JEE Main 2025 Paper 1 (B.E./B.Tech) Jan 22 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 22 January 2025, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The area of the region, inside the circle $(x-2\sqrt{3})^2+y^2=12$ and outside the parabola $y^2=2\sqrt{3}x$ is : [4 marks]

(A) $3\pi-8$

(B) $6\pi-8$

(C) $3\pi+8$

(D) $6\pi-16$

Correct Answer: (D) $6\pi-16$

Solution:

Concept:

- The circle has centre $(2\sqrt{3},0)$ and radius $a=2\sqrt{3}$. Horizontal strips express its left boundary as $x=a-\sqrt{a^2-y^2}$.
- For the right-opening parabola $y^2=ax$, its interior is $x\geq y^2/a$. Thus the part outside it lies to the left of $x=y^2/a$. Symmetry about the x-axis lets us double the upper-half area.

Step 1: Find the boundary intersections Substitute $y^2=2\sqrt{3}x$ in the circle equation. This gives

$$x^2-4\sqrt{3}x+2\sqrt{3}x=0, \qquad x(x-2\sqrt{3})=0.$$

The intersections are $(0,0)$ and $(2\sqrt{3},\pm 2\sqrt{3})$. Hence the relevant vertical range is $-a\leq y\leq a$.

Step 2: Integrate the horizontal width At each height, the desired strip starts at the left semicircle and ends at the parabola. Its width is nonnegative throughout this range. Therefore

$$\begin{aligned} A &= 2\int_0^a \left(\frac{y^2}{a}-a+\sqrt{a^2-y^2}\right)dy \\ &= 2\left(\frac{a^2}{3}-a^2+\frac{\pi a^2}{4}\right). \end{aligned}$$

The last integral is the area of a quarter-circle of radius a.

Step 3: Simplify and compare the choices Using $a^2=12$, we obtain

$$A=8-24+6\pi=6\pi-16.$$

This positive area is smaller than the left semicircle area 6π, as expected. Option B subtracts only 8 instead of 16; options A and C have the incorrect circular-area coefficient 3π.

Step 4: Conclusion The required area is $\boxed{6\pi-16}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: Use horizontal strips for a sideways parabola: write both boundaries as functions of y, then subtract right minus left.

2. Let the foci of a hyperbola be $(1, 14)$ and $(1, -12)$. If it passes through the point $(1, 6)$, then the length of its latus-rectum is : [4 marks]

(A) $\frac{25}{6}$
(B) $\frac{144}{5}$
(C) $\frac{288}{5}$
(D) $\frac{24}{5}$

Correct Answer: (C) $\frac{288}{5}$

Solution:

Concept:

- The centre of a hyperbola is the midpoint of its foci. If the foci lie on a vertical line, its standard equation is $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$.
- The focal distance satisfies $c^2 = a^2 + b^2$, and the full length of the latus rectum is $2b^2/a$. Here a, b, and c are positive lengths.

Step 1: Locate the centre and determine the transverse axis The midpoint of $(1, 14)$ and $(1, -12)$ is $(1, 1)$. Each focus is 13 units from this centre, so $c = 13$. The transverse axis is vertical because both foci have $x = 1$. Thus the equation is

$$\frac{(y-1)^2}{a^2} - \frac{(x-1)^2}{b^2} = 1.$$

Step 2: Use the given point to find the parameters The point $(1, 6)$ has zero horizontal displacement from the centre. Substituting it gives

$$\frac{(6-1)^2}{a^2} = 1, \qquad a^2 = 25, \qquad a = 5.$$

It is the upper vertex, rather than a focus. Now use the hyperbola identity:

$$b^2 = c^2 - a^2 = 169 - 25 = 144.$$

The plus sign in $c^2 = a^2 + b^2$ is essential; the ellipse identity does not apply.

Step 3: Compute the full latus rectum A latus rectum passes through a focus and is perpendicular to the transverse axis. Its full length is twice its semi-length:

$$L = \frac{2b^2}{a} = \frac{2(144)}{5} = \frac{288}{5}.$$

Option B is only the semi-latus rectum b^2/a; options A and D do not use the calculated parameters in the full-length formula.

Step 4: Conclusion The length of the latus rectum is $\boxed{\frac{288}{5}}$ units. Hence, the correct option is (C).

Quick Tip to Solve: A point on the transverse axis of a hyperbola must be a vertex if it lies on the curve; its distance from the centre immediately gives a.

3. Let z_1, z_2 and z_3 be three complex numbers on the circle $|z| = 1$ with $\arg(z_1) = \frac{-\pi}{4}, \arg(z_2) = 0$ and $\arg(z_3) = \frac{\pi}{4}$. If $|z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1|^2 = \alpha + \beta\sqrt{2}, \alpha, \beta \in Z$, then the value of $\alpha^2 + \beta^2$ is : [4 marks]

(A) 41

(B) 29

(C) 24

(D) 31

Correct Answer: (B) 29

Solution:

Concept:

- Every complex number on $|z| = 1$ can be written as $z = e^{i\theta}$, and its conjugate is $\bar{z} = e^{-i\theta}$. Products with conjugates therefore subtract arguments.
- For a complex number $u + iv$, its squared modulus is $u^2 + v^2$, not the square of its real part alone. This converts the final calculation to real algebra.

Step 1: Write the three unit complex numbers explicitly The given arguments give

$$z_1 = \frac{1-i}{\sqrt{2}}, \qquad z_2 = 1, \qquad z_3 = \frac{1+i}{\sqrt{2}}.$$

In particular $\bar{z}_1 = z_3$, while $\bar{z}_3 = z_1$. Their moduli are all one, so there are no additional radial factors in any product.

Step 2: Compute the cyclic sum Let $S = z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1$. The first two terms both equal $(1-i)/\sqrt{2}$. The last term is $z_3^2 = e^{i\pi/2} = i$. Consequently

$$S = 2\frac{1-i}{\sqrt{2}} + i = \sqrt{2} + i(1-\sqrt{2}).$$

Keeping the conjugate signs correct is what produces the negative coefficient of $\sqrt{2}$ below.

Step 3: Find the integer coefficients and the requested value Take the squared modulus:

$$\begin{aligned} |S|^2 &= (\sqrt{2})^2 + (1-\sqrt{2})^2 \\ &= 2 + 1 - 2\sqrt{2} + 2 = 5 - 2\sqrt{2}. \end{aligned}$$

Since $\sqrt{2}$ is irrational, comparison with $\alpha + \beta\sqrt{2}$ uniquely gives $\alpha = 5$, $\beta = -2$. Thus

$$\alpha^2 + \beta^2 = 25 + 4 = 29.$$

The alternatives 41, 24, and 31 all disagree with this direct modulus computation; the sign of β disappears only in the final square.

Step 4: Conclusion We have $\alpha = 5$, $\beta = -2$, and $\boxed{\alpha^2 + \beta^2 = 29}$. Hence, the correct option is (B).

Quick Tip to Solve: For unit complex numbers, turn $z_j\bar{z}_k$ into an argument difference before expanding; conjugation reverses the angle.

4. Let $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ be two lines. Then which of the following points lies on the line of the shortest distance between L_1 and L_2 ? [4 marks]

(A) $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$
(B) $\left(2, 3, \frac{1}{3}\right)$
(C) $\left(\frac{8}{3}, -1, \frac{1}{3}\right)$
(D) $\left(-\frac{5}{3}, -7, 1\right)$

Correct Answer: (A) $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$

Solution:

Concept:

- The line of shortest distance between two skew lines is their common perpendicular. Its direction is perpendicular to both given direction vectors, so their cross product supplies it.
- If P and Q are its feet on the two lines with directions $\mathbf{u}$ and $\mathbf{v}$, then $(Q - P) \cdot \mathbf{u} = 0$ and $(Q - P) \cdot \mathbf{v} = 0$.

Step 1: Parameterise the two feet Choose

$$P = (1 + 2t, 2 + 3t, 3 + 4t), \qquad Q = (2 + 3s, 4 + 4s, 5 + 5s).$$

The respective directions are $\mathbf{u} = (2, 3, 4)$ and $\mathbf{v} = (3, 4, 5)$. Hence

$$Q - P = (1 + 3s - 2t, 2 + 4s - 3t, 2 + 5s - 4t).$$

Step 2: Impose perpendicularity to both lines Taking the two dot products gives

$$29t - 38s = 16, \qquad 38t - 50s = 21.$$

Eliminating t yields $-6s = 1$, so $s = -1/6$ and then $t = 1/3$. Thus the first foot is

$$P = \left(\frac{5}{3}, 3, \frac{13}{3}\right).$$

The cross product is $\mathbf{u} \times \mathbf{v} = (-1, 2, -1)$; the opposite vector is an equally valid direction.

Step 3: Test the proposed points on the common perpendicular The required line is

$$(x, y, z) = \left(\frac{5}{3}, 3, \frac{13}{3}\right) + \lambda(1, -2, 1).$$

For $\lambda = 3$, it gives $(14/3, -3, 22/3)$, exactly option A. Every point on this line satisfies both

$$x - z = -\frac{8}{3}, \qquad y + 2x = \frac{19}{3}.$$

Options B and C fail the first relation; option D satisfies the first but fails the second, so none of them lies on this perpendicular.

Step 4: Conclusion The point is $\boxed{(14/3, -3, 22/3)}$. Hence, the correct option is (A).

Quick Tip to Solve: After finding one foot and the cross-product direction, use two linear invariants of the common perpendicular to test options quickly.

5. Let $A = \{1, 2, 3, \ldots, 10\}$ and $B = \left\{\frac{m}{n} : m, n \in A, m < n \text{ and } \gcd(m, n) = 1\right\}$. Then $n(B)$ is equal to : [4 marks]

(A) 29

(B) 31

(C) 37

(D) 36

Correct Answer: (B) 31

Solution:

Concept:

- The condition $\gcd(m, n) = 1$ means every listed fraction is in lowest terms. A positive reduced fraction has a unique numerator-denominator representation, so fractions counted for different denominators cannot coincide.
- For fixed $n > 1$, the number of positive integers $m < n$ coprime to n is Euler's totient $\varphi(n)$. We can also list them directly because the denominators are at most 10.

Step 1: Organise the fractions by denominator The denominator $n = 1$ contributes nothing because no positive m in A is less than 1. For $n = 2, 3, 4, 5$, the allowed numerators are respectively

$$\{1\}, \quad \{1, 2\}, \quad \{1, 3\}, \quad \{1, 2, 3, 4\}.$$

These contribute $1, 2, 2, 4$ distinct fractions. Numerators such as 2 for denominator 4 are excluded rather than counted again as $1/2$.

Step 2: Count the remaining coprime numerators For the other denominators the eligible numerator sets are

$$\begin{aligned} n = 6 &: \{1, 5\}, \\ n = 7 &: \{1, 2, 3, 4, 5, 6\}, \\ n = 8 &: \{1, 3, 5, 7\}, \\ n = 9 &: \{1, 2, 4, 5, 7, 8\}, \\ n = 10 &: \{1, 3, 7, 9\}. \end{aligned}$$

They contribute $2, 6, 4, 6, 4$, respectively. These lists exclude every numerator sharing a prime factor with its denominator.

Step 3: Add the distinct contributions All fractions have been counted once because each is in lowest terms. Therefore

$$n(B) = 1 + 2 + 2 + 4 + 2 + 6 + 4 + 6 + 4 = 31.$$

Neither 0 nor 1 is present: m is positive and strictly smaller than n. Counting all 45 ordered pairs with $m < n$ would ignore the coprimality condition. The other offered totals 29, 37, and 36 disagree with the explicit denominator-by-denominator count.

Step 4: Conclusion The set contains $\boxed{31}$ distinct fractions. Hence, the correct option is (B).

Quick Tip to Solve: When fractions are required to be reduced, sum the coprime-numerator counts for each denominator; uniqueness of lowest terms prevents duplicates.

6. Let the triangle PQR be the image of the triangle with vertices $(1,3),(3,1)$ and $(2,4)$ in the line $x+2y=2$. If the centroid of $\triangle$PQR is the point (α,β), then $15(\alpha-\beta)$ is equal to : [4 marks]

(A) 21

(B) 19

(C) 22

(D) 24

Correct Answer: (C) 22

Solution:

Concept:

- The centroid of a triangle is the arithmetic mean of its three vertices. Reflection is an affine transformation, so reflecting the centroid gives the centroid of the reflected triangle.
- The reflection of (h,k) in $ax+by+c=0$ is $(h-2ad, k-2bd)$, where $d=(ah+bk+c)/(a^2+b^2)$. This formula changes only the component perpendicular to the mirror.

Step 1: Find the original centroid Average the given coordinates:

$$G=\left(\frac{1+3+2}{3},\frac{3+1+4}{3}\right)=\left(2,\frac{8}{3}\right).$$

Because the same reflection is applied to all three vertices, their mean transforms in exactly the same way. Thus only one point needs to be reflected, rather than all three vertices separately.

Step 2: Reflect the centroid in the given line Write the mirror as $x+2y-2=0$, so $a=1,b=2,c=-2$. The signed factor is

$$d=\frac{2+2(8/3)-2}{1^2+2^2}=\frac{16}{15}.$$

Consequently

$$\alpha=2-2\left(\frac{16}{15}\right)=-\frac{2}{15},$$
$$\beta=\frac{8}{3}-4\left(\frac{16}{15}\right)=-\frac{8}{5}.$$

As a check, the midpoint of G and its reflected point lies on the mirror, and their joining segment is parallel to the normal vector $(1,2)$.

Step 3: Evaluate the requested expression Subtract using the common denominator 15:

$$\alpha-\beta=-\frac{2}{15}+\frac{24}{15}=\frac{22}{15}.$$

Multiplication by 15 gives 22. Options A, B, and D would require the difference to be $21/15$, $19/15$, and $24/15$, respectively; all conflict with the reflected centroid calculated above. The negative coordinates are consistent with reflecting across the specified line.

Step 4: Conclusion The centroid is $(-2/15,-8/5)$, and $\boxed{15(\alpha-\beta)=22}$. Hence, the correct option is (C).

Quick Tip to Solve: Reflect the original centroid directly: every affine map, including a reflection, commutes with taking the mean of the vertices.

7. The product of all solutions of the equation $e^{5(\log_e x)^2+3} = x^8, x > 0$, is : [4 marks]

(A) e^2

(B) e

(C) $e^{6/5}$

(D) $e^{8/5}$

Correct Answer: (D) $e^{8/5}$

Solution:

Concept:

- Taking natural logarithms is reversible for positive quantities. Here $x > 0$, so write $t = \log_e x$, and hence $x = e^t$.
- For a quadratic $at^2 + bt + c = 0$, the sum of its roots is $-b/a$. The product of the corresponding exponential values is $e^{t_1+t_2}$, not $e^{t_1 t_2}$.

Step 1: Convert the equation to a quadratic The left-hand side is an exponential and the right-hand side is positive because $x > 0$. Apply the natural logarithm to both sides without introducing any extraneous values:

$$5(\log_e x)^2 + 3 = 8 \log_e x.$$

Putting $t = \log_e x$ gives

$$5t^2 - 8t + 3 = 0.$$

The substitution maps all real values of t to positive values of x, so every real root of this quadratic supplies an allowed solution.

Step 2: Find the positive solutions and multiply Factor the quadratic to obtain

$$(5t - 3)(t - 1) = 0.$$

Thus $t = 3/5$ or $t = 1$, giving the two distinct positive solutions $x = e^{3/5}$ and $x = e$. Their product is

$$e^{3/5} e = e^{3/5+1} = e^{8/5}.$$

As a check, the sum of the logarithmic roots is $8/5$ by the quadratic coefficient formula. Option (A) would use two copies of the root 1, option (B) keeps only one solution, and option (C) doubles the other root. None includes both distinct solutions correctly.

Step 3: Conclusion The product of all positive solutions is $\boxed{e^{8/5}}$. Hence, the correct option is (D).

Quick Tip to Solve: For products of positive solutions, substitute their logarithms and use the sum of the new roots; this avoids unnecessary exponentiation.

8. From all the English alphabets, five letters are chosen and are arranged in alphabetical order. The total number of ways, in which the middle letter is ' M ', is : [4 marks]

(A) 6084

(B) 5148

(C) 14950

(D) 4356

Correct Answer: (B) 5148

Solution:

Concept:

- An alphabetical arrangement of five distinct chosen letters is uniquely determined by the chosen set. There is no extra permutation factor.
- For M to be the middle (third) letter, exactly two selected letters must precede M and exactly two must follow M in the alphabet. Independent choices multiply.

Step 1: Count the letters on either side of M The letters before M are A through L, giving 12 available letters. The letters after M are N through Z, giving 13 available letters. M itself must be selected and its position is fixed as the third letter. To place it in the middle of the increasing list, we must select two letters from the first group and two from the second group. Selecting a different number from either group would move M away from the third position.

Step 2: Choose the two groups independently The left pair can be chosen in

$$\binom{12}{2} = \frac{12 \cdot 11}{2} = 66$$

ways. Independently, the right pair can be chosen in

$$\binom{13}{2} = \frac{13 \cdot 12}{2} = 78$$

ways. Once these four letters and M have been selected, alphabetical order fixes the entire arrangement. Therefore

$$N = \binom{12}{2}\binom{13}{2} = 66 \cdot 78 = 5148.$$

There is no factor $2!$ for either pair and no factor $5!$ for the whole selection. Option (A), 6084, equals 78^2 and incorrectly uses 13 letters on both sides; option (D), 4356, equals 66^2 and uses 12 on both sides. Option (C) does not satisfy the required two-sided choice count.

Step 3: Conclusion The required number of arrangements is $\boxed{5148}$. Hence, the correct option is (B).

Quick Tip to Solve: Locate the required middle letter first; choose equal numbers before and after it, and let the specified alphabetical order arrange them.

9. A circle C of radius 2 lies in the second quadrant and touches both the coordinate axes. Let r be the radius of a circle that has centre at the point $(2, 5)$ and intersects the circle C at exactly two points. If the set of all possible values of r is the interval (α, β), then $3\beta - 2\alpha$ is equal to : [4 marks]

(A) 10

(B) 12

(C) 14

(D) 15

Correct Answer: (D) 15

Solution:

Concept:

- A circle of radius R tangent to both coordinate axes has its centre at distance R from each axis. The signs of the coordinates are fixed by its quadrant.
- Two circles with centre separation d and radii R, r intersect at exactly two distinct points when $|R - r| < d < R + r$. Equality at either boundary gives tangency, which is excluded.

Step 1: Locate the centres and find their separation Since C lies in the second quadrant and touches both axes, its centre is $O = (-2, 2)$, with radius $R = 2$. The other centre is $P = (2, 5)$. Applying the distance formula gives

$$d = OP = \sqrt{(2+2)^2 + (5-2)^2} = \sqrt{16+9} = 5.$$

The quadrant condition is essential: using $(2, 2)$ would place the first circle on the wrong side of the vertical axis.

Step 2: Impose both strict intersection inequalities External separation must be smaller than the sum of the radii, so $5 < 2 + r$, giving $r > 3$. To prevent one circle from lying wholly inside the other, we also require

$$|r - 2| < 5 \quad \Longrightarrow \quad -5 < r - 2 < 5 \quad \Longrightarrow \quad -3 < r < 7.$$

Combining these requirements and the positivity of a radius gives $3 < r < 7$. At $r = 3$ the circles touch externally, while at $r = 7$ they touch internally; neither endpoint permits two intersection points.

Step 3: Identify the interval endpoints and evaluate the expression Comparing $(3, 7)$ with (α, β), we obtain $\alpha = 3$ and $\beta = 7$. Therefore

$$3\beta - 2\alpha = 3(7) - 2(3) = 21 - 6 = 15.$$

The alternatives 10, 12 and 14 cannot equal this expression for the required endpoints. In particular, 12 is not obtained from the strict two-circle intersection conditions.

Step 4: Conclusion The allowed radii form $(3, 7)$, and hence $3\beta - 2\alpha = 15$. The associated key indicates 12, but the printed centres and radius give $d = 5$, $\alpha = 3$, $\beta = 7$, proving 15 instead. Hence, the correct option is (D).

Quick Tip to Solve: For two distinct circle intersections, use both strict inequalities: the centre distance lies between the difference and the sum of the radii.

10. Let $x = x(y)$ be the solution of the differential equation $y^2\,\mathrm{d}x + \left(x - \frac{1}{y}\right)\mathrm{d}y = 0$. If $x(1) = 1$, then $x\left(\frac{1}{2}\right)$ is : [4 marks]

(A) $\frac{3}{2} + \mathrm{e}$

(B) $\frac{1}{2} + \mathrm{e}$

(C) $3 + e$

(D) $3 - e$

Correct Answer: (D) $3 - e$

Solution:

Concept:

- A linear equation $dx/dy + P(y)x = Q(y)$ has integrating factor $\exp(\int P(y)\,dy)$. Multiplying through turns its left-hand side into a product derivative.
- The initial condition at $y = 1$ determines the constant on the positive interval containing both 1 and $1/2$.

Step 1: Put the equation in linear form Dividing the given equation by dy, and then by y^2, gives

$$\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}.$$

Here $P(y) = y^{-2}$, so the integrating factor is

$$\exp\left(\int y^{-2}\,dy\right) = e^{-1/y}.$$

Consequently,

$$\frac{d}{dy}\left(xe^{-1/y}\right) = y^{-3}e^{-1/y}.$$

The sign in the exponent is negative, since the integral of y^{-2} is $-y^{-1}$.

Step 2: Integrate and impose the initial condition Put $t = 1/y$, so $dt = -y^{-2}dy$. Then

$$\int y^{-3}e^{-1/y}dy = -\int te^{-t}dt = (t+1)e^{-t} + C.$$

It follows that

$$xe^{-1/y} = (1 + 1/y)e^{-1/y} + C, \qquad x = 1 + \frac{1}{y} + Ce^{1/y}.$$

At $y = 1$, the condition $x = 1$ yields $1 = 2 + Ce$, or $C = -e^{-1}$. Thus

$$x(y) = 1 + \frac{1}{y} - e^{1/y-1}.$$

This solution also satisfies the differential equation: its derivative is $-y^{-2} + e^{1/y-1}y^{-2}$, which cancels the terms from x/y^2.

Step 3: Evaluate the requested value Substitute $y = 1/2$, so $1/y = 2$:

$$x(1/2) = 1 + 2 - e^{2-1} = 3 - e.$$

The initial condition forces a negative exponential term, ruling out (A), (B), and (C), each of which adds e. The constant term is 3, not $3/2$ or $1/2$.

Step 4: Conclusion $\boxed{x(1/2) = 3 - e}$. Hence, the correct option is (D).

Quick Tip to Solve: After finding an integrating factor, differentiate the proposed product once to check the exponent sign before using the initial condition.

11. A coin is tossed three times. Let X denote the number of times a tail follows a head. If μ and σ^2 denote the mean and variance of X, then the value of $64\left(\mu + \sigma^2\right)$ is: [4 marks]

(A) 64

(B) 32

(C) 51

(D) 48

Correct Answer: (D) 48

Solution:

Concept:

- For three independent tosses of a fair coin, the eight length-three sequences have equal probability $1/8$. A tail following a head means an adjacent HT pair.
- The mean and variance are $\mu = E[X]$ and $\sigma^2 = E[X^2] - (E[X])^2$. Check the possible values of the variable before computing its distribution.

Step 1: List the sequences with an HT transition The equally likely outcomes are HHH, HHT, HTH, HTT, THH, THT, TTH, and TTT. An adjacent HT occurs in HHT at positions 2–3, in HTH at positions 1–2, in HTT at positions 1–2, and in THT at positions 2–3. In each of these four sequences $X = 1$. The other four sequences contain no HT transition, so $X = 0$. Two HT transitions cannot occur in three tosses: the middle toss would need to be both T (for the first pair) and H (for the second pair).

Step 2: Calculate the mean and variance Thus the distribution is

$$P(X = 0) = \frac{4}{8} = \frac{1}{2}, \qquad P(X = 1) = \frac{4}{8} = \frac{1}{2}.$$

It follows that

$$\mu = 0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = \frac{1}{2}, \qquad E[X^2] = 0^2 \cdot \frac{1}{2} + 1^2 \cdot \frac{1}{2} = \frac{1}{2}.$$

Therefore

$$\sigma^2 = \frac{1}{2} - \left(\frac{1}{2}\right)^2 = \frac{1}{4}.$$

Although two adjacent positions are available, their HT events are not independent, so a binomial model with two independent trials would give a wrong variance.

Step 3: Evaluate the expression and compare options The required value is

$$64(\mu + \sigma^2) = 64\left(\frac{1}{2} + \frac{1}{4}\right) = 64 \cdot \frac{3}{4} = 48.$$

Option (B), 32, accounts for the mean alone and omits the variance. Options (A), 64, and (C), 51, disagree with the explicitly enumerated two-point distribution. Enumeration gives both moments exactly without rounding.

Step 4: Conclusion The required value is $\boxed{48}$. Hence, the correct option is (D).

Quick Tip to Solve: For a short coin sequence, list adjacent patterns explicitly; overlapping pairs usually prevent treating pattern counts as independent binomial trials.

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12. Let $f(x)$ be a real differentiable function such that $f(0) = 1$ and $f(x + y) = f(x)f'(y) + f'(x)f(y)$ for all $x, y \in \mathbf{R}$. Then $\sum_{n=1}^{100} \log_e f(n)$ is equal to: [4 marks]

(A) 2406

(B) 5220

(C) 2525

(D) 2384

Correct Answer: (C) 2525

Solution:

Concept:

- Substituting zero into a functional identity can reveal the derivative at zero and convert it into an ordinary differential equation.
- The equation $f'(x) = cf(x)$, together with $f(0) = 1$, gives $f(x) = e^{cx}$. The logarithm is then defined because this exponential is positive everywhere.

Step 1: Determine the derivative at zero Set $x = y = 0$ in the given identity. Since $f(0) = 1$, it becomes

$$1 = 1 \cdot f'(0) + f'(0) \cdot 1 = 2f'(0).$$

Thus $f'(0) = 1/2$. Next set $y = 0$, leaving x arbitrary:

$$f(x) = f(x)f'(0) + f'(x)f(0) = \frac{1}{2}f(x) + f'(x).$$

Hence $f'(x) = f(x)/2$ for every real x.

Step 2: Solve the resulting differential equation Multiply the differential equation by $e^{-x/2}$. The derivative of $e^{-x/2}f(x)$ is zero, so this expression is constant. The condition at zero fixes that constant as 1. Therefore

$$f(x) = e^{x/2}.$$

This argument does not divide by $f(x)$, so it does not presume positivity or discard possible zeroes.

The resulting solution is positive and satisfies the full identity, since

$$f(x)f'(y) + f'(x)f(y) = \tfrac{1}{2}e^{(x+y)/2} + \tfrac{1}{2}e^{(x+y)/2} = f(x+y).$$

Step 3: Sum the natural logarithms For every positive integer n, $\log_e f(n) = \log_e(e^{n/2}) = n/2$. Thus

$$\sum_{n=1}^{100} \log_e f(n) = \frac{1}{2}\sum_{n=1}^{100} n = \frac{1}{2} \cdot \frac{100 \cdot 101}{2} = 2525.$$

Options (A), 2406, (B), 5220, and (D), 2384, cannot follow from the exact arithmetic progression with first term $1/2$, last term 50, and 100 terms.

Step 4: Conclusion The sum is $\boxed{2525}$. Hence, the correct option is (C).

Quick Tip to Solve: In identities involving both a function and its derivative, put both variables equal to zero first, then set just one variable to zero.

13. Two balls are selected at random one by one without replacement from a bag containing 4 white and 6 black balls. If the probability that the first selected ball is black, given that the second selected ball is also black, is $\frac{m}{n}$, where $\gcd(m, n) = 1$, then $m + n$ is equal to : [4 marks]

(A) 4

(B) 14

(C) 11

(D) 13

Correct Answer: (B) 14

Solution:

Concept:

- Conditional probability restricts the sample space to outcomes satisfying the given event: $P(A \mid B) = P(A \cap B)/P(B)$, provided $P(B) > 0$.
- Without replacement, the first draw changes both the number of black balls and the total number of balls available for the second draw.

Step 1: Define the two events Let B_1 mean that the first ball is black and B_2 mean that the second ball is black. The required probability is $P(B_1 \mid B_2)$, rather than the probability of two black balls without any condition. Initially there are six black balls among ten balls. Thus the probability of a black first ball is $6/10$.

Step 2: Calculate the joint and marginal probabilities If the first ball is black, five black balls remain among nine balls. Therefore

$$P(B_1 \cap B_2) = \frac{6}{10}\frac{5}{9} = \frac{1}{3}.$$

For the second ball to be black, the first can be black or white. These are disjoint possibilities, so

$$P(B_2) = \frac{6}{10}\frac{5}{9} + \frac{4}{10}\frac{6}{9} = \frac{1}{3} + \frac{4}{15} = \frac{3}{5}.$$

This also agrees with symmetry: every ball is equally likely to occupy the second position.

Step 3: Apply the condition and reduce the fraction Divide the probability of the joint event by that of the conditioning event:

$$P(B_1 \mid B_2) = \frac{1/3}{3/5} = \frac{5}{9}.$$

The numerator and denominator have no common factor. Hence $m = 5$ and $n = 9$. The values 4, 11 and 13 cannot be the requested sum because the reduced fraction is fixed uniquely.

Step 4: Conclusion Thus $m + n = 5 + 9 = 14$. Hence, the correct option is (B).

Quick Tip to Solve: Once the second ball is known to be black, imagine reserving that ball: five of the remaining nine balls are black.

14. Let $a_1, a_2, a_3, \ldots$ be a G.P. of increasing positive terms. If $a_1a_5 = 28$ and $a_2 + a_4 = 29$, then a_6 is equal to: [4 marks]

(A) 812

(B) 784

(C) 628

(D) 526

Correct Answer: (B) 784

Solution:

Concept:

- For a geometric progression with first term $a > 0$ and common ratio $r > 1$, the general term is $a_n = ar^{n-1}$. The inequality $r > 1$ encodes the increasing positive terms.
- Terms equally spaced from a central term have equal products: here $a_2a_4 = a_1a_5$. A sum and product determine the two terms as roots of a quadratic.

Step 1: Find the product of the second and fourth terms Using the geometric progression formula gives

$$a_2a_4 = (ar)(ar^3) = a^2r^4 = a_1a_5 = 28.$$

We are also given $a_2 + a_4 = 29$. It is convenient to determine these two terms before attempting to calculate the first term or the ratio separately.

Step 2: Recover the two terms in increasing order If t denotes either a_2 or a_4, their sum and product imply

$$t^2 - 29t + 28 = 0,$$

$$(t - 1)(t - 28) = 0.$$

The roots are 1 and 28. Since the progression is increasing, the earlier term must be smaller, so $a_2 = 1$ and $a_4 = 28$. Reversing these assignments would produce a decreasing progression and violate the hypothesis.

Step 3: Move forward by the same two-term ratio The ratio of terms two places apart is r^2. Consequently

$$r^2 = \frac{a_4}{a_2} = 28, \qquad a_6 = a_4 r^2 = 28 \cdot 28 = 784.$$

As a check, $r = \sqrt{28} > 1$ and $a = 1/\sqrt{28} > 0$, which satisfy both original conditions. None of 812, 628 or 526 equals the uniquely determined sixth term.

Step 4: Conclusion The sixth term is $a_6 = 784$. Hence, the correct option is (B).

Quick Tip to Solve: Use the sum and product of equally spaced GP terms first; the ratio of every alternate pair is the same r^2.

15. Let for $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$, $\mathrm{I}_1 = \int_0^{\pi/4} f(x)\mathrm{d}x$ and $\mathrm{I}_2 = \int_0^{\pi/4} x f(x)\mathrm{d}x$. Then $7\mathrm{I}_1 + 12\mathrm{I}_2$ is equal to : [4 marks]

(A) 2π

(B) 1

(C) π

(D) 2

Correct Answer: (B) 1

Solution:

Concept:

- Group powers of $\tan x$ so that $1 + \tan^2 x = \sec^2 x$ appears. Then $t = \tan x$ converts $\sec^2 x\,\mathrm{d}x$ into $\mathrm{d}t$.
- If $G'(x) = f(x)$, integration by parts gives $\int x f(x)\,\mathrm{d}x = xG(x) - \int G(x)\,\mathrm{d}x$. The endpoint values of G can make the boundary term vanish.

Step 1: Recognise an exact derivative Group the given four terms as

$$f(x) = (7\tan^6 x - 3\tan^2 x)(1 + \tan^2 x) = (7\tan^6 x - 3\tan^2 x)\sec^2 x.$$

Consequently, a primitive is $G(x) = \tan^7 x - \tan^3 x$, since $G'(x) = f(x)$. At the endpoints, $G(0) = 0$ and $G(\pi/4) = 1 - 1 = 0$, so

$$I_1 = [G(x)]_0^{\pi/4} = 0.$$

Step 2: Remove the factor of x by integration by parts Take $u = x$ and $\mathrm{d}v = f(x)\,\mathrm{d}x$, so $v = G(x)$.

The boundary contribution vanishes at both limits, giving

$$I_2 = [xG(x)]_0^{\pi/4} - \int_0^{\pi/4} G(x)\,\mathrm{d}x$$
$$= \int_0^{\pi/4} (\tan^3 x - \tan^7 x)\,\mathrm{d}x.$$

Step 3: Substitute and evaluate the remaining polynomial Set $t = \tan x$. The limits become 0 and 1, and $\mathrm{d}x = \mathrm{d}t/(1+t^2)$. Since $1 - t^4 = (1-t^2)(1+t^2)$,

$$I_2 = \int_0^1 \frac{t^3 - t^7}{1+t^2}\,\mathrm{d}t = \int_0^1 (t^3 - t^5)\,\mathrm{d}t$$
$$= \left[\frac{t^4}{4} - \frac{t^6}{6}\right]_0^1 = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}.$$

Therefore $7I_1 + 12I_2 = 7(0) + 12(1/12) = 1$. The values 2π, π and 2 do not satisfy this exact evaluation.

Step 4: Conclusion $I_1 = 0$ and $I_2 = 1/12$, so $7I_1 + 12I_2 = 1$. The associated key gives 2, but the exact evaluation above proves 1. Hence, the correct option is (B).

Quick Tip to Solve: Factor adjacent even powers of tangent to reveal $\sec^2 x$; check the primitive at both endpoints before using integration by parts.

16. The number of non-empty equivalence relations on the set $\{1, 2, 3\}$ is : [4 marks]

(A) 7

(B) 4

(C) 5

(D) 6

Correct Answer: (C) 5

Solution:

Concept:

- An equivalence relation on a set is reflexive, symmetric and transitive. Its equivalence classes partition the set into nonempty, disjoint blocks covering every element.
- Conversely, every partition determines exactly one equivalence relation: two elements are related precisely when they belong to the same block. Thus counting equivalence relations is equivalent to counting partitions.

Step 1: Count partitions with one block All three elements can be equivalent to one another. This gives the single partition

$$\{\{1, 2, 3\}\}.$$

Its relation contains all nine ordered pairs of the Cartesian product. There is only one way to place all elements in one equivalence class.

Step 2: Count partitions with two blocks The block sizes must be 2 and 1. Choose the singleton element in three ways, giving

$$\{\{1\},\{2,3\}\}, \quad \{\{2\},\{1,3\}\}, \quad \{\{3\},\{1,2\}\}.$$

The blocks have no order, so swapping how they are written does not give a new partition. Each of these three partitions determines its own equivalence relation.

Step 3: Include the discrete partition and add the cases With three blocks, each element is alone, giving just

$$\{\{1\},\{2\},\{3\}\}.$$

This corresponds to the identity relation $\{(1,1),(2,2),(3,3)\}$. Therefore the total is

$$1+3+1=5.$$

Every equivalence relation on this nonempty set already contains all three diagonal pairs by reflexivity, so the word non-empty does not remove any of these five cases. The counts 7, 4 and 6 respectively overcount or omit legitimate partitions.

Step 4: Conclusion There are 5 non-empty equivalence relations. Hence, the correct option is (C).

Quick Tip to Solve: Count equivalence classes by block sizes: for three elements the only patterns are 3, $2+1$ and $1+1+1$.

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17. Let the parabola $y=x^2+\mathrm{p}x-3$, meet the coordinate axes at the points P,Q and R . If the circle C with centre at $(-1,-1)$ passes through the points P,Q and R, then the area of $\triangle PQR$ is : [4 marks]

(A) 4

(B) 6

(C) 5

(D) 7

Correct Answer: (B) 6

Solution:

Concept:

- Intercepts with the coordinate axes are found by setting $x=0$ or $y=0$. A circle consists of points at a fixed distance from its centre.
- When a triangle has two vertices on the x-axis, their separation is a convenient base; the altitude is the absolute y-coordinate of the third vertex. Its area is half the product of the base and altitude.

Step 1: Use the fixed y-intercept to determine the circle Setting $x=0$ in the parabola gives the y-intercept $(0,-3)$, independently of the unknown coefficient p. Since this point lies on the circle with centre $(-1,-1)$, its squared radius is

$$R^2=(0+1)^2+(-3+1)^2=1+4=5.$$

Thus the circle has equation $(x+1)^2+(y+1)^2=5$.

Step 2: Find the two intercepts on the x-axis The other two common points lie on the x-axis. Substituting $y=0$ into the circle gives

$$(x+1)^2+1=5, \qquad (x+1)^2=4.$$

Hence $x=1$ or $x=-3$. The vertices are therefore $(1,0),(-3,0),(0,-3)$, in some order. These roots have product -3, as required by the parabola, and sum -2; thus $p=2$, confirming consistency with its equation.

Step 3: Compute the triangle area The horizontal side has length $|1-(-3)|=4$. The perpendicular distance of $(0,-3)$ from the x-axis is 3. Therefore

$$\text{Area}=\frac{1}{2}\times 4\times 3=6.$$

The altitude is a distance and so is positive, even though the vertex is below the axis. The values 4, 5 and 7 do not equal half the base-altitude product.

Step 4: Conclusion The area of $\triangle PQR$ is 6 square units. Hence, the correct option is (B).

Quick Tip to Solve: The parabola gives the y-intercept immediately; use it to fix the circle before solving for the unknown parabola coefficient.

18. Using the principal values of the inverse trigonometric functions, the sum of the maximum and the minimum values of $16\left(\left(\sec^{-1}x\right)^2+\left(\operatorname{cosec}^{-1}x\right)^2\right)$ is : [4 marks]

(A) $24\pi^2$

(B) $18\pi^2$

(C) $22\pi^2$

(D) $31\pi^2$

Correct Answer: (C) $22\pi^2$

Solution:

Concept:

- For real $|x|\geq 1$, let $t=\sec^{-1}x=\cos^{-1}(1/x)$. Its principal range is $[0,\pi]$ excluding $\pi/2$.
- The identity $\sin^{-1}u+\cos^{-1}u=\pi/2$, valid for $-1\leq u\leq 1$, gives $\operatorname{cosec}^{-1}x=\pi/2-t$. The expression can then be optimized as a quadratic in a single variable.

Step 1: Express the objective using one principal angle Both inverse functions are defined for $|x|\geq 1$. With $t=\cos^{-1}(1/x)$, the given expression becomes

$$F(t)=16\left[t^2+\left(\frac{\pi}{2}-t\right)^2\right]=32\left(t-\frac{\pi}{4}\right)^2+2\pi^2.$$

The domain is $0\leq t\leq\pi$, with $t\neq\pi/2$. Negative values of x give angles above $\pi/2$, and must not be omitted.

Step 2: Determine the minimum and its attainability The squared term is nonnegative and vanishes at $t = \pi/4$, which belongs to the allowed domain. It corresponds to $x = \sqrt{2}$, so the minimum is attained:

$$F_{\min} = 2\pi^2.$$

This is an actual minimum, rather than merely an infimum at an excluded angle.

Step 3: Determine the maximum and add the extremes The allowed angle farthest from $\pi/4$ is $t = \pi$, corresponding to $x = -1$. Therefore

$$F_{\max} = 32\left(\frac{3\pi}{4}\right)^2 + 2\pi^2 = 20\pi^2.$$

For comparison, $t = 0$ gives only $4\pi^2$. Removing $t = \pi/2$ does not affect either extremum. Hence their sum is $20\pi^2 + 2\pi^2 = 22\pi^2$, ruling out $24\pi^2, 18\pi^2$ and $31\pi^2$.

Step 4: Conclusion The sum of the maximum and minimum is $22\pi^2$. Hence, the correct option is (C).

Quick Tip to Solve: Replace inverse secant by $\cos^{-1}(1/x)$; retaining the negative-x branch is essential when finding the maximum.

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19. Let $f : \mathbf{R} \to \mathbf{R}$ be a twice differentiable function such that $f(x+y) = f(x)f(y)$ for all $x, y \in \mathbf{R}$. If $f'(0) = 4\mathrm{a}$ and f satisfies $f''(x) - 3\mathrm{a}f'(x) - f(x) = 0, \mathrm{a} > 0$, then the area of the region $\mathrm{R} = \{(x, y) \mid 0 \le y \le f(ax), 0 \le x \le 2\}$ is : [4 marks]

(A) $\mathrm{e}^2 - 1$

(B) $e^4 + 1$

(C) $\mathrm{e}^2 + 1$

(D) $e^4 - 1$

Correct Answer: (A) $\mathrm{e}^2 - 1$

Solution:

Concept:

- The identity $f(x+y) = f(x)f(y)$, differentiated with respect to y at zero, gives a first-order differential equation for f.
- The area below a positive curve $y = g(x)$ on $[0, 2]$ is $\int_0^2 g(x)\,dx$.

Step 1: Determine the initial value and derivative Setting $y = 0$ gives $f(x) = f(x)f(0)$. The function cannot be identically zero because $f\prime(0) = 4a > 0$; hence $f(0) = 1$. Differentiating the functional equation with respect to y and putting $y = 0$ now gives

$$f\prime(x) = f(x)f\prime(0) = 4af(x).$$

Differentiating again yields $f\prime\prime(x) = 16a^2 f(x)$.

Step 2: Find the positive parameter Substitute these derivatives into the given equation:

$$(16a^2 - 12a^2 - 1)f(x) = 0.$$

At $x = 0$, where $f(0) = 1$, this gives $4a^2 = 1$. Since $a > 0$, $a = 1/2$. Solving $f\prime = 2f$ with $f(0) = 1$ gives $f(x) = e^{2x}$.

Step 3: Integrate the specified curve The upper boundary is $f(ax) = f(x/2) = e^x$, which is positive on the whole interval. Therefore

$$A = \int_0^2 e^x \, dx = [e^x]_0^2 = e^2 - 1.$$

The expressions containing e^4 result from integrating $f(x)$ instead of $f(ax)$; adding rather than subtracting the lower endpoint would give the other incorrect sign.

Step 4: Conclusion The required area is $\boxed{e^2 - 1}$ square units. Hence, the correct option is (A).

Quick Tip to Solve: Find the parameter before substituting the scaled argument: here $f(x) = e^{2x}$, but $f(ax) = e^x$.

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20. If $\sum_{r=1}^{n} T_r = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$, then $\lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{1}{T_r}\right)$ is equal to : [4 marks]

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) 1

(D) 0

Correct Answer: No printed option matches: the literal partial sums give $26/35$, after treating T_1 correctly.

Solution:

Concept:

- For partial sums $S_n = \sum_{r=1}^{n} T_r$, the first term is $T_1 = S_1$, while $T_r = S_r - S_{r-1}$ applies for $r \geq 2$.
- A reciprocal cubic in consecutive odd factors can be written as the difference of two consecutive reciprocal quadratic factors.

Step 1: Keep the first term separate The printed formula gives

$$T_1 = S_1 = \frac{1 \cdot 3 \cdot 5 \cdot 7}{64} = \frac{105}{64}.$$

Its formal extension to $n = 0$ is $-15/64$, whereas an empty partial sum is zero. We must therefore not use that extension to calculate T_1. For $r \geq 2$, subtraction of the actual adjacent partial sums is valid and gives

$$T_r = \frac{(2r-1)(2r+1)(2r+3)}{8}.$$

Step 2: Sum the tail by cancellation For $r \geq 2$,

$$\frac{1}{T_r} = 2\left(\frac{1}{(2r-1)(2r+1)} - \frac{1}{(2r+1)(2r+3)}\right).$$

The intermediate terms cancel. Thus, for $n \geq 2$,

$$\sum_{r=2}^{n} \frac{1}{T_r} = \frac{2}{15} - \frac{2}{(2n+1)(2n+3)}.$$

Add $1/T_1 = 64/105$ and let n tend to infinity.

Step 3: Compare the exact result with the choices The requested limit, taking the printed partial-sum formula literally, is

$$\frac{64}{105} + \frac{2}{15} = \frac{78}{105} = \frac{26}{35}.$$

This is neither $2/3$, nor $1/3$, nor 1, nor 0. The value $2/3$ arises only if the cubic difference formula is incorrectly extended to $r = 1$, replacing the actual $T_1 = 105/64$ by $15/8$.

Step 4: Conclusion The literal printed formula yields $\boxed{26/35}$, so none of the four options is correct. Although the answer key selects $2/3$, that value neglects the required first-term correction proved above.

Quick Tip to Solve: Always check $T_1 = S_1$ before using $T_r = S_r - S_{r-1}$: a nonzero formal S_0 changes a telescoping sum.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let the function,

$$f(x) = \begin{cases} -3ax^2 - 2, & x < 1 \\ a^2 + bx, & x \geqslant 1 \end{cases}$$

be differentiable for all $x \in \mathbf{R}$, where $a > 1$, $b \in \mathbf{R}$. If the area of the region enclosed by $y = f(x)$ and the line $y = -20$ is $\alpha + \beta\sqrt{3}, \alpha, \beta \in Z$, then the value of $\alpha + \beta$ is _____ . [4 marks]

Solution:

Concept:

- Differentiability at a join requires continuity and equality of the left and right derivatives there.
- The bounded area is the integral of upper ordinate minus lower ordinate, split where the formula for the curve changes.

Step 1: Apply the joining conditions At $x = 1$, continuity gives $-3a - 2 = a^2 + b$. The derivative from the left is $-6a$, and the derivative from the right is b, so $b = -6a$. Substituting gives

$$a^2 - 3a + 2 = 0, \qquad (a-1)(a-2) = 0.$$

The condition $a > 1$ selects $a = 2$, and consequently $b = -12$.

Step 2: Find the endpoints of the bounded region The curve is now $f(x) = -6x^2 - 2$ for $x < 1$, and $f(x) = 4 - 12x$ for $x \geq 1$. On the first branch, intersection with $y = -20$ gives $x = \pm\sqrt{3}$, but only $-\sqrt{3} < 1$ belongs to that branch. On the second branch the intersection is $x = 2$. The curve lies above the horizontal line between these endpoints.

Step 3: Integrate the vertical separation Split the area at the joining point:

$$\begin{aligned} A &= \int_{-\sqrt{3}}^{1} (18 - 6x^2)\,dx + \int_{1}^{2} (24 - 12x)\,dx \\ &= [18x - 2x^3]_{-\sqrt{3}}^{1} + [24x - 6x^2]_{1}^{2} \\ &= 16 + 12\sqrt{3} + 6 = 22 + 12\sqrt{3}. \end{aligned}$$

Therefore $\alpha = 22$ and $\beta = 12$; the inadmissible intersection $+\sqrt{3}$ of the first formula must not be used as a region endpoint.

Step 4: Conclusion The area is $22 + 12\sqrt{3}$ square units, so $\boxed{\alpha + \beta = 34}$.

Quick Tip to Solve: Check each intersection against the domain of its own branch before splitting the area integral at the join.

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22. Let $\vec{c}$ be the projection vector of $\vec{b} = \lambda\hat{i} + 4\hat{k}, \lambda > 0$, on the vector $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$. If $|\vec{a} + \vec{c}| = 7$, then the area of the parallelogram formed by the vectors $\vec{b}$ and $\vec{c}$ is _____. [4 marks]

Solution:

Concept:

- The projection of $\vec{b}$ on a nonzero vector $\vec{a}$ is $\vec{c} = (\vec{b} \cdot \vec{a})\vec{a}/|\vec{a}|^2$.
- The area of a parallelogram with adjacent vectors $\vec{b}, \vec{c}$ is $|\vec{b} \times \vec{c}|$.

Step 1: Express the projection as a multiple of the given vector Here $\vec{a} = (1, 2, 2)$, so $|\vec{a}| = 3$, and $\vec{b} = (\lambda, 0, 4)$ gives $\vec{b} \cdot \vec{a} = \lambda + 8$. Thus

$$\vec{c} = \frac{\lambda + 8}{9}\vec{a}, \qquad \vec{a} + \vec{c} = \frac{\lambda + 17}{9}\vec{a}.$$

Because $\lambda > 0$, this coefficient is positive; no negative branch of the absolute value is possible.

Step 2: Use the prescribed length Taking magnitudes yields

$$7 = \frac{\lambda + 17}{9} \cdot 3 = \frac{\lambda + 17}{3}.$$

Hence $\lambda = 4$, $\vec{b} = (4, 0, 4)$, and $\vec{c} = (4/3)\vec{a}$. Notice that the projection is parallel to $\vec{a}$, but the parallelogram in the question is formed with $\vec{b}$, not with $\vec{a}$.

Step 3: Compute the cross-product magnitude Calculate

$$\vec{b} \times \vec{a} = (-8, -4, 8), \qquad |\vec{b} \times \vec{a}| = \sqrt{64 + 16 + 64} = 12.$$

Linearity in the second vector then gives

$$|\vec{b} \times \vec{c}| = \frac{4}{3}|\vec{b} \times \vec{a}| = \frac{4}{3} \cdot 12 = 16.$$

As a check, $|\vec{b}|^2 = 32$ and $|\vec{c}|^2 = 16$, so the component of $\vec{b}$ perpendicular to $\vec{c}$ also has length 4, giving area $4 \cdot 4$.

Step 4: Conclusion The area of the required parallelogram is $\boxed{16}$ square units.

Quick Tip to Solve: Write the projection as a scalar multiple of the direction vector; then pull that scalar out of the cross product.

23. If $\sum_{r=0}^{5} \frac{^{11}C_{2r+1}}{2r+2} = \frac{\mathrm{m}}{\mathrm{n}}, \gcd(\mathrm{m}, \mathrm{n}) = 1$, then $\mathrm{m} - \mathrm{n}$ is equal to _____. [4 marks]

Solution:

Concept:

- The identity $\binom{n}{k}/(k+1) = \binom{n+1}{k+1}/(n+1)$ follows directly from the factorial definition.
- The sum of the even-indexed binomial coefficients in row N is 2^{N-1}, obtained by adding the expansions of $(1+1)^N$ and $(1-1)^N$.

Step 1: Shift the binomial index Put $k = 2r + 1$. The summand becomes

$$\frac{\binom{11}{2r+1}}{2r+2} = \frac{1}{12}\binom{12}{2r+2}.$$

As r runs from 0 to 5, the lower index $2r + 2$ runs through $2, 4, 6, 8, 10, 12$. These are all the positive even indices of row 12, but do not include the zero index.

Step 2: Evaluate the even-index sum Adding the two binomial expansions gives

$$2 \sum_{\substack{j=0 \\ j \text{ even}}}^{12} \binom{12}{j} = 2^{12} + 0 = 4096.$$

Thus the even-index sum including $j = 0$ is 2048. Remove $\binom{12}{0} = 1$, which is absent from our shifted sum, to get

$$\sum_{r=0}^{5} \frac{\binom{11}{2r+1}}{2r+2} = \frac{2048 - 1}{12} = \frac{2047}{12}.$$

Step 3: Reduce the fraction and subtract The denominator factors as $12 = 2^2 \cdot 3$. The numerator 2047 is odd, and its digit sum is 13, so it is divisible by neither 2 nor 3. Hence the fraction is already in lowest terms and its positive coprime numerator and denominator are $m = 2047$, $n = 12$. We subtract

only after establishing this reduced form.

Step 4: Conclusion Therefore $\boxed{m - n = 2047 - 12 = 2035}$.

Quick Tip to Solve: After shifting a binomial coefficient index, list the first and last indices explicitly; here the missing zero-index term is crucial.

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24. Let $L_1 : \frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ **and** $L_2 : \frac{x-2}{2} = \frac{y}{0} = \frac{z+4}{\alpha}, \alpha \in$ **R, be two lines, which intersect at the point** B**. If** P **is the foot of perpendicular from the point** $A(1, 1, -1)$ **on** L_2**, then the value of** $26\alpha(\text{ PB})^2$ **is** _____ . [4 marks]

Solution:

Concept:

- A symmetric line equation with a zero direction component is interpreted parametrically: that coordinate is constant.
- The foot on a line $\vec{r} = \vec{C} + s\vec{d}$ from A has parameter $s_P = (\vec{A} - \vec{C}) \cdot \vec{d}/|\vec{d}|^2$.

Step 1: Locate the intersection and determine the parameter Write L_1 as $(x, y, z) = (1 + 3t, 1 - t, -1)$ and L_2 as $(x, y, z) = (2 + 2s, 0, -4 + \alpha s)$. At the intersection $y = 0$ forces $t = 1$, so $B = (4, 0, -1)$. Its x-coordinate on L_2 gives $s = 1$, and then $-4 + \alpha = -1$ yields $\alpha = 3$.

Step 2: Project the given point onto the second line Now take $C = (2, 0, -4)$ and $\vec{d} = (2, 0, 3)$ on L_2. Since $A - C = (-1, 1, 3)$,

$$(A - C) \cdot \vec{d} = -2 + 9 = 7, \qquad |\vec{d}|^2 = 13.$$

Orthogonality of $A - P$ to $\vec{d}$ gives the projection parameter $s_P = 7/13$. This determines the actual foot without confusing its distance from C with its distance from B.

Step 3: Use the parameter difference to obtain the requested distance The intersection B has parameter 1 on the same line. Thus

$$\overrightarrow{PB} = \left(1 - \frac{7}{13}\right)\vec{d} = \frac{6}{13}\vec{d}, \qquad PB^2 = \frac{36}{169} \cdot 13 = \frac{36}{13}.$$

Substitution into the expression requested in the question yields

$$26\alpha PB^2 = 26 \cdot 3 \cdot \frac{36}{13} = 216.$$

This calculation uses the distance along the line, whereas AP would be the perpendicular distance and would give a different quantity.

Step 4: Conclusion The required value is $\boxed{216}$, with $\alpha = 3$ and $PB^2 = 36/13$ square units.

Quick Tip to Solve: For two points on the same parametrised line, multiply the squared parameter difference by the squared direction-vector length.

---

25. Let A be a square matrix of order 3 such that $\det(A) = -2$ and $\det(3\,\mathrm{adj}(-6\,\mathrm{adj}(3A))) = 2^{m+n} \cdot 3^{mn}, m > n$. Then $4m + 2n$ is equal to _____. [4 marks]

Solution:

Concept:

- For a square matrix X of order three, $\det(cX) = c^3 \det X$: multiplication of the matrix by a scalar multiplies each of its three rows by that scalar.
- The adjugate identity gives $\det(\mathrm{adj}\, X) = (\det X)^2$ for order three. Apply this identity from the innermost matrix outward, keeping the scalar factors separate.

Step 1: Evaluate the first adjugate Start with the given determinant, rather than trying to reconstruct the entries of A. The first scalar multiplication gives

$$\det(3A) = 3^3 \det A = -2 \cdot 3^3.$$

Therefore, on writing $B = \mathrm{adj}(3A)$,

$$\det B = (-2 \cdot 3^3)^2 = 2^2 \cdot 3^6.$$

The negative sign disappears here because the determinant of an adjugate of order three is the square of the original determinant.

Step 2: Include the inner scalar and the outer adjugate Next put $C = -6B$. Since C is still of order three,

$$\det C = (-6)^3 \det B = -2^5 \cdot 3^9.$$

The next adjugate again squares this determinant. Finally the multiplier 3 outside that adjugate contributes its cube:

$$\begin{aligned} \det(3\,\mathrm{adj}\, C) &= 3^3 (\det C)^2 \\ &= 3^3 \left(-2^5 \cdot 3^9\right)^2 \\ &= 2^{10} \cdot 3^{21}. \end{aligned}$$

Each adjugate acts on a complete matrix, so its determinant must be taken only after all inner scalar factors have been included.

Step 3: Determine the two integers Comparison with $2^{m+n}3^{mn}$ gives $m + n = 10$ and $mn = 21$. Thus m, n are the roots of

$$t^2 - 10t + 21 = (t - 7)(t - 3) = 0.$$

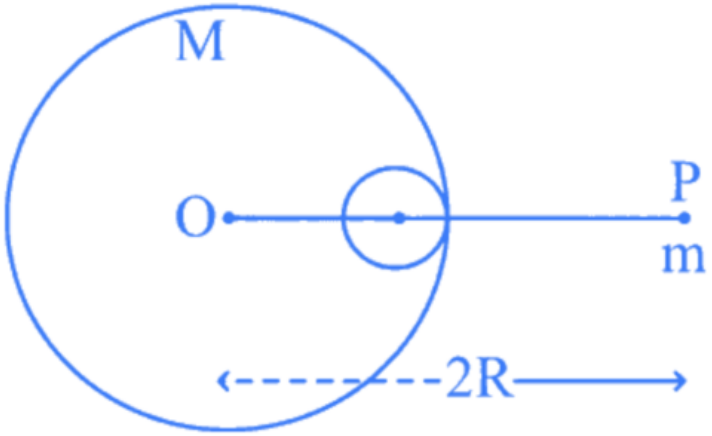
The condition $m > n$ fixes $m = 7$, $n = 3$. Substitution gives $4m + 2n = 28 + 6 = 34$.

Step 4: Conclusion The required numerical value is $\boxed{34}$. It is dimensionless.

Quick Tip to Solve: For nested adjugates of order three, track prime exponents: an adjugate doubles them and a scalar multiplier contributes its cube.

Physics – Section A: Multiple Choice Questions (80 marks)

26. A small point of mass m is placed at a distance $2R$ from the centre ' O ' of a big uniform solid sphere of mass M and radius R . The gravitational force on ' m ' due to M is F_1. A spherical part of radius $\mathrm{R}/3$ is removed from the big sphere as shown in the figure and the gravitational force on m due to remaining part of M is found to be F_2. The value of ratio $F_1 : F_2$ is [4 marks]



(A) 11 : 10

(B) 12 : 11

(C) 16 : 9

(D) 12 : 9

Correct Answer: (B) 12 : 11

Solution:

Concept:

- Outside a uniform sphere, its gravitational attraction equals that of a point mass at its centre: $F = GMm/r^2$. Here G is the gravitational constant and r is the separation of centres.
- Use superposition: the field of the remaining material equals the field of the complete sphere minus the field of the removed spherical portion. For equal density, mass is proportional to radius cubed.

Step 1: Find the force of the complete sphere The point mass lies at a distance $2R$ from O, outside the complete sphere. Hence

$$F_1 = \frac{GMm}{(2R)^2} = \frac{GMm}{4R^2}.$$

The cavity is a sphere of radius $R/3$, internally tangent to the large sphere on the side towards the point mass. Its centre is therefore $R - R/3 = 2R/3$ from O. The centre separation relevant to its own field must be measured from this cavity centre.

Step 2: Subtract the attraction of the removed sphere Uniform density gives the removed mass

$$M_c = M\left(\frac{R/3}{R}\right)^3 = \frac{M}{27}.$$

Its distance from the point mass is

$$r_c = 2R - \frac{2R}{3} = \frac{4R}{3}.$$

The attraction which that portion would have exerted before removal is

$$F_c = \frac{G(M/27)m}{(4R/3)^2} = \frac{GMm}{48R^2} = \frac{F_1}{12}.$$

Both the complete sphere and the removed portion attract the point mass towards the left along the same line. Thus their force magnitudes are subtracted directly, rather than combined by a perpendicular-vector rule.

Step 3: Compare the proposed ratios Consequently,

$$F_2 = F_1 - F_c = \frac{11F_1}{12}, \qquad \frac{F_1}{F_2} = \frac{12}{11}.$$

Option (A) would correspond to removing $1/11$ of the original force, while (C) and (D) would require much larger reductions. None equals the calculated reduction $F_1/12$. The factor $1/27$ in removed mass alone is insufficient because its centre is closer to the point mass.

Step 4: Conclusion The ratio is $\boxed{F_1 : F_2 = 12 : 11}$. Hence, the correct option is (B).

Quick Tip to Solve: For a spherical cavity, subtract the force of its missing mass at the cavity centre; do not place that missing mass at the original centre.

27. If B is magnetic field and μ_0 is permeability of free space, then the dimensions of (B/μ_0) is

[4 marks]

(A) $\mathrm{MT^{-2}\ A^{-1}}$

(B) $\mathrm{L^{-1}\ A}$

(C) $\mathrm{ML^2\ T^{-2}\ A^{-1}}$

(D) $\mathrm{LT^{-2}\ A^{-1}}$

Correct Answer: (B) $\mathrm{L^{-1}\ A}$

Solution:

Concept:

- The magnetic force on a moving charge is $F = qvB$ when the velocity is perpendicular to the field. The dimensions are $[F] = \mathrm{MLT^{-2}}$, $[q] = \mathrm{AT}$, and $[v] = \mathrm{LT^{-1}}$.
- The field of a long straight wire is $B = \mu_0 I/(2\pi r)$, so B/μ_0 must have the dimensions of current divided by length. Numerical constants such as 2π have no dimensions.

Step 1: Obtain the dimensions of the magnetic field Rearrange the magnetic force relation to isolate B:

$$[B] = \frac{[F]}{[q][v]} = \frac{\mathrm{MLT^{-2}}}{(\mathrm{AT})(\mathrm{LT^{-1}})} = \mathrm{MT^{-2}A^{-1}}.$$

Thus the field by itself has no remaining length dimension. This result is exactly the dimensions

displayed in option (A), but the question asks for a quotient involving permeability, so another step is necessary.

Step 2: Obtain the dimensions of permeability Using the straight-wire field relation gives

$$[\mu_0] = \frac{[B][r]}{[I]} = \mathrm{MLT^{-2}A^{-2}}.$$

Equivalently, permeability has the units newtons per ampere squared. Both routes agree since $[\mathrm{N}] = \mathrm{MLT^{-2}}$. Keeping the two current factors is essential: one is already in the denominator of the dimensions of B, and the second enters when dividing by I.

Step 3: Divide and identify the matching expression Now cancel like powers in the quotient:

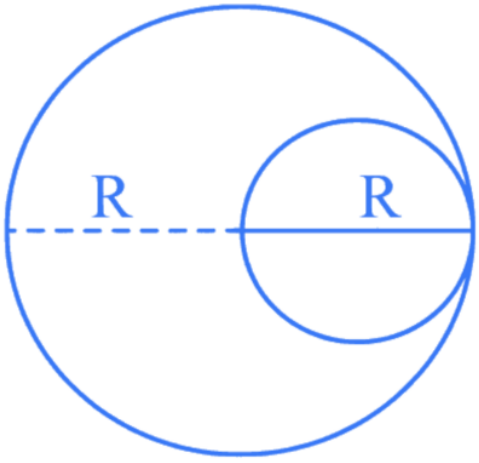
$$\left[\frac{B}{\mu_0}\right] = \frac{\mathrm{MT^{-2}A^{-1}}}{\mathrm{MLT^{-2}A^{-2}}} = \mathrm{L^{-1}A}.$$

The mass and time dimensions cancel entirely. Options (A), (C), and (D) all retain a time factor $\mathrm{T^{-2}}$, so none can represent this quotient. Option (C) also retains mass, while option (D) has the wrong length and current powers.

Step 4: Conclusion $\boxed{[B/\mu_0] = \mathrm{L^{-1}A}}$, or amperes per metre. Hence, the correct option is (B).

Quick Tip to Solve: Recall $B/\mu_0 = I/(2\pi r)$ for a straight wire: this immediately fixes the dimensions as current per unit length.

28. A uniform circular disc of radius R and mass M is rotating about an axis perpendicular to its plane and passing through its centre. A small circular part of radius $R/2$ is removed from the original disc as shown in the figure. Find the moment of inertia of the remaining part of the original disc about the axis as given above. [4 marks]



(A) $\frac{17}{32}\mathrm{MR^2}$
(B) $\frac{13}{32}\mathrm{MR^2}$
(C) $\frac{9}{32}\mathrm{MR^2}$
(D) $\frac{7}{32}\mathrm{MR^2}$

Correct Answer: (B) $\frac{13}{32}\mathrm{MR^2}$

Solution:

Concept:

- A uniform disc of mass M and radius R has moment of inertia $I_0 = MR^2/2$ about its central perpendicular axis. Contributions of separate portions add about a common axis.
- For a uniform disc, mass is proportional to area. To refer the removed disc to the original axis, use the parallel-axis theorem $I = I_{\text{cm}} + md^2$, where d is the separation of parallel axes.

Step 1: Determine the mass and position of the removed part The hole has radius $r = R/2$. Since the surface mass density is unchanged, the mass removed is

$$m = M\frac{\pi(R/2)^2}{\pi R^2} = \frac{M}{4}.$$

The figure shows the small circle passing through the original centre and touching the outer boundary. Its own centre is therefore at a distance $d = R/2$ from the rotation axis. The original symbols M and R refer to the complete disc, so its mass must not be replaced by the remaining mass in the original-disc formula.

Step 2: Calculate the removed inertia about the stated axis First calculate the small disc's central inertia, and then translate its axis:

$$I_c = \frac{1}{2}\left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2 = \frac{MR^2}{32},$$

$$I_{\text{removed}} = I_c + \left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2$$

$$= \frac{MR^2}{32} + \frac{MR^2}{16} = \frac{3MR^2}{32}.$$

The shift term is positive. The hole is not centred on the given rotation axis, so subtracting only its central inertia would underestimate the inertia removed.

Step 3: Subtract from the complete-disc inertia By additivity about the same original axis,

$$I_{\text{remaining}} = \frac{MR^2}{2} - \frac{3MR^2}{32} = \frac{13MR^2}{32}.$$

The answer is smaller than $MR^2/2 = 16MR^2/32$, ruling out (A). Options (C) and (D) subtract too large an inertia for the specified hole; neither follows from its actual mass and offset.

Step 4: Conclusion The moment of inertia is $\boxed{13MR^2/32}$, in units of mass times length squared. Hence, the correct option is (B).

Quick Tip to Solve: For a hole away from the rotation axis, subtract its inertia about that axis, including the parallel-axis term, rather than its central inertia alone.

29. Two spherical bodies of same materials having radii 0.2 m and 0.8 m are placed in same atmosphere. The temperature of the smaller body is 800 K and temperature of the bigger body is 400 K . If the energy radiated from the smaller body is E, the energy radiated from the bigger body is (assume, effect of the surrounding temperature to be negligible), [4 marks]

(A) 64 E

(B) 16 E

(C) E

(D) 256 E

Correct Answer: (C) E

Solution:

Concept:

- With the surrounding temperature negligible, the radiated power is $P = \varepsilon\sigma AT^4$. Here ε is emissivity, σ the Stefan-Boltzmann constant, A the emitting area and T the absolute temperature.
- A sphere radiates through its surface area $A = 4\pi r^2$. For equal time intervals, the ratio of radiated energies equals the ratio of powers because $E = Pt$. The same material gives the same emissivity in this comparison.

Step 1: Set up a ratio that cancels material constants Let the smaller sphere have radius $r_s = 0.2\,\text{m}$, temperature $T_s = 800\,\text{K}$ and radiated energy $E_s = E$. For the bigger sphere, $r_b = 0.8\,\text{m}$ and $T_b = 400\,\text{K}$. For the same emission time,

$$\frac{E_b}{E_s} = \frac{\varepsilon\sigma(4\pi r_b^2)T_b^4 t}{\varepsilon\sigma(4\pi r_s^2)T_s^4 t} = \left(\frac{r_b}{r_s}\right)^2 \left(\frac{T_b}{T_s}\right)^4.$$

The atmosphere does not contribute a significant $T^4_{\text{surroundings}}$ correction under the stated approximation.

Step 2: Evaluate the two competing factors The bigger sphere has four times the radius and therefore sixteen times the radiating area:

$$\left(\frac{0.8}{0.2}\right)^2 = 4^2 = 16.$$

Its absolute temperature is half as large, giving a factor

$$\left(\frac{400}{800}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}.$$

Consequently,

$$\frac{E_b}{E_s} = 16 \times \frac{1}{16} = 1.$$

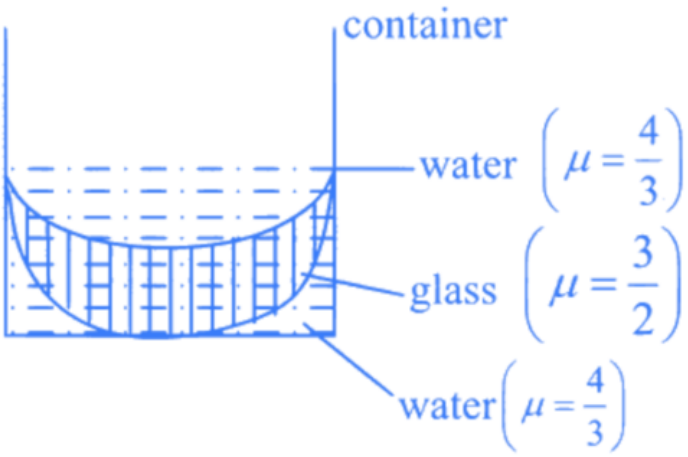
The increase in area exactly compensates for the decrease in radiation per unit area.

Step 3: Eliminate the other choices Option (B), $16E$, accounts for the larger area but ignores the lower temperature. Option (A), $64E$, corresponds to a volume factor rather than the relevant surface factor. Option (D), $256E$, also fails to include the suppressing fourth-power temperature ratio. Radiation must be compared using both area and absolute temperature.

Step 4: Conclusion The bigger sphere radiates $\boxed{E_b = E}$ in the same time. Hence, the correct option is (C).

Quick Tip to Solve: For spherical emitters of the same material, compare r^2T^4; a fourfold radius increase exactly offsets a halved absolute temperature.

30. In the diagram given below, there are three lenses formed. Considering negligible thickness of each of them as compared to $|R_1|$ and $|R_2|$, i.e., the radii of curvature for upper and lower surfaces of the glass lens, the power of the combination is [4 marks]



(A) $-\frac{1}{6}\left(\frac{1}{|R_1|} - \frac{1}{|R_2|}\right)$

(B) $-\frac{1}{6}\left(\frac{1}{|R_1|} + \frac{1}{|R_2|}\right)$

(C) $\frac{1}{6}\left(\frac{1}{|R_1|} + \frac{1}{|R_2|}\right)$

(D) $\frac{1}{6}\left(\frac{1}{|R_1|} - \frac{1}{|R_2|}\right)$

Correct Answer: (A) $-\frac{1}{6}\left(\frac{1}{|R_1|} - \frac{1}{|R_2|}\right)$

Solution:

Concept:

- For a refracting spherical surface, its paraxial power is $P = (n_2 - n_1)/R$, where n_1 and n_2 are the refractive indices before and after the surface, and R has the Cartesian sign.
- Powers of successive surfaces add when their separations are negligible. Plane outer surfaces have zero power because their radii are infinite. The glass index is $3/2$ and each water region has index $4/3$.

Step 1: Assign signed radii using the shown geometry Take the light to travel downward through the water, glass and water layers in the drawing. Both curved glass surfaces are bowl-shaped, so their centres of curvature lie above the surfaces, opposite to this positive propagation direction. Therefore their signed radii are

$$R_{\text{upper}} = -|R_1|, \qquad R_{\text{lower}} = -|R_2|.$$

The upper and lower external surfaces of the combined water-glass-water system are plane and contribute no power. The relevant curved interfaces are water to glass and then glass to water.

Step 2: Add the powers of the two curved interfaces For the upper interface, the refractive index increases by $3/2 - 4/3 = 1/6$. For the lower one, it decreases by the same amount:

$$P_{\text{upper}} = \frac{3/2 - 4/3}{-|R_1|} = -\frac{1}{6|R_1|},$$
$$P_{\text{lower}} = \frac{4/3 - 3/2}{-|R_2|} = \frac{1}{6|R_2|}.$$

Since the layers are thin compared with their curvature radii, propagation-distance corrections are negligible. Their powers can thus be added directly to obtain the combination's power.

Step 3: Express the result and compare signs Adding and factoring gives

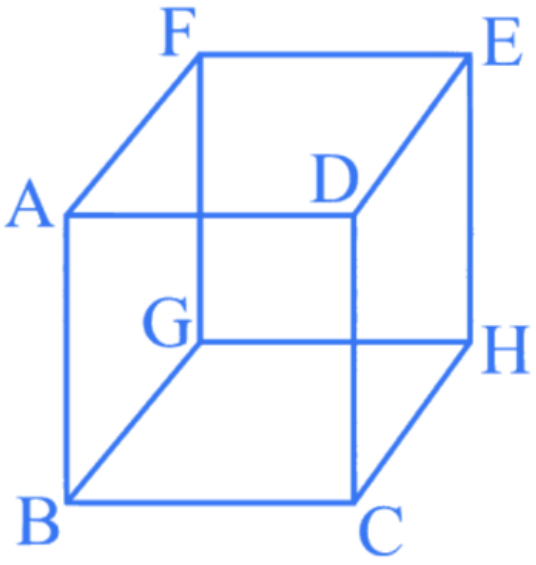
$$P = -\frac{1}{6}\left(\frac{1}{|R_1|} - \frac{1}{|R_2|}\right).$$

The reciprocal curvatures are subtracted because the two index changes have opposite signs while the signed radii have the same sign. This rules out the sums in (B) and (C). Option (D) reverses the overall sign, contrary to the negative power of the upper curved interface. The expression vanishes when the two curvatures are equal, as expected for cancelling thin interfaces.

Step 4: Conclusion The power is $\boxed{-\frac{1}{6}\left(\frac{1}{|R_1|} - \frac{1}{|R_2|}\right)}$, in dioptres when radii are in metres. Hence, the correct option is (A).

Quick Tip to Solve: Add interface powers with signed radii: both bowl-shaped interfaces have the same radius sign, while the index changes are opposite.

31. A line charge of length $\frac{a}{2}$ is kept at the center of an edge BC of a cube ABCDEFGH having edge length ' a ' as shown in the figure. If the density of line charge is λC per unit length, then the total electric flux through all the faces of the cube will be _____ . (Take, ϵ_0 as the free space permittivity) [4 marks]



(A) $\frac{\lambda a}{2\epsilon_0}$
(B) $\frac{\lambda a}{8\epsilon_0}$
(C) $\frac{\lambda a}{4\epsilon_0}$
(D) $\frac{\lambda a}{16\epsilon_0}$

Correct Answer: (B) $\frac{\lambda a}{8\epsilon_0}$

Solution:

Concept:

- Gauss's law gives the total outward electric flux through a closed surface as $\Phi = Q_{\text{enc}}/\epsilon_0$. Here ϵ_0 is the permittivity of free space.
- A charge exactly on an edge shared symmetrically by four cubes contributes one quarter of its full-space flux to each cube. The charge on a uniform segment is its linear density multiplied by its length.

Step 1: Calculate the charge on the segment The charged segment has length $a/2$, not the full edge length a. Its total charge is therefore

$$Q = \lambda\left(\frac{a}{2}\right) = \frac{\lambda a}{2}.$$

The segment is centred on edge BC. Every point of the charged segment lies on the common edge of four identical cubes arranged around that edge. Its endpoints do not lie at cube vertices, so a vertex fraction is not applicable.

Step 2: Use symmetry around the edge Join four identical cubes around BC. The line charge then lies inside the combined closed surface. Reflection in the two planes containing the shared edge divides

space around the charge into four equivalent regions. Consequently each cube receives one quarter of the total outward flux. The two faces containing the line have no normal flux away from the singular line, and the outer faces supply the required flux:

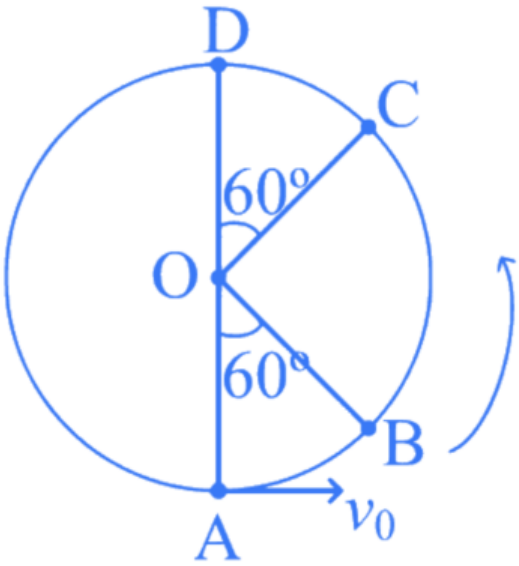
$$4\Phi = \frac{Q}{\epsilon_0}, \qquad \Phi = \frac{\lambda a}{8\epsilon_0}.$$

Step 3: Check the other choices Option (A) is the full flux of the segment and overlooks its position on the edge. Option (C) would assign half of the full flux, appropriate to a flat boundary rather than the present right-angled edge. Option (D) assigns only one eighth, as for a corner shared by eight cubes; this charge is on an edge, not at a vertex.

Step 4: Conclusion The total electric flux through the cube is $\boxed{\lambda a/(8\epsilon_0)}$, in $\mathrm{N\,m^2\,C^{-1}}$ when SI quantities are used. Hence, the correct option is (B).

Quick Tip to Solve: First calculate the actual segment charge, then use the sharing fraction: a face gives one half, an edge one quarter, and a corner one eighth in symmetric cubic arrangements.

32. A bob of mass m is suspended at a point O by a light string of length l and left to perform vertical motion (circular) as shown in figure. Initially, by applying horizontal velocity v_0 at the point ' A ', the string becomes slack when, the bob reaches at the point ' D '. The ratio of the kinetic energy of the bob at the points B and C is _____. [4 marks]



(A) 4

(B) 1

(C) 2

(D) 3

Correct Answer: (C) 2

Solution:

Concept:

- For a bob on a light inextensible string, tension acts radially and does no work while the string is taut. Mechanical energy is conserved, so $K + mgy$ stays constant.
- At the top D, the string just becomes slack when its tension is zero. Gravity alone then supplies the radial force: $mg = mv_D^2/l$. Heights are measured from the centre O.

Step 1: Find the energy at the highest point The radius is the string length l. At D the inward direction is downward. Thus the radial force equation at the limiting instant is

$$T_D + mg = \frac{mv_D^2}{l}, \qquad T_D = 0, \qquad v_D^2 = gl.$$

It follows that $K_D = \frac{1}{2}mgl$. Since D has height l above O, the conserved total mechanical energy relative to O is $E = \frac{3}{2}mgl$. Slackness does not mean zero speed: the bob still moves at D.

Step 2: Read the heights from the figure OB makes an angle of 60° with the downward vertical OA, whereas OC makes 60° with the upward vertical OD. Their vertical coordinates are therefore

$$y_B = -l\cos 60^\circ = -\frac{l}{2}, \qquad y_C = l\cos 60^\circ = \frac{l}{2}.$$

Using the same height reference at both points prevents a sign error. In particular B is below O and C is above O; they do not have equal gravitational potential energies.

Step 3: Calculate and compare the kinetic energies Subtract potential energy from the conserved total energy at each point:

$$K_B = \frac{3}{2}mgl - mg\left(-\frac{l}{2}\right) = 2mgl,$$

$$K_C = \frac{3}{2}mgl - mg\left(\frac{l}{2}\right) = mgl.$$

Hence $K_B/K_C = 2$. The values 4, 1 and 3 do not satisfy these two energy balances; equal speeds at B and C would contradict their different heights.

Step 4: Conclusion The kinetic-energy ratio is $\boxed{K_B : K_C = 2 : 1}$, or $K_B/K_C = 2$. Hence, the correct option is (C).

Quick Tip to Solve: Use zero tension at the top to determine the total energy, then express every other point by its signed vertical height from the centre.

33. The work functions of cesium (Cs) and lithium (Li) metals are 1.9 eV and 2.5 eV , respectively. If we incident a light of wavelength 550 nm on these two metal surfaces, then photo-electric effect is possible for the case of [4 marks]

(A) Both Cs and Li

(B) Neither Cs nor Li

(C) Li only

(D) Cs only

Correct Answer: (D) Cs only

Solution:

Concept:

- Each photon has energy $E_\gamma = hc/\lambda$. In practical units, $hc \simeq 1240\,\text{eV nm}$, with wavelength λ expressed in nanometres.
- Photoelectric emission is possible only if one photon's energy is at least the metal's work function ϕ. Einstein's equation then gives $K_{\max} = E_\gamma - \phi$.

Step 1: Calculate the incident photon energy The same wavelength, $550\,\text{nm}$, illuminates both surfaces. The available energy per photon is therefore identical for Cs and Li:

$$E_\gamma = \frac{1240\,\text{eV nm}}{550\,\text{nm}} \simeq 2.255\,\text{eV}.$$

The wavelength cancels from the units, leaving electronvolts. This energy must be compared with each work function separately; it must not be compared with their difference or their average.

Step 2: Test cesium and lithium separately For cesium, the work function is $1.9\,\text{eV}$, less than the photon energy. Electrons can escape and the largest kinetic energy is

$$K_{\max,\text{Cs}} = 2.255 - 1.9 \simeq 0.355\,\text{eV}.$$

For lithium, the work function is $2.5\,\text{eV}$, greater than the available photon energy. The formal subtraction gives $2.255 - 2.5 < 0$, indicating that emission cannot occur, not that an emitted electron has negative kinetic energy.

Step 3: Eliminate the other options Emission from both metals would require photons with at least $2.5\,\text{eV}$, which the incident light does not provide. Emission from neither is ruled out because Cs has the lower threshold and the photon energy exceeds it. Li-only emission is impossible: any photon energetic enough to liberate an electron from Li would also exceed the smaller Cs work function. Thus only the Cs-only choice is consistent with both threshold tests.

Step 4: Conclusion Light of wavelength $550\,\text{nm}$ produces the photoelectric effect in Cs only. Hence, the correct option is (D).

Quick Tip to Solve: Place the photon energy between the two work functions: $1.9 < 2.255 < 2.5\,\text{eV}$. Only the metal on the lower-threshold side emits.

34. An amount of ice of mass 10^{-3} kg and temperature -10°C is transformed to vapour of temperature 110°C by applying heat. The total amount of work required for this conversion is, (Take, specific heat of ice $= 2100 \text{Jkg}^{-1}\ \text{K}^{-1}$, specific heat of water $= 4180 \text{Jkg}^{-1}\ \text{K}^{-1}$, specific heat of steam $= 1920 \text{Jkg}^{-1}\ \text{K}^{-1}$, Latent heat of ice $= 3.35 \times 10^{5} \text{Jkg}^{-1}$ and Latent heat of steam $= 2.25 \times 10^{6}\ \text{Jkg}^{-1}$) [4 marks]

(A) 3022 J

(B) 3043 J

(C) 3003 J

(D) 3024 J

Correct Answer: (B) 3043 J

Solution:

Concept:

- Within a single phase, the heat required is $Q = mc\Delta T$, where m is mass, c is specific heat capacity and ΔT is temperature rise.
- During melting or boiling at the transition temperature, the energy is $Q = mL$, where L is the relevant latent heat. Temperature remains constant while the phase changes.

Step 1: Separate the conversion into five stages The sample must first warm from ice at -10°C to ice at 0°C, then melt, warm as liquid water to 100°C, boil, and finally warm as steam to 110°C. Both latent heats are necessary because two phase boundaries are crossed. The mass remains 10^{-3} kg throughout. Celsius temperature differences have the same numerical values as kelvin differences.

Step 2: Evaluate the energy for each stage Using the given specific heats and latent heats in SI units gives

$$Q_1 = 10^{-3}(2100)(10) = 21\,\text{J},$$

$$Q_2 = 10^{-3}(3.35 \times 10^{5}) = 335\,\text{J},$$

$$Q_3 = 10^{-3}(4180)(100) = 418\,\text{J},$$

$$Q_4 = 10^{-3}(2.25 \times 10^{6}) = 2250\,\text{J},$$

$$Q_5 = 10^{-3}(1920)(10) = 19.2\,\text{J}.$$

Each contribution is positive because energy is supplied at every stage. A single specific heat cannot be applied across the complete temperature interval because ice, water and steam have different heat capacities.

Step 3: Add the required energy The total supplied energy is

$$Q = 21 + 335 + 418 + 2250 + 19.2 = 3043.2\,\text{J}.$$

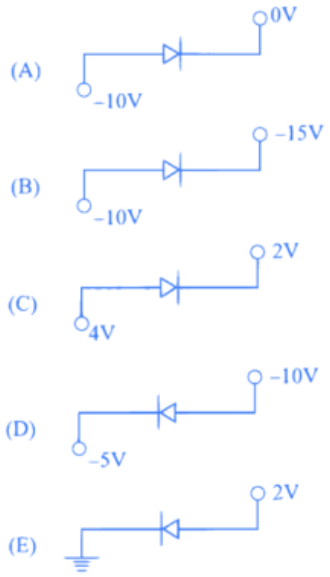
Thus the listed value to the nearest joule is 3043 J. The phrase work required refers here to the energy supplied for this thermal conversion. The other options, 3022 J, 3003 J and 3024 J, do not equal the sum of all five required contributions; omitting the warming of either ice or steam incorrectly lowers the result.

Step 4: Conclusion The required energy is 3043.2 J, approximately $\boxed{3043\,\text{J}}$. Hence, the correct option is (B).

Quick Tip to Solve: List solid warming, melting, liquid warming, boiling and vapour warming before substituting numbers; this prevents either latent heat or an end-stage temperature rise from being omitted.

35. Which of the following circuits represents a forward biased diode?

Choose the correct answer from the options given below: [4 marks]



(A) (C) and (E) only

(B) (A) and (D) only

(C) (B), (D) and (E) only

(D) (B), (C) and (E) only

Correct Answer: (D) (B), (C) and (E) only

Solution:

Concept:

- A diode is forward biased when its anode, or p-side, is at a higher electric potential than its cathode, or n-side: $V_A - V_K > 0$. The bar in the diode symbol marks the cathode.
- Only the potential difference matters. Two negative terminal voltages can forward bias a diode if the anode is less negative than the cathode; a grounded terminal has potential zero.

Step 1: Check circuits A, B and C In these three diagrams, the cathode bar is on the right, so the left terminal is the anode. For circuit (A),

$$V_A - V_K = -10 - 0 = -10\,\text{V},$$

which is reverse bias. For circuit (B),

$$V_A - V_K = -10 - (-15) = +5\,\text{V},$$

so it is forward biased even though both terminal potentials are negative. For circuit (C),

$$V_A - V_K = 4 - 2 = +2\,\text{V},$$

so this circuit is forward biased as well. These tests use the diode orientation, rather than merely checking whether any terminal has a positive voltage.

Step 2: Check the reversed symbols in D and E In circuits (D) and (E), the cathode bar is on the left. The right terminal is therefore the anode. In circuit (D),

$$V_A - V_K = -10 - (-5) = -5\,\text{V},$$

so it is reverse biased. In circuit (E), the left terminal is grounded and the right terminal is at $2\,\text{V}$, giving

$$V_A - V_K = 2 - 0 = +2\,\text{V}.$$

Thus (E) is forward biased. Ground is simply the reference potential; it is not necessarily the negative terminal in every circuit.

Step 3: Select the complete correct group The forward-biased circuits are exactly (B), (C) and (E). The group (C) and (E) leaves out (B). The group (A) and (D) includes only reverse-biased diodes. The group (B), (D) and (E) wrongly includes (D) and leaves out (C). Only the group containing (B), (C) and (E) passes all five potential-difference checks.

Step 4: Conclusion The second, third and fifth circuits are forward biased. Hence, the correct option is (D).

Quick Tip to Solve: Mark the cathode bar first, then subtract cathode potential from anode potential. A positive result means forward bias, regardless of the signs of the separate voltages.

---

36. A closed organ and an open organ tube are filled by two different gases having same bulk modulus but different densities ρ_1 and ρ_2, respectively. The frequency of 9^{th} harmonic of closed tube is identical with 4^{th} harmonic of open tube. If the length of the closed tube is 10 cm and the density ratio of the gases is $\rho_1 : \rho_2 = 1 : 16$, then the length of the open tube is : [4 marks]

(A) $\frac{15}{7}$ cm

(B) $\frac{20}{9}$ cm

(C) $\frac{20}{7}$ cm

(D) $\frac{15}{9}$ cm

Correct Answer: (B) $\frac{20}{9}$ cm

Solution:

Concept:

- The sound speed in a gas is $v = \sqrt{B/\rho}$, where B is bulk modulus and ρ is density. Equal bulk moduli make sound speed inversely proportional to the square root of density.
- A pipe closed at one end supports odd harmonics, with ninth-harmonic frequency $9v/(4L)$. A pipe open at both ends supports every harmonic, with fourth-harmonic frequency $4v/(2L)$.

Step 1: Relate the sound speeds in the two gases Let v_1 be the speed in the closed pipe and v_2 that in

the open pipe. Since both gases have the same bulk modulus and $\rho_1 : \rho_2 = 1 : 16$,

$$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{16} = 4.$$

Thus the lighter gas in the closed pipe carries sound four times faster. Using 16 directly as the speed ratio would miss the square root in the wave-speed formula.

Step 2: Equate the stated harmonic frequencies Write $L_1 = 10\,\text{cm}$ for the closed pipe and L_2 for the open pipe. The equality of the ninth and fourth harmonics requires

$$\frac{9v_1}{4L_1} = \frac{4v_2}{2L_2}.$$

Cross multiplication gives

$$9v_1L_2 = 8v_2L_1, \qquad L_2 = \frac{8}{9}\frac{v_2}{v_1}L_1.$$

The ninth harmonic is the frequency nine times the fundamental, not the ninth allowed mode of a closed pipe. Its use here is valid because nine is odd.

Step 3: Substitute the ratio and reject other lengths Substituting $v_2/v_1 = 1/4$ yields

$$L_2 = \frac{8}{9} \times \frac{1}{4} \times 10 = \frac{20}{9}\,\text{cm}.$$

As a check, the two frequencies become $9(4v_2)/(4 \times 10) = 0.9v_2$ and $2v_2/(20/9) = 0.9v_2$, using centimetres consistently for both lengths. The lengths $15/7$, $20/7$ and $15/9$ cm do not produce this required equality.

Step 4: Conclusion The open tube has length $\boxed{20/9\,\text{cm}}$, approximately $2.22\,\text{cm}$. Hence, the correct option is (B).

Quick Tip to Solve: Keep the two boundary conditions separate: use $4L$ for the closed pipe and $2L$ for the open pipe, then insert the square-root density ratio.

37. Given below are two statements :

Statement-I : The equivalent emf of two nonideal batteries connected in parallel is smaller than either of the two emfs.

Statement-II : The equivalent internal resistance of two nonideal batteries connected in parallel is smaller than the internal resistance of either of the two batteries.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Statement-I is false but Statement-II is true

(B) Statement-I is true but Statement-II is false

(C) Both Statement-I and Statement-II are false

(D) Both Statement-I and Statement-II are true

Correct Answer: (A) Statement-I is false but Statement-II is true

Solution:

Concept:

- A real cell is represented by its emf E in series with its positive internal resistance r. The equivalent emf is the open-circuit terminal voltage of the combination.
- For like-polarity parallel cells, $E_{\text{eq}} = (E_1/r_1 + E_2/r_2)/(1/r_1 + 1/r_2)$, while $r_{\text{eq}} = r_1 r_2/(r_1 + r_2)$.

Step 1: Test Statement-I using the common terminal voltage With no external load, the two branch currents add to zero. Writing each branch current in terms of the common terminal voltage V gives

$$\frac{E_1 - V}{r_1} + \frac{E_2 - V}{r_2} = 0.$$

Consequently,

$$E_{\text{eq}} = V = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}.$$

This is a weighted average with positive weights. If $E_1 < E_2$, the equivalent emf lies between them, rather than below both. If their emfs are equal, it equals that common emf. Either observation disproves the universal claim in Statement-I.

Step 2: Test Statement-II by finding the equivalent resistance To find the internal resistance, replace the ideal emf sources by shorts. Their internal resistances are then in parallel:

$$r_{\text{eq}} = \frac{r_1 r_2}{r_1 + r_2}.$$

Since $r_1 + r_2 > r_2$, this is smaller than r_1; similarly it is smaller than r_2. Thus Statement-II is true for finite positive internal resistances. Options (B) and (D) incorrectly accept Statement-I, while (C) incorrectly rejects Statement-II.

Step 3: Conclusion Statement-I is false and Statement-II is true. Hence, the correct option is (A).

Quick Tip to Solve: In parallel, emf is a conductance-weighted average; only the internal resistance is necessarily smaller than both individual values.

38. An electron in the ground state of the hydrogen atom has the orbital radius of 5.3×10^{-11} m while that for the electron in third excited state is 8.48×10^{-10} m. The ratio of the de Broglie wavelengths of electron in the ground state to that in the excited state is [4 marks]

(A) 4

(B) 3

(C) 9

(D) 16

Correct Answer: The literal ground-to-excited wavelength ratio is $1/4$, absent from the options; keyed option (A), 4, is its reciprocal.

Solution:

Concept:

- The de Broglie wavelength of an electron is $\lambda = h/(mv)$. In a hydrogen orbit, Coulomb attraction provides the centripetal force.
- Using $mv^2/r = ke^2/r^2$ gives $v \propto r^{-1/2}$ and hence $\lambda \propto \sqrt{r}$. Equivalently, Bohr orbits have $r_n \propto n^2$ and $\lambda_n \propto n$.

Step 1: Identify the excited orbit from the given radii The third excited state is $n = 4$, since the ground state is $n = 1$ and the first excited state is $n = 2$. The supplied radii confirm this identification:

$$\frac{r_4}{r_1} = \frac{8.48 \times 10^{-10}}{5.3 \times 10^{-11}} = 16.$$

Thus the larger radius corresponds to four times the principal quantum number and to a lower speed. It does not correspond to sixteen times the electron wavelength.

Step 2: Calculate the ratio in the order actually requested The wording asks for the ground-state wavelength divided by the excited-state wavelength. Therefore,

$$\frac{\lambda_1}{\lambda_4} = \sqrt{\frac{r_1}{r_4}} = \sqrt{\frac{1}{16}} = \frac{1}{4}.$$

As an independent check, the standing-wave condition $2\pi r_n = n\lambda_n$ gives $\lambda_1/\lambda_4 = (r_1/1)/(r_4/4) = 1/4$. The ground-state electron moves faster, so its de Broglie wavelength must be shorter. Option (A), 4, is the reverse ratio; (B), 3, confuses the excitation count with the state number; (C), 9, and (D), 16, are not wavelength ratios for these two states.

Step 3: Conclusion The ratio as printed is $\lambda_{\text{ground}}/\lambda_{\text{excited}} = 1/4$, dimensionless. None of options (A)–(D) gives this value. The answer key selects (A), 4, which is the reciprocal $\lambda_{\text{excited}}/\lambda_{\text{ground}}$; the derivation above establishes why it does not answer the literal order in the question.

Quick Tip to Solve: Write the numerator and denominator before using a radius ratio: wavelength scales as the square root of orbital radius, and the smaller orbit has the shorter wavelength.

39. Given below are two statements :

Statement I: In a vernier callipers, one vernier scale division is always smaller than one main scale division.

Statement II : The vernier constant is given by one main scale division multiplied by the number of vernier scale divisions.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Both Statement I and Statement II are false

(B) Statement I is true but Statement II is false

(C) Both Statement I and Statement II are true

(D) Statement I is false but Statement II is true

Correct Answer: (B) Statement I is true but Statement II is false

Solution:

Concept:

- In the conventional direct vernier calipers, n vernier divisions span $(n-1)$ main-scale divisions. Write the size of one main-scale division as s.
- The vernier constant or least count is the difference between one main-scale and one vernier-scale division, not the product of a division size and the number of divisions.

Step 1: Evaluate Statement I for the direct vernier calipers Let one vernier-scale division have length v. For the ordinary direct vernier used in school-laboratory calipers,

$$nv = (n-1)s, \qquad v = \frac{n-1}{n}s < s.$$

Thus one vernier division is slightly smaller than one main-scale division in this instrument, which is the direct-vernier convention intended here. For example, ten vernier divisions may span nine main-scale divisions, so each vernier division has size $0.9s$. Statement I is true in this conventional calipers construction.

Step 2: Compute the vernier constant and compare with Statement II The difference of the two division sizes is

$$\mathrm{VC} = s - v = s - \frac{n-1}{n}s = \frac{s}{n}.$$

For $s = 1\,\mathrm{mm}$ and $n = 10$, this gives $0.1\,\mathrm{mm}$. Multiplication, as claimed in Statement II, would instead give $10\,\mathrm{mm}$, which is far too coarse to represent the small increment measured by the vernier. Statement II is therefore false. Option (C) accepts the incorrect multiplication formula, option (D) also accepts it while rejecting the direct-vernier relation, and option (A) rejects that relation as well.

Step 3: Conclusion For the conventional direct vernier calipers, Statement I is true but Statement II is false. Hence, the correct option is (B).

Quick Tip to Solve: First write $n\,\mathrm{VSD} = (n-1)\,\mathrm{MSD}$; subtract the division sizes to obtain the least count MSD/n, never $n\,\mathrm{MSD}$.

40. An electron is made to enter symmetrically between two parallel and equally but oppositely charged metal plates, each of 10 cm length. The electron emerges out of the electric field region with a horizontal component of velocity 10^6 m/s. If the magnitude of the electric field between the plates is 9.1 V/cm, then the vertical component of velocity of electron is (mass of electron $= 9.1 \times 10^{-31}$ kg and charge of electron $= 1.6 \times 10^{-19}$C) [4 marks]

(A) 1×10^6 m/s

(B) 16×10^6 m/s

(C) 16×10^4 m/s

(D) 0

Correct Answer: (B) 16×10^6 m/s

Solution:

Concept:

- The electric field between oppositely charged parallel plates is transverse to the incident electron velocity. Therefore there is no horizontal force and the horizontal speed remains constant.
- The electron has vertical acceleration of magnitude $a_y = eE/m$, directed opposite to the electric field. With zero initial vertical velocity, its exit vertical speed is $|v_y| = a_y t$.

Step 1: Convert the data and find the transit time The plate length along the horizontal path and the field in SI units are

$$L = 10\,\text{cm} = 0.10\,\text{m}, \qquad E = 9.1\,\text{V/cm} = 910\,\text{V/m}.$$

The specified horizontal exit speed is also the horizontal entry speed, because the field has no horizontal component. Hence the time within the field region is

$$t = \frac{L}{v_x} = \frac{0.10}{10^6} = 1.0 \times 10^{-7}\,\text{s}.$$

The curved trajectory does not change this result: horizontal displacement still equals the constant horizontal speed multiplied by time.

Step 2: Calculate the vertical acceleration and exit speed Using the given charge magnitude and electron mass,

$$a_y = \frac{eE}{m} = \frac{(1.6 \times 10^{-19})(910)}{9.1 \times 10^{-31}} = 1.6 \times 10^{14}\,\text{m s}^{-2}.$$

The initial motion is symmetric between the plates, so its initial vertical velocity is zero. Therefore,

$$|v_y| = a_y t = (1.6 \times 10^{14})(1.0 \times 10^{-7}) = 1.6 \times 10^7\,\text{m/s}.$$

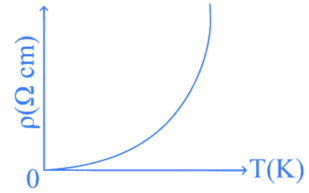
This is 16×10^6 m/s. Option (A) confuses it with the horizontal component; (C) misses the centimetre-to-metre factor in the field; (D) incorrectly neglects the transverse electric force.

Step 3: Conclusion The vertical velocity has magnitude 16×10^6 m/s, directed towards the positive plate. Hence, the correct option is (B).

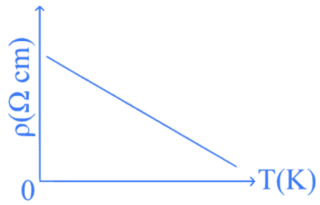
Quick Tip to Solve: Use the unchanged horizontal speed to find $t = L/v_x$, and convert V/cm to V/m before evaluating eEt/m.

41. Which of the following resistivity (ρ) v/s temperature (T) curves is most suitable to be used in wire bound standard resistors? [4 marks]

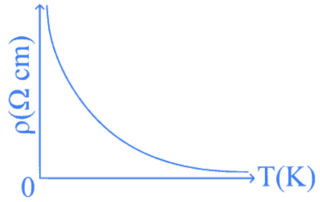
(A)



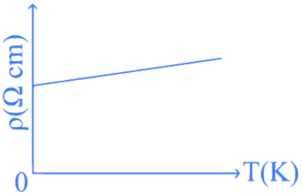
(B)



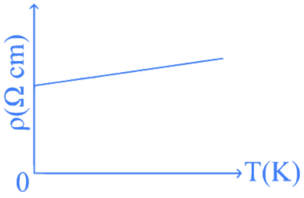
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- The resistance of a uniform wire is $R = \rho L/A$, where ρ is resistivity, L is length and A is cross-sectional area. A standard resistor should preserve its resistance as the ambient temperature varies.
- Over a moderate temperature interval, $\rho(T) \simeq \rho_0[1 + \alpha(T - T_0)]$. The temperature coefficient is $\alpha = (1/\rho_0)(d\rho/dT)$, so a small fractional slope is desirable.

Step 1: Identify the required temperature behaviour A standard resistor is used as a reliable reference. It should have a finite resistivity that changes as little as possible with temperature, otherwise the resistance used in calibration or comparison drifts when the room warms or current heats the wire. Resistance alloys such as manganin are chosen for their small temperature coefficient.

Perfect temperature independence would be represented by a horizontal line in a resistivity-versus-temperature graph; among the supplied curves, the closest practical choice is the gently rising straight line.

Step 2: Compare all four plotted alternatives Curve (A) rises steeply and increasingly rapidly, giving a large and temperature-dependent slope. Curve (B) has a substantial negative slope: decreasing resistivity is still a temperature change, not stability. Curve (C) falls sharply and nonlinearly, likewise unsuitable over an operating interval. Curve (D) starts at a nonzero resistivity and shows only a small positive variation with temperature. It best represents a wire material with a small temperature coefficient. The criterion is stability rather than the sign of the slope, so a negative-temperature-coefficient curve does not become suitable merely because its resistivity decreases.

Step 3: Conclusion The gently rising resistivity curve (D) is most suitable for a wire-wound standard resistor because it indicates the smallest temperature dependence among the choices. Hence, the correct option is (D).

Quick Tip to Solve: For a standard resistor, look for a nearly horizontal ρ-versus-T curve with finite resistivity; a large negative slope is no better than a large positive one.

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42. Given is a thin convex lens of glass (refractive index μ) and each side having radius of curvature R. One side is polished for complete reflection. At what distance from the lens, an object be placed on the optic axis so that the image gets formed on the object itself? [4 marks]

(A) R/μ

(B) $R/(2\mu - 1)$

(C) $\mathrm{R}/(2\mu - 3)$

(D) $\mu\mathrm{R}$

Correct Answer: (B) $R/(2\mu - 1)$

Solution:

Concept:

- A ray traverses the thin convex lens, reflects at the silvered surface and traverses the lens again. The combination behaves as an equivalent concave mirror.
- The lens power is $P_L = 1/f_L = (\mu - 1)(1/R - 1/(-R)) = 2(\mu - 1)/R$. The concave reflecting surface has mirror power $P_M = 2/R$, with converging powers taken positive.

Step 1: Combine the two refractions and the reflection The two sides have equal curvature magnitude R. In the thin-element limit the distance between the refracting and reflecting actions can be neglected. The equivalent power is the sum of the lens power encountered twice and the spherical mirror power:

$$P_{\text{eq}} = 2P_L + P_M = \frac{4(\mu - 1)}{R} + \frac{2}{R} = \frac{4\mu - 2}{R}.$$

Thus the equivalent mirror focal length is

$$F = \frac{1}{P_{\text{eq}}} = \frac{R}{4\mu - 2}.$$

This is a reflecting system; using the ordinary focal length of the unsilvered lens alone would omit both the reflection and the second traversal.

Step 2: Impose the condition that the image coincides with the object For a concave mirror, the object and its real image coincide at the centre of curvature, at distance $2F$ in front of the mirror. Using positive distance magnitudes in the mirror relation gives

$$\frac{1}{F} = \frac{1}{d} + \frac{1}{d} = \frac{2}{d}, \qquad d = 2F = \frac{R}{2\mu - 1}.$$

In the zero-thickness approximation this distance is measured from the lens itself. Option (A) omits the correct double-pass power dependence; option (C) gives an incorrect sign for the reflecting contribution; option (D) has the wrong refractive-index dependence. As a check, $\mu \to 1$ leaves only a mirror of radius R, and the result correctly tends to $d = R$.

Step 3: Conclusion The object must be placed a distance $R/(2\mu - 1)$ in front of the silvered lens on its optic axis. Hence, the correct option is (B).

Quick Tip to Solve: For a silvered thin lens, add twice the lens power to the mirror power; self-imaging requires the equivalent centre of curvature, so use $2F$.

43. A parallel-plate capacitor of capacitance 40μ F is connected to a 100 V power supply. Now the intermediate space between the plates is filled with a dielectric material of dielectric constant $\mathrm{K} = 2$. Due to the introduction of dielectric material, the extra charge and the change in the electrostatic energy in the capacitor, respectively, are [4 marks]

(A) 8 mC and 2.0 J

(B) 4 mC and 0.2 J

(C) 2 mC and 0.2 J

(D) 2 mC and 0.4 J

Correct Answer: (B) 4 mC and 0.2 J

Solution:

Concept:

- The battery remains connected, so the potential difference is fixed at $V = 100\,\mathrm{V}$. The battery can supply extra charge when the capacitance changes.
- A dielectric completely filling the gap multiplies the capacitance by K: $C_f = KC_i$. At fixed voltage, $Q = CV$ and the energy stored in the capacitor is $U = \frac{1}{2}CV^2$.

Step 1: Identify the electrical constraint The initial capacitance is $C_i = 40 \times 10^{-6}\,\mathrm{F}$. Since the capacitor is still connected to the supply, its voltage cannot fall as it would for an isolated charged

capacitor. With $K = 2$, the final capacitance is

$$C_f = 2(40 \times 10^{-6}) = 80 \times 10^{-6}\,\text{F}.$$

Thus $\Delta C = 40 \times 10^{-6}\,\text{F}$.

Step 2: Find the extra charge supplied The initial and final charges are

$$Q_i = C_i V = 4.0 \times 10^{-3}\,\text{C}, \qquad Q_f = C_f V = 8.0 \times 10^{-3}\,\text{C}.$$

The question asks for extra charge, not the final charge. Hence

$$\Delta Q = Q_f - Q_i = 4.0 \times 10^{-3}\,\text{C} = 4\,\text{mC}.$$

Step 3: Calculate the change in stored energy At the same fixed voltage,

$$\Delta U = \frac{1}{2}(C_f - C_i)V^2 = \frac{1}{2}(40 \times 10^{-6})(100)^2 = 0.2\,\text{J}.$$

The stored energy increases from $0.2\,\text{J}$ to $0.4\,\text{J}$. Option (A) confuses final charge with extra charge and gives the wrong energy; (C) and (D) give the wrong charge increment.

Step 4: Conclusion The extra charge is $4\,\text{mC}$, and the electrostatic energy increases by $0.2\,\text{J}$. Hence, the correct option is (B).

Quick Tip to Solve: For a capacitor left on a battery, hold voltage fixed and use $\Delta Q = \Delta C\,V$ and $\Delta U = \frac{1}{2}\Delta C\,V^2$.

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44. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion-(A) : If Young's double slit experiment is performed in an optically denser medium than air, then the consecutive fringes come closer.

Reason-(R) : The speed of light reduces in an optically denser medium than air while its frequency does not change.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) (A) is false but (R) is true

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(C) Both (A) and (R) are true and (R) is the correct explanation of (A)

(D) (A) is true but (R) is false

Correct Answer: (C) Both (A) and (R) are true and (R) is the correct explanation of (A)

Solution:

Concept:

- The fringe width in Young's double-slit experiment is $\beta = \lambda D/d$, where D is the screen distance and d is the slit separation.
- For light entering a stationary transparent medium of refractive index n, its speed becomes $v = c/n$, while its frequency ν stays fixed. Therefore $\lambda = v/\nu = \lambda_{\text{air}}/n$, taking air to have index approximately one.

Step 1: Evaluate the assertion Keep the same experimental geometry, so D and d are unchanged when the surrounding medium changes. In an optically denser medium $n > 1$, and the wavelength decreases. Substitution in the interference formula gives

$$\beta_{\text{medium}} = \frac{\lambda_{\text{air}} D}{nd} = \frac{\beta_{\text{air}}}{n} < \beta_{\text{air}}.$$

Consecutive bright fringes, or consecutive dark fringes, are therefore closer together. The assertion is true.

Step 2: Evaluate the reason The source fixes the oscillation frequency. At the interface, the electromagnetic oscillations must retain that same frequency; otherwise the fields could not remain continuous in time. The propagation speed is smaller in the denser medium, so

$$\lambda_{\text{medium}} = \frac{v}{\nu} = \frac{c}{n\nu}.$$

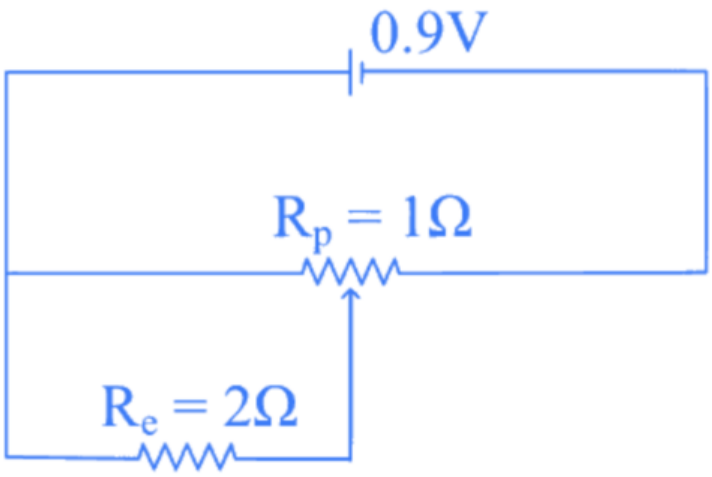
Thus both claims in the reason, reduced speed and unchanged frequency, are true.

Step 3: Test the explanatory link The reason directly implies a reduced wavelength through $v = \nu\lambda$. Because fringe width is proportional to wavelength, it directly explains the reduced separation stated in the assertion. Option (A) incorrectly rejects the assertion; option (D) incorrectly rejects the reason; option (B) overlooks this causal connection.

Step 4: Conclusion Both the assertion and the reason are true, and the reason correctly explains the assertion. Hence, the correct option is (C).

Quick Tip to Solve: When the whole YDSE arrangement is immersed, replace λ by λ/n; the frequency remains fixed and the fringe width shrinks by n.

45. Sliding contact of a potentiometer is in the middle of the potentiometer wire having resistance $R_p = 1\Omega$ as shown in the figure. An external resistance of $R_e = 2\Omega$ is connected via the sliding contact. The electric current in the circuit is : [4 marks]



(A) 1.35 A

(B) 0.9 A

(C) 1.0 A

(D) 0.3 A

Correct Answer: (C) 1.0 A

Solution:

Concept:

- For a uniform resistance wire, resistance is proportional to length. A sliding contact at its midpoint divides the $1\,\Omega$ wire into two equal $0.5\,\Omega$ sections.
- Resistors joining the same two circuit nodes are in parallel. Their equivalent resistance is $R_aR_b/(R_a + R_b)$; series resistances add. The source current then follows from $I = V/R_{\text{eq}}$.

Step 1: Identify the parallel branch In the figure, the external $2\,\Omega$ resistance joins the left end of the potentiometer to its midpoint. It is therefore across the left half of the wire, not across the whole wire. The right half carries the total current and is in series with this parallel combination. The source voltage is $0.9\,\text{V}$.

Step 2: Reduce the resistance network For the left-hand parallel section,

$$R_{\parallel} = \frac{(0.5)(2)}{0.5 + 2} = \frac{1}{2.5} = 0.4\,\Omega.$$

Adding the right half of the wire gives

$$R_{\text{eq}} = 0.4 + 0.5 = 0.9\,\Omega.$$

This is smaller than the original wire resistance, as expected when an extra conducting path is added.

Step 3: Calculate and check the source current Using the voltage across the entire combination,

$$I = \frac{0.9}{0.9} = 1.0\,\text{A}.$$

The series half then drops $I(0.5) = 0.5\,\text{V}$, leaving $0.4\,\text{V}$ across the parallel pair. Its branch currents are $0.4/0.5 = 0.8\,\text{A}$ and $0.4/2 = 0.2\,\text{A}$, adding to $1.0\,\text{A}$. Option (B) would ignore the loading resistor; (A) would put the entire wire in parallel with it; (D) would incorrectly add the full wire and external resistance in series.

Step 4: Conclusion The current supplied by the cell is $1.0\,\text{A}$. Hence, the correct option is (C).

Quick Tip to Solve: Mark the two ends of the external resistor first: it shunts only the wire segment between those same nodes, not necessarily the whole potentiometer.

Physics – Section B: Numerical Value Questions (20 marks)

46. The driver sitting inside a parked car is watching vehicles approaching from behind with the help of his side view mirror, which is a convex mirror with radius of curvature $\text{R} = 2\ \text{m}$. Another car approaches him from behind with a uniform speed of $90\ \text{km/hr}$. When the car is at a distance of 24 m from him, the magnitude of the acceleration of the image of the car in the side view mirror is ' a '. The value of $100a$ is _____ m/s^2. [4 marks]

Solution:

Concept:

- The convex mirror has focal-length magnitude $f = R/2 = 1\,\text{m}$. Let $x > 0$ be the distance of the car in front of the mirror and $y > 0$ the virtual-image distance behind it.
- With Cartesian distances $u = -x$ and $v = y$, the mirror formula gives $1/f = 1/y - 1/x$, or $y = fx/(x + f)$. Image motion must be obtained by differentiating this position relation with respect to time.

Step 1: Express the object motion The approaching car has constant speed

$$V = 90\frac{1000}{3600} = 25\,\text{m}\,\text{s}^{-1}.$$

Hence $\dot{x} = -25\,\text{m}\,\text{s}^{-1}$ and $\ddot{x} = 0$. At the instant requested, $x = 24\,\text{m}$, so $x + f = 25\,\text{m}$. The parked car and its mirror form the stationary reference frame.

Step 2: Differentiate the image position twice From $y = fx/(x + f)$, differentiation gives

$$\frac{dy}{dx} = \frac{f^2}{(x+f)^2}, \qquad \frac{d^2y}{dx^2} = -\frac{2f^2}{(x+f)^3}.$$

The image velocity is $\dot{y} = f^2\dot{x}/(x + f)^2$. Applying the chain rule again,

$$\ddot{y} = \frac{f^2\ddot{x}}{(x+f)^2} - \frac{2f^2\dot{x}^2}{(x+f)^3} = -\frac{2f^2V^2}{(x+f)^3}.$$

Although the object acceleration is zero, the nonlinear image-position relation produces nonzero image acceleration.

Step 3: Evaluate the required magnitude The negative sign means the image accelerates towards the mirror in the coordinate chosen. Its acceleration magnitude is

$$a = \frac{2(1)^2(25)^2}{(24+1)^3} = \frac{1250}{15625} = 0.08\,\mathrm{m\,s^{-2}}.$$

Therefore

$$100a = 8\,\mathrm{m\,s^{-2}}.$$

All lengths were converted to metres before differentiating, so the acceleration is in SI units.

Step 4: Conclusion The image acceleration has magnitude $a = 0.08\,\mathrm{m\,s^{-2}}$; therefore the required value of $100a$ is $8\,\mathrm{m\,s^{-2}}$, and the numerical entry is 8.

Quick Tip to Solve: Differentiate the mirror position relation twice; constant object speed does not imply constant image speed or zero image acceleration.

47. Two soap bubbles of radius 2 cm and 4 cm , respectively, are in contact with each other. The radius of curvature of the common surface, in cm , is _____. [4 marks]

Solution:

Concept:

- A soap bubble has two liquid-air surfaces. For surface tension T and radius r, its pressure above the external air pressure is $4T/r$.
- The common soap film also has two surfaces, so the pressure difference it supports is $4T/R_c$, where R_c is its radius of curvature. The smaller bubble has the higher internal pressure.

Step 1: Write the internal pressures Let the external atmospheric pressure be p_0. For the two bubbles with radii $r_1 = 2\,\mathrm{cm}$ and $r_2 = 4\,\mathrm{cm}$,

$$p_1 = p_0 + \frac{4T}{r_1}, \qquad p_2 = p_0 + \frac{4T}{r_2}.$$

Since $r_1 < r_2$, we have $p_1 > p_2$. The shared film bulges into the larger bubble, which is the lower-pressure side.

Step 2: Balance pressure across the common film The atmospheric term cancels when the internal pressures are subtracted. Thus

$$p_1 - p_2 = 4T\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

This same pressure difference is supplied by the curved common soap film:

$$\frac{4T}{R_c} = 4T\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

The surface tension and the two-surface factor cancel; the result depends only on the two radii.

Step 3: Calculate the radius of curvature Use centimetres consistently because the answer is requested in centimetres:

$$\frac{1}{R_c} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}\,\text{cm}^{-1}, \qquad R_c = 4\,\text{cm}.$$

Equivalently, $R_c = r_1 r_2/(r_2 - r_1)$. As a check, equal bubble radii would give zero pressure difference and hence an infinite curvature radius, corresponding to a plane common film; the formula has that correct limit.

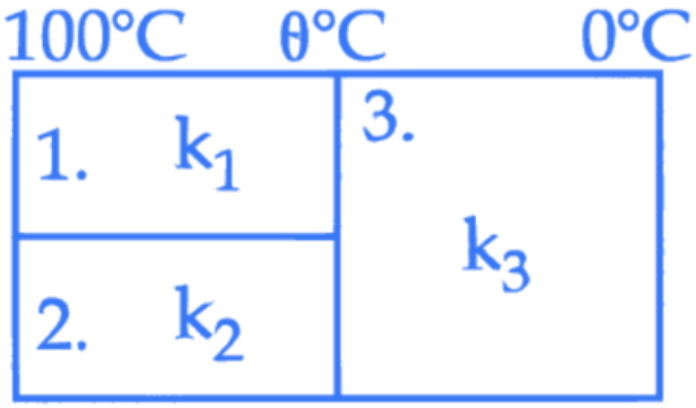
Step 4: Conclusion The common surface has radius of curvature $4\,\text{cm}$. The required numerical entry is 4.

Quick Tip to Solve: For two soap bubbles of the same liquid, subtract their excess pressures: $1/R_c = |1/r_1 - 1/r_2|$; the common film bulges into the larger bubble.

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48. Three conductors of same length having thermal conductivity k_1, k_2 and k_3 are connected as shown in figure.

Area of cross sections of 1^{st} and 2^{nd} conductor are same and for 3^{rd} conductor it is double of the 1^{st} conductor. The temperatures are given in the figure. In steady state condition, the value of θ is _____ °C. (Given : $k_1 = 60\text{Js}^{-1}\,\text{m}^{-1}\,\text{K}^{-1}, k_2 = 120\text{Js}^{-1}\,\text{m}^{-1}\,\text{K}^{-1}, k_3 = 135\text{Js}^{-1}\,\text{m}^{-1}\,\text{K}^{-1}$) [4 marks]



Solution:

Concept:

- The steady heat current through a uniform conductor is $H = kA\Delta T/L$, where k is thermal conductivity, A is cross-sectional area and L is its length.
- At a steady junction, no heat accumulates: the sum of incoming heat currents equals the outgoing heat current. Conductors 1 and 2 are in parallel between $100°\text{C}$ and θ, while conductor 3 connects the junction to $0°\text{C}$.

Step 1: Assign the areas and temperature drops Let the common length of every conductor be L, and let the area of each of conductors 1 and 2 be A. Conductor 3 then has area $2A$. Both left-hand conductors have the same temperature drop $100 - \theta$; the right-hand conductor has drop $\theta - 0 = \theta$.

The factor of two belongs to conductor 3's area, not to its length.

Step 2: Apply heat-current conservation The heat entering through conductors 1 and 2 is

$$H_{\text{in}} = \frac{k_1 A}{L}(100-\theta) + \frac{k_2 A}{L}(100-\theta).$$

The outgoing heat current is

$$H_{\text{out}} = \frac{k_3(2A)}{L}\theta.$$

Equating them and cancelling the common positive factor A/L yields

$$(k_1+k_2)(100-\theta) = 2k_3\theta.$$

No absolute temperature conversion is needed here because only temperature differences enter Fourier's law.

Step 3: Substitute and solve for the junction temperature With the stated conductivities,

$$(60+120)(100-\theta) = 2(135)\theta.$$

Therefore

$$18000 - 180\theta = 270\theta, \qquad 450\theta = 18000, \qquad \theta = 40^\circ\text{C}.$$

The incoming heat current is $180(A/L)(60) = 10800A/L$, equal to the outgoing current $270(A/L)(40)$. The temperature lies between the two boundary temperatures, as required.

Step 4: Conclusion The steady junction temperature is $\theta = 40^\circ\text{C}$. The required numerical entry is 40.

Quick Tip to Solve: Add kA/L for rods in parallel, then equate incoming and outgoing heat currents at the junction; keep the doubled area of the third rod.

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49. The position vectors of two 1 kg particles, (A) and (B), are given by $\vec{r}_A = (\alpha_1 t^2\hat{i} + \alpha_2 t\hat{j} + \alpha_3 t\hat{k})\,\text{m}$ and $\vec{r}_B = (\beta_1 t\hat{i} + \beta_2 t^2\hat{j} + \beta_3 t\hat{k})\,\text{m}$, respectively; $\alpha_1 = 1\,\text{m/s}^2$, $\alpha_2 = 3n\,\text{m/s}$, $\alpha_3 = 2\,\text{m/s}$, $\beta_1 = 2\,\text{m/s}$, $\beta_2 = -1\,\text{m/s}^2$, $\beta_3 = 4p\,\text{m/s}$, where t is time, n and p are constants. At $t = 1\,\text{s}$, $|\vec{V}_A| = |\vec{V}_B|$, and the velocities are orthogonal to each other. At $t = 1\,\text{s}$, the magnitude of angular momentum of particle (A) with respect to the position of particle (B) is $\sqrt{L}\,\text{kg m}^2\,\text{s}^{-1}$. The value of L is _____. [4 marks]

Solution:

Concept:

- The instantaneous velocity is the time derivative of the position vector: $\vec{v} = d\vec{r}/dt$. Equal speeds imply equal squared magnitudes, whereas perpendicular velocities have zero scalar product.
- The angular momentum of A about the specified position B is $\vec{H}_B = (\vec{r}_A - \vec{r}_B) \times m_A\vec{v}_A$. Here the momentum is that of A in the given frame; B supplies the reference position.

Step 1: Differentiate the two position vectors At $t = 1\,\text{s}$, differentiating each component gives

$$\vec{v}_A = (2, 3n, 2)\,\text{m/s}, \qquad \vec{v}_B = (2, -2, 4p)\,\text{m/s}.$$

The constant term in each linear component contributes its coefficient, while the quadratic component contributes twice its coefficient at this instant.

Step 2: Use the speed and perpendicularity conditions Equality of squared speeds gives $8 + 9n^2 = 8 + 16p^2$, hence $3n = \pm 4p$. The scalar product condition is

$$4 - 6n + 8p = 0.$$

The choice $3n = 4p$ would give $4 = 0$, so it is impossible. Therefore $3n = -4p$, and substitution gives $4 + 16p = 0$. Thus $p = -1/4$ and $n = 1/3$. The resulting velocities are $(2, 1, 2)$ and $(2, -2, -1)$, each of magnitude $3\,\mathrm{m/s}$, with zero scalar product.

Step 3: Calculate the displacement and angular momentum The positions at this instant are $\vec{r}_A = (1, 1, 2)\,\mathrm{m}$ and $\vec{r}_B = (2, -1, -1)\,\mathrm{m}$, so

$$\vec{r}_A - \vec{r}_B = (-1, 2, 3)\,\mathrm{m}.$$

For the 1 kg mass,

$$\vec{H}_B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 3 \\ 2 & 1 & 2 \end{vmatrix} = (\hat{i} + 8\hat{j} - 5\hat{k})\,\mathrm{kg\,m^2\,s^{-1}}.$$

Consequently $|\vec{H}_B|^2 = 1 + 64 + 25 = 90$.

Step 4: Conclusion The magnitude is $\sqrt{90}\,\mathrm{kg\,m^2\,s^{-1}}$; hence the requested dimensionless value is $\boxed{L = 90}$.

Quick Tip to Solve: First use equal speed and zero dot product to fix n and p. Use the displacement from B, but the given-frame momentum of A, for this angular momentum.

50. A particle is projected at an angle of 30° from horizontal at a speed of $60\ \mathrm{m/s}$. The height traversed by the particle in the first second is $\mathrm{h_0}$ and height traversed in the last second, before it reaches the maximum height, is h_1. The ratio $h_0 : h_1$ is _____.

[Take, $\mathrm{g} = 10\ \mathrm{m/s^2}$] [4 marks]

Solution:

Concept:

- Horizontal and vertical projectile motions are independent. The upward initial velocity is $u_y = u \sin\theta$, and gravity produces the constant vertical acceleration $-g$.
- The height above the launch level is $y(t) = u_y t - gt^2/2$, while the vertical velocity is $v_y = u_y - gt$. Maximum height occurs when $v_y = 0$.

Step 1: Find the vertical launch speed and ascent time Using the specified launch speed and angle,

$$u_y = 60 \sin 30^\circ = 30\,\mathrm{m/s}.$$

The horizontal component does not enter a calculation of height. At the highest point, $30 - 10T = 0$,

giving $T = 3\,\text{s}$. Therefore the first second is the interval from 0 to 1 s, and the last second of ascent is the interval from 2 to 3 s. The latter is not the final second before landing.

Step 2: Calculate both upward height increments With the given value of gravity,

$$y(t) = 30t - 5t^2.$$

The first-second height is

$$h_0 = y(1) - y(0) = 30 - 5 = 25\,\text{m}.$$

For the final second before the highest point, $y(2) = 60 - 20 = 40\,\text{m}$ and $y(3) = 90 - 45 = 45\,\text{m}$. Hence

$$h_1 = y(3) - y(2) = 45 - 40 = 5\,\text{m}.$$

This agrees with using the average vertical speeds: the speed changes from 30 to 20 m/s in the first second, and from 10 to zero in the last second, giving mean speeds of 25 and 5 m/s respectively.

Step 3: Form the ratio of the two distances Both intervals last one second and both motions are upward, so the heights are positive. Their ratio is

$$h_0 : h_1 = 25 : 5 = 5 : 1.$$

Using the maximum height itself in place of h_1 would compare different quantities; the question asks for the height gained within the last one-second interval.

Step 4: Conclusion $\boxed{h_0 : h_1 = 5 : 1}$. The numerical ratio is 5, with no unit.

Quick Tip to Solve: The last second of ascent ends at zero vertical speed, so its height gain is always g/2 for a one-second interval.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Which of the following electrolyte can be used to obtain $H_2\ S_2O_8$ by the process of electrolysis ?

[4 marks]

(A) Concentrated solution of sulphuric acid

(B) Dilute solution of sodium sulphate.

(C) Acidified dilute solution of sodium sulphate.

(D) Dilute solution of sulphuric acid

Correct Answer: (A) Concentrated solution of sulphuric acid

Solution:

Concept:

- Peroxodisulphuric acid, $H_2S_2O_8$, contains a peroxide linkage connecting two sulphate-derived units. It is prepared by anodic oxidation in concentrated sulphuric acid.
- At the anode oxidation removes electrons. Under suitable electrolysis conditions on platinum, hydrogen sulphate ions couple to give peroxodisulphate: $2\,HSO_4^- \longrightarrow S_2O_8^{2-} + 2\,H^+ + 2\,e^-$.

Step 1: Identify the required anodic reaction The desired product is not the usual oxygen evolved from water during dilute aqueous electrolysis. The sulphate-containing species must instead be oxidised and coupled. In concentrated sulphuric acid, the high concentration of hydrogen sulphate favours the formation of the peroxide bond under appropriate anodic conditions. The oxidation half-reaction is balanced in both atoms and charge: each side contains two sulphur atoms, eight oxygen atoms and two hydrogen atoms, and the total charge is -2.

Step 2: Relate the ions to the molecular acid The peroxodisulphate ion formed at the anode combines with hydrogen ions in the acid medium to represent $H_2S_2O_8$. A molecular form of the anodic change is

$$2\,H_2SO_4 \longrightarrow H_2S_2O_8 + 2\,H^+ + 2\,e^-.$$

Thus concentrated sulphuric acid supplies both the sulphate framework and the strongly acidic medium. Platinum anodes and suitable current conditions are used for this preparation; simply electrolysing any dilute sulphate solution does not describe this method.

Step 3: Eliminate the dilute electrolytes Options (B) and (C) specify dilute sodium sulphate solutions; adding acid to a dilute salt does not make it concentrated sulphuric acid. Option (D) also explicitly specifies dilute sulphuric acid, for which the usual school-level aqueous electrolysis produces hydrogen and oxygen. All three miss the concentration condition required for the stated preparation.

Step 4: Conclusion Use a concentrated solution of sulphuric acid. Hence, the correct option is (A).

Quick Tip to Solve: For persulphuric acid, look for concentrated sulphuric acid and an anodic oxidation; dilute acid points to ordinary water electrolysis.

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52. Given below are two statements:

Statement I: CH_3-O-CH_2-Cl **will undergo** S_N1 **reaction though it is a primary halide.**

Statement II: $(CH_3)_3C-CH_2-Cl$ **will not undergo** S_N2 **reaction very easily though it is a primary halide.**

In the light of the above statements, choose the most appropriate answer from the options given below: [4 marks]

(A) Both Statement I and Statement II are incorrect

(B) Both Statement I and Statement II are correct

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (B) Both Statement I and Statement II are correct

Solution:

Concept:

- An S_N1 reaction proceeds through ionisation to a carbocation. Resonance stabilisation of that intermediate can make an otherwise unusual primary substrate undergo this pathway.
- An S_N2 reaction requires nucleophilic backside approach. Steric crowding close to the carbon carrying the leaving group can slow it markedly, even when that carbon is primary.

Step 1: Test Statement I using the adjacent oxygen In CH_3-O-CH_2-Cl, loss of chloride generates the cation $CH_3-O-CH_2^+$. A lone pair on oxygen can be donated into the empty orbital at the neighbouring carbon, giving the resonance contributor $CH_3-O^+=CH_2$. The positive charge is therefore delocalised over the oxygen-carbon unit. The simple rule that a primary halide disfavors S_N1 applies to an unstabilised primary carbocation and does not remove this stabilisation. Statement I is correct.

Step 2: Test Statement II using the neopentyl structure The second compound is neopentyl chloride, $(CH_3)_3C-CH_2-Cl$. Its carbon bonded to chlorine is attached to only one other carbon, so the halide is indeed primary. However, that adjacent carbon carries three methyl groups. This bulky group obstructs the approach needed to form the S_N2 transition state, producing the neopentyl effect. Consequently its S_N2 reaction is unusually slow compared with an unhindered primary halide. The statement says it does not react very easily, rather than claiming that substitution is impossible.

Step 3: Choose the option matching both evaluations The first statement is true because of electronic stabilisation, and the second is true because of steric hindrance. Option (A) wrongly rejects both, (C) wrongly rejects the neopentyl exception, and (D) wrongly rejects the oxygen-stabilised cation. The two statements illustrate different reasons that primary classification alone cannot determine substitution behaviour.

Step 4: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (B).

Quick Tip to Solve: For an unusual primary halide, check nearby lone-pair resonance for SN1 and bulky beta substitution for SN2.

53. The products formed in the following reaction sequence are:

$$C_6H_5NO_2 \xrightarrow{\text{(i)–(iv)}} A + B$$

The reagents over the arrow, in order, are (i) Br_2**, AcOH; (ii)** Sn, HCl**; (iii)** $NaNO_2$, HCl**, 273 K; (iv)** C_2H_5OH**.** [4 marks]

(A) m-BrC_6H_4OH and m-$BrC_6H_4OC_2H_5$

(B) m-$BrC_6H_4OC_2H_5$ and CH_3COOH

(C) m-BrC_6H_4OH and CH_3CHO

(D) C_6H_5Br and CH_3CHO

Correct Answer: (D) C_6H_5Br and CH_3CHO

Solution:

Concept:

- The nitro group strongly deactivates the benzene ring and directs electrophilic bromination mainly to the meta position. Reduction with tin and hydrochloric acid converts $-NO_2$ into $-NH_2$.
- A primary aromatic amine reacts with nitrous acid generated from $NaNO_2$/HCl at 273 K to form a diazonium salt. Ethanol replaces the diazonium group by hydrogen and is itself oxidised to ethanal.

Step 1: Follow bromination and nitro-group reduction Starting with nitrobenzene, bromination

using Br_2 in acetic acid places bromine meta to the nitro group. This gives m-bromonitrobenzene. Treatment with Sn/HCl reduces the nitro group to an amino group without removing the ring bromine, producing m-bromoaniline. The order matters: bromination takes place while the meta-directing nitro group is still present.

Step 2: Form and reduce the diazonium group At 273 K, $NaNO_2/HCl$ diazotises the amino group to form m-bromobenzenediazonium chloride. Ethanol then acts as a reducing agent, and the diazonium group is replaced by hydrogen:

$$\text{m-BrC}_6\text{H}_4\text{N}_2{}^+\text{Cl}^- + \text{C}_2\text{H}_5\text{OH} \longrightarrow \text{C}_6\text{H}_5\text{Br} + \text{N}_2 + \text{CH}_3\text{CHO} + \text{HCl}.$$

This equation shows both organic products: bromobenzene and ethanal. Nitrogen gas is released, and ethanol loses hydrogen as it is oxidised to the aldehyde.

Step 3: Compare the products with all four options Option (D) contains bromobenzene together with CH_3CHO, exactly as required. Options (A) and (C) retain an OH group at the former diazonium site, which would describe hydrolysis rather than this reduction. Option (B) substitutes an ethoxy group and gives ethanoic acid, missing both the replacement by hydrogen and the aldehyde product of ethanol oxidation. Therefore none of those pairs fits the specified sequence.

Step 4: Conclusion The products are bromobenzene, C_6H_5Br, and ethanal, CH_3CHO. Hence, the correct option is (D).

Quick Tip to Solve: Ethanol reduces an aryl diazonium group to H; remember the paired oxidation product is ethanal, not ethanoic acid.

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54. From the magnetic behaviour of $[NiCl_4]^{2-}$ (paramagnetic) and $[Ni(CO)_4]$ (diamagnetic), choose the correct geometry and oxidation state. [4 marks]

(A) $[NiCl_4]^{2-}$: Ni(0), tetrahedral $[Ni(CO)_4]$: Ni(0), square planar

(B) $[NiCl_4]^{2-}$: Ni^{II}, tetrahedral $[Ni(CO)_4]$: Ni(0), tetrahedral

(C) $[NiCl_4]^{2-}$: Ni^{II}, tetrahedral $[Ni(CO)_4]$: Ni^{II}, square planar

(D) $[NiCl_4]^{2-}$: Ni^{II}, square planar $[Ni(CO)_4]$: Ni(0), square planar

Correct Answer: (B) $[NiCl_4]^{2-}$: Ni^{II}, tetrahedral $[Ni(CO)_4]$: Ni(0), tetrahedral

Solution:

Concept:

- The oxidation state follows from the complex charge and ligand charges. Chloride is an anionic ligand, whereas carbon monoxide is neutral.
- Nickel has the ground-state configuration $[Ar]3d^8 4s^2$. Thus Ni^{2+} has a $3d^8$ configuration, while nickel in $Ni(CO)_4$ has a filled $3d^{10}$ arrangement in the conventional valence-bond description.

Step 1: Analyse the tetrachloridonickelate ion Let the nickel oxidation state in $[NiCl_4]^{2-}$ be x. Four chloride ligands contribute -4, so $x - 4 = -2$, giving $x = +2$. Chloride is a weak-field ligand. In this four-coordinate complex the geometry is tetrahedral, described by sp^3 hybridisation in the elementary model. The d^8 tetrahedral arrangement has two unpaired electrons, so the stated paramagnetism is

consistent with this geometry.

Step 2: Analyse nickel tetracarbonyl Each CO ligand is neutral and the molecule $\mathrm{Ni(CO)_4}$ has zero net charge. Nickel must therefore have oxidation state zero, not +2. In the conventional bonding description the nickel electrons form a filled $3d^{10}$ shell before the four ligand pairs are accommodated in tetrahedral sp^3 orbitals. All electrons are paired, explaining the stated diamagnetism. Diamagnetism alone does not imply square-planar geometry; the electron configuration must also be considered.

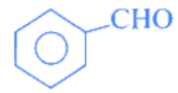
Step 3: Match both complete descriptions Both complexes are tetrahedral, but their metal oxidation states and electron configurations differ. Option (B) gives Ni(II), tetrahedral for the chloride complex and Ni(0), tetrahedral for the carbonyl. Option (A) wrongly assigns Ni(0) to the charged chloride complex, option (C) wrongly assigns Ni(II) to the neutral carbonyl, and option (D) assigns square-planar geometries to both. These charge and geometry errors eliminate all three.

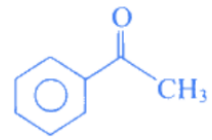
Step 4: Conclusion $[\mathrm{NiCl_4}]^{2-}$: Ni(II), tetrahedral; $\mathrm{Ni(CO)_4}$: Ni(0), tetrahedral. Hence, the correct option is (B).

Quick Tip to Solve: Find oxidation states before using magnetic behaviour: tetrahedral d8 nickel has unpaired electrons, whereas tetrahedral d10 nickel does not.

55. The compounds which give positive Fehling's test are :

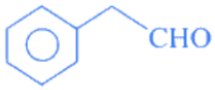
Choose the correct answer from the options given below : [4 marks]

(A) 

(B) 

(C) $HOCH_2 - CO - (CHOH)_3 - CH_2 - OH$

(D) $CH_3 - \overset{\overset{O}{\|}}{C} - H$

(E) 

(A) (A), (C) and (D) Only

(B) (A), (B) and (C) Only

(C) (A), (D) and (E) Only

(D) (C), (D) and (E) Only

Correct Answer: (D) (C), (D) and (E) Only

Solution:

Concept:

- Fehling's reagent is alkaline copper(II) tartrate. A reducing compound converts blue copper(II) into a brick-red precipitate of Cu_2O.
- Aliphatic aldehydes respond positively; ordinary aromatic aldehydes and simple ketones do not. Alpha-hydroxy ketones can also reduce the reagent through alkaline enediol rearrangement.

Step 1: Classify the two aromatic carbonyl compounds Compound (A) is benzaldehyde, C_6H_5CHO: its aldehyde group is directly attached to the benzene ring, so it does not give the ordinary Fehling test. Compound (B) is acetophenone, $C_6H_5COCH_3$, a simple ketone without an adjacent hydroxy group. It also gives a negative test. The presence of a carbonyl group alone is therefore insufficient.

Step 2: Identify the reducing compounds Compound (C), $HOCH_2CO(CHOH)_3CH_2OH$, is an alpha-hydroxy ketose. In the alkaline reagent it forms enediol species and aldoses, and therefore gives a positive test. Compound (D) is ethanal, CH_3CHO, an aliphatic aldehyde that is readily oxidised to ethanoate. Compound (E) is phenylacetaldehyde, $C_6H_5CH_2CHO$. Its aldehyde group is separated from the aromatic ring by a methylene group, so it behaves as an aliphatic aldehyde and gives a positive test.

Step 3: Select the matching combination The positive set is (C), (D) and (E). Options (A), (B) and (C) all incorrectly include benzaldehyde; option (B) additionally includes the non-reducing simple ketone. Only option (D) includes all three reducing compounds and excludes both negative compounds.

Step 4: Conclusion Compounds C, D and E give a positive Fehling test. Hence, the correct option is (D).

Quick Tip to Solve: Check whether the aldehyde carbon is directly bonded to the benzene ring: C_6H_5CHO is negative, but $C_6H_5CH_2CHO$ is positive. Also check ketones for an adjacent hydroxy group.

56. Match List-I with List-II.

	List - I		List - II
(A)	$Al^{3+} < Mg^{2+} < Na^{+} < F^{-}$	**(I)**	**Ionisation Enthalpy**
(B)	$B < C < O < N$	**(II)**	**Metallic character**
(C)	$B < Al < Mg < K$	**(III)**	**Electronegativity**
(D)	$Si < P < S < Cl$	**(IV)**	**Ionic radii**

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(B) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(C) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(D) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

Correct Answer: (A) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

Solution:

Concept:

- For an isoelectronic series, increasing nuclear charge pulls the same number of electrons closer, decreasing ionic radius.
- Ionisation enthalpy, electronegativity and metallic character follow different trends. Subshell stability explains exceptions to the general increase of ionisation enthalpy across a period.

Step 1: Match the ionic-radius sequence The ions Al^{3+}, Mg^{2+}, Na^{+} and F^{-} each contain ten electrons. Their nuclear charges decrease from 13 through 12 and 11 to 9. With the electron count fixed, the attraction weakens and ionic radius increases in that order. Thus List-I (A) matches ionic radii, (IV).

Step 2: Identify ionisation enthalpy For the second-period elements shown, boron has lower first ionisation enthalpy than carbon. Nitrogen has a stable half-filled $2p^3$ subshell, whereas oxygen has a paired electron in $2p^4$ whose repulsion makes removal easier. Hence the increasing order is B < C < O < N. This is ionisation enthalpy, so (B) matches (I).

Step 3: Match the remaining two properties Metallic character increases from B to Al down the group, from Al to Mg to the left across a period, and strongly toward K, an alkali metal. Thus (C) matches (II). Electronegativity generally increases across period 3, giving Si < P < S < Cl; therefore (D) matches (III).

Step 4: Reject the other codes The complete code is (A)-(IV), (B)-(I), (C)-(II), (D)-(III). Option (B) wrongly treats the isoelectronic-ion series as electronegativity, option (C) as metallic character, and option (D) interchanges the final two matches. Only option (A) preserves all four properties.

Step 5: Conclusion The sequence matches ionic radii, ionisation enthalpy, metallic character and electronegativity respectively. Hence, the correct option is (A).

Quick Tip to Solve: Identify an isoelectronic ionic sequence first; then use the N/O half-filled-subshell exception to recognise the ionisation-enthalpy row.

57. A vessel at 1000 K contains CO_2 with a pressure of 0.5 atm . Some of CO_2 is converted into CO on addition of graphite. If total pressure at equilibrium is 0.8 atm , then K_p is : [4 marks]

(A) 0.18 atm

(B) 0.3 atm

(C) 3 atm

(D) 1.8 atm

Correct Answer: (D) 1.8 atm

Solution:

Concept:

- Graphite reacts with carbon dioxide according to $CO_2(g) + C(s) \rightleftharpoons 2\,CO(g)$. The activity of pure solid graphite is unity, so it is omitted from the equilibrium expression.
- At a fixed temperature and vessel volume, partial pressure is proportional to gas amount. If x atm of carbon dioxide is consumed, $2x$ atm of carbon monoxide forms.

Step 1: Express the equilibrium partial pressures Initially the carbon dioxide pressure is $0.5\,\text{atm}$, and carbon monoxide is absent. Let the pressure-equivalent consumption of carbon dioxide be $x\,\text{atm}$. Stoichiometry gives

$$p_{CO_2} = 0.5 - x, \qquad p_{CO} = 2x.$$

The solid does not contribute to the total gas pressure.

Step 2: Use the total equilibrium pressure Adding the gas partial pressures gives

$$P_{eq} = (0.5 - x) + 2x = 0.5 + x.$$

Since the total pressure is $0.8\,\text{atm}$,

$$x = 0.8 - 0.5 = 0.3\,\text{atm}.$$

Consequently

$$p_{CO_2} = 0.2\,\text{atm}, \qquad p_{CO} = 0.6\,\text{atm}.$$

Both partial pressures are positive and sum to the stated total, providing a direct check.

Step 3: Substitute into the equilibrium expression The pressure-based constant in the convention of the options is

$$K_p = \frac{(p_{CO})^2}{p_{CO_2}} = \frac{(0.6)^2}{0.2} = 1.8\,\text{atm}.$$

The $0.3\,\text{atm}$ value in option (B) is merely the reaction extent in pressure units, not the constant. Options (A) and (C) do not satisfy the required squared carbon-monoxide pressure divided by residual carbon-dioxide pressure.

Step 4: Conclusion $K_p = 1.8\,\text{atm}$. Hence, the correct option is (D).

Quick Tip to Solve: For $CO_2 + C \longrightarrow 2\,CO$, the rise in total pressure equals the pressure-equivalent amount of carbon dioxide consumed; carbon monoxide forms at twice that amount.

58. A solution of aluminium chloride is electrolysed for 30 minutes using a current of 2 A . The amount of the aluminium deposited at the cathode is _____ .

[Given : molar mass of aluminium and chlorine are $27\ \text{g mol}^{-1}$ **and** $35.5\ \text{g mol}^{-1}$ **respectively. Faraday constant** $= 96500\text{C mol}^{-1}$**]** [4 marks]

(A) 1.660 g

(B) 1.007 g

(C) 0.336 g

(D) 0.441 g

Correct Answer: (C) 0.336 g

Solution:

Concept:

- Faraday law relates deposited mass to charge: $m = MQ/(zF)$, where M is molar mass, $Q = It$, z is the electrons required per metal ion, and F is the Faraday constant.
- Aluminium deposition corresponds to $\text{Al}^{3+} + 3\,\text{e}^- \longrightarrow \text{Al}$. Thus one mole of aluminium requires three moles of electrons, and the equivalent mass is $27/3 = 9\,\text{g}$.

Step 1: Convert the electrolysis time and find charge The current is $I = 2\,\text{A} = 2\,\text{C s}^{-1}$. Convert the time before applying the charge relation:

$$t = 30 \times 60 = 1800\,\text{s}.$$

Hence

$$Q = It = 2 \times 1800 = 3600\,\text{C}.$$

Using 30 directly would mix minutes with a current measured per second.

Step 2: Account for the trivalent aluminium ion The number of moles of electrons delivered is

$$n_e = \frac{Q}{F} = \frac{3600}{96500}\,\text{mol}.$$

Each aluminium ion needs three electrons, so the amount of deposited aluminium is

$$n_{\text{Al}} = \frac{3600}{3 \times 96500}\,\text{mol}.$$

The chlorine molar mass is not needed because the requested cathodic product is aluminium.

Step 3: Calculate the deposited mass and compare Multiply the aluminium amount by its molar mass:

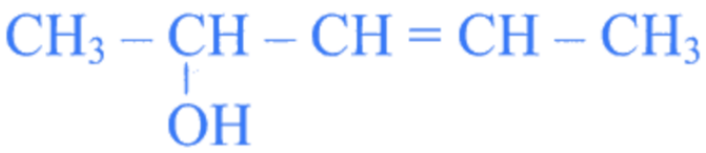
$$m = \frac{27 \times 3600}{3 \times 96500} = \frac{32400}{96500}\,\text{g} \approx 0.33575\,\text{g}.$$

Rounding to the precision of the options gives $0.336\,\text{g}$. Option (B), about $1.007\,\text{g}$, would result from omitting the factor of three for aluminium valency. Options (A) and (D) likewise do not follow from the stated charge and electron stoichiometry.

Step 4: Conclusion The calculated mass of aluminium deposited is $0.336\,\text{g}$. Hence, the correct option is (C).

Quick Tip to Solve: For cathodic metal deposition, divide the molar mass by the metal-ion charge first; for Al^{3+}, use $27/3 = 9$ as the equivalent mass.

59. How many different stereoisomers are possible for the given molecule? [4 marks]



(A) 2

(B) 1

(C) 3

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- A tetrahedral carbon attached to four different groups is a stereogenic centre and allows two configurations, R and S.
- An alkene allows E/Z geometrical isomerism when each double-bonded carbon has two different substituents. Independent binary stereogenic elements multiply the possible configurations unless symmetry makes some identical.

Step 1: Identify the stereogenic carbon The molecule is $CH_3CH(OH)CH{=}CHCH_3$. The carbon bearing the hydroxy group is bonded to OH, H, CH_3, and $CH{=}CHCH_3$. These are four different substituents. Therefore this tetrahedral carbon is stereogenic and can have either R or S configuration. The methyl carbons are not stereogenic because each has identical hydrogen atoms.

Step 2: Check the double bond for geometrical isomerism The left carbon of the double bond is attached to H and $CH(OH)CH_3$; the right carbon is attached to H and CH_3. Each alkene carbon has distinct substituents, so the double bond permits E and Z forms. This restriction is independent of the R/S choice at the hydroxy-bearing carbon.

Step 3: Count the distinct combinations The combinations are (R,E), (S,E), (R,Z), and (S,Z), giving

$$N = 2 \times 2 = 4.$$

There is no internal symmetry that equates an R form with an S form: the two ends of the molecule are not equivalent, and there is only one tetrahedral stereogenic centre. Thus no meso reduction occurs. Option (A) counts just one stereogenic element, option (B) ignores both, and option (C) would require an unjustified equivalence between two of the four combinations.

Step 4: Conclusion There are four different stereoisomers. Hence, the correct option is (D).

Quick Tip to Solve: Count alkene E/Z possibilities separately from tetrahedral R/S centres, then check molecular symmetry before multiplying the counts.

60. A liquid when kept inside a thermally insulated closed vessel at $25^{\circ}\mathrm{C}$ was mechanically stirred from outside. What will be the correct option for the following thermodynamic parameters ? [4 marks]

(A) $\Delta U = 0,\ q < 0,\ w > 0$

(B) $\Delta U > 0,\ q = 0,\ w > 0$

(C) $\Delta U = 0,\ q = 0,\ w = 0$

(D) $\Delta U < 0,\ q = 0,\ w > 0$

Correct Answer: (B) $\Delta U > 0,\ q = 0,\ w > 0$

Solution:

Concept:

- With the chemistry sign convention, $\Delta U = q + w$, where heat entering the system and work done on the system are positive.
- A thermally insulated boundary permits no heat transfer, so $q = 0$. A closed system prevents mass transfer, but it may still receive mechanical work through a stirrer.

Step 1: Use the thermal-insulation condition The vessel is thermally insulated, so no heat flows through its walls during stirring. Therefore

$$q = 0.$$

The initial temperature $25^{\circ}\mathrm{C}$ states the starting condition; it does not mean the temperature is held fixed. Insulation is an adiabatic condition, not an isothermal condition, so a zero heat transfer must not be confused with zero internal-energy change.

Step 2: Determine the sign of the stirring work The outside agency turns the stirrer against viscous resistance and transfers mechanical energy to the liquid. Work is therefore done on the system, and in the stated chemistry convention

$$w > 0.$$

Even if the vessel has rigid walls and there is no pressure-volume expansion work, shaft work through the stirrer is present. Closed does not mean isolated when such a mechanical interaction occurs.

Step 3: Apply the first law and choose the option Substituting into the first law gives

$$\Delta U = q + w = 0 + w > 0.$$

Thus internal energy rises as stirring work is dissipated within the liquid. Option (A) violates thermal insulation by assigning negative heat transfer. Option (C) incorrectly sets the stirring work to zero. Option (D) assigns a decrease of internal energy despite positive work input and zero heat flow. Only option (B) simultaneously respects the boundary condition and energy conservation.

Step 4: Conclusion The parameters are $\Delta U > 0,\ q = 0,\ w > 0$. Hence, the correct option is (B).

Quick Tip to Solve: Insulated means zero heat transfer, not zero energy transfer: stirring adds positive shaft work and raises the internal energy even in a rigid closed vessel.

61. Lanthanoid ions with $4f^7$ configuration are :

(A) Eu^{2+}

(B) Gd^{3+}

(C) Eu^{3+}

(D) Tb^{3+}

(E) Sm^{2+}

Choose the correct answer from the options given below : [4 marks]

(A) (B) and (E) only

(B) (B) and (C) only

(C) (A) and (D) only

(D) (A) and (B) only

Correct Answer: (D) (A) and (B) only

Solution:

Concept:

- The xenon core has 54 electrons. For lanthanoid cations the outer $6s$ electrons are removed before the inner $4f$ electrons; a $5d$ electron, where present, is also removed before an additional $4f$ electron.
- The requested configuration is exactly $[Xe]4f^7$, a half-filled $4f$ subshell. Count the remaining electrons for each ion rather than using the neutral-atom configuration.

Step 1: Determine the configurations of europium and gadolinium Europium has atomic number 63 and neutral configuration $[Xe]4f^7 6s^2$. Removing its two $6s$ electrons gives Eu^{2+} : $[Xe]4f^7$; removing one more gives Eu^{3+} : $[Xe]4f^6$. Thus statement (A) is included and (C) is excluded. Gadolinium, atomic number 64, is $[Xe]4f^7 5d^1 6s^2$. Its tripositive ion loses the two $6s$ electrons and the $5d$ electron, leaving Gd^{3+} : $[Xe]4f^7$. Hence statement (B) is included.

Step 2: Eliminate the remaining ions and option pairs Terbium has atomic number 65, so Tb^{3+} contains $65 - 3 - 54 = 8$ electrons outside the xenon core: its configuration is $[Xe]4f^8$. Samarium has atomic number 62; Sm^{2+} contains $62 - 2 - 54 = 6$ such electrons and is $[Xe]4f^6$. Neither (D) nor (E) qualifies. The required pair is therefore (A) and (B), with no other ion included. Printed options (A), (B) and (C) each include an ion with an incorrect $4f$ count; printed option (D) contains precisely the required pair.

Step 3: Conclusion The ions are Eu^{2+} and Gd^{3+}, the europium(II) and gadolinium(III) pair. Hence, the correct option is (D).

Quick Tip to Solve: For these lanthanoid cations, subtract the charge and the 54-electron xenon core from the atomic number to check the $4f$ count.

62. The incorrect statements regarding geometrical isomerism are :

(A) Propene shows geometrical isomerism.

(B) Trans isomer has identical atoms/groups on the opposite sides of the double bond.

(C) Cis-but-2-ene has higher dipole moment than trans-but-2-ene.

(D) 2-methylbut-2-ene shows two geometrical isomers.

(E) Trans-isomer has lower melting point than cis isomer.

Choose the correct answer from the options given below : [4 marks]

(A) (A) and (E) Only

(B) (A), (D) and (E) Only

(C) (C), (D) and (E) Only

(D) (B) and (C) Only

Correct Answer: (B) (A), (D) and (E) Only

Solution:

Concept:

- An alkene can show geometrical isomerism only when each carbon of its double bond bears two different substituents. Restricted rotation then preserves distinct relative arrangements.
- In the usual cis/trans comparison, identical groups lie on the same side in the cis isomer and opposite sides in the trans isomer. Molecular symmetry affects both dipole cancellation and crystal packing.

Step 1: Test statements (A), (B) and (C) Propene is $CH_3-CH=CH_2$. Its terminal double-bond carbon carries two identical hydrogen atoms, so it cannot give distinct geometrical isomers. Statement (A) is incorrect. Statement (B) correctly describes the trans arrangement: identical groups are on opposite sides of the double bond. In trans-but-2-ene the symmetric arrangement cancels the relevant bond-dipole contributions, whereas in cis-but-2-ene they do not cancel completely. The cis isomer consequently has the higher dipole moment, so statement (C) is correct.

Step 2: Test statements (D) and (E), then select the set In 2-methylbut-2-ene, $(CH_3)_2C=CHCH_3$, one double-bond carbon has two identical methyl groups. The structural requirement for geometrical isomerism fails, making statement (D) incorrect. The trans isomer generally has the higher melting point in the standard comparison because its greater symmetry enables more efficient packing in the crystal. Statement (E), which assigns the lower melting point to the trans isomer, is therefore the incorrect trend. The incorrect set is exactly (A), (D) and (E). Printed option (A) omits (D); printed option (C) wrongly includes the true statement (C) and omits (A); printed option (D) contains two true statements.

Step 3: Conclusion Propene isomerism, 2-methylbut-2-ene isomerism and the stated trans melting-point trend are the three incorrect claims. Hence, the correct option is (B).

Quick Tip to Solve: Check each double-bond carbon for two different groups before comparing cis/trans properties; propene and an alkene with two methyls on one carbon fail immediately.

63. Given below are two statements :

Statement I : One mole of propyne reacts with excess of sodium to liberate half a mole of H_2 gas.

Statement II : Four g of propyne reacts with $NaNH_2$ to liberate NH_3 gas which occupies 224 mL at STP.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are incorrect

(B) Statement I is incorrect but Statement II is correct

(C) Both Statement I and Statement II are correct

(D) Statement I is correct but Statement II is incorrect

Correct Answer: (D) Statement I is correct but Statement II is incorrect

Solution:

Concept:

- Propyne, $CH_3{-}C{\equiv}CH$, is a terminal alkyne with one acidic terminal hydrogen. Sodium replaces this hydrogen with liberation of hydrogen gas.
- Sodium amide deprotonates a terminal alkyne to form its sodium salt and ammonia in a one-to-one molar ratio. At STP, one mole of an ideal gas occupies $22.4\,\text{L}$.

Step 1: Verify the sodium reaction in Statement I Two terminal alkyne molecules supply the two hydrogen atoms required for one molecule of hydrogen. The balanced equation is

$$2\,CH_3{-}C{\equiv}CH + 2\,Na \longrightarrow 2\,CH_3{-}C{\equiv}CNa + H_2.$$

Thus one mole of propyne produces $\frac{1}{2}$ mole of H_2 when sodium is in excess. Only the terminal hydrogen is replaced in this reaction; the hydrogens of the methyl group are not counted as terminal-alkyne acidic hydrogens. Statement I is correct.

Step 2: Calculate the ammonia volume in Statement II The molar mass of propyne is $3(12) + 4(1) = 40\,\text{g mol}^{-1}$, so the given mass corresponds to

$$n(\text{propyne}) = \frac{4}{40} = 0.10\,\text{mol}.$$

The reaction is $CH_3{-}C{\equiv}CH + NaNH_2 \longrightarrow CH_3{-}C{\equiv}CNa + NH_3$. Consequently $n(NH_3) = 0.10\,\text{mol}$, and

$$V(NH_3) = 0.10 \times 22.4 = 2.24\,\text{L} = 2240\,\text{mL}.$$

The claimed $224\,\text{mL}$ is smaller by a factor of ten. Statement II is incorrect. Options (A) and (B) incorrectly reject Statement I, while option (C) incorrectly accepts Statement II.

Step 3: Conclusion Statement I is correct but Statement II is incorrect; ammonia occupies $2240\,\text{mL}$ at STP. Hence, the correct option is (D).

Quick Tip to Solve: For a terminal alkyne, one acidic H gives half a mole of hydrogen with Na but one mole of ammonia with sodium amide per mole of alkyne.

64. Which of the following electronegativity order is incorrect? [4 marks]

(A) $S < Cl < O < F$

(B) $Al < Si < C < N$

(C) $Al < Mg < B < N$

(D) $Mg < Be < B < N$

Correct Answer: (C) $Al < Mg < B < N$

Solution:

Concept:

- Electronegativity measures an atom's tendency to attract the shared electron pair in a bond. It generally increases from left to right across a period and decreases down a group.
- Approximate Pauling values provide a check for elements in different periods: Mg 1.31, Al 1.61, Be 1.57, B 2.04, Si 1.90, C 2.55, N 3.04, S 2.58, Cl 3.16, O 3.44 and F 3.98.

Step 1: Check the first two increasing sequences For option (A), the values follow

$$2.58 < 3.16 < 3.44 < 3.98,$$

which gives $S < Cl < O < F$. Thus (A) is a correct increasing order, even though it compares elements across two periods. For option (B), the values follow

$$1.61 < 1.90 < 2.55 < 3.04,$$

corresponding to $Al < Si < C < N$. This order is also correct. The question asks for the incorrect sequence, so neither is selected.

Step 2: Locate the reversed pair and check the last sequence Across the third period magnesium lies to the left of aluminium, so magnesium is less electronegative. Numerically $1.31 < 1.61$, giving $Mg < Al$. Option (C) instead starts with $Al < Mg$, which reverses this relation. The correct increasing order for its four elements would be $Mg < Al < B < N$. Finally, option (D) gives Mg, Be, B and N, whose values are $1.31 < 1.57 < 2.04 < 3.04$; that option is valid. Hence the only incorrect sequence is (C).

Step 3: Conclusion The inequality $Al < Mg$ is incorrect because magnesium is less electronegative than aluminium. Hence, the correct option is (C).

Quick Tip to Solve: In an ordered chain, one reversed adjacent pair is sufficient to reject the whole chain; check Mg versus Al using their left-to-right positions in period 3.

65. Arrange the following solutions in order of their increasing boiling points.

(i) 10^{-4}MNaCl

(ii) 10^{-4}M **Urea**

(iii) 10^{-3}MNaCl

(iv) 10^{-2}MNaCl [4 marks]

(A) (i) < (ii) < (iii) < (iv)

(B) (ii) < (i) < (iii) < (iv)

(C) (iv) < (iii) < (i) < (ii)

(D) (ii) < (i) ≡ (iii) < (iv)

Correct Answer: (B) (ii) < (i) < (iii) < (iv)

Solution:

Concept:

- Boiling-point elevation obeys $\Delta T_b = iK_b m$, where i is the van't Hoff factor, K_b is the solvent constant and m is molality. More dissolved particles produce a larger elevation.
- For these dilute aqueous solutions, concentration comparisons can use molarity as proportional to molality. Urea is a nonelectrolyte with $i = 1$, whereas dilute sodium chloride dissociates into two ions and has $i \simeq 2$.

Step 1: Compare the effective particle concentrations All solutions have the same solvent, so compare iM, with M denoting the stated molarity. This gives

$$\begin{aligned}(i): &\quad 2 \times 10^{-4},\\ (ii): &\quad 1 \times 10^{-4},\\ (iii): &\quad 2 \times 10^{-3},\\ (iv): &\quad 2 \times 10^{-2}.\end{aligned}$$

Each value is proportional to the concentration of independently moving solute particles. The urea solution has fewer particles than the sodium chloride solution of the same analytical concentration because urea molecules do not dissociate.

Step 2: Convert the particle order to boiling-point order The ascending particle-concentration sequence is

$$10^{-4} < 2 \times 10^{-4} < 2 \times 10^{-3} < 2 \times 10^{-2}.$$

Since K_b is positive and the pure-solvent boiling point is common, the boiling points increase in precisely the same order: (ii), (i), (iii), (iv). Option (A) reverses the two most dilute solutions by overlooking dissociation. Option (C) gives the reverse of the correct sequence. Option (D) makes (i) and (iii) equal even though their sodium chloride concentrations differ by a factor of ten. Thus only option (B) satisfies all four comparisons.

Step 3: Conclusion The increasing boiling-point order is $(ii) < (i) < (iii) < (iv)$. Hence, the correct option is (B).

Quick Tip to Solve: Rank dilute solutions by van't Hoff factor times concentration, rather than concentration alone; dissociation doubles the particle count for NaCl.

66. In which of the following complexes the CFSE, Δ_o will be equal to zero? [4 marks]

(A) $K_3\,[Fe(SCN)_6]$

(B) $[Fe(en)_3]\,Cl_3$

(C) $[Fe\,(NH_3)_6]\,Br_2$

(D) $K_4\,[Fe(CN)_6]$

Correct Answer: (A) $K_3\,[Fe(SCN)_6]$

Solution:

Concept:

- For an octahedral complex, each electron in t_{2g} contributes $-0.4\Delta_o$ and each electron in e_g contributes $+0.6\Delta_o$ relative to the barycentre. Their sum is the orbital crystal-field stabilization energy.
- First find the oxidation state and d-electron count of iron. Weak-field ligands favour high spin; sufficiently strong-field ligands favour low spin. Zero net CFSE does not mean the actual splitting Δ_o vanishes.

Step 1: Find the high-spin half-filled case In $K_3[Fe(SCN)_6]$, the complex ion carries charge -3. Six thiocyanate ligands each contribute -1, so iron is in oxidation state $+3$, with configuration $3d^5$. Thiocyanate is a weak-field ligand in this comparison; the complex is high spin with occupation $t_{2g}^3 e_g^2$. Its net CFSE is

$$3(-0.4\Delta_o) + 2(0.6\Delta_o) = 0.$$

The lowering of three electrons exactly balances the raising of two electrons.

Step 2: Exclude the other complexes In $[Fe(en)_3]Cl_3$, neutral en and three chloride counterions give Fe^{3+}, a d^5 ion; the stronger field of en gives the low-spin t_{2g}^5 arrangement and nonzero CFSE $-2.0\Delta_o$. In $[Fe(NH_3)_6]Br_2$, iron is $+2$, hence d^6. Whether treated as high-spin $t_{2g}^4 e_g^2$ or low-spin t_{2g}^6, its CFSE is nonzero: respectively $-0.4\Delta_o$ or $-2.4\Delta_o$. Finally, $K_4[Fe(CN)_6]$ contains Fe^{2+} with strong-field cyanide; low-spin t_{2g}^6 gives $-2.4\Delta_o$. Thus (B), (C) and (D) do not have zero CFSE.

Step 3: Conclusion $K_3[Fe(SCN)_6]$ has high-spin d^5 occupation and zero net CFSE. Hence, the correct option is (A).

Quick Tip to Solve: For octahedral high-spin d5, remember the cancellation: three lower electrons and two upper electrons give exactly zero net crystal-field stabilization.

67. Which of the following acids is a vitamin ? [4 marks]

(A) Saccharic acid

(B) Ascorbic acid

(C) Aspartic acid

(D) Adipic acid

Correct Answer: (B) Ascorbic acid

Solution:

Concept:

- Vitamins are organic micronutrients required in small amounts for normal physiological functions. Their classification is based on biological identity, not simply on whether their names contain the word acid.
- Vitamin C is ascorbic acid, a water-soluble vitamin. Aspartic acid belongs to the amino acids, whereas saccharic acid and adipic acid are not vitamins.

Step 1: Identify the known vitamin Ascorbic acid is the chemical name of vitamin C. It acts as an antioxidant and is required for the normal formation of connective tissue, including collagen. Its deficiency causes scurvy. These characteristic biological functions identify the acid in option (B) as a vitamin; the acidity of the compound itself is not the deciding property.

Step 2: Distinguish the remaining acids Saccharic acid, also called glucaric acid, is obtained by oxidation of the terminal groups of glucose and is a sugar-derived dicarboxylic acid, not vitamin C. Aspartic acid is an amino acid used in proteins, not a vitamin. Adipic acid is an aliphatic dicarboxylic acid, important in the manufacture of nylon-6,6, rather than a vitamin. Thus options (A), (C) and (D) refer to different chemical categories and do not answer the question.

Step 3: Conclusion Ascorbic acid is vitamin C. Hence, the correct option is (B).

Quick Tip to Solve: Link vitamin C directly with ascorbic acid and scurvy; an acid name alone does not identify a vitamin.

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68. Radius of the first excited state of Helium ion is given as : a_0 → radius of first stationary state of hydrogen atom. [4 marks]

(A) $r = \frac{a_0}{4}$

(B) $r = 2a_0$

(C) $r = 4a_0$

(D) $r = \frac{a_0}{2}$

Correct Answer: (B) $r = 2a_0$

Solution:

Concept:

- A singly ionised helium atom, He^+, has one electron and nuclear charge $Z = 2$, so the Bohr radius formula for a hydrogen-like species applies.

- The radius of orbit n is $r_n = a_0 n^2/Z$, where a_0 is the first-orbit radius of hydrogen. The first excited state has $n = 2$, while the ground state has $n = 1$.

Step 1: Choose the correct state and nuclear charge Helium ion in this hydrogen-like problem means He^+, rather than neutral helium. Its single electron is attracted by a nucleus containing two protons, so use $Z = 2$. First excited state means the electron has moved one level above the ground state: hence $n = 2$, not $n = 1$. No numerical value of a_0 is needed because all the choices are multiples of it.

Step 2: Substitute into the radius relation Using the square dependence on the principal quantum number gives

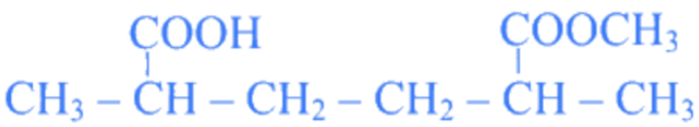
$$r_2 = a_0 \frac{2^2}{2} = 2a_0.$$

Option (D), $a_0/2$, is the ground-state radius of He^+, so it uses the wrong state. Option (C), $4a_0$, would be the first-excited radius of hydrogen and omits the helium nuclear charge. Option (A), $a_0/4$, has neither the required n^2 scaling nor the correct result for $Z = 2$.

Step 3: Conclusion The first-excited-state radius of He^+ is $r = 2a_0$. Hence, the correct option is (B).

Quick Tip to Solve: For a hydrogen-like ion, write n and Z separately before using $r/a_0 = n^2/Z$; first excited always means $n = 2$.

69. The IUPAC name of the following compound is : [4 marks]



(A) 5-Methoxycarbonly-2-methylhexanoic acid

(B) Methyl-5-carboxy-2-methylhexanoate.

(C) 2-Carboxy-5-methoxycarbonylhexane.

(D) Methyl-6-carboxy-2,5-dimethylhexanoate.

Correct Answer: (A) 5-Methoxycarbonly-2-methylhexanoic acid

Solution:

Concept:

- When a molecule contains both $-COOH$ and $-COOCH_3$, the carboxylic acid is the principal functional group and is named by the suffix carboxylic acid or oic acid as appropriate.
- For an alkanoic-acid parent chain, the carbon of $-COOH$ is included in the chain and is carbon 1. An ester substituent $-COOCH_3$ attached through its carbonyl carbon is named methoxycarbonyl.

Step 1: Select and number the acid parent chain Trace a longest chain containing the carboxylic-acid carbon in the given structure. Starting with that carbon, the chain contains, in sequence, the acid carbon, a methyl-bearing carbon, two methylene carbons, an ester-bearing carbon, and a terminal methyl carbon. It contains six carbons, including the acid carbon, so the parent is hexanoic acid. Numbering must begin at the acid group, rather than at the terminal methyl on either side. The

methyl branch is consequently at carbon 2.

Step 2: Name the ester substituent and compare the options The remaining $-COOCH_3$ group is attached to carbon 5 of the hexanoic-acid chain. It is a methoxycarbonyl substituent. Combining the two prefixes with the acid parent gives 5-(methoxycarbonyl)-2-methylhexanoic acid. This corresponds to option (A). Options (B) and (D) instead make a methyl ester the principal named group despite the higher-priority free acid. Option (C) chooses hexane and treats the acid as a carboxy prefix, failing to use the appropriate acid suffix for this chain.

Step 3: Conclusion The name is 5-(methoxycarbonyl)-2-methylhexanoic acid, corresponding to the methoxycarbonyl name in option (A). Hence, the correct option is (A).

Quick Tip to Solve: When acid and ester occur together, begin the parent chain at the $-COOH$ carbon; then locate the $-COOCH_3$ substituent.

70. Which of the following statement is not true for radioactive decay? [4 marks]

(A) Half life is $\ln 2$ times of $\frac{1}{\text{rate constant}}$.

(B) Decay constant increases with increase in temperature.

(C) Decay constant does not depend upon temperature.

(D) Amount of radioactive substance remained after three half lives is $\frac{1}{8}$ th of original amount.

Correct Answer: (B) Decay constant increases with increase in temperature.

Solution:

Concept:

- Radioactive decay follows the first-order law $N = N_0 e^{-\lambda t}$, where N is the number of undecayed nuclei and λ is the decay constant.
- The half-life is $t_{1/2} = \ln 2/\lambda$. In the usual treatment of radioactive decay, the decay constant is a nuclear property and is independent of ordinary changes in temperature.

Step 1: Test the two temperature statements Radioactive decay is a transformation of the nucleus. Heating a sample changes its thermal motion and ordinary chemical reaction rates but does not produce the stated increase in the nuclear decay constant. Thus the claim in option (B), that the decay constant increases with increasing temperature, is not true. Option (C) states the standard temperature independence and is true. Do not apply the Arrhenius temperature dependence of an ordinary chemical reaction to this nuclear-decay question.

Step 2: Verify the half-life and remaining fraction Put $N = N_0/2$ in the decay law. Taking logarithms yields $\lambda t_{1/2} = \ln 2$, so $t_{1/2} = (\ln 2)(1/\lambda)$, exactly as in option (A). After each half-life, one half of the amount present survives; after three half-lives the remaining fraction is

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

Therefore option (D) is also true. Since the question asks for the statement that is not true, the incorrect temperature claim must be selected.

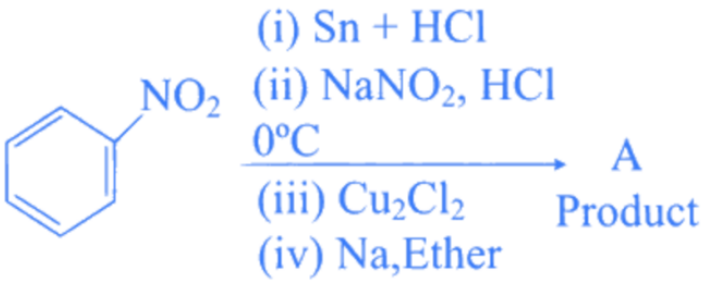
Step 3: Conclusion The statement that the decay constant increases with temperature is not true for radioactive decay in this treatment. Hence, the correct option is (B).

Quick Tip to Solve: Look for the false statement first: $t_{1/2} = 0.693/\lambda$ and repeated halving are valid, while ordinary temperature dependence is not.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. Consider the following sequence of reactions :

Molar mass of the product formed (A) **is** _____ $\mathrm{g\ mol^{-1}}$**.** [4 marks]



Solution:

Concept:

- $\mathrm{Sn/HCl}$ reduces an aromatic nitro group to a primary amino group. Aniline forms a benzenediazonium salt with $\mathrm{NaNO_2/HCl}$ at about $0\,^\circ\mathrm{C}$.
- The Sandmeyer reaction replaces the diazonium group by chlorine using cuprous chloride. An aryl halide couples with sodium in dry ether in the Fittig reaction to form a biaryl.

Step 1: Follow reduction, diazotisation and substitution The starting compound is nitrobenzene. Reduction with $\mathrm{Sn/HCl}$ gives aniline, $\mathrm{C_6H_5NH_2}$. Treatment with nitrous acid generated from $\mathrm{NaNO_2}$ and hydrochloric acid at the stated low temperature converts this to benzenediazonium chloride, $\mathrm{C_6H_5N_2{}^+Cl^-}$. Cuprous chloride, written as $\mathrm{Cu_2Cl_2}$ in the sequence, then replaces the diazonium group by chlorine with loss of nitrogen. The intermediate before the sodium step is therefore chlorobenzene, $\mathrm{C_6H_5Cl}$.

Step 2: Identify the coupling product and calculate its molar mass In dry ether sodium couples two chlorobenzene molecules through the carbons originally bearing chlorine:

$$2\,\mathrm{C_6H_5Cl} + 2\,\mathrm{Na} \longrightarrow \mathrm{C_6H_5{-}C_6H_5} + 2\,\mathrm{NaCl}.$$

The product (A) is biphenyl, with molecular formula $\mathrm{C_{12}H_{10}}$. Counting the atoms in both joined phenyl groups, and using the usual atomic masses $\mathrm{C} = 12$ and $\mathrm{H} = 1$, gives

$$M = 12(12) + 10(1) = 144 + 10 = 154\,\mathrm{g\,mol^{-1}}.$$

The final step couples two rings; stopping at chlorobenzene would miss the product actually requested.

Step 3: Conclusion Product (A) is biphenyl, $C_{12}H_{10}$, of molar mass $\boxed{154\,g\,mol^{-1}}$. The required numerical value is 154.

Quick Tip to Solve: Track the attached group as $NO_2 \longrightarrow NH_2 \longrightarrow N_2^+ \longrightarrow Cl$, then remember that sodium in dry ether couples two aryl groups.

72. The number of molecules/ions that show linear geometry among the following is _____.

$$SO_2, BeCl_2, CO_2,\ N_3^-, NO_2,\ F_2O, XeF_2, NO_2^+, I_3^-, O_3$$

[4 marks]

Solution:

Concept:

- VSEPR theory determines molecular geometry from the arrangement of bonding domains and lone pairs around the central atom. A multiple bond counts as one electron domain for this purpose.
- Two bonding domains without central lone pairs give a linear AX_2 molecule. A species of type AX_2E_3 also has linear molecular geometry: three equatorial lone pairs leave the two bonds in opposite axial positions.

Step 1: Identify the straightforward linear species In $BeCl_2$, the central Be has two bond domains and no lone pair in the molecular VSEPR description, giving a linear arrangement. In CO_2, the two double bonds give two domains about carbon, so the molecule is linear. The azide ion N_3^- has a linear three-nitrogen skeleton described by resonance, with two bond directions around its central nitrogen. The nitronium ion NO_2^+ likewise has two bonding domains about nitrogen and no central lone pair. These supply four linear species.

Step 2: Include lone-pair linear species and exclude the bent ones XeF_2 has two bond pairs and three lone pairs on xenon; its trigonal-bipyramidal electron-domain arrangement places all three lone pairs equatorially, leaving a linear F–Xe–F geometry. I_3^- has the same AX_2E_3 arrangement and is also linear. Conversely, SO_2 and O_3 have bent molecular shapes owing to a central lone pair. F_2O, or OF_2, is bent owing to two lone pairs on oxygen. Neutral NO_2 is a bent odd-electron molecule and must not be confused with linear NO_2^+. Thus the remaining four entries do not increase the count.

Step 3: Conclusion The linear species are $BeCl_2$, CO_2, N_3^-, XeF_2, NO_2^+ and I_3^-. Their number is $\boxed{6}$ (a dimensionless count).

Quick Tip to Solve: Count both AX_2 and AX_2E_3 species; check the charge carefully because NO_2 is bent but NO_2^+ is linear.

73. Some CO_2 gas was kept in a sealed container at a pressure of 1 atm and at 273 K . This entire amount of CO_2 gas was later passed through an aqueous solution of $Ca(OH)_2$. The excess unreacted $Ca(OH)_2$ was later neutralized with 0.1 M of 40 mL HCl . If the volume of the sealed container of CO_2 was x, then x is _____ cm^3 (nearest integer). [Given : The entire amount of $CO_2(g)$ reacted with exactly half the initial amount of $Ca(OH)_2$ present in the aqueous solution.] [4 marks]

Solution:

Concept:

- Carbon dioxide reacts with calcium hydroxide in a one-to-one mole ratio: $CO_2 + Ca(OH)_2 \longrightarrow CaCO_3 + H_2O$. The consumed amount of base therefore measures the amount of gas.
- Back titration measures the base left after absorption. Each mole of $Ca(OH)_2$ requires two moles of hydrochloric acid: $Ca(OH)_2 + 2\,HCl \longrightarrow CaCl_2 + 2\,H_2O$.
- At $273\,K$ and $1\,atm$, an ideal gas occupies approximately $22.4\,L\,mol^{-1}$. The gas volume follows from its mole amount, not from the volume of acid used.

Step 1: Find the amount of unreacted base Convert the acid volume to litres before using its molarity. The titration uses

$$n(HCl) = 0.1\,mol\,L^{-1} \times 0.040\,L = 0.0040\,mol.$$

Since two moles of acid neutralise one mole of calcium hydroxide, the base remaining after the gas has reacted is

$$n_{remaining}(Ca(OH)_2) = \frac{0.0040}{2} = 0.0020\,mol.$$

Step 2: Use the stated half-consumption condition The gas consumed exactly half the initial base. Thus the consumed half and the unreacted half are equal; the initial base was $0.0040\,mol$, but only $0.0020\,mol$ reacted with the gas. From the one-to-one absorption equation,

$$n(CO_2) = n_{consumed}(Ca(OH)_2) = 0.0020\,mol.$$

Using all the initial base instead would double the gas amount incorrectly. The given condition converts the back-titration result into the consumed amount.

Step 3: Calculate and round the gas volume At the stated pressure and temperature, multiply the amount of carbon dioxide by the standard molar volume:

$$V = 0.0020 \times 22.4 = 0.0448\,L = 44.8\,cm^3.$$

Here $1\,L = 1000\,cm^3$. The container held this volume of gas at its original conditions, so round only the final value to the requested nearest integer.

Step 4: Conclusion The volume of the sealed container is $x = \boxed{45\,cm^3}$, to the nearest integer.

Quick Tip to Solve: In this half-consumption back titration, the remaining and consumed base amounts are equal. First divide the HCl amount by two, then use that same amount for carbon dioxide.

74. In Carius method for estimation of halogens, 180 mg of an organic compound produced 143.5 mg of AgCl . The percentage composition of chlorine in the compound is _____ %. (Given : molar mass in $\mathrm{g\ mol^{-1}}$ of $\mathrm{Ag : 108, Cl : 35.5}$) [4 marks]

Solution:

Concept:

- In the Carius method, chlorine from an organic sample is converted quantitatively into silver chloride. Each mole of AgCl contains one mole of chlorine atoms, so its precipitate mass determines the chlorine mass.
- The mass fraction of chlorine in pure silver chloride is $M(\mathrm{Cl})/M(\mathrm{AgCl})$, where M denotes molar mass. The percentage of chlorine in the original sample is its chlorine mass divided by the sample mass, multiplied by 100.

Step 1: Calculate the molar mass of silver chloride Add the supplied atomic molar masses; there is one silver atom and one chlorine atom in each formula unit:

$$M(\mathrm{AgCl}) = 108 + 35.5 = 143.5\,\mathrm{g\,mol^{-1}}.$$

The chlorine contribution is only 35.5 out of 143.5. The full precipitate mass cannot be counted as chlorine, because the silver was supplied by the analytical reagent.

Step 2: Find the chlorine mass in the precipitate Convert the mass of precipitate to grams and then use the one-to-one mole ratio:

$$n(\mathrm{AgCl}) = \frac{143.5 \times 10^{-3}}{143.5} = 1.00 \times 10^{-3}\,\mathrm{mol}.$$

Therefore the sample supplied $1.00 \times 10^{-3}\,\mathrm{mol}$ of chlorine atoms, whose mass is

$$m(\mathrm{Cl}) = 1.00 \times 10^{-3} \times 35.5 = 0.0355\,\mathrm{g} = 35.5\,\mathrm{mg}.$$

This also follows directly by multiplying $143.5\,\mathrm{mg}$ by the mass fraction $35.5/143.5$.

Step 3: Express the chlorine mass as a percentage of the sample The original organic compound weighed $180\,\mathrm{mg}$. Use the same mass units in the numerator and denominator so that they cancel:

$$\%\mathrm{Cl} = \frac{35.5\,\mathrm{mg}}{180\,\mathrm{mg}} \times 100 = 19.7222\ldots\%.$$

This is below 20%, as expected because the chlorine mass is slightly below one fifth of the sample mass. A ratio such as $143.5/180$ would incorrectly include the added silver in the sample composition.

Step 4: Conclusion The chlorine content is $\boxed{19.72\%}$, corresponding to $\boxed{20\%}$ when expressed as an integer numerical answer.

Quick Tip to Solve: Multiply the silver chloride mass by $35.5/143.5$ first. Only this chlorine mass, not the whole precipitate, belongs in the percentage calculation.

75.

$$\mathrm{A} \rightarrow \mathrm{B}$$

The molecule A changes into its isomeric form B by following a first order kinetics at a temperature of 1000 K . If the energy barrier with respect to reactant energy for such isomeric transformation is $191.48\ \mathrm{kJ\ mol^{-1}}$ and the frequency factor is 10^{20}, the time required for 50% molecules of A to become B is _____ picoseconds (nearest integer). $\left[\mathrm{R} = 8.314\ \mathrm{J\ K^{-1}\ mol^{-1}}\right]$ [4 marks]

Solution:

Concept:

- The Arrhenius equation is $k = A\exp[-E_a/(RT)]$, where k is the rate constant, A is the frequency factor, E_a is the activation energy, R is the gas constant and T is absolute temperature.
- For first-order kinetics, the time for half the initial molecules to react is $t_{1/2} = \ln 2/k$. It is independent of initial concentration. With k in $\mathrm{s^{-1}}$, this gives time in seconds; $1\,\mathrm{ps} = 10^{-12}\,\mathrm{s}$.

Step 1: Put the activation energy in consistent units Since the supplied gas constant is in joules, convert the energy barrier before forming the dimensionless exponent:

$$E_a = 191.48\,\mathrm{kJ\,mol^{-1}} = 191480\,\mathrm{J\,mol^{-1}}.$$

The temperature is already absolute, $T = 1000\,\mathrm{K}$. Thus

$$\frac{E_a}{RT} = \frac{191480}{8.314 \times 1000} = 23.03103199\ldots.$$

Step 2: Evaluate the rate constant without premature rounding For a first-order transformation the frequency factor has units of reciprocal seconds. Substitution gives

$$k = 10^{20}\exp(-23.03103199\ldots) = 9.94832334 \times 10^{9}\,\mathrm{s^{-1}}.$$

Retain the calculated rate constant until the final rounding.

Step 3: Find the half-life in picoseconds A conversion of 50% leaves half the original molecules of A, so the requested time is its half-life:

$$t_{1/2} = \frac{0.6931471806}{9.94832334 \times 10^{9}} = 6.96747740 \times 10^{-11}\,\mathrm{s}.$$

Converting seconds to picoseconds gives

$$t_{1/2} = 6.96747740 \times 10^{-11} \times 10^{12} = 69.6747740\,\mathrm{ps}.$$

The fractional part exceeds 0.5, so the nearest integer is 70.

Step 4: Conclusion Using the printed $E_a = 191.48\,\mathrm{kJ\,mol^{-1}}$, $R = 8.314\,\mathrm{J\,K^{-1}\,mol^{-1}}$ and $T = 1000\,\mathrm{K}$, the time is $69.674774\,\mathrm{ps}$, so the nearest-integer answer is $\boxed{70\,\mathrm{ps}}$. The answer key gives 69; that value follows from approximating k as exactly $10^{10}\,\mathrm{s^{-1}}$, whereas direct evaluation of the printed constants gives 70.

Quick Tip to Solve: Retain the full Arrhenius exponent until the final integer rounding: a small rounding of the rate constant here moves the half-life across the $69.5\,\mathrm{ps}$ boundary.