JEE Main 2024 Paper 1 (B.E./B.Tech) Apr 4 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2024, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 4 April 2024, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $f(x) = \begin{cases} -2, & -2 \le x \le 0, \\ x-2, & 0 < x \le 2, \end{cases}$ **and** $h(x) = f(|x|) + |f(x)|$**. Then** $\int_{-2}^{2} h(x)\,dx$ **is equal to:** [4 marks]

(A) 2

(B) 6

(C) 4

(D) 1

Correct Answer: (A) 2

Solution:

Concept:

- The absolute value changes its formula at zero, so split the integral over the two intervals $[-2, 0]$ and $[0, 2]$.
- For $-2 \le x \le 2$, the printed function is nonpositive. Thus $|f(x)| = -f(x)$, but $f(|x|)$ must be evaluated using the nonnegative input $|x|$.

Step 1: Evaluate the negative half For $-2 \le x < 0$, we have $|x| = -x \in (0, 2]$. Therefore the second branch gives $f(|x|) = -x - 2$, whereas the first branch gives $f(x) = -2$. Hence

$$h(x) = (-x - 2) + |-2| = -x.$$

At $x = 0$, both terms are evaluated directly: $f(0) + |f(0)| = -2 + 2 = 0$. This agrees with the limiting expression and contributes no isolated area.

Step 2: Evaluate the positive half For $0 < x \le 2$, the argument $|x|$ equals x, and $f(x) = x - 2 \le 0$. Hence $|f(x)| = 2 - x$, giving

$$h(x) = (x - 2) + (2 - x) = 0.$$

The cancellation on this interval is exact; one cannot assume the same cancellation for negative inputs because $f(|x|)$ and $f(x)$ then use different branches.

Step 3: Integrate each expression The required integral is therefore

$$\int_{-2}^{2} h(x)\,dx = \int_{-2}^{0} (-x)\,dx + \int_{0}^{2} 0\,dx = \left[-\frac{x^2}{2}\right]_{-2}^{0} = 2.$$

Only the negative half contributes. The other listed values $6, 4, 1$ do not equal this evaluated area; in particular doubling the negative-half result would incorrectly treat h as even.

Step 4: Conclusion The integral is $\boxed{2}$. Hence, the correct option is (A).

Quick Tip to Solve: Evaluate $f(|x|)$ and $|f(x)|$ separately before combining them; these two operations are generally different.

2. One of the points of intersection of the curves $y = 1 + 3x - 2x^2$ and $y = \frac{1}{x}$ is $\left(\frac{1}{2}, 2\right)$. Let the area of the region enclosed by these curves be $\frac{1}{24}(l\sqrt{5} + \mathrm{m}) - \mathrm{n}\log_e(1+\sqrt{5})$, where $l,\ \mathrm{m}, \mathrm{n} \in \mathbf{N}$. Then $l + \mathrm{m} + \mathrm{n}$ is equal to [4 marks]

(A) 30

(B) 29

(C) 31

(D) 32

Correct Answer: (A) 30

Solution:

Concept:

- The enclosed area on an interval of intersection is the integral of the upper curve minus the lower curve. Intersections with $y = 1/x$ require $x \neq 0$.
- Use the given root $x = 1/2$ to factor the cubic intersection equation. The negative intersection lies on the other side of the vertical asymptote and does not close a finite region with the positive branch.

Step 1: Find the positive intersection interval Equating the curves and multiplying by $x \neq 0$ gives

$$2x^3 - 3x^2 - x + 1 = (2x - 1)(x^2 - x - 1) = 0.$$

Thus the intersections have abscissae $1/2, (1 - \sqrt{5})/2, (1 + \sqrt{5})/2$. Set $b = (1 + \sqrt{5})/2$. On $(1/2, b)$, testing $x = 1$ gives parabola height 2 and hyperbola height 1, so the parabola is above the hyperbola.

Step 2: Evaluate the area exactly The bounded region on the positive branch has area

$$\mathcal{A} = \int_{1/2}^{b} \left(1 + 3x - 2x^2 - \frac{1}{x}\right) dx.$$

An antiderivative for positive x is

$$F(x) = x + \frac{3}{2}x^2 - \frac{2}{3}x^3 - \log_e x.$$

Since $b^2 = b + 1$ and $b^3 = 2b + 1$, its polynomial part at b is $7b/6 + 5/6$; at $1/2$ it is $19/24$. Therefore their difference is $(15 + 14\sqrt{5})/24$.

Step 3: Match the requested coefficients The logarithmic contribution is

$$-\log_e b + \log_e(1/2) = -\log_e(2b) = -\log_e(1 + \sqrt{5}).$$

Consequently

$$\mathcal{A} = \frac{14\sqrt{5} + 15}{24} - \log_e(1 + \sqrt{5}).$$

Comparing coefficients yields $l = 14, m = 15, n = 1$. Their sum is 30, ruling out $29, 31, 32$. No area is integrated across $x = 0$, where the hyperbola is undefined.

Step 4: Conclusion $l + m + n = 14 + 15 + 1 = \boxed{30}$. Hence, the correct option is (A).

Quick Tip to Solve: Use $b^2 = b + 1$ at the golden-ratio endpoint to simplify powers before inserting radicals into the antiderivative.

3. Let $\alpha \in (0, \infty)$ and $A = \begin{bmatrix} 1 & 2 & \alpha \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$. If $\det\left(\text{adj}\left(2A - A^T\right) \cdot \text{adj}\left(A - 2A^T\right)\right) = 2^8$, then $(\det(A))^2$ is equal to: [4 marks]

(A) 16

(B) 36

(C) 49

(D) 1

Correct Answer: (A) 16

Solution:

Concept:

- For a 3×3 matrix M, $\det(\text{adj}\, M) = (\det M)^2$. Also $\det(XY) = \det X \det Y$.
- Transposition preserves the determinant, while multiplying a 3×3 matrix by -1 reverses its determinant. These facts relate the two matrices inside the adjugates.

Step 1: Relate the two matrices Put $B = 2A - A^T$ and $C = A - 2A^T$. Since $B^T = 2A^T - A$, we have $C = -B^T$, and hence $\det C = -\det B$. Applying multiplicativity and the adjugate identity gives

$$\det(\text{adj}\, B \,\text{adj}\, C) = (\det B)^2 (\det C)^2 = (\det B)^4.$$

The printed condition thus says $(\det B)^4 = 2^8$, or $|\det B| = 4$.

Step 2: Compute the determinant in terms of alpha From the given entries,

$$B = \begin{pmatrix} 1 & 3 & 2\alpha \\ 0 & 0 & 1 \\ -\alpha & 1 & 2 \end{pmatrix}.$$

Expansion along the second row yields

$$\det B = -\begin{vmatrix} 1 & 3 \\ -\alpha & 1 \end{vmatrix} = -(1 + 3\alpha).$$

Because $\alpha > 0$, this determinant is negative. Thus $|1 + 3\alpha| = 4$ reduces to $1 + 3\alpha = 4$, so $\alpha = 1$. The alternative equation would give $\alpha = -5/3$, outside the stated domain.

Step 3: Calculate the requested square Expansion of the original determinant along its first row gives

$$\det A = 1(0 - 1) - 2(2 - 0) + \alpha(1 - 0) = \alpha - 5.$$

At $\alpha = 1$, this is -4, and its square is 16. The choices $36, 49, 1$ do not follow from the admissible parameter; the sign of $\det A$ disappears only in the final square.

Step 4: Conclusion $(\det A)^2 = (-4)^2 = \boxed{16}$. Hence, the correct option is (A).

Quick Tip to Solve: Before expanding multiple determinants, check whether one matrix is a scalar multiple of the transpose of the other.

4. There are 5 points P_1, P_2, P_3, P_4, P_5 on the side AB, excluding A and B, of a triangle ABC. Similarly there are 6 points $\mathrm{P}_6, \mathrm{P}_7, \ldots, \mathrm{P}_{11}$ on the side BC and 7 points $\mathrm{P}_{12}, \mathrm{P}_{13}, \ldots, \mathrm{P}_{18}$ on the side CA of the triangle. The number of triangles, that can be formed using the points $\mathrm{P}_1, \mathrm{P}_2, \ldots, \mathrm{P}_{18}$ as vertices, is: [4 marks]

(A) 751

(B) 776

(C) 796

(D) 771

Correct Answer: (A) 751

Solution:

Concept:

- Three distinct noncollinear points determine one triangle. Unordered selections of three points from n points are counted by $\binom{n}{3}$.
- The disallowed triples here consist of three points on the same side of the original triangle. The marked points exclude the triangle vertices, so each point belongs to just one side.

Step 1: Count all selections of vertices There are $5 + 6 + 7 = 18$ marked points. First count all unordered triples, without imposing noncollinearity:

$$\binom{18}{3} = \frac{18 \cdot 17 \cdot 16}{6} = 816.$$

No permutation factor is needed, since rearranging the three vertices does not produce a new triangle. The vertices A, B, C themselves are excluded and must not be added to the pool.

Step 2: Remove exactly the degenerate triples A triple on side AB can be selected in $\binom{5}{3} = 10$ ways; on BC, in $\binom{6}{3} = 20$ ways; and on CA, in $\binom{7}{3} = 35$ ways. Thus the number to remove is

$$10 + 20 + 35 = 65.$$

There is no overlap among these classes, since no marked point is a common endpoint. Any line other than the three side lines intersects the boundary of a convex triangle in at most two points, so it cannot contain three of the marked boundary points.

Step 3: Subtract and compare the choices The required count is

$$N = 816 - 65 = 751.$$

As a cross-check, the triangles with one vertex from each side contribute $5 \cdot 6 \cdot 7 = 210$; those with two vertices on one side and the third on another contribute $\binom{5}{2}(13) + \binom{6}{2}(12) + \binom{7}{2}(11) = 541$. Their sum is again 751. The other options $776, 796, 771$ omit some degenerate triples in the subtraction.

Step 4: Conclusion The number of triangles is $\boxed{751}$. Hence, the correct option is (A).

Quick Tip to Solve: For points on the sides of a triangle, count all triples first and subtract the triples lying entirely on each side.

5. A square is inscribed in the circle $x^2 + y^2 - 10x - 6y + 30 = 0$. One side of this square is parallel to $y = x + 3$. If (x_i, y_i) are the vertices of the square, then $\Sigma\left(x_i^2 + y_i^2\right)$ is equal to: [4 marks]

(A) 152

(B) 148

(C) 156

(D) 160

Correct Answer: (A) 152

Solution:

Concept:

- Completing squares in the circle equation identifies its centre and radius. Every vertex of an inscribed square is at the same radius from that centre.
- The four radius vectors of a square add to zero, because opposite vertices are diametrically opposite. Consequently sums of squared distances from the origin can be evaluated without finding individual coordinates.

Step 1: Identify the circle Complete the squares:

$$x^2 - 10x + y^2 - 6y + 30 = 0 \quad \Longrightarrow \quad (x-5)^2 + (y-3)^2 = 4.$$

The centre is $C = (5, 3)$ and the squared radius is $r^2 = 4$. The distance of the centre from the origin satisfies $OC^2 = 5^2 + 3^2 = 34$. The slope condition fixes the orientation of the square, but not these quantities.

Step 2: Use cancellation of the vertex offsets Write each vertex as $(x_i, y_i) = (5 + u_i, 3 + v_i)$, where $u_i^2 + v_i^2 = 4$. Since opposite vertices have opposite offsets,

$$\sum_{i=1}^{4} u_i = 0, \qquad \sum_{i=1}^{4} v_i = 0.$$

Expanding a vertex norm gives

$$x_i^2 + y_i^2 = 34 + 10u_i + 6v_i + (u_i^2 + v_i^2).$$

Summing over all vertices cancels both linear terms and retains four copies of the squared radius.

Step 3: Evaluate the sum Therefore

$$\sum_{i=1}^{4}(x_i^2 + y_i^2) = 4(34) + 4(4) = 152.$$

Equivalently, summing the original circle equation over the vertices and using $\sum x_i = 20$, $\sum y_i = 12$ gives $10(20) + 6(12) - 4(30) = 152$. Both calculations show that rotating the inscribed square cannot change the requested sum. Thus the alternatives $148, 156, 160$ are excluded independently of the side slope.

Step 4: Conclusion The sum is $\boxed{152}$. Hence, the correct option is (A).

Quick Tip to Solve: For an inscribed square centred at C, use $\sum OP_i^2 = 4(OC^2 + r^2)$; opposite-vertex symmetry cancels all cross terms.

6. Let $\alpha, \beta \in \mathbf{R}$. Let the mean and the variance of 6 observations $-3, 4, 7, -6, \alpha, \beta$ be 2 and 23, respectively. The mean deviation about the mean of these 6 observations is : [4 marks]

(A) $\frac{16}{3}$
(B) $\frac{11}{3}$
(C) $\frac{14}{3}$
(D) $\frac{13}{3}$

Correct Answer: (D) $\frac{13}{3}$

Solution:

Concept:

- The mean of six observations is their sum divided by six. Their variance is the mean of their squares minus the square of the mean.
- Mean deviation about the mean uses absolute deviations, $\frac{1}{6}\sum|x_i - \bar{x}|$. It is different from variance and standard deviation, so the missing observations must first be determined.

Step 1: Use the prescribed mean The mean is 2, so the sum of the observations is 12. The four known values add to $-3 + 4 + 7 - 6 = 2$. Hence

$$\alpha + \beta = 12 - 2 = 10.$$

This fixes their sum without determining which value is named α; the final statistic is unaffected by interchanging the two.

Step 2: Use the prescribed variance From $23 = \frac{1}{6}\sum x_i^2 - 2^2$, we get $\sum x_i^2 = 6(27) = 162$. The known squares total $9 + 16 + 49 + 36 = 110$, giving $\alpha^2 + \beta^2 = 52$. Since

$$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta,$$

we obtain $100 = 52 + 2\alpha\beta$, or $\alpha\beta = 24$. Thus the missing values are the roots of $t^2 - 10t + 24 = (t - 4)(t - 6) = 0$, namely 4 and 6.

Step 3: Average the absolute deviations Taking deviations from 2, the six nonnegative distances are

$$|-3-2| = 5, \quad |4-2| = 2, \quad |7-2| = 5, \quad |-6-2| = 8, \quad |4-2| = 2, \quad |6-2| = 4.$$

Their sum is 26, so the mean deviation is $26/6 = 13/3$. The fractions $16/3, 11/3, 14/3$ correspond to different totals of absolute deviations and fail this direct sum. The repeated observation 4 is counted twice, as required.

Step 4: Conclusion The mean deviation about the mean is $\boxed{13/3}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the sum and sum of squares of missing observations first; then use absolute values, rather than squares, for mean deviation.

7. Let $f(x) = x^5 + 2e^{x/4}$ for all $x \in \mathbf{R}$. Consider a function $g(x)$ such that $(g \circ f)(x) = x$ for all $x \in \mathbf{R}$. Then the value of $8g'(2)$ is : [4 marks]

(A) 4

(B) 2

(C) 16

(D) 8

Correct Answer: (C) 16

Solution:

Concept:

- The identity $g(f(x)) = x$ makes g an inverse of f on its range. The chain rule gives $g\prime(f(x))f\prime(x) = 1$.
- To find the derivative of the inverse at output 2, first find the input whose image under f is 2. The derivative must be evaluated at that input, not at $x = 2$.

Step 1: Find the preimage of two For $f(x) = x^5 + 2e^{x/4}$, direct substitution yields $f(0) = 0 + 2 = 2$. Moreover,

$$f\prime(x) = 5x^4 + \frac{1}{2}e^{x/4} > 0$$

for every real x, since the exponential term is strictly positive and $5x^4 \geq 0$. Thus f is strictly increasing, and $x = 0$ is the unique input associated with output 2. The nonzero derivative also ensures that the inverse derivative formula applies there.

Step 2: Differentiate the composition Differentiate the given relation with respect to x:

$$g\prime(f(x))\, f\prime(x) = 1.$$

At $x = 0$, $f\prime(0) = 1/2$, so

$$g\prime(2) \cdot \frac{1}{2} = 1 \quad \Longrightarrow \quad g\prime(2) = 2.$$

The reciprocal occurs because small changes in the output of f correspond to inverse changes scaled by the reciprocal of its slope.

Step 3: Multiply by the requested factor The problem asks for $8g\prime(2)$, rather than just $g\prime(2)$. Therefore

$$8g\prime(2) = 8 \cdot 2 = 16.$$

The option 2 gives only the derivative before this final multiplication. The values 4 and 8 result from incorrect handling of the exponential derivative or the inverse slope and do not satisfy the differentiated identity.

Step 4: Conclusion $8g\prime(2) = \boxed{16}$. Hence, the correct option is (C).

Quick Tip to Solve: For an inverse derivative at y_0, solve $f(x_0) = y_0$ first, then use $g\prime(y_0) = 1/f\prime(x_0)$.

8. Let a unit vector which makes an angle of 60° with $2\hat{i}+2\hat{j}-\hat{k}$ and an angle of 45° with $\hat{i}-\hat{k}$ be $\vec{C}$. Then $\vec{C}+\left(-\frac{1}{2}\hat{i}+\frac{1}{3\sqrt{2}}\hat{j}-\frac{\sqrt{2}}{3}\hat{k}\right)$ is: [4 marks]

(A) $-\frac{\sqrt{2}}{3}\hat{i}+\frac{\sqrt{2}}{3}\hat{j}+\left(\frac{1}{2}+\frac{2\sqrt{2}}{3}\right)\hat{k}$

(B) $\left(\frac{1}{\sqrt{3}}+\frac{1}{2}\right)\hat{i}+\left(\frac{1}{\sqrt{3}}-\frac{1}{3\sqrt{2}}\right)\hat{j}+\left(\frac{1}{\sqrt{3}}+\frac{\sqrt{2}}{3}\right)\hat{k}$

(C) $\frac{\sqrt{2}}{3}\hat{i}-\frac{1}{2}\hat{k}$

(D) $\frac{\sqrt{2}}{3}\hat{i}+\frac{1}{3\sqrt{2}}\hat{j}-\frac{1}{2}\hat{k}$

Correct Answer: (C) $\frac{\sqrt{2}}{3}\hat{i}-\frac{1}{2}\hat{k}$

Solution:

Concept:

- For a unit vector $\vec{C}=x\hat{i}+y\hat{j}+z\hat{k}$, the scalar product with $\vec{a}$ equals $|\vec{a}|\cos\theta$, where θ is their angle.
- The unit-length condition is $x^2+y^2+z^2=1$. Both angle conditions and this length condition must hold simultaneously.

Step 1: Translate the angle conditions The two given direction vectors have lengths 3 and $\sqrt{2}$. Therefore the stated acute angles give positive scalar products:

$$2x+2y-z=3\cos 60^\circ=\frac{3}{2}, \qquad x-z=\sqrt{2}\cos 45^\circ=1.$$

Subtracting the second equation from the first gives $x+2y=1/2$. Thus $x=1/2-2y$ and $z=-1/2-2y$.

Step 2: Use the length condition and form the requested vector Substitute these expressions into the unit-length equation:

$$\left(\frac{1}{2}-2y\right)^2+y^2+\left(-\frac{1}{2}-2y\right)^2=1 \quad\Longrightarrow\quad 9y^2=\frac{1}{2}.$$

Hence $y=\pm 1/(3\sqrt{2})$. With the negative sign, $x=1/2+\sqrt{2}/3$ and $z=-1/2+\sqrt{2}/3$. Adding the fixed vector in the question now gives $\sqrt{2}\hat{i}/3-\hat{k}/2$, which is option (C).

Step 3: Check the other possibilities The positive sign for y instead gives the requested sum $-\sqrt{2}\hat{i}/3+\sqrt{2}\hat{j}/3-(1/2+2\sqrt{2}/3)\hat{k}$, which is not listed: option (A) has the opposite sign on its last component. Recovering $\vec{C}$ from option (B) gives an $\hat{i}$ component larger than one, so it cannot be a unit vector. Option (D) would give $y=0$, while the unit-length calculation requires $y^2=1/18$. Thus only (C) is an admissible listed result.

Step 4: Conclusion Among the listed vectors the possible value is $\boxed{\frac{\sqrt{2}}{3}\hat{i}-\frac{1}{2}\hat{k}}$. Hence, the correct option is (C).

Quick Tip to Solve: Convert angles to scalar products first. Keep both roots of the unit-length equation, then test the actual listed vectors, including every component sign.

9. If the domain of the function $\sin^{-1}\left(\frac{3x-22}{2x-19}\right)+\log_e\left(\frac{3x^2-8x+5}{x^2-3x-10}\right)$ is $(\alpha,\beta]$, then $3\alpha+10\beta$ is equal to:

[4 marks]

(A) 95

(B) 100

(C) 97

(D) 98

Correct Answer: (C) 97

Solution:

Concept:

- A real inverse sine requires its argument to lie in $[-1,1]$; a real logarithm requires a strictly positive argument.
- The domain of a sum is the intersection of the domains of its summands. Zeros of logarithm arguments are excluded, while inverse-sine endpoints may be included.

Step 1: Solve the inverse-sine restriction First exclude $2x-19=0$. Squaring the inequality for the absolute value of the inverse-sine argument gives

$$\left|\frac{3x-22}{2x-19}\right|\le 1 \quad\Longleftrightarrow\quad (3x-22)^2\le(2x-19)^2.$$

Factoring the difference of squares yields $(x-3)(5x-41)\le 0$, so $3\le x\le 41/5$. The excluded pole $19/2$ lies outside this interval, so this is the entire inverse-sine domain.

Step 2: Solve the logarithmic restriction Factor numerator and denominator before making a sign chart:

$$\frac{3x^2-8x+5}{x^2-3x-10}=\frac{(3x-5)(x-1)}{(x-5)(x+2)}>0.$$

The ordered critical points are $-2,1,5/3,5$. Checking the sign in each interval gives $(-\infty,-2)\cup(1,5/3)\cup(5,\infty)$. Neither numerator zero is permitted because $\log 0$ is undefined, and neither denominator zero is permitted.

Step 3: Intersect and evaluate the required expression Intersect the logarithmic domain with $[3,41/5]$. Only the last logarithmic interval meets it, giving $(5,41/5]$. The left endpoint is excluded because the logarithmic denominator vanishes there. At the right endpoint the inverse-sine argument is -1, which is allowed, and the logarithmic argument is positive. Thus $\alpha=5$ and $\beta=41/5$, and

$$3\alpha+10\beta=15+82=97.$$

The other numerical options do not equal this endpoint combination.

Step 4: Conclusion The domain is $\boxed{(5,41/5]}$, and $\boxed{3\alpha+10\beta=97}$. Hence, the correct option is (C).

Quick Tip to Solve: Find the inverse-sine interval and logarithm sign intervals separately; intersect them only at the end, preserving open and closed endpoints.

10. Let the point, on the line passing through the points $P(1,-2,3)$ and $Q(5,-4,7)$, farther from the origin and at a distance of 9 units from the point P, be (α,β,γ). Then $\alpha^2+\beta^2+\gamma^2$ is equal to : [4 marks]

(A) 150

(B) 155

(C) 160

(D) 165

Correct Answer: (B) 155

Solution:

Concept:

- Points on a line through P in the direction $\vec{v}$ have position vector $\vec{P}+t\vec{v}$. The distance from P is $|t|\,|\vec{v}|$.
- To compare distances from the origin, compare squared norms. This avoids unnecessary square roots and still identifies the farther point exactly.

Step 1: Find a unit direction along the given line The displacement from $P(1,-2,3)$ to $Q(5,-4,7)$ is

$$\overrightarrow{PQ}=(4,-2,4), \qquad |\overrightarrow{PQ}|=\sqrt{16+4+16}=6.$$

Therefore a unit vector along this line is $(2/3,-1/3,2/3)$. A point at distance nine from P can lie in either direction; both signs must be considered before using the extra condition about the origin.

Step 2: Construct both points at the stated distance The displacement of length nine is

$$9\left(\frac{2}{3},-\frac{1}{3},\frac{2}{3}\right)=(6,-3,6).$$

Adding and subtracting this displacement from P gives the only two possibilities:

$$R_+=(7,-5,9), \qquad R_-=(-5,1,-3).$$

Each lies on the line because its displacement from P is a scalar multiple of $Q-P$. Each displacement also has squared length $36+9+36=81$, confirming the required distance.

Step 3: Select the point farther from the origin The squared distances of these points from the origin are

$$|R_+|^2=7^2+(-5)^2+9^2=49+25+81=155,$$
$$|R_-|^2=(-5)^2+1^2+(-3)^2=25+1+9=35.$$

Since $155>35$, the farther point is $(\alpha,\beta,\gamma)=(7,-5,9)$. The requested sum is precisely its squared norm, so no square root is needed. The other proposed values 150, 160 and 165 do not agree with the exact coordinate calculation.

Step 4: Conclusion $\boxed{\alpha^2+\beta^2+\gamma^2=155}$. Hence, the correct option is (B).

Quick Tip to Solve: A fixed distance from a point on a line gives two candidates. Form both using a unit direction, then compare their squared distances from the origin.

11. The vertices of a triangle are $A(-1,3), B(-2,2)$ and $C(3,-1)$. A new triangle is formed by shifting the sides of the triangle by one unit inwards. Then the equation of the side of the new triangle nearest to origin is : [4 marks]

(A) $-x+y-(2-\sqrt{2})=0$

(B) $x+y-(2-\sqrt{2})=0$

(C) $x+y+(2-\sqrt{2})=0$

(D) $x-y-(2+\sqrt{2})=0$

Correct Answer: (B) $x+y-(2-\sqrt{2})=0$

Solution:

Concept:

- The distance between parallel lines $ax+by+c_1=0$ and $ax+by+c_2=0$ is $|c_1-c_2|/\sqrt{a^2+b^2}$.
- To shift a triangle side inward, identify the half-plane containing the opposite vertex. The origin-to-line distance is $|c|/\sqrt{a^2+b^2}$.

Step 1: Write the original side equations Substitution of the corresponding vertices gives

$$AB:\ y-x-4=0, \qquad BC:\ 3x+5y-4=0, \qquad CA:\ x+y-2=0.$$

For example, both A and B have $y-x=4$, while both A and C have $x+y=2$. These equations also provide the normal lengths $\sqrt{2}, \sqrt{34}, \sqrt{2}$ needed for a shift of one unit; simply changing a constant by one would not give a one-unit perpendicular shift.

Step 2: Shift each side toward its opposite vertex At C, $y-x-4$ is negative, so the inward parallel to AB is $y-x-(4-\sqrt{2})=0$. At A, $3x+5y-4=8>0$, so the inward parallel to BC is $3x+5y-(4+\sqrt{34})=0$. At B, $x+y-2=-2<0$, so the inward parallel to CA is

$$x+y-(2-\sqrt{2})=0.$$

In each equation the constant changes by exactly the norm of its normal vector, ensuring perpendicular displacement one.

Step 3: Compare the new sides with respect to the origin The three distances to the origin are

$$\frac{4-\sqrt{2}}{\sqrt{2}}=2\sqrt{2}-1, \qquad \frac{4+\sqrt{34}}{\sqrt{34}}=1+\frac{4}{\sqrt{34}}, \qquad \frac{2-\sqrt{2}}{\sqrt{2}}=\sqrt{2}-1.$$

The last is smallest (approximately 0.414, compared with 1.828 and 1.686). It is therefore the shifted CA side. Options (A) and (D) have the wrong normal direction for this side; option (C) shifts its constant to the opposite side of the origin instead of giving the inward parallel derived above.

Step 4: Conclusion The required side is $\boxed{x+y-(2-\sqrt{2})=0}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the opposite vertex to choose the inward direction. A one-unit shift changes a line constant by the length of the normal vector, not generally by one.

12. Three urns A, B and C contain 7 red, 5 black; 5 red, 7 black and 6 red, 6 black balls, respectively. One of the urn is selected at random and a ball is drawn from it. If the ball drawn is black, then the probability that it is drawn from urn A is : [4 marks]

(A) $\frac{4}{17}$
(B) $\frac{5}{16}$
(C) $\frac{5}{18}$
(D) $\frac{7}{18}$

Correct Answer: (C) $\frac{5}{18}$

Solution:

Concept:

- Bayes' theorem gives $P(A \mid K) = P(A)P(K \mid A)/P(K)$, where K denotes drawing a black ball.
- The total probability of drawing black is the sum of its joint probabilities over the mutually exclusive, exhaustive urn choices.

Step 1: List the prior and conditional probabilities Each urn contains twelve balls, and each urn is chosen with probability $1/3$. Therefore

$$P(K \mid A) = \frac{5}{12}, \qquad P(K \mid B) = \frac{7}{12}, \qquad P(K \mid C) = \frac{6}{12}.$$

These are probabilities of black after a specified urn choice. They are not the requested posterior probability, because observing black changes the relative likelihoods of the urns.

Step 2: Find the probability of the observed colour Apply the law of total probability to all three urns:

$$P(K) = \frac{1}{3} \cdot \frac{5}{12} + \frac{1}{3} \cdot \frac{7}{12} + \frac{1}{3} \cdot \frac{6}{12} = \frac{18}{36} = \frac{1}{2}.$$

The joint probability of choosing urn A and then drawing black is $P(A \cap K) = (1/3)(5/12) = 5/36$. This joint event is only part of the black-ball event, so it must be divided by the total probability of black.

Step 3: Apply Bayes theorem and check the alternatives The required conditional probability is

$$P(A \mid K) = \frac{P(A \cap K)}{P(K)} = \frac{5/36}{1/2} = \frac{5}{18}.$$

As a direct check, the urns have equal totals and equal selection probabilities, so among the black outcomes their relative weights are simply $5 : 7 : 6$. Urn A accounts for five of eighteen such weighted outcomes. The option $7/18$ belongs to urn B, not A; the denominators 16 and 17 omit some of the eighteen black-ball contributions and are inconsistent with the experiment.

Step 4: Conclusion $\boxed{P(A \mid \text{black}) = 5/18}$. Hence, the correct option is (C).

Quick Tip to Solve: With equally likely urns of equal size, the posterior weights for a colour are proportional to the numbers of balls of that colour in the urns.

13. If the solution $y = y(x)$ of the differential equation $(x^4+2x^3+3x^2+2x+2)\mathrm{d}y-(2x^2+2x+3)\mathrm{d}x=0$ satisfies $y(-1)=-\frac{\pi}{4}$, then $y(0)$ is equal to : [4 marks]

(A) $-\frac{\pi}{12}$

(B) $\frac{\pi}{2}$

(C) 0

(D) $\frac{\pi}{4}$

Correct Answer: (D) $\frac{\pi}{4}$

Solution:

Concept:

- When dy/dx is a function of x only, integrate it with respect to x, then determine the additive constant using the initial value.
- The identity $d(\tan^{-1} u)/dx = u\prime/(1+u^2)$ identifies reciprocal quadratic terms as inverse-tangent derivatives.

Step 1: Factor the denominator and split the rational function The differential equation gives

$$\frac{dy}{dx} = \frac{2x^2+2x+3}{x^4+2x^3+3x^2+2x+2}.$$

The denominator factors as

$$x^4+2x^3+3x^2+2x+2 = (x^2+1)(x^2+2x+2).$$

The numerator is the sum of the two factors: $2x^2+2x+3 = (x^2+1)+(x^2+2x+2)$. Consequently

$$\frac{dy}{dx} = \frac{1}{x^2+1} + \frac{1}{(x+1)^2+1}.$$

Both denominators are positive for real x, so the solution can be evaluated continuously from -1 to zero.

Step 2: Integrate and apply the initial condition Integrating each term gives

$$y(x) = \tan^{-1} x + \tan^{-1}(x+1) + C.$$

Using the principal real inverse tangents at $x=-1$,

$$-\frac{\pi}{4} = \tan^{-1}(-1) + \tan^{-1}(0) + C = -\frac{\pi}{4} + C.$$

Hence $C=0$. Retaining the two inverse tangents separately avoids any branch correction that could arise from combining them through a tangent-addition identity.

Step 3: Evaluate and verify the result At zero, the solution is

$$y(0) = \tan^{-1} 0 + \tan^{-1} 1 = 0 + \frac{\pi}{4} = \frac{\pi}{4}.$$

Differentiating the obtained expression reproduces the two reciprocal quadratics and therefore the

original rational derivative exactly. The values $-\pi/12$, $\pi/2$ and zero are not the value of this initial-value solution at zero; no additional integration constant remains free after imposing the given condition.

Step 4: Conclusion $\boxed{y(0) = \pi/4}$. Hence, the correct option is (D).

Quick Tip to Solve: Before using partial fractions, check whether the numerator is the sum of the quadratic denominator factors; here it splits immediately into two inverse-tangent derivatives.

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14. Let the first three terms 2, p and q, with $q \neq 2$, of a G.P. be respectively the 7^{th}, 8^{th} and 13^{th} terms of an A.P. If the 5^{th} term of the G.P. is the n^{th} term of the A.P., then n is equal to: [4 marks]

(A) 151

(B) 177

(C) 163

(D) 169

Correct Answer: (C) 163

Solution:

Concept:

- The k-th term of an A.P. is $a + (k-1)d$, and the k-th term of a G.P. with first term two is $2r^{k-1}$.
- Three successive G.P. terms satisfy $p^2 = 2q$. Their positions in the A.P. determine the differences between their values.

Step 1: Express the common A.P. difference using the positions The three stated correspondences yield

$$a + 6d = 2, \qquad a + 7d = p, \qquad a + 12d = q.$$

Subtract the first equation from the next two: $p = 2 + d$ and $q = 2 + 6d$. The positions matter: the second term is one A.P. step after the first, but the third is six steps after the first. They are successive terms only in the G.P., not in the A.P.

Step 2: Use the geometric progression condition Because $2, p, q$ are successive G.P. terms,

$$(2 + d)^2 = 2(2 + 6d).$$

Expanding and collecting gives

$$d^2 + 4d + 4 = 4 + 12d \quad \Longrightarrow \quad d(d - 8) = 0.$$

If $d = 0$, then $q = 2$, contrary to the stated exclusion. Hence $d = 8$, which gives $p = 10$, $q = 50$, and common G.P. ratio $r = p/2 = 5$. The A.P. first term follows from $a + 6d = 2$: $a = -46$.

Step 3: Match the fifth geometric term with the arithmetic term The fifth term of the G.P. is

$$T_5 = 2r^4 = 2 \cdot 5^4 = 1250.$$

The general A.P. term is

$$a + (n-1)d = -46 + 8(n-1) = 8n - 54.$$

Equating it to 1250 gives $8n = 1304$, so $n = 163$. However, this would correspond to option (C), whereas the options are 151, 177, 163, 169; checking substitution confirms $8(163) - 54 = 1250$. Thus 151, 177 and 169 do not produce the fifth geometric term.

Step 4: Conclusion $\boxed{n = 163}$. Hence, the correct option is (C).

Quick Tip to Solve: Translate the A.P. positions before applying the G.P. relation. The exclusion on the third term removes the constant-progression root.

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15. The sum of all rational terms in the expansion of $\left(2^{\frac{1}{5}} + 5^{\frac{1}{3}}\right)^{15}$ is equal to : [4 marks]

(A) 633

(B) 6131

(C) 3133

(D) 931

Correct Answer: (C) 3133

Solution:

Concept:

- The general term of $(a+b)^{15}$ is $T_{r+1} = \binom{15}{r} a^{15-r} b^r$, where $r = 0, 1, \ldots, 15$.
- A product of powers of the distinct primes 2 and 5 is rational only when the exponent of each prime is an integer. A fractional power of one prime cannot cancel a fractional power of the other.

Step 1: Write the general term Choose r factors contributing $5^{1/3}$, leaving $15 - r$ factors contributing $2^{1/5}$. Then

$$T_{r+1} = \binom{15}{r} 2^{(15-r)/5} 5^{r/3}.$$

The binomial coefficient is already an integer, so it does not remove either of the fractional prime exponents.

Step 2: Find every admissible index Rationality requires both $(15-r)/5$ and $r/3$ to be integers. The first condition says $r \equiv 0 \pmod 5$, while the second says $r \equiv 0 \pmod 3$. Hence r must be a multiple of 15. Within the permitted inclusive range $0 \le r \le 15$, this gives precisely $r = 0$ and $r = 15$; no intermediate term qualifies.

Step 3: Add the two rational terms At the two endpoints the binomial coefficients are both 1. Their contributions are

$$T_1 = 2^{15/5} = 2^3 = 8, \qquad T_{16} = 5^{15/3} = 5^5 = 3125.$$

Therefore the requested sum is $8 + 3125 = 3133$. The numbers 633, 6131 and 931 do not equal this sum and cannot result from any additional rational terms, since the index conditions already exhaust all possibilities.

Step 4: Conclusion The sum of all rational terms is $\boxed{3133}$. Hence, the correct option is (C).

Quick Tip to Solve: For distinct prime radicals, impose integer exponents separately, then solve the index divisibility conditions before evaluating terms.

16. If 2 and 6 are the roots of the equation $ax^2 + bx + 1 = 0$, then the quadratic equation, whose roots are $\frac{1}{2a+b}$ and $\frac{1}{6a+b}$, is : [4 marks]

(A) $x^2 + 8x + 12 = 0$

(B) $2x^2 + 11x + 12 = 0$

(C) $4x^2 + 14x + 12 = 0$

(D) $x^2 + 10x + 16 = 0$

Correct Answer: (A) $x^2 + 8x + 12 = 0$

Solution:

Concept:

- For $ax^2 + bx + 1 = 0$, the sum and product of its roots are $-b/a$ and $1/a$, respectively; the leading coefficient a is nonzero.
- A monic quadratic with roots p, q is $x^2 - (p+q)x + pq = 0$. The signs of the transformed roots must be retained when forming this equation.

Step 1: Determine the original coefficients Using the given roots 2 and 6, the product relation gives

$$12 = \frac{1}{a} \quad \Rightarrow \quad a = \frac{1}{12}.$$

The sum relation then gives

$$8 = -\frac{b}{a} \quad \Rightarrow \quad b = -8a = -\frac{2}{3}.$$

These coefficients also follow by writing $ax^2 + bx + 1 = a(x-2)(x-6)$ and comparing the constant term.

Step 2: Evaluate the new roots Substitute these exact coefficients into the two denominators:

$$2a + b = \frac{1}{6} - \frac{2}{3} = -\frac{1}{2}, \qquad 6a + b = \frac{1}{2} - \frac{2}{3} = -\frac{1}{6}.$$

Neither denominator is zero, so both reciprocals are defined. Thus the new roots are $p = -2$ and $q = -6$. Their sum is -8 and their product is 12, rather than the positive original sum.

Step 3: Form and check the equation The required monic polynomial is

$$(x+2)(x+6) = x^2 + 8x + 12.$$

It vanishes at both transformed roots. Option (B), after division by 2, has root sum $-11/2$, option (C), after division by 4, has root sum $-7/2$, and option (D) has root sum -10. None has the necessary sum -8, so the alternatives are excluded without confusing a nonmonic equation with its monic form.

Step 4: Conclusion The required quadratic equation is $\boxed{x^2 + 8x + 12 = 0}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the original product first when the constant term is 1; it fixes the leading coefficient immediately, after which the transformed reciprocals are simple.

17. Let the sum of the maximum and the minimum values of the function $f(x) = \frac{2x^2-3x+8}{2x^2+3x+8}$ be $\frac{m}{n}$, where $\gcd(m, n) = 1$. Then $m + n$ is equal to : [4 marks]

(A) 217

(B) 182

(C) 201

(D) 195

Correct Answer: (C) 201

Solution:

Concept:

- A continuous rational function can have a finite extremum at a stationary point where its derivative vanishes. Its denominator must first be checked for zeros on the real domain.
- The quotient rule is $(u/v)\prime = (u\prime v - uv\prime)/v^2$. Global bounds must include behaviour away from the stationary points, not just their values.

Step 1: Check the domain and differentiate The denominator is always positive because

$$2x^2 + 3x + 8 = 2\left(x + \frac{3}{4}\right)^2 + \frac{55}{8} > 0.$$

Consequently the domain is all real numbers and there are no poles. Applying the quotient rule and simplifying gives

$$f'(x) = \frac{12(x^2 - 4)}{(2x^2 + 3x + 8)^2}.$$

The only stationary points are therefore $x = -2$ and $x = 2$.

Step 2: Identify the global extrema The derivative is positive for $x < -2$, negative for $-2 < x < 2$, and positive for $x > 2$. Thus -2 gives a maximum and 2 gives a minimum. Moreover, $f(x) \to 1$ at both infinite ends, so the increasing outer intervals do not produce more extreme values. Direct substitution yields

$$f(-2) = \frac{22}{10} = \frac{11}{5}, \qquad f(2) = \frac{10}{22} = \frac{5}{11}.$$

Step 3: Reduce the sum to lowest terms Adding the two extremal values, rather than their arguments, gives

$$\frac{11}{5} + \frac{5}{11} = \frac{121 + 25}{55} = \frac{146}{55}.$$

The Euclidean algorithm gives remainders $36, 19, 17, 2, 1$ when applied to 146 and 55, so their greatest common divisor is 1. Hence $m = 146$, $n = 55$, and $m + n = 201$. The other proposed totals 217, 182 and 195 do not equal this reduced numerator-plus-denominator sum.

Step 4: Conclusion The required value is $\boxed{201}$. Hence, the correct option is (C).

Quick Tip to Solve: After finding stationary values of a rational function, check its poles and limits at infinity before declaring them global maximum and minimum.

18. If the system of equations

$$x + (\sqrt{2}\sin\alpha)y + (\sqrt{2}\cos\alpha)z = 0$$
$$x + (\cos\alpha)y + (\sin\alpha)z = 0$$
$$x + (\sin\alpha)y - (\cos\alpha)z = 0$$

has a non-trivial solution, then $\alpha \in \left(0, \frac{\pi}{2}\right)$ is equal to : [4 marks]

(A) $\frac{5\pi}{24}$
(B) $\frac{11\pi}{24}$
(C) $\frac{7\pi}{24}$
(D) $\frac{3\pi}{4}$

Correct Answer: (A) $\frac{5\pi}{24}$

Solution:

Concept:

- A square homogeneous linear system has a non-trivial solution exactly when its coefficient determinant vanishes. A nonzero determinant would force all three unknowns to be zero.
- Use $\sin^2\alpha + \cos^2\alpha = 1$, $2\sin\alpha\cos\alpha = \sin 2\alpha$, and $\sin^2\alpha - \cos^2\alpha = -\cos 2\alpha$ to simplify the determinant.

Step 1: Set the determinant equal to zero Put $s = \sin\alpha$ and $c = \cos\alpha$. The coefficients form

$$D = \begin{vmatrix} 1 & \sqrt{2}s & \sqrt{2}c \\ 1 & c & s \\ 1 & s & -c \end{vmatrix}.$$

Expansion along the first row gives

$$D = -c^2 - s^2 + \sqrt{2}s(c+s) + \sqrt{2}c(s-c).$$

Thus $D = -1 + \sqrt{2}(s^2 - c^2 + 2sc)$. The signs in the last row are essential: its last coefficient is $-c$, not c.

Step 2: Convert the condition to one sine The non-trivial-solution condition becomes

$$\sqrt{2}(\sin 2\alpha - \cos 2\alpha) = 1.$$

Using $\sin t - \cos t = \sqrt{2}\sin(t - \pi/4)$, this is equivalent to

$$\sin\left(2\alpha - \frac{\pi}{4}\right) = \frac{1}{2}.$$

This reduction retains all possible solutions because it uses only identities and multiplication by nonzero constants.

Step 3: Enforce the specified interval Since $0 < \alpha < \pi/2$, the shifted angle lies in $(-\pi/4, 3\pi/4)$.

Among the angles giving sine $1/2$, only $\pi/6$ lies in this interval; $5\pi/6$ lies outside it. Therefore

$$2\alpha - \frac{\pi}{4} = \frac{\pi}{6} \quad \Rightarrow \quad \alpha = \frac{5\pi}{24}.$$

The options $11\pi/24$ and $7\pi/24$ do not satisfy the sine equation, while $3\pi/4$ is outside the given interval altogether.

Step 4: Conclusion We obtain $\boxed{\alpha = 5\pi/24}$. Hence, the correct option is (A).

Quick Tip to Solve: For a homogeneous three-variable system, start with determinant zero and carry the angle interval through every trigonometric substitution.

19. Let α and β be the sum and the product of all the non-zero solutions of the equation $(\bar{z})^2 + |z| = 0, z \in C$. Then $4(\alpha^2 + \beta^2)$ is equal to : [4 marks]

(A) 4

(B) 2

(C) 6

(D) 8

Correct Answer: (A) 4

Solution:

Concept:

- For a nonzero complex number $z = re^{i\theta}$, its modulus is $r > 0$ and its conjugate is $\bar{z} = re^{-i\theta}$. Modulus and argument conditions together determine the solutions.
- When two complex numbers are equal, their moduli are equal. Once the radius is found, solve the argument equation modulo 2π, counting distinct complex numbers only once.

Step 1: Find the modulus of a nonzero solution Rewrite the equation as $(\bar{z})^2 = -|z|$. Taking moduli on both sides gives

$$|\bar{z}|^2 = |-|z|| \quad \Rightarrow \quad r^2 = r.$$

The solutions of this real equation are 0 and 1. The question specifically excludes zero, so every retained complex solution has $r = 1$. This step uses only a necessary modulus condition; the original complex equation must still be imposed.

Step 2: Find the allowed arguments With $r = 1$, the original equation reduces to

$$e^{-2i\theta} = -1.$$

Thus $-2\theta \equiv \pi \pmod{2\pi}$, or equivalently $\theta \equiv \pi/2 \pmod{\pi}$. In one full revolution the two distinct numbers are $z = i$ and $z = -i$. They both check directly: conjugating either number and squaring gives -1, which cancels its modulus 1. No other point of the unit circle satisfies the argument condition.

Step 3: Calculate the requested sum and product Their sum and product are

$$\alpha = i + (-i) = 0, \qquad \beta = i(-i) = -i^2 = 1.$$

Consequently

$$4(\alpha^2 + \beta^2) = 4(0^2 + 1^2) = 4.$$

In particular, the product is positive 1, not -1, because $i^2 = -1$. The values 2, 6 and 8 do not follow from this complete list of nonzero solutions, and including the excluded solution zero would change the product incorrectly.

Step 4: Conclusion The requested value is $\boxed{4}$. Hence, the correct option is (A).

Quick Tip to Solve: In equations mixing a conjugate and a modulus, determine the radius first by taking moduli, then return to the original equation for arguments.

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20. Let $f : \mathbf{R} \to \mathbf{R}$ be a function given by

$$f(x) = \begin{cases} \dfrac{1 - \cos 2x}{x^2}, & x < 0, \\ \alpha, & x = 0, \\ \dfrac{\beta\sqrt{1 - \cos x}}{x}, & x > 0. \end{cases}$$

where $\alpha, \beta \in \mathbf{R}$. If f is continuous at $x = 0$, then $\alpha^2 + \beta^2$ is equal to : [4 marks]

(A) 48

(B) 6

(C) 3

(D) 12

Correct Answer: (D) 12

Solution:

Concept:

- Continuity at zero requires the left-hand limit, the assigned value $f(0) = \alpha$, and the right-hand limit to agree. Evaluate the two side limits with their respective branch formulas.
- The standard limits are $\sin x/x \to 1$ and $(1 - \cos x)/x^2 \to 1/2$. For $x > 0$, dividing a nonnegative square root by x allows x to be moved inside that root without changing its sign.

Step 1: Evaluate the left-hand limit For negative x, use $1 - \cos 2x = 2\sin^2 x$. Hence

$$\lim_{x \to 0^-} \frac{1 - \cos 2x}{x^2} = 2 \lim_{x \to 0^-} \left(\frac{\sin x}{x}\right)^2 = 2.$$

The value at zero must equal this limit, which immediately fixes $\alpha = 2$. This is a limit, not a substitution of zero into the original quotient, which would give an undefined expression.

Step 2: Evaluate the right-hand limit The positive branch is $\beta\sqrt{1-\cos x}/x$. Since the approach is from $x > 0$,

$$\lim_{x\to 0^+} \frac{\sqrt{1-\cos x}}{x} = \lim_{x\to 0^+} \sqrt{\frac{1-\cos x}{x^2}} = \frac{1}{\sqrt{2}}.$$

Thus the right-hand limit is $\beta/\sqrt{2}$. Equating it to the common value 2 gives $\beta = 2\sqrt{2}$. The positive-side condition is essential when simplifying the square root of x^2.

Step 3: Combine the parameter values Squaring and adding the determined real parameters gives

$$\alpha^2 + \beta^2 = 2^2 + (2\sqrt{2})^2 = 4 + 8 = 12.$$

Both side limits now equal $f(0) = 2$, so the full continuity requirement is verified. Options 48, 6 and 3 are not this sum; in particular, replacing the square-root limit by $1/2$ would incorrectly omit the outer square root.

Step 4: Conclusion The required value is $\boxed{12}$. Hence, the correct option is (D).

Quick Tip to Solve: When a square-root quotient approaches zero from one side, retain $\sqrt{x^2} = |x|$ until the sign of that side is used.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let the solution $y = y(x)$ of the differential equation $\frac{dy}{dx} - y = 1 + 4\sin x$ satisfy $y(\pi) = 1$. Then $y\left(\frac{\pi}{2}\right) + 10$ is equal to _____. [4 marks]

Solution:

Concept:

- The equation $y' - y = 1 + 4\sin x$ is linear. Its homogeneous solution is Ce^x, and a particular solution can be sought as a constant plus a linear combination of $\sin x$ and $\cos x$.
- An initial condition fixes the remaining homogeneous constant. Verify the candidate by differentiating it in the original equation before evaluating the requested expression.

Step 1: Find a particular solution Take $y_p = A + B\sin x + D\cos x$. Then

$$y_p' - y_p = -A + (-D - B)\sin x + (B - D)\cos x.$$

Comparing with $1 + 4\sin x$ gives $-A = 1$, $-D - B = 4$, and $B - D = 0$. Therefore $A = -1$, $B = D = -2$, so the general solution is

$$y = Ce^x - 1 - 2\sin x - 2\cos x.$$

Step 2: Apply and verify the given condition At $x = \pi$, the sine is zero and the cosine is -1. Consequently

$$1 = y(\pi) = Ce^\pi - 1 + 2 = Ce^\pi + 1,$$

which forces $C = 0$, since $e^{\pi} \neq 0$. We have $y = -1 - 2\sin x - 2\cos x$, with derivative $y' = -2\cos x + 2\sin x$. Subtraction now gives $y' - y = 1 + 4\sin x$, and $y(\pi) = 1$, confirming both requirements exactly.

Step 3: Evaluate the requested combination At $x = \pi/2$, use $\sin(\pi/2) = 1$ and $\cos(\pi/2) = 0$:

$$y\left(\frac{\pi}{2}\right) = -1 - 2 = -3.$$

The question asks for this value plus 10, rather than the function value alone. Therefore

$$y\left(\frac{\pi}{2}\right) + 10 = -3 + 10 = 7.$$

There is no rounding or unit conversion involved; the specified initial condition has removed the exponential term completely.

Step 4: Conclusion The numerical value to enter is $\boxed{7}$.

Quick Tip to Solve: For a linear differential equation forced by a constant and sine, try constant-plus-sine-plus-cosine, then verify by subtraction before applying the condition.

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22. If the shortest distance between the lines $\frac{x+2}{2} = \frac{y+3}{3} = \frac{z-5}{4}$ and $\frac{x-3}{1} = \frac{y-2}{-3} = \frac{z+4}{2}$ is $\frac{38}{3\sqrt{5}}$k, and $\int_0^k [x^2]\,dx = \alpha - \sqrt{\alpha}$, where $[x]$ denotes the greatest integer function, then $6\alpha^3$ is equal to _____.

[4 marks]

Solution:

Concept:

- For lines through position vectors $\mathbf{a}_1, \mathbf{a}_2$ with directions $\mathbf{b}_1, \mathbf{b}_2$, their shortest distance is $d = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|/|\mathbf{b}_1 \times \mathbf{b}_2|$.
- The function $[x^2]$ changes its value when x^2 crosses an integer. Split its integral at those points, and integrate the constant value on each interval.

Step 1: Compute the shortest distance Take $\mathbf{a}_1 = (-2, -3, 5)$, $\mathbf{a}_2 = (3, 2, -4)$, $\mathbf{b}_1 = (2, 3, 4)$ and $\mathbf{b}_2 = (1, -3, 2)$. The displacement and cross product are

$$\mathbf{a}_2 - \mathbf{a}_1 = (5, 5, -9), \qquad \mathbf{b}_1 \times \mathbf{b}_2 = (18, 0, -9).$$

Thus

$$d = \frac{|90 + 81|}{\sqrt{18^2 + 9^2}} = \frac{171}{9\sqrt{5}} = \frac{19}{\sqrt{5}}.$$

The nonzero cross product confirms that the lines are not parallel.

Step 2: Determine the upper limit and evaluate the integral Compare the computed distance with the given expression:

$$\frac{19}{\sqrt{5}} = \frac{38k}{3\sqrt{5}} \quad \Rightarrow \quad k = \frac{3}{2}.$$

On $[0, 1)$, $[x^2] = 0$; on $[1, \sqrt{2})$, it equals 1; and on $[\sqrt{2}, 3/2]$, it equals 2. Endpoint values at isolated

points do not change the integral. Therefore

$$\int_0^{3/2} [x^2]\,dx = (\sqrt{2}-1) + 2\left(\frac{3}{2}-\sqrt{2}\right) = 2-\sqrt{2}.$$

Step 3: Identify the parameter The given form is $\alpha - \sqrt{\alpha}$, so $\alpha = 2$ satisfies the equality. There is no other admissible value: put $u = \sqrt{\alpha} \geq 0$. Then $u^2 - u = 2 - \sqrt{2} > 0$, whose quadratic has exactly one nonnegative root, namely $u = \sqrt{2}$.

Step 4: Conclusion Thus $\alpha = 2$, and the required numerical value is

$$6\alpha^3 = 6(2)^3 = \boxed{48}.$$

Quick Tip to Solve: Find the line distance before integrating; for $[x^2]$, the breakpoints are square roots of integers, not just integers.

23. Let A be a square matrix of order 2 such that $|A| = 2$ and the sum of its diagonal elements is -3 . If the points (x, y) satisfying $\mathrm{A}^2 + x\ \mathrm{A} + y\mathrm{I} = \mathrm{O}$ lie on a hyperbola, whose transverse axis is parallel to the x-axis, eccentricity is e and the length of the latus rectum is l, then $\mathrm{e}^4 + l^4$ is equal to _____.

[4 marks]

Solution:

Concept:

- For a matrix of order two, the characteristic polynomial is $t^2 - (\operatorname{tr} A)t + \det A$. The Cayley–Hamilton theorem says that substituting A into this polynomial gives the zero matrix.
- A matrix equation requires every entry to vanish. Taking its determinant gives only a necessary condition and can introduce points that do not satisfy the original equation.
- A single point does not determine a hyperbola, even when its transverse axis is parallel to the x-axis. Its eccentricity and latus rectum depend on the additional geometric parameters.

Step 1: Apply the characteristic identity The given trace is -3 and determinant is 2, so the characteristic polynomial and its matrix identity are

$$p(t) = t^2 + 3t + 2 = (t+1)(t+2), \qquad A^2 + 3A + 2I = O.$$

Subtracting this identity from the printed equation $A^2 + xA + yI = O$ gives

$$(x-3)A + (y-2)I = O.$$

Step 2: Determine every admissible point The matrices A and I are linearly independent. Indeed, if $A = cI$, its trace would give $2c = -3$, whereas its determinant would then be $c^2 = 9/4$, contradicting $\det A = 2$. Therefore both scalar coefficients in the last matrix equation must vanish:

$$x = 3, \qquad y = 2.$$

Conversely, this pair satisfies the equation by Cayley–Hamilton. Thus the entire solution set is the single point $(3, 2)$, rather than a hyperbola.

Step 3: Test whether the stated geometry determines the result Infinitely many horizontal hyperbolas can pass through $(3, 2)$. For example, for any $0 < a^2 < 9$, take

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \qquad b^2 = \frac{4a^2}{9 - a^2}.$$

Substituting $(3, 2)$ verifies the equation, but $e^2 = 1 + b^2/a^2$ and $l = 2b^2/a$ vary with a. Hence the printed condition does not fix $e^4 + l^4$. Replacing the zero-matrix equation by a determinant equation would be a different problem and cannot supply a value for this one.

Step 4: Conclusion The printed matrix equation has only $(x, y) = (3, 2)$, so $e^4 + l^4$ has no uniquely determined numerical value. The official answer key dropped this question and awarded bonus marks to all.

Quick Tip to Solve: Check whether a matrix expression equals the zero matrix or merely has zero determinant. These conditions are not interchangeable when deriving a locus.

24. If $\lim_{x\to1} \frac{(5x+1)^{1/3}-(x+5)^{1/3}}{(2x+3)^{1/2}-(x+4)^{1/2}} = \frac{\mathrm{m}\sqrt{5}}{\mathrm{n}(2\mathrm{n})^{2/3}}$**, where** $\gcd(\mathrm{m}, \mathrm{n}) = 1$**, then** $8\,\mathrm{m} + 12\mathrm{n}$ **is equal to _____.**

[4 marks]

Solution:

Concept:

- When numerator and denominator both vanish at the limiting point and the denominator has a nonzero derivative there, the ratio limit is the ratio of their derivatives.
- Use $d(u^r)/dx = ru^{r-1}u\prime$, retaining both the outer power and the derivative of the inner linear expression.

Step 1: Check the indeterminate form At $x = 1$, both cube-root expressions in the numerator equal $6^{1/3}$, while both square-root expressions in the denominator equal $\sqrt{5}$. The ratio is therefore of the form $0/0$. All radicands remain positive in a neighbourhood of 1, so the functions are differentiable there. Write the numerator as $f(x)$ and the denominator as $g(x)$.

Step 2: Evaluate the ratio of first derivatives Differentiate both differences, including the inner coefficients:

$$f\prime(1) = \frac{5}{3\,6^{2/3}} - \frac{1}{3\,6^{2/3}} = \frac{4}{3\,6^{2/3}}.$$

$$g\prime(1) = \frac{2}{2\sqrt{5}} - \frac{1}{2\sqrt{5}} = \frac{1}{2\sqrt{5}} \neq 0.$$

Equivalently, their first-order increments are $f\prime(1)(x - 1)$ and $g\prime(1)(x - 1)$; cancel the common increment and take the limit. This gives

$$L = \frac{4}{3\,6^{2/3}}\,(2\sqrt{5}) = \frac{8\sqrt{5}}{3\,6^{2/3}}.$$

Step 3: Match the coprime parameters The prescribed expression is $m\sqrt{5}/[n(2n)^{2/3}]$. Setting $n = 3$ makes $2n = 6$, and the coefficient then gives $m = 8$. These are coprime since $\gcd(8, 3) = 1$. To check uniqueness for positive coprime integers, cubing the coefficient equality gives $m^3/n^5 = 512/243 = 2^9/3^5$; coprimality forces precisely these numerator and denominator prime powers.

Step 4: Conclusion Therefore $m = 8$ and $n = 3$, so

$$8m + 12n = 64 + 36 = \boxed{100}.$$

Quick Tip to Solve: For a difference of two roots vanishing at the same point, differentiate each term carefully; the inner linear coefficients determine the surviving difference.

---

25. Let A be a 3×3 matrix of non-negative real elements such that $A\begin{bmatrix}1\\1\\1\end{bmatrix} = 3\begin{bmatrix}1\\1\\1\end{bmatrix}$. Then the maximum value of $\det(A)$ is _____. [4 marks]

Solution:

Concept:

- The relation $A(1, 1, 1)^T = 3(1, 1, 1)^T$ says that every row sum of A is 3.
- Hadamard's inequality bounds the absolute determinant by the product of the Euclidean lengths of the rows. For nonnegative entries, a row's Euclidean length is no larger than its sum.

Step 1: Translate the matrix equation into row constraints Write the rows as $\mathbf{r}_i = (a_{i1}, a_{i2}, a_{i3})$, for $i = 1, 2, 3$. Matrix multiplication by the column of ones sums the entries of each row. Hence

$$a_{i1} + a_{i2} + a_{i3} = 3, \qquad a_{ij} \geq 0.$$

There is no requirement that every individual entry be positive; zero entries are allowed. This distinction is useful because an extremal matrix can have zeros.

Step 2: Bound every row length and the determinant Because all pairwise products of entries are nonnegative,

$$a_{i1}^2 + a_{i2}^2 + a_{i3}^2 \leq (a_{i1} + a_{i2} + a_{i3})^2 = 9.$$

Thus $\|\mathbf{r}_i\| \leq 3$. Geometrically, the absolute determinant is the volume of the parallelepiped formed by the three row vectors; this volume cannot exceed the product of their lengths. Therefore

$$\det A \leq |\det A| \leq \|\mathbf{r}_1\|\|\mathbf{r}_2\|\|\mathbf{r}_3\| \leq 3 \cdot 3 \cdot 3 = 27.$$

Bounding the absolute value is sufficient even if some admissible matrices have a negative determinant.

Step 3: Exhibit a matrix attaining the bound Take $A = 3I_3 = \mathrm{diag}(3, 3, 3)$. Its entries are nonnegative, every row sums to 3, and it satisfies the given matrix equation. Its row vectors are mutually

perpendicular, so equality in the volume bound is possible. Directly,

$$\det(3I_3) = 3 \cdot 3 \cdot 3 = 27.$$

The upper bound is therefore attained, rather than merely approached.

Step 4: Conclusion The maximum possible determinant is $\boxed{27}$.

Quick Tip to Solve: Convert multiplication by the all-ones column into row sums; a diagonal matrix often attains the determinant bound under nonnegative row constraints.

---

26. If $\int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{1+\sin x \cos x}\, dx = \frac{1}{a}\log_e\left(\frac{a}{3}\right) + \frac{\pi}{b\sqrt{3}}$**, where** $a, b \in N$**, then** $a + b$ **is equal to _____.** [4 marks]

Solution:

Concept:

- Use $t = \tan x$ to replace $\sin^2 x$, $\sin x \cos x$ and dx by rational functions of t.
- For the quadratic $t^2+t+1 = (t+1/2)^2+3/4$, integration separates into a logarithm and an inverse tangent.

Step 1: Convert the integral into a rational integral Let I denote the given integral. On the interval $0 \le x \le \pi/4$, the substitution $t = \tan x$ is one-to-one and changes the limits to 0 and 1. Since

$$\sin^2 x = \frac{t^2}{1+t^2}, \quad \sin x \cos x = \frac{t}{1+t^2}, \quad dx = \frac{dt}{1+t^2},$$

we obtain

$$I = \int_0^1 \frac{t^2}{(t^2+t+1)(t^2+1)}\, dt.$$

Both denominator factors are positive throughout this interval.

Step 2: Split the rational function and integrate The useful decomposition is

$$\frac{t^2}{(t^2+t+1)(t^2+1)} = \frac{t}{t^2+1} - \frac{t}{t^2+t+1}.$$

In the second numerator, write $t = \frac{1}{2}(2t+1) - \frac{1}{2}$. Consequently

$$I = \left[\frac{1}{2}\log(t^2+1) - \frac{1}{2}\log(t^2+t+1)\right]_0^1 + \frac{1}{2}\int_0^1 \frac{dt}{t^2+t+1}.$$

The logarithmic contribution is $\frac{1}{2}\log(2/3)$.

Step 3: Evaluate the inverse-tangent contribution Completing the square gives

$$\int_0^1 \frac{dt}{t^2+t+1} = \frac{2}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right)\right]_0^1 = \frac{2}{\sqrt{3}}\left(\frac{\pi}{3}-\frac{\pi}{6}\right) = \frac{\pi}{3\sqrt{3}}.$$

Hence $I = \frac{1}{2}\log(2/3) + \pi/(6\sqrt{3})$. Comparing with the specified form identifies $a = 2$ and $b = 6$, both natural numbers.

Step 4: Conclusion The required numerical value is $a + b = 2 + 6 = \boxed{8}$.

Quick Tip to Solve: After substituting $\tan x$, subtract fractions with the two quadratic denominators; their difference produces the desired numerator immediately.

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27. In a survey of 220 students of a higher secondary school, it was found that at least 125 and at most 130 students studied Mathematics; at least 85 and at most 95 studied Physics; at least 75 and at most 90 studied Chemistry; 30 studied both Physics and Chemistry; 50 studied both Chemistry and Mathematics; 40 studied both Mathematics and Physics and 10 studied none of these subjects. Let m and n respectively be the least and the most number of students who studied all the three subjects. Then $\mathrm{m}+\mathrm{n}$ is equal to _____. [4 marks]

Solution:

Concept:

- For three sets, $|M \cup P \cup C| = |M| + |P| + |C| - |M \cap P| - |P \cap C| - |C \cap M| + |M \cap P \cap C|$.
- The triple intersection cannot exceed any pairwise intersection. To prove a bound is attainable, check every disjoint Venn-region count is nonnegative.

Step 1: Express the triple intersection using subject totals Let t be the number studying all three subjects, and let $S = |M| + |P| + |C|$. Since 10 of the 220 students study none, the union has size 210. The three given pairwise intersections sum to $40 + 30 + 50 = 120$, and these counts include students in all three. Inclusion-exclusion therefore gives

$$210 = S - 120 + t, \qquad t = 330 - S.$$

The largest allowed total is $130 + 95 + 90 = 315$, so $t \geq 15$. Also $P \cap C$ has only 30 members, so $t \leq 30$.

Step 2: Check the least possible value Take $(|M|, |P|, |C|) = (130, 95, 90)$, which gives $t = 15$. The pair-only regions MP, PC, CM contain respectively $25, 15, 35$ students. The subject-only regions contain

$$130 - 40 - 50 + 15 = 55,$$
$$95 - 40 - 30 + 15 = 40,$$
$$90 - 50 - 30 + 15 = 25.$$

All seven disjoint region counts are nonnegative, and they sum to 210. Thus the lower bound is achieved, so $m = 15$.

Step 3: Check the greatest possible value Take $(|M|, |P|, |C|) = (125, 85, 90)$, all within their given

ranges. Their sum is 300, giving $t = 30$. The pair-only regions now contain $10, 0, 20$, while the subject-only regions contain $65, 45, 40$. Together with the triple region of 30, their total is again 210. Adding the 10 outside all sets recovers the full survey of 220 students. Hence the upper bound is achieved and $n = 30$.

Step 4: Conclusion Therefore $m + n = 15 + 30 = \boxed{45}$.

Quick Tip to Solve: Use inclusion-exclusion first, then impose $t \leq 30$; the smallest subject totals alone would suggest an impossible triple intersection of 45.

28. Let the length of the focal chord PQ of the parabola $y^2 = 12x$ be 15 units. If the distance of PQ from the origin is p, then $10\mathrm{p}^2$ is equal to _____. [4 marks]

Solution:

Concept:

- For $y^2 = 4ax$, the focus is $(a, 0)$, and a focal chord making angle θ with the positive axis has length $4a \csc^2 \theta$.
- The perpendicular distance from the origin to $mx - y - 3m = 0$ is $|3m|/\sqrt{1 + m^2}$.

Step 1: Write the focal-chord line Here $4a = 12$, so $a = 3$ and the focus is $(3, 0)$. A nonvertical focal chord has equation $y = m(x - 3)$. The vertical focal chord is the latus rectum and has length 12, which does not equal the specified 15, so a finite slope is appropriate. A horizontal line through the focus does not cut a finite chord; hence $m \neq 0$.

Step 2: Derive the chord length in terms of its slope Substitute the line into the parabola:

$$m^2(x - 3)^2 = 12x.$$

If the endpoint abscissae are x_1, x_2, the quadratic gives

$$x_1 + x_2 = 6 + \frac{12}{m^2}, \qquad x_1 x_2 = 9.$$

Their squared difference is consequently

$$(x_1 - x_2)^2 = \left(6 + \frac{12}{m^2}\right)^2 - 36 = \frac{144(m^2 + 1)}{m^4}.$$

As $y_1 - y_2 = m(x_1 - x_2)$, the Euclidean chord length is

$$PQ = \sqrt{1 + m^2}\,|x_1 - x_2| = \frac{12(m^2 + 1)}{m^2}.$$

Step 3: Use the length to obtain the distance Setting this length equal to 15 yields

$$12(m^2 + 1) = 15m^2 \quad \Rightarrow \quad m^2 = 4.$$

The chord line is $mx - y - 3m = 0$. Apply the perpendicular-distance formula from $(0, 0)$:

$$p^2 = \frac{(3m)^2}{m^2 + 1} = \frac{9 \cdot 4}{5} = \frac{36}{5}.$$

Both slopes $m = 2$ and $m = -2$ give the same distance, as expected from the symmetry of the parabola about its axis.

Step 4: Conclusion The required value is

$$10p^2 = 10 \cdot \frac{36}{5} = \boxed{72}.$$

Quick Tip to Solve: A focal chord must pass through $(3, 0)$; express its line using the slope, then eliminate that slope with the given chord length.

---

29. Let ABC **be a triangle of area** $15\sqrt{2}$ **and the vectors** $\overrightarrow{\mathrm{AB}} = \hat{i} + 2\hat{j} - 7\hat{k}, \overrightarrow{\mathrm{BC}} = \mathrm{a}\hat{i} + \mathrm{b}\hat{j} + \mathrm{c}\hat{k}$ **and** $\overrightarrow{\mathrm{AC}} = 6\hat{i} + \mathrm{d}\hat{j} - 2\hat{k},\ \mathrm{d} > 0$**. Then the square of the length of the largest side of the triangle** ABC **is _____.** [4 marks]

Solution:

Concept:

- The area of a triangle with adjacent side vectors $\mathbf{u}, \mathbf{v}$ is $\frac{1}{2}|\mathbf{u} \times \mathbf{v}|$. The cross product magnitude accounts for the angle between the sides.
- The third side vector is $\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$. A side length squared is the sum of the squares of its Cartesian components.

Step 1: Use the given area to determine d Take $\mathbf{u} = (1, 2, -7)$ and $\mathbf{v} = (6, d, -2)$, which both start at A. Their cross product is

$$\mathbf{u} \times \mathbf{v} = (7d - 4, -40, d - 12).$$

The area is $15\sqrt{2}$, so its double has squared magnitude 1800. Consequently,

$$(7d - 4)^2 + 1600 + (d - 12)^2 = 1800.$$

On expanding and simplifying,

$$50d^2 - 80d - 40 = 0, \qquad 5d^2 - 8d - 4 = (5d + 2)(d - 2) = 0.$$

The two algebraic roots are 2 and $-2/5$. The stipulated condition $d > 0$ selects $d = 2$.

Step 2: Compare the three squared side lengths With this value the side vectors are $\overrightarrow{AB} = (1, 2, -7)$, $\overrightarrow{AC} = (6, 2, -2)$, and $\overrightarrow{BC} = (5, 0, 5)$. Therefore

$$AB^2 = 1 + 4 + 49 = 54, \qquad AC^2 = 36 + 4 + 4 = 44, \qquad BC^2 = 25 + 0 + 25 = 50.$$

Since lengths are nonnegative, their order is also the order of their squares. The AB side is therefore

longest; taking a square root and then squaring again is unnecessary.

Step 3: Conclusion The square of the largest side length is $\boxed{54}$.

Quick Tip to Solve: Use two vectors with the same initial point in the area formula, impose the sign condition on the parameter, then compare squared lengths directly.

---

30. Let $a = 1 + \frac{{}^2C_2}{3!} + \frac{{}^3C_2}{4!} + \frac{{}^4C_2}{5!} + \ldots$, $b = 1 + \frac{{}^1C_0 + {}^1C_1}{1!} + \frac{{}^2C_0 + {}^2C_1 + {}^2C_2}{2!} + \frac{{}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3}{3!} + \ldots$. **Then** $\frac{2b}{a^2}$ **is equal to _____.** [4 marks]

Solution:

Concept:

- The sum of all binomial coefficients in row n is $\sum_{r=0}^{n} \binom{n}{r} = 2^n$, obtained by putting both terms equal to 1 in the binomial theorem.
- The exponential series is $e^x = \sum_{n=0}^{\infty} x^n/n!$. Expressing each factorial term as a linear combination of reciprocal factorials makes the first series summable.

Step 1: Evaluate the series defining a The nonconstant terms in the first series have general term $\binom{n}{2}/(n+1)!$, starting at $n = 2$. Write

$$\frac{\binom{n}{2}}{(n+1)!} = \frac{1}{2}\left(\frac{1}{(n-1)!} - \frac{2}{n!} + \frac{2}{(n+1)!}\right).$$

This identity follows on multiplying through by $2(n+1)!$, since $n(n+1) - 2(n+1) + 2 = n(n-1)$. Summing the three convergent series gives

$$\begin{aligned} a &= 1 + \frac{1}{2}\left[(e-1) - 2(e-2) + 2\left(e - \tfrac{5}{2}\right)\right] \\ &= 1 + \frac{1}{2}(e-2) = \frac{e}{2}. \end{aligned}$$

Here the three lower indices are respectively 1, 2 and 3, which explain the constants subtracted from e.

Step 2: Evaluate b and form the requested ratio The numerator of the term with denominator $n!$ in the second series is precisely the sum of all binomial coefficients of order n. Thus

$$b = 1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \cdots = e^2.$$

Both series are positive, so dividing by a^2 is valid. Substitute the two exact values before simplifying:

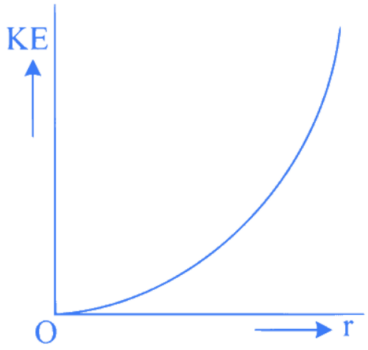
$$\frac{2b}{a^2} = \frac{2e^2}{(e/2)^2} = \frac{2e^2}{e^2/4} = 8.$$

Step 3: Conclusion The required numerical value is $\boxed{8}$.

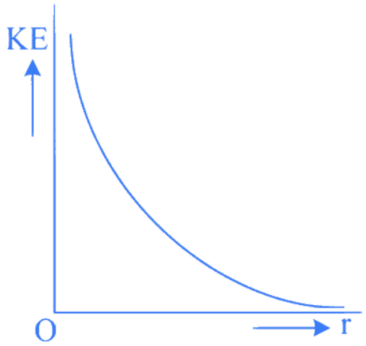
Quick Tip to Solve: For a factorial series, first simplify its general term; for a sum of binomial coefficients, replace the entire numerator by a power of 2.

31. An infinitely long positively charged straight thread has a linear charge density $\lambda\ \mathrm{Cm}^{-1}$. An electron revolves along a circular path having axis along the length of the wire. The graph that correctly represents the variation of the kinetic energy of electron as a function of radius of circular path from the wire is : [4 marks]

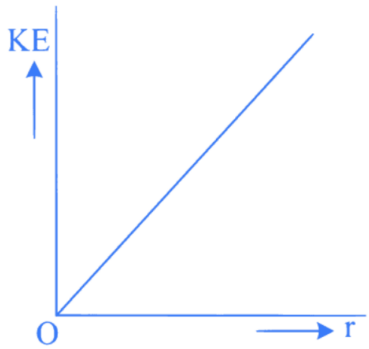
(A)



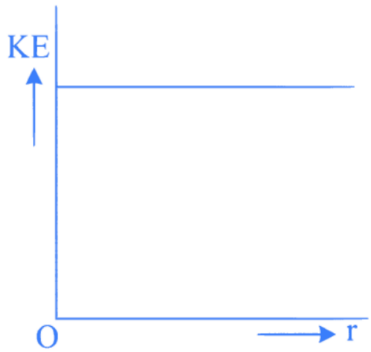
(B)



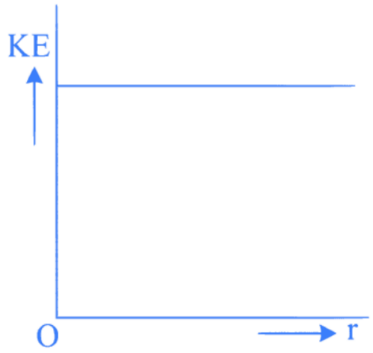
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- Gauss's law gives the electric field magnitude of an infinite line charge at radius r as $E = \lambda/(2\pi\varepsilon_0 r)$. A positive line attracts an electron towards the wire.
- For a circular orbit the inward electric force supplies centripetal force: $eE = mv^2/r$. The orbital kinetic energy is $K = mv^2/2$, with e the magnitude of electron charge.

Step 1: Balance the inward radial force A coaxial cylindrical Gaussian surface has uniform radial field on its curved surface, so $E(2\pi r\ell) = \lambda\ell/\varepsilon_0$. Hence the force magnitude on the electron is

$$F = eE = \frac{e\lambda}{2\pi\varepsilon_0 r}.$$

This force points towards the positively charged wire. Equating it to the centripetal requirement for an orbit of radius r gives

$$\frac{mv^2}{r} = \frac{e\lambda}{2\pi\varepsilon_0 r} \quad \Longrightarrow \quad mv^2 = \frac{e\lambda}{2\pi\varepsilon_0}.$$

The radius cancels because both forces contain the same inverse-radius factor.

Step 2: Identify the kinetic-energy graph It follows that

$$K = \frac{1}{2}mv^2 = \frac{e\lambda}{4\pi\varepsilon_0}.$$

For the same line charge density this is a positive constant, independent of the radius of the orbit. The graph must therefore be a horizontal line above the radius axis, which is the graph in option D. Option A depicts an increasing curved dependence, option B a decreasing dependence and option C a linear increase; each incorrectly introduces a radius dependence into the kinetic energy. The $1/r$ dependence of the electric force must not be confused with a $1/r$ dependence of kinetic energy.

Step 3: Conclusion The kinetic energy is constant: $K = e\lambda/(4\pi\varepsilon_0)$. Hence, the correct option is (D).

Quick Tip to Solve: For a circular orbit write $K = Fr/2$; an inverse-radius central force immediately gives a radius-independent kinetic energy.

32. An electron is projected with uniform velocity along the axis inside a current carrying long solenoid. Then : [4 marks]

(A) the electron will experience a force at 45° to the axis and execute a helical path.

(B) the electron will be accelerated along the axis.

(C) the electron path will be circular about the axis.

(D) the electron will continue to move with uniform velocity along the axis of the solenoid.

Correct Answer: (D) the electron will continue to move with uniform velocity along the axis of the solenoid.

Solution:

Concept:

- The magnetic field inside an ideal long current-carrying solenoid is directed parallel to its axis. Away from the ends it is approximately uniform, with magnitude $B = \mu_0 nI$.
- The magnetic Lorentz force is $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, of magnitude $|q|vB\sin\theta$. Only the velocity component perpendicular to the field produces magnetic deflection.

Step 1: Compare the directions of velocity and field The electron is projected along the solenoid axis. The field inside the long solenoid is also along that axis, so the velocity and field are parallel or antiparallel, depending on the current direction. Either way, $\theta = 0$ or π, giving

$$\sin\theta = 0, \qquad \mathbf{v} \times \mathbf{B} = \mathbf{0}.$$

The negative sign of electron charge changes a nonzero force's direction, but it cannot turn a zero cross product into a nonzero force. Thus the magnetic force on this electron is zero throughout its axial motion.

Step 2: Determine the resulting motion and reject alternatives With no electric field or other force specified, Newton's second law gives

$$m\frac{d\mathbf{v}}{dt} = \mathbf{0} \quad \Longrightarrow \quad \mathbf{v} = \text{constant}.$$

The electron continues in a straight line along the axis at its original speed. A circular path requires a nonzero perpendicular velocity component. A helix requires both parallel and perpendicular components; here the perpendicular component is zero. The claim of a force at 45° is also incompatible with the vanishing Lorentz force. Finally, a magnetic field cannot produce acceleration along itself, so axial acceleration is not a possibility under these conditions.

Step 3: Conclusion The electron continues with uniform velocity along the solenoid axis. Hence, the correct option is (D).

Quick Tip to Solve: Resolve the velocity into components parallel and perpendicular to $\mathbf{B}$; only the perpendicular component can cause circular motion.

33. A metal wire of uniform mass density having length L and mass M is bent to form a semicircular arc and a particle of mass m is placed at the centre of the arc. The gravitational force on the particle by the wire is : [4 marks]

(A) $\frac{\mathrm{GmM}\pi^2}{\mathrm{L}^2}$

(B) $\frac{\mathrm{GMm}\pi}{2\,\mathrm{L}^2}$

(C) 0

(D) $\frac{2\mathrm{GmM}\pi}{\mathrm{L}^2}$

Correct Answer: (D) $\frac{2\mathrm{GmM}\pi}{\mathrm{L}^2}$

Solution:

Concept:

- A small mass element dm at distance R attracts the central particle with magnitude $dF = Gm\,dm/R^2$, directed from the particle towards that element.
- For a uniform semicircular wire, $L = \pi R$ and $dm = (M/\pi)\,d\theta$. Symmetry cancels components perpendicular to the arc's symmetry axis, while components towards the midpoint of the arc add.

Step 1: Express the radius and mass element Let the semicircle occupy polar angles $-\pi/2 \le \theta \le \pi/2$, measured from the line joining its centre to its midpoint. This choice makes the symmetry-axis component of force proportional to $\cos\theta$. From the arc length,

$$R = \frac{L}{\pi}.$$

The linear mass density is M/L, so an element subtending $d\theta$ has mass

$$dm = \frac{M}{L} R\,d\theta = \frac{M}{\pi}\,d\theta.$$

Every element is at the same radius from the particle, but their force directions differ, so their magnitudes cannot simply be added.

Step 2: Integrate the components that survive symmetry The transverse component is odd in θ and integrates to zero. Along the symmetry axis the net force is

$$F = \frac{Gm}{R^2}\frac{M}{\pi}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta = \frac{2GMm}{\pi R^2} = \frac{2GMm\pi}{L^2}.$$

It points from the centre towards the middle of the semicircular arc. Option C would apply to a complete symmetric ring, not a semicircle. Option A effectively adds all elemental force magnitudes without resolving their directions; option B differs from the required result by a factor of four.

Step 3: Conclusion The force magnitude is $\boxed{2GMm\pi/L^2}$, directed towards the midpoint of the arc. Hence, the correct option is (D).

Quick Tip to Solve: At the centre of an arc all source elements are equally distant; use symmetry before integrating the one remaining component.

34. An effective power of a combination of 5 identical convex lenses which are kept in contact along the principal axis is $25\mathrm{D}$. Focal length of each of the convex lens is: [4 marks]

(A) 50 cm

(B) 20 cm

(C) 25 cm

(D) 500 cm

Correct Answer: (B) 20 cm

Solution:

Concept:

- The powers of thin lenses in contact add algebraically: $P_{\text{eq}} = P_1 + \cdots + P_N$. For identical lenses this becomes $P_{\text{eq}} = NP$.
- Lens power in dioptres is the reciprocal of focal length in metres: $P = 1/f$. A convex converging lens has positive power and positive focal length in this convention.

Step 1: Find the power of one lens All five lenses are identical and are kept in contact along their common principal axis, so the same power P occurs five times in the sum. There is no separation correction in this arrangement. Therefore

$$25 = 5P \quad \Longrightarrow \quad P = 5\,\mathrm{D}.$$

The equivalent system is stronger than a single lens. Dividing the total power by five is the appropriate operation; the equivalent focal length itself must not be divided by five to obtain the focal length of one lens.

Step 2: Convert reciprocal power to focal length Since power is stated in dioptres, its reciprocal yields metres:

$$f = \frac{1}{5}\,\mathrm{m} = 0.20\,\mathrm{m} = 20\,\mathrm{cm}.$$

As a check, a 20 cm convex lens has power $+5\,\mathrm{D}$, and five such lenses yield $+25\,\mathrm{D}$, exactly the stated effective power. A 50 cm lens has power 2 D and would give only 10 D in combination. A 25 cm lens has power 4 D and would give 20 D; a 500 cm lens has power 0.2 D and would give only 1 D. Thus none of the other listed focal lengths reproduces the given total power.

Step 3: Conclusion The focal length of each convex lens is $\boxed{20\,\mathrm{cm}}$. Hence, the correct option is (B).

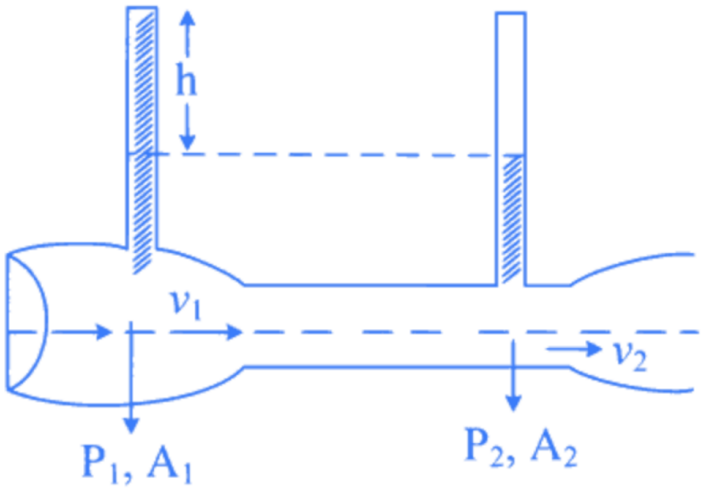
Quick Tip to Solve: For N identical lenses in contact, use $f_{\text{each}} = N/P_{\text{eq}}$ in metres, then convert to centimetres.

35. Given below are two statements :

Statement I : When speed of liquid is zero everywhere, pressure difference at any two points depends on equation $\mathrm{P_1 - P_2 = \rho g\,(\,h_2 - h_1)}$.

Statement II : In ventury tube shown $2gh = v_1^2 - v_2^2$

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]



(A) Statement I is correct but Statement II is incorrect.

(B) Both Statement I and Statement II are correct.

(C) Both Statement I and Statement II are incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (A) Statement I is correct but Statement II is incorrect.

Solution:

Concept:

- For a fluid at rest, $P + \rho gh$ is constant, with height measured upwards from a common datum. Thus $P_1 - P_2 = \rho g(h_2 - h_1)$.
- For steady incompressible ideal flow along a horizontal streamline, Bernoulli's equation is $P + \frac{1}{2}\rho v^2 =$ constant. A piezometer tube measures pressure head through its liquid-column height.

Step 1: Check Statement I using hydrostatic equilibrium When the speed is zero everywhere, there is no kinetic-energy term. The equation of hydrostatic equilibrium between the two locations is

$$P_1 + \rho g h_1 = P_2 + \rho g h_2.$$

Rearranging gives the expression printed in Statement I:

$$P_1 - P_2 = \rho g(h_2 - h_1).$$

A point lower in the liquid has greater pressure, which also confirms the sign. Statement I is therefore correct when the two heights are measured consistently upwards.

Step 2: Check the sign in Statement II against the figure In the shown horizontal Venturi tube, section 1 is wider and has the higher liquid column; section 2 is narrower and has the lower column. Thus $P_1 - P_2 = \rho g h > 0$. Bernoulli's equation between these sections gives

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2.$$

Consequently,

$$\rho g h = \frac{1}{2}\rho(v_2^2 - v_1^2) \quad \Longrightarrow \quad 2gh = v_2^2 - v_1^2.$$

Continuity independently confirms $v_2 > v_1$ because the second cross-section is smaller. The printed expression in Statement II reverses this difference, making its right-hand side negative although h is positive. Statement II is false. Therefore the combinations asserting both true, both false or only II true are all ruled out.

Step 3: Conclusion Statement I is correct and Statement II is incorrect. Hence, the correct option is (A).

Quick Tip to Solve: In a horizontal Venturi tube the narrow section has greater speed and smaller pressure; check the sign of every pressure-head equation against this fact.

---

36. If a rubber ball falls from a height h and rebounds upto the height of $h/2$. The percentage loss of total energy of the initial system as well as velocity ball before it strikes the ground, respectively, are : [4 marks]

(A) $50\%, \sqrt{2gh}$

(B) $50\%, \sqrt{gh}$

(C) $50\%, \sqrt{\frac{gh}{2}}$

(D) $40\%, \sqrt{2gh}$

Correct Answer: (A) $50\%, \sqrt{2gh}$

Solution:

Concept:

- The gravitational potential energy of a mass m at height H, measured from the ground, is mgH. At the highest point the ball is momentarily at rest.
- During the fall, conservation of mechanical energy gives $mgh = \frac{1}{2}mv^2$, where v is the speed immediately before impact. Energy is lost during the collision with the ground.

Step 1: Compare the energies at the two highest points Initially the ball has mechanical energy $E_i = mgh$. After rebounding, its greatest height is $h/2$, so its remaining mechanical energy is $E_f = mg(h/2)$. Taking the ground as the common zero of potential energy makes the comparison direct. Thus the energy lost during impact is

$$\Delta E = E_i - E_f = mgh - \frac{mgh}{2} = \frac{mgh}{2}.$$

The required percentage uses the initial energy in the denominator:

$$\frac{\Delta E}{E_i} \times 100 = \frac{mgh/2}{mgh} \times 100 = 50\%.$$

Step 2: Find the speed just before impact Before the first collision, the ball falls through the full height h, starting from rest. Hence

$$\frac{1}{2}mv^2 = mgh \quad \Rightarrow \quad v = \sqrt{2gh}.$$

This is a speed; the velocity is directed downwards. The speed $\sqrt{gh}$ belongs to the upward motion immediately after the rebound and must not be used for the requested instant. Option (C) also has an incorrect impact speed, while option (D) has an incorrect percentage loss.

Step 3: Conclusion The energy loss is 50%, and the speed before striking the ground is $\sqrt{2gh}$, directed downwards. Hence, the correct option is (A).

Quick Tip to Solve: Use the ratio of rebound height to release height for the retained energy fraction; use the original height for the incident speed.

---

37. In an ac circuit, the instantaneous current is zero, when the instantaneous voltage is maximum. In this case, the source may be connected to :

A. pure inductor.

B. pure capacitor.

C. pure resistor.

D. combination of an inductor and capacitor.

Choose the correct answer from the options given below : [4 marks]

(A) A, B and C only

(B) A and B only

(C) A, B and D only

(D) B, C and D only

Correct Answer: (C) A, B and D only

Solution:

Concept:

- For sinusoidal voltage and current, a zero current at a voltage extremum indicates a phase difference of $\pi/2$. A pure reactance has this phase relation.
- In an ideal inductor current lags voltage by $\pi/2$; in an ideal capacitor current leads voltage by $\pi/2$. In a resistor current and voltage are in phase.

Step 1: Test the pure elements Let the applied voltage be $v = V_m \sin \omega t$. Its positive maximum occurs when $\omega t = \pi/2$. For a pure inductor, $i = I_m \sin(\omega t - \pi/2)$, which is zero at that instant. For a pure capacitor, $i = I_m \sin(\omega t + \pi/2)$, also zero at that instant. Thus statements A and B are both possible. A

pure resistor instead gives $i = (V_m/R)\sin\omega t$; its current is maximum, rather than zero, at the voltage maximum. Statement C must therefore be excluded.

Step 2: Check the inductor-capacitor combination An ideal series combination of an inductor and capacitor has impedance

$$Z = j\left(\omega L - \frac{1}{\omega C}\right).$$

Away from exact resonance this is purely reactive: positive net reactance gives inductive behaviour and negative net reactance gives capacitive behaviour. In either case the current and source voltage differ in phase by $\pi/2$. Thus such a combination may also satisfy the stated condition, making D possible. The question asks what the source may be connected to; it does not assert that every frequency of an LC circuit works. Exact ideal resonance is not required here. Options (A) and (D) include the resistor, while (B) omits the possible LC combination.

Step 3: Conclusion The possible connections are A, B and D only. Hence, the correct option is (C).

Quick Tip to Solve: At a voltage peak, zero current signals a purely reactive circuit; reject a pure resistor immediately.

38. Which of the following nuclear fragments corresponding to nuclear fission between neutron $\left({}_0^1\mathrm{n}\right)$ and uranium isotope $\left({}_{92}^{235}\mathrm{U}\right)$ is correct : [4 marks]

(A) ${}_{56}^{140}\mathrm{Xe} + {}_{38}^{94}\mathrm{Sr} + 3{}_0^1\mathrm{n}$

(B) ${}_{51}^{153}\mathrm{Sb} + {}_{41}^{99}\mathrm{Nb} + 3{}_0^1\mathrm{n}$

(C) ${}_{56}^{144}\mathrm{Ba} + {}_{36}^{89}\mathrm{Kr} + 4{}_0^1\mathrm{n}$

(D) ${}_{56}^{144}\mathrm{Ba} + {}_{36}^{89}\mathrm{Kr} + 3{}_0^1\mathrm{n}$

Correct Answer: (D) ${}_{56}^{144}\mathrm{Ba} + {}_{36}^{89}\mathrm{Kr} + 3{}_0^1\mathrm{n}$

Solution:

Concept:

- In a nuclear reaction, the total mass number A and total atomic number Z are conserved separately. A neutron contributes $A = 1$ and $Z = 0$.
- The incident neutron and uranium nucleus have combined totals $A = 1 + 235 = 236$ and $Z = 0 + 92 = 92$. Every proposed product set must satisfy both totals.

Step 1: Apply the two conservation tests The entrance channel is ${}_0^1\mathrm{n} + {}_{92}^{235}\mathrm{U}$. The outgoing fragments and emitted neutrons must therefore have a combined nucleon count of 236 and a combined proton count of 92. The energy released appears as kinetic energy and radiation, but it does not permit a change in either of these integer balance equations. In particular, one must include the incident neutron when balancing mass numbers; comparing products with 235 alone would be incorrect.

Step 2: Check every proposed product set For (A), the printed mass numbers give $140 + 94 + 3 = 237$, and its printed atomic numbers give $56 + 38 = 94$; neither total is correct. Its xenon symbol also does not match the printed atomic number 56. For (B), $153 + 99 + 3 = 255$, which fails the mass-number test even though $51 + 41 = 92$. For (C), $144 + 89 + 4 = 237$, again one nucleon too many. Finally, (D)

gives

$$144 + 89 + 3 = 236, \qquad 56 + 36 = 92.$$

Both conservation equations hold, and the symbols barium and krypton have atomic numbers 56 and 36 respectively.

Step 3: Conclusion The balanced channel is

$$^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \longrightarrow {}^{144}_{56}\mathrm{Ba} + {}^{89}_{36}\mathrm{Kr} + 3{}^{1}_{0}\mathrm{n}.$$

Hence, the correct option is (D).

Quick Tip to Solve: Balance the upper numbers and lower numbers independently, remembering to count the incoming neutron.

---

39. A body travels $102.5\ \mathrm{m}$ in n^{th} second and $115.0\ \mathrm{m}$ in $(\mathrm{n}+2)^{\mathrm{th}}$ second. The acceleration is : [4 marks]

(A) $6.25\ \mathrm{m/s^2}$

(B) $5\ \mathrm{m/s^2}$

(C) $12.5\ \mathrm{m/s^2}$

(D) $9\ \mathrm{m/s^2}$

Correct Answer: (A) $6.25\ \mathrm{m/s^2}$

Solution:

Concept:

- For motion with constant acceleration a and initial velocity u, displacement in the nth one-second interval is $s_n = u + \frac{a}{2}(2n-1)$ when SI numerical values are used.
- Subtracting displacements over equal one-second intervals eliminates the unknown initial velocity. Intervals whose indices differ by two have displacements differing by $2a$.

Step 1: Write the two one-second displacements Let $S(t) = ut + \frac{1}{2}at^2$ be the total displacement from the start. The displacement in the nth second is

$$s_n = S(n) - S(n-1) = u + \frac{a}{2}(2n-1) = 102.5.$$

Similarly, the displacement in the $(n+2)$th second is

$$s_{n+2} = u + \frac{a}{2}(2n+3) = 115.0.$$

These are distances covered during individual seconds, not the total distances covered up to those times. As usual for this question, uniformly accelerated motion without a reversal during these intervals is understood.

Step 2: Subtract and calculate the acceleration The initial-velocity term and the common n-dependent

contribution cancel. Consequently,

$$115.0 - 102.5 = \frac{a}{2}\big[(2n+3) - (2n-1)\big] = 2a.$$

Therefore

$$a = \frac{12.5}{2} = 6.25\,\mathrm{m\,s^{-2}}.$$

The difference 12.5 m is accumulated over a separation of two interval indices; treating it as the change between adjacent seconds would incorrectly produce option (C). Options (B) and (D) do not reproduce the observed difference $2a = 12.5$ in the two one-second displacements.

Step 3: Conclusion The constant acceleration is $6.25\,\mathrm{m\,s^{-2}}$. Hence, the correct option is (A).

Quick Tip to Solve: For constant acceleration, subtract the distances in two numbered seconds and divide by the difference of their indices.

40. In an experiment to measure focal length (f) of convex lens, the least counts of the measuring scales for the position of object (u) and for the position of image (v) are Δu and Δv, respectively. The error in the measurement of the focal length of the convex lens will be: [4 marks]

(A) $2f\left[\frac{\Delta u}{u} + \frac{\Delta v}{v}\right]$
(B) $f\left[\frac{\Delta u}{u} + \frac{\Delta v}{v}\right]$
(C) $f^2\left[\frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}\right]$
(D) $\frac{\Delta u}{u} + \frac{\Delta v}{v}$

Correct Answer: (C) $f^2\left[\frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}\right]$

Solution:

Concept:

- For a convex lens forming a real image, positive object and image distances obey $1/f = 1/u + 1/v$. Equivalently, the signed Cartesian form gives the same absolute-error result.
- For small independent bounded measurement errors, the maximum error is obtained by adding the magnitudes of the first-order differential contributions, not by allowing them to cancel.

Step 1: Differentiate the lens equation Treat the measured distances u and v as positive magnitudes. Differentiating the reciprocal form of the lens equation gives

$$-\frac{df}{f^2} = -\frac{du}{u^2} - \frac{dv}{v^2}.$$

Thus a change in either measured distance contributes to the inferred focal length through a reciprocal-square coefficient. This differentiation keeps the dependence of both the numerator and denominator that would occur if one instead started from $f = uv/(u+v)$. Applying a product-error rule to only uv would miss the denominator dependence.

Step 2: Convert differentials to maximum absolute errors The least counts supply the magnitudes

Δu and Δv. Taking absolute contributions and adding them yields

$$\frac{\Delta f}{f^2} = \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2},$$

so

$$\Delta f = f^2 \left(\frac{\Delta u}{u^2} + \frac{\Delta v}{v^2} \right).$$

Each term inside the brackets has dimensions of inverse length, making the final result a length, as an absolute focal-length error must be. Option (D) is dimensionless and therefore cannot be the requested error. Options (A) and (B) have the wrong sensitivity coefficients and do not follow from differentiating the lens relation.

Step 3: Conclusion The maximum first-order error is $\Delta f = f^2[\Delta u/u^2 + \Delta v/v^2]$. Hence, the correct option is (C).

Quick Tip to Solve: For errors in a reciprocal relation, differentiate the reciprocal form first; add absolute contributions only after differentiation.

41. The co-ordinates of a particle moving in x-y plane are given by : $x = 2 + 4\text{t}, y = 3\text{t} + 8\text{t}^2$.
The motion of the particle is : [4 marks]

(A) uniform motion along a straight line.

(B) non-uniformly accelerated.

(C) uniformly accelerated having motion along a straight line.

(D) uniformly accelerated having motion along a parabolic path.

Correct Answer: (D) uniformly accelerated having motion along a parabolic path.

Solution:

Concept:

- Velocity components are the first derivatives of coordinates, $v_x = dx/dt$ and $v_y = dy/dt$. Acceleration components are their time derivatives.
- A constant acceleration vector does not require a straight path. Eliminating time from the coordinate equations reveals the geometric trajectory independently of the acceleration test.

Step 1: Differentiate to classify the acceleration From $x = 2 + 4t$ and $y = 3t + 8t^2$, obtain

$$v_x = 4, \qquad v_y = 3 + 16t.$$

Differentiating once more gives

$$a_x = 0, \qquad a_y = 16.$$

Both components are constant, so the vector acceleration is constant and directed along the positive y-axis. The changing y-component of velocity means that the motion is not uniform. In particular, a speed that changes with time must not be confused with an acceleration that changes with time; the latter is constant in this example.

Step 2: Eliminate time to identify the path Since $v_x = 4 \neq 0$, the first equation can be solved for time:

$$t = \frac{x-2}{4}.$$

Substituting this into the second equation gives

$$y = \frac{3}{4}(x-2) + \frac{1}{2}(x-2)^2.$$

This is a quadratic equation for y in terms of x, with a nonzero quadratic coefficient, and hence represents a parabola. Equivalently, the trajectory slope $dy/dx = (3+16t)/4$ varies with time, which rules out a straight line. Option (A) fails both the speed and path tests, (B) incorrectly calls the acceleration nonuniform, and (C) incorrectly calls the path straight.

Step 3: Conclusion The particle has constant acceleration $16\,\hat{\mathbf{j}}$ and moves along a parabolic path. Hence, the correct option is (D).

Quick Tip to Solve: Test constant acceleration by differentiating twice, then test the path by eliminating time; the two classifications are distinct.

42. The equation of stationary wave is :

$$y = 2\mathrm{a}\sin\left(\frac{2\pi \mathrm{nt}}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right).$$

Which of the following is NOT correct : [4 marks]

(A) The dimensions of nt is [L]

(B) The dimensions of n is $[\mathrm{LT}^{-1}]$

(C) The dimensions of x is [L]

(D) The dimensions of n/λ is [T]

Correct Answer: (D) The dimensions of n/λ is [T]

Solution:

Concept:

- The arguments of sine and cosine must be dimensionless. Numerical constants such as 2π carry no dimensions.
- In the given equation, x and wavelength λ have dimension $[L]$, while time t has dimension $[T]$. The meaning of n must be inferred from its actual position in the equation.

Step 1: Find the dimensions required by the sine argument The temporal factor is $\sin(2\pi nt/\lambda)$. Its argument must satisfy

$$\left[\frac{nt}{\lambda}\right] = 1.$$

Therefore

$$[nt] = [\lambda] = [L], \qquad [n] = \frac{[L]}{[T]} = [LT^{-1}].$$

Here the symbol n has the dimensions of speed, even if that letter is sometimes used to denote frequency in other formulas. The particular printed expression, rather than a memorised symbol convention, determines its dimensions. Thus statements (A) and (B) are both correct.

Step 2: Check the spatial argument and the disputed ratio The cosine factor is $\cos(2\pi x/\lambda)$, so $[x] = [\lambda] = [L]$, confirming statement (C). Finally,

$$\left[\frac{n}{\lambda}\right] = \frac{[LT^{-1}]}{[L]} = [T^{-1}].$$

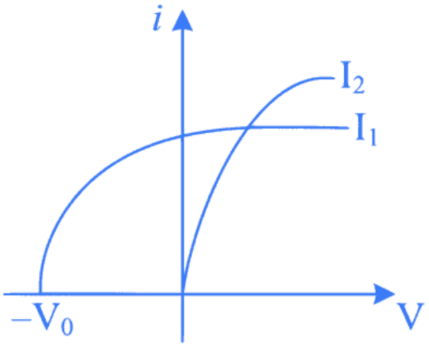
This ratio is a frequency-like quantity, not a time. Statement (D) prints $[T]$, the reciprocal of the correct dimension, and is therefore the false statement requested. The prefactor $2a$ only determines the dimension of displacement amplitude and has no effect on the dimensional conditions on the phases.

Step 3: Conclusion The incorrect statement is that n/λ has dimension $[T]$; its actual dimension is $[T^{-1}]$. Hence, the correct option is (D).

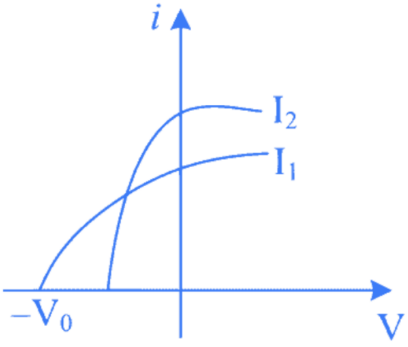
Quick Tip to Solve: Ignore familiar meanings of a symbol: require every trigonometric argument in the printed equation to be dimensionless.

43. Which figure shows the correct variation of applied potential difference (V) with photoelectric current (I) at two different intensities of light $(I_1 < I_2)$ of same wavelengths : [4 marks]

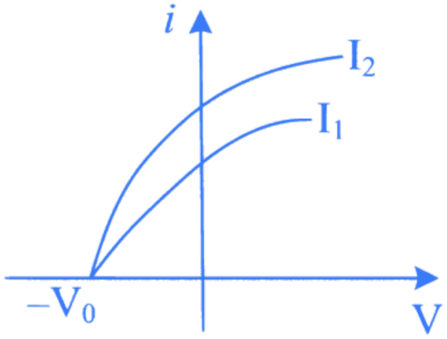
(A)



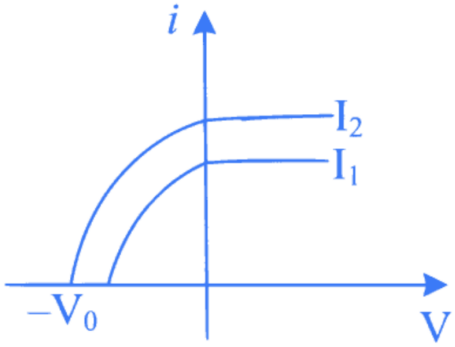
(B)



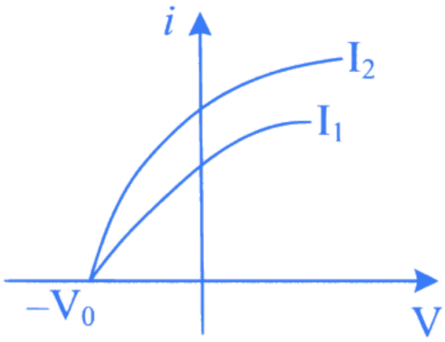
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- The stopping potential is fixed by the largest photoelectron kinetic energy: $eV_0 = h\nu - \phi$, where ν is light frequency and ϕ is the metal work function.
- At fixed wavelength the frequency and energy per photon remain unchanged. Increasing intensity increases the number of photons incident per second and hence the saturation photocurrent.

Step 1: Compare the stopping potentials Both beams have the same wavelength, so $\nu = c/\lambda$ is identical for them. The emitting metal is also unchanged, so the work function is the same. Einstein's equation therefore gives the same maximum electron kinetic energy for both beams. Consequently, both current curves must meet the voltage axis at the same negative voltage $-V_0$; a larger intensity cannot shift that intercept.

Step 2: Compare the saturation currents For intensity $I_2 > I_1$, more photons strike the surface per unit time. Under otherwise identical conditions more electrons are emitted and collected per unit

time. Thus the saturation current for I_2 is greater than for I_1. The upper curve must therefore carry the label I_2. This distinguishes photon number from photon energy: the former affects the current, whereas the latter determines stopping potential.

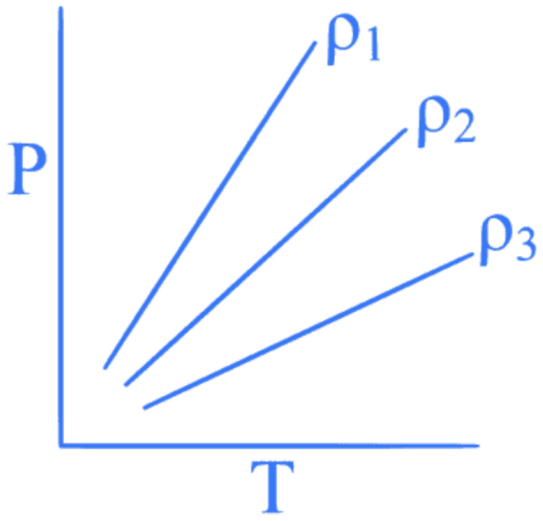
Step 3: Select the graph with both properties Graph (C) shows a common negative-voltage intercept and the larger current for I_2. In (A), the I_2 curve incorrectly starts at zero voltage rather than at the same stopping voltage as I_1. Graphs (B) and (D) also show different stopping voltages for the two intensities. They would require a change in maximum kinetic energy, which changing intensity alone does not produce.

Step 4: Conclusion The two curves have the same stopping potential, while I_2 has the larger saturation photocurrent. Hence, the correct option is (C).

Quick Tip to Solve: For photoelectric graphs, check the voltage-axis intercept first for frequency, then compare saturation current for intensity.

---

44. P-T diagram of an ideal gas having three different densities ρ_1, ρ_2, ρ_3 (in three different cases) is shown in the figure. Which of the following is correct : [4 marks]



(A) $\rho_1 > \rho_2$

(B) $\rho_2 < \rho_3$

(C) $\rho_1 = \rho_2 = \rho_3$

(D) $\rho_1 < \rho_2$

Correct Answer: (A) $\rho_1 > \rho_2$

Solution:

Concept:

- For an ideal gas, $PV = nRT$. Writing $n = m/M$ and density $\rho = m/V$ gives $P = \rho RT/M$, where M is molar mass.

- For the same gas at fixed density, the slope of the pressure-temperature line is $dP/dT = \rho R/M$, so a steeper line represents greater density.

Step 1: Express pressure in terms of density Take a mass m of the gas in volume V. The ideal-gas equation is $PV = (m/M)RT$. Dividing by V gives

$$P = \frac{m}{V}\frac{R}{M}T = \frac{\rho R}{M}T.$$

Temperature in this relation is the absolute temperature. As all three cases concern the same ideal gas, the factor R/M is common to the three lines.

Step 2: Read the slopes in the figure The line labelled ρ_1 is the steepest, the line labelled ρ_2 has the intermediate slope, and the line labelled ρ_3 is the shallowest. Since slope is directly proportional to density,

$$\rho_1 > \rho_2 > \rho_3.$$

One compares the inclinations of the lines, rather than their drawn lengths or the positions of their labels.

Step 3: Check the alternatives Option (A), $\rho_1 > \rho_2$, agrees with the ordering. Option (B) reverses the order of the second and third densities. Option (C) would require three equal slopes, whereas the figure displays different inclinations. Option (D) reverses the comparison of the first two lines. Thus only (A) is consistent with the ideal-gas relation and the graph.

Step 4: Conclusion The density ordering is $\rho_1 > \rho_2 > \rho_3$, in particular $\rho_1 > \rho_2$. Hence, the correct option is (A).

Quick Tip to Solve: For the same gas on a P-T graph, use slope proportional to density; do not confuse it with the inverse-density relation of a V-T graph.

45. The electric field in an electromagnetic wave is given by $\overrightarrow{\mathrm{E}} = \hat{i}40\cos\omega(\mathrm{t} - z/\mathrm{c})\mathrm{NC}^{-1}$. The magnetic field induction of this wave is (in SI unit) : [4 marks]

(A) $\overrightarrow{\mathrm{B}} = \hat{j}\frac{40}{\mathrm{c}}\cos\omega(\mathrm{t} - z/\mathrm{c})$

(B) $\overrightarrow{\mathrm{B}} = \hat{i}\frac{40}{\mathrm{c}}\cos\omega(\mathrm{t} - z/\mathrm{c})$

(C) $\vec{B} = \hat{j}40\cos\omega(t - z/c)$

(D) $\overrightarrow{\mathrm{B}} = \hat{k}\frac{40}{\mathrm{c}}\cos\omega(\mathrm{t} - z/\mathrm{c})$

Correct Answer: (A) $\overrightarrow{\mathrm{B}} = \hat{j}\frac{40}{\mathrm{c}}\cos\omega(\mathrm{t} - z/\mathrm{c})$

Solution:

Concept:

- In a plane electromagnetic wave in vacuum, $\mathbf{E}$, $\mathbf{B}$, and the propagation direction are mutually perpendicular; $\mathbf{E} \times \mathbf{B}$ points along propagation.
- The fields oscillate in phase and their magnitudes obey $E = cB$, where c is the speed of light.

Step 1: Identify the propagation and electric-field directions The phase is $\omega(t - z/c)$. Holding this

phase constant gives $z = ct +$ constant; therefore the wave travels in the positive z-direction. The coefficient $\hat{i}$ shows that the electric field is directed along the x-axis. The magnetic field must be transverse to both the propagation direction and the electric field.

Step 2: Find the magnetic-field direction and amplitude To obtain $+\hat{k}$ for the propagation direction, use $\hat{i} \times \hat{j} = \hat{k}$. Thus the magnetic field is along $+\hat{j}$ when the electric field is along $+\hat{i}$. Its amplitude follows from $B_0 = E_0/c$:

$$B_0 = \frac{40}{c}\,\mathrm{T}.$$

The two fields have the same phase, so no extra phase difference or minus sign is introduced.

Step 3: Write the field and eliminate the other choices The required field is

$$\mathbf{B} = \hat{j}\,\frac{40}{c}\cos[\omega(t - z/c)]\ \mathrm{T}.$$

Option (B) is parallel to the electric field, which violates transversality. Option (C) uses the wrong amplitude: in SI units a magnetic amplitude of 40 would not satisfy $E_0/B_0 = c$. Option (D) makes the magnetic field parallel to propagation. Only option (A) satisfies direction, magnitude and phase simultaneously.

Step 4: Conclusion $\mathbf{B} = \hat{j}\frac{40}{c}\cos[\omega(t - z/c)]\,\mathrm{T}$. Hence, the correct option is (A).

Quick Tip to Solve: Read propagation from the phase first, then use the right-hand rule for E cross B and divide the electric amplitude by c.

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46. The resistances of the platinum wire of a platinum resistance thermometer at the ice point and steam point are 8Ω and 10Ω respectively. After inserting in a hot bath of temperature $400^\circ\mathrm{C}$, the resistance of platinum wire is : [4 marks]

(A) 10 Ω

(B) 16 Ω

(C) 8 Ω

(D) 2 Ω

Correct Answer: (B) 16 Ω

Solution:

Concept:

- A calibrated platinum resistance thermometer uses an approximately linear resistance-temperature relation: $R_t = R_0 + kt$, with t in degrees Celsius.
- The ice and steam points fix the gradient: $k = (R_{100} - R_0)/100$. The resistance increment is proportional to the temperature rise from the ice point.

Step 1: Determine the calibration slope The ice point corresponds to $0^\circ\mathrm{C}$, where $R_0 = 8\,\Omega$. The steam point corresponds to $100^\circ\mathrm{C}$, where $R_{100} = 10\,\Omega$. Hence the resistance rises by $2\,\Omega$ for a temperature

rise of 100°C, giving

$$k = \frac{10 - 8}{100} = 0.02\,\Omega/^\circ\text{C}.$$

The resistance at the ice point is the intercept, not the gradient.

Step 2: Calculate the resistance at the hot-bath temperature Apply the linear calibration to the temperature $t = 400^\circ\text{C}$:

$$R_{400} = R_0 + k(400) = 8 + 0.02(400) = 16\,\Omega.$$

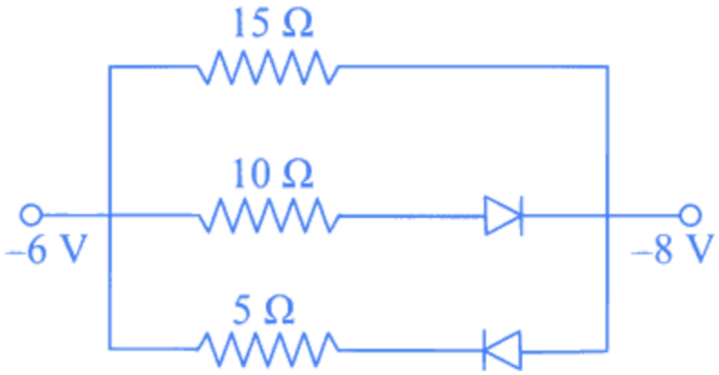
Equivalently, the rise of 400°C is four times the calibration interval, so the resistance increment is four times $2\,\Omega$, namely $8\,\Omega$. Adding this increment to the initial $8\,\Omega$ gives the same result.

Step 3: Check the result against the choices Option (B) gives $16\,\Omega$ and matches the calibrated value. Option (A), $10\,\Omega$, is the steam-point resistance, rather than the resistance at the hotter bath. Option (C), $8\,\Omega$, leaves the resistance at its ice-point value. Option (D), $2\,\Omega$, is only the resistance difference between the two calibration points. None of those alternatives represents the total hot-bath resistance.

Step 4: Conclusion The platinum wire has resistance $16\,\Omega$ at 400°C. Hence, the correct option is (B).

Quick Tip to Solve: Calculate the resistance increase from the two calibration points and add it to the ice-point resistance; the increase alone is not the answer.

47. The value of net resistance of the network as shown in the given figure is : [4 marks]



(A) $(30/11)\Omega$

(B) 6Ω

(C) $(5/2)\Omega$

(D) $(15/4)\Omega$

Correct Answer: (B) 6Ω

Solution:

Concept:

- An ideal forward-biased diode is a short circuit, whereas an ideal reverse-biased diode is an open circuit.

- Current through a conducting resistor branch goes from higher potential to lower potential. Parallel branches satisfy $1/R_{\text{eq}} = \sum 1/R_i$.

Step 1: Determine the higher-potential terminal The left terminal is at $-6\,\text{V}$, and the right terminal is at $-8\,\text{V}$. Although both potentials are negative, -6 is greater than -8. Therefore the left-to-right potential difference is

$$V_L - V_R = (-6) - (-8) = 2\,\text{V}.$$

Conventional current, in a conducting branch, consequently flows from the left terminal toward the right terminal.

Step 2: Identify the conducting diode branch The middle branch contains $10\,\Omega$ and a diode whose cathode bar is on the right. Its anode is on the higher-potential left side, so it is forward biased and conducts. The lower branch contains $5\,\Omega$ and the oppositely directed diode. That diode is reverse biased, so the lower branch is open. The top $15\,\Omega$ branch has no diode and remains connected.

Step 3: Combine only the active resistances The network therefore reduces to $15\,\Omega$ in parallel with $10\,\Omega$:

$$R_{\text{eq}} = \frac{15 \times 10}{15 + 10} = 6\,\Omega.$$

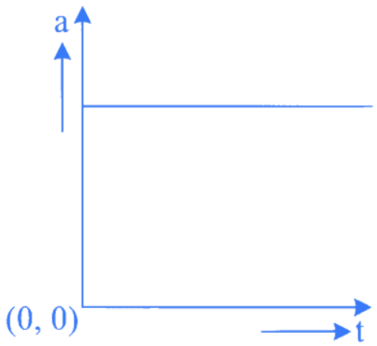
Option (A), $30/11\,\Omega$, comes from incorrectly retaining all three resistors in parallel. Option (D), $15/4\,\Omega$, comes from retaining the reverse-biased $5\,\Omega$ branch instead of the conducting middle branch. Option (C) also does not follow from the correctly selected active branches.

Step 4: Conclusion The $10\,\Omega$ diode branch conducts and the $5\,\Omega$ diode branch is open, giving $R_{\text{eq}} = 6\,\Omega$. Hence, the correct option is (B).

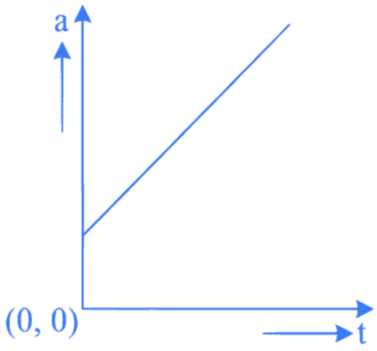
Quick Tip to Solve: With negative terminal potentials, compare their algebraic values before checking diode bias: -6 V is higher than -8 V.

48. A wooden block, initially at rest on the ground, is pushed by a force which increases linearly with time t. Which of the following curve best describes acceleration of the block with time : [4 marks]

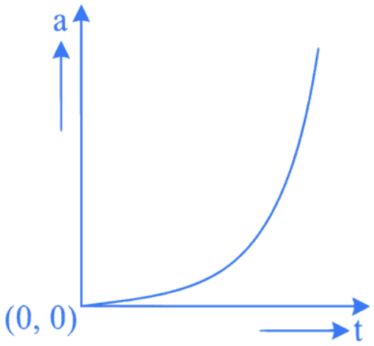
(A)



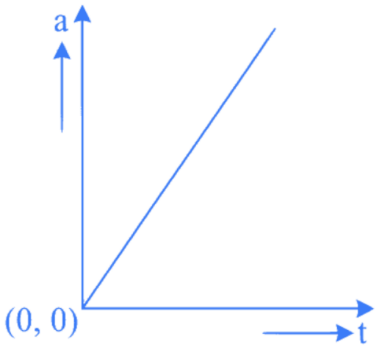
(B)



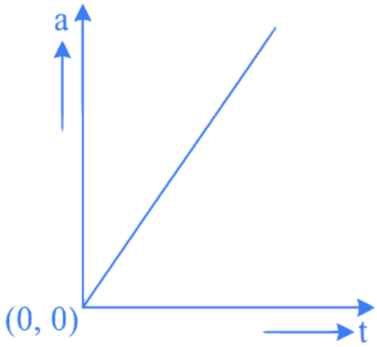
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- Newton's second law gives $ma = F - f$. With a linearly increasing applied force, $F = kt$, where k is a constant rate of increase.

- While the block remains at rest, static friction adjusts to the push. Once sliding begins, kinetic friction is approximately constant and acceleration rises linearly with time.

Step 1: Distinguish the sticking and sliding regimes Initially the wooden block is stationary. As long as the push is below the limiting static friction $\mu_s mg$, static friction balances it and the acceleration is zero. Motion starts when this limit is exceeded. Thus the complete physical graph includes an initial interval on the time axis; the rising branch represents the subsequent sliding motion.

Step 2: Find the acceleration during sliding On a horizontal surface the normal reaction is mg, so kinetic friction has magnitude $\mu_k mg$, approximately independent of speed in this model. Newton's law then gives

$$a = \frac{kt - \mu_k mg}{m} = \frac{k}{m}t - \mu_k g.$$

Hence the acceleration in the moving stage has the constant positive slope k/m. If the time origin for this straight branch is shifted to the corresponding threshold, its equation takes the form $a = (k/m)\tau$.

Step 3: Compare the displayed schematic curves Among the given schematics, (D) represents the required straight increasing branch. It is understood as the linear growth after the initial sticking stage; that schematic suppresses the waiting interval. Option (A) would require constant net force. Option (C) has increasing slope and would require a nonlinear change of net force. Option (B) assigns a positive acceleration at the initial displayed instant, inconsistent with the stated initial rest-and-sticking condition when that instant is the start of pushing. The characteristic increase to choose is therefore the straight rising graph (D).

Step 4: Conclusion Acceleration increases linearly once the force overcomes friction; the displayed straight rising schematic is (D). Hence, the correct option is (D).

Quick Tip to Solve: First account for static friction, then use $a = (F - f_k)/m$ during sliding. A linearly increasing force produces a straight acceleration-time branch.

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49. To measure the internal resistance of a battery, potentiometer is used. For $R = 10\Omega$, the balance point is observed at $l = 500$ cm and for $\mathrm{R} = 1\Omega$ the balance point is observed at $l = 400$ cm. The internal resistance of the battery is approximately : [4 marks]

(A) 0.1Ω

(B) 0.3Ω

(C) 0.2Ω

(D) 0.4Ω

Correct Answer: (B) 0.3Ω

Solution:

Concept:

- With external load resistance R, a cell of emf E and internal resistance r has terminal voltage $V = ER/(R + r)$.
- For an unchanged potentiometer potential gradient, balance length l is proportional to terminal voltage. Ratios eliminate both the gradient and the unknown emf.

Step 1: Write the two loaded terminal voltages Let the potentiometer gradient be k. The balance lengths give $V_1 = 500k$ for $R_1 = 10\,\Omega$ and $V_2 = 400k$ for $R_2 = 1\,\Omega$. These are two loaded-cell measurements, not an open-circuit emf measurement. The terminal voltages are

$$V_1 = \frac{10E}{10+r}, \qquad V_2 = \frac{E}{1+r}.$$

The longer balance length for the larger load resistance is consistent with its smaller internal voltage drop.

Step 2: Eliminate the emf and solve for the internal resistance Take the ratio of the two balance conditions:

$$\frac{500}{400} = \frac{10(1+r)}{10+r}.$$

Cross-multiplying gives

$$5(10+r) = 40(1+r), \qquad 50+5r = 40+40r.$$

Consequently

$$35r = 10, \qquad r = \frac{2}{7}\,\Omega \simeq 0.286\,\Omega.$$

The potentiometer gradient cancels because it remains the same for both observations.

Step 3: Choose the nearest listed resistance To one decimal place the value is $0.3\,\Omega$, which is option (B). The choices $0.1\,\Omega$, $0.2\,\Omega$, and $0.4\,\Omega$ are not the nearest approximation to $2/7\,\Omega$. A common mistake is to apply the open-circuit formula $r = R(l_0/l - 1)$ while treating 500 cm as an emf balance length. Here even the 500 cm reading is obtained with a finite external load, so both loaded terminal voltages must be used.

Step 4: Conclusion The internal resistance is $2/7\,\Omega \simeq 0.286\,\Omega$, approximately $0.3\,\Omega$. Hence, the correct option is (B).

Quick Tip to Solve: When both balance lengths are measured under finite loads, take the ratio of ER/(R+r); neither length is automatically the open-circuit emf length.

50. On celcius scale the temperature of body increases by 40°C. The increase in temperature on Fahrenheit scale is : [4 marks]

(A) 75°F

(B) 70°F

(C) 72°F

(D) 68°F

Correct Answer: (C) 72°F

Solution:

Concept:

- The Celsius and Fahrenheit scales are linear scales of the same temperature, related by $F = \frac{9}{5}C + 32$. The additive constant locates the zero of one scale relative to the other.
- A change of temperature is found by subtracting the initial reading from the final reading. Thus temperature intervals obey $\Delta F = \frac{9}{5}\Delta C$; the offset cancels.

Step 1: Write the initial and final readings Let the initial Celsius reading be C_1 and the final reading be C_2. Their Fahrenheit readings are $F_1 = \frac{9}{5}C_1 + 32$ and $F_2 = \frac{9}{5}C_2 + 32$. Subtracting these two equations removes the common offset and gives

$$F_2 - F_1 = \frac{9}{5}(C_2 - C_1).$$

The problem gives an increase, not an absolute temperature. Therefore the extra 32 must not be added to the numerical change.

Step 2: Convert the stated interval and select the option The Celsius increase is $\Delta C = 40^\circ\text{C}$, so

$$\Delta F = \frac{9}{5} \times 40 = 72^\circ\text{F}.$$

A Fahrenheit degree is smaller than a Celsius degree, so a positive interval has a larger numerical value in Fahrenheit. The exact conversion gives 72, ruling out 75, 70 and 68. The direction of change remains an increase because the slope of the scale conversion is positive.

Step 3: Conclusion The increase is $\boxed{72^\circ\text{F}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a temperature difference, multiply by $9/5$; use the $+32$ offset only when converting an actual temperature reading.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A solid sphere and a hollow cylinder roll up without slipping on same inclined plane with same initial speed v. The sphere and the cylinder reaches upto maximum heights h_1 and h_2 respectively, above the initial level. The ratio $h_1 : h_2$ is $\frac{n}{10}$. The value of n is _____. [4 marks]

Solution:

Concept:

- For pure rolling, $\omega = v/R$, and the initial kinetic energy is $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. Both translation and rotation contribute to the height reached.
- For a solid sphere $I = \frac{2}{5}mR^2$, whereas for a thin hollow cylinder $I = mR^2$. Static friction prevents slipping and does no work on a stationary incline.

Step 1: Apply mechanical energy conservation At the greatest height, the instantaneous translational speed is zero. The rolling constraint makes the angular speed zero as well, so all the initial kinetic energy has become gravitational potential energy. Writing $I = kmR^2$,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}kmR^2\frac{v^2}{R^2} = \frac{1}{2}mv^2(1+k).$$

Consequently $h = v^2(1+k)/(2g)$. The masses, radii and incline angle do not affect the comparison when the initial centre-of-mass speeds are equal.

Step 2: Insert the moments of inertia and compare heights For the solid sphere, $k_1 = 2/5$, giving $h_1 = 7v^2/(10g)$. For the hollow cylinder, $k_2 = 1$, giving $h_2 = v^2/g$. Hence

$$\frac{h_1}{h_2} = \frac{7/10}{1} = \frac{7}{10}.$$

The cylinder rises higher because it initially carries more rotational kinetic energy per unit mass at the same rolling speed. Matching the numerator in the stated ratio $n/10$ now gives the required integer.

Step 3: Conclusion The height ratio is $h_1 : h_2 = 7 : 10$, so $\boxed{n = 7}$.

Quick Tip to Solve: For equal initial rolling speeds, compare $1 + I/(mR^2)$; this directly gives the ratio of the maximum heights.

52. An infinite plane sheet of charge having uniform surface charge density $+\sigma_s \mathrm{C/m^2}$ is placed on x-y plane. Another infinitely long line charge having uniform linear charge density $+\lambda_e \mathrm{C/m}$ is placed at $z = 4$ m plane and parallel to y-axis. If the magnitude values $|\sigma_s| = 2\,|\lambda_e|$ then at point $(0, 0, 2)$, the ratio of magnitudes of electric field values due to sheet charge to that of line charge is $\pi\sqrt{n} : 1$. The value of n is _____. [4 marks]

Solution:

Concept:

- An infinite nonconducting charged sheet produces field magnitude $E_{\text{sheet}} = |\sigma_s|/(2\varepsilon_0)$, independent of distance from the sheet.
- An infinite straight line of charge produces field magnitude $E_{\text{line}} = |\lambda_e|/(2\pi\varepsilon_0 r)$, where r is the perpendicular distance to the line.

Step 1: Identify the perpendicular distance and individual fields The sheet lies in the x-y plane. The positive line charge is parallel to the y-axis at $z = 4\,\mathrm{m}$, while the observation point is $(0, 0, 2)$. Its perpendicular separation from the line is therefore $r = 4 - 2 = 2\,\mathrm{m}$. The sheet field at the point points away from the sheet, while the line field points away from the line. Their directions are opposite here, but the question asks for the ratio of their separate magnitudes, not their resultant.

Step 2: Form the ratio and determine the parameter Dividing the two field expressions cancels $2\varepsilon_0$:

$$\frac{E_{\text{sheet}}}{E_{\text{line}}} = \frac{|\sigma_s|}{2\varepsilon_0}\frac{2\pi\varepsilon_0 r}{|\lambda_e|} = \pi r\frac{|\sigma_s|}{|\lambda_e|}.$$

Using the stated numerical density relation in SI units and $r = 2\,\mathrm{m}$, the ratio becomes 4π. Therefore $E_{\text{sheet}} : E_{\text{line}} = 4\pi : 1$. Comparing with $\pi\sqrt{n} : 1$, we obtain $\sqrt{n} = 4$, whence $n = 16$.

Step 3: Conclusion The required field-magnitude ratio is $4\pi : 1$, giving $\boxed{n = 16}$.

Quick Tip to Solve: Cancel ε_0 before substituting; use the perpendicular distance to the line, and compare magnitudes rather than adding the fields.

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53. A hydrogen atom changes its state from $n = 3$ to $n = 2$. Due to recoil, the percentage change in the wave length of emitted light is approximately 1×10^{-n}. The value of n is _____.
[Given Rhc $= 13.6$ eV, hc $= 1242$ eV nm, h $= 6.6 \times 10^{-34}$ J s mass of the hydrogenatom $= 1.6 \times 10^{-27}$ kg] [4 marks]

Solution:

Concept:

- The energy released in the hydrogen transition is $\Delta E = 13.6(1/2^2 - 1/3^2)\,\mathrm{eV}$. Without recoil the photon wavelength is $\lambda_0 = hc/\Delta E$.
- An emitted photon of energy E_γ has momentum $p = E_\gamma/c = h/\lambda$. Momentum conservation gives the atom equal recoil momentum and kinetic energy $K = p^2/(2M)$.

Step 1: Find the transition energy and recoil scale The transition releases

$$\Delta E = 13.6\left(\frac{1}{4}-\frac{1}{9}\right) = \frac{17}{9}\,\text{eV} \simeq 1.889\,\text{eV}.$$

Thus $\lambda_0 = 1242/1.889 \simeq 657.5\,\text{nm}$. Using $\lambda_0 = 6.575\times10^{-7}\,\text{m}$, the photon momentum to leading order is $p_0 = h/\lambda_0 \simeq 1.004\times10^{-27}\,\text{kg m s}^{-1}$. The atomic recoil energy is therefore

$$K \simeq \frac{p_0^2}{2M} = \frac{(1.004\times10^{-27})^2}{2(1.6\times10^{-27})} \simeq 3.15\times10^{-28}\,\text{J}.$$

Step 2: Calculate the percentage wavelength increase Energy conservation gives $E_\gamma = \Delta E - K$, so recoil slightly decreases the photon energy and increases its wavelength. Since $K \ll \Delta E$,

$$\frac{\lambda-\lambda_0}{\lambda_0} \simeq \frac{K}{\Delta E}.$$

With $\Delta E \simeq 1.889(1.602\times10^{-19}) = 3.026\times10^{-19}\,\text{J}$, the percentage increase is

$$100\frac{K}{\Delta E} \simeq 100\frac{3.15\times10^{-28}}{3.026\times10^{-19}} \simeq 1.04\times10^{-7}.$$

Corrections from using the recoiling photon energy in its momentum are higher order and negligible at this accuracy.

Step 3: Conclusion The wavelength increases by approximately 1×10^{-7} percent. Thus the exponent requested is $\boxed{n=7}$.

Quick Tip to Solve: For recoil, first compute $K = p^2/(2M)$, then use $\Delta\lambda/\lambda \simeq K/\Delta E$. Multiply by 100 only when converting the fraction to percent.

54. A soap bubble is blown to a diameter of $7\ \text{cm}$. $36960\ \text{erg}$ of work is done in blowing it further. If surface tension of soap solution is 40 dyne/cm then the new radius is _____ cm Take $(\pi = \frac{22}{7})$.

[4 marks]

Solution:

Concept:

- A soap bubble has two liquid surfaces, an inner and an outer surface. Its total surface area is approximately $8\pi r^2$, and its surface energy is $U = 8\pi T r^2$.
- The work required to inflate a bubble quasistatically equals its increase in surface energy: $W = 8\pi T(r_2^2 - r_1^2)$, for constant surface tension T.

Step 1: Convert the initial diameter to radius and keep consistent units The initial diameter is $7\,\text{cm}$, so $r_1 = 3.5\,\text{cm}$. The given work is in ergs and the surface tension is in dyne per centimetre, so cgs units can be used directly. Because $1\,\text{erg} = 1\,\text{dyne cm}$, the product of surface tension with area has the correct energy unit. The stated work is for further inflation, and must therefore be equated to the change in energy rather than the final total energy.

Step 2: Find the final radius from the increase in area Substitute $W = 36960\,\text{erg}$, $T = 40\,\text{dyne cm}^{-1}$ and $\pi = 22/7$:

$$36960 = 8\left(\frac{22}{7}\right)40\left(r_2^2 - 3.5^2\right).$$

It follows that

$$r_2^2 - 12.25 = \frac{36960 \times 7}{8 \times 22 \times 40} = 36.75.$$

Hence $r_2^2 = 49\,\text{cm}^2$, and the positive radius is $r_2 = 7\,\text{cm}$. Using a single surface or treating 7 cm as the initial radius would give an incorrect result; both surfaces and the initial diameter must be handled explicitly.

Step 3: Conclusion The new radius is $\boxed{7\,\text{cm}}$, so the numerical entry is 7.

Quick Tip to Solve: For a soap bubble count both surfaces: use $8\pi T(r_2^2 - r_1^2)$, and halve an initial diameter before squaring.

55. An elastic spring under tension of 3 N has a length a. Its length is b under tension 2 N. For its length $(3a - 2b)$, the value of tension will be _____ N. [4 marks]

Solution:

Concept:

- Within the elastic limit, a spring obeys $F = k(L - L_0)$, where L is its stretched length, L_0 its natural length and k its spring constant.
- Tension depends on extension from the natural length, not on total length. Subtracting two force-length equations eliminates the unknown natural length.

Step 1: Use the two given tensions to eliminate the natural length For length a, the equation is $3 = k(a - L_0)$. For length b, it is $2 = k(b - L_0)$, with the forces measured in newtons. Subtracting gives

$$1 = k(a - b).$$

This also shows $a > b$, as expected because the larger tensile force stretches the spring further. The lengths a and b themselves need not be known to determine the requested tension.

Step 2: Express the requested length as an additional extension The new length is $L = 3a - 2b = a + 2(a - b)$. Relative to length a, this adds twice the length difference produced by a force increase of $1\,\text{N}$. Algebraically,

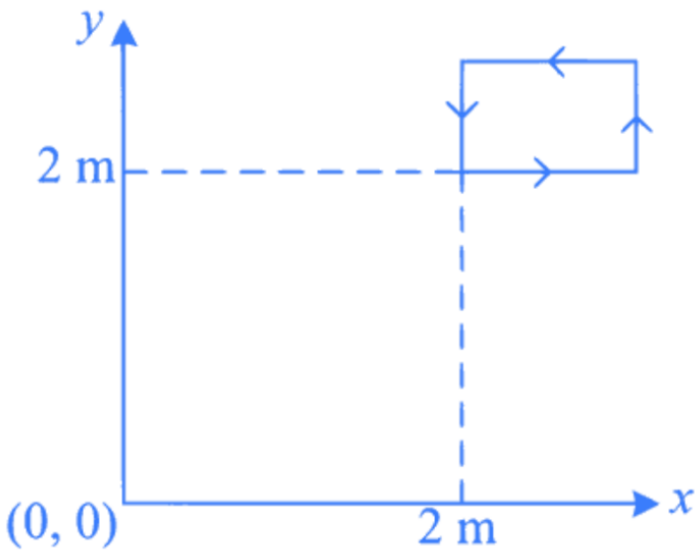
$$\begin{aligned} F &= k(3a - 2b - L_0) \\ &= k(a - L_0) + 2k(a - b) \\ &= 3 + 2(1) = 5\,\text{N}. \end{aligned}$$

Equivalently, the affine force-length law permits $F(3a - 2b) = 3F(a) - 2F(b)$, because the length coefficients sum to one. Ignoring the natural length and taking force proportional to total length would not be justified.

Step 3: Conclusion The tension at length $3a - 2b$ is $\boxed{5\,\text{N}}$. The numerical entry is 5.

Quick Tip to Solve: For spring-length combinations whose coefficients add to one, apply the same combination to the known forces; the natural-length offset cancels.

56. The magnetic field existing in a region is given by $\vec{B} = 0.2(1 + 2x)\hat{k}$. A square loop of edge 50 cm carrying 0.5 A current is placed in x-y plane with its edges parallel to the x-y axes, as shown in figure. The magnitude of the net magnetic force experienced by the loop is _____ mN. [4 marks]



Solution:

Concept:

- The magnetic force on a current element is $d\vec{F} = I\, d\vec{\ell} \times \vec{B}$. Opposite sides of a closed loop cancel exactly only if their corresponding field magnitudes are equal.
- Here $B_z(x) = 0.2(1+2x)\,\mathrm{T}$, so $dB_z/dx = 0.4\,\mathrm{T\,m^{-1}}$. The square side is $a = 0.50\,\mathrm{m}$ and the current is $I = 0.5\,\mathrm{A}$.

Step 1: Cancel the forces on the horizontal sides The top and bottom sides each span the same interval of x, from 2 to $2.5\,\mathrm{m}$, but carry opposite current directions. Since the field depends only on x, each corresponding element sees the same field magnitude and has the opposite magnetic force. Their integrated forces along the y-direction therefore cancel. The two vertical sides, however, lie at different x-coordinates and experience different constant fields.

Step 2: Calculate the difference between the vertical-side forces The left side lies at $x_L = 2\,\mathrm{m}$, giving $B_L = 0.2(1+4) = 1.0\,\mathrm{T}$. The right side lies at $x_R = 2.5\,\mathrm{m}$, giving $B_R = 0.2(1+5) = 1.2\,\mathrm{T}$. The current on the right is upward, so its force is along $+x$; that on the left is downward, giving a force along $-x$. Their difference is

$$F_{\text{net}} = Ia(B_R - B_L) = 0.5 \times 0.50 \times 0.20 = 0.050\,\mathrm{N}.$$

Multiplying by 10^3 converts newtons to millinewtons. Only the field difference matters, so the constant term in B_z contributes no net force.

Step 3: Conclusion The magnitude of the net force is $\boxed{50\,\mathrm{mN}}$. The numerical entry is 50.

Quick Tip to Solve: For a rectangular loop in $B_z(x)$, cancel the horizontal pair first; the remaining magnitude is $I \times$ side length $\times\, \Delta B$.

57. Two forces $\overline{\mathrm{F}}_1$ and $\overline{\mathrm{F}}_2$ are acting on a body. One force has magnitude thrice that of the other force and the resultant of the two forces is equal to the force of larger magnitude. The angle between $\vec{F}_1$ and $\vec{F}_2$ is $\cos^{-1}\left(\frac{1}{n}\right)$. The value of $|n|$ is _____. [4 marks]

Solution:

Concept:

- The magnitude of the resultant of two forces of magnitudes P and Q, inclined at angle θ, satisfies $R^2 = P^2 + Q^2 + 2PQ\cos\theta$. This follows by taking the scalar product of the resultant vector with itself.
- The angle between two nonzero vectors lies between 0 and π. A negative cosine therefore describes an obtuse angle, and its sign must be retained before taking the absolute value requested.

Step 1: Represent the two magnitudes Let the smaller force have magnitude $F > 0$. The larger force then has magnitude $3F$. The condition concerns the magnitude of the resultant, so $R = 3F$, regardless of which force has been named $\vec{F}_1$ or $\vec{F}_2$. The two forces are not necessarily parallel; their inclination is the quantity to determine.

Step 2: Apply the resultant formula Substitute these magnitudes without changing the sign of the scalar-product term:

$$(3F)^2 = F^2 + (3F)^2 + 2(F)(3F)\cos\theta.$$

Cancelling F^2, which is nonzero, gives

$$9 = 1 + 9 + 6\cos\theta, \qquad \cos\theta = -\frac{1}{6}.$$

The negative sign is physically consistent: some cancellation is needed so that adding the smaller force does not increase the larger magnitude.

Step 3: Identify the requested parameter The question writes the angle as $\cos^{-1}(1/n)$. On the usual principal range for the angle between vectors, equal angles have equal cosines. Hence $1/n = -1/6$, so $n = -6$. Taking an absolute value earlier would lose information about the actual obtuse angle; it is taken only now.

Step 4: Conclusion The forces satisfy $\cos\theta = -1/6$, so $n = -6$. The required numerical entry is $\boxed{|n| = 6}$.

Quick Tip to Solve: Use the squared resultant first; when the resultant equals one force, its squared term cancels immediately.

58. A alternating current at any instant is given by $i = [6 + \sqrt{56}\sin(100\pi t + \pi/3)]$ A. The rms value of the current is _____ A. [4 marks]

Solution:

Concept:

- The root mean square of a periodic current is $I_{\text{rms}} = \sqrt{\langle i^2 \rangle}$, where the average is over a complete period. Squaring before averaging is essential because heating is proportional to i^2.

- For any fixed phase ϕ, $\langle \sin(\omega t + \phi) \rangle = 0$ and $\langle \sin^2(\omega t + \phi) \rangle = 1/2$. These identities hold over every full cycle.

Step 1: Separate the constant and oscillating parts Write $i = I_0 + I_m \sin\psi$, with $I_0 = 6\,\text{A}$, $I_m = \sqrt{56}\,\text{A}$, and $\psi = 100\pi t + \pi/3$. The constant part is a DC offset and is not divided by $\sqrt{2}$. Only the sinusoidal amplitude has the usual amplitude-to-rms conversion. The phase merely shifts the starting point of the cycle.

Step 2: Average the square of the complete current Expanding gives

$$i^2 = 36 + 12\sqrt{56}\sin\psi + 56\sin^2\psi.$$

Over a period the cross term averages to zero, since it is proportional to a sine. The last term contributes half its coefficient. Thus

$$\langle i^2 \rangle = 36 + 0 + \frac{56}{2} = 64\,\text{A}^2.$$

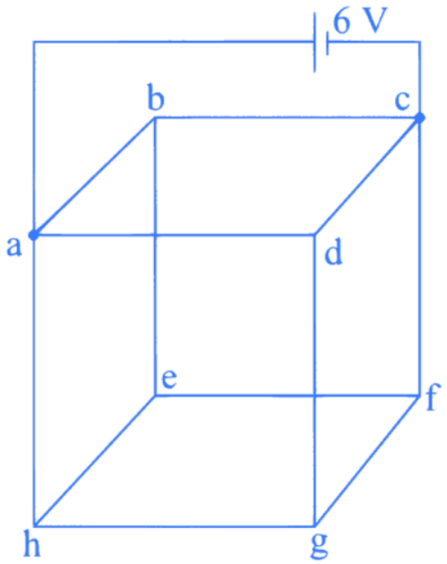
This procedure accounts for both the steady heating contribution and the additional oscillating contribution.

Step 3: Take the positive square root The rms value is nonnegative by definition, even though instantaneous current may change direction. Therefore $I_{\text{rms}} = \sqrt{64}\,\text{A} = 8\,\text{A}$. Equivalently, independent DC and zero-mean AC contributions add in mean square: $I_{\text{rms}}^2 = I_0^2 + (I_m/\sqrt{2})^2$, not as an arithmetic sum of rms currents.

Step 4: Conclusion The rms value of the given current is $\boxed{8\,\text{A}}$.

Quick Tip to Solve: For a sinusoid plus a DC offset, add the squares of the DC value and sinusoidal rms value, then take the square root.

59. Twelve wires each having resistance 2Ω are joined to form a cube. A battery of 6 V emf is joined across point a and c. The voltage difference between e and f is _____ V. [4 marks]



Solution:

Concept:

- In a resistor network, Kirchhoff's current law requires the algebraic sum of currents leaving each internal junction to vanish. For equal resistances this makes a node potential the average of its neighbours' potentials.
- Symmetry may equate potentials only when the interchange leaves both the network and the battery terminals unchanged. Here $b \leftrightarrow d$ and $e \leftrightarrow g$ is such a symmetry.

Step 1: Assign potentials and use the cube symmetry Take $V_a = 6\,\text{V}$ and $V_c = 0$. The battery is across a face diagonal, not a body diagonal. With labels as shown, let $V_b = V_d = x$, $V_e = V_g = y$, $V_h = u$, and $V_f = v$. Every edge is $2\,\Omega$. At node b, the neighbours are a, c, e, giving $3x = 6 + y$. Node d gives the same equation.

Step 2: Write the remaining junction equations Node h joins a, e, g, while node f joins c, e, g. Hence

$$3u = 6 + 2y, \qquad 3v = 2y.$$

Node e joins b, h, f, so $3y = x + u + v$. The common resistance cancels out of these equations because all three currents at each node have the same denominator. We retain all paths through the lower face rather than treating the upper two paths as isolated series branches.

Step 3: Solve for the measured potential difference Substitute $x = (6 + y)/3$, $u = (6 + 2y)/3$, and $v = 2y/3$ into the equation for y:

$$3y = \frac{12 + 5y}{3}, \qquad 9y = 12 + 5y.$$

Thus $y = 3\,\text{V}$, $u = 4\,\text{V}$, and $v = 2\,\text{V}$. Consequently $V_e - V_f = y - v = 1\,\text{V}$. Reversing the battery would reverse the signed difference, but its magnitude would be unchanged.

Step 4: Conclusion The voltage difference between e and f is $\boxed{1\,\text{V}}$.

Quick Tip to Solve: Check which diagonal carries the battery before using cube-network symmetry; for equal edges, node potentials obey simple averaging.

60. Two wavelengths λ_1 and λ_2 are used in Young's double slit experiment. $\lambda_1 = 450\,\text{nm}$ and $\lambda_2 = 650\,\text{nm}$. The minimum order of fringe produced by λ_2 which overlaps with the fringe produced by λ_1 is n. The value of n is _____. [4 marks]

Solution:

Concept:

- In Young's double-slit experiment, a bright fringe of order m has path difference $m\lambda$, with integral m. Its displacement is $y_m = m\lambda D/d$, for screen distance D and slit separation d.
- Bright fringes of two wavelengths coincide when their path differences, or equivalently their screen displacements, are equal. The first noncentral coincidence uses the smallest positive integral orders.

Step 1: Set equal the bright-fringe positions Let m be the order for $\lambda_1 = 450\,\text{nm}$, and let n be the order for $\lambda_2 = 650\,\text{nm}$. Both colours use the same slits and screen, so the common factor D/d cancels. Their coincidence condition is

$$m\lambda_1 = n\lambda_2, \qquad 450m = 650n.$$

The units may remain nanometres because only the ratio is required.

Step 2: Reduce the integer ratio Dividing by 50 yields $9m = 13n$, or $m/n = 13/9$. Since 9 and 13 have no common factor greater than one, every positive solution has $m = 13k$ and $n = 9k$, with positive integer k. The smallest such value is $k = 1$. Larger values would describe later coincidences, rather than the first one.

Step 3: Check which order was requested The question asks for the order of the fringe produced by the longer wavelength λ_2, so the answer is n, not m. Indeed, $13 \times 450 = 9 \times 650 = 5850\,\text{nm}$. The central fringe corresponds to both orders zero and is common to all wavelengths; the stated minimum fringe order means the first nonzero bright-fringe overlap.

Step 4: Conclusion The ninth-order bright fringe of $650\,\text{nm}$ overlaps the thirteenth-order bright fringe of $450\,\text{nm}$. Thus $\boxed{n = 9}$.

Quick Tip to Solve: Reduce the wavelength ratio to coprime integers and reverse the ratio for the corresponding bright-fringe orders.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Number of complexes from the following with even number of unpaired "d" electrons is _____ $[V(H_2O)_6]^{3+}, [Cr(H_2O)_6]^{2+}, [Fe(H_2O)_6]^{3+}, [Ni(H_2O)_6]^{3+}, [Cu(H_2O)_6]^{2+}$ [Given atomic numbers: $V = 23, Cr = 24, Fe = 26, Ni = 28 Cu = 29$] [4 marks]

(A) 1

(B) 5

(C) 2

(D) 4

Correct Answer: (C) 2

Solution:

Concept:

- Water is a neutral ligand, so the charge on each listed complex equals the oxidation state of its metal. Remove outer $4s$ electrons before $3d$ electrons when obtaining the metal-ion configuration.
- An octahedral aqua complex is treated as a weak-field complex in the usual crystal-field counting. Fill the five d orbitals according to the appropriate high-spin arrangement and count singly occupied orbitals.

Step 1: Find the five metal-ion configurations The neutral-atom configurations lead to $V^{3+} : d^2$, $Cr^{2+} : d^4$, $Fe^{3+} : d^5$, $Ni^{3+} : d^7$, and $Cu^{2+} : d^9$. Chromium's exceptional neutral configuration is $3d^5 4s^1$; removing two electrons therefore gives d^4, not d^3. Nickel has lost three electrons, so it is not the

common d^8 nickel(II) ion.

Step 2: Count the unpaired electrons For d^2, the two electrons occupy different t_{2g} orbitals, giving two unpaired electrons. High-spin d^4 has $t_{2g}^3 e_g^1$, giving four. High-spin d^5 has $t_{2g}^3 e_g^2$, giving five. High-spin d^7 has $t_{2g}^5 e_g^2$, giving three. The d^9 configuration $t_{2g}^6 e_g^3$ has one. Thus the sequence is $2, 4, 5, 3, 1$.

Step 3: Select only the even counts The vanadium(III) and chromium(II) complexes satisfy the condition. The iron(III), nickel(III) and copper(II) complexes have odd numbers of unpaired electrons, so they must be excluded. Even if one considers a low-spin alternative for d^7, its one unpaired electron is still odd and does not affect the required count. Options 1, 5 and 4 therefore miscount the qualifying complexes.

Step 4: Conclusion Exactly $\boxed{2}$ complexes have an even number of unpaired d electrons. Hence, the correct option is (C).

Quick Tip to Solve: First obtain each ion's d-electron count; odd d-electron totals always give an odd number of unpaired electrons.

62. What will be the decreasing order of basic strength of the following conjugate bases? $^-OH, R\overline{O}, CH_3CO\overline{O}, Cl$ [4 marks]

(A) $Cl > R\overline{O} > {}^-OH > CH_3CO\overline{O}$

(B) $^-OH > R\overline{O} > CH_3CO\overline{O} > Cl$

(C) $Cl > {}^-OH > R\overline{O} > CH_3CO\bar{O}$

(D) $R\overline{O} > {}^-OH > CH_3CO\overline{O} > Cl$

Correct Answer: (D) $R\overline{O} > {}^-OH > CH_3CO\overline{O} > Cl$

Solution:

Concept:

- The stronger a Brønsted acid is, the weaker its conjugate base is. Thus compare ROH, H_2O, CH_3COOH, and HCl to rank the corresponding anions.
- Electron donation tends to increase the tendency of an anion to accept a proton, whereas resonance stabilization lowers this tendency. These effects distinguish alkoxide, hydroxide, acetate and chloride.

Step 1: Compare alkoxide with hydroxide The alkyl group in RO^- exerts a positive inductive effect, increasing electron density on oxygen. In the standard comparison used here, an alcohol is a weaker acid than water, making alkoxide a stronger conjugate base than hydroxide. Therefore the first part of the decreasing order is $RO^- > OH^-$. This rules out the order that places hydroxide above alkoxide.

Step 2: Compare acetate and chloride with the oxygen bases The negative charge on acetate is delocalized over two oxygen atoms by resonance. It is consequently much more stabilized and less basic than hydroxide or alkoxide. Acetic acid is stronger than water and alcohols. Hydrochloric acid is stronger still, so its conjugate base Cl^- has the weakest proton-accepting tendency among these four species. The chloride entry denotes a conjugate base, rather than a neutral chlorine atom.

Step 3: Assemble the full decreasing order Combining the inductive and resonance comparisons gives

$$RO^- > OH^- > CH_3COO^- > Cl^-.$$

Options that place chloride first invert the strong-acid/weak-conjugate-base relationship and are rejected. The remaining incorrect order swaps alkoxide and hydroxide. Only the sequence with alkoxide first, acetate third and chloride last satisfies all three pairwise comparisons.

Step 4: Conclusion The decreasing basic-strength order is $RO^- > OH^- > CH_3COO^- > Cl^-$. Hence, the correct option is (D).

Quick Tip to Solve: Rank the parent acids in the reverse direction: stronger parent acid means weaker conjugate base.

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63. Given below are two statements :

Statements I : Acidity of α-hydrogens of aldehydes and ketones is responsible for Aldol reaction.

Statement II : Reaction between benzaldehyde and ethanal will NOT give Cross - Aldol product.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are incorrect

(B) Statement I is correct but Statement II is incorrect

(C) Both Statement I and Statement II are correct

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (B) Statement I is correct but Statement II is incorrect

Solution:

Concept:

- An aldehyde or ketone with an α-hydrogen can form an enolate in dilute base. Resonance stabilization of the enolate accounts for the acidity of that hydrogen and enables nucleophilic carbon-carbon bond formation.
- A crossed aldol reaction involves two different carbonyl compounds. Only the enolate donor must have an α-hydrogen; the carbonyl compound acting as electrophile need not have one.

Step 1: Evaluate Statement I In an aldol reaction, base removes an acidic hydrogen from the carbon next to the carbonyl group. The resulting enolate attacks another carbonyl carbon, and protonation forms a β-hydroxy carbonyl compound. Thus the acidity of α-hydrogens makes enolate formation possible and is the reason the relevant aldehydes and ketones undergo this reaction. Statement I is correct.

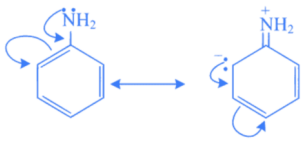
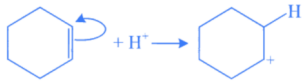
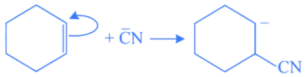
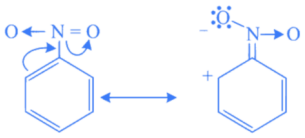
Step 2: Evaluate Statement II using the two reactants Ethanal, CH_3CHO, contains α-hydrogens and can supply the enolate. Benzaldehyde, C_6H_5CHO, lacks an α-hydrogen but still has an electrophilic carbonyl carbon. The ethanal enolate can attack benzaldehyde to form a crossed aldol product. The absence of α-hydrogens in benzaldehyde prevents it from serving as the enolate donor; it does not prevent it from accepting an enolate.

Step 3: Identify the product and eliminate the other choices The crossed addition gives $C_6H_5CH(OH)CH_2CHO$; on dehydration it gives $C_6H_5CH{=}CHCHO$, cinnamaldehyde. Consequently the assertion that no crossed aldol product will form is false. Choices declaring both statements true or both false are excluded, as is the choice making Statement I false and Statement II true.

Step 4: Conclusion Statement I is correct and Statement II is incorrect. Hence, the correct option is (B).

Quick Tip to Solve: For a crossed aldol reaction, check whether at least one reactant can supply an enolate; the other can act only as electrophile.

64. Match List I with List II. Choose the correct answer from the options given below. [4 marks]

	LIST I Mechanism steps		LIST II Effect
A.		I.	– E effect
B.		II.	– R effect
C.		III.	+ E effect
D.		IV.	+ R effect

(A) $(A)-(I),\ (B)-(II),\ (C)-(IV),\ (D)-(III)$
(B) $(A)-(II),\ (B)-(IV),\ (C)-(III),\ (D)-(I)$
(C) $(A)-(III),\ (B)-(I),\ (C)-(II),\ (D)-(IV)$
(D) $(A)-(IV),\ (B)-(III),\ (C)-(I),\ (D)-(II)$
Correct Answer: (D) $(A)-(IV),\ (B)-(III),\ (C)-(I),\ (D)-(II)$

Solution:

Concept:

- Resonance effects describe permanent delocalisation through conjugated orbitals. Electron donation into a conjugated system is the $+R$ effect, while withdrawal is the $-R$ effect.
- The electromeric effect is complete transfer of a multiple-bond electron pair during attack by a reagent. Donation towards an attacking electrophile is $+E$; displacement away from an attacking nucleophile is $-E$.

Step 1: Classify the two resonance drawings In A, the amino nitrogen donates its lone pair into the

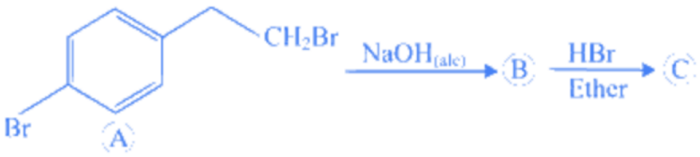
benzene ring. The charge-separated form has positive nitrogen and enhanced electron density in the ring. This is donation by resonance, so A matches IV, the $+R$ effect. In D, the nitro group withdraws ring electrons and the ring acquires positive charge in the resonance form. Therefore D matches II, the $-R$ effect. The resonance double arrows and absence of an attacking reagent distinguish these from electromeric changes.

Step 2: Classify the reagent-induced changes In B, the alkene electron pair forms a bond to the electrophile H^+, leaving a carbocation on the other carbon. Transfer towards the incoming electrophile is $+E$, so B matches III. In C, the incoming CN^- forms a bond at one carbon and the original pi electrons shift to the other carbon, away from the nucleophile. This is $-E$, so C matches I. Options A, B and C each fail at least the amino-resonance match, whereas the fourth option has all four assignments correct.

Step 3: Conclusion The matching is $A-IV,\ B-III,\ C-I,\ D-II$. Hence, the correct option is (D).

Quick Tip to Solve: First distinguish resonance arrows from reagent-induced addition, then use the direction of electron donation to determine the sign.

65. Identify B and C and how are A and C related? [4 marks]



(A) B: 4-bromostyrene; C: 1-bromo-4-(1-bromoethyl)benzene; A and C are position isomers.

(B) B: 4-(2-bromoethyl)phenol; C: 4-(1-bromoethyl)phenol; derivative.

(C) B: 4-(2-bromoethyl)phenol; C: 4-(2-hydroxyethyl)phenol; functional group isomers.

(D) B: 4-bromostyrene; C: 1-bromo-4-(2-bromoethyl)benzene; chain isomers.

Correct Answer: (A) B: 4-bromostyrene; C: 1-bromo-4-(1-bromoethyl)benzene; A and C are position isomers.

Solution:

Concept:

- Hot alcoholic $NaOH$ promotes beta-elimination from an alkyl bromide. An aryl carbon–bromine bond does not undergo the same ordinary elimination.
- Addition of HBr to a styrene double bond in the absence of peroxide follows the pathway through the more stable benzylic carbocation. Position isomers retain the molecular formula and functional groups but differ in substituent position.

Step 1: Eliminate hydrogen bromide from the side chain The starting compound A has a para bromine on the benzene ring and a $-CH_2CH_2Br$ side chain. Alcoholic base removes a beta hydrogen from the carbon attached to the ring while bromide leaves the terminal carbon. The resulting B is para-bromostyrene, $p\text{-}BrC_6H_4CH{=}CH_2$. The bromine directly bonded to the aromatic ring remains

present. Options B and C incorrectly replace this aryl bromine by an OH group under the supplied conditions.

Step 2: Add HBr and identify the isomerism Protonation of the terminal alkene carbon gives the benzylic cation $\text{p-BrC}_6\text{H}_4\text{CH}^+\text{CH}_3$, stabilised by the aromatic ring. Bromide then attacks this carbon, giving C as $\text{p-BrC}_6\text{H}_4\text{CH(Br)CH}_3$. In A the side-chain bromine is at the terminal carbon; in C it is at the benzylic carbon. The carbon skeleton and both bromine atoms are unchanged, so A and C are position isomers. Option D places bromine back on the terminal carbon and incorrectly calls the relation chain isomerism.

Step 3: Conclusion $B = \text{p-BrC}_6\text{H}_4\text{CH=CH}_2$ and $C = \text{p-BrC}_6\text{H}_4\text{CH(Br)CH}_3$; A and C are position isomers. Hence, the correct option is (A).

Quick Tip to Solve: Keep the aryl bromine fixed; follow elimination on the two-carbon side chain and Markovnikov addition through a benzylic carbocation.

66. Identify the correct set of reagents or reaction conditions 'X' and 'Y' in the following set of transformation. [4 marks]

$$CH_3-CH_2-CH_2-Br \xrightarrow{\text{'X'}} \text{Product} \xrightarrow{\text{'Y'}} CH_3-\underset{\underset{Br}{|}}{CH}-CH_3$$

(A) X = dil.aq. $\text{NaOH}, 20^\circ\text{C}$, Y = $\text{HBr}/$ acetic acid

(B) X = conc.alc. $\text{NaOH}, 80^\circ\text{C}$, Y = $\text{Br}_2/\text{CHCl}_3$

(C) X = dil.aq. $\text{NaOH}, 20^\circ\text{C}$, Y = $\text{Br}_2/\text{CHCl}_3$

(D) X = conc.alc. $\text{NaOH}, 80^\circ\text{C}$, Y = $\text{HBr}/$ acetic acid

Correct Answer: (D) X = conc.alc. $\text{NaOH}, 80^\circ\text{C}$, Y = $\text{HBr}/$ acetic acid

Solution:

Concept:

- Concentrated alcoholic NaOH at elevated temperature favours beta-elimination, producing an alkene from a haloalkane.
- HBr adds to an unsymmetrical alkene according to Markovnikov orientation under normal ionic conditions. Aqueous dilute base instead mainly promotes substitution.

Step 1: Make the required alkene intermediate The starting material is 1-bromopropane and the final product is 2-bromopropane. The bromine must therefore move from an end carbon to the central carbon without changing the three-carbon skeleton. Elimination of HBr first forms propene. This requires concentrated alcoholic NaOH at 80°C, the X conditions in options B and D. Dilute aqueous base at 20°C would favour propan-1-ol and cannot provide the required alkene sequence. Thus options A and C are excluded.

Step 2: Choose the addition reagent To convert propene into 2-bromopropane, add HBr in acetic acid. Protonation gives the secondary carbocation on the central carbon rather than a primary terminal carbocation, and bromide attack yields $\text{CH}_3\text{CHBrCH}_3$. The alternative $\text{Br}_2/\text{CHCl}_3$ adds two bromine

atoms across the double bond and gives 1,2-dibromopropane, so option B is also excluded. The desired sequence is elimination followed by ionic hydrohalogenation, rather than substitution followed by bromination.

Step 3: Conclusion X is concentrated alcoholic NaOH at $80^{\circ}\mathrm{C}$, and Y is HBr/acetic acid. Hence, the correct option is (D).

Quick Tip to Solve: To shift a halogen along a carbon chain, check an elimination–addition route; bromine solution adds two bromines, whereas HBr adds one.

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67. In the precipitation of the iron group (III) in qualitative analysis, ammonium chloride is added before adding ammonium hydroxide to : [4 marks]

(A) decrease concentration of $^{-}\mathrm{OH}$ ions

(B) increase concentration of Cl^{-} ions

(C) prevent interference by phosphate ions

(D) increase concentration of $\mathrm{NH_4}^{+}$ ions

Correct Answer: (A) decrease concentration of $^{-}\mathrm{OH}$ ions

Solution:

Concept:

- Ammonium hydroxide is a weak base: $\mathrm{NH_4OH} \rightleftharpoons \mathrm{NH_4}^{+} + \mathrm{OH}^{-}$. Adding a common ion shifts its ionisation equilibrium towards the undissociated base.
- Selective precipitation depends on the hydroxide ion concentration. Group III hydroxides have sufficiently small solubility products to precipitate at a controlled low concentration of OH^{-}.

Step 1: Apply the common-ion effect Ammonium chloride dissociates to supply $\mathrm{NH_4}^{+}$, which is already a product of the weak-base ionisation. Its addition shifts the ammonium hydroxide equilibrium to the left. Equivalently, for a fixed base concentration the expression $K_b = [\mathrm{NH_4}^{+}][\mathrm{OH}^{-}]/[\mathrm{NH_4OH}]$ shows that raising the common-ion concentration lowers the equilibrium hydroxide concentration. The ammonium salt is added before the base to control hydroxide generation as soon as precipitation begins.

Step 2: Connect the concentration change with selective precipitation The lower OH^{-} concentration is still enough for the sparingly soluble group III hydroxides, including ferric hydroxide, but suppresses premature precipitation of hydroxides belonging to later groups. Thus decreased hydroxide concentration is the analytical purpose, stated in option A. The chloride concentration does rise, but that is not the operative common ion for this base equilibrium. Increased ammonium concentration, option D, describes the cause rather than the required selective-precipitation effect. Prevention of phosphate interference is not the role asked for here.

Step 3: Conclusion Ammonium chloride suppresses ionisation of ammonium hydroxide and decreases the concentration of OH^{-} ions. Hence, the correct option is (A).

Quick Tip to Solve: When a salt is added before a weak base, identify its common ion and then track the concentration of the precipitating ion.

---

68. The element which shows only one oxidation state other than its elemental form is : [4 marks]

(A) Nickel

(B) Titanium

(C) Cobalt

(D) Scandium

Correct Answer: (D) Scandium

Solution:

Concept:

- The usual oxidation states of an early transition element follow from loss of its outer ns and, when accessible, its (n-1)d electrons.
- Scandium has configuration $[\mathrm{Ar}]3d^1 4s^2$; loss of all three valence electrons gives the stable argon core. Its characteristic nonzero oxidation state in standard chemistry is $+3$.

Step 1: Inspect scandium and its stable ion Scandium has three electrons outside the argon core. Removal of the two 4s electrons and the single 3d electron produces Sc^{3+}, whose configuration is $[\mathrm{Ar}]$. This gives the one usual nonzero oxidation state of scandium. Removing a further electron would break the closed shell and is energetically unfavourable. This is why scandium contrasts with most transition elements, which have several accessible d-electron counts.

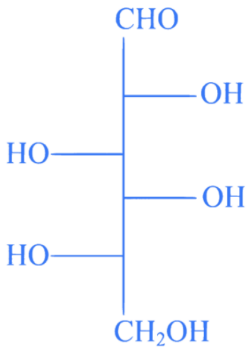
Step 2: Exclude the variable-oxidation-state choices Titanium has familiar $+2$, $+3$ and $+4$ oxidation states, so it cannot meet the stated condition. Cobalt commonly occurs in $+2$ and $+3$, and nickel, though especially stable as $+2$, also has other oxidation states such as $+3$ in suitable compounds. A predominantly common state is different from the single characteristic nonzero state asked for in this periodic-trends question. Thus nickel, titanium and cobalt are excluded and scandium is selected. The elemental state zero is explicitly excluded by the wording.

Step 3: Conclusion Scandium shows the characteristic nonzero oxidation state $+3$. Hence, the correct option is (D).

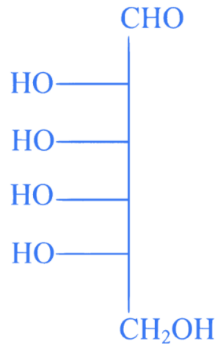
Quick Tip to Solve: For scandium count three electrons outside argon; losing them gives a closed-shell ion and explains its characteristic $+3$ state.

69. Which of the following is the correct structure of L-Glucose? [4 marks]

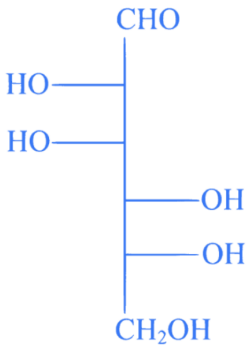
(A) Fischer projection: CHO at top, $\mathrm{CH_2OH}$ at bottom; OH positions from C2 to C5: right, left, right, left.



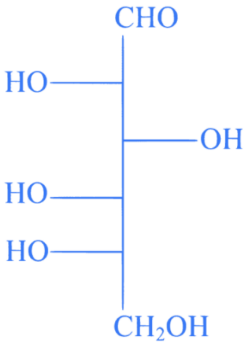
(B) Fischer projection: CHO at top, CH_2OH at bottom; OH positions from C2 to C5: left, left, left, left.



(C) Fischer projection: CHO at top, CH_2OH at bottom; OH positions from C2 to C5: left, left, right, right.

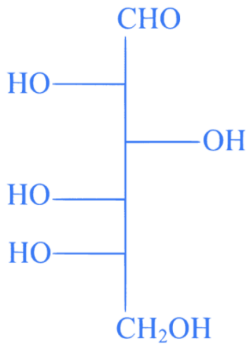


(D) Fischer projection: CHO at top, CH_2OH at bottom; OH positions from C2 to C5: left, right, left, left.



Correct Answer: (D)

Fischer projection: CHO at top, CH_2OH at bottom; OH positions from C2 to C5: left, right, left, left.



Solution:

Concept:

- D-Glucose has OH positions right–left–right–right at C2, C3, C4 and C5 in the standard Fischer projection with the aldehyde group at the top.
- L-Glucose is the mirror image of D-glucose, so all four stereocentres are inverted. The D/L label is fixed by the penultimate carbon, but the identity glucose requires the full stereochemical pattern.

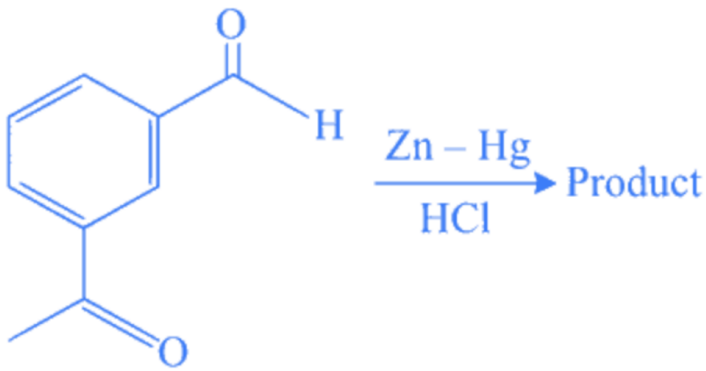
Step 1: Reflect the glucose configuration Number from the aldehyde carbon as C1 to the terminal CH_2OH carbon as C6. Reflection of the D-glucose Fischer projection interchanges left and right at each of C2 through C5. Hence L-glucose must have the OH sequence left–right–left–left. In particular the C5 hydroxyl is on the left, consistent with the L-series. The C2–C4 positions must also match this reflection; merely choosing any structure with C5 OH on the left is insufficient.

Step 2: Compare every printed projection The fourth projection has OH on the left at C2, right at C3, left at C4 and left at C5, so it is exactly L-glucose. The first projection has right–left–right–left: although its final OH is on the left, its other centres have the wrong configuration for glucose. The second has every OH on the left, so it also fails the required C3 orientation. The third has left–left–right–right and is in the D-series at C5, as well as differing at other centres. Only option D reproduces the full mirror-image pattern.

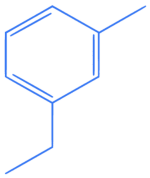
Step 3: Conclusion L-Glucose has the OH sequence left–right–left–left from C2 to C5. Hence, the correct option is (D).

Quick Tip to Solve: Use the penultimate OH only to decide D or L; identify glucose by checking the entire four-centre OH pattern.

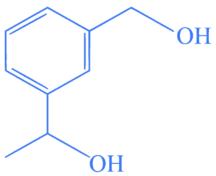
70. Identify the product in the following reaction: [4 marks]



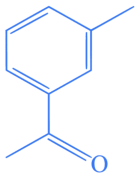
(A) 1-ethyl-3-methylbenzene



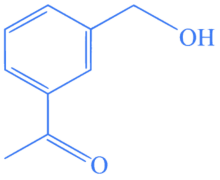
(B) 1-(3-(hydroxymethyl)phenyl)ethan-1-ol



(C) 1-(3-methylphenyl)ethan-1-one

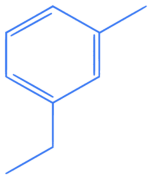


(D) 3-acetylbenzyl alcohol



Correct Answer: (A)

1-ethyl-3-methylbenzene



Solution:

Concept:

- Zinc amalgam and hydrochloric acid are the reagents for Clemmensen reduction. They reduce aldehyde and ketone carbonyl groups to the corresponding hydrocarbon groups.
- An aryl aldehyde group $-CHO$ becomes $-CH_3$, whereas an aryl methyl ketone group $-COCH_3$ becomes $-CH_2CH_3$. The aromatic ring and substituent positions remain unchanged.

Step 1: Apply reduction to each carbonyl group The starting aromatic compound has an aldehyde group and an acetyl group in the meta relation. Clemmensen conditions remove the oxygen of each carbonyl and replace it by hydrogen while retaining the carbon skeleton. The aldehyde carbon is therefore converted to a methyl substituent, and the acetyl carbon becomes the methylene of an ethyl substituent. Both carbonyl groups react; there is no instruction or reagent selectivity that would preserve either one.

Step 2: Match the product against the structures The resulting benzene ring bears a methyl and an ethyl group meta to one another, giving 1-ethyl-3-methylbenzene. This is the first printed structure. Option B shows alcohol functions, which are products of carbonyl-to-alcohol reduction rather than complete Clemmensen deoxygenation. Option C retains the ketone, and option D retains an oxygen-containing substituent, so both are inconsistent with reducing all carbonyl groups under the stated conditions. No migration of substituents or saturation of the benzene ring is required.

Step 3: Conclusion The product is 1-ethyl-3-methylbenzene, with the original meta relationship retained. Hence, the correct option is (A).

Quick Tip to Solve: For Clemmensen reduction, replace each carbonyl oxygen by two hydrogens on its carbon, then recount the side-chain carbons.

71. Number of molecules/ions from the following in which the central atom is involved in sp^3 hybridization is _____.

$$NO_3^-, BCl_3, ClO_2^-, ClO_3^-$$

[4 marks]

(A) 2

(B) 4

(C) 1

(D) 3

Correct Answer: (A) 2

Solution:

Concept:

- For the usual VSEPR assignment, count the sigma bonds and lone pairs on the central atom. Three electron domains correspond to sp^2, and four correspond to sp^3 hybridization.
- A multiple bond counts as one electron domain because its sigma and pi components point towards the same bonded atom. Molecular shape alone is insufficient: lone pairs must also be counted.

Step 1: Classify the two trigonal-planar species In nitrate, NO_3^-, nitrogen forms three sigma bonds with oxygen atoms and has no lone pair. Its steric number is therefore three, giving sp^2 hybridization. The resonance-delocalized pi bonding does not add a fourth sigma direction. In BCl_3, boron likewise forms three sigma bonds and has no lone pair; its electron-deficient octet does not create another electron domain. Boron is also sp^2 hybridized.

Step 2: Count the electron domains of the chlorine oxoanions In ClO_2^-, chlorine has two bonding domains and two lone pairs, so its steric number is $2+2=4$. The electron-pair geometry is tetrahedral and the molecular shape is bent. In ClO_3^-, chlorine has three bonding domains and one lone pair: $3+1=4$. The molecular shape is trigonal pyramidal. Both central chlorine atoms therefore receive the conventional sp^3 assignment. Only these two ions qualify; counting one, three or four species would misclassify at least one of the planar species or neglect a chlorine lone pair.

Step 3: Conclusion The qualifying ions are ClO_2^- and ClO_3^-, so the required number is $\boxed{2}$. Hence, the correct option is (A).

Quick Tip to Solve: Count sigma-bond directions plus central lone pairs; never count each line of a double bond as a separate hybrid orbital.

72. The correct sequence of ligands in the order of decreasing field strength is : [4 marks]

(A) $NCS^- > EDTA^{4-} > CN^- > CO$

(B) $CO > H_2O > F^- > S^{2-}$

(C) $S^{2-} > {}^-OH > EDTA^{4-} > CO$

(D) ${}^-OH > F^- > NH_3 > CN^-$

Correct Answer: (B) $CO > H_2O > F^- > S^{2-}$

Solution:

Concept:

- Ligand field strength describes the splitting Δ of the metal d orbitals caused by ligands. The spectrochemical series orders ligands experimentally from weak to strong field.
- For the ligands needed here, a useful increasing-strength sequence is $S^{2-} < F^- < OH^- < H_2O < NH_3 < CN^- < CO$. Intermediate ligands may be omitted when comparing a subset.

Step 1: Reverse the relevant increasing-strength sequence Carbon monoxide lies at the strong-field end of the series, whereas sulfide is a weak-field ligand. Water produces a greater splitting than fluoride, and fluoride is stronger than sulfide. Reading just these four ligands from strong to weak gives

$$CO > H_2O > F^- > S^{2-}.$$

This is exactly the sequence in option (B); deleting intermediate members from the full series does not change their relative order.

Step 2: Eliminate the other proposed sequences Option (A) puts carbon monoxide below cyanide and EDTA, although carbon monoxide belongs above both. It also places thiocyanate at the strong-field end of that proposed sequence. Option (C) begins with sulfide and ends with carbon monoxide, reversing the decisive weak-field and strong-field endpoints. Option (D) puts hydroxide above ammonia and cyanide, even though these nitrogen- and carbon-donor ligands are stronger-field ligands than hydroxide. Thus each of these sequences contains an immediate contradiction to the spectrochemical order; formal charge alone cannot determine ligand field strength.

Step 3: Conclusion The decreasing field-strength order is $\boxed{CO > H_2O > F^- > S^{2-}}$. Hence, the correct option is (B).

Quick Tip to Solve: First check the endpoints: CO and cyanide are strong-field ligands, whereas sulfide and halide ligands are near the weak-field end.

73. The Molarity (M) of an aqueous solution containing $5.85\ \text{g}$ of NaCl in $500\ \text{mL}$ water is : (Given : Molar Mass $\text{Na} : 23$ and $\text{Cl} : 35.5\ \text{gmol}^{-1}$) [4 marks]

(A) 20

(B) 4

(C) 2

(D) 0.2

Correct Answer: (D) 0.2

Solution:

Concept:

- Molarity is the amount of solute in moles per litre of solution: $M = n/V$. The solute amount is obtained from $n = m/M_{\text{mol}}$, where M_{mol} denotes molar mass.
- Convert millilitres to litres before substitution. For this dilute aqueous calculation, the stated 500 mL water is used as the approximate final solution volume, as implied by the numerical choices.

Step 1: Find the amount of sodium chloride The given atomic molar masses must first be added, since one formula unit contains one sodium and one chlorine atom:

$$M_{\text{mol}}(\text{NaCl}) = 23 + 35.5 = 58.5\,\text{g mol}^{-1}.$$

Therefore the number of moles present is

$$n = \frac{5.85\,\text{g}}{58.5\,\text{g mol}^{-1}} = 0.100\,\text{mol}.$$

The units of grams cancel, leaving an amount in moles; using the mass directly as the numerator of molarity would be incorrect.

Step 2: Use the volume in litres and compare the choices For the intended dilute-solution approximation,

$$V \simeq 500\,\text{mL} = 0.500\,\text{L}.$$

Thus

$$M \simeq \frac{0.100}{0.500} = 0.200\,\text{mol L}^{-1}.$$

Half a litre containing one tenth of a mole must have less than one mole per litre, immediately excluding 2, 4 and 20. Strictly, molarity refers to final solution volume rather than solvent volume; the given information does not quantify a volume change on dissolving. With the approximation implicit in the options, the result is unambiguously the listed 0.2 value.

Step 3: Conclusion The intended molarity is $\boxed{0.2\,\text{mol L}^{-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: Compute moles first, then divide by litres of solution. Here 0.1 mol in approximately half a litre gives 0.2 M.

74. Which among the following is incorrect statement? [4 marks]

(A) The electromeric effect is, temporary effect

(B) Hydrogen ion (H^+) shows negative electromeric effect

(C) The organic compound shows electromeric effect in the presence of the reagent only.

(D) Electromeric effect dominates over inductive effect

Correct Answer: (B) Hydrogen ion (H^+) shows negative electromeric effect

Solution:

Concept:

- The electromeric effect is the temporary complete transfer of a pi-electron pair to one atom of a multiple bond under the influence of an attacking reagent.
- In the positive electromeric effect, the electron pair shifts towards the atom to which the attacking electrophile attaches. In the negative effect, it shifts away from the atom attacked by a nucleophile.

Step 1: Identify the effect associated with a proton A hydrogen ion, H^+, is an electrophile because it accepts an electron pair. When it attacks an alkene, the pi-electron pair forms a bond to the proton at one carbon and the other carbon becomes electron deficient. This illustrates the positive electromeric effect: the pi pair moves towards the carbon that bonds to the attacking electrophile. It is therefore incorrect to attribute a negative electromeric effect to hydrogen ion in the statement supplied.

Step 2: Check the remaining statements individually Option (A) is true because the electron displacement is induced temporarily by an attacking reagent, rather than existing permanently in the unperturbed molecule. Option (C) expresses the same necessary condition: this effect is shown in the presence of the reagent. Option (D) is also true in the usual comparison of these electronic effects. The electromeric displacement transfers the entire pi pair, whereas an inductive effect is only a partial polarization transmitted through sigma bonds. When both influences act in an opposed manner during the reagent attack, the complete electromeric shift dominates. These facts leave only the sign assigned to proton attack as the incorrect assertion.

Step 3: Conclusion Proton attack illustrates the positive electromeric effect, so the statement that H^+ shows the negative effect is incorrect. Hence, the correct option is (B).

Quick Tip to Solve: For the sign of the electromeric effect, identify the attacking reagent: an electrophile such as H+ draws the pi pair towards its new bond.

75. One of the commonly used electrode is calomel electrode. Under which of the following categories, calomel electrode comes? [4 marks]

(A) Metal ion - Metal electrodes

(B) Oxidation - Reduction electrodes

(C) Metal - Insoluble Salt - Anion electrodes

(D) Gas - Ion electrodes

Correct Answer: (C) Metal - Insoluble Salt - Anion electrodes

Solution:

Concept:

- A calomel electrode contains liquid mercury in contact with solid mercurous chloride, Hg_2Cl_2, and a chloride-ion solution, usually aqueous potassium chloride.
- A metal covered by its sparingly soluble salt and immersed in a solution containing the salt anion is a metal–insoluble salt–anion electrode, also called an electrode of the second kind.

Step 1: Write the components and electrode reaction The electrode can be represented as

$$Hg(l) \,|\, Hg_2Cl_2(s) \,|\, Cl^-(aq).$$

Its reversible reduction reaction is

$$Hg_2Cl_2(s) + 2\,e^- \rightleftharpoons 2\,Hg\,(l) + 2\,Cl^-(aq).$$

The pure liquid metal and solid salt have constant activities. As a result, at a fixed temperature its potential is controlled by the chloride-ion activity, which is made reproducible by using a specified potassium chloride concentration. This is why the calomel electrode serves as a convenient reference electrode.

Step 2: Match the physical construction to the category The three essential components are the metal mercury, the insoluble salt mercurous chloride, and the common anion chloride. This exactly matches option (C). A simple metal-ion–metal electrode, such as copper in copper sulfate, instead directly involves the dissolved metal cation and has no required insoluble salt coating. An oxidation–reduction electrode uses an inert conductor with dissolved oxidized and reduced forms; that is not this construction. A gas–ion electrode requires a gas phase in equilibrium with its ions, absent here. The classification is based on the actual interfaces, not merely on the general fact that electron transfer occurs.

Step 3: Conclusion The calomel electrode is a $\boxed{\text{metal–insoluble salt–anion electrode}}$. Hence, the correct option is (C).

Quick Tip to Solve: Remember its three-part notation Hg | Hg2Cl2 | Cl-; the metal, insoluble chloride and chloride ion name the electrode category.

76. The correct order of first ionization enthalpy values of the following elements is :

(A) O

(B) N

(C) Be

(D) F

(E) B

Choose the correct answer from the options given below : [4 marks]

(A) $\mathrm{A} < \mathrm{B} < \mathrm{D} < \mathrm{C} < \mathrm{E}$

(B) $\mathrm{C} < \mathrm{E} < \mathrm{A} < \mathrm{B} < \mathrm{D}$

(C) $\mathrm{E} < \mathrm{C} < \mathrm{A} < \mathrm{B} < \mathrm{D}$

(D) $\mathrm{B} < \mathrm{D} < \mathrm{C} < \mathrm{E} < \mathrm{A}$

Correct Answer: (C) $\mathrm{E} < \mathrm{C} < \mathrm{A} < \mathrm{B} < \mathrm{D}$

Solution:

Concept:

- First ionization enthalpy is the energy required to remove the first electron from an isolated gaseous atom. Across period 2 it generally increases as effective nuclear charge rises.
- Two relevant exceptions are $I_1(\mathrm{B}) < I_1(\mathrm{Be})$ and $I_1(\mathrm{O}) < I_1(\mathrm{N})$, caused by the subshell occupied and by electron pairing respectively.

Step 1: Apply the subshell and pairing exceptions Beryllium has the configuration $1s^2 2s^2$, whereas boron is $1s^2 2s^2 2p^1$. The boron 2p electron is easier to remove than a beryllium 2s electron, despite the increase in nuclear charge. Hence boron precedes beryllium. Nitrogen has a half-filled $2p^3$ subshell with three singly occupied orbitals. Oxygen has $2p^4$, so one orbital contains a pair; repulsion within that pair facilitates electron removal. Oxygen therefore precedes nitrogen.

Step 2: Assemble the complete increasing order Both nitrogen and oxygen are appreciably harder to ionize than beryllium and boron. Fluorine, with still greater effective nuclear charge and a small atomic radius, has the highest first ionization enthalpy among the five. Combining these comparisons gives

$$\mathrm{B} < \mathrm{Be} < \mathrm{O} < \mathrm{N} < \mathrm{F}.$$

Translate elements back to the labels printed in the question: boron is E, beryllium is C, oxygen is A, nitrogen is B, and fluorine is D. The required label order is therefore $E < C < A < B < D$. Option (B) reverses the Be/B exception, while options (A) and (D) also place high-ionization elements before the low-ionization pair.

Step 3: Conclusion The increasing order is $\boxed{E < C < A < B < D}$. Hence, the correct option is (C).

Quick Tip to Solve: Before applying the across-period trend, insert both period-2 exceptions: B is below Be and O is below N.

77. Which of the following nitrogen containing compound does not give Lassaigne's test ? [4 marks]

(A) Glycene

(B) Phenyl hydrazine

(C) Urea

(D) Hydrazine

Correct Answer: (D) Hydrazine

Solution:

Concept:

- Lassaigne's nitrogen test converts covalently bound nitrogen into sodium cyanide during fusion with sodium. Formation of cyanide requires carbon as well as nitrogen in the organic sample.
- The cyanide subsequently forms a ferrocyanide complex with ferrous ions; oxidation and acidification allow the characteristic Prussian-blue ferric ferrocyanide to appear.

Step 1: Identify the requirement for cyanide formation The crucial fusion conversion is commonly summarized as

$$\mathrm{Na} + \mathrm{C} + \mathrm{N} \longrightarrow \mathrm{NaCN}.$$

Thus simply containing nitrogen is not sufficient for the usual nitrogen test: the sample must also supply carbon. If no cyanide forms, the subsequent iron-salt steps cannot generate the blue diagnostic compound. This explains why the question asks which nitrogen-containing compound does not respond, rather than merely asking whether nitrogen occurs in its formula.

Step 2: Check carbon and nitrogen in each compound Glycine, printed as Glycene in the option, has the formula $\mathrm{NH_2CH_2COOH}$, so it contains both carbon and nitrogen. Phenylhydrazine, $\mathrm{C_6H_5NHNH_2}$, also supplies both elements through its phenyl group and hydrazine group. Urea, $\mathrm{NH_2CONH_2}$, contains a carbonyl carbon and two nitrogen atoms. All three can therefore provide the elements required to form cyanide on sodium fusion. Hydrazine, $\mathrm{NH_2NH_2}$, contains nitrogen and hydrogen but no carbon at all. It cannot form sodium cyanide from its own constituents in this test, so it fails to give the usual Prussian-blue nitrogen result. Its lack of carbon, rather than a lack of nitrogen, is the decisive distinction.

Step 3: Conclusion Hydrazine, $\mathrm{N_2H_4}$, does not give the usual Lassaigne nitrogen test because it contains no carbon. Hence, the correct option is (D).

Quick Tip to Solve: For Lassaigne nitrogen detection, check for both C and N in the molecular formula; nitrogen alone cannot produce NaCN.

78. Which one of the following molecules has maximum dipole moment? [4 marks]

(A) CH_4

(B) NF_3

(C) NH_3

(D) PF_5

Correct Answer: (C) NH_3

Solution:

Concept:

- The molecular dipole moment is the vector sum of all bond dipole moments. Molecular geometry therefore matters as much as individual bond polarity.
- A symmetric arrangement of identical surrounding atoms can cancel all bond moments. A lone pair can make the electron distribution asymmetric and alter the resultant moment.

Step 1: Eliminate the symmetric molecules CH_4 has a regular tetrahedral shape. The four equal C–H bond moments cancel by symmetry, so its net dipole moment is zero. PF_5 has a trigonal-bipyramidal shape: the two axial P–F moments cancel each other, and the three equatorial moments at 120° cancel separately. Its resultant is also zero.

Step 2: Compare the two pyramidal molecules Both NH_3 and NF_3 are trigonal pyramidal with a lone pair on nitrogen. In ammonia, the resultant of the N–H bond moments and the lone-pair contribution reinforce one another. In nitrogen trifluoride, fluorine attracts the bonding electrons away from nitrogen, so the N–F resultant opposes the lone-pair contribution. The cancellation makes NF_3 only weakly polar; its dipole moment is about $0.24\,\mathrm{D}$, whereas that of ammonia is about $1.47\,\mathrm{D}$.

Step 3: Conclusion NH_3 has the maximum dipole moment among the four molecules. Hence, the correct option is (C).

Quick Tip to Solve: First remove perfectly symmetric molecules; then compare whether the bond-moment resultant reinforces or opposes the lone-pair contribution.

79. What pressure (bar) of H_2 would be required to make emf of hydrogen electrode zero in pure water at $25^\circ C$? [4 marks]

(A) 0.5

(B) 10^{-14}

(C) 1

(D) 10^{-7}

Correct Answer: (B) 10^{-14}

Solution:

Concept:

- For the reduction $2\,H^+ + 2\,e^- \longrightarrow H_2(g)$, the standard reduction potential is zero. At $25^\circ C$, the Nernst equation uses $0.0591/n$ with base-ten logarithms.

- In pure water at this temperature, $[H^+] = 10^{-7}\,\text{mol}\,L^{-1}$. Gas pressure is measured relative to its standard pressure of one bar.

Step 1: Write the reaction quotient The activities of electrons are not included in the reaction quotient. Taking the dilute aqueous hydrogen-ion activity as its molar concentration relative to the standard concentration,

$$Q = \frac{p_{H_2}/(1\,\text{bar})}{[H^+]^2}.$$

There are two electrons in the half-reaction, so

$$E = 0 - \frac{0.0591}{2}\log Q.$$

The pressure must compensate for the very small concentration of hydrogen ions in neutral water.

Step 2: Impose zero electrode potential Setting $E = 0$ requires $\log Q = 0$, hence $Q = 1$. Substituting the neutral-water hydrogen-ion concentration gives

$$\frac{p_{H_2}/(1\,\text{bar})}{(10^{-7})^2} = 1 \quad \Longrightarrow \quad p_{H_2} = 10^{-14}\,\text{bar}.$$

At one bar the electrode potential is negative, rather than zero. A pressure of 10^{-7} bar would compensate for only one power of hydrogen-ion activity; the balanced reaction requires its square. The value 0.5 bar is also much too large.

Step 3: Conclusion The required hydrogen pressure is 10^{-14} bar. Hence, the correct option is (B).

Quick Tip to Solve: For a zero-potential hydrogen electrode, immediately set the dimensionless hydrogen pressure equal to the square of hydrogen-ion activity.

80. Number of elements from the following that CANNOT form compounds with valencies which match with their respective group valencies is _____. B, C, N, S, O, F, P, Al, Si [4 marks]

(A) 7

(B) 3

(C) 5

(D) 6

Correct Answer: (B) 3

Solution:

Concept:

- For these main-group elements, the group valency being compared is the number of valence-shell electrons: 3 for group 13, 4 for group 14, 5 for group 15, 6 for group 16 and 7 for group 17.
- Second-period elements cannot expand their octet. Heavier main-group elements can exhibit higher covalencies, so the position in a group matters when checking whether its group valency is attainable.

Step 1: Identify elements that attain their group valency Boron and aluminium have group valency 3 and form compounds such as BF_3 and $AlCl_3$. Carbon and silicon have group valency 4, attained in CH_4 and $SiCl_4$. Phosphorus attains valency 5 in PCl_5, while sulfur attains valency 6 in SF_6. Thus B, C, S, P, Al and Si satisfy the condition and must not be counted among the exceptions.

Step 2: Count the second-period exceptions Nitrogen, oxygen and fluorine belong to groups 15, 16 and 17, whose group valencies are respectively 5, 6 and 7. They cannot have these covalencies because their small second-period valence shell has only the $2s$ and $2p$ orbitals and cannot accommodate an expanded octet. Their common covalencies are much smaller: 3 for N, 2 for O and 1 for F. In particular, an oxidation state of $+5$ for nitrogen is not a five-bond covalency. Counting these three exceptions rules out the options 7, 5 and 6.

Step 3: Conclusion The elements are N, O and F, so their number is 3. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish oxidation state from covalency: nitrogen in a $+5$ oxidation state does not thereby form five ordinary covalent bonds.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. 2.5 g **of a non-volatile, non-electrolyte is dissolved in** 100 g **of water at** 25°C**. The solution showed a boiling point elevation by** 2°C**. Assuming the solute concentration is negligible with respect to the solvent concentration, the vapor pressure of the resulting aqueous solution is _____** mm **of** Hg **(nearest integer)**

[Given : Molal boiling point elevation constant of water $(K_b) = 0.52$ K.kgmol^{-1}**,** 1 atm **pressure** $= 760$ mm **of** Hg**, molar mass of water** $= 18$ g mol^{-1}**]** [4 marks]

Solution:

Concept:

- For a non-volatile non-electrolyte, boiling-point elevation gives molality through $\Delta T_b = K_b m$; no dissociation factor is needed.
- Raoult's law relates lowering of solvent vapour pressure to solute mole fraction. Under the stipulated dilute approximation, $(p^0 - p)/p^0 \simeq n_s/n_w$. For the numerical calculation, take the vapour pressure of pure water as $p^0 = 1\,\text{atm} = 760\,\text{mm Hg}$, using the pressure given in the question.

Step 1: Find the amount of dissolved solute The temperature interval 2°C equals 2 K. Therefore

$$m = \frac{2}{0.52} = \frac{50}{13}\,\text{mol kg}^{-1}.$$

The water mass is $100\,\text{g} = 0.100\,\text{kg}$, so the solute amount is

$$n_s = \frac{50}{13}(0.100) = \frac{5}{13}\,\text{mol}.$$

The solute mass need not be used separately because the measured boiling-point rise already deter-

mines its amount in the specified solvent mass.

Step 2: Apply the dilute relative lowering The amount of water is

$$n_w = \frac{100}{18} = \frac{50}{9}\,\text{mol}.$$

Consequently, the stipulated dilute approximation gives

$$\frac{p^0 - p}{p^0} \simeq \frac{n_s}{n_w} = \frac{5/13}{50/9} = \frac{9}{130}.$$

Taking the vapour pressure of pure water as the given one-atmosphere pressure, $p^0 = 760\,\text{mm Hg}$,

$$p = 760\left(1 - \frac{9}{130}\right) = 707.3846\ldots\ \text{mm Hg}.$$

This calculation takes the pure-water vapour pressure as the question's given pressure and uses its negligible-solute-concentration approximation; retaining solute moles in the mole-fraction denominator would not implement that requested approximation.

Step 3: Conclusion Taking pure-water vapour pressure as $760\,\text{mm Hg}$, the vapour pressure rounds to $\boxed{707\,\text{mm Hg}}$.

Quick Tip to Solve: Combine boiling-point elevation with relative lowering directly: $(p^0 - p)/p^0 \simeq (\Delta T_b/K_b)M_w/1000$, using water molar mass in grams.

82. Only $2\ \text{mL}$ of $KMnO_4$ solution of unknown molarity is required to reach the end point of a titration of $20\ \text{mL}$ of oxalic acid (2M) in acidic medium. The molarity of $KMnO_4$ solution should be _____ M. [4 marks]

Solution:

Concept:

- In acidic medium, permanganate is reduced from Mn(VII) to Mn(II), accepting five electrons per mole of MnO_4^-.
- Oxalic acid is oxidised to carbon dioxide and loses two electrons per mole. At the end point, the electron equivalents supplied and accepted are equal.

Step 1: Balance the acidic redox reaction The relevant half-reactions are

$$MnO_4^- + 8\,H^+ + 5\,e^- \longrightarrow Mn^{2+} + 4\,H_2O$$

$$H_2C_2O_4 \longrightarrow 2\,CO_2 + 2\,H^+ + 2\,e^-.$$

Multiplying the first by two and the second by five cancels ten electrons. Thus two moles of permanganate react with five moles of oxalic acid. The acidic medium is essential: the five-electron change belongs to reduction to Mn^{2+}, rather than formation of MnO_2 in a different medium.

Step 2: Calculate the permanganate molarity The oxalic-acid amount is

$$n_{\text{acid}} = 2\,\text{mol}\,\text{L}^{-1} \times 0.020\,\text{L} = 0.040\,\text{mol}.$$

The stoichiometric ratio gives

$$n_{\text{KMnO}_4} = \frac{2}{5}(0.040) = 0.016\,\text{mol}.$$

Only $2\,\text{mL} = 0.002\,\text{L}$ of permanganate solution is required, so

$$M_{\text{KMnO}_4} = \frac{0.016}{0.002} = 8\,\text{mol}\,\text{L}^{-1}.$$

Equivalently, $5M_{\text{KMnO}_4}V_{\text{KMnO}_4} = 2M_{\text{acid}}V_{\text{acid}}$; equal molarities cannot be assumed because the electron factors differ.

Step 3: Conclusion The required molarity of KMnO_4 is $\boxed{8\,\text{M}}$.

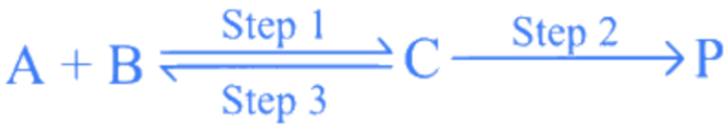
Quick Tip to Solve: In acidic permanganate–oxalic acid titrations, remember the electron factors 5 and 2 before substituting volumes.

83. Consider the following transformation involving first order elementary reaction in each step at constant temperature as shown below.

Some details of the above reactions are listed below.

Step	Rate constant (sec^{-1})	Activation energy (kJ mol^{-1})
1	k_1	**300**
2	k_2	**200**
3	k_3	Ea_3

If the overall rate constant of the above transformation (k) is given as $k = \frac{k_1 k_2}{k_3}$ and the overall activation energy (E_a) is $400\ \text{kJ mol m}^{-1}$, then the value of Ea_3 is _____ integer) [4 marks]



Solution:

Concept:

- For each elementary step, the Arrhenius equation is $k_i = A_i \exp[-E_{a,i}/(RT)]$, where A_i is the pre-exponential factor.
- When an overall rate constant is a product or quotient of elementary rate constants, its activation energy is the same signed sum of their activation energies, under the usual Arrhenius treatment of the prefactors.

Step 1: Insert the three Arrhenius expressions The specified overall rate constant is $k = k_1 k_2 / k_3$.

Substitution gives

$$k = \frac{A_1 A_2}{A_3} \exp\left[-\frac{E_{a,1} + E_{a,2} - E_{a,3}}{RT}\right].$$

The negative sign for the third activation energy arises because k_3 is in the denominator. Taking logarithms gives the equivalent relation

$$\ln k = \ln(A_1 A_2 / A_3) - \frac{E_{a,1} + E_{a,2} - E_{a,3}}{RT}.$$

Thus the coefficient of $-1/(RT)$ is the overall activation energy.

Step 2: Solve for the third activation energy Comparing with the overall Arrhenius equation yields

$$E_a = E_{a,1} + E_{a,2} - E_{a,3}.$$

The table provides $E_{a,1} = 300\,\text{kJ mol}^{-1}$ and $E_{a,2} = 200\,\text{kJ mol}^{-1}$, while the specified overall value is 400 in the activation-energy units of the table. Hence

$$400 = 300 + 200 - E_{a,3}, \qquad E_{a,3} = 100\,\text{kJ mol}^{-1}.$$

A check gives $300 + 200 - 100 = 400$. Adding all three barriers would be incorrect, because the reverse-step rate constant occurs in the denominator of the given expression.

Step 3: Conclusion The activation energy of step 3 is $100\,\text{kJ mol}^{-1}$; the numerical entry is $\boxed{100}$.

Quick Tip to Solve: For an Arrhenius product $k = k_1^a k_2^b / k_3^c$, combine activation energies as $E_a = aE_1 + bE_2 - cE_3$.

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84. The enthalpy of formation of ethane (C_2H_6) from ethylene by addition of hydrogen where the bond-energies of $C-H, C-C, C=C, H-H$ are 414 kJ, 347 kJ, 615 kJ and 435 kJ respectively is $-$ _____ kJ [4 marks]

Solution:

Concept:

- The reaction enthalpy is the energy needed to break reactant bonds minus the energy released in forming product bonds. Thus $\Delta H = E_{\text{broken}} - E_{\text{formed}}$.
- Hydrogenation converts one C=C bond into one C–C bond and adds two C–H bonds. Bonds common to both sides cancel in the bond-energy accounting.

Step 1: Count the bond changes The reaction is

$$CH_2{=}CH_2 + H_2 \longrightarrow CH_3{-}CH_3.$$

Ethylene contains four C–H bonds, and ethane contains six. The four original C–H bonds occur on both sides and cancel. Therefore the net calculation breaks one C=C bond and one H–H bond, while forming one C–C bond and two additional C–H bonds. This bookkeeping avoids counting the double bond as two single bonds: its quoted energy is the energy of the complete C=C bond.

Step 2: Substitute the supplied bond energies Using the values in the order stated,

$$E_{\text{broken}} = 615 + 435 = 1050\,\text{kJ},$$

$$E_{\text{formed}} = 347 + 2(414) = 1175\,\text{kJ}.$$

Consequently,

$$\Delta H = 1050 - 1175 = -125\,\text{kJ}.$$

The negative sign means that bond formation releases more energy than is consumed in breaking the reactant bonds; the hydrogenation is exothermic. This is the enthalpy of the specified hydrogen-addition reaction, rather than an elemental standard enthalpy of formation. The question prints a minus sign before the blank, so the blank itself requires the positive magnitude.

Step 3: Conclusion The reaction enthalpy is $-125\,\text{kJ}$. Since the minus sign is supplied, enter $\boxed{125}$.

Quick Tip to Solve: For hydrogenation, use $D(\text{C=C}) + D(\text{H–H}) - D(\text{C–C}) - 2D(\text{C–H})$; cancel unchanged C–H bonds first.

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85. Xg **of ethylamine is subjected to reaction with** NaNO_2/HCl **followed by water; evolved dinitrogen gas which occupied** $2.24\ \text{L}$ **volume at STP. X is** _____ $\times 10^{-1}$ g. [4 marks]

Solution:

Concept:

- An aliphatic primary amine reacts with nitrous acid to give the corresponding alcohol, water and dinitrogen. Nitrous acid is generated in situ from NaNO_2/HCl.
- The stoichiometric ratio is one mole of ethylamine per mole of N_2. At STP, one mole of an ideal gas occupies $22.4\,\text{L}$.

Step 1: Write the chemical reaction Ethylamine is a primary aliphatic amine. Its unstable aliphatic diazonium intermediate decomposes with loss of nitrogen, and water gives ethanol. The net equation is

$$\text{C}_2\text{H}_5\text{NH}_2 + \text{HNO}_2 \longrightarrow \text{C}_2\text{H}_5\text{OH} + \text{N}_2 + \text{H}_2\text{O}.$$

Thus the number of moles of nitrogen measures the number of moles of ethylamine consumed; a factor of two is not required even though nitrogen gas is diatomic.

Step 2: Convert the gas volume into the amine mass Using the given STP volume,

$$n(\text{N}_2) = \frac{2.24}{22.4} = 0.100\,\text{mol}.$$

Therefore $n(\text{C}_2\text{H}_5\text{NH}_2) = 0.100\,\text{mol}$. Ethylamine is $\text{C}_2\text{H}_7\text{N}$, whose molar mass is

$$M = 2(12) + 7(1) + 14 = 45\,\text{g mol}^{-1}.$$

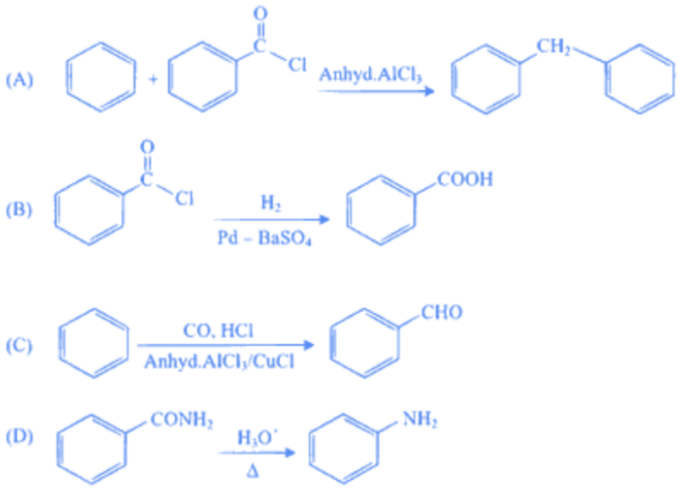
The required mass is

$$m = nM = 0.100(45) = 4.5\,\text{g} = 45 \times 10^{-1}\,\text{g}.$$

Step 3: Conclusion The mass of ethylamine is $4.5\,\text{g}$; the numerical value to enter is $\boxed{45}$.

Quick Tip to Solve: For a primary aliphatic amine, equate its moles to the evolved nitrogen moles before applying the requested power-of-ten scaling.

86. The number of the correct reaction(s) among the following is _____. [4 marks]



Solution:

Concept:

- The reagents determine whether an acyl chloride undergoes acylation, selective reduction or hydrolysis. Compare the functional group in each displayed product with that expected from the reagent.
- Gattermann–Koch formylation uses CO/HCl with anhydrous $\text{AlCl}_3/\text{CuCl}$ to introduce a $-\text{CHO}$ group into benzene.

Step 1: Test reactions A and B In (A), benzene and benzoyl chloride with anhydrous AlCl_3 undergo Friedel–Crafts acylation. The product is benzophenone, $\text{C}_6\text{H}_5\text{COC}_6\text{H}_5$, retaining the carbonyl group; the displayed diphenylmethane, $\text{C}_6\text{H}_5\text{CH}_2\text{C}_6\text{H}_5$, is not obtained. Hence (A) is incorrect. In (B), $\text{H}_2/\text{Pd}-\text{BaSO}_4$ gives Rosenmund reduction of benzoyl chloride to benzaldehyde, $\text{C}_6\text{H}_5\text{CHO}$, rather than the displayed benzoic acid, $\text{C}_6\text{H}_5\text{COOH}$. Hence (B) is incorrect.

Step 2: Test reactions C and D In (C), benzene is treated with carbon monoxide and hydrogen chloride in the presence of anhydrous aluminium chloride and cuprous chloride. This is Gattermann–Koch formylation, producing the displayed benzaldehyde:

$$\text{C}_6\text{H}_6 + \text{CO} + \text{HCl} \xrightarrow{\text{AlCl}_3/\text{CuCl}} \text{C}_6\text{H}_5\text{CHO} + \text{HCl}.$$

Thus (C) is correct. In (D), heating benzamide with aqueous acid hydrolyses its amide group to benzoic acid and an ammonium ion. It does not remove the carbonyl carbon to form aniline. Aniline would require a Hofmann bromamide degradation reagent such as Br_2/KOH, which is absent. Thus (D) is incorrect.

Step 3: Conclusion Only reaction (C) is correct. The required number of correct reactions is $\boxed{1}$.

Quick Tip to Solve: Check the carbonyl carbon: acylation retains it, Rosenmund reduction gives an aldehyde, and acidic amide hydrolysis gives a carboxylic acid.

87. The de-Broglie's wavelength of an electron in the 4^{th} orbit is _____ πa_0. (a_0 = Bohr's radius)

[4 marks]

Solution:

Concept:

- A permitted Bohr orbit contains an integral number of electron matter wavelengths: $2\pi r_n = n\lambda_n$. Here n is the principal quantum number.
- For the hydrogen Bohr model, $r_n = n^2 a_0$, where a_0 is the Bohr radius. The wavelength is consequently proportional to n, rather than to n^2.

Step 1: Relate angular momentum to the wavelength The de Broglie relation is $\lambda = h/(mv)$, while Bohr quantisation gives $mvr_n = nh/(2\pi)$. Eliminating mv between these equations yields

$$\lambda_n = \frac{h}{nh/(2\pi r_n)} = \frac{2\pi r_n}{n}.$$

Equivalently, the circumference must accommodate exactly n complete wavelengths, which ensures a single-valued standing matter wave around the orbit.

Step 2: Insert the fourth-orbit radius The fourth orbit has $n = 4$, and its radius is

$$r_4 = 4^2 a_0 = 16a_0.$$

Substitution gives

$$\lambda_4 = \frac{2\pi(16a_0)}{4} = 8\pi a_0.$$

The factor $32\pi a_0$ would be the circumference, not the electron wavelength. Dividing that circumference by four is essential. The coefficient multiplying the unit of length πa_0 is therefore dimensionless.

Step 3: Conclusion The de Broglie wavelength is $\lambda_4 = 8\pi a_0$. The numerical value to enter is $\boxed{8}$.

Quick Tip to Solve: Use $\lambda_n = 2\pi n a_0$ directly for a hydrogen Bohr orbit; do not confuse the orbit circumference with one wavelength.

88. The number of different chain isomers for C_7H_{16} is _____.

[4 marks]

Solution:

Concept:

- Chain isomers have the same molecular formula but distinct carbon skeletons. Changing the drawing orientation or numbering direction does not create a new isomer.

- For the acyclic saturated formula C_7H_{16}, classify the skeletons by their longest carbon chain: seven, six, five or four carbon atoms.

Step 1: Enumerate chains of length seven and six There is one straight-chain skeleton, heptane. With a six-carbon longest chain, the remaining carbon is a methyl substituent. The distinct positions are 2 and 3, giving 2-methylhexane and 3-methylhexane. Positions 5 and 4 reproduce these structures on reversing the numbering, while a terminal methyl substituent would simply extend the chain to heptane. This class therefore contributes two isomers, for a running total of three.

Step 2: Enumerate the more branched skeletons With a five-carbon longest chain, two methyl groups give 2,2-dimethylpentane, 2,3-dimethylpentane, 2,4-dimethylpentane and 3,3-dimethylpentane. The additional distinct five-carbon-chain skeleton is 3-ethylpentane. An ethyl group at position 2 would produce a longer six-carbon chain and has already been counted under methylhexanes. These give five further isomers. With a four-carbon longest chain, the remaining distinct skeleton is 2,2,3-trimethylbutane. A longest chain of three cannot accommodate seven carbons without exceeding carbon valency or allowing a longer path. The count is

$$1 + 2 + 5 + 1 = 9.$$

Step 3: Conclusion There are $\boxed{9}$ different chain isomers of C_7H_{16}.

Quick Tip to Solve: List alkane isomers by decreasing longest-chain length, then remove duplicates by chain reversal and by checking the actual longest path.

89. Number of molecules/species from the following having one unpaired electron is _____.

$$O_2, O_2^{-1}, NO, CN^{-1}, O_2^{2-}$$

[4 marks]

Solution:

Concept:

- Unpaired electrons are counted by filling degenerate molecular orbitals singly before pairing, according to Hund's rule. The total electron count alone does not equal the number of unpaired electrons.
- Oxygen has two electrons in the degenerate antibonding π^* orbitals. Adding electrons pairs them successively; the closed-shell CN^- ion is isoelectronic with N_2.

Step 1: Count electrons in the oxygen species For O_2, the two degenerate antibonding π^* orbitals each contain one electron, so the molecule has two unpaired electrons and must not be counted. The extra electron in superoxide, $O_2{}^-$, pairs in one of those orbitals; the other orbital retains a single electron. Superoxide therefore has exactly one unpaired electron and is counted. In peroxide, $O_2{}^{2-}$, both antibonding orbitals are doubly occupied, so there are no unpaired electrons.

Step 2: Check NO and the cyanide ion Neutral nitric oxide, NO, has $7 + 8 = 15$ total electrons, or eleven valence electrons. Its highest occupied antibonding molecular orbital contains one electron,

giving exactly one unpaired electron. It is counted. The cyanide ion has $6+7+1=14$ total electrons and a closed-shell molecular-orbital configuration isoelectronic with nitrogen. All its electrons are paired, so it is not counted. The eligible species are therefore only O_2^- and NO, giving

$$N = 1 + 1 = 2.$$

Step 3: Conclusion Exactly O_2^- and NO have one unpaired electron. The required numerical answer is $\boxed{2}$.

Quick Tip to Solve: Distinguish exactly one unpaired electron from merely paramagnetic: oxygen is paramagnetic but has two unpaired electrons, so it is excluded here.

90. Consider the following reaction

$$MnO_2 + KOH + O_2 \rightarrow A + H_2O.$$

Product 'A' in neutral or acidic medium disproportionate to give products 'B' and 'C' along with water. The sum of spin-only magnetic moment values of B and C is _____ BM. (nearest integer) (Given atomic number of Mn is 25) [4 marks]

Solution:

Concept:

- Fusion of manganese dioxide with potassium hydroxide in oxygen produces potassium manganate, K_2MnO_4, containing manganese in oxidation state $+6$.
- Manganate disproportionates into permanganate and manganese dioxide. The spin-only moment for n unpaired electrons is $\mu = \sqrt{n(n+2)}\,\text{BM}$.

Step 1: Identify A and its disproportionation products The first balanced reaction is

$$2\,MnO_2 + 4\,KOH + O_2 \longrightarrow 2\,K_2MnO_4 + 2\,H_2O.$$

Thus A is potassium manganate. In acidic medium its ionic disproportionation is

$$3\,{MnO_4}^{2-} + 4\,H^+ \longrightarrow 2\,{MnO_4}^- + MnO_2 + 2\,H_2O.$$

Manganese in oxidation state $+6$ is oxidised to $+7$ in permanganate and reduced to $+4$ in manganese dioxide. B and C are therefore the permanganate and manganese dioxide products, in either order.

Step 2: Calculate and add the spin-only moments Neutral manganese has configuration $[Ar]3d^5 4s^2$. In permanganate, manganese is $+7$, so its metal-ion configuration is d^0, with no unpaired electrons and zero spin-only moment. In MnO_2, manganese is $+4$, giving d^3 and three unpaired electrons. Consequently,

$$\mu(MnO_2) = \sqrt{3(3+2)} = \sqrt{15}\,\text{BM}.$$

The sum is

$$\mu_B + \mu_C = 0 + \sqrt{15} = 3.873\,\text{BM}.$$

Rounding is performed only after adding the two moments.

Step 3: Conclusion The sum of the spin-only moments is approximately $3.873\,\text{BM}$, which gives $\boxed{4}$ to the nearest integer.

Quick Tip to Solve: First identify oxidation states after disproportionation; the original manganate is d^1, but the requested moments belong to its products.