JEE Main 2025 Paper 1 (B.E./B.Tech) Jan 23 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 23 January 2025, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let a curve $y = f(x)$ pass through the points $(0, 5)$ and $(\log_e 2, k)$. If the curve satisfies the differential equation $2(3 + y)e^{2x}dx - \left(7 + e^{2x}\right) dy = 0$, then k is equal to [4 marks]

(A) 32

(B) 8

(C) 4

(D) 16

Correct Answer: (B) 8

Solution:

Concept:

- A separable differential equation can be integrated after collecting all terms involving y with dy, and all terms involving x with dx.
- The logarithmic integral $\int u^{-1}\, du = \log|u| + C$ applies when the numerator is the differential of the denominator. A point on the curve determines the integration constant.

Step 1: Separate the variables Starting with the printed equation, move the dy term to the other side and divide by the nonzero factors near the initial point:

$$\frac{dy}{y+3} = \frac{2e^{2x}}{7+e^{2x}}\, dx.$$

The right numerator is exactly the derivative of $7 + e^{2x}$, which makes the integration immediate. The initial value $y = 5$ also shows that we are not on the constant solution $y = -3$.

Step 2: Integrate and use the initial point Integrating both sides gives

$$\log|y+3| = \log(7+e^{2x}) + C.$$

Equivalently, $y + 3 = A(7 + e^{2x})$, where the multiplicative constant includes the possible sign. At $(0, 5)$,

$$8 = A(7+1), \qquad A = 1.$$

Thus the required curve is $y = e^{2x} + 4$. Differentiation gives $y' = 2e^{2x}$, which also verifies the original equation.

Step 3: Evaluate at the second point At $x = \log_e 2$, the exponential is $e^{2\log_e 2} = 4$. Therefore

$$k = 4 + 4 = 8.$$

The other displayed numbers, 32, 4 and 16, do not equal the ordinate of this initial-value solution.

Step 4: Conclusion The required value is $k = 8$. Hence, the correct option is (B).

Quick Tip to Solve: Look for a denominator whose derivative is already present in the numerator before attempting a substitution.

2. Let P be the foot of the perpendicular from the point $Q(10,-3,-1)$ on the line $\frac{x-3}{7}=\frac{y-2}{-1}=\frac{z+1}{-2}$. Then the area of the right angled triangle PQR, where R is the point $(3,-2,1)$, is [4 marks]

(A) $\sqrt{30}$

(B) $9\sqrt{15}$

(C) $3\sqrt{30}$

(D) $8\sqrt{15}$

Correct Answer: (C) $3\sqrt{30}$

Solution:

Concept:

- A point on the given line can be written as $P=(3,2,-1)+t(7,-1,-2)$. At the perpendicular foot, $(Q-P)\cdot(7,-1,-2)=0$.
- The area of a right triangle is half the product of its perpendicular sides. The location of the right angle must be checked rather than inferred from the order of its name.

Step 1: Find the perpendicular foot Let $A=(3,2,-1)$ and $\mathbf{d}=(7,-1,-2)$. Since $Q-A=(7,-5,0)$, projection along the line gives

$$t=\frac{(Q-A)\cdot\mathbf{d}}{\mathbf{d}\cdot\mathbf{d}}=\frac{49+5}{49+1+4}=1.$$

Hence $P=A+\mathbf{d}=(10,1,-3)$. In particular, $P-Q=(0,4,-2)$, which is perpendicular to $\mathbf{d}$ as required.

Step 2: Identify the two perpendicular sides The point $R=(3,-2,1)$ need not lie on the given line. Instead,

$$Q-R=(7,-1,-2)=\mathbf{d}.$$

Thus QR is parallel to the given line and QP is perpendicular to it. Directly,

$$(P-Q)\cdot(R-Q)=(0,4,-2)\cdot(-7,1,2)=0.$$

The right angle of triangle PQR is therefore at Q, not at P. Its perpendicular side lengths are $QP=\sqrt{20}=2\sqrt{5}$ and $QR=\sqrt{54}=3\sqrt{6}$.

Step 3: Calculate and compare the area Using these perpendicular sides,

$$[PQR]=\frac{1}{2}(2\sqrt{5})(3\sqrt{6})=3\sqrt{30}.$$

As a check, the squared area is $20\times54/4=270$. The squared areas corresponding to the other options are 30, 1215 and 960; none equals 270.

Step 4: Conclusion The area is $3\sqrt{30}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Check where the right angle actually lies: here the vector from R to Q equals the direction vector of the given line.

3. The number of words, which can be formed using all the letters of the word "DAUGHTER", so that all the vowels never come together, is : [4 marks]

(A) 34000

(B) 37000

(C) 35000

(D) 36000

Correct Answer: (D) 36000

Solution:

Concept:

- For n distinct letters, the number of arrangements using every letter once is $n!$. All eight letters of DAUGHTER are distinct.
- To forbid all vowels from occupying one consecutive block, subtract arrangements containing that complete vowel block from the unrestricted total. The order inside the block must also be counted.

Step 1: Count all unrestricted arrangements The letters are D, A, U, G, H, T, E and R. The vowels are A, U and E, while the other five are consonants. With no restriction there are

$$8! = 40320$$

arrangements. The word in the question need not be a dictionary word; every ordering of the eight distinct letters is counted.

Step 2: Count arrangements with all vowels together Treat the three vowels as one block. The objects to arrange are then this block and the five distinct consonants, giving six distinct objects. They can be ordered in $6!$ ways. Within the vowel block, A, U and E can be ordered in $3!$ ways. Consequently, the unwanted number is

$$6!\,3! = 720 \times 6 = 4320.$$

Each arrangement with three consecutive vowels has exactly one such block, so this multiplication neither misses nor repeats an unwanted arrangement.

Step 3: Subtract the unwanted case The required complement is

$$8! - 6!\,3! = 40320 - 4320 = 36000.$$

The condition forbids all three vowels being together; it does not forbid a pair of adjacent vowels. Thus a stricter gap count requiring all vowels to be separated would answer a different question. The alternative totals 34000, 37000 and 35000 do not equal this complement.

Step 4: Conclusion The number of required arrangements is 36000. Hence, the correct option is (D).

Quick Tip to Solve: For "all vowels never together", subtract the single full vowel block; do not require every pair of vowels to be separated.

4. Let $f(x) = \log_e x$ and $g(x) = \frac{x^4-2x^3+3x^2-2x+2}{2x^2-2x+1}$. Then the domain of $f \circ g$ is [4 marks]

(A) $(0, \infty)$

(B) $[1, \infty)$

(C) $\mathbb{R}$

(D) $[0, \infty)$

Correct Answer: (C) $\mathbb{R}$

Solution:

Concept:

- The real logarithm $\log_e u$ is defined exactly when its argument u is strictly positive. Hence $f \circ g$ requires both a defined rational function $g(x)$ and $g(x) > 0$.
- Completing squares proves positivity on the entire real line without needing roots of a quartic. The numerator and denominator can be checked separately.

Step 1: Show that the denominator never vanishes Rewrite the denominator as

$$2x^2 - 2x + 1 = 2\left(x - \frac{1}{2}\right)^2 + \frac{1}{2}.$$

The squared term is nonnegative, so this expression is at least $1/2$ and is strictly positive for every real x. Therefore the rational expression has no real poles and no exclusion is needed from its denominator.

Step 2: Establish strict positivity of the numerator The numerator admits the useful identity

$$\begin{aligned} x^4 - 2x^3 + 3x^2 - 2x + 2 &= (x^2 - x)^2 + 2(x^2 - x) + 2 \\ &= (x^2 - x + 1)^2 + 1. \end{aligned}$$

This is a square plus 1, so it is strictly positive for every real x. In particular, its sign cannot change as x passes through zero or a negative number. Dividing this positive numerator by the positive denominator proves $g(x) > 0$ everywhere on $\mathbb{R}$.

Step 3: Apply the logarithm condition Because $g(x)$ exists and is positive for every real input, $\log_e(g(x))$ is defined on all of $\mathbb{R}$. The other proposed intervals unnecessarily omit valid inputs. For example, $x = -1$ gives $g(-1) = 10/5 = 2 > 0$, yet it is absent from every other listed domain.

Step 4: Conclusion The domain of $f \circ g$ is $\mathbb{R}$. Hence, the correct option is (C).

Quick Tip to Solve: For logarithms of rational expressions, prove the sign of both polynomials; a square-plus-positive-constant form often settles the whole domain.

5. Let the position vectors of the vertices A, B and C of a tetrahedron ABCD be $\hat{\mathrm{i}}+2\hat{\mathrm{j}}+\hat{\mathrm{k}}, \hat{\mathrm{i}}+3\hat{\mathrm{j}}-2\hat{k}$ and $2\hat{i}+\hat{j}-\hat{k}$ respectively. The altitude from the vertex D to the opposite face ABC meets the median line segment through A of the triangle ABC at the point E. If the length of AD is $\frac{\sqrt{110}}{3}$ and the volume of the tetrahedron is $\frac{\sqrt{805}}{6\sqrt{2}}$, then the position vector of E is [4 marks]

(A) $\frac{1}{6}(7\hat{\mathrm{i}}+12\hat{\mathrm{j}}+\hat{\mathrm{k}})$

(B) $\frac{1}{12}(7\hat{\mathrm{i}}+4\hat{\mathrm{j}}+3\hat{\mathrm{k}})$

(C) $\frac{1}{6}(12\hat{i}+12\hat{j}+\hat{k})$

(D) $\frac{1}{2}(\hat{i}+4\hat{j}+7\hat{k})$

Correct Answer: (A) $\frac{1}{6}(7\hat{\mathrm{i}}+12\hat{\mathrm{j}}+\hat{\mathrm{k}})$

Solution:

Concept:

- The area of face ABC is $\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$, and the tetrahedron volume is one third of this area times its perpendicular altitude DE.
- Since DE is perpendicular to face ABC, triangle AED is right at E, so $AE^2 = AD^2 - DE^2$. The requirement that E lies on the median segment fixes its direction from A.

Step 1: Calculate the face area and altitude The given vertices are $A=(1,2,1)$, $B=(1,3,-2)$ and $C=(2,1,-1)$. Thus

$$\overrightarrow{AB}=(0,1,-3),\quad \overrightarrow{AC}=(1,-1,-2),\quad \overrightarrow{AB}\times\overrightarrow{AC}=(-5,-3,-1).$$

The face area is $\sqrt{35}/2$. Substituting the given volume in $DE = 3V/[ABC]$ gives

$$DE=\frac{\sqrt{805}}{\sqrt{70}}=\sqrt{\frac{23}{2}}.$$

Step 2: Find the distance along the median Apply Pythagoras in triangle AED:

$$AE^2=\frac{110}{9}-\frac{23}{2}=\frac{13}{18}.$$

Let M be the midpoint of BC. Then

$$M=\left(\frac{3}{2},2,-\frac{3}{2}\right),\qquad \overrightarrow{AM}=\left(\frac{1}{2},0,-\frac{5}{2}\right),\quad AM^2=\frac{13}{2}.$$

Writing $E = A + t(M - A)$ with $0 \le t \le 1$, the distance ratio gives $t^2 = AE^2/AM^2 = 1/9$. The segment condition selects $t = 1/3$.

Step 3: Determine the position vector Consequently,

$$E=(1,2,1)+\frac{1}{3}\left(\frac{1}{2},0,-\frac{5}{2}\right)=\left(\frac{7}{6},2,\frac{1}{6}\right).$$

Every point on this median has second coordinate 2. This immediately rejects options (B), whose second coordinate is $1/3$, while (C) and (D) fail the first and third coordinates or the segment

condition. Only (A) has the computed coordinates.

Step 4: Conclusion The position vector is $\frac{1}{6}(7\hat{i} + 12\hat{j} + \hat{k})$. Hence, the correct option is (A).

Quick Tip to Solve: Use the volume to find the perpendicular height first, then Pythagoras to obtain the distance of the foot along the known median.

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6. If $\frac{\pi}{2} \le x \le \frac{3\pi}{4}$, then $\cos^{-1}\left(\frac{12}{13}\cos x + \frac{5}{13}\sin x\right)$ is equal to [4 marks]

(A) $x + \tan^{-1}\frac{5}{12}$

(B) $x - \tan^{-1}\frac{4}{3}$

(C) $x + \tan^{-1}\frac{4}{5}$

(D) $x - \tan^{-1}\frac{5}{12}$

Correct Answer: (D) $x - \tan^{-1}\frac{5}{12}$

Solution:

Concept:

- The addition identity $\cos(x - \alpha) = \cos x \cos\alpha + \sin x \sin\alpha$ combines a linear expression in sine and cosine into a single cosine.
- The principal range of $\cos^{-1}$ is $[0, \pi]$. Therefore $\cos^{-1}(\cos\theta) = \theta$ only after verifying that θ lies in this range.

Step 1: Choose the auxiliary angle The coefficients satisfy $12^2 + 5^2 = 13^2$. Choose the first-quadrant angle

$$\alpha = \tan^{-1}\frac{5}{12}.$$

Its sine and cosine are $\sin\alpha = 5/13$ and $\cos\alpha = 12/13$. The argument of the inverse cosine becomes

$$\frac{12}{13}\cos x + \frac{5}{13}\sin x = \cos(x - \alpha).$$

The minus sign is dictated by the positive sine term in the cosine-difference identity.

Step 2: Check the principal-value interval Since $0 < \alpha < \pi/2$ and $\pi/2 \le x \le 3\pi/4$,

$$0 < \frac{\pi}{2} - \alpha \le x - \alpha \le \frac{3\pi}{4} - \alpha < \pi.$$

Thus the angle $x - \alpha$ already lies strictly inside the principal range. There is no need to replace it by a reflected or periodic angle. This interval check is essential because inverse cosine does not undo cosine globally.

Step 3: Evaluate and compare the options It follows throughout the given interval that

$$\cos^{-1}\left(\frac{12}{13}\cos x + \frac{5}{13}\sin x\right) = x - \tan^{-1}\frac{5}{12}.$$

Options (A) and (C) use the wrong sign of the shift; option (B) uses an angle with the wrong tangent. None reproduces both printed coefficients and the established principal angle.

Step 4: Conclusion The value is $x - \tan^{-1}(5/12)$. Hence, the correct option is (D).

Quick Tip to Solve: After combining sine and cosine into one cosine, check the shifted angle against the principal range before removing the inverse function.

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7. One die has two faces marked 1 , two faces marked 2 , one face marked 3 and one face marked 4 . Another die has one face marked 1 , two faces marked 2 , two faces marked 3 and one face marked 4. The probability of getting the sum of numbers to be 4 or 5 , when both the dice are thrown together, is [4 marks]

(A) $\frac{2}{3}$
(B) $\frac{3}{5}$
(C) $\frac{4}{9}$
(D) $\frac{1}{2}$

Correct Answer: (D) $\frac{1}{2}$

Solution:

Concept:

- Each physical face is equally likely. Repeated labels have probabilities proportional to the number of faces carrying them; the four distinct labels are not equally likely.
- The two dice are independent, giving $6 \times 6 = 36$ equally likely ordered face pairs. The events sum 4 and sum 5 are disjoint, so their favourable counts add.

Step 1: Record the label multiplicities For the first die the multiplicities of labels $1, 2, 3, 4$ are $2, 2, 1, 1$; for the second they are $1, 2, 2, 1$. An ordered label pair (a, b) therefore accounts for the product of its two multiplicities. This weighting is essential: treating the sixteen possible label pairs as equally likely would give an incorrect denominator.

Step 2: Count the pairs giving a sum of four The ordered label pairs are $(1, 3), (2, 2), (3, 1)$. Their respective face-pair counts are

$$N_4 = 2 \cdot 2 + 2 \cdot 2 + 1 \cdot 1 = 4 + 4 + 1 = 9.$$

The first coordinate always belongs to the first die, so no extra symmetry factor is inserted.

Step 3: Count the pairs giving a sum of five The possible ordered labels are $(1, 4), (2, 3), (3, 2), (4, 1)$. Thus

$$N_5 = 2 \cdot 1 + 2 \cdot 2 + 1 \cdot 2 + 1 \cdot 1 = 2 + 4 + 2 + 1 = 9.$$

There is no overlap with the previous event. The favourable count is $9 + 9 = 18$, out of 36 total outcomes, giving $18/36 = 1/2$. The other proposed probabilities $2/3, 3/5, 4/9$ do not equal this weighted count.

Step 4: Conclusion The required probability is $\boxed{\frac{1}{2}}$. Hence, the correct option is (D).

Quick Tip to Solve: For dice with repeated labels, list face multiplicities first and multiply them for each ordered label pair; never count distinct labels as equally likely.

8. The value of $(\sin 70^\circ)\,(\cot 10^\circ \cot 70^\circ - 1)$ **is** [4 marks]

(A) 0

(B) 2/3

(C) 1

(D) 3/2

Correct Answer: (C) 1

Solution:

Concept:

- Use $\cot\theta = \cos\theta/\sin\theta$, retaining the degree convention throughout. Both denominator sines here are nonzero.
- The identity $\cos(A+B) = \cos A\cos B - \sin A\sin B$ combines the two terms after putting them over a common denominator.

Step 1: Rewrite the cotangents as ratios Let the required expression be E. Substituting the definitions gives

$$E = \sin 70^\circ \left(\frac{\cos 10^\circ \cos 70^\circ}{\sin 10^\circ \sin 70^\circ} - 1\right).$$

The factor $\sin 70^\circ$ cancels only in the first term. It does not cancel the separate subtraction of 1 inside the bracket; distributing the factor correctly preserves that subtraction.

Step 2: Combine the terms and apply the identity After multiplying out and taking a common denominator,

$$\begin{aligned} E &= \frac{\cos 10^\circ \cos 70^\circ}{\sin 10^\circ} - \sin 70^\circ \\ &= \frac{\cos 10^\circ \cos 70^\circ - \sin 10^\circ \sin 70^\circ}{\sin 10^\circ} \\ &= \frac{\cos 80^\circ}{\sin 10^\circ}. \end{aligned}$$

The negative product of sines signals the cosine of a sum, with angles adding to 80°.

Step 3: Use complementary angles and compare the options Since $\cos 80^\circ = \sin 10^\circ$, numerator and denominator are identical and positive. Therefore $E = 1$ exactly. In particular it cannot vanish, which rules out 0; the exact identity also rules out $2/3$ and $3/2$. Decimal approximations to small-angle cotangents are unnecessary and could obscure this cancellation.

Step 4: Conclusion The value is $\boxed{1}$. Hence, the correct option is (C).

Quick Tip to Solve: When a cotangent product is followed by minus one, multiply out and seek the numerator $\cos A\cos B - \sin A\sin B = \cos(A+B)$.

9. If the function

$$f(x) = \begin{cases} \frac{2}{x}\{\sin((k_1+1)x) + \sin((k_2-1)x)\}, & x < 0, \\ 4, & x = 0, \\ \frac{2}{x}\log_e\left(\frac{2+k_1x}{2+k_2x}\right), & x > 0 \end{cases}$$

is continuous at $x = 0$, then $k_1^2 + k_2^2$ is equal to: [4 marks]

(A) 5

(B) 10

(C) 20

(D) 8

Correct Answer: (B) 10

Solution:

Concept:

- Continuity at zero requires both one-sided limits to equal the defined value $f(0) = 4$. The two sides supply separate linear equations in k_1, k_2.
- Use $\lim_{x\to 0}\sin(ax)/x = a$ and $\lim_{u\to 0}\log(1+u)/u = 1$, with angles in radians in the sine limit.

Step 1: Evaluate the left-hand limit The sine terms have constant coefficients $k_1 + 1$ and $k_2 - 1$ multiplying x. Applying the standard sine limit separately,

$$\lim_{x\to 0^-} f(x) = 2\{(k_1+1) + (k_2-1)\} = 2(k_1+k_2).$$

Equating this to 4 gives $k_1 + k_2 = 2$. The constants $+1$ and -1 cancel, so they must not be retained as an extra contribution.

Step 2: Evaluate the right-hand limit Write the logarithm of the ratio as a difference and factor 2 out of each argument:

$$\log\frac{2+k_1x}{2+k_2x} = \log(1+k_1x/2) - \log(1+k_2x/2).$$

The arguments are positive for all sufficiently small x, since both approach 1. Consequently,

$$\lim_{x\to 0^+}\frac{2}{x}\log\frac{2+k_1x}{2+k_2x} = 2\left(\frac{k_1}{2} - \frac{k_2}{2}\right) = k_1 - k_2.$$

Continuity now requires $k_1 - k_2 = 4$.

Step 3: Solve the two simultaneous conditions Adding the sum and difference equations yields $2k_1 = 6$, so $k_1 = 3$; substituting back gives $k_2 = -1$. Both one-sided limits are then 4, as required. Thus

$$k_1^2 + k_2^2 = 3^2 + (-1)^2 = 9 + 1 = 10.$$

The values 5, 20 and 8 fail this uniquely determined pair of continuity conditions.

Step 4: Conclusion $\boxed{k_1^2 + k_2^2 = 10}$. Hence, the correct option is (B).

Quick Tip to Solve: For a piecewise function continuous at the joining point, equate each side separately to the assigned middle value before solving for the parameters.

10. If the system of equations

$$(\lambda-1)x+(\lambda-4)y+\lambda z=5$$
$$\lambda x+(\lambda-1)y+(\lambda-4)z=7$$
$$(\lambda+1)x+(\lambda+2)y-(\lambda+2)z=9$$

has infinitely many solutions, then $\lambda^2+\lambda$ is equal to [4 marks]

(A) 20

(B) 10

(C) 6

(D) 12

Correct Answer: (D) 12

Solution:

Concept:

- Infinitely many solutions of three linear equations require $\text{rank}(A) = \text{rank}([A|b]) < 3$. A zero coefficient determinant is necessary but does not alone guarantee consistency.
- An identical linear relation among coefficient rows must also hold among their right-hand sides. A nonzero two-by-two minor confirms rank at least two.

Step 1: Find the singular parameter values The coefficient matrix and its determinant are

$$A=\begin{pmatrix}\lambda-1 & \lambda-4 & \lambda\\ \lambda & \lambda-1 & \lambda-4\\ \lambda+1 & \lambda+2 & -\lambda-2\end{pmatrix}.$$

Expansion and simplification give

$$\det A=-4\lambda^2+10\lambda+6=-2(\lambda-3)(2\lambda+1).$$

Therefore only $\lambda=3$ or $\lambda=-1/2$ can give infinitely many solutions. For other parameters the determinant is nonzero and the solution is unique.

Step 2: Check consistency when the parameter is three For $\lambda=3$, the coefficient rows are $(2,-1,3),(3,2,-1),(4,5,-5)$. The third row equals twice the second minus the first. The right sides obey the same relation: $9=2(7)-5$, so the third equation is redundant. Moreover,

$$\begin{vmatrix}2 & -1\\ 3 & 2\end{vmatrix}=7\neq 0.$$

The coefficient and augmented ranks are both exactly two, leaving one free variable and hence infinitely many solutions.

Step 3: Exclude the other singular value At $\lambda=-1/2$, twice the three coefficient rows are $(-3,-9,-1),(-1,-3,-9),(1,3,-3)$. They satisfy $6R_1-5R_2+13R_3=0$, while their right sides do not: $6(5)-5(7)+13(9)=112\neq 0$. The system is inconsistent there. Thus only $\lambda=3$ qualifies

and $\lambda^2+\lambda=9+3=12$; the options 20, 10 and 6 are excluded.

Step 4: Conclusion $\boxed{\lambda^2+\lambda=12}$. Hence, the correct option is (D).

Quick Tip to Solve: After setting the determinant to zero, test every resulting parameter for consistency; a singular system can have no solution as well as infinitely many.

11. The value of $\int_{e^2}^{e^4}\frac{1}{x}\left(\frac{e^{\left((\log_e x)^2+1\right)^{-1}}}{e^{\left((\log_e x)^2+1\right)^{-1}}+e^{\left((6-\log_e x)^2+1\right)^{-1}}}\right)dx$ **is** [4 marks]

(A) 1

(B) $\log_e 2$

(C) e^2

(D) 2

Correct Answer: (A) 1

Solution:

Concept:

- The substitution $t=\log_e x$ transforms dx/x into dt. The exponential endpoints become $t=2$ and $t=4$.
- For an integral on $[a,b]$, reflection replaces t with $a+b-t$. If an integrand and its reflected partner sum to 1, its integral is half the interval length.

Step 1: Remove the logarithm using substitution Put $t=\log_e x$, so $dt=dx/x$. As x increases from e^2 to e^4, t increases from 2 to 4. Define the positive function $g(t)=\exp(1/(t^2+1))$. The integral becomes

$$I=\int_2^4\frac{g(t)}{g(t)+g(6-t)}\,dt.$$

The denominator is strictly positive throughout, so all the transformations below are valid.

Step 2: Reflect across the midpoint of the interval Use $u=6-t$. The limits 2 and 4 interchange, while $dt=-du$ reverses them back. Thus the same integral is

$$I=\int_2^4\frac{g(6-u)}{g(6-u)+g(u)}\,du.$$

This works because the two logarithmic expressions in the original denominator become t and $6-t$; reflection exchanges them exactly rather than approximating either exponential.

Step 3: Add the equivalent integrals Rename the dummy variable u to t and add the expressions:

$$2I=\int_2^4\frac{g(t)+g(6-t)}{g(t)+g(6-t)}\,dt=\int_2^4 1\,dt=4-2=2.$$

Hence $I=1$. The number $\log_e 2$ would come from different transformed bounds, while e^2 incorrectly retains an original endpoint as an interval length. The option 2 is the value of $2I$, not I.

Step 4: Conclusion The integral equals $\boxed{1}$. Hence, the correct option is (A).

Quick Tip to Solve: After $t = \log x$, check whether the denominator contains a function at t and at $a + b - t$; adding the reflected integrals can eliminate it completely.

12. Marks obtains by all the students of class 12 are presented in a freqency distribution with classes of equal width. Let the median of this grouped data be 14 with median class interval 12-18 and median class frequency 12. If the number of students whose marks are less than 12 is 18 , then the total number of students is : [4 marks]

(A) 52

(B) 44

(C) 40

(D) 48

Correct Answer: (B) 44

Solution:

Concept:

- For equal-width continuous grouped classes, the median formula is $M = L + ((N/2 - C)/f)h$, where L is the lower median-class boundary, C the cumulative frequency before it, f its frequency and h its width.
- The cumulative frequency is a count strictly before the median class. The required median position is $N/2$, which is generally different from both the median value and the class frequency.

Step 1: Identify the entries for the median formula The median class is 12–18, so $L = 12$ and $h = 18 - 12 = 6$. Its frequency is $f = 12$. The 18 students scoring less than 12 constitute the preceding cumulative frequency, so $C = 18$. The median value is $M = 14$, while the unknown total number of students is N. No half-unit boundary correction is required for these adjoining continuous intervals.

Step 2: Substitute and solve for the median position Putting these values in the formula,

$$14 = 12 + \frac{N/2 - 18}{12} \cdot 6.$$

Subtract 12, then multiply by 2:

$$2 = \frac{N/2 - 18}{2}, \qquad 4 = N/2 - 18.$$

Thus $N/2 = 22$ and therefore $N = 44$. Equivalently, the median lies $2/6 = 1/3$ of the way through its class, so 4 of its 12 observations lie before the median position.

Step 3: Check the total against the supplied median For $N = 44$, the median position is 22, lying between the preceding cumulative count 18 and the end-of-class count $18 + 12 = 30$. Substitution returns $12 + (22 - 18)6/12 = 14$, exactly the stated median. Totals 52, 40 and 48 would instead give medians 16, 13 and 15, respectively, and are excluded.

Step 4: Conclusion The total number of students is $\boxed{44}$. Hence, the correct option is (B).

Quick Tip to Solve: When the median is given and the total frequency is unknown, rearrange the median formula to obtain $N/2 = C + f(M - L)/h$ directly.

13. Let $\left|\frac{\bar{z}-i}{2\bar{z}+i}\right| = \frac{1}{3}, z \in C$, be the equation of a circle with center at C. If the area of the triangle, whose vertices are at the points $(0,0), C$ and $(\alpha, 0)$ is 11 square units, then α^2 equals: [4 marks]

(A) $\frac{121}{25}$

(B) 100

(C) $\frac{81}{25}$

(D) 50

Correct Answer: (B) 100

Solution:

Concept:

- Write $z = x + iy$, so that $\bar{z} = x - iy$. The square of a complex modulus is the sum of the squares of its real and imaginary parts.
- A circle in the form $x^2 + y^2 + 2gx + 2fy + k = 0$ has centre $(-g, -f)$. A triangle with a horizontal base has area one-half the base length times its perpendicular height.

Step 1: Convert the modulus equation into Cartesian form The numerator is $x - i(y + 1)$ and the denominator is $2x + i(1 - 2y)$. Squaring the given modulus equation gives

$$9\{x^2 + (y + 1)^2\} = 4x^2 + (1 - 2y)^2.$$

Expand both sides, preserving the sign in $1 - 2y$:

$$5x^2 + 5y^2 + 22y + 8 = 0.$$

After division by 5 the coefficient of y is $22/5$, so the centre is $C = (0, -11/5)$. The excluded denominator zero is not on this locus, so it does not alter the circle.

Step 2: Use the area to determine the base The other two vertices are the origin and $(\alpha, 0)$. Their separation is $|\alpha|$, irrespective of the sign of α. The perpendicular distance of the centre from the horizontal axis is $11/5$. Hence

$$\frac{1}{2}|\alpha|\frac{11}{5} = 11 \quad \Longrightarrow \quad |\alpha| = 10.$$

Squaring removes the possible sign of the horizontal intercept. The remaining choices would give a different base length and therefore fail the prescribed area.

Step 3: Conclusion $\alpha^2 = 100$. Hence, the correct option is (B).

Quick Tip to Solve: For a triangle based on the real axis, use the absolute value of the imaginary coordinate of the circle centre as its height.

14. Let $I(x) = \int \frac{dx}{(x-11)^{\frac{11}{13}}(x+15)^{\frac{15}{13}}}$. **If** $I(37) - I(24) = \frac{1}{4}\left(\frac{1}{b^{\frac{1}{13}}} - \frac{1}{c^{\frac{1}{13}}}\right), b, c \in \mathcal{N}$, **then** $3(b + c)$ **is equal to** [4 marks]

(A) 39

(B) 22

(C) 40

(D) 26

Correct Answer: (A) 39

Solution:

Concept:

- The exponents $11/13$ and $15/13$ add to 2. A ratio substitution can combine the two linear factors and cancel the Jacobian denominator.
- A difference of values of an antiderivative is the corresponding definite integral. The additive constant cancels, so only the transformed endpoint values are needed.

Step 1: Choose a ratio that combines the factors On the interval from 24 to 37 both linear factors are positive, so all the real fractional powers below are unambiguous. Set

$$t = \frac{x-11}{x+15}, \qquad \frac{dt}{dx} = \frac{26}{(x+15)^2}.$$

Since $x - 11 = t(x + 15)$, the denominator becomes

$$(x-11)^{11/13}(x+15)^{15/13} = t^{11/13}(x+15)^2.$$

Thus the substitution exactly cancels the squared factor rather than leaving an additional function of x.

Step 2: Integrate and substitute the endpoints The integral is now a power integral:

$$I(x) = \frac{1}{26}\int t^{-11/13}\,dt = \frac{1}{4}t^{2/13} + K.$$

At $x = 37$, $t = 26/52 = 1/2$, while at $x = 24$, $t = 13/39 = 1/3$. Therefore

$$I(37) - I(24) = \frac{1}{4}\left(\frac{1}{4^{1/13}} - \frac{1}{9^{1/13}}\right).$$

Comparing with the given form identifies $b = 4$ and $c = 9$, in that order. The other numerical choices do not equal three times the sum of these radicands.

Step 3: Conclusion $3(b + c) = 3(4 + 9) = 39$. Hence, the correct option is (A).

Quick Tip to Solve: When the two exponents add to 2, try the ratio of the linear factors; its derivative supplies the squared denominator.

15. If the line $3x - 2y + 12 = 0$ intersects the parabola $4y = 3x^2$ at the points A and B, then at the vertex of the parabola, the line segment AB subtends an angle equal to [4 marks]

(A) $\frac{\pi}{2} - \tan^{-1}\left(\frac{3}{2}\right)$
(B) $\tan^{-1}\left(\frac{9}{7}\right)$
(C) $\tan^{-1}\left(\frac{11}{9}\right)$
(D) $\tan^{-1}\left(\frac{4}{5}\right)$

Correct Answer: (B) $\tan^{-1}\left(\frac{9}{7}\right)$

Solution:

Concept:

- The vertex of $4y = 3x^2$ is the origin. The angle subtended by a chord at the vertex is the angle between the two position vectors to its endpoints.
- For vectors u, v in a plane, an acute angle satisfies $\tan\theta = |u_x v_y - u_y v_x|/(u \cdot v)$ when the dot product is positive. The dot product also resolves the acute/obtuse ambiguity.

Step 1: Find both ends of the chord The line gives $y = (3x + 12)/2$. Substitute this into the parabola:

$$2(3x + 12) = 3x^2 \quad \Longrightarrow \quad x^2 - 2x - 8 = 0.$$

Factoring gives $(x - 4)(x + 2) = 0$. Thus the intersection points are $A = (4, 12)$ and $B = (-2, 3)$. Each point satisfies both equations, so they are the required chord endpoints, not points on an auxiliary line.

Step 2: Calculate the angle between the endpoint vectors The vectors from the vertex are $u = (4, 12)$ and $v = (-2, 3)$. Compute their dot product and the magnitude of their planar determinant:

$$u \cdot v = -8 + 36 = 28 > 0, \qquad |u_x v_y - u_y v_x| = |12 + 24| = 36.$$

The angle is acute and therefore

$$\tan\theta = \frac{36}{28} = \frac{9}{7}, \qquad \theta = \tan^{-1}\left(\frac{9}{7}\right).$$

Option (A) has tangent $2/3$; options (C) and (D) have tangents $11/9$ and $4/5$. None equals $9/7$, and the positive dot product rules out taking the supplementary angle.

Step 3: Conclusion The subtended angle is $\tan^{-1}(9/7)$ radians. Hence, the correct option is (B).

Quick Tip to Solve: Check the dot product before using an inverse tangent: a negative dot product would require the obtuse supplement.

16. If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to [4 marks]

(A) -120

(B) -1200

(C) -1080

(D) -1020

Correct Answer: (C) -1080

Solution:

Concept:

- For an arithmetic progression with first term a and common difference d, $a_n = a + (n-1)d$ and $S_n = \frac{n}{2}[2a + (n-1)d]$.
- The next four terms after the first four are terms 5 through 8. Their sum is $S_8 - S_4$, not S_4 for a new progression with the original first term.

Step 1: Translate the relation between the two blocks Here $a = 3$. Write out the two consecutive groups so that the indices remain clear:

$$S_4 = 3 + (3 + d) + (3 + 2d) + (3 + 3d) = 12 + 6d.$$

The next four terms are $3 + 4d, 3 + 5d, 3 + 6d, 3 + 7d$, with sum $12 + 22d$. The first sum is one-fifth of the second, so

$$5(12 + 6d) = 12 + 22d.$$

Consequently $60+30d = 12+22d$, giving $8d = -48$ and $d = -6$. Negative sums cause no contradiction: the resulting first block sums to -24, and the second sums to -120, exactly five times as large.

Step 2: Evaluate the sum of twenty terms Use the progression sum with the same first term and the derived common difference:

$$S_{20} = \frac{20}{2}\{2(3) + 19(-6)\} = 10(6 - 114) = -1080.$$

As a cross-check, the twentieth term is $3 + 19(-6) = -111$. The average of the first and last terms is $(3 - 111)/2 = -54$, and multiplying by 20 again gives -1080. Thus the other listed negative totals do not satisfy this progression.

Step 3: Conclusion The sum of the first twenty terms is -1080. Hence, the correct option is (C).

Quick Tip to Solve: For equal-length consecutive blocks of an A.P., write their term indices explicitly before applying the stated ratio.

17. Let the arc AC of a circle subtend a right angle at the centre O. If the point B on the arc AC, divides the arc AC such that $\frac{\text{length of arc } AB}{\text{length of arc } BC} = \frac{1}{5}$, and $\overrightarrow{OC} = \alpha\overrightarrow{OA} + \beta\overrightarrow{OB}$, then $\alpha + \sqrt{2}(\sqrt{3}-1)\beta$ is equal to [4 marks]

(A) $2\sqrt{3}$

(B) $5\sqrt{3}$

(C) $2 + \sqrt{3}$

(D) $2 - \sqrt{3}$

Correct Answer: (D) $2 - \sqrt{3}$

Solution:

Concept:

- On one circle, arc lengths are proportional to their central angles. A ratio of arc lengths $1 : 5$ divides the total angle into six equal parts.
- A vector equation can be resolved into two perpendicular Cartesian components. Choosing one radius along the positive horizontal axis simplifies the coordinates without changing the required coefficients.

Step 1: Choose coordinates for the three radii Let the radius be $r > 0$, put O at the origin, and take OA along the positive horizontal axis. Since $\angle AOC = 90°$, we may take $C = (0, r)$. The smaller arc AB occupies one-sixth of the right angle, so $\angle AOB = 15°$. Therefore

$$\overrightarrow{OA} = r(1, 0), \quad \overrightarrow{OB} = r(\cos 15°, \sin 15°), \quad \overrightarrow{OC} = r(0, 1).$$

A reflected coordinate choice gives the same coefficients; only their geometric relationship matters.

Step 2: Resolve the equation and simplify the required combination Comparing vertical and horizontal components in the given equation gives

$$1 = \beta \sin 15°, \qquad 0 = \alpha + \beta \cos 15°.$$

Using the standard exact values yields

$$\beta = \csc 15° = \sqrt{6} + \sqrt{2}, \qquad \alpha = -\cot 15° = -(2 + \sqrt{3}).$$

Now simplify the entire expression before choosing an option:

$$\sqrt{2}(\sqrt{3} - 1)\beta = 2(\sqrt{3} - 1)(\sqrt{3} + 1) = 4.$$

Thus the requested value is $-(2 + \sqrt{3}) + 4 = 2 - \sqrt{3}$. It is positive but less than 1, which also rules out the other three larger positive choices.

Step 3: Conclusion $\alpha + \sqrt{2}(\sqrt{3} - 1)\beta = 2 - \sqrt{3}$. Hence, the correct option is (D).

Quick Tip to Solve: Put one radius on a coordinate axis, then use the perpendicular component first to find the coefficient of the oblique radius.

18. Let the area of a $\triangle PQR$ with vertices $P(5,4), Q(-2,4)$ and $R(a,b)$ be 35 square units. If its orthocenter and centroid are $O\left(2, \frac{14}{5}\right)$ and $C(c,d)$ respectively, then $c+2d$ is equal to [4 marks]

(A) 3

(B) $\frac{7}{3}$

(C) 2

(D) $\frac{8}{3}$

Correct Answer: (A) 3

Solution:

Concept:

- The orthocentre lies on each altitude. An altitude through a vertex is perpendicular to the opposite side; a horizontal side has a vertical altitude.
- The centroid of vertices $(x_1,y_1),(x_2,y_2),(x_3,y_3)$ is $((x_1+x_2+x_3)/3,(y_1+y_2+y_3)/3)$. Area provides an independent check on the resulting third vertex.

Step 1: Use the altitude from the third vertex The points $P=(5,4)$ and $Q=(-2,4)$ form a horizontal side of length 7. Hence the altitude from $R=(a,b)$ is the vertical line $x=a$. Since the orthocentre is $O=(2,14/5)$, it lies on this line and forces $a=2$. Also, the stated area gives

$$\frac{1}{2}(7)|b-4|=35 \quad \Longrightarrow \quad b=14 \text{ or } b=-6.$$

Both candidates satisfy the area, so it is necessary to use another altitude to determine the correct one.

Step 2: Use the altitude through P and find the centroid The slope of PO, the altitude through P, is

$$\frac{14/5-4}{2-5}=\frac{2}{5}.$$

Therefore the perpendicular side QR has slope $-5/2$. Since $a=2$,

$$\frac{b-4}{2-(-2)}=-\frac{5}{2} \quad \Longrightarrow \quad b=-6.$$

The centroid is consequently

$$(c,d)=\left(\frac{5-2+2}{3},\frac{4+4-6}{3}\right)=\left(\frac{5}{3},\frac{2}{3}\right).$$

Then $c+2d=5/3+4/3=3$. The other options do not equal this sum; choosing the other area-compatible vertex would violate the given orthocentre.

Step 3: Conclusion $c+2d=3$. Hence, the correct option is (A).

Quick Tip to Solve: Area gives an absolute height, so use the orthocentre condition to choose which side of the base contains the third vertex.

19. If A, B, and $\left(\operatorname{adj}\left(\mathrm{A}^{-1}\right)+\operatorname{adj}\left(\mathrm{B}^{-1}\right)\right)$ are non-singular matrices of same order, then the inverse of $A\left(\operatorname{adj}\left(A^{-1}\right)+\operatorname{adj}\left(B^{-1}\right)\right)^{-1} B$, is equal to [4 marks]

(A) $\frac{AB^{-1}}{|A|}+\frac{BA^{-1}}{|B|}$

(B) $\operatorname{adj}\left(\mathrm{B}^{-1}\right)+\operatorname{adj}\left(\mathrm{A}^{-1}\right)$

(C) $\mathrm{AB}^{-1}+\mathrm{A}^{-1}\,\mathrm{B}$

(D) $\frac{1}{|AB|}(\operatorname{adj}(B)+\operatorname{adj}(A))$

Correct Answer: (D) $\frac{1}{|AB|}(\operatorname{adj}(B)+\operatorname{adj}(A))$

Solution:

Concept:

- For a nonsingular square matrix M, $\operatorname{adj}(M)=|M|M^{-1}$. In particular, $\operatorname{adj}(A^{-1})=A/|A|$, since $|A^{-1}|=1/|A|$.
- The inverse of a product reverses the factor order: $(XYZ)^{-1}=Z^{-1}Y^{-1}X^{-1}$. Matrix multiplication cannot in general be commuted.

Step 1: Name the middle matrix and invert the product Put $S=\operatorname{adj}(A^{-1})+\operatorname{adj}(B^{-1})$. All required inverses exist by the hypotheses. The matrix in the question is $AS^{-1}B$, so its inverse is

$$(AS^{-1}B)^{-1}=B^{-1}SA^{-1}.$$

This order follows by multiplying the two products: the adjacent factors BB^{-1}, $S^{-1}S$ and AA^{-1} cancel in succession.

Step 2: Expand using the adjoint identity Substitute $S=A/|A|+B/|B|$ without changing the order of the outer factors:

$$\begin{aligned} B^{-1}SA^{-1} &= \frac{B^{-1}AA^{-1}}{|A|}+\frac{B^{-1}BA^{-1}}{|B|} \\ &= \frac{B^{-1}}{|A|}+\frac{A^{-1}}{|B|} \\ &= \frac{\operatorname{adj}(B)+\operatorname{adj}(A)}{|A||B|}. \end{aligned}$$

Since determinants multiply, the denominator is $|AB|$.

Step 3: Check the alternatives Option (B) gives S alone, omitting the outer inverse factors. Option (C) omits determinant factors and has different matrix products. Option (A) also has extra factors of A and B. A quick scalar check with $A=2$, $B=3$ gives the actual inverse $1/3$, whereas these three expressions give different values; thus none is a general identity.

Step 4: Conclusion The required inverse is the sum of the two adjoints divided by the determinant of the product, as simplified above. Hence, the correct option is (D).

Quick Tip to Solve: Write the product inverse in reverse order first; only then replace the adjoints by determinant multiples of inverses.

20. Let $R = \{(1,2),(2,3),(3,3)\}$ be a relation defined on the set $\{1,2,3,4\}$. Then the minimum number of elements, needed to be added in R so that R becomes an equivalence relation, is: [4 marks]

(A) 9

(B) 8

(C) 7

(D) 10

Correct Answer: (C) 7

Solution:

Concept:

- An equivalence relation must be reflexive, symmetric and transitive. Its equivalence classes partition the underlying set, and each class contains every ordered pair of its members.
- If an equivalence class has k elements, its contribution to the relation is k^2 ordered pairs. To minimize additions, merge only elements that the original relation forces together.

Step 1: Find the forced equivalence classes The given pair $(1,2)$ forces 1 and 2 into the same equivalence class. The pair $(2,3)$ then forces 3 into that same class by transitivity. Consequently $1,2,3$ must all be equivalent. The pair $(3,3)$ is already consistent with reflexivity and creates no additional class. There is no pair connecting 4 to another element, so the smallest completion leaves it as the singleton class $\{4\}$.

Step 2: Construct the smallest completion The class $\{1,2,3\}$ must contain all nine pairs in $\{1,2,3\} \times \{1,2,3\}$, and reflexivity on the whole underlying set also requires $(4,4)$. Thus the smallest possible equivalence relation has

$$3^2 + 1^2 = 10$$

pairs. This construction is indeed reflexive, symmetric and transitive because it is the union of the complete relations on two disjoint classes.

Step 3: Count only the new pairs The original relation already contains three of these ten pairs, so the required number of additions is

$$10 - 3 = 7.$$

Explicitly, the new pairs are $(1,1),(2,2),(2,1),(3,2),(1,3),(3,1),(4,4)$. Each is forced, so fewer additions cannot work. Options (A), (B) and (D) give larger counts; they do not represent the minimum. In particular, counting ten confuses the final size with the number to add.

Step 4: Conclusion Exactly $\boxed{7}$ ordered pairs must be added. Hence, the correct option is (C).

Quick Tip to Solve: Connect the elements forced to be equivalent, add the squares of the class sizes, and subtract the number of existing pairs.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. The sum of all rational terms in the expansion of $\left(1+2^{1/3}+3^{1/2}\right)^6$ is equal to _____. [4 marks]

Solution:

Concept:

- A term in $(1+u+v)^6$ is $\dfrac{6!}{a!b!c!}u^b v^c$, where a, b, c are nonnegative integers satisfying $a+b+c=6$.
- Here $u=2^{1/3}$ and $v=3^{1/2}$. Such a term is rational exactly when b is divisible by 3 and c is even; the fractional prime exponents must vanish separately.

Step 1: List the admissible exponent pairs The general term is

$$\frac{6!}{a!b!c!}\,2^{b/3}3^{c/2}, \qquad a=6-b-c.$$

Its multinomial coefficient is an integer and cannot remove a fractional exponent of a prime. Thus the allowed values are $b=0,3,6$, with even c and $b+c\le 6$. For $b=0$, take $c=0,2,4,6$; for $b=3$, take $c=0,2$; for $b=6$, only $c=0$ is possible. This lists every rational term without counting an irrational term.

Step 2: Sum the terms with zero cube-root exponent For $b=0$, the four contributions are

$$\begin{aligned} c=0&: 1,\\ c=2&: \frac{6!}{4!2!}\,3=45,\\ c=4&: \frac{6!}{2!4!}\,9=135,\\ c=6&: 27. \end{aligned}$$

Their sum is $1+45+135+27=208$. The coefficient for each of these terms is the ordinary binomial coefficient because the cube-root term has exponent zero.

Step 3: Include the remaining rational terms For $b=3, c=0$, the contribution is

$$\frac{6!}{3!3!}\,2=40.$$

For $b=3, c=2$, the remaining exponent is $a=1$, giving

$$\frac{6!}{1!3!2!}\,2\cdot 3=360.$$

Finally $b=6, c=0, a=0$ contributes $2^2=4$. Hence the total is

$$208+40+360+4=612.$$

These are sums of the individual rational multinomial terms, with their full multiplicities.

Step 4: Conclusion The sum of all rational terms is $\boxed{612}$.

Quick Tip to Solve: For prime radicals in a multinomial expansion, impose the divisibility conditions on each radical exponent before computing coefficients.

22. If the set of all values of a, for which the equation $5x^3 - 15x - a = 0$ has three distinct real roots, is the interval (α, β), then $\beta - 2\alpha$ is equal to _____. [4 marks]

Solution:

Concept:

- The equation $5x^3 - 15x - a = 0$ can be viewed as the intersections of $y = f(x) = 5x^3 - 15x$ with the horizontal line $y = a$.
- A cubic with two stationary points has three distinct real intersections precisely when the horizontal level lies strictly between its local minimum and local maximum. Equality gives a repeated root.

Step 1: Locate the stationary points Differentiate the cubic function, keeping the parameter as the horizontal level:

$$f\prime(x) = 15x^2 - 15 = 15(x-1)(x+1).$$

Thus the stationary points occur at $x = -1$ and $x = 1$. The derivative is positive on $(-\infty, -1)$, negative on $(-1, 1)$, and positive on $(1, \infty)$. Consequently the first point is a local maximum and the second a local minimum; the three monotone intervals can each supply one root.

Step 2: Evaluate the extreme levels The values at the stationary points are

$$f(-1) = -5 + 15 = 10, \qquad f(1) = 5 - 15 = -10.$$

A horizontal line crosses all three monotone branches exactly when

$$-10 < a < 10.$$

Indeed, the increasing left branch reaches every level below 10, the middle branch reaches precisely the levels between -10 and 10, and the increasing right branch reaches every level above -10. Thus this condition is both necessary and sufficient.

Step 3: Exclude repeated-root endpoints and evaluate At $a = 10$, the line touches the local maximum at $x = -1$; at $a = -10$, it touches the local minimum at $x = 1$. These give a double root and therefore cannot satisfy the demand for three distinct real roots. Levels outside the closed interval intersect only one branch. Hence the requested open interval has endpoints $\alpha = -10$ and $\beta = 10$, giving

$$\beta - 2\alpha = 10 - 2(-10) = 30.$$

Step 4: Conclusion The interval is $(-10, 10)$, so $\boxed{\beta - 2\alpha = 30}$.

Quick Tip to Solve: For a cubic shifted by a parameter, compare that parameter with the two stationary values; strict inequalities ensure distinct roots.

23. If the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ has equal roots, where $a+c=15$ and $b=\frac{36}{5}$, then a^2+c^2 is equal to _____ [4 marks]

Solution:

Concept:

- If the sum of the three coefficients of a quadratic is zero, then $x=1$ is a root, as direct substitution shows.
- A quadratic with equal roots and one known root 1 must have both roots equal to 1. The product of the roots is the constant coefficient divided by the leading coefficient.

Step 1: Identify a root common to the coefficient form Let $P(x)=a(b-c)x^2+b(c-a)x+c(a-b)$. On substituting $x=1$,

$$\begin{aligned} P(1) &= ab-ac+bc-ab+ac-bc \\ &= 0. \end{aligned}$$

Thus 1 is a root regardless of the particular values of a,b,c. The condition that the equation has equal roots means that, for this quadratic, the second root must also be 1. This observation avoids expanding a lengthy discriminant.

Step 2: Use the product of the equal roots The product of the roots is 1, so

$$\frac{c(a-b)}{a(b-c)}=1.$$

Therefore

$$\begin{aligned} c(a-b) &= a(b-c), \\ ac-bc &= ab-ac, \\ 2ac &= b(a+c). \end{aligned}$$

With $a+c=15$ and $b=36/5$, this becomes

$$2ac=\frac{36}{5}\cdot 15=108, \qquad ac=54.$$

The condition can also be read from $P(x)=(x-1)\{a(b-c)x-c(a-b)\}$.

Step 3: Find the requested symmetric expression Use the square of the given sum to avoid determining which of the two numbers is called a:

$$a^2+c^2=(a+c)^2-2ac=15^2-108=117.$$

For consistency, the numbers with sum 15 and product 54 are 6 and 9. For either ordering, $a(b-c)\neq 0$, so the original equation really remains quadratic. Also its leading and constant coefficients agree, with the middle coefficient minus twice the leading coefficient, as required for a multiple of $(x-1)^2$.

Step 4: Conclusion The required value is $\boxed{a^2+c^2=117}$.

Quick Tip to Solve: Before expanding a discriminant, test simple roots such as 1; a zero coefficient sum can turn an equal-root problem into one product identity.

24. Let the circle C touch the line $x - y + 1 = 0$, have the centre on the positive x-axis, and cut off a chord of length $\frac{4}{\sqrt{13}}$ along the line $-3x + 2y = 1$. Let H be the hyperbola $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$, whose one of the foci is the centre of C and the length of the transverse axis is the diameter of C. Then $2\alpha^2 + 3\beta^2$ is equal to _____. [4 marks]

Solution:

Concept:

- For a circle with centre $(h, 0)$, tangency to a line means its radius equals the perpendicular distance from the centre to that line.
- If a line is a distance d from a circle centre and cuts a chord of length L, then $r^2 = d^2 + (L/2)^2$. For the hyperbola, focal distance satisfies $c^2 = \alpha^2 + \beta^2$.

Step 1: Express the radius through the centre Write the centre of C as $(h, 0)$, where $h > 0$ because it is on the positive x-axis. The distance to $x - y + 1 = 0$ is $(h + 1)/\sqrt{2}$, so tangency gives

$$r^2 = \frac{(h+1)^2}{2}.$$

The second line is $-3x + 2y - 1 = 0$, whose distance from the centre is

$$d = \frac{3h + 1}{\sqrt{13}}.$$

Both numerators are positive under the specified condition on h.

Step 2: Apply the chord-length condition The half-chord length is $2/\sqrt{13}$. The right triangle formed by a radius, the perpendicular to the chord and its half-length gives

$$\frac{(h+1)^2}{2} = \frac{(3h+1)^2}{13} + \frac{4}{13}.$$

Clear denominators and simplify:

$$\begin{aligned} 13(h+1)^2 &= 2\{(3h+1)^2 + 4\}, \\ 5h^2 - 14h - 3 &= 0, \\ (5h+1)(h-3) &= 0. \end{aligned}$$

Positivity selects $h = 3$; the other root $-1/5$ violates the centre condition. Therefore $r^2 = 16/2 = 8$.

Step 3: Transfer the circle data to the hyperbola The positive focus is the circle centre $(3, 0)$, so the focal distance of H is $c = 3$. Its transverse-axis length 2α equals the circle diameter $2r$; hence $\alpha = r$ and $\alpha^2 = 8$. Using $c^2 = \alpha^2 + \beta^2$, obtain

$$\beta^2 = 9 - 8 = 1.$$

The requested expression is now

$$2\alpha^2 + 3\beta^2 = 2(8) + 3(1) = 19.$$

The positive value of β^2 also confirms that these parameters define a real hyperbola.

Step 4: Conclusion The hyperbola parameters satisfy $\alpha^2 = 8$, $\beta^2 = 1$, and $\boxed{2\alpha^2 + 3\beta^2 = 19}$.

Quick Tip to Solve: Use the tangent line to eliminate the radius first, then use the half-chord right triangle; the circle centre immediately gives the hyperbola focal distance.

25. If the area of the larger portion bounded between the curves $x^2 + y^2 = 25$ and $\mathrm{y} = |\mathrm{x} - 1|$ is $\frac{1}{4}(\mathrm{b}\pi + \mathrm{c}), \mathrm{b}, \mathrm{c} \in N$, then $\mathrm{b} + \mathrm{c}$ is equal to _____ [4 marks]

Solution:

Concept:

- The circle $x^2 + y^2 = 25$ has radius 5 and total area 25π. The graph $y = |x - 1|$ consists of two straight rays meeting at $(1, 0)$.
- Compute the smaller area above the V-shaped graph and below the upper semicircle by integrating upper minus lower. Subtract this from the circle to obtain the larger portion.

Step 1: Find the two intersection points For $x \le 1$, the line is $y = 1 - x$; for $x \ge 1$, it is $y = x - 1$. In either case,

$$x^2 + (x - 1)^2 = 25 \quad \Longrightarrow \quad x^2 - x - 12 = 0.$$

Thus $x = -3, 4$, giving the intersections $(-3, 4)$ and $(4, 3)$. Both lie on the upper semicircle. The V-shaped line therefore cuts off one upper cap; the rest of the disk is the other bounded portion.

Step 2: Evaluate the smaller portion Using the change in the absolute-value formula at $x = 1$, its area is

$$S = \int_{-3}^{4} \sqrt{25 - x^2}\, dx - \int_{-3}^{1} (1 - x)\, dx - \int_{1}^{4} (x - 1)\, dx.$$

An antiderivative of the circle term is

$$F(x) = \frac{x}{2}\sqrt{25 - x^2} + \frac{25}{2}\sin^{-1}(x/5).$$

Since $\sin^{-1}(4/5) + \sin^{-1}(3/5) = \pi/2$, the first integral equals $12 + 25\pi/4$. The two line integrals are the triangular areas 8 and $9/2$. Consequently,

$$S = 12 + \frac{25\pi}{4} - 8 - \frac{9}{2} = \frac{25\pi - 2}{4}.$$

Step 3: Subtract from the full disk The complementary portion has area

$$L = 25\pi - S = \frac{75\pi + 2}{4}.$$

It is larger, since $L - S = (50\pi + 4)/4 > 0$. Comparing L with the stated form $(b\pi + c)/4$ gives $b = 75$ and $c = 2$. Both are natural numbers, as required.

Step 4: Conclusion The larger area is $\frac{75\pi + 2}{4}$ square units, so $\boxed{b + c = 77}$.

Quick Tip to Solve: Split the absolute-value integral at its vertex $x = 1$, then use complementary inverse-sine angles from the 3-4-5 triangle.

26. Given below are two statements:

Statement I: The hot water flows faster than cold water

Statement II: Soap water has higher surface tension as compared to fresh water. In the light above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are false

(B) Both Statement I and Statement II are true

(C) Statement I is false but Statement II is true

(D) Statement I is true but Statement II is false

Correct Answer: (D) Statement I is true but Statement II is false

Solution:

Concept:

- Viscosity measures a fluid's resistance to relative motion between its layers. For liquids, viscosity generally decreases when temperature rises.
- Surface tension is the force per unit length associated with a liquid surface. Soap acts as a surfactant and reduces the surface tension of water.

Step 1: Assess the statement about hot water Compare flow through the same tube under the same pressure difference. For steady laminar flow, Poiseuille's law gives

$$\dot{V} = \frac{\pi R^4 \Delta P}{8\eta \ell},$$

where R is the tube radius, ℓ its length and η the viscosity. Thus the volume flow rate is inversely proportional to viscosity. Hot water has lower viscosity than cold water under comparable conditions, so it flows faster. Statement I is true; this is a viscosity effect, not an inference from a change in surface tension.

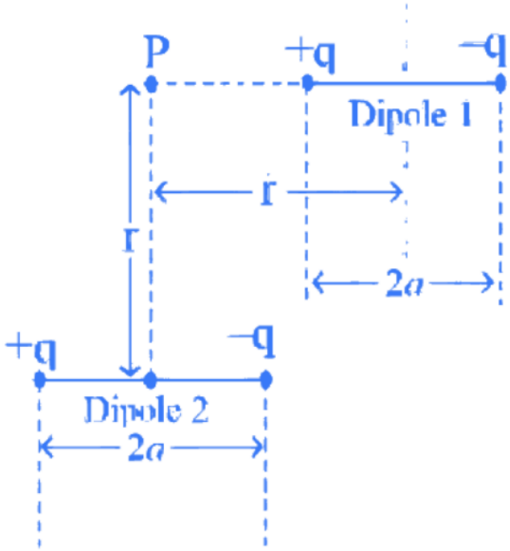
Step 2: Assess the soap-water statement Soap molecules accumulate at the water surface and reduce the cohesive effect responsible for its surface tension. This allows the solution to spread and wet surfaces more readily. Therefore soap water has lower, rather than higher, surface tension than fresh water at the same temperature. Statement II is false.

Step 3: Match the pair of truth values The required pair is true for I and false for II. Option A incorrectly rejects I. Option B incorrectly accepts II, while option C reverses both truth values. Only option D gives the pair established from liquid viscosity and the action of a surfactant. The two statements involve different properties and should be assessed separately.

Step 4: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (D).

Quick Tip to Solve: For liquids, warming usually lowers viscosity; adding soap lowers surface tension. Keep the two fluid properties separate.

27. A point particle of charge Q is located at P along the axis of an electric dipole 1 at a distance r as shown in the figure. The point P is also on the equatorial plane of a second electric dipole 2 at a distance r. The dipoles are made of opposite charge q separated by a distance $2a$. For the charge particle at P not to experience any net force, which of the following correctly describes the situation? [4 marks]



(A) $\frac{a}{r} \sim 0.5$

(B) $\frac{a}{r} \sim 3$

(C) $\frac{a}{r} \sim 10$

(D) $\frac{a}{r} \sim 20$

Correct Answer: No option is correct for the equal dipoles as drawn; the supplied key (B) conflicts with the exact field directions.

Solution:

Concept:

- The force on the nonzero test charge is $\mathbf{F} = Q(\mathbf{E}_1 + \mathbf{E}_2)$. Zero force requires cancellation of the vector fields, including their directions, not just a formal equality of magnitudes.
- Use Coulomb's law for the finite charge separation $2a$; the far-field dipole approximation requires $r \gg a$ and is not valid for the proposed large a/r values.

Step 1: Find the equatorial field direction Take rightward as positive. In dipole 2, the positive charge is on the left and the negative charge on the right. At its equatorial point P, the vertical components cancel while both horizontal components point right. Consequently,

$$E_{2x} = \frac{2kqa}{(r^2 + a^2)^{3/2}} > 0,$$

where $k = 1/(4\pi\varepsilon_0)$. This direction is opposite to that dipole's electric dipole moment. It does not change as the positive ratio a/r varies.

Step 2: Check the external axial region For $r > a$, P is to the left of both charges of dipole 1. The

nearby positive charge produces a leftward field larger than the rightward field from the more distant negative charge:

$$E_{1x} = -kq\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right] = -\frac{4kqar}{(r^2-a^2)^2}.$$

The fields oppose, but their magnitude ratio is

$$\frac{|E_1|}{|E_2|} = \frac{2r(r^2+a^2)^{3/2}}{(r^2-a^2)^2} > 2.$$

Indeed, the numerator exceeds $2r^4$ while the denominator is less than r^4. Hence cancellation is impossible in the external region, including option A with $a/r \sim 0.5$.

Step 3: Check the internal axial region For $a > r$, P lies between the charges of dipole 1. Its positive charge is now to the left of P and its negative charge is to the right. Both fields point right, giving

$$E_{1x} = kq\left[\frac{1}{(a-r)^2} + \frac{1}{(a+r)^2}\right] > 0.$$

Thus E_{1x} adds to the rightward equatorial field; they cannot cancel. Options B, C and D all belong to this region. At $a = r$, P coincides with a point charge and the field is singular. Extending the external difference formula into $a > r$ would incorrectly subtract two fields that actually have the same direction.

Step 4: Conclusion For the equal finite dipoles oriented as printed, no listed ratio gives zero net force on a nonzero charge Q. The supplied key selects (B), but $a/r \sim 3$ places P between the axial charges, where both dipole fields point right. Thus the literal printed geometry has no correct option; the key's (B) does not satisfy vector Coulomb's law.

Quick Tip to Solve: Before equating axial and equatorial magnitudes, check whether the axial point lies outside or between the charges; crossing a charge reverses its field direction.

28. What is the lateral shift of a ray refracted through a parallel-sided glass slab of thickness ' h ' in terms of the angle of incidence ' i ' and angle of refraction ' r ', if the glass slab is placed in air medium? [4 marks]

(A) h
(B) $\frac{h\cos(i-r)}{\sin r}$
(C) $\frac{\text{h}\tan(\text{i}-\text{r})}{\tan \text{r}}$
(D) $\frac{h\sin(i-r)}{\cos r}$

Correct Answer: (D) $\frac{h\sin(i-r)}{\cos r}$

Solution:

Concept:

- A parallel-sided slab placed in air gives an emergent ray parallel to the incident ray, because refraction at the second parallel face reverses the angular deviation at the first.

- The lateral shift is the perpendicular distance between the emergent ray and the original incident direction, not the distance measured along the slab face.

Step 1: Find the ray path inside the slab Let AB be the distance travelled inside the slab between its two plane faces. The normal separation of those faces is the thickness h. Since the ray makes angle r with the normal,

$$AB\cos r = h \quad \Longrightarrow \quad AB = \frac{h}{\cos r}.$$

All the angles here are measured with respect to the normal; using the surface instead would interchange sine and cosine and give the wrong factor.

Step 2: Project onto the perpendicular separation The internal refracted direction differs from the incident direction by $i - r$. Resolving the internal path perpendicular to the original incident direction gives the separation d between the two parallel external rays:

$$d = AB\sin(i-r) = \frac{h\sin(i-r)}{\cos r}.$$

The sine is essential because the shift is a transverse projection. At normal incidence $i = r = 0$, this gives $d = 0$, as the ray passes straight through without displacement.

Step 3: Check the alternatives Option A would give the same nonzero shift even at normal incidence, so it cannot be general. Option B contains the wrong projection and becomes singular at $r = 0$. Option C uses tangent factors that do not follow from the path-length triangle and do not yield the correct small-angle dependence. Option D contains both the internal-path factor $1/\cos r$ and the transverse projection $\sin(i-r)$.

Step 4: Conclusion The lateral shift is $\boxed{d = \frac{h\sin(i-r)}{\cos r}}$, with the same length unit as h. Hence, the correct option is (D).

Quick Tip to Solve: First write the internal path as $h/\cos r$; then project it perpendicular to the incident ray using $\sin(i-r)$.

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29. A sub-atomic particle of mass 10^{-30} kg is moving with a velocity 2.21×10^6 m/s. Under the matter wave consideration, the particle will behave closely like $\left(\text{h} = 6.63 \times 10^{-34}\ \text{J.s}\right)$ [4 marks]

(A) X-rays

(B) Infra-red radiation

(C) Gamma rays

(D) Visible radiation

Correct Answer: (A) X-rays

Solution:

Concept:

- A moving particle has de Broglie wavelength $\lambda = h/p$, where h is Planck's constant and p is its momentum.

- For a nonrelativistic particle, $p = mv$. Compare the resulting wavelength with electromagnetic wavelength bands; this comparison does not turn the particle into a photon.

Step 1: Calculate the momentum The given speed is much smaller than the speed of light: v/c is approximately 0.0074. Hence the nonrelativistic momentum is adequate:

$$p = mv = (10^{-30})(2.21 \times 10^{6}) = 2.21 \times 10^{-24}\,\mathrm{kg\,m\,s^{-1}}.$$

The mass exponent is negative, and it must remain so when combined with the positive speed exponent.

Step 2: Calculate the matter wavelength Substitute the given Planck constant into the de Broglie relation:

$$\lambda = \frac{6.63 \times 10^{-34}}{2.21 \times 10^{-24}} = 3.00 \times 10^{-10}\,\mathrm{m}.$$

Equivalently, this is $0.300\,\mathrm{nm}$ or 3.00 angstrom. The unit is length because a joule-second divided by momentum has units of metres. This is a short wavelength on the atomic-length scale.

Step 3: Identify the closest spectral band A wavelength of $0.300\,\mathrm{nm}$ lies in the X-ray range. Infra-red radiation has much longer wavelengths, beyond the red end of the visible band. Visible light is roughly hundreds of nanometres, also far longer than the calculated value. Gamma rays characteristically have still shorter wavelengths than this X-ray scale. Therefore, among the listed bands, X-rays give the appropriate comparison for the particle's matter wave.

Step 4: Conclusion The de Broglie wavelength is $\boxed{3.00 \times 10^{-10}\,\mathrm{m}}$, corresponding to the X-ray range. Hence, the correct option is (A).

Quick Tip to Solve: Convert $10^{-10}\,\mathrm{m}$ to angstroms or nanometres before comparing wavelength bands; it immediately identifies an atomic-scale X-ray wavelength.

30. The position of a particle moving on x-axis is given by $x(t) = A\sin t + B\cos^2 t + Ct^2 + D$, where t is time. The dimension of $\frac{ABC}{D}$ is [4 marks]

(A) L

(B) $\mathrm{L}^3\,\mathrm{T}^{-2}$

(C) $\mathrm{L}^2\,\mathrm{T}^{-2}$

(D) L^2

Correct Answer: (C) $\mathrm{L}^2\,\mathrm{T}^{-2}$

Solution:

Concept:

- Every additive term in an equation for position must have the dimension of length, L. Dimensionless trigonometric factors do not alter their coefficients' dimensions.
- The arguments of sine and cosine are understood to be dimensionless; the time dependence written as t carries an implicit reciprocal-time scale. The explicit factor t^2 outside a trigonometric function has dimension T^2.

Step 1: Determine the coefficient dimensions Position x has dimension L, so each term on the right must have that same dimension. Because $\sin t$ and $\cos^2 t$ are dimensionless, their coefficients satisfy

$$[A] = \mathrm{L}, \qquad [B] = \mathrm{L}.$$

The constant additive displacement D also has dimension L. One should not assign the dimension of time to the value of a sine or cosine; a trigonometric function returns a pure number.

Step 2: Use the polynomial time term For the term Ct^2, dimensional homogeneity requires

$$[C]\mathrm{T}^2 = \mathrm{L} \quad \Longrightarrow \quad [C] = \mathrm{LT}^{-2}.$$

Thus C has acceleration dimensions. The squared time appears explicitly as a multiplier here, so it must be included, unlike the dimensionless values of the trigonometric functions.

Step 3: Combine the factors in the requested ratio Multiply the numerator dimensions and divide by the denominator dimension:

$$\left[\frac{ABC}{D}\right] = \frac{(\mathrm{L})(\mathrm{L})(\mathrm{LT}^{-2})}{\mathrm{L}} = \mathrm{L}^2\mathrm{T}^{-2}.$$

Option A omits a length and the time factor. Option B retains an extra length by failing to divide by D. Option D loses the time dimension contributed by C. Only option C preserves all the factors correctly.

Step 4: Conclusion The dimension of ABC/D is $\boxed{\mathrm{L}^2\mathrm{T}^{-2}}$. Hence, the correct option is (C).

Quick Tip to Solve: Assign dimensions to each additive term before combining coefficients; the constant D cancels one factor of length.

31. Match the List - I with List - II

	List - I		List - II
(A)	Pressure varies inversely with volume of an ideal gas.	(I)	Adiabatic process
(B)	Heat absorbed goes partly to increase internal energy and partly to do work.	(II)	Isochoric process
(C)	Heat is neither absorbed nor released by a system.	(III)	Isothermal process
(D)	No work is done on or by a gas.	(IV)	Isobaric process

Choose the correct answer from the options given below: [4 marks]

(A) A-I, B-III, C-II, D-IV

(B) A-I, B-IV, C-II, D-III

(C) A-III, B-I, C-IV, D-II

(D) A-III, B-IV, C-I, D-II

Correct Answer: (D) A-III, B-IV, C-I, D-II

Solution:

Concept:

- For a fixed amount of ideal gas, $PV = nRT$. Therefore inverse variation of pressure with volume means that temperature remains constant.
- The first law, with work done by the gas positive, is $Q = \Delta U + W$. Adiabatic means $Q = 0$, while an isochoric change has fixed volume and hence $W = \int P\,dV = 0$.

Step 1: Match the temperature and heat conditions In statement A, $P \propto 1/V$, so PV is constant. The ideal-gas equation then gives constant temperature: A matches III, the isothermal process. In statement C, the explicit absence of heat exchange gives $Q = 0$, so C matches I, the adiabatic process. An adiabatic process need not have constant temperature; its internal energy can change through work.

Step 2: Match the remaining energy and work conditions In an isobaric heating of an ideal gas, pressure remains constant, the temperature rises, and the gas expands. Thus heat supplies both ΔU and the expansion work $P\Delta V$, matching B with IV. With volume fixed there is no boundary displacement, so $W = 0$, matching D with II. The description of B is consistent with the first law; among the listed processes, isobaric heating is the intended match.

Step 3: Compare the complete option combinations The resulting combination is A–III, B–IV, C–I, D–II. Options (A) and (B) already fail by assigning an adiabatic process to inverse pressure-volume variation. Option (C) correctly assigns A and D but interchanges the heat-exchange interpretation of B and C.

Step 4: Conclusion The matching is A–III, B–IV, C–I, D–II. Hence, the correct option is (D).

Quick Tip to Solve: Identify the unambiguous zero quantities first: no heat means adiabatic, and no volume work means isochoric. Then use $PV = nRT$.

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32. A gun fires a lead bullet of temperature 300 K into a wooden block. The bullet having melting temperature of 600 K penetrates into the block and melts down. If the total heat required for the process is 625 J , then the mass of the bullet is _____ grams. (Latent heat of fusion of lead $= 2.5 \times 10^4 \text{JKg}^{-1}$ and specific heat capacity of lead $= 125 \text{JKg}^{-1}\ \text{K}^{-1}$) [4 marks]

(A) 20

(B) 15

(C) 5

(D) 10

Correct Answer: (D) 10

Solution:

Concept:

- Heating a solid to its melting point requires sensible heat $mc\Delta T$, where m is the mass and c the specific heat capacity.
- Melting the solid at its melting point requires latent heat mL_f. Both contributions are necessary

here, so $Q = m[c(T_m - T_i) + L_f]$.

Step 1: Calculate the energy required per kilogram The bullet first warms from $300\,\text{K}$ to $600\,\text{K}$, giving $\Delta T = 300\,\text{K}$. The sensible heat per kilogram is

$$c\Delta T = 125 \times 300 = 37\,500\,\text{J kg}^{-1}.$$

It then melts at constant temperature. Adding the latent heat gives

$$c\Delta T + L_f = 37\,500 + 25\,000 = 62\,500\,\text{J kg}^{-1}.$$

The latent heat is added once; it must not be multiplied by the temperature rise.

Step 2: Use the given heat to find the mass The total required heat is $625\,\text{J}$. Therefore,

$$m = \frac{625}{62\,500} = 0.010\,\text{kg} = 10\,\text{g}.$$

As a direct check, heating this mass requires $0.010 \times 37\,500 = 375\,\text{J}$, and melting requires $0.010 \times 25\,000 = 250\,\text{J}$. Their sum is exactly $625\,\text{J}$.

Step 3: Reject the other proposed masses Since required heat is proportional to mass, $20\,\text{g}$, $15\,\text{g}$ and $5\,\text{g}$ would require $1250\,\text{J}$, $937.5\,\text{J}$ and $312.5\,\text{J}$, respectively. None matches the stated energy. No separate kinetic-energy calculation is needed because the heat available for this process is already specified.

Step 4: Conclusion The mass of the bullet is $\boxed{10\,\text{g}}$. Hence, the correct option is (D).

Quick Tip to Solve: For melting from below the melting point, add $c\Delta T$ and L_f before dividing the heat by this energy per kilogram.

33. A solid sphere of mass ' m ' and radius ' r ' is allowed to roll without slipping from the highest point of an inclined plane of length ' L ' and makes an angle 30° with the horizontal. The speed of the particle at the bottom of the plane is v_1. If the angle of inclination is increased to 45° while keeping L constant. Then the new speed of the sphere at the bottom of the plane is v_2. The ratio $v_1^2 : v_2^2$ is [4 marks]

(A) $1 : 3$

(B) $1 : 2$

(C) $1 : \sqrt{2}$

(D) $1 : \sqrt{3}$

Correct Answer: (C) $1 : \sqrt{2}$

Solution:

Concept:

- A solid sphere has moment of inertia $I = \frac{2}{5}mr^2$. For rolling without slipping, its angular speed and centre-of-mass speed obey $\omega = v/r$.

- Starting from rest, loss of gravitational potential energy becomes translational plus rotational kinetic energy. An incline of length L and angle θ has height $h = L\sin\theta$.

Step 1: Apply energy conservation to the rolling sphere Static friction provides the torque needed for rolling but does no work on the sphere on a fixed plane in ideal rolling. Thus

$$mgL\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\frac{v^2}{r^2} = \frac{7}{10}mv^2.$$

Solving for the squared speed gives

$$v^2 = \frac{10}{7}gL\sin\theta.$$

The mass and radius cancel; the same rotational-energy factor applies at both inclinations.

Step 2: Take the ratio at the two angles The length of the plane stays fixed, so the heights are different. Substituting the angles gives

$$v_1^2 : v_2^2 = \sin 30^\circ : \sin 45^\circ = \frac{1}{2} : \frac{1}{\sqrt{2}} = 1 : \sqrt{2}.$$

The question asks for the ratio of squared speeds, so no additional square root is to be taken. In particular, it is the height change rather than merely the increase in the numerical angle that determines the ratio.

Step 3: Compare with the alternatives The predicted factor by which the squared speed increases is $\sin 45^\circ / \sin 30^\circ = \sqrt{2}$. The factors 3, 2, and $\sqrt{3}$ in options (A), (B), and (D) do not equal this value. A factor of 2 would also incorrectly treat $\sin 45^\circ$ as unity relative to $\sin 30^\circ$.

Step 4: Conclusion The required ratio is $\boxed{v_1^2 : v_2^2 = 1 : \sqrt{2}}$. Hence, the correct option is (C).

Quick Tip to Solve: For the same rolling body on equal-length inclines, v^2 is proportional to $\sin\theta$; the inertia factor cancels in the ratio.

34. A radioactive nucleus n_2 has 3 times the decay constant as compared to the decay constant of another radioactive nucleus n_1. If initial number of both nuclei are the same, what is the ratio of number of nuclei of n_2 to the number of nuclei of n_1, after one half-life of n_1 ? [4 marks]

(A) 1/4

(B) 1/8

(C) 4

(D) 8

Correct Answer: (A) 1/4

Solution:

Concept:

- Radioactive nuclei obey $N(t) = N_0 e^{-\lambda t}$, where λ is the decay constant and N_0 is the initial number.
- The half-life is $T_{1/2} = \ln 2/\lambda$. A larger decay constant therefore gives a shorter half-life and fewer

surviving nuclei after the same elapsed time.

Step 1: Express the two populations at the stated time Let $\lambda_1 = \lambda$, so that $\lambda_2 = 3\lambda$. The initial populations are equal to N_0. After one half-life of the first sample,

$$t = T_1 = \frac{\ln 2}{\lambda}.$$

Its remaining population is

$$N_1 = N_0 e^{-\lambda T_1} = N_0 e^{-\ln 2} = \frac{N_0}{2}.$$

The time is one half-life of sample 1, not one half-life of both samples.

Step 2: Evaluate the faster-decaying population and divide For the second sample at exactly the same time,

$$N_2 = N_0 e^{-3\lambda T_1} = N_0 e^{-3\ln 2} = \frac{N_0}{8}.$$

It has undergone three of its own half-lives, since $T_2 = T_1/3$. Therefore,

$$\frac{N_2}{N_1} = \frac{N_0/8}{N_0/2} = \frac{1}{4}.$$

This ratio is dimensionless and is smaller than unity, as required for the faster-decaying sample.

Step 3: Distinguish a surviving fraction from the requested ratio Option (B), $1/8$, is the fraction of the original second sample that survives; it is not the ratio to the surviving first sample. Options (C) and (D) are greater than one, contradicting the fact that sample 2 decays faster from an equal starting population. Only option (A) compares the two remaining populations in the requested order.

Step 4: Conclusion The ratio of remaining nuclei is $\boxed{N_2/N_1 = 1/4}$. Hence, the correct option is (A).

Quick Tip to Solve: At one half-life of the slower sample, write both surviving fractions first; divide $1/8$ by $1/2$, not by the initial population.

---

35. A light hollow cube of side length 10 cm and mass 10 g , is floating in water. It is pushed down and released to execute simple harmonic oscillations. The time period of oscillations is $y\pi \times 10^{-2}$ s, where the value of y is (Acceleration due to gravity, $g = 10\ \text{m/s}^2$, density of water $= 10^3\ \text{kg/m}^3$) [4 marks]

(A) 2

(B) 4

(C) 1

(D) 6

Correct Answer: (A) 2

Solution:

Concept:

- By Archimedes principle, buoyant force equals $\rho g V_{\text{sub}}$, where ρ is water density and V_{sub} the submerged volume.

- A small downward displacement x of a floating cube increases its submerged volume by Ax, where A is its horizontal cross-sectional area. The extra upward force is therefore proportional to displacement.

Step 1: Find the restoring force about equilibrium At equilibrium the buoyancy balances the weight, $\rho g V_0 = mg$. Let downward displacement from this position be positive. At displacement x, the buoyancy increases by $\rho g A x$, while the weight remains unchanged. Consequently,

$$m\ddot{x} = -\rho g A x.$$

This has the simple harmonic form $\ddot{x} + \omega^2 x = 0$, with

$$\omega^2 = \frac{\rho g A}{m}.$$

The relation applies to the stated oscillations while the waterline remains on the vertical faces.

Step 2: Substitute the mass and face area in SI units The side is $a = 10\,\text{cm} = 0.10\,\text{m}$, hence $A = a^2 = 0.010\,\text{m}^2$. The mass is $m = 10\,\text{g} = 0.010\,\text{kg}$. Thus

$$\omega^2 = \frac{10^3 \times 10 \times 0.010}{0.010} = 10^4\,\text{s}^{-2}, \qquad \omega = 100\,\text{s}^{-1}.$$

It follows that

$$T = \frac{2\pi}{\omega} = 2\pi \times 10^{-2}\,\text{s}.$$

Step 3: Identify the coefficient in the required form Comparing with $T = y\pi \times 10^{-2}\,\text{s}$ gives $y = 2$. The values 4, 1, and 6 would imply different angular frequencies and do not satisfy $\omega^2 = \rho g A/m$. Notice that the mass is that of the hollow cube itself; using the mass of a solid water-filled cube would change the period incorrectly.

Step 4: Conclusion The time period is $\boxed{2\pi \times 10^{-2}\,\text{s}}$, so $\boxed{y = 2}$. Hence, the correct option is (A).

Quick Tip to Solve: For small vertical oscillations of a floating prism, use the effective spring constant $k = \rho g A$, where A is the waterline area.

36. A spherical surface of radius of curvature R, separates air from glass (refractive index $= 1.5$). The centre of curvature is in the glass medium. A point object ' O ' placed in air on the optic axis of the surface, so that its real image is formed at 'I' inside glass. The line OI intersects the spherical surface at P and $PO = PI$. The distance PO equals to [4 marks]

(A) 5R

(B) 2R

(C) 1.5R

(D) 3R

Correct Answer: (A) 5R

Solution:

Concept:

- For refraction from index n_1 to index n_2, the Cartesian spherical-surface formula is $n_2/v - n_1/u = (n_2 - n_1)/R$.
- Take the direction of incident light, from air into glass, as positive. A real object in air has negative object distance; a real image and the centre of curvature in glass have positive distances.

Step 1: Assign signed distances using the equal-distance condition Let the positive geometrical distance $PO = PI$ be d. The real object lies before the refracting pole P and the real image lies after it, so

$$u = -d, \qquad v = +d, \qquad R > 0.$$

The refractive indices are $n_1 = 1$ for air and $n_2 = 1.5$ for glass. Equal geometrical distances do not mean equal signed values of u and v.

Step 2: Substitute in the refraction equation Applying the formula with these signs gives

$$\frac{1.5}{d} - \frac{1}{-d} = \frac{1.5 - 1}{R}.$$

The two contributions on the left add:

$$\frac{2.5}{d} = \frac{0.5}{R} \quad \Rightarrow \quad d = 5R.$$

The result is positive and gives a real image on the glass side, consistent with the stated geometry. The ratio d/R is dimensionless, as the formula requires.

Step 3: Check the candidate distances The equation requires $2.5/d = 0.5/R$. Inserting $d = 5R$ satisfies it exactly. By contrast, $d = 2R$, $1.5R$, and $3R$ give left sides $1.25/R$, $5/(3R)$, and $5/(6R)$, respectively; none equals $0.5/R$. The common sign error of subtracting the two index contributions would miss the sum $n_1 + n_2$ arising from equal distances on opposite sides.

Step 4: Conclusion The distance is $\boxed{PO = 5R}$. Hence, the correct option is (A).

Quick Tip to Solve: For equal real-object and real-image distances across a spherical interface, use opposite signs: $d = (n_1 + n_2)R/(n_2 - n_1)$.

37. Consider a circular disc of radius 20 cm with centre located at the origin. A circular hole of radius 5 cm is cut from this disc in such a way that the edge of the hole touches the edge of the disc. The distance of centre of mass of residual or remaining disc from the origin will be [4 marks]

(A) 1.5 cm

(B) 2.0 cm

(C) 0.5 cm

(D) 1.0 cm

Correct Answer: (D) 1.0 cm

Solution:

Concept:

- For a uniform disc of fixed thickness, mass is proportional to its area. Thus the removed mass fraction is the square of the ratio of radii.
- Treat the circular hole as a negative mass at its own centre. The first moment of the complete disc about its centre is zero; subtracting the hole shifts the remaining centre of mass away from the hole.

Step 1: Locate the centre of the hole Take the positive x-axis from the original centre towards the hole. Internal tangency means that the separation of the two centres is the difference of their radii, not their sum. Therefore

$$d = R - r = 20 - 5 = 15\,\text{cm}.$$

Both centres lie on this axis, so the remaining centre of mass also lies on it, on the opposite side of the origin.

Step 2: Write the mass ratio and first moment Let the original disc have mass M and the removed part have mass m. Uniform areal density gives

$$\frac{m}{M} = \frac{\pi(5)^2}{\pi(20)^2} = \frac{1}{16}.$$

The remaining mass is $M - m = 15M/16$. Subtract the first moment of the missing material from that of the full disc:

$$x_{\text{CM}} = \frac{M(0) - m(15)}{M - m} = \frac{-(M/16)(15)}{15M/16} = -1\,\text{cm}.$$

The negative sign specifies direction; the question asks for distance, which is positive.

Step 3: Compare the listed distances The required magnitude is $1.0\,\text{cm}$. The values 1.5, 2.0 and $0.5\,\text{cm}$ do not satisfy the first-moment equation for the mass fraction $1/16$. In particular, dividing by the original mass rather than the remaining mass would be incorrect.

Step 4: Conclusion The centre of mass is $1.0\,\text{cm}$ from the origin, away from the hole. Hence, the correct option is (D).

Quick Tip to Solve: For an internally tangent hole, use $d = R - r$ and $|x_{\text{CM}}| = r^2 d/(R^2 - r^2)$.

38. Refer to the circuit diagram given in the figure. which of the following observations are correct?

A. Total resistance of circuit is 6Ω

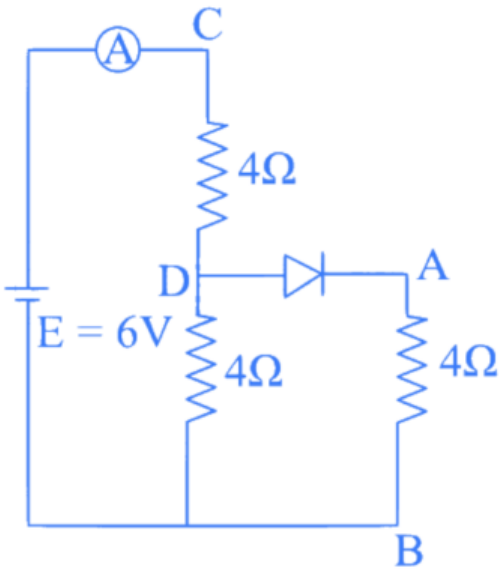
B. Current in Ammeter is 1 A

C. Potential across AB is 4 Volts.

D. Potential across CD is 4 Volts

E. Total resistance of the circuit is 8Ω.

Choose the correct answer from the options given below: [4 marks]



(A) B, C and E Only

(B) A, C and D Only

(C) A, B and C Only

(D) A, B and D Only

Correct Answer: (D) A, B and D Only

Solution:

Concept:

- An ideal forward-biased diode behaves as a wire, while a reverse-biased diode behaves as an open circuit. Determine its bias from the battery polarity before combining resistors.
- Resistors in parallel share the same potential difference. The series current is found using $I = E/R_{\text{eq}}$, and the drop across any resistor is IR.

Step 1: Determine the conducting branch The longer battery plate makes the top node C positive relative to B. The diode is directed from D towards A, so it is forward biased. Taking it as ideal joins D and A electrically. Thus the two lower $4\,\Omega$ resistors, between D and B, and between A and B, are parallel. They are not in series with each other.

Step 2: Calculate resistance and ammeter current The parallel equivalent and total resistance are

$$R_p = \frac{4 \times 4}{4 + 4} = 2\,\Omega, \qquad R_{\text{eq}} = 4 + 2 = 6\,\Omega.$$

The ammeter is in the common series branch, so it measures the total battery current:

$$I = \frac{6\,\text{V}}{6\,\Omega} = 1\,\text{A}.$$

Consequently statements A and B are true, whereas statement E, claiming $8\,\Omega$, is false. The latter value would wrongly neglect the conducting parallel branch.

Step 3: Check both stated voltage drops The upper resistor between C and D carries $1\,\text{A}$, giving

$$V_{CD} = 1 \times 4 = 4\,\text{V}.$$

The remaining drop across the parallel pair is

$$V_{AB} = 6 - 4 = 2\,\text{V}.$$

Hence D is true and C is false. Each lower resistor carries $2/4 = 0.5\,\text{A}$, consistent with the total current. Options containing C or E therefore fail; only the combination A, B and D survives.

Step 4: Conclusion Statements A, B and D only are correct. Hence, the correct option is (D).

Quick Tip to Solve: After checking diode polarity, merge the nodes connected by a forward ideal diode; this reveals the parallel resistors immediately.

39. The electric field of an electromagnetic wave in free space is

$$\overrightarrow{\text{E}} = 57\cos\left[7.5\times10^{6}t - 5\times10^{-3}(3x+4y)\right](4\hat{i}-3\hat{j})\,\text{N/C}.$$

The associated magnetic field in Tesla is [4 marks]

(A) $\overrightarrow{\text{B}} = \frac{57}{3\times10^{8}}\cos\left[7.5\times10^{6}\text{t} - 5\times10^{-3}(3x+4y)\right](\hat{k})$

(B) $\overrightarrow{\text{B}} = \frac{57}{3\times10^{8}}\cos\left[7.5\times10^{6}\text{t} - 5\times10^{-3}(3x+4y)\right](5\hat{k})$

(C) $\overrightarrow{\text{B}} = -\frac{57}{3\times10^{8}}\cos\left[7.5\times10^{6}\text{t} - 5\times10^{-3}(3x+4y)\right](\hat{k})$

(D) $\overrightarrow{\text{B}} = -\frac{57}{3\times10^{8}}\cos\left[7.5\times10^{6}\text{t} - 5\times10^{-3}(3x+4y)\right](5\hat{k})$

Correct Answer: (D) $\overrightarrow{\text{B}} = -\frac{57}{3\times10^{8}}\cos\left[7.5\times10^{6}\text{t} - 5\times10^{-3}(3x+4y)\right](5\hat{k})$

Solution:

Concept:

- For a plane electromagnetic wave in free space, $\mathbf{B} = (\hat{\mathbf{n}}\times\mathbf{E})/c$, where $\hat{\mathbf{n}}$ is the propagation unit vector and $c = 3\times10^{8}\,\text{m}\,\text{s}^{-1}$.
- The phase $\omega t - \mathbf{k}\cdot\mathbf{r}$ gives propagation along $+\mathbf{k}$. Electric and magnetic fields have the same phase and mutually perpendicular directions.

Step 1: Identify the propagation direction Write the common phase as

$$\Phi = 7.5\times10^{6}t - 5\times10^{-3}(3x+4y).$$

The spatial coefficients are proportional to $(3, 4, 0)$, so

$$\hat{\mathbf{n}} = \frac{3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}}{5}.$$

The electric polarization vector is $4\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$. Its dot product with $\hat{\mathbf{n}}$ is $(12 - 12)/5 = 0$, confirming that the field is transverse.

Step 2: Evaluate the cross product with its sign Keeping the full polarization vector, rather than treating it as a unit vector, gives

$$\begin{aligned}\hat{\mathbf{n}} \times (4\hat{\mathbf{i}} - 3\hat{\mathbf{j}}) &= \frac{(3)(-3) - (4)(4)}{5}\hat{\mathbf{k}} \\ &= -5\hat{\mathbf{k}}.\end{aligned}$$

Therefore

$$\mathbf{B} = -\frac{57}{3 \times 10^8} \cos\Phi\,(5\hat{\mathbf{k}})\,\mathrm{T}.$$

Its amplitude is $285/(3 \times 10^8) = 9.5 \times 10^{-7}\,\mathrm{T}$. The negative direction also makes $\mathbf{E} \times \mathbf{B}$ point along $(3, 4, 0)$, as required for energy transport.

Step 3: Rule out the other expressions Options A and B have the wrong sign of the magnetic field and would reverse the propagation direction. Option C has the correct sign but misses the factor 5, since $|4\hat{\mathbf{i}} - 3\hat{\mathbf{j}}| = 5$. Only D has both the correct direction and the amplitude required by $E_0 = cB_0$.

Step 4: Conclusion With $\Phi = 7.5 \times 10^6 t - 5 \times 10^{-3}(3x + 4y)$, the magnetic field is $\mathbf{B} = -\frac{57}{3 \times 10^8} \cos\Phi\,(5\hat{\mathbf{k}})\,\mathrm{T}$. Hence, the correct option is (D).

Quick Tip to Solve: Normalize the propagation vector, but retain the magnitude of the electric polarization vector when applying $\hat{\mathbf{n}} \times \mathbf{E}/c$.

40. The electric flux is $\phi = \alpha\sigma + \beta\lambda$ where λ and σ are linear and surface charge density, respectively. $\left(\frac{\alpha}{\beta}\right)$ represents [4 marks]

(A) displacement

(B) charge

(C) electric field

(D) area

Correct Answer: (A) displacement

Solution:

Concept:

- Only quantities with identical dimensions can be added in a physical equation. Each term of $\phi = \alpha\sigma + \beta\lambda$ must therefore have the dimensions of electric flux.
- Linear charge density is charge per length, $[\lambda] = [Q]L^{-1}$, whereas surface charge density is charge per area, $[\sigma] = [Q]L^{-2}$. The ratio removes the common flux dimension.

Step 1: Apply homogeneity to each term The coefficients compensate for the different dimensions of the charge densities. Equating dimensions separately gives

$$[\alpha][\sigma] = [\phi], \qquad [\beta][\lambda] = [\phi].$$

Thus

$$[\alpha] = \frac{[\phi]}{[\sigma]}, \qquad [\beta] = \frac{[\phi]}{[\lambda]}.$$

It is not necessary to introduce a particular surface or calculate the numerical electric flux; the equation itself supplies the dimensional constraints.

Step 2: Divide the coefficient dimensions Taking the ratio in the order asked, the dimensions of electric flux cancel:

$$\left[\frac{\alpha}{\beta}\right] = \frac{[\phi]/[\sigma]}{[\phi]/[\lambda]} = \frac{[\lambda]}{[\sigma]}.$$

Substitute the charge-density dimensions carefully:

$$\frac{[Q]L^{-1}}{[Q]L^{-2}} = L.$$

In SI units the same calculation is $(\mathrm{C\,m^{-1}})/(\mathrm{C\,m^{-2}}) = \mathrm{m}$. Reversing this density ratio would incorrectly produce an inverse length.

Step 3: Identify the matching physical quantity Displacement has dimensions L, matching the result. Charge has dimension $[Q]$, area has dimension L^2, and electric field has dimension force per charge, $MLT^{-2}[Q]^{-1}$. None of these three alternatives is a length. Dimensional analysis identifies the type of quantity represented here, rather than a numerical displacement or a direction.

Step 4: Conclusion The ratio α/β has dimensions of length and therefore represents displacement among the listed choices. Hence, the correct option is (A).

Quick Tip to Solve: When two coefficients multiply different quantities in a sum, divide their common-result equations; the coefficient ratio has the inverse ratio of the multiplier dimensions.

41. Consider a moving coil galvanomenter (MCG):

A. The torsional constant in moving coil galvanometer has dimensions $\left[\mathrm{ML}^2\ \mathrm{T}^{-2}\right]$

B. Increasing the current sensitivity may not necessarily increase the voltage sensitivity.

C. If we increase number of turns (N) to its double $(2\ \mathrm{N})$, then the voltage sensitivity doubles.

D. MCG can be converted into an ammeter by introducing a shunt resistance of large value in parallel with galvanometer.

E. Current sensitivity of MCG depends inversely on number of turns of coil.

Choose the correct answer from the options given below: [4 marks]

(A) A, D Only

(B) A, B Only

(C) A, B, E Only

(D) B, D, E Only

Correct Answer: (B) A, B Only

Solution:

Concept:

- At equilibrium, magnetic torque equals restoring torque: $NIAB = \kappa\theta$. Thus current sensitivity is $S_I = \theta/I = NAB/\kappa$, with κ the torsional constant.
- Voltage sensitivity is $S_V = \theta/V = S_I/G = NAB/(\kappa G)$, where G is the coil resistance. A shunt for an ammeter must divert most of the current away from the delicate coil.

Step 1: Assess the torsional constant and sensitivities Since $\kappa\theta$ is a torque and angle is dimensionless,

$$[\kappa] = [\text{torque}] = ML^2T^{-2}.$$

Statement A is true. Also $S_V = S_I/G$: an increase in current sensitivity may be offset by an increase in coil resistance, so it need not increase voltage sensitivity. This makes B true. Keeping the resistance in the formula is essential when comparing different coil designs.

Step 2: Check the claim about doubling turns Current sensitivity is directly proportional to the number of turns if A, B and κ are fixed. However, adding turns also increases the length of wire and hence generally increases G. With the same wire and geometry, doubling turns approximately doubles both N and G, leaving N/G, and therefore voltage sensitivity, unchanged. Consequently C does not follow and is false as a general claim. Statement E is also false because $S_I \propto N$, rather than $1/N$.

Step 3: Check the shunt requirement and choose To form an ammeter, use a small resistance S in parallel with the galvanometer so that most of the total current I bypasses it. Equality of branch voltages gives

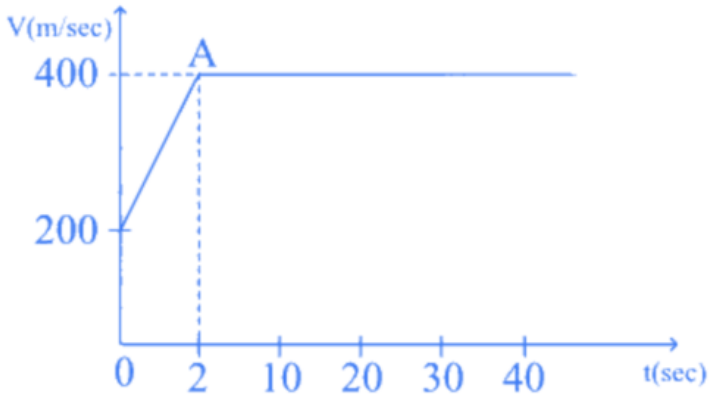
$$I_gG = (I - I_g)S, \qquad S = \frac{I_gG}{I - I_g}.$$

For $I \gg I_g$, this shunt is much smaller than G, not large. Hence D is false. Options A and D contain D, while option C contains E; each is eliminated. The remaining combination is A and B only.

Step 4: Conclusion Statements A and B only are correct. Hence, the correct option is (B).

Quick Tip to Solve: Keep coil resistance G in the voltage-sensitivity formula; increasing turns changes both torque and wire resistance.

42. The motion of an airplane is represented by velocity-time graph as shown below. The distance covered by airplane in the first 30.5 second is _____ km . [4 marks]



(A) 12

(B) 6

(C) 3

(D) 9

Correct Answer: (A) 12

Solution:

Concept:

- Displacement is the signed area under a velocity-time graph. Since the plotted velocity remains positive over the entire interval, this area also equals distance travelled.
- A straight sloping segment contributes a trapezium area, equal to average endpoint velocity times duration. A horizontal segment contributes a rectangle area.

Step 1: Read the two motion intervals At $t = 0$, the airplane already has velocity $200\,\mathrm{m\,s^{-1}}$. Its velocity rises linearly to $400\,\mathrm{m\,s^{-1}}$ at $t = 2\,\mathrm{s}$, then stays at $400\,\mathrm{m\,s^{-1}}$. Thus split the requested interval at $2\,\mathrm{s}$, not at the later tick marks. The first segment starts above the time axis, so its area is a trapezium, not a triangle from zero speed.

Step 2: Calculate each area in metres For the first two seconds the average velocity is $(200+400)/2 = 300\,\mathrm{m\,s^{-1}}$. Therefore

$$s_1 = \frac{200+400}{2} \times 2 = 600\,\mathrm{m}.$$

The remaining duration is

$$\Delta t = 30.5 - 2 = 28.5\,\mathrm{s}.$$

At the constant speed of $400\,\mathrm{m\,s^{-1}}$, its distance is

$$s_2 = 400 \times 28.5 = 11400\,\mathrm{m}.$$

Both areas use seconds and metres per second, so their units are consistently metres before conversion.

Step 3: Add and compare the alternatives The total distance is

$$s = s_1 + s_2 = 600 + 11400 = 12000\,\mathrm{m} = 12\,\mathrm{km}.$$

As a check, moving at $400\,\mathrm{m\,s^{-1}}$ for all $30.5\,\mathrm{s}$ would give $12200\,\mathrm{m}$; the initial speed deficit removes a triangle of area $\frac{1}{2}(2)(200) = 200\,\mathrm{m}$, again giving $12000\,\mathrm{m}$. The other proposed distances, 6, 3 and $9\,\mathrm{km}$, are inconsistent with the plotted speed and duration.

Step 4: Conclusion The airplane covers $12\,\mathrm{km}$ in the first $30.5\,\mathrm{s}$. Hence, the correct option is (A).

Quick Tip to Solve: For a graph starting above zero speed, include the initial-speed rectangle as well as the acceleration triangle.

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43. Regarding self-inductance:

A. The self-inductance of the coil depends on its geometry.

B. Self-inductance does not depend on the permeability of the medium.

C. Self-induced e.m.f. opposes any change in the current in a circuit.

D. Self-inductance is electromagnetic analogue of mass in mechanics.

E. Work needs to be done against self-induced e.m.f. in establishing the current.

Choose the correct answer from the options given below: [4 marks]

(A) A, B, C, E only

(B) A, B, C, D only

(C) A, C, D, E only

(D) B, C, D, E only

Correct Answer: (C) A, C, D, E only

Solution:

Concept:

- Self-inductance relates flux linkage to current: $N\Phi = LI$. For a long solenoid, $L = \mu N^2 A/\ell$, where μ is the permeability, A is the cross-sectional area and ℓ is the length.
- The induced emf is $\mathcal{E} = -L\,dI/dt$. Its negative sign expresses Lenz's law, and the stored magnetic energy is $U = \frac{1}{2}LI^2$.

Step 1: Check the factors controlling inductance For a fixed coil, its number of turns, area, length and shape determine the flux linkage produced by a given current. Thus statement A is true. The expression $L = \mu N^2 A/\ell$ also shows explicit dependence on permeability: changing an air core to a magnetic core changes the inductance. Hence statement B is false, even when the geometrical dimensions remain fixed.

Step 2: Interpret the induced emf and energy When current increases, the induced emf opposes that increase; when current decreases, it acts to maintain the current. Therefore C is true: it opposes the change, rather than necessarily opposing the instantaneous current. In mechanics, mass resists a change of velocity through $F = m\,dv/dt$; inductance resists a change of current through the corresponding voltage term $L\,dI/dt$. This establishes D. The source must supply power $P = LI\,dI/dt$ against the back emf while building current. Integrating gives $W = \int_0^I LI\,dI = \frac{1}{2}LI^2$, so E is true.

Step 3: Match the complete set The true statements are A, C, D and E. Every other listed option includes B, which incorrectly denies the permeability dependence. Consequently none of those

combinations can represent the correct set.

Step 4: Conclusion A, C, D and E are correct; B is incorrect. Hence, the correct option is (C).

Quick Tip to Solve: Use the solenoid formula to test geometry and permeability, then use Lenz's law and $\frac{1}{2}LI^2$ for the remaining statements.

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44. Identify the valid statements relevant to the given circuit at the instant when the key is closed.

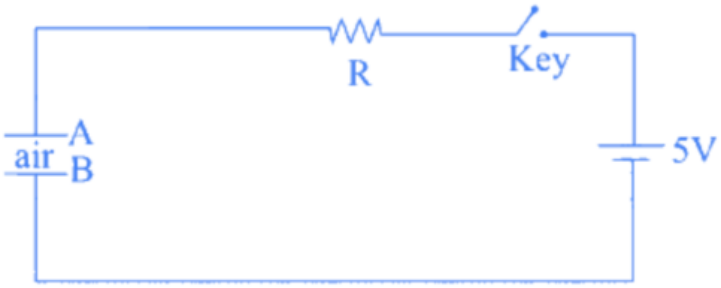
A. There will be no current through resistor R.

B. There will be maximum current in the connecting wires.

C. Potential difference between the capacitor plates A and B is minimum.

D. Charge on the capacitor plates is minimum.

Choose the correct answer from the options given below: [4 marks]



(A) A, C Only

(B) C, D Only

(C) A, B, D Only

(D) B, C, D Only

Correct Answer: (D) B, C, D Only

Solution:

Concept:

- An initially uncharged capacitor cannot acquire a finite charge instantaneously through a finite resistor. Hence $q(0^+) = 0$ and $V_C(0^+) = q/C = 0$.
- For a series charging circuit, Kirchhoff's law gives $E = IR + V_C$. The current decreases as the capacitor voltage increases.

Step 1: Apply the initial capacitor condition Before the key is closed, the capacitor has no stored charge in the circuit shown. At the instant just after closing, its charge remains zero. Thus its potential difference is also zero, which is the minimum during the charging process. Statements C and D are therefore correct. This initial condition concerns the capacitor voltage; it does not mean that the resistor has zero voltage across it.

Step 2: Calculate the initial current At $t = 0^+$, insert $V_C = 0$ and the battery emf $E = 5\,\text{V}$ into the

loop equation:

$$5 = I(0^+)R + 0, \qquad I(0^+) = \frac{5}{R}.$$

The resistor and connecting wires are in the same series loop, so they carry this same nonzero current. Statement A is false. As charge accumulates, V_C increases and $I = (5 - V_C)/R$ decreases; therefore the initial current is maximum, making B true.

Step 3: Check the option combinations The charging expressions $q = 5C(1 - e^{-t/RC})$ and $I = (5/R)e^{-t/RC}$ confirm the limiting behaviour directly. At the start, charge and capacitor voltage are least while current is greatest. The combinations containing A are ruled out because A is false. The option listing C and D alone omits the true statement B, so it is incomplete.

Step 4: Conclusion The valid statements are B, C and D only. Hence, the correct option is (D).

Quick Tip to Solve: At the start of RC charging, use zero capacitor voltage and maximum current; the zero-current condition belongs to the final steady state.

45. Given a thin convex lens (refractive index μ_2), kept in a liquid (refractive index $\mu_1, \mu_1 < \mu_2$) having radii of curvatures $|R_1|$ and $|R_2|$. Its second surface is silver polished. Where should an object be placed on the optic axis so that a real and inverted image is formed at the same place? [4 marks]

(A) $\frac{(\mu_2+\mu_1)|R_1|}{(\mu_2-\mu_1)}$

(B) $\frac{\mu_1|\mathrm{R_1}|\cdot|\mathrm{R_2}|}{\mu_2(2|\mathrm{R_1}|+|\mathrm{R_2}|)-\mu_1\sqrt{|\mathrm{R_1}|\cdot|\mathrm{R_2}|}}$

(C) $\frac{\mu_1|R_1|\cdot|R_2|}{\mu_2(|R_1|+|R_2|)-\mu_1|R_1|}$

(D) $\frac{\mu_1|\mathrm{R_1}|\cdot|\mathrm{R_2}|}{\mu_2(|\mathrm{R_1}|+|\mathrm{R_2}|)-\mu_1|\mathrm{R_2}|}$

Correct Answer: (D) $\frac{\mu_1|\mathrm{R_1}|\cdot|\mathrm{R_2}|}{\mu_2(|\mathrm{R_1}|+|\mathrm{R_2}|)-\mu_1|\mathrm{R_2}|}$

Solution:

Concept:

- For refraction at the first spherical surface, $\mu_2/v - \mu_1/u = (\mu_2 - \mu_1)/R_1$, using Cartesian distances positive towards the incident light's forward direction.
- A concave reflecting surface returns a real object at its centre of curvature to the same point, with transverse inversion. Optical reversibility then makes the return refraction retrace the conjugate imaging relation.

Step 1: Set up the self-conjugate condition Write $r_1 = |R_1|$ and $r_2 = |R_2|$. For light entering from the object side, the first convex surface has $R_1 = +r_1$. The silvered second surface acts as a concave mirror viewed from inside the lens. Its centre of curvature lies a distance r_2 to the left. Since the lens is thin, the surface separation is neglected. To obtain a real inverted image at the original object position after the round trip, the first refraction must make the object conjugate to this mirror centre.

Step 2: Apply refraction at the first surface Let the required positive object distance be s; then $u = -s$. The first surface must form the corresponding virtual image at $v = -r_2$, on the object side.

Substitution gives

$$-\frac{\mu_2}{r_2}+\frac{\mu_1}{s}=\frac{\mu_2-\mu_1}{r_1}.$$

Therefore

$$\frac{\mu_1}{s}=\frac{\mu_2-\mu_1}{r_1}+\frac{\mu_2}{r_2}.$$

This step uses the actual refractive indices and thus includes the liquid correctly.

Step 3: Solve and compare the expressions Taking a common denominator yields

$$s=\frac{\mu_1 r_1 r_2}{\mu_2(r_1+r_2)-\mu_1 r_2}.$$

The denominator is positive because $\mu_2>\mu_1$. Option A lacks the required dependence on the silvered surface radius. B introduces a square-root dependence absent from the surface equation. C subtracts $\mu_1 r_1$, whereas the common-denominator calculation requires $\mu_1 r_2$.

Step 4: Conclusion The object must be placed in the liquid, in front of the lens, at distance $\frac{\mu_1|R_1||R_2|}{\mu_2(|R_1|+|R_2|)-\mu_1|R_2|}$. Hence, the correct option is (D).

Quick Tip to Solve: For an image returning to the object, make the first refracting surface conjugate the object to the silvered surface's centre of curvature.

Physics – Section B: Numerical Value Questions (20 marks)

46. Two particles are located at equal distance from origin. The position vectors of those are represented by $\vec{A}=2\hat{i}+3n\hat{j}+2\hat{k}$ and $\vec{B}=2\hat{i}-2\hat{j}+4p\hat{k}$, respectively. If both the vectors are at right angle to each other, the value of n^{-1} is _____ . [4 marks]

Solution:

Concept:

- Equal distances from the origin mean equal vector magnitudes, so compare $|\vec{A}|^2$ and $|\vec{B}|^2$. Squaring avoids unnecessary square roots.
- Two nonzero perpendicular vectors satisfy $\vec{A}\cdot\vec{B}=0$. Both the magnitude and dot-product conditions must hold simultaneously.

Step 1: Use the equality of distances For $\vec{A}=2\hat{i}+3n\hat{j}+2\hat{k}$, the squared distance is

$$|\vec{A}|^2=2^2+(3n)^2+2^2=8+9n^2.$$

For $\vec{B}=2\hat{i}-2\hat{j}+4p\hat{k}$, it is

$$|\vec{B}|^2=2^2+(-2)^2+(4p)^2=8+16p^2.$$

Hence $9n^2=16p^2$, or $p=\pm 3n/4$. It is necessary to retain both signs at this stage, since magnitudes do not determine component signs.

Step 2: Enforce perpendicularity and select the sign The scalar product is

$$\vec{A} \cdot \vec{B} = (2)(2) + (3n)(-2) + (2)(4p) = 4 - 6n + 8p.$$

Setting this equal to zero gives $4 - 6n + 8p = 0$. If $p = 3n/4$, this reduces to $4 = 0$, which is impossible. Therefore $p = -3n/4$. Substituting the negative branch gives

$$4 - 6n - 6n = 0, \qquad n = \frac{1}{3}.$$

It also follows that $p = -1/4$.

Step 3: Verify the vectors and invert the requested value The resulting vectors are $\vec{A} = (2, 1, 2)$ and $\vec{B} = (2, -2, -1)$. Their squared magnitudes are both 9, while their dot product is $4 - 2 - 2 = 0$. Thus they satisfy both stated requirements. The question asks for the reciprocal of n, rather than for n itself; taking that reciprocal gives $n^{-1} = 3$.

Step 4: Conclusion The required dimensionless numerical value is $\boxed{n^{-1} = 3}$.

Quick Tip to Solve: Keep both signs when taking a square root, and use the dot product to eliminate the branch that makes perpendicularity impossible.

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47. A force $\mathrm{f} = \mathrm{x}^2\mathrm{y}\hat{\mathrm{i}} + \mathrm{y}^2\hat{\mathrm{j}}$ acts on a particle in a plane $\mathrm{x} + \mathrm{y} = 10$. The work done by this force during a displacement from $(0, 0)$ to $(4\ \mathrm{m}, 2\ \mathrm{m})$ is _____ Joule (round off to the nearest integer) [4 marks]

Solution:

Concept:

- The work of $\vec{F} = x^2y\hat{i} + y^2\hat{j}$ along an admissible path is $W = \int (x^2y\,dx + y^2\,dy)$. A nonconservative force requires a consistent path and endpoints.
- Every endpoint of motion constrained to $x + y = 10$ must satisfy that equation. Checking this compatibility precedes evaluating a line integral.

Step 1: Check the stated displacement against the constraint The starting point $(0, 0)$ has $x + y = 0$, and the endpoint $(4, 2)$ has $x + y = 6$. Neither lies on the stated line $x + y = 10$. Consequently no continuous displacement confined to that line can have the two specified endpoints. Also, $\partial F_x/\partial y = x^2$ whereas $\partial F_y/\partial x = 0$, so this force is not conservative. Its work cannot be found from endpoints alone by introducing an arbitrary alternative path.

Step 2: Calculate the formal value associated with the key The listed numerical key corresponds to substituting $y = 10 - x$ into the first term and using the separately supplied limits for the second term:

$$W_{\text{formal}} = \int_0^4 x^2(10 - x)\,dx + \int_0^2 y^2\,dy.$$

The two elementary evaluations give

$$\int_0^4 (10x^2 - x^3)\,dx = \frac{640}{3} - 64 = \frac{448}{3},$$

and

$$\int_0^2 y^2\,dy = \frac{8}{3}.$$

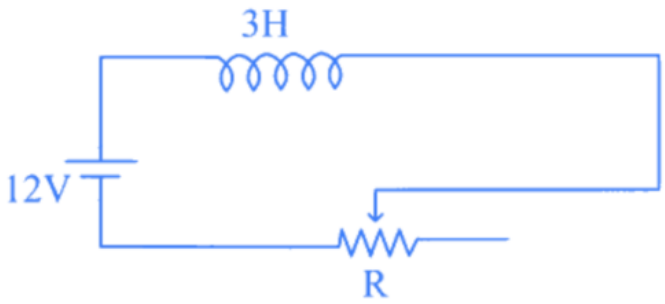
Their sum is $456/3 = 152$.

Step 3: Distinguish the formal number from a valid line integral The first integral assumes points on $y = 10 - x$; as x runs from 0 to 4, that line has y running from 10 to 6, not from 0 to 2. Therefore the two integrals just combined do not describe the same stated path. The formal calculation explains the key, but it cannot resolve the contradiction in the printed endpoints. No unique physical work is determined by all the conditions as written.

Step 4: Conclusion The numerical key is $\boxed{152\ \text{J}}$, obtained by the formal calculation above. Literally, however, the stated constraint and endpoints are inconsistent, so the printed problem does not define a physically admissible path or a unique work value.

Quick Tip to Solve: Before integrating a variable force, substitute both endpoints into the path equation; incompatible endpoints invalidate the proposed line integral.

48. In the given circuit the sliding contact is pulled outwards such that electric current in the circuit changes at the rate of $8\ \text{A/s}$. At an instant when R is 12Ω, the value of the current in the circuit will be _____ A. [4 marks]



Solution:

Concept:

- A self-inductor develops emf $\mathcal{E}_L = -L\,dI/dt$, opposing the change in current. During a decrease it supports the existing current direction.
- Kirchhoff's loop equation for a series battery, resistor and inductor is $E = IR + L\,dI/dt$, with current positive in the battery-driven direction.

Step 1: Determine the sign of the current change Pulling the rheostat contact outwards increases the effective series resistance in the diagram. The current consequently decreases. The quoted rate $8\,\text{A}\,\text{s}^{-1}$ is a magnitude, so in the chosen positive current direction

$$\frac{dI}{dt} = -8\,\text{A}\,\text{s}^{-1}.$$

The negative sign is essential: the induced emf now acts in the same direction as the battery, tending

to maintain the current rather than suppress it further.

Step 2: Find the induced emf and write the loop equation The inductance is $L = 3\,\mathrm{H}$, hence the magnitude of the induced emf is

$$|\mathcal{E}_L| = L\left|\frac{dI}{dt}\right| = 3 \times 8 = 24\,\mathrm{V}.$$

At this instant the battery contributes $12\,\mathrm{V}$, and the inductor contributes another $24\,\mathrm{V}$ in the direction of current. Equivalently, using the signed loop equation,

$$12 = 12I + 3(-8) = 12I - 24.$$

Step 3: Solve for the instantaneous current and check energy flow The resistor voltage drop is therefore $IR = 12 + 24 = 36\,\mathrm{V}$. With $R = 12\,\Omega$,

$$I = \frac{36}{12} = 3\,\mathrm{A}.$$

This exceeds the static value $E/R = 1\,\mathrm{A}$ because the magnetic field is releasing stored energy while the current decreases. Indeed, at this instant the battery supplies $EI = 36\,\mathrm{W}$ and the field supplies $-LI\,dI/dt = 72\,\mathrm{W}$, adding to the resistor loss $I^2R = 108\,\mathrm{W}$.

Step 4: Conclusion The current at the specified instant is $\boxed{3\,\mathrm{A}}$, in agreement with the numerical key.

Quick Tip to Solve: Assign a negative sign to the rate when rheostat resistance increases; the decaying inductor then adds its emf to the battery.

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49. A positive ion A and a negative ion B has charges $6.67 \times 10^{-19}\mathrm{C}$ and $9.6 \times 10^{-10}\mathrm{C}$, and masses 19.2×10^{-27} kg and 9×10^{-27} kg respectively. At an instant, the ions are separated by a certain distance r. At that instant the ratio of the magnitudes of electrostatic force to gravitational force is $\mathrm{P} \times 10^{-13}$, where the value of P is _____.

(Take $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \mathrm{Nm^2C^{-1}}$ and universal gravitational constant as $6.67 \times 10^{-11}\mathrm{Nm^2\,kg^{-2}}$) [4 marks]

Solution:

Concept:

- The magnitudes of the forces between two point particles are $F_e = k|q_Aq_B|/r^2$ and $F_g = Gm_Am_B/r^2$. Here k is the Coulomb constant and G is the gravitational constant.
- The same separation occurs in both inverse-square laws, so the force ratio is independent of r. Opposite charges make the electric force attractive; this does not change its magnitude.

Step 1: Cancel the common distance factor Use the magnitude of each charge, since the question asks for a ratio of magnitudes. The negative sign of the charge on B only specifies attraction. Thus

$$\frac{F_e}{F_g} = \frac{k|q_Aq_B|/r^2}{Gm_Am_B/r^2} = \frac{k|q_Aq_B|}{Gm_Am_B}.$$

No numerical separation is needed, and both forces point towards the other ion.

Step 2: Substitute the printed powers of ten Keeping the exponents exactly as given,

$$\frac{F_e}{F_g} = \frac{(9 \times 10^{9})(6.67 \times 10^{-19})(9.6 \times 10^{-10})}{(6.67 \times 10^{-11})(19.2 \times 10^{-27})(9 \times 10^{-27})}.$$

Cancel 9 and 6.67. The remaining coefficient is $9.6/19.2 = 1/2$, and the net exponent is

$$9 - 19 - 10 - (-11 - 27 - 27) = 45.$$

Therefore $F_e/F_g = 0.5 \times 10^{45} = 5 \times 10^{44}$. This is dimensionless because it is a ratio of two forces.

Step 3: Compare with the requested expression The question writes the ratio as $P \times 10^{-13}$, not as $P \times 10^{13}$. Equating this to the value just obtained gives

$$P \times 10^{-13} = 5 \times 10^{44} \quad \Rightarrow \quad P = 5 \times 10^{57}.$$

The very large value follows from the printed charge and mass exponents; changing those exponents would solve a different question.

Step 4: Conclusion The literal printed data give $\boxed{P = 5 \times 10^{57}}$, a dimensionless number. The official final answer key dropped this question and awarded bonus credit to all candidates.

Quick Tip to Solve: Cancel the separation before substituting, and move the printed power of ten to the other side only after finding the force ratio.

50. An ideal gas initially at $0^{\circ}\mathrm{C}$ temperature, is compressed suddenly to one fourth of its volume. If the ratio of specific heat at constant pressure to that at constant volume is $3/2$, the change in temperature due to the thermodynamic process is _____ K. [4 marks]

Solution:

Concept:

- For the adiabatic ideal-gas relation used for rapid compression, $TV^{\gamma-1}$ is constant, where $\gamma = C_P/C_V$. Temperature in this relation must be in kelvin.
- Rapid compression leaves negligible time for heat exchange with the surroundings. Work done on the gas increases its internal energy and hence its temperature.

Step 1: Convert the initial temperature The initial temperature is $0^{\circ}\mathrm{C}$, so on the absolute temperature scale

$$T_1 = 0 + 273 = 273\,\mathrm{K}.$$

The final volume is $V_2 = V_1/4$. The supplied specific-heat ratio is $\gamma = 3/2$, giving $\gamma - 1 = 1/2$. A Celsius temperature of zero must not be substituted directly into the gas-law relation.

Step 2: Apply the adiabatic temperature-volume relation Writing the relation for the initial and final

states,

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}, \qquad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}.$$

Insert the volume ratio and exponent:

$$\frac{T_2}{273} = 4^{1/2} = 2, \qquad T_2 = 546\,\text{K}.$$

The increase is consistent with positive work being done on the gas. An isothermal relation would instead keep temperature fixed and is not the rapid-compression treatment intended here.

Step 3: Find the change rather than the final value The requested quantity is the change in temperature, so subtract the initial absolute temperature from the final one:

$$\Delta T = T_2 - T_1 = 546 - 273 = 273\,\text{K}.$$

A temperature interval has the same numerical size in kelvin and degrees Celsius, but the gas-law calculation itself requires absolute temperatures. Reporting $546\,\text{K}$ would report the final temperature, not its rise.

Step 4: Conclusion The temperature increases by $\boxed{273\,\text{K}}$, in agreement with the given final answer key.

Quick Tip to Solve: Use kelvin in the adiabatic relation, then distinguish the final temperature from the temperature increase.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Given below are two statements:

Statement I: In Lassaigne's test, the covalent organic molecules are transformed into ionic compounds.

Statement II: The sodium fusion extract of an organic compound having N and S gives prussian blue colour with $FeSO_4$ and $Na_4[Fe(CN)_6]$

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are false

(B) Statement I is false but Statement II is true

(C) Both Statement I and Statement II are true

(D) Statement I is true but Statement II is false

Correct Answer: (D) Statement I is true but Statement II is false

Solution:

Concept:

- Sodium fusion converts covalently bonded heteroatoms in an organic compound into water-soluble ionic sodium salts, enabling ordinary inorganic tests.
- When nitrogen and sulphur occur together, fusion can form sodium thiocyanate, $NaSCN$. Thiocyanate gives a blood-red complex with ferric ions, unlike the Prussian-blue nitrogen test.

Step 1: Test Statement I Organic compounds generally hold nitrogen, sulphur and halogens through covalent bonds. In this form they do not directly respond to the usual ionic tests. Fusion with sodium converts these elements into ionic products such as $NaCN$, Na_2S or NaX, where X denotes a halogen. The water-soluble extract can then be tested. Thus Statement I correctly describes the purpose of sodium fusion and is true.

Step 2: Test Statement II The characteristic product when both nitrogen and sulphur are present together is thiocyanate:

$$Na + C + N + S \longrightarrow NaSCN.$$

With ferric ions it produces the characteristic blood-red thiocyanate complex, often represented in the elementary test as $Fe(SCN)_3$. It is not identified by the Prussian-blue colour claimed in Statement II. The ordinary nitrogen test instead involves cyanide forming ferrocyanide with ferrous ions, followed by formation of ferric ferrocyanide. Sodium ferrocyanide by itself is not a replacement for the required ferric-ion stage.

Step 3: Choose the matching truth combination Statement I is true while Statement II is false. Therefore the combinations in (A) and (B) fail because they mark Statement I false. Option (C) fails because it accepts the incorrect Prussian-blue claim for the combined nitrogen-sulphur test. Only option (D) has the required truth values. Keep the distinction between cyanide and thiocyanate clear: they lead to different characteristic colour tests.

Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (D).

Quick Tip to Solve: For nitrogen and sulphur together, recall thiocyanate and blood red; reserve Prussian blue for the ordinary nitrogen test.

52. Match the List - I with List - II.

	List - I (Classification of molecules based on octet rule)		**List - II (Example)**
(A)	**Molecules obeying octet rule**	**(I)**	NO, NO_2
(B)	**Molecules with incomplete octet**	**(II)**	$BCl_3, AlCl_3$
(C)	**Molecules with incomplete octet with odd electron**	**(III)**	H_2SO_4, PCl_5
(D)	**Molecules with expanded octet**	**(IV)**	CCl_4, CO_2

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-I, C-III, D-II

(B) A-II, B-IV, C-III, D-I

(C) A-IV, B-II, C-I, D-III

(D) A-III, B-II, C-I, D-IV

Correct Answer: (C) A-IV, B-II, C-I, D-III

Solution:

Concept:

- The octet rule describes eight electrons in the valence shell of the central atom in a Lewis structure, counting bonding and nonbonding electrons.
- Important exceptions are electron-deficient molecules, odd-electron molecules and molecules conventionally represented with expanded valence shells.

Step 1: Identify the normal and deficient octets In CCl_4, four shared electron pairs surround carbon, giving it an octet. In CO_2, the two double bonds also give carbon eight electrons. Therefore the molecules obeying the octet rule in List I(A) correspond to List II(IV). In BCl_3 and monomeric $AlCl_3$, the central atom has three covalent bonds and only six electrons in its immediate valence-shell Lewis count. They are the standard incomplete-octet examples, so B corresponds to II.

Step 2: Identify odd-electron and expanded-octet examples NO contains $5+6=11$ valence electrons and NO_2 contains $5+2(6)=17$. These odd totals prevent every atom from having a complete paired-electron octet. Hence the odd-electron category C corresponds to I. The conventional Lewis descriptions of PCl_5 and H_2SO_4 place more than eight electrons around the central third-period atom. These are the expanded-octet examples used in this classification, so D corresponds to III.

Step 3: Check the complete mapping against the options Combining all four independent identifications gives

$$\text{A} - \text{IV}, \quad \text{B} - \text{II}, \quad \text{C} - \text{I}, \quad \text{D} - \text{III}.$$

Option (A) wrongly assigns the normal octet group to the odd-electron examples. Option (B) assigns it to electron-deficient trichlorides. Option (D) assigns it to expanded-octet molecules. Only option (C) preserves all four matches; a single correct pair is insufficient in a matching question.

Step 4: Conclusion The matching is A-IV, B-II, C-I, D-III. Hence, the correct option is (C).

Quick Tip to Solve: Count electrons around the central atom first; use the odd total electron counts of NO and NO2 to identify the odd-electron pair.

53. What amount of bromine will be required to convert 2 g of phenol into 2,4,6-tribromophenol? (Given molar mass in $gmol^{-1}$ of C, H, O, Br are $12, 1, 16, 80$ respectively) [4 marks]

(A) 6.0 g

(B) 10.22 g

(C) 20.44 g

(D) 4.0 g

Correct Answer: (B) 10.22 g

Solution:

Concept:

- The hydroxyl group strongly activates the benzene ring and directs electrophilic substitution to its two ortho positions and its para position.

- Complete conversion of phenol into 2,4,6-tribromophenol consumes three moles of Br_2 for every mole of phenol. Use the molecular mass of bromine, not its atomic mass.

Step 1: Write the substitution stoichiometry Three hydrogen atoms on the phenol ring are replaced by bromine atoms at positions 2, 4 and 6. Each replacement consumes one molecule of bromine and liberates one molecule of hydrogen bromide:

$$C_6H_5OH + 3\,Br_2 \longrightarrow C_6H_2Br_3OH + 3\,HBr.$$

Thus the mole ratio of phenol to bromine is $1 : 3$. The three bromine atoms entering the organic product do not mean that only three bromine atoms are supplied: the other atoms form HBr.

Step 2: Calculate the moles of phenol The formula of phenol is C_6H_6O. Using the supplied atomic masses,

$$M(C_6H_6O) = 6(12) + 6(1) + 16 = 94\,g\,mol^{-1}.$$

The given sample therefore contains

$$n(\text{phenol}) = \frac{2}{94}\,\text{mol}, \qquad n(Br_2) = 3\frac{2}{94} = \frac{6}{94}\,\text{mol}.$$

The required amount is based on complete tribromination, rather than stopping after one substitution.

Step 3: Convert bromine moles to mass Bromine is supplied as diatomic Br_2, whose molar mass is $2(80) = 160\,g\,mol^{-1}$. Consequently,

$$m(Br_2) = \frac{6}{94}(160) = \frac{960}{94} = 10.2127\ldots\,g.$$

This is $10.21\,g$ to two decimal places. The listed value $10.22\,g$ in (B) is the only matching choice, differing slightly from the exact calculation. Options (A) and (D) provide insufficient bromine; (C) is approximately twice the required mass and would result from an extra factor of two. The small rounding difference in (B) does not alter the unique stoichiometric choice.

Step 4: Conclusion The required bromine mass is $10.2127\ldots\,g$, corresponding to the listed $10.22\,g$ choice. Hence, the correct option is (B).

Quick Tip to Solve: For tribromination multiply phenol moles by three, then multiply by the molecular molar mass 160 of Br2.

54. The correct set of ions (aqueous solution) with same colour from the following is: [4 marks]

(A) V^{2+}, Cr^{3+}, Mn^{3+}

(B) Ti^{4+}, V^{4+}, Mn^{2+}

(C) $Sc^{3+}, Ti^{3+}, Cr^{2+}$

(D) Zn^{2+}, V^{3+}, Fe^{3+}

Correct Answer: (A) V^{2+}, Cr^{3+}, Mn^{3+}

Solution:

Concept:

- The colour of an aqueous transition-metal ion depends on its oxidation state and the electronic transitions permitted in its hydrated coordination environment.
- The usual aqueous-ion colour table distinguishes violet, green, blue and nearly colourless ions. Equal or similar d-electron counts alone do not guarantee the same colour.

Step 1: Inspect the ions in option A The standard colours used for these aqueous ions are violet for V^{2+}, violet for the hexaaqua Cr^{3+} ion, and violet for Mn^{3+}. Thus these three form the common-colour set requested. Chromium(III) salts can appear green when chloride or other ligands participate in the coordination sphere, but the aqueous-ion comparison here uses the violet hydrated chromium(III) ion. Oxidation state and coordination environment must both be kept in view when recalling this colour.

Step 2: Eliminate options B and C In option (B), Ti^{4+} is a d^0 ion and is colourless, whereas aqueous vanadium(IV), commonly present as vanadyl species, is blue and Mn^{2+} is pale pink. These do not share one colour. In option (C), Sc^{3+} is d^0 and colourless, Ti^{3+} is purple, and Cr^{2+} is blue. In particular, a colourless scandium ion cannot belong to an all-violet set.

Step 3: Eliminate option D and select the set In option (D), Zn^{2+} is d^{10} and colourless, V^{3+} is green, and aqueous Fe^{3+} is conventionally described as yellow. Hence that group also contains visibly different colours. The alternatives fail by direct comparison of their member colours, rather than by a rule that every transition-metal ion is coloured. Both d^0 and d^{10} ions provide useful colourless checks when eliminating the distractors.

Step 4: Conclusion The common violet set is V^{2+}, Cr^{3+}, Mn^{3+}. Hence, the correct option is (A).

Quick Tip to Solve: Use the colourless d0 or d10 ion to reject mixed sets quickly, then recall the violet hydrated-ion trio.

55.

$$FeO_4^{2-} \xrightarrow{+2.0\ V} Fe^{3+} \xrightarrow{0.8\ V} Fe^{2+} \xrightarrow{-0.5\ V} Fe^0$$

In the above diagram, the standard electrode potentials are given in volts (over the arrow).

The value of $E_{FeO_4^{2-}/Fe^{2+}}$ is : [4 marks]

(A) 1.2 V

(B) 2.1 V

(C) 1.4 V

(D) 1.7 V

Correct Answer: (D) 1.7 V

Solution:

Concept:

- Standard electrode potential is an intensive quantity, so potentials of successive reductions cannot simply be added. Instead, add the standard Gibbs energies, using $\Delta G^\circ = -nFE^\circ$, where n is the number of electrons and F is the Faraday constant.
- For consecutive reduction steps with electron counts n_1, n_2, the combined potential is $E^\circ = (n_1E_1^\circ + n_2E_2^\circ)/(n_1 + n_2)$. The weighting follows conservation of free energy, not the number of arrows.

Step 1: Find the electron changes In $FeO_4{}^{2-}$, oxygen contributes -8, so iron has oxidation state $+6$. Reduction to Fe^{3+} gains three electrons. The next reduction, from Fe^{3+} to Fe^{2+}, gains one electron. The desired overall reduction therefore involves four electrons. The final arrow to metallic iron is outside the requested couple and must not be included.

Step 2: Add Gibbs energies and obtain the potential Use $E_1^\circ = 2.0\,V$ and $E_2^\circ = 0.8\,V$. Their Gibbs energy changes are $-3F(2.0)$ and $-F(0.8)$. Thus

$$-4FE^\circ = -F[3(2.0) + 1(0.8)], \qquad E^\circ = \frac{6.8}{4} = 1.7\,V.$$

This lies between the two contributing potentials and is closer to $2.0\,V$, because that step involves more electrons. Options (A), (B) and (C) fail this electron-weighted calculation; the arithmetic mean $1.4\,V$ incorrectly gives both steps equal weight.

Step 3: Conclusion The standard reduction potential of the $FeO_4{}^{2-}/Fe^{2+}$ couple is $\boxed{1.7\,V}$. Hence, the correct option is (D).

Quick Tip to Solve: In a Latimer diagram, write the oxidation-state change above each arrow before averaging potentials; the electron counts are the weights.

56. $CrCl_3 \cdot xNH_3$ **can exist as a complex. 0.1 molal aqueous solution of this complex shows a depression in freezing point of** $0.558^\circ C$**. Assuming** 100% **ionisation of this complex and coordination number of Cr is 6 , the complex will be (Given** $K_f = 1.86\ K\ kg\ mol^{-1}$ **)** [4 marks]

(A) $[Cr(NH_3)_5 Cl]Cl_2$

(B) $[Cr(NH_3)_4 Cl_2]Cl$

(C) $[Cr(NH_3)_6]Cl_3$

(D) $[Cr(NH_3)_3 Cl_3]$

Correct Answer: (A) $[Cr(NH_3)_5 Cl]Cl_2$

Solution:

Concept:

- Freezing-point depression measures the total number of dissolved particles: $\Delta T_f = iK_f m$. Here i is the van't Hoff factor, K_f the solvent constant and m the analytical molality of the complex.
- On complete ionisation, chloride ions outside the coordination sphere separate in water, whereas coordinated chloride remains bonded to chromium. Both ammonia and chloride are monodentate ligands, so six ligand donor atoms are required.

Step 1: Calculate the particle count Substitute the supplied depression and molality without changing the concentration to molarity:

$$i = \frac{0.558}{1.86 \times 0.1} = \frac{0.558}{0.186} = 3.$$

Since ionisation is stated to be complete, each formula unit must produce exactly three ions. A coordination cation plus two free chloride ions supplies this particle count.

Step 2: Construct the six-coordinate complex Two of the three chloride ions must therefore lie outside the brackets. The remaining chloride is coordinated, leaving five positions for ammonia. The formula is $[Cr(NH_3)_5Cl]Cl_2$, which ionises as

$$[Cr(NH_3)_5Cl]Cl_2 \longrightarrow [Cr(NH_3)_5Cl]^{2+} + 2\,Cl^-.$$

Chromium is $+3$, and the coordinated chloride lowers the complex charge to $+2$, balancing the two counterions. Option (B) yields two ions and predicts $0.372\,K$; option (C) yields four ions and predicts $0.744\,K$; neutral option (D) yields one particle and predicts $0.186\,K$. None reproduces the measured depression.

Step 3: Conclusion The required complex is $\boxed{[Cr(NH_3)_5Cl]Cl_2}$, with $x = 5$. Hence, the correct option is (A).

Quick Tip to Solve: Find the van't Hoff factor first: for one complex cation, subtract one from the ion count to obtain the number of external chloride ions.

57. The major product of the following reaction is :

$$CH_3CH_2CH=O \xrightarrow[\text{reflux}]{\text{excess HCHO}} ?$$ [4 marks]

(A) $CH_2{=}C(CH_3)CHO$

(B) $CH_3CH_2CH_2OH$

(C) $CH_3C(CH_2OH)_3$

(D) $CH_3CH(CH_2OH)CHO$

Correct Answer: (C) $CH_3C(CH_2OH)_3$

Solution:

Concept:

- An aldehyde containing alpha hydrogen forms an enolate in alkali and adds to formaldehyde. Each hydroxymethylation replaces one alpha hydrogen by a CH_2OH group; formaldehyde itself has no alpha hydrogen.
- Once the substrate has no alpha hydrogen, its aldehyde group can undergo crossed Cannizzaro reduction with excess formaldehyde. Formaldehyde is oxidised to formate while the other aldehyde is reduced to a primary alcohol.

Step 1: Replace both alpha hydrogens Propanal is CH_3CH_2CHO. Its carbon next to the aldehyde is CH_2, so it has two alpha hydrogens. With excess formaldehyde in alkali, two successive aldol additions install two hydroxymethyl groups:

$$CH_3CH_2CHO + 2\,HCHO \longrightarrow CH_3C(CH_2OH)_2CHO.$$

The central carbon now has four carbon bonds and no hydrogen. The aldehyde remains, but further enolate formation at that carbon is impossible.

Step 2: Reduce the remaining aldehyde group Under the stated alkaline reflux conditions, the non-enolisable aldehyde undergoes crossed Cannizzaro reaction with another molecule of formaldehyde:

$$CH_3C(CH_2OH)_2CHO + HCHO + OH^- \longrightarrow CH_3C(CH_2OH)_3 + HCOO^-.$$

The former CHO carbon becomes a third CH_2OH group. Option (A) is a dehydrated unsaturated aldehyde and does not represent the final product in excess formaldehyde; option (B) omits the two added carbon atoms; option (D) is only the first hydroxymethylation intermediate and still has an alpha hydrogen. Option (C) alone includes both additions and the final reduction.

Step 3: Conclusion The major product is $\boxed{CH_3C(CH_2OH)_3}$, trimethylolethane. Hence, the correct option is (C).

Quick Tip to Solve: Count the alpha hydrogens before reacting with excess formaldehyde: install one hydroxymethyl group per alpha hydrogen, then check for crossed Cannizzaro reduction.

58. Propane molecule on chlorination under photochemical condition gives two di-chloro products, " x " and " y ". Amongst " x " and " y ", " x " is an optically active molecule. How many tri-chloro products (consider only structural isomers) will be obtained from " x " when it is further treated with chlorine under the photochemical condition? [4 marks]

(A) 4

(B) 2

(C) 3

(D) 5

Correct Answer: (C) 3

Solution:

Concept:

- Photochemical chlorination replaces a carbon-bound hydrogen by chlorine through a free-radical substitution. To count structural products, distinguish carbon connectivities and substitution positions, not different stereoisomers of the same connectivity.
- A tetrahedral carbon is stereogenic only when its four attached groups are different. For a fixed dichloropropane, count the constitutionally different carbon positions that still bear hydrogen; replacing a hydrogen on each gives the possible trichloro structures.

Step 1: Identify the optically active dichloride The optically active dichloropropane is 1,2-dichloropropane:

$$\mathrm{CH_2Cl{-}CHCl{-}CH_3}.$$

Its middle carbon is attached to H, Cl, $\mathrm{CH_3}$ and $\mathrm{CH_2Cl}$, four distinct groups. The other dichloropropane connectivities do not have such a stereogenic centre. Thus the optical-activity condition identifies x uniquely for the required further chlorination.

Step 2: Enumerate further substitution sites All three carbons in x bear hydrogen, and the two terminal carbons are inequivalent because one already bears chlorine. Substitution on carbon 1 gives $\mathrm{CHCl_2{-}CHCl{-}CH_3}$, 1,1,2-trichloropropane. Substitution on carbon 2 gives $\mathrm{CH_2Cl{-}CCl_2{-}CH_3}$, 1,2,2-trichloropropane. Substitution on carbon 3 gives $\mathrm{CH_2Cl{-}CHCl{-}CH_2Cl}$, 1,2,3-trichloropropane. Replacing different hydrogens on the same carbon can change stereochemistry but cannot create another structural isomer. These exhaust the three carbon positions; hence two undercounts the sites, while four and five incorrectly introduce duplicates or stereoisomers.

Step 3: Conclusion Exactly $\boxed{3}$ structurally distinct trichloro products are obtained from x. Hence, the correct option is (C).

Quick Tip to Solve: For one additional halogen substitution, mark each inequivalent hydrogen-bearing carbon once; count stereoisomers only if the question explicitly requests them.

59. The incorrect statement among the following is [4 marks]

(A) PH_3 shows lower proton affinity than NH_3.

(B) SO_2 can act as an oxidizing agent, but not as a reducing agent.

(C) NO_2 can dimerise easily.

(D) PF_3 exists but NF_5 does not.

Correct Answer: (B) SO_2 can act as an oxidizing agent, but not as a reducing agent.

Solution:

Concept:

- An element in an intermediate oxidation state can often be oxidised to a higher state and reduced to a lower state. A species acting as a reducing agent is itself oxidised; an oxidising agent is itself reduced.
- Other p-block trends here involve proton affinity, electron pairing in a radical, and the valence-shell limitation of second-period nitrogen. Evaluate each statement separately because the question asks for the incorrect statement.

Step 1: Test sulfur dioxide for both redox roles Sulfur has oxidation state $+4$ in SO_2. It can be oxidised to $+6$, for example when sulfur dioxide reduces chlorine:

$$SO_2 + Cl_2 + 2\,H_2O \longrightarrow H_2SO_4 + 2\,HCl.$$

It can also be reduced to elemental sulfur, as when it oxidises hydrogen sulfide:

$$SO_2 + 2\,H_2S \longrightarrow 3\,S + 2\,H_2O.$$

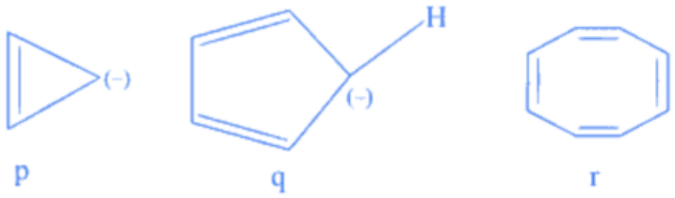
Therefore sulfur dioxide acts as both an oxidising and a reducing agent. The restriction in statement (B), saying it cannot be a reducing agent, is false.

Step 2: Check the remaining alternatives Statement (A) is correct: ammonia has greater proton affinity than phosphine; the nitrogen lone pair bonds to a proton more effectively than the less available lone pair in phosphine. Statement (C) is correct: NO_2 has an odd electron and readily associates to form N_2O_4, pairing those electrons. Statement (D) is also correct: PF_3 is an ordinary trivalent phosphorus compound, whereas second-period nitrogen cannot accommodate five N–F bonds around itself in NF_5. These three true statements must be rejected as answers to an incorrect-statement question.

Step 3: Conclusion The incorrect statement is that SO_2 acts as an oxidising agent but not as a reducing agent. Hence, the correct option is (B).

Quick Tip to Solve: For sulfur in an intermediate oxidation state such as $+4$, check both an oxidation product and a reduction product before declaring that it has only one redox role.

60. The correct stability order of the following species/molecules is : [4 marks]



(A) $q > r > p$

(B) $q > p > r$

(C) $p > q > r$

(D) $r > q > p$

Correct Answer: (A) $q > r > p$

Solution:

Concept:

- Aromaticity requires a cyclic, planar, continuously conjugated system with $4n + 2$ pi electrons. A cyclic planar conjugated system containing $4n$ pi electrons is antiaromatic and destabilised.
- A nonplanar ring can avoid antiaromaticity by interrupting cyclic pi overlap. Such a molecule is nonaromatic, which is normally more stable than an otherwise comparable antiaromatic system but lacks aromatic stabilisation.

Step 1: Classify the two anions Species p is the cyclopropenyl anion. Its double bond contributes two pi electrons and the carbanion lone pair contributes another two, giving four pi electrons in the small conjugated ring. It is antiaromatic and strongly destabilised. Species q is the cyclopentadienyl anion: two double bonds contribute four pi electrons and the negative-charge lone pair adds two. Its six pi electrons satisfy $4n + 2$ for $n = 1$, giving aromatic delocalisation over all five carbon atoms.

Step 2: Classify the eight-membered ring and rank Species r is cyclooctatetraene, containing eight pi electrons. It adopts a nonplanar tub conformation rather than maintaining a planar eight-electron conjugated ring; this avoids the antiaromatic destabilisation that a planar form would have. It is therefore nonaromatic. The relevant stability order is aromatic $>$ nonaromatic $>$ antiaromatic, so

$$q > r > p.$$

Option (B) incorrectly places the antiaromatic anion above the nonaromatic ring; option (C) puts the antiaromatic species first; option (D) places the nonaromatic molecule above the aromatic anion. None follows the electron count and actual geometry.

Step 3: Conclusion The correct stability order is $\boxed{q > r > p}$. Hence, the correct option is (A).

Quick Tip to Solve: Count the carbanion lone pair as two pi electrons only when it participates in ring conjugation, and check planarity before calling a 4n-electron ring antiaromatic.

61. 2.8×10^{-3} mol **of** CO_2 **is left after removing** 10^{21} **molecules from its ' x ' mg sample. The mass of** CO_2 **taken initially is Given:** $N_A = 6.02 \times 10^{23}$ mol^{-1} [4 marks]

(A) 98.3 mg

(B) 196.2 mg

(C) 150.4 mg

(D) 48.2 mg

Correct Answer: (B) 196.2 mg

Solution:

Concept:

- The amount of a substance is related to its number of molecules by $n = N/N_A$, where $N_A = 6.02 \times 10^{23}\,\text{mol}^{-1}$. Removing molecules reduces the amount but does not change the molar mass.
- Conservation of molecules gives $n_{\text{initial}} = n_{\text{remaining}} + n_{\text{removed}}$. For carbon dioxide, $M(CO_2) = 12 + 2(16) = 44\,\text{g mol}^{-1}$, and $1\,\text{g} = 1000\,\text{mg}$.

Step 1: Convert removed molecules into moles The given count is a number of molecules, whereas the remaining sample is specified in moles. Convert the count before adding the two quantities:

$$n_{\text{removed}} = \frac{10^{21}}{6.02 \times 10^{23}} = 1.6611 \times 10^{-3}\,\text{mol}.$$

The removed portion must be added back because the question asks for the mass before removal.

Step 2: Calculate the original mass The initial amount is therefore

$$n_{\text{initial}} = (2.8 + 1.6611) \times 10^{-3} = 4.4611 \times 10^{-3}\,\text{mol}.$$

Multiply by the molar mass and convert grams to milligrams:

$$m = (4.4611 \times 10^{-3})(44)\,\text{g} = 0.19629\,\text{g} = 196.29\,\text{mg}.$$

This corresponds to 196.2 mg among the listed values. The masses 98.3, 150.4 and 48.2 mg all correspond to fewer initial moles than required; none satisfies the molecule balance.

Step 3: Conclusion The initial carbon dioxide mass is approximately $196.2\,\text{mg}$ as represented in the choices. Hence, the correct option is (B).

Quick Tip to Solve: Convert a removed molecule count to moles first, add it to the remaining moles, and only then multiply by molar mass.

62. Given below are two statements:

Statement I: Fructose does not contain an aldehydic group but still reduces Tollen's reagent

Statement II: In the presence of base, fructose undergoes rearrangement to give glucose.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Statement I is true but Statement II is false

(B) Statement I is false but Statement II is true

(C) Both Statement I and Statement II are true

(D) Both Statement I and Statement II are false

Correct Answer: (C) Both Statement I and Statement II are true

Solution:

Concept:

- Fructose is a ketohexose: its open-chain form contains a ketonic carbonyl group rather than an aldehydic group. A reducing-sugar test, however, depends on the species present under the test conditions, not only on the original structural formula.
- Tollens' reagent is ammoniacal silver nitrate and is alkaline. In an alkaline medium, fructose can isomerise through an enediol intermediate to aldoses such as glucose and mannose, which reduce silver(I) to metallic silver.

Step 1: Evaluate Statement I under the test conditions Fructose does not have an aldehyde group in its open-chain structure. Nevertheless, it gives a positive Tollens' test because the alkaline reagent permits interconversion to reducing aldoses. These aldoses are oxidised while Ag^{+} is reduced to Ag, producing the silver mirror. Thus, both clauses of Statement I are correct; the absence of an aldehydic group in the original fructose molecule does not make the test negative.

Step 2: Evaluate Statement II and select the combination Base catalyses removal and transfer of protons adjacent to the carbonyl, giving an enediol intermediate. Reformation of the carbonyl at an adjacent carbon gives glucose, with mannose also possible:

$$\text{fructose} \rightleftharpoons \text{enediol} \rightleftharpoons \text{glucose and mannose.}$$

Statement II is therefore true and provides the chemical basis for the observed reducing behaviour. Option (A) incorrectly denies this rearrangement, option (B) incorrectly denies the positive reducing test, and option (D) denies both established facts.

Step 3: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (C).

Quick Tip to Solve: For reducing-sugar tests, check the medium: alkaline reagents let ketoses form aldoses through an enediol intermediate.

63. Which of the following happens when NH_4OH is added gradually to the solution containing 1 M A^{2+} and $1MB^{3+}$ ions?

Given : K_{sp} $[A(OH)_2] = 9 \times 10^{-10}$ and K_{sp} $[B(OH)_3] = 27 \times 10^{-18}$ at 298 K. [4 marks]

(A) $A(OH)_2$ will precipitate before $B(OH)_3$

(B) $A(OH)_2$ and $B(OH)_3$ will precipitate together

(C) Both $A(OH)_2$ and $B(OH)_3$ do not show precipitation with NH_4OH

(D) $B(OH)_3$ will precipitate before $A(OH)_2$

Correct Answer: (D) $B(OH)_3$ will precipitate before $A(OH)_2$

Solution:

Concept:

- A hydroxide begins to precipitate when its ionic product reaches its solubility product. The threshold hydroxide concentration must be calculated separately for each hydroxide because their stoichiometric powers differ.
- For the salts here, $K_{sp} = [A^{2+}][OH^-]^2$ and $K_{sp} = [B^{3+}][OH^-]^3$. Initially each metal-ion concentration is $1\,M$, so the smaller calculated hydroxide threshold determines the first precipitate.

Step 1: Find the threshold for each hydroxide At the onset of precipitation of the divalent-metal hydroxide,

$$[OH^-]_A = \sqrt{\frac{9 \times 10^{-10}}{1}} = 3 \times 10^{-5}\,M.$$

For the trivalent-metal hydroxide, take the cube root instead:

$$[OH^-]_B = \left(\frac{27 \times 10^{-18}}{1}\right)^{1/3} = 3 \times 10^{-6}\,M.$$

Comparing the numerical solubility products alone would be invalid because the hydroxide exponents are different.

Step 2: Compare the concentrations reached during addition Gradual addition of ammonium hydroxide raises the hydroxide concentration. It reaches $3 \times 10^{-6}\,M$ before $3 \times 10^{-5}\,M$, so the trivalent-metal hydroxide precipitates first. At the first threshold, the ionic product of the other hydroxide is

$$1(3 \times 10^{-6})^2 = 9 \times 10^{-12} < 9 \times 10^{-10},$$

so it remains unsaturated. This excludes simultaneous precipitation, excludes the reverse order, and shows why the statement that neither hydroxide precipitates is incorrect.

Step 3: Conclusion $B(OH)_3$ precipitates before $A(OH)_2$. Hence, the correct option is (D).

Quick Tip to Solve: Compare the required hydroxide concentrations, taking the square or cube root of the appropriate solubility product.

64. Ice at $-5^{\circ}\mathrm{C}$ is heated to become vapor with temperature of $110^{\circ}\mathrm{C}$ at atmospheric pressure. The entropy change associated with this process can be obtained from [4 marks]

(A) $\int_{268\ \mathrm{K}}^{273\ \mathrm{K}} \mathrm{C_{p,m}dT} + \frac{\Delta \mathrm{H_{m,\ fusion}}}{\mathrm{T_f}} + \frac{\Delta \mathrm{H_{m,\ vaporisation}}}{\mathrm{T_b}} + \int_{273\ \mathrm{K}}^{373\ \mathrm{K}} \mathrm{C_{p,m}dT} + \int_{373\ \mathrm{K}}^{383\ \mathrm{K}} \mathrm{C_{p,m}dT}$

(B) $\int_{268\ \mathrm{K}}^{383\ \mathrm{K}} \mathrm{C_p dT} + \frac{\Delta \mathrm{H_{melting}}}{273} + \frac{\Delta \mathrm{H_{boiling}}}{373}$

(C) $\int_{268\ \mathrm{K}}^{383\ \mathrm{K}} \mathrm{C_p dT} + \frac{\mathrm{q_{rev}}}{\mathrm{T}}$

(D) $\displaystyle\int_{268\,\mathrm{K}}^{273\,\mathrm{K}} \frac{C_{p,m}}{T}\,dT + \frac{\Delta H_{m,\mathrm{fusion}}}{T_f} + \frac{\Delta H_{m,\mathrm{vaporisation}}^{373\,\mathrm{K}}}{T_b} + \int_{273\,\mathrm{K}} \frac{C_{p,m}\,dT}{T} + \int_{373\,\mathrm{K}}^{383\,\mathrm{K}} \frac{C_{p,m}\,dT}{T}$

Correct Answer: (D) $\displaystyle\int_{268\,\mathrm{K}}^{273\,\mathrm{K}} \frac{C_{p,m}}{T}\,dT + \frac{\Delta H_{m,\mathrm{fusion}}}{T_f} + \frac{\Delta H_{m,\mathrm{vaporisation}}^{373\,\mathrm{K}}}{T_b} + \int_{273\,\mathrm{K}} \frac{C_{p,m}\,dT}{T} + \int_{373\,\mathrm{K}}^{383\,\mathrm{K}} \frac{C_{p,m}\,dT}{T}$

Solution:

Concept:

- Entropy is a state function, so its change can be evaluated along a reversible path connecting the specified initial and final states. For sensible heating at constant pressure, $dS_{\mathrm{m}} = C_{p,\mathrm{m}}\,dT/T$, with absolute temperature in kelvin.
- For a reversible phase change at its transition temperature, $\Delta S_{\mathrm{m}} = \Delta H_{\mathrm{m}}/T_{\mathrm{transition}}$. Heat capacities belong to the relevant phase; ice, liquid water and steam need separate heating intervals.

Step 1: Separate the path into five stages At atmospheric pressure, heat ice from 268 K to 273 K, melt it at 273 K, heat the liquid to 373 K, vaporise it at 373 K, and heat the vapour to 383 K. The melting and boiling stages are isothermal, so their entropy contributions are the corresponding molar latent heats divided by the transition temperatures. The other three stages have changing temperature and require integration.

Step 2: Add the reversible entropy contributions The complete molar entropy expression is

$$\begin{aligned}\Delta S_{\mathrm{m}} = &\int_{268}^{273} \frac{C_{p,\mathrm{m}}(\mathrm{ice})}{T}\,dT + \frac{\Delta H_{\mathrm{m,fusion}}}{273} \\ &+ \int_{273}^{373} \frac{C_{p,\mathrm{m}}(\mathrm{liquid})}{T}\,dT + \frac{\Delta H_{\mathrm{m,vap}}}{373} \\ &+ \int_{373}^{383} \frac{C_{p,\mathrm{m}}(\mathrm{vapour})}{T}\,dT.\end{aligned}$$

Option (D) is the expression displaying these entropy integrands and the two latent-heat terms. Its placement of the 373 K limit is typographically displaced in the printed expression. Options (A), (B) and (C) use integrals of heat capacity without division by temperature; those terms measure enthalpy changes and cannot be added as entropy contributions. A single unspecified heat-capacity integral also fails to distinguish the three phases.

Step 3: Conclusion The reversible five-stage sum, with $C_{p,\mathrm{m}}/T$ in every heating integral and $\Delta H/T$ for each transition, is represented by option (D). Hence, the correct option is (D).

Quick Tip to Solve: Split a heating path at every phase transition: use an integral of heat capacity divided by temperature within a phase and latent heat divided by temperature at the transition.

65. Match the List - I with List - II

	List - IName reaction		List - IIProduct obtainable
(A)	Swarts reaction	(I)	Ethyl benzene
(B)	Sandmeyer's reaction	(II)	Ethyl iodide
(C)	Wurtz Fittig reaction	(III)	Cyanobenzene
(D)	Finkelstein reaction	(IV)	Ethyl fluoride

Choose the correct answer from the options given below: [4 marks]

(A) A-II, B-III, C-I, D-IV

(B) A-II, B-I, C-III, D-IV

(C) A-IV, B-I, C-III, D-II

(D) A-IV, B-III, C-I, D-II

Correct Answer: (D) A-IV, B-III, C-I, D-II

Solution:

Concept:

- Swarts and Finkelstein reactions are halogen-exchange methods. Swarts reaction replaces chlorine or bromine in an alkyl halide by fluorine; Finkelstein reaction uses sodium iodide in acetone to prepare an alkyl iodide.
- Sandmeyer reaction replaces an aromatic diazonium group using a cuprous salt, including cuprous cyanide for a nitrile. Wurtz–Fittig reaction couples an aryl halide with an alkyl halide using sodium in dry ether.

Step 1: Match the two halogen-exchange reactions Swarts reaction can convert an ethyl chloride or bromide to ethyl fluoride using a suitable metallic fluoride, such as silver fluoride:

$$C_2H_5Cl + AgF \longrightarrow C_2H_5F + AgCl.$$

Therefore A matches IV. Finkelstein reaction gives ethyl iodide from an ethyl chloride or bromide:

$$C_2H_5Br + NaI \longrightarrow C_2H_5I + NaBr.$$

The precipitation of the sodium halide in acetone drives the exchange; hence D matches II.

Step 2: Match the aromatic reactions and compare options Benzenediazonium chloride reacts with cuprous cyanide to form cyanobenzene with loss of nitrogen, so Sandmeyer reaction gives B–III. An aryl halide and an ethyl halide couple under Wurtz–Fittig conditions:

$$C_6H_5Br + C_2H_5Br + 2\,Na \longrightarrow C_6H_5C_2H_5 + 2\,NaBr.$$

Thus C matches I. The complete mapping is A–IV, B–III, C–I, D–II. Options (A) and (B) incorrectly associate Swarts reaction with ethyl iodide; option (C) interchanges the products of Sandmeyer and Wurtz–Fittig reactions. Only option (D) retains all four correct matches.

Step 3: Conclusion The correct matching is A–IV, B–III, C–I and D–II. Hence, the correct option is (D).

Quick Tip to Solve: Remember Swarts for fluorides and Finkelstein for iodides; then distinguish diazonium substitution from aryl–alkyl coupling.

66. The element that does not belong to the same period of the remaining elements (modern periodic table) is: [4 marks]

(A) Platinum

(B) Osmium

(C) Iridium

(D) Palladium

Correct Answer: (D) Palladium

Solution:

Concept:

- A period is a horizontal row in the modern periodic table. Transition elements from different d-block series can have similar properties or belong to adjacent groups without being in the same period.
- The 4d transition series lies in period 5, whereas the 5d transition series lies in period 6. Period membership should be checked from the element's position, rather than inferred only from the number of its group.

Step 1: Locate platinum, osmium and iridium Platinum has atomic number 78, osmium 76 and iridium 77. All three occur in the 5d transition series of period 6. Their respective group numbers are 10, 8 and 9, but different groups do not prevent them from sharing the same horizontal period. Consequently, options (A), (B) and (C) each name an element in the common period of the other two.

Step 2: Locate palladium and check the configuration exception Palladium has atomic number 46 and is a member of the 4d transition series in period 5. It is directly above platinum in group 10. Its exceptional ground-state configuration, $[\mathrm{Kr}]\,4d^{10}$, has an empty 5s subshell; this does not move it out of period 5. Period is determined by its position in the periodic sequence, not by mechanically taking the largest occupied principal quantum number for this exceptional configuration. Thus palladium differs in period from all three of the listed 5d metals.

Step 3: Conclusion Palladium belongs to period 5; platinum, osmium and iridium belong to period 6. Hence, the correct option is (D).

Quick Tip to Solve: For the palladium exception, use its 4d-series position: an empty 5s subshell does not make it a period-4 element.

67. The d-electronic configuration of an octahedral Co (II) complex having magnetic moment of 3.95 BM is: [4 marks]

(A) $e^4t_2^3$

(B) $t_{2g}^6e_g^1$

(C) $t_{2g}^5e_g^2$

(D) $t_{2g^3}^3e_g^0$

Correct Answer: (C) $t_{2g}^5e_g^2$

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons.
- Cobalt has atomic number 27. Removing its two outer $4s$ electrons gives Co^{2+} with a $3d^7$ configuration. In an octahedral field, the lower set is t_{2g} and the upper set is e_g.

Step 1: Find the number of unpaired electrons For three unpaired electrons the spin-only value is $\sqrt{3(3+2)} = \sqrt{15} \approx 3.87\,\text{BM}$. This is close to the given $3.95\,\text{BM}$, allowing for the small orbital contribution in a cobalt complex. One unpaired electron would give only $\sqrt{3} \approx 1.73\,\text{BM}$, so a low-spin assignment is unsuitable.

Step 2: Distribute the seven d electrons For the high-spin octahedral arrangement, fill all five d orbitals singly before pairing. The three lower orbitals receive five electrons in total, and the two upper orbitals receive one electron each:

$$t_{2g}^5e_g^2.$$

There is one unpaired electron in the lower set and two in the upper set. Option A uses the tetrahedral labels e, t_2, so it is not an octahedral configuration. Option B has only one unpaired electron. Option D contains three d electrons rather than the seven required for cobalt(II).

Step 3: Conclusion The configuration is $t_{2g}^5e_g^2$, with three unpaired electrons. Hence, the correct option is (C).

Quick Tip to Solve: First count d electrons from the oxidation state, then use the magnetic moment to choose high spin or low spin before writing the orbital occupancy.

68. The complex that shows Facial - Meridional isomerism is : [4 marks]

(A) $[Co(en)_2Cl_2]^+$

(B) $[Co\,(NH_3)_3\,Cl_3]$

(C) $[Co\,(NH_3)_4\,Cl_2]^+$

(D) $[Co(en)_3]^{3+}$

Correct Answer: (B) $[Co\,(NH_3)_3\,Cl_3]$

Solution:

Concept:

- Facial and meridional isomerism occurs for octahedral complexes of the form MA_3B_3 with two kinds of monodentate ligand.
- In the facial arrangement, the three identical ligands occupy one triangular face and are mutually cis. In the meridional arrangement, they lie along a plane through the metal, including a trans pair.

Step 1: Identify the ligand arrangement required The distinction depends on placing three ligands of each of two types among six octahedral positions. In $[Co(NH_3)_3Cl_3]$, there are three ammonia ligands and three chloride ligands. They are all monodentate, so both the facial arrangement of the three chlorides and the meridional arrangement of those chlorides are possible. The same classification applies to the three ammonia ligands.

Step 2: Eliminate the other complexes In option A, $[Co(en)_2Cl_2]^+$, each ethylenediamine ligand occupies two neighbouring coordination positions. This complex has cis/trans isomerism according to the relative chloride positions, rather than fac/mer isomerism. In option C, $[Co(NH_3)_4Cl_2]^+$, the two chloride ligands can again be cis or trans; there is no group of three chloride ligands to define a face. Option D, $[Co(en)_3]^{3+}$, has three identical bidentate ligands and can exhibit optical isomerism, but it does not have an MA_3B_3 distribution of monodentate ligands.

Step 3: Conclusion $[Co(NH_3)_3Cl_3]$ shows facial-meridional isomerism. Hence, the correct option is (B).

Quick Tip to Solve: For octahedral monodentate complexes, associate MA_4B_2 with cis/trans and MA_3B_3 with fac/mer; do not count a bidentate ligand as a single coordination position.

69. Heat treatment of muscular pain involves radiation of wavelength of about 900 nm . Which spectral line of H atom is suitable for this?

Given : Rydberg constant $\mathrm{R_H} = 10^5\ \mathrm{cm^{-1}},\ \mathrm{h} = 6.6 \times 10^{-34}\ \mathrm{J\ s}, \mathrm{c} = 3 \times 10^8\ \mathrm{m/s}\big)$ [4 marks]

(A) Lyman series, $\infty \to 1$

(B) Balmer series, $\infty \to 2$

(C) Paschen series, $5 \to 3$

(D) Paschen series, $\infty \to 3$

Correct Answer: (D) Paschen series, $\infty \to 3$

Solution:

Concept:

- The hydrogen emission wavelength obeys $1/\lambda = R_H(1/n_1^2 - 1/n_2^2)$, where $n_2 > n_1$ and n_1 identifies the series.
- Here $R_H = 10^5\,\mathrm{cm^{-1}} = 10^7\,\mathrm{m^{-1}}$. Lyman, Balmer and Paschen transitions terminate at $n_1 = 1, 2, 3$, respectively.

Step 1: Calculate the Paschen series limit For the transition from $n_2 = \infty$ to $n_1 = 3$, the upper-level term is zero. Thus

$$\frac{1}{\lambda} = \frac{10^7}{9}\,\mathrm{m^{-1}}, \qquad \lambda = 9 \times 10^{-7}\,\mathrm{m} = 900\,\mathrm{nm}.$$

This is exactly the approximate wavelength stated for the treatment. It lies in the infrared, as expected for thermal radiation. The given Planck constant is unnecessary because the question asks for a wavelength rather than photon energy.

Step 2: Check the competing spectral lines The Lyman series limit has $\lambda = 1/R_H = 100\,\mathrm{nm}$. The Balmer limit gives $\lambda = 4/R_H = 400\,\mathrm{nm}$. Neither is near $900\,\mathrm{nm}$. For the finite Paschen transition $5 \to 3$,

$$\frac{1}{\lambda} = 10^7\left(\frac{1}{9} - \frac{1}{25}\right) = 10^7\frac{16}{225},$$

so $\lambda = 1.40625 \times 10^{-6}\,\mathrm{m}$, or approximately $1406\,\mathrm{nm}$, which is longer than the specified radiation.

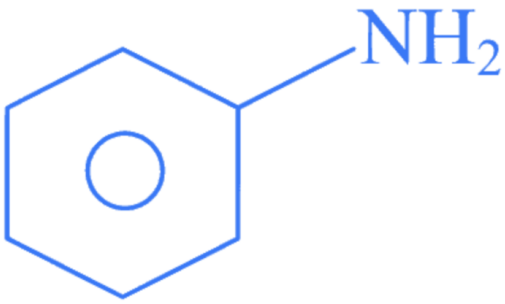
Step 3: Conclusion The suitable line is the Paschen series limit, $\infty \to 3$, of wavelength $900\,\mathrm{nm}$. Hence, the correct option is (D).

Quick Tip to Solve: At a hydrogen series limit, set $1/n_2^2 = 0$, giving $\lambda = n_1^2/R_H$; convert the Rydberg constant to the same length units before substituting.

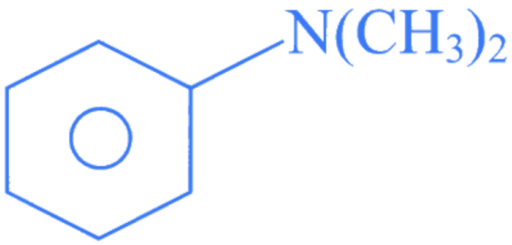
70. Which among the following react with Hinsberg's reagent?

Choose the correct answer from the options given below : [4 marks]

A.



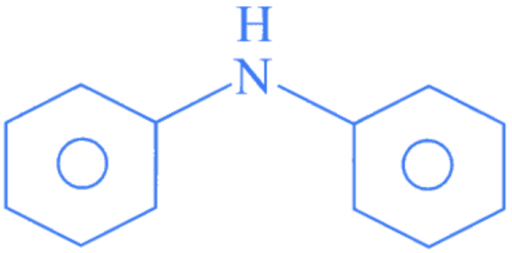
B.



C. $CH_3 - NH_2$

D. $N(CH_3)_3$

E.



(A) A, C and E only

(B) A, B and E only

(C) C and D only

(D) B and D only

Correct Answer: (A) A, C and E only

Solution:

Concept:

- Hinsberg's reagent is benzenesulphonyl chloride, $C_6H_5SO_2Cl$. Primary and secondary amines form sulphonamides by substitution at sulphur.
- The amine must have an N–H bond for ordinary sulphonamide formation. Tertiary amines have no N–H bond and do not undergo this reaction.

Step 1: Classify each listed amine Compound A is aniline, $C_6H_5NH_2$, and C is methylamine, CH_3NH_2. Both are primary amines and contain two N–H bonds. Compound E is diphenylamine, $(C_6H_5)_2NH$, which is a secondary amine containing one N–H bond. Compound B is N,N-dimethylaniline and D is trimethylamine. In both B and D, nitrogen is attached to three carbon groups and bears no hydrogen.

Step 2: Apply sulphonamide formation and compare options For a primary amine the substitution is represented by

$$RNH_2 + C_6H_5SO_2Cl \longrightarrow C_6H_5SO_2NHR + HCl.$$

The product still contains an N–H bond and can dissolve in alkali through deprotonation. For a secondary amine,

$$R_2NH + C_6H_5SO_2Cl \longrightarrow C_6H_5SO_2NR_2 + HCl,$$

giving a product with no N–H bond. Thus A, C and E react, while B and D do not. Option B incorrectly includes tertiary amine B and excludes primary amine C. Options C and D each include tertiary amine D and therefore cannot be correct.

Step 3: Conclusion A, C and E react with Hinsberg's reagent. Hence, the correct option is (A).

Quick Tip to Solve: Count the hydrogens directly bonded to nitrogen: an aryl group does not prevent the Hinsberg reaction, but the absence of every N–H bond does.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. For the thermal decomposition of $N_2O_5(g)$ at constant volume, the following table can be formed, for the reaction mentioned below.

$$2\,N_2O_5(g) \rightarrow 2\,N_2O_4(g) + O_2(g)$$

Sr. No.	Time/s	Total pressure/(atm)
1	0	0.6
2	100	'x'

$x =$ _____ $\times 10^{-3}$ atm **[nearest integer]**

Given : Rate constant for the reaction is $4.606 \times 10^{-2}\ s^{-1}$. [4 marks]

Solution:

Concept:

- For first-order decomposition, the fraction of reactant remaining after time t is e^{-kt}. At constant temperature and volume, a partial pressure is proportional to the number of moles.
- For $2\,N_2O_5(g) \longrightarrow 2\,N_2O_4(g) + O_2(g)$, one mole of reactant decomposed forms one mole of N_2O_4 and half a mole of O_2. Total pressure therefore increases as the reaction proceeds.

Step 1: Find the remaining reactant pressure Initially the gas is N_2O_5 alone at $P_0 = 0.6\,atm$. The rate constant has units s^{-1}, consistent with first-order kinetics. At $t = 100\,s$,

$$kt = (4.606 \times 10^{-2})(100) = 4.606, \qquad e^{-kt} \approx 0.009992.$$

Hence the partial pressure of the undecomposed reactant is $p = 0.6e^{-4.606} \approx 0.005995\,\text{atm}$.

Step 2: Sum the pressures using stoichiometry The decrease in reactant partial pressure is $P_0 - p$. The product partial pressures are $P_0 - p$ for N_2O_4 and $(P_0 - p)/2$ for O_2. Adding the three gases gives

$$P_t = p + (P_0 - p) + \frac{P_0 - p}{2} = \frac{3}{2}P_0 - \frac{1}{2}p.$$

Substitution gives

$$P_{100} = 0.9 - 0.3e^{-4.606} \approx 0.897002\,\text{atm} = 897.002 \times 10^{-3}\,\text{atm}.$$

This approaches, but does not exceed, the complete-decomposition limit of $0.9\,\text{atm}$.

Step 3: Conclusion The total pressure is approximately $897 \times 10^{-3}\,\text{atm}$. The requested nearest-integer entry is $\boxed{897}$.

Quick Tip to Solve: For a gas decomposition, use the remaining reactant pressure in the kinetic equation and only then convert it to total pressure using the balanced mole ratio.

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72. The standard enthalpy and standard entropy of decomposition of N_2O_4 to NO_2 are $55.0\ \text{kJ mol}^{-1}$ and $175.0\ \text{J/K/mol}$ respectively. The standard free energy change for this reaction at 25°C in J mol^{-1} is _____ (Nearest integer) [4 marks]

Solution:

Concept:

- The standard Gibbs energy change at temperature T is $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$. Temperature must be expressed in kelvin.
- Before subtracting the terms, put enthalpy and entropy into compatible energy units. With entropy in $\text{J K}^{-1}\,\text{mol}^{-1}$, use enthalpy in J mol^{-1}.

Step 1: Convert the supplied quantities The reaction is the decomposition $N_2O_4(g) \longrightarrow 2\,NO_2(g)$. Its stated standard enthalpy change is positive, so retain $\Delta H^\circ = +55.0\,\text{kJ mol}^{-1} = 55000\,\text{J mol}^{-1}$. The entropy change is $\Delta S^\circ = +175.0\,\text{J K}^{-1}\,\text{mol}^{-1}$, consistent with an increase in gas molecules. Using the customary integer-kelvin conversion for this question, $25^\circ\text{C} = 298\,\text{K}$.

Step 2: Subtract the entropy contribution Calculate the entropy term first, keeping its sign:

$$T\Delta S^\circ = (298)(175.0) = 52150\,\text{J mol}^{-1}.$$

Now substitute into the Gibbs equation:

$$\begin{aligned}\Delta G^\circ &= 55000 - 52150 \\ &= 2850\,\text{J mol}^{-1}.\end{aligned}$$

The subtraction is essential: the positive entropy change favours decomposition and offsets most of its positive enthalpy change. The result remains positive at the stated temperature, so the standard-state

forward process is not thermodynamically favoured there; this does not mean that no equilibrium dissociation occurs.

Step 3: Conclusion The standard free energy change is $\boxed{2850\,\mathrm{J\,mol^{-1}}}$. The requested nearest-integer entry is 2850.

Quick Tip to Solve: Convert kJ to J before calculating $\Delta H^\circ - T\Delta S^\circ$; a positive entropy term reduces Gibbs energy and must be subtracted.

73. If 1 mM solution of ethylamine produces $\mathrm{pH} = 9$, then the ionization constant $(\mathrm{K_b})$ of ethylamine is 10^{-x}. The value of x is _____ (nearest integer).

[The degree of ionization of ethylamine can be neglected with respect to unity.] [4 marks]

Solution:

Concept:

- Ethylamine is a weak base: it accepts a proton from water and forms ethylammonium and hydroxide ions. Its ionization constant is $K_b = [\mathrm{C_2H_5NH_3^+}][\mathrm{OH^-}]/[\mathrm{C_2H_5NH_2}]$.
- For aqueous solutions under the usual conditions, $\mathrm{pH} + \mathrm{pOH} = 14$ and $[\mathrm{OH^-}] = 10^{-\mathrm{pOH}}\,\mathrm{mol\,L^{-1}}$. The given small-ionization approximation lets us retain the initial base concentration in the denominator.

Step 1: Convert the initial concentration The prefix milli means 10^{-3}, so the initial concentration is

$$C = 1\,\mathrm{mM} = 1.0 \times 10^{-3}\,\mathrm{mol\,L^{-1}}.$$

This is the concentration of ethylamine before its slight reaction with water; it is not the concentration of the hydroxide produced.

Step 2: Obtain the hydroxide concentration from pH A pH of 9 gives

$$\mathrm{pOH} = 14 - 9 = 5, \qquad [\mathrm{OH^-}] = 10^{-5}\,\mathrm{mol\,L^{-1}}.$$

Each ionized ethylamine molecule produces one ethylammonium ion and one hydroxide ion:

$$\mathrm{C_2H_5NH_2} + \mathrm{H_2O} \rightleftharpoons \mathrm{C_2H_5NH_3^+} + \mathrm{OH^-}.$$

Thus their equilibrium concentrations are both $10^{-5}\,\mathrm{mol\,L^{-1}}$, neglecting the much smaller contribution from water.

Step 3: Evaluate the base ionization constant The ionized fraction is $\alpha = 10^{-5}/10^{-3} = 0.01$, consistent with the question's instruction to neglect it relative to unity. Therefore,

$$K_b = \frac{C\alpha^2}{1-\alpha} \simeq C\alpha^2 = \frac{(10^{-5})^2}{10^{-3}} = 10^{-7}.$$

Even retaining the small concentration decrease gives $K_b \simeq 1.01 \times 10^{-7}$, so the required nearest-integer exponent is unchanged.

Step 4: Conclusion Comparing $K_b = 10^{-7}$ with 10^{-x}, the required dimensionless exponent is $\boxed{x = 7}$, to the nearest integer.

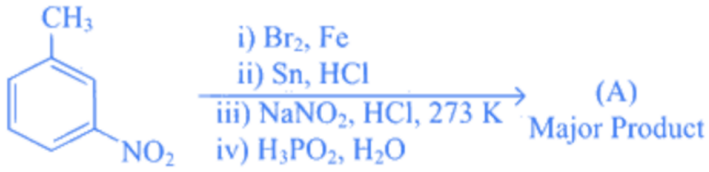
Quick Tip to Solve: Convert mM to molarity before using $K_b = [OH^-]^2/C$; obtain hydroxide from pOH, not directly from pH.

74. Consider the following sequence of reactions to produce major product (A)

Molar mass of product (A) **is** g mol^{-1}.

(Given molar mass in g mol^{-1} **of** C : 12, H : 1, O : 16, Br : 80, N : 14, P : 31 **)**

[4 marks]



Solution:

Concept:

- Electrophilic bromination with Br_2/Fe replaces a ring hydrogen by bromine. Reduction with Sn/HCl converts an aromatic nitro group into an amino group without removing the methyl or bromo substituent.
- Diazotization of a primary aromatic amine with $NaNO_2/HCl$ at 273 K forms an arenediazonium salt. Hypophosphorous acid, H_3PO_2, then replaces its diazonium group by hydrogen.

Step 1: Follow bromination of nitrotoluene The starting ring bears a methyl group and a nitro group at meta positions. The first reagent, Br_2/Fe, introduces one bromine atom into the aromatic ring by electrophilic substitution. The methyl group remains attached. For the molar-mass calculation, the precise ring position of bromine is immaterial: the monobrominated nitrotoluene has formula $C_7H_6BrNO_2$. This counts three ring hydrogens and three methyl hydrogens.

Step 2: Convert the nitro group into the diazonium group Tin and hydrochloric acid reduce $-NO_2$ to $-NH_2$, giving a bromo-substituted toluidine. Subsequent reaction with nitrous acid generated from $NaNO_2$ and HCl at 273 K converts the amino group into $-N_2{}^+Cl-$. Neither of these operations removes the ring bromine or changes the carbon skeleton.

Step 3: Identify the final product and add atomic masses In the last step, H_3PO_2/H_2O replaces the diazonium substituent by hydrogen, with evolution of nitrogen. Thus the original nitro substituent is absent from product (A), while one methyl group and one bromine remain. Product (A) is a bromotoluene with molecular formula C_7H_7Br. Its seven hydrogens comprise four on the ring and three in the methyl group. Therefore,

$$M(A) = 7(12) + 7(1) + 1(80) = 84 + 7 + 80 = 171\,\text{g mol}^{-1}.$$

Nitrogen, oxygen and phosphorus do not enter this sum because none is retained in the final organic product; their presence in reagents does not imply incorporation.

Step 4: Conclusion Product (A) has formula C_7H_7Br, so its molar mass is $\boxed{171\,\text{g mol}^{-1}}$. The required numerical entry is 171.

Quick Tip to Solve: The sequence $-NO_2 \longrightarrow -NH_2 \longrightarrow -N_2{}^+ \longrightarrow -H$ removes the original nitro group. Count only the methyl group and bromine retained on the final ring.

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75. During " S " estimation, 160 mg of an organic compound gives 466 mg of barium sulphate. The percentage of Sulphur in the given compound is _____ %.

(Given molar mass in g mol^{-1} **of** $\text{Ba} : 137,\ \text{S} : 32, \text{O} : 16$**)** [4 marks]

Solution:

Concept:

- In the gravimetric estimation of sulphur, sulphur in the organic compound is converted into sulphate and precipitated as $BaSO_4$. Each mole of the precipitate contains one mole of sulphur atoms.
- The sulphur mass fraction in the precipitate is $M(\text{S})/M(BaSO_4)$. Multiply this by the precipitate mass to recover the sulphur mass, then divide by the original sample mass to obtain its percentage.

Step 1: Calculate the molar mass of the precipitate Use the atomic masses supplied in the question. Barium sulphate contains one barium atom, one sulphur atom and four oxygen atoms per formula unit:

$$M(BaSO_4) = 137 + 32 + 4(16) = 233\,\text{g mol}^{-1}.$$

Only $32\,\text{g}$ out of every $233\,\text{g}$ of this precipitate represents sulphur originally present in the organic sample. The barium and oxygen masses must not be counted as sample sulphur.

Step 2: Determine the sulphur mass in the sample The measured precipitate mass is $466\,\text{mg} = 0.466\,\text{g}$. Hence,

$$n(BaSO_4) = \frac{0.466}{233} = 0.00200\,\text{mol}.$$

Since the mole ratio of sulphur atoms to barium sulphate is $1 : 1$, the sulphur mass is

$$m(\text{S}) = 0.00200(32) = 0.0640\,\text{g} = 64.0\,\text{mg}.$$

There is no additional factor from the four oxygen atoms: the sulphur count per formula unit remains one.

Step 3: Express the result as a percentage The original organic sample has mass $160\,\text{mg}$, so use that mass, rather than the precipitate mass, as the percentage denominator:

$$\%\text{S} = \frac{64.0}{160} \times 100 = 40.0.$$

Both masses use milligrams here, so their units cancel. The same calculation with both masses in grams gives the identical result.

Step 4: Conclusion The percentage of sulphur in the given organic compound is $\boxed{40\%}$; the numerical entry is 40.

Quick Tip to Solve: For sulphur estimation, use $\%\mathrm{S} = (m_{\mathrm{BaSO_4}}/m_{\mathrm{sample}})(32/233) \times 100$, with both masses in the same unit.