JEE Main 2025 Paper 1 (B.E./B.Tech) Jan 22 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 22 January 2025, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let α_θ and β_θ be the distinct roots of $2x^2 + (\cos\theta)x - 1 = 0, \theta \in (0, 2\pi)$. If m and M are the minimum and the maximum values of $\alpha_\theta^4 + \beta_\theta^4$, then $16(M + m)$ equals : [4 marks]

(A) 27

(B) 17

(C) 25

(D) 24

Correct Answer: (C) 25

Solution:

Concept:

- For a quadratic $ax^2 + bx + c = 0$, the sum of its roots is $-b/a$, and their product is c/a. Symmetric powers can therefore be computed without finding the individual roots.
- If $S_2 = \alpha^2 + \beta^2$ and $p = \alpha\beta$, then $S_2 = (\alpha + \beta)^2 - 2p$ and $\alpha^4 + \beta^4 = S_2^2 - 2p^2$.

Step 1: Express the fourth-power sum using the coefficients Put $t = \cos^2\theta$. The given quadratic has $\alpha_\theta + \beta_\theta = -\cos\theta/2$ and $\alpha_\theta\beta_\theta = -1/2$. Consequently,

$$S_2 = 1 + \frac{t}{4}, \qquad \alpha_\theta^4 + \beta_\theta^4 = \left(1 + \frac{t}{4}\right)^2 - \frac{1}{2}.$$

Its discriminant is $\cos^2\theta + 8 > 0$, so the roots are indeed distinct for every permitted angle.

Step 2: Find the attained minimum and maximum The interval $\theta \in (0, 2\pi)$ gives $0 \le t \le 1$. Both endpoints occur: $t = 0$ at $\theta = \pi/2$, and $t = 1$ at $\theta = \pi$. The expression increases with t, since $1 + t/4$ is positive and increasing. Thus

$$m = 1 - \frac{1}{2} = \frac{1}{2}, \qquad M = \left(\frac{5}{4}\right)^2 - \frac{1}{2} = \frac{17}{16}.$$

The value 17 in option (B) is only $16M$; the question requires the sum of both extrema. Options (A) and (D) likewise do not equal that sum.

Step 3: Conclusion

$$16(M + m) = 16\left(\frac{17}{16} + \frac{8}{16}\right) = 25.$$

Hence, the correct option is (C).

Quick Tip to Solve: For even powers of quadratic roots, compute the sum of squares first; then optimize in $\cos^2\theta$, checking that the endpoint angles are allowed.

2. Let a line pass through two distinct points $P(-2,-1,3)$ and Q, and be parallel to the vector $3\hat{i}+2\hat{j}+2\hat{k}$. If the distance of the point Q from the point $\mathrm{R}(1,3,3)$ is 5 , then the square of the area of $\triangle PQR$ is equal to : [4 marks]

(A) 148

(B) 144

(C) 136

(D) 140

Correct Answer: (C) 136

Solution:

Concept:

- A line through position vector $\mathbf{p}$, parallel to $\mathbf{d}$, has points $\mathbf{p}+t\mathbf{d}$. A distance condition determines the parameter t.
- The area of triangle PQR is $\frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{PR}|$, so its square is one fourth of the squared magnitude of the cross product.

Step 1: Determine the point on the line Take $\mathbf{d}=(3,2,2)$. Then $Q=(-2+3t,-1+2t,3+2t)$ and $\overrightarrow{RQ}=(3t-3,2t-4,2t)$. Using the given distance,

$$(3t-3)^2+(2t-4)^2+(2t)^2=25.$$

Expanding gives $17t^2-34t+25=25$, hence $17t(t-2)=0$. The root $t=0$ would make $Q=P$, contrary to the distinct-point condition. Therefore $t=2$, and $Q=(4,3,7)$.

Step 2: Compute and square the triangular area Now $\overrightarrow{PQ}=(6,4,4)$, while $\overrightarrow{PR}=(3,4,0)$. Their cross product is

$$\overrightarrow{PQ}\times\overrightarrow{PR}=(-16,12,12).$$

It follows that

$$[\triangle PQR]^2=\frac{1}{4}(256+144+144)=136.$$

As a check, $|PQ|^2=68$, $|PR|^2=25$, and $PQ\cdot PR=34$. The cross-product identity gives the same result:

$$|PQ\times PR|^2=68\cdot 25-34^2=544.$$

Thus 148, 144 and 140 in the other options fail the cross-product identity.

Step 3: Conclusion The square of the area of $\triangle PQR$ is 136 in fourth-power coordinate units. Hence, the correct option is (C).

Quick Tip to Solve: Do not keep the parameter root that makes the two stated distinct points coincide. For an area squared, square the cross product directly to avoid unnecessary radicals.

3. If the system of linear equations :

$$x + y + 2z = 6$$
$$2x + 3y + \mathrm{az} = \mathrm{a} + 1$$
$$-x - 3y + \mathrm{b}z = 2\ \mathrm{b}$$

where $a, b \in$ R, has infinitely many solutions, then $7a + 3b$ is equal to : [4 marks]

(A) 12

(B) 9

(C) 22

(D) 16

Correct Answer: (D) 16

Solution:

Concept:

- A system of three equations in three unknowns has infinitely many solutions when the coefficient rank equals the augmented rank and is less than three.
- Elimination reveals both requirements: a zero coefficient row must also have zero on the right-hand side. A zero determinant alone can instead describe an inconsistent system.

Step 1: Eliminate the first unknown Subtract twice the first equation from the second, and add the first equation to the third. This gives

$$y + (a-4)z = a - 11, \qquad -2y + (b+2)z = 2b + 6.$$

Adding twice the first of these reduced equations to the second eliminates y:

$$(2a + b - 6)z = 2a + 2b - 16.$$

The first two original coefficient rows are independent for every a: their first two entries are $(1, 1)$ and $(2, 3)$, which are not proportional. Thus their rank is always at least two.

Step 2: Enforce dependence without contradiction To leave a free variable, the final reduced equation must be the identity $0 = 0$. Therefore

$$2a + b - 6 = 0, \qquad 2a + 2b - 16 = 0.$$

The second relation is $a + b = 8$. Subtracting this from $2a + b = 6$ yields $a = -2$, and then $b = 10$. These values are sufficient: the reduced equations become $y - 6z = -13$ and $-2y + 12z = 26$, so one is minus twice the other and z remains free. Consequently both ranks equal two, proving infinitely many solutions rather than no solution.

Step 3: Conclusion

$$7a + 3b = 7(-2) + 3(10) = 16.$$

The other offered values 12, 9 and 22 do not satisfy this evaluation for the uniquely determined parameter pair. Hence, the correct option is (D).

Quick Tip to Solve: For infinitely many solutions, set both the last coefficient and its right-hand side to zero after elimination; setting only the determinant to zero misses the consistency condition.

4. The sum of all values of $\theta \in [0, 2\pi]$ satisfying $2\sin^2\theta = \cos 2\theta$ and $2\cos^2\theta = 3\sin\theta$ is [4 marks]

(A) π

(B) $\frac{5\pi}{6}$

(C) $\frac{\pi}{2}$

(D) 4π

Correct Answer: (A) π

Solution:

Concept:

- Both given equations must hold for the same angle. Solve one to restrict $\sin\theta$, then use the other to eliminate any extraneous possibilities.
- The identities $\cos 2\theta = 1 - 2\sin^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$ express the two equations in one trigonometric quantity.

Step 1: Use the first equation to restrict the sine From $2\sin^2\theta = \cos 2\theta$, substitute the double-angle identity:

$$2\sin^2\theta = 1 - 2\sin^2\theta, \qquad 4\sin^2\theta = 1.$$

Thus $\sin\theta = \pm 1/2$, and necessarily $\cos^2\theta = 3/4$. The first equation alone allows angles in all four quadrants, so these candidates cannot yet be added. We must retain only those that satisfy the second equation as well.

Step 2: Apply the second equation and list the common angles Substitute $\cos^2\theta = 3/4$ into $2\cos^2\theta = 3\sin\theta$. It gives

$$\frac{3}{2} = 3\sin\theta, \qquad \sin\theta = \frac{1}{2}.$$

The negative sine candidates are excluded. In the closed interval $[0, 2\pi]$, the two remaining angles are $\pi/6$ and $5\pi/6$. The endpoints 0 and 2π have zero sine and fail the first equation. Both retained angles have $\sin^2\theta = 1/4$, $\cos 2\theta = 1/2$ and $\cos^2\theta = 3/4$, verifying the two original equations directly. Option (B) gives only one angle rather than their sum, while (C) and (D) do not equal the sum of the common solutions.

Step 3: Conclusion

$$\sum\theta = \frac{\pi}{6} + \frac{5\pi}{6} = \pi.$$

Hence, the correct option is (A).

Quick Tip to Solve: With simultaneous trigonometric equations, intersect the solution sets before adding the angles. Squared trigonometric functions can hide a sign restriction supplied by the other equation.

5. Let α, β, γ and δ be the coefficients of x^7, x^5, x^3 and x respectively in the expansion of

$$\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5, x>1.$$ **If u and v satisfy the equations**

$$\alpha u+\beta v=18,$$

$$\gamma u+\delta v=20,$$

then $\mathrm{u}+\mathrm{v}$ **equals :** [4 marks]

(A) 4

(B) 3

(C) 5

(D) 8

Correct Answer: (C) 5

Solution:

Concept:

- In $(x+y)^5+(x-y)^5$, all terms involving odd powers of y cancel, while the even-power terms double.
- Set $y=\sqrt{x^3-1}$. Since $x>1$, this radical is real, and $y^2=x^3-1$, $y^4=(x^3-1)^2$. These substitutions turn the sum into a polynomial.

Step 1: Expand the symmetric expression The surviving binomial terms are

$$(x+y)^5+(x-y)^5=2\left(x^5+10x^3y^2+5xy^4\right).$$

Substitute for y^2 and y^4, and expand:

$$2x^5+20x^3(x^3-1)+10x(x^3-1)^2=10x^7+20x^6+2x^5 \\ -20x^4-20x^3+10x.$$

Reading the requested powers, rather than consecutive coefficients, gives $\alpha=10$, $\beta=2$, $\gamma=-20$ and $\delta=10$. In particular, the coefficient of x^3 is negative.

Step 2: Solve for the two parameters The two given equations therefore become

$$10u+2v=18, \qquad -20u+10v=20.$$

Divide by 2 and 10 respectively to get $5u+v=9$ and $-2u+v=2$. Subtracting the latter from the former gives $7u=7$, so $u=1$ and $v=4$. Substitution checks both original equations: $10+8=18$ and $-20+40=20$. Option (A), 4, is merely v, not the requested sum. The alternatives 3 and 8 also disagree with the uniquely determined values of u and v.

Step 3: Conclusion

$$u + v = 1 + 4 = 5.$$

Hence, the correct option is (C).

Quick Tip to Solve: Add the conjugate binomial expansions before substituting the radical. Then identify coefficients by the specified powers; do not mistake the polynomial's x^6 term for the requested x^5 term.

6. The perpendicular distance, of the line $\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+3}{2}$ from the point $\mathrm{P}(2, -10, 1)$, is : [4 marks]

(A) 6

(B) $4\sqrt{3}$

(C) $3\sqrt{5}$

(D) $5\sqrt{2}$

Correct Answer: (C) $3\sqrt{5}$

Solution:

Concept:

- For a line through A, parallel to a nonzero vector $\mathbf{d}$, the perpendicular distance from P is $|\overrightarrow{AP} \times \mathbf{d}|/|\mathbf{d}|$.
- Equivalently, distance squared is $|\overrightarrow{AP}|^2 - (\overrightarrow{AP} \cdot \mathbf{d})^2/|\mathbf{d}|^2$: subtract the squared projection along the line from the full squared displacement.

Step 1: Identify a point and the direction vector From the symmetric equation, choose $A = (1, -2, -3)$ on the line and $\mathbf{d} = (2, -1, 2)$ as its direction. For $P = (2, -10, 1)$,

$$\overrightarrow{AP} = P - A = (1, -8, 4), \qquad |\mathbf{d}|^2 = 4 + 1 + 4 = 9.$$

The signs follow from $y + 2 = y - (-2)$ and $z + 3 = z - (-3)$; the line's base point does not have coordinates $(1, 2, 3)$. Also, the direction vector is not a unit vector, so its magnitude must be included in the distance formula.

Step 2: Evaluate the perpendicular component Compute the cross product:

$$\overrightarrow{AP} \times \mathbf{d} = (-12, 6, 15), \qquad |\overrightarrow{AP} \times \mathbf{d}|^2 = 144 + 36 + 225 = 405.$$

Hence the required distance squared is $405/9 = 45$. To check independently, $|AP|^2 = 1 + 64 + 16 = 81$ and $AP \cdot \mathbf{d} = 2 + 8 + 8 = 18$. The projection formula gives $81 - 18^2/9 = 81 - 36 = 45$, in agreement. The squared distances corresponding to (A), (B) and (D) are 36, 48 and 50, respectively, so none has the required perpendicular component.

Step 3: Conclusion The perpendicular distance is $\sqrt{45} = 3\sqrt{5}$ coordinate units. Hence, the correct option is (C).

Quick Tip to Solve: Read the base point with its signs from the symmetric line equation. Compare squared distances first, and take the square root only after subtracting the parallel projection.

7. If $x = f(y)$ is the solution of the differential equation $(1+y^2) + \left(x - 2e^{\tan^{-1} y}\right)\frac{dy}{dx} = 0, y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ with $f(0) = 1$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is equal to : [4 marks]

(A) $e^{\pi/4}$

(B) $e^{\pi/12}$

(C) $e^{\pi/6}$

(D) $e^{\pi/3}$

Correct Answer: (C) $e^{\pi/6}$

Solution:

Concept:

- Treat x as a function of y. A first-order linear equation $x' + P(y)x = Q(y)$ is solved using the integrating factor $\exp(\int P(y)\,dy)$.
- Here the substitution $t = \tan^{-1} y$ is useful because $dt = dy/(1+y^2)$. The initial condition fixes the one arbitrary constant.

Step 1: Rewrite the equation in linear form Multiply the given differential equation by dx/dy. Since $1 + y^2$ is positive, the resulting equation is

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{2e^{\tan^{-1} y}}{1+y^2}.$$

We have not interchanged the dependent variables arbitrarily: the question already specifies $x = f(y)$. The integrating factor is therefore $e^{\tan^{-1} y}$, with positive rather than negative exponent.

Step 2: Integrate and apply the initial condition Put $t = \tan^{-1} y$ and multiply the linear equation by e^t. Its left side becomes an exact product derivative:

$$\frac{d}{dy}(xe^t) = \frac{2e^{2t}}{1+y^2}.$$

Integration with respect to y, using $dt = dy/(1+y^2)$, gives

$$xe^t = \int 2e^{2t}\,dt = e^{2t} + C.$$

At $y = 0$, we have $t = 0$ and $x = 1$, so $1 = 1 + C$ and $C = 0$. Consequently $f(y) = e^{\tan^{-1} y}$. This also directly satisfies the original equation.

Step 3: Evaluate at the required argument The principal inverse tangent gives $\tan^{-1}(1/\sqrt{3}) = \pi/6$. Thus

$$f(1/\sqrt{3}) = e^{\pi/6}.$$

The exponents $\pi/4$, $\pi/12$, and $\pi/3$ in the other choices do not equal this inverse-tangent value; there is no extra factor of two after division by the integrating factor.

Step 4: Conclusion The required value is $e^{\pi/6}$. Hence, the correct option is (C).

Quick Tip to Solve: When the equation is linear in x rather than y, first use dx/dy; then recognise $dy/(1+y^2) = d(\tan^{-1} y)$.

8. If A and B are two events such that $P(A \cap B) = 0.1$, and $P(A \mid B)$ and $P(B \mid A)$ are the roots of the equation $12x^2 - 7x + 1 = 0$, then the value of $\frac{P(\bar{A} \cup \bar{B})}{P(\bar{A} \cap \bar{B})}$ is : [4 marks]

(A) $\frac{4}{3}$

(B) $\frac{7}{4}$

(C) $\frac{9}{4}$

(D) $\frac{5}{3}$

Correct Answer: (C) $\frac{9}{4}$

Solution:

Concept:

- Conditional probability gives $P(A \mid B) = P(A \cap B)/P(B)$ and $P(B \mid A) = P(A \cap B)/P(A)$.
- De Morgan's laws give $\bar{A} \cup \bar{B} = \overline{A \cap B}$ and $\bar{A} \cap \bar{B} = \overline{A \cup B}$. Also, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Step 1: Find the two conditional probabilities Factor the quadratic exactly:

$$12x^2 - 7x + 1 = (3x - 1)(4x - 1).$$

Its two roots are $1/3$ and $1/4$. The statement does not specify which root belongs to which conditional probability. There is no need to choose an ordering because the final expression is symmetric in A and B.

Step 2: Recover the marginal and union probabilities Using $P(A \cap B) = 1/10$, the two marginal probabilities are

$$\{P(A), P(B)\} = \left\{\frac{1/10}{1/3}, \frac{1/10}{1/4}\right\} = \{3/10, 4/10\}.$$

Therefore their sum is $7/10$, regardless of the ordering. Subtract the intersection once when forming the union:

$$P(A \cup B) = \frac{3}{10} + \frac{4}{10} - \frac{1}{10} = \frac{6}{10}.$$

This subtraction prevents the outcomes common to both events from being counted twice.

Step 3: Apply the complement identities and divide The numerator is the complement of the intersection, whereas the denominator is the complement of the union. Hence

$$P(\bar{A} \cup \bar{B}) = 1 - \frac{1}{10} = \frac{9}{10},$$

$$P(\bar{A} \cap \bar{B}) = 1 - \frac{6}{10} = \frac{4}{10}.$$

Their ratio is $(9/10)/(4/10) = 9/4$. The other values $4/3$, $7/4$, and $5/3$ cannot result from these two complement probabilities; in particular, $7/10$ is the sum of marginals and is not the required numerator.

Step 4: Conclusion The requested probability ratio is $9/4$. Hence, the correct option is (C).

Quick Tip to Solve: Before calculating a ratio involving barred events, use De Morgan's laws: the union of complements complements the intersection, and conversely.

9. For a 3×3 matrix M, let trace (M) denote the sum of all the diagonal elements of M. Let A be a 3×3 matrix such that $|A| = \frac{1}{2}$ and trace $(A) = 3$. If $B = \text{adj}(\text{adj}(2A))$, then the value of $|B|+$ trace (B) equals : [4 marks]

(A) 56

(B) 132

(C) 174

(D) 280

Correct Answer: (D) 280

Solution:

Concept:

- For an invertible $n \times n$ matrix C, $\text{adj}\, C = |C|C^{-1}$, and $\text{adj}(\text{adj}\, C) = |C|^{n-2}C$.
- For a 3×3 matrix, $|kA| = k^3|A|$, while $\text{trace}(kA) = k\,\text{trace}(A)$. Determinant and trace therefore scale differently.

Step 1: Apply the double-adjugate identity Set $C = 2A$. Since $|A| = 1/2 \neq 0$, both A and C are invertible, so the adjugate identities apply. For order three,

$$\text{adj}(\text{adj}\, C) = |C|C.$$

For completeness, this follows from $|\,\text{adj}\, C| = |C|^2$ and $(\text{adj}\, C)^{-1} = C/|C|$. Multiplying these two expressions yields $|C|C$, with no inverse of C remaining.

Step 2: Express the matrix B as a multiple of A Calculate the determinant of $2A$ using its order, not just the scalar 2:

$$|C| = |2A| = 2^3 \cdot \frac{1}{2} = 4.$$

Consequently

$$B = |C|C = 4(2A) = 8A.$$

This identity determines everything asked even though the individual entries of A have not been supplied. No assumption that A is diagonal or symmetric is necessary.

Step 3: Find the determinant and trace separately The determinant of B is

$$|B| = |8A| = 8^3|A| = 512 \cdot \frac{1}{2} = 256.$$

Its trace is instead linear:

$$\text{trace}(B) = 8\,\text{trace}(A) = 8 \cdot 3 = 24.$$

Adding gives $256 + 24 = 280$. The values 56, 132 and 174 disagree with this sum. Scaling the determinant linearly, or scaling the trace cubically, would violate their respective definitions and produces spurious alternatives.

Step 4: Conclusion The requested determinant-plus-trace sum is 280. Hence, the correct option is (D).

Quick Tip to Solve: Reduce a double adjugate to a scalar times the original matrix first; then use cubic scaling for a third-order determinant and linear scaling for trace.

10. If $\int e^x \left(\frac{x\sin^{-1}x}{\sqrt{1-x^2}} + \frac{\sin^{-1}x}{(1-x^2)^{3/2}} + \frac{x}{1-x^2}\right) dx = g(x) + C$, where C is the constant of integration, then $g\left(\frac{1}{2}\right)$ equals : [4 marks]

(A) $\frac{\pi}{6}\sqrt{\frac{e}{3}}$
(B) $\frac{\pi}{6}\sqrt{\frac{e}{2}}$
(C) $\frac{\pi}{4}\sqrt{\frac{e}{3}}$
(D) $\frac{\pi}{4}\sqrt{\frac{e}{2}}$

Correct Answer: (A) $\frac{\pi}{6}\sqrt{\frac{e}{3}}$

Solution:

Concept:

- The product rule gives $\frac{d}{dx}(e^x h(x)) = e^x(h(x) + h\prime(x))$. An integrand containing e^x often has this structure.
- For $|x| < 1$, $\frac{d}{dx}(\sin^{-1}x) = 1/\sqrt{1-x^2}$ and $\frac{d}{dx}[x/\sqrt{1-x^2}] = (1-x^2)^{-3/2}$.

Step 1: Identify the function paired with its derivative Take

$$h(x) = \frac{x\sin^{-1}x}{\sqrt{1-x^2}}.$$

This is exactly the first term inside the parentheses of the integrand. Differentiate it as the product of $\sin^{-1}x$ and $x(1-x^2)^{-1/2}$. In the second factor, combining the two derivative terms gives

$$\frac{d}{dx}\left(\frac{x}{\sqrt{1-x^2}}\right) = \frac{1-x^2+x^2}{(1-x^2)^{3/2}} = \frac{1}{(1-x^2)^{3/2}}.$$

Step 2: Verify the exact antiderivative The product rule now yields

$$h\prime(x) = \frac{\sin^{-1}x}{(1-x^2)^{3/2}} + \frac{x}{1-x^2}.$$

Thus all three printed terms are precisely $h + h\prime$, and the integral is

$$e^x h(x) + C = \frac{e^x x\sin^{-1}x}{\sqrt{1-x^2}} + C.$$

The function g is the displayed antiderivative with its arbitrary additive constant represented separately by C. Differentiation checks the result exactly on $(-1, 1)$, which includes the required argument.

Step 3: Substitute one-half and compare the choices Use $\sin^{-1}(1/2) = \pi/6$ and $\sqrt{1-1/4} = \sqrt{3}/2$. Then

$$g(1/2) = \frac{\sqrt{e}\,(1/2)(\pi/6)}{\sqrt{3}/2} = \frac{\pi}{6}\sqrt{\frac{e}{3}}.$$

The factor $\pi/4$ would use an incorrect inverse-sine value, while the denominator $\sqrt{2}$ would use an incorrect value for $\sqrt{1-x^2}$. This excludes each of the other three expressions.

Step 4: Conclusion The required value is $\frac{\pi}{6}\sqrt{e/3}$. Hence, the correct option is (A).

Quick Tip to Solve: For e^x times several terms, try taking one term as h and check whether the rest form $h\prime$; this avoids repeated integration by parts.

11. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{3}$. If $\lambda\vec{a}+2\vec{b}$ and $3\vec{a}-\lambda\vec{b}$ are perpendicular to each other, then the number of values of λ in $[-1,3]$ is : [4 marks]

(A) 1

(B) 3

(C) 2

(D) 0

Correct Answer: (D) 0

Solution:

Concept:

- Two vectors are perpendicular when their scalar product is zero. Dot products distribute over vector addition and scalar multiplication.
- For unit vectors with angle $\pi/3$, $\vec{a}\cdot\vec{a}=\vec{b}\cdot\vec{b}=1$ and $\vec{a}\cdot\vec{b}=\cos(\pi/3)=1/2$.

Step 1: Translate perpendicularity into a scalar equation The given condition is

$$(\lambda\vec{a}+2\vec{b})\cdot(3\vec{a}-\lambda\vec{b})=0.$$

Expand all four products before substituting the magnitudes:

$$3\lambda(\vec{a}\cdot\vec{a})-\lambda^2(\vec{a}\cdot\vec{b})+6(\vec{b}\cdot\vec{a})-2\lambda(\vec{b}\cdot\vec{b})=0.$$

The mixed products are equal and positive because the angle is acute. The two pure products equal one because both given vectors are unit vectors.

Step 2: Solve for all real parameter values Substitution reduces the equation to

$$3\lambda-\frac{\lambda^2}{2}+3-2\lambda=0,$$

so $\lambda^2-2\lambda-6=0$. The quadratic formula gives exactly

$$\lambda=\frac{2\pm\sqrt{4+24}}{2}=1\pm\sqrt{7}.$$

There are two real algebraic solutions. However, the question asks for the number lying in a particular closed interval, rather than the total number of real solutions.

Step 3: Check the interval endpoints exactly Since $\sqrt{7}>2$, the smaller root satisfies

$$1-\sqrt{7}<-1,$$

and the larger root satisfies

$$1+\sqrt{7}>3.$$

Both roots therefore lie outside $[-1,3]$, and neither is an endpoint. The answer is zero. Option (C) counts both real roots without imposing the interval; option (A) would require exactly one root inside it; option (B) is impossible even before the interval check because the equation is quadratic.

Step 4: Conclusion There are 0 admissible values of λ in $[-1, 3]$. Hence, the correct option is (D).

Quick Tip to Solve: After solving a parameter quadratic, check every root against the stated interval; the number of real roots need not equal the number of admissible values.

12. The area of the region enclosed by the curves $y = x^2 - 4x + 4$ and $y^2 = 16 - 8x$ is : [4 marks]

(A) $\frac{8}{3}$

(B) 5

(C) 8

(D) $\frac{4}{3}$

Correct Answer: (A) $\frac{8}{3}$

Solution:

Concept:

- For horizontal slices, the enclosed area is $\int (x_{\text{right}} - x_{\text{left}})\, dy$. Choose the direction that gives simple boundaries.
- Writing $y = (x-2)^2$ gives $x = 2 \pm \sqrt{y}$, with $y \geq 0$. The other parabola is $x = 2 - y^2/8$, which never exceeds 2.

Step 1: Find the branches that bound the enclosed region The right branch $x = 2 + \sqrt{y}$ of the first parabola can meet $x = 2 - y^2/8$ only at $y = 0$, since the former is at least 2 while the latter is at most 2. The nontrivial enclosed region therefore uses the left branch

$$x_{\text{left}} = 2 - \sqrt{y}.$$

The other boundary will be $x = 2 - y^2/8$. The restriction $y \geq 0$ is essential: the first parabola has no points below the x-axis, so the negative-y branch of the second curve does not create another enclosed region.

Step 2: Find the intersections and order the boundaries Equating the two x-coordinates gives

$$2 - \sqrt{y} = 2 - \frac{y^2}{8}, \qquad \sqrt{y} = \frac{y^2}{8}.$$

This gives $y = 0$, or, for $y > 0$, $y^{3/2} = 8$ and hence $y = 4$. The intersection points are $(2, 0)$ and $(0, 4)$. For $0 < y < 4$, we have $y^2/8 < \sqrt{y}$; thus $2 - y^2/8$ is the right boundary, as required for a positive slice width.

Step 3: Integrate the positive horizontal width The area in square units is

$$\begin{aligned}\mathcal{A} &= \int_0^4 \left[\left(2 - \frac{y^2}{8}\right) - (2 - \sqrt{y})\right] dy \\ &= \left[\frac{2}{3}y^{3/2} - \frac{y^3}{24}\right]_0^4 \\ &= \frac{16}{3} - \frac{8}{3} = \frac{8}{3}.\end{aligned}$$

There is only one such enclosed region, so no factor of two is needed. The choices 5, 8 and $4/3$ disagree with this exact integral; in particular, $4/3$ would incorrectly halve the area.

Step 4: Conclusion The enclosed area is $\frac{8}{3}$ square units. Hence, the correct option is (A).

Quick Tip to Solve: When one parabola is already easy to express as x in terms of y, use horizontal strips and identify the correct square-root branch before integrating.

13. In a group of 3 girls and 4 boys, there are two boys B_1 and B_2. The number of ways, in which these girls and boys can stand in a queue such that all the girls stand together, all the boys stand together, but B_1 and B_2 are not adjacent to each other, is : [4 marks]

(A) 120

(B) 96

(C) 72

(D) 144

Correct Answer: (D) 144

Solution:

Concept:

- When designated people must stand together, regard that entire group as a single block, then arrange the people within the block.
- Use complementary counting inside the boys block: count all orders, then subtract those in which B_1 and B_2 are adjacent.

Step 1: Arrange the two gender blocks The three girls form one block and the four boys form another. The two blocks can appear as girls–boys or boys–girls, giving $2! = 2$ orders. Within the girls block, the three distinct girls can be arranged in $3! = 6$ ways. This enforces both togetherness conditions before we impose the restriction on the boys.

Step 2: Count the allowed boys orders There are $4! = 24$ orders of the four distinct boys without the non-adjacency restriction. To count forbidden orders, combine B_1, B_2 into a temporary pair-block. This block and the two other boys are three objects, with $3!$ orders. The pair can internally be B_1B_2 or B_2B_1, so the forbidden count is $2!\,3! = 12$. Hence the permitted boys orders number $24 - 12 = 12$. The restriction concerns adjacency within the boys block; placing the girls block on either side does not change it.

Step 3: Multiply the independent choices Each block order, girls order and allowed boys order produces a different queue, and every valid queue is obtained once. Thus

$$N = 2! \times 3! \times (4! - 2!\,3!) = 2 \times 6 \times 12 = 144.$$

Options (A), (B) and (C) give 120, 96 and 72, none equal to this complete count. In particular, 72 would count only one of the two orders of the gender blocks.

Step 4: Conclusion The required number of queues is $\boxed{144}$. Hence, the correct option is (D).

Quick Tip to Solve: Apply non-adjacency counting within the boys block first, and remember the separate factor of 2 for the order of the two gender blocks.

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14. If $\lim_{x\to\infty}\left(\left(\frac{\mathrm{e}}{1-\mathrm{e}}\right)\left(\frac{1}{\mathrm{e}}-\frac{x}{1+x}\right)\right)^{x}=\alpha$, then the value of $\frac{\log_{\mathrm{e}}\alpha}{1+\log_{\mathrm{e}}\alpha}$ equals : [4 marks]

(A) e^{-2}

(B) e^2

(C) e

(D) e^{-1}

Correct Answer: (C) e

Solution:

Concept:

- For a positive base tending to 1, logarithms turn a power limit into a product limit: $\log\alpha = \lim x\log b(x)$.
- The standard limit $\lim_{u\to0}\log(1+u)/u = 1$ evaluates forms of type 1^{∞}. Here $\log$ denotes the natural logarithm.

Step 1: Rewrite the base as one plus a small term Let $b(x)$ denote the base in the given power. Combine the fractions exactly:

$$\begin{aligned} b(x) &= \frac{\mathrm{e}}{1-\mathrm{e}}\left(\frac{1}{\mathrm{e}}-\frac{x}{1+x}\right) \\ &= \frac{1+(1-\mathrm{e})x}{(1-\mathrm{e})(1+x)} \\ &= 1-\frac{\mathrm{e}}{(\mathrm{e}-1)(1+x)}. \end{aligned}$$

For sufficiently large positive x, this base is positive and approaches 1 from below, so taking its real logarithm is valid.

Step 2: Take the logarithm and evaluate its limit Put $c = \mathrm{e}/(\mathrm{e}-1) > 0$ and $u = -c/(1+x)$. Then $u \to 0$ and $xu \to -c$. Therefore

$$\log\alpha = \lim_{x\to\infty} x\log(1+u) = \lim_{x\to\infty}(xu)\frac{\log(1+u)}{u} = -\frac{\mathrm{e}}{\mathrm{e}-1}.$$

This gives the natural logarithm needed in the final ratio.

Step 3: Substitute into the requested ratio The denominator is nonzero because

$$1+\log\alpha = 1-\frac{\mathrm{e}}{\mathrm{e}-1} = -\frac{1}{\mathrm{e}-1}.$$

Consequently,

$$\frac{\log\alpha}{1+\log\alpha} = \frac{-\mathrm{e}/(\mathrm{e}-1)}{-1/(\mathrm{e}-1)} = \mathrm{e}.$$

Both negatives cancel. The values $\mathrm{e}^{-2}, \mathrm{e}^{2}, \mathrm{e}^{-1}$ in (A), (B) and (D) are different from this exact ratio.

Step 4: Conclusion The requested value is $\boxed{\mathrm{e}}$. Hence, the correct option is (C).

Quick Tip to Solve: First express the base as $1-c/(x+1)$. Compute $\log\alpha = -c$ directly instead of finding the power limit numerically.

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15. Let $f(x)=\int_0^{x^2}\frac{t^2-8t+15}{e^t}dt, x\in\mathbf{R}$. Then the numbers of local maximum and local minimum points of f, respectively, are : [4 marks]

(A) 3 and 2

(B) 2 and 2

(C) 2 and 3

(D) 1 and 3

Correct Answer: (C) 2 and 3

Solution:

Concept:

- The fundamental theorem of calculus and the chain rule give $\frac{d}{dx}\int_0^{g(x)} h(t)\,dt = h(g(x))g'(x)$.
- A derivative changing from negative to positive gives a local minimum; a positive-to-negative change gives a local maximum. A positive exponential factor does not affect its sign.

Step 1: Differentiate the integral Here the upper limit is x^2, so its derivative contributes the factor $2x$. Therefore

$$f'(x) = \frac{2x(x^4-8x^2+15)}{e^{x^2}} = 2x(x^2-3)(x^2-5)\mathrm{e}^{-x^2}.$$

The integrand is continuous for every real argument, and f is differentiable on all of $\mathbf{R}$. Thus there are no endpoints or nondifferentiable points to add to the critical-point list.

Step 2: List and classify all critical points The exponential factor never vanishes. The derivative is zero precisely at

$$-\sqrt{5},\quad -\sqrt{3},\quad 0,\quad \sqrt{3},\quad \sqrt{5}.$$

Each zero is simple in the factored derivative, so the sign changes at every one. On $(\sqrt{5},\infty)$, all three polynomial factors x, x^2-3, x^2-5 are positive. Reading the intervals from left to right, the signs of f' are consequently

$$-,\quad +,\quad -,\quad +,\quad -,\quad +.$$

This also follows by choosing one test point in each interval.

Step 3: Count maxima and minima separately The changes $- \to +$ occur at $-\sqrt{5}, 0, \sqrt{5}$, giving three local minima. The changes $+ \to -$ occur at $-\sqrt{3}, \sqrt{3}$, giving two local maxima. These are all extrema since the derivative has no other zeros or singularities. Option (A) reverses the two counts; (B) misses the minimum at zero, and (D) misses one of the two symmetric maxima. The requested order is maximum first, minimum second.

Step 4: Conclusion There are $\boxed{2}$ local maximum points and $\boxed{3}$ local minimum points. Hence, the correct option is (C).

Quick Tip to Solve: Do not omit the factor $2x$ from the moving limit x^2; it creates the additional local minimum at $x = 0$.

16. Let $\mathrm{A} = \{1, 2, 3, 4\}$ and $\mathrm{B} = \{1, 4, 9, 16\}$. Then the number of many-one functions $f : \mathrm{A} \to \mathrm{B}$ such that $1 \in f(\mathrm{A})$ is equal to : [4 marks]

(A) 151

(B) 139

(C) 163

(D) 127

Correct Answer: (A) 151

Solution:

Concept:

- A function from a four-element set to a four-element set has 4^4 possible assignments, because each input chooses its image independently.
- A many-one function is a function that is not one-one: at least two distinct inputs share an image. With equal finite domain and codomain sizes, each one-one function is a bijection.

Step 1: Count functions whose range contains one The requirement $1 \in f(A)$ means that at least one input has image 1; it does not require $f(1) = 1$. Count these functions by subtracting their complement. There are $4^4 = 256$ unrestricted functions. If 1 is absent from the range, each of the four inputs must choose its image from $\{4, 9, 16\}$, giving $3^4 = 81$ functions. Thus

$$N_{\text{contains } 1} = 4^4 - 3^4 = 256 - 81 = 175.$$

This count includes both one-one and many-one functions.

Step 2: Remove the one-one functions A one-one function uses four distinct images. Since B has exactly four elements, such a function uses every element of B, including 1. Therefore all one-one functions are already among the 175 functions just counted. Their number is the number of permutations of the four images:

$$N_{\text{one-one}} = 4! = 24.$$

No additional subtraction for avoiding 1 is needed, because a bijection cannot avoid any codomain element.

Step 3: Find the many-one count and compare the options The two categories, one-one and many-one, are disjoint and exhaust all functions. Hence

$$N_{\text{many-one, contains 1}} = 175 - 24 = 151.$$

Constant functions are included when they meet the range condition: the constant function with value 1 is indeed many-one. The three other displayed counts, 139, 163 and 127, cannot follow from the exhaustive complement calculation. No arithmetic property of the values 1, 4, 9 and 16 changes the counting; only their distinctness matters.

Step 4: Conclusion The required number of functions is $\boxed{151}$. Hence, the correct option is (A).

Quick Tip to Solve: Count functions hitting the required codomain element first, then subtract all $4!$ bijections; every bijection necessarily hits that element.

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17. Let the curve $z(1+i)+\bar{z}(1-i)=4, z \in C$, divide the region $|z-3| \leq 1$ into two parts of areas α and β. Then $|\alpha-\beta|$ equals : [4 marks]

(A) $1+\frac{\pi}{3}$

(B) $1+\frac{\pi}{6}$

(C) $1+\frac{\pi}{2}$

(D) $1+\frac{\pi}{4}$

Correct Answer: (C) $1+\frac{\pi}{2}$

Solution:

Concept:

- Writing $z = x + iy$ and $\bar{z} = x - iy$ converts a conjugate-linear equation into a straight line in the Argand plane.
- A circle of radius r cut by a chord subtending angle θ at its centre has minor segment area $\frac{1}{2}r^2(\theta - \sin\theta)$, with θ in radians.

Step 1: Identify the line and circle Expand the equation using real and imaginary parts:

$$z(1+i)+\bar{z}(1-i) = 2(x-y) = 4.$$

Thus the dividing line is $x - y - 2 = 0$. The region $|z-3| \leq 1$ is the disk $(x-3)^2 + y^2 \leq 1$, with centre $C = (3, 0)$ and radius 1. The perpendicular distance from its centre to the line is

$$d = \frac{|3 - 0 - 2|}{\sqrt{1^2 + (-1)^2}} = \frac{1}{\sqrt{2}} < 1.$$

Thus the line cuts the disk into two parts.

Step 2: Calculate the smaller circular segment Let θ be the smaller central angle subtended by the

chord. The perpendicular from the centre bisects the chord and its angle, so

$$\cos\frac{\theta}{2} = \frac{d}{1} = \frac{1}{\sqrt{2}}, \qquad \theta = \frac{\pi}{2}.$$

Subtract the triangle area $1/2$ from the sector area $\pi/4$ to get

$$s = \frac{\pi}{4} - \frac{1}{2}.$$

Step 3: Take the difference of the two areas The whole disk has area π. Its two parts therefore have areas s and $\pi - s$. Their positive difference is

$$|\alpha - \beta| = (\pi - s) - s = \pi - 2\left(\frac{\pi}{4} - \frac{1}{2}\right) = 1 + \frac{\pi}{2}.$$

The absolute value makes the labelling of the parts irrelevant. Options (A), (B) and (D) differ from this exact value.

Step 4: Conclusion The area difference is $\boxed{1 + \frac{\pi}{2}}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: For a chord-cut disk, compute only the smaller segment s; the area difference is the total disk area minus $2s$.

18. Suppose that the number of terms in an A.P. is $2k, k \in N$. If the sum of all odd terms of the A.P. is 40 , the sum of all even terms is 55 and the last term of the A.P. exceeds the first term by 27 , then k is equal to: [4 marks]

(A) 8

(B) 6

(C) 4

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- An A.P. with first term a and common difference d has nth term $a + (n-1)d$.
- Pair each odd-positioned term with the following even-positioned term. Every such even-minus-odd difference equals the common difference d.

Step 1: Subtract the odd-position and even-position sums There are $2k$ terms, so there are exactly k pairs $(T_1, T_2), (T_3, T_4), \ldots, (T_{2k-1}, T_{2k})$. For every pair, $T_{2j} - T_{2j-1} = d$. Summing these differences gives

$$kd = S_{\text{even positions}} - S_{\text{odd positions}} = 55 - 40 = 15.$$

Here odd and even refer to the term positions, rather than to the numerical parity of their values. Consequently $d = 15/k$, which is positive since k is a positive integer.

Step 2: Use the difference between the last and first terms The last term has index $2k$, so

$$T_{2k} - T_1 = (2k-1)d = 27.$$

Substitute the expression for d:

$$(2k-1)\frac{15}{k} = 27, \qquad 30k - 15 = 27k, \qquad 3k = 15.$$

It follows that $k = 5$, and then $d = 3$. The factor is $2k - 1$, since the first and last of $2k$ terms are separated by that many common-difference increments.

Step 3: Check that both given sums can hold The odd-positioned terms themselves form an A.P. with $k = 5$ terms and difference $2d = 6$. Their sum is

$$40 = \frac{5}{2}[2a + 4(6)] = 5(a + 12),$$

so $a = -4$. The odd-positioned terms are $-4, 2, 8, 14, 20$, summing to 40; the even-positioned terms are $-1, 5, 11, 17, 23$, summing to 55. Also $23 - (-4) = 27$, confirming consistency. The alternatives 8, 6 and 4 fail the derived necessary equation $3k = 15$.

Step 4: Conclusion The value is $\boxed{k = 5}$, corresponding to ten terms in the A.P. Hence, the correct option is (D).

Quick Tip to Solve: Subtract the sums of alternate positions to obtain kd immediately; this removes the unknown first term before any sum formula is needed.

19. Let $\mathrm{P}(4, 4\sqrt{3})$ be a point on the parabola $y^2 = 4ax$ and PQ be a focal chord of the parabola. If M and N are the foot of perpendiculars drawn from P and Q respectively on the directrix of the parabola, then the area of the quadrilateral PQMN is equal to : [4 marks]

(A) $\frac{34\sqrt{3}}{3}$

(B) $\frac{343\sqrt{3}}{8}$

(C) $17\sqrt{3}$

(D) $\frac{263\sqrt{3}}{8}$

Correct Answer: (B) $\frac{343\sqrt{3}}{8}$

Solution:

Concept:

- The standard parametrisation of $y^2 = 4ax$ is $(at^2, 2at)$. Its directrix is $x = -a$.
- For the endpoints of a focal chord, the parameters satisfy $t_1 t_2 = -1$. Perpendicular distances to the vertical directrix are horizontal lengths.
- The two perpendicular segments form the parallel sides of a trapezium; its area is half their sum times the vertical separation.

Step 1: Find the parabola and the first parameter Substitute the coordinates of P into the parabola:

$$(4\sqrt{3})^2 = 4a(4) \quad \Longrightarrow \quad a = 3.$$

Thus the focus is $(3, 0)$, the directrix is $x = -3$, and

$$t_1 = \frac{4\sqrt{3}}{2(3)} = \frac{2\sqrt{3}}{3}.$$

The sign is positive because P is above the horizontal axis.

Step 2: Locate the second endpoint of the focal chord The chord passes through the focus, so its other parameter is

$$t_2 = -\frac{1}{t_1} = -\frac{\sqrt{3}}{2}.$$

Consequently

$$Q = (3t_2^2, 6t_2) = \left(\frac{9}{4}, -3\sqrt{3}\right).$$

The negative ordinate puts Q on the opposite side of the axis, as expected for this focal chord.

Step 3: Use the trapezium area The perpendicular feet are $M = (-3, 4\sqrt{3})$ and $N = (-3, -3\sqrt{3})$. Therefore

$$PM = 4 + 3 = 7, \qquad QN = \frac{9}{4} + 3 = \frac{21}{4}, \qquad MN = 7\sqrt{3}.$$

The region bounded by the chord, its perpendiculars and the directrix has area

$$\mathcal{A} = \frac{1}{2}\left(7 + \frac{21}{4}\right) 7\sqrt{3} = \frac{343\sqrt{3}}{8}.$$

This is the sum-of-parallel-sides formula, not the product of the chord length and a horizontal distance. The other three offered values do not equal this exact area.

Step 4: Conclusion The area is $\boxed{\frac{343\sqrt{3}}{8}}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: For a focal chord of $y^2 = 4ax$, find the second endpoint using $t_2 = -1/t_1$; the directrix perpendiculars then give a trapezium immediately.

20. Let $\mathrm{E}: \frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \mathrm{a}>\mathrm{b}$ **and** $\mathrm{H}: \frac{x^2}{A^2}-\frac{y^2}{B^2}=1$**. Let the distance between the foci of E and the foci of** H **be** $2\sqrt{3}$**. If** $a-A=2$**, and the ratio of the eccentricities of** E **and** H **is** $\frac{1}{3}$**, then the sum of the lengths of their latus rectums is equal to :** [4 marks]

(A) 10

(B) 7

(C) 9

(D) 8

Correct Answer: (D) 8

Solution:

Concept:

- For the ellipse, the focal semidistance is $c=\sqrt{a^2-b^2}$ and eccentricity is c/a. Its latus rectum has length $2b^2/a$.
- For the hyperbola, $c=\sqrt{A^2+B^2}$, eccentricity is c/A, and its latus rectum has length $2B^2/A$.

Step 1: Express the common focal semidistance The given separation of the foci is $2\sqrt{3}$, so the focal semidistance for each conic is $c=\sqrt{3}$. Therefore

$$a^2-b^2=3, \qquad A^2+B^2=3.$$

The subtraction belongs to the ellipse; for the hyperbola the squares add. Interchanging these identities would give incorrect transverse and conjugate parameters.

Step 2: Use the ratio of eccentricities Using the same focal semidistance in both eccentricities gives

$$\frac{e_E}{e_H}=\frac{\sqrt{3}/a}{\sqrt{3}/A}=\frac{A}{a}=\frac{1}{3}.$$

Hence $a=3A$. Combining this with the given difference,

$$a-A=2 \quad\Longrightarrow\quad 2A=2 \quad\Longrightarrow\quad A=1,\ a=3.$$

These positive values also agree with the requirement $a>b$.

Step 3: Calculate and add both latus rectums The remaining squares are

$$b^2=a^2-3=6, \qquad B^2=3-A^2=2.$$

The full latus rectum lengths, each twice its semilatus rectum, are

$$L_E=\frac{2(6)}{3}=4, \qquad L_H=\frac{2(2)}{1}=4.$$

Thus $L_E+L_H=4+4=8$. The values 10, 7 and 9 cannot be obtained from these parameters; in particular the answer is a sum of full chord lengths, not of semilatus rectums.

Step 4: Conclusion The required sum is $\boxed{8}$ units. Hence, the correct option is (D).

Quick Tip to Solve: When an ellipse and a hyperbola have the same focal separation, their eccentricity ratio is the inverse ratio of their transverse semiaxes.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let $\mathrm{A}(6,8), \mathrm{B}(10\cos\alpha, -10\sin\alpha)$ and $\mathrm{C}(-10\sin\alpha, 10\cos\alpha)$, be the vertices of a triangle. If $L(a,9)$ and $G(h,k)$ be its orthocenter and centroid respectively, then $(5a-3h+6k+100\sin 2\alpha)$ is equal to _____. [4 marks]

Solution:

Concept:

- All three vertices lie on the circle $x^2+y^2=100$, whose circumcentre is the origin.
- For a triangle with circumcentre at the origin, the orthocentre position vector is the sum of the three vertex vectors. The centroid is one third of that sum.

Step 1: Verify the circumcircle and obtain the orthocentre For A, $6^2+8^2=100$. For B and C, the squared distance from the origin is $100(\cos^2\alpha+\sin^2\alpha)=100$. Write $d=\cos\alpha-\sin\alpha$. The sum of the vertex vectors is then

$$(6+10d,\ 8+10d).$$

To justify the orthocentre identity directly, if $H=A+B+C$, then $(H-A)\cdot(B-C)=(B+C)\cdot(B-C)=|B|^2-|C|^2=0$. Thus AH is perpendicular to BC, and the same argument works for the other altitudes.

Step 2: Use the prescribed ordinate and the centroid The orthocentre has ordinate 9, so

$$8+10d=9 \quad\Longrightarrow\quad d=\frac{1}{10}.$$

Its abscissa is therefore

$$a=6+10d=7.$$

The centroid is one third of the vertex sum, giving

$$h=\frac{a}{3}=\frac{7}{3}, \qquad k=\frac{9}{3}=3.$$

These relations avoid having to determine an individual angle α.

Step 3: Evaluate the trigonometric term and combine Square the difference already known:

$$(\cos\alpha-\sin\alpha)^2=1-\sin 2\alpha=\frac{1}{100}.$$

Hence $100\sin 2\alpha=99$. Now substitute each term separately:

$$\begin{aligned}5a-3h+6k+100\sin 2\alpha &= 5(7)-3\left(\frac{7}{3}\right)+6(3)+99\\ &= 35-7+18+99=145.\end{aligned}$$

The requested expression is dimensionless; no coordinate unit needs to be attached to this numerical entry.

Step 4: Conclusion The required numerical value is $\boxed{145}$.

Quick Tip to Solve: If all vertices have the same distance from the origin, use $H=A+B+C=3G$ before attempting any altitude equations.

22. Let $y = f(x)$ be the solution of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2-1} = \frac{x^6+4x}{\sqrt{1-x^2}}, -1 < x < 1$ such that $f(0) = 0$. If $6\int_{-1/2}^{1/2} f(x)dx = 2\pi - \alpha$ then α^2 is equal to _____ . [4 marks]

Solution:

Concept:

- For $y' + P(x)y = Q(x)$, multiplication by the integrating factor $e^{\int P(x)\,dx}$ converts the left side into a product derivative.
- On $(-1, 1)$, $\sqrt{1-x^2}$ is positive. An odd integrand integrates to zero over symmetric limits.

Step 1: Determine the integrating factor and solve Here $P(x) = x/(x^2-1)$. Since

$$\int \frac{x}{x^2-1}\,dx = \frac{1}{2}\log(1-x^2),$$

an integrating factor on the stated interval is $\sqrt{1-x^2}$. Multiplying the equation by this factor gives

$$\frac{d}{dx}\left(y\sqrt{1-x^2}\right) = x^6 + 4x.$$

Integrate and use $f(0) = 0$:

$$y\sqrt{1-x^2} = \frac{x^7}{7} + 2x^2 + C, \qquad C = 0.$$

Thus $f(x) = (x^7/7 + 2x^2)/\sqrt{1-x^2}$.

Step 2: Exploit symmetry in the required integral The term $x^7/\sqrt{1-x^2}$ is odd, so its contribution vanishes. Accordingly

$$6\int_{-1/2}^{1/2} f(x)\,dx = 12\int_{-1/2}^{1/2} \frac{x^2}{\sqrt{1-x^2}}\,dx.$$

Put $x = \sin t$. The limits become $-\pi/6$ and $\pi/6$, and the integrand becomes $\sin^2 t$. The positive square root equals $\cos t$ on this interval, so the substitution introduces no sign change.

Step 3: Evaluate and identify the constant Using $\int \sin^2 t\,dt = t/2 - \sin(2t)/4$, we obtain

$$\int_{-\pi/6}^{\pi/6} \sin^2 t\,dt = \frac{\pi}{6} - \frac{\sqrt{3}}{4}.$$

Therefore

$$6\int_{-1/2}^{1/2} f(x)\,dx = 12\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) = 2\pi - 3\sqrt{3}.$$

Comparison with $2\pi - \alpha$ gives $\alpha = 3\sqrt{3}$. Squaring the whole value, including the factor 3, gives $\alpha^2 = 9(3) = 27$.

Step 4: Conclusion The required value is $\boxed{\alpha^2 = 27}$.

Quick Tip to Solve: Find the integrating factor using the sign valid on the stated interval, then discard the odd part before evaluating a symmetric integral.

23. If $\sum_{r=1}^{30} \frac{r^2\binom{30}{r}^2}{\binom{30}{r-1}} = \alpha \times 2^{29}$, then α is equal to _____. [4 marks]

Solution:

Concept:

- Adjacent binomial coefficients satisfy $\binom{n}{r}/\binom{n}{r-1} = (n-r+1)/r$. This removes the quotient of coefficients.
- Differentiating $(1+x)^n$ gives $\sum r\binom{n}{r} = n2^{n-1}$ at $x=1$, and a second derivative gives $\sum r(r-1)\binom{n}{r} = n(n-1)2^{n-2}$.

Step 1: Simplify the summand For $1 \le r \le 30$, the adjacent-coefficient identity gives

$$\frac{r^2\binom{30}{r}^2}{\binom{30}{r-1}} = r^2\binom{30}{r}\frac{31-r}{r} = r(31-r)\binom{30}{r}.$$

Hence the required sum is

$$S = 31\sum_{r=1}^{30} r\binom{30}{r} - \sum_{r=1}^{30} r^2\binom{30}{r}.$$

No endpoint term is lost: the omitted term at $r=0$ would be zero in both weighted sums.

Step 2: Calculate the first two moments From the differentiated binomial theorem,

$$\sum r\binom{30}{r} = 30\,2^{29}.$$

Write $r^2 = r(r-1)+r$ before using the second derivative. This yields

$$\begin{aligned}\sum r^2\binom{30}{r} &= 30(29)2^{28} + 30\,2^{29}\\ &= 30(31)2^{28}.\end{aligned}$$

The extra first-moment term is necessary because the second derivative produces $r(r-1)$, not r^2.

Step 3: Collect the coefficient of the prescribed power Substitution gives

$$\begin{aligned}S &= 31(30)2^{29} - 30(31)2^{28}\\ &= 30(31)2^{28}\\ &= 465\,2^{29}.\end{aligned}$$

The last equality divides the coefficient 930 by 2 while increasing the power from 2^{28} to 2^{29}. Comparing this exact expression with $\alpha\,2^{29}$ determines α directly; it is a dimensionless coefficient.

Step 4: Conclusion The required coefficient is $\boxed{\alpha = 465}$.

Quick Tip to Solve: When a summand contains adjacent binomial coefficients in a quotient, simplify their ratio first; then express powers of r using binomial moments.

24. Let the distance between two parallel lines be 5 units and a point P lie between the lines at a unit distance from one of them. An equilateral triangle PQR is formed such that Q lies on one of the parallel lines, while R lies on the other. Then $(QR)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- A translation and rotation of axes preserve all distances, so take the parallel lines as $y = 0$ and $y = 5$, with $P = (0, 1)$.
- In an equilateral triangle, the vectors from one vertex to the other two have equal lengths and differ by a rotation of $60°$.

Step 1: Choose convenient coordinates Place the line nearer P at $y = 0$, the other at $y = 5$, and set $P = (0, 1)$. We may call the vertex on the nearer line Q, so $Q = (u, 0)$. If the original names Q and R are interchanged, the side length QR is unchanged. Then

$$\overrightarrow{PQ} = (u, -1), \qquad PQ^2 = u^2 + 1.$$

This construction accounts for both perpendicular distances, 1 and 4, without introducing an unnecessary slope for either line.

Step 2: Rotate the side through sixty degrees The vector $\overrightarrow{PR}$ is obtained from $(u, -1)$ by a clockwise or anticlockwise rotation through $60°$. Its vertical component is therefore

$$\pm\frac{\sqrt{3}}{2}u - \frac{1}{2}.$$

Since R lies on $y = 5$ and P has ordinate 1, the same vertical component must be 4. Thus

$$\pm\frac{\sqrt{3}}{2}u - \frac{1}{2} = 4 \quad \Longrightarrow \quad \pm\sqrt{3}\,u = 9.$$

Squaring removes the orientation sign and gives $u^2 = 27$. The two possible signs describe reflected triangles and give the same side length.

Step 3: Use equality of the three sides Every side of an equilateral triangle has the same length. Hence

$$QR^2 = PQ^2 = u^2 + 1 = 27 + 1 = 28.$$

As a direct coordinate check, take $u = 3\sqrt{3}$ and the anticlockwise rotation. Its horizontal component is $u/2 + \sqrt{3}/2 = 2\sqrt{3}$, so $R = (2\sqrt{3}, 5)$. Then

$$QR^2 = (2\sqrt{3} - 3\sqrt{3})^2 + (5 - 0)^2 = 3 + 25 = 28.$$

Thus the computed configuration satisfies both line constraints and the equal-side condition.

Step 4: Conclusion The required value is $\boxed{(QR)^2 = 28}$ square units.

Quick Tip to Solve: With parallel lines horizontal, rotate one side vector by $60°$; the known vertical separation fixes its horizontal component immediately.

25. Let $A = \{1, 2, 3\}$. The number of relations on A, containing $(1, 2)$ and $(2, 3)$, which are reflexive and transitive but not symmetric, is _____. [4 marks]

Solution:

Concept:

- A reflexive relation on A contains every diagonal pair (a, a). Transitivity requires (a, c) whenever (a, b) and (b, c) are present.
- Symmetry requires the reverse of every pair. We must count relations satisfying the first two conditions while excluding those that are symmetric.

Step 1: Identify the pairs that are compulsory Reflexivity gives $(1, 1), (2, 2), (3, 3)$. The question also includes $(1, 2), (2, 3)$, so transitivity forces $(1, 3)$. Thus every admissible relation contains

$$R_0 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}.$$

This relation is already transitive, since its only nontrivial forward chain, from 1 through 2 to 3, has its endpoint pair included. The only pairs still available are $(2, 1), (3, 1), (3, 2)$.

Step 2: Analyse the optional reverse pairs If $(3, 1)$ is included, combining it with $(1, 2)$ forces $(3, 2)$, while combining $(2, 3)$ with $(3, 1)$ forces $(2, 1)$. All nine pairs are then present, giving the symmetric relation $A \times A$, which is excluded.

Without $(3, 1)$, the pairs $(2, 1)$ and $(3, 2)$ cannot both be present: their chain would force $(3, 1)$. Either can be included separately, however, without requiring any further pair. They create the tied classes $\{1, 2\}$ or $\{2, 3\}$, respectively, with all forward pairs already in R_0.

Step 3: List the admissible relations The complete list is

$$R_0, \qquad R_0 \cup \{(2, 1)\}, \qquad R_0 \cup \{(3, 2)\}.$$

Each is reflexive and transitive. Each contains $(1, 3)$ but omits $(3, 1)$, so none is symmetric. The optional-pair analysis exhausts all possibilities.

Step 4: Conclusion The number of relations is $\boxed{3}$. This is a count, so it has no physical unit.

Quick Tip to Solve: First take the transitive closure of the compulsory pairs. Then test only the remaining reverse pairs; do not count all subsets of the Cartesian product.

26. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : A simple pendulum is taken to a planet of mass and radius, 4 times and 2 times, respectively, than the Earth. The time period of the pendulum remains same on earth and the planet.

Reason (R) : The mass of the pendulum remains unchanged at Earth and the other planet.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) (A) is false but (R) is true

(C) (A) is true but (R) is false

(D) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

Correct Answer: (D) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

Solution:

Concept:

- For a simple pendulum of fixed length l and small amplitude, $T = 2\pi\sqrt{l/g}$, where g is the local gravitational acceleration.
- At the surface of a spherical planet, $g = GM/R^2$. Inertial and gravitational mass of the bob cancel from the pendulum equation.

Step 1: Compare the surface gravitational accelerations Write Earth's mass and radius as M_E, R_E. The other planet has $M_P = 4M_E$ and $R_P = 2R_E$, hence

$$\frac{g_P}{g_E} = \frac{M_P}{M_E}\left(\frac{R_E}{R_P}\right)^2 = 4\left(\frac{1}{2}\right)^2 = 1.$$

The increased mass strengthens gravity by a factor four, while the doubled radius weakens it by the same factor. Thus the local accelerations are equal.

Step 2: Evaluate the assertion The same pendulum retains the same length, and its small-oscillation period depends only on that length and the local acceleration. Therefore

$$\frac{T_P}{T_E} = \sqrt{\frac{g_E}{g_P}} = 1.$$

The time period remains unchanged. Assertion (A) is true under the usual simple-pendulum conditions.

Step 3: Evaluate the reason and its explanatory role The bob's mass is an intrinsic property and remains unchanged on moving between planets, so Reason (R) is also true. However, unchanged bob mass does not establish equal periods: the period is independent of the bob's mass in the first place. Moving the same bob to a planet with a different g would change its period despite its unchanged mass. Here the equality follows specifically from $4/(2^2) = 1$ for surface gravity.

Option (A) incorrectly makes R the explanation; (B) incorrectly calls the assertion false; and (C)

incorrectly calls the reason false.

Step 4: Conclusion The assertion and the reason are both true, but the reason does not explain the assertion. Hence, the correct option is (D).

Quick Tip to Solve: For a pendulum moved to another planet, compare M/R^2 before calculating the period; the bob mass never enters the period formula.

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27. Given are statements for certain thermodynamic variables,

(A) Internal energy, volume (V) **and mass** (M) **are extensive variables.**

(B) Pressure (P), temperature (T) and density (ρ **) are intensive variables.**

(C) Volume (V), temperature (T) and density (ρ **) are intensive variables.**

(D) Mass (M), temperature (T) and internal energy are extensive variables.

Choose the correct answer from the options given below : [4 marks]

(A) (C) and (D) Only

(B) (A) and (B) Only

(C) (D) and (A) Only

(D) (B) and (C) Only

Correct Answer: (B) (A) and (B) Only

Solution:

Concept:

- An extensive variable scales with the amount of matter. On combining identical equilibrium subsystems, their masses, volumes and internal energies add.
- An intensive variable is independent of the total system size for a homogeneous equilibrium state; temperature, pressure and density are examples.

Step 1: Test the extensive-variable statement Imagine dividing a homogeneous sample into two equal parts without changing its state. Each half has half the original mass, half the volume and half the internal energy. Conversely, joining two identical samples doubles these three quantities. Therefore internal energy, volume V and mass M are extensive variables, so statement (A) is correct. The classification concerns how a quantity scales with system size, not whether it can change in a physical process.

Step 2: Test the intensive-variable statement The two halves remain at the same temperature T and pressure P as the original sample. Their density also remains unchanged, since

$$\rho_{\text{half}} = \frac{M/2}{V/2} = \frac{M}{V} = \rho.$$

Thus pressure, temperature and density are intensive variables, making statement (B) correct. Density is intensive even though both its numerator and denominator are extensive: their common scaling factor cancels.

Step 3: Reject the mixed classifications Statement (C) wrongly includes volume among intensive

variables. Volume changes in proportion to sample size, whereas temperature and density do not. Statement (D) wrongly includes temperature among extensive variables. Temperature is not obtained by adding the temperatures of two identical samples; their combined equilibrium temperature remains the same.

Accordingly, statements (C) and (D) are false. Option (A) contains both false statements, option (C) includes the false statement (D), and option (D) includes the false statement (C). Only the pair (A), (B) contains two true statements.

Step 4: Conclusion The first two statements are correct; the last two are false. Hence, the correct option is (B).

Quick Tip to Solve: Double the amount of a homogeneous system at unchanged state: quantities that double are extensive, while those that stay the same are intensive.

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28. The torque due to the force $(2\hat{i}+\hat{j}+2\hat{k})$ about the origin, acting on a particle whose position vector is $(\hat{i}+\hat{j}+\hat{k})$, would be [4 marks]

(A) $\hat{j}+\hat{k}$

(B) $\hat{i}-\hat{k}$

(C) $\hat{i}-\hat{j}+\hat{k}$

(D) $\hat{i}+\hat{k}$

Correct Answer: (B) $\hat{i}-\hat{k}$

Solution:

Concept:

- Torque about the origin is $\boldsymbol{\tau}=\mathbf{r}\times\mathbf{F}$, with the position vector first and force second.
- For a cross product, obtain each component by subtracting the two cross-products of the other components, cycling through x, y, z. The order fixes the sign by the right-hand rule.

Step 1: Write the given Cartesian components The position vector and force are

$$\mathbf{r}=(1,1,1), \qquad \mathbf{F}=(2,1,2).$$

The position is measured from the origin, exactly the point about which torque is requested. Use the cross product rather than a dot product: torque is a vector perpendicular to the plane of $\mathbf{r}$ and $\mathbf{F}$. Its magnitude is $rF\sin\theta$, but evaluating components avoids having to find the intervening angle.

Step 2: Evaluate the cross product Using the determinant expansion,

$$\begin{aligned}\boldsymbol{\tau}&=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&1\\2&1&2\end{vmatrix}\\&=\hat{i}(1\cdot 2-1\cdot 1)-\hat{j}(1\cdot 2-1\cdot 2)+\hat{k}(1\cdot 1-1\cdot 2)\\&=\hat{i}+0\hat{j}-\hat{k}.\end{aligned}$$

The middle component vanishes because the corresponding two products are equal. The final component is negative; reversing the order of the cross product would reverse the entire torque.

Step 3: Check its direction and compare the options The resulting vector $(1, 0, -1)$ is perpendicular to both input vectors:

$$(1, 0, -1) \cdot (1, 1, 1) = 0, \qquad (1, 0, -1) \cdot (2, 1, 2) = 0.$$

Option (A) has the wrong components, option (C) has a nonzero middle component and the wrong final sign, and option (D) has the wrong final sign. Only option (B) agrees with the component calculation and the perpendicularity check.

Step 4: Conclusion The torque is $\boxed{\hat{i} - \hat{k}}$, in torque units corresponding to the supplied force and position units. Hence, the correct option is (B).

Quick Tip to Solve: Always use $\mathbf{r} \times \mathbf{F}$. Check the answer by taking its dot product with both input vectors; each should vanish.

29. Which one of the following is the correct dimensional formula for the capacitance in F ? $\mathrm{M}, \mathrm{L}, \mathrm{T}$ **and** C **stand for unit of mass, length, time and charge,** [4 marks]

(A) $[\mathrm{F}] = \left[\mathrm{CM}^{-1}\ \mathrm{L}^{-2}\ \mathrm{T}^{2}\right]$
(B) $[\mathrm{F}] = \left[\mathrm{C}^{2}\mathrm{M}^{-2}\ \mathrm{L}^{2}\ \mathrm{T}^{2}\right]$
(C) $[\mathrm{F}] = \left[\mathrm{C}^{2}\mathrm{M}^{-1}\ \mathrm{L}^{-2}\ \mathrm{T}^{2}\right]$
(D) $[\mathrm{F}] = \left[\mathrm{CM}^{-2}\ \mathrm{L}^{-2}\ \mathrm{T}^{-2}\right]$
Correct Answer: (C) $[\mathrm{F}] = \left[\mathrm{C}^{2}\mathrm{M}^{-1}\ \mathrm{L}^{-2}\ \mathrm{T}^{2}\right]$

Solution:

Concept:

- Capacitance is charge divided by potential difference: $\mathcal{C} = Q/V$. The letter $\mathcal{C}$ here denotes capacitance, while C in the question denotes the charge dimension.
- Potential difference is work per unit charge. Work has dimensions ML^2T^{-2}; therefore voltage has dimensions $ML^2T^{-2}C^{-1}$.

Step 1: Express voltage in the stated base dimensions The question uses charge as a base dimension rather than electric current. Mechanical work is force times displacement. Since force has dimensions MLT^{-2}, work has dimensions

$$[W] = [MLT^{-2}][L] = ML^2T^{-2}.$$

Potential difference is the work required per unit charge, so

$$[V] = \frac{[W]}{[Q]} = ML^2T^{-2}C^{-1}.$$

This keeps the electrical dimension consistent with the symbols specified in the question.

Step 2: Divide charge by voltage Substitute the voltage dimensions into the defining relation for capacitance:

$$\begin{aligned}[C] &= \frac{[Q]}{[V]} \\ &= \frac{C}{ML^2T^{-2}C^{-1}} \\ &= C^2M^{-1}L^{-2}T^2.\end{aligned}$$

The two charge powers arise because one charge factor is in the numerator and another comes from the reciprocal of voltage. The negative mass and length powers follow from dividing by the mechanical energy dimension.

Step 3: Compare every offered expression Option (A) has only one power of charge rather than two. Option (B) has mass exponent -2 and length exponent $+2$, both inconsistent with the definition. Option (D) has incorrect charge, mass and time powers. Option (C) alone contains charge squared, inverse mass, inverse length squared and time squared. As another check, $\frac{1}{2}CV^2$ must have the dimensions of energy; the expression obtained above satisfies that condition.

Step 4: Conclusion The dimensional formula of capacitance in farads is $\boxed{C^2M^{-1}L^{-2}T^2}$. Hence, the correct option is (C).

Quick Tip to Solve: With charge as a base dimension, write capacitance as Q^2/energy; this gives all four exponents directly.

30. The maximum percentage error in the measurment of density of a wire is

[Given, mass of wire $= (0.60 \pm 0.003)\text{g}$

radius of wire $= (0.50 \pm 0.01)\text{cm}$

length of wire $= (10.00 \pm 0.05)\text{cm}]$ [4 marks]

(A) 7

(B) 8

(C) 5

(D) 4

Correct Answer: (C) 5

Solution:

Concept:

- For a cylindrical wire of radius r, length l and mass m, density is $\rho = m/(\pi r^2 l)$.
- For maximum uncertainty in a product or quotient, absolute fractional errors add, weighted by the magnitudes of the powers. Thus $\Delta\rho/\rho = \Delta m/m + 2\Delta r/r + \Delta l/l$.

Step 1: Identify how each measured quantity enters density The wire volume is $\pi r^2 l$, so the measured radius appears squared in the denominator. The constant π contributes no measurement error. For maximum possible uncertainty, choose the error signs so their effects reinforce one another; fractional

uncertainties therefore add even though mass and volume appear on opposite sides of the quotient.

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 2\frac{\Delta r}{r} + \frac{\Delta l}{l}.$$

This is the standard first-order propagation rule for limiting measurement errors.

Step 2: Calculate the three percentage contributions Use each absolute uncertainty with the corresponding central value in the same units:

$$\frac{\Delta m}{m} \times 100 = \frac{0.003}{0.60} \times 100 = 0.5\%,$$
$$\frac{\Delta r}{r} \times 100 = \frac{0.01}{0.50} \times 100 = 2\%,$$
$$\frac{\Delta l}{l} \times 100 = \frac{0.05}{10.00} \times 100 = 0.5\%.$$

The radius contribution must be counted twice because the volume contains r^2, giving a total radius contribution of 4%.

Step 3: Add the maximum contributions The resulting percentage uncertainty is

$$\frac{\Delta\rho}{\rho} \times 100 = 0.5 + 2(2) + 0.5 = 5\%.$$

This is a maximum-error calculation, so adding errors in quadrature would answer a different statistical question. Option (D), 4, retains only the doubled radius contribution and neglects mass and length. Options (A) and (B), 7 and 8, exceed the correctly weighted sum. Only option (C) matches all three stated measurements.

Step 4: Conclusion The maximum percentage error in the measured density is $\boxed{5\%}$. Hence, the correct option is (C).

Quick Tip to Solve: Write the measured quantity as a product of powers before propagating errors. Here the radius error is doubled because density contains r^{-2}.

31. An electron projected perpendicular to a uniform magnetic field B moves in a circle. If Bohr's quantization is applicable, then the radius of the electronic orbit in the first excited state is : [4 marks]

(A) $\sqrt{\frac{h}{\pi eB}}$

(B) $\sqrt{\frac{4h}{\pi eB}}$

(C) $\sqrt{\frac{\mathrm{h}}{2\pi e\mathrm{eB}}}$

(D) $\sqrt{\frac{2h}{\pi eB}}$

Correct Answer: (A) $\sqrt{\frac{h}{\pi eB}}$

Solution:

Concept:

- The magnetic Lorentz force provides centripetal force when velocity is perpendicular to the field: $evB = m_e v^2/r$, where e is the magnitude of the electron charge.
- Bohr quantization requires $m_e vr = n\hbar = nh/(2\pi)$. The ground state has $n = 1$, so the first excited state has $n = 2$.

Step 1: Relate momentum to the orbit radius The magnetic force bends the trajectory without changing the speed. Equating its magnitude to the required radial force gives

$$evB = \frac{m_e v^2}{r} \quad \Longrightarrow \quad m_e v = eBr.$$

The charge sign determines the direction of rotation, but only its magnitude enters this equation for radius. Thus a negative electron charge does not imply a negative radius.

Step 2: Apply the quantization condition Multiply the momentum equation by the radius, then impose the allowed angular momentum:

$$m_e vr = eBr^2 = \frac{nh}{2\pi}.$$

Therefore the possible orbit radii satisfy

$$r_n = \sqrt{\frac{nh}{2\pi eB}}.$$

The mass cancels between force balance and quantization. This is the magnetic orbit described in the question, so the Coulomb-force formula for a hydrogen atom is not applicable.

Step 3: Select the first excited orbit Substitute $n = 2$, rather than the ground-state value $n = 1$:

$$r_2 = \sqrt{\frac{2h}{2\pi eB}} = \sqrt{\frac{h}{\pi eB}}.$$

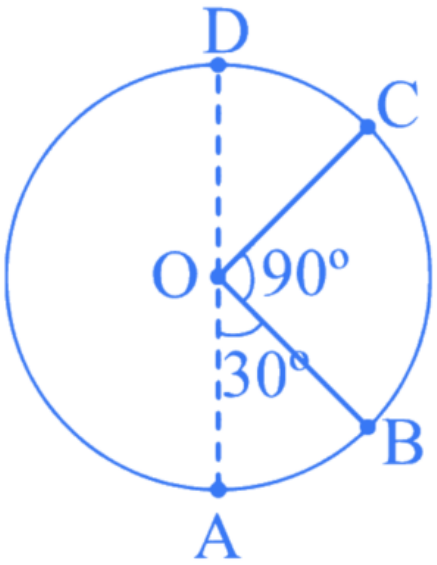
Option (B) has squared radius four times this value, and option (D) has squared radius twice this value. Neither corresponds to the first excited state. Option (C) also fails the required expression for $n = 2$. The inverse square-root dependence on field agrees with a stronger field confining the orbit more tightly.

Step 4: Conclusion The first excited orbit has radius $\boxed{r = \sqrt{h/(\pi eB)}}$. Hence, the correct option is (A).

Quick Tip to Solve: First excited means $n = 2$. Combine $m_e v = eBr$ with angular momentum quantization before substituting the quantum number.

---

32. A body of mass 100 g is moving in circular path of radius 2 m on vertical plane as shown in figure. The velocity of the body at point A is $10\ \mathrm{m/s}$. The ratio of its kinetic energies at point B and C is : (Take acceleration due to gravity as $10\ \mathrm{m/s^2}$) [4 marks]



(A) $\frac{3-\sqrt{2}}{2}$

(B) $\frac{2+\sqrt{3}}{3}$

(C) $\frac{2+\sqrt{2}}{3}$

(D) $\frac{3+\sqrt{3}}{2}$

Correct Answer: (D) $\frac{3+\sqrt{3}}{2}$

Solution:

Concept:

- A radial constraint force does no work during circular motion, so mechanical energy gives $K + mgh =$ constant, with the bottom point A taken as zero height.
- If a radius makes angle θ with the downward radius OA, the height above A is $h = r(1 - \cos\theta)$. The central angles in the figure determine the gravitational energy changes.

Step 1: Read the heights of B and C The diagram gives $\angle AOB = 30^\circ$ and $\angle BOC = 90^\circ$. Hence $\angle AOC = 120^\circ$. Using $r = 2\,\mathrm{m}$,

$$h_B = 2(1 - \cos 30^\circ) = (2 - \sqrt{3})\,\mathrm{m},$$

$$h_C = 2(1 - \cos 120^\circ) = 3\,\text{m}.$$

B lies below the centre while C lies above it. Using a complementary angle would incorrectly change either height and therefore the ratio.

Step 2: Calculate both kinetic energies Convert the mass: $m = 100\,\text{g} = 0.100\,\text{kg}$. At the lowest point,

$$K_A = \tfrac{1}{2}(0.100)(10)^2 = 5\,\text{J}.$$

Since $mg = (0.100)(10) = 1\,\text{N}$, energy conservation gives

$$K_B = 5 - 1(2 - \sqrt{3}) = (3 + \sqrt{3})\,\text{J},$$

$$K_C = 5 - 1(3) = 2\,\text{J}.$$

Both energies are positive, and the higher point has the smaller kinetic energy, as expected when gravitational potential energy increases.

Step 3: Form the ratio in the requested order Dividing the energy at B by that at C gives

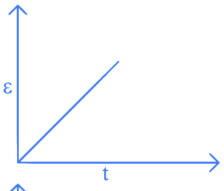
$$\frac{K_B}{K_C} = \frac{3 + \sqrt{3}}{2} \approx 2.366.$$

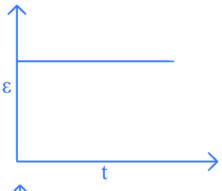
The alternatives (A), (B) and (C) give approximately 0.793, 1.244 and 1.138 respectively, so none equals this ratio. The mass cancels if one uses $K_B/K_C = v_B^2/v_C^2$, but the changes in height must still be evaluated correctly.

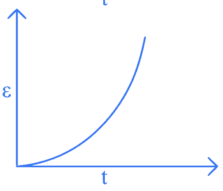
Step 4: Conclusion $\boxed{K_B/K_C = (3 + \sqrt{3})/2}$. Hence, the correct option is (D).

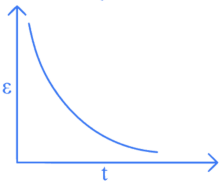
Quick Tip to Solve: Convert each central angle into height above the bottom using $r(1 - \cos\theta)$, then subtract mgh from the bottom kinetic energy.

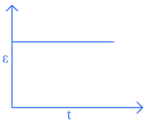
33. A rectangular metallic loop is moving out of a uniform magnetic field region to a field free region with a constant speed. When the loop is partially inside the magnate field, the plot of magnitude of induced emf (ε) with time (t) is given by [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- Faraday's law gives the induced emf magnitude as $\varepsilon = |d\Phi_B/dt|$. In a uniform perpendicular field, $\Phi_B = BA$, where A is the area of the loop still in the field.
- A rectangular loop crossing a straight field boundary at constant speed loses area at a constant rate. Thus flux decreases linearly with time, and its time derivative has constant magnitude.

Step 1: Express the area remaining in the field Let ℓ be the length of the side parallel to the field boundary, and let $x(t)$ be the width still inside the field. While the loop is partially inside,

$$A(t) = \ell x(t), \qquad x(t) = x_0 - vt.$$

The side length, speed and magnetic field are constant. Consequently the area lost per second is the fixed value ℓv. The area itself changes, but the rate of its change does not.

Step 2: Differentiate the magnetic flux The flux through the loop during this interval is

$$\Phi_B(t) = B\ell(x_0 - vt).$$

Applying Faraday's law,

$$\varepsilon = |-B\ell v| = B\ell v.$$

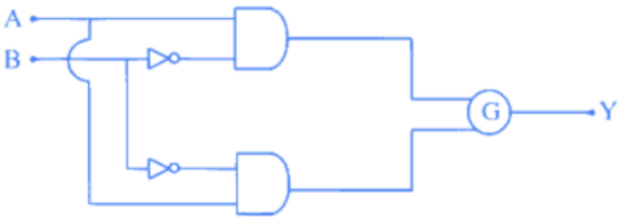
This result contains no time dependence. Lenz's law determines the direction of the induced current but does not alter this magnitude. Once the loop completely leaves the field, the emf becomes zero, but the question specifically asks about the interval in which it is only partially inside.

Step 3: Compare the four graphs Graph (B) is a horizontal line at a positive emf and therefore matches the constant value. Graph (A) rises linearly, and graph (C) rises nonlinearly; each would require an increasing rate of flux loss. Graph (D) decreases with time and would require a decreasing rate of flux loss. None of those changes occurs when the loop has constant speed across a uniform field boundary. A decreasing flux must not be confused with a decreasing magnitude of its derivative.

Step 4: Conclusion The induced emf remains $\boxed{\varepsilon = B\ell v}$ throughout the partial-exit interval. Hence, the correct option is (B).

Quick Tip to Solve: Differentiate the flux before choosing an emf graph: linearly decreasing flux gives constant emf, rather than linearly decreasing emf.

34. To obtain the given truth table, following logic gate should be placed at G : [4 marks]



A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(A) NOR Gate

(B) OR Gate

(C) AND Gate

(D) NAND Gate

Correct Answer: (A) NOR Gate

Solution:

Concept:

- An AND gate outputs 1 only when both inputs are 1. A NOT gate reverses an input. The two preliminary AND gates therefore produce $P = A\overline{B}$ and $Q = \overline{A}B$.
- The required truth table is 1 for equal inputs and 0 for unequal inputs. This is the equivalence or XNOR function, which must be formed from the signals supplied to G.

Step 1: Identify the inputs of the unknown gate The upper AND gate receives A and inverted B, so its output is $P = A\overline{B}$. The lower AND gate receives inverted A and B, so its output is $Q = \overline{A}B$. These products identify the two cases in which A and B differ. Their OR is

$$P + Q = A\overline{B} + \overline{A}B.$$

This is XOR. The unknown gate acts on P and Q rather than directly on A and B, so selecting a gate from the final truth table alone would miss the preceding circuit.

Step 2: Match every row using NOR A NOR gate complements the OR of its inputs:

$$Y = \overline{P + Q} = \overline{A\overline{B} + \overline{A}B}.$$

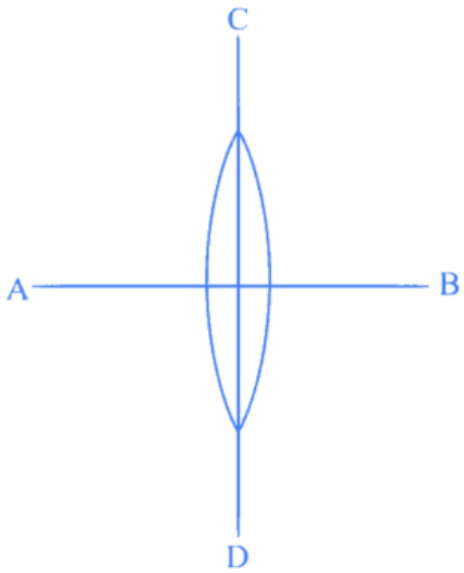
When $(A, B) = (0, 0)$, both products vanish and NOR gives 1. At $(0, 1)$, P is 0 and Q is 1, giving 0. At $(1, 0)$, P is 1 and Q is 0, again giving 0. Finally, at $(1, 1)$, both products vanish and the output returns to 1. The resulting output sequence $1, 0, 0, 1$ agrees with every row of the printed table.

Step 3: Eliminate the remaining gate choices OR would give $0, 1, 1, 0$, the opposite of the required output. Since P and Q can never simultaneously equal 1, an AND gate would always give 0, while a NAND gate would always give 1. These fail the equal-input rows or the unequal-input rows. NOR alone gives the required output with the preceding gates unchanged.

Step 4: Conclusion The gate required at G is $\boxed{\text{NOR}}$. Hence, the correct option is (A).

Quick Tip to Solve: Write the signals entering the unknown gate first. Here they are the two XOR product terms, so NOR converts their OR into XNOR.

35. A symmetric thin biconvex lens is cut into four equal parts by two planes AB and CD as shown in figure. If the power of original lens is 4D then the power of a part of the divided lens is [4 marks]



(A) 2D

(B) 8D

(C) 4D

(D) D

Correct Answer: (A) 2D

Solution:

Concept:

- The thin lens-maker formula in air is $P = (\mu - 1)(1/R_1 - 1/R_2)$, where P is the power in dioptres when the radii are in metres and μ is the refractive index.
- For a symmetric biconvex lens, $R_1 = R$ and $R_2 = -R$. A perpendicular central cut replaces one curved surface by a plane, while an axial cut reduces aperture without changing the remaining surface curvatures.

Step 1: Separate the two surface contributions AB is the principal-axis direction in the diagram. For the original symmetric lens, the two spherical surfaces contribute equally:

$$P = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{2(\mu - 1)}{R} = 4\,\mathrm{D}.$$

Thus each curved surface contributes $(\mu - 1)/R = 2\,\mathrm{D}$ to the original total. Power depends on refractive index and surface curvature, rather than on the amount of glass or the aperture area.

Step 2: Apply the cuts in sequence CD passes through the centre perpendicular to the principal axis. This cut makes two plano-convex halves, each with one original curved surface and one plane surface. The plane has infinite radius, so

$$P_{\text{half}} = (\mu - 1)\left(\frac{1}{R} - 0\right) = \frac{P}{2}.$$

The cut in plane AB then divides each half along the principal axis. It reduces aperture, but preserves the curved-face radius R and the plane-face radius infinity. It therefore does not halve the optical power again.

Step 3: Evaluate the power and reject alternatives Every quarter-piece has power

$$P_{\text{part}} = \frac{4}{2} = 2\,\mathrm{D}.$$

The corresponding focal length is $f = 1/P_{\text{part}} = 0.50\,\mathrm{m}$, still positive because each piece remains converging. The value $4\,\mathrm{D}$ would apply to an axial cut alone; $1\,\mathrm{D}$ incorrectly divides the power by the number of pieces. The value $8\,\mathrm{D}$ wrongly assumes that reducing lens size increases curvature.

Step 4: Conclusion Each divided part has power $\boxed{2\,\mathrm{D}}$. Hence, the correct option is (A).

Quick Tip to Solve: Track which cut changes the optical surfaces: a perpendicular central cut halves this symmetric lens power, while an axial cut changes aperture only.

36. A tube of length L is shown in the figure. The radius of cross section at the point (1) is 2 cm and at the point (2) is 1 cm , respectively. If the velocity of water entering at point (1) is $2\ \mathrm{m/s}$, then velocity of water leaving the point (2) will be [4 marks]



(A) 4 m/s

(B) 6 m/s

(C) 2 m/s

(D) 8 m/s

Correct Answer: (D) 8 m/s

Solution:

Concept:

- For steady incompressible flow with no leakage, volume flow rate is conserved: $Q = A_1 v_1 = A_2 v_2$. Water is treated as incompressible in this calculation.
- The area of a circular cross-section is $A = \pi r^2$. Thus the speed is inversely proportional to radius squared, rather than inversely proportional to radius itself.

Step 1: Calculate the area ratio The inlet radius is $r_1 = 2\,\mathrm{cm}$ and the outlet radius is $r_2 = 1\,\mathrm{cm}$. Therefore,

$$\frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \left(\frac{2}{1}\right)^2 = 4.$$

The common factor π cancels. Since both radii are in the same unit, conversion to metres is unnecessary for this dimensionless ratio. Converting both first would yield exactly the same ratio.

Step 2: Use conservation of volume flow Steady flow requires the volume entering the tube per second to equal the volume leaving it per second. Otherwise water would accumulate inside. Thus

$$A_1 v_1 = A_2 v_2 \quad \Longrightarrow \quad v_2 = \frac{A_1}{A_2} v_1.$$

Substitute $v_1 = 2\,\mathrm{m\,s^{-1}}$:

$$v_2 = 4(2) = 8\,\mathrm{m\,s^{-1}}.$$

The smaller outlet area requires greater speed. Tube length L does not enter this relation because no pressure drop or viscous-flow calculation is required.

Step 3: Check each proposed speed The speed $4\,\mathrm{m\,s^{-1}}$ follows from incorrectly using the radius ratio instead of the area ratio. A speed of $2\,\mathrm{m\,s^{-1}}$ gives an outlet volume flow rate only one quarter of the

inlet rate. At $6\,\mathrm{m\,s^{-1}}$, it is only three quarters of the inlet rate. Only $8\,\mathrm{m\,s^{-1}}$ makes the product of area and speed equal at both ends and satisfies the continuity condition.

Step 4: Conclusion Water leaves point (2) with velocity $\boxed{8\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (D).

Quick Tip to Solve: Square the radius ratio before multiplying the inlet speed: halving a circular tube's radius makes the speed four times as large.

37. A series LCR circuit is connected to an alternating source of emf E. The current amplitude at resonant frequency is I_0. If the value of resistance R becomes twice of its initial value then amplitude of current at resonance will be [4 marks]

(A) $\frac{I_0}{2}$

(B) $\frac{I_0}{\sqrt{2}}$

(C) $2I_0$

(D) I_0

Correct Answer: (A) $\frac{I_0}{2}$

Solution:

Concept:

- For a sinusoidal source of emf amplitude E, the current amplitude in a series LCR circuit is $I = E/Z$, where $Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}$.
- At resonance, the inductive and capacitive reactances cancel: $\omega_0 L = 1/(\omega_0 C)$. The impedance is then purely resistive, $Z_0 = R$.

Step 1: Write the initial resonant current The word amplitude means the peak value; the same amplitude convention must be used for both the initial and final circuits. At the resonant frequency the reactive voltages cancel, so only the resistance limits the current. Thus

$$I_0 = \frac{E}{R}.$$

The source emf amplitude, inductance and capacitance remain unchanged in the stated change.

Step 2: Double the resistance and compare The new resistance is $R' = 2R$. Since L and C are unchanged, the resonance condition and resonant frequency are unchanged. The new current amplitude is therefore

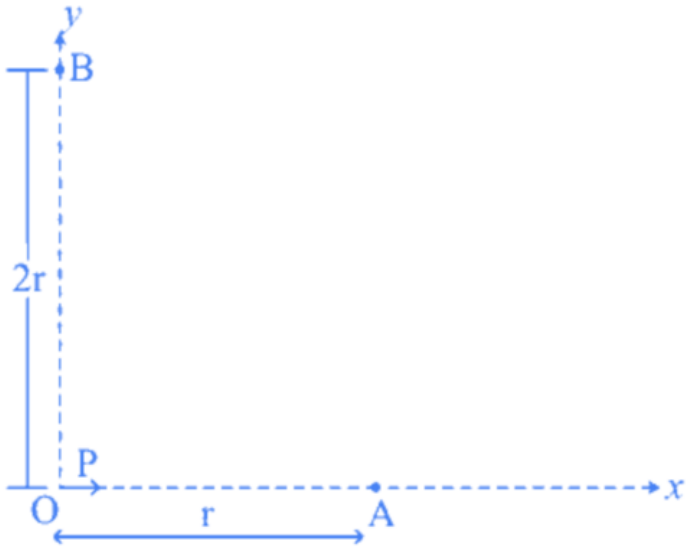
$$I' = \frac{E}{2R} = \frac{1}{2}\left(\frac{E}{R}\right) = \frac{I_0}{2}.$$

This inverse dependence is on resistance itself, not on its square root. Option (B) would represent a square-root scaling and does not follow from Ohm's law at resonance. Option (C) incorrectly makes current increase with resistance, while option (D) ignores the resistance change.

Step 3: Conclusion The current amplitude at resonance becomes $\boxed{I_0/2}$. Hence, the correct option is (A).

Quick Tip to Solve: At series resonance replace the entire LCR impedance by R; with fixed source amplitude, multiplying R by a factor divides the current by that factor.

38. For a short dipole placed at origin O , the dipole moment P is along x-axis, as shown in the figure. If the electric potential and electric field at A are V_0 and E_0, respectively, then the correct combination of the electric potential and electric field, respectively, at point B on the y-axis is given by [4 marks]



(A) $\frac{V_0}{2}$ and $\frac{E_0}{16}$
(B) zero and $\frac{E_0}{8}$
(C) V_0 and $\frac{E_0}{4}$
(D) zero and $\frac{E_0}{16}$

Correct Answer: (D) zero and $\frac{E_0}{16}$

Solution:

Concept:

- For a short dipole of moment magnitude p, the potential at distance s and polar angle θ is $V = kp\cos\theta/s^2$, where $k = 1/(4\pi\varepsilon_0)$.
- The axial field magnitude is $2kp/s^3$, while the equatorial field magnitude is kp/s^3. On the equatorial line the field points opposite to the dipole moment.

Step 1: Use the axial point A as the reference The figure places A on the positive dipole axis with $OA = r$. Hence

$$V_0 = \frac{kp}{r^2}, \qquad E_0 = \frac{2kp}{r^3}.$$

The factor 2 in the axial field is essential. The symbols V_0 and E_0 refer to this axial location, so neither can be transported to a point on a different line using distance scaling alone.

Step 2: Evaluate the potential and field at B Point B lies on the equatorial line, at $OB = 2r$. Its polar angle is 90°, and therefore

$$V_B = \frac{kp\cos 90^\circ}{(2r)^2} = 0.$$

The individual charge potentials cancel there, but their electric fields do not cancel. The field

magnitude is

$$E_B = \frac{kp}{(2r)^3} = \frac{kp}{8r^3} = \frac{1}{16}\left(\frac{2kp}{r^3}\right) = \frac{E_0}{16}.$$

It points along negative x. Options (A) and (C) fail because their potentials are nonzero. Option (B) includes only the distance-cubed factor and misses the additional factor of two between axial and equatorial fields.

Step 3: Conclusion At B, the potential is $\boxed{0}$ and the electric-field magnitude is $\boxed{E_0/16}$, directed opposite to the dipole moment. Hence, the correct option is (D).

Quick Tip to Solve: For an axial-to-equatorial comparison, first include the factor $1/2$ in field strength, then apply the inverse-cube distance factor.

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39. A transparent film of refractive index, 2.0 is coated on a glass slab of refractive index, 1.45. What is the minimum thickness of transparent film to be coated for the maximum transmission of Green light of wavelength 550 nm . [Assume that the light is incident nearly perpendicular to the glass surface.] [4 marks]

(A) 68.7 nm

(B) 137.5 nm

(C) 94.8 nm

(D) 275 nm

Correct Answer: (B) 137.5 nm

Solution:

Concept:

- At near-normal incidence, the optical path difference between successive reflected rays in a film of refractive index n and thickness t is $2nt$. Its propagation phase difference is $4\pi nt/\lambda$.
- Reflection from a lower to a higher refractive index introduces a phase reversal of π; reflection from a higher to a lower index does not. For a transparent nonabsorbing film, a reflection minimum corresponds to a transmission maximum.

Step 1: Count the reflection phase reversals The light first meets the air-to-film boundary, where the index increases from approximately 1 to 2.0. This reflected ray undergoes a phase reversal. The ray reflected at the film-to-glass boundary travels from index 2.0 to 1.45, so it has no reflection phase reversal. Thus the two reflected rays have one relative phase reversal, in addition to the propagation phase accumulated inside the film.

Step 2: Apply the condition for minimum reflection Because there is one reflection phase reversal, destructive interference of the reflected rays occurs when the propagation phase is an integral multiple of 2π:

$$2nt = m\lambda.$$

The zero-thickness value $m = 0$ is not a coating. The smallest positive thickness corresponds to $m = 1$,

giving

$$t_{\min} = \frac{\lambda}{2n} = \frac{550\,\text{nm}}{2(2.0)} = 137.5\,\text{nm}.$$

The familiar quarter-wave thickness, about 68.7 nm here, would instead maximize reflection because the film has a larger index than both surrounding media. The 94.8 nm option incorrectly uses the glass index, while 275 nm is a larger allowed thickness, not the minimum.

Step 3: Conclusion The minimum positive film thickness for maximum transmission is $\boxed{137.5\,\text{nm}}$. Hence, the correct option is (B).

Quick Tip to Solve: Before using a quarter-wave coating formula, compare all three indices. With exactly one reflection phase reversal, minimum reflection requires a half-wave optical thickness.

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40. A light source of wavelength λ illuminates a metal surface and electrons are ejected with maximum kinetic energy of 2 eV . If the same surface is illuminated by a light source of wavelength $\frac{\lambda}{2}$, then the maximum kinetic energy of ejected electrons will be (The work function of metal is 1 eV) [4 marks]

(A) 5 eV

(B) 3 eV

(C) 2 eV

(D) 6 eV

Correct Answer: (A) 5 eV

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi = hc/\lambda - \phi$, where ϕ is the work function of the metal.
- Photon energy varies inversely with wavelength. Halving the wavelength doubles the photon energy, while the work function stays fixed for the same metal surface.

Step 1: Recover the original photon energy The original maximum electron kinetic energy is $K_1 = 2\,\text{eV}$, and the work function is $\phi = 1\,\text{eV}$. The photon must provide both the energy needed to release the electron and its maximum kinetic energy:

$$\frac{hc}{\lambda} = K_1 + \phi = 2 + 1 = 3\,\text{eV}.$$

Since every energy is expressed in electron volts, no conversion to joules is required.

Step 2: Halve the wavelength and subtract the same work function For the new source,

$$E_{\gamma,2} = \frac{hc}{\lambda/2} = 2\frac{hc}{\lambda} = 6\,\text{eV}.$$

The work function is a property of the unchanged metal, so it is subtracted once again:

$$K_2 = E_{\gamma,2} - \phi = 6 - 1 = 5\,\text{eV}.$$

Equivalently, $K_2 = 2(K_1 + \phi) - \phi = 2K_1 + \phi$. Option (D) gives the new photon energy rather than the electron kinetic energy. Option (B) gives the old photon energy, and option (C) leaves the kinetic energy unchanged despite the higher incident photon energy.

Step 3: Conclusion The maximum kinetic energy of the emitted electrons is $\boxed{5\,\text{eV}}$. Hence, the correct option is (A).

Quick Tip to Solve: Double $K_{\max} + \phi$, not $K_{\max}$, when the wavelength is halved; then subtract the same work function.

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41. For a diatomic gas, if $\gamma_1 = \left(\frac{Cp}{Cv}\right)$ for rigid molecules and $\gamma_2 = \left(\frac{Cp}{Cv}\right)$ for another diatomic molecules, but also having vibrational modes. Then, which one of the following options is correct? (Cp and Cv are specific heats of the gas at constant pressure and volume) [4 marks]

(A) $\gamma_2 < \gamma_1$

(B) $\gamma_2 > \gamma_1$

(C) $\gamma_2 = \gamma_1$

(D) $2\gamma_2 = \gamma_1$

Correct Answer: (A) $\gamma_2 < \gamma_1$

Solution:

Concept:

- For an ideal gas with f active quadratic degrees of freedom, molar heat capacities are $C_V = fR/2$ and $C_P = C_V + R$. Consequently, $\gamma = C_P/C_V = 1 + 2/f$.
- A rigid diatomic molecule has three translational and two rotational degrees of freedom, so $f = 5$. An active vibrational mode adds both kinetic and potential energy contributions to the heat capacity.

Step 1: Calculate the ratio for rigid molecules For a rigid diatomic gas, vibration is absent from the energy accounting. The five active degrees of freedom give

$$C_{V,1} = \frac{5}{2}R, \qquad C_{P,1} = \frac{7}{2}R.$$

Therefore

$$\gamma_1 = \frac{C_{P,1}}{C_{V,1}} = \frac{7}{5} = 1.4.$$

The ratio is the same whether molar or mass-specific heat capacities are used, since the conversion factor cancels.

Step 2: Include vibration and compare the ratios For a fully active diatomic vibrational mode there are two additional quadratic contributions, so $f = 7$. Thus

$$C_{V,2} = \frac{7}{2}R, \qquad C_{P,2} = \frac{9}{2}R, \qquad \gamma_2 = \frac{9}{7}.$$

Comparing directly,

$$\gamma_1 - \gamma_2 = \frac{7}{5} - \frac{9}{7} = \frac{4}{35} > 0.$$

More generally, activating vibration increases C_V while the difference $C_P - C_V = R$ stays unchanged; hence $1 + R/C_V$ decreases. Option (B) reverses this effect, option (C) ignores vibration, and option (D) incorrectly makes the ratio half its rigid value. Indeed both ratios must remain greater than one.

Step 3: Conclusion The gas with active vibrational motion has the smaller specific-heat ratio: $\boxed{\gamma_2 < \gamma_1}$. Hence, the correct option is (A).

Quick Tip to Solve: Extra active degrees of freedom increase C_V, so use $\gamma = 1 + R/C_V$ to see immediately that γ decreases.

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42. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : In Young's double slit experiment, the fringes produced by red light are closer as compared to those produced by blue light.

Reason (R) : The fringe width is directly proportional to the wavelength of light.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) (A) is true but (R) is false

(B) Both (A) and (R) are true and (R) is the correct explanation of (A)

(C) (A) is false but (R) is true

(D) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

Correct Answer: (C) (A) is false but (R) is true

Solution:

Concept:

- In Young's double-slit experiment, the fringe width is $\beta = \lambda D/d$, where λ is the wavelength, D is the slit-to-screen distance, and d is the slit separation.
- For unchanged geometry, fringe spacing is directly proportional to wavelength. Red light has a longer wavelength than blue light.

Step 1: Test the assertion using the fringe-width formula The assertion says that fringes produced by red light are closer together than those produced by blue light. Let their wavelengths be λ_r and λ_b. Since $\lambda_r > \lambda_b$ and the apparatus is unchanged,

$$\frac{\beta_r}{\beta_b} = \frac{\lambda_r}{\lambda_b} > 1.$$

Thus red fringes are farther apart, not closer. The assertion is false. The word closer refers to the smaller distance between adjacent fringes; it is blue light that produces that smaller distance.

Step 2: Test the reason independently The reason states that fringe width is directly proportional to wavelength. Holding D and d constant in

$$\beta = \lambda \frac{D}{d}$$

shows that this statement is true. Increasing wavelength increases the width by exactly the same factor. Hence the reason predicts the opposite of the assertion rather than supporting it.

Step 3: Select the matching truth combination The required combination is a false assertion and a true reason. Option (A) reverses both truth values. Options (B) and (D) both claim the assertion is true, so neither can be selected, regardless of their explanation clauses. Option (C) alone identifies the two statements correctly.

Step 4: Conclusion The assertion is false and the reason is true. Hence, the correct option is (C).

Quick Tip to Solve: For a fixed double-slit setup, longer wavelength means wider fringes: red spreads them out, while blue packs them closer.

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43. A ball of mass 100 g is projected with velocity $20\ \mathrm{m/s}$ at 60° with horizontal. The decrease in kinetic energy of the ball during the motion from point of projection to highest point is [4 marks]

(A) 20 J

(B) 5 J

(C) 15 J

(D) zero

Correct Answer: (C) 15 J

Solution:

Concept:

- Without air resistance, gravity changes only the vertical component of a projectile's velocity. The horizontal component remains $u_x = u\cos\theta$.
- Kinetic energy is $K = \frac{1}{2}mv^2$. At the highest point, the vertical velocity vanishes, but horizontal motion continues.

Step 1: Resolve the launch velocity Convert the mass before using SI units: $m = 100\,\mathrm{g} = 0.100\,\mathrm{kg}$. The initial speed is $u = 20\,\mathrm{m\,s^{-1}}$, at $\theta = 60^\circ$. Thus

$$u_x = 20\cos 60^\circ = 10\,\mathrm{m\,s^{-1}}, \qquad u_y = 20\sin 60^\circ = 10\sqrt{3}\,\mathrm{m\,s^{-1}}.$$

At the top, the vertical component is zero because the upward motion has just ended; it does not follow that the whole velocity is zero.

Step 2: Calculate the lost kinetic energy The launch kinetic energy includes both velocity components, whereas at the highest point only the horizontal component remains:

$$K_i = \tfrac{1}{2}(0.100)(20)^2 = 20\,\mathrm{J}, \qquad K_h = \tfrac{1}{2}(0.100)(10)^2 = 5\,\mathrm{J}.$$

Therefore the decrease, taken as a positive quantity, is

$$K_i - K_h = 20 - 5 = 15\,\mathrm{J}.$$

Equivalently, the lost energy is the initial vertical kinetic energy, $\frac{1}{2}mu^2\sin^2 60^\circ$. This energy becomes

gravitational potential energy.

Step 3: Check the options The value $20\,\text{J}$ is the initial total energy, so option (A) would incorrectly stop the horizontal motion too. The value $5\,\text{J}$ in option (B) is the energy still present at the top, not its decrease. Zero in option (D) ignores the work done against gravity during the rise.

Step 4: Conclusion The kinetic energy decreases by $\boxed{15\,\text{J}}$. Hence, the correct option is (C).

Quick Tip to Solve: At a projectile's highest point retain $u\cos\theta$; the kinetic-energy decrease is $\frac{1}{2}mu^2\sin^2\theta$.

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44. A force $\overrightarrow{\text{F}} = 2\hat{i} + \text{b}\hat{j} + \hat{k}$ is applied on a particle and it undergoes a displacement $\hat{i} - 2\hat{j} - \hat{k}$ What will be the value of b, if work done on the particle is zero. [4 marks]

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) 0

(D) 2

Correct Answer: (B) $\frac{1}{2}$

Solution:

Concept:

- For a constant force, work is the scalar product $W = \vec{F}\cdot\vec{s} = F_x s_x + F_y s_y + F_z s_z$. Only matching Cartesian components contribute.
- Unit vectors along mutually perpendicular axes have zero scalar product; each unit vector has scalar product one with itself. Zero work means the force has no net component along the displacement.

Step 1: Identify corresponding components The force components are $(2, b, 1)$, while the displacement components are $(1, -2, -1)$. Keep the minus signs of the displacement; they specify motion opposite to the positive y and z axes. Pair the i, j, and k components with their counterparts rather than multiplying the vector magnitudes.

Step 2: Set the work equal to zero Expanding the scalar product gives

$$W = (2\hat{i} + b\hat{j} + \hat{k})\cdot(\hat{i} - 2\hat{j} - \hat{k}) = 2(1) + b(-2) + 1(-1).$$

Thus $W = 1 - 2b$. The condition in the question is $W = 0$, so

$$1 - 2b = 0 \quad\Longrightarrow\quad b = \frac{1}{2}.$$

The positive x-contribution to work is cancelled exactly by the two negative contributions from motion along the other axes.

Step 3: Verify and exclude the other values For $b = \frac{1}{2}$, direct substitution gives $2 - 1 - 1 = 0$, as required. For option (A), $b = \frac{1}{3}$ gives $W = \frac{1}{3}$, while option (C), $b = 0$, gives $W = 1$. Option (D), $b = 2$, gives $W = -3$. None of these satisfies zero work. The result also confirms that a nonzero force can do zero work when it is perpendicular to the displacement.

Step 4: Conclusion $\boxed{b = \frac{1}{2}}$ makes the work zero. Hence, the correct option is (B).

Quick Tip to Solve: For zero work with Cartesian vectors, write one scalar equation by multiplying corresponding components; retain every displacement sign.

45. A small rigid spherical ball of mass M is dropped in a long vertical tube containing glycerine. The velocity of the ball becomes constant after some time. If the density of glycerine is half of the density of the ball, then the viscous force acting on the ball will be (consider g as acceleration due to gravity) [4 marks]

(A) 2 Mg

(B) $\frac{3}{2}$Mg

(C) $\frac{\text{Mg}}{2}$

(D) Mg

Correct Answer: (C) $\frac{\text{Mg}}{2}$

Solution:

Concept:

- At terminal speed a falling body has zero acceleration. Its weight is balanced by the upward buoyant force plus the upward viscous drag.
- Archimedes' principle gives buoyancy $B = \rho_f V g$, where ρ_f is the fluid density and V is the submerged volume. For the ball, $M = \rho_b V$.

Step 1: Express buoyancy using the ball mass Let ρ_b be the density of the rigid ball and ρ_f the density of glycerine. Once the ball is completely immersed, the displaced glycerine volume equals the ball volume, $V = M/\rho_b$. The stated density relation is $\rho_f = \rho_b/2$. Therefore

$$B = \rho_f V g = \frac{\rho_b}{2}\frac{M}{\rho_b} g = \frac{Mg}{2}.$$

This force acts upward and already supports half the weight.

Step 2: Balance the forces at constant velocity During downward motion the viscous force opposes the velocity, so it also acts upward. The downward force is Mg. The velocity becomes constant after some time, so Newton's second law gives

$$Mg - B - F_v = 0.$$

Substitute the buoyancy found above:

$$F_v = Mg - \frac{Mg}{2} = \frac{Mg}{2}.$$

No value of the viscosity or ball radius is required: those quantities determine the terminal speed, whereas force balance directly determines the drag at that speed.

Step 3: Check the alternatives physically Options (A) and (B) exceed the entire weight; together

with buoyancy they would produce an upward net force instead of constant downward speed. Option (D), Mg, would balance weight only if buoyancy were absent, contrary to the given finite glycerine density. Only option (C) leaves the total upward force equal to the weight.

Step 4: Conclusion The viscous force has magnitude $\boxed{Mg/2}$ and acts upward. Hence, the correct option is (C).

Quick Tip to Solve: At terminal speed, viscous drag equals apparent weight: $F_v = Mg(1 - \rho_f/\rho_b)$, not the full weight.

Physics – Section B: Numerical Value Questions (20 marks)

46. A proton is moving undeflected in a region of crossed electric and magnetic fields at a constant speed of 2×10^5 ms^{-1}. When the electric field is switched off, the proton moves along a circular path of radius 2 cm . The magnitude of electric field is $x \times 10^4$ N/C. The value of x is _____. Take the mass of the proton $= 1.6 \times 10^{-27}$ kg. [4 marks]

Solution:

Concept:

- For undeflected motion through crossed fields, electric and magnetic forces cancel: $qE = qvB$, so $E = vB$. The speed is perpendicular to the magnetic field in the circular motion described.
- When the electric field is removed, magnetic force supplies centripetal force: $qvB = mv^2/r$. A magnetic force changes the direction of velocity without changing its speed.

Step 1: Use the circular path to eliminate the magnetic field With the electric field switched off, the proton speed remains $v = 2 \times 10^5\,\mathrm{m\,s^{-1}}$, because magnetic force is perpendicular to velocity and does no work. The orbit radius is $r = 2\,\mathrm{cm} = 2 \times 10^{-2}\,\mathrm{m}$. From the radial force equation,

$$qvB = \frac{mv^2}{r} \quad \Longrightarrow \quad B = \frac{mv}{qr}.$$

This relation uses the speed and radius after switching off the electric field, so it avoids needing the magnetic-field value in advance.

Step 2: Recover the electric field Before switching off, undeflected motion requires equal and opposite electric and magnetic forces. Hence

$$E = vB = \frac{mv^2}{qr}.$$

Use the supplied proton mass $m = 1.6 \times 10^{-27}\,\mathrm{kg}$ and proton charge magnitude $q = 1.6 \times 10^{-19}\,\mathrm{C}$:

$$E = \frac{(1.6 \times 10^{-27})(2 \times 10^5)^2}{(1.6 \times 10^{-19})(2 \times 10^{-2})} = \frac{6.4 \times 10^{-17}}{3.2 \times 10^{-21}} = 2 \times 10^4\,\mathrm{N\,C^{-1}}.$$

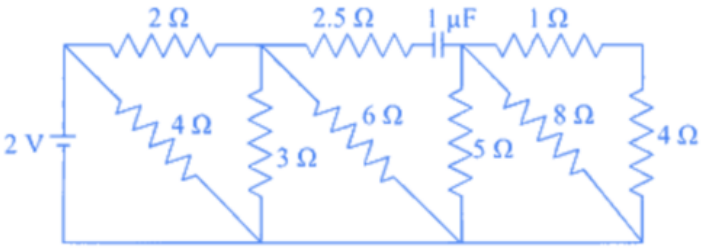
Step 3: Identify the requested coefficient The field is expressed in the question as $x \times 10^4\,\mathrm{N\,C^{-1}}$. Comparing its coefficient with the computed field gives $x = 2$. As a separate check, $B = mv/(qr) = 0.10\,\mathrm{T}$, and $vB = (2 \times 10^5)(0.10) = 2 \times 10^4\,\mathrm{N\,C^{-1}}$, consistent with the direct calculation.

Step 4: Conclusion The electric-field magnitude is $\boxed{2 \times 10^4\,\mathrm{N\,C^{-1}}}$; therefore the required dimensionless coefficient is $\boxed{x = 2}$.

Quick Tip to Solve: Combine $E = vB$ with $r = mv/(qB)$ to obtain $E = mv^2/(qr)$, and convert centimetres to metres first.

---

47. The net current flowing in the given circuit is _____ A. [4 marks]



Solution:

Concept:

- After a capacitor is charged by a steady DC source, its branch carries no current: the capacitor behaves as an open circuit.
- Resistors in parallel share the same voltage and satisfy $1/R_p = 1/R_1 + 1/R_2$. Resistors in series add directly; the source current is $I = V/R_{\mathrm{eq}}$.

Step 1: Remove the open capacitor branch The $1\,\mu\mathrm{F}$ capacitor lies in the upper connection after the $2.5\,\Omega$ resistor. In the steady DC state that connection is open. Consequently no current flows through the $2.5\,\Omega$ resistor or the right-hand network containing $5\,\Omega$, $8\,\Omega$, $1\,\Omega$, and the rightmost $4\,\Omega$ resistor. That network has no closed path to the positive terminal once the capacitor connection is opened.

Step 2: Reduce the conducting left-hand network The $3\,\Omega$ vertical resistor and the $6\,\Omega$ diagonal resistor join the same upper junction and the common lower wire. They are therefore in parallel:

$$R_{36} = \frac{3 \times 6}{3 + 6} = 2\,\Omega.$$

This combination is in series with the upper-left $2\,\Omega$ resistor, giving $2 + 2 = 4\,\Omega$. The resulting branch is in parallel with the left diagonal $4\,\Omega$ resistor, which is connected directly across the source.

Step 3: Find the equivalent resistance and source current The two conducting source branches each have resistance $4\,\Omega$. Their equivalent resistance and the total current are

$$R_{\mathrm{eq}} = \frac{4 \times 4}{4 + 4} = 2\,\Omega, \qquad I = \frac{2\,\mathrm{V}}{2\,\Omega} = 1\,\mathrm{A}.$$

Each branch draws $0.5\,\mathrm{A}$, so adding the branch currents also gives $1\,\mathrm{A}$. Treating the capacitor as a wire would incorrectly include the right-hand network and change the source current.

Step 4: Conclusion The net steady current supplied by the $2\,\mathrm{V}$ cell is $\boxed{1\,\mathrm{A}}$.

Quick Tip to Solve: In steady DC, open the capacitor branch first; then identify parallel resistors by their two common nodes, rather than by their visual positions.

48. A parallel plate capacitor of area $A = 16\ \text{cm}^2$ and separation between the plates 10 cm , is charged by a DC current. Consider a hypothetical plane surface of area $\text{A}_0 = 3.2\ \text{cm}^2$ inside the capacitor and parallel to the plates. At an instant, the current through the circuit is 6A. At the same instant the displacement current through A_0 is _____ mA . [4 marks]

Solution:

Concept:

- Displacement current is $I_d = \varepsilon_0\, d\Phi_E/dt$. Between ideal parallel plates the electric field is uniform, so a parallel surface of area A_0 has flux $\Phi_E = EA_0$.
- The displacement current through the full plate area equals the conduction current in the charging leads. For a smaller parallel area, the displacement current scales in proportion to area.

Step 1: Relate the changing field to charging current Let Q be the instantaneous charge on a plate. Neglecting edge effects, the electric field inside the parallel-plate capacitor is

$$E = \frac{Q}{\varepsilon_0 A}.$$

Since the plate area is fixed, differentiating with respect to time gives

$$\frac{dE}{dt} = \frac{1}{\varepsilon_0 A}\frac{dQ}{dt} = \frac{I}{\varepsilon_0 A}.$$

Here $I = 6\,\text{A}$ is the current charging the plates at the specified instant. A DC charging current still changes the electric field while charge accumulates.

Step 2: Evaluate the current through the smaller area The hypothetical surface is parallel to the plates, so its normal is along the electric field. Its displacement current magnitude is

$$I_{d0} = \varepsilon_0 A_0 \frac{dE}{dt} = I\frac{A_0}{A}.$$

Both given areas are in cm^2, so their units cancel in the ratio:

$$I_{d0} = 6\frac{3.2}{16} = 6(0.20) = 1.2\,\text{A}.$$

The plate separation does not enter this area ratio; it is unnecessary to calculate the capacitance or potential difference.

Step 3: Convert the result to the requested unit Using $1\,\text{A} = 1000\,\text{mA}$, the result is

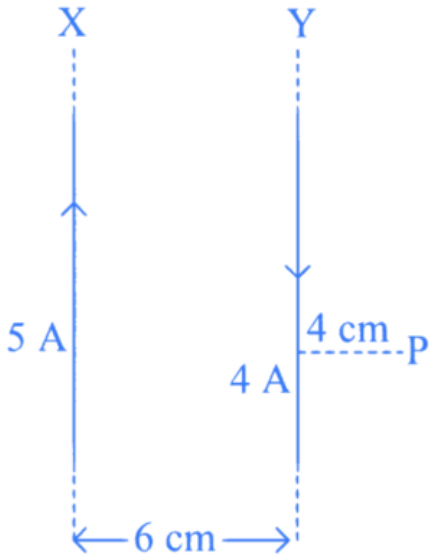
$$1.2\,\text{A} = 1200\,\text{mA}.$$

The smaller surface covers one fifth of the full plate area, so its displacement current is one fifth of the full $6\,\text{A}$, which provides a direct consistency check. The question asks for milliamperes, so entering 1.2 would miss the required conversion.

Step 4: Conclusion The displacement current through A_0 is $\boxed{1200\,\text{mA}}$, so the numerical entry is $\boxed{1200}$.

Quick Tip to Solve: For a parallel surface between ideal capacitor plates use $I_d = IA_0/A$; calculate the area fraction before converting amperes to milliamperes.

49. Two long parallel wires X and Y, separated by a distance of 6 cm , carry currents of 5 A and 4A, respectively, in opposite directions as shown in the figure. Magnitude of the resultant magnetic field at point P at a distance of 4 cm from wire Y is $x \times 10^{-5}$ T. The value of x is _____ . Take permeability of free space as $\mu_0 = 4\pi \times 10^{-7}$ SI units. [4 marks]



Solution:

Concept:

- The field of a long straight wire at perpendicular distance r is $B = \mu_0 I/(2\pi r)$. Its direction follows the right-hand thumb rule.
- Magnetic fields add as vectors. At a point outside two wires carrying opposite currents, their fields point oppositely, so their magnitudes must be subtracted.

Step 1: Read both distances from the figure Point P is on the right of wire Y, rather than between the wires. Thus $r_Y = 4\,\text{cm} = 0.04\,\text{m}$, whereas $r_X = 6 + 4 = 10\,\text{cm} = 0.10\,\text{m}$. The currents are $I_X = 5\,\text{A}$ upward and $I_Y = 4\,\text{A}$ downward. Using only the 6 cm wire separation as a field distance would evaluate the field at the wrong point.

Step 2: Calculate and combine the fields For X, the right-hand rule gives a field into the page at P; for Y it gives a field out of the page. Their magnitudes are

$$B_X = \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 0.10} = 1.0 \times 10^{-5}\,\text{T},$$

$$B_Y = \frac{4\pi \times 10^{-7} \times 4}{2\pi \times 0.04} = 2.0 \times 10^{-5}\,\text{T}.$$

Therefore $B_{\text{net}} = B_Y - B_X = (2.0 - 1.0) \times 10^{-5} = 1.0 \times 10^{-5}\,\text{T}$, directed out of the page. This is a difference, not the sum, even though the currents are opposite, because P lies outside the pair.

Step 3: Conclusion The resultant field has magnitude $1.0 \times 10^{-5}\,\text{T}$. Comparing with $x \times 10^{-5}\,\text{T}$, the required dimensionless value is $\boxed{x = 1}$.

Quick Tip to Solve: First locate the observation point: outside opposite-current wires, subtract their fields and use each wire's own perpendicular distance.

50. A tube of length 1 m is filled completely with an ideal liquid of mass 2 M , and closed at both ends. The tube is rotated uniformly in horizontal plane about one of its ends. If the force exerted by the liquid at the other end is F then angular velocity of the tube is $\sqrt{\frac{F}{\alpha M}}$ in SI unit. The value of α is _____.

[4 marks]

Solution:

Concept:

- A liquid element at distance r from the rotation axis needs inward force $dF = \omega^2 r\, dm$. Different elements have different radii, so use an integral rather than assigning all the liquid to the far end.
- For a uniform tube of length L, the linear mass density is $\lambda = 2M/L$. The resultant centrifugal loading on its outer closure is the sum $\int_0^L \lambda\omega^2 r\, dr$.

Step 1: Set up the distributed force The tube rotates about one end, so the liquid extends from radius zero to radius $L = 1\,\text{m}$. Its total mass is $2M$, giving $dm = (2M/L)\, dr$. In the rotating frame, each element presses outward, and the force transmitted to the outer closed end is found by adding these loads. Equivalently, rotational hydrostatic balance gives $dp/dr = \rho\omega^2 r$, with the closure loading given by the resulting centrifugal pressure difference.

Step 2: Integrate and identify the coefficient Using the mass distribution,

$$F = \int_0^L \frac{2M}{L}\omega^2 r\, dr = \frac{2M\omega^2}{L}\left[\frac{r^2}{2}\right]_0^L = M\omega^2 L.$$

For the stated one-metre tube this gives $\omega = \sqrt{F/(M \times 1\,\text{m})}$. The question expresses lengths in SI numerical values, so its expression becomes $\omega = \sqrt{F/(\alpha M)}$. The numerical coefficient is therefore one. Treating the entire mass $2M$ as if it were at the outer end would overestimate the centrifugal load by a factor of two; its mean radius is $L/2$.

Step 3: Conclusion For $L = 1\,\text{m}$, the requested coefficient is $\boxed{\alpha = 1}$, and the angular velocity is in $\text{rad}\,\text{s}^{-1}$.

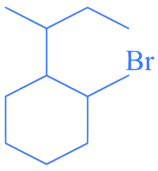
Quick Tip to Solve: For a uniformly filled radial tube, use its mean radius L/2: total mass times omega squared times L/2 gives the distributed centrifugal load.

Chemistry – Section A: Multiple Choice Questions (80 marks)

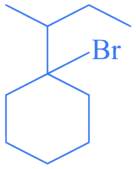
51. When sec-butylcyclohexane reacts with bromine in the presence of sunlight, the major product is :

[4 marks]

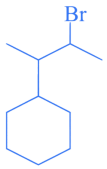
(A) 1-Bromo-2-(butan-2-yl)cyclohexane



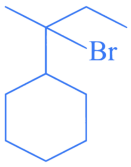
(B) 1-Bromo-1-(butan-2-yl)cyclohexane



(C) $C_6H_{11}-CH(CH_3)-CH(Br)-CH_3$

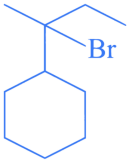


(D) $C_6H_{11}-C(Br)(CH_3)-CH_2-CH_3$



Correct Answer: (D)

$C_6H_{11}-C(Br)(CH_3)-CH_2-CH_3$



Solution:

Concept:

- Bromine in sunlight undergoes homolytic cleavage, $Br_2 \xrightarrow{h\nu} 2\,Br\cdot$. Alkane bromination proceeds by a radical-chain substitution, not electrophilic addition.
- Bromination is highly selective: abstraction of a tertiary hydrogen is favoured over secondary and primary sites. Between tertiary radicals, compare hyperconjugative stabilisation by adjacent carbon–hydrogen bonds.

Step 1: Identify the competing tertiary sites Write the substrate as $C_6H_{11}-CH(CH_3)-CH_2-CH_3$. It has a tertiary hydrogen at the side-chain carbon bonded directly to the cyclohexyl group and another tertiary hydrogen at the substituted ring carbon. Removal of the side-chain hydrogen produces $C_6H_{11}-C\cdot(CH_3)-CH_2-CH_3$. The carbons adjacent to this radical supply six hydrogens in total: three from the methyl group, two from the methylene group and one from the attached ring carbon.

Step 2: Select the most stabilised radical and product The radical at the substituted ring carbon has five adjacent hydrogens: two on each neighbouring ring methylene and one on the side-chain methine. The side-chain tertiary radical therefore has the greater hyperconjugative stabilisation and is favoured. It abstracts bromine from Br_2, giving $C_6H_{11}-C(Br)(CH_3)-CH_2-CH_3$. Option A substitutes at a secondary ring position, and option C at a secondary side-chain position; both are less favoured. Option B brominates the other tertiary site, whose radical has fewer hyperconjugating adjacent hydrogens.

Step 3: Conclusion The major product has Br on the tertiary side-chain carbon directly attached to the ring, $C_6H_{11}-C(Br)(CH_3)-CH_2-CH_3$. Hence, the correct option is (D).

Quick Tip to Solve: In selective bromination, first rank radical substitution; if two tertiary sites compete, count hydrogens on the carbons adjacent to each radical centre.

52. Given below are two statements :

Statement (I) : Corrosion is an electrochemical phenomenon in which pure metal acts as an anode and impure metal as a cathode.

Statement (II) : The rate of corrosion is more in alkaline medium than in acidic medium.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- Corrosion is an electrochemical oxidation: the anodic region loses metal atoms as ions, while a cathodic region supports a reduction reaction.
- In the usual microscopic corrosion cell, the metal region acts as anode and an impurity-rich region acts as cathode. Acidity generally accelerates ordinary corrosion by assisting the cathodic reaction and attacking protective films.

Step 1: Assess Statement I At an anodic iron site, oxidation can be written $\mathrm{Fe} \longrightarrow \mathrm{Fe}^{2+} + 2\,\mathrm{e}^{-}$. The electrons flow through the metal to a cathodic impurity site, where oxygen or hydrogen ions can accept them. Thus the pure-metal region is the anodic dissolving region, whereas the impurity-associated region is cathodic in the ordinary local-cell description used here. Statement I correctly identifies both the electrochemical nature and the roles of these regions.

Step 2: Assess Statement II and compare the options An acidic medium provides hydrogen ions and often dissolves oxide layers, thereby favouring corrosion. For example, the cathodic reduction may be represented by $2\,\mathrm{H}^{+} + 2\,\mathrm{e}^{-} \longrightarrow \mathrm{H}_2$. In neutral aerated water, oxygen reduction is $\mathrm{O}_2 + 2\,\mathrm{H_2O} + 4\,\mathrm{e}^{-} \longrightarrow 4\,\mathrm{OH}^{-}$. An alkaline medium commonly encourages formation of protective oxide or hydroxide films and does not generally corrode ordinary metals faster than an acidic medium. Hence Statement II reverses the usual trend and is false. Option A incorrectly accepts II; option B incorrectly rejects I; option D reverses both truth values.

Step 3: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: Locate oxidation at the anode, then test the medium separately: ordinary acid corrosion is faster because acidity supports reduction and removes protective layers.

53. The most stable carbocation from the following is : [4 marks]

(A) m-MeO$-C_6H_4-CH_2^+$

(B) p-$CH_3-C_6H_4-CH_2^+$

(C) p-MeO$-C_6H_4-CH_2^+$

(D) $C_6H_5-CH_2^+$

Correct Answer: (C) p-MeO$-C_6H_4-CH_2^+$

Solution:

Concept:

- A benzyl carbocation is resonance stabilised: its positive charge is delocalised from the benzylic carbon to the ortho and para ring positions.
- A methoxy group donates an oxygen lone pair by resonance when it occupies an ortho or para position relative to the benzylic centre. At meta it cannot provide this direct resonance stabilisation, while its inductive effect is withdrawing.

Step 1: Locate the substituents and the resonance pathway The options show a meta-methoxybenzyl cation (A), a para-methylbenzyl cation (B), a para-methoxybenzyl cation (C), and unsubstituted benzyl cation (D). In a benzyl-cation resonance contributor with positive charge at the para carbon, the oxygen lone pair of the para methoxy group can form a bond to that carbon. This gives an additional contributor in which oxygen carries the positive charge and the ring carbon has a complete octet. The para methoxy group therefore provides strong stabilisation through its positive mesomeric effect.

Step 2: Compare all four cations Option C has this direct lone-pair donation and is the most stable. Option B is stabilised by methyl hyperconjugation and its weak electron-releasing inductive effect, but methyl cannot provide the same lone-pair resonance donation. Option D has only the usual benzyl resonance contributors. In A, the methoxy group is meta, so it is not directly conjugated with the positions at which the benzyl positive charge is delocalised; its negative inductive effect also does not favour the cation. Simply noticing a methoxy group is therefore insufficient: its position must be checked.

Step 3: Conclusion The para-methoxybenzyl carbocation receives the strongest additional resonance stabilisation. Hence, the correct option is (C).

Quick Tip to Solve: For benzyl cations, map the charge to ortho and para positions; a methoxy substituent stabilises it strongly by resonance only when attached at one of those positions.

54. The correct order of the following complexes in terms of their crystal field stabilization energies is : [4 marks]

(A) $[Co\,(NH_3)_4]^{2+} < [Co\,(NH_3)_6]^{2+} < [Co\,(NH_3)_6]^{3+} < [Co(en)_3]^{3+}$

(B) $[Co\,(NH_3)_4]^{2+} < [Co\,(NH_3)_6]^{2+} < [Co(en)_3]^{3+} < [Co\,(NH_3)_6]^{3+}$

(C) $[Co\,(NH_3)_6]^{2+} < [Co\,(NH_3)_6]^{3+} < [Co\,(NH_3)_4]^{2+} < [Co(en)_3]^{3+}$

(D) $[Co(en)_3]^{3+} < [Co\,(NH_3)_6]^{3+} < [Co\,(NH_3)_6]^{2+} < [Co\,(NH_3)_4]^{2+}$

Correct Answer: (A) $[Co\,(NH_3)_4]^{2+} < [Co\,(NH_3)_6]^{2+} < [Co\,(NH_3)_6]^{3+} < [Co(en)_3]^{3+}$

Solution:

Concept:

- Crystal-field splitting increases with metal oxidation state and ligand-field strength. For the same metal and comparable ligands, tetrahedral splitting is smaller than octahedral splitting, approximately $\Delta_t = 4\Delta_o/9$.
- Ammonia and ethylenediamine are neutral ligands, so the ionic charges give cobalt oxidation states directly: Co(II) is d^7, and Co(III) is d^6. Ethylenediamine lies above ammonia in the spectrochemical series.

Step 1: Compare coordination geometry and oxidation state The four-coordinate $[Co(NH_3)_4]^{2+}$ has the smaller tetrahedral field and hence the least stabilisation magnitude in the listed comparison. The six-coordinate $[Co(NH_3)_6]^{2+}$ is octahedral, giving greater splitting. For the high-spin d^7 case, tetrahedral filling $e^4t_2^3$ gives stabilisation magnitude $1.2\Delta_t$, whereas octahedral filling $t_{2g}^5e_g^2$ gives $0.8\Delta_o$. Since $1.2(4/9)\Delta_o < 0.8\Delta_o$, this confirms the first inequality. Raising cobalt from +2 to +3 increases the metal–ligand interaction and the octahedral splitting.

Step 2: Compare the two Co(III) ligand sets The Co(III) ammine and ethylenediamine complexes are low-spin d^6, with t_{2g}^6 configuration and crystal-field stabilisation magnitude $2.4\Delta_o$. Ethylenediamine produces greater splitting than ammonia, so its complex has the greatest stabilisation magnitude. Therefore the increasing order is tetraammine Co(II), hexaammine Co(II), hexaammine Co(III), then tris(ethylenediamine) Co(III). B incorrectly reverses the last pair; C places the tetrahedral complex above both octahedral ammines; D reverses the expected overall trend. Here the stated order concerns the magnitude of stabilisation, as conventionally used in these options.

Step 3: Conclusion The increasing stabilisation order is $[Co(NH_3)_4]^{2+} < [Co(NH_3)_6]^{2+} < [Co(NH_3)_6]^{3+} < [Co(en)_3]^{3+}$. Hence, the correct option is (A).

Quick Tip to Solve: Check geometry first, oxidation state second, and the spectrochemical series last; en produces a stronger field than ammonia for the same Co(III) ion.

55. The maximum covalency of a non-metallic group 15 element ' E ' with weakest $E - E$ bond is : [4 marks]

(A) 5

(B) 3

(C) 4

(D) 6

Correct Answer: (C) 4

Solution:

Concept:

- The strength of a single element–element bond depends on orbital overlap and lone-pair repulsion; nitrogen is anomalous among group 15 elements.
- Covalency counts shared electron pairs around the central atom. A second-period nitrogen atom is restricted to an octet, so its maximum covalency is four.

Step 1: Identify the element from its single bond The non-metallic group 15 elements include nitrogen and phosphorus. Because nitrogen is small, the nonbonding electron pairs on adjacent nitrogen atoms lie close together across an $N-N$ single bond, producing strong lone-pair repulsion between the bonded atoms. This makes its single bond weaker than the corresponding $P-P$ bond, despite the excellent overlap that makes the nitrogen triple bond very strong. Thus the element E indicated by the weakest single $E - E$ bond is nitrogen.

Step 2: Count the largest allowed covalency Nitrogen has valence configuration $2s^2 2p^3$. It usually makes three bonds, as in NH_3, but its lone pair can participate in a fourth bond, as in $NH_4{}^+$. The nitrogen atom then has four shared pairs and eight electrons in its valence shell. Hence its maximum covalency is four, not merely its usual valency of three.

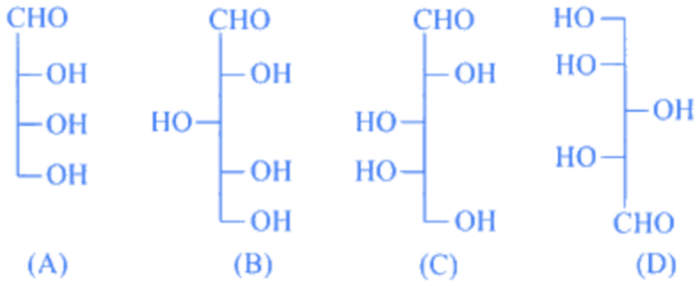
Step 3: Eliminate the remaining values Option (B) gives the common covalency in ammonia rather than the maximum. Options (A) and (D), five and six, would place more than eight bonding electrons around second-period nitrogen and are therefore excluded. The weakness of a single $N-N$ bond must not be confused with the strength of $N \equiv N$.

Step 4: Conclusion The element is nitrogen, whose maximum covalency is $\boxed{4}$. Hence, the correct option is (C).

Quick Tip to Solve: When nitrogen is identified, test the word maximum using $NH_4{}^+$, rather than stopping at the three bonds in NH_3.

56. Identify the number of structure/s from the following which can be corelated to D-glyceraldehyde.

[4 marks]



(A) three

(B) two

(C) one

(D) four

Correct Answer: (A) three

Solution:

Concept:

- A carbohydrate belongs to the D series if the configuration at its highest-numbered chiral carbon is related to D-glyceraldehyde.
- In a standard Fischer projection with the aldehyde group at the top, an OH group on the right at the penultimate carbon indicates D; an OH group on the left indicates L.
- A Fischer projection may be rotated through 180° in its plane without changing its configuration; a 90° rotation is not an allowed equivalent representation.

Step 1: Inspect structures A and B Use the chiral centre farthest from the aldehyde group, not every OH group along the chain. In structure (A), the lowest horizontal OH is on the terminal carbon; the chiral centre immediately above it has OH on the right. Structure (A) therefore has D configuration. In structure (B), the penultimate chiral centre also has OH on the right when CHO is at the top, so (B) is D as well.

Step 2: Inspect structures C and D In structure (C), the penultimate chiral centre has its OH on the left with CHO at the top. Thus (C) belongs to the L series. Structure (D) is drawn with CHO at the bottom. Rotate its entire Fischer projection through 180° in the plane of the paper: CHO then comes to the top, and the OH at the chiral centre farthest from it appears on the right. Hence (D) belongs to the D series.

Step 3: Count only the D structures Structures (A), (B) and (D) can be correlated to D-glyceraldehyde; structure (C) cannot. The count is therefore three. The choices one and two omit at least one valid D structure, while four incorrectly includes the L structure (C). The configurations at the other chiral centres do not alter the D/L designation.

Step 4: Conclusion Three of the displayed structures belong to the D series. Hence, the correct option is (A).

Quick Tip to Solve: Orient CHO at the top first, then read only the highest-numbered chiral centre; ignore the terminal, non-chiral carbon.

57. Given below are two statements :

Statement (I) : Nitrogen, sulphur, halogen and phosphorus present in an organic compound are detected by Lassaigne's Test.

Statement (II) : The elements present in the compound are converted from covalent form into ionic form by fusing the compound with Magnesium in Lassaigne's test.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are false

(B) Statement I is true but Statement II is false

(C) Statement I is false but Statement II is true

(D) Both Statement I and Statement II are true

Correct Answer: (B) Statement I is true but Statement II is false

Solution:

Concept:

- In qualitative elemental analysis, covalently bound heteroatoms must be converted into ionic compounds that can be tested in an aqueous extract.
- Lassaigne's fusion uses metallic sodium. Nitrogen, sulphur and halogens form sodium cyanide, sodium sulphide and sodium halides; phosphorus is detected after conversion to phosphate.

Step 1: Evaluate Statement I Nitrogen, sulphur, halogens and phosphorus are among the heteroatoms investigated in the elemental analysis of organic compounds using the sodium-fusion procedure and its associated tests. Conversion into suitable ionic species permits their detection. Nitrogen gives cyanide and sulphur gives sulphide in the sodium extract, while halogens give halide ions. Phosphorus is converted into phosphate and detected through the appropriate phosphate test. Thus Statement I is true.

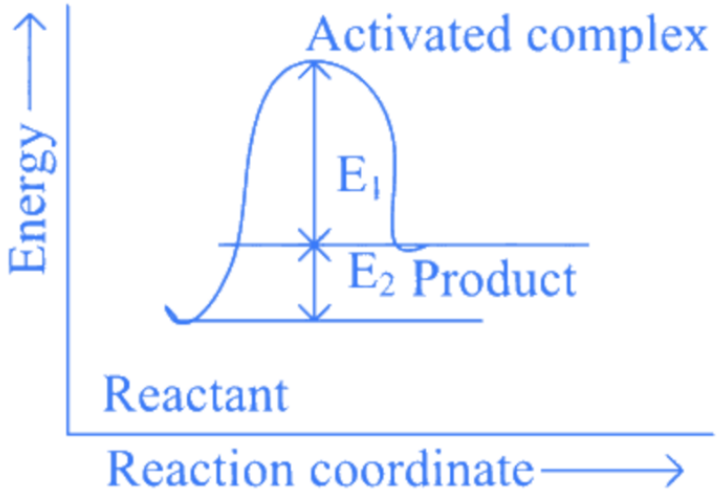
Step 2: Evaluate Statement II The general idea of changing covalently bound elements into ionic species is correct, but the named fusion metal is wrong. The organic compound is fused with sodium, not magnesium, in Lassaigne's test. Typical conversions are represented schematically by $\mathrm{Na + C + N \longrightarrow NaCN}$, $\mathrm{2\,Na + S \longrightarrow Na_2S}$ and $\mathrm{Na + X \longrightarrow NaX}$, where X is a halogen. Since Statement II specifically names magnesium, it is false.

Step 3: Match the truth values to the choices The required combination is I true and II false. Option (A) incorrectly rejects I, option (C) reverses both truth values, and option (D) incorrectly accepts magnesium as the fusion reagent. Only option (B) has the correct pair. A statement containing a correct principle followed by an incorrect reagent is still false as a whole.

Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (B).

Quick Tip to Solve: For Lassaigne's test, check the metal before judging a long statement: sodium fusion produces the soluble ionic extract.

58. Consider the given figure and choose the correct option : [4 marks]



(A) Activation energy of both forward and backward reaction is $E_1 + E_2$ and reactant is more stable than product.

(B) Activation energy of forward reaction is $E_1 + E_2$ and product is more stable than reactant.

(C) Activation energy of backward reaction is E_1 and product is more stable than reactant.

(D) Activation energy of forward reaction is $E_1 + E_2$ and product is less stable than reactant.

Correct Answer: (D) Activation energy of forward reaction is $E_1 + E_2$ and product is less stable than reactant.

Solution:

Concept:

- Activation energy is the vertical energy difference between the activated complex and the starting state for the direction under consideration.
- Let U_R, U_P and $U^{\ddagger}$ be the energies of reactants, products and the activated complex. Then $E_{a,f} = U^{\ddagger} - U_R$ and $E_{a,b} = U^{\ddagger} - U_P$.
- A lower energy state is more stable. The sign of $U_P - U_R$ therefore determines which side is more stable.

Step 1: Read the two labelled energy gaps The product level lies above the reactant level by E_2, and the activated-complex peak lies above the product level by E_1. Therefore $U_P - U_R = E_2 > 0$ and $U^{\ddagger} - U_P = E_1$. The reaction profile is endothermic in the forward direction because the products have greater energy than the reactants.

Step 2: Calculate both activation energies For the forward reaction, climb from the reactant level to the product level and then from the product level to the peak. Adding the two vertical gaps gives

$$E_{a,f} = (U^{\ddagger} - U_P) + (U_P - U_R) = E_1 + E_2.$$

For the backward reaction, the starting level is already the product level, so only the upper gap is crossed:

$$E_{a,b} = U^{\ddagger} - U_P = E_1.$$

Step 3: Compare stability and reject the distractors The products lie at higher energy and hence are less stable than the reactants. Option (A) incorrectly makes both activation energies equal. Options (B) and (C) incorrectly call the product more stable, although (C) correctly identifies the backward barrier. Option (D) gives both the forward activation energy and the stability comparison correctly.

Step 4: Conclusion The forward activation energy is $E_1 + E_2$, and the product is less stable than the reactant. Hence, the correct option is (D).

Quick Tip to Solve: Trace a vertical climb from the starting energy level to the peak; use the starting level for the chosen reaction direction.

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59. Arrange the following compounds in increasing order of their dipole moment :

HBr, H_2S, NF_3 **and** $CHCl_3$ [4 marks]

(A) $HBr < H_2S < NF_3 < CHCl_3$

(B) $H_2S < HBr < NF_3 < CHCl_3$

(C) $NF_3 < HBr < H_2S < CHCl_3$

(D) $CHCl_3 < NF_3 < HBr < H_2S$

Correct Answer: (C) $NF_3 < HBr < H_2S < CHCl_3$

Solution:

Concept:

- A molecular dipole moment is the vector sum of its bond dipoles. Molecular geometry and the direction of lone-pair effects matter as well as bond polarity.
- Dipole moments are commonly expressed in debye (D). For the given molecules, the approximate magnitudes are $NF_3 : 0.24$, $HBr : 0.79$, $H_2S : 0.97$ and $CHCl_3 : 1.04$ D.

Step 1: Identify the unusually small dipole moment Although the $N-F$ bonds are polar, NF_3 has a trigonal-pyramidal geometry in which the resultant of the bond dipoles opposes the contribution associated with the nitrogen lone pair. Their partial cancellation produces a very small molecular dipole moment, about $0.24\,D$. It therefore comes first; high fluorine electronegativity alone does not make its molecular dipole the largest.

Step 2: Order the other three molecules The heteronuclear diatomic molecule HBr has a dipole moment of about $0.79\,D$. Bent H_2S has a larger resultant, about $0.97\,D$, since its bond dipoles do not cancel. Tetrahedral $CHCl_3$ is unsymmetrical because one substituent is hydrogen, so its dipoles also do not cancel; its magnitude is about $1.04\,D$. Thus the last three increase in the order $HBr < H_2S < CHCl_3$.

Step 3: Compare with every proposed order Combining the two comparisons gives

$$NF_3 < HBr < H_2S < CHCl_3.$$

Option (A) places NF_3 too high. Option (B) also reverses the order of HBr and H_2S. Option (D) wrongly starts with $CHCl_3$, the largest of these four. Only (C) matches the increasing magnitudes.

Step 4: Conclusion The increasing order is $\boxed{NF_3 < HBr < H_2S < CHCl_3}$. Hence, the correct option

is (C).

Quick Tip to Solve: Do not rank molecular dipoles by electronegativity alone: first check geometry and cancellation, especially for NF_3.

60. Density of 3 M NaCl solution is $1.25\ \mathrm{g/mL}$. The molality of the solution is : [4 marks]

(A) 2.79 m

(B) 2 m

(C) 1.79 m

(D) 3 m

Correct Answer: (A) 2.79 m

Solution:

Concept:

- Molarity M is moles of solute per litre of solution, whereas molality m is moles of solute per kilogram of solvent.
- Density gives the mass of the whole solution: $m_{\text{solution}} = \rho V$. Subtract the solute mass to obtain the solvent mass before calculating molality.
- The molar mass of NaCl is $23 + 35.5 = 58.5\,\mathrm{g\,mol^{-1}}$.

Step 1: Choose one litre of solution Take $1.000\,\mathrm{L} = 1000\,\mathrm{mL}$ of the solution. Since its molarity is $3\,\mathrm{mol\,L^{-1}}$, this contains

$$n(\mathrm{NaCl}) = 3 \times 1 = 3\,\mathrm{mol}.$$

Using the density, its total mass is

$$m_{\text{solution}} = 1.25 \times 1000 = 1250\,\mathrm{g}.$$

This is the mass of solute plus solvent, not the mass of solvent alone.

Step 2: Find the mass of the solvent The three moles of sodium chloride have mass

$$m_{\text{solute}} = 3 \times 58.5 = 175.5\,\mathrm{g}.$$

Consequently the water mass is

$$m_{\text{solvent}} = 1250 - 175.5 = 1074.5\,\mathrm{g} = 1.0745\,\mathrm{kg}.$$

The subtraction is essential because molality refers specifically to solvent mass.

Step 3: Calculate and compare the molality Divide the number of moles by the solvent mass in kilograms:

$$m = \frac{3}{1.0745} = 2.792\ldots\ \mathrm{mol\,kg^{-1}} \simeq 2.79\,\mathrm{m}.$$

Thus option (A) agrees. The values $2\,\mathrm{m}$ and $1.79\,\mathrm{m}$ do not follow from the given masses. The value $3\,\mathrm{m}$ incorrectly treats the numerical molarity as molality, ignoring the density and the distinction

between solution volume and solvent mass.

Step 4: Conclusion The solution molality is $\boxed{2.79\,\text{mol kg}^{-1}}$, or $2.79\,\text{m}$. Hence, the correct option is (A).

Quick Tip to Solve: Take one litre for a molarity question, convert its volume to total mass using density, and subtract solute mass before dividing.

61. The molar solubility(s) of zirconium phosphate with molecular formula $\left(Zr^{4+}\right)_3\left(PO_4^{3-}\right)_4$ is given by relation : [4 marks]

(A) $\left(\frac{K_{sp}}{5348}\right)^{\frac{1}{6}}$

(B) $\left(\frac{K_{sp}}{8435}\right)^{\frac{1}{7}}$

(C) $\left(\frac{K_{sp}}{6912}\right)^{\frac{1}{7}}$

(D) $\left(\frac{K_{sp}}{9612}\right)^{\frac{1}{3}}$

Correct Answer: (C) $\left(\frac{K_{sp}}{6912}\right)^{\frac{1}{7}}$

Solution:

Concept:

- The molar solubility s is the number of moles of a sparingly soluble salt that dissolve per litre of its saturated solution.
- The solubility product K_{sp} is the product of the dissolved-ion concentrations raised to their stoichiometric powers. Concentrations must therefore include the number of ions released per formula unit.

Step 1: Write the dissolution equilibrium One formula unit releases three zirconium ions and four phosphate ions:

$$Zr_3(PO_4)_4(s) \rightleftharpoons 3\,Zr^{4+}(aq) + 4\,{PO_4}^{3-}(aq).$$

Thus, when the salt has molar solubility s, its saturated solution contains

$$[Zr^{4+}] = 3s, \qquad [{PO_4}^{3-}] = 4s.$$

The solid does not appear in the equilibrium expression because its activity is taken as unity. The coefficients 3 and 4 follow from the formula and also balance total charge: $3(4) = 4(3)$.

Step 2: Form the product and solve for solubility Substitute both ion concentrations into the solubility-product expression:

$$\begin{aligned} K_{sp} &= [Zr^{4+}]^3[{PO_4}^{3-}]^4 \\ &= (3s)^3(4s)^4 \\ &= 3^3 \cdot 4^4\, s^{3+4} \\ &= 27 \cdot 256\, s^7 = 6912 s^7. \end{aligned}$$

Rearranging and taking the positive seventh root gives the physical solubility. The sixth-root and cube-root options have the wrong total exponent. The other seventh-root expression uses 8435 instead of the required factor 6912, so it also fails the stoichiometric calculation.

Step 3: Conclusion

$$s = \left(\frac{K_{\text{sp}}}{6912}\right)^{1/7}.$$

Hence, the correct option is (C).

Quick Tip to Solve: For a salt A_mB_n, first write ion concentrations as ms and ns; the power of s is $m+n$, not the number of distinct ions.

62. Match List - I with List - II.

	List - I (Partial Derivatives)		List - II (Thermodynamic Quantity)
(A)	$\left(\frac{\partial G}{\partial T}\right)_P$	**(I)**	C_p
(B)	$\left(\frac{\partial H}{\partial T}\right)_P$	**(II)**	$-S$
(C)	$\left(\frac{\partial G}{\partial P}\right)_T$	**(III)**	C_v
(D)	$\left(\frac{\partial U}{\partial T}\right)_V$	**(IV)**	V

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(I), (B)-(II), (C)-(IV), (D)-(III)

(B) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(C) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(D) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

Correct Answer: (C) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

Solution:

Concept:

- For a simple closed system of fixed composition doing only pressure-volume work, $dG = -S\,dT + V\,dP$. Here G is Gibbs energy, S entropy and V volume.
- The heat capacities are $C_p = (\partial H/\partial T)_P$ and $C_v = (\partial U/\partial T)_V$, where H and U are enthalpy and internal energy.

Step 1: Read the derivatives of Gibbs energy In $dG = -S\,dT + V\,dP$, hold pressure constant to set $dP = 0$. The coefficient of the temperature change is then

$$\left(\frac{\partial G}{\partial T}\right)_P = -S.$$

Therefore list entry (A) matches (II), including the essential minus sign. Next hold temperature constant, so $dT = 0$. This gives

$$\left(\frac{\partial G}{\partial P}\right)_T = V,$$

and entry (C) matches (IV). These two derivatives measure different responses of the same state

function and must not be interchanged.

Step 2: Use the definitions of heat capacities At constant pressure the temperature derivative of enthalpy is the constant-pressure heat capacity:

$$\left(\frac{\partial H}{\partial T}\right)_P = C_p.$$

Hence (B) matches (I). Likewise, at fixed volume the temperature derivative of internal energy is

$$\left(\frac{\partial U}{\partial T}\right)_V = C_v,$$

so (D) matches (III). Combining all four gives (A)-(II), (B)-(I), (C)-(IV), (D)-(III). Option (A) incorrectly makes the Gibbs temperature derivative a heat capacity. Option (B) assigns the wrong derivatives to heat capacity and volume. Option (D) interchanges the final two matches, assigning a heat capacity to the pressure derivative of Gibbs energy.

Step 3: Conclusion The four matches, in list order, are II, I, IV and III. Hence, the correct option is (C).

Quick Tip to Solve: Start with $dG = -S\,dT + V\,dP$; the two Gibbs derivatives follow immediately, leaving the standard definitions of C_p and C_v.

63. Identify the homoleptic complex(es) that is/are low spin.

(A) $[Fe(CN)_5NO]^{2-}$

(B) $[CoF_6]^{3-}$

(C) $[Fe(CN)_6]^{4-}$

(D) $[Co\,(NH_3)_6]^{3+}$

(E) $[Cr\,(H_2O)_6]^{2+}$

Choose the correct answer from the options given below : [4 marks]

(A) (C) and (D) only

(B) (C) only

(C) (A) and (C) only

(D) (B) and (E) only

Correct Answer: (A) (C) and (D) only

Solution:

Concept:

- A homoleptic complex has only one kind of ligand. A heteroleptic complex contains two or more different kinds, even when its electron configuration is low spin.
- For octahedral d^4 to d^7 ions, low spin occurs when ligand-field splitting Δ_o exceeds pairing energy. Both the ligand and the metal oxidation state influence this competition.

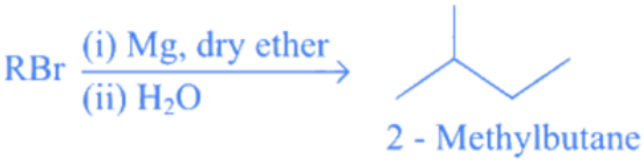
Step 1: Select the homoleptic complexes Complex (A), $[Fe(CN)_5NO]^{2-}$, contains cyanido and nitrosyl ligands. It is heteroleptic and therefore cannot satisfy the question, irrespective of its spin state. Complexes (B), (C), (D) and (E) each contain six ligands of one kind and are homoleptic. The two conditions in the question must be satisfied together; simply choosing every low-spin species would incorrectly retain (A).

Step 2: Check the metal configurations and ligand fields In (B), $[CoF_6]^{3-}$, cobalt is +3 and d^6, but weak-field fluorido ligands give a high-spin $t_{2g}^4 e_g^2$ configuration. In (C), $[Fe(CN)_6]^{4-}$, iron is +2 and d^6. Strong-field cyanido ligands produce low-spin $t_{2g}^6 e_g^0$. In (D), $[Co(NH_3)_6]^{3+}$, cobalt is +3 and d^6; the splitting for this cobalt(III) ammine complex is large enough to give low-spin $t_{2g}^6 e_g^0$. Finally, (E), $[Cr(H_2O)_6]^{2+}$, contains chromium(II), d^4, with weak-field aqua ligands and is high spin, $t_{2g}^3 e_g^1$. Thus (C) and (D) pass both tests. The option containing (C) alone omits (D); the option containing (A) includes a heteroleptic species; the option containing (B) and (E) selects high-spin complexes.

Step 3: Conclusion Only $[Fe(CN)_6]^{4-}$ and $[Co(NH_3)_6]^{3+}$ are both homoleptic and low spin. Hence, the correct option is (A).

Quick Tip to Solve: Apply the ligand-identity test before the spin test. For d^6, a strong octahedral field fills t_{2g} completely and leaves e_g empty.

64. The maximum number of RBr producing 2-methylbutane by above sequence of reactions is _____ . (Consider the structural isomers only) [4 marks]



(A) 5

(B) 3

(C) 4

(D) 1

Correct Answer: (C) 4

Solution:

Concept:

- Magnesium inserts into an alkyl bromide in dry ether to form a Grignard reagent: $RBr + Mg \longrightarrow RMgBr$. Hydrolysis replaces the carbon-magnesium bond by a carbon-hydrogen bond.
- The carbon skeleton remains unchanged in this sequence: $RMgBr + H_2O \longrightarrow RH + Mg(OH)Br$. Distinct precursors are found by replacing one hydrogen at each inequivalent carbon position of the product with bromine.

Step 1: Identify the inequivalent positions in the alkane Write 2-methylbutane as $(CH_3)_2CHCH_2CH_3$. The two methyl groups bonded directly to the methine carbon are equivalent by symmetry; substitution in either gives the same constitutional bromide. The remaining terminal methyl group is different

because it is bonded to the methylene group. There are also the methylene position and the central methine position. Thus the product has four distinct carbon environments bearing hydrogen: two inequivalent kinds of primary position, one secondary position and one tertiary position.

Step 2: List the bromides and check the count Replacing hydrogen by bromine at those four positions gives:

- $BrCH_2CH(CH_3)CH_2CH_3$: 1-bromo-2-methylbutane;
- $BrCH_2CH_2CH(CH_3)CH_3$: 1-bromo-3-methylbutane;
- $CH_3CH(Br)CH(CH_3)CH_3$: 2-bromo-3-methylbutane;
- $(CH_3)_2C(Br)CH_2CH_3$: 2-bromo-2-methylbutane.

Each forms the corresponding carbon-magnesium bond, whose hydrolysis regenerates 2-methylbutane. Only structural isomers are counted, so enantiomers of any chiral bromide are not additional cases. Five double-counts the equivalent methyl groups; three omits a distinct environment; one ignores the different substitution positions.

Step 3: Conclusion The maximum number of structurally distinct alkyl bromides is 4. Hence, the correct option is (C).

Quick Tip to Solve: For Grignard formation followed by hydrolysis, work backwards from RH: replace one H by Br at each symmetry-inequivalent carbon, counting no stereoisomers.

65. The alkane from below having two secondary hydrogens is : [4 marks]

(A) 2,2,4,5-Tetramethylheptane

(B) 2,2,4,4-Tetramethylhexane

(C) 4-Ethyl-3,4-dimethyloctane

(D) 2,2,3,3-Tetramethylpentane

Correct Answer: (D) 2,2,3,3-Tetramethylpentane

Solution:

Concept:

- A secondary hydrogen is attached to a secondary carbon, which is bonded to exactly two other carbon atoms. In an acyclic saturated alkane, each secondary carbon is a CH_2 group.
- Therefore two secondary hydrogens mean exactly one CH_2 group. Hydrogens on terminal or branch CH_3 groups are primary, and those on CH groups bonded to three carbons are tertiary.

Step 1: Count the methylene groups in the first three choices For (A), 2,2,4,5-tetramethylheptane, the main-chain carbons C3 and C6 are CH_2 groups. The methyl substituents are primary groups; hence there are $2 \times 2 = 4$ secondary hydrogens. For (B), 2,2,4,4-tetramethylhexane, C3 and C5 are CH_2, again giving four secondary hydrogens. For (C), 4-ethyl-3,4-dimethyloctane, the main-chain C2, C5, C6 and C7 are methylene groups. The carbon of the ethyl substituent attached to C4 is also CH_2, because it is bonded to C4 and the ethyl terminal methyl carbon. Thus (C) has five methylene groups and ten secondary hydrogens.

Step 2: Verify the remaining structure directly For (D), 2,2,3,3-tetramethylpentane, the condensed

main-chain structure is

$$CH_3-C(CH_3)_2-C(CH_3)_2-CH_2-CH_3.$$

Both C2 and C3 are quaternary and carry no hydrogen. Every methyl carbon is primary. Only C4 is bonded to two other carbons while retaining two hydrogens, so it is the sole secondary carbon. Its two hydrogens are therefore the only secondary hydrogens in the molecule. This explicit count eliminates the common mistake of treating all nonterminal carbons as secondary: carbons bearing methyl branches can be tertiary or quaternary instead.

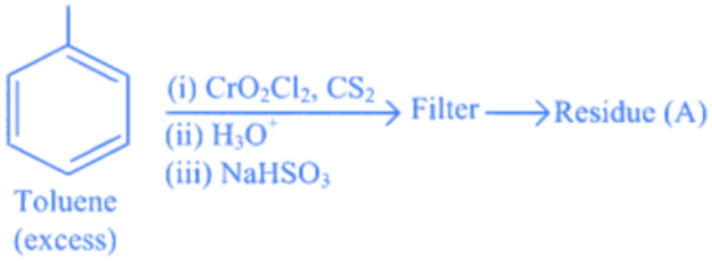
Step 3: Conclusion 2,2,3,3-Tetramethylpentane has exactly two secondary hydrogens. Hence, the correct option is (D).

Quick Tip to Solve: Count CH_2 groups, including those in side chains, then multiply by two. A carbon carrying two methyl branches may be quaternary and has no H.

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66. Residue (A) + HCl (dil) $\rightarrow$ Compound (B)

Structure of residue (A) and compound (B) formed respectively is : [4 marks]



(A) (A): $C_6H_5CH(OCrOHCl_2)_2$; (B): C_6H_5CHO

(B) (A): C_6H_5CHO; (B): C_6H_5COONa

(C) (A): $C_6H_5CH(OH)SO_3Na$; (B): C_6H_5CHO

(D) (A): C_6H_5CHO; (B): C_6H_5COOH

Correct Answer: (C) (A): $C_6H_5CH(OH)SO_3Na$; (B): C_6H_5CHO

Solution:

Concept:

- Chromyl chloride in carbon disulfide oxidises the benzylic methyl group of toluene by the Etard reaction. Hydrolysis of the intermediate complex gives benzaldehyde, C_6H_5CHO.
- Aldehydes form crystalline sodium bisulfite addition compounds with $NaHSO_3$. Dilute acid decomposes this addition compound and regenerates the carbonyl compound.

Step 1: Follow the oxidation and bisulfite treatment The first two stages convert the methyl side chain of toluene into an aldehyde group without changing the aromatic ring:

$$C_6H_5CH_3 \xrightarrow[CS_2]{CrO_2Cl_2} \text{Etard complex} \xrightarrow{H_3O^+} C_6H_5CHO.$$

The subsequent $NaHSO_3$ treatment adds bisulfite across the carbonyl group:

$$C_6H_5CHO + NaHSO_3 \longrightarrow C_6H_5CH(OH)SO_3Na.$$

The carbonyl carbon now carries H, OH, SO_3Na and the phenyl group. This solid addition product is separated by filtration and is residue (A). It is not the chromium-containing intermediate, since hydrolysis and bisulfite addition have already occurred before filtration.

Step 2: Regenerate the carbonyl compound with acid Dilute hydrochloric acid breaks up the bisulfite addition compound:

$$C_6H_5CH(OH)SO_3Na + HCl \longrightarrow C_6H_5CHO + SO_2 + H_2O + NaCl.$$

Thus compound (B) is benzaldehyde again. Option (A) incorrectly identifies the residue as the earlier chromium complex. Options (B) and (D) start with free benzaldehyde as the residue and respectively predict sodium benzoate or benzoic acid as the final compound. Neither salt formation nor further oxidation is caused by this dilute-acid decomposition. Only the pair consisting of the bisulfite adduct followed by benzaldehyde follows every reagent in its stated order.

Step 3: Conclusion The residue is $C_6H_5CH(OH)SO_3Na$, and the final compound is C_6H_5CHO. Hence, the correct option is (C).

Quick Tip to Solve: When $NaHSO_3$ is followed by filtration and dilute acid, recognise carbonyl purification: isolate the solid bisulfite adduct, then regenerate the aldehyde.

67. Given below are two statements :

Statement (I) : An element in the extreme left of the periodic table forms acidic oxides.

Statement (II) : Acid is formed during the reaction between water and oxide of a reactive element present in the extreme right of the periodic table.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are false

(B) Both Statement I and Statement II are true

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Concept:

- Across a period, metallic character generally decreases and nonmetallic character increases. Strongly electropositive metals form basic oxides, while many reactive nonmetals form acidic oxides.
- An acidic oxide reacts with water, when such a reaction occurs, to produce an oxyacid; a basic

oxide produces a hydroxide. The two statements must be judged separately before choosing their truth combination.

Step 1: Test the oxide of a left-side element The extreme left contains the alkali metals. Sodium, for example, forms the basic oxide sodium oxide, whose reaction with water gives an alkali:

$$\mathrm{Na_2O + H_2O \longrightarrow 2\,NaOH.}$$

Thus the claim that a left-side element forms an acidic oxide reverses the usual periodic trend. Statement I is false.

Step 2: Test the oxide of a reactive right-side element The reactive nonmetals toward the right side form acidic oxides. A chlorine oxide illustrates the reaction with water:

$$\mathrm{Cl_2O_7 + H_2O \longrightarrow 2\,HClO_4.}$$

The product is perchloric acid, so Statement II is true. The word reactive excludes the unreactive noble gases, which are not the intended oxide-forming elements here.

Step 3: Compare the four truth combinations Option (A) wrongly makes Statement II false. Option (B) wrongly accepts Statement I, while option (C) gets both truth values reversed. Only option (D) combines a false first statement with a true second statement. The classification concerns oxide acid-base character rather than whether the element itself is called an acid.

Step 4: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: Use a concrete oxide to check each trend: sodium oxide gives a hydroxide, whereas a reactive nonmetal oxide can give an oxyacid.

68. The species which does not undergo disproportionation reaction is : [4 marks]

(A) ClO_4^-

(B) ClO_3^-

(C) ClO^-

(D) ClO_2^-

Correct Answer: (A) ClO_4^-

Solution:

Concept:

- In disproportionation, the same element in a single reactant species is both oxidised and reduced. Its initial oxidation state must permit a higher as well as a lower accessible oxidation state.
- For a chlorine oxyanion $ClO_n{}^-$, assigning oxygen its usual oxidation state -2 gives $x - 2n = -1$, so chlorine has oxidation state $x = 2n - 1$. The maximum usual oxidation state of chlorine is $+7$.

Step 1: Calculate all four chlorine oxidation states Apply charge balance to each option:

$$\begin{aligned} \mathrm{ClO_4^-}: &\quad x-8=-1 \Rightarrow x=+7, \\ \mathrm{ClO_3^-}: &\quad x-6=-1 \Rightarrow x=+5, \\ \mathrm{ClO^-}: &\quad x-2=-1 \Rightarrow x=+1, \\ \mathrm{ClO_2^-}: &\quad x-4=-1 \Rightarrow x=+3. \end{aligned}$$

Perchlorate alone already contains chlorine at its highest oxidation state.

Step 2: Apply the oxidation-state requirement Chlorine in $\mathrm{ClO_4^-}$ can be reduced, but it cannot be oxidised beyond $+7$ in ordinary chlorine chemistry. Therefore two portions of perchlorate cannot supply the oxidation and reduction branches required for disproportionation. This rules out disproportionation of option (A).

Step 3: Check why the other species are not the answer The intermediate states $+1, +3, +5$ leave room for both upward and downward changes. Examples of balanced overall disproportionation changes are

$$3\,\mathrm{ClO^-} \longrightarrow \mathrm{ClO_3^-} + 2\,\mathrm{Cl^-},$$

$$3\,\mathrm{ClO_2^-} \longrightarrow 2\,\mathrm{ClO_3^-} + \mathrm{Cl^-},$$

$$4\,\mathrm{ClO_3^-} \longrightarrow 3\,\mathrm{ClO_4^-} + \mathrm{Cl^-}.$$

These equations illustrate the two directions of chlorine oxidation-state change; appropriate reaction conditions may be needed. Options (B), (C) and (D) therefore do not satisfy the requested exclusion.

Step 4: Conclusion $\mathrm{ClO_4^-}$, with chlorine at $+7$, does not undergo disproportionation. Hence, the correct option is (A).

Quick Tip to Solve: Before writing a disproportionation equation, check whether the element is already at its maximum or minimum oxidation state.

69. Given below are two statements :

Statement (I) : A spectral line will be observed for a $2p_x \rightarrow 2p_y$ transition.

Statement (II) : $2\mathrm{p}_x$ and $2\mathrm{p}_y$ are degenerate orbitals.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Both Statement I and Statement II are true

Correct Answer: (A) Statement I is false but Statement II is true

Solution:

Concept:

- Degenerate orbitals have the same energy. In an isolated atom without an external field, the $2p_x$, $2p_y$ and $2p_z$ orbitals differ in spatial orientation but have equal energy.
- A spectral line corresponds to absorption or emission of a photon of nonzero frequency, with energy $h\nu = |E_f - E_i|$, where h is Planck's constant and E_i, E_f are the initial and final electronic energies.

Step 1: Compare the energies of the two orbitals Both orbitals belong to the same $2p$ subshell: they have identical principal quantum number $n = 2$ and azimuthal quantum number $l = 1$. Their different orientations do not create an energy difference in the ordinary field-free atomic situation. Thus

$$E(2p_x) = E(2p_y),$$

and Statement II is true.

Step 2: Apply the photon-energy condition For the proposed change of orbital orientation,

$$\Delta E = E(2p_y) - E(2p_x) = 0.$$

Therefore

$$\nu = \frac{|\Delta E|}{h} = 0.$$

There is no finite-energy photon and hence no ordinary spectral line associated with this transition. A zero energy separation must not be mistaken for a very low but nonzero transition energy. Statement I is false.

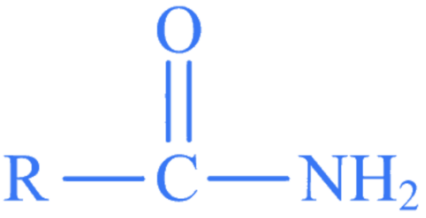
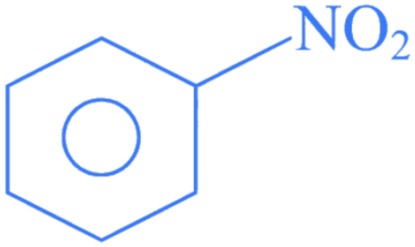
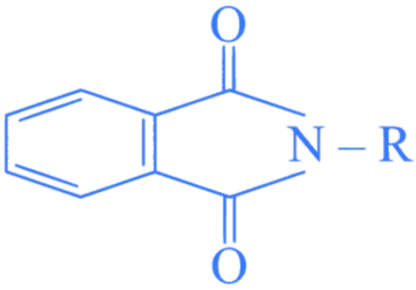
Step 3: Select the matching statement combination Option (A) states exactly the derived truth values: Statement I false and Statement II true. Option (B) incorrectly denies degeneracy; option (C) reverses both conclusions; option (D) incorrectly predicts a line. No splitting by an external magnetic or electric field is specified, so such additional effects must not be introduced into the question.

Step 4: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (A).

Quick Tip to Solve: Check the energy gap before discussing spectral lines: different orbital orientations within one unsplit subshell do not imply different energies.

70. Match the Compounds (List - I) with the appropriate Catalyst/Reagents (List - II) for their reduction into corresponding amines.

Choose the correct answer from the options given below: [4 marks]

	List - I (Compounds)		List - II (Catalyst/Reagents)
(A)		(I)	NaOH (aqueous)
(B)		(II)	H_2/Ni
(C)	$R-C\equiv N$	(III)	$LiAlH_4, H_2O$
(D)		(IV)	Sn, HCl

(A) (A)-(II), (B)-(IV), (C)-(III), (D)-(I)

(B) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(C) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(D) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

Correct Answer: (B) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Solution:

Concept:

- Amides and nitriles can be reduced to primary amines, but reagent selection depends on the functional group. $LiAlH_4$, followed by water, reduces an amide carbonyl to a methylene group.
- Nitro compounds are commonly reduced with tin and hydrochloric acid. Catalytic hydrogenation reduces nitriles to primary amines, while aqueous alkali hydrolyses an N-alkylphthalimide to release the amine.

Step 1: Match the amide and nitro compound Compound (A) is $RCONH_2$. Lithium aluminium

hydride followed by water gives RCH_2NH_2, retaining the carbonyl carbon in the chain. Therefore (A) matches (III). Compound (B) is nitrobenzene; Sn/HCl reduces its nitro group to the amino group, giving aniline after the usual work-up. Therefore (B) matches (IV).

Step 2: Match the nitrile and protected amine Compound (C), $R-C\equiv N$, undergoes catalytic hydrogenation:

$$RCN + 2\,H_2 \xrightarrow{Ni} RCH_2NH_2.$$

Thus (C) matches (II). Compound (D) is an N-alkylphthalimide. Hydrolysis with aqueous $NaOH$ breaks the imide acyl-nitrogen linkages and releases RNH_2, so (D) matches (I). This last conversion is hydrolysis in the Gabriel synthesis work-up, although the question broadly groups the preparations as reductions.

Step 3: Check the mapping against every option The combined matching is

$$(A):(III), \quad (B):(IV), \quad (C):(II), \quad (D):(I).$$

Option (A) interchanges the assigned reagents for the amide and nitrile. Option (C) assigns the nitro compound and nitrile incorrectly; option (D) also assigns aqueous alkali to nitrobenzene and tin/HCl to the imide. Only option (B) contains all four required matches.

Step 4: Conclusion The amide, nitrobenzene, nitrile and N-alkylphthalimide match reagents III, IV, II and I, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Identify the functional group first: amide to hydride, nitro to Sn/HCl, nitrile to catalytic hydrogenation, and phthalimide to hydrolysis.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. Consider the following cases of standard enthalpy of reaction $(\Delta H_r^\circ$ in $kJmol^{-1})$

$$C_2H_6(g) + \frac{7}{2}O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l) \Delta H_1^\circ = -1550$$
$$C(\textbf{graphite}) + O_2(g) \rightarrow CO_2(g) \Delta H_2^\circ = -393.5$$
$$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l) \Delta H_3^\circ = -286$$

The magnitude of $\Delta H_{f C_2H_6(g)}^\circ$ is _____ $kJmol^{-1}$ (Nearest integer). [4 marks]

Solution:

Concept:

- Hess's law states that reaction enthalpy depends only on the initial and final states. Reversing a thermochemical equation reverses its enthalpy, and multiplying the equation multiplies its enthalpy by the same factor.
- The standard enthalpy of formation of ethane is the enthalpy for $2\,C(\text{graphite}) + 3\,H_2(g) \longrightarrow C_2H_6(g)$. Elements in their standard reference states have zero standard formation enthalpy.

Step 1: Form the combustion products from their elements Multiply the carbon combustion equation by two and the hydrogen combustion equation by three. Their sum forms two moles of carbon dioxide and three moles of liquid water from graphite, hydrogen and oxygen:

$$\Delta H_a^\circ = 2(-393.5) + 3(-286).$$

Hence

$$\Delta H_a^\circ = -787 - 858 = -1645\,\text{kJ mol}^{-1}.$$

The mole basis here is one mole of the ethane formation reaction to be obtained.

Step 2: Reverse ethane combustion and cancel common species The reverse of the given ethane combustion has enthalpy $+1550\,\text{kJ mol}^{-1}$. Add it to the preceding equation. The CO_2, H_2O and oxygen terms cancel, leaving

$$2\,\text{C(graphite)} + 3\,\text{H}_2\text{(g)} \longrightarrow \text{C}_2\text{H}_6\text{(g)}.$$

Therefore

$$\Delta H_f^\circ(\text{C}_2\text{H}_6\text{(g)}) = -1645 + 1550 = -95\,\text{kJ mol}^{-1}.$$

Step 3: Report the requested magnitude The negative sign means ethane formation is exothermic for the supplied thermochemical data. The question asks for the magnitude, so take the absolute value only after completing the Hess-law calculation:

$$|\Delta H_f^\circ| = |-95| = 95\,\text{kJ mol}^{-1}.$$

This is already an integer; rounding does not change it. Use the printed combustion enthalpy rather than replacing it with a tabulated value.

Step 4: Conclusion The required magnitude is $\boxed{95\,\text{kJ mol}^{-1}}$. The numerical entry is 95.

Quick Tip to Solve: Write products minus reactants for the combustion enthalpy, then solve for the unknown formation enthalpy; take its magnitude last.

72. The complex of Ni^{2+} ion and dimethyl glyoxime contains _____ number of Hydrogen (H) atoms. [4 marks]

Solution:

Concept:

- Dimethylglyoxime has structure $CH_3{-}C(=NOH){-}C(=NOH){-}CH_3$ and molecular formula $C_4H_8N_2O_2$. Each neutral ligand contains six methyl hydrogens and two oxime hydroxyl hydrogens.
- Nickel(II) forms the neutral bis(dimethylglyoximato) complex $Ni(dmgH)_2$. Two monoanionic ligands coordinate to Ni^{2+}, each losing one proton from a neutral dimethylglyoxime molecule.

Step 1: Count hydrogen atoms in the neutral ligand The two methyl groups contribute $2 \times 3 = 6$

hydrogen atoms. The two N−OH groups contribute two more, giving

$$N_H(\text{neutral dimethylglyoxime}) = 6 + 2 = 8.$$

Counting only the methyl groups would miss the oxime hydrogens and give the wrong starting formula.

Step 2: Account for deprotonation and charge balance Each dimethylglyoxime ligand loses one proton to become $C_4H_7N_2O_2^-$. Two such ligands balance the $+2$ charge of nickel, producing the familiar neutral square-planar precipitate:

$$Ni^{2+} + 2\,C_4H_8N_2O_2 \longrightarrow Ni(C_4H_7N_2O_2)_2 + 2\,H^+.$$

The remaining oxime hydrogens participate in the characteristic intramolecular hydrogen bonding; they are still present in the complex.

Step 3: Add the hydrogens in the complete complex There are two ligands, each with seven hydrogens, so

$$N_H = 2 \times 7 = 14.$$

Equivalently, two neutral ligand molecules initially contain $2 \times 8 = 16$ hydrogens and lose a total of two protons, leaving $16 - 2 = 14$. The coordination formula is $NiC_8H_{14}N_4O_4$, which provides an independent check on the count. A number of atoms is a dimensionless count and requires no physical unit.

Step 4: Conclusion The complex contains $\boxed{14}$ hydrogen atoms. The numerical entry is 14.

Quick Tip to Solve: Count the neutral ligand hydrogens, subtract one proton per coordinated glyoxime ligand, and then multiply by the two ligands.

73. Niobium (Nb) and ruthenium (Ru) have " x " and " y " number of electrons in their respective 4 d orbitals. The value of $x + y$ is _____. [4 marks]

Solution:

Concept:

- The number of electrons in a specified subshell is read from its exponent in the actual ground-state electronic configuration.
- For the second transition series, the relevant valence subshells are $4d$ and $5s$, outside the krypton core [Kr]. Their energies are close, so the observed configurations can differ from a simple filling-order prediction.

Step 1: Write the configuration of niobium Niobium has atomic number 41. Removing the 36 electrons of the krypton core leaves five valence electrons. Its observed ground-state configuration is $[Kr]\,4d^4 5s^1$, rather than the simple predicted arrangement $4d^3 5s^2$. Hence the number of electrons in its $4d$ subshell is $x = 4$. The superscript on $5s$ must not be included, because the question asks only about $4d$.

Step 2: Write the configuration of ruthenium Ruthenium has atomic number 44 and therefore eight electrons beyond the krypton core. Its observed ground-state configuration is $[\mathrm{Kr}]\,4d^7 5s^1$. The electron count checks: $36 + 7 + 1 = 44$. Consequently the number of $4d$ electrons is $y = 7$, not 6 from a naive $4d^6 5s^2$ filling. These are configurations of neutral atoms; no electrons are removed as in the formation of metal ions.

Step 3: Add the two subshell populations Using the actual $4d$ occupancies gives

$$x + y = 4 + 7 = 11.$$

This sum counts electrons in the two $4d$ subshells only; adding both $5s$ electrons as well would instead count all valence electrons and answer a different question.

Step 4: Conclusion The required dimensionless numerical value is $\boxed{11}$.

Quick Tip to Solve: For Nb and Ru, recall the actual configurations $4d^4 5s^1$ and $4d^7 5s^1$; read only the $4d$ superscripts before adding.

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74. 20 mL of 2 M NaOH solution is added to 400 mL of 0.5 M NaOH solution. The final concentration of the solution is _____ $\times 10^{-2}$M. (Nearest integer) [4 marks]

Solution:

Concept:

- Molarity is moles of solute per litre of solution: $M = n/V$, so the number of moles in an aliquot is $n = MV$.
- When two solutions of the same solute are mixed without reaction, add their moles and divide by the combined volume. For the stated dilute solutions, use additive volumes.

Step 1: Find the moles in each solution Convert the volumes from millilitres to litres before using molarity. The concentrated aliquot contains

$$n_1 = (2\,\mathrm{mol\,L^{-1}})(0.020\,\mathrm{L}) = 0.040\,\mathrm{mol}.$$

The second solution contains

$$n_2 = (0.5\,\mathrm{mol\,L^{-1}})(0.400\,\mathrm{L}) = 0.200\,\mathrm{mol}.$$

Both quantities refer to NaOH, so they can be added directly; there is no neutralisation reaction between them.

Step 2: Calculate the final molarity The total amount of solute and final solution volume are

$$n_{\mathrm{total}} = 0.040 + 0.200 = 0.240\,\mathrm{mol},$$

$$V_{\mathrm{total}} = 0.020 + 0.400 = 0.420\,\mathrm{L}.$$

Therefore

$$M_f = \frac{0.240}{0.420} = \frac{4}{7}\,\text{mol}\,\text{L}^{-1} \approx 0.5714286\,\text{M}.$$

This lies between 0.5 M and 2 M and is much closer to 0.5 M, as expected because the larger volume is the more dilute solution.

Step 3: Express the requested coefficient and round Write the concentration as $k \times 10^{-2}\,\text{M}$. Then

$$k = \frac{0.5714286}{0.01} = 57.14286.$$

Rounding this coefficient to the nearest integer gives 57. Rounding the molarity itself to an integer before rescaling would discard the precision needed for the answer.

Step 4: Conclusion The final concentration is approximately $57 \times 10^{-2}\,\text{M}$; the required entry is $\boxed{57}$.

Quick Tip to Solve: For the same solute, use $M_f = (M_1V_1 + M_2V_2)/(V_1 + V_2)$. Round only the coefficient of the requested power of ten.

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75. The compound with molecular formula C_6H_6, which gives only one monobromo derivative and takes up four moles of hydrogen per mole for complete hydrogenation has _____ π electrons.

[4 marks]

Solution:

Concept:

- One carbon-carbon double bond contains one π bond and consumes one mole of H_2 on complete hydrogenation. A triple bond contains two π bonds and consumes two moles of H_2.
- Each π bond contains two electrons. Only one monobromo substitution derivative indicates that all replaceable hydrogen atoms are equivalent by molecular symmetry.

Step 1: Use the hydrogen uptake to count pi bonds The compound consumes four moles of H_2 per mole. Complete addition across its multiple bonds therefore removes four π bonds. Each such bond contributes two electrons, giving

$$N_{\pi\ \text{electrons}} = 4 \times 2 = 8.$$

The molecular formula also has degree of unsaturation

$$(2 \times 6 + 2 - 6)/2 = 4,$$

consistent with four units of multiple-bond unsaturation in an acyclic structure.

Step 2: Check a structure against both conditions Hexa-2,4-diyne, $CH_3{-}C{\equiv}C{-}C{\equiv}C{-}CH_3$, has formula C_6H_6 and two triple bonds. Each triple bond contains two π bonds, so the molecule has four in total. Hydrogenating both triple bonds completely takes four equivalents of H_2 and gives hexane. Its two terminal methyl groups are symmetry-equivalent; the six hydrogens are equivalent for monobromo substitution, so only one constitutional monobromo derivative results.

Step 3: Distinguish the benzene interpretation Benzene also has formula C_6H_6, equivalent hydrogens

and one monobromo substitution product, but it has only three π bonds and six π electrons. It requires three, rather than four, moles of hydrogen to form cyclohexane. Thus 6 describes benzene but does not satisfy the stated hydrogen uptake; the four-mole condition selects 8.

Step 4: Conclusion For the stated four-mole hydrogen uptake, the compound has $\boxed{8}$ π electrons. The final answer key also accepts 6, corresponding to the benzene interpretation, although benzene consumes only three moles of H_2.

Quick Tip to Solve: Use hydrogen uptake as the direct count of π bonds: multiply the moles of H_2 added per mole of an ordinary unsaturated hydrocarbon by two to obtain its π-electron count.