JEE Main 2024 Paper 1 (B.E./B.Tech) Apr 9 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 9 April 2024, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If the domain of the function $f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is $\mathbf{R} - (\alpha, \beta)$, then $12\alpha\beta$ is equal to : [4 marks]

(A) 40

(B) 36

(C) 24

(D) 32

Correct Answer: (D) 32

Solution:

Concept:

- The real function $\sin^{-1} t$ is defined exactly when $-1 \le t \le 1$, including both endpoints.
- For a rational argument, exclude zeros of its denominator first. With a nonzero denominator, $\left|\frac{p}{q}\right| \le 1$ is equivalent to $p^2 \le q^2$; squaring is legitimate because both sides are nonnegative.

Step 1: Impose the inverse-sine condition The denominator must be nonzero, so initially $x \neq -\frac{3}{2}$. Now require

$$\left|\frac{x-1}{2x+3}\right| \le 1 \quad \Longleftrightarrow \quad (x-1)^2 \le (2x+3)^2.$$

Using a squared inequality avoids multiplying by a denominator whose sign is unknown. Expand and bring the terms to one side:

$$x^2 - 2x + 1 \le 4x^2 + 12x + 9 \quad \Longleftrightarrow \quad 3x^2 + 14x + 8 \ge 0.$$

Step 2: Find the excluded interval and compare the choices Factor the quadratic and use its upward-opening sign pattern:

$$(3x+2)(x+4) \ge 0 \quad \Longleftrightarrow \quad x \le -4 \text{ or } x \ge -\frac{2}{3}.$$

The denominator's zero $-\frac{3}{2}$ lies strictly between these endpoints, so it is already outside the allowed set. At $x = -4$ and $x = -\frac{2}{3}$, the argument equals 1 and -1, respectively, and both endpoints are included. Therefore the domain is

$$\mathbf{R} - \left(-4, -\frac{2}{3}\right).$$

Thus $\alpha = -4$, $\beta = -\frac{2}{3}$, and

$$12\alpha\beta = 12(-4)\left(-\frac{2}{3}\right) = 32.$$

The values 40, 36 and 24 in options (A), (B) and (C) disagree with this product.

Step 3: Conclusion $12\alpha\beta = 32$. Hence, the correct option is (D).

Quick Tip to Solve: For an inverse sine of a rational expression, compare the squared numerator and denominator, then separately check any denominator zero and the interval endpoints.

2. Let three vectors , $\overrightarrow{a} = \alpha\hat{i} + 4\hat{j} + 2\hat{k}, \overrightarrow{b} = 5\hat{i} + 3\hat{j} + 4\hat{k}, \overrightarrow{c} = x\hat{i} + y\hat{j} + z\hat{k}$ **form a triangle such that** $\vec{c} = \vec{a} - \vec{b}$ **and the area of the triangle is** $5\sqrt{6}$**. If** α **is a positive real number, then** $|\vec{c}|^2$ **is equal to:** [4 marks]

(A) 14

(B) 12

(C) 16

(D) 10

Correct Answer: (A) 14

Solution:

Concept:

- If two side vectors of a triangle are $\vec{a}$ and $\vec{b}$, its area is $\frac{1}{2}|\vec{a} \times \vec{b}|$. Reversing a side changes the cross product sign but leaves the area unchanged.
- The squared length of $u\hat{i} + v\hat{j} + w\hat{k}$ is $u^2 + v^2 + w^2$. Apply the stated positivity condition to select the allowed parameter.

Step 1: Use the area to determine the parameter The cross product of the two given vectors is

$$\begin{aligned}\vec{a} \times \vec{b} &= (4 \cdot 4 - 2 \cdot 3)\hat{i} + (2 \cdot 5 - 4\alpha)\hat{j} + (3\alpha - 4 \cdot 5)\hat{k} \\ &= 10\hat{i} + (10 - 4\alpha)\hat{j} + (3\alpha - 20)\hat{k}.\end{aligned}$$

Since the area is $5\sqrt{6}$, the cross-product magnitude is $10\sqrt{6}$. Squaring therefore gives

$$100 + (10 - 4\alpha)^2 + (3\alpha - 20)^2 = 600.$$

Expanding carefully, the constant terms total 600, and hence

$$25\alpha^2 - 200\alpha + 600 = 600 \quad \Longrightarrow \quad 25\alpha(\alpha - 8) = 0.$$

The possible parameters are 0 and 8. Only $\alpha = 8$ is positive.

Step 2: Calculate the third side and its squared length Use the vector relation in the question, rather than adding the vectors:

$$\vec{c} = \vec{a} - \vec{b} = (8 - 5)\hat{i} + (4 - 3)\hat{j} + (2 - 4)\hat{k} = 3\hat{i} + \hat{j} - 2\hat{k}.$$

Consequently,

$$|\vec{c}|^2 = 3^2 + 1^2 + (-2)^2 = 9 + 1 + 4 = 14.$$

As a check, at $\alpha = 8$ the cross product is $(10, -22, 4)$, whose squared length is $100 + 484 + 16 = 600$, recovering the prescribed area. The other offered squared lengths, 12, 16 and 10, do not equal the sum of the three component squares.

Step 3: Conclusion $|\vec{c}|^2 = 14$. Hence, the correct option is (A).

Quick Tip to Solve: Square twice the triangle area to obtain the cross-product equation, select the allowed parameter, and only then subtract the vectors to find the requested side.

3. Let $f(x) = ax^3 + bx^2 + cx + 41$ be such that $f(1) = 40, f'(1) = 2$ and $f''(1) = 4$. Then $a^2 + b^2 + c^2$ is equal to: [4 marks]

(A) 54

(B) 51

(C) 73

(D) 62

Correct Answer: (B) 51

Solution:

Concept:

- For $f(x) = ax^3 + bx^2 + cx + 41$, differentiation gives $f'(x) = 3ax^2 + 2bx + c$ and $f''(x) = 6ax + 2b$.
- Each prescribed value at $x = 1$ is a linear equation in the coefficients. Solve these equations together before computing the sum of squares.

Step 1: Translate all three conditions into equations Substitute $x = 1$ into the polynomial. The constant 41 must be retained:

$$a + b + c + 41 = 40 \quad \Longrightarrow \quad a + b + c = -1. \tag{1}$$

Substitute into the first derivative to obtain

$$3a + 2b + c = 2. \tag{2}$$

The second derivative condition gives

$$6a + 2b = 4 \quad \Longrightarrow \quad 3a + b = 2. \tag{3}$$

Subtracting (1) from (2) eliminates c:

$$2a + b = 3. \tag{4}$$

This uses all the information in the question without differentiating the constant incorrectly or replacing $f(1)$ by a coefficient.

Step 2: Solve and verify the coefficients Subtract (4) from (3): $a = -1$. Substitution into (4) yields $b = 5$, and (1) then gives $c = -5$. Check all three original conditions directly:

$$\begin{aligned} f(1) &= -1 + 5 - 5 + 41 = 40, \\ f'(1) &= -3 + 10 - 5 = 2, \\ f''(1) &= -6 + 10 = 4. \end{aligned}$$

All conditions hold, so the coefficient values are consistent. Now square the coefficients, including their negative signs inside parentheses:

$$a^2 + b^2 + c^2 = (-1)^2 + 5^2 + (-5)^2 = 1 + 25 + 25 = 51.$$

The choices 54, 73 and 62 fail this directly verified coefficient calculation; they cannot be obtained

from another valid triple because the three independent linear equations determine the triple uniquely.

Step 3: Conclusion $a^2+b^2+c^2=51$. Hence, the correct option is (B).

Quick Tip to Solve: Subtract the function-value equation from the first-derivative equation to remove the linear coefficient, then use the second derivative to isolate the cubic coefficient.

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4. The parabola $y^2=4x$ divides the area of the circle $x^2+y^2=5$ in two parts. The area of the smaller part is equal to : [4 marks]

(A) $\frac{2}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

(B) $\frac{2}{3}+\sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

(C) $\frac{1}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

(D) $\frac{1}{3}+\sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

Correct Answer: (A) $\frac{2}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

Solution:

Concept:

- For horizontal strips, bounded area is $\int(x_{\text{right}}-x_{\text{left}})\,dy$. Here the circle gives $x=\sqrt{5-y^2}$ on its right half and the parabola gives $x=y^2/4$.
- Both curves are symmetric about the x-axis. Integrate only for $y\geq 0$ and double the result; use the standard circular-segment integral for the circle term.

Step 1: Locate the intersections and select the smaller region Substitute $y^2=4x$ into the circle:

$$x^2+4x=5 \quad\Longrightarrow\quad (x-1)(x+5)=0.$$

Since the parabola requires $x\geq 0$, only $x=1$ is admissible, giving $y=\pm 2$. The region to the right of the parabola is therefore described by

$$-2\leq y\leq 2, \qquad \frac{y^2}{4}\leq x\leq\sqrt{5-y^2}.$$

This region lies strictly within the circle's right semicircle, so its area is less than half the total circle area 5π. It is thus the smaller of the two parts; the other part contains the whole left semicircle.

Step 2: Integrate horizontal strips and compare the options Symmetry gives the area

$$A=2\int_0^2\left(\sqrt{5-y^2}-\frac{y^2}{4}\right)dy.$$

Evaluate the two terms:

$$A = \left[y\sqrt{5-y^2} + 5\sin^{-1}\left(\frac{y}{\sqrt{5}}\right) - \frac{y^3}{6} \right]_0^2$$
$$= 2 + 5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right) - \frac{8}{6}$$
$$= \frac{2}{3} + 5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right).$$

The coefficient of the inverse sine is the radius squared, namely 5, so options (B) and (D), which use $\sqrt{5}$, are incorrect. The nontrigonometric contribution is $2 - 4/3 = 2/3$, not $1/3$, which also excludes (C).

Step 3: Conclusion The smaller area is $\frac{2}{3} + 5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ square units. Hence, the correct option is (A).

Quick Tip to Solve: When the parabola is given as $y^2 = 4x$, horizontal strips avoid splitting the area into multiple integrals; symmetry then halves the working.

5. The solution of the differential equation $(x^2 + y^2)\mathrm{d}x - 5xy\ \mathrm{d}y = 0, y(1) = 0$, is : [4 marks]

(A) $\left|x^2 - 4y^2\right|^5 = x^2$
(B) $\left|x^2 - 2y^2\right|^6 = x$
(C) $\left|x^2 - 2y^2\right|^5 = x^2$
(D) $\left|x^2 - 4y^2\right|^6 = x$

Correct Answer: (A) $\left|x^2 - 4y^2\right|^5 = x^2$

Solution:

Concept:

- The substitution $u = y^2$ converts the product $y\,dy$ to $du/2$, making the given differential relation linear in u as a function of x.
- For $u' + P(x)u = Q(x)$, multiply by the integrating factor $e^{\int P(x)\,dx}$. Since the initial point has $x = 1$, use the interval $x > 0$.

Step 1: Convert the relation and integrate The equation becomes

$$(x^2 + u)\,dx - \frac{5}{2}x\,du = 0.$$

For $x > 0$, rearrange it as

$$\frac{du}{dx} - \frac{2u}{5x} = \frac{2x}{5}.$$

Its integrating factor is $x^{-2/5}$, so

$$\frac{d}{dx}\left(ux^{-2/5}\right) = \frac{2}{5}x^{3/5}.$$

Integrating both sides gives

$$ux^{-2/5} = \frac{1}{4}x^{8/5} + C, \qquad y^2 = \frac{x^2}{4} + Cx^{2/5}.$$

Keeping the fractional power on the positive interval avoids an unnecessary absolute-value ambiguity during integration.

Step 2: Apply the initial condition and test the implicit form At $x = 1, y = 0$, the equation gives $C = -1/4$. Consequently,

$$4y^2 = x^2 - x^{2/5}, \qquad x^2 - 4y^2 = x^{2/5}.$$

Raise this last relation to the fifth power:

$$\left|x^2 - 4y^2\right|^5 = x^2.$$

The absolute value is compatible with the positive expression on the branch through $(1, 0)$. To verify the differential relation before taking powers, differentiate $F = x^2 - 4y^2 = x^{2/5}$:

$$2x\,dx - 8y\,dy = \frac{2F}{5x}\,dx.$$

Multiplication by $5x/2$ and substitution of F recover $4(x^2 + y^2)dx - 20xy\,dy = 0$, exactly four times the given equation. The curve has a vertical tangent at the initial point and is naturally expressed implicitly there. Options (B) and (C) have the wrong coefficient of y^2; (D) has the wrong power and right-hand side.

Step 3: Conclusion The required implicit solution through $(1, 0)$ is $\left|x^2 - 4y^2\right|^5 = x^2$. Hence, the correct option is (A).

Quick Tip to Solve: When a differential equation contains $y\,dy$ and y^2, try $u = y^2$ first; it often reduces a homogeneous equation to a short linear one.

6. Let $|\cos\theta\cos(60-\theta)\cos(60+\theta)| \le \frac{1}{8}, \theta\epsilon[0, 2\pi]$. Then, the sum of all $\theta \in [0, 2\pi]$, where $\cos 3\theta$ attains its maximum value, is : [4 marks]

(A) 6π

(B) 9π

(C) 18π

(D) 15π

Correct Answer: (A) 6π

Solution:

Concept:

- The product identity $4\cos\theta\cos(60^\circ - \theta)\cos(60^\circ + \theta) = \cos 3\theta$ reduces the constraint to a bound on one cosine.
- Under $|\cos 3\theta| \le 1/2$, the greatest permitted value is $1/2$. Solve $\cos 3\theta = 1/2$ across the full stated

interval, including every period.

Step 1: Reduce the product constraint Use the product formula with $60^\circ = \pi/3$:

$$\begin{aligned}\cos(60^\circ - \theta)\cos(60^\circ + \theta) &= \cos^2\theta - \sin^2 60^\circ \\ &= \cos^2\theta - \frac{3}{4}.\end{aligned}$$

Multiplying by $\cos\theta$ gives $\cos^3\theta - \frac{3}{4}\cos\theta = \frac{1}{4}\cos 3\theta$. Thus the inequality is

$$\frac{1}{4}|\cos 3\theta| \le \frac{1}{8} \quad \Longleftrightarrow \quad |\cos 3\theta| \le \frac{1}{2}.$$

The maximum must obey this constraint, so it is $1/2$, rather than the unrestricted cosine maximum 1. The angles written as 60 use the standard degree-angle identity; the variable interval is expressed in radians.

Step 2: List all maximizers and add them Since $0 \le \theta \le 2\pi$, the angle 3θ ranges from 0 to 6π. The solutions of $\cos 3\theta = 1/2$ in these three full periods are

$$3\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}.$$

After dividing by 3,

$$\theta = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}.$$

Neither endpoint of the original interval satisfies the constraint. The six listed angles pair to 2π three times, so their total is

$$\frac{(1+5+7+11+13+17)\pi}{9} = 6\pi.$$

Thus 9π, 18π and 15π in the other choices do not equal the sum over the complete set of permitted maximizers.

Step 3: Conclusion The sum of all angles attaining the constrained maximum is 6π. Hence, the correct option is (A).

Quick Tip to Solve: First find the largest cosine value permitted by the inequality. Then solve for all angles over the expanded interval for 3θ; symmetric pairs provide a quick sum check.

7. If the sum of the series $\frac{1}{1\cdot(1+d)} + \frac{1}{(1+d)(1+2\ d)} + \ldots + \frac{1}{(1+9\ d)(1+10\ d)}$ is equal to 5, then $50\ d$ is equal to : [4 marks]

(A) 5

(B) 10

(C) 15

(D) 20

Correct Answer: (A) 5

Solution:

Concept:

- For $d \neq 0$, consecutive factors give $\frac{1}{(1+kd)(1+(k+1)d)} = \frac{1}{d}\left(\frac{1}{1+kd} - \frac{1}{1+(k+1)d}\right)$.
- A telescoping sum cancels every intermediate reciprocal, leaving only the first and last. Check separately any value excluded by a division made during the method.

Step 1: Rewrite the ten terms as differences There are ten terms, indexed by $k = 0, 1, \ldots, 9$. First consider $d = 0$: every term would be 1, giving sum 10, so $d = 0$ cannot satisfy the question. For $d \neq 0$, subtract adjacent reciprocals:

$$\frac{1}{1+kd} - \frac{1}{1+(k+1)d} = \frac{d}{(1+kd)(1+(k+1)d)}.$$

All denominators in the original series must also remain nonzero. Applying the identity term by term gives

$$S = \frac{1}{d}\sum_{k=0}^{9}\left(\frac{1}{1+kd} - \frac{1}{1+(k+1)d}\right).$$

The negative reciprocal in one term cancels the positive reciprocal in the next, so no approximation of the series is needed.

Step 2: Use the stated sum and check the parameter Only the endpoints survive:

$$S = \frac{1}{d}\left(1 - \frac{1}{1+10d}\right) = \frac{1}{d}\frac{10d}{1+10d} = \frac{10}{1+10d}.$$

The given sum is 5, hence

$$\frac{10}{1+10d} = 5 \quad \Longrightarrow \quad 1+10d = 2 \quad \Longrightarrow \quad d = \frac{1}{10}.$$

This value makes all factors $1 + kd$, for $0 \leq k \leq 10$, positive, so it violates no original denominator restriction. Substitution into the telescoped result gives $S = 10/2 = 5$, verifying the answer. Finally,

$$50d = 50 \cdot \frac{1}{10} = 5.$$

The other proposed values 10, 15 and 20 would require $d = 1/5, 3/10, 2/5$, which instead give sums $10/3, 5/2, 2$, respectively, and are therefore excluded.

Step 3: Conclusion $50d = 5$. Hence, the correct option is (A).

Quick Tip to Solve: For products of adjacent arithmetic-progression factors, split each reciprocal product into a difference; count the terms carefully to identify the final surviving denominator.

8. The coefficient of x^{70} in $x^2(1+x)^{98}+x^3(1+x)^{97}+x^4(1+x)^{96}+\ldots+x^{54}(1+x)^{46}$ is ${}^{99}C_p-{}^{46}C_q$. Then a possible value of $p+q$ is : [4 marks]

(A) 61

(B) 83

(C) 55

(D) 68

Correct Answer: (B) 83

Solution:

Concept:

- The coefficient of x^r in $(1+x)^n$ is $\binom{n}{r}$. A prefactor x^k reduces the required exponent by k.
- Pascal's identity gives $\binom{n}{30}=\binom{n+1}{31}-\binom{n}{31}$, allowing a finite sum to telescope.

Step 1: Extract the coefficient from each term The general term is $x^k(1+x)^{100-k}$, with $k=2,3,\ldots,54$. Its contribution to the coefficient of x^{70} is

$$\binom{100-k}{70-k}=\binom{100-k}{30}.$$

Here the complementary lower index is constant because $(100-k)-(70-k)=30$. Thus the coefficient sought is the sum of $\binom{n}{30}$ for every integer n from 46 to 98, inclusive.

Step 2: Telescope and use binomial symmetry Applying Pascal's identity term by term gives

$$\begin{aligned}\sum_{n=46}^{98}\binom{n}{30}&=\sum_{n=46}^{98}\left(\binom{n+1}{31}-\binom{n}{31}\right)\\&=\binom{99}{31}-\binom{46}{31}.\end{aligned}$$

All intermediate terms cancel. Since $\binom{99}{31}=\binom{99}{68}$, take $p=68$ and $q=31$. Alternatively $q=15$ by symmetry; the four natural index sums are 62, 46, 99 and 83, of which only 83 appears among the choices. The alternatives 61, 55 and 68 do not result from these indices.

Step 3: Conclusion A possible value is $p+q=68+31=83$. Hence, the correct option is (B).

Quick Tip to Solve: When both indices of a binomial coefficient decrease together, replace the lower index by its complement before applying the hockey-stick identity.

9. The frequency distribution of the age of students in a class of 40 students is given below.

Age	15	16	17	18	19	20
No of Students	5	8	5	12	x	y

If the mean deviation about the median is 1.25, then $4x + 5y$ is equal to : [4 marks]

(A) 43

(B) 46

(C) 44

(D) 47

Correct Answer: (C) 44

Solution:

Concept:

- The median of 40 ordered observations is the average of the 20th and 21st observations.
- Mean deviation about a median M is $\frac{1}{N}\sum f_i|a_i - M|$, where a_i is an age and f_i its frequency.

Step 1: Locate the median and total frequency The known frequencies sum to $5 + 8 + 5 + 12 = 30$, so $x + y = 10$. The cumulative frequencies up to ages 15, 16, 17 and 18 are 5, 13, 18 and 30. Consequently both the 20th and 21st students have age 18, regardless of the values of the remaining nonnegative frequencies. The median is therefore $M = 18$; using the arithmetic mean here would not answer the stated question.

Step 2: Evaluate the absolute deviations The respective distances from 18 are 3, 2, 1, 0, 1 and 2. Hence

$$\frac{5(3) + 8(2) + 5(1) + 12(0) + x + 2y}{40} = 1.25.$$

The known contribution is $15+16+5 = 36$, while the required total is $40(1.25) = 50$. Thus $x+2y = 14$. Subtracting $x + y = 10$ gives $y = 4$, and then $x = 6$. Both are valid frequencies, and they restore the class total to 40.

Step 3: Calculate the requested expression Substitution gives

$$4x + 5y = 4(6) + 5(4) = 24 + 20 = 44.$$

As a check, the deviation sum becomes $36 + 6 + 8 = 50$, giving $50/40 = 1.25$ exactly. The options 43, 46 and 47 disagree with the uniquely determined frequencies.

Step 4: Conclusion The required value is 44. Hence, the correct option is (C).

Quick Tip to Solve: Locate the middle observations using cumulative frequencies first; then absolute deviations provide a second linear equation for the missing frequencies.

10. Let a circle passing through $(2,0)$ have its centre at the point (h,k). Let (x_c, y_c) be the point of intersection of the lines $3x+5y=1$ and $(2+\mathrm{c})x+5\mathrm{c}^2y=1$. If $\mathrm{h}=\lim_{\mathrm{c}\to 1} x_c$ and $\mathrm{k}=\lim_{\mathrm{c}\to 1} y_c$, then the equation of the circle is : [4 marks]

(A) $5x^2+5y^2-4x-2y-12=0$

(B) $25x^2+25y^2-20x+2y-60=0$

(C) $25x^2+25y^2-2x+2y-60=0$

(D) $5x^2+5y^2-4x+2y-12=0$

Correct Answer: (B) $25x^2+25y^2-20x+2y-60=0$

Solution:

Concept:

- A circle with centre (h,k) through $(2,0)$ has equation $(x-h)^2+(y-k)^2=(2-h)^2+k^2$.
- When two lines coincide at the limiting parameter value, solve their intersection for nearby values before taking limits.

Step 1: Solve the two line equations before taking the limit Multiply $3x_c+5y_c=1$ by c^2 and subtract $(2+c)x_c+5c^2y_c=1$. This gives

$$(3c^2-c-2)x_c=c^2-1.$$

Factor both sides for $c\neq 1$ near 1:

$$x_c=\frac{(c-1)(c+1)}{(c-1)(3c+2)}=\frac{c+1}{3c+2}.$$

Substitution in the first line then gives

$$y_c=\frac{1-3x_c}{5}=-\frac{1}{5(3c+2)}.$$

The common factor must be cancelled before evaluation; direct substitution into the original pair would give the same line twice.

Step 2: Find the centre and form the circle Taking the limits now yields $h=2/5$ and $k=-1/25$. Expand the circle equation using the given point:

$$x^2+y^2-2hx-2ky+4h-4=0.$$

Therefore

$$x^2+y^2-\frac{4}{5}x+\frac{2}{25}y-\frac{12}{5}=0.$$

Multiplying by 25 produces the required integer coefficients. Options (A) and (D) have a centre with a different vertical coordinate; option (C) has a different horizontal centre. As a direct check, $(2,0)$ satisfies the resulting equation.

Step 3: Conclusion The equation is $25x^2+25y^2-20x+2y-60=0$. Hence, the correct option is (B).

Quick Tip to Solve: For parameter-dependent lines that coincide in the limit, first factor the intersection formula; the nearby intersection selects the limiting point uniquely.

11. Let $\int \frac{2-\tan x}{3+\tan x}\,dx = \frac{1}{2}\left(\alpha x + \log_e|\beta\sin x + \gamma\cos x|\right) + C$, where C is the constant of integration. Then $\alpha + \frac{\gamma}{\beta}$ is equal to : [4 marks]

(A) 3

(B) 7

(C) 1

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- A quotient with a linear combination of sine and cosine in its denominator can be split into a constant and a logarithmic derivative.
- For nonzero $D(x)$, $\int D'(x)/D(x)\,dx = \log_e|D(x)| + C$.

Step 1: Rewrite and split the numerator Multiply numerator and denominator by $\cos x$, on any interval where the original expression is defined:

$$\frac{2-\tan x}{3+\tan x} = \frac{2\cos x - \sin x}{\sin x + 3\cos x}.$$

Let $D = \sin x + 3\cos x$; then $D' = \cos x - 3\sin x$. Seek constants A, B such that the numerator is $AD + BD'$. Comparing coefficients of sine and cosine gives $A - 3B = -1$ and $3A + B = 2$, so $A = B = 1/2$.

Step 2: Integrate and compare coefficients The decomposition is therefore

$$\frac{2\cos x - \sin x}{\sin x + 3\cos x} = \frac{1}{2} + \frac{1}{2}\frac{\cos x - 3\sin x}{\sin x + 3\cos x}.$$

Integration gives

$$\int \frac{2-\tan x}{3+\tan x}\,dx = \frac{1}{2}\left(x + \log_e|\sin x + 3\cos x|\right) + C.$$

Comparison with the specified form gives $\alpha = 1$ and $\gamma/\beta = 3$. Multiplying both β and γ by a common nonzero constant only changes the constant of integration, leaving this ratio unchanged. Thus the alternatives 3, 7 and 1 cannot equal the required sum.

Step 3: Conclusion We obtain $\alpha + \gamma/\beta = 1 + 3 = 4$. Hence, the correct option is (D).

Quick Tip to Solve: Convert a tangent quotient to sine and cosine, then express its numerator as a combination of the denominator and its derivative.

12. Let α, β be the roots of the equation $x^2 + 2\sqrt{2}x - 1 = 0$. The quadratic equation, whose roots are $\alpha^4 + \beta^4$ and $\frac{1}{10}(\alpha^6 + \beta^6)$, is: [4 marks]

(A) $x^2 - 180x + 9506 = 0$

(B) $x^2 - 195x + 9506 = 0$

(C) $x^2 - 190x + 9466 = 0$

(D) $x^2 - 195x + 9466 = 0$

Correct Answer: (B) $x^2 - 195x + 9506 = 0$

Solution:

Concept:

- For roots α, β of $x^2 + 2\sqrt{2}x - 1 = 0$, Vieta's formulae give $\alpha + \beta = -2\sqrt{2}$, $\alpha\beta = -1$.
- If new roots are u, v, their monic quadratic is $x^2 - (u + v)x + uv = 0$.

Step 1: Calculate the even power sums First find the sum and product of the squared roots:

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 8 + 2 = 10, \qquad \alpha^2\beta^2 = 1.$$

It follows that

$$\alpha^4 + \beta^4 = 10^2 - 2(1) = 98.$$

For the sixth powers, apply $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$ to $a = \alpha^2, b = \beta^2$:

$$\alpha^6 + \beta^6 = 10^3 - 3(1)(10) = 970.$$

This avoids separately evaluating the irrational original roots.

Step 2: Form and check the new quadratic The requested new roots are therefore $u = 98$ and $v = 970/10 = 97$. Their sum is 195, and their product is

$$98(97) = (100 - 2)(100 - 3) = 10000 - 500 + 6 = 9506.$$

The required polynomial is $(x - 98)(x - 97) = x^2 - 195x + 9506$. Option (A) has the wrong sum 180; option (C) has both an incorrect sum and product; option (D) has the correct sum but the incorrect product 9466. The discriminant of the obtained equation is $195^2 - 4(9506) = 1$, confirming that its roots differ by one as required.

Step 3: Conclusion The quadratic equation is $x^2 - 195x + 9506 = 0$. Hence, the correct option is (B).

Quick Tip to Solve: For expressions involving only even powers, treat the squared original roots as a new pair with known sum and product.

13. Let $f(x) = x^2 + 9, g(x) = \frac{x}{x-9}$ and $\mathrm{a} = f \circ g(10), \mathrm{b} = g \circ f(3)$. If e and l denote the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{\mathrm{a}} + \frac{y^2}{\mathrm{b}} = 1$, then $8\mathrm{e}^2 + l^2$ is equal to. [4 marks]

(A) 6

(B) 12

(C) 8

(D) 16

Correct Answer: (C) 8

Solution:

Concept:

- Function composition is evaluated from the inside outward: $(f \circ g)(t) = f(g(t))$.
- For an ellipse with squared semiaxes $A^2 > B^2$, $e^2 = 1 - B^2/A^2$, and the full latus rectum length is $l = 2B^2/A$.

Step 1: Evaluate the denominators of the ellipse For the first composition, $g(10) = 10/(10-9) = 10$, so $a = f(10) = 10^2 + 9 = 109$. For the second, $f(3) = 3^2 + 9 = 18$, so $b = g(18) = 18/(18-9) = 2$. Thus the ellipse is

$$\frac{x^2}{109} + \frac{y^2}{2} = 1.$$

Both denominators are positive and 109 is larger, so the major axis is along the x-axis. Notice that these denominators are squared semiaxes, not the semiaxes themselves.

Step 2: Calculate the eccentricity and full latus rectum Take $A = \sqrt{109}$ and $B = \sqrt{2}$. The focal-distance relation gives $e^2 = (109-2)/109 = 107/109$. The full length of the chord through a focus perpendicular to the major axis is

$$l = \frac{2B^2}{A} = \frac{4}{\sqrt{109}}, \qquad l^2 = \frac{16}{109}.$$

Consequently

$$8e^2 + l^2 = \frac{8(107) + 16}{109} = \frac{872}{109} = 8.$$

Using the semilatus rectum would miss a factor of four in the squared length. The alternatives 6, 12 and 16 do not match the exact sum.

Step 3: Conclusion The required value is 8. Hence, the correct option is (C).

Quick Tip to Solve: Keep ellipse denominators as squared semiaxes until the latus rectum formula is applied, and distinguish the full chord length from half its length.

14. The shortest distance between the lines $\frac{x-3}{4}=\frac{y+7}{-11}=\frac{z-1}{5}$ and $\frac{x-5}{3}=\frac{y-9}{-6}=\frac{z+2}{1}$ is: [4 marks]

(A) $\frac{185}{\sqrt{563}}$

(B) $\frac{187}{\sqrt{563}}$

(C) $\frac{178}{\sqrt{563}}$

(D) $\frac{179}{\sqrt{563}}$

Correct Answer: (B) $\frac{187}{\sqrt{563}}$

Solution:

Concept:

- For lines through points $\mathbf{a}_1, \mathbf{a}_2$ with nonparallel direction vectors $\mathbf{b}_1, \mathbf{b}_2$, the shortest distance is $d = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|/|\mathbf{b}_1 \times \mathbf{b}_2|$.
- The cross product is normal to both directions; projection of the connecting vector onto this common normal gives the perpendicular separation.

Step 1: Read the points and common normal The first line passes through $\mathbf{a}_1 = (3, -7, 1)$ with direction $\mathbf{b}_1 = (4, -11, 5)$. The second passes through $\mathbf{a}_2 = (5, 9, -2)$ with direction $\mathbf{b}_2 = (3, -6, 1)$. Thus $\mathbf{a}_2 - \mathbf{a}_1 = (2, 16, -3)$. Compute

$$\mathbf{b}_1 \times \mathbf{b}_2 = \big(-11 + 30,\ 15 - 4,\ -24 + 33\big) = (19, 11, 9).$$

Its magnitude is $\sqrt{19^2 + 11^2 + 9^2} = \sqrt{563}$, so the two lines are not parallel.

Step 2: Project the connecting vector onto the normal The scalar triple product is

$$(2, 16, -3) \cdot (19, 11, 9) = 38 + 176 - 27 = 187.$$

Taking its absolute value and dividing by the magnitude of the common normal gives

$$d = \frac{187}{\sqrt{563}}.$$

The positive triple product also shows that the lines do not intersect. In reading their points, $y + 7$ means $y = -7$, and $z + 2$ means $z = -2$; missing these signs changes the numerator. Options (A), (C) and (D) retain the same denominator but have incorrect numerators 185, 178 and 179.

Step 3: Conclusion The shortest distance is $187/\sqrt{563}$ units. Hence, the correct option is (B).

Quick Tip to Solve: Read the signs of each point carefully from symmetric line equations, and take the absolute value of the scalar triple product only at the end.

15. A variable line L passes through the point $(3,5)$ and intersects the positive coordinate axes at the points A and B. The minimum area of the triangle OAB, where O is the origin, is : [4 marks]

(A) 35

(B) 25

(C) 30

(D) 40

Correct Answer: (C) 30

Solution:

Concept:

- For positive axis intercepts a, b, the intercept form of a line is $x/a + y/b = 1$. The area of its coordinate triangle is $ab/2$.
- The arithmetic-geometric mean inequality gives $uv \leq (u+v)^2/4$ for positive u, v, with equality exactly when $u = v$.

Step 1: Express the condition on the intercepts Let $A = (a, 0)$ and $B = (0, b)$, where $a, b > 0$. Since the line passes through $(3, 5)$, its intercepts satisfy

$$\frac{3}{a} + \frac{5}{b} = 1.$$

Define $u = 3/a$ and $v = 5/b$. Both are positive and their sum is one. This change of variables converts the geometric constraint into a fixed-sum condition.

Step 2: Bound the product and find equality By AM-GM,

$$uv \leq \left(\frac{u+v}{2}\right)^2 = \frac{1}{4}.$$

But $uv = 15/(ab)$, so

$$\frac{15}{ab} \leq \frac{1}{4} \quad \Longrightarrow \quad ab \geq 60.$$

Consequently the triangle area satisfies $ab/2 \geq 30$. Equality is achievable: $u = v = 1/2$ gives $a = 6$ and $b = 10$. These intercepts are positive, and $3/6 + 5/10 = 1$, so the corresponding line is permitted.

Step 3: Check the listed areas The value 30 is attained and is therefore the minimum, not merely a lower bound. The proposed values 35 and 40 are larger than the attained minimum, while 25 is impossible because it would require $ab = 50 < 60$. The equality check is essential because the intercepts must remain positive.

Step 4: Conclusion The minimum area of $\triangle OAB$ is $\boxed{30}$ square units, attained for intercepts 6 and 10. Hence, the correct option is (C).

Quick Tip to Solve: For a positive-intercept line through (p, q), apply AM-GM to $p/a + q/b = 1$; the least triangle area is $2pq$.

16. The solution curve, of the differential equation $2y\frac{dy}{dx}+3=5\frac{dy}{dx}$, passing through the point $(0,1)$ is a conic, whose vertex lies on the line : [4 marks]

(A) $2x+3y=-9$

(B) $2x+3y=-6$

(C) $2x+3y=9$

(D) $2x+3y=6$

Correct Answer: (C) $2x+3y=9$

Solution:

Concept:

- Rearrange the differential equation as $(2y-5)\,dy=-3\,dx$, then integrate both sides and apply the given point to determine the constant.
- A conic written as $(y-k)^2=4a(x-h)$ is a horizontal parabola with vertex (h,k). Completing the square identifies this vertex directly.

Step 1: Integrate the differential equation The given equation is

$$2y\frac{dy}{dx}+3=5\frac{dy}{dx}.$$

Move the derivative terms together and integrate:

$$\begin{aligned}(2y-5)\frac{dy}{dx}&=-3,\\ \int(2y-5)\,dy&=\int-3\,dx,\\ y^2-5y&=-3x+C.\end{aligned}$$

The coefficient of the constant term must be retained until the initial point is used; differentiating this implicit equation recovers the stated differential equation.

Step 2: Use the initial point and complete the square At $(x,y)=(0,1)$,

$$1-5=C=-4.$$

Thus $y^2-5y=-3x-4$. Add $25/4$ to both sides to obtain

$$\left(y-\frac{5}{2}\right)^2=-3x+\frac{9}{4}=-3\left(x-\frac{3}{4}\right).$$

This is a parabola opening towards decreasing x. Its vertex is $V=(3/4,5/2)$. The curve through the initial point is a branch of this conic, so it is the conic vertex that the question requests.

Step 3: Test the vertex against the lines Substitute the vertex coordinates into the common left-hand side of the four proposed lines:

$$2\left(\frac{3}{4}\right)+3\left(\frac{5}{2}\right)=\frac{3}{2}+\frac{15}{2}=9.$$

Therefore the line with right-hand side 9 passes through the vertex. The right-hand sides $-9, -6, 6$ do not equal this value and hence their lines cannot contain it.

Step 4: Conclusion The vertex lies on $\boxed{2x + 3y = 9}$. Hence, the correct option is (C).

Quick Tip to Solve: After integrating, complete the square before testing the options; the initial point determines the constant, whereas the square identifies the vertex.

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17. Let $\lambda, \mu \in \mathbf{R}$. If the system of equations

$$3x + 5y + \lambda z = 3$$
$$7x + 11y - 9z = 2$$
$$97x + 155y - 189z = \mu$$

has infinitely many solutions, then $\mu + 2\lambda$ is equal to : [4 marks]

(A) 24

(B) 25

(C) 27

(D) 22

Correct Answer: (B) 25

Solution:

Concept:

- Infinitely many solutions require consistency and coefficient rank less than the number of unknowns. A dependent equation must have the same linear dependence on both coefficients and constants.
- The first two equations are independent because the determinant of their x, y coefficients is $3(11) - 5(7) = -2 \neq 0$. Thus their rank is already two.

Step 1: Find the necessary row combination If the third equation is dependent, write its left-hand side as α times the first plus β times the second. Matching the x and y coefficients gives

$$3\alpha + 7\beta = 97, \qquad 5\alpha + 11\beta = 155.$$

Multiply the first equality by 5 and the second by 3, then subtract:

$$2\beta = 20.$$

Therefore $\beta = 10$, and $3\alpha = 97 - 70 = 27$, so $\alpha = 9$.

Step 2: Match the remaining coefficient and constant The z coefficient must obey the identical combination:

$$9\lambda + 10(-9) = -189.$$

Hence $9\lambda = -99$, giving $\lambda = -11$. Consistency also requires the third constant to be the same

combination of the first two constants:

$$\mu = 9(3) + 10(2) = 47.$$

With these values the third equation is exactly nine times the first plus ten times the second. There are only two independent equations in three unknowns, so one variable is free and infinitely many solutions really do occur.

Step 3: Evaluate the requested expression The question asks for a combination of the parameters, not either parameter alone:

$$\mu + 2\lambda = 47 + 2(-11) = 47 - 22 = 25.$$

The other listed values 24, 27 and 22 fail this substitution. In particular, the minus sign of λ is needed; treating it as positive would violate the z coefficient of the third equation.

Step 4: Conclusion $\lambda = -11$, $\mu = 47$, and $\boxed{\mu + 2\lambda = 25}$. Hence, the correct option is (B).

Quick Tip to Solve: Match two convenient coefficients first to find a row combination, then impose it on the remaining coefficient and on the constants.

18. A ray of light coming from the point $\mathrm{P}(1, 2)$ gets reflected from the point Q on the x-axis and then passes through the point $R(4, 3)$. If the point $S(h, k)$ is such that $PQRS$ is a parallelogram, then hk^2 is equal to: [4 marks]

(A) 60

(B) 70

(C) 80

(D) 90

Correct Answer: (B) 70

Solution:

Concept:

- Reflection at the x-axis can be unfolded by replacing $P = (x, y)$ with its mirror point $P' = (x, -y)$; the reflected path then becomes a straight line.
- For consecutive vertices P, Q, R, S of a parallelogram, its diagonals bisect each other, so $P + R = Q + S$ coordinatewise.

Step 1: Unfold the reflected ray Reflect $P(1, 2)$ in the x-axis to get $P'(1, -2)$. Equal angles of incidence and reflection imply that P', Q, R are collinear. Parameterise the line from P' to $R(4, 3)$:

$$(x, y) = (1, -2) + t(3, 5).$$

At Q, the ordinate is zero, so $-2+5t=0$ and $t=2/5$. It follows that

$$Q=\left(1+\frac{6}{5},0\right)=\left(\frac{11}{5},0\right).$$

This lies between the mirror point and R, as the unfolded path requires.

Step 2: Find the fourth vertex Because the stated order is P,Q,R,S, the opposite vertices are P,R and Q,S. Their diagonal midpoints coincide, yielding

$$S=P+R-Q=\left(1+4-\frac{11}{5},2+3-0\right)=\left(\frac{14}{5},5\right).$$

Thus $h=14/5$ and $k=5$. Using $S=P+Q-R$ instead would correspond to a different vertex order and would not solve the printed question.

Step 3: Calculate and compare the requested value Square the ordinate before multiplying:

$$hk^2=\frac{14}{5}\times 5^2=\frac{14}{5}\times 25=70.$$

The alternatives 60, 80 and 90 do not result from the reflected point and the diagonal identity. As a coordinate check, $P+R=(5,5)$, while $Q+S=(11/5+14/5,5)=(5,5)$, confirming the parallelogram.

Step 4: Conclusion $S=(14/5,5)$, so $\boxed{hk^2=70}$. Hence, the correct option is (B).

Quick Tip to Solve: Reflect one endpoint across the mirror to locate the reflection point by a straight line, then use opposite-vertex sums for the parallelogram.

19. Let the line L intersect the lines $x-2=-y=z-1, 2(x+1)=2(y-1)=z+1$ and be parallel to the line $\frac{x-2}{3}=\frac{y-1}{1}=\frac{z-2}{2}$. Then which of the following points lies on L ? [4 marks]

(A) $\left(-\frac{1}{3},1,-1\right)$
(B) $\left(-\frac{1}{3},-1,1\right)$
(C) $\left(-\frac{1}{3},-1,-1\right)$
(D) $\left(-\frac{1}{3},1,1\right)$

Correct Answer: (A) $\left(-\frac{1}{3},1,-1\right)$

Solution:

Concept:

- A line parallel to a line with symmetric denominators $3,1,2$ has direction vector $(3,1,2)$. Intersection points on the two other lines must differ by a multiple of this vector.
- Use a separate parameter on each given line; an equality such as $2(x+1)=2(y-1)=z+1$ gives direction proportional to $(1,1,2)$.

Step 1: Parameterise the intersection points Let A be the intersection with the first line. Setting

$x-2=-y=z-1=t$ gives

$$A=(2+t,-t,1+t).$$

On the second line, set the common value equal to $2u$, giving

$$B=(-1+u,1+u,-1+2u).$$

These parameterisations retain the minus sign in $-y$ and the factor two in the second line, both of which affect the transversal.

Step 2: Impose the parallel direction Write $B-A=s(3,1,2)$. Comparing coordinates produces

$$\begin{aligned}-3+u-t&=3s,\\1+u+t&=s,\\-2+2u-t&=2s.\end{aligned}$$

Use $s=1+u+t$ in the first equation to obtain $u+2t=-3$. Use it in the third equation to obtain $-2-t=2+2t$, hence $t=-4/3$. Consequently $u=-1/3$. Thus

$$A=\left(\frac{2}{3},\frac{4}{3},-\frac{1}{3}\right),\quad B=\left(-\frac{4}{3},\frac{2}{3},-\frac{5}{3}\right).$$

Their difference is $(-2/3)(3,1,2)$, as required.

Step 3: Test the options on the transversal The desired line is

$$(x,y,z)=\left(\frac{2}{3},\frac{4}{3},-\frac{1}{3}\right)+r(3,1,2).$$

All four options have $x=-1/3$, which forces $r=-1/3$. That parameter gives $y=1$ and $z=-1$. Only the first option has both these coordinates; the other three fail at least one coordinate equation.

Step 4: Conclusion The point $\boxed{(-1/3,1,-1)}$ lies on L. Hence, the correct option is (A).

Quick Tip to Solve: For a transversal with known direction, subtract parameterised intersection points and equate the difference to a scalar multiple of that direction.

20. Let $\overrightarrow{OA} = 2\vec{a}, \overrightarrow{OB} = 6\vec{a} + 5\vec{b}$ and $\overrightarrow{OC} = 3\vec{b}$, where O is the origin. If the area of the parallelogram with adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is 15 sq. units, then the area (in sq. units) of the quadrilateral $OABC$ is equal to: [4 marks]

(A) 32

(B) 38

(C) 35

(D) 40

Correct Answer: (C) 35

Solution:

Concept:

- The area of a parallelogram spanned by vectors $\vec{u}, \vec{v}$ is $|\vec{u} \times \vec{v}|$, while a triangle has half that area.
- The diagonal OB divides the quadrilateral into triangles OAB and OBC. Their cross products point the same way here, so their positive areas add.

Step 1: Find the basic area Its adjacent side vectors are $2\vec{a}$ and $3\vec{b}$. Therefore

$$15 = |(2\vec{a}) \times (3\vec{b})| = 6|\vec{a} \times \vec{b}|,$$

which gives

$$|\vec{a} \times \vec{b}| = \frac{5}{2}.$$

The cross product is nonzero, so $\vec{a}, \vec{b}$ span a genuine plane and the quadrilateral is nondegenerate.

Step 2: Calculate the two triangle areas For triangle OAB, use the two position vectors from O:

$$\begin{aligned}[OAB] &= \frac{1}{2}|(2\vec{a}) \times (6\vec{a} + 5\vec{b})| \\ &= \frac{1}{2}|10\vec{a} \times \vec{b}| = 5 \cdot \frac{5}{2} = \frac{25}{2}.\end{aligned}$$

For triangle OBC,

$$\begin{aligned}[OBC] &= \frac{1}{2}|(6\vec{a} + 5\vec{b}) \times (3\vec{b})| \\ &= \frac{1}{2}|18\vec{a} \times \vec{b}| = 9 \cdot \frac{5}{2} = \frac{45}{2}.\end{aligned}$$

We used $\vec{a} \times \vec{a} = \vec{b} \times \vec{b} = 0$, so only the mixed terms contribute.

Step 3: Add the areas and compare In the $(\vec{a}, \vec{b})$ coordinate plane the vertices, in boundary order, are

$$(0,0), \quad (2,0), \quad (6,5), \quad (0,3).$$

The diagonal OB lies inside this convex quadrilateral, and the triangle orientations agree. Hence

$$[OABC] = \frac{25}{2} + \frac{45}{2} = 35.$$

The values 32, 38 and 40 differ from this exact sum. Adding cross products without the triangle factor $1/2$ would double the answer.

Step 4: Conclusion The quadrilateral has area $\boxed{35}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: For a quadrilateral with one vertex at the origin, split along a diagonal and use half the sum of the two appropriate cross-product magnitudes.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $A = \{2, 3, 6, 7\}$ and $B = \{4, 5, 6, 8\}$. Let R be a relation defined on $A \times B$ by $(a_1, b_1)R(a_2, b_2)$ if and only if $a_1 + a_2 = b_1 + b_2$. Then the number of elements in R is _____. [4 marks]

Solution:

Concept:

- An element of R is an ordered pair $((a_1, b_1), (a_2, b_2))$. Its membership condition is $a_1 + a_2 = b_1 + b_2$.
- Count ordered pairs from A^2 and B^2 by their sum. If the two counts for a sum s are $n_A(s), n_B(s)$, their contribution is $n_A(s)n_B(s)$.

Step 1: List the possible common sums For $A = \{2, 3, 6, 7\}$, the sums of ordered pairs, with multiplicities, are

s	4	5	6	8	9	10	12	13	14
$n_A(s)$	1	2	1	2	4	2	1	2	1

For example, sum 9 comes from $(2, 7), (7, 2), (3, 6), (6, 3)$, so its multiplicity is four. Distinct elements contribute two orders; equal elements contribute only one. The multiplicities add to 16, as expected for four choices of each entry.

Step 2: Count the matching sums from the second set For $B = \{4, 5, 6, 8\}$, the ordered-pair sums are

s	8	9	10	11	12	13	14	16
$n_B(s)$	1	2	3	2	3	2	2	1

Here sum 10 arises from $(4, 6), (6, 4), (5, 5)$; sum 12 arises from $(4, 8), (8, 4), (6, 6)$. Again the total count is 16. Only the common sums $8, 9, 10, 12, 13, 14$ can satisfy the defining equality.

Step 3: Multiply the independent counts and add For each common sum, choose an ordered A-pair and independently an ordered B-pair. These choices uniquely specify $((a_1, b_1), (a_2, b_2))$; no extra factor is needed. Therefore

$$\begin{aligned} |R| &= 2(1) + 4(2) + 2(3) + 1(3) + 2(2) + 1(2) \\ &= 2 + 8 + 6 + 3 + 4 + 2 = 25. \end{aligned}$$

Ordered pairs with unequal entries have already been counted in both orders, so multiplying by two again would overcount.

Step 4: Conclusion The number of elements in the relation is $\boxed{25}$ (a dimensionless count).

Quick Tip to Solve: Group ordered pairs by the sum in the defining condition, then take the dot product of the two frequency lists.

22. Let $\lim_{n\to\infty}\left(\frac{n}{\sqrt{n^4+1}}-\frac{2n}{(n^2+1)\sqrt{n^4+1}}+\frac{n}{\sqrt{n^4+16}}-\frac{8n}{(n^2+4)\sqrt{n^4+16}}+\ldots+\frac{n}{\sqrt{n^4+n^4}}-\frac{2n\cdot n^2}{(n^2+n^2)\sqrt{n^4+n^4}}\right)$ **be** $\frac{\pi}{k}$**, using only the principal values of the inverse trigonometric functions. Then** k^2 **is equal to _____.**

[4 marks]

Solution:

Concept:

- A continuous function on $[0,1]$ satisfies the Riemann-sum identity $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n} g(r/n) = \int_0^1 g(x)\,dx$.
- The substitution $u = \sqrt{2}x/(1+x^2)$ converts a denominator involving $\sqrt{1+x^4}$ into $\sqrt{1-u^2}$. The principal inverse sine lies in $[-\pi/2, \pi/2]$.

Step 1: Combine each pair of terms Denote the given limit by L. In the pair indexed by r, the terms combine as

$$\frac{n}{\sqrt{n^4+r^4}}-\frac{2nr^2}{(n^2+r^2)\sqrt{n^4+r^4}}$$
$$=\frac{n(n^2-r^2)}{(n^2+r^2)\sqrt{n^4+r^4}}=\frac{1}{n}\frac{1-(r/n)^2}{[1+(r/n)^2]\sqrt{1+(r/n)^4}}.$$

This accounts for all pairs from $r=1$ through $r=n$, including the final zero pair. Therefore, with a continuous integrand,

$$L=\int_0^1 \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx.$$

Step 2: Reduce the integral to an inverse sine Put $u = \sqrt{2}x/(1+x^2)$. On $[0,1]$, this increases from 0 to $1/\sqrt{2}$, and

$$du=\frac{\sqrt{2}(1-x^2)}{(1+x^2)^2}\,dx, \qquad \sqrt{1-u^2}=\frac{\sqrt{1+x^4}}{1+x^2}.$$

The positive square root is appropriate because $1+x^2>0$. Thus

$$L=\frac{1}{\sqrt{2}}\int_0^{1/\sqrt{2}}\frac{du}{\sqrt{1-u^2}}=\frac{1}{\sqrt{2}}\left[\sin^{-1}u\right]_0^{1/\sqrt{2}}=\frac{\pi}{4\sqrt{2}}.$$

Here the principal value $\sin^{-1}(1/\sqrt{2})=\pi/4$, so no alternative inverse-trigonometric branch enters the answer.

Step 3: Conclusion Comparing $L=\pi/(4\sqrt{2})$ with π/k gives $k=4\sqrt{2}$. Hence $\boxed{k^2=32}$, a dimensionless value.

Quick Tip to Solve: Combine the two terms before taking the limit; the factor $1-x^2$ points to differentiating $x/(1+x^2)$.

23. Let $f:(0,\pi)\to$ R be a function given by $f(x)=\begin{cases}\left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}, & 0<x<\frac{\pi}{2}\\ \mathrm{a}-8, & x=\frac{\pi}{2}\\ (1+|\cot x)^{\frac{\mathrm{b}}{\mathrm{a}}|\tan x|}, & \frac{\pi}{2}<x<\pi\end{cases}$ where $\mathrm{a},\mathrm{b}\in$ Z. If f is continuous at $x=\frac{\pi}{2}$, then $\mathrm{a}^2+\mathrm{b}^2$ is equal to _____. [4 marks]

Solution:

Concept:

- Continuity at $x=\pi/2$ requires the left-hand limit, the right-hand limit and the assigned value $f(\pi/2)=a-8$ to be equal.
- For $u\to 0^+$, $\log(1+u)/u\to 1$. Taking logarithms evaluates variable powers of the form $(1+u)^{c/u}$.

Step 1: Evaluate the left-hand limit Write $x=\pi/2-t$, with $t\to 0^+$. The tangent functions reduce to

$$\tan 8x=-\tan 8t,\qquad \tan 7x=\cot 7t.$$

Consequently,

$$\frac{\tan 8x}{\tan 7x}=-\tan 8t\tan 7t\longrightarrow 0.$$

Since $8/7$ is a fixed positive base, its zeroth limiting power is 1. Continuity therefore gives

$$a-8=1,\qquad a=9.$$

This substitution also confirms that the denominator a in the right-hand exponent is nonzero.

Step 2: Evaluate the right-hand limit For $x\to\pi/2^+$, $\cot x<0$ and $|\tan x|=-1/\cot x$. With the right-hand base as displayed, put $v=\cot x\to 0^-$. The exponent is $-(b/a)/v$, so the logarithm of that branch has limit

$$\lim_{v\to 0^-}-\frac{b}{a}\frac{\log(1+v)}{v}=-\frac{b}{a}.$$

Thus its limit is $e^{-b/a}$. The base remains positive sufficiently close to $\pi/2$, which justifies the real logarithm. Setting the right-hand limit equal to 1 gives

$$e^{-b/9}=1\quad\Longrightarrow\quad b=0.$$

If the vertical stroke on the cotangent term is read as a magnitude, the logarithmic limit is instead b/a; the same continuity condition still forces $b=0$.

Step 3: Conclusion The integer parameters are $a=9$ and $b=0$. Therefore the required dimensionless value is $\boxed{a^2+b^2=81}$.

Quick Tip to Solve: Use the left-hand limit to find a first; a right-hand exponential limit equals one only when its real exponent is zero.

24. The remainder when 428^{2024} is divided by 21 is _____. [4 marks]

Solution:

Concept:

- If two integers are congruent modulo 21, their positive integer powers are also congruent modulo 21. Reduce the base before attempting a large power.
- A remainder on division by 21 must be an integer from 0 to 20. A short repeating cycle of powers determines which of these values occurs.

Step 1: Reduce the base modulo 21 Divide 428 by 21. Since $21 \times 20 = 420$,

$$428 = 21 \times 20 + 8, \qquad 428 \equiv 8 \pmod{21}.$$

Raising the congruence to the required positive integer exponent preserves it:

$$428^{2024} \equiv 8^{2024} \pmod{21}.$$

This eliminates the large base without changing the remainder. Computing the full power would be unnecessary and would obscure the simple cycle.

Step 2: Find the cycle and use the even exponent Square the reduced base and reduce once more:

$$8^2 = 64 = 3 \times 21 + 1, \qquad 8^2 \equiv 1 \pmod{21}.$$

Hence even powers of 8 have remainder 1, whereas odd powers have remainder 8. The exponent $2024 = 2 \times 1012$ is even, so

$$8^{2024} = (8^2)^{1012} \equiv 1^{1012} = 1 \pmod{21}.$$

As a direct divisibility check, $428^2 - 1 = (428 - 1)(428 + 1) = 427 \times 429$. The first factor is divisible by 7, and the second by 3, so their product is divisible by 21. Raising this congruence to the power 1012 gives the same result.

Step 3: Conclusion The integer 428^{2024} is congruent to 1 modulo 21, and 1 lies in the required remainder range. Therefore the remainder is $\boxed{1}$, with no physical unit.

Quick Tip to Solve: After reducing the base, test its square: when the square is one modulo the divisor, only the parity of the exponent matters.

25. If a function f satisfies $f(m+n) = f(m) + f(n)$ for all $m, n \in \mathbf{N}$ and $f(1) = 1$, then the largest natural number λ such that $\sum_{k=1}^{2022} f(\lambda + k) \leq (2022)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- Additivity on natural numbers, together with $f(1) = 1$, fixes every value of f: repeated addition or induction gives $f(r) = r$ for $r \in \mathbf{N}$.

- The sum of the first N natural numbers is $N(N+1)/2$. After solving an inequality over real numbers, take the largest integer allowed by the bound.

Step 1: Determine the function on its domain Apply the given relation with $m = r$ and $n = 1$. It gives

$$f(r+1) = f(r) + f(1) = f(r) + 1.$$

Starting from $f(1) = 1$, we obtain $f(2) = 2$, $f(3) = 3$, and, by induction,

$$f(r) = r \quad \text{for every } r \in \mathbf{N}.$$

No extension of the function to real or negative arguments is required: $\lambda + k$ is always a natural number for the values in the question.

Step 2: Sum the progression and isolate the parameter Replace each function value by its argument. There are 2022 terms, hence

$$\sum_{k=1}^{2022} f(\lambda + k) = \sum_{k=1}^{2022} (\lambda + k)$$
$$= 2022\lambda + \frac{2022 \times 2023}{2}.$$

The required inequality becomes, after dividing by the positive number 2022,

$$\lambda + \frac{2023}{2} \le 2022, \qquad \lambda \le \frac{2021}{2} = 1010.5.$$

Therefore the greatest natural-number candidate is 1010. It really satisfies the bound because $1010 + 1011.5 = 2021.5 \le 2022$. The next integer fails because $1011 + 1011.5 = 2022.5 > 2022$, confirming maximality rather than merely finding one admissible value.

Step 3: Conclusion The largest natural number satisfying the inequality is $\boxed{\lambda = 1010}$, a dimensionless integer.

Quick Tip to Solve: First use additivity to remove f; then divide the progression sum by its number of terms before taking the integer bound.

26. Let the centre of a circle, passing through the points $(0,0), (1,0)$ and touching the circle $x^2 + y^2 = 9$, be (h,k). Then for all possible values of the coordinates of the centre (h,k), $4\left(h^2 + k^2\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- All points on a circle are at the same distance from its centre. Passing through the origin makes its radius $r = \sqrt{h^2 + k^2}$.
- Two distinct circles with radii r, R and centre separation d touch externally when $d = r + R$, or internally when $d = |r - R|$.

Step 1: Use both points to locate the centre Let the required circle have centre $C = (h, k)$ and radius $r > 0$. Its passage through $(0, 0)$ and $(1, 0)$ gives

$$h^2 + k^2 = r^2, \qquad (1 - h)^2 + k^2 = r^2.$$

Subtract these equations:

$$(1 - h)^2 - h^2 = 0 \quad \Longrightarrow \quad 1 - 2h = 0 \quad \Longrightarrow \quad h = \frac{1}{2}.$$

The given circle $x^2 + y^2 = 9$ has centre at the origin and radius 3. Since the required circle passes through the origin, its centre separation from the given circle is $d = \sqrt{h^2 + k^2} = r$.

Step 2: Apply the possible tangency conditions External tangency would require $r = r + 3$, which is impossible. For internal tangency,

$$r = |r - 3|.$$

If $r \geq 3$, this would give $r = r - 3$, again impossible. Thus $r < 3$, and

$$r = 3 - r \quad \Longrightarrow \quad r = \frac{3}{2}.$$

It follows that

$$h^2 + k^2 = \frac{9}{4}, \qquad k^2 = \frac{9}{4} - \frac{1}{4} = 2, \qquad k = \pm\sqrt{2}.$$

Both centres $(1/2, \sqrt{2})$ and $(1/2, -\sqrt{2})$ work: in either case $d + r = 3$, so the smaller circle touches the larger one from inside. The sign of k does not change the requested expression.

Step 3: Conclusion For every possible centre, $4(h^2 + k^2) = 4(9/4)$. The required numerical value is therefore $\boxed{9}$ in the coordinate units used in the question.

Quick Tip to Solve: A circle through the origin has radius equal to its centre's distance from the origin; combine that equality with tangency before solving for both coordinates.

27. The sum of the square of the modulus of the elements in the set $\{z = \mathrm{a} + \mathrm{ib} : \mathrm{a}, \mathrm{b} \in \mathbf{Z}, z \in \mathbf{C}, |z - 1| \leq 1, |z - 5| \leq |z - 5\mathrm{i}|\}$ is _____. [4 marks]

Solution:

Concept:

- For $z = a + ib$, the squared modulus is $|z|^2 = a^2 + b^2$. Integer real and imaginary parts restrict the points to the integer lattice.
- Both sides of a modulus inequality are nonnegative, so squaring preserves its direction. Distances from two fixed points can then be compared algebraically.

Step 1: Enumerate the integer points in the disk The first restriction is

$$|z - 1| \leq 1 \quad \Longleftrightarrow \quad (a - 1)^2 + b^2 \leq 1.$$

The summands are nonnegative integers. Thus b can only be $-1, 0, 1$. For $b = 0$, the permitted values are $a = 0, 1, 2$; for either $b = 1$ or $b = -1$, the term $b^2 = 1$ forces $a = 1$. The complete candidate list is therefore

$$z \in \{0, 1, 2, 1 + i, 1 - i\}.$$

These include the boundary points because the original inequality is non-strict.

Step 2: Impose the second distance restriction Squaring the second inequality gives

$$\begin{aligned} |z - 5| \le |z - 5i| &\iff (a - 5)^2 + b^2 \le a^2 + (b - 5)^2 \\ &\iff -10a \le -10b \\ &\iff a \ge b. \end{aligned}$$

Check the five pairs: $(0, 0), (1, 0), (2, 0), (1, 1), (1, -1)$ all satisfy $a \ge b$. None is removed, including $1 + i$, which lies on the equality boundary. Their squared moduli are respectively $0, 1, 4, 2, 2$, so

$$\sum |z|^2 = 0 + 1 + 4 + 2 + 2 = 9.$$

This is a sum over distinct complex numbers, so each lattice point contributes exactly once.

Step 3: Conclusion The set contains $0, 1, 2, 1 + i, 1 - i$, and the sum of their squared moduli is the dimensionless value $\boxed{9}$.

Quick Tip to Solve: Enumerate the tiny integer disk first; then square the distance comparison to obtain a simple linear inequality for the remaining filter.

28. Let the set of all positive values of λ, for which the point of local minimum of the function $(1 + x(\lambda^2 - x^2))$ satisfies $\frac{x^2+x+2}{x^2+5x+6} < 0$, be (α, β). Then $\alpha^2 + \beta^2$ is equal to _____. [4 marks]

Solution:

Concept:

- Stationary points satisfy $f'(x) = 0$. At a stationary point, $f''(x) > 0$ proves a local minimum, while $f''(x) < 0$ proves a local maximum.
- A rational expression with an everywhere positive numerator is negative exactly where its denominator is negative. Roots of the denominator are excluded from its domain.

Step 1: Identify the local minimum correctly Write the given function as $f(x) = 1 + \lambda^2 x - x^3$, with $\lambda > 0$. Its derivatives are

$$f'(x) = \lambda^2 - 3x^2, \qquad f''(x) = -6x.$$

The stationary points are

$$x = \pm \frac{\lambda}{\sqrt{3}}.$$

At the negative point, $f''(-\lambda/\sqrt{3}) = 2\sqrt{3}\lambda > 0$, so the local minimum occurs at

$$x_{\min} = -\frac{\lambda}{\sqrt{3}}.$$

The positive stationary point has negative second derivative and is a local maximum; substituting that point would impose the wrong condition.

Step 2: Solve the inequality and translate it to lambda Complete the square in the numerator:

$$x^2 + x + 2 = \left(x + \frac{1}{2}\right)^2 + \frac{7}{4} > 0.$$

Therefore the displayed fraction is negative precisely when its denominator is negative:

$$x^2 + 5x + 6 = (x+2)(x+3) < 0 \quad \Longleftrightarrow \quad -3 < x < -2.$$

Insert the minimum point into this interval:

$$-3 < -\frac{\lambda}{\sqrt{3}} < -2 \quad \Longleftrightarrow \quad 2\sqrt{3} < \lambda < 3\sqrt{3}.$$

Multiplication by the negative number $-\sqrt{3}$ reverses both inequalities. Both endpoints must remain excluded because they make the original denominator zero. Thus the full permitted set is the open interval $(2\sqrt{3}, 3\sqrt{3})$.

Step 3: Conclusion We have $\alpha = 2\sqrt{3}$ and $\beta = 3\sqrt{3}$. Hence the required dimensionless sum is $\boxed{\alpha^2 + \beta^2 = 12 + 27 = 39}$.

Quick Tip to Solve: Classify the two stationary points before using the inequality, and exploit the positive numerator to reduce the sign test to the denominator alone.

29. Let A be a non-singular matrix of order 3. If $\det(3\,\mathrm{adj}(2\,\mathrm{adj}((\det A)A))) = 3^{-13} \cdot 2^{-10}$ and $\det(3\,\mathrm{adj}(2\mathrm{A})) = 2^{\mathrm{m}} \cdot 3^{\mathrm{n}}$, then $|3\,\mathrm{m} + 2\mathrm{n}|$ is equal to _____. [4 marks]

Solution:

Concept:

- For a square matrix X of order three, $\det(cX) = c^3 \det X$, because each of its three rows is multiplied by c.
- For a nonsingular matrix of order three, $\mathrm{adj}\, X = (\det X)X^{-1}$, hence $\det(\mathrm{adj}\, X) = (\det X)^2$. Apply these identities from the innermost matrix outward.

Step 1: Express the nested determinant in one variable Put $d = \det A$, where $d \neq 0$. First handle the scalar multiplying A:

$$\det(dA) = d^3 \det A = d^4.$$

Taking the adjugate squares this determinant, so

$$\det(\operatorname{adj}(dA)) = d^8.$$

The next scalar is 2. It is outside the adjugate already taken, so it contributes its cube:

$$\det(2\operatorname{adj}(dA)) = 2^3 d^8.$$

Step 2: Apply the outer adjugate and scalar Now square the last determinant when taking its adjugate, then multiply by 3^3 for the outer scalar 3:

$$\det(3\operatorname{adj}(2\operatorname{adj}(dA))) = 3^3(2^3 d^8)^2 = 3^3 2^6 d^{16}.$$

Equating this with the given value gives

$$d^{16} = 3^{-16}2^{-16} = 6^{-16}.$$

The sign of d need not be determined: the second expression involves only its square. In particular, $d^2 = 1/36$.

Step 3: Evaluate the second determinant and read the exponents For the matrix $2A$, its determinant is $2^3 d$. Therefore

$$\det(3\operatorname{adj}(2A)) = 3^3(2^3 d)^2 = 3^3 2^6 \frac{1}{36} = 2^4 \cdot 3.$$

Thus $m = 4$ and $n = 1$. The required absolute value is

$$|3m + 2n| = |3(4) + 2(1)| = 14.$$

Step 4: Conclusion The required dimensionless numerical value is $\boxed{14}$.

Quick Tip to Solve: For a third-order matrix, every outside scalar contributes a cube, whereas every adjugate squares the current determinant. Work from the inside outward.

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30. Let a, b and c denote the outcome of three independent rolls of a fair tetrahedral die, whose four faces are marked $1, 2, 3, 4$. If the probability that $ax^2 + bx + c = 0$ has all real roots is $\frac{m}{n}, \gcd(\mathrm{m}, \mathrm{n}) = 1$, then $\mathrm{m} + \mathrm{n}$ is equal to _____. [4 marks]

Solution:

Concept:

- The three independent fair rolls give $4^3 = 64$ equally likely ordered triples (a, b, c). Different orders of a and c are separate outcomes.
- Since $a \in \{1, 2, 3, 4\}$, the equation is always quadratic. Its roots are both real, including a repeated root, exactly when its discriminant satisfies $b^2 - 4ac \geq 0$.

Step 1: Convert the root condition to a counting condition For a fixed middle coefficient b, the

favourable pairs (a, c) satisfy

$$ac \le \frac{b^2}{4}, \qquad a, c \in \{1, 2, 3, 4\}.$$

We include equality, because a zero discriminant gives two coincident real roots. Counting by the value of b keeps the possible events disjoint, so their counts can be added.

Step 2: Count the outcomes for each middle coefficient For $b = 1$, we need $ac \le 1/4$; there are no pairs because $ac \ge 1$. For $b = 2$, we need $ac \le 1$; only $(1, 1)$ works, giving 1 pair. For $b = 3$, we need $ac \le 9/4$; the integer products allowed are 1 and 2. The pairs are $(1, 1), (1, 2), (2, 1)$, giving 3 pairs. For $b = 4$, we need $ac \le 4$. With $a = 1$, there are four choices of c; with $a = 2$, there are two; with $a = 3$ or 4, only $c = 1$ works. This gives

$$4 + 2 + 1 + 1 = 8$$

favourable pairs. These lists also include every permitted equality case.

Step 3: Reduce the probability and form the requested sum The total favourable count and probability are

$$N = 0 + 1 + 3 + 8 = 12, \qquad P = \frac{12}{64} = \frac{3}{16}.$$

Because 3 and 16 have no common factor, the reduced numerator and denominator are $m = 3$ and $n = 16$. Hence

$$m + n = 3 + 16 = 19.$$

The probability is dimensionless, and the final requested sum is an integer.

Step 4: Conclusion The probability is $3/16$, so the required dimensionless value is $\boxed{19}$.

Quick Tip to Solve: Fix the coefficient of the linear term first and count ordered pairs with product at most $b^2/4$. Include discriminant zero; repeated roots are real.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A plane EM wave is propagating along x direction. It has a wavelength of 4 mm. If electric field is in y direction with the maximum magnitude of $60\ \mathrm{Vm}^{-1}$, the equation for magnetic field is : [4 marks]

(A) $\mathrm{B}_z = 2 \times 10^{-7} \sin\left[\frac{\pi}{2}\left(x - 3 \times 10^8 \mathrm{t}\right)\right] \hat{\mathrm{k}}\mathrm{T}$

(B) $\mathrm{B}_z = 2 \times 10^{-7} \sin\left[\frac{\pi}{2} \times 10^3\left(x - 3 \times 10^8 \mathrm{t}\right)\right] \hat{\mathrm{k}}\mathrm{T}$

(C) $\mathrm{B}_z = 60 \sin\left[\frac{\pi}{2}\left(x - 3 \times 10^8 \mathrm{t}\right)\right] \hat{\mathrm{k}}\mathrm{T}$

(D) $\mathrm{B}_x = 60 \sin\left[\frac{\pi}{2}\left(x - 3 \times 10^8 \mathrm{t}\right)\right] \hat{\mathrm{i}}\mathrm{T}$

Correct Answer: (B) $\mathrm{B}_z = 2 \times 10^{-7} \sin\left[\frac{\pi}{2} \times 10^3\left(x - 3 \times 10^8 \mathrm{t}\right)\right] \hat{\mathrm{k}}\mathrm{T}$

Solution:

Concept:

- For a plane electromagnetic wave in vacuum, the peak electric and magnetic fields satisfy $E_0 = cB_0$, where $c = 3 \times 10^8\ \mathrm{m\,s}^{-1}$. The fields oscillate in phase.

- The propagation direction is the direction of $\mathbf{E} \times \mathbf{B}$, and both fields are transverse. A wave moving along positive x has phase $k(x - ct)$, with $k = 2\pi/\lambda$.

Step 1: Find the magnetic amplitude and direction Substitute the given electric amplitude into the amplitude relation:

$$B_0 = \frac{E_0}{c} = \frac{60}{3 \times 10^8} = 2 \times 10^{-7}\,\mathrm{T}.$$

The electric field points along the y axis. To obtain propagation along positive x, the magnetic field must point along the z axis, because $\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}$. A magnetic field parallel to x would not be transverse to the propagation direction.

Step 2: Calculate the wave number with the wavelength in metres Convert millimetres to metres before using the wave-number formula:

$$\lambda = 4 \times 10^{-3}\,\mathrm{m}, \qquad k = \frac{2\pi}{4 \times 10^{-3}} = \frac{\pi}{2} \times 10^3\,\mathrm{m}^{-1}.$$

The factor 10^3 is essential; without it, the wavelength would be 4 metres rather than 4 millimetres. Multiplication of k by $x - ct$, measured in metres, makes the sine argument dimensionless.

Step 3: Assemble the field and check the alternatives Combining the amplitude, wave number and direction gives

$$\mathbf{B} = 2 \times 10^{-7} \sin\left[\frac{\pi}{2} \times 10^3 (x - 3 \times 10^8 t)\right] \hat{\mathbf{k}}\,\mathrm{T}.$$

Option (A) has the correct amplitude and direction but the wrong wave number. Option (C) has both the wrong amplitude and the wrong wave number. Option (D) also directs the magnetic field along the propagation axis, which is impossible for this plane transverse wave.

Step 4: Conclusion The magnetic field is the expression in option (B), with amplitude $2 \times 10^{-7}\,\mathrm{T}$ along z. Hence, the correct option is (B).

Quick Tip to Solve: Check an EM-wave option in three independent ways: $B_0 = E_0/c$, $k = 2\pi/\lambda$ in SI units, and the direction of $\mathbf{E} \times \mathbf{B}$.

32. A galvanometer has a coil of resistance 200Ω with a full scale deflection at $20\mu A$. The value of resistance to be added to use it as an ammeter of range $(0-20)\text{mA}$ is : [4 marks]

(A) 0.40Ω

(B) 0.10Ω

(C) 0.20Ω

(D) 0.50Ω

Correct Answer: (C) 0.20Ω

Solution:

Concept:

- An ammeter must carry the large measured current while only the full-scale galvanometer current passes through its delicate coil. Connect a small shunt resistance S in parallel with the coil resistance G.
- The parallel branches have the same potential difference. If the desired full-scale current is I and the coil current is I_g, then $I_gG = (I - I_g)S$, so $S = I_gG/(I - I_g)$.

Step 1: Convert both current values to amperes The given coil resistance and its full-scale current are

$$G = 200\,\Omega, \quad I_g = 20 \times 10^{-6}\,\text{A}, \quad I = 20 \times 10^{-3}\,\text{A}.$$

At full-scale reading, the current diverted through the shunt is therefore

$$I_s = I - I_g = 0.020 - 0.000020 = 0.01998\,\text{A}.$$

The shunt must bypass almost all of the measured current, rather than increasing the current through the coil.

Step 2: Use the common voltage across the two branches The potential difference across the coil at its full-scale deflection is

$$V_g = I_gG = (20 \times 10^{-6})(200) = 0.004\,\text{V}.$$

The same voltage acts across the shunt, giving

$$S = \frac{V_g}{I_s} = \frac{0.004}{0.01998} = \frac{200}{999}\,\Omega \simeq 0.2002\,\Omega.$$

To the precision of the options, this is $0.20\,\Omega$. A series resistance would instead be useful in a voltmeter conversion and would not protect this coil from a large input current.

Step 3: Check the listed values For a shunt of $0.20\,\Omega$, the approximate shunt current at $0.004\,\text{V}$ is $0.020\,\text{A}$, as required. Values $0.40\,\Omega$ and $0.50\,\Omega$ divert too little current at this voltage; $0.10\,\Omega$ diverts too much. Thus options (A), (B) and (D) do not give the stated range.

Step 4: Conclusion Connect a shunt of approximately $\boxed{0.20\,\Omega}$ in parallel with the galvanometer. Hence, the correct option is (C).

Quick Tip to Solve: For ammeter conversion, use the current through the shunt, $I - I_g$, in the denominator. Convert microamperes and milliamperes before substituting.

33. The dimensional formula of latent heat is : [4 marks]

(A) $\left[\mathrm{M^0LT^{-2}}\right]$

(B) $\left[\mathrm{MLT^{-2}}\right]$

(C) $\left[\mathrm{M^0\ L^2\ T^{-2}}\right]$

(D) $\left[\mathrm{ML^2\ T^{-2}}\right]$

Correct Answer: (C) $\left[\mathrm{M^0\ L^2\ T^{-2}}\right]$

Solution:

Concept:

- In the phase-change relation $Q = mL$, latent heat L denotes the heat required per unit mass without a temperature change. Thus $L = Q/m$, with SI unit $\mathrm{J\,kg^{-1}}$.
- The dimensions of energy follow from work: force times displacement. Force has dimensions $[MLT^{-2}]$, and displacement has dimensions $[L]$, so heat energy has dimensions $[ML^2T^{-2}]$.

Step 1: Identify the physical quantity in the heat relation During melting or boiling, the absorbed heat is proportional to the mass undergoing the change of state:

$$Q = mL.$$

Here Q is a quantity of energy, m is mass, and L is the substance-dependent latent heat per unit mass. There is no temperature difference multiplying L; this distinguishes it from specific heat capacity. Solving for the requested quantity gives

$$L = \frac{Q}{m}.$$

Step 2: Obtain the dimensional formula Use the dimensions of work for heat and divide by mass:

$$[Q] = [\text{force}][\text{distance}] = [MLT^{-2}][L] = [ML^2T^{-2}],$$

$$[L] = \frac{[ML^2T^{-2}]}{[M]} = [M^0L^2T^{-2}].$$

The mass exponent cancels completely. Equivalently, one joule is one $\mathrm{kg\,m^2\,s^{-2}}$, so one joule per kilogram is one $\mathrm{m^2\,s^{-2}}$. This unit check gives the same exponents.

Step 3: Compare with the alternatives Option (A), $[M^0LT^{-2}]$, is the dimension of acceleration and has only one power of length. Option (B), $[MLT^{-2}]$, is force. Option (D), $[ML^2T^{-2}]$, is total heat or energy, before division by the mass. Only option (C) represents energy per unit mass and therefore describes the latent heat coefficient in $Q = mL$.

Step 4: Conclusion The dimensional formula is $\boxed{[M^0L^2T^{-2}]}$, corresponding to $\mathrm{J\,kg^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: Start from $Q = mL$, rather than identifying latent heat with the total heat Q. Dividing the energy dimension by mass removes the mass exponent.

34. A sphere of relative density σ and diameter D has concentric cavity of diameter d. The ratio of $\frac{D}{d}$, if it just floats on water in a tank is : [4 marks]

(A) $\left(\frac{\sigma-2}{\sigma+2}\right)^{1/3}$

(B) $\left(\frac{\sigma+1}{\sigma-1}\right)^{1/3}$

(C) $\left(\frac{\sigma-1}{\sigma}\right)^{1/3}$

(D) $\left(\frac{\sigma}{\sigma-1}\right)^{1/3}$

Correct Answer: (D) $\left(\frac{\sigma}{\sigma-1}\right)^{1/3}$

Solution:

Concept:

- For a closed hollow sphere, the material volume equals the outer sphere volume minus the concentric cavity volume. The density of the material is $\sigma\rho_w$, where ρ_w is water density.
- At the just-floating limit, the sphere is fully immersed and its weight equals the largest buoyant force. Archimedes' principle gives that force as $\rho_w g$ times the complete external volume, including the enclosed cavity.

Step 1: Write the material mass and weight The external diameter is D and the cavity diameter is d. Using the volume formula in terms of diameter gives

$$V_{\text{outer}} = \frac{\pi}{6}D^3, \qquad V_{\text{cavity}} = \frac{\pi}{6}d^3.$$

The cavity reduces the mass but does not reduce the external volume displaced by the closed sphere. Thus its material mass and weight are

$$m = \sigma\rho_w\frac{\pi}{6}(D^3 - d^3), \qquad W = \sigma\rho_w g\frac{\pi}{6}(D^3 - d^3).$$

Step 2: Apply the limiting floating condition When it just floats, water can provide a buoyant force equal to the weight at complete immersion:

$$\rho_w g\frac{\pi}{6}D^3 = \sigma\rho_w g\frac{\pi}{6}(D^3 - d^3).$$

Cancel the common factors to obtain

$$D^3 = \sigma D^3 - \sigma d^3, \qquad (\sigma - 1)D^3 = \sigma d^3.$$

Therefore

$$\frac{D}{d} = \left(\frac{\sigma}{\sigma - 1}\right)^{1/3}.$$

A nonzero closed cavity with this limiting behaviour requires $\sigma > 1$; otherwise the solid material itself is no denser than water.

Step 3: Check the direction of the requested ratio The requested ratio is external diameter divided by cavity diameter, so it must exceed one. Option (C) is its reciprocal and is less than one for $\sigma > 1$.

Options (A) and (B) introduce factors absent from the exact volume balance. As an independent limiting check, very dense material requires a thin shell: as σ increases, the correct ratio tends to one from above, consistent with the derived expression.

Step 4: Conclusion The diameter ratio is $\boxed{D/d = (\sigma/(\sigma-1))^{1/3}}$. Hence, the correct option is (D).

Quick Tip to Solve: The mass depends on the shell volume $D^3 - d^3$, while the limiting displaced water depends on the whole outer volume D^3. Keep these volumes distinct.

35. The volume of an ideal gas $(\gamma = 1.5)$ is changed adiabatically from 5 litres to 4 litres. The ratio of initial pressure to final pressure is : [4 marks]

(A) $\frac{4}{5}$
(B) $\frac{8}{5\sqrt{5}}$
(C) $\frac{2}{\sqrt{5}}$
(D) $\frac{16}{25}$

Correct Answer: (B) $\frac{8}{5\sqrt{5}}$

Solution:

Concept:

- For the adiabatic change of an ideal gas, $PV^\gamma =$ constant, where $\gamma = C_p/C_v$. Apply the relation between the stated initial and final equilibrium states.
- The required ratio is initial pressure divided by final pressure. The volume ratio is therefore V_f/V_i: $P_i/P_f = (V_f/V_i)^\gamma$. Since the gas is compressed, this ratio must be less than one.

Step 1: Apply the adiabatic relation to the two states Let P_i, V_i denote the initial state and P_f, V_f the final state. The given values are

$$V_i = 5\,\text{L}, \qquad V_f = 4\,\text{L}, \qquad \gamma = 1.5 = \frac{3}{2}.$$

The adiabatic equation gives

$$P_i V_i^{3/2} = P_f V_f^{3/2}.$$

Divide both sides by $P_f V_i^{3/2}$, taking care to retain the order of the pressure ratio requested in the question:

$$\frac{P_i}{P_f} = \left(\frac{V_f}{V_i}\right)^{3/2}.$$

Both volumes have the same unit, so the litres cancel and no further conversion is needed.

Step 2: Simplify the fractional power exactly Substitute and separate the rational power into its linear and square-root factors:

$$\frac{P_i}{P_f} = \left(\frac{4}{5}\right)^{3/2} = \frac{4}{5}\sqrt{\frac{4}{5}} = \frac{4}{5}\frac{2}{\sqrt{5}} = \frac{8}{5\sqrt{5}}.$$

Numerically this is about 0.716. Its being less than one is consistent with the pressure increasing when the gas is compressed. Taking the reciprocal would instead answer the reverse pressure ratio.

Step 3: Rule out the other powers Option (A), $4/5$, would result from using exponent 1, as in an isothermal ideal-gas process. Option (C), $2/\sqrt{5}$, uses only exponent $1/2$ and omits the factor $4/5$. Option (D), $16/25$, uses exponent 2 rather than the stated $3/2$. Only option (B) uses the given adiabatic exponent and the correct order of states.

Step 4: Conclusion The initial-to-final pressure ratio is $\boxed{8/(5\sqrt{5})}$, a dimensionless quantity. Hence, the correct option is (B).

Quick Tip to Solve: Write $P_iV_i^{\gamma} = P_fV_f^{\gamma}$ before rearranging. Compression means $P_f > P_i$, so the requested P_i/P_f must be below one.

36. A light emitting diode (LED) is fabricated using GaAs semiconducting material whose band gap is 1.42 eV. The wavelength of light emitted from the LED is : [4 marks]

(A) 1243 nm

(B) 875 nm

(C) 650 nm

(D) 1400 nm

Correct Answer: (B) 875 nm

Solution:

Concept:

- In an LED, an electron recombines with a hole across the semiconductor band gap. The emitted photon has energy approximately equal to the band-gap energy E_g.
- Photon energy and wavelength obey $E = hc/\lambda$, where h is Planck's constant and c is the speed of light. In convenient units, $hc \simeq 1240\,\text{eV nm}$.

Step 1: Identify the photon energy The given GaAs band gap is $E_g = 1.42\,\text{eV}$. The radiative transition releases this energy as one photon. Thus use $E = 1.42\,\text{eV}$; this is an energy, not a frequency. No conversion to joules is needed when hc is expressed in electron-volt nanometres.

Step 2: Calculate the wavelength Applying the photon relation gives

$$\lambda = \frac{hc}{E_g} = \frac{1240\,\text{eV nm}}{1.42\,\text{eV}} \simeq 873.2\,\text{nm}.$$

With the usual rounded physical constants this is approximately $875\,\text{nm}$. The electron-volt units cancel, leaving nanometres. This wavelength lies in the near infrared.

Step 3: Check the alternatives The offered wavelengths 1243, 650 and $1400\,\text{nm}$ correspond to photon energies approximately 1.00, 1.91 and $0.886\,\text{eV}$, respectively. None matches $1.42\,\text{eV}$; the small rounding difference between 873 and $875\,\text{nm}$ does not change the choice.

Step 4: Conclusion The emitted wavelength is approximately $875\,\text{nm}$. Hence, the correct option is (B).

Quick Tip to Solve: For a band gap stated in electron-volts, use $\lambda(\text{nm}) \simeq 1240/E_g(\text{eV})$ and choose the nearest offered wavelength.

37. A particle of mass m moves on a straight line with its velocity increasing with distance according to the equation $v = \alpha\sqrt{x}$, where α is a constant. The total work done by all the forces applied on the particle during its displacement from $x = 0$ to $x = \mathrm{d}$, will be : [4 marks]

(A) $\frac{\mathrm{m}}{2\alpha^2\ \mathrm{d}}$

(B) $\frac{\mathrm{md}}{2\alpha^2}$

(C) $\frac{\mathrm{m}\alpha^2\ \mathrm{d}}{2}$

(D) $2\ \mathrm{m}\alpha^2\ \mathrm{d}$

Correct Answer: (C) $\frac{\mathrm{m}\alpha^2\ \mathrm{d}}{2}$

Solution:

Concept:

- The net work done by all forces equals the change in kinetic energy: $W_{\text{net}} = \frac{1}{2}m(v_f^2 - v_i^2)$. This includes every force through its net effect.
- When speed is specified as a function of position, its endpoint values are sufficient for net work. The alternative relation $a = v\,dv/dx$ gives a useful force check.

Step 1: Find the endpoint speeds The supplied relation is $v = \alpha\sqrt{x}$, so $v^2 = \alpha^2 x$. At the starting position $x = 0$, the particle has $v_i = 0$. At $x = d$, its final squared speed is $v_f^2 = \alpha^2 d$. Squaring the full coefficient is essential; the result contains α^2, not $1/\alpha^2$.

Step 2: Apply the work-energy theorem Substitute these endpoint values into the net-work expression:

$$W_{\text{net}} = \frac{1}{2}m\left(\alpha^2 d - 0\right) = \frac{m\alpha^2 d}{2}.$$

The work is positive because the particle gains kinetic energy. Individual forces need not be identified because the question asks for their total work.

Step 3: Verify the force and reject the other expressions Differentiating $v^2 = \alpha^2 x$ gives $a = \frac{1}{2}d(v^2)/dx = \alpha^2/2$. Hence the constant net force is $m\alpha^2/2$, whose work through d gives the same answer. Since $[\alpha^2] = \mathrm{m\,s^{-2}}$, options (A) and (B), which divide by α^2, do not have the required energy dependence or units. Option (D) is four times the computed work.

Step 4: Conclusion The total work is $\boxed{m\alpha^2 d/2}$. Hence, the correct option is (C).

Quick Tip to Solve: For net work with $v(x)$ given, evaluate v^2 at the two endpoints before trying to calculate forces or time.

38. Given below are two statements :

Statement (I) : When currents vary with time, Newton's third law is valid only if momentum carried by the electromagnetic field is taken into account

Statement (II) : Ampere's circuital law does not depend on Biot-Savart's law.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are false

(B) Statement I is false but Statement II is true

(C) Both Statement I and Statement II are true

(D) Statement I is true but Statement II is false

Correct Answer: (D) Statement I is true but Statement II is false

Solution:

Concept:

- An electromagnetic field can carry momentum. Conservation of momentum applies to matter and the field together; the mechanical forces on separated charges need not be an instantaneous equal-and-opposite pair.
- For steady currents, Ampere's circuital law is $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enclosed}}$. In magnetostatics it can be obtained from the magnetic field given by the Biot–Savart law.

Step 1: Assess Statement I When currents change with time, the electromagnetic interaction propagates through the field. A change in the particles' mechanical momentum can be balanced by an opposite change in the electromagnetic-field momentum. Thus the complete accounting is

$$\frac{d}{dt}\left(\mathbf{P}_{\text{matter}} + \mathbf{P}_{\text{field}}\right) = 0$$

for an isolated system. The intended action-reaction or momentum balance requires including the field contribution. Statement I is true.

Step 2: Assess Statement II The steady-current circuital relation is consistent with, and derivable from, the Biot–Savart magnetic field. One may express this connection as $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ in magnetostatics, and then integrate over a surface using Stokes' theorem. Therefore the claim that Ampere's circuital law does not depend on Biot–Savart's law is false in the magnetostatic context of this statement. Time-varying fields require Maxwell's displacement-current addition.

Step 3: Match the truth values The pair is therefore true for Statement I and false for Statement II. Options (A) and (B) incorrectly reject the need for electromagnetic-field momentum. Option (C) incorrectly accepts the asserted independence in Statement II. Only option (D) has the correct combination.

Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (D).

Quick Tip to Solve: For electromagnetic action-reaction questions, include field momentum; for circuital-law statements, distinguish the steady-current law from its Maxwell extension.

39. A particle moving in a straight line covers half the distance with speed $6\ \mathrm{m/s}$. The other half is covered in two equal time intervals with speeds $9\ \mathrm{m/s}$ and $15\ \mathrm{m/s}$ respectively. The average speed of the particle during the motion is : [4 marks]

(A) 9.2 m/s

(B) 8.8 m/s

(C) 10 m/s

(D) 8 m/s

Correct Answer: (D) 8 m/s

Solution:

Concept:

- Average speed is total distance divided by total elapsed time: $v_{\text{av}} = D/t_{\text{total}}$. It is not generally the arithmetic mean of all speeds listed.
- Equal-time intervals have an arithmetic-mean speed, whereas two equal-distance portions combine through the harmonic mean of their individual average speeds.

Step 1: Combine the equal-time intervals Let each time interval in the second half last τ. The distances travelled in these intervals are 9τ and 15τ, so that half covers 24τ in time 2τ. Its average speed is therefore

$$v_2 = \frac{9\tau + 15\tau}{2\tau} = 12\,\mathrm{m\,s^{-1}}.$$

The two distances within this half are unequal, even though the times are equal.

Step 2: Find the total travel time Let the full journey length be D. The first half takes $t_1 = (D/2)/6 = D/12$, and the second half takes $t_2 = (D/2)/12 = D/24$, with SI units understood. Thus

$$t_{\text{total}} = \frac{D}{12} + \frac{D}{24} = \frac{D}{8}.$$

The slower first half uses twice the time of the faster second half.

Step 3: Calculate and check the average Dividing total distance by total time gives

$$v_{\text{av}} = \frac{D}{D/8} = 8\,\mathrm{m\,s^{-1}}.$$

Equivalently the harmonic mean of 6 and 12 is $2(6)(12)/(6+12) = 8$. None of 9.2, 8.8, or $10\,\mathrm{m\,s^{-1}}$ equals the distance-over-time result, so options (A), (B), and (C) fail. A direct mean of $6, 9, 15$ would wrongly assign them equal durations.

Step 4: Conclusion The average speed for the whole journey is $\boxed{8\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (D).

Quick Tip to Solve: First average the 9 and $15\,\mathrm{m\,s^{-1}}$ equal-time segments; then take the harmonic mean with 6 for the equal-distance halves.

40. Given below are two statements :

Statement (I) : When an object is placed at the centre of curvature of a concave lens, image is formed at the centre of curvature of the lens on the other side.

Statement (II) : Concave lens always forms a virtual and erect image.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are false

(C) Both Statement I and Statement II are true

(D) Statement I is true but Statement II is false

Correct Answer: (A) Statement I is false but Statement II is true

Solution:

Concept:

- A concave lens is a diverging lens. For a real object in front of an isolated concave lens, the emerging rays diverge and their backward extensions meet on the object side.
- With Cartesian signs, $f < 0$, $u < 0$, and the thin-lens formula is $1/v - 1/u = 1/f$. The magnification is $m = v/u$.

Step 1: Test Statement I using the lens formula Write the focal length as $f = -F$ and the real-object distance as $u = -a$, where $F, a > 0$. The image position satisfies

$$\frac{1}{v} = -\frac{1}{F} - \frac{1}{a}, \qquad v = -\frac{Fa}{F+a}.$$

Thus the image is on the object side, not at an equal distance on the other side. A lens has two curved surfaces rather than the single centre of curvature of a spherical mirror; the proposed other-side image rule is incorrect for a concave lens. Statement I is false.

Step 2: Test Statement II The result $v = -Fa/(F+a)$ is negative for every real-object position. Its magnitude is less than F, so the image lies between the lens and its near focus and is virtual. Moreover,

$$m = \frac{v}{u} = \frac{F}{F+a} > 0,$$

which means it is erect; $m < 1$ also shows it is diminished. Thus Statement II is true for the usual real-object situation in elementary lens imaging.

Step 3: Choose the correct pair Option (A) states exactly that Statement I is false and Statement II is true. Option (B) wrongly calls the virtual, erect-image statement false. Option (C) wrongly accepts the other-side image assertion. Option (D) reverses both truth values. The concave-lens result must not be confused with the real image made by a concave mirror at its centre of curvature.

Step 4: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (A).

Quick Tip to Solve: For a real object and concave lens, set both u and f negative; the resulting negative v and positive magnification immediately identify a virtual, erect image.

41. A sample of 1 mole gas at temperature T is adiabatically expanded to double its volume. If adiab constant for the gas is $\gamma = \frac{3}{2}$, then the work done by the gas in the process is : [4 marks]

(A) $\mathrm{RT}[2+\sqrt{2}]$

(B) $\mathrm{RT}[2-\sqrt{2}]$

(C) $\frac{\mathrm{R}}{\mathrm{T}}[2-\sqrt{2}]$

(D) $\frac{T}{R}[2+\sqrt{2}]$

Correct Answer: (B) $\mathrm{RT}[2-\sqrt{2}]$

Solution:

Concept:

- For an ideal gas in a reversible adiabatic process, $TV^{\gamma-1}$ is constant, where $\gamma = C_P/C_V$. Expansion lowers the temperature without heat transfer.
- Using the convention that work done by the gas is positive, the first law with $Q = 0$ gives $W = nC_V(T_i - T_f)$, with $C_V = R/(\gamma - 1)$.

Step 1: Find the final temperature Here $n = 1$, $T_i = T$, $V_f = 2V_i$, and $\gamma = 3/2$. Applying the adiabatic temperature-volume relation gives

$$\frac{T_f}{T} = \left(\frac{V_i}{V_f}\right)^{\gamma-1} = \left(\frac{1}{2}\right)^{1/2} = \frac{1}{\sqrt{2}}.$$

Therefore the gas cools to $T/\sqrt{2}$. The temperature is not constant, so an isothermal-work expression would be inappropriate.

Step 2: Apply the first law For the given adiabatic constant, $C_V = R/(3/2 - 1) = 2R$. The work supplied by the decrease in internal energy is

$$W = 2R\left(T - \frac{T}{\sqrt{2}}\right) = RT(2-\sqrt{2}).$$

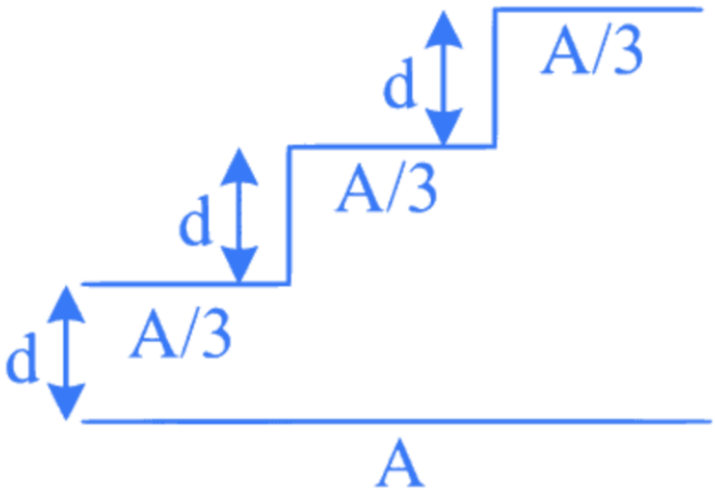
This is positive, as required for expansion, and is less than the initial internal energy $2RT$.

Step 3: Check dimensions and the other options For one mole, the factor RT has units of energy. Options (C) and (D), containing R/T or T/R, lack the correct dimensions. Option (A), $RT(2+\sqrt{2})$, would exceed the initial internal energy despite zero heat input; it also arises from adding, rather than subtracting, the final temperature. Only option (B) agrees with both the energy balance and the cooling relation.

Step 4: Conclusion The gas does work $\boxed{RT(2-\sqrt{2})}$, in joules when SI quantities are used. Hence, the correct option is (B).

Quick Tip to Solve: For adiabatic expansion, calculate T_f from $TV^{\gamma-1} = \text{constant}$, then use the fall in internal energy rather than an isothermal logarithm.

42. A capacitor is made of a flat plate of area A and a second plate having a stair-like structure as shown in figure. If the area of each stair is $\frac{A}{3}$ and the height is d, the capacitance of the arrangement is : [4 marks]



(A) $\frac{11\epsilon_o A}{20\ d}$
(B) $\frac{13\epsilon_o A}{17\ d}$
(C) $\frac{18\epsilon_o A}{11\ d}$
(D) $\frac{11\epsilon_o A}{18\ d}$

Correct Answer: (D) $\frac{11\epsilon_o A}{18\ d}$

Solution:

Concept:

- Ignoring edge effects, two parallel conducting surfaces of overlap area a and separation s have capacitance $C = \varepsilon_0 a/s$.
- All portions of either connected conducting plate share the same potential. Side-by-side overlap regions therefore act as capacitors in parallel, so their capacitances add.

Step 1: Read the three plate separations The figure divides the upper conducting surface into three horizontal regions, each of area $A/3$. The lowest region is a distance d above the flat plate. The next two rises are also d, making the other gaps $2d$ and $3d$. The label d on a rise is an increment, not the common separation of all three regions.

Step 2: Write the component capacitances The flat lower plate forms one common terminal, while all three steps belong to the other terminal. They all experience the same voltage and store separate portions of the total charge. Thus

$$C_1 = \frac{\varepsilon_0 A}{3d}, \qquad C_2 = \frac{\varepsilon_0 A}{6d}, \qquad C_3 = \frac{\varepsilon_0 A}{9d}.$$

These regions are not connected end-to-end, so the series-capacitance rule does not apply.

Step 3: Add and compare with the options For the parallel combination,

$$C = C_1 + C_2 + C_3 = \frac{\varepsilon_0 A}{3d}\left(1 + \frac{1}{2} + \frac{1}{3}\right) = \frac{11\varepsilon_0 A}{18d}.$$

The numerical coefficient is $11/18$; it is neither $11/20$, $13/17$, nor its reciprocal $18/11$. This rules out (A), (B), and (C). The result also lies below $\varepsilon_0 A/d$, the value if the entire upper area were at the smallest gap, as physically expected.

Step 4: Conclusion The equivalent capacitance is $\boxed{11\varepsilon_0 A/(18d)}$. Hence, the correct option is (D).

Quick Tip to Solve: For a stepped conducting plate, list each overlap area and its full gap from the opposite plate; then add $\varepsilon_0 a/s$ because every region has the same voltage.

43. One main scale division of a vernier caliper is equal to m units. If n^{th} division of main scale coincides with $(n+1)^{\mathrm{th}}$ division of vernier scale, the least count of the vernier caliper is : [4 marks]

(A) $\frac{1}{(\mathrm{n}+1)}$
(B) $\frac{m}{(n+1)}$
(C) $\frac{n}{(n+1)}$
(D) $\frac{\mathrm{m}}{\mathrm{n}(\mathrm{n}+1)}$

Correct Answer: (B) $\frac{m}{(n+1)}$

Solution:

Concept:

- The least count of a direct vernier is the difference between one main-scale division (MSD) and one vernier-scale division (VSD). Both are lengths in the same units.
- If n main-scale divisions coincide with $n+1$ vernier divisions, their total physical lengths are equal: $n\,\mathrm{MSD} = (n+1)\,\mathrm{VSD}$.

Step 1: Translate the coincidence into lengths The length of one main-scale division is given as m units. Thus the length occupied by n main-scale divisions is nm units. Exactly the same length contains $n+1$ vernier divisions, so

$$1\,\mathrm{VSD} = \frac{nm}{n+1}\ \text{units}.$$

This is smaller than one MSD, as required for a direct vernier. Do not equate one VSD to the total coincident length; division by the number of vernier intervals is essential.

Step 2: Subtract the two division lengths The small difference between the two interval lengths determines the smallest measurable displacement. Therefore

$$\mathrm{LC} = 1\,\mathrm{MSD} - 1\,\mathrm{VSD}$$

$$= m - \frac{nm}{n+1} = \frac{m(n+1) - nm}{n+1} = \frac{m}{n+1}\ \text{units}.$$

Options (A) and (C) omit the main-scale length m, so they cannot represent the general length requested. Option (D) introduces an additional factor n in the denominator that is absent from the subtraction. These alternatives do not satisfy the stated coincidence relation.

Step 3: Conclusion The least count is $\boxed{m/(n+1)}$ units. Hence, the correct option is (B).

Quick Tip to Solve: First convert the coincidence statement into an equality of total lengths, then subtract one VSD from one MSD; never subtract the counts themselves.

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44. A heavy iron bar, of weight W is having its one end on the ground and the other on the shoulder of a person. The bar makes an angle θ with the horizontal. The weight experienced by the person is : [4 marks]

(A) $W \sin\theta$

(B) W

(C) $\frac{W}{2}$

(D) $W \cos\theta$

Correct Answer: (C) $\frac{W}{2}$

Solution:

Concept:

- A uniform bar has its centre of gravity at its midpoint. Its weight W acts vertically downward there, while the ground and shoulder provide upward support.
- For static equilibrium, both the net vertical force and the net torque are zero. The lever arm of a vertical force is its horizontal distance from the pivot.

Step 1: Take moments about the end on the ground Let the bar length be L, and let the upward force exerted by the person on its upper end be R. Choose the ground contact as the pivot, eliminating the unknown ground reaction from the torque equation. The horizontal distance of the shoulder contact from this pivot is $L\cos\theta$. The midpoint is at half that distance, $(L/2)\cos\theta$. Hence the shoulder force and gravity produce opposite torques:

$$RL\cos\theta = W\frac{L}{2}\cos\theta.$$

Step 2: Find the force exerted on the shoulder For the supported inclined bar, cancelling the common geometric factor gives

$$R = \frac{W}{2}.$$

By Newton's third law, the bar exerts an equal downward force on the person. Thus the weight experienced by the person is $W/2$. The remaining $W/2$ is supported by the ground, so the result also satisfies vertical-force equilibrium. Option (B), W, would apply if the person alone supported the entire bar. Options (A) and (D) incorrectly retain an angle factor: the same $\cos\theta$ multiplies both torque arms and cancels.

Step 3: Conclusion The person supports half the bar's weight, $\boxed{W/2}$. Hence, the correct option is (C).

Quick Tip to Solve: For a uniform bar supported at its two ends by vertical forces, take moments about one end: the midpoint weight gives equal vertical reactions at the two ends.

45. A light unstretchable string passing over a smooth light pulley connects two blocks of masses m_1 and m_2. If the acceleration of the system is $\frac{g}{8}$, then the ratio of the masses $\frac{m_2}{m_1}$ is : [4 marks]

(A) $5:3$

(B) $8:1$

(C) $9:7$

(D) $4:3$

Correct Answer: (C) $9:7$

Solution:

Concept:

- A light inextensible string over a smooth light pulley gives the same tension T on both sides, and the connected masses have equal acceleration magnitudes.
- Newton's second law applied separately to the masses gives $a = g(m_2 - m_1)/(m_2 + m_1)$ when $m_2 > m_1$.

Step 1: Eliminate the common string tension The listed ratios all take $m_2 > m_1$, so let m_2 move downward and m_1 upward with acceleration a. For the heavier mass, weight exceeds tension; for the lighter mass, tension exceeds weight:

$$m_2 g - T = m_2 a, \qquad T - m_1 g = m_1 a.$$

Adding these equations cancels T, giving

$$(m_2 - m_1)g = (m_2 + m_1)a.$$

This uses the sum of the masses as the inertial mass, whereas the difference provides the unbalanced driving force.

Step 2: Insert the acceleration and solve the ratio With $a = g/8$, cancel g and rearrange:

$$\frac{m_2 - m_1}{m_2 + m_1} = \frac{1}{8},$$

$$8m_2 - 8m_1 = m_2 + m_1, \qquad 7m_2 = 9m_1.$$

Thus $m_2/m_1 = 9/7$. Directly checking the alternatives, ratios $5:3$, $8:1$, and $4:3$ yield accelerations $g/4$, $7g/9$, and $g/7$, respectively. None equals the prescribed $g/8$. The ratio $9:7$ gives $(9-7)/(9+7) = 1/8$, as required.

Step 3: Conclusion The required mass ratio is $\boxed{m_2 : m_1 = 9:7}$. Hence, the correct option is (C).

Quick Tip to Solve: For an Atwood machine write $a/g = (r-1)/(r+1)$, where r is the heavier-to-lighter mass ratio; it keeps the driving force and inertia distinct.

46. The energy equivalent of $1\ \mathrm{g}$ of substance is : [4 marks]

(A) $5.6 \times 10^{26}\ \mathrm{MeV}$

(B) $5.6 \times 10^{12}\ \mathrm{MeV}$

(C) $5.6\ \mathrm{eV}$

(D) $11.2 \times 10^{24}\ \mathrm{MeV}$

Correct Answer: (A) $5.6 \times 10^{26}\ \mathrm{MeV}$

Solution:

Concept:

- Einstein's mass-energy relation is $E = mc^2$, where m is in kilograms and $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$ for SI energy in joules.
- One electronvolt equals $1.6 \times 10^{-19}\,\mathrm{J}$; therefore $1\,\mathrm{MeV} = 1.6 \times 10^{-13}\,\mathrm{J}$.

Step 1: Calculate the rest energy in joules The given mass is $1\,\mathrm{g}$, which must first be converted to $10^{-3}\,\mathrm{kg}$. Substituting into the mass-energy relation gives

$$E = (10^{-3})(3.0 \times 10^8)^2\,\mathrm{J}$$

$$= 10^{-3} \times 9.0 \times 10^{16}\,\mathrm{J} = 9.0 \times 10^{13}\,\mathrm{J}.$$

This is the energy equivalent of the complete rest mass, not its kinetic energy. Squaring the speed of light doubles its power of ten and squares its coefficient.

Step 2: Convert joules to megaelectronvolts Since each MeV represents $1.6 \times 10^{-13}\,\mathrm{J}$, divide the energy in joules by that conversion factor:

$$E = \frac{9.0 \times 10^{13}}{1.6 \times 10^{-13}}\,\mathrm{MeV}$$

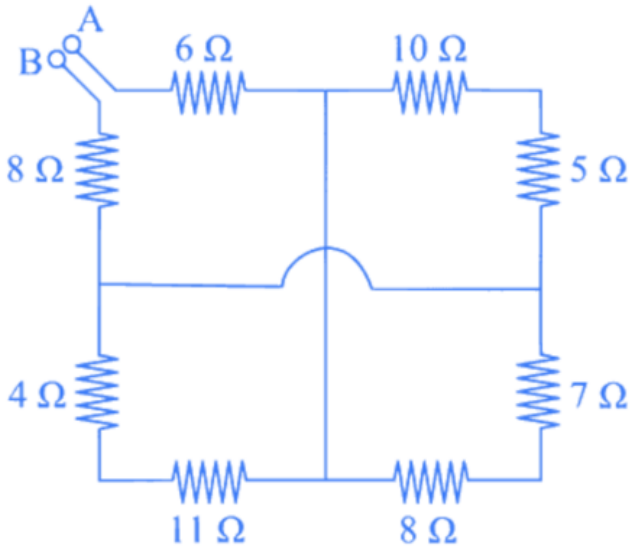
$$= 5.625 \times 10^{26}\,\mathrm{MeV} \simeq 5.6 \times 10^{26}\,\mathrm{MeV}.$$

The exponent becomes $13 - (-13) = 26$. Option (B) is far too small and corresponds to a wrong joule-to-MeV exponent. Option (C) is an atomic-scale energy, inconsistent with a gram of rest mass. Option (D) also fails the explicit conversion, giving a value fifty times smaller than the required rounded value.

Step 3: Conclusion The energy equivalent is $\boxed{5.6 \times 10^{26}\,\mathrm{MeV}}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert grams to kilograms before applying mc^2, then use $1\,\mathrm{MeV} = 1.6 \times 10^{-13}\,\mathrm{J}$; keeping these as separate steps prevents exponent errors.

47. The equivalent resistance between A and B is : [4 marks]



(A) 18 Ω

(B) 27 Ω

(C) 19 Ω

(D) 25 Ω

Correct Answer: (C) 19 Ω

Solution:

Concept:

- Ideal wires connect points at the same potential. A crossing with a bridge-shaped jump is not a junction; the horizontal and vertical wires remain electrically separate.
- Resistors in one unbranched path add in series. Separate paths connecting the same pair of nodes combine in parallel: $1/R = \sum_i 1/R_i$.

Step 1: Identify the nodes and three internal paths Call the central vertical wire node X, joining the top and bottom middle points. Call the middle horizontal wire node Y, joining the two side midpoints. The bridge at the crossing shows that X and Y are not directly joined. Between these nodes there are three paths. The upper-right path has $10\,\Omega$ and $5\,\Omega$ in series; the lower-right path has $8\,\Omega$ and $7\,\Omega$; the lower-left path has $11\,\Omega$ and $4\,\Omega$. Each path therefore has resistance

$$10 + 5 = 8 + 7 = 11 + 4 = 15\,\Omega.$$

Step 2: Combine the internal network and terminal resistors The three $15\,\Omega$ paths have the same endpoints X and Y, so they are in parallel:

$$\frac{1}{R_{XY}} = \frac{1}{15} + \frac{1}{15} + \frac{1}{15} = \frac{1}{5}, \qquad R_{XY} = 5\,\Omega.$$

Terminal A connects to X through $6\,\Omega$, while terminal B connects to Y through $8\,\Omega$. These two

terminal resistors are in series with the internal equivalent. Thus

$$R_{AB} = 6 + 5 + 8 = 19\,\Omega.$$

Options (A), (B), and (D) give 18, 27, and $25\,\Omega$, respectively, and do not equal this node-based reduction. Treating the crossing as a junction would wrongly short the internal network.

Step 3: Conclusion The equivalent resistance is $\boxed{19\,\Omega}$. Hence, the correct option is (C).

Quick Tip to Solve: Redraw a network mentally by nodes before combining resistors. Here the wire bridge keeps two distinct nodes, exposing three equal $15\,\Omega$ parallel branches.

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48. An astronaut takes a ball of mass m from earth to space. He throws the ball into a circular orbit about earth at an altitude of 318.5 km. From earth's surface to the orbit, the change in total mechanical energy of the ball is $x\frac{GM_em}{21R_e}$. The value of x is (take $R_e = 6370$ km) : [4 marks]

(A) 12

(B) 11

(C) 9

(D) 10

Correct Answer: (B) 11

Solution:

Concept:

- With gravitational potential zero at infinity, a mass at rest on Earth's surface has mechanical energy $E_i = -GM_em/R_e$. Earth's rotation is neglected.
- A circular orbit of radius r has kinetic energy $GM_em/(2r)$, potential energy $-GM_em/r$, and total energy $E_f = -GM_em/(2r)$.

Step 1: Express the orbital radius using the Earth radius The height is measured above the surface, so the orbital radius is the Earth radius plus the altitude, not the altitude alone. Here

$$\frac{h}{R_e} = \frac{318.5}{6370} = \frac{1}{20}, \qquad r = R_e + h = \frac{21}{20}R_e.$$

For circular motion, $GM_em/r^2 = mv^2/r$, giving $mv^2/2 = GM_em/(2r)$. Adding this kinetic energy to the gravitational potential produces the negative circular-orbit total energy.

Step 2: Subtract the initial energy from the final energy Using the orbital radius found above,

$$E_f = -\frac{GM_em}{2(21R_e/20)} = -\frac{10GM_em}{21R_e}.$$

The change relative to the initially stationary ball at the surface is

$$\Delta E = E_f - E_i = -\frac{10GM_em}{21R_e} + \frac{GM_em}{R_e}$$

$$= \frac{11GM_em}{21R_e}.$$

It is positive: energy must be supplied to lift the ball and give it orbital speed. Comparing with the stated form identifies $x = 11$. Options (A), (C), and (D) do not follow this subtraction. In particular, the coefficient 10 is associated with the magnitude of the final energy, rather than the increase in energy.

Step 3: Conclusion The coefficient is $\boxed{x = 11}$, a dimensionless value. Hence, the correct option is (B).

Quick Tip to Solve: Use $-GMm/R$ for a stationary surface mass and $-GMm/(2r)$ for a circular-orbit mass; the factor of two already accounts for orbital kinetic energy.

49. A bulb and a capacitor are connected in series across an ac supply. A dielectric is then placed between the plates of the capacitor. The glow of the bulb : [4 marks]

(A) becomes zero

(B) remains same

(C) increases

(D) decreases

Correct Answer: (C) increases

Solution:

Concept:

- A dielectric increases a capacitor's capacitance: for complete filling C becomes KC, with dielectric constant $K > 1$.
- At fixed angular frequency ω, capacitive reactance is $X_C = 1/(\omega C)$. The series RC impedance is $Z = \sqrt{R^2 + X_C^2}$, and bulb power is $P = I_{\text{rms}}^2 R$.

Step 1: Determine how the capacitive reactance changes The bulb provides the resistive part of the series circuit, and the capacitor limits the alternating current through its reactance. Inserting a dielectric increases the capacitance because the polarized medium permits a greater charge for the same capacitor voltage. At the same ac supply frequency,

$$C \longrightarrow KC, \qquad X_C \longrightarrow \frac{1}{\omega KC} = \frac{X_C}{K}.$$

Since $K > 1$, the magnitude of the capacitive reactance decreases. A capacitor does not block ac completely as it does steady dc; its impedance depends on both capacitance and frequency.

Step 2: Relate the current change to the bulb power For a fixed rms supply voltage V, the current is

$$I_{\text{rms}} = \frac{V}{\sqrt{R^2 + X_C^2}}.$$

After insertion, the denominator becomes $\sqrt{R^2 + X_C^2/K^2}$, which is smaller. Hence the rms current through the series bulb increases. Its heating power therefore increases:

$$P = I_{\text{rms}}^2 R = \frac{V^2 R}{R^2 + X_C^2}.$$

This increased power makes the bulb brighter. Option (A) would require zero current, which does not occur. Option (B) ignores the change in reactance, and option (D) predicts the opposite of the current and power change.

Step 3: Conclusion The glow of the bulb increases after inserting the dielectric. Hence, the correct option is (C).

Quick Tip to Solve: In a series RC circuit at fixed voltage and frequency, increasing capacitance lowers reactance and raises current; check bulb brightness through I^2R.

50. A proton, an electron and an alpha particle have the same energies. Their de-Broglie wavelengths will be compared as : [4 marks]

(A) $\lambda_{\text{p}} > \lambda_{\text{e}} > \lambda_{\alpha}$

(B) $\lambda_{\alpha} < \lambda_{\text{p}} < \lambda_{\text{e}}$

(C) $\lambda_{\text{e}} > \lambda_{\alpha} > \lambda_{\text{p}}$

(D) $\lambda_{\text{p}} < \lambda_{\text{e}} < \lambda_{\alpha}$

Correct Answer: (B) $\lambda_{\alpha} < \lambda_{\text{p}} < \lambda_{\text{e}}$

Solution:

Concept:

- For a non-relativistic particle of mass m, momentum p and kinetic energy K, the de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant.
- The relation $K = p^2/(2m)$ gives $\lambda = h/\sqrt{2mK}$. At the same kinetic energy, wavelength varies inversely as the square root of mass.

Step 1: Express the wavelengths at equal energy Let the common kinetic energy be K. Using the momentum corresponding to this energy gives

$$\lambda_{\text{e}} = \frac{h}{\sqrt{2m_{\text{e}}K}}, \qquad \lambda_{\text{p}} = \frac{h}{\sqrt{2m_{\text{p}}K}}, \qquad \lambda_{\alpha} = \frac{h}{\sqrt{2m_{\alpha}K}}.$$

All three expressions have the same numerator and the same energy factor. Therefore, only their masses determine the order; comparing their charges would not answer a question about equal energies.

Step 2: Order the masses and reject the alternatives The electron is much lighter than a proton, while an alpha particle contains two protons and two neutrons and has approximately four times the proton mass:

$$m_{\text{e}} < m_{\text{p}} < m_{\alpha}.$$

Taking reciprocal square roots reverses this ordering:

$$\lambda_e > \lambda_p > \lambda_\alpha.$$

In particular, $\lambda_\alpha \simeq \lambda_p/2$. Option (A) incorrectly makes the proton wavelength larger than the electron wavelength. Option (C) incorrectly places the alpha wavelength above the proton wavelength. Option (D) makes the heaviest particle have the largest wavelength, contrary to the inverse-square-root dependence.

Step 3: Conclusion The required comparison is $\lambda_\alpha < \lambda_p < \lambda_e$. Hence, the correct option is (B).

Quick Tip to Solve: First identify what is equal: for equal kinetic energies use $\lambda \propto m^{-1/2}$, whereas equal momenta would give equal wavelengths.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A square loop of edge length 2 m carrying current of 2 A is placed with its edges parallel to the x-y axis. A magnetic field is passing through the x-y plane and expressed as $\vec{B} = B_0(1+4x)\hat{k}$, where $B_o = 5T$. The net magnetic force experienced by the loop is _____ N. [4 marks]

Solution:

Concept:

- The magnetic force on a current element is $d\vec{F} = I\, d\vec{\ell} \times \vec{B}$. The direction follows the cross product and the force must be added over all four edges.
- Here $B_z = B_0(1+4x)$ varies only with x. Opposite edges parallel to the x-axis sample identical field distributions and have opposite current directions.

Step 1: Cancel the forces on the horizontal edges Let the side length be $L = 2\,\text{m}$, and let the left and right edges have coordinates x_0 and $x_0 + L$. Each horizontal edge spans the same interval of x. The field has no dependence on y, so corresponding elements of the two horizontal edges experience equal field magnitudes. Since their current directions are opposite, their forces are opposite along y and cancel after integration. Thus these edges do not contribute to the resultant force. The unspecified absolute position x_0 will also cancel from the remaining field difference.

Step 2: Subtract the forces on the vertical edges Each vertical edge has constant x, so its field is uniform along that edge. The forces on these edges are opposite along x, with magnitudes $ILB_z(x_0)$ and $ILB_z(x_0+L)$. Hence the net magnitude is

$$F = IL\,|B_z(x_0+L) - B_z(x_0)|\,.$$

The field difference is

$$\Delta B = B_0\,[1+4(x_0+L) - (1+4x_0)] = 4B_0L.$$

Therefore,

$$F = 4IB_0L^2 = 4\times 2\times 5\times 2^2 = 160\,\text{N}.$$

The loop current direction is not specified, so the direction of the resultant cannot be chosen uniquely; reversing the current reverses it. Its magnitude is independent of that direction.

Step 3: Conclusion The net magnetic force has magnitude $160\,\mathrm{N}$, so the required numerical entry is 160.

Quick Tip to Solve: For a rectangular loop in $B_z(x)$, cancel the two horizontal sides first and use $F = IL_y\Delta B$ on the two vertical sides.

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52. At the centre of a half ring of radius $\mathrm{R} = 10\ \mathrm{cm}$ and linear charge density $4\ \mathrm{nC\ m^{-1}}$, the potential is $x\pi\mathrm{V}$. The value of x is _____. [4 marks]

Solution:

Concept:

- Electric potential is a scalar: a charge element dq at distance r contributes $dV = k\,dq/r$, taking zero potential at infinity. Here $k = 1/(4\pi\varepsilon_0) = 9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$.
- For a uniformly charged semicircular wire of radius R and linear density λ, its length is πR, so the total charge is $Q = \lambda\pi R$.

Step 1: Integrate the potential contributions Every point of the half ring is at the same distance R from its centre. Thus the denominator in the potential expression is constant, and the scalar contributions add directly:

$$V = \int \frac{k\,dq}{R} = \frac{k}{R}\int dq = \frac{kQ}{R}.$$

There is no need to resolve contributions into components; such a resolution applies to the electric field, which is a vector, rather than to the potential. Since the stated charge density is positive, the potential relative to infinity is positive as well.

Step 2: Substitute the semicircular charge and compare coefficients Using the semicircle length rather than the full circumference,

$$V = \frac{k(\lambda\pi R)}{R} = k\lambda\pi.$$

The factor R cancels because increasing the radius increases both the charge on the wire and the distance of every charge element by the same factor when λ is fixed. Convert the charge density to SI units:

$$\lambda = 4\,\mathrm{nC\,m^{-1}} = 4 \times 10^{-9}\,\mathrm{C\,m^{-1}}.$$

Then

$$V = (9 \times 10^9)(4 \times 10^{-9})\pi = 36\pi\,\mathrm{V}.$$

Comparison with the specified form $V = x\pi\,\mathrm{V}$ gives the dimensionless coefficient $x = 36$. Using the full-ring length would incorrectly double this result.

Step 3: Conclusion The potential is $36\pi\,\mathrm{V}$; hence the requested dimensionless value is $x = 36$.

Quick Tip to Solve: At the centre of an arc with angle θ in radians, directly use $V = k\lambda\theta$. A semicircle has $\theta = \pi$.

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53. In a Young's double slit experiment, the intensity at a point is $\left(\frac{1}{4}\right)^{\text{th}}$ of the maximum intensity, the minimum distance of the point from the central maximum is _____ μm. (Given : $\lambda = 600$ nm, d $= 1.0$ mm, D $= 1.0$ m) [4 marks]

Solution:

Concept:

- For equal-intensity coherent slits, the intensity is $I = I_{\max}\cos^2(\phi/2)$, where $\phi = 2\pi\delta/\lambda$ and δ is the path difference.
- For small angles, a screen displacement y from the central maximum gives $\delta = dy/D$. Thus $I/I_{\max} = \cos^2(\pi y/\beta)$, where $\beta = \lambda D/d$ is the fringe width.

Step 1: Find the smallest positive phase displacement The required intensity is one-fourth of the central maximum intensity, so

$$\cos^2\left(\frac{\pi y}{\beta}\right) = \frac{1}{4}.$$

Starting from the central maximum at $y = 0$, the cosine decreases from 1. Its first value with magnitude $1/2$ occurs at argument $\pi/3$, rather than at $2\pi/3$. Hence the minimum positive displacement satisfies

$$\frac{\pi y_{\min}}{\beta} = \frac{\pi}{3}, \qquad y_{\min} = \frac{\beta}{3}.$$

The equally close point on the opposite side has coordinate $-y_{\min}$; the question asks for distance, which is positive.

Step 2: Calculate the fringe width and the distance Convert all given quantities into SI units before substitution:

$$\lambda = 600 \times 10^{-9}\,\text{m}, \quad d = 1.0 \times 10^{-3}\,\text{m}, \quad D = 1.0\,\text{m}.$$

The fringe width is

$$\beta = \frac{\lambda D}{d} = \frac{600 \times 10^{-9} \times 1.0}{1.0 \times 10^{-3}} = 600 \times 10^{-6}\,\text{m} = 600\,\mu\text{m}.$$

Consequently,

$$y_{\min} = \frac{600}{3}\,\mu\text{m} = 200\,\mu\text{m}.$$

As a check, this position has path difference $\lambda/3$, phase difference $2\pi/3$, and intensity ratio $\cos^2(\pi/3) = 1/4$, exactly as required.

Step 3: Conclusion The minimum distance from the central maximum is $200\,\mu$m, so the numerical entry is 200.

Quick Tip to Solve: Solve for the smallest positive half-phase in the cosine-squared intensity law; do not confuse a quarter of the intensity with a quarter fringe width.

54. If $\vec{a}$ and $\vec{b}$ makes an angle $\cos^{-1}\left(\frac{5}{9}\right)$ with each other, then $|\vec{a}+\vec{b}| = \sqrt{2}|\vec{a}-\vec{b}|$ for $|\vec{a}| = n|\vec{b}|$ The integer value of n is _____. [4 marks]

Solution:

Concept:

- For vectors of magnitudes a and b separated by angle θ, $|\vec{a} \pm \vec{b}|^2 = a^2 + b^2 \pm 2ab\cos\theta$.
- The given angle implies $\cos\theta = 5/9$. Write the magnitude ratio as $a = nb$; the nonzero reference magnitude b then cancels from the equation.

Step 1: Square the given vector-magnitude equation Both magnitudes are nonnegative, so squaring the stated equation introduces no sign choice:

$$|\vec{a}+\vec{b}|^2 = 2|\vec{a}-\vec{b}|^2.$$

Apply the scalar-product expansions on the two sides:

$$a^2 + b^2 + 2ab\cos\theta = 2(a^2 + b^2 - 2ab\cos\theta).$$

Collecting the squared terms and cross terms gives

$$a^2 + b^2 = 6ab\cos\theta.$$

The factor of 2 on the entire squared difference is essential: it multiplies its cross term as well as the two squared magnitudes.

Step 2: Determine both ratios and retain the integer Insert $a = nb$ and $\cos\theta = 5/9$, then divide by b^2:

$$n^2 + 1 = 6n\left(\frac{5}{9}\right) = \frac{10n}{3}.$$

Thus

$$3n^2 - 10n + 3 = 0, \qquad (3n-1)(n-3) = 0.$$

The two possible positive ratios are $n = 1/3$ and $n = 3$. The question specifically asks for an integer value, so retain 3. The reciprocal roots are expected because the squared-magnitude relation is symmetric in a and b. To check the retained root, take $a = 3b$: the squared sum is $(40/3)b^2$, whereas the squared difference is $(20/3)b^2$. Their ratio is 2, recovering the original factor $\sqrt{2}$.

Step 3: Conclusion The integer magnitude ratio is $n = 3$. This is a dimensionless numerical value.

Quick Tip to Solve: Expand the squared sum and difference before inserting the ratio. Keep both reciprocal roots until the condition that the ratio is an integer is applied.

55. Two persons pull a wire towards themselves. Each person exerts a force of $200\ \mathrm{N}$ on the wire. Young's modulus of the material of wire is $1 \times 10^{11}\ \mathrm{N\ m^{-2}}$. Original length of the wire is $2\ \mathrm{m}$ and the area of cross section is $2\ \mathrm{cm^2}$. The wire will extend in length by _____ μm. [4 marks]

Solution:

Concept:

- Young's modulus is $Y = (F/A)/(\Delta L/L)$, relating tensile stress to longitudinal strain in the elastic range. Here F is the internal tension, A is cross-sectional area and L is original length.
- For equal and opposite pulls on a wire in equilibrium, the tension equals either end force. Adding the magnitudes of the external forces would double-count the force transmitted through a cross-section.

Step 1: Identify the tension and convert the area Each person pulls an end with $200\,\mathrm{N}$, so the tension throughout the wire is $F = 200\,\mathrm{N}$. A cut through the wire would require a force of this magnitude to balance either separated portion. Although the total external force on the whole wire is zero, its tensile stress is not zero. The area must be converted as a squared length:

$$A = 2\,\mathrm{cm^2} = 2(10^{-2}\,\mathrm{m})^2 = 2 \times 10^{-4}\,\mathrm{m^2}.$$

Use the given original length $L = 2\,\mathrm{m}$ and modulus $Y = 1 \times 10^{11}\,\mathrm{N\,m^{-2}}$.

Step 2: Calculate the strain and extension The tensile stress is

$$\sigma = \frac{F}{A} = \frac{200}{2 \times 10^{-4}} = 1.0 \times 10^{6}\,\mathrm{N\,m^{-2}}.$$

The dimensionless longitudinal strain is therefore

$$\epsilon = \frac{\sigma}{Y} = \frac{1.0 \times 10^{6}}{1.0 \times 10^{11}} = 1.0 \times 10^{-5}.$$

Multiplying by the original length gives

$$\Delta L = \epsilon L = (1.0 \times 10^{-5})(2) = 2.0 \times 10^{-5}\,\mathrm{m}.$$

Since $1\,\mu\mathrm{m} = 10^{-6}\,\mathrm{m}$, this is $20\,\mu\mathrm{m}$. The small positive strain is consistent with a slight elongation under a tensile pull, not a contraction.

Step 3: Conclusion The wire extends by $20\,\mu\mathrm{m}$, so the required numerical entry is 20.

Quick Tip to Solve: In a wire pulled equally at both ends, use one end force as its tension, and convert $\mathrm{cm^2}$ with a factor of 10^{-4}.

56. The position, velocity and acceleration of a particle executing simple harmonic motion are found to have magnitudes of $4\ \text{m}, 2\ \text{ms}^{-1}$ and $16\ \text{ms}^{-2}$ at a certain instant. The amplitude of the motion is $\sqrt{x},\ \text{m}$ where x is _____. [4 marks]

Solution:

Concept:

- In simple harmonic motion, signed acceleration and displacement obey $a = -\omega^2 s$. Thus their magnitudes satisfy $|a| = \omega^2|s|$, where ω is angular frequency.
- The speed-displacement relation is $v^2 = \omega^2(A^2 - s^2)$. It follows either from conservation of oscillator energy or from the sine and cosine forms of SHM, with A the amplitude.

Step 1: Find the angular frequency from acceleration Use a displacement symbol s to distinguish the instantaneous displacement from the unknown coefficient x in the requested amplitude. At the stated instant, $|s| = 4\,\text{m}$ and $|a| = 16\,\text{m}\,\text{s}^{-2}$. Therefore,

$$\omega^2 = \frac{|a|}{|s|} = \frac{16}{4} = 4\,\text{s}^{-2}, \qquad \omega = 2\,\text{s}^{-1}.$$

The restoring acceleration points opposite to displacement, but the question gives only magnitudes. Taking their positive ratio is therefore the appropriate way to determine ω^2.

Step 2: Use the speed to determine the amplitude Rearrange the speed-displacement relation to isolate the amplitude squared:

$$A^2 = s^2 + \frac{v^2}{\omega^2}.$$

Substitute the instantaneous speed $v = 2\,\text{m}\,\text{s}^{-1}$, displacement and angular frequency:

$$A^2 = (4\,\text{m})^2 + \frac{(2\,\text{m}\,\text{s}^{-1})^2}{4\,\text{s}^{-2}} = 16\,\text{m}^2 + 1\,\text{m}^2 = 17\,\text{m}^2.$$

Hence $A = \sqrt{17}\,\text{m}$. This amplitude is slightly greater than the instantaneous displacement magnitude, as required because the particle has nonzero speed and has not yet reached a turning point. Inserting $A^2 = 17\,\text{m}^2$ back into the speed equation yields $v^2 = 4(17 - 16) = 4\,\text{m}^2\,\text{s}^{-2}$, confirming the given speed.

Step 3: Conclusion The amplitude is $\sqrt{17}\,\text{m}$, so the requested dimensionless coefficient is $x = 17$.

Quick Tip to Solve: Use $\omega^2 = |a|/|s|$ first, then $A^2 = s^2 + v^2/\omega^2$. The displacement at one instant is not generally the amplitude.

57. A star has 100% helium composition. It starts to convert three $^4\mathrm{He}$ into one $^{12}\mathrm{C}$ via triple alpha process as $^4\mathrm{He} + {}^4\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \mathrm{Q}$. The mass of the star is 2.0×10^{32} kg and it generates energy at the rate of 5.808×10^{30} W. The rate of converting these $^4\mathrm{He}$ to $^{12}\mathrm{C}$ is $\mathrm{n} \times 10^{42}\ \mathrm{s}^{-1}$, where n is _____. [Take, mass of $^4\mathrm{He} = 4.0026\mathrm{u}$, mass of $^{12}\mathrm{C} = 12\mathrm{u}$] [4 marks]

Solution:

Concept:

- The triple-alpha reaction consumes three helium nuclei and produces one carbon nucleus. Its released energy is the mass defect multiplied by c^2.
- For one reaction, $Q = \Delta m c^2$; the luminosity is $P = \dot{N}Q$, where $\dot{N}$ counts complete fusion reactions per second. Use $1\,\mathrm{u}\,c^2 \simeq 931\,\mathrm{MeV}$ and $1\,\mathrm{MeV} = 1.6 \times 10^{-13}\,\mathrm{J}$.

Step 1: Find the mass defect of one fusion event Three helium atoms supply the mass of the reactants. Carbon is the product, so subtract its mass from their combined mass. The atomic electrons balance on both sides, allowing the quoted atomic masses to be used directly.

$$\Delta m = 3(4.0026) - 12 = 0.0078\,\mathrm{u}.$$

This positive difference is the mass converted into released energy, rather than the full mass of the three helium atoms.

Step 2: Convert the released energy into joules Apply mass-energy equivalence to this mass difference:

$$Q = 0.0078 \times 931\,\mathrm{MeV} = 7.2618\,\mathrm{MeV}.$$

$$Q = 7.2618 \times 1.6 \times 10^{-13}\,\mathrm{J} \simeq 1.162 \times 10^{-12}\,\mathrm{J}.$$

Each complete triple-alpha event releases this energy. The total stellar mass is unnecessary for the instantaneous reaction rate; it would matter for estimating how long the fuel lasts.

Step 3: Divide the power by the energy per reaction Power measures energy released per second. Therefore,

$$\dot{N} = \frac{5.808 \times 10^{30}}{1.162 \times 10^{-12}} \simeq 5.0 \times 10^{42}\,\mathrm{s}^{-1}.$$

The rate in the question is expressed as the number of complete conversions forming carbon nuclei, not as the number of individual helium nuclei consumed. The latter would be three times as large.

Step 4: Conclusion The conversion rate is $5.0 \times 10^{42}\,\mathrm{s}^{-1}$, so the required dimensionless coefficient is $\boxed{n = 5}$.

Quick Tip to Solve: Find the energy of one complete triple-alpha event first; divide luminosity by that energy, without multiplying the event rate by three.

58. A string is wrapped around the rim of a wheel of moment of inertia $0.40\ \text{kgm}^2$ and radius $10\ \text{cm}$. The wheel is free to rotate about its axis. Initially the wheel is at rest. The string is now pulled by a force of $40\ \text{N}$. The angular velocity of the wheel after $10\ \text{s}$ is $x\ \text{rad/s}$, where x is _____. [4 marks]

Solution:

Concept:

- A tangential force on a wheel produces torque $\tau = Fr$, where r is the radius perpendicular to the force.
- For fixed moment of inertia, $\tau = I\alpha$. A constant torque produces constant angular acceleration, so $\omega = \omega_0 + \alpha t$.

Step 1: Convert the radius and calculate torque The string pulls along the tangent to the rim, giving a perpendicular lever arm equal to the radius. First express the radius in metres:

$$r = 10\,\text{cm} = 0.10\,\text{m}.$$

Then the constant applied torque is

$$\tau = Fr = 40 \times 0.10 = 4.0\,\text{N m}.$$

The specified freedom to rotate about the axis allows this torque to accelerate the wheel; no opposing frictional torque is given.

Step 2: Find the angular acceleration Use the rotational form of Newton's second law with the given moment of inertia about the wheel's own axis:

$$\alpha = \frac{\tau}{I} = \frac{4.0}{0.40} = 10\,\text{rad s}^{-2}.$$

Both force and radius remain constant as the string unwinds, so this angular acceleration remains constant. There is no need to infer a wheel mass or assume a solid-disc mass distribution, because the moment of inertia is supplied directly.

Step 3: Calculate the angular velocity after ten seconds The wheel starts from rest, so its initial angular velocity is zero. Apply the constant-acceleration equation:

$$\omega = \omega_0 + \alpha t = 0 + (10)(10) = 100\,\text{rad s}^{-1}.$$

This is angular velocity, not tangential speed. As a check, the rim speed would be $r\omega = 10\,\text{m s}^{-1}$, and the rotational kinetic energy would be $I\omega^2/2 = 2000\,\text{J}$, equal to the work $\tau(\alpha t^2/2)$.

Step 4: Conclusion The wheel reaches $\boxed{\omega = 100\,\text{rad s}^{-1}}$; hence the requested numerical coefficient is $\boxed{x = 100}$.

Quick Tip to Solve: For a string pulled tangentially at constant force, combine the two laws directly as $\omega = Frt/I$ when the wheel starts at rest.

59. When a coil is connected across a $20\ \text{V}$ dc supply, it draws a current of $5\ \text{A}$. When it is connected across $20\ \text{V}, 50\ \text{Hz}$ ac supply, it draws a current of $4\ \text{A}$. The self inductance of the coil is _____ mH. (Take $\pi = 3$) [4 marks]

Solution:

Concept:

- A real coil has resistance R and self-inductance L. At steady dc, its induced emf vanishes and the current is determined by R.
- On an ac supply the impedance magnitude is $Z = \sqrt{R^2 + X_L^2}$, with inductive reactance $X_L = 2\pi f L$. Ac voltage and current are compared using rms values.

Step 1: Determine the resistance from steady dc Once the dc current is steady, there is no changing magnetic flux through the coil. Thus the inductive part does not contribute to the steady voltage drop. Ohm's law gives

$$R = \frac{V_{\text{dc}}}{I_{\text{dc}}} = \frac{20}{5} = 4\,\Omega.$$

This resistance remains the resistive part of the coil when it is connected to the ac source; the lower ac current is caused by the additional inductive reactance.

Step 2: Separate reactance from ac impedance For the ac connection, first calculate the magnitude of the total impedance:

$$Z = \frac{V_{\text{ac}}}{I_{\text{ac}}} = \frac{20}{4} = 5\,\Omega.$$

The resistive and inductive voltages are perpendicular phasors. Therefore their magnitudes add in quadrature, not arithmetically:

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{25 - 16} = 3\,\Omega.$$

Subtracting $Z - R$ would incorrectly ignore this phase difference.

Step 3: Calculate the self-inductance Insert the stated frequency and the required approximation $\pi = 3$ into the reactance formula:

$$L = \frac{X_L}{2\pi f} = \frac{3}{2 \times 3 \times 50} = 0.010\,\text{H}.$$

Finally convert henries into millihenries using $1\,\text{H} = 1000\,\text{mH}$:

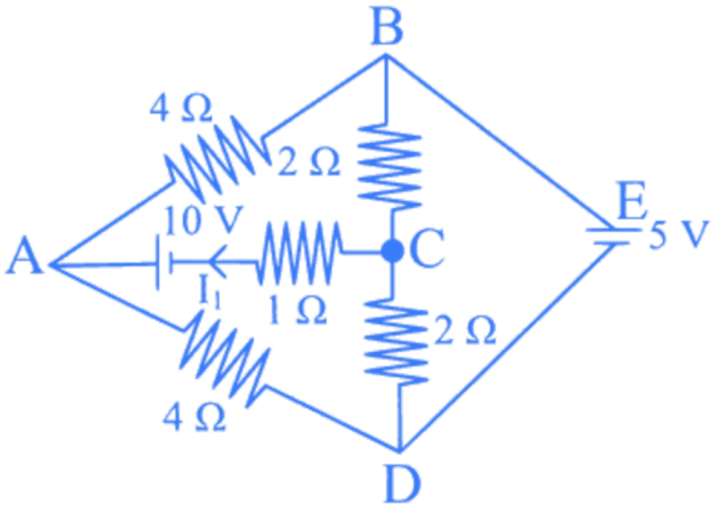
$$L = 0.010 \times 1000 = 10\,\text{mH}.$$

The result is positive, as required for the passive coil.

Step 4: Conclusion The self-inductance of the coil is $\boxed{10\,\text{mH}}$, so the numerical entry is $\boxed{10}$.

Quick Tip to Solve: Use dc to find resistance, ac to find impedance, and the right triangle $Z^2 = R^2 + X_L^2$ to isolate the inductive part.

60. The current flowing through the 1Ω resistor is $\frac{n}{10}$ A. The value of n is _____. [4 marks]



Solution:

Concept:

- Kirchhoff's junction rule requires the algebraic sum of currents leaving a node to vanish. A branch current is the potential difference across its resistor divided by its resistance.
- A battery causes a potential change fixed by its polarity. The 10 V cell has its positive terminal toward A; the 5 V source maintains the potential difference between B and D.

Step 1: Assign node potentials and the central current Take $V_D = 0$ and $V_B = 5\,\text{V}$, following the 5 V battery in the diagram. Let the remaining node potentials be V_A and V_C. Define the current i in the central branch as positive from A toward C. Going from A toward C crosses the 10 V battery from positive to negative, so

$$i = \frac{V_A - V_C - 10}{1} = V_A - V_C - 10.$$

A negative value will indicate that the actual current follows the arrow from C toward A.

Step 2: Apply the junction rule at A and C At A, currents into the two outer resistors and the central branch satisfy

$$\frac{V_A - 5}{4} + \frac{V_A}{4} + (V_A - V_C - 10) = 0.$$

Collecting terms gives

$$\frac{3}{2}V_A - V_C = \frac{45}{4}.$$

At C, include both 2 ohm resistors and the reverse of the central-branch current:

$$\frac{V_C - 5}{2} + \frac{V_C}{2} + (V_C - V_A + 10) = 0,$$

$$-V_A + 2V_C = -\frac{15}{2}.$$

All potentials are measured in volts and the resulting currents in amperes.

Step 3: Solve for the current and its requested coefficient The second equation gives $V_A = 2V_C + 15/2$.

Substitute this into the first equation:

$$\frac{3}{2}\left(2V_C + \frac{15}{2}\right) - V_C = \frac{45}{4},$$

$$2V_C = 0, \qquad V_C = 0, \qquad V_A = 7.5\,\text{V}.$$

Consequently,

$$i = 7.5 - 0 - 10 = -2.5\,\text{A}.$$

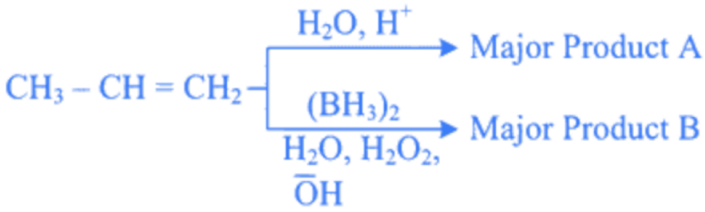
Its magnitude is $2.5\,\text{A}$, directed from C toward A. Equating $n/10 = 2.5$ gives $n = 25$.

Step 4: Conclusion The current through the $1\,\Omega$ resistor is $2.5\,\text{A}$ from C toward A, so $\boxed{n = 25}$.

Quick Tip to Solve: Keep the central battery's 10 V drop in the branch equation; applying Ohm's law only to $V_A - V_C$ would give the wrong current.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Identify the product A and product B in the following set of reactions. [4 marks]



(A) A: $CH_3CH_2CH_2OH$; B: $CH_3CH(OH)CH_3$

(B) A: $CH_3CH(OH)CH_3$; B: $CH_3CH_2CH_2OH$

(C) A: $CH_3CH_2CH_2OH$; B: $CH_3CH_2CH_2OH$

(D) A: $CH_3CH_2CH_3$; B: $CH_3CH_2CH_3$

Correct Answer: (B) A: $CH_3CH(OH)CH_3$; B: $CH_3CH_2CH_2OH$

Solution:

Concept:

- Acid-catalysed hydration of an unsymmetrical alkene follows Markovnikov addition: the hydroxyl group reaches the more substituted carbon.
- Hydroboration followed by alkaline hydrogen peroxide gives anti-Markovnikov hydration without a carbocation intermediate. The hydroxyl group reaches the less substituted carbon.

Step 1: Identify product A from acid-catalysed hydration The substrate is propene, $CH_3CH{=}CH_2$. In the acid-catalysed reaction, protonation preferentially places the positive charge on the middle carbon, forming the more stable secondary carbocation rather than a primary one. Water attacks this

carbon, and loss of a proton gives the alcohol:

$$CH_3CH{=}CH_2 + H_2O \xrightarrow{H^+} CH_3CH(OH)CH_3.$$

Thus product A is propan-2-ol. The carbon skeleton remains unchanged; the reaction adds the elements of water across the double bond.

Step 2: Identify product B from hydroboration and oxidation In hydroboration, boron bonds preferentially to the terminal, less substituted carbon of propene while hydrogen adds to the middle carbon. Oxidation with hydrogen peroxide in alkaline medium replaces the carbon-boron bond by a carbon-oxygen bond. The overall alcohol therefore has its hydroxyl group at the terminal carbon:

$$CH_3CH{=}CH_2 \xrightarrow[2.\ H_2O_2/OH^-]{1.\ B_2H_6} CH_3CH_2CH_2OH.$$

Hence product B is propan-1-ol. The reaction does not produce propane, because oxidation introduces an oxygen-containing hydroxyl group.

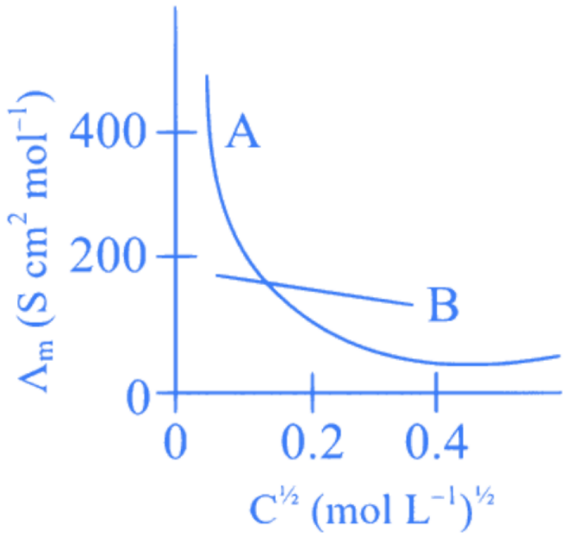
Step 3: Compare the ordered product pair with the options The order matters: A belongs to the acidic-water route and B to the hydroboration-oxidation route. Option (A) reverses these alcohols and is incorrect. Option (C) gives propan-1-ol for both reactions, so it misses the Markovnikov product A. Option (D) gives propane for both reactions and does not describe either hydration. Only option (B) places propan-2-ol first and propan-1-ol second.

Step 4: Conclusion Product A is $CH_3CH(OH)CH_3$ and product B is $CH_3CH_2CH_2OH$. Hence, the correct option is (B).

Quick Tip to Solve: For propene hydration, remember acid gives the middle-carbon alcohol, whereas hydroboration-oxidation gives the terminal-carbon alcohol.

62. The molar conductivity for electrolytes A and B are plotted against $C^{3/2}$ as shown below. Electrolytes A and B respectively are:

[4 marks]



(A) A - weak electrolyte B - weak electrolyte

(B) A - weak electrolyte B - strong electrolyte

(C) A - strong electrolyte B - strong electrolyte

(D) A - strong electrolyte B - weak electrolyte

Correct Answer: (B) A - weak electrolyte B - strong electrolyte

Solution:

Concept:

- A strong electrolyte is almost completely ionised even before dilution. Its molar conductivity varies relatively gently with concentration because interionic effects change.
- A weak electrolyte ionises more extensively on dilution, producing a steep increase of molar conductivity near zero concentration. The characteristic curved trace distinguishes it from a strong electrolyte.

Step 1: Read the behaviour of curve A Curve A rises very sharply as the concentration approaches zero. This is the characteristic behaviour of a weak electrolyte: dilution shifts its ionisation equilibrium toward the separated ions, markedly increasing the number of charge carriers per mole. Therefore, A represents a weak electrolyte. The identifying feature is the pronounced non-linear increase on dilution; the absolute value of conductivity by itself does not establish whether an electrolyte is weak or strong.

Step 2: Read the behaviour of trace B Trace B shows a much smaller, approximately linear variation along the plotted square-root concentration axis. For a dilute strong electrolyte the usual relation is

$$\Lambda_m = \Lambda_m^\circ - K\sqrt{C},$$

where Λ_m° is the limiting molar conductivity and K is positive for the given electrolyte and solvent. Ionisation is already nearly complete, so dilution chiefly reduces interionic hindrance. Thus B represents a strong electrolyte. The supplied graph itself labels its horizontal axis as $C^{1/2}$; its characteristic traces decide the classification.

Step 3: Eliminate the other combinations Option (A) incorrectly calls B weak despite its gently varying strong-electrolyte trace. Option (C) incorrectly calls A strong despite the sharp increase from enhanced ionisation. Option (D) interchanges both classifications. Option (B) is the only combination consistent with A being weak and B being strong. The distinction concerns ionisation behaviour, rather than merely choosing the curve with the greater conductivity at a single concentration.

Step 4: Conclusion A is a weak electrolyte and B is a strong electrolyte. Hence, the correct option is (B).

Quick Tip to Solve: Identify a weak electrolyte by the steep rise of molar conductivity on dilution; do not classify it from one conductivity value alone.

63. Given below are two statements :

Statement (I) : The oxidation state of an element in a particular compound is the charge acquired by its atom on the basis of electron gain enthalpy consideration from other atoms in the molecule.

Statement (II) : $\mathrm{p}\pi - \mathrm{p}\pi$ **bond formation is more prevalent in second period elements over other periods.**

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is correct but Statement II is incorrect

(B) Both Statement I and Statement II are correct

(C) Statement I is incorrect but Statement II is correct

(D) Both Statement I and Statement II are incorrect

Correct Answer: (C) Statement I is incorrect but Statement II is correct

Solution:

Concept:

- Oxidation state is the formal charge assigned after allocating the bonding electrons to the more electronegative atom, with equal sharing for bonds between identical atoms.
- Effective sideways overlap of compact 2p orbitals favours $p\pi$-$p\pi$ bonding among second-period elements. Larger diffuse orbitals in later periods generally give less effective overlap.

Step 1: Evaluate the first statement Statement I replaces the electronegativity rule for assigning bonding electrons by electron gain enthalpy. These are different ideas. Electronegativity describes an atom's tendency to attract shared electrons in a bond; electron gain enthalpy refers to the enthalpy change when an isolated gaseous atom gains an electron. Formal oxidation-state assignment does not directly use that gas-phase enthalpy. For example, in a bond between carbon and oxygen the electrons are assigned formally to the more electronegative oxygen. Thus Statement I is incorrect.

Step 2: Evaluate the second statement Second-period atoms such as carbon, nitrogen and oxygen have small sizes and compact 2p orbitals. Parallel p orbitals on neighbouring atoms can overlap sideways efficiently, forming a pi bond in addition to a sigma bond. Common examples include the carbon-carbon double bond of ethene and the multiple bond in N_2. In heavier congeners the valence orbitals are more diffuse and the bond distances generally larger, reducing efficient sideways overlap. Hence $p\pi$-$p\pi$ bonding is more prevalent in the second period, making Statement II correct.

Step 3: Select the matching truth-value combination The required combination is false for I and true for II. Option (A) gives the opposite combination, so it fails both tests. Option (B) incorrectly accepts the electron-gain-enthalpy wording of I. Option (D) incorrectly rejects the effective 2p sideways-overlap argument in II. Option (C) alone records the two statements correctly. The second statement expresses a general trend rather than asserting that heavier elements can never form pi bonds.

Step 4: Conclusion Statement I is incorrect but Statement II is correct. Hence, the correct option is (C).

Quick Tip to Solve: For oxidation-state definitions, look for electronegativity; for second-period pi bonding, look for efficient overlap of compact 2p orbitals.

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64. $0.05M\ CuSO_4$ **when treated with** $0.01M\ K_2Cr_2O_7$ **gives green colour solution of** $Cu_2Cr_2O_7$**. The two solutions are separated as shown below : [SPM : Semi Permeable Membrane]**

Due to osmosis : [4 marks]



(A) Molarity of $K_2Cr_2O_7$ solution is lowered.

(B) Green colour formation observed on side Y.

(C) Molarity of $CuSO_4$ solution is lowered.

(D) Green colour formation observed on side X.

Correct Answer: (C) Molarity of $CuSO_4$ solution is lowered.

Solution:

Concept:

- A semipermeable membrane permits solvent molecules to pass but prevents the dissolved solute particles from crossing. Osmosis therefore changes concentration without mixing the two salts.
- At the same temperature the osmotic pressure is $\pi = iCRT$, where i counts particles produced per formula unit and C is the molarity. Solvent flows towards the side with the higher osmotic pressure.

Step 1: Compare the two osmotic concentrations Side X contains $0.01\,\mathrm{M}$ potassium dichromate, while side Y contains $0.05\,\mathrm{M}$ copper sulphate. Using their ideal ionic dissociations,

$$\mathrm{K_2Cr_2O_7} \longrightarrow 2\,\mathrm{K^+} + \mathrm{Cr_2O_7{}^{2-}}, \qquad \mathrm{CuSO_4} \longrightarrow \mathrm{Cu^{2+}} + \mathrm{SO_4{}^{2-}}.$$

Thus $i_X = 3$, $i_Y = 2$, and

$$i_X C_X = 3(0.01) = 0.03, \qquad i_Y C_Y = 2(0.05) = 0.10.$$

The copper sulphate side has the larger osmotic pressure; comparing molarity alone is unnecessary because the particle concentrations give the direction directly.

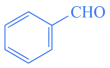
Step 2: Track solvent flow and reject mixing Water moves from X to Y through the membrane. The number of moles of copper sulphate on Y stays the same while its solution volume increases, so $C_Y = n_Y/V_Y$ decreases. Side X loses water and its dichromate solution becomes more concentrated, ruling out (A). Neither copper ions nor dichromate ions traverse the SPM; they cannot meet across it to generate the stated green product. Consequently neither (B) nor (D) follows from osmosis.

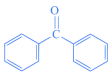
Step 3: Conclusion The molarity of $\mathrm{CuSO_4}$ on side Y is lowered because that side gains solvent. Hence, the correct option is (C).

Quick Tip to Solve: Compare iC on the two sides first; the side with the larger value gains water and becomes diluted, without solute transfer.

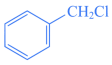
65. Identify major product "X" formed in the following reaction : [4 marks]

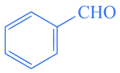


(A) Benzaldehyde, $\mathrm{C_6H_5CHO}$. 

(B) Benzophenone, $\mathrm{C_6H_5COC_6H_5}$. 

(C) Benzoyl chloride, $\mathrm{C_6H_5COCl}$. 

(D) Benzyl chloride, $\mathrm{C_6H_5CH_2Cl}$. 

Correct Answer: (A) Benzaldehyde, $\mathrm{C_6H_5CHO}$. 

Solution:

Concept:

- Benzene reacts with carbon monoxide and hydrogen chloride in the presence of anhydrous $\mathrm{AlCl_3}$ and CuCl by the Gattermann–Koch reaction. This introduces a formyl group, $-\mathrm{CHO}$, onto the aromatic ring.
- The reaction is electrophilic aromatic substitution: an aromatic hydrogen is replaced by the formyl carbon, while the aromatic ring is retained after deprotonation.

Step 1: Recognise the reagent combination The reagents above the arrow are CO and HCl, and the catalysts below the arrow are anhydrous $AlCl_3/CuCl$. Together they provide a formylating electrophile. The Lewis acid activates the reagents and makes the carbon derived from CO electrophilic. This is a specific named reaction, so the presence of HCl must not be mistaken for ordinary chlorination of the ring or side chain.

Step 2: Identify the installed group and compare products Benzene attacks the activated formyl species, giving a sigma complex. Loss of a proton restores aromaticity and hydrolysis of the complex gives benzaldehyde. The overall organic transformation is

$$C_6H_6 + CO \xrightarrow{HCl,\ AlCl_3/CuCl} C_6H_5CHO.$$

The product therefore contains one phenyl group and an aldehyde hydrogen at the carbonyl carbon. Benzophenone in (B) has two phenyl groups and would require a different acylating reagent. Benzoyl chloride in (C) retains chlorine at the acyl carbon, unlike this formylation product. Benzyl chloride in (D) has a saturated $-CH_2Cl$ group and no carbonyl; it is not formed by the given reagent combination.

Step 3: Conclusion X is benzaldehyde, C_6H_5CHO, shown in option (A). Hence, the correct option is (A).

Quick Tip to Solve: The combination CO/HCl with anhydrous $AlCl_3/CuCl$ signals Gattermann–Koch formylation: attach $-CHO$ to the aromatic ring.

66. The F^- ions make the enamel on teeth much harder by converting hydroxyapatite (the enamel on the surface of teeth) into much harder fluoroapatite having the formula. [4 marks]

(A) $[3\,(Ca_3\,(PO_4)_2)\cdot CaF_2]$

(B) $[3\,(Ca_3\,(PO_4)_2)\cdot Ca(OH)_2]$

(C) $[3\,(Ca_3\,(PO_4)_3)\cdot CaF_2]$

(D) $[3\,(Ca_2\,(PO_4)_2)\cdot Ca(OH)_2]$

Correct Answer: (A) $[3\,(Ca_3\,(PO_4)_2)\cdot CaF_2]$

Solution:

Concept:

- Hydroxyapatite is the calcium phosphate mineral represented by $Ca_{10}(PO_4)_6(OH)_2$, equivalently $3Ca_3(PO_4)_2\cdot Ca(OH)_2$.
- Fluoride replaces hydroxide in the apatite lattice, producing the harder, less acid-soluble fluoroapatite $Ca_{10}(PO_4)_6F_2$, without changing the calcium-to-phosphate framework.

Step 1: Replace hydroxide with fluoride Each F^- ion replaces an OH^- ion, so the charge balance of the mineral is maintained. The ion-exchange representation is

$$Ca_{10}(PO_4)_6(OH)_2 + 2\,F^- \longrightarrow Ca_{10}(PO_4)_6F_2 + 2\,OH^-.$$

The calcium ions and phosphate groups remain in the apatite framework; only the anion at the

hydroxide site changes. This is why a formula still containing hydroxide alone cannot represent the fluorinated mineral.

Step 2: Express the product in the printed notation Expanding the formula in option (A) gives

$$3Ca_3(PO_4)_2 \cdot CaF_2 = Ca_{10}(PO_4)_6F_2.$$

It contains ten calcium ions, six phosphate ions and two fluoride ions. Their charges sum to $10(2) - 6(3) - 2 = 0$, as required for a neutral solid. Option (B) is the original hydroxyapatite because its final component is $Ca(OH)_2$. Option (C) uses $Ca_3(PO_4)_3$, an incorrect calcium phosphate composition and charge ratio. Option (D) also has the wrong calcium phosphate ratio and retains hydroxide rather than fluoride. Thus only (A) preserves the correct mineral framework and introduces fluorine.

Step 3: Conclusion Fluoroapatite has formula $Ca_{10}(PO_4)_6F_2$, or $[3Ca_3(PO_4)_2 \cdot CaF_2]$. Hence, the correct option is (A).

Quick Tip to Solve: Keep the calcium phosphate skeleton unchanged and replace the two hydroxide groups of hydroxyapatite by two fluoride ions.

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67. Identify the incorrect statements regarding primary standard of titrimetric analysis.

(A) It should be purely available in dry form.

(B) It should not undergo chemical change in air.

(C) It should be hygroscopic and should react with another chemical instantaneously and stoichiometrically.

(D) It should be readily soluble in water.

(E) $KMnO_4$ **&** $NaOH$ **can be used as primary standard.**

Choose the correct answer from the options given below : [4 marks]

(A) (C) and (E) only

(B) (A) and (B) only

(C) (B) and (E) only

(D) (C) and (D) only

Correct Answer: (A) (C) and (E) only

Solution:

Concept:

- A primary standard must have a reliably known composition and high purity, so its weighed mass fixes its amount of substance accurately. It must be stable during storage and weighing.
- A useful primary standard is non-hygroscopic, readily soluble in the titration medium and capable of a rapid, complete reaction of known stoichiometry. Hygroscopic substances take up variable amounts of water.

Step 1: Check statements A to D Statement (A) is correct: a pure, dry substance can be weighed to calculate the moles present. Statement (B) is correct because chemical alteration in air would

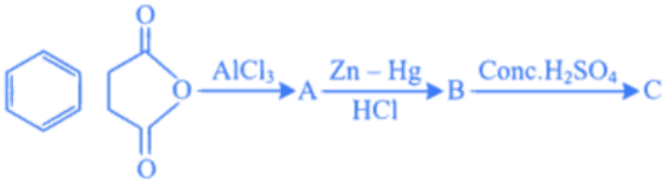
change its composition and make its weighed mass unreliable. Statement (C) is incorrect: although rapid stoichiometric reaction is desirable, the standard must not be hygroscopic. Absorbed water contributes to the measured mass without contributing the expected amount of analyte. Statement (D) is correct for the aqueous titrations considered here; ready dissolution allows preparation of a solution of accurately known concentration.

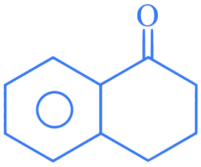
Step 2: Assess the named reagents Statement (E) is incorrect. $NaOH$ absorbs moisture and also reacts with atmospheric CO_2 to form carbonate, so its pellets cannot normally be weighed directly to establish an exact standard concentration. $KMnO_4$ solutions require standardisation because of impurities and changes such as formation of manganese dioxide; it is not used as a primary standard. Both are ordinarily used after standardisation against suitable primary standards. The incorrect statements are therefore (C) and (E). The combinations (A),(B), (B),(E), and (C),(D) each include at least one statement that is actually correct.

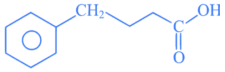
Step 3: Conclusion Only the hygroscopicity statement and the claim about the two named reagents are incorrect. Their pair appears in the first printed option. Hence, the correct option is (A).

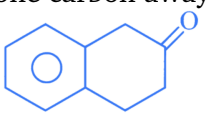
Quick Tip to Solve: For a primary standard, think pure, stable and non-hygroscopic. A reagent that absorbs water or carbon dioxide cannot provide a reliable concentration simply by weighing and needs standardisation.

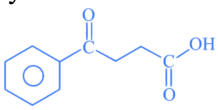
68. What is the structure of C? [4 marks]

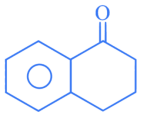


(A) 1-Tetralone (carbonyl carbon directly bonded to the aromatic fusion carbon). 

(B) 4-Phenylbutanoic acid, $C_6H_5CH_2CH_2CH_2COOH$. 

(C) 2-Tetralone (carbonyl carbon one carbon away from the aromatic fusion carbon). 

(D) 4-Oxo-4-phenylbutanoic acid, $C_6H_5COCH_2CH_2COOH$. 

Correct Answer: (A) 1-Tetralone (carbonyl carbon directly bonded to the aromatic fusion carbon). 

Solution:

Concept:

- Benzene undergoes Friedel–Crafts acylation with succinic anhydride in the presence of $AlCl_3$, opening the anhydride to give $C_6H_5COCH_2CH_2COOH$.
- Clemmensen reduction, $Zn-Hg/HCl$, converts a ketonic $C=O$ into CH_2. Subsequent intramolecular acylation of the carboxylic acid side chain closes a six-membered ring.

Step 1: Determine intermediate A from the anhydride The cyclic anhydride shown is succinic anhydride. Lewis-acid activation allows benzene to attack one acyl end. Opening of the anhydride followed by work-up produces the keto acid

$$A = C_6H_5COCH_2CH_2COOH.$$

The chain contains the benzene-bound ketone carbonyl, two methylene carbons and the terminal carboxylic acid carbon. This identifies the open-chain keto acid pictured in option (D) as A, not the final C.

Step 2: Reduce the ketone to obtain B With $Zn-Hg/HCl$, the ketone group next to the benzene ring is reduced to a methylene group, whereas the carboxylic acid end is retained:

$$C_6H_5COCH_2CH_2COOH \xrightarrow{Zn-Hg/HCl} C_6H_5CH_2CH_2CH_2COOH.$$

Thus B is 4-phenylbutanoic acid, the structure shown in option (B). It is an intermediate before the concentrated sulphuric acid step.

Step 3: Close the ring and locate the carbonyl Concentrated H_2SO_4 promotes intramolecular acylation at an ortho position of the benzene ring. The terminal acid carbon forms the new bond to the aromatic carbon; the three methylene groups complete a six-membered ring fused to benzene. The product is 1-tetralone. Its carbonyl carbon is directly attached to an aromatic fusion carbon, as in option (A). Option (C) is 2-tetralone, with the carbonyl shifted away from that fusion position; this connectivity cannot result from closure of B in the stated way.

Step 4: Conclusion C is 1-tetralone, the fused six-membered cyclic ketone in option (A). Hence, the correct option is (A).

Quick Tip to Solve: Label the terminal carboxyl carbon before cyclisation: it becomes the ring carbonyl carbon and bonds directly to the ortho aromatic carbon.

69. The electronic configuration of $Cu(II)$ is $3\ d^9$ whereas that of $Cu(I)$ is $3\ d^{10}$. Which of the following is correct? [4 marks]

(A) $Cu(II)$ is more stable

(B) Cu (II) is less stable

(C) $Cu(I)$ and $Cu(II)$ are equally stable

(D) Stability of $Cu(I)$ and $Cu(II)$ depends on nature of copper salts

Correct Answer: (A) $Cu(II)$ is more stable

Solution:

Concept:

- A filled d^{10} shell favours the isolated Cu^+ configuration, but stability in a solution depends on the complete energy balance, including hydration of the ions.
- In the usual aqueous comparison, the greater hydration enthalpy of the smaller, doubly charged Cu^{2+} ion compensates for the energy required to remove an additional electron. Thus the d-electron count alone is insufficient.

Step 1: Distinguish electronic configuration from overall stability Neutral copper has configuration $[Ar]3d^{10}4s^1$. Removing its outer electron gives Cu^+, with $3d^{10}$; removal of one more gives Cu^{2+}, with $3d^9$. A closed subshell may suggest that the monovalent ion should always be preferred, but that argument considers only one contribution to stability. Formation of ions in water also includes hydration, which is much more favourable for the divalent ion because of its larger charge density.

Step 2: Use the aqueous disproportionation behaviour The standard aqueous behaviour supports the greater stability of Cu^{2+}: cuprous ions tend to disproportionate according to

$$2\,Cu^+ \longrightarrow Cu^{2+} + Cu.$$

This produces a cupric ion and copper metal rather than leaving both ions as Cu^+. In this conventional comparison, option (A) is correct; (B) reverses the order and (C) incorrectly asserts equal stability. Specific ligands and lattice energies can stabilise particular copper(I) compounds, so those special environments must not be confused with the ordinary aqueous-ion trend being tested. Option (D) does not state the expected general aqueous comparison.

Step 3: Conclusion In the usual aqueous-ion comparison, Cu^{2+} is more stable than Cu^+ because its hydration stabilisation outweighs the filled-subshell advantage of Cu^+. Hence, the correct option is (A).

Quick Tip to Solve: Do not rank aqueous transition-metal ions using only filled subshells: include hydration energy, especially when the ionic charge increases.

70. In which one of the following pairs the central atoms exhibit sp^2 hybridization ? [4 marks]

(A) BF_3 and NO_2^-

(B) NH_2^- and BF_3

(C) NH_2^- and H_2O

(D) H_2O and NO_2

Correct Answer: (A) BF_3 and NO_2^-

Solution:

Concept:

- The steric number is the number of sigma-bonding domains plus lone pairs on the central atom. A multiple bond contributes one domain to this count, because its pi component does not add a new bond direction.
- Three electron domains correspond to sp^2 hybridisation; four correspond to sp^3. Electron-domain geometry and molecular shape must be distinguished when lone pairs are present.

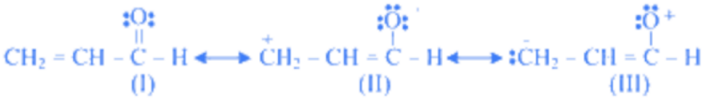
Step 1: Count domains in boron trifluoride and nitrite In BF_3, boron makes three sigma bonds to fluorine and has no lone pair. Its steric number is $3 + 0 = 3$, so it is trigonal planar and sp^2-hybridised. In $NO_2{}^-$, the Lewis structure contains two N–O sigma bonds and one lone pair on nitrogen; resonance distributes the pi bonding over both oxygens. Hence its steric number is $2 + 1 = 3$, so nitrogen is also sp^2-hybridised. Its molecular shape is bent, despite the trigonal-planar arrangement of electron domains.

Step 2: Eliminate pairs containing four-domain centres For $NH_2{}^-$, nitrogen has two N–H sigma bonds and two lone pairs, giving steric number $2+2 = 4$ and sp^3 hybridisation. In water, oxygen likewise has two O–H sigma bonds and two lone pairs, giving steric number four and sp^3 hybridisation. Therefore option (B) fails because of $NH_2{}^-$, option (C) contains two sp^3 centres, and option (D) fails because of H_2O, regardless of the description of the odd-electron species NO_2. Only option (A) has two central atoms with exactly three electron domains.

Step 3: Conclusion The central atoms of both BF_3 and $NO_2{}^-$ are sp^2-hybridised. Hence, the correct option is (A).

Quick Tip to Solve: Count sigma bonds plus lone pairs, not total bond order: three domains mean sp^2, even when a lone pair makes the molecular shape bent.

71. Relative stability of the contributing structures is : [4 marks]



(A) (II) > (I) > (III)

(B) (III) > (II) > (I)

(C) (I) > (III) > (II)

(D) (I) > (II) > (III)

Correct Answer: (D) (I) > (II) > (III)

Solution:

Concept:

- Resonance contributors have the same arrangement of atoms but differ in electron placement. A more stable contributor makes a greater contribution to the resonance hybrid.
- For comparable structures, complete octets and fewer formal charges favour stability. When charge separation is present, negative charge is better accommodated on the more electronegative atom, here oxygen rather than carbon.

Step 1: Examine the neutral contributor Structure (I) is the neutral conjugated carbonyl form. Its carbon and oxygen atoms have complete octets and it introduces no separated positive and negative charges. It is therefore the principal contributor. This comparison concerns individual canonical forms; the real molecule is the resonance hybrid rather than an equilibrium mixture of the three drawings.

Step 2: Compare the charge-separated contributors Structure (II) places negative charge on oxygen and positive charge on the terminal carbon. Oxygen stabilises the negative charge through its high electronegativity. Structure (III) reverses this favourable distribution: its terminal carbon carries negative charge whereas oxygen carries positive charge. This is much less favourable. Thus (II) contributes more than (III), although neither exceeds the neutral form (I).

Step 3: Check the offered sequences Combining these comparisons gives

$$\text{(I)} > \text{(II)} > \text{(III)}.$$

Option (A) wrongly puts a charge-separated form ahead of the neutral form. Option (B) makes the unfavourable positive-oxygen form most stable. Option (C) correctly starts with (I) but reverses the relative order of the two charged forms.

Step 4: Conclusion The relative stability is (I) > (II) > (III). Hence, the correct option is (D).

Quick Tip to Solve: First compare octets and charge separation; only then use electronegativity to rank contributors carrying comparable formal charges.

72. Correct order of basic strength of Pyrrole , Pyridine and Piperidine is : [4 marks]



(A) Pyridine > Piperidine > Pyrrole

(B) Pyrrole > Piperidine > Pyridine

(C) Pyrrole > Pyridine > Piperidine

(D) Piperidine > Pyridine > Pyrrole

Correct Answer: (D) Piperidine > Pyridine > Pyrrole

Solution:

Concept:

- A nitrogen base accepts a proton through its lone pair. Basicity increases when that pair is more available and decreases when protonation destroys an important stabilising interaction.
- Aromaticity and hybridisation distinguish these compounds: pyrrole uses its nitrogen lone pair in the aromatic sextet, pyridine has a non-aromatic lone pair on an sp^2 nitrogen, and piperidine is a saturated secondary amine with an sp^3 nitrogen.

Step 1: Identify the weakest base In pyrrole the two carbon-carbon double bonds provide four pi electrons and the nitrogen lone pair provides two more. The lone pair is therefore essential to the six-electron aromatic system. Binding a proton through that pair would sacrifice this aromatic stabilisation. Consequently the lone pair is poorly available for ordinary nitrogen protonation, and

pyrrole is the weakest base of the three.

Step 2: Compare pyridine with piperidine The nitrogen lone pair of pyridine lies in an sp^2 orbital outside the aromatic pi system, so it can accept a proton while retaining ring aromaticity. Pyridine is therefore more basic than pyrrole. Piperidine has an sp^3 nitrogen whose lone pair is more available, together with electron-releasing alkyl groups attached to nitrogen. As a saturated secondary amine it is more basic than pyridine.

Step 3: Arrange and test the options The decreasing basic-strength order is

$$\text{Piperidine} > \text{Pyridine} > \text{Pyrrole}.$$

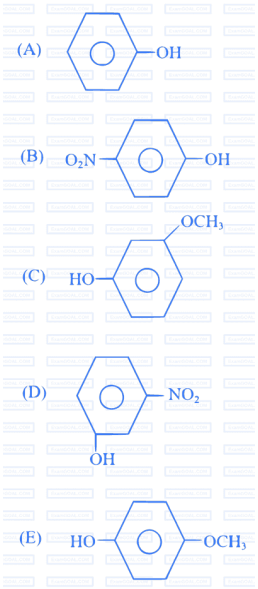
Option (A) incorrectly puts pyridine ahead of the saturated amine. Options (B) and (C) both incorrectly place aromatic pyrrole first, ignoring the participation of its lone pair in aromaticity. Only option (D) preserves both independently established comparisons.

Step 4: Conclusion The basic-strength order is Piperidine $>$ Pyridine $>$ Pyrrole. Hence, the correct option is (D).

Quick Tip to Solve: Ask whether the nitrogen lone pair belongs to an aromatic sextet before comparing nitrogen hybridisation or alkyl electron donation.

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73. For the given compounds, the correct order of increasing pK_a value :
Choose the correct answer from the options given below : [4 marks]



(A) (E) $<$ (D) $<$ (C) $<$ (B) $<$ (A)
(B) (B) $<$ (D) $<$ (A) $<$ (C) $<$ (E)
(C) (D) $<$ (E) $<$ (C) $<$ (B) $<$ (A)
(D) (E) $<$ (D) $<$ (B) $<$ (A) $<$ (C)

Correct Answer: None of the printed options matches the increasing order $B < D < C < A < E$.

Solution:

Concept:

- The acidity of a phenol depends on the stability of its phenoxide ion. Electron-withdrawing substituents stabilise the negative charge and lower $\mathrm{p}K_a$; electron donors generally have the opposite effect.
- An ortho or para substituent can interact by resonance with the phenoxide charge. At a meta position its inductive effect is decisive because the principal phenoxide resonance structures do not place charge at that carbon.

Step 1: Identify the substituent positions The structures are A: phenol, B: para-nitrophenol, C: meta-methoxyphenol, D: meta-nitrophenol and E: para-methoxyphenol. In C, the OH group and methoxy group are separated by one ring carbon, so they are meta, rather than para. The positional assignment must precede the comparison of electronic effects.

Step 2: Compare the nitro derivatives The nitro group withdraws electrons inductively. In B it additionally stabilises the phenoxide ion by resonance, whereas in D the meta nitro group acts mainly through its inductive effect. Thus B is more acidic than D, and both are more acidic than phenol. In increasing $\mathrm{p}K_a$, B therefore precedes D and A.

Step 3: Compare the methoxy derivatives and test the options The methoxy group has a withdrawing inductive effect and a donating resonance effect. At the meta position in C, the inductive effect makes C slightly more acidic than A. At the para position in E, resonance donation destabilises the phenoxide ion and makes E less acidic than A. Combining these comparisons gives

$$B < D < C < A < E.$$

Options (A), (C) and (D) place E among the most acidic compounds, contrary to its para methoxy donation. Option (B) correctly places B and D first and E last, but incorrectly places A before C. Thus no listed sequence is the literal answer.

Step 4: Conclusion The increasing order for the structures as drawn is $\boxed{B < D < C < A < E}$. None of the printed options matches it. Although the key identifies option (B), that option reverses A and the meta-methoxy compound C, so a unique correct printed option cannot be selected.

Quick Tip to Solve: Locate the substituent relative to OH before using resonance: a meta methoxy group does not have the same acidity effect as a para methoxy group.

74. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Both rhombic and monoclinic sulphur exist as S_8 while oxygen exists as O_2.

Reason (R) : Oxygen forms $p\pi - p\pi$ multiple bonds with itself and other elements having small size and high electronegativity like C, N, which is not possible for sulphur.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(B) (A) is correct but (R) is not correct

(C) (A) is not correct but (R) is correct

(D) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Correct Answer: (D) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Solution:

Concept:

- Oxygen readily forms strong $p\pi - p\pi$ bonds because its compact valence $2p$ orbitals overlap effectively sideways. The stable ordinary elemental molecule is O_2.
- Sulphur has larger, more diffuse $3p$ orbitals and weaker sideways overlap. Its common allotropes consist of puckered S_8 rings held together by single covalent bonds.

Step 1: Test the assertion Rhombic sulphur and monoclinic sulphur both contain cyclic S_8 molecules. They are different crystalline allotropes because their molecular packing differs; the molecular formula of the ring does not change between them. Under ordinary conditions oxygen exists as the diatomic molecule O_2. Therefore Assertion (A) is correct in the ordinary elemental forms described by the question.

Step 2: Test the reason Oxygen is a second-period element whose small size permits effective lateral overlap of its $2p$ orbitals. Strong multiple bonding is possible with itself and with small atoms such as carbon and nitrogen. In contrast, the larger $3p$ orbitals of sulphur overlap less effectively in this way, so the analogous strong $p\pi - p\pi$ multiple bonding is not favoured. This is the periodic trend meant by Reason (R), so the reason is correct.

Step 3: Connect the bonding to the molecular forms The strong multiple bond stabilises a discrete O_2 molecule. Sulphur preferentially forms S–S single bonds and shows appreciable catenation; joining eight atoms in a ring gives its common stable molecular form. Thus the contrasting ability to form strong multiple bonds explains why the question compares O_2 with S_8. Option (A) incorrectly denies the explanatory link; options (B) and (C) incorrectly reject one of the two true statements.

Step 4: Conclusion Both The assertion and the reason are correct, and R is the correct explanation of A. Hence, the correct option is (D).

Quick Tip to Solve: For oxygen versus sulphur, connect compact 2p overlap with multiple bonding and diffuse 3p overlap with single-bonded sulphur rings.

75. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): The total number of geometrical isomers shown by $[Co(en)_2Cl_2]^+$ complex ion is three.

Reason (R): $[Co(en)_2Cl_2]^+$ complex ion has an octahedral geometry.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) (A) is correct but (R) is not correct

(B) (A) is not correct but (R) is correct

(C) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(D) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Correct Answer: (B) (A) is not correct but (R) is correct

Solution:

Concept:

- Ethylenediamine, en, is a bidentate ligand: each molecule donates through two nitrogen atoms. Two en ligands and two chloride ligands give coordination number six.
- Geometrical isomers differ in the relative positions of ligands. Optical enantiomers of one geometrical form do not count as additional geometrical isomers.

Step 1: Determine the coordination geometry In $[Co(en)_2Cl_2]^+$, the two en ligands occupy four coordination positions and the two chloride ligands occupy the remaining two. The coordination number is therefore $2 \times 2 + 2 = 6$, consistent with the octahedral geometry stated in Reason (R). The chelate rings constrain the two donor atoms of each en ligand to adjacent positions. Thus the reason is correct.

Step 2: Count the geometrical arrangements Classify the possible structures by the positions of the two identical chloride ligands. In the cis form they occupy adjacent octahedral vertices, making a 90° angle at cobalt. In the trans form they occupy opposite vertices, making a 180° angle. Rotating the octahedron produces no third distinct chloride relationship. Hence there are exactly two geometrical isomers, cis and trans, and Assertion (A), which says three, is false.

Step 3: Distinguish the optical pair from the geometry count The cis form can exist as two non-superimposable mirror images, conventionally designated Δ and Λ. These have the same cis arrangement and are optical isomers of that one geometrical form. The trans form is optically inactive. Counting individual stereoisomers gives a total of three, but counting geometrical forms gives only two. Option (A) reverses the truth values; options (C) and (D) wrongly accept the assertion. Only option (B) has the required false-assertion, true-reason combination.

Step 4: Conclusion The assertion is incorrect because the complex has two geometrical isomers; the reason is correct because its geometry is octahedral. Hence, the correct option is (B).

Quick Tip to Solve: First count cis/trans arrangements, then count optical partners separately. The two enantiomers of a cis form still represent only one geometrical isomer.

76. Methods used for purification of organic compounds are based on : [4 marks]

(A) neither on nature of compound nor on the impurity present.

(B) presence of impurity only.

(C) nature of compound and presence of impurity.

(D) nature of compound only.

Correct Answer: (C) nature of compound and presence of impurity.

Solution:

Concept:

- Purification separates a desired organic compound from accompanying impurities by exploiting differences in physical properties such as solubility, volatility, sublimability or adsorption.
- A useful method must suit the desired compound and distinguish it from the impurity. The mere presence of an impurity does not determine which separation will work.

Step 1: Consider the properties of the desired compound A solid that dissolves readily in hot solvent but only sparingly in cold solvent may be purified by crystallisation. A volatile liquid may be suitable for distillation, whereas a sublimable solid can sometimes be separated by sublimation. These examples show that the nature of the compound matters: a method suitable for one type of material may be ineffective or destructive for another.

Step 2: Include the properties of the impurity The same desired compound can require different treatment with different impurities. Crystallisation is useful only when the impurity can be removed by hot filtration or retained in the mother liquor. Distillation separates components when their volatility differs sufficiently. Differential extraction uses contrasting distribution between immiscible solvents, while chromatography uses different affinities for stationary and mobile phases. Thus the impurity and its contrast with the product also govern the choice.

Step 3: Eliminate incomplete descriptions Option (A) denies dependence on both factors and cannot explain any selective separation. Option (B) ignores the properties and stability of the compound being purified. Option (D) ignores whether the impurity behaves similarly to the product under the chosen conditions. Option (C) includes both necessary considerations and is the complete statement.

Step 4: Conclusion Purification methods depend on the nature of the compound and the impurity present. Hence, the correct option is (C).

Quick Tip to Solve: Choose a purification method by naming the property that differs between the product and the impurity, rather than by identifying the product alone.

77. Compare the energies of following sets of quantum numbers for multielectron system.

(A) $\mathrm{n} = 4, 1 = 1$

(B) $\mathrm{n} = 4, 1 = 2$

(C) $\mathrm{n} = 3, \mathrm{l} = 1$

(D) $\mathrm{n} = 3, 1 = 2$

(E) $\mathrm{n} = 4, 1 = 0$

Choose the correct answer from the options given below : [4 marks]

(A) (E) > (C) > (A) > (D) > (B)

(B) (B) > (A) > (C) > (E) > (D)

(C) (C) < (E) < (D) < (A) < (B)

(D) (E) < (C) < (D) < (A) < (B)

Correct Answer: (C) (C) < (E) < (D) < (A) < (B)

Solution:

Concept:

- For the ordering used for ground-state filling in multielectron atoms, orbitals are compared by the $(n + \ell)$ rule: smaller $n + \ell$ corresponds to lower energy.
- If two orbitals have equal $n + \ell$, the orbital with the smaller principal quantum number n is lower in energy. The azimuthal values $\ell = 0, 1, 2$ denote s, p and d orbitals, respectively.

Step 1: Translate the quantum numbers into orbitals The listed sets correspond to (A) $4p$, (B) $4d$, (C) $3p$, (D) $3d$ and (E) $4s$. The character appearing as 1 in some entries denotes the azimuthal quantum number ℓ, as is clear from the pairs of quantum numbers. Compute the sums separately:

$$(A) : n + \ell = 4 + 1 = 5,$$
$$(B) : n + \ell = 4 + 2 = 6,$$
$$(C) : n + \ell = 3 + 1 = 4,$$
$$(D) : n + \ell = 3 + 2 = 5,$$
$$(E) : n + \ell = 4 + 0 = 4.$$

Step 2: Resolve the tied sums Among the orbitals with sum four, $3p$ has the smaller principal quantum number, so (C) lies below (E). Among those with sum five, $3d$ has smaller n than $4p$, so (D) lies below (A). The $4d$ orbital has sum six and therefore comes last. Combining the groups gives

$$3p < 4s < 3d < 4p < 4d.$$

Step 3: Compare the complete order with the choices The corresponding labels are $(C) < (E) < (D) < (A) < (B)$. Option (A) reverses several required comparisons and even places (B) lowest. Option (B) places (C) above (E) and (D) below both, contrary to the sums. Option (D) swaps the first tied pair by putting $4s$ below $3p$. Option (C) alone follows both stages of the rule.

Step 4: Conclusion The orbital energies increase as $3p < 4s < 3d < 4p < 4d$. Hence, the correct option is (C).

Quick Tip to Solve: Write $n + \ell$ beside every orbital first; use the smaller n only to break a tie, never as the primary ordering rule.

78. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : S_N2 **reaction of** $C_6H_5CH_2Br$ **occurs more readily than the** S_N2 **reaction of** CH_3CH_2Br**.**

Reason (R) : The partially bonded unhybridized p-orbital that develops in the trigonal bipyramidal transition state is stabilized by conjugation with the phenyl ring.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(B) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(C) (A) is not correct but (R) is correct

(D) (A) is correct but (R) is not correct

Correct Answer: (B) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Solution:

Concept:

- An S_N2 reaction is a concerted substitution: nucleophile attachment and leaving-group departure occur through one transition state. A lower transition-state energy gives a faster reaction.
- At a benzylic reaction centre, the developing orbital arrangement in this transition state can overlap with the adjacent phenyl π system. Conjugation stabilizes the transition state relative to an ordinary primary alkyl halide.

Step 1: Assess the assertion Both benzyl bromide, $C_6H_5CH_2Br$, and ethyl bromide, CH_3CH_2Br, have a primary carbon bearing bromine. Benzyl bromide nevertheless undergoes backside nucleophilic displacement more readily, because the adjacent aromatic ring provides extra stabilization of the developing transition state. Therefore assertion (A) is correct. The phenyl group is attached to the neighbouring carbon framework, not directly to the leaving bromine; this is a benzylic halide, not an unreactive aryl halide.

Step 2: Assess the reason In the approximately trigonal-bipyramidal transition-state arrangement, the incoming nucleophile and departing bromide occupy the two opposite axial directions. The partially bonded orbital at the reacting carbon can conjugate with the phenyl ring. This delocalization lowers the activation barrier and increases the substitution rate. Thus reason (R) is also correct in the transition-state description used here; no separate, freely formed carbocation is required for this concerted mechanism.

Step 3: Connect the two statements and eliminate alternatives The transition-state conjugation stated in (R) is precisely the cause of the higher benzylic S_N2 reactivity in (A). Option (A) incorrectly denies this causal link. Option (C) incorrectly rejects the true assertion, and option (D) incorrectly rejects the true reason. Only option (B) accepts both statements and their explanatory connection.

Step 4: Conclusion The assertion and reason are both correct, and the reason correctly explains the assertion. Hence, the correct option is (B).

Quick Tip to Solve: For a benzylic S_N2 comparison, compare transition-state stabilization as well as steric crowding; do not invoke a free carbocation.

79. On reaction of Lead Sulphide with dilute nitric acid which of the following is not formed? [4 marks]

(A) Sulphur

(B) Nitric oxide

(C) Lead nitrate

(D) Nitrous oxide

Correct Answer: (D) Nitrous oxide

Solution:

Concept:

- Dilute nitric acid is an oxidizing acid. It oxidizes sulphide sulphur from oxidation state -2 to elemental sulphur, while nitrate nitrogen is reduced from $+5$ to $+2$ in nitric oxide.
- Lead remains in its $+2$ oxidation state and forms soluble lead nitrate. The products are found by balancing both atoms and electron transfer, rather than by treating this as a simple acid–sulphide reaction.

Step 1: Identify oxidation and reduction In PbS, lead is $+2$ and sulphur is -2. Oxidation to S releases two electrons per sulphur atom. Reduction of nitrate to NO consumes three electrons per nitrogen atom. Taking three sulphide units and two reduced nitrate units balances six electrons. The remaining nitrate ions supply the counterions for the three lead(II) ions.

Step 2: Write the balanced reaction The overall reaction with dilute nitric acid is

$$3\,\mathrm{PbS} + 8\,\mathrm{HNO_3} \longrightarrow 3\,\mathrm{Pb(NO_3)_2} + 3\,\mathrm{S} + 2\,\mathrm{NO} + 4\,\mathrm{H_2O}.$$

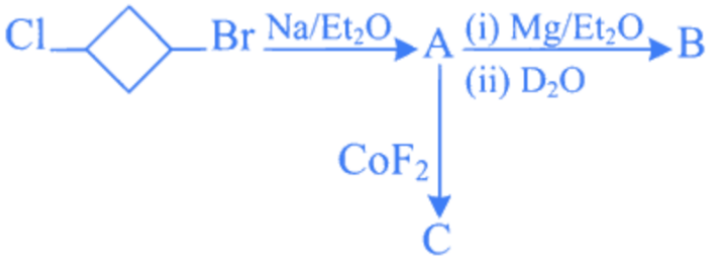
There are three lead atoms, three sulphur atoms, eight nitrogen atoms, eight hydrogen atoms and twenty-four oxygen atoms on each side. This confirms the balanced equation and identifies the actual nitrogen oxide as nitric oxide, NO.

Step 3: Check each listed substance Sulphur, option (A), occurs explicitly among the products. Nitric oxide, option (B), is the nitrate-reduction product. Lead nitrate, option (C), is the salt formed from the lead(II) ions. Nitrous oxide, $\mathrm{N_2O}$, option (D), is a different oxide of nitrogen and is not a product of this reaction.

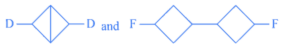
Step 4: Conclusion Nitrous oxide, $\mathrm{N_2O}$, is not formed. Hence, the correct option is (D).

Quick Tip to Solve: For dilute $\mathrm{HNO_3}$ oxidizing a metal sulphide, balance the sulphur oxidation and nitrate reduction together; distinguish NO from $\mathrm{N_2O}$.

80. In the following sequence of reaction, the major products B and C respectively are : [4 marks]



(A) B: a bicyclo[1.1.0]butane skeleton with D on the two non-bridgehead carbons; C: two cyclobutane rings joined by a single bond, each bearing F opposite the joining bond.



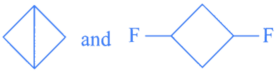
(B) B: two cyclobutane rings joined by a single bond, each bearing D opposite the joining bond; C: the corresponding two-ring product with F in place of each D.



(C) B: 1,3-dideuteriocyclobutane; C: 1,3-difluorocyclobutane.



(D) B: bicyclo[1.1.0]butane; C: 1,3-difluorocyclobutane.



Correct Answer: (B)

B: two cyclobutane rings joined by a single bond, each bearing D opposite the joining bond; C: the corresponding two-ring product with F in place of each D.



Solution:

Concept:

- The Wurtz reaction joins two carbon centres bearing reactive halogen atoms using sodium in dry ether. In a substrate containing both bromine and chlorine, the carbon–bromine bond reacts preferentially.
- A retained carbon–chlorine bond can form a Grignard reagent with magnesium in dry ether. Quenching $\mathrm{R{-}MgCl}$ with $\mathrm{D_2O}$ replaces the magnesium-containing group by deuterium.
- Swarts halogen exchange replaces organic chlorine by fluorine using a metallic fluoride such as $\mathrm{CoF_2}$; it does not rearrange or cleave the carbon skeleton.

Step 1: Find the intermediate formed with sodium The starting cyclobutane has chlorine and bromine on opposite ring carbons. Sodium in dry ether couples two molecules at their brominated carbons. Consequently, intermediate A contains two cyclobutane rings joined through a new single carbon–carbon bond, while one chlorine remains on the opposite carbon of each ring. This is intermolecular coupling at the more reactive bromide sites, not closure across the diagonal of one ring.

Step 2: Follow the magnesium and heavy-water branch Magnesium inserts into each of the two retained carbon–chlorine bonds of A. For each site the sequence is

$$\mathrm{R{-}Cl} \xrightarrow{\mathrm{Mg/Et_2O}} \mathrm{R{-}MgCl} \xrightarrow{\mathrm{D_2O}} \mathrm{R{-}D}.$$

Thus B retains both cyclobutane rings and their connecting bond. Each terminal chlorine is replaced by deuterium. Deuterium is introduced by the heavy-water quench; the ring carbon skeleton is unchanged.

Step 3: Follow the cobalt fluoride branch and compare options In the separate branch from A, CoF_2 replaces both retained chlorine atoms by fluorine. Hence C also has two linked cyclobutane rings, with fluorine opposite the connecting bond on each ring. Option (B) shows exactly these dimeric D and F products. Option (A) incorrectly gives a bridged single-ring B. Option (C) omits the Wurtz coupling in both products. Option (D) gives a bridged hydrocarbon without the required deuterium and an uncoupled fluorinated ring.

Step 4: Conclusion B is the linked cyclobutane dimer bearing two terminal D atoms, and C is the corresponding dimer bearing two terminal F atoms. Hence, the correct option is (B).

Quick Tip to Solve: First preserve the carbon skeleton made in the sodium/ether step; then replace only the remaining halogens in the two independent branches.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The standard reduction potentials at $298\ K$ for the following half cells are given below :

$$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O, \quad E^\circ = 1.33\ V$$

$$\begin{aligned} Fe^{3+}(aq) + 3e^- &\rightarrow Fe \quad E^\circ = -0.04\ V \\ Ni^{2+}(aq) + 2e^- &\rightarrow Ni \quad E^\circ = -0.25\ V \\ Ag^+(aq) + e^- &\rightarrow Ag \quad E^\circ = 0.80\ V \\ Au^{3+}(aq) + 3e^- &\rightarrow Au \quad E^\circ = 1.40\ V \end{aligned}$$

Consider the given electrochemical reactions,

The number of metal(s) which will be oxidized be $Cr_2O_7^{2-}$, in aqueous solution is _____. [4 marks]

Solution:

Concept:

- A metal is oxidized by an oxidizing agent when the combined standard cell potential is positive. Using tabulated reduction potentials, $E^\circ_{cell} = E^\circ_{oxidant} - E^\circ_{metal}$.
- The dichromate reduction potential is $1.33\ V$. Standard electrode potentials are intensive quantities: multiplying a half-reaction to balance electrons does not multiply its potential.

Step 1: Set the oxidation criterion Dichromate accepts electrons and is reduced to Cr^{3+} in the acidic medium specified by its given half-reaction. Each tested metal supplies those electrons by undergoing the reverse of its listed reduction half-reaction. The reaction is thermodynamically favourable if

$$1.33 - E^\circ_{metal} > 0.$$

Equivalently, the metal couple must have a reduction potential lower than that of dichromate.

Step 2: Calculate the cell potential for every metal For the four supplied couples, the potentials are

$$E^\circ_{\text{cell}}(\text{Fe}) = 1.33 - (-0.04) = 1.37\,\text{V},$$
$$E^\circ_{\text{cell}}(\text{Ni}) = 1.33 - (-0.25) = 1.58\,\text{V},$$
$$E^\circ_{\text{cell}}(\text{Ag}) = 1.33 - 0.80 = 0.53\,\text{V},$$
$$E^\circ_{\text{cell}}(\text{Au}) = 1.33 - 1.40 = -0.07\,\text{V}.$$

The first three values are positive, whereas the gold value is negative. The different electron counts in the half-reactions do not alter these comparisons.

Step 3: Count the metals that satisfy the criterion Iron, nickel and silver can therefore be oxidized by dichromate under the stated standard-potential comparison. Gold cannot, because its reduction couple lies above dichromate and the proposed oxidation would give a negative standard cell potential. We count metals, rather than adding the electrons transferred or the coefficients used to balance the overall equations.

Step 4: Conclusion The oxidized metals are Fe, Ni and Ag. The required number is $\boxed{3}$ (a dimensionless count).

Quick Tip to Solve: For an oxidant with known reduction potential, count the metal couples below it; never multiply electrode potentials by balancing coefficients.

82. Total number of essential amino acid among the given list of amino acids is _____.

Arginine, Phenylalanine, Aspartic acid, Cysteine, Histidine, Valine, Proline [4 marks]

Solution:

Concept:

- Essential amino acids must be supplied through food because the body cannot synthesize them in adequate amounts. Classifying them is a nutritional property, not simply a consequence of their charge or functional group.
- In the conventional classification used for this question, arginine and histidine are included with the essential amino acids; arginine is often described more specifically as conditionally essential, especially during growth.

Step 1: Identify the amino acids counted as essential From the stated list, phenylalanine, histidine and valine belong to the essential amino-acid group. Arginine is also counted in the classification used here, which includes the requirement during growth. Thus the selected names are arginine, phenylalanine, histidine and valine. This distinction matters because a list restricted to indispensable amino acids in healthy adults can treat arginine separately.

Step 2: Exclude the remaining names Aspartic acid is a non-essential amino acid that can be produced through normal metabolic pathways. Cysteine can be synthesized from methionine and is classified as non-essential in the conventional examination list, although it may become conditionally essential in particular circumstances. Proline is also synthesized by the body. Hence aspartic acid, cysteine and

proline are not included in the requested count.

Step 3: Add the qualifying entries Count each selected amino acid once:

$$N = 1\ (\text{arginine}) + 1\ (\text{phenylalanine}) + 1\ (\text{histidine}) + 1\ (\text{valine}) = 4.$$

The answer is a number of entries, so it has no physical unit. No additional amino acids outside the seven listed names are needed, and the three excluded entries must not be included merely because they are constituents of proteins.

Step 4: Conclusion The four qualifying amino acids are arginine, phenylalanine, histidine and valine. The required number is $\boxed{4}$ (a dimensionless count), using the essential/conditionally essential classification specified above.

Quick Tip to Solve: When counting essential amino acids in this convention, include arginine and histidine; do not count acidic amino acids simply because they contain extra functional groups.

83. Number of ambidentate ligands among the following is _____.

$$NO_2^-, SCN^-, C_2O_4^{2-}, NH_3, CN^-, SO_4^{2-}, H_2O.$$

[4 marks]

Solution:

Concept:

- An ambidentate ligand has two alternative donor atoms and can coordinate through either one, usually one donor at a time. Alternative attachment sites must be distinguished from simultaneous attachment through two sites.
- A bidentate chelating ligand binds through two donor atoms to the same metal simultaneously. Having two donor atoms therefore does not, by itself, make a ligand ambidentate.

Step 1: Identify the ligands with alternative donor atoms The nitrite ion, NO_2^-, can bind through nitrogen or oxygen, giving nitro or nitrito linkage. Thiocyanate, SCN^-, can bind through sulphur or nitrogen. Cyanide, CN^-, can attach through carbon or nitrogen, giving the corresponding alternative linkage. These are the three ambidentate ligands in the list.

Step 2: Classify the remaining ligands Oxalate, $C_2O_4^{2-}$, is the familiar bidentate chelating ligand: it normally coordinates through two oxygen atoms together, rather than through two chemically different alternative donor ends. Ammonia, NH_3, donates through nitrogen, and water, H_2O, through oxygen. Sulphate, SO_4^{2-}, coordinates through oxygen; its several oxygen atoms do not make it an ambidentate ligand in this classification.

Step 3: Count the ambidentate set The qualifying set is

$$\{NO_2^-,\ SCN^-,\ CN^-\}.$$

It contains three members. In particular, do not add oxalate merely because its denticity is two, and do

not count equivalent oxygen atoms of sulphate as different kinds of alternative donor sites. Denticity and ambidentate behaviour answer different questions about how a ligand binds.

Step 4: Conclusion The ambidentate ligands are NO_2^-, SCN^- and CN^-. Their number is $\boxed{3}$ (a dimensionless count).

Quick Tip to Solve: Ask whether the ligand can choose between different donor atoms, rather than whether it has two atoms available for simultaneous chelation.

84. Number of colourless lanthanoid ions among the following is _____.

$$Eu^{3+}, Lu^{3+}, Nd^{3+}, La^{3+}, Sm^{3+}$$

[4 marks]

Solution:

Concept:

- The characteristic colours of most lanthanoid(III) ions arise from electronic transitions within a partially filled $4f$ subshell. Empty and completely filled $4f$ subshells do not allow such f-f transitions.
- Find the configuration of the tripositive ion, not the neutral atom. Lanthanoid ions with $4f^0$ or $4f^{14}$ configurations are colourless in this comparison.

Step 1: Find the configurations of the listed ions The relevant tripositive configurations, omitting the common xenon core, are

$$Eu^{3+}: 4f^6, \quad Lu^{3+}: 4f^{14},$$
$$Nd^{3+}: 4f^3, \quad La^{3+}: 4f^0,$$
$$Sm^{3+}: 4f^5.$$

In particular, the stable half-filled configuration of Eu^{2+} must not be assigned to Eu^{3+}, and lutetium loses its outer $5d$ electron on forming the tripositive ion.

Step 2: Apply the colour criterion La^{3+} has an empty $4f$ subshell, so there is no $4f$ electron available for an intra-subshell transition. Lu^{3+} has a completely filled $4f$ subshell, so an ordinary f-f excitation into a vacant state of that subshell is unavailable. Both are colourless. Europium(III), neodymium(III) and samarium(III) have partially filled $4f$ subshells and are the coloured members of the list.

Step 3: Count only the colourless ions The qualifying ions are therefore Lu^{3+} and La^{3+}. Counting these gives two, not three: a partially filled subshell should not be counted as colourless merely because its absorption may be weak. The question asks for the usual colour classification of these lanthanoid ions, rather than a ranking of colour intensity.

Step 4: Conclusion The colourless ions are Lu^{3+} and La^{3+}. The required number is $\boxed{2}$ (a dimensionless count).

Quick Tip to Solve: For lanthanoid(III) colour questions, write the $4f$ electron count first: the empty and full endpoints, $4f^0$ and $4f^{14}$, are colourless.

85. The total number of species from the following in which one unpaired electron is present, is _____.

$$N_2, O_2, C_2^-, O_2^-, O_2^{2-}, H_2^+, CN^-, He_2^+$$

[4 marks]

Solution:

Concept:

- Fill molecular orbitals in increasing energy and apply Hund's rule to degenerate orbitals. An unpaired electron occupies an orbital without a second electron of opposite spin.
- Count the electrons after adding one for a negative charge or subtracting one for a positive charge. A species with an odd electron count has at least one unpaired electron, but the actual orbital filling must determine how many.

Step 1: Check the nitrogen, carbon and cyanide species N_2 has 14 electrons and its occupied molecular orbitals are all doubly occupied, so it has zero unpaired electrons. Neutral C_2 has paired electrons in its two bonding π orbitals; adding one electron to make $C_2{}^-$ places a single electron in the next bonding σ orbital. Thus $C_2{}^-$ has exactly one unpaired electron. CN^- has $6 + 7 + 1 = 14$ electrons and is isoelectronic with N_2; it also has zero unpaired electrons.

Step 2: Fill the antibonding orbitals of the oxygen species The two highest electrons of O_2 occupy separate degenerate π^* orbitals, giving two unpaired electrons. In $O_2{}^-$, one added electron pairs in one of these orbitals, leaving exactly one unpaired electron. In $O_2{}^{2-}$, the second added electron completes the other pair, leaving zero unpaired electrons. A paramagnetic species is not automatically included: the question demands exactly one, so O_2 must be excluded.

Step 3: Check the light diatomic ions and count $H_2{}^+$ has one electron, in the bonding σ_{1s} orbital, so its unpaired count is one. $He_2{}^+$ has three electrons with configuration $\sigma_{1s}^2\sigma_{1s}^{*1}$, again leaving one. The qualifying species are therefore $C_2{}^-$, $O_2{}^-$, $H_2{}^+$ and $He_2{}^+$.

Step 4: Conclusion The total number of species with exactly one unpaired electron is $\boxed{4}$ (a dimensionless count).

Quick Tip to Solve: For oxygen species, use the sequence of unpaired counts $2, 1, 0$ for O_2, $O_2{}^-$, $O_2{}^{2-}$; then check the odd-electron ions separately.

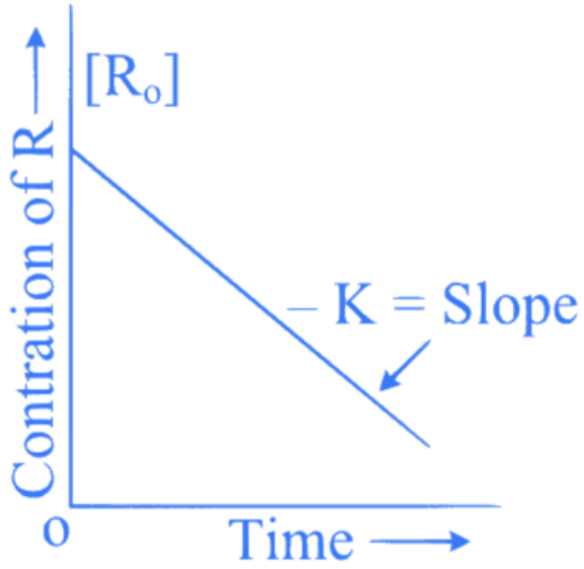
86. Given below are two statements :

Statement I : The rate law for the reaction $A + B \rightarrow C$ is rate $(r) = k[A]^2[B]$. When the concentration of both A and B is doubled, the reaction rate is increased "x" times.

Statement II :

The figure is showing "the variation in concentration against time plot" for a "y" order reaction.

The Value of $x + y$ is _____. [4 marks]



Solution:

Concept:

- For a rate law $r = k[A]^m[B]^n$, multiplying the two concentrations by factors a and b multiplies the rate by $a^m b^n$, at unchanged temperature.
- A zero-order reaction has integrated law $[R]_t = [R]_0 - kt$. Its concentration versus time graph is a straight line of negative slope $-k$.

Step 1: Calculate the rate multiplication factor The given rate law is $r = k[A]^2[B]$. Both concentrations are doubled while k remains fixed. Substitution gives

$$r_{\text{new}} = k(2[A])^2(2[B]) = 2^2 \times 2\, k[A]^2[B] = 8r.$$

Hence the multiplication factor is $x = 8$. The powers in the measured rate law determine this factor; the coefficients in the overall reaction do not replace those powers. In particular, doubling both concentrations is not merely a factor of four, because the dependence on $[A]$ is quadratic.

Step 2: Identify the reaction order from the graph The vertical axis in the supplied figure is the concentration of reactant R, not its logarithm or reciprocal. The plot falls linearly from $[R]_0$, and the marked slope is negative. Comparing with

$$[R]_t = [R]_0 - kt$$

identifies a zero-order reaction, so $y = 0$. For a first-order reaction it would instead be $\ln[R]$ versus time

that is linear; for a second-order reaction, $1/[R]$ versus time is linear. Thus the straight concentration plot cannot be classified as first order just because it decreases.

Step 3: Combine the two independent results The two statements supply separate quantities: x is the rate factor from concentration changes, while y is the order represented by the graph. Therefore

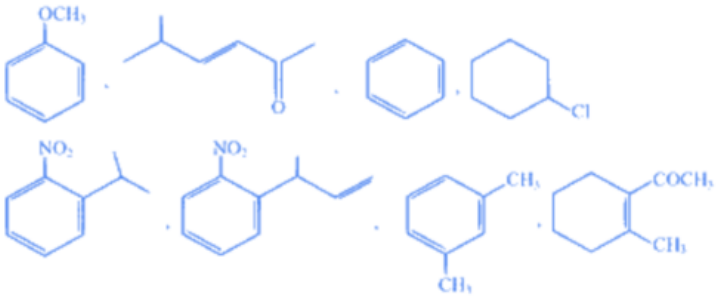
$$x + y = 8 + 0 = 8.$$

Neither quantity has units, so their requested sum is dimensionless.

Step 4: Conclusion The required value is $\boxed{x + y = 8}$.

Quick Tip to Solve: Always read the vertical axis before identifying order: straight $[R]$, $\ln[R]$, and $1/[R]$ plots indicate zero, first, and second order respectively.

87. How many compounds among the following compounds show inductive, mesomeric as well as hyperconjugation effects? [4 marks]



Solution:

Concept:

- An inductive effect is electron displacement through sigma bonds caused by a polar substituent. A mesomeric effect requires conjugation of a substituent lone pair or multiple bond with a neighbouring pi system.
- Ordinary hyperconjugation requires an alpha carbon–hydrogen sigma bond adjacent to a pi bond or an aromatic ring. All three effects must occur within the same compound; showing only aromatic resonance is not enough.

Step 1: Identify the compounds with all three effects Number the structures from left to right in the top row as 1–4 and in the bottom row as 5–8. Structure 2 is the conjugated unsaturated ketone: its carbonyl produces an inductive effect and is conjugated with the carbon–carbon double bond, while adjacent alkyl carbon–hydrogen bonds allow hyperconjugation. Structures 5 and 6 are the two nitro-substituted alkylbenzenes. In each, the nitro group exerts both $-I$ and $-M$ effects, while the alkyl side chain has a benzylic carbon–hydrogen bond that can hyperconjugate with the ring. Structure 8 has the conjugated alkene–carbonyl unit, providing inductive and mesomeric effects, and a methyl group attached to the alkene, providing hyperconjugation.

Step 2: Exclude structures that lack a required effect Structure 1 is anisole: oxygen gives an inductive

effect and donates a lone pair by the mesomeric effect, but the methoxy methyl group is separated from the aromatic pi system by oxygen and does not provide the required alpha carbon–hydrogen hyperconjugation. Structure 3 is unsubstituted benzene and lacks the necessary polar substituent and alpha alkyl carbon–hydrogen bonds. Structure 4 is chlorocyclohexane: its carbon–chlorine bond has an inductive effect, but there is no adjacent pi system for mesomeric conjugation. Structure 7 is dimethylbenzene: the methyl groups supply inductive donation and hyperconjugation, but no substituent supplies a separate mesomeric effect; resonance within the benzene ring itself is not a substituent mesomeric effect.

Step 3: Count the qualifying structures The selected structures are 2, 5, 6 and 8, giving

$$N = 1 + 1 + 1 + 1 = 4.$$

All three tests must hold for each selected compound.

Step 4: Conclusion Exactly $\boxed{4}$ compounds show all three effects.

Quick Tip to Solve: Apply three separate tests to each structure: a polar sigma-bond effect, a conjugated substituent for mesomerism, and an alpha carbon–hydrogen bond for hyperconjugation.

88. When equal volume of $1\ \mathrm{M\ HCl}$ and $1\ \mathrm{M\ H_2SO_4}$ are separately neutralised by excess volume of $1\mathrm{M}$ NaOH solution. x and y kJ of heat is liberated respectively. The value of y/x is _____. [4 marks]

Solution:

Concept:

- For strong acid–strong base neutralisation, heat released is proportional to the number of moles of water formed: $q = n(\mathrm{H_2O})|\Delta H_{\mathrm{neut}}|$.
- HCl supplies one acidic equivalent per mole whereas complete neutralisation of $\mathrm{H_2SO_4}$ consumes two moles of NaOH per mole of acid. The excess base ensures that all acidic equivalents are neutralised.

Step 1: Express the equal acid volumes in moles Let the volume of each acid be V litres. Both solutions have molarity $1\,\mathrm{mol\,L^{-1}}$, so each contains V moles of acid. Equal volume and equal molarity guarantee equal formula-unit amounts, not equal numbers of acidic equivalents. The heat comparison must therefore be made after considering each balanced neutralisation equation.

Step 2: Compare the amounts of water formed For hydrochloric acid,

$$\mathrm{HCl + NaOH \longrightarrow NaCl + H_2O}.$$

Thus V moles of HCl form V moles of water. For sulfuric acid,

$$\mathrm{H_2SO_4 + 2\,NaOH \longrightarrow Na_2SO_4 + 2\,H_2O}.$$

Thus V moles of $\mathrm{H_2SO_4}$ form $2V$ moles of water on complete neutralisation. There is sufficient NaOH in each experiment, so the base is not a limiting reagent. The stated $1\,\mathrm{M}$ base concentration

consequently does not change this two-to-one ratio.

Step 3: Form the ratio of released heats Let h be the magnitude of heat liberated per mole of water formed in strong acid–strong base neutralisation. Then

$$x = Vh, \qquad y = 2Vh,$$

$$\frac{y}{x} = \frac{2Vh}{Vh} = 2.$$

The heats x and y are positive liberated-heat magnitudes; both reaction enthalpies are negative. Using either consistent convention gives the same positive ratio.

Step 4: Conclusion The required ratio is $\boxed{y/x = 2}$, which is dimensionless.

Quick Tip to Solve: For equal molar amounts of strong acids neutralised by excess strong base, compare the acidic equivalents: monoprotic HCl gives one and diprotic H_2SO_4 gives two.

89. The heat of solution of anhydrous $CuSO_4$ and $CuSO_4 \cdot 5H_2O$ are -70 kJ mol^{-1} and $+12$ kJ mol^{-1} respectively.

The heat of hydration of $CuSO_4$ to $CuSO_4 \cdot 5H_2O$ is $-x$ kJ. The value of x is _____. (nearest integer). [4 marks]

Solution:

Concept:

- Hess's law states that enthalpy change depends only on the initial and final states, so two paths to the same aqueous solution have equal total enthalpy changes.
- Hydration here converts solid anhydrous $CuSO_4$ into solid $CuSO_4 \cdot 5\,H_2O$. This is different from dissolving either solid; the signs of both supplied solution enthalpies must be retained.

Step 1: Write the direct dissolution path For one mole of anhydrous salt, dissolution in sufficient water has the supplied enthalpy

$$CuSO_4(s) \longrightarrow CuSO_4(aq), \qquad \Delta H_1 = -70\,\text{kJ mol}^{-1}.$$

The negative sign means the direct dissolution releases heat. The aqueous product denotes the same final dissolved state used for the pentahydrate solution enthalpy; solvent water is included as required in the thermochemical cycle.

Step 2: Write the path through the pentahydrate First hydrate the anhydrous solid:

$$CuSO_4(s) + 5\,H_2O\,(l) \longrightarrow CuSO_4 \cdot 5\,H_2O\,(s),$$

$$\Delta H_{\text{hyd}} = -x\,\text{kJ mol}^{-1}.$$

Then dissolve the resulting pentahydrate:

$$CuSO_4 \cdot 5\,H_2O\,(s) \longrightarrow CuSO_4(aq) + 5\,H_2O\,(l),$$

$$\Delta H_2 = +12\,\text{kJ mol}^{-1}.$$

This second step absorbs heat. On adding these equations, the pentahydrate and five water molecules cancel, leaving the direct dissolution equation.

Step 3: Apply Hess law with the given signs Equating the two paths gives

$$\Delta H_{\text{hyd}} + \Delta H_2 = \Delta H_1,$$

$$-x + 12 = -70, \qquad x = 82.$$

Equivalently, $\Delta H_{\text{hyd}} = -70 - 12 = -82\,\text{kJ mol}^{-1}$. Hydration must release enough heat to supply the subsequent endothermic dissolution and still yield the net release of $70\,\text{kJ}$ per mole; this checks the subtraction sign.

Step 4: Conclusion The heat of hydration is $-82\,\text{kJ mol}^{-1}$, or $-82\,\text{kJ}$ for the stated one-mole conversion. Hence $\boxed{x = 82}$ to the nearest integer.

Quick Tip to Solve: Arrange the cycle as anhydrous solid $\rightarrow$ hydrate $\rightarrow$ solution, so hydration enthalpy equals anhydrous solution enthalpy minus hydrate solution enthalpy.

90. Molarity (M) of an aqueous solution containing x g of anhyd. $CuSO_4$ in 500 mL solution at 32°C is 2×10^{-1}M. Its molality will be _____ $\times 10^{-3}$ m. (nearest integer). [Given density of the solution $= 1.25$ g/mL] [4 marks]

Solution:

Concept:

- Molarity is moles of solute per litre of solution: $M = n/V_{\text{solution}}$. Molality is moles of solute per kilogram of solvent: $m = n/m_{\text{solvent}}$.
- Density converts solution volume into solution mass. Subtract the solute mass to obtain solvent mass; using the total solution mass in the molality denominator would be incorrect.

Step 1: Find moles and mass of anhydrous salt The molarity is $2 \times 10^{-1} = 0.200\,\text{mol L}^{-1}$ and the solution volume is $500\,\text{mL} = 0.500\,\text{L}$. Therefore

$$n = MV = 0.200 \times 0.500 = 0.100\,\text{mol}.$$

Using conventional atomic masses Cu $= 63.5$, S $= 32$, and O $= 16$, the molar mass of anhydrous $CuSO_4$ is

$$M_r = 63.5 + 32 + 4(16) = 159.5\,\text{g mol}^{-1}.$$

Thus the solute mass is $x = 0.100 \times 159.5 = 15.95\,\text{g}$. No water of crystallisation is included because the salt is specified as anhydrous.

Step 2: Calculate the solvent mass The density is $1.25\,\text{g mL}^{-1}$, so the mass of the complete solution is

$$m_{\text{solution}} = \rho V = 1.25 \times 500 = 625\,\text{g}.$$

Subtracting the dissolved salt gives

$$m_{\text{solvent}} = 625 - 15.95 = 609.05\,\text{g} = 0.60905\,\text{kg}.$$

The given temperature needs no separate correction because both the stated solution volume and its density refer to the solution under the stated conditions.

Step 3: Convert to molality and round the requested coefficient Now divide the solute amount by the solvent mass in kilograms:

$$m = \frac{0.100}{0.60905} = 0.164190\ldots\ \text{mol}\,\text{kg}^{-1}.$$

Thus

$$m = 164.190\ldots \times 10^{-3}\,\text{mol}\,\text{kg}^{-1}.$$

Rounding the coefficient to the nearest integer gives 164. Using $160\,\text{g}\,\text{mol}^{-1}$ as a rounded molar mass also gives the same requested integer.

Step 4: Conclusion The molality is approximately $\boxed{164 \times 10^{-3}\,\text{mol}\,\text{kg}^{-1}}$, so the number to enter is $\boxed{164}$.

Quick Tip to Solve: Use density to find solution mass, subtract the mass of $CuSO_4$, and only then convert grams of solvent into kilograms for molality.