ammeter reads $\boxed{160 \text{ mA}}$. The numerical entry is 160.

Quick Tip to Solve: First combine the coil and shunt in parallel; then add that equivalent resistance to the external series resistor.

54. A coil having 100 turns, area of $5 \times 10^{-3} \text{ m}^2$, carrying current of 1 mA is placed in uniform magnetic field of 0.20 T such a way that plane of coil is perpendicular to the magnetic field. The work done in turning the coil through 90° is _____ μJ . [4 marks]

Solution:

Concept:

- A coil of N turns, each of area A , carrying current I , has magnetic dipole moment $\mu = NIA$, directed normal to its plane by the right-hand rule.
- The potential energy in a uniform magnetic field is $U = -\mu B \cos \theta$, where θ is the angle between the magnetic moment and the field, not between the coil plane and the field. Quasistatic external work equals the increase in potential energy.

Step 1: Set the initial and final orientations Initially the coil plane is perpendicular to the field, so the coil normal, and hence its moment, is parallel to the field in the stable orientation. Therefore $\theta_i = 0^\circ$. Turning the coil through 90° makes its normal perpendicular to the field, giving $\theta_f = 90^\circ$. The required work against the magnetic torque is

$$W = U_f - U_i = -\mu B \cos 90^\circ + \mu B \cos 0^\circ = \mu B.$$

One must use the angle of the normal; using the plane angle in the cosine would reverse the energy interpretation.

Step 2: Substitute the magnetic moment Convert $I = 1 \text{ mA} = 10^{-3} \text{ A}$. With $N = 100$ and $A = 5 \times 10^{-3} \text{ m}^2$,

$$\mu = 100(10^{-3})(5 \times 10^{-3}) = 5 \times 10^{-4} \text{ A m}^2.$$

Consequently

$$W = (5 \times 10^{-4})(0.20) = 1.0 \times 10^{-4} \text{ J} = 100 \mu\text{J}.$$

The torque varies with orientation as $\mu B \sin \theta$; multiplying its maximum value by the full angle would not give the correct work. The energy difference automatically accounts for this variation.

Step 3: Conclusion The work required to rotate the coil is $\boxed{100 \mu\text{J}}$. The numerical entry is 100.

Quick Tip to Solve: For coil-rotation work, use $\Delta(-NIA B \cos \theta)$ with θ measured from the coil normal, rather than multiplying a torque by the angle.

55. A capacitor of $10\mu\text{F}$ capacitance whose plates are separated by 10 mm through air and each plate has area 4 cm^2 is now filled equally with two dielectric media of $K_1 = 2$, $K_2 = 3$ respectively as shown in figure. If new force between the plates is 8 N . The supply voltage is _____ V. [4 marks]



Solution:

Concept:

- When two dielectrics occupy equal halves of the plate area and each spans the full separation, the regions are equivalent to parallel capacitors. Thus $C_{\text{new}} = C_0(K_1 + K_2)/2$, where C_0 is the stated capacitance with air.
- At fixed supply voltage V , the attraction between the plates has magnitude $F = C_{\text{new}}V^2/(2d)$. This follows either by adding the electric pressures in the two regions or from the derivative of capacitance with separation.

Step 1: Identify the dielectric arrangement The figure places $K_1 = 2$ and $K_2 = 3$ side by side across the plate area, with the same separation d for both. Each half experiences the same potential difference, so the contributions add in parallel. Using the given air capacitance $C_0 = 10\mu\text{F}$,

$$C_1 = \frac{K_1 C_0}{2} = 10\mu\text{F}, \quad C_2 = \frac{K_2 C_0}{2} = 15\mu\text{F},$$

$$C_{\text{new}} = 25\mu\text{F}.$$

The dielectric constants are averaged arithmetically for this geometry. A series formula would apply if the dielectrics instead occupied successive layers along the separation.

Step 2: Relate force to supply voltage For fixed area and dielectric arrangement, capacitance varies inversely with separation. The plate force is therefore $F = C_{\text{new}}V^2/(2d)$. Convert $d = 10\text{ mm} = 10^{-2}\text{ m}$, then solve:

$$V^2 = \frac{2Fd}{C_{\text{new}}} = \frac{2(8)(10^{-2})}{25 \times 10^{-6}} = 6400\text{ V}^2, \quad V = 80\text{ V}.$$

The positive root is the magnitude of the supply voltage. Since the original capacitance is supplied directly, it already supplies the factor involving plate area and permittivity needed in this calculation.

Step 3: Conclusion The supply voltage is 80 V. The numerical entry is 80.

Quick Tip to Solve: Dielectrics side by side share the same voltage: add their capacitances, then use $F = CV^2/(2d)$ with the filled capacitance.

56. A particle moves in a straight line so that its displacement x at any time t is given by $x^2 = 1 + t^2$. Its acceleration at any time t is x^{-n} where $n =$ _____. [4 marks]

Solution:

Concept:

- Velocity and acceleration are time derivatives: $v = dx/dt$ and $a = dv/dt$. An implicit equation relating displacement and time can be differentiated without first solving explicitly for either variable.
- The product rule gives $d(xv)/dt = v^2 + xa$. Alternatively, after writing v as a function of position, one may use $a = v dv/dx$. Both methods must retain the original relation $x^2 - t^2 = 1$.

Step 1: Differentiate the displacement equation Begin with $x^2 = 1 + t^2$. Differentiating both sides with respect to time gives

$$2x \frac{dx}{dt} = 2t, \quad xv = t, \quad v = \frac{t}{x}.$$

The displacement cannot vanish because $x^2 = 1 + t^2 \geq 1$. Thus dividing by x is permissible on either continuous branch of the motion. Differentiate $xv = t$ once more. The product rule on the left yields

$$v^2 + xa = 1.$$

This step includes the change in both the displacement and velocity; treating x as a constant would lose the v^2 term.

Step 2: Eliminate time and compare the power Substitute $v = t/x$ into the second derivative equation:

$$a = \frac{1 - v^2}{x} = \frac{1 - t^2/x^2}{x} = \frac{x^2 - t^2}{x^3}.$$

The original displacement law gives $x^2 - t^2 = 1$, so

$$a = \frac{1}{x^3} = x^{-3}.$$

Comparing with the required form x^{-n} gives $n = 3$. A direct check using the positive branch $x = \sqrt{1 + t^2}$ gives $a = (1 + t^2)^{-3/2}$, exactly the same expression. On the negative branch the sign is also correctly retained by x^{-3} .

Step 3: Conclusion The acceleration is $a = x^{-3}$, so the required exponent is $\boxed{n = 3}$.

Quick Tip to Solve: For a relation like $x^2 - t^2 = \text{constant}$, differentiate implicitly twice; substitute the original constant only after forming the acceleration.

57. Two coherent monochromatic light beams of intensities I and $4I$ are superimposed. The difference between maximum and minimum possible intensities in the resulting beam is xI . The value of x is _____.

[4 marks]

Solution:

Concept:

- For two coherent beams, the resultant intensity is $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$, where ϕ is their phase difference. Intensities are proportional to squared amplitudes.
- Constructive interference occurs for $\cos \phi = 1$, while destructive interference occurs for $\cos \phi = -1$. Thus unequal beam amplitudes generally leave a nonzero minimum intensity.

Step 1: Write the two extreme intensities The intensity of the first beam is $I_1 = I$, and that of the second is $I_2 = 4I$. Their amplitude ratio is therefore 1 : 2, not 1 : 4. Adding amplitudes for the maximum and subtracting them for the minimum gives

$$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2, \quad I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2.$$

Both expressions refer to the same two incident beams; only the relative phase changes.

Step 2: Substitute and subtract the extremes Using $\sqrt{4I} = 2\sqrt{I}$, we obtain

$$I_{\max} = (\sqrt{I} + 2\sqrt{I})^2 = 9I,$$

$$I_{\min} = (\sqrt{I} - 2\sqrt{I})^2 = I.$$

Hence the requested difference, rather than the sum of the extremes, is

$$I_{\max} - I_{\min} = 9I - I = 8I.$$

Equivalently, subtraction of the general formulas gives $4\sqrt{I_1 I_2} = 4\sqrt{4I^2} = 8I$. This also checks that the unequal intensities prevent complete darkness at the minimum.

Step 3: Conclusion Comparing $I_{\max} - I_{\min} = 8I$ with xI , the required dimensionless value is $x = 8$.

Quick Tip to Solve: For the difference between interference extremes, use $4\sqrt{I_1 I_2}$ directly; do not subtract the incident intensities.

58. Two open organ pipes of lengths 60 cm and 90 cm resonate at 6th and 5th harmonics respectively. The difference of frequencies for the given modes is _____ Hz. (Velocity of sound in air = 333 m/s)

[4 marks]

Solution:

Concept:

- An organ pipe open at both ends supports every integer harmonic. Neglecting end correction, the frequency of its n th harmonic is $f_n = nv/(2L)$, where v is sound speed and L is pipe length.
- The harmonic number multiplies the fundamental frequency. Lengths must be expressed in metres when the speed is in metres per second; the resulting frequency is in hertz.

Step 1: Find the sixth harmonic of the shorter pipe The first pipe has length $L_1 = 60 \text{ cm} = 0.60 \text{ m}$, and its specified mode is $n_1 = 6$. With the given sound speed $v = 333 \text{ m s}^{-1}$,

$$f_1 = \frac{n_1 v}{2L_1} = \frac{6 \times 333}{2 \times 0.60} = 1665 \text{ Hz}.$$

Here the factor of two arises from the open-open boundary condition; using the closed-pipe formula would give the wrong mode frequencies.

Step 2: Find the fifth harmonic and take the difference For the second open pipe, $L_2 = 90 \text{ cm} = 0.90 \text{ m}$ and $n_2 = 5$. Therefore

$$f_2 = \frac{n_2 v}{2L_2} = \frac{5 \times 333}{2 \times 0.90} = 925 \text{ Hz}.$$

The first specified frequency is larger because $6/0.60 > 5/0.90$. The positive separation is consequently

$$\Delta f = |f_1 - f_2| = 1665 - 925 = 740 \text{ Hz}.$$

The two harmonic numbers belong to different pipes, so they must not be subtracted before accounting for their different lengths.

Step 3: Conclusion The difference between the specified harmonic frequencies is 740 Hz; the numerical entry is 740.

Quick Tip to Solve: For two open pipes, calculate n/L for each first, then multiply their difference by $v/2$.

59. In Franck-Hertz experiment, the first dip in the current-voltage graph for hydrogen is observed at 10.2 V. The wavelength of light emitted by hydrogen atom when excited to the first excitation level is _____ nm. (Given $hc = 1245 \text{ eV nm}$, $e = 1.6 \times 10^{-19} \text{ C}$). [4 marks]

Solution:

Concept:

- In the Franck–Hertz experiment, an electron accelerated through potential difference V acquires kinetic energy eV . The first inelastic threshold measures the energy gap from the atomic ground state to its first excited state.
- When the atom returns from that first excited state to the ground state, the emitted photon has energy $\Delta E = hc/\lambda$. If hc is supplied in eV nm , use photon energy in electronvolts to obtain wavelength in nanometres.

Step 1: Obtain the first excitation energy The first current dip occurs at $V = 10.2 \text{ V}$. At this threshold an incident electron can transfer the excitation energy to a hydrogen atom, so

$$\Delta E = eV = 10.2 \text{ eV}.$$

In joules this is $10.2 \times 1.6 \times 10^{-19} = 1.632 \times 10^{-18} \text{ J}$. The energy in electronvolts is already convenient for the supplied value of hc ; the charge factor must not be inserted a second time.

Step 2: Convert the transition energy to wavelength The atom loses the same energy on its return

to the ground state, and that energy is carried by the emitted photon. Using the constant exactly as given,

$$\lambda = \frac{hc}{\Delta E} = \frac{1245 \text{ eV nm}}{10.2 \text{ eV}} = 122.0588 \dots \text{ nm}.$$

The electronvolt units cancel. This is an ultraviolet wavelength, consistent with the first excited-to-ground transition of hydrogen. Rounding the numerical value to the nearest integer gives 122.

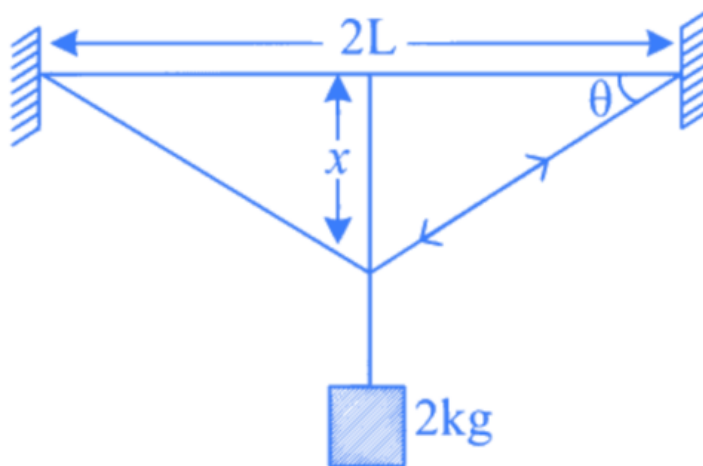
Step 3: Conclusion The emitted wavelength is approximately 122 nm, so the required numerical entry is 122.

Quick Tip to Solve: When hc is in eV nm, divide directly by the excitation voltage expressed as an energy in eV.

60. A wire of cross sectional area A , modulus of elasticity $2 \times 10^{11} \text{ Nm}^{-2}$ and length 2 m is stretched between two vertical rigid supports. When a mass of 2 kg is suspended at the middle it sags lower from its original position making angle $\theta = \frac{1}{100}$ radian on the points of support. The value of A is $\underline{\hspace{2cm}} \times 10^{-4} \text{ m}^2$ (consider $x \ll L$).

(given : $g = 10 \text{ m/s}^2$)

[4 marks]



Solution:

Concept:

- Young's modulus is $Y = (T/A)/\epsilon$, where T is wire tension, A is cross-sectional area and ϵ is longitudinal strain. Both inclined halves carry the same tension by symmetry.
- For a small inclination θ , $\sin \theta \simeq \theta$ and $\sec \theta - 1 \simeq \theta^2/2$. The strain is second order in the angle, whereas the vertical component of tension is first order.

Step 1: Use vertical force balance The two wire tensions support the suspended mass through their upward components. Their horizontal components cancel. Hence

$$2T \sin \theta = mg.$$

For $m = 2 \text{ kg}$, $g = 10 \text{ m s}^{-2}$ and $\theta = 1/100 = 0.01 \text{ rad}$,

$$T \simeq \frac{mg}{2\theta} = \frac{20}{0.02} = 1000 \text{ N}.$$

It would be incorrect to equate the tension itself to half the weight, since the wire is almost horizontal.

Step 2: Find the geometrical strain and area Let the original length of each half be ℓ . The stretched length is $\ell \sec \theta$, so

$$\epsilon = \frac{\ell \sec \theta - \ell}{\ell} = \sec \theta - 1 \simeq \frac{\theta^2}{2} = 5 \times 10^{-5}.$$

The length cancels, as expected for this small-angle relation. Now $T = YA\epsilon$, which gives

$$A = \frac{T}{Y\epsilon} = \frac{1000}{(2 \times 10^{11})(5 \times 10^{-5})} = 1.0 \times 10^{-4} \text{ m}^2.$$

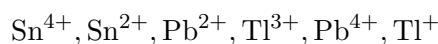
The approximation is justified by the stated small sag and the angle 0.01 radian.

Step 3: Conclusion The cross-sectional area is $A = 1 \times 10^{-4} \text{ m}^2$. Thus the coefficient requested in the blank is 1.

Quick Tip to Solve: In a sagging-wire problem, use $T \simeq mg/(2\theta)$ together with strain $\theta^2/2$; strain is not θ .

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. The number of ions from the following that are expected to behave as oxidising agent is :



[4 marks]

- (A) 3
- (B) 4
- (C) 2
- (D) 1

Correct Answer: (C) 2

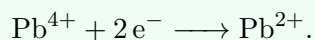
Solution:

Concept:

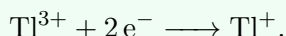
- An oxidising agent accepts electrons and is itself reduced. For the heavier p-block elements, compare the relative stability of the oxidation states before deciding whether a species tends to accept or donate electrons.
- The inert-pair effect stabilises the lower oxidation state down a group. Thus Pb^{2+} is more stable than Pb^{4+} , and Tl^{+} is more stable than Tl^{3+} ; for tin the +4 state is relatively stable.

Step 1: Identify the unstable higher oxidation states The inert-pair effect becomes stronger down the p-block groups. Lead favours its lower +2 state over +4, so lead(IV) readily accepts electrons and

acts as an oxidising agent:



Similarly, the +1 state is substantially more stable for thallium than +3. Thallium(III) therefore shows oxidising behaviour:



In both cases, reduction leads to the lower state stabilised by the inert pair.

Step 2: Compare the remaining ions and count Tin differs from lead: the +4 state is relatively stable, while tin(II) readily undergoes oxidation to tin(IV) and is characteristically a reducing agent. Thus Sn^{4+} is not counted among the ions expected to show the characteristic oxidising behaviour tested here. Merely having a high positive charge is not enough to classify it with the unstable higher states of lead and thallium. The stable lower-state ions Pb^{2+} and Tl^{+} are likewise excluded. The two expected oxidising ions are therefore

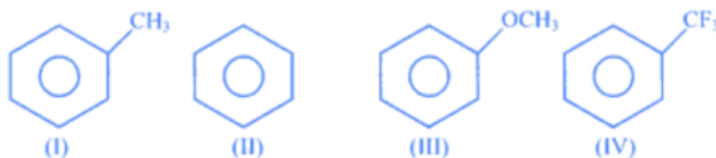


The alternatives one, three and four do not match this oxidation-state stability classification; counting tin(IV) as another characteristic oxidant would give the tempting but inappropriate count of three.

Step 3: Conclusion The number of ions expected to show oxidising behaviour is 2. Hence, the correct option is (C).

Quick Tip to Solve: For heavy p-block ions, check whether reduction leads to the oxidation state stabilised by the inert-pair effect before counting oxidants.

62. The correct arrangement for decreasing order of electrophilic substitution for above compounds is : [4 marks]



- (A) (III) > (IV) > (II) > (I)
(B) (III) > (I) > (II) > (IV)
(C) (II) > (IV) > (III) > (I)
(D) (IV) > (I) > (II) > (III)

Correct Answer: (B) (III) > (I) > (II) > (IV)

Solution:

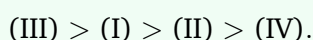
Concept:

- Electrophilic aromatic substitution is faster when a substituent raises electron density in the aromatic ring and stabilises the positively charged sigma complex. Electron-withdrawing groups have the opposite effect.

- An alkoxy group donates a lone pair by resonance; an alkyl group donates more weakly through hyperconjugation and its inductive effect. A trifluoromethyl group strongly withdraws electron density by induction.

Step 1: Identify the substituents in the four structures The structures are (I) toluene, with $-\text{CH}_3$; (II) benzene, with no substituent; (III) anisole, with $-\text{OCH}_3$; and (IV) trifluoromethylbenzene, with $-\text{CF}_3$. Oxygen in the methoxy group has a lone pair directly conjugated with the ring. Its resonance donation makes anisole strongly activated, even though oxygen also has an electron-withdrawing inductive effect. In this reaction the resonance contribution controls the overall activation.

Step 2: Rank activation and compare the alternatives The methyl group activates the ring less strongly than methoxy because hyperconjugative donation is weaker than direct lone-pair resonance donation. Benzene supplies the unsubstituted reference. In $-\text{CF}_3$, the highly electronegative fluorine atoms draw electron density through sigma bonds, and the carbon attached to the ring provides no donating lone pair. This makes compound (IV) less reactive than benzene. Combining these effects gives



Option (A) wrongly puts the strongly withdrawing group ahead of benzene and toluene. Option (C) puts benzene ahead of anisole, while (D) reverses the strongest activation and deactivation; neither is consistent with the electronic effects.

Step 3: Conclusion The decreasing reactivity order is $(III) > (I) > (II) > (IV)$. Hence, the correct option is (B).

Quick Tip to Solve: For this set, remember the activation sequence: methoxy resonance donation, methyl donation, unsubstituted benzene, then trifluoromethyl withdrawal.

63. During the detection of acidic radical present in a salt, a student gets a pale yellow precipitate soluble with difficulty in NH_4OH solution when sodium carbonate extract was first acidified with dil. HNO_3 and then AgNO_3 solution was added. This indicates presence of : [4 marks]

- (A) I^-
 (B) CO_3^{2-}
 (C) Cl^-
 (D) Br^-

Correct Answer: (D) Br^-

Solution:

Concept:

- Acidified silver nitrate precipitates halide ions as their sparingly soluble silver salts. Precipitate colour and solubility in aqueous ammonia distinguish chloride, bromide and iodide.
- Aqueous ammonia can dissolve a silver halide by forming the soluble complex $[\text{Ag}(\text{NH}_3)_2]^+$. Silver chloride dissolves readily, silver bromide dissolves only with difficulty in sufficiently concentrated ammonia, and silver iodide remains insoluble.

Step 1: Interpret the pale yellow precipitate Addition of AgNO_3 supplies Ag^+ . A bromide-containing extract gives the pale yellow or cream precipitate silver bromide:



The colour already distinguishes it from white silver chloride. Silver iodide is a deeper yellow and, crucially, is insoluble in ammonia. The reported difficulty of dissolution is therefore a diagnostic feature of silver bromide rather than silver iodide.

Step 2: Use acidification and ammonia to eliminate other ions The extract was first acidified with dilute HNO_3 . This removes carbonate introduced by sodium carbonate extraction as carbon dioxide and water:



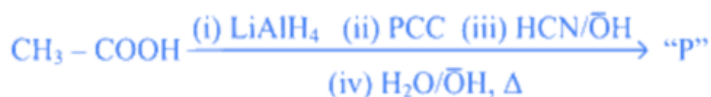
Consequently carbonate is not responsible for the observed silver salt. Chloride would give white AgCl , readily soluble in dilute ammonia, whereas iodide would give yellow AgI , insoluble even in concentrated ammonia. Bromide alone fits both observations: a pale yellow AgBr precipitate that dissolves with difficulty through silver-ammonia complex formation. Thus options (A), (B) and (C) each conflict with a stated test condition.

Step 3: Conclusion The acidic radical is the bromide ion, $\boxed{\text{Br}^-}$. Hence, the correct option is (D).

Quick Tip to Solve: Use both colour and ammonia solubility: white/readily soluble means chloride, cream/difficultly soluble means bromide, and yellow/insoluble means iodide.

64. Consider the given reaction, identify the major product P.

[4 marks]



- (A) $\text{CH}_3\text{CH}(\text{OH})\text{COOH}$
- (B) $\text{CH}_3\text{CH}_2\text{CONH}_2$
- (C) $\text{CH}_3\text{COCH}_2\text{CH}_3$
- (D) $\text{CH}_3\text{CH}_2\text{CH}_2\text{OH}$

Correct Answer: (A) $\text{CH}_3\text{CH}(\text{OH})\text{COOH}$

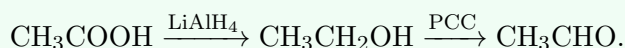
Solution:

Concept:

- LiAlH_4 reduces a carboxylic acid to a primary alcohol, preserving its carbon skeleton; PCC oxidises a primary alcohol to an aldehyde without continuing to the acid.
- Addition of HCN to an aldehyde forms a cyanohydrin, $\text{RCH}(\text{OH})\text{CN}$. Hydrolysis of the nitrile group converts its carbon into a carboxyl carbon, increasing the carbon chain by one.

Step 1: Reduce the acid and oxidise the alcohol The starting acid is ethanoic acid, CH_3COOH . Lithium aluminium hydride reduces its carboxyl group to $-\text{CH}_2\text{OH}$, giving ethanol. PCC then

removes hydrogen to form ethanal. The sequence is therefore



PCC is a controlled oxidant, so the second intermediate is the aldehyde and not regenerated ethanoic acid.

Step 2: Add cyanide to the carbonyl group In the presence of base, cyanide attacks the electrophilic carbonyl carbon of ethanal. Protonation of the resulting alkoxide gives the cyanohydrin:



The original carbonyl oxygen is now the hydroxyl group on the middle carbon; the incoming cyanide supplies the third carbon.

Step 3: Hydrolyse the nitrile and identify the product Heating the cyanohydrin with aqueous base hydrolyses $-\text{CN}$, through the amide stage, to carboxylate. The corresponding hydroxy acid represented in the options is $\text{CH}_3\text{CH}(\text{OH})\text{COOH}$, lactic acid. Strictly, the alkaline reaction mixture contains its carboxylate salt before acidification; option (A) shows the parent acid. Option (B) lacks the retained hydroxyl group and stops at an unrelated amide. Option (C) is a four-carbon ketone, while option (D) would require reduction rather than nitrile hydrolysis.

Step 4: Conclusion The product represented is 2-hydroxypropanoic acid (lactic acid). Under basic hydrolysis conditions its carboxylate is present. Hence, the correct option is (A).

Quick Tip to Solve: Track the original carbonyl oxygen and count carbons: HCN adds one carbon, and hydrolysis changes $-\text{CN}$ into $-\text{COOH}$ without removing the adjacent $-\text{OH}$.

65. Match List I with List II.

	List I: Reaction		List II: Type of redox reaction
A.	$\text{N}_2(\text{g}) + \text{O}_2(\text{g}) \longrightarrow 2\text{NO}(\text{g})$	I.	Decomposition
B.	$2\text{Pb}(\text{NO}_3)_2(\text{s}) \longrightarrow 2\text{PbO}(\text{s}) + 4\text{NO}_2(\text{g}) + \text{O}_2(\text{g})$	II.	Displacement
C.	$2\text{Na}(\text{s}) + 2\text{H}_2\text{O}(\text{l}) \longrightarrow 2\text{NaOH}(\text{aq}) + \text{H}_2(\text{g})$	III.	Disproportionation
D.	$2\text{NO}_2(\text{g}) + 2\text{OH}^-(\text{aq}) \longrightarrow \text{NO}_2^-(\text{aq}) + \text{NO}_3^-(\text{aq}) + \text{H}_2\text{O}(\text{l})$	IV.	Combination

Choose the correct answer from the options given below:

[4 marks]

(A) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(B) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(C) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(D) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Correct Answer: (A) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

Solution:

Concept:

- A combination reaction joins reactants into one product, decomposition breaks a compound into simpler products, and displacement replaces an element in a compound.
- Disproportionation oxidises and reduces the same element from one initial oxidation state to two different final states. Classify the overall reaction and use oxidation numbers to confirm the electron transfer.

Step 1: Classify reactions A and B In reaction A, nitrogen and oxygen combine to form only nitric oxide: $\text{N}_2 + \text{O}_2 \longrightarrow 2\text{NO}$. Nitrogen changes from 0 to +2 and oxygen from 0 to -2, so it is a redox combination reaction. Thus A matches IV. In reaction B, one compound, lead nitrate, breaks down into lead oxide, nitrogen dioxide and oxygen. Nitrogen is reduced from +5 to +4, while some oxygen is oxidised from -2 to 0. It is therefore a redox decomposition reaction, giving B-I.

Step 2: Classify the metal-water reaction Sodium displaces hydrogen from water in $2\text{Na} + 2\text{H}_2\text{O} \longrightarrow 2\text{NaOH} + \text{H}_2$. Sodium changes from 0 to +1 and hydrogen from +1 in water to 0 in the hydrogen molecule. This is displacement, so C matches II. Although several substances appear on both sides, the distinguishing feature is replacement of hydrogen by a more reactive metal.

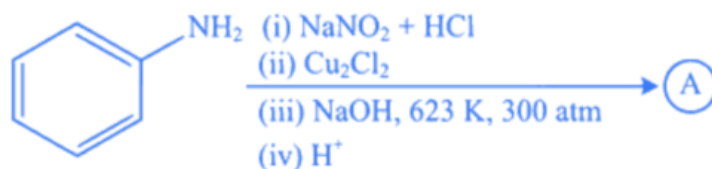
Step 3: Check the two nitrogen products in reaction D In NO_2 , nitrogen has oxidation state +4. In NO_2^- , it is +3, and in NO_3^- , it is +5. The same starting species therefore undergoes reduction and oxidation simultaneously. This identifies disproportionation, so D matches III. The complete mapping A-IV, B-I, C-II, D-III is option (A). Options (B), (C) and (D) each misclassify the combination reaction A and fail the individual checks above.

Step 4: Conclusion The correct matching is A-IV, B-I, C-II and D-III. Hence, the correct option is (A).

Quick Tip to Solve: For disproportionation, locate one element in a single reactant oxidation state and verify that its products contain both a higher and a lower oxidation state.

66. Identify the product A in the following reaction.

[4 marks]



- (A) 4-Chloroaniline ($\text{p-ClC}_6\text{H}_4\text{NH}_2$)
(B) Phenol ($\text{C}_6\text{H}_5\text{OH}$)
(C) 4-Chlorophenol ($\text{p-ClC}_6\text{H}_4\text{OH}$)
(D) 4-Aminophenol ($\text{p-H}_2\text{NC}_6\text{H}_4\text{OH}$)

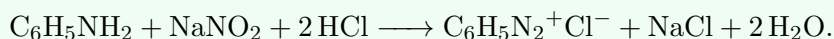
Correct Answer: (B) Phenol ($\text{C}_6\text{H}_5\text{OH}$)

Solution:

Concept:

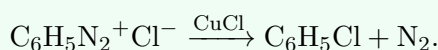
- Aniline reacts with nitrous acid generated from NaNO_2/HCl to give benzenediazonium chloride. The diazonium group can be replaced by chlorine using cuprous chloride (Sandmeyer reaction).
- Chlorobenzene undergoes hydrolysis with aqueous NaOH at high temperature and pressure to form sodium phenoxide; acidification gives phenol. Follow substitution at the carbon bearing the original amino group.

Step 1: Convert aniline into the diazonium salt The starting aromatic compound is aniline, $\text{C}_6\text{H}_5\text{NH}_2$. Sodium nitrite and hydrochloric acid generate nitrous acid in situ; diazotisation replaces the amino group by the diazonium group:



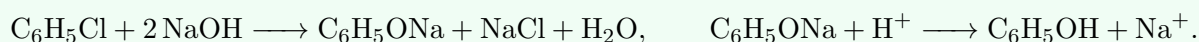
No second substituent is added to the benzene ring in this step.

Step 2: Replace the diazonium group by chlorine The cuprous chloride reagent, written as Cu_2Cl_2 , carries out the Sandmeyer reaction. The diazonium group leaves as exceptionally stable nitrogen gas, and chlorine replaces it:



The product at this stage is chlorobenzene, not para-chloroaniline. The original amino group has already been removed.

Step 3: Hydrolyse chlorobenzene and acidify At 623 K and 300 atm, aqueous sodium hydroxide converts chlorobenzene into sodium phenoxide. The final H^+ treatment protonates phenoxide:



Thus only one hydroxyl substituent remains on the ring. Options (A), (C) and (D) require an additional chlorine or amino group that the sequence does not introduce or retain.

Step 4: Conclusion Product A is phenol, $\text{C}_6\text{H}_5\text{OH}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the intermediate after each reagent: aniline becomes a diazonium salt, then chlorobenzene, then phenoxide, and finally phenol.

67. The correct statement among the following, for a “chromatography” purification method is :

[4 marks]

- (A) R_f of a polar compound is smaller than that of a non-polar compound.
- (B) Non-polar compounds are retained at top and polar compounds come down in column chromatography.
- (C) Organic compounds run faster than solvent in the thin layer chromatographic plate.
- (D) R_f is an integral value.

Correct Answer: (A) R_f of a polar compound is smaller than that of a non-polar compound.

Solution:

Concept:

- In ordinary adsorption chromatography, the stationary phase is a polar solid such as silica gel or alumina. Stronger adsorption slows a solute relative to the mobile phase.
- The retention factor is $R_f = \frac{\text{distance travelled by the solute spot}}{\text{distance travelled by the solvent front}}$, measured from the same origin on a thin-layer plate. It is a dimensionless ratio.

Step 1: Compare polar and nonpolar solutes A polar compound interacts more strongly with the polar stationary phase through dipole interactions and, where possible, hydrogen bonding. It consequently spends more time adsorbed and travels a shorter distance. A nonpolar compound is held less strongly and moves farther with the solvent under the same conditions. Hence R_f for the polar compound is smaller, which establishes statement (A). The comparison refers to the conventional polar adsorbents used in the stated purification method.

Step 2: Test the column-chromatography claim In a column packed with a polar adsorbent, polar compounds are retained more strongly near the top, whereas less strongly adsorbed nonpolar compounds move down and elute earlier. Option (B) reverses these roles, so it is false. The governing issue is relative affinity for the stationary phase, rather than the mass or boiling point of the solute.

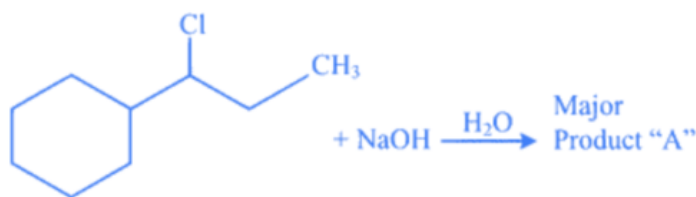
Step 3: Check the distance ratio and remaining options The solvent front advances ahead of the dissolved spots. A solute cannot run beyond that front in a normal TLC development, so option (C) is false. Normally a moving spot has $0 < R_f < 1$; a spot remaining at the origin has $R_f = 0$. Values such as 0.25 or 0.70 are entirely ordinary, so R_f is not restricted to an integer. Thus option (D) is false as well, leaving the adsorption comparison in (A) as the correct statement.

Step 4: Conclusion A polar compound has a smaller R_f than a nonpolar compound on a conventional polar adsorbent. Hence, the correct option is (A).

Quick Tip to Solve: On silica or alumina, remember: stronger polarity usually means stronger retention, a shorter travel distance and therefore a smaller R_f .

68. Consider the above chemical reaction. Product "A" is :

[4 marks]



- (A) 1-Propylcyclohexan-1-ol
- (B) 1-Cyclohexylpropan-2-ol
- (C) 1-Cyclohexylpropan-1-ol
- (D) 2-Propylcyclohexan-1-ol

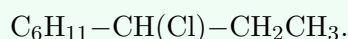
Correct Answer: (A) 1-Propylcyclohexan-1-ol

Solution:

Concept:

- In aqueous polar conditions an ionising alkyl halide can react through a carbocation. A 1,2-hydride shift can convert a secondary carbocation into a more stable tertiary carbocation before nucleophilic capture.
- A hydride shift transfers hydrogen together with its bonding electron pair; the positive charge moves to the carbon that lost the hydride. Water or hydroxide then supplies the hydroxyl group at that new cationic centre.

Step 1: Locate the initially formed carbocation The substrate has the connectivity



Chlorine is on the side-chain carbon immediately next to the cyclohexane ring. Ionisation of the carbon-chlorine bond produces the secondary side-chain carbocation $\text{C}_6\text{H}_{11}-\text{CH}^+-\text{CH}_2\text{CH}_3$. Direct capture would give the unrearranged secondary alcohol (C); first check for rearrangement.

Step 2: Shift a hydride from the adjacent ring carbon The ring carbon bonded to the side chain has one hydrogen and three carbon neighbours. Transfer of that hydrogen as hydride to the adjacent positively charged side-chain carbon fills its electron deficiency. The side-chain carbon becomes $-\text{CH}_2-$, while the ring carbon becomes a tertiary carbocation bonded to two ring carbons and the propyl chain. The favourable shift preserves the ring and carbon count.

Step 3: Capture the rearranged cation and compare the options Nucleophilic capture at the tertiary ring carbon, followed by proton transfer if water attacks, gives 1-propylcyclohexan-1-ol. The hydroxyl group and the straight propyl chain are therefore attached to the same ring carbon, exactly as in (A). Option (B) places hydroxyl at a different side-chain carbon and does not represent the favourable hydride shift. Option (C) is the unrearranged product, while (D) puts hydroxyl on the neighbouring ring carbon rather than the tertiary cationic centre.

Step 4: Conclusion The rearranged tertiary alcohol is 1-propylcyclohexan-1-ol. Hence, the correct option is (A).

Quick Tip to Solve: Before replacing a leaving group by $-\text{OH}$, check its neighbouring carbon for a hydride shift that changes a secondary carbocation into a tertiary one.

69. How can an electrochemical cell be converted into an electrolytic cell ?

[4 marks]

- (A) Applying an external opposite potential greater than E_{cell}^0
- (B) Exchanging the electrodes at anode and cathode.
- (C) Applying an external opposite potential lower than E_{cell}^0
- (D) Reversing the flow of ions in salt bridge.

Correct Answer: (A) Applying an external opposite potential greater than E_{cell}^0

Solution:

Concept:

- A galvanic cell converts the free energy of a spontaneous redox reaction into electrical work. For a reaction transferring n electrons, $\Delta G = -nFE_{\text{cell}}$, where F is the Faraday constant.
- An electrolytic cell uses an external electrical source to drive a nonspontaneous reaction. The external voltage must oppose the galvanic emf and exceed it to reverse the net current and cell reaction.

Step 1: Identify what the external source must oppose An operating galvanic cell develops a potential difference that drives electrons through its external circuit. Connecting a source in the same driving direction reinforces the spontaneous reaction and does not turn it into electrolysis. Instead, connect the source so that its emf opposes that of the cell. The source then attempts to send electrons opposite to the direction favoured by the spontaneous cell reaction.

Step 2: Compare the magnitudes of the two potentials Let E_{ext} denote the magnitude of the opposing source. If $E_{\text{ext}} < E_{\text{cell}}$, the galvanic driving force still wins, although the current decreases. At equality, the ideal net driving force is zero and no net cell reaction proceeds. When $E_{\text{ext}} > E_{\text{cell}}$, the external source drives the reverse reaction; electrical energy is consumed, so the system functions as an electrolytic cell. Under the standard conditions used in the options, the comparison is with E_{cell}° .

Step 3: Eliminate the other proposed operations Option (C) supplies an opposing potential that is too small to reverse the reaction. Merely exchanging electrode positions, option (B), does not supply the energy required for a nonspontaneous reaction. Ion flow in the salt bridge maintains electroneutrality as a consequence of electrode reactions; reversing it by itself, as in option (D), is not the external electrical driving force needed for electrolysis. In a practical cell, additional voltage can also be required to overcome resistance and electrode overpotentials.

Step 4: Conclusion Apply an external potential in opposition to the cell emf with magnitude greater than E_{cell}° under standard conditions. Hence, the correct option is (A).

Quick Tip to Solve: Opposing voltage below the cell emf slows the galvanic reaction; equality stops it ideally; a larger opposing voltage drives electrolysis.

70. Given below are two statements :

Statement I : PF_5 and BrF_5 both exhibit $sp^3 d$ hybridisation.

Statement II : Both SF_6 and $[\text{Co}(\text{NH}_3)_6]^{3+}$ exhibit $sp^3 d^2$ hybridisation.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are false

(B) Statement I is true but Statement II is false

(C) Statement I is false but Statement II is true

(D) Both Statement I and Statement II are true

Correct Answer: (A) Both Statement I and Statement II are false

Solution:

Concept:

- In the usual school-level hybridisation model, count bonding pairs and lone pairs around a main-group central atom: five electron domains correspond to $sp^3 d$, while six correspond to $sp^3 d^2$.
- For octahedral transition-metal complexes, distinguish outer-orbital $sp^3 d^2$ from inner-orbital $d^2 sp^3$ hybridisation. Strong-field pairing can make two inner d orbitals available.

Step 1: Evaluate Statement I using electron domains In PF_5 , phosphorus forms five bonds and has no lone pair. Its five electron domains give trigonal-bipyramidal geometry and the conventional $sp^3 d$ description. In BrF_5 , bromine forms five bonds and retains one lone pair. There are therefore six electron domains, giving an octahedral electron-pair arrangement and a square-pyramidal molecular shape, described by $sp^3 d^2$. Since these two species do not both have $sp^3 d$ hybridisation, Statement I is false.

Step 2: Determine the hybridisation of the cobalt complex In $[\text{Co}(\text{NH}_3)_6]^{3+}$, ammonia is neutral, so cobalt is in the +3 oxidation state. Cobalt has atomic number 27; Co^{3+} is $3d^6$. In this cobalt(III) ammine complex, the ligand field produces a low-spin octahedral configuration $t_{2g}^6 e_g^0$. Pairing makes two inner $3d$ orbitals available for the valence-bond description, giving $d^2 sp^3$ hybridisation rather than $sp^3 d^2$.

Step 3: Evaluate Statement II and select the combination Sulfur hexafluoride has six bonding domains around sulfur and is conventionally assigned $sp^3 d^2$ hybridisation. However, the cobalt complex is an inner-orbital $d^2 sp^3$ complex. The claim that both have $sp^3 d^2$ hybridisation is therefore false. Both statements are false, which matches (A). Option (B) incorrectly accepts Statement I, option (C) incorrectly accepts Statement II, and option (D) incorrectly accepts both.

Step 4: Conclusion Statement I is false and Statement II is false. Hence, the correct option is (A).

Quick Tip to Solve: Do not infer hybridisation from octahedral shape alone: an octahedral complex may be inner-orbital $d^2 sp^3$ or outer-orbital $sp^3 d^2$.

71. Match List I with List II.

	LIST I Tetrahedral Complex		LIST II Electronic configuration
A.	TiCl_4	I.	e^2, t_2^0
B.	$[\text{FeO}_4]^{2-}$	II.	e^4, t_2^3
C.	$[\text{FeCl}_4]^-$	III.	e^0, t_2^0
D.	$[\text{CoCl}_4]^{2-}$	IV.	e^2, t_2^3

Choose the correct answer from the options given below :

[4 marks]

(A) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(B) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(C) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

(D) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Correct Answer: (B) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Solution:

Concept:

- The oxidation state of the central metal follows from the overall charge and the ligand charges. For a transition metal, the d-electron count is the group number minus its oxidation state.
- In a tetrahedral crystal field, the two e orbitals lie below the three t_2 orbitals. The relatively small splitting gives high-spin filling, so electrons occupy available orbitals singly before pairing.

Step 1: Determine each metal oxidation state In TiCl_4 , four chloride ligands require titanium to be +4. Titanium is in group 4, so Ti^{4+} is d^0 . For $[\text{FeO}_4]^{2-}$, oxide contributes a total charge of -8, giving iron the oxidation state +6 and hence d^2 . In $[\text{FeCl}_4]^-$, iron is +3, hence d^5 . Finally, cobalt in $[\text{CoCl}_4]^{2-}$ is +2; group-9 cobalt therefore gives d^7 .

Step 2: Fill the tetrahedral orbitals and match the lists The d^0 titanium species has $e^0 t_2^0$, which is III. The two electrons of d^2 occupy separate lower e orbitals, producing $e^2 t_2^0$, which is I. High-spin d^5 occupies all five orbitals singly, giving $e^2 t_2^3$, which is IV. For high-spin d^7 , the sixth and seventh electrons pair in the lower e set, giving $e^4 t_2^3$, which is II. Thus A matches III, B matches I, C matches IV and D matches II. Options (A) and (C) incorrectly assign a nonzero d count to titanium, while (D) gives the wrong configurations to iron and cobalt.

Step 3: Conclusion The configurations are titanium: III, ferrate: I, ferric chloride complex: IV, and cobalt chloride complex: II. Hence, the correct option is (B).

Quick Tip to Solve: Find oxidation states before filling tetrahedral levels: d^0, d^2, d^5, d^7 immediately identify these four entries.

72. Molality (m) of 3M aqueous solution of NaCl is : (Given : Density of solution = 1.25 g mL^{-1} , Molar mass in g mol^{-1} : Na – 23, Cl – 35.5) [4 marks]

(A) 2.90 m

(B) 3.85 m

(C) 1.90 m

(D) 2.79 m

Correct Answer: (D) 2.79 m

Solution:

Concept:

- Molarity is the number of moles of solute per litre of solution, whereas molality is the number of moles of solute per kilogram of solvent. The denominator must therefore be the mass of water, not the solution mass.
- Density converts a chosen solution volume to its total mass. Subtracting the solute mass leaves the solvent mass; using one litre makes the number of solute moles numerically equal to the molarity.

Step 1: Choose one litre and calculate its component masses Take $1.000 \text{ L} = 1000 \text{ mL}$ of solution. Its mass is

$$m_{\text{solution}} = 1.25 \times 1000 = 1250 \text{ g.}$$

Since the concentration is 3 mol L^{-1} , this volume contains 3 mol of NaCl. The molar mass is $23 + 35.5 = 58.5 \text{ g mol}^{-1}$; hence the dissolved salt weighs

$$m_{\text{solute}} = 3 \times 58.5 = 175.5 \text{ g.}$$

Step 2: Use solvent mass in the molality definition The water mass is the difference between total solution mass and salt mass:

$$m_{\text{water}} = 1250 - 175.5 = 1074.5 \text{ g} = 1.0745 \text{ kg.}$$

Therefore

$$m = \frac{3}{1.0745} = 2.792 \text{ mol kg}^{-1} \simeq 2.79 \text{ m.}$$

The other values 2.90, 3.85 and 1.90 do not follow from the given density and solute mass. In particular, taking the full 1.250 kg as solvent would give 2.40, which is also incorrect because that mass includes the salt.

Step 3: Conclusion The molality is 2.79 mol kg^{-1} , or 2.79 m. Hence, the correct option is (D).

Quick Tip to Solve: For one litre, use $m = 1000M / (1000\rho - MM_s)$, with density in g mL^{-1} and molar mass M_s in g mol^{-1} .

73. Evaluate the following statements related to group 14 elements for their correctness.

(A) Covalent radius decreases down the group from C to Pb in a regular manner.

(B) Electronegativity decreases from C to Pb down the group gradually.

(C) Maximum covalance of C is 4 whereas other elements can expand their covalance due to presence of d orbitals.

(D) Heavier elements do not form $p\pi - p\pi$ bonds.

(E) Carbon can exhibit negative oxidation states.

Choose the correct answer from the options given below :

[4 marks]

(A) (A), (B) and (C) Only

(B) (A) and (B) Only

(C) (C), (D) and (E) Only

(D) (C) and (D) Only

Correct Answer: (C) (C), (D) and (E) Only

Solution:

Concept:

- Group 14 atoms have valence configuration ns^2np^2 . Atomic size, electronegativity, covalence and multiple bonding must be assessed separately rather than assuming that every property varies regularly down a group.
- Carbon has only four valence-shell orbitals and is particularly effective at forming $p\pi - p\pi$ multiple bonds. The conventional NCERT description attributes expanded covalence of heavier congeners to the availability of d orbitals.

Step 1: Test the two statements about regular trends Statement (A) is false. Covalent radius increases substantially from carbon to silicon as a new principal shell is added; radii do not decrease regularly from carbon to lead. Statement (B) is also false. Electronegativity drops from carbon to silicon, but the heavier group-14 elements have nearly similar values, with small irregularities. It therefore does not decrease gradually throughout the entire sequence to lead.

Step 2: Test covalence, multiple bonding and oxidation states Statement (C) is correct in the conventional syllabus treatment: carbon cannot expand beyond covalence four, whereas heavier members can show expanded covalence, as in hexacoordinate fluorosilicate. Statement (D) expresses the standard group trend and is treated as correct: larger, more diffuse p orbitals of heavier members have ineffective lateral overlap and do not form the strong conventional $p\pi - p\pi$ bonds characteristic of carbon. Statement (E) is correct because carbon can have negative oxidation states; for example its oxidation state in CH_4 is -4. Thus (C), (D) and (E) are the accepted statements. The first two options wrongly include (A) and (B), while the last omits the true statement (E).

Step 3: Conclusion The correct statements are C, D and E only. Hence, the correct option is (C).

Quick Tip to Solve: Use methane to check negative carbon oxidation states, and remember that group-14 electronegativities are nearly constant after silicon.

74. Arrange the following elements in the increasing order of number of unpaired electrons in it.

- (A) Sc
- (B) Cr
- (C) V
- (D) Ti
- (E) Mn

Choose the correct answer from the options given below :

[4 marks]

- (A) (A) < (D) < (C) < (B) < (E)
- (B) (A) < (D) < (C) < (E) < (B)
- (C) (B) < (C) < (D) < (E) < (A)
- (D) (C) < (E) < (B) < (A) < (D)

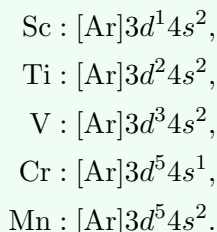
Correct Answer: (B) (A) < (D) < (C) < (E) < (B)

Solution:

Concept:

- Hund's rule places electrons singly in degenerate orbitals before pairing. The number of unpaired electrons must be counted in the complete ground-state atomic configuration.
- Chromium is an exception to the simple filling pattern: its ground state is $[\text{Ar}]3d^54s^1$. Both the five unpaired d electrons and the unpaired 4s electron contribute.

Step 1: Write the ground-state configurations The relevant neutral atoms have the following configurations:



The argon core is completely paired and does not affect the count. In Sc, Ti, V and Mn, the two 4s electrons are also paired.

Step 2: Count the unpaired electrons and arrange the atoms The $3d^1, 3d^2, 3d^3$ configurations give one, two and three unpaired electrons for Sc, Ti and V respectively. Manganese has five unpaired d electrons and no unpaired s electron, giving five in total. Chromium has five unpaired d electrons plus one unpaired s electron, giving six. Hence

$$1 < 2 < 3 < 5 < 6 \implies \text{Sc} < \text{Ti} < \text{V} < \text{Mn} < \text{Cr}.$$

Translating to the labels gives (A) < (D) < (C) < (E) < (B). Option (A) interchanges chromium and manganese, missing the unpaired 4s electron of chromium; options (C) and (D) also contradict the direct counts.

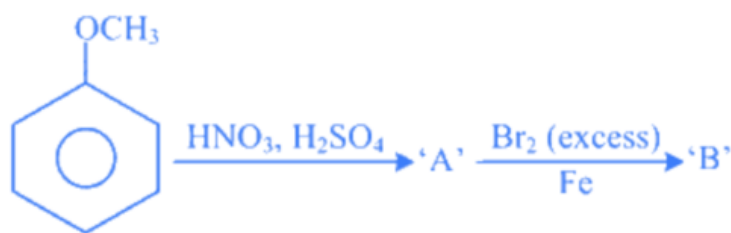
Step 3: Conclusion The increasing order is $\text{Sc} < \text{Ti} < \text{V} < \text{Mn} < \text{Cr}$. Hence, the correct option is (B).

Quick Tip to Solve: For neutral chromium, count the $4s^1$ electron as well as $3d^5$; chromium has six unpaired electrons, while manganese has five.

75. The major products formed :

A and B respectively are :

[4 marks]



(A) 4-Nitroanisole and 2,6-dibromo-4-nitroanisole

(B) 2-Nitroanisole and 4-bromo-2-nitroanisole

(C) 4-Nitroanisole and 2-bromo-4-nitroanisole

(D) 2-Nitroanisole and 4,6-dibromo-2-nitroanisole

Correct Answer: (A) 4-Nitroanisole and 2,6-dibromo-4-nitroanisole

Solution:

Concept:

- The methoxy group donates a lone pair to an aromatic ring by resonance. It activates the ring towards electrophilic aromatic substitution and directs incoming electrophiles to ortho and para positions.
- The nitro group is deactivating and meta-directing. When two substituents are present, identify positions favoured by both groups and account for the stated excess of the second reagent.

Step 1: Identify the major nitration product A The starting compound is anisole, $\text{C}_6\text{H}_5\text{OCH}_3$. The mixture of nitric acid and sulfuric acid generates the nitronium electrophile, NO_2^+ . Resonance donation by $-\text{OCH}_3$ stabilizes attack at ortho and para positions. The para product is the major product because the incoming nitro group is farther from the methoxy group and experiences less steric crowding. Thus A is 4-nitroanisole; choices with 2-nitroanisole as the major A are excluded.

Step 2: Apply both directing effects during bromination Number the anisole carbon bearing $-\text{OCH}_3$ as 1 and the nitro-bearing carbon as 4. The methoxy group favours positions 2 and 6, since its para position is occupied. Positions 2 and 6 are also meta to the nitro group, so both directing effects favour these same sites. Bromine in the presence of iron produces the electrophilic brominating species. Because bromine is supplied in excess, substitution occurs at both equivalent favoured positions, producing 2,6-dibromo-4-nitroanisole as B. The option containing only one bromine does not represent the product under the stated excess-bromine conditions. The options beginning with ortho-nitroanisole already fail the first step.

Step 3: Conclusion A is 4-nitroanisole and B is 2,6-dibromo-4-nitroanisole. Hence, the correct option is (A).

Quick Tip to Solve: After para nitration of anisole, positions 2 and 6 are simultaneously ortho to methoxy and meta to nitro; excess bromine fills both.

76. The incorrect statements regarding enzymes are :

- (A) Enzymes are biocatalysts.
- (B) Enzymes are non-specific and can catalyse different kinds of reactions.
- (C) Most Enzymes are globular proteins.
- (D) Enzyme - oxidase catalyses the hydrolysis of maltose into glucose.

Choose the correct answer from the option given below :

[4 marks]

- (A) (B) and (D)
- (B) (A), (B) and (C)
- (C) (B) and (C)
- (D) (B), (C) and (D)

Correct Answer: (A) (B) and (D)

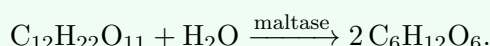
Solution:

Concept:

- Enzymes are biological catalysts: they accelerate biochemical reactions by lowering activation energy and are regenerated after the catalytic cycle. Most enzymes are globular proteins with a specific three-dimensional active site.
- Enzyme specificity concerns both the substrate and the type of transformation. Names such as maltase and oxidase identify different catalytic functions rather than interchangeable general catalysts.

Step 1: Evaluate the statements about nature and specificity Statement (A) is correct because enzymes catalyse reactions in living systems; describing them as biocatalysts is appropriate. Statement (B) is incorrect: enzymes generally show high specificity for their substrates and for the reactions they catalyse. The shape and chemical environment of an active site select a suitable substrate and reaction pathway; an enzyme is not a non-specific catalyst for arbitrary kinds of reactions. Statement (C) is correct because most enzymes are globular proteins. The word most also allows for catalytic RNA, so such exceptions do not make this statement false.

Step 2: Identify the enzyme for maltose hydrolysis Statement (D) is incorrect. Maltose is hydrolysed by maltase, which cleaves its glycosidic bond using water:

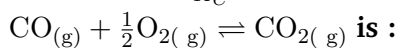


Oxidases catalyse oxidation reactions, typically involving oxygen as the electron acceptor, rather than the specified hydrolytic cleavage. Thus the two incorrect statements are precisely (B) and (D). The remaining answer choices include the correct statement (C), or the correct statements (A) and (C), and therefore cannot be the required set of incorrect statements.

Step 3: Conclusion The incorrect statements are B and D. Hence, the correct option is (A).

Quick Tip to Solve: Keep the enzyme name tied to the reaction: maltase hydrolyses maltose, while an oxidase catalyses an oxidation.

77. The ratio $\frac{K_P}{K_C}$ for the reaction :



[4 marks]

(A) 1

(B) $(RT)^{1/2}$

(C) RT

(D) $\frac{1}{\sqrt{RT}}$

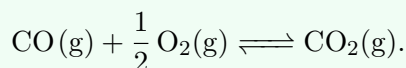
Correct Answer: (D) $\frac{1}{\sqrt{RT}}$

Solution:

Concept:

- For an ideal-gas equilibrium, $K_P = K_C(RT)^{\Delta n_g}$, where Δn_g is the sum of gaseous-product stoichiometric coefficients minus the sum of gaseous-reactant coefficients.
- The equilibrium constants must refer to the reaction exactly as written. Fractional coefficients contribute their fractional values to Δn_g ; multiplying the equation would change both equilibrium constants.

Step 1: Calculate the change in gaseous stoichiometric amount The given equation is



There is one stoichiometric mole of gaseous product. The gaseous reactants have total coefficient $1 + 1/2 = \frac{3}{2}$. Hence

$$\Delta n_g = 1 - \frac{3}{2} = -\frac{1}{2}.$$

The sign is negative because the written reaction reduces the total gaseous amount; it must not be reversed when forming products minus reactants.

Step 2: Substitute in the equilibrium-constant relation Therefore

$$\frac{K_P}{K_C} = (RT)^{-1/2} = \frac{1}{\sqrt{RT}}.$$

Equivalently, substituting $p_i = c_i RT$ directly into $K_P = p_{\text{CO}_2} / (p_{\text{CO}} p_{\text{O}_2}^{1/2})$ leaves one factor RT in the numerator and $(RT)^{3/2}$ in the denominator. Their ratio is $(RT)^{-1/2}$. Option (A) would require no change in gaseous coefficients, option (B) uses the opposite sign, and option (C) uses a positive unit exponent instead of the required negative half.

Step 3: Conclusion The required ratio is $K_P/K_C = 1/\sqrt{RT}$. Hence, the correct option is (D).

Quick Tip to Solve: Calculate Δn_g from the equation exactly as printed, including the half-mole coefficient of oxygen, before using $(RT)^{\Delta n_g}$.

78. The correct IUPAC name of $[\text{PtBr}_2(\text{PMe}_3)_2]$ is :

[4 marks]

- (A) bis(trimethylphosphine)dibromoplatinum(II)
- (B) dibromodi(trimethylphosphine)platinum(II)
- (C) dibromobis(trimethylphosphine)platinum(II)
- (D) bis[bromo(trimethylphosphine)]platinum(II)

Correct Answer: (C) dibromobis(trimethylphosphine)platinum(II)

Solution:

Concept:

- In a neutral coordination entity the sum of the metal oxidation state and all ligand charges is zero. Bromide is an anionic ligand, whereas trimethylphosphine, $\text{P}(\text{CH}_3)_3$, is neutral.
- Ligand names precede the metal name and are arranged alphabetically, ignoring multiplicative prefixes. For an already complex ligand name, use bis, tris and related prefixes with parentheses.

Step 1: Determine the oxidation state of platinum Let the oxidation state of platinum be z . The two bromide ligands each contribute -1 , while both phosphine ligands have zero charge. Since no overall charge is shown,

$$z + 2(-1) + 2(0) = 0 \Rightarrow z = +2.$$

The metal is therefore named platinum(II). The neutral complex does not require the anionic metal-name ending -ate.

Step 2: Arrange and count the ligands The ligand names are bromo and trimethylphosphine. Bromo comes before trimethylphosphine in alphabetical order, so the name begins dibromo. There are two trimethylphosphine ligands; because their name already contains the prefix tri, they are written as bis(trimethylphosphine). Combining these portions gives dibromobis(trimethylphosphine)platinum(II). Option (A) reverses the alphabetical order, (B) uses di instead of bis for the complex ligand name, and (D) incorrectly groups bromo and phosphine together as a repeated composite ligand.

Step 3: Conclusion The correct name is dibromobis(trimethylphosphine)platinum(II). Hence, the correct option is (C).

Quick Tip to Solve: Find the metal oxidation state first; alphabetise the ligand names without counting di or bis as part of their names.

79. The incorrect statement regarding the geometrical isomers of 2-butene is :

[4 marks]

- (A) cis-2-butene and trans-2-butene are not interconvertible at room temperature.
- (B) trans-2-butene is more stable than cis-2-butene.
- (C) cis-2-butene has less dipole moment than trans-2-butene.
- (D) cis-2-butene and trans-2-butene are stereoisomers.

Correct Answer: (C) cis-2-butene has less dipole moment than trans-2-butene.

Solution:

Concept:

- Geometrical isomers have the same connectivity but differ in the arrangement of substituents around a rigid double bond. Rotation requires disruption of the π bond.
- The molecular dipole moment is the vector sum of bond dipoles. Symmetry in trans-2-butene causes cancellation, whereas cis-2-butene has a nonzero resultant.

Step 1: Compare the dipole moments In cis-2-butene the two methyl groups lie on the same side of the carbon-carbon double bond. The bond dipoles do not cancel completely, so the molecule has a small but nonzero dipole moment. In trans-2-butene the methyl groups lie on opposite sides; the symmetric arrangement produces cancellation. Thus,

$$\mu_{\text{cis}} > \mu_{\text{trans}} = 0.$$

The claim in (C), that the cis isomer has the smaller dipole moment, is therefore reversed.

Step 2: Check the remaining statements At room temperature ordinary thermal motion does not provide free rotation about the double bond, so the cis and trans forms are not ordinarily interconverted; (A) is correct. In the trans form the two methyl groups are farther apart, reducing steric repulsion relative to the cis arrangement, so (B) is correct. Both molecules have the same atom-to-atom connections and differ only in spatial arrangement; they are geometrical stereoisomers, making (D) correct. Since the question asks for the incorrect statement, these true statements must not be selected.

Step 3: Conclusion The incorrect statement is that cis-2-butene has less dipole moment than trans-2-butene. Hence, the correct option is (C).

Quick Tip to Solve: For a symmetric trans alkene, check cancellation of bond dipoles before comparing polarity with the cis form.

80. Match List I with List II.

	List I: Alkali Metal		List II: Wavelength (nm)
A.	Li	I.	589.2
B.	Na	II.	455.5
C.	Rb	III.	670.8
D.	Cs	IV.	780.0

Choose the correct answer from the options given below:

[4 marks]

(A) (A)-(II), (B)-(IV), (C)-(III), (D)-(I)

(B) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(C) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)

(D) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

Correct Answer: (B) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Solution:

Concept:

- Heating an alkali-metal salt excites its electrons. As the electrons return to lower energy levels, they emit characteristic wavelengths that identify the metal.
- The characteristic flame-test wavelengths used here are lithium 670.8 nm, sodium 589.2 nm, rubidium 780.0 nm, and caesium 455.5 nm.

Step 1: Match lithium and sodium Lithium gives the crimson-red flame associated with 670.8 nm; this is entry III in List II. Sodium gives the intense golden-yellow emission near 589 nm, so the listed 589.2 nm is entry I. Hence the first two matches are

$$A \rightarrow \text{III}, \quad B \rightarrow \text{I}.$$

Already these two assignments identify the second option uniquely. Sodium must not be assigned to the red or violet characteristic wavelengths.

Step 2: Complete and verify the matching The rubidium entry corresponds to the characteristic 780.0 nm wavelength, so C matches IV. Caesium corresponds to 455.5 nm, so D matches II. The complete association is therefore

$$A \rightarrow \text{III}, \quad B \rightarrow \text{I}, \quad C \rightarrow \text{IV}, \quad D \rightarrow \text{II}.$$

Options (A), (C), and (D) all fail the lithium and sodium assignments and cannot be the correct complete matching. The wavelength entries must be matched as given; arranging them solely by atomic number is not a valid shortcut.

Step 3: Conclusion The correct matching is A-III, B-I, C-IV, D-II. Hence, the correct option is (B).

Quick Tip to Solve: Remember sodium by its strong yellow line near 589 nm and lithium by its crimson-red line near 671 nm; these often settle the matching immediately.

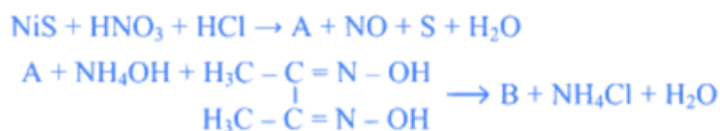
Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Consider the following reactions

The number of protons that do not involve in hydrogen bonding in the product B is _____.

[4 marks]



Solution:

Concept:

- Nickel(II) ions react with dimethylglyoxime in ammoniacal medium to give the characteristic red bis(dimethylglyoximate)nickel(II) precipitate.
- An O-H hydrogen can participate in hydrogen bonding, whereas methyl C-H hydrogens are not counted as hydrogen-bond donors in this coordination-complex question.

Step 1: Identify the nickel complex formed The first reaction dissolves nickel sulfide in the acidic oxidising mixture, giving a nickel(II) salt A. In the second reaction ammonium hydroxide provides the ammoniacal medium and dimethylglyoxime supplies the chelating ligand. Each dimethylglyoxime molecule loses one oxime proton on complex formation. Two such monoanionic ligands bind one Ni^{2+} ion through nitrogen donors, giving neutral $\text{Ni}(\text{dmgH})_2$, the product B. Each ligand retains two methyl groups and one oxime O-H proton.

Step 2: Count the hydrogens outside hydrogen bonding There are two dimethylglyoximate ligands and two methyl groups per ligand. Consequently, the complex has four methyl groups. Their carbon-bound hydrogens are not involved in the $\text{O-H} \cdots \text{O}$ hydrogen bonds characteristic of the complex. The retained oxime protons participate in these hydrogen bonds and are excluded from the requested count. Thus,

$$N_{\text{non-hydrogen-bonded H}} = 2 \times 2 \times 3 = 12.$$

Equivalently, B has fourteen hydrogens in total: twelve methyl hydrogens and two hydrogen-bonded oxime hydrogens. Counting only one ligand would incorrectly give six rather than the total for the product.

Step 3: Conclusion The number of protons not involved in hydrogen bonding in B is 12.

Quick Tip to Solve: Count both dimethylglyoximate ligands: their four methyl groups supply all twelve non-hydrogen-bonded protons.

82. For the reaction at 298 K, $2A + B \rightarrow C$, $\Delta H = 400 \text{ kJ mol}^{-1}$ and $\Delta S = 0.2 \text{ kJ mol}^{-1} \text{ K}^{-1}$. The reaction will become spontaneous above _____ K. [4 marks]

Solution:

Concept:

- At constant temperature and pressure the sign of Gibbs energy change determines spontaneity: $\Delta G = \Delta H - T\Delta S$. A forward reaction is spontaneous when $\Delta G < 0$.
- For a reaction with both $\Delta H > 0$ and $\Delta S > 0$, the entropy contribution favours the reaction increasingly as temperature rises. The crossover temperature is $T_0 = \Delta H/\Delta S$.

Step 1: Write the condition for the forward reaction The enthalpy change is positive, so the reaction is endothermic. The supplied data are

$$\Delta H = 400 \text{ kJ mol}^{-1}, \quad \Delta S = 0.2 \text{ kJ mol}^{-1} \text{ K}^{-1}.$$

Keeping both values in kJ units avoids an unnecessary conversion. Substitution gives

$$\Delta G = 400 - 0.2T \text{ kJ mol}^{-1}.$$

The stated temperature 298 K does not itself make the process spontaneous: at that temperature $\Delta G = 400 - 59.6 = 340.4 \text{ kJ mol}^{-1} > 0$.

Step 2: Find the threshold temperature At the boundary between nonspontaneous and spontaneous behaviour, $\Delta G = 0$. Hence

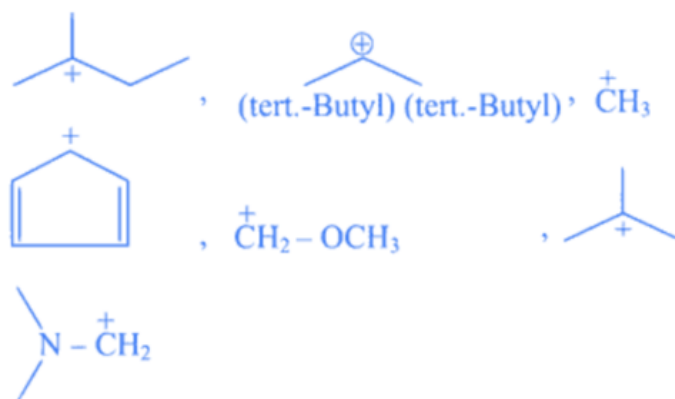
$$400 - 0.2T_0 = 0, \quad T_0 = \frac{400}{0.2} = 2000 \text{ K}.$$

For spontaneous forward change we require $400 - 0.2T < 0$, which gives $T > 2000 \text{ K}$. At exactly 2000 K the Gibbs energy change vanishes; it is the threshold, not a temperature at which the inequality is already strictly negative. The calculation uses the supplied enthalpy and entropy as temperature independent over the intended comparison.

Step 3: Conclusion The reaction becomes spontaneous above 2000 K; the required numerical entry is 2000.

Quick Tip to Solve: Keep enthalpy and entropy in matching energy units, set Gibbs energy to zero for the boundary, then apply the strict sign inequality.

83. Number of carbocations from the following that are not stabilized by hyperconjugation is _____. [4 marks]



Solution:

Concept:

- Classical alkyl hyperconjugation stabilises a carbocation by donation from a suitably oriented adjacent saturated-carbon C-H σ bond into its vacant p orbital.
- Resonance donation from a heteroatom lone pair or an adjacent π system is distinct from alkyl C-H hyperconjugation. A stabilised cation need not be hyperconjugatively stabilised.

Step 1: Identify the cations with adjacent alkyl hydrogens The first carbocation has two methyl groups and an ethyl group attached to the positively charged carbon. Its adjacent saturated carbons have C-H bonds and can donate by hyperconjugation. The tert-butyl carbocation shown in the second row likewise has three adjacent methyl groups, hence nine adjacent C-H bonds. These two species are stabilised by the mechanism asked about and must be left out of the required count.

Step 2: Examine and count the remaining five species In the cation flanked by two tert-butyl groups, the directly adjacent carbons are quaternary and carry no hydrogen. The methyl cation has no adjacent carbon at all. The cyclopentadienyl cation has adjacent unsaturated carbons; its cyclic conjugated π system is not the classical alkyl C-H hyperconjugation counted here. In $^+\text{CH}_2\text{--OCH}_3$, oxygen is directly adjacent to the cationic carbon and can donate a lone pair by resonance; the methyl group beyond oxygen does not provide an adjacent carbon C-H bond. The same distinction applies to $(\text{CH}_3)_2\text{N--CH}_2^+$, whose nitrogen lone pair gives resonance stabilisation. Thus five of the seven displayed carbocations do not have the required alkyl hyperconjugation.

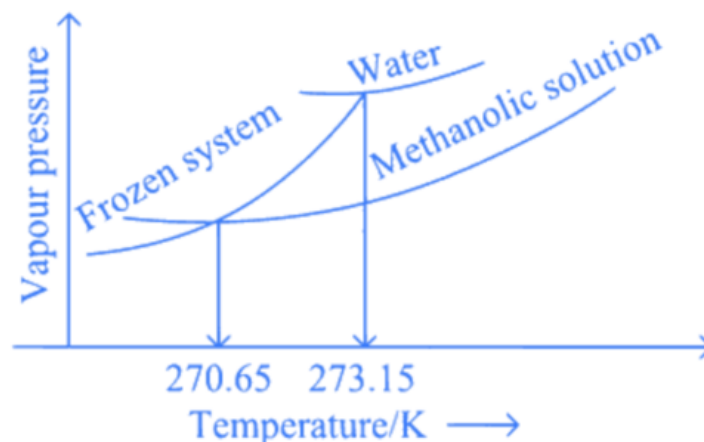
Step 3: Conclusion The number of displayed carbocations not stabilised by hyperconjugation is 5.

Quick Tip to Solve: Inspect only the atoms directly attached to the positively charged carbon; separate alkyl C-H hyperconjugation from heteroatom-lone-pair resonance.

84. When ' x ' $\times 10^{-2}$ mL methanol (molar mass = 32 g, density = 0.792 g/cm³) is added to 100 mL water (density = 1 g/cm³), the following diagram is obtained.

$x = \underline{\hspace{2cm}}$ (nearest integer).

[Given : Molal freezing point depression constant of water at 273.15 K is 1.86 K kg mol⁻¹] [4 marks]



Solution:

Concept:

- For a dilute solution of a nonelectrolyte, freezing-point depression is $\Delta T_f = K_f m$, where m is moles of solute per kilogram of solvent.
- Methanol is a nonelectrolyte, so the van't Hoff factor is one. Convert the solvent volume to kilograms and the solute mass to volume using its density.

Step 1: Read the freezing-point depression from the graph The graph gives the freezing point of pure water as 273.15 K and that of the methanolic solution as 270.65 K. The depression is the positive difference,

$$\Delta T_f = 273.15 - 270.65 = 2.50 \text{ K.}$$

The water mass is $100 \text{ mL} \times 1 \text{ g mL}^{-1} = 100 \text{ g} = 0.100 \text{ kg}$. This is the solvent mass used in molality, rather than the mass or volume of the final solution.

Step 2: Calculate the methanol volume and the requested coefficient Using the given molal constant,

$$m = \frac{2.50}{1.86} \text{ mol kg}^{-1}, \quad n = \frac{2.50}{1.86} \times 0.100 \text{ mol.}$$

The methanol mass is $32n$ grams, so its volume is

$$V = \frac{(2.50/1.86)(0.100)(32)}{0.792} = 5.4308 \text{ mL.}$$

The question expresses this as $x \times 10^{-2}$ mL, so $x = 100V = 543.08$. Rounding the coefficient to the nearest integer gives 543. The factor of 100 belongs to the requested volume notation; the methanol volume itself is about 5.43 mL.

Step 3: Conclusion The requested nearest-integer coefficient is $x = 543$.

Quick Tip to Solve: Use the graph's two freezing temperatures to find their difference, then remember to convert the final volume into the stated units of 0.01 mL.

85. For hydrogen atom, energy of an electron in first excited state is -3.4 eV, K.E. of the same electron of hydrogen atom is x eV. Value of x is _____ $\times 10^{-1}$ eV. (Nearest integer) [4 marks]

Solution:

Concept:

- For an electron bound by a Coulomb attraction, the kinetic energy K and potential energy U obey $U = -2K$. The total orbital energy is $E = K + U = -K$.
- The energy zero is a free electron at rest at infinite separation. Thus a bound state has negative total energy, although its kinetic energy is positive. In hydrogen the first excited state is $n = 2$.

Step 1: Relate the orbital energies The electrostatic attraction supplies the centripetal force in the Bohr orbit. Hence

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}.$$

Multiplying by $r/2$ gives $K = mv^2/2 = e^2/(8\pi\epsilon_0 r)$. The electrostatic potential energy is $U = -e^2/(4\pi\epsilon_0 r) = -2K$, so their sum is negative and has magnitude equal to the kinetic energy.

Step 2: Use the stated total energy The given -3.4 eV is the total energy of the first excited state, not its potential energy. Therefore

$$K = -E = -(-3.4) = 3.4 \text{ eV}.$$

As a consistency check, the corresponding potential energy would be -6.8 eV; adding 3.4 and -6.8 reproduces the stated total energy. No conversion to joules is needed because the answer is requested in electronvolts.

Step 3: Express the answer in the requested scale The blank multiplies 10^{-1} eV. Rewrite the positive kinetic energy as

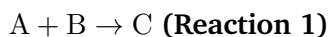
$$3.4 \text{ eV} = 34 \times 10^{-1} \text{ eV}.$$

The coefficient is exactly an integer, so rounding to the nearest integer does not alter it.

Step 4: Conclusion The kinetic energy is $3.4 \text{ eV} = 34 \times 10^{-1} \text{ eV}$. The required numerical entry is 34.

Quick Tip to Solve: For a hydrogenic bound orbit, remember $K = -E$ and $U = 2E$; apply the requested power-of-ten scale only after finding the energy.

86. Consider the two different first order reactions given below



The ratio of the half life of Reaction 1 : Reaction 2 is 5 : 2. If t_1 and t_2 represent the time taken to complete $2/3^{\text{rd}}$ and $4/5^{\text{th}}$ of Reaction 1 and Reaction 2, respectively, then the value of the ratio $t_1 : t_2$ is _____ $\times 10^{-1}$ (nearest integer). [Given : $\log_{10}(3) = 0.477$ and $\log_{10}(5) = 0.699$] [4 marks]

Solution:

Concept:

- For a first-order reaction, $t = (2.303/k) \log_{10}([R]_0/[R]_t)$, where k is its rate constant and $[R]_t$ is the remaining reactant concentration.
- The first-order half-life is $t_{1/2} = \ln 2/k$. Therefore a ratio of half-lives gives the inverse ratio of rate constants, irrespective of the initial concentrations.

Step 1: Determine the rate-constant ratio Both reactions are explicitly specified as first order; the number of reactants in the written reaction does not change that given kinetic order. Write their rate constants as k_1 and k_2 . From the half-lives,

$$\frac{t_{1/2,1}}{t_{1/2,2}} = \frac{k_2}{k_1} = \frac{5}{2}.$$

Thus Reaction 1 has the smaller rate constant and the longer half-life.

Step 2: Convert completion fractions to remaining fractions Completion of $2/3$ of Reaction 1 leaves $1 - 2/3 = 1/3$ of its initial reactant. Completion of $4/5$ of Reaction 2 leaves $1/5$. Applying the integrated law separately gives

$$t_1 = \frac{2.303}{k_1} \log_{10} 3, \quad t_2 = \frac{2.303}{k_2} \log_{10} 5.$$

Use remaining fractions, rather than substituting the reacted fractions into the logarithm.

Step 3: Evaluate and scale the time ratio The common logarithm-conversion factor cancels, giving

$$\frac{t_1}{t_2} = \frac{k_2 \log_{10} 3}{k_1 \log_{10} 5} = \frac{5 \cdot 0.477}{2 \cdot 0.699} = 1.7060 \dots$$

The ratio is dimensionless because both times are expressed in the same units. Since it is requested as a coefficient of 10^{-1} , that coefficient is $17.060 \dots$. Rounding this coefficient, rather than rounding the ratio prematurely, gives 17.

Step 4: Conclusion $t_1/t_2 = 1.7060 \dots = 17.060 \dots \times 10^{-1}$. The nearest-integer numerical entry is 17.

Quick Tip to Solve: First turn each reacted fraction into the fraction left; then combine the logarithm ratio with the inverse rate-constant ratio from the half-lives.

87. An amine (X) is prepared by ammonolysis of benzyl chloride. On adding p-toluenesulphonyl chloride to it the solution remains clear. Molar mass of the amine (X) formed is _____ g mol^{-1} .
(Given molar mass in g mol^{-1} C : 12, H : 1, O : 16, N : 14) [4 marks]

Solution:

Concept:

- Ammonolysis of $\text{C}_6\text{H}_5\text{CH}_2\text{Cl}$ can give successively benzylamine, dibenzylamine and tribenzylamine because the initially formed amines can undergo further alkylation.
- In the Hinsberg test, a tertiary amine has no nitrogen-bound hydrogen and does not form a sulphonamide with p-toluenesulphonyl chloride. A secondary amine forms an insoluble sulphonamide; a primary amine forms a sulphonamide that dissolves in alkali.

Step 1: Identify the amine consistent with the test The stated clear solution on adding the sulphonyl chloride is used here to identify the tertiary amine, which does not give a sulphonamide precipitate. Repeated benzylation replaces all three hydrogens of ammonia by benzyl groups. Thus the amine is tribenzylamine, $(\text{C}_6\text{H}_5\text{CH}_2)_3\text{N}$. The benzyl group is $\text{C}_6\text{H}_5\text{CH}_2^-$, not a phenyl group bonded directly to nitrogen.

Step 2: Check the nitrogen substitution Tribenzylamine has three carbon groups attached to nitrogen and no N–H bond. It consequently cannot undergo the usual sulphonamide formation that requires replacement of an N-bound hydrogen. This distinguishes it from dibenzylamine, whose sulphonamide is insoluble. The solubility of a primary-amine sulphonamide specifically in alkali should not be confused with the tertiary amine failing to form that derivative.

Step 3: Count the atoms and calculate molar mass Each benzyl group has formula C_7H_7 . Three such groups and one nitrogen give $\text{C}_{21}\text{H}_{21}\text{N}$. Using only the supplied atomic molar masses,

$$M = 21(12) + 21(1) + 1(14) = 252 + 21 + 14 = 287 \text{ g mol}^{-1}.$$

There is no oxygen in the amine itself: the oxygen atoms of the test reagent are not added to its molecular formula. The calculation is for the neutral amine, not its hydrochloride or a quaternary ammonium salt.

Step 4: Conclusion The amine is tribenzylamine, $(\text{C}_6\text{H}_5\text{CH}_2)_3\text{N}$, with molar mass 287 g mol^{-1} . The numerical entry is 287.

Quick Tip to Solve: Count each benzyl substituent as C_7H_7 ; three benzyl groups on nitrogen give $\text{C}_{21}\text{H}_{21}\text{N}$, not triphenylamine.

88. Among VO_2^+ , MnO_4^- and $\text{Cr}_2\text{O}_7^{2-}$, the spin-only magnetic moment value of the species with least oxidising ability is _____ BM (Nearest integer).

(Given atomic number $V = 23$, $Mn = 25$, $Cr = 24$)

[4 marks]

Solution:

Concept:

- Oxidising ability is compared by the reduction tendency of the oxidant. The usual acidic-medium reduction-potential order is $\text{MnO}_4^- > \text{Cr}_2\text{O}_7^{2-} > \text{VO}_2^+$.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)} \text{ BM}$, where n is the number of unpaired electrons. A d^0 metal ion has $n = 0$.

Step 1: Choose the weakest oxidising species Permanganate and dichromate are stronger oxidising agents than the vanadium(V) oxocation in the usual acidic comparison. Their standard reduction potentials are approximately 1.51, 1.33, and 1.00 V, respectively. Therefore the species of least oxidising ability among those listed is VO_2^+ . The formula here contains two oxygen atoms and an overall charge of +1.

Step 2: Find the vanadium oxidation state and electron count Let the oxidation state of vanadium in VO_2^+ be z . Taking each oxygen as -2 ,

$$z + 2(-2) = +1 \Rightarrow z = +5.$$

Neutral vanadium, atomic number 23, has configuration $[\text{Ar}] 3d^3 4s^2$. Removing the two $4s$ electrons and then three $3d$ electrons gives V^{5+} as $[\text{Ar}] 3d^0$. It has no unpaired d electrons.

Step 3: Calculate the spin-only moment Substitute $n = 0$ into the spin-only expression,

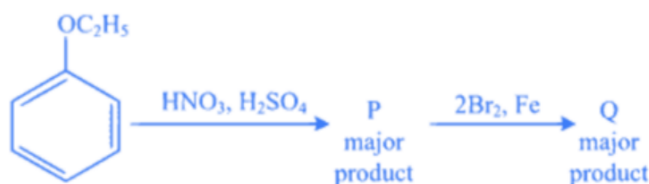
$$\mu = \sqrt{0(0+2)} = 0 \text{ BM}.$$

This is consistent with the diamagnetic electron count. As an additional check, manganese(VII) in permanganate and chromium(VI) in dichromate also have d^0 configurations, so their electron counts do not introduce a competing nonzero value. The requested nearest-integer operation leaves zero unchanged.

Step 4: Conclusion For the weakest oxidant VO_2^+ , vanadium is d^0 and its spin-only magnetic moment is $\boxed{0 \text{ BM}}$. The numerical entry is 0.

Quick Tip to Solve: Determine the metal oxidation state before counting d electrons; the neutral atom configuration is not the configuration of the oxoanion or oxocation.

89. The ratio of number of oxygen atoms to bromine atoms in the product Q is _____ $\times 10^{-1}$.
[4 marks]



Solution:

Concept:

- The ethoxy group $-\text{OC}_2\text{H}_5$ donates electron density to a benzene ring by resonance and directs electrophilic substitution to the ortho and para positions.
- The nitro group $-\text{NO}_2$ is deactivating and meta-directing. To count atoms in a product, retain the original substituents and add only the groups introduced in each substitution.

Step 1: Identify the major nitration product The starting compound in the reaction scheme is ethoxybenzene (phenetole). Nitration with concentrated $\text{HNO}_3/\text{H}_2\text{SO}_4$ introduces $-\text{NO}_2$. The ethoxy substituent favours ortho and para attack; the less crowded para position gives the major product P. Thus P is 1-ethoxy-4-nitrobenzene. Nitration adds two oxygen atoms in the nitro group while retaining the ether oxygen already present.

Step 2: Locate the two bromination sites With the ethoxy carbon labelled position 1 and the nitro group at position 4, positions 2 and 6 are ortho to ethoxy and meta to nitro. The directing effects of both groups therefore favour these positions. The two equivalents of Br_2 , with iron as the halogenation catalyst precursor, replace the two corresponding ring hydrogens. Product Q is 2,6-dibromo-1-ethoxy-4-nitrobenzene; bromination does not replace an oxygen atom or remove the nitro group.

Step 3: Count oxygen and bromine atoms Q contains one oxygen atom in $-\text{OC}_2\text{H}_5$ and two in $-\text{NO}_2$, for a total of three oxygen atoms. It contains two bromine substituents. Therefore

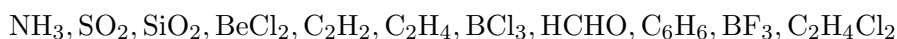
$$\frac{N_{\text{O}}}{N_{\text{Br}}} = \frac{1 + 2}{2} = \frac{3}{2} = 1.5 = 15 \times 10^{-1}.$$

This is an atom-number ratio, so no atomic molar masses are used. Both bromine molecules contribute one bromine atom to the ring; their other atoms leave as hydrogen bromide.

Step 4: Conclusion The oxygen-to-bromine atom ratio in Q is 3 : 2, or 15×10^{-1} . The required numerical entry is 15.

Quick Tip to Solve: In successive aromatic substitutions, track each substituent separately: here ether oxygen stays, nitration adds two oxygens, and two brominations add two bromines.

90. Total number of species from the following with central atom utilising sp^2 hybrid orbitals for bonding is _____.



[4 marks]

Solution:

Concept:

- In the usual hybridisation model, three electron domains around a bonding centre correspond to sp^2 hybridisation. A multiple bond counts as one domain for this purpose.
- A carbon participating in a double bond is normally sp^2 ; a carbon in a triple bond is sp , while a saturated carbon with four sigma bonds is sp^3 . Lone pairs must also be included when counting domains.

Step 1: Classify the noncarbon centres In NH_3 , nitrogen has three sigma bonds and one lone pair, giving sp^3 , so it is excluded. Sulfur in SO_2 has two bonding domains and one lone pair and is sp^2 , so it is included. Ordinary solid SiO_2 is a tetrahedral network with sp^3 silicon, not a discrete carbon-dioxide-like molecule. Linear BeCl_2 is sp . Both BCl_3 and BF_3 have three sigma bonds at boron and are trigonal planar, so both are included.

Step 2: Classify the carbon-containing species Each carbon in C_2H_2 has two electron domains and is sp , so acetylene is excluded. Each carbon in C_2H_4 has three sigma-bonding directions and a p orbital forming the pi bond, giving sp^2 . The carbonyl carbon in HCHO is also trigonal planar and sp^2 . Every ring carbon in benzene, C_6H_6 , is sp^2 . In $\text{C}_2\text{H}_4\text{Cl}_2$, the carbon atoms are saturated with four sigma bonds and are sp^3 , regardless of the dichloroethane isomer.

Step 3: List and count the qualifying species The complete selected list is



There are six species. Count each molecular formula once, rather than counting individual sp^2 atoms: ethene and benzene each contribute one species to the requested total even though they contain multiple such carbon atoms.

Step 4: Conclusion The total number of listed species with the relevant central atom using sp^2 hybrid orbitals is 6.

Quick Tip to Solve: Count electron domains, including lone pairs, and count species rather than atoms. Treat ordinary SiO_2 as a tetrahedral network solid.