JEE Main 2025 Paper 1 (B.E./B.Tech) Jan 29 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 29 January 2025, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $\mathrm{L}_1 : \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{2}$ **and** $\mathrm{L}_2 : \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z}{1}$ **be two lines.**

Let L_3 **be a line passing through the point** (α, β, γ) **and be perpendicular to both** L_1 **and** L_2**. If** L_3 **intersects** L_1**, then** $|5\alpha - 11\beta - 8\gamma|$ **equals :** [4 marks]

(A) 25

(B) 20

(C) 16

(D) 18

Correct Answer: (A) 25

Solution:

Concept:

- The direction vectors of the given lines are $\mathbf{u} = (1, -1, 2)$ and $\mathbf{v} = (-1, 2, 1)$. A direction perpendicular to both is their cross product.
- A linear expression $\mathbf{n} \cdot \mathbf{r}$ remains constant along a line whenever $\mathbf{n}$ is perpendicular to its direction vector. This avoids finding the unspecified intersection point.

Step 1: Find the direction of the perpendicular line Take the cross product of the two independent direction vectors:

$$\mathbf{u} \times \mathbf{v} = (-5, -3, 1) = \mathbf{w}.$$

Thus every line perpendicular to both given lines has direction proportional to $\mathbf{w}$. A reversal of this direction describes the same line and does not change the argument.

Step 2: Use the intersection with the first line Let the intersection of L_3 and L_1 be P. A point on L_1 is $(1, 2, 1)$, so for some real parameters s, t, the given point on L_3 satisfies

$$(\alpha, \beta, \gamma) = (1, 2, 1) + s(1, -1, 2) + t(-5, -3, 1).$$

This representation uses only the stated intersection with L_1; an intersection with L_2 is not required.

Step 3: Evaluate the invariant expression Set $\mathbf{n} = (5, -11, -8)$. Direct scalar products give

$$\mathbf{n} \cdot \mathbf{u} = 5 + 11 - 16 = 0, \qquad \mathbf{n} \cdot \mathbf{w} = -25 + 33 - 8 = 0.$$

Consequently both parameter terms disappear:

$$5\alpha - 11\beta - 8\gamma = 5 - 22 - 8 = -25.$$

The other proposed values 20, 16 and 18 cannot be obtained, since the expression is independent of both free parameters.

Step 4: Conclusion Therefore $|5\alpha - 11\beta - 8\gamma| = 25$. Hence, the correct option is (A).

Quick Tip to Solve: When a linear expression is requested for a variable point on intersecting directions, test its coefficient vector against those directions before solving for parameters.

2. Let P be the set of seven digit numbers with sum of their digits equal to 11. If the numbers in P are formed by using the digits 1, 2 and 3 only, then the number of elements in the set P is : [4 marks]

(A) 164

(B) 158

(C) 161

(D) 173

Correct Answer: (C) 161

Solution:

Concept:

- Let x, y, z count the digits 1, 2 and 3. The number of positions gives $x + y + z = 7$, while the digit sum gives $x + 2y + 3z = 11$.
- For specified counts of repeated digits, the number of distinct arrangements is the multinomial number $7!/(x!y!z!)$. None starts with zero because zero is not an allowed digit.

Step 1: Determine the possible digit counts Subtract the position-count equation from the digit-sum equation to obtain

$$y + 2z = 4.$$

Since y, z are nonnegative integers, z can only be 0, 1 or 2. These give, respectively,

$$(x, y, z) = (3, 4, 0), \quad (4, 2, 1), \quad (5, 0, 2).$$

This list is exhaustive: a third digit 3 would already contribute six extra units above the baseline sum of seven ones, exceeding the allowed extra sum of four.

Step 2: Count arrangements in each disjoint case With no digit 3, choose the four places occupied by 2:

$$N_0 = \frac{7!}{3!4!} = 35.$$

With one digit 3 and two digits 2, choose its place and then the places of the two 2s:

$$N_1 = \frac{7!}{4!2!1!} = 7\binom{6}{2} = 105.$$

With two digits 3 and no digit 2, choose their two places:

$$N_2 = \frac{7!}{5!2!} = 21.$$

The remaining places in each case contain 1, so no further choices are needed.

Step 3: Add the case totals The three cases have different numbers of digit 3 and therefore do not overlap. Their total is

$$|P| = 35 + 105 + 21 = 161.$$

Every counted string has exactly seven nonzero digits and sum 11. Conversely, the count equations force every admissible number into one of these cases. Thus 164, 158 and 173 are excluded by the exhaustive total.

Step 4: Conclusion The number of elements of P is 161. Hence, the correct option is (C).

Quick Tip to Solve: Start with all digits equal to 1; the excess digit sum is then shared as one unit per 2 and two units per 3.

---

3. Let $A = \left[a_{ij}\right] = \begin{bmatrix} \log_5 128 & \log_4 5 \\ \log_5 8 & \log_4 25 \end{bmatrix}$**. If** A_{ij} **is the cofactor of** a_{ij}, $C_{ij} = \sum_{k=1}^{2} a_{ik}A_{jk}, 1 \le i, j \le 2$, **and** $C = [C_{ij}]$, **then** $8|C|$ **is equal to :** [4 marks]

(A) 288

(B) 262

(C) 222

(D) 242

Correct Answer: (D) 242

Solution:

Concept:

- The cofactor identity is $\sum_k a_{ik}A_{jk} = |A|\delta_{ij}$. Thus the matrix defined by these sums is $C = A\,\mathrm{adj}(A) = |A|I_2$.
- Use logarithm power and reciprocal-base identities to evaluate $|A|$ exactly. For a two-dimensional scalar matrix, $|cI_2| = c^2$.

Step 1: Identify the matrix formed by the cofactor sums The second index of the cofactor is summed against the column index of A. This is exactly multiplication by the transpose of the cofactor matrix, namely the adjugate. For the diagonal entries the cofactor expansion gives $|A|$, whereas an off-diagonal entry is the corresponding expansion with two equal rows and is zero. Hence

$$C = \begin{bmatrix} |A| & 0 \\ 0 & |A| \end{bmatrix}, \qquad |C| = |A|^2.$$

In particular, C is not merely the original cofactor matrix.

Step 2: Evaluate the determinant of the given matrix Write $t = \log_5 2$, which is nonzero. The entries become

$$\log_5 128 = 7t, \quad \log_5 8 = 3t, \quad \log_4 5 = \frac{1}{2t}, \quad \log_4 25 = \frac{1}{t}.$$

Therefore

$$|A| = (7t)\frac{1}{t} - (3t)\frac{1}{2t} = 7 - \frac{3}{2} = \frac{11}{2}.$$

All logarithms have positive arguments and valid bases, so the reciprocal and power laws apply directly.

Step 3: Compute the requested quantity Substitute this determinant into the scalar-matrix result:

$$8|C| = 8\left(\frac{11}{2}\right)^2 = \frac{8 \cdot 121}{4} = 242.$$

The square is essential because C has two diagonal entries equal to $|A|$. Values 288, 262 and 222 do not satisfy this exact identity and evaluation; no logarithmic approximations are needed.

Step 4: Conclusion The required value is $8|C| = 242$. Hence, the correct option is (D).

Quick Tip to Solve: Look for the cofactor identity before expanding every entry of a matrix defined through a sum; it often reduces the whole matrix to a scalar identity.

4. Consider an A. P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11^{th} term is : [4 marks]

(A) 108

(B) 90

(C) 122

(D) 84

Correct Answer: (B) 90

Solution:

Concept:

- An arithmetic progression with first term a and common difference d has $T_n = a + (n-1)d$ and $S_n = n[2a + (n-1)d]/2$.
- All terms here are integers, so a and $d = T_2 - T_1$ are integers. The interval for a sum can therefore single out the common difference.

Step 1: Use the sum of the first three terms The three terms are $a, a+d, a+2d$, and their sum is 54. Thus

$$3a + 3d = 54 \quad \Longrightarrow \quad a + d = 18, \qquad a = 18 - d.$$

The middle of the first three terms is therefore fixed, but the first term and common difference must still satisfy the information about the longer sum.

Step 2: Restrict the integer common difference For the first twenty terms,

$$S_{20} = 10(2a + 19d) = 10\{2(18 - d) + 19d\} = 360 + 170d.$$

The given bounds imply

$$1600 < 360 + 170d < 1800,$$

so that

$$\frac{124}{17} < d < \frac{144}{17}.$$

These endpoints are about 7.29 and 8.47, respectively, leaving only $d = 8$ among integers. Even if the word between included endpoints, neither endpoint would give another integer value.

Step 3: Find and check the eleventh term Now $a = 18 - 8 = 10$, and hence

$$T_{11} = a + 10d = 10 + 80 = 90.$$

The first three terms 10, 18 and 26 sum to 54, while

$$S_{20} = 360 + 170 \cdot 8 = 1720,$$

which lies in the required interval. All progression terms are positive since the first term and difference are both positive. The other listed values 108, 122 and 84 conflict with the uniquely determined a, d.

Step 4: Conclusion The eleventh term is 90. Hence, the correct option is (B).

Quick Tip to Solve: For three consecutive terms of an A.P., their sum is three times the middle term; use that first to eliminate the first term from a later sum.

---

5. Let the ellipse $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$ and $E_2 : \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$, $A < B$ have same eccentricity $\frac{1}{\sqrt{3}}$. Let the product of their lengths of latus rectums be $\frac{32}{\sqrt{3}}$ and the distance between the foci of E_1 be 4. If E_1 and E_2 meet at A, B, C and D, then the area of the quadrilateral ABCD equals : [4 marks]

(A) $\frac{24\sqrt{6}}{5}$

(B) $\frac{18\sqrt{6}}{5}$

(C) $6\sqrt{6}$

(D) $\frac{12\sqrt{6}}{5}$

Correct Answer: (A) $\frac{24\sqrt{6}}{5}$

Solution:

Concept:

- For a horizontal ellipse, $e^2 = 1 - b^2/a^2$, focal separation is $2ae$, and the length of the latus rectum is $2b^2/a$.
- For a vertical ellipse, interchange the roles of the major and minor semiaxes: the focal parameter uses B, while the latus rectum has length $2A^2/B$. Symmetric intersections form a rectangle.

Step 1: Determine both ellipse equations The eccentricity condition gives $b^2 = 2a^2/3$. From $2ae = 4$ we obtain $a = 2\sqrt{3}$, so $a^2 = 12$ and $b^2 = 8$. The first latus rectum is

$$\ell_1 = \frac{16}{2\sqrt{3}} = \frac{8}{\sqrt{3}}.$$

The given product gives $\ell_2 = 4$. For the second ellipse, $A^2 = 2B^2/3$, whence

$$\frac{2A^2}{B} = \frac{4B}{3} = 4 \quad \Longrightarrow \quad B = 3, \quad A^2 = 6.$$

Step 2: Find the coordinates of an intersection The simultaneous equations are

$$\frac{x^2}{12} + \frac{y^2}{8} = 1, \qquad \frac{x^2}{6} + \frac{y^2}{9} = 1.$$

Multiplying to clear denominators gives

$$2x^2 + 3y^2 = 24, \qquad 3x^2 + 2y^2 = 18.$$

Elimination yields $5y^2 = 36$, and substitution then gives $x^2 = 6/5$. Both squares are positive, so there are four intersections, with independent signs of their coordinates.

Step 3: Calculate the rectangle area Write the intersection coordinates as $(\pm x_0, \pm y_0)$, where $x_0 = \sqrt{6/5}$ and $y_0 = 6/\sqrt{5}$. In cyclic order the quadrilateral has horizontal and vertical sides of lengths $2x_0$ and $2y_0$. Thus

$$\text{Area} = 4x_0y_0 = 4\sqrt{\frac{6}{5}}\frac{6}{\sqrt{5}} = \frac{24\sqrt{6}}{5}.$$

Using only one quadrant would omit the factor four. The other three options differ from the rectangle area obtained from all four symmetric intersections.

Step 4: Conclusion The area of $ABCD$ is $\frac{24\sqrt{6}}{5}$ square units. Hence, the correct option is (A).

Quick Tip to Solve: Solve intersecting axis-aligned ellipses as linear equations in the variables x^2 and y^2, then use all four sign combinations for the rectangle area.

6. Two parabolas have the same focus (4, 3) and their directrices are the x-axis and the y-axis, respectively. If these parabolas intersect at the points A and B, then $(AB)^2$ is equal to : [4 marks]

(A) 384

(B) 392

(C) 96

(D) 192

Correct Answer: (D) 192

Solution:

Concept:

- A parabola is the locus of points equidistant from its focus and directrix. Distances to the x-axis and y-axis are $|y|$ and $|x|$, respectively.
- At a common point of two parabolas with the same focus, the distances to both directrices must be equal. Thus their common chord lies on one of the angle bisectors $y = x$ or $y = -x$.

Step 1: Write the two distance equations For a point (x, y), the squared distance to the common focus $(4, 3)$ is $(x-4)^2 + (y-3)^2$. The two parabola equations are therefore

$$(x-4)^2 + (y-3)^2 = y^2, \qquad (x-4)^2 + (y-3)^2 = x^2.$$

At an intersection, subtracting these equations gives $x^2 = y^2$. Both possible signs must be checked; choosing a sign without this check could omit intersections.

Step 2: Find which angle bisector has real intersections For $y = x$, substitution into either distance

equation gives

$$(x-4)^2+(x-3)^2=x^2 \quad \Longrightarrow \quad x^2-14x+25=0.$$

Its roots are $x=7\pm 2\sqrt{6}$. For $y=-x$, the equation instead becomes

$$(x-4)^2+(-x-3)^2=x^2 \quad \Longrightarrow \quad x^2-2x+25=0.$$

This second quadratic has discriminant $4-100=-96<0$, so there are no real intersections on that bisector.

Step 3: Compute the squared length of the common chord Thus the endpoints are $A=(7+2\sqrt{6},7+2\sqrt{6})$ and $B=(7-2\sqrt{6},7-2\sqrt{6})$. The differences of both coordinates equal $4\sqrt{6}$, giving

$$(AB)^2=(4\sqrt{6})^2+(4\sqrt{6})^2=96+96=192.$$

The value 96 counts only one coordinate contribution. Values 384 and 392 likewise do not agree with the Euclidean distance formula for these endpoints.

Step 4: Conclusion The required squared distance is $(AB)^2=192$. Hence, the correct option is (D).

Quick Tip to Solve: For two parabolas with a common focus, equate distances to the directrices first; this often gives the common chord line before any quadratic solving.

7. Let $\triangle ABC$ be a triangle formed by the lines $7x-6y+3=0$, $x+2y-31=0$ and $9x-2y-19=0$. Let the point (h, k) be the image of the centroid of $\triangle ABC$ in the line $3x+6y-53=0$. Then h^2+k^2+hk is equal to : [4 marks]

(A) 47

(B) 37

(C) 40

(D) 36

Correct Answer: (B) 37

Solution:

Concept:

- The centroid of a triangle with vertices $(x_1,y_1),(x_2,y_2),(x_3,y_3)$ is $G=((x_1+x_2+x_3)/3,(y_1+y_2+y_3)/3)$. The vertices here are intersections of pairs of the given lines.
- The reflection of (u,v) in $ax+by+c=0$ is $(u-2ad,v-2bd)$, where $d=(au+bv+c)/(a^2+b^2)$. The displacement is normal to the mirror line.

Step 1: Find the three vertices Intersect $7x-6y+3=0$ with $x+2y-31=0$. Substituting $x=31-2y$ gives $220-20y=0$, hence $(9,11)$. Intersect the second and third lines: $9(31-2y)-2y-19=0$ gives $(5,13)$. For the first and third, substitute $x=(19+2y)/9$ into the first to obtain $160-40y=0$, giving $(3,4)$. Each point satisfies the pair used to obtain it.

Step 2: Reflect the centroid The centroid is therefore

$$G = \left(\frac{9+5+3}{3}, \frac{11+13+4}{3}\right) = \left(\frac{17}{3}, \frac{28}{3}\right).$$

For the mirror line, $a = 3, b = 6, c = -53$, so

$$d = \frac{17 + 56 - 53}{9 + 36} = \frac{4}{9}.$$

Thus

$$h = \frac{17}{3} - 6\left(\frac{4}{9}\right) = 3, \qquad k = \frac{28}{3} - 12\left(\frac{4}{9}\right) = 4.$$

As a check, the midpoint of G and $(3, 4)$ lies on the mirror line, and their joining segment has the normal direction $(3, 6)$.

Step 3: Evaluate the expression Substitute the reflected coordinates, rather than the original centroid:

$$h^2 + k^2 + hk = 3^2 + 4^2 + 3 \cdot 4 = 9 + 16 + 12 = 37.$$

The values 47, 40 and 36 in the remaining options do not equal this exact evaluation.

Step 4: Conclusion The required value is 37. Hence, the correct option is (B).

Quick Tip to Solve: Find all three intersections before averaging; in a reflection formula keep the signed line value, and verify that the midpoint lies on the mirror.

8. The number of solutions of the equation $\left(\frac{9}{x} - \frac{9}{\sqrt{x}} + 2\right)\left(\frac{2}{x} - \frac{7}{\sqrt{x}} + 3\right) = 0$ **is :** [4 marks]

(A) 3

(B) 2

(C) 1

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- The original expression is real and defined only for $x > 0$, since both x and $\sqrt{x}$ occur in denominators.
- The substitution $t = 1/\sqrt{x}$ converts $1/x$ to t^2. It is a one-to-one correspondence between positive x and positive t, with inverse $x = 1/t^2$.

Step 1: Transform and factor the equation With $t > 0$, the equation becomes

$$(9t^2 - 9t + 2)(2t^2 - 7t + 3) = 0.$$

A product is zero if at least one factor is zero. Factor each quadratic separately:

$$9t^2 - 9t + 2 = (3t - 1)(3t - 2),$$

$$2t^2 - 7t + 3 = (2t - 1)(t - 3).$$

Multiplication verifies both factorizations, so no roots have been introduced by a squaring operation.

Step 2: List and check the roots The first quadratic gives $t = 1/3, 2/3$, while the second gives $t = 1/2, 3$. All four numbers are positive, distinct and nonzero, and therefore satisfy the substitution domain. Converting back gives

$$x = 9, \quad \frac{9}{4}, \quad 4, \quad \frac{1}{9}.$$

These four values are also distinct and positive. For each value the associated factor in the transformed product vanishes, which verifies the original equation because the substitution was reversible.

Step 3: Count distinct original solutions The four positive transformed roots produce exactly four original solutions. The factored polynomial has no further roots, and the one-to-one map cannot produce two different x values from a single positive t. Thus the choices 1, 2 and 3 undercount the solution set; counting only one quadratic would miss valid roots of the other quadratic.

Step 4: Conclusion The number of solutions is 4. Hence, the correct option is (D).

Quick Tip to Solve: For expressions containing both reciprocal square roots and reciprocals, put $t = 1/\sqrt{x} > 0$, then count distinct positive roots before converting back.

9. The least value of n for which the number of integral terms in the Binomial expansion of $(\sqrt[3]{7} + \sqrt[12]{11})^n$ is 183, is : [4 marks]

(A) 2184

(B) 2172

(C) 2196

(D) 2148

Correct Answer: (A) 2184

Solution:

Concept:

- The general term in $(a + b)^n$ is $T_{r+1} = \binom{n}{r} a^{n-r} b^r$, where $r = 0, 1, \ldots, n$. End terms must be included when counting.
- For distinct primes 7 and 11, a rational product $7^u 11^v$ with rational exponents is an integer only when both prime exponents are nonnegative integers; fractional powers of different primes cannot cancel.

Step 1: Impose integrality on the general term Here

$$T_{r+1} = \binom{n}{r} 7^{(n-r)/3} 11^{r/12}.$$

The binomial coefficient is a nonzero integer. Its integer prime factors cannot remove a fractional part of either prime exponent. Hence an integral term requires

$$r \equiv 0 \pmod{12}, \qquad n - r \equiv 0 \pmod{3}.$$

Since a multiple of 12 is already a multiple of 3, the second condition reduces to $n \equiv 0 \pmod{3}$. If n is not divisible by 3, there are no integral terms.

Step 2: Count the admissible indices For $3 \mid n$, the allowable values are

$$r = 0, 12, 24, \ldots, 12\left\lfloor \frac{n}{12} \right\rfloor.$$

Therefore the number of integral terms is $\lfloor n/12 \rfloor + 1$. The extra 1 counts the index $r = 0$; overlooking it would shift the answer by 12. Requiring 183 terms gives

$$\left\lfloor \frac{n}{12} \right\rfloor = 182, \qquad 2184 \le n < 2196.$$

Step 3: Select the least permissible exponent The smallest integer in this interval is 2184, and it is divisible by 3. It therefore meets both integrality conditions and yields $2184/12 + 1 = 183$ terms. Among the other options, 2172 gives 182 terms, 2196 gives 184, and 2148 gives 180. Thus none of those has the required count.

Step 4: Conclusion The least value of n is 2184. Hence, the correct option is (A).

Quick Tip to Solve: Use separate divisibility conditions for the exponents of distinct primes; count the indices from zero, including both permissible endpoints.

---

10. The value of $\lim\limits_{n\to\infty}\left(\sum\limits_{k=1}^{n}\frac{k^3+6k^2+11k+5}{(k+3)!}\right)$ **is :** [4 marks]

(A) 5/3

(B) 2

(C) 4/3

(D) 7/3

Correct Answer: (A) 5/3

Solution:

Concept:

- A series with factorial denominators often telescopes after expressing its numerator as a product of consecutive factors plus a remainder.
- For integer $k \ge 1$, $(k+3)! = (k+3)(k+2)(k+1)k!$. Also $1/n! \to 0$ as $n \to \infty$, so any fixed finite collection of terminal factorial reciprocals tends to zero.

Step 1: Rewrite the summand Expand the consecutive product:

$$(k+1)(k+2)(k+3) = k^3 + 6k^2 + 11k + 6.$$

The given numerator is one less than this product, so

$$\frac{k^3+6k^2+11k+5}{(k+3)!}=\frac{1}{k!}-\frac{1}{(k+3)!}.$$

This identity is exact for every summation index; it avoids estimating a slowly accumulated numerical sum.

Step 2: Cancel the overlapping factorial terms Let the partial sum be S_n. For $n \geq 3$, write its two sums with matching factorial indices:

$$S_n=\sum_{j=1}^{n}\frac{1}{j!}-\sum_{j=4}^{n+3}\frac{1}{j!}.$$

Terms with indices 4 through n cancel, leaving

$$S_n=\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}-\frac{1}{(n+1)!}-\frac{1}{(n+2)!}-\frac{1}{(n+3)!}.$$

Only the first three positive terms and the last three negative terms remain.

Step 3: Take the limit Each terminal reciprocal factorial tends to zero. Consequently

$$\lim_{n\to\infty}S_n=1+\frac{1}{2}+\frac{1}{6}=\frac{10}{6}=\frac{5}{3}.$$

The finite partial sums approach this number from below because the remaining three factorial terms are subtracted and positive. In particular, the values 2, 4/3 and 7/3 cannot equal the limit obtained from the exact cancellation.

Step 4: Conclusion The required limit is $\frac{5}{3}$. Hence, the correct option is (A).

Quick Tip to Solve: Compare a cubic numerator over $(k+3)!$ with $(k+1)(k+2)(k+3)$; a difference of 1 produces a three-index telescoping sum.

11. Let $y = y(x)$ be the solution of the differential equation :

$\cos x\,(\log_e(\cos x))^2\,dy + (\sin x - 3y\sin x\log_e(\cos x))\,dx = 0$, **x** $\in$ **(0,** $\frac{\pi}{2}$ **). If** $y(\frac{\pi}{4}) = -\frac{1}{\log_e 2}$, **then** $y(\frac{\pi}{6})$ **is equal to :** [4 marks]

(A) $\frac{2}{\log_e(3)-\log_e(4)}$

(B) $-\frac{1}{\log_e(4)}$

(C) $\frac{1}{\log_e(4)-\log_e(3)}$

(D) $\frac{1}{\log_e(3)-\log_e(4)}$

Correct Answer: (D) $\frac{1}{\log_e(3)-\log_e(4)}$

Solution:

Concept:

- Put $t = \ln(\cos x)$. Since $0 < x < \pi/2$, one has $\cos x > 0$, $t < 0$, and $dt = -\tan x\,dx$. Division by t is therefore legitimate throughout the stated interval.
- A first-order linear equation $dy/dt + P(t)y = Q(t)$ is solved by an integrating factor. For $P(t) = 3/t$, the nonzero factor t^3 converts the left-hand side into the derivative of t^3y.

Step 1: Transform the differential equation Dividing the given equation by $\cos x$ gives

$$t^2\,dy + (\tan x - 3yt\tan x)\,dx = 0.$$

Use $\tan x\,dx = -dt$, retaining its negative sign:

$$t^2\,dy + (3yt - 1)\,dt = 0.$$

Thus

$$\frac{dy}{dt} + \frac{3}{t}y = \frac{1}{t^2}.$$

Multiplying by t^3 and integrating yields

$$\frac{d}{dt}(t^3y) = t, \qquad t^3y = \frac{t^2}{2} + C,$$

so $y = 1/(2t) + C/t^3$.

Step 2: Apply the initial value At $x = \pi/4$, $t = \ln(1/\sqrt{2}) = -(\ln 2)/2$. The first term $1/(2t)$ is already $-1/\ln 2$, precisely the prescribed initial value. Since $t \neq 0$, this forces $C = 0$. The solution is consequently

$$y(x) = \frac{1}{2\ln(\cos x)}.$$

Substitution in the transformed equation checks it: its derivative is $-1/(2t^2)$, and adding $3y/t$ gives $1/t^2$.

Step 3: Evaluate at the required angle At $x = \pi/6$, $\cos x = \sqrt{3}/2$, hence

$$2\ln(\cos x) = 2\ln(\sqrt{3}/2) = \ln 3 - \ln 4.$$

Therefore $y(\pi/6) = 1/(\ln 3 - \ln 4)$. This is negative, excluding the positive reciprocal in (C). Option

(A) has twice the required magnitude, while (B) is a different negative value.

Step 4: Conclusion $y(\pi/6) = \dfrac{1}{\ln 3 - \ln 4}$. Hence, the correct option is (D).

Quick Tip to Solve: When $\ln(\cos x)$ accompanies $\tan x\, dx$, substitute the logarithm itself and keep the minus sign in its differential.

12. Let $|z_1 - 8 - 2i| \le 1$ and $|z_2 - 2 + 6i| \le 2$, $z_1, z_2 \in \mathbb{C}$. Then the minimum value of $|z_1 - z_2|$ is : [4 marks]

(A) 3

(B) 10

(C) 7

(D) 13

Correct Answer: (C) 7

Solution:

Concept:

- The inequality $|z - c| \le r$ describes the closed disc with centre c and radius r in the Argand plane; the equality boundary is included.
- For two disjoint discs with centre separation $d > r_1 + r_2$, the minimum distance between their points is $d - r_1 - r_2$. The reverse triangle inequality proves the bound, and points on the line of centres attain it.

Step 1: Identify the two centres and radii Write the first modulus as $|z_1 - (8 + 2i)| \le 1$, so $c_1 = 8 + 2i$ and $r_1 = 1$. Similarly, $|z_2 - (2 - 6i)| \le 2$, so $c_2 = 2 - 6i$ and $r_2 = 2$. In particular, the second centre has imaginary coordinate -6, not $+6$. Their separation is

$$d = |c_1 - c_2| = |6 + 8i| = \sqrt{36 + 64} = 10.$$

Since $10 > 1 + 2$, the discs do not overlap.

Step 2: Prove the lower bound and attain it The triangle inequality gives

$$|c_1 - c_2| \le |c_1 - z_1| + |z_1 - z_2| + |z_2 - c_2|.$$

Hence every admissible pair satisfies $|z_1 - z_2| \ge 10 - 1 - 2 = 7$. To show this is the minimum, let $u = (c_2 - c_1)/10 = -3/5 - 4i/5$, a unit vector from the first centre to the second. Choose $z_1 = c_1 + u$ and $z_2 = c_2 - 2u$. These are permitted boundary points, and

$$z_2 - z_1 = (c_2 - c_1) - 3u = 7u,$$

whose modulus is exactly 7.

Step 3: Compare the listed values The value 10 is the centre separation, not the minimum between disc points. Adding both radii gives 13, the maximum separation. The value 3 is merely the sum of the radii and omits the centre distance. Only 7 is both a universal lower bound and attained by valid

points.

Step 4: Conclusion The minimum value of $|z_1 - z_2|$ is 7. Hence, the correct option is (C).

Quick Tip to Solve: Rewrite each modulus as $|z - c|$ before reading its centre; subtract the sum of the radii from the centre distance only after checking the discs are disjoint.

---

13. Let $x_1, x_2, ..., x_{10}$ be ten observations such that $\sum_{i=1}^{10}(x_i - 2) = 30$, $\sum_{i=1}^{10}(x_i - \beta)^2 = 98$, $\beta > 2$, and their variance is $\frac{4}{5}$. If μ and σ^2 are respectively the mean and the variance of $2(x_1 - 1) + 4\beta, 2(x_2 - 1) + 4\beta, ..., 2(x_{10} - 1) + 4\beta$, then $\frac{\beta\mu}{\sigma^2}$ is equal to : [4 marks]

(A) 100

(B) 90

(C) 120

(D) 110

Correct Answer: (A) 100

Solution:

Concept:

- For observations with mean $\bar{x}$, $\sum(x_i - b)^2 = \sum(x_i - \bar{x})^2 + n(\bar{x} - b)^2$. The cross term vanishes because $\sum(x_i - \bar{x}) = 0$.
- For $Y = aX + c$, the mean is $a\bar{x} + c$, whereas the variance is $a^2\,\mathrm{Var}(X)$. Adding a constant changes location but not spread.

Step 1: Find the original mean The first condition gives $\sum x_i - 20 = 30$, hence $\sum x_i = 50$. There are ten observations, so their mean is $\bar{x} = 50/10 = 5$. Since the variance uses the divisor ten, the sum of squared deviations about this mean is $10(4/5) = 8$.

Step 2: Determine the permitted value of beta Apply the squared-deviation identity about the mean, rather than treating β as the mean itself:

$$98 = 8 + 10(5 - \beta)^2.$$

Thus $(5 - \beta)^2 = 9$, yielding $\beta = 2$ or 8. The strict restriction $\beta > 2$ eliminates the first root and leaves $\beta = 8$.

Step 3: Transform the mean and variance The new observations are $Y_i = 2x_i - 2 + 4\beta$. Consequently

$$\mu = 2(5) - 2 + 4(8) = 40, \qquad \sigma^2 = 2^2\left(\frac{4}{5}\right) = \frac{16}{5}.$$

The constant $-2 + 4\beta$ does not contribute to the variance. Therefore

$$\frac{\beta\mu}{\sigma^2} = \frac{8(40)}{16/5} = 100.$$

Values 90, 120 and 110 do not satisfy this substitution; retaining the excluded root or scaling variance

linearly would give an incorrect result.

Step 4: Conclusion The required ratio is 100. Hence, the correct option is (A).

Quick Tip to Solve: Find the original mean first; use the displacement-of-origin identity to recover beta, and square the multiplier when transforming variance.

---

14. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 7\hat{j} + 3\hat{k}$. Let $L_1 : \vec{r} = (-\hat{i} + 2\hat{j} + \hat{k}) + \lambda\vec{a}, \lambda \in \mathbf{R}$ and $L_2 : \vec{r} = (\hat{j} + \hat{k}) + \mu\vec{b}, \mu \in \mathrm{R}$ be two lines. If the line L_3 passes through the point of intersection of L_1 and L_y and is parallel to $\vec{a} + \vec{b}$, then L_3 passes through the point : [4 marks]

(A) $(-1, -1, 1)$

(B) $(2, 8, 5)$

(C) $(8, 26, 12)$

(D) $(5, 17, 4)$

Correct Answer: (C) $(8, 26, 12)$

Solution:

Concept:

- A common point of two lines has identical Cartesian coordinates in both parametric descriptions. The parameters on the two lines need not be equal.
- A line through P parallel to a nonzero vector $\vec{d}$ consists of points $P + t\vec{d}$, with the same parameter t in all three coordinates.

Step 1: Find the intersection of the two given lines The second line supplied in the question is L_2; the reference to L_y identifies that second line. Write its coordinates beside those of L_1:

$$L_1 : (-1 + \lambda,\ 2 + 2\lambda,\ 1 + \lambda), \qquad L_2 : (2\mu,\ 1 + 7\mu,\ 1 + 3\mu).$$

Equating the third coordinates gives $\lambda = 3\mu$. The first coordinates then give $-1 + 3\mu = 2\mu$, so $\mu = 1$ and $\lambda = 3$. Both second coordinates equal 8, confirming that this is a genuine intersection. Thus $P = (2, 8, 4)$.

Step 2: Write the required line The direction vector is the sum, not either original direction separately:

$$\vec{a} + \vec{b} = 3\hat{i} + 9\hat{j} + 4\hat{k}.$$

Therefore

$$L_3 : (x, y, z) = (2 + 3t,\ 8 + 9t,\ 4 + 4t), \qquad t \in \mathbb{R}.$$

This representation checks both requirements: at $t = 0$ the line contains the intersection, and its displacement vector is parallel to the prescribed sum.

Step 3: Test the proposed points For $(8, 26, 12)$, the three parameter values are $(8 - 2)/3 = 2$, $(26 - 8)/9 = 2$, and $(12 - 4)/4 = 2$, so it lies on the line. For $(-1, -1, 1)$, the first two give -1 but the third gives $-3/4$. For $(2, 8, 5)$, the first two give 0 but the third gives $1/4$. For $(5, 17, 4)$, the first two give 1 but the third gives 0. These three points fail the common-parameter test.

Step 4: Conclusion The line passes through $(8, 26, 12)$. Hence, the correct option is (C).

Quick Tip to Solve: After finding the intersection, test each candidate with one common parameter; agreement in just two coordinates does not establish membership.

15. Let the area of the region

$(x, y) : 2y \le x^2 + 3,\ y + |x| \le 3,\ y \ge |x - 1|$ **be** A**. Then** $6A$ **is equal to :** [4 marks]

(A) 14

(B) 18

(C) 16

(D) 12

Correct Answer: (A) 14

Solution:

Concept:

- For vertical strips, the region has lower boundary $y = |x - 1|$ and upper boundary $y = \min\{(x^2 + 3)/2,\ 3 - |x|\}$. Integrate upper minus lower only where it is nonnegative.
- Absolute values and intersections determine the breakpoints. A change in the active upper boundary must be distinguished from a change in the formula for the lower boundary.

Step 1: Locate the horizontal extent of the region The two V-shaped boundaries require $|x-1|+|x| \le 3$. Splitting at 0 and 1 shows that this is equivalent to $-1 \le x \le 2$. On this interval the lower boundary is $1 - x$ up to 1 and $x - 1$ thereafter. We still have to compare both proposed upper boundaries before integrating.

Step 2: Choose the active upper boundary For $-1 \le x \le 0$, compare the parabola with $3 + x$: their difference is $(x^2 - 2x - 3)/2 = (x - 3)(x + 1)/2 \le 0$. For $0 \le x \le 2$, the difference from $3 - x$ is $(x^2 + 2x - 3)/2 = (x + 3)(x - 1)/2$. Thus the parabola is the upper boundary through $x = 1$; the descending line is the upper boundary from 1 to 2. On $[-1, 1]$, the strip height simplifies to $(x + 1)^2/2$, so no further exclusion is necessary.

Step 3: Integrate the two strip heights The required area is

$$\begin{aligned} A &= \int_{-1}^{1} \left(\frac{x^2 + 3}{2} - (1 - x) \right) dx + \int_{1}^{2} \left((3 - x) - (x - 1) \right) dx \\ &= \left[\frac{x^3}{6} + \frac{x^2}{2} + \frac{x}{2} \right]_{-1}^{1} + \left[4x - x^2 \right]_{1}^{2} \\ &= \frac{4}{3} + 1 = \frac{7}{3}. \end{aligned}$$

Hence $6A = 14$. The alternatives 18, 16 and 12 are incompatible with the exact integrated area; using only one of the two upper constraints would overcount the region.

Step 4: Conclusion The area is $A = 7/3$ square units, giving $6A = 14$. Hence, the correct option is (A).

Quick Tip to Solve: For simultaneous upper inequalities, use the smaller upper curve. Solve its crossings first, then integrate the resulting nonnegative vertical heights.

16. Let $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} - 5\hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} = \vec{a} \times \vec{b} = \vec{c} \times \vec{b}$ and $(\vec{a} + \vec{c}) \cdot (\vec{b} + \vec{c}) = 168$. Then the maximum value of $|\vec{c}|^2$ is : [4 marks]

(A) 77

(B) 154

(C) 308

(D) 462

Correct Answer: (A) 77

Solution:

Concept:

- If $\vec{u} \times \vec{v} = \vec{0}$ with $\vec{u} \neq \vec{0}$, then $\vec{v}$ is a scalar multiple of $\vec{u}$. Equal cross products can therefore impose a strong linear restriction.
- The squared magnitude is $|\vec{v}|^2 = \vec{v} \cdot \vec{v}$. Bilinearity of both products allows the conditions to be checked directly after finding the vector.

Step 1: Use the first cross-product equality From $\vec{a} \times \vec{c} = \vec{a} \times \vec{b}$, subtract to obtain

$$\vec{a} \times (\vec{c} - \vec{b}) = \vec{0}.$$

Therefore $\vec{c} = \vec{b} + t\vec{a}$ for a real scalar t. This retains every vector satisfying the first equality.

Step 2: Use the second equality to determine the vector Substituting into $\vec{c} \times \vec{b} = \vec{a} \times \vec{b}$ gives

$$t(\vec{a} \times \vec{b}) = \vec{a} \times \vec{b}.$$

Here $\vec{a} \times \vec{b} = 14\hat{i} + 7\hat{j} - 7\hat{k} \neq \vec{0}$, so $t = 1$. Thus

$$\vec{c} = \vec{a} + \vec{b} = 5\hat{i} - 6\hat{j} + 4\hat{k}.$$

The cross-product conditions already specify a unique vector; the word maximum does not introduce another unconstrained family.

Step 3: Verify the remaining condition and calculate the magnitude Compute $|\vec{a}|^2 = 14$, $|\vec{b}|^2 = 35$, and $\vec{a} \cdot \vec{b} = 6 + 5 + 3 = 14$. With $\vec{c} = \vec{a} + \vec{b}$,

$$\begin{aligned}(\vec{a} + \vec{c}) \cdot (\vec{b} + \vec{c}) &= (2\vec{a} + \vec{b}) \cdot (\vec{a} + 2\vec{b}) \\ &= 2(14) + 5(14) + 2(35) = 168.\end{aligned}$$

Hence the unique candidate satisfies every condition. Its squared magnitude is

$$|\vec{c}|^2 = 5^2 + (-6)^2 + 4^2 = 25 + 36 + 16 = 77.$$

The other three values require different vectors, excluded by the cross-product conditions.

Step 4: Conclusion The printed cross-product equalities force the unique value 77, which is therefore the maximum. The supplied key indicates 308, but that value does not satisfy these literal equalities. Hence, the correct option is (A).

Quick Tip to Solve: Before treating a vector problem as an optimization, subtract equal cross products and check whether they already determine the vector uniquely.

17. Let the line x+y=1 meet the circle $x^2 + y^2 = 4$ at the points A and B. If the line perpendicular to AB and passing through the mid-point of the chord AB intersects the circle at C and D, then the area of the quadrilateral ABCD is equal to : [4 marks]

(A) $\sqrt{14}$

(B) $3\sqrt{7}$

(C) $2\sqrt{14}$

(D) $5\sqrt{7}$

Correct Answer: (C) $2\sqrt{14}$

Solution:

Concept:

- For a circle of radius r, a chord whose line is a perpendicular distance d from the centre has length $2\sqrt{r^2 - d^2}$. The perpendicular from the centre bisects the chord.
- The area enclosed by two intersecting perpendicular diagonals is half the product of their lengths. This follows by adding the areas of the four right triangles around their intersection.

Step 1: Find the length of the chord The circle has centre $O = (0, 0)$ and radius $r = 2$. The distance of O from the chord line $x + y - 1 = 0$ is

$$d = \frac{|-1|}{\sqrt{1^2 + 1^2}} = \frac{1}{\sqrt{2}}.$$

Let M be the midpoint of AB. Then $OM \perp AB$, and the right triangle with hypotenuse 2 gives

$$AM = \sqrt{4 - \frac{1}{2}} = \sqrt{\frac{7}{2}}, \qquad AB = 2AM = \sqrt{14}.$$

This uses half of the chord in the right triangle, not the full chord.

Step 2: Identify the second chord as a diameter The line through M perpendicular to AB is the perpendicular bisector of that chord. It passes through O, so its intersections C, D with the circle are the endpoints of a diameter. Thus $CD = 4$ and $AB \perp CD$. Since M lies inside the circle, it lies between C, D, so $MC + MD = 4$.

Step 3: Add the areas on the two sides of the chord The four points enclose a quadrilateral whose diagonals are AB and CD. Add the triangles with common base AB and third vertices C and D:

$$\begin{aligned} \text{Area} &= \frac{1}{2} AB \cdot MC + \frac{1}{2} AB \cdot MD \\ &= \frac{1}{2} AB(MC + MD) = \frac{1}{2}\sqrt{14}(4) = 2\sqrt{14}. \end{aligned}$$

This is the ordinary enclosed quadrilateral area. The alternative $\sqrt{14}$ is half this result; $3\sqrt{7}$ and $5\sqrt{7}$ also fail the chord-and-diameter area calculation.

Step 4: Conclusion The area is $2\sqrt{14}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: When a chord is paired with its perpendicular bisector, the second chord is a diameter. Use half the product of these perpendicular diagonals.

18. Define a relation R on the interval $\left[0, \frac{\pi}{2}\right)$ by x R y if and only if $\sec^2 x - \tan^2 y = 1$. Then R is : [4 marks]

(A) both reflexive and symmetric but not transitive

(B) both reflexive and transitive but not symmetric

(C) reflexive but neither symmetric not transitive

(D) an equivalence relation

Correct Answer: (D) an equivalence relation

Solution:

Concept:

- A relation is an equivalence relation when it is reflexive, symmetric and transitive on its stated set. Each property must be checked for all elements in the domain.
- The identity $\sec^2 t = 1 + \tan^2 t$ converts the given mixed trigonometric condition into equality of squared tangents. On $[0, \pi/2)$, tangent is nonnegative and strictly increasing.

Step 1: Simplify the defining condition Use the identity at x:

$$\sec^2 x - \tan^2 y = 1 \iff \tan^2 x = \tan^2 y.$$

Both tangents are defined because the excluded endpoint $\pi/2$ is the only pole at the edge of this interval. Nonnegativity gives $\tan x = \tan y$, and strict monotonicity gives $x = y$. Conversely, if $x = y$, the original equation holds by the same trigonometric identity. Therefore R is precisely the equality relation on this interval.

Step 2: Check reflexivity and symmetry For each x in the set, $\sec^2 x - \tan^2 x = 1$; hence xRx and the relation is reflexive. If xRy, the equivalent condition $x = y$ immediately implies $y = x$, and therefore yRx. Thus it is symmetric as well. In particular, the mixed appearance of secant and tangent in the original equation does not destroy symmetry after the identity is applied.

Step 3: Check transitivity and compare the choices If xRy and yRz, the equality characterization gives $x = y$ and $y = z$. Thus $x = z$, which implies xRz. The relation is transitive. All three required properties hold. Option (A) incorrectly denies transitivity, option (B) incorrectly denies symmetry, and option (C) incorrectly denies both. Only the option stating that the relation is an equivalence relation includes the complete list of established properties.

Step 4: Conclusion The relation is reflexive, symmetric and transitive, hence an equivalence relation. Hence, the correct option is (D).

Quick Tip to Solve: Use the trigonometric identity before checking relation properties. Here it reduces the condition to equality, whose three equivalence properties follow immediately.

19. Let M and m respectively be the maximum and the minimum values of

$$f(x)=\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 4x \\ \sin^2 x & 1+\cos^2 x & 4\sin 4x \\ \sin^2 x & \cos^2 x & 1+4\sin 4x \end{vmatrix}, x \in R$$

Then $M^4 - m^4$ is equal to : [4 marks]

(A) 1280

(B) 1040

(C) 1215

(D) 1295

Correct Answer: (A) 1280

Solution:

Concept:

- Adding a multiple of one row to another does not change a determinant. Subtracting equal variable parts reveals the simple identity-matrix structure here.
- For real angles, $-1 \le \sin 4x \le 1$. Both endpoint values occur, so the resulting bounds are actual extrema, not just estimates.

Step 1: Simplify the determinant Put $a = \sin^2 x$, $b = \cos^2 x$ and $c = 4\sin 4x$, so $a + b = 1$. Apply $R_1 \leftarrow R_1 - R_3$ and $R_2 \leftarrow R_2 - R_3$:

$$f(x)=\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ a & b & 1+c \end{vmatrix}.$$

Expand along the first row. The first minor contributes $1 + c + b$, and the third-entry contribution is $+a$. Therefore

$$f(x) = 1 + a + b + c = 2 + 4\sin 4x.$$

Step 2: Find the extrema and compare their powers The coefficient of $\sin 4x$ is positive. Hence the maximum is reached when $\sin 4x = 1$, while the minimum is reached when $\sin 4x = -1$:

$$M = 2 + 4 = 6, \qquad m = 2 - 4 = -2.$$

Evaluate the requested difference:

$$M^4 - m^4 = 6^4 - (-2)^4 = 1296 - 16 = 1280.$$

The other three options, 1040, 1215 and 1295, do not equal this difference. In particular, retaining the even fourth power of the negative minimum is essential.

Step 3: Conclusion The maximum and minimum are 6 and -2, respectively, giving $\boxed{M^4 - m^4 = 1280}$. Hence, the correct option is (A).

Quick Tip to Solve: When all rows share the same variable entries plus a diagonal 1, subtract a convenient row first; do not expand nine trigonometric products.

20. The integral $80\int_0^{\frac{\pi}{4}}\left(\frac{\sin\theta+\cos\theta}{9+16\sin 2\theta}\right)d\theta$ **is equal to :** [4 marks]

(A) $3\log 4$

(B) $4\log 3$

(C) $6\log\frac{4}{3}$

(D) $2\log 3$

Correct Answer: (B) $4\log 3$

Solution:

Concept:

- A useful substitution matches the derivative to the numerator. Here $\frac{d}{d\theta}(\sin\theta-\cos\theta)=\sin\theta+\cos\theta$.
- The identity $(\sin\theta-\cos\theta)^2=1-\sin 2\theta$ converts the denominator to a difference of squares. Natural logarithms arise from its partial fractions.

Step 1: Transform the integral and its limits Write the required value as I, and set $u=\sin\theta-\cos\theta$. Then $du=(\cos\theta+\sin\theta)\,d\theta$ and

$$9+16\sin 2\theta=9+16(1-u^2)=25-16u^2.$$

At $\theta=0$, $u=-1$; at $\theta=\pi/4$, $u=0$. Thus

$$I=80\int_{-1}^{0}\frac{du}{25-16u^2}.$$

The denominator is positive throughout this interval, so the integral is proper and no absolute-value sign changes occur.

Step 2: Integrate the rational expression The standard partial-fraction antiderivative gives

$$\int\frac{du}{25-16u^2}=\frac{1}{40}\log\left|\frac{5+4u}{5-4u}\right|.$$

This coefficient can be checked by differentiating the logarithm: its derivative is $40/(25-16u^2)$. Consequently

$$\begin{aligned}I&=2\left[\log\left(\frac{5+4u}{5-4u}\right)\right]_{-1}^{0}\\&=2\left(\log 1-\log\frac{1}{9}\right)\\&=2\log 9=4\log 3.\end{aligned}$$

Option D loses a factor of two. Options A and C involve different logarithmic arguments and do not match the evaluated endpoint ratio.

Step 3: Conclusion The value of the given integral is $\boxed{4\log 3}$. Hence, the correct option is (B).

Quick Tip to Solve: For a numerator $\sin\theta+\cos\theta$, try $u=\sin\theta-\cos\theta$; it also replaces $\sin 2\theta$ by $1-u^2$.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let [t] be the greatest integer less than or equal to t. Then the least value of p $\in$ N for which $\lim_{x\to 0^+}\left(x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{p}{x}\right]\right)-x^2\left(\left[\frac{1}{x^2}\right]+\left[\frac{2^2}{x^2}\right]+\ldots+\left[\frac{9^2}{x^2}\right]\right)\geq 1$ is equal to _____. [4 marks]

Solution:

Concept:

- The floor inequality $t-1<[t]\leq t$ controls the error made by replacing a greatest integer by its argument.
- For a fixed positive constant c, squeezing gives $x[c/x]\to c$ and $x^2[c/x^2]\to c$ as $x\to 0^+$. Finite sums preserve these limits.

Step 1: Evaluate both finite sums in the limit For each fixed integer k with $1\leq k\leq p$, multiply the floor inequality by $x>0$:

$$k-x<x\left[\frac{k}{x}\right]\leq k.$$

Therefore each term tends to k. Similarly, for $1\leq k\leq 9$,

$$k^2-x^2<x^2\left[\frac{k^2}{x^2}\right]\leq k^2,$$

so each term of the second sum tends to k^2. The number of terms is finite for any candidate p, hence the required limit is exactly

$$L(p)=\sum_{k=1}^{p}k-\sum_{k=1}^{9}k^2.$$

Step 2: Solve the integer threshold Use the standard sum formulae:

$$\sum_{k=1}^{p}k=\frac{p(p+1)}{2},\qquad \sum_{k=1}^{9}k^2=\frac{9\cdot 10\cdot 19}{6}=285.$$

The condition is therefore

$$\frac{p(p+1)}{2}-285\geq 1\quad\Longleftrightarrow\quad p(p+1)\geq 572.$$

Check the consecutive natural numbers around the threshold:

$$23\cdot 24=552<572,\qquad 24\cdot 25=600\geq 572.$$

Since $p(p+1)$ strictly increases for natural p, all smaller values fail and 24 is the first one that succeeds. The corresponding limits are $L(23)=-9$ and $L(24)=15$.

Step 3: Conclusion The least natural number satisfying the stated limit inequality is $\boxed{24}$.

Quick Tip to Solve: Replace each scaled floor by its argument only after bounding its error. Here the total errors are at most px and $9x^2$, which vanish for fixed p.

22. Let $S = \left\{m \in \mathbf{Z} : A^{m^2} + A^m = 3I - A^{-6}\right\}$, where $A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$. Then $n(S)$ is equal to _____.

[4 marks]

Solution:

Concept:

- If $A = I + N$ with $N^2 = 0$, the binomial formula truncates to $A^k = I + kN$. This identity also holds for negative integers because $(I + N)^{-1} = I - N$.
- A matrix equality becomes a scalar equation when both sides are linear combinations of I and the same nonzero matrix N. Count distinct integer solutions only.

Step 1: Identify the nilpotent part From the given matrix,

$$N = A - I = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}, \qquad N^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Thus $A = I + N$. For a nonnegative integer k, every binomial term beyond the linear term contains N^2 and vanishes. For negative powers, use $(I + N)(I - N) = I$, which gives the same formula. Therefore, for every integer k,

$$A^k = I + kN.$$

In particular, $A^{-6} = I - 6N$; the inverse in the question exists since $\det A = 1$.

Step 2: Reduce the condition to a quadratic Apply the power formula to the defining equation for S:

$$A^{m^2} + A^m = 2I + (m^2 + m)N,$$
$$3I - A^{-6} = 3I - (I - 6N) = 2I + 6N.$$

After subtracting $2I$, equality requires

$$(m^2 + m - 6)N = 0.$$

Because N has a nonzero entry, its scalar coefficient must vanish. Factor:

$$m^2 + m - 6 = (m - 2)(m + 3) = 0.$$

Both $m = 2$ and $m = -3$ are integers and each gives $m^2 + m = 6$, so both satisfy the original matrix equation. There are no additional roots or restrictions.

Step 3: Conclusion We obtain $S = \{-3, 2\}$, and therefore $\boxed{n(S) = 2}$.

Quick Tip to Solve: For a repeated eigenvalue 1, inspect $A - I$. If its square is zero, use $A^k = I + k(A - I)$ even for negative integer powers.

23. Let S = $\{x : \cos^{-1} x = \pi + \sin^{-1} x + \sin^{-1}[2x+1]\}$. Then $\sum_{x\in S}(2x-1)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- Principal inverse functions satisfy $\cos^{-1} x = \pi/2 - \sin^{-1} x$ for $-1 \le x \le 1$. Also $\sin^{-1} y \in [-\pi/2, \pi/2]$.
- Taking sine can introduce extraneous candidates because sine is not one-to-one on the real line. Every algebraic root must meet the original domains and principal-value range.

Step 1: Restrict the domain and transform the equation The two inverse sines require $-1 \le x \le 1$ and $-1 \le 2x+1 \le 1$, whose intersection is $[-1, 0]$. Put $t = \sin^{-1} x$; then $t \in [-\pi/2, 0]$. Using the complementary inverse-function identity, the equation becomes

$$\frac{\pi}{2} - t = \pi + t + \sin^{-1}(2x+1),$$

so

$$\sin^{-1}(2x+1) = -\frac{\pi}{2} - 2t.$$

The right-hand side lies in $[-\pi/2, \pi/2]$ throughout the permitted range of t, which will allow us to recover the inverse sine after solving.

Step 2: Find and verify the admissible root Take sine on both sides and use $\cos 2t = 1 - 2\sin^2 t$:

$$2x + 1 = -\cos 2t = 2x^2 - 1.$$

Hence

$$x^2 - x - 1 = 0, \qquad x = \frac{1 \pm \sqrt{5}}{2}.$$

Only $x_0 = (1 - \sqrt{5})/2$ belongs to $[-1, 0]$. For this root, the derived sine equality holds and $-\pi/2 - 2\sin^{-1} x_0$ is in the principal inverse-sine range. Taking the inverse sine therefore recovers the equation, confirming that this root is valid. Thus $S = \{x_0\}$, with no other members.

Step 3: Evaluate the required sum There is only one term. For the accepted root,

$$2x_0 - 1 = -\sqrt{5}, \qquad \sum_{x\in S}(2x-1)^2 = (-\sqrt{5})^2 = 5.$$

The rejected positive root cannot contribute because $\cos^{-1} x$ is not real at that value.

Step 4: Conclusion The requested sum is $\boxed{5}$.

Quick Tip to Solve: Combine the domains before taking sine. Checking the principal-value interval then proves sufficiency and avoids accepting extraneous quadratic roots.

24. Let $f:(0,\infty)\to\mathbf{R}$ be a twice differentiable function. If for some $a\neq 0$, $\int_0^1 f(\lambda x)\mathrm{d}\lambda = af(x)$, $f(1)=1$ and $f(16)=\frac{1}{8}$, then $16-f'\left(\frac{1}{16}\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- The substitution $t=\lambda x$, with $x>0$, converts the scaled integral into $x^{-1}\int_0^x f(t)\,dt$. The original integral implies the required improper integral exists at zero.
- Differentiating an integral with a variable upper endpoint gives its integrand. The resulting separable equation determines a power function whose exponent is fixed by the supplied values.

Step 1: Derive the differential equation Write $F(x)=\int_0^x f(t)\,dt$. The substitution $t=\lambda x$ in the given identity yields

$$\frac{1}{x}F(x)=af(x), \qquad F(x)=axf(x).$$

For $x>0$, differentiability of f justifies differentiation at the upper endpoint. Therefore

$$f(x)=a\{f(x)+xf\prime(x)\},$$

and since $a\neq 0$,

$$xf\prime(x)=\left(\frac{1}{a}-1\right)f(x).$$

Set $r=1/a-1$. Solving this linear equation on $(0,\infty)$ gives $f(x)=Cx^r$. The condition $f(1)=1$ fixes $C=1$.

Step 2: Determine the exponent and check the integral The second supplied value gives

$$16^r=\frac{1}{8}, \qquad 2^{4r}=2^{-3}, \qquad r=-\frac{3}{4}.$$

Consequently $f(x)=x^{-3/4}$ and $a=1/(r+1)=4$. This also confirms convergence at the lower endpoint: $\int_0^1 \lambda^{-3/4}\,d\lambda=4$. Hence the resulting function satisfies the original integral relation, rather than merely its differentiated form.

Step 3: Evaluate the derivative at the stated point Differentiate the obtained function and substitute $x=1/16$:

$$f\prime(x)=-\frac{3}{4}x^{-7/4},$$
$$f\prime\left(\frac{1}{16}\right)=-\frac{3}{4}16^{7/4}=-\frac{3}{4}\cdot 128=-96.$$

Finally, preserve the minus sign in the quantity requested:

$$16-f\prime\left(\frac{1}{16}\right)=16-(-96)=112.$$

Step 4: Conclusion The required value is $\boxed{112}$.

Quick Tip to Solve: In $\int_0^1 f(\lambda x)\,d\lambda$, substitute $t=\lambda x$ first, multiply by x, and only then differentiate.

25. The number of 6-letter words, with or without meaning, that can be formed using the letters of the word MATHS such that any letter that appears in the word must appear at least twice, is _____. [4 marks]

Solution:

Concept:

- A word is an ordered sequence, so different arrangements of the same chosen letters count separately. Repetition is permitted, but every letter used has frequency at least two.
- For a fixed multiset of six letters with frequencies $n_1, n_2, \ldots$, the number of arrangements is $6!/(n_1!n_2!\cdots)$. The five available letters are distinct.

Step 1: List the possible frequency patterns If r different letters occur, then $2r \le 6$, so only one, two or three distinct letters can occur. The complete list of frequency patterns is (6), $(4,2)$, $(3,3)$, and $(2,2,2)$. Patterns such as $(5,1)$ or $(3,2,1)$ are excluded because they contain a letter appearing only once.

Step 2: Count words using one or two letters For pattern (6), choose the repeated letter in 5 ways; each choice gives one word. For $(4,2)$, choose the letter appearing four times in 5 ways and the letter appearing twice in 4 ways, then choose the two positions of the latter. This gives

$$5 \cdot 4 \cdot \frac{6!}{4!2!} = 300.$$

For $(3,3)$, the two letters have equal frequencies, so choose them as an unordered pair. Their arrangements number

$$\binom{5}{2}\frac{6!}{3!3!} = 10 \cdot 20 = 200.$$

Step 3: Count words using three letters and add Three distinct letters must each occur exactly twice. Choose the three letters without ordering them, then arrange the repeated multiset:

$$\binom{5}{3}\frac{6!}{2!2!2!} = 10 \cdot 90 = 900.$$

The four patterns are disjoint and exhaust all legal words. Therefore the total is

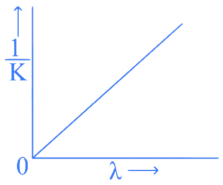
$$5 + 300 + 200 + 900 = 1405.$$

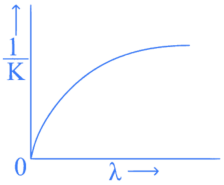
Step 4: Conclusion The required number of six-letter words is $\boxed{1405}$.

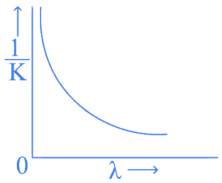
Quick Tip to Solve: Partition the length into allowed frequencies first. Choose equal-frequency letters as a set, but distinguish letters assigned unequal frequencies.

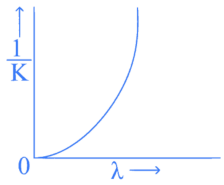
Physics – Section A: Multiple Choice Questions (80 marks)

26. If λ and K are de Broglie wavelength and kinetic energy, respectively, of a particle with constant mass. The correct graphical representation for the particle will be : [4 marks]

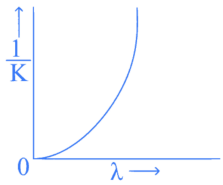
(A) 

(B) 

(C) 

(D) 

Correct Answer: (D)



Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$, where h is Planck's constant and p is the magnitude of momentum.
- For a particle of fixed mass m in the nonrelativistic kinetic-energy relation, $K = p^2/(2m)$. Eliminating momentum gives the dependence between the two plotted quantities.

Step 1: Eliminate the momentum Substitute $p = h/\lambda$ into the kinetic energy:

$$K = \frac{h^2}{2m\lambda^2}.$$

Invert this expression because the vertical axis of each proposed graph is $1/K$, whereas the horizontal axis is λ:

$$\frac{1}{K} = \frac{2m}{h^2}\lambda^2.$$

The factor $2m/h^2$ is positive and constant for the given particle; it changes the scale of the graph, not its shape.

Step 2: Compare the shapes and slopes Writing $y = 1/K$ and $x = \lambda$, the relation is $y = ax^2$ with $a > 0$. For positive wavelengths the graph rises and is convex upward. Its slope $dy/dx = 2ax$ increases with x, and $d^2y/dx^2 = 2a > 0$. Thus the increasing curve with increasing steepness is the required representation. Option (A) is a straight line and would incorrectly imply $1/K \propto \lambda$. Option (B) increases with decreasing slope, and option (C) decreases; both contradict the derived square dependence. The origin in the sketch represents the limiting form of the relation, rather than a finite-energy state with zero wavelength.

Step 3: Conclusion $1/K = (2m/h^2)\lambda^2$, so the upward-curving graph is correct. Hence, the correct option is (D).

Quick Tip to Solve: Check the actual axis labels before choosing a curve: kinetic energy varies inversely as wavelength squared, but its reciprocal varies directly as wavelength squared.

---

27. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Electromagnetic waves carry energy but not momentum.

Reason (R) : Mass of a photon is zero.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(B) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**

(C) **(A)** is false but **(R)** is true

(D) **(A)** is true but **(R)** is false

Correct Answer: (C) **(A)** is false but **(R)** is true

Solution:

Concept:

- A photon has energy $E = h\nu$ and momentum $p = h\nu/c = E/c$, where ν is its frequency and c is the speed of light in vacuum.
- A photon has zero rest mass. Zero rest mass does not mean zero energy or zero momentum; its energy and momentum obey $E^2 = p^2c^2 + m_0^2c^4$.

Step 1: Test the assertion The assertion contains two claims joined together: electromagnetic waves carry energy, but do not carry momentum. The first claim is true, whereas the second is false. Photons composing electromagnetic radiation possess momentum $p = E/c$. On absorption, radiation transfers this momentum to a surface and exerts radiation pressure. A beam of total energy U transfers momentum U/c on complete absorption. Consequently the complete assertion is false.

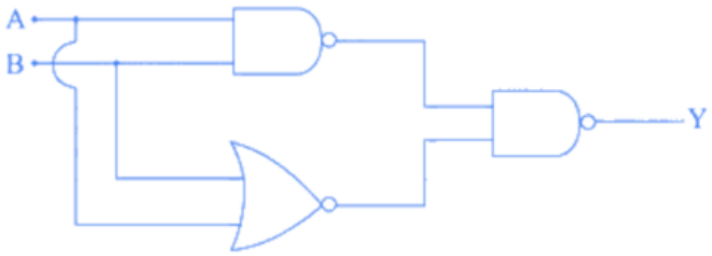
Step 2: Test the reason The photon has zero rest mass, $m_0 = 0$. In the standard meaning of photon mass in this question, the reason is therefore true. Substitution into the energy-momentum relation gives $E = pc$, rather than $p = 0$. The massive-particle expression $p = mv$ with a fixed Newtonian mass cannot be used to infer that a photon has no momentum.

Step 3: Select the truth combination The required combination is a false assertion with a true reason. Options (A) and (B) both require the assertion to be true, so neither can apply. Option (D) makes the reason false as well as making the assertion true, contradicting both independent checks. The presence of zero rest mass does not establish the false claim about momentum, so there is no valid explanatory link that could rescue the assertion.

Step 4: Conclusion The assertion is false, while the reason is true. Hence, the correct option is (C).

Quick Tip to Solve: For a photon use $E = pc$, not the Newtonian formula $p = mv$. Masslessness is compatible with nonzero momentum and radiation pressure.

28. For the circuit shown above, equivalent GATE is : [4 marks]



(A) NOT gate

(B) NAND gate

(C) AND gate

(D) OR gate

Correct Answer: (D) OR gate

Solution:

Concept:

- A NAND gate produces the complement of a logical product: $\overline{AB}$. A NOR gate produces the complement of a logical sum: $\overline{A+B}$.
- De Morgan's law gives $\overline{XY} = \overline{X} + \overline{Y}$. The absorption identity $X + XY = X$ simplifies redundant Boolean terms.

Step 1: Write the intermediate outputs Trace the two inputs carefully through the circuit. The upper gate is an AND-shaped gate with an output bubble, hence a NAND gate. Its output is $X = \overline{AB}$. The lower gate is OR-shaped with an output bubble, hence a NOR gate. It receives the same two inputs and produces $Z = \overline{A+B}$. The curved crossover on the left denotes crossing wires, so it must not be treated as a short joining the two inputs.

Step 2: Simplify the final NAND output The final gate is also NAND, with inputs X and Z. Therefore

$$Y = \overline{XZ} = \overline{\overline{AB}\,\overline{A+B}}.$$

Apply De Morgan's law to the outer complement:

$$Y = AB + (A + B) = A + B.$$

The product AB contributes no additional true case because whenever both inputs are true, their logical sum is already true.

Step 3: Check the input-output behaviour For inputs $(A, B) = (0, 0)$, the two intermediate outputs are both 1, giving final output 0. For $(0, 1)$ and $(1, 0)$, the upper output is 1 and lower output is 0, giving final output 1. For $(1, 1)$, both intermediate outputs are 0, again giving final output 1. Thus the output is high whenever either input is high. It is not AND, which is low for unequal inputs, nor NAND, which is low at $(1, 1)$; a NOT gate has only one independent input.

Step 4: Conclusion The equivalent relation is $Y = A + B$, which is an OR gate. Hence, the correct option is (D).

Quick Tip to Solve: Write one Boolean expression per gate and simplify the final output with De Morgan's law. Output bubbles represent complements.

29. The pair of physical quantities not having the same dimensions is : [4 marks]

(A) Angular momentum and Planck's constant

(B) Torque and energy

(C) Surface tension and impulse

(D) Pressure and Young's modulus

Correct Answer: (C) Surface tension and impulse

Solution:

Concept:

- Dimensions express a physical quantity using mass M, length L, and time T. Quantities can have the same dimensions even when their physical meanings differ.
- Force has dimensions $[MLT^{-2}]$, momentum has $[MLT^{-1}]$, and work or energy has $[ML^2T^{-2}]$. These relations generate the required comparisons.

Step 1: Compare angular momentum and Planck's constant Angular momentum is position crossed with linear momentum, so its dimensions are $L(MLT^{-1}) = ML^2T^{-1}$. From $E = h\nu$, with frequency dimensions T^{-1}, Planck's constant has dimensions $[h] = [E]/[\nu] = ML^2T^{-1}$. Thus the pair in option (A) has equal dimensions.

Step 2: Compare torque, energy, pressure and modulus Torque is force times perpendicular distance, giving $[\tau] = ML^2T^{-2}$, the same as energy. Their mathematical character differs, but their dimensional formula is equal, so option (B) is not the requested exception. Pressure is force divided by area:

$$[P] = ML^{-1}T^{-2}.$$

Young's modulus is longitudinal stress divided by strain. Strain is a ratio of lengths and is dimensionless,

while stress has pressure dimensions. Hence pressure and Young's modulus in option (D) also match.

Step 3: Compare surface tension and impulse Surface tension is force per unit length, so

$$[S] = \frac{MLT^{-2}}{L} = MT^{-2}.$$

Impulse is force integrated over time, equal to change in momentum:

$$[J] = MLT^{-2}\,T = MLT^{-1}.$$

The length and time powers differ between these two quantities. Surface tension may also be written as energy per area, which independently gives $ML^2T^{-2}/L^2 = MT^{-2}$, confirming the comparison.

Step 4: Conclusion Surface tension and impulse do not have the same dimensions. Hence, the correct option is (C).

Quick Tip to Solve: To compare dimensions, reduce both quantities to M, L, T; do not reject torque and energy simply because one is a vector and the other is a scalar.

30. The expression given below shows the variation of velocity (v) with time (t),
$v = \mathrm{At}^2 + \frac{\mathrm{Bt}}{\mathrm{C+t}}$.
The dimension of ABC is : [4 marks]

(A) $[\mathrm{M^0L^1T^{-2}}]$
(B) $[\mathrm{M^0L^2T^{-3}}]$
(C) $[\mathrm{M^0L^2T^{-2}}]$
(D) $[\mathrm{M^0L^1T^{-3}}]$

Correct Answer: (B) $[\mathrm{M^0L^2T^{-3}}]$

Solution:

Concept:

- Every additive term in a physical equation must have the same dimensions. Since v is velocity, each complete term has dimensions LT^{-1}.
- Quantities added in a denominator must also have equal dimensions. Constants appearing in a formula need not be dimensionless; their dimensions are determined by the equation.

Step 1: Determine the dimensions of C and A The denominator is $C + t$. Since t is time, dimensional homogeneity requires

$$[C] = T.$$

The first velocity term is At^2, so

$$[A]T^2 = LT^{-1} \quad \Rightarrow \quad [A] = LT^{-3}.$$

There is no mass factor in either result. The square on time is essential: treating At^2 as At would give an incorrect power of time for A.

Step 2: Determine the dimensions of B The second complete term $Bt/(C+t)$ must have the dimensions of velocity. Its numerator has dimensions $[B]T$, while its denominator has dimensions T. Therefore

$$\left[\frac{Bt}{C+t}\right] = \frac{[B]T}{T} = [B] = LT^{-1}.$$

The ratio $t/(C+t)$ is dimensionless, so B itself is a velocity scale. One must not equate the dimensions of the numerator alone to velocity.

Step 3: Multiply the three dimensional formulae Dimensions of a product multiply, so the powers of each fundamental dimension add:

$$[ABC] = (LT^{-3})(LT^{-1})(T) = M^0L^2T^{-3}.$$

Option (B) has exactly these powers. Option (A) contains only one power of length and has the wrong time exponent; option (C) gives T^{-2}, omitting one inverse-time factor; option (D) also contains only one power of length. The two length factors come independently from A and B, while C supplies one compensating power of time.

Step 4: Conclusion The dimensional formula is $\boxed{[M^0L^2T^{-3}]}$. Hence, the correct option is (B).

Quick Tip to Solve: Start with sums such as $C+t$ to identify dimensions immediately, then check each complete term before multiplying the constants.

31. The fractional compression $\left(\frac{\Delta V}{V}\right)$ of water at the depth of 2.5 km below the sea level is _____ %. Given, the Bulk modulus of water = 2×10^9 N m^{-2}, density of water = 10^3 kg m^{-3}, acceleration due to gravity $g = 10$ m s^{-2}. [4 marks]

(A) 1.0

(B) 1.25

(C) 1.75

(D) 1.5

Correct Answer: (B) 1.25

Solution:

Concept:

- The bulk modulus is the pressure increase divided by the fractional decrease in volume: $K = \Delta P/(|\Delta V|/V)$. Compression is reported as a positive magnitude.
- At depth h in water of density ρ, the additional pressure is $\Delta P = \rho gh$. The requested percentage equals the fractional compression multiplied by 100.

Step 1: Convert the depth and calculate the pressure The depth must be expressed in metres before using the SI values of density and acceleration. Thus $h = 2.5 \times 10^3$ m. The water column produces the pressure increase relative to sea level:

$$\Delta P = (10^3)(10)(2.5 \times 10^3) = 2.5 \times 10^7 \text{ Pa}.$$

Atmospheric pressure need not be added because the compression is relative to water at sea level; the change in pressure is the hydrostatic contribution.

Step 2: Divide by the bulk modulus and express a percentage Using the given modulus $K = 2\times10^9$ Pa,

$$\frac{|\Delta V|}{V} = \frac{2.5 \times 10^7}{2 \times 10^9} = 1.25 \times 10^{-2}.$$

This is a dimensionless fraction, not yet a percentage. Therefore the percentage compression is $100(1.25 \times 10^{-2}) = 1.25\%$. The volume change itself is negative, but the amount of compression is positive. The values 1.0, 1.75 and 1.5 do not equal the pressure-to-modulus ratio expressed as a percentage.

Step 3: Conclusion The fractional compression is 0.0125, corresponding to $\boxed{1.25\%}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert kilometres to metres first, then compute $100\rho gh/K$ directly to obtain the percentage compression.

---

32. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Time period of a simple pendulum is longer at the top of a mountain than that at the base of the mountain.

Reason (R) : Time period of a simple pendulum decreases with increasing value of acceleration due to gravity and vice-versa.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**.

(B) **(A)** is true but **(R)** is false.

(C) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**.

(D) **(A)** is false but **(R)** is true.

Correct Answer: (C) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**.

Solution:

Concept:

- For small oscillations, a simple pendulum of fixed length L has period $T = 2\pi\sqrt{L/g}$, where g is the local gravitational acceleration.
- Above the Earth, $g = GM/r^2$ decreases as distance r from its centre increases. Thus an increase in elevation increases the pendulum period when its length is unchanged.

Step 1: Evaluate the assertion Let the base and summit of the mountain have gravitational accelerations g_b and g_t. The summit is farther from the Earth's centre, so $g_t < g_b$. With the same pendulum length at both places,

$$\frac{T_t}{T_b} = \sqrt{\frac{g_b}{g_t}} > 1.$$

The period at the top is therefore longer than at the base. Assertion (A) is true; the comparison concerns the same pendulum and small-amplitude motion.

Step 2: Evaluate the reason independently Keeping L fixed in the pendulum formula gives $T \propto g^{-1/2}$. Increasing gravitational acceleration increases the restoring torque and makes each oscillation faster, reducing its period. Conversely, decreasing g makes the period longer. This is exactly the dependence stated in Reason (R), so the reason is true.

Step 3: Check the explanatory connection The decrease of gravitational acceleration on going to the summit, together with the inverse square-root relation in Reason (R), directly predicts the increase in period claimed in Assertion (A). Therefore the two statements are not merely independently true: the reason explains the assertion. Option (A) wrongly denies this connection, while options (B) and (D) wrongly declare one of the statements false.

Step 4: Conclusion The Assertion and Reason are both true, and the Reason correctly explains the Assertion. Hence, the correct option is (C).

Quick Tip to Solve: For the same pendulum, compare local gravity: smaller g means a larger T, because $T\sqrt{g}$ remains constant.

33. Two projectiles are fired with same initial speed from same point on ground at angles of $(45^\circ - \alpha)$ and $(45^\circ + \alpha)$, respectively, with the horizontal direction. The ratio of their maximum heights attained is : [4 marks]

(A) $\frac{1+\sin\alpha}{1-\sin\alpha}$
(B) $\frac{1+\sin 2\alpha}{1-\sin 2\alpha}$
(C) $\frac{1-\tan\alpha}{1+\tan\alpha}$
(D) $\frac{1-\sin 2\alpha}{1+\sin 2\alpha}$

Correct Answer: (D) $\frac{1-\sin 2\alpha}{1+\sin 2\alpha}$

Solution:

Concept:

- A projectile launched at speed u and elevation θ has initial vertical velocity $u\sin\theta$. At maximum height its vertical velocity becomes zero.
- With constant downward acceleration g, the maximum height above launch level is $H = u^2\sin^2\theta/(2g)$. Equal speeds make the ratio depend only on the launch angles.

Step 1: Write the ratio in the stated order Let H_1 correspond to $45^\circ - \alpha$ and H_2 to $45^\circ + \alpha$. Applying $v_y^2 = u_y^2 - 2gH$ with $v_y = 0$ gives

$$\frac{H_1}{H_2} = \frac{\sin^2(45^\circ - \alpha)}{\sin^2(45^\circ + \alpha)}.$$

Both the common speed squared and the factor $2g$ cancel. The ratio must retain the order in which the projectiles are described; reversing that order would give its reciprocal.

Step 2: Simplify using the double-angle identity Use $\sin^2 x = (1 - \cos 2x)/2$, together with $\cos(90^\circ -$

$2\alpha) = \sin 2\alpha$ and $\cos(90^\circ + 2\alpha) = -\sin 2\alpha$. Therefore

$$\sin^2(45^\circ - \alpha) = \frac{1 - \sin 2\alpha}{2}, \qquad \sin^2(45^\circ + \alpha) = \frac{1 + \sin 2\alpha}{2}.$$

Dividing these expressions produces the required ratio. For positive $\alpha < 45^\circ$, it is less than one, as the first projectile is launched at the smaller elevation. Option (B) is the reciprocal; option (A) has the wrong angle and order. Option (C) omits the square: the equivalent tangent expression would be $((1 - \tan\alpha)/(1 + \tan\alpha))^2$.

Step 3: Conclusion

$$\boxed{\frac{H_1}{H_2} = \frac{1 - \sin 2\alpha}{1 + \sin 2\alpha}}.$$

Hence, the correct option is (D).

Quick Tip to Solve: Maximum height depends on the square of the vertical velocity; complementary angles give equal ranges, but generally unequal heights.

---

34. An electric dipole of mass m, charge q, and length l is placed in a uniform electric field $\vec{E} = E_0\hat{i}$. When the dipole is rotated slightly from its equilibrium position and released, the time period of its oscillations will be : [4 marks]

(A) $\frac{1}{2\pi}\sqrt{\frac{ml}{2qE_0}}$

(B) $2\pi\sqrt{\frac{ml}{2qE_0}}$

(C) $2\pi\sqrt{\frac{ml}{qE_0}}$

(D) $\frac{1}{2\pi}\sqrt{\frac{2\ ml}{qE_0}}$

Correct Answer: The keyed option (B) assumes mass m for each endpoint; the stated total dipole mass m gives a different period absent from the options.

Solution:

Concept:

- A dipole with charge magnitude q and separation l has moment $p = ql$. In a uniform field the restoring torque near aligned stable equilibrium is $\tau = -pE_0\sin\theta \simeq -pE_0\theta$.
- Rotational dynamics gives $I\ddot{\theta} = -pE_0\theta$, so the angular frequency is $\omega_0 = \sqrt{pE_0/I}$ and the period is $T = 2\pi\sqrt{I/(pE_0)}$. The mass distribution is needed to determine I.

Step 1: Derive the period for the stated total mass The wording calls m the mass of the dipole, rather than the mass of each charge. In the usual model of two identical point particles connected by a massless rod of length l, each particle then has mass $m/2$. About the centre,

$$I = 2\left(\frac{m}{2}\right)\left(\frac{l}{2}\right)^2 = \frac{ml^2}{4}.$$

Substitution into the small-oscillation formula yields

$$T = 2\pi\sqrt{\frac{ml^2/4}{qlE_0}} = \pi\sqrt{\frac{ml}{qE_0}}.$$

None of the four printed choices is this expression.

Step 2: Identify the interpretation behind the listed answer If instead m denotes the mass of each endpoint particle, the total dipole mass is $2m$, and $I = 2m(l/2)^2 = ml^2/2$. This gives

$$T = 2\pi\sqrt{\frac{ml}{2qE_0}},$$

which is option (B), the keyed interpretation. Options (A) and (D) also use the incorrect period prefactor $1/(2\pi)$, while (C) requires a different inertia. A massless connector is also implicit in the endpoint model. Thus choosing (B) requires a mass convention the printed wording does not specify correctly; the distinction cannot be removed by algebra.

Step 3: Conclusion The keyed option is (B) if m is the mass of each charged particle. With m interpreted as the total dipole mass, the usual equal-particle model instead gives $\boxed{T = \pi\sqrt{ml/(qE_0)}}$, absent from the choices. The printed mass description is therefore ambiguous.

Quick Tip to Solve: For dipole oscillations, start with $T = 2\pi\sqrt{I/(pE)}$ and establish whether the stated mass belongs to each particle or to the whole dipole.

35. Consider a long straight wire of a circular cross-section (radius *a*) carrying a steady current *I*. The current is uniformly distributed across this cross-section. The distances from the centre of the wire's cross-section at which the magnetic field [inside the wire, outside the wire] is half of the maximum possible magnetic field, any where due to the wire, will be : [4 marks]

(A) [*a*/2, 3*a*]

(B) [*a*/4, 3*a*/2]

(C) [*a*/2, *2a*]

(D) [*a*/4, 2*a*]

Correct Answer: (C) [*a*/2, *2a*]

Solution:

Concept:

- Ampere's law for a circular path of radius r centred on a long wire gives $B(2\pi r) = \mu_0 I_{\text{enc}}$. Cylindrical symmetry makes B constant along the path.
- Uniform current density means the current enclosed inside the wire is proportional to enclosed cross-sectional area. Outside the wire, the full current I is enclosed.

Step 1: Find the field on either side of the surface For $r \leq a$, uniform current density gives

$I_{\text{enc}} = Ir^2/a^2$. Hence

$$B_{\text{in}}(r) = \frac{\mu_0 I r}{2\pi a^2}.$$

For $r \geq a$, the enclosed current is simply I, giving

$$B_{\text{out}}(r) = \frac{\mu_0 I}{2\pi r}.$$

The first expression rises linearly from zero at the centre; the second falls inversely with radius. Both agree at the surface, so the largest field anywhere is $B_{\max} = \mu_0 I/(2\pi a)$, at $r = a$.

Step 2: Locate the two half-maximum positions Inside, divide the field by its surface value to obtain $B_{\text{in}}/B_{\max} = r/a$. Setting this equal to $1/2$ gives $r_{\text{in}} = a/2$. Outside, $B_{\text{out}}/B_{\max} = a/r$, so half maximum requires $r_{\text{out}} = 2a$. Both positions lie in their assumed regions. The other proposed inside radius, $a/4$, gives one quarter of the maximum. The proposed outside radii $3a$ and $3a/2$ give one third and two thirds, respectively, rather than one half.

Step 3: Conclusion The distances, in the requested order [inside, outside], are $\boxed{[a/2,\ 2a]}$. Hence, the correct option is (C).

Quick Tip to Solve: For uniform current density, normalize to the surface field: $B/B_{\max} = r/a$ inside and a/r outside.

---

36. A coil of area A and N turns is rotating with angular velocity ω in a uniform magnetic field $\vec{B}$ about an axis perpendicular to $\vec{B}$. Magnetic flux φ and induced emf ε across it, at an instant when $\vec{B}$ is parallel to the plane of coil, are : [4 marks]

(A) $\phi = AB$, $\phi = NAB\omega$

(B) $\phi = AB$, $\phi = 0$

(C) $\phi = 0$, $\varepsilon = 0$

(D) $\phi = 0$, $\varepsilon = NAB\omega$

Correct Answer: (D) $\phi = 0$, $\varepsilon = NAB\omega$

Solution:

Concept:

- The magnetic flux through one turn is $\varphi = BA\cos\theta$, where θ is the angle between the magnetic field and the coil's area normal, not its plane.
- For N identical turns rotating uniformly with angular speed ω, Faraday's law gives $\varepsilon = -d(N\varphi)/dt$. The emf magnitude is $NAB\omega|\sin\theta|$.

Step 1: Determine the orientation of the area normal When the magnetic field is parallel to the coil's plane, it is perpendicular to the normal vector of that plane. Thus $\theta = 90^\circ$ (or the equivalent opposite orientation), and

$$\varphi = BA\cos 90^\circ = 0.$$

The total flux linkage $N\varphi$ also vanishes at this instant. In contrast, the flux magnitude is maximum

when the field is perpendicular to the plane, because the area normal is then parallel to the field.

Step 2: Differentiate the flux before evaluating the instant Write $\theta = \omega t + \theta_0$, where the phase θ_0 accounts for the initial orientation. Then

$$\varepsilon = -\frac{d}{dt}(NAB\cos\theta) = NAB\omega\sin\theta.$$

At the stated instant, $|\sin\theta| = 1$, so the emf magnitude is $NAB\omega$, its maximum value. Zero flux does not imply zero emf: emf depends on how quickly flux changes, which is greatest as the flux passes through zero. Options (A) and (B) incorrectly give nonzero flux in this orientation, and (C) incorrectly makes the emf zero. The emf polarity depends on the chosen terminal orientation, while the options compare its magnitude.

Step 3: Conclusion At that instant, $\boxed{\varphi = 0, \quad |\varepsilon| = NAB\omega}$. Hence, the correct option is (D).

Quick Tip to Solve: In a rotating coil, flux and emf are a quarter cycle apart: zero flux coincides with maximum emf magnitude, not zero emf.

37. Consider I_1 and I_2 are the currents flowing simultaneously in two nearby coils 1 & 2, respectively. If L_1 = self inductance of coil 1, M_{12} = mutual inductance of coil 1 with respect to coil 2, then the value of induced emf in coil 1 will be : [4 marks]

(A) $e_1 = -L_1\frac{dI_2}{dt} - M_{12}\frac{dI_1}{dt}$
(B) $e_1 = -L_1\frac{dI_1}{dt} + M_{12}\frac{dI_2}{dt}$
(C) $e_1 = -L_1\frac{dI_1}{dt} - M_{12}\frac{dI_1}{dt}$
(D) $e_1 = -L_1\frac{dI_1}{dt} - M_{12}\frac{dI_2}{dt}$

Correct Answer: (D) $e_1 = -L_1\frac{dI_1}{dt} - M_{12}\frac{dI_2}{dt}$

Solution:

Concept:

- Faraday's law relates induced emf to the rate of change of total flux linkage: $e_1 = -d\lambda_1/dt$. Flux linkage includes the number of turns.
- The self-inductance L_1 multiplies the current in coil 1, while mutual inductance M_{12} multiplies the current in coil 2. For the common positive flux convention, $\lambda_1 = L_1I_1 + M_{12}I_2$.

Step 1: Identify the two sources of flux Coil 1 produces a magnetic field when its own current I_1 flows. Its self-produced flux linkage is L_1I_1. The neighbouring coil produces an additional field through coil 1 when its current I_2 flows, giving linkage $M_{12}I_2$. These contributions must be added before applying Faraday's law. A current that is constant produces a constant contribution and hence no emf from that contribution.

Step 2: Differentiate the total flux linkage For fixed coils with constant inductance coefficients, differentiate each current separately:

$$e_1 = -\frac{d}{dt}(L_1I_1 + M_{12}I_2) = -L_1\frac{dI_1}{dt} - M_{12}\frac{dI_2}{dt}.$$

The overall minus sign is Lenz's law. It indicates opposition to the change in linked flux, rather than opposition to the mere presence of a current.

Step 3: Compare the offered expressions Option (A) incorrectly pairs self-inductance with the other coil's current. Option (C) incorrectly uses dI_1/dt in the mutual term. Option (B) reverses the mutual contribution for the positive linkage convention used in the question. The expression containing the correct current derivative in each negative term is (D).

Step 4: Conclusion

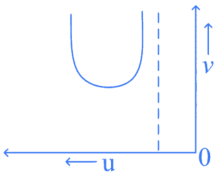
$$e_1 = -L_1\frac{dI_1}{dt} - M_{12}\frac{dI_2}{dt}.$$

Hence, the correct option is (D).

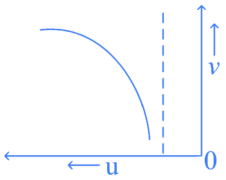
Quick Tip to Solve: Write total flux linkage first: self means the same coil's current, and mutual means the other coil's current. Then differentiate once with the Faraday minus sign.

38. Let u and v be the distances of the object and the image from a lens of focal length f. The correct graphical representation of u and v for a convex lens when $|u| > f$, is [4 marks]

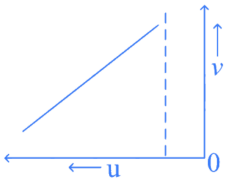
(A)



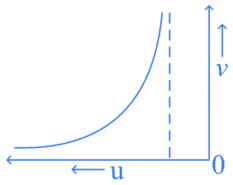
(B)



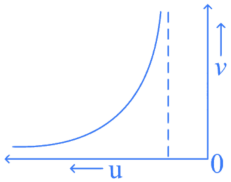
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- For a convex lens under the Cartesian sign convention, $1/v - 1/u = 1/f$, with $f > 0$. A real object beyond the focus has $u < -f$ and gives a real image with $v > 0$.
- The lens equation gives $v = fu/(u + f)$. Its vertical asymptote is $u = -f$, and its limiting image distance for a very distant object is $v = f$.

Step 1: Express image distance in terms of object distance Use the signed object distance rather than treating the leftward axis as an ordinary positive horizontal axis. Rearranging the lens formula gives

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{u+f}{fu}, \qquad v = \frac{fu}{u+f}.$$

For $u < -f$, numerator and denominator are both negative, so the image distance is positive, as required for a real image on the other side of the lens.

Step 2: Check the limiting behaviour When the object approaches the focus from beyond it, $u \to -f^-$, the denominator approaches zero from below and $v \to +\infty$. When the object moves very far to the left, $u \to -\infty$, the image approaches the focal plane, $v \to f^+$. At $u = -2f$, the equation gives $v = 2f$, which supplies a useful intermediate check.

Step 3: Identify the matching curve The required branch therefore rises sharply near the dashed focal-position line on its left and levels out at a finite positive height far to the left. This is the graph in (D). Option (A) has an unwanted turning point; (B) approaches a small image distance near the focus instead of diverging; (C) is a straight line and cannot represent the reciprocal lens relation.

Step 4: Conclusion The physically allowed branch has $u < -f$, $v > f$, with $v \to +\infty$ near $u = -f$. Hence, the correct option is (D).

Quick Tip to Solve: For a lens graph, test the object at infinity and just beyond the focus. The image must go from the focal plane to infinity, with the correct signs.

---

39. The work done in an adiabatic change in an ideal gas depends upon only : [4 marks]

(A) change in its pressure

(B) change in its temperature

(C) change in its specific heat

(D) change in its volume

Correct Answer: (B) change in its temperature

Solution:

Concept:

- An adiabatic process transfers no heat: $Q = 0$. With work W defined as work done by the gas, the first law is $\Delta U = Q - W$.
- An ideal gas has internal energy determined only by temperature. For a fixed quantity of gas with constant molar heat capacity, $\Delta U = nC_V(T_f - T_i)$.

Step 1: Apply the adiabatic condition Heat exchange and mechanical work are separate ways of transferring energy. Adiabatic means that heat transfer is absent, even though the gas may expand or be compressed. Setting $Q = 0$ in the first law gives

$$\Delta U = -W, \qquad W = U_i - U_f.$$

Thus the energy delivered as work is supplied by the gas's internal energy; an expansion cools the gas, and a compression heats it.

Step 2: Relate internal energy to temperature For a fixed amount of an ideal gas, write

$$W = nC_V(T_i - T_f).$$

Here n is the number of moles and C_V is the molar heat capacity at constant volume. For a fixed gas these are given properties, so the state change determining the work is its temperature change. If heat capacity varies with temperature, the more general expression is $W = -n\int_{T_i}^{T_f} C_V(T)\,dT$, which still depends on the temperature endpoints and the gas properties.

Step 3: Exclude the other state changes Pressure or volume alone does not determine the internal-energy change of an ideal gas: the state equation involves both. The specific heat is a property used in the coefficient and need not change during the process. Consequently a change in pressure, volume, or specific heat is not the requested single determining change. Reversing the work sign convention changes the sign of work, but does not change this dependence.

Step 4: Conclusion Adiabatic work is determined by the change in temperature of the fixed amount of ideal gas. Hence, the correct option is (B).

Quick Tip to Solve: Use the first law before integrating pressure: for an adiabatic ideal gas, no heat enters, so work comes directly from the temperature-dependent internal energy.

40. A body of mass 'm' connected to a massless and unstretchable string goes in vertical circle of radius 'R' under gravity g. The other end of the string is fixed at the center of circle. If velocity at top of circular path is $n\sqrt{gR}$, where, n $\geq$ 1, then ratio of kinetic energy of the body at bottom to that at top of the circle is : [4 marks]

(A) $\frac{n^2+4}{n^2}$

(B) $\frac{n+4}{n}$

(C) $\frac{n^2}{n^2+4}$

(D) $\frac{n}{n+4}$

Correct Answer: (A) $\frac{n^2+4}{n^2}$

Solution:

Concept:

- Kinetic energy is $K = \frac{1}{2}mv^2$, so a speed proportional to n gives kinetic energy proportional to n^2.
- String tension is radial and is perpendicular to the instantaneous displacement along the circle. It

does no work, so gravity alone transfers energy between the top and bottom.

Step 1: Find the kinetic energy at the top The specified speed at the highest point is $v_t = n\sqrt{gR}$. Squaring before substituting into kinetic energy gives

$$v_t^2 = n^2 gR, \qquad K_t = \frac{1}{2}mn^2 gR.$$

The condition $n \geq 1$ ensures that the string can remain taut at the top: $T_t + mg = mv_t^2/R$ gives $T_t = mg(n^2 - 1) \geq 0$. It does not change the energy calculation.

Step 2: Use the height difference The bottom is lower than the top by the diameter $2R$, rather than by the radius. The decrease in gravitational potential energy is therefore $2mgR$. Conservation of mechanical energy gives

$$K_b = K_t + 2mgR = \frac{1}{2}mgR(n^2 + 4).$$

Equivalently, $v_b^2 = v_t^2 + 4gR$. The string redirects the motion but supplies none of this increase in speed.

Step 3: Form the ratio in the stated order Divide the bottom kinetic energy by the top kinetic energy:

$$\frac{K_b}{K_t} = \frac{\frac{1}{2}mgR(n^2 + 4)}{\frac{1}{2}mgRn^2} = \frac{n^2 + 4}{n^2}.$$

Mass, gravity and radius cancel. The ratio must exceed one because the body accelerates while descending. This immediately excludes the reversed ratio (C); (B) and (D) incorrectly use n instead of n^2, forgetting that kinetic energy depends on speed squared.

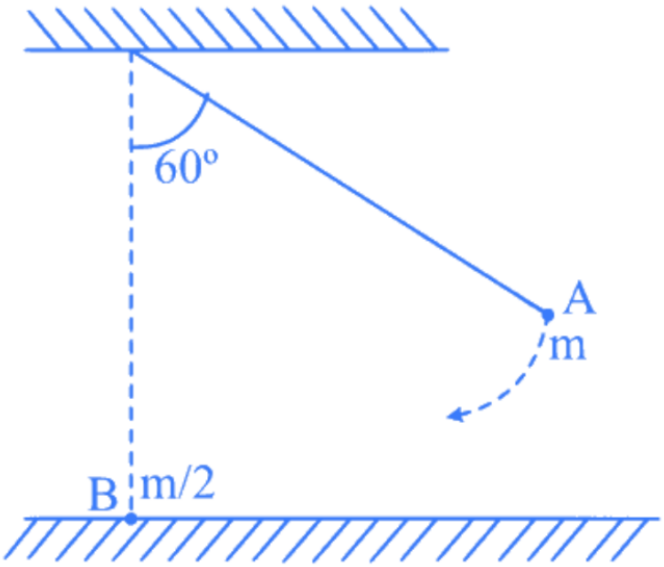
Step 4: Conclusion

$$K_b : K_t = (n^2 + 4) : n^2.$$

Hence, the correct option is (A).

Quick Tip to Solve: Between the top and bottom of a vertical circle use a height change of $2R$. Square the given speed before comparing kinetic energies.

41. As shown below, bob A of a pendulum having massless string of length 'R' is released from 60° to the vertical. It hits another bob B of half the mass that is at rest on a frictionless table in the center. Assuming elastic collision, the magnitude of the velocity of bob A after the collision will be (take g as acceleration due to gravity.) [4 marks]



(A) $\frac{4}{3}\sqrt{Rg}$
(B) $\frac{1}{3}\sqrt{Rg}$
(C) $\sqrt{Rg}$
(D) $\frac{2}{3}\sqrt{Rg}$

Correct Answer: (B) $\frac{1}{3}\sqrt{Rg}$

Solution:

Concept:

- Mechanical energy is conserved during the frictionless pendulum swing. A release angle θ gives a vertical fall $h = R(1 - \cos\theta)$.
- For a one-dimensional elastic collision, both momentum and kinetic energy are conserved. If the target is initially at rest, the incident mass has final velocity $v_1 = (m_1 - m_2)u_1/(m_1 + m_2)$.

Step 1: Find the speed just before impact Let bob A have mass m. It starts from rest at 60° to the downward vertical and reaches the lowest point before meeting B. Its fall is

$$h = R(1 - \cos 60^\circ) = \frac{R}{2}.$$

The string tension is perpendicular to the motion, so it does no work during the swing. Consequently

$$\frac{1}{2}mu^2 = mgh = \frac{1}{2}mgR, \qquad u = \sqrt{gR}.$$

This is the speed before the collision, rather than the requested final speed.

Step 2: Apply the elastic-collision equations At the lowest point the incident velocity is horizontal,

along the line of impact. The target mass is $m/2$, initially stationary. With final velocities v_A, v_B, momentum conservation gives $mu = mv_A + (m/2)v_B$. Elasticity gives the relative-speed relation $v_B - v_A = u$. Combining these equations yields

$$v_A = \frac{m - m/2}{m + m/2}u = \frac{1}{3}u.$$

The positive sign means A continues in its original horizontal direction immediately after impact.

Step 3: Check the requested mass and speed Substitute the incident speed to obtain $v_A = \sqrt{gR}/3$. Bob B instead has speed $v_B = 4\sqrt{gR}/3$, explaining the tempting option (A). Option (C) is A's speed before impact, and (D) does not satisfy the two collision conservation laws for these masses. The table is frictionless, so there is no horizontal external impulse to invalidate the momentum calculation.

Step 4: Conclusion The magnitude of bob A's velocity immediately after collision is $\frac{1}{3}\sqrt{Rg}$. Hence, the correct option is (B).

Quick Tip to Solve: Separate the two stages: use the pendulum's height loss to find its impact speed, then use the unequal-mass elastic-collision formula for the named bob.

---

42. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Emission of electrons in photoelectric effect can be suppressed by applying a sufficiently negative electron potential to the photoemissive substance.

Reason (R) : A negative electric potential, which stops the emission of electrons from the surface of a photoemissive substance, varies linearly with frequency of incident radiation.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is false but (R) is true

(D) (A) is true but (R) is false

Correct Answer: (B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where ϕ is the work function and ν is the incident frequency above threshold.
- A retarding potential stops the detected photoelectric current when $eV_s = K_{\max}$, where V_s is the positive magnitude of the stopping voltage and e is the elementary charge.

Step 1: Evaluate the assertion using stopping bias The assertion refers to suppression by a sufficiently negative retarding potential in a photoelectric arrangement. Electrons leaving the emitting surface have kinetic energies up to $K_{\max}$. An opposing electric field can prevent all of them from reaching

the collecting electrode when the opposing electric energy is at least that maximum. Thus the photoelectric current can be suppressed, which is the stopping-potential meaning intended here. Strictly, this suppresses collection of the emitted electrons rather than turning off the photon-induced surface process.

Step 2: Evaluate the reason independently At the point where even the most energetic electrons fail to reach the collector,

$$eV_s = h\nu - \phi, \qquad V_s = \frac{h}{e}\nu - \frac{\phi}{e}.$$

For a given photosensitive material, ϕ is fixed. The stopping-potential magnitude is consequently linear in frequency, with slope h/e. The corresponding signed negative retarding voltage also has a linear dependence, with the opposite slope. Hence the stated frequency dependence in the reason is true.

Step 3: Check whether the reason explains the assertion The frequency dependence tells us how much stopping voltage is needed when the illumination frequency changes. It does not explain why a retarding field stops the current at a fixed frequency; that follows from electric work opposing the electron kinetic energy. Therefore both statements, in their stopping-current usage, are true but the reason is not the explanation of the assertion. Option (A) confuses a dependence with a cause; (C) and (D) reject one of the two true statements.

Step 4: Conclusion The assertion and reason are both true in the stated stopping-potential sense, but the reason does not explain the assertion. Hence, the correct option is (B).

Quick Tip to Solve: In assertion-reason questions, test the causal link separately. A formula for how stopping voltage varies with frequency need not explain the stopping action itself.

43. At the interface between two materials having refractive indices n_1 and n_2, the critical angle for reflection of an em wave is θ_{1C}. The n_2 material is replaced by another material having refractive index n_3 such that the critical angle at the interface between n_1 and n_3 materials is θ_{2C}. If $n_3 > n_2 > n_1$; $\frac{n_2}{n_3} = \frac{2}{5}$ and $\sin\theta_{2C} - \sin\theta_{1C} = \frac{1}{2}$, then θ_{1C} is : [4 marks]

(A) $\sin^{-1}\left(\frac{1}{6n_1}\right)$

(B) $\sin^{-1}\left(\frac{1}{3n_1}\right)$

(C) $\sin^{-1}\left(\frac{5}{6n_1}\right)$

(D) $\sin^{-1}\left(\frac{2}{3n_1}\right)$

Correct Answer: No physically admissible critical angle satisfies the printed positive sine difference; none of the options is correct.

Solution:

Concept:

- At the critical angle, a ray travels from the optically denser medium to the rarer medium and the refracted angle is 90°. Snell's law therefore gives $\sin\theta_c = n_{\text{rarer}}/n_{\text{denser}}$.
- Refractive indices of ordinary transparent materials are positive. For a fixed rarer medium,

increasing the denser medium's index decreases the critical angle.

Step 1: Identify the two denser media Since $n_3 > n_2 > n_1$, the critical angle at the first interface must be measured in medium 2 and that at the second interface in medium 3. Thus

$$\sin\theta_{1C} = \frac{n_1}{n_2}, \qquad \sin\theta_{2C} = \frac{n_1}{n_3}.$$

Reversing the direction of incidence does not provide another positive critical angle: incidence from the rarer medium cannot produce total internal reflection.

Step 2: Test the printed difference Using $n_2/n_3 = 2/5$, we obtain

$$\sin\theta_{2C} = \frac{2}{5}\sin\theta_{1C}.$$

Consequently

$$\sin\theta_{2C} - \sin\theta_{1C} = -\frac{3}{5}\sin\theta_{1C} < 0.$$

The question instead sets this difference equal to $+1/2$. Algebra would require $\sin\theta_{1C} = -5/6$, which is impossible for a physical critical angle between 0 and 90°.

Step 3: Check the offered answers Every offered inverse sine has a positive argument for positive n_1; none can satisfy the negative-sine condition forced by the printed equations. Even changing only the sign of the difference would give $\sin\theta_{1C} = 5/6$, independent of n_1, rather than the general expression $5/(6n_1)$ in option (C).

Step 4: Conclusion As printed, the data are inconsistent and no listed option is physically correct. The supplied answer key lists (C), but it does not follow from the stated refractive-index ordering and sine difference.

Quick Tip to Solve: Before solving critical-angle equations, check the sign: replacing the denser material by one of higher refractive index must reduce the critical angle.

44. Match List - I with List - II.

List - I	List - II
(A) Electric field inside (distance r $>$ 0 from center) of a uniformly charged spherical shell with surface charge density σ, and radius R.	**(I) σ/ε_0**
(B) Electric field at distance r$>$0 from a uniformly charged infinite plane sheet with surface charge density σ.	**(II) $\sigma/2\varepsilon_0$**
(C) Electric field outside (distance r$>$0 from center) of a uniformly charged spherical shell with surface charge density σ, and radius R.	**(III) 0**
(D) Electric field between 2 oppositely charged infinite plane parallel sheets with uniform surface charge density σ.	**(IV) $\frac{\sigma}{\epsilon_0 r^2}$**

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

(B) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(C) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(D) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

Correct Answer: Intended option (C); no option is fully correct as printed because List II (IV) omits the shell-radius factor R squared.

Solution:

Concept:

- Gauss's law states $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enclosed}}/\varepsilon_0$. Spherical and planar symmetry make the field constant on a suitable Gaussian surface.
- Fields of several sheets add by vector superposition. A spherical shell of radius R and surface charge density σ carries total charge $Q = 4\pi R^2\sigma$.

Step 1: Find the shell fields Inside a uniformly charged spherical shell, a concentric Gaussian sphere encloses no charge, so $E = 0$. Hence (A) matches (III). Outside, for $r > R$,

$$E(4\pi r^2) = \frac{4\pi R^2\sigma}{\varepsilon_0}, \qquad E = \frac{\sigma R^2}{\varepsilon_0 r^2}.$$

The printed expression (IV) is $\sigma/(\varepsilon_0 r^2)$; it is missing the factor R^2 and is not the outside field for the stated surface charge density.

Step 2: Find the plane-sheet fields A pillbox through a single infinite sheet gives $2EA = \sigma A/\varepsilon_0$, so $E = \sigma/(2\varepsilon_0)$. Thus (B) matches (II). Between equal and opposite parallel infinite sheets the two fields point from the positive sheet to the negative sheet, and add:

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}.$$

Thus (D) matches (I).

Step 3: Compare the complete codes The three unambiguous matches are (A)–(III), (B)–(II), and (D)–(I). They identify option (C) as the intended matching code. Options (A), (B), and (D) already fail one or more of these matches. However, no code is entirely correct literally, because List II contains

no correct expression for (C). The omitted R^2 also makes (IV) dimensionally inconsistent with an electric field.

Step 4: Conclusion The intended code is (C): (A)–(III), (B)–(II), (C)–(IV), (D)–(I), as in the supplied key. Literally, none is fully correct because (IV) should contain $\sigma R^2/(\varepsilon_0 r^2)$.

Quick Tip to Solve: For a charged shell, first calculate total charge as surface density times $4\pi R^2$; this prevents loss of the radius factor in the outside field.

---

45. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Choke coil is simply a coil having a large inductance but a small resistance. Choke coils are used with fluorescent mercury-tube fittings. If household electric power is directly connected to a mercury tube, the tube will be damaged.

Reason (R): By using the choke coil, the voltage across the tube is reduced by a factor $\left(R/\sqrt{R^2+\omega^2L^2}\right)$, where ω is frequency of the supply across resistor R and inductor L. If the choke coil were not used, the voltage across the resistor would be the same as the applied voltage.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) (A) is true but (R) is false

(B) Both (**A**) and (**R**) are true and (**R**) is the correct explanation of (**A**) 3.

(C) Both (**A**) and (**R**) are true but (**R**) is not the correct explanation of (**A**)

(D) (A) is false but (R) is true

Correct Answer: (B) Both (**A**) and (**R**) are true and (**R**) is the correct explanation of (**A**) 3.

Solution:

Concept:

- A choke coil has large inductance L and small winding resistance, so it limits alternating current through inductive reactance $X_L = \omega L$ with little resistive power loss.
- In the series resistor-inductor model used here, impedance is $Z = \sqrt{R^2+\omega^2L^2}$. The same current passes through the tube, represented by resistance R, and the choke.

Step 1: Evaluate the assertion A fluorescent mercury tube needs a ballast to control its operating current. A choke provides this control with a large inductive reactance while its small resistance avoids the power dissipation of a large series resistor. Connecting the tube directly across the household supply removes this current-limiting action and can damage the tube. The assertion is therefore true.

Step 2: Evaluate the reason Let V be the applied rms voltage, with ω understood as its angular frequency. For the stated series model,

$$I = \frac{V}{\sqrt{R^2+\omega^2L^2}}, \qquad V_R = IR.$$

Hence

$$\frac{V_R}{V} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} < 1.$$

Without the inductor, $L = 0$, giving $V_R = V$. Thus the reason correctly describes the voltage reduction in this model. The printed word frequency for ω uses the conventional angular-frequency notation in the formula.

Step 3: Connect the reason to the assertion The reduced voltage across the resistive load accompanies the reduced series current; increasing L increases the impedance and protects the tube. This is precisely why the choke is used in the fitting. Therefore the reason explains the assertion. Option (A) wrongly rejects the reason, option (C) wrongly denies the explanatory connection, and option (D) wrongly rejects the assertion.

Step 4: Conclusion The assertion and reason are both true, and the reason correctly explains the assertion. Hence, the correct option is (B).

Quick Tip to Solve: For a series RL circuit use a phasor sum: $V^2 = V_R^2 + V_L^2$, rather than adding the two voltage magnitudes directly.

Physics – Section B: Numerical Value Questions (20 marks)

46. A container of fixed volume contains a gas at 27°C. To double the pressure of the gas, the temperature of gas should be raised to _____ °C. [4 marks]

Solution:

Concept:

- For a fixed amount of ideal gas, $PV = nRT$, where pressure P is absolute, volume V is fixed and temperature T must be measured in kelvin.
- At constant n and V, $P/T = nR/V$ is constant. The Celsius and kelvin scales have the same interval size but different zero points: $T = t + 273$ at the precision used here.

Step 1: Convert the initial temperature The initial gas temperature is $27^\circ\mathrm{C}$, so its absolute temperature is

$$T_1 = 27 + 273 = 300\,\mathrm{K}.$$

The container is fixed in volume and there is no addition or removal of gas. Thus doubling pressure is governed by the ratio of absolute temperatures, not by the ratio of Celsius readings.

Step 2: Apply the pressure-temperature ratio With $P_2 = 2P_1$, the ideal-gas equation gives

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} = 2.$$

Therefore

$$T_2 = 2(300) = 600\,\mathrm{K}.$$

This also has a kinetic interpretation: mean translational kinetic energy is proportional to absolute temperature, and at fixed number density the pressure increases in the same ratio.

Step 3: Express the final temperature in Celsius The requested blank is the temperature to which the gas is raised. Converting the final absolute temperature,

$$t_2 = T_2 - 273 = 600 - 273 = 327^\circ\text{C}.$$

The required rise itself is $327 - 27 = 300^\circ\text{C}$, but that is not the final Celsius temperature asked for. Doubling 27 to get 54°C would incorrectly use the arbitrary Celsius zero in a gas-law ratio.

Step 4: Conclusion The gas should be raised to $\boxed{327^\circ\text{C}}$. The required numerical entry is 327.

Quick Tip to Solve: Use kelvin in pressure-temperature ratios, then convert back only at the end. Distinguish the final temperature from the temperature rise.

---

47. The maximum speed of a boat in still water is 27 km/h. Now this boat is moving downstream in a river flowing at 9 km/h. A man in the boat throws a ball vertically upwards with speed of 10 m/s. Range of the ball as observed by an observer at rest on the bank is _____ cm. (Take $g = 10$ m/s^2) [4 marks]

Solution:

Concept:

- Velocity relative to the bank is the vector sum of velocity relative to the boat and the boat's velocity relative to the bank. Downstream boat and river velocities add.
- For a projectile returning to its launch height, $T = 2u_y/g$. With no horizontal acceleration, its horizontal displacement is $x = u_x T$.

Step 1: Find the downstream speed Taking the boat to move at its stated maximum speed relative to still water, its speed relative to the bank is

$$v_b = 27 + 9 = 36\,\text{km}\,\text{h}^{-1}.$$

Converting units,

$$v_b = 36\frac{1000}{3600} = 10\,\text{m}\,\text{s}^{-1}.$$

The water carries the boat downstream, so subtracting the current speed would describe upstream motion, not the situation given.

Step 2: Resolve the ball velocity in the bank frame The man throws the ball vertically relative to the boat with speed $10\,\text{m}\,\text{s}^{-1}$. It already shares the boat's horizontal velocity at release. Thus an observer on the stationary bank measures

$$u_x = 10\,\text{m}\,\text{s}^{-1}, \qquad u_y = 10\,\text{m}\,\text{s}^{-1}.$$

Horizontal velocity is unchanged throughout the flight when air resistance is neglected. Gravity acts only in the vertical direction.

Step 3: Calculate the flight time and range The ball returns to its launch height when

$$0 = u_y T - \frac{1}{2} g T^2.$$

Taking the nonzero root with $g = 10\,\mathrm{m\,s^{-2}}$,

$$T = \frac{2(10)}{10} = 2\,\mathrm{s}.$$

Its bank-frame range is therefore

$$x = u_x T = 10(2) = 20\,\mathrm{m} = 2000\,\mathrm{cm}.$$

The boat observer sees vertical motion, but the bank observer sees a parabola and a nonzero horizontal range.

Step 4: Conclusion The range observed from the bank is $\boxed{2000\,\mathrm{cm}}$. The required numerical entry is 2000.

Quick Tip to Solve: A vertical throw from a moving boat retains the boat's horizontal ground speed; compute the flight time from vertical motion before finding the range.

48. The coordinates of a particle with respect to origin in a given reference frame is (1, 1, 1) meters. If a force of $\vec{F} = \hat{i} - \hat{j} + \hat{k}$ acts on the particle, then the magnitude of torque (with respect to origin) in z-direction is _____. [4 marks]

Solution:

Concept:

- Torque about an origin is $\vec{\tau} = \vec{r} \times \vec{F}$, where $\vec{r}$ points from that origin to the point of application of the force.
- The component about the z-axis is $\tau_z = xF_y - yF_x$. Its magnitude is $|\tau_z|$; this differs from the magnitude of the complete torque vector.

Step 1: Identify the position and force components The position vector is

$$\vec{r} = (\hat{i} + \hat{j} + \hat{k})\,\mathrm{m},$$

so $x = y = z = 1\,\mathrm{m}$. The force is

$$\vec{F} = (\hat{i} - \hat{j} + \hat{k})\,\mathrm{N},$$

giving $F_x = 1\,\mathrm{N}$, $F_y = -1\,\mathrm{N}$, and $F_z = 1\,\mathrm{N}$. Keeping the negative sign of the y-component is essential; the force and position vectors are not parallel.

Step 2: Evaluate the z-component of torque Only the x- and y-components enter the torque component about z:

$$\tau_z = xF_y - yF_x.$$

Substitution gives

$$\tau_z = (1)(-1) - (1)(1) = -2\,\mathrm{N\,m}.$$

The sign follows the right-hand rule and indicates a torque component directed along the negative z-axis. No factor involving zF_z is included in this component of a cross product.

Step 3: Distinguish component magnitude from total magnitude As a check, the full cross product is

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 2\hat{i} - 2\hat{k} \quad \mathrm{N\,m}.$$

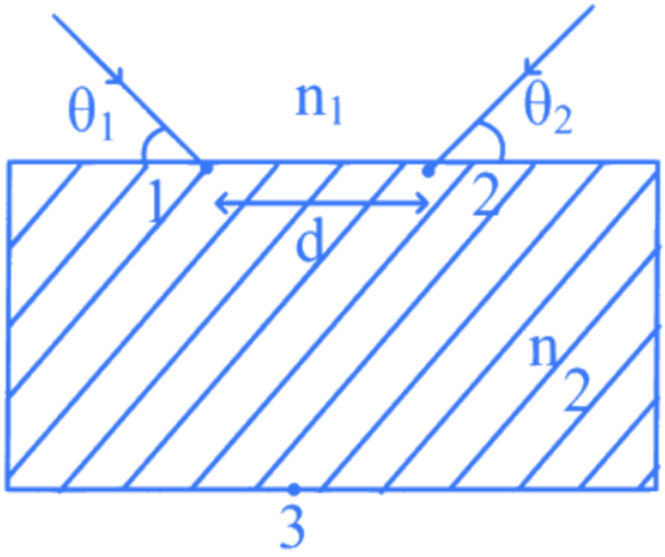
Its total magnitude is $2\sqrt{2}\,\mathrm{N\,m}$, but the question specifically asks for the magnitude in the z-direction. Therefore take the absolute value of the z-component alone: $|-2| = 2$.

Step 4: Conclusion The magnitude of torque in the z-direction is $\boxed{2\,\mathrm{N\,m}}$, so the numerical entry is 2.

Quick Tip to Solve: When a torque component about z is requested, use $xF_y - yF_x$ directly and take an absolute value only if the question asks for its magnitude.

---

49. Two light beams fall on a transparent material block at point 1 and 2 with angle θ_1 and θ_2, respectively, as shown in figure. After refraction, the beams intersect at point 3 which is exactly on the interface at other end of the block. Given : the distance between 1 and 2, $\mathrm{d} = 4\sqrt{3}$ cm and $\theta_1 = \theta_2 = \cos^{-1}\left(\frac{n_2}{2n_1}\right)$. where refractive index of the block n_2 > refractive index of the outside medium n_1, then the thickness of the block is _____ cm . [4 marks]



Solution:

Concept:

- Snell's law relates angles measured from the normal: $n_1 \sin i = n_2 \sin r$, where i is the incident angle and r is the refracted angle.

- The angles θ_1 and θ_2 in the figure are measured from the surface, so $i = 90^\circ - \theta$. Equal incident angles give equal refracted angles inside the uniform block.
- For block thickness t, the horizontal displacement of either refracted beam is $t\tan r$. The two displacements together equal the separation d.

Step 1: Convert the given surface angle to the normal angle The rays approach the top face symmetrically. It is essential to use the normal angle in the refraction law, rather than substituting the labelled surface angle directly. With $\theta = \theta_1 = \theta_2$, the given relation gives

$$\cos\theta = \frac{n_2}{2n_1}, \qquad \sin i = \cos\theta = \frac{n_2}{2n_1}.$$

Consequently,

$$n_1 \sin i = n_2 \sin r \quad \Longrightarrow \quad \sin r = \frac{1}{2}.$$

Both beams therefore make $r = 30^\circ$ with the downward normal inside the block.

Step 2: Use the two equal horizontal displacements Because both refracted rays travel through the same vertical thickness and at the same angle to the normal, their meeting point lies halfway between the entry points in the horizontal direction. Each beam covers $d/2$ horizontally. The resulting right triangle has vertical side t, so

$$\tan 30^\circ = \frac{d/2}{t}.$$

Substitute the separation, retaining centimetres throughout:

$$t = \frac{4\sqrt{3}}{2\tan 30^\circ} = \frac{4\sqrt{3}\,\sqrt{3}}{2} = 6 \text{ cm}.$$

The refractive indices cancel because their ratio was already built into the specified incident angle.

Step 3: Conclusion The thickness of the transparent block is $\boxed{6 \text{ cm}}$. The numerical entry is $\boxed{6}$.

Quick Tip to Solve: Check whether a marked ray angle is measured from the surface or the normal before applying Snell's law; here $\sin i = \cos\theta$.

---

50. In a hydraulic lift, the surface area of the input piston is 6 cm^2 and that of the output piston is 1500 cm^2. If 100 N force is applied to the input piston to raise the output piston by 20 cm, then the work done is _____ kJ. [4 marks]

Solution:

Concept:

- Pascal's law gives equal pressure increments in an ideal confined fluid: $F_{\text{in}}/A_{\text{in}} = F_{\text{out}}/A_{\text{out}}$.
- Work done by a constant force parallel to the displacement is $W = Fs$. An ideal hydraulic lift increases force at the expense of input displacement, conserving work.

Step 1: Find the amplified output force The two piston areas must first be expressed in the same units. Since both are supplied in square centimetres, their ratio can be used directly; converting each

separately to square metres would give the same result. Pressure transmission through the fluid gives

$$F_{\text{out}} = F_{\text{in}} \frac{A_{\text{out}}}{A_{\text{in}}} = 100 \frac{1500}{6} = 25\,000 \text{ N}.$$

This is the upward force available to move the larger piston. The applied 100 N is the force on the smaller piston and must not be multiplied by the larger piston's displacement.

Step 2: Calculate the output work and check energy conservation The output piston rises by $20 \text{ cm} = 0.20 \text{ m}$, in the direction of the output force. Thus,

$$W = F_{\text{out}} s_{\text{out}} = 25\,000(0.20) = 5000 \text{ J} = 5 \text{ kJ}.$$

As an independent consistency check, equal displaced fluid volumes require

$$A_{\text{in}} s_{\text{in}} = A_{\text{out}} s_{\text{out}}, \qquad s_{\text{in}} = \frac{1500}{6}(0.20) = 50 \text{ m}.$$

The input work is therefore $100 \times 50 = 5000$ J, equal to the output work. A hydraulic lift does not multiply energy even though it multiplies the force.

Step 3: Conclusion The work done in raising the output piston is $\boxed{5 \text{ kJ}}$. The required numerical entry is $\boxed{5}$.

Quick Tip to Solve: Multiply the output force by the output displacement, or the input force by the input displacement; never mix the two pistons' quantities.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. 1.24 g of AX_2 (molar mass 124 g mol^{-1}) is dissolved in 1 kg of water to form a solution with boiling point of 100.015°C, while 25.4 g of AY_2 (molar mass 250 g mol^{-1}) in 2 kg of water constitutes a solution with a boiling point of 100.0260°C.

$K_b(H_2O)$ = 0.52 kg mol^{-1}

Which of the following is correct? [4 marks]

(A) AX_2 and AY_2 (both) are completely unionised.

(B) AX_2 and AY_2 (both) are fully ionised.

(C) AX_2 is completely unionised while AY_2 is fully ionised.

(D) AX_2 is fully ionised while AY_2 is completely unionised.

Correct Answer: (D) AX_2 is fully ionised while AY_2 is completely unionised.

Solution:

Concept:

- For a dilute solution, elevation of boiling point is $\Delta T_b = iK_b m$, with m the molality of solute formula units and i the van't Hoff factor.
- An undissociated solute has $i = 1$. Complete dissociation of a salt AX_2 into A^{2+} and $2X^-$ gives

three particles and ideal $i = 3$.

Step 1: Calculate the factor for the first solution The first solution contains

$$n(\mathrm{AX_2}) = \frac{1.24}{124} = 0.0100 \text{ mol}, \qquad m_1 = 0.0100 \text{ mol kg}^{-1}.$$

Its elevation above the boiling point of pure water is $100.015 - 100 = 0.015$ K. Hence,

$$i_1 = \frac{0.015}{0.52(0.0100)} \simeq 2.885.$$

This is close to the three-particle value and far from unity. The appropriate classification among the alternatives is ionised $\mathrm{AX_2}$, rather than unionised $\mathrm{AX_2}$.

Step 2: Calculate the factor for the second solution Use the stated molar mass and the mass of solvent, rather than the mass of the complete solution:

$$n(\mathrm{AY_2}) = \frac{25.4}{250} = 0.1016 \text{ mol}, \qquad m_2 = \frac{0.1016}{2} = 0.0508 \text{ mol kg}^{-1}.$$

Then

$$i_2 = \frac{0.0260}{0.52(0.0508)} \simeq 0.9843 \simeq 1.$$

Thus the second solution has approximately one solute particle per formula unit. The first three alternatives either make the first solute unionised or the second solute fully ionised, contrary to these factors.

Step 3: Interpret the precision of the supplied numbers The printed data give factors close to, rather than exactly, the ideal values 3 and 1. Strict numerical equality with complete ideal dissociation is not obtained: ideal values would predict elevations 0.0156 K and 0.026416 K. The listed classification is therefore understood at the approximate level of the supplied boiling-point data, and only the fourth alternative has the corresponding particle counts.

Step 4: Conclusion $\mathrm{AX_2}$ is classified as fully ionised and $\mathrm{AY_2}$ as completely unionised, using $i_1 \approx 3$ and $i_2 \approx 1$. Hence, the correct option is (D).

Quick Tip to Solve: Compute $i = \Delta T_b/(K_b m)$ separately for each salt; its proximity to 1 or 3 identifies the particle count.

52. For a Mg | Mg^{2+} (aq) || Ag^{+} (aq) | Ag the correct Nernst Equation is : [4 marks]

(A) $E_{cell} = E^{o}_{cell} - \frac{RT}{2F} \ln \frac{[Ag^{+}]^2}{[Mg^{2+}]}$

(B) $E_{cell} = E^{o}_{cell} - \frac{RT}{2F} \ln \frac{[Mg^{2+}]}{[Ag^{+}]}$

(C) $E_{cell} = E^{o}_{cell} - \frac{RT}{2F} \ln \frac{[Ag^{+}]}{[Mg^{2+}]}$

(D) $E_{cell} = E^{o}_{cell} + \frac{RT}{2F} \ln \frac{[Ag^{+}]^2}{[Mg^{2+}]}$

Correct Answer: (D) $E_{cell} = E^{o}_{cell} + \frac{RT}{2F} \ln \frac{[Ag^{+}]^2}{[Mg^{2+}]}$

Solution:

Concept:

- The cell notation places the oxidation half-cell on the left and the reduction half-cell on the right. Magnesium supplies electrons and silver ions accept them.
- The Nernst equation is $E_{cell} = E^{\circ}_{cell} - (RT/nF) \ln Q$, where n is the number of electrons in the balanced reaction and Q is its reaction quotient.

Step 1: Balance the cell reaction At the magnesium electrode, oxidation is

$$Mg(s) \longrightarrow Mg^{2+}(aq) + 2e^{-}.$$

The silver reduction must be doubled to consume those two electrons:

$$2\,Ag^{+}(aq) + 2e^{-} \longrightarrow 2\,Ag(s).$$

Adding gives $Mg(s) + 2\,Ag^{+}(aq) \longrightarrow Mg^{2+}(aq) + 2\,Ag(s)$. Therefore the electron-transfer number is $n = 2$, while the stoichiometric coefficient of dissolved silver ion is also two. These two appearances of the number two have distinct roles in the Nernst expression.

Step 2: Form the quotient and compare the options Pure solid magnesium and silver have unit activity and do not appear in Q. Using concentrations for the aqueous activities as in the options,

$$Q = \frac{[Mg^{2+}]}{[Ag^{+}]^2}.$$

Substitution yields

$$E_{cell} = E^{\circ}_{cell} - \frac{RT}{2F} \ln \frac{[Mg^{2+}]}{[Ag^{+}]^2}.$$

Invert the logarithm argument to write the same expression as

$$E_{cell} = E^{\circ}_{cell} + \frac{RT}{2F} \ln \frac{[Ag^{+}]^2}{[Mg^{2+}]}.$$

Option (A) reverses the quotient without reversing the sign. Options (B) and (C) omit the required square on silver-ion concentration. An increase in silver-ion concentration should increase the cell potential, consistent with the last expression.

Step 3: Conclusion The plus sign must accompany $\ln([Ag^+]^2/[Mg^{2+}])$. Hence, the correct option is (D).

Quick Tip to Solve: Balance the reaction first: stoichiometric coefficients become powers in Q, while the electron count becomes the denominator nF.

53. Match List - I with List - II.

List - I (Carbohydrate)	List - II (Linkage Source)
(A) Amylose	(I) β - $C_1 - C_4$, plant
(B) Cellulose	(II) α - $C_1 - C_4$, animal
(C) Glycogen	(III) α - $C_1 - C_4$, α - $C_1 - C_6$, plant
(D) Amylopectin	(IV) α - $C_1 - C_4$, plant

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(B) (A)-(III), (B)-(III), (C)-(I), (D)-(IV)

(C) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

(D) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

Correct Answer: (D) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

Solution:

Concept:

- Glucose units in polysaccharides are joined by glycosidic linkages. The α or β configuration and the carbon positions distinguish the chains.
- Amylose and amylopectin are components of plant starch, cellulose is a plant structural polysaccharide, and glycogen is the storage polysaccharide of animals.

Step 1: Match the two unbranched plant polymers Amylose is the essentially unbranched component of starch. Its glucose residues are connected through α-(1→4) glycosidic bonds. Thus amylose corresponds to the entry α-$C_1 - C_4$, plant, namely (IV).

Cellulose consists of glucose units connected by β-(1→4) linkages, producing extended chains suited to the structural role of plant cell walls. It therefore corresponds to (I), not to either of the entries specifying an α linkage.

$$\text{A} \longrightarrow \text{IV}, \qquad \text{B} \longrightarrow \text{I}.$$

Step 2: Distinguish the branched storage polysaccharides Glycogen is found in animals and contains α-(1→4) linked chains, with α-(1→6) bonds at branch points. The animal-source entry in List II is (II). Although that entry mentions only the chain linkage, it identifies glycogen among the given carbohydrates.

Amylopectin is the branched plant-starch component. Its chains have α-(1→4) bonds and its branch points have α-(1→6) bonds, matching (III).

$$\text{C} \longrightarrow \text{II}, \qquad \text{D} \longrightarrow \text{III}.$$

The first two alternatives already fail the amylose and cellulose matches. The third exchanges the plant amylopectin entry and the animal glycogen entry; the fourth retains both correct sources and linkages.

Step 3: Conclusion The matching is $\boxed{\text{A} - \text{IV},\ \text{B} - \text{I},\ \text{C} - \text{II},\ \text{D} - \text{III}}$. Hence, the correct option is (D).

Quick Tip to Solve: Identify cellulose by its β-(1→4) linkage first; then distinguish glycogen from amylopectin using animal versus plant source.

---

54. The correct increasing order of stability of the complexes based on Δ_0 value is :

I. $[\mathbf{Mn(CN)_6}]^{3-}$

II. $[\mathbf{Co(CN)_6}]^{4-}$

III. $[\mathbf{Fe(CN)_6}]^{4-}$

IV. $[\mathbf{Fe(CN)_6}]^{3-}$ [4 marks]

(A) I < II < IV < III

(B) IV < III < II < I

(C) III < II < IV < I

(D) II < III < I < IV

Correct Answer: (A) I < II < IV < III

Solution:

Concept:

- In an octahedral field, each t_{2g} electron contributes $-0.4\Delta_o$ and each e_g electron contributes $+0.6\Delta_o$ to the crystal-field energy.
- Cyanide is a strong-field ligand, so the given complexes are treated as low spin: electrons pair in t_{2g} orbitals before occupying e_g. Greater crystal-field stabilisation magnitude means greater stability in the requested comparison.

Step 1: Determine the metal oxidation states and electron counts Six CN^- ligands contribute charge -6. Hence the metal oxidation number is the overall complex charge plus six. For I, manganese is $+3$, giving d^4. For II, cobalt is $+2$, giving d^7. The two iron complexes contain Fe^{2+}, d^6, in III and Fe^{3+}, d^5, in IV.

$$\text{I} : d^4, \quad \text{II} : d^7,$$
$$\text{III} : d^6, \quad \text{IV} : d^5.$$

The charges on the whole complexes must not be mistaken for the oxidation states of their metal centres.

Step 2: Calculate the low-spin crystal-field stabilisation Fill the lower t_{2g} level first and evaluate the

orbital contribution in units of Δ_o:

$$\begin{aligned} \text{I}: \quad & t_{2g}^4 e_g^0: \quad 4(-0.4) = -1.6, \\ \text{II}: \quad & t_{2g}^6 e_g^1: \quad 6(-0.4) + 0.6 = -1.8, \\ \text{III}: \quad & t_{2g}^6 e_g^0: \quad 6(-0.4) = -2.4, \\ \text{IV}: \quad & t_{2g}^5 e_g^0: \quad 5(-0.4) = -2.0. \end{aligned}$$

A more negative crystal-field energy represents a larger lowering relative to the barycentre. Therefore the positive stabilisation magnitudes increase as $1.6 < 1.8 < 2.0 < 2.4$. The question compares stability based on these Δ_o contributions; it does not ask for a ranking of the metal ions by their absolute splitting parameters or a full thermodynamic formation constant. The other alternatives fail this sequence of orbital stabilisation contributions.

Step 3: Conclusion The increasing order of crystal-field stabilisation is $\boxed{\text{I} < \text{II} < \text{IV} < \text{III}}$. Hence, the correct option is (A).

Quick Tip to Solve: For strong-field octahedral complexes, write $t_{2g}^a e_g^b$, compute $(-0.4a+0.6b)\Delta_o$, and compare stabilisation magnitudes.

55. 500 J of energy is transferred as heat to 0.5 mol of Argon gas at 298 K and 1.00 atm. The final temperature and the change in internal energy respectively are: Given: R = 8.3 J K^{-1} mol^{-1} [4 marks]

(A) 368 K and 500 J

(B) 348 K and 300 J

(C) 378 K and 300 J

(D) 378 K and 500 J

Correct Answer: (B) 348 K and 300 J

Solution:

Concept:

- Argon is a monatomic ideal gas, so its molar heat capacities are $C_{V,m} = 3R/2$ and $C_{P,m} = 5R/2$. Only translational degrees of freedom contribute here.
- For heating at constant pressure, $q_P = nC_{P,m}\Delta T$, whereas the change in internal energy is $\Delta U = nC_{V,m}\Delta T$. The difference is expansion work.

Step 1: Calculate the temperature rise Take the stated pressure as the constant pressure for the heating process. The gas absorbs heat, so its temperature increases. Using the amount $n = 0.5\,\text{mol}$ and the supplied gas constant gives

$$\Delta T = \frac{q_P}{n(5R/2)} = \frac{500}{0.5 \times (5/2) \times 8.3} = 48.19\,\text{K}.$$

Thus the final temperature is $T_f = 298 + 48.19 = 346.19\,\text{K}$. The listed approximate temperature is 348 K; it is also obtained if R is rounded to 8 in this elementary estimate.

Step 2: Find the internal-energy increase For an ideal gas, internal energy depends only on tempera-

ture. Hence

$$\Delta U = n\frac{3R}{2}\Delta T = \frac{C_{V,m}}{C_{P,m}}q_P = \frac{3}{5}(500) = 300\,\text{J}.$$

The remaining $200\,\text{J}$ is used in expansion against the surroundings. Options (A) and (D) incorrectly make all the absorbed heat internal energy. Option (C) has the correct energy but an incompatible temperature rise. The only intended pair is therefore option (B); the numerical temperature in the options is approximate and differs slightly from the result using the stated value of R.

Step 3: Conclusion The internal energy increases by $300\,\text{J}$; the stated constant gives $T_f \simeq 346\,\text{K}$, corresponding to the listed approximate pair 348 K and 300 J. Hence, the correct option is (B).

Quick Tip to Solve: For a monatomic gas heated at constant pressure, use $\Delta U = 3q_P/5$ first; do not set $q_P = \Delta U$.

56. The molar conductivity of a weak electrolyte when plotted against the square root of its concentration, which of the following is expected to be observed?

[4 marks]

(A) Molar conductivity increases sharply with increase in concentration.

(B) Molar conductivity decreases sharply with increase in concentration.

(C) A small increase in molar conductivity is observed at infinite dilution.

(D) A small decrease in molar conductivity is observed at infinite dilution.

Correct Answer: (B) Molar conductivity decreases sharply with increase in concentration.

Solution:

Concept:

- Molar conductivity Λ_m measures the conductance associated with one mole of electrolyte. For a weak electrolyte, the number of ions produced per mole depends strongly on its degree of ionisation α.
- On dilution, ionisation increases appreciably, so Λ_m rises steeply toward its limiting value Λ_m°. For a simple weak electrolyte, $\alpha = \Lambda_m/\Lambda_m^\circ$.

Step 1: Follow the ionisation change Consider a weak electrolyte $\text{AB} \rightleftharpoons \text{A}^+ + \text{B}^-$. Most formula units remain undissociated at appreciable concentration. Adding solvent favours dissociation, increasing the fraction of ions available to carry current per mole of electrolyte. Consequently the molar conductivity rises markedly as concentration decreases. Conversely, increasing concentration reduces the degree of ionisation and produces a sharp fall in molar conductivity. This concerns conductivity per mole; it must not be confused with the specific conductivity of the whole solution.

Step 2: Interpret the horizontal axis and compare options The horizontal coordinate is $\sqrt{c}$, which increases whenever c increases. Therefore the curve slopes downwards strongly as one moves to the right from the dilute region. Its variation is curved rather than the approximately linear strong-electrolyte relation. Option (A) reverses this trend. Options (C) and (D) misdescribe the limiting region: dilution of a weak electrolyte brings a substantial increase, not merely a small change or a decrease. The sharp decrease on increasing concentration is exactly the statement in option (B).

Step 3: Conclusion The molar conductivity of a weak electrolyte decreases sharply with increasing concentration, or equivalently increases sharply on dilution. Hence, the correct option is (B).

Quick Tip to Solve: For weak electrolytes, track ionisation per mole: dilution produces many more ions, so molar conductivity rises sharply as $\sqrt{c}$ falls.

57. The correct option with order of melting points of the pairs (Mn, Fe), (Tc, Ru) and (Re, Os) is : [4 marks]

(A) Fe $<$ Mn, Ru $<$ Tc and Re $<$ Os

(B) Fe $<$ Mn, Ru $<$ Tc and Os $<$ Re

(C) Mn $<$ Fe, Tc $<$ Ru and Re $<$ Os

(D) Mn $<$ Fe, Tc $<$ Ru and Os $<$ Re

Correct Answer: (D) Mn $<$ Fe, Tc $<$ Ru and Os $<$ Re

Solution:

Concept:

- Melting points of transition metals depend on metallic bonding and crystal structure. Participation of d and s electrons in bonding produces high but irregular melting points across a transition series.
- The middle-of-series trend is not a strict rule for every neighbouring pair. The experimentally established order of the particular elements must be compared separately in each row.

Step 1: Compare manganese with iron and technetium with ruthenium Manganese has an unusually low melting point in the first transition series compared with its neighbours. Its melting point is about $1246\,^{\circ}\mathrm{C}$, whereas iron melts near $1538\,^{\circ}\mathrm{C}$. Hence $\mathrm{Mn} < \mathrm{Fe}$. This immediately eliminates options (A) and (B), which reverse the first pair. In the second transition series, technetium melts at roughly $2157\,^{\circ}\mathrm{C}$, below ruthenium at roughly $2334\,^{\circ}\mathrm{C}$. Thus $\mathrm{Tc} < \mathrm{Ru}$.

Step 2: Distinguish rhenium from osmium The third pair does not follow the direction of the first two pairs. Rhenium has a particularly high melting point, about $3186\,^{\circ}\mathrm{C}$, whereas osmium melts near $3033\,^{\circ}\mathrm{C}$. Therefore $\mathrm{Os} < \mathrm{Re}$. Option (C) fails precisely at this final comparison. Combining all three observed comparisons gives

$$\mathrm{Mn} < \mathrm{Fe}, \qquad \mathrm{Tc} < \mathrm{Ru}, \qquad \mathrm{Os} < \mathrm{Re}.$$

One should not extrapolate the direction of a pair from one transition series to another: detailed bonding and solid-state structure introduce exceptions to a simple electronic-configuration argument.

Step 3: Conclusion The three orders are $\mathrm{Mn} < \mathrm{Fe}$, $\mathrm{Tc} < \mathrm{Ru}$, and $\mathrm{Os} < \mathrm{Re}$. Hence, the correct option is (D).

Quick Tip to Solve: Check all three pairs separately: manganese is the low first-row member, while rhenium melts above osmium in the third row.

58. The reaction $A_2 + B_2 \rightarrow 2AB$ follows the mechanism:

$A_2 \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} A + A$ **(fast)**

$A + B_2 \xrightarrow{k_2} AB + B$ **(slow)**

$A + B \rightarrow AB$ **(fast)**

The overall order of the reaction is: [4 marks]

(A) 3

(B) 2.5

(C) 1.5

(D) 2

Correct Answer: (C) 1.5

Solution:

Concept:

- The slow elementary step determines the reaction rate. Its rate law is written using the reactants in that step, including any intermediate that must subsequently be eliminated.
- A fast equilibrium before the slow step relates the intermediate concentration to stable reactant concentrations. Overall order is the sum of exponents in the resulting rate law, not the sum of overall stoichiometric coefficients.

Step 1: Write the rate of the slow step The rate-determining step is $\mathrm{A} + \mathrm{B}_2 \longrightarrow \mathrm{AB} + \mathrm{B}$, so its rate is

$$r = k_2[A][B_2].$$

The atom A is an intermediate formed by dissociation of A_2. It is not an independently supplied reactant, so leaving the rate law in this form would not give the required overall order in terms of A_2 and B_2. The final fast step removes the intermediates and does not independently determine the slow rate.

Step 2: Eliminate the intermediate using the fast equilibrium For the first step, forward and reverse rates are equal at pre-equilibrium:

$$k_1[A_2] = k_{-1}[A]^2.$$

Hence

$$[A] = \left(\frac{k_1}{k_{-1}}\right)^{1/2} [A_2]^{1/2}.$$

Substitution yields

$$r = k_2 \left(\frac{k_1}{k_{-1}}\right)^{1/2} [A_2]^{1/2}[B_2] = k_{\text{eff}}[A_2]^{1/2}[B_2].$$

The partial orders are therefore $1/2$ and 1. Their sum is $3/2$, not 2, 2.5 or 3. A constant factor from the definition of product-formation rate would change the effective constant but never these concentration exponents.

Step 3: Conclusion The overall order is $\frac{1}{2} + 1 = \frac{3}{2} = 1.5$. Hence, the correct option is (C).

Quick Tip to Solve: When a fast step dissociates one molecule into two identical radicals, its equilibrium often introduces a square-root concentration and a half-order contribution.

59. If a_0 is denoted as the Bohr radius of hydrogen atom, then what is the de-Broglie wavelength (λ) of the electron present in the second orbit of hydrogen atom? [n : any integer] [4 marks]

(A) $\frac{8\pi a_0}{n}$
(B) $\frac{2a_0}{n\pi}$
(C) $\frac{na_0}{4\pi}$
(D) $\frac{4\pi a_0}{n}$

Correct Answer: (A) $\frac{8\pi a_0}{n}$

Solution:

Concept:

- A stationary orbit accommodates an integral number of electron wavelengths around its circumference: $2\pi r = n\lambda$, where n counts the wavelengths.
- The radius of the second Bohr orbit of hydrogen is $r_2 = 2^2 a_0 = 4a_0$. Keep this fixed orbit radius distinct from the integer symbol used in the wavelength condition.

Step 1: Find the circumference of the specified orbit For a hydrogen atom, the radius of orbit number j is $r_j = j^2 a_0$. The question specifies the second orbit, so set $j = 2$ before using the standing-wave condition. This gives

$$r_2 = 4a_0, \qquad 2\pi r_2 = 8\pi a_0.$$

The circumference, rather than the radius or diameter alone, is the distance over which the electron matter wave must close on itself without a phase mismatch.

Step 2: Apply the integral-wavelength condition Using the integer notation requested in the question, n wavelengths fit around this circumference:

$$n\lambda = 8\pi a_0 \quad \Longrightarrow \quad \lambda = \frac{8\pi a_0}{n}.$$

This is option (A). For the physical electron state in the second Bohr orbit the wave count is specifically $n = 2$, and the expression reduces to $\lambda_2 = 4\pi a_0$, agreeing with the usual general result $\lambda_j = 2\pi j a_0$. Option (D) would give half the required wavelength at the same wave count. Options (B) and (C) also have incorrect circumference factors. The arbitrary-integer expression in the choices is the standing-wave relation; the specified physical orbit fixes its allowed wave count.

Step 3: Conclusion In the notation of the options, $\lambda = 8\pi a_0/n$; for the actual second-orbit state, $n = 2$ and $\lambda_2 = 4\pi a_0$. Hence, the correct option is (A).

Quick Tip to Solve: First insert the specified orbit radius, then divide its circumference by the wavelength count; do not confuse the orbit index with a free integer.

60. The standard reduction potential values of some of the p-block ions are given below. Predict the one with the strongest oxidising capacity. [4 marks]

(A) $E^o_{Sn^{4+}/Sn^{2+}} = +1.15\ V$
(B) $E^o_{Al^{3+}/Al} = -1.66\ V$
(C) $E^o_{Pb^{4+}/Pb^{2+}} = +1.67\ V$
(D) $E^o_{Tl^{3+}/Tl} = +1.26\ V$
Correct Answer: (C) $E^o_{Pb^{4+}/Pb^{2+}} = +1.67\ V$

Solution:

Concept:

- An oxidising agent accepts electrons and is itself reduced. A larger standard reduction potential means a stronger thermodynamic tendency to accept electrons under standard conditions.
- Compare reduction potentials as intensive quantities. Do not multiply an electrode potential by the number of electrons when comparing half-reactions.

Step 1: Rank the supplied reduction potentials The problem supplies four values, all written as reduction potentials. Put them in increasing order:

$$-1.66 < +1.15 < +1.26 < +1.67 \quad (\mathrm{V}).$$

The largest value is $+1.67\,\mathrm{V}$, associated with the Pb^{4+}/Pb^{2+} couple. Thus Pb^{4+} has the greatest tendency among the stated ions to take electrons from a reducing partner. This comparison uses the values supplied in the question; no sign reversal is needed because all four are already reduction potentials.

Step 2: Identify the oxidant and reject the other choices The relevant half-reaction is

$$Pb^{4+} + 2\,e^- \longrightarrow Pb^{2+}.$$

Lead(IV) accepts electrons, so it is the oxidising species, while lead(II) is its reduction product. Tin(IV), aluminium(III), and thallium(III) have lower supplied reduction potentials and therefore weaker standard oxidising tendencies in this comparison. In particular, the negative aluminium reduction potential does not make aluminium(III) a stronger oxidant: it indicates that reduction to the metal is much less favourable than for the positive-potential couples. The magnitude alone is insufficient; the algebraic value determines the ordering.

Step 3: Conclusion Pb^{4+}, with $E^\circ_{Pb^{4+}/Pb^{2+}} = +1.67\,\mathrm{V}$, has the strongest oxidising capacity. Hence, the correct option is (C).

Quick Tip to Solve: To select the strongest oxidant, choose the oxidised member of the couple with the most positive reduction potential; keep the signs.

61. Choose the correct statements.

(A) Weight of a substance is the amount of matter present in it.

(B) Mass is the force exerted by gravity on an object.

(C) Volume is the amount of space occupied by a substance.

(D) Temperatures below 0°C are possible in Celsius scale, but in Kelvin scale negative temperature is not possible.

(E) Precision refers to the closeness of various measurements for the same quantity.

Choose the correct answer from the options given below : [4 marks]

(A) (B), (C) and (D) Only

(B) (A), (B) and (C) Only

(C) (C), (D) and (E) Only

(D) (A), (D) and (E) Only

Correct Answer: (C) (C), (D) and (E) Only

Solution:

Concept:

- Mass measures the amount of matter in a body. Weight is the gravitational force on it, $W = mg$, where m is mass and g is gravitational acceleration.
- Volume is the space occupied by a substance. Precision describes agreement among repeated measurements, whereas accuracy describes agreement with the accepted value.
- For the ordinary thermodynamic temperature scale used here, $T(\mathrm{K}) = t(^\circ\mathrm{C}) + 273.15$; absolute zero is the lower limit.

Step 1: Test statements (A) and (B) Statement (A) assigns the definition of mass to weight, so it is false. Weight depends on the local gravitational acceleration and is measured in newtons. Statement (B) assigns the definition of weight to mass, so it is also false. Mass is measured in kilograms and does not change merely because the body is moved to a place with different gravity.

Step 2: Test the remaining statements Statement (C) correctly identifies volume as occupied space. Statement (D) is correct in the ordinary temperature range of chemistry: a Celsius reading can be negative, such as $-10^\circ\mathrm{C} = 263.15\,\mathrm{K}$, while absolute thermodynamic temperature is nonnegative. Statement (E) correctly defines precision by the closeness of repeated measurements of the same quantity.

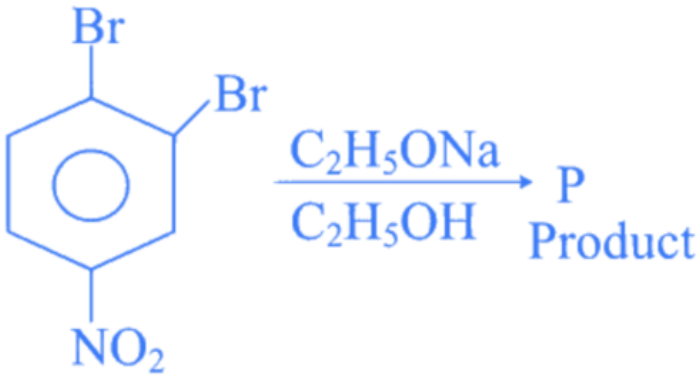
Step 3: Compare the combinations The true set is therefore (C), (D) and (E). The first alternative incorrectly includes (B), the second includes both (A) and (B), and the fourth incorrectly includes (A). Repeated readings can be precise yet inaccurate if the instrument has a systematic offset; thus statement (E) must not be confused with accuracy.

Step 4: Conclusion The correct set comprises the statements about volume, absolute temperature and precision. Hence, the correct option is (C).

Quick Tip to Solve: Keep the pairs separate: mass–matter, weight–gravity, precision–repeatability, and accuracy–accepted value.

62. In the following substitution reaction :

Product 'P' formed is : [4 marks]



(A) 1-Bromo-2-ethoxy-4-nitrobenzene

(B) 1,2-Dibromo-4-ethoxybenzene

(C) 1,2-Diethoxy-4-nitrobenzene

(D) 2-Bromo-1-ethoxy-4-nitrobenzene

Correct Answer: (D) 2-Bromo-1-ethoxy-4-nitrobenzene

Solution:

Concept:

- An electron-withdrawing NO_2 group activates an aryl halide toward nucleophilic substitution when it is ortho or para to the leaving group.
- Ethoxide, $C_2H_5O^-$, replaces the activated bromine to form an aryl ethyl ether. The nitro group stabilises the anionic addition intermediate by resonance.

Step 1: Locate each bromine relative to the nitro group In the starting ring, the bromine at the top of the drawing is para to NO_2. The neighbouring bromine at the upper-right position is meta to NO_2. Only the former has the resonance activation required for this addition–elimination substitution. The two bromines are therefore chemically inequivalent even though they are on the same benzene ring.

Step 2: Replace the activated leaving group Ethoxide attacks the carbon bearing the para bromine. The resulting intermediate can delocalise its negative charge toward the nitro substituent; elimination of Br^- then restores aromaticity. Consequently the top Br becomes OC_2H_5, while the other bromine and the nitro group remain unchanged. The product is 2-bromo-1-ethoxy-4-nitrobenzene.

Step 3: Reject the other substitution patterns The first alternative substitutes the bromine meta to nitro instead of the activated para bromine. The second replaces the nitro group itself, which is not the leaving group in this reaction. The third replaces both bromines, although the remaining meta bromine lacks nitro resonance activation. The fourth structure alone preserves the correct selectivity and all other substituents.

Step 4: Conclusion The bromine para to NO_2 is replaced by OC_2H_5, giving 2-bromo-1-ethoxy-4-nitrobenzene. Hence, the correct option is (D).

Quick Tip to Solve: For nitro-activated aromatic substitution, first mark the positions ortho and para to nitro; a meta halogen is not activated by its resonance effect.

63. At temperature T, compound $AB_{2(g)}$ dissociates as $AB_{2(g)} \rightleftharpoons AB_{(g)} + \frac{1}{2}B_{2(g)}$ having degree of dissociation x (small compared to unity). The correct expression for x in terms of K_p and p is:

[4 marks]

(A) $\sqrt{K_p}$

(B) $\sqrt[3]{\frac{2K_p^2}{p}}$

(C) $\sqrt[3]{\frac{2K_p}{p}}$

(D) $\sqrt[4]{\frac{2K_p}{p}}$

Correct Answer: (B) $\sqrt[3]{\frac{2K_p^2}{p}}$

Solution:

Concept:

- For $\mathrm{AB_2(g)} \rightleftharpoons \mathrm{AB\,(g)} + \frac{1}{2}\,\mathrm{B_2(g)}$, the pressure equilibrium constant is $K_p = p_{\mathrm{AB}}\sqrt{p_{\mathrm{B_2}}}/p_{\mathrm{AB_2}}$.
- Partial pressure equals mole fraction times total pressure p. Start with one mole of $\mathrm{AB_2}$, and let x be its fraction dissociated; use $x \ll 1$ only after forming the equilibrium expression.

Step 1: Write the equilibrium mole amounts Initially take one mole of pure $\mathrm{AB_2}$. Dissociation of a fraction x leaves $1-x$ moles of $\mathrm{AB_2}$, produces x moles of AB, and produces $x/2$ moles of $\mathrm{B_2}$. The total is $1+x/2$ moles. The factor one-half is essential: the second product is not formed in the same amount as AB.

Step 2: Substitute the partial pressures Using mole fractions gives

$$p_{\mathrm{AB_2}} = \frac{1-x}{1+x/2}p, \quad p_{\mathrm{AB}} = \frac{x}{1+x/2}p, \quad p_{\mathrm{B_2}} = \frac{x/2}{1+x/2}p.$$

Thus

$$K_p = \frac{x}{1-x}\sqrt{\frac{xp}{2(1+x/2)}}.$$

This expression retains both the reactant depletion and the increase in total mole number.

Step 3: Apply the small-dissociation approximation Since $x \ll 1$, set $1-x \simeq 1$ and $1+x/2 \simeq 1$. Then

$$K_p \simeq x\sqrt{\frac{xp}{2}}, \qquad K_p^2 \simeq \frac{x^3 p}{2}.$$

Solving gives $x = (2K_p^2/p)^{1/3}$. The square-root option omits pressure dependence. The other cube-root option misses the square on K_p, and a fourth root is incompatible with the power $x^{3/2}$ in the equilibrium expression.

Step 4: Conclusion For small dissociation,

$$\boxed{x = \sqrt[3]{\frac{2K_p^2}{p}}}.$$

Hence, the correct option is (B).

Quick Tip to Solve: Keep fractional stoichiometric coefficients in both the mole table and the exponents of the equilibrium constant before squaring the expression.

64. Match List - I with List - II.

List - I (Complex)	List - II (Hybridisation & Magnetic characters)
(A) $[MnBr_4]^{2-}$	(I) d^2sp^3 & diamagnetic
(B) $[FeF_6]^{3-}$	(II) sp^3d^2 & paramagnetic
(C) $[Co(C_2O_4)_3]^{3-}$	(III) sp^3 & diamagnetic
(D) $[Ni(CO)_4]$	(IV) sp^3 & paramagnetic

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(III), (B)-(I), (C)-(I), (D)-(IV)

(B) (A)-(III), (B)-(II), (C)-(III), (D)-(IV)

(C) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(D) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

Correct Answer: (C) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

Solution:

Concept:

- Find the metal oxidation state first, then its d-electron count. Unpaired electrons make a complex paramagnetic; complete pairing gives diamagnetism.
- In the valence-bond description, tetrahedral complexes use sp^3, outer-orbital octahedral complexes use sp^3d^2, and inner-orbital octahedral complexes use d^2sp^3.

Step 1: Classify the manganese and iron complexes In $[MnBr_4]^{2-}$, manganese is $+2$, hence $3d^5$. Bromide is a weak-field ligand, so the tetrahedral complex has five unpaired electrons, sp^3 hybridisation and paramagnetism: (A)-(IV). In $[FeF_6]^{3-}$, iron is $+3$, again $3d^5$. Weak-field fluoride gives a high-spin octahedral arrangement with five unpaired electrons and outer-orbital sp^3d^2 hybridisation: (B)-(II).

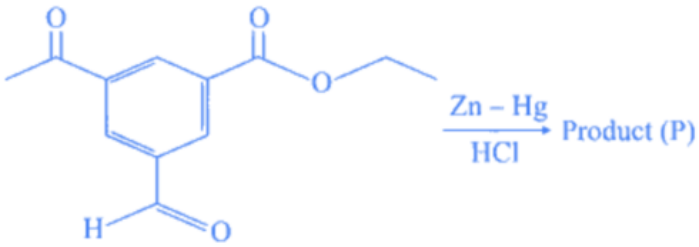
Step 2: Classify the cobalt and nickel complexes In $[Co(C_2O_4)_3]^{3-}$, the three oxalate ions contribute -6, so cobalt is $+3$, or $3d^6$. The octahedral cobalt(III) complex is low spin, with configuration t_{2g}^6, all electrons paired, and d^2sp^3 hybridisation: (C)-(I). In $Ni(CO)_4$, neutral carbon monoxide leaves nickel in oxidation state zero. Its paired d^{10} arrangement gives a tetrahedral, sp^3, diamagnetic complex: (D)-(III).

Step 3: Match the complete sequence The required sequence is IV, II, I, III. The first two alternatives assign diamagnetic tetrahedral behaviour to the high-spin manganese complex and paramagnetism to nickel carbonyl. The fourth incorrectly treats fluoride-bound iron(III) as an inner-orbital diamagnetic complex and cobalt(III) oxalate as outer orbital. Only the third alternative matches all four entries.

Step 4: Conclusion The hybridisation and magnetic-character sequence is IV, II, I, III. Hence, the correct option is (C).

Quick Tip to Solve: Check the distinctive endpoints first: tetrahedral high-spin Mn^{2+} is paramagnetic, whereas $Ni(CO)_4$ is diamagnetic.

65. The product (P) formed in the following reaction is : [4 marks]



(A) Ethyl 3-acetyl-5-(hydroxymethyl)benzoate

(B) Ethyl 3-(2-hydroxyethyl)-5-(hydroxymethyl)benzoate

(C) A benzene ring bearing a 2-hydroxyethyl group and two hydroxymethyl groups at alternating positions

(D) Ethyl 3-ethyl-5-methylbenzoate

Correct Answer: (D) Ethyl 3-ethyl-5-methylbenzoate

Solution:

Concept:

- Clemmensen reduction uses zinc amalgam and concentrated hydrochloric acid, $Zn(Hg)/HCl$, to replace an aldehyde or ketone carbonyl by a methylene unit.
- An aldehyde substituent $-CHO$ becomes $-CH_3$, while an acetyl substituent $-COCH_3$ becomes $-CH_2CH_3$. An ester is not reduced to an alcohol by this reagent.

Step 1: Identify the three carbonyl-containing groups The aromatic starting compound has an acetyl group, a formyl group and an ethyl ester at alternating ring positions. These groups must be distinguished by their connectivity: the ketone carbonyl is bonded to the ring and a methyl group, the aldehyde carbonyl to the ring and hydrogen, and the ester carbonyl to the ring and an ethoxy oxygen.

Step 2: Reduce the aldehyde and ketone The zinc amalgam–acid reagent removes the carbonyl oxygen from the acetyl and formyl groups without removing their carbon atoms. The substitutions are

$$Ar-COCH_3 \longrightarrow Ar-CH_2CH_3, \qquad Ar-CHO \longrightarrow Ar-CH_3.$$

The benzene ring and the ester group remain in the product shown by the appropriate alternative. Thus the acetyl side chain becomes ethyl and the formyl side chain becomes methyl.

Step 3: Compare the proposed products The first product reduces only the aldehyde to a primary alcohol and leaves the ketone unchanged, which is not Clemmensen reduction. The second and third products likewise contain alcohol groups rather than the deoxygenated carbon chains; the third also reduces the ester. The fourth retains the ethyl ester while showing the ethyl and methyl groups expected from the other two carbonyls. It therefore satisfies both functional-group selectivity and carbon conservation.

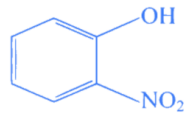
Step 4: Conclusion The product is ethyl 3-ethyl-5-methylbenzoate, with the ester retained and the aldehyde and ketone deoxygenated. Hence, the correct option is (D).

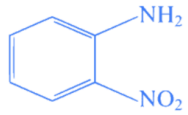
Quick Tip to Solve: Distinguish carbonyl-to-alcohol reduction from Clemmensen deoxygenation: $-CO-$ becomes $-CH_2-$, not $-CHOH-$.

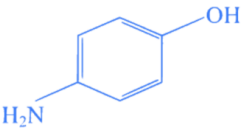
66. The steam volatile compounds among the following are :

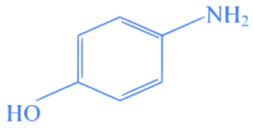
Choose the correct answer from the options given below :

[4 marks]

(A) 

(B) 

(C) 

(D) 

(A) (B) and (D) Only

(B) (A), (B) and (C) Only

(C) (A) and (B) Only

(D) (A) and (C) Only

Correct Answer: (C) (A) and (B) Only

Solution:

Concept:

- Intramolecular hydrogen bonding occurs within a molecule when donor and acceptor groups can approach, commonly at neighbouring positions on a benzene ring.
- Intramolecular bonding reduces intermolecular association and can favour steam volatility. Extensive intermolecular hydrogen bonding raises association and suppresses volatility under the same conditions.

Step 1: Identify the ortho-substituted compounds Compound (A) is ortho-nitrophenol. Its phenolic $O-H$ hydrogen can bond to an oxygen of the adjacent nitro group, producing an intramolecular hydrogen-bonded ring. Compound (B) is ortho-nitroaniline; a hydrogen of NH_2 can similarly bond to an oxygen of the neighbouring nitro group. Both have neighbouring donor and acceptor sites, so hydrogen bonding can take place within one molecule.

Step 2: Examine the para-substituted compounds Compounds (C) and (D) show hydroxyl and amino groups opposite one another on the benzene ring; they depict the para-aminophenol arrangement. These distant sites cannot close a short intramolecular hydrogen-bonded ring. Instead, their polar groups form hydrogen bonds between different molecules. The resulting association lowers their volatility relative to the ortho-nitro compounds and prevents their selection as the steam-volatile pair

here.

Step 3: Select the correct combination The steam-volatile compounds are consequently (A) and (B). The first alternative incorrectly includes (D) and excludes (A). The second incorrectly adds (C), and the fourth incorrectly replaces (B) with (C). The third contains exactly the two ortho-nitro compounds capable of the relevant intramolecular association. Group positions, rather than simply the presence of oxygen or nitrogen, decide the comparison.

Step 4: Conclusion The steam-volatile pair comprises ortho-nitrophenol and ortho-nitroaniline. Hence, the correct option is (C).

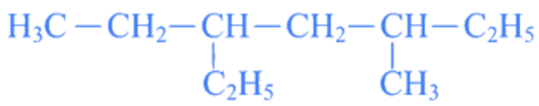
Quick Tip to Solve: For steam-volatility comparisons, mark ortho donor–acceptor pairs first; internal hydrogen bonding usually means less association between molecules.

67. Match List - I with List - II.

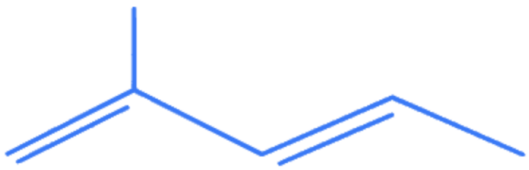
List - I (Structure)	List - II (IUPAC Name)
(A)	(I) 4-Methylpent-1-ene
(B) $(CH_3)_2C(C_3H_7)_2$	(II) 3-Ethyl-5-methylheptane
(C)	(III) 4,4-Dimethylheptane
(D)	(IV) 2-Methyl-1,3-pentadiene

Choose the correct answer from the options given below : [4 marks]

(A)



(C)



(D)



(A) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(B) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(C) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(D) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Correct Answer: (A) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

Solution:

Concept:

- Choose the longest continuous carbon chain containing the maximum number of multiple bonds. Number an unsaturated chain to give its double bonds the lowest possible locants.
- For equally numbered substituent positions, the substituent cited first alphabetically receives the lower locant. Two identical methyl groups are denoted by dimethyl.

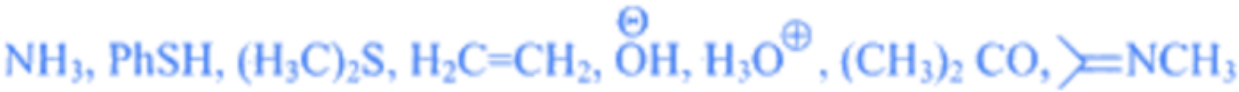
Step 1: Name the two saturated structures In structure (A), the longest chain contains seven carbons. Numbering in either direction gives substituent positions 3 and 5. Since ethyl precedes methyl alphabetically, ethyl receives position 3: the name is 3-ethyl-5-methylheptane, matching (II). In (B), the chain passes through both propyl groups and their common central carbon. It therefore contains seven carbons, with both methyl groups attached at carbon 4. Thus (B) is 4,4-dimethylheptane, matching (III).

Step 2: Name the unsaturated structures Structure (C) contains a five-carbon chain with two double bonds. Numbering from the terminal double bond gives double-bond positions 1 and 3 and the methyl substituent at carbon 2. It is 2-methylpenta-1,3-diene, the compound called 2-methyl-1,3-pentadiene in list (IV). Structure (D) has a five-carbon chain with its double bond at carbon 1 and methyl at carbon 4, so it is 4-methylpent-1-ene, matching (I). Option (B) incorrectly interchanges the monoene and diene; options (C) and (D) already misidentify the saturated structures.

Step 3: Conclusion The complete matching is A–II, B–III, C–IV, D–I. Hence, the correct option is (A).

Quick Tip to Solve: Name the diene and monoene first: their numbers of double bonds immediately distinguish structures (C) and (D).

68. Total number of nucleophiles from the following is : [4 marks]



(A) 4

(B) 6

(C) 5

(D) 7

Correct Answer: (D) 7

Solution:

Concept:

- A nucleophile donates an available electron pair to an electron-deficient centre. The donated electrons can belong to a lone pair or to a carbon-carbon pi bond.
- Neutral compounds can be nucleophiles; a negative charge is not required. In contrast, a protonated species whose reactive atom has no available lone pair is not counted here.

Step 1: Count the lone-pair donors NH_3 has an available lone pair on nitrogen. Thiophenol, $PhSH$, and dimethyl sulfide, $(CH_3)_2S$, have lone pairs on sulfur and can donate them. Hydroxide, OH^-, is an anionic oxygen donor. Acetone, $(CH_3)_2CO$, has oxygen lone pairs and can donate at oxygen, even though its carbonyl carbon is electrophilic. The imine, $(CH_3)_2C{=}NCH_3$, has an available lone pair on nitrogen. These constitute six lone-pair nucleophiles.

Step 2: Include the pi donor and exclude hydronium Ethene, $H_2C{=}CH_2$, supplies its pi-electron pair to an electrophile, so it contributes one more nucleophile. Hydronium, H_3O^+, functions as a proton donor rather than a nucleophile in this classification: its positive charge strongly suppresses electron-pair donation. The total is therefore

$$6 + 1 = 7.$$

Counts of 4, 5 or 6 omit one or more neutral donors, commonly ethene or the oxygen donor acetone. The presence of a carbonyl group does not prevent oxygen itself from being a Lewis base.

Step 3: Conclusion There are 7 electron-pair donors, including oxygen in acetone and nitrogen in the imine. The supplied key lists five, which counts only the conventional strongly nucleophilic reagents and omits these two available lone-pair donors. For the literal question counting nucleophiles, the donor-site analysis gives seven. Hence, the correct option is (D).

Quick Tip to Solve: Inspect each atom separately: acetone can be electrophilic at carbon and nucleophilic at oxygen, while an alkene donates its pi electrons.

---

69. An element 'E' has the ionisation enthalpy value of 374 kJ mol^{-1}. 'E' reacts with elements A, B, C and D with electron gain enthalpy values of -328, -349, -325 and -295 kJ mol^{-1}, respectively. The correct order of the products EA, EB, EC and ED in terms of ionic character is : [4 marks]

(A) EB $>$ EA $>$ EC $>$ ED

(B) EA $>$ EB $>$ EC $>$ ED

(C) ED $>$ EC $>$ EA $>$ EB

(D) ED $>$ EC $>$ EB $>$ EA

Correct Answer: (B) EA $>$ EB $>$ EC $>$ ED

Solution:

Concept:

- With the same electropositive element E, a larger electronegativity difference between E and the halogen generally gives a bond of greater ionic character.
- Electron gain enthalpy and electronegativity are distinct properties. Chlorine has a more negative electron gain enthalpy than fluorine, but fluorine is the most electronegative element.

Step 1: Identify the elements from the data The low ionisation enthalpy, approximately $374\,\text{kJ}\,\text{mol}^{-1}$, identifies E as the strongly electropositive alkali metal caesium. The listed electron gain enthalpies

correspond to the halogens:

$$A = \mathrm{F} \quad (-328), \qquad B = \mathrm{Cl} \quad (-349),$$
$$C = \mathrm{Br} \quad (-325), \qquad D = \mathrm{I} \quad (-295).$$

The values in parentheses are in $\mathrm{kJ\,mol^{-1}}$. Chlorine exceeds fluorine in the magnitude of electron gain enthalpy because electron addition to the compact fluorine atom encounters greater electron-electron repulsion.

Step 2: Compare the ionic characters The halogen electronegativity order is $\mathrm{F} > \mathrm{Cl} > \mathrm{Br} > \mathrm{I}$. Since E is unchanged, the E-halogen electronegativity difference decreases in exactly this order. The larger, more polarisable anions down the group also favour increasing covalent character. Therefore

$$\mathrm{EF} > \mathrm{ECl} > \mathrm{EBr} > \mathrm{EI}$$

in ionic character. Option (A) incorrectly ranks the products only by electron gain enthalpy, placing chloride ahead of fluoride. Options (C) and (D) wrongly place the iodide first.

Step 3: Conclusion The required ionic-character order is $\mathrm{EA} > \mathrm{EB} > \mathrm{EC} > \mathrm{ED}$. The supplied key ranks EB before EA by electron gain enthalpy; that would rank the fluoride below the chloride despite its greater electronegativity difference and smaller, less polarisable anion. For ionic character as asked, the halogen identification gives fluoride first. Hence, the correct option is (B).

Quick Tip to Solve: Use electron gain enthalpies to identify the halogens, then use electronegativity and polarisation to compare the ionic character of their salts.

---

70. Given below are two statements :

Statement (I) : The radii of isoelectronic species increases in the order.

$Mg^{2+} < Na^{+} < F^{-} < O^{2-}$

Statement (II) : The magnitude of electron gain enthalpy of halogen decreases in the order.

Cl > F > Br > I

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is incorrect but Statement II is correct

(B) Both Statement I and Statement II are correct

(C) Statement I is correct but Statement II is incorrect

(D) Both Statement I and Statement II are incorrect

Correct Answer: (B) Both Statement I and Statement II are correct

Solution:

Concept:

- Isoelectronic species have the same electron count. Within such a series, increasing nuclear charge attracts the same electron cloud more strongly and decreases its radius.

- The magnitude of electron gain enthalpy measures the energy released on adding an electron to an isolated gaseous atom; its halogen trend has a fluorine-chlorine exception.

Step 1: Check Statement I using nuclear charges Each listed ion has ten electrons: Mg^{2+} has $12-2=10$, Na^{+} has $11-1=10$, F^{-} has $9+1=10$, and O^{2-} has $8+2=10$. Their nuclear charges decrease from 12 to 11 to 9 to 8. Therefore the attraction experienced by the isoelectronic electron clouds decreases in this sequence, and radii increase:

$$r(Mg^{2+}) < r(Na^{+}) < r(F^{-}) < r(O^{2-}).$$

Statement I is correct; comparison by ionic charge alone is unnecessary when nuclear charges and electron counts are available.

Step 2: Check Statement II including the fluorine exception Approximate electron gain enthalpy magnitudes in $kJ\,mol^{-1}$ are 349 for chlorine, 328 for fluorine, 325 for bromine and 295 for iodine. Thus the printed sequence $Cl > F > Br > I$ is correct. Fluorine has a very small 2p shell, so electron-electron repulsion makes electron addition less favourable than to the larger chlorine atom. Down from chlorine, increasing atomic size reduces the energy released. Both statements are independently true, excluding every option that declares either statement false.

Step 3: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (B).

Quick Tip to Solve: For isoelectronic radii, rank nuclear charges; for halogen electron gain enthalpy, remember that chlorine, not fluorine, has the greatest magnitude.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The molar mass of the water insoluble product formed from the fusion of chromite ore ($FeCr_2O_4$) with Na_2CO_3 in presence of O_2 is _____ g mol^{-1}. [4 marks]

Solution:

Concept:

- Heating chromite with sodium carbonate in oxygen oxidises chromium(III) to chromium(VI), forming sodium chromate. Iron remains in an insoluble iron(III) oxide residue.
- The molar mass of a compound is the sum of its constituent atomic masses multiplied by their subscripts; use $M(Fe)=56$ and $M(O)=16$ in $g\,mol^{-1}$.

Step 1: Identify the products of oxidative fusion Chromite is represented by $FeCr_2O_4$. Its alkaline oxidative fusion produces sodium chromate, iron(III) oxide and carbon dioxide. A balanced equation is

$$4\,FeCr_2O_4 + 8\,Na_2CO_3 + 7\,O_2 \longrightarrow 8\,Na_2CrO_4 + 2\,Fe_2O_3 + 8\,CO_2.$$

The equation conserves four Fe atoms, eight Cr atoms, sixteen Na atoms and eight C atoms. The oxygen atoms total 54 on each side. Oxygen oxidises chromium to the chromate oxidation state and converts the iron component to ferric oxide.

Step 2: Select the insoluble product and calculate its mass Sodium chromate, Na_2CrO_4, is water

soluble and is extracted by leaching the fused mass with water. Carbon dioxide escapes as a gas; it is not the insoluble residue. The water-insoluble solid is Fe_2O_3. One mole of ferric oxide contains two moles of iron atoms and three moles of oxygen atoms. Hence

$$M(Fe_2O_3) = 2(56) + 3(16)$$
$$= 112 + 48 = 160\,g\,mol^{-1}.$$

Using chromite itself or sodium chromate would give the mass of the starting material or of the soluble product, rather than the product requested.

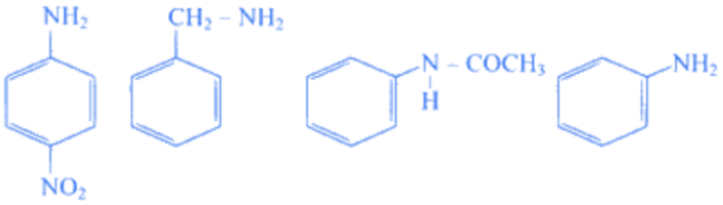
Step 3: Conclusion The insoluble product is Fe_2O_3, with molar mass $160\,g\,mol^{-1}$. The numerical entry is $\boxed{160}$.

Quick Tip to Solve: In chromite processing, distinguish the soluble chromate extracted by water from the insoluble ferric oxide residue before calculating a molar mass.

72. Given below are some nitrogen containing compounds :

Each of them is treated with HCl separately. 1.0 g of the most basic compound will consume _____ mg of HCl.

(Given molar mass in g mol^{-1} C : 12, H : 1, O : 16, Cl : 35.5) [4 marks]



Solution:

Concept:

- An amine accepts a proton using the nitrogen lone pair. Resonance delocalisation of that lone pair reduces its availability and therefore decreases basicity.
- A primary amine with one basic nitrogen consumes one mole of HCl per mole of amine: $RNH_2 + HCl \longrightarrow RNH_3^+\,Cl^-$. Convert mass to moles before using this ratio.

Step 1: Identify the most basic compound The four structures are p-nitroaniline, benzylamine, acetanilide and aniline. In benzylamine, $C_6H_5CH_2NH_2$, the nitrogen is separated from the aromatic ring by a saturated CH_2 group. Its lone pair therefore cannot delocalise directly into the ring and remains available for protonation. In aniline, that pair is delocalised into the ring; a para nitro group withdraws electrons further and makes p-nitroaniline still less basic. In acetanilide, the nitrogen lone pair is strongly delocalised into the carbonyl group. Thus benzylamine is the most basic of the four.

Step 2: Calculate the hydrochloric acid consumed Benzylamine has molecular formula C_7H_9N. Using the standard nitrogen atomic mass 14, its molar mass is

$$M = 7(12) + 9(1) + 14 = 107\,g\,mol^{-1}.$$

Thus $1.0\,\mathrm{g}$ contains $1/107\,\mathrm{mol}$. There is one proton-accepting amino group per molecule, so the same amount of HCl is required. Since $M(\mathrm{HCl}) = 1 + 35.5 = 36.5\,\mathrm{g\,mol^{-1}}$,

$$m(\mathrm{HCl}) = \frac{36.5}{107}\,\mathrm{g} = 341.12\,\mathrm{mg}.$$

Reporting the numerical value to the nearest integer gives 341; using aniline instead would ignore the basicity comparison required first.

Step 3: Conclusion The most basic compound is benzylamine, and $1.0\,\mathrm{g}$ consumes approximately $\boxed{341}\,\mathrm{mg}$ of HCl.

Quick Tip to Solve: A methylene group between nitrogen and a benzene ring interrupts lone-pair resonance; select the strongest base before applying the 1:1 acid ratio.

73. If A_2B is 30% ionised in an aqueous solution, then the value of van't Hoff factor (i) is _____ $\times 10^{-1}$. [4 marks]

Solution:

Concept:

- The van't Hoff factor i is the ratio of the actual number of dissolved solute particles to the number of formula units initially dissolved. Dissociation increases this ratio because one formula unit produces several ions.
- For a solute producing ν ions per dissociated formula unit, $i = 1 + (\nu - 1)\alpha$, where α is its degree of ionisation expressed as a fraction, not as a percentage.

Step 1: Write the ionisation and count particles One formula unit of A_2B produces two A ions and one B ion on complete ionisation, so $\nu = 3$. Take initially one mole of the compound. Since its degree of ionisation is 30%, the amount remaining undissociated is $1 - 0.30 = 0.70$ mol. The ionised fraction produces $2(0.30) = 0.60$ mol of A ions and 0.30 mol of B ions. The charges need not be specified to count the particles.

Step 2: Calculate the factor and convert to the requested form The total amount of dissolved particles is therefore

$$n_{\text{particles}} = 0.70 + 0.60 + 0.30 = 1.60\ \text{mol}.$$

Relative to the initial one mole of formula units, this gives

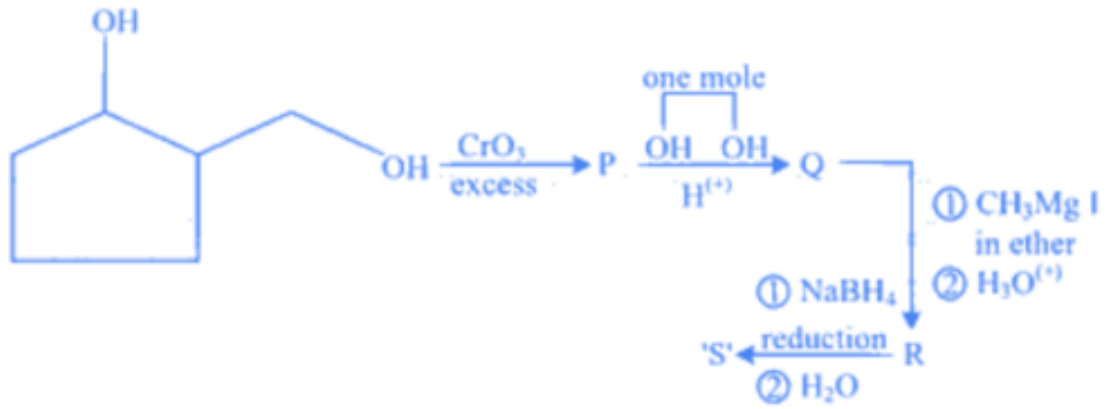
$$i = \frac{1.60}{1} = 1.60 = 16 \times 10^{-1}.$$

Equivalently, the dissociation formula gives $i = 1 + (3 - 1)(0.30) = 1.60$. The blank asks for the coefficient multiplying 10^{-1}, so entering 1.6 would fail to account for that printed scale factor.

Step 3: Conclusion The van't Hoff factor is $i = 1.6$, so the required numerical entry is $\boxed{16}$.

Quick Tip to Solve: Count all ions from one formula unit, use the ionised fraction 0.30, and only then adjust for the printed power of ten.

74. 0.1 mole of compound 'S' will weigh _____ g.
(Given molar mass in g mol^{-1} C : 12, H : 1, O : 16) [4 marks]



Solution:

Concept:

- Excess CrO_3 oxidises a secondary alcohol to a ketone and a primary alcohol to a carboxylic acid. Ethylene glycol protects a ketone as a cyclic ketal in acidic medium.
- A Grignard reagent reacts first with the acidic proton of a carboxylic acid; it does not simply convert that unprotected acid into a tertiary alcohol. A ketal survives the basic Grignard conditions but is hydrolysed during acidic work-up.
- $NaBH_4$ reduces an aldehyde or ketone to an alcohol under ordinary conditions but leaves a carboxylic acid group unchanged. The required mass follows from $m = nM$.

Step 1: Identify the oxidation product and protect its ketone The starting cyclopentane bears an OH group on one ring carbon and a hydroxymethyl group, $-CH_2OH$, on the adjacent carbon. Excess CrO_3 converts the ring alcohol into a ketone and the side-chain primary alcohol into $-COOH$. Thus P is a cyclopentanone bearing an adjacent carboxylic acid substituent. One mole of ethylene glycol reacts with the ketone carbonyl to form its cyclic ketal Q; the acid group remains available.

Step 2: Follow the Grignard step and the final reduction In Q, the ketone is protected, so CH_3MgI cannot add to that carbonyl. Instead, the carboxylic acid proton quenches the Grignard reagent:

$$RCOOH + CH_3MgI \longrightarrow RCOOMgI + CH_4.$$

Acidic work-up restores the carboxylic acid and hydrolyses the ketal, regenerating the ring ketone in R. The subsequent $NaBH_4$, followed by water, reduces this ketone to a secondary alcohol. Consequently S is a cyclopentane with adjacent OH and $-COOH$ substituents, not a methyl-addition product.

Step 3: Count atoms and calculate the mass Cyclopentane has formula C_5H_{10}. Substitution of two ring hydrogens by OH and $-COOH$ gives

$$C_5H_8 + OH + COOH = C_6H_{10}O_3.$$

Using the supplied atomic molar masses,

$$M(S) = 6(12) + 10(1) + 3(16) = 130 \text{ g mol}^{-1},$$

so the mass of the stated amount is

$$m = 0.1 \times 130 = 13 \text{ g}.$$

Any stereochemical mixture formed on reducing the ketone has the same molecular formula and hence the same molar mass.

Step 4: Conclusion The compound S has molecular formula $C_6H_{10}O_3$; 0.1 mole weighs $\boxed{13 \text{ g}}$. The required numerical entry is $\boxed{13}$.

Quick Tip to Solve: In a multistep sequence, track protection and deprotection separately. An unprotected COOH group consumes Grignard reagent by acid-base reaction before any proposed carbonyl addition.

75. The sum of sigma (σ) and pi (π) bonds in Hex-1,3-dien-5-yne is _____. [4 marks]

Solution:

Concept:

- A single bond contains one sigma bond. A double bond contains one sigma and one pi bond, while a triple bond contains one sigma and two pi bonds. A multiple bond must therefore contribute only one sigma bond to the total.
- The name hex-1,3-dien-5-yne specifies a six-carbon open chain, double bonds at positions 1 and 3, and a triple bond at position 5. Carbon completes four covalent bonds, which determines the number of attached hydrogens.

Step 1: Expand the named carbon chain and assign hydrogens The connectivity specified by the locants is

$$CH_2{=}CH{-}CH{=}CH{-}C{\equiv}CH.$$

The first carbon carries two hydrogens. The second, third and fourth carbons each carry one hydrogen. Carbon 5 has a single bond to carbon 4 and a triple bond to carbon 6, so it carries no hydrogen. The terminal carbon 6 carries one hydrogen. Thus there are $2+1+1+1+0+1 = 6$ C–H bonds, consistent with molecular formula C_6H_6.

Step 2: Count sigma and pi contributions separately A six-carbon unbranched chain has five links between neighbouring carbons. Each link contains exactly one sigma bond, irrespective of its multiplicity. All six C–H bonds are also sigma bonds, so

$$N_\sigma = 5 + 6 = 11.$$

The two C=C double bonds provide two pi bonds altogether, and the one triple bond supplies two more:

$$N_\pi = 2(1) + 1(2) = 4.$$

Counting a double bond as two sigma bonds or a triple bond as three sigma bonds would overcount the sigma framework. We require the sum of the two kinds, rather than either count separately.

Step 3: Conclusion The molecule has 11 sigma bonds and 4 pi bonds. Their sum is $11 + 4 = \boxed{15}$, the required numerical entry.

Quick Tip to Solve: First count one sigma bond per linked atom pair, including every C–H bond; then add one pi bond per double bond and two per triple bond.