JEE Main 2025 Paper 1 (B.E./B.Tech) Jan 28 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 28 January 2025, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $\mathrm{A}(x, y, z)$ be a point in xy-plane, which is equidistant from three points $(0, 3, 2), (2, 0, 3)$ and $(0, 0, 1)$.

Let $\mathrm{B} = (1, 4, -1)$ and $\mathrm{C} = (2, 0, -2)$. Then among the statements

(S1) : $\triangle\mathrm{ABC}$ is an isosceles right angled triangle, and

(S2) : the area of $\triangle\mathrm{ABC}$ is $\frac{9\sqrt{2}}{2}$, [4 marks]

(A) both are false

(B) only (S2) is true

(C) only (S1) is true

(D) both are true

Correct Answer: (C) only (S1) is true

Solution:

Concept:

- A point in the xy-plane has third coordinate zero. Equal distances can be compared using their squares, avoiding square roots.
- Two nonzero vectors are perpendicular when their dot product is zero. The area of a right triangle is half the product of its perpendicular sides.

Step 1: Find the point in the plane Write $A = (x, y, 0)$. Its squared distances from the three given points must be equal. Comparing each with the distance from $(0, 0, 1)$ gives

$$x^2 + (y-3)^2 + 4 = x^2 + y^2 + 1 \quad \Rightarrow \quad y = 2.$$

$$(x-2)^2 + y^2 + 9 = x^2 + y^2 + 1 \quad \Rightarrow \quad x = 3.$$

Therefore the required point is $A = (3, 2, 0)$; the condition of lying in the plane has removed the third unknown.

Step 2: Check the sides and the angle The vectors from A to the other two vertices are

$$\overrightarrow{AB} = (-2, 2, -1), \qquad \overrightarrow{AC} = (-1, -2, -2).$$

Their squared lengths are $4 + 4 + 1 = 9$ and $1 + 4 + 4 = 9$, respectively, so $AB = AC = 3$. Also,

$$\overrightarrow{AB} \cdot \overrightarrow{AC} = 2 - 4 + 2 = 0.$$

Thus the equal sides meet at a right angle at A, proving statement (S1). As a check, $BC^2 = 1+16+1 = 18 = AB^2 + AC^2$.

Step 3: Evaluate the area and compare the statements Use the two perpendicular sides, rather than the hypotenuse, as base and height:

$$\text{Area}(\triangle ABC) = \frac{1}{2}(3)(3) = \frac{9}{2}.$$

This is not $9\sqrt{2}/2$, so statement (S2) is false. Consequently the options claiming both statements true, both false, or only the area statement true are excluded.

Step 4: Conclusion Only (S1) is true; the area is $9/2$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Find the equal squared distances first, then use the dot product of the two equal sides to test a right angle.

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2. If $f(x) = \frac{2^x}{2^x+\sqrt{2}}$, x $\in \mathbb{R}$, then $\sum_{k=1}^{81} f\left(\frac{k}{82}\right)$ is equal to [4 marks]

(A) 82

(B) $81\sqrt{2}$

(C) 41

(D) $\frac{81}{2}$

Correct Answer: (D) $\frac{81}{2}$

Solution:

Concept:

- For a finite sum with arguments equally spaced between zero and one, pairing x with $1-x$ can give a constant.
- The identity $2^{1-x} = 2/2^x$ allows both terms of such a pair to be written using the same positive variable.

Step 1: Establish a complementary-argument identity Put $t = 2^x > 0$ and $s = \sqrt{2}$, so $s^2 = 2$. The given denominator is always positive, so every real argument used in the sum is allowed. Then

$$f(x) = \frac{t}{t+s}, \qquad f(1-x) = \frac{2}{2+st} = \frac{s}{s+t}.$$

Adding the fractions yields the exact identity

$$f(x) + f(1-x) = \frac{t+s}{t+s} = 1.$$

The complement is $1-x$, rather than $-x$, because the constant in the denominator is $2^{1/2}$.

Step 2: Pair the terms without losing the middle one Let $S = \sum_{k=1}^{81} f(k/82)$. The index substitution $k \mapsto 82-k$ permutes precisely the integers from 1 to 81. Thus the reversed sum has the same value, and

$$2S = \sum_{k=1}^{81} \left[f\left(\frac{k}{82}\right) + f\left(1 - \frac{k}{82}\right) \right] = 81.$$

Equivalently, there are 40 distinct pairs contributing 1 each and the central term $f(41/82) = f(1/2) = 1/2$. This explains why an odd number of terms does not give an integer total.

Step 3: Check the listed alternatives Therefore $S = 40 + 1/2 = 81/2$. Since each term lies strictly between zero and one, the total is less than 81, excluding 82 and $81\sqrt{2}$ immediately. The middle term

is one half, not one, so the alternative 41 also fails.

Step 4: Conclusion The required sum is $\frac{81}{2}$. Hence, the correct option is (D).

Quick Tip to Solve: When a denominator contains $2^x + 2^{1/2}$, test $f(x) + f(1-x)$ before evaluating individual terms.

3. The sum, of the squares of all the roots of the equation $x^2 + |2x - 3| - 4 = 0$, is [4 marks]

(A) $6(2 - \sqrt{2})$

(B) $3(3 - \sqrt{2})$

(C) $3(2 - \sqrt{2})$

(D) $6(3 - \sqrt{2})$

Correct Answer: (A) $6(2 - \sqrt{2})$

Solution:

Concept:

- The expression $|2x - 3|$ changes formula at $x = 3/2$. Each quadratic root must satisfy the inequality for the case that produced it.
- Only roots of the original absolute-value equation contribute to the requested sum; roots introduced by ignoring a case restriction must be discarded.

Step 1: Solve on the nonnegative branch For $x \geq 3/2$, replace $|2x - 3|$ by $2x - 3$. The equation becomes

$$x^2 + 2x - 7 = 0, \qquad x = -1 \pm 2\sqrt{2}.$$

The root $-1 + 2\sqrt{2}$ is approximately 1.828 and satisfies the branch condition. The other root is negative and hence cannot belong to this branch. We retain only $-1 + 2\sqrt{2}$.

Step 2: Solve on the negative branch For $x < 3/2$, replace $|2x - 3|$ by $3 - 2x$. This gives

$$x^2 - 2x - 1 = 0, \qquad x = 1 \pm \sqrt{2}.$$

Here $1 - \sqrt{2}$ is negative and meets the condition, whereas $1 + \sqrt{2} > 3/2$ does not. The boundary $x = 3/2$ is not a root, since its substitution in the original expression gives $9/4 - 4 = -7/4$. Thus no boundary root has been missed.

Step 3: Square only the accepted roots The two accepted roots are $-1 + 2\sqrt{2}$ and $1 - \sqrt{2}$. Therefore their squared sum is

$$\begin{aligned}(-1 + 2\sqrt{2})^2 + (1 - \sqrt{2})^2 &= (9 - 4\sqrt{2}) + (3 - 2\sqrt{2}) \\ &= 12 - 6\sqrt{2} = 6(2 - \sqrt{2}).\end{aligned}$$

The other listed expressions have different rational or radical coefficients and cannot equal this exact value. Applying a quadratic root-sum formula to either branch without filtering its roots would include an inadmissible value.

Step 4: Conclusion The sum of the squares of all roots is $6(2-\sqrt{2})$. Hence, the correct option is (A).

Quick Tip to Solve: For an absolute-value equation, write the branch inequality beside each quadratic and filter its roots before computing a sum.

4. The relation $R = \{(x, y) : x, y \in \mathbb{Z}$ and $x + y$ is even $\}$ is: [4 marks]

(A) reflexive and transitive but not symmetric

(B) reflexive and symmetric but not transitive

(C) an equivalence relation

(D) symmetric and transitive but not reflexive

Correct Answer: (C) an equivalence relation

Solution:

Concept:

- A relation is an equivalence relation precisely when it is reflexive, symmetric and transitive on its stated underlying set.
- For integers, $x+y$ is even exactly when x and y have the same parity. The relation therefore groups even integers together and odd integers together.

Step 1: Verify reflexivity on every integer Take any $x \in \mathbb{Z}$. To test reflexivity, the pair to check is (x, x). Its coordinate sum is

$$x + x = 2x,$$

which is even for every integer x, including negative integers and zero. Hence $(x, x) \in R$ for every element of the underlying set, so the relation is reflexive. The option which denies reflexivity is therefore false.

Step 2: Verify symmetry by reversing a related pair Suppose $(x, y) \in R$. By the definition of the relation, $x + y = 2m$ for some integer m. Since addition is commutative,

$$y + x = x + y = 2m,$$

so $(y, x) \in R$. This establishes symmetry for every related pair, rather than merely for a few examples. Consequently the option which says the relation is not symmetric is excluded.

Step 3: Verify transitivity with three integers Suppose $(x, y) \in R$ and $(y, z) \in R$. There are integers m, n with $x + y = 2m$ and $y + z = 2n$. Subtract the repeated middle contribution:

$$x + z = (x + y) + (y + z) - 2y = 2(m + n - y).$$

Because $m + n - y$ is an integer, $x + z$ is even and $(x, z) \in R$. Thus transitivity also holds; the alternative denying it is false. These checks establish all three required properties, with the two parity classes forming the equivalence classes.

Step 4: Conclusion The relation is reflexive, symmetric and transitive, and is therefore an equivalence relation. Hence, the correct option is (C).

Quick Tip to Solve: Recognise the condition of an even sum as equal parity; then prove transitivity by subtracting twice the middle integer.

5. Let $\langle a_n \rangle$ be a sequence such that $a_0 = 0, a_1 = \frac{1}{2}$ and $2a_{n+2} = 5a_{n+1} - 3a_n, n = 0, 1, 2, 3, \ldots$. Then $\sum_{k=1}^{100} a_k$ is equal to [4 marks]

(A) $3a_{100} + 100$

(B) $3a_{100} - 100$

(C) $3a_{99} - 100$

(D) $3a_{99} + 100$

Correct Answer: (B) $3a_{100} - 100$

Solution:

Concept:

- A linear recurrence can often be simplified by considering first differences. A geometric progression has sum $r(r^N - 1)/(r - 1)$ from powers 1 through N, for $r \neq 1$.
- The initial conditions determine the particular sequence; a relation between its sum and last term must preserve those initial values.

Step 1: Convert the recurrence into geometric differences Rearrange the given recurrence as

$$2(a_{n+2} - a_{n+1}) = 3(a_{n+1} - a_n).$$

With $d_n = a_{n+1} - a_n$, this reads $d_{n+1} = \frac{3}{2}d_n$, and $d_0 = a_1 - a_0 = 1/2$. Hence

$$d_n = \frac{1}{2}\left(\frac{3}{2}\right)^n.$$

Summing the first n differences, with $a_0 = 0$, gives

$$a_n = \frac{1}{2}\frac{(3/2)^n - 1}{(3/2) - 1} = \left(\frac{3}{2}\right)^n - 1.$$

For example, this gives $a_1 = 1/2$, confirming the starting condition.

Step 2: Sum the explicit expression Using the expression just obtained, write the requested sum as

$$\begin{aligned}\sum_{k=1}^{100} a_k &= \sum_{k=1}^{100}\left(\frac{3}{2}\right)^k - 100 \\ &= \frac{(3/2)((3/2)^{100} - 1)}{(3/2) - 1} - 100 \\ &= 3\left[\left(\frac{3}{2}\right)^{100} - 1\right] - 100.\end{aligned}$$

There are exactly 100 subtracted copies of 1; losing this count would give the wrong sign on the constant term.

Step 3: Express the result through the last term The quantity in square brackets is a_{100}, so the answer is $3a_{100} - 100$. The alternative with $+100$ has the opposite constant correction. Also $a_{100} - a_{99} =$

$\frac{1}{2}(3/2)^{99} \neq 0$, so replacing a_{100} by a_{99} does not preserve the derived sum. The explicit geometric formula therefore selects the stated last-term expression.

Step 4: Conclusion $\sum_{k=1}^{100} a_k = 3a_{100} - 100$. Hence, the correct option is (B).

Quick Tip to Solve: If the characteristic roots include 1, try first differences: here their ratio immediately becomes $3/2$.

6. $\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{33}{65}\right)$ **is equal to:** [4 marks]

(A) $\frac{33}{65}$

(B) 1

(C) $\frac{32}{65}$

(D) 0

Correct Answer: (D) 0

Solution:

Concept:

- The principal value of $\sin^{-1} t$ belongs to $[-\pi/2, \pi/2]$. For each positive fraction here, the corresponding angle is acute and has positive cosine.
- Use angle-addition identities with exact right-triangle values. Branch information is essential when recognising two angles as complementary.

Step 1: Compute the sine and cosine of the first two angles Let $\alpha = \sin^{-1}(3/5)$, $\beta = \sin^{-1}(5/13)$, and $\gamma = \sin^{-1}(33/65)$. All three angles are acute. Thus

$$\cos\alpha = \frac{4}{5}, \qquad \cos\beta = \frac{12}{13}.$$

The addition formula gives

$$\sin(\alpha+\beta) = \frac{3}{5}\frac{12}{13} + \frac{4}{5}\frac{5}{13} = \frac{56}{65},$$
$$\cos(\alpha+\beta) = \frac{4}{5}\frac{12}{13} - \frac{3}{5}\frac{5}{13} = \frac{33}{65}.$$

Both values are positive, and $0 < \alpha + \beta < \pi$, so their sum is actually acute.

Step 2: Combine with the third principal angle Since $\sin\gamma = 33/65$ and γ is acute,

$$\cos\gamma = \sqrt{1 - \left(\frac{33}{65}\right)^2} = \frac{56}{65},$$

using $65^2 - 33^2 = 56^2$. Therefore γ is the complement of the acute angle $\alpha + \beta$. Alternatively, apply the cosine addition formula directly:

$$\cos(\alpha+\beta+\gamma) = \frac{33\cdot 56 - 56\cdot 33}{65^2} = 0.$$

This exact cancellation avoids numerical inverse-trigonometric approximations.

Step 3: Select the value of the entire expression The requested expression is the cosine of all three angles together, not the cosine of only the first two. The value $33/65$ is the latter intermediate result and so is a distractor. The total angle is $\pi/2$, giving zero rather than 1 or $32/65$. The principal-angle check ensures that no different inverse-sine branch has been used.

Step 4: Conclusion The given expression equals 0. Hence, the correct option is (D).

Quick Tip to Solve: Compute both sine and cosine of the first angle sum; an exchanged pair of values identifies the complementary third angle.

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7. If $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{96x^2\cos^2 x}{(1+e^x)}dx = \pi\left(\alpha\pi^2+\beta\right), \alpha,\beta\in\mathbb{Z}$, then $(\alpha+\beta)^2$ equals [4 marks]

(A) 196

(B) 100

(C) 64

(D) 144

Correct Answer: (B) 100

Solution:

Concept:

- For an even numerator $g(x)$, substitution $x \mapsto -x$ pairs the factors $1/(1+e^x)$ and $1/(1+e^{-x})$, whose sum is 1.
- Use $\cos^2 x = (1+\cos 2x)/2$, and integration by parts to integrate $x^2\cos 2x$.

Step 1: Exploit symmetry before integrating Write the given integral as I, and put $a = \pi/2$. Substitution of $-x$ for x leaves $x^2\cos^2 x$ unchanged and gives a second expression for the same integral. Adding the expressions yields

$$2I = 96\int_{-a}^{a} x^2\cos^2 x\,dx, \qquad I = 96\int_{0}^{a} x^2\cos^2 x\,dx.$$

The exponential denominator disappears; directly integrating it would obscure this symmetry.

Step 2: Evaluate the remaining elementary integral The double-angle identity gives

$$I = 48\left[\frac{a^3}{3} + \int_0^a x^2\cos 2x\,dx\right].$$

Twice integrating by parts gives

$$\int x^2\cos 2x\,dx = \frac{x^2\sin 2x}{2} + \frac{x\cos 2x}{2} - \frac{\sin 2x}{4}.$$

At $a = \pi/2$, the sine terms vanish and $\cos 2a = -1$, so the definite integral in brackets is $-a/2$. Hence

$$I = 48\left(\frac{\pi^3}{24} - \frac{\pi}{4}\right) = \pi(2\pi^2 - 12).$$

Step 3: Identify the integer coefficients Comparison with $\pi(\alpha\pi^2 + \beta)$ gives $\alpha = 2$ and $\beta = -12$. Thus their sum is -10, whose square is 100. Values 196, 64 and 144 do not equal this square; in particular, the negative sign of the cosine boundary contribution must be retained.

Step 4: Conclusion $(\alpha + \beta)^2 = (2 - 12)^2 = 100$. Hence, the correct option is (B).

Quick Tip to Solve: On symmetric limits, first pair the values at x and $-x$; the logistic denominators then add to 1.

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8. Let $\mathrm{T_r}$ be the $\mathrm{r^{th}}$ term of an A.P. If for some $\mathrm{m}, \mathrm{T_m} = \frac{1}{25}$, $\mathrm{T_{25}} = \frac{1}{20}$, and $20\sum_{\mathrm{r=1}}^{25} \mathrm{T_r} = 13$, then $5\,\mathrm{m}\sum_{\mathrm{r=m}}^{2\,\mathrm{m}} \mathrm{T_r}$ is equal to [4 marks]

(A) 98

(B) 126

(C) 112

(D) 142

Correct Answer: (B) 126

Solution:

Concept:

- For an arithmetic progression with first term a and common difference d, $T_r = a + (r - 1)d$.
- The sum of a consecutive block is its number of terms multiplied by the average of its first and last terms. Both endpoints are included.

Step 1: Find the first term and common difference The supplied sum is $S_{25} = 13/20$, while $T_{25} = 1/20$. Therefore

$$\frac{25}{2}\left(a + \frac{1}{20}\right) = \frac{13}{20}, \qquad a + \frac{1}{20} = \frac{13}{250}.$$

Consequently $a = 1/500$. Now use $T_{25} = a + 24d$:

$$24d = \frac{1}{20} - \frac{1}{500} = \frac{24}{500}, \qquad d = \frac{1}{500}.$$

Thus the progression has the particularly simple general term $T_r = r/500$.

Step 2: Determine the unknown index The condition $T_m = 1/25$ becomes $m/500 = 1/25$, so $m = 20$. This agrees with the relative sizes of the terms: the twentieth term is smaller than the twenty-fifth because the common difference is positive. The required block therefore runs from index 20 through index 40, rather than from 1 through 40.

Step 3: Sum the block with both endpoints included There are $40 - 20 + 1 = 21$ terms. Hence

$$\sum_{r=m}^{2m} T_r = \frac{21}{2}\left(\frac{20}{500} + \frac{40}{500}\right) = \frac{63}{50}.$$

Multiplying by the requested factor gives

$$5m \sum_{r=m}^{2m} T_r = 100 \cdot \frac{63}{50} = 126.$$

The alternatives 98, 112 and 142 fail this direct calculation. Counting only 20 terms would incorrectly omit one endpoint.

Step 4: Conclusion The required value is 126. Hence, the correct option is (B).

Quick Tip to Solve: After finding $a = d$, replace T_r by rd; a block from m to $2m$ contains $m + 1$ terms.

9. Let ${}^nC_{r-1} = 28, {}^nC_r = 56$ and ${}^nC_{r+1} = 70$.
Let $A(4\cos t, 4\sin t)$, $B(2\sin t, -2\cos t)$ and $C\left(3r - n, r^2 - n - 1\right)$ be the vertices of a triangle ABC, where t is a parameter. If $(3x-1)^2 + (3y)^2 = \alpha$, is the locus of the centroid of triangle ABC , then α equals [4 marks]

(A) 18

(B) 8

(C) 20

(D) 6

Correct Answer: (C) 20

Solution:

Concept:

- Adjacent binomial coefficients satisfy $\binom{n}{r}/\binom{n}{r-1} = (n-r+1)/r$. Likewise, $\binom{n}{r+1}/\binom{n}{r} = (n-r)/(r+1)$.
- The centroid coordinates are the averages of the three vertex coordinates. Eliminate a trigonometric parameter using $\sin^2 t + \cos^2 t = 1$.

Step 1: Find the integers in the binomial coefficients Taking ratios avoids factorial expansion:

$$\frac{n-r+1}{r} = \frac{56}{28} = 2, \qquad \frac{n-r}{r+1} = \frac{70}{56} = \frac{5}{4}.$$

The first equation gives $n = 3r - 1$, while the second gives $4n = 9r + 5$. Substitution yields $12r - 4 = 9r + 5$, so $r = 3$ and $n = 8$. These satisfy the original values: $\binom{8}{2} = 28$, $\binom{8}{3} = 56$, and $\binom{8}{4} = 70$.

Step 2: Express the centroid in terms of the parameter The fixed vertex is

$$C = (3r - n, r^2 - n - 1) = (1, 0).$$

Writing the centroid as (x, y), the three vertices give

$$3x = 4\cos t + 2\sin t + 1, \qquad 3y = 4\sin t - 2\cos t.$$

Thus $3x - 1 = 4\cos t + 2\sin t$; the translation by 1 exactly removes the contribution of the fixed vertex.

Step 3: Eliminate the parameter Square and add the two equations:

$$\begin{aligned}(3x-1)^2 + (3y)^2 &= (4\cos t + 2\sin t)^2 \\ &\quad + (4\sin t - 2\cos t)^2 \\ &= 20\cos^2 t + 20\sin^2 t = 20.\end{aligned}$$

The mixed terms cancel because they have opposite signs. Therefore the locus has the requested constant 20. None of 18, 8 or 6 equals the sum of the squared sine and cosine coefficients.

Step 4: Conclusion $\alpha = 20$. Hence, the correct option is (C).

Quick Tip to Solve: Use ratios of consecutive binomial coefficients first; then square and add the shifted centroid coordinates to cancel mixed terms.

10. Two number k_1 and k_2 are randomly chosen from the set of natural numbers. Then, the probability that the value of $i^{k_1} + i^{k_2}, (i = \sqrt{-1})$ is non-zero, equals [4 marks]

(A) $\frac{3}{4}$
(B) $\frac{1}{2}$
(C) $\frac{1}{4}$
(D) $\frac{2}{3}$

Correct Answer: (A) $\frac{3}{4}$

Solution:

Concept:

- The powers of i repeat every four exponents: $i, -1, -i, 1$. Under random choice, the four residue classes modulo 4 have equal limiting frequency.
- Use the complement: a sum of two fourth roots of unity is zero precisely when the two terms are opposites. The two exponent choices are independent.

Step 1: Reduce the exponent choices modulo four Only the residues of k_1 and k_2 modulo 4 matter. For example, exponents congruent to 1 and 3 give i and $-i$, respectively, while residues 0 and 2 give 1 and -1. Equally likely residue pairs therefore form a sample space of $4 \times 4 = 16$ ordered possibilities. The exponents themselves need not be enumerated.

Step 2: Count the pairs whose sum vanishes For each first value there is exactly one opposite second value. The zero-sum ordered pairs of values are

$$(1, -1), \quad (-1, 1), \quad (i, -i), \quad (-i, i).$$

Thus four of the sixteen residue pairs give zero. Equivalently, the exponents must differ by 2 modulo 4. Equal residues give twice a nonzero fourth root, and the other residue differences give nonzero combinations of a real and an imaginary part.

Step 3: Take the complementary probability The zero-sum probability is $4/16 = 1/4$. Therefore

$$P(i^{k_1} + i^{k_2} \neq 0) = 1 - \frac{1}{4} = \frac{3}{4}.$$

The option $1/4$ is the probability of the complementary event. The value $1/2$ would incorrectly count two opposite partners for each first root, while $2/3$ does not arise from the sixteen equally likely ordered residue pairs. The usual random-natural-number statement is interpreted through these uniform residue frequencies.

Step 4: Conclusion The required probability is $3/4$. Hence, the correct option is (A).

Quick Tip to Solve: For periodic powers, work with exponent residues. Once the first root is fixed, exactly one of the four possible second roots cancels it.

11. Three defective oranges are accidently mixed with seven good ones and on looking at them, it is not possible to differentiate between them. Two oranges are drawn at random from the lot. If x denote the number of defective oranges, then the variance of x is [4 marks]

(A) $26/75$

(B) $14/25$

(C) $18/25$

(D) $28/75$

Correct Answer: (D) $28/75$

Solution:

Concept:

- Two oranges are selected without replacement from ten, of which three are defective. Thus the count X has a hypergeometric distribution, not a binomial distribution.
- Compute $\mathrm{Var}(X) = E(X^2) - [E(X)]^2$, using $P(X = j) = \binom{3}{j}\binom{7}{2-j}/\binom{10}{2}$.

Step 1: Write the probability distribution The total number of unordered pairs is $\binom{10}{2} = 45$. A pair has zero, one, or two defective oranges. Counting these cases gives

$$P(X = 0) = \frac{\binom{7}{2}}{45} = \frac{21}{45},$$

$$P(X = 1) = \frac{\binom{3}{1}\binom{7}{1}}{45} = \frac{21}{45}, \qquad P(X = 2) = \frac{\binom{3}{2}}{45} = \frac{3}{45}.$$

The numerators add to 45, confirming that all possible pairs have been counted exactly once.

Step 2: Calculate the first and second moments Multiply each possible value by its probability:

$$E(X) = \frac{0 \cdot 21 + 1 \cdot 21 + 2 \cdot 3}{45} = \frac{3}{5}.$$

For the second moment, square the values before weighting:

$$E(X^2) = \frac{0^2 \cdot 21 + 1^2 \cdot 21 + 2^2 \cdot 3}{45} = \frac{11}{15}.$$

The distinction between $E(X^2)$ and $[E(X)]^2$ is essential; the variance measures the difference.

Step 3: Subtract the square of the mean Therefore

$$\mathrm{Var}(X) = \frac{11}{15} - \left(\frac{3}{5}\right)^2 = \frac{55 - 27}{75} = \frac{28}{75}.$$

As a check, the hypergeometric formula gives $2(3/10)(7/10)(8/9) = 28/75$. The factor $8/9$ corrects for sampling without replacement. The other proposed fractions do not equal the variance obtained from this exact distribution.

Step 4: Conclusion $\mathrm{Var}(X) = 28/75$. Hence, the correct option is (D).

Quick Tip to Solve: For a small sample, list the possible defective counts and compute both moments; include the without-replacement correction if using a formula.

12. Let ABCD be a trapezium whose vertices lie on the parabola $y^2 = 4x$. Let the sides AD and BC of the trapezium be parallel to y-axis. If the diagonal AC is of length $\frac{25}{4}$ and it passes through the point $(1, 0)$, then the area of $ABCD$ is [4 marks]

(A) $\frac{75}{8}$
(B) $\frac{125}{8}$
(C) $\frac{25}{2}$
(D) $\frac{75}{4}$

Correct Answer: (D) $\frac{75}{4}$

Solution:

Concept:

- For $y^2 = 4x$, a point has coordinates $(t^2, 2t)$. A chord through the focus $(1, 0)$ has endpoint parameters t and $-1/t$.
- The area of a trapezium is half the sum of its parallel sides multiplied by the perpendicular separation between them. Here the parallel sides are vertical chords.

Step 1: Represent the focal chord endpoints The diagonal passes through the focus, so take

$$A = (t^2, 2t), \qquad C = (t^{-2}, -2/t), \qquad t \neq 0.$$

Indeed the chord joining parameters t, u has equation $2x - (t + u)y + 2tu = 0$; insertion of $(1, 0)$

requires $tu = -1$. Its length is

$$AC = \sqrt{(t^2 - t^{-2})^2 + 4(t + t^{-1})^2} = (t + t^{-1})^2.$$

The last equality follows by factoring out $(t + t^{-1})^2$ and using $(t - t^{-1})^2 + 4 = (t + t^{-1})^2$.

Step 2: Find the side sum and separation Put $s = |t + t^{-1}|$. The given diagonal length yields $s^2 = 25/4$, hence $s = 5/2$. Since t and $1/t$ have the same sign,

$$|t| + |t|^{-1} = \frac{5}{2}, \qquad |t - t^{-1}| = \sqrt{\frac{25}{4} - 4} = \frac{3}{2}.$$

The other endpoint on each vertical chord has the opposite ordinate. Thus $AD = 4|t|$ and $BC = 4/|t|$, giving $AD + BC = 10$. Their horizontal separation is

$$|t^2 - t^{-2}| = |t + t^{-1}|\,|t - t^{-1}| = \frac{15}{4}.$$

Step 3: Apply the trapezium area formula The area is therefore

$$\frac{1}{2}(AD + BC)|x_A - x_C| = \frac{1}{2} \cdot 10 \cdot \frac{15}{4} = \frac{75}{4}.$$

The diagonal length is not the perpendicular distance between the vertical sides; using it as the height would be incorrect. The values $75/8$, $125/8$ and $25/2$ fail the side-and-separation calculation.

Step 4: Conclusion The area of $ABCD$ is $75/4$ square units. Hence, the correct option is (D).

Quick Tip to Solve: A diagonal through the focus suggests parameters $t, -1/t$. Extract $|t + 1/t|$ from its length before finding the vertical side sum.

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13. If the image of the point $(4, 4, 3)$ in the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-1}{3}$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to [4 marks]

(A) 12

(B) 9

(C) 7

(D) 8

Correct Answer: (B) 9

Solution:

Concept:

- The foot of the perpendicular from a point to a line is its orthogonal projection. If the line is $\mathbf{r} = \mathbf{A} + t\mathbf{d}$, its parameter is $t = ((\mathbf{P} - \mathbf{A}) \cdot \mathbf{d})/(\mathbf{d} \cdot \mathbf{d})$.
- For reflection in a line, the original point and its image lie on opposite sides of the perpendicular foot, which is their midpoint. Thus $\mathbf{P}' = 2\mathbf{H} - \mathbf{P}$.

Step 1: Find the perpendicular foot Take $\mathbf{A} = (1, 2, 1)$, $\mathbf{d} = (2, 1, 3)$ and $\mathbf{P} = (4, 4, 3)$. Then

$\mathbf{P}-\mathbf{A}=(3,2,2)$, so

$$t=\frac{3(2)+2(1)+2(3)}{2^2+1^2+3^2}=\frac{14}{14}=1.$$

Therefore the foot is $\mathbf{H}=(1,2,1)+(2,1,3)=(3,3,4)$. As a check, $(\mathbf{P}-\mathbf{H})\cdot\mathbf{d}=(1,1,-1)\cdot(2,1,3)=0$, as required.

Step 2: Reflect the point and add its coordinates The image is obtained by moving the same distance beyond the foot:

$$(\alpha,\beta,\gamma)=2(3,3,4)-(4,4,3)=(2,2,5).$$

Consequently,

$$\alpha+\beta+\gamma=2+2+5=9.$$

The sum of the coordinates of the foot is 10, whereas the original sum is 11; reflection gives $2(10)-11=9$ independently. The alternatives 12, 7 and 8 do not equal this reflected coordinate sum.

Step 3: Conclusion The image is $(2,2,5)$, and $\alpha+\beta+\gamma=9$. Hence, the correct option is (B).

Quick Tip to Solve: Project onto the line first; then use image = twice the foot minus the original point. Do not confuse the foot with the reflected image.

14. Let for some function $y=f(x), \int_0^x tf(t)dt=x^2f(x), x>0$ **and** $f(2)=3$. **Then** $f(6)$ **is equal to** [4 marks]

(A) 1

(B) 6

(C) 2

(D) 3

Correct Answer: (A) 1

Solution:

Concept:

- The fundamental theorem of calculus gives $\frac{d}{dx}\int_0^x tf(t)\,dt=xf(x)$. The right side must be differentiated by the product rule.
- After differentiation, the resulting equation can be written $(xf(x))'=0$, so $xf(x)$ is constant on $x>0$. The given value determines that constant.

Step 1: Differentiate the stated identity For every $x>0$, the equation is

$$\int_0^x tf(t)\,dt=x^2f(x).$$

Differentiating both sides gives

$$xf(x)=2xf(x)+x^2f'(x).$$

Since $x > 0$, division by x is valid. Hence

$$xf'(x) + f(x) = 0, \qquad \frac{d}{dx}[xf(x)] = 0.$$

Therefore $xf(x) = C$, or $f(x) = C/x$. This step uses a product derivative, so it avoids any unnecessary logarithmic sign restrictions.

Step 2: Use the given value and verify the integral The condition $f(2) = 3$ yields $C = 2(3) = 6$. Therefore

$$f(6) = \frac{6}{6} = 1.$$

The differentiated solution also satisfies the original integral: for $t > 0$, $tf(t) = 6$, and so

$$\int_0^x tf(t)\,dt = 6x = x^2\frac{6}{x}.$$

The endpoint value at zero does not affect this integral. The options 6, 2 and 3 confuse the constant, another input, or the initial value with the requested value at six.

Step 3: Conclusion We obtain $f(x) = 6/x$ for $x > 0$, so $f(6) = 1$. Hence, the correct option is (A).

Quick Tip to Solve: After differentiating an integral equation, check the answer in the original identity; differentiation can otherwise leave an unwanted constant.

15. The sum of all local minimum values of the function

$$f(x) = \begin{cases} 1 - 2x, & x < -1, \\ \frac{1}{3}(7 + 2|x|), & -1 \le x \le 2, \\ \frac{11}{18}(x-4)(x-5), & x > 2 \end{cases}$$

is [4 marks]

(A) $\frac{167}{72}$
(B) $\frac{157}{72}$
(C) $\frac{171}{72}$
(D) $\frac{131}{72}$

Correct Answer: (B) $\frac{157}{72}$

Solution:

Concept:

- A local minimum value is the function value at a point where nearby values are no smaller. For a piecewise function, check stationary points, absolute-value corners and all junctions.
- The quadratic $\frac{11}{18}(x-4)(x-5)$ opens upwards. Its vertex lies halfway between its roots, at $x = 9/2$. On the middle interval the minimum of $|x|$ occurs at zero.

Step 1: Examine the linear pieces and their junctions For $x < -1$, $f(x) = 1 - 2x$ decreases strictly,

so it has no interior local minimum. On $[-1, 0]$, $f(x) = (7 - 2x)/3$ also decreases, while on $[0, 2]$, $f(x) = (7 + 2x)/3$ increases. Thus zero is a local minimum and

$$f(0) = \frac{7}{3}.$$

At $x = -1$, both adjacent formulas give 3 and both sides are decreasing through the junction; this is not an extremum. At $x = 2$, both formulas give $11/3$, but the function increases immediately to the left and decreases immediately to the right. Thus this junction is a local maximum, not a minimum.

Step 2: Find the quadratic minimum and add the values For $x > 2$, completing the square gives

$$f(x) = \frac{11}{18}\left[\left(x - \frac{9}{2}\right)^2 - \frac{1}{4}\right].$$

Since $9/2 > 2$, the vertex lies inside this branch and is a local minimum. Its value is

$$f\left(\frac{9}{2}\right) = -\frac{11}{72}.$$

There are no further stationary points or junctions. The requested sum of minimum values, rather than their input locations, is

$$\frac{7}{3} - \frac{11}{72} = \frac{168 - 11}{72} = \frac{157}{72}.$$

The other listed numerators do not follow from adding these two values; including the value at a junction maximum would be incorrect.

Step 3: Conclusion The local minimum values are $7/3$ and $-11/72$, whose sum is $157/72$. Hence, the correct option is (B).

Quick Tip to Solve: For piecewise extrema, list branch interiors and junctions separately. Add the minimum heights, not the x-coordinates where they occur.

16. The number of different 5 digit numbers greater than 50000 that can be formed using the digits 0 , $1, 2, 3, 4, 5, 6, 7$, such that the sum of their first and last digits should not be more than 8 , is [4 marks]

(A) 5720

(B) 5719

(C) 4608

(D) 4607

Correct Answer: (D) 4607

Solution:

Concept:

- The multiplication principle counts independent choices of digit positions, and mutually exclusive leading-digit cases are added. Repetition is permitted because the question does not prohibit repeating digits.

- A five-digit number greater than 50000 must start with 5, 6 or 7 when the available digits are 0 through 7. The first-plus-last restriction then determines how many units digits each case allows.

Step 1: Count the permitted endpoint pairs Let the first digit be a and last digit be e. The condition is $a + e \leq 8$, with $e \in \{0, 1, \ldots, 7\}$. If $a = 5$, then $e = 0, 1, 2, 3$, giving four choices. If $a = 6$, then $e = 0, 1, 2$, giving three choices. If $a = 7$, then $e = 0, 1$, giving two choices. Thus there are $4+3+2 = 9$ admissible ordered endpoint pairs. The order matters because the leading and units digits occupy distinct positions.

Step 2: Choose the middle digits and enforce the strict inequality Each of the three middle positions independently permits all eight digits. Therefore the endpoint count initially gives

$$9 \times 8^3 = 9 \times 512 = 4608.$$

This includes exactly one number that fails the strict lower bound: 50000. Its first digit is 5, its last digit is 0, and all three middle digits are zero. Remove it once:

$$4608 - 1 = 4607.$$

Option (C), 4608, would be correct for a lower bound of at least 50000. The counts 5720 and 5719 do not correspond to the nine allowed endpoint pairs, with repetition in the middle positions.

Step 3: Conclusion The number of admissible integers strictly greater than 50000 is 4607. Hence, the correct option is (D).

Quick Tip to Solve: First count allowed first-and-last pairs, then fill the middle digits. When a bound is strict, check whether the boundary number was counted.

---

17. Let the equation of the circle, which touches x-axis at the point $(a, 0), a > 0$ and cuts off an intercept of length b on $y - axis$ be $x^2 + y^2 - \alpha x + \beta y + \gamma = 0$. If the circle lies below $x - axis$, then the ordered pair $(2a, b^2)$ is equal to [4 marks]

(A) $(\alpha, \beta^2 + 4\gamma)$
(B) $(\alpha, \beta^2 - 4\gamma)$
(C) $(\gamma, \beta^2 - 4\alpha)$
(D) $(\gamma, \beta^2 + 4\alpha)$

Correct Answer: (B) $(\alpha, \beta^2 - 4\gamma)$

Solution:

Concept:

- For $x^2 + y^2 - \alpha x + \beta y + \gamma = 0$, the centre is $(\alpha/2, -\beta/2)$. The point of tangency with the x-axis lies vertically above or below the centre.
- To find the intercept on the y-axis, put $x = 0$. If the resulting quadratic has roots y_1, y_2, the intercept length is $|y_1 - y_2|$, whose square equals its discriminant.

Step 1: Identify the tangency coordinate The circle touches the x-axis at $(a, 0)$. The radius to a point

of tangency is perpendicular to the tangent, so the centre has x-coordinate a. Comparing with the standard centre yields

$$a = \frac{\alpha}{2}, \qquad 2a = \alpha.$$

Because the circle lies below the axis, its centre has negative y-coordinate, so $\beta > 0$, and the radius equals $\beta/2$. Substituting $(a, 0)$ in the circle also gives $\gamma = a^2$, consistent with the tangency condition.

Step 2: Compute the squared y-axis intercept At the y-axis, $x = 0$, and the circle equation becomes

$$y^2 + \beta y + \gamma = 0.$$

Let its two intersection coordinates be y_1 and y_2. Their sum is $-\beta$ and product is γ. Therefore

$$b^2 = (y_1 - y_2)^2 = (y_1 + y_2)^2 - 4y_1y_2 = \beta^2 - 4\gamma.$$

The fact that both intersections lie below the x-axis does not change their positive separation. A plus sign before 4γ would contradict the root-difference identity, and γ cannot replace α as twice the tangency coordinate.

Step 3: Conclusion The ordered pair is $(2a, b^2) = (\alpha, \beta^2 - 4\gamma)$. Hence, the correct option is (B).

Quick Tip to Solve: An axis-intercept length is the difference between two quadratic roots; its square is the discriminant, not the sum of the roots squared alone.

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18. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = (2 + 3a)x^2 + \left(\frac{a+2}{a-1}\right)x + b, a \neq 1$. If $f(x + y) = f(x) + f(y) + 1 - \frac{2}{7}xy$, then the value of $28\sum_{i=1}^{5} |f(i)|$ is [4 marks]

(A) 735

(B) 675

(C) 715

(D) 545

Correct Answer: (B) 675

Solution:

Concept:

- For a quadratic $f(x) = Ax^2 + Bx + b$, the expression $f(x+y) - f(x) - f(y)$ simplifies to $2Axy - b$; all linear terms cancel.
- Compare coefficients in an identity valid for all real x and y. After determining the polynomial, check its sign at the required positive integers before removing the absolute-value signs.

Step 1: Compare the constant and mixed terms Put $A = 2 + 3a$ and $B = (a + 2)/(a - 1)$. Expanding the given relation gives

$$2Axy - b = 1 - \frac{2}{7}xy.$$

Comparison yields $A = -1/7$ and $b = -1$. Therefore

$$2 + 3a = -\frac{1}{7}, \quad a = -\frac{5}{7}, \quad B = \frac{9/7}{-12/7} = -\frac{3}{4}.$$

The restriction $a \neq 1$ is satisfied. Thus

$$f(x) = -\frac{x^2}{7} - \frac{3x}{4} - 1.$$

In particular, substituting zero into the original relation gives $f(0) = -1$, providing an immediate check on the constant term.

Step 2: Remove the absolute values and evaluate the sum For each positive integer i, every term of $f(i)$ is negative. Hence

$$|f(i)| = \frac{i^2}{7} + \frac{3i}{4} + 1.$$

Using $\sum_{i=1}^{5} i = 15$ and $\sum_{i=1}^{5} i^2 = 55$, obtain

$$\begin{aligned} 28\sum_{i=1}^{5} |f(i)| &= 28\left(\frac{55}{7} + \frac{45}{4} + 5\right) \\ &= 220 + 315 + 140 = 675. \end{aligned}$$

All three contributions are positive after taking absolute values. The other listed totals cannot be obtained from these coefficient comparisons and the two standard finite sums.

Step 3: Conclusion The required value is $28\sum_{i=1}^{5} |f(i)| = 675$. Hence, the correct option is (B).

Quick Tip to Solve: For a quadratic functional equation, compute f(x+y)-f(x)-f(y) symbolically first. Linear terms cancel, leaving the mixed coefficient and constant immediately.

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19. Let O be the origin, the point A be $z_1 = \sqrt{3} + 2\sqrt{2}i$, the point $B\,(z_2)$ be such that $\sqrt{3}\,|z_2| = |z_1|$ and $\arg(z_2) = \arg(z_1) + \frac{\pi}{6}$. Then [4 marks]

(A) area of triangle ABO is $\frac{11}{4}$

(B) area of triangle ABO is $\frac{11}{\sqrt{3}}$

(C) ABO is a scalene triangle

(D) ABO is an obtuse angled isosceles triangle

Correct Answer: (D) ABO is an obtuse angled isosceles triangle

Solution:

Concept:

- The modulus of a complex number is its distance from the origin, and the difference of arguments gives the included angle between its position vectors.
- For two sides a, b enclosing an angle θ, the cosine rule gives the third side squared as $a^2 + b^2 - 2ab\cos\theta$. Equal side lengths establish an isosceles triangle.

Step 1: Find the two radii and included angle The point A represents $z_1 = \sqrt{3} + 2\sqrt{2}\,i$, so

$$OA = |z_1| = \sqrt{3+8} = \sqrt{11}.$$

The given modulus relation and argument relation imply

$$OB = |z_2| = \frac{\sqrt{11}}{\sqrt{3}}, \qquad \angle AOB = \frac{\pi}{6}.$$

The argument difference is the included angle; its value does not depend on the separate arguments of the two points.

Step 2: Classify the triangle and check the areas Apply the cosine rule to the side opposite the angle at the origin:

$$AB^2 = 11 + \frac{11}{3} - 2\sqrt{11}\sqrt{\frac{11}{3}}\frac{\sqrt{3}}{2} = \frac{11}{3}.$$

Thus $AB = OB$; the triangle is isosceles, so the angles opposite these equal sides, at O and A, are each 30°. Consequently

$$\angle ABO = 180^\circ - 30^\circ - 30^\circ = 120^\circ.$$

For comparison, its area is

$$\frac{1}{2}OA\,OB\sin 30^\circ = \frac{11}{4\sqrt{3}}.$$

This is neither area stated in (A) or (B), and the equal sides rule out the scalene description in (C).

Step 3: Conclusion $AB = OB = \sqrt{11/3}$ and $\angle ABO = 120^\circ$, so ABO is an obtuse angled isosceles triangle. Hence, the correct option is (D).

Quick Tip to Solve: Use the argument difference as the angle at the origin, then apply the cosine rule before deciding the triangle type.

20. The area (in sq. units) of the region $\{(x, y) : 0 \le y \le 2|x| + 1, 0 \le y \le x^2 + 1, |x| \le 3\}$ is [4 marks]

(A) $\frac{32}{3}$
(B) $\frac{64}{3}$
(C) $\frac{17}{3}$
(D) $\frac{80}{3}$

Correct Answer: (B) $\frac{64}{3}$

Solution:

Concept:

- When a region satisfies two upper bounds on y, its upper boundary is their minimum, not their sum. Its area is the integral of that minimum above the lower boundary $y = 0$.
- Both $2|x| + 1$ and $x^2 + 1$ are even functions. The region is symmetric about the y-axis, so its area is twice the area in $0 \le x \le 3$.

Step 1: Locate the change in upper boundary For $x \geq 0$, compare the two upper bounds using

$$(x^2+1)-(2x+1)=x(x-2).$$

Their intersections in this interval are at $x=0$ and $x=2$. On $0 \leq x \leq 2$, the difference is nonpositive, so the parabola is the lower upper bound. On $2 \leq x \leq 3$, the line is the lower upper bound. The common lower boundary throughout is $y=0$.

Step 2: Integrate the lower upper bound on each interval The area on the positive side is therefore

$$\begin{aligned} A_+ &= \int_0^2 (x^2+1)\,dx + \int_2^3 (2x+1)\,dx \\ &= \left[\frac{x^3}{3}+x\right]_0^2 + \left[x^2+x\right]_2^3 \\ &= \frac{8}{3}+2+(12-6) = \frac{32}{3}. \end{aligned}$$

Reflection across the y-axis contributes an equal area, giving $A = 2A_+ = 64/3$. The value $32/3$ in (A) counts only one half. Options (C) and (D) do not equal the sum of these two required integrals and their reflected contribution.

Step 3: Conclusion The required area is $\boxed{64/3}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: For simultaneous inequalities below two curves, integrate the lower curve; compare their difference first and use symmetry only after locating the switch.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let M denote the set of all real matrices of order 3×3 and let $S = \{-3,-2,-1,1,2\}$. Let

$$\begin{aligned} S_1 &= \left\{A=[a_{ij}] \in M : A = A^T \text{ and } a_{ij} \in S, \forall i,j\right\}, \\ S_2 &= \left\{A=[a_{ij}] \in M : A = -A^T \text{ and } a_{ij} \in S, \forall i,j\right\}, \\ S_3 &= \left\{A=[a_{ij}] \in M : a_{11}+a_{22}+a_{33}=0 \text{ and } a_{ij} \in S, \forall i,j\right\}. \end{aligned}$$

If $n(S_1 \cup S_2 \cup S_3) = 125\alpha$, then α equls _____. [4 marks]

Solution:

Concept:

- A symmetric 3×3 matrix has six independent entries: three diagonal entries and three entries above the diagonal. A skew-symmetric real matrix has all diagonal entries zero.
- Count trace-zero matrices by first counting their ordered diagonal triples. The six off-diagonal entries are then independent. Inclusion-exclusion prevents counting symmetric trace-zero matrices twice.

Step 1: Count symmetric and skew-symmetric matrices Each independent entry of a symmetric

matrix can be chosen from the five-element set S, so

$$n(S_1) = 5^6 = 15625.$$

For a real skew-symmetric matrix, $a_{ii} = -a_{ii}$ forces $a_{ii} = 0$. Since zero is not in $S = \{-3, -2, -1, 1, 2\}$, such a matrix cannot satisfy the entry restriction. Hence S_2 is empty.

Step 2: Enumerate the permitted trace-zero diagonals We need ordered triples from S whose sum is zero. Ignoring their order, the possibilities are exactly

$$(-3, 1, 2), \qquad (-2, 1, 1), \qquad (-1, -1, 2).$$

For one negative entry, the positive entries must be either $1, 1$ or $1, 2$; for two negative entries, only $-1, -1, 2$ works. Three entries of the same sign cannot sum to zero. The first triple has $3! = 6$ permutations; each remaining triple has $3!/2! = 3$. Thus there are $6 + 3 + 3 = 12$ ordered diagonals. The six off-diagonal entries are unrestricted, giving

$$n(S_3) = 12 \cdot 5^6.$$

Step 3: Subtract the intersection and extract the parameter A matrix in $S_1 \cap S_3$ has one of these twelve diagonal triples and three independently chosen upper-triangular entries; symmetry fixes the lower entries. Therefore

$$n(S_1 \cap S_3) = 12 \cdot 5^3.$$

Since S_2 is empty, inclusion-exclusion yields

$$\begin{aligned} n(S_1 \cup S_2 \cup S_3) &= 5^6 + 12 \cdot 5^6 - 12 \cdot 5^3 \\ &= 13 \cdot 15625 - 1500 = 201625. \end{aligned}$$

Equating this to 125α gives $\alpha = 201625/125 = 1613$.

Step 4: Conclusion The required numerical value is $\boxed{1613}$.

Quick Tip to Solve: Check whether zero belongs to the allowed entry set before counting skew-symmetric matrices; then separate diagonal constraints from free off-diagonal choices.

22. Let

$$f(x) = \begin{cases} 3x, & x < 0, \\ \min\{1 + x + [x],\ x + 2[x]\}, & 0 \le x \le 2, \\ 5, & x > 2. \end{cases}$$

where $[\cdot]$ denotes the greatest integer function. If α and β are the number of points where f is not continuous and is not differentiable, respectively, then $\alpha + \beta$ equals _____. [4 marks]

Solution:

Concept:

- The greatest integer function is constant between consecutive integers, so the minimum can be simplified separately on each such interval.
- Continuity requires the left limit, right limit and function value to agree. Differentiability requires continuity as well as equal one-sided derivatives; every jump is automatically a point of nondifferentiability.

Step 1: Simplify the middle branch For $0 \le x < 1$, we have $[x] = 0$, giving $\min\{1 + x, x\} = x$. For $1 \le x < 2$, we have $[x] = 1$, so both expressions equal $x + 2$. At $x = 2$, the integer part is two and the minimum is $\min\{5, 6\} = 5$. Thus

$$f(x) = \begin{cases} 3x, & x < 0, \\ x, & 0 \le x < 1, \\ x + 2, & 1 \le x < 2, \\ 5, & x \ge 2. \end{cases}$$

Every open interval displayed here has a linear or constant function, so only $0, 1, 2$ need to be checked.

Step 2: Check continuity at all junctions At zero, both one-sided limits and the function value are zero, so the function is continuous. At one,

$$\lim_{x \to 1^-} f(x) = 1, \qquad f(1) = \lim_{x \to 1^+} f(x) = 3,$$

which gives a jump. At two,

$$\lim_{x \to 2^-} f(x) = 4, \qquad f(2) = \lim_{x \to 2^+} f(x) = 5,$$

which gives another jump. Hence precisely two points are discontinuities, and $\alpha = 2$.

Step 3: Count the nondifferentiable points At zero, the left derivative is 3 and the right derivative is 1. The function is continuous there but has a corner, so it is not differentiable. At one and two it is not differentiable because it is not continuous. At every other real point it is represented locally by one fixed linear or constant formula and hence is differentiable. Therefore

$$\beta = 3, \qquad \alpha + \beta = 2 + 3 = 5.$$

Step 4: Conclusion There are two discontinuities and three points of nondifferentiability. The required

sum is $\boxed{5}$.

Quick Tip to Solve: After resolving each floor-function interval, check junction values separately; a continuous corner contributes only to nondifferentiability, while a jump contributes to both counts.

23. If $\alpha = 1 + \sum_{r=1}^{6} (-3)^{r-1} \ {}^{12}C_{2r-1}$, then the distance of the point $(12, \sqrt{3})$ from the line $\alpha x - \sqrt{3}y + 1 = 0$ is _____. [4 marks]

Solution:

Concept:

- The imaginary part of $(1 + i\sqrt{3})^{12}$ collects exactly its odd-power binomial terms. Since $(i\sqrt{3})^{2r-1} = i\sqrt{3}(-3)^{r-1}$, it produces the sum in the question.
- The distance of (x_0, y_0) from $ax + by + c = 0$ is $|ax_0 + by_0 + c|/\sqrt{a^2 + b^2}$. The absolute value makes the distance nonnegative.

Step 1: Identify the binomial sum as an imaginary part Write

$$T = \sum_{r=1}^{6} (-3)^{r-1} \binom{12}{2r-1}.$$

Expanding $(1 + i\sqrt{3})^{12}$, all even powers of $i\sqrt{3}$ are real. The odd powers give

$$\mathrm{Im}(1 + i\sqrt{3})^{12} = \sqrt{3} \sum_{r=1}^{6} (-3)^{r-1} \binom{12}{2r-1} = \sqrt{3}\, T.$$

This isolates exactly the terms needed without calculating six large binomial coefficients separately.

Step 2: Evaluate the sum using polar form The complex number $1 + i\sqrt{3}$ has modulus two and argument $\pi/3$. By De Moivre's theorem,

$$(1 + i\sqrt{3})^{12} = 2^{12} \left(\cos 4\pi + i \sin 4\pi\right) = 4096.$$

Its imaginary part is zero, hence $\sqrt{3}\, T = 0$ and $T = 0$. The parameter defined in the question is therefore $\alpha = 1 + T = 1$.

Step 3: Apply the point-to-line distance formula Substituting $\alpha = 1$ reduces the line to $x - \sqrt{3}y + 1 = 0$. For the point $(12, \sqrt{3})$, the numerator of the distance is

$$|12 - \sqrt{3} \cdot \sqrt{3} + 1| = |12 - 3 + 1| = 10.$$

The denominator is $\sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{4} = 2$. Consequently the required perpendicular distance is $10/2 = 5$.

Step 4: Conclusion The distance of the point from the given line is $\boxed{5}$ units.

Quick Tip to Solve: An alternating sum of odd binomial coefficients often comes from the imaginary part of a complex binomial; choose a complex number whose square introduces the alternating factor.

24. Let $\mathrm{E}_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ be an ellipse. Ellipses E_i 's are constructed such that their centres and eccentricities are same as that of E_1, and the length of minor axis of E_i is the length of major axis of $E_{i+1}(i \geq 1)$. If A_i is the area of the ellipse E_i, then $\frac{5}{\pi}\left(\sum_{i=1}^{\infty} A_i\right)$, is equal to _____. [4 marks]

Solution:

Concept:

- For an ellipse with semimajor axis a and semiminor axis b, $e^2 = 1 - b^2/a^2$ and its area is πab. Equal eccentricities imply the same ratio b/a.
- If successive ellipses are similar with linear scale factor k, their areas have ratio k^2. An infinite geometric series of first term A_1 and ratio r, with $|r| < 1$, has sum $A_1/(1-r)$.

Step 1: Determine the common shape and linear scaling For $E_1 : x^2/9 + y^2/4 = 1$, the semiaxes are $a_1 = 3$ and $b_1 = 2$. Thus $b_1/a_1 = 2/3$, and every subsequent ellipse, having the same eccentricity, satisfies

$$\frac{b_i}{a_i} = \frac{2}{3}.$$

The given equality of the minor axis of E_i and major axis of E_{i+1} means

$$2b_i = 2a_{i+1}, \quad \text{so} \quad a_{i+1} = b_i = \frac{2}{3}a_i.$$

The factor of two belongs to both full axis lengths and cancels; it must not become an extra scaling factor.

Step 2: Find the area ratio and sum the series Since $b_{i+1} = (2/3)a_{i+1} = (2/3)b_i$, both semiaxes shrink by $2/3$. Consequently

$$\frac{A_{i+1}}{A_i} = \frac{\pi a_{i+1} b_{i+1}}{\pi a_i b_i} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}.$$

The first area is $A_1 = \pi \cdot 3 \cdot 2 = 6\pi$. The positive area series converges because $4/9 < 1$, and

$$\sum_{i=1}^{\infty} A_i = \frac{6\pi}{1 - 4/9} = \frac{54\pi}{5}.$$

Finally, multiplying by the factor requested in the question gives

$$\frac{5}{\pi}\sum_{i=1}^{\infty} A_i = \frac{5}{\pi} \cdot \frac{54\pi}{5} = 54.$$

Step 3: Conclusion The required numerical value is $\boxed{54}$.

Quick Tip to Solve: For a chain of equal-eccentricity ellipses, find the semiaxis scale factor first and square it to get the area ratio.

25. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+2\hat{j}+\hat{k}$ and $\vec{d}=\vec{a}\times\vec{b}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c}=|\vec{c}|$, $|\vec{c}-2\vec{a}|^2=8$ and the angle between $\vec{d}$ and $\vec{c}$ is $\frac{\pi}{4}$, then $|10-3\vec{b}\cdot\vec{c}|+|\vec{d}\times\vec{c}|^2$ is equal to _____. [4 marks]

Solution:

Concept:

- For vectors, $|\vec{c}-2\vec{a}|^2=|\vec{c}|^2-4\vec{a}\cdot\vec{c}+4|\vec{a}|^2$. Here $|\vec{a}|^2=3$.
- The dot and cross products satisfy $|\vec{d}\times\vec{c}|^2=|\vec{d}|^2|\vec{c}|^2-(\vec{d}\cdot\vec{c})^2$. The given angle determines the dot product.

Step 1: Determine the magnitude of the unknown vector Put $s=|\vec{c}|$, so the first condition gives $\vec{a}\cdot\vec{c}=s$. Substitute into the distance condition:

$$s^2-4s+12=8 \quad \Rightarrow \quad (s-2)^2=0.$$

Hence $|\vec{c}|=2$ and $\vec{a}\cdot\vec{c}=2$. This uses both conditions without assuming a direction for the unknown vector.

Step 2: Find the possible components Calculate $\vec{d}=\vec{a}\times\vec{b}=(-1,1,0)$, with $|\vec{d}|=\sqrt{2}$. The acute angle gives

$$\vec{d}\cdot\vec{c}=\sqrt{2}(2)\cos(\pi/4)=2.$$

Write $\vec{c}=(x,y,z)$. Then $-x+y=2$ and $x+y+z=2$, so $y=x+2$ and $z=-2x$. The magnitude condition becomes

$$x^2+(x+2)^2+4x^2=4, \qquad 2x(3x+2)=0.$$

Thus $x=0$ or $-2/3$; both possible vectors must be considered.

Step 3: Evaluate both terms In either case $\vec{b}\cdot\vec{c}=2x+2y+z=2x+4$, giving 4 or $8/3$. Therefore $|10-3\vec{b}\cdot\vec{c}|=2$ for both vectors. Also,

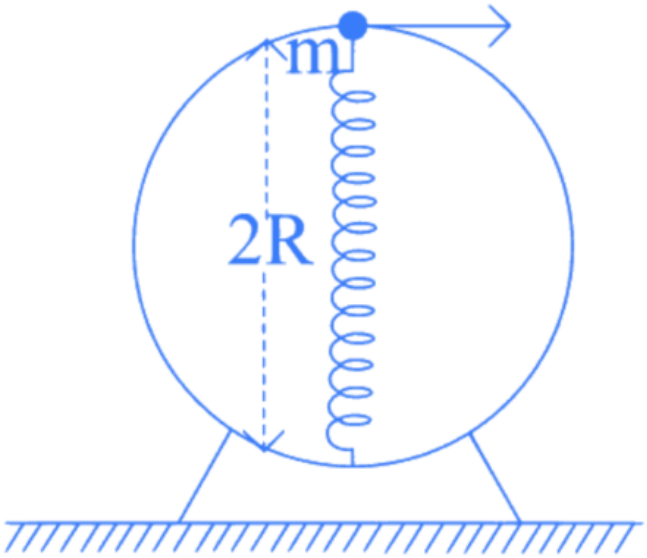
$$|\vec{d}\times\vec{c}|^2=(2)(4)-2^2=4.$$

Thus the requested expression has the same value for both admissible vectors; the two possible directions do not make the answer ambiguous.

Step 4: Conclusion The required value is $2+4=\boxed{6}$.

Quick Tip to Solve: Use the magnitude condition first; then compute the squared cross product from dot products rather than expanding its three components.

26. A bead of mass ' m ' slides without friction on the wall of a vertical circular hoop of radius ' R ' as shown in figure. The bead moves under the combined action of gravity and a massless spring (k) attached to the bottom of the hoop. The equilibrium length of the spring is ' R '. If the bead is released from top of the hoop with (negligible) zero initial speed, velocity of bead, when the length of spring becomes ' R ', would be (spring constant is ' k ', g is accleration due to gravity) [4 marks]



(A) $\sqrt{2Rg + \frac{kR^2}{m}}$

(B) $\sqrt{3Rg + \frac{kR^2}{m}}$

(C) $\sqrt{2Rg + \frac{4kR^2}{m}}$

(D) $2\sqrt{gR + \frac{kR^2}{m}}$

Correct Answer: (B) $\sqrt{3Rg + \frac{kR^2}{m}}$

Solution:

Concept:

- The hoop is frictionless, so its normal force does no work. Gravity and an ideal spring are conservative, and total mechanical energy is conserved.
- The elastic energy is $U_s = \frac{1}{2}k(\ell - R)^2$, where ℓ is the spring length and R is its natural length.

Step 1: Find the final height from the chord geometry Take the bottom of the hoop as height zero. A point at height h and horizontal coordinate x satisfies $x^2 + (h - R)^2 = R^2$. Consequently its distance ℓ from the bottom obeys

$$\ell^2 = x^2 + h^2 = 2Rh.$$

When the spring length is R, the bead is therefore at $h = R/2$, rather than at the centre height R. It has fallen through $2R - R/2 = 3R/2$.

Step 2: Apply conservation of mechanical energy Initially the bead is at height $2R$, its speed is negligible, and the spring length is $2R$. Its initial extension is therefore R, so its initial elastic energy is $kR^2/2$. At the final point the spring has its natural length, so its elastic energy vanishes. Energy conservation gives

$$2mgR + \frac{1}{2}kR^2 = \frac{1}{2}mv^2 + \frac{1}{2}mgR.$$

Hence

$$v^2 = 3gR + \frac{kR^2}{m}.$$

The coefficient of the gravitational term rules out A and C. Option D also overcounts the released spring energy and the gravitational fall.

Step 3: Conclusion The speed is $\boxed{\sqrt{3gR + kR^2/m}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a chord joining the bottom of a vertical circle to the bead, use $\ell^2 = 2Rh$ to determine the height before writing the energy equation.

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27. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: A sound wave has higher speed in solids than gases.

Reason R: Gases have higher value of Bulk modulus than solids.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**

(B) **A** is false but **R** is true

(C) **A** is true but **R** is false

(D) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

Correct Answer: (C) **A** is true but **R** is false

Solution:

Concept:

- Sound is a mechanical disturbance propagated by elastic restoring forces. Its speed depends on the ratio of an appropriate elastic modulus to density.
- For a fluid, $v = \sqrt{B/\rho}$, where B is bulk modulus and ρ is density. In a solid, the relevant longitudinal elastic modulus also reflects its rigidity.

Step 1: Evaluate Assertion A In the usual comparison of solids and gases, sound travels faster in solids. Particles in a solid are strongly coupled by elastic forces, so a compression can be transmitted rapidly. For example, the speed of longitudinal sound in a typical metal is several kilometres per second, whereas in air it is only a few hundred metres per second. The larger elastic modulus of the solid more than compensates for its greater density in this standard comparison. Thus Assertion A is true.

Step 2: Evaluate Reason R The bulk modulus measures resistance to volume compression: $B = -V\,dP/dV$. Gases are much more readily compressed than solids, so their bulk modulus is much

smaller, not larger. For an ideal gas the adiabatic bulk modulus is γP; the modulus of a solid under ordinary conditions is far larger. Therefore Reason R reverses the physical comparison and is false.

Step 3: Select the truth combination Since A is true and R is false, the reason cannot explain the assertion. Options A and D both require a true reason and are excluded. Option B requires a false assertion and a true reason, so it is also excluded. The true-assertion, false-reason combination is precisely option C.

Step 4: Conclusion Assertion A is true, but Reason R is false. Hence, the correct option is (C).

Quick Tip to Solve: Check compressibility when judging a bulk-modulus statement: easy compression means a small bulk modulus, which immediately rejects the stated reason.

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28. In the experiment for measurement of viscosity ' η ' of given liquid with a ball having radius R, consider following statements.

A. Graph between terminal velocity V and R will be a parabola.

B. The terminal velocities of different diameter balls are constant for a given liquid.

C. Measurement of terminal velocity is dependent on the temperature.

D. This experiment can be utilized to assess the density of a given liquid.

E. If balls are dropped with some initial speed, the value of η will change.

Choose the correct answer from the options given below: [4 marks]

(A) C, D and E Only

(B) A, B and E Only

(C) A, C and D Only

(D) B, D and E Only

Correct Answer: (C) A, C and D Only

Solution:

Concept:

- In the Stokes regime, a small sphere reaches terminal speed when its effective weight equals viscous drag: $6\pi\eta RV = \frac{4}{3}\pi R^3(\rho_s - \rho_l)g$.
- Thus $V = 2R^2(\rho_s - \rho_l)g/(9\eta)$. Here ρ_s and ρ_l are the sphere and liquid densities; temperature affects liquid viscosity.

Step 1: Check statements A and B using radius dependence For spheres of the same material in the same liquid at a fixed temperature, all factors other than R^2 are constant. Hence $V \propto R^2$: a graph with terminal velocity on the vertical axis and radius on the horizontal axis is a parabola. Statement A is true. Different radii generally give different terminal velocities, so statement B is false. The word terminal means constant in time for a particular falling ball, not the same value for every ball.

Step 2: Check statements C and D The coefficient of viscosity of a liquid depends on temperature, and so does the measured terminal speed. The liquid density can also vary with temperature. Thus C is true. If the ball density and radius are known, and viscosity is known or independently determined,

the measured speed gives

$$\rho_l = \rho_s - \frac{9\eta V}{2R^2 g}.$$

This allows the experiment to assess the liquid density, so D is true.

Step 3: Check the effect of initial speed An initial speed changes only the transient approach to terminal motion. It does not change the viscosity, which is a material property at the specified temperature. Thus E is false. The true statements are A, C and D; every other offered combination contains B or E, which the terminal-speed equation rules out.

Step 4: Conclusion A, C and D only are correct. Hence, the correct option is (C).

Quick Tip to Solve: Write $V \propto R^2(\rho_s - \rho_l)/\eta$ first: it separates size effects, temperature effects and material properties in a single relation.

29. The center of mass of a thin rectangular plate (fig - x) with sides of length a and b, whose mass per unit area (σ) varies as $\sigma = \frac{\sigma_0 x}{ab}$ (where σ_0 is a constant), would be _____. [4 marks]

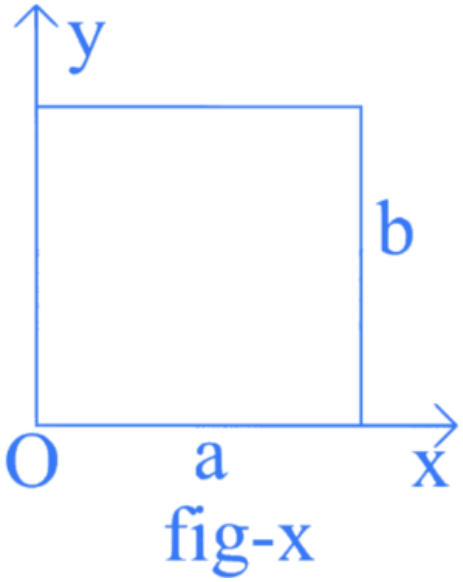


fig-x

(A) $\left(\frac{2}{3}a, \frac{2}{3}b\right)$
(B) $\left(\frac{2}{3}a, \frac{b}{2}\right)$
(C) $\left(\frac{1}{3}a, \frac{b}{2}\right)$
(D) $\left(\frac{a}{2}, \frac{b}{2}\right)$

Correct Answer: (B) $\left(\frac{2}{3}a, \frac{b}{2}\right)$

Solution:

Concept:

- For a thin lamina, $dm = \sigma(x, y)\, dx\, dy$, and $x_{\text{cm}} = \int x\, dm/M$, $y_{\text{cm}} = \int y\, dm/M$.
- The rectangle occupies $0 \le x \le a$, $0 \le y \le b$. Its density is proportional to x, so it is uniform in the vertical direction but increases toward the right.

Step 1: Calculate the total mass Use the stated surface density rather than treating the plate as uniform. Its total mass is

$$M = \int_0^a \int_0^b \frac{\sigma_0 x}{ab}\, dy\, dx = \frac{\sigma_0}{a} \int_0^a x\, dx = \frac{\sigma_0 a}{2}.$$

The constant σ_0 cancels from the centre-of-mass ratios, as expected: multiplying every mass element by one factor cannot change the position.

Step 2: Calculate the horizontal coordinate The first moment about the vertical axis is

$$\int x\, dm = \frac{\sigma_0}{ab} \int_0^a x^2\, dx \int_0^b dy = \frac{\sigma_0 a^2}{3}.$$

Dividing by M gives

$$x_{\text{cm}} = \frac{\sigma_0 a^2/3}{\sigma_0 a/2} = \frac{2a}{3}.$$

The result is to the right of the geometric centre, which is consistent with the larger density at larger x.

Step 3: Calculate the vertical coordinate and compare options The density has no y-dependence, so the distribution is symmetric about $y = b/2$. Directly,

$$\int y\, dm = \frac{\sigma_0}{ab} \frac{a^2}{2} \frac{b^2}{2} = \frac{\sigma_0 ab}{4}, \qquad y_{\text{cm}} = \frac{b}{2}.$$

Option A wrongly shifts the vertical coordinate; C shifts the horizontal coordinate toward the lighter side; D incorrectly assumes uniform density.

Step 4: Conclusion The centre of mass is $\boxed{(2a/3,\ b/2)}$. Hence, the correct option is (B).

Quick Tip to Solve: When the density depends on only one coordinate, use symmetry for the other coordinate and integrate only the coordinate that carries the density variation.

30. Consider following statements:

A. Surface tension arises due to extra energy of the molecules at the interior as compared to the molecules at the surface, of a liquid.

B. As the temperature of liquid rises, the coefficient of viscosity increases.

C. As the temperature of gas increases, the coefficient of viscosity increases

D. The onset of turbulence is determined by Reynold's number.

E. In a steady flow two stream lines never intersect.

Choose the correct answer from the options given below: [4 marks]

(A) B, C, D Only

(B) C, D, E Only

(C) A, D, E Only

(D) A, B, C Only

Correct Answer: (B) C, D, E Only

Solution:

Concept:

- Surface tension reflects the excess free energy of molecules at a liquid surface compared with molecules in the interior.
- Temperature affects viscosity differently in liquids and gases. Flow stability is characterised by Reynolds number $\mathrm{Re} = \rho v L/\eta$, while a streamline is tangent to the local velocity.

Step 1: Evaluate the surface-energy and liquid-viscosity claims A molecule in the interior is surrounded by neighbours and experiences cohesive attraction in all directions. A surface molecule has fewer neighbours and higher energy; creating extra surface therefore requires work. Statement A reverses this energy comparison and is false. In an ordinary liquid, raising the temperature allows molecules to move past one another more readily, so viscosity decreases. Statement B, which claims an increase, is false.

Step 2: Evaluate the gas-viscosity and turbulence claims In a gas, viscosity arises predominantly from transport of molecular momentum between adjacent layers. Increasing temperature increases molecular speeds and normally increases gas viscosity. Hence C is true. Reynolds number compares inertial and viscous effects; its magnitude indicates when laminar flow becomes unstable and turbulence can occur, subject to the geometry and disturbances. Thus D is true.

Step 3: Evaluate the streamline statement and choose the set At a point in steady flow, the fluid has one definite velocity direction. If two streamlines intersected with different tangents, they would assign two directions to that same velocity, which is impossible. Hence E is true. The true statements are therefore C, D and E. Option A includes the false B; option C includes the false A; and option D includes both false statements. Only option B contains exactly the three true statements.

Step 4: Conclusion C, D and E only are correct. Hence, the correct option is (B).

Quick Tip to Solve: Remember the opposite temperature trends: liquid viscosity generally decreases on heating, whereas gas viscosity increases; surface molecules have the excess energy.

31. Find the equivalent resistance between two ends of the following circuit [4 marks]



(A) r

(B) $\frac{r}{6}$

(C) $\frac{r}{3}$

(D) $\frac{r}{9}$

Correct Answer: (D) $\frac{r}{9}$

Solution:

Concept:

- An ideal connecting wire has negligible resistance, so every point on it is at the same potential. Nodes are identified by connections, not by their position on the page.
- Resistors connected between the same two nodes are in parallel. Their conductances add: $1/R_{\text{eq}} = \sum_i 1/R_i$.

Step 1: Identify the two distinct nodes Call the left terminal A and the right terminal B. The upper connecting wire joins A to the junction between the second and third resistors. These two points therefore form one node. The lower connecting wire joins B to the junction between the first and second resistors. Those two points form the other node. Neither connecting wire joins A directly to B, so the entire circuit is not short-circuited.

Step 2: Combine the three parallel branches Each resistor marked $r/3$ is now seen to have one end at A and the other at B. The middle resistor is drawn in the opposite orientation, which does not change its resistance. For terminal voltage V, each branch carries $3V/r$. Thus

$$I = 3\left(\frac{3V}{r}\right) = \frac{9V}{r}, \qquad R_{\text{eq}} = \frac{V}{I} = \frac{r}{9}.$$

Adding the three resistances as a series chain would give r, but the bridging wires make that interpretation incorrect. Two parallel branches alone would give $r/6$; here there are three branches. The value $r/3$ is just one branch resistance.

Step 3: Conclusion The equivalent resistance between the terminals is $\boxed{r/9}$. Hence, the correct option is (D).

Quick Tip to Solve: First colour all points joined by ideal wire as one node; only then decide whether the resistors are in series or parallel.

32. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: In a central force field, the work done is independent of the path chosen.

Reason R: Every force encountered in mechanics does not have an associated potential energy.

In the light of the above statements, choose the most appropriate answer from the options given below [4 marks]

(A) **A** is false but **R** is true

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(C) **A** is true but **R** is false

(D) Both **A** and **R** are true and **R** is the correct explanation of **A**

Correct Answer: (B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

Solution:

Concept:

- A central force field in the usual mechanics sense has the form $\mathbf{F} = f(r)\hat{\mathbf{r}}$, depending on separation r from a fixed centre.
- For a conservative force, work equals the negative change of potential energy. Dissipative forces such as friction do not have a position-only potential energy.

Step 1: Evaluate Assertion A For an infinitesimal displacement, only its radial component contributes to work because the central force is radial. Consequently

$$dW = \mathbf{F} \cdot d\mathbf{r} = f(r)\, dr, \qquad W_{i\to f} = \int_{r_i}^{r_f} f(r)\, dr.$$

This expression depends on the initial and final radial positions and not on the route taken between them. Equivalently, define $U(r) = -\int f(r)\, dr$, so $W = U(r_i) - U(r_f)$. Assertion A is therefore true.

Step 2: Evaluate Reason R The reason means that not every force encountered in mechanics has an associated potential energy. This is true: kinetic friction dissipates mechanical energy and its work depends on the path length, so a position-only potential cannot describe it. Conservative forces such as gravity have potential energies, but this property does not extend to all forces.

Step 3: Test whether the reason explains the assertion The existence of nonconservative forces elsewhere in mechanics does not establish the path independence of a central force. The assertion follows from the radial, separation-dependent form of the force and the integral above. Thus both statements are true, but R does not explain A. The options declaring either statement false are excluded, and the option claiming an explanatory connection is also excluded.

Step 4: Conclusion Both A and R are true, but R is not the correct explanation of A. Hence, the correct option is (B).

Quick Tip to Solve: For assertion-reason questions, decide the truth of each statement first, then test the explanatory connection as a separate step.

33. A proton of mass ' m_P ' has same energy as that of a photon of wavelength ' λ '. If the proton is moving at non-relativistic speed, then ratio of its de Broglie wavelength to the wavelength of photon is. [4 marks]

(A) $\frac{1}{c}\sqrt{\frac{E}{m_p}}$

(B) $\frac{1}{c}\sqrt{\frac{2E}{m_p}}$

(C) $\frac{1}{2c}\sqrt{\frac{E}{m_p}}$

(D) $\frac{1}{c}\sqrt{\frac{E}{2\ m_p}}$

Correct Answer: (D) $\frac{1}{c}\sqrt{\frac{E}{2\ m_p}}$

Solution:

Concept:

- The de Broglie wavelength of a particle of momentum p is $\lambda_{dB} = h/p$, where h is Planck's constant.
- A nonrelativistic proton of kinetic energy E has $p = \sqrt{2m_pE}$. A photon of energy E has wavelength $\lambda_\gamma = hc/E$.

Step 1: Express both wavelengths using the common energy The energy specified for the moving nonrelativistic proton is its kinetic energy. Using $E = p^2/(2m_p)$ gives

$$p_p = \sqrt{2m_pE}, \qquad \lambda_p = \frac{h}{\sqrt{2m_pE}}.$$

For the photon the energy-momentum relation is different: $E = p_\gamma c = hc/\lambda_\gamma$. Hence

$$\lambda_\gamma = \frac{hc}{E}.$$

The same energy does not imply the same momentum for the photon and proton.

Step 2: Divide and simplify the requested ratio Dividing the proton wavelength by the photon wavelength eliminates Planck's constant:

$$\frac{\lambda_p}{\lambda_\gamma} = \frac{h}{\sqrt{2m_pE}}\frac{E}{hc} = \frac{1}{c}\sqrt{\frac{E}{2m_p}}.$$

The quantity $\sqrt{E/m_p}$ has the dimensions of speed, so division by c gives the required dimensionless ratio. As a further check, $E = m_pv^2/2$ makes the ratio $v/(2c)$, which is much smaller than one in the stated nonrelativistic regime. Option A omits the factor $1/\sqrt{2}$; option B is twice the required value; option C puts the factor 2 outside the root in the wrong way.

Step 3: Conclusion The requested ratio is $\boxed{\lambda_p/\lambda_\gamma = c^{-1}\sqrt{E/(2m_p)}}$. Hence, the correct option is (D).

Quick Tip to Solve: At equal energy, use the separate momentum laws $p_\gamma = E/c$ and $p_p = \sqrt{2m_pE}$; the wavelength ratio is the inverse momentum ratio.

34. A wire of resistance R is bent into an equilateral triangle and an identical wire is bent into a square. The ratio of resistance between the two end points of an edge of the triangle to that of the square is [4 marks]

(A) $8/9$

(B) $9/8$

(C) $32/27$

(D) $27/32$

Correct Answer: (C) $32/27$

Solution:

Concept:

- For a uniform wire, resistance is proportional to length. Bending the wire without stretching it leaves its total resistance R unchanged.
- Between adjacent vertices of a closed polygon there are two paths: the chosen edge and all remaining edges in series. These paths are in parallel.

Step 1: Find the equivalent resistance of the triangle The equilateral triangle has three equal sides, so each side has resistance $R/3$. Between the endpoints of one edge, the direct path has resistance $R/3$, while the other two sides form a series path of resistance $2R/3$. Therefore

$$R_{\triangle} = \frac{(R/3)(2R/3)}{R/3 + 2R/3} = \frac{2R}{9}.$$

The result is smaller than the resistance of either path, as a parallel combination must be.

Step 2: Find the equivalent resistance of the square The identical wire used for the square also has total resistance R. Its four equal edges each have resistance $R/4$. Between adjacent vertices, the two paths have resistances $R/4$ and $3R/4$, giving

$$R_{\square} = \frac{(R/4)(3R/4)}{R/4 + 3R/4} = \frac{3R}{16}.$$

The different side lengths matter: one must not assign the same edge resistance to both polygons.

Step 3: Take the ratio in the stated order The triangle resistance is requested in the numerator:

$$\frac{R_{\triangle}}{R_{\square}} = \frac{2R/9}{3R/16} = \frac{32}{27}.$$

The total wire resistance cancels. The reciprocal $27/32$ reverses the order; the other offered ratios do not include the correct parallel combinations for both shapes.

Step 4: Conclusion The ratio of the triangle resistance to the square resistance is $\boxed{32/27}$. Hence, the correct option is (C).

Quick Tip to Solve: For a uniform wire bent into an n-sided regular polygon, adjacent vertices have equivalent resistance $R(n-1)/n^2$.

35. Consider a long thin conducting wire carrying a uniform current I. A particle having mass "M" and charge " q " is released at a distance " a " from the wire with a speed v_0 along the direction of current in the wire. The particle gets attracted to the wire due to magnetic force. The particle turns round when it is at distance x from the wire. The value of x is [μ_0 is vacuum permeability] [4 marks]

(A) $a\left[1-\frac{mv_o}{2q\mu_0 \mathrm{I}}\right]$

(B) $ae^{-\frac{4\pi \mathrm{mv}_o}{q\mu_o \mathrm{I}}}$

(C) $a\left[1-\frac{\mathrm{mv}}{\mathrm{q}}\mu_\mathrm{o}\mathrm{I}\right]$

(D) $\frac{a}{2}$

Correct Answer: (B) $ae^{-\frac{4\pi \mathrm{mv}_o}{q\mu_o \mathrm{I}}}$

Solution:

Concept:

- At radial distance r from a long straight wire, $B(r)=\mu_0 I/(2\pi r)$. The field is azimuthal around the wire.
- The magnetic force does no work, so the particle speed stays v_0. Its radial and axial velocity components change while $v_r^2+v_z^2=v_0^2$.

Step 1: Relate the axial velocity to the radial position Take the wire along the positive z-axis, with the initial particle velocity $v_z=v_0$. Use M for the particle mass. The stated initial attraction selects the sign of charge for which $qI>0$. Since $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}}=\hat{\mathbf{z}}$, the axial equation of motion is

$$M\frac{dv_z}{dt}=qv_rB(r)=\frac{q\mu_0 I}{2\pi r}\frac{dr}{dt}.$$

Integrating from the initial position a gives

$$v_z-v_0=\frac{q\mu_0 I}{2\pi M}\ln\left(\frac{r}{a}\right).$$

This uses the varying wire field rather than treating it as uniform.

Step 2: Apply the radial turning condition At the closest approach $r=x$, the radial velocity vanishes. Conservation of speed then gives $|v_z|=v_0$. The initial value $+v_0$ belongs to r=a; after inward motion, the axial velocity decreases, so the inner turning point has $v_z=-v_0$. Substitution yields

$$-2v_0=\frac{q\mu_0 I}{2\pi M}\ln\left(\frac{x}{a}\right).$$

Hence

$$x=a\exp\left(-\frac{4\pi M v_0}{q\mu_0 I}\right).$$

At this point the reversed axial velocity produces an outward radial magnetic force, confirming a closest-approach turning point. The exponential is dimensionless and lies between zero and one. The linear alternatives do not follow from the logarithmic integral, and a fixed a/2 cannot describe arbitrary initial speed or current.

Step 3: Conclusion The turning distance is $\boxed{x=a\exp[-4\pi Mv_0/(q\mu_0 I)]}$, with m in the printed option denoting the same particle mass M. Hence, the correct option is (B).

Quick Tip to Solve: At the closest approach the radial velocity is zero, but the particle is not at rest: its axial velocity has reversed to $-v_0$.

36. Due to presence of an em-wave whose electric component is given by $E = 100\sin(\omega t - kx)\mathrm{NC}^{-1}$ a cylinder of length 200 cm holds certain amount of em-energy inside it. If another cylinder of same length but half diameter than previous one holds same amount of em-energy, the magnitude of the electric field of the corresponding em-wave should be modified as [4 marks]

(A) $50\sin(\omega\mathrm{t} - \mathrm{kx})\mathrm{NC}^{-1}$

(B) $400\sin(\omega\mathrm{t} - \mathrm{kx})\mathrm{NC}^{-1}$

(C) $200\sin(\omega t - kx)\mathrm{NC}^{-1}$

(D) $25\sin(\omega\mathrm{t} - \mathrm{kx})\mathrm{NC}^{-1}$

Correct Answer: (C) $200\sin(\omega t - kx)\mathrm{NC}^{-1}$

Solution:

Concept:

- For an electromagnetic wave in vacuum, the cycle-averaged total energy density is $\langle u\rangle = \frac{1}{2}\varepsilon_0 E_0^2$, where E_0 is the electric-field amplitude.
- Energy contained in an equal-length cylinder scales as field-amplitude squared times cross-sectional area: $U \propto E_0^2 d^2 L$.

Step 1: Compare the cylinder volumes Let the first cylinder have diameter d and length L. Its volume is

$$V_1 = \frac{\pi d^2 L}{4}.$$

The second cylinder has the same length, $L = 200\,\mathrm{cm}$, but diameter d/2. Its volume is therefore

$$V_2 = \frac{\pi (d/2)^2 L}{4} = \frac{V_1}{4}.$$

Halving the diameter quarters the area and volume; it does not halve them. The numerical length cancels because the two lengths are identical.

Step 2: Impose equality of electromagnetic energies Write the original amplitude as $E_{01} = 100\,\mathrm{N\,C^{-1}}$ and the new amplitude as E_{02}. Equality of the contained energies requires

$$\frac{1}{2}\varepsilon_0 E_{01}^2 V_1 = \frac{1}{2}\varepsilon_0 E_{02}^2 \frac{V_1}{4}.$$

Thus $E_{02}^2 = 4E_{01}^2$, so the positive amplitude doubles: $E_{02} = 200\,\mathrm{N\,C^{-1}}$. The frequency, wave number and phase pattern are unchanged; the question asks only for an amplitude modification. An amplitude 50 would reduce the energy density to one quarter instead of increasing it fourfold. An amplitude 400 would increase it sixteenfold, and 25 would reduce it sixteenfold, so none of those alternatives preserves the energy in the smaller cylinder.

Step 3: Conclusion The modified field is $\boxed{E = 200\sin(\omega t - kx)\,\mathrm{N\,C^{-1}}}$. Hence, the correct option is (C).

Quick Tip to Solve: For equal wave energy and equal cylinder length, the field amplitude is inversely proportional to cylinder diameter.

37. Three infinitely long wires with linear charge density λ are placed along the $x-axis, y-axis$ and $z-$ axis respectively. Which of the following denotes an equipotential surface? [4 marks]

(A) $\left(x^2+y^2\right)\left(y^2+z^2\right)\left(z^2+x^2\right)=$ constant

(B) $xyz=$ constant

(C) $xy+yz+zx=$ constant

(D) $(x+y)(y+z)(z+x)=$ constant

Correct Answer: (A) $\left(x^2+y^2\right)\left(y^2+z^2\right)\left(z^2+x^2\right)=$ constant

Solution:

Concept:

- An infinite straight wire produces a radial field $E=\lambda/(2\pi\varepsilon_0 r)$, where r is the perpendicular distance from that wire.
- Potential differences follow $dV=-E\,dr$; potentials of different charge distributions add algebraically. Equal potential therefore means a constant sum of logarithms of the three perpendicular distances.

Step 1: Write the three perpendicular distances At a point $P(x,y,z)$, the distances from the wires on the coordinate axes are

$$r_x=\sqrt{y^2+z^2}, \qquad r_y=\sqrt{z^2+x^2}, \qquad r_z=\sqrt{x^2+y^2}.$$

Coordinates along a given wire do not affect its own potential. Thus using x, y, z themselves as the respective distances would be incorrect.

Step 2: Add the logarithmic potentials Integrating the line-charge field gives $V_i=-\lambda\ln r_i/(2\pi\varepsilon_0)+C_i$. An absolute zero at infinity is unavailable for an infinite line, but reference constants cancel in potential comparisons. Hence

$$V=C-\frac{\lambda}{2\pi\varepsilon_0}\ln(r_x r_y r_z).$$

For fixed nonzero common λ, an equipotential requires $r_x r_y r_z$ to be constant. Squaring this condition removes the square roots without changing a positive distance product.

Step 3: Identify the matching surface The resulting equation is

$$(x^2+y^2)(y^2+z^2)(z^2+x^2)=\text{constant}.$$

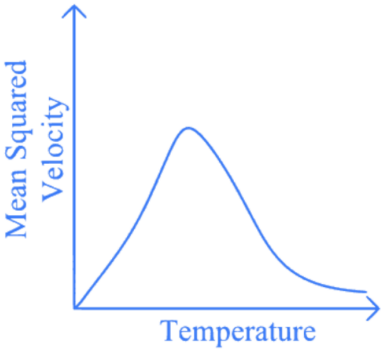
The remaining options involve coordinates or sums of coordinates rather than the perpendicular-distance product. In particular, they do not remain unchanged under all independent coordinate sign reversals, whereas this charge configuration and its potential do.

Step 4: Conclusion The equipotential surface is $(x^2+y^2)(y^2+z^2)(z^2+x^2)=$ constant, away from the charged axes. Hence, the correct option is (A).

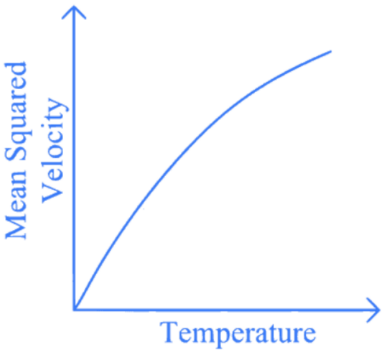
Quick Tip to Solve: For equal infinite line charges, add logarithmic potentials: their perpendicular distances multiply on an equipotential.

38. For a particular ideal gas which of the following graphs represents the variation of mean squared velocity of the gas molecules with temperature? [4 marks]

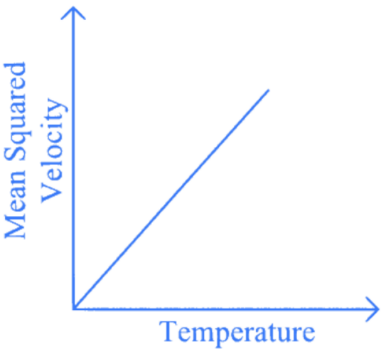
(A)



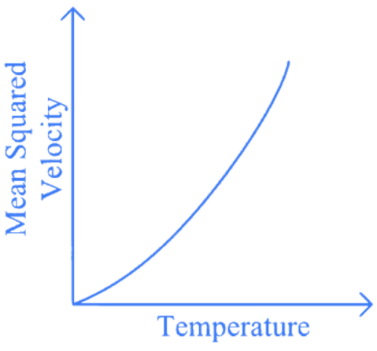
(B)



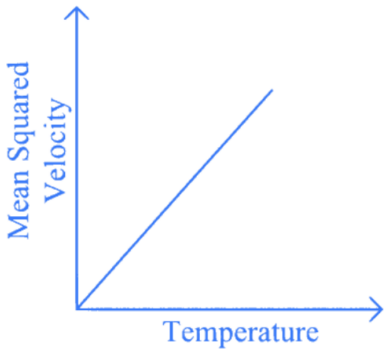
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- The translational kinetic energy per ideal-gas molecule is $\frac{1}{2}m\langle v^2\rangle = \frac{3}{2}k_{\mathrm{B}}T$, with temperature measured on the absolute Kelvin scale.
- For a particular gas the molecular mass m is fixed, so $\langle v^2\rangle = (3k_{\mathrm{B}}/m)T$. The mean squared speed, unlike the root mean square speed, is directly proportional to temperature.

Step 1: Relate molecular speed to temperature The pressure relation from kinetic theory is $PV = Nm\langle v^2\rangle/3$, while the ideal-gas equation is $PV = Nk_{\mathrm{B}}T$. Equating these expressions and cancelling the number of molecules gives

$$\langle v^2\rangle = \frac{3k_{\mathrm{B}}}{m}T.$$

The proportionality factor is positive and independent of temperature for the specified gas. Equivalently it is $3R/M$, where M is the molar mass.

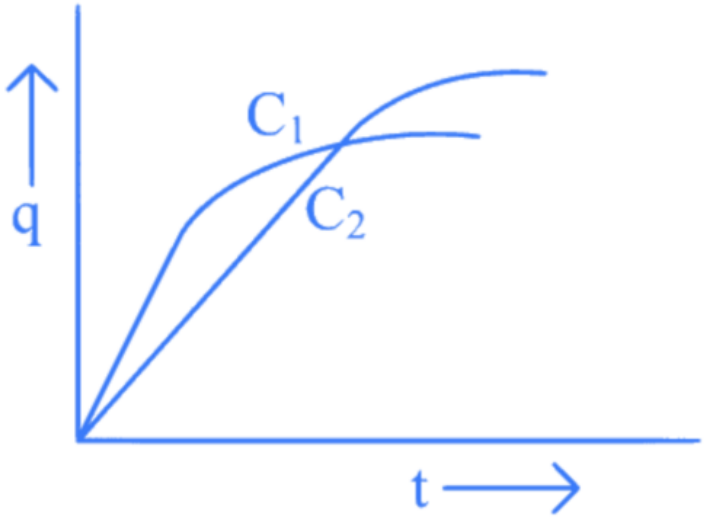
Step 2: Choose the straight-line graph A plot of $\langle v^2\rangle$ vertically against absolute temperature horizontally must therefore be a straight line through the origin with positive slope. Option (C) has precisely this form. The falling part of (A) would make the mean kinetic energy decrease as the temperature increases. Options (B) and (D) have changing slopes and therefore do not represent the linear equation.

Step 3: Distinguish the square from its square root The rms speed obeys $v_{\mathrm{rms}} = \sqrt{3k_{\mathrm{B}}T/m}$, which would be a curve concave down. That relation cannot be used for the ordinate here: the question explicitly asks for mean squared velocity, namely $\langle v^2\rangle$, rather than its square root. The physical ideal-gas model is used for $T > 0$; the intercept is its formal continuation to zero Kelvin.

Step 4: Conclusion Mean squared molecular speed increases linearly with absolute temperature. Hence, the correct option is (C).

Quick Tip to Solve: Check the vertical axis first: mean square speed varies as T, whereas rms speed varies as $\sqrt{T}$.

39. Two capacitors C_1 and C_2 are connected in parallel to a battery. Charge-time graph is shown below for the two capacitors. The energy stored with them are U_1 and U_2, respectively. Which of the given statements is true? [4 marks]



(A) $C_2 > C_1, U_2 > U_1$

(B) $C_1 > C_2, U_1 > U_2$

(C) $C_2 > C_1, U_2 < U_1$

(D) $C_1 > C_2, U_1 < U_2$

Correct Answer: (A) $C_2 > C_1, U_2 > U_1$

Solution:

Concept:

- Capacitors connected in parallel to a battery have the same final potential difference V. Their equilibrium charges are $Q_i = C_i V$.
- The stored electrostatic energy is $U_i = \frac{1}{2} C_i V^2 = \frac{1}{2} Q_i V$. At common voltage the larger capacitance stores both more charge and more energy.

Step 1: Read the final charges from the graph The charge-time curves approach horizontal limiting values after charging. The curve labelled C_2 reaches the higher final charge, even though the C_1 curve is initially above it. Therefore

$$Q_{2,\infty} > Q_{1,\infty}.$$

The initial slope describes the charging current and is not by itself a comparison of equilibrium capacitances. The crossing of the curves makes this distinction particularly important.

Step 2: Compare the capacitances at the common voltage Both capacitors are attached in parallel to the same battery, so their final voltages are equal. Dividing their limiting charges by this common positive voltage gives

$$\frac{C_2}{C_1} = \frac{Q_{2,\infty}}{Q_{1,\infty}} > 1.$$

Thus $C_2 > C_1$. No numerical time constant or charging resistance is needed: the final plateau heights already supply the required comparison.

Step 3: Compare the stored energies With the battery voltage unchanged,

$$\frac{U_2}{U_1} = \frac{C_2V^2/2}{C_1V^2/2} = \frac{C_2}{C_1} > 1.$$

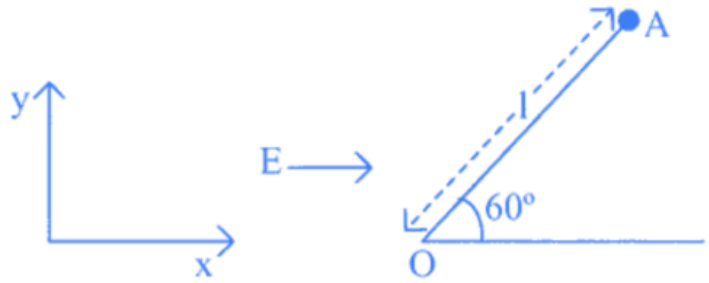
Hence $U_2 > U_1$. Option (B) reverses both inequalities. Option (C) gets the capacitance ordering right but reverses the energy ordering; option (D) reverses the capacitance ordering and combines it with an inconsistent energy comparison at fixed voltage.

Step 4: Conclusion $C_2 > C_1$ and $U_2 > U_1$. Hence, the correct option is (A).

Quick Tip to Solve: For parallel capacitors, compare the final charge plateaux, not the initial slopes; at fixed voltage, Q, C and U have the same ordering.

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40. A particle of mass ' m ' and charge ' q ' is fastened to one end ' A ' of a massless string having equilibrium length l, whose other end is fixed at point ' O '. The whole system is placed on a frictionless horizontal plane and is initially at rest. If uniform electric field is switched on along the direction as shown in figure, then the speed of the particle when it crosses the x-axis is [4 marks]



(A) $\sqrt{\frac{\mathrm{qEl}}{2\ \mathrm{m}}}$

(B) $\sqrt{\frac{\mathrm{qEl}}{4\ \mathrm{m}}}$

(C) $\sqrt{\frac{\mathrm{qEl}}{\mathrm{m}}}$

(D) $\sqrt{\frac{2\mathrm{qEl}}{\mathrm{m}}}$

Correct Answer: (C) $\sqrt{\frac{\mathrm{qEl}}{\mathrm{m}}}$

Solution:

Concept:

- In a uniform electric field directed along the positive x-axis, the work on a charge is $W = qE\Delta x$, depending on horizontal displacement rather than path length.
- The fixed-length string constrains the particle to a circle of radius l about O. Tension is radial and perpendicular to the instantaneous displacement, so it does no work.

Step 1: Find the initial and final horizontal coordinates The figure shows the string initially making 60° with the positive x-axis. The initial position therefore has $x_i = l\cos 60^\circ = l/2$. When the particle first crosses the positive x-axis the taut string lies along that axis, so $x_f = l$. Its displacement along

the field is consequently

$$\Delta x = l - \frac{l}{2} = \frac{l}{2}.$$

The arc travelled is different from this horizontal displacement and must not be inserted into $qE\Delta x$.

Step 2: Apply the work-energy theorem The plane is frictionless, and the particle starts at rest. Gravity and the normal force have no component along its horizontal displacement, and the tension contributes zero work. All the change in kinetic energy is therefore due to the field:

$$\frac{1}{2}mv^2 - 0 = qE\left(\frac{l}{2}\right).$$

Cancelling the factors of two gives $mv^2 = qEl$. This uses the direction of motion indicated by the positive charge-field product in the offered expressions.

Step 3: Check the speed and eliminate alternatives Taking the positive square root,

$$v = \sqrt{\frac{qEl}{m}}.$$

Its dimensions are those of speed because qEl is an energy. Option (D) would use a field-direction displacement of l, ignoring the nonzero initial coordinate. Options (A) and (B) introduce additional factors of two or four not present in the work-energy equation.

Step 4: Conclusion The particle has speed $v = \sqrt{qEl/m}$ when it crosses the x-axis. Hence, the correct option is (C).

Quick Tip to Solve: With a uniform field, compute the change in the coordinate parallel to the field; tension changes direction but contributes no work.

41. A thin prism P_1 with angle 4° made of glass having refractive index 1.54 , is combined with another thin prism P_2 made of glass having refractive index 1.72 to get dispersion without deviation. The angle of the prism P_2 in degrees is [4 marks]

(A) 1.5

(B) 16/3

(C) 3

(D) 4

Correct Answer: (C) 3

Solution:

Concept:

- For a thin prism of angle A and refractive index μ, the small-angle deviation is $\delta = (\mu - 1)A$. Consistent angular units may be used on both sides.
- Dispersion without net deviation uses oppositely oriented prisms. Their mean deviations cancel, requiring $(\mu_1 - 1)A_1 = (\mu_2 - 1)A_2$; their chromatic dispersions need not cancel.

Step 1: Write the condition for zero mean deviation The first prism deflects light towards its base. To cancel that overall deviation, the second prism must have its base in the opposite orientation. Taking the deviation of the first prism as positive,

$$\delta_{\text{net}} = (\mu_1 - 1)A_1 - (\mu_2 - 1)A_2 = 0.$$

The given refractive indices determine the required cancellation of mean deviation. Equality of prism angles is not the condition unless the refractive indices are equal.

Step 2: Substitute the prism data Here $A_1 = 4^\circ$, $\mu_1 = 1.54$, and $\mu_2 = 1.72$. Thus

$$(1.54 - 1)4^\circ = (1.72 - 1)A_2.$$

Solving gives

$$A_2 = \frac{0.54 \times 4^\circ}{0.72} = \frac{2.16}{0.72}^\circ = 3^\circ.$$

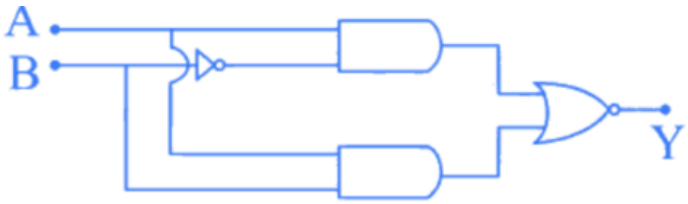
The result is smaller than 4°, as expected: the material with the larger refractive index requires a smaller angle to produce an equal magnitude of deviation.

Step 3: Check the cancellation and the options For $A_2 = 3^\circ$, the deviation magnitudes are both 2.16°, so their opposite directions give zero mean deviation. The proposed angles 1.5°, $16/3$ degrees and 4° give respective second-prism deviation magnitudes 1.08°, 3.84° and 2.88°, none equal to 2.16°. Dispersion can remain because the wavelength-dependence of the two glasses is different.

Step 4: Conclusion The second prism angle is 3°, with opposite orientation to the first prism. Hence, the correct option is (C).

Quick Tip to Solve: For zero deviation, equate $(\mu - 1)A$ for opposite prisms; do not equate their refracting angles directly.

42. Which of the following circuits has the same output as that of the given circuit? [4 marks]



(A)



(B)



(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- An AND gate outputs the product of its inputs; an inverter complements its input; the final NOR gate complements the sum of its two inputs.
- The Boolean identities $B + \overline{B} = 1$ and $A \cdot 1 = A$ simplify complementary branches. A NAND gate with both inputs joined to A acts as a NOT gate.

Step 1: Trace the two AND-gate inputs The upper horizontal input is A. The B line passes through a NOT gate before reaching the upper AND gate; the crossing hump denotes a crossing without a junction to the vertical branch of A. Thus the upper AND output is $A\overline{B}$. The lower AND gate receives the uncomplemented A branch and the uncomplemented B branch, so its output is AB. Reading that crossing correctly is essential to the circuit.

Step 2: Simplify the final NOR output The two AND outputs enter an OR-shaped gate with an output bubble, namely a NOR gate. Therefore

$$Y = \overline{A\overline{B} + AB} = \overline{A(\overline{B} + B)} = \overline{A}.$$

The result is independent of B. In particular, $A = 0$ makes both AND outputs zero and gives $Y = 1$, while $A = 1$ makes exactly one branch high for each value of B, giving $Y = 0$.

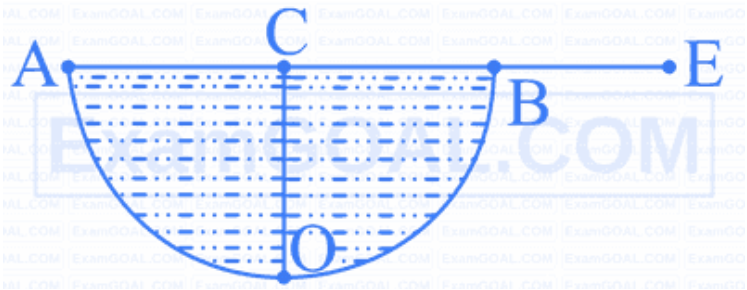
Step 3: Match the equivalent option Option (A) is a NAND gate with both inputs tied to A; its output is $\overline{A \cdot A} = \overline{A}$, exactly as required. Option (B) gives AB. Option (C), with both NAND inputs tied to B, gives $\overline{B}$. Option (D) inverts an AND output and gives $\overline{AB}$. Those outputs depend on B or have the wrong truth values, so they are not equivalent to the given circuit.

Step 4: Conclusion The given network has output $Y = \overline{A}$, which is the tied-input NAND circuit in option (A). Hence, the correct option is (A).

Quick Tip to Solve: Factor a shared input before expanding a truth table: $A\overline{B} + AB = A$; then account for the final output bubble.

43. A hemispherical vessel is completely filled with a liquid of refractive index μ. A small coin is kept at the lowest point (O) of the vessel as shown in figure. The minimum value of the refractive index of the liquid so that a person can see the coin from point E (at the level of the vessel) is _____.

[4 marks]



(A) $\frac{3}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\sqrt{3}$

(D) $\sqrt{2}$

Correct Answer: (D) $\sqrt{2}$

Solution:

Concept:

- For refraction from liquid to air, Snell's law is $\mu \sin i = \sin r$, where both angles are measured from the normal to the free surface.
- A ray travelling along the liquid surface after refraction has $r = 90^\circ$; its incidence angle is the critical angle i_c, with $\sin i_c = 1/\mu$.

Step 1: Locate the largest available incidence angle Let the hemispherical radius be R. The coin is at O, directly below the centre C of the horizontal circular surface, so $OC = R$. A surface point P on the observer's side has $CP = x$, with $0 \le x \le R$. For ray OP, the normal is vertical and the right triangle gives

$$\tan i = \frac{x}{R}.$$

Thus the greatest incidence angle available at the free surface occurs at the rim B, where $CB = OC = R$, and is 45°.

Step 2: Apply the grazing-emergence condition The eye E is at the same height as the liquid surface, so the ray that reaches it from the surface is horizontal. It must therefore emerge at $r = 90^\circ$. Such a ray exists only if the critical angle is no greater than the largest available incidence angle:

$$i_c \le 45^\circ, \qquad \mu \ge \frac{1}{\sin 45^\circ} = \sqrt{2}.$$

At the limiting minimum the emergence point is the rim. For larger indices it moves inward. Option B is below unity and cannot produce the required critical refraction. Option A, though greater than the minimum, is not the minimum; option C is also larger than necessary.

Step 3: Conclusion The minimum refractive index is $\boxed{\mu_{\min} = \sqrt{2}}$. Hence, the correct option is (D).

Quick Tip to Solve: For an eye at the surface level, impose grazing emergence first; then find the largest incidence angle allowed by the vessel geometry.

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44. Choose the correct nuclear process from the below options [p : proton, n : neutron, e^-: electron, e^+: positron, v : neutrino, $\bar{v}$: antineutrino] [4 marks]

(A) $n \to p + e^+ + v$

(B) $\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \bar{v}$

(C) $\mathrm{n} \to \mathrm{p} + \mathrm{e}^+ + \bar{v}$

(D) $\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + v$

Correct Answer: (B) $\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \bar{v}$

Solution:

Concept:

- Nuclear processes conserve electric charge, baryon number, energy and lepton number. A neutron and proton each have baryon number 1 and lepton number 0.
- An electron has charge $-e$ and electron-lepton number $+1$; an electron antineutrino has zero charge and lepton number -1.

Step 1: Check electric charge and baryon number A neutron is neutral. If it becomes a proton, the proton contributes charge $+e$, so the accompanying charged lepton must be an electron of charge $-e$. The total final charge is then zero. A positron instead contributes $+e$, producing a final charge $+2e$, which cannot equal the initial charge. Consequently options A and C fail charge conservation immediately. The neutron-to-proton conversion retains baryon number 1, while neither the electron nor neutrino contributes baryon number.

Step 2: Select the neutral lepton using lepton conservation The initial neutron has lepton number zero. Emitting an electron contributes electron-lepton number $+1$, so the neutral particle must have lepton number -1: it is the electron antineutrino, not the neutrino. The permitted beta-minus process is

$$\mathrm{n} \longrightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}_e.$$

The final lepton sum is $1 - 1 = 0$. Option D has an electron and a neutrino, giving lepton number $+2$,

so it is incorrect even though it conserves charge. In the allowed process the electron and antineutrino share the released energy and momentum with the recoiling proton; no conservation law requires their energies to be individually fixed.

Step 3: Conclusion The correct process is $\boxed{\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \bar{\nu}_e}$. Hence, the correct option is (B).

Quick Tip to Solve: Choose the sign of the charged beta particle by charge conservation, then choose neutrino or antineutrino by lepton-number conservation.

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45. A Carnot engine (E) is working between two temperatures 473 K and 273 K . In a new system two engines - engine E_1 works between 473 K to 373 K and engine E_2 works between 373 K to 273 K . If η_{12}, η_1 and η_2 are the efficiencies of the engines E, E_1 and E_2, respectively, then [4 marks]

(A) $\eta_{12} = \eta_1\eta_2$

(B) $\eta_{12} = \eta_1 + \eta_2$

(C) $\eta_{12} \geq \eta_1 + \eta_2$

(D) $\eta_{12} < \eta_1 + \eta_2$

Correct Answer: (D) $\eta_{12} < \eta_1 + \eta_2$

Solution:

Concept:

- A reversible Carnot engine operating between absolute temperatures $T_h > T_c$ has efficiency $\eta = 1 - T_c/T_h$. Use kelvin temperatures in this ratio.
- In a cascade the second engine receives the first engine's rejected heat, rather than the entire initial heat input. Therefore their efficiencies do not simply add.

Step 1: Write the three efficiencies For the direct engine E and the two engines with the intermediate reservoir at $373\,\mathrm{K}$, the efficiencies are

$$\eta_{12} = 1 - \frac{273}{473} = \frac{200}{473},$$

$$\eta_1 = 1 - \frac{373}{473} = \frac{100}{473}, \qquad \eta_2 = 1 - \frac{273}{373} = \frac{100}{373}.$$

All three numbers are positive, because each engine has a hotter source and a colder sink. Numerically these are about 0.4228, 0.2114 and 0.2681, respectively.

Step 2: Compare the sum with the direct efficiency The product of the unconverted heat fractions telescopes:

$$(1-\eta_1)(1-\eta_2) = \frac{373}{473}\frac{273}{373} = \frac{273}{473} = 1 - \eta_{12}.$$

Expanding gives

$$\eta_{12} = \eta_1 + \eta_2 - \eta_1\eta_2.$$

Since $\eta_1\eta_2 > 0$, the direct efficiency is strictly smaller than the sum. Thus options B and C fail. Option A would give only about 0.0567, much less than the actual direct efficiency. The subtraction accounts for the fact that the second engine converts only a fraction of the heat rejected by the first, and cannot

act on the original heat input again.

Step 3: Conclusion We obtain $\boxed{\eta_{12} < \eta_1 + \eta_2}$. Hence, the correct option is (D).

Quick Tip to Solve: For two reversible stages in cascade, multiply the heat-rejection fractions: $1 - \eta_{\text{total}} = (1 - \eta_1)(1 - \eta_2)$.

Physics – Section B: Numerical Value Questions (20 marks)

46. Two iron solid discs of negligible thickness have radii R_1 and R_2 and moment of intertia I_1 and I_2, respectively. For $R_2 = 2R_1$, the ratio of I_1 and I_2 would be $1/x$, where x = _____ . [4 marks]

Solution:

Concept:

- A homogeneous thin disc of mass M and radius R has central perpendicular moment of inertia $I = \frac{1}{2}MR^2$.
- For discs of the same material and common thickness t, $M = \rho\pi R^2 t$, so mass is proportional to R^2, not independent of radius.

Step 1: Find how the masses scale Use the usual uniform thin-disc comparison, with both iron discs having the same thickness and density. The negligible thickness means that the thin-disc formula applies; the mass still depends on the amount of iron present. Writing the common density as ρ and thickness as t,

$$M_1 = \rho\pi R_1^2 t, \qquad M_2 = \rho\pi R_2^2 t.$$

Since $R_2 = 2R_1$, this gives

$$\frac{M_2}{M_1} = \left(\frac{R_2}{R_1}\right)^2 = 4.$$

The larger disc has four times the mass, so using equal masses in the moment-of-inertia ratio would be inappropriate.

Step 2: Combine the mass and radius factors For corresponding central perpendicular axes, substitute the mass dependence into the disc formula:

$$I = \frac{1}{2}(\rho\pi R^2 t)R^2 = \frac{\rho\pi t}{2}R^4.$$

Consequently,

$$\frac{I_1}{I_2} = \left(\frac{R_1}{R_2}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}.$$

Comparing with the stated form $I_1/I_2 = 1/x$ gives $x = 16$. For corresponding diametral axes the coefficient becomes $1/4$, but it cancels in the ratio and the same radius-to-the-fourth scaling results. The common-thickness assumption is implicit in this standard comparison; unequal unspecified thicknesses would introduce an additional thickness ratio.

Step 3: Conclusion The ratio is $I_1/I_2 = 1/16$, and the requested dimensionless value is $\boxed{x = 16}$.

Quick Tip to Solve: For similar thin discs of common material and thickness, doubling radius quadruples mass and multiplies moment of inertia by sixteen.

47. In a measurement, it is asked to find modulus of elasticity per unit torque applied on the system. The measured quantity has dimension of $\left[M^a L^b T^c\right]$. If $b = 3$, the value of c is _____. [4 marks]

Solution:

Concept:

- A modulus of elasticity is stress divided by strain. Strain is dimensionless, so a modulus has the dimensions of pressure: $[ML^{-1}T^{-2}]$.
- Torque is force multiplied by a perpendicular lever arm, with dimensions $[ML^2T^{-2}]$. Dimensional exponents subtract when one quantity is divided by another.

Step 1: Derive the dimensions of each factor Let the modulus be Y and applied torque be τ. From force $F = ma$,

$$[F] = [MLT^{-2}].$$

Stress is force per area, so

$$[Y] = \frac{[F]}{[L^2]} = [ML^{-1}T^{-2}].$$

The torque contains one further length factor:

$$[\tau] = [F][L] = [ML^2T^{-2}].$$

This derivation applies whether the elastic modulus describes tensile or shear stress: in either case strain is a ratio without dimensions. The torque is a mechanical moment, not a force alone.

Step 2: Divide and identify the time exponent The dimensions of modulus per unit torque are

$$\left[\frac{Y}{\tau}\right] = \frac{[ML^{-1}T^{-2}]}{[ML^2T^{-2}]} = [M^0L^{-3}T^0].$$

Thus the time powers cancel exactly: $c = -2 - (-2) = 0$, and the mass powers cancel as well. The literal ratio gives $b = -3$, whereas the question states $b = 3$; this sign inconsistency does not affect the requested time exponent. Even if the reciprocal ratio were considered, its dimensions would be $[L^3]$, again with time exponent zero. Hence the value of c is unambiguous without changing the question or assigning a nonzero time dependence to either ratio.

Step 3: Conclusion The measured ratio is independent of time dimension, so the requested exponent is $\boxed{c = 0}$.

Quick Tip to Solve: When two quantities contain the same time power, their ratio has time exponent zero; subtract exponents before using any stated auxiliary exponent.

48. The moment of inertia of a solid disc rotating along its diameter is 2.5 times higher than the moment of inertia of a ring rotating in similar way. The moment of inertia of a solid sphere which has same radius as the disc and rotating in similar way, is n times higher than the moment of inertia of the given ring. Here, n = _____ Consider all the bodies have equal masses. [4 marks]

Solution:

Concept:

- For a thin solid disc about a diameter, $I_d = \frac{1}{4}MR_d^2$. For a thin ring about a diameter, $I_r = \frac{1}{2}MR_r^2$.
- For a solid sphere about any diameter, $I_s = \frac{2}{5}MR_s^2$. Equal masses do not imply equal moments: the radius and distribution of mass both matter.

Step 1: Use the disc-to-ring ratio All bodies have the same mass M. Interpret the stated factor of 2.5 as the ratio of the disc moment to the ring moment, so $I_d = 2.5I_r$. The axes are diameters, as specified; using $\frac{1}{2}MR_d^2$ for the disc would instead describe the perpendicular symmetry axis and would be incorrect here. Substituting the diametral expressions gives

$$\frac{MR_d^2}{4} = \frac{5}{2}\left(\frac{MR_r^2}{2}\right), \qquad R_d^2 = 5R_r^2.$$

Thus the disc radius is $\sqrt{5}$ times the ring radius, not equal to it. This radius information is what the given inertia ratio supplies.

Step 2: Replace the disc by the solid sphere The sphere has the same radius as the disc, $R_s = R_d$, and rotates about a diameter. Its inertia relative to the ring is therefore

$$\frac{I_s}{I_r} = \frac{(2/5)MR_d^2}{(1/2)MR_r^2} = \frac{4}{5}\frac{R_d^2}{R_r^2} = \frac{4}{5} \times 5 = 4.$$

Equivalently, at the same mass and radius the sphere-to-disc inertia ratio is $(2/5)/(1/4) = 8/5$. Multiplying this by the given disc-to-ring factor $5/2$ again gives 4. This independent cancellation check avoids mixing up the two different ring and disc radii. The numerical factors are dimensionless; the mass and squared-radius units cancel in the final ratio.

Step 3: Conclusion The sphere has four times the moment of inertia of the ring. The requested value is $\boxed{n = 4}$.

Quick Tip to Solve: First compare sphere and disc at their common mass and radius: their diametral moments are in the ratio $8 : 5$; then use the given disc-to-ring ratio.

49. A tiny metallic rectangular sheet has length and breadth of 5 mm and 2.5 mm , respectively. Using a specially designed screw gauge which has pitch of 0.75 mm and 15 divisions in the circular scale, you are asked to find the area of the sheet. In this measurement, the maximum fractional error will be $\frac{x}{100}$ where x is _____ . [4 marks]

Solution:

Concept:

- The least count of a screw gauge is its pitch divided by the number of circular-scale divisions. It gives the smallest measured displacement and is used as the limiting absolute error here.
- For an area $A = lb$, the maximum fractional error, to first order in the small measurement errors, is $\Delta A/A = \Delta l/l + \Delta b/b$. The positive contributions are added for a maximum error; they are not subtracted.

Step 1: Calculate the least count The pitch is the advance of the screw in one complete revolution. Dividing the given pitch by the 15 equal divisions gives the displacement corresponding to one division:

$$\text{LC} = \frac{0.75}{15} = 0.05\,\text{mm}.$$

Thus the limiting absolute error in each independently measured dimension is $0.05\,\text{mm}$. Use the same unit for this error, the length and the breadth.

Step 2: Propagate the errors into the area The dimensions are $l = 5\,\text{mm}$ and $b = 2.5\,\text{mm}$, so the nominal area is $12.5\,\text{mm}^2$. The fractional contributions are $0.05/5 = 0.01$ and $0.05/2.5 = 0.02$. Therefore

$$\frac{\Delta A}{A} = \frac{0.05}{5} + \frac{0.05}{2.5} = 0.03 = \frac{3}{100}.$$

The smaller breadth contributes twice the fractional uncertainty of the length. This is the usual first-order maximum-error rule; retaining a product of errors is not part of this rule.

Step 3: Conclusion The maximum fractional error is $3/100$, or 3%. Comparing with $x/100$, the required numerical answer is $\boxed{x = 3}$.

Quick Tip to Solve: Find the least count first, then add its ratios to the two measured dimensions. For a product, maximum fractional errors add.

50. A double slit interference experiment performed with a light of wavelength 600 nm forms an interference fringe pattern on a screen with 10 th bright fringe having its centre at a distance of 10 mm from the central maximum. Distance of the centre of the same 10 th bright fringe from the central maximum when the source of light is replaced by another source of wavelength 660 nm would be _____ mm . [4 marks]

Solution:

Concept:

- The centre of the nth bright fringe in Young's double-slit experiment is at $y_n = n\lambda D/d$, where D is the slit-to-screen distance and d is the slit separation.

- For the same bright-fringe order and unchanged apparatus, n, D, d are fixed. Hence the displacement from the central maximum is directly proportional to the wavelength: $y_2/y_1 = \lambda_2/\lambda_1$.

Step 1: Keep the fringe order and geometry fixed The question tracks the centre of the same tenth bright fringe, not the fringe at the old screen position. Its order remains $n = 10$. Replacing the light source changes the wavelength but does not change the slit separation or the screen position. Write the two fringe positions as

$$y_1 = \frac{10\lambda_1 D}{d}, \qquad y_2 = \frac{10\lambda_2 D}{d}.$$

The central maximum remains the reference point, since its path difference is zero for either wavelength.

Step 2: Scale the measured position by the wavelength ratio Dividing the equations eliminates all unknown apparatus dimensions. Both wavelengths are in nanometres, so they can be used directly in their ratio:

$$y_2 = y_1 \frac{\lambda_2}{\lambda_1} = 10\,\text{mm} \times \frac{660}{600} = 11\,\text{mm}.$$

The wavelength has increased by ten percent; the fringe width and the displacement of every fixed-order bright fringe therefore increase by ten percent. The shift of this fringe is $1\,\text{mm}$, whereas the requested distance from the central maximum is its full new position.

Step 3: Conclusion The centre of the tenth bright fringe is now $\boxed{11\,\text{mm}}$ from the central maximum. The numerical entry is $\boxed{11}$.

Quick Tip to Solve: For a fixed-order fringe in the same apparatus, multiply its old distance by the new-to-old wavelength ratio. Distinguish its new position from its shift.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. The incorrect decreasing order of atomic radii is [4 marks]

(A) Si > P > Cl > F

(B) Mg > Al > C > O

(C) Al > B > N > F

(D) Be > Mg > Al > Si

Correct Answer: (D) Be > Mg > Al > Si

Solution:

Concept:

- Atomic radius generally decreases from left to right across a period because increasing effective nuclear charge draws electrons in the same principal shell closer to the nucleus.
- Atomic radius increases down a group because an additional principal shell is occupied and shielding increases. Apply the down-group comparison before accepting a proposed mixed-period sequence.

Step 1: Check the first three decreasing sequences For option (A), silicon, phosphorus and chlorine

lie in period 3, so $\mathrm{Si} > \mathrm{P} > \mathrm{Cl}$. Fluorine lies above chlorine in group 17 and is smaller, giving the stated final comparison. In option (B), magnesium is larger than aluminium across period 3; the period-2 atoms carbon and oxygen are smaller, with $\mathrm{C} > \mathrm{O}$. Thus $\mathrm{Mg} > \mathrm{Al} > \mathrm{C} > \mathrm{O}$ is consistent. For option (C), aluminium is below boron in group 13 and hence larger, while within period 2 the size decreases from boron through nitrogen to fluorine. The entire sequence is again consistent.

Step 2: Identify the reversed down-group comparison Option (D) begins with $\mathrm{Be} > \mathrm{Mg}$. This reverses the group-2 trend: magnesium has an additional occupied electron shell compared with beryllium, so $\mathrm{Mg} > \mathrm{Be}$. The following comparisons $\mathrm{Mg} > \mathrm{Al} > \mathrm{Si}$ do follow the period-3 trend, but one incorrect adjacent comparison is enough to invalidate the whole decreasing sequence. Since the question asks for the incorrect order, the reversed first pair decides the answer.

Step 3: Conclusion The incorrect decreasing order is $\mathrm{Be} > \mathrm{Mg} > \mathrm{Al} > \mathrm{Si}$, because magnesium is larger than beryllium. Hence, the correct option is (D).

Quick Tip to Solve: In a proposed decreasing sequence, test each adjacent pair. The pair Be and Mg immediately reveals the reversed down-group trend.

52. What is the freezing point depression constant of a solvent, 50 g of which contain 1 g non volatile solute (molar mass $256\ \mathrm{g\ mol^{-1}}$) and the decrease in freezing point is 0.40 K ? [4 marks]

(A) $4.43\ \mathrm{K\ kg\ mol^{-1}}$

(B) $3.72\ \mathrm{K\ kg\ mol^{-1}}$

(C) $5.12\ \mathrm{K\ kg\ mol^{-1}}$

(D) $1.86\ \mathrm{K\ kg\ mol^{-1}}$

Correct Answer: (C) $5.12\ \mathrm{K\ kg\ mol^{-1}}$

Solution:

Concept:

- For a dilute solution of a nonvolatile, non-electrolyte solute, freezing point depression obeys $\Delta T_f = K_f m$, where m is the solute molality.
- Molality is moles of solute per kilogram of solvent, not per kilogram of solution. Thus $m = (w/M)/W$, where w is solute mass in grams, M is molar mass and W is solvent mass in kilograms.

Step 1: Calculate the solution molality The solute amount is obtained using its given molar mass:

$$n = \frac{1\,\mathrm{g}}{256\,\mathrm{g\,mol^{-1}}} = \frac{1}{256}\,\mathrm{mol}.$$

The solvent mass is $50\,\mathrm{g} = 0.050\,\mathrm{kg}$. Therefore

$$m = \frac{1/256}{0.050} = 0.078125\,\mathrm{mol\,kg^{-1}}.$$

Only the 50 g solvent belongs in this denominator; adding the 1 g solute would calculate a different concentration and is inappropriate for molality.

Step 2: Obtain the constant and check the options Rearrange the depression law and substitute the

positive magnitude of the temperature decrease:

$$K_f = \frac{\Delta T_f}{m} = \frac{0.40}{0.078125} = 5.12\,\mathrm{K\,kg\,mol^{-1}}.$$

The units follow by dividing kelvin by moles per kilogram. As a direct check, $5.12 \times 0.078125 = 0.40$, exactly recovering the given depression. The other listed constants, 4.43, 3.72 and 1.86, do not reproduce the stated decrease for this molality. No dissociation factor is specified, so the ordinary molecular-solute relation is used.

Step 3: Conclusion The freezing point depression constant is $\boxed{K_f = 5.12\,\mathrm{K\,kg\,mol^{-1}}}$. Hence, the correct option is (C).

Quick Tip to Solve: Convert solvent grams to kilograms before computing molality. A quick check is to multiply your proposed constant by the molality and recover the measured depression.

53. Given below are two statements:

Statement I : D-glucose pentaacetate reacts with 2,4 -dinitrophenylhydrazine

Statement II : Starch, on heating with concentrated sulfuric acid at $100^\circ\mathrm{C}$ **and** $2-3\mathrm{atmosphere}$ **pressure produces glucose.**

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Statement I is true but Statement II is false

(B) Both Statement I and Statement II are true

(C) Statement I is false but Statement II is true

(D) Both Statement I and Statement II are false

Correct Answer: (D) Both Statement I and Statement II are false

Solution:

Concept:

- 2,4-Dinitrophenylhydrazine detects a free aldehyde or ketone carbonyl by formation of a hydrazone. A protected anomeric centre that cannot open to a free aldehyde does not give this reaction.
- Starch is a polysaccharide whose glycosidic bonds yield glucose on hydrolysis with dilute acid. Concentrated sulfuric acid is strongly dehydrating; it is not the dilute-acid hydrolysis condition.

Step 1: Evaluate the glucose pentaacetate statement In glucose pentaacetate, the five hydroxyl groups of cyclic glucose are acetylated, including the anomeric hydroxyl group. The resulting protected cyclic form cannot establish the normal ring-chain equilibrium that exposes the aldehyde group of free glucose. Its acetate ester carbonyl groups are not free aldehyde or ketone groups and do not give the usual 2,4-dinitrophenylhydrazine test. Therefore D-glucose pentaacetate does not react as stated, and Statement I is false.

Step 2: Evaluate the stated conditions for starch Acid-catalysed hydrolysis of starch can be represented as

$$(\mathrm{C_6H_{10}O_5})_n + n\mathrm{H_2O} \longrightarrow n\mathrm{C_6H_{12}O_6}.$$

This conversion requires water and the usual dilute-acid hydrolysis conditions. The statement specifically says concentrated sulfuric acid. Concentrated H_2SO_4 acts as a powerful dehydrating agent toward carbohydrates and promotes dehydration and charring rather than the claimed preparation of glucose under these conditions. Heating and the quoted pressure do not replace the need for the proper hydrolysis medium. Thus Statement II is also false. Options (A), (B) and (C) each incorrectly mark at least one of these statements true.

Step 3: Conclusion Statement I is false and Statement II is false. Hence, the correct option is (D). The listed final key treats Statement II as true, but the explicitly stated concentrated sulfuric acid makes it false. The stated answer follows the literal reagent; glucose preparation instead requires the appropriate dilute-acid hydrolysis conditions.

Quick Tip to Solve: For carbohydrate statements, inspect the exact reagent and its concentration: a protected anomeric centre suppresses the aldehyde test, and dilute acid hydrolyses whereas concentrated sulfuric acid dehydrates.

54. A weak acid HA has degree of dissociation x . Which option gives the correct expression of (pH - pK_a)? [4 marks]

(A) $\log\left(\frac{1-x}{x}\right)$

(B) 0

(C) $\log(1+2x)$

(D) $\log\left(\frac{x}{1-x}\right)$

Correct Answer: (D) $\log\left(\frac{x}{1-x}\right)$

Solution:

Concept:

- For $HA \rightleftharpoons H^+ + A^-$, the acid dissociation constant is $K_a = [H^+][A^-]/[HA]$. The degree of dissociation x is the fraction of the initial acid molecules that ionise.
- The definitions $pH = -\log[H^+]$ and $pK_a = -\log K_a$ imply $pH - pK_a = \log(K_a/[H^+])$, using base-10 logarithms.

Step 1: Express the equilibrium concentrations Let the initial analytical concentration of the weak acid be C, and use the ordinary weak-acid treatment in which water ionisation is negligible. At equilibrium, dissociation of the fraction x gives

$$[H^+] = Cx, \quad [A^-] = Cx, \quad [HA] = C(1-x).$$

The remaining acid is proportional to $1-x$, not to $1+x$. Substitution into the equilibrium expression gives

$$K_a = \frac{C^2x^2}{C(1-x)} = \frac{Cx^2}{1-x}.$$

No approximation $1-x \simeq 1$ is required.

Step 2: Form the logarithmic difference Divide the last expression by the hydrogen-ion concentration.

The unknown initial concentration cancels:

$$\frac{K_a}{[\mathrm{H}^+]} = \frac{Cx^2}{(1-x)Cx} = \frac{x}{1-x}.$$

Taking logarithms therefore yields

$$\mathrm{pH} - \mathrm{p}K_a = \log\left(\frac{x}{1-x}\right).$$

Option (A) is the negative of this result; option (B) applies only at the special case $x = 1/2$, not for general dissociation. Option (C) is not the equilibrium concentration ratio. For a small degree of dissociation the ratio is below one, so the negative logarithm also gives a useful sign check.

Step 3: Conclusion The required expression is $\boxed{\log\left(\frac{x}{1-x}\right)}$. Hence, the correct option is (D).

Quick Tip to Solve: Write the equilibrium ratio of conjugate base to undissociated acid as x/(1-x); the pH minus pKa is its logarithm. This also prevents reversing the ratio.

55. For a given reaction $\mathrm{R} \rightarrow \mathrm{P}, t_{1/2}$ is related to $[\mathrm{A}]_0$ as given in table.

Given: $\log 2 = 0.30$

Which of the following is true?

A. The order of the reaction is $1/2$.

B. If $[\mathrm{A}]_0$ is 1 M , then $t_{1/2}$ is $200\sqrt{10}$ min

C. The order of the reaction changes to 1 if the concentration of reactant changes from 0.100 M to 0.500 M.

D. $t_{1/2}$ is 800 min for $[\mathrm{A}]_0 = 1.6\mathrm{M}$

Choose the correct answer from the options given below: [4 marks]

$[A]_o / \mathrm{mol\ L^{-1}}$	$t_{1/2}$ / min
0.100	200
0.025	100

(A) A and C Only

(B) A, B and D Only

(C) A and B Only

(D) C and D Only

Correct Answer: (B) A, B and D Only

Solution:

Concept:

- For a reaction of order $n \neq 1$, the half-life at fixed temperature obeys $t_{1/2} \propto [A]_0^{1-n}$. The rate constant remains fixed when only the initial concentration is changed.
- For half-order kinetics, integration of $-d[A]/dt = k[A]^{1/2}$ gives $t_{1/2} = 2(1 - 1/\sqrt{2})\sqrt{[A]_0}/k$.

Step 1: Determine the reaction order The concentration decreases from 0.100 M to 0.025 M, a factor of four, while the half-life decreases from 200 min to 100 min, a factor of two. Therefore

$$\frac{200}{100} = \left(\frac{0.100}{0.025}\right)^{1-n} = 4^{1-n}.$$

Taking logarithms gives $0.30 = (1 - n)(0.60)$, hence $n = 1/2$. Statement A is true.

Step 2: Check the predicted half-lives Use the first observation as the reference so that the unknown rate constant cancels:

$$t_{1/2} = 200\sqrt{\frac{[A]_0}{0.100}}\ \text{min}.$$

At 1 M this is $200\sqrt{10}$ min, confirming B. At 1.6 M it is $200\sqrt{16} = 800$ min, confirming D.

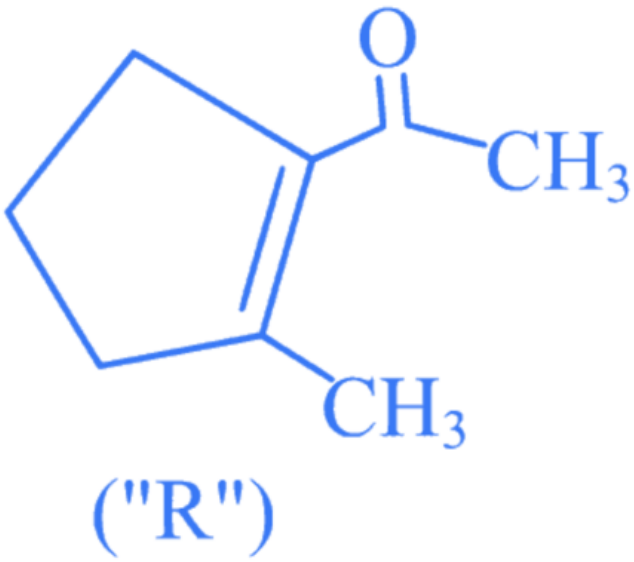
Step 3: Reject the concentration-dependent order claim Changing initial concentration changes the half-life according to the same rate law; it does not change the reaction order from one-half to one. There is no evidence of a mechanism change. Thus C is false. Options A and D include C, while option C omits the independently verified statement D.

Step 4: Conclusion A, B and D are true; C is false. Hence, the correct option is (B).

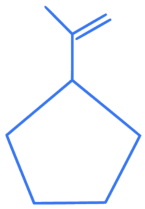
Quick Tip to Solve: Compare half-life ratios before calculating a rate constant: a fourfold concentration increase giving a twofold half-life increase identifies half-order kinetics.

56. A molecule (“P”) on treatment with acid undergoes rearrangement and gives (“Q”). (“Q”) on ozonolysis followed by reflux under alkaline condition gives (” R “). The structure of (” R “) is given below.

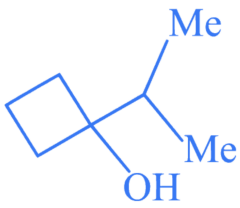
The structure of (“P”) is [4 marks]



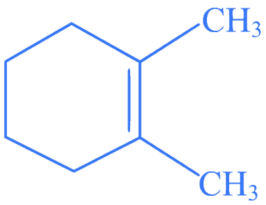
(A) Isopropenylcyclopentane



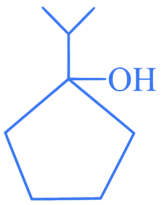
(B) 1-(Propan-2-yl)cyclobutan-1-ol



(C) 1,2-Dimethylcyclohex-1-ene

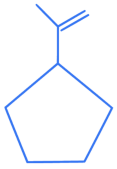


(D) 1-(Propan-2-yl)cyclopentan-1-ol



Correct Answer: (A)

Isopropenylcyclopentane



Solution:

Concept:

- Acid-induced protonation or water loss produces carbocations; a cyclopentylcarbinyl cation can expand to a six-membered ring by migration of an adjacent carbon-carbon bond.
- Ozonolysis cleaves an alkene into carbonyl compounds. A suitable diketone undergoes intramolecular aldol condensation on heating with base, forming a conjugated cyclic enone.

Step 1: Work backwards from the final enone The product contains a five-membered ring, a methyl substituent on its double bond and an acetyl group on the adjacent double-bond carbon. The diketone that provides this skeleton is $CH_3CO(CH_2)_4COCH_3$, octane-2,7-dione. Enolate formation at an internal methylene adjacent to one carbonyl, attack on the other carbonyl, and dehydration give the shown five-membered conjugated product.

Step 2: Identify the alkene before ozonolysis Joining the two carbonyl carbons of this diketone by a double bond reconstructs 1,2-dimethylcyclohex-1-ene as Q. Cleavage of that endocyclic double bond opens the six-membered ring, leaving four methylene groups between the two ketone carbonyls. Thus the alkene precursor must have the correct eight-carbon skeleton.

Step 3: Trace the acid-induced rearrangement In A, Markovnikov protonation of the isopropenyl group gives a tertiary cyclopentylcarbinyl cation. Ring expansion followed by a 1,2-methyl shift and loss of a proton can give the more substituted six-membered alkene Q. In D, protonation of the alcohol and loss of water, followed by hydride transfer from the isopropyl carbon, accesses the same cyclopentylcarbinyl cation and hence the same rearranged alkene. B has a different carbon skeleton. C already depicts the alkene Q and does not supply the acid-induced skeletal rearrangement specified for P.

Step 4: Conclusion Option (A) provides the required rearrangement sequence. The official final answer key accepts both A and D, since both can reach the same rearranged alkene and final product. Hence, the correct option is (A).

Quick Tip to Solve: Reverse an intramolecular aldol condensation first: restore the two carbonyl groups, then reverse ozonolysis to identify the alkene required before examining carbocation rearrangements.

57. The products A and B in the following reactions, respectively are

$$A \overset{Ag-NO_2}{\longleftrightarrow} CH_3 - CH_2 - CH_2 - Br \xrightarrow{AgCN} B$$

[4 marks]

(A) $CH_3 - CH_2 - CH_2 - ONO, CH_3 - CH_2 - CH_2 - CN$

(B) $CH_3 - CH_2 - CH_2 - NO_2, CH_3 - CH_2 - CH_2 - CN$

(C) $CH_3 - CH_2 - CH_2 - ONO, CH_3 - CH_2 - CH_2 - NC$

(D) $CH_3 - CH_2 - CH_2 - NO_2, CH_3 - CH_2 - CH_2 - NC$

Correct Answer: (D) $CH_3 - CH_2 - CH_2 - NO_2, CH_3 - CH_2 - CH_2 - NC$

Solution:

Concept:

- An ambident nucleophile can form a new bond through either of two atoms. The reagent determines which atom is available for bonding to the alkyl group.
- Silver nitrite predominantly gives nitroalkanes, $R-NO_2$, while silver cyanide predominantly gives isocyanides, $R-NC$.

Step 1: Determine the product with silver nitrite The starting haloalkane is 1-bromopropane, $CH_3CH_2CH_2Br$. With silver nitrite, the relatively covalent silver-oxygen interaction makes nitrogen the effective attacking site. Replacement of bromide therefore gives a carbon-nitrogen bond:

$$CH_3CH_2CH_2Br + AgNO_2 \longrightarrow CH_3CH_2CH_2NO_2 + AgBr.$$

Product A is 1-nitropropane, not propyl nitrite. The latter contains the different linkage $R-O-N=O$.

Step 2: Determine the product with silver cyanide Silver cyanide is also substantially covalent. Its carbon end is associated with silver, so reaction of the haloalkane occurs through nitrogen and gives propyl isocyanide:

$$CH_3CH_2CH_2Br + AgCN \longrightarrow CH_3CH_2CH_2NC + AgBr.$$

Product B therefore has the connectivity $R-NC$, rather than the nitrile connectivity $R-CN$.

Step 3: Compare all the listed pairs Option A assigns both the wrong nitrite linkage and the wrong cyanide linkage. B gives the correct nitro product but incorrectly assigns a nitrile to silver cyanide. C gives the correct isocyanide but the wrong nitrite product. Only D gives both carbon-nitrogen linkages correctly. No rearrangement or change of the three-carbon propyl skeleton is needed.

Step 4: Conclusion $A=CH_3CH_2CH_2NO_2$ and $B=CH_3CH_2CH_2NC$. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish silver salts from alkali-metal salts: AgNO2 favours nitroalkanes and AgCN favours isocyanides; the linkage, rather than the molecular formula alone, decides the answer.

58. In a multielectron atom, which of the following orbitals described by three quantum numbers will have same energy in absence of electric and magnetic fields?

A. $\mathrm{n}=1, \mathrm{l}=0,\ \mathrm{m_l}=0$

B. $\mathrm{n}=2, \mathrm{l}=0,\ \mathrm{m_l}=0$

C. $\mathrm{n}=2, \mathrm{I}=1,\ \mathrm{m_l}=1$

D. $\mathrm{n}=3, \mathrm{l}=2,\ \mathrm{m_l}=1$

E. $\mathrm{n}=3, \mathrm{l}=2,\ \mathrm{m_l}=0$

Choose the correct answer from the options given below: [4 marks]

(A) B and C Only

(B) C and D Only

(C) A and B Only

(D) D and E Only

Correct Answer: (D) D and E Only

Solution:

Concept:

- In a multielectron atom, orbital energy depends on both the principal quantum number n and the azimuthal quantum number l, because shielding and penetration differ between subshells.
- In the absence of electric and magnetic fields, orbitals within a fixed (n, l) subshell are degenerate: changing m_l changes orientation but not energy.

Step 1: Identify the subshell for each set A has $n = 1, l = 0$, so it is a 1s orbital. B has $n = 2, l = 0$, so it is 2s. C has $n = 2, l = 1$, so it is a 2p orbital. D and E both have $n = 3, l = 2$, identifying them as 3d orbitals. Their different magnetic quantum numbers, respectively 1 and 0, specify two orientations of the same subshell.

Step 2: Apply the field-free degeneracy condition D and E share both quantum numbers that determine the subshell energy. With no external field to select a spatial direction, these orientations have identical energies. Their m_l values are also allowed because a d subshell permits $m_l = -2, -1, 0, 1, 2$. Thus they form a valid degenerate pair rather than merely two algebraically similar labels.

Step 3: Eliminate the other proposed pairs B and C share $n = 2$ but differ in l; unlike a hydrogenic atom, a multielectron atom does not generally make 2s and 2p equal in energy. C and D are different principal shells and different subshells. A and B are 1s and 2s, which also differ in energy. Consequently options A, B and C all pair nondegenerate orbitals. The condition about absence of fields is essential because a field can lift orientational degeneracy.

Step 4: Conclusion Only D and E are orbitals of the same field-free 3d subshell. Hence, the correct option is (D).

Quick Tip to Solve: For field-free multielectron atoms, match both n and l; different magnetic quantum numbers alone do not split a subshell into different energies.

59. Given below are two statements:

Statement I: In the oxalic acid vs $KMnO_4$ (in the presence of dil H_2SO_4) titration the solution needs to be heated initially to $60°C$, but no heating is required in Ferrous ammonium sulphate (FAS) vs $KMnO_4$ titration (in the presence of dil H_2SO_4)

Statement II: In oxalic acid vs $KMnO_4$ titration, the initial formation of $MnSO_4$ takes place at high temperature, which then acts as catalyst for further reaction. In the case of FAS vs $KMnO_4$, heating oxidizes Fe^{2+} into Fe^{3+} by oxygen of air and error may be introduced in the experiment.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Statement I is false but Statement II is true

(B) Statement I is true but Statement II is false

(C) Both Statement I and Statement II are true

(D) Both Statement I and Statement II are false

Correct Answer: (C) Both Statement I and Statement II are true

Solution:

Concept:

- Acidified permanganate oxidises oxalate to carbon dioxide and ferrous ions to ferric ions, while manganese is reduced from +7 to +2.
- The oxalate reaction is initially slow and is autocatalysed by the manganese(II) ions produced. Ferrous-ion oxidation by permanganate is sufficiently fast at room temperature.

Step 1: Assess the required temperatures For the oxalic-acid titration, warming the solution initially to about $60°C$ increases the rate of the slow initial reaction. Once sufficient Mn^{2+} has formed, autocatalysis speeds up subsequent additions. In a FAS titration the oxidation of Fe^{2+} is already rapid, so this initial heating is unnecessary. These observations establish that Statement I is true.

Step 2: Explain the catalytic behaviour The oxalate redox equation in acidic solution is

$$2\,MnO_4^- + 5\,C_2O_4^{2-} + 16\,H^+ \longrightarrow 2\,Mn^{2+} + 10\,CO_2 + 8\,H_2O.$$

In the sulphuric-acid medium the manganese(II) product is present as manganese sulphate. It promotes further reaction, explaining why the first decolourisation is slower and later portions react more readily. Heating initiates a practical titration; the catalyst then sustains a convenient rate.

Step 3: Check why heating FAS is avoided In FAS, unwanted oxidation of Fe^{2+} by dissolved or atmospheric oxygen reduces the amount left for reaction with permanganate. Heating favours this side oxidation and can therefore give an erroneous titre. The intended titration is

$$MnO_4^- + 5\,Fe^{2+} + 8\,H^+ \longrightarrow Mn^{2+} + 5\,Fe^{3+} + 4\,H_2O.$$

Both explanations in Statement II are correct. Options A, B and D each incorrectly declare at least one true statement false.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (C).

Quick Tip to Solve: Remember the contrast: warm oxalate to overcome its slow start, but keep ferrous titrations unheated to minimise oxidation of Fe(II) by air.

60. Consider the following elements $\mathrm{In}, \mathrm{Tl}, \mathrm{Al}, \mathrm{Pb}, \mathrm{Sn}$ and Ge . The most stable oxidation states of elements with highest and lowest first ionisation enthalpies, respectively, are [4 marks]

(A) +4 and +1

(B) +4 and +3

(C) +1 and +4

(D) +2 and +3

Correct Answer: (B) +4 and +3

Solution:

Concept:

- First ionisation enthalpy is the energy needed to remove the first electron from an isolated gaseous atom. Its ordering must be established before assigning oxidation states.
- The inert-pair effect stabilises lower oxidation states down the p block, but germanium generally favours +4 and indium generally favours +3.

Step 1: Locate the largest and smallest first ionisation enthalpies Among the six listed elements, germanium has the highest first ionisation enthalpy, approximately $762\ \mathrm{kJ\,mol^{-1}}$. The other values are smaller: Pb about 716, Sn about 709, Tl about 589, Al about 578 and In about $558\ \mathrm{kJ\,mol^{-1}}$. Thus the required pair of elements, in the question's order, is Ge followed by In. Simply choosing the heaviest element would be incorrect.

Step 2: Assign the stable oxidation states Germanium has the valence configuration $4s^2 4p^2$. Participation of all four valence electrons makes +4 its most characteristic stable oxidation state. Indium has $5s^2 5p^1$; although a +1 state exists and becomes more significant down Group 13, indium commonly favours +3. The +1 state is especially stabilised for thallium, which is not the element of lowest first ionisation enthalpy in this list.

Step 3: Match the requested order to the options The first entry must therefore be +4 for Ge, and the second +3 for In. Option A incorrectly replaces indium's stable +3 state with +1. Option C reverses the oxidation-state pattern and does not match either selected element. Option D assigns +2 to the highest-ionisation element, whereas the selected Ge favours +4. The ionisation-enthalpy ranking and oxidation-state trend are separate decisions; keeping them separate avoids an inert-pair-effect shortcut applied to the wrong atom.

Step 4: Conclusion The elements are Ge and In, whose most stable oxidation states are respectively +4 and +3. Hence, the correct option is (B).

Quick Tip to Solve: Identify the extremal ionisation enthalpies first; only then use group trends to assign oxidation states. The strongest inert-pair effect in a list need not identify its lowest-ionisation element.

61. Both acetaldehyde and acetone (individually) undergo which of the following reactions?

A. Iodoform Reaction

B. Cannizaro Reaction

C. Aldol Condensation

D. Tollen's Test

E. Clemmensen Reduction

Choose the correct answer from the options given below: [4 marks]

(A) C and E Only

(B) A, B and D Only

(C) A, C and E Only

(D) B, C and D Only

Correct Answer: (C) A, C and E Only

Solution:

Concept:

- Acetaldehyde is CH_3CHO, whereas acetone is CH_3COCH_3. Both contain alpha hydrogen atoms and the methyl carbonyl arrangement needed for the iodoform test.
- Aldol condensation requires an enolisable carbonyl compound. Clemmensen reduction converts the carbonyl group into a methylene group using zinc amalgam and concentrated hydrochloric acid.

Step 1: Check the reactions common to both compounds In the iodoform reaction, acetaldehyde and the methyl ketone acetone both give yellow CHI_3 with iodine and alkali. Both also undergo base-catalysed aldol addition followed by dehydration: acetaldehyde gives an aldol that dehydrates to crotonaldehyde, while acetone gives diacetone alcohol and then mesityl oxide. Thus statements A and C apply separately to each compound.

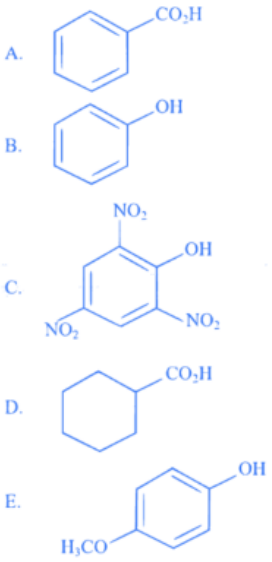
Step 2: Eliminate the distinguishing tests Cannizzaro disproportionation is characteristic of aldehydes lacking alpha hydrogen. Acetaldehyde has alpha hydrogen, and acetone is a ketone, so B is not a reaction common to both. Tollens reagent oxidises acetaldehyde and gives a silver mirror, but ordinary acetone does not respond, excluding D. Clemmensen reduction reduces acetaldehyde to ethane and acetone to propane, so E is common to both. Option A misses the iodoform reaction, while options B and D include reactions that fail the requirement that both compounds react.

Step 3: Conclusion The shared reactions are A, C and E: iodoform reaction, aldol condensation and Clemmensen reduction. Hence, the correct option is (C).

Quick Tip to Solve: For a pair of carbonyl compounds, check alpha hydrogens and the methyl carbonyl group before applying aldehyde-specific oxidation tests.

62. The compounds that produce CO_2 with aqueous $NaHCO_3$ solution are:

Choose the correct answer from the options given below: [4 marks]



(A) A and C Only

(B) A, B and E Only

(C) A and B Only

(D) A, C and D Only

Correct Answer: (D) A, C and D Only

Solution:

Concept:

- An acid liberates CO_2 from aqueous $NaHCO_3$ when it is sufficiently stronger than carbonic acid. Carboxylic acids normally satisfy this requirement; ordinary phenols do not.
- Electron-withdrawing nitro groups stabilise a phenoxide conjugate base and markedly increase phenolic acidity. Electron-donating methoxy groups have the opposite effect in the displayed para-substituted phenol.

Step 1: Identify the carboxylic acids Compound A is benzoic acid and compound D is cyclohexanecarboxylic acid. Each contains the carboxyl group, whose conjugate base is resonance-stabilised over two oxygen atoms. Both therefore react with bicarbonate, forming their sodium carboxylate, water and carbon dioxide:

$$RCO_2H + NaHCO_3 \longrightarrow RCO_2Na + H_2O + CO_2\uparrow.$$

The aliphatic ring in D does not remove the acidity of its attached carboxyl group.

Step 2: Compare the three phenolic compounds B is phenol, which is too weak an acid to give the bicarbonate effervescence test. E is para-methoxyphenol; its methoxy group donates electron density by resonance and does not make it acidic enough to release carbon dioxide. C is 2,4,6-trinitrophenol, or picric acid. Its three nitro groups strongly stabilise the conjugate base by inductive and resonance withdrawal, making C sufficiently acidic to react. Thus A and C alone omit D; the combinations

containing B or E incorrectly treat ordinary phenols as carboxylic acids.

Step 3: Conclusion Carbon dioxide is liberated by A, C and D only. Hence, the correct option is (D).

Quick Tip to Solve: In the bicarbonate test, look for a carboxyl group first; then check whether a phenol has several strong electron-withdrawing groups, as in picric acid.

63. The metal ion whose electronic configuration is not affected by the nature of the ligand and which gives a violet colour in non-luminous flame under hot condition in borax bead test is [4 marks]

(A) Cr^{3+}

(B) Ti^{3+}

(C) Ni^{2+}

(D) Mn^{2+}

Correct Answer: (C) Ni^{2+}

Solution:

Concept:

- The electronic configuration of a transition-metal ion is obtained by removing the outer $4s$ electrons before the $3d$ electrons. In an octahedral field, a d^8 ion has the configuration $t_{2g}^6 e_g^2$, irrespective of ligand strength.
- Borax-bead colours depend on both the metal and the observation conditions, including whether the bead is hot or cold. The violet bead under the stated hot condition is associated with nickel.

Step 1: Apply the electronic-configuration clue Nickel has atomic number 28 and ground-state configuration $[Ar]3d^8 4s^2$. Loss of its two outer $4s$ electrons gives Ni^{2+} : $[Ar]3d^8$. In an octahedral ligand field its lower three d orbitals contain six electrons, and the remaining two occupy the two upper orbitals separately: $t_{2g}^6 e_g^2$. Increasing the splitting does not produce a different high-spin or low-spin octahedral arrangement for this electron count.

Step 2: Use the hot-bead observation to identify the ion The question specifically asks for the violet borax bead observed while hot in the non-luminous flame, rather than a colour remembered without the temperature condition. This identifies nickel and selects Ni^{2+}. Chromium and titanium do not match this hot-violet observation. Although manganese is also associated with violet or amethyst beads in qualitative analysis, Mn^{2+} has a d^5 configuration whose spin arrangement can change with ligand strength; it therefore does not satisfy the combined clues. The temperature qualifier and the configuration requirement must be used together, rather than choosing from bead colour alone.

Step 3: Conclusion The ion satisfying the stated hot-bead colour and configuration clues is Ni^{2+}. Hence, the correct option is (C).

Quick Tip to Solve: For borax-bead questions, retain the hot/cold qualifier and cross-check the ion with its d-electron configuration before selecting the option.

64. Consider ' n ' is the number of lone pair of electrons present in the equatorial position of the most stable structure of ClF_3. The ions from the following with ' n ' number of unpaired electrons are

A. V^{3+}

B. Ti^{3+}

C. Cu^{2+}

D. Ni^{2+}

E. Ti^{2+}

Choose the correct answer from the options given below: [4 marks]

(A) B and D Only

(B) A, D and E Only

(C) B and C Only

(D) A and C Only

Correct Answer: (B) A, D and E Only

Solution:

Concept:

- In ClF_3, chlorine has five electron domains: three bond pairs and two lone pairs. The electron-domain arrangement is trigonal bipyramidal and the molecular shape is T-shaped.
- Lone pairs preferentially occupy equatorial sites because an equatorial site has fewer close 90° interactions. For the free ions listed, Hund's rule determines the number of unpaired d electrons.

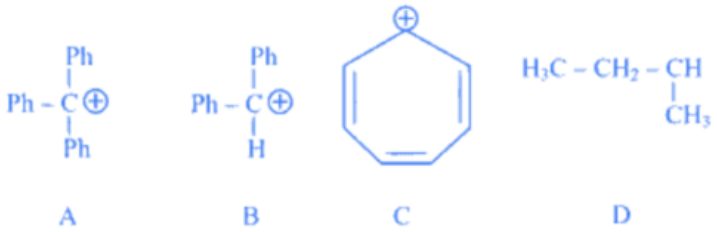
Step 1: Find the number of equatorial lone pairs Chlorine contributes seven valence electrons. Three electrons form three chlorine-fluorine bonds, leaving four nonbonding electrons as two lone pairs. Both lone pairs occupy equatorial positions in the most stable trigonal-bipyramidal arrangement. Therefore $n = 2$. The remaining equatorial site and the two axial sites are occupied by fluorine atoms; counting the three bonds as lone pairs would give the wrong target.

Step 2: Count the unpaired electrons of every ion Remove the outer $4s$ electrons before the $3d$ electrons. The configurations and unpaired counts are A, V^{3+} : $3d^2$, two; B, Ti^{3+} : $3d^1$, one; C, Cu^{2+} : $3d^9$, one; D, Ni^{2+} : $3d^8$, two; and E, Ti^{2+} : $3d^2$, two. In a d subshell, the first five electrons occupy separate orbitals and subsequent electrons pair, giving two unpaired electrons for d^8. Thus A, D and E meet the required count. Options containing B or C incorrectly include ions with only one unpaired electron.

Step 3: Conclusion Both equatorial lone pairs give $n = 2$. The matching ions are A, D and E only. Hence, the correct option is (B).

Quick Tip to Solve: For this combined question, finish the VSEPR count first and then compare that integer with each ion's unpaired-electron count.

65. The correct order of stability of following carbocations is : [4 marks]



(A) A > B > C > D

(B) C > B > A > D

(C) B > C > A > D

(D) C > A > B > D

Correct Answer: (D) C > A > B > D

Solution:

Concept:

- Aromatic carbocations receive cyclic delocalisation stabilisation when they are planar, fully conjugated and contain $4k + 2$ pi electrons. The tropylium cation is a standard six-pi-electron example.
- Benzylic carbocations are stabilised by delocalisation into attached phenyl rings. Additional phenyl rings provide more resonance pathways, whereas an ordinary secondary alkyl carbocation relies mainly on hyperconjugation and the inductive effect.

Step 1: Recognise the structures in the figure A is the triphenylmethyl cation, Ph_3C^+, and B is the diphenylmethyl cation, Ph_2CH^+. C is the seven-membered tropylium cation: three double bonds supply six pi electrons, and the positively charged carbon supplies an empty p orbital to complete cyclic conjugation. D is a secondary butyl carbocation, with no aromatic ring directly conjugated to its vacant p orbital.

Step 2: Apply aromaticity and resonance stabilisation C has a delocalised positive charge around an aromatic ring and satisfies $4k + 2 = 6$ with $k = 1$, so it is placed first. Between A and B, A has three phenyl rings available for charge delocalisation whereas B has two; the conventional resonance comparison therefore gives A greater stability than B. D has only alkyl hyperconjugation and electron donation, making it the least stable of these structures. These comparisons give $C > A > B > D$. The first option incorrectly puts the aromatic tropylium ion below both benzylic ions, and the second and third incorrectly rank B above A.

Step 3: Conclusion The stability order is $C > A > B > D$. Hence, the correct option is (D).

Quick Tip to Solve: Identify an aromatic carbocation before counting alkyl substituents: tropylium stabilisation outweighs the ordinary secondary-carbocation effect.

66. Match List I with List II.

	List I (Redox reaction)		List II (Type of redox reaction)
(A)	$CH_4(g) + 2\,O_2(g) \xrightarrow{\Delta} CO_2(g) + 2\,H_2O\,(l)$	**(I)**	**Disproportionation reaction**
(B)	$2\,NaH\,(s) \xrightarrow{\Delta} 2\,Na\,(s) + H_2(g)$	**(II)**	**Combination reaction**
(C)	$V_2O_5(s) + 5\,Ca\,(s) \xrightarrow{\Delta} 2\,V\,(s) + 5\,CaO\,(s)$	**(III)**	**Decomposition reaction**
(D)	$2\,H_2O_2(aq) \xrightarrow{\Delta} 2\,H_2O\,(l) + O_2(g)$	**(IV)**	**Displacement reaction**

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-IV, C-I, D-II

(B) A-IV, B-I, C-II, D-III

(C) A-II, B-III, C-I, D-IV

(D) A-II, B-III, C-IV, D-I

Correct Answer: (D) A-II, B-III, C-IV, D-I

Solution:

Concept:

- Redox classifications distinguish decomposition, displacement and disproportionation by the pattern of reactants and products. Oxidation numbers identify the simultaneous electron loss and gain.
- In disproportionation, one element in a single oxidation state is both oxidised and reduced. In the supplied classification, combustion of methane is placed under combination with oxygen.

Step 1: Match decomposition and displacement In B, a single compound, sodium hydride, breaks into sodium and hydrogen on heating, so B is a decomposition reaction and matches III. Sodium changes from $+1$ to zero while hydride hydrogen changes from -1 to zero, confirming redox. In C, calcium removes oxygen from vanadium pentoxide, displacing vanadium as the free metal. Calcium changes from zero to $+2$ and vanadium from $+5$ to zero, so C matches IV, displacement.

Step 2: Identify disproportionation and complete the matching In D, oxygen in hydrogen peroxide has oxidation state -1. It becomes -2 in water and zero in oxygen gas, so the same element undergoes reduction and oxidation: D matches I, disproportionation. This leaves A matched with II in the supplied list. A is specifically methane combustion, with carbon oxidised from -4 to $+4$ and oxygen reduced from zero to -2; it is described here as combination with oxygen. Strictly it is not a simple synthesis reaction giving only one product, since both carbon dioxide and water form. The unmistakable B-III, C-IV and D-I assignments determine the intended option uniquely, and the other options contradict those assignments.

Step 3: Conclusion The intended matching is A-II, B-III, C-IV and D-I. Hence, the correct option is (D).

Quick Tip to Solve: Fix the unequivocal disproportionation, displacement and decomposition matches first; peroxide oxygen changing both upward and downward is the strongest clue.

67. Given below are two statements:

Statement I: $Et_2N-CH_2-CH_2Cl$ **will undergo alkaline hydrolysis at a faster rate than** $(Et)_2CH-CH_2Cl$.

Statement II: In $Et_2N-CH_2-CH_2Cl$**, intramolecular substitution takes place first by involving lone pair of electrons on nitrogen.**

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]

(A) Both Statement I and Statement II are correct

(B) Statement I is incorrect but Statement II is correct

(C) Both Statement I and Statement II are incorrect

(D) Statement I is correct but Statement II is incorrect

Correct Answer: (A) Both Statement I and Statement II are correct

Solution:

Concept:

- A neighbouring atom bearing a lone pair can assist nucleophilic substitution by attacking the carbon bearing the leaving group within the same molecule.
- In a beta-chloro tertiary amine, nitrogen can form a three-membered cyclic aziridinium ion. Hydroxide then opens this ring to give the amino alcohol.

Step 1: Identify the assisting group The first structure is $Et_2N-CH_2-CH_2Cl$. Nitrogen has a nonbonding electron pair and is separated from the carbon bonded to chlorine by one carbon. Its lone pair can attack that carbon intramolecularly as the chloride ion leaves. The resulting cyclic ion contains nitrogen and the two methylene carbons; nitrogen carries a positive charge after forming its fourth bond.

Step 2: Compare the hydrolysis pathways The second structure has a carbon group in place of the amino group. It has no comparable neighbouring nitrogen lone pair to supply this assisted pathway. Hydroxide must therefore substitute without this particular intramolecular assistance. The neighbouring-group pathway lowers the activation barrier for the amino compound, so its alkaline hydrolysis is faster. This makes Statement I correct.

Step 3: Evaluate Statement II and select the pair For the amino compound, formation of the cyclic aziridinium intermediate is the initial intramolecular substitution. Hydroxide subsequently attacks a ring carbon and breaks a carbon–nitrogen bond to open the ring. Thus Statement II correctly describes the first step. Options (B) and (D) wrongly reject one true statement, while option (C) rejects both.

Step 4: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (A).

Quick Tip to Solve: Look for a lone-pair-bearing atom two bonds from the leaving-group carbon: it may accelerate substitution through a cyclic intermediate.

68. Which of the following oxidation reactions are carried out by both $K_2Cr_2O_7$ and $KMnO_4$ in acidic medium?

A. $\Gamma^- \rightarrow I_2$

B. $S^{2-} \rightarrow S$

C. $Fe^{2+} \rightarrow Fe^{3+}$

D. $I^- \rightarrow IO_3^-$

E. $S_2O_3{}^{2-} \rightarrow SO_4^{2-}$

Choose the correct answer from the options given below : [4 marks]

(A) A, B and C Only

(B) C, D and E Only

(C) B, C and D Only

(D) A, D and E Only

Correct Answer: (A) A, B and C Only

Solution:

Concept:

- Acidified $K_2Cr_2O_7$ and $KMnO_4$ are oxidising agents; their characteristic reduction products are Cr^{3+} and Mn^{2+}, respectively.
- The products of oxidation depend on the reagent and the medium. A reaction common to both reagents must have the stated final product with each reagent.

Step 1: Test the three familiar common oxidations In reaction A, the printed Γ^- symbol is read as iodide, I^-, since the stated product is iodine, I_2. This resolves the symbol in the oxidation without changing the listed question. Iodide is oxidised to iodine:

$$2\,I^- \longrightarrow I_2 + 2\,e^-.$$

In reaction B, sulphide is oxidised to elemental sulphur:

$$S^{2-} \longrightarrow S + 2\,e^-.$$

In reaction C, iron(II) loses one electron:

$$Fe^{2+} \longrightarrow Fe^{3+} + e^-.$$

These are the standard acidic-medium oxidations carried out by both dichromate and permanganate. Each oxidation supplies electrons to the oxidising agent.

Step 2: Check the specified products in D and E Reaction D requires further oxidation of iodide all the way to iodate rather than the iodine product characteristic of these acidic-medium reactions. It is therefore not included in the common set. Reaction E specifies complete oxidation of thiosulphate to sulphate; this is not a common characteristic reaction of both reagents under the stated conditions. One must not select an oxidation merely because the oxidation state increases: the named reagent and medium must produce the stated species.

Step 3: Eliminate the other combinations Only the set A, B and C contains all three common reactions without D or E. Option (B) includes both excluded reactions, option (C) includes D, and option (D) includes D and E while omitting the common iron(II) oxidation.

Step 4: Conclusion The common oxidations are A, B and C only. Hence, the correct option is (A).

Quick Tip to Solve: For acidic dichromate and permanganate, recall the common products: iodide to iodine, sulphide to sulphur, and iron(II) to iron(III).

69. The molecules having square pyramidal geometry are [4 marks]

(A) BrF_5 & PCl_5

(B) SbF_5 & PCl_5

(C) BrF_5 & $XeOF_4$

(D) SbF_5 & $XeOF_4$

Correct Answer: (C) BrF_5 & $XeOF_4$

Solution:

Concept:

- VSEPR predicts shape from the number of bonding domains and lone pairs on the central atom. A multiple bond counts as one spatial domain.
- An AX_5E arrangement has six electron domains, with octahedral electron-pair geometry and square pyramidal molecular geometry after one lone-pair position is omitted.

Step 1: Count the domains around bromine In BrF_5, bromine has seven valence electrons. Five are used in its five single bonds with fluorine, leaving one lone pair. Thus there are five bonding domains and one lone pair. The six domains adopt an octahedral arrangement. Four fluorine atoms form a square base and the fifth forms the apex; the lone pair occupies the position opposite the apical bond.

Step 2: Count the domains around xenon In $XeOF_4$, xenon is bonded to four fluorine atoms and one oxygen atom. The xenon–oxygen double bond constitutes one domain, so there are five bonding domains altogether. Xenon also has one lone pair, giving the same AX_5E classification. The molecular geometry is square pyramidal, with oxygen at the apex and the fluorine atoms forming the square base.

Step 3: Exclude the pentahalides without lone pairs In molecular PCl_5 and SbF_5, the central atom forms five bonds and has no lone pair. Their five bonding domains give the AX_5 trigonal bipyramidal geometry, with three equatorial and two axial bonds. Consequently every option containing either pentahalide is unsuitable; only option (C) contains two square pyramidal species.

Step 4: Conclusion The square pyramidal molecules are BrF_5 and $XeOF_4$. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish five bonds alone from five bonds plus one lone pair: the first is trigonal bipyramidal, the second square pyramidal.

70. Ice and water are placed in a closed container at a pressure of 1 atm and temperature 273.15 K . If pressure of the system is increased 2 times, keeping temperature constant, then identify correct observation from following [4 marks]

(A) Liquid phase disappears completely.

(B) The amount of ice decreases.

(C) The solid phase (ice) disappears completely.

(D) Volume of system increases .

Correct Answer: (C) The solid phase (ice) disappears completely.

Solution:

Concept:

- At a fixed temperature, increased pressure favours the phase with smaller molar volume. Liquid water has a smaller molar volume than ordinary ice near the melting point.
- The melting curve follows $\mathrm{d}T/\mathrm{d}P = T\Delta V/\Delta H_{fus}$. For ice melting, $\Delta V = V_{liquid} - V_{solid} < 0$ and $\Delta H_{fus} > 0$, so increasing pressure lowers the melting temperature.

Step 1: Identify the initial coexistence state At $1\,\mathrm{atm}$ and $273.15\,\mathrm{K}$, the ice and water are initially at their melting equilibrium. The two phases can coexist because their chemical potentials are equal. Ice is less dense than liquid water, so converting a given mass of ice into water reduces the volume occupied by that mass.

Step 2: Apply the pressure increase at constant temperature The pressure rises to $2\,\mathrm{atm}$, while the temperature remains $273.15\,\mathrm{K}$. Because the melting curve has negative slope, the new equilibrium melting temperature is slightly below the maintained temperature. Thus the system is now above the melting temperature at its new pressure. Melting is favoured, and the external maintenance of temperature allows the latent heat needed for melting to be supplied.

Step 3: Determine the final equilibrium observation At the new pressure and fixed temperature, the stable phase is liquid water: ice disappears on reaching equilibrium. Saying only that the amount of ice decreases describes the direction of change but does not specify this final equilibrium outcome. Liquid does not disappear, since pressure favours its smaller volume. The volume decreases on melting rather than increasing.

Step 4: Conclusion The solid phase (ice) disappears completely at the final equilibrium. Hence, the correct option is (C).

Quick Tip to Solve: For water near freezing, remember that pressure promotes melting because ice occupies more volume than the same mass of liquid water.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The formation enthalpies, $\Delta H_f^{\ominus}$ for $H_{(g)}$ and $O_{(g)}$ are 220.0 and $250.0\ \text{kJ mol}^{-1}$, respectively, at 298.15 K , and $\Delta H_f^{\ominus}$ for $H_2O_{(g)}$ is $-242.0\ \text{kJ mol}^{-1}$ at the same temperature. The average bond enthalpy of the $O-H$ bond in water at 298.15 K is _____ kJ mol^{-1} (nearest integer). [4 marks]

Solution:

Concept:

- The average O–H bond enthalpy in gaseous water is half the enthalpy needed to convert one mole of $H_2O(g)$ into two moles of $H(g)$ and one mole of $O(g)$.
- Hess's law gives a reaction enthalpy as the sum of product formation enthalpies minus the sum of reactant formation enthalpies, including stoichiometric coefficients.

Step 1: Write the complete atomisation process The required gas-phase process breaks both O–H bonds:

$$H_2O(g) \longrightarrow 2\,H(g) + O(g).$$

The given $220.0\,\text{kJ mol}^{-1}$ is the formation enthalpy per mole of hydrogen atoms, so its contribution must be multiplied by two. The oxygen atom contribution is counted once. No liquid-water vaporisation term is needed because the reactant is explicitly gaseous water.

Step 2: Calculate the total bond-breaking enthalpy Using the supplied formation enthalpies at the same temperature,

$$\begin{aligned}\Delta H_{atom} &= 2\Delta H_f^{\circ}[H(g)] + \Delta H_f^{\circ}[O(g)] \\ &\quad - \Delta H_f^{\circ}[H_2O(g)] \\ &= 2(220.0) + 250.0 - (-242.0) \\ &= 932.0\,\text{kJ mol}^{-1}.\end{aligned}$$

The negative formation enthalpy of water is subtracted, producing a positive contribution. The positive total is consistent with energy being required to break bonds.

Step 3: Average over the two O–H bonds The atomisation process breaks two O–H bonds per water molecule. Therefore the average enthalpy per mole of O–H bonds is

$$\overline{D}(O-H) = \frac{932.0}{2} = 466.0\,\text{kJ mol}^{-1}.$$

This is an average of the successive bond dissociation enthalpies, which need not individually be equal.

Step 4: Conclusion The average O–H bond enthalpy is $466\,\text{kJ mol}^{-1}$, so the required nearest integer is $\boxed{466}$.

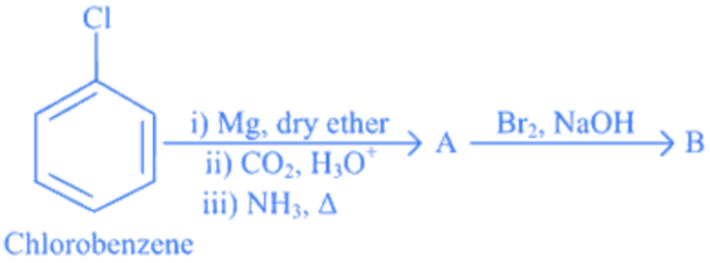
Quick Tip to Solve: First atomise the entire gaseous molecule using Hess's law, then divide by the number of bonds being averaged.

72. Consider the following sequence of reactions:

11.25 mg of chlorobenzene will produce _____$\times 10^{-1}$mg of product B.

(Consider the reactions result in complete conversion.)

[Given molar mass of C, H, O, N and Cl as $12, 1, 16, 14$ and 35.5 g mol^{-1} respectively] [4 marks]



Solution:

Concept:

- Aryl magnesium halides react with CO_2, followed by acidic work-up, to give carboxylic acids containing one extra carbon atom.
- A primary amide undergoes Hofmann bromamide rearrangement with $Br_2/NaOH$, yielding a primary amine with one fewer carbon atom than the amide. Complete conversion allows the mole ratio to be used directly.

Step 1: Identify A and B through the reaction sequence Chlorobenzene first forms phenylmagnesium chloride with magnesium in dry ether. Carbon dioxide followed by acidic work-up gives benzoic acid. Heating with ammonia converts the acid through its ammonium salt into benzamide, so A is $C_6H_5CONH_2$. Treatment with bromine and sodium hydroxide removes the carbonyl carbon by the Hofmann reaction, giving B as aniline, $C_6H_5NH_2$.

Step 2: Calculate the amount of chlorobenzene Using the supplied atomic masses,

$$M(C_6H_5Cl) = 6(12) + 5(1) + 35.5 = 112.5\,\text{g mol}^{-1}.$$

The starting mass is $11.25\,\text{mg} = 0.01125\,\text{g}$, hence

$$n(C_6H_5Cl) = \frac{0.01125}{112.5} = 1.00 \times 10^{-4}\,\text{mol}.$$

Every stage preserves one aromatic ring per molecule, so complete conversion produces the same number of moles of aniline.

Step 3: Convert the product amount into the requested mass unit The molar mass of aniline is

$$M(C_6H_7N) = 6(12) + 7(1) + 14 = 93\,\text{g mol}^{-1}.$$

Thus

$$m_B = (1.00 \times 10^{-4})(93)\,\text{g} = 9.3\,\text{mg}.$$

Writing this in the requested form gives

$$9.3\,\text{mg} = 93 \times 10^{-1}\,\text{mg}.$$

The blank is the coefficient 93, rather than the mass value 9.3.

Step 4: Conclusion Product B is aniline and its mass is $9.3\,\text{mg} = 93 \times 10^{-1}\,\text{mg}$. The required answer is $\boxed{93}$.

Quick Tip to Solve: Track moles through the one-to-one sequence, then check the power of ten printed after the answer blank before entering the number.

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73. The molarity of a 70% (mass/mass) aqueous solution of a monobasic acid (X) is _____ $\times 10^{-1}$ M (Nearest integer)

[Given: Density of aqueous solution of (X) is $1.25\ \text{g mL}^{-1}$

Molar mass of the acid is $70\ \text{g mol}^{-1}$] [4 marks]

Solution:

Concept:

- Mass percentage is the mass of solute per 100 g of solution. Thus a 70 percent mass/mass solution contains 70 g of acid and 30 g of water in each 100 g portion.
- Molarity is moles of solute divided by litres of solution: $M = n/V$. Density connects the mass and volume of the entire solution through $V = m/\rho$.

Step 1: Choose a convenient volume of solution Take exactly $1.000\,\text{L}$, or $1000\,\text{mL}$, of solution. The density refers to the solution rather than the pure acid, so the mass of this chosen portion is

$$m_{\text{solution}} = \rho V = (1.25\,\text{g mL}^{-1})(1000\,\text{mL}) = 1250\,\text{g}.$$

This basis makes the final number of moles equal numerically to the molarity.

Step 2: Calculate the mass and amount of acid Only 70 percent of the solution mass is solute. Therefore

$$m_X = \frac{70}{100}(1250) = 875\,\text{g}.$$

Using the given molar mass,

$$n_X = \frac{875\,\text{g}}{70\,\text{g mol}^{-1}} = 12.5\,\text{mol}.$$

The remaining 375 g is water; it is already included in the solution volume and must not be used as the denominator of molarity.

Step 3: Express the molarity in the requested scaled form The 12.5 mol of acid occupies the chosen one litre of solution, giving

$$M = \frac{12.5\,\text{mol}}{1.000\,\text{L}} = 12.5\,\text{mol L}^{-1} = 125 \times 10^{-1}\,\text{M}.$$

The blank asks for the coefficient of 10^{-1}, not for 12.5 rounded directly. The monobasic nature would matter for equivalent concentration, but introduces no additional factor into molarity.

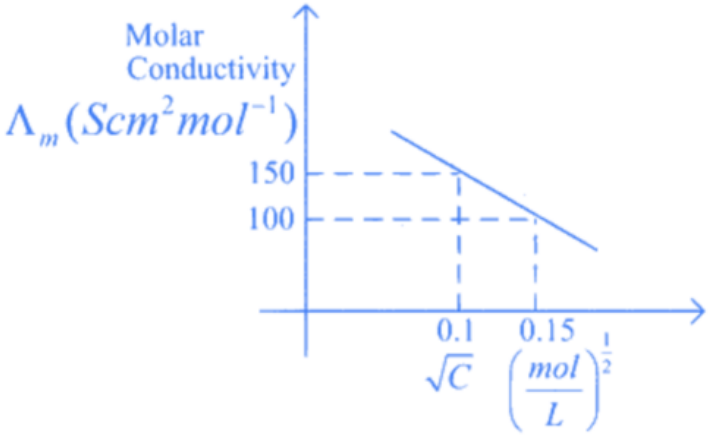
Step 4: Conclusion The molarity is $12.5\,\mathrm{M} = 125 \times 10^{-1}\,\mathrm{M}$. Hence the required nearest integer is $\boxed{125}$.

Quick Tip to Solve: For mass percent and density in g/mL, use $M = 10\rho w/M_r$, with w the numerical percentage; then convert to the power of ten requested by the blank.

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74. Given below is the plot of the molar conductivity vs $\sqrt{\text{concentration}}$ for KCl in aqueous solution. If, for the higher concentration of KCl solution, the resistance of the conductivity cell is 100Ω, then the resistance of the same cell with the dilute solution is ' x ' Ω

The value of x is _____ (Nearest integer) [4 marks]



Solution:

Concept:

- Molar conductivity and conductivity are related by $\Lambda_m = 1000\kappa/c$ when concentration c is in mol/L and κ is in S/cm. Hence conductivity is proportional to $\Lambda_m c$.
- For the same conductivity cell, $\kappa = K_{\text{cell}}/R$, where the cell constant $K_{\text{cell}} = l/A$ remains fixed. Thus resistance is inversely proportional to conductivity, not directly to molar conductivity.

Step 1: Read both coordinates from the graph The dilute solution has $\sqrt{c_d} = 0.10$ and $\Lambda_d = 150\,\mathrm{S\,cm^2\,mol^{-1}}$. The more concentrated solution has $\sqrt{c_h} = 0.15$ and $\Lambda_h = 100\,\mathrm{S\,cm^2\,mol^{-1}}$. Because the horizontal axis is the square root of concentration, square its readings:

$$c_d = (0.10)^2 = 0.0100\,\mathrm{mol\,L^{-1}}, \qquad c_h = (0.15)^2 = 0.0225\,\mathrm{mol\,L^{-1}}.$$

Step 2: Find the ratio of conductivities Apply $\kappa = \Lambda_m c/1000$ separately to the two solutions:

$$\kappa_d = \frac{150(0.0100)}{1000} = 1.50 \times 10^{-3}\,\mathrm{S\,cm^{-1}},$$

$$\kappa_h = \frac{100(0.0225)}{1000} = 2.25 \times 10^{-3}\,\mathrm{S\,cm^{-1}}.$$

Although dilution raises molar conductivity, it reduces the number of charge carriers per unit volume enough to lower conductivity here.

Step 3: Use the unchanged cell constant The geometry of the cell is unchanged, so $R_d\kappa_d = R_h\kappa_h$. With the given $R_h = 100\,\Omega$,

$$R_d = 100\frac{2.25 \times 10^{-3}}{1.50 \times 10^{-3}} = 150\,\Omega.$$

This is greater than the resistance of the concentrated solution, as expected from its smaller conductivity. The requested value is already an integer.

Step 4: Conclusion The resistance of the dilute solution is $\boxed{150\,\Omega}$, so the required nearest-integer value is $\boxed{x = 150}$.

Quick Tip to Solve: When a graph uses $\sqrt{c}$, square the horizontal readings before forming $\Lambda_m c$. For a fixed cell, invert that conductivity ratio to obtain the resistance ratio.

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75. Quantitative analysis of an organic compound (X) shows following % composition.

C : 14.5%

Cl : 64.46%

H: 1.8 %

(Empirical formula mass of the compound (X) **is** _____ $\times 10^{-1}$

(Given molar mass in $\mathrm{g\ mol^{-1}}$ **of** $\mathrm{C}:12, \mathrm{H}:1, \mathrm{O}:16, \mathrm{Cl}:35.5$**)** [4 marks]

Solution:

Concept:

- An empirical formula gives the simplest whole-number ratio of atoms. On a 100 g basis, the numerical percentage of each element is its mass in grams, which is divided by its atomic molar mass to obtain a mole ratio.
- The listed carbon, hydrogen and chlorine percentages do not total 100 percent. The remaining mass belongs to oxygen, whose atomic molar mass is supplied. Small departures from exact ratios arise from the rounded percentage data.

Step 1: Find the oxygen percentage by difference For a 100 g sample, the masses of carbon, chlorine and hydrogen are 14.5 g, 64.46 g and 1.8 g respectively. Their total is 80.76 g, so

$$m_{\mathrm{O}} = 100 - (14.5 + 64.46 + 1.8) = 19.24\,\mathrm{g}.$$

Include this oxygen before normalising the mole ratios; otherwise both the empirical formula and its mass would be incorrect.

Step 2: Convert masses into the simplest atomic ratio Divide by the given atomic molar masses:

$$n_{\mathrm{C}} = 14.5/12 = 1.2083,$$
$$n_{\mathrm{H}} = 1.8/1 = 1.8000,$$
$$n_{\mathrm{Cl}} = 64.46/35.5 = 1.8158,$$
$$n_{\mathrm{O}} = 19.24/16 = 1.2025.$$

Dividing by the smallest value, 1.2025, gives approximately

$$\mathrm{C} : \mathrm{H} : \mathrm{Cl} : \mathrm{O} = 1.005 : 1.497 : 1.510 : 1 \simeq 1 : 1.5 : 1.5 : 1.$$

The half-integer entries should not be rounded individually to integers. Multiply the entire ratio by two to preserve the composition.

Step 3: Calculate the empirical formula mass and its coefficient The simplest integer ratio is $2 : 3 : 3 : 2$, giving the empirical formula $\mathrm{C_2H_3Cl_3O_2}$. Its mass is

$$M_{\mathrm{emp}} = 2(12) + 3(1) + 3(35.5) + 2(16) = 165.5.$$

In the requested form this is 1655×10^{-1}. No molecular-mass information is required because the question asks for the empirical formula mass rather than the molecular formula.

Step 4: Conclusion The empirical formula is $\mathrm{C_2H_3Cl_3O_2}$, with empirical formula mass 165.5. Therefore the required coefficient is $\boxed{1655}$.

Quick Tip to Solve: First include oxygen by difference. If normalisation gives ratios near 1.5, multiply every ratio by two before writing the empirical formula; never round 1.5 to 2 alone.