JEE Main 2025 Paper 1 (B.E./B.Tech) Jan 28 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 28 January 2025, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The area of the region bounded by the curves $x(1+y^2)=1$ and $y^2=2x$ is: [4 marks]

(A) $\frac{\pi}{4}-\frac{1}{3}$
(B) $\frac{\pi}{2}-\frac{1}{3}$
(C) $2\left(\frac{\pi}{2}-\frac{1}{3}\right)$
(D) $\frac{1}{2}\left(\frac{\pi}{2}-\frac{1}{3}\right)$

Correct Answer: (B) $\frac{\pi}{2}-\frac{1}{3}$

Solution:

Concept:

- For curves expressed as x in terms of y, integrate the horizontal width: $A=\int(x_{\text{right}}-x_{\text{left}})\,dy$.
- Both curves depend on y^2, so the enclosed region is symmetric about the x-axis. The identity $\int(1+y^2)^{-1}\,dy=\tan^{-1}y+C$ evaluates the curved boundary.

Step 1: Find the intersection limits Write the boundaries as $x=1/(1+y^2)$ and $x=y^2/2$. At an intersection,

$$\frac{1}{1+y^2}=\frac{y^2}{2} \quad\Longrightarrow\quad y^4+y^2-2=0.$$

Thus $(y^2-1)(y^2+2)=0$. Since y is real, $y^2=1$, giving the limits $y=-1,1$. The corresponding intersections both have $x=1/2$.

Step 2: Integrate the positive horizontal width For $-1<y<1$, the rational curve lies to the right of the parabola; at $y=0$, for example, their coordinates are 1 and 0. Therefore

$$A=2\int_0^1\left(\frac{1}{1+y^2}-\frac{y^2}{2}\right)dy=2\left[\tan^{-1}y-\frac{y^3}{6}\right]_0^1.$$

This gives $A=2(\pi/4-1/6)=\pi/2-1/3$. Option (A) misses part of the inverse-tangent contribution, while (C) doubles the full area and (D) halves it. The factor 2 has already included both sides of the symmetry axis.

Step 3: Conclusion The enclosed area is $\boxed{\pi/2-1/3}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: When both boundaries are simple functions of y, use horizontal strips and apply symmetry only once.

2. Bag B_1 contains 6 white and 4 blue balls, Bag B_2 contains 4 white and 6 blue balls, and Bag B_3 contains 5 white and 5 blue balls. One of the bags is selected at random and a ball is drawn from it. If the ball is white, then the probability that the ball is drawn from Bag B_2 is: [4 marks]

(A) $\frac{2}{5}$
(B) $\frac{4}{15}$
(C) $\frac{1}{3}$
(D) $\frac{2}{3}$

Correct Answer: (B) $\frac{4}{15}$

Solution:

Concept:

- Bayes theorem gives $P(B_2 \mid W) = P(B_2)P(W \mid B_2)/P(W)$, where W denotes drawing a white ball.
- The bags are selected at random, so each prior probability is $1/3$. The total probability of white is obtained by adding the three mutually exclusive bag-and-white outcomes.

Step 1: Calculate the probability of a white ball Each bag contains 10 balls. Hence

$$P(W \mid B_1) = \frac{6}{10}, \qquad P(W \mid B_2) = \frac{4}{10}, \qquad P(W \mid B_3) = \frac{5}{10}.$$

Weight each conditional probability by its bag-selection probability:

$$P(W) = \frac{1}{3}\frac{6}{10} + \frac{1}{3}\frac{4}{10} + \frac{1}{3}\frac{5}{10} = \frac{15}{30} = \frac{1}{2}.$$

The observation of white does not leave the bags equally likely, because their white-ball proportions differ.

Step 2: Condition on the observed colour The joint probability of choosing the second bag and obtaining white is $P(B_2 \cap W) = (1/3)(4/10) = 2/15$. Divide this by the probability of the observed event:

$$P(B_2 \mid W) = \frac{2/15}{1/2} = \frac{4}{15}.$$

Option (A), $2/5$, is the probability of white given the second bag, which reverses the conditioning. Option (C) is merely the initial bag-selection probability. Option (D) is also excluded by the computed conditional probability; the second bag has the smallest white proportion and must have posterior probability below $1/3$.

Step 3: Conclusion The required probability is $\boxed{4/15}$. Hence, the correct option is (B).

Quick Tip to Solve: For equally likely bags of equal size, the posterior bag probabilities are proportional to their white-ball counts: $6 : 4 : 5$.

3. Let $\mathrm{f}:\mathrm{R}\to\mathrm{R}$ **be a twice differentiable function such that** $f(2)=1$**. If** $\mathrm{F(x)}=\mathrm{x}f(\mathrm{x})$ **for all** $\mathrm{x}\in\mathrm{R}$**,** $\int_0^2 xF'(x)dx=6$ **and** $\int_0^2 x^2F''(x)dx=40$**, then** $F'(2)+\int_0^2 F(x)dx$ **is equal to :** [4 marks]

(A) 13

(B) 11

(C) 9

(D) 15

Correct Answer: (B) 11

Solution:

Concept:

- Integration by parts states $\int u\,dv=uv-\int v\,du$; use the powers of x as u to reduce both given integrals.
- The relation $F(x)=xf(x)$ and $f(2)=1$ gives $F(2)=2$. Endpoint terms containing x or x^2 vanish at zero because the functions are differentiable there.

Step 1: Find the integral of the function Apply integration by parts to the first given integral:

$$6=\int_0^2 xF'(x)\,dx=[xF(x)]_0^2-\int_0^2 F(x)\,dx.$$

Since $F(2)=2$, the boundary term equals $2\cdot 2=4$, so

$$\int_0^2 F(x)\,dx=4-6=-2.$$

The negative value is consistent: the question does not require F to remain positive on the interval.

Step 2: Extract the endpoint derivative For the second integral, take $u=x^2$ and $dv=F''(x)\,dx$:

$$40=[x^2F'(x)]_0^2-2\int_0^2 xF'(x)\,dx=4F'(2)-12.$$

Consequently $F'(2)=52/4=13$. The required expression is therefore $13+(-2)=11$. Option (A) gives the derivative alone and omits the integral. Options (C) and (D) disagree with the signed sum of the two independently determined quantities; replacing the negative integral by its magnitude would incorrectly give 15.

Step 3: Conclusion $\boxed{F'(2)+\int_0^2 F(x)\,dx=11}$. Hence, the correct option is (B).

Quick Tip to Solve: Reduce the two supplied integrals in sequence: first determine $\int F$, then use the known $\int xF'$ to obtain the endpoint derivative.

4. If the midpoint of a chord of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ is $(\sqrt{2},4/3)$, and the length of the chord is $\frac{2\sqrt{\alpha}}{3}$, then α is : [4 marks]

(A) 26

(B) 18

(C) 22

(D) 20

Correct Answer: (C) 22

Solution:

Concept:

- For the ellipse $x^2/a^2+y^2/b^2=1$, a chord with midpoint (h,k) has equation $hx/a^2+ky/b^2=h^2/a^2+k^2/b^2$.
- Write the endpoints symmetrically as midpoint plus or minus a multiple of a direction vector. Substitution into the ellipse determines the half-chord length.

Step 1: Obtain a direction for the chord With $a^2=9$, $b^2=4$, $h=\sqrt{2}$, and $k=4/3$, the midpoint chord equation is

$$\frac{\sqrt{2}\,x}{9}+\frac{y}{3}=\frac{2}{9}+\frac{4}{9}=\frac{2}{3}.$$

Equivalently, $\sqrt{2}\,x+3y=6$. A direction vector is therefore $(3,-\sqrt{2})$, whose squared length is $9+2=11$. Its dot product with the normal $(\sqrt{2},3)$ is zero, as required.

Step 2: Calculate the two endpoint parameters Parameterise the chord by $(x,y)=(\sqrt{2}+3t,4/3-\sqrt{2}t)$. Substituting in the ellipse gives

$$\frac{(\sqrt{2}+3t)^2}{9}+\frac{(4/3-\sqrt{2}t)^2}{4}=\frac{2}{3}+\frac{3}{2}t^2=1.$$

The terms linear in t cancel precisely because the chosen point is the midpoint. Thus $t^2=2/9$, and the endpoints correspond to $t=\pm\sqrt{2}/3$. Their separation squared is

$$L^2=4\left(\frac{2}{9}\right)11=\frac{88}{9}.$$

Compare this with $L^2=4\alpha/9$, obtaining $\alpha=22$. The alternatives 26, 18 and 20 would give different endpoint separations and cannot satisfy the ellipse equation.

Step 3: Conclusion The chord length is $2\sqrt{22}/3$, so $\boxed{\alpha=22}$. Hence, the correct option is (C).

Quick Tip to Solve: Use an unnormalised direction vector for the chord, but include its squared norm when converting the endpoint parameter difference into a length.

5. The square of the distance of the point $\left(\frac{15}{7}, \frac{32}{7}, 7\right)$ from the line $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ in the direction of the vector $\hat{i} + 4\hat{j} + 7\hat{k}$ is: [4 marks]

(A) 66

(B) 54

(C) 41

(D) 44

Correct Answer: (A) 66

Solution:

Concept:

- Distance in a specified direction is found by intersecting the given line with a line through the point parallel to that direction.
- If the displacement is $s(1, 4, 7)$, its squared length is $s^2(1^2 + 4^2 + 7^2) = 66s^2$. This is a directed-intersection calculation rather than the perpendicular-distance formula.

Step 1: Parameterise both intersecting lines Let $P = (15/7, 32/7, 7)$. A point on the given line is

$$Q = (-1 + 3t, -3 + 5t, -5 + 7t).$$

For the line through P parallel to $(1, 4, 7)$, write $Q = P + s(1, 4, 7)$. Equating the first and third coordinates gives

$$3t - s = \frac{22}{7}, \qquad t - s = \frac{12}{7}.$$

Subtracting gives $2t = 10/7$, hence $t = 5/7$ and $s = -1$. The negative parameter only means that the intersection is along the opposite orientation of the same parallel direction.

Step 2: Check the intersection and compute the distance At $t = 5/7$, the point on the given line is $Q = (8/7, 4/7, 0)$. Its displacement from P is

$$Q - P = (-1, -4, -7),$$

which also verifies the second-coordinate condition: $32/7 - 4 = 4/7$. Therefore

$$PQ^2 = (-1)^2 + (-4)^2 + (-7)^2 = 1 + 16 + 49 = 66.$$

The other options, 54, 41 and 44, do not equal the squared norm of this parallel displacement. In particular, using a perpendicular projection onto the original line would answer a different distance question and would ignore the direction explicitly specified here.

Step 3: Conclusion The square of the required distance is $\boxed{66}$ square units. Hence, the correct option is (A).

Quick Tip to Solve: For a distance measured parallel to a vector, solve the intersection with that parallel line first; only then take the displacement norm.

6. Two equal sides of an isosceles triangle are along $-x+2y=4$ and $x+y=4$. If m is the slope of its third side, then the sum, of all possible distinct values of m, is: [4 marks]

(A) $-2\sqrt{10}$

(B) 12

(C) -6

(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- In an isosceles triangle the base makes equal angles with the two equal sides. Thus its line direction must satisfy equality of the two angle magnitudes.
- For finite slopes m and r, the tangent of the angle between their lines is $|(m-r)/(1+mr)|$, provided the denominator is nonzero.

Step 1: Impose equal angles with the two side lines The slopes of $-x+2y=4$ and $x+y=4$ are $1/2$ and -1, respectively. If m is the base slope, equality of its angles with these lines gives

$$\left|\frac{2m-1}{m+2}\right| = \left|\frac{m+1}{1-m}\right|.$$

Neither $m=-2$ nor $m=1$ works: each makes the base perpendicular to just one of the two distinct equal-side lines. A vertical base also makes unequal angles with their directions.

Step 2: Solve for all possible directions Removing the absolute values gives two sign cases. For the plus sign,

$$(2m-1)(1-m) = (m+1)(m+2),$$

which simplifies to $-3m^2-3=0$ and has no real root. For the minus sign,

$$(2m-1)(1-m) = -(m+1)(m+2),$$

Expanding, the left side is $-2m^2+3m-1$; moving the right side to the left gives $-m^2+6m+1=0$. Thus the distinct roots are $m=3+\sqrt{10}$ and $m=3-\sqrt{10}$. Their sum is 6. Both directions are attainable as bases by joining equal distances along suitable rays of the two given side lines. Options (A), (B) and (C) do not equal this sum; the difference of the roots involves $2\sqrt{10}$, not their sum.

Step 3: Conclusion The sum of all possible distinct slopes is $\boxed{6}$. Hence, the correct option is (D).

Quick Tip to Solve: The base directions are perpendicular to the two angle-bisector directions. Use the sum of roots after deriving their quadratic, instead of calculating both angles.

7. Let the coefficients of three consecutive terms T_r, T_{r+1} and T_{r+2} in the binomial expansion of $(a+b)^{12}$ be in a G.P. and let p be the number of all possible values of r. Let q be the sum of all rational terms in the binomial expansion of $(\sqrt[4]{3}+\sqrt[3]{4})^{12}$. Then $p+q$ is equal to: [4 marks]

(A) 295

(B) 283

(C) 299

(D) 287

Correct Answer: (B) 283

Solution:

Concept:

- Three consecutive coefficients form a geometric progression only if the square of the middle coefficient equals the product of its neighbours. The coefficients of a binomial expansion are strictly log-concave, so this condition must be tested rather than assumed.
- The general term of the second expansion is $\binom{12}{k}3^{(12-k)/4}4^{k/3}$, with $0 \le k \le 12$. Rationality requires integral exponents of each prime in its prime factorisation.

Step 1: Test the geometric-progression condition Using the usual convention that the first term is T_1, the three coefficients are $\binom{12}{r-1}, \binom{12}{r}, \binom{12}{r+1}$. Their successive ratios would have to agree:

$$\frac{13-r}{r} = \frac{12-r}{r+1}.$$

Cross-multiplication gives

$$(13-r)(r+1) = r(12-r) \quad \Longrightarrow \quad 13 = 0.$$

This is impossible for every admissible integer r. Thus there are no such triples and $p = 0$. The impossibility is consistent with the strictly decreasing successive ratios of binomial coefficients.

Step 2: Identify and add the rational terms Write $4^{k/3} = 2^{2k/3}$. As 2 and 3 are distinct primes, the term is rational exactly when $3 \mid k$ and $4 \mid (12-k)$. The latter is equivalent to $4 \mid k$; together they require $12 \mid k$. Hence only $k = 0, 12$ contribute:

$$q = 3^3 + 4^4 = 27 + 256 = 283.$$

No intermediate term satisfies both divisibility conditions. Therefore adding a positive count of possible triples would be an error; the other proposed totals arise from such an extra contribution.

Step 3: Conclusion $p + q = 0 + 283 = 283$. Hence, the correct option is (B).

Quick Tip to Solve: For rational radical terms, track the exponent of each distinct prime separately; for coefficient G.P. conditions, compare adjacent binomial ratios.

8. If the components of $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ along and perpendicular to $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ respectively, are $\frac{16}{11}(3\hat{i} + \hat{j} - \hat{k})$ and $\frac{1}{11}(-4\hat{i} - 5\hat{j} - 17\hat{k})$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to : [4 marks]

(A) 16

(B) 23

(C) 26

(D) 18

Correct Answer: (C) 26

Solution:

Concept:

- A vector is the sum of its parallel and perpendicular vector components relative to a nonzero direction. These are vectors, so their Cartesian coordinates add directly.
- For $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, its squared magnitude is $|\vec{a}|^2 = \alpha^2 + \beta^2 + \gamma^2$. Orthogonal components also obey the Pythagorean identity.

Step 1: Reconstruct the original vector The problem supplies the two vector components, so no further projection calculation is required. Add like unit-vector coefficients:

$$\begin{aligned}\vec{a} &= \frac{1}{11}\left[16(3\hat{i} + \hat{j} - \hat{k}) - 4\hat{i} - 5\hat{j} - 17\hat{k}\right] \\ &= \frac{1}{11}(44\hat{i} + 11\hat{j} - 33\hat{k}) \\ &= 4\hat{i} + \hat{j} - 3\hat{k}.\end{aligned}$$

Thus $\alpha = 4,\ \beta = 1,\ \gamma = -3$. The negative sign on the last coefficient comes from adding both negative contributions; changing it before the addition would give an incorrect vector.

Step 2: Compute and independently check the squared magnitude The requested sum is

$$\alpha^2 + \beta^2 + \gamma^2 = 4^2 + 1^2 + (-3)^2 = 16 + 1 + 9 = 26.$$

To check consistency, the supplied perpendicular numerator has zero dot product with the direction: $3(-4) + (-5) + (-1)(-17) = 0$. Therefore the components really are orthogonal. Their squared magnitudes give the same result:

$$\frac{256}{121}(9 + 1 + 1) + \frac{16 + 25 + 289}{121} = \frac{2816 + 330}{121} = 26.$$

The values 16, 23 and 18 fail this explicit norm calculation; 16 alone is just the square of the first Cartesian coefficient.

Step 3: Conclusion $\alpha^2 + \beta^2 + \gamma^2 = 26$. Hence, the correct option is (C).

Quick Tip to Solve: When both vector components are already given, add them first; alternatively add their squared magnitudes only after confirming perpendicularity.

9. Let $f : \mathbf{R} - \{0\} \to (-\infty, 1)$ be a polynomial of degree 2 , satisfying $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$. If $f(\mathrm{K}) = -2\,\mathrm{K}$, then the sum of squares of all possible values of K is : [4 marks]

(A) 9

(B) 1

(C) 6

(D) 7

Correct Answer: (C) 6

Solution:

Concept:

- The identity $uv = u + v$ can be rewritten as $(u-1)(v-1) = 1$. Subtracting 1 from the polynomial therefore converts the functional equation into a reciprocal-product identity.
- A polynomial identity holding for all nonzero real inputs can be compared coefficient by coefficient after denominators are cleared. The given range then selects the admissible sign.

Step 1: Determine the polynomial from the identity Put $g(x) = f(x) - 1 = ax^2 + bx + c$, with $a \neq 0$. The functional equation becomes $g(x)g(1/x) = 1$. Its Laurent expansion is

$$ac(x^2 + x^{-2}) + b(a + c)(x + x^{-1}) + (a^2 + b^2 + c^2) = 1.$$

Comparing the coefficients gives $ac = 0$, $b(a + c) = 0$ and $a^2 + b^2 + c^2 = 1$. Since $a \neq 0$, first $c = 0$, then $b = 0$, and finally $a = \pm 1$. Thus $f(x) = 1 \pm x^2$. The range is contained in $(-\infty, 1)$, so the positive sign is impossible at every nonzero real input. Consequently $f(x) = 1 - x^2$.

Step 2: Solve for the possible values and sum their squares Substituting into the required equation yields

$$1 - K^2 = -2K \quad \Longrightarrow \quad K^2 - 2K - 1 = 0.$$

Its two real roots are $K_1 = 1 + \sqrt{2}$ and $K_2 = 1 - \sqrt{2}$, both nonzero and hence in the domain. By the root-coefficient relations,

$$K_1 + K_2 = 2, \qquad K_1 K_2 = -1.$$

Therefore

$$K_1^2 + K_2^2 = (K_1 + K_2)^2 - 2K_1K_2 = 4 + 2 = 6.$$

The values 9, 1 and 7 do not equal this sum. Both roots must be retained: the range restriction selects the polynomial, not a positive sign for the input.

Step 3: Conclusion The sum of the squares of all possible values of K is 6. Hence, the correct option is (C).

Quick Tip to Solve: First shift a reciprocal identity by 1; once the polynomial is known, use the sum and product of roots rather than expanding surds.

10. Let [x] denote the greatest integer less than or equal to x. Then the domain of $f(x) = \sec^{-1}(2[x]+1)$ is: [4 marks]

(A) $(-\infty, \infty)$

(B) $(-\infty, \infty) - \{0\}$

(C) $(-\infty, -1] \cup [0, \infty)$

(D) $(-\infty, -1] \cup [1, \infty)$

Correct Answer: (A) $(-\infty, \infty)$

Solution:

Concept:

- The real inverse secant $\sec^{-1} t$ is defined when $|t| \geq 1$. The endpoints $t = 1$ and $t = -1$ are included.
- The greatest-integer function $[x]$ is an integer for every real x. Therefore $2[x] + 1$ is always an odd integer, never a number strictly between -1 and 1.

Step 1: Apply the inverse-secant restriction Set $n = [x]$. The argument must satisfy

$$|2n + 1| \geq 1.$$

Equivalently,

$$2n + 1 \geq 1 \quad \text{or} \quad 2n + 1 \leq -1,$$

which gives $n \geq 0$ or $n \leq -1$. These alternatives exhaust every integer. There is no integer between -1 and 0, so no possible floor value is excluded by the restriction. In particular, the inverse function permits its endpoint arguments; a strict inequality here would incorrectly remove whole intervals of real inputs.

Step 2: Translate the floor values back to real inputs For $x \geq 0$, the floor satisfies $[x] \geq 0$, so the argument is at least 1. For $x < 0$, it satisfies $[x] \leq -1$, so the argument is at most -1. Thus both half-lines are allowed:

$$(-\infty, 0) \cup [0, \infty) = \mathbb{R}.$$

At $x = 0$, the argument is 1, which is valid; this directly rules out option (B). For a test in the intervals omitted by options (C) and (D), choose $x = -\frac{1}{2}$. Its floor is -1, giving argument -1, also valid. Likewise $x = \frac{1}{2}$ gives argument 1. These checks confirm that artificial gaps around zero must not be introduced.

Step 3: Conclusion The domain is $(-\infty, \infty)$. Hence, the correct option is (A).

Quick Tip to Solve: If an inverse trigonometric argument contains a floor function, test the allowed integer values first, then translate the result into intervals.

11. Let S be the set of all the words that can be formed by arranging all the letters of the word GARDEN. From the set S, one word is selected at random. The probability that the selected word will NOT have vowels in alphabetical order is: [4 marks]

(A) $\frac{1}{4}$
(B) $\frac{1}{2}$
(C) $\frac{1}{3}$
(D) $\frac{2}{3}$

Correct Answer: (B) $\frac{1}{2}$

Solution:

Concept:

- All six letters of GARDEN are distinct, so the sample space contains $6!$ equally likely arrangements. Words here mean all letter arrangements, whether or not they are dictionary words.
- The only vowels are A and E. Alphabetical order means A appears somewhere before E; they need not occupy adjacent positions. Swapping their positions pairs the two possible relative orders.

Step 1: Count arrangements with alphabetical vowel order Choose the two positions occupied by the vowels in $\binom{6}{2} = 15$ ways. For alphabetical order, the earlier chosen position must contain A and the later one E, so there is exactly one permitted filling of these positions. The four distinct consonants G, R, D and N can then be arranged in $4! = 24$ ways. Therefore

$$N(\text{A before E}) = \binom{6}{2} 4! = 15 \times 24 = 360.$$

The total number of arrangements is $6! = 720$. This counting includes arrangements with consonants between the vowels, as required; imposing adjacency would count a different event.

Step 2: Take the complement and check by symmetry The event requested is that the vowels are not in alphabetical order. As A and E occupy different positions, this means precisely that E occurs before A. Its probability is

$$1 - \frac{360}{720} = 1 - \frac{1}{2} = \frac{1}{2}.$$

As an independent check, exchange A and E in any arrangement. This is a reversible pairing between an A-before-E arrangement and an E-before-A arrangement, with no arrangement paired to itself. Hence both events have equal counts and equal probability. The options $1/4$, $1/3$ and $2/3$ fail this equal-pairing check; the positions of the consonants cannot favour either ordering of the two vowels.

Step 3: Conclusion The required probability is $\frac{1}{2}$. Hence, the correct option is (B).

Quick Tip to Solve: For two distinct specified letters in a uniformly random permutation, either relative order has probability one-half, regardless of the other letters.

12. If A and B are the points of intersection of the circle $x^2+y^2-8x=0$ and the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ and a point P moves on the line $2x-3y+4=0$, then the centroid of ΔPAB lies on the line : [4 marks]

(A) $x+9y=36$

(B) $9x-9y=32$

(C) $4x-9y=12$

(D) $6x-9y=20$

Correct Answer: (D) $6x-9y=20$

Solution:

Concept:

- The centroid of vertices $(u,v),(x_A,y_A),(x_B,y_B)$ is the coordinatewise average. If A and B are fixed, this is an affine transformation of the moving point P.
- Both curves are symmetric about the x-axis. Their real intersection points therefore come in a pair with equal x-coordinate and opposite y-coordinates once the admissible intersection abscissa is found.

Step 1: Find the fixed intersection points The hyperbola gives $y^2=\frac{4}{9}x^2-4$. Substitute into the circle:

$$x^2+\frac{4}{9}x^2-4-8x=0 \quad\Longrightarrow\quad 13x^2-72x-36=0.$$

Factoring,

$$(x-6)(13x+6)=0.$$

The candidate $x=-6/13$ gives $y^2<0$ in the hyperbola equation and hence no real intersection. The circle also has $0\le x\le 8$. Thus the real intersections have $x=6$, and $y^2=12$. We may take

$$A=(6,2\sqrt{3}), \qquad B=(6,-2\sqrt{3}).$$

In particular, $x_A+x_B=12$ and $y_A+y_B=0$; these sums are all the centroid calculation needs.

Step 2: Transform the equation of the moving line Let $P=(u,v)$ and denote the centroid by (X,Y). Then

$$X=\frac{u+12}{3}, \qquad Y=\frac{v}{3}, \qquad u=3X-12, \quad v=3Y.$$

Since P lies on $2u-3v+4=0$, substitution gives

$$2(3X-12)-3(3Y)+4=0 \quad\Longrightarrow\quad 6X-9Y=20.$$

The coefficient ratio must remain $2:-3$ under this uniform scaling and translation, immediately excluding options (A), (B) and (C). The constant follows from the fixed coordinate sum 12; treating both algebraic roots for x as real intersection coordinates would incorrectly change it.

Step 3: Conclusion The centroid lies on $6x-9y=20$. Hence, the correct option is (D).

Quick Tip to Solve: For a centroid locus with two fixed vertices, calculate their coordinate sums, express P through the centroid, and substitute into the line equation.

13. Let A, B, C be three points in xy-plane, whose position vector are given by $\sqrt{3}\hat{i}+\hat{j}, \hat{i}+\sqrt{3}\hat{j}$ and $a\hat{i}+(1-a)\hat{j}$ respectively with respect to the origin O . If the distance of the point C from the line bisecting the angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is $\frac{9}{\sqrt{2}}$, then the sum of all the possible values of a is : [4 marks]

(A) 2

(B) 0

(C) $\frac{9}{2}$

(D) 1

Correct Answer: (D) 1

Solution:

Concept:

- The internal angle bisector of two vectors has the direction of the sum of their unit vectors. Both given vectors have length 2, so their sum itself gives this direction.
- The perpendicular distance of (x_0, y_0) from $Ax + By + C = 0$ is $|Ax_0 + By_0 + C|/\sqrt{A^2 + B^2}$. Absolute values produce two possible signed equations.

Step 1: Identify the bisector The two position vectors make angles of $30°$ and $60°$ with the positive x-axis. Their internal bisector therefore makes an angle of $45°$. Equivalently, adding their unit vectors gives a vector with equal positive components:

$$\frac{\overrightarrow{OA}}{2}+\frac{\overrightarrow{OB}}{2}=\frac{\sqrt{3}+1}{2}(\hat{i}+\hat{j}).$$

Thus the required line through the origin is $x - y = 0$. The external bisector is not the line inside the angle between these two vectors.

Step 2: Apply the distance and solve both signs The point is $C = (a, 1 - a)$. Substituting both coordinates, rather than treating the second coordinate as constant, gives

$$\frac{|a-(1-a)|}{\sqrt{2}}=\frac{9}{\sqrt{2}}, \qquad |2a-1|=9.$$

Consequently, $2a - 1 = 9$ or $2a - 1 = -9$, giving $a = 5$ or $a = -4$. Both points have the stated perpendicular distance. The question asks for their sum, not either individual value or the positive difference. The other listed numbers do not equal $5 + (-4)$.

Step 3: Conclusion The sum of all possible values is $5 - 4 = 1$. Hence, the correct option is (D).

Quick Tip to Solve: Find the angle bisector before using the distance formula, and retain both signs when removing an absolute value.

14. If $\alpha + i\beta$ and $\gamma + i\delta$ are the roots of $x^2 - (3 - 2i)x - (2i - 2) = 0$, $i = \sqrt{-1}$, then $\alpha\gamma + \beta\delta$ is equal to: [4 marks]

(A) 2

(B) -6

(C) 6

(D) -2

Correct Answer: (A) 2

Solution:

Concept:

- The quadratic formula applies over the complex numbers. Its discriminant is $\Delta = b^2 - 4ac$; a complex square root can be checked by squaring it.
- The expression $\alpha\gamma + \beta\delta$ is the dot product of the real-imaginary coordinate pairs. It is not the real part $\alpha\gamma - \beta\delta$ of the ordinary product of the roots.

Step 1: Evaluate the discriminant Write the equation as $x^2 - (3 - 2i)x + (2 - 2i) = 0$. The constant term is $2 - 2i$, because the minus sign outside the printed bracket changes both terms. Hence

$$\Delta = (3 - 2i)^2 - 4(2 - 2i) = -3 - 4i.$$

A convenient square root is $1 - 2i$, since $(1 - 2i)^2 = 1 - 4i - 4 = -3 - 4i$. The other square root changes only the ordering of the two roots, so it cannot change the requested symmetric expression.

Step 2: Find the roots and combine their components Using the quadratic formula gives

$$x = \frac{3 - 2i \pm (1 - 2i)}{2},$$

so the two roots are $2 - 2i$ and 1. We may take $\alpha = 2,\ \beta = -2,\ \gamma = 1,\ \delta = 0$. Their sum is $3 - 2i$ and their product is $2 - 2i$, verifying both signs in the polynomial. Therefore

$$\alpha\gamma + \beta\delta = 2(1) + (-2)(0) = 2.$$

Interchanging the roots gives the same value. In particular, neither a negative product nor the square of the root sum is the quantity requested, which rules out the other numerical options.

Step 3: Conclusion We obtain $\alpha\gamma + \beta\delta = 2$. Hence, the correct option is (A).

Quick Tip to Solve: For roots written as real plus imaginary parts, distinguish the coordinate dot product from the real part of their ordinary product.

15. If $\sum_{r=1}^{13}\left\{\frac{1}{\sin\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)}\right\}=a\sqrt{3}+b, a, b \in Z$, **then** a^2+b^2 **is equal to :** [4 marks]

(A) 10

(B) 4

(C) 8

(D) 2

Correct Answer: (C) 8

Solution:

Concept:

- The identity $\cot u - \cot v = \sin(v-u)/(\sin u \sin v)$ converts reciprocals of products of sines into successive differences.
- A telescoping sum cancels all intermediate cotangents. Use the period π of cotangent only after keeping the first and last arguments correctly.

Step 1: Convert each summand Let $u_r = \pi/4 + (r-1)\pi/6$, so the second angle in the denominator is u_{r+1} and their difference is $\pi/6$. Since $\sin(\pi/6) = 1/2$,

$$\frac{1}{\sin u_r \sin u_{r+1}} = 2(\cot u_r - \cot u_{r+1}).$$

None of these sine factors vanishes: their angles are odd multiples of $\pi/12$, rather than integer multiples of π. Thus the identity is valid for every term, including terms whose sine factors have different signs.

Step 2: Telescope and identify the integers Summing from 1 through 13 leaves only the endpoints:

$$S = 2\left[\cot\frac{\pi}{4} - \cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)\right].$$

The last angle is $29\pi/12 = 2\pi + 5\pi/12$. Using $\cot(5\pi/12) = \cot 75^\circ = 2-\sqrt{3}$, we obtain

$$S = 2[1-(2-\sqrt{3})] = 2\sqrt{3}-2.$$

Hence $a = 2$ and $b = -2$. Their signs matter when matching the sum, even though their squares are positive. The desired number is $4+4=8$, so 10, 4 and 2 do not equal the required sum of squares.

Step 3: Conclusion We have $a^2+b^2 = 2^2+(-2)^2 = 8$. Hence, the correct option is (C).

Quick Tip to Solve: When sine arguments differ by a fixed angle, try a cotangent difference; count the last endpoint as one index beyond the final summand.

16. If $f(x)=\int \frac{1}{x^{1/4}\left(1+x^{1/4}\right)}\mathrm{d}x, f(0)=-6$, then $f(1)$ is equal to : [4 marks]

(A) $4\left(\log_{\mathrm{e}} 2-2\right)$

(B) $\log_{e^2} 2+2$

(C) $2-\log \mathrm{e}^2$

(D) $4\left(\log_e 2+2\right)$

Correct Answer: (A) $4\left(\log_{\mathrm{e}} 2-2\right)$

Solution:

Concept:

- For a rational expression in $x^{1/4}$, put $t=x^{1/4}$, so $x=t^4$ and $dx=4t^3\,dt$. This turns the integral into a rational function of t.
- The additive constant is fixed by $f(0)=-6$. Although the derivative is singular at zero, its antiderivative has a finite right-hand limit there, which supplies this boundary value.

Step 1: Substitute and divide the polynomial For $x>0$, take the positive fourth root $t=x^{1/4}$. Including the differential correctly gives

$$f(x)=4\int \frac{t^2}{1+t}\,dt.$$

The division identity $t^2/(1+t)=t-1+1/(1+t)$ is checked by multiplying by $1+t$. Therefore

$$f(x)=2t^2-4t+4\ln(1+t)+C.$$

In terms of x this is $2\sqrt{x}-4x^{1/4}+4\ln(1+x^{1/4})+C$. The logarithm has a positive argument throughout the nonnegative domain under consideration.

Step 2: Use the condition and evaluate at one As x tends to zero from the right, all three displayed variable terms tend to zero. Thus the stated boundary value gives $C=-6$. At $x=1$, the fourth root and square root are both 1, so

$$f(1)=2-4+4\ln 2-6=4\ln 2-8.$$

This also follows by adding $\int_0^1 f'(x)\,dx=4\ln 2-2$ to $f(0)$. The negative boundary constant must be retained; dropping it would produce a different value. The resulting expression matches the first option, whereas the remaining choices have different logarithmic coefficients or constant terms.

Step 3: Conclusion The required value is $f(1)=4(\log_e 2-2)$. Hence, the correct option is (A).

Quick Tip to Solve: After a root substitution, transform dx as well as the denominator; then fix the integration constant before substituting the final argument.

17. Let $\mathrm{A} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -2 \\ 0 & 1 \end{bmatrix}$ **and** $\mathrm{P} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \theta > 0$**. If** $\mathrm{B} = \mathrm{PAP}^{\top}, \mathrm{C} = \mathrm{P}^{\top}\mathrm{B}^{10}\mathrm{P}$ **and the sum of the diagonal elements of** C **is** $\frac{m}{n}$**, where** $\gcd(m, n) = 1$**, then** $m + n$ **is :** [4 marks]

(A) 127

(B) 2049

(C) 258

(D) 65

Correct Answer: (D) 65

Solution:

Concept:

- The rotation matrix is orthogonal: $P^{\top}P = PP^{\top} = I$. Therefore powers of $PAP^{\top}$ simplify by cancelling each adjacent $P^{\top}P$.
- The diagonal entries of a power of an upper triangular matrix are the corresponding powers of its diagonal entries. Its trace is the sum of these diagonal entries, without needing the off-diagonal entry.

Step 1: Remove the orthogonal similarity Using orthogonality repeatedly,

$$B^{10} = (PAP^{\top})^{10} = PA^{10}P^{\top}.$$

This identity follows already at the second power: $B^2 = PA(P^{\top}P)AP^{\top} = PA^2P^{\top}$, and the same cancellation continues for every further factor. Hence

$$C = P^{\top}B^{10}P = (P^{\top}P)A^{10}(P^{\top}P) = A^{10}.$$

Thus neither the positive value of θ nor the entries of P need to be evaluated explicitly.

Step 2: Read the diagonal and reduce the fraction The matrix A is upper triangular with diagonal entries $1/\sqrt{2}$ and 1. Multiplication preserves triangular form, so the diagonal entries of A^{10} are

$$\left(\frac{1}{\sqrt{2}}\right)^{10} = \frac{1}{32}, \qquad 1^{10} = 1.$$

Its off-diagonal entry, though affected by the printed -2, contributes nothing to the trace. Consequently the requested diagonal sum is $33/32$. Since consecutive integers 33 and 32 are coprime, this is already the stipulated reduced fraction: $m = 33$, $n = 32$. The sum is 65, excluding all other listed numbers without computing any full matrix power.

Step 3: Conclusion The diagonal sum is $33/32$, and $m + n = 33 + 32 = 65$. Hence, the correct option is (D).

Quick Tip to Solve: Cancel orthogonal similarity matrices before raising powers; for a triangular matrix, calculate the trace directly from the diagonal.

18. Let f be a real valued continuous function defined on the positive real axis such that $g(x) = \int_0^x tf(t)dt$. If $g\left(x^3\right) = x^6 + x^7$, then value of $\sum_{r=1}^{15} f\left(r^3\right)$ is : [4 marks]

(A) 270

(B) 340

(C) 310

(D) 320

Correct Answer: (C) 310

Solution:

Concept:

- For a continuous integrand, the fundamental theorem gives $g\prime(u) = uf(u)$. When differentiating $g(x^3)$, the chain rule adds the factor $3x^2$.
- The sum of the first n positive integers is $n(n+1)/2$. Once $f(r^3)$ is found as a linear expression in r, the required sum separates into a constant and an arithmetic sum.

Step 1: Differentiate the composite function For $x > 0$, differentiate the given identity directly:

$$g\prime(x^3)\,3x^2 = 6x^5 + 7x^6.$$

At the argument x^3, the fundamental theorem gives $g\prime(x^3) = x^3 f(x^3)$. Substituting it yields

$$3x^5 f(x^3) = 6x^5 + 7x^6.$$

The factor x^5 can be cancelled because the domain here is positive. It is essential to retain both the upper-limit factor x^3 from the integrand and the derivative $3x^2$ of that upper limit.

Step 2: Evaluate and sum at the required arguments Division by $3x^5$ gives $f(x^3) = 2 + 7x/3$. Thus, for each integer $r = 1, \ldots, 15$,

$$f(r^3) = 2 + \frac{7}{3}r.$$

Now sum all 15 terms:

$$\sum_{r=1}^{15} f(r^3) = 2(15) + \frac{7}{3}\frac{15(16)}{2} = 30 + 280 = 310.$$

As a check, writing $f(u) = 2 + (7/3)u^{1/3}$ and integrating $uf(u)$ gives $g(u) = u^2 + u^{7/3}$, which indeed satisfies the original condition. The other options do not equal this exact arithmetic sum.

Step 3: Conclusion The value of the sum is 310. Hence, the correct option is (C).

Quick Tip to Solve: Differentiate the supplied composite identity first; there is no need to solve separately for g before obtaining the values of f at cubes.

19. Let $f : [0,3] \to$ A be defined by $f(x) = 2x^3 - 15x^2 + 36x + 7$ and $g : [0,\infty) \to B$ be defined by $g(x) = \frac{x^{2025}}{x^{2025}+1}$, If both the functions are onto and $S = \{x \in Z; x \in A$ or $x \in B\}$, then $n(S)$ is equal to : [4 marks]

(A) 29

(B) 31

(C) 30

(D) 36

Correct Answer: (C) 30

Solution:

Concept:

- An onto function has codomain equal to its actual range. Thus find the ranges of both functions before counting integers.
- A continuous function on a closed interval attains its maximum and minimum at endpoints or stationary points. For g, monotonicity and the limiting value determine its range.

Step 1: Find the range of the cubic Differentiate and factor to locate all interior candidates:

$$f\prime(x) = 6x^2 - 30x + 36 = 6(x-2)(x-3).$$

The only interior stationary point is $x = 2$; $x = 3$ is already an endpoint. The derivative is positive on $(0,2)$ and negative on $(2,3)$. Evaluate the candidate values:

$$f(0) = 7, \qquad f(2) = 16 - 60 + 72 + 7 = 35, \qquad f(3) = 34.$$

Continuity gives every value between the minimum and maximum, so onto-ness implies $A = [7,35]$.

Step 2: Find the second range and count the union For $x \geq 0$, put $t = x^{2025}$. Then t takes all values in $[0,\infty)$, and

$$g(x) = \frac{t}{t+1} = 1 - \frac{1}{t+1}.$$

This increases from 0 towards 1, never attaining 1 at a finite input. Thus $B = [0,1)$, whose sole integer is 0. The integers in A are $7, 8, \ldots, 35$, giving $35 - 7 + 1 = 29$ integers. This set is disjoint from $\{0\}$, so the union has $29 + 1$ members. Option (A) forgets the integer from B; (B) and (D) do not equal the count of the actual integer union.

Step 3: Conclusion

$$n(S) = 30.$$

Hence, the correct option is (C).

Quick Tip to Solve: For an onto-function range question, determine endpoint inclusion before counting integers: here 0 belongs to the second range but 1 does not.

20. For positive integers n, if $4a_n = (n^2 + 5n + 6)$ and $S_n = \sum_{k=1}^{n} \left(\frac{1}{a_k}\right)$, then the value of $507S_{2025}$ is : [4 marks]

(A) 540

(B) 675

(C) 1350

(D) 135

Correct Answer: (B) 675

Solution:

Concept:

- Factor a quadratic denominator before summing its reciprocal. Consecutive linear factors lead to a telescoping partial-fraction decomposition.
- The given equation is $4a_k = (k+2)(k+3)$, so its reciprocal contains a factor of 4, rather than $1/4$.

Step 1: Decompose the reciprocal From the stated definition,

$$a_k = \frac{(k+2)(k+3)}{4}, \qquad \frac{1}{a_k} = \frac{4}{(k+2)(k+3)}.$$

Because the two factors differ by one,

$$\frac{1}{k+2} - \frac{1}{k+3} = \frac{1}{(k+2)(k+3)}.$$

Therefore each summand equals $4\big(1/(k+2) - 1/(k+3)\big)$. The factor of four must remain outside the entire sum.

Step 2: Cancel the intermediate terms Writing the first and last terms makes the endpoint indices explicit:

$$S_n = 4\left[\left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \cdots + \left(\frac{1}{n+2} - \frac{1}{n+3}\right)\right].$$

Every intermediate reciprocal occurs once with each sign, leaving

$$S_n = 4\left(\frac{1}{3} - \frac{1}{n+3}\right) = \frac{4n}{3(n+3)}.$$

As a check, $n = 1$ gives $S_1 = 1/3 = 1/a_1$, consistent with the original sequence.

Step 3: Evaluate the requested multiple At $n = 2025$, the final denominator is 2028, not 2025 or 2027. Since $2028 = 4 \cdot 507$,

$$507S_{2025} = 507\frac{4 \cdot 2025}{3 \cdot 2028} = \frac{2025}{3} = 675.$$

The other listed numbers, 540, 1350 and 135, fail this exact cancellation; in particular 1350 is twice the required value.

Step 4: Conclusion

$$507S_{2025} = 675.$$

Hence, the correct option is (B).

Quick Tip to Solve: After telescoping, check the first surviving denominator and the final one directly from k=1 and k=n before substituting a large index.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let $f(x) = \lim\limits_{n\to\infty} \sum\limits_{r=0}^{n} \left(\frac{\tan\left(x/2^{r+1}\right)+\tan^3\left(x/2^{r+1}\right)}{1-\tan^2(x/2^{r+1})} \right)$ **Then** $\lim\limits_{x\to 0} \frac{e^x - e^{f(x)}}{(x-f(x))}$ **is equal to** _____. [4 marks]

Solution:

Concept:

- The double-angle identity $\tan(2t) = 2\tan t/(1-\tan^2 t)$ converts the summand into a difference of consecutive tangent terms.
- The exponential divided difference tends to its derivative when both arguments tend to the same point. Here both arguments will tend to zero.

Step 1: Rewrite the summand Work in a sufficiently small neighbourhood $|x| < \pi/4$; all denominators in the sum are then nonzero. Put $u = \tan t$, where $t = x/2^{r+1}$. The identity gives

$$\tan(2t) - \tan t = \frac{2u - u(1-u^2)}{1-u^2} = \frac{u+u^3}{1-u^2}.$$

Hence the term with index r is precisely

$$\tan\left(\frac{x}{2^r}\right) - \tan\left(\frac{x}{2^{r+1}}\right).$$

This equality preserves the cubic numerator term and the minus sign in the denominator.

Step 2: Sum before taking the limit The finite sum cancels in adjacent pairs:

$$\sum_{r=0}^{n} \left[\tan\left(\frac{x}{2^r}\right) - \tan\left(\frac{x}{2^{r+1}}\right)\right] = \tan x - \tan\left(\frac{x}{2^{n+1}}\right).$$

For fixed such x, the second argument tends to zero as $n \to \infty$, giving $f(x) = \tan x$. In particular, $f(x) \to 0$ as $x \to 0$.

Step 3: Evaluate the divided difference For nonzero sufficiently small x, $\tan x \neq x$: the function $\tan x - x$ is strictly increasing away from its zero at the origin, since its derivative is $\tan^2 x$. Apply the mean value theorem to the exponential function between x and $\tan x$:

$$\frac{e^x - e^{\tan x}}{x - \tan x} = e^{c_x},$$

where c_x lies between those two arguments. Both endpoints tend to zero, so $c_x \to 0$ and $e^{c_x} \to 1$.

Step 4: Conclusion The required numerical value is $\boxed{1}$.

Quick Tip to Solve: Recognise (tan t + tan cubed t)/(1 - tan squared t) as tan(2t) - tan t; dyadic arguments then telescope immediately.

22. Let A and B be the two points of intersection of the line $y+5=0$ and the mirror image of the parabola $y^2=4x$ with respect to the line $x+y+4=0$. If d denotes the distance between A and B, and a denotes the area of $\triangle SAB$, where S is the focus of the parabola $y^2=4x$, then the value of $(a+d)$ is _____. [4 marks]

Solution:

Concept:

- Reflection of (u,v) in $x+y+4=0$ maps it to $(-v-4,-u-4)$. Reflection is its own inverse.
- The focus of $y^2=4x$ is $(1,0)$. For a horizontal base, the triangle altitude is the vertical distance from the vertex to its supporting line.

Step 1: Obtain the reflected equation Let (u,v) lie on the original parabola, so $v^2=4u$, and let (x,y) be its reflected point. The standard line-reflection formula subtracts $u+v+4$ from each coordinate:

$$x=u-(u+v+4)=-v-4, \qquad y=v-(u+v+4)=-u-4.$$

Thus $u=-y-4$, $v=-x-4$. Substitution into the original equation yields

$$(-x-4)^2=4(-y-4), \qquad (x+4)^2=-4(y+4).$$

This equation describes the mirror image, rather than the original parabola.

Step 2: Find the chord endpoints and length The line is $y=-5$. On inserting this into the reflected equation,

$$(x+4)^2=-4(-5+4)=4.$$

Thus $x+4=\pm 2$, giving the two intersection points $A=(-6,-5)$ and $B=(-2,-5)$, in either order. They share the same ordinate, so their distance is

$$d=|-2-(-6)|=4.$$

There is no additional vertical component in this distance.

Step 3: Use the specified original focus The question specifies the focus of $y^2=4x$, which is $S=(1,0)$, not the reflected focus. The perpendicular distance of S from the base line $y=-5$ is 5. Its foot may lie outside the segment; the usual base-altitude area formula still applies. Hence

$$a=\frac{1}{2}\cdot AB\cdot 5=\frac{1}{2}\cdot 4\cdot 5=10.$$

Adding the chord length gives $a+d=10+4$.

Step 4: Conclusion The required numerical value is $\boxed{14}$.

Quick Tip to Solve: For reflection in x+y+c=0, use (u,v) maps to (-v-c,-u-c), then substitute the inverse map into the curve equation.

23. If $y = y(x)$ is the solution of the differential equation, $\sqrt{4-x^2}\frac{dy}{dx} = \left(\left(\sin^{-1}\left(\frac{x}{2}\right)\right)^2 - y\right)\sin^{-1}\left(\frac{x}{2}\right), -2 \leq x \leq 2, y(2) = \frac{\pi^2-8}{4}$, then $y^2(0)$ is equal to _____. [4 marks]

Solution:

Concept:

- The substitution $t = \sin^{-1}(x/2)$ satisfies $dt/dx = 1/\sqrt{4-x^2}$ on $-2 < x < 2$, eliminating the square-root coefficient.
- A first-order linear equation $dy/dt + ty = t^3$ has integrating factor $e^{t^2/2}$. Apply the given endpoint value by continuity.

Step 1: Transform the independent variable Set $t = \sin^{-1}(x/2)$, taking the principal inverse sine so that $-\pi/2 \leq t \leq \pi/2$. The chain rule on the open interval gives

$$\sqrt{4-x^2}\frac{dy}{dx} = \frac{dy}{dt}.$$

The differential equation therefore becomes

$$\frac{dy}{dt} = (t^2 - y)t, \qquad \frac{dy}{dt} + ty = t^3.$$

The condition at $x = 2$ corresponds to $t = \pi/2$.

Step 2: Integrate the linear equation Multiplying by $e^{t^2/2}$ turns the left side into a product derivative:

$$\frac{d}{dt}\left(ye^{t^2/2}\right) = t^3e^{t^2/2}.$$

For the integral, put $z = t^2/2$, so $t\,dt = dz$ and $t^2 = 2z$. Then

$$\int t^3e^{t^2/2}\,dt = 2\int ze^z\,dz = 2e^z(z-1) + C.$$

Consequently

$$y = t^2 - 2 + Ce^{-t^2/2}.$$

Differentiating this expression checks that $y\prime + ty = t^3$.

Step 3: Apply the condition and evaluate At $t = \pi/2$, the particular part already equals the prescribed value:

$$t^2 - 2 = \frac{\pi^2 - 8}{4}.$$

Thus $Ce^{-\pi^2/8} = 0$, and the positive exponential factor forces $C = 0$. The resulting expression extends continuously to the stated endpoints and is $y(x) = [\sin^{-1}(x/2)]^2 - 2$. At $x = 0$, $t = 0$, so

$$y(0) = -2, \qquad y^2(0) = (-2)^2 = 4.$$

The square is taken after finding the signed value of $y(0)$.

Step 4: Conclusion The required numerical value is $\boxed{4}$.

Quick Tip to Solve: When a square root of 4-x squared multiplies dy/dx alongside inverse sine of x/2, switch the independent variable to that inverse sine.

24. The interior angles of a polygon with *n* sides, are in an A.P. with common difference 6°. If the largest interior angle of the polygon is 219°, then *n* is equal to _____. [4 marks]

Solution:

Concept:

- The interior angles of a simple polygon with n sides sum to $180(n-2)$ degrees, including for a concave polygon.
- An arithmetic progression with first term a, common difference d, and n terms has last term $a+(n-1)d$ and sum $n(a+l)/2$, where l is its last term.

Step 1: Express the smallest angle in terms of n The common difference is positive, $6°$, so the largest angle $219°$ is the last term. If the smallest angle is a degrees, then

$$a+6(n-1)=219, \qquad a=225-6n.$$

There are n angles, one at each vertex; the number of intervals between them is $n-1$, which explains this subtraction. A reflex largest angle is consistent with a concave polygon and does not change the interior-angle sum formula.

Step 2: Equate the two expressions for the sum The arithmetic-progression sum and the polygon sum refer to the same set of interior angles. Equating their numerical degree values gives

$$\frac{n}{2}\left[(225-6n)+219\right]=180(n-2).$$

Simplify without approximating:

$$222n-3n^2=180n-360,$$

$$3n^2-42n-360=0, \qquad n^2-14n-120=0.$$

Factoring,

$$(n-20)(n+6)=0.$$

Both algebraic roots must be considered before applying the geometric constraint.

Step 3: Select and check the valid root The roots are $n=20$ and $n=-6$. A polygon must have a positive integer number of sides, at least three, so the negative root is inadmissible. For $n=20$, the first angle is $225-120=105°$. The final angle is $105+19\cdot 6=219°$, as given. The total is

$$\frac{20}{2}(105+219)=3240°=180(20-2)°.$$

Thus the accepted root satisfies both the progression and the polygon condition.

Step 4: Conclusion The polygon has $\boxed{20}$ sides.

Quick Tip to Solve: Use the largest angle as the last A.P. term, not the first; there are n-1 common-difference increments among n interior angles.

25. The number of natural numbers, between 212 and 999, such that the sum of their digits is 15, is _____.

[4 marks]

Solution:

Concept:

- Write a three-digit number as $100a + 10b + c$, where a is the hundreds digit and $0 \le b, c \le 9$. The digit-sum condition is $a + b + c = 15$.
- For a fixed hundreds digit, count ordered pairs (b, c); interchanging the tens and units digits generally produces a different number.

Step 1: Identify the allowed hundreds digits The required interval lies within the three-digit numbers with hundreds digit from 2 to 9. When $a = 2$, the condition is $b + c = 13$. Its smallest admissible number is 249, which is already greater than 212. Thus no digit-sum solution with hundreds digit 2 is excluded by the lower boundary. Also 999 has digit sum 27, so whether the endpoints are included does not change the count.

Step 2: Count the pairs for each digit For $b + c = s$ with $10 \le s \le 18$, the tens digit ranges from $s - 9$ to 9, giving $19 - s$ pairs. For $0 \le s \le 9$, it ranges from 0 to s, giving $s + 1$ pairs. The required sums and counts are therefore

a	2	3	4	5	6	7	8	9
$s = 15 - a$	13	12	11	10	9	8	7	6
count	6	7	8	9	10	9	8	7

Zero is allowed in the tens or units place, so these endpoint pairs must be retained.

Step 3: Add the disjoint cases Each number has exactly one hundreds digit, and each ordered pair specifies exactly one number. The cases neither overlap nor omit any admissible number. Hence

$$N = 6 + 7 + 8 + 9 + 10 + 9 + 8 + 7 = 64.$$

Step 4: Conclusion The number of natural numbers satisfying the condition is $\boxed{64}$.

Quick Tip to Solve: Fix the hundreds digit first and count ordered digit pairs; check the boundary only in the first hundreds block.

26. A concave mirror produces an image of an object such that the distance between the object and image is 20 cm. If the magnification of the image is –3, then the magnitude of the radius of curvature of the mirror is : [4 marks]

(A) 3.75 cm

(B) 15 cm

(C) 7.5 cm

(D) 30 cm

Correct Answer: (B) 15 cm

Solution:

Concept:

- For a spherical mirror, the Cartesian sign convention gives $m = -v/u$, where u and v are the signed object and image distances.
- The mirror equation is $1/f = 1/u + 1/v$, and the magnitude of the radius of curvature is $|R| = 2|f|$.

Step 1: Use the inverted magnification The given negative magnification means that the image is inverted and real. Both the object and this real image lie in front of the concave mirror. Put $u = -x$, with $x > 0$. Then

$$-\frac{v}{u} = -3 \quad \Longrightarrow \quad v = 3u = -3x.$$

The image is three times as far from the mirror as the object; the minus sign in the magnification must not be treated as a negative physical size.

Step 2: Use the separation of object and image As both positions are on the same side of the mirror, their physical separation is the difference of their distances from it:

$$|v - u| = |-3x + x| = 2x = 20\,\text{cm}.$$

Thus $x = 10\,\text{cm}$, so $u = -10\,\text{cm}$ and $v = -30\,\text{cm}$. Adding 10 and 30 would give the separation only for points on opposite sides, which is not this real-image configuration.

Step 3: Calculate the curvature radius Substitute these signed distances into the mirror equation:

$$\frac{1}{f} = -\frac{1}{10} - \frac{1}{30} = -\frac{2}{15}, \qquad f = -7.5\,\text{cm}.$$

Consequently $|R| = 2(7.5) = 15\,\text{cm}$. Option (C) gives the focal-length magnitude instead of the radius; (A) halves it again, while (D) is the image-distance magnitude. None of those quantities answers the question.

Step 4: Conclusion The magnitude of the radius of curvature is $\boxed{15\,\text{cm}}$. Hence, the correct option is (B).

Quick Tip to Solve: For an inverted magnified mirror image, first put object and real image on the same side; use their distance difference before the mirror formula.

27. A uniform rod of mass 250 g having length 100 cm is balanced on a sharp edge at 40 cm mark. A mass of 400 g is suspended at 10 cm mark. To maintain the balance of the rod, the mass to be suspended at 90 cm mark, is [4 marks]

(A) 290 g

(B) 200 g

(C) 190 g

(D) 300 g

Correct Answer: (C) 190 g

Solution:

Concept:

- A rod in rotational equilibrium has zero net torque about the support. A downward force on the left and a downward force on the right produce opposite torques.
- The weight of a uniform rod acts at its midpoint, the 50 cm mark here. Taking moments about the sharp edge eliminates its unknown support reaction.

Step 1: Locate every lever arm The pivot is at the 40 cm mark. The 400 g load at the 10 cm mark is therefore 30 cm to its left. The rod has mass 250 g and its centre of mass is at 50 cm, which is 10 cm to the right of the pivot. The unknown mass M, suspended at 90 cm, has a lever arm of 50 cm on the right. These distances must be measured from the support, rather than from the left end of the rod.

Step 2: Write the torque equation The left-hand load produces an anticlockwise torque, while the rod's own weight and the unknown right-hand load produce clockwise torques. All forces are vertical, so the horizontal separations are the perpendicular lever arms. Equating their magnitudes gives

$$400g(30) = 250g(10) + Mg(50).$$

Using masses in grams and lengths in centimetres is consistent because the same conversion factors and gravitational acceleration multiply every term and cancel.

Step 3: Solve and check the candidate masses After cancelling the common acceleration due to gravity,

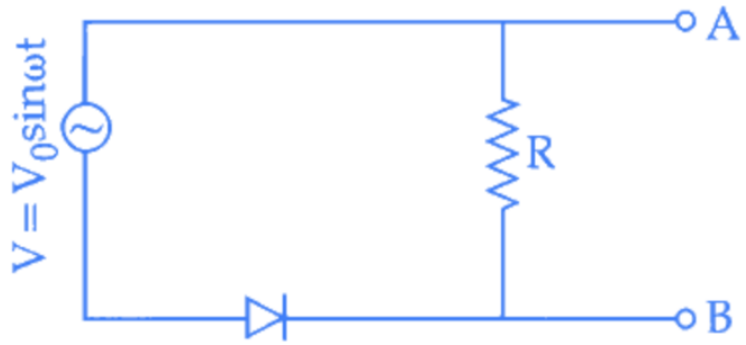
$$12000 = 2500 + 50M, \qquad M = \frac{9500}{50} = 190\,\text{g}.$$

Its right-hand moment is $2500 + 190(50) = 12000$, exactly matching the left-hand moment. The candidate 200 g would result from an incorrect treatment of the rod weight, while 290 g and 300 g also give excessive clockwise moments. Ignoring the rod's own weight would wrongly give 240 g.

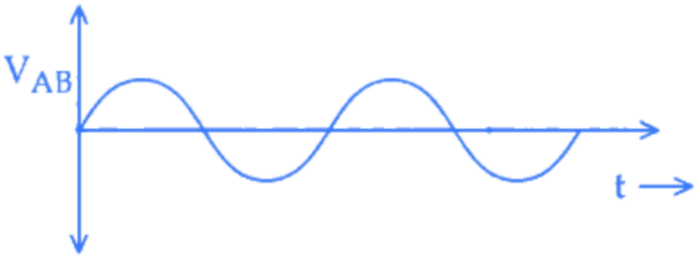
Step 4: Conclusion The required suspended mass is $\boxed{190\,\text{g}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a uniform rod, place its entire weight at the midpoint before taking moments about the actual support.

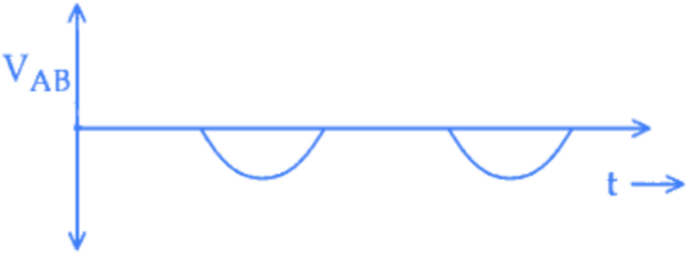
28. In the circuit shown here, assuming threshold voltage of diode is negligibly small, then voltage V_{AB} is correctly represented by: [4 marks]



(A) Full sinusoidal waveform with positive and negative half-cycles.

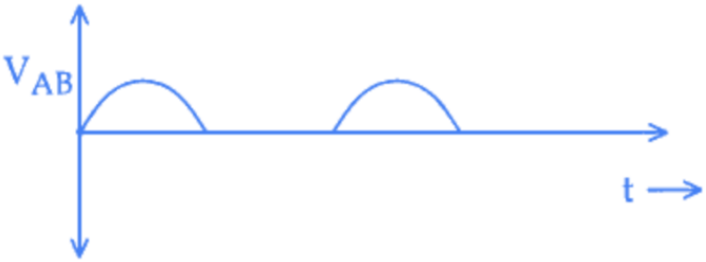


(B) Negative half-wave waveform, with zero output during positive half-cycles.



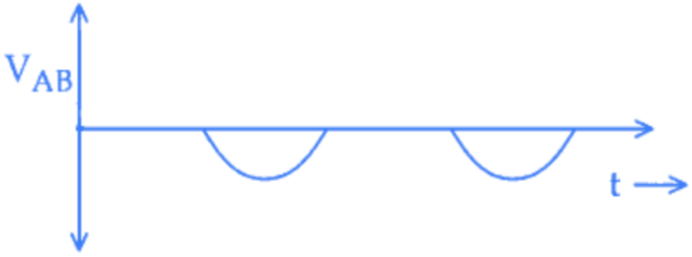
(C) V_{AB} would be zero at all times.

(D) Positive half-wave waveform, with zero output during negative half-cycles.



Correct Answer: (B)

Negative half-wave waveform, with zero output during positive half-cycles.



Solution:

Concept:

- An ideal diode conducts from its anode to its cathode when forward biased, and blocks current when reverse biased. Its conducting voltage drop is zero here.
- The measured output is $V_{AB} = V_A - V_B$. The polarity of this output must be determined from the circuit connections rather than from the usual positive rectifier sketch.

Step 1: Determine the conducting half-cycle The diode in the lower wire has its anode at the left and cathode at the right, towards terminal B. Suppose first that the source makes A positive relative to its lower terminal. A possible load current would have to travel from A down through R to B and then through the diode from right to left. That is the blocked direction. Thus the diode is reverse biased during this positive source half-cycle, and no steady current flows through the resistor.

Step 2: Find the output in both states In the blocked state, the current through R is zero, so $V_{AB} = IR = 0$. When the source reverses polarity, its lower terminal becomes positive relative to A. Current can then pass from the lower terminal through the diode from left to right, into B and upwards through R to A. Therefore B is positive relative to A during conduction and

$$V_{AB} = V_A - V_B < 0.$$

With a negligible diode threshold, the conducting negative half-cycle follows the applied sinusoid without a voltage reduction.

Step 3: Compare the plotted waveforms The output is zero over the positive half-cycles and consists of negative sinusoidal pulses over the negative half-cycles. This is negative half-wave rectification. Option (A) retains both polarities and would require the diode not to block; option (C) incorrectly suppresses even the conducting half-cycles; option (D) has positive pulses, the opposite output polarity for the diode orientation shown.

Step 4: Conclusion The output is the negative half-wave waveform: $V_{AB} = 0$ on the blocked half-cycle and negative on the conducting half-cycle. Hence, the correct option is (B).

Quick Tip to Solve: Trace conventional current through the diode arrow direction, then mark which resistor terminal is at higher potential before choosing a rectified waveform.

29. A uniform magnetic field of 0.4 T acts perpendicular to a circular copper disc 20 cm in radius. The disc is having a uniform angular velocity of 10 π rad s^{-1} about an axis through its centre and perpendicular to the disc. What is the potential difference developed between the axis of the disc and the rim? $(\pi = 3.14)$ [4 marks]

(A) 0.5024 V

(B) 0.0628 V

(C) 0.2512 V

(D) 0.1256 V

Correct Answer: (C) 0.2512 V

Solution:

Concept:

- A point of a rigid disc at distance r from its axis moves at tangential speed $v = \omega r$. A perpendicular magnetic field produces a radial magnetic force on its mobile charges.
- The magnitude of the motional emf across a radial element is $d\mathcal{E} = Bv\,dr = B\omega r\,dr$; the full centre-to-rim emf is found by integration.

Step 1: Express the local motional emf Take a narrow conducting radial strip of the disc. Its inner elements move more slowly than its outer elements, so the full strip cannot be assigned the rim speed throughout. For a charge moving with the material, the magnetic force has magnitude qvB, because the velocity and the field are perpendicular. Charge separation establishes a compensating electric field along the radius with magnitude $E(r) = B\omega r$.

Step 2: Integrate from the axis to the rim The potential difference is the integral of this radial electric field. With disc radius R,

$$|\Delta V| = \int_0^R B\omega r\,dr = \frac{1}{2}B\omega R^2.$$

The factor one-half expresses the variation of speed from zero at the centre to ωR at the rim. The question asks for the magnitude, so no choice of magnetic-field sense is necessary.

Step 3: Substitute the quantities in SI units Convert the radius: $R = 20\,\text{cm} = 0.20\,\text{m}$, and use $\omega = 10\pi = 31.4\,\text{rad}\,\text{s}^{-1}$. Then

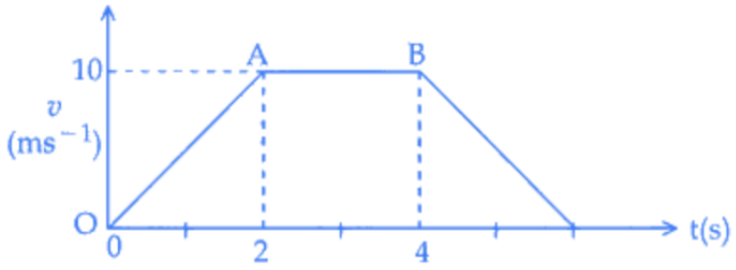
$$|\Delta V| = \frac{1}{2}(0.4)(31.4)(0.20)^2 = 0.2512\,\text{V}.$$

Option (A) doubles this value by omitting the one-half factor. Options (B) and (D) are smaller than the correctly integrated value and do not follow from the given radius and angular speed.

Step 4: Conclusion The potential difference between the axis and rim is $\boxed{0.2512\,\text{V}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a rotating disc, integrate the linearly increasing speed along a radius: the emf is half of $Bv_{\text{rim}}R$.

30. The velocity-time graph of an object moving along a straight line is shown in the figure. What is the distance covered by the object between $t = 0$ to $t = 4s$? [4 marks]



(A) 11 m

(B) 30 m

(C) 13 m

(D) 10 m

Correct Answer: (B) 30 m

Solution:

Concept:

- Distance travelled is the area under a speed-time graph, $s = \int |v|\, dt$. If velocity stays nonnegative, the area under the velocity-time curve gives both distance and displacement.
- The plotted motion has uniform acceleration from 0 to 2 s, followed by constant velocity from 2 to 4 s. These intervals form a triangle and a rectangle.

Step 1: Read only the requested interval The graph starts at $(0, 0)$, rises linearly to A at $(2\,\text{s}, 10\,\text{m s}^{-1})$, and remains horizontal until B at $(4\,\text{s}, 10\,\text{m s}^{-1})$. The later downward sloping part is outside the specified interval and contributes no distance to this question. Velocity is positive throughout the interval except at its starting instant, so there is no need to separate positive and negative areas.

Step 2: Calculate the triangular and rectangular areas Between 0 and 2 s, the area is that of a triangle with base 2 s and height $10\,\text{m s}^{-1}$:

$$s_1 = \frac{1}{2}(2)(10) = 10\,\text{m}.$$

Between 2 and 4 s, the velocity is constant and the area is a rectangle:

$$s_2 = (4 - 2)(10) = 20\,\text{m}.$$

Each area has units of velocity multiplied by time, which are metres.

Step 3: Add and compare with the options The two time intervals are consecutive and exhaustive, so their distances add:

$$s = s_1 + s_2 = 10 + 20 = 30\,\text{m}.$$

The 10 m option counts the first triangle alone and omits the constant-velocity segment. The 11 m and 13 m choices do not equal the sum of the graph areas. Taking the entire interval as a rectangle would also be wrong because the initial velocity is not already $10\,\text{m s}^{-1}$.

Step 4: Conclusion The distance covered from $t = 0$ to $t = 4\,\text{s}$ is $\boxed{30\,\text{m}}$. Hence, the correct option is

(B).

Quick Tip to Solve: Mark the requested end time first, then split the velocity graph into simple areas; ignore every segment beyond that time.

31. The magnetic field of an E.M. wave is given by $\vec{B} = \left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right) 30 \sin\left[\omega\left(t - \frac{z}{c}\right)\right]$ (S.I. Units). The corresponding electric field in S.I. units is: [4 marks]

(A) $\overrightarrow{\mathrm{E}} = \left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) 30\mathrm{c} \sin\left[\omega\left(\mathrm{t} + \frac{z}{\mathrm{c}}\right)\right]$

(B) $\overrightarrow{\mathrm{E}} = \left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right) 30\mathrm{c} \sin\left[\omega\left(\mathrm{t} - \frac{z}{\mathrm{c}}\right)\right]$

(C) $\overrightarrow{\mathrm{E}} = \left(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right) 30\mathrm{c} \sin\left[\omega\left(\mathrm{t} + \frac{z}{\mathrm{c}}\right)\right]$

(D) $\overrightarrow{\mathrm{E}} = \left(\frac{3}{4}\hat{i} + \frac{1}{4}\hat{j}\right) 30\mathrm{c} \cos\left[\omega\left(\mathrm{t} - \frac{z}{\mathrm{c}}\right)\right]$

Correct Answer: (B) $\overrightarrow{\mathrm{E}} = \left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right) 30\mathrm{c} \sin\left[\omega\left(\mathrm{t} - \frac{z}{\mathrm{c}}\right)\right]$

Solution:

Concept:

- In a plane electromagnetic wave in vacuum, the electric and magnetic fields are transverse, mutually perpendicular and in phase. Their magnitudes satisfy $E = cB$.
- The propagation direction is along $\vec{E} \times \vec{B}$. A phase of the form $\omega(t - z/c)$ represents travel along the positive z-axis.

Step 1: Identify the direction of propagation For a fixed phase, $t - z/c$ is constant, so increasing time increases z. Thus the wave travels along $+\hat{k}$. The electric field must have exactly the same sine phase as the magnetic field; changing to $t + z/c$ would reverse the propagation direction.

Step 2: Find the transverse electric vector Write $\vec{B} = B_x\hat{i} + B_y\hat{j}$. For propagation along $+\hat{k}$, the required relation is $\vec{E} = c(\vec{B} \times \hat{k})$. Since $\hat{i} \times \hat{k} = -\hat{j}$ and $\hat{j} \times \hat{k} = \hat{i}$, this gives

$$\vec{E} = 30c\left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right) \sin\left[\omega\left(t - \frac{z}{c}\right)\right].$$

The polarization vectors have zero dot product and unit magnitude. Their cross product points along $+\hat{k}$, as required.

Step 3: Eliminate the other options Options (A) and (C) use the opposite travelling-wave phase. Option (D) has a cosine instead of the common sine phase, and its polarization vector is not perpendicular to the given magnetic vector. Hence none can describe this wave.

Step 4: Conclusion The electric field is $\vec{E} = 30c(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j})\sin[\omega(t - z/c)]$ in SI units. Hence, the correct option is (B).

Quick Tip to Solve: Use $\vec{E} = c(\vec{B} \times \hat{n})$, with $\hat{n}$ read from the travelling-wave phase; then check $\vec{E} \times \vec{B}$.

32. A balloon and its content having mass M is moving up with an acceleration 'a'. The mass that must be released from the content so that the balloon starts moving up with an acceleration '3a' will be

(Take 'g' as acceleration due to gravity) [4 marks]

(A) $\frac{3Ma}{2a+g}$
(B) $\frac{2Ma}{3a+g}$
(C) $\frac{3Ma}{2a-g}$
(D) $\frac{2Ma}{3a-g}$

Correct Answer: (B) $\frac{2Ma}{3a+g}$

Solution:

Concept:

- Newton's second law applies to the balloon and remaining contents as one system. Taking upward as positive, buoyancy minus weight equals mass times upward acceleration.
- The upthrust is unchanged when ballast is released: the balloon displaces the same external volume of air. Its weight and inertial mass both decrease.

Step 1: Find the initial upthrust Let the constant upward buoyant force be F. The initial total mass is M, so

$$F - Mg = Ma, \qquad F = M(g + a).$$

This includes the weight of the balloon and all contents; Ma alone is the net force, not the buoyant force.

Step 2: Apply the new acceleration Let the released mass be m. Immediately afterwards the remaining total mass is $M - m$, while the upthrust remains F. The new upward acceleration is $3a$, hence

$$F - (M - m)g = 3a(M - m).$$

Substitute the first expression for F:

$$M(g + a) = (M - m)(g + 3a).$$

Step 3: Solve and check the mass Rearranging gives

$$M - m = \frac{M(g + a)}{g + 3a}, \qquad m = M\frac{(g + 3a) - (g + a)}{g + 3a} = \frac{2Ma}{g + 3a}.$$

For positive a and g, this mass is positive and smaller than M. The numerator is set by the increase $3a - a = 2a$; this rules out (A) and (C). The weight and inertial terms add in the denominator, so the minus sign in (D) is incorrect.

Step 4: Conclusion The mass to release is $\boxed{2Ma/(3a + g)}$. Hence, the correct option is (B).

Quick Tip to Solve: Keep buoyancy fixed but replace the mass in both the weight and the inertia term after releasing ballast.

33. The ratio of vapour densities of two gases at the same temperature is $\frac{4}{25}$, then the ratio of r.m.s. velocities will be : [4 marks]

(A) $\frac{5}{2}$
(B) $\frac{25}{4}$
(C) $\frac{4}{25}$
(D) $\frac{2}{5}$

Correct Answer: (A) $\frac{5}{2}$

Solution:

Concept:

- The root mean square molecular speed of an ideal gas is $v_{\text{rms}} = \sqrt{3RT/\mathcal{M}}$, where $\mathcal{M}$ is its molar mass and T its absolute temperature.
- Vapour density is measured relative to hydrogen and is proportional to molar mass: $\mathcal{M} = 2D$ in the usual relative units. Ratios of vapour densities therefore equal ratios of molar masses.

Step 1: Convert vapour density into mass ratio Label the gases 1 and 2 in the order given. Since the same reference gas defines both vapour densities,

$$\frac{\mathcal{M}_1}{\mathcal{M}_2} = \frac{D_1}{D_2} = \frac{4}{25}.$$

The first gas has the smaller molecular mass. Equal temperature means equal average translational kinetic energy per molecule, so the first gas must move faster.

Step 2: Take the inverse square root Dividing the two rms-speed expressions cancels the common factors $3R$ and T:

$$\frac{v_1}{v_2} = \sqrt{\frac{3RT/\mathcal{M}_1}{3RT/\mathcal{M}_2}} = \sqrt{\frac{\mathcal{M}_2}{\mathcal{M}_1}} = \sqrt{\frac{25}{4}} = \frac{5}{2}.$$

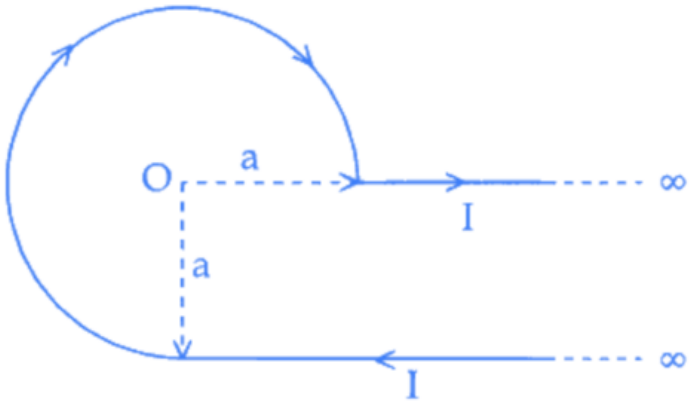
Thus speed scales inversely with the square root of vapour density, rather than inversely with density itself.

Step 3: Compare with the alternatives Option (B), $25/4$, omits the square root. Option (C) repeats the density ratio instead of converting it to speed. Option (D), $2/5$, takes the square root without reversing the ratio and would incorrectly make the lighter gas slower. The physical ordering and the calculation both select (A).

Step 4: Conclusion The ratio of rms speeds, in the same order as the vapour densities, is $\boxed{5/2}$. Hence, the correct option is (A).

Quick Tip to Solve: For two gases at equal temperature, reverse the molar-mass or vapour-density ratio first, then take its square root.

34. An infinite wire has a circular bend of radius *a*, and carrying a current *I* as shown in the figure. The magnitude of magnetic field at the origin O of the arc is given by: [4 marks]



(A) $\frac{\mu_0}{2\pi}\frac{\mathrm{I}}{\mathrm{a}}\left[\frac{\pi}{2}+2\right]$
(B) $\frac{\mu_0}{4\pi}\frac{\mathrm{I}}{\mathrm{a}}\left[\frac{3\pi}{2}+1\right]$
(C) $\frac{\mu_0}{4\pi}\frac{\mathrm{I}}{\mathrm{a}}\left[\frac{\pi}{2}+1\right]$
(D) $\frac{\mu_0}{4\pi}\frac{\mathrm{I}}{\mathrm{a}}\left[\frac{3\pi}{2}+2\right]$

Correct Answer: (B) $\frac{\mu_0}{4\pi}\frac{\mathrm{I}}{\mathrm{a}}\left[\frac{3\pi}{2}+1\right]$

Solution:

Concept:

- The magnetic field at the centre of a circular arc of radius a and angle θ radians is $B_{\text{arc}} = \mu_0 I\theta/(4\pi a)$.
- A semi-infinite straight wire at perpendicular distance a, beginning at the perpendicular foot, produces $\mu_0 I/(4\pi a)$. A radial straight segment gives zero field at its own line because $d\vec{\ell} \times \vec{r} = 0$.

Step 1: Identify the contributing pieces The illustrated bend runs clockwise from the bottom of the circle, around its left and top sides, to the rightmost point. It therefore subtends $270^\circ = 3\pi/2$, not $\pi/2$. The upper straight extension lies along the radius through O, so its contribution at O is zero. The lower horizontal wire extends from the bottom tangent point to infinity.

Step 2: Calculate the arc and straight-wire fields The clockwise arc gives a field into the plane, of magnitude

$$B_{\text{arc}} = \frac{\mu_0 I}{4\pi a}\frac{3\pi}{2}.$$

The lower wire carries current to the left. Since O lies above that wire, its right-hand-rule field is also into the plane. Its magnitude is

$$B_{\text{straight}} = \frac{\mu_0 I}{4\pi a}.$$

Step 3: Add fields with their directions The two nonzero contributions point in the same direction, so their magnitudes add:

$$B_O = \frac{\mu_0 I}{4\pi a}\left(\frac{3\pi}{2}+1\right).$$

The radial extension supplies no extra straight-wire term. Option (C) uses the missing quarter-circle

rather than the actual three-quarter arc; (D) counts two tangent contributions; (A) also has an incorrect combination of coefficients.

Step 4: Conclusion The magnetic-field magnitude is $\boxed{\frac{\mu_0 I}{4\pi a}\left(\frac{3\pi}{2}+1\right)}$, directed into the plane. Hence, the correct option is (B).

Quick Tip to Solve: Before adding arc and wire fields, mark radial segments as zero and check each remaining direction with the right-hand rule.

35. The frequency of revolution of the electron in Bohr's orbit varies with *n*, the principal quantum number as: [4 marks]

(A) $\frac{1}{n^4}$
(B) $\frac{1}{n^2}$
(C) $\frac{1}{n^3}$
(D) $\frac{1}{n}$

Correct Answer: (C) $\frac{1}{n^3}$

Solution:

Concept:

- For a fixed hydrogen-like atom, the Bohr orbit radius scales as $r_n \propto n^2$, while the electron speed scales as $v_n \propto 1/n$.
- The frequency of revolution is the number of complete orbits per unit time: $f_n = v_n/(2\pi r_n)$. It is a mechanical orbital frequency, distinct from a photon frequency for a transition.

Step 1: Obtain the radius and speed dependences Bohr's quantization condition is $m_e v_n r_n = n\hbar$. The electrostatic force supplies the centripetal force, so

$$\frac{m_e v_n^2}{r_n} = \frac{Ze^2}{4\pi\varepsilon_0 r_n^2}.$$

Combining these relations for fixed nuclear charge Z yields $r_n \propto n^2$ and $v_n \propto n^{-1}$. Both changes matter when calculating how many circuits are completed in one second.

Step 2: Form the orbital frequency The distance travelled during one revolution is the circumference $2\pi r_n$. Therefore

$$T_n = \frac{2\pi r_n}{v_n} \propto \frac{n^2}{n^{-1}} = n^3, \qquad f_n = \frac{1}{T_n} \propto \frac{1}{n^3}.$$

As n increases, the orbit becomes larger and the electron becomes slower, so its period grows by both effects.

Step 3: Check the competing powers Option (D) includes only the decrease of speed, ignoring the longer circumference. Option (B) corresponds to the radius or energy-magnitude scaling, not the revolution frequency. Option (A) introduces one extra inverse power. For example, going from n to $2n$ makes the radius four times larger and speed half as large, so the frequency becomes one-eighth, consistent only with n^{-3}.

Step 4: Conclusion The electron's frequency of revolution varies as $\boxed{f_n \propto n^{-3}}$. Hence, the correct option is (C).

Quick Tip to Solve: For orbital frequency use speed divided by circumference: combine $v_n \propto n^{-1}$ with $r_n \propto n^2$.

36. Earth has mass 8 times and radius 2 times that of a planet. If the escape velocity from the earth is 11.2 km/s, the escape velocity in km/s from the planet will be: [4 marks]

(A) 8.4

(B) 11.2

(C) 5.6

(D) 2.8

Correct Answer: (C) 5.6

Solution:

Concept:

- The escape speed from the surface of a spherical body is $v_{\text{esc}} = \sqrt{2GM/R}$, neglecting atmospheric resistance and rotation.
- Escape speed depends on the square root of the mass-to-radius ratio. When two bodies are compared, the common gravitational constant cancels.

Step 1: Write the mass and radius relations Let M_E, R_E refer to Earth and M_P, R_P to the planet. The wording gives

$$M_E = 8M_P, \qquad R_E = 2R_P.$$

Thus the planet's mass is one-eighth of Earth's, while its radius is one-half. Both factors must be included rather than scaling escape speed with mass alone.

Step 2: Compare the escape-speed squares Apply the same surface escape formula to both objects:

$$\frac{v_E^2}{v_P^2} = \frac{2GM_E/R_E}{2GM_P/R_P} = \frac{M_E}{M_P}\frac{R_P}{R_E} = 8 \times \frac{1}{2} = 4.$$

Taking the positive square root, since escape speed is a nonnegative magnitude, gives $v_E/v_P = 2$. Hence the planet needs half Earth's escape speed.

Step 3: Substitute and test the choices Using $v_E = 11.2\,\text{km}\,\text{s}^{-1}$,

$$v_P = \frac{11.2}{2} = 5.6\,\text{km}\,\text{s}^{-1}.$$

Option (B) would require equal M/R, which is not the case. Option (D) divides by four, confusing the ratio of squared speeds with that of speeds. Option (A) is not consistent with the square-root dependence. The smaller mass more than compensates for the smaller radius, giving the lower escape speed.

Step 4: Conclusion The escape velocity from the planet is $\boxed{5.6\,\text{km}\,\text{s}^{-1}}$. Hence, the correct option is

(C).

Quick Tip to Solve: Compare M/R first; the ratio obtained is the ratio of escape speeds squared, so take its square root.

37. Which of the following phenomena cannot be explained by wave theory of light? [4 marks]

(A) Refraction of light

(B) Reflection of light

(C) Diffraction of light

(D) Compton effect

Correct Answer: (D) Compton effect

Solution:

Concept:

- Classical wave theory treats light as an electromagnetic wave whose energy is distributed continuously across its wavefront.
- Compton scattering is explained using photons with energy $E = h\nu$ and momentum $p = h/\lambda$. Energy and momentum are conserved in a photon–electron collision.

Step 1: Identify phenomena accounted for by waves Reflection follows from the boundary conditions on an electromagnetic wave, and Huygens' construction gives the equality of incidence and reflection angles. Refraction follows from the change of wave speed across a boundary and gives Snell's law. Diffraction results from the superposition of secondary wavelets, so it is a characteristic wave effect. Thus options (A), (B) and (C) can all be described within wave theory.

Step 2: Use the observed Compton wavelength shift An X-ray photon scattered by an electron emerges with a wavelength that depends on its scattering angle. Conservation of energy and momentum for a photon colliding with an initially stationary free electron gives

$$\lambda\prime - \lambda = \frac{h}{m_e c}(1 - \cos\theta).$$

The finite, angle-dependent shift requires transfer of a discrete photon's energy and momentum. Classical scattering of a continuous electromagnetic wave by a free electron does not predict this Compton shift; it predicts radiation at the incident frequency instead.

Step 3: Conclusion The Compton effect cannot be explained by classical wave theory of light. Hence, the correct option is (D).

Quick Tip to Solve: When a question asks which effect needs particle nature, look for an energy or wavelength change caused by a photon collision.

38. A body of mass 4 kg is placed on a plane at a point P having coordinate $(3, 4)\text{m}$. Under the action of force $\vec{F} = (2\hat{i} + 3\hat{j})\text{N}$, it moves to a new point Q having coordinates $(6, 10)\text{m}$ in 4 sec . The average power and instanteous power at the end of 4 sec are in the ratio of : [4 marks]

(A) 4 : 3

(B) 13 : 6

(C) 1 : 2

(D) 6 : 13

Correct Answer: The literal data give 12:25, which is absent from all four options.

Solution:

Concept:

- For a constant force, work is $W = \vec{F} \cdot \Delta\vec{r}$, average power is W/t, and instantaneous power is $\vec{F} \cdot \vec{v}$.
- With no other in-plane force specified, $\vec{a} = \vec{F}/m$ is constant. Initial rest is not stated, so initial velocity must be obtained from the given displacement.

Step 1: Calculate the displacement and average power The displacement from $P(3, 4)$ to $Q(6, 10)$ is

$$\Delta\vec{r} = (3\hat{i} + 6\hat{j})\,\text{m}.$$

Therefore

$$W = (2)(3) + (3)(6) = 24\,\text{J}, \qquad P_{\text{av}} = \frac{24}{4} = 6\,\text{W}.$$

Coordinates are positions, so their difference must be used rather than the coordinates of Q alone.

Step 2: Determine the final velocity from the full data The mass is $4\,\text{kg}$, hence

$$\vec{a} = (0.5\hat{i} + 0.75\hat{j})\,\text{m s}^{-2}.$$

For constant acceleration, $\vec{v}_f = \Delta\vec{r}/t + \vec{a}t/2$. Substitution gives

$$\vec{v}_f = (0.75\hat{i} + 1.5\hat{j}) + (1\hat{i} + 1.5\hat{j}) = (1.75\hat{i} + 3\hat{j})\,\text{m s}^{-1}.$$

Thus the final instantaneous power is

$$P_f = 2(1.75) + 3(3) = 12.5\,\text{W}.$$

The resulting initial velocity is $(-0.25\hat{i})\,\text{m s}^{-1}$, so the body is not initially at rest under the literal data.

Step 3: Compare the ratio with the choices The ratio is

$$P_{\text{av}} : P_f = 6 : 12.5 = 12 : 25.$$

This equals none of (A) $4 : 3$, (B) $13 : 6$, (C) $1 : 2$, or (D) $6 : 13$. Assuming initial rest would produce the familiar ratio $1 : 2$, but then the force predicts displacement $(4, 6)\,\text{m}$, contradicting the stated $(3, 6)\,\text{m}$. If unspecified additional forces are allowed, the endpoint velocity is instead undetermined; that also cannot establish a unique listed choice.

Step 4: Conclusion The printed data give $\boxed{12 : 25}$ under the stated constant force, so no listed option

is correct. Option (C), the intended key choice, requires an extra initial-rest assumption inconsistent with the given displacement.

Quick Tip to Solve: Before using the standard 1:2 power ratio for constant acceleration, verify that the body starts from rest and that the displacement agrees with the force and mass.

39. Match List - I with List - II.

List - I	List - II
(A) Angular Impulse	(I) $M^0 L^2 T^{-2}$
(B) Latent Heat	(II) $M L^2 T^{-3} A^{-1}$
(C) Electrical resistivity	(III) $M L^2 T^{-1}$
(D) Electromotive force	(IV) $M L^3 T^{-3} A^{-2}$

Choose the correct answer from the options given below: [4 marks]

(A) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(B) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

(C) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(D) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

Correct Answer: (C) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Solution:

Concept:

- Dimensions follow from defining equations. Write mechanical dimensions in M, L, T and electric current in A.
- Angular impulse is change in angular momentum; latent heat here is specific latent heat, energy per unit mass. Resistivity follows from $R = \rho L/S$, where S is cross-sectional area.

Step 1: Find the mechanical dimensions Angular impulse is torque multiplied by time. Since force has dimensions MLT^{-2}, torque has dimensions ML^2T^{-2}, and therefore

$$[\text{angular impulse}] = ML^2T^{-1}.$$

This is entry (III). Specific latent heat is energy required per unit mass for a change of phase, so

$$[\text{latent heat}] = \frac{ML^2T^{-2}}{M} = M^0L^2T^{-2}.$$

Thus (A) matches (III), and (B) matches (I). The mass exponent distinguishes specific latent heat from total heat energy.

Step 2: Find the electrical dimensions Electromotive force is work per unit charge, with $[Q] = AT$. Hence

$$[\mathcal{E}] = \frac{ML^2T^{-2}}{AT} = ML^2T^{-3}A^{-1},$$

which is (II). Resistance has dimensions $[R] = [\mathcal{E}]/A = ML^2T^{-3}A^{-2}$. Since resistivity is $\rho = RS/L$,

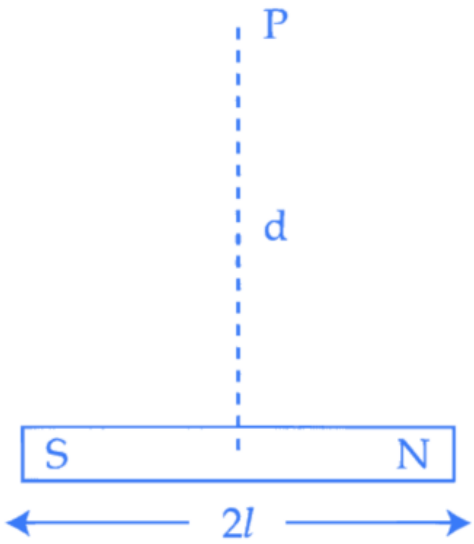
$$[\rho] = ML^2T^{-3}A^{-2}\frac{L^2}{L} = ML^3T^{-3}A^{-2}.$$

This is (IV). Options (A) and (B) assign the wrong angular-impulse dimension; option (D) interchanges the electrical dimensions.

Step 3: Conclusion The four matches are angular impulse to III, specific latent heat to I, resistivity to IV and electromotive force to II. Hence, the correct option is (C).

Quick Tip to Solve: Resistivity has one extra power of length compared with resistance because resistivity equals resistance times area divided by length.

40. A bar magnet has total length $2l = 20$ units and the field point P is at a distance $\mathrm{d} = 10$ units from the centre of the magnet. If the relative uncertainty of length measurement is 1%, then uncertainty of the magnetic field at point P is : [4 marks]



(A) 10%

(B) 5%

(C) 3%

(D) 4%

Correct Answer: (D) 4%

Solution:

Concept:

- The figure places P on the equatorial line of a finite bar magnet. In the pole model, $B = (\mu_0/4\pi)\,2pl/(d^2 + l^2)^{3/2}$, with pole strength p.
- For a product or quotient, a conservative maximum relative uncertainty is obtained by adding

the positive fractional contributions. The question states a 1% relative uncertainty for length measurements.

Step 1: Identify the geometrical quantities The total magnet length is $2l = 20$, so $l = 10$, while the equatorial distance is $d = 10$. The finite-length expression is essential because d is not much larger than l. Set $X = d^2 + l^2$. Applying the stated 1% uncertainty separately to the measured lengths gives

$$\frac{\Delta X}{X} = \frac{2d^2(0.01) + 2l^2(0.01)}{d^2 + l^2} = 0.02.$$

Thus the maximum fractional uncertainty of the sum of the two squares is 2%.

Step 2: Propagate the uncertainties through the field formula With pole strength treated as fixed, the numerator $2pl$ contributes 1%, and the denominator $X^{3/2}$ contributes $(3/2)(2\%) = 3\%$. The usual conservative product-and-quotient rule therefore gives

$$\frac{\Delta B}{B} \times 100\% = 1\% + 3\% = 4\%.$$

This yields option (D). The question does not specify how magnetic moment or pole strength is held fixed, and it gives no correlation model for the length errors, so uncertainty conventions require care.

Step 3: Account for the accepted alternative If the magnetic moment $\mathcal{M} = 2pl$ is instead regarded as a fixed parameter, the numerator has no length uncertainty to propagate. The denominator contribution alone is then 3%, giving option (C). This explains the two accepted choices without adding an unsupported measured uncertainty. The stated key accepts both (C) and (D). Neither 5% nor 10% follows from either of these conventional treatments for the geometry shown.

Step 4: Conclusion Using the usual conservative rule with fixed pole strength gives $\boxed{4\%}$. Hence, the correct option is (D). The official key accepts both choices C and D, corresponding to 3% and 4%, respectively.

Quick Tip to Solve: For a finite magnet use the full equatorial-field expression, and state whether the pole strength or the magnetic moment is being treated as fixed.

41. In a long glass tube, mixture of two liquids A and B with refractive indices 1.3 and 1.4 respectively, forms a convex refractive meniscus towards A . If an object placed at 13 cm from the vertex of the meniscus in A forms an image with a magnification of $'-2'$ then the radius of curvature of meniscus is : [4 marks]

(A) $\frac{2}{3}$ cm

(B) $\frac{4}{3}$ cm

(C) $\frac{1}{3}$ cm

(D) 1 cm

Correct Answer: (A) $\frac{2}{3}$ cm

Solution:

Concept:

- For refraction from medium n_1 to medium n_2, $n_2/v - n_1/u = (n_2 - n_1)/R$ using the Cartesian sign convention.
- The lateral magnification of a single refracting surface is $m = n_1 v/(n_2 u)$, not simply v/u. Distances are positive along the incident light direction.

Step 1: Obtain the image distance from magnification Light travels from liquid A into liquid B, so $n_1 = 1.3$, $n_2 = 1.4$, and the object distance is $u = -13\,\text{cm}$. The stated magnification is $m = -2$, indicating an inverted image. Therefore

$$-2 = \frac{1.3v}{1.4(-13)} \quad \Longrightarrow \quad v = 28\,\text{cm}.$$

The positive image distance is consistent with a real image on the B side. Using the thin-lens magnification relation here would incorrectly omit the refractive-index factor.

Step 2: Apply the spherical refraction formula The surface is convex towards A; its centre of curvature lies on the B side, so R is positive. Substitution gives

$$\frac{1.4}{28} - \frac{1.3}{-13} = \frac{1.4 - 1.3}{R}.$$

Thus

$$0.05 + 0.10 = \frac{0.10}{R}, \qquad R = \frac{0.10}{0.15} = \frac{2}{3}\,\text{cm}.$$

The dimensions are centimetres because both object and image distances were expressed in centimetres. This equals option (A); the other three radii do not satisfy both the surface equation and the given magnification.

Step 3: Conclusion The radius of curvature is $\boxed{\frac{2}{3}\,\text{cm}}$. Hence, the correct option is (A).

Quick Tip to Solve: At a refracting surface, first use the magnification containing both refractive indices, then substitute the signed distances into the refraction equation.

42. The kinetic energy of translation of the molecules in 50 g of CO_2 gas at 17°C is : [4 marks]

(A) 4205.5 J

(B) 3582.7 J

(C) 3986.3 J

(D) 4102.8 J

Correct Answer: (D) 4102.8 J

Solution:

Concept:

- Each ideal-gas molecule has three translational degrees of freedom. Equipartition gives average translational kinetic energy $3k_BT/2$ per molecule.
- For n moles, the total translational kinetic energy is $K_{\text{tr}} = 3nRT/2$. Rotational and vibrational energies are excluded because only translation is requested.

Step 1: Convert the mass to moles and temperature to kelvin The molar mass of CO_2 is $12 + 2(16) = 44\,\text{g}\,\text{mol}^{-1}$. Hence the number of moles in the given sample is

$$n = \frac{50}{44}\,\text{mol}.$$

The absolute temperature is

$$T = 17 + 273 = 290\,\text{K}.$$

The Celsius value cannot be substituted directly into an equipartition formula, since thermal energy is proportional to absolute temperature.

Step 2: Evaluate the translational energy Using $R = 8.3\,\text{J}\,\text{mol}^{-1}\text{K}^{-1}$, consistent with the option precision,

$$K_{\text{tr}} = \frac{3}{2} \times \frac{50}{44} \times 8.3 \times 290 = 4102.84\,\text{J}.$$

Thus the answer rounds to $4102.8\,\text{J}$. Using the more precise value $R = 8.314$ changes the result slightly to about $4109.8\,\text{J}$, leaving the same nearest choice. Although carbon dioxide is a polyatomic molecule with additional internal degrees of freedom, those do not enter the energy of centre-of-mass translation. The other choices fail the $3nRT/2$ calculation.

Step 3: Conclusion The translational kinetic energy is approximately $\boxed{4102.8\,\text{J}}$. Hence, the correct option is (D).

Quick Tip to Solve: Translational energy is always 3nRT/2 for an ideal gas, regardless of whether its molecules are monatomic, diatomic or polyatomic.

43. A 400 g solid cube having an edge of length 10 cm floats in water. How much volume of the cube is outside the water?

(Given: density of water = 1000 kg m^{-3}) [4 marks]

(A) 400 cm^3

(B) 600 cm^3

(C) 1400 cm^3

(D) 4000 cm^3

Correct Answer: (B) 600 cm^3

Solution:

Concept:

- For a floating body in equilibrium, its weight equals the buoyant force: $mg = \rho_w V_{\text{sub}} g$. Here V_{sub} is only the submerged volume.
- The geometric volume of a cube of edge a is a^3. Its exposed volume is the total volume minus the displaced water volume.

Step 1: Calculate the whole cube volume The edge is $10\,\text{cm}$, so the total volume is

$$V = (10)^3 = 1000\,\text{cm}^3.$$

The mass is $400\,\text{g}$. The supplied water density $1000\,\text{kg}\,\text{m}^{-3}$ is $1\,\text{g}\,\text{cm}^{-3}$, allowing the calculation to remain in centimetre units.

Step 2: Apply the floating condition The cube floats freely, so the displaced water must have the same mass as the cube. The factors of gravitational acceleration cancel from the force balance:

$$V_{\text{sub}} = \frac{m}{\rho_w} = \frac{400}{1} = 400\,\text{cm}^3.$$

This is the volume inside the water, not the volume asked for. It is smaller than the full cube volume, as floating requires.

Step 3: Subtract the submerged part The portion above the water is the complementary volume:

$$V_{\text{out}} = 1000 - 400 = 600\,\text{cm}^3.$$

Option (A) gives the displaced volume instead. Options (C) and (D) both exceed the entire cube volume and cannot be exposed portions of this cube. The submerged fraction is 0.4, leaving fraction 0.6 outside.

Step 4: Conclusion The volume outside water is $\boxed{600\,\text{cm}^3}$. Hence, the correct option is (B).

Quick Tip to Solve: For a floating solid, first find displaced volume as mass divided by liquid density; subtract it from the body volume only when the exposed volume is requested.

44. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Knowing initial position x_0 and initial momentum p_0 is enough to determine the position and momentum at any time t for a simple harmonic motion with a given angular frequency ω.

Reason (R) : The amplitude and phase can be expressed in terms of X_0 an p_0.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) (A) is true but (R) is false

(B) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(C) (A) is false but (R) is true

(D) Both (A) and (R) are true and (R) is the correct explanation of (A)

Correct Answer: (D) Both (A) and (R) are true and (R) is the correct explanation of (A)

Solution:

Concept:

- A specified oscillator obeys $\ddot{x} + \omega^2 x = 0$, whose solution has two constants. Initial position and initial velocity fix these constants.
- For the oscillator of mass m, momentum and velocity satisfy $p = m\dot{x}$. Thus initial momentum supplies the second initial condition.

Step 1: Test the assertion For the given oscillator, write the solution directly in terms of its initial data:

$$x(t) = x_0 \cos \omega t + \frac{p_0}{m\omega} \sin \omega t.$$

Differentiation gives

$$p(t) = p_0 \cos \omega t - m\omega x_0 \sin \omega t.$$

At $t = 0$ these reduce to x_0, p_0. Both later quantities are therefore determined; the assertion is true.

Step 2: Test the reason Alternatively write $x = A\cos(\omega t + \phi)$. Initially, $x_0 = A\cos\phi$ and $p_0 = -m\omega A \sin\phi$. Hence

$$A = \sqrt{x_0^2 + \left(\frac{p_0}{m\omega}\right)^2}.$$

The two signed relations fix the phase quadrant as well as its tangent. The reason is true for the specified oscillator. If both initial data vanish, the motion is identically zero and the irrelevant phase need not be fixed.

Step 3: Check the explanatory connection Once amplitude and phase are fixed, the displacement formula fixes position at every time; differentiating it fixes momentum. This is exactly why the two initial data suffice. Thus the reason explains the assertion, rather than merely stating an unrelated property. Options (A) and (C) incorrectly reject one statement, and option (B) incorrectly denies their direct logical connection.

Step 4: Conclusion Both statements are true, and the reason correctly explains the assertion. Hence, the correct option is (D).

Quick Tip to Solve: For SHM initial-data questions, use the sine-cosine form first; it avoids phase-quadrant ambiguity and displays the dependence on position and momentum explicitly.

45. A parallel plate capacitor of capacitance 1 μF is charged to a potential difference of 20 V. The distance between plates is 1 μm. The energy density between plates of capacitor is : [4 marks]

(A) 1.8×10^3 J/m^3

(B) 2×10^2 J/m^3

(C) 2×10^{-4} J/m^3

(D) 1.8×10^5 J/m^3

Correct Answer: (A) 1.8×10^3 J/m^3

Solution:

Concept:

- The electric field between ideal parallel plates is $E = V/d$. The field energy per unit volume in air or vacuum is $u = \frac{1}{2}\varepsilon_0 E^2$.
- Total stored energy $U = \frac{1}{2}CV^2$ is different from energy density. Dividing by the field-filled volume gives the same density formula.

Step 1: Convert the separation and find the field The potential difference is $20\,\mathrm{V}$, while the plate separation is $1\,\mu\mathrm{m} = 10^{-6}\,\mathrm{m}$. Therefore

$$E = \frac{20}{10^{-6}} = 2.0 \times 10^7\,\mathrm{V\,m^{-1}}.$$

Converting the micrometre separation before squaring is essential; a missing conversion would alter the density by many powers of ten.

Step 2: Evaluate the energy per unit volume Use $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$:

$$u = \frac{1}{2}(8.85 \times 10^{-12})(2.0 \times 10^7)^2.$$

Thus

$$u = \frac{1}{2}(8.85 \times 10^{-12})(4.0 \times 10^{14}) = 1.77 \times 10^3\,\mathrm{J\,m^{-3}}.$$

Rounding to the precision of the listed choices gives $1.8 \times 10^3\,\mathrm{J\,m^{-3}}$.

Step 3: Distinguish density from total energy The capacitance would give total energy $U = \frac{1}{2}(10^{-6})(20)^2 = 2 \times 10^{-4}\,\mathrm{J}$. This is the numerical value in option (C), but it is an energy, not an energy density. Option (B) has the wrong magnitude, while option (D) is a hundred times too large. The capacitance is unnecessary once the voltage and spacing determine the field.

Step 4: Conclusion The energy density is $\boxed{1.8 \times 10^3\,\mathrm{J\,m^{-3}}}$. Hence, the correct option is (A).

Quick Tip to Solve: When voltage and plate spacing are given, calculate energy density with $\frac{1}{2}\varepsilon_0(V/d)^2$; do not confuse it with $\frac{1}{2}CV^2$.

Physics – Section B: Numerical Value Questions (20 marks)

46. A thin transparent film with refractive index 1.4 , is held on circular ring of radius 1.8 cm . The fluid in the film evaporates such that transmission through the film at wavelength 560 nm goes to a minimum every 12 seconds. Assuming that the film is flat on its two sides, the rate of evaporation is _____ $\pi \times 10^{-13}$ m^3/s. [4 marks]

Solution:

Concept:

- At normal incidence, the optical path change corresponding to a thickness change Δt in a film of index μ is $2\mu\Delta t$.
- Successive transmission minima occur when this optical path changes by one wavelength: $2\mu\Delta t = \lambda$. With fixed circular area, volume loss is area times thickness loss.

Step 1: Find the thickness lost between successive minima The transmission returns to a minimum after each $12\,s$. These are successive minima of the same type, so the change is a full interference order, not half an order. Consequently

$$\Delta t = \frac{\lambda}{2\mu} = \frac{560 \times 10^{-9}}{2(1.4)} = 2.0 \times 10^{-7}\,m.$$

The film becomes thinner as the liquid evaporates; this positive quantity is the magnitude of the thickness decrease.

Step 2: Calculate the film area and volume decrease The ring radius is $1.8\,cm = 1.8 \times 10^{-2}\,m$. Since both film faces are flat, its liquid volume is area times thickness. Thus

$$A = \pi r^2 = 3.24\pi \times 10^{-4}\,m^2,$$

and

$$\Delta V = A\Delta t = 6.48\pi \times 10^{-11}\,m^3.$$

Although evaporation can occur from two faces, the measured thickness decrease already includes their combined effect; no extra factor of two is needed.

Step 3: Divide by the elapsed time The rate of loss of liquid volume is

$$\left|\frac{dV}{dt}\right| = \frac{6.48\pi \times 10^{-11}}{12} = 5.4\pi \times 10^{-12}\,m^3\,s^{-1}.$$

Expressing this in the units specified in the blank gives

$$5.4\pi \times 10^{-12} = 54\pi \times 10^{-13}.$$

The requested coefficient is therefore 54, consistent with the numerical answer key.

Step 4: Conclusion The evaporation rate is $54\pi \times 10^{-13}\,m^3\,s^{-1}$; the blank is $\boxed{54}$.

Quick Tip to Solve: Between consecutive minima, use $\Delta t = \lambda/(2\mu)$, then multiply by the fixed film area and divide by the recurrence time.

47. The volume contraction of a solid copper cube of edge length 10 cm , when subjected to a hydraulic pressure of 7×10^6 Pa, would be _____ mm^3.

(Given bulk modulus of copper $= 1.4 \times 10^{11}$ N m^{-2}) [4 marks]

Solution:

Concept:

- Bulk modulus is the applied pressure increase divided by the magnitude of volumetric strain: $B = \Delta P/(|\Delta V|/V)$.
- For small compression, $|\Delta V| = V\Delta P/B$. Volume change itself is negative, but the contraction requested is its positive magnitude.

Step 1: Calculate the original volume in SI units The edge of the copper cube is $10\,\text{cm} = 0.10\,\text{m}$. Its volume before compression is therefore

$$V = (0.10)^3 = 1.0 \times 10^{-3}\,\text{m}^3.$$

The hydraulic pressure acts from all sides, so bulk modulus, rather than Young modulus or shear modulus, describes the response.

Step 2: Apply the volumetric strain relation The given pressure is 7×10^6 Pa and the bulk modulus is 1.4×10^{11} Pa. Their ratio is dimensionless:

$$\frac{|\Delta V|}{V} = \frac{7 \times 10^6}{1.4 \times 10^{11}} = 5.0 \times 10^{-5}.$$

This very small strain supports the use of the linear elastic relation. Multiplying by the initial volume yields

$$|\Delta V| = (1.0 \times 10^{-3})(5.0 \times 10^{-5}) = 5.0 \times 10^{-8}\,\text{m}^3.$$

Step 3: Convert the contraction to cubic millimetres Since $1\,\text{m} = 10^3\,\text{mm}$, cubing the conversion gives $1\,\text{m}^3 = 10^9\,\text{mm}^3$. Hence

$$|\Delta V| = (5.0 \times 10^{-8})(10^9) = 50\,\text{mm}^3.$$

Using only a linear conversion factor would be incorrect because the requested quantity is a volume. The actual signed change is negative, while the blank asks how much the volume contracts.

Step 4: Conclusion The volume contraction is $\boxed{50\,\text{mm}^3}$; enter 50.

Quick Tip to Solve: Compute pressure divided by bulk modulus first to obtain the fractional volume decrease, and cube the length-conversion factor when changing volume units.

48. An electric dipole of dipole moment $6 \times 10^{-6}\text{Cm}$ is placed in uniform electric field of magnitude 10^6V/m. Initially, the dipole moment is parallel to electric field. The work that needs to be done on the dipole to make its dipole moment opposite to the field, will be _____ J. [4 marks]

Solution:

Concept:

- The potential energy of a permanent dipole in a uniform electric field is $U = -pE\cos\theta$, where θ is the angle between dipole moment and field.
- For a controlled rotation with no net kinetic-energy change, the work supplied by an external agent equals the increase of potential energy: $W_{\text{ext}} = U_f - U_i$.

Step 1: Identify the initial and final orientations Initially the dipole moment is parallel to the field, so $\theta_i = 0$ and $\cos\theta_i = 1$. Finally it is opposite to the field, so $\theta_f = \pi$ and $\cos\theta_f = -1$. These are respectively the minimum- and maximum-energy orientations. Merely changing the angle by π must not be treated as multiplication by a constant torque, because the torque varies throughout the rotation.

Step 2: Calculate the energy increase The product of dipole moment and field strength is

$$pE = (6 \times 10^{-6})(10^6) = 6\,\text{J}.$$

The initial and final potential energies are

$$U_i = -pE = -6\,\text{J}, \qquad U_f = +pE = +6\,\text{J}.$$

Their difference is

$$\Delta U = 6 - (-6) = 12\,\text{J}.$$

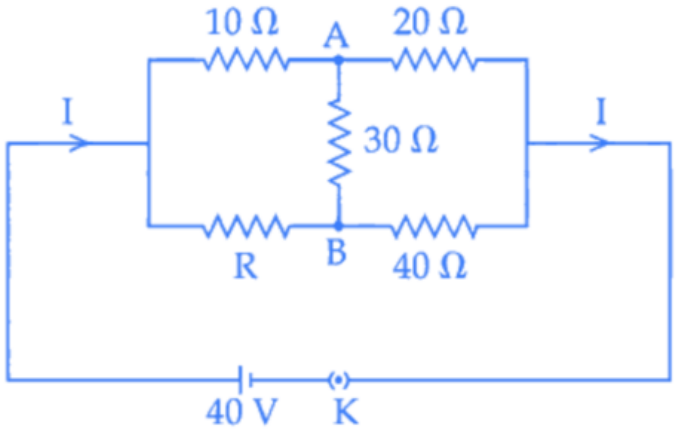
The positive sign is physical: an external agent must supply energy to move the dipole away from its stable aligned orientation.

Step 3: Interpret the requested work The question asks for work done on the dipole to reverse it, so the relevant quantity is positive external work. Work done by the electric field during this reversal would instead be $-12\,\text{J}$. The result can also be checked by integrating the opposing torque: $W_{\text{ext}} = \int_0^{\pi} pE\sin\theta\, d\theta = 2pE$. This confirms the factor of two and the sign without assuming a constant torque.

Step 4: Conclusion The required work is $\boxed{12\,\text{J}}$; enter 12.

Quick Tip to Solve: For complete reversal of a dipole from parallel to antiparallel, the external work is $2pE$, since its potential energy changes from $-pE$ to $+pE$.

49. The value of current I in the electrical circuit as given below, when potential at A is equal to the potential at B, will be _____ A. [4 marks]



Solution:

Concept:

- Equal potentials at A and B mean that the potential difference across the central $30\,\Omega$ resistor is zero. Ohm's law therefore gives zero current through that resistor.
- For a balanced Wheatstone bridge, the ratios of the left and right arm resistances are equal. After establishing balance, the two series arms are parallel across the battery.

Step 1: Use the equal-potential condition Take the right-hand junction as the zero of potential. Let the currents in the upper and lower arms be i_1 and i_2, respectively. Because the bridge resistor carries no current, these currents remain the same along their respective arms. Equality of the potentials at A and B gives

$$20i_1 = 40i_2 \quad \Rightarrow \quad i_1 = 2i_2.$$

The potential drops from the left junction to A and B must also be equal, so $10i_1 = Ri_2$. Combining these two relations yields $R = 20\,\Omega$.

Step 2: Find the equivalent resistance and source current The upper branch has resistance $10 + 20 = 30\,\Omega$, whereas the lower branch has resistance $20 + 40 = 60\,\Omega$. These branches are in parallel; the central resistor must not be added in series with either one. Thus

$$R_{\text{eq}} = \frac{30 \times 60}{30 + 60} = 20\,\Omega.$$

The indicated current I is the total current supplied by the $40\,\text{V}$ source, not the current in either individual arm. Therefore

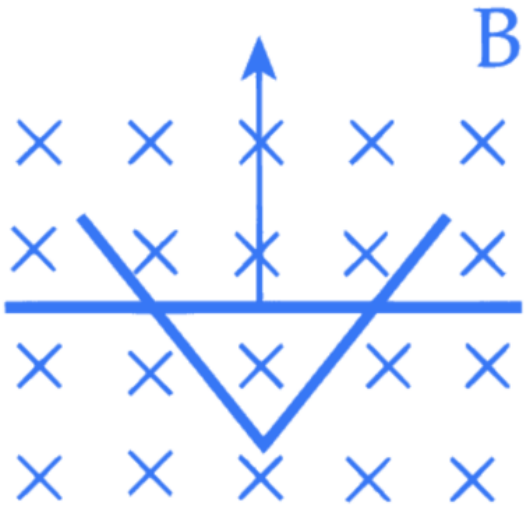
$$I = \frac{40}{20} = 2\,\text{A}.$$

As a check, the branch currents are $4/3\,\text{A}$ and $2/3\,\text{A}$; they add to $2\,\text{A}$ and give identical potentials at A and B.

Step 3: Conclusion The total current is $\boxed{I = 2\,\text{A}}$. The numerical entry is $\boxed{2}$.

Quick Tip to Solve: When two bridge junctions have the same potential, first remove the zero-current bridge branch from the calculation, then combine the two series arms in parallel.

50. A conducting bar moves on two conducting rails as shown in the figure. A constant magnetic field B exists into the page. The bar starts to move from the vertex at time t = 0 with a constant velocity. If the induced EMF is E $\propto t^n$, then value of n is _____. [4 marks]



Solution:

Concept:

- Faraday's law gives the magnitude of the induced emf as $E = B\,dA/dt$ in a uniform, steady field perpendicular to the plane of a loop.
- The rails diverge linearly from their vertex. Consequently the length of the conducting segment between the rails is proportional to its distance from the vertex.

Step 1: Express the changing area geometrically Let the bar move a distance $x = vt$ from the vertex, where v is its constant speed. If each rail makes a fixed angle θ with the direction of motion, the separation of the rails at the bar is

$$\ell = 2x \tan\theta.$$

The conducting loop is triangular. Its area is one-half of the base times the perpendicular height, so

$$A = \frac{1}{2}\ell x = x^2 \tan\theta = v^2 t^2 \tan\theta.$$

The precise rail angle changes the constant coefficient of area but cannot change its time exponent. The important geometric fact is that both the triangle's base and its height grow linearly with time.

Step 2: Differentiate the magnetic flux The field has constant magnitude and points into the page, perpendicular to the triangular loop. The magnitude of the magnetic flux is therefore $\Phi = BA$. Applying Faraday's law gives

$$E = \left|\frac{d\Phi}{dt}\right| = 2Bv^2 \tan\theta\, t.$$

All factors except t are constants, so the induced emf is proportional to the first power of time. Equivalently, the motional-emf formula $E = B\ell v$ gives the same result because $\ell \propto t$. Although the flux grows as t^2, emf is its time derivative and grows as t. Lenz's law fixes the direction of the induced

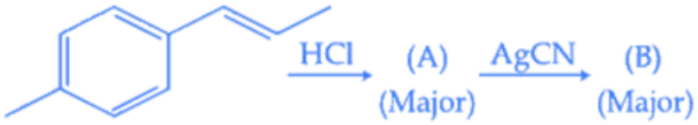
current, but does not alter this power law.

Step 3: Conclusion Comparing $E \propto t$ with $E \propto t^n$, the required numerical value is $\boxed{n = 1}$.

Quick Tip to Solve: For rails that diverge linearly and a bar moving at constant speed, the loop area grows as time squared; differentiate once to obtain the emf exponent.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. The product B formed in the following reaction sequence is: [4 marks]



(A) p-$CH_3C_6H_4CH_2CH(NC)CH_3$

(B) p-$CH_3C_6H_4CH(CN)CH_3$

(C) p-$CH_3C_6H_4CH_2CH(NC)CH_3$

(D) p-$CH_3C_6H_4CH(NC)CH_2CH_3$

Correct Answer: (D) p-$CH_3C_6H_4CH(NC)CH_2CH_3$

Solution:

Concept:

- Addition of HCl to an unsymmetrical alkene proceeds through the more stable carbocation. A benzylic carbocation is stabilised by resonance with the aromatic ring.
- $AgCN$ is largely covalent and gives an isocyanide, $R-NC$, rather than the nitrile $R-CN$ normally favoured by ionic cyanides.

Step 1: Locate chlorine in the major intermediate Write the aromatic substituent as $Ar = p$-$CH_3C_6H_4$. The starting side chain is $Ar-CH=CH-CH_3$. Addition of a proton to the carbon farther from the ring leaves the positive charge at the carbon directly attached to the ring:

$$Ar-CH=CH-CH_3 + H^+ \longrightarrow Ar-CH^+-CH_2CH_3.$$

This benzylic cation is resonance stabilised; the alternative orientation would place the positive charge on a non-benzylic carbon. Chloride capture therefore produces the major intermediate $Ar-CH(Cl)-CH_2CH_3$. The carbon skeleton and the para methyl group remain unchanged.

Step 2: Apply the linkage selectivity of silver cyanide Treatment of the benzylic chloride with $AgCN$ replaces chlorine by the nitrogen end of the cyanide group. The resulting product is

$$Ar-CH(Cl)-CH_2CH_3 \xrightarrow{AgCN} Ar-CH(NC)-CH_2CH_3.$$

The product thus contains an isocyano group attached to the benzylic carbon, not to the next carbon along the chain. Options (A) and (C), as printed, place NC away from the benzylic position. Option

(B) shows CN, the wrong linkage for this reagent, and also has a different side-chain carbon count. Option (D) alone preserves the side chain and gives the correct benzylic isocyanide.

Step 3: Conclusion Product B is p-$CH_3C_6H_4CH(NC)CH_2CH_3$, the benzylic isocyanide. Hence, the correct option is (D).

Quick Tip to Solve: Use two separate checks: stabilise the carbocation to place chlorine, then distinguish AgCN giving R-NC from ionic cyanide giving R-CN.

52. The major product of the following reaction is: [4 marks]

(A) 6-Phenylhepta-3,5-diene

(B) 6-Phenylhepta-2,4-diene

(C) 2-Phenylhepta-2,4-diene

(D) 2-Phenylhepta-2,5-diene

Correct Answer: (C) 2-Phenylhepta-2,4-diene

Solution:

Concept:

- Hot alcoholic KOH favours elimination of HBr from a bromoalkane. With excess reagent, both bromine-bearing centres can undergo elimination.
- Among competing alkene products, greater substitution and conjugation stabilise the product. A conjugated diene directly conjugated with a phenyl group is especially favoured.

Step 1: Identify the chain and possible eliminations Read the substrate along its seven-carbon chain as

$$CH_3-CH(Ph)-CH(Br)-CH_2-CH(Br)-CH_2-CH_3.$$

The two bromine atoms occur at the third and fifth positions in this orientation. Each elimination removes a bromine and a hydrogen from an adjacent carbon, introducing a double bond. For the first brominated centre, removal of the hydrogen from the phenyl-bearing adjacent carbon produces a substituted double bond conjugated with the aromatic ring. Elimination toward the other side would miss that additional stabilisation.

Step 2: Choose the second double bond and name the product At the other brominated centre, removal of a hydrogen from the intervening carbon gives the second double bond next to the first one. The resulting skeleton is

$$CH_3-C(Ph)=CH-CH=CH-CH_2-CH_3.$$

This arrangement contains a conjugated C=C−C=C system, and the phenyl ring is also conjugated with that system. Numbering the parent chain to give the double bonds the lowest locants yields

hepta-2,4-diene with phenyl at carbon 2. Option (D) places the double bonds at 2 and 5, losing diene conjugation. Reading option (A) from the opposite end gives the same connectivity, but its double-bond locants 3,5 violate the lowest-locant numbering rule; the correctly numbered product must use 2,4. Thus (A) is not the correct systematic name. Option (B) has the same diene locants as (C) but places the phenyl group at the other end, which describes a different product.

Step 3: Conclusion The major product is 2-phenylhepta-2,4-diene. Hence, the correct option is (C).

Quick Tip to Solve: For double elimination, compare the complete diene products: select the arrangement that combines alkene substitution with conjugation, then number the chain for the lowest double-bond locants.

---

53. Given below are two statements:

Statement I: Methylcyclopentane and hex-3-ene are isomeric compounds.

Statement II: Pentan-1-amine and N-ethylpropan-1-amine are functional group isomers.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are true

(C) Statement I is true but Statement II is false

(D) Both Statement I and Statement II are false

Correct Answer: (B) Both Statement I and Statement II are true

Solution:

Concept:

- Isomers have the same molecular formula but different arrangements of atoms. Ring-chain isomerism compares an open-chain compound with a cyclic compound having the same formula.
- Primary and secondary amines with the same molecular formula are functional isomers: they differ in how many carbon groups are directly bonded to nitrogen.

Step 1: Compare the structures in Statement I The first structure is methylcyclopentane, a five-membered saturated ring with one methyl substituent. It contains six carbon atoms. A saturated monocyclic hydrocarbon has the formula C_nH_{2n}, giving C_6H_{12} here. The second structure is hex-3-ene, an acyclic six-carbon chain containing one carbon-carbon double bond; its molecular formula is also C_6H_{12}. They differ in connectivity: one uses a ring and the other a double bond to account for one degree of unsaturation. They are therefore ring-chain structural isomers, so Statement I is true.

Step 2: Compare the nitrogen environments in Statement II The first amine is pentan-1-amine, $CH_3CH_2CH_2CH_2CH_2NH_2$. It is primary because nitrogen has one carbon substituent and two hydrogen atoms. The second is N-ethylpropan-1-amine, $CH_3CH_2NHCH_2CH_2CH_3$, which is secondary because nitrogen has two carbon substituents and one hydrogen atom. Both contain five carbon atoms and have the molecular formula $C_5H_{13}N$. In the standard classification of amine isomerism, primary and secondary amines of the same formula are functional isomers. Hence Statement II is true. Merely observing different chain lengths on either side of nitrogen must not lead to rejecting isomerism; the

total atom counts are what determine the common formula. Options (A), (C) and (D) each incorrectly reject at least one of the two verified statements.

Step 3: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (B).

Quick Tip to Solve: Count all carbon atoms in each line structure before classifying isomerism; for amines, also count the carbon groups attached directly to nitrogen.

54. Match List - I with List - II.

List - I (Complex)	List - II (Hybridisation of central metal ion)
(A) $[CoF_6]^{3-}$	(I) d^2sp^3
(B) $[NiCl_4]^{2-}$	(II) sp^3
(C) $[Co(NH_3)_6]^{3+}$	(III) sp^3d^2
(D) $[Ni(CN)_4]^{2-}$	(IV) dsp^2

Choose the correct answer from the options given below: [4 marks]

(A) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(B) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(C) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)

(D) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Correct Answer: (D) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Solution:

Concept:

- The oxidation state and d-electron configuration of the metal are found before assigning hybridisation. In these complexes cobalt is Co^{3+}, a d^6 ion, and nickel is Ni^{2+}, a d^8 ion.
- In valence bond theory, octahedral complexes may use inner-orbital d^2sp^3 or outer-orbital sp^3d^2 hybridisation; tetrahedral complexes use sp^3 and square-planar complexes use dsp^2.

Step 1: Assign the cobalt complexes For $[CoF_6]^{3-}$, the six fluoride ions give cobalt oxidation state +3. Fluoride is weak field and does not pair the $3d$ electrons of Co^{3+} sufficiently to provide two vacant inner d orbitals. This high-spin octahedral complex therefore uses outer d orbitals, giving sp^3d^2: (A)-(III). In $[Co(NH_3)_6]^{3+}$, cobalt is again +3, but the ligand field for the cobalt(III)-ammine complex produces a low-spin d^6 arrangement. Two inner d orbitals are available, giving d^2sp^3: (C)-(I). The cobalt oxidation state alone does not determine the hybridisation; ligand-induced pairing must also be considered.

Step 2: Assign the nickel complexes and match the list In $[NiCl_4]^{2-}$, nickel is +2 and chloride is weak field. The complex is tetrahedral with sp^3 hybridisation, so (B)-(II). In contrast, CN^- is a strong-field ligand. The d^8 nickel(II) ion in $[Ni(CN)_4]^{2-}$ forms a square-planar complex using dsp^2 hybridisation, so (D)-(IV). Combining all four assignments gives

$$(A)-(III), \quad (B)-(II), \quad (C)-(I), \quad (D)-(IV).$$

Options (A) and (C) interchange the cobalt hybridisations, and option (B) assigns the nickel geometries incorrectly. Only the fourth option contains all four correct matches.

Step 3: Conclusion The four complexes in list order have hybridisations sp^3d^2, sp^3, d^2sp^3 and dsp^2, respectively. Hence, the correct option is (D).

Quick Tip to Solve: For Ni(II), remember the useful contrast: weak-field chloride gives tetrahedral sp3, while strong-field cyanide gives square-planar dsp2.

55. Arrange the following in increasing order of solubility product :

$$\mathrm{Ca(OH)_2, AgBr, PbS, HgS}$$

[4 marks]

(A) $\mathrm{PbS < HgS < Ca(OH)_2 < AgBr}$

(B) $\mathrm{HgS < AgBr < PbS < Ca(OH)_2}$

(C) $\mathrm{HgS < PbS < AgBr < Ca(OH)_2}$

(D) $\mathrm{Ca(OH)_2 < AgBr < HgS < PbS}$

Correct Answer: (C) $\mathrm{HgS < PbS < AgBr < Ca(OH)_2}$

Solution:

Concept:

- The solubility product K_{sp} is the equilibrium product of the dissolved-ion activities, each raised to its stoichiometric coefficient, for a saturated solution at a fixed temperature.
- For a 1:1 salt such as AgBr, $K_{sp} = [\mathrm{Ag^+}][\mathrm{Br^-}]$; for $\mathrm{Ca(OH)_2}$, $K_{sp} = [\mathrm{Ca^{2+}}][\mathrm{OH^-}]^2$. Compare the constants themselves, rather than assuming identical solubility formulas.

Step 1: Compare the orders of magnitude The familiar room-temperature solubility products have widely separated orders: HgS is around 10^{-53}, PbS around 10^{-28}, AgBr around 10^{-13}, and $\mathrm{Ca(OH)_2}$ around 10^{-6}. Exact tabulated values vary with conditions and solid form, but these separations make the required order unambiguous. Mercury(II) sulfide has the smallest ion product, followed by lead(II) sulfide.

Step 2: Arrange the constants and eliminate alternatives For positive powers of ten with negative exponents, a more negative exponent means a smaller value. Thus

$$K_{sp}(\mathrm{HgS}) < K_{sp}(\mathrm{PbS}) < K_{sp}(\mathrm{AgBr}) < K_{sp}(\mathrm{Ca(OH)_2}).$$

Option (A) reverses the two sulfides and incorrectly places the hydroxide below silver bromide. Option (B) places silver bromide below lead sulfide, contrary to their orders of magnitude. Option (D) starts with calcium hydroxide, which actually has the largest constant of this group. The constants do not follow simply from the number of ions produced on dissolution.

Step 3: Conclusion The increasing order is $\mathrm{HgS < PbS < AgBr < Ca(OH)_2}$ in terms of K_{sp}. Hence, the correct option is (C).

Quick Tip to Solve: Remember the scale: extremely insoluble metal sulfides lie below silver halides, while calcium hydroxide has a much larger solubility product.

56. Match List-I with List-II.

List - I (Saccharides)	List - II (Glycosidic-linkages found)
(A) Sucrose	(I) α 1 − 4
(B) Maltose	(II) α 1 − 4 and α 1 − 6
(C) Lactose	(III) α 1 − β 2
(D) Amylopectin	(IV) β 1 − 4

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(B) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(C) (A)-(II), (B)-(IV), (C)-(III), (D)-(I)

(D) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

Correct Answer: (A) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Solution:

Concept:

- A glycosidic linkage identifies the anomeric configuration and the carbon positions joined through oxygen. The linkage is a structural property, not determined merely by whether a sugar is reducing.
- Sucrose joins both anomeric centres; maltose and lactose each retain one free anomeric centre. Amylopectin is a branched glucose polymer, so it contains both chain and branch linkages.

Step 1: Assign the three disaccharides Sucrose contains an α-D-glucose unit joined from C1 to C2 of a β-D-fructose unit. Its linkage is therefore $\alpha(1 \rightarrow 2)\beta$, represented by entry (III). Maltose contains two glucose units connected by an $\alpha(1 \rightarrow 4)$ linkage, giving (B)-(I). Lactose contains galactose joined to glucose through a $\beta(1 \rightarrow 4)$ linkage, giving (C)-(IV). In particular, the difference between maltose and lactose is not just their constituent monosaccharides: their glycosidic configurations also differ.

Step 2: Identify the branched polysaccharide Amylopectin has $\alpha(1 \rightarrow 4)$ links along its glucose chains and $\alpha(1 \rightarrow 6)$ links at branch points, so (D)-(II). Combining the assignments gives

$$(A):(III), \quad (B):(I), \quad (C):(IV), \quad (D):(II).$$

Only option (A) contains all four matches. Option (B) incorrectly assigns sucrose a simple 1-to-4 linkage. Option (C) assigns the branched linkage to sucrose and the lactose linkage to maltose. Option (D) makes sucrose beta-1-to-4 and fails to give amylopectin both of its required linkages.

Step 3: Conclusion Sucrose matches III, maltose matches I, lactose matches IV and amylopectin matches II. Hence, the correct option is (A).

Quick Tip to Solve: Use three anchors: maltose is alpha-1,4; lactose is beta-1,4; amylopectin adds alpha-1,6 branches.

57. An ideal gas undergoes a cyclic transformation starting from the point A and coming back to the same point by tracing the path A→B→C→D→A as shown in the three cases above.

Choose the correct option regarding ΔU : [4 marks]

(A) $\Delta U(\text{ Case-III }) > \Delta U(\text{ Case-II }) > \Delta U(\text{ Case-I })$

(B) $\Delta U(\text{ Case-I }) > \Delta U(\text{ Case-II }) > \Delta U(\text{ Case-III })$

(C) $\Delta U(\text{ Case-I }) = \Delta U(\text{ Case-II }) = \Delta U(\text{ Case-III })$

(D) $\Delta U(\text{ Case-I }) > \Delta U(\text{ Case-III }) > \Delta U(\text{ Case-II })$

Correct Answer: (C) $\Delta U(\text{ Case-I }) = \Delta U(\text{ Case-II }) = \Delta U(\text{ Case-III })$

Solution:

Concept:

- Internal energy U is a state function: its change depends only on the initial and final equilibrium states, irrespective of the path joining them.
- For an ideal gas of fixed amount, internal energy depends only on temperature: $\Delta U = nC_V\Delta T$ when C_V is constant. A complete cycle returns pressure, volume and temperature to their starting values.

Step 1: Identify the initial and final states In each diagram the gas starts at A, follows A to B to C to D, and finally returns to A. Although the shapes, coordinates and enclosed areas differ, the final state within each cycle is exactly its initial state. The ideal-gas relation $PV = nRT$ confirms that returning to the same pressure and volume for the same amount of gas restores the original temperature. Consequently,

$$T_f = T_i, \qquad \Delta U = U(A) - U(A) = 0.$$

This conclusion does not require evaluating any individual segment of the path.

Step 2: Distinguish energy change from cyclic work The area enclosed by a pressure-volume loop can determine net work, and different loops can therefore perform different amounts of work. That area does not determine the change in internal energy. In the chemistry sign convention, the first law is $\Delta U = q + w$; over each cycle $q = -w$, even if neither quantity is zero. Thus

$$\Delta U_{\text{I}} = \Delta U_{\text{II}} = \Delta U_{\text{III}} = 0.$$

Options (A), (B) and (D) all assert strict inequalities, confusing path-dependent transfers with the change of a state function. No ordering based on the apparent loop areas can make any of those options correct.

Step 3: Conclusion All three cycles have zero internal-energy change, so $\Delta U(\text{Case-I}) =$

$\Delta U(\text{Case-II}) = \Delta U(\text{Case-III}) = 0$. Hence, the correct option is (C).

Quick Tip to Solve: When a process returns to its initial state, set the change of every state function to zero before considering the loop area.

58. Which of the following is/are not correct with respect to energy of atomic orbitals of hydrogen atom?

(A) 1s < 2p < 3d < 4s

(B) 1s < 2s = 2p < 3s = 3p

(C) 1s < 2s < 2p < 3s < 3p

(D) 1s < 2s < 4s < 3d

Choose the correct answer from the options given below : [4 marks]

(A) (A) and (B) only

(B) (A) and (C) only

(C) (B) and (D) only

(D) (C) and (D) only

Correct Answer: (D) (C) and (D) only

Solution:

Concept:

- For the isolated hydrogen atom in the usual nonrelativistic treatment, $E_n = -13.6/n^2$ eV. The orbital energy depends on the principal quantum number n, not on l.
- Orbitals with the same n, such as 2s and 2p or 3s, 3p and 3d, are degenerate. Higher n gives a less negative and therefore higher energy.

Step 1: Check statements (A) and (B) Statement (A) compares orbitals with $n = 1, 2, 3, 4$, respectively. Since

$$-13.6 < -3.4 < -13.6/9 < -0.85 \quad (\text{eV}),$$

the sequence $1s < 2p < 3d < 4s$ is correct. Statement (B) correctly groups 2s with 2p and 3s with 3p. Its inequalities then simply compare $n = 1$, $n = 2$ and $n = 3$, so it is also correct. The equality of different orbital types within one principal shell is particular to the hydrogenic energy expression used here.

Step 2: Identify the two incorrect statements Statement (C) asserts $2s < 2p$ and $3s < 3p$, but each pair must have equal energy in hydrogen. It is therefore incorrect. Statement (D) puts 4s below 3d, whereas

$$E(3d) = -13.6/9 \text{ eV} < -13.6/16 \text{ eV} = E(4s).$$

Thus (D) is also incorrect. The familiar filling of 4s before 3d in many-electron atoms must not be imported into this one-electron question. Options (A), (B) and (C) each include a correct statement among those labelled incorrect; only the pair (C), (D) meets the demand.

Step 3: Conclusion The third and fourth statements are incorrect, whereas the first two are correct.

Hence, the correct option is (D).

Quick Tip to Solve: For hydrogen, replace every orbital label by its principal quantum number; ignore the many-electron Aufbau order.

59. The amphoteric oxide among V_2O_3, V_2O_4 and V_2O_5, upon reaction with alkali leads to formation of an oxide anion. The oxidation state of V in the oxide anion is : [4 marks]

(A) + 4

(B) + 5

(C) + 3

(D) + 7

Correct Answer: (B) + 5

Solution:

Concept:

- For a transition metal, increasing oxidation state generally increases the acidic character of its oxide. An amphoteric oxide reacts with both acid and alkali.
- An acid-base reaction forming an oxoanion does not by itself change the oxidation state of the central metal. In an ordinary oxide anion, oxygen is assigned oxidation number -2.

Step 1: Identify the amphoteric oxide Among the listed vanadium oxides, V_2O_3 has vanadium in the $+3$ state and is basic. V_2O_4 contains vanadium in the $+4$ state and is less basic. V_2O_5, with vanadium in the $+5$ state, is amphoteric and reacts with alkali to produce vanadate. The required starting oxide is therefore the pentoxide, rather than the lower oxide. A representative reaction with sodium hydroxide forms sodium metavanadate:

$$V_2O_5 + 2\,NaOH \longrightarrow 2\,NaVO_3 + H_2O.$$

The equation balances V, Na, O and H without any electron-transfer reagent.

Step 2: Calculate the oxidation number in vanadate For the metavanadate ion VO_3^-, let the oxidation number of vanadium be x. The sum must equal the ion charge:

$$x + 3(-2) = -1 \quad \Rightarrow \quad x = +5.$$

Depending on solution conditions, other vanadate species can form, but vanadium remains $+5$ in the absence of a redox change; for example, VO_4^{3-} also gives $x - 8 = -3$, hence $+5$. Option (A) corresponds to the lower $+4$ oxide, and option (C) to the basic $+3$ oxide. Option (D), $+7$, does not result from the reaction and exceeds vanadium's usual maximum oxidation state of $+5$.

Step 3: Conclusion Vanadium is in oxidation state $+5$ in the oxide anion formed with alkali. Hence, the correct option is (B).

Quick Tip to Solve: First identify the amphoteric oxide; then use charge balance in its oxoanion. Alkali alone does not imply oxidation or reduction.

60. Identify product [A], [B] and [C] in the following reaction sequence.

$$CH_3 - C \equiv CH \xrightarrow[H_2]{Pd/C} [A] \xrightarrow[\textbf{(ii)}\ Zn,H_2O]{\textbf{(i)}\ O_3} [\ B] + [C]$$

[4 marks]

(A) [A]: $CH_3CH{=}CH_2$, [B]: CH_3CHO, [C]: CH_3CH_2OH

(B) [A]: $CH_3CH{=}CH_2$, [B]: CH_3CHO, [C]: $HCHO$

(C) [A]: $CH_3CH_2CH_3$, [B]: CH_3CHO, [C]: $HCHO$

(D) [A]: $CH_2{=}CH_2$, [B]: CH_3COCH_3, [C]: $HCHO$

Correct Answer: (B) [A]: $CH_3CH{=}CH_2$, [B]: CH_3CHO, [C]: $HCHO$

Solution:

Concept:

- Partial hydrogenation of propyne gives propene after addition of one equivalent of hydrogen. Complete catalytic hydrogenation instead gives propane.
- Reductive ozonolysis cleaves an alkene double bond and converts each of its two carbons into a carbonyl carbon, preserving the groups attached to that carbon.

Step 1: Infer the intermediate needed for ozonolysis To obtain the carbonyl products in the displayed sequence, the intended intermediate [A] must be propene, $CH_3CH{=}CH_2$. This corresponds to addition of one equivalent of hydrogen to propyne:

$$CH_3C{\equiv}CH + H_2 \longrightarrow CH_3CH{=}CH_2.$$

There is a reagent qualification in the printed question: ordinary unpoisoned Pd/C can hydrogenate an alkyne further to the alkane, and the question does not explicitly specify one equivalent of hydrogen or a selective catalyst. Thus the propene intermediate describes the intended controlled reduction, not a generally selective property of Pd/C.

Step 2: Split the alkene and compare the options For the intended propene intermediate, the substituted double-bond carbon carries CH_3 and H, giving ethanal; the terminal carbon carries two H atoms, giving methanal. Thus

$$CH_3CH{=}CH_2 \xrightarrow[2.\ Zn,\ H_2O]{1.\ O_3} CH_3CHO + HCHO.$$

These products and the propene intermediate match option (B). Option (A) gives ethanol instead of methanal, even though reductive ozonolysis gives carbonyl compounds. Option (C) gives propane as [A], which has no double bond for this alkene-cleavage reaction. Option (D) both loses a carbon in [A] and assigns acetone to cleavage of ethene; ethene instead gives methanal only.

Step 3: Conclusion The intended sequence gives [A] = $CH_3CH{=}CH_2$, [B] = CH_3CHO, [C] = $HCHO$. Hence, the correct option is (B) under controlled partial hydrogenation. As printed, Pd/C without a hydrogen-quantity or selectivity qualification does not uniquely guarantee this intermediate.

Quick Tip to Solve: Cut the C=C bond into two C=O bonds to identify ozonolysis products, and check whether the preceding reduction actually stops at the alkene.

61. Identify correct conversion during acidic hydrolysis from the following :

(A) starch gives galactose.

(B) cane sugar gives equal amount of glucose and fructose.

(C) milk sugar gives glucose and galactose.

(D) amylopectin gives glucose and fructose.

(E) amylose gives only glucose.

Choose the correct answer from the options given below : [4 marks]

(A) (A), (B) and (C) only

(B) (B), (C) and (D) only

(C) (B), (C) and (E) only

(D) (C), (D) and (E) only

Correct Answer: (C) (B), (C) and (E) only

Solution:

Concept:

- Acidic hydrolysis breaks the glycosidic bonds of carbohydrates. The products depend on the monosaccharide units present, not merely on whether the carbohydrate tastes sweet.
- Sucrose contains glucose and fructose units, lactose contains glucose and galactose units, and both components of starch are polymers of glucose.

Step 1: Test the two disaccharides Cane sugar is sucrose. Its hydrolysis gives one mole each of glucose and fructose, so statement (B) is correct. Since these sugars have the same molar mass, their masses are also equal. Milk sugar is lactose, which gives glucose and galactose on hydrolysis; hence statement (C) is correct. Both products must be identified rather than classifying all sugars as glucose sources.

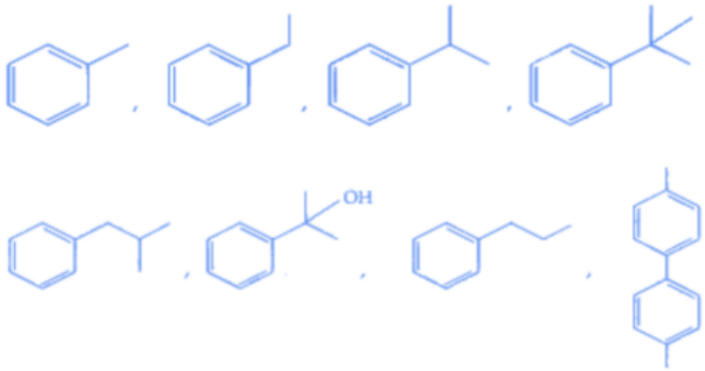
Step 2: Test starch and its components Starch consists of amylose and amylopectin. Amylose is an unbranched polymer of glucose, whereas amylopectin is branched, but branching does not introduce fructose or galactose. Complete hydrolysis of either gives glucose only. Therefore statement (E) is correct, statement (A) is false because starch does not give galactose, and statement (D) is false because amylopectin does not give fructose. The branching changes linkages, not the identity of the monomer.

Step 3: Match the complete statement set The correct set is (B), (C) and (E). Option (A) wrongly includes the galactose claim for starch; option (B) wrongly includes the fructose claim for amylopectin; option (D) also includes that false statement. Only option (C) includes all three correct statements and excludes both false ones.

Step 4: Conclusion The correct conversions are statements B, C and E only. Hence, the correct option is (C).

Quick Tip to Solve: Remember the three hydrolysis pairs: sucrose gives glucose plus fructose, lactose gives glucose plus galactose, and starch gives glucose only.

62. The total number of compounds from below when treated with hot $KMnO_4$, giving benzoic acid is: [4 marks]



(A) 4

(B) 3

(C) 5

(D) 6

Correct Answer: (C) 5

Solution:

Concept:

- Hot $KMnO_4$ oxidises an alkyl side chain attached to a benzene ring to $-COOH$ when the benzylic carbon has at least one hydrogen.
- To count compounds giving benzoic acid, check both the benzylic hydrogen and the complete final aromatic skeleton. Oxidation of two separate methyl groups on biphenyl does not give benzoic acid.

Step 1: Count the eligible alkylbenzenes Read the structures from left to right in each row. The first three top-row compounds are toluene, ethylbenzene and isopropylbenzene. Each has at least one hydrogen on the carbon directly bonded to the benzene ring. Thus each undergoes side-chain oxidation to C_6H_5COOH, irrespective of how many additional carbon atoms the side chain contains. These contribute three compounds.

Step 2: Inspect the second row and the blocked side chains The first and third compounds in the second row also have a benzylic CH_2 group: they are isobutylbenzene and propylbenzene. Each gives benzoic acid, adding two. The top-right tert-butylbenzene has no benzylic hydrogen. The second compound in the lower row is a tertiary benzylic alcohol, $C_6H_5C(OH)(CH_3)_2$; its benzylic carbon likewise lacks hydrogen, so it is not counted by the stated side-chain oxidation rule.

Step 3: Exclude the biphenyl derivative and select the count The last lower-row structure contains two benzene rings joined together, with a methyl substituent on each outer end. Oxidising those methyl groups gives a biphenyl dicarboxylic acid; the bond joining the two rings is not an alkyl side chain removed to form benzoic acid. Therefore this compound does not count. The total is $3 + 2 = 5$. The values 3 and 4 omit eligible side chains, while 6 includes an ineligible product.

Step 4: Conclusion Five of the displayed compounds give benzoic acid on the specified oxidation. Hence, the correct option is (C).

Quick Tip to Solve: First circle every benzylic hydrogen, then check the final ring skeleton; a biphenyl dicarboxylic acid is not benzoic acid.

63. Assume a living cell with 0.9% (w/w) of glucose solution (aqueous). This cell is immersed in another solution having equal mole fraction of glucose and water.

(Consider the data upto first decimal place only)

The cell will : [4 marks]

(A) shrink since solution is $0.45\%(\omega/\omega)$ as a result of association of glucose molecules (due to hydrogen bonding)

(B) shrink since solution is $0.5\%(\omega/\omega)$

(C) show no change in volume since solution is $0.9\%(\omega/\omega)$

(D) swell up since solution is $1\%(\omega/\omega)$

Correct Answer: The literal equimolar outside solution contains 90.9% glucose by mass and shrinks the cell; none of the offered options gives this result.

Solution:

Concept:

- Osmosis is solvent transfer across a selectively permeable membrane caused by a difference in solvent chemical potential. Equal glucose compositions at the same temperature are isotonic.
- For a glucose mass of $0.9\,\mathrm{g}$ in $100\,\mathrm{g}$ solution, use $M_{\text{glucose}} = 180\,\mathrm{g\,mol^{-1}}$ and $M_{\text{water}} = 18\,\mathrm{g\,mol^{-1}}$.

Step 1: Calculate the composition inside the cell Take $100\,\mathrm{g}$ of the cell solution. It contains $0.9\,\mathrm{g}$ glucose and $99.1\,\mathrm{g}$ water. Thus

$$n_g = 0.9/180 = 0.005\,\mathrm{mol}, \qquad n_w = 99.1/18 = 5.5056\,\mathrm{mol}.$$

The glucose mole fraction is approximately 0.000907, and the water mole fraction is approximately 0.999093. These two mole fractions are clearly not equal to each other.

Step 2: Interpret equality of the corresponding mole fractions The keyed answer reads the other solution as having the same corresponding glucose and water mole fractions as the cell solution. On that reading, both solutions have the same composition: 0.9% glucose by mass to the requested precision. There is no concentration gradient causing net solvent flow, so the cell neither shrinks nor swells. Hydrogen bonding is not a basis for arbitrarily halving the stated concentration.

Step 3: Check the wording and the alternatives Under the equal-composition reading, (C) is correct; (A) and (B) claim shrinking despite the same composition, while (D) claims swelling. The wording also admits a different literal reading: if it means $x_g = x_w = 0.5$ in the outside solution, take one mole of each. Then the glucose mass percentage is

$$100\frac{180}{180+18} = 90.9\%.$$

That solution would draw water out and shrink the cell; none of the listed options states this result. Therefore the equality must refer to corresponding compositions for the keyed answer to apply.

Step 4: Conclusion As literally printed, equal mole fractions of glucose and water give an outside glucose concentration of 90.9% by mass, so the cell shrinks. None of the four complete options is correct. The final key selects (C), which applies only if the wording is read as equal corresponding

compositions inside and outside, rather than equal glucose and water mole fractions within the external solution.

Quick Tip to Solve: Distinguish equal compositions of two solutions from equal mole fractions of two components within one solution; the latter would be a 90.9% glucose mixture.

64. The purification method based on the following physical transformation is :

$$\underset{(X)}{\text{Solid}} \xrightarrow{\text{Heat}} \underset{(X)}{\text{Vapour}} \xrightarrow{\text{Cool}} \underset{(X)}{\text{Solid}}$$

[4 marks]

(A) Extraction

(B) Crystallization

(C) Distillation

(D) Sublimation

Correct Answer: (D) Sublimation

Solution:

Concept:

- Sublimation is the direct conversion of a solid into vapour without passing through a liquid phase. On cooling, the vapour deposits as a solid.
- A sublimable solid can be purified from non-sublimable impurities because the desired substance enters the vapour and is collected separately on a cool surface.

Step 1: Follow the phases of the same substance The species labelled X starts as a solid, becomes vapour on heating, and returns to solid on cooling. The label X is unchanged, so this is a physical change rather than chemical decomposition or formation of a new compound. No liquid phase appears in the sequence. The direct solid-to-vapour step is the identifying feature of sublimation, and the reverse vapour-to-solid step allows collection of the purified solid.

Step 2: Relate the sequence to the purification process In a sublimation apparatus, heat the impure solid gently. The sublimable component vaporises while suitable nonvolatile impurities remain behind. A cooler collecting surface converts the vapour directly into crystals of the same substance. Compounds such as camphor, naphthalene and ammonium chloride illustrate this separation when the impurity does not undergo the same transformation under the chosen conditions.

Step 3: Eliminate the other methods Extraction separates a substance by its different solubilities in two immiscible solvents; it does not require this phase sequence. Crystallization usually dissolves a solid in a solvent and then deposits crystals as solubility decreases. Distillation involves vaporisation of a liquid followed by condensation into a liquid. None of these corresponds to a direct solid-vapour-solid path. Sublimation alone matches both changes shown in the question.

Step 4: Conclusion The indicated solid-to-vapour-to-solid purification is sublimation. Hence, the correct option is (D).

Quick Tip to Solve: Look for an absent liquid phase: solid directly becoming vapour and then solid identifies sublimation immediately.

65. Identify correct statements :

(A) Primary amines do not give diazonium salts when treated with $NaNO_2$ in acidic condition.

(B) Aliphatic and aromatic primary amines on heating with $CHCl_3$ and ethanolic KOH form carbylamines.

(C) Secondary and tertiary amines also give carbylamine test.

(D) Benzenesulfonyl chloride is known as Hinsberg's reagent.

(E) Tertiary amines reacts with benzenesulfonyl chloride very easily.

Choose the correct answer from the options given below : [4 marks]

(A) (B) and (C) only

(B) (D) and (E) only

(C) (A) and (B) only

(D) (B) and (D) only

Correct Answer: (D) (B) and (D) only

Solution:

Concept:

- The carbylamine test is specific to primary amines, both aliphatic and aromatic: heating with $CHCl_3$ and alcoholic KOH produces an isocyanide.
- Hinsberg's reagent is benzenesulfonyl chloride, $C_6H_5SO_2Cl$. Primary and secondary amines contain an N-H bond needed for sulfonamide formation, whereas tertiary amines do not.

Step 1: Assess diazotisation and the carbylamine statements Statement (A) is false as a general claim. Primary aromatic amines form diazonium salts with nitrous acid generated from $NaNO_2$ and acid at low temperature. Aliphatic diazonium ions are unstable, but that does not justify denying diazonium-salt formation for all primary amines. Statement (B) is correct: primary amines undergo

$$RNH_2 + CHCl_3 + 3\,KOH \longrightarrow RNC + 3\,KCl + 3\,H_2O.$$

Statement (C) is false because secondary and tertiary amines do not give the carbylamine test.

Step 2: Assess the Hinsberg statements Statement (D) correctly names benzenesulfonyl chloride as Hinsberg's reagent. A primary amine forms a sulfonamide with an acidic N-H hydrogen; a secondary amine forms an N,N-disubstituted sulfonamide without that acidic hydrogen. A tertiary amine has no N-H bond and does not readily form such a sulfonamide. Therefore statement (E), which claims easy reaction of tertiary amines with the reagent, is false.

Step 3: Select the statement pair Only (B) and (D) survive all the tests. Option (A) includes the false carbylamine statement (C); option (B) includes the false tertiary-amine statement (E); option (C) includes the false general statement (A). The fourth option is the only pair containing both true statements and neither false statement.

Step 4: Conclusion Statements B and D only are correct. Hence, the correct option is (D).

Quick Tip to Solve: Carbylamine means primary amine; Hinsberg reagent means benzenesulfonyl chloride. Check the availability of N-H before predicting sulfonamide formation.

66. Identify the inorganic sulphides that are yellow in colour :

(A) $(NH_4)_2S$

(B) PbS

(C) CuS

(D) As_2S_3

(E) As_2S_5

Choose the correct answer from the options given below : [4 marks]

(A) (D) and (E) only

(B) (A) and (B) only

(C) (A), (D) and (E) only

(D) (A) and (C) only

Correct Answer: (A) (D) and (E) only

Solution:

Concept:

- Qualitative salt analysis identifies precipitates partly by characteristic colours. Both arsenic(III) sulphide, As_2S_3, and arsenic(V) sulphide, As_2S_5, are yellow.
- The colour belongs to the particular sulphide and its composition; the presence of sulphur alone does not make every sulphide yellow.

Step 1: Identify the yellow arsenic sulphides Statement (D) names arsenic trisulphide, As_2S_3, the yellow sulphide associated with arsenic in the +3 oxidation state. Statement (E) names arsenic pentasulphide, As_2S_5, also yellow, with arsenic in the +5 state. Both must therefore be included. The different sulphur-to-arsenic ratios do not change the requested classification of these two precipitates as yellow.

Step 2: Check the remaining sulphides Lead sulphide, PbS, is black, so (B) is excluded. Copper(II) sulphide, CuS, is also black, so (C) is excluded. Ordinary ammonium sulphide, $(NH_4)_2S$, is not the yellow sulphide intended in this list. Yellow ammonium sulphide reagent contains polysulphides, $(NH_4)_2S_x$, and should not be confused with the simple sulphide formula in (A). The word yellow must not be transferred from the polysulphide reagent to the listed pure compound.

Step 3: Compare the four combinations The desired set is (D) and (E) only, printed as the first option. The second and fourth options include one of the black sulphides. The third option contains both arsenic sulphides but also incorrectly includes ammonium sulphide, so it is not the correct complete set. Testing every member matters: a combination containing the right arsenic compounds can still be wrong because of an extra compound.

Step 4: Conclusion The yellow sulphides are As_2S_3 and As_2S_5, corresponding to statements D and E only. Hence, the correct option is (A).

Quick Tip to Solve: Remember black PbS and CuS, yellow arsenic sulphides; distinguish colourless simple ammonium sulphide from yellow polysulphide reagent.

67. Consider an elementary reaction

$$\mathrm{A(\ g) + B(\ g) \rightarrow C(\ g) + D(\ g)}$$

If the volume of reaction mixture is suddenly reduced to $\frac{1}{3}$ of its initial volume, the reaction rate will become ' x' times of the original reaction rate. The value of x is : [4 marks]

(A) 3

(B) 9

(C) $\frac{1}{3}$

(D) $\frac{1}{9}$

Correct Answer: (B) 9

Solution:

Concept:

- For an elementary bimolecular reaction, the reactant stoichiometric coefficients directly give the powers in the rate law: $r = k[\mathrm{A}][\mathrm{B}]$. This direct identification is justified by the word elementary.
- Concentration is the number of moles divided by volume. An instantaneous compression at unchanged temperature changes concentrations while leaving the rate constant k unchanged.

Step 1: Write the initial rate Let the initial amounts be n_A, n_B, and let the initial volume be V. Before compression the concentrations are $[\mathrm{A}] = n_A/V$ and $[\mathrm{B}] = n_B/V$, so the rate is

$$r = k\frac{n_A}{V}\frac{n_B}{V}.$$

Both reactants contribute separately; the overall order is two, even though each reactant individually has exponent one.

Step 2: Apply the new volume Immediately after the volume becomes $V/3$, the amounts have not yet changed by reaction. Therefore each concentration is multiplied by three:

$$r\prime = k\frac{n_A}{V/3}\frac{n_B}{V/3} = 9k\frac{n_A n_B}{V^2} = 9r.$$

The factor 3 would account for only one reactant. The reciprocal factors $1/3$ and $1/9$ describe a decrease and are incompatible with compression of this elementary gas reaction.

Step 3: Conclusion The new reaction rate is nine times the original rate, so $x = 9$. Hence, the correct option is (B).

Quick Tip to Solve: For a sudden volume change at constant temperature, multiply the concentration factor once for each power in the elementary rate law.

68. Given below are two statements :

Statement I : According to the Law of Octaves, the elements were arranged in the increasing order of their atomic number.

Statement II : Meyer observed a periodically repeated pattern upon plotting physical properties of certain elements against their respective atomic numbers.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both **Statement I** and **Statement II** are false

(B) **Statement I** is false but **Statement II** is true

(C) **Statement I** is true but **Statement II** is false

(D) Both **Statement I** and **Statement II** are true

Correct Answer: (A) Both **Statement I** and **Statement II** are false

Solution:

Concept:

- Newlands' Law of Octaves arranges elements by increasing atomic mass; it proposes repetition of properties at every eighth element.
- Lothar Meyer related repeating physical properties, notably atomic volume, to atomic mass. Atomic number is the basis of the modern periodic law, not these historical arrangements.

Step 1: Assess Statement I The statement says that Newlands arranged elements in increasing atomic number. The actual criterion was increasing atomic mass. His octave analogy referred to a resemblance between an element and the eighth following its position in the mass-based sequence. Thus the use of atomic number changes the historical claim and makes Statement I false. The existence of a periodic pattern does not make the stated ordering variable correct.

Step 2: Assess Statement II and select the combination Meyer plotted physical properties against atomic mass (then called atomic weight). For example, his atomic-volume curve showed recurring maxima associated with related elements. Although the observation of periodic repetition is correct, the statement incorrectly replaces atomic mass with atomic number. Statement II is therefore also false. Option B incorrectly accepts Statement II, option C incorrectly accepts Statement I, and option D incorrectly accepts both. Only the option rejecting both statements has the correct truth-value pair.

Step 3: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (A).

Quick Tip to Solve: In historical periodic-table questions, distinguish atomic mass in Newlands and Meyer from atomic number in the modern periodic law.

69. Concentrated nitric acid is labelled as 75% by mass. The volume in mL of the solution which contains 30 g of nitric acid is _____.

Given : Density of nitric acid solution is $1.25\ \mathrm{g/mL}$. [4 marks]

(A) 32

(B) 40

(C) 55

(D) 45

Correct Answer: (A) 32

Solution:

Concept:

- Mass percentage is $100\, m_{\text{solute}}/m_{\text{solution}}$. A 75 percent by mass solution contains 75 g of HNO_3 in every 100 g of solution.
- Density connects the mass of the complete solution to its volume: $\rho = m_{\text{solution}}/V_{\text{solution}}$. The stated density belongs to the solution, rather than to its solute alone.

Step 1: Find the required mass of solution Let m be the solution mass containing the required $30\,\mathrm{g}$ of nitric acid. Use the mass fraction, not a volume fraction:

$$0.75 = \frac{30}{m}, \qquad m = \frac{30}{0.75} = 40\,\mathrm{g}.$$

This sample consequently contains $10\,\mathrm{g}$ of the other component. The 40 g result is a mass and cannot yet be reported as a volume in mL.

Step 2: Convert solution mass into volume Using the given density of $1.25\,\mathrm{g\,mL^{-1}}$,

$$V = \frac{m}{\rho} = \frac{40\,\mathrm{g}}{1.25\,\mathrm{g\,mL^{-1}}} = 32\,\mathrm{mL}.$$

As a check, 32 mL weighs 40 g, of which 75 percent is exactly 30 g. Option B confuses solution mass with volume. Options C and D give volumes with larger solution masses and hence more than 30 g of acid; neither satisfies both the percentage and density conditions.

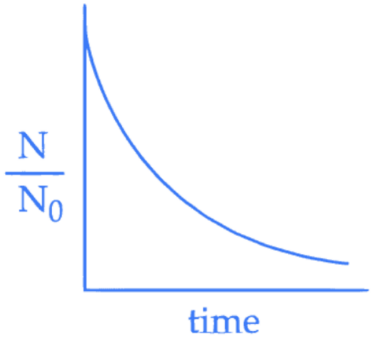
Step 3: Conclusion The required solution volume is $32\,\mathrm{mL}$. Hence, the correct option is (A).

Quick Tip to Solve: First divide the solute mass by the mass fraction; then divide the resulting solution mass by its density.

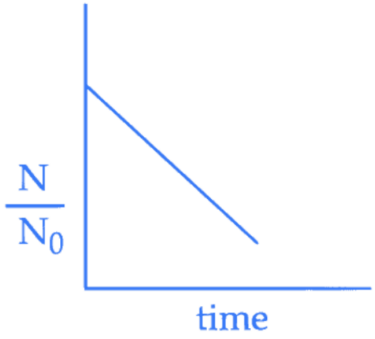
70. For bacterial growth in a cell culture, growth law is very similar to the law of radioactive decay. Which of the following graphs is most suitable to represent bacterial colony growth ?

Where N - Number of Bacteria at any time, N_0 - Initial number of Bacteria. [4 marks]

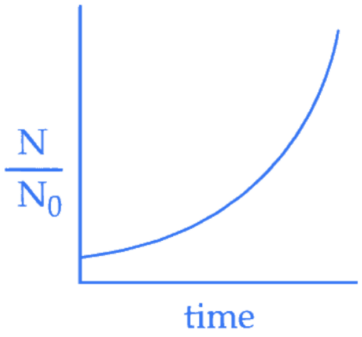
(A)



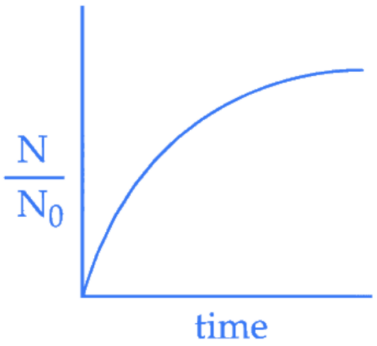
(B)



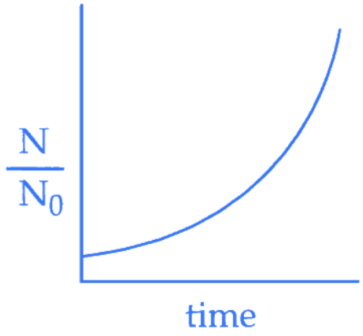
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- For ideal growth with a constant per-capita reproduction rate, the rate of increase is proportional to the current population: $dN/dt = kN$, with $k > 0$.
- The integrated growth law is $N = N_0 e^{kt}$. It has the same first-order mathematical form as radioactive decay, but growth uses a positive exponent instead of the negative decay exponent.

Step 1: Determine the direction and curvature Separating variables and integrating from N_0 at time zero to N at time t gives

$$\ln\left(\frac{N}{N_0}\right) = kt, \qquad \frac{N}{N_0} = e^{kt}.$$

Thus the ratio starts at one and increases with time. Its slope is $ke^{kt} > 0$, and its second derivative is $k^2 e^{kt} > 0$. The graph must therefore rise increasingly steeply; it is concave upwards in the ideal growth regime.

Step 2: Compare the four plotted shapes Option A is a falling exponential, appropriate to radioactive decay rather than bacterial multiplication. Option B is a straight decreasing line, which has neither the direction nor the curvature required. Option C rises with an increasing slope and represents exponential growth. Option D rises while flattening, implying a decreasing slope and a limiting population; that is incompatible with the ideal constant per-capita growth law specified here. Resource limitation can change real bacterial growth later, but it is outside the first-order growth model in this question.

Step 3: Conclusion The increasing, concave-up curve represents $N/N_0 = e^{kt}$. Hence, the correct option is (C).

Quick Tip to Solve: Radioactive decay and bacterial growth share proportional-rate equations; check the sign of the rate before choosing the graph.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The spin only magnetic moment (μ) value (B.M.) of the compound with strongest oxidising power among Mn_2O_3, TiO and VO is _____ B.M. (Nearest integer). [4 marks]

Solution:

Concept:

- The strongest oxidising species is the one most readily reduced. A particularly stable electron configuration in the reduced product can favour that reduction.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{B.M.}$, where n is the number of unpaired electrons on the relevant transition-metal ion.

Step 1: Compare the metal oxidation states With oxygen assigned oxidation state -2, the metal ions are Mn^{3+} in Mn_2O_3, Ti^{2+} in TiO, and V^{2+} in VO. The manganese(III) ion has a $3d^4$ configuration and is readily reduced to Mn^{2+}, whose half-filled $3d^5$ configuration is especially stable. Titanium(II) and vanadium(II), in contrast, tend to be oxidised to more stable higher oxidation states and behave

as reducing species. Therefore Mn_2O_3 is the strongest oxidising compound among the three.

Step 2: Count unpaired electrons in the original ion The question asks for the magnetic moment of the oxidising compound, so use Mn^{3+}, not its reduction product Mn^{2+}. The oxide has the high-spin d^4 arrangement, with four unpaired electrons. Consequently,

$$\mu = \sqrt{4(4+2)} = \sqrt{24}\,\text{B.M.} \approx 4.90\,\text{B.M.}$$

Rounding this value to the nearest integer gives 5. Using five unpaired electrons would incorrectly calculate the moment of manganese(II) instead of the stated compound.

Step 3: Conclusion The strongest oxidiser is Mn_2O_3, and its spin-only magnetic moment is $\sqrt{24} \approx 4.90\,\text{B.M.}$. The required nearest integer is $\boxed{5}$.

Quick Tip to Solve: Use the stable reduced configuration to identify the oxidiser, but count unpaired electrons in the original metal oxidation state.

72. Total number of molecules/species from following which will be paramagnetic is _____.

$$O_2,\ O_2^+,\ O_2^-,\ NO,\ NO_2,\ CO,\ K_2\,[NiCl_4],\ [Co\,(NH_3)_6]\,Cl_3,\ \ K_2\,[Ni(CN)_4]$$

[4 marks]

Solution:

Concept:

- A species is paramagnetic if it has at least one unpaired electron. Molecular orbital occupation determines the unpaired electrons in the small molecules and ions.
- For complexes, identify the metal oxidation state and its d-electron count, then account for ligand strength and geometry before deciding whether electrons remain unpaired.

Step 1: Classify the molecular species O_2 has two unpaired electrons in the degenerate antibonding π^* orbitals. Removing one electron gives $O_2{}^+$ with one unpaired electron; adding one gives $O_2{}^-$, also with one. Both NO and NO_2 have an odd number of electrons, and each has an unpaired electron. These five species are paramagnetic. Carbon monoxide, CO, has a closed-shell molecular-orbital arrangement and is diamagnetic, so it does not increase the count.

Step 2: Classify the three complexes In $K_2[NiCl_4]$, nickel is $+2$, hence d^8. Weak-field chloride gives a tetrahedral complex with two unpaired electrons, making it paramagnetic. In $[Co(NH_3)_6]Cl_3$, cobalt is $+3$, hence d^6; the low-spin octahedral configuration is $t_{2g}^6 e_g^0$, with all electrons paired. Finally, $K_2[Ni(CN)_4]$ contains d^8 nickel(II) with strong-field cyanide in a square-planar arrangement, also with all electrons paired. Only the chloride complex is added to the five molecular species.

Step 3: Conclusion The paramagnetic species are O_2, $O_2{}^+$, $O_2{}^-$, NO, NO_2 and $K_2[NiCl_4]$. Their total number is $\boxed{6}$.

Quick Tip to Solve: For nickel(II), remember the contrast: tetrahedral chloride is paramagnetic, while square-planar cyanide is diamagnetic.

73. Electrolysis of 600 mL aqueous solution of NaCl for 5 min changes the pH of the solution to 12 . The current in Amperes used for the given electrolysis is _____ . (Nearest integer). [4 marks]

Solution:

Concept:

- The pH fixes the hydroxide concentration through $\mathrm{pH}+\mathrm{pOH}=14$, using the usual aqueous-solution convention at room temperature.
- At the cathode, water gains electrons to produce hydrogen and hydroxide: $2\,\mathrm{H_2O}+2\,\mathrm{e^-}\longrightarrow \mathrm{H_2}+2\,\mathrm{OH^-}$. Thus each mole of electrons generates one mole of $\mathrm{OH^-}$.
- Faraday's law gives $Q=n_eF=It$, where n_e is the amount of electrons, $F=96500\,\mathrm{C\,mol^{-1}}$, and t is in seconds.

Step 1: Convert the final pH to hydroxide concentration The initially neutral salt solution becomes alkaline because electrolysis produces hydroxide. Its final pH is 12, so

$$\mathrm{pOH}=14-12=2,\qquad [\mathrm{OH^-}]=10^{-2}\,\mathrm{mol\,L^{-1}}.$$

Use hydroxide rather than the hydrogen-ion concentration: the latter would describe the small residual acidity rather than the amount of base produced. The initial hydroxide concentration of neutral water is negligible compared with $10^{-2}\,\mathrm{mol\,L^{-1}}$.

Step 2: Find the electron requirement for the whole solution The solution volume is $600\,\mathrm{mL}=0.600\,\mathrm{L}$. Taking its volume as unchanged, the amount of hydroxide produced is

$$n(\mathrm{OH^-})=0.600\times10^{-2}=6.00\times10^{-3}\,\mathrm{mol}.$$

The cathode equation has equal coefficients for electrons and hydroxide. Consequently, $n_e=6.00\times10^{-3}\,\mathrm{mol}$; no extra factor of two is needed even though one mole of hydrogen requires two moles of electrons.

Step 3: Calculate the current and round at the end The elapsed time is $5\times60=300\,\mathrm{s}$. Hence

$$Q=0.00600\times96500=579\,\mathrm{C},\qquad I=\frac{579}{300}=1.93\,\mathrm{A}.$$

This is the usual quantitative-electrolysis calculation, with the generated hydroxide retained in the solution. Rounding the current only after completing the charge and time conversions gives the requested integer.

Step 4: Conclusion The current is approximately $1.93\,\mathrm{A}$, so the required nearest-integer answer is $\boxed{2}$.

Quick Tip to Solve: For aqueous sodium chloride electrolysis, count one electron per hydroxide ion; convert both millilitres to litres and minutes to seconds before using $I=nF/t$.

74. Consider the following data :

Heat of formation of $CO_2(g) = -393.5\ kJ\ mol^{-1}$

Heat of formation of $H_2O(l) = -286.0\ kJ\ mol^{-1}$

Heat of combustion of benzene $= -3267.0\ kJ\ mol^{-1}$

The heat of formation of benzene is _____ $kJmol^{-1}$. (Nearest integer) [4 marks]

Solution:

Concept:

- Hess's law expresses a reaction enthalpy as the sum of formation enthalpies of products minus that of reactants, each multiplied by its stoichiometric coefficient.
- The standard enthalpy of formation of an element in its standard state is zero. Therefore the oxygen reactant contributes zero to the combustion calculation.
- Combustion of one mole of benzene produces six moles of carbon dioxide and three moles of liquid water. The phase of water matters because the supplied formation enthalpy is for liquid water.

Step 1: Balance combustion for one mole of benzene Benzene contains six carbon atoms and six hydrogen atoms, so complete combustion gives six carbon dioxide molecules and three water molecules:

$$C_6H_6(l) + \frac{15}{2}O_2(g) \longrightarrow 6\,CO_2(g) + 3\,H_2O\,(l).$$

This equation uses one mole of benzene, matching the molar heat of combustion supplied in the question. Using a doubled balanced equation without doubling its enthalpy would introduce a factor-of-two error.

Step 2: Substitute the supplied formation enthalpies Let the molar heat of formation of benzene be x. With all enthalpies in $kJ\,mol^{-1}$, Hess's law gives

$$-3267.0 = 6(-393.5) + 3(-286.0) - x.$$

The oxygen term is zero because oxygen gas is an element in its reference state. The benzene contribution is subtracted because benzene is a reactant, whereas both combustion products contribute with positive stoichiometric coefficients to the product sum.

Step 3: Isolate the formation enthalpy with its sign The product sum and rearrangement are

$$\begin{aligned} 6(-393.5) + 3(-286.0) &= -2361.0 - 858.0 = -3219.0, \\ -3267.0 &= -3219.0 - x, \\ x &= -3219.0 + 3267.0 = +48.0. \end{aligned}$$

As a sign check, subtracting the positive benzene formation enthalpy from the product sum makes combustion more negative, reproducing the given -3267.0. A negative value of x would instead make combustion less negative and would fail this check.

Step 4: Conclusion The heat of formation of benzene is $\boxed{+48\,kJ\,mol^{-1}}$. The numerical answer to the nearest integer is $\boxed{48}$.

Quick Tip to Solve: Write combustion for exactly one mole of fuel, then use products minus reactants; keep the negative combustion sign until the final rearrangement.

75. A group 15 element forms $\mathrm{d}\pi - \mathrm{d}\pi$ bond with transition metals. It also forms hydride, which is a strongest base among the hydrides of other group members that form $d\pi - d\pi$ bond.

The atomic number of the element is _____ . [4 marks]

Solution:

Concept:

- Group 15 contains nitrogen, phosphorus, arsenic, antimony and bismuth. Their simple hydrides are $\mathrm{NH_3}$, $\mathrm{PH_3}$, $\mathrm{AsH_3}$, $\mathrm{SbH_3}$ and $\mathrm{BiH_3}$.
- In the conventional bonding description used for this question, the heavier group 15 elements can participate in $d\pi$-$d\pi$ bonding with transition metals. Nitrogen has no valence-shell d orbitals and is excluded by that condition.
- The basicity of group 15 hydrides decreases down the group: $\mathrm{NH_3} > \mathrm{PH_3} > \mathrm{AsH_3} > \mathrm{SbH_3} > \mathrm{BiH_3}$. Basicity concerns donation of the central atom's lone pair to an acid.

Step 1: Apply the bonding condition before comparing bases The question supplies two linked restrictions. First, the required element must belong to group 15 and satisfy the stated $d\pi$-$d\pi$ bonding description with transition metals. Nitrogen is a second-period element, whose valence shell consists of 2s and 2p orbitals. It therefore does not meet this particular d-orbital condition, even though its hydride ammonia is the strongest base in the full group.

Step 2: Compare hydrides only among the eligible elements After excluding nitrogen, the remaining familiar hydrides are phosphine, arsine, stibine and bismuthine. Their basic strengths follow

$$\mathrm{PH_3} > \mathrm{AsH_3} > \mathrm{SbH_3} > \mathrm{BiH_3}.$$

Thus phosphine is the strongest base within the subset specified by the question. The answer is not obtained by choosing ammonia from the unrestricted hydride series: the phrase about the other members that form the stated bond limits which hydrides must be compared.

Step 3: Identify the element and report its atomic number Phosphine is $\mathrm{PH_3}$, so its central element is phosphorus, symbol P. Its ground-state configuration is

$$1s^2\,2s^2\,2p^6\,3s^2\,3p^3 = [\mathrm{Ne}]\,3s^2\,3p^3.$$

Counting these electrons gives $2+2+6+2+3=15$. The question asks for the atomic number rather than the group number or the mass number, so it is this electron count for the neutral atom that must be entered.

Step 4: Conclusion The element is phosphorus, whose hydride $\mathrm{PH_3}$ is the strongest base among the eligible heavier group 15 hydrides. Its atomic number is $\boxed{15}$.

Quick Tip to Solve: Apply every qualifying condition first: ammonia is strongest in the full series, but excluding nitrogen makes phosphine the strongest hydride in the remaining subset.