JEE Main 2025 Paper 1 (B.E./B.Tech) Jan 23 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 23 January 2025, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If $\mathrm{I}=\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x+\cos^{\frac{3}{2}} x}\,\mathrm{d}x$, **then** $\int_0^{2I} \frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}\,\mathrm{d}x$ **equals :** [4 marks]

(A) $\frac{\pi^2}{12}$
(B) $\frac{\pi^2}{4}$
(C) $\frac{\pi^2}{16}$
(D) $\frac{\pi^2}{8}$

Correct Answer: (C) $\frac{\pi^2}{16}$

Solution:

Concept:

- For an integrable function on $[0,a]$, reflection gives $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$. Adding the two forms often removes a symmetric denominator.
- If $g(a-x)=g(x)$, then $\int_0^a xg(x)\,dx=\frac{a}{2}\int_0^a g(x)\,dx$. The substitution $t=\tan x$ then converts the remaining trigonometric integral into a rational integral.

Step 1: Determine the first integral Reflect the given integral about $\pi/4$. Since sine and cosine interchange, the numerator in the reflected form is $\cos^{3/2} x$, with the same denominator. Therefore

$$2I=\int_0^{\pi/2}\frac{\sin^{3/2}x+\cos^{3/2}x}{\sin^{3/2}x+\cos^{3/2}x}\,dx=\frac{\pi}{2}.$$

Thus $I=\pi/4$, and the upper limit of the requested integral is $\pi/2$.

Step 2: Use symmetry to remove the factor x Put $g(x)=\sin x\cos x/(\sin^4 x+\cos^4 x)$. Both its numerator and denominator are unchanged by $x\mapsto\pi/2-x$. If the required integral is J, adding its reflected expression gives

$$2J=\frac{\pi}{2}\int_0^{\pi/2} g(x)\,dx,\qquad J=\frac{\pi}{4}K,$$

where $K=\int_0^{\pi/2} g(x)\,dx$.

Step 3: Evaluate the remaining integral For $t=\tan x$, the factors of $1+t^2$ cancel, and the limits become zero and infinity. Hence

$$K=\int_0^{\infty}\frac{t}{1+t^4}\,dt=\frac{1}{2}\int_0^{\infty}\frac{du}{1+u^2}=\frac{1}{2}[\tan^{-1}u]_0^{\infty}=\frac{\pi}{4},$$

using $u=t^2$. Thus $J=(\pi/4)(\pi/4)$. The other options are different multiples of this value; in particular, keeping the factor $\pi/2$ instead of $\pi/4$ would incorrectly double the answer.

Step 4: Conclusion The required integral is $\boxed{\pi^2/16}$. Hence, the correct option is (C).

Quick Tip to Solve: Apply reflection twice: first to find the upper limit, then to replace the factor x by its midpoint value $\pi/4$.

2. The number of complex numbers z, satisfying $|z| = 1$ and $\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$, is : [4 marks]

(A) 8

(B) 10

(C) 4

(D) 6

Correct Answer: (A) 8

Solution:

Concept:

- A complex number with $|z| = 1$ can be written uniquely as $z = e^{i\theta}$ for $0 \le \theta < 2\pi$. Its conjugate is $\bar{z} = e^{-i\theta}$, and neither denominator in the question vanishes.
- The identity $e^{it} + e^{-it} = 2\cos t$ converts conjugate exponential pairs into a real expression. Absolute value one permits both signs of that real expression.

Step 1: Express the condition in terms of an angle Using the unit-circle parametrisation, the two ratios are

$$\frac{z}{\bar{z}} = e^{2i\theta}, \qquad \frac{\bar{z}}{z} = e^{-2i\theta}.$$

Consequently the condition becomes

$$|2\cos 2\theta| = 1, \qquad \cos 2\theta = \frac{1}{2} \text{ or } -\frac{1}{2}.$$

It is essential to retain the negative possibility, since the outer modulus removes its sign.

Step 2: Count every distinct point once As θ varies over $[0, 2\pi)$, the angle 2θ varies over $[0, 4\pi)$, covering exactly two full cosine periods. In each period, $\cos t = 1/2$ has two solutions and $\cos t = -1/2$ has two further solutions. Thus the total is $2(2 + 2) = 8$. Equivalently, the allowed angles are

$$\theta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}.$$

They all lie in the selected half-open interval and are distinct, so they give eight distinct complex numbers. Counting only one sign would yield four, while six and ten do not match the complete angle list.

Step 3: Conclusion There are $\boxed{8}$ such complex numbers. Hence, the correct option is (A).

Quick Tip to Solve: Keep both signs after removing an absolute value, and track the full range of 2θ before counting roots.

3. The length of the chord of the ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$, whose mid-point is $\left(1,\frac{1}{2}\right)$, is : [4 marks]

(A) $\frac{2}{3}\sqrt{15}$

(B) $\frac{1}{3}\sqrt{15}$

(C) $\sqrt{15}$

(D) $\frac{5}{3}\sqrt{15}$

Correct Answer: (A) $\frac{2}{3}\sqrt{15}$

Solution:

Concept:

- A chord centred at (h,k) has endpoints $(h+u,k+v)$ and $(h-u,k-v)$. Subtracting their ellipse equations gives a linear relation that determines the chord direction.
- The distance between these endpoints is $2\sqrt{u^2+v^2}$. This is the full chord length; $\sqrt{u^2+v^2}$ is only its half-length.

Step 1: Find the direction of the chord Let the endpoints be $(1+u,1/2+v)$ and $(1-u,1/2-v)$. Both satisfy the given ellipse. Subtracting their equations yields

$$\frac{(1+u)^2-(1-u)^2}{4}+\frac{(1/2+v)^2-(1/2-v)^2}{2}=0.$$

This simplifies to $u+v=0$, so $v=-u$. Thus the chord has equation $x+y=3/2$, not the tangent line at the midpoint, which lies inside the ellipse.

Step 2: Determine the displacement to an endpoint Substitute $(x,y)=(1+u,1/2-u)$ into the ellipse:

$$\frac{(1+u)^2}{4}+\frac{(1/2-u)^2}{2}=1.$$

The linear terms cancel, as expected for a chord centred at the chosen midpoint. The remaining equation is

$$\frac{3}{8}+\frac{3}{4}u^2=1, \qquad u^2=\frac{5}{6}.$$

The positive value of u^2 confirms that two real endpoints occur.

Step 3: Compute the full endpoint distance Since $v^2=u^2$, the squared half-chord length is $u^2+v^2=5/3$. Therefore

$$L=2\sqrt{\frac{5}{3}}=\frac{2\sqrt{15}}{3}.$$

Option (B) is only the half-chord length. Options (C) and (D) give different distances and fail the endpoint calculation. The factor two must be included because the midpoint-to-endpoint displacement covers only half the chord.

Step 4: Conclusion The length of the chord is $\boxed{2\sqrt{15}/3}$ units. Hence, the correct option is (A).

Quick Tip to Solve: Write the endpoints as midpoint plus and minus a displacement; subtracting their conic equations immediately fixes the chord direction.

4. $\lim\limits_{x\to\infty} \frac{\left(2x^2-3x+5\right)(3x-1)^{\frac{x}{2}}}{(3x^2+5x+4)\sqrt{(3x+2)^x}}$ **is equal to :** [4 marks]

(A) $\frac{2e}{3}$

(B) $\frac{2}{3\sqrt{e}}$

(C) $\frac{2e}{\sqrt{3}}$

(D) $\frac{2}{\sqrt{3e}}$

Correct Answer: (B) $\frac{2}{3\sqrt{e}}$

Solution:

Concept:

- A ratio of polynomials of the same degree tends to the ratio of their leading coefficients. Here the quadratic factor therefore tends to $2/3$.
- For a positive base tending to one, evaluate a variable power by taking its logarithm. The standard limit $\log(1+u)/u \to 1$ as $u \to 0$ determines the exponential factor.

Step 1: Separate the polynomial and power factors For sufficiently large positive x, both $3x-1$ and $3x+2$ are positive. Thus the square root in the denominator equals $(3x+2)^{x/2}$, and the expression can be written as

$$\frac{2x^2-3x+5}{3x^2+5x+4}\left(\frac{3x-1}{3x+2}\right)^{x/2}.$$

Dividing the polynomial numerator and denominator by x^2 gives the limit $2/3$ for the first factor. The second factor is an indeterminate power of the form 1^∞, so direct substitution is insufficient.

Step 2: Evaluate the power by logarithms Let $E_x = (1-3/(3x+2))^{x/2}$. Its logarithm is

$$\log E_x = \frac{x}{2}\log\left(1-\frac{3}{3x+2}\right).$$

Set $u = -3/(3x+2)$. Then $u \to 0$, and

$$\log E_x = \frac{x}{2}\,u\,\frac{\log(1+u)}{u} \longrightarrow -\frac{1}{2}.$$

Continuity of the exponential function gives $E_x \to e^{-1/2}$. Since the base is below one for large positive x, a limit smaller than one is also consistent with the sign of this exponent.

Step 3: Combine the limits and compare options The required limit is

$$\frac{2}{3}e^{-1/2} = \frac{2}{3\sqrt{e}}.$$

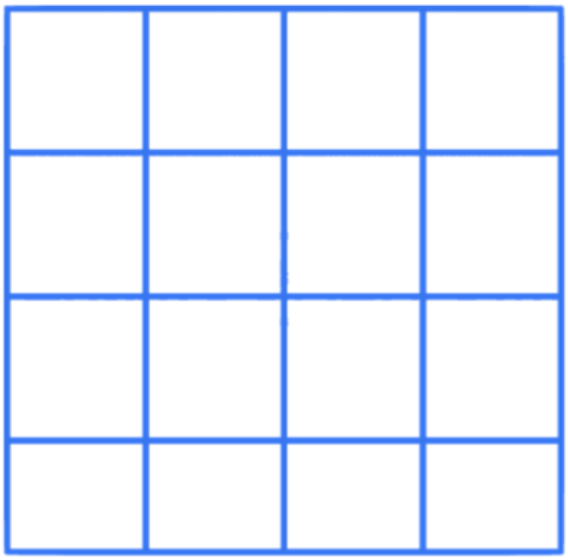
A positive exponent, as in options (A) and (C), has the wrong sign for the power factor. Option (D) has a different constant multiplier, because the polynomial ratio is $2/3$, not $2/\sqrt{3}$. Both the exponential factor and the leading-coefficient ratio must be retained.

Step 4: Conclusion The limit is $\boxed{2/(3\sqrt{e})}$. Hence, the correct option is (B).

Quick Tip to Solve: Combine powers with the same exponent before taking logarithms; the small change in the base multiplied by the exponent gives $-1/2$.

5. A board has 16 squares as shown in the figure :

Out of these 16 squares, two squares are chosen at random. The probability that they have no side in common is : [4 marks]



(A) $\frac{3}{5}$

(B) $\frac{4}{5}$

(C) $\frac{23}{30}$

(D) $\frac{7}{10}$

Correct Answer: (B) $\frac{4}{5}$

Solution:

Concept:

- Two distinct squares chosen at random from sixteen give $\binom{16}{2}$ equally likely unordered pairs. Their order of selection does not affect whether they share a side.
- The complementary event is that the two chosen squares share a full side. In the shown 4×4 board, count horizontal and vertical adjacent pairs separately; corner contact alone is allowed in the required event.

Step 1: Count all possible pairs of squares The board consists of four rows and four columns of unit squares. Choosing any two of the sixteen cells gives

$$N = \binom{16}{2} = \frac{16 \cdot 15}{2} = 120$$

equally likely pairs. The sixteen squares in the question are the displayed cells, so larger composite squares are not additional choices. Using unordered pairs avoids counting each physical selection twice.

Step 2: Count pairs sharing a side In each row there are three horizontally adjacent pairs, giving $4 \cdot 3 = 12$ pairs. Likewise each column has three vertically adjacent pairs, giving another $4 \cdot 3 = 12$. A pair of distinct cells cannot be both horizontally and vertically adjacent, so these counts do not

overlap. Hence

$$N_{\text{side}} = 12 + 12 = 24.$$

Diagonally neighbouring cells meet at a corner but share no side; they are correctly excluded from this complementary count.

Step 3: Subtract from the total The number of pairs with no common side is $120 - 24 = 96$. Therefore

$$P(\text{no common side}) = 1 - \frac{24}{120} = \frac{96}{120} = \frac{4}{5}.$$

The probability of sharing a side is only $1/5$, and its complement is the required answer. Options (A), (C) and (D) would correspond to different counts of forbidden pairs and disagree with the exact horizontal-plus-vertical count.

Step 4: Conclusion The required probability is $\boxed{4/5}$. Hence, the correct option is (B).

Quick Tip to Solve: For an $r \times c$ board, count side-sharing pairs as $r(c-1) + c(r-1)$, then divide by $\binom{rc}{2}$ and subtract from one.

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6. If the square of the shortest distance between the lines $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z+3}{-3}$ and $\frac{x+1}{2} = \frac{y+3}{4} = \frac{z+5}{-5}$ is $\frac{m}{n}$, where m, n are coprime numbers, then $m+n$ is equal to : [4 marks]

(A) 14

(B) 6

(C) 21

(D) 9

Correct Answer: (D) 9

Solution:

Concept:

- For nonparallel lines through position vectors $\mathbf{a}_1, \mathbf{a}_2$ with direction vectors $\mathbf{b}_1, \mathbf{b}_2$, the shortest distance is $d = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)| / |\mathbf{b}_1 \times \mathbf{b}_2|$.
- The cross product is perpendicular to both lines, so the scalar triple product extracts the component of their separation in the common normal direction. Squaring the formula removes the absolute value and square root.

Step 1: Identify the points and direction vectors Reading the symmetric equations carefully gives

$$\mathbf{a}_1 = (2, 1, -3), \qquad \mathbf{b}_1 = (1, 2, -3),$$
$$\mathbf{a}_2 = (-1, -3, -5), \quad \mathbf{b}_2 = (2, 4, -5).$$

In particular, $z+3$ and $z+5$ correspond to negative coordinates of the chosen points. Their separation is $\mathbf{a}_2 - \mathbf{a}_1 = (-3, -4, -2)$. The directions are not proportional, because the third components do not have the same ratio as the first two.

Step 2: Compute the common normal and triple product The cross product is

$$\mathbf{b}_1 \times \mathbf{b}_2 = (2(-5) - (-3)4,\ (-3)2 - 1(-5),\ 1(4) - 2(2)) = (2, -1, 0).$$

Its squared magnitude is $2^2 + (-1)^2 = 5$. The scalar triple product is

$$(-3, -4, -2) \cdot (2, -1, 0) = -6 + 4 = -2.$$

The nonzero triple product also confirms that the lines do not intersect.

Step 3: Reduce the squared distance Using the shortest-distance formula,

$$d^2 = \frac{(-2)^2}{5} = \frac{4}{5}.$$

The numerator and denominator are already coprime. Thus $m = 4$, $n = 5$, and $m + n = 9$. Reversing the cross-product order changes only its sign and leaves this squared distance unchanged. The other three options, 14, 6 and 21, do not equal the sum obtained from the reduced fraction.

Step 4: Conclusion We obtain $d^2 = 4/5$ and $\boxed{m + n = 9}$. Hence, the correct option is (D).

Quick Tip to Solve: Square the scalar-triple-product distance formula directly when the question asks for d^2; this avoids unnecessary square roots.

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7. Let $\mathrm{X} = \mathbf{R} \times \mathbf{R}$. Define a relation R on X as :

$$(a_1, b_1)\, R\, (a_2, b_2) \Leftrightarrow b_1 = b_2$$

Statement I: R **is an equivalence relation.**

Statement II : For some $(\mathrm{a}, \mathrm{b}) \in \mathrm{X}$**, the** $\operatorname{set} \mathrm{S} = \{(x, y) \in \mathrm{X} : (x, y)\mathrm{R}(\mathrm{a}, \mathrm{b})\}$ **represents a line parallel to** $y = x$**.**

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are true

(B) Statement I is true but Statement II is false

(C) Both Statement I and Statement II are false

(D) Statement I is false but Statement II is true

Correct Answer: (B) Statement I is true but Statement II is false

Solution:

Concept:

- A relation is an equivalence relation precisely when it is reflexive, symmetric and transitive. Here two ordered pairs are related when their second coordinates are equal.
- The equivalence class of a point consists of all points related to it. Fixing the second coordinate produces a horizontal line, whereas a line parallel to $y = x$ must have slope 1.

Step 1: Verify reflexivity and symmetry For any $u = (a, b) \in X$, its second coordinate equals itself, so uRu. Thus the relation is reflexive. If $(a_1, b_1)R(a_2, b_2)$, then $b_1 = b_2$. Equality is symmetric, so $b_2 = b_1$, which gives $(a_2, b_2)R(a_1, b_1)$. Hence the relation is also symmetric; the first coordinates need not be equal.

Step 2: Verify transitivity If $(a_1, b_1)R(a_2, b_2)$ and $(a_2, b_2)R(a_3, b_3)$, then $b_1 = b_2$ and $b_2 = b_3$. Therefore $b_1 = b_3$, giving $(a_1, b_1)R(a_3, b_3)$. All three defining properties hold, so Statement I is true.

Step 3: Identify the set in Statement II Fix any $(a, b) \in X$. By the definition of the relation,

$$(x, y)R(a, b) \iff y = b.$$

Thus $S = \{(x, b) : x \in \mathbf{R}\}$, a horizontal line of slope zero, parallel to the x-axis. Changing a has no effect on this set, and changing b merely changes its height. No such line is parallel to $y = x$, whose slope is one. Statement II is therefore false for every choice of the fixed point. Options (A) and (D) wrongly make Statement II true, while option (C) wrongly makes Statement I false.

Step 4: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (B).

Quick Tip to Solve: To identify an equivalence class, fix the reference point and rewrite the relation as an equation in the variable coordinates.

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8. If the area of the region $\{(x, y) : -1 \le x \le 1, 0 \le y \le \mathrm{a} + \mathrm{e}^{|x|} - \mathrm{e}^{-x}, \mathrm{a} > 0\}$ is $\frac{\mathrm{e}^2+8\mathrm{e}+1}{\mathrm{e}}$, then the value of a is : [4 marks]

(A) 7

(B) 5

(C) 6

(D) 8

Correct Answer: (B) 5

Solution:

Concept:

- The required area is the integral of the upper boundary minus the lower boundary over $[-1, 1]$. Since $a > 0$, the upper boundary is positive on this interval.
- The absolute value changes its expression at zero: $|x| = -x$ on $[-1, 0]$ and $|x| = x$ on $[0, 1]$. Split the area integral at this point.

Step 1: Split the boundary at zero On the negative half of the interval, $e^{|x|} = e^{-x}$, so the two exponential terms cancel and the upper boundary is simply a. On the positive half it is $a + e^x - e^{-x}$. Consequently,

$$A = \int_{-1}^{0} a\,dx + \int_{0}^{1} (a + e^x - e^{-x})\,dx.$$

This is an ordinary area integral, with no additional absolute value needed: both heights are positive.

Step 2: Evaluate the two areas The first integral gives a. In the second, the constant contributes

another a, and an antiderivative of $e^x - e^{-x}$ is $e^x + e^{-x}$. Hence

$$A = 2a + \left[e^x + e^{-x}\right]_0^1 = 2a + e + e^{-1} - 2.$$

In particular, the negative exponential contributes $e^{-1} - 1$, which is negative; keeping this sign is essential.

Step 3: Use the given area and compare the options The specified area simplifies to

$$\frac{e^2 + 8e + 1}{e} = e + 8 + e^{-1}.$$

Equating the two expressions cancels the exponential terms:

$$2a + e + e^{-1} - 2 = e + 8 + e^{-1} \quad \Longrightarrow \quad 2a = 10.$$

The unique positive solution is $a = 5$. The proposed values $7, 6, 8$ would give constant contributions $2a - 2 = 12, 10, 14$, respectively, rather than the required value 8.

Step 4: Conclusion The required value is $a = 5$. Hence, the correct option is (B).

Quick Tip to Solve: For an area containing $e^{|x|}$, split at zero before integrating; on the negative side it may cancel another exponential exactly.

9. Let the range of the function $f(x) = 6 + 16\cos x \cdot \cos\left(\frac{\pi}{3} - x\right) \cdot \cos\left(\frac{\pi}{3} + x\right) \cdot \sin 3x \cdot \cos 6x, x \in \mathbf{R}$ be $[\alpha, \beta]$. Then the distance of the point (α, β) from the line $3x + 4y + 12 = 0$ is : [4 marks]

(A) 11

(B) 10

(C) 8

(D) 9

Correct Answer: (A) 11

Solution:

Concept:

- Use product identities to replace a product of shifted cosines by a multiple-angle expression. For real x, the sine of $12x$ takes every value in $[-1, 1]$.
- The perpendicular distance from (x_0, y_0) to $Ax + By + C = 0$ is $|Ax_0 + By_0 + C|/\sqrt{A^2 + B^2}$.

Step 1: Simplify the three cosine factors Using the product-to-sum identity gives

$$\cos\left(\frac{\pi}{3} - x\right)\cos\left(\frac{\pi}{3} + x\right) = \frac{\cos(2\pi/3) + \cos 2x}{2} = \cos^2 x - \frac{3}{4}.$$

Multiplying by $\cos x$ and using $\cos 3x = 4\cos^3 x - 3\cos x$, we obtain

$$\cos x \cos\left(\frac{\pi}{3} - x\right)\cos\left(\frac{\pi}{3} + x\right) = \frac{1}{4}\cos 3x.$$

This identity holds for all real x, so no possible range values are lost.

Step 2: Find both endpoints of the range Substitute the product identity into the given function and apply the double-angle identity twice:

$$\begin{aligned} f(x) &= 6 + 4\cos 3x \sin 3x \cos 6x \\ &= 6 + 2\sin 6x \cos 6x \\ &= 6 + \sin 12x. \end{aligned}$$

Thus the smallest value is 5 and the largest is 7. Both are attained, for example when $12x$ has sine -1 or 1, respectively. Therefore $\alpha = 5$ and $\beta = 7$.

Step 3: Calculate the point-to-line distance The point specified in the question is consequently $(5, 7)$, not a point obtained by substituting x into the original function. Its distance from the given line is

$$d = \frac{|3(5) + 4(7) + 12|}{\sqrt{3^2 + 4^2}} = \frac{15 + 28 + 12}{5} = 11.$$

The other listed distances $10, 8, 9$ disagree with this direct substitution.

Step 4: Conclusion The distance is 11 units. Hence, the correct option is (A).

Quick Tip to Solve: In products with angles $x, \pi/3 - x, \pi/3 + x$, look for $\cos 3x$; then successive double-angle identities often reveal the range immediately.

10. The system of equations

$$\begin{aligned} x + y + z &= 6, \\ x + 2y + 5z &= 9, \\ x + 5y + \lambda z &= \mu, \end{aligned}$$

has no solution if [4 marks]

(A) $\lambda = 17, \mu = 18$

(B) $\lambda = 17, \mu \neq 18$

(C) $\lambda = 15, \mu \neq 17$

(D) $\lambda \neq 17, \mu \neq 18$

Correct Answer: (B) $\lambda = 17, \mu \neq 18$

Solution:

Concept:

- Elementary elimination preserves the solution set of a linear system. A contradiction of the form $0 = c$, with $c \neq 0$, proves inconsistency.
- A zero coefficient alone does not imply no solution: the corresponding right-hand side must be nonzero. When both sides vanish, one equation becomes dependent.

Step 1: Eliminate the variable common to all equations Label the displayed equations E_1, E_2, E_3.

Subtract E_1 from each of the other two to remove x:

$$E_2 - E_1 : y + 4z = 3,$$
$$E_3 - E_1 : 4y + (\lambda - 1)z = \mu - 6.$$

These differences are equivalent constraints together with E_1; they retain every solution of the original system.

Step 2: Isolate the decisive consistency condition Subtract four times the first reduced equation from the second. The y-terms cancel and give

$$(\lambda - 1 - 16)z = \mu - 6 - 12,$$

or

$$(\lambda - 17)z = \mu - 18.$$

If $\lambda \neq 17$, this equation determines a unique z. Then $y = 3 - 4z$ and $x = 6 - y - z$, so the original system has a unique solution for every μ.

Step 3: Classify the singular cases and reject the distractors When $\lambda = 17$, the last equation becomes $0 = \mu - 18$. It is impossible exactly when $\mu \neq 18$. If instead $\mu = 18$, the last constraint is an identity and z remains free, giving infinitely many solutions, not no solution. Thus option (A) describes the dependent consistent case. Option (C), with $\lambda = 15$, and option (D), with $\lambda \neq 17$, both have a nonzero coefficient of z and therefore unique solutions.

Step 4: Conclusion The system has no solution precisely when $\lambda = 17$ and $\mu \neq 18$. Hence, the correct option is (B).

Quick Tip to Solve: After elimination, inspect both sides of the final equation: zero equals nonzero means no solution, whereas zero equals zero means dependence.

11. The distance of the line $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ from the point $(1, 4, 0)$ along the line $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ is : [4 marks]

(A) $\sqrt{17}$
(B) $\sqrt{13}$
(C) $\sqrt{15}$
(D) $\sqrt{14}$

Correct Answer: (D) $\sqrt{14}$

Solution:

Concept:

- To measure the distance along a given line, find where that line meets the target line and then measure from the supplied point to the intersection.
- A line with direction vector $\mathbf{d}$ has parametric form $\mathbf{r} = \mathbf{r}_0 + t\mathbf{d}$. A parameter change Δt corresponds to length $|\Delta t|\,|\mathbf{d}|$.

Step 1: Parameterise the line along which distance is measured Write the second given line as

$$(x, y, z) = (t, 2 + 2t, -3 + 3t).$$

The supplied point $P = (1, 4, 0)$ lies on this line at $t = 1$, since all three coordinates agree. Its direction vector is $(1, 2, 3)$. The question asks for distance along this direction, so the perpendicular point-to-line distance formula is not applicable here.

Step 2: Find the intersection with the target line Parameterise the first line independently by s:

$$(x, y, z) = (2 + 2s, 6 + 3s, 3 + 4s).$$

At an intersection the first two coordinates give

$$t = 2 + 2s, \qquad 2 + 2t = 6 + 3s.$$

Substituting for t yields $6 + 4s = 6 + 3s$, hence $s = 0$ and $t = 2$. The third coordinates also match: $-3 + 3(2) = 3 + 4(0) = 3$. Therefore the lines really intersect at $T = (2, 6, 3)$; they are not skew.

Step 3: Measure the required segment and check the options The displacement from P to the intersection is

$$\overrightarrow{PT} = (2 - 1, 6 - 4, 3 - 0) = (1, 2, 3).$$

It is parallel to the prescribed line, as required. Its length is

$$PT = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}.$$

Equivalently the parameter changes from 1 to 2, so the distance is $|2 - 1|\sqrt{14}$. The squared distances in options (A), (B) and (C) are $17, 13, 15$, respectively, while the actual squared displacement is 14.

Step 4: Conclusion The distance measured along the specified line is $\sqrt{14}$ units. Hence, the correct option is (D).

Quick Tip to Solve: When a distance is requested along a line, parameterise that line first and check that the supplied point lies on it before locating the intersection.

12. Let the point A divide the line segment joining the points $\mathrm{P}(-1,-1,2)$ and $\mathrm{Q}(5,5,10)$ internally in the ratio $r:1(r>0)$. If O is the origin and $(\overrightarrow{\mathrm{OQ}}\cdot\overrightarrow{\mathrm{OA}})-\frac{1}{5}|\overrightarrow{\mathrm{OP}}\times\overrightarrow{\mathrm{OA}}|^2=10$, then the value of r is : [4 marks]

(A) $\sqrt{7}$

(B) 14

(C) 7

(D) 3

Correct Answer: (C) 7

Solution:

Concept:

- If A divides PQ internally in the ratio $r:1$, then $\overrightarrow{OA}=(\overrightarrow{OP}+r\overrightarrow{OQ})/(r+1)$, with $r>0$.
- The dot product is the sum of coordinatewise products. Bilinearity gives $\overrightarrow{OP}\times\overrightarrow{OA}=r(\overrightarrow{OP}\times\overrightarrow{OQ})/(r+1)$.

Step 1: Express the position vector of the dividing point Use the internal section formula with the coefficient r attached to Q:

$$\overrightarrow{OA}=\frac{(5r-1,\,5r-1,\,10r+2)}{r+1}.$$

Taking the dot product with $\overrightarrow{OQ}=(5,5,10)$ gives

$$\overrightarrow{OQ}\cdot\overrightarrow{OA}=\frac{5(5r-1)+5(5r-1)+10(10r+2)}{r+1}=\frac{150r+10}{r+1}.$$

The denominator is positive because the division is internal and $r>0$.

Step 2: Evaluate the squared cross product First compute the fixed vector product:

$$(-1,-1,2)\times(5,5,10)=(-20,20,0).$$

Its squared magnitude is 800. The component of $\overrightarrow{OA}$ parallel to $\overrightarrow{OP}$ contributes zero to the cross product, so

$$|\overrightarrow{OP}\times\overrightarrow{OA}|^2=\frac{800r^2}{(r+1)^2}.$$

Thus the coefficient $1/5$ in the question changes this term to $160r^2/(r+1)^2$.

Step 3: Solve the scalar condition and impose internal division Substitute both expressions and multiply by $(r+1)^2$:

$$\frac{150r+10}{r+1}-\frac{160r^2}{(r+1)^2}=10,$$
$$(150r+10)(r+1)-160r^2=10(r+1)^2,$$
$$-20r^2+140r=0.$$

Hence $r(r-7)=0$. The root $r=0$ is excluded by the printed condition $r>0$; it would place A at the endpoint P. The only admissible ratio is $7:1$. None of $\sqrt{7},14,3$ satisfies this factored equation with positive r.

Step 4: Conclusion The required value is $r = 7$. Hence, the correct option is (C).

Quick Tip to Solve: Before expanding a cross product involving a section point, use bilinearity; the cross product of a vector with itself vanishes and saves most of the algebra.

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13. Let $A = [a_{ij}]$ be a 3×3 matrix such that $A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, A \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $A \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, then a_{23} equals : [4 marks]

(A) 2

(B) -1

(C) 1

(D) 0

Correct Answer: (B) -1

Solution:

Concept:

- Multiplication by a column vector forms a linear combination of the columns of the matrix. In particular, multiplying by the second standard basis vector selects the second column.
- To find a_{23}, only the second row is needed. Write that row as (u, v, w), so the requested entry is w.

Step 1: Read the second component of each equation The second component of the first given product is zero. Since its input is $(0, 1, 0)^T$, it follows that $v = 0$. The other two second components give

$$4u + v + 3w = 1, \qquad 2u + v + 2w = 0.$$

These equations use the second row throughout; the third component of the first output does not give the requested entry.

Step 2: Eliminate the other row entries Substitute $v = 0$. Dividing the last equation by two gives $u + w = 0$, hence $u = -w$. Therefore

$$4(-w) + 3w = 1 \quad \Longrightarrow \quad -w = 1 \quad \Longrightarrow \quad w = -1.$$

For a direct check, the second row is $(1, 0, -1)$. Its scalar products with the three input vectors are respectively 0, $4 - 3 = 1$, and $2 - 2 = 0$, exactly the required second components.

Step 3: Compare the candidate entries The alternatives 2, 1, and 0 cannot satisfy both $u + w = 0$ and $4u + 3w = 1$: after using the first equation, the second requires specifically $-w = 1$. There is therefore a unique possible third entry in the second row; recovering the entire matrix would be unnecessary.

Step 4: Conclusion Thus $a_{23} = \boxed{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: When only one matrix entry is requested, use the corresponding output component in every given matrix equation instead of calculating the whole matrix.

---

14. Let $A = \{(x, y) \in \mathbf{R} \times \mathbf{R} : |x + y| \geqslant 3\}$ **and** $B = \{(x, y) \in \mathbf{R} \times \mathbf{R} : |x| + |y| \leq 3\}$**. If** $C = \{(x, y) \in A \cap B : x = 0$ **or** $y = 0\}$**, then** $\sum_{(x,y)\in C} |x + y|$ **is :** [4 marks]

(A) 18

(B) 24

(C) 15

(D) 12

Correct Answer: (D) 12

Solution:

Concept:

- The triangle inequality gives $|x + y| \leq |x| + |y|$. Thus the conditions defining the intersection force equality at the boundary value 3.
- The extra condition defining C restricts the ordered pairs to the coordinate axes. Each distinct ordered pair must be counted once in the sum.

Step 1: Restrict the intersection to the axes For a point in $A \cap B$, combine the two defining inequalities with the triangle inequality:

$$3 \leq |x + y| \leq |x| + |y| \leq 3.$$

Consequently every quantity in this chain equals 3. In particular, the origin cannot belong to the intersection, since its value of $|x + y|$ is zero.

Step 2: List the admissible ordered pairs On the vertical axis, set $x = 0$. The two original conditions become $|y| \geq 3$ and $|y| \leq 3$, so $y = 3$ or $y = -3$. Similarly, on the horizontal axis, $y = 0$ forces $x = 3$ or $x = -3$. Hence

$$C = \{(0, 3), (0, -3), (3, 0), (-3, 0)\}.$$

These are four different ordered pairs. Although their absolute-value contributions coincide, they are four separate members of the set.

Step 3: Add the four contributions At each listed point the absolute value of the coordinate sum is 3. Therefore

$$\sum_{(x,y)\in C} |x + y| = 3 + 3 + 3 + 3 = 12.$$

A sum of 15, 18, or 24 would require five, six, or eight contributions of size three, respectively; none agrees with the explicit list of four admissible points.

Step 4: Conclusion The required sum is $\boxed{12}$. Hence, the correct option is (D).

Quick Tip to Solve: When an intersection includes an axis condition, substitute each axis directly; remember that positive and negative axis intercepts are distinct ordered pairs.

15. If in the expansion of $(1+x)^p(1-x)^q$, the coefficients of x and x^2 are 1 and -2 , respectively, then p^2+q^2 is equal to : [4 marks]

(A) 8

(B) 20

(C) 13

(D) 18

Correct Answer: (C) 13

Solution:

Concept:

- In a product of two power series, the coefficient of a given power is obtained by summing products whose exponents add to that power.
- The linear and quadratic coefficients of the first factor are p and $p(p-1)/2$. Those of the second factor are $-q$ and $q(q-1)/2$, respectively.

Step 1: Use the linear coefficient The coefficient of x comes from multiplying the linear term of either factor by the constant term of the other. It is therefore $p-q$. The stated coefficient gives

$$p-q=1.$$

The minus sign in the second factor matters: replacing this difference by a sum would discard the sign of its linear term.

Step 2: Use the quadratic coefficient The coefficient of x^2 includes the two quadratic terms and also the product of the two linear terms. Thus

$$\frac{p(p-1)}{2}+\frac{q(q-1)}{2}-pq=-2.$$

Collecting terms expresses the left side using the known difference:

$$\frac{(p-q)^2-(p+q)}{2}=-2.$$

Substituting $p-q=1$ gives $1-(p+q)=-4$, so $p+q=5$.

Step 3: Calculate and check the exponents Adding and subtracting the equations $p+q=5$ and $p-q=1$ gives $p=3$, $q=2$. Indeed, the quadratic coefficient is $3+1-6=-2$, and the linear coefficient is $3-2=1$. Finally,

$$p^2+q^2=3^2+2^2=9+4=13.$$

The other proposed values 8, 20, and 18 do not equal the sum of squares of these uniquely determined exponents.

Step 4: Conclusion Therefore $p^2+q^2=\boxed{13}$. Hence, the correct option is (C).

Quick Tip to Solve: For this product, remember the quadratic coefficient as $\big((p-q)^2-(p+q)\big)/2$; it immediately connects the two supplied coefficients.

16. A rod of length eight units moves such that its ends A and B always lie on the lines $x - y + 2 = 0$ and $y + 2 = 0$, respectively. If the locus of the point P, that divides the rod AB internally in the ratio $2 : 1$ is $9\left(x^2 + \alpha y^2 + \beta xy + \gamma x + 28y\right) - 76 = 0$, then $\alpha - \beta - \gamma$ is equal to : [4 marks]

(A) 24

(B) 22

(C) 21

(D) 23

Correct Answer: (D) 23

Solution:

Concept:

- An internal division point with $AP : PB = 2 : 1$ has coordinates $P = (A + 2B)/3$. The larger coefficient multiplies the endpoint farther from P.
- The endpoint constraints reduce the motion to two parameters. The fixed rod length then supplies the equation of the locus after those parameters are eliminated.

Step 1: Express the endpoints using the moving point Let $A = (a, a+2)$, since A lies on $x - y + 2 = 0$, and let $B = (b, -2)$. If $P = (x, y)$, the section formula gives

$$x = \frac{a + 2b}{3}, \qquad y = \frac{a - 2}{3}.$$

Thus $a = 3y + 2$ and $b = (3x - a)/2$. These relations encode both line constraints and the prescribed division ratio.

Step 2: Impose the fixed distance The horizontal separation is $a - b = 3(a - x)/2$, and the vertical separation is $a + 4$. Using $AB = 8$,

$$\frac{9}{4}(a - x)^2 + (a + 4)^2 = 64.$$

Substitute $a = 3y + 2$ and multiply by four:

$$9(x - 3y - 2)^2 + 36(y + 2)^2 = 256.$$

Both squared separations are required; using only the vertical separation would ignore the rod's inclination.

Step 3: Expand and compare coefficients Combining the squares gives

$$(x - 3y - 2)^2 + 4(y + 2)^2 = x^2 + 13y^2 - 6xy - 4x + 28y + 20.$$

Consequently the locus is

$$9(x^2 + 13y^2 - 6xy - 4x + 28y) - 76 = 0.$$

This matches the printed constant and the coefficient of y, providing checks on the expansion. Hence $\alpha = 13$, $\beta = -6$, and $\gamma = -4$. Their required combination is $13 - (-6) - (-4) = 23$, excluding 24, 22,

and 21.

Step 4: Conclusion The required value is $\alpha - \beta - \gamma = \boxed{23}$. Hence, the correct option is (D).

Quick Tip to Solve: Parameterise each endpoint on its own line, eliminate the parameters with the section formula, and use the fixed length only after that substitution.

---

17. Let $\int x^3 \sin x \, dx = g(x) + C$, where C is the constant of integration. If $8\left(g\left(\frac{\pi}{2}\right) + g'\left(\frac{\pi}{2}\right)\right) = \alpha\pi^3 + \beta\pi^2 + \gamma, \alpha, \beta, \gamma \in Z$, then $\alpha + \beta - \gamma$ equals : [4 marks]

(A) 47

(B) 55

(C) 62

(D) 48

Correct Answer: (B) 55

Solution:

Concept:

- Integration by parts uses $\int u\,dv = uv - \int v\,du$. Choosing the polynomial as u reduces its degree at every repetition.
- The derivative of the nonconstant antiderivative term is the integrand: $g\prime(x) = x^3 \sin x$. The separate constant of integration is represented by C.

Step 1: Find the antiderivative term Integrate by parts with $u = x^3$ and $dv = \sin x\,dx$. Repeating on the remaining polynomial-trigonometric integrals gives

$$\int x^3 \sin x\,dx = -x^3 \cos x + 3\int x^2 \cos x\,dx,$$
$$\int x^2 \cos x\,dx = x^2 \sin x + 2x \cos x - 2\sin x.$$

Therefore take

$$g(x) = -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x.$$

The arbitrary additive constant belongs to the separately displayed C.

Step 2: Evaluate the function and its derivative At $x = \pi/2$, $\sin x = 1$ and $\cos x = 0$. Hence

$$g(\pi/2) = \frac{3\pi^2}{4} - 6, \qquad g\prime(\pi/2) = \frac{\pi^3}{8}.$$

The derivative value can be read directly from the integrand, without differentiating the expanded answer. As a check, differentiating that expansion cancels all terms except $x^3 \sin x$.

Step 3: Identify the integer coefficients Multiply the sum by eight:

$$8\big(g(\pi/2) + g\prime(\pi/2)\big) = \pi^3 + 6\pi^2 - 48.$$

Thus $\alpha = 1$, $\beta = 6$, and $\gamma = -48$. The requested subtraction is

$$\alpha + \beta - \gamma = 1 + 6 - (-48) = 55.$$

The alternatives 47, 62, and 48 do not follow from these coefficients; in particular, subtracting the negative constant adds 48.

Step 4: Conclusion The required value is $\boxed{55}$. Hence, the correct option is (B).

Quick Tip to Solve: For a combination of an antiderivative and its derivative at one point, obtain the derivative value straight from the original integrand.

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18. A spherical chocolate ball has a layer of ice-cream of uniform thickness around it. When the thickness of the ice-cream layer is 1 cm , the ice-cream melts at the rate of $81\ \text{cm}^3/\text{min}$ and the thickness of the ice-cream layer decreases at the rate of $\frac{1}{4\pi}\ \text{cm/min}$. The surface area (in cm^2) of the chocolate ball (without the ice-cream layer) is : [4 marks]

(A) 128π

(B) 196π

(C) 225π

(D) 256π

Correct Answer: (D) 256π

Solution:

Concept:

- If the chocolate radius is the constant r and the ice-cream thickness is h, the outer radius is $R = r + h$. Only h, and therefore R, varies with time.
- The ice-cream volume is $V = \frac{4\pi}{3}(R^3 - r^3)$, so $dV/dt = 4\pi R^2\, dh/dt$. The chocolate surface area is $4\pi r^2$.

Step 1: Assign signs to the melting rates The ice-cream volume and thickness both decrease. Their signed rates are therefore

$$\frac{dV}{dt} = -81\ \text{cm}^3\,\text{min}^{-1}, \qquad \frac{dh}{dt} = -\frac{1}{4\pi}\ \text{cm}\,\text{min}^{-1}.$$

The radius r of the chocolate ball stays fixed, so $dR/dt = dh/dt$. Differentiating the shell volume does not introduce any derivative of r.

Step 2: Determine the outer radius at that instant Substitute the given rates into the volume-rate equation:

$$-81 = 4\pi R^2\left(-\frac{1}{4\pi}\right) = -R^2.$$

Thus $R^2 = 81\ \text{cm}^2$ and $R = 9\ \text{cm}$, taking the positive root because a radius is positive. The two negative signs are consistent: an inward moving spherical surface produces decreasing volume.

Step 3: Remove the coating thickness and find the area At the specified instant the coating is $1\ \text{cm}$

thick. Hence the chocolate radius is

$$r = R - h = 9 - 1 = 8 \text{ cm}.$$

Its surface area is

$$4\pi r^2 = 4\pi(8)^2 = 256\pi \text{ cm}^2.$$

The values 128π, 196π, and 225π do not correspond to radius eight. Using the outer radius would instead give 324π, which is the area of the ice-cream surface, not the requested chocolate surface.

Step 4: Conclusion The chocolate ball has surface area $\boxed{256\pi \text{ cm}^2}$. Hence, the correct option is (D).

Quick Tip to Solve: The volume rate determines the outer radius of a melting spherical coating; subtract its thickness before calculating the area of the inner ball.

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19. Let the shortest distance from $(a, 0), a > 0$, to the parabola $y^2 = 4x$ be 4 . Then the equation of the circle passing through the point $(a, 0)$ and the focus of the parabola, and having its centre on the axis of the parabola is : [4 marks]

(A) $x^2 + y^2 - 8x + 7 = 0$

(B) $x^2 + y^2 - 6x + 5 = 0$

(C) $x^2 + y^2 - 4x + 3 = 0$

(D) $x^2 + y^2 - 10x + 9 = 0$

Correct Answer: (B) $x^2 + y^2 - 6x + 5 = 0$

Solution:

Concept:

- A general point of $y^2 = 4x$ is $(t^2, 2t)$, where t is real. Minimise the square of the distance, which has the same minimiser as the distance itself.
- A circle whose centre lies on the axis $y = 0$, and passes through two distinct points on that axis, has the segment joining those points as a diameter.

Step 1: Express the squared distance as a quadratic For the point $(a, 0)$, the distance to $(t^2, 2t)$ satisfies

$$d^2 = (t^2 - a)^2 + 4t^2.$$

Put $s = t^2 \geq 0$. Completing the square gives

$$d^2 = (s - a + 2)^2 + 4a - 4.$$

The restriction $s \geq 0$ matters: an unconstrained stationary point is not always a point on the parabola.

Step 2: Use the minimum distance to determine a If $0 < a \leq 2$, the vertex of the quadratic in s is nonpositive, so its minimum on $[0, \infty)$ occurs at $s = 0$. The minimum distance is then $a \leq 2$, contradicting the given distance 4. Thus $a > 2$; the minimiser $s = a - 2$ is allowed. Hence

$$4a - 4 = 4^2 = 16 \quad \Longrightarrow \quad a = 5.$$

In fact, $s = 3$ gives points $(3, \pm 2\sqrt{3})$, each exactly 4 units from $(5, 0)$.

Step 3: Construct the circle and compare the choices The focus is $(1, 0)$, and the other specified point is $(5, 0)$. Their midpoint is $(3, 0)$, and their separation is 4. Therefore the radius is 2 and

$$(x - 3)^2 + y^2 = 4.$$

On expansion this gives $x^2 + y^2 - 6x + 5 = 0$. Options (A), (C) and (D) have centres $(4, 0)$, $(2, 0)$ and $(5, 0)$; none is the required midpoint.

Step 4: Conclusion The circle is $\boxed{x^2 + y^2 - 6x + 5 = 0}$. Hence, the correct option is (B).

Quick Tip to Solve: When minimising distance to a parametrised parabola, replace t^2 by s and retain the constraint $s \geq 0$.

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20. Let $x = x(y)$ be the solution of the differential equation $y = \left(x - y\frac{dx}{dy}\right)\sin\left(\frac{x}{y}\right), y > 0$ and $x(1) = \frac{\pi}{2}$. Then $\cos(x(2))$ is equal to : [4 marks]

(A) $2\left(\log_e 2\right) - 1$

(B) $1 - 2\left(\log_e 2\right)^2$

(C) $1 - 2\left(\log_e 2\right)$

(D) $2\left(\log_e 2\right)^2 - 1$

Correct Answer: (D) $2\left(\log_e 2\right)^2 - 1$

Solution:

Concept:

- The expression $x - y\,dx/dy$, together with x/y, suggests the substitution $v = x/y$. Since $x = yv$, differentiation gives $dx/dy = v + y\,dv/dy$.
- After separation, integrate using $d(\cos v) = -\sin v\,dv$. The final quantity involves twice the angle, so use $\cos(2v) = 2\cos^2 v - 1$.

Step 1: Transform and separate the equation With $x = yv$, we have

$$x - y\frac{dx}{dy} = yv - y\left(v + y\frac{dv}{dy}\right) = -y^2\frac{dv}{dy}.$$

Substituting in the given equation and dividing by the positive quantity y gives

$$1 = -y\sin v\frac{dv}{dy}, \qquad \sin v\,dv = -\frac{dy}{y}.$$

The original equation itself requires $\sin v \neq 0$ along a differentiable solution; the initial angle $\pi/2$ satisfies this condition.

Step 2: Integrate and impose the initial value Integration yields

$$-\cos v = -\log_e y + C, \qquad \cos(x/y) = \log_e y + K.$$

At $y = 1$, the prescribed value $x(1) = \pi/2$ gives $v = \pi/2$. Consequently $0 = 0 + K$, so

$$\cos\left(\frac{x(y)}{y}\right) = \log_e y.$$

On the interval from 1 to 2, $\log_e y$ lies between 0 and $\log_e 2 < 1$, so the branch continuing from the initial angle remains admissible.

Step 3: Evaluate the required double angle At $y = 2$, write $u = x(2)/2$. Then $\cos u = \log_e 2$. The question asks for $\cos(x(2)) = \cos(2u)$, not simply $\cos u$. Thus

$$\cos(x(2)) = 2(\log_e 2)^2 - 1.$$

Option (B) has the opposite sign; options (A) and (C) omit the square required by the double-angle identity. Numerically the result is about -0.0391, consistent with the exact expression.

Step 4: Conclusion $\boxed{\cos(x(2)) = 2(\log_e 2)^2 - 1}$. Hence, the correct option is (D).

Quick Tip to Solve: Keep track of the angle: the substitution gives $\cos(x/y)$; at $y = 2$, the requested $\cos x$ requires a double-angle identity.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let α, β be the roots of the equation $x^2 - ax - b = 0$ with $\mathrm{Im}(\alpha) < \mathrm{Im}(\beta)$. Let $P_n = \alpha^n - \beta^n$. If $P_3 = -5\sqrt{7}i, P_4 = -3\sqrt{7}i, P_5 = 11\sqrt{7}i$ and $P_6 = 45\sqrt{7}i$, then $|\alpha^4 + \beta^4|$ is equal to _____. [4 marks]

Solution:

Concept:

- Every root r of $r^2 - ar - b = 0$ satisfies $r^n = ar^{n-1} + br^{n-2}$. Subtracting the identities for the two roots gives $P_n = aP_{n-1} + bP_{n-2}$.
- By Vieta's relations, $\alpha + \beta = a$ and $\alpha\beta = -b$. Symmetric even-power sums can then be calculated without writing the individual complex roots.

Step 1: Obtain equations for the coefficients Use the recurrence at $n = 5$ and $n = 6$. Cancelling the nonzero common factor $\sqrt{7}\,i$ from the given values gives

$$11 = -3a - 5b, \qquad 45 = 11a - 3b.$$

Equivalently, $3a + 5b = -11$ and $11a - 3b = 45$. Multiply these equations by 3 and 5 respectively and add:

$$64a = 192 \quad \Longrightarrow \quad a = 3.$$

Substitution into $3a + 5b = -11$ yields $b = -4$.

Step 2: Calculate the symmetric fourth-power sum The resulting quadratic is $x^2 - 3x + 4 = 0$, so the

root sum is 3 and the root product is 4. Therefore

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 9 - 8 = 1.$$

Squaring once more, while subtracting the cross term, gives

$$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = 1 - 2(16) = -31.$$

The required modulus is thus 31, rather than the signed power sum -31.

Step 3: Check the complex-root ordering and data The roots are $\alpha = (3 - i\sqrt{7})/2$ and $\beta = (3 + i\sqrt{7})/2$, in the required order of imaginary parts. Hence $P_1 = -i\sqrt{7}$ and $P_2 = -3i\sqrt{7}$. Using $P_n = 3P_{n-1} - 4P_{n-2}$ gives successively $-5i\sqrt{7}, -3i\sqrt{7}, 11i\sqrt{7}, 45i\sqrt{7}$, exactly the four supplied values. This confirms that the recovered coefficients satisfy all the data.

Step 4: Conclusion The fourth-power sum is -31; therefore $\boxed{|\alpha^4 + \beta^4| = 31}$, a dimensionless value.

Quick Tip to Solve: Use the difference-of-powers data to recover the recurrence coefficients, then switch to symmetric sums for the expression being asked.

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22. The focus of the parabola $y^2 = 4x + 16$ is the centre of the circle C of radius 5 . If the values of λ, for which C passes through the point of intersection of the lines $3x - y = 0$ and $x + \lambda y = 4$, are λ_1 and $\lambda_2, \lambda_1 < \lambda_2$, then $12\lambda_1 + 29\lambda_2$ is equal to _____ . [4 marks]

Solution:

Concept:

- The standard parabola $y^2 = 4a(x - h)$ has vertex $(h, 0)$ and focus $(h + a, 0)$. A circle of centre (u, v) and radius r satisfies $(x - u)^2 + (y - v)^2 = r^2$.
- First intersect the fixed line with the circle. Each resulting point determines a value of the parameter in the second line; order those values only after finding both.

Step 1: Write the circle from the focus Rewrite the parabola as

$$y^2 = 4(x + 4).$$

Here $a = 1$ and the vertex is $(-4, 0)$, so the focus is $(-3, 0)$. This is the centre of the circle, whose radius is 5. Its equation is therefore

$$(x + 3)^2 + y^2 = 25.$$

The shift of the vertex must be included: the focus is not $(1, 0)$.

Step 2: Intersect the fixed line with the circle The first line gives $y = 3x$. Substitute this into the circle:

$$(x + 3)^2 + 9x^2 = 25,$$

$$10x^2 + 6x - 16 = 0, \qquad (5x + 8)(x - 1) = 0.$$

Consequently the intersection points of the fixed line and the circle are

$$(1,3) \quad \text{and} \quad \left(-\frac{8}{5}, -\frac{24}{5}\right).$$

These exhaust the possible intersections, since a line meets a circle in at most two distinct points.

Step 3: Find and order the parameter values For a point with $y \neq 0$, the other line $x + \lambda y = 4$ gives $\lambda = (4 - x)/y$. At $(1,3)$, this is $\lambda = 1$. At the second point,

$$\lambda = \frac{4 + 8/5}{-24/5} = \frac{28}{-24} = -\frac{7}{6}.$$

Thus $\lambda_1 = -7/6 < \lambda_2 = 1$, and

$$12\lambda_1 + 29\lambda_2 = 12\left(-\frac{7}{6}\right) + 29 = -14 + 29 = 15.$$

Neither value makes the two given lines parallel, so both intersections used are well defined.

Step 4: Conclusion $\boxed{12\lambda_1 + 29\lambda_2 = 15}$, a dimensionless value.

Quick Tip to Solve: Intersect the circle with the fixed line first; this turns a parameter problem into two direct substitutions and prevents unnecessary rational algebra.

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23. The number of ways, 5 boys and 4 girls can sit in a row so that either all the boys sit together or no two boys sit together, is _____. [4 marks]

Solution:

Concept:

- Treat all five boys as one block when they must sit together, but still account for the $5!$ internal orders of that block. All nine individuals are distinct.
- To prevent two boys from sitting together, arrange the four girls first and place boys in the gaps. Add counts only after checking whether the two cases overlap.

Step 1: Count arrangements with all boys together Combine the five boys into one block. Alongside the four individual girls, this creates five objects to arrange in a row. These objects have $5!$ orders. Within the boys' block, the five distinct boys can be arranged in $5!$ ways independently. Hence the first case contributes

$$N_1 = 5! \times 5! = 120 \times 120 = 14400.$$

The girls need not be adjacent in this case: only the boys are required to form one consecutive group.

Step 2: Count arrangements with no adjacent boys Arrange the four girls in $4!$ ways. Every such arrangement creates exactly five available gaps: one at each end and three between consecutive girls. Symbolically, the positions are

$$_\, G_1 \,_\, G_2 \,_\, G_3 \,_\, G_4 \,_.$$

At most one boy can occupy each gap, or two boys would be adjacent. Because there are five boys and

exactly five gaps, all gaps must be used. Assign the distinct boys to these five gaps in $5!$ ways. Thus

$$N_2 = 4! \times 5! = 24 \times 120 = 2880.$$

No extra choice of gaps is needed: the gap selection factor is $\binom{5}{5} = 1$.

Step 3: Combine the disjoint cases An arrangement in the first case has adjacent boys, since the block contains five of them. An arrangement in the second case has no adjacent boys. The two sets of arrangements therefore have empty intersection. The word either permits their union, and no subtraction term is necessary. The required number is

$$N = N_1 + N_2 = 14400 + 2880 = 17280.$$

This counts each admissible seating order once, including the internal orders of every distinct person.

Step 4: Conclusion The required number of seating arrangements is $\boxed{17280}$.

Quick Tip to Solve: For five separated boys and four girls, every gap must be filled; for five consecutive boys, replace them by one block and multiply by their internal permutations.

24. The roots of the quadratic equation $3x^2 - px + q = 0$ are 10^{th} and 11^{th} terms of an arithmetic progression with common difference $\frac{3}{2}$. If the sum of the first 11 terms of this arithmetic progression is 88 , then $q - 2p$ is equal to _____ . [4 marks]

Solution:

Concept:

- For an arithmetic progression with first term A and common difference d, $T_n = A + (n-1)d$ and $S_n = \frac{n}{2}[2A + (n-1)d]$.
- For $3x^2 - px + q = 0$, Vieta's relations give the root sum $p/3$ and root product $q/3$. The leading coefficient 3 must be retained.

Step 1: Determine the first term from the sum Here $d = 3/2$ and $S_{11} = 88$. Substituting into the sum formula gives

$$88 = \frac{11}{2}\left[2A + 10\left(\frac{3}{2}\right)\right].$$

Multiplying by $2/11$, we obtain

$$16 = 2A + 15 \quad \Longrightarrow \quad A = \frac{1}{2}.$$

Equivalently, the middle term T_6 equals the average $88/11 = 8$, and moving five common differences backwards gives $A = 8 - 15/2 = 1/2$.

Step 2: Calculate the two specified roots The tenth term uses nine common differences, while the eleventh uses ten:

$$T_{10} = \frac{1}{2} + 9\left(\frac{3}{2}\right) = 14,$$

$$T_{11} = \frac{1}{2} + 10\left(\frac{3}{2}\right) = \frac{31}{2}.$$

Their difference is $31/2 - 14 = 3/2$, as required. These two numbers, rather than the first and last terms of a ten-term progression, are the roots of the given quadratic.

Step 3: Use the coefficients of the quadratic The root sum and product are

$$14 + \frac{31}{2} = \frac{59}{2}, \qquad 14\left(\frac{31}{2}\right) = 217.$$

Thus

$$p = 3\left(\frac{59}{2}\right) = \frac{177}{2}, \qquad q = 3(217) = 651.$$

Indeed, expanding $3(x - 14)(x - 31/2)$ gives $3x^2 - (177/2)x + 651$, confirming the coefficient signs. Finally,

$$q - 2p = 651 - 2\left(\frac{177}{2}\right) = 651 - 177 = 474.$$

Although p is fractional, the required combination is an integer.

Step 4: Conclusion $\boxed{q - 2p = 474}$, a dimensionless value.

Quick Tip to Solve: Obtain the AP terms first, then apply Vieta with the leading coefficient: here $p = 3 \times$ sum and $q = 3 \times$ product.

25. The variance of the numbers $8, 21, 34, 47, \ldots, 320$ is _____. [4 marks]

Solution:

Concept:

- The variance of equally weighted observations is the mean squared deviation from their arithmetic mean: $\sigma^2 = \frac{1}{n}\sum(x_i - \bar{x})^2$. It uses division by n, not the sample correction $n - 1$.
- For an arithmetic progression with common difference d and n terms, shifting every observation leaves variance unchanged, while multiplying every observation by d multiplies variance by d^2.

Step 1: Count the observations The common difference is $21 - 8 = 13$. Including both endpoints, the last term satisfies

$$320 = 8 + (n - 1)13.$$

Thus $n - 1 = 312/13 = 24$, so there are 25 observations. Their mean is the average of the endpoints, $\bar{x} = (8 + 320)/2 = 164$.

Step 2: Sum the squared deviations The deviations from 164 are $-156, -143, \ldots, 0, \ldots, 143, 156$, or $13j$ for $j = -12, \ldots, 12$. Pairing opposite deviations gives

$$\sigma^2 = \frac{2 \cdot 13^2}{25} \sum_{j=1}^{12} j^2.$$

Using $\sum_{j=1}^{12} j^2 = 12 \cdot 13 \cdot 25/6 = 650$, we obtain

$$\sigma^2 = \frac{338 \cdot 650}{25} = 8788.$$

Step 3: Check with the progression formula The general expression gives $d^2(n^2-1)/12 = 169(625-1)/12 = 169 \cdot 52 = 8788$, confirming the direct calculation. The first term changes the mean but contributes nothing separately to the variance.

Step 4: Conclusion The variance of the given numbers is $\boxed{8788}$ (in squared units of the observations), agreeing with the stated numerical key.

Quick Tip to Solve: For consecutive equally spaced observations, count both endpoints and use $d^2(n^2-1)/12$; the starting value does not affect variance.

Physics – Section A: Multiple Choice Questions (80 marks)

26. The equation of a transverse wave travelling along a string is $y(x,t) = 4.0 \sin\left[20 \times 10^{-3}x + 600t\right]$ mm, where x is in mm and t is in second. The velocity of the wave is : [4 marks]

(A) -60 m/s

(B) $+60$ m/s

(C) $+30$ m/s

(D) -30 m/s

Correct Answer: (D) -30 m/s

Solution:

Concept:

- A travelling wave transports a fixed phase along the string. For $y = A\sin(kx+\omega t)$, constant phase implies $kx + \omega t =$ constant, so the signed propagation velocity is $-\omega/k$.
- The units attached to the coordinate determine the units of the wave number. Here x is in millimetres, so $k = 20 \times 10^{-3}\,\text{mm}^{-1}$, while $\omega = 600\,\text{s}^{-1}$.

Step 1: Determine the direction from the phase The phase is $20 \times 10^{-3}x + 600t$. Differentiating the constant-phase condition with respect to time gives

$$20 \times 10^{-3}\frac{dx}{dt} + 600 = 0.$$

Hence the crest moves toward decreasing x. The plus sign between the space and time terms therefore means negative propagation velocity, independent of the positive amplitude.

Step 2: Calculate and convert the speed The signed velocity is

$$v = -\frac{600}{20 \times 10^{-3}} = -30000\,\text{mm}\,\text{s}^{-1} = -30\,\text{m}\,\text{s}^{-1}.$$

Equivalently, the wave number is $20\,\mathrm{m}^{-1}$, giving $-600/20 = -30\,\mathrm{m\,s}^{-1}$. The displacement amplitude $4.0\,\mathrm{mm}$ does not enter this ratio.

Step 3: Eliminate the other choices Options (A) and (B) both have magnitude $60\,\mathrm{m\,s}^{-1}$, twice the calculated speed, and cannot satisfy the constant-phase equation. Option (C) has the correct magnitude but the wrong direction. Option (D) satisfies both the phase sign and the unit conversion.

Step 4: Conclusion The velocity is $\boxed{-30\,\mathrm{m\,s}^{-1}}$, directed along the negative x-axis. Hence, the correct option is (D).

Quick Tip to Solve: First hold the sine argument constant to find the sign; then convert the coordinate unit before reporting the wave velocity.

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27. A plane electromagnetic wave of frequency 20 MHz travels in free space along the $+x$ direction. At a particular point in space and time, the electric field vector of the wave is $\mathrm{E}_y = 9.3\mathrm{Vm}^{-1}$. Then, the magnetic field vector of the wave at that point is [4 marks]

(A) $\mathrm{B}_z = 1.55 \times 10^{-8}$ T

(B) $\mathrm{B}_z = 6.2 \times 10^{-8}$ T

(C) $\mathrm{B}_z = 3.1 \times 10^{-8}$ T

(D) $\mathrm{B}_z = 9.3 \times 10^{-8}$ T

Correct Answer: (C) $\mathrm{B}_z = 3.1 \times 10^{-8}$ T

Solution:

Concept:

- In a plane electromagnetic wave in free space, the instantaneous field magnitudes obey $E = cB$, where $c = 3.0 \times 10^8\,\mathrm{m\,s}^{-1}$. The electric and magnetic fields are in phase.
- The direction of energy propagation is along $\mathbf{E} \times \mathbf{B}$. The electric field, magnetic field and propagation direction are mutually perpendicular.

Step 1: Find the magnetic-field magnitude The given instantaneous electric field is $E_y = 9.3\,\mathrm{V\,m}^{-1}$. Applying the free-space field ratio at the same point and time,

$$B = \frac{E}{c} = \frac{9.3}{3.0 \times 10^8} = 3.1 \times 10^{-8}\,\mathrm{T}.$$

The stated frequency is not needed: the ratio of these field amplitudes, and of their simultaneous values, is the speed of light.

Step 2: Fix the direction with the right-hand rule The electric field points along $+y$, and the wave travels along $+x$. Since $\hat{\mathbf{y}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}}$, the magnetic field must point along $+z$. A negative z-component would instead give energy propagation along $-x$, contrary to the question.

Step 3: Compare the listed values All four choices name a z-component, so their magnitudes decide the selection after the direction check. Option (A) is half the required field; option (B) is twice it; option (D) is three times it. Only option (C) obeys $cB = 9.3\,\mathrm{V\,m}^{-1}$. The magnetic field is small because the conversion factor is c, not the wave frequency.

Step 4: Conclusion The magnetic field is $\boxed{\mathbf{B} = 3.1 \times 10^{-8}\hat{\mathbf{z}}\,\mathrm{T}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $B = E/c$ for magnitude and $\mathbf{E} \times \mathbf{B}$ for direction; frequency is irrelevant to this field ratio.

28. If a satellite orbiting the Earth is 9 times closer to the Earth than the Moon, what is the time period of rotation of the satellite? Given rotational time period of Moon $= 27$ days and gravitational attraction between the satellite and the moon is neglected. [4 marks]

(A) 3 days

(B) 27 days

(C) 81 days

(D) 1 day

Correct Answer: (D) 1 day

Solution:

Concept:

- For bodies orbiting the same central mass under gravity, Kepler's third law states $T^2 \propto r^3$, with r the orbital radius (or semimajor axis). Thus $T \propto r^{3/2}$.
- The gravitational force of the Moon on the satellite is neglected, so both the satellite and Moon are compared as bodies orbiting Earth. Their own masses cancel from the orbital-period relation.

Step 1: Express the orbital-radius ratio Let r_m and T_m be the Moon's orbital radius and period, and let r_s, T_s denote those of the satellite. Being nine times closer corresponds to $r_s = r_m/9$. The relevant distance is from Earth as the central body, not a change in the satellite's physical size. We are given $T_m = 27$ days.

Step 2: Apply Kepler third law to both orbits Taking a ratio removes the common gravitational parameter GM_E:

$$\left(\frac{T_s}{T_m}\right)^2 = \left(\frac{r_s}{r_m}\right)^3 = \left(\frac{1}{9}\right)^3.$$

The positive period is therefore

$$\frac{T_s}{T_m} = \frac{1}{9^{3/2}} = \frac{1}{27}, \qquad T_s = \frac{27}{27} = 1\,\text{day}.$$

Step 3: Check the scale and the alternatives A closer orbit must have a shorter period. Option (C), 81 days, gives the opposite trend, and option (B), 27 days, ignores the radius change. Option (A), 3 days, follows a linear radius-period relation rather than the required power $3/2$. The one-day result correctly accounts for both the smaller path and the increased orbital speed.

Step 4: Conclusion The satellite's orbital period is $\boxed{1\,\text{day}}$. Hence, the correct option is (D).

Quick Tip to Solve: For two orbits about the same central body, take the radius ratio to the power $3/2$; a factor of nine in radius gives a factor of 27 in period.

29. A massless spring gets elongated by amount x_1 under a tension of 5 N . Its elongation is x_2 under the tension of 7 N . For the elongation of $(5x_1 - 2x_2)$, the tension in the spring will be, [4 marks]

(A) 20 N

(B) 39 N

(C) 11 N

(D) 15 N

Correct Answer: (C) 11 N

Solution:

Concept:

- Within the elastic limit, a spring obeys Hooke's law $F = kx$, where k is its constant stiffness and x is extension from natural length.
- Because the same spring is used in both measurements, the same k relates each tension to its extension. Consequently, a linear combination of extensions corresponds to the same linear combination of tensions.

Step 1: Write the two measured force-extension relations For the first load, $kx_1 = 5\,\text{N}$, and for the second load, $kx_2 = 7\,\text{N}$. In particular, $x_1 = 5/k$ and $x_2 = 7/k$ when the forces are expressed in newtons. There is no need to find k separately: it will cancel when the requested extension is multiplied by the stiffness.

Step 2: Evaluate the force for the specified extension The required extension is the algebraic combination $x = 5x_1 - 2x_2$. Therefore

$$F = k(5x_1 - 2x_2) = 5(kx_1) - 2(kx_2).$$

Substituting the known tensions gives

$$F = 5(5) - 2(7) = 25 - 14 = 11\,\text{N}.$$

The result is positive, so the specified extension is still an elongation and requires tension, rather than compression.

Step 3: Reject incorrect combinations Option (B), 39 N, results from adding $25 + 14$, which loses the minus sign in the specified extension. Options (A), 20 N, and (D), 15 N, do not equal the Hooke-law combination $5 \cdot 5 - 2 \cdot 7$. Multiplying an extension coefficient by the corresponding force is valid because the spring constant is unchanged throughout.

Step 4: Conclusion The required tension is $\boxed{11\,\text{N}}$. Hence, the correct option is (C).

Quick Tip to Solve: For one Hookean spring, replace each kx_i by its known force; preserve the coefficients and their signs.

30. Water of mass m gram is slowly heated to increase the temperature from T_1 to T_γ. The change in entropy of the water, given specific heat of water is $1 \mathrm{Jkg}^{-1}\ \mathrm{K}^{-1}$, is : [4 marks]

(A) $\mathrm{m}\,(\mathrm{T}_2 - \mathrm{T}_1)$

(B) zero

(C) $\mathrm{m} \ln\left(\frac{\mathrm{T}_1}{\mathrm{T}_2}\right)$

(D) $\mathrm{m} \ln\left(\frac{\mathrm{T}_2}{\mathrm{T}_1}\right)$

Correct Answer: The printed mass is in grams but the specific heat is per kilogram; all options omit the required factor 10^{-3}. Option (D) gives only the intended functional form.

Solution:

Concept:

- Entropy change for reversible heating is $dS = \delta Q_{\text{rev}}/T$. With constant specific heat c and mass M, $\delta Q_{\text{rev}} = Mc\,dT$. Temperatures in the logarithm must be absolute temperatures.
- The printed mass is m grams, whereas the printed specific heat is $1\,\mathrm{J\,kg^{-1}\,K^{-1}}$. Consistent SI substitution therefore requires $M = m \times 10^{-3}\,\mathrm{kg}$.

Step 1: Integrate the entropy differential Write T_f for the final temperature denoted by T_γ in the question and by T_2 in the options. For slow heating between positive absolute temperatures,

$$\Delta S = \int_{T_1}^{T_f} \frac{Mc}{T}\,dT = Mc \ln\left(\frac{T_f}{T_1}\right).$$

No extra factor of temperature multiplies the logarithm; its argument is a dimensionless ratio.

Step 2: Substitute the printed mass and specific heat consistently Using the units actually supplied gives

$$\Delta S = 10^{-3} m \ln\left(\frac{T_f}{T_1}\right)\ \mathrm{J\,K^{-1}}.$$

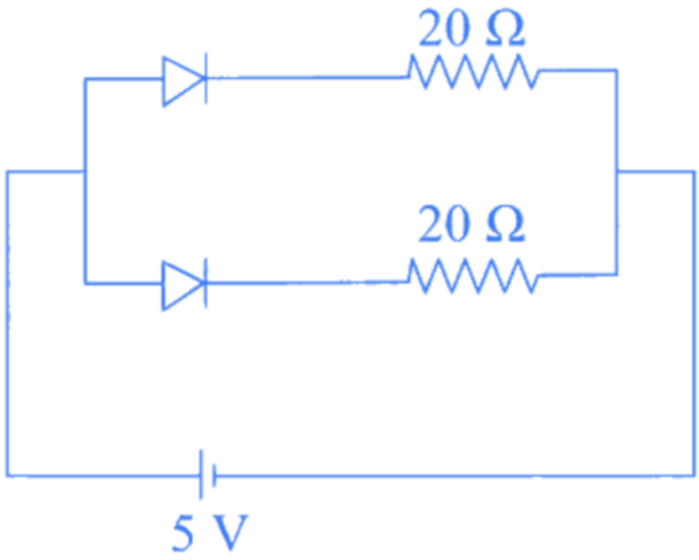
For heating, $T_f > T_1$, so this entropy change is positive. The factor 10^{-3} is essential because the mass is in grams while the specific heat is per kilogram.

Step 3: Explain the option discrepancy Option (A) has a temperature difference instead of the logarithmic dependence. Option (B) would mean no entropy change, despite heating. Option (C) reverses the ratio and gives a negative value. Option (D) has the intended logarithmic form and positive sign, but as printed it omits the factor 10^{-3}. It would be correct if the mass were m kilograms or the specific heat were per gram instead. Neither alteration is stated, so no listed expression is literally correct with the supplied units.

Step 4: Conclusion With the printed units, $\boxed{\Delta S = 10^{-3} m \ln(T_f/T_1)\,\mathrm{J\,K^{-1}}}$. No option is exactly correct. Option (D) is the intended form, but it lacks the gram-to-kilogram conversion; this is why a unique listed answer cannot be assigned literally.

Quick Tip to Solve: Before using $\Delta S = Mc\ln(T_f/T_i)$, put mass and specific heat in matching units and use Kelvin temperatures.

31. What is the current through the battery in the circuit shown below? [4 marks]



(A) 1.5 A

(B) 0.25 A

(C) 1.0 A

(D) 0.5 A

Correct Answer: (D) 0.5 A

Solution:

Concept:

- An ideal diode conducts when its anode is at a higher potential than its cathode. A conducting ideal diode has zero potential drop; a reverse-biased diode is an open circuit.
- Resistors connected across the same two nodes are in parallel. Their branch currents add at the battery: $I = I_1 + I_2$, with each current determined by Ohm's law.

Step 1: Identify the conducting paths The longer plate of the 5 V battery is on the left, so the left-hand node is positive relative to the right-hand node. In both branches the diode anode is on the left and its cathode bar is on the right. Both diodes are therefore forward biased and conduct. Using the ideal-diode model implicit in the circuit, neither diode removes any voltage from its series resistor.

Step 2: Combine the parallel resistors Each conducting branch contains one $20\,\Omega$ resistor, and both branches have the same endpoints. Thus the resistors are in parallel, rather than in series.

$$R_{\text{eq}} = \frac{20 \times 20}{20 + 20} = 10\,\Omega.$$

The entire battery voltage is across each branch, giving $I_1 = I_2 = 5/20 = 0.25\,\text{A}$.

Step 3: Calculate the battery current and check the alternatives Kirchhoff's junction rule gives the current leaving the battery as the sum of the two equal branch currents:

$$I = 0.25 + 0.25 = 0.50\,\text{A}.$$

Equivalently, $I = 5/10 = 0.50\,\text{A}$. The 0.25 A choice counts just one branch. Neither 1.0 A nor 1.5 A satisfies Ohm's law for two parallel 20-ohm resistors driven by 5 V.

Step 4: Conclusion The current through the battery is $\boxed{0.5\,\text{A}}$. Hence, the correct option is (D).

Quick Tip to Solve: Check the battery polarity and each diode direction first; then add branch currents, rather than confusing a branch current with the battery current.

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32. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : The binding energy per nucleon is found to be practically independent of the atomic number A , for nuclei with mass numbers between 30 and 170 .

Reason (R) : Nuclear force is long range. In the light of the above statements, choose the correct answer from the options given below : [4 marks]

(A) is true but (R) is false

(B) Both (A) and (R) are true and (R) is the correct explanation of (A)

(C) is false but (R) is true

(D) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

Correct Answer: (A) is true but (R) is false

Solution:

Concept:

- Binding energy per nucleon is the total binding energy divided by the mass number, B/A. It measures the mean energy needed to separate one nucleon from a nucleus.
- Nuclear forces are short range and exhibit saturation: a nucleon interacts strongly with only a small number of nearby nucleons. This makes total binding energy roughly proportional to nucleon number in medium-mass nuclei.

Step 1: Evaluate the assertion The binding-energy-per-nucleon curve is nearly flat over the broad middle range of nuclei, including mass numbers about 30 to 170. The binding energy per nucleon is of order 8 MeV and changes relatively slowly in this range. The statement says practically independent, not exactly constant, so the small rise towards the iron region and gradual fall afterwards do not invalidate it. Assertion (A) is true. The symbol A here denotes mass number, as the specified range makes clear.

Step 2: Evaluate the reason The strong nuclear force is appreciable only over nuclear distances, of order a few femtometres. It is therefore a short-range force, not a long-range force. Nucleons deep inside a larger nucleus do not attract every other nucleon equally strongly; mainly their close neighbours contribute. Reason (R), which states that nuclear force is long range, is false.

Step 3: Test the explanatory link and the options The approximate constancy of B/A is connected with saturation of the short-range nuclear interaction. It cannot be explained by the incorrect long-range claim. Options (B) and (D) both require the reason to be true, while option (C) additionally declares the true assertion false. Only the combination of a true assertion and a false reason is consistent with

both tests.

Step 4: Conclusion Assertion (A) is true, but Reason (R) is false. Hence, the correct option is (A).

Quick Tip to Solve: For an assertion about nearly constant binding energy per nucleon, think of short range and saturation; first judge each statement separately before testing the explanation.

33. A ball having kinetic energy KE, is projected at an angle of 60° from the horizontal. What will be the kinetic energy of ball at the highest point of its flight? [4 marks]

(A) $\frac{(\text{KE})}{2}$
(B) $\frac{(\text{KE})}{8}$
(C) $\frac{(\text{KE})}{4}$
(D) $\frac{(\text{KE})}{16}$

Correct Answer: (C) $\frac{(\text{KE})}{4}$

Solution:

Concept:

- In ideal projectile motion, gravity acts vertically, so the horizontal velocity remains constant: $v_x = u\cos\theta$, where u is the launch speed.
- Kinetic energy depends on the square of the total speed: $K = \frac{1}{2}mv^2$. At the highest point only the vertical component of velocity is zero.

Step 1: Resolve the initial velocity Let the mass of the ball be m and its initial speed be u. Its given initial kinetic energy is $\text{KE} = \frac{1}{2}mu^2$. The launch angle is 60°, so the velocity components are

$$u_x = u\cos 60^\circ = \frac{u}{2}, \qquad u_y = u\sin 60^\circ = \frac{\sqrt{3}u}{2}.$$

The vertical component decreases under gravity, while the horizontal component has no acceleration.

Step 2: Find the speed and energy at the highest point At the top of the trajectory the vertical component momentarily vanishes: $v_y = 0$. The horizontal velocity is still $u/2$, so the ball has nonzero speed $v = u/2$. Substituting this speed gives

$$K_{\text{top}} = \frac{1}{2}m\left(\frac{u}{2}\right)^2 = \frac{1}{4}\left(\frac{1}{2}mu^2\right) = \frac{\text{KE}}{4}.$$

The remaining three quarters of the original kinetic energy has been converted into gravitational potential energy.

Step 3: Check the energy fraction against the options For any launch angle the fraction of kinetic energy remaining at the top is $\cos^2\theta$. Here $\cos^2 60^\circ = (1/2)^2 = 1/4$. The fraction 1/2 incorrectly uses the speed ratio directly as the energy ratio, while 1/8 and 1/16 introduce additional factors not present in $\frac{1}{2}mv^2$.

Step 4: Conclusion The kinetic energy at the highest point is $\boxed{\text{KE}/4}$. Hence, the correct option is (C).

Quick Tip to Solve: At a projectile's highest point remove only the vertical velocity component. The remaining kinetic-energy fraction is the square of the horizontal speed fraction.

34. In photoelectric effect an em-wave is incident on a metal surface and electrons are ejected from the surface. If the work function of the metal is 2.14 eV and stopping potential is 2 V , what is the wavelength of the em-wave? (Given $\mathrm{hc} = 1242\mathrm{eVnm}$ where h is the Planck's constant and c is the speed of light in vaccum.) [4 marks]

(A) 400 nm

(B) 600 nm

(C) 300 nm

(D) 200 nm

Correct Answer: (C) 300 nm

Solution:

Concept:

- Einstein's photoelectric equation is $hc/\lambda = \phi + K_{\max}$, where ϕ is the work function and $K_{\max}$ is the greatest emitted-electron kinetic energy.
- The stopping potential obeys $K_{\max} = eV_s$. For an electron, a stopping potential of one volt corresponds to one electronvolt of maximum kinetic energy.

Step 1: Obtain the maximum electron kinetic energy The stopping potential is $V_s = 2\,\mathrm{V}$. The retarding field removes the entire maximum kinetic energy of an emitted electron when it just fails to reach the collector. Therefore

$$K_{\max} = e(2\,\mathrm{V}) = 2\,\mathrm{eV}.$$

The numerical value in electronvolts is 2, not 2 joules; this unit choice permits direct addition to the supplied work function.

Step 2: Find the incident photon energy and wavelength Each absorbed photon must supply the 2.14 eV work function as well as the 2 eV maximum kinetic energy. Hence

$$E_\gamma = 2.14 + 2 = 4.14\,\mathrm{eV}.$$

Using the given $hc = 1242\,\mathrm{eV\,nm}$,

$$\lambda = \frac{hc}{E_\gamma} = \frac{1242}{4.14}\,\mathrm{nm} = 300\,\mathrm{nm}.$$

The electronvolt units cancel, leaving nanometres as required.

Step 3: Check the alternative wavelengths A wavelength of 400 nm would give photon energy 3.105 eV and maximum kinetic energy 0.965 eV, rather than 2 eV. At 600 nm the photon energy is only 2.07 eV, below the work function, so emission would not occur. At 200 nm the photon energy is 6.21 eV and the maximum kinetic energy is 4.07 eV. Thus none of those alternatives yields the specified stopping potential.

Step 4: Conclusion The wavelength of the incident electromagnetic wave is $\boxed{300\,\mathrm{nm}}$. Hence, the correct option is (C).

Quick Tip to Solve: When the work function is in eV and stopping potential in volts, add their numerical values to get photon energy in eV, then divide the given hc by that energy.

35. Two point charges $-4\mu c$ and $4\mu c$, constituting an electric dipole, are placed at $(-9, 0, 0)\text{cm}$ and $(9, 0, 0)\text{cm}$ in a uniform electric field of strength 10^4NC^{-1}. The work done on the dipole in rotating it from the equilibrium through $180°$ is : [4 marks]

(A) 18.4 mJ

(B) 12.4 mJ

(C) 16.4 mJ

(D) 14.4 mJ

Correct Answer: (D) 14.4 mJ

Solution:

Concept:

- The electric dipole moment has magnitude $p = q\ell$, where q is the magnitude of either charge and ℓ is the full separation of the charges.
- In a uniform electric field the dipole potential energy is $U = -pE\cos\theta$, where θ is the angle between the dipole moment and the field. Quasistatic external work equals the change in this potential energy.

Step 1: Calculate the charge separation and dipole moment The charges are located at $x = -9\,\text{cm}$ and $x = +9\,\text{cm}$, so their separation is the difference of these coordinates:

$$\ell = 9 - (-9) = 18\,\text{cm} = 0.18\,\text{m}.$$

The magnitude of each charge is $q = 4 \times 10^{-6}\,\text{C}$. Therefore

$$p = q\ell = (4 \times 10^{-6})(0.18) = 7.2 \times 10^{-7}\,\text{C m}.$$

Using 9 cm here would incorrectly use the half-separation.

Step 2: Evaluate the energy change on rotation The equilibrium starting orientation is the stable one, with the dipole aligned with the field: $\theta_i = 0$, giving $U_i = -pE$. After rotation through $180°$, it is antiparallel, so $\theta_f = \pi$ and $U_f = +pE$. Thus the required external work is

$$W = U_f - U_i = 2pE.$$

The field does negative work during this rotation; the work done on the dipole by the external agent is positive.

Step 3: Substitute and convert to millijoules With $E = 10^4\,\text{N C}^{-1}$,

$$W = 2(7.2 \times 10^{-7})(10^4) = 1.44 \times 10^{-2}\,\text{J} = 14.4\,\text{mJ}.$$

The unit is joules because $(\text{C m})(\text{N/C}) = \text{N m}$. The other values 18.4, 12.4 and 16.4 mJ do not equal this energy change for the given charge, separation and field.

Step 4: Conclusion The work required to rotate the dipole through $180°$ from stable equilibrium is $\boxed{14.4\,\text{mJ}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the full distance between the charges for p. Turning a dipole from parallel to antiparallel increases its energy by 2pE, not pE.

36. A concave mirror of focal length f in air is dipped in a liquid of refractive index μ. Its focal length in the liquid will be: [4 marks]

(A) μf

(B) f

(C) $\frac{f}{\mu}$

(D) $\frac{f}{(\mu-1)}$

Correct Answer: (B) f

Solution:

Concept:

- For a spherical mirror under the paraxial approximation, the focal length is $f = R/2$, where R is its radius of curvature.
- The law of reflection states that the angle of incidence equals the angle of reflection in the same medium. Changing the surrounding medium does not change the geometry of reflected rays from an ideal mirror.

Step 1: Identify the process determining the focus A concave mirror focuses rays by reflection at its reflecting surface. At each point on that surface, the incident ray and reflected ray make equal angles with the local normal. The normal is a radius of the spherical surface. Consequently the convergence of paraxial rays follows from the surface geometry and does not require a refractive-index contrast across a transmitting surface.

Step 2: Apply the mirror focal-length relation Dipping the mirror in liquid changes the speed and wavelength of light in the surrounding medium, but it does not change the mirror's radius of curvature. Let its focal length in air be $f_{\text{air}} = R/2 = f$. For the same mirror immersed in a homogeneous liquid, the reflection geometry still gives

$$f_{\text{liquid}} = \frac{R}{2} = f.$$

This relation concerns an ideal reflecting surface; there is no lens-like refraction to introduce a factor of μ.

Step 3: Compare the alternatives The options μf and f/μ incorrectly make the mirror's focus scale with the optical properties of the surrounding medium. The expression $f/(\mu - 1)$ similarly imports a refractive-index dependence appropriate to refraction problems; it would even become singular as $\mu \to 1$, although the mirror has the finite focal length f in air. Only the unchanged value agrees with reflection and with the air limit.

Step 4: Conclusion The focal length of the concave mirror in the liquid remains $\boxed{f}$. Hence, the correct option is (B).

Quick Tip to Solve: For a mirror, start from f = R/2. Refractive index changes lens power through refraction, but it does not change the focus of the same ideal reflecting surface.

37. The width of one of the two slits in Young's double slit experiment is d while that of the other slit is x d. If the ratio of the maximum to the minimum intensity in the interference pattern on the screen is $9:4$ then what is the value of x ? (Assume that the field strength varies according to the slit width.) [4 marks]

(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- The electric-field amplitudes add coherently. For amplitudes a_1, a_2, constructive interference gives $I_{\max} \propto (a_1 + a_2)^2$ and destructive interference gives $I_{\min} \propto (a_1 - a_2)^2$.
- The stated assumption makes field amplitude proportional to slit width, so the amplitude ratio is $a_2/a_1 = x$, rather than its square root.

Step 1: Convert the intensity ratio to an amplitude ratio Let the amplitude from the slit of width d be a. The other slit then supplies amplitude xa. At a bright fringe the fields reinforce, while at a dark fringe their difference remains. Hence

$$\frac{I_{\max}}{I_{\min}} = \frac{(1+x)^2}{(1-x)^2} = \frac{9}{4}.$$

Taking the positive square root is essential because an intensity measures the square of a field: $(1+x)/|1-x| = 3/2$. The nonzero minimum confirms that the slit amplitudes are unequal.

Step 2: Solve for the listed width ratio Every listed value is greater than one. Thus $|1-x| = x-1$, giving

$$2(1+x) = 3(x-1), \qquad 2+2x = 3x-3, \qquad x = 5.$$

Interchanging which slit is wider would instead produce the reciprocal $x = 1/5$; that is not among the choices. A direct check for the listed result gives $I_{\max} : I_{\min} = 6^2 : 4^2 = 36 : 16 = 9 : 4$. The other listed values $2, 3, 4$ give ratios $9, 4, 25/9$, respectively, rather than $9/4$.

Step 3: Conclusion The listed slit-width multiplier is $\boxed{x = 5}$, a dimensionless ratio. Hence, the correct option is (D).

Quick Tip to Solve: Take the square root of the fringe intensity ratio first; use the question's explicit amplitude-versus-width assumption before solving.

38. The refractive index of the material of a glass prism is $\sqrt{3}$. The angle of minimum deviation is equal to the angle of the prism. What is the angle of the prism? [4 marks]

(A) 60°

(B) 50°

(C) 58°

(D) 48°

Correct Answer: (A) 60°

Solution:

Concept:

- At minimum deviation, the light path inside a prism is symmetric: the incidence and emergence angles are equal, and each refraction angle equals half the prism angle A.
- The refractive index relative to air is $\mu = \sin[(A+\delta_{\min})/2]/\sin(A/2)$, where $\delta_{\min}$ denotes minimum deviation.

Step 1: Apply the given equality of angles The question specifies $\delta_{\min} = A$. Substituting this condition and $\mu = \sqrt{3}$ into the minimum-deviation relation gives

$$\sqrt{3} = \frac{\sin A}{\sin(A/2)}.$$

The prism is a genuine refracting wedge with nonzero apex angle, so $\sin(A/2) \neq 0$. The symmetric path also gives $i = (A+\delta_{\min})/2 = A$, which is consistent with the same formula through Snell's law.

Step 2: Reduce the trigonometric relation Use the double-angle identity $\sin A = 2\sin(A/2)\cos(A/2)$ and cancel the nonzero factor:

$$\sqrt{3} = 2\cos(A/2), \qquad \cos(A/2) = \frac{\sqrt{3}}{2}.$$

For the physical prism angle, $0 < A < 180^\circ$, the half-angle lies between 0° and 90°. Thus $A/2 = 30^\circ$, giving $A = 60^\circ$. As a check, $\sin 60^\circ / \sin 30^\circ = \sqrt{3}$. Angles $50^\circ, 58^\circ, 48^\circ$ have half-angles other than 30°, so none satisfies the required cosine value.

Step 3: Conclusion The angle of the prism is $\boxed{60^\circ}$. Hence, the correct option is (A).

Quick Tip to Solve: When minimum deviation equals the prism angle, the prism formula immediately becomes $\mu = 2\cos(A/2)$.

39. Two charges $7\mu c$ and $-4\mu c$ are placed at $(-7\text{ cm}, 0, 0)$ and $(7\text{ cm}, 0, 0)$ respectively. Given, $\epsilon_0 = 8.85 \times 10^{-12} \text{C}^2\text{ N}^{-1}\text{ m}^{-2}$, the electrostatic potential energy of the charge configuration is : [4 marks]

(A) -2.0 J

(B) -1.5 J

(C) -1.2 J

(D) -1.8 J

Correct Answer: (D) -1.8 J

Solution:

Concept:

- The potential energy of two point charges, with zero energy at infinite separation, is $U = q_1 q_2/(4\pi\epsilon_0 r)$. Here r is their separation, not either charge's distance from the origin.
- Opposite signs make $q_1 q_2 < 0$, so the interaction is attractive and the configuration has negative potential energy.

Step 1: Find the separation and convert the charges The charges lie on the same coordinate axis at $-7\,\text{cm}$ and $+7\,\text{cm}$. Their separation is therefore

$$r = |7 - (-7)|\,\text{cm} = 14\,\text{cm} = 0.14\,\text{m}.$$

Convert microcoulombs to coulombs before using SI permittivity:

$$q_1 = 7 \times 10^{-6}\,\text{C}, \quad q_2 = -4 \times 10^{-6}\,\text{C}, \quad q_1 q_2 = -28 \times 10^{-12}\,\text{C}^2.$$

Using only $7\,\text{cm}$ would incorrectly halve the separation and double the magnitude of the energy.

Step 2: Evaluate the interaction energy The supplied permittivity gives

$$k = \frac{1}{4\pi(8.85 \times 10^{-12})} \simeq 8.99 \times 10^{9}\,\text{N m}^2\,\text{C}^{-2}.$$

Therefore

$$U = \frac{(8.99 \times 10^{9})(-28 \times 10^{-12})}{0.14}\,\text{J} \simeq -1.798\,\text{J}.$$

To the precision of the options this is $-1.8\,\text{J}$. The minus sign indicates energy is released on assembling the opposite charges from infinity. The alternatives $-2.0, -1.5, -1.2\,\text{J}$ do not agree with the calculated magnitude for the stated separation and charges.

Step 3: Conclusion The electrostatic potential energy is $\boxed{-1.8\,\text{J}}$. Hence, the correct option is (D).

Quick Tip to Solve: For charges on opposite sides of the origin, add their coordinate magnitudes to find separation; retain the signs in $q_1 q_2$.

40. Water flows in a horizontal pipe whose one end is closed with a valve. The reading of the pressure gauge attached to the pipe is P_1. The reading of the pressure gauge falls to P_2 when the valve is opened. The speed of water flowing in the pipe is proportional to [4 marks]

(A) $(P_1 - P_2)^2$

(B) $\sqrt{P_1 - P_2}$

(C) $P_1 - P_2$

(D) $(P_1 - P_2)^4$

Correct Answer: (B) $\sqrt{P_1 - P_2}$

Solution:

Concept:

- For steady incompressible flow, Bernoulli's equation conserves $P + \frac{1}{2}\rho v^2 + \rho g h$ along a streamline when dissipative losses are neglected.
- In a horizontal pipe, the elevation term is unchanged. A reduction in static pressure supplies the kinetic energy per unit volume of moving water.

Step 1: Compare the closed and open valve states With the valve closed, water is at rest and the gauge reads the static supply pressure P_1. On opening the valve, water flows and the static gauge pressure decreases to P_2. For the same pressure supply and height, the pressure decrease becomes the dynamic pressure. The atmospheric reference in a gauge reading cancels when subtracting the readings, so either absolute or gauge pressures lead to the same difference. Bernoulli's relation is

$$P_1 + 0 = P_2 + \frac{1}{2}\rho v^2.$$

Here ρ is the density of water and v is its flow speed.

Step 2: Isolate the speed and identify its dependence Rearranging the energy equation gives

$$P_1 - P_2 = \frac{1}{2}\rho v^2, \qquad v = \sqrt{\frac{2(P_1 - P_2)}{\rho}}.$$

The density is constant for the same water, so the prefactor $\sqrt{2/\rho}$ does not vary. Consequently v is proportional to the square root of the pressure fall. Dimensionally, $\Delta P/\rho$ has units $\mathrm{m^2 s^{-2}}$, confirming the square root produces speed. A linear, quadratic or fourth-power dependence would contradict the quadratic kinetic-energy term; they also do not give the required dimensions with this fixed-density factor.

Step 3: Conclusion The speed satisfies $\boxed{v \propto \sqrt{P_1 - P_2}}$. Hence, the correct option is (B).

Quick Tip to Solve: A pressure drop converts into kinetic energy per unit volume: write $\Delta P = \rho v^2/2$ before reading off the power.

41. The energy of a system is given as $\mathrm{E(t)} = \alpha^3 \mathrm{e}^{-\beta t}$, where t is the time and $\beta = 0.3\ \mathrm{s}^{-1}$. The errors in the measurement of α and t are 1.2% and 1.6%, respectively. At $t = 5$ s, maximum percentage error in the energy is : [4 marks]

(A) 6%

(B) 11.6%

(C) 4%

(D) 8.4%

Correct Answer: (A) 6%

Solution:

Concept:

- For $E = \alpha^3 e^{-\beta t}$, logarithmic differentiation gives $dE/E = 3\,d\alpha/\alpha - \beta\,dt$ when β is exact.
- The maximum first-order uncertainty adds the absolute magnitudes of independent contributions. The negative exponent does not cause cancellation in a worst-case error estimate.

Step 1: Calculate the absolute uncertainty in time The time uncertainty is given as a percentage of the measured time, not as a number of seconds. At $t = 5\,\mathrm{s}$,

$$\Delta t = \frac{1.6}{100}(5\,\mathrm{s}) = 0.08\,\mathrm{s}.$$

Thus the exponential factor contributes the relative uncertainty

$$\beta\Delta t = (0.3\,\mathrm{s}^{-1})(0.08\,\mathrm{s}) = 0.024,$$

or 2.4%. The product is dimensionless, as required for a relative uncertainty; multiplying β by the percentage directly would miss the measured time.

Step 2: Add the maximum fractional contributions The cubic dependence on α multiplies its fractional uncertainty by three:

$$3\frac{\Delta\alpha}{\alpha} = 3(0.012) = 0.036.$$

For maximum error, the two magnitudes add, giving

$$\left(\frac{\Delta E}{E}\right)_{\max} = 0.036 + 0.024 = 0.060.$$

Hence the maximum percentage error is 6%. A signed subtraction would describe one particular direction of perturbation, not the largest possible magnitude. The choices $11.6\%, 4\%, 8.4\%$ do not equal the sum of the correctly weighted contributions 3.6% and 2.4%. This is the standard linear propagation convention for small measurement errors.

Step 3: Conclusion The maximum percentage error in the energy is $\boxed{6\%}$. Hence, the correct option is (A).

Quick Tip to Solve: For an exponential in time, convert the percentage time error into Δt in seconds, then use $\beta\Delta t$.

42. Match List - I with List - II

	List - I		List - II
(A)	**Permeability of free space**	**(I)**	$[\mathrm{ML^2\ T^{-2}}]$
(B)	**Magnetic field**	**(II)**	$[\mathrm{MT^{-2}\ A^{-1}}]$
(C)	**Magnetic moment**	**(III)**	$[\mathrm{MLT^{-2}\ A^{-2}}]$
(D)	**Torsional constant**	**(IV)**	$[\mathrm{L^2\ A}]$

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(B) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

(C) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

(D) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Correct Answer: (A) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

Solution:

Concept:

- Base dimensions are mass M, length L, time T and electric current A. Dimensions follow from a defining physical equation, with numerical constants dimensionless.
- Use magnetic force for the dimensions of magnetic field, the force between parallel currents for permeability, current-loop area for magnetic moment, and torque per angular twist for torsional constant.

Step 1: Determine the two magnetic field dimensions From $F = BIl$, the magnetic field has dimensions

$$[B] = \frac{\mathrm{MLT^{-2}}}{\mathrm{AL}} = \mathrm{MT^{-2}A^{-1}},$$

so List-I (B) matches List-II (II). The force per unit length between parallel currents obeys $F/l = \mu_0 I_1 I_2/(2\pi r)$. Therefore

$$[\mu_0] = \frac{[F][r]}{[l][I]^2} = \mathrm{MLT^{-2}A^{-2}},$$

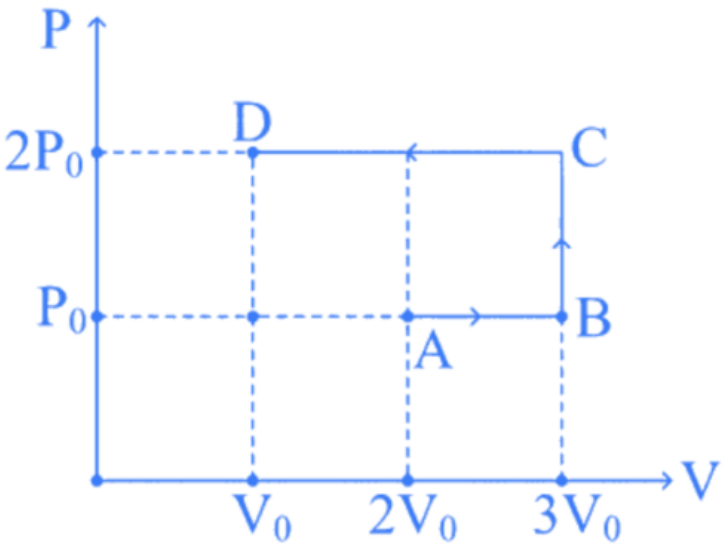
which makes List-I (A) match List-II (III). The extra inverse current power distinguishes permeability from the field itself.

Step 2: Match moment and torsional constant The magnetic moment of a current loop is current times area: $m = IS$. Its dimensions are therefore $[m] = \mathrm{AL^2}$, so (C) matches (IV). For a twisted wire, restoring torque is $\tau = C\theta$, where C is torsional constant and the angular displacement θ is dimensionless. Thus $[C] = [\tau] = \mathrm{ML^2T^{-2}}$, giving (D) with (I). Combining all four independent matches yields A-III, B-II, C-IV, D-I. The other three options already mismatch permeability and also interchange quantities with different powers of current or length, so none is dimensionally consistent.

Step 3: Conclusion The correct matching is $\boxed{\text{A-III, B-II, C-IV, D-I}}$. Hence, the correct option is (A).

Quick Tip to Solve: For magnetic dimensions, begin with $F = BIl$; remember that magnetic moment is current times area and torque has energy dimensions.

43. Using the given $P-V$ diagram, the work done by an ideal gas along the path $ABCD$ is : [4 marks]



(A) $3P_0\ V_0$

(B) $-3P_0\ V_0$

(C) $-4P_0\ V_0$

(D) $4P_0\ V_0$

Correct Answer: (B) $-3P_0\ V_0$

Solution:

Concept:

- Work done by a gas is the signed area under its pressure-volume path: $W = \int P\,dV$. Expansion gives positive work and compression gives negative work.
- For a horizontal segment, pressure is constant and $W = P(V_f - V_i)$. For a vertical segment, volume is constant, so its contribution to work is zero.

Step 1: Read the coordinates and directions The arrows show expansion from $A(2V_0, P_0)$ to $B(3V_0, P_0)$, followed by the vertical rise to $C(3V_0, 2P_0)$. The last segment goes left from C to $D(V_0, 2P_0)$, so this is compression. The specified path is open; it must not be replaced by a closed rectangular cycle.

Step 2: Calculate the work on each segment On AB, the volume rises by V_0, so

$$W_{AB} = P_0(3V_0 - 2V_0) = P_0V_0.$$

On BC, $dV = 0$, giving $W_{BC} = 0$. On CD, the volume falls by $2V_0$ at pressure $2P_0$:

$$W_{CD} = 2P_0(V_0 - 3V_0) = -4P_0V_0.$$

Step 3: Add the signed contributions and compare Therefore,

$$W_{ABCD} = P_0V_0 + 0 - 4P_0V_0 = -3P_0V_0.$$

The negative sign means that net work is done on the gas. Option (A) reverses this sign. Option (C) counts only compression and misses AB, while option (D) also assigns compression the wrong sign.

Step 4: Conclusion The work done by the gas is $\boxed{-3P_0V_0}$, in joules when pressure and volume are in SI units. Hence, the correct option is (B).

Quick Tip to Solve: Use the arrow direction before computing each rectangle: leftward motion contributes negative work even when pressure is positive.

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44. A circular disk of radius R meter and mass M kg is rotating around the axis perpendicular to the disk. An external torque is applied to the disk such that $\theta(t) = 5t^2 - 8t$, where $\theta(t)$ is the angular position of the rotating disc as a function of time t. How much power is delivered by the applied torque, when $t = 2$ s ? [4 marks]

(A) 60MR^2

(B) 72MR^2

(C) 8MR^2

(D) 108MR^2

Correct Answer: (A) 60MR^2

Solution:

Concept:

- For rotation about the central axis perpendicular to a uniform disk, the moment of inertia is $I = \frac{1}{2}MR^2$. The applied torque is $\tau = I\alpha$.
- Angular velocity and acceleration follow by differentiating angular position: $\omega = d\theta/dt$, $\alpha = d\omega/dt$. Instantaneous power is $P = \tau\omega$, not torque times angular position.

Step 1: Differentiate the given angular position The coefficients of $\theta(t) = 5t^2 - 8t$ are in the corresponding SI units. Differentiating once and then twice gives

$$\omega(t) = 10t - 8, \qquad \alpha(t) = 10 \text{ rad s}^{-2}.$$

At $t = 2$ s, the angular speed is $\omega = 20 - 8 = 12 \text{ rad s}^{-1}$. The speed is positive at this instant, so the positive torque adds rotational kinetic energy.

Step 2: Determine the torque and power The angular acceleration is constant; hence the required torque is

$$\tau = \left(\frac{1}{2}MR^2\right)(10) = 5MR^2.$$

Now use the angular speed at the requested time:

$$P = \tau\omega = (5MR^2)(12) = 60MR^2 \text{ W}.$$

Here M and R have their numerical values in kilograms and metres respectively.

Step 3: Check the expression and eliminate alternatives As an independent check, differentiating

$K = \frac{1}{2}I\omega^2$ gives $dK/dt = I\omega\alpha$, the same power. The kinetic energy is $\frac{1}{4}MR^2(12)^2 = 36MR^2$, not the instantaneous power. The candidate $72MR^2$ is neither this energy nor its time derivative; it can arise by using the incorrect disk inertia MR^2 in the energy formula. The values $8MR^2$ and $108MR^2$ do not follow from the required instantaneous product $I\alpha\omega$.

Step 4: Conclusion The instantaneous power delivered at $t = 2$ s is $\boxed{60MR^2\ \text{W}}$. Hence, the correct option is (A).

Quick Tip to Solve: Differentiate the angular position twice for torque and once for angular speed; multiply these through I only after inserting the requested time.

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45. A galvanometer having a coil of resistance 30Ω need 20 mA of current for full-scale deflection. If a maximum current of 3 A is to be measured using this galvanometer, the resistance of the shunt to be added to the galvanometer should be $\frac{30}{X}\Omega$, where X is [4 marks]

(A) 447

(B) 298

(C) 149

(D) 596

Correct Answer: (C) 149

Solution:

Concept:

- A shunt resistor is connected in parallel with a galvanometer so that only its full-scale current I_g enters the coil while the remaining current $I - I_g$ bypasses it.
- Parallel branches have the same potential difference. If the coil resistance is G and the shunt is S, then $I_g G = (I - I_g)S$, giving $S = I_g G/(I - I_g)$.

Step 1: Identify the currents in the two branches At full-scale reading, the galvanometer carries

$$I_g = 20\ \text{mA} = 0.020\ \text{A}.$$

The total current is $I = 3.00$ A. Thus the shunt carries $I_s = 3.00 - 0.020 = 2.980$ A. It would be incorrect to assign the entire 3 A to the shunt, because the galvanometer branch also carries a small but definite current.

Step 2: Equate the potential differences The voltage across the coil at full-scale deflection is

$$V_g = I_g G = 0.020 \times 30 = 0.600\ \text{V}.$$

The shunt has the same voltage. Its resistance must therefore be

$$S = \frac{0.600}{2.980} = \frac{30}{149}\ \Omega.$$

This is a small resistance, approximately $0.2013\ \Omega$, as required to divert almost all the current away from the sensitive coil.

Step 3: Match the specified form and verify Comparison with $S = 30/X\ \Omega$ gives $X = 149$. Equivalently,

$$X = \frac{I - I_g}{I_g} = \frac{2.980}{0.020} = 149.$$

The other candidate denominators 447, 298 and 596 would imply shunt currents 8.94, 5.96 and 11.92 A, respectively, at the required coil voltage. None gives the stipulated total current of 3 A.

Step 4: Conclusion The required shunt is $\boxed{S = 30/149\ \Omega}$, so $\boxed{X = 149}$. Hence, the correct option is (C).

Quick Tip to Solve: For a shunt written as G/X, the denominator is simply the current ratio $(I - I_g)/I_g$; convert milliamperes to amperes first.

Physics – Section B: Numerical Value Questions (20 marks)

46. A time varying potential difference is applied between the plates of a parallel plate capacitor of capacitance 2.5μ F. The dielectric constant of the medium between the capacitor plates is 1 . It produces an instantaneous displacement current of 0.25 mA in the intervening space between the capacitor plates, the magnitude of the rate of change of the potential difference will be _____ Vs^{-1}. [4 marks]

Solution:

Concept:

- The displacement current between capacitor plates is $I_d = \varepsilon_0\, d\Phi_E/dt$. It equals the charging current in the connecting wires.
- For a parallel-plate capacitor with constant geometry and capacitance, $Q = CV$, so $I_d = dQ/dt = C\, dV/dt$. The dielectric constant is 1, so the given capacitance needs no additional dielectric factor.

Step 1: Relate the electric flux to the applied voltage For plate area A and separation d, neglecting edge effects, the field is $E = V/d$ and the electric flux is $\Phi_E = AV/d$. Therefore,

$$I_d = \frac{\varepsilon_0 A}{d}\frac{dV}{dt} = C\frac{dV}{dt}.$$

This relation is instantaneous and does not require knowing the waveform or frequency of the applied voltage.

Step 2: Convert the given quantities to SI units Use the capacitance and current as given, without confusing microfarads and milliamperes:

$$C = 2.5 \times 10^{-6}\ \text{F}, \qquad |I_d| = 0.25 \times 10^{-3}\ \text{A}.$$

Since the question asks for a magnitude, divide the positive current magnitude by the positive capacitance. No information about whether the voltage is rising or falling is required.

Step 3: Evaluate the voltage rate and check dimensions Rearranging the current relation gives

$$\left|\frac{dV}{dt}\right| = \frac{|I_d|}{C} = \frac{0.25 \times 10^{-3}}{2.5 \times 10^{-6}} = 0.1 \times 10^3 = 100\ \mathrm{V\,s^{-1}}.$$

The unit check is $\mathrm{A/F} = (\mathrm{C/s})/(\mathrm{C/V}) = \mathrm{V/s}$. The dielectric constant has already been accounted for and must not be multiplied in a second time.

Step 4: Conclusion The magnitude of the rate of change of potential difference is $\boxed{100\ \mathrm{V\,s^{-1}}}$. The required numerical entry is $\boxed{100}$.

Quick Tip to Solve: For a fixed capacitor, use $I = C\,dV/dt$ directly; displacement current does not require finding the plate area or separation.

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47. In a series LCR circuit, a resistor of 300Ω, a capacitor of 25 nF and an inductor of 100 mH are used. For maximum current in the circuit, the angular frequency of the ac source is _____ $\times 10^4$ radians $\mathrm{s^{-1}}$ [4 marks]

Solution:

Concept:

- The current amplitude in a driven series LCR circuit is $I_0 = V_0/Z$, where

$$Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}.$$

- For a fixed source-voltage amplitude, current is maximum when inductive and capacitive reactances cancel. The resonance condition $\omega_0 L = 1/(\omega_0 C)$ gives $\omega_0 = 1/\sqrt{LC}$.

Step 1: Identify the condition for maximum current The squared reactive term in the impedance cannot be negative. It has its smallest value, zero, when $X_L = X_C$. Then the impedance becomes R alone and the current is largest. The resistor $300\ \Omega$ determines the height of the current maximum, but it does not enter the resonance frequency for a series circuit with the specified ideal L and C.

Step 2: Convert the inductance and capacitance Write both quantities in SI units before taking the square root:

$$L = 100\ \mathrm{mH} = 0.100\ \mathrm{H}, \qquad C = 25\ \mathrm{nF} = 25 \times 10^{-9}\ \mathrm{F}.$$

Their product is

$$LC = (0.100)(25 \times 10^{-9}) = 2.5 \times 10^{-9}\ \mathrm{s^2}.$$

Equivalently this is $25 \times 10^{-10}\ \mathrm{s^2}$, whose square root is especially simple.

Step 3: Evaluate the angular frequency and requested coefficient The square root and its reciprocal are

$$\sqrt{LC} = 5 \times 10^{-5}\ \mathrm{s}, \qquad \omega_0 = \frac{1}{5 \times 10^{-5}} = 2 \times 10^4\ \mathrm{rad\,s^{-1}}.$$

As a check, $\omega_0 L = 2000\ \Omega$, while $1/(\omega_0 C) = 2000\ \Omega$; the reactances cancel exactly. This is angular frequency, so no division by 2π should be made when filling the stated blank.

Step 4: Conclusion The resonant angular frequency is $\boxed{2 \times 10^4 \text{ rad s}^{-1}}$. The coefficient required in the blank is $\boxed{2}$.

Quick Tip to Solve: Use $1/\sqrt{LC}$ for angular frequency; insert $1/(2\pi\sqrt{LC})$ only if ordinary frequency in hertz is asked.

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48. An air bubble of radius 1.0 mm is observed at a depth 20 cm below the free surface of a liquid having surface tension 0.095 J/m^2 and density 10^3 kg/m^3. The difference between pressure inside the bubble and atmospheric pressure is _____ N/m^2. (Take $g = 10$ m/s^2) [4 marks]

Solution:

Concept:

- At depth h in a liquid open to the atmosphere, the pressure just outside a bubble is $P_{\text{out}} = P_{\text{atm}} + \rho g h$.
- An air bubble in liquid has one liquid-gas interface. Its surface-tension pressure jump is $P_{\text{in}} - P_{\text{out}} = 2T/r$, where T is surface tension and r is radius; the factor $4T/r$ applies to a soap bubble with two interfaces.

Step 1: Separate the two contributions to the pressure The question compares the pressure inside the bubble with atmospheric pressure, not with the surrounding liquid at the same depth. We must therefore include both hydrostatic pressure and the surface-tension jump:

$$P_{\text{in}} - P_{\text{atm}} = \rho g h + \frac{2T}{r}.$$

The surface-tension term is positive because pressure on the concave, gaseous side of the spherical interface is greater than that immediately outside it.

Step 2: Calculate the hydrostatic pressure Convert the depth and radius to metres:

$$h = 20 \text{ cm} = 0.20 \text{ m}, \qquad r = 1.0 \text{ mm} = 1.0 \times 10^{-3} \text{ m}.$$

Using the specified acceleration due to gravity,

$$\rho g h = (10^3)(10)(0.20) = 2000 \text{ Pa}.$$

This is the excess pressure already present in the liquid before accounting for the bubble curvature.

Step 3: Add the Laplace pressure of the single interface The surface tension 0.095 J m^{-2} is equivalently 0.095 N m^{-1}. Thus

$$\frac{2T}{r} = \frac{2(0.095)}{1.0 \times 10^{-3}} = 190 \text{ Pa}.$$

Adding the contributions gives

$$P_{\text{in}} - P_{\text{atm}} = 2000 + 190 = 2190 \text{ N m}^{-2}.$$

Using only 190 Pa would compare with the nearby liquid, and using the soap-bubble factor would

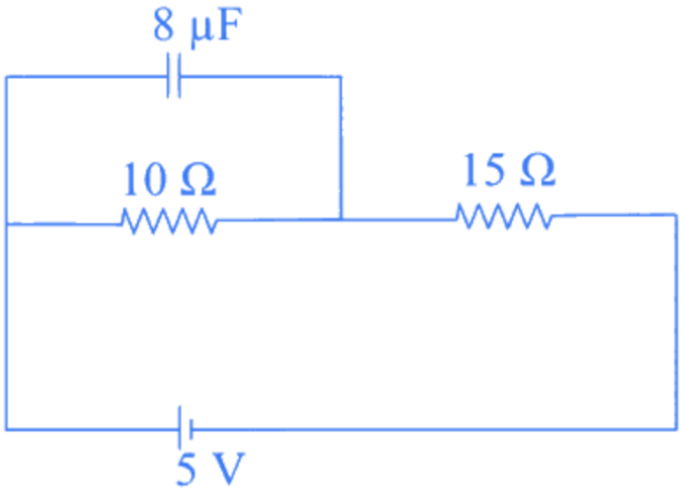
incorrectly double the surface contribution.

Step 4: Conclusion The required pressure difference is $\boxed{2190\ \mathrm{N\,m^{-2}}}$. The numerical entry is $\boxed{2190}$.

Quick Tip to Solve: For a submerged bubble, first add ρgh to atmospheric pressure, then add $2T/r$; an air bubble in liquid has only one interface.

49. At steady state the charge on the capacitor, as shown in the circuit below, is _____ μC.

[4 marks]



Solution:

Concept:

- At steady state an ideal capacitor carries no conduction current. It is an open branch for finding the DC current, although its plates can hold equal and opposite charges.
- Resistors carry current according to Ohm's law, $V = IR$. The capacitor charge magnitude is $Q = CV$, where V is the potential difference between its two terminals.

Step 1: Identify the capacitor terminals The $8\,\mu\mathrm{F}$ capacitor is connected between the left-hand node and the junction of the $10\,\Omega$ and $15\,\Omega$ resistors. It is therefore in parallel with the $10\,\Omega$ resistor, rather than across the entire battery. After charging is complete no current flows through the capacitor branch. The two resistors then form a series path across the $5\,\mathrm{V}$ source.

Step 2: Find the steady current and capacitor voltage The same current passes through both resistors, since the junction has no current entering the capacitor at steady state. Thus

$$R_{\mathrm{eq}} = 10 + 15 = 25\,\Omega, \qquad I = \frac{5}{25} = 0.20\,\mathrm{A}.$$

The voltage across the $10\,\Omega$ resistor, and hence across the capacitor, is

$$V_C = I(10) = 0.20 \times 10 = 2.0\,\mathrm{V}.$$

The remaining $3.0\,\mathrm{V}$ drops across the $15\,\Omega$ resistor, consistent with the battery voltage.

Step 3: Calculate the charge magnitude Use the capacitance with the voltage across its own terminals:

$$Q = CV_C = (8 \times 10^{-6})(2.0) = 16 \times 10^{-6}\,\text{C}.$$

Multiplying capacitance in microfarads by voltage in volts directly gives charge in microcoulombs. Each plate therefore carries charge of magnitude $16\,\mu$C; the two plate charges have opposite signs.

Step 4: Conclusion The steady-state capacitor charge is $\boxed{16\,\mu\text{C}}$. The numerical entry is $\boxed{16}$.

Quick Tip to Solve: At steady DC, open the capacitor branch first; then find the voltage between its two nodes before using $Q = CV$.

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50. A satellite of mass $\frac{M}{2}$ is revolving around earth in a circular orbit at a height of $\frac{R}{3}$ from earth surface. The angular momentum of the satellite is $\text{M}\sqrt{\frac{\text{GMR}}{x}}$. The value of x is _____ , where M and R are the mass and radius of earth, respectively. (G is the gravitational constant) [4 marks]

Solution:

Concept:

- The orbital radius is measured from Earth's centre: $r = R + h$, where R is Earth's radius and h is height above its surface.
- Gravity supplies the centripetal force in a circular orbit. Hence $v = \sqrt{GM/r}$, and the orbital angular momentum magnitude is $L = mvr = m\sqrt{GMr}$.

Step 1: Write the orbital radius and satellite mass The stated height is $R/3$, so the radius of the circular orbit is

$$r = R + \frac{R}{3} = \frac{4R}{3}.$$

The satellite mass is $m = M/2$, while the gravitating Earth has mass M. These are different roles for the mass: Earth's mass determines the orbital speed and the satellite's mass multiplies that speed when calculating angular momentum.

Step 2: Use the circular-orbit condition Equate the gravitational force to the centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \quad \Rightarrow \quad v = \sqrt{\frac{GM}{r}}.$$

The satellite mass cancels from the speed. Since its radius vector is perpendicular to its tangential velocity, the angular momentum has magnitude mrv, with no additional sine factor required.

Step 3: Compare with the supplied expression Substitute $m = M/2$ and $r = 4R/3$:

$$\begin{aligned} L &= \frac{M}{2}\sqrt{GM\frac{4R}{3}} \\ &= \frac{M}{2}2\sqrt{\frac{GMR}{3}} \\ &= M\sqrt{\frac{GMR}{3}}. \end{aligned}$$

The given expression is $M\sqrt{GMR/x}$. All quantities outside and inside the radical now match, so comparison of denominators gives $x = 3$. The result is dimensionless; $M\sqrt{GMR}$ itself has the dimensions of angular momentum.

Step 4: Conclusion The required dimensionless value is $\boxed{x = 3}$.

Quick Tip to Solve: Use distance from Earth's centre in orbital formulae. Keep satellite mass outside the speed formula until forming $L = m\sqrt{GMr}$.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. pH of water is 7 at $25^\circ C$. If water is heated to $80^\circ C$., it's pH will : [4 marks]

(A) Decrease

(B) Remains the same

(C) Increase

(D) H^+concentration increases, OH^-concentration decreases

Correct Answer: (A) Decrease

Solution:

Concept:

- Water self-ionizes according to $2\,H_2O \rightleftharpoons H_3O^+ + OH^-$. This ionization is endothermic, so its equilibrium constant K_w increases when temperature increases.
- For pure water, $[H_3O^+] = [OH^-]$ at every temperature. Therefore $[H_3O^+] = \sqrt{K_w}$, and $pH = \frac{1}{2}pK_w$.

Step 1: Apply the effect of heating At $25^\circ C$, K_w is approximately 10^{-14}, so each ion concentration is $10^{-7}\,mol\,L^{-1}$, giving pH 7. Increasing the temperature to $80^\circ C$ favours the forward, heat-absorbing ionization process. More water molecules produce ions, and K_w becomes larger than its value at $25^\circ C$.

Step 2: Determine the change in pH Since the water remains pure, electrical neutrality requires the hydronium and hydroxide concentrations to rise equally. The hydronium concentration consequently exceeds $10^{-7}\,mol\,L^{-1}$. Because

$$pH = -\log_{10}[H_3O^+],$$

an increase in this concentration means a decrease in pH. An exact value is unnecessary: the direction follows directly from the endothermic ionization equilibrium.

Step 3: Check the alternatives and neutrality Option (A) states the required decrease. Option (B) incorrectly treats K_w as independent of temperature. Option (C) would require a decrease in hydronium concentration, opposite to the equilibrium shift. Option (D) is also incorrect because hydroxide does not decrease: both ion concentrations increase together. Despite pH falling below 7, pure water remains neutral, since neutrality means equal hydronium and hydroxide concentrations rather than a universal pH of 7.

Step 4: Conclusion The pH decreases on heating the water to $80^\circ C$. Hence, the correct option is (A).

Quick Tip to Solve: For pure water use $pH = pK_w/2$; neutral pH is 7 specifically near $25^\circ C$, not at every temperature.

52. When a non-volatile solute is added to the solvent, the vapour pressure of the solvent decreases by 10 mm of Hg . The mole fraction of the solute in the solution is 0.2 . What would be the mole fraction of the solvent if decrease in vapour pressure is 20 mm of Hg ? [4 marks]

(A) 0.2

(B) 0.4

(C) 0.8

(D) 0.6

Correct Answer: (D) 0.6

Solution:

Concept:

- For a non-volatile solute in an ideal solution, Raoult's law gives $P = x_{\text{solvent}} P^0$, where P^0 is the vapour pressure of the pure solvent at the same temperature.
- The lowering is $\Delta P = P^0 - P = P^0 x_{\text{solute}}$, and the two mole fractions add to unity.

Step 1: Find the pure-solvent vapour pressure Initially, the solute mole fraction is 0.2 and the lowering of vapour pressure is $10\,\text{mm Hg}$. Apply the relative-lowering relation:

$$\frac{\Delta P}{P^0} = x_{\text{solute}} \quad \Rightarrow \quad P^0 = \frac{10}{0.2} = 50\,\text{mm Hg}.$$

This reference pressure belongs to the pure solvent; it stays the same when the solution composition is changed at the same temperature.

Step 2: Calculate the new mole fractions For a lowering of $20\,\text{mm Hg}$, the new solute mole fraction is

$$x'_{\text{solute}} = \frac{20}{50} = 0.4.$$

The question asks for the solvent mole fraction, so take the complement:

$$x'_{\text{solvent}} = 1 - 0.4 = 0.6.$$

Equivalently, the final solution vapour pressure is $50 - 20 = 30\,\text{mm Hg}$, giving $x'_{\text{solvent}} = 30/50 = 0.6$.

Step 3: Reject the common substitutions Option (D) agrees with both calculations. Option (B), 0.4, is the new solute mole fraction and is therefore the wrong component. Option (C), 0.8, is the original solvent mole fraction, before the lowering was doubled. Option (A), 0.2, is the original solute mole fraction. Increasing the amount of a non-volatile solute must decrease the solvent mole fraction, consistent with its change from 0.8 to 0.6.

Step 4: Conclusion The mole fraction of the solvent is $\boxed{0.6}$. Hence, the correct option is (D).

Quick Tip to Solve: Vapour-pressure lowering gives the solute mole fraction first; subtract from 1 when the question asks for the solvent.

53. Match List - I with List - II

	List - I		List - II
(A)	Bronze	(I)	Cu, Ni
(B)	Brass	(II)	Fe, Cr, Ni, C
(C)	UK silver coin	(III)	Cu, Zn
(D)	Stainless steel	(IV)	Cu, Sn

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(B) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(C) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(D) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Correct Answer: (B) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Solution:

Concept:

- An alloy is a metallic mixture with a characteristic composition and useful properties different from those of its individual components. Common alloy names identify standard combinations of elements.
- Bronze is primarily copper and tin, whereas brass is copper and zinc. Cupro-nickel contains copper and nickel, and stainless steel contains iron with chromium, nickel and carbon.

Step 1: Match the two copper alloys List I gives bronze as (A). Its constituent metals are copper and tin, so it matches List II entry (IV), $\mathrm{Cu, Sn}$. Brass, listed as (B), is composed mainly of copper and zinc, so it matches (III), $\mathrm{Cu, Zn}$. Distinguishing tin from zinc is essential here: both alloys contain copper, so copper alone cannot decide between them.

Step 2: Match the coin alloy and stainless steel The UK silver-coloured coin alloy in this list is cupro-nickel, a combination of copper and nickel. Thus (C) matches (I), $\mathrm{Cu, Ni}$. Stainless steel is an iron-based alloy containing chromium, nickel and a controlled amount of carbon. Chromium promotes corrosion resistance through a protective surface layer. Thus (D) matches (II), $\mathrm{Fe, Cr, Ni, C}$. The term silver coin in this matching question refers to the coin's colour, not a requirement that its alloy contain elemental silver.

Step 3: Assemble and check the full matching The complete correspondence is

$$\text{(A)–(IV), (B)–(III), (C)–(I), (D)–(II).}$$

This is option (B). Option (A) incorrectly assigns the steel constituents to brass and the brass constituents to the coin. Options (C) and (D) start by assigning copper-zinc to bronze, which already rules them out. Checking all four pairs confirms that the selected option has no residual mismatch.

Step 4: Conclusion Bronze matches copper-tin, brass copper-zinc, the coin copper-nickel, and stainless steel iron-chromium-nickel-carbon. Hence, the correct option is (B).

Quick Tip to Solve: Remember bronze as copper-tin and brass as copper-zinc; these two anchors usually fix the entire alloy matching.

54. Consider a binary solution of two volatile liquid components 1 and $2.x_1$ and y_1 are the mole fractions of component 1 in liquid and vapour phase, respectively. The slope and intercept of the linear plot of $\frac{1}{x_1}$ vs $\frac{1}{y_1}$ are given respectively as : [4 marks]

(A) $\frac{P_1^0}{P_2^0}, \frac{P_1^0 - P_2^0}{P_2^0}$

(B) $\frac{P_2^0}{P_1^0}, \frac{P_2^0 - P_1^0}{P_2^0}$

(C) $\frac{P_2^0}{P_1^0}, \frac{P_1^0 - P_2^0}{P_2^0}$

(D) $\frac{P_1^0}{P_2^0}, \frac{P_2^0 - P_1^0}{P_2^0}$

Correct Answer: (D) $\frac{P_1^0}{P_2^0}, \frac{P_2^0 - P_1^0}{P_2^0}$

Solution:

Concept:

- For an ideal binary liquid solution, Raoult's law gives $p_1 = x_1 P_1^0$ and $p_2 = (1 - x_1)P_2^0$, where P_i^0 is the pure-component vapour pressure.
- Dalton's law gives the vapour mole fraction $y_1 = p_1/(p_1 + p_2)$. A plot of $1/x_1$ versus $1/y_1$ has $1/x_1$ as its vertical coordinate.

Step 1: Write the vapour mole fraction The total pressure over the ideal mixture is

$$P = x_1 P_1^0 + (1 - x_1)P_2^0.$$

Therefore,

$$y_1 = \frac{x_1 P_1^0}{x_1 P_1^0 + (1 - x_1)P_2^0}.$$

Both liquid components are volatile, so the second partial pressure must be retained. Dropping it would incorrectly make the vapour contain only component 1.

Step 2: Rearrange into the requested straight-line form Take the reciprocal and separate the terms:

$$\frac{1}{y_1} = \frac{x_1(P_1^0 - P_2^0) + P_2^0}{x_1 P_1^0} = \frac{P_1^0 - P_2^0}{P_1^0} + \frac{P_2^0}{P_1^0}\frac{1}{x_1}.$$

Now isolate the vertical variable:

$$\frac{1}{x_1} = \frac{P_1^0}{P_2^0}\frac{1}{y_1} + \frac{P_2^0 - P_1^0}{P_2^0}.$$

The pressure ratios are dimensionless, as required for reciprocal mole fractions.

Step 3: Read the slope and intercept Comparing with $Y = mX + c$, using $Y = 1/x_1$ and $X = 1/y_1$, gives slope P_1^0/P_2^0 and intercept $(P_2^0 - P_1^0)/P_2^0$. Option (A) has the opposite intercept sign. Options (B) and (C) invert the slope. Option (D) has both quantities in the required form.

Step 4: Conclusion The slope and intercept are $\frac{P_1^0}{P_2^0}$ and $\frac{P_2^0 - P_1^0}{P_2^0}$, respectively. Hence, the correct option is (D).

Quick Tip to Solve: For a plot of A versus B, isolate A on the left before reading the slope; reciprocating the composition relation can reverse the apparent ratio.

55. Given below are the atomic numbers of some group 14 elements. The atomic number of the element with lowest melting point is : [4 marks]

(A) 14

(B) 50

(C) 6

(D) 82

Correct Answer: (B) 50

Solution:

Concept:

- The melting point depends on the strength and nature of bonding in the solid, as well as its crystal structure. It is not determined by atomic number alone.
- Carbon and silicon form giant covalent structures with strong bonds throughout the solid. Tin and lead are metallic solids; the group does not show a strictly monotonic melting-point trend.

Step 1: Identify the elements The atomic numbers correspond to carbon, $Z = 6$; silicon, $Z = 14$; tin, $Z = 50$; and lead, $Z = 82$. Carbon and silicon have very high melting or sublimation temperatures because disrupting their extended covalent networks requires considerable energy. They cannot be the lowest-melting members in this list.

Step 2: Compare tin with lead Tin melts at approximately $232\,^{\circ}\mathrm{C}$, whereas lead melts at approximately $327\,^{\circ}\mathrm{C}$. Thus tin has the lower melting point even though lead lies below it in the group. The increase from tin to lead is an exception to a simple decreasing trend; differences in metallic bonding and packing matter.

Step 3: Eliminate the alternatives Option (A) denotes silicon and option (C) denotes carbon, both network solids with far higher transition temperatures. Option (D) denotes lead, which melts above tin. Option (B), atomic number 50, is therefore the unique choice among the four listed elements.

Step 4: Conclusion The element is tin, Sn, whose atomic number is 50. Hence, the correct option is (B).

Quick Tip to Solve: For group 14 melting points, remember the tin-to-lead rise; do not select lead merely because it is lower in the group.

56. Given below are two statements :

Statement (I) : The boiling points of alcohols and phenols increase with increase in the number of C -atoms.

Statement (II) : The boiling points of alcohols and phenols are higher in comparison to other class of compounds such as ethers, haloalkanes.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are true

(C) Statement I is true but Statement II is false

(D) Both Statement I and Statement II are false

Correct Answer: (B) Both Statement I and Statement II are true

Solution:

Concept:

- Boiling requires overcoming intermolecular attractions. Stronger attractions generally lead to a higher boiling point for substances of comparable molecular mass and structure.
- Alcohols and phenols have an O–H group and form intermolecular hydrogen bonds. Increasing carbon-chain length generally increases molecular size, polarizability and dispersion forces.

Step 1: Assess Statement I Within a comparable homologous series, an increase in the number of carbon atoms increases the size and surface area of the molecule. London dispersion attractions become stronger, so more energy is needed to separate the molecules into the vapour. Therefore the boiling points generally increase with carbon number, and Statement I is true. This is the usual homologous-series trend, rather than a comparison of every possible branched isomer.

Step 2: Assess Statement II Alcohol and phenol molecules can both donate and accept hydrogen bonds through their hydroxyl groups. Ethers lack an O–H bond and cannot form such hydrogen-bonded associations with themselves; haloalkanes also lack this donor group. Consequently, for comparable molecular masses, alcohols and phenols have higher boiling points. Statement II is true.

Step 3: Choose the statement pair Since both statements describe the standard intermolecular-force trends, option (B) follows. Option (A) incorrectly rejects the chain-length trend, option (C) incorrectly rejects the hydrogen-bonding effect, and option (D) rejects both valid statements.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (B).

Quick Tip to Solve: Compare similar molecular masses first: an O–H group raises boiling point through hydrogen bonding, while a longer carbon skeleton strengthens dispersion forces.

57. The α-Helix and β - Pleated sheet structures of protein are associated with its : [4 marks]

(A) tertiary structure

(B) quaternary structure

(C) primary structure

(D) secondary structure

Correct Answer: (D) secondary structure

Solution:

Concept:

- A protein is a polypeptide made of amino-acid residues linked by peptide bonds. Its structural levels distinguish amino-acid sequence, local backbone folding, complete-chain folding and association of chains.
- The secondary structure consists of regular local arrangements of the peptide backbone, chiefly the α-helix and β-pleated sheet, stabilized by hydrogen bonding between peptide groups.

Step 1: Identify the common feature In an α-helix the polypeptide backbone coils regularly and hydrogen bonds form between suitable carbonyl and N–H groups along the chain. In a β-pleated sheet, extended backbone segments lie alongside one another and are held together by hydrogen bonds. Both describe an organized arrangement of the backbone rather than a change in the sequence of amino acids.

Step 2: Distinguish the other levels The primary structure is the exact amino-acid sequence, so option (C) does not describe these folds. The tertiary structure is the overall three-dimensional conformation of an entire polypeptide chain, including interactions among side chains; it is broader than the local helix or sheet, excluding option (A). The quaternary structure describes the arrangement of two or more polypeptide subunits, excluding option (B).

Step 3: Select the relevant level The named motifs are therefore the characteristic examples of secondary structure. Their hydrogen bonds involve backbone carbonyl oxygen and amide hydrogen atoms; recognizing these interactions distinguishes them from peptide bonds that fix the primary sequence and from side-chain interactions important in tertiary folding.

Step 4: Conclusion The α-helix and β-pleated sheet are secondary structures of proteins. Hence, the correct option is (D).

Quick Tip to Solve: Sequence means primary; local helix or sheet means secondary; whole-chain folding means tertiary; multiple subunits mean quaternary.

58. The effect of temperature on spontaneity of reactions are represented as:

	ΔH	ΔS	Temperature	Spontaneity
(A)	$+$	$-$	**any** T	**Non spontaneous**
(B)	$+$	$+$	**low** T	**spontaneous**
(C)	$-$	$-$	**low** T	**Non spontaneous**
(D)	$-$	$+$	**any** T	**spontaneous**

The incorrect combinations are: [4 marks]

(A) (A) and (C) only

(B) (B) and (D) only

(C) (A) and (D) only

(D) (B) and (C) only

Correct Answer: (D) (B) and (C) only

Solution:

Concept:

- At constant temperature and pressure, a reaction is spontaneous in the forward direction when its Gibbs energy change is negative: $\Delta G = \Delta H - T\Delta S < 0$. Temperature is measured on the absolute scale.
- The enthalpy term is temperature independent in the usual sign analysis, while the magnitude of the entropy contribution grows with temperature. For like signs of ΔH and ΔS, temperature can therefore change spontaneity.

Step 1: Check rows A and D For row (A), $\Delta H > 0$ and $\Delta S < 0$, so $\Delta G = \Delta H + T|\Delta S| > 0$ at every positive temperature. Its description as nonspontaneous is correct. For row (D), $\Delta H < 0$ and $\Delta S > 0$, both terms in $\Delta G = -|\Delta H| - T\Delta S$ are negative. Its description as spontaneous at any temperature is also correct.

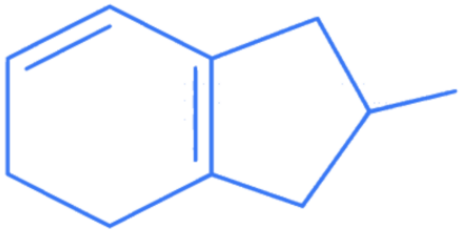
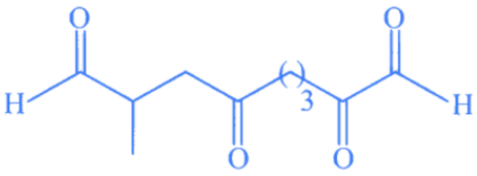
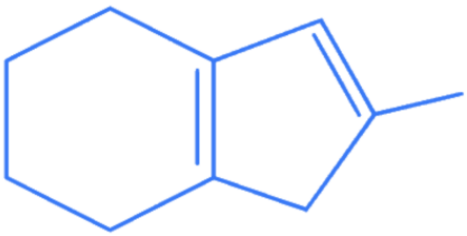
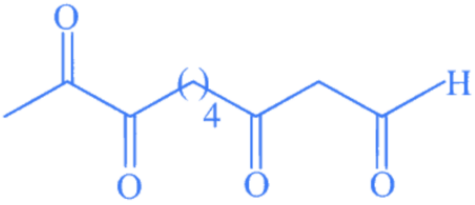
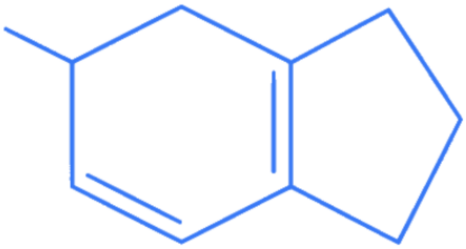
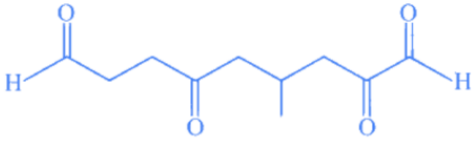
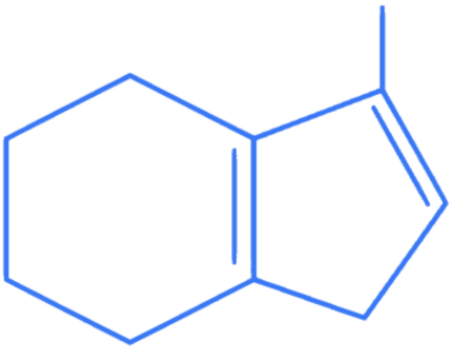
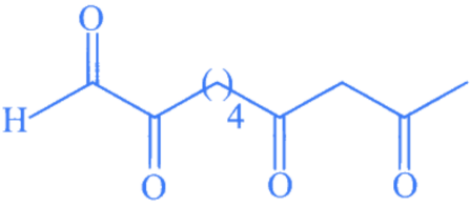
Step 2: Check the two temperature-dependent rows For row (B), both changes are positive. Spontaneity requires $T > \Delta H/\Delta S$, so it occurs at sufficiently high temperature, not low temperature as listed. For row (C), both changes are negative: $\Delta G = -|\Delta H| + T|\Delta S|$. At sufficiently low temperature the negative enthalpy term dominates and the reaction is spontaneous, contrary to the printed description.

Step 3: Select the incorrect pair The incorrect rows are (B) and (C). Option (A) wrongly includes the valid row (A); option (B) wrongly includes the valid row (D); option (C) consists entirely of valid rows. Only option (D) selects both incorrect temperature-dependent cases.

Step 4: Conclusion The positive-enthalpy, positive-entropy low-temperature row and the negative-enthalpy, negative-entropy low-temperature row are incorrect. Hence, the correct option is (D).

Quick Tip to Solve: Write the signs directly into Gibbs energy before reading the table: positive enthalpy and entropy need high temperature; both negative need low temperature.

59. **Match List I (isomers of $C_{10}H_{14}$) with List II (ozonolysis product). Choose the correct answer from the options given below:** [4 marks]

	List - I (Isomers of $C_{10}H_{14}$)		List - II (Ozonolysis product)
(A)		(I)	
(B)		(II)	
(C)		(III)	
(D)		(IV)	

(A) (A)–(III), (B)–(IV), (C)–(I), (D)–(II)

(B) (A)–(I), (B)–(IV), (C)–(III), (D)–(II)

(C) (A)–(III), (B)–(II), (C)–(I), (D)–(IV)

(D) (A)–(II), (B)–(II), (C)–(I), (D)–(IV)

Correct Answer: (A) (A)–(III), (B)–(IV), (C)–(I), (D)–(II)

Solution:

Concept:

- Ozonolysis cleaves each carbon–carbon double bond and converts each of its two carbon atoms into a carbonyl carbon. An alkene carbon carrying hydrogen gives an aldehyde; a carbon without hydrogen gives a ketone.
- For a fused-ring diene, follow the carbon skeleton after breaking both double bonds. A methyl substituent remains attached to the same carbon, so its position relative to the new carbonyl groups identifies the product.

Step 1: Match the six-membered-ring dienes A and C In (A), cleavage of the outer double bond gives two aldehyde termini, while cleavage of the fused double bond gives two internal ketones because the fusion carbons have no hydrogens. The methyl substituent lies on the surviving three-carbon bridge of the five-membered ring, between the two ketone regions. This places (A) with product (III). In (C), the methyl group lies instead on the outer six-membered-ring path, adjacent to an aldehyde end after cleavage. That connectivity is shown in product (I), so (C)–(I).

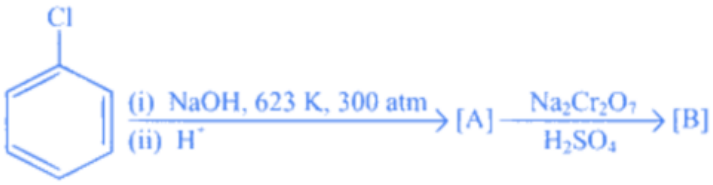
Step 2: Distinguish the five-membered-ring dienes B and D In (B), one carbon of the outer five-membered-ring double bond carries the methyl group and becomes a methyl ketone. Its other carbon bears hydrogen and becomes an aldehyde. Tracing the retained bonds places the aldehyde adjacent to a fusion-derived ketone and the methyl ketone beyond the remaining methylene: product (IV). In (D), the methyl-bearing alkene carbon is adjacent to the upper fusion carbon, so the methyl-ketone end is adjacent to that fusion-derived carbonyl; the aldehyde lies beyond the methylene. This is product (II).

Step 3: Compare all four assignments with the options The complete matching is (A)–(III), (B)–(IV), (C)–(I), (D)–(II). Option (B) interchanges the products of (A) and (C). Option (C) interchanges those of (B) and (D). Option (D) also fails these connectivity tests and repeats product (II). Only option (A) preserves every substituent and carbonyl position.

Step 4: Conclusion The products in the order of the four listed isomers are III, IV, I and II. Hence, the correct option is (A).

Quick Tip to Solve: Mark every alkene carbon as aldehyde or ketone before tracing the opened skeleton; the methyl position distinguishes products with the same carbonyl count.

60. Identify the products [A] and [B], respectively in the following reaction: [4 marks]



(A) [A] Benzene; [B] Phenol

(B) [A] Phenol; [B] 1,4-Benzoquinone

(C) [A] Phenol; [B] Sodium phenoxide

(D) [A] Benzene; [B] Cyclohexa-2,4-dien-1-one

Correct Answer: (B) [A] Phenol; [B] 1,4-Benzoquinone

Solution:

Concept:

- Chlorobenzene requires vigorous conditions for replacement of chlorine by hydroxyl. Heating with aqueous sodium hydroxide at $623\,\mathrm{K}$ and high pressure gives sodium phenoxide; acidification produces phenol.
- Phenol is oxidized by acidified dichromate to a quinone. The product shown among the choices is 1,4-benzoquinone, which contains carbonyl groups at opposite ring positions.

Step 1: Determine intermediate A The first reagent set is the Dow-process conversion of chlorobenzene. Under the stated conditions, hydroxide replaces chlorine to form sodium phenoxide. The second part of the first step is acid treatment, which protonates the phenoxide oxygen. Thus the isolated intermediate [A] is phenol, $\mathrm{C_6H_5OH}$, rather than benzene or sodium phenoxide. The high temperature and pressure are characteristic of the relatively unreactive aryl chloride.

Step 2: Determine oxidation product B The next reagent is $\mathrm{Na_2Cr_2O_7/H_2SO_4}$, an oxidizing combination. Oxidation of phenol gives 1,4-benzoquinone, also called para-benzoquinone, $\mathrm{C_6H_4O_2}$. The hydroxy-substituted aromatic ring is converted into a conjugated diketone with two opposite carbonyl groups. This is a chemical oxidation, not merely removal of the phenolic proton.

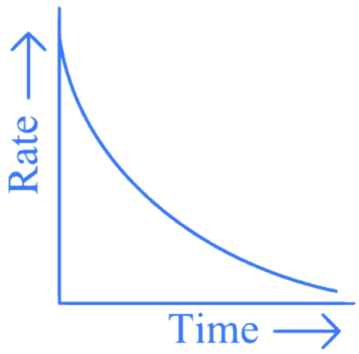
Step 3: Eliminate the other product pairs Options (A) and (D) assign benzene to [A], although the reagents hydrolyse the aryl chloride rather than remove chlorine reductively. Option (C) assigns sodium phenoxide to [B], but acidified dichromate oxidizes phenol and is not a base that isolates its sodium salt. Only option (B) contains phenol followed by 1,4-benzoquinone.

Step 4: Conclusion [A] is phenol and [B] is 1,4-benzoquinone. Hence, the correct option is (B).

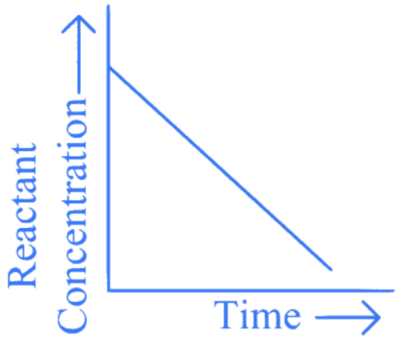
Quick Tip to Solve: Separate the sequence into hydrolysis plus acidification, then oxidation: aryl chloride gives phenol first, and acidified dichromate converts phenol to a quinone.

61. Which of the following graphs most appropriately represents a zero order reaction? [4 marks]

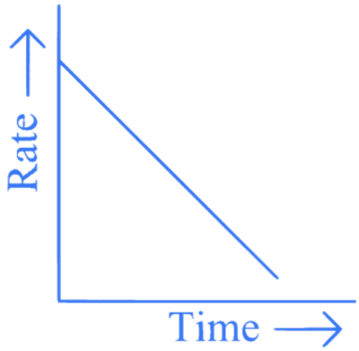
(A)



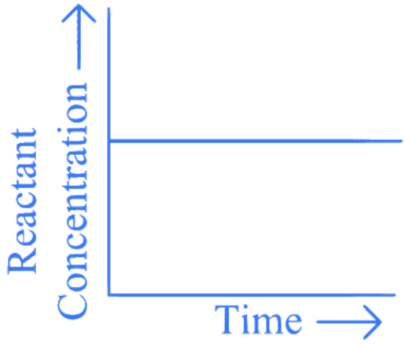
(B)



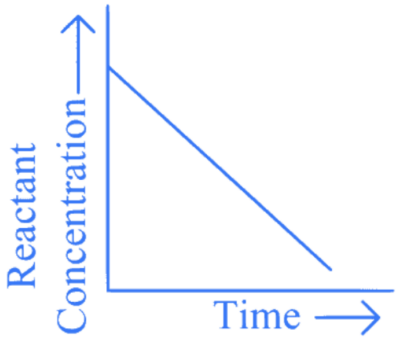
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- For a zero-order reaction, the rate is independent of reactant concentration: $-d[A]/dt = k[A]^0 = k$, where k is the rate constant.
- Integration gives $[A]_t = [A]_0 - kt$. Therefore a plot of reactant concentration against time is straight, with slope $-k$ and intercept $[A]_0$.

Step 1: Apply the integrated rate equation At fixed temperature the rate constant does not change as the reaction proceeds. Over a time interval Δt, the zero-order reaction consumes $k\Delta t$ concentration units, regardless of the instantaneous concentration. Thus equal time intervals produce equal decreases in concentration:

$$[A]_{t_2} - [A]_{t_1} = -k(t_2 - t_1).$$

The concentration-time line consequently slopes downwards until the reactant is exhausted; the linear law is not extrapolated to negative concentration.

Step 2: Match the axes and reject the other graphs Graph (B) has reactant concentration on the vertical axis and time on the horizontal axis, with a straight decreasing line, exactly as required. Graph (A) shows the rate decreasing nonlinearly with time, whereas a zero-order rate must remain constant. Graph (C) also shows a decreasing rate, this time linearly, so it fails the same test. Graph (D) has constant reactant concentration; it would describe no consumption of that reactant, rather than a nonzero zero-order reaction. The distinction between a constant rate and a constant concentration is essential: the former makes concentration fall linearly.

Step 3: Conclusion The decreasing straight concentration-time graph is (B). Hence, the correct option is (B).

Quick Tip to Solve: Read the vertical axis before judging the line: zero order means constant rate, but a straight decreasing reactant concentration.

62. Identify the coordination complexes in which the central metal ion has d^4 configuration.

(A) $[FeO_4]^{2-}$

(B) $[Mn(CN)_6]^{3-}$

(C) $[Fe(CN)_6]^{3-}$

(D) $Cr_2(CH_3COO)_4(H_2O)_2$

(E) $[NiF_6]^{2-}$

Choose the correct answer from the options given below: [4 marks]

(A) (B), (C) and (D) only

(B) (C) and (E) only

(C) (A), (B) and (E) only

(D) (B) and (D) only

Correct Answer: (D) (B) and (D) only

Solution:

Concept:

- The oxidation state follows from the sum of metal and ligand charges equalling the complex charge. For transition-metal ions, the d-electron count is the group number minus the oxidation state.
- Oxo ligands have charge -2; cyanido, fluorido and acetate ligands have charge -1; water is neutral. The ligand field changes orbital occupation, but not the total d-electron count.

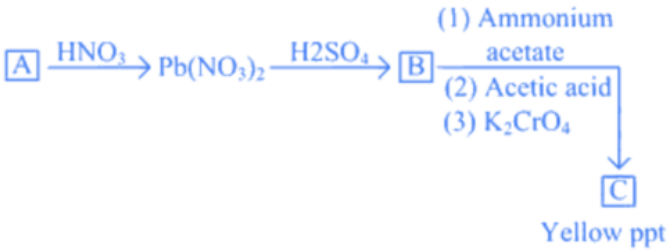
Step 1: Count the electrons in complexes A, B and C For (A), let the iron oxidation state be z. Then $z + 4(-2) = -2$, so iron is $+6$. Iron belongs to group 8, hence Fe^{6+} is $d^{8-6} = d^2$, not d^4. For (B), $z + 6(-1) = -3$ gives manganese in oxidation state $+3$. Manganese belongs to group 7, so Mn^{3+} is $d^{7-3} = d^4$. For (C), the identical ligand-charge calculation gives iron in oxidation state $+3$, hence Fe^{3+} is $d^{8-3} = d^5$.

Step 2: Count the electrons in D and E and compare the choices In (D), the neutral complex contains four singly negative acetate ligands and two neutral water ligands. The two equivalent chromium atoms together balance charge -4, so each chromium is $+2$. Chromium is in group 6, giving $d^{6-2} = d^4$ for each metal. In (E), $z + 6(-1) = -2$, hence nickel is $+4$. Group-10 nickel therefore has $d^{10-4} = d^6$. Only (B) and (D) meet the condition. Option (A) incorrectly includes the d^5 iron complex (C); option (B) includes (C) and the d^6 nickel complex; option (C) includes both the d^2 ferrate and d^6 nickel complex while omitting (D).

Step 3: Conclusion The manganese(III) cyanido complex and chromium(II) acetate complex alone have d^4 metal centres. Hence, the correct option is (D).

Quick Tip to Solve: Find oxidation state first, then use group number minus oxidation state; strong-field and weak-field labels do not change the total d-electron count.

63. Identify A, B and C in the given below reaction sequence [4 marks]



(A) $PbCl_2, PbSO_4, PbCrO_4$

(B) $PbS, PbSO_4, Pb(CH_3COO)_2$

(C) $PbS, PbSO_4, PbCrO_4$

(D) $PbCl_2, Pb(SO_4)_2, PbCrO_4$

Correct Answer: (C) $PbS, PbSO_4, PbCrO_4$

Solution:

Concept:

- In qualitative analysis of lead, black PbS is converted to soluble lead nitrate by nitric acid. Sulfuric acid then precipitates white $PbSO_4$.
- Lead sulfate dissolves on treatment with ammonium acetate. The resulting lead-containing solution gives yellow $PbCrO_4$ with chromate in acetic-acid medium.

Step 1: Identify the starting sulfide and sulfate intermediate The sequence is the standard confirmatory test for lead from its sulfide precipitate. Nitric acid oxidises the sulfide and transfers lead into nitrate solution, so $A = PbS$. A representative balanced equation with dilute nitric acid is

$$3\,PbS + 8\,HNO_3 \longrightarrow 3\,Pb(NO_3)_2 + 3\,S + 2\,NO + 4\,H_2O.$$

The exact nitrogen oxide formed depends on the acid conditions; the necessary observation here is the production of lead nitrate. Addition of sulfuric acid then gives

$$Pb(NO_3)_2 + H_2SO_4 \longrightarrow PbSO_4\downarrow + 2\,HNO_3.$$

Thus the second unknown is $B = PbSO_4$, with lead still in oxidation state $+2$.

Step 2: Use the yellow precipitate to identify the final product Ammonium acetate brings the sparingly soluble lead sulfate into solution as soluble lead acetate. Subsequent treatment with potassium chromate gives the characteristic yellow lead chromate precipitate:

$$Pb^{2+} + {CrO_4}^{2-} \longrightarrow PbCrO_4\downarrow.$$

Therefore $C = PbCrO_4$. Option (C) contains all three substances in their correct sequence. Option (A) substitutes lead chloride for the initial sulfide of this standard sequence. Option (B) incorrectly identifies the yellow final precipitate as lead acetate, which is the soluble intermediate instead. Option (D) both substitutes chloride at the start and writes $Pb(SO_4)_2$, inconsistent with the divalent lead sulfate formed here.

Step 3: Conclusion $A = \mathrm{PbS}$, $B = \mathrm{PbSO_4}$ and $C = \mathrm{PbCrO_4}$. Hence, the correct option is (C).

Quick Tip to Solve: Work backwards from the yellow chromate precipitate: identify lead, then trace its sulfate and the initial black sulfide.

64. Standard electrode potentials for a few half cells are mentioned below :

$$\mathrm{E^\circ_{Cu^{2+}/Cu}} = 0.34\ \mathrm{V}, \mathrm{E^\circ_{Zn^{2+}/Zn}} = -0.76\ \mathrm{V}$$
$$\mathrm{E^\circ_{Ag^{+}/Ag}} = 0.80\ \mathrm{V}, \mathrm{E^\circ_{Mg^{2+}/Mg}} = -2.37\ \mathrm{V}$$

Which one of the following cells gives the most negative value of ΔG° ? [4 marks]

(A) $\mathrm{Zn}\left|\mathrm{Zn^{2+}(1M)}\right\|\mathrm{Ag^{+}(1M)}\left|\mathrm{Ag}\right.$
(B) $\mathrm{Ag}\left|\mathrm{Ag^{+}(1M)}\right|\left|\mathrm{Mg^{2+}(1M)}\right|\mathrm{Mg}$
(C) $\mathrm{Zn}\left|\mathrm{Zn^{2+}(1M)}\right|\left|\mathrm{Mg^{2+}(1M)}\right|\mathrm{Mg}$
(D) $\mathrm{Cu}\left|\mathrm{Cu^{2+}(1M)}\right|\left|\mathrm{Ag^{+}(1M)}\right|\mathrm{Ag}$

Correct Answer: (A) $\mathrm{Zn}\left|\mathrm{Zn^{2+}(1M)}\right\|\mathrm{Ag^{+}(1M)}\left|\mathrm{Ag}\right.$

Solution:

Concept:

- For a cell written with the left electrode as anode and the right electrode as cathode, $E^\circ_{\text{cell}} = E^\circ_{\text{right}} - E^\circ_{\text{left}}$, using reduction potentials.
- The standard Gibbs energy change is $\Delta G^\circ = -nFE^\circ_{\text{cell}}$. Here F is the Faraday constant and n is the number of electrons transferred in the balanced cell reaction.

Step 1: Calculate the cell potentials in their written directions For option (A), zinc is oxidised and silver ions are reduced:

$$E^\circ_A = 0.80 - (-0.76) = 1.56\,\mathrm{V}.$$

For option (B), the written direction oxidises silver and reduces magnesium ions:

$$E^\circ_B = -2.37 - 0.80 = -3.17\,\mathrm{V}.$$

For option (C), zinc is on the left and magnesium on the right:

$$E^\circ_C = -2.37 - (-0.76) = -1.61\,\mathrm{V}.$$

For option (D), copper is oxidised and silver ions are reduced:

$$E^\circ_D = 0.80 - 0.34 = 0.46\,\mathrm{V}.$$

The large magnitude in option (B) must not be mistaken for a favourable forward cell potential: its sign is negative.

Step 2: Include electron transfer and compare Gibbs energies Each balanced reaction transfers two electrons. In (A), one zinc atom supplies the electrons needed by two silver ions; the other cells

likewise require $n = 2$. Therefore the four Gibbs energies, expressed using F, are

$$\Delta G^\circ_A = -3.12F, \quad \Delta G^\circ_B = +6.34F,$$
$$\Delta G^\circ_C = +3.22F, \quad \Delta G^\circ_D = -0.92F.$$

Options (B) and (C) have positive Gibbs energy changes in the written direction. Option (D) is favourable but less negative than (A). Thus the zinc-silver cell gives the greatest decrease in standard Gibbs energy among the choices.

Step 3: Conclusion The zinc-silver cell in (A) gives the most negative value: $\Delta G^\circ = -2F(1.56\,\mathrm{V})$, approximately $-301\,\mathrm{kJ\,mol^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: Keep the cell direction as written and compare nE, not the absolute electrode-potential difference.

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65. Consider the reaction

$$\mathrm{X_2Y(\,g)} \rightleftharpoons \mathrm{X_2(\,g)} + \frac{1}{2}\mathrm{Y_2(\,g)}$$

The equation representing correct relationship between the degree of dissociation (x) of $\mathrm{X_2Y(g)}$ with its equilibrium constant Kp is _____.

Assume x to be very very small. [4 marks]

(A) $x = \sqrt[3]{\frac{\mathrm{Kp}}{\mathrm{p}}}$

(B) $x = \sqrt[3]{\frac{\mathrm{Kp}}{2\mathrm{p}}}$

(C) $x = \sqrt[3]{\frac{2\mathrm{Kp}^2}{\mathrm{p}}}$

(D) $x = \sqrt[3]{\frac{2\mathrm{Kp}}{\mathrm{p}}}$

Correct Answer: (C) $x = \sqrt[3]{\frac{2\mathrm{Kp}^2}{\mathrm{p}}}$

Solution:

Concept:

- For gaseous dissociation, each partial pressure equals its equilibrium mole fraction times the total pressure p. The equilibrium expression uses the stoichiometric powers.
- The degree of dissociation x is the fraction of the initial reactant that dissociates. If $x \ll 1$, factors $1 - x$ and $1 + x/2$ can be approximated by one after writing the exact expression.

Step 1: Write equilibrium amounts and partial pressures Start with one mole of $\mathrm{X_2Y}$ and no products. For

$$\mathrm{X_2Y\,(g)} \rightleftharpoons \mathrm{X_2(g)} + \tfrac{1}{2}\mathrm{Y_2(g)},$$

the equilibrium amounts are $1-x$, x and $x/2$, respectively. The total is $1+x/2$. Thus

$$p_{X_2Y} = \frac{1-x}{1+x/2}p,$$
$$p_{X_2} = \frac{x}{1+x/2}p,$$
$$p_{Y_2} = \frac{x/2}{1+x/2}p.$$

The half coefficient changes both the amount of Y_2 and its power in the equilibrium expression.

Step 2: Apply the small-dissociation approximation and solve for x Using the pressure convention of the options,

$$K_p = \frac{p_{X_2}\sqrt{p_{Y_2}}}{p_{X_2Y}} = \frac{x}{1-x}\sqrt{\frac{xp}{2(1+x/2)}}.$$

For very small x, this becomes

$$K_p \simeq x\sqrt{\frac{xp}{2}}.$$

Squaring and rearranging yields

$$K_p^2 \simeq \frac{x^3 p}{2}, \qquad x^3 \simeq \frac{2K_p^2}{p}.$$

The correct relation must contain K_p^2, so options (A), (B) and (D), each containing only the first power of K_p under the cube root, are ruled out. The factor of two comes from the half mole of diatomic product, and is in the numerator after rearrangement.

Step 3: Conclusion

$$x = \sqrt[3]{\frac{2K_p^2}{p}} \quad (x \ll 1).$$

Hence, the correct option is (C).

Quick Tip to Solve: Write the half mole of Y2 and its square-root pressure factor explicitly before making the small-x approximation.

66. Given below are two statements :

Statement (I) : For a given shell, the total number of allowed orbitals is given by n^2.

Statement (II) : For any subshell, the spatial orientation of the orbitals is given by $-l$ to $+l$ values including zero.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is false but Statement II is true

(D) Statement I is true but Statement II is false

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- A shell with principal quantum number n contains subshells with $l = 0, 1, \ldots, n-1$. Each subshell contains $2l + 1$ orbitals.
- The magnetic quantum number m_l specifies spatial orientation. Its allowed integer values are $-l, -l+1, \ldots, 0, \ldots, l-1, l$.

Step 1: Verify the number of orbitals in a shell Sum the orbital counts over all allowed subshells:

$$N = \sum_{l=0}^{n-1}(2l+1) = 2\frac{n(n-1)}{2} + n = n^2.$$

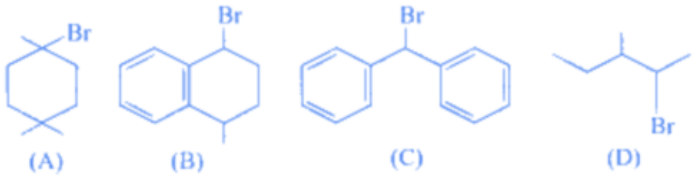
For example, the $n = 3$ shell contains the $3s$, $3p$ and $3d$ subshells, with $1 + 3 + 5 = 9$ orbitals, which agrees with 3^2. An orbital can accommodate two electrons, so the electron capacity $2n^2$ must not be confused with the orbital count n^2. Statement I correctly concerns orbitals and is therefore true.

Step 2: Verify the allowed orientations and classify both statements For a fixed subshell, l is fixed, while m_l takes every integral value from $-l$ through $+l$, including zero. There are $2l + 1$ such values, corresponding to the independent orbital orientations. For a p subshell, $l = 1$, so $m_l = -1, 0, +1$, giving three orbitals. For a d subshell, $l = 2$, so $m_l = -2, -1, 0, +1, +2$, giving five. Statement II is the usual abbreviated description of this magnetic-quantum-number rule and is true. It does not claim that the azimuthal quantum number itself ranges over negative values. Since both statements are true, option (A) applies; (B) incorrectly rejects both, while (C) and (D) each reject one valid statement.

Step 3: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: Use n to count a shell, l to select a subshell, and m_l to count its orientations; do not confuse n squared orbitals with twice that many electrons.

67. The ascending order of relative rate of solvolysis of following compounds is : [4 marks]



(A) (C) < (D) < (B) < (A)
(B) (D) < (A) < (B) < (C)
(C) (D) < (B) < (A) < (C)
(D) (C) < (B) < (A) < (D)

Correct Answer: (B) (D) < (A) < (B) < (C)

Solution:

Concept:

- Solvolysis in a polar protic medium proceeds predominantly through ionisation of the carbon–bromine bond. For an analogous leaving group, a more stable developing carbocation lowers the activation barrier and increases the rate.
- Resonance stabilisation spreads a positive charge over an aromatic system. Alkyl groups stabilise a carbocation by hyperconjugation and their electron-releasing inductive effect, but this is weaker than benzylic resonance stabilisation.

Step 1: Identify the cation from each substrate Compound (D) has bromine on a secondary aliphatic carbon. Compound (A) has bromine on the ring carbon that is also bonded to a methyl group, so it gives a tertiary aliphatic carbocation. Compound (B) loses bromide from a carbon directly adjacent to its benzene ring and gives a benzylic carbocation. Compound (C) gives a diphenylmethyl carbocation, in which the positively charged carbon is bonded to two phenyl groups.

Step 2: Compare stabilisation and arrange the rates The secondary aliphatic cation from (D) is least stabilised. The tertiary cation from (A) benefits from more alkyl substitution, so (A) reacts faster than (D). The benzylic cation from (B) can delocalise its charge into one aromatic ring and is more stable than the aliphatic tertiary cation. The cation from (C) can delocalise the charge into either of two phenyl rings, giving the greatest stabilisation. Thus the ascending rate order is

$$(D) < (A) < (B) < (C).$$

The order in (A) incorrectly puts the diphenyl compound first; (C) reverses the benzylic and tertiary comparison; (D) incorrectly makes the secondary substrate fastest.

Step 3: Conclusion The rate increases from the secondary aliphatic substrate to the tertiary aliphatic substrate, then the singly benzylic substrate and finally the diphenylmethyl substrate. Hence, the correct option is (B).

Quick Tip to Solve: Locate the carbon bonded to the leaving group before counting alkyl groups or phenyl rings; only rings conjugated with that developing cation stabilise it by resonance.

68. Given below are two statements about X-ray spectra of elements :

Statement (I) : A plot of $\sqrt{v}$ (v = frequency of X-rays emitted) vs atomic mass is a straight line.

Statement (II) : A plot of $v(\nu$ = frequency of X-rays emitted) vs atomic number is a straight line. In the light of the above statements, choose the correct answer from the options given below : [4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are true

(C) Statement I is true but Statement II is false

(D) Both Statement I and Statement II are false

Correct Answer: (D) Both Statement I and Statement II are false

Solution:

Concept:

- Moseley's law for a specified characteristic X-ray line is $\sqrt{\nu} = a(Z - b)$, where Z is atomic number, b is a screening constant and a is a constant for that series.
- Consequently, characteristic frequency is $\nu = a^2(Z - b)^2$. The straight-line relation uses the square root of frequency and atomic number, not atomic mass.

Step 1: Test Statement I against the correct variable The nuclear charge, expressed through atomic number, controls the inner-shell electronic energy levels that produce characteristic X-rays. Atomic mass is not the independent variable in Moseley's relation. Although mass usually increases with atomic number, their relationship is not an exact linear law; isotopes have different masses but the same atomic number and essentially the same characteristic X-ray frequencies. Therefore a straight line of $\sqrt{\nu}$ against atomic mass is not the stated physical law. Statement I is false.

Step 2: Test Statement II and choose the combination Squaring Moseley's relation gives

$$\nu = a^2Z^2 - 2a^2bZ + a^2b^2.$$

This is a quadratic dependence on atomic number, so a plot of frequency itself against Z is curved rather than a straight line. Statement II is also false. Option (A) would require the unsquared frequency to be linear; option (B) asserts both incorrect relations; option (C) incorrectly substitutes mass for atomic number. Only option (D) identifies both statements as false.

Step 3: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (D).

Quick Tip to Solve: Remember the pair in Moseley's graph: square root of frequency on the vertical axis and atomic number on the horizontal axis.

69. Consider the following reactions

$$K_2Cr_2O_7 \xrightarrow[-H_2O]{KOH} [\,A\,] \xrightarrow[-H_2O]{H_2SO_4} [\,B\,] + K_2SO_4$$

The products $[A]$ and $[B]$, respectively are : [4 marks]

(A) $K_2Cr(OH)_6$ and Cr_2O_3

(B) K_2CrO_4 and Cr_2O_3

(C) K_2CrO_4 and $K_2Cr_2O_7$

(D) K_2CrO_4 and CrO

Correct Answer: (C) K_2CrO_4 and $K_2Cr_2O_7$

Solution:

Concept:

- Chromate and dichromate ions interconvert through an acid–base equilibrium: $2\,CrO_4^{2-} + 2\,H^+ \rightleftharpoons Cr_2O_7^{2-} + H_2O$. Chromium remains in oxidation state $+6$ on both sides.
- An alkaline medium favours the yellow chromate ion, while addition of acid favours the orange dichromate ion. These changes do not require a reducing agent.

Step 1: Apply alkali to potassium dichromate Potassium hydroxide consumes the acidity associated with the dichromate side and produces potassium chromate. The balanced molecular reaction is

$$K_2Cr_2O_7 + 2\,KOH \longrightarrow 2\,K_2CrO_4 + H_2O.$$

Thus the labelled product [A] is K_2CrO_4. The potassium and chromium atom counts show why two chromate formula units are formed from each dichromate formula unit. The indicated removal of water does not change the identity of this chromium(VI) product.

Step 2: Acidify the chromate and reject reduced oxides Adding sulphuric acid reverses the chromate–dichromate equilibrium:

$$2\,K_2CrO_4 + H_2SO_4 \longrightarrow K_2Cr_2O_7 + K_2SO_4 + H_2O.$$

Therefore [B] is $K_2Cr_2O_7$. Potassium sulphate is the accompanying salt, exactly as shown in the reaction sequence. Options (A) and (B) contain Cr_2O_3, where chromium is $+3$, and option (D) contains CrO, where it is $+2$. Such products would require reduction, whereas the reagents here change only the acidity of the medium. Option (A) also fails to identify the chromate formed in alkali.

Step 3: Conclusion The products [A] and [B] are K_2CrO_4 and $K_2Cr_2O_7$, respectively. Hence, the correct option is (C).

Quick Tip to Solve: For chromate–dichromate questions, first identify whether the medium becomes acidic or basic; check chromium oxidation states to rule out spurious reduction products.

70. Given below are two statements:

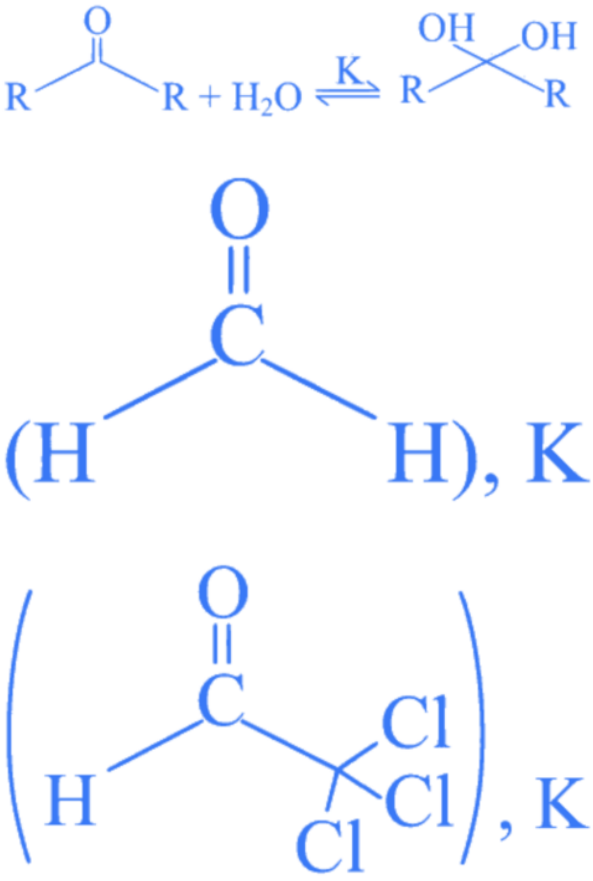
Consider the following reaction

Statement (I): In the case of formaldehyde is about 2280, due to small substituents, hydration is faster.

Statement (II) : In the case of trichloro acetaldehyde is about 2000 due to $-$I effect of $-$Cl .

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]



(A) Statement I is true but Statement II is false

(B) Both Statement I and Statement II are true

(C) Statement I is false but Statement II is true

(D) Both Statement I and Statement II are false

Correct Answer: (B) Both Statement I and Statement II are true

Solution:

Concept:

- Water adds to a carbonyl group to form a geminal diol: $R_2CO + H_2O \rightleftharpoons R_2C(OH)_2$. A large hydration equilibrium constant means the hydrate is strongly favoured.
- Hydration is promoted by low steric hindrance around the carbonyl carbon and by electron-withdrawing substituents, which increase its electrophilicity. Electron-donating alkyl groups oppose nucleophilic addition.

Step 1: Evaluate the formaldehyde statement Formaldehyde, $HCHO$, has two hydrogen substituents on the carbonyl carbon. These offer minimal steric resistance to the approach of water, and there are no electron-releasing alkyl groups to reduce the positive character of that carbon. Formation of methanediol, $CH_2(OH)_2$, is therefore highly favourable. The large stated hydration constant, about 2280, is consistent with extensive hydration and the small substituents facilitate addition. Hence Statement I is true. The equilibrium constant describes the equilibrium composition; the statement also identifies the steric reason that facilitates the hydration process.

Step 2: Evaluate the chloral statement and compare options Trichloroacetaldehyde, or chloral, is CCl_3CHO. Its three chlorine atoms withdraw electron density through sigma bonds, producing a strong negative inductive effect. This makes the neighbouring carbonyl carbon more electrophilic and promotes attack by water. The product $CCl_3CH(OH)_2$, chloral hydrate, is consequently strongly stabilised relative to the unhydrated aldehyde under hydration conditions. The stated large constant, about 2000, accords with this tendency, so Statement II is true as well. Options (A) and (C) each reject one valid explanation, while option (D) rejects both; neither steric accessibility in formaldehyde nor the inductive effect in chloral should be ignored.

Step 3: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (B).

Quick Tip to Solve: Use two separate checks for carbonyl hydration: how easily water reaches the carbonyl carbon, and whether substituents withdraw or donate electron density.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. 0.01 mole of an organic compound (X) containing 10% hydrogen, on complete combustion produced 0.9 gH_2O. Molar mass of (X) is _____ g mol^{-1}. [4 marks]

Solution:

Concept:

- On complete combustion, the hydrogen present in the organic sample appears in the water formed. One mole of H_2O contains two moles of hydrogen atoms, corresponding to $2\,g$ of hydrogen.
- Mass percentage is $100m_H/m_{sample}$. Once the sample mass is determined from its hydrogen content, its molar mass follows from $M = m/n$.

Step 1: Find the hydrogen mass from the water produced The given $0.9\,g$ of water corresponds to

$$n(H_2O) = \frac{0.9}{18} = 0.05\,\text{mol}.$$

Since each water molecule contains two hydrogen atoms, the combustion product contains $0.10\,\text{mol}$ of hydrogen atoms. Taking the molar mass of hydrogen atoms as $1\,g\,mol^{-1}$, the original organic sample therefore contained

$$m_H = 0.10 \times 1 = 0.10\,g.$$

The oxygen mass in the water is not included in this hydrogen mass; it can originate from the oxygen used for combustion.

Step 2: Recover the sample mass and calculate molar mass The compound contains 10 percent hydrogen by mass. Thus

$$0.10 = \frac{m_{\mathrm{H}}}{m_{\mathrm{sample}}}, \qquad m_{\mathrm{sample}} = \frac{0.10}{0.10} = 1.0\,\mathrm{g}.$$

This is the mass of the stated $0.01\,\mathrm{mol}$ of compound. Hence

$$M(X) = \frac{1.0\,\mathrm{g}}{0.01\,\mathrm{mol}} = 100\,\mathrm{g\,mol^{-1}}.$$

As a check, one mole of compound contains $10\,\mathrm{g}$ hydrogen, so its 0.01-mole sample contains 0.10 g hydrogen, which gives the specified 0.90 g water on complete combustion.

Step 3: Conclusion The molar mass of X is $\boxed{100\,\mathrm{g\,mol^{-1}}}$. The numerical entry is 100.

Quick Tip to Solve: Convert water mass to hydrogen mass using the factor $2/18$ before applying the hydrogen percentage; water mass itself is not the hydrogen mass.

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72. The bond dissociation enthalpy of $\mathrm{X_2}\Delta\mathrm{H^\circ_{bond}}$ calculated from the given data is _____ $\mathrm{kJmol^{-1}}$. (Nearest integer)

$$\mathrm{M^+X^-(s) \rightarrow M^+(g) + X^-(g)}\Delta\mathrm{H^\circ_{lattice}} = 800\ \mathrm{kJ\ mol^{-1}}$$
$$\mathrm{M(\ s) \rightarrow M(\ g)}\Delta\mathrm{H^\circ_{sub}} = 100\ \mathrm{kJ\ mol^{-1}}$$

$$\mathrm{M(\ g) \rightarrow M^+(g) + e^-(g)}\Delta\mathrm{H^\circ_i} = 500\ \mathrm{kJ\ mol^{-1}}$$

$$\mathrm{X(\ g) + e^-(g) \rightarrow X^-(g)}\Delta\mathrm{H^\circ_{eg}} = -300\ \mathrm{kJ\ mol^{-1}}$$

$$\mathrm{M(\ s) + \frac{1}{2}X_2(\ g) \rightarrow M^+X^-(s)}\Delta\mathrm{H^\circ_f} = -400\ \mathrm{kJ\ mol^{-1}}$$

[Given : $\mathrm{M^+X^-}$ is a pure ionic compound and X forms a diatomic molecule $\mathrm{X_2}$ in gaseous state]

[4 marks]

Solution:

Concept:

- Hess's law makes the enthalpy of formation independent of the chosen path. A Born–Haber cycle resolves formation of an ionic solid into atomisation, ionisation, electron gain and lattice formation.
- The quoted lattice enthalpy is for dissociation into gaseous ions and is positive. Lattice formation is its reverse, with enthalpy $-800\,\mathrm{kJ\,mol^{-1}}$. Formation of one mole of MX requires only half a mole of $\mathrm{X_2}$.

Step 1: Write the formation cycle with the correct signs Let D be the bond dissociation enthalpy for $\mathrm{X_2(g) \longrightarrow 2\,X(g)}$, in $\mathrm{kJ\,mol^{-1}}$. The steps for one mole of solid are sublimation of M, ionisation of gaseous M, atomisation of half a mole of $\mathrm{X_2}$, electron gain by X, and assembly of the gaseous ions

into the lattice. Their enthalpies are, respectively,

$$100, \quad 500, \quad \frac{D}{2}, \quad -300, \quad -800.$$

Electron gain is exothermic as explicitly given; the lattice term must be reversed because the question supplies lattice dissociation rather than formation.

Step 2: Equate the cycle to the enthalpy of formation Adding the five contributions and setting the result equal to the given formation enthalpy gives

$$-400 = 100 + 500 + \frac{D}{2} - 300 - 800.$$

The known terms sum to -500, so

$$-400 = -500 + \frac{D}{2}, \qquad \frac{D}{2} = 100, \qquad D = 200\,\text{kJ mol}^{-1}.$$

Substitution checks the result: $100 + 500 + 100 - 300 - 800 = -400$, as required. The intermediate value 100 is the energy for dissociating half a mole of the diatomic gas, not its full molar bond dissociation enthalpy.

Step 3: Conclusion The bond dissociation enthalpy of X_2 is $\boxed{200\,\text{kJ mol}^{-1}}$. To the nearest integer, the required entry is 200.

Quick Tip to Solve: Write the direction of the lattice process and the coefficient of the diatomic gas before substituting: the common errors are using $+800$ or forgetting the factor $1/2$.

73. When 81.0 g of aluminium is allowed to react with 128.0 g of oxygen gas, the mass of aluminium oxide produced in grams is _____ . (Nearest integer)

Given :

Molar mass of Al is $27.0\ \text{g mol}^{-1}$

Molar mass of O is $16.0\ \text{g mol}^{-1}$ [4 marks]

Solution:

Concept:

- The balanced equation fixes mole ratios, not mass ratios. For aluminium oxide formation, 4 mol of Al react with 3 mol of O_2 to produce 2 mol of Al_2O_3.
- The limiting reagent is consumed first and determines the maximum product amount. Oxygen gas is diatomic, so its molar mass is $2(16.0) = 32.0\,\text{g mol}^{-1}$, whereas $M(Al_2O_3) = 102.0\,\text{g mol}^{-1}$.

Step 1: Convert the reactant masses to moles Balance the combination reaction before using the supplied masses:

$$4\,Al + 3\,O_2 \longrightarrow 2\,Al_2O_3.$$

The initial amounts are

$$n(\mathrm{Al}) = \frac{81.0}{27.0} = 3.00\,\mathrm{mol}, \qquad n(\mathrm{O_2}) = \frac{128.0}{32.0} = 4.00\,\mathrm{mol}.$$

The atomic molar mass of oxygen cannot be used directly for oxygen gas; doing so would double its calculated mole amount.

Step 2: Identify the limiting reagent Three moles of aluminium require only

$$3.00 \times \frac{3}{4} = 2.25\,\mathrm{mol\ O_2}.$$

Since 4.00 mol of oxygen is available, oxygen is in excess and aluminium is completely consumed. Equivalently, the stoichiometric quotients are $3/4 = 0.75$ for aluminium and $4/3$ for oxygen; the smaller quotient belongs to aluminium.

Step 3: Calculate the oxide mass From the balanced equation, four moles of aluminium give two moles of oxide. Therefore,

$$n(\mathrm{Al_2O_3}) = 3.00 \times \frac{2}{4} = 1.50\,\mathrm{mol},$$

$$m(\mathrm{Al_2O_3}) = 1.50 \times 102.0 = 153.0\,\mathrm{g}.$$

As a mass check, $2.25 \times 32 = 72$ g of oxygen is consumed, and $81 + 72 = 153$ g of oxide forms; the remaining 56 g of oxygen is unreacted.

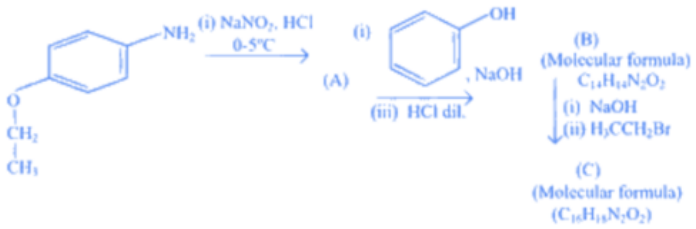
Step 4: Conclusion The mass of aluminium oxide produced is $\boxed{153\,\mathrm{g}}$. The nearest-integer entry is $\boxed{153}$.

Quick Tip to Solve: Compare moles divided by balanced coefficients to identify the limiting reagent; never choose it merely because its starting mass is smaller.

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74. Consider the following sequence of reactions.

Total number of sp^3 hybridised carbon atoms in the major product C formed is _____. [4 marks]



Solution:

Concept:

- A primary aromatic amine undergoes diazotisation with $\mathrm{NaNO_2/HCl}$ at 0–5 °C. The resulting aryl diazonium salt couples with phenol in alkaline medium to form an azo compound.
- Phenoxide preferentially undergoes azo coupling at its free para position. Subsequent treatment of a phenolic group with NaOH, then $\mathrm{CH_3CH_2Br}$, gives an ethoxy group by Williamson ether

synthesis.

Step 1: Identify the diazonium intermediate A The starting aromatic amine carries an ethoxy group para to its amino group. Nitrous acid generated from sodium nitrite and hydrochloric acid converts the amino group into $-N_2^+Cl-$, giving 4-ethoxybenzenediazonium chloride. This changes the nitrogen functionality but preserves the original $-OCH_2CH_3$ substituent and all its carbon atoms.

Step 2: Form the azo-coupled product B In alkaline solution, phenol forms phenoxide, which strongly activates its ortho and para positions. Its para position is free and is preferred for coupling with the diazonium ion. Acid work-up gives the neutral azo phenol:

$$C_2H_5O-C_6H_4-N=N-C_6H_4-OH.$$

Both substituents are para to the azo linkage on their respective rings. The formula is $C_{14}H_{14}N_2O_2$, consistent with the displayed formula of B. Each carbon in either benzene ring remains sp^2 hybridised.

Step 3: Ethylate the phenolic oxygen and count saturated carbons Sodium hydroxide converts the remaining phenolic group into phenoxide. It attacks the primary carbon of bromoethane, replacing bromide and forming an ether:

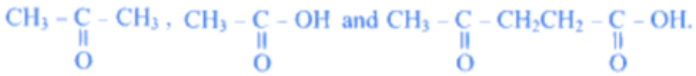
$$C_2H_5O-C_6H_4-N=N-C_6H_4-OC_2H_5.$$

Thus C contains two ethoxy groups. Each has one $O-CH_2$ carbon and one terminal CH_3 carbon; all four have four sigma bonds and tetrahedral, sp^3 hybridisation. The twelve aromatic carbons are sp^2, and the azo linkage adds no carbon atoms. The resulting formula, $C_{16}H_{18}N_2O_2$, also confirms addition of C_2H_4 to B.

Step 4: Conclusion The major product C has two ethoxy groups and therefore $2 \times 2 = \boxed{4}$ sp^3-hybridised carbon atoms. The numerical entry is $\boxed{4}$.

Quick Tip to Solve: Track the phenolic oxygen through Williamson ether synthesis: replacing its hydrogen by an ethyl group adds exactly two saturated carbon atoms.

75. A compound ' X′ absorbs 2 moles of hydrogen and ' X ' upon oxidation with $KMnO_4 \mid H^+$ gives The total number of σ bonds present in the compound ' X' ' is _____. [4 marks]



Solution:

Concept:

- Absorption of two moles of hydrogen indicates two reducible pi bonds here. Oxidative cleavage by hot acidic $KMnO_4$ converts an alkene carbon bearing hydrogen into a carboxylic acid; an alkene carbon bearing no hydrogen gives a ketone.
- Every single, double or triple bond contains exactly one sigma bond. For a connected open-chain hydrocarbon with n carbon atoms there are $n-1$ carbon–carbon sigma bonds, to which its

carbon–hydrogen bonds must be added.

Step 1: Read the carbonyl fragments The products shown are acetone, acetic acid and $CH_3COCH_2CH_2COOH$. Acetone identifies an original alkene carbon bonded to two methyl groups, namely $(CH_3)_2C=$. Acetic acid identifies the terminal alkene fragment $=CHCH_3$, since its alkene carbon originally carried a hydrogen and is oxidised to the acid. The third product has one methyl-ketone end and one acid end separated by two methylene groups. Its ketone carbon therefore originally had a methyl substituent and no hydrogen, while its acid carbon originally carried a hydrogen.

Step 2: Reconstruct the diene and its formula Joining the corresponding cleavage carbons restores an open-chain diene consistent with all three products:

$$(CH_3)_2C=CHCH_2CH_2C(CH_3)=CHCH_3.$$

Cleavage of its first double bond releases acetone; cleavage of its second releases acetic acid. The remaining middle fragment becomes $HOOCCH_2CH_2COCH_3$, exactly the third displayed product. The structure contains ten carbon atoms and two carbon–carbon double bonds, so complete hydrogenation consumes two moles of H_2 per mole of X. Its molecular formula is $C_{10}H_{18}$, using the acyclic diene relation C_nH_{2n-2}.

Step 3: Count each kind of sigma bond once The carbon skeleton is connected and has no ring, so its ten carbon atoms form nine carbon–carbon connections. Each connection contributes one sigma bond, including each of the two double bonds. Its eighteen hydrogen atoms contribute eighteen carbon–hydrogen sigma bonds. Therefore,

$$N_\sigma = N_{C-C,\sigma} + N_{C-H,\sigma} = 9 + 18 = 27.$$

The two additional bonds in the double bonds are pi bonds and must not be included in the sigma count. Counting them would give the total number of bonding components rather than the quantity asked for.

Step 4: Conclusion Compound X contains $\boxed{27}$ sigma bonds, so the required numerical entry is $\boxed{27}$.

Quick Tip to Solve: After finding an acyclic hydrocarbon formula, count sigma bonds as carbon atoms minus one plus hydrogen atoms; a double bond still contributes only one sigma bond.