

JEE Main 2024 Paper 1 (B.E./B.Tech) Apr 8 Shift 2

Question Paper with Solutions PDF



Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours

Maximum Marks: 300

General Instructions

- (i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 8 April 2024, Shift 2. The total duration is 3 hours.
- (ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.
- (iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.
- (iv) Marking scheme (multiple choice questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (v) Marking scheme (numerical value questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $y = y(x)$ be the solution curve of the differential equation $\sec y \frac{dy}{dx} + 2x \sin y = x^3 \cos y$, $y(1) = 0$. Then $y(\sqrt{3})$ is equal to: [4 marks]

- (A) $\frac{\pi}{6}$
(B) $\frac{\pi}{12}$
(C) $\frac{\pi}{3}$
(D) $\frac{\pi}{4}$

Correct Answer: (D) $\frac{\pi}{4}$

Solution:

Concept:

- The substitution $u = \tan y$ gives $u' = \sec^2 y y'$. Dividing the given equation by $\cos y$ converts it into a linear equation for u .
- For $u' + P(x)u = Q(x)$, multiplication by the integrating factor $e^{\int P(x) dx}$ makes the left side an exact derivative. The initial value determines the integration constant.

Step 1: Convert to a linear equation Divide by $\cos y$, which is nonzero near the initial value $y = 0$:

$$\sec^2 y \frac{dy}{dx} + 2x \tan y = x^3.$$

Thus, with $u = \tan y$, the equation is $u' + 2xu = x^3$. Its integrating factor is e^{x^2} , so

$$\frac{d}{dx} (ue^{x^2}) = x^3 e^{x^2}.$$

Step 2: Integrate and impose the initial condition Put $t = x^2$, so $x^3 dx = \frac{1}{2}t dt$. Integration by parts gives

$$ue^{x^2} = \frac{1}{2} \int te^t dt = \frac{1}{2}(x^2 - 1)e^{x^2} + C.$$

Consequently $u = (x^2 - 1)/2 + Ce^{-x^2}$. Since $y(1) = 0$, we have $u(1) = 0$, which forces $C = 0$.

Step 3: Evaluate on the continuous solution branch At $x = \sqrt{3}$, $\tan y = (3 - 1)/2 = 1$. The branch continuous from $y(1) = 0$ is

$$y(x) = \tan^{-1} \left(\frac{x^2 - 1}{2} \right).$$

It stays between 0 and $\pi/4$ for $1 \leq x \leq \sqrt{3}$, so division by $\cos y$ remains valid throughout. The tangents of options (A), (B) and (C) are respectively $1/\sqrt{3}$, $2 - \sqrt{3}$, and $\sqrt{3}$, none of which equals 1.

Step 4: Conclusion $y(\sqrt{3}) = \pi/4$. Hence, the correct option is (D).

Quick Tip to Solve: When a differential equation combines $\sec^2 y y'$ and $\tan y$, replace $\tan y$ by one variable before finding an integrating factor.

2. If the shortest distance between the lines $\frac{x-\lambda}{2} = \frac{y-4}{3} = \frac{z-3}{4}$ and $\frac{x-2}{4} = \frac{y-4}{6} = \frac{z-7}{8}$ is $\frac{13}{\sqrt{29}}$, then a value of λ is : [4 marks]

- (A) $\frac{13}{25}$
 (B) 1
 (C) -1
 (D) $-\frac{13}{25}$

Correct Answer: (B) 1

Solution:

Concept:

- The direction vectors (2, 3, 4) and (4, 6, 8) are proportional. These are parallel lines, so their distance must be computed using a point-to-line perpendicular distance.
- For points A, B on parallel lines with direction vector $\mathbf{d}$, the separation is $D = |\vec{AB} \times \mathbf{d}|/|\mathbf{d}|$. This removes the component along the lines.

Step 1: Choose the points and common direction Take $A = (\lambda, 4, 3)$ on the first line and $B = (2, 4, 7)$ on the second. Then

$$\vec{AB} = (2 - \lambda, 0, 4), \quad \mathbf{d} = (2, 3, 4), \quad |\mathbf{d}| = \sqrt{29}.$$

The skew-line scalar-triple-product formula cannot be used directly because the cross product of the two direction vectors is zero.

Step 2: Find the perpendicular distance The cross product is

$$\vec{AB} \times \mathbf{d} = (-12, 4\lambda, 6 - 3\lambda).$$

Hence its squared magnitude is

$$144 + 16\lambda^2 + (6 - 3\lambda)^2 = 25\lambda^2 - 36\lambda + 180.$$

Equating the required distance to $13/\sqrt{29}$ gives

$$\frac{25\lambda^2 - 36\lambda + 180}{29} = \frac{169}{29}.$$

Step 3: Solve and compare the listed values Therefore

$$25\lambda^2 - 36\lambda + 11 = (\lambda - 1)(25\lambda - 11) = 0.$$

The possible values are 1 and $11/25$. The question asks for a value among the options, so 1 is selected. For a direct check, substitution of $\lambda = 1$ gives squared cross-product magnitude $25 - 36 + 180 = 169$. Neither $13/25$, -1 , nor $-13/25$ is a root of the factored equation; the unlisted second root does not create ambiguity among the options.

Step 4: Conclusion A listed value is $\lambda = 1$. Hence, the correct option is (B).

Quick Tip to Solve: Check proportionality of the direction ratios first: parallel lines need the point-to-line distance formula, rather than the skew-line formula.

3. There are three bags X, Y and Z . Bag X contains 5 one-rupee coins and 4 five-rupee coins; Bag Y contains 4 one-rupee coins and 5 five-rupee coins and Bag Z contains 3 one-rupee coins and 6 five-rupee coins. A bag is selected at random and a coin drawn from it at random is found to be a one-rupee coin. Then the probability, that it came from bag Y , is : [4 marks]

- (A) $\frac{1}{2}$
 (B) $\frac{1}{3}$
 (C) $\frac{5}{12}$
 (D) $\frac{1}{4}$

Correct Answer: (B) $\frac{1}{3}$

Solution:

Concept:

- Conditional probability reverses the given selection process. If E is the event of drawing a one-rupee coin, Bayes' theorem gives $P(Y | E) = P(Y)P(E | Y)/P(E)$.
- The three bags are selected with equal probability $1/3$, and each bag contains nine coins. The likelihoods of observing a one-rupee coin are therefore proportional to their one-rupee counts.

Step 1: Write the prior and likelihood probabilities Let X, Y, Z also denote selection of the corresponding bag. Random selection means

$$P(X) = P(Y) = P(Z) = \frac{1}{3}.$$

Because the coin is drawn randomly within the selected bag,

$$P(E | X) = \frac{5}{9}, \quad P(E | Y) = \frac{4}{9}, \quad P(E | Z) = \frac{3}{9}.$$

The five-rupee coins contribute to the bag totals but do not satisfy the observed event.

Step 2: Find the total probability of the observation The bag-selection events are mutually exclusive and exhaustive. Thus the law of total probability gives

$$P(E) = \frac{1}{3} \frac{5}{9} + \frac{1}{3} \frac{4}{9} + \frac{1}{3} \frac{3}{9} = \frac{12}{27} = \frac{4}{9}.$$

Also, the joint probability of selecting bag Y and obtaining a one-rupee coin is $P(Y \cap E) = (1/3)(4/9) = 4/27$.

Step 3: Apply Bayes' theorem and compare the options Conditioning on the observed coin gives

$$P(Y | E) = \frac{4/27}{12/27} = \frac{4}{12} = \frac{1}{3}.$$

Equivalently, after the observation the bag probabilities are in the ratio $5 : 4 : 3$, giving $5/12, 4/12, 3/12$. Hence $5/12$, option (C), belongs to bag X , and $1/4$, option (D), belongs to bag Z . The value $1/2$, option (A), is not the normalized likelihood for any bag.

Step 4: Conclusion The probability that the coin came from bag Y is $1/3$. Hence, the correct option is (B).

Quick Tip to Solve: For equally likely bags of equal size, the posterior probabilities are proportional to the numbers of coins matching the observation: here use $5 : 4 : 3$.

4. The area of the region in the first quadrant inside the circle $x^2 + y^2 = 8$ and outside the parabola $y^2 = 2x$ is equal to : [4 marks]

- (A) $\frac{\pi}{2} - \frac{1}{3}$
(B) $\pi - \frac{1}{3}$
(C) $\pi - \frac{2}{3}$
(D) $\frac{\pi}{2} - \frac{2}{3}$

Correct Answer: (C) $\pi - \frac{2}{3}$

Solution:

Concept:

- The circle has radius $2\sqrt{2}$. The parabola $y^2 = 2x$ opens to the right; its interior is $x \geq y^2/2$, so the required exterior in the first quadrant has $x \leq y^2/2$.
- Horizontal strips give the width as the smaller of $y^2/2$ and $\sqrt{8 - y^2}$. The intersection determines where these bounds exchange roles.

Step 1: Locate the first-quadrant intersection At an intersection, substitute $y^2 = 2x$ into the circle:

$$x^2 + 2x = 8, \quad (x + 4)(x - 2) = 0.$$

The first-quadrant intersection is $(2, 2)$. For $0 \leq y \leq 2$, the parabola is the right boundary of the required region. For $2 \leq y \leq 2\sqrt{2}$, the circle is the right boundary, while the left boundary is the y -axis throughout.

Step 2: Set up and evaluate the strip integrals The area in square units is

$$A = \int_0^2 \frac{y^2}{2} dy + \int_2^{2\sqrt{2}} \sqrt{8 - y^2} dy.$$

The first term is $[y^3/6]_0^2 = 4/3$. For the second term use

$$F(y) = \frac{y}{2} \sqrt{8 - y^2} + 4 \sin^{-1} \left(\frac{y}{2\sqrt{2}} \right).$$

At the upper endpoint $F(2\sqrt{2}) = 2\pi$, while $F(2) = 2 + \pi$. Thus the second integral equals $\pi - 2$.

Step 3: Combine the contributions and compare the options Adding both positive strip areas gives

$$A = \frac{4}{3} + \pi - 2 = \pi - \frac{2}{3}.$$

This also checks geometrically: the first part is the region to the left of the parabola below the intersection, and the second is the cap above it. Neither contribution should be subtracted from the other. Numerically the result is about 2.475 square units. Options (A), (B), and (D) are approximately 1.237, 2.808, and 0.904, respectively, so none equals the evaluated area.

Step 4: Conclusion The required area is $\pi - 2/3$ square units. Hence, the correct option is (C).

Quick Tip to Solve: For a right-opening parabola, describe its exterior as $x \leq y^2/2$; horizontal strips make the boundary switch at the intersection especially clear.

5. Let $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 11\hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $(\vec{a} + \vec{b}) \times \vec{c} = \vec{c} \times (-2\vec{a} + 3\vec{b})$. If $(2\vec{a} + 3\vec{b}) \cdot \vec{c} = 1670$, then $|\vec{c}|^2$ is equal to: [4 marks]

- (A) 1600
(B) 1618
(C) 1627
(D) 1609

Correct Answer: (B) 1618

Solution:

Concept:

- The cross product is anticommutative: $\mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u}$. A zero cross product with a nonzero vector establishes parallelism.
- Once the direction of $\vec{c}$ is known, write $\vec{c} = t\mathbf{d}$. The given scalar product then determines its scale, after which $|\vec{c}|^2 = t^2|\mathbf{d}|^2$.

Step 1: Reduce the cross-product condition Reverse the order of the right-hand cross product:

$$(\vec{a} + \vec{b}) \times \vec{c} = (2\vec{a} - 3\vec{b}) \times \vec{c}.$$

Moving all terms to the left yields

$$(-\vec{a} + 4\vec{b}) \times \vec{c} = \vec{0}.$$

Here $-\vec{a} + 4\vec{b} = (40, -3, 3)$, which is nonzero. Therefore $\vec{c} = t(40, -3, 3)$ for a real number t .

Step 2: Determine the scalar multiplier Using the supplied vectors gives

$$2\vec{a} + 3\vec{b} = (41, -5, 5).$$

Consequently the prescribed dot product becomes

$$(41, -5, 5) \cdot t(40, -3, 3) = t(1640 + 15 + 15) = 1670t.$$

Since it must equal 1670, we obtain $t = 1$. This also shows that $\vec{c}$ is not the zero vector and fixes its orientation rather than merely its line of direction.

Step 3: Calculate the squared magnitude and check alternatives Thus

$$\vec{c} = 40\hat{i} - 3\hat{j} + 3\hat{k}, \quad |\vec{c}|^2 = 40^2 + (-3)^2 + 3^2 = 1618.$$

All three orthogonal components contribute positively. In particular, option (A), 1600, keeps only the $\hat{i}$ component, and option (D), 1609, omits one transverse component. Option (C), 1627, would add an extra 9, which is absent from the vector. As a consistency check, the dot product is exactly $1640 + 15 + 15 = 1670$, as required.

Step 4: Conclusion $|\vec{c}|^2 = 1618$. Hence, the correct option is (B).

Quick Tip to Solve: First put both cross products in the same order; only then combine their vector coefficients to identify the direction of the unknown vector.

6. If the line segment joining the points $(5, 2)$ and $(2, a)$ subtends an angle $\frac{\pi}{4}$ at the origin, then the absolute value of the product of all possible values of a is : [4 marks]

- (A) 4
- (B) 8
- (C) 6
- (D) 2

Correct Answer: (A) 4

Solution:

Concept:

- The angle subtended at the origin is the angle between $\mathbf{u} = (5, 2)$ and $\mathbf{v} = (2, a)$. Their determinant is $5a - 4$, and their dot product is $10 + 2a$.
- For an acute angle θ , $\tan \theta = |\det(\mathbf{u}, \mathbf{v})|/(\mathbf{u} \cdot \mathbf{v})$. The dot product must be positive; this condition excludes supplementary-angle solutions.

Step 1: Translate the angle condition Since the given angle is $\pi/4$, its tangent is 1. Therefore

$$\frac{|5a - 4|}{10 + 2a} = 1, \quad 10 + 2a > 0.$$

The absolute value allows the two possible orientations of the second ray relative to the first. Neither point is the origin, so both position vectors are nonzero and this angle formula is applicable.

Step 2: Solve both sign cases The equation $|5a - 4| = 10 + 2a$ gives two linear equations:

$$5a - 4 = 10 + 2a \implies a = \frac{14}{3},$$

$$5a - 4 = -(10 + 2a) \implies a = -\frac{6}{7}.$$

For the first root the dot product is $58/3 > 0$; for the second it is $58/7 > 0$. Thus both roots indeed give the acute angle $\pi/4$, rather than the supplementary angle $3\pi/4$.

Step 3: Form the required absolute product The product of all permissible values is

$$\frac{14}{3} \left(-\frac{6}{7} \right) = -4.$$

Its absolute value is 4. One may also square the angle equation to obtain $21a^2 - 80a - 84 = 0$, whose root product is $-84/21 = -4$; the positivity check still has to be retained when using this squared equation. Options (B), (C), and (D) give 8, 6, 2, respectively, none equal to the absolute product. The sign of the product is removed only at the final step, exactly as the question requests.

Step 4: Conclusion The absolute value of the product is 4. Hence, the correct option is (A).

Quick Tip to Solve: After using the tangent formula or squaring an angle equation, check the dot-product sign so that supplementary angles are not accidentally included.

7. If $\alpha \neq a, \beta \neq b, \gamma \neq c$ and $\begin{vmatrix} \alpha & b & c \\ a & \beta & c \\ a & b & \gamma \end{vmatrix} = 0$, then $\frac{a}{\alpha-a} + \frac{b}{\beta-b} + \frac{\gamma}{\gamma-c}$ is equal to : [4 marks]

- (A) 2
(B) 3
(C) 1
(D) 0

Correct Answer: (D) 0

Solution:

Concept:

- Write $A = \alpha - a$, $B = \beta - b$, and $C = \gamma - c$. The three stated inequalities ensure $ABC \neq 0$, so division by this product is valid.
- The matrix is the sum of $\text{diag}(A, B, C)$ and a matrix whose three rows are all (a, b, c) . Multilinearity of a determinant makes terms containing two identical rows vanish.

Step 1: Expand using the diagonal differences The determinant can be written as

$$\begin{vmatrix} A+a & b & c \\ a & B+b & c \\ a & b & C+c \end{vmatrix}.$$

Choose in each row either its diagonal-only part or its (a, b, c) part. The all-diagonal term is ABC . Choosing the common row only in the first, second, or third row gives aBC , bAC , or cAB , respectively. Every term choosing it in at least two rows has identical rows and is zero.

Step 2: Use the zero determinant The given condition therefore says

$$ABC + aBC + bAC + cAB = 0.$$

Since $ABC \neq 0$, divide through to obtain

$$1 + \frac{a}{A} + \frac{b}{B} + \frac{c}{C} = 0, \quad \frac{a}{A} + \frac{b}{B} + \frac{c}{C} = -1.$$

This conclusion does not require any of a, b, c to be nonzero; only the three diagonal differences occur in denominators.

Step 3: Account for the numerator in the final fraction The requested last numerator is γ , not c . Because $\gamma = C + c$,

$$\frac{a}{\alpha-a} + \frac{b}{\beta-b} + \frac{\gamma}{\gamma-c} = \frac{a}{A} + \frac{b}{B} + \frac{C+c}{C} = -1 + 1 = 0.$$

The extra 1 in the last fraction is essential. The values 2, 3, and 1 in options (A), (B), and (C) do not satisfy this identity, whereas zero follows for every set of parameters allowed by the question.

Step 4: Conclusion The value of the expression is 0. Hence, the correct option is (D).

Quick Tip to Solve: When a determinant has constant off-diagonal columns, introduce the diagonal differences; then carefully check whether the final numerator is c or γ .

8. Let $\int_{\alpha}^{\log_e 4} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}$. Then e^{α} and $e^{-\alpha}$ are the roots of the equation:

[4 marks]

(A) $2x^2 - 5x + 2 = 0$

(B) $x^2 - 2x - 8 = 0$

(C) $2x^2 - 5x - 2 = 0$

(D) $x^2 + 2x - 8 = 0$

Correct Answer: (A) $2x^2 - 5x + 2 = 0$

Solution:

Concept:

- Convert an exponential radical into a rational integrand by setting $t = \sqrt{e^x - 1}$. Since the integral is real, the relevant values satisfy $e^x \geq 1$.
- The antiderivative of $1/(1+t^2)$ is $\tan^{-1} t$. For roots u, v , the monic quadratic is $x^2 - (u+v)x + uv = 0$.

Step 1: Transform the definite integral Put $t = \sqrt{e^x - 1}$, so $e^x = 1 + t^2$ and $e^x dx = 2t dt$. Therefore

$$\frac{dx}{\sqrt{e^x - 1}} = \frac{2 dt}{1 + t^2}.$$

The upper endpoint $x = \log_e 4$ becomes $t = \sqrt{3}$; the lower endpoint becomes $\sqrt{e^{\alpha} - 1}$. Keep the endpoints attached to the substitution rather than treating the integral as indefinite.

Step 2: Use the given value to determine the endpoint Integrating and substituting the endpoints gives

$$2 [\tan^{-1} t]_{\sqrt{e^{\alpha} - 1}}^{\sqrt{3}} = \frac{\pi}{6}.$$

Since $\tan^{-1} \sqrt{3} = \pi/3$, this is

$$\tan^{-1} \sqrt{e^{\alpha} - 1} = \frac{\pi}{3} - \frac{\pi}{12} = \frac{\pi}{4}.$$

The square root is nonnegative, so the principal inverse tangent is the appropriate branch. Thus $\sqrt{e^{\alpha} - 1} = 1$, and $e^{\alpha} = 2$.

Step 3: Form the quadratic and check the options The second root is $e^{-\alpha} = 1/2$. Their sum is $5/2$ and their product is 1, hence

$$x^2 - \frac{5}{2}x + 1 = 0.$$

Multiplication by 2 yields the listed equation. Option (C) has root product -1 , while (B) and (D) have product -8 ; none can have two positive reciprocal roots. This product check independently excludes all three alternatives.

Step 4: Conclusion The required equation is $2x^2 - 5x + 2 = 0$. Hence, the correct option is (A).

Quick Tip to Solve: For roots e^{α} and $e^{-\alpha}$, check that their product is 1 before evaluating the integral; then find only their sum.

9. If the system of equations $x + 4y - z = \lambda$, $7x + 9y + \mu z = -3$, $5x + y + 2z = -1$ has infinitely many solutions, then $(2\mu + 3\lambda)$ is equal to :

[4 marks]

- (A) -2
- (B) 2
- (C) 3
- (D) -3

Correct Answer: (D) -3

Solution:

Concept:

- Infinitely many solutions require a dependent coefficient equation and the same dependence among the constants. A zero determinant alone does not guarantee consistency.
- Use the coefficients of x and y to determine the possible row dependence before comparing the z coefficient and right-hand side.

Step 1: Locate the dependence of the first two columns Let E_1, E_2, E_3 denote the three equations in printed order. Express the coefficients of E_2 as a linear combination $uE_1 + vE_3$. The first two columns give

$$u + 5v = 7, \quad 4u + v = 9.$$

Substituting $u = 7 - 5v$ into the second relation gives $28 - 19v = 9$, so $v = 1$ and $u = 2$. Thus a dependence, if present, must be $E_2 = 2E_1 + E_3$.

Step 2: Impose dependence and consistency together The z coefficient in $2E_1 + E_3$ is $2(-1) + 2 = 0$. Therefore infinite solutions require $\mu = 0$. The same row combination must also reproduce the constant -3 :

$$2\lambda + (-1) = -3 \implies \lambda = -1.$$

At these values the second equation is exactly redundant, rather than contradictory. This explicitly checks the augmented column and rules out an inconsistent system with no solution.

Step 3: Check that a free variable remains and evaluate The coefficient pairs $(1, 4)$ and $(5, 1)$ in E_1, E_3 are independent because their determinant is $1 - 20 = -19 \neq 0$. Hence the rank is exactly 2 for three unknowns, leaving one free variable and therefore infinitely many solutions, as required. Finally,

$$2\mu + 3\lambda = 2(0) + 3(-1) = -3.$$

The other listed values $-2, 2, 3$ do not equal this combination; in particular, $+3$ would result from losing the negative sign of λ .

Step 4: Conclusion We obtain $\mu = 0$, $\lambda = -1$, and $2\mu + 3\lambda = -3$. Hence, the correct option is (D).

Quick Tip to Solve: When one coefficient row is a simple combination of the others, apply that same combination to the constants; determinant zero is only the first check.

10. Let $f(x) = \begin{cases} -a & \text{if } -a \leq x \leq 0 \\ x+a & \text{if } 0 < x \leq a \end{cases}$ where $a > 0$ and $g(x) = (f(|x|) - |f(x)|)/2$. Then the function $g : [-a, a] \rightarrow [-a, a]$ is [4 marks]

- (A) neither one-one nor onto.
 (B) both one-one and onto.
 (C) one-one.
 (D) onto

Correct Answer: (A) neither one-one nor onto.

Solution:

Concept:

- A function is one-one if distinct inputs always have distinct images. It is onto only when its range equals the entire specified codomain.
- Evaluate $f(|x|)$ and $|f(x)|$ separately in each interval, treating $x = 0$ explicitly because it belongs to the negative branch of f .

Step 1: Evaluate the function on the negative interval For $-a \leq x < 0$, we have $|x| = -x > 0$. Thus the positive-input branch gives $f(|x|) = a - x$, whereas $f(x) = -a$ and $|f(x)| = a$. Consequently,

$$g(x) = \frac{a - x - a}{2} = -\frac{x}{2} \quad (-a \leq x < 0).$$

These values fill $(0, a/2]$: the endpoint $x = -a$ is included, while the limit 0 from this interval is not attained there.

Step 2: Evaluate the remaining inputs At the origin, $f(0) = -a$, so

$$g(0) = \frac{-a - |-a|}{2} = -a.$$

For $0 < x \leq a$, both $f(|x|)$ and $|f(x)|$ equal $x + a$, which is positive. Therefore $g(x) = 0$ throughout this interval. In particular, the point $x = 0$ must not be grouped with positive inputs: its image is $-a$, not 0.

Step 3: Compare images and the range with the definitions Since $a > 0$, the distinct inputs $a/2$ and a both map to 0. This proves that g is not one-one. Combining the three pieces gives

$$\text{Range}(g) = \{-a\} \cup [0, a/2].$$

This is a proper subset of the codomain $[-a, a]$. For example, $-a/2$ is in the codomain but is never an output. Hence g is not onto either. Options (B) and (C) fail the repeated-image test, and option (D) fails the missing-output test.

Step 4: Conclusion The function g is neither one-one nor onto. Hence, the correct option is (A).

Quick Tip to Solve: For a piecewise function involving absolute values, isolate the origin before testing injectivity or range; its value can differ from both nearby formulas.

11. In an increasing geometric progression of positive terms, the sum of the second and sixth terms is $\frac{70}{3}$ and the product of the third and fifth terms is 49. Then the sum of the 4th, 6th and 8th terms is equal to: [4 marks]

- (A) 78
(B) 96
(C) 91
(D) 84

Correct Answer: (C) 91

Solution:

Concept:

- In a geometric progression with positive first term u and common ratio r , the n th term is $T_n = ur^{n-1}$. An increasing positive progression has $r > 1$.
- Terms equidistant from a middle term satisfy $T_{k-j}T_{k+j} = T_k^2$. Use the product condition to determine T_4 before finding the ratio.

Step 1: Determine the middle term from the product The third and fifth terms are equally spaced about the fourth, so

$$T_3T_5 = (ur^2)(ur^4) = u^2r^6 = T_4^2 = 49.$$

Every term is positive, hence $T_4 = 7$, rather than -7 . In terms of this middle term, $T_2 = 7/r^2$ and $T_6 = 7r^2$. This avoids solving separately for the first term before it is needed.

Step 2: Use the sum and the increasing condition The given sum becomes

$$7(r^{-2} + r^2) = \frac{70}{3}, \quad r^2 + \frac{1}{r^2} = \frac{10}{3}.$$

Let $s = r^2 > 0$. Multiplication by $3s$ gives

$$3s^2 - 10s + 3 = (3s - 1)(s - 3) = 0.$$

Thus $s = 3$ or $s = 1/3$. Because the progression is increasing and positive, $r > 1$, and only $r^2 = 3$ is allowed. The other root belongs to a decreasing progression.

Step 3: Sum the requested alternate terms Moving forward by two positions multiplies a term by $r^2 = 3$. Therefore

$$T_4 = 7, \quad T_6 = 21, \quad T_8 = 63,$$

so their sum is $7 + 21 + 63 = 91$. As a check, $T_2 = 7/3$, and $T_2 + T_6 = 7/3 + 21 = 70/3$, exactly as printed. Options 78, 96 and 84 disagree with the three terms fixed by both given conditions; no rounding is involved.

Step 4: Conclusion The requested sum is 91. Hence, the correct option is (C).

Quick Tip to Solve: When the product of two equally spaced GP terms is known, take its positive square root to find their middle term, then solve for the squared ratio.

12. For $a, b > 0$, let

$$f(x) = \begin{cases} \frac{\tan((a+1)x) + b \tan x}{x}, & x < 0, \\ 3, & x = 0, \\ \frac{\sqrt{ax + b^2x^2} - \sqrt{ax}}{b\sqrt{a}x\sqrt{x}}, & x > 0 \end{cases}$$

be a continuous function at $x = 0$. Then $\frac{b}{a}$ is equal to:

[4 marks]

(A) 4

(B) 5

(C) 8

(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- Continuity at $x = 0$ requires both one-sided limits to equal the printed value $f(0) = 3$. The parameters a, b are positive.
- Use $\lim_{x \rightarrow 0} \tan(kx)/x = k$ on the left. On the right, rationalize a difference of square roots before cancelling powers of x .

Step 1: Apply the left-hand limit For $x < 0$, split the quotient into its two tangent terms. Taking the limit gives

$$\lim_{x \rightarrow 0^-} f(x) = (a+1) + b.$$

Equality with $f(0) = 3$ therefore requires $a + b = 2$. This first equation uses the left branch only; it does not by itself determine the requested ratio because the right-hand expression imposes a second condition.

Step 2: Rationalize the right-hand branch For $x > 0$, multiply numerator and denominator by $\sqrt{ax + b^2x^2} + \sqrt{ax}$. The numerator becomes b^2x^2 . Since all square roots are real and $x > 0$, factor $\sqrt{x}$ out of both radicals in the new denominator:

$$\begin{aligned} f(x) &= \frac{b^2x^2}{b\sqrt{a}x\sqrt{x}[\sqrt{ax + b^2x^2} + \sqrt{ax}]} \\ &= \frac{b}{\sqrt{a}[\sqrt{a + b^2x} + \sqrt{a}]}. \end{aligned}$$

The cancellation is valid for the limiting positive values of x ; no value at $x = 0$ is cancelled directly.

Step 3: Enforce the right-hand limit and check positivity Now

$$\lim_{x \rightarrow 0^+} f(x) = \frac{b}{2a} = 3,$$

so $b/a = 6$. Together with $a + b = 2$, this gives $a = 2/7$ and $b = 12/7$, both positive, confirming that the two conditions are compatible. Ratios 4, 5 and 8 would make the right-hand limit 2, $5/2$ and 4 respectively, rather than the required value 3.

Step 4: Conclusion The required ratio is $b/a = 6$. Hence, the correct option is (D).

Quick Tip to Solve: In a continuity problem with a radical difference, rationalize before taking the limit; retain the separate equation supplied by the other one-sided branch.

13. Let $A = \{2, 3, 6, 8, 9, 11\}$ and $B = \{1, 4, 5, 10, 15\}$. Let R be a relation on $A \times B$ defined by $(a, b)R(c, d)$ if and only if $3ad - 7bc$ is an even integer. Then the relation R is [4 marks]

- (A) reflexive but not symmetric.
(B) an equivalence relation.
(C) reflexive and symmetric but not transitive.
(D) transitive but not symmetric.

Correct Answer: (C) reflexive and symmetric but not transitive.

Solution:

Concept:

- Reflexivity requires each element to be related to itself; symmetry reverses every related pair; transitivity combines two consecutive related pairs.
- The factors 3 and 7 are odd, so parity of $3ad - 7bc$ is the same as parity of $ad - bc$. Work with even and odd values, but use elements of the stated Cartesian product in counterexamples.

Step 1: Verify reflexivity for every element An element of the underlying set is an ordered pair $(a, b) \in A \times B$. Relating it to itself gives

$$3ab - 7ba = -4ab,$$

which is even for every integer a, b . Therefore $(a, b)R(a, b)$ for every element, and R is reflexive. This is a universal check, rather than checking only one selected pair.

Step 2: Verify symmetry using parity Suppose $(a, b)R(c, d)$, so $3ad - 7bc$ is even. Modulo 2, this says $ad - bc \equiv 0$. On reversing the two ordered pairs, the tested expression is $3cb - 7da$, whose parity is $bc - ad$. It is the negative of the previous expression modulo 2, hence also even. Thus $(c, d)R(a, b)$ always follows, proving symmetry. Both options (A) and (D), which deny symmetry, are thereby excluded.

Step 3: Test transitivity with an explicit counterexample Choose $P = (3, 1)$, $Q = (2, 4)$ and $S = (2, 1)$, all belonging to $A \times B$. Then

$$PRQ : 3(3)(4) - 7(1)(2) = 22,$$

$$QRS : 3(2)(1) - 7(4)(2) = -50,$$

$$PRS : 3(3)(1) - 7(1)(2) = -5.$$

The first two integers are even, but the third is odd. Thus PRQ and QRS do not force PRS ; transitivity fails. Consequently the relation cannot be an equivalence relation, excluding option (B).

Step 4: Conclusion The relation is reflexive and symmetric but not transitive. Hence, the correct

option is (C).

Quick Tip to Solve: For a parity relation on ordered pairs, an even-even intermediate pair often relates to many pairs; use it to look for a failure of transitivity.

14. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + \lambda\hat{k}$ be three vectors. Let $\vec{r}$ be a unit vector along $\vec{b} + \vec{c}$. If $\vec{r} \cdot \vec{a} = 3$, then 3λ is equal to: [4 marks]

(A) 21

(B) 25

(C) 27

(D) 30

Correct Answer: (B) 25

Solution:

Concept:

- A unit vector along a nonzero vector $\vec{v}$ is $\vec{v}/|\vec{v}|$. The scalar product of Cartesian vectors is the sum of the products of corresponding components.
- When an equation contains a square root, record its sign condition before squaring and check the resulting value in the original equation.

Step 1: Form the vector in the required direction Adding the two given vectors yields

$$\vec{b} + \vec{c} = 5\hat{i} + 2\hat{j} + (\lambda - 5)\hat{k}.$$

Its magnitude is $\sqrt{25 + 4 + (\lambda - 5)^2} = \sqrt{29 + (\lambda - 5)^2}$, which is strictly positive for every real λ . Hence the required unit vector exists and is

$$\vec{r} = \frac{5\hat{i} + 2\hat{j} + (\lambda - 5)\hat{k}}{\sqrt{29 + (\lambda - 5)^2}}.$$

Step 2: Apply the scalar-product condition Taking the scalar product with $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, the numerator is

$$5 + 2(2) + 3(\lambda - 5) = 3\lambda - 6.$$

Thus the stated condition is

$$\frac{3\lambda - 6}{\sqrt{29 + (\lambda - 5)^2}} = 3, \quad \lambda - 2 = \sqrt{29 + (\lambda - 5)^2}.$$

The square root is positive, so $\lambda > 2$ is necessary. The normalization factor must be retained; the vector $\vec{b} + \vec{c}$ itself is not a unit vector.

Step 3: Solve and verify before selecting the option Squaring now gives

$$\lambda^2 - 4\lambda + 4 = 29 + \lambda^2 - 10\lambda + 25,$$

so $6\lambda = 50$ and $\lambda = 25/3$. It satisfies $\lambda > 2$. In the original expression, the numerator is 19 and the denominator is $19/3$, giving the required dot product 3. Therefore $3\lambda = 25$. The remaining candidate values 21, 27 and 30 would not satisfy the derived equation $6\lambda = 50$, so they are excluded.

Step 4: Conclusion We find $\lambda = 25/3$, hence $3\lambda = 25$. Hence, the correct option is (B).

Quick Tip to Solve: For a unit vector along a sum of vectors, normalize the sum before taking its dot product; then check the sign condition after any squaring.

15. The sum of all possible values of $\theta \in [-\pi, 2\pi]$, for which $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is purely imaginary, is equal to : [4 marks]

- (A) 4π
- (B) 3π
- (C) 2π
- (D) 5π

Correct Answer: (B) 3π

Solution:

Concept:

- A complex number is purely imaginary when its real part is zero. Multiplying numerator and denominator by the conjugate of the denominator separates the real and imaginary parts.
- For real u , the denominator $1 + 4u^2$ is strictly positive, so a zero real part comes only from the numerator. Solutions of a trigonometric equation must be restricted to the specified closed interval.

Step 1: Rationalise the complex expression Put $u = \cos\theta$, so u is real. The conjugate of $1 - 2iu$ is $1 + 2iu$. Thus

$$\frac{1 + iu}{1 - 2iu} = \frac{(1 + iu)(1 + 2iu)}{1 + 4u^2} = \frac{1 - 2u^2 + 3iu}{1 + 4u^2}.$$

The term $(iu)(2iu) = -2u^2$ supplies the minus sign in the real part.

Step 2: Set the real part equal to zero Since $1 + 4u^2 > 0$, the real part vanishes precisely when

$$1 - 2u^2 = 0 \implies \cos^2\theta = \frac{1}{2}.$$

Hence $\theta = \pi/4 + k\pi/2$, where k is an integer. At these points $u \neq 0$, so the imaginary part is also nonzero.

Step 3: List the admissible angles and add them In the interval $[-\pi, 2\pi]$, the complete list is

$$-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}.$$

Neither endpoint satisfies the equation. The first four values cancel in opposite pairs, leaving

$$\sum \theta = \frac{5\pi}{4} + \frac{7\pi}{4} = 3\pi.$$

Therefore the alternatives $4\pi, 2\pi, 5\pi$ cannot be the sum over the specified interval.

Step 4: Conclusion The required sum is 3π . Hence, the correct option is (B).

Quick Tip to Solve: After finding $\cos^2 \theta = 1/2$, list every angle in the entire given interval; using only one standard period misses valid solutions.

16. The number of ways five alphabets can be chosen from the alphabets of the word MATHEMATICS, where the chosen alphabets are not necessarily distinct, is equal to: [4 marks]

- (A) 179
- (B) 177
- (C) 175
- (D) 181

Correct Answer: (A) 179

Solution:

Concept:

- MATHEMATICS contains eight distinct letter types: M, A, T, H, E, I, C, S . The types M, A, T occur twice each, while the other five occur once.
- A selection is unordered, so identical copies are not distinguished. Classify selections by the number of letter types used twice, then use combinations to choose the remaining distinct types.

Step 1: Identify the possible repetition patterns We select five letters from the actual letters available in the word. No type can occur three times, since no letter occurs three times in MATHEMATICS. At most two types can be doubled, because three doubled types would already require six letters. Thus the disjoint cases are: no repeated type; one doubled type and three singles; or two doubled types and one single. We count these separately to avoid either omissions or overcounting.

Step 2: Count selections with zero or one doubled type If all five selected letters are distinct, choose any five of the eight types:

$$N_0 = \binom{8}{5} = 56.$$

If exactly one type is doubled, choose it from M, A, T , then choose three distinct single types from the other seven types:

$$N_1 = \binom{3}{1} \binom{7}{3} = 3 \cdot 35 = 105.$$

A single type may itself have two copies available, but choosing it only once is still one selection, not two labelled choices.

Step 3: Count two doubled types and total the cases Choose the two doubled types from the three repeatable types. The remaining fifth letter is one of the six types not already chosen:

$$N_2 = \binom{3}{2} \binom{6}{1} = 3 \cdot 6 = 18.$$

The cases cannot overlap, since they have different numbers of doubled types. Their total is

$$N = N_0 + N_1 + N_2 = 56 + 105 + 18 = 179.$$

The totals 177, 175 and 181 differ from this complete count and are therefore rejected.

Step 4: Conclusion There are $\boxed{179}$ selections. Hence, the correct option is (A).

Quick Tip to Solve: For repeated-letter selections, count by multiplicity patterns first. Do not choose labelled positions, because identical copies would then be overcounted.

17. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, $a > 0$ has a local maximum at $x = \alpha$ and a local minimum at $x = \alpha^2$, then α and α^2 are the roots of the equation : [4 marks]

(A) $x^2 - 6x + 8 = 0$

(B) $8x^2 - 6x + 1 = 0$

(C) $8x^2 + 6x - 1 = 0$

(D) $x^2 + 6x + 8 = 0$

Correct Answer: (A) $x^2 - 6x + 8 = 0$

Solution:

Concept:

- At an interior local extremum of a differentiable function, $f'(x) = 0$. The second derivative distinguishes a local maximum from a local minimum when it is nonzero.
- Once the two roots are known, the monic quadratic with roots p, q is $x^2 - (p + q)x + pq = 0$. The condition $a > 0$ fixes the order of the two critical points.

Step 1: Find and classify the stationary points Differentiate the given cubic with respect to x , treating a as a constant:

$$f'(x) = 6x^2 - 18ax + 12a^2 = 6(x - a)(x - 2a).$$

The stationary points are therefore $x = a$ and $x = 2a$. Also,

$$f''(x) = 12x - 18a.$$

At $x = a$, this is $-6a < 0$, giving a local maximum. At $x = 2a$, it is $6a > 0$, giving a local minimum.

Step 2: Use the specified locations of the extrema The local maximum is at $x = \alpha$, so classification gives $\alpha = a$. The local minimum is at $x = \alpha^2$, so $\alpha^2 = 2a$. Combining the two identities,

$$a^2 = 2a \implies a(a - 2) = 0.$$

The solution $a = 0$ is forbidden by the given strict inequality. Hence $a = 2$, and the two required roots are $\alpha = 2$ and $\alpha^2 = 4$.

Step 3: Form and check the quadratic Their sum is $2 + 4 = 6$, and their product is $2 \cdot 4 = 8$.

Consequently,

$$(x - 2)(x - 4) = x^2 - 6x + 8.$$

Thus the required equation is $x^2 - 6x + 8 = 0$. The second option has sum of roots $3/4$; the third has negative product; and the fourth has negative sum. None can have the two positive roots 2 and 4.

Step 4: Conclusion The equation is $x^2 - 6x + 8 = 0$. Hence, the correct option is (A).

Quick Tip to Solve: Classify the stationary points before matching them with α and α^2 ; reversing maximum and minimum gives a different, incorrect parameter.

18. If the value of $\frac{3 \cos 36^\circ + 5 \sin 18^\circ}{5 \cos 36^\circ - 3 \sin 18^\circ}$ is $\frac{a\sqrt{5}-b}{c}$, where a, b, c are natural numbers and $\gcd(a, c) = 1$, then $a + b + c$ is equal to : [4 marks]

- (A) 54
(B) 52
(C) 50
(D) 40

Correct Answer: (B) 52

Solution:

Concept:

- The exact values $\cos 36^\circ = (\sqrt{5}+1)/4$ and $\sin 18^\circ = (\sqrt{5}-1)/4$ convert the trigonometric expression into an expression in $\sqrt{5}$.
- Rationalising a denominator $4 + \sqrt{5}$ uses its conjugate $4 - \sqrt{5}$. The resulting rational denominator is $16 - 5 = 11$; coefficients can then be compared with the prescribed form.

Step 1: Substitute the exact trigonometric values Let E denote the given expression. Substituting the standard values, the common factors of $1/4$ cancel:

$$E = \frac{3(\sqrt{5} + 1) + 5(\sqrt{5} - 1)}{5(\sqrt{5} + 1) - 3(\sqrt{5} - 1)}.$$

Collect the radical terms and the constant terms separately. The numerator becomes $8\sqrt{5} - 2$, and the denominator becomes $2\sqrt{5} + 8$. The minus sign before the last bracket is essential.

Step 2: Reduce and rationalise the fraction Divide numerator and denominator by 2, then multiply by the conjugate of the denominator:

$$E = \frac{4\sqrt{5} - 1}{\sqrt{5} + 4} = \frac{(4\sqrt{5} - 1)(4 - \sqrt{5})}{(4 + \sqrt{5})(4 - \sqrt{5})}.$$

Expanding the numerator gives $16\sqrt{5} - 20 - 4 + \sqrt{5} = 17\sqrt{5} - 24$. Therefore

$$E = \frac{17\sqrt{5} - 24}{11}.$$

Step 3: Read the coefficients and add them Comparison with $(a\sqrt{5} - b)/c$ gives $a = 17$, $b = 24$, $c = 11$. These are natural numbers, and $\gcd(17, 11) = 1$, as required. The coprimality condition rules out multiplying all three coefficients by a common factor. Hence

$$a + b + c = 17 + 24 + 11 = 52.$$

The other listed sums 54, 50 and 40 do not follow from the reduced exact expression.

Step 4: Conclusion The required value is $\boxed{52}$. Hence, the correct option is (B).

Quick Tip to Solve: Collect constants carefully before rationalising: $-3(\sqrt{5} - 1)$ contributes $+3$ to the denominator, not -3 .

19. If the image of the point $(-4, 5)$ in the line $x + 2y = 2$ lies on the circle $(x + 4)^2 + (y - 3)^2 = r^2$, then r is equal to: [4 marks]

- (A) 2
- (B) 3
- (C) 4
- (D) 1

Correct Answer: (A) 2

Solution:

Concept:

- The reflection of (x_0, y_0) in $Ax + By + C = 0$ is $(x_0 - 2Ad, y_0 - 2Bd)$, where $d = (Ax_0 + By_0 + C)/(A^2 + B^2)$.
- The circle $(x - h)^2 + (y - k)^2 = r^2$ has centre (h, k) . Its radius is the positive distance from the centre to any point on the circle.

Step 1: Calculate the reflection of the given point Write the mirror line as $x + 2y - 2 = 0$, so $A = 1$, $B = 2$, and $C = -2$. For $P = (-4, 5)$,

$$d = \frac{-4 + 2(5) - 2}{1^2 + 2^2} = \frac{4}{5}.$$

Hence its image $P' = (x', y')$ is

$$x' = -4 - 2\left(\frac{4}{5}\right) = -\frac{28}{5}, \quad y' = 5 - 4\left(\frac{4}{5}\right) = \frac{9}{5}.$$

The denominator 5 is the squared length of the line's normal vector, not its length.

Step 2: Check the reflected coordinates geometrically The midpoint of PP' is $(-24/5, 17/5)$. Substitution in the line gives $-24/5 + 34/5 = 2$, so this midpoint lies on the mirror. Also, the displacement from P to P' is $(-8/5, -16/5)$, parallel to the normal $(1, 2)$. Thus the line is the perpendicular bisector, confirming the reflection rather than merely a projection.

Step 3: Find the distance from the circle centre The circle has centre $C = (-4, 3)$. Since P' lies on it,

$$r^2 = \left(-\frac{28}{5} + 4\right)^2 + \left(\frac{9}{5} - 3\right)^2 = \left(-\frac{8}{5}\right)^2 + \left(-\frac{6}{5}\right)^2 = \frac{64 + 36}{25} = 4.$$

A radius is nonnegative, so $r = 2$. The other listed radii 3, 4 and 1 have squares 9, 16 and 1, none equal to the computed squared distance.

Step 4: Conclusion The radius is $r = 2$. Hence, the correct option is (A).

Quick Tip to Solve: After reflecting the point, use the centre read from the circle equation; the distance to the mirror line is not the circle radius.

20. If the term independent of x in the expansion of $(\sqrt{a}x^2 + \frac{1}{2x^3})^{10}$ is 105, then a^2 is equal to :

[4 marks]

- (A) 6
- (B) 4
- (C) 2
- (D) 9

Correct Answer: (B) 4

Solution:

Concept:

- In $(u + v)^n$, the general term is $T_{k+1} = \binom{n}{k} u^{n-k} v^k$, with integer $0 \leq k \leq n$.
- A term is independent of x precisely when its net exponent of x is zero. First find that term, then equate its coefficient to the prescribed constant.

Step 1: Write the general term with powers separated Choose $u = \sqrt{a}x^2$, $v = 1/(2x^3)$, and $n = 10$. The term obtained by choosing v exactly k times is

$$T_{k+1} = \binom{10}{k} (\sqrt{a}x^2)^{10-k} \left(\frac{1}{2x^3}\right)^k = \binom{10}{k} \frac{a^{(10-k)/2}}{2^k} x^{20-5k}.$$

Here $x \neq 0$, as required by the original expression. Both the coefficient and the power of x must be retained when identifying the constant term.

Step 2: Select the term independent of the variable Set the exponent of x equal to zero:

$$20 - 5k = 0 \implies k = 4.$$

This is an allowed integer index, so the fifth term is the unique constant term. Substituting $k = 4$, its coefficient is

$$\binom{10}{4} \frac{a^3}{2^4} = \frac{210a^3}{16}.$$

In particular, the exponent of a is 3 because six factors of $\sqrt{a}$ remain.

Step 3: Use the given coefficient and obtain the requested power The constant term is given to be 105. Therefore

$$\frac{210a^3}{16} = 105 \implies a^3 = 8.$$

For the real square root in the question, $a \geq 0$; the resulting value is $a = 2$. The question asks for a^2 , not for a , so

$$a^2 = 2^2 = 4.$$

The value 2 gives a itself rather than the requested square. The values 6 and 9 also fail the coefficient condition.

Step 4: Conclusion The required value is $a^2 = 4$. Hence, the correct option is (B).

Quick Tip to Solve: Find the zero exponent of x before simplifying coefficients, and distinguish the binomial index k from the term number $k + 1$.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $a, b, c \in \mathbb{N}$ and $a < b < c$. Let the mean, the mean deviation about the mean and the variance of the 5 observations 9, 25, a, b, c be 18, 4 and $\frac{136}{5}$, respectively. Then $2a + b - c$ is equal to _____

[4 marks]

Solution:

Concept:

- For five observations with mean $\bar{x}$, their sum is $5\bar{x}$. Mean deviation about the mean is $\frac{1}{5} \sum |x_i - \bar{x}|$.
- Variance is $\frac{1}{5} \sum (x_i - \bar{x})^2$. Using signed deviations from 18 combines the three statistical conditions without expanding large squares.

Step 1: Use the mean and the mean deviation The mean is 18, so the sum of all observations is 90. Consequently,

$$a + b + c = 90 - 9 - 25 = 56.$$

The mean deviation is 4, so the absolute deviations sum to 20. The known observations contribute $|9 - 18| + |25 - 18| = 9 + 7 = 16$. Thus

$$|a - 18| + |b - 18| + |c - 18| = 4.$$

Step 2: Determine the negative and positive deviations Put $u = a - 18$, $v = b - 18$, $w = c - 18$. Then $u < v < w$ are integers, and $u + v + w = 56 - 54 = 2$. Let P be the sum of the positive deviations and N the absolute sum of the negative deviations. The two equations give

$$P + N = 4, \quad P - N = 2, \quad P = 3, \quad N = 1.$$

There is therefore exactly one negative deviation, namely -1 . By the ordering, $u = -1$, hence $a = 17$. The remaining deviations satisfy $0 \leq v < w$ and $v + w = 3$.

Step 3: Apply the variance to distinguish the possibilities Multiplying the variance by five gives the sum of squared deviations as 136. The observations 9 and 25 contribute $81 + 49 = 130$, so

$$u^2 + v^2 + w^2 = 6.$$

With $u = -1$, we get $v^2 + w^2 = 5$. Combined with $v + w = 3$, this gives $9 = 5 + 2vw$, hence $vw = 2$. The ordered pair is $(v, w) = (1, 2)$, giving $b = 19, c = 20$. These values satisfy both the strict ordering and all three statistics.

Step 4: Evaluate the requested combination Substitute the determined observations into the required expression:

$$2a + b - c = 2(17) + 19 - 20 = 33.$$

The value is dimensionless, since it is a numerical combination of the supplied observations.

Step 5: Conclusion The required numerical value is $\boxed{33}$.

Quick Tip to Solve: Work with deviations from the given mean: the sum and absolute sum immediately give the total positive and negative deviations, greatly reducing the integer possibilities.

22. The number of distinct real roots of the equation $|x + 1||x + 3| - 4|x + 2| + 5 = 0$, is _____ [4 marks]

Solution:

Concept:

- The identity $|a||b| = |ab|$ combines the two moduli. A translation $u = x + 2$ turns their product into $|u^2 - 1|$.
- Putting $t = |u|$ gives $t \geq 0$. Solve separately on the intervals where $t^2 - 1$ has a fixed sign, then retain only roots belonging to the interval used.

Step 1: Reduce the equation to a nonnegative variable Since $x + 1 = u - 1$ and $x + 3 = u + 1$, the given equation becomes

$$|u^2 - 1| - 4|u| + 5 = 0.$$

Thus, with $t = |u|$, we must solve $|t^2 - 1| - 4t + 5 = 0$ for $t \geq 0$. Working with this nonnegative variable also makes it easy to count the original roots without confusing repeated factors with distinct roots.

Step 2: Check both absolute-value branches For $t \geq 1$, the expression inside the modulus is nonnegative, so

$$t^2 - 1 - 4t + 5 = (t - 2)^2 = 0.$$

Hence $t = 2$, which belongs to this branch. For $0 \leq t < 1$, instead obtain

$$1 - t^2 - 4t + 5 = 0, \quad t = -2 \pm \sqrt{10}.$$

The negative root is inadmissible, and $-2 + \sqrt{10} > 1$, since $\sqrt{10} > 3$. Neither root belongs to the second branch. At the boundary $t = 1$, the original reduced expression equals 1, so no boundary root has been missed.

Step 3: Recover and count the real roots The only admissible value is $|x + 2| = 2$. Therefore

$$x + 2 = 2 \quad \text{or} \quad x + 2 = -2, \quad x = 0 \quad \text{or} \quad x = -4.$$

Both give $3 - 8 + 5 = 0$ in the printed equation. The double factor in the equation for t does not create extra distinct values of x .

Step 4: Conclusion The distinct real roots are 0 and -4 . Therefore the required number is $\boxed{2}$.

Quick Tip to Solve: When two absolute values involve numbers equally spaced about one point, translate to that midpoint and use a nonnegative variable before counting roots.

23. If $\int \frac{1}{\sqrt[5]{(x-1)^4(x+3)^6}} dx = A \left(\frac{\alpha x - 1}{\beta x + 3} \right)^B + C$, where C is the constant of integration, then the value of $\alpha + \beta + 20AB$ is _____. [4 marks]

Solution:

Concept:

- For factors $x - 1$ and $x + 3$ whose powers sum to 2, their ratio is a useful substitution: its derivative supplies a factor $(x + 3)^{-2}$.
- The power rule gives $\int u^{-4/5} du = 5u^{1/5} + C$. Fifth roots are real roots, so this calculation applies on intervals avoiding $x = 1, -3$.

Step 1: Introduce the ratio of the linear factors Put

$$u = \frac{x - 1}{x + 3}, \quad \frac{du}{dx} = \frac{(x + 3) - (x - 1)}{(x + 3)^2} = \frac{4}{(x + 3)^2}.$$

The denominator can be rewritten using $(x - 1)^4(x + 3)^6 = u^4(x + 3)^{10}$. Its real fifth root is $u^{4/5}(x + 3)^2$. Thus the substitution matches both the fractional power and the remaining quadratic denominator exactly.

Step 2: Integrate the resulting single power For $x \neq 1, -3$, transform the integral as

$$\begin{aligned} I &= \int u^{-4/5} \frac{dx}{(x + 3)^2} = \frac{1}{4} \int u^{-4/5} du \\ &= \frac{5}{4} u^{1/5} + C = \frac{5}{4} \left(\frac{x - 1}{x + 3} \right)^{1/5} + C. \end{aligned}$$

Differentiating the last expression gives $u^{-4/5}/(x + 3)^2$, which is the original integrand. The constant may be chosen separately on each interval of definition.

Step 3: Identify the constants in the given form Comparing the evaluated integral with the expression in the question gives

$$\alpha = 1, \quad \beta = 1, \quad A = \frac{5}{4}, \quad B = \frac{1}{5}.$$

In particular, the derivative of the ratio contains the factor 4; retaining it is essential for the correct

coefficient. Therefore

$$\alpha + \beta + 20AB = 1 + 1 + 20 \left(\frac{5}{4} \right) \left(\frac{1}{5} \right) = 7.$$

Step 4: Conclusion The required numerical value is $\boxed{7}$.

Quick Tip to Solve: For an integral containing two linear factors with fractional powers, test their ratio; keep its constant derivative factor when identifying the prefactor.

24. Let S be the focus of the hyperbola $\frac{x^2}{3} - \frac{y^2}{5} = 1$, on the positive x -axis. Let C be the circle with its centre at $A(\sqrt{6}, \sqrt{5})$ and passing through the point S . If O is the origin and SAB is a diameter of C , then the square of the area of the triangle OSB is equal to _____. [4 marks]

Solution:

Concept:

- For the hyperbola $x^2/a^2 - y^2/b^2 = 1$, a focus on the positive axis is $(c, 0)$, where $c^2 = a^2 + b^2$.
- The centre of a circle is the midpoint of every diameter. A triangle whose base lies on the x -axis has area equal to half the base length times the absolute ordinate of its third vertex.

Step 1: Locate the positive focus Here $a^2 = 3$ and $b^2 = 5$. The distance of either focus from the origin is therefore

$$c = \sqrt{a^2 + b^2} = \sqrt{8} = 2\sqrt{2}.$$

The required focus is $S = (2\sqrt{2}, 0)$, and $O = (0, 0)$. Hence the segment OS is horizontal and has length $2\sqrt{2}$. The negative focus is not used because the question explicitly selects the positive x -axis.

Step 2: Use the midpoint of the diameter The notation SAB describes a diameter through the centre $A = (\sqrt{6}, \sqrt{5})$. Its endpoints are S and B , so

$$\frac{S + B}{2} = A, \quad B = 2A - S.$$

Consequently,

$$B = (2\sqrt{6} - 2\sqrt{2}, 2\sqrt{5}).$$

This reflection across A automatically gives $AS = AB$ and places all three points on one straight line, as required for a diameter. There is no need to find the circle's full equation or radius.

Step 3: Evaluate the area and then square it Take OS as the base. The perpendicular distance from B to its supporting line $y = 0$ is $2\sqrt{5}$, irrespective of the abscissa of B . Thus

$$[OSB] = \frac{1}{2}(2\sqrt{2})(2\sqrt{5}) = 2\sqrt{10}.$$

The question asks for the square of this area, rather than the area itself:

$$[OSB]^2 = (2\sqrt{10})^2 = 40.$$

Step 4: Conclusion The square of the area of triangle OSB is $\boxed{40}$ in fourth-power coordinate units.

Quick Tip to Solve: When a circle diameter is given through its centre, reflect the known endpoint in the centre; an axis-aligned triangle base then avoids the determinant formula.

25. Let $P(\alpha, \beta, \gamma)$ be the image of the point $Q(1, 6, 4)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Then $2\alpha + \beta + \gamma$ is equal to _____ [4 marks]

Solution:

Concept:

- The image of a point in a line is obtained by a half-turn about the line. The perpendicular foot H on the line is the midpoint of the point and its image, so $P = 2H - Q$.
- A direction vector of the line is $\mathbf{d} = (1, 2, 3)$. The foot satisfies $(H - Q) \cdot \mathbf{d} = 0$, which determines its line parameter.

Step 1: Parameterise the foot of the perpendicular Writing the common value of the three symmetric ratios as t , a general point of the line is

$$H = (t, 1 + 2t, 2 + 3t).$$

Since $Q = (1, 6, 4)$, the displacement from Q to this point is $(t - 1, 2t - 5, 3t - 2)$. At the perpendicular foot its dot product with $(1, 2, 3)$ vanishes. Therefore

$$(t - 1) + 2(2t - 5) + 3(3t - 2) = 0, \quad 14t - 17 = 0.$$

Thus $t = 17/14$. Using the direction vector, rather than separately reflecting coordinates, is necessary because the line is oblique.

Step 2: Reflect the point through the foot The perpendicular foot is

$$H = \left(\frac{17}{14}, \frac{24}{7}, \frac{79}{14} \right).$$

Because H is the midpoint of QP , the image has coordinates

$$\begin{aligned} P &= 2H - Q \\ &= \left(\frac{17}{7} - 1, \frac{48}{7} - 6, \frac{79}{7} - 4 \right) \\ &= \left(\frac{10}{7}, \frac{6}{7}, \frac{51}{7} \right). \end{aligned}$$

The displacement from H to P is the negative of that from H to Q ; hence the two points are equally far from the reflecting line.

Step 3: Form the requested linear combination Substitute $\alpha = 10/7$, $\beta = 6/7$ and $\gamma = 51/7$:

$$2\alpha + \beta + \gamma = \frac{20 + 6 + 51}{7} = \frac{77}{7} = 11.$$

The use of a common denominator also provides a direct arithmetic check on the reflected coordinates.

Step 4: Conclusion The requested value is $\boxed{11}$.

Quick Tip to Solve: For reflection in a three-dimensional line, first project onto its direction vector; then use image = twice the foot minus the original point.

26. Let A be the region enclosed by the parabola $y^2 = 2x$ and the line $x = 24$. Then the maximum area of the rectangle inscribed in the region A is _____. [4 marks]

Solution:

Concept:

- The parabola $y^2 = 2x$ is symmetric about the x -axis. For the inscribed rectangle with a side on $x = 24$, write its horizontal sides as $y = \pm t$.
- Its height is $2t$ and its width is $24 - t^2/2$. A maximum of the resulting area function is found from its derivative, with the allowable endpoints checked.

Step 1: Express the dimensions using one variable Let the upper and lower left vertices on the parabola have ordinates t and $-t$, where $t \geq 0$. Both then have abscissa $t^2/2$. The right vertices lie on the vertical boundary $x = 24$, so the rectangle has dimensions

$$\text{height} = 2t, \quad \text{width} = 24 - \frac{t^2}{2}.$$

For these dimensions to be nonnegative, $0 \leq t \leq \sqrt{48} = 4\sqrt{3}$. Placing a horizontal side closer to the axis or a right side short of the line would only reduce area for the same left abscissa.

Step 2: Differentiate and locate the stationary point The area in terms of t is

$$F(t) = 2t \left(24 - \frac{t^2}{2} \right) = 48t - t^3.$$

Hence

$$F'(t) = 48 - 3t^2 = 0 \implies t^2 = 16.$$

The admissible stationary point is $t = 4$; the negative solution is excluded because t measures a nonnegative height. Furthermore, $F'(t) > 0$ before 4 and $F'(t) < 0$ after 4 within the allowable interval.

Step 3: Check the maximum and evaluate its area At both endpoints of the interval the rectangle degenerates and $F = 0$. Thus the interior stationary point gives the absolute maximum. At $t = 4$, the left abscissa is $4^2/2 = 8$, giving

$$\text{width} = 24 - 8 = 16, \quad \text{height} = 8.$$

The maximum area is therefore $16 \times 8 = 128$. Equivalently, $F''(4) = -24 < 0$ confirms a strict local maximum, consistent with the endpoint comparison.

Step 4: Conclusion The maximum area of the rectangle is 128 square coordinate units.

Quick Tip to Solve: For a rectangle inside a horizontal parabola cut by a vertical line, use its half-height as the variable; the area becomes a simple cubic.

27. Let $\alpha|x| = |y|e^{xy-\beta}$, $\alpha, \beta \in \mathbf{N}$ be the solution of the differential equation $x \, dy - y \, dx + xy(x \, dy + y \, dx) = 0$, $y(1) = 2$. Then $\alpha + \beta$ is equal to _____ [4 marks]

Solution:

Concept:

- After division by xy , the combination $(x \, dy - y \, dx)/(xy)$ becomes $d \log |y| - d \log |x|$.
- The remaining differential $x \, dy + y \, dx$ is exactly $d(xy)$. Integrating these exact differentials gives an implicit relation, and the initial value fixes its constant.

Step 1: Recognise the exact differentials In a neighbourhood of the initial point $(1, 2)$, both x and y are nonzero. Divide the given equation by xy :

$$\frac{dy}{y} - \frac{dx}{x} + x \, dy + y \, dx = 0.$$

This can be written without expanding an explicit derivative as

$$d \log \left| \frac{y}{x} \right| + d(xy) = 0.$$

The absolute values are needed for the real logarithms; at the initial point they can subsequently be evaluated using positive arguments.

Step 2: Integrate and apply the initial value Integration gives

$$\log \left| \frac{y}{x} \right| + xy = C.$$

Since $y(1) = 2$,

$$C = \log 2 + 2.$$

Consequently the solution through this initial point satisfies

$$\log \left| \frac{y}{x} \right| = \log 2 + 2 - xy.$$

Differentiating this relation returns the divided differential equation, so no extra integration term has been introduced. The initial point also selects the branch with $y/x > 0$ locally.

Step 3: Rearrange into the printed form Exponentiate the last equation:

$$\left| \frac{y}{x} \right| = 2e^{2-xy}.$$

Multiplying by $|x|e^{xy-2}$ yields

$$2|x| = |y|e^{xy-2}.$$

Comparison with $\alpha|x| = |y|e^{xy-\beta}$ gives $\alpha = 2$ and $\beta = 2$, both natural numbers as required. As a check at the initial point, each side equals 2.

Step 4: Conclusion Therefore $\alpha + \beta = 2 + 2 = \boxed{4}$.

Quick Tip to Solve: When a differential equation contains both $x dy$ minus $y dx$ and $x dy$ plus $y dx$, divide by xy and look for a logarithmic ratio together with $d(xy)$.

28. If $\alpha = \lim_{x \rightarrow 0^+} \left(\frac{e^{\sqrt{\tan x}} - e^{\sqrt{x}}}{\sqrt{\tan x} - \sqrt{x}} \right)$ and $\beta = \lim_{x \rightarrow 0} (1 + \sin x)^{\frac{1}{2} \cot x}$ are the roots of the quadratic equation $ax^2 + bx - \sqrt{e} = 0$, then $12 \log_e(a + b)$ is equal to _____. [4 marks]

Solution:

Concept:

- The divided difference $(e^u - e^v)/(u - v)$ tends to 1 as $u, v \rightarrow 0$, by the mean value theorem for the exponential function.
- For a positive base tending to 1, take logarithms to evaluate a variable power. If α, β are roots of $ax^2 + bx + c = 0$, then $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$.

Step 1: Evaluate the first limit Take $u = \sqrt{\tan x}$ and $v = \sqrt{x}$. For sufficiently small positive x , $\tan x > x$, so $u \neq v$, and both tend to zero. The mean value theorem gives

$$\frac{e^u - e^v}{u - v} = e^{\xi_x},$$

where ξ_x lies between u and v . Thus $\xi_x \rightarrow 0$, and the first root is $\alpha = 1$. This avoids separately approximating two small differences.

Step 2: Evaluate the variable-power limit Let $L = (1 + \sin x)^{\frac{1}{2} \cot x}$. The base is positive near zero, and

$$\log L = \frac{1}{2} \frac{\cos x}{\sin x} \log(1 + \sin x).$$

Since $\log(1 + z)/z \rightarrow 1$ as $z \rightarrow 0$ and $\cos x \rightarrow 1$, this expression tends to $1/2$. Exponentiating gives $\beta = e^{1/2} = \sqrt{e}$. The same limit holds from either side of zero.

Step 3: Determine the coefficients from the roots The product of the roots gives

$$\alpha\beta = \sqrt{e} = \frac{-\sqrt{e}}{a}, \quad a = -1.$$

Their sum then gives

$$1 + \sqrt{e} = -\frac{b}{a} = b, \quad a + b = \sqrt{e}.$$

The negative leading coefficient is essential because the printed constant term is negative while both

roots are positive. Finally,

$$12 \log_e(a + b) = 12 \log_e \sqrt{e} = 12 \left(\frac{1}{2} \right) = 6.$$

Step 4: Conclusion The required value is $\boxed{6}$.

Quick Tip to Solve: For a variable-power limit take logarithms first; afterwards use both the sum and product of the roots so the sign of the leading coefficient is preserved.

29. An arithmetic progression is written in the following way

The sum of all the terms of the 10th row is _____.

[4 marks]

			2			
		5		8		
	11		14		17	
20		23		26		29

Solution:

Concept:

- The displayed numbers form one arithmetic progression with first term $a = 2$ and common difference $d = 3$. Its n th term is $t_n = a + (n - 1)d = 3n - 1$.
- Row r contains r consecutive terms. The number of terms in the preceding rows is the triangular number $1 + 2 + \dots + (r - 1) = r(r - 1)/2$.
- The sum of consecutive terms of an arithmetic progression is the number of terms multiplied by the mean of its first and last terms.

Step 1: Locate the terms in the tenth row The first four rows have respectively one, two, three and four entries: 2; 5, 8; 11, 14, 17; and 20, 23, 26, 29. Thus the arrangement continues the progression without resetting its first term at each row. Before row ten there are

$$1 + 2 + \dots + 9 = \frac{9 \times 10}{2} = 45$$

terms. The tenth row therefore consists of terms numbered 46 through 55, inclusive. It has exactly ten entries.

Step 2: Find the two endpoints of the row Using the progression formula separately at each end gives

$$t_{46} = 2 + 45(3) = 137, \quad t_{55} = 2 + 54(3) = 164.$$

The endpoint difference is $164 - 137 = 27 = 9d$, as required for a row of ten equally spaced entries. This check prevents confusing the tenth row with the tenth individual term.

Step 3: Add the ten entries Pairing the first entry with the last, the second with the second-last, and

so on produces five identical pair sums. Each pair totals $137 + 164 = 301$. Equivalently, the average entry is $301/2$, and hence

$$S_{10\text{th row}} = \frac{10}{2}(137 + 164) = 5(301) = 1505.$$

Step 4: Conclusion The sum of all terms in the tenth row is $\boxed{1505}$. This is a dimensionless sum of numbers.

Quick Tip to Solve: For a triangular row arrangement, find the term indices from triangular numbers first; then use only the first and last terms of that row.

30. Let a ray of light passing through the point $(3, 10)$ reflects on the line $2x + y = 6$ and the reflected ray passes through the point $(7, 2)$. If the equation of the incident ray is $ax + by + 1 = 0$, then $a^2 + b^2 + 3ab$ is equal to _____. [4 marks]

Solution:

Concept:

- Reflect the point on the reflected ray across the mirror line. Joining its mirror image to the given point on the incident ray unfolds the reflected path into a straight line.
- For the mirror $Ax + By + C = 0$, the reflection of (x_0, y_0) is $(x_0 - 2A\lambda, y_0 - 2B\lambda)$, where $\lambda = (Ax_0 + By_0 + C)/(A^2 + B^2)$.
- The equation of a line can be multiplied by a nonzero constant, so its coefficients must finally be normalized to the specified constant term $+1$.

Step 1: Reflect the point on the outgoing ray Let $P = (3, 10)$ and $Q = (7, 2)$. The mirror is $2x + y - 6 = 0$, so $A = 2$, $B = 1$, and $C = -6$. For point Q ,

$$\lambda = \frac{2(7) + 2 - 6}{2^2 + 1^2} = \frac{10}{5} = 2.$$

Its reflected point is therefore

$$Q' = (7 - 2(2)(2), 2 - 2(1)(2)) = (-1, -2).$$

The mirror is the perpendicular bisector of QQ' , which is exactly the symmetry needed for the law of reflection.

Step 2: Find and normalize the incident line In the unfolded path the incident line passes through P and Q' . Their slope is

$$m = \frac{10 - (-2)}{3 - (-1)} = 3.$$

The line through P is $y - 10 = 3(x - 3)$, or $y = 3x + 1$. Written with the required constant term this becomes

$$3x - y + 1 = 0.$$

Thus $a = 3$ and $b = -1$. As a geometry check, its mirror intersection is $(1, 4)$; the outgoing segment from this point to Q has direction $(6, -2)$, consistent with reflecting incident direction $(-2, -6)$ across the mirror.

Step 3: Evaluate the requested coefficient expression Substitute the normalized coefficients, retaining the negative sign of b :

$$a^2 + b^2 + 3ab = 3^2 + (-1)^2 + 3(3)(-1) = 9 + 1 - 9 = 1.$$

The normalization matters because scaling both coefficients changes this quadratic expression.

Step 4: Conclusion The required dimensionless value is $\boxed{1}$.

Quick Tip to Solve: Reflect a known point on the outgoing ray across the mirror, join it to the incident point, and normalize the line equation before using its coefficients.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A long straight wire of radius a carries a steady current I . The current is uniformly distributed across its cross section. The ratio of the magnetic field at $\frac{a}{2}$ and $2a$ from axis of the wire is : [4 marks]

- (A) 4 : 1
- (B) 3 : 4
- (C) 1 : 1
- (D) 1 : 4

Correct Answer: (C) 1 : 1

Solution:

Concept:

- For a long wire with cylindrical symmetry, Ampere's circuital law gives $B(2\pi r) = \mu_0 I_{\text{enc}}$, where r is the distance from its axis.
- Uniform current density means $J = I/(\pi a^2)$. Inside the conductor only the current through the enclosed area πr^2 contributes to I_{enc} .
- Outside the conductor the Amperian loop encloses the entire current I . Consequently B increases linearly inside and decreases inversely with distance outside.

Step 1: Calculate the field at the interior point At $r = a/2$, the enclosed current is proportional to area, not radius:

$$I_{\text{enc}} = I \frac{(a/2)^2}{a^2} = \frac{I}{4}.$$

Ampere's law then yields

$$B_{\text{in}} = \frac{\mu_0(I/4)}{2\pi(a/2)} = \frac{\mu_0 I}{4\pi a}.$$

Equivalently, the interior formula $B(r) = \mu_0 I r / (2\pi a^2)$ gives the same result. The field at half the radius is half the surface field.

Step 2: Calculate the field at the exterior point At $r = 2a$, the loop lies outside the wire and encloses all its current. Thus

$$B_{\text{out}} = \frac{\mu_0 I}{2\pi(2a)} = \frac{\mu_0 I}{4\pi a}.$$

Both values have the same dimensions and the same magnitude, even though the two points lie at very different radial distances. The change of formula at $r = a$ is essential.

Step 3: Compare the magnitudes and the choices Dividing the two expressions cancels μ_0 , I , and a :

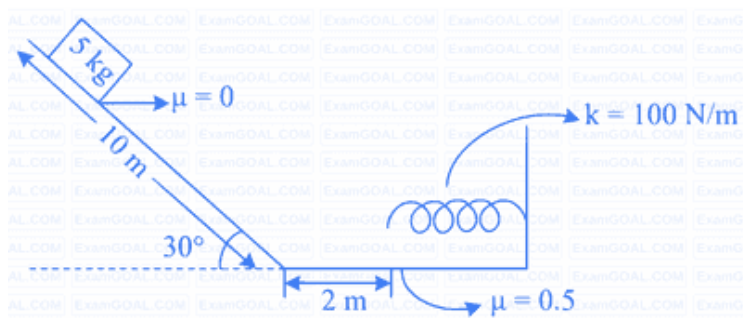
$$\frac{B(a/2)}{B(2a)} = 1.$$

Option (A), 4 : 1, would follow from incorrectly using the exterior inverse-distance law at both points. Option (D), 1 : 4, would follow from extending the interior linear law outside the conductor. Option (B), 3 : 4, agrees with neither enclosed-current calculation.

Step 4: Conclusion The magnetic-field ratio is $\boxed{1 : 1}$. Hence, the correct option is (C).

Quick Tip to Solve: Always decide whether each radius is inside or outside the wire before applying Ampere's law; inside, the enclosed current scales as the square of the radius.

32. A block is simply released from the top of an inclined plane as shown in the figure above. The maximum compression in the spring when the block hits the spring is : [4 marks]



- (A) $\sqrt{6}$ m
- (B) $\sqrt{5}$ m
- (C) 1 m
- (D) 2 m

Correct Answer: (D) 2 m

Solution:

Concept:

- On the smooth incline the gravitational potential energy lost becomes kinetic energy: $K = mgh$, where $h = L \sin \theta$. The released block starts from rest.
- On the marked rough horizontal stretch, friction removes energy $W_f = \mu mg\ell$. At maximum spring compression the block is momentarily at rest and the spring stores $U_s = kx^2/2$.
- The diagram gives $m = 5$ kg, $L = 10$ m, $\theta = 30^\circ$, a rough stretch $\ell = 2$ m with $\mu = 0.5$, and spring

constant $k = 100 \text{ N m}^{-1}$.

Step 1: Find the energy gained on the incline The vertical descent is the component of the inclined length along gravity, not the full 10 m:

$$h = 10 \sin 30^\circ = 5 \text{ m.}$$

Taking $g = 10 \text{ m s}^{-2}$, the gravitational energy released is

$$mgh = 5(10)(5) = 250 \text{ J.}$$

Since the incline is smooth, its normal reaction does no work and there is no dissipative loss on this part of the path.

Step 2: Subtract the loss on the rough stretch The horizontal normal force is $N = mg = 50 \text{ N}$, and the friction force on the marked rough stretch is

$$f = \mu N = 0.5(50) = 25 \text{ N.}$$

Its work over the indicated 2 m distance removes

$$W_f = 25(2) = 50 \text{ J.}$$

Thus the energy available for compression of the spring is $250 - 50 = 200 \text{ J}$. The friction loss is evaluated on the marked rough stretch before the spring.

Step 3: Convert the remaining energy into spring energy At maximum compression, the kinetic energy vanishes, so

$$\frac{1}{2}(100)x^2 = 200 \quad \Rightarrow \quad x^2 = 4 \text{ m}^2.$$

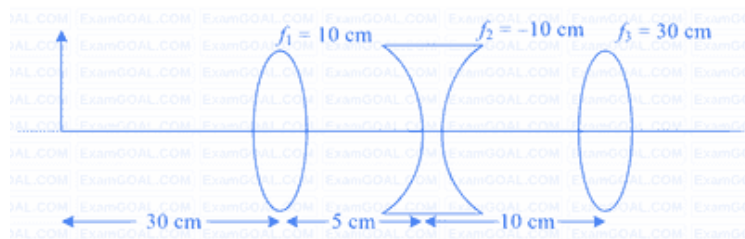
Compression is a nonnegative length, giving $x = 2 \text{ m}$. Options (A) and (B) would require spring energies of 300 J and 250 J, respectively; both ignore the available-energy calculation, and the former even exceeds the initial gravitational energy. Option (C) stores only 50 J, leaving unaccounted energy at the turning point.

Step 4: Conclusion The maximum spring compression is $\boxed{2 \text{ m}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the vertical drop to find the available energy, subtract friction work on the marked rough length, and equate the remainder to $kx^2/2$.

33. The position of the image formed by the combination of lenses is :

[4 marks]



- (A) 15 cm (right of second lens)
- (B) 30 cm (right of third lens)
- (C) 30 cm (left of third lens)
- (D) 15 cm (left of second lens)

Correct Answer: (B) 30 cm (right of third lens)

Solution:

Concept:

- Use the Cartesian thin-lens formula $1/v - 1/u = 1/f$, taking the direction of incident light as positive. A real object to the left of a lens has $u < 0$.
- The image produced by one lens serves as the object for the next lens. If the rays reaching a lens are converging toward a point on its right, that point is a virtual object and has $u > 0$.
- The diagram gives $f_1 = 10$ cm, $f_2 = -10$ cm, $f_3 = 30$ cm, object distance 30 cm, and separations 5 cm and 10 cm.

Step 1: Locate the image produced by the first lens For the first converging lens, $u_1 = -30$ cm. Hence

$$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{10} - \frac{1}{30} = \frac{1}{15}.$$

The rays emerging from the first lens would meet 15 cm to its right if no other lens intercepted them. This is an intermediate image position, not yet the final image of the whole combination.

Step 2: Treat that point as a virtual object for the second lens The second lens is only 5 cm to the right of the first, so the converging rays at that lens are directed toward a point $15 - 5 = 10$ cm to its right. Thus $u_2 = +10$ cm, and

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{10} + \frac{1}{10} = 0.$$

The second lens therefore makes the rays parallel. Using $u_2 = -10$ cm here would give an incorrect answer because the object for this lens is virtual.

Step 3: Focus the parallel rays with the third lens Parallel rays correspond to an object at infinity. The third converging lens consequently brings them to its right focal point:

$$v_3 = f_3 = +30 \text{ cm}.$$

The 10 cm gap before this lens does not change the vergence of a parallel beam. Option (A) confuses an intermediate image with the final image; option (C) puts the image on the wrong side of the

converging third lens; option (D) also gives the wrong position and ignores the complete combination.

Step 4: Conclusion The final image is 30 cm to the right of the third lens. Hence, the correct option is (B).

Quick Tip to Solve: Carry each intermediate image position into the coordinate system of the next lens; a point on its right is a virtual object with positive u .

34. If ϵ_0 is the permittivity of free space and E is the electric field, then $\epsilon_0 E^2$ has the dimensions : [4 marks]

- (A) $[ML^{-1} T^{-2}]$
- (B) $[ML^2 T^{-2}]$
- (C) $[M^0 L^{-2} T A]$
- (D) $[M^{-1} L^{-3} T^4 A^2]$

Correct Answer: (A) $[ML^{-1} T^{-2}]$

Solution:

Concept:

- The dimensions of a physical product are obtained by multiplying the dimensions of its factors and adding the corresponding exponents of mass, length, time and electric current.
- Electric field is force per unit charge, $E = F/q$, while the dimensions of charge are $[q] = [A T]$.
- From Coulomb's law $F = q_1 q_2 / (4\pi \epsilon_0 r^2)$, the factor 4π is dimensionless and $[\epsilon_0] = [q]^2 / ([F][r]^2)$.

Step 1: Find the dimensions of each factor Force has dimensions $[ML T^{-2}]$. Dividing it by charge gives

$$[E] = [ML T^{-3} A^{-1}].$$

Coulomb's law independently gives the permittivity dimensions:

$$[\epsilon_0] = \frac{[A^2 T^2]}{[ML T^{-2}][L^2]} = [M^{-1} L^{-3} T^4 A^2].$$

The charge and current factors must be retained until the two quantities are multiplied.

Step 2: Multiply by the square of the field Squaring the field dimensions doubles every exponent, so

$$[E^2] = [M^2 L^2 T^{-6} A^{-2}].$$

Multiplying this by the permittivity dimensions produces

$$[\epsilon_0 E^2] = [M^{-1+2} L^{-3+2} T^{4-6} A^{2-2}] = [ML^{-1} T^{-2}].$$

The electric-current dimensions cancel completely.

Step 3: Check the physical meaning and the other choices The electric energy density is $u = \epsilon_0 E^2 / 2$. Energy divided by volume has dimensions $[ML^2 T^{-2}] / [L^3] = [ML^{-1} T^{-2}]$, confirming the result.

Option (B) gives energy rather than energy density. Option (C) retains an electric-current factor and has no mass factor. Option (D) gives the dimensions of permittivity alone, omitting the factor E^2 .

Step 4: Conclusion The dimensions are $[M L^{-1} T^{-2}]$. Hence, the correct option is (A).

Quick Tip to Solve: Remember that $\epsilon_0 E^2/2$ is energy per unit volume, so its dimensions equal those of pressure; the numerical factor does not affect dimensions.

35. Two satellite A and B go round a planet in circular orbits having radii $4R$ and R respectively. If the speed of A is $3v$, the speed of B will be : [4 marks]

- (A) $6v$
- (B) $\frac{4}{3}v$
- (C) $3v$
- (D) $12v$

Correct Answer: (A) $6v$

Solution:

Concept:

- For a circular orbit about a planet of mass M , the gravitational force supplies the centripetal force: $GMm/r^2 = mv_{\text{orb}}^2/r$.
- Thus the orbital speed is $v_{\text{orb}} = \sqrt{GM/r}$. For the same planet, orbital speed is inversely proportional to the square root of the orbital radius.
- The satellite mass cancels from the speed relation. Radius is measured from the planet's centre, as specified for the two circular orbits.

Step 1: Derive the relation between the two speeds Writing the force balance separately for either satellite gives

$$v_{\text{orb}}^2 = \frac{GM}{r}.$$

For satellites A and B, $r_A = 4R$ and $r_B = R$. Since G and the central planetary mass are identical in both equations, their ratio is

$$\frac{v_B^2}{v_A^2} = \frac{GM/R}{GM/(4R)} = 4.$$

Taking the positive square root is appropriate because speed is a nonnegative magnitude.

Step 2: Substitute the stated speed of satellite A The speed ratio is $v_B/v_A = 2$; the satellite closer to the planet moves faster. The question states $v_A = 3v$, where v is the reference speed used in the options. Therefore

$$v_B = 2v_A = 2(3v) = 6v.$$

As a check, the orbital angular speed varies as $r^{-3/2}$. Satellite B has eight times A's angular speed but one-quarter its orbital radius, giving twice its linear speed, consistently.

Step 3: Eliminate the competing answers Option (B), $4v/3$, would make the inner satellite slower than A, contrary to the force balance. Option (C), $3v$, incorrectly treats circular orbital speed as

independent of radius. Option (D), $12v$, uses an inverse-radius law for speed rather than for speed squared: reducing radius by a factor four doubles speed instead of quadrupling it. Only option (A) has both the correct direction of change and the correct square-root factor.

Step 4: Conclusion The speed of satellite B is $6v$. Hence, the correct option is (A).

Quick Tip to Solve: For satellites orbiting the same planet, use $v_B/v_A = \sqrt{r_A/r_B}$; do not replace the square-root relation with a direct inverse-radius ratio.

36. Water boils in an electric kettle in 20 minutes after being switched on. Using the same main supply, the length of the heating element should be _____ to _____ times of its initial length if the water is to be boiled in 15 minutes. [4 marks]

- (A) increased, $\frac{3}{4}$
- (B) increased, $\frac{4}{3}$
- (C) decreased, $\frac{3}{4}$
- (D) decreased, $\frac{4}{3}$

Correct Answer: (C) decreased, $\frac{3}{4}$

Solution:

Concept:

- For a uniform heating wire, resistance is $R = \rho L/A$, where ρ , L and A are its resistivity, length and cross-sectional area. Cutting its length changes resistance in direct proportion.
- At fixed supply voltage V , Joule heating power is $P = V^2/R$. The same amount of water heated through the same temperature interval needs the same energy $Q = Pt$.

Step 1: Relate boiling time to resistance The water and its initial and final temperatures are unchanged, so the required heating energy is fixed. With the same mains voltage,

$$Q = \frac{V^2 t}{R}.$$

Consequently t/R remains constant, or $t \propto R$. A shorter boiling time therefore requires a smaller resistance, not a larger one. We use the usual ideal kettle model with the same heating conditions and neglect heat losses.

Step 2: Calculate the required wire length For unchanged material and cross-sectional area, $R \propto L$, so

$$\frac{L_2}{L_1} = \frac{R_2}{R_1} = \frac{t_2}{t_1} = \frac{15}{20} = \frac{3}{4}.$$

Thus the element must be shortened to three-fourths of its original length. Its resistance falls to $3R_1/4$, and its power rises to $4P_1/3$. This supplies the same energy in $3/4$ of the original time. Options (A) and (B) wrongly call for an increase; option (D) gives a length ratio greater than one while describing a decrease.

Step 3: Conclusion The heating-element length should be decreased to $\frac{3}{4}$ of its initial length. Hence,

the correct option is (C).

Quick Tip to Solve: At fixed voltage, shortening a heater lowers resistance and increases power: for the same heat requirement, $t \propto R \propto L$.

37. A diatomic gas ($\gamma = 1.4$) does 100 J of work in an isobaric expansion. The heat given to the gas is : [4 marks]

- (A) 150 J
- (B) 490 J
- (C) 350 J
- (D) 250 J

Correct Answer: (C) 350 J

Solution:

Concept:

- For an ideal gas expanding at constant pressure, the work done by the gas is $W = P\Delta V = \nu R\Delta T$, where ν is the amount of gas.
- The first law with work done by the gas positive is $Q = \Delta U + W$. The heat capacities satisfy $C_P - C_V = R$ and $\gamma = C_P/C_V$, so $C_V = R/(\gamma - 1)$.

Step 1: Find the rise in internal energy The positive work in an isobaric expansion implies a positive temperature change. Using $W = \nu R\Delta T$,

$$\Delta U = \nu C_V \Delta T = \frac{\nu R \Delta T}{\gamma - 1} = \frac{W}{\gamma - 1}.$$

For $\gamma = 1.4$ and $W = 100$ J,

$$\Delta U = \frac{100}{0.4} = 250 \text{ J}.$$

This is the increase in internal energy, not the full heat supplied; some of the heat also produces expansion work.

Step 2: Add the expansion work Applying the first law to the gas gives

$$Q = \Delta U + W = 250 + 100 = 350 \text{ J}.$$

Equivalently, $Q/W = C_P/R = \gamma/(\gamma - 1) = 3.5$, which gives the same result. The value 250 J in option (D) omits the work done on the surroundings. The values in (A) and (B) do not obey this heat-capacity ratio or the first-law energy balance. The supplied heat must exceed 100 J because the gas both warms and expands.

Step 3: Conclusion The heat given to the gas is 350 J. Hence, the correct option is (C).

Quick Tip to Solve: In an ideal-gas isobaric process use $Q = \gamma W/(\gamma - 1)$; do not confuse the internal-energy change with the heat supplied.

38. If M_0 is the mass of isotope ${}^{12}_5\text{B}$, M_p and M_n are the masses of proton and neutron, then nuclear binding energy of isotope is:

[4 marks]

(A) $(5M_p + 7M_n - M_0)C^2$

(B) $(M_0 - 5M_p - 7M_n)C^2$

(C) $(M_0 - 5M_p)C^2$

(D) $(M_0 - 12M_n)C^2$

Correct Answer: (A) $(5M_p + 7M_n - M_0)C^2$

Solution:

Concept:

- A nucleus ${}^A_ZX$ contains Z protons and $A - Z$ neutrons. Its mass number counts all nucleons, while its atomic number counts only protons.
- The positive mass defect is the mass of the separated nucleons minus the bound nuclear mass: $\Delta m = ZM_p + (A - Z)M_n - M_0$. Binding energy is $E_b = \Delta m c^2$.

Step 1: Count the constituent nucleons For the isotope ${}^{12}_5\text{B}$, the atomic number is $Z = 5$ and the mass number is $A = 12$. Therefore it contains five protons and

$$N = A - Z = 12 - 5 = 7$$

neutrons. If these nucleons were separated and at rest, their total rest mass would be

$$M_{\text{free}} = 5M_p + 7M_n.$$

The neutron mass cannot be substituted for all twelve nucleons because the nucleus has two different kinds of constituents.

Step 2: Convert the mass defect into energy When the nucleons bind, energy is released and the bound nuclear mass is lower than the sum of the free nucleon masses. Hence

$$\Delta m = 5M_p + 7M_n - M_0,$$

$$E_b = (5M_p + 7M_n - M_0)c^2.$$

This is the positive energy required to separate the nucleus completely. Option (B) reverses the mass-defect sign. Option (C) omits the seven neutrons as well as using the wrong subtraction. Option (D) treats every nucleon as a neutron and also reverses the binding-energy subtraction. The factor denoted C^2 in the options represents the square of the speed of light.

Step 3: Conclusion The nuclear binding energy is $(5M_p + 7M_n - M_0)c^2$. Hence, the correct option is (A).

Quick Tip to Solve: Write the free-nucleon mass first, using Z protons and $A - Z$ neutrons, then subtract the bound nuclear mass before multiplying by c^2 .

39. A given object takes n times the time to slide down 45° rough inclined plane as it takes the time to slide down an identical perfectly smooth 45° inclined plane. The coefficient of kinetic friction between the object and the surface of inclined plane is : [4 marks]

- (A) $1 - \frac{1}{n^2}$
 (B) $\sqrt{1 - \frac{1}{n^2}}$
 (C) $1 - n^2$
 (D) $\sqrt{1 - n^2}$

Correct Answer: (A) $1 - \frac{1}{n^2}$

Solution:

Concept:

- For a body released from rest and travelling a fixed distance s with constant acceleration, $s = \frac{1}{2}at^2$. Thus the travel time is inversely proportional to $\sqrt{a}$.
- Down a rough incline of angle θ , gravity contributes $mg \sin \theta$ down the plane and kinetic friction $\mu_k mg \cos \theta$ acts up the plane. Hence $a = g(\sin \theta - \mu_k \cos \theta)$.

Step 1: Use the ratio of sliding times Let t_s, a_s denote the time and acceleration on the smooth plane, and t_r, a_r those on the rough plane. The planes have identical lengths and inclinations, and the object starts from rest in each case. Therefore

$$\frac{1}{2}a_s t_s^2 = \frac{1}{2}a_r t_r^2.$$

Since $t_r = nt_s$,

$$\frac{a_r}{a_s} = \frac{1}{n^2}.$$

The square is essential: time is proportional to the inverse square root of acceleration, not its inverse.

Step 2: Substitute the incline accelerations For the smooth plane, $a_s = g \sin 45^\circ = g/\sqrt{2}$. For the rough plane,

$$a_r = g(\sin 45^\circ - \mu_k \cos 45^\circ) = \frac{g}{\sqrt{2}}(1 - \mu_k).$$

Taking their ratio gives

$$1 - \mu_k = \frac{1}{n^2}, \quad \mu_k = 1 - \frac{1}{n^2}.$$

For frictional retardation $n \geq 1$, giving $0 \leq \mu_k < 1$ for finite downhill sliding time. Options (C) and (D) become negative or non-real for $n > 1$. Option (B) incorrectly takes a second square root after the time ratio has already been squared.

Step 3: Conclusion The coefficient of kinetic friction is $\mu_k = 1 - \frac{1}{n^2}$. Hence, the correct option is (A).

Quick Tip to Solve: First square the time ratio to obtain the inverse acceleration ratio; at 45° , the rough-to-smooth acceleration ratio reduces to $1 - \mu_k$.

40. There are 100 divisions on the circular scale of a screw gauge of pitch 1 mm. With no measuring quantity in between the jaws, the zero of the circular scale lies 5 divisions below the reference line. The diameter of a wire is then measured using this screw gauge. It is found that 4 linear scale divisions are clearly visible while 60 divisions on circular scale coincide with the reference line. The diameter of the wire is : [4 marks]

- (A) 4.65 mm
- (B) 4.60 mm
- (C) 4.55 mm
- (D) 3.35 mm

Correct Answer: (C) 4.55 mm

Solution:

Concept:

- A screw gauge has least count $LC = \text{pitch}/\text{number of circular-scale divisions}$. The observed reading is the main-scale reading plus the circular-scale contribution.
- The corrected dimension is observed reading – zero error. A zero below the reference line when the jaws are closed denotes a positive zero error in the standard screw-gauge convention.

Step 1: Calculate the least count and observed reading One full turn advances the screw by 1 mm, and the circular scale contains 100 divisions. Hence

$$LC = \frac{1}{100} \text{ mm} = 0.01 \text{ mm}.$$

The four visible linear-scale divisions give 4 mm, and the coincident circular-scale reading contributes

$$60 \times 0.01 = 0.60 \text{ mm}.$$

Thus the observed diameter before zero correction is $4 + 0.60 = 4.60 \text{ mm}$. This is the uncorrected reading in option (B).

Step 2: Apply the signed zero correction With no wire between the jaws the instrument should read zero, but its circular zero is five divisions below the reference line. The resulting positive zero error is

$$e = +5(0.01) = +0.05 \text{ mm}.$$

The instrument therefore overestimates every measurement by 0.05 mm. Subtract it:

$$d = 4.60 - 0.05 = 4.55 \text{ mm}.$$

Adding the error instead would give 4.65 mm, option (A), and would worsen the overestimate. Option (D) does not follow either the main-scale reading or the circular-scale contribution.

Step 3: Conclusion The corrected diameter of the wire is 4.55 mm. Hence, the correct option is (C).

Quick Tip to Solve: Calculate the raw screw-gauge reading first, then subtract the signed zero error; a positive closed-jaw reading always requires a negative correction.

41. A capacitor has air as dielectric medium and two conducting plates of area 12 cm^2 and they are 0.6 cm apart. When a slab of dielectric having area 12 cm^2 and 0.6 cm thickness is inserted between the plates, one of the conducting plates has to be moved by 0.2 cm to keep the capacitance same as in previous case. The dielectric constant of the slab is : (Given $\epsilon_0 = 8.834 \times 10^{-12} \text{ F/m}$) [4 marks]

- (A) 1.50
(B) 0.66
(C) 1.33
(D) 1

Correct Answer: (A) 1.50

Solution:

Concept:

- An air-filled parallel-plate capacitor of plate area A and separation d has capacitance $C_0 = \epsilon_0 A/d$, neglecting edge effects.
- For a dielectric slab of thickness t and relative permittivity K together with an air gap g , the layers act in series. The capacitance is $C = \epsilon_0 A/(g + t/K)$.

Step 1: Identify the final air gap Initially $d = 0.6 \text{ cm}$. The inserted slab has this same thickness, $t = 0.6 \text{ cm}$, and covers the full plate area. Inserting an ordinary dielectric increases capacitance if the separation is kept fixed. To restore the original capacitance, one plate must move away by 0.2 cm . The final plate separation is therefore 0.8 cm , of which 0.6 cm is dielectric and $g = 0.2 \text{ cm}$ is air. Treating the entire final separation as dielectric would incorrectly fill the added gap.

Step 2: Equate the effective separations Equal capacitances with the same area imply equal effective electrical separations:

$$\frac{\epsilon_0 A}{0.6} = \frac{\epsilon_0 A}{0.2 + 0.6/K}$$

All lengths here are expressed in the same unit, so the common conversion to metres cancels. Therefore

$$0.6 = 0.2 + \frac{0.6}{K}, \quad \frac{0.6}{K} = 0.4,$$

$$K = \frac{0.6}{0.4} = 1.50.$$

The area and ϵ_0 cancel, so their numerical values are unnecessary. Option (B) is less than unity, option (D) would produce no dielectric increase, and option (C) results from confusing the geometric separation with the series-layer effective separation.

Step 3: Conclusion The dielectric constant of the slab is $K = 1.50$, a dimensionless ratio. Hence, the correct option is (A).

Quick Tip to Solve: For a slab of fixed thickness, use effective separation $g + t/K$; moving a plate adds an air layer, not additional dielectric.

42. Least count of a vernier caliper is $\frac{1}{20N}$ cm. The value of one division on the main scale is 1 mm. Then the number of divisions of main scale that coincide with N divisions of vernier scale is : [4 marks]

- (A) $\left(\frac{2N-1}{2N}\right)$
(B) $\left(\frac{2N-1}{20N}\right)$
(C) $(2N-1)$
(D) $\left(\frac{2N-1}{2}\right)$

Correct Answer: (D) $\left(\frac{2N-1}{2}\right)$

Solution:

Concept:

- For a direct vernier, the least count is the difference between one main-scale division and one vernier-scale division: $LC = 1 \text{ MSD} - 1 \text{ VSD}$.
- To find how many main-scale divisions coincide with N vernier divisions, compare their total lengths in the same unit, then divide by the length of one main-scale division.

Step 1: Determine the length of one vernier division One main-scale division equals $1 \text{ mm} = 0.1 \text{ cm}$. The least count is $1/(20N) \text{ cm}$. Hence

$$1 \text{ VSD} = \frac{1}{10} - \frac{1}{20N} = \frac{2N-1}{20N} \text{ cm.}$$

This is the physical length of one vernier division, not the requested count of main-scale divisions. Option (B) reproduces this length numerically without making the conversion from a length to a count.

Step 2: Compare the lengths of the coincident divisions The length occupied by N vernier divisions is

$$N \text{ VSD} = \frac{2N-1}{20} \text{ cm.}$$

If it coincides with m main-scale divisions, the same length is $m/10 \text{ cm}$. Equating the two gives

$$\frac{m}{10} = \frac{2N-1}{20}, \quad m = \frac{2N-1}{2}.$$

Option (A) is the ratio of one vernier division to one main-scale division; it lacks the factor N needed for the full vernier length. Option (C) introduces an extra factor of two. Only option (D) represents the requested number, which is dimensionless.

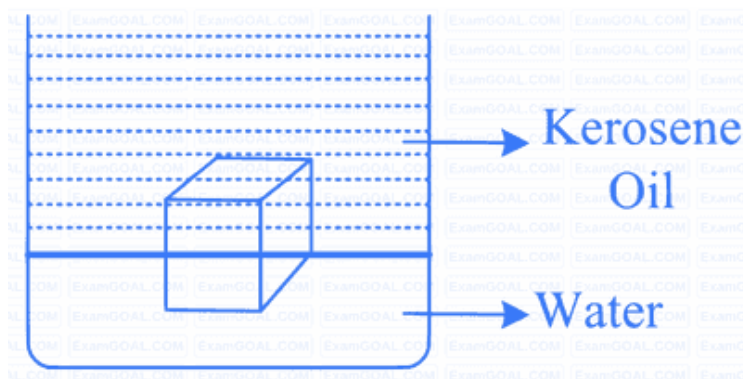
Step 3: Conclusion The number of main-scale divisions coinciding with N vernier divisions is

$$\boxed{\frac{2N-1}{2}}. \text{ Hence, the correct option is (D).}$$

Quick Tip to Solve: Convert the least count and main-scale division to the same unit, find one vernier division, and multiply by N before counting main-scale divisions.

43. A cube of ice floats partly in water and partly in kerosene oil. The ratio of volume of ice immersed in water to that in kerosene oil (specific gravity of Kerosene oil = 0.8, specific gravity of ice = 0.9):

[4 marks]



- (A) 5 : 4
- (B) 9 : 10
- (C) 8 : 9
- (D) 1 : 1

Correct Answer: (D) 1 : 1

Solution:

Concept:

- Archimedes' principle gives an upward buoyant force equal to the weight of displaced liquid. When a body displaces two liquids, add the two displaced weights.
- Specific gravity is density divided by water density. Thus $\rho_w = \rho$, $\rho_k = 0.8\rho$ and $\rho_i = 0.9\rho$, where the subscripts denote water, kerosene and ice.

Step 1: Define the immersed volumes Let V_w and V_k be the ice volumes immersed in water and kerosene respectively. The figure shows the ice entirely within these two liquid layers, so its total volume is $V = V_w + V_k$. The ice is at rest, hence its total upward buoyant force balances its weight. The two displaced volumes must be treated separately because the liquid densities differ.

Step 2: Balance weight and buoyancy Using the displaced weight in each liquid gives

$$\rho_w g V_w + \rho_k g V_k = \rho_i g (V_w + V_k).$$

Divide by the common nonzero factor ρg and collect terms:

$$\begin{aligned} V_w + 0.8V_k &= 0.9V_w + 0.9V_k, \\ 0.1V_w &= 0.1V_k \Rightarrow \frac{V_w}{V_k} = 1. \end{aligned}$$

The same volume must therefore lie on either side of the interface. This also follows because the ice density is exactly the arithmetic mean of the two liquid densities.

Step 3: Check the offered ratios The required ratio is water volume to kerosene volume, not the ratio of liquid densities. Options (A), (B) and (C) give unequal immersed volumes and would make the

weighted mean displaced-liquid density different from 0.9ρ ; none satisfies the force balance. Option (D) alone gives equal contributions in volume and the correct mean density.

Step 4: Conclusion The ratio of the ice volume immersed in water to that immersed in kerosene is $1 : 1$. Hence, the correct option is (D).

Quick Tip to Solve: At a liquid interface use $V_w/V_k = (\rho_i - \rho_k)/(\rho_w - \rho_i)$; a density midway between the liquids gives equal immersed volumes.

44. Given below are two statements :

Statement (I) : The mean free path of gas molecules is inversely proportional to square of molecular diameter.

Statement (II) : Average kinetic energy of gas molecules is directly proportional to absolute temperature of gas.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Statement I is false but Statement II is true
- (B) Both Statement I and Statement II are true
- (C) Statement I is true but Statement II is false
- (D) Both Statement I and Statement II are false

Correct Answer: (B) Both Statement I and Statement II are true

Solution:

Concept:

- For a dilute gas of hard spherical molecules, the mean free path is $\lambda = 1/(\sqrt{2}\pi d^2 n)$, where d is the molecular collision diameter and $n = N/V$ is number density.
- The mean translational kinetic energy per molecule of an ideal gas is $\langle K \rangle = \frac{3}{2}k_B T$, where T is absolute temperature and k_B is Boltzmann's constant.

Step 1: Evaluate Statement I Keep the number density fixed when comparing the dependence on molecular diameter. From the collision formula,

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2} \Rightarrow \lambda \propto d^{-2}.$$

The effective area for a collision is πd^2 . Larger molecules therefore collide more frequently and travel a shorter mean distance between collisions. For example, doubling the diameter at the same number density reduces the mean free path to one fourth. Statement I correctly states this inverse-square dependence and is true.

Step 2: Evaluate Statement II Kinetic theory relates temperature to random translational molecular motion:

$$\frac{1}{2}m\langle v^2 \rangle = \frac{3}{2}k_B T.$$

Here the left side is the average kinetic energy per molecule. Since the multiplying constant is

independent of temperature, this energy is directly proportional to absolute temperature. Doubling the kelvin temperature doubles the average translational kinetic energy. The molecular mass changes the corresponding rms speed, but it does not change this energy-temperature relation. Statement II is therefore true.

Step 3: Choose the combined verdict The statements concern two separate standard kinetic-theory results, and both have been verified directly. Option (A) incorrectly rejects the mean-free-path formula; option (C) incorrectly rejects the energy-temperature relation; option (D) rejects both true results. The option stating that both statements are true is the required choice.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (B).

Quick Tip to Solve: Remember the two dependences separately: collision geometry gives $\lambda \propto d^{-2}$ at fixed number density, while thermal motion gives $\langle K \rangle \propto T$ in kelvin.

45. The angle of projection for a projectile to have same horizontal range and maximum height is : [4 marks]

- (A) $\tan^{-1} \left(\frac{1}{2} \right)$
- (B) $\tan^{-1}(2)$
- (C) $\tan^{-1} \left(\frac{1}{4} \right)$
- (D) $\tan^{-1}(4)$

Correct Answer: (D) $\tan^{-1}(4)$

Solution:

Concept:

- For launch speed u at angle θ to the horizontal, a projectile returning to its launch level has horizontal range $R = u^2 \sin(2\theta)/g$. Air resistance is neglected.
- The maximum height above the launch point is $H = u^2 \sin^2 \theta / (2g)$. The relevant nontrivial projection angle lies between 0 and $\pi/2$.

Step 1: Set the required range equal to the height Use the usual level-ground projectile formulae and impose the stated condition $R = H$:

$$\frac{u^2 \sin(2\theta)}{g} = \frac{u^2 \sin^2 \theta}{2g}.$$

The launch speed and gravitational acceleration are common factors, so the required angle is independent of either of them. Cancelling u^2/g gives

$$2 \sin \theta \cos \theta = \frac{1}{2} \sin^2 \theta.$$

Step 2: Solve for the projection angle Multiply by 2 and divide by the nonzero $\sin \theta$ for a projectile that rises above its launch point:

$$4 \sin \theta \cos \theta = \sin^2 \theta,$$

$$4 \cos \theta = \sin \theta \Rightarrow \tan \theta = 4.$$

Hence $\theta = \tan^{-1}(4)$, approximately 76° . The zero-angle case would give zero height and zero flight range in this ideal equal-level formula; it is the degenerate case and is not the projected trajectory asked for.

Step 3: Reject the other angle choices A useful direct check is

$$\frac{R}{H} = \frac{u^2(2 \sin \theta \cos \theta)/g}{u^2 \sin^2 \theta / (2g)} = 4 \cot \theta.$$

For the tangent values $1/2$, 2 , and $1/4$ offered in (A), (B), and (C), this ratio is respectively 8 , 2 , and 16 , rather than 1 . The tangent value 4 in option (D) alone makes the range equal the height.

Step 4: Conclusion The required angle of projection is $\theta = \tan^{-1}(4)$. Hence, the correct option is (D).

Quick Tip to Solve: Form $R/H = 4 \cot \theta$ before substituting a condition on range and height; the launch speed and g cancel immediately.

46. A proton and an electron have the same de Broglie wavelength. If K_p and K_e be the kinetic energies of proton and electron respectively, then choose the correct relation : [4 marks]

- (A) $K_p > K_e$
- (B) $K_p = K_e$
- (C) $K_p < K_e$
- (D) $K_p = K_e^2$

Correct Answer: (C) $K_p < K_e$

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$, where h is Planck's constant and p is the magnitude of linear momentum. Equal wavelengths therefore mean equal momentum magnitudes.
- In the nonrelativistic treatment, $K = p^2/(2m)$. At fixed momentum the kinetic energy is inversely proportional to the particle mass; a proton is much more massive than an electron.

Step 1: Translate equal wavelength into equal momentum The proton and electron have the same value of λ . Applying the de Broglie formula separately,

$$p_p = \frac{h}{\lambda} = p_e = p.$$

This fixes their momenta, not their speeds. Because $p = mv$ in the usual nonrelativistic treatment, the heavier proton moves more slowly for this same momentum. Equal wavelength must not be confused with equal kinetic energy.

Step 2: Compare the kinetic energies at fixed momentum For the common momentum p ,

$$K_p = \frac{p^2}{2m_p}, \quad K_e = \frac{p^2}{2m_e}.$$

Divide these expressions to remove the common momentum:

$$\frac{K_p}{K_e} = \frac{m_e}{m_p} \simeq \frac{1}{1836} < 1.$$

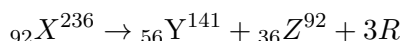
Thus the kinetic energy of the proton is smaller. This is a comparison at the same momentum; at the same speed a heavier particle would instead have the larger kinetic energy, which is a different condition.

Step 3: Check the alternative relations Option (A) reverses the inverse-mass dependence. Option (B) would require equal masses for nonzero common momentum, whereas the proton and electron masses differ greatly. Option (D), $K_p = K_e^2$, is not dimensionally a relation between energies because its right side has dimensions of energy squared. Only the strict inequality in option (C) agrees with the de Broglie condition and the energy formula.

Step 4: Conclusion For the same de Broglie wavelength, $K_p < K_e$. Hence, the correct option is (C).

Quick Tip to Solve: First ask what is held fixed: equal de Broglie wavelength fixes momentum, so use $K = h^2/(2m\lambda^2)$ and compare inverse masses.

47. In a hypothetical fission reaction



The identity of emitted particles (R) is :

[4 marks]

- (A) Proton
- (B) Neutron
- (C) Electron
- (D) γ -radiations

Correct Answer: (B) Neutron

Solution:

Concept:

- A nuclear reaction conserves total mass number A and total atomic number Z . These count nucleons and proton charge respectively, rather than the atomic masses in kilograms.
- A neutron has $A = 1$ and $Z = 0$; a proton has $A = 1$, $Z = 1$; an electron has $A = 0$, $Z = -1$; and a gamma photon has $A = 0$, $Z = 0$.

Step 1: Balance the mass numbers Let each emitted particle R have mass number a . The printed

reaction contains three such particles. Conservation of nucleon number gives

$$236 = 141 + 92 + 3a.$$

The two heavy fragments contain $141 + 92 = 233$ nucleons in total. Therefore

$$3a = 236 - 233 = 3 \Rightarrow a = 1.$$

Each emitted particle must carry one nucleon. This already excludes electrons and gamma radiation, neither of which carries a nucleon mass number.

Step 2: Balance the atomic numbers Let the atomic number of each emitted particle be z . Charge conservation requires

$$92 = 56 + 36 + 3z.$$

The heavy fragments already account for all $56 + 36 = 92$ units of positive nuclear charge, giving

$$3z = 0 \Rightarrow z = 0.$$

Thus the emitted particle has mass number 1 but no electric charge. The required nuclear symbol is ${}_0^1\text{n}$, which identifies a neutron.

Step 3: Compare every offered particle A proton would supply the missing nucleon number but would add three units of positive charge and violate the atomic-number balance. An electron would alter the charge and supply no missing nucleons. Gamma radiation carries energy and can accompany fission, but cannot account for the three missing nucleons in this reaction. Neutrons satisfy both conservation equations at once, so checking only the mass-number deficit is not sufficient without the charge check.

Step 4: Conclusion The emitted particles R are neutrons, each represented by ${}_0^1\text{n}$. Hence, the correct option is (B).

Quick Tip to Solve: In a nuclear equation balance the superscripts and subscripts independently; missing mass number with no missing charge signals neutron emission.

48. A thin circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with angular velocity ω . If another disc of same dimensions but of mass $M/2$ is placed gently on the first disc co-axially, then the new angular velocity of the system is :

[4 marks]

- (A) $\frac{4}{5}\omega$
- (B) $\frac{5}{4}\omega$
- (C) $\frac{3}{2}\omega$
- (D) $\frac{2}{3}\omega$

Correct Answer: (D) $\frac{2}{3}\omega$

Solution:

Concept:

- For a uniform thin disc rotating about its central perpendicular axis, the moment of inertia is $I = \frac{1}{2}mR^2$. Moments of inertia about the same axis add.
- With negligible external torque about the rotation axis, total angular momentum $L = I\omega$ is conserved. Friction between the discs transfers angular momentum internally until they share one angular velocity.

Step 1: Find the initial and final moments of inertia The first disc has mass M , radius R , and initial angular velocity ω , so

$$I_1 = \frac{1}{2}MR^2.$$

The second disc has the same radius but half the mass and is initially at rest about the common axis. Its moment of inertia is therefore

$$I_2 = \frac{1}{2} \left(\frac{M}{2} \right) R^2 = \frac{1}{4}MR^2.$$

After contact friction has brought them to a common angular speed, their combined moment of inertia is $I_1 + I_2 = \frac{3}{4}MR^2$.

Step 2: Apply conservation of angular momentum The initial angular momentum comes entirely from the first disc. Denote the common final angular speed by ω_f . Then

$$I_1\omega = (I_1 + I_2)\omega_f,$$

$$\omega_f = \frac{\frac{1}{2}MR^2}{\frac{3}{4}MR^2} \omega = \frac{2}{3}\omega.$$

The contact forces between the discs provide equal and opposite torques on the two-disc system, so they do not change its total axial angular momentum. Kinetic energy need not be conserved: slipping friction converts some mechanical energy to heat.

Step 3: Check the options physically Adding a stationary disc increases the total moment of inertia, so the common angular speed must be smaller than the original speed. Options (B) and (C), both larger than ω , are impossible. Option (A), $4\omega/5$, would require a different added moment of inertia and fails the explicit angular-momentum equation. Option (D) gives the correct ratio. Using conservation of

rotational kinetic energy instead would be inappropriate because the contact is dissipative.

Step 4: Conclusion The final common angular velocity is $\omega_f = \frac{2}{3}\omega$. Hence, the correct option is (D).

Quick Tip to Solve: For gently coupled coaxial rotors use $\omega_f = I_{\text{rotating}}\omega / (I_1 + I_2)$; the stationary rotor adds inertia but no initial angular momentum.

49. A coil of negligible resistance is connected in series with 90Ω resistor across 120 V, 60 Hz supply. A voltmeter reads 36 V across resistance. Inductance of the coil is : [4 marks]

- (A) 0.91 H
- (B) 0.76 H
- (C) 2.86 H
- (D) 0.286 H

Correct Answer: (B) 0.76 H

Solution:

Concept:

- For an ideal series RL circuit, the same rms current flows through both elements: $V_R = IR$, $V_L = IX_L$, and $X_L = 2\pi fL$. The coil's negligible resistance means its impedance is purely inductive.
- The resistor voltage is in phase with current, while the inductor voltage leads by 90° . Hence the supply rms voltage satisfies $V^2 = V_R^2 + V_L^2$, rather than an arithmetic sum.

Step 1: Find the current using the resistor reading The resistor has resistance 90Ω , and the voltmeter measures 36 V across it. Consequently the rms current through the whole series circuit is

$$I = \frac{V_R}{R} = \frac{36}{90} = 0.4 \text{ A.}$$

The total impedance magnitude is also

$$Z = \frac{V}{I} = \frac{120}{0.4} = 300 \Omega.$$

This impedance exceeds the resistance because of the inductive reactance.

Step 2: Obtain the inductive reactance The resistive and reactive voltage drops are perpendicular in a phasor diagram. Equivalently $Z^2 = R^2 + X_L^2$, so

$$X_L = \sqrt{300^2 - 90^2} = \sqrt{81900} \Omega \simeq 286.18 \Omega.$$

As a voltage check, the coil voltage is $IX_L \simeq 114.47 \text{ V}$; this combines with 36 V in quadrature to give 120 V. Subtracting 36 directly from 120 would give the wrong reactance.

Step 3: Calculate the inductance and identify the option Use the supplied frequency $f = 60 \text{ Hz}$:

$$L = \frac{X_L}{2\pi f} = \frac{286.18}{2\pi \times 60} \text{ H} \simeq 0.759 \text{ H}.$$

This rounds to 0.76 H . The other proposed inductances, 0.91 , 2.86 , and 0.286 H , would give different reactances at the same frequency and fail the measured supply/resistor voltage relation. The reactance is in ohms; it must still be divided by angular frequency to obtain henries.

Step 4: Conclusion The coil inductance is $L \simeq 0.76 \text{ H}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the resistor voltmeter to find current first, then combine AC drops in quadrature:

$$L = \sqrt{V^2 - V_R^2} / (2\pi f I).$$

50. A plane progressive wave is given by $y = 2 \cos 2\pi(330t - x)\text{m}$. The frequency of the wave is :

[4 marks]

- (A) 660 Hz
- (B) 340 Hz
- (C) 330 Hz
- (D) 165 Hz

Correct Answer: (C) 330 Hz

Solution:

Concept:

- A sinusoidal progressive wave can be written as $y = A \cos(\omega t - kx + \phi)$, where A is displacement amplitude, ω is angular frequency and k is wave number.
- Frequency counts complete cycles per second and satisfies $\omega = 2\pi f$. Equivalently, $y = A \cos[2\pi(ft - x/\lambda)]$.

Step 1: Compare the temporal phase At any fixed position, the given wave has phase $2\pi(330t - x)$. The coefficient of time in the entire phase is therefore $\omega = 2\pi \times 330 \text{ rad s}^{-1}$. The number outside the cosine, 2, specifies the displacement amplitude in metres; it does not multiply the frequency.

Step 2: Convert angular frequency into frequency The phase must advance by 2π radians for one complete oscillation. Thus

$$f = \frac{\omega}{2\pi} = \frac{2\pi \times 330}{2\pi} = 330 \text{ Hz}.$$

The corresponding period is $T = 1/330 \text{ s}$, so the wave repeats 330 times each second at a fixed position.

Step 3: Check the listed alternatives Option (A), 660 Hz, incorrectly doubles the temporal coefficient, as if the amplitude 2 affected the rate of oscillation. Option (D), 165 Hz, incorrectly halves that coefficient. Option (B), 340 Hz, does not follow from the phase at all. The spatial term fixes wavelength, and must not be substituted for the time coefficient when finding frequency.

Step 4: Conclusion The frequency is 330 Hz . Hence, the correct option is (C).

Quick Tip to Solve: When the phase is already written as $2\pi(\dots)$, read the coefficient of time inside the bracket directly as frequency in hertz.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. An alternating emf $E = 110\sqrt{2}\sin 100t$ volt is applied to a capacitor of $2\mu\text{F}$, the rms value of current in the circuit is _____ mA. [4 marks]

Solution:

Concept:

- For a capacitor, the instantaneous current is $i = C dE/dt$; a sinusoidal voltage therefore produces a sinusoidal current leading the voltage by $\pi/2$.
- For an ideal capacitor, $X_C = 1/(\omega C)$ and $I_{\text{rms}} = E_{\text{rms}}/X_C = \omega C E_{\text{rms}}$. Peak and rms values differ by $\sqrt{2}$.

Step 1: Read the voltage and angular frequency Compare $E = 110\sqrt{2}\sin 100t$ with $E = E_0 \sin \omega t$. The peak voltage is $E_0 = 110\sqrt{2}$ V, so $E_{\text{rms}} = 110$ V. The angular frequency is 100 rad s^{-1} ; it is not the frequency in hertz. Also, $C = 2 \times 10^{-6}$ F.

Step 2: Calculate the capacitive reactance The opposition offered by the capacitor at this angular frequency is

$$X_C = \frac{1}{100(2 \times 10^{-6})} = 5000 \Omega.$$

This reactance relates rms voltage directly to rms current. An ideal capacitor has a phase difference between these quantities, but their magnitudes still satisfy this relation.

Step 3: Find and convert the rms current Using the rms voltage gives

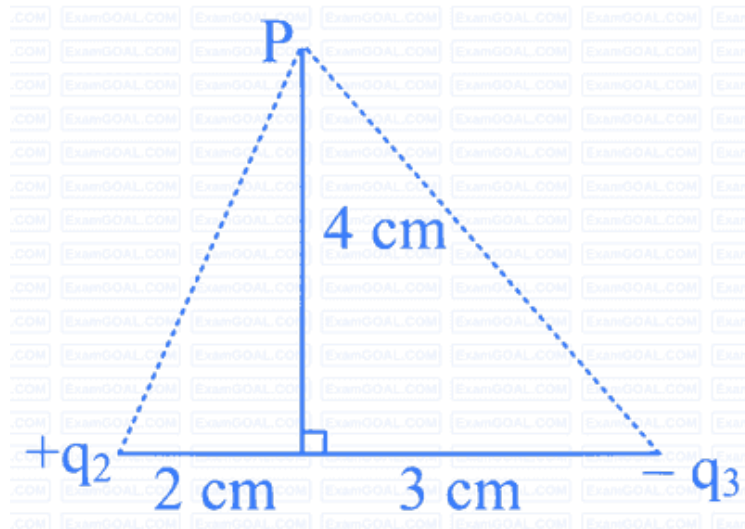
$$I_{\text{rms}} = \frac{110}{5000} = 0.022 \text{ A} = 22 \text{ mA}.$$

As a check, differentiating the applied voltage gives $i = 0.022\sqrt{2}\cos 100t$ A. Its peak is $0.022\sqrt{2}$ A, whose rms value is again 0.022 A. Thus no extra factor of $\sqrt{2}$ remains in the requested value.

Step 4: Conclusion The rms current is 22 mA, so the numerical entry is 22.

Quick Tip to Solve: Convert the voltage amplitude to rms once, then use $I_{\text{rms}} = \omega C E_{\text{rms}}$; convert microfarads to farads before multiplying.

52. If the net electric field at point P along Y axis is zero, then the ratio of $\left| \frac{q_2}{q_3} \right|$ is $\frac{8}{5\sqrt{x}}$, where $x =$ _____ . [4 marks]



Solution:

Concept:

- A point charge produces a field of magnitude $k|q|/r^2$, along the line joining the charge and the observation point; $k = 1/(4\pi\epsilon_0)$.
- If the perpendicular height of the observation point above a charge is h , its vertical field component has magnitude $k|q|h/r^3$. Only the vertical components are required to cancel here.

Step 1: Find both charge-to-point distances The figure puts P 4 cm above the horizontal line containing the charges. The horizontal separations from the foot of this perpendicular are 2 cm for $+q_2$ and 3 cm for $-q_3$. Pythagoras gives

$$r_2 = \sqrt{2^2 + 4^2} = \sqrt{20} \text{ cm}, \quad r_3 = \sqrt{3^2 + 4^2} = 5 \text{ cm}.$$

Distances may remain in the same unit because only their ratio will enter.

Step 2: Cancel the vertical components The positive charge produces an upward component at P, whereas the negative charge produces a downward component. Both projections use the common vertical height $h = 4$ cm. Therefore the condition is

$$\frac{k|q_2|h}{r_2^3} = \frac{k|q_3|h}{r_3^3}.$$

The horizontal components need not cancel; the question specifies zero field along the Y axis only.

Step 3: Match the required ratio Cancel the common factors and simplify:

$$\left| \frac{q_2}{q_3} \right| = \left(\frac{\sqrt{20}}{5} \right)^3 = \frac{40\sqrt{5}}{125} = \frac{8}{5\sqrt{5}}.$$

Comparing with $8/(5\sqrt{x})$ gives $\sqrt{x} = \sqrt{5}$. Since the square root is positive, the required value is

uniquely $x = 5$.

Step 4: Conclusion $x = 5$, a dimensionless numerical value.

Quick Tip to Solve: When cancelling one field component at a common perpendicular height, write $E_y = kqh/r^3$; this avoids losing the projection factor.

53. Two slits are 1 mm apart and the screen is located 1 m away from the slits. A light of wavelength 500 nm is used. The width of each slit to obtain 10 maxima of the double slit pattern within the central maximum of the single slit pattern is _____ $\times 10^{-4}$ m. [4 marks]

Solution:

Concept:

- Two slits separated by d produce an interference fringe width $\beta = \lambda D/d$, where D is the distance to the screen.
- For individual slit width a , the central single-slit diffraction envelope has width $W = 2\lambda D/a$. The stated number of interference maxima is obtained using the width ratio W/β .

Step 1: Calculate the interference fringe width Convert the given dimensions to SI units: $d = 10^{-3}$ m, $D = 1$ m and $\lambda = 500 \times 10^{-9}$ m. Hence

$$\beta = \frac{500 \times 10^{-9} \times 1}{10^{-3}} = 5 \times 10^{-4} \text{ m.}$$

This is the separation between successive bright fringes in the small-angle approximation.

Step 2: Relate the diffraction width to the fringe width The first minima of the single-slit pattern lie at distances $\lambda D/a$ on either side of the centre. Thus the full width of its central maximum is $2\lambda D/a$. For ten fringe widths across this envelope,

$$\frac{W}{\beta} = \frac{2\lambda D/a}{\lambda D/d} = \frac{2d}{a} = 10.$$

The wavelength and screen distance cancel because both effects use the same light and screen.

Step 3: Determine the width of each slit Rearrange the ratio to obtain

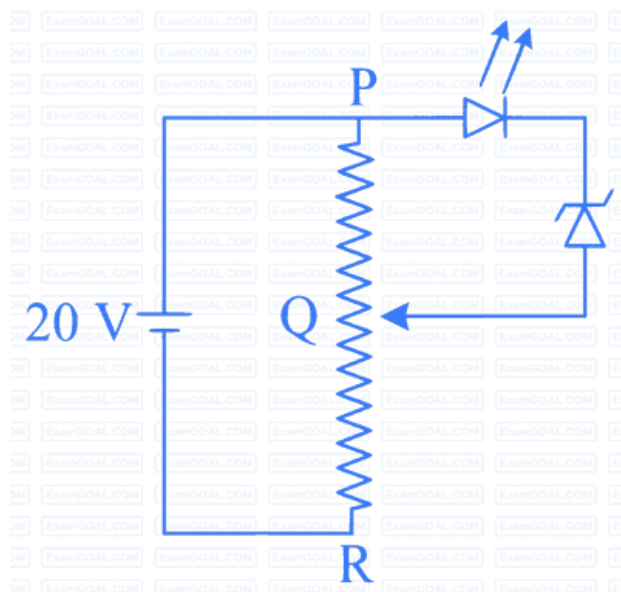
$$a = \frac{2d}{10} = \frac{2 \times 10^{-3}}{10} = 2 \times 10^{-4} \text{ m.}$$

As a scale check, this makes $W = 5 \times 10^{-3}$ m, ten times the calculated fringe width. This standard width-ratio calculation gives the required numerical coefficient in the units specified by the question.

Step 4: Conclusion The slit width is 2×10^{-4} m, so the numerical entry is 2 .

Quick Tip to Solve: Use the full central diffraction width $2\lambda D/a$, rather than its half-width; dividing by $\lambda D/d$ immediately gives $2d/a$.

54. A potential divider circuit is connected with a dc source of 20 V, a light emitting diode of glow in voltage 1.8 V and a zener diode of breakdown voltage of 3.2 V. The length (PR) of the resistive wire is 20 cm. The minimum length of PQ to just glow the LED is _____ cm. [4 marks]



Solution:

Concept:

- A uniform resistive wire connected across a fixed voltage has a uniform potential gradient when the output current is negligible at the turn-on threshold.
- In the indicated series branch the LED is forward biased and the zener is reverse biased. To just light the LED, the applied branch voltage must reach the sum of their threshold voltages.

Step 1: Identify the voltage needed across the diode branch The branch connects P to the slider Q. Current from P passes through the forward-biased LED and then through the reverse-biased zener toward Q. The LED needs 1.8 V, while the zener must reach its breakdown voltage 3.2 V. Thus

$$V_{PQ,\min} = 1.8 + 3.2 = 5.0 \text{ V.}$$

Using either diode voltage alone would leave the other series device below its required operating threshold.

Step 2: Find the potential gradient along the wire The complete uniform wire PR has length 20 cm and is connected across 20 V. At the instant the diode branch just begins conducting, its loading tends to zero, giving

$$\frac{V_{PR}}{PR} = \frac{20}{20} = 1 \text{ V cm}^{-1}.$$

The relevant voltage is the drop from P to Q, since those are the terminals of the LED-zener branch.

Step 3: Calculate the minimum slider length Voltage drop is proportional to wire length, so

$$PQ_{\min} = \frac{V_{PQ,\min}}{1 \text{ V cm}^{-1}} = 5 \text{ cm.}$$

This position uses one quarter of the full wire and therefore supplies one quarter of 20 V. For a shorter PQ the total drop is less than 5 V and cannot simultaneously satisfy both diode thresholds.

Step 4: Conclusion The minimum length is $PQ = 5 \text{ cm}$, so the numerical entry is $\boxed{5}$.

Quick Tip to Solve: First add the series LED forward threshold and zener breakdown voltage, then convert that voltage to a length using the divider gradient.

55. An object of mass 0.2 kg executes simple harmonic motion along x axis with frequency of $(\frac{25}{\pi})$ Hz. At the position $x = 0.04 \text{ m}$ the object has kinetic energy 0.5 J and potential energy 0.4 J. The amplitude of oscillation is _____ cm. [4 marks]

Solution:

Concept:

- For SHM, angular frequency and frequency satisfy $\omega = 2\pi f$, and the effective spring constant is $k = m\omega^2$.
- The total mechanical energy is constant: $E = K + U = \frac{1}{2}m\omega^2 A^2$. Potential energy at displacement x is $U = \frac{1}{2}m\omega^2 x^2$.

Step 1: Calculate angular frequency and total energy From $f = 25/\pi \text{ Hz}$,

$$\omega = 2\pi \frac{25}{\pi} = 50 \text{ rad s}^{-1}.$$

At the stated position the kinetic and potential energies add to $E = 0.5 + 0.4 = 0.9 \text{ J}$. The amplitude must be calculated from this total, because kinetic energy is nonzero at the specified displacement.

Step 2: Determine the amplitude Using $m = 0.2 \text{ kg}$ in the total-energy formula gives

$$0.9 = \frac{1}{2}(0.2)(50)^2 A^2 = 250 A^2.$$

Therefore

$$A^2 = \frac{0.9}{250} = 0.0036 \text{ m}^2, \quad A = 0.06 \text{ m}.$$

Take the positive square root because amplitude is a nonnegative maximum displacement.

Step 3: Check against the supplied potential energy At $x = 0.04 \text{ m}$, the potential energy predicted by these parameters is

$$U = 250(0.04)^2 = 0.4 \text{ J},$$

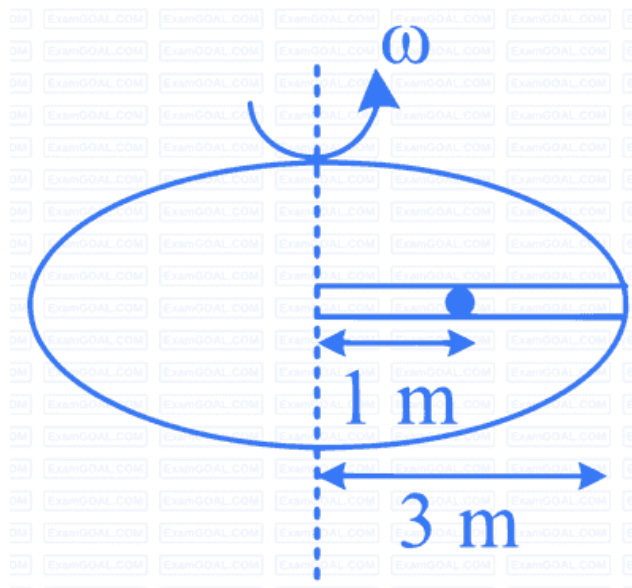
exactly as given. The remaining energy is $0.9 - 0.4 = 0.5 \text{ J}$, also consistent. The amplitude exceeds the current displacement, as it must when the particle is moving. Finally, $0.06 \text{ m} = 6 \text{ cm}$.

Step 4: Conclusion The amplitude is $\boxed{6 \text{ cm}}$, so the numerical entry is $\boxed{6}$.

Quick Tip to Solve: Add kinetic and potential energies before finding amplitude. As an alternative check, use $A/x = \sqrt{(K + U)/U}$ when the potential-energy zero is at equilibrium.

56. A circular table is rotating with an angular velocity of ω rad/s about its axis (see figure). There is a smooth groove along a radial direction on the table. A steel ball is gently placed at a distance of 1 m on the groove. All the surfaces are smooth. If the radius of the table is 3 m, the radial velocity of the ball w.r.t. the table at the time ball leaves the table is $x\sqrt{2}\omega$ m/s, where the value of x is _____.

[4 marks]



Solution:

Concept:

- The groove rotates at the stated constant angular speed ω . In the inertial frame, radial acceleration for a particle with $\dot{\theta} = \omega$ is $\ddot{r} - r\omega^2$.
- The smooth groove exerts no force along its radial direction, giving $\ddot{r} = \omega^2 r$. Radial speed relative to the table is $v_r = \dot{r}$.

Step 1: Set the radial equation and initial condition The ball is gently placed at $r_0 = 1$ m, so its initial radial velocity relative to the table is zero. The groove constrains its angle to rotate with the table, but its contact force acts perpendicular to the groove. With no radial real force,

$$m(\ddot{r} - r\omega^2) = 0.$$

Thus the ball accelerates outward as its radius increases; its radial speed cannot be found by assuming conserved angular momentum of the ball.

Step 2: Integrate with respect to the radial position Write $\ddot{r} = v_r dv_r/dr$ and integrate from r_0 to r :

$$\int_0^{v_r} v_r dv_r = \omega^2 \int_{r_0}^r r dr.$$

This gives

$$v_r^2 = \omega^2(r^2 - r_0^2).$$

The mass cancels. The groove can do work through its sideways force as it rotates, so ordinary conservation of the ball's inertial kinetic energy is not the appropriate shortcut.

Step 3: Evaluate the speed at the table edge The ball leaves the groove at the outer radius $r = 3$ m. Hence

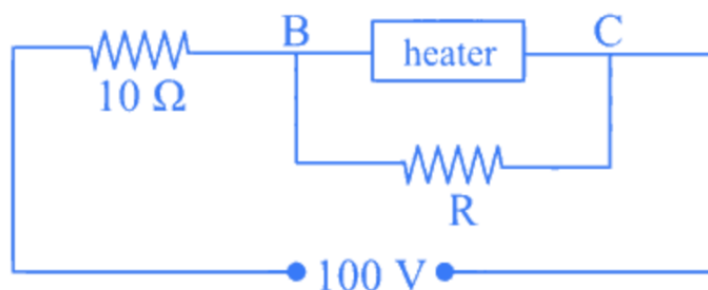
$$v_r = \omega \sqrt{3^2 - 1^2} = \sqrt{8} \omega \text{ m s}^{-1} = 2\sqrt{2} \omega \text{ m s}^{-1}.$$

This is only the radial velocity relative to the table; the inertial velocity also includes a transverse component. Matching the specified form $x\sqrt{2}\omega \text{ m s}^{-1}$ determines the required coefficient.

Step 4: Conclusion $x = 2$, a dimensionless coefficient in the radial speed $2\sqrt{2}\omega \text{ m s}^{-1}$.

Quick Tip to Solve: For a smooth radial groove rotating at fixed angular speed, integrate $\ddot{r} = \omega^2 r$ to get $v_r^2 = \omega^2(r^2 - r_0^2)$; do not conserve the ball's angular momentum.

57. A heater is designed to operate with a power of 1000 W in a 100 V line. It is connected in combination with a resistance of 10Ω and a resistance R , to a 100 V mains as shown in figure. For the heater to operate at 62.5 W, the value of R should be _____ Ω . [4 marks]



Solution:

Concept:

- Treat the heater as an ohmic resistance whose value is determined by its rated power and voltage: $R_h = V_0^2/P_0$. The same resistance is used when its operating power changes.
- The heater and R are connected in parallel between B and C, so their potential differences are equal. This parallel combination is in series with the separate 10Ω resistance.

Step 1: Find the resistance of the heater The rating gives the resistance of the heating element, not the power it must consume in every circuit. Using the rated values,

$$R_h = \frac{(100)^2}{1000} = 10\Omega.$$

This resistance remains the value to use in the new circuit. The separate resistor on the left is also 10Ω , but it is a distinct component and must not be combined in series with the heater alone.

Step 2: Determine the required parallel-branch voltage The desired power in the heater is 62.5 W. Since its resistance is known, its potential difference must satisfy

$$V_{BC} = \sqrt{P_h R_h} = \sqrt{62.5 \times 10} = 25\text{ V}.$$

The resistor R has this same voltage because it is connected across the heater. The heater current is

therefore

$$I_h = \frac{25}{10} = 2.5 \text{ A.}$$

Step 3: Use the total current to find the unknown branch The series resistor has the remaining voltage, $100 - 25 = 75 \text{ V}$, across it. Its current, which is the total current supplied to the parallel branches, is

$$I = \frac{75}{10} = 7.5 \text{ A.}$$

Kirchhoff's junction rule at B gives

$$I_R = I - I_h = 7.5 - 2.5 = 5 \text{ A}, \quad R = \frac{V_{BC}}{I_R} = \frac{25}{5} = 5 \Omega.$$

As a check, 10Ω in parallel with 5Ω gives $10/3 \Omega$; the voltage divider with the series resistor then gives exactly 25 V .

Step 4: Conclusion The required resistance is $R = 5 \Omega$. The numerical entry is $\boxed{5}$.

Quick Tip to Solve: Find the heater voltage from its desired power first; subtract it from the supply voltage before calculating the series current.

58. Small water droplets of radius 0.01 mm are formed in the upper atmosphere and falling with a terminal velocity of 10 cm/s . Due to condensation, if 8 such droplets are coalesced and formed a larger drop, the new terminal velocity will be _____ cm/s . [4 marks]

Solution:

Concept:

- For a small sphere falling through a viscous fluid in the Stokes regime, the terminal speed is $v_t = 2r^2(\rho_d - \rho_f)g/(9\eta)$, where r is its radius and η is the fluid viscosity.
- Coalescence conserves the total liquid volume. For the same drop material and surrounding fluid, terminal speed varies as the square of the drop radius, not as its volume.

Step 1: Find the radius after coalescence Let each original drop have radius r , and let the combined drop have radius R . Equating the volume of eight equal spherical drops to the volume of the new sphere gives

$$8 \left(\frac{4}{3} \pi r^3 \right) = \frac{4}{3} \pi R^3.$$

Thus $R^3 = 8r^3$, so

$$R = 2r = 2(0.01 \text{ mm}) = 0.02 \text{ mm}.$$

The radius doubles; it does not become eight times its original value.

Step 2: Compare the terminal speeds At terminal speed, the downward weight minus buoyancy is balanced by the viscous drag $6\pi\eta r v_t$. Weight minus buoyancy is proportional to r^3 , so dividing by the drag's radius factor yields $v_t \propto r^2$. The density difference, gravitational acceleration and viscosity are

common to both drops, so they cancel in the ratio:

$$\frac{V_t}{v_t} = \left(\frac{R}{r}\right)^2 = 2^2 = 4.$$

This comparison uses the terminal speeds after each drop has settled into steady motion, rather than the transient speed immediately after merging.

Step 3: Insert the given original speed The original terminal speed is 10 cm s^{-1} . Therefore

$$V_t = 4 \times 10 = 40 \text{ cm s}^{-1}.$$

Keeping the speed in centimetres per second directly gives the unit requested by the question. Equivalently, the new speed is 0.40 m s^{-1} , which converts back to the same result. The absolute radius is useful for identifying the size change, but its numerical value cancels from the speed ratio.

Step 4: Conclusion The new terminal velocity is 40 cm s^{-1} , directed downward. The numerical entry is 40 .

Quick Tip to Solve: When n equal small drops merge, use $R/r = n^{1/3}$ and then $V_t/v_t = n^{2/3}$; do not multiply the speed directly by n .

59. The coercivity of a magnet is $5 \times 10^3 \text{ A/m}$. The amount of current required to be passed in a solenoid of length 30 cm and the number of turns 150, so that the magnet gets demagnetised when inside the solenoid is _____ A. [4 marks]

Solution:

Concept:

- Coercivity H_c is the magnitude of the reverse magnetising field strength required to reduce the residual magnetisation to zero.
- For an ideal solenoid, the magnetising field strength is $H = nI = (N/L)I$, where N is its number of turns, L its length in metres and I its current. The unit of H is A m^{-1} .

Step 1: Identify the reverse field required The magnet has coercivity

$$H_c = 5 \times 10^3 \text{ A m}^{-1}.$$

To demagnetise it, the solenoid must provide a field of this magnitude in the direction opposite to the original magnetisation. Coercivity is a field strength H , rather than a magnetic flux density B , so it should be compared directly with nI . No factor of μ_0 is needed in this comparison.

Step 2: Convert the solenoid data to turn density The solenoid has $N = 150$ turns and length

$$L = 30 \text{ cm} = 0.30 \text{ m}.$$

Its number of turns per metre is

$$n = \frac{N}{L} = \frac{150}{0.30} = 500 \text{ m}^{-1}.$$

Converting centimetres to metres is essential because the given coercivity is measured in amperes per metre. Using the number 30 as a length in metres would give a current larger by a factor of one hundred.

Step 3: Equate the applied field to the coercivity Set the magnitude of the solenoid's reverse field equal to the coercive field:

$$nI = H_c, \quad I = \frac{H_c}{n} = \frac{5 \times 10^3}{500} = 10 \text{ A}.$$

A direct check gives $(150/0.30) \times 10 = 5000 \text{ A m}^{-1}$, exactly the specified coercivity. The direction of the current must be chosen by the solenoid right-hand rule to oppose the magnet's initial magnetisation; changing that direction changes whether the field demagnetises or reinforces it.

Step 4: Conclusion The required current magnitude is $\boxed{10 \text{ A}}$, with its direction chosen to produce the reverse field. The numerical entry is $\boxed{10}$.

Quick Tip to Solve: Coercivity is given as H in amperes per metre. Use $H = NI/L$ directly and convert the solenoid length to metres.

60. A body of mass M thrown horizontally with velocity v from the top of the tower of height H touches the ground at a distance of 100 m from the foot of the tower. A body of mass $2M$ thrown at a velocity $\frac{v}{2}$ from the top of the tower of height $4H$ will touch the ground at a distance of _____ m. [4 marks]

Solution:

Concept:

- For horizontal projection from height h , the initial vertical velocity is zero and the flight time is $t = \sqrt{2h/g}$. Horizontal motion remains uniform with range $x = u_x t$.
- Neglecting air resistance, gravitational acceleration is independent of mass. Doubling the projectile mass therefore changes neither its fall time nor its range when height and velocity are otherwise fixed.

Step 1: Express the range of the first projectile For the first body, vertical displacement from the tower top to the ground is H . Its horizontal launch gives zero initial vertical speed, so

$$H = \frac{1}{2}gt_1^2, \quad t_1 = \sqrt{\frac{2H}{g}}.$$

Its horizontal speed is v throughout the flight. The stated horizontal distance is consequently

$$x_1 = vt_1 = v\sqrt{\frac{2H}{g}} = 100 \text{ m}.$$

There is no need to determine v or H individually, because only a comparison with the second throw is required.

Step 2: Compare the new time of flight The second body is projected horizontally from height $4H$, so its time of fall is

$$t_2 = \sqrt{\frac{2(4H)}{g}} = 2\sqrt{\frac{2H}{g}} = 2t_1.$$

Although the height is four times larger, the fall time is only twice as large because height is proportional to the square of time. The mass is $2M$, but mass does not appear in the vertical equation of motion and hence does not change this result.

Step 3: Combine the speed and time ratios The new horizontal speed is $v/2$. Therefore

$$x_2 = \frac{v}{2} t_2 = \frac{v}{2} (2t_1) = vt_1 = x_1.$$

Equivalently,

$$\frac{x_2}{x_1} = \frac{v/2}{v} \sqrt{\frac{4H}{H}} = \frac{1}{2} \times 2 = 1.$$

The decrease in horizontal speed is exactly compensated by the increase in flight time. This cancellation explains why the horizontal distance remains unchanged even though the tower height and body mass differ.

Step 4: Conclusion The second body touches the ground 100 m from the foot of the tower. The numerical entry is 100.

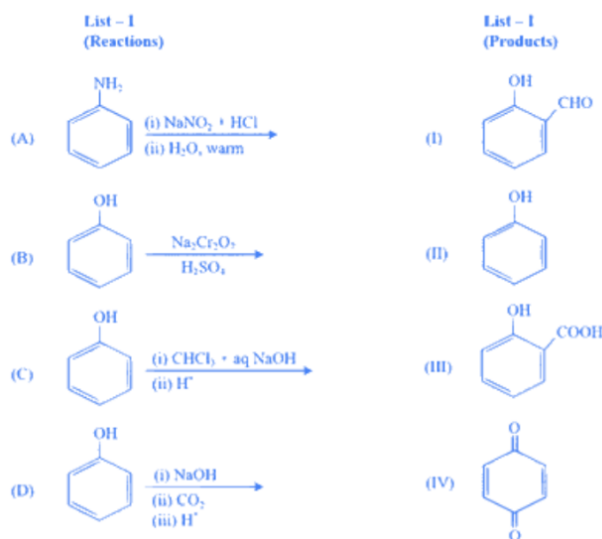
Quick Tip to Solve: For horizontal launches compare $x \propto v\sqrt{H}$; mass cancels, and a fourfold height doubles the time of flight.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Match List I with List II.

Choose the correct answer from the options given below:

[4 marks]



(A) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(B) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

(C) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(D) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

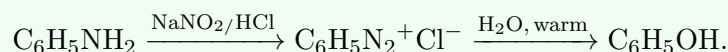
Correct Answer: (A) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

Solution:

Concept:

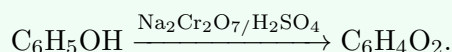
- Match a reaction by identifying the functional-group change produced by its reagents. Aniline undergoes diazotisation followed by hydrolysis to give phenol.
- Phenol is oxidised to a quinone, whereas Reimer–Tiemann formylation and Kolbe–Schmitt carboxylation introduce different carbon-containing groups into its ring.

Step 1: Hydrolyse the diazonium salt in reaction A Aniline reacts with nitrous acid generated from NaNO_2 and HCl to form benzenediazonium chloride. Warming with water replaces the diazonium group by $-\text{OH}$, with nitrogen gas released:



The product is phenol, labelled (II), so (A)-(II). No extra carbon atom is introduced in this conversion.

Step 2: Recognise the oxidation in reaction B Sodium dichromate in sulphuric acid is an oxidising mixture. Phenol is oxidised to the displayed para-benzoquinone, whose ring has two opposite carbonyl groups:



This is product (IV), giving (B)-(IV). It is not a formylation or carboxylation because the reagents do not supply an additional carbon atom to the ring.

Step 3: Identify Reimer–Tiemann formylation in reaction C Chloroform and aqueous sodium hydroxide, followed by acidification, formylate phenol mainly at an ortho position. The phenolic group remains and a neighbouring –CHO group appears. Thus the displayed product is salicylaldehyde, or 2-hydroxybenzaldehyde, labelled (I): (C)-(I). Distinguish its aldehyde group from the –COOH group in product (III).

Step 4: Identify Kolbe–Schmitt carboxylation in reaction D Sodium hydroxide first converts phenol to sodium phenoxide. Reaction with carbon dioxide introduces a carboxylate group mainly ortho to the oxygen substituent; acidification gives salicylic acid, or 2-hydroxybenzoic acid. Its adjacent –OH and –COOH groups identify product (III), so (D)-(III). Only option (A) contains all four matches. Options (B), (C) and (D) already assign an incorrect product to reaction A and cannot satisfy the complete mapping.

Step 5: Conclusion The four products, in reaction order, are phenol (II), para-benzoquinone (IV), salicylaldehyde (I) and salicylic acid (III). Hence, the correct option is (A). This follows from the reactions and products printed in the question; the question-sheet key's different option does not fit this printed reaction table.

Quick Tip to Solve: Separate the two carbon-introducing phenol reactions: chloroform gives an ortho aldehyde, whereas carbon dioxide gives an ortho carboxylic acid after acidification.

62. Given below are two statements :

Statement (I) : A Buffer solution is the mixture of a salt and an acid or a base mixed in any particular quantities

Statement (II) : Blood is naturally occurring buffer solution whose pH is maintained by $\text{H}_2\text{CO}_3/\text{HCO}_3^-$ concentrations.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Both Statement I and Statement II are false
- (B) Both Statement I and Statement II are true
- (C) Statement I is false but Statement II is true
- (D) Statement I is true but Statement II is false

Correct Answer: (C) Statement I is false but Statement II is true

Solution:

Concept:

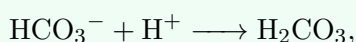
- A buffer resists a substantial pH change when a small amount of acid or base is added. A typical acidic buffer contains a weak acid and a significant amount of its conjugate base; a basic buffer contains a weak base and its conjugate acid.
- The carbonic acid–hydrogen carbonate system is an important physiological buffer. Its acid component consumes added base, while its conjugate-base component consumes added acid.

Step 1: Evaluate Statement I using the buffer requirement Merely mixing a salt with an acid or a base does not guarantee buffering. The components must provide a suitable weak acid–conjugate base pair or weak base–conjugate acid pair in appreciable amounts. For example, CH_3COOH with CH_3COONa is an acidic buffer, whereas an arbitrary strong acid and an unrelated salt do not give such a pair. Arbitrary proportions can also exhaust one buffer component. Statement I therefore gives an overgeneralised description and is false.

Step 2: Evaluate the naturally occurring buffer in Statement II Blood contains the carbonic acid–hydrogen carbonate buffer pair,



The hydrogen carbonate ion can consume added acid,



and carbonic acid can consume added hydroxide,



These reactions limit pH changes. The relative concentrations of the two components influence the pH, consistently with the Henderson–Hasselbalch relationship. Statement II correctly describes this naturally occurring buffer and is true.

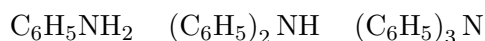
Step 3: Select the option matching the two truth values The separate evaluations give Statement I false and Statement II true. Option (A) incorrectly calls Statement II false; option (B) incorrectly accepts Statement I; and option (D) reverses both truth values. Only option (C) states the pair of truth values obtained from the chemical requirements. The word buffer must therefore be associated with the conjugate pair and its ability to remove small additions of acid or base, rather than with any mixture containing a salt.

Step 4: Conclusion Statement I is false, while Statement II is true. Hence, the correct option is (C).

Quick Tip to Solve: To recognise a buffer, identify its weak component and conjugate partner; the presence of a salt alone is insufficient.

63. Given below are two statements :

Statement (I) : All the following compounds react with p-toluenesulfonyl chloride.



Statement (II) : Their products in the above reaction are soluble in aqueous NaOH.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Statement I is false but Statement II is true
(B) Both Statement I and Statement II are true
(C) Both Statement I and Statement II are false
(D) Statement I is true but Statement II is false

Correct Answer: (C) Both Statement I and Statement II are false

Solution:

Concept:

- Arenesulfonyl chlorides react with primary and secondary amines through replacement of an N-H hydrogen, producing sulfonamides. A tertiary amine has no N-H hydrogen and does not form the corresponding sulfonamide in this test.
- A sulfonamide retaining an N-H bond is acidic enough to dissolve in aqueous alkali by salt formation. A sulfonamide with no N-H bond cannot undergo that deprotonation.

Step 1: Check the reaction of each amine Aniline, $\text{C}_6\text{H}_5\text{NH}_2$, is a primary amine, and diphenylamine, $(\text{C}_6\text{H}_5)_2\text{NH}$, is a secondary amine. Both contain an N-H bond and can form sulfonamide derivatives with p-toluenesulfonyl chloride. Triphenylamine, $(\text{C}_6\text{H}_5)_3\text{N}$, is a tertiary amine and has no N-H bond. It does not form the analogous sulfonamide under the test conditions. Therefore the claim that all three compounds react is false: Statement I is false.

Step 2: Compare the solubility of the sulfonamide products Writing Ts for the p-toluenesulfonyl group, the aniline product has the form $\text{Ts} - \text{NH} - \text{C}_6\text{H}_5$. Its remaining N-H proton can be removed by aqueous NaOH, producing a soluble sulfonamide salt. In contrast, the product from diphenylamine has the form $\text{Ts} - \text{N}(\text{C}_6\text{H}_5)_2$. It has no N-H proton and does not dissolve through this acid-base salt-forming mechanism. Thus even the two reacting amines do not both yield alkali-soluble products. Triphenylamine also supplies no sulfonamide product. Statement II is consequently false.

Step 3: Select the combined verdict and reject the alternatives Both statements are false, which corresponds to option (C). Option (A) wrongly accepts the claim about the solubility of the products. Option (B) wrongly accepts both the universal reaction claim and the universal solubility claim. Option (D) wrongly accepts the reaction of the tertiary amine. Counting the N-H bonds before and after sulfonylation gives a direct check of both conclusions: the primary amine retains one, the secondary amine retains none, and the tertiary amine cannot undergo this substitution.

Step 4: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (C).

Quick Tip to Solve: Count N-H bonds twice: before sulfonylation to test reactivity, and in the sulfonamide product to test solubility in alkali.

64. Match List I with List II.

	List I: Complex ion		List II: Spin-only moment (B.M.)
A.	$[\text{Cr}(\text{NH}_3)_6]^{3+}$	I.	4.90
B.	$[\text{NiCl}_4]^{2-}$	II.	3.87
C.	$[\text{CoF}_6]^{3-}$	III.	0.0
D.	$[\text{Ni}(\text{CN})_4]^{2-}$	IV.	2.83

Choose the correct answer from the options given below:

[4 marks]

(A) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(B) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(C) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

(D) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

Correct Answer: (D) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)**Solution:****Concept:**

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}$ B.M., where n is the number of unpaired electrons. First find the metal oxidation state and its d -electron count.
- Ligand strength and geometry determine electron pairing: tetrahedral chloride complexes are normally high spin, whereas the nickel(II) cyanide complex is square planar and diamagnetic.

Step 1: Find the chromium and tetrahedral nickel moments Ammonia is neutral, so chromium is Cr^{3+} , with configuration $3d^3$. Its three electrons occupy the three lower octahedral orbitals singly; there is no choice of high-spin versus low-spin arrangement for this count. Therefore

$$\mu_A = \sqrt{3(3+2)} = \sqrt{15} = 3.87 \text{ B.M.}$$

Thus A matches II. In $[\text{NiCl}_4]^{2-}$, nickel is +2, hence $3d^8$. The weak-field tetrahedral arrangement leaves two unpaired electrons:

$$\mu_B = \sqrt{2(2+2)} = \sqrt{8} = 2.83 \text{ B.M.}$$

Thus B matches IV.

Step 2: Find the cobalt and square-planar nickel moments In $[\text{CoF}_6]^{3-}$, six fluoride ions contribute -6, so cobalt is +3, with $3d^6$. Fluoride is a weak-field ligand; the octahedral high-spin distribution is $t_{2g}^4 e_g^2$, with four unpaired electrons. Hence

$$\mu_C = \sqrt{4(4+2)} = \sqrt{24} = 4.90 \text{ B.M.}$$

C matches I. In $[\text{Ni}(\text{CN})_4]^{2-}$, nickel is again $3d^8$, but strong-field cyanide favours square-planar pairing. All electrons are paired, so $n = 0$ and $\mu_D = 0$. Thus D matches III. The other listed combinations each misassign at least one of these four independently determined moments.

Step 3: Conclusion The complete matching is A-II, B-IV, C-I, D-III. Hence, the correct option is (D).

Quick Tip to Solve: Determine the metal oxidation state before counting unpaired electrons; the two nickel(II) complexes have the same electron count but different geometries and moments.

65. Which one the following compounds will readily react with dilute NaOH ?

[4 marks]

- (A) $\text{C}_6\text{H}_5\text{CH}_2\text{OH}$
- (B) $\text{C}_2\text{H}_5\text{OH}$
- (C) $\text{C}_6\text{H}_5\text{OH}$
- (D) $(\text{CH}_3)_3\text{COH}$

Correct Answer: (C) $\text{C}_6\text{H}_5\text{OH}$

Solution:

Concept:

- An acid reacts readily with aqueous sodium hydroxide when hydroxide can remove its proton to form a sufficiently stable conjugate base. The relevant proton here is the hydrogen attached to oxygen.
- Phenol forms a resonance-stabilised phenoxide ion. Ordinary alcohols form alkoxide ions whose negative charge remains localised mainly on oxygen, so dilute aqueous hydroxide does not convert them appreciably into alkoxides.

Step 1: Identify the resonance-stabilised conjugate base In phenol, $\text{C}_6\text{H}_5\text{OH}$, the oxygen atom is directly attached to the aromatic ring. Loss of the hydroxyl proton produces $\text{C}_6\text{H}_5\text{O}^-$. An oxygen lone pair can overlap with the ring π system, spreading the negative charge over oxygen and the ortho and para ring positions. This stabilisation makes phenol substantially more acidic than water and than the other alcohols listed. Thus the acid-base equilibrium with hydroxide lies strongly toward phenoxide and water:



The product is sodium phenoxide, a salt; the aromatic ring itself is not substituted in this reaction.

Step 2: Rule out the three ordinary alcohols In benzyl alcohol, $\text{C}_6\text{H}_5\text{CH}_2\text{OH}$, the intervening saturated CH_2 group separates oxygen from the ring. Its alkoxide therefore lacks the direct phenoxide-type resonance stabilisation. Ethanol, $\text{C}_2\text{H}_5\text{OH}$, likewise forms an ordinary alkoxide. The third alcohol, $(\text{CH}_3)_3\text{COH}$, is tert-butanol; its alkyl groups donate electron density and do not provide a resonance sink for the oxygen charge. None of these three gives the ready acid-base reaction with dilute aqueous NaOH characteristic of phenol. Their ability to react with sodium metal must not be confused with reaction with sodium hydroxide.

Step 3: Conclusion Phenol, $\text{C}_6\text{H}_5\text{OH}$, readily reacts with dilute NaOH to give sodium phenoxide. Hence, the correct option is (C).

Quick Tip to Solve: Check whether the hydroxyl oxygen is directly bonded to the benzene ring: phenol is acidic enough for NaOH, but benzyl alcohol is an ordinary alcohol.

66. The shape of carbocation is :

[4 marks]

- (A) tetrahedral
- (B) diagonal pyramidal
- (C) diagonal
- (D) trigonal planar

Correct Answer: (D) trigonal planar

Solution:

Concept:

- A conventional tricoordinate carbon carbocation has three sigma bonds and an empty orbital. Its positively charged carbon has only six electrons in its valence shell rather than a complete octet.
- Three bonding directions are accommodated by sp^2 hybrid orbitals in one plane. The remaining unhybridised p orbital is perpendicular to this plane and is empty.

Step 1: Count the electron domains at the cationic carbon For a simple carbocation written R_3C^+ , the central carbon forms three sigma bonds to its attached groups. Each bond counts as one electron domain. There is no lone pair on that carbon: the positive charge reflects electron deficiency, not an additional occupied orbital. Consequently there are three occupied bonding directions around the centre. These directions arrange themselves as far apart as possible in a common plane, with ideal bond angles close to 120° . In the usual valence-bond description the carbon uses three sp^2 hybrid orbitals for those bonds.

Step 2: Connect hybridisation to the listed shapes The three sp^2 orbitals point toward the corners of a triangle, so the attached atoms and cationic carbon are trigonal planar. A vacant p orbital projects above and below the plane. This empty orbital also explains why a carbocation accepts an electron pair from a nucleophile and can be stabilised by conjugation or hyperconjugation. Tetrahedral geometry would correspond to four bonding directions, which are absent here. A pyramidal arrangement would require a different occupied-orbital pattern, such as three bonds and a lone pair; that is not the carbocation electron count. The printed alternatives described as diagonal or diagonal pyramidal therefore do not represent the standard three-coordinate carbocation geometry.

Step 3: Conclusion The positively charged carbon is sp^2 -hybridised with an empty perpendicular p orbital; its shape is trigonal planar. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish an empty orbital from a lone pair: a simple carbocation has three bonds and no lone pair, giving a planar centre rather than a pyramidal one.

67. When ψ_A and ψ_B are the wave functions of atomic orbitals, then σ^* is represented by : [4 marks]

- (A) $\psi_A + 2\psi_B$
- (B) $\psi_A - \psi_B$
- (C) $\psi_A - 2\psi_B$
- (D) $\psi_A + \psi_B$

Correct Answer: (B) $\psi_A - \psi_B$

Solution:

Concept:

- In the linear combination of atomic orbitals, compatible atomic wave functions combine with either the same phase or opposite phases. Addition produces enhanced amplitude between the nuclei; subtraction produces cancellation there.
- The star in σ^* denotes an antibonding sigma molecular orbital. A normalisation constant may multiply the linear combination without changing its bonding or antibonding character.

Step 1: Form the antibonding combination For the equivalent atomic orbitals implied by these alternatives, the bonding and antibonding combinations have the forms

$$\psi_{\sigma} = N_+(\psi_A + \psi_B),$$

$$\psi_{\sigma^*} = N_-(\psi_A - \psi_B),$$

where N_+ and N_- are normalisation constants. In the difference combination the lobes facing one another have opposite signs. Their destructive interference gives a node between the nuclei and reduces electron density in the internuclear region. An electron occupying this orbital therefore opposes bonding, which is why the orbital is labelled with a star.

Step 2: Compare the four expressions Ignoring the common normalisation factor, the required expression is $\psi_A - \psi_B$. The sum $\psi_A + \psi_B$ is the symmetric bonding combination, so it cannot describe σ^* . The two alternatives containing an arbitrary factor of 2 are not the equal-magnitude, opposite-phase combination of equivalent orbitals: merely inserting a minus sign is not enough to justify an unequal coefficient. Thus $\psi_A + 2\psi_B$ also has the wrong phase relationship, and $\psi_A - 2\psi_B$ has the wrong relative coefficient for the standard combination being tested. Multiplying the whole difference by -1 would represent the same orbital, but changing only one coefficient does not.

Step 3: Conclusion Up to normalisation, σ^* is represented by $\psi_A - \psi_B$. Hence, the correct option is (B).

Quick Tip to Solve: Read the star as destructive overlap: for equivalent orbitals, the antibonding combination uses equal coefficients with opposite signs.

68. The emf of cell $\text{Tl} \left| \text{Tl}^+_{(0.001\text{M})} \right| \text{Cu}^{2+}_{(0.01\text{M})} \mid \text{Cu}$ is 0.83 V at 298 K. It could be increased by : [4 marks]

- (A) increasing concentration of Tl^+ ions
 (B) increasing concentration of Cu^{2+} ions
 (C) increasing concentration of both Tl^+ and Cu^{2+} ions
 (D) decreasing concentration of both Tl^+ and Cu^{2+} ions

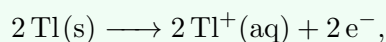
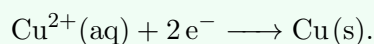
Correct Answer: (B) increasing concentration of Cu^{2+} ions

Solution:

Concept:

- For the cell as written, the left electrode is the oxidation electrode and the right electrode is the reduction electrode. Electrons lost by thallium must balance the two electrons gained by each copper(II) ion.
- At 298 K, the Nernst equation is $E = E^\circ - (0.0591/n) \log Q$, with solid-metal activities equal to one. Lowering the reaction quotient Q increases the cell emf.

Step 1: Write the balanced cell reaction and quotient The electrode reactions are



Adding them gives $2 \text{Tl(s)} + \text{Cu}^{2+}(\text{aq}) \longrightarrow 2 \text{Tl}^+(\text{aq}) + \text{Cu(s)}$. Therefore $n = 2$, and, using the dilute-solution concentration approximation,

$$Q = \frac{[\text{Tl}^+]^2}{[\text{Cu}^{2+}]}, \quad E = E^\circ - \frac{0.0591}{2} \log Q.$$

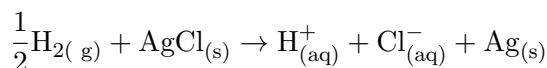
For the given concentrations, $Q = (0.001)^2/0.01 = 10^{-4}$. The numerical emf of 0.83 V fixes the starting point, but the direction of change follows directly from this quotient.

Step 2: Choose the definite way to raise the emf Increasing $[\text{Cu}^{2+}]$ while retaining the thallium concentration increases the denominator and lowers Q . Its logarithm becomes more negative, so E rises. Increasing $[\text{Tl}^+]$ alone instead raises the numerator and lowers E , eliminating option (A). Changing both concentrations, as in (C) or (D), is not specified quantitatively: if the thallium and copper concentrations change by factors a and b , then $Q'/Q = a^2/b$. Merely saying both increase or both decrease does not determine whether this ratio is below one. Those statements therefore do not provide a definite increase without specifying their relative changes. In particular, equal proportional dilution would increase the emf, but that additional condition is not stated in (D).

Step 3: Conclusion Increasing only the concentration of Cu^{2+} definitely lowers Q and raises the cell emf above 0.83 V. Hence, the correct option is (B).

Quick Tip to Solve: Balance electrons before writing the reaction quotient: the thallium-ion concentration is squared, so simultaneous concentration changes require their relative factors.

69. The reaction;



occurs in which of the following galvanic cell :

[4 marks]

- (A) $\text{Pt} | \text{H}_{2(\text{g})} | \text{HCl}_{(\text{soln.})} | \text{AgNO}_{3(\text{aq})} | \text{Ag}$
(B) $\text{Ag} | \text{AgCl}_{(\text{s})} | \text{KCl}_{(\text{soln.})} | \text{AgNO}_{3(\text{aq.})} | \text{Ag}$
(C) $\text{Pt} | \text{H}_{2(\text{g})} | \text{KCl}_{(\text{soln.})} | \text{AgCl}_{(\text{s})} | \text{Ag}$
(D) $\text{Pt} | \text{H}_{2(\text{g})} | \text{HCl}_{(\text{soln.})} | \text{AgCl}_{(\text{s})} | \text{Ag}$

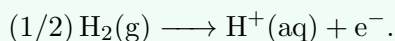
Correct Answer: (D) $\text{Pt} | \text{H}_{2(\text{g})} | \text{HCl}_{(\text{soln.})} | \text{AgCl}_{(\text{s})} | \text{Ag}$

Solution:

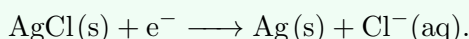
Concept:

- A galvanic cell is written with oxidation at the left and reduction at the right. The overall reaction is obtained by adding the two balanced half-reactions after cancelling electrons.
- An inert platinum conductor is required for the hydrogen electrode because the reacting hydrogen gas and hydrogen ions do not themselves supply a metallic terminal. The silver-silver chloride electrode contains both solid silver and solid silver chloride in contact with chloride ions.

Step 1: Separate the reaction into electrode processes The hydrogen in the stated overall reaction is oxidised:

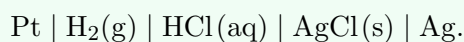


This is the left-hand hydrogen electrode and requires a platinum surface in contact with hydrogen gas and a solution containing hydrogen ions. The reduction process at the right is



On adding the two equations, one electron cancels and the given reaction is recovered exactly. Notice that AgCl , rather than dissolved Ag^{+} alone, is consumed. Both this solid phase and the chloride-containing electrolyte must appear in the appropriate electrode arrangement.

Step 2: Assemble and check the cell notation Hydrochloric acid supplies the H^{+} required by the hydrogen electrode and the Cl^{-} required by the silver-silver chloride electrode. The arrangement is consequently



Option (A) uses a silver nitrate electrode without the required silver chloride solid, leading to a different reduction reaction. Option (B) does not contain the hydrogen electrode and therefore cannot oxidise hydrogen as demanded. Option (C) places the hydrogen electrode in potassium chloride rather than the specified acidic electrolyte supplying hydrogen ions. Option (D) contains the hydrogen gas, acidic chloride solution, silver chloride phase and silver terminal in the correct sequence.

Step 3: Conclusion The required galvanic cell is $\text{Pt} | \text{H}_{2(\text{g})} | \text{HCl}_{(\text{aq})} | \text{AgCl}_{(\text{s})} | \text{Ag}$. Hence, the correct option is (D).

Quick Tip to Solve: Write and add the two half-reactions before matching cell notation; the solid AgCl in the overall reaction identifies a silver-silver chloride electrode.

70. The correct sequence of acidic strength of the following aliphatic acids in their decreasing order is:



[4 marks]

(A) $\text{HCOOH} > \text{CH}_3\text{CH}_2\text{CH}_2\text{COOH} > \text{CH}_3\text{CH}_2\text{COOH} > \text{CH}_3\text{COOH}$

(B) $\text{HCOOH} > \text{CH}_3\text{COOH} > \text{CH}_3\text{CH}_2\text{COOH} > \text{CH}_3\text{CH}_2\text{CH}_2\text{COOH}$

(C) $\text{CH}_3\text{CH}_2\text{CH}_2\text{COOH} > \text{CH}_3\text{CH}_2\text{COOH} > \text{CH}_3\text{COOH} > \text{HCOOH}$

(D) $\text{CH}_3\text{COOH} > \text{CH}_3\text{CH}_2\text{COOH} > \text{CH}_3\text{CH}_2\text{CH}_2\text{COOH} > \text{HCOOH}$

Correct Answer: (B) $\text{HCOOH} > \text{CH}_3\text{COOH} > \text{CH}_3\text{CH}_2\text{COOH} > \text{CH}_3\text{CH}_2\text{CH}_2\text{COOH}$

Solution:

Concept:

- Carboxylic acids ionise by $\text{RCOOH} \rightleftharpoons \text{RCOO}^- + \text{H}^+$. A factor stabilising the carboxylate conjugate base favours ionisation and increases acidity.
- Alkyl groups have an electron-releasing inductive effect. In the standard comparison of these simple acids, increasing electron donation toward the carboxyl group decreases acidic strength; formic acid has no electron-releasing alkyl group.

Step 1: Place formic acid at the strongest end All four compounds contain the same carboxyl functional group, so compare their substituents rather than the mere presence of an acidic hydrogen. In HCOOH , hydrogen is attached to the carboxyl carbon. The other three acids have alkyl groups in that position. Their electron donation makes formation of the negatively charged carboxylate less favourable relative to formic acid. Formic acid is therefore the strongest of the four. This immediately rules out options (C) and (D), since each puts HCOOH last instead of first.

Step 2: Apply the expected alkyl-group trend The school-level inductive-effect order used for this series compares methyl, ethyl and normal propyl groups as increasingly electron releasing. Acetic acid has a methyl group, propionic acid an ethyl group, and butanoic acid a propyl group. The corresponding decreasing acidic-strength sequence is



Option (A) reverses the expected order of the three alkyl-substituted acids, placing butanoic acid above acetic acid, so it does not follow the stated inductive comparison. Option (B) has both formic acid at the strongest end and the correct order for the alkyl-substituted members. This question tests the qualitative substituent-effect trend rather than a comparison requiring experimentally specified solvent-dependent dissociation constants.

Step 3: Conclusion The expected decreasing order is $\text{HCOOH} > \text{CH}_3\text{COOH} > \text{CH}_3\text{CH}_2\text{COOH} > \text{CH}_3\text{CH}_2\text{CH}_2\text{COOH}$. Hence, the correct option is (B).

Quick Tip to Solve: First put formic acid ahead of alkyl carboxylic acids; then use increasing alkyl electron donation to select the expected qualitative order.

71. For a reaction $A \xrightarrow{K_1} B \xrightarrow{K_2} C$ If the rate of formation of B is set to be zero then the concentration of B is given by : [4 marks]

- (A) $(K_1 - K_2)[A]$
(B) $(K_1 + K_2)[A]$
(C) $K_1 K_2 [A]$
(D) $(K_1/K_2)[A]$

Correct Answer: (D) $(K_1/K_2)[A]$

Solution:

Concept:

- For consecutive first-order reactions $A \xrightarrow{K_1} B \xrightarrow{K_2} C$, the intermediate is produced in the first step and consumed in the second step.
- The net rate of change of an intermediate is its rate of formation minus its rate of consumption. Here these rates are $K_1[A]$ and $K_2[B]$, respectively.

Step 1: Write the net rate for the intermediate The first elementary step converts A into B, so it contributes positively to the concentration of B. The second step removes B to form C, so it contributes negatively. Therefore,

$$\frac{d[B]}{dt} = K_1[A] - K_2[B].$$

The phrase that the rate of formation of B is set to zero refers to its net accumulation, not to stopping the first reaction. B can continue to be formed while an equal amount is consumed.

Step 2: Apply the zero-accumulation condition Set the derivative equal to zero and rearrange:

$$0 = K_1[A] - K_2[B], \quad K_2[B] = K_1[A].$$

For the nonzero rate constant K_2 , division gives

$$[B] = \frac{K_1}{K_2}[A].$$

The larger the removal constant, the smaller the concentration of intermediate needed to balance a given production rate. This is the steady-state relation at the specified condition, rather than a statement that the individual reaction rates vanish.

Step 3: Check the alternatives dimensionally Both first-order rate constants have dimensions of inverse time. Their ratio is dimensionless, so multiplying it by $[A]$ gives a concentration. In contrast, the difference or sum of the constants times $[A]$ has dimensions of concentration per time; their product times $[A]$ has concentration per time squared. Thus options (A), (B) and (C) cannot represent $[B]$.

Step 4: Conclusion The concentration is $[B] = (K_1/K_2)[A]$. Hence, the correct option is (D).

Quick Tip to Solve: For an intermediate at zero net accumulation, equate its production and removal rates before solving for its concentration.

72. The equilibrium $\text{Cr}_2\text{O}_7^{2-} \rightleftharpoons 2\text{CrO}_4^{2-}$ is shifted to the right in :

[4 marks]

- (A) a weakly acidic medium
- (B) a basic medium
- (C) a neutral medium
- (D) an acidic medium

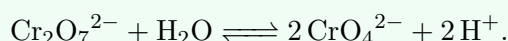
Correct Answer: (B) a basic medium

Solution:

Concept:

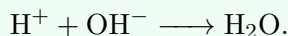
- Dichromate and chromate ions are interconverted by an acid-base equilibrium. The complete balanced equilibrium contains water and hydrogen ions, which determine the direction of its pH response.
- Le Chatelier's principle predicts that removal of a product shifts an equilibrium towards that product. Hydroxide ions remove hydrogen ions by forming water.

Step 1: Restore the acid-base species in the equilibrium The abbreviated equilibrium in the question compares the two chromium(VI) oxyanions. Its balanced aqueous form is



Atoms and charge balance on both sides: two chromium atoms, eight oxygen atoms and two hydrogen atoms are retained, and the net charge is minus two. Hydrogen ions occur on the right, so their concentration is essential even though the abbreviated expression omits them.

Step 2: Examine the effect of a basic medium A base supplies hydroxide ions, which neutralise the hydrogen ions:



Removal of H^+ makes the system generate more of it. The equilibrium therefore moves to the right, converting orange dichromate into yellow chromate. Combining the two processes also gives



This form directly shows hydroxide being consumed when chromate is produced.

Step 3: Reject the other media An acidic medium raises the hydrogen-ion concentration, favouring the reverse reaction and hence dichromate; that excludes (D). A weakly acidic medium is still not the medium that drives the reaction right by removing hydrogen ions, so (A) is unsuitable. A neutral medium supplies no comparable excess of hydroxide to cause the stated shift, so (C) is not the required answer. The explicit basic-medium choice describes the decisive chemical change.

Step 4: Conclusion A basic medium favours CrO_4^{2-} and shifts the equilibrium to the right. Hence, the correct option is (B).

Quick Tip to Solve: Write the balanced chromium(VI) equilibrium with hydrogen ions: acid favours dichromate, whereas base favours chromate.

73. Identify the correct statements about p-block elements and their compounds.

(A) Non metals have higher electronegativity than metals.

(B) Non metals have lower ionisation enthalpy than metals.

(C) Compounds formed between highly reactive nonmetals and highly reactive metals are generally ionic.

(D) The non-metal oxides are generally basic in nature.

(E) The metal oxides are generally acidic or neutral in nature.

Choose the correct answer from the options given below :

[4 marks]

(A) (B) and (D) only

(B) (B) and (E) only

(C) (D) and (E) only

(D) (A) and (C) only

Correct Answer: (D) (A) and (C) only

Solution:

Concept:

- Electronegativity measures an atom's attraction for shared electrons, while ionisation enthalpy measures the energy required to remove an electron from a gaseous atom.
- Reactive metals tend to lose electrons and reactive nonmetals tend to gain them. The resulting oppositely charged ions can form ionic compounds; typical metallic and nonmetallic oxides differ in acid-base character.

Step 1: Assess electronegativity and ionisation enthalpy Statement (A) is correct as a general periodic trend. Nonmetals lie towards the upper right of the periodic table and usually attract bonding electrons more strongly than metals. Statement (B) reverses the usual ionisation-enthalpy trend: nonmetals generally resist electron loss more strongly, whereas metals lose valence electrons relatively easily. Thus nonmetals generally have higher, not lower, ionisation enthalpy. The comparison is a broad classification trend, as indicated by the wording of the question, rather than a claim that every individual pair behaves identically.

Step 2: Assess ionic-compound formation Statement (C) is correct. A highly reactive metal readily donates electrons, and a highly reactive nonmetal readily accepts them. This large tendency towards electron transfer produces cations and anions held by electrostatic attraction. For example, sodium chloride consists of Na^+ and Cl^- . The word generally allows exceptions without altering the standard classification of such compounds.

Step 3: Assess the oxide statements and select the pair Statement (D) is false: nonmetal oxides are generally acidic, although some are neutral. For instance, CO_2 is an acidic oxide. Statement (E) is also false: metallic oxides are generally basic, with amphoteric exceptions such as Al_2O_3 , rather than generally acidic or neutral. The true set is therefore exactly (A) and (C). Option (A) contains the false statements (B) and (D), option (B) contains (B) and (E), and option (C) contains (D) and (E); none is acceptable.

Step 4: Conclusion Only statements A and C are correct. Hence, the correct option is (D).

Quick Tip to Solve: Keep the usual paired trends together: metals lose electrons and form basic oxides; nonmetals attract electrons and usually form acidic oxides.

74. Given below are two statements :

Statement (I) : Fusion of MnO_2 with KOH and an oxidising agent gives dark green K_2MnO_4 .

Statement (II) : Manganate ion on electrolytic oxidation in alkaline medium gives permanganate ion.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Statement I is false but Statement II is true
(B) Statement I is true but Statement II is false
(C) Both Statement I and Statement II are false
(D) Both Statement I and Statement II are true

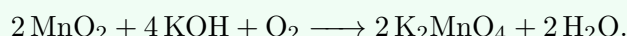
Correct Answer: (D) Both Statement I and Statement II are true

Solution:

Concept:

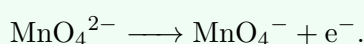
- Manganese changes oxidation state during the preparation of manganate and permanganate: it is +4 in MnO_2 , +6 in MnO_4^{2-} , and +7 in MnO_4^- .
- Fusion with alkali and an oxidising agent raises the oxidation state to manganate. Further electrolytic oxidation removes one electron from manganate to produce permanganate.

Step 1: Verify the first statement In an alkaline fusion, manganese dioxide reacts with potassium hydroxide and an oxidising agent such as oxygen to form potassium manganate. One balanced preparation equation is



Manganese is oxidised from +4 to +6, while oxygen acts as the oxidant. Potassium manganate is dark green, so both the product and its stated colour in Statement (I) are correct. This is manganate, with two potassium ions per tetraoxoanion, not purple permanganate.

Step 2: Verify the electrolytic oxidation In alkaline solution, electrolytic oxidation converts manganate into permanganate at the anode:



The anion loses an electron, and manganese increases its oxidation state from +6 to +7. Oxygen atoms are unchanged and the charges balance: minus two on the left equals minus one plus the electron's minus one on the right. Hence Statement (II) describes a chemically valid oxidation step. The fact that the medium is alkaline does not prevent this externally driven electrolytic oxidation.

Step 3: Choose the correct truth combination Statement (I) correctly describes the preparation and colour of manganate, and Statement (II) correctly describes its further oxidation to permanganate. Therefore neither statement should be rejected. Option (A) wrongly rejects I, option (B) wrongly rejects II, and option (C) wrongly rejects both. The only option retaining both verified facts is (D).

Step 4: Conclusion Both Statement (I) and Statement (II) are true. Hence, the correct option is (D).

Quick Tip to Solve: Remember the oxidation-state ladder: manganese dioxide (+4), green manganate (+6), then purple permanganate (+7).

75. Given below are two statements :

Statement (I) : S_N2 reactions are 'stereospecific', indicating that they result in the formation of only one stereo-isomer as the product.

Statement (II) : S_N1 reactions generally result in formation of product as racemic mixtures.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Both Statement I and Statement II are false
(B) Statement I is false but Statement II is true
(C) Statement I is true but Statement II is false
(D) Both Statement I and Statement II are true

Correct Answer: (D) Both Statement I and Statement II are true

Solution:

Concept:

- An S_N2 substitution is a concerted displacement: the nucleophile attacks from the side opposite the leaving group and produces inversion at the reacting stereogenic carbon.
- An S_N1 substitution proceeds through a planar carbocation. Nucleophilic attack can occur from either face, giving both configurations when the product remains chiral.

Step 1: Judge the stereospecificity of the S_N2 pathway The incoming nucleophile approaches the carbon from the back side because this geometry permits overlap with the carbon-leaving-group antibonding orbital. Bond formation and bond cleavage occur in one step. The three other substituents invert their spatial arrangement, the usual Walden inversion. A specified configuration of a suitable chiral substrate therefore gives a definite inverted configuration of the substitution product, rather than a random mixture of both configurations. This fixed stereochemical relationship makes the reaction stereospecific. Statement (I) is true in the substitution context described.

Step 2: Judge the stereochemical outcome of the S_N1 pathway Ionisation of the substrate first produces a carbocation with a planar arrangement at the positively charged carbon. Since the nucleophile can approach either side of this plane, products of the two possible configurations are formed. The elementary textbook description is racemisation, explaining Statement (II). In real reactions a leaving-group ion pair can shield one face, so equal amounts are not an absolute rule for every substrate and solvent. The statement uses generally, and expresses the standard comparison of an S_N1 pathway with the stereospecific S_N2 pathway.

Step 3: Select the combination supported by both mechanisms Both statements agree with the mechanistic origin of the stereochemistry: concerted back-side attack leads to inversion, whereas the planar intermediate allows attack from either face. Option (A) wrongly rejects both statements. Option (B) rejects the correctly stated S_N2 behaviour, and option (C) rejects the usual S_N1 racemisation. Hence only the choice in which both statements are true remains.

Step 4: Conclusion Both Statement (I) and Statement (II) are true. Hence, the correct option is (D).

Quick Tip to Solve: Use the intermediate to predict stereochemistry: one-step back-side attack gives inversion; a planar carbocation permits both faces of attack.

76. Match List I with List II.

List I (Test)	List II (Identification)
A. Bayer's test	I. Phenol
B. Ceric ammonium nitrate test	II. Aldehyde
C. Phthalein dye test	III. Alcoholic-OH group
D. Schiff's test	IV. Unsaturation

Choose the correct answer from the options given below:

[4 marks]

(A) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(B) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(C) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(D) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Correct Answer: (D) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Solution:

Concept:

- Qualitative tests identify characteristic functional groups by a chemical change such as oxidation, complex formation or production of a coloured compound.
- The named tests have different targets: cold alkaline permanganate detects unsaturation, ceric ammonium nitrate detects alcoholic hydroxyl groups, phthalein dye formation detects phenols, and Schiff's reagent detects aldehydes.

Step 1: Match the oxidation and alcohol tests Bayer's test (usually called Baeyer's test) uses cold dilute alkaline potassium permanganate. A reactive carbon-carbon double or triple bond is oxidised and the purple reagent is decolourised, often with brown manganese dioxide formation. This detects unsaturation: A-IV. Ceric ammonium nitrate gives a characteristic red colour by complex formation with an alcoholic hydroxyl group: B-III.

Step 2: Match the phenol and aldehyde tests The phthalein dye test involves reaction with phthalic anhydride and concentrated sulphuric acid, followed by making the solution alkaline. The coloured phthalein product identifies phenol: C-I. Schiff's reagent develops a pink or magenta colour with an aldehyde: D-II. Thus,

$$A - IV, \quad B - III, \quad C - I, \quad D - II.$$

Every identification is used once, as required for the matching.

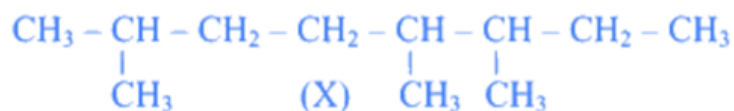
Step 3: Compare all the printed combinations The first combination incorrectly sends the alcohol test to phenol, phthalein to aldehyde and Schiff's test to alcoholic hydroxyl. The second wrongly assigns Bayer's test to alcohol and phthalein to unsaturation. The third wrongly assigns Bayer's test to aldehyde, phthalein to unsaturation and Schiff's test to phenol. Only the fourth combination agrees with all four tests.

Step 4: Conclusion The matching is A-IV, B-III, C-I, D-II. Hence, the correct option is (D). The supplied key gives the first choice, but that printed combination misidentifies three tests; the chemistry above proves the fourth printed choice.

Quick Tip to Solve: Anchor the easy pairs first: Schiff means aldehyde, and Baeyer means unsaturation. Then distinguish the alcohol and phenol tests.

77. IUPAC name of following hydrocarbon(X) is :

[4 marks]



- (A) 2,5,6-Trimethyloctane
(B) 2-Ethyl-2,6-diethylheptane
(C) 3,4,7-Trimethyloctane
(D) 2-Ethyl-3,6-dimethylheptane

Correct Answer: (A) 2,5,6-Trimethyloctane

Solution:

Concept:

- The parent chain of an acyclic saturated hydrocarbon is the longest continuous carbon chain. Its number of carbon atoms determines the alkane parent name.
- Number the selected chain from the end giving the lowest set of substituent locants at the first point of difference. Identical substituents use multiplicative prefixes such as tri-.

Step 1: Find the longest continuous chain The displayed horizontal chain contains eight carbon atoms:



The three branches are each single-carbon methyl groups. Including a terminal branch in a possible continuous path does not create a chain longer than eight carbon atoms. The correct parent hydrocarbon is therefore octane. A seven-carbon parent would unnecessarily leave part of this longer chain outside the parent and cannot be chosen merely to name an ethyl substituent.

Step 2: Compare numbering from the two ends Starting at the left end of the displayed structure puts the methyl groups at positions 2, 5 and 6. Numbering from the right instead gives positions 3, 4 and 7. Compare the sets in increasing order:

$$(2, 5, 6) \quad \text{and} \quad (3, 4, 7).$$

The first entries already differ, and 2 is lower than 3. Therefore the first set is chosen. One must use this first-point-of-difference rule rather than comparing the sums of the locants; in this case both sums happen to be 13.

Step 3: Construct the name and reject the alternatives There are three identical methyl substituents, so combine their locants with trimethyl and the parent octane to obtain 2,5,6-trimethyloctane. Option (C) identifies an eight-carbon chain but numbers it from the less favourable end. Options (B) and (D) choose heptane as the parent and introduce ethyl branches, even though an eight-carbon continuous chain is present. They therefore fail the longest-chain rule before any further numbering comparison is needed.

Step 4: Conclusion The IUPAC name is 2,5,6-trimethyloctane. Hence, the correct option is (A).

Quick Tip to Solve: Choose the longest chain first, then compare locant lists from left to right; the first unequal locant decides the numbering.

78. Given below are two statements :

Statement (I) : Kjeldahl method is applicable to estimate nitrogen in pyridine.

Statement (II) : The nitrogen present in pyridine can easily be converted into ammonium sulphate in Kjeldahl method.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Both Statement I and Statement II are true
- (B) Both Statement I and Statement II are false
- (C) Statement I is true but Statement II is false
- (D) Statement I is false but Statement II is true

Correct Answer: (B) Both Statement I and Statement II are false

Solution:

Concept:

- Kjeldahl estimation depends on converting organic nitrogen quantitatively into ammonium sulphate during digestion with concentrated sulphuric acid.
- The method is unsuitable for nitrogen in a heterocyclic ring, as in pyridine, because this nitrogen is not quantitatively converted into $(\text{NH}_4)_2\text{SO}_4$.

Step 1: Identify the nitrogen environment Pyridine is a six-membered aromatic heterocycle with one ring nitrogen. Its nitrogen atom forms part of the ring skeleton; it is not an ordinary amino group attached to an alkyl chain. In the standard Kjeldahl procedure this form of nitrogen does not give the required quantitative conversion to ammonium sulphate. The exclusion is a limitation of the analytical method, rather than evidence that pyridine contains no nitrogen.

Step 2: Evaluate Statement I Statement I claims that the Kjeldahl method is applicable to pyridine. Since the digestion does not quantitatively convert pyridine nitrogen into the ammonium salt used for measurement, the method cannot reliably estimate its nitrogen content. Therefore Statement I is false. A method such as Dumas estimation can be used for nitrogen compounds outside the scope of Kjeldahl digestion.

Step 3: Evaluate Statement II and select the combination Statement II claims easy conversion of pyridine nitrogen into ammonium sulphate. This is precisely the conversion that fails in the standard procedure, so Statement II is also false. Option (A) incorrectly accepts both statements. Option (C) accepts the applicability claim, and option (D) accepts the conversion claim; neither is consistent with the limitation of the method.

Step 4: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (B).

Quick Tip to Solve: Before using Kjeldahl estimation, check how nitrogen is bonded: heterocyclic ring nitrogen, nitro nitrogen and azo nitrogen are standard exclusions.

79. Identify the incorrect statements about group 15 elements :

- (A) Dinitrogen is a diatomic gas which acts like an inert gas at room temperature.
(B) The common oxidation states of these elements are -3 , $+3$ and $+5$.
(C) Nitrogen has unique ability to form $p\pi - p\pi$ multiple bonds.
(D) The stability of $+5$ oxidation states increases down the group.
(E) Nitrogen shows a maximum covalency of 6.

Choose the correct answer from the options given below :

[4 marks]

- (A) (A), (C), (E) only
(B) (D) and (E) only
(C) (A), (B), (D) only
(D) (B), (D), (E) only

Correct Answer: (B) (D) and (E) only

Solution:

Concept:

- Group 15 elements have the valence configuration ns^2np^3 , giving common oxidation states -3 , $+3$, $+5$.
- The inert-pair effect strengthens down the group: the ns^2 pair participates less readily in bonding, stabilising $+3$ relative to $+5$.
- Nitrogen has no vacant valence-shell d orbitals; its maximum covalency is four, as in NH_4^+ .

Step 1: Check statements A and B Statement (A) is correct. Dinitrogen is diatomic and relatively inert at room temperature because its strong triple bond has a high bond dissociation enthalpy; ordinary reactions must overcome a large activation barrier. Statement (B) is also correct: the common oxidation states of the family are -3 , $+3$, $+5$. These are trends for the group, not a claim that each state has equal stability for every member.

Step 2: Check the bonding statement C Statement (C) is correct in the group-15 comparison. The small size of nitrogen permits effective lateral overlap of its compact 2p orbitals, producing strong $p\pi - p\pi$ multiple bonds such as the bonds in N_2 . The heavier members have more diffuse orbitals and a much weaker tendency to form such multiple bonds with themselves.

Step 3: Identify the incorrect trends D and E Statement (D) reverses the oxidation-state trend: the stability of $+5$ decreases, rather than increases, down the group as the inert-pair effect grows. Statement (E) is false because nitrogen cannot expand its octet to attain covalency six; its maximum covalency is four. Thus only (D) and (E) are incorrect. Options (A), (C) and (D) include one or more correct statements among the allegedly incorrect ones.

Step 4: Conclusion The two incorrect claims are increasing stability of the pentavalent state down the group and covalency six for nitrogen. Hence, the correct option is (B).

Quick Tip to Solve: For group 15, pair the two trends: down the group the lower oxidation state becomes more stable, while nitrogen retains a strict second-period octet.

80. In qualitative test for identification of presence of phosphorous, the compound is heated with an oxidising agent. Which is further treated with nitric acid and ammonium molybdate respectively. The yellow coloured precipitate obtained is :

[4 marks]

- (A) $\text{Na}_3\text{PO}_4 \cdot 12\text{MoO}_3$
(B) $(\text{NH}_4)_3\text{PO}_4 \cdot 12(\text{NH}_4)_2\text{MoO}_4$
(C) $\text{MoPO}_4 \cdot 21\text{NH}_4\text{NO}_3$
(D) $(\text{NH}_4)_3\text{PO}_4 \cdot 12\text{MoO}_3$

Correct Answer: (D) $(\text{NH}_4)_3\text{PO}_4 \cdot 12\text{MoO}_3$

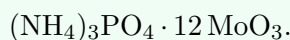
Solution:

Concept:

- Organic phosphorus is first oxidised to phosphate; this converts the element into an inorganic form suitable for a characteristic precipitation test.
- In nitric-acid medium, phosphate gives a yellow precipitate of ammonium phosphomolybdate with ammonium molybdate. Its conventional formula is $(\text{NH}_4)_3\text{PO}_4 \cdot 12\text{MoO}_3$.

Step 1: Identify the product of oxidation Heating the organic compound with the oxidising agent converts its phosphorus into phosphate. This step removes the complication of the original organic bonding environment. The subsequent test is therefore a phosphate-ion test, not direct precipitation of the unchanged organic compound. Nitric acid provides the acidic medium required for formation of the characteristic molybdate complex.

Step 2: Recognise the yellow precipitate Ammonium molybdate reacts with the acidified phosphate solution to form ammonium phosphomolybdate, an insoluble yellow compound. Its composition is commonly written as



The same composition may be represented by the heteropoly salt $(\text{NH}_4)_3[\text{PMo}_{12}\text{O}_{40}]$. The three ammonium ions balance the charge of the phosphomolybdate anion; twelve molybdenum atoms belong to the polyoxo framework.

Step 3: Eliminate the other formulae Option (A) contains sodium in place of ammonium and is not the characteristic product of this test. Option (B) retains twelve units of ammonium molybdate as though the reagents merely formed an addition mixture; that is not the precipitate composition. Option (C), with its ammonium nitrate term, also does not represent ammonium phosphomolybdate. Only option (D) gives the recognised yellow precipitate.

Step 4: Conclusion The yellow precipitate is ammonium phosphomolybdate, $(\text{NH}_4)_3\text{PO}_4 \cdot 12\text{MoO}_3$. Hence, the correct option is (D).

Quick Tip to Solve: For the phosphorus test, remember the sequence: oxidation to phosphate, acidification with nitric acid, then yellow ammonium phosphomolybdate.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. $\Delta_{\text{vap}} H^\ominus$ for water is $+40.79 \text{ kJ mol}^{-1}$ at 1 bar and 100°C . Change in internal energy for this vapourisation under same condition is _____ kJ mol^{-1} . (Integer answer) (Given $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$)

[4 marks]

Solution:

Concept:

- For a process involving ideal-gas formation, $\Delta H = \Delta U + \Delta n_g RT$, where Δn_g is the change in moles of gas.
- Vaporising one mole of liquid water forms one mole of vapour. Neglecting the small liquid-volume contribution gives $\Delta n_g = 1$, so $\Delta U = \Delta H - RT$.

Step 1: Write the molar phase change The process is $\text{H}_2\text{O}(\text{l}) \longrightarrow \text{H}_2\text{O}(\text{g})$. For each mole vaporised, one mole of gas is produced and no gaseous reactant is consumed. The enthalpy change includes both the increase in internal energy and the pressure-volume expansion term. Thus the internal-energy increase must be slightly smaller than the positive enthalpy of vaporisation.

Step 2: Calculate the expansion contribution Use absolute temperature in the gas term:

$$T = 100 + 273 = 373 \text{ K}.$$

The gas constant is supplied in joules, whereas the enthalpy is in kilojoules, so convert the product consistently:

$$RT = 8.3 \times 373 = 3095.9 \text{ J mol}^{-1} = 3.0959 \text{ kJ mol}^{-1}.$$

Using 100 directly instead of 373 would incorrectly substitute Celsius temperature into a gas-law expression.

Step 3: Subtract and round to the required integer Applying the enthalpy relation gives

$$\begin{aligned}\Delta_{\text{vap}} U^\ominus &= 40.79 - 3.0959 \\ &= 37.6941 \text{ kJ mol}^{-1}.\end{aligned}$$

It remains positive because vaporisation requires energy to overcome intermolecular attractions. Since the question asks for an integer, round the final result rather than rounding the intermediate gas correction. The nearest integer is 38.

Step 4: Conclusion The internal-energy change is approximately $+38 \text{ kJ mol}^{-1}$. The integer to enter is 38.

Quick Tip to Solve: For liquid-to-gas vaporisation, subtract one RT per mole from the molar enthalpy and convert joules to kilojoules before subtracting.

82. Total number of aromatic compounds among the following compounds is ____.

[4 marks]



Solution:

Concept:

- An aromatic compound needs a planar cyclic conjugated system containing $4n + 2$ pi electrons, where $n = 0, 1, 2, \dots$
- A count of double bonds alone is insufficient: an sp^3 atom interrupts the continuous ring of p orbitals. A planar fully conjugated $4n$ -electron ring is antiaromatic rather than aromatic.

Step 1: Test the first two structures Number the six displayed compounds from left to right. In the first fused-ring structure, the double bonds do not form a complete conjugated aromatic circuit; saturated carbon atoms interrupt the possible ring paths. It is therefore nonaromatic. In the second structure, the central connecting double bond and the double bond in each five-membered ring likewise do not supply a continuously conjugated cyclic path through either ring. This structure is also nonaromatic.

Step 2: Test the third and fourth structures The third compound is the cyclopentadienyl cation. Its two double bonds contribute four pi electrons, and the positively charged carbon supplies an empty p orbital, so it does not satisfy the aromatic $4n + 2$ count. In the planar conjugated description it is antiaromatic. The fourth compound is cyclooctatetraene with eight pi electrons. Its nonplanar tub conformation avoids antiaromaticity, but the absence of a planar conjugated circuit makes it nonaromatic.

Step 3: Identify the aromatic ring and check the last structure The fifth compound is pyridine. All six ring atoms have p orbitals, and the three alternating double bonds supply six pi electrons, satisfying $4n + 2 = 6$ with $n = 1$. The nitrogen lone pair lies in an sp^2 orbital and is not included in that pi count. The last compound is cycloheptatriene: although it contains three double bonds, its saturated carbon interrupts conjugation, so it is not aromatic. Thus precisely one of the six qualifies.

Step 4: Conclusion Only pyridine, the fifth compound from the left, is aromatic. The total number is 1 (a dimensionless count).

Quick Tip to Solve: Check cyclic conjugation before applying the electron-count rule: cycloheptatriene has six pi electrons but an interrupting saturated carbon.

83. Total number of unpaired electrons in the complex ions $[\text{Co}(\text{NH}_3)_6]^{3+}$ and $[\text{NiCl}_4]^{2-}$ is _____.

[4 marks]

Solution:

Concept:

- Find the metal oxidation state before counting d electrons. Neutral ammonia contributes zero charge, while each chloride ligand contributes -1 .
- Octahedral Co^{3+} with ammonia is low-spin d^6 ; tetrahedral Ni^{2+} with chloride is d^8 and retains two unpaired electrons.

Step 1: Count the cobalt d electrons In $[\text{Co}(\text{NH}_3)_6]^{3+}$, the six ammonia ligands are neutral, so cobalt has oxidation state $+3$. Neutral cobalt has configuration $[\text{Ar}]3d^74s^2$; removing the two 4s electrons and one 3d electron gives d^6 . The high metal charge and ammonia ligand field favour pairing in the lower octahedral orbitals. Its configuration is

$$t_{2g}^6 e_g^0.$$

All three lower orbitals are doubly occupied, so the cobalt complex contributes zero unpaired electrons.

Step 2: Count the nickel d electrons For $[\text{NiCl}_4]^{2-}$, write the charge equation as

$$x + 4(-1) = -2, \quad x = +2.$$

Nickel(II) is d^8 . Chloride is a weak-field ligand and this four-coordinate ion is tetrahedral, rather than the square-planar low-spin arrangement associated with some strong-field nickel complexes. The tetrahedral configuration is

$$e^4 t_2^4.$$

The three upper orbitals receive one electron each before pairing, so four electrons in them leave two singly occupied orbitals.

Step 3: Add the unpaired-electron counts The lower tetrahedral e set contains two filled orbitals and contributes no unpaired electrons. The upper set contributes two. Adding the counts from the two separate complex ions gives

$$N_{\text{unpaired}} = 0 + 2 = 2.$$

Counting the free-ion electrons without considering the ligand field would incorrectly assign a high-spin configuration to the cobalt complex.

Step 4: Conclusion The total number of unpaired electrons in the two complex ions is 2 (a dimensionless electron count).

Quick Tip to Solve: For these common examples remember the contrast: octahedral cobalt(III)-ammine is paired, whereas tetrahedral nickel(II)-chloride has two unpaired electrons.

84. The total number of carbon atoms present in tyrosine, an amino acid, is _____.

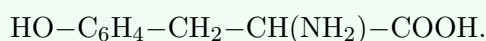
[4 marks]

Solution:

Concept:

- Tyrosine is an aromatic alpha-amino acid with side chain $-\text{CH}_2-\text{C}_6\text{H}_4-\text{OH}$, with the phenolic hydroxyl group para to the side-chain attachment.
- The amino-acid backbone contains both the alpha carbon and the carboxyl carbon. Count these separately from the carbons in the side chain.

Step 1: Write the condensed carbon skeleton The condensed structure of tyrosine is



The six-membered aromatic ring contains six carbon atoms even though it carries only four ring hydrogens after the two substitutions. Replacing ring hydrogens by hydroxyl and the amino-acid side chain does not remove any ring carbon. The phenolic hydroxyl group adds oxygen and hydrogen, not another carbon.

Step 2: Count the carbons outside the ring The side-chain methylene group $-\text{CH}_2-$ supplies one carbon between the aromatic ring and the alpha carbon. The alpha carbon in $-\text{CH}(\text{NH}_2)-$ supplies one more. Finally, the carbonyl carbon in $-\text{COOH}$ supplies a third carbon outside the ring. Neither the amino group nor the oxygen atoms introduce any additional carbon atoms. Thus the count outside the aromatic ring is three.

Step 3: Add and check the molecular formula The total is therefore

$$N_{\text{C}} = 6 + 1 + 1 + 1 = 9.$$

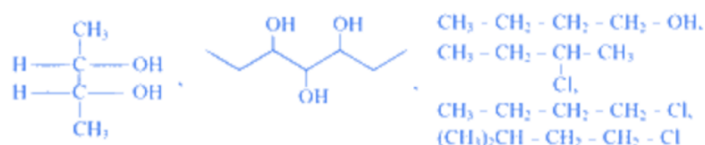
This agrees with the molecular formula $\text{C}_9\text{H}_{11}\text{NO}_3$. It also agrees with the amino-acid name 2-amino-3-(4-hydroxyphenyl)propanoic acid: the propanoic-acid chain contains three carbons and the hydroxyphenyl substituent contains six. The phenolic substitution distinguishes tyrosine from phenylalanine by an extra oxygen, without changing the number of carbon atoms.

Step 4: Conclusion Tyrosine contains 9 carbon atoms per molecule. The required answer is 9, a dimensionless atom count.

Quick Tip to Solve: For tyrosine, count a six-carbon benzene ring plus the three-carbon amino-acid chain; remember to include the carboxyl carbon.

85. The number of optically active compounds from the following is _____.

[4 marks]



Solution:

Concept:

- An optically active molecule is chiral: its mirror image cannot be superimposed on it. A tetrahedral carbon with four different substituents is a useful first test for a stereogenic centre.
- A molecule with stereogenic centres can still be achiral when an internal symmetry plane relates its two halves. Such a meso structure must not be counted as optically active.

Step 1: Examine the Fischer projection The first compound is the Fischer projection of butane-2,3-diol with both hydroxyl groups on the same side and identical terminal methyl groups. The upper and lower halves are interchanged by an internal plane passing between the two central carbon atoms. The two centres have opposite configurations, so this is the meso form. It is optically inactive despite having two stereogenic carbon atoms.

Step 2: Check the symmetric triol and the alcohol The next structure has equal ethyl ends and symmetrically placed hydroxyl groups: its left and right arms are mirror-related in the configuration drawn. This internal symmetry makes the depicted triol achiral. The straight-chain alcohol, butan-1-ol, also has no stereogenic centre: each internal methylene carbon has two identical hydrogen atoms, and the terminal hydroxyl-bearing carbon is a methylene carbon too.

Step 3: Inspect each chlorine-containing compound In 2-chlorobutane, the chlorine-bearing carbon is attached to Cl, H, CH_3 and CH_2CH_3 . These four groups differ, so this compound has a chiral centre and is optically active in an individual enantiomeric form. Butan-1-ol and 1-chlorobutane have no such centre. In $(\text{CH}_3)_2\text{CHCH}_2\text{CH}_2\text{Cl}$, the branched carbon has two identical methyl groups, while the chlorine-bearing carbon is CH_2 . It is therefore achiral as well.

Step 4: Conclusion Only 2-chlorobutane satisfies the chirality test among the depicted compounds. The required number of optically active compounds is 1 (dimensionless).

Quick Tip to Solve: After locating possible chiral carbons, check the whole molecule for internal symmetry; counting stereogenic centres alone incorrectly includes meso compounds.

86. Two moles of benzaldehyde and one mole of acetone under alkaline conditions using aqueous NaOH after heating gives x as the major product. The number of π bonds in the product x is _____.

[4 marks]

Solution:

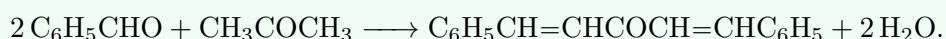
Concept:

- Benzaldehyde has no α -hydrogen and acts as the electrophilic carbonyl partner. Acetone has α -hydrogens on both sides of its carbonyl and supplies the enolate nucleophile.

- Heating an aldol addition product promotes dehydration, introducing a carbon–carbon double bond. Each double bond contributes one π bond, including each carbonyl double bond.

Step 1: Identify the first condensation product Under aqueous alkaline conditions, an acetone enolate attacks benzaldehyde. The initially formed beta-hydroxy ketone loses water on heating, giving benzylideneacetone. The acetone carbonyl is retained, while one carbon–carbon double bond is introduced next to it. Benzaldehyde itself does not undergo self-aldol condensation because its carbonyl-adjacent aromatic carbon has no removable alpha-hydrogen.

Step 2: Condense the second molecule of benzaldehyde The other methyl side of the acetone-derived ketone still has alpha-hydrogens, so it reacts with the second equivalent of benzaldehyde. A second dehydration gives dibenzalacetone. The overall stoichiometry is



The two phenyl rings remain aromatic; neither is consumed by the aldol reaction.

Step 3: Count the distinct double bonds Each phenyl ring contributes three π bonds, so the two rings contribute six. The two newly formed alkene groups contribute two more, and the retained ketone carbonyl contributes one. Hence

$$N_\pi = 2(3) + 2 + 1 = 9.$$

Conjugation delocalises these electrons but does not change the count of π bonds in the usual structural representation.

Step 4: Conclusion The major product is dibenzalacetone, containing 9 π bonds. The numerical answer is 9 (dimensionless).

Quick Tip to Solve: For dibenzalacetone, count the two benzene rings, the two new alkene bonds and the single retained carbonyl separately.

87. A solution is prepared by adding 1 mole ethyl alcohol in 9 mole water. The mass percent of solute in the solution is _____ (Integer answer) (Given : Molar mass in g mol^{-1} Ethyl alcohol : 46 water : 18) [4 marks]

Solution:

Concept:

- Mass percentage is $100 m_{\text{solute}}/m_{\text{solution}}$. The denominator is the combined mass of solute and solvent, not just the mass of water.
- Convert the supplied mole amounts to masses using $m = nM$, where n is amount in moles and M is molar mass in g mol^{-1} .

Step 1: Convert the amounts to masses Ethyl alcohol is the solute being added to the water. With the stated molar mass, its mass is

$$m_{\text{alcohol}} = 1 \text{ mol} \times 46 \text{ g mol}^{-1} = 46 \text{ g}.$$

The solvent contains nine moles of water, not nine grams, so its mass is

$$m_{\text{water}} = 9 \text{ mol} \times 18 \text{ g mol}^{-1} = 162 \text{ g}.$$

This conversion is essential because a mole fraction would compare amounts, whereas the question asks for a mass percentage.

Step 2: Form the mass of the complete solution Mixing conserves total mass. Consequently, the mass of solution is the sum of the two component masses:

$$m_{\text{solution}} = 46 + 162 = 208 \text{ g}.$$

No volume or density is needed for this concentration measure. In particular, dividing by 162 g would give a solute-to-solvent mass ratio, not the requested percentage of the solution.

Step 3: Calculate and round the percentage Substitute the solute mass and the complete solution mass into the definition:

$$w_{\text{alcohol}} = \frac{46}{208} \times 100 = 22.115 \dots \%$$

The problem requests an integer answer, so round the calculated percentage to the nearest integer. The unrounded mass fraction is about 0.221, which also confirms that the percentage should be near 22 rather than near the 10 percent mole percentage.

Step 4: Conclusion The mass percentage of ethyl alcohol is approximately 22.115%, or 22% to the nearest integer. Enter 22.

Quick Tip to Solve: Always convert both solute and solvent to masses before using mass percent; the mole ratio of 1:9 is not the mass ratio.

88. Molality of an aqueous solution of urea is 4.44 m. Mole fraction of urea in solution is $x \times 10^{-3}$, Value of x is _____. (Integer answer) [4 marks]

Solution:

Concept:

- Molality is the number of moles of solute per kilogram of solvent. A convenient basis for a 4.44 mol kg^{-1} solution is exactly 1 kg of water.
- The mole fraction of urea is $X_{\text{urea}} = n_{\text{urea}} / (n_{\text{urea}} + n_{\text{water}})$. Both denominator terms must be amounts in moles.

Step 1: Choose the solvent basis Take 1.000 kg of water. Since the molality is 4.44 mol per kilogram of solvent, this basis contains

$$n_{\text{urea}} = 4.44 \text{ mol}.$$

The chosen kilogram is the mass of the solvent, not the mass of the resulting solution. Any other amount of water would give the same final mole fraction, because both component amounts scale together.

Step 2: Find the amount of water Using the molar mass of water, 18 g mol^{-1} , convert the chosen

solvent mass to moles:

$$n_{\text{water}} = \frac{1000 \text{ g}}{18 \text{ g mol}^{-1}} = 55.555 \dots \text{ mol.}$$

Thus the total amount of molecules in the solution is

$$n_{\text{total}} = 4.44 + 55.555 \dots = 59.995 \dots \text{ mol.}$$

Urea is a molecular solute here, so there is no dissociation factor to insert in this count.

Step 3: Extract the requested coefficient The mole fraction is therefore

$$X_{\text{urea}} = \frac{4.44}{4.44 + 1000/18} = 0.074005 \dots$$

The question expresses this quantity as $x \times 10^{-3}$, which means that the requested coefficient is

$$x = 1000X_{\text{urea}} = 74.005 \dots$$

Round this coefficient only at the end to obtain the integer answer. Confusing the coefficient with the mole fraction itself would miss the factor of one thousand.

Step 4: Conclusion The urea mole fraction is approximately 74.005×10^{-3} . Thus $x = 74$ to the nearest integer; the coefficient is dimensionless.

Quick Tip to Solve: Use one kilogram of water: then the solute amount equals the molality numerically and the water amount is 1000/18 moles.

89. Wavenumber for a radiation having 5800 Å wavelength is $x \times 10 \text{ cm}^{-1}$. The value of x is _____.
(Integer answer) [4 marks]

Solution:

Concept:

- The spectroscopic wavenumber is $\tilde{\nu} = 1/\lambda$. It counts wavelengths per unit length and differs from the angular wavenumber $2\pi/\lambda$.
- The wavelength must first be expressed in centimetres when the requested wavenumber is in cm^{-1} . One angstrom equals 10^{-8} cm .

Step 1: Convert the wavelength to centimetres The supplied wavelength is 5800 angstrom. Using $1 \text{ Å} = 10^{-10} \text{ m} = 10^{-8} \text{ cm}$, obtain

$$\lambda = 5800 \times 10^{-8} \text{ cm} = 5.8 \times 10^{-5} \text{ cm.}$$

Keeping the conversion factor in centimetres at this stage avoids the extra factor of one hundred that would arise if the reciprocal were calculated in metres and then left unconverted.

Step 2: Take the reciprocal wavelength Apply the definition of spectroscopic wavenumber:

$$\tilde{\nu} = \frac{1}{5.8 \times 10^{-5} \text{ cm}} = \frac{10^5}{5.8} \text{ cm}^{-1} = 17241.379 \dots \text{ cm}^{-1}.$$

The result has inverse-length units, as required. No frequency calculation or value of the speed of light is needed, since the question asks directly for the reciprocal wavelength.

Step 3: Match the stated power of ten The question writes the wavenumber as $x \times 10 \text{ cm}^{-1}$. Therefore equate the numerical coefficients:

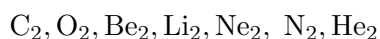
$$10x = 17241.379 \dots, \quad x = 1724.1379 \dots$$

The requested integer is the nearest integer to this coefficient. The factor printed in the question is 10, so the answer is not the full numerical value 17241 of the wavenumber itself.

Step 4: Conclusion The wavenumber is $1.7241379 \times 10^4 \text{ cm}^{-1}$. In the form $x \times 10 \text{ cm}^{-1}$, the nearest integer coefficient is $x = 1724$ (dimensionless).

Quick Tip to Solve: Convert angstrom to centimetres before taking the reciprocal, then divide by the exact power of ten used in the question.

90. Number of molecules having bond order 2 from the following molecules is _____.



[4 marks]

Solution:

Concept:

- In molecular orbital theory, bond order is $\text{BO} = (N_b - N_a)/2$, where N_b and N_a are the numbers of bonding and antibonding electrons.
- Filled core bonding and antibonding orbitals cancel in this difference. For second-period diatomic molecules it is therefore sufficient to count the valence orbital electrons correctly.

Step 1: Evaluate the lighter diatomic species For Li_2 , the two valence electrons occupy the bonding σ_{2s} orbital, giving $(2 - 0)/2 = 1$. In Be_2 , two further electrons occupy σ_{2s}^* , so its bond order is $(2 - 2)/2 = 0$. In C_2 , the remaining four valence electrons fill the bonding π_{2p} orbitals after the $2s$ bonding and antibonding pair. Consequently, its bond order is $(6 - 2)/2 = 2$.

Step 2: Evaluate nitrogen, oxygen and neon For N_2 , the valence bonding count is eight and the antibonding count is two, giving $(8 - 2)/2 = 3$. For O_2 , the two additional electrons occupy antibonding π_{2p}^* orbitals, giving $(8 - 4)/2 = 2$. In Ne_2 , all valence bonding and antibonding orbitals are filled equally, giving $(8 - 8)/2 = 0$. The change in the relative σ_{2p} and π_{2p} ordering across the period does not alter these completed electron counts.

Step 3: Check helium and select the required molecules For He_2 , the configuration has two electrons in σ_{1s} and two in σ_{1s}^* . Thus its bond order is zero. In the exact order of the question, the bond orders

are

Species	C ₂	O ₂	Be ₂	Li ₂	Ne ₂	N ₂	He ₂
BO	2	2	0	1	0	3	0

Only the first two entries equal the specified bond order of two.

Step 4: Conclusion The molecules with bond order 2 are C₂ and O₂. Their number is 2 (dimensionless).

Quick Tip to Solve: For these homonuclear diatomics, remember the sequence Li₂, Be₂, C₂, N₂, O₂, Ne₂: bond orders 1, 0, 2, 3, 2, 0.