JEE Main 2025 Paper 1 (B.E./B.Tech) Jan 24 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 24 January 2025, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. $\lim_{x\to 0} \operatorname{cosec} x \left(\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4}\right)$ **is:** [4 marks]

(A) $\frac{1}{\sqrt{15}}$

(B) $\frac{1}{2\sqrt{5}}$

(C) 0

(D) $-\frac{1}{2\sqrt{5}}$

Correct Answer: (D) $-\frac{1}{2\sqrt{5}}$

Solution:

Concept:

- When two square roots approach the same nonzero value, rationalise their difference before using standard trigonometric limits.
- Use $\lim_{x\to 0} \sin x/x = 1$, continuity of cosine, and $\cos^2 x - 1 = -\sin^2 x$. Angles are measured in radians.

Step 1: Check the indeterminate form As $x \to 0$, both radicands tend to 5, so the difference of roots tends to zero while $\operatorname{cosec} x$ becomes unbounded. Substituting zero into these factors separately is invalid. Both radicands remain positive in a neighbourhood of zero, allowing multiplication by the conjugate.

Step 2: Multiply by the conjugate Write the whole expression as one quotient and rationalise its numerator:

$$\frac{\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4}}{\sin x} = \frac{\cos^2 x + 3\cos x - \sin x - 4}{\sin x\left(\sqrt{2\cos^2 x + 3\cos x} + \sqrt{\cos^2 x + \sin x + 4}\right)}.$$

Separate the numerator into $(\cos^2 x - 1) + 3(\cos x - 1) - \sin x$. This retains the negative sign on the sine term.

Step 3: Evaluate the remaining factors The numerator after division by $\sin x$ becomes

$$-\sin x + 3\frac{\cos x - 1}{\sin x} - 1.$$

Using $(\cos x - 1)/\sin x = -\sin x/(1 + \cos x)$, its limit is -1. The sum of square roots tends to $2\sqrt{5}$, which is nonzero. The quotient therefore tends to $-1/(2\sqrt{5})$. Options (A) and (B) are positive, whereas the surviving linear term is negative; option (C) incorrectly discards that term.

Step 4: Conclusion The required limit is $\boxed{-\frac{1}{2\sqrt{5}}}$. Hence, the correct option is (D).

Quick Tip to Solve: Rationalise first: cosine changes only quadratically near zero, so the linear $-\sin x$ term determines the sign and value.

2. Let $f : \mathbb{R}-\{0\} \to \mathbb{R}$ be a function such that $f(x)-6f\left(\frac{1}{x}\right) = \frac{35}{3x}-\frac{5}{2}$. If the $\lim\limits_{x\to 0}\left(\frac{1}{\alpha x}+f(x)\right) = \beta; \alpha, \beta \in \mathbb{R}$, then $\alpha + 2\beta$ is equal to [4 marks]

(A) 6

(B) 5

(C) 3

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- Replacing x by $1/x$ gives a second equation in the same unknowns $f(x)$ and $f(1/x)$, valid because $x \neq 0$.
- A finite two-sided limit of $c/x + h(x)$, with h continuous at zero, requires $c = 0$.

Step 1: Form the reciprocal equation Let $u = f(x)$ and $v = f(1/x)$. The given equation and the equation obtained by replacing x with $1/x$ are

$$u - 6v = \frac{35}{3x} - \frac{5}{2}, \qquad v - 6u = \frac{35x}{3} - \frac{5}{2}.$$

These are simultaneous linear equations, so no continuity assumption about the initially unknown function is needed.

Step 2: Eliminate the reciprocal value Multiply the second equation by six and add it to the first. The coefficients of v cancel:

$$-35u = \frac{35}{3x} + 70x - \frac{35}{2}.$$

Hence, for every nonzero x,

$$f(x) = -\frac{1}{3x} - 2x + \frac{1}{2}.$$

Substitution into the original relation gives the same $35/(3x) - 5/2$, which checks both coefficients and the constant.

Step 3: Impose the finite limit The expression in the limit is

$$\frac{1}{\alpha x} + f(x) = \frac{1/\alpha - 1/3}{x} - 2x + \frac{1}{2}.$$

Its definition requires $\alpha \neq 0$. If the coefficient of $1/x$ were nonzero, the expression would diverge with opposite signs on the two sides of zero. Thus $1/\alpha = 1/3$, giving $\alpha = 3$. The remaining polynomial tends to $1/2$, so $\beta = 1/2$. Therefore $\alpha + 2\beta = 3 + 1 = 4$. The other listed numbers 6, 5 and 3 do not satisfy these uniquely determined values.

Step 4: Conclusion We obtain $\alpha = 3$, $\beta = \frac{1}{2}$, and $\boxed{\alpha + 2\beta = 4}$. Hence, the correct option is (D).

Quick Tip to Solve: Solve reciprocal functional equations as two linear equations, then cancel the singular $1/x$ coefficient before evaluating the limit.

3. The product of all the rational roots of the equation $\left(x^2-9x+11\right)^2-(x-4)(x-5)=3$, is equal to [4 marks]

(A) 7

(B) 21

(C) 28

(D) 14

Correct Answer: (D) 14

Solution:

Concept:

- A repeated quadratic expression can reduce a quartic equation to a quadratic in a new variable.
- A quadratic with integer coefficients has rational roots only when its discriminant is a perfect square; Vieta gives the product once the correct factor is identified.

Step 1: Identify the repeated quadratic Put $t=x^2-9x+11$. The other factor expands to

$$(x-4)(x-5)=x^2-9x+20=t+9.$$

Therefore the given equation becomes $t^2-(t+9)=3$, or

$$t^2-t-12=(t-4)(t+3)=0.$$

Both possible values $t=4$ and $t=-3$ must be examined; retaining only one without checking rationality could lose valid roots.

Step 2: Separate the rational and irrational roots For $t=4$, the original variable satisfies

$$x^2-9x+7=0, \qquad x=\frac{9\pm\sqrt{53}}{2}.$$

Since 53 is not a perfect square, neither root is rational. For $t=-3$, we get

$$x^2-9x+14=(x-2)(x-7)=0,$$

so the rational roots are exactly 2 and 7. Each satisfies the given equation: $t=-3$ makes $t^2-(t+9)=9-6=3$.

Step 3: Multiply only the required roots The requested product is the product of the rational roots, so

$$2\times 7=14.$$

The product 7 is that of the two irrational roots from the other quadratic and answers a different question. The values 21 and 28 are not the product of the verified rational roots. Equivalently, the full quartic factors as $(x^2-9x+7)(x^2-9x+14)=0$, making the classification transparent.

Step 4: Conclusion The rational roots are 2 and 7, whose product is $\boxed{14}$. Hence, the correct option is (D).

Quick Tip to Solve: Before multiplying roots of a reducible quartic, classify the roots of each quadratic factor; the question may ask only for rational roots.

4. For some $n \neq 10$, let the coefficients of the 5 th, 6 th and 7 th terms in the binomial expansion of $(1+x)^{n+4}$ be in A.P. Then the largest coefficient in the expansion of $(1+x)^{n+4}$ is: [4 marks]

(A) 10

(B) 35

(C) 70

(D) 20

Correct Answer: (B) 35

Solution:

Concept:

- The $(r+1)$-th term of $(1+x)^N$ has coefficient $\binom{N}{r}$; therefore the fifth, sixth and seventh coefficients have lower indices 4, 5 and 6.
- Three numbers a, b, c are in arithmetic progression when $a + c = 2b$. For odd N, the two equal central binomial coefficients are the largest.

Step 1: Translate the arithmetic progression Set $N = n + 4$. Since the seventh term is present, $N \geq 6$. The stated condition gives

$$\binom{N}{4} + \binom{N}{6} = 2\binom{N}{5}.$$

Divide by the positive quantity $\binom{N}{4}$. Consecutive coefficient ratios yield

$$1 + \frac{(N-4)(N-5)}{30} = \frac{2(N-4)}{5}.$$

The term numbers must be reduced by one for their binomial indices; using indices 5, 6 and 7 would produce a different equation.

Step 2: Solve and apply the exclusion Multiplication by 30 gives

$$30 + (N-4)(N-5) = 12(N-4).$$

Expanding and collecting terms gives

$$N^2 - 21N + 98 = (N-7)(N-14) = 0.$$

Thus $N = 7$ or $N = 14$, corresponding to $n = 3$ or $n = 10$. The question explicitly excludes $n = 10$, so the only admissible expansion is $(1+x)^7$.

Step 3: Locate the central coefficients The coefficients increase while $\binom{N}{r+1}/\binom{N}{r} = (N-r)/(r+1) > 1$ and then decrease by symmetry. With $N = 7$, the maxima occur at $r = 3, 4$:

$$\binom{7}{3} = \binom{7}{4} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35.$$

Indeed the three given coefficients are $35, 21, 7$, with common difference -14. Options 10, 70 and 20 are not central coefficients of this admissible expansion.

Step 4: Conclusion The largest coefficient is $\boxed{35}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert term numbers to indices first, and apply the excluded value before choosing the central binomial coefficient.

5. Let $f(x)=\frac{2^{x+2}+16}{2^{2x+1}+2^{x+4}+32}$. Then the value of $8\left(f\left(\frac{1}{15}\right)+f\left(\frac{2}{15}\right)+\ldots+f\left(\frac{59}{15}\right)\right)$ is equal to [4 marks]

(A) 108

(B) 92

(C) 118

(D) 102

Correct Answer: (C) 118

Solution:

Concept:

- Factor an exponential rational function by setting $t=2^x>0$; a common factor can reveal a simple complementary relation.
- For equally spaced inputs, pair the first with the last. Here paired arguments sum to 4, so compare $f(x)$ with $f(4-x)$.

Step 1: Simplify the function With $t=2^x$, the numerator is $4t+16=4(t+4)$ and the denominator is

$$2t^2+16t+32=2(t+4)^2.$$

Since $t>0$, cancellation is valid and

$$f(x)=\frac{2}{2^x+4}.$$

This simplification also shows that all terms in the requested sum are positive.

Step 2: Find a constant pair sum The complementary argument has $2^{4-x}=16/t$, so

$$f(4-x)=\frac{2}{16/t+4}=\frac{t}{2(t+4)}.$$

Consequently,

$$f(x)+f(4-x)=\frac{2}{t+4}+\frac{t}{2(t+4)}=\frac{1}{2}.$$

This equality is exact for every real x, avoiding the need to approximate any fractional power of two.

Step 3: Count the pairs and the middle term Write $S=\sum_{k=1}^{59}f(k/15)$. The partner of $k/15$ is $(60-k)/15$. There are 29 distinct pairs, namely indices $(1,59)$ through $(29,31)$, and the unpaired index is $k=30$. Its argument is 2 and $f(2)=2/(4+4)=1/4$. Therefore

$$S=29\left(\frac{1}{2}\right)+\frac{1}{4}=\frac{59}{4},\qquad 8S=118.$$

As a check, reversing all 59 indices gives $2S=59/2$ directly. The other options 108, 92 and 102 conflict with this constant-pair identity.

Step 4: Conclusion The requested value is $\boxed{118}$. Hence, the correct option is (C).

Quick Tip to Solve: After factoring the denominator, pair indices whose arguments sum to 4; count the central term once, not twice.

6. If $I(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}dx, m,n>0$, then $I(9,14)+I(10,13)$ is [4 marks]

(A) $I(9,1)$

(B) $I(1,13)$

(C) I$(19,27)$

(D) I$(9,13)$

Correct Answer: (D) I$(9,13)$

Solution:

Concept:

- Definite integrals with the same limits can be combined by adding their integrands, provided the integrals converge.
- For $m,n>0$, $I(m,n)$ converges. Factor the common powers of x and $1-x$ before comparing the resulting exponents with the definition.

Step 1: Write both integrands explicitly Using the definition with its exponent offsets,

$$I(9,14)=\int_0^1 x^8(1-x)^{13}\,dx, \qquad I(10,13)=\int_0^1 x^9(1-x)^{12}\,dx.$$

All powers here are nonnegative integers, so both integrands are continuous on the closed interval. Adding the integrals is therefore legitimate without any endpoint convergence complication.

Step 2: Factor the sum under one integral Combine the integrands and remove their common factor:

$$\begin{aligned} I(9,14)+I(10,13) &= \int_0^1 \left[x^8(1-x)^{13}+x^9(1-x)^{12}\right]\,dx \\ &= \int_0^1 x^8(1-x)^{12}\left[(1-x)+x\right]\,dx \\ &= \int_0^1 x^8(1-x)^{12}\,dx. \end{aligned}$$

The bracket is exactly 1, which is why one exponent is reduced without introducing a numerical coefficient.

Step 3: Identify the parameters and exclude the distractors Comparison with $x^{m-1}(1-x)^{n-1}$ gives $m-1=8$ and $n-1=12$, hence $m=9$, $n=13$. Thus the result is $I(9,13)$. Option (A) would omit the factor $(1-x)^{12}$; option (B) would omit x^8. Option (C) would have powers 18 and 26 and is not obtained by adding integrals. Parameters of these integrals cannot simply be added as though the integral itself were a product.

Step 4: Conclusion The sum equals $\boxed{I(9,13)}$. Hence, the correct option is (D).

Quick Tip to Solve: For adjacent parameter pairs, combine the integrands first: the factor $(1-x)+x=1$ often yields the answer immediately.

7. If α and β are the roots of the equation $2z^2-3z-2i=0$, where $i=\sqrt{-1}$, then $16\cdot\mathrm{Re}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right)\cdot \mathrm{lm}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right)$ is equal to [4 marks]

(A) 441

(B) 312

(C) 409

(D) 398

Correct Answer: (A) 441

Solution:

Concept:

- For a quadratic, the sum and product of its roots are obtained from the coefficients. Here $\alpha+\beta=3/2$ and $\alpha\beta=-i$.
- If $S_n=\alpha^n+\beta^n$, then $S_2=(\alpha+\beta)^2-2\alpha\beta$ and $S_4=S_2^2-2(\alpha\beta)^2$.
- The fourth power of the product is $(\alpha\beta)^4=1$. Consequently, fourth powers of the two roots are reciprocal; this simplifies the high powers without expanding them.

Step 1: Find the fourth-power sum Vieta's relations give $\alpha\beta=-i$, so $(\alpha\beta)^2=-1$. Therefore

$$S_2=\frac{9}{4}+2i.$$

Squaring this expression and subtracting twice the squared product gives

$$S_4=\left(\frac{9}{4}+2i\right)^2+2=\frac{81}{16}+9i-4+2=\frac{49}{16}+9i.$$

The real and imaginary parts must be kept separate during this calculation.

Step 2: Reduce the high-power quotient Since $\alpha^4\beta^4=1$, we have $\alpha^{11}=\alpha^{15}\beta^4$ and $\beta^{11}=\beta^{15}\alpha^4$. Thus the numerator factors exactly as

$$\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}=(\alpha^{15}+\beta^{15})(\alpha^4+\beta^4).$$

The quotient in the question is consequently $49/16+9i$; no approximation to either complex root is required.

Step 3: Multiply the required components The real part is $49/16$ and the imaginary part is 9. Substituting them in the requested expression,

$$16\times\frac{49}{16}\times 9=441.$$

The other proposed values, 312, 409 and 398, do not equal this exact product, so options (B), (C) and (D) are excluded.

Step 4: Conclusion The required value is 441. Hence, the correct option is (A).

Quick Tip to Solve: Before expanding large powers of two roots, check whether a small power of their product equals 1; it may make the powers reciprocal.

8. Let the product of the focal distances of the point $\left(\sqrt{3}, \frac{1}{2}\right)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$, be $\frac{7}{4}$. Then the absolute difference of the eccentricities of two such ellipses is [4 marks]

(A) $\frac{1-2\sqrt{2}}{\sqrt{3}}$

(B) $\frac{1-\sqrt{3}}{\sqrt{2}}$

(C) $\frac{3-2\sqrt{2}}{2\sqrt{3}}$

(D) $\frac{3-2\sqrt{2}}{3\sqrt{2}}$

Correct Answer: (C) $\frac{3-2\sqrt{2}}{2\sqrt{3}}$

Solution:

Concept:

- For a horizontal ellipse with semimajor axis a, the focal distances of a point with abscissa x are $a + ex$ and $a - ex$. Their product is $a^2 - e^2x^2$.
- The eccentricity satisfies $e^2 = 1 - b^2/a^2$. The specified point must also satisfy the ellipse equation, so both conditions are needed to determine the possible ellipses.

Step 1: Express the semiaxes using the given point Put $A = a^2$ and $B = b^2$, with $A > B > 0$. Substitution of $(\sqrt{3}, 1/2)$ in the ellipse gives

$$\frac{3}{A} + \frac{1}{4B} = 1, \qquad B = \frac{A}{4(A-3)}.$$

In particular $A > 3$, which is necessary for a positive value of B.

Step 2: Use the product of focal distances At the given point the focal-distance product is

$$A - 3e^2 = A - 3 + \frac{3B}{A} = \frac{7}{4}.$$

Substitute the expression for B:

$$A + \frac{3}{4(A-3)} = \frac{19}{4}.$$

Multiplication by $4(A-3)$ gives

$$4A^2 - 31A + 60 = 0 = (A-4)(4A-15).$$

Thus the two possible values are $A = 4$ and $A = 15/4$, both of which exceed 3.

Step 3: Find and compare the eccentricities When $A = 4$, we obtain $B = 1$, hence $e_1 = \sqrt{3}/2$. When $A = 15/4$, we obtain $B = 5/4$, hence $e_2 = \sqrt{2/3}$. Both satisfy $A > B > 0$, so both are valid ellipses. Their difference is positive because $3/4 > 2/3$:

$$|e_1 - e_2| = \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{\sqrt{3}} = \frac{3-2\sqrt{2}}{2\sqrt{3}}.$$

Options (A) and (B) are negative and cannot represent an absolute difference. Option (D) has a different denominator and does not equal the computed result.

Step 4: Conclusion The absolute difference is $\frac{3-2\sqrt{2}}{2\sqrt{3}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the product $(a+ex)(a-ex)$ for focal distances; introduce squared semiaxes to avoid square roots until the final step.

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9. Let $\mathrm{y} = \mathrm{y(x)}$ **be the solution of the differential equation** $\left(xy - 5x^2\sqrt{1+x^2}\right)dx + \left(1+x^2\right)dy = 0, y(0) = 0$**. Then** $y(\sqrt{3})$ **is equal to** [4 marks]

(A) $\frac{5\sqrt{3}}{2}$

(B) $\sqrt{\frac{15}{2}}$

(C) $\sqrt{\frac{14}{3}}$

(D) $2\sqrt{2}$

Correct Answer: (A) $\frac{5\sqrt{3}}{2}$

Solution:

Concept:

- A first-order linear differential equation has the form $y' + P(x)y = Q(x)$. Its integrating factor is $\exp\left(\int P(x)\,dx\right)$.
- After multiplication by the integrating factor, the left side becomes the derivative of its product with y. The initial value fixes the integration constant uniquely.

Step 1: Put the equation in linear form Divide the equation by $(1+x^2)\,dx$. As $1+x^2 > 0$ for every real x, this is valid throughout the interval from 0 to $\sqrt{3}$. We obtain

$$\frac{dy}{dx} + \frac{x}{1+x^2}y = \frac{5x^2}{\sqrt{1+x^2}}.$$

Here $P(x) = x/(1+x^2)$; it is not a separable equation in the displayed form.

Step 2: Multiply by the integrating factor and integrate The integrating factor is

$$\exp\left(\int \frac{x}{1+x^2}\,dx\right) = \exp\left(\frac{1}{2}\log(1+x^2)\right) = \sqrt{1+x^2}.$$

Therefore

$$\frac{d}{dx}\left(y\sqrt{1+x^2}\right) = 5x^2.$$

Integration gives

$$y\sqrt{1+x^2} = \frac{5}{3}x^3 + C.$$

At $x = 0$, the condition $y(0) = 0$ yields $C = 0$.

Step 3: Evaluate the solution at the required point The solution on the real line is

$$y(x) = \frac{5x^3}{3\sqrt{1+x^2}}.$$

For $x = \sqrt{3}$, the numerator is $5(3\sqrt{3})$ and the square root in the denominator is 2. Thus

$$y(\sqrt{3}) = \frac{5\sqrt{3}}{2}.$$

The square of this positive value is $75/4$, whereas the squares of options (B), (C) and (D) are respectively $15/2$, $14/3$ and 8. Thus none of those alternatives agrees with the initial-value solution.

Step 4: Conclusion $y(\sqrt{3}) = 5\sqrt{3}/2$. Hence, the correct option is (A).

Quick Tip to Solve: When the coefficient of y is x/(1+x squared), try the integrating factor square root of (1+x squared); it also cancels the radical here.

10. Let in a $\triangle ABC$, the length of the side AC be 6 , the vertex B be $(1, 2, 3)$ and the vertices A, C lie on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Then the area (in sq. units) of $\triangle ABC$ is: [4 marks]

(A) 42

(B) 17

(C) 56

(D) 21

Correct Answer: (D) 21

Solution:

Concept:

- The area of a triangle is one half of a chosen base times the perpendicular distance of the opposite vertex from the line containing that base.
- If a line passes through R and has direction vector $\mathbf{d}$, the distance of B from it is $\|\overrightarrow{RB} \times \mathbf{d}\|/\|\mathbf{d}\|$. This avoids finding the unknown endpoints of the base.

Step 1: Choose the base and read the line data Take $AC = 6$ as the base. Both endpoints lie on the specified line, so the altitude from B is its distance from this line, independently of the individual positions of A and C. Read a convenient point and a direction vector from its symmetric equation:

$$R = (6, 7, 7), \qquad \mathbf{d} = (3, 2, -2).$$

With $B = (1, 2, 3)$,

$$\overrightarrow{RB} = (-5, -5, -4), \qquad \|\mathbf{d}\| = \sqrt{17}.$$

Step 2: Compute the perpendicular height The cross product is

$$\overrightarrow{RB} \times \mathbf{d} = \big(10 + 8,\ -12 - 10,\ -10 + 15\big) = (18, -22, 5).$$

Its norm is

$$\sqrt{18^2 + (-22)^2 + 5^2} = \sqrt{833} = 7\sqrt{17}.$$

Consequently the perpendicular distance, which is the required altitude, equals

$$h = \frac{7\sqrt{17}}{\sqrt{17}} = 7.$$

The cross-product numerator alone is not a distance; division by the direction-vector norm is essential.

Step 3: Apply the triangle-area formula The base has length 6 and the altitude is 7, so

$$\text{Area} = \frac{1}{2} \times 6 \times 7 = 21 \text{ square units.}$$

Option (A), 42, omits the factor one half. Option (B), 17, is the squared norm of the line direction and is not the area. Option (C), 56, also differs from the base-altitude computation. Only option (D) has the correct area.

Step 4: Conclusion The triangle has area 21 square units. Hence, the correct option is (D).

Quick Tip to Solve: If two triangle vertices lie on a known line and their separation is given, use that separation as the base and find only the third vertex's distance from the line.

11. Let the line passing through the points $(-1, 2, 1)$ and parallel to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$ intersect the line $\frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1}$ at the point P. Then the distance of P from the point $Q(4, -5, 1)$ is [4 marks]

(A) $5\sqrt{6}$

(B) 5

(C) $5\sqrt{5}$

(D) 10

Correct Answer: (C) $5\sqrt{5}$

Solution:

Concept:

- A line through a point $\mathbf{r}_0$ parallel to a vector $\mathbf{d}$ is $\mathbf{r} = \mathbf{r}_0 + t\mathbf{d}$. Parallel lines share a direction vector, not necessarily a point.
- Two lines intersect only when all three coordinate equations agree for the same pair of parameters. Once the intersection is known, use the three-dimensional Euclidean distance formula.

Step 1: Parametrise both lines The given parallel line has direction vector $(2, 3, 4)$. Hence the line through $(-1, 2, 1)$ is

$$(x, y, z) = (-1 + 2t,\ 2 + 3t,\ 1 + 4t).$$

The line it intersects is

$$(x, y, z) = (-2 + 3s,\ 3 + 2s,\ 4 + s).$$

Use independent parameters s and t, because the two directions and starting points differ.

Step 2: Solve for and verify the intersection Equality of the first two coordinates gives

$$2t - 3s = -1, \qquad 3t - 2s = 1.$$

Multiplying the first equation by 3 and the second by 2, then subtracting, gives $-5s = -5$, so $s = 1$ and $t = 1$. The third-coordinate check is

$$1 + 4(1) = 4 + 1 = 5.$$

Thus the lines really meet, rather than being skew, and their point of intersection is

$$P = (1, 5, 5).$$

Step 3: Measure the distance to the specified point For $Q = (4, -5, 1)$, the coordinate differences from Q to P are $(-3, 10, 4)$. Therefore

$$PQ = \sqrt{(-3)^2 + 10^2 + 4^2} = \sqrt{125} = 5\sqrt{5}.$$

To check the alternatives directly, the squared distances in options (A), (B), (C) and (D) are 150, 25, 125 and 100. The computed squared distance is 125, so only the third alternative works. No rounding is needed.

Step 4: Conclusion The required distance is $5\sqrt{5}$ units. Hence, the correct option is (C).

Quick Tip to Solve: After finding parameters from two coordinate equations, always check the third equation before declaring that two spatial lines intersect.

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12. A and B alternately throw a pair of dice. A wins if he throws a sum of 5 before B throws a sum of 8 , and B wins if he throws a sum of 8 before A throws a sum of 5 . The probability, that A wins if A makes the first throw, is [4 marks]

(A) $\frac{8}{19}$
(B) $\frac{9}{19}$
(C) $\frac{8}{17}$
(D) $\frac{9}{17}$

Correct Answer: (B) $\frac{9}{19}$

Solution:

Concept:

- Two fair dice have 36 equally likely ordered outcomes. The chance of a specified sum is the number of its ordered outcomes divided by 36.
- When a complete pair of turns ends without a winner, the game restarts in the same state with A to play. This gives either a geometric series or a recurrence for A's winning probability.

Step 1: Find the success probabilities on individual turns A wins on a sum of 5, whose ordered

outcomes are $(1,4),(2,3),(3,2),(4,1)$. Thus

$$p = \frac{4}{36} = \frac{1}{9}.$$

B wins on a sum of 8, whose outcomes are $(2,6),(3,5),(4,4),(5,3),(6,2)$. Thus

$$q = \frac{5}{36}.$$

A failure on a turn means only that the relevant player did not obtain their own target sum; any other sum simply passes the turn.

Step 2: Write the recurrence after a complete unsuccessful round Let P be the probability that A eventually wins when A is about to throw. A either succeeds immediately, with probability p, or both players fail and the same situation starts again. The latter event has probability

$$(1-p)(1-q) = \frac{8}{9} \times \frac{31}{36} = \frac{62}{81}.$$

The throws are independent, so

$$P = \frac{1}{9} + \frac{62}{81}P.$$

There is no additional winning term when B succeeds, since in that case A has lost.

Step 3: Solve and compare with the proposed values Rearrange the recurrence:

$$\frac{19}{81}P = \frac{1}{9}, \qquad P = \frac{9}{19}.$$

Equivalently this is the sum $p\sum_{k=0}^{\infty}[(1-p)(1-q)]^k$, which converges because $62/81 < 1$. Options (A), (C) and (D), namely $8/19$, $8/17$ and $9/17$, do not satisfy the recurrence. Using only the ratio of the one-turn success probabilities would overlook A's first-turn advantage.

Step 4: Conclusion The probability that A wins is $9/19$. Hence, the correct option is (B).

Quick Tip to Solve: For alternating games, group one turn of each player into a round; after both fail, reuse the original winning probability.

13. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c}$ be three vectors such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$. If the vector $\vec{C}$ is perpendicular to $\vec{b}$ and $\vec{a} \cdot \vec{c} = 5$, then $|\vec{c}|$ is equal to [4 marks]

(A) $\sqrt{\frac{11}{6}}$

(B) $\frac{1}{3\sqrt{2}}$

(C) 18

(D) 16

Correct Answer: (A) $\sqrt{\frac{11}{6}}$

Solution:

Concept:

- Coplanarity allows us to write $\vec{c} = s\vec{a} + t\vec{b}$, so there are only two scalar unknowns.
- The dot product gives perpendicularity through $\vec{b} \cdot \vec{c} = 0$. The squared magnitude is $|\vec{c}|^2 = \vec{c} \cdot \vec{c}$.

Step 1: Calculate the required dot products Using the three components of the given vectors, obtain

$$\vec{a} \cdot \vec{a} = 1 + 4 + 9 = 14, \qquad \vec{b} \cdot \vec{b} = 9 + 1 + 1 = 11, \qquad \vec{a} \cdot \vec{b} = 3 + 2 - 3 = 2.$$

These are sufficient to impose both constraints without introducing three separate components for the unknown vector.

Step 2: Use perpendicularity and the prescribed scalar product Substitute $\vec{c} = s\vec{a} + t\vec{b}$. Perpendicularity to $\vec{b}$ gives

$$2s + 11t = 0, \qquad t = -\frac{2s}{11}.$$

The other condition gives

$$14s + 2t = 5 \quad \Longrightarrow \quad \frac{150s}{11} = 5.$$

Hence $s = 11/30$, $t = -1/15$, and $\vec{c} = (11\vec{a} - 2\vec{b})/30$. Both conditions are now satisfied simultaneously.

Step 3: Evaluate the magnitude and compare the alternatives Since $\vec{b} \cdot \vec{c} = 0$, take the dot product of $\vec{c} = s\vec{a} + t\vec{b}$ with $\vec{c}$:

$$|\vec{c}|^2 = s(\vec{a} \cdot \vec{c}) + t(\vec{b} \cdot \vec{c}) = \frac{11}{30}(5) = \frac{11}{6}.$$

Its positive square root is required. Option B has square $1/18$, while options C and D have squares 324 and 256; none can satisfy this magnitude condition.

Step 4: Conclusion The magnitude is $\boxed{\sqrt{11/6}}$. Hence, the correct option is (A).

Quick Tip to Solve: Express a coplanar vector as a linear combination first; after finding the coefficients, dot it with itself using the given constraints to avoid lengthy component squaring.

14. Consider the region $R = \{(x, y) : x \le y \le 9 - \frac{11}{3}x^2, x \ge 0\}$. The area, of the largest rectangle of sides parallel to the coordinate axes and inscribed in R , is: [4 marks]

(A) $\frac{821}{123}$
(B) $\frac{567}{121}$
(C) $\frac{730}{119}$
(D) $\frac{625}{111}$

Correct Answer: (B) $\frac{567}{121}$

Solution:

Concept:

- An axis-parallel rectangle inside a region must satisfy the upper and lower boundary inequalities at every horizontal position.
- Once its width is fixed, maximize its possible height before differentiating the resulting one-variable area.

Step 1: Locate the best rectangle of a fixed right edge Let the rectangle occupy $l \le x \le t$, where $0 \le l < t$. Because the lower boundary $y = x$ increases, its bottom must be at least t. Because $9 - 11x^2/3$ decreases on $x \ge 0$, its top can be at most $9 - 11t^2/3$. Neither height bound depends on l, so the largest width is obtained at $l = 0$.

Step 2: Form and maximize the area The optimal rectangle with right edge $x = t$ therefore has area

$$A(t) = t\left(9 - t - \frac{11}{3}t^2\right).$$

Differentiating yields

$$A\prime(t) = 9 - 2t - 11t^2 = -(11t - 9)(t + 1).$$

The only nonnegative stationary point is $t = 9/11$. Moreover, $A\prime\prime(t) = -2 - 22t < 0$ for $t \ge 0$, so it is a strict maximum.

Step 3: Calculate the maximum and check the choices At the stationary point, the height is

$$9 - \frac{9}{11} - \frac{11}{3}\left(\frac{9}{11}\right)^2 = \frac{63}{11}.$$

Thus

$$A_{\max} = \frac{9}{11} \cdot \frac{63}{11} = \frac{567}{121}.$$

The feasible interval ends where the height vanishes, and its area is zero at either endpoint, confirming the global maximum. The other three fractions differ from this exact product and cannot be the extremal area.

Step 4: Conclusion The largest area is $\boxed{567/121}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: For increasing lower and decreasing upper boundaries on the first quadrant, the right edge controls both height limits; extend the left edge to the axis before differentiating.

15. The area of the region $\{(x, y) : x^2 + 4x + 2 \le y \le |x + 2|\}$ is equal to [4 marks]

(A) 7

(B) 24/5

(C) 20/3

(D) 5

Correct Answer: (C) 20/3

Solution:

Concept:

- The area between two curves is the integral of upper minus lower over the interval where that difference is nonnegative.
- The translation $u = x + 2$ converts the quadratic to $u^2 - 2$ and exposes the symmetry of $|u|$.

Step 1: Find the translated boundaries and their intersections Complete the square:

$$x^2 + 4x + 2 = (x + 2)^2 - 2 = u^2 - 2, \qquad |x + 2| = |u|.$$

Hence a vertical strip has height $|u| - u^2 + 2$. On the nonnegative half-axis the intersections satisfy

$$u^2 - 2 = u \quad \Longrightarrow \quad (u - 2)(u + 1) = 0.$$

The nonnegative intersection is $u = 2$. By symmetry, the other is $u = -2$, corresponding to $x = -4$ and $x = 0$.

Step 2: Integrate the strip height using symmetry Both bounding curves are even functions of u, and translating the coordinate does not change area. On $0 \le u \le 2$, replace $|u|$ by u. Therefore

$$A = 2\int_0^2 (2 + u - u^2)\, du = 2\left[2u + \frac{u^2}{2} - \frac{u^3}{3}\right]_0^2.$$

The height is nonnegative throughout this interval because $2 + u - u^2 = (2 - u)(u + 1)$.

Step 3: Simplify and eliminate the remaining options Insert the endpoints without dropping the factor of two:

$$A = 2\left(4 + 2 - \frac{8}{3}\right) = 2 \cdot \frac{10}{3} = \frac{20}{3}.$$

The choices 7, $24/5$, and 5 are unequal to this definite integral. Integrating only one side would give $10/3$, so doubling is essential for the entire region.

Step 4: Conclusion The required area is $\boxed{20/3}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: When both expressions involve the same shifted variable, translate first. An absolute value and an even quadratic then give a symmetric area, so integrate one half and double.

16. Let $S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \ldots$ upto n terms. If the sum of the first six terms of an A.P. with first term -p and common difference p is $\sqrt{2026\ S_{2025}}$, then the absolute difference betwen 20^{th} and 15^{th} terms of the A.P. is [4 marks]

(A) 20

(B) 45

(C) 90

(D) 25

Correct Answer: (D) 25

Solution:

Concept:

- The denominator pattern is $k(k+1)$, and $1/[k(k+1)] = 1/k - 1/(k+1)$ makes the sum telescope.
- For an arithmetic progression, $T_n = a + (n-1)d$ and $S_n = n[2a + (n-1)d]/2$. Here $a = -p$ and $d = p$.

Step 1: Evaluate the given reciprocal sum The first four denominators are $1 \cdot 2, 2 \cdot 3, 3 \cdot 4, 4 \cdot 5$. Consequently,

$$S_n = \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{k+1} \right) = 1 - \frac{1}{n+1} = \frac{n}{n+1}.$$

In particular, $S_{2025} = 2025/2026$, so the specified positive sum of the first six progression terms is

$$\sqrt{2026 S_{2025}} = \sqrt{2025} = 45.$$

Step 2: Determine the common difference from the six-term sum The progression starts $-p, 0, p, 2p, 3p, 4p$. Adding these six terms, or applying the standard sum formula, gives

$$S_6 = \frac{6}{2}(2(-p) + 5p) = 9p.$$

The stated condition is therefore $9p = 45$, giving $p = 5$. The positive square root fixes its sign; no negative value of p satisfies that equality.

Step 3: Subtract the two specified terms Using $T_n = (n-2)p$, the two terms are $T_{20} = 18p = 90$ and $T_{15} = 13p = 65$. Thus

$$|T_{20} - T_{15}| = |5p| = 25.$$

Option C, 90, is only the twentieth term, not the requested difference. The other values 20 and 45 also fail the term-difference formula; 45 is the given six-term sum.

Step 4: Conclusion The absolute difference is $\boxed{25}$. Hence, the correct option is (D).

Quick Tip to Solve: Telescope the reciprocal series before using the A.P. condition. For any A.P., the difference of the twentieth and fifteenth terms is five common differences.

17. For a statistical data $x_1, x_2, \ldots, x_{10}$ of 10 values, a student obtained the mean as 5.5 and $\sum_{i=1}^{10} x_i^2 = 371$. He later found that he had noted two values in the data incorrectly as 4 and 5 , instead of the correct values 6 and 8 , respectively. The variance of the corrected data is [4 marks]

(A) 5

(B) 7

(C) 9

(D) 4

Correct Answer: (B) 7

Solution:

Concept:

- For n equally weighted observations, the population variance is $\sigma^2 = \sum x_i^2/n - (\sum x_i/n)^2$.
- Correct the total and the sum of squares separately: remove each incorrect observation and insert its replacement. The number of observations remains ten.

Step 1: Correct the sum and calculate the new mean The original mean was calculated from the incorrectly recorded observations. Its corresponding total is

$$\sum x_i = 10(5.5) = 55.$$

Replace the incorrectly entered 4 and 5 with 6 and 8:

$$\sum x_i\big|_{\text{corrected}} = 55 - 4 - 5 + 6 + 8 = 60.$$

Thus the corrected mean is $60/10 = 6$. Adding two new entries would be incorrect because these are replacements, not extra observations.

Step 2: Correct the sum of squared observations The original sum of squares, 371, also contains the two mistaken values. Remove their squares and insert the correct squares:

$$\sum x_i^2\big|_{\text{corrected}} = 371 - 4^2 - 5^2 + 6^2 + 8^2 = 371 - 16 - 25 + 36 + 64 = 430.$$

Notice that the increase in the square sum is 59, not the increase 5 in the ordinary sum. Squaring a change is not the same as changing the squares.

Step 3: Apply the variance identity and compare the options The corrected mean square is $430/10 = 43$, whereas the square of the corrected mean is $6^2 = 36$. Hence

$$\sigma^2 = 43 - 36 = 7.$$

Options A, C, and D give 5, 9, and 4; each differs from the result of the corrected first and second moments. The denominator remains 10 because the question asks for the variance of the complete data, not an unbiased sample estimate.

Step 4: Conclusion The corrected variance is $\boxed{7}$. Hence, the correct option is (B).

Quick Tip to Solve: Keep two running totals in a data-correction problem: the ordinary sum and the square sum. Replace the wrong values in both before recalculating the variance.

18. Let the lines $3x - 4y - \alpha = 0, 8x - 11y - 33 = 0$, and $2x - 3y + \lambda = 0$ be concurrent. If the image of the point $(1, 2)$ in the line $2x - 3y + \lambda = 0$ is $\left(\frac{57}{13}, \frac{-40}{13}\right)$, then $|\alpha\lambda|$ is equal to [4 marks]

(A) 91

(B) 113

(C) 101

(D) 84

Correct Answer: (A) 91

Solution:

Concept:

- A point and its reflection have a midpoint on the reflecting line, and their joining segment is perpendicular to that line.
- Concurrent lines pass through the same point. First determine the reflecting line, then intersect it with the known second line to obtain the remaining constant.

Step 1: Use the midpoint to determine the reflecting line Let $P = (1, 2)$ and $P\prime = (57/13, -40/13)$. Their midpoint is

$$M = \left(\frac{1 + 57/13}{2}, \frac{2 - 40/13}{2}\right) = \left(\frac{35}{13}, -\frac{7}{13}\right).$$

Because it lies on $2x - 3y + \lambda = 0$,

$$\frac{70}{13} + \frac{21}{13} + \lambda = 0, \qquad \lambda = -7.$$

As a check, $P\prime - P = (44/13, -66/13)$ is parallel to the line normal $(2, -3)$, as reflection requires.

Step 2: Find the concurrency point from the two determined lines The last two lines now give

$$8x - 11y = 33, \qquad 2x - 3y = 7.$$

Multiply the second equation by 4 and subtract the first equation to obtain $-y = -5$, so $y = 5$. Back-substitution gives $2x - 15 = 7$, hence $x = 11$. Thus every line in the concurrent family must pass through $(11, 5)$.

Step 3: Recover the other constant and calculate the product Substitute this point into $3x - 4y - \alpha = 0$:

$$\alpha = 3(11) - 4(5) = 13.$$

Therefore

$$|\alpha\lambda| = |13(-7)| = 91.$$

The absolute value removes the negative sign of the product. Options 113, 101, and 84 are inconsistent with the independently fixed constants $\alpha = 13$ and $\lambda = -7$.

Step 4: Conclusion The required value is $\boxed{91}$. Hence, the correct option is (A).

Quick Tip to Solve: A reflected pair gives a midpoint on the mirror line. Use this to find its constant before solving the concurrency equations; check the joining vector against the line normal.

19. If the system of equations

$$2x - y + z = 4$$
$$5x + \lambda y + 3z = 12$$
$$100x - 47y + \mu z = 212$$

has infinitely many solutions, then $\mu - 2\lambda$ is equal to [4 marks]

(A) 56

(B) 59

(C) 57

(D) 55

Correct Answer: (C) 57

Solution:

Concept:

- An infinite family of solutions requires the coefficient and augmented matrices to have the same rank, smaller than the number of unknowns.
- The first two equations are independent because their coefficients of x and their right sides are not proportional. Thus their intersection is a line; the third equation must contain that entire line.

Step 1: Express the third equation as a combination Let the third equation equal a times the first plus b times the second. Matching the coefficient of x and the constant gives

$$2a + 5b = 100, \qquad 4a + 12b = 212.$$

Subtract twice the first equality from the second:

$$2b = 12, \quad b = 6, \quad a = 35.$$

Using the constants here is essential: dependence of coefficient rows alone would allow an inconsistent third equation.

Step 2: Match the remaining coefficients and check consistency The coefficient of y in the combination is $-35 + 6\lambda$. Hence

$$-35 + 6\lambda = -47 \quad \Rightarrow \quad \lambda = -2.$$

Similarly, the coefficient of z must be

$$\mu = 35 + 6(3) = 53.$$

With these values the whole third equation, including its right side, is exactly the stated combination. The first two coefficient rows remain independent, so the common rank is two and one variable is free. Therefore the system really does have infinitely many solutions, rather than merely a zero determinant.

Step 3: Evaluate the requested expression Substitute both parameters, taking care with the negative

sign:

$$\mu - 2\lambda = 53 - 2(-2) = 57.$$

The other numerical options 56, 59 and 55 do not equal this expression; none is compatible with the parameters forced by the row relation.

Step 4: Conclusion The parameters are $\lambda = -2$ and $\mu = 53$, giving $\mu - 2\lambda = 57$. Hence, the correct option is (C).

Quick Tip to Solve: For infinitely many solutions, match an entire augmented row as a linear combination; using only the determinant misses the consistency condition.

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20. Let circle C be the image of $x^2 + y^2 - 2x + 4y - 4 = 0$ in the line $2x - 3y + 5 = 0$ and A be the point on C such that OA is parallel to x-axis and A lies on the right hand side of the centre O of C. If $B(\alpha, \beta)$, with $\beta < 4$, lies on C such that the length of the arc AB is $(1/6)^{\text{th}}$ of the perimeter of C, then $\beta - \sqrt{3}\alpha$ is equal to [4 marks]

(A) $4 - \sqrt{3}$

(B) 3

(C) 4

(D) $3 + \sqrt{3}$

Correct Answer: (C) 4

Solution:

Concept:

- Reflection preserves the radius of a circle and sends its centre to the reflected centre. For a point (h, k) reflected in $ax + by + c = 0$, subtract $2(ah + bk + c)(a, b)/(a^2 + b^2)$.
- An arc that is one sixth of the circumference subtends an angle $2\pi/6 = \pi/3$ at the centre. The condition on the vertical coordinate selects the direction of rotation.

Step 1: Reflect the centre of the given circle Complete squares in the original equation:

$$(x - 1)^2 + (y + 2)^2 = 9.$$

Its centre is $(1, -2)$, and its radius is 3. For the reflecting line, $a = 2$, $b = -3$, $c = 5$, so

$$ah + bk + c = 2 + 6 + 5 = 13, \qquad a^2 + b^2 = 13.$$

The image centre is consequently

$$O = (1, -2) - 2(2, -3) = (-3, 4).$$

The reflected circle still has radius 3; the point at its rightmost end is therefore $A = (0, 4)$.

Step 2: Locate the point with the required arc length The radius OA points along the positive horizontal direction. The two points reached by a one-sixth-circumference arc have angles $\pi/3$ and

$-\pi/3$ relative to it. Because $\beta < 4$, choose the lower point, with angle $-\pi/3$. Thus

$$\alpha = -3 + 3\cos(\pi/3) = -\frac{3}{2},$$

$$\beta = 4 - 3\sin(\pi/3) = 4 - \frac{3\sqrt{3}}{2}.$$

The upper point violates the inequality and cannot be used.

Step 3: Combine the coordinates Now the radical terms cancel:

$$\beta - \sqrt{3}\alpha = 4 - \frac{3\sqrt{3}}{2} + \frac{3\sqrt{3}}{2} = 4.$$

In particular, $4 - \sqrt{3}$, 3 and $3 + \sqrt{3}$ do not result from the reflected centre and the required lower endpoint.

Step 4: Conclusion The required value of $\beta - \sqrt{3}\alpha$ is 4. Hence, the correct option is (C).

Quick Tip to Solve: Reflect the centre first, retain the radius, and use the inequality on the endpoint to choose clockwise or anticlockwise rotation.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let A be a 3×3 matrix such that $\mathrm{X^TAX} = \mathrm{O}$ for all nonzero 3×1 matrices $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. If $\mathrm{A}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}, \mathrm{A}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$, and $\det(\mathrm{adj}(2(\mathrm{A}+\mathrm{I}))) = 2^\alpha 3^\beta 5^\gamma, \alpha, \beta, \gamma \in N$, then $\alpha^2 + \beta^2 + \gamma^2$ is [4 marks]

Solution:

Concept:

- For a real matrix, $X^TAX = 0$ for every vector implies that its symmetric part vanishes: $A^T = -A$. This follows by testing coordinate vectors and their pairwise sums.
- For a three-dimensional matrix M, $\det(\mathrm{adj}\, M) = (\det M)^2$. Also, multiplying all three rows by 2 multiplies the determinant by 2^3.

Step 1: Determine the entries of the skew-symmetric matrix Write

$$A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}.$$

Subtracting the two given matrix-vector equalities isolates the second column:

$$A\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}-1\\0\\-3\end{pmatrix}.$$

Thus $a=-1$ and $c=3$. From the first component of $A(1,1,1)^T=(1,4,-5)^T$, obtain $a+b=1$, hence $b=2$. The other components verify $-a+c=4$ and $-b-c=-5$, so all the supplied conditions agree.

Step 2: Compute the determinant before taking the adjugate Therefore

$$A+I=\begin{pmatrix}1&-1&2\\1&1&3\\-2&-3&1\end{pmatrix}.$$

Expanding along its first row gives

$$\det(A+I)=1(1+9)+1(1+6)+2(-3+2)=15.$$

Consequently, for $M=2(A+I)$,

$$\det M=2^3\cdot 15=120.$$

This scale factor is cubed because M is a 3×3 matrix; using a factor of 2 alone would give wrong prime exponents.

Step 3: Identify the prime exponents Apply the adjugate determinant identity:

$$\det(\operatorname{adj} M)=120^2=(2^3\cdot 3\cdot 5)^2=2^6 3^2 5^2.$$

Unique prime factorisation gives $\alpha=6$, $\beta=2$, $\gamma=2$. Finally,

$$\alpha^2+\beta^2+\gamma^2=36+4+4=44.$$

Step 4: Conclusion The required integer is $\boxed{44}$. The exponents are $(\alpha,\beta,\gamma)=(6,2,2)$.

Quick Tip to Solve: Subtract the given matrix-vector products to extract one column directly, then use skew symmetry before evaluating any determinant.

22. Let f be a differentiable function such that $2(x+2)^2f(x)-3(x+2)^2=10\int_0^x(t+2)f(t)dt, x\geq 0$. Then $f(2)$ is equal to _____ . [4 marks]

Solution:

Concept:

- The fundamental theorem of calculus gives the derivative of $\int_0^x(t+2)f(t)\,dt$ as $(x+2)f(x)$, since the given differentiable function is continuous.
- The original equation at the lower endpoint supplies an initial condition. After differentiating,

retain that condition so that no extraneous integration constant survives.

Step 1: Find the initial value Set $x = 0$. The definite integral has identical upper and lower limits and is zero, so

$$2(2)^2 f(0) - 3(2)^2 = 0.$$

Therefore $8f(0) - 12 = 0$, giving $f(0) = 3/2$. The condition is obtained from the original integral equation, not from its derivative.

Step 2: Differentiate and solve the resulting equation Apply the product rule to both factors on the left and the fundamental theorem to the right:

$$4(x+2)f(x) + 2(x+2)^2 f'(x) - 6(x+2) = 10(x+2)f(x).$$

Because $x \geq 0$, the factor $x + 2$ is nonzero. Divide by $2(x + 2)$ and rearrange:

$$(x+2)f'(x) - 3f(x) = 3.$$

Set $g(x) = f(x) + 1$. Then $(x + 2)g'(x) = 3g(x)$, whose solution is

$$g(x) = C(x+2)^3, \qquad f(x) = C(x+2)^3 - 1.$$

This can also be verified by direct substitution into the differential equation, including the case $C = 0$.

Step 3: Apply the initial condition and evaluate Using $f(0) = 3/2$, obtain

$$\frac{3}{2} = 8C - 1 \quad \Rightarrow \quad C = \frac{5}{16}.$$

Thus

$$f(2) = \frac{5}{16}(4)^3 - 1 = 20 - 1 = 19.$$

Differentiation made the difference between the two original sides constant. Since the initial condition makes that difference zero at $x = 0$, the resulting function satisfies the original equation throughout the given domain.

Step 4: Conclusion The requested value is $\boxed{f(2) = 19}$.

Quick Tip to Solve: Evaluate an integral equation at its lower endpoint before differentiating; that initial value determines the constant in the resulting differential equation.

23. Let $S = \{p_1, p_2 \ldots, p_{10}\}$ be the set of first ten prime numbers. Let $A = S \cup P$, where P is the set of all possible products of distinct elements of S. Then the number of all ordered pairs $(x, y), x \in S$, $y \in A$, such that x divides y, is _____ . [4 marks]

Solution:

Concept:

- Unique prime factorisation associates every product of distinct primes from S with exactly one subset of S. Different subsets cannot give the same product.
- For a fixed prime p_i, a square-free product is divisible by p_i exactly when its subset contains p_i. Each of the remaining nine primes may independently be present or absent.

Step 1: Describe the possible second coordinates Every element of A is a nonempty product of distinct primes drawn from the ten-element set S. The singleton products are already the elements of S; the union ensures that they are counted once, regardless of whether singleton products are included in the definition of P. Thus subsets, rather than permutations of their factors, describe the distinct possible values of y. If an empty product is allowed by convention, its value is 1 and it contributes no divisible pair, so the answer is unaffected.

Step 2: Count the choices for a fixed first coordinate Fix $x = p_i$. For $x \mid y$, the prime p_i must occur in y, since all factors of y belong to S and unique prime factorisation permits no other way for p_i to divide it. With this one prime included, choose any subset of the other nine primes:

$$\#\{y \in A : p_i \mid y\} = \sum_{r=0}^{9} \binom{9}{r} = 2^9 = 512.$$

The choice of no other prime gives $y = p_i$, so no singleton case has been lost.

Step 3: Sum over the possible first coordinates There are ten choices of x, each supplying 512 pairs. The required objects are ordered pairs: a product containing several primes contributes once for each permitted first coordinate. This is legitimate, because different first coordinates give different ordered pairs even if their second coordinates coincide. Hence

$$N = 10 \cdot 512 = 5120.$$

Step 4: Conclusion The total number of ordered pairs is $\boxed{5120}$.

Quick Tip to Solve: Fix the divisor prime first. Include it obligatorily in the product and make independent include-or-omit choices for all remaining primes.

24. If for some $\alpha, \beta; \alpha \leq \beta, \alpha+\beta = 8$ and $\sec^2\left(\tan^{-1}\alpha\right)+\text{cosec}^2\left(\cot^{-1}\beta\right) = 36$, then $\alpha^2+\beta$ is _____

[4 marks]

Solution:

Concept:

- If $\theta = \tan^{-1} u$, then $\sec^2\theta = 1+u^2$. Likewise, if $\phi = \cot^{-1} v$, then $\csc^2\phi = 1+v^2$; these follow directly from the Pythagorean identities.
- Given the sum and sum of squares of two numbers, use $(\alpha+\beta)^2 = \alpha^2+\beta^2+2\alpha\beta$ to find their product. The inequality determines their order.

Step 1: Remove the inverse trigonometric functions Since $\tan(\tan^{-1}\alpha) = \alpha$, the first term is $1+\alpha^2$. Similarly, $\cot(\cot^{-1}\beta) = \beta$, so the second term is $1+\beta^2$. Substituting in the given equality gives

$$(1+\alpha^2)+(1+\beta^2) = 36,$$

$$\alpha^2+\beta^2 = 34.$$

Both identities hold for real inputs with the usual principal inverse functions, so no additional branch choices are introduced by this conversion.

Step 2: Determine the two numbers and assign their order The other given condition is $\alpha+\beta = 8$. Squaring it and using the sum of squares,

$$64 = 34+2\alpha\beta \quad \Rightarrow \quad \alpha\beta = 15.$$

Therefore α and β are the roots of

$$t^2-8t+15 = 0 \quad \Rightarrow \quad (t-3)(t-5) = 0.$$

The unordered pair is $\{3,5\}$, and the condition $\alpha \leq \beta$ selects $\alpha = 3$, $\beta = 5$. Interchanging them would preserve the two symmetric conditions but would change the requested asymmetric expression, so the inequality is essential.

Step 3: Evaluate and check the result For the selected values, the original trigonometric sum is $(1+9)+(1+25) = 36$, and their sum is 8, confirming both conditions. Hence

$$\alpha^2+\beta = 3^2+5 = 14.$$

The quantity is dimensionless, as are the original inverse trigonometric inputs.

Step 4: Conclusion The requested value is $\boxed{14}$, with $\alpha = 3$ and $\beta = 5$.

Quick Tip to Solve: Use the Pythagorean identities before manipulating inverse functions, then apply the ordering condition only after finding the unordered roots.

25. The number of 3 -digit numbers, that are divisible by 2 and 3 , but not divisible by 4 and 9 , is _____.

[4 marks]

Solution:

Concept:

- A number divisible by both 2 and 3 is divisible by their least common multiple, 6. Count multiples in a closed interval by subtracting floor functions.
- The combined exclusion in this question is divisibility by both 4 and 9 together. These common multiples are multiples of their least common multiple, 36.

Step 1: Identify the numbers satisfying the positive condition The three-digit integers run from 100 through 999. Being divisible by 2 and by 3 is equivalent to being a multiple of 6. The first permitted multiple is 102 and the last is 996. Equivalently, the number of candidates is

$$N_6 = \left\lfloor \frac{999}{6} \right\rfloor - \left\lfloor \frac{99}{6} \right\rfloor = 166 - 16 = 150.$$

Using 99 in the second floor function ensures that all multiples below 100, rather than the first three-digit multiple, are removed.

Step 2: Remove the joint divisibility condition The phrase involving 4 and 9 is used here as a joint restriction: exclude a candidate when it is divisible by both of those numbers. Since 4 and 9 are coprime, their common multiples are exactly the multiples of 36. Every such number is already among the multiples of 6, so it can be subtracted directly.

$$N_{36} = \left\lfloor \frac{999}{36} \right\rfloor - \left\lfloor \frac{99}{36} \right\rfloor = 27 - 2 = 25.$$

The first excluded number is 108 and the last is 972. Therefore the required count is

$$N = N_6 - N_{36} = 150 - 25 = 125.$$

This does not exclude a number merely for being divisible by 4 alone or by 9 alone. If the restriction were instead interpreted as requiring divisibility by neither 4 nor 9, inclusion-exclusion would give $150 - 75 - 50 + 25 = 50$; that is a different interpretation of the compound condition.

Step 3: Conclusion With the joint exclusion of numbers divisible by both 4 and 9, the required number of three-digit integers is $\boxed{125}$. This is a count and has no physical unit.

Quick Tip to Solve: Translate each combined divisibility condition into an LCM, then count its multiples using the upper bound minus the count through 99.

26. An object of mass ' m ' is projected from origin in a vertical xy plane at an angle 45° with the x− axis with an initial velocity v_0. The magnitude and direction of the angular momentum of the object with respect to origin, when it reaches at the maximum height, will be [g is acceleration due to gravity] [4 marks]

(A) $\frac{mv_o^3}{2\sqrt{2}g}$ along negative z-axis

(B) $\frac{mv_o^3}{2\sqrt{2}g}$ along positive z-axis

(C) $\frac{mv_o^3}{4\sqrt{2}g}$ along positive z-axis

(D) $\frac{mv_o^3}{4\sqrt{2}g}$ along negative z-axis

Correct Answer: (D) $\frac{mv_o^3}{4\sqrt{2}g}$ along negative z-axis

Solution:

Concept:

- Angular momentum about the origin is $\mathbf{L} = \mathbf{r} \times m\mathbf{v}$. In the xy-plane its only component is $L_z = m(xv_y - yv_x)$.
- At the highest point of projectile motion, vertical velocity is zero, horizontal velocity is unchanged, and maximum height is $v_{0y}^2/(2g)$.

Step 1: Determine the position and velocity at the highest point Resolve the initial velocity at 45° into its horizontal and vertical components:

$$v_{0x} = v_{0y} = \frac{v_0}{\sqrt{2}}.$$

Gravity acts downward, so the vertical velocity becomes zero after $t_H = v_0/(\sqrt{2}g)$. The height and horizontal position at this instant are

$$y_H = \frac{v_0^2}{4g}, \qquad x_H = \frac{v_0^2}{2g}.$$

The instantaneous velocity is entirely horizontal: $\mathbf{v}_H = (v_0/\sqrt{2})\hat{\mathbf{i}}$. In particular, the horizontal momentum is positive even though the projectile has stopped rising.

Step 2: Evaluate the cross product and compare the options At the top, $v_y = 0$. Hence the xv_y contribution vanishes, while the $-yv_x$ contribution remains:

$$\mathbf{L}_H = m\,(x_H v_y - y_H v_x)\,\hat{\mathbf{k}} = -m\frac{v_0^2}{4g}\frac{v_0}{\sqrt{2}}\hat{\mathbf{k}}.$$

Thus

$$\mathbf{L}_H = -\frac{mv_0^3}{4\sqrt{2}g}\hat{\mathbf{k}}.$$

The negative sign follows directly from $\hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}$: the position is above the horizontal momentum line. Options A and B have twice the correct magnitude. Option C has the correct magnitude but

reverses the direction. Only D matches both the cross-product sign and its magnitude. The dimensions are $\mathrm{kg\,m^2\,s^{-1}}$, as required for angular momentum.

Step 3: Conclusion The magnitude is $\boxed{mv_0^3/(4\sqrt{2}g)}$, directed along the negative z-axis. Hence, the correct option is (D).

Quick Tip to Solve: At a projectile's highest point, use $L_z = -my_H v_x$; horizontal displacement makes no contribution because $v_y = 0$.

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27. The Young's double slit interference experiment is performed using light consisting of 480 nm and 600 nm wavelengths to form interference patterns. The least number of the bright fringes of 480 nm light that are required for the first coincidence with the bright fringes formed by 600 nm light is [4 marks]

(A) 8

(B) 4

(C) 5

(D) 6

Correct Answer: (C) 5

Solution:

Concept:

- In Young's experiment the position of the nth bright fringe is $y_n = n\lambda D/d$, where D is the screen distance and d is the slit separation.
- Bright fringes of two wavelengths coincide when their integer orders satisfy $n_1\lambda_1 = n_2\lambda_2$. The central fringe is the common zero-order fringe.

Step 1: Set up the condition for a common bright fringe Both wavelengths pass through the same double slit arrangement, so the geometrical factor D/d is common. Let n denote the order for the 480 nm light and m the order for the 600 nm light. At a coincidence their positions from the central maximum are equal:

$$n(480\,\mathrm{nm})\frac{D}{d} = m(600\,\mathrm{nm})\frac{D}{d}.$$

Cancelling the nonzero common factors gives

$$480n = 600m \quad \Longrightarrow \quad 4n = 5m.$$

The required fringe counts therefore depend only on the wavelength ratio, and not on the actual screen distance or slit separation.

Step 2: Find the smallest positive integer orders Because 4 and 5 are relatively prime, the least positive solution of $4n = 5m$ is

$$n = 5, \qquad m = 4.$$

The fifth bright fringe of the shorter wavelength coincides with the fourth bright fringe of the longer

wavelength. Their common path difference is

$$5(480\,\text{nm}) = 4(600\,\text{nm}) = 2400\,\text{nm}.$$

No lower positive order of the 480 nm pattern makes $480n/600$ an integer. Option B gives the order of the 600 nm light instead of the requested 480 nm light. For options A and D, $n = 8$ or 6 would require $m = 6.4$ or 4.8, which are not integer bright-fringe orders. The common central maximum is not counted as the first noncentral coincidence.

Step 3: Conclusion The first noncentral coincidence requires the fifth bright fringe of 480 nm light. Thus the least fringe order is $\boxed{5}$. Hence, the correct option is (C).

Quick Tip to Solve: Reduce the wavelength ratio first: $480 : 600 = 4 : 5$. The coincidence orders are the reversed coprime ratio, $5 : 4$.

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28. What is the relative decrease in focal length of a lens for an increase in optical power by 0.1 D from 2.5D ? ['D' stands for dioptre] [4 marks]

(A) 0.04

(B) 0.1

(C) 0.40

(D) 0.01

Correct Answer: (A) 0.04

Solution:

Concept:

- Lens power in dioptres and focal length in metres obey $P = 1/f$. An increase in positive power therefore decreases the positive focal length.
- The relative decrease is $(f_i - f_f)/f_i$. For a small power increment, its first-order estimate is $\Delta P/P_i$; the finite-change value uses the final power in the denominator.

Step 1: Calculate the initial and final focal lengths The initial power is $P_i = 2.5\,\text{D}$, and the increase is $\Delta P = 0.1\,\text{D}$. Thus

$$P_f = P_i + \Delta P = 2.6\,\text{D}.$$

Using focal length in metres gives

$$f_i = \frac{1}{2.5} = 0.4000\,\text{m}, \qquad f_f = \frac{1}{2.6} \approx 0.384615\,\text{m}.$$

The positive focal length has decreased by approximately $0.015385\,\text{m}$. This decrease in metres is different from the dimensionless relative decrease requested in the question.

Step 2: Form the fractional decrease and select its rounded value Divide the absolute decrease by the initial focal length:

$$\frac{f_i - f_f}{f_i} = \frac{1/P_i - 1/P_f}{1/P_i} = 1 - \frac{P_i}{P_f} = \frac{P_f - P_i}{P_f}.$$

Substituting the powers,

$$\frac{f_i - f_f}{f_i} = \frac{0.1}{2.6} = \frac{1}{26} \approx 0.03846.$$

At the precision of the listed choices, this is 0.04. The common small-change estimate also gives $\Delta P/P_i = 0.1/2.5 = 0.04$, consistent with that choice. The exact finite-change value is slightly smaller than the estimate. Option B is merely the numerical power increment without normalization; option C corresponds to a 40 percent decrease and is much too large; option D corresponds to only a 1 percent decrease, inconsistent with the calculated focal lengths.

Step 3: Conclusion The relative decrease is $1/26 \approx 0.03846$, or approximately $\boxed{0.04}$ as listed. It is dimensionless. Hence, the correct option is (A).

Quick Tip to Solve: For a finite increase in power, the exact fractional focal-length decrease is $\Delta P/(P_i + \Delta P)$; use $\Delta P/P_i$ only as a small-change estimate.

29. A car of mass ' m ' moves on a banked road having radius ' r ' and banking angle θ. To avoid slipping from banked road, the maximum permissible speed of the car is v_0. The coefficient of friction μ between the wheels of the car and the banked road is [4 marks]

(A) $\mu = \frac{v_0^2 + rg\tan\theta}{rg + v_0^2\tan\theta}$

(B) $\mu = \frac{v_0^2 - rg\tan\theta}{\mathrm{rg} - \mathrm{v}_o^2\tan\theta}$

(C) $\mu = \frac{v_0^2 - rg\tan\theta}{rg + v_0^2\tan\theta}$

(D) $\mu = \frac{v_o^2 + rg\tan\theta}{rg - v_o^2\tan\theta}$

Correct Answer: (C) $\mu = \frac{v_0^2 - rg\tan\theta}{rg + v_0^2\tan\theta}$

Solution:

Concept:

- At the maximum permitted speed a car tends to slip up the bank, so limiting friction acts down the bank with magnitude $f = \mu N$.
- Vertical forces balance, while the net horizontal inward force supplies centripetal acceleration v_0^2/r. The normal reaction is tilted inward by the banking angle.

Step 1: Resolve the normal reaction and limiting friction Take the horizontal inward direction as radial positive and the vertical upward direction as positive. The normal reaction has inward component $N\sin\theta$ and upward component $N\cos\theta$. Down-slope friction has inward component $\mu N\cos\theta$ and downward component $\mu N\sin\theta$. Therefore the two force equations are

$$N\sin\theta + \mu N\cos\theta = \frac{mv_0^2}{r},$$

$$N\cos\theta - \mu N\sin\theta = mg.$$

The negative sign in the vertical friction term is essential: at the upper speed limit, friction acts downward along the incline rather than upward.

Step 2: Eliminate the normal reaction and solve for friction Dividing the radial equation by the vertical equation eliminates both the normal reaction and the mass:

$$\frac{v_0^2}{rg} = \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{\tan\theta + \mu}{1 - \mu\tan\theta}.$$

Cross multiplication and collection of terms give

$$v_0^2 - v_0^2\mu\tan\theta = rg\tan\theta + rg\mu,$$

$$v_0^2 - rg\tan\theta = \mu\left(rg + v_0^2\tan\theta\right).$$

Thus μ is the ratio in option C. Options A and D have the wrong plus sign in the numerator: they do not give zero friction at the frictionless banking speed $v_0^2 = rg\tan\theta$. Option B incorrectly subtracts the denominator term, which results from mishandling the friction contribution when rearranging the equations.

Step 3: Conclusion The required dimensionless coefficient is $\boxed{\mu = \frac{v_0^2 - rg\tan\theta}{rg + v_0^2\tan\theta}}$. Hence, the correct option is (C).

Quick Tip to Solve: At maximum speed, friction points down the bank. Check your final formula by setting $\mu = 0$: it must recover $v_0^2 = rg\tan\theta$.

30. During the transition of electron from state A to state C of a Bohr atom, the wavelength of emitted radiation is $2000\,\overset{o}{A}$ and it becomes $6000\,\overset{o}{A}$ when the electron jumps from state B to state C. Then the wavelength of the radiation emitted during the transition of electrons from state A to state B is [4 marks]

(A) $4000\,\overset{o}{A}$
(B) $6000\,\overset{o}{A}$
(C) $2000\,\overset{o}{A}$
(D) $3000\,\overset{o}{A}$

Correct Answer: (D) $3000\,\overset{o}{A}$

Solution:

Concept:

- A photon emitted in an atomic transition has energy equal to the level-energy difference: $E_\gamma = hc/\lambda$.
- Energy differences add along successive transitions. Since both A and B emit to C and the A-to-C photon has higher energy, the levels satisfy $E_A > E_B > E_C$.

Step 1: Relate the three energy differences The A-to-C transition has wavelength $\lambda_{AC} = 2000\,\text{Å}$, while the B-to-C transition has wavelength $\lambda_{BC} = 6000\,\text{Å}$. Because a shorter wavelength means a

larger photon energy, A is above B. Subtract the B-to-C gap from the A-to-C gap to find the A-to-B gap:

$$E_A - E_B = (E_A - E_C) - (E_B - E_C).$$

Replacing each positive energy difference by its emitted photon energy yields

$$\frac{hc}{\lambda_{AB}} = \frac{hc}{2000\,\text{Å}} - \frac{hc}{6000\,\text{Å}}.$$

One must subtract energies or reciprocal wavelengths, not the wavelengths themselves.

Step 2: Evaluate the reciprocal wavelength and test the choices Cancel the common factor hc and use a common denominator:

$$\frac{1}{\lambda_{AB}} = \frac{1}{2000} - \frac{1}{6000} = \frac{3-1}{6000} = \frac{1}{3000} \quad \text{in Å}^{-1}.$$

Taking the reciprocal gives

$$\lambda_{AB} = 3000\,\text{Å} = 300\,\text{nm}.$$

As a consistency check, the A-to-B photon energy is twice the B-to-C photon energy, and those two energies sum to the A-to-C photon energy. Option A, 4000 angstrom, comes from directly subtracting the given wavelengths, which is not an energy-conservation relation. Options B and C simply repeat the wavelengths of the already specified transitions; neither satisfies the reciprocal-wavelength difference. No principal quantum numbers or Rydberg constant are needed because the energy differences are already encoded in the supplied wavelengths.

Step 3: Conclusion The A-to-B emission wavelength is $\boxed{3000\,\text{Å}}$, equivalently $300\,\text{nm}$. Hence, the correct option is (D).

Quick Tip to Solve: For transitions sharing a lower level, subtract $1/\lambda$, because photon energy is proportional to reciprocal wavelength, not to wavelength.

31. A satellite is launched into a circular orbit of radius ' R ' around the earth. A second satellite is launched into an orbit of radius 1.03 R . The time period of revolution of the second satellite is larger than the first one approximately by [4 marks]

(A) 3%

(B) 2.5%

(C) 4.5%

(D) 9%

Correct Answer: (C) 4.5%

Solution:

Concept:

- For satellites orbiting the same central mass M, gravitational force provides centripetal force: $GMm/r^2 = mv^2/r$. Thus the satellite mass cancels.

- The orbital period is $T = 2\pi\sqrt{r^3/(GM)}$, so $T \propto r^{3/2}$. A small fractional change obeys $\Delta T/T \simeq (3/2)\Delta r/r$.

Step 1: Form the ratio of periods The radii are measured from the same centre of the Earth. Using their ratio avoids needing either the Earth's mass or the radius in metres.

$$\frac{T_2}{T_1} = \left(\frac{1.03R}{R}\right)^{3/2} = (1.03)^{3/2}.$$

The radius increase is $0.03R$, or 3%, rather than an increase of 1.03 per cent.

Step 2: Estimate the fractional increase Apply the first-order binomial expansion $(1+x)^n \simeq 1 + nx$, valid here because $x = 0.03 \ll 1$:

$$\frac{T_2 - T_1}{T_1} \simeq \frac{3}{2}(0.03) = 0.045.$$

Thus the increase is about 4.5%. The exact increase is about 4.53%, confirming the approximation. The 3% option incorrectly makes period proportional to radius; 9% uses the exponent for period squared; 2.5% does not follow Kepler's law.

Step 3: Conclusion The second satellite has a period approximately 4.5% greater than the first. Hence, the correct option is (C).

Quick Tip to Solve: For small orbital-radius changes around the same planet, multiply the percentage radius change by $3/2$ to obtain the percentage period change.

32. A particle is executing simple harmonic motion with time period 2 s and amplitude 1 cm . If D and d are the total distance and displacement covered by the particle in 12.5 s , then $\frac{D}{d}$ is [4 marks]

(A) 10

(B) $\frac{16}{5}$

(C) 25

(D) $\frac{15}{4}$

Correct Answer: Initial phase is unspecified; the keyed answer (C), 25, follows for a mean/extreme start using displacement magnitude.

Solution:

Concept:

- One complete oscillation covers distance $4A$ and produces zero net displacement. The remaining fraction of a cycle must be evaluated using the initial phase.
- Here $t/T = 12.5/2 = 6.25$: six complete cycles plus one quarter-cycle. With $x = A\sin(\omega t + \phi)$, the unspecified phase ϕ affects both the final displacement and the remaining distance.

Step 1: Calculate the conventional keyed result Six full cycles contribute $6(4A) = 24A$ to the distance. If motion starts at the mean position, the next quarter-cycle takes the particle to an extreme, adding

distance A and displacement magnitude A. Hence

$$D = 25A = 25\,\text{cm}, \qquad |d| = A = 1\,\text{cm}, \qquad \frac{D}{|d|} = 25.$$

Starting at an extreme gives the same magnitude ratio. This is the intended keyed choice; 10, 16/5 and 15/4 are inconsistent with this complete-cycle count.

Step 2: Check whether the printed data determine that result The initial position and direction are not specified. For example, take $\phi = \pi/4$. In the remaining quarter-cycle the particle goes from $A/\sqrt{2}$ to A and back to $A/\sqrt{2}$, so its net displacement is zero but its distance is positive:

$$d = 0, \qquad D = 24A + 2A\left(1 - \frac{1}{\sqrt{2}}\right).$$

The ratio is then undefined, although all stated data are satisfied. The sign of a signed displacement creates a further convention issue; distance divided by displacement magnitude is the intended positive ratio.

Step 3: Conclusion The answer key selects (C), 25, obtained when starting at a mean or extreme position and using displacement magnitude. As printed, the initial phase is missing, so a unique ratio is not determined.

Quick Tip to Solve: Count full SHM cycles first, then use the starting phase for any leftover fraction; a quarter-cycle need not always cover distance A.

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33. An air bubble of radius 0.1 cm lies at a depth of 20 cm below the free surface of a liquid of density $1000\ \text{kg/m}^3$. If the pressure inside the bubble is $2100\ \text{N/m}^2$ greater than the atmospheric pressure, then the surface tension of the liquid in SI unit is (use $g = 10\ \text{m/s}^2$) [4 marks]

(A) 0.02

(B) 0.05

(C) 0.25

(D) 0.1

Correct Answer: (B) 0.05

Solution:

Concept:

- At depth h, the pressure in a liquid is $p_{\text{atm}} + \rho g h$. This pressure is external to the air bubble.
- An air bubble in a liquid has one curved liquid-air interface, so its Laplace excess pressure is $2S/r$, where S is surface tension. The $4S/r$ formula instead applies to a thin soap bubble with two interfaces.

Step 1: Subtract the hydrostatic contribution Convert the radius and depth before substitution:

$$r = 0.1\,\text{cm} = 10^{-3}\,\text{m}, \qquad h = 20\,\text{cm} = 0.20\,\text{m}.$$

The surrounding liquid exceeds atmospheric pressure by

$$\rho g h = (1000)(10)(0.20) = 2000\,\text{Pa}.$$

The stated 2100 Pa is the excess over atmospheric pressure, not the excess over the liquid immediately outside the bubble.

Step 2: Find the interfacial excess pressure The pressure balance is

$$p_{\text{in}} - p_{\text{atm}} = \rho g h + \frac{2S}{r}.$$

Consequently

$$\frac{2S}{r} = 2100 - 2000 = 100\,\text{Pa}, \qquad S = \frac{100(10^{-3})}{2} = 0.05\,\text{N m}^{-1}.$$

The other values 0.02, 0.25 and 0.1 do not satisfy this pressure balance. In particular, 0.1 omits the factor of two in the single-interface Laplace pressure.

Step 3: Conclusion The surface tension is $0.05\,\text{N m}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: First subtract $\rho g h$ from the stated excess over atmospheric pressure; only the remainder equals the bubble's Laplace pressure.

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34. An ideal gas goes from an initial state to final state. During the process, the pressure of gas increases linearly with temperature.

A. The work done by gas during the process is zero.

B. The heat added to gas is different from change in its internal energy.

C. The volume of the gas is increased.

D. The internal energy of the gas is increased.

E. The process is isochoric (constant volume process)

Choose the correct answer from the options given below: [4 marks]

(A) A, C Only

(B) A, D, E Only

(C) E Only

(D) A, B, C, D Only

Correct Answer: (B) A, D, E Only

Solution:

Concept:

- For a fixed quantity of ideal gas, $PV = nRT$, with temperature measured on the absolute scale. The proportional pressure-temperature relation of an isochoric process is $P/T = nR/V$.
- Work done by the gas is $W = \int P\,dV$, and the first law in this sign convention is $Q = \Delta U + W$. Ideal-gas internal energy increases with temperature.

Step 1: Identify the constant-volume process The pressure-temperature statement is used in its

intended ideal-gas sense, $P = kT$, for a constant k. Therefore

$$V = \frac{nRT}{P} = \frac{nR}{k} = \text{constant}.$$

The process is isochoric, so statement E is true and C is false. As pressure increases along this relation, absolute temperature increases. A general affine relation $P = aT + b$ with nonzero intercept would not establish constant volume; the offered statements use the standard proportional relation.

Step 2: Test work, heat and internal energy Constant volume means $dV = 0$, giving

$$W = \int P\, dV = 0.$$

Thus A is true. With $W = 0$, the first law gives $Q = \Delta U$, so B is false. Since the temperature rises and the heat capacity is positive, $\Delta U = nC_V \Delta T > 0$; D is true. The combination A and C fails because C is false; E alone omits two true statements; the combination including B and C includes two false statements.

Step 3: Conclusion The correct statements are A, D and E. Hence, the correct option is (B).

Quick Tip to Solve: For an ideal gas, a straight P-versus-T line through the origin means constant volume; then immediately use $W = 0$ and $Q = \Delta U$.

35. Consider a parallel plate capacitor of area A (of each plate) and separation 'd' between the plates. If E is the electric field and ε_0 is the permittivity of free space between the plates, then potential energy stored in the capacitor is [4 marks]

(A) $\varepsilon_0 \mathrm{E}^2 \mathrm{Ad}$

(B) $\frac{3}{4}\varepsilon_0 \mathrm{E}^2 \mathrm{Ad}$

(C) $\frac{1}{4}\varepsilon_0 \mathrm{E}^2 \mathrm{Ad}$

(D) $\frac{1}{2}\varepsilon_0 \mathrm{E}^2 \mathrm{Ad}$

Correct Answer: (D) $\frac{1}{2}\varepsilon_0 \mathrm{E}^2 \mathrm{Ad}$

Solution:

Concept:

- For a vacuum parallel-plate capacitor with negligible edge effects, $C = \varepsilon_0 A/d$. The uniform field and voltage obey $V = Ed$.
- The energy accumulated during charging is $U = \frac{1}{2}CV^2$, equivalently $U = \frac{1}{2}QV$. The factor one-half arises because the capacitor voltage grows from zero while charge is supplied.

Step 1: Express voltage in terms of the field The given E is the electric field between the plates. Since this field is uniform across the separation d, its potential difference has magnitude

$$V = \int_0^d E\, dx = Ed.$$

The symbol A is the area of one plate; the field-filled volume is Ad, not $2Ad$. Thus no extra factor of

two is introduced by the two conducting plates.

Step 2: Substitute into the capacitor energy Combine the capacitance and voltage expressions:

$$U = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\varepsilon_0 E^2 Ad.$$

Equivalently the electric-field energy density $u = \frac{1}{2}\varepsilon_0 E^2$ multiplied by the volume Ad yields the same result. The coefficient 1 would incorrectly use the final voltage for every increment of charge; the coefficients 3/4 and 1/4 do not match either charging integration or the field-energy density.

Step 3: Conclusion The stored potential energy is $U = \frac{1}{2}\varepsilon_0 E^2 Ad$. Hence, the correct option is (D).

Quick Tip to Solve: Remember electric-field energy as density $\frac{1}{2}\varepsilon_0 E^2$ times the field-filled volume; here that volume is Ad.

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36. A force $\mathrm{F} = \alpha + \beta \mathrm{x}^2$ acts on an object in the x -direction. The work done by the force is 5 J when the object is displaced by 1 m . If the constant $\alpha = 1$ N then β will be [4 marks]

(A) $15\ \mathrm{N/m^2}$

(B) $10\ \mathrm{N/m^2}$

(C) $12\ \mathrm{N/m^2}$

(D) $8\ \mathrm{N/m^2}$

Correct Answer: (C) $12\ \mathrm{N/m^2}$

Solution:

Concept:

- For a force along the displacement, work equals the position integral $W = \int F(x)\,dx$. A varying force must be integrated rather than multiplied by the final displacement.
- The constant term contributes αL, whereas the quadratic term contributes $\beta L^3/3$ for travel from $x = 0$ to $x = L$. Since βx^2 is a force, β has units $\mathrm{N\,m^{-2}}$.

Step 1: Integrate over the stated displacement Using the coordinate interval $x = 0$ to $x = 1\,\mathrm{m}$ corresponding to the given displacement,

$$W = \int_0^L (\alpha + \beta x^2)\,dx = \alpha L + \frac{\beta L^3}{3}.$$

The term $\alpha L = (1\,\mathrm{N})(1\,\mathrm{m}) = 1\,\mathrm{J}$. Thus the quadratic-force contribution must supply the remaining $5 - 1 = 4\,\mathrm{J}$.

Step 2: Determine and check the coefficient Substitute $L = 1\,\mathrm{m}$:

$$4\,\mathrm{J} = \frac{\beta(1\,\mathrm{m})^3}{3}, \qquad \beta = 12\,\mathrm{N\,m^{-2}}.$$

A check gives $W = 1 + 12/3 = 5\,\mathrm{J}$. The values 15, 10 and 8 instead give 6, 13/3 and 11/3 joules on this interval, so none fits. Multiplying only the final force by 1 m would overestimate the work

because the force increases throughout the motion. The intended interval starts at the coordinate origin; a different initial coordinate would require that extra datum.

Step 3: Conclusion The force coefficient is $\beta = 12\,\mathrm{N\,m^{-2}}$. Hence, the correct option is (C).

Quick Tip to Solve: Integrate each power of position: an x^2 force contributes $\beta L^3/3$, so do not replace it by its endpoint value.

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37. Consider the following statements :

A. The junction area of solar cell is made very narrow compared to a photo diode.

B. Solar cells are not connected with any external bias.

C. LED is made of lightly doped p-n junction.

D. Increase of forward current results in continuous increase of LED light intensity.

E. LEDs have to be connected in forward bias for emission of light.

Choose the correct answer from the options given below : [4 marks]

(A) A, C, E Only

(B) B, E Only

(C) A, C Only

(D) B, D, E Only

Correct Answer: (B) B, E Only

Solution:

Concept:

- A solar cell converts incident light into electrical energy without an externally applied bias. A large illuminated junction area helps collect more incident light.
- An LED is a heavily doped p-n junction. Under forward bias, injected electrons and holes recombine and emit photons in a suitable direct-band-gap semiconductor.

Step 1: Assess the solar-cell statements Statement A is false: a solar cell needs a comparatively large junction area to intercept light and produce useful electrical power. Making its active junction very narrow is contrary to this requirement. Statement B is true: the light-generated carriers are separated by the built-in junction field, so the cell supplies power without an external bias source. A photodiode used as a detector, in contrast, is commonly operated under reverse bias.

Step 2: Assess the LED statements Statement C is false because an LED is heavily doped, rather than lightly doped. Statement E is true: forward bias injects majority carriers across the junction, enabling radiative recombination. Statement D is false as an unrestricted statement. Light output initially rises as forward current increases, but it reaches a maximum; further increase does not produce a continuous increase and can reduce light output because of heating and loss mechanisms.

Step 3: Match the true statements to the choices Only B and E survive all the tests. Option A includes the false statements A and C, while option C consists entirely of those false statements. Option D includes B and E but additionally includes the false statement D, so it is also rejected. The word “continuous in D must not be replaced by “initial when deciding its truth.

Step 4: Conclusion The correct statements are B and E only. Hence, the correct option is (B).

Quick Tip to Solve: For LED intensity statements, distinguish the initial rise with current from a claim of unlimited or continuous increase.

38. A parallel plate capacitor was made with two rectangular plates, each with a length of $l = 3$ cm and breath of $\mathrm{b} = 1$ cm. The distance between the plates is 3μ m. Out of the following, which are the ways to increase the capacitance by a factor of 10 ?

A. $l = 30$ cm, $\mathrm{b} = 1$ cm, $\mathrm{d} = 1\mu$ m

B. $l = 3$ cm, $\mathrm{b} = 1$ cm, $\mathrm{d} = 30\mu$ m

C. $l = 6$ cm, $\mathrm{b} = 5$ cm, $\mathrm{d} = 3\mu$ m

D. $l = 1$ cm, $\mathrm{b} = 1$ cm, $\mathrm{d} = 10\mu$ m

E. $l = 5$ cm, $\mathrm{b} = 2$ cm, $\mathrm{d} = 1\mu$ m

Choose the correct answer from the options given below: [4 marks]

(A) C only

(B) A only

(C) B and D only

(D) C and E only

Correct Answer: (D) C and E only

Solution:

Concept:

- For a parallel-plate capacitor with unchanged dielectric, $C = \varepsilon A/d = \varepsilon lb/d$, where l, b are plate dimensions and d is their separation.
- Compare each proposed capacitor with the original using a dimensionless ratio. Lengths may remain in centimetres and separations in micrometres because each ratio uses the same units in numerator and denominator.

Step 1: Construct the capacitance ratio The original plate area is $A_0 = 3 \times 1 = 3\,\mathrm{cm}^2$ and the original separation is $d_0 = 3\,\mu\mathrm{m}$. Thus, for each proposal,

$$\frac{C}{C_0} = \frac{lb}{3 \times 1}\frac{3}{d},$$

where the numerical value of d is expressed in micrometres. The required result is exactly ten, so changing just one dimension cannot be judged without also checking the separation.

Step 2: Test all five proposed changes For A, $C/C_0 = (30/3)(3/1) = 30$, which exceeds the required factor. For B, $C/C_0 = (3/3)(3/30) = 1/10$, so the capacitance decreases. For C, $C/C_0 = (30/3)(3/3) = 10$. For D, $C/C_0 = (1/3)(3/10) = 1/10$. Finally, for E, $C/C_0 = (10/3)(3/1) = 10$. Therefore both C and E give the specified increase, although they use different combinations of plate area and separation.

Step 3: Select the complete combination Option A omits E, which also meets the condition. Option B

incorrectly accepts proposal A, whose increase is thirtyfold. Option C contains B and D, both of which reduce capacitance to one tenth of its initial value. Only option D lists exactly the two successful proposals, C and E.

Step 4: Conclusion Proposals C and E each increase capacitance by a factor of 10. Hence, the correct option is (D).

Quick Tip to Solve: Evaluate lb/d relative to its original value; an increased plate area can be offset or amplified by a simultaneous change in separation.

39. A thin plano convex lens made of glass of refractive index 1.5 is immersed in a liquid of refractive index 1.2. When the plane side of the lens is silver coated for complete reflection, the lens immersed in the liquid behaves like a concave mirror of focal length 0.2 m . The radius of curvature of the curved surface of the lens is [4 marks]

(A) 0.15 m

(B) 0.10 m

(C) 0.25 m

(D) 0.20 m

Correct Answer: (B) 0.10 m

Solution:

Concept:

- The focal length f of a thin plano-convex lens in a medium obeys $1/f = (n_g/n_l - 1)/R$, where R is the magnitude of the curved-surface radius.
- With the plane face silvered, light traverses the lens twice. The plane mirror adds no focusing power, so the equivalent reflecting system has $1/F = 2/f$, using positive magnitudes for the converging focal lengths.

Step 1: Calculate the relative refractive index The surrounding liquid must be included in the lens-maker formula. The relative refractive index of glass is

$$n_{\text{rel}} = \frac{1.5}{1.2} = 1.25.$$

The plane face has infinite radius of curvature and therefore contributes zero to the curvature difference. Consequently, the one-pass focal length satisfies

$$\frac{1}{f} = \frac{1.25 - 1}{R} = \frac{0.25}{R}, \qquad f = 4R.$$

Using the refractive index relative to air here would overestimate the lens power.

Step 2: Account for the return traversal An incident paraxial beam refracts at the curved face, reflects from the silvered plane face, and refracts again on returning through the curved face. In the thin-lens limit the two lens powers add. The plane reflector has infinite focal length and hence zero optical

power of its own. Therefore,

$$\frac{1}{F} = \frac{2}{f} = \frac{0.50}{R}, \qquad F = 2R.$$

The system behaves as a converging concave mirror, with the given focal-length magnitude $F = 0.20\,\text{m}$.

Step 3: Find and check the radius Substituting gives $R = F/2 = 0.10\,\text{m}$. As a check, this radius makes the one-pass focal length $0.40\,\text{m}$; doubling its power gives a reflecting focal length of $0.20\,\text{m}$. Radii 0.15 m, 0.25 m and 0.20 m would instead give reflecting focal lengths 0.30 m, 0.50 m and 0.40 m, so the remaining choices fail.

Step 4: Conclusion The curved surface has radius $R = 0.10\,\text{m}$. Hence, the correct option is (B).

Quick Tip to Solve: For a plane-silvered thin lens, first find its focal length in the surrounding medium, then halve it for the double-pass reflecting system.

40. A plano-convex lens having radius of curvature of first surface 2 cm exhibits focal length of f_1 in air. Another plano-convex lens with first surface radius of curvature 3 cm has focal length of f_2 when it is immersed in a liquid of refractive index 1.2. If both the lenses are made of same glass of refractive index 1.5 , the ratio of f_1 and f_2 will be [4 marks]

(A) $2 : 3$

(B) $1 : 3$

(C) $1 : 2$

(D) $3 : 5$

Correct Answer: (B) $1 : 3$

Solution:

Concept:

- For a thin plano-convex lens, $1/f = (n_g/n_m - 1)/R$. Here n_g is the glass index, n_m is the surrounding-medium index and the plane surface has infinite curvature radius.
- The focal length therefore depends on both curvature and relative refractive index. Comparing radii alone is valid only when the relative indices are also the same.

Step 1: Find the first focal length in air For the first lens the surrounding index is one, and the curved-surface radius is $R_1 = 2\,\text{cm}$. Using the lens-maker relation with positive magnitudes for this converging lens,

$$\frac{1}{f_1} = \frac{1.5 - 1}{2\,\text{cm}} = \frac{1}{4\,\text{cm}}, \qquad f_1 = 4\,\text{cm}.$$

The absence of curvature at the second, plane face means that there is no additional finite-radius term to include.

Step 2: Find the second focal length in liquid The refractive index that controls refraction for the immersed lens is the ratio $n_g/n_m = 1.5/1.2 = 1.25$, not the glass index 1.5 by itself. Its radius is $R_2 = 3\,\text{cm}$, so

$$\frac{1}{f_2} = \frac{1.25 - 1}{3\,\text{cm}} = \frac{1}{12\,\text{cm}}, \qquad f_2 = 12\,\text{cm}.$$

The liquid reduces the index contrast and consequently weakens the lens compared with its behaviour in air.

Step 3: Form the ratio and exclude the distractors The requested order is $f_1 : f_2$, hence

$$f_1 : f_2 = 4 : 12 = 1 : 3.$$

The choice 2:3 compares the curvature radii while ignoring the changed surrounding medium. The choices 1:2 and 3:5 do not satisfy the two independently computed focal lengths. In particular, taking the ratio in reverse would also be incorrect because the question explicitly specifies the first focal length before the second.

Step 4: Conclusion The focal-length ratio is $f_1 : f_2 = 1 : 3$. Hence, the correct option is (B).

Quick Tip to Solve: Write $f = R/(n_g/n_m - 1)$ separately for each lens before forming the ratio; never use the glass index alone for an immersed lens.

41. An electron of mass ' m ' with an initial velocity $\vec{v} = v_0\hat{i}\,(v_0 > 0)$ enters an electric field $\vec{E} = -E_o\hat{k}$. If the initial de Broglie wavelength is λ_0, the value after time t would be [4 marks]

(A) $\frac{\lambda_o}{\sqrt{1-\frac{e^2E_o^2t^2}{m^2v_o^2}}}$

(B) λ_0

(C) $\frac{\lambda_o}{\sqrt{1+\frac{e^2E_o^2t^2}{m^2v_o^2}}}$

(D) $\lambda_o\sqrt{1+\frac{e^2E_o^2t^2}{m^2v_o^2}}$

Correct Answer: (C) $\frac{\lambda_o}{\sqrt{1+\frac{e^2E_o^2t^2}{m^2v_o^2}}}$

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, with momentum magnitude $p = mv$ in the nonrelativistic description used here.
- An electron has charge $-e$, where $e > 0$. A field $\vec{E} = -E_0\hat{k}$ therefore produces force $\vec{F} = +eE_0\hat{k}$, perpendicular to its initial momentum.

Step 1: Obtain the momentum components Initially the electron moves along the positive x-axis, so $p_x = mv_0$, $p_y = 0$, and $p_z = 0$. The electric force has no x-component, so the initial x-momentum remains unchanged. The constant z-force changes the z-momentum according to

$$p_z = eE_0t.$$

The sign follows from multiplying the negative electron charge by the negative z-directed electric field; overlooking either negative sign reverses the acceleration direction, although it does not change the magnitude calculated next.

Step 2: Calculate the momentum magnitude The two nonzero components are perpendicular, so their magnitudes must be added in quadrature, rather than algebraically:

$$p(t) = \sqrt{m^2v_0^2 + e^2E_0^2t^2} = mv_0\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}}.$$

Since $\lambda_0 = h/(mv_0)$, division of Planck's constant by this new momentum directly yields the required wavelength.

Step 3: Express the wavelength and check the choices Thus

$$\lambda(t) = \frac{\lambda_0}{\sqrt{1 + \dfrac{e^2E_0^2t^2}{m^2v_0^2}}}.$$

At $t = 0$ this gives λ_0, and at later times the momentum grows, so the wavelength decreases. Option A incorrectly subtracts the squared impulse; option B ignores the additional momentum; option D increases the wavelength when momentum increases. Only option C has both the correct vector sum and the correct inverse dependence on momentum.

Step 4: Conclusion The wavelength is $\lambda_0/\sqrt{1 + e^2E_0^2t^2/(m^2v_0^2)}$. Hence, the correct option is (C).

Quick Tip to Solve: When a force is perpendicular to the initial velocity, add the new and initial momentum components in quadrature before applying $\lambda = h/p$.

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42. The amount of work done to break a big water drop of radius ' R ' into 27 small drops of equal radius is 10 J . The work done required to break the same big drop into 64 small drops of equal radius will be [4 marks]

(A) 20 J

(B) 10 J

(C) 5 J

(D) 15 J

Correct Answer: (D) 15 J

Solution:

Concept:

- Breaking a liquid drop creates new surface. For constant surface tension T, the work required is the increase in surface energy, $W = T(A_f - A_i)$.
- Liquid volume is conserved: if one drop of radius R forms N equal drops of radius r, then $Nr^3 = R^3$, so $r = R/N^{1/3}$.

Step 1: Find the surface-area increase for an arbitrary number of drops The original area is $A_i = 4\pi R^2$. After division, the total area of all drops is

$$A_f = N\,4\pi r^2 = 4\pi R^2 N^{1/3}.$$

It follows that

$$W_N = 4\pi T R^2 \left(N^{1/3} - 1\right).$$

The subtraction of one represents the surface area already present on the original drop. Work is needed for the additional surface, rather than for the entire final surface area.

Step 2: Use the known work for twenty-seven drops For $N = 27$, the cube root is three and each small radius is $R/3$. Hence

$$W_{27} = 4\pi T R^2(3 - 1) = 8\pi T R^2 = 10\,\text{J}.$$

There is no need to determine either T or R separately, because the same original water drop and the same surface tension appear in the second fragmentation process.

Step 3: Calculate the work for sixty-four drops For $N = 64$, the cube root is four and each small radius is $R/4$. Comparing with the first process gives

$$\frac{W_{64}}{W_{27}} = \frac{4-1}{3-1} = \frac{3}{2}, \qquad W_{64} = \frac{3}{2} \times 10 = 15\,\text{J}.$$

The options 20 J, 10 J and 5 J do not have this ratio. In particular, equal work would ignore the greater area created in forming sixty-four drops, while direct proportionality to the number of drops would ignore the change in each drop's radius.

Step 4: Conclusion The required work is $15\,\text{J}$. Hence, the correct option is (D).

Quick Tip to Solve: For equal daughter drops, use $W \propto N^{1/3} - 1$, not N or $N^{1/3}$; the original surface must be subtracted.

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43. A uniform solid cylinder of mass ' m ' and radius ' r ' rolls along an inclined rough plane of inclination 45°. If it starts to roll from rest from the top of the plane then the linear acceleration of the cylinder's axis will be [4 marks]

(A) $\sqrt{2}$ g

(B) $\frac{1}{\sqrt{2}}$ g

(C) $\frac{1}{3\sqrt{2}}$ g

(D) $\frac{\sqrt{2}g}{3}$

Correct Answer: (D) $\frac{\sqrt{2}g}{3}$

Solution:

Concept:

- For rolling without slipping, translation of the centre and rotation about the centre are coupled by $a = \alpha r$. Static friction supplies the torque.
- The moment of inertia of a uniform solid cylinder about its symmetry axis is $I = \frac{1}{2}mr^2$. Its acceleration on an incline is $a = g\sin\theta/(1 + I/(mr^2))$.

Step 1: Write the translational and rotational equations Take downhill as positive. The component of

weight down the incline is $mg\sin\theta$; static friction f acts uphill and turns the cylinder. Thus

$$mg\sin\theta - f = ma, \qquad fr = I\alpha.$$

Using $\alpha = a/r$ and $I = mr^2/2$, the friction required for rolling is

$$f = \frac{Ia}{r^2} = \frac{ma}{2}.$$

The friction does not simply cancel the downhill force: part of the gravitational effect produces rotational acceleration.

Step 2: Substitute the angle and compare the options Eliminating friction gives

$$mg\sin\theta = ma + \frac{ma}{2} = \frac{3}{2}ma.$$

At $\theta = 45^\circ$,

$$a = \frac{2}{3}g\sin 45^\circ = \frac{2g}{3\sqrt{2}} = \frac{\sqrt{2}g}{3}.$$

The mass and radius cancel because the shape fixes $I/(mr^2)$. Option (B) is the frictionless sliding acceleration $g\sin 45^\circ$, so it neglects rotation. Option (C) is half the required result. Option (A) even exceeds g, whereas the downhill component of gravity is smaller than g.

Step 3: Conclusion The cylinder axis accelerates down the incline with magnitude $\boxed{a = \sqrt{2}g/3}$. Hence, the correct option is (D).

Quick Tip to Solve: For rolling bodies, insert the shape factor $I/(mr^2)$ before substituting the angle; a solid cylinder has shape factor $1/2$.

44. An alternating current is given by $\mathrm{I} = \mathrm{I_A}\sin\omega\mathrm{t} + \mathrm{I_B}\cos\omega\mathrm{t}$. The r.m.s current will be [4 marks]

(A) $\frac{\sqrt{\mathrm{I_A^2}+\mathrm{I_B^2}}}{2}$

(B) $\sqrt{\frac{\mathrm{I_A^2}+\mathrm{I_B^2}}{2}}$

(C) $\frac{|\mathrm{I_A}+\mathrm{I_B}|}{\sqrt{2}}$

(D) $\sqrt{\mathrm{I_A^2}+\mathrm{I_B^2}}$

Correct Answer: (B) $\sqrt{\frac{\mathrm{I_A^2}+\mathrm{I_B^2}}{2}}$

Solution:

Concept:

- The root mean square current is $I_{\text{rms}} = \sqrt{\langle I^2\rangle}$, where the average is taken over one complete cycle.
- Over a complete cycle, $\langle\sin^2\omega t\rangle = \langle\cos^2\omega t\rangle = 1/2$ and $\langle\sin\omega t\cos\omega t\rangle = 0$.

Step 1: Square the complete expression before averaging The two components are at the same

frequency but differ in phase by 90°. Their instantaneous values add algebraically:

$$I = I_A \sin \omega t + I_B \cos \omega t.$$

Squaring this expression retains the cross term:

$$I^2 = I_A^2 \sin^2 \omega t + I_B^2 \cos^2 \omega t + 2 I_A I_B \sin \omega t \cos \omega t.$$

The current may be positive or negative during the cycle, but the heating effect and the RMS definition depend on its square.

Step 2: Average over one period and take the square root The cross term has zero mean because $\sin \omega t \cos \omega t = \frac{1}{2} \sin 2\omega t$, whose integral over one period vanishes. Therefore

$$\langle I^2 \rangle = \frac{I_A^2}{2} + \frac{I_B^2}{2}, \qquad I_{\text{rms}} = \sqrt{\frac{I_A^2 + I_B^2}{2}}.$$

Option (C) adds amplitudes as if they were in phase, which they are not. Option (D) is the peak amplitude of the resultant sinusoid, rather than its RMS value. Option (A) divides that peak by 2 rather than $\sqrt{2}$. Setting $I_B = 0$ also confirms that the answer must reduce to $|I_A|/\sqrt{2}$.

Step 3: Conclusion The required RMS current is $\boxed{\sqrt{(I_A^2 + I_B^2)/2}}$. Hence, the correct option is (B).

Quick Tip to Solve: Sine and cosine components of one frequency are in quadrature: add their squared amplitudes, then divide the resulting amplitude by $\sqrt{2}$ for RMS.

45. For an experimental expression $y = \frac{32.3 \times 1125}{27.4}$, where all the digits are significant. Then to report the value of y we should write [4 marks]

(A) $y = 1326.186$

(B) $y = 1326.2$

(C) $y = 1326.19$

(D) $y = 1330$

Correct Answer: (D) $y = 1330$

Solution:

Concept:

- For a product or quotient of measured quantities, report the result with the number of significant figures of the least precise input.
- Significant figures describe relative precision. Decimal places are used for a different rule, namely addition and subtraction. Keep guard digits until the final rounding.

Step 1: Identify the limiting input precision The expression is $y = (32.3 \times 1125)/27.4$. The factor 32.3 has three significant figures, 1125 has four, and the divisor 27.4 has three. All their displayed digits are significant as stated in the question. Since multiplication and division are involved, the

final result must be reported to three significant figures. The four-digit input does not increase the precision supplied by either three-digit input.

Step 2: Calculate first and round once Without rounding intermediate values,

$$y = \frac{32.3 \times 1125}{27.4} = \frac{36337.5}{27.4} = 1326.18613\ldots.$$

The first three significant digits are 1, 3 and 2. The next digit is 6, so the third significant digit rounds upward:

$$y = 1.33 \times 10^3 = 1330.$$

Writing scientific notation makes the intended three significant figures unambiguous. Among the printed choices, (D) is this rounded value. Option (A) retains seven significant figures, (B) retains five, and (C) retains six; all claim more precision than the measured factors support. Rounding to a fixed number of decimal places would therefore apply the wrong reporting rule.

Step 3: Conclusion Report $\boxed{y = 1.33 \times 10^3}$, written as 1330 in the options. Hence, the correct option is (D).

Quick Tip to Solve: Count significant figures in each input before calculating; carry extra digits through the arithmetic and round the final product or quotient only once.

Physics – Section B: Numerical Value Questions (20 marks)

46. A wire of resistance 9Ω is bent to form an equilateral triangle. Then the equivalent resistance across any two vertices will be _____ ohm. [4 marks]

Solution:

Concept:

- For a uniform wire of fixed material and cross-section, resistance is proportional to length: $R = \rho L/A$. Equal lengths therefore have equal resistances.
- Resistors in series add directly, while two branches between the same pair of nodes combine in parallel: $R_{\text{eq}} = R_1R_2/(R_1 + R_2)$.

Step 1: Divide the wire resistance among the three sides The entire wire has resistance $9\,\Omega$. Bending it into an equilateral triangle divides its length into three equal sides without changing the material or cross-section. Hence each side has resistance

$$R_{\text{side}} = \frac{9}{3} = 3\,\Omega.$$

Choose two vertices A and B as the terminals, and name the remaining vertex C. By symmetry the result will be the same for any other pair of vertices.

Step 2: Identify the two paths between the terminals Current entering at A can flow directly along side AB, whose resistance is $3\,\Omega$, or around $A \to C \to B$. The latter path contains two side resistances in series, so it has resistance $3 + 3 = 6\,\Omega$. Both paths start at A and end at B, and consequently have

the same potential difference: they are parallel branches. Thus

$$\frac{1}{R_{AB}} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}, \qquad R_{AB} = \frac{3 \times 6}{3+6} = 2\,\Omega.$$

The third vertex is simply an internal junction; it is not a third external terminal. The result being smaller than $3\,\Omega$ is a useful check, since the longer path provides an additional route for current.

Step 3: Conclusion The equivalent resistance between any two vertices is $\boxed{2\,\Omega}$, so the value to enter is $\boxed{2}$.

Quick Tip to Solve: For a triangular loop measured across two vertices, put one side in parallel with the other two sides in series; do not add all three sides.

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47. A current of 5 A exists in a square loop of side $\frac{1}{\sqrt{2}}$ m. Then the magnitude of the magnetic field B at the centre of the square loop will be $p \times 10^{-6}$ T. where, value of p is _____ [Take $\mu_0 = 4\pi \times 10^{-7}$ T mA^{-1}]. [4 marks]

Solution:

Concept:

- A finite straight wire carrying current I produces field $B = \mu_0 I(\sin\theta_1 + \sin\theta_2)/(4\pi d)$, where d is the perpendicular distance from the wire.
- At the centre of a square loop, the fields due to all four sides have equal magnitudes and the same direction normal to the loop, so they add.

Step 1: Find the geometry for one side Let the square side be $a = 1/\sqrt{2}\,\text{m}$. The centre lies at perpendicular distance $d = a/2$ from each side. The perpendicular bisects that side, so the two endpoint angles measured from the perpendicular satisfy

$$\tan\theta_1 = \tan\theta_2 = \frac{a/2}{a/2} = 1.$$

Therefore $\theta_1 = \theta_2 = 45^\circ$. The right-hand rule shows that for a single circulating current, all four sides produce fields in the same sense at the centre.

Step 2: Add the four contributions and substitute the data For one side,

$$B_{\text{side}} = \frac{\mu_0 I}{4\pi(a/2)}(\sin 45^\circ + \sin 45^\circ) = \frac{\sqrt{2}\mu_0 I}{2\pi a}.$$

Thus the field of the complete loop is

$$B = 4B_{\text{side}} = \frac{2\sqrt{2}\mu_0 I}{\pi a}.$$

Using $I = 5\,\text{A}$, $a = 1/\sqrt{2}\,\text{m}$, and the specified permeability gives

$$B = \frac{2\sqrt{2}(4\pi \times 10^{-7})(5)}{\pi(1/\sqrt{2})} = 80 \times 10^{-7}\,\text{T} = 8 \times 10^{-6}\,\text{T}.$$

The finite-wire endpoint angles are essential; treating each side as an infinitely long wire would overestimate its contribution. The coefficient p is obtained only after expressing the answer in the requested power of ten.

Step 3: Conclusion The magnetic field magnitude is $\boxed{8 \times 10^{-6}\,\text{T}}$. Hence, in $B = p \times 10^{-6}\,\text{T}$, the requested value is $\boxed{p = 8}$.

Quick Tip to Solve: At a square loop centre, each side subtends endpoint angles of $45°$; calculate one finite-side field and multiply by four.

48. The least count of a screw guage is 0.01 mm . If the pitch is increased by 75% and number of divisions on the circular scale is reduced by 50%, the new least count will be _____ $\times 10^{-3}$ mm

[4 marks]

Solution:

Concept:

- The least count of a screw gauge is $L = P/N$, where P is the pitch, the axial advance in one complete turn, and N is the number of circular-scale divisions.
- For simultaneous percentage changes, use multiplicative factors for the numerator and denominator separately. A larger pitch and fewer divisions both increase the least count.

Step 1: Convert the two percentage changes into factors Let the original pitch and number of divisions be P and N. The original least count is

$$L = \frac{P}{N} = 0.01\,\text{mm}.$$

Increasing the pitch by 75% makes the new pitch

$$P' = P + 0.75P = 1.75P.$$

Reducing the circular-scale divisions by 50% leaves half of them, so $N' = 0.50N$. The percentage changes act on different quantities and cannot simply be added to obtain the percentage change of their ratio.

Step 2: Calculate the new least count and convert the scale The new least count is the axial motion corresponding to one of the new circular divisions:

$$L' = \frac{P'}{N'} = \frac{1.75P}{0.50N} = 3.5\frac{P}{N} = 3.5L.$$

Therefore

$$L' = 3.5(0.01\,\text{mm}) = 0.035\,\text{mm} = 35 \times 10^{-3}\,\text{mm}.$$

The factor is greater than unity, as expected: in one turn the screw now travels farther, while the turn is split into fewer readings. Notice that 0.035 is the value in millimetres, whereas the blank asks for the coefficient multiplying $10^{-3}\,\text{mm}$. Confusing these two representations would shift the answer by a factor of one thousand.

Step 3: Conclusion The new least count is $\boxed{0.035\,\text{mm} = 35 \times 10^{-3}\,\text{mm}}$. The requested value in the blank is $\boxed{35}$.

Quick Tip to Solve: Use $L'/L = (P'/P)/(N'/N)$; a reduction of divisions by half doubles the least count before the pitch change is included.

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49. The temperature of 1 mole of an ideal monoatomic gas is increased by 50°C at constant pressure. The total heat added and change in internal energy are E_1 and E_2, respectively. If $\frac{E_1}{E_2} = \frac{x}{9}$ then the value of x is _____. [4 marks]

Solution:

Concept:

- For an ideal monatomic gas, only three translational degrees of freedom contribute, so the molar heat capacity at constant volume is $C_V = 3R/2$.
- The internal-energy change is $\Delta U = nC_V\Delta T$. At constant pressure, supplied heat is $Q = nC_P\Delta T$, with $C_P = C_V + R = 5R/2$.

Step 1: Identify the relevant heat capacities The gas has one mole and its temperature rises by 50°C. A temperature difference has the same numerical value in kelvin and Celsius, so $\Delta T = 50\,\text{K}$, not $323\,\text{K}$. Monatomic ideal-gas molecules have no rotational or vibrational contribution in this treatment. Therefore the energy stored as microscopic kinetic energy is determined by $C_V = 3R/2$, even though the actual process takes place at constant pressure.

Step 2: Calculate heat supplied and energy stored The piston or boundary can move in a constant-pressure heating process, so some supplied heat performs expansion work. Substituting $n = 1$ gives

$$E_1 = nC_P\Delta T = 1 \times \frac{5R}{2} \times 50 = 125R\,\text{J},$$

$$E_2 = nC_V\Delta T = 1 \times \frac{3R}{2} \times 50 = 75R\,\text{J}.$$

The difference is $E_1 - E_2 = 50R\,\text{J} = nR\Delta T$, exactly the constant-pressure work, which checks the first law.

Step 3: Compare the ratio with the stated form Both the amount of gas and its temperature rise cancel in the ratio:

$$\frac{E_1}{E_2} = \frac{125R}{75R} = \frac{5}{3} = \frac{x}{9}.$$

Cross-multiplication gives $3x = 45$, hence $x = 15$. This ratio exceeds unity because heat supplied

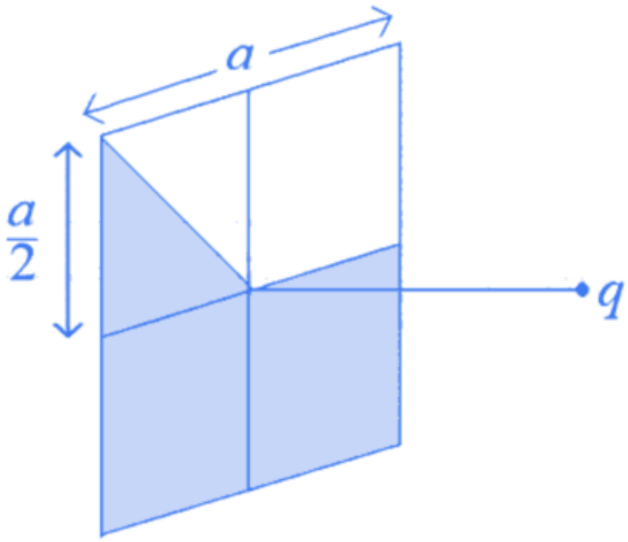
includes work as well as the internal-energy increase.

Step 4: Conclusion The heat-to-internal-energy ratio is $5/3$; therefore the required dimensionless numerical value is $\boxed{x = 15}$.

Quick Tip to Solve: For constant-pressure heating of a monatomic ideal gas, use $Q/\Delta U = C_P/C_V = 5/3$; the temperature rise cancels.

50. A square loop of sides $a = 1$ m is held normally in front of a point charge $\mathrm{q} = 1\mathrm{C}$ at a distance $\frac{\mathrm{a}}{2}$. The flux of the electric field through the shaded region is $\frac{5}{\mathrm{p}} \times \frac{1}{\varepsilon_0}\frac{\mathrm{Nm}^2}{\mathrm{C}}$, where the value of p is _____ .

[4 marks]



Solution:

Concept:

- The electric flux of a point charge through a surface is $\Phi = q\Omega/(4\pi\varepsilon_0)$, where Ω is the solid angle subtended at the charge.
- A square of side a, whose centre is a perpendicular distance $a/2$ from the charge, can be treated as one face of a cube with the charge at its centre. Each face carries flux $q/(6\varepsilon_0)$.

Step 1: Complete a cube around the charge The charge lies on the normal through the square centre, a distance $a/2$ away. Imagine the given square as a face of a cube of side a. The point charge is then at the cube centre. By Gauss's law, the total outward flux through all six faces is q/ε_0. The faces are equivalent by symmetry, giving

$$\Phi_{\text{square}} = \frac{q}{6\varepsilon_0}.$$

Only this geometric relation is needed; the side length drops out.

Step 2: Identify the equal-flux pieces in the shading The horizontal and vertical lines through the square centre divide it into four equivalent quadrants. The shaded lower half occupies two of them. In the upper-left quadrant, its diagonal from the upper-left corner to the centre cuts that quadrant

into two congruent triangles. Reflection in this diagonal leaves the charge position unchanged, so these two triangles have equal flux. Thus the shaded region contains four small equal-flux triangles in the lower half and one more in the upper-left quadrant: five of the eight symmetry-equivalent triangles covering the square. This argument uses symmetry of the field, rather than assuming that arbitrary equal areas must receive equal flux.

Step 3: Evaluate the flux and compare coefficients The shaded flux is consequently

$$\Phi_{\text{shaded}} = \frac{5}{8}\Phi_{\text{square}} = \frac{5}{8}\frac{q}{6\varepsilon_0} = \frac{5q}{48\varepsilon_0}.$$

Putting $q = 1\,\text{C}$ and comparing with $(5/p)(1/\varepsilon_0)$ in the stated flux units gives $p = 48$.

Step 4: Conclusion The shaded electric flux is $\dfrac{5}{48\varepsilon_0}\,\text{N}\,\text{m}^2/\text{C}$ in the numerical form specified; hence $\boxed{p = 48}$.

Quick Tip to Solve: First obtain the whole-square flux from a cube. Then divide it only into pieces related by symmetries that also preserve the charge position.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. For a reaction, $N_2O_{5(\,g)} \rightarrow 2NO_{2(\,g)} + \frac{1}{2}O_{2(\,g)}$ in a constant volume container, no products were present initially. The final pressure of the system when 50% of reaction gets completed is [4 marks]

(A) $5/2$ times of initial pressure

(B) $7/2$ times of initial pressure

(C) $7/4$ times of initial pressure

(D) 5 times of initial pressure

Correct Answer: (C) $7/4$ times of initial pressure

Solution:

Concept:

- For ideal gases at fixed temperature and volume, pressure is proportional to total moles: $P = nRT/V$.
- For each mole of N_2O_5 decomposed, 2 moles of NO_2 and $1/2$ mole of O_2 form. Unreacted starting gas must also be counted.

Step 1: Choose a convenient initial amount Let the initial quantity of N_2O_5 be n_0 moles and its initial pressure be P_0. There are initially no gaseous products. At fifty per cent completion, $n_0/2$ moles have decomposed and $n_0/2$ moles remain. All gaseous species contribute to the measured total pressure; the final mixture cannot be treated as pure product.

Step 2: Count the final gaseous moles Use the printed stoichiometric equation

$$\text{N}_2\text{O}_5(\text{g}) \longrightarrow 2\,\text{NO}_2(\text{g}) + \frac{1}{2}\,\text{O}_2(\text{g}).$$

The amount of NO_2 formed is $2(n_0/2) = n_0$; the amount of O_2 formed is $(1/2)(n_0/2) = n_0/4$. Adding these to the undecomposed reactant gives

$$n_f = \frac{n_0}{2} + n_0 + \frac{n_0}{4} = \frac{7n_0}{4}.$$

Step 3: Convert the mole ratio into the pressure ratio For the usual isothermal pressure comparison at the stated constant volume, $P_f/P_0 = n_f/n_0 = 7/4$. The value $5/2$ would be the total-mole factor for complete decomposition, rather than fifty per cent conversion. The value $7/2$ doubles the correct result by losing the initial-mole normalisation, and 5 likewise cannot follow from the half-conversion mole balance. If temperature were allowed to change, a further factor T_f/T_0 would be needed; the standard kinetic pressure comparison holds temperature fixed.

Step 4: Conclusion The final pressure is $\boxed{P_f = \frac{7}{4}P_0}$. Hence, the correct option is (C).

Quick Tip to Solve: At fractional conversion α, count reactant plus products: here $n_f/n_0 = 1 + (3/2)\alpha$, then insert $\alpha = 1/2$.

52. The large difference between the melting and boiling points of oxygen and sulphur may be explained on the basis of [4 marks]

(A) Atomicity

(B) Electron gain enthalpy

(C) Atomic size

(D) Electronegativity

Correct Answer: (A) Atomicity

Solution:

Concept:

- Atomicity is the number of atoms present in a molecule. Ordinary molecular oxygen is O_2, whereas sulphur commonly exists as puckered S_8 rings.
- Melting and boiling separate molecules against intermolecular attractions. Larger molecular units with more electrons generally have stronger London dispersion attractions.

Step 1: Identify the molecular units being compared Oxygen is a small diatomic molecular substance, O_2. Sulphur in its common molecular allotropes consists of eight-membered S_8 rings. The relevant contrast is therefore between molecules of atomicity two and eight, rather than between isolated oxygen and sulphur atoms. A phase change does not normally require breaking each covalent bond inside those molecular units.

Step 2: Relate molecular size to the phase-change temperatures The larger S_8 unit contains many more electrons and has a much more polarisable electron cloud than O_2. Instantaneous and induced dipoles therefore create stronger dispersion attractions between sulphur molecules. More thermal energy is required to overcome the attractions and separate sulphur molecules, so sulphur has substantially higher melting and boiling temperatures than oxygen. The molecular atomicity difference supplies

the direct explanation asked for.

Step 3: Compare the other suggested properties Electron gain enthalpy concerns adding an electron to an isolated gaseous atom, so it is not a direct measure of intermolecular attraction in these neutral molecular substances. Atomic size does influence polarisability, but citing it alone misses the major O_2-versus-S_8 molecular distinction. Electronegativity describes attraction for bonding electrons; both elemental molecules have bonds between identical atoms, so this is not the explanation for the large phase-change temperature difference.

Step 4: Conclusion The large difference is explained primarily by the different atomicities, O_2 and S_8, and their resulting intermolecular attractions. Hence, the correct option is (A).

Quick Tip to Solve: For elemental molecular solids and gases, identify the actual molecules first: O_2 versus S_8, rather than comparing isolated atoms.

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53. Given below are two statements I and II.

Statement I : Dumas method is used for estimation of "Nitrogen" in an organic compound.

Statement II : Dumas method involves the formation of ammonium sulphate by heating the organic compound with conc H_2SO_4.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Statement I is true but Statement II is false

(B) Statement I is false but Statement II is true

(C) Both Statement I and Statement II are true

(D) Both Statement I and Statement II are false

Correct Answer: (A) Statement I is true but Statement II is false

Solution:

Concept:

- Dumas' method estimates nitrogen by converting nitrogen in an organic compound into molecular nitrogen and measuring the gas produced.
- Kjeldahl' method uses digestion with concentrated H_2SO_4 to convert suitable organic nitrogen into ammonium sulphate, followed by ammonia liberation and estimation.

Step 1: Evaluate the first statement using Dumas method In Dumas method the organic compound is heated with copper(II) oxide in a carbon dioxide atmosphere. Carbon and hydrogen are oxidised to CO_2 and H_2O, and nitrogen is ultimately collected as N_2. Hot copper removes oxygen and reduces nitrogen oxides formed during combustion to nitrogen. The measured nitrogen volume, after the usual corrections, determines the nitrogen content. Thus the assertion that Dumas method estimates nitrogen is true.

Step 2: Identify the digestion described in the second statement Heating an organic compound with concentrated sulphuric acid to produce $(NH_4)_2SO_4$ describes the digestion stage of Kjeldahl method.

Subsequent treatment with alkali liberates ammonia:

$$(NH_4)_2SO_4 + 2\,NaOH \longrightarrow 2\,NH_3 + Na_2SO_4 + 2\,H_2O.$$

The ammonia is absorbed in a known quantity of standard acid for estimation. This ammonium-sulphate route is not the defining chemistry of Dumas combustion method.

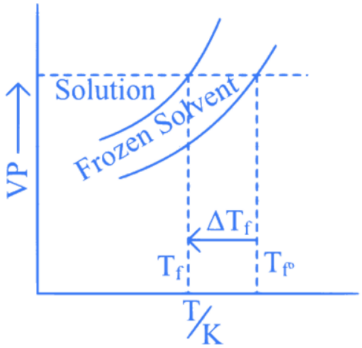
Step 3: Combine the two independent truth values Statement I correctly assigns nitrogen estimation to Dumas method, so it must be retained as true. Statement II attributes the Kjeldahl digestion reaction to Dumas method, so it is false. Option (B) reverses both truth values; option (C) incorrectly accepts the method attribution in Statement II; option (D) incorrectly rejects Statement I. Only the true-false combination agrees with the chemistry of both procedures.

Step 4: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (A).

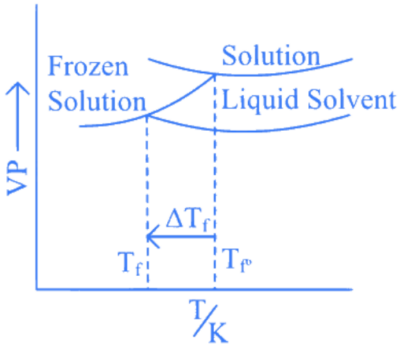
Quick Tip to Solve: Remember the measured product: Dumas collects N_2; Kjeldahl forms ammonium sulphate during acid digestion and later releases NH_3.

54. Consider the given plots of vapour pressure (VP) vs temperature(T/K). Which amongst the following options is correct graphical representation showing ΔT_f, depression in the freezing point of a solvent in a solution? [4 marks]

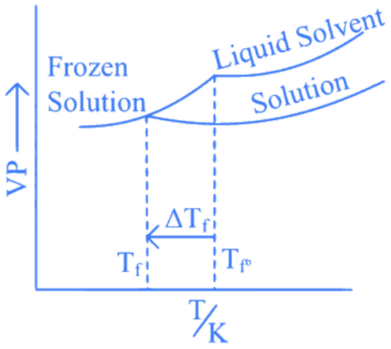
(A)



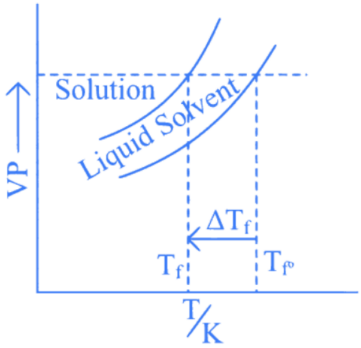
(B)



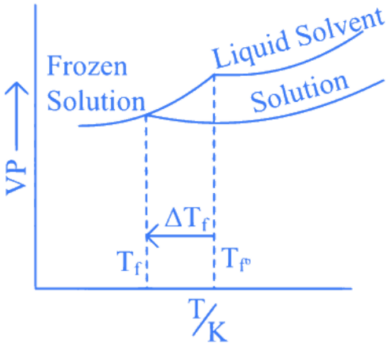
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- At freezing equilibrium, the vapour pressure of the solid solvent equals that of the liquid phase. The intersection of the appropriate vapour-pressure curves gives the freezing temperature.
- A nonvolatile solute lowers the solvent vapour pressure over the liquid solution. Its freezing point is therefore below that of the pure liquid solvent: $\Delta T_f = T_f^\circ - T_f > 0$.

Step 1: Locate the two freezing equilibria The pure solvent freezes where its liquid-solvent vapour-pressure curve meets the solid-solvent curve. Call this temperature T_f°. On adding a nonvolatile solute, the vapour pressure of the liquid solution is lower than that of pure liquid solvent at a given temperature. The solid phase in the usual depression-of-freezing-point treatment remains pure solvent, so the solid-solvent reference is common to both equilibria.

Step 2: Apply the curve-order and temperature-order tests The solution curve must lie below the liquid-solvent curve. Its intersection with the solid-solvent curve must occur to the left, at a lower

temperature T_f. The separation on the temperature axis is

$$\Delta T_f = T_f^\circ - T_f.$$

Graph (C) has the liquid-solvent curve above the solution curve and marks the solution freezing temperature to the left of the pure-solvent freezing temperature. These are the two required physical checks.

Step 3: Eliminate the remaining graphical options Graph (B) places the solution above the liquid solvent, contradicting vapour-pressure lowering. Graph (D) likewise puts the solution curve above the liquid-solvent curve and compares a fixed-pressure crossing rather than the required solid-liquid equilibrium intersections. Graph (A) also relies on a horizontal fixed-pressure construction and fails to show the two freezing points as intersections with one common solid-solvent curve. Such a constant-pressure construction is inappropriate for identifying freezing equilibrium. Thus the curve positions and the marked depression are consistent only in graph (C).

Step 4: Conclusion The graph consistent with vapour-pressure lowering and $T_f < T_f^\circ$ is (C). Hence, the correct option is (C).

Quick Tip to Solve: Check both conditions: liquid solution below pure liquid solvent, and its solid-solvent intersection at the lower temperature.

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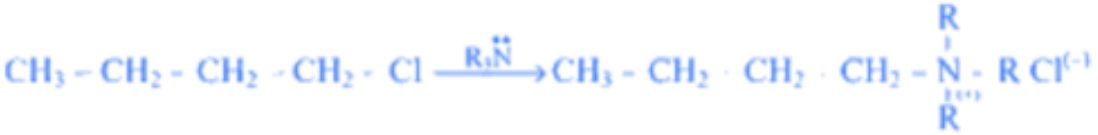
55. Given below are two statements:

Statement I: The conversion proceeds well in the less polar medium.

$$CH_3 - CH_2 - CH_2 - CH_2 - Cl \xrightarrow{HO^-} CH_3 - CH_2 - CH_2 - CH_2 - OH + Cl^{(-)}$$

Statement II: The conversion proceeds well in the more polar medium.

In the light of the above statements, choose the correct answer from the options given below [4 marks]



(A) Statement I is false but Statement II is true

(B) Statement I is true but Statement II is false

(C) Both Statement I and Statement II are false

(D) Both Statement I and Statement II are true

Correct Answer: (D) Both Statement I and Statement II are true

Solution:

Concept:

- Both reactions involve substitution at a primary carbon by an S_N2 pathway. The effect of solvent polarity depends on the charge distribution in the reactants and transition state.

- A polar solvent preferentially stabilises strongly localised ionic charge. If charge becomes more dispersed in the transition state, this stabilisation can increase the activation barrier.
- When two neutral reactants develop separated charges during bond formation and bond breaking, a more polar medium stabilises the developing charges and favours the reaction.

Step 1: Assess the hydroxide substitution The first conversion is $CH_3CH_2CH_2CH_2Cl + OH^- \longrightarrow CH_3CH_2CH_2CH_2OH + Cl^-$. Hydroxide is already an anion. In the transition state, the incoming oxygen and departing chlorine share the negative charge while the carbon is partially bonded to both. A strongly polar, especially protic, medium stabilises the concentrated charge on the entering hydroxide more strongly and also solvates it. This reduces its effective nucleophilicity and raises the relative activation barrier. Thus the stated preference for a less polar medium is the expected comparison for this ionic-nucleophile substitution. Statement I is true.

Step 2: Assess the neutral amine substitution shown The second conversion shown uses a neutral tertiary amine, R_3N, with the primary alkyl chloride to produce a quaternary ammonium chloride, $[R_3NCH_2CH_2CH_2CH_3]^+ Cl^-$. The initially neutral reactants develop positive charge on nitrogen and negative charge on the leaving chloride in the transition state. A more polar medium stabilises this developing charge separation and lowers the activation barrier. Statement II is therefore true.

Step 3: Compare all four statement combinations Option (A) incorrectly rejects the hydroxide result, while (B) incorrectly rejects the amine result. Option (C) rejects both and is inconsistent with both charge analyses. Option (D) is the only combination that retains both valid statements. Here the two statements refer to the two conversions displayed, rather than contradictory solvent prescriptions for one unchanged reaction.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (D).

Quick Tip to Solve: Compare the charges before and during substitution: dispersed anionic charge and newly generated charge separation give opposite solvent-polarity trends.

56. Preparation of potassium permanganate from MnO_2 involves two step process in which the 1st step is a reaction with KOH and KNO_3 to produce [4 marks]

(A) K_3MnO_4

(B) $K_4[Mn(OH)_6]$

(C) K_2MnO_4

(D) $KMnO_4$

Correct Answer: (C) K_2MnO_4

Solution:

Concept:

- Fusion of MnO_2 with alkali and an oxidising agent first gives green potassium manganate, K_2MnO_4.
- Manganese changes from oxidation state $+4$ in MnO_2 to $+6$ in manganate; further oxidation gives the $+7$ permanganate ion.

Step 1: Identify the first oxidation product The first step uses hot potassium hydroxide and potassium

nitrate. The alkaline medium supplies potassium ions, while nitrate serves as the oxidising agent. Manganese dioxide does not jump directly to the final permanganate in this stated two-stage preparation: the first isolable oxyanion is manganate, MnO_4^{2-}. Its charge follows from $x + 4(-2) = -2$, giving $x = +6$. Two potassium ions are needed to balance its charge.

Step 2: Write and verify the first-step equation The nitrate-assisted alkaline fusion can be represented by the balanced equation

$$MnO_2 + 2\,KOH + KNO_3 \longrightarrow K_2MnO_4 + KNO_2 + H_2O.$$

There are three potassium atoms, one manganese atom, one nitrogen atom, two hydrogen atoms and seven oxygen atoms on either side. Manganese loses two electrons as it goes from $+4$ to $+6$, and nitrogen gains two as nitrate is reduced to nitrite. The product asked for is therefore K_2MnO_4, not the product of the subsequent oxidation step.

Step 3: Eliminate the other formulae Option (A), K_3MnO_4, would assign manganese oxidation state $+5$, so it is not the manganate obtained in this fusion. Option (B) is a hydroxide coordination formula rather than the oxidised tetraoxomanganate product. Option (D), $KMnO_4$, contains manganese in $+7$ and belongs to the final stage of permanganate preparation. These distinctions isolate option (C).

Step 4: Conclusion The first step produces potassium manganate, K_2MnO_4. Hence, the correct option is (C).

Quick Tip to Solve: Remember the preparation sequence: black manganese dioxide gives green manganate before purple permanganate.

57. Which one of the carbocations from the following is most stable? [4 marks]

(A) $CH_2^+ - CH{=}CH{-}CH_2{-}O{-}CH_3$

(B) $CH_2^+ - CH{=}CH{-}O{-}C(=O){-}CH_3$

(C) $CH_2^+ - CH{=}CH{-}O{-}CH_3$

(D) $CH_2^+ - CH{=}CH{-}F$

Correct Answer: (C) $CH_2^+ - CH{=}CH{-}O{-}CH_3$

Solution:

Concept:

- An allylic carbocation delocalises its positive charge over the terminal carbon atoms of the three-carbon conjugated unit.
- A heteroatom directly attached to that conjugated unit can donate a lone pair and generate an additional resonance form with complete carbon octets.
- Oxygen donation is most effective when its lone pair is not simultaneously withdrawn by an adjacent carbonyl group; fluorine has a strong electron-withdrawing inductive effect.

Step 1: Locate the conjugated unit in each option All four structures begin with the allylic sequence

$CH_2{}^+ - CH{=}CH^-$. Its common resonance relation is

$$CH_2{}^+ - CH{=}CH{-}X \longleftrightarrow CH_2{=}CH{-}CH^+ - X.$$

The identity and position of X therefore determine the extra stabilisation. In (A), the ether oxygen is separated from the alkene by a saturated CH_2 group. That intervening tetrahedral carbon interrupts continuous conjugation, so the oxygen cannot directly donate into the allylic cation.

Step 2: Compare the two oxygen-conjugated alternatives In (C), $X = OCH_3$, and the ether oxygen is directly bonded to the terminal carbon of the allylic unit. Donation gives the additional oxonium contributor

$$CH_2{=}CH{-}CH^+ - OCH_3 \longleftrightarrow CH_2{=}CH{-}CH{=}O^+ - CH_3.$$

In the right-hand contributor every carbon has a complete octet, and the positive charge is also delocalised onto oxygen. Option (B) also has an adjacent oxygen, but this oxygen is part of an ester, $-O-C(=O)CH_3$. Its lone pair also participates in conjugation with the carbonyl, reducing its availability to stabilise the allylic positive charge.

Step 3: Compare fluorine and select the strongest donor In (D), fluorine can show lone-pair donation, but its high electronegativity makes it strongly withdrawing by induction and makes donation less favourable than from a methoxy oxygen. Thus (D) cannot exceed the directly conjugated ether in (C). The nonconjugated ether (A), carbonyl-competing oxygen (B), and strongly inductively withdrawing fluorine (D) all offer less effective overall stabilisation than the methoxy-substituted allylic cation.

Step 4: Conclusion The carbocation $CH_2{}^+ - CH{=}CH{-}OCH_3$ is most stable through direct oxygen-lone-pair conjugation. Hence, the correct option is (C).

Quick Tip to Solve: First trace an uninterrupted conjugated path to the positive charge; then compare how freely each heteroatom can donate its lone pair.

58. The product (A) formed in the following reaction sequence is [4 marks]

$$CH_3 - C \equiv CH \xrightarrow[\text{iii) } H_2/Ni]{\text{i) } Hg^{2+} \,.\, H_2SO_4 \text{ ii) HCN}} \underset{\text{Product}}{(A)}$$

(A) $CH_3CH_2CH(OH)CH_2NH_2$

(B) $CH_3CH_2CH(NH_2)CH_2NH_2$

(C) $(CH_3)_2C(OH)CH_2NH_2$

(D) $(CH_3)_2C(NH_2)CH_2OH$

Correct Answer: (C) $(CH_3)_2C(OH)CH_2NH_2$

Solution:

Concept:

- Mercuric-ion-catalysed acidic hydration of a terminal alkyne follows Markovnikov orientation and gives a methyl ketone after enol-keto tautomerism.
- Addition of hydrogen cyanide to a ketone forms a cyanohydrin: the carbonyl carbon acquires OH and CN groups.
- Catalytic hydrogenation reduces a nitrile group, $-C{\equiv}N$, to $-CH_2NH_2$ while retaining its carbon atom in the product.

Step 1: Hydrate propyne to the ketone The initial alkyne is propyne, $CH_3{-}C{\equiv}CH$. With Hg^{2+} in dilute sulphuric acid, water adds with its oxygen at the more substituted alkyne carbon. The initially produced enol tautomerises to propanone:

$$CH_3{-}C{\equiv}CH \xrightarrow[H_2O]{Hg^{2+},\,H_2SO_4} CH_3{-}C(OH){=}CH_2 \longrightarrow CH_3COCH_3.$$

The two methyl groups of this ketone are crucial: subsequent addition does not turn this branched carbon skeleton into a straight four-carbon chain.

Step 2: Form the cyanohydrin with hydrogen cyanide Cyanide attacks the electrophilic carbonyl carbon, and protonation of the alkoxide gives acetone cyanohydrin:

$$(CH_3)_2CO + HCN \longrightarrow (CH_3)_2C(OH)CN.$$

The central carbon now bears two methyl groups, one hydroxyl group and one nitrile carbon. This step adds one carbon atom, increasing the total from three to four, without moving the hydroxyl group from the former carbonyl carbon.

Step 3: Reduce the nitrile and match the structure Hydrogen in the presence of nickel reduces the nitrile to a primary amino group:

$$(CH_3)_2C(OH)CN + 2\,H_2 \xrightarrow{Ni} (CH_3)_2C(OH)CH_2NH_2.$$

Option (C) has exactly this connectivity. Option (A) has an unbranched skeleton, which would require an aldehyde rather than propanone as the hydration product. Option (B) similarly has an unbranched

chain and replaces the retained hydroxyl with an amino group. Option (D) swaps the hydroxyl and amino positions, contrary to cyanohydrin formation followed by reduction only at the nitrile group.

Step 4: Conclusion The final product is $(CH_3)_2C(OH)CH_2NH_2$, the amino alcohol shown in option (C). Hence, the correct option is (C).

Quick Tip to Solve: Track the original carbonyl carbon: it keeps its two methyl groups and becomes the carbon bearing OH; the new cyanide carbon becomes CH2NH2.

59. K_{sp} **for** $Cr(OH)_3$ **is** 1.6×10^{-30}**. What is the molar solubility of this salt in water?** [4 marks]

(A) $\sqrt[5]{1.8 \times 10^{-30}}$

(B) $\frac{1.8\times10^{-30}}{27}$

(C) $\sqrt[4]{\frac{1.6\times10^{-30}}{27}}$

(D) $\sqrt[2]{1.6 \times 10^{-30}}$

Correct Answer: (C) $\sqrt[4]{\frac{1.6\times10^{-30}}{27}}$

Solution:

Concept:

- The solubility product is the product of the equilibrium ion concentrations, each raised to its stoichiometric coefficient.
- For an ideal dilute dissolution calculation, if the molar solubility of $Cr(OH)_3$ is s, it supplies s of Cr^{3+} and $3s$ of OH^-.

Step 1: Set up the dissolution equilibrium Write the dissociation of one formula unit rather than treating this hydroxide as a one-to-one salt:

$$Cr(OH)_3(s) \rightleftharpoons Cr^{3+}(aq) + 3\,OH^-(aq).$$

If s moles dissolve per litre, the stoichiometric ion concentrations are $[Cr^{3+}] = s$ and $[OH^-] = 3s$. The solid is omitted from the equilibrium expression because its activity is constant. We use the conventional solubility-product treatment intended by the given choices, without introducing additional hydrolysis or complex-formation equilibria.

Step 2: Substitute the ion concentrations The solubility-product expression is

$$K_{sp} = [Cr^{3+}][OH^-]^3 = s(3s)^3 = 27s^4.$$

Using the stated value gives

$$27s^4 = 1.6 \times 10^{-30}, \qquad s^4 = \frac{1.6 \times 10^{-30}}{27}.$$

Taking the positive fourth root is required because a concentration cannot be negative. Numerically, this root is approximately $1.56 \times 10^{-8}\,mol\,L^{-1}$ within this textbook dissociation model.

Step 3: Check the exponent and all options Option (C) retains both the factor 27 from the three

hydroxide ions and the fourth root from the total power $1+3$. Option (A) uses a fifth root and the wrong stated constant. Option (B) omits the fourth-root operation and also uses the wrong numerator. Option (D) uses a square root, as if this were a one-to-one dissociation, and loses the stoichiometric factor. Only option (C) follows the specified hydroxide formula.

Step 4: Conclusion The intended molar solubility is $s = \sqrt[4]{\frac{1.6 \times 10^{-30}}{27}}\,\text{mol}\,\text{L}^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: For a salt giving one trivalent ion and three monovalent ions, write $s(3s)^3$ before taking the fourth root.

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60. Which of the following linear combination of atomic orbitals will lead to formation of molecular orbitals in homonuclear diatomic molecules [internuclear axis in z-direction] ?

A. 2p_z **and** 2p_x

B. 2 s and 2p_x

C. $3d_{xy}$ **and** $3d_{x^2-y^2}$

D. 2 s and 2p_z

E. $2p_z$ **and** $3d_{x^2-y^2}$

Choose the correct answer from the options given below: [4 marks]

(A) D Only

(B) E Only

(C) C and D Only

(D) A and B Only

Correct Answer: (A) D Only

Solution:

Concept:

- Atomic orbitals combine effectively when they have compatible energies, nonzero overlap and the same symmetry with respect to the internuclear axis.
- With the internuclear axis chosen as z, an s orbital and a p_z orbital both possess sigma symmetry. The p_x orbital has pi symmetry relative to this axis.
- Transverse d_{xy} and $d_{x^2-y^2}$ orbitals have different angular patterns; their mutual overlap cancels by symmetry.

Step 1: Check combinations A and B Combination A pairs $2p_z$ with $2p_x$. The former points along the bond axis, whereas the latter changes sign under reflection in a plane containing that axis. Positive and negative overlap contributions therefore cancel. Combination B, $2s$ with $2p_x$, fails for the same reason: the spherical s orbital overlaps equally with the oppositely signed transverse p lobes. Neither A nor B has the required nonzero net overlap for the stated molecular-orbital combination.

Step 2: Check combinations C and E In combination C, the $3d_{xy}$ lobes lie between the transverse coordinate axes, while those of $3d_{x^2-y^2}$ lie along them. Reflection that leaves one orbital unchanged

reverses the other, so their overlap integral vanishes. In E, $2p_z$ has sigma symmetry, while $3d_{x^2-y^2}$ has a transverse delta-type angular pattern. Those different symmetries also preclude the desired overlap. Thus C and E must both be excluded.

Step 3: Identify the allowed axial combination Combination D pairs $2s$ with $2p_z$. Both are symmetric about the z-axis and can overlap head-on; there is no symmetry cancellation of their overlap. Accordingly D is the only listed pairing that satisfies the symmetry criterion. Option (B) wrongly selects E, option (C) includes the forbidden pair C, and option (D) selects the two forbidden transverse pairings A and B. Option (A), D only, matches the complete symmetry analysis.

Step 4: Conclusion Only combination D, $2s$ and $2p_z$, is symmetry-allowed for overlap along the z-axis. Hence, the correct option is (A).

Quick Tip to Solve: Choose the bond axis first: s and p along that axis are sigma-compatible, while perpendicular p components cancel against a sigma orbital.

---

61. Which of the following statements are NOT true about the periodic table?

A. The properties of elements are function of atomic weights.

B. The properties of elements are function of atomic numbers.

C. Elements having similar outer electronic configurations are arranged in same period.

D. An element's location reflects the quantum numbers of the last filled orbital.

E. The number of elements in a period is same as the number of atomic orbitals available in energy level that is being filled.

Choose the correct answer from the options given below: [4 marks]

(A) A, C and E Only

(B) A and E Only

(C) B, C and E Only

(D) D and E Only

Correct Answer: (A) A, C and E Only

Solution:

Concept:

- Modern periodic law states that the physical and chemical properties of elements are periodic functions of their atomic numbers, rather than their atomic weights.
- Elements with similar valence-shell electronic configurations occupy a group. The capacity of a set of orbitals is twice its orbital count because each orbital accommodates two electrons.

Step 1: Test the periodic-law statements Statement A is false for the modern periodic table: ordering by atomic weight belongs to the earlier Mendeleev formulation, whereas the modern ordering uses atomic number. Statement B is therefore true. The atomic number fixes the nuclear charge and, for a neutral atom, the electron count, which determines electronic configuration and explains the periodic recurrence of chemical properties.

Step 2: Distinguish groups from periods Statement C is false. Similar outer electronic configurations

recur vertically in the same group, such as the alkali metals with an outer ns^1 configuration. A period is a horizontal row in which the electronic configuration changes as atomic number increases. Statement D is true in the usual electronic-configuration interpretation: the shell and subshell being filled indicate the period and block, and the valence configuration helps identify the group.

Step 3: Check orbital capacity and reject the other choices Statement E is false because it equates the number of elements to the number of available orbitals, omitting the two-electron capacity of an orbital. For example, the first period fills one $1s$ orbital but contains two elements; the second fills one $2s$ and three $2p$ orbitals but contains eight elements. Option B omits the false statement C. Option C incorrectly includes the true statement B, while option D incorrectly includes the true statement D and omits A and C.

Step 4: Conclusion The statements that are NOT true are A, C and E only. Hence, the correct option is (A).

Quick Tip to Solve: For periodic-table statement sets, separate the modern ordering variable, atomic number, from group similarity and the two-electron capacity of each orbital.

62. One mole of the octahedral complex compound $\mathrm{Co\,(NH_3)_5\,Cl_3}$ gives 3 moles of ions on dissolution in water. One mole of the same complex reacts with excess of $\mathrm{AgNO_3}$ solution to yield two moles of $\mathrm{AgCl_{(s)}}$. The structure of the complex is: [4 marks]

(A) $\mathrm{[Co\,(NH_3)_5\,Cl]\,Cl_2}$

(B) $\mathrm{[Co\,(NH_3)_4\,Cl]\cdot Cl_2\cdot NH_3}$

(C) $\mathrm{[Co\,(NH_3)_3\,Cl_3]\cdot 2NH_3}$

(D) $\mathrm{[Co\,(NH_3)_4\,Cl_2]\cdot Cl\cdot NH_3}$

Correct Answer: (A) $\mathrm{[Co\,(NH_3)_5\,Cl]\,Cl_2}$

Solution:

Concept:

- Only counterions outside the coordination sphere dissociate freely in water and are immediately precipitated by a suitable reagent. Coordinated chloride remains inside the complex ion in the usual ionisation test.
- An octahedral mononuclear complex has coordination number six. Each ammine and each chlorido ligand here occupies one coordination site.

Step 1: Use the silver nitrate precipitation result Two moles of AgCl are obtained per mole of compound, so two of the three chlorides are ionisable counterions. The remaining chloride belongs to the inner coordination sphere. Since ammonia is neutral and the two free chlorides have total charge -2, the complex cation must have charge $+2$. Cobalt consequently has oxidation state $+3$, because its one coordinated chlorido ligand contributes -1.

Step 2: Satisfy the octahedral coordination number The coordination sphere must contain six donor atoms. All five ammonia molecules and one chloride give exactly six monodentate ligands, so the

formula is $[Co(NH_3)_5Cl]Cl_2$. Its dissolution is represented by

$$[Co(NH_3)_5Cl]Cl_2 \longrightarrow [Co(NH_3)_5Cl]^{2+} + 2\,Cl^-.$$

One complex cation plus two chloride anions gives three moles of ions per mole, agreeing with the independent ion-count observation.

Step 3: Check every alternative against both observations For the two free chlorides, $2\,Ag^+ + 2\,Cl^- \longrightarrow 2\,AgCl(s)$, giving the required two moles of precipitate. Option B places only four ammines and one chlorido ligand in brackets, making a five-coordinate rather than octahedral species. Option C has all three chlorides coordinated and therefore cannot furnish two free chloride ions. Option D has only one chloride outside the sphere and would yield one mole of silver chloride rather than two. Thus merely matching the overall empirical composition is insufficient; both coordination number and ionisation must agree.

Step 4: Conclusion The complex is $[Co(NH_3)_5Cl]Cl_2$, with five ammine ligands, one chlorido ligand and two chloride counterions. Hence, the correct option is (A).

Quick Tip to Solve: First count precipitable chloride ions to locate the outer-sphere chlorides; then use the coordination number to place the remaining ligands inside brackets.

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63. Which of the following statement is true with respect to H_2O, NH_3 and CH_4 ?

A. The central atoms of all the molecules are sp^3 hybridized.

B. The $H-O-H, H-N-H$ and $H-C-H$ angles in the above molecules are $104.5°, 107.5°$ and $109.5°$, respectively.

C. The increasing order of dipole moment is $CH_4 < NH_3 < H_2O$.

D. Both H_2O and NH_3 are Lewis acids and CH_4 is a Lewis base.

E. A solution of NH_3 in H_2O is basic. In this solution NH_3 and H_2O act as Lowry-Bronsted acid and base respectively.

Choose the correct answer from the options given below: [4 marks]

(A) A, D and E Only

(B) C, D and E Only

(C) A, B and C Only

(D) A, B, C and E Only

Correct Answer: (C) A, B and C Only

Solution:

Concept:

- Four electron-pair domains around a central atom correspond to the conventional sp^3 hybridisation description. Lone pairs compress bond angles more strongly than bonding pairs.
- A molecular dipole moment is the vector sum of bond moments. A Lewis base donates an electron pair, while a Bronsted base accepts a proton.

Step 1: Assess hybridisation and bond angles Carbon in CH_4 has four bond pairs, nitrogen in NH_3 has three bond pairs and one lone pair, and oxygen in H_2O has two bond pairs and two lone pairs. Each central atom therefore has four electron domains, making statement A true. Methane is tetrahedral, ammonia trigonal pyramidal and water bent. Increasing lone-pair repulsion gives the stated approximate angle sequence $104.5^\circ < 107.5^\circ < 109.5^\circ$, so B is accepted as true with the values provided in the question.

Step 2: Compare polarity and Lewis behaviour The tetrahedral symmetry of methane cancels its bond dipoles, whereas the pyramidal ammonia and bent water molecules have nonzero resultant moments. Their familiar order is

$$\mu(CH_4) = 0 < \mu(NH_3) < \mu(H_2O).$$

Statement C is true. In contrast to D, water and ammonia have lone pairs that can be donated and normally act as Lewis bases; methane is not an ordinary electron-pair-donating Lewis base. Thus D is false.

Step 3: Identify the proton donor and acceptor in water The aqueous equilibrium is

$$NH_3 + H_2O \rightleftharpoons NH_4^+ + OH^-.$$

Ammonia accepts a proton and is the Bronsted base; water donates that proton and is the Bronsted acid. The solution is basic because hydroxide ions are produced, but statement E reverses the acid and base roles and is false as a whole. Options A and B each include false statements D and E. Option D also includes E. Only option C contains exactly the three true statements, A, B and C.

Step 4: Conclusion A, B and C only are true; D and E are false. Hence, the correct option is (C).

Quick Tip to Solve: In an acid-base statement, verify the named roles with a written proton-transfer equation; a correct claim of basicity does not rescue reversed donor and acceptor roles.

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64. Let us consider an endothermic reaction which is non-spontaneous at the freezing point of water. However, the reaction is spontaneous at boiling point of water. Choose the correct option. [4 marks]

(A) Both ΔH and ΔS are (-ve)

(B) ΔH is $(+ve)$ but ΔS is (-ve)

(C) ΔH is $(-ve)$ but ΔS is (+ve)

(D) Both ΔH and ΔS are (+ve)

Correct Answer: (D) Both ΔH and ΔS are (+ve)

Solution:

Concept:

- At constant temperature and pressure, spontaneity is determined by $\Delta G = \Delta H - T\Delta S$, where temperature is measured on the absolute Kelvin scale.
- An endothermic reaction has $\Delta H > 0$. A spontaneous process has $\Delta G < 0$, whereas a non-spontaneous process has $\Delta G > 0$ under the stated conditions.

Step 1: Determine the enthalpy sign from the wording The reaction absorbs heat, so its enthalpy change is positive. This immediately eliminates options A and C, both of which assign a negative enthalpy change. The sign of enthalpy alone does not decide spontaneity: the entropy contribution must be included in the Gibbs energy. In particular, an endothermic reaction can be spontaneous when a sufficiently favourable entropy change outweighs its heat requirement.

Step 2: Use the change in spontaneity to find the entropy sign Let the freezing and boiling temperatures be approximately $T_f = 273\,\text{K}$ and $T_b = 373\,\text{K}$. The given behaviour requires

$$\Delta H - T_f \Delta S > 0, \qquad \Delta H - T_b \Delta S < 0.$$

At the higher temperature the positive enthalpy must be overcome by subtracting a positive entropy term. Therefore $\Delta S > 0$. If instead $\Delta S < 0$, then $-T\Delta S$ would be positive, making $\Delta G = \Delta H + T|\Delta S| > 0$ at every positive temperature. Such a reaction could not become spontaneous on heating.

Step 3: Check the threshold and remaining option Treating the enthalpy and entropy changes as approximately constant over this interval, the inequalities can also be written

$$T_f < \frac{\Delta H}{\Delta S} < T_b.$$

Below this threshold the enthalpy term dominates and the reaction is non-spontaneous; above it the entropy term dominates. Option B has positive enthalpy but negative entropy and predicts non-spontaneity at both temperatures. Option D supplies both positive signs and correctly allows the observed crossover. No numerical enthalpy or entropy value is needed to determine these signs.

Step 4: Conclusion Both ΔH and ΔS are positive. Hence, the correct option is (D).

Quick Tip to Solve: When heating makes an endothermic reaction spontaneous, the entropy change must be positive: the increasingly negative term $-T\Delta S$ pays the enthalpy cost.

65. Which of the following arrangements with respect to their reactivity in nucleophilic addition reaction is correct? [4 marks]

(A) acetophenone < benzaldehyde < p-tolualdehyde < p- nitrobenzaldehyde

(B) acetophenone < p-tolualdehyde < benzaldehyde < p- nitrobenzaldehyde

(C) p- nitrobenzaldehyde < benzaldehyde < p-tolualdehyde < acetophenone

(D) benzaldehyde < acetophenone < p- nitrobenzaldehyde < p-tolualdehyde

Correct Answer: (B) acetophenone < p-tolualdehyde < benzaldehyde < p- nitrobenzaldehyde

Solution:

Concept:

- A nucleophile attacks the electrophilic carbonyl carbon. Electron-withdrawing substituents increase its electrophilicity, whereas electron-donating substituents reduce it.
- Aldehydes are generally more reactive than comparable ketones in nucleophilic addition because

they have less steric hindrance and less electron donation from alkyl groups.

Step 1: Compare acetophenone with the aromatic aldehydes Acetophenone is an aromatic ketone, $C_6H_5COCH_3$, while benzaldehyde and its para-substituted derivatives are aldehydes. The methyl group directly attached to the carbonyl carbon in acetophenone supplies electron density and increases crowding relative to the hydrogen of an aldehyde. Both effects make attack by a nucleophile less favourable. Acetophenone therefore belongs at the low-reactivity end of the listed sequence.

Step 2: Rank the three aromatic aldehydes by substituent effects In p-tolualdehyde the para methyl group donates electron density to the aromatic system, lowering the electrophilicity of the conjugated carbonyl relative to unsubstituted benzaldehyde. The para nitro group in p-nitrobenzaldehyde is strongly electron withdrawing by inductive and resonance effects; it increases the electron deficiency of the carbonyl carbon and promotes nucleophilic attack. Thus the aldehyde ordering is

$$p\text{-tolualdehyde} < \text{benzaldehyde} < p\text{-nitrobenzaldehyde}.$$

The comparison concerns the same type of addition reaction, so the substituent effects can be compared directly.

Step 3: Assemble the sequence and reject the reversals Combining the ketone-versus-aldehyde comparison with the para-substituent effects gives

$$\begin{aligned}\text{acetophenone} &< p\text{-tolualdehyde} \\ &< \text{benzaldehyde} < p\text{-nitrobenzaldehyde}.\end{aligned}$$

Option A reverses the relative positions of p-tolualdehyde and benzaldehyde. Option C puts the electron-poor nitro-substituted aldehyde at the least reactive end and acetophenone at the most reactive end. Option D incorrectly places acetophenone above benzaldehyde and p-tolualdehyde above the nitro-substituted aldehyde. Only option B agrees with both the steric and electronic comparisons.

Step 4: Conclusion The correct increasing order is acetophenone, p-tolualdehyde, benzaldehyde, then p-nitrobenzaldehyde. Hence, the correct option is (B).

Quick Tip to Solve: Rank carbonyl compounds in two passes: compare aldehyde with ketone first, then place electron-donating ring groups below hydrogen and electron-withdrawing groups above it.

66. The carbohydrate "Ribose" present in DNA, is

A. A pentose sugar

B. present in pyranose from

C. in "D" configuration

D. a reducing sugar, when free

E. in α-anomeric form

Choose the correct answer from the options given below: [4 marks]

(A) A, D and E Only

(B) B, D and E Only

(C) A, C and D Only

(D) A, B and E Only

Correct Answer: (C) A, C and D Only

Solution:

Concept:

- Ribose is a five-carbon aldose. DNA contains 2-deoxy-D-ribose, whereas RNA contains D-ribose; both nucleic-acid sugars have a five-carbon skeleton.
- D/L configuration and alpha/beta anomerism describe different stereochemical relationships. A free reducing sugar has an anomeric centre that can equilibrate with an open-chain carbonyl form.

Step 1: Identify the sugar class and absolute configuration Statement A is true because ribose is a pentose: its carbon skeleton contains five carbon atoms. The sugar in DNA is more precisely 2-deoxy-D-ribose, in which the hydroxyl group at the second carbon of ribose is replaced by hydrogen. This distinction does not alter the five-carbon classification. Statement C is also true: the naturally occurring sugar has the D configuration, defined by its stereochemical relationship to D-glyceraldehyde, rather than by the sign of optical rotation.

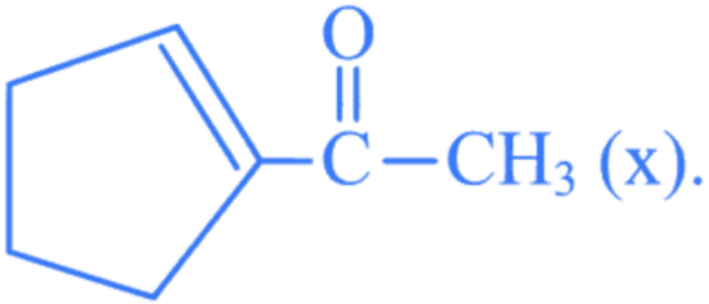
Step 2: Check ring size and the anomeric linkage The sugar residue in DNA is in the furanose form, with a five-membered ring containing four carbon atoms and one oxygen atom. The remaining sugar carbon is outside that ring. Statement B, which calls this a pyranose form, is therefore false; a pyranose ring is six-membered. The nucleic-acid glycosidic linkage has the beta configuration at the anomeric carbon, so the alpha-anomeric assertion in E is false. D configuration does not imply alpha configuration: these are independent labels.

Step 3: Evaluate reducing behaviour and compare the answer sets The phrase "when free" is decisive in statement D. Free ribose or free deoxyribose has a hemiacetal anomeric centre and can open to an aldehyde-containing form, giving reducing-sugar behaviour. In a nucleoside the anomeric centre is tied up in an N-glycosidic bond; the claim here explicitly concerns the free sugar. Thus D is true. Option A includes the false statement E, option B includes both false B and false E, and option D also includes B and E while omitting the true C and D. Only the set A, C and D contains all and only the true statements.

Step 4: Conclusion A, C and D only are true. DNA specifically contains 2-deoxy-D-ribose in a beta-furanose residue. Hence, the correct option is (C).

Quick Tip to Solve: Keep three labels separate: pentose counts carbons, furanose counts ring members, and beta describes the anomeric linkage; D describes a different stereochemical reference.

67. Aman has been asked to synthesise the molecule He thought of preparing the molecule using an aldol condensation reaction. He found a few cyclic alkenes in his laboratory. He thought of performing ozonolysis reaction on alkene to produce a dicarbonyl compound followed by aldol reaction to prepare " x ". Predict the suitable alkene that can lead to the formation of " x ". [4 marks]



(A) 4-Methylcyclohex-1-ene

(B) 3-Methylcyclohex-1-ene

(C) Methylenecyclohexane

(D) 1-Methylcyclohex-1-ene

Correct Answer: (D) 1-Methylcyclohex-1-ene

Solution:

Concept:

- Ozonolysis cleaves a carbon–carbon double bond into carbonyl groups. A double-bond carbon carrying hydrogen gives an aldehyde; one carrying an alkyl substituent gives a ketone.
- An intramolecular aldol reaction makes a new carbon–carbon bond between an enolate alpha-carbon and a carbonyl carbon. Subsequent loss of water gives an alpha,beta-unsaturated carbonyl compound.

Step 1: Identify the carbonyl precursor required by the target The target contains a cyclopentene ring with an acetyl group attached to a double-bond carbon. Its carbonyl must therefore survive outside the ring while the ring double bond arises by aldol dehydration. Reversing that condensation gives the keto-aldehyde $CH_3CO(CH_2)_4CHO$. The methylene immediately beside the ketone attacks the terminal aldehyde, closing a five-membered ring. The aldehyde carbon becomes the beta-carbon of the new double bond.

Step 2: Choose the alkene that gives this keto-aldehyde In 1-methylcyclohex-1-ene, one alkene carbon bears a methyl group and no hydrogen, while the other bears hydrogen. Ozonolysis opens the six-membered ring and gives exactly $CH_3CO(CH_2)_4CHO$. Enolate formation on the chain side of the ketone, cyclisation and dehydration then give the required product. Options A and B instead have hydrogen on both alkene carbons, so they form substituted dialdehydes rather than this methyl keto-aldehyde. Option C cleaves to cyclohexanone and formaldehyde, two separate molecules, rather than the required connected dicarbonyl chain.

Step 3: Conclusion The suitable precursor is 1-methylcyclohex-1-ene. Hence, the correct option is (D).

Quick Tip to Solve: For an intramolecular aldol synthesis, count the atoms in the new ring and trace the carbonyl that remains unchanged back through ozonolysis.

68. For the given cell

$$Fe^{2+}(aq) + Ag^{+}_{(aq)} \rightarrow Fe^{3+}(aq) + Ag_{(s)}$$

The standard cell potential of the above reaction is Given:

$$\begin{aligned} Ag^{+} + e^{-} &\rightarrow Ag \quad E^{\theta} = xV \\ Fe^{2+} + 2e^{-} &\rightarrow Fe \quad E^{\theta} = yV \\ Fe^{3+} + 3e^{-} &\rightarrow Fe \quad E^{\theta} = zV \end{aligned}$$

[4 marks]

(A) $x + 2y - 3z$

(B) $x + 2y$

(C) $y - 2x$

(D) $x + y - z$

Correct Answer: (A) $x + 2y - 3z$

Solution:

Concept:

- Standard reduction potentials are intensive quantities and cannot be added directly when electron numbers differ. Combine their Gibbs energies using $\Delta G^{\circ} = -nFE^{\circ}$.
- For a galvanic reaction written as oxidation plus reduction, $E^{\circ}_{\text{cell}} = E^{\circ}_{\text{cathode}} - E^{\circ}_{\text{anode}}$, with both entries expressed as reduction potentials.

Step 1: Find the iron(III)/iron(II) reduction potential Let the reduction potential for $Fe^{3+} + e^{-} \longrightarrow Fe^{2+}$ be u. Adding this reduction to $Fe_2{}^{2+} + e^{-} \longrightarrow Fe$ gives the three-electron reduction $Fe_3{}^{3+} + e^{-} \longrightarrow Fe$. Add the free-energy changes, preserving the electron counts:

$$-Fu - 2Fy = -3Fz.$$

Dividing by $-F$ gives $u + 2y = 3z$, hence $u = 3z - 2y$. This thermodynamic cycle accounts for the fact that the supplied iron half-reactions exchange different numbers of electrons.

Step 2: Combine silver reduction with iron oxidation In the required cell, Ag^{+} gains one electron and Fe^{2+} loses one electron. The silver reduction potential is x, while the corresponding iron reduction potential is $3z - 2y$. Therefore,

$$E^{\circ}_{\text{cell}} = x - (3z - 2y) = x + 2y - 3z.$$

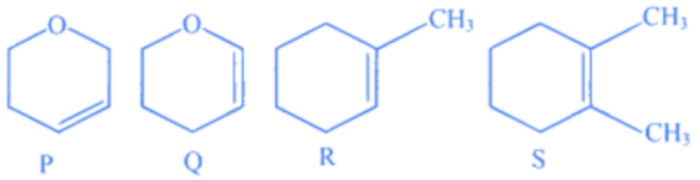
Option B omits the contribution of the three-electron iron reduction. Option C uses an incorrect silver coefficient and misses the iron(III) term. Option D incorrectly combines the iron potentials without weighting them by their electron numbers.

Step 3: Conclusion The standard cell potential is $\boxed{x + 2y - 3z}$ V. Hence, the correct option is (A).

Quick Tip to Solve: When electrode reactions involve different electron numbers, combine nE° through Gibbs energies before using the cathode-minus-anode formula.

69. Following are the four molecules "P", "Q", "R" and "S".

Which one among the four molecules will react with $H - Br_{(aq)}$ at the fastest rate? [4 marks]



(A) Q

(B) S

(C) P

(D) R

Correct Answer: (A) Q

Solution:

Concept:

- Addition of aqueous HBr to an alkene begins with electrophilic protonation of its pi bond. The stability of the developing carbocation strongly affects the rate.
- An oxygen lone pair conjugated with an adjacent double bond raises the electron density of that bond. The resulting alpha-oxy carbocation is stabilised by resonance as an oxocarbenium ion.

Step 1: Locate the oxygen-conjugated double bond In molecule Q, the double bond is directly adjacent to the ring oxygen: it is an enol ether unit. Protonation at the carbon farther from oxygen places the positive charge on the carbon directly bonded to oxygen. A lone pair on oxygen can then form a pi bond to this carbon, delocalising the charge. Thus the cationic intermediate is stabilised by two resonance contributors, one with charge on carbon and one with charge on oxygen.

Step 2: Compare the other three alkenes Molecule P also contains oxygen, but its double bond is separated from oxygen by a saturated carbon, so it does not receive the same direct lone-pair conjugation. Molecules R and S are ordinary alkyl-substituted cyclohexenes. Methyl substitution can stabilise their protonation intermediates by hyperconjugation and inductive donation, but neither has the strong adjacent-oxygen resonance available to Q. Although S has the more substituted double bond, counting alkyl groups alone would overlook the decisive enol-ether activation. Bromide attack follows formation of the cationic intermediate and does not remove this advantage of Q.

Step 3: Conclusion Molecule Q reacts with aqueous HBr at the fastest rate. Hence, the correct option is (A).

Quick Tip to Solve: Before ranking alkene substitution, check whether a heteroatom lone pair is directly conjugated with the double bond; enol ethers are especially reactive toward acids.

70. Which of the following ions is the strongest oxidizing agent? (Atomic Number of $\mathrm{Ce} = 58, \mathrm{Eu} = 63,\ \mathrm{Tb} = 65, \mathrm{Lu} = 71$) [4 marks]

(A) Lu^{3+}

(B) Eu^{2+}

(C) Tb^{4+}

(D) Ce^{3+}

Correct Answer: (C) Tb^{4+}

Solution:

Concept:

- An oxidising agent accepts electrons and is itself reduced. A high oxidation-state lanthanoid ion is strongly oxidising when reduction restores its stable trivalent state.
- The usual stable lanthanoid oxidation state is $+3$. Half-filled $4f^7$ and fully filled $4f^{14}$ configurations give additional stability in suitable ions.

Step 1: Examine the tetravalent terbium ion Terbium has atomic number 65. Its trivalent ion has the configuration $[\mathrm{Xe}]4f^8$, whereas Tb^{4+} has $[\mathrm{Xe}]4f^7$. Despite the half-filled configuration of the tetravalent ion, the high positive charge makes it a strong electron acceptor and it is readily reduced to the usual trivalent state:

$$\mathrm{Tb}^{4+} + \mathrm{e}^- \longrightarrow \mathrm{Tb}^{3+}.$$

Half-filled-shell stability must not be used alone to dismiss the strong oxidising tendency of this high oxidation state.

Step 2: Eliminate ions that are stable or reducing Lu^{3+} has the closed $4f^{14}$ configuration and is not readily reduced. Eu^{2+} has a half-filled $4f^7$ configuration but, among these choices, behaves as a reducing agent by losing an electron to form Eu^{3+}; it is not the strongest electron acceptor. Ce^{3+} is already in the common trivalent state and must not be confused with the well-known oxidant Ce^{4+}. Thus options A, B and D do not have the oxidising tendency of the tetravalent terbium ion given in option C.

Step 3: Conclusion $\boxed{\mathrm{Tb}^{4+}}$ is the strongest oxidising agent among the listed ions. Hence, the correct option is (C).

Quick Tip to Solve: Identify which species is reduced in an oxidising action; do not confuse Ce^{3+} with Ce^{4+}, or a stable divalent ion with an oxidant.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. 37.8 g N_2O_5 **was taken in a 1 L reaction vessel and allowed to undergo the following reaction at 500 K**

$$2\ N_2O_{5(\ g)} \rightleftharpoons 2\ N_2O_{4(\ g)} + O_{2(\ g)}$$

The total pressure at equilibrium was found to be 18.65 bar.

Then, Kp = _____ $\times 10^{-2}$ **[nearest integer]**

Assume N_2O_5 **to behave ideally under these conditions.**

Given: R $= 0.082$ **bar** $\text{Lmol}^{-1}\ \text{K}^{-1}$ [4 marks]

Solution:

Concept:

- For an ideal gas mixture, total moles follow $n_{\text{total}} = PV/(RT)$, and each partial pressure is $p_i = n_i RT/V$.
- For $2\,N_2O_5(g) \rightleftharpoons 2\,N_2O_4(g) + O_2(g)$, the pressure equilibrium expression is $K_p = p^2_{N_2O_4} p_{O_2} / p^2_{N_2O_5}$.

Step 1: Use the increase in total moles to find the extent The molar mass of N_2O_5 is $108\ \text{g mol}^{-1}$, so initially there are $37.8/108 = 0.350$ mol. Let the reaction extent be ξ mol. The equilibrium amounts are $0.350 - 2\xi$, 2ξ and ξ mol for N_2O_5, N_2O_4 and O_2, respectively. Their sum is $0.350 + \xi$. Using the supplied value of R,

$$0.350 + \xi = \frac{18.65(1)}{0.082(500)} = \frac{18.65}{41}, \qquad \xi = 0.104878 \text{ mol}.$$

Step 2: Calculate the partial pressures and equilibrium constant Here $RT/V = 41\ \text{bar mol}^{-1}$. Hence

$$\begin{aligned} p_{N_2O_5} &= (0.350 - 2\xi)41 = 5.75 \text{ bar}, \\ p_{N_2O_4} &= 2\xi(41) = 8.60 \text{ bar}, \\ p_{O_2} &= \xi(41) = 4.30 \text{ bar}. \end{aligned}$$

They sum to 18.65 bar, checking the calculation. Substitution gives

$$K_p = \frac{(8.60)^2(4.30)}{(5.75)^2} = 9.61899 = 961.899 \times 10^{-2}.$$

The requested blank is the coefficient multiplying 10^{-2}, so round 961.899 rather than 9.61899 to the nearest integer.

Step 3: Conclusion The required integer is $\boxed{962}$. The corresponding pressure-form K_p is approximately 9.62 bar when pressures are entered in bar (or 9.62 dimensionless with standard-state pressure factors).

Quick Tip to Solve: First find the extra total moles: they equal the reaction extent here. Round only the coefficient of the power of ten printed beside the answer blank.

72. Among the following cations, the number of cations which will give characteristic precipitate in their identification tests with $K_4\,[Fe(CN)_6]$ is _____.

$$Cu^{2+}, Fe^{3+}, Ba^{2+}, Ca^{2+}, NH_4^+, Mg^{2+}, Zn^{2+}$$

[4 marks]

Solution:

Concept:

- Potassium ferrocyanide supplies the complex anion $[Fe(CN)_6]^{4-}$. Characteristic insoluble ferrocyanides can identify selected metal cations under the prescribed test conditions.
- Count cations giving the specified identification test, rather than counting every cation that forms a precipitate with some other analytical reagent.

Step 1: Identify the cations with characteristic ferrocyanide precipitates Cu^{2+} gives a chocolate-brown precipitate of cupric ferrocyanide, and Fe^{3+} gives a deep-blue Prussian-blue precipitate. Their simplified ionic equations are

$$2\,Cu^{2+} + [Fe(CN)_6]^{4-} \longrightarrow Cu_2[Fe(CN)_6](s),$$

$$4\,Fe^{3+} + 3\,[Fe(CN)_6]^{4-} \longrightarrow Fe_4[Fe(CN)_6]_3(s).$$

Zn^{2+} gives a white zinc ferrocyanide precipitate. Ca^{2+} also gives the characteristic white calcium ammonium ferrocyanide precipitate in the ammonium-containing medium used for its ferrocyanide identification test.

Step 2: Exclude the remaining cations and count The four qualifying ions are therefore Cu^{2+}, Fe^{3+}, Ca^{2+} and Zn^{2+}. The characteristic identification of Ba^{2+} is instead carried out with reagents such as chromate or sulfate; it is not counted in this specified ferrocyanide test. Mg^{2+} is commonly confirmed as magnesium ammonium phosphate, while $NH_4{}^+$ is identified by liberation of ammonia with alkali. Those are different tests, so none adds another qualifying cation here. The colour distinction between brown copper, blue iron and the white precipitates is useful for recognition but each qualifying cation is counted only once.

Step 3: Conclusion There are $\boxed{4}$ qualifying cations. This is a dimensionless count.

Quick Tip to Solve: Associate ferrocyanide with brown copper, Prussian-blue iron, white zinc and the calcium test in ammonium medium; keep phosphate, sulfate and ammonia tests separate.

73. Consider the following reaction occurring in the blast furnace:

$$Fe_3O_{4(\ s)} + 4CO_{(g)} \rightarrow 3Fe_{(l)} + 4CO_{2(\ g)}$$

' x ' kg of iron is produced when 2.32×10^3 kgFe_3O_4 and 2.8×10^2 kgCO are brought together in the furnace. The value of ' x ' is _____ . (nearest integer)

{Given: molar mass of $Fe_3O_4 = 232$ g mol^{-1}

molar mass of $CO = 28$ g mol^{-1}

molar mass of $Fe = 56$ g mol^{-1}} [4 marks]

Solution:

Concept:

- A balanced reaction fixes the mole ratios of reactants and products. Here one mole of Fe_3O_4 consumes four moles of CO and forms three moles of iron.
- The limiting reagent is the reactant with the smaller value of n/ν, where n is its amount and ν its stoichiometric coefficient. It fixes the maximum product yield.

Step 1: Convert both reactant masses into amounts Use kilomoles so that the given kilogram masses can be divided by numerically equal molar masses in kg kmol^{-1}. This avoids mixing kilogram masses with gram-per-mole units.

$$n(Fe_3O_4) = \frac{2320}{232} = 10\,\text{kmol}, \qquad n(CO) = \frac{280}{28} = 10\,\text{kmol}.$$

Although the two amounts are equal, they are not required in an equal ratio in the reaction.

Step 2: Identify the reagent that is exhausted first The reaction is

$$Fe_3O_4 + 4\,CO \longrightarrow 3\,Fe + 4\,CO_2.$$

Complete reduction of $10\,\text{kmol}$ of oxide would require $40\,\text{kmol}$ of carbon monoxide, whereas only $10\,\text{kmol}$ is available. Equivalently, the reaction capacities are $10/1 = 10$ and $10/4 = 2.5\,\text{kmol}$. Therefore carbon monoxide is limiting and the oxide remains in excess.

Step 3: Calculate the mass of iron from the limiting amount Every four kilomoles of carbon monoxide produce three kilomoles of iron. Hence

$$n(Fe) = 10 \times \frac{3}{4} = 7.5\,\text{kmol}, \qquad m(Fe) = 7.5 \times 56 = 420\,\text{kg}.$$

Only $2.5\,\text{kmol}$ of oxide is consumed, less than its available $10\,\text{kmol}$, confirming the limiting-reagent choice. Using all the oxide would overestimate the iron yield because insufficient carbon monoxide is present.

Step 4: Conclusion The iron produced is $\boxed{420\,\text{kg}}$. Therefore the requested nearest-integer value is $\boxed{x = 420}$.

Quick Tip to Solve: Compare reactant moles divided by their balanced coefficients before computing yield; equal reactant moles do not imply equal reaction capacities.

74. Xg of benzoic acid on reaction with aq $NaHCO_3$ released CO_2 that occupied 11.2 L volume at STP. X is _____ g. [4 marks]

Solution:

Concept:

- A carboxylic acid reacts with aqueous sodium hydrogen carbonate to give its sodium salt, carbon dioxide and water. Benzoic acid has one carboxyl group, so each mole releases one mole of CO_2.
- At STP, the conventional ideal-gas molar volume used in this problem is $22.4\,\mathrm{L\,mol^{-1}}$. The amount of evolved gas is its volume divided by this molar volume.

Step 1: Write the balanced reaction The acidic hydrogen of the carboxyl group is replaced by sodium. The bicarbonate ion accepts this proton, forming carbonic acid, which decomposes into water and carbon dioxide. The net reaction is

$$C_6H_5COOH + NaHCO_3 \longrightarrow C_6H_5COONa + CO_2 + H_2O.$$

Thus the mole ratio of benzoic acid to the evolved gas is $1:1$. The aromatic hydrogens do not provide additional carbon dioxide in this acid-base reaction.

Step 2: Find the amount of carbon dioxide and acid Divide the stated gas volume by the STP molar volume:

$$n(CO_2) = \frac{11.2\,\mathrm{L}}{22.4\,\mathrm{L\,mol^{-1}}} = 0.500\,\mathrm{mol}.$$

The balanced equation therefore gives $n(C_6H_5COOH) = 0.500\,\mathrm{mol}$. No factor of two is introduced: benzoic acid is monocarboxylic and its single acidic proton produces one molecule of gas per acid molecule.

Step 3: Convert the amount of benzoic acid into mass Benzoic acid has molecular formula $C_7H_6O_2$. Using the usual atomic masses of carbon, hydrogen and oxygen, its molar mass is

$$M = 7(12) + 6(1) + 2(16) = 122\,\mathrm{g\,mol^{-1}}.$$

Consequently the initial acid mass is

$$m = nM = 0.500 \times 122 = 61\,\mathrm{g}.$$

As a check, one mole, or $122\,\mathrm{g}$, would release $22.4\,\mathrm{L}$; half that acid mass must release the stated half-volume.

Step 4: Conclusion The required benzoic acid mass is $\boxed{61\,\mathrm{g}}$, so $\boxed{X = 61}$.

Quick Tip to Solve: Count carboxyl groups before using the gas volume: each reacting carboxyl group produces one molecule of carbon dioxide with bicarbonate.

75. Standard entropies of X_2, Y_2 and XY_5 are 70, 50 and 110 J K^{-1} mol^{-1} respectively. The temperature in Kelvin at which the reaction

$$\frac{1}{2}X_2 + \frac{5}{2}Y_2 \rightleftharpoons XY_5 \Delta H^{\ominus} = -35 \text{ kJ mol}^{-1}$$

will be at equilibrium is _____ (Nearest integer) [4 marks]

Solution:

Concept:

- For a reaction, the standard entropy change is the sum of standard molar entropies of products minus that of reactants, each multiplied by its balanced coefficient: $\Delta S^\circ = \sum \nu S^\circ_{\text{products}} - \sum \nu S^\circ_{\text{reactants}}$.
- The Gibbs relation is $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$. For the equilibrium-temperature calculation requested here, set this Gibbs change to zero and use consistent energy units.

Step 1: Apply the fractional reaction coefficients to the entropies The reaction is written for formation of one mole of XY_5 from half a mole of X_2 and five-halves of a mole of Y_2. The entropy change for exactly this reaction is therefore

$$\begin{aligned}\Delta S^\circ &= 110 - \left(\frac{1}{2} \times 70 + \frac{5}{2} \times 50\right) \\ &= 110 - (35 + 125) \\ &= -50 \,\text{J K}^{-1} \,\text{mol}^{-1}.\end{aligned}$$

The negative sign means that the product side has a lower standard entropy for the specified reaction amounts. Omitting the fractional coefficients would give an incorrect temperature.

Step 2: Express the enthalpy in matching energy units The stated enthalpy change is negative because the forward reaction is exothermic. Convert kilojoules to joules before combining it with the entropy:

$$\Delta H^\circ = -35 \,\text{kJ mol}^{-1} = -35000 \,\text{J mol}^{-1}.$$

Both ΔH° and $T\Delta S^\circ$ now have the same units of joules per mole.

Step 3: Set the Gibbs energy change to zero At the requested balance between enthalpy and entropy contributions,

$$0 = -35000 - T(-50) = -35000 + 50T.$$

Thus

$$T = \frac{-35000}{-50} = 700 \,\text{K}.$$

The ratio is positive because both changes are negative. Substitution checks the result: at 700 K, $T\Delta S^\circ = -35000 \,\text{J mol}^{-1}$, which exactly cancels the enthalpy in the Gibbs expression.

Step 4: Conclusion The requested equilibrium temperature, to the nearest integer, is $\boxed{700 \,\text{K}}$. The numerical entry is $\boxed{700}$.

Quick Tip to Solve: Multiply each entropy by its reaction coefficient and convert kJ to J before using the ratio of enthalpy change to entropy change.