JEE Main 2024 Paper 1 (B.E./B.Tech) Apr 8 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 8 April 2024, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $A = \begin{bmatrix} 2 & a & 0 \\ 1 & 3 & 1 \\ 0 & 5 & b \end{bmatrix}$. If $A^3 = 4A^2 - A - 21I$, where I is the identity matrix of order 3×3, then $2a + 3b$ is equal to [4 marks]

(A) -10

(B) -12

(C) -13

(D) -9

Correct Answer: (C) -13

Solution:

Concept:

- A polynomial identity in a matrix is an equality of corresponding entries. Carefully selected entries can determine the unknown parameters without multiplying every entry.
- For a product, $(A^2)_{ij} = \sum_k A_{ik}A_{kj}$. Here the sparse first and third rows make the corner entries particularly convenient.

Step 1: Use the bottom-left entry to determine the diagonal parameter First compute the entries needed for the third row: $(A^2)_{31} = 5$ and $(A^2)_{32} = 15 + 5b$. Multiplying that row by the first column of A gives

$$(A^3)_{31} = 5(2) + (15 + 5b)(1) = 25 + 5b.$$

The same entry of $4A^2 - A - 21I$ is $4(5) - 0 - 0 = 20$. Therefore $25 + 5b = 20$, which gives $b = -1$. This calculation does not divide by the other unknown and so does not discard any possible case.

Step 2: Use the top-left entry to find the remaining parameter The first row of A^2 begins with $4+a, 5a$. Consequently

$$(A^3)_{11} = 2(4 + a) + 5a = 8 + 7a.$$

The prescribed polynomial gives the corresponding entry as

$$4(4 + a) - 2 - 21 = 4a - 7.$$

Equating these expressions yields $8 + 7a = 4a - 7$, hence $a = -5$. Thus

$$2a + 3b = 2(-5) + 3(-1) = -13.$$

As a consistency check, the $(1, 3)$ entry requires $a(5 + b) = 4a$, which is indeed satisfied by $b = -1$. The other three proposed numbers differ from this unique value.

Step 3: Conclusion The required value is $\boxed{-13}$. Hence, the correct option is (C).

Quick Tip to Solve: For a sparse matrix polynomial, inspect corner entries first: they often isolate one unknown before the diagonal entries are used.

2. The value of $k \in \mathbb{N}$ for which the integral $I_n = \int_0^1 \left(1 - x^k\right)^n dx, n \in \mathbb{N}$, satisfies $147I_{20} = 148I_{21}$ is [4 marks]

(A) 8

(B) 14

(C) 7

(D) 10

Correct Answer: (C) 7

Solution:

Concept:

- Integration by parts relates consecutive powers without expanding the binomial. The endpoint term vanishes for a positive integer power.
- Use $x^k = 1 - (1 - x^k)$ to express the remaining integral as a difference of two consecutive members of the given sequence.

Step 1: Derive a recurrence for consecutive integrals Apply integration by parts to I_{n+1}, taking $u = (1 - x^k)^{n+1}$ and $dv = dx$. Since $x(1 - x^k)^{n+1}$ vanishes at both endpoints,

$$I_{n+1} = k(n + 1) \int_0^1 x^k (1 - x^k)^n \, dx.$$

Replacing x^k by $1 - (1 - x^k)$ gives

$$I_{n+1} = k(n + 1)(I_n - I_{n+1}).$$

Thus $[k(n + 1) + 1]I_{n+1} = k(n + 1)I_n$. Both integrals are positive, so forming their ratio is legitimate.

Step 2: Insert the printed relation and solve for the integer For $n = 20$, the recurrence gives

$$\frac{I_{21}}{I_{20}} = \frac{21k}{21k + 1}.$$

The given equality $147I_{20} = 148I_{21}$ requires this ratio to equal $147/148$. Cross-multiplying carefully,

$$148(21k) = 147(21k + 1), \qquad 21k = 147.$$

Therefore $k = 7$, a natural number as required. Conversely, substituting $k = 7$ in the recurrence gives $I_{21}/I_{20} = 147/148$, reproducing the given relation exactly. The values 8, 14 and 10 would instead give different ratios, since $21k/(21k + 1)$ is strictly increasing for positive k.

Step 3: Conclusion The required natural number is $\boxed{7}$. Hence, the correct option is (C).

Quick Tip to Solve: When consecutive powers occur under an integral, integration by parts followed by writing the extra factor as a difference often produces a short recurrence.

3. Let $f(x) = 4\cos^3 x + 3\sqrt{3}\cos^2 x - 10$. The number of points of local maxima of f in interval $(0, 2\pi)$ is [4 marks]

(A) 1

(B) 3

(C) 4

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

- An interior local maximum requires the derivative to change from positive to negative. A zero derivative alone does not establish a maximum.
- Writing $u = \cos x$ reduces the algebra to $g(u) = 4u^3 + 3\sqrt{3}u^2 - 10$, but turning points of the cosine must also be checked.

Step 1: Find every stationary point in the open interval Differentiation and factorisation give

$$f\prime(x) = -6\sin x\cos x(2\cos x + \sqrt{3}).$$

Its zeros in $(0, 2\pi)$ are

$$\frac{\pi}{2}, \quad \frac{5\pi}{6}, \quad \pi, \quad \frac{7\pi}{6}, \quad \frac{3\pi}{2}.$$

The endpoints 0 and 2π are excluded by the question and must not be counted. These five points exhaust all zeros because the displayed derivative is a product of the three listed factors.

Step 2: Classify the stationary points before counting For the polynomial in u,

$$g\prime(u) = 6u(2u + \sqrt{3}), \qquad g\prime\prime(u) = 24u + 6\sqrt{3}.$$

At $u = -\sqrt{3}/2$, $g\prime\prime(u) = -6\sqrt{3} < 0$. Since $\sin x \neq 0$ at both corresponding points, $x = 5\pi/6$ and $7\pi/6$ are strict local maxima. At $u = 0$, $g\prime\prime(0) = 6\sqrt{3} > 0$; hence $\pi/2$ and $3\pi/2$ are minima. Finally, at $x = \pi$, the second derivative is

$$f\prime\prime(\pi) = 6(2 - \sqrt{3}) > 0,$$

so that point is also a minimum. There are exactly two maxima. Counting all stationary points, or treating the cosine turning point as a maximum automatically, would give an incorrect count.

Step 3: Conclusion There are $\boxed{2}$ points of local maximum in the specified open interval. Hence, the correct option is (D).

Quick Tip to Solve: After substituting $u = \cos x$, remember to check $\sin x = 0$ separately; these stationary points arise from the inner function.

4. The set of all α, for which the vectors $\vec{a} = \alpha t\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{b} = t\hat{i} - 2\hat{j} - 2\alpha t\hat{k}$ are inclined at an obtuse angle for all $t \in \mathbb{R}$, is [4 marks]

(A) $[0, 1)$

(B) $\left(-\frac{4}{3}, 0\right]$

(C) $(-2, 0]$

(D) $\left(-\frac{4}{3}, 1\right)$

Correct Answer: (B) $\left(-\frac{4}{3}, 0\right]$

Solution:

Concept:

- Two nonzero vectors make an obtuse angle precisely when their scalar product is strictly negative, because $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$.
- A quadratic must be negative for every real input. Completing the square identifies its maximum directly when its leading coefficient is negative.

Step 1: Convert the angle condition into a quadratic inequality The fixed $\hat{j}$ components ensure that neither vector can vanish for any real t. Compute the scalar product component by component:

$$\vec{a} \cdot \vec{b} = \alpha t^2 - 12 + 6\alpha t = \alpha(t + 3)^2 - 9\alpha - 12.$$

It must be strictly below zero for every $t \in \mathbb{R}$. If $\alpha > 0$, the square term tends to positive infinity as $|t|$ increases, so no positive value can work. The case $\alpha = 0$ gives the constant -12, which is always negative and therefore must be included.

Step 2: Impose strict negativity at the largest value For $\alpha < 0$, the square term is nonpositive and the maximum occurs at $t = -3$. Thus the required necessary and sufficient condition is

$$-9\alpha - 12 < 0 \quad \Longleftrightarrow \quad \alpha > -\frac{4}{3}.$$

Combining it with $\alpha < 0$, then adding the valid constant case, gives $(-4/3, 0]$. At $\alpha = -4/3$ and $t = -3$, the dot product is zero, giving a right angle rather than an obtuse angle, so that endpoint is excluded. Option (C) incorrectly admits more negative values, for which the maximum is positive; (A) and (D) incorrectly admit positive values. This checks both the strict endpoint and the unbounded-input requirement.

Step 3: Conclusion The complete set is $\boxed{(-4/3, 0]}$. Hence, the correct option is (B).

Quick Tip to Solve: For a condition holding for every real parameter, complete the square and test the extreme value; check a zero leading coefficient separately.

5. Let the circles $C_1 : (x-\alpha)^2 + (y-\beta)^2 = r_1^2$ and $C_2 : (x-8)^2 + \left(y-\frac{15}{2}\right)^2 = r_2^2$ touch each other externally at the point $(6,6)$. If the point $(6,6)$ divides the line segment joining the centres of the circles C_1 and C_2 internally in the ratio $2:1$, then $(\alpha+\beta)+4\left(r_1^2+r_2^2\right)$ equals [4 marks]

(A) 130

(B) 110

(C) 145

(D) 125

Correct Answer: (A) 130

Solution:

Concept:

- For externally touching circles, the point of contact lies between the two centres. Distances from that point to the centres equal their respective radii.
- If P divides O_1O_2 internally in the ratio $2:1$, then $P = (O_1 + 2O_2)/3$. The coefficient of each centre is the opposite segment length.

Step 1: Determine the unknown centre from the division ratio The centres are $O_1 = (\alpha, \beta)$ and $O_2 = (8, 15/2)$, while the point of contact is $P = (6,6)$. The stated ratio means $O_1P : PO_2 = 2:1$. Apply the section formula to each coordinate:

$$6 = \frac{\alpha + 2(8)}{3}, \qquad 6 = \frac{\beta + 2(15/2)}{3}.$$

Solving gives $\alpha = 2$ and $\beta = 3$. In particular, $\alpha + \beta = 5$. Reversing the coefficients would reverse the ratio and would not represent the printed internal division.

Step 2: Calculate both radii and the requested expression Use the distances from the contact point to each centre:

$$r_1^2 = (6-2)^2 + (6-3)^2 = 16 + 9 = 25,$$

$$r_2^2 = (6-8)^2 + (6-15/2)^2 = 4 + \frac{9}{4} = \frac{25}{4}.$$

Hence $r_1 = 5$ and $r_2 = 5/2$, which indeed have the required ratio $2:1$. The centre separation is $15/2 = r_1 + r_2$, confirming external tangency. Finally,

$$(\alpha+\beta) + 4(r_1^2 + r_2^2) = 5 + 4\left(25 + \frac{25}{4}\right) = 5 + 125 = 130.$$

The other numerical options cannot satisfy this value obtained from the unique centre and the contact distances.

Step 3: Conclusion The required expression equals $\boxed{130}$. Hence, the correct option is (A).

Quick Tip to Solve: At external contact, the division ratio is also the radius ratio; use it to check the section formula before simplifying the final expression.

6. Let $f(x)$ be a positive function such that the area bounded by $y = f(x), y = 0$ from $x = 0$ to $x = a > 0$ is $e^{-a} + 4a^2 + a - 1$. Then the differential equation, whose general solution is $y = c_1 f(x) + c_2$, where c_1 and c_2 are arbitrary constants, is [4 marks]

(A) $(8e^x + 1)\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

(B) $(8e^x + 1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

(C) $(8e^x - 1)\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

(D) $(8e^x - 1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

Correct Answer: (B) $(8e^x + 1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

Solution:

Concept:

- For a positive function, the stated area is $\int_0^a f(x)\,dx$; the fundamental theorem of calculus recovers $f(a)$ by differentiating with respect to the upper limit.
- A family with two arbitrary constants generally leads to a second-order equation. Differentiating removes the additive constant, and a ratio of derivatives removes the multiplicative one.

Step 1: Recover the function from its given area The area condition is

$$\int_0^a f(x)\,dx = e^{-a} + 4a^2 + a - 1.$$

Differentiate with respect to a, then rename the variable x:

$$f(x) = -e^{-x} + 8x + 1.$$

The area at $a = 0$ is zero as required. Its derivative is positive for $x > 0$, consistent with the prescribed positive function over the integration interval. The derivatives needed to eliminate constants are

$$f\prime(x) = 8 + e^{-x}, \qquad f\prime\prime(x) = -e^{-x}.$$

Step 2: Eliminate the arbitrary constants and verify the equation For $y = c_1 f(x) + c_2$, differentiating twice gives

$$y\prime = c_1(8 + e^{-x}), \qquad y\prime\prime = -c_1 e^{-x}.$$

Multiply the second equation by $8e^x + 1$. Its right side becomes $-c_1(8 + e^{-x}) = -y\prime$, so

$$(8e^x + 1)y\prime\prime + y\prime = 0.$$

This identity also includes $c_1 = 0$, for which both derivatives vanish. Conversely, setting $v = y\prime$ yields $v\prime/v = -1/(8e^x + 1)$; its solutions are $v = C(8 + e^{-x})$, and integration recovers the stated two-constant family. The negative sign before $y\prime$ in (A), or replacing the plus inside the coefficient by a minus in (C) and (D), fails direct substitution.

Step 3: Conclusion The differential equation is $\boxed{(8e^x + 1)y\prime\prime + y\prime = 0}$. Hence, the correct option is (B).

Quick Tip to Solve: Differentiate an area formula once to obtain the function, then differentiate the given family twice and eliminate the multiplicative constant without unnecessary expansion.

7. Let $H : \frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the hyperbola, whose eccentricity is $\sqrt{3}$ and the length of the latus rectum is $4\sqrt{3}$. Suppose the point $(\alpha, 6), \alpha > 0$ lies on H. If β is the product of the focal distances of the point $(\alpha, 6)$, then $\alpha^2 + \beta$ is equal to [4 marks]

(A) 170

(B) 171

(C) 169

(D) 172

Correct Answer: (B) 171

Solution:

Concept:

- The positive term y^2/b^2 identifies a vertical transverse axis. Thus $c^2 = a^2 + b^2$, eccentricity $e = c/b$, and latus-rectum length $2a^2/b$.
- The foci of this centred vertical hyperbola are $(0, \pm c)$. Squared distances avoid unnecessary radicals when finding their product.

Step 1: Find the semi-axes and the focus coordinates From $e = \sqrt{3}$,

$$\frac{a^2 + b^2}{b^2} = 3 \quad \Longrightarrow \quad a^2 = 2b^2.$$

The latus rectum condition then gives

$$\frac{2a^2}{b} = 4b = 4\sqrt{3}.$$

Since lengths are positive, $b = \sqrt{3}$, $b^2 = 3$, and $a^2 = 6$. Therefore $c^2 = 9$, so the two foci are $(0, 3)$ and $(0, -3)$. Using a as the transverse semi-axis would exchange the denominators incorrectly; the positive term of the printed equation determines the orientation.

Step 2: Locate the point and multiply its focal distances Substitute $(\alpha, 6)$ into the hyperbola:

$$-\frac{\alpha^2}{6} + \frac{36}{3} = 1 \quad \Longrightarrow \quad \alpha^2 = 66.$$

The condition $\alpha > 0$ selects $\alpha = \sqrt{66}$, though only its square is needed. The distances from this point to the two foci are

$$d_1 = \sqrt{66 + (6-3)^2} = \sqrt{75}, \qquad d_2 = \sqrt{66 + (6+3)^2} = \sqrt{147}.$$

Their positive product is

$$\beta = \sqrt{75 \cdot 147} = \sqrt{11025} = 105.$$

As a check, $d_2 - d_1 = 7\sqrt{3} - 5\sqrt{3} = 2\sqrt{3} = 2b$, the expected focal-distance difference. Consequently $\alpha^2 + \beta = 66 + 105 = 171$; the neighbouring options arise from arithmetic errors and do not equal this expression.

Step 3: Conclusion The requested value is $\boxed{171}$. Hence, the correct option is (B).

Quick Tip to Solve: Read the hyperbola orientation from the positive squared term; then compute focal distances directly and check that their difference is twice the transverse semi-axis.

8. The number of critical points of the function $f(x) = (x-2)^{2/3}(2x+1)$ is [4 marks]

(A) 2

(B) 1

(C) 0

(D) 3

Correct Answer: (A) 2

Solution:

Concept:

- A critical point is a point in the domain where the derivative is zero or does not exist. Both possibilities must be tested.
- The real power $(x-2)^{2/3}$ means the square of the real cube root, so the function is defined for every real x.

Step 1: Differentiate away from the exceptional point For $x \neq 2$, apply the product rule, retaining the factor with negative exponent:

$$\begin{aligned} f\prime(x) &= \frac{2(2x+1)}{3(x-2)^{1/3}} + 2(x-2)^{2/3} \\ &= \frac{2(2x+1)+6(x-2)}{3(x-2)^{1/3}} \\ &= \frac{10(x-1)}{3(x-2)^{1/3}}. \end{aligned}$$

The denominator is nonzero on this part of the domain. Consequently the only stationary point here is $x = 1$.

Step 2: Check where the derivative is undefined The excluded value $x = 2$ belongs to the domain, since $f(2) = 0$. Its difference quotient is

$$\frac{f(2+h)-f(2)}{h} = \frac{h^{2/3}(5+2h)}{h} = \frac{5+2h}{h^{1/3}}.$$

This has no finite limit as $h \to 0$. Thus $x = 2$ is also critical, although its derivative is not zero. No other denominator can vanish, and the numerator has only the one zero already found. Counting just stationary points would incorrectly give option (B); options (C) and (D) respectively omit both points or add an unsupported third point.

Step 3: Conclusion The critical points occur at $x = 1$ and $x = 2$, so their number is $\boxed{2}$. Hence, the correct option is (A).

Quick Tip to Solve: For fractional powers, list domain points with undefined derivative separately before counting zeros of the derivative.

9. Let $y = y(x)$ be the solution of the differential equation $(1+y^2)e^{\tan x}dx + \cos^2 x(1+e^{2\tan x})dy = 0, y(0) = 1$. Then $y\left(\frac{\pi}{4}\right)$ is equal to [4 marks]

(A) $\frac{1}{e^2}$

(B) $\frac{2}{e^2}$

(C) $\frac{2}{e}$

(D) $\frac{1}{e}$

Correct Answer: (D) $\frac{1}{e}$

Solution:

Concept:

- Separate the variables and use $\int dy/(1+y^2) = \tan^{-1} y$. The initial condition determines the integration constant.
- The substitution $t = e^{\tan x}$ gives $dt = e^{\tan x}\sec^2 x\, dx$, which matches the entire remaining numerator.

Step 1: Separate and integrate On $0 \le x \le \pi/4$, $\cos x \neq 0$, so division gives

$$\frac{dy}{1+y^2} = -\frac{e^{\tan x}\sec^2 x}{1+e^{2\tan x}}\,dx.$$

Set $t = e^{\tan x}$; then the right side becomes $-dt/(1+t^2)$. Integrating both sides yields

$$\tan^{-1} y + \tan^{-1}(e^{\tan x}) = C.$$

No factor of two arises: the derivative of the inner exponential is exactly the numerator after division by $\cos^2 x$.

Step 2: Apply the initial condition and evaluate At $x = 0$, both y and $e^{\tan x}$ equal 1. Hence $C = \pi/4 + \pi/4 = \pi/2$, and

$$\tan^{-1} y = \frac{\pi}{2} - \tan^{-1}(e^{\tan x}).$$

The exponential is positive, so the complementary-angle identity has no branch ambiguity and gives $y = e^{-\tan x}$. This expression also directly satisfies the initial value. Finally $\tan(\pi/4) = 1$, so $y(\pi/4) = e^{-1}$. The choices involving e^{-2} use an incorrect exponent, while $2/e$ has an extra multiplier inconsistent with the initial condition.

Step 3: Conclusion $y\left(\frac{\pi}{4}\right) = \boxed{\frac{1}{e}}$. Hence, the correct option is (D).

Quick Tip to Solve: After separation, look for the whole derivative of an inner expression; here it is the derivative of $e^{\tan x}$.

10. Let the sum of two positive integers be 24 . If the probability, that their product is not less than $\frac{3}{4}$ times their greatest possible product, is $\frac{m}{n}$, where $\gcd(m, n) = 1$, then n-m equals [4 marks]

(A) 10

(B) 11

(C) 9

(D) 8

Correct Answer: (A) 10

Solution:

Concept:

- Write the two positive integers as a and $24 - a$, with $a = 1, 2, \ldots, 23$. These give 23 equally likely ordered possibilities.
- For a fixed sum, the product is greatest at equal values. Completing the square also gives the exact acceptable range.

Step 1: Find the threshold product The product is

$$P(a) = a(24 - a) = 144 - (a - 12)^2.$$

Its greatest possible value is therefore 144, attained at the positive integer pair $(12, 12)$. The phrase not less than includes equality, so the required threshold is

$$\frac{3}{4} \times 144 = 108.$$

It is essential to use positive integers: allowing a zero would enlarge the sample space and change the probability.

Step 2: Count the favourable integers The event is equivalent to

$$\begin{aligned} 144 - (a - 12)^2 &\geq 108, \\ (a - 12)^2 &\leq 36, \\ 6 \leq a &\leq 18. \end{aligned}$$

Every integer in this closed interval is allowed. In particular, its endpoints give product exactly 108 and must be counted. There are $18 - 6 + 1 = 13$ favourable choices among the 23 possible values of a. Thus

$$\frac{m}{n} = \frac{13}{23}.$$

Since 23 is prime and does not divide 13, this fraction is already in lowest terms. Consequently $n - m = 23 - 13 = 10$; values 11, 9 and 8 do not result from the inclusive count. Counting the endpoints incorrectly is a tempting source of error.

Step 3: Conclusion The probability is $13/23$, so $\boxed{n - m = 10}$. Hence, the correct option is (A).

Quick Tip to Solve: For integers with fixed sum S, rewrite their product as $S^2/4 - (a - S/2)^2$, then count integer endpoints inclusively.

11. For the function $f(x) = (\cos x) - x + 1, x \in \mathbb{R}$, between the following two statements

(S1) $f(x) = 0$ for only one value of x in $[0, \pi]$.

(S2) $f(x)$ is decreasing in $\left[0, \frac{\pi}{2}\right]$ and increasing in $\left[\frac{\pi}{2}, \pi\right]$. [4 marks]

(A) Both (S1) and (S2) are incorrect.

(B) Only (S1) is correct.

(C) Only (S2) is correct.

(D) Both (S1) and (S2) are correct.

Correct Answer: (B) Only (S1) is correct.

Solution:

Concept:

- A continuous function with opposite signs at two endpoints has a zero between them by the intermediate value theorem.
- A strictly negative derivative on an interval makes the function strictly decreasing there, which also limits it to at most one zero.

Step 1: Determine the derivative on the whole interval Differentiating the complete expression, including the term $-x$, gives

$$f\prime(x) = -\sin x - 1.$$

For $0 \le x \le \pi$, $\sin x \ge 0$, hence

$$f\prime(x) \le -1 < 0.$$

Thus f is strictly decreasing throughout $[0, \pi]$. In particular, it decreases on both halves of the interval; the point $\pi/2$ causes no change of sign. Statement (S2), which asserts increase on the second half, is therefore false.

Step 2: Verify existence and uniqueness for statement S1 The function is continuous, being the sum of a cosine and a polynomial. At the endpoints,

$$f(0) = 1 - 0 + 1 = 2 > 0, \qquad f(\pi) = -1 - \pi + 1 = -\pi < 0.$$

The intermediate value theorem therefore supplies at least one root in $(0, \pi)$. Strict decrease supplies at most one root, since any two distinct arguments in increasing order must have strictly decreasing function values. Together these facts prove exactly one root, so (S1) is true. The options saying that (S1) is false are ruled out by the endpoint and uniqueness checks; the option saying both are true ignores the negative derivative on the second half.

Step 3: Conclusion Statement (S1) is true and statement (S2) is false. Hence, the correct option is (B).

Quick Tip to Solve: For a claim of one zero, establish both a sign change for existence and strict monotonicity for uniqueness.

12. Let $P(x, y, z)$ be a point in the first octant, whose projection in the xy-plane is the point Q. Let $OP = \gamma$; the angle between OQ and the positive x-axis be θ; and the angle between OP and the positive z-axis be ϕ, where O is the origin. Then the distance of P from the x-axis is [4 marks]

(A) $\gamma\sqrt{1 - \sin^2\phi\cos^2\theta}$
(B) $\gamma\sqrt{1 + \cos^2\theta\sin^2\phi}$
(C) $\gamma\sqrt{1 + \cos^2\phi\sin^2\theta}$
(D) $\gamma\sqrt{1 - \sin^2\theta\cos^2\phi}$

Correct Answer: (A) $\gamma\sqrt{1 - \sin^2\phi\cos^2\theta}$

Solution:

Concept:

- The distance of $P(x, y, z)$ from the x-axis is $\sqrt{y^2 + z^2}$, because its perpendicular foot is $(x, 0, 0)$.
- The angle ϕ is measured from the positive z-axis. Thus $z = \gamma\cos\phi$ and the projection length in the xy-plane is $OQ = \gamma\sin\phi$.

Step 1: Resolve the position vector into coordinates The right triangle formed by OP, its projection OQ, and the vertical segment QP gives

$$OQ = \gamma\sin\phi, \qquad z = \gamma\cos\phi.$$

Resolving the projection in the xy-plane using its angle θ with the positive x-axis gives

$$x = \gamma\sin\phi\cos\theta, \qquad y = \gamma\sin\phi\sin\theta.$$

These expressions use the stated angular references; interchanging the roles of θ and ϕ would describe different coordinates.

Step 2: Remove the component along the x-axis Since $x^2 + y^2 + z^2 = \gamma^2$, the required perpendicular distance d satisfies

$$\begin{aligned} d^2 &= y^2 + z^2 = \gamma^2 - x^2 \\ &= \gamma^2\left(1 - \sin^2\phi\cos^2\theta\right). \end{aligned}$$

Taking the nonnegative square root gives the required distance. The plus-sign expressions in (B) and (C) would generically exceed $OP = \gamma$, which no distance to an axis through the origin can do. Option (D) subtracts the wrong coordinate component, arising from exchanging the two different angles.

Step 3: Conclusion The distance is $\boxed{\gamma\sqrt{1 - \sin^2\phi\cos^2\theta}}$. Hence, the correct option is (A).

Quick Tip to Solve: For distance to an axis through the origin, subtract the square of the coordinate along that axis from the squared radius.

13. Let z be a complex number such that $|z+2|=1$ and $\mathrm{Im}\left(\frac{z+1}{z+2}\right)=\frac{1}{5}$. Then the value of $|\mathrm{Re}(z+2)|$ is [4 marks]

(A) $\frac{2\sqrt{6}}{5}$

(B) $\frac{24}{5}$

(C) $\frac{\sqrt{6}}{5}$

(D) $\frac{1+\sqrt{6}}{5}$

Correct Answer: (A) $\frac{2\sqrt{6}}{5}$

Solution:

Concept:

- Write $w=z+2=a+ib$. The condition $|w|=1$ gives $a^2+b^2=1$ and $1/w=\overline{w}=a-ib$.
- The real part is unchanged by conjugation. Thus the requested absolute real part is $|a|$, whether written using w or $\overline{w}$.

Step 1: Rewrite the quotient in terms of a unit complex number Because $z+1=(z+2)-1=w-1$, the quotient becomes

$$\frac{z+1}{z+2}=\frac{w-1}{w}=1-\frac{1}{w}.$$

The denominator is nonzero since $|w|=1$. Using the reciprocal identity on the unit circle,

$$1-\frac{1}{w}=1-(a-ib)=1-a+ib.$$

Its imaginary part is therefore exactly b, not $-b$. The stated imaginary-part condition gives $b=1/5$.

Step 2: Use the modulus to recover the real component Substitute $b=1/5$ in the unit-modulus equation:

$$a^2+\frac{1}{25}=1,$$
$$a^2=\frac{24}{25},$$
$$|a|=\frac{\sqrt{24}}{5}=\frac{2\sqrt{6}}{5}.$$

The sign of a is not determined, but the absolute value requested makes both signs give the same answer. Option (B) omits the square-root operation and is larger than the modulus 1. Option (C) loses the factor 2 in $\sqrt{24}$; option (D) incorrectly adds a real shift to the component of $z+2$. Neither is consistent with $a^2+b^2=1$.

Step 3: Conclusion The requested value is $\boxed{2\sqrt{6}/5}$. Hence, the correct option is (A).

Quick Tip to Solve: Whenever $|w|=1$, replace $1/w$ by $\overline{w}$ before expanding a rational complex expression.

14. The sum of all the solutions of the equation $(8)^{2x} - 16 \cdot (8)^x + 48 = 0$ is : [4 marks]

(A) $1 + \log_8(6)$

(B) $1 + \log_6(8)$

(C) $\log_8(6)$

(D) $\log_8(4)$

Correct Answer: (A) $1 + \log_8(6)$

Solution:

Concept:

- The substitution $t = 8^x > 0$ turns the exponential equation into a quadratic. Each positive root produces one real value of x.
- For positive arguments, $\log_8 u + \log_8 v = \log_8(uv)$. This permits summing the exponents without separately approximating them.

Step 1: Factor the quadratic after substitution Let $t = 8^x$. Then $8^{2x} = (8^x)^2 = t^2$, so the given equation is

$$t^2 - 16t + 48 = 0.$$

Factorising yields

$$(t - 4)(t - 12) = 0.$$

Both roots $t = 4$ and $t = 12$ are positive. Therefore neither is discarded by the condition $8^x > 0$. The exponential function with base 8 is strictly increasing and maps the real line onto the positive numbers, so these roots correspond to exactly two distinct real solutions.

Step 2: Sum the logarithmic solutions Taking logarithms to base 8 gives

$$x_1 = \log_8 4, \qquad x_2 = \log_8 12.$$

Their sum is

$$\begin{aligned} x_1 + x_2 &= \log_8 4 + \log_8 12 \\ &= \log_8 48 \\ &= \log_8(8 \cdot 6) \\ &= 1 + \log_8 6. \end{aligned}$$

Option (B) reverses the logarithm base and argument without taking the required reciprocal. Option (C) omits the factor 8 in 48 and hence the added 1. Option (D) is only the first solution, rather than the sum of both. As a check, the answer is greater than 1, consistent with the two positive exponents obtained.

Step 3: Conclusion The sum of all solutions is $\boxed{1 + \log_8 6}$. Hence, the correct option is (A).

Quick Tip to Solve: After substituting $t = a^x$, the sum of the real exponents is the logarithm of the product of all positive quadratic roots.

15. The equations of two sides AB and AC of a triangle ABC are $4x + y = 14$ and $3x - 2y = 5$, respectively. The point $\left(2, -\frac{4}{3}\right)$ divides the third side BC internally in the ratio $2:1$, the equation of the side BC is [4 marks]

(A) $x + 6y + 6 = 0$

(B) $x - 3y - 6 = 0$

(C) $x + 3y + 2 = 0$

(D) $x - 6y - 10 = 0$

Correct Answer: (C) $x + 3y + 2 = 0$

Solution:

Concept:

- For a point P dividing BC internally in the ratio 2:1, the section formula gives $P = (B + 2C)/3$. The coefficient of C is the larger one because P is nearer C.
- A vertex on a given side satisfies its line equation. Determine B and C using their respective side equations and the two coordinate equations for P, then find the line through them.

Step 1: Express the vertices using the two known sides Write $B = (u, 14 - 4u)$ from $4x + y = 14$, and $C = (v, (3v - 5)/2)$ from $3x - 2y = 5$. These points lie on AB and AC respectively; their unknown coordinates are determined by the division point, rather than by assuming the direction of BC.

Step 2: Apply the internal section formula Since $BP : PC = 2 : 1$, the point P has coordinates $(B + 2C)/3$. Equating its coordinates to $(2, -4/3)$ gives

$$u + 2v = 6, \qquad 14 - 4u + 3v - 5 = -4.$$

Thus $-4u + 3v = -13$. Substitute $u = 6 - 2v$: $-24 + 11v = -13$, so $v = 1$, $u = 4$. Consequently $B = (4, -2)$ and $C = (1, -1)$.

Step 3: Find and check the third side The slope of BC is

$$\frac{-1 - (-2)}{1 - 4} = -\frac{1}{3}.$$

Hence $y + 2 = -(x - 4)/3$, or $x + 3y + 2 = 0$. It contains B, C and the specified division point. Options A, B and D do not contain both recovered vertices; merely testing the division point would not uniquely identify the side.

Step 4: Conclusion The third side is $\boxed{x + 3y + 2 = 0}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the ratio with opposite vertex weights: BP:PC = 2:1 means P = (B+2C)/3. Testing P alone can leave several options.

16. If $\sin x = -\frac{3}{5}$, where $\pi < x < \frac{3\pi}{2}$, then $80\left(\tan^2 x - \cos x\right)$ is equal to [4 marks]

(A) 109

(B) 108

(C) 19

(D) 18

Correct Answer: (A) 109

Solution:

Concept:

- The identity $\sin^2 x + \cos^2 x = 1$ determines the magnitude of cosine. The quadrant fixes its sign, so it must be used before evaluating an expression containing an unsquared cosine.
- For $\pi < x < 3\pi/2$, x is in quadrant III: sine and cosine are negative, while $\tan x = \sin x / \cos x$ is positive.

Step 1: Find cosine with the correct sign Given $\sin x = -3/5$, the Pythagorean identity gives

$$\cos^2 x = 1 - \frac{9}{25} = \frac{16}{25}.$$

As x lies strictly in the third quadrant, choose $\cos x = -4/5$, rather than the positive square root. This sign is essential because the requested expression subtracts cosine itself.

Step 2: Calculate the squared tangent Using the ratio definition,

$$\tan x = \frac{-3/5}{-4/5} = \frac{3}{4}, \qquad \tan^2 x = \frac{9}{16}.$$

Squaring removes the tangent sign, but it does not remove the sign of the separate cosine term. Keep the rational values until the final arithmetic.

Step 3: Substitute and compare the options Now

$$80(\tan^2 x - \cos x) = 80\left(\frac{9}{16} + \frac{4}{5}\right) = 80\left(\frac{45+64}{80}\right) = 109.$$

The contributions are 45 and 64, so their sum is exactly 109, excluding 108, 19 and 18. In particular, taking a positive cosine would subtract 64 and contradict the quadrant condition.

Step 4: Conclusion The required value is $\boxed{109}$. Hence, the correct option is (A).

Quick Tip to Solve: Fix the sign of cosine from the quadrant before substituting; the square on tangent does not apply to the whole bracket.

17. Let $[t]$ be the greatest integer less than or equal to t. Let A be the set of all prime factors of 2310 and $f : A \to \mathbb{Z}$ be the function $f(x) = \left[\log_2\left(x^2 + \left[\frac{x^3}{5}\right]\right)\right]$. The number of one-to-one functions from A to the range of f is [4 marks]

(A) 20

(B) 120

(C) 25

(D) 24

Correct Answer: (B) 120

Solution:

Concept:

- The prime-factor set contains distinct prime divisors, not their powers or repeated factors. Compute the range of f by evaluating its nested floor operations in the stated order.
- An injective function from a set of n elements to a set of n elements is a bijection, so there are $n!$ such functions. The codomain here is the range of f, not all integers.

Step 1: List the domain and evaluate the inner expression Factor $2310 = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11$, so $A = \{2, 3, 5, 7, 11\}$. For these inputs, the values of $\lfloor x^3/5 \rfloor$ are respectively $1, 5, 25, 68, 266$. Thus $x^2 + \lfloor x^3/5 \rfloor$ gives $5, 14, 50, 117, 387$. Both floor symbols matter; one cannot replace the inner integer part by an approximation.

Step 2: Find the distinct values of f Compare each positive number with consecutive powers of 2:

$$2^2 \le 5 < 2^3, \qquad 2^3 \le 14 < 2^4,$$
$$2^5 \le 50 < 2^6, \qquad 2^6 \le 117 < 2^7,$$
$$2^8 \le 387 < 2^9.$$

Hence $f(2) = 2$, $f(3) = 3$, $f(5) = 5$, $f(7) = 6$, $f(11) = 8$. The range is $\{2, 3, 5, 6, 8\}$, containing five distinct integers.

Step 3: Count all injective assignments For a one-to-one function from A to this range, the first input has 5 choices of image, the next 4, then 3, 2 and 1. Thus the number is

$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! = 120.$$

This counts all such functions, not only the displayed f. The values 20, 25 and 24 do not count all assignments of five distinct images.

Step 4: Conclusion There are $\boxed{120}$ one-to-one functions. Hence, the correct option is (B).

Quick Tip to Solve: Find the size of the actual range before applying a permutation formula; different domain elements can sometimes have equal floor-function images.

18. Let $I(x) = \int \frac{6}{\sin^2 x(1-\cot x)^2}dx$. If $I(0) = 3$, then $I\left(\frac{\pi}{12}\right)$ is equal to [4 marks]

(A) $\sqrt{3}$

(B) $2\sqrt{3}$

(C) $6\sqrt{3}$

(D) $3\sqrt{3}$

Correct Answer: (D) $3\sqrt{3}$

Solution:

Concept:

- First simplify $\sin^2 x(1-\cot x)^2 = (\sin x - \cos x)^2$ wherever the original expression is defined. This reveals a simple shifted trigonometric square.
- Use $\sin x - \cos x = -\sqrt{2}\cos(x+\pi/4)$, and $\int \sec^2 u\, du = \tan u + C$. The condition at zero fixes the continuous extension of the antiderivative.

Step 1: Simplify the integrand The denominator becomes $(\sin x - \cos x)^2 = 2\cos^2(x+\pi/4)$. Therefore

$$I(x) = \int 3\sec^2(x+\pi/4)\,dx = 3\tan(x+\pi/4) + C.$$

Differentiating this result gives $3\sec^2(x+\pi/4) = 6/(\sin x - \cos x)^2$, which verifies the antiderivative against the simplified integrand.

Step 2: Apply the value at zero The original cotangent form is undefined exactly at zero, but its simplified integrand and the displayed primitive have a continuous extension there. The given condition is applied to this extension:

$$3 = I(0) = 3\tan(\pi/4) + C = 3 + C.$$

Hence $C = 0$. The interval from 0 to $\pi/12$ contains no zero of $\cos(x+\pi/4)$, so this same primitive applies throughout.

Step 3: Evaluate the requested argument At $x = \pi/12$, the shifted angle is $\pi/12 + \pi/4 = \pi/3$. Consequently

$$I(\pi/12) = 3\tan(\pi/3) = 3\sqrt{3}.$$

Retaining the factor 3 from the integrand is necessary. The other offered multiples $\sqrt{3}$, $2\sqrt{3}$, and $6\sqrt{3}$ do not follow from the verified primitive and initial condition.

Step 4: Conclusion Thus $\boxed{I(\pi/12) = 3\sqrt{3}}$. Hence, the correct option is (D).

Quick Tip to Solve: Combine the cotangent factor with sin squared before substituting; the denominator becomes a shifted cosine square and also clarifies the value at zero.

19. If the shortest distance between the lines

$$L_1 : \vec{r} = (2+\lambda)\hat{i} + (1-3\lambda)\hat{j} + (3+4\lambda)\hat{k}, \quad \lambda \in \mathbb{R}$$
$$L_2 : \vec{r} = 2(1+\mu)\hat{i} + 3(1+\mu)\hat{j} + (5+\mu)\hat{k}, \quad \mu \in \mathbb{R}$$

is $\frac{m}{\sqrt{n}}$, where $\gcd(m,n)=1$, then the value of $m+n$ equals [4 marks]

(A) 384

(B) 387

(C) 390

(D) 377

Correct Answer: (B) 387

Solution:

Concept:

- For lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$, their shortest distance is $d = |(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)|/|\vec{b}_1 \times \vec{b}_2|$.
- The cross product provides a common normal. Projecting the displacement between arbitrary points on the two lines onto that normal removes the components along either line.

Step 1: Read a point and direction from each line The first line has $\vec{a}_1 = (2,1,3)$ and $\vec{b}_1 = (1,-3,4)$. Expanding the second line gives $\vec{a}_2 = (2,3,5)$ and $\vec{b}_2 = (2,3,1)$. In particular, the first two components of its direction are 2 and 3, not 1 and 1. Thus $\vec{a}_2 - \vec{a}_1 = (0,2,2)$.

Step 2: Calculate the common normal and its magnitude The cross product is

$$\vec{b}_1 \times \vec{b}_2 = (-3-12,\ 8-1,\ 3+6) = (-15,7,9).$$

It is nonzero, so the lines are not parallel. Its squared magnitude is

$$(-15)^2 + 7^2 + 9^2 = 225 + 49 + 81 = 355.$$

The scalar triple product is $(0,2,2)\cdot(-15,7,9) = 14+18 = 32$.

Step 3: Form the distance and identify the integers The shortest distance is therefore

$$d = \frac{|32|}{\sqrt{355}}.$$

Since $355 = 5\cdot 71$ and 32 is a power of 2, $\gcd(32,355) = 1$. The requested representation has $m = 32$, $n = 355$, giving $m+n = 387$. The alternatives 384, 390 and 377 disagree with this exact scalar-triple-product calculation.

Step 4: Conclusion The required sum is $\boxed{387}$. Hence, the correct option is (B).

Quick Tip to Solve: Expand a parametric line first, then separate its constant point and direction vector; misplaced coefficients change both the numerator and denominator.

20. If the set $R = \{(a, b) : a + 5b = 42, a, b \in \mathbb{N}\}$ has m elements and $\sum_{n=1}^{m} \left(1 - i^{n!}\right) = x + iy$, where $i = \sqrt{-1}$, then the value of $m + x + y$ is [4 marks]

(A) 12

(B) 4

(C) 8

(D) 5

Correct Answer: (A) 12

Solution:

Concept:

- Count positive integer solutions of $a + 5b = 42$ by bounding b; each allowable b determines exactly one a. Here $\mathbb{N}$ denotes the positive integers.
- The powers of i have period 4. For $n \geq 4$, the factorial $n!$ is divisible by 4, so $i^{n!} = 1$ and the corresponding summand vanishes.

Step 1: Count the ordered pairs Because $a = 42 - 5b \geq 1$ and $b \geq 1$, we have $1 \leq b \leq 8$. Each integer b in this range produces a positive integer a: $37, 32, 27, 22, 17, 12, 7, 2$. Thus there are exactly $m = 8$ elements of R. The case $b = 0$ is excluded because b is natural and positive.

Step 2: Evaluate the exceptional factorial powers The first three factorials are $1!, 2!, 3! = 1, 2, 6$, respectively. Hence

$$i^{1!} = i, \qquad i^{2!} = -1, \qquad i^{3!} = i^6 = -1.$$

Their summands $1 - i^{n!}$ are $1 - i, 2, 2$. For every n from 4 through 8, $4 \mid n!$, and periodicity gives $i^{n!} = 1$, contributing zero.

Step 3: Extract the real and imaginary parts The full sum therefore reduces to

$$\sum_{n=1}^{8}(1 - i^{n!}) = (1 - i) + 2 + 2 = 5 - i.$$

Comparing with $x + iy$, we obtain $x = 5$ and $y = -1$, not $+1$. Consequently $m + x + y = 8 + 5 - 1 = 12$. The other options 4, 8 and 5 reflect incomplete counting or using a component in place of the required sum.

Step 4: Conclusion Thus $\boxed{m + x + y = 12}$. Hence, the correct option is (A).

Quick Tip to Solve: For factorial powers of i, evaluate n = 1, 2, 3 separately; every later factorial is divisible by four and gives a zero summand.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. If the orthocentre of the triangle formed by the lines $2x+3y-1=0, x+2y-1=0$ and $ax+by-1=0$, is the centroid of another triangle, whose circumcentre and orthocentre respectively are $(3,4)$ and $(-6,-8)$, then the value of $|a-b|$ is _____. [4 marks]

Solution:

Concept:

- The centroid G, circumcentre O and orthocentre H satisfy the Euler-line vector identity $3G = 2O+H$. It determines the orthocentre of the first triangle from the centres of the second.
- An altitude passes through a vertex and is perpendicular to the opposite side. If the orthocentre is the origin, each vertex position vector is perpendicular to its opposite side.

Step 1: Find the orthocentre of the first triangle For the second triangle, $O = (3,4)$ and $H = (-6,-8)$. Thus

$$G = \frac{2(3,4)+(-6,-8)}{3} = (0,0).$$

The problem states that this centroid is the first triangle's orthocentre, so that orthocentre is the origin. Let its two given sides be AB and AC, with A at their intersection.

Step 2: Locate the common vertex and third-side direction Solve $2x+3y=1$ and $x+2y=1$: subtracting twice the second equation from the first gives $y=1$, then $x=-1$. Hence $A=(-1,1)$. The altitude AO has slope -1, so the opposite side BC has slope +1. Since BC is $ax+by=1$, its slope is $-a/b=1$, giving $a=-b$.

Step 3: Use a second altitude to determine the scale The side AC, $x+2y=1$, has slope $-1/2$. The altitude from B therefore has slope 2 and passes through the origin: $y=2x$. Intersect it with AB to get $2x+3(2x)=1$, so $B=(1/8,1/4)$. Substitution into BC gives

$$\frac{a}{8}+\frac{b}{4}=1.$$

With $b=-a$, this becomes $-a/8=1$. Therefore $a=-8$, $b=8$. As a check, C is $(1/4,3/8)$; its altitude to AB has slope $3/2$, consistent with the origin.

Step 4: Conclusion It follows that $\boxed{|a-b| = |-8-8| = 16}$.

Quick Tip to Solve: Use Euler's centre relation first. Once the orthocentre is the origin, the slope of each altitude can be read directly from its vertex.

22. If the range of $f(\theta) = \frac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta}, \theta \in \mathbb{R}$ is $[\alpha, \beta]$, then the sum of the infinite G.P., whose first term is 64 and the common ratio is $\frac{\alpha}{\beta}$, is equal to _____. [4 marks]

Solution:

Concept:

- A trigonometric expression involving even powers can be reduced to one variable: put $t = \cos^2\theta$, so $0 \le t \le 1$ and $\sin^4\theta = (1-t)^2$. Every value of this interval is attainable.
- An infinite geometric progression with first term a and common ratio r has sum $S = a/(1-r)$, provided $|r| < 1$. Its ratio here is determined by the minimum and maximum of the function.

Step 1: Reduce the range problem to a rational function Substitute the identities before trying to bound the numerator and denominator separately:

$$f(t) = \frac{1+t+t^2}{1-t+t^2} = 1 + \frac{2t}{1-t+t^2}.$$

The denominator is $(t-\frac{1}{2})^2 + \frac{3}{4} > 0$, so the function is continuous throughout the allowed interval and no value is excluded by a zero denominator.

Step 2: Locate the endpoints of the range Differentiate with respect to the new variable:

$$f'(t) = \frac{2(1-t^2)}{(1-t+t^2)^2} \ge 0 \qquad (0 \le t \le 1).$$

Thus the minimum occurs at $t = 0$, giving $f = 1$, and the maximum at $t = 1$, giving $f = 3$. Continuity ensures all intermediate values occur. Hence $\alpha = 1$ and $\beta = 3$.

Step 3: Sum the specified geometric progression The common ratio is $r = \alpha/\beta = 1/3$; its magnitude is less than one, so the infinite sum exists. Using the first term 64,

$$S = \frac{64}{1 - 1/3} = 64 \cdot \frac{3}{2} = 96.$$

The first term is used directly; multiplying it by the ratio first would incorrectly omit the initial term.

Step 4: Conclusion The range is $[1, 3]$, and the required infinite G.P. sum is $\boxed{96}$.

Quick Tip to Solve: Put $t = \cos^2\theta$ first and check monotonicity on $[0, 1]$; then use the range endpoints, not arbitrary function values, for the GP ratio.

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23. Let $\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}, \vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}$ and $\vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = (\vec{b} + \vec{c}) \times \vec{a}$ and $\vec{r} \cdot (\vec{b} - \vec{c}) = 0$, then $\frac{|593\vec{r} + 67\vec{a}|^2}{(593)^2}$ is equal to _____. [4 marks]

Solution:

Concept:

- If $\mathbf{u} \times \mathbf{a} = \mathbf{0}$ with $\mathbf{a} \neq \mathbf{0}$, then $\mathbf{u}$ is parallel to $\mathbf{a}$. Consequently the cross-product equation gives a one-parameter family of possible vectors.

- The additional dot-product equation determines that parameter. For $\mathbf{v} = x\hat{i} + y\hat{j} + z\hat{k}$, its squared magnitude is $|\mathbf{v}|^2 = x^2 + y^2 + z^2$.

Step 1: Use the cross product to express the unknown vector Move both cross products to one side and use linearity:

$$[\vec{r} - (\vec{b} + \vec{c})] \times \vec{a} = \vec{0}.$$

Since $\vec{a}$ is nonzero, there is a real scalar λ such that

$$\vec{r} = \vec{b} + \vec{c} + \lambda\vec{a}.$$

This keeps the component parallel to $\vec{a}$, which a cross product cannot determine.

Step 2: Determine the parallel component with the dot product Here $\vec{b} + \vec{c} = (20, 5, -12)$ and $\vec{b} - \vec{c} = (-14, 9, -14)$. Their dot product is

$$20(-14) + 5(9) + (-12)(-14) = -67.$$

Also

$$\vec{a} \cdot (\vec{b} - \vec{c}) = 9(-14) - 13(9) + 25(-14) = -593.$$

The condition $\vec{r} \cdot (\vec{b} - \vec{c}) = 0$ therefore gives $-67 - 593\lambda = 0$, so $\lambda = -67/593$.

Step 3: Simplify the required vector before taking its norm The chosen coefficients now cancel the undetermined-direction correction exactly:

$$593\vec{r} + 67\vec{a} = 593(\vec{b} + \vec{c}).$$

It follows that

$$\frac{|593\vec{r} + 67\vec{a}|^2}{593^2} = |\vec{b} + \vec{c}|^2 = 20^2 + 5^2 + (-12)^2 = 569.$$

There is no need to calculate three fractional components of $\vec{r}$ separately.

Step 4: Conclusion The required squared-magnitude quotient is $\boxed{569}$.

Quick Tip to Solve: For equal cross products with the same nonzero vector, add an unknown parallel component and use the dot-product condition to find it.

24. Let the positive integers be written in the form:

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 2 & & 3 & & \\ & 4 & & 5 & & 6 & \\ 7 & & 8 & & 9 & & 10 \\ \vdots & & & & & & \ddots \end{array}$$

If the k^{th} row contains exactly k numbers for every natural number k, then the row in which the number 5310 will be, is _____. [4 marks]

Solution:

Concept:

- The positive integers are arranged consecutively, and row k contains k entries. Hence the number of entries through row k is the triangular number $T_k = 1 + 2 + \cdots + k = k(k+1)/2$.
- A positive integer N belongs to row k precisely when $T_{k-1} < N \leq T_k$. The strict inequality at the lower end ensures that the last entry of the preceding row is not counted again.

Step 1: Translate the displayed arrangement into row endpoints The arrangement starts with row 1 containing 1, row 2 containing 2 and 3, and row 3 containing 4, 5 and 6. Thus row k begins at $T_{k-1} + 1$ and ends at T_k. For the given integer we need

$$\frac{k(k-1)}{2} < 5310 \leq \frac{k(k+1)}{2}.$$

The number sought is a row index, rather than the position within that row.

Step 2: Bracket the integer using adjacent triangular numbers The equation $k(k+1)/2 = 5310$ suggests k close to $\sqrt{10620}$, approximately 103. Check the neighbouring complete rows exactly:

$$T_{102} = \frac{102 \cdot 103}{2} = 5253, \qquad T_{103} = \frac{103 \cdot 104}{2} = 5356.$$

Since $5253 < 5310 \leq 5356$, row 102 has finished before 5310 is reached, while row 103 has enough places to include it.

Step 3: Check the position inside the selected row Row 103 therefore contains the consecutive numbers from 5254 to 5356, inclusive. The position of 5310 in that row is

$$5310 - 5253 = 57.$$

This lies between 1 and 103, confirming the row assignment. No subsequent row can contain 5310 because the arrangement uses every positive integer exactly once in increasing order.

Step 4: Conclusion The number 5310 occurs in the $\boxed{103}$rd row.

Quick Tip to Solve: Find the first triangular number at least as large as the target, and check the preceding triangular number to avoid an off-by-one row error.

25. The number of 3-digit numbers, formed using the digits 2, 3, 4, 5 and 7, when the repetition of digits is not allowed, and which are not divisible by 3 , is equal to _____. [4 marks]

Solution:

Concept:

- A number is divisible by 3 exactly when its digit sum is divisible by 3. The order of a fixed set of three different digits does not affect this divisibility test.
- With no repeated digits and no zero among the available digits, each selection of three digits produces $3! = 6$ valid three-digit numbers. Count all possibilities and subtract those divisible by 3.

Step 1: Count all three-digit numbers without repetition There are five choices for the hundreds digit, four for the tens digit after the first is used, and three for the units digit. Hence the total is

$$^5P_3 = 5 \cdot 4 \cdot 3 = 60.$$

All these arrangements are genuine three-digit numbers, because none of the supplied digits is zero. The restriction on divisibility has not yet been imposed.

Step 2: Classify allowable digit triples by residues Group the digits according to their remainders modulo 3:

$$R_0 = \{3\}, \qquad R_1 = \{4, 7\}, \qquad R_2 = \{2, 5\}.$$

A sum of three residues is zero modulo 3 if the residues are all equal, or if they are $0, 1, 2$. An all-equal selection would require three digits from one group, and every group here has fewer than three. Thus only one digit from each group is possible.

Step 3: Subtract the divisible arrangements There are $1 \cdot 2 \cdot 2 = 4$ qualifying unordered triples, namely $\{3, 4, 2\}$, $\{3, 4, 5\}$, $\{3, 7, 2\}$ and $\{3, 7, 5\}$. Each has six distinct permutations, so

$$N_{\text{divisible}} = 4 \cdot 6 = 24.$$

The required complement is

$$N_{\text{not divisible}} = 60 - 24 = 36.$$

The factor of six is essential because the question counts numbers, rather than digit selections.

Step 4: Conclusion The number of permitted three-digit numbers not divisible by 3 is $\boxed{36}$.

Quick Tip to Solve: Use residues to count unordered digit triples first; multiply by $3!$ only after applying the divisibility test, then take the complement.

26. Three balls are drawn at random from a bag containing 5 blue and 4 yellow balls. Let the random variables X and Y respectively denote the number of blue and yellow balls. If $\bar{X}$ and $\bar{Y}$ are the means of X and Y respectively, then $7\bar{X} + 4\bar{Y}$ is equal to _____. [4 marks]

Solution:

Concept:

- The expected number of successes in n draws without replacement from N objects, of which K are successes, is nK/N. Independence between the draws is not required for this mean.
- Linearity of expectation says that the mean of a sum is the sum of its means. Here every draw produces one blue or one yellow ball, so $X + Y = 3$ for every sample and consequently $\bar{X} + \bar{Y} = 3$.

Step 1: Find the expected count of blue balls For each of the three draw positions, let its blue indicator be 1 for a blue ball and 0 otherwise. Each position has marginal blue probability $5/9$, because the sample is taken uniformly from the same nine balls. Adding the three indicators gives X, hence

$$\bar{X} = E(X) = 3 \cdot \frac{5}{9} = \frac{5}{3}.$$

The draws are dependent, but expectation may still be added across positions.

Step 2: Find the expected count of yellow balls The same reasoning gives yellow probability $4/9$ at each position, and therefore

$$\bar{Y} = E(Y) = 3 \cdot \frac{4}{9} = \frac{4}{3}.$$

As an immediate consistency check,

$$\bar{X} + \bar{Y} = \frac{5}{3} + \frac{4}{3} = 3,$$

which agrees with the fixed total number of balls drawn. The expectations may be fractional even though observed ball counts are integers.

Step 3: Evaluate the weighted combination of the means Insert the two means into the expression in the question:

$$7\bar{X} + 4\bar{Y} = 7 \cdot \frac{5}{3} + 4 \cdot \frac{4}{3} = \frac{35 + 16}{3} = 17.$$

Equivalently, use $\bar{Y} = 3 - \bar{X}$ to obtain $12 + 3\bar{X} = 12 + 5$, confirming the same result without constructing the full hypergeometric probability distribution.

Step 4: Conclusion The required weighted sum of the means is $\boxed{17}$.

Quick Tip to Solve: For expected colour counts, multiply sample size by the population proportion; use $X + Y = 3$ as a check instead of listing every possible sample.

27. Let $\alpha = \sum_{r=0}^{n}(4r^2+2r+1)\binom{n}{r}$ and $\beta = \left(\sum_{r=0}^{n}\frac{\binom{n}{r}}{r+1}\right) + \frac{1}{n+1}$. If $140 < \frac{2\alpha}{\beta} < 281$, then the value of n is _____. [4 marks]

Solution:

Concept:

- The binomial identities $\sum_{r=0}^{n}\binom{n}{r} = 2^n$, $\sum r\binom{n}{r} = n2^{n-1}$, and $\sum r(r-1)\binom{n}{r} = n(n-1)2^{n-2}$ evaluate polynomially weighted coefficient sums.
- The identity $\binom{n}{r}/(r+1) = \binom{n+1}{r+1}/(n+1)$ converts the denominator-weighted sum into an ordinary binomial sum. Strict inequalities must be preserved when selecting an integer value of n.

Step 1: Evaluate the polynomially weighted sum Write $r^2 = r(r-1)+r$. Applying the binomial identities to the definition of α gives

$$\begin{aligned}\alpha &= 4n(n-1)2^{n-2} + 6n2^{n-1} + 2^n \\ &= 2^n\{n(n-1)+3n+1\} \\ &= 2^n(n+1)^2.\end{aligned}$$

These identities follow by differentiating $(1+x)^n$ once and twice and then setting $x = 1$.

Step 2: Evaluate the denominator-weighted sum Shift the binomial index by one:

$$\begin{aligned}\sum_{r=0}^{n}\frac{\binom{n}{r}}{r+1} &= \frac{1}{n+1}\sum_{s=1}^{n+1}\binom{n+1}{s} \\ &= \frac{2^{n+1}-1}{n+1}.\end{aligned}$$

The additional $1/(n+1)$ in the printed definition cancels the subtraction of 1. Thus $\beta = 2^{n+1}/(n+1)$.

Step 3: Solve the resulting cube inequality The quotient simplifies before any numerical trial is needed:

$$\frac{2\alpha}{\beta} = (n+1)^3.$$

The condition is therefore $140 < (n+1)^3 < 281$. The adjacent integer cubes are $5^3 = 125$, $6^3 = 216$, and $7^3 = 343$. Only 216 lies strictly between the bounds, so $n+1 = 6$ and $n = 5$.

Step 4: Conclusion The only admissible integer value is $\boxed{n = 5}$.

Quick Tip to Solve: In weighted binomial sums, replace r^2 by $r(r-1)+r$; for $1/(r+1)$, raise the binomial upper index and shift the lower index.

28. Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$. If the sum of the diagonal elements of A^{13} is 3^n, then n is equal to _____.

[4 marks]

Solution:

Concept:

- The sum of a square matrix's diagonal entries is its trace. Trace is linear under scalar multiplication: $\text{tr}(cA) = c\,\text{tr}(A)$.
- A matrix satisfying a low-degree polynomial can have large powers reduced efficiently. Here direct multiplication gives $A^2 - 3A + 3I = 0$, where I is the 2×2 identity matrix.

Step 1: Find a useful relation for small powers For the supplied matrix,

$$A^2 = \begin{pmatrix} 3 & -3 \\ 3 & 0 \end{pmatrix} = 3A - 3I.$$

Multiplying this equality by A and substituting for A^2 once more gives

$$A^3 = 3A^2 - 3A = 6A - 9I = \begin{pmatrix} 3 & -6 \\ 6 & -3 \end{pmatrix}.$$

Reducing powers through this relation is much shorter than multiplying out thirteen copies individually.

Step 2: Reduce the sixth and twelfth powers to scalar matrices Square the last matrix:

$$A^6 = \begin{pmatrix} 3 & -6 \\ 6 & -3 \end{pmatrix}^2 = \begin{pmatrix} -27 & 0 \\ 0 & -27 \end{pmatrix} = -27I.$$

The diagonal products are $9 - 36 = -27$ and the off-diagonal products cancel. Squaring again gives

$$A^{12} = (-27I)^2 = 729I = 3^6 I.$$

The negative sign disappears at the twelfth power, a necessary sign check before taking the trace.

Step 3: Take the trace of the thirteenth power Multiply the scalar matrix by A:

$$A^{13} = A^{12}A = 729A.$$

Since the diagonal entries of A are 2 and 1, its trace is 3. Therefore

$$\text{tr}(A^{13}) = 729(2 + 1) = 2187 = 3^7.$$

Comparing this with the specified form 3^n yields $n = 7$. Raising the trace of A itself to the thirteenth power would not be a valid matrix identity.

Step 4: Conclusion The sum of the diagonal elements of A^{13} is 3^7, so $\boxed{n = 7}$.

Quick Tip to Solve: Look for a small power that becomes a scalar multiple of I; here $A^6 = -27I$ makes $A^{13} = 729A$ immediate.

29. Let the area of the region enclosed by the curve $y = \min\{\sin x, \cos x\}$ and the x axis between $x = -\pi$ to $x = \pi$ be A. Then A^2 is equal to _____. [4 marks]

Solution:

Concept:

- Geometrical area between a graph and the x-axis is $A = \int |y|\, dx$; portions below the axis contribute positively.
- The lower of $\sin x$ and $\cos x$ changes where $\sin x = \cos x$. Their signs must then be checked before integrating.

Step 1: Locate the changes of the lower curve Within $[-\pi, \pi]$, equality occurs at $x = -3\pi/4$ and $x = \pi/4$. Testing one point in each interval shows

$$y = \begin{cases} \cos x, & -\pi \le x \le -3\pi/4, \\ \sin x, & -3\pi/4 \le x \le \pi/4, \\ \cos x, & \pi/4 \le x \le \pi. \end{cases}$$

For example, at $-\pi$, cosine is the smaller value, whereas at $-\pi/2$, sine is smaller. Thus the minimum is not one fixed trigonometric function throughout.

Step 2: Split at the zeros and calculate positive areas The sine piece crosses the axis at zero, and the final cosine piece crosses it at $\pi/2$. Therefore

$$\begin{aligned} A = & -\int_{-\pi}^{-3\pi/4} \cos x\, dx - \int_{-3\pi/4}^{0} \sin x\, dx \\ & + \int_{0}^{\pi/4} \sin x\, dx + \int_{\pi/4}^{\pi/2} \cos x\, dx - \int_{\pi/2}^{\pi} \cos x\, dx \\ = & \frac{\sqrt{2}}{2} + \left(1 + \frac{\sqrt{2}}{2}\right) + \left(1 - \frac{\sqrt{2}}{2}\right) + \left(1 - \frac{\sqrt{2}}{2}\right) + 1 = 4. \end{aligned}$$

The negative signs on the first, second and fifth integrals convert their negative signed integrals to positive geometrical areas. The radical terms cancel exactly, so no decimal approximation is required.

Step 3: Conclusion The enclosed area is $A = 4$ square units. Hence the required numerical value is $\boxed{A^2 = 16}$.

Quick Tip to Solve: First find the intersections of the two curves, then split the chosen lower curve at its zeros; integrating the minimum directly gives signed area.

30. The value of $\lim\limits_{x\to 0} 2\left(\dfrac{1-\cos x\sqrt{\cos 2x}\sqrt[3]{\cos 3x}\cdots\sqrt[10]{\cos 10x}}{x^2}\right)$ **is _____.** [4 marks]

Solution:

Concept:

- For small real u, $\log(\cos u) = -u^2/2 + O(u^4)$. Every cosine is positive sufficiently close to zero, so logarithms and the indicated roots are defined.
- If $z \to 0$, then $e^z = 1 + z + O(z^2)$. A finite product is conveniently expanded by adding its logarithms.

Step 1: Expand the logarithm of the product Put $P(x) = \prod_{k=1}^{10}(\cos kx)^{1/k}$. This includes the ordinary cosine factor for $k = 1$, the square root for $k = 2$, and the tenth root for $k = 10$, exactly as in the question. For $|x| < \pi/20$, every factor is positive. Then

$$\begin{aligned}\log P(x) &= \sum_{k=1}^{10}\frac{1}{k}\log(\cos kx)\\ &= \sum_{k=1}^{10}\frac{1}{k}\left(-\frac{k^2x^2}{2}+O(x^4)\right)\\ &= -\frac{x^2}{2}\sum_{k=1}^{10}k + O(x^4) = -\frac{55}{2}x^2 + O(x^4).\end{aligned}$$

Because the number of factors is finite, their fourth-order remainder terms remain fourth order after summation.

Step 2: Recover the product and take the limit Exponentiating the preceding expression gives

$$P(x) = \exp\left(-\frac{55}{2}x^2 + O(x^4)\right) = 1 - \frac{55}{2}x^2 + O(x^4).$$

Consequently

$$2\frac{1-P(x)}{x^2} = 2\frac{(55/2)x^2 + O(x^4)}{x^2} = 55 + O(x^2) \longrightarrow 55.$$

Each root changes the contribution of its cosine from k^2 to k; ignoring the root exponents would incorrectly produce a sum of squares. The outer factor two also cancels the factor one-half in the cosine expansion.

Step 3: Conclusion The required limit is $\boxed{55}$.

Quick Tip to Solve: For a finite product $\prod(\cos kx)^{1/k}$, add the quadratic logarithm coefficients: the required limit here is $1 + 2 + \cdots + 10$.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A stationary particle breaks into two parts of masses m_A and m_B which move with velocities v_A and v_B respectively. The ratio of their kinetic energies $(K_B : K_A)$ is : [4 marks]

(A) $v_B : v_A$

(B) $1 : 1$

(C) $m_B v_B : m_A v_A$

(D) $m_B : m_A$

Correct Answer: (A) $v_B : v_A$

Solution:

Concept:

- In the absence of external impulse, total linear momentum is conserved during an explosion. Initially the particle is stationary, so its momentum is zero.
- For a fragment of mass m, speed v and momentum magnitude $p = mv$, kinetic energy is $K = mv^2/2 = pv/2$.

Step 1: Relate the fragment momenta The vector conservation equation is

$$m_A\vec{v}_A + m_B\vec{v}_B = 0.$$

Thus the fragments travel in opposite directions with equal momentum magnitudes:

$$m_A v_A = m_B v_B = p.$$

The velocities in the requested energy ratio are used through their speed magnitudes; kinetic energy cannot be negative. Equal and opposite momenta do not mean equal speeds unless the fragment masses happen to be equal.

Step 2: Calculate and compare the kinetic-energy ratios Using the same momentum magnitude in each kinetic energy,

$$K_B = \frac{1}{2}pv_B, \qquad K_A = \frac{1}{2}pv_A,$$

so

$$\frac{K_B}{K_A} = \frac{v_B}{v_A} = \frac{m_A}{m_B}.$$

Option (A) is therefore correct. Option (B) would hold only for equal masses. Option (C) is the momentum ratio and is always one, so it cannot generally represent the kinetic-energy ratio. Option (D) gives the direct mass ratio, whereas at fixed momentum energy is inversely proportional to mass. As a check, if fragment B has half the mass of A, it has twice the speed and twice the kinetic energy, exactly as option (A) predicts. The energy released by the breakup supplies the final kinetic energies; kinetic energy itself is not required to remain equal to its initial zero value.

Step 3: Conclusion $\boxed{K_B : K_A = v_B : v_A}$. Hence, the correct option is (A).

Quick Tip to Solve: For fragments of an initially stationary body, set their momentum magnitudes equal first; their kinetic energies are then proportional to speeds and inversely proportional to masses.

32. Two charged conducting spheres of radii a and b are connected to each other by a conducting wire. The ratio of charges of the two spheres respectively is: [4 marks]

(A) ab

(B) $\frac{b}{a}$

(C) $\frac{a}{b}$

(D) $\sqrt{ab}$

Correct Answer: (C) $\frac{a}{b}$

Solution:

Concept:

- Conductors connected by a conducting wire reach electrostatic equilibrium at the same electric potential; charge flows until the potential difference vanishes.
- For a charged spherical conductor of radius R, the surface potential is $V = kQ/R$, with $k = 1/(4\pi\varepsilon_0)$.

Step 1: Equate the final sphere potentials Let the final charges on the spheres of radii a and b be Q_a and Q_b. In the standard connected-spheres model, the wire is thin and mutual influence is neglected when writing the isolated-sphere potentials. Electrostatic equilibrium requires

$$V_a = V_b, \qquad \frac{kQ_a}{a} = \frac{kQ_b}{b}.$$

The connection fixes equality of potential, not equality of surface charge density or equality of the two charges. A larger sphere can hold more charge while remaining at the same potential.

Step 2: Obtain the ratio and test the options Cancel the common electrostatic constant and rearrange:

$$\frac{Q_a}{Q_b} = \frac{a}{b}.$$

Thus the first-to-second charge ratio is the first-to-second radius ratio. Option (C) gives this result. Option (B) reverses the requested ordering. Options (A) and (D) have dimensions of area and length, respectively, and therefore cannot represent a dimensionless ratio of charges. If $a = b$, the result becomes one, as symmetry requires. If the first radius is doubled while the second is fixed, its charge at a fixed common potential doubles; the reciprocal option would incorrectly decrease it. The unknown total charge is unnecessary because it cancels in the ratio.

Step 3: Conclusion The ratio of charges is $\boxed{Q_a/Q_b = a/b}$. Hence, the correct option is (C).

Quick Tip to Solve: A wire equalises potentials: use $Q/R =$ constant for spherical conductors, rather than setting charges or charge densities equal.

33. Two planets A and B having masses m_1 and m_2 move around the sun in circular orbits of r_1 and r_2 radii respectively. If angular momentum of A is L and that of B is $3\ \mathrm{L}$, the ratio of time period $\left(\frac{T_A}{T_B}\right)$ is: [4 marks]

(A) $\left(\frac{r_2}{r_1}\right)^{\frac{3}{2}}$

(B) $27\left(\frac{m_1}{m_2}\right)^3$

(C) $\left(\frac{r_1}{r_2}\right)^3$

(D) $\frac{1}{27}\left(\frac{m_2}{m_1}\right)^3$

Correct Answer: (D) $\frac{1}{27}\left(\frac{m_2}{m_1}\right)^3$

Solution:

Concept:

- For a circular orbit around a Sun of mass M, gravitational attraction supplies centripetal force, giving $v = \sqrt{GM/r}$.
- The orbital angular momentum is $L = mvr = m\sqrt{GMr}$, while the period is $T = 2\pi\sqrt{r^3/(GM)}$.

Step 1: Use the angular-momentum ratio to relate the radii The planets share the same central mass, so

$$\frac{L_B}{L_A} = \frac{m_2\sqrt{GMr_2}}{m_1\sqrt{GMr_1}} = \frac{m_2}{m_1}\sqrt{\frac{r_2}{r_1}} = 3.$$

Hence

$$\sqrt{\frac{r_2}{r_1}} = \frac{3m_1}{m_2}, \qquad \sqrt{\frac{r_1}{r_2}} = \frac{m_2}{3m_1}.$$

The masses must be retained because the given angular momenta belong to whole planets, rather than being angular momenta per unit mass.

Step 2: Apply the period-radius law and compare Kepler's third law for a common central body gives

$$\frac{T_A}{T_B} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{m_2}{3m_1}\right)^3 = \frac{1}{27}\left(\frac{m_2}{m_1}\right)^3.$$

Option (D) matches. Option (A) reverses the radii and represents T_B/T_A; option (C) uses exponent three instead of three-halves. Option (B) is the reciprocal of the correct mass expression. For a simple check, take equal planet masses: B then has nine times A's orbital radius and twenty-seven times its period, so $T_A/T_B = 1/27$, not 27. This also verifies the placement of the factor 27 independently of the algebra.

Step 3: Conclusion $\boxed{T_A/T_B = \frac{1}{27}(m_2/m_1)^3}$. Hence, the correct option is (D).

Quick Tip to Solve: For circular solar orbits combine $L/m \propto \sqrt{r}$ with $T \propto r^{3/2}$; therefore $T \propto (L/m)^3$.

34. Average force exerted on a non-reflecting surface at normal incidence is 2.4×10^{-4} N. If $360\ \mathrm{W/cm^2}$ is the light energy flux during span of 1 hour 30 minutes, Then the area of the surface is: [4 marks]

(A) $20\ \mathrm{m^2}$

(B) $0.2\ \mathrm{m^2}$

(C) $0.1\ \mathrm{m^2}$

(D) $0.02\ \mathrm{m^2}$

Correct Answer: (D) $0.02\ \mathrm{m^2}$

Solution:

Concept:

- For normal incidence on a fully absorbing, non-reflecting surface, radiation pressure is $p = I/c$, where I is energy flux per unit area and c is the speed of light.
- The resulting force is $F = pA = IA/c$. With constant flux, the duration cancels when impulse is divided by time.

Step 1: Convert the intensity and identify the force law The given flux is power per unit area, not total energy:

$$I = 360\,\mathrm{W\,cm^{-2}} = 360 \times 10^4\,\mathrm{W\,m^{-2}} = 3.60 \times 10^6\,\mathrm{W\,m^{-2}}.$$

Here $1\,\mathrm{cm^2} = 10^{-4}\,\mathrm{m^2}$, so the conversion multiplies by 10^4. The light absorbed over time t has energy IAt and transfers momentum IAt/c. Dividing this impulse by t yields $F = IA/c$, explaining why the stated one hour thirty minutes need not be inserted.

Step 2: Solve for area and check the options Using $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$,

$$A = \frac{Fc}{I} = \frac{(2.4 \times 10^{-4})(3.0 \times 10^8)}{3.60 \times 10^6}\,\mathrm{m^2}$$
$$= \frac{7.2 \times 10^4}{3.60 \times 10^6}\,\mathrm{m^2} = 2.0 \times 10^{-2}\,\mathrm{m^2}.$$

The pressure is $I/c = 0.012\,\mathrm{N\,m^{-2}}$; multiplying by $0.02\,\mathrm{m^2}$ reproduces the given force. Options (A), (B) and (C) would give forces 1000, 10 and 5 times the stated force, respectively. Reflection would introduce a factor two, but the question explicitly specifies a non-reflecting surface.

Step 3: Conclusion The surface area is $\boxed{0.02\,\mathrm{m^2}}$. Hence, the correct option is (D).

Quick Tip to Solve: For absorbed light use $A = Fc/I$; convert $\mathrm{W/cm^2}$ to $\mathrm{W/m^2}$ before substituting, and do not multiply a steady force by the exposure time.

35. In an expression $a \times 10^b$: [4 marks]

(A) a is order of magnitude for $b \leq 5$

(B) b is order of magnitude for $a \leq 5$

(C) b is order of magnitude for $a \geq 5$

(D) b is order of magnitude for $5 < a \leq 10$

Correct Answer: (B) b is order of magnitude for $a \leq 5$

Solution:

Concept:

- In scientific notation a positive quantity is written $a \times 10^b$, with a normalised coefficient a and integer exponent b.
- Under the coefficient-five convention used by the printed options, the order exponent is b for $a \leq 5$, and it is increased to $b + 1$ when $a > 5$.

Step 1: Distinguish the coefficient from the power exponent The coefficient a determines whether the scientific-notation power should be retained or advanced for the stated order-of-magnitude convention. The exponent b, rather than the coefficient, names the power of ten. For example, under this convention, a quantity written as 3×10^4 has order exponent 4, while one written as 7×10^4 is placed at the next order, with exponent 5. Thus a condition on the coefficient is needed; a bound on b does not decide this rounding.

Step 2: Apply the printed convention and eliminate alternatives For $a \leq 5$, the exponent remains b, exactly as option (B) states. Option (A) incorrectly identifies the coefficient a as the order and uses a restriction on the exponent. Options (C) and (D) incorrectly retain b for coefficients above five, where this convention increases the order exponent. The endpoint wording in option (B) places $a = 5$ with the lower order. In some mathematical treatments, order is instead defined by logarithmic rounding with threshold $\sqrt{10}$; the question's explicit coefficient-five alternatives identify the elementary convention being tested here. Keeping the convention clear avoids confusing the coefficient, the exponent and the power 10^b itself. No numerical value of a or b is supplied or needed to select the conditional statement.

Step 3: Conclusion With the convention used in the question, b is the order-of-magnitude exponent when $a \leq 5$. Hence, the correct option is (B).

Quick Tip to Solve: Read the condition on the scientific-notation coefficient: under the stated five-threshold convention, retain b up to five and use $b + 1$ above five.

36. A clock has $75\ \text{cm}, 60\ \text{cm}$ long second hand and minute hand respectively. In 30 minutes duration the tip of second hand will travel x distance more than the tip of minute hand. The value of x in meter is nearly (Take $\pi = 3.14$) : [4 marks]

(A) 118.9

(B) 140.5

(C) 139.4

(D) 220.0

Correct Answer: (C) 139.4

Solution:

Concept:

- The distance travelled by the tip of a clock hand is the arc length, not its displacement. For radius r and N complete revolutions, $s = 2\pi r N$.
- The second hand completes one revolution per minute, whereas the minute hand completes one revolution in 60 minutes. Convert the lengths from centimetres to metres before calculating.

Step 1: Count the revolutions during the given interval In 30 minutes the second hand completes $N_s = 30$ revolutions. The minute hand completes only $N_m = 30/60 = 1/2$ revolution. Their radii are $r_s = 75/100 = 0.75\,\text{m}$ and $r_m = 60/100 = 0.60\,\text{m}$, respectively. Even though the second hand returns to its starting position, its distance travelled is the sum of all 30 circumferences.

Step 2: Calculate the two distances Using the prescribed value $\pi = 3.14$, the second-hand tip travels

$$s_s = 30(2\pi \times 0.75) = 45\pi = 141.30\,\text{m}.$$

The minute-hand tip travels half its circumference:

$$s_m = \tfrac{1}{2}(2\pi \times 0.60) = 0.60\pi = 1.884\,\text{m}.$$

Step 3: Subtract and compare the choices The requested excess is

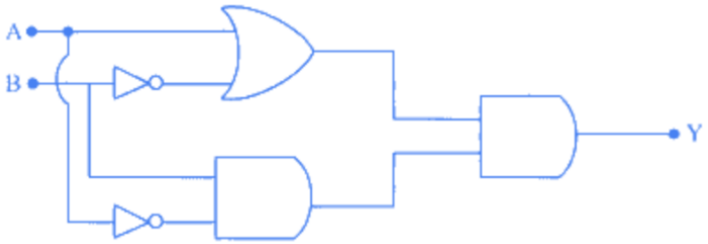
$$x = s_s - s_m = 141.30 - 1.884 = 139.416\,\text{m}.$$

To one decimal place this is $139.4\,\text{m}$. The other proposed values, 118.9, 140.5 and 220.0, do not equal the difference of the two actual arc lengths; in particular, using displacement instead would incorrectly disregard the repeated turns.

Step 4: Conclusion $x \approx 139.4\,\text{m}$. Hence, the correct option is (C).

Quick Tip to Solve: Count turns separately for each hand, then multiply by its own circumference; a whole number of turns has zero displacement but nonzero distance.

37. The output Y of following circuit for given inputs is : [4 marks]



(A) $A \cdot B$

(B) $\bar{A} \cdot B$

(C) $\Lambda \cdot B(\Lambda + B)$

(D) 0

Correct Answer: (D) 0

Solution:

Concept:

- An OR gate outputs the Boolean sum, an AND gate outputs the Boolean product, and a NOT gate complements its input. Here $+$ denotes OR and a product denotes AND.
- Boolean complements satisfy $A\overline{A} = 0$ and $B\overline{B} = 0$. Track each wire through its inverter before combining the intermediate outputs.

Step 1: Read the upper branch The direct input to the upper OR gate is A. The other input is obtained by passing B through the upper NOT gate, so it is $\overline{B}$. Denote this upper output by U. Then

$$U = A + \overline{B}.$$

The crossing near the left of the diagram is a wire crossover: it does not join the two distinct input signals.

Step 2: Read the lower branch The lower AND gate receives B directly and the complement of A from the lower NOT gate. Thus its output is

$$L = \overline{A}B.$$

The final gate is also an AND gate. Its inputs are U and L, so

$$Y = (A + \overline{B})\overline{A}B.$$

Step 3: Simplify and check all possible inputs Distributing the product gives

$$Y = A\overline{A}B + \overline{B}\overline{A}B = 0 + 0 = 0.$$

An independent input check gives the same result: the lower branch can be 1 only when $A = 0, B = 1$, but then the upper branch is $0 + 0 = 0$. For each of the other three input pairs the lower branch is already 0. Hence neither the product AB nor $\overline{A}B$, nor the nonconstant third expression, represents

the final circuit output.

Step 4: Conclusion The output remains $Y = 0$ for every pair of input values. Hence, the correct option is (D).

Quick Tip to Solve: For a final AND gate, first find when either input branch can be 1; if those conditions are incompatible, the output is always zero.

38. A player caught a cricket ball of mass $150\ \mathrm{g}$ moving at a speed of $20\ \mathrm{m/s}$. If the catching process is completed in $0.1\ \mathrm{s}$, the magnitude of force exerted by the ball on the hand of the player is: [4 marks]

(A) 150 N

(B) 3 N

(C) 30 N

(D) 300 N

Correct Answer: (C) 30 N

Solution:

Concept:

- The average net force acting during a collision equals the change in linear momentum divided by the interaction time: $\vec{F}_{\text{av}} = \Delta\vec{p}/\Delta t$.
- Newton's third law gives equal magnitudes and opposite directions for the force of the hand on the ball and the force of the ball on the hand. Use the ball's mass in kilograms.

Step 1: Identify the momentum change The ball has mass $m = 150\,\mathrm{g} = 0.150\,\mathrm{kg}$. Choose its initial direction of motion as positive. Its initial speed is $u = 20\,\mathrm{m\,s^{-1}}$, and a completed catch brings it to rest, so $v = 0$. The change in the ball's momentum is

$$\Delta p = m(v - u) = 0.150(0 - 20) = -3.0\,\mathrm{kg\,m\,s^{-1}}.$$

The negative sign shows that the stopping impulse opposes the incoming motion.

Step 2: Divide the impulse by the catching time The catching time is $\Delta t = 0.1\,\mathrm{s}$. Therefore the average stopping force on the ball is

$$F_{\text{av}} = \frac{-3.0}{0.1} = -30\,\mathrm{N}.$$

Only its magnitude is requested. The force need not be constant at every instant: the supplied interval determines its average value during the catch.

Step 3: Apply the action and reaction relation The ball exerts an opposite force on the hand with the same average magnitude, $30\,\mathrm{N}$. The value 3 is the momentum change in magnitude, rather than the force; 150 uses the mass numeral without the required calculation; and 300 would correspond to a ten-times shorter stopping interval. Thus these alternatives are incompatible with the given mass, speed and time.

Step 4: Conclusion The magnitude of the average force exerted by the ball on the hand is $30\,\mathrm{N}$.

Hence, the correct option is (C).

Quick Tip to Solve: Convert grams to kilograms, calculate the stopping impulse mu, and divide by the actual stopping time before using Newton's third law.

39. Three bodies A, B and C have equal kinetic energies and their masses are $400\ \text{g}, 1.2\ \text{kg}$ and $1.6\ \text{kg}$ respectively. The ratio of their linear momenta is : [4 marks]

(A) $1 : \sqrt{3} : 2$

(B) $\sqrt{3} : \sqrt{2} : 1$

(C) $1 : \sqrt{3} : \sqrt{2}$

(D) $\sqrt{2} : \sqrt{3} : 1$

Correct Answer: (A) $1 : \sqrt{3} : 2$

Solution:

Concept:

- For nonrelativistic translation, $K = \frac{1}{2}mv^2$ and $p = mv$; eliminating speed gives $K = p^2/(2m)$.
- When kinetic energies are equal, the momentum magnitude varies as the square root of mass: $p = \sqrt{2mK}$. Equal energy does not imply equal momentum or equal speed.

Step 1: Express every momentum using the common energy Let the common kinetic energy of the three bodies be K. Their momenta are

$$p_A = \sqrt{2m_AK}, \quad p_B = \sqrt{2m_BK}, \quad p_C = \sqrt{2m_CK}.$$

The positive square roots give magnitudes, as required for a ratio without specified directions. The common factor $\sqrt{2K}$ cancels from the ratio.

Step 2: Convert the masses and simplify In the same unit, the masses are $m_A = 0.4\,\text{kg}$, $m_B = 1.2\,\text{kg}$, and $m_C = 1.6\,\text{kg}$. Their ratio is

$$m_A : m_B : m_C = 0.4 : 1.2 : 1.6 = 1 : 3 : 4.$$

Taking square roots therefore gives

$$p_A : p_B : p_C = \sqrt{1} : \sqrt{3} : \sqrt{4} = 1 : \sqrt{3} : 2.$$

Step 3: Check the ordering and reject the alternatives Body C is the heaviest and therefore must have the greatest momentum at the common kinetic energy. Body A is the lightest and must have the least. This immediately rules out choices that place C below another body. The proposed $1 : \sqrt{3} : \sqrt{2}$ also fails because C has four times, rather than twice, the mass of A; its momentum must consequently be twice that of A. No numerical value of the common energy is needed.

Step 4: Conclusion The linear momentum ratio is $1 : \sqrt{3} : 2$. Hence, the correct option is (A).

Quick Tip to Solve: At equal kinetic energy, take square roots of the mass ratio; at equal momentum, by contrast, kinetic energy varies inversely with mass.

40. Young's modulus is determined by the equation given by $\mathrm{Y} = 49000 \frac{\mathrm{m}}{\mathrm{l}} \frac{\mathrm{dyne}}{\mathrm{cm}^2}$ where M is the mass and l is the extension of wire used in the experiment. Now error in Young modules (Y) is estimated by taking data from $M - l$ plot in graph paper. The smallest scale divisions are $5\ \mathrm{g}$ and $0.02\ \mathrm{cm}$ along load axis and extension axis respectively. If the value of M and l are $500\ \mathrm{g}$ and $2\ \mathrm{cm}$ respectively then percentage error of Y is : [4 marks]

(A) 2%

(B) 0.02%

(C) 0.5%

(D) 0.2%

Correct Answer: (A) 2%

Solution:

Concept:

- For a measured quantity proportional to a quotient, $Y = kM/l$, the maximum fractional errors add: $\Delta Y/Y = \Delta M/M + \Delta l/l$.
- The numerical constant $k = 49000$ is fixed in the supplied relation and contributes no measurement uncertainty. Use consistent units within each fractional error.

Step 1: Read the two measurement uncertainties The smallest scale division along the load axis is $5\,\mathrm{g}$, so take $\Delta M = 5\,\mathrm{g}$ for the stated graph reading. Along the extension axis it is $0.02\,\mathrm{cm}$, giving $\Delta l = 0.02\,\mathrm{cm}$. The measured values are $M = 500\,\mathrm{g}$ and $l = 2\,\mathrm{cm}$. Neither conversion to SI nor the calculation of Y itself is necessary, since each ratio has cancelling units.

Step 2: Calculate each relative error The load contributes

$$\frac{\Delta M}{M} = \frac{5}{500} = 0.01,$$

while the extension contributes

$$\frac{\Delta l}{l} = \frac{0.02}{2} = 0.01.$$

Thus each reading individually contributes a one-percent fractional uncertainty. The denominator does not produce a subtraction when calculating the maximum possible error.

Step 3: Add the errors and express the result as a percentage For a quotient, an overestimated numerator and an underestimated denominator both increase the result. Hence the maximum fractional error is

$$\frac{\Delta Y}{Y} = 0.01 + 0.01 = 0.02.$$

Multiplying by 100 gives

$$100\frac{\Delta Y}{Y} = 2\%.$$

The alternative 0.02 percent confuses a fractional error with a percentage, while 0.5 percent and 0.2 percent omit or mis-scale the stated uncertainties.

Step 4: Conclusion The percentage error in Young's modulus is 2%. Hence, the correct option is (A).

Quick Tip to Solve: For a product or quotient, add the magnitudes of the fractional errors first, and multiply by 100 only when reporting a percentage.

41. Paramagnetic substances:

A. align themselves along the directions of external magnetic field.

B. attract strongly towards external magnetic field.

C. has susceptibility little more than zero.

D. move from a region of strong magnetic field to weak magnetic field.

Choose the most appropriate answer from the options given below: [4 marks]

(A) A, B, C Only

(B) A, C Only

(C) A, B, C, D

(D) B, D Only

Correct Answer: (B) A, C Only

Solution:

Concept:

- A paramagnetic substance has a small positive magnetic susceptibility χ_m, so its induced magnetisation $\vec{M} = \chi_m \vec{H}$ is parallel to the applied field.
- Paramagnetic materials are weakly attracted toward a stronger magnetic field. Strong attraction is characteristic of ferromagnetism, whereas motion toward weaker field is characteristic of diamagnetism.

Step 1: Judge alignment and strength of attraction Statement A is correct: paramagnetic magnetisation is in the direction of the applied magnetic field, and an elongated sample tends to align along it. Statement B is incorrect because it says the attraction is strong. Ordinary paramagnets respond weakly; they should not be confused with ferromagnets, whose domain alignment produces a much stronger response.

Step 2: Judge the susceptibility and direction of motion Statement C is correct because paramagnetic susceptibility is positive but small:

$$0 < \chi_m \ll 1.$$

Equivalently, the relative permeability $\mu_r = 1 + \chi_m$ is slightly greater than unity. Statement D is incorrect. In a nonuniform field a paramagnetic specimen is pulled toward the region of greater field strength, so its motion is from weaker field to stronger field, opposite to the motion described in D.

Step 3: Select the option containing exactly the true statements The truth pattern is therefore A true, B false, C true and D false. Only the combination A and C contains both correct statements and excludes both incorrect ones. The alternatives containing B incorrectly assign strong attraction to a paramagnet, and those containing D incorrectly assign it the direction of motion of a diamagnetic substance. The sign of susceptibility consistently explains both the alignment and the force direction.

Step 4: Conclusion Only statements A and C are correct. Hence, the correct option is (B).

Quick Tip to Solve: Remember the linked paramagnetic properties: small positive susceptibility, weak attraction, and motion toward stronger field; a negative susceptibility reverses the last behaviour.

42. A mixture of one mole of monoatomic gas and one mole of a diatomic gas (rigid) are kept at room temperature $(27^\circ C)$. The ratio of specific heat of gases at constant volume respectively is: [4 marks]

(A) $\frac{3}{2}$
(B) $\frac{3}{5}$
(C) $\frac{7}{5}$
(D) $\frac{5}{3}$

Correct Answer: (B) $\frac{3}{5}$

Solution:

Concept:

- For an ideal gas with f active quadratic degrees of freedom, equipartition gives molar internal energy $U_m = fRT/2$ and molar heat capacity $C_V = fR/2$.
- A monoatomic gas has three translational degrees of freedom. A rigid diatomic gas at room temperature has three translational and two rotational degrees of freedom; vibrational modes are excluded by the rigid-molecule description.

Step 1: Find the monoatomic constant-volume heat capacity A monoatomic molecule has no rotational energy modes relevant to this ideal-gas treatment, so $f_{\text{mono}} = 3$. For one mole its internal energy is

$$U_{\text{mono}} = \frac{3}{2}RT,$$

and differentiating with respect to temperature at constant volume gives

$$C_{V,\text{mono}} = \frac{3}{2}R.$$

This heat capacity concerns the monoatomic component, rather than the sum for the mixture.

Step 2: Find the diatomic constant-volume heat capacity For the rigid diatomic gas, $f_{\text{dia}} = 3 + 2 = 5$. Therefore

$$U_{\text{dia}} = \frac{5}{2}RT, \qquad C_{V,\text{dia}} = \frac{5}{2}R.$$

At the specified room temperature, the rigid model does not include vibrational excitation. Since there is one mole of each gas, the respective total constant-volume heat capacities have the same ratio as their molar heat capacities.

Step 3: Form the ratio in the stated order Taking monoatomic first and diatomic second gives

$$\frac{C_{V,\text{mono}}}{C_{V,\text{dia}}} = \frac{3R/2}{5R/2} = \frac{3}{5}.$$

The value $5/3$ reverses that order and is also the monoatomic heat-capacity ratio C_P/C_V, a different quantity. The value $7/5$ is the diatomic C_P/C_V, and $3/2$ is the monoatomic coefficient of R, not the requested ratio.

Step 4: Conclusion The ratio of the respective constant-volume heat capacities is $3/5$. Hence, the correct option is (B).

Quick Tip to Solve: For equal mole amounts at constant volume, the heat-capacity ratio equals the ratio of active degrees of freedom: monoatomic to rigid diatomic is $3 : 5$.

43. A LCR circuit is at resonance for a capacitor C, inductance L and resistance R. Now the value of resistance is halved keeping all other parameters same. The current amplitude at resonance will be now: [4 marks]

(A) halved

(B) same

(C) Zero

(D) double

Correct Answer: (D) double

Solution:

Concept:

- For a series LCR circuit driven at angular frequency ω, the impedance is $Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}$. Current amplitude is the applied voltage amplitude divided by this impedance.
- At resonance, $\omega_0 = 1/\sqrt{LC}$, so the inductive and capacitive reactances cancel. The circuit then presents only its resistance to the source.

Step 1: Write the initial resonant current Let the unchanged applied voltage amplitude be V_0. At resonance, the net reactance is zero, hence the magnitude of the impedance is $Z_0 = R$. Therefore

$$I_0 = \frac{V_0}{R}.$$

The capacitor and inductor may individually have nonzero voltage amplitudes, but their opposite reactive voltage contributions cancel in the series impedance. The source current is consequently limited by resistance.

Step 2: Replace the resistance and compare The new resistance is $R' = R/2$. Since neither L nor C changes, the resonant frequency is unchanged and the same source remains resonant. Thus

$$I_0' = \frac{V_0}{R/2} = 2\frac{V_0}{R} = 2I_0.$$

Halving would require doubling the resistance. The same amplitude would require unchanged impedance, while zero current would require an open circuit or zero applied voltage; neither occurs here. Lower resistance instead reduces dissipation per squared current and permits a larger current for the same source voltage.

Step 3: Conclusion The resonant current amplitude becomes double its original value: $I_0' = 2I_0$. Hence, the correct option is (D).

Quick Tip to Solve: At series resonance immediately replace the impedance by R; with a fixed source amplitude the current varies inversely with R.

44. The diameter of a sphere is measured using a vernier caliper whose 9 divisions of main scale are equal to 10 divisions of vernier scale. The shortest division on the main scale is equal to $1\ \text{mm}$. The main scale reading is $2\ \text{cm}$ and second division of vernier scale coincides with a division on main scale. If mass of the sphere is 8.635 g, the density of the sphere is: [4 marks]

(A) $2.2\ \text{g/cm}^3$

(B) $2.0\ \text{g/cm}^3$

(C) $1.7\ \text{g/cm}^3$

(D) $2.5\ \text{g/cm}^3$

Correct Answer: (B) $2.0\ \text{g/cm}^3$

Solution:

Concept:

- The least count of a direct vernier is one main-scale division minus one vernier-scale division. A coincidence at division n adds n times the least count to the main-scale reading.
- A sphere of measured diameter d has volume $V = \pi d^3/6$, and its density is $\rho = m/V$. Use diameter and mass in compatible centimetre and gram units.

Step 1: Calculate the diameter from the vernier Ten vernier divisions equal nine main-scale divisions, and one main-scale division is $1\,\text{mm}$. Thus

$$1\,\text{VSD} = 0.9\,\text{mm}, \qquad \text{LC} = 1 - 0.9 = 0.1\,\text{mm}.$$

The second vernier division coincides, so its contribution is $2(0.1) = 0.2\,\text{mm} = 0.02\,\text{cm}$. No zero error is specified. The measured diameter is therefore

$$d = 2.00 + 0.02 = 2.02\,\text{cm}.$$

This is a diameter, not a radius, so the radius for the volume calculation would be $1.01\,\text{cm}$.

Step 2: Find the volume and density Using the diameter form avoids an additional radius conversion:

$$V = \frac{\pi}{6}(2.02)^3 \approx 4.316\,\text{cm}^3.$$

With the given mass,

$$\rho = \frac{8.635}{4.316} \approx 2.00\,\text{g cm}^{-3}.$$

Equivalently, using $4\pi(1.01)^3/3$ gives the same volume. The other values 2.2, 1.7 and 2.5 do not result from the measured diameter and mass. Because volume depends on the cube of diameter, the vernier contribution must be added before cubing, rather than used as an additive volume correction.

Step 3: Conclusion The density is approximately $2.0\,\text{g cm}^{-3}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert the vernier correction from millimetres to centimetres before adding it to the main-scale reading, and cube the final diameter.

45. Critical angle of incidence for a pair of optical media is 45°. The refractive indices of first and second media are in the ratio: [4 marks]

(A) $1 : 2$

(B) $1 : \sqrt{2}$

(C) $2 : 1$

(D) $\sqrt{2} : 1$

Correct Answer: (D) $\sqrt{2} : 1$

Solution:

Concept:

- Total internal reflection is possible only for light incident from an optically denser medium into a rarer medium. At the critical angle C, the refracted ray just grazes the interface, making a refraction angle of 90 degrees.
- Snell's law is $n_1 \sin i = n_2 \sin r$, with the first medium the incident medium. At the critical angle this reduces to $\sin C = n_2/n_1$.

Step 1: Apply the critical-angle condition Take the first medium as the medium of incidence and the second as the medium of refraction. For a critical angle to exist in that direction, $n_1 > n_2$. At the limiting incidence angle $i = C$, the angle of refraction is $r = 90^\circ$. Substitution in Snell's law gives

$$n_1 \sin C = n_2 \sin 90^\circ = n_2.$$

Consequently the refractive-index ratio is the reciprocal of the sine of the critical angle. This is a ratio of absolute refractive indices, not a ratio of the ray angles.

Step 2: Insert the angle and identify the ratio For the given value $C = 45^\circ$,

$$\frac{n_2}{n_1} = \sin 45^\circ = \frac{1}{\sqrt{2}}, \qquad \frac{n_1}{n_2} = \sqrt{2}.$$

Therefore $n_1 : n_2 = \sqrt{2} : 1$. The inverse ratio $1 : \sqrt{2}$ would reverse the incident and refracting media, while $1 : 2$ also puts the smaller refractive index first. The ratio $2 : 1$ would give $\sin C = 1/2$, hence a critical angle of 30°, rather than the specified angle. Only one listed ratio meets both Snell's law and the denser-to-rarer requirement.

Step 3: Conclusion The refractive indices of the first and second media are in the ratio $\sqrt{2} : 1$. Hence, the correct option is (D).

Quick Tip to Solve: For a critical-angle question write denser index divided by rarer index as cosec C; this prevents reversing the required ratio.

46. A proton and an electron are associated with same de-Broglie wavelength. The ratio of their kinetic energies is:

(Assume h = 6.63 $\times 10^{-34}$ J s, $m_e = 9.0 \times 10^{-31}$ kg **and** $m_p = 1836$ **times** m_e **)** [4 marks]

(A) $1 : \frac{1}{1836}$

(B) $1 : \sqrt{1836}$

(C) $1 : 1836$

(D) $1 : \frac{1}{\sqrt{1836}}$

Correct Answer: (C) $1 : 1836$

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$, where p is the magnitude of momentum. Equal wavelengths therefore mean equal momentum magnitudes, regardless of particle mass.
- For nonrelativistic particles, $K = p^2/(2m)$. At a fixed momentum the kinetic energy is inversely proportional to the mass, so the heavier particle has less kinetic energy.

Step 1: Use the equality of wavelengths Let the common wavelength be λ. Applying the de Broglie relation separately to the proton and electron gives

$$p_p = \frac{h}{\lambda} = p_e.$$

It is momentum, rather than speed or kinetic energy, that is made equal by this condition. In particular, $m_p v_p = m_e v_e$, so the proton moves more slowly. The numerical value of Planck's constant is common to both particles and will cancel from their energy ratio.

Step 2: Form the kinetic-energy ratio With the common momentum denoted by p,

$$\frac{K_p}{K_e} = \frac{p^2/(2m_p)}{p^2/(2m_e)} = \frac{m_e}{m_p} = \frac{1}{1836}.$$

Thus the requested proton-to-electron ratio is $1 : 1836$. The inverse ratio in option A would give the heavier proton more kinetic energy at fixed momentum, contradicting $K \propto 1/m$. The square-root ratios in B and D have no place in this comparison: squaring the common momentum leaves a first power of mass in the denominator, not a square root. Neither the wavelength nor a speed value is required.

Step 3: Conclusion The ratio of proton kinetic energy to electron kinetic energy is $1 : 1836$. Hence, the correct option is (C).

Quick Tip to Solve: Equal de Broglie wavelengths mean equal momenta; use K = p squared divided by twice the mass rather than assuming equal speeds.

47. Correct Bernoulli's equation is (symbols have their usual meaning) : [4 marks]

(A) $P + \frac{1}{2}\rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

(B) $P + mgh + \frac{1}{2}mv^2 = \text{constant}$

(C) $P + \rho gh + \rho v^2 = \text{constant}$

(D) $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

Correct Answer: (D) $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

Solution:

Concept:

- For steady flow of an incompressible, nonviscous fluid along a streamline, the sum of pressure energy, gravitational potential energy and kinetic energy per unit volume remains constant.
- The three contributions per unit volume are P, ρgh and $\frac{1}{2}\rho v^2$. Here density is ρ, height is h and flow speed is v.

Step 1: Express all energy terms per unit volume Consider a fluid element of volume ΔV and mass $m = \rho \Delta V$. Pressure work per unit volume contributes P. Gravitational potential energy divided by the element volume is

$$\frac{mgh}{\Delta V} = \rho gh.$$

Similarly, kinetic energy per unit volume is

$$\frac{\frac{1}{2}mv^2}{\Delta V} = \frac{1}{2}\rho v^2.$$

The potential term does not have a factor of one half; that factor belongs to the kinetic energy alone. Each resulting term has units of energy per volume, or equivalently pressure.

Step 2: Combine the terms and compare the options Conservation of mechanical energy for the fluid element gives

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}.$$

Option A incorrectly halves the gravitational term. Option B combines pressure with the total energies mgh and $mv^2/2$, which do not have the same dimensions as pressure; those energies would first need division by a volume. Option C omits the factor of one half from the kinetic term. Option D alone contains all three correct contributions in a common per-unit-volume form. The constant has the same value at different points on the chosen streamline under the stated ideal-flow assumptions.

Step 3: Conclusion The correct relation is $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$. Hence, the correct option is (D).

Quick Tip to Solve: Check both dimensions and coefficients: pressure pairs with energy per unit volume, and only the kinetic term carries a factor of one half.

48. Binding energy of a certain nucleus is 18×10^8 J. How much is the difference between total mass of all the nucleons and nuclear mass of the given nucleus: [4 marks]

(A) 20 μg

(B) 2 μg

(C) 10 μg

(D) 0.2 μg

Correct Answer: (A) 20 μg

Solution:

Concept:

- The mass defect of a bound nucleus is the total mass of its separated nucleons minus its nuclear mass. This positive mass difference corresponds to the released binding energy.
- Einstein's mass-energy relation gives $E_{\mathrm{b}} = \Delta m c^2$. With energy in joules and c in metres per second, the resulting mass is in kilograms.

Step 1: Calculate the mass defect in SI units The requested difference is

$$\Delta m = \left(\sum m_{\text{nucleons}}\right) - m_{\text{nucleus}}.$$

The nucleus has less mass than its separated constituents because energy is released when they bind. Using the supplied binding energy and $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$,

$$\Delta m = \frac{E_{\mathrm{b}}}{c^2} = \frac{18 \times 10^8}{(3.0 \times 10^8)^2}\,\mathrm{kg}.$$

Squaring the speed gives 9.0×10^{16}, so

$$\Delta m = 2.0 \times 10^{-8}\,\mathrm{kg}.$$

This value is the total nuclear mass defect, not a mass defect per nucleon; no nucleon count is needed.

Step 2: Convert to the option unit One kilogram equals 10^3 grams and one gram equals 10^6 micrograms. Therefore

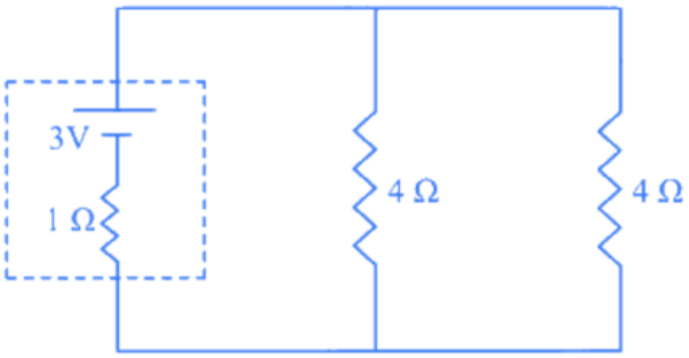
$$1\,\mathrm{kg} = 10^9\,\mu\mathrm{g}, \qquad \Delta m = (2.0 \times 10^{-8})(10^9) = 20\,\mu\mathrm{g}.$$

The alternatives 2, 10 and 0.2 micrograms disagree with this conversion. As a check, $20\,\mu\mathrm{g} = 20 \times 10^{-9}\,\mathrm{kg}$, which on multiplication by $9 \times 10^{16}\,\mathrm{m^2 s^{-2}}$ restores 18×10^8 J. The positive sign agrees with the fact that a bound nucleus weighs less than its free nucleons.

Step 3: Conclusion The difference between the total nucleon mass and the nuclear mass is $20\,\mu\mathrm{g}$. Hence, the correct option is (A).

Quick Tip to Solve: After using E = delta m times c squared in SI units, remember that one microgram equals ten to the power minus nine kilograms.

49. In the given circuit, the terminal potential difference of the cell is : [4 marks]



(A) 3 V

(B) 4 V

(C) 2 V

(D) 1.5 V

Correct Answer: (C) 2 V

Solution:

Concept:

- A real cell is modelled as an ideal emf $\mathcal{E}$ in series with an internal resistance r. During discharge, the terminal potential difference is $V = \mathcal{E} - Ir$.
- Resistors connected between the same two terminals are in parallel. Their equivalent resistance is found from $1/R = 1/R_1 + 1/R_2$, and the total cell current is $I = \mathcal{E}/(R + r)$.

Step 1: Reduce the external circuit The figure shows a cell of emf $3\,\text{V}$ and internal resistance $1\,\Omega$. The two external $4\,\Omega$ resistors share the top and bottom rails, so they have the same potential difference and are connected in parallel. Their equivalent resistance is

$$R = \frac{4 \times 4}{4 + 4} = 2\,\Omega.$$

This external equivalent is in series with the internal resistance as seen by the ideal emf. Consequently the total resistance limiting the cell current is $R + r = 2 + 1 = 3\,\Omega$. Treating the two external resistors as series elements would not match the common-rail connections.

Step 2: Calculate current and terminal voltage The total current supplied by the cell is

$$I = \frac{\mathcal{E}}{R + r} = \frac{3}{3} = 1\,\text{A}.$$

There is an internal voltage drop $Ir = 1\,\text{V}$, so

$$V = \mathcal{E} - Ir = 3 - 1 = 2\,\text{V}.$$

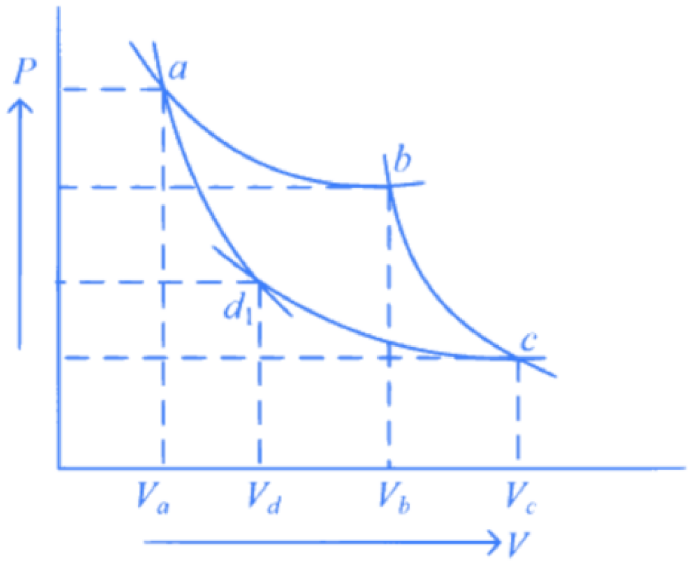
An independent check is $V = IR = 1(2) = 2\,\text{V}$. Each external branch then carries $2/4 = 0.5\,\text{A}$, and the branch currents add to the calculated cell current. The 3 V choice neglects the internal drop; 4 V

exceeds the emf of a discharging cell. The 1.5 V choice would require a different load resistance and does not satisfy this circuit.

Step 3: Conclusion The terminal potential difference of the cell is $2\,\mathrm{V}$. Hence, the correct option is (C).

Quick Tip to Solve: First combine the external resistors in parallel, then use E minus Ir; the terminal voltage also equals the current times the external equivalent resistance.

50. Two different adiabatic paths for the same gas intersect two isothermal curves as shown in P-V diagram. The relation between the ratio $\frac{V_a}{V_d}$ and the ratio $\frac{V_b}{V_c}$ is: [4 marks]



(A) $\frac{V_a}{V_d} \neq \frac{V_b}{V_c}$

(B) $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^{-1}$

(C) $\frac{V_a}{V_d} = \frac{V_b}{V_c}$

(D) $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^{2}$

Correct Answer: (C) $\frac{V_a}{V_d} = \frac{V_b}{V_c}$

Solution:

Concept:

- For the same ideal gas undergoing a reversible adiabatic change, $TV^{\gamma-1} = \text{constant}$, where $\gamma = C_P/C_V$. The constants may differ on different adiabatic paths.
- Points on one isothermal curve have the same absolute temperature. In the figure, $T_a = T_b = T_H$ and $T_d = T_c = T_L$, with $T_H > T_L$.

Step 1: Apply the adiabatic relation to each path The first adiabatic path connects a to d, while the second connects b to c. The gas is the same, so the exponent $\gamma - 1$ is the same for both paths.

Therefore,

$$T_H V_a^{\gamma-1} = T_L V_d^{\gamma-1}, \qquad T_H V_b^{\gamma-1} = T_L V_c^{\gamma-1}.$$

The temperature decreases during either expansion; it is the common pair of isotherms that fixes the temperature ratio.

Step 2: Eliminate the common temperature ratio Rearrange both equations to express the ratio of the final temperature to the initial temperature:

$$\left(\frac{V_a}{V_d}\right)^{\gamma-1} = \frac{T_L}{T_H} = \left(\frac{V_b}{V_c}\right)^{\gamma-1}.$$

All volumes are positive and $\gamma > 1$, so taking the positive $1/(\gamma-1)$ power gives equal volume ratios. Both ratios are less than one. Thus the reciprocal in option B cannot agree with the first ratio; option A denies the derived equality, and the square in option D introduces an unjustified extra power.

Step 3: Conclusion The required relation is $\frac{V_a}{V_d} = \frac{V_b}{V_c}$. Hence, the correct option is (C).

Quick Tip to Solve: For adiabats connecting the same two isotherms, eliminate the common temperature ratio before comparing the volumes.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A closed and an open organ pipe have same lengths. If the ratio of frequencies of their seventh overtones is $\left(\frac{a-1}{a}\right)$ then the value of a is _____. [4 marks]

Solution:

Concept:

- An open organ pipe of length L has harmonics $f_n = nv/(2L)$, where v is the speed of sound. Every positive integer harmonic is allowed.
- A pipe closed at one end has only odd harmonics: $f_n = (2n-1)v/(4L)$. An overtone is counted above the fundamental, not from it.

Step 1: Identify the seventh overtone in each pipe For the open pipe, the fundamental is the first harmonic and the first overtone is the second harmonic. Its seventh overtone is consequently the eighth harmonic:

$$f_{\text{open},7} = \frac{8v}{2L} = \frac{16v}{4L}.$$

For the closed pipe, the allowed sequence is $1, 3, 5, 7, 9, 11, 13, 15, \ldots$ times its fundamental. The seventh overtone is the eighth allowed mode, corresponding to the fifteenth harmonic:

$$f_{\text{closed},7} = \frac{15v}{4L}.$$

The closed pipe must not be assigned the eighth harmonic, because that even harmonic is absent.

Step 2: Form the stated ratio and determine a The pipes have equal lengths and the same sound speed, so the common factors cancel from the ratio. In the order closed pipe to open pipe,

$$\frac{f_{\text{closed},7}}{f_{\text{open},7}} = \frac{15v/(4L)}{16v/(4L)} = \frac{15}{16}.$$

Equate this to the expression given in the question:

$$\frac{a-1}{a} = \frac{15}{16} \quad \Rightarrow \quad 16a - 16 = 15a \quad \Rightarrow \quad a = 16.$$

This ratio is less than one, as expected from the two frequencies just calculated.

Step 3: Conclusion The required dimensionless value is $\boxed{a = 16}$.

Quick Tip to Solve: Write out the odd-harmonic sequence for a closed pipe; the seventh overtone is its fifteenth harmonic.

---

52. An electric field, $\vec{E} = \frac{2\hat{i}+6\hat{j}+8\hat{k}}{\sqrt{6}}$ passes through the surface of $4\ \text{m}^2$ area having unit vector $\hat{n} = \left(\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)$. The electric flux for that surface is _____ Vm. [4 marks]

Solution:

Concept:

- Electric flux through a plane surface in a uniform electric field is $\Phi = \vec{E} \cdot \vec{A}$, where $\vec{A} = A\hat{n}$ is its vector area.
- The unit normal selects the component of the field perpendicular to the surface. A dot product multiplies corresponding Cartesian components and adds them.

Step 1: Check the normal and form the vector area The specified normal is $\hat{n} = (2\hat{i} + \hat{j} + \hat{k})/\sqrt{6}$. Its magnitude is

$$|\hat{n}| = \frac{\sqrt{2^2 + 1^2 + 1^2}}{\sqrt{6}} = 1,$$

so no further normalization is needed. Since the scalar area is $A = 4\,\text{m}^2$, the vector area is

$$\vec{A} = \frac{4(2\hat{i} + \hat{j} + \hat{k})}{\sqrt{6}}\ \text{m}^2.$$

The area is multiplied by the normal; using the scalar area alone would lose the orientation information essential for the flux.

Step 2: Evaluate the dot product Substitute the electric-field components and take the dot product:

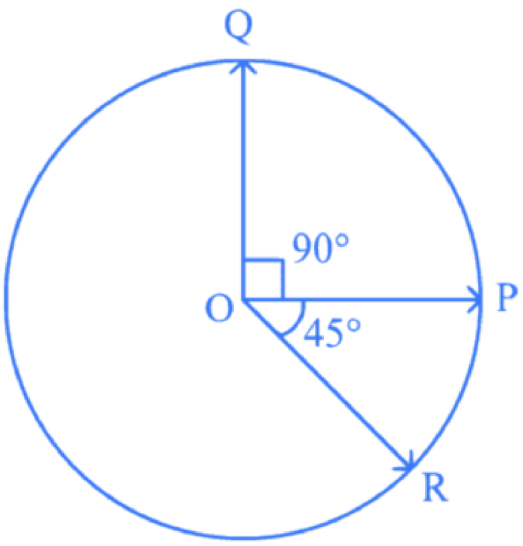
$$\Phi = 4\left(\frac{2\hat{i}+6\hat{j}+8\hat{k}}{\sqrt{6}}\right)\cdot\left(\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)$$
$$= \frac{4}{6}(2\times 2+6\times 1+8\times 1)$$
$$= \frac{4}{6}(18) = 12\,\mathrm{V\,m}.$$

The two denominators each contribute a factor $\sqrt{6}$, giving 6 in the product. Cross terms vanish because the Cartesian unit vectors are perpendicular. The positive value means the net field component is along the stated outward normal.

Step 3: Conclusion The electric flux through the surface is $\boxed{12\,\mathrm{V\,m}}$, so the required entry is 12.

Quick Tip to Solve: Multiply the component dot product by the scalar area, and include both normalization denominators.

53. Three vectors $\overrightarrow{\mathrm{OP}}, \overrightarrow{\mathrm{OQ}}$ and $\overrightarrow{\mathrm{OR}}$ each of magnitude A are acting as shown in figure. The resultant of the three vectors is $\mathrm{A}\sqrt{x}$. The value of x is _____. [4 marks]



Solution:

Concept:

- A vector sum can be found by adding its Cartesian components, then using $R = \sqrt{R_x^2 + R_y^2}$ for its magnitude.
- The figure has $\overrightarrow{OP}$ along positive x, $\overrightarrow{OQ}$ along positive y, and $\overrightarrow{OR}$ at 45° below positive x. Each magnitude is A.

Step 1: Resolve the three vectors Choose O as the origin, OP as the positive x-axis and OQ as the

positive y-axis. Their components are

$$\overrightarrow{OP} = A\hat{i}, \qquad \overrightarrow{OQ} = A\hat{j}.$$

Since OR is in the fourth quadrant, its x-component is positive and its y-component is negative:

$$\overrightarrow{OR} = A\cos 45^\circ \hat{i} - A\sin 45^\circ \hat{j} = \frac{A}{\sqrt{2}}\hat{i} - \frac{A}{\sqrt{2}}\hat{j}.$$

The minus sign is required by the actual orientation of R in the diagram.

Step 2: Find the magnitude of the resultant Add components before taking the magnitude:

$$R_x = A\left(1 + \frac{1}{\sqrt{2}}\right), \qquad R_y = A\left(1 - \frac{1}{\sqrt{2}}\right).$$

Therefore,

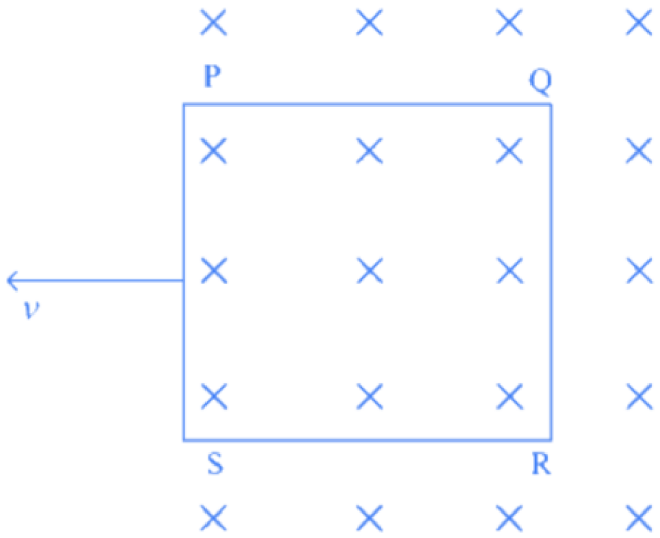
$$\begin{aligned} R^2 &= A^2\left[\left(1 + \frac{1}{\sqrt{2}}\right)^2 + \left(1 - \frac{1}{\sqrt{2}}\right)^2\right] \\ &= A^2\left[(1 + \sqrt{2} + \tfrac{1}{2}) + (1 - \sqrt{2} + \tfrac{1}{2})\right] \\ &= 3A^2. \end{aligned}$$

The mixed terms cancel because the diagonal vector contributes equal and opposite changes to the two components. Thus $R = A\sqrt{3}$, which can be compared directly with the given expression $A\sqrt{x}$.

Step 3: Conclusion The resultant has magnitude $A\sqrt{3}$; hence the required value is $\boxed{x = 3}$.

Quick Tip to Solve: Keep the negative y-component of OR; the two squared component expressions then have cancelling cross terms.

54. A square loop PQRS having 10 turns, area 3.6×10^{-3} m^2 and resistance 100Ω is slowly and uniformly being pulled out of a uniform magnetic field of magnitude $\mathrm{B} = 0.5$ T as shown. Work done in pulling the loop out of the field in 1.0 s is _____ $\times 10^{-6}$ J. [4 marks]



Solution:

Concept:

- For a coil of N turns, Faraday's law gives the induced-emf magnitude $\mathcal{E} = N|d\Phi/dt|$, where Φ is the flux through one turn.
- Uniform withdrawal from a uniform field gives a constant rate of flux loss. During slow motion, the work against magnetic drag becomes Joule heat: $W = I^2Rt$.

Step 1: Determine the constant induced emf The square coil initially encloses its full area $A = 3.6 \times 10^{-3}\,\mathrm{m}^2$ within the field, and is completely withdrawn in $t = 1.0\,\mathrm{s}$. Since the speed is uniform, its overlap area decreases at a constant rate. The flux per turn changes from BA to zero, so

$$\mathcal{E} = \frac{NBA}{t} = \frac{10(0.5)(3.6 \times 10^{-3})}{1.0} = 1.8 \times 10^{-2}\,\mathrm{V}.$$

Here the factor of ten is included in the emf, because all turns contribute to the flux linkage.

Step 2: Calculate the mechanical work from the heating The resistance of the loop is $R = 100\,\Omega$, giving a current

$$I = \frac{\mathcal{E}}{R} = \frac{1.8 \times 10^{-2}}{100} = 1.8 \times 10^{-4}\,\mathrm{A}.$$

The magnetic force opposes withdrawal by Lenz's law. As the coil is pulled slowly at constant speed, there is no change in kinetic energy, and external work supplies the electrical energy dissipated:

$$W = I^2Rt = \frac{\mathcal{E}^2 t}{R} = \frac{(1.8 \times 10^{-2})^2(1.0)}{100} = 3.24 \times 10^{-6}\,\mathrm{J}.$$

The requested coefficient is 3.24 before rounding; its nearest integer is 3, consistent with the numerical entry.

Step 3: Conclusion The work is $3.24 \times 10^{-6}\,\mathrm{J}$. The required nearest-integer coefficient of $10^{-6}\,\mathrm{J}$ is $\boxed{3}$.

Quick Tip to Solve: Calculate the total-coil emf first; the dissipated energy involves its square, so the number of turns enters quadratically.

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55. A liquid column of height $0.04\ \mathrm{cm}$ balances excess pressure of a soap bubble of certain radius. If density of liquid is $8 \times 10^3\ \mathrm{kg\ m^{-3}}$ and surface tension of soap solution is $0.28\ \mathrm{Nm^{-1}}$, then diameter of the soap bubble is _____ cm. (if $\mathrm{g} = 10\ \mathrm{m\ s^{-2}}$) [4 marks]

Solution:

Concept:

- The pressure difference supported by a liquid column of height h and density ρ is $\Delta P = \rho g h$. Use SI units before substitution.
- A soap bubble has two liquid-air surfaces, so its excess pressure is $\Delta P = 4S/r = 8S/d$, where S is surface tension, r radius and d diameter.

Step 1: Convert the height and find the pressure difference The given height is in centimetres, so

$$h = 0.04\,\text{cm} = 0.04 \times 10^{-2}\,\text{m} = 4.0 \times 10^{-4}\,\text{m}.$$

With $\rho = 8 \times 10^3\,\text{kg}\,\text{m}^{-3}$ and $g = 10\,\text{m}\,\text{s}^{-2}$, the liquid column balances a pressure

$$\Delta P = \rho g h = (8 \times 10^3)(10)(4 \times 10^{-4}) = 32\,\text{Pa}.$$

The height conversion is essential: treating 0.04 as a height in metres would make the pressure one hundred times too large.

Step 2: Equate the pressure to the bubble excess pressure For a soap bubble, both its inner and outer surfaces contribute to the excess pressure. Equating this pressure to the column pressure gives

$$\frac{4S}{r} = 32 \quad \Rightarrow \quad r = \frac{4(0.28)}{32} = 0.035\,\text{m}.$$

The requested quantity is the diameter, so

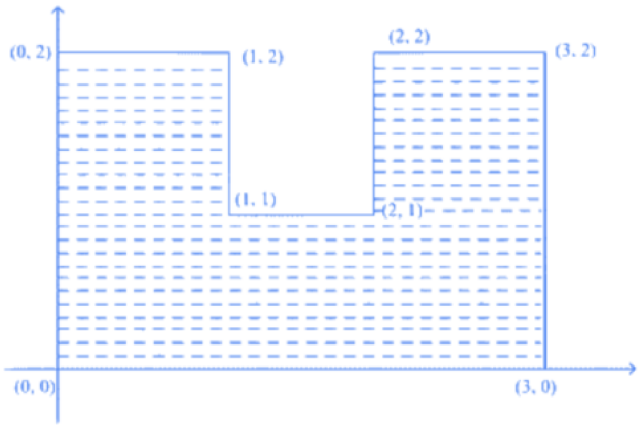
$$d = 2r = 0.070\,\text{m} = 7.0\,\text{cm}.$$

Equivalently, the direct diameter formula is $d = 8S/(\rho g h)$. Using the pressure law for a liquid drop, which has only one surface, would miss a factor of two; reporting the radius instead of the diameter would miss another.

Step 3: Conclusion The soap bubble has diameter $\boxed{7\,\text{cm}}$, so the required entry is 7.

Quick Tip to Solve: For a soap bubble diameter, use $\Delta P = 8S/d$; convert the liquid-column height to metres first.

56. A uniform thin metal plate of mass $10\ \text{kg}$ with dimensions is shown. The ratio of x and y coordinates of center of mass of plate in $\frac{n}{9}$. The value of n is _____. [4 marks]



Solution:

Concept:

- For a uniform thin plate, mass is proportional to area. Its centre of mass can be computed using a complete rectangle minus a removed rectangular region.
- The centroid of a rectangle is its geometric centre. A removed area contributes negative area and negative first moments to the composite-lamina calculation.

Step 1: Identify the complete and removed rectangles The displayed outer rectangle extends from $x = 0$ to $x = 3$ and from $y = 0$ to $y = 2$. Its area is 6 and its centroid is $(3/2, 1)$. The missing top central square extends from $x = 1$ to $x = 2$ and from $y = 1$ to $y = 2$; it has area 1 and centroid $(3/2, 3/2)$. Thus the remaining plate has area $6 - 1 = 5$. Both the complete rectangle and the cutout are symmetric about $x = 3/2$, so the remaining plate has the same symmetry line. The given total mass does not need to be substituted, since the common areal density cancels.

Step 2: Subtract the first moments and form the ratio Using area-weighted centroids with a negative contribution for the missing square gives

$$x_{\mathrm{CM}} = \frac{6(3/2) - 1(3/2)}{5} = \frac{3}{2},$$

$$y_{\mathrm{CM}} = \frac{6(1) - 1(3/2)}{5} = \frac{9}{10}.$$

The vertical centroid lies below the mid-height 1 because material was removed from the top, providing a useful sign check. Hence

$$\frac{x_{\mathrm{CM}}}{y_{\mathrm{CM}}} = \frac{3/2}{9/10} = \frac{5}{3} = \frac{15}{9}.$$

Comparing the last expression with $n/9$ determines the requested numerator without requiring the individual component masses.

Step 3: Conclusion The ratio is $x_{\mathrm{CM}}/y_{\mathrm{CM}} = 15/9$, so $\boxed{n = 15}$.

Quick Tip to Solve: Use the outer rectangle minus its top central square; symmetry immediately fixes the x-coordinate of the centre of mass.

---

57. Resistance of a wire at $0^\circ\mathrm{C}, 100^\circ\mathrm{C}$ and $t^\circ\mathrm{C}$ is found to be $10\Omega, 10.2\Omega$ and 10.95Ω respectively. The temperature t in Kelvin scale is _____. [4 marks]

Solution:

Concept:

- Over the stated temperature range, take the resistance to vary linearly: $R_t = R_0(1 + \alpha t)$, where t is measured in degrees Celsius and α is the temperature coefficient referred to zero Celsius.
- For the same wire, resistance increments are proportional to Celsius temperature increments. Convert the resulting temperature using $T = t + 273$ for the integer Kelvin value requested.

Step 1: Determine the resistance change per degree The initial resistance is $R_0 = 10\,\Omega$. At $100^\circ\mathrm{C}$, the

increase is only $10.2 - 10 = 0.2\,\Omega$, so the slope is

$$\frac{\Delta R}{\Delta t} = \frac{0.2}{100} = 0.002\,\Omega\,^{\circ}\mathrm{C}^{-1}.$$

Equivalently, $\alpha = 0.002/10 = 2 \times 10^{-4}\,^{\circ}\mathrm{C}^{-1}$. The reference resistance must be the value at zero Celsius, rather than the value at 100 Celsius.

Step 2: Find the Celsius temperature and convert it The resistance at the unknown temperature exceeds the zero-Celsius value by $10.95 - 10 = 0.95\,\Omega$. Therefore

$$t = \frac{0.95}{0.002} = 475^{\circ}\mathrm{C}.$$

As a check, $10[1 + (2 \times 10^{-4})(475)] = 10.95\,\Omega$, which reproduces the given measurement. Now add the Celsius-to-Kelvin offset:

$$T = 475 + 273 = 748\,\mathrm{K}.$$

Using the exact offset 273.15 would give 748.15 K, with the same nearest integer.

Step 3: Conclusion The required temperature on the Kelvin scale is $\boxed{748\,\mathrm{K}}$. The numerical value to enter is $\boxed{748}$.

Quick Tip to Solve: Use the ratio $t/100 = (R_t - R_0)/(R_{100} - R_0)$, then add 273; do not use the full resistance in place of its increase.

58. A parallel beam of monochromatic light of wavelength 600 nm passes through single slit of 0.4 mm width. Angular divergence corresponding to second order minima would be _____ $\times 10^{-3}$ rad.

[4 marks]

Solution:

Concept:

- A single-slit minimum of order n satisfies $a \sin\theta_n = n\lambda$, where a is the slit width and θ_n is measured from the forward central direction.
- Angular divergence between the two symmetric minima is $2\theta_n$. At the small angles here, $\sin\theta_n \simeq \theta_n$, giving $2n\lambda/a$.

Step 1: Calculate the position of one second-order minimum Convert both lengths into metres before forming their ratio:

$$\lambda = 600 \times 10^{-9}\,\mathrm{m}, \qquad a = 0.4 \times 10^{-3}\,\mathrm{m}.$$

For the second-order minimum set $n = 2$, not 1. The diffraction condition gives

$$\sin\theta_2 = \frac{2(600 \times 10^{-9})}{0.4 \times 10^{-3}} = 3 \times 10^{-3}.$$

The angle is very small, so $\theta_2 \simeq 3 \times 10^{-3}\,\mathrm{rad}$. This is the angle on one side of the central axis.

Step 2: Obtain the full angular divergence The negative-angle second minimum occurs at $-\theta_2$, because

the diffraction pattern is symmetric about the forward direction. The divergence corresponding to these two minima is their angular separation, rather than the one-sided displacement:

$$\Delta\theta = \theta_2 - (-\theta_2) = 2\theta_2.$$

Hence

$$\Delta\theta = 2(3 \times 10^{-3}) = 6 \times 10^{-3}\,\text{rad}.$$

Using the exact inverse sine instead changes only insignificant digits. A result of 3 would omit the second side, whereas the central maximum bounded by the first minima would have a different width.

Step 3: Conclusion The angular divergence is $\boxed{6 \times 10^{-3}\,\text{rad}}$, so the blank is $\boxed{6}$.

Quick Tip to Solve: First find the one-sided angle from $a \sin\theta = n\lambda$; double it only when the question asks for divergence between symmetric minima.

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59. An electron with kinetic energy 5 eV enters a region of uniform magnetic field of 3 μT perpendicular to its direction. An electric field E is applied perpendicular to the direction of velocity and magnetic field. The value of E, so that electron moves along the same path, is _____ NC^{-1}.
(Given, mass of electron $= 9 \times 10^{-31}$ kg, electric charge $= 1.6 \times 10^{-19}$C) [4 marks]

Solution:

Concept:

- For a charged particle to remain undeflected in mutually perpendicular electric and magnetic fields, the electric force and magnetic force must cancel: $q\mathbf{E} + q\mathbf{v} \times \mathbf{B} = 0$.
- The magnitudes satisfy $E = vB$. Determine the electron speed from the nonrelativistic kinetic-energy relation $K = mv^2/2$, converting electronvolts to joules first.

Step 1: Find the speed from the kinetic energy An electronvolt is the energy acquired by one elementary charge through a potential difference of one volt. Thus

$$K = 5(1.6 \times 10^{-19}) = 8.0 \times 10^{-19}\,\text{J}.$$

With the stated electron mass,

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(8.0 \times 10^{-19})}{9 \times 10^{-31}}} = \frac{4}{3} \times 10^{6}\,\text{m s}^{-1}.$$

This is much smaller than the speed of light, so the classical kinetic-energy expression is appropriate.

Step 2: Balance the two transverse forces The electron enters perpendicular to the magnetic field, so the magnetic-force magnitude is evB, with no additional sine factor. The electric-force magnitude is eE. A straight path requires equal magnitudes in opposite directions:

$$eE = evB \quad \Rightarrow \quad E = vB.$$

Substitute $B = 3 \times 10^{-6}\,\mathrm{T}$:

$$E = \left(\frac{4}{3} \times 10^{6}\right)(3 \times 10^{-6}) = 4\,\mathrm{N\,C^{-1}}.$$

The field direction is selected so that $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$. The negative electron charge reverses both forces and therefore does not change this cancellation condition.

Step 3: Conclusion The required electric-field magnitude is $\boxed{4\,\mathrm{N\,C^{-1}}}$. The numerical value to enter is $\boxed{4}$.

Quick Tip to Solve: Calculate v from energy in joules, then use $E = vB$; the charge cancels from the force balance but is needed to convert eV.

60. In an alpha particle scattering experiment distance of closest approach for the α particle is 4.5×10^{-14} m. If target nucleus has atomic number 80 , then maximum velocity of α-particle is _____ $\times 10^5$ m/s approximately.

($\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$SI unit, mass of α particle $= 6.72 \times 10^{-27}$ kg) [4 marks]

Solution:

Concept:

- In a head-on encounter, the incident alpha particle momentarily stops at its distance of closest approach. Conservation of mechanical energy converts its initial kinetic energy into electrostatic potential energy.
- The alpha-particle charge is $+2e$ and the target nuclear charge is $+Ze$. Thus $mv^2/2 = k(2e)(Ze)/r_0$, taking the initial separation as effectively infinite.

Step 1: Evaluate the electrostatic energy at closest approach Use $Z = 80$, $r_0 = 4.5 \times 10^{-14}\,\mathrm{m}$, $e = 1.6 \times 10^{-19}\,\mathrm{C}$, and the given $k = 9 \times 10^9$. The potential energy is

$$U = \frac{(9 \times 10^9)(2)(80)(1.6 \times 10^{-19})^2}{4.5 \times 10^{-14}} = 8.192 \times 10^{-13}\,\mathrm{J}.$$

Both charges are positive, so the potential energy is positive and increases as the separation decreases. The factor of 2 is essential because an alpha particle is a helium nucleus, not a singly charged ion.

Step 2: Convert this energy to the incident speed At the turning point the alpha particle has zero speed. The maximum speed, far from the repulsive nucleus, therefore satisfies

$$\frac{1}{2}(6.72 \times 10^{-27})v^2 = 8.192 \times 10^{-13}.$$

Solving gives

$$v = \sqrt{\frac{2(8.192 \times 10^{-13})}{6.72 \times 10^{-27}}} \simeq 1.5614 \times 10^{7}\,\mathrm{m\,s^{-1}}.$$

Expressing the answer in the stated units,

$$v \simeq 156.14 \times 10^5\,\mathrm{m\,s^{-1}}.$$

This is the incident speed; the speed at closest approach is zero, so substituting that zero as the requested maximum would confuse two different positions.

Step 3: Conclusion The maximum speed is approximately $\boxed{156 \times 10^5\,\mathrm{m\,s^{-1}}}$, giving the required numerical value $\boxed{156}$.

Quick Tip to Solve: At closest approach set the kinetic energy to zero and use the product $(2e)(Ze)$; keep the square root until the final numerical substitution.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Combustion of glucose $(C_6H_{12}O_6)$ produces CO_2 and water. The amount of oxygen (in g) required for the complete combustion of 900 g of glucose is :

[Molar mass of glucose in g mol^{-1} = 180] [4 marks]

(A) 32

(B) 480

(C) 800

(D) 960

Correct Answer: (D) 960

Solution:

Concept:

- Balance the combustion equation before converting masses. Glucose combustion is $C_6H_{12}O_6 + 6\,O_2 \longrightarrow 6\,CO_2 + 6\,H_2O$, so one mole of glucose consumes six moles of oxygen molecules.
- Use $n = m/M$ for glucose and $m = nM$ for oxygen. The molar mass of molecular oxygen O_2 is $32\,\mathrm{g\,mol^{-1}}$, not the atomic mass 16.

Step 1: Establish the oxygen requirement per mole of glucose Six carbon atoms produce six molecules of CO_2, and twelve hydrogen atoms produce six molecules of H_2O. These products contain eighteen oxygen atoms in total. Glucose supplies six of them internally, so twelve more atoms, or six molecules of O_2, are needed. Therefore the balanced equation is

$$C_6H_{12}O_6 + 6\,O_2 \longrightarrow 6\,CO_2 + 6\,H_2O.$$

The ratio 1:6 relates amounts in moles, not directly the masses of the two reactants.

Step 2: Calculate the mass and test the choices The amount of glucose in the sample is

$$n_{\text{glucose}} = \frac{900}{180} = 5\,\mathrm{mol}.$$

It consumes

$$n_{O_2} = 6(5) = 30\,\text{mol}, \qquad m_{O_2} = 30(32) = 960\,\text{g}.$$

Option A, 32 g, is only one mole of oxygen. Option B, 480 g, results from using the oxygen atomic mass 16 instead of the molecular molar mass 32. Option C, 800 g, does not satisfy the balanced 1:6 molar ratio. The mass 960 g is also consistent with each 180 g of glucose consuming 192 g of oxygen.

Step 3: Conclusion The required oxygen mass is $\boxed{960\,\text{g}}$. Hence, the correct option is (D).

Quick Tip to Solve: For glucose, first remember the balanced 1:6 glucose-to-oxygen molar ratio; multiply moles of oxygen by 32, since the reagent is O_2.

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62. Thiosulphate reacts differently with iodine and bromine in the reactions given below:

$$2\,S_2O_3^{2-} + I_2 \rightarrow S_4O_6^{2-} + 2I^-$$
$$S_2O_3^{2-} + 5Br_2 + 5H_2O \rightarrow 2SO_4^{2-} + 4Br^- + 10H^+$$

Which of the following statement justifies the above dual behaviour of thiosulphate? [4 marks]

(A) Thiosulphate undergoes oxidation by bromine and reduction by iodine in these reactions

(B) Bromine is a weaker oxidant than iodine

(C) Bromine is a stronger oxidant than iodine

(D) Bromine undergoes oxidation and iodine undergoes reduction in these reactions

Correct Answer: (C) Bromine is a stronger oxidant than iodine

Solution:

Concept:

- An oxidising agent accepts electrons and is itself reduced. Both iodine and bromine accept electrons from thiosulphate, while sulphur in thiosulphate is oxidised.
- A stronger oxidising agent can oxidise sulphur further. Bromine oxidises thiosulphate to sulphate, whereas the milder oxidant iodine produces tetrathionate.

Step 1: Compare the oxidation changes of sulphur The sum of the two sulphur oxidation numbers in $S_2O_3{}^{2-}$ is +4, giving an average of +2. In $S_4O_6{}^{2-}$, their sum is +10 and the average is +2.5. In $SO_4{}^{2-}$, sulphur is at +6. Thus both reactions oxidise sulphur, but the oxidation by bromine proceeds much further. The unequal sulphur sites in thiosulphate can also be described as +5 and -1; the average is used here only to compare the overall changes.

Step 2: Identify the stronger oxidant and reject the alternatives In the iodine reaction, I_2 becomes I^-; bromine similarly becomes bromide. Both halogens are reduced. Bromine has the greater oxidising power and can therefore form sulphate rather than stopping at tetrathionate. Option A incorrectly calls the iodine reaction a reduction of thiosulphate. Option B reverses the relative oxidising strengths. Option D incorrectly calls the bromine change oxidation. The second printed equation has a coefficient error: atom and charge balance require eight bromide ions and four bromine molecules. Its balanced form is

$$S_2O_3{}^{2-} + 4\,Br_2 + 5\,H_2O \longrightarrow 2\,SO_4{}^{2-} + 8\,Br^- + 10\,H^+.$$

This correction to the stoichiometry does not alter the identity of the stronger oxidant or the product-based reasoning.

Step 3: Conclusion Bromine is a stronger oxidising agent than iodine, explaining the greater oxidation to sulphate. Hence, the correct option is (C).

Quick Tip to Solve: Compare how far sulphur is oxidised in the products; sulphate formation indicates the stronger oxidant, while both halogens themselves are reduced.

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63. Give below are two statements: One is labelled as Assertion A and the other is labelled as Reason R:

Assertion A: The stability order of +1 oxidation state of Ga, In and Tl is Ga $<$ In $<$ Tl.

Reason R: The inert pair effect stabilizes the lower oxidation state down the group.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) A is true but R is false.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is false but R is true.

(D) Both A and R are true and R is the correct explanation of A.

Correct Answer: (D) Both A and R are true and R is the correct explanation of A.

Solution:

Concept:

- Group 13 elements have the valence configuration ns^2np^1. The $+1$ state uses the np electron while retaining the ns^2 pair; the $+3$ state additionally involves that pair.
- The inert pair effect is the increasing reluctance of the ns^2 pair to participate in bonding down the heavier p-block elements. It makes the lower oxidation state relatively more stable.

Step 1: Evaluate Assertion A For the heavier Group 13 elements, the $+1$ oxidation state becomes progressively more stable from gallium through indium to thallium:

$$\mathrm{Ga}(+1) < \mathrm{In}(+1) < \mathrm{Tl}(+1).$$

Gallium more readily exhibits the $+3$ state, while thallium strongly favours retention of its $6s^2$ pair and hence the $+1$ state. The assertion concerns the stability of the same $+1$ state for these three elements, not a comparison of their elemental reactivities. Assertion A is therefore true.

Step 2: Evaluate Reason R Down the group the filled inner d and f subshells shield imperfectly, and the outer s pair becomes less available for bonding relative to the p electron. This is the inert pair effect. Retaining the s pair yields the lower oxidation state, so the reason correctly states that this effect stabilises the lower state down the group. Reason R is true.

Step 3: Check the explanatory connection The lower oxidation state here is specifically $+1$, because only the single outer p electron is involved. Increasing retention of the s pair therefore directly

explains the stated gallium-to-thallium stability order. Option A wrongly rejects the reason; option C wrongly rejects the assertion. Option B accepts both statements but denies their direct causal connection, which is precisely the connection supplied by the inert pair effect.

Step 4: Conclusion Both A and R are true, and R correctly explains A. Hence, the correct option is (D).

Quick Tip to Solve: For Group 13, retaining ns^2 gives $+1$; stronger inert pair behaviour down the group therefore means greater stability of the $+1$ state.

64. Iron (III) catalyses the reaction between iodide and persulphate ions, in which

A. Fe^{3+} **oxidises the iodide ion**

B. Fe^{3+} **oxidises the persulphate ion**

C. Fe^{2+} **reduces the iodide ion**

D. Fe^{2+} **reduces the persulphate ion**

Choose the most appropriate answer from the options given below: [4 marks]

(A) B and C only

(B) A only

(C) A and D only

(D) B only

Correct Answer: (C) A and D only

Solution:

Concept:

- A redox catalyst provides an alternative electron-transfer pathway and is regenerated. Iron cycles between Fe^{3+} and Fe^{2+}, so it can connect an oxidant with a reductant.
- An oxidising agent accepts electrons and is reduced; a reducing agent supplies electrons and is oxidised. Persulphate, $S_2O_8{}^{2-}$, is the oxidant and iodide is the reductant.

Step 1: Identify the iodide-oxidation step Iron(III) accepts an electron from iodide and becomes iron(II). Combining two one-electron transfers gives

$$2\,Fe^{3+} + 2\,I^- \longrightarrow 2\,Fe^{2+} + I_2.$$

Iodide loses electrons to form iodine; therefore Fe^{3+} oxidises iodide. Statement A is true. Statement B reverses the role of persulphate: it is already the electron acceptor, not the species oxidised by iron(III).

Step 2: Regenerate iron(III) with persulphate The iron(II) produced above donates electrons to persulphate:

$$S_2O_8{}^{2-} + 2\,Fe^{2+} \longrightarrow 2\,SO_4{}^{2-} + 2\,Fe^{3+}.$$

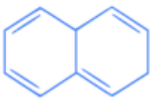
Thus iron(II) reduces persulphate and statement D is true. It does not reduce iodide, whose iodine is already in the -1 oxidation state, so C is false. Adding the steps cancels both forms of iron and yields

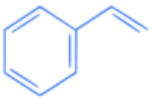
$S_2O_8^{2-} + 2I^- \longrightarrow 2SO_4^{2-} + I_2$, confirming the catalytic cycle.

Step 3: Conclusion Only statements A and D describe the two electron-transfer steps correctly. Hence, the correct option is (C).

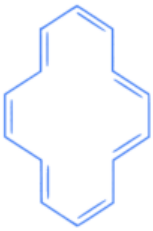
Quick Tip to Solve: Write the two iron half-cycles first: Fe^{3+} accepts electrons from iodide, and Fe^{2+} gives electrons to persulphate.

65. Which of the following are aromatic? [4 marks]

A. 

B. 

C.

D. 

(A) B and D only

(B) A and C only

(C) C and D only

(D) A and B only

Correct Answer: (A) B and D only

Solution:

Concept:

- Aromaticity requires a cyclic, continuously conjugated and sufficiently planar array of p orbitals containing $4n + 2$ pi electrons. A count of double bonds alone does not establish aromaticity.
- A molecule containing an intact benzene ring is aromatic even if its side chain is unsaturated. In an annulene, the geometry of the entire conjugated ring must also permit delocalisation.

Step 1: Examine structures A and B In A, the fused-ring drawing contains only four double bonds and does not have an uninterrupted aromatic p-orbital circuit: the saturated ring-junction carbon interrupts conjugation. It must not be mistaken for naphthalene, which has five double bonds. In B, the six-membered ring has three alternating double bonds. Its six pi electrons satisfy $4n + 2 = 6$, with $n = 1$, and the vinyl substituent does not destroy the benzene aromatic sextet. Thus B is aromatic whereas A is not.

Step 2: Check the annulene rings C and D C is the ten-membered conjugated ring. Although its five double bonds provide ten pi electrons, steric interaction of the inward-facing hydrogens prevents the required planar arrangement; it is non-aromatic. D is the larger fourteen-membered conjugated ring. It has seven double bonds, giving fourteen pi electrons, and can sustain the aromatic delocalised circuit. Here $4n + 2 = 14$ gives $n = 3$. Therefore D is aromatic. Options containing A or C fail either the continuous-conjugation test or the geometrical requirement.

Step 3: Conclusion The aromatic compounds are B and D only. Hence, the correct option is (A).

Quick Tip to Solve: Apply all three tests together: continuous cyclic conjugation, suitable planarity, and $4n + 2$ pi electrons; a ten-electron count alone does not make [10]annulene aromatic.

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66. Among the following halogens

$$F_2, Cl_2, Br_2 \textbf{ and } I_2$$

Which can undergo disproportionation reactions? [4 marks]

(A) F_2 and Cl_2

(B) Cl_2, Br_2 and I_2

(C) Only I_2

(D) F_2, Cl_2 and Br_2

Correct Answer: (B) Cl_2, Br_2 and I_2

Solution:

Concept:

- Disproportionation is a redox reaction in which the same element in one oxidation state is simultaneously oxidised and reduced. An elemental halogen begins at oxidation state zero.
- Fluorine, the most electronegative element, has only the -1 oxidation state in its compounds. Chlorine, bromine and iodine can have both negative and positive oxidation states.

Step 1: Exclude elemental fluorine For F_2 to disproportionate, part of its fluorine would have to be oxidised from zero to a positive oxidation state while another part became fluoride. Fluorine cannot show the required positive oxidation state, so it cannot undergo disproportionation. Its reactions with water or alkali must instead oxidise another element while fluorine is reduced. This excludes both options that contain F_2.

Step 2: Identify the halogens with both pathways Chlorine, bromine and iodine can each be reduced to a halide and oxidised to an oxyanion. A standard example is

$$Cl_2 + 2\,OH^- \longrightarrow Cl^- + ClO^- + H_2O.$$

The chlorine oxidation states change from zero to -1 and $+1$. Under suitable alkaline conditions, bromine and iodine also disproportionate; in hot concentrated alkali the general reaction is

$$3\,X_2 + 6\,OH^- \longrightarrow 5\,X^- + XO_3{}^- + 3\,H_2O.$$

For $X = \mathrm{Cl}, \mathrm{Br}, \mathrm{I}$, the oxidised halogen has state $+5$. Hence restricting the answer to iodine alone omits two valid halogens.

Step 3: Conclusion $\mathrm{Cl_2}$, $\mathrm{Br_2}$ and $\mathrm{I_2}$ can undergo disproportionation; $\mathrm{F_2}$ cannot. Hence, the correct option is (B).

Quick Tip to Solve: An element at oxidation state zero can disproportionate only if it can reach both a higher and a lower oxidation state; fluorine fails the higher-state requirement.

67. Which among the following compounds will undergo fastest S_N2 reaction. [4 marks]

(A) (1-Bromoethyl)cyclobutane

(B) (2-Bromopropan-2-yl)cyclobutane

(C) (Bromomethyl)cyclobutane

(D) 1-Bromo-1-methylcyclobutane

Correct Answer: (C) (Bromomethyl)cyclobutane

Solution:

Concept:

- An $\mathrm{S_N2}$ reaction is a concerted substitution in which the nucleophile attacks the carbon bearing the leaving group from the side opposite that group.
- With the same leaving group and comparable conditions, steric crowding controls accessibility: methyl and primary halides react faster than secondary halides, while tertiary centres strongly suppress the $\mathrm{S_N2}$ pathway.

Step 1: Classify the carbon bonded to bromine The ring itself is a cyclobutyl group, but the classification must use the carbon directly bonded to Br. In A, this carbon is attached to a methyl group and the ring, making it secondary. In B, it is attached to two methyl groups and the ring, so it is tertiary. In C, the bromine is bonded to the exocyclic $\mathrm{CH_2}$ group, whose carbon has only one carbon neighbour; C is a primary bromide. In D, the brominated ring carbon is bonded to the two neighbouring ring carbons and a methyl group, so D is tertiary.

Step 2: Compare backside access to the reacting centre All four compounds have bromide as the leaving group, so a change in leaving-group ability cannot explain their relative rates. C has the least crowded reacting carbon, allowing the nucleophile to approach opposite the C–Br bond with the smallest steric obstruction. A has an additional alkyl group directly at that carbon and is slower. B and D have three carbon substituents around the reacting centre and are particularly unfavourable for the concerted backside attack. The possible stability of a carbocation is not the deciding criterion because an $\mathrm{S_N2}$ reaction has no carbocation intermediate.

Step 3: Conclusion (Bromomethyl)cyclobutane, structure C, undergoes the fastest $\mathrm{S_N2}$ reaction. Hence, the correct option is (C).

Quick Tip to Solve: Locate the C–Br carbon before calling a halide primary, secondary or tertiary; a ring attached to a $\mathrm{CH_2Br}$ group does not make that carbon tertiary.

68. Match List I with List II

	LIST I (Molecule)		LIST II (Shape)
A.	NH_3	**I.**	**Square pyramid**
B.	BrF_5	**II.**	**Tetrahedral**
C.	PCl_5	**III.**	**Trigonal pyramidal**
D.	CH_4	**IV.**	**Trigonal bipyramidal**

Choose the correct answer from the options given below: [4 marks]

(A) A-II, B-IV, C-I, D-III

(B) A-III, B-I, C-IV, D-II

(C) A-IV, B-III, C-I, D-II

(D) A-III, B-IV, C-I, D-II

Correct Answer: (B) A-III, B-I, C-IV, D-II

Solution:

Concept:

- VSEPR theory counts bonding pairs and lone pairs on the central atom. Electron-pair geometry includes both, whereas molecular shape describes only the positions of the bonded atoms.
- Lone pairs occupy space and change the molecular shape even when the electron-pair arrangement is tetrahedral or octahedral. Use the central atom valence count to distinguish these cases.

Step 1: Assign the shapes of ammonia and bromine pentafluoride In NH_3, nitrogen has five valence electrons. Three N–H bonds leave one lone pair, giving four electron domains. The electron-pair arrangement is tetrahedral, but the molecular shape is trigonal pyramidal, so A matches III. In BrF_5, bromine has seven valence electrons; its five bonds leave one lone pair. Six electron domains give an octahedral arrangement, and removing the lone-pair vertex leaves a square pyramid. Therefore B matches I.

Step 2: Assign the shapes without central lone pairs In PCl_5, phosphorus forms five bonds and has no central lone pair. The five bond directions form a trigonal bipyramid, with three equatorial and two axial positions, so C matches IV. In CH_4, carbon forms four bonds and has no lone pair. Its four equivalent bond directions form a tetrahedron, so D matches II. The completed sequence is thus III, I, IV, II. Every other option misassigns ammonia or bromine pentafluoride; in particular, treating ammonia as tetrahedral confuses electron-domain geometry with molecular shape.

Step 3: Conclusion The matching is A–III, B–I, C–IV and D–II. Hence, the correct option is (B).

Quick Tip to Solve: Count lone pairs before naming the shape: NH_3 and BrF_5 each have one, while CH_4 and PCl_5 have none.

69. Match List I with List II

	LIST I (Elements)		LIST II (Properties in their respective groups)
A.	Cl, S	**I.**	**Elements with highest electronegativity**
B.	Ge, As	**II.**	**Elements with largest atomic size**
C.	Fr, Ra	**III.**	**Elements which show properties of both metals and non-metal**
D.	F, O	**IV.**	**Elements with highest negative electron gain enthalpy**

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-II, C-I, D-IV

(B) A-II, B-I, C-IV, D-III

(C) A-II, B-III, C-IV, D-I

(D) A-IV, B-III, C-II, D-I

Correct Answer: (D) A-IV, B-III, C-II, D-I

Solution:

Concept:

- Within a group, atomic size generally increases downwards, while electronegativity generally increases upwards. Germanium and arsenic are metalloids, showing both metallic and non-metallic properties.
- Electron gain enthalpy is the enthalpy change on adding an electron to an isolated gaseous atom. A more negative value means a more exothermic process; this is not identical to electronegativity.

Step 1: Match the size and metalloid entries Francium and radium are at the bottom of their respective groups, with the largest number of occupied shells among the listed group members. They therefore represent the largest atomic sizes: C matches II. Germanium and arsenic lie near the boundary separating metals and non-metals. Their intermediate character identifies them as metalloids, so B matches III. These assignments already eliminate options A and B, which place these pairs in unsuitable categories.

Step 2: Distinguish electronegativity from electron gain enthalpy Fluorine and oxygen have the highest electronegativities in their respective groups, so D matches I. However, their compact $2p$ shells cause substantial electron-electron repulsion when an extra electron is added. Chlorine and sulphur, with larger $3p$ orbitals, have more negative electron gain enthalpies than fluorine and oxygen respectively. Thus A matches IV, describing the highest negative electron gain enthalpies within those groups. Option C interchanges the size and electron-gain assignments and is incorrect. The final sequence is IV, III, II, I.

Step 3: Conclusion The correct matching is A–IV, B–III, C–II and D–I. Hence, the correct option is (D).

Quick Tip to Solve: Remember the $2p$-shell exceptions: chlorine accepts an extra electron more exothermically than fluorine, and sulphur more exothermically than oxygen.

70. Match List I with List II

	LIST I (Name of the test)		LIST II (Reaction sequence involved) [M is metal]
A.	**Borax bead test**	**I.**	$MCO_3 \rightarrow MO \xrightarrow[+\Delta]{Co(NO_3)_2} CoO \cdot MO$
B.	**Charcoal cavity test**	**II.**	$MCO_3 \rightarrow MCl_2 \rightarrow M^{2+}$
C.	**Cobalt nitrate test**	**III.**	$MSO_4 \xrightarrow[\Delta]{Na_2B_4O_7} M(BO_2)_2 \rightarrow MBO_2 \rightarrow M$
D.	**Flame test**	**IV.**	$MSO_4 \xrightarrow[\Delta]{Na_2CO_3} MCO_3 \rightarrow MO \rightarrow M$

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-I, C-IV, D-II

(B) A-III, B-IV, C-I, D-II

(C) A-III, B-II, C-IV, D-I

(D) A-III, B-I, C-II, D-IV

Correct Answer: (B) A-III, B-IV, C-I, D-II

Solution:

Concept:

- Identify each qualitative test from its characteristic reagent and final product. Borax produces metal borates, charcoal promotes reduction, and cobalt nitrate produces a coloured mixed oxide.
- In the flame test, conversion to a volatile chloride helps introduce the metal into the flame. Excitation and subsequent electronic relaxation produce its characteristic flame colour.

Step 1: Identify the borax and charcoal sequences Sequence III explicitly uses $Na_2B_4O_7$, the borax reagent. Heating borax forms a glassy borate medium, and metal oxides dissolve to form coloured metaborates; the printed sequence also shows subsequent reduction. Therefore the borax bead test A matches III. Sequence IV starts with Na_2CO_3, converting the salt to carbonate and then oxide. On charcoal in a reducing flame, that oxide is reduced to the metal. This is the charcoal cavity test, so B matches IV.

Step 2: Identify the cobalt nitrate and flame sequences Sequence I decomposes the carbonate to the metal oxide and then heats it with $Co(NO_3)_2$. The cobalt nitrate gives cobalt oxide, which combines with the sample oxide to form the characteristic coloured mixed oxide written as $CoO \cdot MO$. Hence C matches I. Sequence II converts the sample to a chloride before producing metal species in the flame, identifying D as II. The identities of the reagents make the matching unique: exchanging I and IV would confuse the cobalt nitrate treatment with charcoal reduction, while exchanging II with either would misidentify the chloride preparation.

Step 3: Conclusion The matching is A–III, B–IV, C–I and D–II. Hence, the correct option is (B).

Quick Tip to Solve: Use the reagent as a fingerprint: borax gives III, sodium carbonate on charcoal gives IV, cobalt nitrate gives I, and chloride formation identifies the flame test II.

71. Number of Complexes with even number of electrons in t_{2g} orbitals is -

$$[Fe\,(H_2O)_6]^{2+}\,,[Co\,(H_2O)_6]^{2+}\,,[Co\,(H_2O)_6]^{3+}\,,[Cu\,(H_2O)_6]^{2+}\,,[Cr\,(H_2O)_6]^{2+}$$

[4 marks]

(A) 3

(B) 2

(C) 1

(D) 5

Correct Answer: (A) 3

Solution:

Concept:

- In an octahedral field, the five d orbitals split into lower-energy t_{2g} and higher-energy e_g sets. Count electrons in the lower set, not unpaired electrons.
- Water is generally a weak-field ligand. Determine the metal ion d count from its oxidation state, then fill the octahedral orbitals using Hund's rule.

Step 1: Find the metal-ion configurations Water is neutral, so the charges on the complexes are the oxidation states of their metals. The five ions are Fe^{2+} : d^6, Co^{2+} : d^7, Co^{3+} : d^6, Cu^{2+} : d^9, and Cr^{2+} : d^4. The neutral-atom outer s electrons are removed before d electrons when obtaining these configurations.

Step 2: Count the lower-set electrons For the high-spin aqua ions, the occupancies are $d^6 : t_{2g}^4 e_g^2$, $d^7 : t_{2g}^5 e_g^2$, $d^9 : t_{2g}^6 e_g^3$, and $d^4 : t_{2g}^3 e_g^1$. The cobalt(III) d^6 ion also has an even lower-set population: it is 6 if low spin and 4 if high spin. Consequently, the parity required here is independent of that distinction.

Step 3: Select the qualifying complexes The iron(II), cobalt(III), and copper(II) complexes have even t_{2g} populations. Cobalt(II) has five and chromium(II) has three, both odd. Thus exactly three complexes qualify; counting only one or two misses an even population, while five incorrectly includes the two odd populations.

Step 4: Conclusion The required number of complexes is $\boxed{3}$. Hence, the correct option is (A).

Quick Tip to Solve: For a parity question, tabulate the t_{2g} electron count; do not substitute the number of unpaired electrons.

72. For the given hypothetical reactions, the equilibrium constants are as follows :

$$\mathrm{X} \rightleftharpoons \mathrm{Y}; \mathrm{K}_1 = 1.0$$
$$\mathrm{Y} \rightleftharpoons \mathrm{Z}; \mathrm{K}_2 = 2.0$$
$$\mathrm{Z} \rightleftharpoons \mathrm{W}; \mathrm{K}_3 = 4.0$$

The equilibrium constant for the reaction $\mathrm{X} \rightleftharpoons \mathrm{W}$ is [4 marks]

(A) 12.0

(B) 8.0

(C) 6.0

(D) 7.0

Correct Answer: (B) 8.0

Solution:

Concept:

- When chemical equations are added, their equilibrium constants are multiplied. This follows either from cancelling concentration ratios or adding standard Gibbs energy changes.
- For a reaction written in the forward direction, use its given constant. Reversing a reaction would require its reciprocal, but none of these steps is reversed.

Step 1: Combine the elementary equilibria The successive reactions are $\mathrm{X} \rightleftharpoons \mathrm{Y}$, $\mathrm{Y} \rightleftharpoons \mathrm{Z}$, and $\mathrm{Z} \rightleftharpoons \mathrm{W}$. Adding them cancels the intermediate species Y and Z because each is produced once and consumed once. The resulting net equation is exactly $\mathrm{X} \rightleftharpoons \mathrm{W}$; every reaction has coefficient one, so no square root or power of a constant is involved.

Step 2: Multiply the concentration ratios Using the same equilibrium convention for all steps,

$$K_1 = \frac{[Y]}{[X]}, \qquad K_2 = \frac{[Z]}{[Y]}, \qquad K_3 = \frac{[W]}{[Z]}.$$

Multiplication cancels the intermediate concentrations and gives

$$K_1 K_2 K_3 = \frac{[W]}{[X]} = K.$$

Substitution yields $K = 1.0 \times 2.0 \times 4.0 = 8.0$.

Step 3: Check the alternatives The value 7.0 is the sum of the three constants, but constants are not added when reactions are added. Values 6.0 and 12.0 likewise do not follow from multiplication in the printed directions. A constant greater than one is also consistent with the net forward-favoured conversion.

Step 4: Conclusion The equilibrium constant is $\boxed{8.0}$. Hence, the correct option is (B).

Quick Tip to Solve: Add equations on paper first; then multiply their constants, taking a reciprocal only for an equation you reverse.

73. An octahedral complex with the formula $CoCl_3 \cdot nNH_3$ upon reaction with excess of $AgNO_3$ solution gives 2 moles of $AgCl$. Consider the oxidation state of Co in the complex is 'x'. The value of "$x + n$" is _____. [4 marks]

(A) 6

(B) 5

(C) 3

(D) 8

Correct Answer: (D) 8

Solution:

Concept:

- Excess $AgNO_3$ precipitates ionisable chloride outside the coordination sphere as $AgCl$; coordinated chloride does not contribute to this immediate precipitation.
- An octahedral complex has coordination number six. Both ammonia and chloride are monodentate, while ammonia is electrically neutral.

Step 1: Locate the chloride ions One mole of the complex gives two moles of $AgCl$, so two of its three chlorides are outside the coordination sphere. Only one chloride is coordinated to cobalt. The compound must therefore have the form $[Co(NH_3)_nCl]Cl_2$. This uses all three chloride ions and distinguishes total chlorine content from ionisable chloride content.

Step 2: Use octahedral coordination The one coordinated chloride supplies one donor atom. To reach coordination number six, ammonia must supply the other five donor atoms. Since each NH_3 is monodentate,

$$n + 1 = 6 \quad \Rightarrow \quad n = 5.$$

The coordination entity is therefore $[Co(NH_3)_5Cl]^{2+}$.

Step 3: Calculate the oxidation state and sum The two external chlorides give the complex cation a charge of $+2$. With neutral ammonia and one inner-sphere chloride of charge -1, charge balance gives

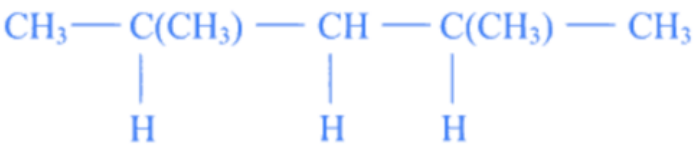
$$x - 1 = +2 \quad \Rightarrow \quad x = 3.$$

Hence $x + n = 3 + 5 = 8$. The other choices confuse the number of ammonia ligands, oxidation state, or total coordination number with the requested sum.

Step 4: Conclusion The value is $\boxed{x + n = 8}$. Hence, the correct option is (D).

Quick Tip to Solve: Use precipitated chloride to find the outer sphere first, then use coordination number and charge balance as separate equations.

74. In the given compound, the number of 2° carbon atom/s is _____. [4 marks]



(A) Two

(B) Three

(C) Four

(D) One

Correct Answer: (D) One

Solution:

Concept:

- A secondary carbon is directly bonded to exactly two other carbon atoms. The degree counts carbon neighbours, not the total number of bonds or attached hydrogen atoms.
- Each terminal or branch methyl carbon is primary. A carbon attached to two chain carbons and an additional methyl branch is tertiary.

Step 1: Read the connectivity The displayed carbon skeleton can be written as

$$CH_3-CH(CH_3)-CH_2-CH(CH_3)-CH_3$$

The central printed CH with its additional downward H is the CH_2 group. Reading the explicit downward hydrogens prevents mistaking that central atom for a branch point. There are five carbons in the horizontal chain and two methyl branches.

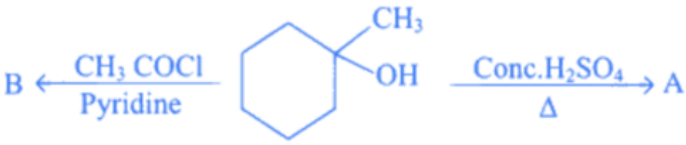
Step 2: Classify the seven carbons The first and last chain carbons each have one carbon neighbour, as do the two branch methyl carbons. These four are primary. The second and fourth chain carbons are attached to three carbons each: a terminal carbon, the central carbon, and a methyl branch. They are tertiary. The central chain carbon alone is bonded to exactly two carbon atoms, namely those two branched neighbours.

Step 3: Count only secondary centres There is thus one secondary carbon, four primary carbons and two tertiary carbons, totalling seven carbons as required by the structure. The alternatives two, three and four overcount by including carbons bearing three carbon neighbours or by counting branch methyl groups. The number of hydrogens is useful as a check here, but connectivity is the defining criterion.

Step 4: Conclusion The compound has $\boxed{1}$ secondary carbon atom. Hence, the correct option is (D).

Quick Tip to Solve: Mark the number of directly bonded carbon neighbours beside every vertex before counting primary, secondary or tertiary carbons.

75. Identify the major products A and B respectively in the following set of reactions. [4 marks]



(A) A: methylenecyclohexane; B: 2-acetyl-1-methylcyclohexan-1-ol.

(B) A: 1-methylcyclohex-1-ene; B: 1-methylcyclohexyl ethanoate.

(C) A: methylenecyclohexane; B: 1-acetyl-1-methylcyclohexane.

(D) A: 1-methylcyclohex-1-ene; B: 4-acetyl-1-methylcyclohexan-1-ol.

Correct Answer: (B) A: 1-methylcyclohex-1-ene; B: 1-methylcyclohexyl ethanoate.

Solution:

Concept:

- Hot concentrated H_2SO_4 dehydrates tertiary alcohols by an E1 pathway; the more substituted alkene is normally the major product.
- Acetyl chloride in pyridine acylates the alcohol oxygen, forming an ethanoate ester. It does not introduce an acetyl group into the saturated carbon skeleton.

Step 1: Identify the alcohol and eliminate water The starting compound is 1-methylcyclohexan-1-ol: the carbon carrying OH is also attached to a methyl group and two ring carbons, so it is tertiary. Protonation and loss of water give a tertiary carbocation at carbon 1. Removal of a beta hydrogen from either adjacent ring carbon produces the same endocyclic product, 1-methylcyclohex-1-ene.

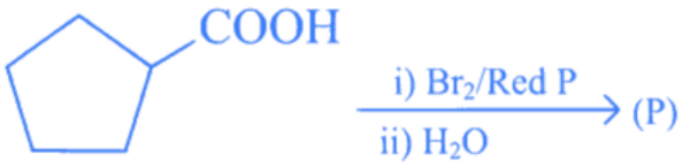
Step 2: Compare the possible alkenes The endocyclic alkene has three carbon substituents on the double bond and is more substituted than the exocyclic methylenecyclohexane alternative. Zaitsev elimination therefore makes 1-methylcyclohex-1-ene the major product A. The options showing the exocyclic double bond consequently fail the major-product requirement.

Step 3: Determine the separate acylation product In the other reaction, the alcohol oxygen attacks acetyl chloride, and pyridine removes the liberated acid. The O–H hydrogen is replaced by the acetyl group, giving an $-OCOCH_3$ substituent on the original carbon. Product B is 1-methylcyclohexyl ethanoate. The ring and methyl substituent remain intact; a carbon-bonded $-COCH_3$ group with the OH retained would be a different transformation. The ester has an O–C bond to the acetyl carbon, which identifies option B unambiguously.

Step 4: Conclusion A is 1-methylcyclohex-1-ene and B is 1-methylcyclohexyl ethanoate. Hence, the correct option is (B).

Quick Tip to Solve: For acetyl chloride/pyridine, trace the original alcohol oxygen into the ester; for hot acid, compare alkene substitution.

76. Identify the product (P) in the following reaction: [4 marks]



(A) Cyclopentanecarbonyl bromide (ring bonded to COBr).

(B) 1-Bromocyclopentane-1-carbaldehyde (Br and CHO on the same ring carbon).

(C) 2-Bromocyclopentane-1-carboxylic acid (Br on the adjacent ring carbon).

(D) 1-Bromocyclopentane-1-carboxylic acid (Br and COOH on the same ring carbon).

Correct Answer: (D) 1-Bromocyclopentane-1-carboxylic acid (Br and COOH on the same ring carbon).

Solution:

Concept:

- Br_2 with red phosphorus followed by water is the Hell–Volhard–Zelinsky sequence, which brominates a carboxylic acid at its alpha carbon.
- The alpha carbon is the carbon directly attached to the carboxyl carbon. In cyclopentanecarboxylic acid it is the ring carbon that already bears the COOH group.

Step 1: Locate the reactive hydrogen The substrate is cyclopentanecarboxylic acid. Its carboxyl group is bonded directly to a ring carbon bearing one hydrogen. That hydrogen is alpha to the carbonyl and is available for replacement. The neighbouring ring carbons are beta to the carboxyl carbon, so placing bromine there would not be alpha halogenation.

Step 2: Follow bromination and hydrolysis Red phosphorus and bromine generate the phosphorus bromide reagent that converts the acid into an acyl bromide. Enolisation permits bromination at the alpha carbon, replacing its hydrogen with bromine. The water in the second step then hydrolyses the acyl bromide back to the carboxylic acid. Thus the final functional group is COOH, not COBr or CHO.

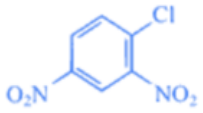
Step 3: Match the connectivity to the choices The product has both Br and COOH attached to the same ring carbon: 1-bromocyclopentane-1-carboxylic acid. Option A is an acyl bromide intermediate without alpha bromination. Option B changes the acid to an aldehyde, for which no reducing reagent is present. Option C places bromine on the adjacent ring carbon and therefore gives the wrong site of substitution.

Step 4: Conclusion P is 1-bromocyclopentane-1-carboxylic acid. Hence, the correct option is (D).

Quick Tip to Solve: Locate the alpha carbon by starting at the carboxyl carbon; the first skeletal carbon, even within a ring, receives bromine.

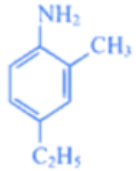
77. Given below are two statements :

In the light of the above statements, choose the most appropriate answer from the options given below: [4 marks]

Statement I :  IPUAC name of Compound A is

Compound A

4-chloro-1,3-dinitrobenzene.

Statement II :  IPUAC name of Compound B is

Compound B

4-ethyl-2-methylaniline.

(A) Both Statement I and Statement II are incorrect.

(B) Both Statement I and Statement II are correct.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (D) Statement I is incorrect but Statement II is correct.

Solution:

Concept:

- For substituted benzene without a principal suffix group, choose the numbering giving the lowest set of substituent locants; alphabetical order alone does not override that rule.
- For an aniline parent, the carbon bearing NH_2 is carbon 1. Other substituents receive the lowest possible locants and are listed alphabetically.

Step 1: Check Statement I The first ring has chlorine adjacent to one nitro group, with the other nitro group para to chlorine. Numbering from the chlorine-bearing carbon towards the adjacent nitro group gives locants 1,2,4. Numbering to obtain the printed name 4-chloro-1,3-dinitrobenzene instead gives the set 1,3,4. At the first point of difference, 2 is smaller than 3, so the correct name is 1-chloro-2,4-dinitrobenzene. Statement I is incorrect.

Step 2: Check Statement II In the second structure the parent is aniline, and the amino-bearing carbon is carbon 1. Numbering towards the adjacent methyl group assigns methyl locant 2 and the para ethyl group locant 4; the reverse direction would give methyl locant 6. Listing substituents alphabetically places ethyl before methyl, producing 4-ethyl-2-methylaniline. Statement II is correct.

Step 3: Choose the combined verdict The required truth pattern is therefore false for I and true for II. Options A and B assign the same truth value to both statements and therefore fail. Option C reverses the required evaluations. Only option D agrees with both names.

Step 4: Conclusion Statement I is incorrect but Statement II is correct. Hence, the correct option is (D).

Quick Tip to Solve: Compare locant sets at their first difference before alphabetising substituent names; aniline fixes the amino carbon as position 1.

78. Given below are two statements:

Statement I: $\mathrm{N(CH_3)_3}$ **and** $\mathrm{P(CH_3)_3}$ **can act as ligands to form transition metal complexes.**

Statement II: As N and P are from same group, the nature of bonding of $\mathrm{N(CH_3)_3}$ **and** $\mathrm{P(CH_3)_3}$ **is always same with transition metals.**

In the light of the above statements, choose the most appropriate answer from the options given below: [4 marks]

(A) Both Statement I and Statement II are incorrect.

(B) Both Statement I and Statement II are correct.

(C) Statement I is incorrect but Statement II is correct.

(D) Statement I is correct but Statement II is incorrect.

Correct Answer: (D) Statement I is correct but Statement II is incorrect.

Solution:

Concept:

- A ligand donates an available lone pair to a transition-metal centre, forming a coordinate sigma bond. Both tertiary amines and phosphines possess such a donor lone pair.
- The ability to donate electron density does not imply identical bonding. Phosphines can also accept metal electron density into suitable ligand antibonding orbitals, producing a pi-backbonding contribution.

Step 1: Assess Statement I In trimethylamine, $N(CH_3)_3$, nitrogen has three carbon–nitrogen bonds and one lone pair. In trimethylphosphine, $P(CH_3)_3$, phosphorus likewise has a lone pair. Each can supply this electron pair to a metal orbital and act as a neutral monodentate ligand. Their donor atoms are nitrogen and phosphorus, respectively; no ionisation of a methyl group is required. Statement I is therefore correct.

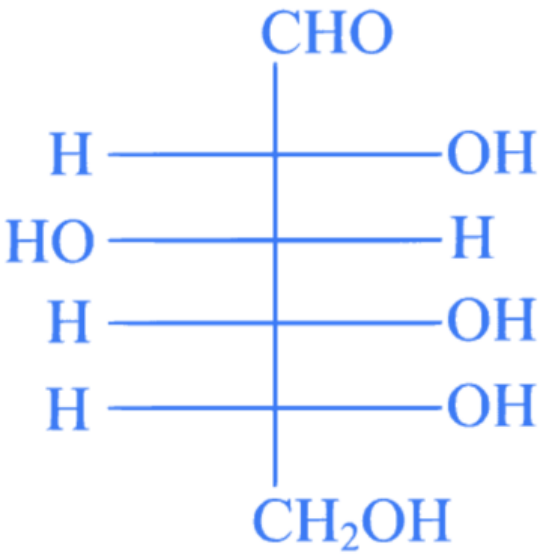
Step 2: Assess Statement II Although nitrogen and phosphorus belong to the same periodic group, their ligand orbitals and acceptor properties differ. Trimethylamine behaves predominantly as a sigma donor. A coordinated phosphine can participate in sigma donation together with metal-to-ligand pi back donation, involving appropriate phosphorus–carbon antibonding orbitals. Thus the metal–ligand interaction need not have the same character for the two ligands. The word “always makes Statement II false.

Step 3: Select the statement combination The required truth pattern is I true and II false. Option A incorrectly rejects the donor ability of both ligands; option B incorrectly makes the bonding universally identical; option C reverses both assessments. Only option D contains the two independently established truth values.

Step 4: Conclusion Statement I is correct but Statement II is incorrect. Hence, the correct option is (D).

Quick Tip to Solve: For amine versus phosphine ligands, check sigma donation first and then ask whether pi acceptance can also contribute; group membership alone does not settle the bonding.

79. The incorrect statement regarding the given structure is [4 marks]



(A) can be oxidized to a dicarboxylic acid with Br_2 water

(B) despite the presence of $-CHO$ does not give Schiff's test

(C) will coexist in equilibrium with 2 other cyclic structure

(D) has 4 asymmetric carbon atom

Correct Answer: (A) can be oxidized to a dicarboxylic acid with Br_2 water

Solution:

Concept:

- The Fischer projection is open-chain D-glucose, an aldohexose. Its aldehyde carbon is C1, while C6 is the terminal primary-alcohol carbon.
- Mild oxidation with bromine water oxidises an aldehyde to a carboxylic acid. Oxidising both terminal groups to obtain a dicarboxylic acid requires a stronger oxidant, such as nitric acid.

Step 1: Identify the structure and its stereocentres The six-carbon chain has $-CHO$ at the upper end and $-CH_2OH$ at the lower end. The hydroxyl arrangement from C2 to C5 is right, left, right, right, identifying D-glucose. Each of C2, C3, C4 and C5 has four different substituents and is asymmetric. The carbonyl carbon C1 and terminal carbon C6 are not asymmetric. Thus the statement that there are four asymmetric carbon atoms is correct.

Step 2: Test the oxidation statement Bromine water is a mild oxidant and converts the aldehyde group into $-COOH$, forming gluconic acid. It does not simultaneously oxidise the terminal $-CH_2OH$ group to another carboxyl group. The dicarboxylic oxidation product, saccharic or glucaric acid, is instead obtained using strong oxidation with nitric acid. Consequently, option A attributes the wrong product to bromine water.

Step 3: Check the remaining statements Glucose exists predominantly as cyclic hemiacetals in solution. Its open-chain form is in equilibrium with the alpha and beta cyclic anomers, accounting for the two cyclic structures in option C. Consistent with its cyclic behaviour, glucose does not show the ordinary Schiff test expected of a free simple aldehyde, so option B is accepted. Together with the

four stereocentres, these observations leave only the oxidation statement incorrect.

Step 4: Conclusion The incorrect statement is that bromine water gives a dicarboxylic acid. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish mild bromine-water oxidation to gluconic acid from strong nitric-acid oxidation to the dicarboxylic saccharic acid.

80. Match List I with List II.

	List I (Compound)		List II (Colour)
A.	$Fe_4[Fe(CN)_6]_3 \cdot xH_2O$	I.	Violet
B.	$[Fe(CN)_5NOS]^{4-}$	II.	Blood Red
C.	$[Fe(SCN)]^{2+}$	III.	Prussian Blue
D.	$(NH_4)_3PO_4 \cdot 12\,MoO_3$	IV.	Yellow

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-I, C-II, D-III

(B) A-I, B-II, C-III, D-IV

(C) A-II, B-III, C-IV, D-I

(D) A-III, B-I, C-II, D-IV

Correct Answer: (D) A-III, B-I, C-II, D-IV

Solution:

Concept:

- Characteristic colours of coordination compounds are useful qualitative-analysis identifiers. Match each compound individually rather than choosing an option from only one remembered colour.
- Prussian blue, the violet nitroprusside–sulfide complex, blood-red ferric thiocyanate and yellow ammonium phosphomolybdate are standard colour associations.

Step 1: Match the two cyanide-containing compounds Compound A, $Fe_4[Fe(CN)_6]_3 \cdot xH_2O$, is hydrated ferric ferrocyanide, commonly called Prussian blue. It therefore matches entry III, rather than yellow, violet or blood red. Compound B, $[Fe(CN)_5NOS]^{4-}$, is the violet complex obtained in the nitroprusside test for sulfide. It therefore matches entry I. These are chemically distinct coordination species, so their colours cannot be interchanged merely because both contain cyanide ligands.

Step 2: Match the remaining analytical compounds Compound C, $[Fe(SCN)]^{2+}$, is the blood-red iron(III)–thiocyanate complex used in the qualitative test for ferric ions. Hence C matches II. Compound D, $(NH_4)_3PO_4 \cdot 12\,MoO_3$, is ammonium phosphomolybdate, the yellow precipitate associated with the phosphate test. Hence D matches IV. The Roman numerals refer to the colour entries, while the letters refer to compounds.

Step 3: Compare all four pairs with the choices The complete matching is A–III, B–I, C–II, D–IV. Option A incorrectly assigns the yellow and Prussian-blue entries to A and D. Option B assigns violet to A and shifts the iron-complex colours. Option C disagrees with all these characteristic associations. Option D reproduces all four matches simultaneously.

Step 4: Conclusion The matching is A–III, B–I, C–II, D–IV. Hence, the correct option is (D).

Quick Tip to Solve: Anchor this list with Prussian blue for ferric ferrocyanide and blood red for ferric thiocyanate, then check the violet and yellow pairs.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. If 279 g of aniline is reacted with one equivalent of benzenediazonium chloride, the maximum amount of aniline yellow formed will be _____ g. (nearest integer)
(consider complete conversion). [4 marks]

Solution:

Concept:

- Aniline undergoes azo coupling with benzenediazonium chloride, predominantly at the para position of the activated aniline ring, to form p-aminoazobenzene (aniline yellow).
- The coupling consumes one mole of aniline and one mole of diazonium salt per mole of azo dye. The maximum mass follows from the limiting amount and the molar mass of the dye.

Step 1: Write the coupling stoichiometry The product contains two benzene rings joined by an azo linkage, with an amino group on one ring: $C_6H_5-N=N-C_6H_4-NH_2$. Its molecular formula is $C_{12}H_{11}N_3$. The balanced coupling can be written as

$$C_6H_5N_2Cl + C_6H_5NH_2 \longrightarrow C_6H_5N{=}NC_6H_4NH_2 + HCl.$$

The hydrochloric acid accounts for one hydrogen lost from the aniline ring and the chloride counterion; it is not part of the dye mass.

Step 2: Calculate the amount of aniline Using the usual atomic masses C = 12, H = 1 and N = 14, aniline C_6H_7N has molar mass

$$M_{\text{aniline}} = 6(12) + 7(1) + 14 = 93\,\text{g mol}^{-1}.$$

Therefore the supplied amount is

$$n_{\text{aniline}} = \frac{279}{93} = 3.00\,\text{mol}.$$

One equivalent of diazonium chloride supplies the same reacting mole amount, so the 1:1 stoichiometry permits formation of three moles of dye under complete conversion.

Step 3: Convert the product amount to mass The azo dye has molar mass

$$M_{\text{dye}} = 12(12) + 11(1) + 3(14) = 197\,\text{g mol}^{-1}.$$

Thus

$$m_{\text{dye}} = 3.00 \times 197 = 591\,\text{g}.$$

The dye can weigh more than the initial aniline because it incorporates the phenylazo portion supplied by the diazonium reagent. The calculated mass is already an integer, so the requested rounding does not change it.

Step 4: Conclusion The maximum amount of aniline yellow is $591\ \mathrm{g}$; the required numerical entry is $\boxed{591}$.

Quick Tip to Solve: Use the 1:1 azo-coupling mole ratio, then the molar mass of the complete dye; do not treat the starting aniline mass as the product mass.

82. Consider the following reaction

$$\mathrm{A} + \mathrm{B} \rightarrow \mathrm{C}$$

The time taken for A to become $1/4^{\text{th}}$ of its initial concentration is twice the time taken to become $1/2$ of the same. Also, when the change of concentration of B is plotted against time, the resulting graph gives a straight line with a negative slope and a positive intercept on the concentration axis. The overall order of the reaction is _____. [4 marks]

Solution:

Concept:

- For a first-order dependence on A, the integrated relation is $t = k_A^{-1} \ln([A]_0/[A]_t)$. Successive halvings take equal time.
- A straight line of concentration versus time with a negative slope obeys $[B]_t = [B]_0 - k_B t$, the zero-order integrated form with respect to B. Overall order is the sum of the individual orders.

Step 1: Use the successive-halving information for A Let the time to reach half of the initial concentration be $t_{1/2}$. The stated time to reach one quarter is $2t_{1/2}$, so reducing the concentration from half to one quarter takes another $t_{1/2}$. The equal times for successive halvings identify first-order behaviour in A. Explicitly,

$$t_{1/2} = \frac{\ln 2}{k_A}, \qquad t_{1/4} = \frac{\ln 4}{k_A} = 2t_{1/2}.$$

This is a logarithmic concentration dependence rather than the linear depletion expected for zero order.

Step 2: Read the concentration–time graph for B The given line has a positive concentration intercept and a constant negative slope. Write its equation as

$$[B]_t = [B]_0 - k_B t.$$

Its rate of disappearance, $-d[B]/dt = k_B$, is independent of the value of $[B]$ in this dependence test. Thus the order with respect to B is zero. A first-order dependence in B would give a straight line for the logarithm of concentration, not for concentration itself.

Step 3: Add the individual orders The rate-law exponents inferred from the stated tests are one for A

and zero for B. Accordingly, the dependence is

$$r = k[A]^1[B]^0 = k[A],$$

and the overall order is

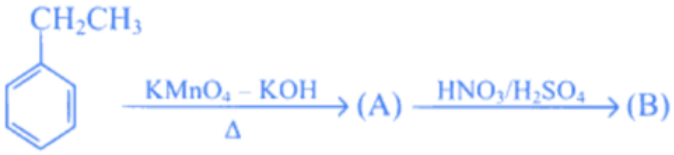
$$n = 1 + 0 = 1.$$

The stoichiometric coefficients in the written reaction do not determine these exponents; the supplied kinetic observations do. In particular, adding the two reactant coefficients would incorrectly suggest order two.

Step 4: Conclusion The overall order of the reaction is $\boxed{1}$.

Quick Tip to Solve: Equal successive half-lives indicate first order; a straight concentration–time line indicates zero order. Add the kinetic exponents rather than stoichiometric coefficients.

83. Major product B of the following reaction has _____ π-bond. [4 marks]



Solution:

Concept:

- Hot alkaline permanganate oxidises an alkylbenzene with a benzylic hydrogen to a benzoate group; the subsequent strongly acidic medium gives the corresponding carboxylic acid.
- The carboxyl group is deactivating and meta directing in electrophilic aromatic substitution. Count one pi bond for each independent double-bond unit in the final product.

Step 1: Identify the oxidation product The starting material is ethylbenzene, $C_6H_5CH_2CH_3$. Its benzylic carbon has hydrogen atoms, so hot $KMnO_4$/KOH oxidises the entire alkyl side chain to a carboxylate substituent attached to the ring. The carbon directly joined to the ring becomes the carboxyl carbon; the terminal side-chain carbon is not retained as an extra substituent. In the acid mixture used in the next step, the group is represented as $-COOH$, giving the benzoic-acid framework.

Step 2: Predict the major nitration product The $-COOH$ substituent withdraws electron density and directs incoming electrophiles mainly to the meta position. Nitration with HNO_3/H_2SO_4 therefore produces m-nitrobenzoic acid as the major product B. The ring remains aromatic after substitution: one ring hydrogen is replaced by $-NO_2$. Nitration does not consume a benzene double-bond unit permanently, and the carboxyl carbonyl is retained.

Step 3: Count the pi bonds of B The benzene ring contributes three pi-bond units. The carboxyl group contributes one carbonyl C=O pi bond. The nitro group contributes one delocalised pi-bond unit, represented by one N=O double bond in either principal resonance form; the two resonance

forms are not counted as two simultaneous double bonds. Thus

$$N_\pi = 3_{\text{ring}} + 1_{\text{carbonyl}} + 1_{\text{nitro}} = 5.$$

Moving the nitro group to another ring position would not change this count, but the meta position identifies the correct major product.

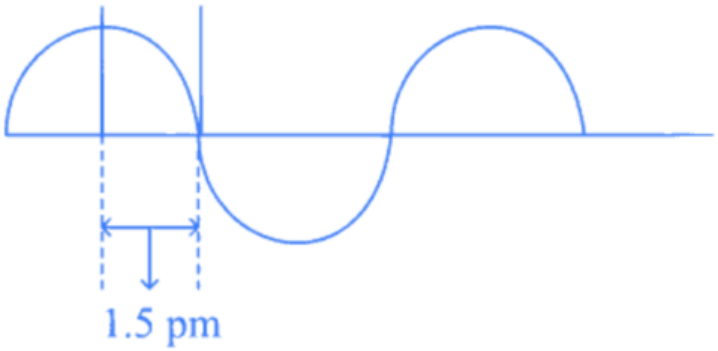
Step 4: Conclusion Major product B is m-nitrobenzoic acid and contains $\boxed{5}$ pi bonds.

Quick Tip to Solve: After side-chain oxidation and nitration, count three pi units in benzene, one in the carboxyl carbonyl and one in the nitro group; do not double-count nitro resonance forms.

84. A hypothetical electromagnetic wave is show below.

The frequency of the wave is $\mathrm{x} \times 10^{19}$ **Hz.**

$\mathrm{x} =$ **_____ (nearest integer)** [4 marks]



Solution:

Concept:

- The wavelength is the spatial separation of two successive points in the same phase, such as crest to crest. A crest and its adjacent zero crossing are separated by one quarter wavelength.
- For an electromagnetic wave in free space, $c = \nu\lambda$, where $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$, ν is frequency and λ is wavelength. One picometre is $10^{-12}\,\mathrm{m}$.

Step 1: Interpret the labelled horizontal distance The left dashed vertical line passes through the first crest. The right dashed line passes through the immediately following downward zero crossing. The marked distance of $1.5\,\mathrm{pm}$ therefore represents the crest-to-zero interval, not the full wavelength and not half a wavelength. For a sinusoidal profile, that phase change is one quarter cycle, so

$$\frac{\lambda}{4} = 1.5\,\mathrm{pm}, \qquad \lambda = 6.0\,\mathrm{pm}.$$

The amplitude is vertical and does not enter the frequency calculation.

Step 2: Convert the wavelength and calculate frequency Express the wavelength in metres before substituting the wave speed:

$$\lambda = 6.0 \times 10^{-12}\,\mathrm{m}.$$

The frequency is

$$\nu = \frac{c}{\lambda} = \frac{3.0 \times 10^{8}}{6.0 \times 10^{-12}} = 0.5 \times 10^{20}\,\mathrm{s}^{-1}.$$

Since one inverse second is one hertz, this is 5.0×10^{19} Hz. The exponent follows by subtracting the negative exponent in the denominator: $8 - (-12) = 20$.

Step 3: Extract the requested coefficient The question expresses the frequency as $x \times 10^{19}$ Hz. Comparing with the computed value gives $x = 5.0$, which is 5 to the nearest integer. Using the labelled distance directly as a full wavelength would overestimate the frequency by a factor of four. Doubling it as though it were half a wavelength would also misread the two marked phases.

Step 4: Conclusion The frequency is 5.0×10^{19} Hz, so the required value is $\boxed{x = 5}$.

Quick Tip to Solve: Identify the phases at both ends of a marked wave interval before using c divided by wavelength; crest to the next zero is a quarter wavelength.

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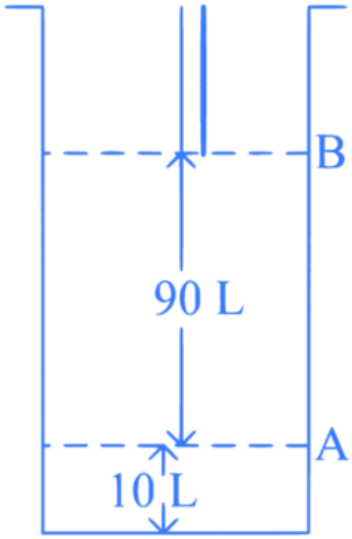
85. Consider the figure provided.

1 mol **of an ideal gas is kept in a cylinder, fitted with a piston, at the position A, at** $18^{\circ}\mathrm{C}$**. If the piston is moved to position** B**, keeping the temperature unchanged, then 'x'** L **atm work is done in this reversible process.**

x = _____ L **atm. (nearest integer)**

[Given : Absolute temperature $= {}^{\circ}\mathrm{C} + 273.15, \mathrm{R} = 0.08206$ L atm mol^{-1} K^{-1}**]** [4 marks]



Solution:

Concept:

- For a reversible expansion of an ideal gas at constant temperature, the work done by the gas is $W_{\mathrm{by}} = nRT\ln(V_f/V_i)$. The chemistry sign convention instead gives work on the gas as $w = -W_{\mathrm{by}}$.
- The volume ratio uses the total gas volume at each piston position. An indicated increase in volume must first be added to the initial volume; it is not itself the final volume.

Step 1: Read the two volumes and convert the temperature The gas occupies $10\,\mathrm{L}$ at A. The figure labels the additional volume between A and B as $90\,\mathrm{L}$. Thus the final volume is the sum, not $90\,\mathrm{L}$ alone.

$$V_i = 10\,\mathrm{L}, \qquad V_f = 10 + 90 = 100\,\mathrm{L}.$$

The temperature stays unchanged during the displacement, so this is an isothermal expansion. Use the precise Celsius-to-kelvin conversion supplied in the question:

$$T = 18 + 273.15 = 291.15\,\mathrm{K}.$$

Step 2: Calculate the reversible work and round once At every stage of a reversible process the opposing pressure is infinitesimally below the gas pressure, $p = nRT/V$. Integrating this pressure over the change in volume gives

$$W_{\text{by}} = \int_{10}^{100} \frac{nRT}{V}\,dV = nRT \ln 10.$$

Substitute the given gas constant in litre-atmosphere units, so no conversion into joules is required:

$$W_{\text{by}} = 1(0.08206)(291.15)\ln 10 = 55.01\,\mathrm{L\,atm}.$$

Expansion means the gas does positive work on its surroundings. Equivalently, work on the gas is approximately $-55.01\,\mathrm{L\,atm}$; the requested numerical magnitude is positive.

Step 3: Conclusion The work done by the gas is approximately $55.01\,\mathrm{L\,atm}$. Therefore the nearest-integer entry is $\boxed{x = 55}$.

Quick Tip to Solve: Add the labelled 90 L increase to the initial 10 L before forming the logarithm: here the volume ratio is 10, not 9.

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86. A solution containing 10 g of an electrolyte AB_2 in 100 g of water boils at 100.52°C. The degree of ionization of the electrolyte (α) is _____ $\times 10^{-1}$. (nearest integer)

[Given : Molar mass of $AB_2 = 200\ \mathrm{g\ mol^{-1}}$, K_b (molal boiling point elevation const. of water) $= 0.52\ \mathrm{K\ kg\ mol^{-1}}$, boiling point of water $= 100^\circ$C; AB_2 ionises as $AB_2 \rightarrow A^{2+} + 2\,B^-$] [4 marks]

Solution:

Concept:

- For a dilute electrolyte solution, boiling-point elevation is $\Delta T_b = iK_bm$, where m is the molality of undissociated formula units and i is the van't Hoff factor.
- If one formula unit produces u ions on dissociation, $i = 1 + (u-1)\alpha$. For $AB_2 \longrightarrow A^{2+} + 2\,B^-$, $u = 3$, hence $i = 1 + 2\alpha$.

Step 1: Find molality and the observed elevation The number of moles of electrolyte follows from its given molar mass:

$$n = \frac{10\,\mathrm{g}}{200\,\mathrm{g\,mol^{-1}}} = 0.050\,\mathrm{mol}.$$

The solvent is $100\,\mathrm{g} = 0.100\,\mathrm{kg}$ of water. Molality uses the mass of solvent, not the total mass of solution:

$$m = \frac{0.050}{0.100} = 0.50\,\mathrm{mol\,kg^{-1}}.$$

The increase above the boiling point of pure water is

$$\Delta T_b = 100.52 - 100.00 = 0.52\,\mathrm{K}.$$

Step 2: Determine the particle factor and degree of ionization Without ionization, this molality would give $K_b m = 0.52(0.50) = 0.26\,\mathrm{K}$. The actual elevation is twice as large, so

$$i = \frac{0.52}{0.52(0.50)} = 2.$$

Starting with one mole of electrolyte, the undissociated amount is $1-\alpha$, while the cation and anion amounts are α and 2α. The total is $1+2\alpha$ moles of solute particles. Consequently,

$$2 = 1 + 2\alpha \quad \Longrightarrow \quad \alpha = 0.50 = 5 \times 10^{-1}.$$

This is physically consistent: it lies between no ionization and complete ionization, for which the particle factor would be 1 and 3 respectively.

Step 3: Conclusion The degree of ionization is $\alpha = 0.50 = \boxed{5} \times 10^{-1}$. The requested nearest-integer coefficient is $\boxed{5}$.

Quick Tip to Solve: First extract the van't Hoff factor from the boiling-point data; then use three ions per formula unit, so divide $i-1$ by 2.

87. Number of molecules from the following which are exceptions to octet rule is _____.

$$\mathrm{CO_2, NO_2, H_2SO_4, BF_3, CH_4, SiF_4, ClO_2, PCl_5, BeF_2, C_2H_6, CHCl_3, CBr_4}$$

[4 marks]

Solution:

Concept:

- The octet rule asks whether a main-group atom has eight electrons in its valence shell in the conventional Lewis structure. Exceptions include electron-deficient species, odd-electron species and expanded-octet species.
- Count electrons around the central atom, including bonding and nonbonding electrons. A multiple bond contributes more than one shared electron pair, so a small number of neighbouring atoms does not automatically imply an incomplete octet.

Step 1: Identify electron-deficient and odd-electron molecules In $\mathrm{BF_3}$, boron forms three single bonds and has only six electrons around it. In $\mathrm{BeF_2}$, beryllium forms two single bonds and has only four. Both are incomplete-octet exceptions. $\mathrm{NO_2}$ has $5 + 2(6) = 17$ valence electrons and $\mathrm{ClO_2}$ has

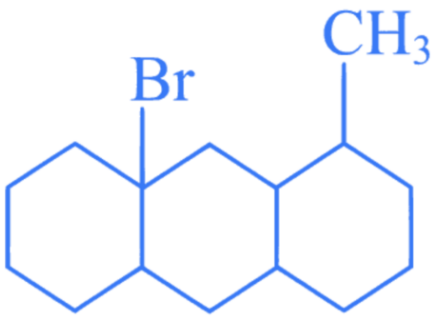
$7 + 2(6) = 19$. Their odd totals prevent an ordinary all-paired octet description; these are the two odd-electron exceptions. This gives four exceptions so far.

Step 2: Add expanded-octet species and check the remainder In the usual Lewis structure of PCl_5, phosphorus has five shared pairs, corresponding to ten surrounding electrons. Sulfur in the conventional low-formal-charge Lewis structure of H_2SO_4 is represented with two double bonds to oxygen and two single bonds to hydroxyl groups, giving an expanded octet. These add two more exceptions. The remaining species, CO_2, CH_4, SiF_4, C_2H_6, $CHCl_3$ and CBr_4, have ordinary octets at carbon or silicon. In particular, carbon in carbon dioxide has two double bonds and therefore eight surrounding electrons; the fact that it has only two oxygen neighbours does not make it electron-deficient. Thus no further molecule is added.

Step 3: Conclusion The exceptions are NO_2, H_2SO_4, BF_3, ClO_2, PCl_5 and BeF_2. Their number is $\boxed{6}$.

Quick Tip to Solve: Sort the list into incomplete, odd-electron and expanded octets: here the three groups each contribute two molecules.

88. The number of optical isomers in following compound is : _____. [4 marks]



Solution:

Concept:

- A tetrahedral carbon is stereogenic when its four attached groups are different. For a ring carbon, compare the two paths around the ring rather than treating both neighbours simply as carbon atoms.
- With n independent stereogenic centres and no symmetry reduction, the number of configurational stereoisomers is 2^n. A meso reduction occurs only when the complete substituted structure has an appropriate internal symmetry.

Step 1: Locate the stereogenic carbon atoms The drawing contains three fused six-membered rings. The two carbons at the left ring junction are stereogenic. The upper one bears bromine and three different carbon paths; the lower one bears hydrogen and three different paths. Paths through the outer left ring and through the substituted fused-ring system are different. The two carbons at the right ring junction are also stereogenic: each has hydrogen and three distinct carbon paths. The methyl substituent on the right ring distinguishes the paths around that ring from those through the central ring. Finally, the carbon carrying the methyl group is stereogenic because it is attached to

hydrogen, methyl and two inequivalent ring paths. The count is therefore

$$n = 2 + 2 + 1 = 5.$$

Step 2: Check symmetry before applying the power rule The other ring carbons are methylene groups and cannot be stereogenic because each bears two identical hydrogen atoms. The methyl carbon likewise has three identical hydrogens. Only the five centres already identified contribute. The bromine substituent near the left junction and the methyl substituent on the right ring destroy any interchange symmetry of the whole skeleton. There is no internal mirror plane that pairs equivalent halves to give a meso configuration. Therefore the usual independent-centre count is not reduced:

$$N = 2^5 = 32.$$

These configurations form 16 enantiomeric pairs; the question asks for the number of optical isomers, rather than the number of pairs.

Step 3: Conclusion There are five stereogenic centres and no meso reduction. Hence the number of optical isomers is $\boxed{32}$.

Quick Tip to Solve: In fused-ring questions, test every junction carbon separately, then check the substituents for symmetry before using 2^n.

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89. The 'spin only' magnetic moment value of MO_4^{2-} is _____ BM. (Where M is a metal having least metallic radii. among Sc, Ti, V, Cr, Mn and Zn).

(Given atomic number: $Sc = 21, Ti = 22,\ V = 23, Cr = 24, Mn = 25$ **and** $Zn = 30$**)** [4 marks]

Solution:

Concept:

- Across the early first transition series, increasing effective nuclear charge generally contracts the metallic radius; the detailed trend has irregularities, so the listed metals must be compared using their metallic radii.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons in the metal ion, not in the neutral atom.

Step 1: Choose the metal using metallic radii Among the listed metals, the standard metallic radii are approximately 164, 147, 135, 129, 137 and $134\,\text{pm}$ for scandium, titanium, vanadium, chromium, manganese and zinc respectively. The smallest of these is chromium. The unusually larger value for manganese means that blindly extending a monotonically decreasing trend would select the wrong metal. Thus $M = Cr$, and the ion in the question is chromate, CrO_4^{2-}.

Step 2: Find the oxidation state and count unpaired electrons Assign the usual oxidation number -2 to each oxygen. If the chromium oxidation state is z, charge balance gives

$$z + 4(-2) = -2 \quad \Longrightarrow \quad z = +6.$$

Neutral chromium has configuration $[\mathrm{Ar}]\,3d^5 4s^1$. Forming chromium(VI) removes the one $4s$ electron and the five $3d$ electrons, leaving a d^0 configuration. Therefore there are no unpaired d electrons:

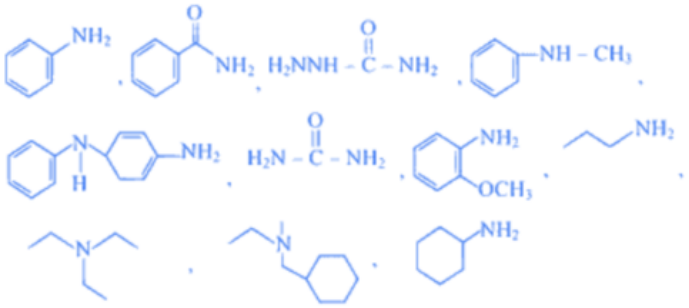
$$n = 0, \qquad \mu = \sqrt{0(0+2)} = 0\,\mathrm{BM}.$$

No high-spin versus low-spin choice is needed for a d^0 ion. Counting the unpaired electrons of neutral chromium would be inappropriate because the magnetic species is its hexavalent oxoanion.

Step 3: Conclusion The selected metal is chromium and $\mathrm{CrO_4}^{2-}$ contains a d^0 metal centre. Its spin-only magnetic moment is $\boxed{0\,\mathrm{BM}}$, so the numerical entry is $\boxed{0}$.

Quick Tip to Solve: Use the metallic-radius comparison first, then determine the oxidation state. The neutral atom's electron count is not the ion's count.

90. Number of amine compounds from the following giving solids which are soluble in NaOH upon reaction with Hinsberg's reagent is _____. [4 marks]



Solution:

Concept:

- Hinsberg's reagent is benzenesulfonyl chloride, $\mathrm{C_6H_5SO_2Cl}$. A primary amine gives a sulfonamide retaining an acidic nitrogen-bound hydrogen; its conjugate base dissolves in aqueous sodium hydroxide.
- A secondary amine gives a sulfonamide with no nitrogen-bound hydrogen and is insoluble in alkali. A tertiary amine has no suitable nitrogen-bound hydrogen for this sulfonamide formation. Amides are distinguished from ordinary amines because their nitrogen lone pair is delocalized into a carbonyl group.

Step 1: Select compounds containing an ordinary primary amino group Read the structures in the displayed list and look for an amino nitrogen directly attached to one carbon group and two hydrogens. The qualifying compounds are aniline, the para-amino-substituted diphenylamine, ortho-methoxyaniline, propan-1-amine and cyclohexylamine. The diphenylamine derivative is included because its separate primary amino group can react even though its bridging nitrogen is secondary. The methoxy substituent on the anisidine ring does not remove the primary character of its amino group. These are five compounds.

Step 2: Exclude the other nitrogen classes and apply the solubility test Benzamide and urea contain

amide nitrogen rather than ordinary primary amino nitrogen. The semicarbazide structure is a hydrazine/carboxamide derivative and is not counted as an ordinary carbon-substituted primary amine in this Hinsberg classification. N-Methylaniline is secondary. Triethylamine is tertiary; the bottom-middle amine also has nitrogen bonded to three carbon groups and has no N-H bond, so it is tertiary. They do not add to the selected five. For each selected primary amine, the relevant sequence is

$$RNH_2 + C_6H_5SO_2Cl \longrightarrow C_6H_5SO_2NHR + HCl.$$

The remaining sulfonamide hydrogen is made acidic by the electron-withdrawing sulfonyl group, so alkali removes it:

$$C_6H_5SO_2NHR + OH^- \longrightarrow C_6H_5SO_2NR^- + H_2O.$$

The resulting ionic salt accounts for the stated solubility in sodium hydroxide.

Step 3: Conclusion The five selected compounds give the alkali-soluble sulfonamide products in the primary-amine Hinsberg test. The required number is $\boxed{5}$.

Quick Tip to Solve: Count molecules with a free primary amino group, not all occurrences of NH_2: an amide $-CONH_2$ is a different functional group.