JEE Main 2025 Paper 1 (B.E./B.Tech) Jan 29 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 29 January 2025, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $\mathrm{S} = \mathbf{N} \cup \{0\}$. Define a relation R from S to R by :

$$\mathrm{R} = \left\{ (x, y) : \log_e y = x \log_e \left(\frac{2}{5} \right), x \in \mathrm{S}, y \in \mathbf{R} \right\}.$$

Then, the sum of all the elements in the range of R is equal to : [4 marks]

(A) $\frac{3}{2}$

(B) $\frac{10}{9}$

(C) $\frac{5}{2}$

(D) $\frac{5}{3}$

Correct Answer: (D) $\frac{5}{3}$

Solution:

Concept:

- The range is the set of distinct second coordinates produced by the allowed inputs. Exponentiating a natural logarithm gives a positive output.
- An infinite geometric series with first term a and ratio r, where $|r| < 1$, has sum $a/(1 - r)$.

Step 1: Determine the range Exponentiate the defining equation to obtain

$$y = e^{x \log_e (2/5)} = \left(\frac{2}{5} \right)^x.$$

Since $S = \{0, 1, 2, \ldots\}$, the range is $\{1, 2/5, 4/25, 8/125, \ldots\}$. These outputs are distinct because the powers of a positive number less than one decrease strictly. In particular, the input zero contributes the first term 1; it must not be omitted.

Step 2: Sum the geometric series The ratio of successive terms is $2/5$, whose absolute value is less than one. Hence the required sum converges and equals

$$1 + \frac{2}{5} + \left(\frac{2}{5} \right)^2 + \cdots = \frac{1}{1 - 2/5} = \frac{5}{3}.$$

None of the other offered values equals this geometric-series sum; using only positive inputs would instead miss the initial term.

Step 3: Conclusion The sum of all elements in the range is $\boxed{5/3}$. Hence, the correct option is (D).

Quick Tip to Solve: Include the input zero first: it supplies 1, the initial term of the geometric series.

2. Let $A = [a_{ij}]$ be a 2×2 matrix such that $a_{ij} \in \{0, 1\}$ for all i and j. Let the random variable X denote the possible values of the determinant of the matrix A. Then, the variance of X is: [4 marks]

(A) $\frac{5}{8}$
(B) $\frac{1}{4}$
(C) $\frac{3}{4}$
(D) $\frac{3}{8}$

Correct Answer: (D) $\frac{3}{8}$

Solution:

Concept:

- Each of the four binary entries has two choices, so there are $2^4 = 16$ equally likely matrices under the usual random selection.
- For $X = ad - bc$, variance is $E(X^2) - [E(X)]^2$; equal determinant values must be weighted by their numbers of matrices.

Step 1: Find the determinant distribution Write $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. The products ad and bc use disjoint entries and are independent. A product equals 1 only when both factors equal 1, with probability $1/4$; it equals 0 with probability $3/4$. Thus

$$P(X = 1) = P(ad = 1, bc = 0) = \frac{3}{16},$$

$$P(X = -1) = \frac{3}{16}, \qquad P(X = 0) = \frac{10}{16}.$$

The corresponding counts are 3, 3 and 10.

Step 2: Compute the variance The positive and negative determinant contributions cancel, giving

$$E(X) = \frac{3 - 3}{16} = 0.$$

Squaring removes the sign, so

$$E(X^2) = \frac{3 + 3}{16} = \frac{3}{8}, \qquad \text{Var}(X) = \frac{3}{8} - 0 = \frac{3}{8}.$$

The alternatives $5/8, 1/4, 3/4$ do not equal the weighted second moment; treating the three determinant values as equally probable would also be incorrect.

Step 3: Conclusion The variance is $\boxed{3/8}$. Hence, the correct option is (D).

Quick Tip to Solve: Count matrices rather than distinct determinant values; the zero determinant occurs ten times out of sixteen.

3. Let a straight line L pass through the point $P(2,-1,3)$ and be perpendicular to the lines $\frac{x-1}{2}=\frac{y+1}{1}=\frac{z-3}{-2}$ and $\frac{x-3}{1}=\frac{y-2}{3}=\frac{z+2}{4}$. If the line L intersects the yz-plane at the point Q, then the distance between the points P and Q is: [4 marks]

(A) $\sqrt{10}$

(B) 2

(C) $2\sqrt{3}$

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- The denominators of a symmetric line equation are its direction ratios. A direction perpendicular to two nonparallel directions is their cross product.
- The yz-plane has equation $x=0$. A line through P with direction $\mathbf{d}$ is $\mathbf{r}=\mathbf{P}+t\mathbf{d}$.

Step 1: Find a direction for the required line The two given directions are $\mathbf{u}=(2,1,-2)$ and $\mathbf{v}=(1,3,4)$. Their cross product is

$$\mathbf{u}\times\mathbf{v}=(10,-10,5)=5(2,-2,1).$$

We may therefore use $\mathbf{d}=(2,-2,1)$. As a check, its dot products with both given directions are zero: $4-2-2=0$ and $2-6+4=0$.

Step 2: Locate the plane intersection and measure the distance The line is

$$x=2+2t,\qquad y=-1-2t,\qquad z=3+t.$$

At the yz-plane, $2+2t=0$, giving $t=-1$ and $Q=(0,1,2)$. Hence

$$PQ=\sqrt{(0-2)^2+(1+1)^2+(2-3)^2}=\sqrt{4+4+1}=3.$$

The three coordinate changes must all enter the distance. The other options give squared lengths 10, 4 and 12, whereas the actual squared length is 9.

Step 3: Conclusion The distance is $\boxed{3}$. Hence, the correct option is (D).

Quick Tip to Solve: Use a simplified cross product as the line direction, then set x equal to zero to reach the yz-plane.

4. If the domain of the function $\log_5(18x - x^2 - 77)$ is (α, β) and the domain of the function $\log_{(x-1)}\left(\frac{2x^2+3x-2}{x^2-3x-4}\right)$ is (γ, δ), then $\alpha^2 + \beta^2 + \gamma^2$ is equal to: [4 marks]

(A) 186

(B) 179

(C) 195

(D) 174

Correct Answer: (A) 186

Solution:

Concept:

- A real logarithm requires a strictly positive argument. A variable base must additionally be positive and different from 1.
- Factor a rational argument and use its sign intervals without including any zero of its numerator or denominator.

Step 1: Find the first domain The base 5 already satisfies the base conditions. Therefore

$$18x - x^2 - 77 > 0 \iff -(x-7)(x-11) > 0.$$

The argument is positive strictly between the roots, so the domain is $(7, 11)$. Thus $\alpha = 7$ and $\beta = 11$. Both endpoints are excluded because the logarithm of zero is undefined.

Step 2: Combine the second argument and base conditions Factor the second argument:

$$\frac{2x^2 + 3x - 2}{x^2 - 3x - 4} = \frac{(2x-1)(x+2)}{(x-4)(x+1)}.$$

Its positivity intervals are $(-\infty, -2)$, $(-1, 1/2)$ and $(4, \infty)$. The base condition $x - 1 > 0$ requires $x > 1$, and $x - 1 \neq 1$ excludes $x = 2$. Intersecting these conditions leaves only $(4, \infty)$, so $\gamma = 4$.

Step 3: Evaluate the requested expression Only the finite lower endpoint γ is needed; there is no need to assign a finite value to the upper endpoint. We get

$$\alpha^2 + \beta^2 + \gamma^2 = 7^2 + 11^2 + 4^2 = 49 + 121 + 16 = 186.$$

The other numerical options do not equal this sum.

Step 4: Conclusion The requested value is $\boxed{186}$. Hence, the correct option is (A).

Quick Tip to Solve: For a variable logarithm base, intersect the argument positivity set with base positivity and base not equal to one.

5. Let $\hat{a}$ be a unit vector perpendicular to the vectors $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} - \hat{k}$, and $\hat{a}$ makes an angle of $\cos^{-1}\left(-\frac{1}{3}\right)$ with the vector $\hat{i} + \hat{j} + \hat{k}$. If $\hat{a}$ makes an angle of $\frac{\pi}{3}$ with the vector $\hat{i} + \alpha\hat{j} + \hat{k}$, then the value of a is: [4 marks]

(A) $\sqrt{3}$

(B) $\sqrt{6}$

(C) $-\sqrt{6}$

(D) $-\sqrt{3}$

Correct Answer: (C) $-\sqrt{6}$

Solution:

Concept:

- A unit vector perpendicular to two nonparallel vectors is either sign of their normalized cross product.
- The angle formula is $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos\theta$. Keep its sign before squaring an equation.

Step 1: Choose the correct perpendicular unit vector For the given vectors,

$$\mathbf{b} \times \mathbf{c} = (-7, 7, 7).$$

The two possible unit vectors are $\pm(-1, 1, 1)/\sqrt{3}$. The specified cosine with $(1, 1, 1)$ is $-1/3$. The vector $(-1, 1, 1)/\sqrt{3}$ gives cosine $+1/3$, so the required choice is

$$\hat{a} = \frac{(1, -1, -1)}{\sqrt{3}}.$$

Its length is one and its dot products with $\mathbf{b}$ and $\mathbf{c}$ vanish.

Step 2: Use the second angle condition With $\mathbf{v} = (1, \alpha, 1)$, we have $\hat{a} \cdot \mathbf{v} = -\alpha/\sqrt{3}$ and $|\mathbf{v}| = \sqrt{\alpha^2 + 2}$. Thus

$$\frac{-\alpha}{\sqrt{3}\sqrt{\alpha^2 + 2}} = \frac{1}{2}.$$

The right side is positive, so $\alpha < 0$. Squaring yields

$$4\alpha^2 = 3(\alpha^2 + 2), \qquad \alpha^2 = 6.$$

Therefore the negative root must be selected. The positive root would give the supplementary angle, while the options involving $\sqrt{3}$ fail the magnitude equation.

Step 3: Conclusion The required value is $\boxed{\alpha = -\sqrt{6}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the first angle to fix the cross-product sign, and retain the second cosine equation sign before squaring.

6. Let α, β $(\alpha \neq \beta)$ be the values of m, for which the equations $x+y+z=1$, $x+2y+4z=m$ and $x+4y+10z=m^2$ have infinitely many solutions. Then the value of $\sum_{n=1}^{10}(n^\alpha+n^\beta)$ is equal to : [4 marks]

(A) 3410

(B) 560

(C) 3080

(D) 440

Correct Answer: (D) 440

Solution:

Concept:

- Infinitely many solutions require the coefficient and augmented matrices to have the same rank, strictly smaller than the number of unknowns.
- When one coefficient row is a linear combination of the others, its right-hand side must obey exactly the same combination.

Step 1: Impose the consistency condition The third coefficient row satisfies

$$(1,4,10) = 3(1,2,4) - 2(1,1,1).$$

Therefore the third equation can agree with the first two only when

$$m^2 = 3m - 2 \iff (m-1)(m-2) = 0.$$

Thus the two distinct possible values of m are 1 and 2. The first two coefficient rows are independent, so their rank is 2. For either value the augmented rank is also 2, leaving one free variable among three unknowns; hence the solutions are indeed infinite.

Step 2: Evaluate the power sum The order of α and β is immaterial because they appear symmetrically. Taking them as 1 and 2 gives

$$\sum_{n=1}^{10}(n^\alpha+n^\beta) = \sum_{n=1}^{10} n + \sum_{n=1}^{10} n^2.$$

Use the standard finite sums:

$$\frac{10\cdot 11}{2} + \frac{10\cdot 11\cdot 21}{6} = 55 + 385 = 440.$$

None of 3410, 560 or 3080 equals this sum. Merely setting the coefficient determinant to zero would be insufficient, since that determinant is already zero for every m.

Step 3: Conclusion The required sum is $\boxed{440}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the row dependence first; the same relation on the right-hand sides determines the admissible parameter values.

7. If the set of all $a \in \mathbf{R}$, for which the equation $2x^2 + (a-5)x + 15 = 3a$ has no real root, is the interval (α, β), and $X = |x \in Z; \alpha < x < \beta|$, then $\sum_{x \in X} x^2$ is equal to: [4 marks]

(A) 2139

(B) 2119

(C) 2109

(D) 2129

Correct Answer: (A) 2139

Solution:

Concept:

- A real quadratic has no real roots exactly when its discriminant is negative. Here the coefficient of x^2 is the fixed nonzero number 2.
- The strict interval excludes its endpoints. The sum of the first n positive squares is $n(n+1)(2n+1)/6$.

Step 1: Apply the discriminant condition Bring every term to the left before identifying the constant term:

$$2x^2 + (a-5)x + (15-3a) = 0.$$

Thus

$$D = (a-5)^2 - 8(15-3a) = a^2 + 14a - 95 = (a+19)(a-5).$$

A monic quadratic in a is negative between its two roots, so $\alpha = -19$, $\beta = 5$. At either endpoint a repeated real root exists, so the inequalities must remain strict.

Step 2: Sum the squares of the allowed integers The allowed integers are $-18, -17, \ldots, -1, 0, 1, 2, 3, 4$. Negative signs disappear after squaring, but the positive integers 1 through 4 occur in addition to their negative counterparts. Hence

$$\sum_{x \in X} x^2 = \sum_{k=1}^{18} k^2 + \sum_{k=1}^{4} k^2 = \frac{18 \cdot 19 \cdot 37}{6} + 30 = 2109 + 30 = 2139.$$

Option (C) omits the additional positive squares; (B) and (D) also fail this exact sum.

Step 3: Conclusion The requested sum is 2139. Hence, the correct option is (A).

Quick Tip to Solve: After finding an open interval, list its extreme allowed integers before applying the sum-of-squares formula; this prevents endpoint and double-counting errors.

8. Let the line $x + y = 1$ meet the axes of x and y at A and B, respectively. A right angled triangle AMN is inscribed in the triangle OAB, where O is the origin and the points M and N lie on the lines OB and AB, respectively. If the area of the triangle AMN is $\frac{4}{9}$ of the area of the triangle OAB and $AN : NB = \lambda : 1$, then the sum of all possible value(s) of λ is: [4 marks]

(A) $\frac{1}{2}$

(B) $\frac{5}{2}$

(C) 2

(D) $\frac{13}{6}$

Correct Answer: The right-angle vertex is unspecified; counting every inscribed right triangle gives a sum absent from the options.

Solution:

Concept:

- An inscribed triangle has its vertices on the sides of the containing triangle. The location of its right angle must be checked rather than silently prescribed.
- Use coordinates $A = (1, 0), B = (0, 1), M = (0, m), N = (1 - t, t)$, with $0 < m, t < 1$. Then $\lambda = t/(1 - t)$, and the area ratio is $t(1 - m)$.

Step 1: Apply the area condition and test the angle at N The determinant of $\overrightarrow{AM} = (-1, m)$ and $\overrightarrow{AN} = (-t, t)$ gives

$$t(1 - m) = \frac{4}{9}.$$

A right angle at N requires $\overrightarrow{NA} \cdot \overrightarrow{NM} = t(2t - 1 - m) = 0$, hence $m = 2t - 1$. Substitution gives $2t(1 - t) = 4/9$, so $t = 1/3$ or $2/3$. Only $t = 2/3$ has $m \geq 0$. This yields $m = 1/3$, $\lambda = 2$. If the right angle were specified at N, option (C) would follow.

Step 2: Include the other permitted right-angle vertex A right angle at M is also possible. The perpendicularity condition is

$$(1, -m) \cdot (1 - t, t - m) = 1 + m^2 - t(1 + m) = 0.$$

Thus $t = (1 + m^2)/(1 + m)$. Together with the area condition this gives

$$9m^3 - 9m^2 + 13m - 5 = 0.$$

This polynomial is negative at 0 and positive at 1, and its derivative $27m^2 - 18m + 13 = 27(m - 1/3)^2 + 10$ is strictly positive. It therefore has exactly one root in $(0, 1)$, giving a valid second inscribed triangle. Its t is also strictly between 0 and 1.

Step 3: Compare the literal sum with every offered choice Eliminating m using $m = (5\lambda - 4)/(9\lambda)$ gives

$$10\lambda^3 - 51\lambda^2 + 12\lambda - 8 = 0.$$

For the unique admissible M-case root, this polynomial changes sign between $24/5$ and 5, so $24/5 < \lambda_M < 5$. The angle at A cannot be right, since $\overrightarrow{AM} \cdot \overrightarrow{AN} = t(1 + m) > 0$. Thus all cases have been

checked, and their sum obeys

$$\frac{34}{5} < 2 + \lambda_M < 7.$$

Every listed option is at most $5/2$, so none equals the literal sum.

Step 4: Conclusion As worded, the question has no correct listed option: its sum is $2 + \lambda_M$, where λ_M is the unique admissible root above, and lies between $34/5$ and 7. Option (C), 2, results only if the right angle is additionally restricted to N.

Quick Tip to Solve: When a right triangle is named without specifying its right-angle vertex, test all three vertices with dot products before selecting an option.

9. Bag 1 contains 4 white balls and 5 black balls, and Bag 2 contains *n* white balls and 3 black balls. One ball is drawn randomly from Bag 1 and transferred to Bag 2. A ball is then drawn randomly from Bag 2. If the probability, that the ball drawn is white, is $\frac{29}{45}$, then *n* is equal to: [4 marks]

(A) 5

(B) 6

(C) 4

(D) 3

Correct Answer: (B) 6

Solution:

Concept:

- Condition on the colour transferred from Bag 1. These two events are mutually exclusive and exhaustive.
- After either transfer Bag 2 contains $n + 4$ balls. Only a white transfer increases its white-ball count.

Step 1: Calculate the two conditional probabilities The transfer is white with probability $4/9$, and then Bag 2 has $n + 1$ white balls. The transfer is black with probability $5/9$, and then Bag 2 still has n white balls. Therefore the law of total probability gives

$$P(W) = \frac{4}{9}\frac{n+1}{n+4} + \frac{5}{9}\frac{n}{n+4} = \frac{9n+4}{9(n+4)}.$$

The denominator is the same in both cases because exactly one ball is transferred, whatever its colour.

Step 2: Use the given probability and check the result Equating the expression to the stated probability yields

$$\frac{9n+4}{9(n+4)} = \frac{29}{45} \quad \Longrightarrow \quad 5(9n+4) = 29(n+4).$$

$$45n + 20 = 29n + 116 \quad \Longrightarrow \quad 16n = 96 \quad \Longrightarrow \quad n = 6.$$

For this value, the conditional white probabilities are $7/10$ and $6/10$, giving $(4 \cdot 7 + 5 \cdot 6)/90 = 58/90 = 29/45$. The linear equation has only one solution, excluding 5, 4 and 3 in the other options.

Step 3: Conclusion The initial number of white balls in Bag 2 is $n = 6$. Hence, the correct option is (B).

Quick Tip to Solve: Write the total number of balls after transfer first; the colour changes the numerator, while the denominator remains the same.

10. The remainder, when 7^{103} is divided by 23, is equal to: [4 marks]

(A) 9

(B) 6

(C) 14

(D) 17

Correct Answer: (C) 14

Solution:

Concept:

- Congruent integers give the same remainder under addition and multiplication; reducing powers early keeps the numbers small.
- The required remainder is the unique integer from 0 to 22 congruent to 7^{103} modulo 23.

Step 1: Find a short power congruent to minus one Compute successive convenient powers modulo 23:

$$7^2 = 49 \equiv 3, \qquad 7^3 \equiv 21 \equiv -2 \pmod{23}.$$

Cubing the second relation gives $7^9 \equiv -8$. Multiplying this by $7^2 \equiv 3$ gives

$$7^{11} \equiv -24 \equiv -1 \pmod{23}.$$

This small-power cycle avoids evaluating the large integer directly.

Step 2: Separate the exponent and reduce the final value Since $103 = 9 \cdot 11 + 4$,

$$7^{103} = (7^{11})^9\, 7^4 \equiv (-1)^9 (7^2)^2 \equiv -9 \equiv 14 \pmod{23}.$$

We add 23 to the negative representative to obtain the ordinary nonnegative remainder. The residue 9 is the value of 7^4, but it misses the minus sign contributed by the odd power of 7^{11}. Neither 6 nor 17 is congruent to -9 modulo 23.

Step 3: Conclusion The remainder is 14. Hence, the correct option is (C).

Quick Tip to Solve: Look for a small power congruent to -1; an odd number of such factors changes the sign of the remaining residue.

11. If $\alpha x + \beta y = 109$ is the equation of the chord of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, whose mid point is $\left(\frac{5}{2}, \frac{1}{2}\right)$. then $\alpha + \beta$ is equal to : [4 marks]

(A) 37

(B) 46

(C) 72

(D) 58

Correct Answer: (D) 58

Solution:

Concept:

- For the ellipse $x^2/a^2 + y^2/b^2 = 1$, the chord with midpoint (h, k) has equation $hx/a^2 + ky/b^2 = h^2/a^2 + k^2/b^2$.
- This is the midpoint form $T = S_1$; the constant on the right is evaluated at the midpoint, not set equal to 1.

Step 1: Substitute the midpoint into the chord formula Here $a^2 = 9$, $b^2 = 4$, $h = 5/2$, and $k = 1/2$. Consequently

$$\frac{5x}{18} + \frac{y}{8} = \frac{25}{36} + \frac{1}{16} = \frac{100 + 9}{144} = \frac{109}{144}.$$

The midpoint is inside the ellipse, since $109/144 < 1$, so it indeed defines a nondegenerate chord.

Step 2: Normalize the constant and compare coefficients Multiply the entire equation, including both terms on the left, by 144:

$$40x + 18y = 109.$$

Its right-hand side now matches the prescribed constant, so the coefficients can be compared directly: $\alpha = 40$, $\beta = 18$. Their sum is

$$\alpha + \beta = 40 + 18 = 58.$$

As a check, substitution of $(5/2, 1/2)$ gives $100 + 9 = 109$. The other proposed sums 37, 46 and 72 cannot arise from these uniquely normalized coefficients.

Step 3: Conclusion We obtain $\alpha + \beta = 58$. Hence, the correct option is (D).

Quick Tip to Solve: Use the midpoint chord equation rather than the tangent equation; its right side is the ellipse expression evaluated at the midpoint.

12. Let a circle C pass through the points (4, 2) and (0, 2), and its centre lie on 3x + 2y + 2 = 0. Then the length of the chord, of the circle C, whose mid-point is (1, 2), is: [4 marks]

(A) $4\sqrt{2}$

(B) $2\sqrt{2}$

(C) $2\sqrt{3}$

(D) $\sqrt{3}$

Correct Answer: (C) $2\sqrt{3}$

Solution:

Concept:

- The centre of a circle lies on the perpendicular bisector of every chord. The two given points have the same ordinate, so their perpendicular bisector is vertical.
- If the radius is r and the distance from the centre to a chord midpoint is d, its full length is $2\sqrt{r^2 - d^2}$.

Step 1: Determine the centre and radius The midpoint of the given points $(4, 2)$ and $(0, 2)$ is $(2, 2)$. Their joining line is horizontal, so the centre has abscissa 2. Substitution into its given line gives

$$3(2) + 2y + 2 = 0 \quad \Longrightarrow \quad y = -4.$$

Thus the centre is $C_0 = (2, -4)$. Its squared distance from either given point is

$$r^2 = (4 - 2)^2 + (2 + 4)^2 = 4 + 36 = 40.$$

Step 2: Use the midpoint of the requested chord Let $P = (1, 2)$ be the midpoint of the requested chord, which is distinct from the chord joining the original pair. The centre-to-midpoint segment is perpendicular to this chord, and

$$d^2 = C_0P^2 = (1 - 2)^2 + (2 + 4)^2 = 1 + 36 = 37.$$

The right triangle formed by a radius and half the chord gives half-length $\sqrt{40 - 37} = \sqrt{3}$. Hence the full length is $2\sqrt{3}$. Option (D) gives only the half-length, while (A) and (B) use an incorrect centre-to-chord distance.

Step 3: Conclusion The chord has length $2\sqrt{3}$. Hence, the correct option is (C).

Quick Tip to Solve: Find the centre from the perpendicular bisector first, then use the specified chord midpoint; do not confuse it with the midpoint of the original two points.

13. If for the solution curve $y = f(x)$ of the differential equation $\frac{dy}{dx} + (\tan x)y = \frac{2+\sec x}{(1+2\sec x)^2}$, $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, $f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{10}$, then $f\left(\frac{\pi}{4}\right)$ is equal to: [4 marks]

(A) $\frac{5-\sqrt{3}}{2\sqrt{2}}$

(B) $\frac{4-\sqrt{2}}{14}$

(C) $\frac{9\sqrt{3}+3}{10(4+\sqrt{3})}$

(D) $\frac{\sqrt{3}+1}{10(4+\sqrt{3})}$

Correct Answer: (B) $\frac{4-\sqrt{2}}{14}$

Solution:

Concept:

- A linear equation $y' + P(x)y = Q(x)$ is solved by multiplying by the integrating factor $e^{\int P(x)\,dx}$.
- On the given interval, $\cos x > 0$, so the integrating factor for $P(x) = \tan x$ is $\sec x$, without an absolute-value ambiguity.

Step 1: Apply the integrating factor Multiplying the differential equation by $\sec x$ makes its left side an exact derivative:

$$\frac{d}{dx}(y\sec x) = \frac{\sec x(2+\sec x)}{(1+2\sec x)^2} = \frac{2\cos x + 1}{(2+\cos x)^2}.$$

The denominator suggests differentiating a quotient involving $\sin x$, since differentiating $2 + \cos x$ gives $-\sin x$. Indeed,

$$\frac{d}{dx}\left(\frac{\sin x}{2+\cos x}\right) = \frac{\cos x(2+\cos x) + \sin^2 x}{(2+\cos x)^2} = \frac{2\cos x + 1}{(2+\cos x)^2}.$$

Thus $y\sec x = \sin x/(2+\cos x) + C$.

Step 2: Use the initial value and evaluate At $x = \pi/3$, the given value yields

$$y\sec x = \frac{\sqrt{3}}{10}\cdot 2 = \frac{\sqrt{3}}{5}, \qquad \frac{\sin x}{2+\cos x} = \frac{\sqrt{3}/2}{5/2} = \frac{\sqrt{3}}{5}.$$

Consequently $C = 0$, and the required solution is

$$f(x) = \frac{\sin x \cos x}{2+\cos x}.$$

Substitute $x = \pi/4$, then rationalise:

$$f(\pi/4) = \frac{1/2}{2+1/\sqrt{2}} = \frac{1}{4+\sqrt{2}} = \frac{4-\sqrt{2}}{14}.$$

This exact value matches only (B); the expressions in (A), (C) and (D) do not equal this rationalised form.

Step 3: Conclusion $f(\pi/4) = (4-\sqrt{2})/14$. Hence, the correct option is (B).

Quick Tip to Solve: After multiplying by $\sec x$, rewrite everything in terms of $\cos x$; recognise the derivative of $\sin x/(2+\cos x)$.

14. Let $A = [a_{ij}]$ be a matrix of order 3×3, with $a_{ij} = (\sqrt{2})^{i+j}$. If the sum of all the elements in the third row of A^2 is $\alpha + \beta\sqrt{2}, \alpha, \beta \in$ Z, then $\alpha + \beta$ is equal to : [4 marks]

(A) 210

(B) 280

(C) 224

(D) 168

Correct Answer: (C) 224

Solution:

Concept:

- If $A = uu^T$, then $A^2 = u(u^Tu)u^T = (u^Tu)A$. Here u^Tu is a scalar.
- The entry $(\sqrt{2})^{i+j}$ factors as $(\sqrt{2})^i(\sqrt{2})^j$, so the matrix has precisely this outer-product form.

Step 1: Factor the matrix and square it Put $u = (\sqrt{2}, 2, 2\sqrt{2})^T$. The three components are the first, second and third powers of $\sqrt{2}$, respectively, hence

$$A = uu^T = \begin{pmatrix} 2 & 2\sqrt{2} & 4 \\ 2\sqrt{2} & 4 & 4\sqrt{2} \\ 4 & 4\sqrt{2} & 8 \end{pmatrix}.$$

When multiplying two copies of this outer product, the middle row-vector times column-vector is a number:

$$u^Tu = 2 + 4 + 8 = 14, \qquad A^2 = 14A.$$

This reduces nine separate entry calculations to one scalar multiplication. It is matrix multiplication, rather than squaring each entry independently.

Step 2: Add the required row and compare coefficients The third row of A is $(4, 4\sqrt{2}, 8)$. Therefore the third row of A^2 is $(56, 56\sqrt{2}, 112)$, whose sum is

$$56 + 56\sqrt{2} + 112 = 168 + 56\sqrt{2}.$$

The representation in the form $\alpha + \beta\sqrt{2}$, with integral coefficients, is unique because $\sqrt{2}$ is irrational. Thus $\alpha = 168$ and $\beta = 56$, giving

$$\alpha + \beta = 168 + 56 = 224.$$

Option (D) is only the rational part α, and omits the contribution from β. Options (A) and (B) give neither the sum of these coefficients nor the computed third-row sum.

Step 3: Conclusion $\alpha = 168$, $\beta = 56$, and $\alpha + \beta = 224$. Hence, the correct option is (C).

Quick Tip to Solve: Whenever matrix entries separate as $a_{ij} = u_iu_j$, use $A^2 = (\sum u_i^2)A$ before calculating the requested row.

15. If $\sin x + \sin^2 x = 1$, $x \in \left(0, \frac{\pi}{2}\right)$, then
$(\cos^{12} x + \tan^{12} x) + 3(\cos^{10} x + \tan^{10} x + \cos^8 x + \tan^8 x) + (\cos^6 x + \tan^6 x)$ **is equal to:** [4 marks]

(A) 3

(B) 4

(C) 2

(D) 1

Correct Answer: (C) 2

Solution:

Concept:

- Use $\sin^2 x + \cos^2 x = 1$ together with the given relation before expanding any high powers.
- The identity $a^3 + 3a^2 + 3a + 1 = (a+1)^3$ groups the resulting polynomial efficiently.

Step 1: Reduce both trigonometric bases to one variable Let $s = \sin x$. Since $0 < x < \pi/2$, $s > 0$, so division by s is valid. The hypothesis says

$$s + s^2 = 1, \qquad \cos^2 x = 1 - s^2 = s.$$

Also,

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{s^2}{s} = s.$$

Thus both even-power bases $\cos^2 x$ and $\tan^2 x$ are equal to the same positive number. In particular, $\cos^{2k} x + \tan^{2k} x = 2s^k$. There is no need to calculate the angle or substitute a decimal approximation to the positive root of the quadratic.

Step 2: Factor the complete expression Apply the even-power identity separately to the powers twelve, ten, eight and six. The requested expression becomes

$$E = 2s^6 + 3(2s^5 + 2s^4) + 2s^3 = 2s^3(s^3 + 3s^2 + 3s + 1).$$

The coefficients inside the parentheses are those of a cube, so

$$E = 2s^3(s+1)^3 = 2\{s(s+1)\}^3.$$

But the original equation gives $s(s+1) = s^2 + s = 1$. Hence $E = 2$. Retaining the coefficient two is essential: both $\cos^{2k} x$ and $\tan^{2k} x$ contribute. Dropping one would incorrectly give option (D). Neither 3 in (A) nor 4 in (B) agrees with the exact factorisation.

Step 3: Conclusion The complete expression equals 2. Hence, the correct option is (C).

Quick Tip to Solve: First prove $\cos^2 x = \tan^2 x = \sin x$, then look for the binomial cube in the high-power expression.

16. Let $f(x) = \int_0^x t\left(t^2 - 9t + 20\right) dt, 1 \le x \le 5$. If the range of f is $[\alpha, \beta]$, then $4(\alpha + \beta)$ equals : [4 marks]

(A) 253

(B) 157

(C) 154

(D) 125

Correct Answer: (B) 157

Solution:

Concept:

- By the fundamental theorem of calculus, $f\prime(x)$ equals the integrand evaluated at the upper limit.
- A continuous function on a closed interval attains its extrema at endpoints or interior critical points; all such candidates must be compared.

Step 1: Find the derivative and critical points The integrand factors as $t(t-4)(t-5)$. Therefore

$$f\prime(x) = x(x-4)(x-5).$$

Within $1 \le x \le 5$, the only interior stationary point is $x = 4$. The root $x = 0$ lies outside the interval, while $x = 5$ is already an endpoint. For $1 < x < 4$, both $x - 4$ and $x - 5$ are negative, so $f\prime(x) > 0$. For $4 < x < 5$, the derivative is negative. Thus the function rises to $x = 4$ and then falls; its minimum must still be decided by checking both endpoints.

Step 2: Evaluate every candidate and obtain the range Integrate the polynomial using the fixed lower limit zero:

$$f(x) = \left[\frac{t^4}{4} - 3t^3 + 10t^2\right]_0^x = \frac{x^4}{4} - 3x^3 + 10x^2.$$

The candidate values are

$$f(1) = \frac{29}{4}, \qquad f(4) = 32, \qquad f(5) = \frac{125}{4}.$$

Hence the smallest is $29/4$ and the largest is 32. Continuity ensures every value between these extremes occurs, so $\alpha = 29/4$, $\beta = 32$. Consequently

$$4(\alpha + \beta) = 4\left(\frac{29}{4} + 32\right) = 29 + 128 = 157.$$

The numbers 253, 154 and 125 in the other options do not result from the global minimum and maximum; in particular 125 is just $4f(5)$.

Step 3: Conclusion The range is $[29/4, 32]$, and $4(\alpha + \beta) = 157$. Hence, the correct option is (B).

Quick Tip to Solve: For an integral with variable upper limit, find extrema from the integrand, but compare both endpoints as well as interior stationary points.

17. Let the area enclosed between the curves $|y| = 1 - x^2$ and $x^2 + y^2 = 1$ be α. If $9\alpha = \beta\pi + \gamma; \beta, \gamma$ are integers, then the value of $|\beta - \gamma|$ equals: [4 marks]

(A) 15

(B) 18

(C) 33

(D) 27

Correct Answer: (C) 33

Solution:

Concept:

- The equation $|y| = 1 - x^2$ requires $-1 \le x \le 1$ and comprises the two branches $y = \pm(1 - x^2)$.
- The circle has upper and lower branches $y = \pm\sqrt{1 - x^2}$. Symmetry lets us calculate the total area between the curves from one quadrant.

Step 1: Locate intersections and identify the outer boundary Set $u = 1 - x^2$, where $0 \le u \le 1$. Substitution of $y = \pm u$ into the circle gives

$$x^2 + y^2 = 1 - u + u^2 = 1 \quad \Longrightarrow \quad u(u - 1) = 0.$$

Thus intersections occur at $(\pm 1, 0)$ and $(0, \pm 1)$. For interior values, $\sqrt{u} \ge u$, so the circular arc lies outside the corresponding parabolic arc. The area between the curves is the part inside the circle and outside the region enclosed by the two parabola branches. The same relation holds in all four quadrants, with equality only at their boundary intersections.

Step 2: Integrate the difference and extract the integers The total area enclosed by the parabola branches is

$$A_p = \int_{-1}^{1} \{(1 - x^2) - [-(1 - x^2)]\}\, dx = 4 \int_{0}^{1} (1 - x^2)\, dx = \frac{8}{3}.$$

Since this entire region is inside the unit circle, subtract it from the circle's area:

$$\alpha = \pi - \frac{8}{3}, \qquad 9\alpha = 9\pi - 24.$$

Equivalently, integrating $4[\sqrt{1 - x^2} - (1 - x^2)]$ from 0 to 1 gives the same result. Comparing with $\beta\pi + \gamma$ gives $\beta = 9$ and $\gamma = -24$, including the negative sign. Therefore

$$|\beta - \gamma| = |9 - (-24)| = 33.$$

Option (A), 15, results from overlooking the negative sign of γ. The other values, 18 and 27, likewise do not match these exact coefficients.

Step 3: Conclusion $\alpha = \pi - 8/3$, so $\beta = 9$, $\gamma = -24$, and $|\beta - \gamma| = 33$. Hence, the correct option is (C).

Quick Tip to Solve: For these symmetric curves, subtract the area between the two parabolas from the unit-circle area, and keep the sign of the constant coefficient.

18. Let P be the foot of the perpendicular from the point $(1,2,2)$ on the line $\mathrm{L}: \frac{x-1}{1}=\frac{y+1}{-1}=\frac{z-2}{2}$. Let the line $\vec{r}=(-\hat{i}+\hat{j}-2\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k}), \lambda \in$ R, intersect the line L at Q . Then $2(\mathrm{PQ})^2$ is equal to : [4 marks]

(A) 25

(B) 27

(C) 19

(D) 29

Correct Answer: (B) 27

Solution:

Concept:

- A point on L can be written $B+t\mathbf{d}$, with $B=(1,-1,2)$ and direction $\mathbf{d}=(1,-1,2)$.
- The foot of a perpendicular from A satisfies $(P-A)\cdot\mathbf{d}=0$. Distances between points on one line can be calculated from their parameter difference.

Step 1: Find the perpendicular foot Let $A=(1,2,2)$. Since $P=B+t\mathbf{d}$, its coordinates are $(1+t,-1-t,2+2t)$. The perpendicularity condition gives

$$(P-A)\cdot\mathbf{d}=(t,-3-t,2t)\cdot(1,-1,2)=6t+3=0.$$

Hence $t_P=-1/2$, and

$$P=\left(\frac{1}{2},-\frac{1}{2},1\right).$$

The direction vector's squared length is $1+1+4=6$. Notice that the given point A itself is not on L; using it in place of the foot would change the distance asked for.

Step 2: Find the intersection and compute the squared distance The other line has coordinates $(-1+\lambda,1-\lambda,-2+\lambda)$. At its intersection with L, equality of the first and third coordinates yields

$$\lambda=t+2, \qquad \lambda=4+2t.$$

Thus $t_Q=-2$ and $\lambda=0$, giving $Q=(-1,1,-2)$. The second coordinate also agrees, verifying the intersection. The parameter separation is

$$t_Q-t_P=-2+\frac{1}{2}=-\frac{3}{2}.$$

Both points lie on the same line, so

$$PQ^2=(t_Q-t_P)^2\|\mathbf{d}\|^2=\frac{9}{4}\cdot 6=\frac{27}{2}, \qquad 2PQ^2=27.$$

The coordinate difference $Q-P=(-3/2,3/2,-3)$ gives the same squared length directly. Hence 25, 19 and 29, the other offered values, are excluded by both distance calculations.

Step 3: Conclusion $PQ^2=27/2$, and therefore $2(PQ)^2=27$. Hence, the correct option is (B).

Quick Tip to Solve: Once both points are parametrised on the same line, use squared parameter difference times squared direction length instead of expanding three coordinate squares.

19. If all the words with or without meaning made using all the letters of the word "KANPUR" are arranged as in a dictionary, then the word at 440th position in this arrangement is : [4 marks]

(A) PRNAKU

(B) PRKAUN

(C) PRKANU

(D) PRNAUK

Correct Answer: (B) PRKAUN

Solution:

Concept:

- Dictionary order compares letters from left to right; the alphabetic order here is A, K, N, P, R, U.
- With a fixed prefix and r distinct unused letters, there are $r!$ completions. Subtract whole preceding blocks, keeping positions one-based.

Step 1: Locate the first two letters Each first-letter block contains $5! = 120$ words. The blocks beginning with A, K and N occupy positions 1 through 360. Since $360 < 440 \leq 480$, the first letter is P and the required position within the P block is

$$440 - 3(120) = 80.$$

The remaining letters are A, K, N, R and U. Each second-letter block has $4! = 24$ words. The PA, PK and PN blocks account for 72 words, so the prefix is PR, with position $80 - 72 = 8$ within that block.

Step 2: Resolve the remaining four letters The remaining letters A, K, N and U give $3! = 6$ words per third-letter block. The six PRA words come first, so the eighth word begins PRK and is second among its completions. With A, N and U left, each fourth-letter block contains $2! = 2$ words. Thus the fourth letter is A; the first completion is PRKANU and the second is PRKAUN. Hence the required arrangement is PRKAUN. The ordering within the final two-letter block matters: NU precedes UN because N precedes U.

Step 3: Check the global rank directly For PRKAUN, three letters precede P, three unused letters precede R, one precedes K and one precedes U at the fifth position. Therefore its rank is

$$1 + 3(5!) + 3(4!) + 1(3!) + 0(2!) + 1(1!) = 440.$$

By contrast, PRKANU has rank 439; PRNAKU and PRNAUK have ranks 445 and 446, respectively. The direct rank check removes any possible off-by-one error in the block method.

Step 4: Conclusion The word at the 440th position is PRKAUN. Hence, the correct option is (B).

Quick Tip to Solve: Convert the required position to rank minus one before using factorial blocks, or verify your final word by adding all preceding prefix blocks and then one.

20. Let the function $f(x) = \left(x^2 - 1\right)\left|x^2 - ax + 2\right| + \cos|x|$ be not differentiable at the two points $x = \alpha = 2$ and $x = \beta$. Then the distance of the point (α, β) from the line $12x + 5y + 10 = 0$ is equal to : [4 marks]

(A) 5

(B) 2

(C) 4

(D) 3

Correct Answer: The printed premise is inconsistent: the function is differentiable at $x = 1$, so no second point β exists. Option (D) results only if this cancellation is overlooked.

Solution:

Concept:

- A polynomial and $\cos|x| = \cos x$ are differentiable for every real x. Possible corners therefore occur only at real zeros of the polynomial inside the absolute value.
- A zero of an absolute-value argument is only a candidate for non-differentiability. Multiplication by a factor vanishing at that point can remove the corner, so one-sided derivatives must be checked.

Step 1: Use the stated point to determine the parameter Since the function is stated to be non-differentiable at $x = 2$, the absolute-value argument must vanish there. Otherwise it has a fixed sign near 2 and every term is smooth. Thus

$$4 - 2a + 2 = 0 \quad \Longrightarrow \quad a = 3.$$

The function therefore becomes

$$f(x) = (x^2 - 1)|(x - 1)(x - 2)| + \cos x.$$

Its only candidate exceptional points are 1 and 2.

Step 2: Check whether the other root really produces a corner Write $g(x) = (x^2 - 1)|(x - 1)(x - 2)|$. Near $x = 1$, the factorization gives

$$g(x) = (x - 1)|x - 1|(x + 1)|x - 2|.$$

Since $g(1) = 0$, its derivative there is

$$g'(1) = \lim_{h \to 0} |h|(2 + h)|h - 1| = 0.$$

Consequently $f'(1) = -\sin 1$ exists: the root at 1 does not supply a second non-differentiable point. At 2, however, the nonzero multiplier $2^2 - 1 = 3$ preserves the corner; the one-sided derivatives of g are -3 and 3.

Step 3: Assess the requested distance and the options There is only one non-differentiable point, $x = 2$. Therefore the pair (α, β) is not defined under the printed condition and none of the four

distances is a valid answer. Treating both roots as corners would incorrectly set $\beta = 1$ and produce

$$\frac{|12(2) + 5(1) + 10|}{\sqrt{12^2 + 5^2}} = 3.$$

This explains option (D), but it does not fix the differentiability contradiction; options (A), (B) and (C) likewise cannot follow from the literal premise.

Step 4: Conclusion As printed, no second non-differentiable point β exists, so the requested distance has no defined value. No option is mathematically valid; the value 3 in option (D) follows only from overlooking the cancelled corner at $x = 1$.

Quick Tip to Solve: After locating zeros inside an absolute value, test the complete product at each zero. A vanishing prefactor can make an apparent corner differentiable.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. If $24\int_0^{\frac{\pi}{4}} \left[\sin\left|4x - \frac{\pi}{12}\right| + [2\sin x]\right] dx = 2\pi + \alpha$**, where** $[\cdot]$ **denotes the greatest integer function, then** α **is equal to _____.** [4 marks]

Solution:

Concept:

- Split an absolute-value expression where its argument changes sign. Here $4x - \pi/12 = 0$ at $x = \pi/48$.
- The greatest integer $[u]$ is constant between consecutive integers. Since $2\sin x$ increases from 0 to $\sqrt{2}$, its only interior integer crossing on this interval is 1.

Step 1: Evaluate the sine contribution Put $I_1 = \int_0^{\pi/4} \sin|4x - \pi/12|\, dx$. On the first interval the argument inside the absolute value is negative; on the second it is nonnegative. Therefore

$$\begin{aligned} I_1 &= \int_0^{\pi/48} \sin(\pi/12 - 4x)\, dx + \int_{\pi/48}^{\pi/4} \sin(4x - \pi/12)\, dx \\ &= \frac{1 - \cos(\pi/12)}{4} + \frac{1 - \cos(11\pi/12)}{4}. \end{aligned}$$

Since $\cos(11\pi/12) = -\cos(\pi/12)$, the two cosine terms cancel and $I_1 = 1/2$. The outer square brackets in the integrand group the sum; the inner brackets on $2\sin x$ denote the greatest integer.

Step 2: Integrate the greatest integer term The threshold is $2\sin x = 1$, or $x = \pi/6$. Thus the greatest integer is 0 for $0 \le x < \pi/6$, and it is 1 for $\pi/6 \le x \le \pi/4$. The value at a single endpoint does not change the integral. Hence

$$I_2 = \int_0^{\pi/4} [2\sin x]\, dx = 0 + \int_{\pi/6}^{\pi/4} 1\, dx = \frac{\pi}{12}.$$

Step 3: Compare with the required expression Add the two independently evaluated contributions

before multiplying by 24:

$$24(I_1 + I_2) = 24\left(\frac{1}{2} + \frac{\pi}{12}\right) = 12 + 2\pi.$$

Comparing this with $2\pi + \alpha$ identifies the constant term directly.

Step 4: Conclusion The required value is $\boxed{\alpha = 12}$, a dimensionless number.

Quick Tip to Solve: Mark both kinds of breakpoints first: zeros inside absolute values and integer crossings inside greatest integer functions. They need not occur at the same point.

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22. Let $y^2 = 12x$ be the parabola and S be its focus. Let PQ be a focal chord of the parabola such that $(SP)(SQ) = \frac{147}{4}$. Let C be the circle described taking PQ as a diameter. If the equation of a circle C is $64x^2 + 64y^2 - \alpha x - 64\sqrt{3}y = \beta$, then $\beta - \alpha$ is equal to . [4 marks]

Solution:

Concept:

- On $y^2 = 4ax$, a point has coordinates $(at^2, 2at)$, and the focus is $(a, 0)$. For endpoints of a focal chord, their parameters satisfy $tu = -1$.
- The circle with endpoints (x_1, y_1) and (x_2, y_2) as a diameter is $(x-x_1)(x-x_2)+(y-y_1)(y-y_2) = 0$. Its linear coefficients use the coordinate sums, not the midpoint coordinates.

Step 1: Express the focal distances in parameters Here $a = 3$, so write $P = (3t^2, 6t)$ and $Q = (3u^2, 6u)$, with $u = -1/t$. The focal distances are $SP = 3(1 + t^2)$ and $SQ = 3(1 + u^2)$; these follow either by the distance formula or by equality of focal and directrix distances. Their product gives

$$9(1 + t^2)(1 + u^2) = \frac{147}{4}.$$

Using $t^2u^2 = 1$, obtain

$$t^2 + u^2 + 2 = \frac{49}{12}, \qquad t^2 + u^2 = \frac{25}{12}.$$

Step 2: Find the coordinate sums and products The sum of the endpoint abscissae is

$$x_1 + x_2 = 3(t^2 + u^2) = \frac{25}{4}.$$

Also $(t + u)^2 = t^2 + u^2 + 2tu = 1/12$, so $y_1 + y_2 = 6(t + u) = \pm\sqrt{3}$. The given negative coefficient of y selects $+\sqrt{3}$. Finally,

$$x_1x_2 + y_1y_2 = 9t^2u^2 + 36tu = 9 - 36 = -27.$$

These quantities determine the circle without finding either endpoint separately.

Step 3: Form the diameter circle and compare coefficients Expanding the diameter equation now gives

$$x^2 + y^2 - \frac{25}{4}x - \sqrt{3}\,y - 27 = 0.$$

Multiply through by 64 and move the constant to the right:

$$64x^2 + 64y^2 - 400x - 64\sqrt{3}\,y = 1728.$$

Hence $\alpha = 400$ and $\beta = 1728$, with the sign of the constant checked by the original diameter formula.

Step 4: Conclusion The required dimensionless difference is $\boxed{\beta - \alpha = 1728 - 400 = 1328}$.

Quick Tip to Solve: For a focal chord, use $tu = -1$ immediately. The diameter circle needs only sums and products of endpoint coordinates, so solving for the individual endpoints is unnecessary.

23. Let integers $\mathrm{a}, \mathrm{b} \in [-3, 3]$ **be such that** $\mathrm{a} + \mathrm{b} \neq 0$**. Then the number of all possible ordered pairs (a, b), for which** $\left|\frac{z-\mathrm{a}}{z+\mathrm{b}}\right| = 1$ **and** $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 1, z \in \mathrm{C}$**, where** ω **and** ω^2 **are the roots of** $x^2 + x + 1 = 0$**, is equal to _____ .** [4 marks]

Solution:

Concept:

- The nonreal cube roots of unity obey $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$, so powers of ω reduce modulo 3.
- The equality $|z - a| = |z + b|$, with real a, b and $a + b \neq 0$, fixes $\mathrm{Re}\, z = (a - b)/2$. The vertical bars around a matrix denote its determinant.

Step 1: Simplify the determinant exactly Denote the displayed determinant by $D(z)$. Expansion along permutations gives

$$\begin{aligned} D(z) = (z+1)(z+\omega^2)(z+\omega) + 2\omega^3 \\ - \omega^4(z+\omega^2) - \omega^2(z+\omega) - (z+1). \end{aligned}$$

The first product is $z^3 + 1$, since its three factors are associated with the cube roots of -1. The three subtracted terms sum to

$$z(\omega^4 + \omega^2 + 1) + \omega^6 + \omega^3 + 1 = 0 + 3.$$

As $2\omega^3 = 2$, the determinant is $z^3 + 1 + 2 - 3 = z^3$. Hence the equation is $z^3 = 1$, so z must be $1, \omega$ or ω^2.

Step 2: Translate the distance condition into an integer difference Set $z = x + iy$. Squaring the modulus equality yields

$$(x - a)^2 + y^2 = (x + b)^2 + y^2,$$

and hence $(a + b)(a - b - 2x) = 0$. The excluded case $a + b = 0$ therefore leaves $a - b = 2x$. For $z = 1$, the required difference is 2; for either nonreal root, $x = -1/2$, and the difference is -1. Count each ordered pair only once even when both nonreal roots work.

Step 3: Count the two disjoint sets of pairs If $a = b + 2$, the bounds $-3 \leq a, b \leq 3$ allow $b = -3, -2, -1, 0, 1$, giving five pairs. The pair $(1, -1)$ has $a + b = 0$ and is excluded, leaving four. If $a = b - 1$, the allowed values are $b = -2, -1, 0, 1, 2, 3$, giving six pairs. Here $a + b = 2b - 1$ cannot

vanish for integral b, so all six qualify. The two difference conditions are distinct and their pair sets do not overlap. The modulus denominator is also nonzero: for $z = 1$ the only problematic pair is the already excluded $(1, -1)$, while the nonreal roots cannot equal the real number $-b$.

Step 4: Conclusion The number of possible ordered pairs is $\boxed{4 + 6 = 10}$, a dimensionless count.

Quick Tip to Solve: Simplify a roots-of-unity determinant before counting pairs. Then count distinct pairs, rather than multiplying by the number of complex roots that realize each pair.

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24. If $\lim\limits_{t\to 0}\left(\int\limits_0^1 (3x+5)^t dx\right)^{\frac{1}{t}} = \frac{\alpha}{5e}\left(\frac{8}{5}\right)^{\frac{2}{3}}$**, then** α **is equal to _____.** [4 marks]

Solution:

Concept:

- An expression of type 1^∞ is evaluated by taking logarithms. If $F(0) = 1$, then $\lim_{t\to 0} \log F(t)/t = F'(0)$, provided the derivative exists.
- Here $(3x + 5)^t$ and its derivative in t are continuous on a rectangle around $[0, 1] \times \{0\}$. Thus differentiation under this finite integral is valid, and the limit is the exponential of an average logarithm.

Step 1: Take the logarithm of the limiting expression Define $F(t) = \int_0^1 (3x + 5)^t\, dx$. Then $F(0) = 1$, and $F(t) > 0$ for real t. If the requested limit is L, continuity of the exponential gives

$$\log L = \lim_{t\to 0} \frac{\log F(t)}{t} = \frac{F'(0)}{F(0)} = \int_0^1 \log(3x + 5)\, dx.$$

This step evaluates the removable $0/0$ limit by the derivative at zero; it does not substitute $t = 0$ into the original exponent.

Step 2: Evaluate the logarithmic integral Substitute $u = 3x + 5$, so $dx = du/3$, with lower and upper limits 5 and 8. Using $\int \log u\, du = u \log u - u$, obtain

$$\begin{aligned}\log L &= \frac{1}{3}\int_5^8 \log u\, du \\ &= \frac{8\log 8 - 8 - (5\log 5 - 5)}{3} \\ &= \frac{8}{3}\log 8 - \frac{5}{3}\log 5 - 1.\end{aligned}$$

Both bases are positive, so the logarithms and resulting real powers are unambiguous.

Step 3: Match the prescribed form Exponentiate and separate the powers that occur in the question:

$$L = \frac{8^{8/3}}{e\,5^{5/3}} = \frac{8^2}{5e}\left(\frac{8}{5}\right)^{2/3} = \frac{64}{5e}\left(\frac{8}{5}\right)^{2/3}.$$

Comparison with $\frac{\alpha}{5e}(8/5)^{2/3}$ now determines the coefficient. As a check, this value lies between 5 and

8, as expected for the exponential of the average of $\log(3x+5)$ over an interval of length one.

Step 4: Conclusion The required dimensionless value is $\boxed{\alpha = 64}$.

Quick Tip to Solve: For a positive integral raised to $1/t$, first check that the integral tends to one. Its derivative at zero then gives the logarithm of the limit.

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25. Let $a_1, a_2, \ldots, a_{2024}$ be an Arithmetic Progression such that $a_1+(a_5+a_{10}+a_{15}+\ldots+a_{2020})+a_{2024} = 2233$. Then $a_1+a_2+a_3+\ldots+a_{2024}$ is equal to _____. [4 marks]

Solution:

Concept:

- In an arithmetic progression, $a_k = a_1 + (k-1)d$, where d is the common difference.
- Terms whose indices add to 2025 have the same sum $T = a_1 + a_{2024}$. The full sum is $S_{2024} = 1012T$.

Step 1: Count and pair the selected terms The indices $5, 10, \ldots, 2020$ give $2020/5 = 404$ terms. Their first and last indices add to 2025, as do the next pair 10 and 2015. Thus all 202 pairs have sum T, and their total is $202T$. The two outside terms a_1 and a_{2024} contribute one more T.

Step 2: Use the given sum and find the complete sum The supplied condition therefore becomes

$$203T = 2233, \qquad T = 11.$$

The full progression contains $2024/2 = 1012$ such pairs, so

$$S_{2024} = 1012(a_1 + a_{2024}) = 1012 \times 11 = 11132.$$

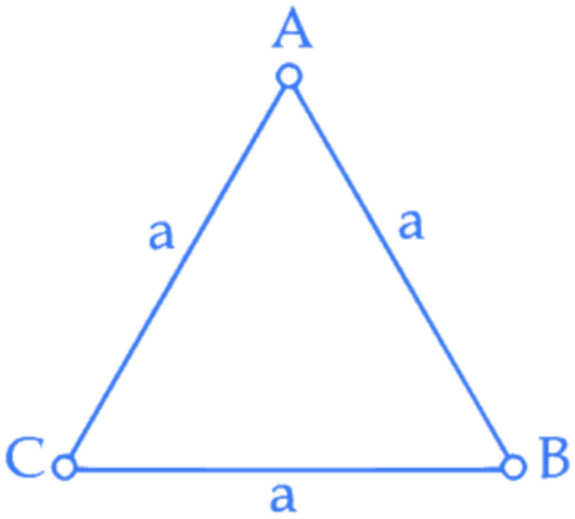
No separate determination of the first term or common difference is necessary, because the condition already fixes the endpoint sum.

Step 3: Conclusion The required sum is $\boxed{11132}$. It is a sum of terms of the progression, with no physical unit.

Quick Tip to Solve: Pair terms by complementary indices: every pair whose indices total 2025 has the same sum.

Physics – Section A: Multiple Choice Questions (80 marks)

26. Three equal masses m are kept at vertices (A, B, C) of an equilateral triangle of side a in free space. At $t = 0$, they are given an initial velocity $\overrightarrow{V_A} = V_0\overrightarrow{AC}, \overrightarrow{V_B} = V_0\overrightarrow{BA}$ and $\overrightarrow{V_C} = V_0\overrightarrow{CB}$. Here, $\overrightarrow{AC}, \overrightarrow{CB}$ and $\overrightarrow{BA}$ are unit vectors along the edges of the triangle. If the three masses interact gravitationally, then the magnitude of the net angular momentum of the system at the point of collision is : [4 marks]



(A) $\frac{1}{2}$ a mV_0

(B) 3 a mV_0

(C) $\frac{3}{2}$ a mV_0

(D) $\frac{\sqrt{3}}{2}$ a mV_0

Correct Answer: (D) $\frac{\sqrt{3}}{2}$ a mV_0

Solution:

Concept:

- Gravitational forces between particles are central internal forces; their net torque on the complete isolated system is zero. Total angular momentum is conserved.
- The three equal masses have their centre of mass at the centroid O. For an equilateral triangle its perpendicular distance from each side is the inradius $r = a/(2\sqrt{3})$.

Step 1: Find angular momentum about the centroid The three initial velocity vectors form a closed cycle along the sides, so their vector sum and hence total linear momentum are zero. The centre of mass remains at rest. Each momentum has magnitude mV_0 and acts parallel to a side at distance r from O. All three circulate in the same sense, so their angular momenta add rather than cancel.

Step 2: Add the contributions and apply conservation The magnitude is

$$L_O = 3mV_0r = 3mV_0\frac{a}{2\sqrt{3}} = \frac{\sqrt{3}}{2}amV_0.$$

Since total momentum is zero, shifting the origin adds no term $\boldsymbol{R} \times \boldsymbol{P}$. Internal gravity cannot change this total. The quantities $amV_0/2$, $3amV_0$ and $3amV_0/2$ use an incorrect perpendicular distance or omit the three-body geometry.

Step 3: Conclusion The conserved net angular momentum has magnitude $\boxed{\frac{\sqrt{3}}{2}amV_0}$. Hence, the correct option is (D).

Quick Tip to Solve: For momentum along a triangle side, use the centroid-to-side perpendicular distance, not the centroid-to-vertex distance.

27. In an experiment with photoelectric effect, the stopping potential, [4 marks]

(A) is $\left(\frac{1}{e}\right)$ times the maximum kinetic energy of the emitted photoelectrons

(B) increases with increase in the intensity of the incident light

(C) decreases with increase in the intensity of the incident light

(D) increases with increase in the wavelength of the incident light

Correct Answer: (A) is $\left(\frac{1}{e}\right)$ times the maximum kinetic energy of the emitted photoelectrons

Solution:

Concept:

- The stopping potential V_s is the magnitude of the retarding potential that just prevents the fastest photoelectrons from reaching the collector.
- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, and the stopping condition is $eV_s = K_{\max}$, where e is the positive elementary charge.

Step 1: Relate stopping potential to electron energy At the stopping potential, the maximum initial kinetic energy is entirely used against the electric potential energy barrier. Hence

$$V_s = \frac{K_{\max}}{e} = \frac{h\nu - \phi}{e}.$$

Thus option (A) states the defining energy relation. Kinetic energy must be expressed in joules when dividing by charge in coulombs to obtain volts.

Step 2: Check intensity and wavelength dependence At fixed frequency, increasing intensity increases the number of emitted electrons and the saturation current, but does not change their maximum kinetic energy. Therefore neither (B) nor (C) is correct. Since $\nu = c/\lambda$,

$$V_s = \frac{hc}{e\lambda} - \frac{\phi}{e}.$$

An increase of wavelength reduces the stopping potential while photoemission remains possible, contradicting (D).

Step 3: Conclusion The stopping potential is $1/e$ times the maximum kinetic energy of the emitted photoelectrons. Hence, the correct option is (A).

Quick Tip to Solve: Intensity controls photoelectron count; frequency controls maximum kinetic energy and stopping potential.

28. The number of spectral lines emitted by atomic hydrogen that is in the 4th energy level, is

[4 marks]

(A) 3

(B) 6

(C) 1

(D) 0

Correct Answer: (B) 6

Solution:

Concept:

- A photon is emitted when an electron moves from a higher to a lower allowed energy level, with $h\nu = E_i - E_f$.
- For an ensemble excited up to level n, the possible downward transitions among levels $1, 2, \ldots, n$ number $n(n-1)/2$.

Step 1: List the transitions from the accessible levels For hydrogen initially excited to $n = 4$, a transition may occur directly to levels 3, 2 or 1. Electrons that first reach level 3 can subsequently make transitions $3 \to 2$ or $3 \to 1$, and those reaching level 2 can make $2 \to 1$. Thus the possible transitions are

$$4 \to 3, \quad 4 \to 2, \quad 4 \to 1, \quad 3 \to 2, \quad 3 \to 1, \quad 2 \to 1.$$

Step 2: Count the distinct spectral lines These six energy differences give six spectral lines, equivalently

$$N = \frac{4(4-1)}{2} = 6.$$

The value 3 counts only direct transitions from level 4 and misses lines arising during cascades. The value 1 describes a selected single transition, not the whole possible spectrum; zero would wrongly exclude de-excitation altogether. A single atom follows one cascade, whereas the spectral-line count includes all allowed cascades.

Step 3: Conclusion The number of possible emitted spectral lines is $\boxed{6}$. Hence, the correct option is (B).

Quick Tip to Solve: Count every pair of accessible energy levels, not only transitions that start directly at the initial level.

29. The difference of temperature in a material can convert heat energy into electrical energy. To harvest the heat energy, the material should have [4 marks]

(A) low thermal conductivity and high electrical conductivity

(B) low thermal conductivity and low electrical conductivity

(C) high thermal conductivity and high electrical conductivity

(D) high thermal conductivity and low electrical conductivity

Correct Answer: (A) low thermal conductivity and high electrical conductivity

Solution:

Concept:

- A thermoelectric generator uses a maintained temperature difference to produce an emf through the Seebeck effect.
- A useful thermoelectric material must limit heat leakage while allowing charge to move readily: low thermal conductivity κ and high electrical conductivity σ.

Step 1: Preserve the temperature difference If thermal conductivity is large, heat flows rapidly from the hot side to the cold side without producing useful electrical work. This tends to equalise their temperatures, weakening the driving temperature difference. A low thermal conductivity suppresses this direct heat leakage and helps maintain the hot and cold junctions.

Step 2: Reduce electrical losses and compare options The generated current must pass through the material. High electrical conductivity means low internal resistance, reducing resistive losses I^2R and allowing useful power delivery. Thus option (A) combines the two desired properties. Option (B) preserves the thermal gradient but obstructs current; option (C) conducts current well but also leaks heat rapidly. Option (D) combines strong heat leakage with poor electrical transport, so both of its tendencies are unfavourable. The standard figure of merit $ZT = S^2\sigma T/\kappa$ expresses the same preference when other properties are fixed.

Step 3: Conclusion The material should have low thermal conductivity and high electrical conductivity. Hence, the correct option is (A).

Quick Tip to Solve: A thermoelectric generator should keep heat from bypassing conversion while letting the generated electric current flow easily.

30. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : With the increase in the pressure of an ideal gas, the volume falls off more rapidly in an isothermal process in comparison to the adiabatic process.

Reason (R) : In isothermal process, PV = constant, while in adiabatic process PV^{γ} = constant. Here γ is the ratio of specific heats, P is the pressure and V is the volume of the ideal gas.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) (A) is true but (R) is false

(B) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(C) (A) is false but (R) is true

(D) Both (A) and (R) are true and (R) is the correct explanation of (A)

Correct Answer: (D) Both (A) and (R) are true and (R) is the correct explanation of (A)

Solution:

Concept:

- For an ideal gas, an isothermal change obeys $PV =$ constant; a reversible adiabatic change obeys $PV^{\gamma} =$ constant.
- The ratio $\gamma = C_P/C_V$ exceeds one. Compare the magnitude of dV/dP at the same state to determine how rapidly volume falls with pressure.

Step 1: Evaluate the assertion Differentiating the isothermal equation gives

$$\left(\frac{dV}{dP}\right)_T = -\frac{V}{P}.$$

For the adiabatic equation, logarithmic differentiation gives $dP/P + \gamma\, dV/V = 0$, hence

$$\left(\frac{dV}{dP}\right)_{\text{ad}} = -\frac{V}{\gamma P}.$$

At the same initial state, $V/P > V/(\gamma P)$, so an equal small pressure increase produces a larger volume decrease isothermally. The assertion is true.

Step 2: Evaluate the reason The two equations quoted in the reason are exactly the ideal-gas relations used above. Thus the reason is also true. For a finite pressure ratio $r = P/P_0 > 1$, these relations give $V_T = V_0/r$ and $V_{\text{ad}} = V_0 r^{-1/\gamma}$, with $V_T < V_{\text{ad}}$, confirming the comparison for compression from a common state.

Step 3: Test the explanatory link The reason directly yields the different pressure-volume dependences and therefore explains the assertion. Option (A) incorrectly rejects the reason, option (C) incorrectly rejects the assertion, and option (B) misses their direct mathematical connection.

Step 4: Conclusion Both the assertion and the reason are true, and the reason correctly explains the assertion. Hence, the correct option is (D).

Quick Tip to Solve: An adiabatic curve is steeper on a P-versus-V plot, but this question asks about volume change per pressure change, the inverse slope.

31. A sand dropper drops sand of mass m(t) on a conveyer belt at a rate proportional to the square root of speed (v) of the belt, i.e., $\frac{dm}{dt} \propto \sqrt{v}$. If P is the power delivered to run the belt at constant speed then which of the following relationship is true? [4 marks]

(A) P $\propto \sqrt{v}$

(B) P $\propto v$

(C) $P^2 \propto v^3$

(D) $P^2 \propto v^5$

Correct Answer: (D) $P^2 \propto v^5$

Solution:

Concept:

- A force is needed to give freshly deposited sand the horizontal velocity of the belt. The incoming sand has zero horizontal velocity.
- For a steady mass supply, the horizontal momentum supplied per second is $F = v\,dm/dt$. The mechanical power supplied to a belt moving at speed v is $P = Fv$.

Step 1: Calculate the force on the sand Write the mass flow rate as $\dot{m} = a\sqrt{v}$, where a is the constant of proportionality. In a short interval dt, a mass dm lands on the belt. Its horizontal momentum changes from zero to $v\,dm$. Therefore the belt exerts the forward force

$$F = \frac{v\,dm}{dt} = av^{3/2}.$$

The sand exerts an equal backward force on the belt; the drive must counter this force to maintain constant speed.

Step 2: Convert the force into drive power The point at which the drive acts moves with belt speed v, so

$$P = Fv = av^{5/2}, \qquad P^2 = a^2v^5.$$

It would be incorrect to identify drive power with only the rate of increase of the sand kinetic energy, $\frac{1}{2}\dot{m}v^2$. The loading process is inelastic and the other half is dissipated while the sand slips and is accelerated. This does not change the exponent, but explains the force method directly. Options (A) and (B) omit the momentum and work factors, while (C) would require $P \propto v^{3/2}$, the scaling of force rather than power.

Step 3: Conclusion The drive power obeys $P \propto v^{5/2}$, hence $P^2 \propto v^5$. Hence, the correct option is (D).

Quick Tip to Solve: For material landing on a moving belt, first use $F = \dot{m}v$, then multiply by the belt speed once more to obtain the drive power.

32. A plane electromagnetic wave propagates along the + x direction in free space. The components of the electric field, $\vec{E}$ and magnetic field, $\vec{B}$ vectors associated with the wave in Cartesian frame are [4 marks]

(A) E_x, B_y

(B) E_y, B_x

(C) E_y, B_z

(D) E_z, B_y

Correct Answer: (C) E_y, B_z

Solution:

Concept:

- In free space the electric and magnetic fields of a plane wave are transverse to the propagation direction and perpendicular to each other.
- The energy-flow direction is given by the Poynting vector $\vec{S} = \vec{E} \times \vec{B}/\mu_0$. Thus the field directions and propagation direction form a right-handed set.

Step 1: Exclude longitudinal components The propagation vector is along the positive x-axis. Transversality gives

$$\vec{E} \cdot \hat{x} = 0, \qquad \vec{B} \cdot \hat{x} = 0.$$

Consequently the wave has no longitudinal components E_x or B_x. Option (A) contains E_x, and option (B) contains B_x; these cannot represent the specified free-space plane wave. The remaining pairs lie in the transverse yz-plane, so the direction of their cross product must now be checked.

Step 2: Use the right-hand rule Taking the field directions in the positive coordinate sense used in the options,

$$\hat{y} \times \hat{z} = \hat{x}.$$

Therefore an electric field along $+y$ and a magnetic field along $+z$ carry energy along $+x$. Their magnitudes also satisfy $E = cB$, and they oscillate in phase. In contrast, $\hat{z} \times \hat{y} = -\hat{x}$, so positive E_z, B_y directions give propagation along negative x, eliminating (D) under this directional convention. Reversing one of those directions would describe a different signed pair; simply checking that the two fields are perpendicular is therefore insufficient.

Step 3: Conclusion The right-handed field pair for the stated positive propagation direction is E_y, B_z. Hence, the correct option is (C).

Quick Tip to Solve: After checking transversality, use the cyclic order x, y, z: $\hat{y} \times \hat{z} = \hat{x}$. The order electric field cross magnetic field matters.

33. A cup of coffee cools from 90°C to 80°C in t minutes when the room temperature is 20°C. The time taken by the similar cup of coffee to cool from 80°C to 60°C at the same room temperature is: [4 marks]

(A) $\frac{13}{5}t$
(B) $\frac{10}{13}t$
(C) $\frac{5}{13}t$
(D) $\frac{13}{10}t$

Correct Answer: (A) $\frac{13}{5}t$

Solution:

Concept:

- Newton cooling states that the cooling rate is proportional to the excess temperature over the surroundings: $-dT/d\tau = k(T - T_r)$.
- For a finite temperature interval, the usual mean-temperature approximation is $(T_1 - T_2)/\Delta\tau \simeq k[(T_1 + T_2)/2 - T_r]$. Here the identical cups and room conditions give the same k.

Step 1: Determine the cooling constant from the first interval The temperature fall is $10°\text{C}$ in t minutes. The mean coffee temperature over this interval is $85°\text{C}$; its mean excess over the room is $85 - 20 = 65°\text{C}$. Hence

$$\frac{10}{t} \simeq 65k, \qquad k \simeq \frac{2}{13t}.$$

Temperature differences have the same numerical values in kelvin and in degrees Celsius, so no offset conversion is needed here.

Step 2: Apply the mean-temperature approximation to the second interval Let the required time be τ. The coffee now loses $20°\text{C}$, and its mean temperature is $70°\text{C}$, giving a mean excess of $50°\text{C}$. Thus

$$\frac{20}{\tau} \simeq 50k, \qquad \tau \simeq \frac{20}{50}\frac{13t}{2} = \frac{13t}{5}.$$

This interval has both a larger temperature fall and a lower cooling rate, so the time must exceed $2t$; the other three options fail this check. Strict integration gives $\tau/t = \ln(60/40)/\ln(70/60) \simeq 2.630$, whereas the listed answer 2.6 uses the finite-interval mean-temperature approximation.

Step 3: Conclusion Using the mean-temperature cooling approximation represented by the options, the time is $\tau \simeq \frac{13}{5}t$. Hence, the correct option is (A).

Quick Tip to Solve: Compare fall divided by mean excess temperature in each interval; the later, cooler interval must take longer for the same temperature fall.

34. A convex lens made of glass (refractive index = 1.5) has focal length 24 cm in air. When it is totally immersed in water (refractive index = 1.33), its focal length changes to [4 marks]

(A) 96 cm

(B) 72 cm

(C) 24 cm

(D) 48 cm

Correct Answer: (A) 96 cm

Solution:

Concept:

- For a thin lens in a surrounding medium, $1/f = (n_g/n_m - 1)(1/R_1 - 1/R_2)$, where n_g is the glass index and n_m is the surrounding index.
- The lens shape and radii remain unchanged on immersion. Only its relative refractive index and therefore its optical power change.

Step 1: Eliminate the unchanged curvature factor Put $K = 1/R_1 - 1/R_2$. In air,

$$\frac{1}{24} = (1.5 - 1)K = \frac{1}{2}K.$$

In water,

$$\frac{1}{f_w} = \left(\frac{1.5}{n_w} - 1\right) K.$$

Dividing these two relations eliminates the unknown radii:

$$f_w = 24\frac{0.5}{1.5/n_w - 1}.$$

The water index must occur in the denominator of the relative index; subtracting 1.33 directly from 1.5 without dividing by it would give the wrong power.

Step 2: Evaluate and compare the given choices The stated water index 1.33 is the rounded conventional value $4/3$. With this standard approximation,

$$\frac{n_g}{n_w} - 1 = \frac{3/2}{4/3} - 1 = \frac{1}{8}, \qquad f_w = 24\frac{1/2}{1/8} = 96\,\text{cm}.$$

Directly retaining the rounded decimal gives $f_w = 12(1.33)/0.17 \simeq 93.88\,\text{cm}$; the listed value $96\,\text{cm}$ therefore reflects $n_w = 4/3$. Immersion reduces the index contrast and increases the focal length. The unchanged value 24 cm ignores the medium; 48 cm and 72 cm do not satisfy the relative-index calculation.

Step 3: Conclusion The listed focal length, using the conventional $n_w = 4/3$ represented by 1.33, is $96\,\text{cm}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the relative index $n_{\text{lens}}/n_{\text{medium}}$, and cancel the curvature factor by taking the ratio of the two lens-maker equations.

35. Two bodies A and B of equal mass are suspended from two massless springs of spring constant k_1 and k_2, respectively. If the bodies oscillate vertically such that their amplitudes are equal, the ratio of the maximum velocity of A to the maximum velocity of B is [4 marks]

(A) $\sqrt{\frac{k_2}{k_1}}$

(B) $\sqrt{\frac{k_1}{k_2}}$

(C) $\frac{k_2}{k_1}$

(D) $\frac{k_1}{k_2}$

Correct Answer: (B) $\sqrt{\frac{k_1}{k_2}}$

Solution:

Concept:

- For a mass m on a massless spring of constant k, the angular frequency of small vertical oscillations about equilibrium is $\omega = \sqrt{k/m}$.
- In simple harmonic motion of amplitude a, the maximum speed is $v_{\max} = a\omega$, attained at the equilibrium position.

Step 1: Write the speed for each oscillator Use a for their common amplitude and m for their common mass. Their angular frequencies and maximum speeds are

$$\omega_A = \sqrt{\frac{k_1}{m}}, \quad v_{A,\max} = a\sqrt{\frac{k_1}{m}},$$

$$\omega_B = \sqrt{\frac{k_2}{m}}, \quad v_{B,\max} = a\sqrt{\frac{k_2}{m}}.$$

Gravity shifts the static equilibrium by mg/k, but it does not change these angular frequencies. The amplitudes are measured from the respective equilibrium positions, rather than from the natural lengths of the springs.

Step 2: Form the required ratio Divide the speed of A by that of B, keeping the stated order:

$$\frac{v_{A,\max}}{v_{B,\max}} = \frac{a\sqrt{k_1/m}}{a\sqrt{k_2/m}} = \sqrt{\frac{k_1}{k_2}}.$$

Equal amplitudes and masses cancel. An energy check gives the same result: at equilibrium the kinetic energy $\frac{1}{2}mv_{\max}^2$ equals the oscillatory energy $\frac{1}{2}ka^2$. If $k_1 > k_2$, A must have the greater speed, which rules out the reciprocal in (A). Options (C) and (D) omit the square root required when converting from energy to speed. All four choices give unity for equal springs, so that limiting case alone cannot distinguish them.

Step 3: Conclusion The ratio is $v_{A,\max}/v_{B,\max} = \sqrt{k_1/k_2}$. Hence, the correct option is (B).

Quick Tip to Solve: At fixed amplitude and mass, speed scales as $\sqrt{k}$. Check the order of the requested ratio before substituting spring constants.

36. Match List - I with List - II.

List - I	List - II
(A) Magnetic induction	(I) Ampere meter^2
(B) Magnetic intensity	(II) Weber
(C) Magnetic flux	(III) Gauss
(D) Magnetic moment	(IV) Ampere meter

Choose the correct answer from the options given below: [4 marks]

(A) -(III), (B)-(IV), (C)-(II), (D)-(I)

(B) -(III), (B)-(II), (C)-(I), (D)-(IV)

(C) -(III), (B)-(IV), (C)-(I), (D)-(II)

(D) -(I), (B)-(II), (C)-(III), (D)-(IV)

Correct Answer: (A) -(III), (B)-(IV), (C)-(II), (D)-(I)

Solution:

Concept:

- Magnetic induction B, magnetic field intensity H, magnetic flux Φ, and magnetic dipole moment m are different physical quantities.
- Their standard units follow from $F = qvB$, $B = \mu_0 H$, $\Phi = \int \vec{B} \cdot d\vec{A}$, and $m = IA$.

Step 1: Match induction and flux Magnetic induction is measured in tesla in SI, or gauss in the centimetre-gram-second convention; $1\,\mathrm{G} = 10^{-4}\,\mathrm{T}$. Thus induction matches (III). Magnetic flux is field times area, so its unit is $\mathrm{T\,m^2}$, named the weber. Therefore flux matches (II). The two statements already demand $A \to III$ and $C \to II$, which select the first option among the printed combinations.

Step 2: Match intensity and magnetic moment For a solenoid, $H = nI$, with n the number of turns per metre. Its standard SI unit is consequently $\mathrm{A\,m^{-1}}$. Magnetic dipole moment of a current loop is current times loop area, with unit $\mathrm{A\,m^2}$, so moment matches (I). The remaining match is intensity to (IV). The table prints "Ampere meter for (IV); the physically correct unit for intensity is ampere per metre, so that entry has a missing inverse-metre qualification. This unit wording does not make intensity a flux or a moment. Options (B), (C), and (D) assign either flux or moment to units belonging to another quantity; none preserves the verified induction, flux and moment matches.

Step 3: Conclusion The intended unit sequence is (III), (IV), (II), (I), with (IV) understood as ampere per metre for magnetic intensity. Hence, the correct option is (A).

Quick Tip to Solve: Remember the defining products: flux is B times area, while dipole moment is current times area. Field intensity instead carries current per length.

37. Match List - I with List - II.

List - I	List - II
(A) Young's Modulus	(I) $M L^{-1} T^{-1}$
(B) Torque	(II) $M L^{-1} T^{-2}$
(C) Coefficient of Viscosity	(III) $M^{-1} L^{3} T^{-2}$
(D) Gravitational Constant	(IV) $M L^{2} T^{-2}$

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

(B) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(C) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(D) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

Correct Answer: (D) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

Solution:

Concept:

- Dimensions express a physical quantity in powers of mass M, length L and time T. Equal dimensions do not imply the same physical meaning.
- Young's modulus is stress divided by strain; torque is force times perpendicular distance. Dynamic viscosity follows Newton's law of viscous force, and G follows the gravitational force law.

Step 1: Find the mechanical dimensions Strain is a ratio of lengths and is dimensionless. Thus Young's modulus has the dimensions of pressure:

$$[Y] = [F/A] = ML^{-1}T^{-2}.$$

This matches (II). The moment of a force is $\tau = Fr_{\perp}$, so

$$[\tau] = (MLT^{-2})L = ML^{2}T^{-2}.$$

Hence torque matches (IV), even though energy has these same dimensions.

Step 2: Obtain the transport and gravitational dimensions For viscous flow, $F = \eta A(dv/dy)$. The velocity gradient has dimension T^{-1}, giving

$$[\eta] = \frac{MLT^{-2}}{L^{2}T^{-1}} = ML^{-1}T^{-1}.$$

Therefore viscosity matches (I). From $F = Gm_1m_2/r^2$,

$$[G] = \frac{[F][r]^2}{[m]^2} = M^{-1}L^{3}T^{-2},$$

which matches (III).

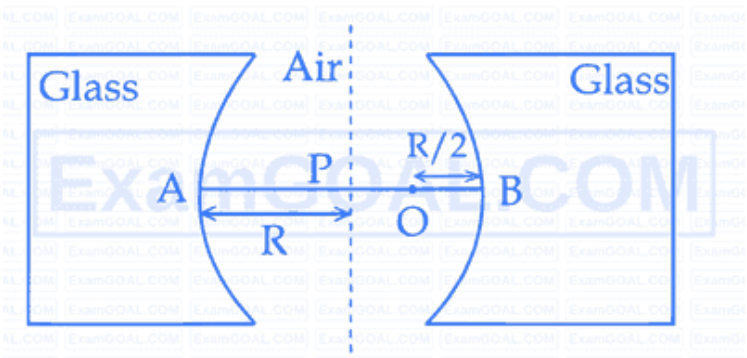
Step 3: Compare all four matches The complete sequence is A–II, B–IV, C–I, D–III. Only the fourth option contains this sequence. The first assigns the viscosity dimension to Young's modulus; the

second assigns torque dimensions to Young's modulus. The third gets Young's modulus and G right but interchanges the required matches for torque and viscosity.

Step 4: Conclusion The matching is $\mathrm{A}-\mathrm{II},\ \mathrm{B}-\mathrm{IV},\ \mathrm{C}-\mathrm{I},\ \mathrm{D}-\mathrm{III}$. Hence, the correct option is (D).

Quick Tip to Solve: Start with Young's modulus as pressure and torque as force times length; these two matches already identify the only viable option.

38. Two concave refracting surfaces of equal radii of curvature and refractive index 1.5 face each other in air as shown in figure. A point object O is placed midway, between P and B. The separation between the images of O, formed by each refracting surface is : [4 marks]



(A) 0.124R

(B) 0.114R

(C) 0.411R

(D) 0.214R

Correct Answer: (B) 0.114R

Solution:

Concept:

- For refraction from air into glass, use $n_2/v - n_1/u = (n_2 - n_1)/R_s$, measuring distances in the direction of incident light for each surface separately.
- Here $n_1 = 1$, $n_2 = 1.5$. The figure gives $AP = R$, $PB = R$, and $OB = R/2$, so $AO = 3R/2$. Both surfaces are concave as viewed by the incident rays.

Step 1: Locate the image formed by the left surface Rays travelling from O toward A move leftward. Choose leftward positive for this calculation. The object is on the incident side, so $u_A = -3R/2$, while the centre of curvature lies on the incident side, giving $R_A = -R$. Therefore

$$\frac{1.5}{v_A} - \frac{1}{-3R/2} = \frac{0.5}{-R}.$$

Simplifying,

$$\frac{1.5}{v_A} = -\frac{7}{6R}, \qquad v_A = -\frac{9R}{7}.$$

The negative sign places this virtual image to the right of A, at coordinate $x_A = 9R/7$ when A is the

origin.

Step 2: Locate the image formed by the right surface For rays travelling toward B, take rightward positive. Now $u_B = -R/2$ and $R_B = -R$. The refraction equation becomes

$$\frac{1.5}{v_B} + \frac{2}{R} = -\frac{0.5}{R}, \qquad v_B = -\frac{3R}{5}.$$

This image is to the left of B. Since $AB = 2R$, its coordinate measured from A is

$$x_B = 2R - \frac{3R}{5} = \frac{7R}{5}.$$

Both images are virtual; their signed distances cannot be directly subtracted without converting to a common origin.

Step 3: Subtract positions using one origin The separation is

$$d = \left|\frac{7R}{5} - \frac{9R}{7}\right| = \frac{4R}{35} = 0.114285\ldots R.$$

The options 0.124R, 0.411R and 0.214R do not equal this value; 0.114R is the listed rounded result.

Step 4: Conclusion The two images are separated by approximately $0.114R$. Hence, the correct option is (B).

Quick Tip to Solve: Reset the positive direction for each incident beam, then convert both image positions to the same fixed origin before subtracting.

39. The truth table for the circuit given below is: [4 marks]



(A)

A	B	Y
0	0	0
1	1	1
1	0	1
0	1	1

(B)

A	B	Y
0	0	0
1	0	1
0	1	0
1	1	0

(C)

A	B	Y
0	0	0
1	0	0
1	1	0
0	1	1

(D)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Correct Answer: (D)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Solution:

Concept:

- An inverter gives the complement of its input; an AND gate is high only when both its inputs are high. An OR gate is high when either input is high.
- The upper branch forms $A\overline{B}$, and the lower branch forms $\overline{A}B$. Their outputs feed the final OR gate.

Step 1: Translate the circuit into Boolean algebra Follow the upper branch first: the direct A wire and the inverted B wire enter its AND gate, giving $A\overline{B}$. In the lower branch A is inverted while B enters directly, giving $\overline{A}B$. The wire crossing does not join the two independent input signals. The OR gate combines the two terms:

$$Y = A\overline{B} + \overline{A}B.$$

This is the exclusive-OR operation, which detects unequal inputs.

Step 2: Evaluate each input combination For $A = B = 0$, both products vanish, so $Y = 0$. For $A = 0, B = 1$, the lower product is one and $Y = 1$. For $A = 1, B = 0$, the upper product is one and $Y = 1$. For $A = B = 1$, each branch contains an inverted zero, so $Y = 0$. Thus

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Input row order may vary between the options; compare the output associated with each pair, rather than its row position.

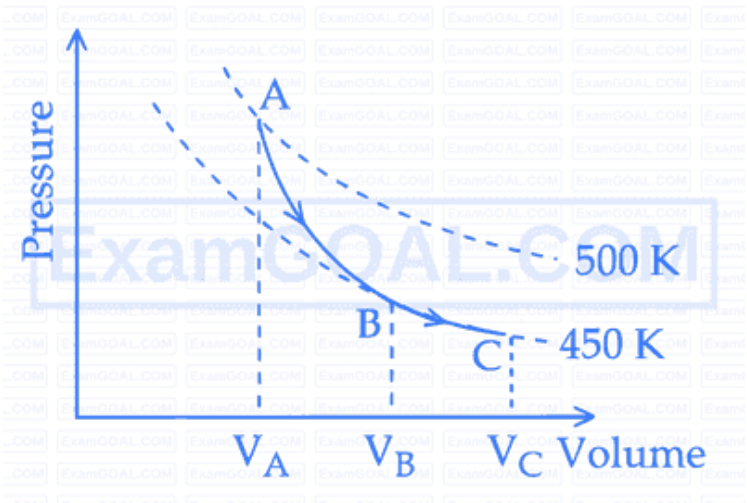
Step 3: Reject the other truth tables Option (A) is the inclusive OR table: it incorrectly gives one for equal high inputs. Option (B) retains only $A\overline{B}$, missing the lower branch's response to 01. Option (C) retains only $\overline{A}B$, missing the upper branch's response to 10. Option (D) alone includes both unequal-input cases while rejecting both equal-input cases.

Step 4: Conclusion The circuit is an XOR gate, with output sequence $0, 1, 1, 0$ for inputs $00, 01, 10, 11$. Hence, the correct option is (D).

Quick Tip to Solve: Two AND terms with opposite complemented inputs followed by OR form XOR: the output is high exactly when the inputs differ.

40. A poly-atomic molecule ($C_V = 3R, C_P = 4R$, where R is gas constant) goes from phase space point A $(\mathrm{P_A} = 10^5\ \mathrm{Pa},\ \mathrm{V_A} = 4 \times 10^{-6}\ \mathrm{m^3})$ to point B $(\mathrm{P_B} = 5 \times 10^4\ \mathrm{Pa},\ \mathrm{V_B} = 6 \times 10^{-6}\ \mathrm{m^3})$ to point C $(\mathrm{P_C} = 10^4\ \mathrm{Pa}, \mathrm{V}_C = 8 \times 10^{-6}\ \mathrm{m^3})$. A to B is an adiabatic path and B to C is an isothermal path.

The net heat absorbed per unit mole by the system is : [4 marks]



(A) $500\mathrm{R}(\ln 3 + \ln 4)$

(B) $450\mathrm{R}(\ln 4 - \ln 3)$

(C) $500\mathrm{R}\ln 2$

(D) $400\mathrm{R}\ln 4$

Correct Answer: (B) $450\mathrm{R}(\ln 4 - \ln 3)$

Solution:

Concept:

- An adiabatic process exchanges no heat: $Q_{AB} = 0$. For one mole of an ideal gas expanding isothermally, $Q = W = RT\ln(V_f/V_i)$.
- The plotted isotherm through B and C is labelled $450\,\mathrm{K}$. Thus the graph gives the temperature needed for the isothermal heat calculation.

Step 1: Separate the two stages The path A to B is explicitly adiabatic, so its heat contribution is zero even though work is done and the temperature decreases. The heat capacities give $\gamma = C_P/C_V = 4/3$, but no adiabatic work calculation is required when finding heat absorbed. Only the B to C isothermal expansion contributes to the net heat. The graph labels this common temperature $T_B = T_C = 450\,\mathrm{K}$.

Step 2: Calculate the heat on the isothermal stage For a fixed quantity of ideal gas at constant temperature, internal energy does not change, so the first law gives heat equal to expansion work. Per mole,

$$Q_{BC} = \int_{V_B}^{V_C} \frac{RT}{V}\,dV = RT\ln\frac{V_C}{V_B}.$$

Substituting the plotted temperature and the two given volumes,

$$Q_{BC} = 450R \ln \frac{8 \times 10^{-6}}{6 \times 10^{-6}} = 450R(\ln 4 - \ln 3)\ \text{J mol}^{-1}.$$

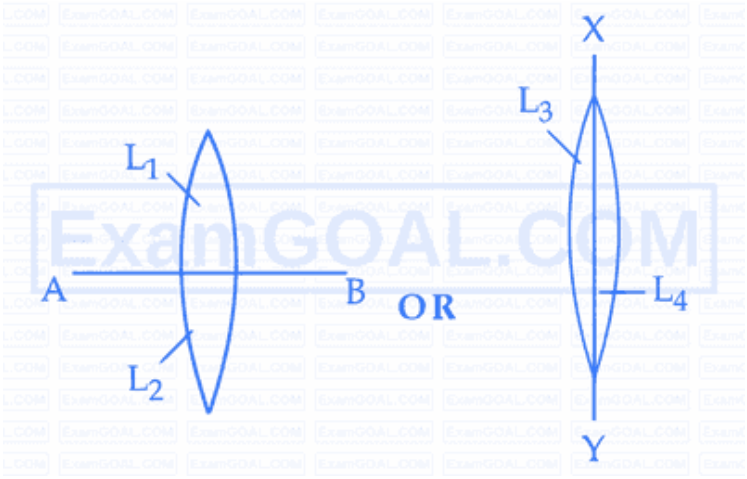
It is positive because volume increases.

Step 3: Check the numerical data and the options The stated pressures and volumes are internally inconsistent with a single isotherm: $P_B V_B = 0.30\,\text{J}$, whereas $P_C V_C = 0.08\,\text{J}$. Therefore all the printed coordinates cannot simultaneously describe the stated ideal-gas path. Using the explicit process description and the plotted 450 K isotherm yields option (B). The other choices use a different temperature or an incorrect logarithmic volume ratio; none represents the indicated B to C expansion.

Step 4: Conclusion From the stated adiabatic/isothermal path and the 450 K label in the graph, the net heat absorbed per mole is $450R(\ln 4 - \ln 3)\,\text{J mol}^{-1}$. Hence, the correct option is (B). The printed pressure coordinates do not satisfy the stated isothermal condition.

Quick Tip to Solve: For a path made of an adiabat followed by an isotherm, only the isothermal segment contributes heat; use its temperature and volume ratio.

41. Two identical symmetric double convex lenses of focal length f are cut into two equal parts L_1, L_2 by AB plane and L_3, L_4 by XY plane as shown in figure respectively. The ratio of focal lengths of lenses L_1 and L_3 is [4 marks]



(A) 1 : 2

(B) 1 : 1

(C) 2 : 1

(D) 1 : 4

Correct Answer: (A) 1 : 2

Solution:

Concept:

- The thin-lens power is $P = 1/f = (\mu - 1)(1/R_1 - 1/R_2)$. Power depends on surface curvatures and refractive index, rather than the available aperture.
- For the original symmetric biconvex lens, $R_1 = R$, $R_2 = -R$, so $1/f = 2(\mu - 1)/R$.

Step 1: Analyse the cut along the principal axis The AB plane divides the first lens into upper and lower portions. Each portion retains the same two curved refracting surfaces with their original radii of curvature. Its aperture is reduced, but its optical power is unchanged:

$$\frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{2(\mu - 1)}{R}.$$

Therefore $f_1 = f$. Such a cut reduces the light admitted and can displace the image relative to the remaining lens's aperture, but it does not double the focal length.

Step 2: Analyse the cut perpendicular to the principal axis The XY plane cuts the second lens through its middle, making each piece plano-convex. For L3, one refracting surface has radius R and the new plane face has infinite radius. Consequently

$$\frac{1}{f_3} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{\mu - 1}{R} = \frac{1}{2f}.$$

Thus $f_3 = 2f$. A plane surface contributes zero power in the lens-maker expression.

Step 3: Form the ratio and compare The required ratio is

$$f_1 : f_3 = f : 2f = 1 : 2.$$

The value 1:1 incorrectly treats both cuts as changes of aperture only. The ratio 2:1 reverses which lens loses a curved surface. The ratio 1:4 would require reducing L3's power to one quarter, whereas removing one of two equal curved-surface contributions only halves its power.

Step 4: Conclusion The focal lengths are $f_1 = f$ and $f_3 = 2f$, giving $f_1 : f_3 = 1 : 2$. Hence, the correct option is (A).

Quick Tip to Solve: A cut along the principal axis changes aperture; a central cut perpendicular to it replaces one curved face by a plane face and halves the power.

42. A point charge causes an electric flux of $-2 \times 10^4 \mathrm{Nm^2C^{-1}}$ to pass through a spherical Gaussian surface of 8.0 cm radius, centred on the charge. The value of the point charge is :
(Given $\epsilon_0 = 8.85 \times 10^{-12} \mathrm{C^2\ N^{-1}\ m^{-2}}$) [4 marks]

(A) $17.7 \times 10^{-8}\mathrm{C}$

(B) $-17.7 \times 10^{-8}\mathrm{C}$

(C) $15.7 \times 10^{-8}\mathrm{C}$

(D) $-15.7 \times 10^{-8}\mathrm{C}$

Correct Answer: (B) $-17.7 \times 10^{-8}\mathrm{C}$

Solution:

Concept:

- Gauss's law gives the total outward electric flux through a closed surface: $\Phi_E = q_{\text{enclosed}}/\epsilon_0$. Thus $q = \epsilon_0 \Phi_E$.
- Flux is signed using the outward area normal. Negative flux means a net inward field, and therefore a negative enclosed charge.

Step 1: Choose the flux relation The charge is enclosed at the centre of the Gaussian sphere. Gauss's law connects the total flux directly with that charge; there is no need to calculate the surface electric field first. The radius does not enter the final relation. Although the field magnitude falls as $1/r^2$, the sphere's area grows as r^2, so these factors cancel when the total flux is computed. Hence use

$$q = \epsilon_0 \Phi_E.$$

The given radius of 8.0 cm provides no extra multiplier.

Step 2: Substitute with the correct sign and exponents The supplied flux is $-2 \times 10^4\,\mathrm{N\,m^2\,C^{-1}}$, and the permittivity is $8.85 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$. Their product is

$$q = (8.85 \times 10^{-12})(-2 \times 10^4)\,\mathrm{C} = -17.7 \times 10^{-8}\,\mathrm{C}.$$

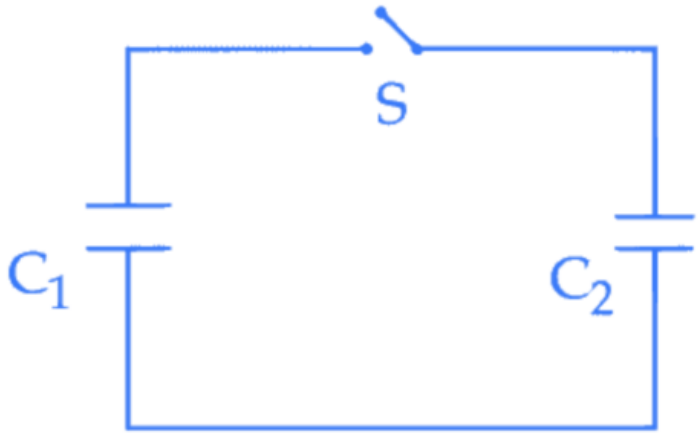
Equivalently, this is $-1.77 \times 10^{-7}\,\mathrm{C}$. The units cancel to coulombs, as required for charge.

Step 3: Compare the candidate charges Both positive options contradict the negative outward flux and are excluded immediately. Of the negative options, $-15.7 \times 10^{-8}\,\mathrm{C}$ has the wrong magnitude: multiplying 8.85 by 2 gives 17.7, not 15.7. Thus the sign and magnitude together identify the second option uniquely, independently of the Gaussian sphere's radius.

Step 4: Conclusion The enclosed point charge is $-17.7 \times 10^{-8}\,\mathrm{C}$. Hence, the correct option is (B).

Quick Tip to Solve: For total flux through any closed surface, multiply by permittivity directly; do not multiply by the sphere's area a second time.

43. A capacitor, $C_1 = 6\mu F$ is charged to a potential difference of $V_0 = 5V$ using a 5V battery. The battery is removed and another capacitor, $C_2 = 12\mu F$ is inserted in place of the battery. When the switch 'S' is closed, the charge flows between the capacitors for some time until equilibrium condition is reached. What are the charges (q_1 and q_2) on the capacitors C_1 and C_2 when equilibrium condition is reached. [4 marks]



(A) $q_1 = 10\mu C,\ q_2 = 20\mu C$

(B) $q_1 = 15\mu C,\ q_2 = 30\mu C$

(C) $q_1 = 20\mu C,\ q_2 = 10\mu C$

(D) $q_1 = 30\mu C,\ q_2 = 15\mu C$

Correct Answer: (A) $q_1 = 10\mu C,\ q_2 = 20\mu C$

Solution:

Concept:

- The battery initially supplies charge $Q_0 = C_1V_0$. After the battery is disconnected, the isolated capacitor combination cannot receive additional charge.
- At electrostatic equilibrium, capacitors connected across the same two nodes have a common potential difference V. Their charges are $q_1 = C_1V$ and $q_2 = C_2V$.

Step 1: Find the initially stored charge The second capacitor is initially uncharged. The charge available on the connected positive plates is therefore entirely the charge initially stored on the first capacitor. Using microfarads directly gives the charge in microcoulombs:

$$Q_0 = (6\,\mu\text{F})(5\,\text{V}) = 30\,\mu\text{C}.$$

An equal negative charge remains on the connected negative plates; charge is redistributed, rather than created, when the switch closes.

Step 2: Apply charge conservation and equal voltage The final connection places the capacitors in parallel. Thus their capacitances add for the purpose of finding the common voltage:

$$V = \frac{Q_0}{C_1 + C_2} = \frac{30}{6 + 12} = \frac{5}{3}\,\text{V}.$$

The individual charges are then

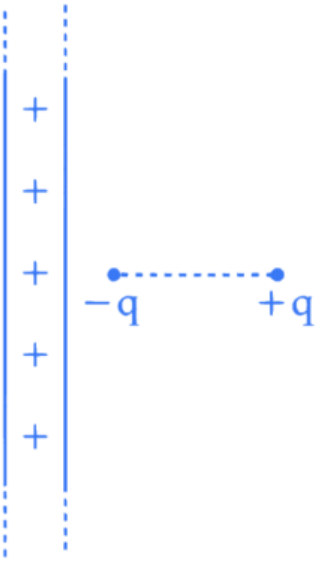
$$q_1 = 6\left(\frac{5}{3}\right) = 10\,\mu\text{C}, \qquad q_2 = 12\left(\frac{5}{3}\right) = 20\,\mu\text{C}.$$

Their sum is the original $30\,\mu$C, and their ratio is $1:2$, matching the capacitance ratio. Option (B) and option (D) require more charge than was initially available. Option (C) conserves the total charge but incorrectly gives the smaller capacitance the larger charge at equilibrium.

Step 3: Conclusion The equilibrium charges are $q_1 = 10\,\mu$C and $q_2 = 20\,\mu$C. Hence, the correct option is (A).

Quick Tip to Solve: For isolated parallel capacitors, conserve the total charge first, then divide it in the ratio of the capacitances.

44. An electric dipole is placed at a distance of 2 cm from an infinite plane sheet having positive charge density σ_o. Choose the correct option from the following. [4 marks]



(A) Torque on dipole is zero and net force is directed away from the sheet.

(B) Potential energy of dipole is minimum and torque is zero.

(C) Potential energy and torque both are maximum.

(D) Torque on dipole is zero and net force acts towards the sheet.

Correct Answer: (B) Potential energy of dipole is minimum and torque is zero.

Solution:

Concept:

- An infinite uniformly charged plane sheet produces a uniform electric field $E = \sigma_0/(2\varepsilon_0)$, directed normally away from a positively charged sheet.
- For a dipole of moment **p**, the torque magnitude is $pE\sin\theta$, and its potential energy is $U =$

$-pE\cos\theta$, where θ is the angle between the dipole moment and the field.

Step 1: Determine the direction of the dipole moment The figure places the negative charge closer to the sheet and the positive charge farther away. The dipole moment points from the negative charge to the positive charge, so it points away from the sheet. This is also the direction of the electric field on this side of the positively charged sheet. Consequently, $\theta = 0$. The stated distance of $2\,\text{cm}$ does not change the field magnitude, since the ideal infinite sheet produces a field independent of distance.

Step 2: Evaluate torque, energy and net force Substituting the alignment gives

$$\tau = pE\sin 0 = 0, \qquad U = -pE\cos 0 = -pE.$$

Since $-1 \le \cos\theta \le 1$, this is the minimum possible dipole energy. Both charges experience fields of equal magnitude, so their forces are equal and opposite:

$$\mathbf{F}_{\text{net}} = q\mathbf{E} - q\mathbf{E} = \mathbf{0}.$$

The charges therefore do not produce a net force either toward or away from the sheet. This rules out options (A) and (D). Option (C) fails twice: the torque is zero rather than maximum, and the energy is minimum rather than maximum. The alignment is stable with respect to small angular displacements.

Step 3: Conclusion The dipole has minimum potential energy and zero torque. Hence, the correct option is (B).

Quick Tip to Solve: Read the dipole arrow from negative to positive. Parallel alignment with a uniform field gives minimum energy and zero torque.

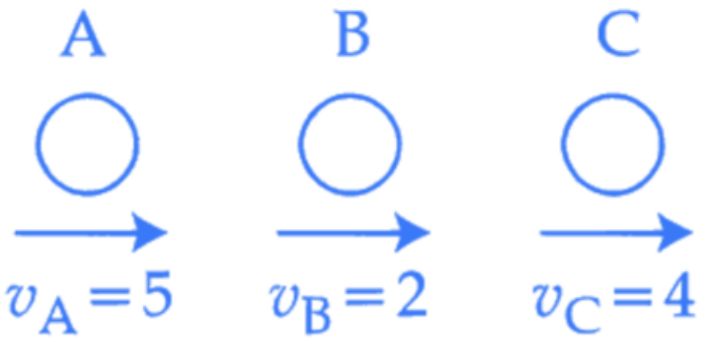
45. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): Three identical spheres of same mass undergo one dimensional motion as shown in figure with initial velocities $v_A = 5\ \text{m/s}, v_B = 2\ \text{m/s}, v_C = 4\ \text{m/s}$. If we wait sufficiently long for elastic collision to happen, then $v_A = 4\ \text{m/s}, v_B = 2\ \text{m/s}$, $v_C = 5\ \text{m/s}$ will be the final velocities.

Reason (R): In an elastic collision between identical masses, two objects exchange their velocities.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]



(A) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(B) Both (A) and (R) are true and (R) is the correct explanation of (A)

(C) (A) is false but (R) is true

(D) (A) is true but (R) is false

Correct Answer: (C) (A) is false but (R) is true

Solution:

Concept:

- In a one-dimensional perfectly elastic collision between two identical masses, conservation of momentum and kinetic energy imply that their velocities are exchanged.
- For spheres arranged from left to right, collisions have ceased only when no trailing sphere has a larger rightward velocity than the sphere immediately ahead of it.

Step 1: Verify the reason from the collision equations For equal masses, momentum conservation gives $u_1 + u_2 = v_1 + v_2$. Elasticity gives the relative-speed relation $v_2 - v_1 = u_1 - u_2$. Adding and subtracting these equations yields $v_1 = u_2$ and $v_2 = u_1$. Thus Reason (R), which states that the objects exchange velocities, is true for the head-on elastic collisions specified here.

Step 2: Follow the spheres until no collision remains Initially the ordered velocities of (A, B, C) are $(5, 2, 4)\ \text{m s}^{-1}$. Sphere A catches B first, producing

$$(5, 2, 4) \longrightarrow (2, 5, 4).$$

B now catches C, and their exchange gives

$$(2, 5, 4) \longrightarrow (2, 4, 5).$$

These velocities increase from left to right, so every separation subsequently increases. They are the final velocities. The assertion instead proposes $(4, 2, 5)\,\mathrm{m\,s^{-1}}$. In that arrangement A would still catch B because $4 > 2$; it cannot be the final collision-free state. The assertion is therefore false. Options (A) and (B) incorrectly declare the assertion true, while option (D) also incorrectly rejects the valid exchange rule.

Step 3: Conclusion The final velocities are $v_A = 2\,\mathrm{m\,s^{-1}}$, $v_B = 4\,\mathrm{m\,s^{-1}}$ and $v_C = 5\,\mathrm{m\,s^{-1}}$. The assertion is false and the reason is true. Hence, the correct option is (C).

Quick Tip to Solve: For identical spheres in a line, exchange velocities at each collision and stop only when the left-to-right velocities are nondecreasing.

Physics – Section B: Numerical Value Questions (20 marks)

46.

A physical quantity Q is related to four observables a, b, c, d as follows :

$$Q = \frac{ab^4}{cd}$$

where, $\mathrm{a} = (60 \pm 3)\mathrm{Pa}; \mathrm{b} = (20 \pm 0.1)\mathrm{m}; \mathrm{c} = (40 \pm 0.2)\mathrm{Nsm}^{-2}$ **and** $\mathrm{d} = (50 \pm 0.1)\mathrm{m}$**, then the percentage error in Q is** $\frac{x}{1000}$**, where** $x =$ **_____ .** [4 marks]

Solution:

Concept:

- For a product or quotient $Q = a^\alpha b^\beta c^\gamma d^\delta$, the maximum fractional error is the sum of the absolute exponent-weighted fractional errors.
- Percentage error is $100\Delta Q/Q$. A denominator contributes a positive amount to the maximum uncertainty even though its exponent in the expression is negative.

Step 1: Calculate the individual relative uncertainties The uncertainties attached to the four measured quantities are absolute uncertainties. Divide each by its central measured value:

$$\frac{\Delta a}{a} = \frac{3}{60} = 0.05, \qquad \frac{\Delta b}{b} = \frac{0.1}{20} = 0.005,$$

$$\frac{\Delta c}{c} = \frac{0.2}{40} = 0.005, \qquad \frac{\Delta d}{d} = \frac{0.1}{50} = 0.002.$$

These ratios are dimensionless because each uncertainty has the same unit as its corresponding observable. The fourth power on b must be included when its uncertainty is propagated.

Step 2: Add the weighted uncertainties and determine x For the stated relation $Q = ab^4/(cd)$, the maximum fractional uncertainty is

$$\frac{\Delta Q}{Q} = \frac{\Delta a}{a} + 4\frac{\Delta b}{b} + \frac{\Delta c}{c} + \frac{\Delta d}{d}.$$

Substitution gives

$$\frac{\Delta Q}{Q} = 0.05 + 4(0.005) + 0.005 + 0.002 = 0.077.$$

Multiplying by 100, the percentage error is 7.7%. The question expresses the numerical percentage error as $x/1000$, hence

$$\frac{x}{1000} = 7.7 \quad \Longrightarrow \quad x = 7700.$$

One must not equate $x/1000$ to the fractional error 0.077, since that would miss the conversion from fraction to percentage.

Step 3: Conclusion The maximum percentage error is 7.7%, so the required dimensionless integer is $\boxed{x = 7700}$.

Quick Tip to Solve: Work entirely in fractional errors, include the power four on b, and convert to a percentage only once at the end.

47. Two cars P and Q are moving on a road in the same direction. Acceleration of car P increases linearly with time whereas car Q moves with a constant acceleration. Both cars cross each other at time t = 0, for the first time. The maximum possible number of crossing(s) (including the crossing at t = 0) is _____. [4 marks]

Solution:

Concept:

- Two cars cross when their positions are equal; equivalently, their relative displacement is zero. Each distinct nonnegative root of this displacement represents one meeting.
- A linearly increasing acceleration integrates to a quadratic velocity and a cubic displacement. Subtracting a motion with constant acceleration preserves a cubic relative displacement.

Step 1: Write the relative displacement polynomial Choose a common coordinate along the road. Write $a_P = a_0 + kt$, with $k > 0$, and $a_Q = b$, where a_0 and b are constants. Let the initial velocity difference be $u = v_P(0) - v_Q(0)$. Since the cars meet at $t = 0$, their initial relative position is zero. Integrating the relative acceleration twice gives

$$x_P - x_Q = ut + \frac{a_0 - b}{2}t^2 + \frac{k}{6}t^3.$$

Thus the crossing condition factors as

$$t\left[u + \frac{a_0 - b}{2}t + \frac{k}{6}t^2\right] = 0.$$

There is the stated root at zero, followed by at most two additional real roots from the quadratic.

Step 2: Show that two later crossings are attainable The cubic upper bound is actually achievable with both later roots positive. For example, in a consistent choice of SI units take the relative displacement to be $t(t-1)(t-2)$. Its three distinct zeros are $t = 0, 1, 2$. It corresponds to relative acceleration $6t - 6$. Choose $a_Q = 10$ and $a_P = 4 + 6t$, so P has linearly increasing acceleration and Q has constant

acceleration as required. Initial velocities $v_Q(0) = 10$ and $v_P(0) = 12$ keep both cars moving forward for all $t \geq 0$. Hence this example respects the same-direction condition and realizes three crossings; none can exceed the cubic bound.

Step 3: Conclusion Including the first crossing at $t = 0$, the maximum possible number of crossings is $\boxed{3}$. This is a dimensionless count.

Quick Tip to Solve: Count crossings using zeros of relative position, not relative velocity. The acceleration degree increases by two after two integrations.

48. Two planets, A and B are orbiting a common star in circular orbits of radii R_A and R_B, respectively, with $R_B = 2R_A$. The planet B is $4\sqrt{2}$ times more massive than planet A. The ratio $\left(\frac{L_B}{L_A}\right)$ of angular momentum (L_B) of planet B to that of planet $A\,(L_A)$ is closest to integer _____. [4 marks]

Solution:

Concept:

- For a circular orbit around a star of mass M, gravitational force supplies centripetal force: $GMm/R^2 = mv^2/R$. Therefore $v = \sqrt{GM/R}$.
- For motion perpendicular to the radius, the orbital angular momentum is $L = mvR = m\sqrt{GMR}$. At fixed central mass it is proportional to $m\sqrt{R}$, not to mR or mR^2.

Step 1: Express each angular momentum using its orbital radius Apply the circular-orbit condition separately to the two planets. Their speeds are

$$v_A = \sqrt{\frac{GM}{R_A}}, \qquad v_B = \sqrt{\frac{GM}{R_B}}.$$

The same central star appears in both expressions. Although planet B has the greater orbital radius, its speed is smaller: with $R_B = 2R_A$, the speed ratio is $v_B/v_A = 1/\sqrt{2}$. The planet masses cancel in the equations for their individual speeds, but they do not cancel from their angular momenta.

Step 2: Form the ratio and substitute the given mass factor Using $L = mvR$, divide B by A before inserting any quantities:

$$\frac{L_B}{L_A} = \frac{m_B}{m_A}\frac{v_B}{v_A}\frac{R_B}{R_A} = \frac{m_B}{m_A}\sqrt{\frac{R_B}{R_A}}.$$

The stated mass factor is $m_B/m_A = 4\sqrt{2}$, and the radius factor is two. Consequently,

$$\frac{L_B}{L_A} = 4\sqrt{2}\,\sqrt{2} = 8.$$

Both angular momenta have units of $\mathrm{kg\,m^2\,s^{-1}}$, so their ratio is dimensionless. The result is already an exact integer; no rounding adjustment is required. Using equal speeds for the two orbits would incorrectly introduce an extra factor of $\sqrt{2}$.

Step 3: Conclusion The angular momentum ratio is $L_B/L_A = 8$. The closest integer is therefore $\boxed{8}$, a dimensionless ratio.

Quick Tip to Solve: For circular orbits around the same star, use L proportional to mass times the square root of orbital radius; do not assume equal orbital speeds.

49. A parallel plate capacitor consisting of two circular plates of radius 10 cm is being charged by a constant current of 0.15 A . If the rate of change of potential difference between the plates is 7×10^8 V/s then the integer value of the distance between the parallel plates is

(**Take,** $\epsilon_0 = 9 \times 10^{-12} \frac{\text{F}}{\text{m}}, \pi = \frac{22}{7}$) _____ μm. [4 marks]

Solution:

Concept:

- For a capacitor of fixed geometry, charge and potential difference obey $Q = CV$. Thus the charging current is $I = C\,dV/dt$.
- For two parallel plates separated by air or vacuum, neglecting edge effects, $C = \epsilon_0 A/d$, where $A = \pi r^2$ is the overlapping area and d is the separation.

Step 1: Find the capacitance from the charging rate The current is the rate at which charge reaches either plate. Since the capacitance does not change during charging, differentiating $Q = CV$ gives

$$C = \frac{I}{dV/dt} = \frac{0.15}{7 \times 10^8} = \frac{3}{14} \times 10^{-9}\,\text{F}.$$

This step uses the potential difference between the plates, so no extra factor of two is needed.

Step 2: Calculate the plate area and separation Convert the radius before finding the area: $r = 10\,\text{cm} = 0.10\,\text{m}$. With the specified value of π,

$$A = \frac{22}{7}(0.10)^2 = \frac{22}{700}\,\text{m}^2.$$

Rearranging the capacitance formula,

$$d = \frac{\epsilon_0 A(dV/dt)}{I} = \frac{(9 \times 10^{-12})(22/700)(7 \times 10^8)}{0.15} = 1.32 \times 10^{-3}\,\text{m}.$$

Step 3: Express the result in the requested unit One metre contains 10^6 micrometres. Therefore

$$d = (1.32 \times 10^{-3}) \times 10^6 = 1320\,\mu\text{m}.$$

The small separation relative to the plate radius also agrees with using the parallel-plate expression.

Step 4: Conclusion The distance between the plates is $1320\,\mu$m; the integer to enter is $\boxed{1320}$.

Quick Tip to Solve: Combine the two capacitor equations first: $d = \epsilon_0 \pi r^2 (dV/dt)/I$. Convert the radius to metres before squaring it.

50. The magnetic field inside a 200 turns solenoid of radius 10 cm is 2.9×10^{-4} Tesla. If the solenoid carries a current of 0.29 A , then the length of the solenoid is _____ πcm. [4 marks]

Solution:

Concept:

- For an ideal long air-core solenoid, $B = \mu_0 nI$, where $n = N/L$ is the number of turns per unit length.
- Thus $L = \mu_0 NI/B$. Use $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T\,m\,A^{-1}}$, and express the resulting length in the unit requested in the question.

Step 1: Identify the quantities in the solenoid formula The solenoid has $N = 200$ turns, carries $I = 0.29\,\mathrm{A}$, and produces $B = 2.9 \times 10^{-4}\,\mathrm{T}$. The number of turns per unit length is not 200; it is $200/L$. Therefore

$$B = \mu_0 \frac{200}{L}(0.29).$$

The stated radius does not enter the standard ideal-solenoid field formula.

Step 2: Substitute and simplify the length Rearranging before inserting numerical values gives

$$L = \frac{(4\pi \times 10^{-7})(200)(0.29)}{2.9 \times 10^{-4}}.$$

Since $0.29/(2.9 \times 10^{-4}) = 10^3$, this reduces to

$$L = (4\pi \times 10^{-7})(200)(10^3) = 0.08\pi\,\mathrm{m}.$$

Step 3: Convert metres to centimetres Multiply the length in metres by 100:

$$L = 0.08\pi \times 100 = 8\pi\,\mathrm{cm}.$$

The blank is the coefficient multiplying $\pi\,\mathrm{cm}$, rather than the full numerical length in centimetres. Consequently the coefficient is 8, and there is no need to approximate π.

Step 4: Conclusion The solenoid length is $8\pi\,\mathrm{cm}$. The required integer is $\boxed{8}$.

Quick Tip to Solve: Keep π symbolic when the requested unit is $\pi\,\mathrm{cm}$; compute the coefficient only after converting metres to centimetres.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Given below are two statements :

Statement (I): On nitration of m-xylene with HNO_3, H_2SO_4 followed by oxidation, 4-nitrobenzene-1,3-dicarboxylic acid is obtained as the major product.

Statement (II) : $-CH_3$ group is o/p-directing while $-NO_2$ group is m-directing group.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is true but Statement II is false

(B) Both Statement I and Statement II are true

(C) Both Statement I and Statement II are false

(D) Statement I is false but Statement II is true

Correct Answer: (B) Both Statement I and Statement II are true

Solution:

Concept:

- A methyl group activates an aromatic ring and directs an entering electrophile predominantly to its ortho and para positions through electron donation and hyperconjugation.
- A nitro group withdraws electrons strongly and is meta directing. Oxidation of an alkylbenzene side chain containing benzylic hydrogen converts that side chain into a carboxylic acid group.

Step 1: Locate the favoured nitration position In m-xylene the methyl groups occupy positions 1 and 3. Position 4 is para to the methyl group at 1 and ortho to the methyl group at 3; position 6 is symmetry equivalent. Both methyl groups therefore favour substitution at these positions. Position 2 is ortho to both methyl groups and is more sterically crowded, while position 5 is meta to both. The major mononitration product is consequently 4-nitro-m-xylene.

Step 2: Follow oxidation of the two side chains Oxidation changes each benzylic $-CH_3$ group to $-COOH$, while retaining the substituent positions on the benzene ring. The two carboxyl groups remain at positions 1 and 3 and the nitro group is at position 4. Thus the stated product, 4-nitrobenzene-1,3-dicarboxylic acid, is the major product; Statement I is true.

Step 3: Judge the second statement and eliminate the alternatives The $-CH_3$ group is indeed ortho/-para directing, whereas $-NO_2$ is meta directing because its electron-withdrawing effect destabilises the ortho/para substitution intermediates more strongly. Statement II is also true. Options (A) and (D) each incorrectly reject one true statement, and option (C) incorrectly rejects both.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (B).

Quick Tip to Solve: For m-xylene, check which vacant positions are favoured by both methyl groups, then use steric crowding to distinguish position 2 from positions 4 and 6.

52. Identify the essential amino acids from below:

(A) Valine

(B) Proline

(C) Lysine

(D) Threonine

(E) Tyrosine

Choose the correct answer from the options given below: [4 marks]

(A) (A), (C) and (D) only

(B) (C), (D) and (E) only

(C) (A), (C) and (E) only

(D) (B), (C) and (E) only

Correct Answer: (A) (A), (C) and (D) only

Solution:

Concept:

- Essential amino acids are those the human body cannot synthesise in adequate amounts; they must be supplied by the diet.
- Valine, lysine and threonine are essential amino acids. Proline and tyrosine belong to the usual non-essential category in this classification.

Step 1: Classify each amino acid in the list Valine, labelled (A), is an essential branched-chain amino acid. Lysine, labelled (C), and threonine, labelled (D), are also essential and must be obtained from dietary protein. Thus the three entries that meet the definition are $(A), (C), (D)$. The criterion is nutritional requirement, not whether an amino acid is present in proteins.

Step 2: Exclude the non-essential entries Proline, labelled (B), can be synthesised by the body and is not included in the standard list of essential amino acids. Tyrosine, labelled (E), can be formed from phenylalanine and is likewise classified as non-essential for this question. Calling tyrosine non-essential does not mean it has no biological function; it means dietary supply of tyrosine itself is not normally obligatory.

Step 3: Compare the combinations with the options The first option selects valine, lysine and threonine and excludes both proline and tyrosine. The other combinations include tyrosine, which must be excluded, and omit at least one of the essential entries; the last combination also includes proline. Therefore only the combination $(A), (C), (D)$ is correct. Distinguish the letters assigned to the amino acids from the letters assigned to the answer options.

Step 4: Conclusion The essential amino acids are valine, lysine and threonine only. Hence, the correct option is (A).

Quick Tip to Solve: Check each name against the dietary-essential list before examining the combinations; do not confuse the amino-acid labels with the answer-option labels.

53. Given below are two statements :

Statement (I): NaCl is added to the ice at 0°C, present in the ice cream box to prevent the melting of ice cream.

Statement (II): On addition of NaCl to ice at 0°C, there is a depression in freezing point.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are false

(B) Statement I is true but Statement II is false

(C) Both Statement I and Statement II are true

(D) Statement I is false but Statement II is true

Correct Answer: (C) Both Statement I and Statement II are true

Solution:

Concept:

- Adding a non-volatile solute lowers the freezing point of the solvent. For a dilute solution, $\Delta T_f = iK_f m$, where i accounts for the number of dissolved particles.
- An ice-salt mixture can cool below $0^\circ\mathrm{C}$. Melting ice absorbs latent heat, so this mixture serves as a cooling bath for ice cream.

Step 1: Evaluate the freezing-point statement Pure water freezes at $0^\circ\mathrm{C}$ at ordinary pressure. When NaCl dissolves in the liquid water in contact with ice, it produces dissolved ions and lowers the temperature at which the liquid can coexist with solid ice. This is freezing-point depression, so Statement II is true. The salt is mixed with the surrounding ice rather than being added as an ingredient to the ice cream.

Step 2: Explain how the mixture keeps ice cream cold At the original temperature of $0^\circ\mathrm{C}$, some ice melts because the new equilibrium freezing temperature is lower. Melting requires latent heat, which is taken from the mixture and its surroundings. The resulting cold brine-and-ice bath can be below zero degrees Celsius and absorb heat from the ice-cream container. This helps prevent the ice cream from melting, so Statement I is true.

Step 3: Select the statement combination The word ice in the cooling bath and the ice cream in the container refer to different materials. Salt can promote melting of the bath ice while keeping the ice cream frozen through cooling. Thus there is no contradiction between the two statements. Options (A), (B) and (D) reject one or both true statements and are eliminated.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish bath ice from ice cream: salt melts some bath ice, and that melting absorbs heat to produce a colder cooling mixture.

54. Given below are two statements :

Statement (I): In partition chromatography, stationary phase is thin film of liquid present in the inert support.

Statement (II): In paper chromatography, the material of paper acts as a stationary phase.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is true but Statement II is false

(B) Statement I is false but Statement II is true

(C) Both Statement I and Statement II are false

(D) Both Statement I and Statement II are true

Correct Answer: (A) Statement I is true but Statement II is false

Solution:

Concept:

- Partition chromatography separates components through their different distributions between two liquid phases, one stationary and one moving.
- In ordinary paper chromatography, water retained in the cellulose fibres is the stationary liquid phase; the paper supports this liquid rather than itself being the stationary phase.

Step 1: Evaluate Statement I using the partition mechanism In partition chromatography a thin film of liquid is held on an inert solid support. A moving phase passes over it, and each solute repeatedly partitions between the stationary liquid and the mobile phase. A component that is relatively more soluble in the stationary liquid moves more slowly. This description agrees with Statement I, which is therefore true.

Step 2: Identify what remains stationary in paper chromatography Cellulose paper retains water in and around its fibres. This retained water provides the stationary liquid phase in the standard partition description of paper chromatography. The solvent travelling along the paper is the mobile phase. The paper material is the support; naming the paper itself as the stationary phase confuses the support with the liquid held by it. Therefore Statement II is false.

Step 3: Eliminate the incorrect combinations Only the combination in which Statement I is true and Statement II is false matches the phase assignments. Option (B) reverses both truth values. Option (C) wrongly rejects the liquid-film description in Statement I, while option (D) wrongly identifies the solid paper material as the stationary phase. Separation here is explained by liquid-liquid partition rather than simply by adsorption on dry cellulose.

Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (A).

Quick Tip to Solve: In paper chromatography, remember the three roles separately: paper supports, retained water stays, and the developing solvent moves.

55. 0.1 M solution of KI reacts with excess of H_2SO_4 and KIO_3 solutions. According to equation

$$5I^- + IO_3^- + 6H^+ \rightarrow 3I_2 + 3H_2O$$

Identify the correct statements :

(A) 200 mL of KI solution reacts with 0.004 mol of KIO_3

(B) 200 mL of KI solution reacts with 0.006 mol of H_2SO_4

(C) 0.5 L of KI solution produced 0.005 mol of I_2

(D) Equivalent weight of KIO_3 is equal to ($\frac{\text{Molecular weight}}{5}$)

Choose the correct answer from the options given below : [4 marks]

(A) (C) and (D) only

(B) (A) and (B) only

(C) (B) and (C) only

(D) (A) and (D) only

Correct Answer: (D) (A) and (D) only

Solution:

Concept:

- Molarity gives the amount of iodide: $n = MV$, with volume in litres. KI supplies one iodide ion per formula unit.
- Use the coefficients of the balanced redox equation to obtain the reactant and product mole ratios. Equivalent weight is molar mass divided by the number of electrons gained or lost per formula unit.

Step 1: Check the reactant amounts For 200 mL of the solution, $n(I^-) = 0.1(0.200) = 0.020$ mol. The equation requires one mole of iodate per five moles of iodide, so

$$n(KIO_3) = 0.020/5 = 0.004 \text{ mol.}$$

Statement (A) is true. Six moles of hydrogen ions are needed for these five moles of iodide. Hence the acid demand is $0.020(6/5) = 0.024$ mol of hydrogen ions, supplied by 0.012 mol of sulphuric acid. Statement (B), which gives 0.006 mol, is false.

Step 2: Check iodine production and equivalent weight For 0.5 L, iodide amounts to $0.1(0.5) = 0.050$ mol. Therefore

$$n(I_2) = \frac{3}{5}(0.050) = 0.030 \text{ mol,}$$

not 0.005 mol, so (C) is false. In iodate, iodine has oxidation number +5; in molecular iodine it has oxidation number zero. Thus one mole of iodate gains five moles of electrons and its reaction-specific n-factor is 5. Its equivalent weight is consequently molar mass divided by 5, making (D) true. Do not use the six hydrogen ions as the iodate n-factor: electron transfer fixes that factor.

Step 3: Conclusion The iodate requirement and the equivalent-weight expression are correct; the acid and iodine amounts are incorrect. Hence, the correct option is (D).

Quick Tip to Solve: Calculate iodide moles first, then use 5:1:6:3 for iodide, iodate, hydrogen ions and iodine; halve the hydrogen-ion amount to obtain sulphuric acid.

56. The type of oxide formed by the element among Li, Na, Be, Mg, B and Al that has the least atomic radius is : [4 marks]

(A) AO

(B) AO_2

(C) A_2O

(D) A_2O_3

Correct Answer: (D) A_2O_3

Solution:

Concept:

- Across a period, effective nuclear charge increases and atomic radius generally decreases because electrons enter the same principal shell.
- Down a group the added electron shell increases atomic radius. In a normal oxide oxygen has oxidation number -2, and the formula must be electrically neutral.

Step 1: Identify the smallest element Arrange the listed second-period elements in the direction of decreasing atomic radius:

$$\mathrm{Li} > \mathrm{Be} > \mathrm{B}.$$

Sodium, magnesium and aluminium are their third-period counterparts and have an additional occupied shell. Thus Na is larger than Li, Mg is larger than Be, and Al is larger than B. Boron is therefore the element with the least atomic radius among all six. The comparison uses both the across-period contraction and the down-group increase; it is not enough merely to choose an element from the right side of the third period.

Step 2: Write the neutral oxide formula Boron is a group 13 element and forms its normal oxide in oxidation state $+3$. Let A denote this element, as in the choices. Charge balance between A^{3+} and O^{2-} requires

$$2(+3) + 3(-2) = 0.$$

Hence the formula is $\mathrm{A_2O_3}$, represented by $\mathrm{B_2O_3}$ for boron. AO would require oxidation state $+2$, AO2 would require $+4$ in a normal oxide, and A2O would require $+1$. None is the normal boron oxide asked for. The question asks for the formula type, rather than whether the oxide is acidic or basic; boron oxide is acidic but that classification does not replace the formula.

Step 3: Conclusion The smallest element is boron, whose oxide is $\mathrm{B_2O_3}$, of type $\mathrm{A_2O_3}$. Hence, the correct option is (D).

Quick Tip to Solve: Choose the smallest atom before balancing its oxide; group 13 gives the 2:3 element-to-oxygen ratio in the normal oxide.

57. Which one of the following reaction sequences will give an azo dye? [4 marks]

(A) Benzenesulphonic acid: (i) $SOCl_2$, (ii) NH_3, (iii) benzyl chloride.

(B) Benzonitrile: (i) 70% H_2SO_4, (ii) PCl_5, (iii) aniline.

(C) Nitrobenzene: (i) Sn/HCl, (ii) $NaNO_2/HCl$, (iii) β-naphthol, $NaOH$.

(D) Aniline: (i) $HCl/NaNO_2$, (ii) toluene.

Correct Answer: (C) Nitrobenzene: (i) Sn/HCl, (ii) $NaNO_2/HCl$, (iii) β-naphthol, $NaOH$.

Solution:

Concept:

- An azo dye contains the conjugated linkage $-N{=}N-$ connecting aromatic systems. It is formed by coupling an aryl diazonium salt with an activated aromatic ring.
- A primary aromatic amine undergoes diazotisation with $NaNO_2/HCl$ at about 273–278 K. Phenoxide and naphthoxide ions activate their aromatic rings for electrophilic coupling.

Step 1: Follow the nitrobenzene sequence In option (C), tin and hydrochloric acid reduce nitrobenzene to aniline:

$$C_6H_5NO_2 \xrightarrow{Sn/HCl} C_6H_5NH_2.$$

Nitrous acid generated from sodium nitrite and hydrochloric acid then converts aniline to benzenediazonium chloride:

$$C_6H_5NH_2 + NaNO_2 + 2\,HCl \longrightarrow C_6H_5N_2Cl + NaCl + 2\,H_2O.$$

This supplies the required diazonium electrophile. In the final stage, sodium hydroxide converts beta-naphthol into its activated naphthoxide form, which couples with the diazonium ion to form a strongly conjugated azo compound.

Step 2: Reject sequences lacking the coupling partners Option (A) converts the sulphonic acid into a sulphonyl chloride and then a sulphonamide, followed by reaction with benzyl chloride; it does not generate an aryl diazonium ion. Option (B) involves nitrile hydrolysis, conversion of the resulting acid into an acyl chloride, and reaction with aniline to give an amide rather than an azo linkage. Option (D) does generate a diazonium salt from aniline, but toluene is not the strongly activated phenol or aromatic amine coupling partner required in this standard dye preparation. Only (C) combines diazotisation with effective coupling to beta-naphthol in alkaline medium.

Step 3: Conclusion Nitrobenzene followed by Sn/HCl, $NaNO_2/HCl$, and alkaline β-naphthol gives an azo dye. Hence, the correct option is (C).

Quick Tip to Solve: Look for both stages: formation of an aryl diazonium salt and coupling with an activated phenolic or amino aromatic ring.

58. Identify the homoleptic complexes with odd number of d electrons in the central metal :

(A) $[FeO_4]^{2-}$

(B) $[Fe(CN)_6]^{3-}$

(C) $[Fe(CN)_5NO]^{2-}$

(D) $[CoCl_4]^{2-}$

(E) $[Co\,(H_2O)_3\ F_3]$

Choose the correct answer from the options given below : [4 marks]

(A) (C) and (E) only

(B) (A), (B) and (D) only

(C) (B) and (D) only

(D) (A), (C) and (E) only

Correct Answer: (C) (B) and (D) only

Solution:

Concept:

- A homoleptic complex has only one kind of ligand coordinated to its central metal. A complex containing different ligand types is heteroleptic.
- The d-electron count of a transition-metal ion is its group number minus its oxidation state; odd total d count does not mean the same thing as an odd number of unpaired electrons.

Step 1: Count electrons in the homoleptic candidates For (A), four oxide ligands contribute -8; the complex has charge -2, so iron is $+6$. Iron belongs to group 8, giving $d^{8-6} = d^2$, an even count. For (B), six cyanide ions contribute -6, and the overall charge is -3. Thus iron is $+3$ and has $d^{8-3} = d^5$, an odd count. All six ligands are cyanide, so (B) is homoleptic. For (D), four chloride ions contribute -4; charge -2 gives cobalt(II). Cobalt is in group 9, so it has $d^{9-2} = d^7$, again odd. Its four ligands are identical chloride ions.

Step 2: Apply the ligand-type condition Complex (C), $[Fe(CN)_5NO]^{2-}$, has cyanide and nitrosyl ligands and is therefore heteroleptic. It is excluded regardless of how its electronic structure is represented. Complex (E), $[Co(H_2O)_3F_3]$, contains water and fluoride ligands, so it too is heteroleptic; charge balance also gives cobalt(III), or d^6. Consequently (A) fails the odd-electron requirement, while (C) and (E) fail the homoleptic requirement. Choices including any of these cannot satisfy both conditions. The qualifying pair is exactly (B) and (D), not the larger set containing ferrate.

Step 3: Conclusion Only $[Fe(CN)_6]^{3-}$ and $[CoCl_4]^{2-}$ meet both conditions. Hence, the correct option is (C).

Quick Tip to Solve: First remove mixed-ligand complexes, then calculate group number minus oxidation state for the remaining metal ions.

59. **Match List - I with List - II :**

List - I (Applications)	List - II (Batteries/Cell)
(A) Transistors	(I) Anode - Zn/Hg; Cathode - HgO + C
(B) Hearing aids	(II) Hydrogen fuel cell
(C) Inverters	(III) Anode - Zn; Cathode - Carbon
(D) Apollo space ship	(IV) Anode - Pb; Cathode - Pb | PbO_2

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(B) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(C) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(D) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

Correct Answer: (A) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Solution:

Concept:

- Different cells are selected for different applications according to their size, voltage stability, rechargeability and energy supply.
- The zinc-carbon dry cell, mercury cell, lead storage battery and hydrogen fuel cell have characteristic electrode materials that identify them.

Step 1: Match portable applications A transistor radio commonly uses the ordinary zinc-carbon dry cell. Its zinc container is the anode and its carbon rod serves as the cathodic current collector in contact with the cathode mixture. This is List II entry (III), so (A) matches (III). A mercury cell is suitable for small devices such as hearing aids because it provides a relatively constant potential over its operating life. It uses zinc amalgam, Zn/Hg, at the anode and a mixture containing mercury(II) oxide and carbon at the cathode. Therefore (B) matches (I), not the zinc-carbon or fuel-cell descriptions.

Step 2: Match the high-demand applications An inverter requires a rechargeable energy store, supplied here by the lead storage battery. Its negative plate is lead and its positive plate contains lead dioxide supported on lead; these correspond to entry (IV). Thus (C) matches (IV). The Apollo spacecraft used hydrogen-oxygen fuel cells to supply electricity, with water as the useful reaction product. Entry (II), the hydrogen fuel cell, therefore matches (D). Together the complete mapping is

A–III, B–I, C–IV, D–II.

Option (B) incorrectly assigns a fuel cell to transistors; option (C) assigns a lead battery to transistors and a fuel cell to inverters; option (D) assigns a fuel cell to hearing aids and a mercury cell to Apollo. Each conflicts with the electrode identities and applications.

Step 3: Conclusion Transistors use III, hearing aids use I, inverters use IV, and the Apollo spacecraft uses II. Hence, the correct option is (A).

Quick Tip to Solve: Anchor the pair Apollo–hydrogen fuel cell and inverter–lead storage battery, then distinguish the dry cell from the compact mercury cell.

60. Which one of the following, with HBr will give a phenol? [4 marks]

(A) Anisole, $C_6H_5-O-CH_3$.

(B) $C_6H_5-CH_2-CH(OCH_3)_2$.

(C) $C_6H_5-CH_2-O-C(CH_3)_3$.

(D) $C_6H_5-CH_2-O-CH_3$.

Correct Answer: (A) Anisole, $C_6H_5-O-CH_3$.

Solution:

Concept:

- Protonation of an ether oxygen makes the ether susceptible to cleavage by bromide. An alkyl carbon can undergo substitution, whereas the aryl carbon of an aryl ether does not undergo normal nucleophilic substitution.
- The aryl–oxygen bond has partial double-bond character through resonance. Cleaving the alkyl–oxygen bond in an aryl alkyl ether leaves the oxygen attached directly to the ring, giving a phenol.

Step 1: Identify the aryl alkyl ether Option (A) is anisole, $C_6H_5-O-CH_3$. Here oxygen is directly bonded to an aromatic carbon on one side and a methyl group on the other. After protonation, bromide attacks the methyl carbon by an S_N2 process. The methyl–oxygen bond breaks while the aryl–oxygen bond remains intact:

$$C_6H_5OCH_3 + HBr \longrightarrow C_6H_5OH + CH_3Br.$$

The aromatic product is phenol because the hydroxyl oxygen is directly attached to the benzene ring.

Step 2: Compare the other oxygen-containing compounds In (B), the two methoxy groups belong to an acetal side chain, $C_6H_5CH_2CH(OCH_3)_2$. The aromatic ring is connected through carbon atoms, not directly to oxygen; acidic cleavage or hydrolysis therefore does not create phenol. In (C), $C_6H_5CH_2OC(CH_3)_3$, the oxygen is attached to a benzyl carbon and a tert-butyl carbon. Ether cleavage yields side-chain products such as benzyl alcohol and tert-butyl bromide, rather than a ring hydroxyl group. Option (D), $C_6H_5CH_2OCH_3$, is also a benzyl ether; its cleavage gives benzyl or methyl substitution products and an alcohol, but no direct aromatic C–OH bond. Thus the presence of a benzene ring somewhere in a molecule is insufficient: oxygen must already be attached to that ring.

Step 3: Conclusion Anisole in option (A) gives phenol and methyl bromide on cleavage with HBr. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish aryl–O–alkyl from aryl–CH2–O–alkyl: only the first retains oxygen directly on the aromatic ring to give phenol.

61. The calculated spin-only magnetic moments of $K_3[Fe(OH)_6]$ and $K_4[Fe(OH)_6]$ respectively are :

[4 marks]

(A) 4.90 and 4.90 B.M.

(B) 4.90 and 5.92 B.M.

(C) 5.92 and 4.90 B.M.

(D) 3.87 and 4.90 B.M.

Correct Answer: (C) 5.92 and 4.90 B.M.

Solution:

Concept:

- The oxidation state of iron follows from electrical neutrality; potassium is $+1$ and each hydroxide ligand has charge -1.
- Hydroxide is a weak-field ligand, so these octahedral iron complexes are high spin. For u unpaired electrons, the spin-only moment is $\mu = \sqrt{u(u+2)}$ B.M.

Step 1: Find the oxidation states and d counts For $\mathrm{K_3[Fe(OH)_6]}$, the complex ion is $[\mathrm{Fe(OH)_6}]^{3-}$. If iron has oxidation number x, then $x - 6 = -3$, giving $x = +3$. Iron has atomic number 26, and its neutral configuration is $[\mathrm{Ar}]3d^6 4s^2$. Remove the two 4s electrons before the 3d electrons: $\mathrm{Fe^{3+}}$ is therefore $3d^5$. Similarly, the second ion has charge -4, so $x - 6 = -4$, giving $\mathrm{Fe^{2+}}$, with configuration $3d^6$.

Step 2: Count unpaired electrons in the weak field The octahedral splitting caused by $\mathrm{OH^-}$ is insufficient to force low-spin pairing. The five electrons of high-spin d^5 occupy all five d orbitals singly, giving $t_{2g}^3 e_g^2$ and five unpaired electrons. In high-spin d^6, the sixth electron pairs in one of the lower orbitals, giving $t_{2g}^4 e_g^2$ and four unpaired electrons. Thus the two ions do not have equal magnetic moments.

Step 3: Calculate and compare the moments In the order asked,

$$\mu_1 = \sqrt{5(5+2)} = \sqrt{35} = 5.92 \text{ B.M.}$$

$$\mu_2 = \sqrt{4(4+2)} = \sqrt{24} = 4.90 \text{ B.M.}$$

Option A incorrectly assigns four unpaired electrons to both ions; B reverses the correct order, and D would require only three unpaired electrons for the first ion.

Step 4: Conclusion The moments are 5.92 and 4.90 B.M., respectively. Hence, the correct option is (C).

Quick Tip to Solve: Find the metal oxidation state before counting d electrons; with weak-field hydroxide, use high-spin populations.

62. Consider the equilibrium

$$CO(g) + 3H_2(\ g) \rightleftharpoons CH_4(\ g) + H_2O(\ g)$$

If the pressure applied over the system increases by two fold at constant temperature then

(A) Concentration of reactants and products increases.

(B) Equilibrium will shift in forward direction.

(C) Equilibrium constant increases since concentration of products increases.

(D) Equilibrium constant remains unchanged as concentration of reactants and products remain same.

Choose the correct answer from the options given below : [4 marks]

(A) (A) and (B) only

(B) (A), (B) and (D) only

(C) (B) and (C) only

(D) (A), (B) and (C) only

Correct Answer: (A) (A) and (B) only

Solution:

Concept:

- Compression at constant temperature initially increases all gaseous concentrations; Le Chatelier's principle then favours the side with fewer gas molecules.
- The equilibrium constant of a specified reaction depends on temperature. A pressure change at fixed temperature changes the equilibrium composition, not the constant.

Step 1: Count the gas molecules The reaction is $CO(g) + 3\,H_2(g) \rightleftharpoons CH_4(g) + H_2O(g)$. There are four moles of gaseous reactants for every two moles of gaseous products. Increasing the pressure by decreasing the volume makes the system favour the forward reaction, which reduces the number of gas molecules. Consequently statement B is correct. No change of temperature is specified, so the temperature dependence of the equilibrium constant cannot be invoked.

Step 2: Check the concentration and reaction quotient At the instant the volume is halved, each concentration doubles, which is the compression effect described in statement A. With

$$Q_c = \frac{[CH_4][H_2O]}{[CO][H_2]^3},$$

the immediate new value is

$$Q_c' = \frac{2^2}{2^4} K_c = \frac{K_c}{4}.$$

Since this is less than K_c, the reaction proceeds forward to restore the same equilibrium constant. Statement A and the forward-shift statement B are therefore the intended correct statements.

Step 3: Reject the incorrect explanations Statement C is false: product concentration can rise without increasing K_c. Statement D combines a true assertion, that the constant is unchanged, with the false explanation that reactant and product concentrations remain the same. Compression and the subsequent shift do change concentrations. Thus D as a complete statement is false. Any option

containing C or D must be rejected.

Step 4: Conclusion Only the concentration-increase and forward-shift statements are correct. Hence, the correct option is (A).

Quick Tip to Solve: After compression, compare the powers of the concentration multiplier in the numerator and denominator of the reaction quotient.

63. First ionisation enthalpy values of first four group 15 elements are given below. Choose the correct value for the element that is a main component of apatite family : [4 marks]

(A) 834 kJ mol^{-1}

(B) 947 kJ mol^{-1}

(C) 1402 kJ mol^{-1}

(D) 1012 kJ mol^{-1}

Correct Answer: (D) 1012 kJ mol^{-1}

Solution:

Concept:

- Apatite minerals are calcium phosphates, commonly represented by $Ca_5(PO_4)_3X$, where X can be F^-, Cl^-, or OH^-. The group 15 element to identify is phosphorus.
- First ionisation enthalpy is the energy needed to remove one electron from each atom of a mole of isolated gaseous atoms: $M(g) \longrightarrow M^+(g) + e^-$.

Step 1: Identify the group 15 constituent The first four members of group 15 are nitrogen, phosphorus, arsenic and antimony. In apatite, the characteristic anion is phosphate, $PO_4{}^{3-}$. Its central atom is phosphorus; calcium is a group 2 element and oxygen belongs to group 16. We must therefore select the first ionisation enthalpy of phosphorus, rather than the value belonging to nitrogen merely because nitrogen heads the group.

Step 2: Match the values to the elements The approximate first ionisation enthalpies of these four elements are

$$\begin{aligned} \mathrm{N} &: 1402\ \mathrm{kJ\,mol^{-1}}, \\ \mathrm{P} &: 1012\ \mathrm{kJ\,mol^{-1}}, \\ \mathrm{As} &: 947\ \mathrm{kJ\,mol^{-1}}, \\ \mathrm{Sb} &: 834\ \mathrm{kJ\,mol^{-1}}. \end{aligned}$$

Moving down the group increases the principal shell number and atomic size, while shielding weakens the attraction on the outer electron. Thus the listed values decrease in this order. Phosphorus has outer configuration $3s^2 3p^3$; it is the second member of the group.

Step 3: Eliminate the other numerical choices Option A, 834, corresponds to antimony and option B, 947, to arsenic; neither is the phosphorus present in phosphate minerals. Option C, 1402, corresponds to the much smaller nitrogen atom. The remaining value 1012 belongs to phosphorus and has the required units of energy per mole of gaseous atoms.

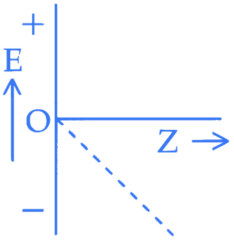
Step 4: Conclusion The required first ionisation enthalpy is $1012\ \mathrm{kJ\,mol^{-1}}$. Hence, the correct option

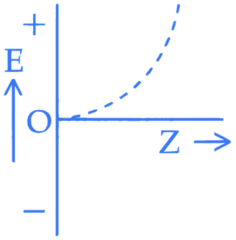
is (D).

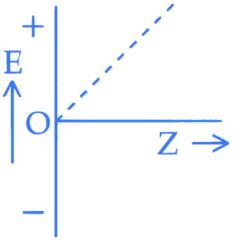
Quick Tip to Solve: Identify the element from the mineral formula first: apatite is a phosphate, so select the phosphorus value.

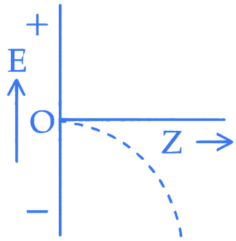
64. For hydrogen like species, which of the following graphs provides the most appropriate representation of E vs Z plot for a constant n?

[E: Energy of the stationary state, Z: atomic number, n: principal quantum number] [4 marks]

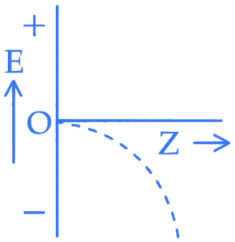
(A) 

(B) 

(C) 

(D) 

Correct Answer: (D)



Solution:

Concept:

- For a one-electron hydrogen-like species, the stationary-state energy is $E_n = -13.6Z^2/n^2$ eV, where Z is the atomic number and n is the principal quantum number.
- At fixed n, the factor $13.6/n^2$ is positive and constant, so E is proportional to $-Z^2$, not to Z or $-Z$.

Step 1: Determine the sign and dependence The zero of energy is the separated nucleus and electron, so a bound stationary state has negative energy. Substitution of different positive atomic numbers at the same n gives

$$E(1) = -\frac{13.6}{n^2}, \quad E(2) = -\frac{54.4}{n^2}, \quad E(3) = -\frac{122.4}{n^2} \quad \text{eV}.$$

The magnitude grows by the squares 1, 4 and 9. Thus increasing nuclear charge binds the electron more strongly and moves the energy downward, farther below zero.

Step 2: Determine the curvature Writing $a = 13.6/n^2 > 0$, the continuous curve through the allowed

integer atomic numbers is $E = -aZ^2$. Its slope is $dE/dZ = -2aZ$, which becomes more negative as Z grows, and its curvature is $d^2E/dZ^2 = -2a < 0$. The required plot therefore bends downward into the negative-energy region. The formal origin helps identify the curve shape, although actual hydrogen-like species have positive integer Z.

Step 3: Compare the four graphs Graph A decreases linearly, so it incorrectly describes a dependence proportional to $-Z$. Graph B curves upwards into positive energies, giving the wrong sign. Graph C also gives positive energy and is linear. Graph D alone is a downward-curving plot below the axis, with the magnitude of its negative slope increasing as Z increases. It therefore represents the quadratic negative dependence.

Step 4: Conclusion The required plot is the negative, downward-curving quadratic in graph D. Hence, the correct option is (D).

Quick Tip to Solve: For a Bohr-energy graph, check both the minus sign and the square in $E_n \propto -Z^2/n^2$; sign alone cannot distinguish A from D.

---

65. If C **(diamond** $) \rightarrow C$ **(graphite)** $+X$ kJ mol^{-1}

C (diamond) $+\mathrm{O}_2(\mathrm{~g}) \rightarrow \mathrm{CO}_2(\mathrm{~g}) + \mathrm{Y kJ mol}^{-1}$

C (graphite) $+\mathrm{O}_2(\mathrm{~g}) \rightarrow \mathrm{CO}_2(\mathrm{~g}) + \mathrm{Z kJ mol}^{-1}$

at constant temperature. Then [4 marks]

(A) $-$X = Y + Z

(B) X = Y $-$ Z

(C) X = $-$Y + Z

(D) X = Y + Z

Correct Answer: (B) X = Y $-$ Z

Solution:

Concept:

- Hess's law states that an enthalpy change depends only on the initial and final states, so the enthalpies of successive reaction steps can be added.
- Heat written on the product side is released: the three reaction enthalpies here are $-X$, $-Y$ and $-Z$, respectively. Keep that sign convention consistent.

Step 1: Choose the direct and indirect paths The direct path converts diamond plus oxygen into carbon dioxide, releasing Y kJ mol^{-1}. The alternative path first converts diamond into graphite, releasing X kJ mol^{-1}, and then burns that graphite in oxygen, releasing Z kJ mol^{-1}. Both paths consume one mole of diamond and one mole of oxygen overall and produce one mole of carbon dioxide.

Step 2: Add the reaction equations The two steps of the indirect path are

$$\mathrm{C(diamond)} \longrightarrow \mathrm{C(graphite)}, \qquad \Delta H_1 = -X,$$

$$\mathrm{C(graphite)} + \mathrm{O_2(g)} \longrightarrow \mathrm{CO_2(g)}, \qquad \Delta H_3 = -Z.$$

Adding cancels graphite, which is produced in the first step and consumed in the second. The overall equation is exactly the diamond-combustion equation, whose enthalpy is $-Y$. Hence

$$-Y = (-X) + (-Z).$$

Each symbol represents the heat term in the same molar units, so no extra stoichiometric multiplier is needed.

Step 3: Rearrange and eliminate the alternatives Multiplying the last relation by -1 gives $Y = X + Z$, and subtracting Z gives $X = Y - Z$. The reverse difference in C would reverse the diamond-to-graphite step. D incorrectly adds the two combustion heats rather than subtracting them; A also uses an inconsistent sign for the conversion heat. Only B follows the cancellation of graphite and conservation of enthalpy.

Step 4: Conclusion The correct relation is $X = Y - Z$. Hence, the correct option is (B).

Quick Tip to Solve: Add diamond-to-graphite conversion to graphite combustion; cancel the graphite before comparing the heat terms.

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66. Given below are two statements :

Statement (I): It is impossible to specify simultaneously with arbitrary precision, both the linear momentum and the position of a particle.

Statement (II) : If the uncertainty in the measurement of position and uncertainty in measurement of momentum are equal for an electron, then the uncertainty in the measurement of velocity is $\geqslant \sqrt{\frac{h}{\pi}} \times \frac{1}{2m}$.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) **Statement I** is false but **Statement II** is true

(B) Both **Statement I** and **Statement II** are true

(C) Both **Statement I** and **Statement II** are false

(D) **Statement I** is true but **Statement II** is false

Correct Answer: (B) Both **Statement I** and **Statement II** are true

Solution:

Concept:

- Heisenberg's principle gives $\Delta x\, \Delta p \geq h/(4\pi)$. It limits simultaneous precision in position and the corresponding momentum component.
- For a nonrelativistic electron of mass m, $p = mv$, so the uncertainties satisfy $\Delta p = m\Delta v$. The question equates the numerical uncertainties in position and momentum.

Step 1: Evaluate Statement I Arbitrarily precise simultaneous position and momentum would mean taking both Δx and Δp towards zero. Their product would then be less than $h/(4\pi)$, contradicting

the uncertainty principle. The limitation is intrinsic to a quantum state and is not removed by improving the measuring instrument. Statement I is therefore true; it correctly states the impossibility of specifying both quantities with arbitrary precision.

Step 2: Apply the equal-uncertainty condition Using the stated numerical equality, put $\Delta x = \Delta p = u$. The uncertainty relation becomes

$$u^2 \geq \frac{h}{4\pi}, \qquad u \geq \sqrt{\frac{h}{4\pi}}.$$

Take the positive square root because an uncertainty cannot be negative. The equality case gives the minimum value allowed by the principle; larger uncertainties remain possible, so the answer must retain the inequality.

Step 3: Convert momentum uncertainty into velocity uncertainty Since $\Delta v = \Delta p/m = u/m$,

$$\Delta v \geq \frac{1}{m}\sqrt{\frac{h}{4\pi}} = \sqrt{\frac{h}{\pi}}\,\frac{1}{2m}.$$

This is the expression in Statement II, so that statement is true in the numerical convention of its given equal-uncertainty condition. Options A and D each incorrectly reject one of the verified statements, and C rejects both. The common error is to forget the square root or to omit the electron mass when converting momentum into velocity.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (B).

Quick Tip to Solve: First apply $\Delta x \Delta p \geq h/(4\pi)$, then convert Δp to $m\Delta v$; do not substitute velocity directly for momentum.

67. Total number of sigma (σ) _____ and pi(π) _____ bonds respectively present in hex-1-en-4-yne are: [4 marks]

(A) 14 and 3

(B) 11 and 3

(C) 13 and 3

(D) 3 and 13

Correct Answer: (C) 13 and 3

Solution:

Concept:

- A single bond contains one σ bond. A double bond contains one σ and one π, while a triple bond contains one σ and two π bonds.
- Carbon is tetravalent. Count all carbon-carbon connections once for the sigma skeleton and then add every carbon-hydrogen bond.

Step 1: Write the six-carbon chain Hex-1-en-4-yne has a double bond between C1 and C2 and a triple bond between C4 and C5. Its condensed structure is $CH_2{=}CH{-}CH_2{-}C{\equiv}C{-}CH_3$. The hydrogens on

the six carbons are respectively 2, 1, 2, 0, 0 and 3. Thus it has eight carbon-hydrogen bonds, each of which is a sigma bond.

Step 2: Count the two kinds of bonds The open chain has five carbon-carbon connections. Even at a double or triple bond, precisely one bond along the internuclear axis is sigma. Therefore

$$N_\sigma = 5 + 8 = 13.$$

The double bond contributes one pi bond and the triple bond contributes two:

$$N_\pi = 1 + 2 = 3.$$

Option (A) overcounts sigma bonds by one; (B) undercounts them by two; (D) reverses the two totals.

Step 3: Conclusion The molecule contains 13 sigma bonds and 3 pi bonds. Hence, the correct option is (C).

Quick Tip to Solve: Count the carbon skeleton connections first, then add C–H bonds; multiple bonds contribute extra pi bonds, never extra sigma bonds.

68. Which among the following halides will generate the most stable carbocation in the nucleophilic substitution reaction? [4 marks]

(A) $CH_2{=}CH{-}CH_2Br$

(B) $(C_6H_5)_3CBr$

(C) $C_6H_5CH_2Br$

(D) 3-bromocyclohex-1-ene

Correct Answer: (B) $(C_6H_5)_3CBr$

Solution:

Concept:

- In ionisation of an alkyl bromide, heterolytic cleavage of the carbon-bromine bond gives Br^- and a carbocation. Compare the stability of the carbocations rather than the strength of the nucleophile.
- Resonance stabilises a carbocation by distributing its positive charge over a conjugated system. More extensive delocalisation generally makes ion formation more favourable.

Step 1: Identify the cation formed by each halide Option (A), allyl bromide, gives the allyl cation $CH_2{=}CH{-}CH_2^+$, whose charge is shared by two terminal carbon atoms. Option (B), triphenylmethyl bromide, gives $(C_6H_5)_3C^+$. Option (C), benzyl bromide, gives $C_6H_5CH_2^+$. Option (D), the cyclic allylic bromide, gives a cyclohexenyl allylic carbocation. The double bonds next to the charged carbon in (A) and (D) allow allylic resonance.

Step 2: Compare the extent of resonance stabilisation In the triphenylmethyl or trityl cation, the empty p orbital of the central carbon can interact with the aromatic pi systems of all three phenyl rings. The positive charge is spread over a much larger framework than in an allyl cation or a benzyl cation with only one phenyl ring. Thus (B) gives the most resonance-stabilised carbocation. The

allylic cations from (A) and (D) and the benzyl cation from (C) are stabilised, but their delocalisation is less extensive.

Step 3: Conclusion Triphenylmethyl bromide generates the most stable carbocation, $(C_6H_5)_3C^+$. Hence, the correct option is (B).

Quick Tip to Solve: For a substitution question asking about the carbocation, remove the halide first and compare how many conjugated rings or double bonds can share the positive charge.

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69. Drug X becomes ineffective after 50% decomposition. The original concentration of drug in a bottle was 16mg/mL which becomes 4mg/mL in 12 months. The expiry time of the drug in months is _____.

Assume that the decomposition of the drug follows first order kinetics. [4 marks]

(A) 12

(B) 3

(C) 6

(D) 2

Correct Answer: (C) 6

Solution:

Concept:

- For first-order decomposition, $[X]_t = [X]_0 e^{-kt}$, where k is the rate constant and t is time.
- The first-order half-life is $t_{1/2} = \ln 2/k$, independent of initial concentration. Fifty per cent decomposition means that half of the initial drug remains.

Step 1: Determine the rate constant The concentration decreases from $16\,\text{mg}\,\text{mL}^{-1}$ to $4\,\text{mg}\,\text{mL}^{-1}$ in 12 months. The ratio of the initial and remaining concentrations is 4. Thus

$$k = \frac{1}{12}\ln\frac{16}{4} = \frac{\ln 4}{12} = \frac{\ln 2}{6}\,\text{month}^{-1}.$$

Equivalently, the changes $16 \to 8 \to 4$ represent two successive half-lives.

Step 2: Apply the expiry criterion The drug becomes ineffective at 50 per cent decomposition, when the concentration is $8\,\text{mg}\,\text{mL}^{-1}$. Hence the expiry time is one half-life:

$$t_{\text{expiry}} = \frac{\ln(16/8)}{k} = \frac{\ln 2}{\ln 2/6} = 6\ \text{months}.$$

Twelve months corresponds to 75 per cent decomposition, not the expiry threshold. Three months is half a half-life, and two months is one-sixth of the observed interval; neither leaves exactly half the drug.

Step 3: Conclusion The expiry time is 6 months. Hence, the correct option is (C).

Quick Tip to Solve: For first-order decay, a fall to one-quarter means two half-lives. Expiry at 50 per cent loss occurs after just one of them.

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70. O_2 **gas will be evolved as a product of electrolysis of :**

(A) an aqueous solution of $AgNO_3$ **using silver electrodes.**

(B) an aqueous solution of $AgNO_3$ **using platinum electrodes.**

(C) a dilute solution of H_2SO_4 **using platinum electrodes.**

(D) a high concentration solution of H_2SO_4 **using platinum electrodes.**

Choose the correct answer from the options given below : [4 marks]

(A) (B) and (C) only

(B) (B) and (D) only

(C) (A) and (D) only

(D) (A) and (C) only

Correct Answer: (A) (B) and (C) only

Solution:

Concept:

- Oxidation occurs at the anode. Its product depends both on the electrolyte and on whether the anode is inert or itself undergoes oxidation.
- At an inert platinum anode in water or dilute acid, oxygen evolution can occur by $2\,H_2O \longrightarrow O_2 + 4\,H^+ + 4\,e^-$. Concentrated sulfate can instead undergo oxidation to peroxodisulfate.

Step 1: Check the silver nitrate solutions In (A), the silver anode is active and dissolves: $Ag \longrightarrow Ag^+ + e^-$. This supplies the anode oxidation process, so oxygen is not the expected product. In (B), platinum does not dissolve. Nitrate is not readily oxidised further, and water is oxidised to oxygen. Thus (A) is excluded and (B) is included. Silver ions are reduced to silver at the cathode; that cathode reaction does not determine whether oxygen is produced at the anode.

Step 2: Distinguish dilute and concentrated sulfuric acid In (C), dilute sulfuric acid with platinum electrodes undergoes electrolysis of water, producing oxygen at the anode and hydrogen at the cathode. Thus (C) is included. For the high-concentration acid in (D), the standard concentrated-sulfate anode process forms peroxodisulfate:

$$2\,HSO_4^- \longrightarrow S_2O_8^{2-} + 2\,H^+ + 2\,e^-.$$

Therefore (D) is not selected as the oxygen-producing case. Options containing (A) or replacing (C) by (D) fail the electrode or concentration test.

Step 3: Conclusion Oxygen is evolved with platinum electrodes in aqueous silver nitrate and in dilute sulfuric acid. Hence, the correct option is (A).

Quick Tip to Solve: Check whether the anode is active before choosing water oxidation; then distinguish dilute acid from concentrated sulfate, which can form a peroxide linkage.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. Total number of non-bonded electrons present in NO_2^- ion based on Lewis theory is _____.

[4 marks]

Solution:

Concept:

- For a Lewis structure, total valence electrons equal the sum from the constituent atoms, with one additional electron for each unit of negative charge.
- Non-bonded electrons occur as lone pairs. A single bond uses two electrons and a double bond uses four; resonance relocates bonds and lone pairs without changing their total number.

Step 1: Find the total and bonding electrons Nitrogen contributes five valence electrons and the two oxygen atoms contribute $2 \times 6 = 12$. The negative charge adds one more electron, so

$$N_{\text{valence}} = 5 + 12 + 1 = 18.$$

A Lewis contributor for NO_2^- has nitrogen in the centre, one nitrogen-oxygen double bond and one nitrogen-oxygen single bond. The three shared electron pairs therefore contain $4 + 2 = 6$ bonding electrons. This arrangement allows octets on all three atoms.

Step 2: Count the lone-pair electrons The double-bonded oxygen has two lone pairs, contributing four non-bonded electrons. The single-bonded oxygen has three lone pairs, contributing six, and nitrogen has one lone pair, contributing two. Therefore

$$N_{\text{nonbonded}} = 4 + 6 + 2 = 12.$$

As a cross-check, subtract the six bonding electrons from all eighteen valence electrons: $18 - 6 = 12$. The other resonance contributor exchanges the two oxygen atoms but leaves this total unchanged. The required number is electrons, not lone pairs; there are six lone pairs in total.

Step 3: Conclusion The total number of non-bonded electrons in NO_2^- is $\boxed{12}$. This is a dimensionless electron count.

Quick Tip to Solve: Count all valence electrons first, then subtract two for every shared bond pair; remember to include the extra electron of the negative charge.

72. Consider the following low-spin complexes

$$K_3\left[Co\left(NO_2\right)_6\right], K_4\left[Fe(CN)_6\right], K_3\left[Fe(CN)_6\right], Cu_2\left[Fe(CN)_6\right] \text{ and } Zn_2\left[Fe(CN)_6\right]$$

The sum of the spin-only magnetic moment values of complexes having yellow colour is _____ B.M. (answer in nearest integer) [4 marks]

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{B.M.}$, where n is the number of unpaired electrons.
- In an octahedral low-spin d^6 complex, electrons occupy $t_{2g}^6 e_g^0$ and are all paired. Observed colour identifies which listed compounds must be included in the requested sum.

Step 1: Identify the yellow compounds Potassium hexanitritocobaltate(III), $K_3[Co(NO_2)_6]$, is yellow. Potassium hexacyanidoferrate(II), $K_4[Fe(CN)_6]$, is also yellow. Potassium hexacyanidoferrate(III), $K_3[Fe(CN)_6]$, forms red crystals, while copper(II) ferrocyanide is a reddish-brown precipitate and zinc ferrocyanide is white. Thus the yellow compounds to include are the cobalt(III) nitrito complex and the iron(II) cyanido complex.

Step 2: Calculate and sum their moments In $[Co(NO_2)_6]^{3-}$, six ligands each carry charge -1, so cobalt is $+3$. Cobalt(III) has a d^6 configuration. In $[Fe(CN)_6]^{4-}$, iron is $+2$, again d^6. Both are specified to be low spin, so both have $n = 0$ and $t_{2g}^6 e_g^0$ occupancy. Each moment is therefore

$$\mu = \sqrt{0(0+2)} = 0\,\text{B.M.}$$

Their sum is exactly $0 + 0 = 0\,\text{B.M.}$; rounding to the nearest integer leaves zero. Colour does not by itself imply unpaired electrons: the spin configuration must be checked separately.

Step 3: Conclusion The sum of the spin-only magnetic moments of the yellow complexes is $\boxed{0}\,\text{B.M.}$. The requested nearest integer is 0.

Quick Tip to Solve: After identifying the yellow salts, calculate oxidation state and d count; low-spin octahedral d-six ions have all six electrons paired and zero spin-only moment.

73. In the sulphur estimation, 0.20 g of a pure organic compound gave 0.40 g of barium sulphate. The percentage of sulphur in the compound is _____ $\times 10^{-1}\%$.
(Molar mass : $O = 16,\ S = 32, Ba = 137$ in g mol^{-1}) [4 marks]

Solution:

Concept:

- In Carius estimation, the sulphur in the organic sample is converted quantitatively into sulphate and precipitated as $BaSO_4$. Each mole of this precipitate contains exactly one mole of sulphur atoms.
- The mass fraction of sulphur in the precipitate is $M(S)/M(BaSO_4)$. The percentage in the original

compound must use the original sample mass, rather than the precipitate mass, in the denominator.

Step 1: Calculate the molar mass of the precipitate Using the atomic masses given in the question, add the contributions of one barium, one sulphur and four oxygen atoms:

$$M(\mathrm{BaSO_4}) = 137 + 32 + 4(16) = 233\,\mathrm{g\,mol^{-1}}.$$

The barium and oxygen contribute to the measured precipitate mass but are not sulphur originally present in the sample.

Step 2: Convert precipitate mass into sulphur mass The stoichiometric ratio of sulphur to barium sulphate is one to one. Therefore,

$$m(\mathrm{S}) = 0.40 \times \frac{32}{233} = 0.05494\,\mathrm{g}.$$

This is smaller than the original 0.20 g sample, as required for a constituent of that sample.

Step 3: Calculate the percentage and the requested coefficient Divide by the mass of the organic compound and multiply by 100:

$$\%\mathrm{S} = \frac{0.40 \times 32}{233 \times 0.20} \times 100 = 27.4678\%.$$

Writing this as $x \times 10^{-1}\%$ gives $x = 274.678$, which rounds to the integer 275. The coefficient is ten times the numerical percentage, not the percentage itself.

Step 4: Conclusion The sulphur content is approximately $27.47\% = 274.68 \times 10^{-1}\%$. Hence the requested integer is $\boxed{275}$ (the coefficient of $10^{-1}\%$).

Quick Tip to Solve: First calculate the percentage using the original sample mass; only then convert to the power-of-ten format printed beside the blank.

74. In the Claisen-Schmidt reaction to prepare, dibenzalacetone from 5.3 g of benzaldehyde, a total of 3.51 g of product was obtained. The percentage yield in this reaction was _____ %. [4 marks]

Solution:

Concept:

- Dibenzalacetone is formed by double aldol condensation: two molecules of benzaldehyde react with one molecule of acetone, eliminating two molecules of water.
- Percentage yield is actual isolated mass divided by theoretical product mass, multiplied by 100. The theoretical product amount is obtained from the limiting reagent and the balanced stoichiometric ratio.

Step 1: Write the balanced condensation Benzaldehyde has no alpha hydrogen; acetone supplies the enolate for successive condensations at its two methyl groups. The overall reaction is

$$2\,\mathrm{C_6H_5CHO} + \mathrm{CH_3COCH_3} \longrightarrow \mathrm{C_6H_5CH{=}CHCOCH{=}CHC_6H_5} + 2\,\mathrm{H_2O}.$$

Thus two moles of benzaldehyde give one mole of dibenzalacetone, taking acetone as sufficiently available for the stated preparation.

Step 2: Find the theoretical product mass The molar masses are $M(\mathrm{C_7H_6O}) = 106\,\mathrm{g\,mol^{-1}}$ and $M(\mathrm{C_{17}H_{14}O}) = 234\,\mathrm{g\,mol^{-1}}$. Therefore,

$$n(\text{benzaldehyde}) = \frac{5.3}{106} = 0.050\,\text{mol},$$

$$n(\text{dibenzalacetone})_{\text{theoretical}} = \frac{0.050}{2} = 0.025\,\text{mol}.$$

The theoretical mass is $0.025 \times 234 = 5.85\,\mathrm{g}$.

Step 3: Compare actual and theoretical masses The isolated product weighs 3.51 g, so

$$\%\text{yield} = \frac{3.51}{5.85} \times 100 = 60\%.$$

The denominator must be the theoretical mass of dibenzalacetone; dividing by the 5.3 g reactant mass would compare different substances and ignore the acetone incorporated into the product.

Step 4: Conclusion The percentage yield of dibenzalacetone is $\boxed{60\%}$. The numerical entry is $\boxed{60}$.

Quick Tip to Solve: For dibenzalacetone, remember the 2:1 benzaldehyde-to-product mole ratio; using a 1:1 ratio doubles the theoretical yield and halves the answer.

75. Isomeric hydrocarbons → negative Baeyer's test

(Molecular formula C_9H_{12})

The total number of isomers from above with four different non-aliphatic substitution sites is -

[4 marks]

Solution:

Concept:

- For $\mathrm{C_9H_{12}}$, the degree of unsaturation is $(2 \times 9 + 2 - 12)/2 = 4$, consistent with a benzene ring carrying saturated alkyl groups. Aromatic rings do not give the ordinary Baeyer's test for alkenes.
- Different non-aliphatic substitution sites are inequivalent hydrogen-bearing positions on the aromatic ring. Positions related by molecular symmetry give the same substitution product and must be counted only once.

Step 1: List the possible alkylbenzene families The benzene ring accounts for six carbon atoms, leaving three carbons in saturated substituents. They can be distributed as one propyl group, one ethyl plus one methyl group, or three methyl groups. These give two propylbenzenes (normal and isopropyl), three ethyltoluenes (ortho, meta and para), and three trimethylbenzenes (1,2,3; 1,2,4; and 1,3,5).

Step 2: Count sites in the ethylmethylbenzenes In 1-ethyl-2-methylbenzene the four remaining aromatic hydrogens occupy positions 3, 4, 5 and 6. Since the adjacent substituents differ, no symmetry interchanges these positions while preserving the molecule, so all four sites are distinct. The same is

true for 1-ethyl-3-methylbenzene: its hydrogen positions 2, 4, 5 and 6 are all inequivalent. In the para isomer, a symmetry plane pairs positions 2 with 6 and 3 with 5, leaving only two distinct sites.

Step 3: Exclude the other families Each monosubstituted propylbenzene has three kinds of aromatic hydrogen: ortho, meta and para. The three trimethylbenzene arrangements have respectively two, three and one kinds of aromatic hydrogen; none can have four because each has only three aromatic hydrogen atoms in total. Hence only the ortho and meta ethylmethylbenzenes satisfy the specified count. Aliphatic side-chain hydrogens are excluded by the wording.

Step 4: Conclusion The qualifying isomers are 1-ethyl-2-methylbenzene and 1-ethyl-3-methylbenzene. The total number is $\boxed{2}$, a dimensionless count.

Quick Tip to Solve: Count only aromatic hydrogen positions, and use symmetry before counting them as different. Unlike substituents remove symmetries that would interchange methyl and ethyl groups.