JEE Main 2024 Paper 1 (B.E./B.Tech) Apr 9 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 9 April 2024, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let the foci of a hyperbola H coincide with the foci of the ellipse $E : \frac{(x-1)^2}{100} + \frac{(y-1)^2}{75} = 1$ and the eccentricity of the hyperbola H be the reciprocal of the eccentricity of the ellipse E. If the length of the transverse axis of H is α and the length of its conjugate axis is β, then $3\alpha^2 + 2\beta^2$ is equal to [4 marks]

(A) 225

(B) 237

(C) 242

(D) 205

Correct Answer: (A) 225

Solution:

Concept:

- For an ellipse with semi-axes $a_E > b_E$, the focal distance satisfies $c^2 = a_E^2 - b_E^2$ and its eccentricity is $e_E = c/a_E$.
- For a hyperbola, $e_H = c/a_H$ and $c^2 = a_H^2 + b_H^2$. The transverse and conjugate axis lengths are $2a_H$ and $2b_H$, respectively.

Step 1: Find the common focal distance The centre of the ellipse is $(1, 1)$, and its major axis is horizontal because the larger denominator is under the squared x-coordinate. Read the squares of its semi-axes directly from the denominators, rather than interpreting these as full axis lengths:

$$a_E^2 = 100, \quad b_E^2 = 75, \quad c^2 = 100 - 75 = 25.$$

Thus $a_E = 10$, $c = 5$, and $e_E = 5/10 = 1/2$. The common foci ensure that the hyperbola has this same value of c; their translation away from the origin has no effect on the axis lengths.

Step 2: Recover the hyperbola axes The reciprocal-eccentricity condition gives $e_H = 2$. Hence

$$a_H = \frac{c}{e_H} = \frac{5}{2}, \qquad a_H^2 = \frac{25}{4}.$$

For a hyperbola the focal-distance identity has a plus sign, so

$$b_H^2 = c^2 - a_H^2 = 25 - \frac{25}{4} = \frac{75}{4}.$$

The question asks for complete axis lengths. Therefore $\alpha^2 = 4a_H^2 = 25$ and $\beta^2 = 4b_H^2 = 75$. Substitution yields

$$3\alpha^2 + 2\beta^2 = 3(25) + 2(75) = 225.$$

The other three numbers do not satisfy these common-focus and eccentricity conditions simultaneously.

Step 3: Conclusion The requested value is 225. Hence, the correct option is (A).

Quick Tip to Solve: Keep semi-axis symbols separate from full axis lengths: square the factor 2 when converting an axis length to its square.

2. Let z be a complex number such that the real part of $\frac{z-2i}{z+2i}$ is zero. Then, the maximum value of $|z-(6+8i)|$ is equal to [4 marks]

(A) 8

(B) 12

(C) 10

(D) ∞

Correct Answer: (B) 12

Solution:

Concept:

- Writing a complex number as $z = x + iy$ turns its real-part condition into a Cartesian locus. A quotient is defined only when its denominator is nonzero.
- The modulus $|z - a|$ is the distance between the points representing z and a. For a circle centred at the origin, the triangle inequality gives $|z - a| \leq |z| + |a|$.

Step 1: Determine the locus Put $z = x + iy$, where x, y are real. Since $z \neq -2i$, rationalising the denominator is legitimate:

$$\frac{z-2i}{z+2i} = \frac{(x+i(y-2))(x-i(y+2))}{x^2+(y+2)^2}.$$

The real numerator is $x^2 + (y-2)(y+2) = x^2 + y^2 - 4$. The denominator is strictly positive wherever the quotient exists. Therefore the given zero real part implies $x^2 + y^2 = 4$: the locus is the radius-2 circle, with the point $-2i$ excluded.

Step 2: Maximise the distance Let $a = 6 + 8i$, so $|a| = \sqrt{36+64} = 10$. Every permitted point satisfies $|z| = 2$, hence

$$|z - a| \leq |z| + |a| = 12.$$

Equality occurs on the ray opposite to a, at $z = -2a/|a| = -6/5 - 8i/5$. This point lies on the radius-2 circle and is not the excluded point $-2i$, so the bound is actually attained. Thus it is a maximum rather than merely a supremum. The value 8 is the minimum distance, 10 is the distance of the fixed point from the centre, and infinity is impossible on this bounded locus.

Step 3: Conclusion The maximum value is $\boxed{12}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert the real-part condition into a circle first; then use centre distance plus radius, checking that the farthest point is not excluded by the original denominator.

3. Let the range of the function $f(x) = \frac{1}{2+\sin 3x+\cos 3x}, x \in \mathbb{R}$ be $[a, b]$. If α and β ar respectively the A.M. and the G.M. of a and b, then $\frac{\alpha}{\beta}$ is equal to [4 marks]

(A) π

(B) $\sqrt{\pi}$

(C) $\sqrt{2}$

(D) 2

Correct Answer: (C) $\sqrt{2}$

Solution:

Concept:

- For a real angle t, $\sin t + \cos t = \sqrt{2}\sin(t + \pi/4)$, so its range is $[-\sqrt{2}, \sqrt{2}]$.
- On positive numbers the reciprocal reverses order. For positive endpoints a, b, their arithmetic and geometric means are $(a + b)/2$ and $\sqrt{ab}$.

Step 1: Determine the range endpoints As x ranges over all real numbers, so does the angle $3x$. Therefore every value between the extreme values of $\sin 3x + \cos 3x$ is attained. Adding 2 gives

$$2 - \sqrt{2} \le 2 + \sin 3x + \cos 3x \le 2 + \sqrt{2}.$$

The lower bound is strictly positive, so there is neither a pole nor a sign change in the reciprocal. Reversing the inequalities on taking reciprocals, we obtain

$$a = \frac{1}{2+\sqrt{2}}, \qquad b = \frac{1}{2-\sqrt{2}}.$$

Both endpoints belong to the range, because the corresponding trigonometric extrema are attained.

Step 2: Calculate the means through sum and product Compute the symmetric quantities instead of approximating the two endpoints separately:

$$a + b = \frac{(2-\sqrt{2}) + (2+\sqrt{2})}{4-2} = 2, \qquad ab = \frac{1}{4-2} = \frac{1}{2}.$$

It follows that

$$\alpha = \frac{a+b}{2} = 1, \qquad \beta = \sqrt{ab} = \frac{1}{\sqrt{2}}, \qquad \frac{\alpha}{\beta} = \sqrt{2}.$$

The factor 3 in the angle changes the period but not the attained range. Neither π nor $\sqrt{\pi}$ enters this algebra, and option (D) would incorrectly replace the geometric mean by the product.

Step 3: Conclusion The ratio is $\sqrt{2}$. Hence, the correct option is (C).

Quick Tip to Solve: After finding reciprocal endpoints, compute their sum and product before calculating the arithmetic and geometric means.

4. Let $\int_0^x \sqrt{1-(y'(t))^2}dt = \int_0^x y(t)dt, 0 \le x \le 3, y \ge 0, y(0) = 0$. Then at $x = 2, y'' + y + 1$ is equal to [4 marks]

(A) $\sqrt{2}$

(B) 2

(C) 1/2

(D) 1

Correct Answer: Both (B) and (D) are possible without a global twice-continuously-differentiable requirement.

Solution:

Concept:

- The fundamental theorem of calculus turns the integral equality into $\sqrt{1-(y\prime)^2} = y$, hence $(y\prime)^2 + y^2 = 1$.
- Division by $y\prime$ is valid only where it is nonzero. At a turning point the first-order equation can admit a constant continuation, so uniqueness must be checked before choosing an option.

Step 1: Derive the relation and the usual smooth solution Differentiating the two integrals gives $\sqrt{1-(y\prime)^2} = y$. Since $y \ge 0$, this is equivalent to $(y\prime)^2 + y^2 = 1$. The initial data give $y\prime(0) = 1$, using nonnegativity immediately to the right of zero. A solution on the whole interval is $y(x) = \sin x$, because $0 \le x \le 3 < \pi$ implies $\sqrt{1-\cos^2 x} = \sin x$. It obeys the original integral equality pointwise and gives

$$y\prime\prime(2) + y(2) + 1 = -\sin 2 + \sin 2 + 1 = 1.$$

Thus the usual globally smooth interpretation selects (D).

Step 2: Check whether the printed assumptions force uniqueness The printed assumptions do not require a second derivative at every point or continuity of that derivative throughout the interval. Consider instead

$$y(x) = \begin{cases} \sin x, & 0 \le x \le \pi/2, \\ 1, & \pi/2 \le x \le 3. \end{cases}$$

This function is continuously differentiable at $\pi/2$, since both pieces have value 1 and derivative zero there. On the first piece $\sqrt{1-(y\prime)^2} = \sin x = y$, and on the second piece it equals $\sqrt{1-0} = 1 = y$. Therefore integrating this pointwise equality proves the given integral identity for every stated x. Its second derivative exists at $x = 2 > \pi/2$ and is zero, so

$$y\prime\prime(2) + y(2) + 1 = 0 + 1 + 1 = 2.$$

This selects (B). The second derivative fails to exist only at the join, which the printed question does not explicitly exclude. The standard derivation $y\prime(y\prime\prime + y) = 0$ cannot eliminate this constant branch by division.

Step 3: Conclusion As printed, the conditions allow both 1, option (D), and 2, option (B), so there is no unique defensible option. Requiring a globally twice-continuously-differentiable solution would give the usual answer 1, option (D).

Quick Tip to Solve: Never divide a differentiated identity by y-prime at a turning point without checking whether a constant continuation also satisfies the original equation.

5. Two vertices of a triangle ABC are $\mathrm{A}(3,-1)$ and $\mathrm{B}(-2,3)$, and its orthocentre is $\mathrm{P}(1,1)$. If the coordinates of the point C are (α,β) and the centre of the of the circle circumscribing the triangle PAB is (h,k), then the value of $(\alpha+\beta)+2(\mathrm{h}+\mathrm{k})$ equals [4 marks]

(A) 81

(B) 15

(C) 51

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- An orthocentre lies on each altitude. Thus the line from a vertex to the orthocentre is perpendicular to the opposite side.
- A circumcentre is equidistant from the three vertices of its triangle. Subtracting two squared-distance equations gives a linear equation for its coordinates.

Step 1: Find the third vertex using two altitudes The slope of AP is $(1+1)/(1-3)=-1$, so BC, perpendicular to it, has slope 1. Since it passes through $B(-2,3)$,

$$BC:\ y=x+5.$$

Similarly the slope of BP is $(1-3)/(1+2)=-2/3$, so the slope of AC is $3/2$. Passing through $A(3,-1)$, its equation is

$$AC:\ y+1=\frac{3}{2}(x-3).$$

Their intersection satisfies $x+5=(3x-11)/2$, giving $C=(21,26)$. Hence $\alpha+\beta=47$. These lines use the orthocentre of ABC, without confusing it with a circumcentre.

Step 2: Find the circumcentre of the specified triangle The requested circumcentre belongs to PAB. Let it be (h,k). Equating squared distances to $P(1,1)$ and $A(3,-1)$, then to $B(-2,3)$, gives

$$h-k=2, \qquad 6h-4k=-11.$$

Solving, $k=-23/2$ and $h=-19/2$; thus $h+k=-21$. Substitute into the expression in the question:

$$(\alpha+\beta)+2(h+k)=47+2(-21)=5.$$

The distant negative circumcentre is consistent with an obtuse triangle; assuming it must lie inside would lead to an incorrect choice. The other numerical options fail these two equal-distance equations.

Step 3: Conclusion The requested value is 5. Hence, the correct option is (D).

Quick Tip to Solve: Label the triangle whose circumcentre is requested: here it is PAB, while the altitude information belongs to ABC.

6. The integral $\int_{1/4}^{3/4} \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)\,dx$ is equal to [4 marks]

(A) $-1/2$

(B) $-1/4$

(C) $1/4$

(D) $1/2$

Correct Answer: (B) $-1/4$

Solution:

Concept:

- If $t = \cot^{-1} u$ for positive u, then $t \in (0, \pi/2)$, $\cot t = u$, and $\cos 2t = (\cot^2 t - 1)/(\cot^2 t + 1)$.
- An inverse trigonometric expression should be simplified algebraically before integration. After simplification, the fundamental theorem evaluates a continuous integrand by subtracting its antiderivative at the two limits.

Step 1: Simplify the double angle On the entire integration interval $1/4 \le x \le 3/4$, both $1 - x$ and $1 + x$ are positive. Set $t = \cot^{-1}\sqrt{(1-x)/(1+x)}$. There is no ambiguity from the square root or the principal inverse cotangent. Then

$$\cos 2t = \frac{\frac{1-x}{1+x} - 1}{\frac{1-x}{1+x} + 1} = \frac{-2x/(1+x)}{2/(1+x)} = -x.$$

In particular, the integrand is negative, as the angle $2t$ lies between $\pi/2$ and π. This also checks the sign independently of the algebra.

Step 2: Integrate the resulting polynomial The original expression therefore reduces exactly to

$$I = -\int_{1/4}^{3/4} x\,dx = -\frac{1}{2}\left[\left(\frac{3}{4}\right)^2 - \left(\frac{1}{4}\right)^2\right] = -\frac{1}{2}\frac{8}{16} = -\frac{1}{4}.$$

The positive choices are excluded by the sign of the integrand. The interval has width $1/2$ and the linear integrand has mean value $-1/2$, so its integral is also checked as $(-1/2)(1/2) = -1/4$; the value $-1/2$ confuses the mean value with the area.

Step 3: Conclusion The integral equals $\boxed{-1/4}$. Hence, the correct option is (B).

Quick Tip to Solve: For a cosine of twice an inverse cotangent, use the double-angle identity in cotangent squared; here the radicals disappear immediately and leave the integrand $-x$.

7. Let $\alpha, \beta; \alpha > \beta$, be the roots of the equation $x^2 - \sqrt{2}x - \sqrt{3} = 0$. Let $\mathrm{P}_n = \alpha^n - \beta^n, n \in \mathrm{N}$. Then $(11\sqrt{3} - 10\sqrt{2})\mathrm{P}_{10} + (11\sqrt{2} + 10)\mathrm{P}_{11} - 11\mathrm{P}_{12}$ is equal to [4 marks]

(A) $10\sqrt{3}\mathrm{P}_9$

(B) $11\sqrt{3}\mathrm{P}_9$

(C) $11\sqrt{2}\mathrm{P}_9$

(D) $10\sqrt{2}\mathrm{P}_9$

Correct Answer: (A) $10\sqrt{3}\mathrm{P}_9$

Solution:

Concept:

- Every root r of $x^2 - \sqrt{2}x - \sqrt{3} = 0$ satisfies $r^2 = \sqrt{2}r + \sqrt{3}$. Multiplying by powers of the root generates a recurrence.
- Subtracting the identities for the two roots gives $P_n = \sqrt{2}P_{n-1} + \sqrt{3}P_{n-2}$, with $P_n = \alpha^n - \beta^n$.

Step 1: Establish the recurrence for the sequence For either root $r = \alpha$ or $r = \beta$, multiply its quadratic identity by r^{n-2}. For $n \geq 2$, this gives

$$r^n = \sqrt{2}\, r^{n-1} + \sqrt{3}\, r^{n-2}.$$

Subtracting the equation for β from the one for α proves

$$P_n = \sqrt{2}P_{n-1} + \sqrt{3}P_{n-2}.$$

We need only the cases $n = 12$ and $n = 11$. Computing the roots explicitly or raising radicals to the twelfth power would add work without using the structure of the requested expression.

Step 2: Eliminate the highest powers successively Denote the requested expression by S, and first substitute $P_{12} = \sqrt{2}P_{11} + \sqrt{3}P_{10}$:

$$\begin{aligned} S &= (11\sqrt{3} - 10\sqrt{2})P_{10} + (11\sqrt{2} + 10)P_{11} \\ &\quad - 11(\sqrt{2}P_{11} + \sqrt{3}P_{10}) \\ &= 10P_{11} - 10\sqrt{2}P_{10}. \end{aligned}$$

Now use $P_{11} = \sqrt{2}P_{10} + \sqrt{3}P_9$, obtaining

$$S = 10(P_{11} - \sqrt{2}P_{10}) = 10\sqrt{3}P_9.$$

The coefficient 11 cancels with the coefficient of P_{12}; it does not survive in the answer. The remaining recurrence term carries $\sqrt{3}$, ruling out the two options with $\sqrt{2}$.

Step 3: Conclusion The expression equals $10\sqrt{3}P_9$. Hence, the correct option is (A).

Quick Tip to Solve: Use the polynomial satisfied by each root to reduce high-power combinations; eliminate the largest index first.

8. If $\log_e y = 3\sin^{-1} x$, then $(1-x^2)y'' - xy'$ at $x = \frac{1}{2}$ is equal to [4 marks]

(A) $9e^{\pi/2}$

(B) $9e^{\pi/6}$

(C) $3e^{\pi/2}$

(D) $3e^{\pi/6}$

Correct Answer: (A) $9e^{\pi/2}$

Solution:

Concept:

- Use the principal inverse sine, whose derivative is $1/\sqrt{1-x^2}$ for $-1 < x < 1$. The given logarithm also implies $y > 0$.
- The chain rule gives $d(\log_e y)/dx = y'/y$. Differentiating a useful first-derivative identity avoids expanding the second derivative into several fractions.

Step 1: Differentiate the logarithmic relation From $\log_e y = 3\sin^{-1} x$, we obtain

$$\frac{y'}{y} = \frac{3}{\sqrt{1-x^2}}, \qquad \sqrt{1-x^2}\, y' = 3y.$$

This identity holds on an open interval containing $x = 1/2$, so it can be differentiated there. The factor 3 multiplies the entire inverse-sine function.

Step 2: Obtain the required combination Differentiate the last equality using the product rule:

$$\sqrt{1-x^2}\, y'' - \frac{x}{\sqrt{1-x^2}} y' = 3y'.$$

Multiplying by $\sqrt{1-x^2}$ and using the first identity again yields

$$(1-x^2)y'' - xy' = 3\sqrt{1-x^2}\, y' = 9y.$$

Thus the requested expression reduces to a multiple of the original function.

Step 3: Evaluate the function At $x = 1/2$, the principal value is $\sin^{-1}(1/2) = \pi/6$, giving $\log_e y = \pi/2$ and $y = e^{\pi/2}$. The options with exponent $\pi/6$ omit the factor 3 in the logarithmic equation; those with coefficient 3 omit the second factor introduced by differentiation.

Step 4: Conclusion The required value is $\boxed{9e^{\pi/2}}$. Hence, the correct option is (A).

Quick Tip to Solve: For an exponential of an inverse trigonometric function, first isolate a square-root factor times the first derivative; differentiating that identity often produces the requested combination directly.

9. If an unbiased dice is rolled thrice, then the probability of getting a greater number in the i^{th} roll than the number obtained in the $(i-1)^{th}$ roll, $i = 2, 3$, is equal to [4 marks]

(A) 5/54

(B) 2/54

(C) 1/54

(D) 3/54

Correct Answer: (A) 5/54

Solution:

Concept:

- Three independent rolls of a fair six-sided die give $6^3 = 216$ equally likely ordered triples.
- The conditions for $i = 2, 3$ mean strict increase throughout: if the results are a, b, c, the event is $a < b < c$. Equal results are excluded.

Step 1: Translate both inequalities The second roll must exceed the first, and the third must exceed the second. Thus all three results must be distinct. It is insufficient to require only that the final result exceed the first: for example, $(1, 4, 3)$ would satisfy that weaker condition but fails the question. Independence allows us to count triples directly rather than incorrectly multiply two dependent comparison probabilities.

Step 2: Count the increasing triples Choose any three distinct faces from the six available faces. Each chosen set has exactly one increasing arrangement. Consequently the favourable count is

$$\binom{6}{3} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20.$$

For instance, the set $\{1, 3, 6\}$ contributes only $(1, 3, 6)$, not its other five permutations. This explains why an extra factor of $3!$ must not be inserted.

Step 3: Divide by the sample-space size Every ordered triple has probability $1/216$. Therefore

$$P(a < b < c) = \frac{20}{216} = \frac{5}{54}.$$

As a check, the probability that all three results differ is $6 \cdot 5 \cdot 4/216$; symmetry divides this among the six possible strict orderings, again giving $20/216$. The smaller listed fractions do not count all choices of three distinct faces.

Step 4: Conclusion The probability is $\boxed{5/54}$. Hence, the correct option is (A).

Quick Tip to Solve: For strict increase in repeated fair-die rolls, choose distinct faces and use their unique sorted ordering; do not count all permutations as favourable.

10. $\lim_{x\to\frac{\pi}{2}}\left(\frac{\int_{x^3}^{(\pi/2)^3}\left(\sin\left(2t^{1/3}\right)+\cos\left(t^{1/3}\right)\right)dt}{\left(x-\frac{\pi}{2}\right)^2}\right)$ **is equal to** [4 marks]

(A) $\frac{3\pi^2}{2}$

(B) $\frac{9\pi^2}{8}$

(C) $\frac{5\pi^2}{9}$

(D) $\frac{11\pi^2}{10}$

Correct Answer: (B) $\frac{9\pi^2}{8}$

Solution:

Concept:

- The fundamental theorem of calculus differentiates an integral with a variable endpoint; the endpoint derivative must be included.
- For a twice differentiable function F with $F(a) = F\prime(a) = 0$, Taylor expansion gives $\lim_{x\to a} F(x)/(x-a)^2 = F\prime\prime(a)/2$.

Step 1: Transform the endpoint Put $a = \pi/2$ and substitute $t = u^3$, so $dt = 3u^2du$. The numerator becomes

$$F(x) = \int_x^a 3u^2(\sin 2u + \cos u)\,du.$$

This substitution is valid near the positive point a, and changes both endpoints exactly. The upper endpoint is constant; hence the derivative of this integral carries a minus sign.

Step 2: Find the first two derivatives Let $g(x) = 3x^2(\sin 2x + \cos x)$. Then $F\prime(x) = -g(x)$. Since $\sin\pi = \cos(\pi/2) = 0$, both $F(a)$ and $F\prime(a)$ vanish. Differentiate the integrand:

$$g\prime(x) = 6x(\sin 2x + \cos x) + 3x^2(2\cos 2x - \sin x).$$

At a, its first term is zero and the second is $3a^2(-2-1) = -9a^2$. Thus $F\prime\prime(a) = 9a^2$.

Step 3: Evaluate the quadratic quotient The integrand is smooth near a, so the second-order expansion applies on both sides:

$$F(x) = \frac{9a^2}{2}(x-a)^2 + o((x-a)^2).$$

Dividing gives $9a^2/2 = 9\pi^2/8$. The other options do not preserve both the endpoint chain factor and the factor $1/2$ from the quadratic Taylor term.

Step 4: Conclusion The limit equals $\boxed{9\pi^2/8}$. Hence, the correct option is (B).

Quick Tip to Solve: When the integrand vanishes at the limiting endpoint, expect a quadratic numerator; compute its second derivative and retain the Taylor factor one-half.

11. The value of the integral $\int_{-1}^{2} \log_e\left(x+\sqrt{x^2+1}\right)\,dx$ is [4 marks]

(A) $\sqrt{5}-\sqrt{2}+\log_e\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

(B) $\sqrt{2}-\sqrt{5}+\log_e\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

(C) $\sqrt{5}-\sqrt{2}+\log_e\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

(D) $\sqrt{2}-\sqrt{5}+\log_e\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

Correct Answer: (D) $\sqrt{2}-\sqrt{5}+\log_e\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

Solution:

Concept:

- The derivative of $\log_e(x+\sqrt{x^2+1})$ is $1/\sqrt{x^2+1}$. Its argument is positive for every real x, since $\sqrt{x^2+1}>|x|$.
- Integration by parts, $\int u\,dv = uv - \int v\,du$, converts the logarithmic integral into an elementary square-root integral.

Step 1: Find an antiderivative Take $u=\log_e(x+\sqrt{x^2+1})$ and $dv=dx$. Then

$$\int \log_e(x+\sqrt{x^2+1})\,dx = x\log_e(x+\sqrt{x^2+1}) - \sqrt{x^2+1}+C.$$

Indeed, the product derivative contributes $x/\sqrt{x^2+1}$, exactly cancelled by the derivative of the square-root term. This checks the sign before evaluating the endpoints.

Step 2: Substitute both limits At $x=2$, the antiderivative is $2\log_e(2+\sqrt{5})-\sqrt{5}$. At $x=-1$, it is $-\log_e(\sqrt{2}-1)-\sqrt{2}$. Their difference is therefore

$$\sqrt{2}-\sqrt{5}+2\log_e(2+\sqrt{5})+\log_e(\sqrt{2}-1).$$

The negative lower limit multiplies the logarithm before the lower-endpoint value is subtracted.

Step 3: Combine the logarithms Use $(2+\sqrt{5})^2=9+4\sqrt{5}$ and $\sqrt{2}-1=1/(1+\sqrt{2})$. All logarithm arguments are positive, so the logarithm laws give

$$\sqrt{2}-\sqrt{5}+\log_e\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right).$$

Options with $7+4\sqrt{5}$ square the binomial incorrectly, while the remaining incorrect option reverses the nonlogarithmic endpoint contribution.

Step 4: Conclusion The integral is $\boxed{\sqrt{2}-\sqrt{5}+\log_e\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)}$. Hence, the correct option is (D).

Quick Tip to Solve: For this logarithmic primitive, integrate by parts and use conjugates to simplify the lower-endpoint logarithm only after subtracting the endpoint values.

12. Between the following two statements:

Statement I : Let $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ **and** $\vec{b}=2\hat{i}+\hat{j}-\hat{k}$**. Then the vector** $\vec{r}$ **satisfying** $\vec{a}\times\vec{r}=\vec{a}\times\vec{b}$ **and** $\vec{a}\cdot\vec{r}=0$ **is of magnitude** $\sqrt{10}$**.**

Statement II : In a triangle $ABC, \cos 2A+\cos 2B+\cos 2C\geq -\frac{3}{2}$**.** [4 marks]

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (D) Statement I is incorrect but Statement II is correct.

Solution:

Concept:

- If $\vec{a}\times(\vec{r}-\vec{b})=0$ and $\vec{a}\neq 0$, then $\vec{r}=\vec{b}+\lambda\vec{a}$. The dot-product constraint fixes λ.
- For triangle angles, $\cos 2A+\cos 2B+\cos 2C=-1-4\cos A\cos B\cos C$. An upper bound on the product proves the stated lower bound.

Step 1: Test Statement I Compute $\vec{a}\cdot\vec{b}=2+2+3=7$ and $|\vec{a}|^2=1+4+9=14$. From $\vec{a}\cdot(\vec{b}+\lambda\vec{a})=0$, we get $7+14\lambda=0$, hence $\lambda=-1/2$. Consequently

$$\vec{r}=\tfrac{3}{2}\hat{i}-\tfrac{1}{2}\hat{k},\qquad |\vec{r}|=\sqrt{\tfrac{9}{4}+\tfrac{1}{4}}=\frac{\sqrt{10}}{2}.$$

The vector is unique and its magnitude is not $\sqrt{10}$; Statement I is false.

Step 2: Bound the cosine product If a triangle is right or obtuse, the cosine product is nonpositive, so it is already at most $1/8$. For an acute triangle,

$$\cos A\cos B=\frac{\cos(A+B)+\cos(A-B)}{2}\leq\frac{1-\cos C}{2}.$$

Writing $u=\cos C\in(0,1)$ gives $\cos A\cos B\cos C\leq u(1-u)/2\leq 1/8$, because $u(1-u)=1/4-(u-1/2)^2$.

Step 3: Test Statement II and select the pair Substituting this bound into the triangle identity yields

$$\cos 2A+\cos 2B+\cos 2C\geq -1-4\left(\frac{1}{8}\right)=-\frac{3}{2}.$$

Equality occurs for the equilateral triangle, confirming that the bound is sharp. Statement II is

therefore true. Both-true and both-false options each misclassify a statement, and the option asserting only Statement I reverses both conclusions.

Step 4: Conclusion Statement I is incorrect and Statement II is correct. Hence, the correct option is (D).

Quick Tip to Solve: The cross-product equation fixes the component perpendicular to a vector; impose the dot-product condition to remove its parallel component before computing the magnitude.

13. If the variance of the frequency distribution

x	c	$2c$	$3c$	$4c$	$5c$	$6c$
f	**2**	**1**	**1**	**1**	**1**	**1**

is 160, then the value of $c \in N$ is [4 marks]

(A) 5

(B) 8

(C) 6

(D) 7

Correct Answer: (D) 7

Solution:

Concept:

- For total frequency N, the variance is $\sigma^2 = \sum fx^2/N - (\sum fx/N)^2$. Frequency weights must be included in both sums.
- Multiplying every observation by c multiplies the variance by c^2. Here the distribution has seven observations because the first value occurs twice.

Step 1: Calculate the weighted sums The total frequency is $N = 2 + 1 + 1 + 1 + 1 + 1 = 7$. The first moment numerator is

$$\sum fx = 2c + 2c + 3c + 4c + 5c + 6c = 22c.$$

The second moment numerator is

$$\sum fx^2 = 2c^2 + 4c^2 + 9c^2 + 16c^2 + 25c^2 + 36c^2 = 92c^2.$$

Although there are six displayed columns of values, the first contributes twice to both sums.

Step 2: Apply the variance formula The mean is $22c/7$ and the mean square is $92c^2/7$. Thus

$$\sigma^2 = \frac{92c^2}{7} - \left(\frac{22c}{7}\right)^2 = \frac{644 - 484}{49}c^2 = \frac{160c^2}{49}.$$

The subtraction uses the square of the mean, not the mean itself; keeping the denominator 49 until the last step avoids premature rounding.

Step 3: Use the given variance Equating the exact expression to 160 gives

$$\frac{160c^2}{49} = 160 \quad \Longrightarrow \quad c^2 = 49.$$

The algebraic roots are ± 7, but the condition $c \in \mathbb{N}$ selects $c = 7$. The other listed positive integers would give variances $160(25/49)$, $160(64/49)$, or $160(36/49)$, respectively, and hence fail the supplied value.

Step 4: Conclusion The required natural number is $\boxed{c = 7}$. Hence, the correct option is (D).

Quick Tip to Solve: Factor a common scale out of all observations before calculating variance, and count repeated observations using their frequencies rather than the number of table columns.

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14. Let $B = \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix}$ and A be a 2×2 matrix such that $AB^{-1} = A^{-1}$. If $BCB^{-1} = A$ and $C^4+\alpha C^2+\beta I = O$, then $2\beta - \alpha$ is equal to [4 marks]

(A) 16

(B) 10

(C) 8

(D) 2

Correct Answer: (B) 10

Solution:

Concept:

- A matrix obeys its characteristic polynomial: for a 2×2 matrix B, $B^2 - (\operatorname{tr} B)B + (\det B)I = O$.
- Similarity preserves polynomial identities: if $A = BCB^{-1}$, then $p(A) = Bp(C)B^{-1}$ for any polynomial p. The supplied inverse relation also determines A^2.

Step 1: Relate the square of A to B The inverse A^{-1} exists by the given relation. Multiplying $AB^{-1} = A^{-1}$ on the left by A and then on the right by B gives

$$A^2B^{-1} = I, \qquad A^2 = B.$$

Matrix multiplication order has been retained throughout; no general commutativity assumption is needed.

Step 2: Obtain a polynomial identity For $B = \begin{pmatrix} 1 & 3 \\ 1 & 5 \end{pmatrix}$, we have $\operatorname{tr} B = 6$ and $\det B = 5 - 3 = 2$. The characteristic polynomial is $t^2 - 6t + 2$, so

$$B^2 - 6B + 2I = O.$$

Substitute $B = A^2$ to obtain $A^4 - 6A^2 + 2I = O$. This avoids finding a particular square root of B.

Step 3: Transfer the identity to C Conjugating by B^{-1} and B yields $C^4 - 6C^2 + 2I = O$. Comparing

with the given identity gives $\alpha = -6$, $\beta = 2$. These coefficients are unique: C^2 is similar to $A^2 = B$, which is not a scalar matrix, so C^2 and I are linearly independent. Therefore

$$2\beta - \alpha = 2(2) - (-6) = 10.$$

The other options do not satisfy the trace-and-determinant coefficients.

Step 4: Conclusion The value is $\boxed{10}$. Hence, the correct option is (B).

Quick Tip to Solve: An inverse relation can reveal a matrix square; use the smaller matrix characteristic polynomial and similarity instead of calculating its square root explicitly.

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15. $\lim_{x\to 0} \frac{e-(1+2x)^{1/(2x)}}{x}$ **is equal to** [4 marks]

(A) $-\frac{2}{e}$

(B) $e - e^2$

(C) 0

(D) e

Correct Answer: (D) e

Solution:

Concept:

- To analyse a variable power, take its logarithm. Near zero, $\log(1+u) = u - u^2/2 + O(u^3)$, valid for real u sufficiently small with $1 + u > 0$.
- The linear term after exponentiation follows from $e^v = 1 + v + O(v^2)$. A quotient with denominator x requires the first-order change in the numerator, not just its limiting value.

Step 1: Expand the logarithm of the power Let $f(x) = (1+2x)^{1/(2x)}$, defined for nonzero real x near zero with $1 + 2x > 0$. Its logarithm satisfies

$$\log f(x) = \frac{\log(1+2x)}{2x} = \frac{2x - 2x^2 + O(x^3)}{2x} = 1 - x + O(x^2).$$

Thus $f(x) \to e$, but we retain the additional term $-x$. Stopping at $f(x) \to e$ only identifies the indeterminate form $0/0$, and does not determine the requested limit.

Step 2: Exponentiate and form the quotient Applying the exponential expansion to the small exponent $-x + O(x^2)$ gives

$$f(x) = e\exp(-x + O(x^2)) = e(1 - x + O(x^2)).$$

Consequently,

$$\frac{e - f(x)}{x} = \frac{ex + O(x^2)}{x} = e + O(x) \longrightarrow e.$$

The expansions hold on both sides of zero inside the real domain, so this is a two-sided limit. The coefficient is positive because the numerator subtracts $f(x)$ from e; reversing that subtraction would give $-e$. Options involving $-2/e$ or $e - e^2$ do not equal this coefficient, while zero would incorrectly discard the first-order term.

Step 3: Conclusion The limit is $\boxed{e}$. Hence, the correct option is (D).

Quick Tip to Solve: For this variable-power limit, expand its logarithm one order beyond the constant term, then exponentiate; the coefficient of x gives the answer after subtraction.

---

16. Consider the line L passing through the points $(1, 2, 3)$ and $(2, 3, 5)$. The distance of the point $\left(\frac{11}{3}, \frac{11}{3}, \frac{19}{3}\right)$ from the line L along the line $\frac{3x-11}{2} = \frac{3y-11}{1} = \frac{3z-19}{2}$ is equal to [4 marks]

(A) 6

(B) 3

(C) 5

(D) 4

Correct Answer: (B) 3

Solution:

Concept:

- The line through two points has a direction vector equal to their difference. Parametric equations provide a direct way to find its intersection with another line.
- Distance measured along a specified line is the length from the given point to the intersection on that line. It need not be the perpendicular distance to the target line.

Step 1: Parameterise the target line The direction from $(1, 2, 3)$ to $(2, 3, 5)$ is $(1, 1, 2)$. Hence a point on L is

$$Q = (1 + t,\ 2 + t,\ 3 + 2t), \qquad t \in \mathbb{R}.$$

The point from which distance is measured is $P = (11/3, 11/3, 19/3)$.

Step 2: Parameterise the prescribed direction The equal-ratio equation in the question has direction ratios $(2, 1, 2)$. Absorbing a common scale into the parameter, write its points as

$$R = \left(\frac{11}{3} + 2s, \frac{11}{3} + s, \frac{19}{3} + 2s\right).$$

At $s = 0$, this point is exactly P, so s directly gives the displacement along that line.

Step 3: Solve for the intersection and verify all coordinates At the intersection, the difference of the first two coordinates on L is -1, whereas on the prescribed line it is s. Thus $s = -1$. The second-coordinate equality then gives $2 + t = 8/3$, so $t = 2/3$. The intersection is

$$Q = \left(\frac{5}{3}, \frac{8}{3}, \frac{13}{3}\right).$$

Its third coordinate is $3 + 2(2/3) = 13/3$, agreeing with $19/3 + 2(-1)$. Thus the lines really intersect; they are not skew.

Step 4: Calculate the length along the specified line The displacement is $Q - P = (-2, -1, -2)$. Its length is therefore

$$PQ = \sqrt{(-2)^2 + (-1)^2 + (-2)^2} = \sqrt{9} = 3.$$

The alternatives 6, 5 and 4 do not equal this displacement length. Using a perpendicular-distance formula would answer a different question.

Step 5: Conclusion The required distance is $\boxed{3}$ units. Hence, the correct option is (B).

Quick Tip to Solve: When distance is requested along a particular line, first find the intersection in that direction, then take the ordinary length of the displacement.

17. The area (in square units) of the region enclosed by the ellipse $x^2 + 3y^2 = 18$ in the first quadrant below the line $y = x$ is [4 marks]

(A) $\sqrt{3}\pi + 1$

(B) $\sqrt{3}\pi$

(C) $\sqrt{3}\pi - \frac{3}{4}$

(D) $\sqrt{3}\pi + \frac{3}{4}$

Correct Answer: (B) $\sqrt{3}\pi$

Solution:

Concept:

- A linear scaling $x = au,\ y = bv$ changes the ellipse $x^2/a^2 + y^2/b^2 = 1$ into a unit circle and multiplies every area by the Jacobian ab.
- In a unit circle a sector of angle θ, measured in radians, has area $\theta/2$. The straight boundary through the centre becomes another radial boundary under this scaling.

Step 1: Convert the ellipse to a unit circle The given ellipse can be written as

$$\frac{x^2}{18} + \frac{y^2}{6} = 1.$$

Set $x = 3\sqrt{2}\,u$ and $y = \sqrt{6}\,v$. The first quadrant of the ellipse becomes the first quadrant of $u^2+v^2 \le 1$. The area multiplier is

$$\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = 3\sqrt{2}\,\sqrt{6} = 6\sqrt{3}.$$

Step 2: Transform the condition below the line The requested region satisfies $0 \le y \le x$. Under the scaling this is

$$0 \le v \le \frac{3\sqrt{2}}{\sqrt{6}}u = \sqrt{3}\,u.$$

In polar coordinates in the (u, v)-plane, the ray $v = \sqrt{3}u$ makes angle $\arctan(\sqrt{3}) = \pi/3$ with the positive u-axis. Thus the region is the unit circular sector between angles 0 and $\pi/3$, rather than half of a quadrant.

Step 3: Integrate the sector area and scale it back Writing $u = r\cos\theta$, $v = r\sin\theta$, the desired area

equals

$$\mathcal{A} = 6\sqrt{3}\int_0^{\pi/3}\int_0^1 r\,dr\,d\theta$$
$$= 6\sqrt{3}\left(\frac{1}{2}\right)\left(\frac{\pi}{3}\right) = \sqrt{3}\pi.$$

The two radial boundaries pass through the centre, so no separate triangular area needs to be added or removed. The nonzero additive corrections in the other three options therefore do not arise.

Step 4: Conclusion The enclosed area below $y = x$ in the first quadrant is $\boxed{\sqrt{3}\pi}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: Scale an axis-aligned ellipse to a circle before finding an area cut by a line through its centre; remember that the line angle changes under unequal scaling.

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18. Let a, ar, ar^2, ... be an infinite G.P. If $\sum_{n=0}^{\infty} ar^n = 57$ and $\sum_{n=0}^{\infty} a^3r^{3n} = 9747$, then $a + 18r$ is equal to [4 marks]

(A) 27

(B) 38

(C) 31

(D) 46

Correct Answer: (C) 31

Solution:

Concept:

- An infinite geometric series with first term a and ratio r converges for $|r| < 1$, with sum $a/(1 - r)$.
- Cubing every term produces a new geometric progression with first term a^3 and ratio r^3. The second given sum is therefore $a^3/(1 - r^3)$, not the cube of the first sum.

Step 1: Express the first term using the first sum The first equality gives

$$\frac{a}{1-r} = 57 \quad \Longrightarrow \quad a = 57(1 - r).$$

Since the sum is nonzero and finite, $a \neq 0$ and the convergence condition $|r| < 1$ applies. In particular, $1 - r$ does not vanish.

Step 2: Combine the two independent sum conditions The series formed from the cubes gives

$$\frac{a^3}{1-r^3} = 9747.$$

Substituting $a = 57(1 - r)$, using $1 - r^3 = (1 - r)(1 + r + r^2)$, and noting that $57^3 = 185193$ and $185193/9747 = 19$, we obtain

$$\frac{(1-r)^2}{1+r+r^2} = \frac{1}{19}.$$

This simplification is valid because $1 - r \neq 0$.

Step 3: Solve for the ratio and retain the convergent value Cross-multiplication yields

$$19(1 - 2r + r^2) = 1 + r + r^2.$$

Thus

$$18r^2 - 39r + 18 = 0 \quad \Longrightarrow \quad (3r - 2)(2r - 3) = 0.$$

The two algebraic roots are $r = 2/3$ and $r = 3/2$. The latter makes the nonzero infinite series divergent and must be discarded. Consequently $r = 2/3$ and $a = 57(1 - 2/3) = 19$.

Step 4: Evaluate the requested expression and check the sums The value is

$$a + 18r = 19 + 18\left(\frac{2}{3}\right) = 31.$$

As a verification, $19/(1 - 2/3) = 57$, and

$$\frac{19^3}{1 - (2/3)^3} = \frac{6859 \cdot 27}{19} = 9747.$$

Both original conditions hold. The other listed values 27, 38 and 46 do not result from the admissible ratio.

Step 5: Conclusion The requested value is $\boxed{31}$. Hence, the correct option is (C).

Quick Tip to Solve: For a sum of cubes of G.P. terms, use ratio r^3; after solving, reject every ratio that violates $|r| < 1$.

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19. The sum of the coefficient of $x^{2/3}$ and $x^{-2/5}$ in the binomial expansion of $\left(x^{2/3} + \frac{1}{2}x^{-2/5}\right)^9$ is

[4 marks]

(A) 19/4

(B) 69/16

(C) 63/16

(D) 21/4

Correct Answer: (D) 21/4

Solution:

Concept:

- The general term of $(p + q)^9$ is $\binom{9}{r}p^{9-r}q^r$, where r is an integer from 0 to 9.
- For fractional powers, combine the powers of x first. Equating that total exponent to the desired power identifies the unique term, while the remaining numerical factor is its coefficient.

Step 1: Write the general term with its total exponent Choose r factors of $\frac{1}{2}x^{-2/5}$ and $9 - r$ factors of

$x^{2/3}$. The resulting term is

$$T_{r+1} = \binom{9}{r}\left(x^{2/3}\right)^{9-r}\left(\frac{1}{2}x^{-2/5}\right)^{r}$$
$$= \frac{\binom{9}{r}}{2^r}x^{\frac{2}{3}(9-r)-\frac{2}{5}r} = \frac{\binom{9}{r}}{2^r}x^{6-16r/15}.$$

The exponent decreases by $16/15$ for each unit increase in r, so each requested exponent can occur at most once.

Step 2: Find the coefficient of the positive fractional power For the term in $x^{2/3}$, equate exponents:

$$6 - \frac{16r}{15} = \frac{2}{3} \quad \Longrightarrow \quad r = 5.$$

This lies in the allowed range. Hence its coefficient is

$$\frac{\binom{9}{5}}{2^5} = \frac{126}{32} = \frac{63}{16}.$$

It is the sixth term because the general term is indexed by $r + 1$.

Step 3: Find the coefficient of the negative fractional power For the term in $x^{-2/5}$, solve

$$6 - \frac{16r}{15} = -\frac{2}{5} \quad \Longrightarrow \quad r = 6.$$

The coefficient of this seventh term is

$$\frac{\binom{9}{6}}{2^6} = \frac{84}{64} = \frac{21}{16}.$$

The negative exponent does not make the coefficient negative: both terms of the binomial have positive numerical coefficients.

Step 4: Add the two coefficients The required sum is

$$\frac{63}{16} + \frac{21}{16} = \frac{84}{16} = \frac{21}{4}.$$

The option $63/16$ is only the first coefficient and omits the second. Neither $19/4$ nor $69/16$ equals the correctly combined sum.

Step 5: Conclusion The sum of the coefficients is $\boxed{\frac{21}{4}}$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the coefficient 2^{-r} separate from the exponent of x; solving for the power alone does not give the numerical coefficient.

20. Let $\vec{a} = 2\hat{i} + \alpha\hat{j} + \hat{k}, \vec{b} = -\hat{i} + \hat{k}, \vec{c} = \beta\hat{j} - \hat{k}$, where α and β are integers and $\alpha\beta = -6$. Let the values of the ordered pair (α, β), for which the area of the parallelogram of diagonals $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$ is $\frac{\sqrt{21}}{2}$, be (α_1, β_1) and (α_2, β_2). Then $\alpha_1^2 + \beta_1^2 - \alpha_2\beta_2$ is equal to [4 marks]

(A) 21

(B) 24

(C) 19

(D) 17

Correct Answer: (C) 19

Solution:

Concept:

- If $\mathbf{d}_1$ and $\mathbf{d}_2$ are the diagonal vectors of a parallelogram, its area is $\frac{1}{2}|\mathbf{d}_1 \times \mathbf{d}_2|$. The factor one-half distinguishes diagonals from adjacent sides.
- The scalar constraint $\alpha\beta = -6$, together with integrality, limits the possible ordered pairs. It can be combined with a squared cross-product equation without introducing a sign ambiguity in the area.

Step 1: Form the two diagonal vectors Using the vectors in the question,

$$\mathbf{d}_1 = \vec{a} + \vec{b} = (1, \alpha, 2), \qquad \mathbf{d}_2 = \vec{b} + \vec{c} = (-1, \beta, 0).$$

Their cross product is

$$\mathbf{d}_1 \times \mathbf{d}_2 = (-2\beta, -2, \alpha + \beta).$$

Thus its squared magnitude is $4\beta^2 + 4 + (\alpha + \beta)^2$.

Step 2: Use the prescribed area The stated area $\sqrt{21}/2$ requires the cross-product magnitude to be $\sqrt{21}$. Squaring gives

$$4\beta^2 + 4 + (\alpha + \beta)^2 = 21.$$

Expand and insert $\alpha\beta = -6$:

$$\alpha^2 + 5\beta^2 + 2\alpha\beta = 17 \quad \Longrightarrow \quad \alpha^2 + 5\beta^2 = 29.$$

Step 3: Find all integral ordered pairs The last equation implies $\beta^2 \leq 29/5$. Since β is a nonzero integer, $|\beta|$ is 1 or 2. If $|\beta| = 1$, the product condition gives $|\alpha| = 6$, leading to $\alpha^2 + 5\beta^2 = 41$, which is impossible. If $|\beta| = 2$, then $|\alpha| = 3$, and $9 + 5(4) = 29$, as required. The two pairs are therefore

$$(\alpha, \beta) = (-3, 2), \quad (3, -2).$$

Step 4: Evaluate the expression independently of ordering Whichever pair is called the first, $\alpha_1^2 + \beta_1^2 = 9 + 4 = 13$. The second pair has product $\alpha_2\beta_2 = -6$. Therefore

$$\alpha_1^2 + \beta_1^2 - \alpha_2\beta_2 = 13 - (-6) = 19.$$

This result does not depend on interchanging the two pairs. The other options 21, 24 and 17 disagree with the two conditions used together.

Step 5: Conclusion The required value is $\boxed{19}$. Hence, the correct option is (C).

Quick Tip to Solve: For diagonals, halve the cross-product magnitude before comparing areas; then use the given product to simplify the squared norm.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $A = \{(x, y) : 2x + 3y = 23, x, y \in \mathbb{N}\}$ and $B = \{x : (x, y) \in A\}$. Then the number of one-one functions from A to B is equal to _____. [4 marks]

Solution:

Concept:

- A one-one function assigns different images to distinct domain elements. Between two finite sets of the same cardinality n, every injection is a bijection, and there are $n!$ such functions.
- The elements of A are ordered pairs, whereas the elements of B are their first coordinates. Counting these sets separately prevents confusing the relation with its projection.

Step 1: List the positive-integer solutions The equation is $2x + 3y = 23$, with $x, y \in \mathbb{N}$. Since the right side is odd and $2x$ is even, y must be odd. Positivity of x gives

$$3y \leq 21 \quad \Longrightarrow \quad y \leq 7.$$

Thus the possible positive values are $y = 1, 3, 5, 7$. Substitution gives $x = 10, 7, 4, 1$, respectively. Every pair satisfies the original equation.

Step 2: Determine the cardinalities of the domain and codomain Consequently,

$$A = \{(10, 1), (7, 3), (4, 5), (1, 7)\}, \qquad B = \{10, 7, 4, 1\}.$$

Both sets have four elements. Distinct solutions have distinct first coordinates because for a fixed x, the equation uniquely determines $y = (23 - 2x)/3$. Therefore taking the first-coordinate set has not reduced the cardinality. Even if a convention included zero in $\mathbb{N}$, neither $x = 0$ nor $y = 0$ would give an integer solution for the other variable here.

Step 3: Count the one-one assignments Order the four elements of A in any fixed way. The first element has four choices of image in B. Once that image is used, the second element has three choices, the third has two and the fourth has one. The multiplication principle gives

$$4 \cdot 3 \cdot 2 \cdot 1 = 4! = 24.$$

Every assignment counted this way is one-one, and every one-one function appears once. There is no further restriction that the function send an ordered pair to its own first coordinate; the problem asks for all injections between the sets.

Step 4: Conclusion The number of one-one functions from A to B is $\boxed{24}$.

Quick Tip to Solve: First count the ordered-pair solutions, then count distinct first coordinates; use a permutation count for injections rather than $|B|^{|A|}$, which counts all functions.

22. **Let A, B and C be three points on the parabola $y^2 = 6x$ and let the line segment AB meet the line L through C parallel to the x-axis at the point D. Let M and N respectively be the feet of the perpendiculars from A and B on L. Then $\left(\frac{AM \cdot BN}{CD}\right)^2$ is equal to _____.** [4 marks]

Solution:

Concept:

- On the parabola $y^2 = 6x$, a point with ordinate t has coordinates $(t^2/6, t)$. Thus each point can be described using only its ordinate.
- The perpendicular distance to a horizontal line is the absolute difference of the ordinates. Distances along that line are differences of abscissae.

Step 1: Write the chord equation Let the ordinates of A, B, C be u, v, w, respectively. The line through $A = (u^2/6, u)$ and $B = (v^2/6, v)$ has equation

$$6x = (u + v)y - uv.$$

Substitution of both endpoints verifies this equation. The horizontal line through C is $y = w$, so $D = ((u + v)w/6 - uv/6, w)$.

Step 2: Compare the three distances Since $C = (w^2/6, w)$, factor the horizontal displacement to obtain

$$CD = \frac{|w^2 - (u + v)w + uv|}{6} = \frac{|(w - u)(w - v)|}{6}.$$

Also $AM = |u - w|$ and $BN = |v - w|$. Consequently the numerator and denominator contain exactly the same factors. For the nondegenerate configuration in which the displayed ratio is defined,

$$\frac{AM \cdot BN}{CD} = 6.$$

Step 3: Conclusion Therefore the required squared ratio is $6^2 = \boxed{36}$.

Quick Tip to Solve: For a horizontal chord-intersection construction, parameterise the parabola by the ordinate and factor the difference of the abscissae.

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23. **Consider the matrices : $A = \begin{bmatrix} 2 & -5 \\ 3 & m \end{bmatrix}, B = \begin{bmatrix} 20 \\ m \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \end{bmatrix}$. Let the set of all m, for which the system of equations $AX = B$ has a negative solution (i.e., $x < 0$ and $y < 0$), be the interval (a, b). Then $8\int_a^b |A|dm$ is equal to _____.** [4 marks]

Solution:

Concept:

- A two-by-two linear system has a unique solution when its determinant is nonzero. Cramer's rule expresses each unknown as a quotient with that determinant.
- To require both unknowns to be negative, intersect their strict sign intervals. The notation $|A|$ here denotes the determinant of the matrix A.

Step 1: Find the solution and singular case The determinant and Cramer numerators are

$$|A| = 2m + 15, \qquad D_x = 25m, \qquad D_y = 2m - 60.$$

Thus $x = 25m/(2m + 15)$ and $y = (2m - 60)/(2m + 15)$. At $m = -15/2$, the second coefficient row is $3/2$ times the first, but the right-hand side is not 30; the system is inconsistent, so this value contributes no solution.

Step 2: Intersect the negative-solution intervals The sign changes of x occur at $-15/2$ and 0, giving $x < 0$ exactly on $(-15/2, 0)$. The sign changes of y occur at $-15/2$ and 30, giving $y < 0$ on $(-15/2, 30)$. Their intersection is therefore $(a, b) = (-15/2, 0)$.

Step 3: Evaluate the determinant integral On this interval the determinant is positive. Integrate it directly:

$$8 \int_{-15/2}^{0} (2m + 15)\, dm = 8 \left[m^2 + 15m\right]_{-15/2}^{0} = 8 \cdot \frac{225}{4} = 450.$$

Step 4: Conclusion The requested value is $\boxed{450}$.

Quick Tip to Solve: Find the signs of the Cramer quotients before integrating; remember that the determinant bars are not absolute-value bars around a scalar function.

24. For a differentiable function $f : \mathbb{R} \to \mathbb{R}$, suppose $f'(x) = 3f(x) + \alpha$, where $\alpha \in \mathbb{R}$, $f(0) = 1$ and $\lim_{x \to -\infty} f(x) = 7$. Then $9f(-\log_e 3)$ is equal to _____. [4 marks]

Solution:

Concept:

- The differential equation $f' - 3f = \alpha$ has a constant particular solution $-\alpha/3$ and complementary solution Ce^{3x}.
- As $x \to -\infty$, the exponential term tends to zero. The prescribed limit therefore determines the constant particular solution before the initial condition is used.

Step 1: Solve for the constants The general solution of the given differential equation is

$$f(x) = Ce^{3x} - \frac{\alpha}{3}.$$

Taking the stated limit gives $-\alpha/3 = 7$, hence $\alpha = -21$. The initial condition $f(0) = 1$ now gives $C + 7 = 1$, or $C = -6$. Therefore $f(x) = 7 - 6e^{3x}$. This also verifies $f'(x) = -18e^{3x} = 3f(x) - 21$.

Step 2: Substitute the logarithmic argument At $x = -\log_e 3 = -\ln 3$, the exponential has exponent $-3 \ln 3$, not just $-\ln 3$. Hence

$$e^{3x} = e^{-3 \ln 3} = 3^{-3} = \frac{1}{27}, \qquad f(-\ln 3) = 7 - \frac{6}{27} = \frac{61}{9}.$$

Multiplying by the factor requested in the question gives $9f(-\ln 3) = 61$.

Step 3: Conclusion The numerical answer is $\boxed{61}$.

Quick Tip to Solve: For a linear equation with a constant forcing term, use the limit to find the equilibrium level, then use the initial value to find the exponential coefficient.

25. Consider the circle $C : x^2 + y^2 = 4$ and the parabola $P : y^2 = 8x$. If the set of all values of α, for which three chords of the circle C on three distinct lines passing through the point $(\alpha, 0)$ are bisected by the parabola P is the interval (p, q), then $(2q - p)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- If $M = (h, k)$ is the midpoint of a nondegenerate chord of a circle centred at the origin, then the chord is perpendicular to OM. Thus its equation is $hx + ky = h^2 + k^2$.
- A midpoint must lie strictly inside the circle. A midpoint on the circumference gives only a tangent point, not a nondegenerate chord.

Step 1: Impose the midpoint conditions For the chord to pass through $(\alpha, 0)$, its midpoint satisfies $\alpha h = h^2 + k^2$. Since the midpoint also lies on the parabola, $k^2 = 8h$. Combining these equations gives

$$h(h + 8 - \alpha) = 0.$$

For $\alpha \neq 0$, the solution $h = k = 0$ gives the unique horizontal diameter through the origin and $(\alpha, 0)$. At $\alpha = 0$, every diameter has midpoint at the origin, giving infinitely many such lines rather than exactly three; this exceptional case does not belong to the stated interval. The other possibilities have $h = \alpha - 8$ and $k = \pm\sqrt{8(\alpha - 8)}$.

Step 2: Require two additional distinct chords Two distinct nonzero midpoints require $h > 0$. They must also lie inside $x^2 + y^2 = 4$, so

$$h^2 + 8h < 4 \quad \Longrightarrow \quad 0 < h < -4 + 2\sqrt{5}.$$

Thus $8 < \alpha < 4 + 2\sqrt{5}$. The two nonzero midpoint vectors have opposite nonzero ordinates, giving two different chord directions, each different from the horizontal diameter. No other midpoint can satisfy the factored equation.

Step 3: Evaluate the required expression Consequently $p = 8$ and $q = 4 + 2\sqrt{5}$. Therefore

$$(2q - p)^2 = \left(2(4 + 2\sqrt{5}) - 8\right)^2 = (4\sqrt{5})^2 = 80.$$

Step 4: Conclusion The required answer is $\boxed{80}$.

Quick Tip to Solve: Use the perpendicular-from-centre property to describe a chord by its midpoint, and impose strict interior inequalities to exclude tangent degeneracy.

26. The square of the distance of the image of the point $(6, 1, 5)$ in the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z-2}{4}$, from the origin is _____. [4 marks]

Solution:

Concept:

- The image of a point P in a line is obtained by a half-turn about that line. If H is the perpendicular projection of P onto the line, the image is $P' = 2H - P$.
- For a line $H = A + t\mathbf{d}$, perpendicularity is $(P - H) \cdot \mathbf{d} = 0$. This determines the foot without solving three separate coordinate equations.

Step 1: Find the perpendicular foot The line passes through $A = (1, 0, 2)$ and has direction vector $\mathbf{d} = (3, 2, 4)$. With $P = (6, 1, 5)$, compute

$$P - A = (5, 1, 3), \quad (P - A) \cdot \mathbf{d} = 15 + 2 + 12 = 29, \quad \mathbf{d} \cdot \mathbf{d} = 9 + 4 + 16 = 29.$$

Hence $t = 29/29 = 1$, and the foot is $H = A + \mathbf{d} = (4, 2, 6)$. The vector $P - H = (2, -1, -1)$ has dot product $6 - 2 - 4 = 0$ with the line direction, confirming perpendicularity.

Step 2: Reflect and calculate the squared distance The midpoint condition gives the reflected point

$$P' = 2(4, 2, 6) - (6, 1, 5) = (2, 3, 7).$$

Its squared distance from the origin is therefore

$$OP'^2 = 2^2 + 3^2 + 7^2 = 4 + 9 + 49 = 62.$$

The question asks for the square of the distance, so no square root is needed.

Step 3: Conclusion The required squared distance is $\boxed{62}$.

Quick Tip to Solve: Project onto the line first and use image = twice the foot minus the original point; do not confuse reflection in a line with reflection in a plane.

27. Let the set of all values of p, for which $f(x) = \left(p^2 - 6p + 8\right)\left(\sin^2 2x - \cos^2 2x\right) + 2(2 - p)x + 7$ does not have any critical point, be the interval (a, b). Then $16ab$ is equal to _____. [4 marks]

Solution:

Concept:

- The given function is differentiable for every real x. Therefore a critical point occurs exactly where its derivative vanishes.
- The range of $\sin 4x$ is the entire closed interval $[-1, 1]$. An equation prescribing a sine value has no real solution precisely when that prescribed value lies outside this interval.

Step 1: Differentiate and factor the parameter Use $\sin^2 2x - \cos^2 2x = -\cos 4x$ and $p^2 - 6p + 8 =$

$(p-2)(p-4)$. The derivative is

$$f'(x) = 4(p-2)(p-4)\sin 4x - 2(p-2) = 2(p-2)\big(2(p-4)\sin 4x - 1\big).$$

When $p = 2$, this derivative is identically zero, so every point is critical and the parameter is excluded.

Step 2: Exclude all zeros of the derivative For $p \neq 2$, a zero exists exactly when $2(p-4)\sin 4x = 1$. There are no zeros when $|2(p-4)| < 1$, including $p = 4$, where the left side is zero for every x. Equality must be excluded because $\sin 4x = 1$ or -1 is attainable. Thus

$$|p-4| < \frac{1}{2} \quad \Longrightarrow \quad \frac{7}{2} < p < \frac{9}{2}.$$

Step 3: Compute the endpoint product The required endpoints are $a = 7/2$ and $b = 9/2$. Thus

$$16ab = 16 \cdot \frac{7}{2} \cdot \frac{9}{2} = 4 \cdot 63 = 252.$$

Step 4: Conclusion The required numerical value is $\boxed{252}$.

Quick Tip to Solve: Factor parameter-dependent derivatives before dividing, and test any cancelled parameter separately; sine reaches both endpoints of its range.

28. The number of integers, between 100 and 1000 having the sum of their digits equals to 14 , is _____. [4 marks]

Solution:

Concept:

- Write a three-digit integer as $100a + 10b + c$, where $1 \le a \le 9$ and $0 \le b, c \le 9$. The digit-sum condition is $a + b + c = 14$.
- The stars-and-bars count for three nonnegative integers with sum s is $\binom{s+2}{2}$. Digit upper bounds are imposed by subtracting the solutions that violate them.

Step 1: Remove the nonzero leading-digit condition Put $u = a - 1$. Then

$$u + b + c = 13, \qquad 0 \le u \le 8, \qquad 0 \le b, c \le 9.$$

Without the upper bounds, the number of nonnegative triples is $\binom{15}{2} = 105$. The substitution guarantees that the leading digit is nonzero, so the counted integers all have three digits.

Step 2: Subtract the forbidden upper-bound cases For $u \ge 9$, replace u by $u\prime + 9$; the remaining sum is 4, giving $\binom{6}{2} = 15$ forbidden triples. For $b \ge 10$, the remaining sum is 3, giving $\binom{5}{2} = 10$. The same count applies to $c \ge 10$. No two violations can occur simultaneously, since even the smallest combined forbidden contribution, $9 + 10$, exceeds 13. Thus inclusion-exclusion has no intersection terms.

Step 3: Complete the count The required number is

$$105 - 15 - 10 - 10 = 70.$$

The boundary integers 100 and 1000 have digit sum 1, so neither would affect the count under either interpretation of the word between.

Step 4: Conclusion There are $\boxed{70}$ such integers.

Quick Tip to Solve: For a digit sum, shift the leading digit down by one, then use stars and bars with separate upper bounds of 8 for that shifted digit and 9 for the others.

29. Let the inverse trigonometric functions take principal values. The number of real solutions of the equation $2\sin^{-1}x + 3\cos^{-1}x = \frac{2\pi}{5}$, is _____. [4 marks]

Solution:

Concept:

- For real inverse trigonometric values the argument must satisfy $-1 \le x \le 1$. On this domain, $\sin^{-1}x \in [-\pi/2, \pi/2]$ and $\cos^{-1}x \in [0, \pi]$.
- The principal branches satisfy $\sin^{-1}x + \cos^{-1}x = \pi/2$. This identity allows the entire left-hand side to be expressed using a single angle whose range is known.

Step 1: Introduce a principal angle Put $t = \cos^{-1}x$. Then $0 \le t \le \pi$, and the complementary inverse-function identity gives $\sin^{-1}x = \pi/2 - t$. Substitution into the printed equation yields

$$2(\pi/2 - t) + 3t = \frac{2\pi}{5}.$$

Combining the terms on the left gives $\pi + t$. The coefficient of t is positive, so its smallest possible value occurs at $t = 0$.

Step 2: Compare the required angle with its allowed range The equation would require

$$t = \frac{2\pi}{5} - \pi = -\frac{3\pi}{5}.$$

This is outside the principal range $[0, \pi]$ of an inverse cosine. Equivalently, the left-hand side $\pi + t$ ranges from π to 2π, whereas $2\pi/5 < \pi$. Neither endpoint $x = 1$ nor $x = -1$, and no intermediate admissible real argument, can satisfy the equation. Values outside $[-1, 1]$ do not supply extra real inverse-trigonometric values.

Step 3: Conclusion There are no admissible real solutions. The required number is $\boxed{0}$.

Quick Tip to Solve: Combine complementary inverse functions before trying to take a sine or cosine; the principal-angle range can rule out every solution immediately.

30. **If** $\left(\frac{1}{\alpha+1}+\frac{1}{\alpha+2}+\ldots..+\frac{1}{\alpha+1012}\right)-\left(\frac{1}{2\cdot1}+\frac{1}{4\cdot3}+\frac{1}{6\cdot5}+\ldots\ldots+\frac{1}{2024\cdot2023}\right)=\frac{1}{2024}$**, then** α **is equal to** _____. [4 marks]

Solution:

Concept:

- The identity $1/[(2k)(2k-1)] = 1/(2k-1) - 1/(2k)$ converts the product-denominator sum into the difference between odd and even reciprocal sums.
- Write $H_n = \sum_{j=1}^{n} 1/j$. Separating its odd and even terms gives $\sum_{k=1}^{1012} 1/[(2k)(2k-1)] = H_{2024} - H_{1012}$.

Step 1: Rewrite the known sum Let T denote the second bracket in the question. Resolving each term into partial fractions gives

$$T = \sum_{k=1}^{1012}\left(\frac{1}{2k-1}-\frac{1}{2k}\right).$$

The sum of all odd reciprocals through $1/2023$ is $H_{2024} - \frac{1}{2}H_{1012}$, while the even reciprocal sum is $\frac{1}{2}H_{1012}$. Hence

$$T = H_{2024} - H_{1012}.$$

Therefore the first bracket must equal

$$T + \frac{1}{2024} = H_{2023} - H_{1011}.$$

Step 2: Match the consecutive denominators The difference $H_{2023} - H_{1011}$ is precisely

$$\frac{1}{1012}+\frac{1}{1013}+\cdots+\frac{1}{2023}.$$

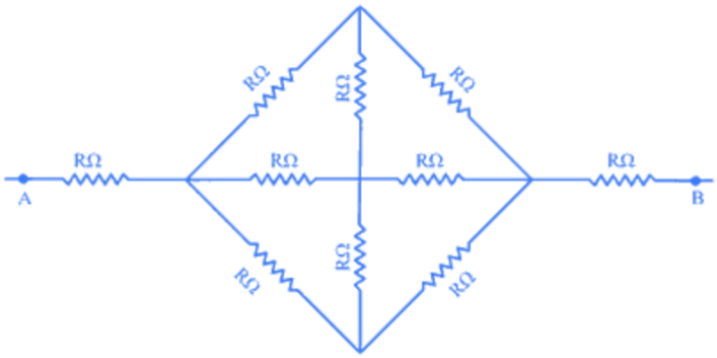
There are $2023 - 1012 + 1 = 1012$ terms. The first bracket also has 1012 terms, with denominators $\alpha+1, \ldots, \alpha+1012$. Choosing $\alpha = 1011$ makes these the same denominators in the same order, so direct substitution verifies the printed equality. On the positive domain the sum is strictly decreasing, since each reciprocal decreases as α increases; consequently this is its unique positive solution. For completeness, between successive negative poles $-j-1$ and $-j$, the continuous reciprocal sum decreases from $+\infty$ to $-\infty$, so it also attains the required finite right-hand side once in each such interval.

Step 3: Conclusion The positive solution, corresponding to the numerical answer key, is $\boxed{\alpha = 1011}$. The printed question does not state a domain restriction: if arbitrary real α are allowed, additional negative solutions occur between the poles, so 1011 is not the unique real solution.

Quick Tip to Solve: Split $1/[2k(2k-1)]$ into consecutive reciprocal terms, then use harmonic numbers to identify the shifted block without adding 1012 fractions.

Physics – Section A: Multiple Choice Questions (80 marks)

31. The effective resistance between A and B, if resistance of each resistor is R, will be : [4 marks]



(A) $\frac{8R}{3}$

(B) $\frac{4R}{3}$

(C) $\frac{2}{3}R$

(D) $\frac{5R}{3}$

Correct Answer: (A) $\frac{8R}{3}$

Solution:

Concept:

- Resistors in series carry the same current, so their resistances add. For parallel branches, the reciprocal equivalent resistance is the sum of the reciprocal branch resistances.
- Nodes at equal potential have no potential difference across a connecting resistor. Such a resistor carries zero current and can be removed when computing the resistance seen by the terminals.

Step 1: Use symmetry in the central network Call the left and right vertices of the diamond L and M. The top, central and bottom nodes each connect to L through one resistance R, and to M through one resistance R. With a voltage applied between L and M, each of these three intermediate nodes is therefore at the midpoint potential $(V_L + V_M)/2$. The two vertical resistors join nodes at this identical potential and consequently carry no current. This conclusion concerns the internal bridge resistors only; the resistors next to the external terminals still carry the full terminal current.

Step 2: Reduce the parallel branches and add the outer resistors After removing the zero-current vertical branches, three paths remain between L and M: the upper path, the horizontal middle path and the lower path. Each contains two resistors in series, giving $2R$. Thus

$$\frac{1}{R_{LM}} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{2R} = \frac{3}{2R}, \qquad R_{LM} = \frac{2R}{3}.$$

The resistor from A to L and that from M to B are in series with this entire network. Hence

$$R_{AB} = R + \frac{2R}{3} + R = \frac{8R}{3}.$$

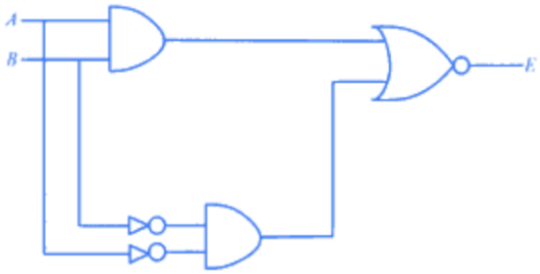
Option (C) counts only the diamond. Options (B) and (D) omit part of the necessary external series

resistance.

Step 3: Conclusion The effective resistance is $\boxed{8R/3}$. Hence, the correct option is (A).

Quick Tip to Solve: Identify equal-potential bridge nodes before combining resistors, and remember to add the two terminal resistors after simplifying the diamond.

32. In the truth table of the above circuit the value of X and Y are : [4 marks]



A	B	E
0	0	0
0	1	X
1	0	Y
1	1	0

(A) 1, 1

(B) 0, 1

(C) 1, 0

(D) 0, 0

Correct Answer: (A) 1, 1

Solution:

Concept:

- An AND gate outputs 1 only when both inputs are 1. A NOT gate complements its input; a NOR gate complements the OR of its inputs.
- The upper branch of the circuit produces AB, while the lower AND gate receives the complemented inputs and produces $\overline{A}\,\overline{B}$.

Step 1: Write the output of the final gate The two branch outputs enter an OR-shaped gate with a small circle at its output. The circle represents inversion, so this final device is a NOR gate, not an ordinary OR gate. Its output is

$$E = \overline{AB + \overline{A}\,\overline{B}}.$$

The expression inside the bar is 1 when the two inputs are equal: the upper product is 1 for $A = B = 1$, and the lower product is 1 for $A = B = 0$. The final inversion makes the circuit output 0 for both equal-input cases, agreeing with the two supplied entries in the truth table.

Step 2: Evaluate the two missing rows For the row $A = 0, B = 1$, the upper product is $AB = 0$, and the lower product is $\overline{A}\,\overline{B} = 1 \times 0 = 0$. Hence

$$X = \overline{0 + 0} = 1.$$

For $A = 1, B = 0$, the products again vanish: $AB = 0$ and $\overline{A}\,\overline{B} = 0 \times 1 = 0$. Therefore

$$Y = \overline{0 + 0} = 1.$$

Both unequal-input rows give output 1; the circuit implements exclusive OR. Options (B) and (C) incorrectly treat the two symmetric rows differently, while option (D) misses the final inversion.

Step 3: Conclusion The missing truth-table entries are $\boxed{X = 1,\ Y = 1}$. Hence, the correct option is (A).

Quick Tip to Solve: Check the output bubble first: complementing the equal-input detector $AB + \overline{A}\,\overline{B}$ produces XOR.

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33. Two cars are travelling towards each other at speed of $20\ \text{m s}^{-1}$ each. When the cars are $300\ \text{m}$ apart, both the drivers apply brakes and the cars retard at the rate of $2\ \text{m s}^{-2}$. The distance between them when they come to rest is : [4 marks]

(A) 25 m

(B) 100 m

(C) 50 m

(D) 200 m

Correct Answer: (B) 100 m

Solution:

Concept:

- For rectilinear motion with constant acceleration, $v^2 = u^2 + 2as$, where u and v are initial and final velocities and s is displacement.
- While the cars approach each other, the reduction in their separation is the sum of the forward distances each travels. Retardation is opposite to the direction of motion of each car.

Step 1: Find the stopping distance of either car Choose the positive direction separately along the initial velocity of each car. For either car the data are $u = 20\,\text{m s}^{-1}$, $a = -2\,\text{m s}^{-2}$, and $v = 0$. Using the equation that does not involve time,

$$0 = (20)^2 + 2(-2)s, \qquad s = \frac{400}{4} = 100\,\text{m}.$$

Both cars have identical initial speeds and retardations, so each covers this same stopping distance. Each takes $t = (0 - 20)/(-2) = 10\,\text{s}$ to stop, which confirms that their stopping instants coincide.

Step 2: Subtract the total approach from the initial gap The first car travels 100 m towards the second, and the second travels 100 m towards the first. Therefore the total reduction of the original separation is

$$s_1 + s_2 = 100 + 100 = 200\,\text{m}.$$

Starting from a 300 m gap, the final gap is

$$d = 300 - 200 = 100\,\text{m}.$$

Because this is positive, the cars stop before meeting and the separate stopping calculations remain applicable. The 200 m option is the distance by which the gap decreases, not the gap remaining. The 25 m and 50 m options do not follow from the individual or relative-motion stopping equation.

Step 3: Conclusion The distance between the stationary cars is $\boxed{100\,\text{m}}$. Hence, the correct option is (B).

Quick Tip to Solve: Calculate each stopping distance and subtract their sum from the initial gap; do not subtract just one car's displacement.

34. A nucleus at rest disintegrates into two smaller nuclei with their masses in the ratio of $2:1$. After disintegration they will move : [4 marks]

(A) in opposite directions with speed in the ratio of $1:2$ respectively.

(B) in the same direction with same speed.

(C) in opposite directions with speed in the ratio of $2:1$ respectively.

(D) in opposite directions with the same speed.

Correct Answer: (A) in opposite directions with speed in the ratio of $1:2$ respectively.

Solution:

Concept:

- In an isolated disintegration the total linear momentum is conserved. A nucleus initially at rest has zero total momentum before decay.
- For a two-fragment decay with no other products carrying momentum, the vector momenta of the two nuclei must be equal in magnitude and opposite in direction.

Step 1: Apply vector momentum conservation Let the fragment masses be $m_1 = 2m$ and $m_2 = m$, in the order specified. Since the parent nucleus is initially at rest,

$$m_1\vec{v}_1 + m_2\vec{v}_2 = \vec{0}.$$

Thus

$$2m\vec{v}_1 = -m\vec{v}_2, \qquad \vec{v}_2 = -2\vec{v}_1.$$

The minus sign has physical significance: the fragments move in opposite directions. Their momenta cancel even though each fragment has a nonzero momentum. Moving in the same direction could not give zero total momentum for two positive masses with nonzero speeds.

Step 2: Extract the speed ratio and compare the choices Taking magnitudes of the momentum equation gives

$$m_1v_1 = m_2v_2, \qquad \frac{v_1}{v_2} = \frac{m_2}{m_1} = \frac{1}{2}.$$

The heavier fragment consequently moves more slowly. In the same order as the mass ratio $2:1$, the speed ratio is $1:2$. Option (C) uses the mass ratio directly instead of its inverse, so the two momenta would not cancel. Option (D) gives equal speeds to unequal masses and also fails momentum conservation. Option (B) fails the opposite-direction requirement as well as the speed condition. Equal momentum magnitudes do not imply equal kinetic energies or equal speeds.

Step 3: Conclusion The nuclei move in opposite directions with speeds in the ratio $\boxed{1:2}$, respectively. Hence, the correct option is (A).

Quick Tip to Solve: For two fragments from a stationary parent, write $m_1v_1 = m_2v_2$: speeds are inversely proportional to masses, and their directions are opposite.

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35. A satellite of 10^3 kg mass is revolving in circular orbit of radius $2R$. If $\frac{10^4R}{6}$ J energy is supplied to the satellite, it would revolve in a new circular orbit of radius
(use $g = 10\ \mathrm{m/s^2}$, $R =$ radius of earth) [4 marks]

(A) 4 R

(B) 6 R

(C) 2.5 R

(D) 3 R

Correct Answer: (B) 6 R

Solution:

Concept:

- The total mechanical energy of a satellite of mass m in a circular orbit of radius r is $E = -GMm/(2r)$. It includes both gravitational potential energy and orbital kinetic energy.
- Using surface gravity $g = GM/R^2$, write $GM = gR^2$. Supplying energy increases the total energy, making its initially negative value less negative and increasing the circular-orbit radius.

Step 1: Evaluate the initial orbital energy Here $m = 10^3\,\mathrm{kg}$, $g = 10\,\mathrm{m\,s^{-2}}$, and the initial radius is $r_i = 2R$. Therefore

$$E_i = -\frac{mgR^2}{2(2R)} = -\frac{mgR}{4} = -\frac{10^4R}{4}\,\mathrm{J}.$$

This is the total mechanical energy. Using only potential energy would introduce a factor-of-two error, because the kinetic energy of a circular orbit is half the magnitude of its potential energy.

Step 2: Add the supplied energy and find the final radius The supplied amount is $\Delta E = 10^4R/6\,\mathrm{J}$. Thus

$$E_f = E_i + \Delta E = 10^4R\left(-\frac{1}{4} + \frac{1}{6}\right) = -\frac{10^4R}{12}\,\mathrm{J}.$$

For the final circular orbit,

$$-\frac{mgR^2}{2r_f} = -\frac{10^4R}{12}.$$

Since $mg = 10^4$, cancelling the common factors yields

$$\frac{R^2}{2r_f} = \frac{R}{12}, \qquad r_f = 6R.$$

The final energy is one third of the initial energy in magnitude, so the radius must be three times the initial $2R$. Options $4R$, $2.5R$, and $3R$ do not produce the specified energy increase. The energy remains negative, confirming a bound orbit rather than escape.

Step 3: Conclusion The radius of the new circular orbit is $\boxed{6R}$. Hence, the correct option is (B).

Quick Tip to Solve: Use total orbital energy $-GMm/(2r)$, add the supplied energy with its positive sign, and invert the resulting energy ratio to obtain the radius ratio.

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36. The magnetic field in a plane electromagnetic wave is $\mathrm{B_y} = \left(3.5 \times 10^{-7}\right) \sin\left(1.5 \times 10^3 x + 0.5 \times 10^{11} t\right)$ T. The corresponding electric field will be : [4 marks]

(A) $E_z = 105 \sin\left(1.5 \times 10^3 x + 0.5 \times 10^{11} t\right) \mathrm{Vm}^{-1}$

(B) $E_y = 10.5 \sin\left(1.5 \times 10^3 x + 0.5 \times 10^{11} t\right) \mathrm{Vm}^{-1}$

(C) $E_y = 1.17 \sin\left(1.5 \times 10^3 x + 0.5 \times 10^{11} t\right) \mathrm{Vm}^{-1}$

(D) $E_z = 1.17 \sin\left(1.5 \times 10^3 x + 0.5 \times 10^{11} t\right) \mathrm{Vm}^{-1}$

Correct Answer: No printed option has the amplitude 11.67 V/m required by the stated phase; A is obtained only using the inconsistent vacuum-speed interpretation.

Solution:

Concept:

- For a plane electromagnetic wave, the electric field, magnetic field and direction of propagation are mutually perpendicular. The energy-flow direction is that of $\mathbf{E} \times \mathbf{B}$.
- Faraday's law gives $E_0/B_0 = \omega/k = v$ for a common sinusoidal phase. In vacuum this speed must also equal $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$.

Step 1: Determine the direction The phase is $kx + \omega t$, so a point of constant phase moves towards negative x. With the magnetic field along $+y$, the electric field is along $+z$: $\hat{\mathbf{z}} \times \hat{\mathbf{y}} = -\hat{\mathbf{x}}$. An electric field along y, as in (B) and (C), would be parallel to the magnetic field and cannot describe this transverse wave.

Step 2: Use the printed angular frequency and wave number The phase specifies $k = 1.5 \times 10^3\,\mathrm{m^{-1}}$ and $\omega = 0.5 \times 10^{11}\,\mathrm{s^{-1}}$. Thus

$$v = \frac{\omega}{k} = \frac{0.5 \times 10^{11}}{1.5 \times 10^3} = 3.333 \times 10^7\,\mathrm{m\,s^{-1}},$$

$$E_0 = vB_0 = (3.333 \times 10^7)(3.5 \times 10^{-7}) = 11.67\,\mathrm{V\,m^{-1}}.$$

The corresponding electric field with the stated phase is therefore

$$E_z = 11.67 \sin(1.5 \times 10^3 x + 0.5 \times 10^{11} t)\,\mathrm{V\,m^{-1}}.$$

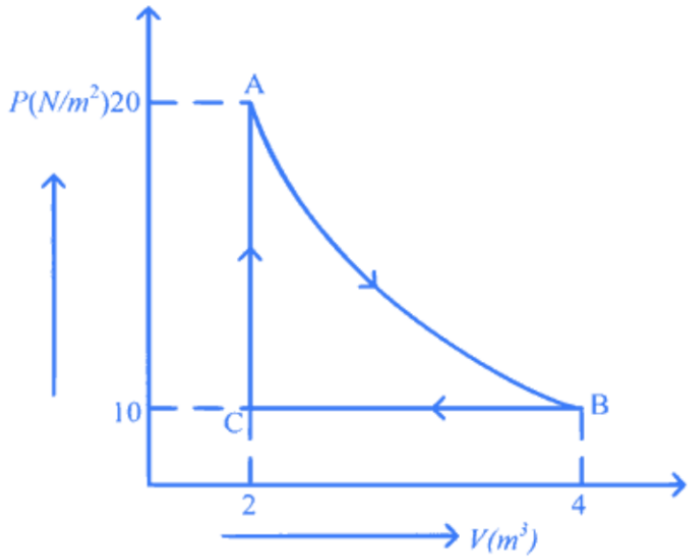
Neither z-directed option has this amplitude.

Step 3: Check the usual vacuum interpretation If the intended relation is instead the vacuum formula $E_0 = cB_0$, it gives $105\,\mathrm{V\,m^{-1}}$, the amplitude in option (A). However, the stated ω/k is not c; using that amplitude together with the printed phase fails Faraday's law. Option (D), with amplitude 1.17, also differs by a factor of ten from the value obtained using the phase. The numerical inconsistency cannot be removed by changing the direction.

Step 4: Conclusion As printed, no option satisfies both the phase and the plane-wave field relation: the required amplitude is $11.67\,\mathrm{V\,m^{-1}}$ along z. Option (A) is the usual intended vacuum answer, but requires phase coefficients consistent with c.

Quick Tip to Solve: Check both $\mathbf{E} \times \mathbf{B}$ and ω/k before using $E_0 = cB_0$; the latter relation requires a vacuum wave.

37. A real gas within a closed chamber at $27^\circ\mathrm{C}$ undergoes the cyclic process as shown in figure. The gas obeys $PV^3 = RT$ equation for the path A to B. The net work done in the complete cycle is (assuming $R = 8\ \mathrm{J/molK}$): [4 marks]



(A) -20J

(B) 205J

(C) 225J

(D) 20J

Correct Answer: (B) 205J

Solution:

Concept:

- The work done by the gas in a quasistatic process is $W = \int P\,dV$. Expansion gives positive work, compression negative work, and an isochoric leg contributes zero.

- Using the stated temperature $27^\circ\mathrm{C} = 300\,\mathrm{K}$ in the supplied path equation gives $P = RT/V^3$. The closed-cycle work is the algebraic sum over the three directed legs.

Step 1: Integrate the stated curved path The arrows show expansion from $V_A = 2\,\mathrm{m}^3$ to $V_B = 4\,\mathrm{m}^3$. Using the supplied path equation with $R = 8$ and $T = 300$,

$$W_{AB} = \int_2^4 \frac{8(300)}{V^3}\,dV = 2400\left[-\frac{1}{2V^2}\right]_2^4.$$

Hence

$$W_{AB} = 1200\left(\frac{1}{4} - \frac{1}{16}\right) = 225\,\mathrm{J}.$$

The sign is positive because the volume increases along the arrow.

Step 2: Add the straight return paths The lower horizontal leg is at $P = 10\,\mathrm{N\,m^{-2}}$ and runs from $V = 4$ to $V = 2\,\mathrm{m}^3$. Therefore

$$W_{BC} = 10(2 - 4) = -20\,\mathrm{J}.$$

On $C \to A$, the volume stays at $2\,\mathrm{m}^3$, so $dV = 0$ and $W_{CA} = 0$. The formal sum is

$$W_{\text{cycle}} = 225 - 20 + 0 = 205\,\mathrm{J}.$$

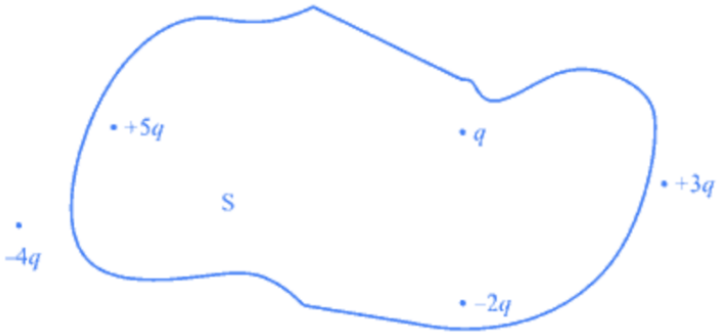
Option (A) gives only the compression work, option (C) gives only the curved-path integral, and option (D) reverses the sign of the compression work.

Step 3: Check the consistency of the supplied data This calculation uses the stated path law and the given temperature as a constant on AB, which is the interpretation yielding the listed answer. There is a numerical inconsistency in the printed diagram: its marked endpoint pressures 20 at $V = 2$ and 10 at $V = 4$ do not satisfy $PV^3 = 2400$. They would give 160 and 640, respectively. Thus the result follows the algebraic path as supplied; the endpoint pressure labels cannot simultaneously describe that same constant-temperature path.

Step 4: Conclusion Using the supplied $PV^3 = RT$ path with $T = 300\,\mathrm{K}$, the net work done by the gas is $205\,\mathrm{J}$. Hence, the correct option is (B). The endpoint pressure labels are numerically inconsistent with that stated path.

Quick Tip to Solve: Calculate the curved-leg integral first; then subtract the rectangular compression work. A vertical leg in a P–V diagram always contributes zero work.

38. Five charges $+q, +5q, -2q, +3q$ and $-4q$ are situated as shown in the figure. The electric flux due to this configuration through the surface S is : [4 marks]



(A) $\frac{q}{\epsilon_0}$
(B) $\frac{3q}{\epsilon_0}$
(C) $\frac{5q}{\epsilon_0}$
(D) $\frac{4q}{\epsilon_0}$

Correct Answer: (D) $\frac{4q}{\epsilon_0}$

Solution:

Concept:

- Gauss's law relates the net outward electric flux through a closed surface to its enclosed charge: $\Phi_E = \oint_S \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enclosed}}/\epsilon_0$.
- Charge is added algebraically. Charges outside the Gaussian surface can change the field at individual points, but contribute zero net flux through the complete closed surface.

Step 1: Identify the enclosed charges The boundary marked S encloses $+5q$, $+q$ and $-2q$. The charge $-4q$ lies outside the left side of the boundary, and $+3q$ lies outside the right side. Only the three enclosed charges enter Q_{enclosed}. The position of a charge relative to the drawn closed boundary, rather than its distance from the centre, determines whether it is counted.

Step 2: Add the charges with their signs Adding the enclosed charges gives

$$Q_{\text{enclosed}} = 5q + q - 2q = 4q.$$

The negative charge reduces the enclosed total; it must not be replaced by its magnitude. Nor should the two exterior charges be added, because their entering and leaving field lines give cancelling contributions to the net closed-surface flux. Summing all five charges would give $3q$, but that is the total charge of the whole configuration, not the charge within S.

Step 3: Apply the closed-surface law Substitution into Gauss's law gives

$$\Phi_E = \frac{4q}{\epsilon_0}.$$

The positive sign means that the net outward flux is positive. No assumption of a spherical surface or a uniform electric field is needed: the law applies to the irregular closed surface in the figure. Options (A) and (C) do not equal the algebraic enclosed total; option (B) results from including the exterior

charges. Only option (D) corresponds to the actual enclosed charge.

Step 4: Conclusion The electric flux through S is $\boxed{4q/\epsilon_0}$, outward in net sign. Hence, the correct option is (D).

Quick Tip to Solve: Trace the Gaussian boundary before adding charges. Ignore every exterior charge when calculating the net flux, even though it contributes to the local electric field.

39. UV light of 4.13 eV is incident on a photosensitive metal surface having work function 3.13 eV. The maximum kinetic energy of ejected photoelectrons will be: [4 marks]

(A) 4.13 eV

(B) 1 eV

(C) 7.26 eV

(D) 3.13 eV

Correct Answer: (B) 1 eV

Solution:

Concept:

- A single photon transfers its energy $E_\gamma = h\nu$ to one electron. The work function ϕ is the minimum energy needed for an electron to escape the metal surface.
- For the most energetic emitted electrons, Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, provided the incident photon energy exceeds the work function.

Step 1: Check that photoemission occurs The incident photon energy is $4.13\,\text{eV}$, and the work function is $3.13\,\text{eV}$. Since $4.13 > 3.13$, the photons supply enough energy to eject electrons. The work function is a property of the metal and represents energy consumed in overcoming the surface binding, rather than an additional contribution to the outgoing electron's kinetic energy.

Step 2: Subtract the escape energy Both energies are already expressed in electronvolts, so no conversion to joules is required for an answer in electronvolts:

$$K_{\max} = E_\gamma - \phi = 4.13 - 3.13 = 1.00\,\text{eV}.$$

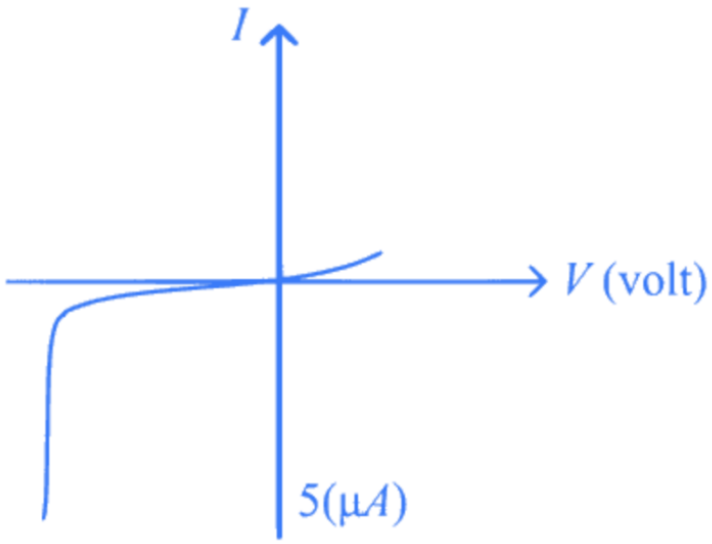
If expressed in SI units, this is 1.602×10^{-19} J. Electrons that lose some energy before escape have smaller kinetic energies, which is why the equation gives the maximum rather than the energy of every emitted electron.

Step 3: Distinguish the listed energies Option (A), $4.13\,\text{eV}$, is the incoming photon energy and neglects the energy required for escape. Option (C), $7.26\,\text{eV}$, incorrectly adds the work function to the photon energy and violates energy conservation. Option (D), $3.13\,\text{eV}$, is the escape threshold itself, not the kinetic energy remaining after emission. The difference $1\,\text{eV}$ is option (B). Increasing the light intensity would change the number of emitted electrons, not this maximum energy for a fixed photon energy.

Step 4: Conclusion The maximum kinetic energy of the photoelectrons is $\boxed{1\,\text{eV}}$. Hence, the correct option is (B).

Quick Tip to Solve: When photon energy and work function use the same units, subtract them directly. The work function is spent on escape; the remainder is the maximum kinetic energy.

40. The I-V characteristics of an electronic device shown in the figure. The device is : [4 marks]



(A) a solar cell

(B) a transistor which can be used as an amplifier

(C) a diode which can be used as a rectifier

(D) a zener diode which can be used as a voltage regulator

Correct Answer: (D) a zener diode which can be used as a voltage regulator

Solution:

Concept:

- A junction diode conducts readily in forward bias after its turn-on region. In reverse bias, a small leakage current persists until breakdown is reached.
- A Zener diode is designed to operate safely in reverse breakdown. Its steep reverse-current characteristic allows the voltage across it to remain nearly constant over an appreciable change in current.

Step 1: Read the two bias regions The right-hand portion of the curve is the forward-bias characteristic, where positive current increases with positive voltage. On the left, the current is initially small for negative voltage, then rises sharply in magnitude near a particular reverse voltage. This steep negative-current branch is the important identifying feature. The voltage changes little while the reverse current changes greatly, so the differential resistance in that region is small.

Step 2: Connect reverse breakdown to regulation In a Zener regulator, the diode is connected in reverse bias across the output and operated at its breakdown voltage. A series resistance limits current and absorbs changes in supply voltage. Because the diode voltage stays close to the breakdown value, the output remains approximately constant when the input or load changes within the operating limits. The plotted steep reverse branch therefore matches a Zener diode used as a voltage regulator.

Step 3: Exclude the other devices A solar cell under illumination has a power-delivering part of its characteristic and a nonzero short-circuit photocurrent; that is not the defining shape shown here.

A transistor amplifier requires an appropriate family of transistor characteristics and biasing, rather than this single junction-like forward/reverse curve. An ordinary rectifier diode is used for its one-way forward conduction and is normally kept out of reverse breakdown. The graph specifically emphasises the controlled reverse-breakdown behaviour relevant to regulation, so the Zener option is the best identification.

Step 4: Conclusion The device is a Zener diode, operated in reverse breakdown for voltage regulation. Hence, the correct option is (D).

Quick Tip to Solve: In a diode I–V graph, a steep reverse branch at nearly fixed negative voltage points to Zener regulation; ordinary rectification uses the forward branch.

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41. The energy released in the fusion of $2\ \text{kg}$ of hydrogen deep in the sun is E_H and the energy released in the fission of $2\ \text{kg}$ of ^{235}U is E_U. The ratio $\frac{E_H}{E_U}$ is approximately: (Consider the fusion reaction as $4_1^1H + 2e^- \rightarrow {}_2^4\text{He} + 2v + 6\gamma + 26.7\ \text{MeV}$, energy released in the fission reaction of ^{235}U is $200\ \text{MeV}$ per fission nucleus and $\text{N}_\text{A} = 6.023 \times 10^{23}$) [4 marks]

(A) 7.62

(B) 25.6

(C) 9.13

(D) 15.04

Correct Answer: (A) 7.62

Solution:

Concept:

- Total nuclear energy equals the number of completed reactions multiplied by the energy released in one reaction. Equal fuel masses do not contain equal numbers of nuclei.
- One helium-producing fusion reaction consumes four hydrogen nuclei and releases $26.7\,\text{MeV}$. One uranium-235 fission consumes one uranium nucleus and releases $200\,\text{MeV}$.

Step 1: Count the fusion reactions The hydrogen mass is $2\,\text{kg} = 2000\,\text{g}$. Taking hydrogen's molar mass as $1\,\text{g mol}^{-1}$, the number of hydrogen nuclei is $2000N_A$. Four are consumed in each fusion event, so

$$N_H^{\text{events}} = \frac{2000N_A}{4}, \qquad E_H = \frac{2000N_A}{4}(26.7)\,\text{MeV}.$$

The factor of four is essential: the quoted $26.7\,\text{MeV}$ belongs to the full reaction, not to each proton separately.

Step 2: Count the fissions The same mass of uranium-235 contains $2000/235$ moles. As each uranium nucleus can undergo one fission,

$$N_U^{\text{events}} = \frac{2000N_A}{235}, \qquad E_U = \frac{2000N_A}{235}(200)\,\text{MeV}.$$

Both expressions use the same Avogadro constant, fuel mass and energy unit, which will cancel in the requested dimensionless ratio. There is consequently no need to convert MeV into joules.

Step 3: Form the ratio and compare the options Dividing gives

$$\frac{E_H}{E_U} = \frac{(2000N_A/4)26.7}{(2000N_A/235)200} = \frac{235(26.7)}{4(200)} = 7.843125.$$

Thus the supplied reaction energies give approximately 7.84. Among the numerical choices, 7.62 is the closest and is the listed approximate answer. The other values, 25.6, 9.13 and 15.04, are farther from the calculated ratio. The small discrepancy between 7.84 and 7.62 should not be hidden by changing the quoted reaction energies.

Step 4: Conclusion The supplied numbers give $E_H/E_U \simeq 7.84$; the nearest listed approximation is 7.62. Hence, the correct option is (A).

Quick Tip to Solve: For equal fuel masses, compare energy per unit mass: divide fusion energy by four hydrogen mass units and fission energy by 235 mass units before taking the ratio.

42. A hydrogen atom in ground state is given an energy of 10.2 eV. How many spectral lines will be emitted due to transition of electrons? [4 marks]

(A) 3

(B) 6

(C) 10

(D) 1

Correct Answer: (D) 1

Solution:

Concept:

- The hydrogen energy levels are $E_n = -13.6/n^2\,\text{eV}$, with $n = 1, 2, \ldots$. An absorbed energy must equal a difference between allowed energy levels.
- If atoms are excited up to level n, the possible downward transitions among levels 1 through n give at most $n(n-1)/2$ distinct emission lines.

Step 1: Find the excited level The initial ground-state energy is $E_1 = -13.6\,\text{eV}$. After absorbing $10.2\,\text{eV}$, the atom has

$$E_f = -13.6 + 10.2 = -3.4\,\text{eV}.$$

Setting this equal to the hydrogen level energy gives

$$-\frac{13.6}{n^2} = -3.4 \quad \Longrightarrow \quad n^2 = 4, \qquad n = 2.$$

The supplied energy therefore excites the electron only to the first excited state, not to a higher level.

Step 2: Enumerate the possible emissions Between the accessible levels $n = 2$ and $n = 1$, there is only one distinct downward transition: $2 \to 1$. Its photon energy is

$$E_2 - E_1 = (-3.4) - (-13.6) = 10.2\,\text{eV}.$$

There are no intermediate stationary levels between 1 and 2, so no alternate cascade can introduce further distinct spectral lines. Equivalently, the line-count formula gives $2(2-1)/2 = 1$.

Step 3: Check the alternative counts Three possible lines require excitation through $n = 3$, six require excitation through $n = 4$, and ten require excitation through $n = 5$. Those upper levels require energies greater than the stated $10.2\,\mathrm{eV}$. Consequently options (A), (B) and (C) count transitions from excited states that the given energy cannot reach. The correct count is the single $n = 2 \to 1$ line, the Lyman-alpha transition. The question counts distinct spectral lines, not the number of photons from a large collection of atoms.

Step 4: Conclusion Only the $2 \to 1$ transition is possible, so the number of spectral lines is $\boxed{1}$. Hence, the correct option is (D).

Quick Tip to Solve: First convert the absorbed energy into the upper principal quantum number; only then apply $n(n-1)/2$. Never use the energy value itself as a line count.

---

43. A proton and a deutron $(q = +\mathrm{e}, m = 2.0\mathrm{u})$ having same kinetic energies enter a region of uniform magnetic field $\vec{B}$, moving perpendicular to $\vec{B}$. The ratio of the radius r_d of deutron path to the radius r_p of the proton path is: [4 marks]

(A) $1:2$

(B) $1:1$

(C) $\sqrt{2}:1$

(D) $1:\sqrt{2}$

Correct Answer: (C) $\sqrt{2}:1$

Solution:

Concept:

- A magnetic force perpendicular to the velocity changes the direction of motion without changing kinetic energy. Its magnitude is qvB, and it supplies the centripetal force.
- The orbit radius is $r = mv/(qB) = p/(qB)$. For a nonrelativistic particle of mass m and kinetic energy K, the momentum is $p = \sqrt{2mK}$.

Step 1: Express the radius in terms of kinetic energy For a particle entering perpendicular to the uniform field, equate the magnetic and centripetal forces. This gives

$$qvB = \frac{mv^2}{r}, \qquad r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}.$$

The two radii must be compared at equal kinetic energy, rather than at equal speed. The charge magnitudes are equal because both particles have charge $+e$.

Step 2: Use the proton and deuteron masses The proton mass is approximately $1\,\mathrm{u}$, whereas the given deuteron mass is $2\,\mathrm{u}$. The common kinetic energy, charge and magnetic field cancel:

$$\frac{r_d}{r_p} = \sqrt{\frac{m_d}{m_p}} = \sqrt{2}.$$

The heavier particle has smaller speed but larger momentum at fixed energy. Therefore its path has the larger radius. Option (A) incorrectly makes the deuteron radius smaller and linear in mass; (B) overlooks the mass dependence; (D) reverses the required ratio.

Step 3: Conclusion The required ratio is $r_d : r_p = \sqrt{2} : 1$. Hence, the correct option is (C).

Quick Tip to Solve: At equal kinetic energy and equal charge, magnetic radius scales as $\sqrt{m}$; at equal speed it instead scales as m.

44. The temperature of a gas is $-78^\circ\mathrm{C}$ and the average translational kinetic energy of its molecules is K. The temperature at which the average translational kinetic energy of the molecules of the same gas becomes $2\ \mathrm{K}$ is : [4 marks]

(A) $-78^\circ\mathrm{C}$

(B) $127^\circ\mathrm{C}$

(C) $-39^\circ\mathrm{C}$

(D) $117^\circ\mathrm{C}$

Correct Answer: (D) $117^\circ\mathrm{C}$

Solution:

Concept:

- The mean translational kinetic energy per molecule is $\overline{K} = \frac{3}{2}k_BT$, where T is the absolute temperature in kelvin.
- Temperature ratios in kinetic theory use the kelvin scale. The conversion is $T(\mathrm{K}) = t(^\circ\mathrm{C}) + 273$ to the precision needed here.

Step 1: Convert the initial temperature to kelvin The initial Celsius temperature is negative, but its absolute temperature is positive. Thus

$$T_1 = -78 + 273 = 195\,\mathrm{K}.$$

The symbol K in the question denotes the initial kinetic energy, rather than the kelvin unit. The condition that the energy becomes $2K$ means the energy per molecule doubles. No gas-dependent molar mass is needed because this is translational energy at a specified temperature.

Step 2: Double the absolute temperature and convert back Take the ratio of the two kinetic-energy equations for the same gas:

$$\frac{\overline{K}_2}{\overline{K}_1} = \frac{T_2}{T_1} = 2, \qquad T_2 = 390\,\mathrm{K}.$$

Therefore

$$t_2 = 390 - 273 = 117^\circ\mathrm{C}.$$

Using 273.15 gives $117.15^\circ\mathrm{C}$, which selects the same option. Option (A) leaves the energy unchanged. Option (C) halves the magnitude of the Celsius reading, which is not an absolute-temperature ratio. Option (B) would give $400\,\mathrm{K}$, inconsistent with doubling $195\,\mathrm{K}$.

Step 3: Conclusion The required temperature is $117^\circ\mathrm{C}$. Hence, the correct option is (D).

Quick Tip to Solve: Convert Celsius to kelvin before multiplying a temperature by an energy ratio, then convert the final kelvin value back to Celsius.

45. The excess pressure inside a soap bubble is thrice the excess pressure inside a second soap bubble. The ratio between the volume of the first and the second bubble is: [4 marks]

(A) $1 : 9$

(B) $1 : 27$

(C) $1 : 81$

(D) $1 : 3$

Correct Answer: (B) $1 : 27$

Solution:

Concept:

- A soap bubble has two liquid surfaces. Its excess internal pressure is $\Delta P = 4S/r$, where S is surface tension and r is its radius.
- For bubbles made from the same soap solution, surface tension is common. Thus excess pressure is inversely proportional to radius, while volume is proportional to the cube of radius.

Step 1: Find the ratio of the bubble radii Let the subscripts 1 and 2 denote the first and second bubbles. The given statement concerns excess pressure, so write

$$\Delta P_1 = 3\Delta P_2.$$

Using the soap-bubble pressure relation on both bubbles,

$$\frac{\Delta P_1}{\Delta P_2} = \frac{4S/r_1}{4S/r_2} = \frac{r_2}{r_1} = 3.$$

Consequently $r_1/r_2 = 1/3$. A higher excess pressure corresponds to a more sharply curved, smaller bubble.

Step 2: Cube the radius ratio to obtain the volume ratio Each bubble encloses a spherical volume $V = 4\pi r^3/3$. Common numerical factors cancel, giving

$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}.$$

Option (A), $1 : 9$, would be the ratio of surface areas rather than volumes. Option (D), $1 : 3$, is only the radius ratio. Option (C), $1 : 81$, incorrectly uses a fourth power. The factor 4 in the pressure law cancels, but retaining the inverse dependence on radius is essential.

Step 3: Conclusion The volume ratio of the first bubble to the second bubble is $1 : 27$. Hence, the correct option is (B).

Quick Tip to Solve: For bubbles of the same liquid, combine $\Delta P \propto r^{-1}$ and $V \propto r^3$ to get $V \propto (\Delta P)^{-3}$.

46. A spherical ball of radius 1×10^{-4} m and density 10^5 kg/m^3 falls freely under gravity through a distance h before entering a tank of water, If after entering in water the velocity of the ball does not change, then the value of h is approximately:

(The coefficient of viscosity of water is 9.8×10^{-6} N s/m^2) [4 marks]

(A) 2518 m

(B) 2396 m

(C) 2249 m

(D) 2296 m

Correct Answer: (A) 2518 m

Solution:

Concept:

- In the Stokes-law model, a sphere of radius r moving at speed v through a liquid of viscosity η experiences drag $6\pi\eta r v$.
- If its speed does not change after entering water, its entry speed equals the terminal speed. Free fall from rest through height h gives $v^2 = 2gh$.

Step 1: Find the terminal speed in water Balance the downward weight against viscous drag and buoyancy. With sphere density ρ_s and water density ρ_w,

$$v_t = \frac{2r^2(\rho_s - \rho_w)g}{9\eta}.$$

Here $\rho_s = 10^5\,\mathrm{kg\,m^{-3}}$, about 100 times the density of water. For the approximate answer requested, neglect buoyancy compared with weight. Taking $g = 9.8\,\mathrm{m\,s^{-2}}$,

$$v_t \simeq \frac{2(10^{-4})^2(10^5)(9.8)}{9(9.8 \times 10^{-6})} = 222.2\,\mathrm{m\,s^{-1}}.$$

Step 2: Match the free-fall speed to the terminal speed The speed acquired before entry must equal v_t, otherwise there would initially be a net force in water and the speed would change. Therefore

$$h \simeq \frac{(222.2)^2}{2(9.8)} = 2.52 \times 10^3\,\mathrm{m}.$$

The listed value closest to this approximation is $2518\,\mathrm{m}$. Retaining the standard water density $10^3\,\mathrm{kg\,m^{-3}}$ gives $v_t = 220\,\mathrm{m\,s^{-1}}$ and $h \approx 2469\,\mathrm{m}$, for which (A) remains the nearest listed answer. The other heights do not match the stated high-density approximation as closely.

Step 3: Conclusion The approximate fall height is $2.52 \times 10^3\,\mathrm{m}$, corresponding to $2518\,\mathrm{m}$ among the listed choices. Hence, the correct option is (A).

Quick Tip to Solve: Unchanged speed on entering a viscous liquid means entry speed equals terminal speed; combine the force balance with $v^2 = 2gh$.

47. The de-Broglie wavelength associated with a particle of mass m and energy E is $h/\sqrt{2mE}$. The dimensional formula for Planck's constant is : [4 marks]

(A) $\left[\mathrm{M}^2\ \mathrm{L}^2\ \mathrm{T}^{-2}\right]$
(B) $\left[\mathrm{ML}^{-1}\ \mathrm{T}^{-2}\right]$
(C) $\left[\mathrm{ML}^2\ \mathrm{T}^{-1}\right]$
(D) $\left[\mathrm{MLT}^{-2}\right]$

Correct Answer: (C) $\left[\mathrm{ML}^2\ \mathrm{T}^{-1}\right]$

Solution:

Concept:

- The de Broglie relation is $\lambda = h/\sqrt{2mE}$. Numerical constants such as 2 have no dimensions.
- Wavelength has dimension L, mass has dimension M, and kinetic energy has dimension $\mathrm{ML}^2\mathrm{T}^{-2}$. A valid physical equation must be dimensionally homogeneous.

Step 1: Rearrange the wavelength relation Isolate Planck's constant before taking dimensions:

$$h = \lambda\sqrt{2mE}.$$

The factor under the square root contains mass multiplied by energy, not energy alone. Since the numerical factor 2 does not affect dimensions,

$$[mE] = \mathrm{M} \times \mathrm{ML}^2\mathrm{T}^{-2} = \mathrm{M}^2\mathrm{L}^2\mathrm{T}^{-2}.$$

Its square root has dimensions MLT^{-1}, the dimensions of linear momentum.

Step 2: Multiply by the wavelength dimension Now multiply momentum by length:

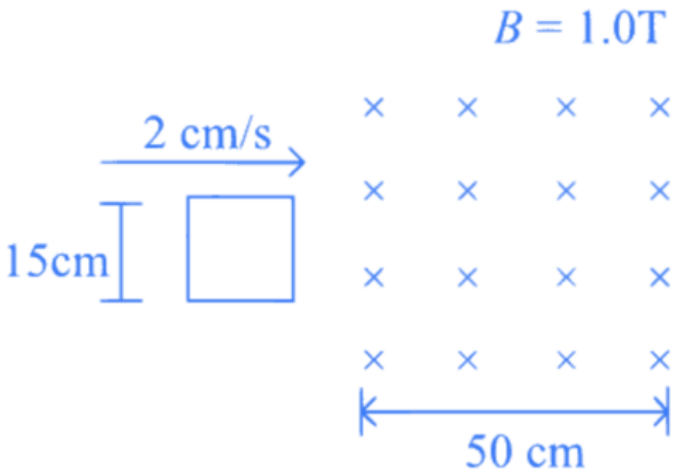
$$[h] = \mathrm{L} \times \mathrm{MLT}^{-1} = \mathrm{ML}^2\mathrm{T}^{-1}.$$

This also agrees with $E = h\nu$: energy divided by frequency has dimensions of energy multiplied by time. Option (A) gives the dimensions before the square root in the denominator expression, rather than those of h. Option (B) is the dimension of pressure. Option (D) is the dimension of force. None of these has the action dimension required for Planck's constant.

Step 3: Conclusion The dimensional formula is $[\mathrm{ML}^2\mathrm{T}^{-1}]$. Hence, the correct option is (C).

Quick Tip to Solve: Planck's constant has the dimensions of action: either energy times time or momentum times length gives $\mathrm{ML}^2\mathrm{T}^{-1}$.

48. A square loop of side $15\ \text{cm}$ being moved towards right at a constant speed of $2\ \text{cm/s}$ as shown in figure. The front edge enters the $50\ \text{cm}$ wide magnetic field at $t = 0$. The value of induced emf in the loop at $t = 10\ \text{s}$ will be : [4 marks]



(A) zero

(B) 4.5 mV

(C) 0.3 mV

(D) 3 mV

Correct Answer: (A) zero

Solution:

Concept:

- Faraday's law gives the net induced emf around a closed loop as $\mathcal{E} = -d\Phi_B/dt$, where Φ_B is magnetic flux.
- For a loop translating through a uniform field, emf is produced while its overlap area with the field changes. Once it is completely inside, that overlap area is constant.

Step 1: Locate both edges at the stated instant Take the near boundary of the magnetic strip as $x = 0$. The front edge enters this boundary at $t = 0$, and the loop translates right at $2\,\text{cm}\,\text{s}^{-1}$. At $t = 10\,\text{s}$,

$$x_{\text{front}} = vt = 20\,\text{cm}, \qquad x_{\text{rear}} = 20 - 15 = 5\,\text{cm}.$$

Both edges lie between the field boundaries $x = 0$ and $x = 50\,\text{cm}$. Thus the whole loop is already inside the uniform field.

Step 2: Evaluate the rate of change of magnetic flux The loop finishes entering at $15/2 = 7.5\,\text{s}$, and its front edge first reaches the far boundary at $50/2 = 25\,\text{s}$. Hence at 10 seconds its enclosed area in the field is fixed:

$$\Phi_B = B(0.15)^2, \qquad \frac{d\Phi_B}{dt} = 0, \qquad \mathcal{E} = 0.$$

The nonzero value $Blv = (1)(0.15)(0.02) = 3\,\text{mV}$ applies during entry or exit, not at this instant. This rules out (D); (B) and (C) are also incompatible with constant flux. Individual side contributions cancel in the fully immersed translating loop.

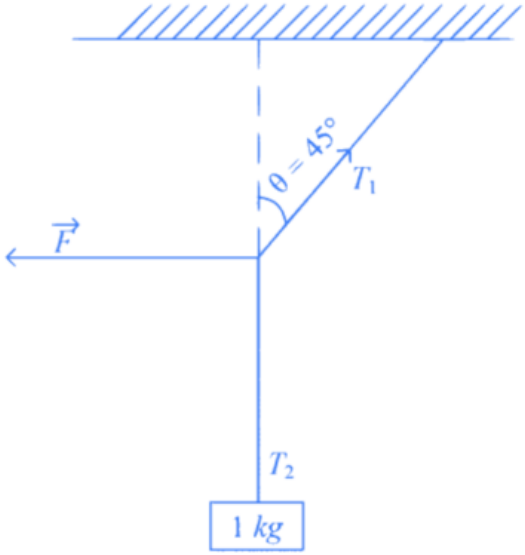
Step 3: Conclusion The induced emf at $t = 10\,\text{s}$ is zero. Hence, the correct option is (A).

Quick Tip to Solve: Before using Blv, check whether only one side crosses a boundary. A rigid loop wholly within a uniform field has constant flux during translation.

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49. A $1\ \text{kg}$ mass is suspended from the ceiling by a rope of length $4\ \text{m}$. A horizontal force 'F' is applied at the mid point of the rope so that the rope makes an angle of 45° with respect to the vertical axis as shown in figure. The magnitude of F is :

(Assume that the system is in equilibrium and $g = 10\ \text{m/s}^2$) [4 marks]



(A) 1 N

(B) 10 N

(C) $\frac{10}{\sqrt{2}}N$

(D) $\frac{1}{10\times\sqrt{2}}N$

Correct Answer: (B) 10 N

Solution:

Concept:

- For equilibrium, the vector sum of all forces acting on each body or massless junction is zero. Resolve forces horizontally and vertically.
- Tension acts along each straight rope segment. The lower segment is vertical, while the upper segment makes 45° with the vertical.

Step 1: Find the tension in the lower rope segment The suspended mass has weight $mg = (1)(10) = 10\,\text{N}$ downward. Its only upward force is the lower-segment tension T_2. Equilibrium of the mass therefore gives

$$T_2 = mg = 10\,\text{N}.$$

At the midpoint junction this same lower segment pulls downward. The upper segment pulls upward

and right, and the applied horizontal force pulls left, as shown in the question figure.

Step 2: Balance the components at the midpoint Let the upper tension be T_1 and the angle to the vertical be $\theta = 45^\circ$. Vertical balance at the junction requires

$$T_1 \cos\theta = T_2.$$

Horizontal balance gives

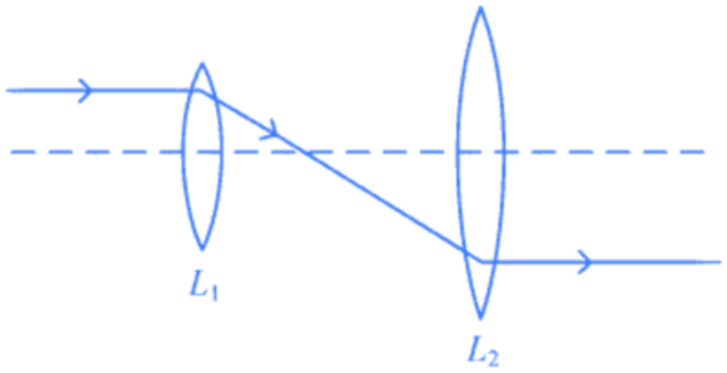
$$F = T_1 \sin\theta = T_2 \tan\theta = 10 \tan 45^\circ = 10\,\text{N}.$$

The upper tension is $10\sqrt{2}\,\text{N}$, so it must not be confused with either of its components. Option (C) incorrectly resolves the lower tension instead of the upper one; (A) omits the factor g, and (D) is far too small to balance the horizontal component. The rope length does not enter the force balance once the angle is specified.

Step 3: Conclusion The required horizontal force has magnitude $F = 10\,\text{N}$. Hence, the correct option is (B).

Quick Tip to Solve: Balance the hanging mass first, then the junction. At angle θ to the vertical, the horizontal force is $mg\tan\theta$.

50. The following figure represents two biconvex lenses L_1 and L_2 having focal length 10 cm and 15 cm respectively. The distance between L_1 & L_2 is : [4 marks]



(A) 25 cm

(B) 35 cm

(C) 10 cm

(D) 15 cm

Correct Answer: (A) 25 cm

Solution:

Concept:

- A ray parallel to the principal axis of a converging lens passes through its second principal focus. The distance from the first lens to this crossing point is therefore its focal length.

- A ray emerging parallel from a second converging lens must have arrived along a line through its first principal focus. For the arrangement shown, these two focal points coincide between the lenses.

Step 1: Locate the focus of the first lens The incident ray is parallel to the common principal axis. After refraction at L_1, it slopes down and crosses that axis at the second focus of L_1. Thus the crossing point is $f_1 = 10\,\text{cm}$ to the right of L_1.

Step 2: Apply the parallel-emergence condition The ray then reaches L_2 and emerges parallel to the axis, as the figure shows. Its incident straight-line segment must consequently pass through the first focus of L_2, located $f_2 = 15\,\text{cm}$ to its left. The axis crossing is between the lenses, so the separation is the sum, rather than the difference, of the focal lengths.

$$d = f_1 + f_2 = 10 + 15 = 25\,\text{cm}.$$

The other proposed separations do not make these focal points coincide: neither 35 cm nor either individual focal length gives parallel output for the depicted parallel input.

Step 3: Conclusion The distance between the lenses is $\boxed{25\,\text{cm}}$. Hence, the correct option is (A).

Quick Tip to Solve: For two converging lenses with parallel input and parallel output, identify their shared internal focal point and add their focal lengths.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. The resultant of two vectors $\vec{A}$ and $\vec{B}$ is perpendicular to $\vec{A}$ and its magnitude is half that of $\vec{B}$. The angle between vectors $\vec{A}$ and $\vec{B}$ is _____°. [4 marks]

Solution:

Concept:

- Let the resultant be $\vec{R} = \vec{A} + \vec{B}$, with magnitudes A, B, R, and let θ be the angle between the nonzero vectors $\vec{A}$ and $\vec{B}$.
- Perpendicular vectors have zero scalar product. Also, the magnitude of a vector sum satisfies $R^2 = A^2 + B^2 + 2AB\cos\theta$. Here $R = B/2$.

Step 1: Use perpendicularity to relate the components The condition $\vec{R} \perp \vec{A}$ gives

$$(\vec{A} + \vec{B}) \cdot \vec{A} = 0 \quad \Longrightarrow \quad A^2 + AB\cos\theta = 0.$$

Therefore $AB\cos\theta = -A^2$, so the component of $\vec{B}$ along $\vec{A}$ cancels $\vec{A}$. In particular, the angle sought must be obtuse.

Step 2: Use the resultant magnitude Substitute this scalar-product result in the squared magnitude:

$$R^2 = A^2 + B^2 - 2A^2 = B^2 - A^2.$$

Since $R = B/2$,

$$\frac{B^2}{4} = B^2 - A^2 \quad \Longrightarrow \quad A = \frac{\sqrt{3}}{2}B.$$

Consequently,

$$\cos\theta = -\frac{A}{B} = -\frac{\sqrt{3}}{2}.$$

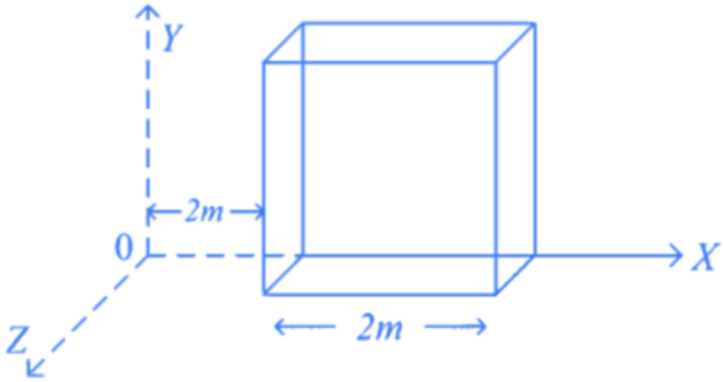
The angle between vectors is chosen in the interval from zero to $180°$, which selects $150°$.

Step 3: Conclusion The angle between $\vec{A}$ and $\vec{B}$ is $\boxed{150°}$; the required numerical value is 150.

Quick Tip to Solve: First use the perpendicularity condition to eliminate the cross term; the magnitude equation then becomes a simple difference of squares.

---

52. An electric field $\vec{E} = (2x\hat{i})NC^{-1}$ exists in space. A cube of side 2 m is placed in the space as per figure given below. The electric flux through the cube is _____ $\mathrm{Nm^2/C}$. [4 marks]



Solution:

Concept:

- Electric flux is $\Phi = \oint \vec{E} \cdot d\vec{A}$, with each area vector directed outward. For $\vec{E} = 2x\hat{i}$, only faces whose normals have an x component contribute.
- The cube extends from $x = 2\,\mathrm{m}$ to $x = 4\,\mathrm{m}$ in the figure, and the area of each face is $S = (2\,\mathrm{m})^2 = 4\,\mathrm{m}^2$.

Step 1: Identify the contributing faces On the four faces parallel to the x direction, the outward area vector is along $\pm\hat{j}$ or $\pm\hat{k}$. Its scalar product with the electric field is zero. On each of the two remaining faces the field is uniform, because x is constant over that face. The right face has outward normal $+\hat{i}$, whereas the left face has normal $-\hat{i}$.

Step 2: Subtract the incoming flux from the outgoing flux At the left face, $E(2) = 4\,\mathrm{N\,C^{-1}}$; at the right face, $E(4) = 8\,\mathrm{N\,C^{-1}}$. Thus

$$\Phi = E(4)S - E(2)S = (8-4) \times 4 = 16\,\mathrm{N\,m^2\,C^{-1}}.$$

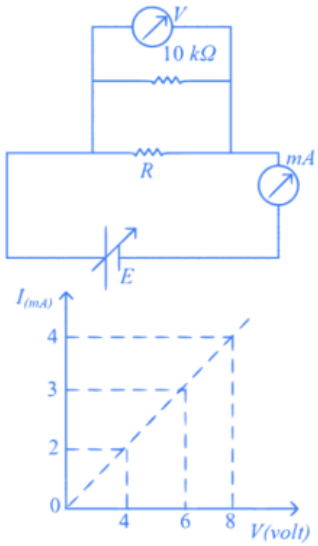
As a check, $\nabla \cdot \vec{E} = \partial(2x)/\partial x = 2$, and the cube has volume $8\,\mathrm{m^3}$. The divergence theorem gives the

same flux, $2 \times 8 = 16$. The nonzero value occurs because the field grows across the cube.

Step 3: Conclusion The outward electric flux is $\boxed{16\,\mathrm{N\,m^2/C}}$. The required numerical value is 16.

Quick Tip to Solve: For a field along x that varies only with x, use the difference of the field on the two x faces multiplied by their common area.

53. To determine the resistance (R) of a wire, a circuit is designed below. The V-I characteristic curve for this circuit is plotted for the voltmeter and the ammeter readings as shown in figure. The value of R is _____ Ω. [4 marks]



Solution:

Concept:

- The ammeter in the figure measures the total current through the parallel combination of the unknown resistance R and the voltmeter. The voltmeter resistance is finite: $R_V = 10\,\mathrm{k\Omega}$.
- Ohm's law gives $I = V/R + V/R_V$. Therefore the measured ratio V/I is the equivalent parallel resistance, not the resistance of the wire alone.

Step 1: Read the equivalent resistance from the graph The vertical graph axis is current in milliamperes and the horizontal axis is voltage in volts. The straight line passes through $(V, I) = (8\,\mathrm{V}, 4\,\mathrm{mA})$, also through $(4\,\mathrm{V}, 2\,\mathrm{mA})$. Thus

$$R_{\mathrm{eq}} = \frac{V}{I} = \frac{8}{4 \times 10^{-3}} = 2000\,\Omega.$$

Reading the axes and converting mA to A are essential before forming this ratio.

Step 2: Remove the voltmeter contribution The two branches have the same voltage, while their currents add. Hence

$$\frac{1}{2000} = \frac{1}{R} + \frac{1}{10000}.$$

Solving,

$$\frac{1}{R} = \frac{5}{10000} - \frac{1}{10000} = \frac{4}{10000}, \qquad R = 2500\,\Omega.$$

At 8 V, the voltmeter draws $0.8\,\text{mA}$, leaving $3.2\,\text{mA}$ in the wire; $8/(3.2 \times 10^{-3}) = 2500\,\Omega$, confirming the correction.

Step 3: Conclusion The resistance of the wire is $\boxed{2500\,\Omega}$, so the required numerical value is 2500.

Quick Tip to Solve: When the ammeter is outside the voltmeter branch, subtract the voltmeter current before calculating the unknown wire resistance.

54. A circular disc reaches from top to bottom of an inclined plane of length l. When it slips down the plane, if takes t s. When it rolls down the plane then it takes $\left(\frac{\alpha}{2}\right)^{1/2} t$ s, where α is _____. [4 marks]

Solution:

Concept:

- For free sliding down a smooth incline of angle θ, the acceleration is $a_s = g\sin\theta$. For pure rolling, translational and rotational motion share the gravitational energy.
- A uniform circular disc has moment of inertia $I = mr^2/2$ about its symmetry axis. Its acceleration in rolling without slipping is $a_r = g\sin\theta/(1 + I/(mr^2))$.

Step 1: Compare the two accelerations The slipping case is the usual free-sliding reference, for which there is no rotational energy gain from a rolling constraint. For the rolling disc, inserting its moment of inertia gives

$$a_r = \frac{g\sin\theta}{1 + 1/2} = \frac{2}{3} g\sin\theta = \frac{2}{3} a_s.$$

Static friction supplies the torque in pure rolling; it reduces the translational acceleration while the disc gains angular speed.

Step 2: Convert the acceleration ratio to a time ratio Both descents start from rest and cover the same distance l, so $l = at^2/2$. Therefore descent time is inversely proportional to the square root of the acceleration:

$$\frac{t_r}{t_s} = \sqrt{\frac{a_s}{a_r}} = \sqrt{\frac{3}{2}}.$$

With $t_s = t$, the rolling time is $t_r = \sqrt{3/2}\,t$. Comparing this with the printed expression $\sqrt{\alpha/2}\,t$ gives $\alpha/2 = 3/2$. The longer rolling time is consistent with its smaller acceleration.

Step 3: Conclusion The rolling time is $\sqrt{3/2}\,t$, and therefore $\boxed{\alpha = 3}$.

Quick Tip to Solve: For the same distance from rest, compare times using the square root of the inverse acceleration ratio, not the acceleration ratio itself.

55. A force $(3x^2 + 2x - 5)\text{N}$ displaces a body from $x = 2\text{ m}$ to $x = 4\text{ m}$. Work done by this force is _____ J. [4 marks]

Solution:

Concept:

- For a force along the displacement direction whose magnitude varies with position, work is the signed area under the force-position graph: $W = \int_{x_i}^{x_f} F(x)\,dx$.
- Here $F(x) = 3x^2 + 2x - 5$ in newtons and x is measured in metres. Integrate the polynomial term by term rather than multiplying one endpoint force by the total displacement.

Step 1: Find an antiderivative of the force The limits are $x_i = 2\,\text{m}$ and $x_f = 4\,\text{m}$. Applying the power rule yields

$$W = \int_2^4 (3x^2 + 2x - 5)\,dx = \left[x^3 + x^2 - 5x\right]_2^4.$$

Each term represents a contribution to work over the stated interval, and the lower-limit value must be subtracted after evaluating the entire bracket.

Step 2: Evaluate the two endpoints At the final position the bracket is $4^3+4^2-5(4) = 64+16-20 = 60$. At the initial position it is $2^3 + 2^2 - 5(2) = 8 + 4 - 10 = 2$. Thus

$$W = 60 - 2 = 58\,\text{J}.$$

As a separate check, the three integrated terms contribute $56\,\text{J}$, $12\,\text{J}$, and $-10\,\text{J}$, respectively, which sum to 58 J. The force is positive throughout this interval, so positive work agrees with its direction relative to the positive displacement.

Step 3: Conclusion The work done by the force is $\boxed{58\,\text{J}}$; the required numerical value is 58.

Quick Tip to Solve: For a polynomial force, integrate first and use one bracket for all terms before substituting the upper and lower positions.

---

56. A particle of mass 0.50 kg executes simple harmonic motion under force $F = -50(\text{Nm}^{-1})x$. The time period of oscillation is $\frac{x}{35}s$. The value of x is _____.
(Given $\pi = \frac{22}{7}$) [4 marks]

Solution:

Concept:

- The restoring force in simple harmonic motion has the form $F = -kx = -m\omega^2 x$. Its negative sign indicates that the force always points toward equilibrium.
- The angular frequency is $\omega = \sqrt{k/m}$, and the time period is $T = 2\pi/\omega = 2\pi\sqrt{m/k}$. Use the spring constant 50 in SI units and mass $0.50\,\text{kg}$.

Step 1: Identify the angular frequency Comparing the stated force $F = -50x$ with $-kx$ gives $k = 50\,\text{N m}^{-1}$. Hence

$$\omega = \sqrt{\frac{50}{0.50}} = \sqrt{100} = 10\,\text{rad s}^{-1}.$$

Equivalently, the equation of motion is $\ddot{x} = -100x$, whose coefficient is ω^2, not ω.

Step 2: Calculate the period and match the requested parameter The time for one complete oscillation is

$$T = \frac{2\pi}{10} = \frac{\pi}{5}\,\text{s}.$$

Using the explicitly supplied value $\pi = 22/7$,

$$T = \frac{22}{35}\,\text{s}.$$

The question writes the same period as $x/35$ seconds. Here this x is the requested numerical coefficient, distinct from the instantaneous displacement symbol in the force law. Equating the numerators gives $x = 22$. The period depends on k and m, not on the oscillation amplitude.

Step 3: Conclusion The period is $22/35\,\text{s}$, so the requested value is $\boxed{x = 22}$.

Quick Tip to Solve: Match the force coefficient with k first; then calculate the period using the supplied value of pi before comparing with the stated fraction.

57. At room temperature (27°C), the resistance of a heating element is 50Ω. The temperature coefficient of the material is $2.4 \times 10^{-4}{}^\circ\text{C}^{-1}$. The temperature of the element, when its resistance is 62Ω, is _____ $^\circ$C. [4 marks]

Solution:

Concept:

- The temperature coefficient relates the fractional increase in resistance to the rise in temperature. Using the stated room-temperature resistance as the reference, $R = R_{27}[1 + \alpha(T - 27)]$.
- Here $R_{27} = 50\,\Omega$, $R = 62\,\Omega$, and $\alpha = 2.4 \times 10^{-4}\,^\circ\text{C}^{-1}$. A Celsius temperature difference has the same numerical value as a kelvin difference.

Step 1: Find the fractional resistance increase The resistance increases by $62 - 50 = 12\,\Omega$. Divide by the initial resistance before applying the coefficient:

$$\frac{R - R_{27}}{R_{27}} = \frac{12}{50} = 0.24.$$

This is a fractional increase, not a temperature and not the final resistance. Since the coefficient is positive, the element must be hotter than the initial room temperature.

Step 2: Calculate the rise and add the reference temperature Rearrange the linear resistance relation to find the temperature change:

$$T - 27 = \frac{0.24}{2.4 \times 10^{-4}} = 1000\,^\circ\text{C}.$$

The requested temperature is measured from the Celsius zero, whereas the calculated change is

measured from 27°C. Therefore add the reference value:

$$T = 27 + 1000 = 1027\,^\circ\text{C}.$$

As a check, the multiplier is $1 + 0.00024(1000) = 1.24$, and $50(1.24) = 62\,\Omega$, exactly the final resistance stated in the question.

Step 3: Conclusion The temperature of the element is $\boxed{1027\,^\circ\text{C}}$. The numerical entry is $\boxed{1027}$.

Quick Tip to Solve: First calculate the temperature rise using the fractional resistance change, then add the initial temperature; a rise of 1000 degrees is not the final temperature.

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58. A straight magnetic strip has a magnetic moment of $44\ \text{Am}^2$. If the strip is bent in a semicircular shape, its magnetic moment will be _____ Am^2.
(given $\pi = \frac{22}{7}$) [4 marks]

Solution:

Concept:

- The magnetic dipole moment of a thin uniformly magnetised strip is its pole strength p multiplied by the straight-line separation between its end poles: $M = p\ell$.
- Bending without changing the magnetisation preserves the pole strength and the material length. For a semicircle of radius r, the arc length is πr, but the pole separation is the diameter $2r$.

Step 1: Relate the original moment to the strip length Let the straight strip have length ℓ and pole strength p. Its original moment is

$$M_0 = p\ell = 44\,\text{A m}^2.$$

After bending, the same length forms a semicircular arc, so $\ell = \pi r$. Therefore the original moment can also be written $M_0 = p\pi r$. The length of material is conserved, but it no longer equals the separation between the two end poles.

Step 2: Use the new pole separation The end poles lie at opposite ends of the semicircle's diameter. Hence their vector separation has magnitude $2r$, giving

$$M' = p(2r) = \frac{2}{\pi}(p\pi r) = \frac{2M_0}{\pi}.$$

With the given value $\pi = 22/7$,

$$M' = \frac{2(44)}{22/7} = 88\frac{7}{22} = 28\,\text{A m}^2.$$

The moment decreases because the poles move closer together, although the pole strength does not change. Using the curved arc length again would incorrectly leave the moment equal to its original value.

Step 3: Conclusion The magnetic moment of the semicircular strip is $\boxed{28\,\text{A m}^2}$. The required numerical entry is $\boxed{28}$.

Quick Tip to Solve: For a bent magnet, use the straight chord joining its poles, not the length measured along the material; a semicircle gives the factor $2/\pi$.

59. Monochromatic light of wavelength $500\ \text{nm}$ is used in Young's double slit experiment. An interference pattern is obtained on a screen. When one of the slits is covered with a very thin glass plate (refractive index $= 1.5$), the central maximum is shifted to a position previously occupied by the 4^{th} bright fringe. The thickness of the glass-plate is _____ μm. [4 marks]

Solution:

Concept:

- A plate of thickness t and refractive index μ replaces an equal thickness of air. The additional optical path in that arm is $\delta = (\mu - 1)t$, since optical path length is refractive index times geometrical length.
- In Young's experiment, one fringe displacement corresponds to a path difference of one wavelength. A shift of n fringe widths therefore obeys $(\mu - 1)t = n\lambda$.

Step 1: Translate the observed shift into optical path The new central maximum lies at the old fourth bright fringe. The number of fringe widths shifted is therefore $n = 4$, not three: the central maximum is the zero-order bright fringe. The required added optical path is

$$\delta = 4\lambda = 4(500\,\text{nm}) = 2000\,\text{nm}.$$

The plate delays the light in the covered arm; the new central maximum occurs where the geometrical path difference cancels that delay. Only the magnitude of the shift is needed for the thickness.

Step 2: Determine the plate thickness The optical path gained is the path in glass minus the path through the same thickness of air, rather than μt alone. Substituting $\mu = 1.5$ gives

$$(1.5 - 1)t = 2000\,\text{nm}, \qquad t = 4000\,\text{nm} = 4\,\mu\text{m}.$$

Equivalently, $t = 4(0.5)/(0.5) = 4\,\mu$m. The slit separation and screen distance cancel because the displacement is specified as a fringe order. Checking the result gives $(0.5)(4) = 2\,\mu\text{m} = 4\lambda$, as required.

Step 3: Conclusion The thickness of the glass plate is $\boxed{4\,\mu\text{m}}$; the numerical entry is $\boxed{4}$.

Quick Tip to Solve: When the displacement is stated in fringe widths, directly use $(\mu - 1)t = n\lambda$; screen distance and slit separation are unnecessary.

60. A capacitor of reactance $4\sqrt{3}\Omega$ and a resistor of resistance 4Ω are connected in series with an ac source of peak value $8\sqrt{2}$ V. The power dissipation in the circuit is _____ W. [4 marks]

Solution:

Concept:

- The impedance magnitude of a series resistor-capacitor circuit is $Z = \sqrt{R^2 + X_C^2}$. Resistance and capacitive reactance add as perpendicular phasor components, not as scalar magnitudes.
- Only the resistor dissipates average power: $P = I_{\text{rms}}^2 R$. A sinusoidal source with peak voltage V_0

has $V_{\text{rms}} = V_0/\sqrt{2}$.

Step 1: Convert the voltage and calculate impedance The source peak is $8\sqrt{2}\,\text{V}$, so its effective voltage is

$$V_{\text{rms}} = \frac{8\sqrt{2}}{\sqrt{2}} = 8\,\text{V}.$$

With $R = 4\,\Omega$ and $X_C = 4\sqrt{3}\,\Omega$,

$$Z = \sqrt{4^2 + (4\sqrt{3})^2} = \sqrt{16 + 48} = 8\,\Omega.$$

The negative imaginary sign of capacitive reactance affects the phase, but its square enters the impedance magnitude positively.

Step 2: Find the current and average dissipated power The same rms current flows through both series components:

$$I_{\text{rms}} = \frac{8}{8} = 1\,\text{A}.$$

Thus the heat dissipated per second is

$$P = I_{\text{rms}}^2 R = (1)^2(4) = 4\,\text{W}.$$

An ideal capacitor alternately stores and returns electrical energy, with zero average dissipation over a complete cycle. As a second check, the power factor is $R/Z = 1/2$; hence $VI\cos\phi = 8(1)(1/2) = 4\,\text{W}$. Using the peak voltage directly in an rms power expression would wrongly double the answer.

Step 3: Conclusion The average power dissipation is $\boxed{4\,\text{W}}$. Enter $\boxed{4}$.

Quick Tip to Solve: Convert peak voltage to rms before calculating power, and multiply I_{rms}^2 by the resistance rather than the impedance.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. For the reaction

$$C_6H_5-CH(CH_3)-CH(Br)-OCH_3 \xrightarrow[\Delta]{KCN\ (alc)} P,$$

the major product P is: [4 marks]

(A) $C_6H_5-CH(CH_3)-CH(CN)-OCH_3$

(B) $C_6H_5-CH(CN)-CH(CH_3)-OCH_3$

(C) $C_6H_5-C(CH_3)(CN)-CH_2-OCH_3$

(D) $C_6H_5-CH(OCH_3)-CH(CN)-CH_3$

Correct Answer: (A) $C_6H_5-CH(CH_3)-CH(CN)-OCH_3$

Solution:

Concept:

- Alcoholic potassium cyanide supplies free CN^-, an ambident nucleophile. With ionic cyanide, carbon attack is favoured, producing a nitrile with a carbon-carbon bond to the substrate.
- For nucleophilic replacement of bromide by cyanide, identify the carbon originally bonded to Br. Direct substitution preserves the carbon skeleton and places CN at that same carbon.

Step 1: Identify the leaving group and nucleophilic site Bromine is attached to the carbon that also bears the methoxy group. Potassium cyanide provides a strong carbon nucleophile; replacement of the bromide therefore forms

$$C_6H_5-CH(CH_3)-CH(CN)-OCH_3.$$

The pre-existing phenyl, methyl and methoxy attachments remain at their respective carbons. The reaction does not require migration of any of these groups to describe the direct substitution product.

Step 2: Compare the product connectivities Option (A) is exactly the structure obtained by replacing Br with CN on the same carbon. Option (B) places the nitrile group on the adjacent benzylic carbon and relocates the methyl attachment. Option (C) also places cyanide at the benzylic centre, yielding a different carbon substitution pattern. Option (D) moves the methoxy group to the benzylic carbon and places the methyl group on the other carbon. These rearranged connectivities do not represent the cyanide replacement shown. The distinction from AgCN is also relevant: covalent silver cyanide favours isocyanide formation, whereas the reagent here is KCN and gives the nitrile.

Step 3: Conclusion The major product is $C_6H_5-CH(CH_3)-CH(CN)-OCH_3$. Hence, the correct option is (A).

Quick Tip to Solve: Mark the carbon carrying the leaving group first, then replace only that group with carbon-bonded CN for a potassium-cyanide substitution.

62. The correct stability order of the following resonance structures of $CH_3-CH=CH-CHO$ is:

$$\begin{aligned} \text{I}: &\quad CH_3-CH^{-}-CH=CH-O^{+}, \\ \text{II}: &\quad CH_3-CH^{+}-CH=CH-O^{-}, \\ \text{III}: &\quad CH_3-CH=CH-CH=O. \end{aligned}$$

[4 marks]

(A) II > III > I

(B) II > I > III

(C) III > II > I

(D) I > II > III

Correct Answer: (C) III > II > I

Solution:

Concept:

- Resonance contributors are compared by the number of covalent bonds, satisfaction of octets, charge separation and placement of formal charges on atoms of different electronegativities.
- For otherwise comparable charge-separated forms, negative charge is more favourable on electronegative oxygen than on carbon, and positive charge is less favourable on oxygen than on carbon. A neutral covalent contributor is generally preferred when it has normal bonding.

Step 1: Identify the most stable contributor Structure III is the ordinary neutral conjugated aldehyde, $CH_3-CH=CH-CH=O$. It has no separated formal charges and normal covalent bonding. Structures I and II each separate positive and negative charge across the conjugated chain. Consequently III is more stable and contributes more strongly to the resonance hybrid than either charge-separated structure. This immediately excludes options (A), (B) and (D), because none of them places III first.

Step 2: Compare the two charge-separated forms In II the oxygen carries a negative formal charge while the carbon carries the positive charge. In I those charge placements are reversed: the negative charge is on carbon and the positive charge on oxygen. Oxygen is more electronegative than carbon, so the placement in II is the more favourable of these two arrangements. Thus II is more stable than I. The full decreasing order is

$$\text{III} > \text{II} > \text{I}.$$

This order ranks resonance contributors of one molecule; it does not mean three separate compounds are in a chemical equilibrium. The arrows in the question represent delocalisation of electrons without movement of the atomic framework.

Step 3: Conclusion The correct stability order is $\boxed{\text{III} > \text{II} > \text{I}}$. Hence, the correct option is (C).

Quick Tip to Solve: Rank the neutral contributor first, then compare charge placement: negative charge belongs preferentially on oxygen rather than carbon.

63. Match List I with List II.

	List I (Element)		List II (Electronic configuration)
A.	N	**I.**	$[Ar]3d^{10}4s^24p^5$
B.	S	**II.**	$[Ne]3s^23p^4$
C.	Br	**III.**	$[He]2s^22p^3$
D.	Kr	**IV.**	$[Ar]3d^{10}4s^24p^6$

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-III, C-II, D-I

(B) A-I, B-IV, C-III, D-II

(C) A-III, B-II, C-I, D-IV

(D) A-II, B-I, C-IV, D-III

Correct Answer: (C) A-III, B-II, C-I, D-IV

Solution:

Concept:

- A neutral atom has as many electrons as its atomic number. The noble-gas core abbreviates the filled inner shells; the remaining electrons occupy the indicated subshells.
- The relevant atomic numbers are nitrogen 7, sulfur 16, bromine 35 and krypton 36. A filled d subshell holds ten electrons and a filled p subshell holds six.

Step 1: Match nitrogen and sulfur Nitrogen has seven electrons. Removing the helium core of two electrons leaves five, arranged as

$$\text{N}: \quad [\text{He}]2s^22p^3.$$

This is List II entry III, so A matches III. Sulfur has sixteen electrons; the neon core contains ten, leaving six:

$$\text{S}: \quad [\text{Ne}]3s^23p^4.$$

This is entry II, so B matches II. These two matches already identify the candidate option without confusing an element's atomic number with its group number.

Step 2: Match bromine and krypton and check all entries The argon core accounts for eighteen electrons. For bromine, the remaining seventeen occupy $3d^{10}4s^24p^5$, giving $18 + 10 + 2 + 5 = 35$. Therefore C matches I. Krypton has one additional electron, completing the $4p$ shell:

$$\text{Kr}: \quad [\text{Ar}]3d^{10}4s^24p^6,$$

so D matches IV. The complete mapping is A-III, B-II, C-I, D-IV, exactly option (C). Options (A), (B) and (D) assign nitrogen an incompatible core or outer-shell occupation and also fail the complete mapping. Checking all four entries avoids selecting a code from only a single guessed match.

Step 3: Conclusion The matching is $\boxed{\text{A-III, B-II, C-I, D-IV}}$. Hence, the correct option is (C).

Quick Tip to Solve: Count core plus subshell electrons to verify each match; bromine and krypton differ only by the final electron in $4p$.

64. The electronic configuration of Einsteinium is :

(Given atomic number of Einsteinium $= 99$) [4 marks]

(A) $[\mathrm{Rn}]\,5f^{13}\,6d^{0}\,7s^{2}$

(B) $[\mathrm{Rn}]\,5f^{12}\,6d^{0}\,7s^{2}$

(C) $[\mathrm{Rn}]\,5f^{11}\,6d^{0}\,7s^{2}$

(D) $[\mathrm{Rn}]\,5f^{10}\,6d^{0}\,7s^{2}$

Correct Answer: (C) $[\mathrm{Rn}]\,5f^{11}\,6d^{0}\,7s^{2}$

Solution:

Concept:

- A neutral atom contains as many electrons as its atomic number. Einsteinium therefore has 99 electrons; the radon core, $[\mathrm{Rn}]$, accounts for 86 of them.
- The actinoids progressively occupy the $5f$ subshell. In the ground state of einsteinium the outer configuration is $5f^{11}7s^{2}$, with no electron in $6d$.

Step 1: Count electrons outside the noble-gas core Subtract the atomic number of radon from the given atomic number:

$$99 - 86 = 13.$$

Thus the part written after the bracket must contain exactly thirteen electrons. The symbol in the $6d$ exponent denotes zero occupancy, so it adds no electrons. All four options contain $7s^{2}$; these two electrons leave eleven electrons to be placed in the $5f$ subshell.

Step 2: Check the configuration and all alternatives The ground-state configuration is therefore

$$[\mathrm{Rn}]\,5f^{11}\,6d^{0}\,7s^{2}.$$

Its electron count is $86 + 11 + 0 + 2 = 99$, as required. Option (A) instead gives $86 + 13 + 2 = 101$ electrons, option (B) gives 100, and option (D) gives 98. These describe different atomic numbers and cannot be configurations of the neutral atom asked for. The count also agrees with einsteinium being an actinoid, rather than an element whose differentiating electron occupies an outer p subshell.

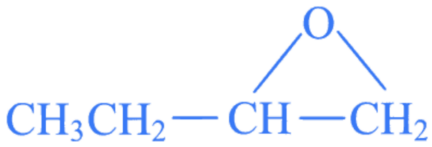
Step 3: Conclusion Einsteinium has the electronic configuration $[\mathrm{Rn}]\,5f^{11}\,6d^{0}\,7s^{2}$. Hence, the correct option is (C).

Quick Tip to Solve: When all proposed configurations share a noble-gas core and outer subshell, first count the remaining electrons; here $99 - 86 - 2 = 11$.

65. Which of the following compound can give positive iodoform test when treated with aqueous KOH solution followed by potassium hypoiodite. [4 marks]

(A) $CH_3CH_2COCH_2CH_3$

(B) 2-Ethyloxirane



(C) $CH_3CH_2CCl_2CH_3$

(D) $CH_3CH_2CH_2CHO$

Correct Answer: (C) $CH_3CH_2CCl_2CH_3$

Solution:

Concept:

- A methyl ketone, $RCOCH_3$, gives the iodoform reaction with hypoiodite in alkaline medium, producing yellow CHI_3. Suitable secondary alcohols can also respond after oxidation to methyl ketones.
- Aqueous alkali hydrolyses a geminal dihalide to a carbonyl compound. Two halogens attached to the same carbon are replaced through a geminal diol, which loses water.

Step 1: Identify what aqueous alkali produces Option (C) is $CH_3CH_2CCl_2CH_3$, a geminal dichloride. Hydrolysis replaces both chlorine atoms by hydroxyl groups on the same carbon; the resulting unstable geminal diol gives butan-2-one:

$$CH_3CH_2CCl_2CH_3 \xrightarrow{\text{aq. KOH}} CH_3CH_2COCH_3.$$

The product contains the required $-COCH_3$ group. Subsequent treatment with potassium hypoiodite therefore gives iodoform, with the remaining carbon chain forming a carboxylate.

Step 2: Exclude the other starting compounds Option (A) is pentan-3-one, $CH_3CH_2COCH_2CH_3$. Its carbonyl carbon is bonded to two ethyl groups, so it is not a methyl ketone. Option (B) is the epoxide shown; alkaline ring opening gives a vicinal diol with the secondary alcohol attached to an ethyl group and a hydroxymethyl group, rather than the required $CH_3CH(OH)^-$ unit. Option (D), butanal, is not ethanal and does not have a methyl group directly bonded to its aldehyde carbonyl. Thus neither its aldehyde group alone nor the mere presence of a terminal methyl group ensures a positive test.

Step 3: Conclusion $CH_3CH_2CCl_2CH_3$ hydrolyses to the methyl ketone butan-2-one and then gives a positive iodoform test. Hence, the correct option is (C).

Quick Tip to Solve: Apply any preliminary reagent before testing the functional group: a geminal dihalide can hide a carbonyl compound that is revealed by hydrolysis.

66. Give below are two statements :

Statement I : The higher oxidation states are more stable down the group among transition elements unlike p-block elements.

Statement II : Copper can not liberate hydrogen from weak acids.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is true but Statement II is false

(B) Both Statement I and Statement II are false

(C) Both Statement I and Statement II are true

(D) Statement I is false but Statement II is true

Correct Answer: (C) Both Statement I and Statement II are true

Solution:

Concept:

- Among transition elements, the heavier members of a group commonly stabilise higher oxidation states more effectively; for example, the higher oxidation state is more persistent for the heavier group-6 metals.
- A metal liberates hydrogen from a non-oxidising acid only if oxidation of the metal can be coupled favourably with reduction of hydrogen ions. Copper lies below hydrogen in the activity series.

Step 1: Evaluate the oxidation-state statement Statement I states the usual contrast between the transition elements and the p-block. The more extended d orbitals of the heavier transition metals participate effectively in bonding, making high oxidation states increasingly stable down a group. By comparison, the inert-pair effect often stabilises lower oxidation states down a p-block group. Thus the direction of the trend given in Statement I is correct. This is a trend in stability, not a claim that every compound of every transition metal has the maximum possible oxidation state.

Step 2: Evaluate hydrogen liberation by copper For copper to displace hydrogen, the coupled process would oxidise Cu and reduce H^+ to H_2. Using $E^\circ(\mathrm{Cu}^{2+}/\mathrm{Cu}) = +0.34\,\mathrm{V}$ and the standard hydrogen value zero gives

$$E^\circ_{\text{cell}} = 0 - 0.34 = -0.34\,\mathrm{V}.$$

The displacement is not spontaneous under the standard comparison. Weak non-oxidising acids do not liberate hydrogen with copper; oxidising acids may attack copper by a different reduction process. Hence Statement II is also true. Choices (A), (B), and (D) each wrongly mark at least one of the two statements false.

Step 3: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (C).

Quick Tip to Solve: For metal–acid reactions, distinguish hydrogen displacement from oxidation by the acid anion; copper may dissolve in oxidising acids without producing hydrogen.

67. Which of the following compounds will give silver mirror with ammoniacal silver nitrate?

A. Formic acid

B. Formaldehyde

C. Benzaldehyde

D. Acetone

Choose the correct answer from the options given below : [4 marks]

(A) B and C only

(B) C and D only

(C) A, B and C only

(D) A only

Correct Answer: (C) A, B and C only

Solution:

Concept:

- Ammoniacal silver nitrate is Tollens reagent. A reducing substrate converts silver(I) in the complex $[Ag(NH_3)_2]^+$ into metallic silver, which deposits as a mirror.
- Aldehydes are oxidised to carboxylates by Tollens reagent, whereas ordinary ketones are not. Formic acid is an important reducing exception among carboxylic acids.

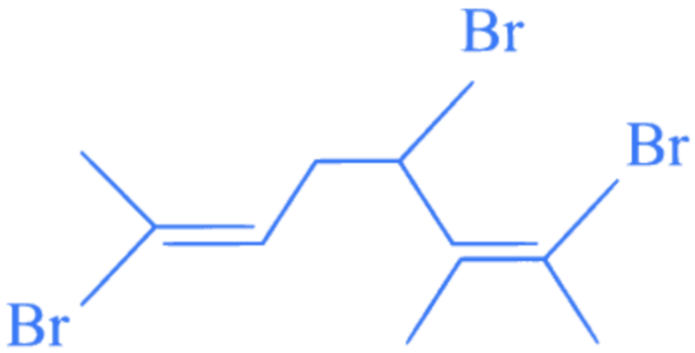
Step 1: Test the aldehydes and formic acid Compound A is formic acid, $HCOOH$. It is readily oxidised further to carbon dioxide and therefore reduces the silver reagent. Compound B is formaldehyde, $HCHO$, an aldehyde that can be oxidised to formate. Compound C is benzaldehyde, C_6H_5CHO, which is oxidised to benzoate in the alkaline reagent. Thus all three compounds A, B, and C can supply reducing equivalents for silver deposition. The aromatic ring of benzaldehyde does not prevent the aldehyde group from giving the test.

Step 2: Test acetone and select the combination Compound D is acetone, CH_3COCH_3, an ordinary ketone. It has no aldehydic hydrogen, and it is not one of the special reducing hydroxyketones. Tollens reagent under the test conditions does not oxidise it to give a silver mirror. Consequently the required set is A, B, and C only. Choice (A) is incomplete because it omits formic acid; choice (B) incorrectly includes acetone and omits two reducing compounds; choice (D) omits both aldehydes. This is distinct from the iodoform test, which acetone does give because it is a methyl ketone.

Step 3: Conclusion Formic acid, formaldehyde, and benzaldehyde give the silver mirror; acetone does not. Hence, the correct option is (C).

Quick Tip to Solve: For Tollens reagent, include aldehydes and remember formic acid as the reducing carboxylic acid; do not confuse this test with the methyl-ketone test.

68. Total number of stereo isomers possible for the given structure : [4 marks]



(A) 4

(B) 2

(C) 8

(D) 3

Correct Answer: (C) 8

Solution:

Concept:

- An alkene exhibits two geometrical arrangements when each carbon of its double bond has two different substituents. Each independent such double bond contributes a factor of two.
- A tetrahedral carbon bearing four different groups is a stereogenic centre and can have two configurations. Multiply independent choices and then check whether molecular symmetry makes any combinations identical.

Step 1: Identify the stereogenic units in the structure The left-hand double bond has a methyl group and bromine on its left carbon, and hydrogen and the rest of the chain on its right carbon. It therefore permits E/Z isomerism. The right-hand double bond also has different groups on both carbon atoms: its right carbon bears bromine and methyl, while its left carbon bears methyl and the remaining chain. This supplies a second independent E/Z choice. Between these units the saturated carbon bearing bromine is attached to hydrogen and to two different carbon chains, so it is one chiral centre.

Step 2: Multiply the choices and check symmetry The two double bonds supply $2 \times 2 = 4$ geometrical combinations. For each combination the bromine-bearing tetrahedral carbon can have either of its two configurations, giving

$$N = 2^2 \times 2^1 = 8.$$

The two sides of this centre are constitutionally different: one side leads through a methylene group towards an alkene, while the other is directly attached to a substituted alkene carbon. Hence there is no internal symmetry interchanging the sides and no meso reduction. Option (A) counts only the double-bond arrangements; option (B) counts only one binary unit; option (D) does not count all independent configurations.

Step 3: Conclusion There are two independent E/Z units and one chiral centre, giving 8 stereoisomers. Hence, the correct option is (C).

Quick Tip to Solve: Mark E/Z units and tetrahedral chiral centres separately before multiplying; then inspect the actual connectivity for symmetry rather than automatically subtracting a meso form.

69. Match List I with List II.

	LIST I (Cell)		LIST II (Use/Property/Reaction)
A.	Leclanche cell	I.	Converts energy of combustion into electrical energy
B.	Ni–Cd cell	II.	Does not involve any ion in solution and is used in hearing aids
C.	Fuel cell	III.	Rechargeable
D.	Mercury cell	IV.	Reaction at anode $\mathrm{Zn} \longrightarrow \mathrm{Zn}^{2+} + 2\,\mathrm{e}^{-}$

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-I, C-IV, D-II

(B) A-I, B-II, C-III, D-IV

(C) A-IV, B-III, C-I, D-II

(D) A-II, B-III, C-IV, D-I

Correct Answer: (C) A-IV, B-III, C-I, D-II

Solution:

Concept:

- A primary cell is used without regeneration by charging, whereas a secondary cell has a substantially reversible cell reaction and can be recharged.
- Fuel cells turn the chemical energy of a continuously supplied fuel and oxidant into electrical energy. Mercury cells have nearly constant voltage because the overall cell reaction does not involve dissolved ions.

Step 1: Match the primary cells In a Leclanche dry cell, the zinc container acts as the anode and is oxidised:

$$\mathrm{Zn} \longrightarrow \mathrm{Zn}^{2+} + 2\,\mathrm{e}^{-}.$$

Thus A matches IV. The mercury cell is a compact primary cell used in hearing aids and similar low-current devices. Its overall reaction can be written $\mathrm{Zn(Hg)} + \mathrm{HgO} \longrightarrow \mathrm{ZnO} + \mathrm{Hg}$; no dissolved ion occurs in this net reaction. Therefore D matches II. The absence of ions in the overall equation should not be confused with the absence of an electrolyte inside the cell.

Step 2: Match the rechargeable and fuel cells The nickel–cadmium cell is a secondary cell. Its chemical reaction can be reversed by passing current in the charging direction, so B matches III, rechargeable. A fuel cell converts the energy of the combustion reaction into electrical energy through separated electrode reactions instead of direct burning. Therefore C matches I. Combining these four independent identifications gives

$$\text{A–IV, B–III, C–I, D–II.}$$

Choice (A) wrongly treats Leclanche as rechargeable; choice (B) assigns the fuel-cell property to Leclanche; choice (D) swaps both the Leclanche and fuel-cell identifications. Only choice (C) agrees with all four matches.

Step 3: Conclusion The matching is A–IV, B–III, C–I, D–II. Hence, the correct option is (C).

Quick Tip to Solve: Use the distinctive matches first: Ni–Cd means rechargeable, and a fuel cell means continuous fuel-to-electricity conversion; then check the two primary cells.

70. The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 'A' $\times 10^{12}$ hertz and that has a radiant intensity in that direction of $\frac{1}{'B'}$, watt per steradian. 'A' and 'B' are respectively [4 marks]

(A) 450 and 683

(B) 540 and $\frac{1}{683}$

(C) 540 and 683

(D) 450 and $\frac{1}{683}$

Correct Answer: (C) 540 and 683

Solution:

Concept:

- The candela is the SI base unit of luminous intensity. Its stated monochromatic reference frequency is $540 \times 10^{12}\,\text{Hz}$, in the visible region.
- At this reference frequency, the photometric conversion is $683\,\text{lm}\,\text{W}^{-1}$. A radiant intensity of $1/683\,\text{W}\,\text{sr}^{-1}$ consequently corresponds to a luminous intensity of one candela.

Step 1: Read the frequency coefficient Compare the question with the reference-frequency definition of the candela. The monochromatic radiation has frequency

$$\nu = 540 \times 10^{12}\,\text{Hz}.$$

The blank A is the coefficient before 10^{12}, so $A = 540$, not 450. The frequency specifies which radiation is being used; the radiant intensity is then converted to luminous intensity using the standard photometric efficacy at that frequency.

Step 2: Identify the denominator and check the options The radiant intensity required for one candela is

$$I_e = \frac{1}{683}\,\text{W}\,\text{sr}^{-1}.$$

The question already prints this as $1/B$, so the number to substitute for B is 683. Taking $B = 1/683$ would instead make the stated radiant intensity $683\,\text{W}\,\text{sr}^{-1}$, inverting the intended value a second time. Thus choices (B) and (D) have the denominator wrong, while choices (A) and (D) have the reference-frequency coefficient wrong. Only choice (C) uses both the correct coefficient and the correct denominator. The unit candela describes luminous intensity rather than total luminous flux, which would be measured in lumens.

Step 3: Conclusion The required values are $A = 540$ and $B = 683$. Hence, the correct option is (C).

Quick Tip to Solve: When a definition is written as $1/B$, enter the denominator itself for B; do not enter the reciprocal a second time.

71. Match List I with List II

	LIST I		LIST II
A.	**Melting Point** [K]	**I.**	$Tl > In > Ga > Al > B$
B.	**Ionic Radius** $[M^{+3}/pm]$	**II.**	$B > Tl > Al \approx Ga > In$
C.	$\Delta_i H_1[\text{ kJ mol}^{-1}]$	**III.**	$Tl > In > Al > Ga > B$
D.	**Atomic Radius [pm]**	**IV.**	$B > Al > Tl > In > Ga$

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-I, C-II, D-III

(B) A-I, B-II, C-III, D-IV

(C) A-III, B-IV, C-I, D-II

(D) A-II, B-III, C-IV, D-I

Correct Answer: (A) A-IV, B-I, C-II, D-III

Solution:

Concept:

- The group 13 elements are B, Al, Ga, In and Tl. Their radii, melting points and first ionization enthalpies show distinct trends; one universal down-group order cannot describe all four properties.
- Ionic radius is compared for ions with the same charge, here M^{3+}. Poor shielding by filled d and f shells causes deviations in atomic radii and ionization enthalpies.

Step 1: Match the melting point and ionic radius Boron has a very high melting point because of its strong covalent network. Aluminium follows it, whereas gallium melts near room temperature. The order is $B > Al > Tl > In > Ga$, so A matches IV. For the tripositive ions, size generally increases down the group: $Tl^{3+} > In^{3+} > Ga^{3+} > Al^{3+} > B^{3+}$. Thus B matches I; this is not the order of the neutral atomic radii.

Step 2: Match the ionization enthalpy and atomic radius The first ionization enthalpy order is $B > Tl > Al \approx Ga > In$. Boron is small, while the weak shielding of intervening d and f electrons raises the effective nuclear attraction in the heavier members. Hence C matches II. The neutral atomic radii follow $Tl > In > Al > Ga > B$; the slightly smaller radius of Ga than Al reflects d-block contraction. Thus D matches III. The other options misassign at least one of these established sequences.

Step 3: Conclusion The matching is A-IV, B-I, C-II, D-III. Hence, the correct option is (A).

Quick Tip to Solve: For group 13, distinguish neutral atomic radius from tripositive ionic radius, and remember the unusually low melting point of gallium.

72. Which out of the following is a correct equation to show change in molar conductivity with respect to concentration for a weak electrolyte, if the symbols carry their usual meaning : [4 marks]

(A) $\Lambda_m - \Lambda_m^\circ + AC^{\frac{1}{2}} = 0$

(B) $\Lambda_m^2 C + K_a \Lambda_m^{\circ 2} - K_a \Lambda_m \Lambda_m^\circ = 0$

(C) $\Lambda_m - \Lambda_m^\circ - AC^{\frac{1}{2}} = 0$

(D) $\Lambda_m^2 C - K_a \Lambda_m^{\circ 2} + K_a \Lambda_m \Lambda_m^\circ = 0$

Correct Answer: (D) $\Lambda_m^2 C - K_a \Lambda_m^{\circ 2} + K_a \Lambda_m \Lambda_m^\circ = 0$

Solution:

Concept:

- For a weak electrolyte, the degree of dissociation is $\alpha = \Lambda_m/\Lambda_m^\circ$, where Λ_m° is the molar conductivity at infinite dilution.
- Ostwald's dilution law for a weak 1:1 electrolyte of concentration C is $K_a = C\alpha^2/(1-\alpha)$. The extent of dissociation changes substantially on dilution.

Step 1: Substitute conductivity into the equilibrium law Writing $L = \Lambda_m$ and $L_0 = \Lambda_m^\circ$ for brevity, substitution gives

$$K_a = \frac{C(L/L_0)^2}{1 - L/L_0} = \frac{CL^2}{L_0(L_0 - L)}.$$

The denominator retains the undissociated fraction; replacing it by unity would give only an approximation rather than the required equation.

Step 2: Rearrange and compare the options Multiply by the denominator and bring all terms to one side:

$$K_a L_0^2 - K_a L L_0 = CL^2, \qquad CL^2 - K_a L_0^2 + K_a L L_0 = 0.$$

This is option D. Options A and C have the square-root concentration form associated with strong electrolytes, so they do not express weak-electrolyte dissociation. Option B has the two equilibrium terms with their signs reversed. As a check, at infinite dilution $C \to 0$ and $L \to L_0$, the two remaining terms cancel in the derived equation.

Step 3: Conclusion The required relation is $\Lambda_m^2 C - K_a(\Lambda_m^\circ)^2 + K_a \Lambda_m \Lambda_m^\circ = 0$. Hence, the correct option is (D).

Quick Tip to Solve: For a weak electrolyte, begin with Ostwald's law and replace the degree of dissociation by the ratio of molar conductivities.

73. The correct increasing order for bond angles among BF_3, PF_3 and ClF_3 is : [4 marks]

(A) $BF_3 = PF_3 < ClF_3$

(B) $BF_3 < PF_3 < ClF_3$

(C) $ClF_3 < PF_3 < BF_3$

(D) $PF_3 < BF_3 < ClF_3$

Correct Answer: (C) $ClF_3 < PF_3 < BF_3$

Solution:

Concept:

- VSEPR theory relates molecular shape to the arrangement of bonding and lone electron pairs around the central atom. Lone-pair repulsions can reduce the observed bond angle from an ideal geometry.
- BF_3 is trigonal planar, PF_3 is trigonal pyramidal, and ClF_3 is T-shaped. For the last molecule, compare the smaller F–Cl–F angle between adjacent bonds.

Step 1: Determine the geometry of each molecule In BF_3, boron has three bond pairs and no lone pair, giving a planar arrangement with $120°$ angles. In PF_3, phosphorus has three bond pairs and one lone pair. Its pyramidal geometry and the electronegative fluorine atoms give an F–P–F angle of about $97.8°$, substantially below the tetrahedral ideal of $109.5°$.

Step 2: Compare the relevant angles In ClF_3, chlorine has three bond pairs and two lone pairs. Five electron domains have a trigonal-bipyramidal arrangement, with the two lone pairs in equatorial positions; the molecular shape is T-shaped. The adjacent F–Cl–F angle is slightly less than $90°$, approximately $87.5°$. Therefore it is smaller than the angle in PF_3, which in turn is smaller than the $120°$ angle of BF_3. Options putting ClF_3 last confuse the adjacent angle with its approximately straight axial bond pair; equality of the boron and phosphorus angles also contradicts their shapes.

Step 3: Conclusion The increasing order is $ClF_3 < PF_3 < BF_3$. Hence, the correct option is (C).

Quick Tip to Solve: Write the shapes before comparing angles: T-shaped gives an adjacent angle near 90 degrees, pyramidal is intermediate here, and trigonal planar gives 120 degrees.

74. For a sparingly soluble salt AB_2, the equilibrium concentrations of A^{2+} ions and B^- ions are 1.2×10^{-4}M and 0.24×10^{-3}M, respectively. The solubility product of AB_2 is : [4 marks]

(A) 0.069×10^{-12}

(B) 0.276×10^{-12}

(C) 6.91×10^{-12}

(D) 27.65×10^{-12}

Correct Answer: (C) 6.91×10^{-12}

Solution:

Concept:

- For $AB_2(s) \rightleftharpoons A^{2+}(aq) + 2\,B^-(aq)$, the solubility product is $K_{sp} = [A^{2+}][B^-]^2$. The pure solid does not enter the expression.
- Use the given equilibrium ion concentrations directly. The stoichiometric coefficient two becomes an exponent on the anion concentration; it is not an additional factor multiplying that concentration.

Step 1: Express both concentrations with consistent powers of ten The stated concentrations are $[A^{2+}] = 1.2 \times 10^{-4}\,M$ and $[B^-] = 0.24 \times 10^{-3}\,M = 2.4 \times 10^{-4}\,M$. Their ratio is 1:2, consistent with the dissolution equation. No further conversion to molar solubility is needed because both equilibrium concentrations have already been supplied.

Step 2: Substitute and simplify Calculate the square first and then multiply:

$$\begin{aligned} K_{sp} &= (1.2 \times 10^{-4})(2.4 \times 10^{-4})^2 \\ &= 1.2 \times 5.76 \times 10^{-12} \\ &= 6.912 \times 10^{-12} \approx 6.91 \times 10^{-12}. \end{aligned}$$

Option C is the rounded numerical value. Option D is approximately four times this value and would result from incorrectly introducing the factor two into an already specified anion concentration. The much smaller values in A and B have incorrect powers or concentration factors. Keeping the square on the entire anion concentration avoids these errors.

Step 3: Conclusion The solubility product is $K_{sp} = 6.91 \times 10^{-12}$. Hence, the correct option is (C).

Quick Tip to Solve: When equilibrium ion concentrations are given, substitute them directly in the product; apply stoichiometric coefficients only as exponents.

75. Match List I with List II

	LIST I		LIST II
A.	$K_2\,[Ni(CN)_4]$	**I.**	sp^3
B.	$[Ni(CO)_4]$	**II.**	sp^3d^2
C.	$[Co\,(NH_3)_6]\,Cl_3$	**III.**	dsp^2
D.	$Na_3\,[CoF_6]$	**IV.**	d^2sp^3

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-II, C-IV, D-I

(B) A-III, B-I, C-II, D-IV

(C) A-I, B-III, C-II, D-IV

(D) A-III, B-I, C-IV, D-II

Correct Answer: (D) A-III, B-I, C-IV, D-II

Solution:

Concept:

- In valence bond theory, the metal oxidation state fixes its d-electron count. Ligand field strength and coordination number then determine whether inner or outer orbitals are used.
- A four-coordinate square-planar complex uses dsp^2, a tetrahedral complex uses sp^3, and an octahedral complex uses either inner-orbital d^2sp^3 or outer-orbital sp^3d^2.

Step 1: Assign the two nickel complexes In $K_2[Ni(CN)_4]$, the complex ion has charge -2 and each cyanide has charge -1, so nickel is +2 with a $3d^8$ configuration. The strong-field cyanide ligands produce the square-planar dsp^2 complex: A-III. In $[Ni(CO)_4]$, neutral carbon monoxide gives nickel oxidation state zero and a filled $3d^{10}$ shell. Four-coordinate nickel carbonyl is tetrahedral and uses sp^3: B-I.

Step 2: Assign the two cobalt complexes In $[Co(NH_3)_6]Cl_3$, cobalt is +3 and hence $3d^6$. With ammonia and the highly charged cobalt(III) ion, pairing gives a low-spin inner-orbital octahedral complex, d^2sp^3: C-IV. In $Na_3[CoF_6]$, cobalt is again +3, but fluoride is weak field. The d electrons remain high spin and the octahedral complex uses outer orbitals, sp^3d^2: D-II. Thus options B and C wrongly exchange cobalt assignments, while A incorrectly makes nickel carbonyl octahedral. Only the complete matching in option D fits all oxidation states and geometries.

Step 3: Conclusion The matching is A-III, B-I, C-IV, D-II. Hence, the correct option is (D).

Quick Tip to Solve: Calculate the oxidation state before counting d electrons; for the two cobalt(III) complexes, the ligand strength distinguishes inner- and outer-orbital octahedral forms.

76. Match List I with List II

	LIST I (Test)		LIST II (Observation)
A.	Br_2 water test	I.	Yellow orange or orange red precipitate formed
B.	Ceric ammonium nitrate test	II.	Reddish orange colour disappears
C.	Ferric chloride test	III.	Red colour appears
D.	2, 4 - DNP test	IV.	Blue, Green, Violet or Red colour appear

Choose the correct answer from the options given below: [4 marks]

(A) A-II, B-III, C-IV, D-I

(B) A-III, B-IV, C-I, D-II

(C) A-I, B-II, C-III, D-IV

(D) A-IV, B-I, C-II, D-III

Correct Answer: (A) A-II, B-III, C-IV, D-I

Solution:

Concept:

- Qualitative tests identify functional groups through characteristic colour changes or precipitates. Match each reagent with its own diagnostic observation rather than treating all colour tests alike.
- Bromine water is reddish orange; ceric ammonium nitrate tests alcohols, ferric chloride tests phenols, and 2,4-dinitrophenylhydrazine (2,4-DNP) detects aldehyde and ketone carbonyl groups.

Step 1: Match bromine water and ceric ammonium nitrate When bromine water reacts with an appropriate unsaturated compound, bromine is consumed and its reddish orange colour disappears. Thus A matches II. Ceric ammonium nitrate gives a characteristic red colour with an alcohol through formation of a coloured complex. Thus B matches III. These observations are colour loss and colour formation respectively, so exchanging them would misidentify the tests.

Step 2: Match ferric chloride and 2,4-DNP Phenolic compounds can give blue, green, violet or red colours with ferric chloride, depending on the phenol and the complex formed. Therefore C matches IV; the question correctly lists a range rather than one universal colour. Aldehydes and ketones react with 2,4-DNP to form their corresponding hydrazones, observed as a yellow-orange or orange-red precipitate, so D matches I. That precipitate is a carbonyl test and should not be confused with the red solution in the ceric ammonium nitrate test. Checking all four assignments eliminates B, C and D because each pairs a reagent with a different reagent's observation.

Step 3: Conclusion The matching is A-II, B-III, C-IV, D-I. Hence, the correct option is (A).

Quick Tip to Solve: Associate 2,4-DNP with a precipitate, ferric chloride with coloured phenol complexes, and bromine water with disappearance of its original colour.

77. The incorrect statement about Glucose is : [4 marks]

(A) Glucose remains in multiple isomeric form in its aqueous solution

(B) Glucose is soluble in water because of having aldehyde functional group

(C) Glucose is an aldohexose

(D) Glucose is one of the monomer unit in sucrose

Correct Answer: (B) Glucose is soluble in water because of having aldehyde functional group

Solution:

Concept:

- Glucose is an aldohexose: its open-chain form has six carbon atoms and an aldehyde group. In water, most molecules exist as cyclic hemiacetals rather than as the open-chain aldehyde.
- The several hydroxyl groups in glucose form extensive hydrogen bonds with water. This interaction is the main reason glucose dissolves readily; an aldehyde group alone does not explain its high water solubility.

Step 1: Check the statements about structure and aqueous forms Option A is correct: aqueous glucose contains interconverting forms, principally the alpha and beta cyclic anomers, together with a small amount of the open-chain form. Their interconversion is responsible for mutarotation. Option C is also correct because glucose contains six carbons and is classified by the aldehyde group of its open-chain structure. Calling it an aldohexose does not imply that all molecules remain open chain in solution.

Step 2: Identify the incorrect explanation of solubility Option B attributes water solubility to the aldehyde functional group. Glucose has five hydroxyl groups in its open-chain representation and remains rich in hydroxyl groups in the cyclic forms; these groups establish many hydrogen bonds with water. The aldehyde functionality is not the principal explanation, especially because the free aldehyde form is present only to a small extent. Option D is correct in the intended constituent-unit sense: sucrose consists of a glucose unit joined to a fructose unit through a glycosidic linkage. Thus the incorrect statement is B; the other three express recognized structural or compositional facts.

Step 3: Conclusion The incorrect statement is that glucose is soluble in water because it has an aldehyde functional group; its multiple hydroxyl groups principally account for the solubility. Hence, the correct option is (B).

Quick Tip to Solve: For carbohydrate solubility, count hydroxyl groups and their hydrogen bonding with water; do not attribute the behaviour solely to the carbonyl classification.

78. Major product of the following reaction is: p-$NC-C_6H_4-CO_2CH_3$ treated with (i) excess CH_3MgBr, then (ii) H_3O^+. [4 marks]

(A) p-$NC-C_6H_4-C(OH)(CH_3)_2$

(B) p-$CH_3CO-C_6H_4-C(OH)(CH_3)_2$

(C) p-$CH_3CO-C_6H_4-CO_2CH_3$

(D) p-$NC-C_6H_4-COCH_3$

Correct Answer: (B) p-$CH_3CO-C_6H_4-C(OH)(CH_3)_2$

Solution:

Concept:

- A Grignard reagent supplies a nucleophilic alkyl group. With a nitrile, one addition gives an imine magnesium salt, which yields a ketone on acidic hydrolysis.
- An ester undergoes two successive additions of a Grignard reagent. Excess reagent therefore converts its acyl carbon into a tertiary alcohol after acidic work-up.

Step 1: Identify the two reactive groups The starting benzene ring bears para $-CN$ and $-CO_2CH_3$ substituents. Both contain electrophilic carbon atoms and both must be considered because excess CH_3MgBr is specified. The aromatic ring and the para relationship of the substituents remain intact; the reagent reacts at the two functional groups rather than hydrogenating the ring.

Step 2: Convert the nitrile group The methyl group attacks the nitrile carbon, giving an imine magnesium salt. Only during the subsequent acid treatment is this intermediate hydrolysed to a ketone:

$$Ar-CN \xrightarrow{CH_3MgBr} Ar-C(CH_3)=N-MgBr \xrightarrow{H_3O^+} Ar-COCH_3.$$

The ketone is generated during work-up, after the Grignard reagent is quenched. Consequently the nitrile-derived group ends as $-COCH_3$, not as a tertiary alcohol.

Step 3: Convert the ester and compare the options At the ester, the first methyl addition followed by methoxide elimination generates a ketone while Grignard reagent is still present. A second methyl addition then gives a tertiary alkoxide. Acid work-up protonates it to $-C(OH)(CH_3)_2$. Thus both groups change: the nitrile becomes an acetyl group and the ester becomes a tertiary alcohol. A leaves the nitrile unchanged, C leaves the ester unchanged, and D leaves the nitrile unchanged while stopping the ester at the ketone stage; each misses a reaction required by excess reagent.

Step 4: Conclusion The product is p-$CH_3CO-C_6H_4-C(OH)(CH_3)_2$. Hence, the correct option is (B).

Quick Tip to Solve: Track nitrile and ester groups separately: the nitrile ketone appears only at work-up, whereas an ester-derived ketone forms early enough for a second Grignard addition.

79. The coordination environment of Ca^{2+} ion in its complex with $EDTA^{4-}$ is : [4 marks]

(A) trigonal prismatic

(B) octahedral

(C) square planar

(D) tetrahedral

Correct Answer: (B) octahedral

Solution:

Concept:

- EDTA is a hexadentate ligand: a single $EDTA^{4-}$ ion can bind a metal through six donor atoms.
- The usual six-donor coordination environment of a metal-EDTA chelate is described as octahedral; coordination number counts donor atoms, not the number of ligand molecules.

Step 1: Count the donor atoms in EDTA EDTA has two amine nitrogen atoms and four carboxylate groups. In its hexadentate binding mode, each nitrogen contributes a lone pair and one oxygen atom from each carboxylate group coordinates to the metal. The donor count is therefore

$$2 \text{ nitrogen donors} + 4 \text{ oxygen donors} = 6.$$

The negative charge on EDTA does not itself give the coordination number; the available donor atoms do.

Step 2: Assign the coordination environment In the calcium-EDTA chelate, the six donor atoms surround the Ca^{2+} ion. The standard coordination description used for this complex is octahedral, with six metal-donor connections. A chelate need not be a perfectly regular geometric solid: the linked donor atoms constrain bond angles, but such distortions do not change the conventional octahedral assignment in this question.

Step 3: Eliminate the alternative geometries Square-planar and tetrahedral environments have four primary coordination positions, so neither accounts for the six EDTA donor atoms. A trigonal prism also has six vertices, but six-coordinate complexes are not automatically trigonal prismatic; the familiar metal-EDTA arrangement here is octahedral. The fact that only one EDTA molecule is attached must not be interpreted as one-coordinate calcium, since that single molecule attaches through several atoms.

Step 4: Conclusion The coordination environment of calcium in its EDTA complex is octahedral. Hence, the correct option is (B).

Quick Tip to Solve: For a polydentate ligand, draw up the donor count first: EDTA contributes two N and four O atoms, giving six coordination sites.

80. The incorrect statement regarding ethyne is [4 marks]

(A) The carbon - carbon bonds in ethyne is weaker than that in ethene

(B) Both carbons are sp hybridised

(C) The $C - C$ bonds in ethyne is shorter than that in ethene

(D) Ethyne is linear

Correct Answer: (A) The carbon - carbon bonds in ethyne is weaker than that in ethene

Solution:

Concept:

- Ethyne, $HC{\equiv}CH$, contains a carbon-carbon triple bond: one sigma bond and two pi bonds. Its carbon atoms are sp hybridised.
- A carbon-carbon triple bond has a greater total bond strength and a shorter bond length than the corresponding double bond in ethene.

Step 1: Compare the carbon-carbon bonds Ethyne has bond order three, whereas ethene has bond order two. The additional bonding interaction raises the total energy required to separate the two carbon atoms. Typical total carbon-carbon bond dissociation energies are approximately $839\,\text{kJ}\,\text{mol}^{-1}$ for a triple bond and $614\,\text{kJ}\,\text{mol}^{-1}$ for a double bond. Thus the statement that the carbon-carbon bond in ethyne is weaker than that in ethene reverses the correct comparison.

Step 2: Check hybridisation and molecular shape Each ethyne carbon has two sigma-bonding directions: one toward hydrogen and one toward the other carbon. These use sp hybrid orbitals arranged at 180°. Its two remaining unhybridised p orbitals form the two mutually perpendicular pi bonds. Consequently both carbons are sp hybridised and the molecular skeleton $H{-}C{\equiv}C{-}H$ is linear. Options B and D correctly describe these features.

Step 3: Check the bond length and select the false statement The stronger, higher-order carbon-carbon bond in ethyne is shorter, about $120\,\text{pm}$, compared with roughly $134\,\text{pm}$ in ethene. Greater s-character of the sp carbon orbitals is consistent with this contraction. Therefore C is also correct. The question asks for the incorrect statement, so the true descriptions in B, C and D must be rejected as answers; A is the sole false claim.

Step 4: Conclusion The carbon-carbon bond in ethyne is stronger, not weaker, than that in ethene. Hence, the correct option is (A).

Quick Tip to Solve: For an incorrect-statement question, mark each claim true or false before selecting; a triple bond is shorter and stronger overall than a double bond.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Based on Heisenberg's uncertainty principle, the uncertainty in the velocity of the electron to be found within an atomic nucleus of diameter 10^{-15} m is _____ $\times 10^9$ ms^{-1} (nearest integer)
[Given : mass of electron $= 9.1 \times 10^{-31}$ kg, Plank's constant $(h) = 6.626 \times 10^{-34}$Js] (Value of $\pi = 3.14$)

[4 marks]

Solution:

Concept:

- The position-momentum uncertainty relation is $\Delta x\,\Delta p \geq h/(4\pi)$, where Δx is position uncertainty and Δp is momentum uncertainty.
- In the standard uncertainty estimate, $\Delta p = m\Delta v$, so the limiting velocity uncertainty is $\Delta v = h/(4\pi m\Delta x)$.

Step 1: Use the confinement length as position uncertainty The question locates the electron within a nucleus of diameter $10^{-15}\,\text{m}$. Use that entire specified confinement length as Δx, rather than replacing it by the radius:

$$\Delta x = 10^{-15}\,\text{m}, \qquad m = 9.1 \times 10^{-31}\,\text{kg}.$$

The uncertainty relation concerns a spread in possible position and momentum, not the electron travelling across that diameter in a known time.

Step 2: Substitute the supplied constants At the limiting value of the uncertainty inequality,

$$\begin{aligned}\Delta v &= \frac{6.626 \times 10^{-34}}{4(3.14)(9.1 \times 10^{-31})(10^{-15})} \\ &= \frac{6.626}{114.296} \times 10^{12}\,\text{m s}^{-1} \\ &= 5.7972 \times 10^{10}\,\text{m s}^{-1}.\end{aligned}$$

The units are correct because $\text{J s}/(\text{kg m}) = \text{m s}^{-1}$. Keeping the powers of ten separate avoids losing a factor of ten.

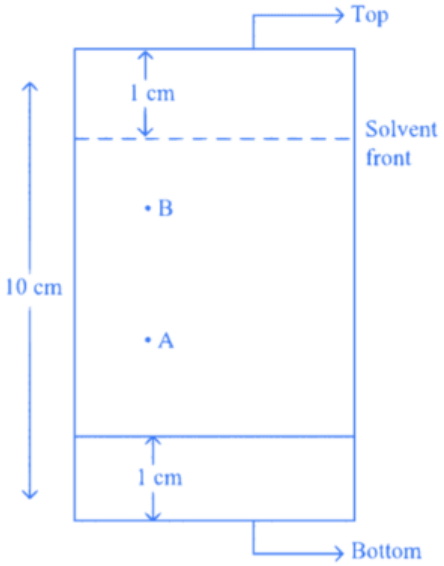
Step 3: Express the requested coefficient The result is $57.972 \times 10^9\,\text{m s}^{-1}$, so the nearest integer coefficient is 58. This very large formal uncertainty is the familiar consequence of applying the elementary nonrelativistic confinement estimate on nuclear scales; it is not a prediction of a measurable electron speed exceeding light. The question asks for that uncertainty-principle estimate using its given mass and constants.

Step 4: Conclusion The estimated minimum uncertainty is approximately $58 \times 10^9\,\text{m s}^{-1}$. The required nearest integer is $\boxed{58}$.

Quick Tip to Solve: Use the printed nuclear diameter as the confinement length, then convert the final velocity into a coefficient of the exact power of ten requested.

82. In the given TLC, the distance of spot A & B are 5 cm & 7 cm, from the bottom of TLC plate, respectively.

R_f value of B is $x \times 10^{-1}$ times more than A. The value of x is _____. [4 marks]



Solution:

Concept:

- The retention factor is $R_f = d_{\text{solute}}/d_{\text{solvent}}$, with both travel distances measured from the application baseline.
- When two spots are on the same TLC plate, their solvent travel distance is identical and cancels in the ratio of their retention factors.

Step 1: Read distances from the actual starting line The plate is 10 cm tall. The application baseline is 1 cm above the bottom, and the solvent front is 1 cm below the top. The stated 5 cm and 7 cm spot positions are measured from the bottom of the plate, not from the baseline. The relevant travel distances are therefore

$$d_A = 5 - 1 = 4\,\text{cm}, \qquad d_B = 7 - 1 = 6\,\text{cm}.$$

The solvent moves from height 1 cm to height 9 cm, giving $d_s = 9 - 1 = 8\,\text{cm}$.

Step 2: Calculate the retention factors Apply the same solvent-front distance to both spots:

$$R_f(A) = \frac{4}{8} = 0.50, \qquad R_f(B) = \frac{6}{8} = 0.75.$$

Both numbers lie between zero and one, as required for spots below the solvent front. Using 5 and 7 directly would incorrectly count the unused strip below the application line as migration distance.

Step 3: Compare the factors in the requested form The multiplicative comparison is

$$\frac{R_f(B)}{R_f(A)} = \frac{0.75}{0.50} = \frac{3}{2} = 1.5.$$

Interpreting the question's stated multiplier as $R_f(B) = x \times 10^{-1} R_f(A)$, its coefficient obeys $x \times 10^{-1} =$

1.5. Hence $x = 15$. The comparison is a ratio of dimensionless retention factors; there is no remaining centimetre unit and no need to use the unused top margin as a separate distance.

Step 4: Conclusion $R_f(B) = 1.5R_f(A) = 15 \times 10^{-1}R_f(A)$. Thus the required value is $\boxed{15}$.

Quick Tip to Solve: Always subtract the TLC application-line height from spot heights measured from the plate bottom; for two spots on one plate the solvent distance cancels in their ratio.

83. Number of compounds from the following which cannot undergo Friedel-Crafts reactions is: _____

toluene, nitrobenzene, xylene, cumene, aniline, chlorobenzene, m-nitroaniline, m-dinitrobenzene

[4 marks]

Solution:

Concept:

- Friedel-Crafts alkylation and acylation are electrophilic aromatic substitutions, typically using a Lewis acid such as $AlCl_3$.
- Strongly electron-withdrawing nitro groups prevent the usual Friedel-Crafts reaction. An amino group also prevents it under ordinary Lewis-acid conditions because it complexes with the catalyst.

Step 1: Classify the alkyl-substituted aromatic compounds Toluene, xylene and cumene contain alkyl substituents that donate electron density to the aromatic ring. They are suitable substrates for normal Friedel-Crafts substitution, subject to the familiar regioselectivity and possible multiple alkylation issues. These three compounds therefore do not enter the list of compounds that cannot undergo the reaction. Chlorobenzene is deactivating in electrophilic substitution but is not treated like the strongly deactivating nitro substrates in this standard classification.

Step 2: Identify the nitro and amino cases Nitrobenzene is strongly deactivated by the $-NO_2$ group and does not undergo the usual Friedel-Crafts reaction. Aniline forms a Lewis acid-base complex through its nitrogen lone pair with $AlCl_3$, producing a strongly deactivated nitrogen-containing substituent and consuming catalyst. Thus aniline is also unsuitable. The compound m-nitroaniline has both obstructing features, while m-dinitrobenzene bears two strongly deactivating nitro groups.

Step 3: Count each unsuitable molecule once The excluded compounds are nitrobenzene, aniline, m-nitroaniline and m-dinitrobenzene. Count compounds rather than the number of nitro or amino substituents: nitroaniline is one molecule in the list even though it has two reasons for exclusion, and dinitrobenzene is also counted once. Toluene, xylene, cumene and chlorobenzene are the remaining four entries, so the required count is four out of the eight supplied compounds.

Step 4: Conclusion The four compounds that cannot undergo the usual Friedel-Crafts reactions are nitrobenzene, aniline, m-nitroaniline and m-dinitrobenzene. The answer is $\boxed{4}$.

Quick Tip to Solve: Flag strong nitro deactivation and amino-Lewis-acid complex formation, then count each flagged compound once rather than counting its substituents.

84. Total number of electrons present in (π^*) molecular orbitals of O_2, O_2^+ and O_2^- is _____.

[4 marks]

Solution:

Concept:

- For oxygen, the two degenerate antibonding π^*2p molecular orbitals are populated after the bonding $\pi 2p$ and $\sigma 2p$ orbitals.
- Ionisation removes an electron from the highest occupied molecular orbital; adding an electron puts it into the lowest available orbital. Here both changes involve the π^*2p pair.

Step 1: Count antibonding pi electrons in oxygen Neutral O_2 has 12 valence electrons. Its valence configuration is

$$(\sigma 2s)^2 (\sigma^* 2s)^2 (\sigma 2p_z)^2 (\pi 2p_x)^2 (\pi 2p_y)^2 (\pi^* 2p_x)^1 (\pi^* 2p_y)^1 .$$

Thus there are two electrons in the antibonding pi orbitals together. The asterisk matters: the four electrons in bonding pi orbitals must not be included in this count.

Step 2: Adjust the count for the two ions The highest occupied levels in neutral oxygen are the degenerate π^*2p orbitals. Forming $O_2{}^+$ removes one of their electrons, reducing the total pi-antibonding occupancy from two to one. Forming $O_2{}^-$ adds one electron to that same pair of orbitals, increasing the total occupancy from two to three. Hund's rule affects the distribution and spin of these electrons, but not the total number requested.

Step 3: Add the three occupancies The species are counted separately: neutral oxygen contributes 2, the cation contributes 1 and the anion contributes 3. Therefore

$$N_{\pi^*} = 2 + 1 + 3 = 6.$$

The corresponding bond orders would be 2, 2.5 and 1.5, respectively, which supplies a useful consistency check: removing an antibonding electron strengthens the bond, whereas adding one weakens it. These bond orders are a check only; the question asks for an electron count.

Step 4: Conclusion The total number of electrons in the π^* molecular orbitals of O_2, $O_2{}^+$ and $O_2{}^-$ is $\boxed{6}$.

Quick Tip to Solve: Start with two pi-antibonding electrons in oxygen; subtract one for its cation and add one for its anion before taking the total.

85. A transition metal 'M' among Sc, Ti, V, Cr, Mn and Fe has the highest second ionisation enthalpy. The spin-only magnetic moment value of M^+ ion is _____ BM (Near integer)

(Given atomic number Sc : 21, Ti : 22, V : 23, Cr : 24, Mn : 25, Fe : 26)

[4 marks]

Solution:

Concept:

- The second ionisation enthalpy is the energy required for the gaseous process $M^+ \longrightarrow M^{2+} + e^-$; it depends on the stability of the singly charged ion.

- A half-filled $3d^5$ subshell is especially stable. The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons.

Step 1: Identify the stable singly charged ion Chromium has atomic number 24 and the exceptional ground-state configuration $[\text{Ar}]3d^5 4s^1$. The first ionisation removes its outer $4s$ electron, producing Cr^+ with configuration $[\text{Ar}]3d^5$. This ion therefore has a half-filled d subshell. Removing its next electron disrupts this stable arrangement, explaining the unusually high second ionisation enthalpy.

Step 2: Compare the other candidates The special feature is that the second ionisation of chromium breaks its stable half-filled d subshell. Vanadium(I) instead has configuration $[\text{Ar}]3d^4$, with no half-filled stability to disrupt. In comparison, Mn^+ is $[\text{Ar}]3d^5 4s^1$, so its next ionisation produces, rather than destroys, a half-filled d subshell. Thus the metal specified is chromium, not manganese, although neutral manganese itself has a half-filled d subshell.

Step 3: Count unpaired electrons and calculate For Cr^+, the five $3d$ electrons occupy the five d orbitals singly according to Hund's rule. Therefore $n = 5$, and

$$\mu = \sqrt{5(5+2)} = \sqrt{35} = 5.92\,\text{BM}.$$

The question asks for the nearest integer, so this value must be rounded only after evaluating the square root.

Step 4: Conclusion The metal is chromium and its Cr^+ ion has spin-only moment $5.92\,\text{BM}$. The required nearest-integer answer is $\boxed{6}$.

Quick Tip to Solve: For a second-ionisation question, first write the configuration of the $+1$ ion; the stability of the neutral atom alone can mislead you.

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86. Consider the following first order gas phase reaction at constant temperature $\text{A(g)} \rightarrow 2\text{B(g)} + \text{C(g)}$ **If the total pressure of the gases is found to be 200 torr after 23** sec. **and 300 torr upon the complete decomposition of A after a very long time, then the rate constant of the given reaction is _____** $\times 10^{-2}\ \text{s}^{-1}$ **(nearest integer)**

[Given : $\log_{10}(2) = 0.301$**]** [4 marks]

Solution:

Concept:

- At fixed temperature and volume, a gas partial pressure is proportional to its number of moles. A first-order reaction obeys $k = (2.303/t)\log_{10}(P_0/P_A)$.
- For $\text{A(g)} \longrightarrow 2\,\text{B(g)} + \text{C(g)}$, complete decomposition triples the initial number of moles when only A is present initially.

Step 1: Find the initial pressure of A Let the initial pressure of pure A be P_0. Every mole of A produces three moles of gaseous products, so after complete decomposition the total pressure is $P_\infty = 3P_0$. Using the final pressure given,

$$P_0 = \frac{300}{3} = 100\,\text{torr}.$$

This initial pressure is the pressure before decomposition; the given 200 torr is the total pressure at a later time, not the remaining pressure of A.

Step 2: Obtain the remaining partial pressure Let the pressure equivalent of A decomposed after 23 s be x. The partial pressures of A, B and C then equal $P_0 - x$, $2x$ and x, respectively. Their sum is $P_t = P_0 + 2x$, hence

$$200 = 100 + 2x \quad \Rightarrow \quad x = 50\,\text{torr}.$$

The unreacted A therefore has partial pressure $P_A = 100 - 50 = 50\,\text{torr}$. Half of A remains.

Step 3: Apply the integrated rate equation Substitute the initial and remaining partial pressures into the first-order law:

$$k = \frac{2.303}{23}\log_{10}\frac{100}{50} = \frac{2.303(0.301)}{23} = 0.03014\,\text{s}^{-1}.$$

Thus $k = 3.014 \times 10^{-2}\,\text{s}^{-1}$. The numerical blank is the coefficient of 10^{-2}, so round 3.014 to the nearest integer.

Step 4: Conclusion The rate constant is approximately $3.014 \times 10^{-2}\,\text{s}^{-1}$; the required entry is $\boxed{3}$.

Quick Tip to Solve: Use the stoichiometric change in total gas moles to recover the pressure of unreacted A before applying a first-order pressure formula.

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87. When $\Delta H_{vap} = 30$ kJ/mol and $\Delta S_{vap} = 75$ J mol^{-1} K^{-1}, then the temperature of vapour, at one atmosphere is _____ K. [4 marks]

Solution:

Concept:

- At liquid-vapour equilibrium at the boiling temperature, the Gibbs energy change of vaporisation is zero: $\Delta G_{vap} = \Delta H_{vap} - T\Delta S_{vap} = 0$.
- The enthalpy and entropy must use consistent energy units before evaluating $T = \Delta H_{vap}/\Delta S_{vap}$.

Step 1: Set the equilibrium condition The temperature sought at one atmosphere is the temperature at which the liquid and its vapour coexist in equilibrium at that pressure. Transferring an infinitesimal quantity from one phase to the other then has no net Gibbs energy change. Consequently, the energetic cost of vaporisation is balanced by its temperature-weighted entropy gain:

$$\Delta H_{vap} = T\Delta S_{vap}.$$

Both supplied changes refer to the same vaporisation process.

Step 2: Convert the enthalpy to joules The enthalpy is given in kilojoules per mole, while the entropy is in joules per mole per kelvin. Convert the former before dividing:

$$\Delta H_{vap} = 30\,\text{kJ mol}^{-1} = 30000\,\text{J mol}^{-1}, \qquad \Delta S_{vap} = 75\,\text{J mol}^{-1}\text{K}^{-1}.$$

Using 30 directly with 75 would introduce a factor-of-1000 error. The molar units cancel in their ratio.

Step 3: Calculate the equilibrium temperature Rearrange the equilibrium relation and substitute:

$$T = \frac{\Delta H_{\text{vap}}}{\Delta S_{\text{vap}}} = \frac{30000}{75} = 400\,\text{K}.$$

The ratio has kelvin units because energy per mole is divided by energy per mole per kelvin. No Celsius conversion is needed, since the question explicitly requests an absolute temperature in kelvin.

Step 4: Conclusion The temperature of liquid-vapour equilibrium at one atmosphere is $\boxed{400\,\text{K}}$. The required numerical entry is 400.

Quick Tip to Solve: At a phase-transition equilibrium use $\Delta G = 0$, and convert kJ to J before dividing enthalpy by entropy.

88. Consider the following test for a group-IV cation.

$$\text{M}^{2+} + \text{H}_2\text{S} \rightarrow \text{A}\ \textbf{(Black precipitate)} + \textbf{byproduct}$$

$$\text{A} + \ \textbf{aqua regia}\ \rightarrow \text{B} + \text{NOCl} + \text{S} + \text{H}_2\text{O}$$

$$\text{B} + \text{KNO}_2 + \text{CH}_3\text{COOH} \rightarrow \text{C} + \ \textbf{byproduct}$$

The spin-only magnetic moment value of the metal complex C is _____ BM (Nearest integer)

[4 marks]

Solution:

Concept:

- Cobalt(II) is a group-IV cation in qualitative inorganic analysis and forms black cobalt(II) sulphide, CoS.
- The nitrite test produces yellow potassium hexanitritocobaltate(III), $\text{K}_3[\text{Co}(\text{NO}_2)_6]$. Its metal ion is low-spin d^6, so all electrons are paired.

Step 1: Identify the black sulphide and metal salt The group-IV cation giving the specified confirmatory nitrite test is Co^{2+}. Its black sulphide is A, CoS:

$$\text{Co}^{2+} + \text{H}_2\text{S} \longrightarrow \text{CoS}\,(\text{s}) + 2\,\text{H}^{+}.$$

Aqua regia dissolves this sulphide, yielding a cobalt salt in chloride medium (B), while oxidising sulphide sulphur to elemental sulphur. The diagnostic final step, rather than the black colour alone, distinguishes cobalt from other black sulphide-forming ions.

Step 2: Identify the complex and oxidation state Treatment with potassium nitrite in acetic acid gives the yellow cobaltinitrite complex C, $\text{K}_3[\text{Co}(\text{NO}_2)_6]$. Cobalt is oxidised from +2 to +3 during this test. Three K^+ ions require a complex-ion charge of -3. If the cobalt oxidation state is z, the six nitrite

ligands give

$$z + 6(-1) = -3 \quad \Rightarrow \quad z = +3.$$

Cobalt has atomic number 27, so Co^{3+} has the $3d^6$ electron configuration.

Step 3: Fill the octahedral levels and calculate The six nitrite ligands surround cobalt(III) octahedrally. Their field together with the high +3 metal charge leads to the low-spin arrangement $t_{2g}^6 e_g^0$. Each of the three lower-energy orbitals contains a pair, leaving no unpaired electrons. Hence $n = 0$ in the spin-only expression:

$$\mu = \sqrt{n(n+2)} = \sqrt{0(0+2)} = 0\,\text{BM}.$$

The complex is therefore diamagnetic; using the initial cobalt(II) configuration would give the wrong result.

Step 4: Conclusion Complex C is potassium hexanitritocobaltate(III), $K_3[Co(NO_2)_6]$, with spin-only moment $\boxed{0\,\text{BM}}$. The entry is 0.

Quick Tip to Solve: Calculate the metal oxidation state in the final complex before counting unpaired electrons; a qualitative test may also oxidise the metal.

89. Number of oxygen atoms present in chemical formula of fuming sulphuric acid is _____.

[4 marks]

Solution:

Concept:

- Fuming sulphuric acid, also called oleum, is represented in this question by disulphuric acid (pyrosulphuric acid), $H_2S_2O_7$.
- Its formula follows by adding sulphur trioxide to sulphuric acid: $H_2SO_4 + SO_3 \longrightarrow H_2S_2O_7$. The subscript on O counts the oxygen atoms in one formula unit.

Step 1: Recall the meaning of fuming sulphuric acid Oleum is obtained by absorbing sulphur trioxide into concentrated sulphuric acid. The conventional chemical formula used for fuming sulphuric acid in this question is $H_2S_2O_7$, also written as $H_2SO_4 \cdot SO_3$. This is distinct from ordinary sulphuric acid, H_2SO_4. The adjective fuming signals the presence of additional sulphur trioxide, so counting only four oxygen atoms would refer to the wrong substance.

Step 2: Combine the atom counts Write the addition relation without removing a water molecule:

$$H_2SO_4 + SO_3 \longrightarrow H_2S_2O_7.$$

The reactant formula units together contain two hydrogen atoms, two sulphur atoms and $4 + 3 = 7$ oxygen atoms. This atom count directly checks the product formula. No oxidation-state calculation is required, since the question asks for an atom count, rather than the charge or oxidation number of oxygen.

Step 3: Read the oxygen subscript In the molecular formula $H_2S_2O_7$, the oxygen subscript is 7, meaning seven oxygen atoms in each formula unit. Equivalently, the structure contains two sulphur

centres connected through one bridging oxygen, with three additional oxygen atoms associated with each centre; this gives $1 + 3 + 3 = 7$. Count the common bridging atom once, rather than counting it separately for both sulphur atoms.

Step 4: Conclusion Fuming sulphuric acid is represented as $H_2S_2O_7$, containing $\boxed{7}$ oxygen atoms per formula unit.

Quick Tip to Solve: Remember oleum as $H_2SO_4 \cdot SO_3$: its oxygen count is immediately $4 + 3 = 7$, unlike the four in ordinary sulphuric acid.

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90. The vapor pressure of pure benzene and methyl benzene at $27^\circ C$ is given as 80 Torr and 24 Torr, respectively. The mole fraction of methyl benzene in vapor phase, in equilibrium with an equimolar mixture of those two liquids (ideal solution) at the same temperature is _____ $\times 10^{-2}$ (nearest integer) [4 marks]

Solution:

Concept:

- For an ideal liquid solution, Raoult's law gives each partial pressure: $p_i = x_i p_i^\circ$, where x_i is its liquid mole fraction.
- Dalton's law gives the vapour mole fraction $y_i = p_i/P$, where P is the sum of the component partial pressures. Liquid and vapour mole fractions are generally different.

Step 1: Write the liquid mole fractions The liquid mixture is equimolar in benzene and methyl benzene (toluene). Thus

$$x_{\text{benzene}} = x_{\text{methylbenzene}} = \frac{1}{2}.$$

At the stated temperature the pure-component vapour pressures are respectively 80 torr and 24 torr. The smaller pure vapour pressure of methyl benzene means it is less volatile, so its fraction in the equilibrium vapour should be less than one-half. This provides a useful check on the calculation.

Step 2: Calculate the two partial pressures Apply Raoult's law separately to each component:

$$p_{\text{benzene}} = \frac{1}{2}(80) = 40\,\text{torr}, \qquad p_{\text{methylbenzene}} = \frac{1}{2}(24) = 12\,\text{torr}.$$

The total pressure is therefore

$$P = 40 + 12 = 52\,\text{torr}.$$

The requested composition is a vapour composition, so retaining the liquid mole fraction of 0.5 as the answer would ignore the unequal volatilities.

Step 3: Find and scale the vapour mole fraction By Dalton's law, the methyl benzene mole fraction in the vapour is

$$y_{\text{methylbenzene}} = \frac{12}{52} = \frac{3}{13} = 0.230769\ldots.$$

The question writes the result as a coefficient times 10^{-2}. Thus

$$0.230769\ldots = 23.0769\ldots \times 10^{-2}.$$

Round this coefficient, rather than the mole fraction itself, to obtain the nearest-integer entry 23.

Step 4: Conclusion The vapour mole fraction of methyl benzene is 0.23077, or approximately 23×10^{-2}. The required entry is $\boxed{23}$.

Quick Tip to Solve: For an equimolar ideal binary liquid, each vapour fraction is its pure vapour pressure divided by the sum of the two pure vapour pressures.