JEE Main 2025 Paper 1 (B.E./B.Tech) Apr 2 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 2 April 2025, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $\mathrm{A} = \begin{bmatrix} \alpha & -1 \\ 6 & \beta \end{bmatrix}, \alpha > 0$**, such that** $\det(\mathrm{A}) = 0$ **and** $\alpha + \beta = 1$**. If I denotes** 2×2 **identity matrix, then the matrix** $(I + A)^8$ **is :** [4 marks]

(A) $\begin{bmatrix} 257 & -64 \\ 514 & -127 \end{bmatrix}$

(B) $\begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$

(C) $\begin{bmatrix} 1025 & -511 \\ 2024 & -1024 \end{bmatrix}$

(D) $\begin{bmatrix} 4 & -1 \\ 6 & -1 \end{bmatrix}$

Correct Answer: (B) $\begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$

Solution:

Concept:

- The determinant and trace determine the characteristic equation of a matrix of order two: $A^2 - (\operatorname{tr} A)A + (\det A)I = 0$.
- If $A^2 = A$, the matrix is idempotent, so every positive integer power of A equals A. The binomial expansion of $(I + A)^n$ then reduces to $I + (2^n - 1)A$.

Step 1: Determine the entries of the matrix The determinant condition gives $\alpha\beta + 6 = 0$, while the trace condition gives $\beta = 1 - \alpha$. Therefore

$$\alpha(1 - \alpha) = -6 \quad \Longrightarrow \quad (\alpha - 3)(\alpha + 2) = 0.$$

Since $\alpha > 0$, choose $\alpha = 3$, and hence $\beta = -2$. Thus

$$A = \begin{pmatrix} 3 & -1 \\ 6 & -2 \end{pmatrix}.$$

The negative root is inadmissible because it contradicts the stated sign condition.

Step 2: Reduce the eighth power using idempotence Here $\operatorname{tr} A = 1$ and $\det A = 0$, so the characteristic equation gives $A^2 - A = 0$. Direct multiplication also gives $A^2 = A$, confirming that no diagonalisation is required. Because I commutes with A, expand by the binomial theorem:

$$(I + A)^8 = I + \left(\sum_{k=1}^{8} \binom{8}{k} \right) A = I + (2^8 - 1)A = I + 255A.$$

Therefore

$$(I+A)^8 = \begin{pmatrix} 1+765 & -255 \\ 1530 & 1-510 \end{pmatrix} = \begin{pmatrix} 766 & -255 \\ 1530 & -509 \end{pmatrix}.$$

Option (D) is only $I+A$, rather than its eighth power. Options (A) and (C) have entries inconsistent with $I+255A$; in particular their upper-right entries are not -255.

Step 3: Conclusion The required matrix is $\begin{pmatrix} 766 & -255 \\ 1530 & -509 \end{pmatrix}$. Hence, the correct option is (B).

Quick Tip to Solve: Before multiplying matrices repeatedly, use their trace and determinant to check whether the characteristic equation makes the matrix idempotent or nilpotent.

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2. Let the vertices Q and R of the triangle PQR lie on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}, \mathrm{QR} = 5$ and the coordinates of the point P be $(0,2,3)$. If the area of the triangle PQR is $\frac{m}{n}$ then : [4 marks]

(A) $2\,\mathrm{m} - 5\sqrt{21}\mathrm{n} = 0$

(B) $\mathrm{m} - 5\sqrt{21}\mathrm{n} = 0$

(C) $5\,\mathrm{m} - 21\sqrt{2}\mathrm{n} = 0$

(D) $5\,\mathrm{m} - 2\sqrt{21}\mathrm{n} = 0$

Correct Answer: (A) $2\,\mathrm{m} - 5\sqrt{21}\mathrm{n} = 0$

Solution:

Concept:

- The area of a triangle is half the product of its base and the perpendicular height. Since Q and R lie on a fixed line, the height from P is the distance from P to that line.
- For a line through a point A with direction vector $\mathbf{d}$, the perpendicular distance of P is $h = |\overrightarrow{AP} \times \mathbf{d}|/|\mathbf{d}|$. The denominator makes the formula independent of the length chosen for the direction vector.

Step 1: Identify the line and the displacement vector Equating the three ratios to a parameter t gives

$$(x,y,z) = (-3,1,-4) + t(5,2,3).$$

Thus choose $A = (-3,1,-4)$ on the line and $\mathbf{d} = (5,2,3)$. With $P = (0,2,3)$, the displacement from A to P is

$$\overrightarrow{AP} = (3,1,7), \qquad |\mathbf{d}| = \sqrt{25+4+9} = \sqrt{38}.$$

There is no need to determine the positions of Q and R individually: their separation, which is the base length, is already given as 5.

Step 2: Calculate the height and the area Take the cross product in the same coordinate order throughout:

$$\overrightarrow{AP} \times \mathbf{d} = (3,1,7) \times (5,2,3) = (-11,26,1).$$

Its squared magnitude is $121 + 676 + 1 = 798 = 38 \cdot 21$. Therefore

$$h = \frac{\sqrt{798}}{\sqrt{38}} = \sqrt{21}, \qquad \Delta = \frac{1}{2} QR\, h = \frac{5\sqrt{21}}{2}.$$

Since $\Delta = m/n$, multiply $m/n = 5\sqrt{21}/2$ by $2n$ to obtain $2m - 5\sqrt{21}\, n = 0$. Option (B) omits the factor one-half in the triangle area. Options (C) and (D) give different ratios m/n and do not satisfy the calculated area.

Step 3: Conclusion The relation is $2m - 5\sqrt{21}\, n = 0$. Hence, the correct option is (A).

Quick Tip to Solve: When two vertices lie on a known line and their separation is given, use that separation as the base and compute just one point-to-line distance.

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3. Let $a \in R$ and A be a matrix of order 3×3 such that $\det(A) = -4$ and $A + I = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}$, where I is the identity matrix of order 3×3. If $\det((a+1)\,\mathrm{adj}((a-1)A))$ is $2^{\mathrm{m}}3^{\mathrm{n}}, \mathrm{m}, \mathrm{n} \in \{0, 1, 2, \ldots, 20\}$, then $\mathrm{m} + \mathrm{n}$ is equal to : [4 marks]

(A) 14

(B) 17

(C) 15

(D) 16

Correct Answer: (D) 16

Solution:

Concept:

- For a matrix B of order three and a scalar k, $\det(kB) = k^3 \det B$. The scalar multiplies each of the three rows, so its contribution is cubed.
- For a nonsingular matrix B of order three, $\det(\mathrm{adj}\, B) = (\det B)^2$. This follows by taking determinants in $B\,\mathrm{adj}\, B = (\det B)I$.

Step 1: Find the parameter from the determinant of A Subtract the identity from the stated matrix to obtain

$$A = \begin{pmatrix} 0 & a & 1 \\ 2 & 0 & 0 \\ a & 1 & 1 \end{pmatrix}.$$

Expansion along the second row is simplest because only its first entry is nonzero:

$$\det A = -2 \begin{vmatrix} a & 1 \\ 1 & 1 \end{vmatrix} = -2(a-1) = 2 - 2a.$$

As $\det A = -4$, we get $2 - 2a = -4$, or $a = 3$. The minus sign in the expansion comes from the cofactor position $(2, 1)$.

Step 2: Apply the determinant identities in the correct order Substitute $a = 3$ into the requested expression. The inner matrix is $(a-1)A = 2A$, whose determinant is

$$\det(2A) = 2^3 \det A = 8(-4) = -32.$$

Its adjugate therefore has determinant $(-32)^2 = 1024 = 2^{10}$. The scalar outside the adjugate is $a + 1 = 4$, and it multiplies a matrix of order three. Consequently

$$\begin{aligned}\det\big(4\,\mathrm{adj}(2A)\big) &= 4^3 \det\big(\mathrm{adj}(2A)\big) \\ &= 2^6 \cdot 2^{10} = 2^{16}3^0.\end{aligned}$$

Unique prime factorisation now gives $m = 16$, $n = 0$; zero is allowed by the stated set. The choices 14, 17 and 15 cannot be the sum of these exponents. In particular, no factor 3 is introduced by either scalar or by $\det A$.

Step 3: Conclusion We obtain $m + n = 16 + 0 = 16$. Hence, the correct option is (D).

Quick Tip to Solve: Track scalar factors at two separate stages: a scalar inside the adjugate affects its inner determinant, while a scalar outside is cubed when taking the final determinant.

4. If $\theta \in [-2\pi, 2\pi]$, then the number of solutions of $2\sqrt{2}\cos^2\theta + (2-\sqrt{6})\cos\theta - \sqrt{3} = 0$, is equal to: [4 marks]

(A) 8

(B) 6

(C) 10

(D) 12

Correct Answer: (A) 8

Solution:

Concept:

- A quadratic equation in $\cos\theta$ can be solved as an algebraic quadratic by setting $u = \cos\theta$. Only roots in $[-1, 1]$ correspond to real angles.
- For a cosine value strictly between -1 and 1, there are two angles per complete period. On a closed interval, list the angles explicitly to avoid counting endpoints twice or missing a negative angle.

Step 1: Factor the quadratic in cosine Write the given equation in terms of $u = \cos\theta$. Its left side factors as

$$(\sqrt{2}\,u + 1)(2u - \sqrt{3}) = 2\sqrt{2}\,u^2 + (2-\sqrt{6})u - \sqrt{3}.$$

Thus the two possible cosine values are

$$\cos\theta = -\frac{1}{\sqrt{2}} \quad \text{or} \quad \cos\theta = \frac{\sqrt{3}}{2}.$$

Both values belong to $[-1, 1]$, so neither may be discarded. They are different, and hence the two

resulting sets of angles cannot overlap.

Step 2: List the angles in the specified interval The general solution of $\cos\theta = \sqrt{3}/2$ is $\theta = 2k\pi \pm \pi/6$, with integer k. Its members in $[-2\pi, 2\pi]$ are

$$-\frac{11\pi}{6}, \quad -\frac{\pi}{6}, \quad \frac{\pi}{6}, \quad \frac{11\pi}{6}.$$

Similarly, $\cos\theta = -1/\sqrt{2}$ gives $\theta = 2k\pi \pm 3\pi/4$, with the four permitted angles

$$-\frac{5\pi}{4}, \quad -\frac{3\pi}{4}, \quad \frac{3\pi}{4}, \quad \frac{5\pi}{4}.$$

The endpoints $\pm 2\pi$ have cosine 1 and are not solutions. There are therefore $4 + 4 = 8$ distinct angles. The alternatives 6, 10 and 12 respectively omit valid angles or count extra angles not satisfying the equation on the stated interval.

Step 3: Conclusion The equation has exactly 8 solutions in $[-2\pi, 2\pi]$. Hence, the correct option is (A).

Quick Tip to Solve: Factor in cosine first, then list the angles for each root over the actual interval; a closed endpoint contributes only if it satisfies the original equation.

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5. If S and S' are the foci of the ellipse $\frac{x^2}{18} + \frac{y^2}{9} = 1$ and P be a point on the ellipse, then $\min(SP \cdot S'P) + \max(SP \cdot S'P)$ is equal to : [4 marks]

(A) $3(6 + \sqrt{2})$

(B) $3(1 + \sqrt{2})$

(C) 27

(D) 9

Correct Answer: (C) 27

Solution:

Concept:

- For the ellipse $x^2/a^2 + y^2/b^2 = 1$, with $a > b > 0$, its foci are $(\pm c, 0)$, where $c^2 = a^2 - b^2$, and its eccentricity is $e = c/a$.
- A point $P = (a\cos t, b\sin t)$ has distances $a - c\cos t$ and $a + c\cos t$ from the two foci. Their product is $a^2 - c^2\cos^2 t$, which is bounded using $0 \le \cos^2 t \le 1$.

Step 1: Express the product of the two distances Here $a^2 = 18$, $b^2 = 9$, so $a = 3\sqrt{2}$ and $c = 3$. Take $S = (c, 0)$ and $S' = (-c, 0)$. To check the focal distance formula directly, use $b^2 = a^2 - c^2$:

$$\begin{aligned} SP^2 &= (a\cos t - c)^2 + b^2\sin^2 t \\ &= a^2 - 2ac\cos t + c^2\cos^2 t \\ &= (a - c\cos t)^2. \end{aligned}$$

As $a > c$, the expression $a - c\cos t$ is always positive. Hence $SP = a - c\cos t$, and similarly

$S'P = a + c\cos t$. Multiplying gives

$$SP \cdot S'P = a^2 - c^2\cos^2 t = 18 - 9\cos^2 t.$$

Step 2: Find and add the attainable extreme values Since $\cos^2 t$ ranges from 0 to 1, the product is smallest when $\cos^2 t = 1$, at the major-axis vertices $(\pm a, 0)$. It is largest when $\cos t = 0$, at the minor-axis vertices $(0, \pm b)$. Therefore

$$\min(SP \cdot S'P) = 18 - 9 = 9, \qquad \max(SP \cdot S'P) = 18.$$

Both extremes occur on the ellipse, so these are an actual minimum and maximum, not merely bounds. Their required sum is $9 + 18 = 27$. Option (D) gives only the minimum. The two expressions containing $\sqrt{2}$ in (A) and (B) do not equal the sum of the product extremes.

Step 3: Conclusion The required sum is 27. Hence, the correct option is (C).

Quick Tip to Solve: For focal distances of an ellipse, use their product $a^2 - c^2\cos^2 t$: its minimum is b^2 and its maximum is a^2.

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6. Let the focal chord PQ of the parabola $y^2 = 4x$ make an angle of 60° with the positive x axis, where P lies in the first quadrant. If the circle, whose one diameter is PS, S being the focus of the parabola, touches the y-axis at the point $(0, \alpha)$, then $5\alpha^2$ is equal to: [4 marks]

(A) 15

(B) 25

(C) 20

(D) 30

Correct Answer: (A) 15

Solution:

Concept:

- The focus of $y^2 = 4ax$ is $(a, 0)$. A focal chord passes through the focus, so its line is determined by the stated inclination as well as that point.
- The circle with endpoints of a diameter P and S has centre equal to their midpoint and radius $PS/2$. When it touches the y-axis, the point of contact has the same ordinate as the centre.

Step 1: Locate the first-quadrant end of the focal chord Here $a = 1$, so the focus is $S = (1, 0)$. The chord has slope $\tan 60^\circ = \sqrt{3}$; hence its line is

$$y = \sqrt{3}(x - 1).$$

Substitute into $y^2 = 4x$ to find the two ends of the chord:

$$3(x-1)^2 = 4x \quad \Longrightarrow \quad 3x^2 - 10x + 3 = (3x - 1)(x - 3) = 0.$$

For $x = 3$, the ordinate is $2\sqrt{3} > 0$. For $x = 1/3$, the ordinate is $-2\sqrt{3}/3 < 0$. The first-quadrant condition therefore identifies

$$P = (3, 2\sqrt{3}).$$

Choosing the other intersection would ignore the stated location of P.

Step 2: Determine the circle and its point of tangency The centre C of the circle with diameter PS is the midpoint of $P = (3, 2\sqrt{3})$ and $S = (1, 0)$:

$$C = (2, \sqrt{3}), \qquad PS = \sqrt{(3-1)^2 + (2\sqrt{3})^2} = 4.$$

Its radius is $r = PS/2 = 2$. The distance of C from the y-axis is also 2, which confirms the stipulated tangency rather than merely assuming it. The perpendicular from C to that axis is horizontal, so the contact point is $(0, \sqrt{3})$. Thus

$$\alpha = \sqrt{3}, \qquad 5\alpha^2 = 5 \cdot 3 = 15.$$

The alternatives 25, 20 and 30 would require α^2 to be 5, 4 and 6 respectively, whereas the midpoint and tangency calculation give exactly 3.

Step 3: Conclusion The required value of $5\alpha^2$ is 15. Hence, the correct option is (A).

Quick Tip to Solve: For a circle specified by a diameter, find the midpoint first; tangency to an axis then fixes the contact point by perpendicular projection of the centre.

7. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its local maximum and local minimum values at p and q , respectively, such that $p^2 = q$, then $f(3)$ is equal to : [4 marks]

(A) 55

(B) 37

(C) 10

(D) 23

Correct Answer: (B) 37

Solution:

Concept:

- A differentiable function can have a local extremum at an interior point only where its derivative vanishes. The second derivative distinguishes a local maximum from a local minimum when it is nonzero.
- Here $a > 0$, so the critical points a and $2a$ are distinct and ordered. The quantities p, q denote the points at which the respective extrema occur.

Step 1: Find and classify the stationary points Differentiate with respect to x, keeping a fixed, and factor the derivative:

$$f\prime(x) = 6x^2 - 18ax + 12a^2 = 6(x - a)(x - 2a).$$

Thus the critical points are $x = a$ and $x = 2a$. To identify which is the maximum, use $f\prime\prime(x) = 12x - 18a$.

We obtain $f''(a) = -6a < 0$ and $f''(2a) = 6a > 0$. Therefore $p = a$ is the local-maximum point and $q = 2a$ is the local-minimum point.

Step 2: Use the relation and evaluate the function The condition $p^2 = q$ now gives

$$a^2 = 2a \quad \Longrightarrow \quad a(a-2) = 0.$$

The value $a = 0$ is excluded by the hypothesis; hence $a = 2$. Substitute into the original function before evaluating:

$$f(x) = 2x^3 - 18x^2 + 48x + 1.$$

At $x = 3$, this becomes

$$f(3) = 2(27) - 18(9) + 48(3) + 1 = 54 - 162 + 144 + 1 = 37.$$

The other listed numbers, 55, 10 and 23, do not equal this substitution result; in particular, 55 would omit both parameter-dependent terms.

Step 3: Conclusion The allowed parameter is $a = 2$, and $f(3) = 37$. Hence, the correct option is (B).

Quick Tip to Solve: Classify both critical points before applying a relation between them; reversing the maximum and minimum changes the parameter.

8. Let $ABCD$ be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the areas of the triangles ABC, ACD and ADB be 5,6 and 7 square units respectively. Then the area (in square units) of the $\triangle BCD$ is equal to : [4 marks]

(A) $\sqrt{110}$

(B) 12

(C) $\sqrt{340}$

(D) $7\sqrt{3}$

Correct Answer: (A) $\sqrt{110}$

Solution:

Concept:

- The area of a triangle with two side vectors $\mathbf{u}, \mathbf{v}$ from the same vertex is $\frac{1}{2}|\mathbf{u} \times \mathbf{v}|$.
- Mutually perpendicular edges can be placed along three coordinate axes. Their opposite-face area is found from the cross product, avoiding the need to determine the edge lengths separately.

Step 1: Choose coordinates using the perpendicular edges Put $A = (0,0,0)$, $B = (u,0,0)$, $C = (0,v,0)$ and $D = (0,0,w)$, where the positive numbers u, v, w are the three edge lengths. The three faces through A are right triangles. Their given areas yield

$$\frac{uv}{2} = 5, \qquad \frac{vw}{2} = 6, \qquad \frac{wu}{2} = 7.$$

Consequently $uv = 10$, $vw = 12$, and $wu = 14$. These products contain exactly the information needed

for the fourth face.

Step 2: Calculate the area of the opposite face Two vectors in triangle BCD, both starting at B, are $\overrightarrow{BC} = (-u, v, 0)$ and $\overrightarrow{BD} = (-u, 0, w)$. Their cross product is

$$(-u, v, 0) \times (-u, 0, w) = (vw, uw, uv).$$

Therefore its magnitude is $\sqrt{(vw)^2 + (uw)^2 + (uv)^2}$. Dividing by two gives

$$[BCD] = \frac{1}{2}\sqrt{12^2 + 14^2 + 10^2} = \frac{1}{2}\sqrt{440} = \sqrt{110}.$$

Equivalently, the square of this area is $5^2 + 6^2 + 7^2 = 110$. The other options have squares $144, 340, 147$, respectively, so none satisfies the cross-product result.

Step 3: Conclusion The area of $\triangle BCD$ is $\sqrt{110}$ square units. Hence, the correct option is (A).

Quick Tip to Solve: For a tetrahedron with three mutually perpendicular edges at one vertex, square the three adjoining face areas and add them to obtain the opposite-face area squared.

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9. Let z be a complex number such that $|z| = 1$. If $\frac{2+k^2 z}{k+\bar{z}} = kz, k \in$ R, then the maximum distance of $k + ik^2$ from the circle $|z - (1 + 2i)| = 1$ is : [4 marks]

(A) $\sqrt{5} + 1$

(B) 3

(C) $\sqrt{3} + 1$

(D) 2

Correct Answer: (A) $\sqrt{5} + 1$

Solution:

Concept:

- For a complex number on the unit circle, $z\bar{z} = |z|^2 = 1$. This reduces products of z and its conjugate without choosing a particular argument.
- For a fixed point P and a circle with centre C and radius r, the greatest distance from P to a point on the circle is $PC + r$, attained on the line through P, C beyond C.

Step 1: Find the real parameter from the complex equation The fraction is defined, so its denominator $k + \bar{z}$ is nonzero. Multiply through by it:

$$2 + k^2 z = kz(k + \bar{z}) = k^2 z + kz\bar{z}.$$

Since $|z| = 1$, the last term is k. Cancelling the common term $k^2 z$ on both sides leaves $2 = k$. This value is admissible: $2 + \bar{z}$ cannot be zero because $|\bar{z}| = 1$, whereas $|-2| = 2$. Thus the point represented by $k + ik^2$ is $P = (2, 4)$.

Step 2: Find the farthest point on the circle The circle $|z - (1 + 2i)| = 1$ has centre $C = (1, 2)$ and

radius 1. The distance from the fixed point to its centre is

$$PC = \sqrt{(2-1)^2 + (4-2)^2} = \sqrt{5}.$$

For any point X on the circle, the triangle inequality gives $PX \le PC + CX = \sqrt{5} + 1$. Equality occurs when P, C, X are collinear with C between P and X, so this upper bound is attained. The other options, 3, $\sqrt{3}+1$ and 2, are smaller and cannot be the maximum; they do not use the correct centre distance.

Step 3: Conclusion The maximum distance from the point to the circle is $\sqrt{5}+1$. Hence, the correct option is (A).

Quick Tip to Solve: Use $z\bar{z} = 1$ before parametrising a unit complex number. For a farthest-point distance to a circle, add the radius to the centre distance.

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10. Let one focus of the hyperbola $\mathrm{H} : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ **be at** $(\sqrt{10}, 0)$ **and the corresponding directrix be** $x = \frac{9}{\sqrt{10}}$**. If** e **and** l **respectively are the eccentricity and the length of the latus rectum of H , then** $9\left(e^2 + l\right)$ **is equal to :** [4 marks]

(A) 12

(B) 14

(C) 15

(D) 16

Correct Answer: (D) 16

Solution:

Concept:

- For $x^2/a^2 - y^2/b^2 = 1$, the positive focus is $(c, 0)$, with $c = ae$ and $c^2 = a^2 + b^2$.
- The corresponding directrix is $x = a/e = a^2/c$. The full length of a latus rectum is $l = 2b^2/a$, rather than its half-length.

Step 1: Recover the semiaxes from the focus and directrix The focus gives $c = \sqrt{10}$. The corresponding positive directrix is at a^2/c. Hence

$$\frac{a^2}{\sqrt{10}} = \frac{9}{\sqrt{10}} \quad \Longrightarrow \quad a^2 = 9.$$

Take the positive semiaxis length $a = 3$. Now use the hyperbola identity, whose sign differs from the ellipse identity:

$$b^2 = c^2 - a^2 = 10 - 9 = 1.$$

Both squares are positive, as required for the stated real hyperbola.

Step 2: Compute the eccentricity and latus rectum From $e = c/a$, the eccentricity squared is $e^2 = 10/9$,

which exceeds one as it should for a hyperbola. The full latus rectum length is

$$l = \frac{2b^2}{a} = \frac{2}{3}.$$

For a direct check, set $x = c = \sqrt{10}$ in the hyperbola: $10/9 - y^2 = 1$, giving the two endpoints at $y = \pm 1/3$, separated by $2/3$. Finally,

$$9(e^2 + l) = 9\left(\frac{10}{9} + \frac{2}{3}\right) = 10 + 6 = 16.$$

The options 12, 14 and 15 do not match the expression with both correct values; using a half-latus rectum would also be incorrect.

Step 3: Conclusion The required value is 16. Hence, the correct option is (D).

Quick Tip to Solve: Multiply the focal distance by the directrix distance to get a^2, then use $b^2 = c^2 - a^2$. Remember that the full latus rectum is $2b^2/a$.

11. The largest $n \in N$ such that 3^n divides 50 ! is : [4 marks]

(A) 22

(B) 20

(C) 21

(D) 23

Correct Answer: (A) 22

Solution:

Concept:

- The greatest exponent $v_p(N!)$ of a prime p dividing a factorial is the total number of copies of p contributed by its factors.
- Legendre's formula counts these copies in layers: $v_p(N!) = \lfloor N/p \rfloor + \lfloor N/p^2 \rfloor + \cdots$, stopping once the power exceeds N.

Step 1: Count the factors of three in successive layers In the product $50! = 1 \cdot 2 \cdots 50$, every multiple of 3 contributes at least one factor of 3. There are $\lfloor 50/3 \rfloor = 16$ such multiples. Every multiple of 9 contributes an additional factor, and there are $\lfloor 50/9 \rfloor = 5$ of these. Every multiple of 27 contributes one further factor, with $\lfloor 50/27 \rfloor = 1$. No factor between 1 and 50 is divisible by 81, so higher powers add nothing.

Step 2: Add the contributions and test maximality The full exponent is therefore

$$v_3(50!) = 16 + 5 + 1 = 22.$$

The extra counts are intentional: for instance, 9 contributes two copies of 3 and 27 contributes three, so counting each multiple only once would miss factors. After removing all 22 copies, the remaining integer is not divisible by 3. Hence 3^{22} divides $50!$, but 3^{23} does not. The listed exponents 20 and 21

also give divisors, but are not the largest possible; the listed exponent 23 is too large. This establishes the required maximum rather than merely finding one power that divides the factorial.

Step 3: Conclusion The largest natural number is $n = 22$. Hence, the correct option is (A).

Quick Tip to Solve: For the exponent of a prime in a factorial, repeatedly divide the factorial argument by that prime, take integer parts and add until the quotient is zero.

12. Let $P_n = \alpha^n + \beta^n, n \in N$. If $P_{10} = 123, P_9 = 76, P_8 = 47$ and $P_1 = 1$, then the quadratic equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is : [4 marks]

(A) $x^2 + x - 1 = 0$

(B) $x^2 - x + 1 = 0$

(C) $x^2 + x + 1 = 0$

(D) $x^2 - x - 1 = 0$

Correct Answer: (A) $x^2 + x - 1 = 0$

Solution:

Concept:

- If $s = \alpha + \beta$ and $t = \alpha\beta$, each root satisfies $r^2 = sr - t$. Thus the power sums obey $P_n = sP_{n-1} - tP_{n-2}$.
- When $t \neq 0$, the reciprocal roots have sum s/t and product $1/t$. Their monic quadratic is obtained from these two quantities using Vieta's relations.

Step 1: Determine the sum and product of the original roots The condition $P_1 = 1$ gives $s = \alpha + \beta = 1$. Multiply $\alpha^2 = s\alpha - t$ by α^8, and the corresponding identity for β by β^8, then add. This gives

$$P_{10} = sP_9 - tP_8.$$

Substituting the supplied values yields

$$123 = 76 - 47t \quad \Longrightarrow \quad 47t = -47 \quad \Longrightarrow \quad t = -1.$$

In particular neither original root is zero, so both reciprocal roots exist.

Step 2: Form the equation for the reciprocal roots The required sum and product are

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = -1, \qquad \frac{1}{\alpha\beta} = -1.$$

A monic quadratic with root sum S and product T is $x^2 - Sx + T = 0$. Taking $S = -1, T = -1$ gives $x^2 + x - 1 = 0$. As a check, the original equation is $r^2 - r - 1 = 0$; replacing r by $1/x$ and multiplying by x^2 gives $1 - x - x^2 = 0$, the same result. Option (B) has the wrong sum and product, (C) the wrong product, and (D) the wrong sum for the reciprocal roots.

Step 3: Conclusion The required quadratic equation is $x^2 + x - 1 = 0$. Hence, the correct option is (A).

Quick Tip to Solve: Use three consecutive power sums to find the root product, then divide the original sum by that product to obtain the reciprocal-root sum.

13. If $\vec{a}$ is a nonzero vector such that its projections on the vectors $2\hat{i}-\hat{j}+2\hat{k}, \hat{i}+2\hat{j}-2\hat{k}$ and $\hat{k}$ are equal, then a unit vector along $\vec{a}$ is : [4 marks]

(A) $\frac{1}{\sqrt{155}}(-7\hat{i}+9\hat{j}+5\hat{k})$

(B) $\frac{1}{\sqrt{155}}(-7\hat{i}+9\hat{j}-5\hat{k})$

(C) $\frac{1}{\sqrt{155}}(7\hat{i}+9\hat{j}-5\hat{k})$

(D) $\frac{1}{\sqrt{155}}(7\hat{i}+9\hat{j}+5\hat{k})$

Correct Answer: (D) $\frac{1}{\sqrt{155}}(7\hat{i}+9\hat{j}+5\hat{k})$

Solution:

Concept:

- The scalar projection of a vector $\mathbf{a}$ on a nonzero vector $\mathbf{b}$ is $\mathbf{a}\cdot\mathbf{b}/|\mathbf{b}|$. The length in the denominator is essential when the reference vectors have different lengths.
- Write $\mathbf{a} = x\hat{i}+y\hat{j}+z\hat{k}$. A unit vector along it is obtained by dividing its components by $\sqrt{x^2+y^2+z^2}$.

Step 1: Translate the equal projections into equations The first two reference vectors both have magnitude $\sqrt{4+1+4}=3$, whereas $|\hat{k}|=1$. Their common scalar projection is therefore z, giving

$$\frac{2x-y+2z}{3}=z, \qquad \frac{x+2y-2z}{3}=z.$$

Multiplying by three and collecting terms yields $2x-y=z$ and $x+2y=5z$. We compare scalar projections, rather than equating the three unnormalised dot products.

Step 2: Find the direction ratios and normalise From the first equation, $y=2x-z$. Substitute this into the second:

$$x+2(2x-z)=5z \quad \Longrightarrow \quad 5x=7z.$$

Thus $x=7z/5$ and $y=9z/5$, so the direction ratios are $7:9:5$. Since the original vector is nonzero, $z\neq 0$. The corresponding positive unit direction is

$$\frac{7\hat{i}+9\hat{j}+5\hat{k}}{\sqrt{49+81+25}}=\frac{7\hat{i}+9\hat{j}+5\hat{k}}{\sqrt{155}}.$$

For this option the three projections are all $5/\sqrt{155}$. Options A, B and C have different relative signs, so they fail at least one of the two linear equations. The opposite direction also satisfies equal signed projections, but it is not among those three options.

Step 3: Conclusion A listed unit vector satisfying the projection condition is $\frac{7\hat{i}+9\hat{j}+5\hat{k}}{\sqrt{155}}$. Hence, the correct option is (D).

Quick Tip to Solve: Before comparing projections, divide every dot product by the magnitude of its reference vector; equal dot products alone would give the wrong direction.

14. If the system of linear equations

$$3x + y + \beta z = 3$$
$$2x + \alpha y - z = -3$$
$$x + 2y + z = 4$$

has infinitely many solutions, then the value of $22\beta - 9\alpha$ is : [4 marks]

(A) 31

(B) 37

(C) 43

(D) 49

Correct Answer: (A) 31

Solution:

Concept:

- Infinitely many solutions require equal coefficient and augmented ranks, both smaller than the number of unknowns. A vanishing coefficient determinant alone does not establish consistency.
- Eliminate the same variable from two equations, then require the resulting equations to be proportional including their constant terms.

Step 1: Eliminate x using the third equation The third equation gives $x = 4 - 2y - z$. Substitution into the first equation produces

$$3(4 - 2y - z) + y + \beta z = 3 \quad \Longrightarrow \quad -5y + (\beta - 3)z = -9.$$

The second equation similarly becomes

$$2(4 - 2y - z) + \alpha y - z = -3 \quad \Longrightarrow \quad (\alpha - 4)y - 3z = -11.$$

These are exact elimination operations, so they preserve the solutions of the original system.

Step 2: Impose dependence without losing consistency If these two equations were independent, they would determine y, z uniquely and the third original equation would then determine x uniquely. Thus infinitely many solutions require proportional reduced equations. Their nonzero constants fix the proportionality factor as $11/9$, hence

$$\alpha - 4 = \frac{11}{9}(-5), \qquad -3 = \frac{11}{9}(\beta - 3).$$

Consequently $\alpha = -19/9$ and $\beta = 6/11$. For these values, the two reduced equations coincide, leaving one free parameter; they are consistent and have rank one in y, z. Together with the equation for x, the original system has rank two and infinitely many solutions.

Step 3: Evaluate the requested expression Substituting both parameter values gives

$$22\beta - 9\alpha = 22\left(\frac{6}{11}\right) - 9\left(-\frac{19}{9}\right) = 12 + 19 = 31.$$

The other numerical options, $37, 43, 49$, disagree with this value. In particular, keeping the minus sign in -9α is necessary because α is negative.

Step 4: Conclusion The required value is $\boxed{31}$. Hence, the correct option is (A).

Quick Tip to Solve: For infinitely many solutions, check proportionality of the constants as well as the coefficients after elimination; determinant zero can also mean no solution.

15. Let $f : \mathbf{R} \to \mathbf{R}$ be a twice differentiable function such that $(\sin x \cos y)(f(2x + 2y) - f(2x - 2y)) = (\cos x \sin y)(f(2x + 2y) + f(2x - 2y))$, for all $x, y \in \mathbf{R}$. If $f'(0) = \frac{1}{2}$, then the value of $24f''\left(\frac{5\pi}{3}\right)$ is :

[4 marks]

(A) 2

(B) 3

(C) -3

(D) -2

Correct Answer: (C) -3

Solution:

Concept:

- The identities $\sin x \cos y - \cos x \sin y = \sin(x - y)$ and $\sin x \cos y + \cos x \sin y = \sin(x + y)$ separate the two function arguments.
- After identifying the function, apply the chain rule twice. The condition on $f'(0)$ determines the multiplicative constant.

Step 1: Separate the two independent arguments Bring the terms involving $f(2x + 2y)$ to one side and those involving $f(2x - 2y)$ to the other. The given identity becomes

$$\sin(x - y)f(2x + 2y) = \sin(x + y)f(2x - 2y).$$

Put $u = x + y$ and $v = x - y$. Every real pair u, v arises from $x = (u + v)/2$, $y = (u - v)/2$. Thus

$$\sin v\, f(2u) = \sin u\, f(2v) \quad \text{for all real } u, v.$$

Step 2: Determine the function from its derivative Choose $v = \pi/2$, so that $\sin v = 1$, without dividing by a possibly zero variable sine. It follows that $f(2u) = f(\pi) \sin u$. Therefore, for a constant $C = f(\pi)$,

$$f(t) = C \sin(t/2), \qquad f'(t) = \frac{C}{2} \cos(t/2).$$

The condition $f'(0) = 1/2$ gives $C = 1$. This formula is valid for all real t, including the zeros of $\sin(t/2)$, and substitution verifies the original functional relation.

Step 3: Differentiate again and evaluate The second derivative is

$$f''(t) = -\frac{1}{4} \sin(t/2).$$

At $t = 5\pi/3$, the half-angle is $5\pi/6$, whose sine is $1/2$. Hence

$$24f''(5\pi/3) = 24\left(-\frac{1}{4}\right)\left(\frac{1}{2}\right) = -3.$$

Option B has the wrong sign, while A and D have the wrong magnitude. Both the negative sign from the second differentiation and the half-angle chain-rule factors matter.

Step 4: Conclusion The requested value is $\boxed{-3}$. Hence, the correct option is (C).

Quick Tip to Solve: In a two-variable functional identity, look for sine sum and difference identities; fixing one transformed variable at $\pi/2$ often isolates the entire function.

16. Let A be the set of all functions $f : \mathbf{Z} \to \mathbf{Z}$ and R be a relation on A such that $\mathrm{R} = \{(\mathrm{f}, \mathrm{g}) : f(0) = \mathrm{g}(1)$ and $f(1) = \mathrm{g}(0)\}$. Then R is : [4 marks]

(A) Symmetric and transitive but not reflective

(B) Symmetric but neither reflective nor transitive

(C) Transitive but neither reflexive nor symmetric

(D) Reflexive but neither symmetric nor transitive

Correct Answer: (B) Symmetric but neither reflective nor transitive

Solution:

Concept:

- A relation is reflexive if every element is related to itself, symmetric if reversing a related pair preserves membership, and transitive if two successive related pairs imply the outer pair.
- Here membership depends only on the ordered pair of values $(f(0), f(1))$. Passing to a related function swaps those two entries; values at other integers impose no restriction.

Step 1: Test reflexivity and symmetry separately For fRf, the defining conditions require $f(0) = f(1)$. This is not true for every integer-valued function on $\mathbf{Z}$: the function $f(n) = n$ has $f(0) = 0$ and $f(1) = 1$. Hence the relation is not reflexive. Next, suppose fRg. Then $f(0) = g(1)$ and $f(1) = g(0)$. Reversing these equalities gives $g(0) = f(1)$ and $g(1) = f(0)$, which are exactly the conditions for gRf. Thus the relation is symmetric for all functions in the set.

Step 2: Disprove transitivity with explicit functions Take $f(n) = n$, $g(n) = 1 - n$, and $h(n) = n$ for every integer n. The relevant values are

	0	1
f	0	1
g	1	0
h	0	1

We have fRg, because $0 = g(1)$ and $1 = g(0)$. We also have gRh, because $1 = h(1)$ and $0 = h(0)$. However, fRh is false since $f(0) = 0 \neq 1 = h(1)$. These functions are all valid members of the stated set; the fact that $h = f$ is allowed. This single chain of related pairs disproves transitivity.

Step 3: Compare the classifications The tests establish symmetry, failure of reflexivity, and failure

of transitivity. Option A incorrectly asserts transitivity; option C incorrectly denies symmetry and asserts transitivity; option D incorrectly asserts reflexivity and denies symmetry. Only the second option agrees with all three direct checks.

Step 4: Conclusion The relation is symmetric but neither reflexive nor transitive. Hence, the correct option is (B).

Quick Tip to Solve: For a relation that swaps two function values, applying it twice restores the original ordered pair; this helps construct a counterexample to transitivity.

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17. The number of sequences of ten terms, whose terms are either 0 or 1 or 2 , that contain exactly five 1 s and exactly three 2 s , is equal to : [4 marks]

(A) 360

(B) 2520

(C) 1820

(D) 45

Correct Answer: (B) 2520

Solution:

Concept:

- A sequence distinguishes positions, but equal symbols are indistinguishable. The number of arrangements with multiplicities n_0, n_1, n_2 is the multinomial coefficient $(n_0+n_1+n_2)!/(n_0!n_1!n_2!)$.
- One may equivalently select positions for each symbol successively. After the positions for the first two symbols are fixed, the remaining positions are forced.

Step 1: Determine all three multiplicities There are ten positions, exactly five of which contain 1 and exactly three of which contain 2. Since the only allowed symbols are $0, 1, 2$, the remaining $10 - 5 - 3 = 2$ positions must contain 0. Thus every admissible sequence has the same multiset of entries: five ones, three twos and two zeros. We must count arrangements of this multiset, rather than all unrestricted ternary sequences.

Step 2: Choose the positions without overcounting First choose five of the ten positions for the ones. There are $\binom{10}{5} = 252$ choices. Next choose three of the five still-empty positions for the twos; this gives $\binom{5}{3} = 10$ choices. The last two positions necessarily hold zeros. Hence

$$N = \binom{10}{5}\binom{5}{3} = 252 \times 10 = 2520.$$

Each sequence determines exactly one pair of these position choices, and conversely every such pair gives exactly one sequence. This establishes that no sequence is missed or counted twice.

Step 3: Verify with the factorial form Treating all ten entries as distinct would give $10!$, but permutations within the five identical ones, three identical twos and two identical zeros leave a sequence unchanged. Therefore

$$N = \frac{10!}{5!\,3!\,2!} = \frac{3628800}{120 \times 6 \times 2} = 2520.$$

This agrees with the independent position-selection count. Options 360, 1820 and 45 do not include the required number of distinct placements of all three symbol groups, whereas option B matches both calculations.

Step 4: Conclusion The number of sequences is $\boxed{2520}$. Hence, the correct option is (B).

Quick Tip to Solve: When exact counts of all but one allowed symbol are specified, first find the leftover count; then choose positions or use the corresponding multinomial coefficient.

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18. For $\alpha, \beta, \gamma \in \mathbf{R}$, if $\lim_{x\to 0} \frac{x^2 \sin \alpha x + (\gamma - 1)e^{x^2}}{\sin 2x - \beta x} = 3$, then $\beta + \gamma - \alpha$ is equal to : [4 marks]

(A) -1

(B) 4

(C) 6

(D) 7

Correct Answer: (D) 7

Solution:

Concept:

- For a quotient with denominator tending to zero to have a finite limit, any nonzero constant term in the numerator must first vanish.
- Near zero, $\sin t = t - t^3/6 + O(t^5)$ and $\sin(\alpha x) = \alpha x + O(x^3)$. Compare the first nonzero powers after all necessary cancellations.

Step 1: Remove the constant term of the numerator As $x \to 0$, the denominator $\sin 2x - \beta x$ tends to zero, while the numerator tends to $\gamma - 1$, because $x^2 \sin(\alpha x) \to 0$ and $e^{x^2} \to 1$. A nonzero value of $\gamma - 1$ cannot give the stated finite limit 3. Therefore $\gamma = 1$, and the numerator reduces exactly to $x^2 \sin(\alpha x)$.

Step 2: Cancel the linear denominator term The expansions now give

$$x^2 \sin(\alpha x) = \alpha x^3 + O(x^5), \qquad \sin 2x - \beta x = (2 - \beta)x - \frac{4}{3}x^3 + O(x^5).$$

If $\beta \neq 2$, the denominator has order x, so the quotient tends to zero, contradicting the specified limit. Hence $\beta = 2$. The cubic coefficient of the denominator is then $-4/3$, which is nonzero and fixes the relevant leading power.

Step 3: Use the nonzero limit to find alpha With these necessary parameter values,

$$\lim_{x\to 0} \frac{x^2 \sin(\alpha x)}{\sin 2x - 2x} = \frac{\alpha}{-4/3} = -\frac{3\alpha}{4} = 3.$$

Thus $\alpha = -4$. Substitution checks the leading numerator coefficient as -4 and denominator coefficient as $-4/3$, whose ratio is indeed 3. Finally,

$$\beta + \gamma - \alpha = 2 + 1 - (-4) = 7.$$

The alternatives $-1, 4, 6$ do not equal this sum; the subtraction of the negative α contributes positive four.

Step 4: Conclusion The required value is $\boxed{7}$. Hence, the correct option is (D).

Quick Tip to Solve: In parameter-dependent limits, cancel terms in order: first the numerator constant, then any lower-order denominator term, and only then compare the surviving leading coefficients.

19. The term independent of x in the expansion of $\left(\frac{(x+1)}{(x^{2/3}+1-x^{1/3})} - \frac{(x-1)}{(x-x^{1/2})}\right)^{10}, x > 1$, is : [4 marks]

(A) 240

(B) 120

(C) 150

(D) 210

Correct Answer: (D) 210

Solution:

Concept:

- Factor the numerators before expanding. The identities $u^3 + 1 = (u + 1)(u^2 - u + 1)$ and $v^2 - 1 = (v - 1)(v + 1)$ simplify the two fractions.
- In a binomial expansion, the term independent of x has total exponent zero. Its coefficient includes both the binomial coefficient and the sign of the selected negative term.

Step 1: Simplify both rational expressions Put $u = x^{1/3}$ in the first fraction and $v = x^{1/2}$ in the second. Since $x > 1$, the cancelled factors and denominators are nonzero. Then

$$\frac{x+1}{x^{2/3} - x^{1/3} + 1} = x^{1/3} + 1, \qquad \frac{x-1}{x - x^{1/2}} = \frac{(v-1)(v+1)}{v(v-1)} = 1 + x^{-1/2}.$$

Thus the entire expression reduces exactly to $(x^{1/3} - x^{-1/2})^{10}$. The constant terms inside the brackets cancel; they must not be expanded separately.

Step 2: Set the general exponent equal to zero Choose the negative term r times in the expansion. The corresponding term is

$$T_{r+1} = \binom{10}{r}(-1)^r x^{(10-r)/3 - r/2}.$$

For this term to be constant, $2(10 - r) - 3r = 0$, giving $r = 4$. This is an allowed integer between 0 and 10, so there is exactly one constant term. Its coefficient is

$$(-1)^4\binom{10}{4} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = 210.$$

The other proposed numbers 240, 120 and 150 do not equal this coefficient; no additional constant contributions exist to change it.

Step 3: Conclusion The term independent of x is 210. Hence, the correct option is (D).

Quick Tip to Solve: First factor quotients involving fractional powers; after cancellation, solve one linear equation for the zero exponent.

20. Let $a_1, a_2, a_3, \ldots$ be in an A.P. such that $\sum_{k=1}^{12} a_{2k-1} = -\frac{72}{5}a_1$, $a_1 \neq 0$. If $\sum_{k=1}^{n} a_k = 0$, then n is: [4 marks]

(A) 18

(B) 17

(C) 11

(D) 10

Correct Answer: (C) 11

Solution:

Concept:

- For an arithmetic progression with first term a_1 and common difference d, $a_j = a_1 + (j-1)d$. Odd-indexed terms form a new A.P. with common difference $2d$.
- The sum of the first n terms is $S_n = \frac{n}{2}[2a_1 + (n-1)d]$. A positive number of terms has zero sum only when the bracket vanishes.

Step 1: Use the sum of the odd-indexed terms The twelve terms in the given sum are $a_1, a_3, \ldots, a_{23}$; the last index is 23, not 12. As a progression with initial term a_1 and difference $2d$, their sum is

$$\sum_{k=1}^{12} a_{2k-1} = \frac{12}{2}[2a_1 + 11(2d)] = 12a_1 + 132d.$$

Use the given relation and collect the a_1 terms:

$$12a_1 + 132d = -\frac{72}{5}a_1 \quad \Longrightarrow \quad 132d = -\frac{132}{5}a_1 \quad \Longrightarrow \quad d = -\frac{a_1}{5}.$$

The condition $a_1 \neq 0$ will justify division by a_1 in the next step.

Step 2: Find the positive index with zero partial sum Substitute the common difference into the ordinary partial-sum formula:

$$0 = S_n = \frac{n}{2}\left[2a_1 - \frac{n-1}{5}a_1\right] = \frac{na_1}{10}(11-n).$$

Here n is a positive integer and $a_1 \neq 0$, so $11 - n = 0$, giving $n = 11$. As a direct check, $a_{11} = a_1 + 10d = -a_1$, making the first and last terms opposites and the mean zero. The other choices 18, 17 and 10 leave $11 - n \neq 0$ and hence cannot give a zero sum.

Step 3: Conclusion The required number of terms is $n = 11$. Hence, the correct option is (C).

Quick Tip to Solve: Treat odd-indexed terms as an A.P. with difference twice the original difference; then use first-plus-last to check a zero sum.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Three distinct numbers are selected randomly from the set $\{1, 2, 3, \ldots, 40\}$. If the probability, that the selected numbers are in an increasing G.P., is $\frac{m}{n}, \gcd(m, n) = 1$, then $m + n$ is equal to _____ . [4 marks]

Solution:

Concept:

- Three distinct elements are selected without order from 40 elements, so the equally likely sample space has $\binom{40}{3}$ outcomes. Each favourable set has one increasing arrangement.
- Every increasing integer geometric progression has the unique form (du^2, duv, dv^2), where d, u, v are positive integers, $u < v$, and $\gcd(u, v) = 1$. The upper bound is $dv^2 \leq 40$.

Step 1: Parameterise every favourable triple without duplication Let the common ratio in lowest terms be $v/u > 1$, with $\gcd(u, v) = 1$. If the first term is a, the third term is av^2/u^2. Its integrality forces $u^2 \mid a$; write $a = du^2$. The middle and last terms are then duv and dv^2. Conversely, every such choice gives three distinct positive integers in increasing G.P. Thus this parametrisation is both exhaustive and unique, and avoids counting the same set several times. Since $dv^2 \leq 40$, only $v = 2, 3, 4, 5, 6$ occur.

Step 2: Count the admissible parameters For each v, count integers $u < v$ coprime to v, and multiply by the number $\lfloor 40/v^2 \rfloor$ of scales d:

v	2	3	4	5	6
$\#\{u < v : \gcd(u, v) = 1\}$	1	2	2	4	2
$\lfloor 40/v^2 \rfloor$	10	4	2	1	1
Number of triples	10	8	4	4	2

Hence there are $10+8+4+4+2 = 28$ favourable selections. Non-coprime choices such as $(u, v) = (2, 4)$ must not be included, because they duplicate a ratio already counted.

Step 3: Reduce the probability The number of all selections is

$$\binom{40}{3} = \frac{40 \cdot 39 \cdot 38}{6} = 9880.$$

Therefore $P = 28/9880 = 7/2470$. The numerator 7 is coprime to 2470, so this is the required reduced form: $m = 7$, $n = 2470$.

Step 4: Conclusion Thus $m + n = 7 + 2470 = \boxed{2477}$, a dimensionless integer.

Quick Tip to Solve: For integer G.P. triples use coprime ratio parameters and square endpoints; this counts rational common ratios as well as integer ones.

22. Let $f : \mathbf{R} \to \mathbf{R}$ be a thrice differentiable odd function satisfying $f'(x) \geq 0, f''(x) = f(x), f(0) = 0, f'(0) = 3$. Then $9f(\log_e 3)$ is equal to _____ . [4 marks]

Solution:

Concept:

- The linear differential equation $f'' = f$ has general real solution $f(x) = Ae^x + Be^{-x}$, because the characteristic roots are 1 and -1.
- The initial conditions determine the two constants. The oddness and the nonnegative derivative can then be checked directly on the resulting function rather than imposed as extra guesses.

Step 1: Determine the constants from the initial data Write the equation as $f'' - f = 0$. The functions e^x and e^{-x} are linearly independent solutions, so

$$f(x) = Ae^x + Be^{-x}, \qquad f'(x) = Ae^x - Be^{-x}.$$

At zero the given values give $A + B = 0$ and $A - B = 3$. Adding and subtracting these equations yields $A = 3/2$, $B = -3/2$. Consequently,

$$f(x) = \frac{3}{2}(e^x - e^{-x}).$$

This construction uses both initial conditions, so it selects the unique solution of the second-order equation.

Step 2: Check the stated properties and evaluate the function Replacing x by $-x$ in the expression gives $f(-x) = -f(x)$, confirming oddness. Also

$$f'(x) = \frac{3}{2}(e^x + e^{-x}) > 0$$

for every real x, so the required monotonicity condition is satisfied. Differentiating a second time returns f, which also checks the differential equation. Finally, since $e^{\log_e 3} = 3$ and $e^{-\log_e 3} = 1/3$,

$$f(\log_e 3) = \frac{3}{2}\left(3 - \frac{1}{3}\right) = \frac{3}{2} \cdot \frac{8}{3} = 4.$$

The question asks for nine times this value, not just for the function value itself; multiplying only after evaluating avoids losing that factor.

Step 3: Conclusion We obtain $9f(\log_e 3) = 9 \cdot 4 = \boxed{36}$. The requested value is dimensionless.

Quick Tip to Solve: For an odd solution of $f'' = f$, the exponential coefficients are opposites; use the derivative at zero to fix their size.

23. If the area of the region $\{(x, y) : |4 - x^2| \le y \le x^2, y \le 4, x \ge 0\}$ is $\left(\frac{80\sqrt{2}}{\alpha} - \beta\right), \alpha, \beta \in \mathbf{N}$, then $\alpha + \beta$ is equal to _____. [4 marks]

Solution:

Concept:

- The vertical height of the region is its upper boundary minus its lower boundary. Here the lower boundary is $|4 - x^2|$ and the upper boundary is $\min(x^2, 4)$.
- Both boundaries change form at $x = 2$. Solve the feasibility inequalities first to find the actual horizontal interval, then integrate separately on the two sides of 2.

Step 1: Find the admissible intervals and their boundaries For $0 \le x \le 2$, the lower boundary is $4 - x^2$ and the upper boundary is x^2. A nonempty vertical segment requires $4 - x^2 \le x^2$, or $x \ge \sqrt{2}$. Hence the first interval is $[\sqrt{2}, 2]$, with height $2x^2 - 4$. For $x \ge 2$, the lower boundary becomes $x^2 - 4$ and the upper boundary becomes 4. The feasibility condition $x^2 - 4 \le 4$ gives $x \le 2\sqrt{2}$. The second interval is therefore $[2, 2\sqrt{2}]$, with height $8 - x^2$. No region exists outside these two intervals.

Step 2: Integrate the two nonnegative heights The required area is

$$A = \int_{\sqrt{2}}^{2} (2x^2 - 4)\, dx + \int_{2}^{2\sqrt{2}} (8 - x^2)\, dx.$$

Evaluate each piece at its own endpoints:

$$A_1 = \left[\frac{2x^3}{3} - 4x\right]_{\sqrt{2}}^{2} = -\frac{8}{3} + \frac{8\sqrt{2}}{3},$$

$$A_2 = \left[8x - \frac{x^3}{3}\right]_{2}^{2\sqrt{2}} = \frac{32\sqrt{2}}{3} - \frac{40}{3}.$$

Thus $A = 40\sqrt{2}/3 - 16$, which is positive as required for an area.

Step 3: Match the specified expression Rewrite the result with exactly the irrational numerator printed in the question:

$$A = \frac{80\sqrt{2}}{6} - 16.$$

Comparison with $80\sqrt{2}/\alpha - \beta$ gives $\alpha = 6$ and $\beta = 16$. The rational and irrational parts must match separately; they cannot compensate for each other because $\sqrt{2}$ is irrational.

Step 4: Conclusion Therefore $\alpha + \beta = 6 + 16 = \boxed{22}$, a dimensionless integer.

Quick Tip to Solve: For a region involving both an absolute value and two upper bounds, split wherever either boundary changes, then check where upper exceeds lower.

24. The absolute difference between the squares of the radii of the two circles passing through the point $(-9, 4)$ and touching the lines $x + y = 3$ and $x - y = 3$, is equal to _____ . [4 marks]

Solution:

Concept:

- A circle tangent to a line has radius equal to the perpendicular distance of its centre from that line. Centres equidistant from two intersecting lines lie on their angle bisectors.
- For a centre (h, k), tangency to $x + y = 3$ and $x - y = 3$ gives $r^2 = (h + k - 3)^2/2 = (h - k - 3)^2/2$. Passing through $(-9, 4)$ additionally gives $r^2 = (h + 9)^2 + (k - 4)^2$.

Step 1: Identify and test both angle bisectors Equate the two squared line distances and subtract:

$$(h + k - 3)^2 - (h - k - 3)^2 = 4k(h - 3) = 0.$$

Thus every possible centre has either $k = 0$ or $h = 3$. Both branches must be checked because the two lines have two angle bisectors. On the branch $h = 3$, the radius squared is $k^2/2$. The point condition would require

$$144 + (k - 4)^2 = \frac{k^2}{2} \quad \Longrightarrow \quad k^2 - 16k + 320 = 0.$$

Its discriminant is $256 - 1280 = -1024 < 0$, so this branch produces no real centre.

Step 2: Find the two centres on the remaining bisector On $k = 0$, use $r^2 = (h - 3)^2/2$ and the given point:

$$(h + 9)^2 + 16 = \frac{(h - 3)^2}{2}.$$

Expanding and collecting gives

$$h^2 + 42h + 185 = 0 = (h + 5)(h + 37).$$

The two centres are therefore $(-5, 0)$ and $(-37, 0)$. Their squared radii from the line-distance formula are

$$r_1^2 = \frac{(-5 - 3)^2}{2} = 32, \qquad r_2^2 = \frac{(-37 - 3)^2}{2} = 800.$$

The point distances independently verify these values: $(-9 + 5)^2 + 4^2 = 32$ and $(-9 + 37)^2 + 4^2 = 800$. Thus both circles really satisfy every condition, and no centre was lost.

Step 3: Conclusion The absolute difference is $|r_2^2 - r_1^2| = |800 - 32| = \boxed{768}$ square coordinate units.

Quick Tip to Solve: Find both angle bisectors before solving for the centre, and work with squared radii throughout to avoid unnecessary square roots.

25. Let [.] denote the greatest integer function. If $\int_0^{e^3} \left[\frac{1}{e^{x-1}}\right] dx = \alpha - \log_e 2$, then α^3 is equal to _____. [4 marks]

Solution:

Concept:

- The greatest integer $[y]$ is the largest integer not exceeding y. Its integral is evaluated by separating the intervals on which it is constant.
- Here the argument is e^{1-x}, which decreases continuously from e as x increases from zero. Only the integer levels 2 and 1 can contribute a positive interval to the integral.

Step 1: Locate the integer-level crossings Write the integrand as $[e^{1-x}]$. Since $2 < e < 3$, its initial value is 2. The changes of its constant value occur when the argument reaches an integer:

$$e^{1-x} = 2 \;\Rightarrow\; x = 1 - \ln 2,$$

$$e^{1-x} = 1 \;\Rightarrow\; x = 1.$$

Both crossing points are inside the integration interval because $0 < 1 - \ln 2 < 1 < e^3$. No crossing at 3 occurs because the largest argument is $e < 3$.

Step 2: Integrate the constant pieces Thus the greatest integer equals 2 between 0 and $1 - \ln 2$, equals 1 between $1 - \ln 2$ and 1, and equals 0 after 1 up to e^3. The values at isolated endpoints do not affect a definite integral. Therefore

$$\begin{aligned} I &= 2\int_0^{1-\ln 2} dx + \int_{1-\ln 2}^{1} dx \\ &= 2(1 - \ln 2) + \ln 2 = 2 - \ln 2. \end{aligned}$$

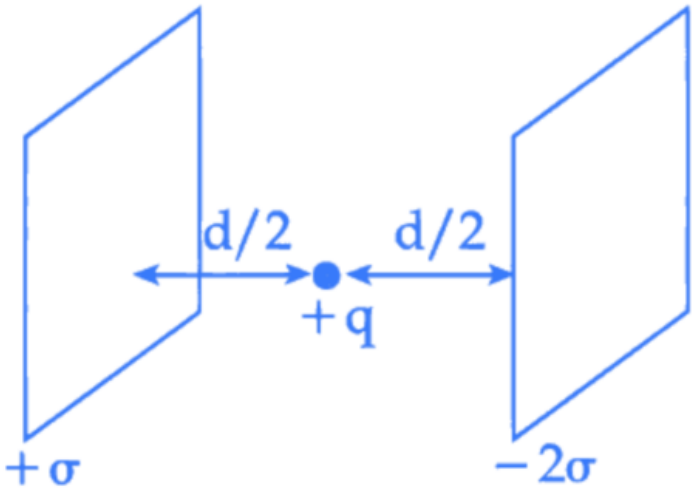
The long upper interval contributes nothing: its exponential argument lies strictly between zero and one.

Step 3: Identify the requested constant Comparing the obtained value $2 - \ln 2$ with the given form $\alpha - \log_e 2$, and using $\log_e 2 = \ln 2$, gives $\alpha = 2$. The question asks for the cube of this constant, rather than the integral itself, so we calculate $\alpha^3 = 2^3 = 8$.

Step 4: Conclusion The required dimensionless numerical value is $\boxed{8}$.

Quick Tip to Solve: Find where the exponential crosses each integer first; the greatest-integer integral then becomes a sum of rectangle areas.

26. Consider two infinitely large plane parallel conducting plates as shown below. The plates are uniformly charged with a surface charge density $+\sigma$ and -2σ. The force experienced by a point charge +q placed at the mid point between two plates will be: [4 marks]



(A) $\frac{3\sigma q}{4\epsilon_0}$
(B) $\frac{3\sigma q}{2\epsilon_0}$
(C) $\frac{\sigma q}{4\epsilon_0}$
(D) $\frac{\sigma q}{2\epsilon_0}$

Correct Answer: (B) $\frac{3\sigma q}{2\epsilon_0}$

Solution:

Concept:

- For an infinite uniformly charged plane with charge per unit area s, the field due to that plane has magnitude $|s|/(2\varepsilon_0)$ on each side.
- The net electric field is the vector sum of the fields of the planes. A charge $+q$ experiences $\vec{F} = q\vec{E}$, in the same direction as the electric field.

Step 1: Determine the field directions in the gap The left plate carries positive charge density $+\sigma$, so its field at the midpoint points away from it, toward the right. The right plate carries negative charge density -2σ, so its field at the midpoint points toward it, also toward the right. Consequently the two fields reinforce each other between the plates. Subtracting their magnitudes would give the field in an exterior region, rather than in this gap.

Step 2: Add the plane-field contributions The charge densities label the total charge per unit area of the respective large plates. Superposition gives

$$E_1 = \frac{\sigma}{2\varepsilon_0}, \qquad E_2 = \frac{2\sigma}{2\varepsilon_0}.$$

Hence

$$E = E_1 + E_2 = \frac{3\sigma}{2\varepsilon_0}.$$

The field of an infinite plane is independent of distance, so the distances $d/2$ do not enter this result. The conducting character does not change the superposition result for the specified total charges; charges redistribute over the faces consistently with zero field within each conductor.

Step 3: Calculate the force and check the options Multiplying by the positive test charge gives

$$F = qE = \frac{3\sigma q}{2\varepsilon_0},$$

directed toward the plate carrying -2σ. Option A is half this value and incorrectly introduces an extra factor of two. Options C and D correspond to smaller fields and do not include the reinforcing contributions of both plates. The force also vanishes linearly with either q or σ, as expected.

Step 4: Conclusion The force is $\boxed{3\sigma q/(2\varepsilon_0)}$, toward the negatively charged plate. Hence, the correct option is (B).

Quick Tip to Solve: Between oppositely charged infinite plates, draw the field direction from each plate before adding their magnitudes.

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27. A monochromatic light is incident on a metallic plate having work function ϕ. An electron, emitted normally to the plate from a point A with maximum kinetic energy, enters a constant magnetic field, perpendicular to the initial velocity of electron. The electron passes through a curve and hits back the plate at a point B. The distance between A and B is : (Given : The magnitude of charge of an electron is e and mass is m, h is Planck's constant and c is velocity of light. Take the magnetic field exists throughout the path of electron) [4 marks]

(A) $\sqrt{2m\left(\frac{hc}{\lambda} - \phi\right)}/\mathrm{eB}$

(B) $\sqrt{8\ \mathrm{m}\left(\frac{\mathrm{hc}}{\lambda} - \phi\right)}/\mathrm{eB}$

(C) $\sqrt{\mathrm{m}(\mathrm{hc}/\lambda - \phi)}/\mathrm{eB}$

(D) $2\sqrt{\mathrm{m}(\mathrm{hc}/\lambda - \phi)}/\mathrm{eB}$

Correct Answer: (B) $\sqrt{8\ \mathrm{m}\left(\frac{\mathrm{hc}}{\lambda} - \phi\right)}/\mathrm{eB}$

Solution:

Concept:

- Einstein's photoelectric equation gives the maximum electron kinetic energy as $K_{\max} = hc/\lambda - \phi$, where λ is the incident wavelength.
- For velocity perpendicular to a uniform magnetic field, the magnetic force provides centripetal force: $evB = mv^2/r$. The magnetic field does no work, so the speed stays constant.

Step 1: Find the emitted electron momentum An incident photon has energy hc/λ. Of this energy, ϕ

is needed to release the electron, and the remainder is its maximum kinetic energy. Thus

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi, \qquad mv = \sqrt{2m\left(\frac{hc}{\lambda} - \phi\right)}.$$

The premise that an electron is emitted requires the photon energy to exceed the work function. We use the nonrelativistic kinetic-energy expression appropriate to the stated photoelectric setup.

Step 2: Obtain the radius and the return geometry Equating the magnetic and centripetal forces gives

$$r = \frac{mv}{eB} = \frac{\sqrt{2m(hc/\lambda - \phi)}}{eB}.$$

Initially the velocity is normal to the plate, so the circle's center lies along a direction parallel to the plate through A. The next intersection of this circle with the plate is diametrically opposite A. The electron therefore traverses a semicircle before hitting B, and the straight separation is $AB = 2r$. It is the diameter, not the arc length πr, that the question requests.

Step 3: Simplify and distinguish the alternatives Substituting the radius yields

$$AB = \frac{2\sqrt{2m(hc/\lambda - \phi)}}{eB} = \frac{\sqrt{8m(hc/\lambda - \phi)}}{eB}.$$

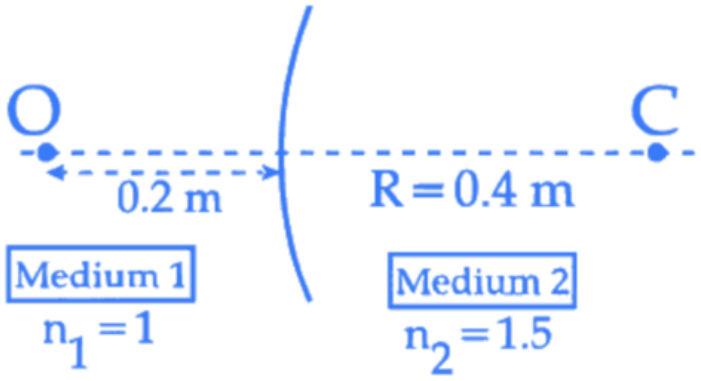
Option A gives only the radius. Options C and D omit the factor of 2 in the kinetic-energy relation inside the radical, so neither equals the diameter. Increasing the field reduces the separation in inverse proportion, consistent with a tighter circular orbit.

Step 4: Conclusion $\boxed{AB = \sqrt{8m(hc/\lambda - \phi)}/(eB)}$. Hence, the correct option is (B).

Quick Tip to Solve: For an electron emitted normally from a flat plate into a perpendicular magnetic field, the next impact is one diameter away.

28. A spherical surface separates two media of refractive indices 1 and 1.5 as shown in figure. Distance of the image of an object ' O ', is :

(C is the center of curvature of the spherical surface and R is the radius of curvature) [4 marks]



(A) 0.24 m left to the spherical surface

(B) 0.24 m right to the spherical surface

(C) 0.4 m left to the spherical surface

(D) 0.4 m right to the spherical surface

Correct Answer: (C) 0.4 m left to the spherical surface

Solution:

Concept:

- Refraction at a spherical surface obeys $n_2/v - n_1/u = (n_2 - n_1)/R$, using the Cartesian sign convention.
- Take the incident-light direction, from left to right, as positive. Distances are measured from the pole of the refracting surface; a negative image distance denotes a virtual image on the object side.

Step 1: Assign the signed distances from the figure The object O lies 0.2 m to the left of the surface, and the center of curvature C lies 0.4 m to its right. Therefore

$$u = -0.2\,\text{m}, \qquad R = +0.4\,\text{m}.$$

The incident medium has $n_1 = 1$, and the refracted medium has $n_2 = 1.5$. The sign of R follows the position of C, regardless of how the curved line appears on the page. Using an unsigned object distance here would reverse the physical interpretation of the result.

Step 2: Apply the spherical-refraction equation Substitute the signed quantities directly into the formula:

$$\frac{1.5}{v} - \frac{1}{-0.2} = \frac{1.5 - 1}{0.4}.$$

Thus

$$\frac{1.5}{v} + 5 = 1.25, \qquad \frac{1.5}{v} = -3.75.$$

Dividing gives

$$v = \frac{1.5}{-3.75}\,\text{m} = -0.4\,\text{m}.$$

All reciprocal distances in the intermediate equation are in inverse metres, so the final distance is in metres.

Step 3: Interpret the sign and compare the options The negative result places the image on the left of the spherical surface, the same side as the object. It is virtual: the refracted rays have backward extensions that meet there. Therefore the distance asked for is 0.4 m to the left. Options A and B give 0.24 m, which does not satisfy the refraction equation. Option D has the correct magnitude but the wrong side; its positive distance would give the wrong signed left-hand side when substituted.

Step 4: Conclusion The image lies $\boxed{0.4\,\text{m}}$ to the left of the spherical surface. Hence, the correct option is (C).

Quick Tip to Solve: Locate the object and curvature center relative to the pole before substituting; the sign of the computed image distance then gives its side.

---

29. A river is flowing from west to east direction with speed of $9\ \text{km h}^{-1}$. If a boat capable of moving at a maximum speed of $27\ \text{km h}^{-1}$ in still water, crosses the river in half a minute, while moving with maximum speed at an angle of 150° to direction of river flow, then the width of the river is : [4 marks]

(A) 112.5 m

(B) 75 m

(C) 300 m

(D) $112.5 \times \sqrt{3}$ m

Correct Answer: (A) 112.5 m

Solution:

Concept:

- The boat's velocity relative to the bank is the vector sum of its velocity relative to water and the water's velocity relative to the bank.
- Only the velocity component perpendicular to the banks contributes to crossing the river. The river current is parallel to the banks and changes the downstream displacement, not the crossing component.

Step 1: Resolve the boat velocity across the river Choose east, the current direction, as the horizontal axis and the perpendicular crossing direction as the vertical axis. The boat moves at $27\,\text{km h}^{-1}$ relative to water, making an angle 150° with east. Its perpendicular component is

$$v_\perp = 27 \sin 150^\circ = 27\left(\frac{1}{2}\right) = 13.5\,\text{km h}^{-1}.$$

The $9\ \text{km h}^{-1}$ current has zero component along this perpendicular direction, so adding the current leaves this crossing speed unchanged.

Step 2: Convert units and calculate the width Use seconds and metres consistently. The crossing time is half a minute, or $30\,\text{s}$, and

$$v_\perp = 13.5 \times \frac{5}{18} = 3.75\,\text{m s}^{-1}.$$

Therefore the width is

$$w = v_{\perp} t = 3.75 \times 30 = 112.5\,\text{m}.$$

This is the perpendicular distance between the banks, not the length of the boat's slanted trajectory relative to the ground.

Step 3: Check the other numerical choices Option B, 75 m, would imply a crossing speed of only $2.5\,\text{m s}^{-1}$, contrary to the resolved component. Option C, 300 m, would imply $10\,\text{m s}^{-1}$, greater than the boat's entire still-water speed of $7.5\,\text{m s}^{-1}$. Option D uses the cosine-related magnitude instead of the sine component needed across the river. The stated current speed affects where the boat lands along the bank but is unnecessary for its width.

Step 4: Conclusion The river width is $\boxed{112.5\,\text{m}}$. Hence, the correct option is (A).

Quick Tip to Solve: For river width, use the component of boat velocity normal to the banks; a current parallel to the banks contributes no crossing speed.

30. Moment of inertia of a rod of mass ' M ' and length ' L ' about an axis passing through its center and normal to its length is ' α '. Now the rod is cut into two equal parts and these parts are joined symmetrically to form a cross shape. Moment of inertia of cross about an axis passing through its center and normal to plane containing cross is : [4 marks]

(A) $\alpha/4$

(B) $\alpha/8$

(C) α

(D) $\alpha/2$

Correct Answer: (A) $\alpha/4$

Solution:

Concept:

- A uniform thin rod of mass M and length L has moment of inertia $ML^2/12$ about an axis through its midpoint and perpendicular to the rod.
- Moments of inertia of separate parts about the same axis add. Cutting a uniform rod into equal halves halves both the mass and the length of each piece.

Step 1: Express the original moment of inertia For the original rod, the specified transverse axis passes through its center. Hence

$$\alpha = \frac{ML^2}{12}.$$

After cutting, each piece has mass $M/2$ and length $L/2$. When the pieces are joined symmetrically into a cross, their midpoints coincide at the center of the cross. The final rotation axis is normal to the plane of the cross and therefore perpendicular to each of the two rods.

Step 2: Calculate the contribution of one half Because the axis passes through the midpoint of each

piece, its moment of inertia is directly the central transverse value:

$$I_{\text{one half}} = \frac{1}{12}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{ML^2}{96} = \frac{\alpha}{8}.$$

No parallel-axis displacement term is needed: neither piece has its center away from the final axis. The factor of eight comes from a factor of two in mass together with a factor of four in the square of length.

Step 3: Add both halves and compare the choices There are two identical pieces, so the total is

$$I_{\text{cross}} = 2\frac{ML^2}{96} = \frac{ML^2}{48} = \frac{\alpha}{4}.$$

Option B, $\alpha/8$, counts only one half. Option C retains the original inertia despite moving all the material closer to the new axis. Option D accounts for neither the full squared-length reduction nor the sum correctly. The new cross extends only $L/4$ from its center along each arm, compared with the original rod's $L/2$, consistent with the reduced inertia.

Step 4: Conclusion The moment of inertia of the cross is $\boxed{\alpha/4}$. Hence, the correct option is (A).

Quick Tip to Solve: For a symmetric cross made from two half-rods, calculate each half about its own midpoint and then add the two equal contributions.

---

31. Let B_1 be the magnitude of magnetic field at center of a circular coil of radius R carrying current I. Let B_2 be the magnitude of magnetic field at an axial distance 'x' from the center. For $x : R = 3 : 4$, $\frac{B_2}{B_1}$ is : [4 marks]

(A) $64 : 125$

(B) $25 : 16$

(C) $4 : 5$

(D) $16 : 25$

Correct Answer: (A) $64 : 125$

Solution:

Concept:

- The field on the axis of a circular coil follows the Biot–Savart law. For radius R, current I, and N turns, $B(x) = \mu_0 NIR^2/[2(R^2 + x^2)^{3/2}]$.
- At the centre, $x = 0$, so $B_1 = \mu_0 NI/(2R)$. Dividing the axial field by the central field cancels the current and number of turns.

Step 1: Form the ratio of the two fields The coil is unchanged between the two observations, so use the same I, N and R. Dividing the expressions gives

$$\frac{B_2}{B_1} = \frac{\mu_0 NIR^2}{2(R^2 + x^2)^{3/2}} \frac{2R}{\mu_0 NI} = \frac{R^3}{(R^2 + x^2)^{3/2}}.$$

The ratio is dimensionless, as required, and decreases as the observation point moves away from the centre.

Step 2: Insert the specified axial distance From $x : R = 3 : 4$, write $x = 3R/4$. Then

$$R^2 + x^2 = R^2\left(1 + \frac{9}{16}\right) = \frac{25R^2}{16},$$

$$\frac{B_2}{B_1} = \left(\frac{16}{25}\right)^{3/2} = \left(\frac{4}{5}\right)^3 = \frac{64}{125}.$$

Option (B) exceeds one and would incorrectly make the axial field larger than the central field. Options (C) and (D) use only the first and second powers of $4/5$; the field expression requires its third power.

Step 3: Conclusion The magnetic-field ratio is $B_2 : B_1 = 64 : 125$. Hence, the correct option is (A).

Quick Tip to Solve: For an axial point, immediately use $B(x)/B(0) = [1 + (x/R)^2]^{-3/2}$; remember the exponent is $-3/2$, not -1.

32. A light wave is propagating with plane wave fronts of the type $x + y + z =$ constant. Th angle made by the direction of wave propagation with the x-axis is : [4 marks]

(A) $\cos^{-1}(2/3)$

(B) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(C) $\cos^{-1}\left(\frac{1}{3}\right)$

(D) $\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

Correct Answer: (B) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Solution:

Concept:

- A wave propagates along the normal to its wavefront. For a plane $ax + by + cz =$ constant, a normal vector has components (a, b, c).
- The direction cosine with the positive x-axis is the x-component of the unit propagation vector: $\cos\theta = n_x/|\boldsymbol{n}|$.

Step 1: Identify a normal to the wavefront The equal-phase surfaces are $x + y + z =$ constant. Their coefficients are all one, so a normal is

$$\boldsymbol{n} = \hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}.$$

It has magnitude $\sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$. Thus the unit normal in the positive direction is

$$\hat{\boldsymbol{n}} = \frac{\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}}{\sqrt{3}}.$$

The three coordinate axes have equal status, so all three direction cosines must have equal magnitudes.

Step 2: Calculate the angle and compare the options Taking the scalar product with the unit vector along the x-axis gives

$$\cos\theta = \hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{i}} = \frac{1}{\sqrt{3}}, \qquad \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right).$$

This is approximately 54.7°. Check that the squared direction cosines sum to one: $3(1/\sqrt{3})^2 = 1$. The values $2/3$, $1/3$ and $\sqrt{2/3}$ in (A), (C) and (D) fail that equal-component normalization. The options specify the acute angle made by the normal direction with the axis.

Step 3: Conclusion The angle is $\cos^{-1}(1/\sqrt{3})$. Hence, the correct option is (B).

Quick Tip to Solve: Read a plane wavefront's coefficients as a normal vector, then normalize them before using a component as a direction cosine.

33. Match List I with List II.

	List - I		List - II
(A)	**Coefficient of viscosity**	**(I)**	$[\mathrm{ML^0\ T^{-3}}]$
(B)	**Intensity of wave**	**(II)**	$[\mathrm{ML^{-2}\ T^{-2}}]$
(C)	**Pressure gradient**	**(III)**	$[\mathrm{M^{-1}LT^2}]$
(D)	**Compressibility**	**(IV)**	$[\mathrm{ML^{-1}\ T^{-1}}]$

Choose the correct answer from the options given below: [4 marks]

(A) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(B) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(C) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(D) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)

Correct Answer: (C) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

Solution:

Concept:

- Dimensions follow from the defining physical equations, rather than from the numerical values of a quantity. Force has dimensions $[MLT^{-2}]$, and pressure has dimensions $[ML^{-1}T^{-2}]$.
- Dynamic viscosity relates shear stress to a velocity gradient. Wave intensity is power per area, pressure gradient is pressure per length, and compressibility is fractional volume change per pressure change.

Step 1: Find viscosity and intensity dimensions For laminar flow, $\tau = \eta\, dv/dy$. Shear stress has pressure dimensions, while $[dv/dy] = [T^{-1}]$. Therefore

$$[\eta] = \frac{[ML^{-1}T^{-2}]}{[T^{-1}]} = [ML^{-1}T^{-1}],$$

which is (IV). Intensity is energy transferred per unit time per unit area. Hence

$$[\mathcal{I}] = \frac{[ML^2T^{-2}]}{[T][L^2]} = [ML^0T^{-3}],$$

which is (I).

Step 2: Find pressure gradient and compressibility dimensions A spatial pressure gradient contains one extra inverse length compared with pressure:

$$\left[\frac{dP}{dx}\right] = \frac{[ML^{-1}T^{-2}]}{[L]} = [ML^{-2}T^{-2}],$$

so (C) matches (II). Compressibility is $\kappa = -(1/V)(dV/dP)$. The volume dimensions cancel, leaving

$$[\kappa] = [P]^{-1} = [M^{-1}LT^2],$$

so (D) matches (III). Thus the complete matching is A–IV, B–I, C–II, D–III. Option (A) already mismatches viscosity; (B) interchanges intensity and pressure gradient; (D) gives viscosity the intensity dimensions. Only (C) has all four assignments correct.

Step 3: Conclusion The dimensions give viscosity–IV, intensity–I, pressure gradient–II, and compressibility–III. Hence, the correct option is (C).

Quick Tip to Solve: Compressibility is inverse pressure; intensity is power per area. These two quick checks often distinguish nearly identical matching options.

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34. A particle is subjected to two simple harmonic motions as : $x_1 = \sqrt{7}\sin 5t$cm and $x_2 = 2\sqrt{7}\sin\left(5t + \frac{\pi}{3}\right)$ cm where x is displacement and t is time in seconds. The maximum acceleration of the particle is $x \times 10^{-2}$ ms^{-2}. The value of x is : [4 marks]

(A) $5\sqrt{7}$

(B) 125

(C) $25\sqrt{7}$

(D) 175

Correct Answer: (D) 175

Solution:

Concept:

- Two collinear simple harmonic motions of the same angular frequency combine into another SHM. If their amplitudes are A_1, A_2 and phase difference is ϕ, the resultant amplitude satisfies $A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$.
- For SHM, $a = -\omega^2 x$, so the maximum magnitude of acceleration is $a_{\max} = \omega^2 A$. Convert centimetres to metres before expressing acceleration in SI units.

Step 1: Calculate the resultant amplitude The two angular frequencies are both $5\,\text{rad}\,\text{s}^{-1}$, and their

phase difference is $\pi/3$. Use $A_1 = \sqrt{7}\,\text{cm}$, $A_2 = 2\sqrt{7}\,\text{cm}$, and $\cos(\pi/3) = 1/2$:

$$A^2 = 7 + 28 + 2(\sqrt{7})(2\sqrt{7})\left(\frac{1}{2}\right)$$
$$= 7 + 28 + 14 = 49\,\text{cm}^2.$$

Therefore $A = 7\,\text{cm} = 7 \times 10^{-2}\,\text{m}$. Adding the amplitudes directly would require zero phase difference, which is not the case here.

Step 2: Obtain the acceleration coefficient The maximum acceleration occurs at the largest displacement magnitude, not at the equilibrium position. Thus

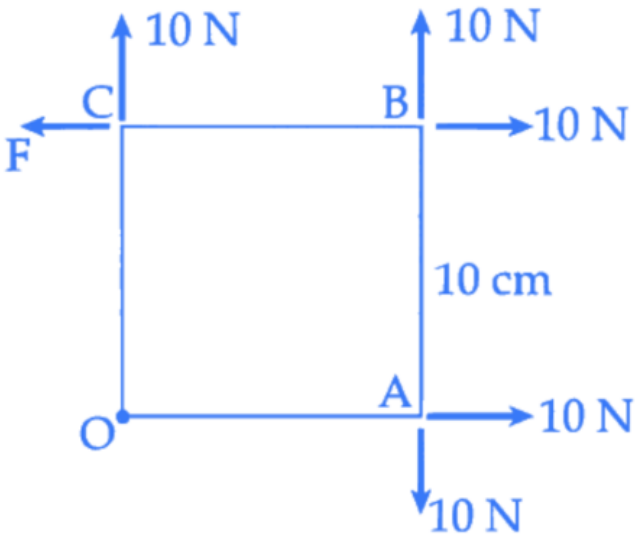
$$a_{\max} = 5^2(7 \times 10^{-2}) = 175 \times 10^{-2}\,\text{m s}^{-2}.$$

Comparing with the specified form $x \times 10^{-2}\,\text{m s}^{-2}$ gives $x = 175$. The first and third options retain an unnecessary $\sqrt{7}$ because they do not combine the two oscillations correctly. The value 125 in option (B) is incompatible with the resultant amplitude of $7\,\text{cm}$.

Step 3: Conclusion The maximum acceleration is $1.75\,\text{m s}^{-2}$, giving $x = 175$. Hence, the correct option is (D).

Quick Tip to Solve: Combine same-frequency SHMs using the phase-dependent amplitude formula first; only then multiply the resultant amplitude by ω^2.

35. A square Lamina OABC of length 10 cm is pivoted at ' O′. Forces act at Lamina as shown in figure. If Lamina remains stationary, then the magnitude of F is : [4 marks]



(A) 10 N

(B) 0 (zero)

(C) $10\sqrt{2}$ N

(D) 20 N

Correct Answer: (A) 10 N

Solution:

Concept:

- A lamina stationary about a pivot has zero net torque about that pivot. The signed torque in the plane is $\tau_z = xF_y - yF_x$; equivalently, use force times perpendicular distance.
- A force whose line of action passes through the pivot produces no torque. The pivot can supply a reaction force, so torque balance must be applied about O without imposing an unnecessary condition on the shown forces alone.

Step 1: Identify the torques of the known forces Let the side length be $a = 10\,\mathrm{cm} = 0.10\,\mathrm{m}$, with $O = (0,0)$, $A = (a,0)$, $B = (a,a)$, and $C = (0,a)$. Take anticlockwise torque as positive. The upward $10\,\mathrm{N}$ force at C and rightward $10\,\mathrm{N}$ force at A have lines of action through O, so each has zero torque. At B, the upward force gives $+10a$, and the rightward force gives $-10a$; these cancel. The downward $10\,\mathrm{N}$ force at A gives the remaining clockwise torque $-10a$.

Step 2: Balance the torque of the unknown force The leftward force F is applied at C, a vertical distance a above the pivot. It therefore gives an anticlockwise torque $+Fa$. The stationary condition is

$$Fa + 10a - 10a - 10a = 0,$$

$$Fa = 10a \quad \Longrightarrow \quad F = 10\,\mathrm{N}.$$

Numerically, the known net torque is $-1.0\,\mathrm{N\,m}$, balanced by $F(0.10\,\mathrm{m})$. A zero force would leave that clockwise torque uncompensated, while $10\sqrt{2}\,\mathrm{N}$ and $20\,\mathrm{N}$ would overcompensate it. No diagonal lever arm is needed because the force is horizontal.

Step 3: Conclusion The required force magnitude is $F = 10\,\mathrm{N}$. Hence, the correct option is (A).

Quick Tip to Solve: Eliminate forces whose lines pass through the pivot, then assign a sign to each remaining torque before combining them.

36. A point charge $+q$ is placed at the origin. A second point charge $+9q$ is placed at $(\mathrm{d},0,0)$ in Cartesian coordinate system. The point in between them where the electric field vanishes is: [4 marks]

(A) $(3\mathrm{d}/4, 0, 0)$

(B) $(\mathrm{d}/4, 0, 0)$

(C) $(4\mathrm{d}/3, 0, 0)$

(D) $(\mathrm{d}/3, 0, 0)$

Correct Answer: (B) $(\mathrm{d}/4, 0, 0)$

Solution:

Concept:

- The electric field of a positive point charge points away from the charge, with magnitude $E = kQ/r^2$. Between two positive charges their fields are opposite, so cancellation requires equal magnitudes.
- The cancellation point lies nearer the smaller charge. Along the line of centres, distances from the

charges must add to their separation d.

Step 1: Express both fields at an intermediate point Let the point be $(x, 0, 0)$, with $0 < x < d$. Its distance from $+q$ at the origin is x, and its distance from $+9q$ is $d - x$. The first field points along the positive x-direction and the second along the negative x-direction. Their magnitudes are

$$E_1 = \frac{kq}{x^2}, \qquad E_2 = \frac{9kq}{(d-x)^2}.$$

Thus equating magnitudes is justified only after checking that these directions oppose each other.

Step 2: Solve the equality and check the choices Cancellation gives

$$\frac{kq}{x^2} = \frac{9kq}{(d-x)^2} \quad \Longrightarrow \quad (d-x)^2 = 9x^2.$$

Both distances are positive in the stated interval, so take $d - x = 3x$, obtaining

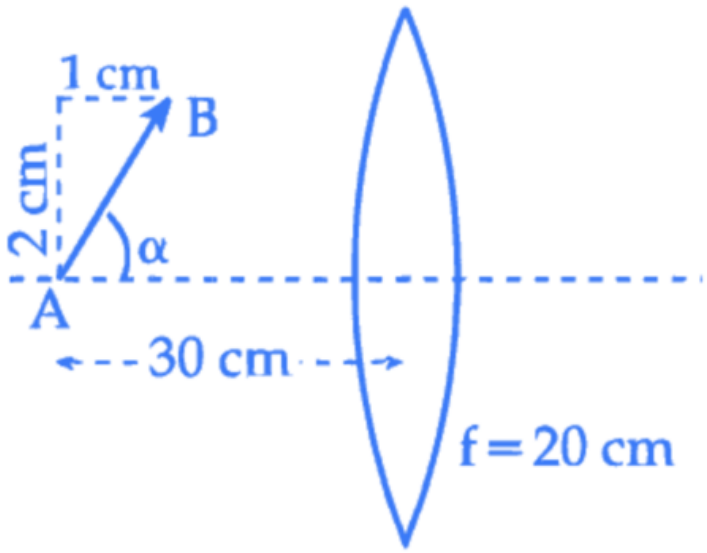
$$d = 4x, \qquad x = \frac{d}{4}.$$

At this point, the distances are $d/4$ and $3d/4$, exactly the required ratio $1 : 3$. Option (A) places the point nearer the larger charge and cannot cancel the fields. Option (D) gives distance ratio $1 : 2$, which would balance charges in ratio $1 : 4$, not $1 : 9$. Option (C) lies outside the charges, where both fields point in the same direction.

Step 3: Conclusion The electric field vanishes at $(d/4, 0, 0)$. Hence, the correct option is (B).

Quick Tip to Solve: Between like charges, cancellation distances are proportional to the square roots of the charges; the point is closer to the smaller charge.

37. A slanted object AB is placed on one side of convex lens as shown in the diagram. The image is formed on the opposite side. Angle made by the image with principal axis is : [4 marks]



(A) $-\alpha$

(B) $-\frac{\alpha}{2}$

(C) -45°

(D) $+45^\circ$

Correct Answer: (C) -45°

Solution:

Concept:

- Treat the two ends of the inclined object separately because their distances from the lens differ. For each point the thin-lens formula is $1/v - 1/u = 1/f$, using the Cartesian sign convention.
- Transverse magnification is $m = v/u$, so an object point at height y has image height $y\prime = my$. The slope between the two image points determines the inclination of the image.

Step 1: Locate the two object points Take the optical centre as the origin and the positive horizontal direction towards the image. From the figure, $f = 20\,\text{cm}$, $A = (-30, 0)\,\text{cm}$, and $B = (-29, 2)\,\text{cm}$. The horizontal separation of the ends is $1\,\text{cm}$, not the slant length. Thus the object inclination satisfies $\tan\alpha = 2/1 = 2$.

Step 2: Find the images of the endpoints For point A, put $u_A = -30\,\text{cm}$ in the lens formula:

$$\frac{1}{v_A} = \frac{1}{20} - \frac{1}{30} = \frac{1}{60}, \qquad A\prime = (60, 0)\,\text{cm}.$$

For B, the object distance is $-29\,\text{cm}$:

$$\frac{1}{v_B} = \frac{1}{20} - \frac{1}{29} = \frac{9}{580}, \qquad v_B = \frac{580}{9}\,\text{cm}.$$

Its magnification is $m_B = (580/9)/(-29) = -20/9$, giving $y_{B\prime} = -40/9\,\text{cm}$. The negative height

confirms an inverted real image.

Step 3: Calculate the image slope The horizontal displacement from $A\prime$ to $B\prime$ is $580/9 - 60 = 40/9\,\text{cm}$. Therefore

$$\tan\theta = \frac{-40/9}{40/9} = -1, \qquad \theta = -45^\circ.$$

The image tilts down as we move right. Hence the positive-angle option has the wrong sign. Also $-\alpha$ would have tangent -2, while $-\alpha/2$ does not have tangent -1; neither describes the calculated image.

Step 4: Conclusion The image makes an angle $\boxed{-45^\circ}$ with the principal axis. Hence, the correct option is (C).

Quick Tip to Solve: For a tilted object, image its two ends independently; a single transverse magnification cannot preserve the object angle.

---

38. Considering Bohr's atomic model for hydrogen atom :

(A) the energy of H atom in ground state is same as energy of He^+ion in its first excited state.

(B) the energy of H atom in ground state is same as that for Li^{++} ion in its second excited state.

(C) the energy of H atom in its ground state is same as that of He^+ion for its ground state.

(D) the energy of He^+ion in its first excited state is same as that for Li^{++}ion in its ground state.

Choose the correct answer from the options given below : [4 marks]

(A) (A), (B) only

(B) (A), (D) only

(C) (A), (C) only

(D) (B), (D) only

Correct Answer: (A) (A), (B) only

Solution:

Concept:

- For a one-electron atom or ion of nuclear charge Z, Bohr theory gives $E_n = -13.6Z^2/n^2\,\text{eV}$, with zero energy at ionisation.
- The ground, first excited and second excited states have $n = 1, 2, 3$, respectively. Equal energies require equal ratios Z^2/n^2, not equal quantum numbers alone.

Step 1: Compare the states in statements A and B For hydrogen in its ground state, $Z = 1$ and $n = 1$, hence

$$E_{\text{H},1} = -13.6\,\text{eV}.$$

For He^+ in the first excited state, $Z = 2, n = 2$. Therefore

$$E_{\text{He}^+,2} = -13.6\frac{2^2}{2^2} = -13.6\,\text{eV}.$$

Statement A is true. For Li^{2+} in the second excited state, $Z = 3, n = 3$, and

$$E_{Li^{2+},3} = -13.6\frac{3^2}{3^2} = -13.6\,\text{eV}.$$

Thus statement B is also true: in all these states $Z/n = 1$.

Step 2: Test statements C and D The helium ion in its ground state has

$$E_{He^+,1} = -13.6(4) = -54.4\,\text{eV},$$

which is different from hydrogen ground-state energy. Statement C is false. The lithium ion in its ground state has

$$E_{Li^{2+},1} = -13.6(9) = -122.4\,\text{eV}.$$

This does not equal $-13.6\,\text{eV}$, the energy of the first excited helium ion, so statement D is false. The negative signs signify bound electrons; equality must be checked for the signed energies, not just the state names.

Step 3: Select the correct statement combination Only A and B survive the energy comparison. The second option includes false statement D, the third includes false statement C, and the fourth also includes false statement D. Consequently none of those combinations gives exactly the two correct statements.

Step 4: Conclusion Only the first two statements are correct; the remaining two are false. Hence, the correct option is (A).

Quick Tip to Solve: In hydrogen-like ions, compare Z/n: equal values give equal Bohr energies, even when the ions and excited-state names differ.

39. In an adiabatic process, which of the following statements is true? [4 marks]

(A) The internal energy of the gas decreases as the temperature increases

(B) The molar heat capacity is zero

(C) Work done by the gas equals the increase in internal energy

(D) The molar heat capacity is infinite

Correct Answer: (B) The molar heat capacity is zero

Solution:

Concept:

- An adiabatic process has no heat transfer across the system boundary: $\delta Q = 0$. It need not have a constant temperature.
- Using work done by the gas as positive, the first law is $\mathrm{d}U = \delta Q - \delta W$. The molar heat capacity of a specified process is $C = \delta Q/(n\,\mathrm{d}T)$, where n is the amount of gas.

Step 1: Apply the definition of process heat capacity For an adiabatic change with a nonzero

temperature change, the numerator of the process heat capacity is zero:

$$C_{\text{adiabatic}} = \frac{0}{n\,\mathrm{d}T} = 0.$$

This is a property of the path: no heat is supplied to produce the temperature change. It does not mean that the gas has zero constant-volume or constant-pressure heat capacity. Those quantities refer to different processes and remain finite for an ordinary ideal gas.

Step 2: Check the energy and work statements Substitution of $\delta Q = 0$ into the first law gives

$$\mathrm{d}U = -\delta W.$$

Thus positive work done by the gas is supplied by a decrease of internal energy, rather than an increase. This rules out the statement that work done by the gas equals an increase in internal energy. For an ideal gas, $\mathrm{d}U = nC_V\,\mathrm{d}T$ with $C_V > 0$, so raising the temperature increases internal energy; the opposite relation in option A is incorrect.

Step 3: Distinguish adiabatic from isothermal behaviour An infinite process heat capacity can arise in the isothermal limit, where heat is transferred while the temperature stays fixed. That is not the adiabatic condition. In an adiabatic expansion the gas cools as it performs work, and in an adiabatic compression it warms as work is done on it, in both cases without heat transfer. Hence the zero-capacity choice is the one consistent with the definition.

Step 4: Conclusion The molar heat capacity for the adiabatic process is $\boxed{0}$. Hence, the correct option is (B).

Quick Tip to Solve: Use $C = \delta Q/(n\,\mathrm{d}T)$ for the stated path; adiabatic means a zero heat numerator, not a zero temperature change.

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40. The equation for real gas is given by $\left(\mathrm{P} + \frac{\mathrm{a}}{\mathrm{V}^2}\right)(\mathrm{V} - \mathrm{b}) = \mathrm{RT}$, where $\mathrm{P}, \mathrm{V}, \mathrm{T}$ and R are the pressure, volume, temperature and gas constant, respectively. The dimension of ab^{-2} is equivalent to that of : [4 marks]

(A) Compressibility

(B) Planck's constant

(C) Energy density

(D) Strain

Correct Answer: (C) Energy density

Solution:

Concept:

- Quantities added or subtracted in a physical equation must have identical dimensions. Thus in $(P + a/V^2)(V - b) = RT$, a/V^2 has the dimensions of pressure and b has the dimensions of volume.
- Pressure is force per unit area, with dimensions $[P] = ML^{-1}T^{-2}$. Energy density is energy per unit volume and has the same dimensions.

Step 1: Obtain the dimensions of each constant The pressure correction a/V^2 must be commensurate with P; otherwise the first bracket could not represent a physical quantity. Therefore

$$[a] = [P][V]^2.$$

Similarly, since $V - b$ is a difference of volumes,

$$[b] = [V].$$

With $[V] = L^3$, this also gives $[a] = ML^5T^{-2}$ and $[b] = L^3$. These statements follow directly from the printed equation without using any numerical value of R or T.

Step 2: Form the required combination Now cancel the volume dimensions before expanding the fundamental units:

$$[ab^{-2}] = \frac{[P][V]^2}{[V]^2} = [P] = ML^{-1}T^{-2}.$$

For comparison, energy has dimensions ML^2T^{-2}. Dividing it by volume gives

$$\left[\frac{\text{energy}}{\text{volume}}\right] = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}.$$

The required combination thus has exactly the dimensions of energy density.

Step 3: Eliminate the remaining quantities Compressibility is a fractional volume change per pressure change, so its dimensions are inverse pressure, $M^{-1}LT^2$. Planck's constant has dimensions energy times time, ML^2T^{-1}, which also differ. Strain is a ratio of lengths and is dimensionless. Neither an inverse pressure, an action nor a dimensionless ratio matches the pressure dimension found above.

Step 4: Conclusion The dimensions of ab^{-2} are equivalent to those of energy density. Hence, the correct option is (C).

Quick Tip to Solve: First cancel $[V]^2$ against $[b]^2$; the target becomes a pressure, and pressure equals energy per unit volume dimensionally.

41. The relationship between the magnetic susceptibility (x) and the magnetic permeability (μ) is given by :

(μ_0 is the permeability of free space and μ_T is relative permeability) [4 marks]

(A) $X = \frac{\mu}{\mu_0} - 1$

(B) $X = \frac{\mu_r}{\mu_0} + 1$

(C) $\chi = \mu_r + 1$

(D) $X = 1 - \frac{\mu}{\mu_0}$

Correct Answer: (A) $X = \frac{\mu}{\mu_0} - 1$

Solution:

Concept:

- For a linear isotropic magnetic medium, the magnetisation is $M = \chi H$, where χ is dimensionless magnetic susceptibility and H is magnetising field strength.
- The magnetic induction obeys $B = \mu_0(H + M) = \mu H$. Relative permeability is $\mu_r = \mu/\mu_0$, and it is dimensionless.

Step 1: Express induction using susceptibility Substitute $M = \chi H$ in the relation between induction, applied field and magnetisation:

$$B = \mu_0(H + \chi H) = \mu_0(1 + \chi)H.$$

The factor $1+\chi$ describes how the medium changes the induction relative to free space. This definition includes weakly positive susceptibility for paramagnetic materials and weakly negative susceptibility for diamagnetic materials; no reversal of the sign convention is required.

Step 2: Compare with the definition of permeability By definition the same induction is $B = \mu H$. Comparing the coefficients of H gives

$$\mu = \mu_0(1 + \chi).$$

Divide by the permeability of free space and rearrange:

$$\frac{\mu}{\mu_0} = 1 + \chi, \qquad \chi = \frac{\mu}{\mu_0} - 1 = \mu_r - 1.$$

Both sides of the last equation are dimensionless, as required for a susceptibility.

Step 3: Check the signs and the limiting case In vacuum $\mu = \mu_0$, and this expression gives $\chi = 0$, correctly representing no material magnetisation. Option B divides dimensionless relative permeability by dimensional μ_0, making its sum with 1 inconsistent. Option C would give $\chi = 2$ in vacuum, so its plus sign is wrong. Option D has the negative of the required expression and would incorrectly assign negative susceptibility to an ordinary paramagnetic medium with $\mu > \mu_0$.

Step 4: Conclusion The required relation is $\boxed{\chi = \mu/\mu_0 - 1}$. Hence, the correct option is (A).

Quick Tip to Solve: Remember $\mu_r = 1 + \chi$, then check vacuum: $\mu_r = 1$ must correspond to zero susceptibility.

42. The battery of a mobile phone is rated as $4.2\ \text{V}, 5800\ \text{mAh}$. How much energy is stored ir it when fully charged? [4 marks]

(A) 87.7 kJ

(B) 43.8 kJ

(C) 48.7 kJ

(D) 24.4 kJ

Correct Answer: (A) 87.7 kJ

Solution:

Concept:

- The energy delivered by a battery at potential difference V while transferring charge Q is $E = VQ$. With V in volts and Q in coulombs, E is in joules.
- A battery rating in ampere-hours is a charge capacity, not an energy. Since $1\,\text{A} = 1\,\text{C}\,\text{s}^{-1}$, $1\,\text{Ah} = 3600\,\text{C}$.

Step 1: Convert the charge capacity to SI units The stated capacity is $5800\,\text{mAh}$. Convert milliamperes to amperes first:

$$5800\,\text{mAh} = 5.8\,\text{Ah}.$$

One hour contains $3600\,\text{s}$, so the charge available at full capacity is

$$Q = 5.8 \times 3600 = 20880\,\text{C}.$$

The unit conversion is essential: multiplying volts by a number in milliampere-hours directly would not give joules.

Step 2: Calculate the stored electrical energy Use the rated potential difference $V = 4.2\,\text{V}$, as intended by the rating given in the question:

$$E = VQ = 4.2 \times 20880 = 87696\,\text{J}.$$

Convert to kilojoules by dividing by 10^3:

$$E = 87.696\,\text{kJ} \simeq 87.7\,\text{kJ}.$$

The same result follows from the watt-hour form: $4.2 \times 5.8 = 24.36\,\text{Wh}$, and each watt-hour equals $3.6\,\text{kJ}$.

Step 3: Compare the numerical options The value 24.4 is approximately the energy in watt-hours, not kilojoules; attaching kJ to that number loses the factor 3.6. The option $43.8\,\text{kJ}$ is about half the computed result and would wrongly introduce a factor of one-half. A battery charge rating uses VQ, not the capacitor expression $VQ/2$. The remaining $48.7\,\text{kJ}$ also fails the direct product of the supplied voltage and charge. Only the first option has the correct magnitude and units.

Step 4: Conclusion The stored energy is $\boxed{87.7\,\text{kJ}}$. Hence, the correct option is (A).

Quick Tip to Solve: Multiply volts by ampere-hours to obtain watt-hours, then multiply by 3.6 for kJ; do not use the capacitor factor of one-half.

43. A zener diode with 5 V zener voltage is used to regulate an unregulated dc voltage input of 25 V . For a 400Ω resistor connected in series, the zener current is found to be 4 times load current. The load current (I_L) and load resistance (R_L) are : [4 marks]

(A) $\mathrm{I_L} = 0.02\ \mathrm{mA}; \mathrm{R_L} = 250\Omega$

(B) $\mathrm{I_L} = 10\ \mathrm{A}; \mathrm{R_L} = 0.5\Omega$

(C) $\mathrm{I_L} = 10\ \mathrm{mA}; \mathrm{R_L} = 500\Omega$

(D) $\mathrm{I_L} = 20\ \mathrm{mA}; \mathrm{R_L} = 250\Omega$

Correct Answer: (C) $\mathrm{I_L} = 10\ \mathrm{mA}; \mathrm{R_L} = 500\Omega$

Solution:

Concept:

- A reverse-biased zener diode in breakdown maintains the load voltage at its zener voltage, so $V_L = V_Z = 5\,\mathrm{V}$. The load and zener are parallel branches.
- The series resistor carries the sum of the load and zener currents: $I_s = I_L + I_Z$. Ohm's law gives $I_s = (V_{\text{in}} - V_Z)/R_s$ and $R_L = V_L/I_L$.

Step 1: Calculate the total current through the series resistor The resistor does not have the full input voltage across it, because the regulated output remains at 5 V. Its voltage drop is therefore $25 - 5 = 20\,\mathrm{V}$. With $R_s = 400\,\Omega$, the current supplied to both branches is

$$I_s = \frac{20}{400} = 0.050\,\mathrm{A} = 50\,\mathrm{mA}.$$

This is the total current, not the load current alone.

Step 2: Use the specified current division The given relation is $I_Z = 4I_L$. Applying the junction rule at the output node gives

$$50 = I_L + 4I_L = 5I_L \quad \text{(currents in mA)}.$$

Hence $I_L = 10\,\mathrm{mA} = 0.010\,\mathrm{A}$, while $I_Z = 40\,\mathrm{mA}$. Their sum is indeed 50 mA, consistent with the resistor calculation.

Step 3: Find the load resistance and compare all options The load shares the regulated 5 V, giving

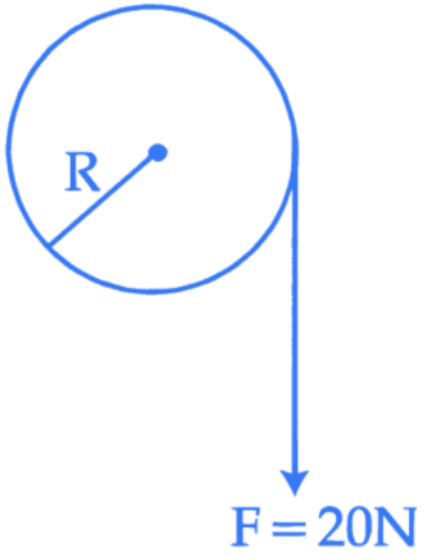
$$R_L = \frac{5}{0.010} = 500\,\Omega.$$

Option A has a current and resistance that do not produce the required output voltage. Option B demands much more than the available 50 mA. Option D has a consistent 5 V load pair, but its 20 mA current would require 100 mA total when $I_Z = 4I_L$, contradicting the series current.

Step 4: Conclusion The load current is $I_L = 10\,\mathrm{mA}$ and the load resistance is $R_L = 500\,\Omega$. Hence, the correct option is (C).

Quick Tip to Solve: First subtract the zener voltage from the supply voltage. Then divide the series current in the ratio $I_L : I_Z = 1 : 4$; the load gets one fifth.

44. A cord of negligible mass is wound around the rim of a wheel supported by spokes with negligible mass. The mass of wheel is 10 kg and radius is 10 cm and it can freely rotate without any friction. Initially the wheel is at rest. If a steady pull of 20 N is applied on the cord, the angular velocity of the wheel, after the cord is unwound by 1 m , would be: [4 marks]



(A) 20 rad/s

(B) 30 rad/s

(C) 10 rad/s

(D) 0 rad/s

Correct Answer: (A) 20 rad/s

Solution:

Concept:

- A wheel whose spokes have negligible mass has its mass concentrated at the rim. Its moment of inertia about the fixed central axis is $I = MR^2$, as for a thin ring.
- The work done by the pulling force equals the gain in rotational kinetic energy when there is no friction: $Fs = \frac{1}{2}I\omega^2$. The centre is supported, so there is no translational kinetic energy.

Step 1: Find the moment of inertia of the wheel Convert the radius to SI units before squaring it: $R = 10\,\text{cm} = 0.10\,\text{m}$. The spokes contribute negligible inertia, so

$$I = MR^2 = 10(0.10)^2 = 0.10\,\text{kg m}^2.$$

Using the solid-disc value $MR^2/2$ would be inappropriate for the rim-and-massless-spokes model in the question.

Step 2: Calculate the work supplied by unwinding the cord The point where the pulling force acts moves through the unwound length $s = 1\,\text{m}$. The steady force is tangential, so its work is

$$W = Fs = 20 \times 1 = 20\,\text{J}.$$

Equivalently, the torque is $FR = 2\,\mathrm{N\,m}$, and the angular displacement is $s/R = 10\,\mathrm{rad}$. Their product gives the same 20 J.

Step 3: Apply the energy equation and test the options The initial angular velocity is zero. Therefore,

$$20 = \frac{1}{2}(0.10)\omega^2, \qquad \omega^2 = 400.$$

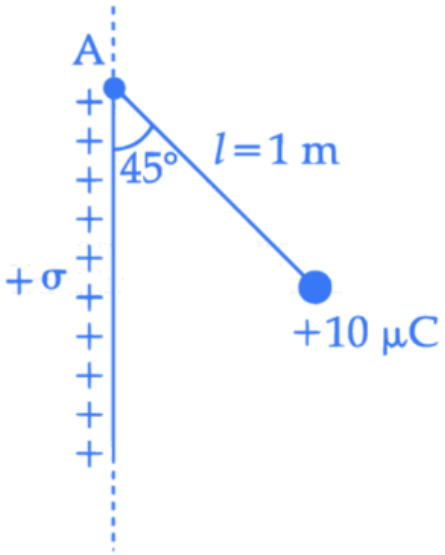
The angular speed is $20\,\mathrm{rad\,s^{-1}}$. At 30 rad/s the rotational energy would be 45 J, while at 10 rad/s it would be 5 J; neither equals the supplied 20 J. Zero angular speed is impossible after positive work on a frictionless wheel.

Step 4: Conclusion The wheel attains angular speed $\boxed{20\,\mathrm{rad\,s^{-1}}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a cord unwound from a fixed wheel, use $Fs = I\omega^2/2$. Identify whether the mass lies in a rim or a solid disc before choosing I.

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45. A small bob of mass 100 mg and charge $+10\mu\mathrm{C}$ is connected to an insulating string of length 1 m . It is brought near to an infinitely long non-conducting sheet of charge density ' σ ' as shown in figure. If string subtends an angle of 45° with the sheet at equilibrium the charge density of sheet will be. (Given, $\epsilon_0 = 8.85 \times 10^{-12}\frac{\mathrm{F}}{\mathrm{m}}$ and acceleration due to gravity, $\mathrm{g} = 10\frac{\mathrm{m}}{\mathrm{s}^2}$) [4 marks]



(A) $1.77\ \mathrm{nC/m^2}$

(B) $0.885\ \mathrm{nC/m^2}$

(C) $885\ \mathrm{nC/m^2}$

(D) $17.7\ \mathrm{nC/m^2}$

Correct Answer: (A) $1.77\ \mathrm{nC/m^2}$

Solution:

Concept:

- An infinite uniformly charged non-conducting sheet produces a field $E = \sigma/(2\epsilon_0)$, directed normally away from a positively charged sheet. The factor 2 is essential.
- For the suspended charged bob in equilibrium, the tension balances weight vertically and electric force horizontally: $T\cos\theta = mg$ and $T\sin\theta = qE$, where θ is measured from the vertical sheet.

Step 1: Resolve the equilibrium forces The sheet is vertical and the string makes 45° with it, so the string also makes 45° with the vertical. Dividing the horizontal balance by the vertical balance removes the unknown tension:

$$\tan 45^\circ = \frac{qE}{mg} = 1.$$

Thus the horizontal electric force has the same magnitude as the weight, and $E = mg/q$. The string length is not needed because the field of an infinite sheet is independent of distance.

Step 2: Convert the mass and determine the field The given mass is in milligrams, not grams:

$$m = 100\,\text{mg} = 10^{-4}\,\text{kg}, \qquad q = 10\,\mu\text{C} = 10^{-5}\,\text{C}.$$

Therefore the weight is $10^{-3}\,\text{N}$, and

$$E = \frac{10^{-4} \times 10}{10^{-5}} = 100\,\text{N C}^{-1}.$$

Step 3: Find the surface charge density and exclude the alternatives For this non-conducting sheet,

$$\sigma = 2\epsilon_0 E = 2(8.85 \times 10^{-12})(100) = 1.77 \times 10^{-9}\,\text{C m}^{-2}.$$

Thus the density is positive and equals 1.77 nC per square metre. Option B loses the factor 2 in the sheet-field relation. Options C and D are respectively 500 and 10 times the required density, so their electric forces cannot balance the specified weight at this angle.

Step 4: Conclusion The sheet has charge density $\boxed{\sigma = 1.77\,\text{nC m}^{-2}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use $\tan\theta = qE/(mg)$, then $\sigma = 2\epsilon_0 E$. Convert 100 mg to 10^{-4} kg before calculating the weight.

Physics – Section B: Numerical Value Questions (20 marks)

46. A steel wire of length 2 m and Young's modulus $2.0 \times 10^{11}\ \mathrm{N\ m^{-2}}$ is stretched by a force. If Poisson ratio and transverse strain for the wire are 0.2 and 10^{-3} respectively, then the elastic potential energy density of the wire is _____ $\times 10^5$ (in SI units). [4 marks]

Solution:

Concept:

- Poisson's ratio is the magnitude of transverse strain divided by longitudinal strain: $\nu = |\epsilon_t|/\epsilon_l$. The transverse contraction has the opposite sign to the longitudinal extension.
- For linear elastic stretching, stress is $\sigma = Y\epsilon_l$ and elastic energy per unit volume is $u = \frac{1}{2}\sigma\epsilon_l = \frac{1}{2}Y\epsilon_l^2$. Both strains are dimensionless.

Step 1: Recover the longitudinal strain The wire contracts laterally as it stretches, so the given transverse strain 10^{-3} is used as a magnitude in the Poisson-ratio formula. With $\nu = 0.2$,

$$\epsilon_l = \frac{|\epsilon_t|}{\nu} = \frac{10^{-3}}{0.2} = 5 \times 10^{-3}.$$

It would be incorrect to multiply the transverse strain by 0.2: the longitudinal strain must be larger because the Poisson ratio is less than unity.

Step 2: Evaluate the elastic energy per unit volume The stress rises linearly from zero to its final value during stretching. Hence the stored energy density is half the final stress times the longitudinal strain:

$$u = \frac{1}{2}(2.0 \times 10^{11})(5 \times 10^{-3})^2.$$

Squaring the strain gives 25×10^{-6}, so

$$u = 10^{11} \times 25 \times 10^{-6} = 25 \times 10^5\ \mathrm{J\,m^{-3}}.$$

Step 3: Check the scale and identify the requested coefficient As a check, the final stress is $Y\epsilon_l = 10^9\ \mathrm{Pa}$. Half this value multiplied by 0.005 is $2.5 \times 10^6\ \mathrm{J\,m^{-3}}$, agreeing with the energy calculation. The question asks for the coefficient multiplying 10^5, not the entire energy density. The length 2 m would be required to find extension, but it cancels from energy per unit volume and is not needed here.

Step 4: Conclusion The elastic potential energy density is $\boxed{25 \times 10^5\ \mathrm{J\,m^{-3}}}$. The numerical entry is $\boxed{25}$.

Quick Tip to Solve: When transverse strain is given, divide it by Poisson's ratio first. Use the resulting longitudinal strain, squared, in $u = Y\epsilon_l^2/2$.

47. γ_A **is the specific heat ratio of monoatomic gas A having 3 translational degrees of freedom.** γ_B **is the specific heat ratio of polyatomic gas** B **having 3 translational, 3 rotational degrees of freedom and 1 vibrational mode. If** $\frac{\gamma_A}{\gamma_B} = \left(1 + \frac{1}{n}\right)$**, then the value of** n **is _____ .** [4 marks]

Solution:

Concept:

- For an ideal gas with f active quadratic energy degrees of freedom per molecule, $C_V = fR/2$, $C_P = C_V + R$, and $\gamma = C_P/C_V = (f+2)/f$. These heat capacities are molar.
- Each translational or rotational degree contributes $R/2$ to C_V. One active vibrational mode contributes R, because it has both kinetic and potential energy terms.

Step 1: Calculate the ratio for the monatomic gas Gas A has the three translational degrees stated in the question. Consequently,

$$f_A = 3, \qquad C_{V,A} = \frac{3}{2}R, \qquad C_{P,A} = \frac{5}{2}R.$$

Its heat-capacity ratio is therefore

$$\gamma_A = \frac{C_{P,A}}{C_{V,A}} = \frac{5}{3}.$$

There is no rotational or vibrational contribution to include for this monatomic gas.

Step 2: Count the active energy terms for gas B The three translational and three rotational degrees contribute six quadratic terms. The one vibrational mode supplies two additional terms, one kinetic and one potential. Thus

$$f_B = 3 + 3 + 2 = 8.$$

Accordingly,

$$C_{V,B} = 4R, \qquad C_{P,B} = 5R, \qquad \gamma_B = \frac{5}{4}.$$

Counting the vibrational mode only once would give seven terms and would not implement equipartition correctly. Its activity is specified in the question, so it is included here.

Step 3: Compare the two ratios and solve for n Taking the ratio in the order requested gives

$$\frac{\gamma_A}{\gamma_B} = \frac{5/3}{5/4} = \frac{4}{3} = 1 + \frac{1}{3}.$$

Comparison with $1 + 1/n$ yields $1/n = 1/3$, hence $n = 3$. This result is dimensionless, as it must be, because it comes from ratios of heat capacities. Both individual values of γ exceed unity, and the polyatomic gas has the smaller value because more energy is stored internally.

Step 4: Conclusion The ratio is $\gamma_A/\gamma_B = 4/3$, so the required dimensionless numerical value is $\boxed{n = 3}$.

Quick Tip to Solve: An active vibrational mode counts twice in the energy degree count. Once f is known, use $\gamma = 1 + 2/f$ for each gas before taking their ratio.

48. A vessel with square cross-section and height of 6 m is vertically partitioned. A small window of $100\ \text{cm}^2$ with hinged door is fitted at a depth of 3 m in the partition wall. One part of the vessel is filled completely with water and the other side is filled with the liquid having density $1.5 \times 10^3\ \text{kg/m}^3$. What force one needs to apply on the hinged door so that it does not get opened ?

(Acceleration due to gravity $= 10\ \text{m/s}^2$)

[4 marks]

Solution:

Concept:

- The pressure at depth h below an open liquid surface is $p = p_{\text{atm}} + \rho g h$. At the same depth on two sides, the atmospheric terms cancel from the pressure difference.
- The net hydrostatic force on a small plane window is $F = \Delta p\, A$, using the pressure difference at its centre. For equal free-surface levels, $\Delta p = (\rho_2 - \rho_1)gh$.

Step 1: Identify the pressure on each side of the window The window is at depth 3 m below the top liquid level. Both sides are filled to the vessel's full height, so the hydrostatic columns above the window have the same depth. Taking water density as $\rho_w = 1000\,\text{kg}\,\text{m}^{-3}$, the two gauge pressures are

$$p_w = 1000 \times 10 \times 3 = 3.0 \times 10^4\,\text{Pa},$$

$$p_l = 1500 \times 10 \times 3 = 4.5 \times 10^4\,\text{Pa}.$$

The vessel's total height 6 m is not the depth to use at the window.

Step 2: Find the net pressure and convert the window area The denser liquid pushes more strongly toward the water side. The difference in pressure is

$$\Delta p = p_l - p_w = 1.5 \times 10^4\,\text{Pa}.$$

The area must be expressed in square metres:

$$A = 100\,\text{cm}^2 = 100 \times 10^{-4}\,\text{m}^2 = 1.0 \times 10^{-2}\,\text{m}^2.$$

The conversion involves the square of the centimetre-to-metre factor.

Step 3: Calculate the balancing force The resultant fluid force on the small door is

$$F = \Delta p\, A = (1.5 \times 10^4)(1.0 \times 10^{-2}) = 150\,\text{N}.$$

An equal opposing force is required to balance this hydrostatic resultant and prevent opening. Its direction is from the water side toward the denser liquid side. The common atmospheric pressure does not add to this result, because it acts on both sides and produces no net pressure difference.

Step 4: Conclusion The required balancing force has magnitude $\boxed{150\,\text{N}}$, directed against the force from the denser liquid. The numerical entry is $\boxed{150}$.

Quick Tip to Solve: For two liquids at the same depth, calculate $\Delta\rho\, ghA$ directly. Use the window depth, and remember that $100\,\text{cm}^2 = 0.01\,\text{m}^2$.

49. If the measured angular separation between the second minimum to the left of the central maximum and the third minimum to the right of the central maximum is 30° in a single slit diffraction pattern recorded using 628 nm light, then the width of the slit is _____ μm. [4 marks]

Solution:

Concept:

- For a slit of width a, a diffraction minimum of order m obeys $a \sin\theta_m = m\lambda$, where θ_m is measured from the central maximum and λ is the wavelength.
- Minima on opposite sides have angular distances that add. At small angles, $\sin\theta \simeq \theta$ with θ in radians; numerical answers are reported to the requested integer value.

Step 1: Identify the two minimum orders The second minimum on the left is at angular coordinate $-\theta_2$, whereas the third on the right is at $+\theta_3$. Consequently their separation is $\theta_2 + \theta_3 = 30^\circ = \pi/6$ radians. Their orders must be added, not subtracted: both angular distances are measured away from the same central maximum.

Step 2: Estimate the slit width Using $\theta_m \simeq m\lambda/a$, add the angular distances for the two specified minima:

$$\frac{(2+3)\lambda}{a} = \frac{\pi}{6}.$$

With $\lambda = 628\,\text{nm} = 0.628\,\mu\text{m}$, this gives

$$a \simeq \frac{30(0.628)}{\pi}\,\mu\text{m} \simeq 5.997\,\mu\text{m}.$$

The conversion to radians is essential; substituting the numerical value 30 directly would give the wrong width.

Step 3: Check the integer using the exact minimum condition The approximation can be checked without treating 30° as a very small angle. The exact equations give $\sin\theta_3/\sin\theta_2 = 3/2$ and $\theta_3 = 30^\circ - \theta_2$. Thus

$$\tan\theta_2 = \frac{\sin 30^\circ}{3/2 + \cos 30^\circ}.$$

This gives $\theta_2 \simeq 11.932^\circ$ and $a = 2(0.628)/\sin\theta_2 \simeq 6.075\,\mu\text{m}$, which has the same nearest-integer value.

Step 4: Conclusion The slit width is approximately $6\,\mu\text{m}$; the numerical value to enter is $\boxed{6}$.

Quick Tip to Solve: For minima on opposite sides, add the orders and convert the total angular separation to radians before using the small-angle formula.

50. A person travelling on a straight line moves with a uniform velocity v_1 for a distance x and with a uniform velocity v_2 for the next $\frac{3}{2}x$ distance. The average velocity in this motion is $\frac{50}{7}$ m/s. If v_1 is 5 m/s then $v_2 =$ _____ m/s. [4 marks]

Solution:

Concept:

- Average velocity equals total displacement divided by total elapsed time: $v_{\text{av}} = \Delta x/\Delta t$. For a uniform-velocity segment, the time is its displacement divided by its velocity.
- For two consecutive distances in the same direction, add their displacements and their travel times separately. An arithmetic mean of the two velocities is valid only for equal time intervals.

Step 1: Write the displacement and time for each segment The first displacement is x and the second is $3x/2$, in the same direction. With $v_1 = 5\,\text{m}\,\text{s}^{-1}$, the times are

$$t_1 = \frac{x}{5}, \qquad t_2 = \frac{3x}{2v_2}.$$

Hence the total displacement is $5x/2$ and the total time is $x/5 + 3x/(2v_2)$. Here x represents a nonzero distance; it will cancel, so its numerical value is unnecessary.

Step 2: Use the specified average velocity Insert the given average velocity rather than averaging v_1 and v_2 directly:

$$\frac{50}{7} = \frac{5x/2}{x/5 + 3x/(2v_2)} = \frac{5/2}{1/5 + 3/(2v_2)}.$$

Dividing the total displacement by the specified velocity also gives $t_{\text{total}} = 7x/20$. The time remaining after the first segment is therefore

$$t_2 = \frac{7x}{20} - \frac{x}{5} = \frac{3x}{20}.$$

Step 3: Determine and check the second velocity The second displacement is $3x/2$, so its velocity is

$$v_2 = \frac{3x/2}{3x/20} = 10\,\text{m}\,\text{s}^{-1}.$$

As a check, these velocities give total time $x/5 + 3x/20 = 7x/20$; dividing $5x/2$ by this time reproduces $50/7\,\text{m}\,\text{s}^{-1}$. The average also lies between 5 and 10, as it should for motion in one direction.

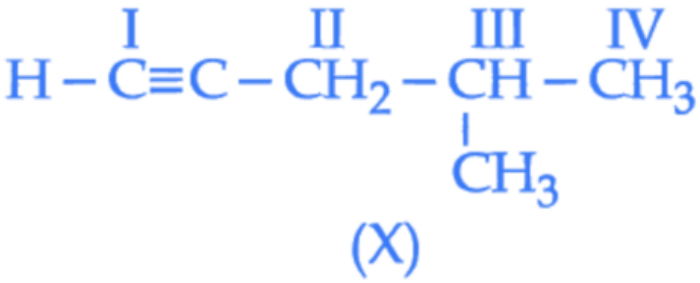
Step 4: Conclusion $\boxed{v_2 = 10\,\text{m}\,\text{s}^{-1}}$. The numerical value to enter is 10.

Quick Tip to Solve: For unequal distances, calculate each travel time first; the unknown common distance often cancels from the average-velocity equation.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Consider the following compound (X)

The most stable and least stable carbon radicals, respectively, produced by homolytic cleavage of corresponding $C-H$ bond are : [4 marks]



(X)

(A) III, II

(B) II, IV

(C) I, IV

(D) II, I

Correct Answer: (D) II, I

Solution:

Concept:

- Homolytic cleavage of a $C-H$ bond leaves one bonding electron on carbon and one on hydrogen, producing a carbon-centred radical and $H^{\bullet}$.
- Radicals are stabilised by delocalisation of the unpaired electron and by hyperconjugation. A radical adjacent to a triple bond is propargylic and has resonance stabilisation; a radical directly on an alkynyl carbon lacks this usual propargylic stabilisation.

Step 1: Classify each labelled carbon The structure is $HC{\equiv}C{-}CH_2{-}CH(CH_3){-}CH_3$. Site I is the terminal carbon of the triple bond, so removal of its hydrogen gives an alkynyl radical. Site II is the CH_2 carbon immediately next to the triple bond, giving a propargylic radical. Site III is the branched CH carbon, giving a tertiary alkyl radical; site IV gives a primary alkyl radical from the terminal methyl group.

Step 2: Identify the most stable radical The unpaired electron at II can delocalise through the adjacent triple-bond system. One resonance description is

$$R{-}C{\equiv}C{-}CH_2^{\bullet} \longleftrightarrow R{-}C^{\bullet}{=}C{=}CH_2.$$

This resonance stabilisation makes the propargylic radical at II more stable than the simple tertiary alkyl radical at III. The primary radical at IV has still less alkyl stabilisation. Thus the first member of the required pair is II.

Step 3: Identify the least stable radical and compare all options At I the radical is directly on an sp-hybridised carbon of the triple bond, rather than adjacent to it. It is an unstable alkynyl radical

and does not gain the propargylic delocalisation available at II. Therefore I is the least stable site. Option (A) wrongly makes II least stable; (B) wrongly places IV below I; and (C) wrongly makes I most stable. Only (D) has both extreme sites in the correct order.

Step 4: Conclusion The most stable radical is formed at II and the least stable at I. Hence, the correct option is (D).

Quick Tip to Solve: First distinguish a radical adjacent to a multiple bond from one directly on the multiple-bond carbon; only the former has the usual allylic or propargylic resonance advantage.

52.

$$\mathrm{CaCO_3(\ s) + 2HCl(aq) \rightarrow CaCl_2(aq) + CO_2(\ g) + H_2O(l)}$$

Consider the above reaction, what mass of $\mathrm{CaCl_2}$ will be formed if 250 mL of 0.76 M HCl reacts with 1000 g of $\mathrm{CaCO_3}$?

(Given : Molar mass of $\mathrm{Ca, C, O, H}$ and Cl are $40, 12, 16, 1$ and $35.5\ \mathrm{g\ mol^{-1}}$, respectively) [4 marks]

(A) 3.908 g

(B) 2.636 g

(C) 10.545 g

(D) 5.272 g

Correct Answer: (C) 10.545 g

Solution:

Concept:

- The balanced reaction is $\mathrm{CaCO_3 + 2\,HCl \longrightarrow CaCl_2 + CO_2 + H_2O}$. Two moles of hydrochloric acid are required per mole of calcium chloride formed.
- Moles in a solution equal molarity times volume in litres: $n = MV$. The limiting reagent is the reactant with the smaller available number of reaction equivalents, and product mass is $m = nM_{\mathrm{molar}}$.

Step 1: Calculate the available reactant amounts Convert the acid volume before multiplying by its molarity:

$$V_{\mathrm{HCl}} = 250\,\mathrm{mL} = 0.250\,\mathrm{L}, \qquad n_{\mathrm{HCl}} = 0.76(0.250) = 0.190\,\mathrm{mol}.$$

The molar mass of calcium carbonate is $40 + 12 + 3(16) = 100\,\mathrm{g\,mol^{-1}}$. Therefore

$$n_{\mathrm{CaCO_3}} = \frac{1000}{100} = 10.0\,\mathrm{mol}.$$

Step 2: Find the limiting reagent and product moles Complete reaction of 10.0 mol of carbonate would require 20.0 mol of acid, whereas only 0.190 mol is available. Thus hydrochloric acid is limiting and calcium carbonate is in excess. Apply the coefficient ratio $2 : 1$ from acid to product:

$$n_{\mathrm{CaCl_2}} = \frac{n_{\mathrm{HCl}}}{2} = \frac{0.190}{2} = 0.0950\,\mathrm{mol}.$$

Only this amount of carbonate reacts; the remaining solid does not increase the product yield.

Step 3: Convert to mass and test the options The product molar mass, using the supplied atomic masses, is

$$M_{\mathrm{CaCl_2}} = 40 + 2(35.5) = 111\,\mathrm{g\,mol^{-1}}.$$

Hence

$$m_{\mathrm{CaCl_2}} = 0.0950(111) = 10.545\,\mathrm{g}.$$

Option (D) is approximately half this mass and would incorrectly halve the stoichiometric product amount again. The masses in (A) and (B) also fail the acid-to-product mole ratio and the product molar mass. Only (C) matches both requirements.

Step 4: Conclusion The mass of $\mathrm{CaCl_2}$ formed is $\boxed{10.545\,\mathrm{g}}$. Hence, the correct option is (C).

Quick Tip to Solve: Compare moles divided by stoichiometric coefficients to locate the limiting reagent, then convert its reaction equivalents to product mass.

53. If equal volumes of AB_2 and XY (both are salts) aqueous solutions are mixed, which of the following combination will give a precipitate of $\mathrm{AY_2}$ at 300 K ? (Given $\mathrm{K_{sp}}$ (at 300 K) for $\mathrm{AY_2} = 5.2 \times 10^{-7}$) [4 marks]

(A) $2.0 \times 10^{-4}\mathrm{MAB_2}, 0.8 \times 10^{-3}\mathrm{MXY}$

(B) $2.0 \times 10^{-2}\mathrm{MAB_2}, 2.0 \times 10^{-2}\mathrm{MXY}$

(C) $1.5 \times 10^{-4}\mathrm{MAB_2}, 1.5 \times 10^{-3}\mathrm{MXY}$

(D) $3.6 \times 10^{-3}\mathrm{MAB_2}, 5.0 \times 10^{-4}\mathrm{MXY}$

Correct Answer: (B) $2.0 \times 10^{-2}\mathrm{MAB_2}, 2.0 \times 10^{-2}\mathrm{MXY}$

Solution:

Concept:

- For $\mathrm{AY_2(s)} \rightleftharpoons \mathrm{A^{2+}(aq)} + 2\,\mathrm{Y^-(aq)}$, the ionic product before precipitation is $Q = [\mathrm{A^{2+}}][\mathrm{Y^-}]^2$. A precipitate forms when $Q > K_{\mathrm{sp}}$.
- Mixing equal volumes doubles the total volume, so each original ion concentration is halved. Complete dissociation of $\mathrm{AB_2}$ supplies one $\mathrm{A^{2+}}$ per formula unit, and XY supplies one $\mathrm{Y^-}$.

Step 1: Account for dilution in the mixture Let the original molarities of $\mathrm{AB_2}$ and XY be c_1 and c_2. Equal volumes give $[\mathrm{A^{2+}}] = c_1/2$ and $[\mathrm{Y^-}] = c_2/2$ before any precipitation. Consequently

$$Q = \frac{c_1}{2}\left(\frac{c_2}{2}\right)^2 = \frac{c_1 c_2^2}{8}.$$

The factor of eight results from the three powers of concentration in the solubility-product expression.

Step 2: Evaluate the ionic product for every combination Calculate using the diluted concentrations:

$$Q_A = (1.0 \times 10^{-4})(4.0 \times 10^{-4})^2 = 1.6 \times 10^{-11},$$
$$Q_B = (1.0 \times 10^{-2})(1.0 \times 10^{-2})^2 = 1.0 \times 10^{-6},$$
$$Q_C = (7.5 \times 10^{-5})(7.5 \times 10^{-4})^2 = 4.21875 \times 10^{-11},$$
$$Q_D = (1.8 \times 10^{-3})(2.5 \times 10^{-4})^2 = 1.125 \times 10^{-10}.$$

These are initial ionic products, so they are the quantities to test before deciding whether the solid appears.

Step 3: Compare with the solubility product At 300 K the given value is $K_{sp} = 5.2 \times 10^{-7}$. Combination (B) has $Q_B = 1.0 \times 10^{-6} > K_{sp}$, so it is supersaturated and AY_2 precipitates. In each of (A), (C), and (D), the ionic product is several orders of magnitude smaller than K_{sp}; these solutions remain unsaturated and do not form this precipitate.

Step 4: Conclusion Only the combination 2.0×10^{-2} M AB_2 and 2.0×10^{-2} M XY gives a precipitate. Hence, the correct option is (B).

Quick Tip to Solve: Apply dilution before testing precipitation: equal-volume mixing divides the ionic product of an AB2-type salt by eight, not merely by two.

54. Given below are two statements :

Statement (I) : The metallic radius of Al is less than that of Ga .

Statement (II) : The ionic radius of Al^{3+} is less than that of Ga^{3+}.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is correct but Statement II is incorrect

(B) Both Statement I and Statement II are correct

(C) Statement I is incorrect but Statement II is correct

(D) Both Statement I and Statement II are incorrect

Correct Answer: (C) Statement I is incorrect but Statement II is correct

Solution:

Concept:

- Atomic size generally increases down a group because a new electron shell is added, but poor shielding by filled d orbitals can cause exceptions in metallic radii.
- Metallic radii and ionic radii refer to different species and environments. Compare Al with Ga separately from the comparison of Al^{3+} with Ga^{3+}.

Step 1: Evaluate the metallic-radius statement Gallium follows the filling of the $3d$ subshell. The $3d$ electrons shield the increased nuclear charge imperfectly, so the outer electrons experience an enhanced effective nuclear attraction. This produces a contraction that offsets the simple down-group size increase. The commonly used metallic radii are about 143 pm for Al and 135 pm for Ga. Hence

$r_{\text{metallic}}(\text{Al}) > r_{\text{metallic}}(\text{Ga})$, and Statement I, which asserts the reverse inequality, is incorrect.

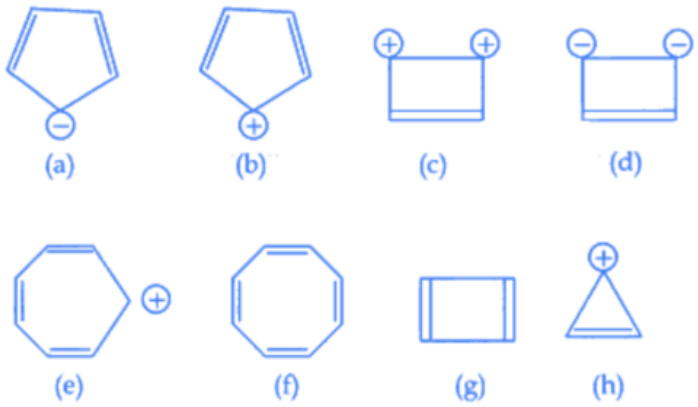
Step 2: Evaluate the ionic-radius statement The trivalent ions have configurations $\text{Al}^{3+} : [\text{Ne}]$ and $\text{Ga}^{3+} : [\text{Ar}]3d^{10}$. The gallium ion retains an additional occupied shell relative to the aluminium ion. For the same coordination convention, the ionic radius of Ga^{3+} is larger than that of Al^{3+}; representative values are approximately 62 and 53.5 pm, respectively. Thus $r(\text{Al}^{3+}) < r(\text{Ga}^{3+})$, so Statement II is correct.

Step 3: Choose the statement combination The two comparisons therefore have different outcomes: Statement I is false and Statement II is true. Option (A) gives exactly the opposite pair; (B) incorrectly accepts the metallic-radius statement; and (D) incorrectly rejects the ionic-radius statement. Option (C) alone states that I is incorrect while II is correct. One must not transfer the metallic-radius exception directly to the sizes of the trivalent ions.

Step 4: Conclusion Statement I is incorrect but Statement II is correct. Hence, the correct option is (C).

Quick Tip to Solve: In the Al-Ga comparison, remember the d-block contraction for metallic atoms, then make a fresh comparison for the trivalent ions.

55. Designate whether each of the following compounds is aromatic or not aromatic. [4 marks]



(A) a, c, d, e, h aromatic and b, f, g not aromatic

(B) b, e, f, g aromatic and a, c, d, h not aromatic

(C) a, b, c, d aromatic and e, f, g, h not aromatic

(D) e, g aromatic and a, b, c, d, f, h not aromatic

Correct Answer: (A) a, c, d, e, h aromatic and b, f, g not aromatic

Solution:

Concept:

- An aromatic ring must be cyclic, planar and continuously conjugated, with $4n + 2$ delocalised π electrons, where n is a nonnegative integer.
- A carbon anion contributes its lone pair, hence two electrons, to a conjugated ring; a carbocation contributes an empty p orbital and no electrons. A planar conjugated $4n$-electron ring is

antiaromatic, hence is not aromatic.

Step 1: Count electrons in the charged rings In (a), the two double bonds contribute four electrons and the anionic lone pair contributes two. The cyclopentadienyl anion therefore has six π electrons and is aromatic. In (b), the corresponding cation has only four electrons, so it is not aromatic. In (c), one double bond supplies two electrons and both positively charged carbons have empty p orbitals: its continuously conjugated ring has two electrons and is aromatic. In (d), the double bond supplies two electrons and the two anionic lone pairs supply four more. Its total is six, so it is aromatic.

Step 2: Classify the remaining rings and compare the lists The tropylium ion (e) has three double bonds and an empty p orbital at the positive centre. Its six electrons satisfy Huckel's rule, so it is aromatic. Cyclooctatetraene (f) has eight electrons and adopts a nonplanar tub shape, so it is nonaromatic. Cyclobutadiene (g) has four electrons and is antiaromatic rather than aromatic. The cyclopropenyl cation (h) has two electrons in a planar conjugated ring and is aromatic. Thus (a), (c), (d), (e) and (h) form the aromatic group. Option B reverses several charged-ring classifications; C wrongly includes (b) and excludes (e) and (h); D wrongly makes (g) aromatic.

Step 3: Conclusion The aromatic compounds are a, c, d, e, h, while b, f, g are not aromatic. Hence, the correct option is (A).

Quick Tip to Solve: Count a negative carbon's participating lone pair as two electrons and a positive carbon's empty p orbital as zero; then check planarity before applying $4n + 2$.

56. Choose the correct tests with respective observations.

(A) $CuSO_4$ **(acidified with acetic acid)** $+K_4\,[Fe(CN)_6] \rightarrow$ **Chocolate brown precipitate.**

(B) $FeCl_3 + K_4\,[Fe(CN)_6] \rightarrow$ **Prussian blue precipitate.**

(C) $ZnCl_2 + K_4\,[Fe(CN)_6]$**, neutralised with** $NH_4OH \rightarrow$ **White or bluish white precipitate.**

(D) $MgCl_2 + K_4\,[Fe(CN)_6] \rightarrow$ **Blue precipitate.**

(E) $BaCl_2 + K_4\,[Fe(CN)_6]$**, neutralised with** $NaOH \rightarrow$ **White precipitate.**

Choose the correct answer from the options given below : [4 marks]

(A) A, D and E only

(B) A, B and C only

(C) B, D and E only

(D) C, D and E only

Correct Answer: (B) A, B and C only

Solution:

Concept:

- Potassium ferrocyanide, $K_4[Fe(CN)_6]$, supplies the hexacyanidoferrate(II) ion. Its characteristic precipitates identify certain metal ions in qualitative analysis.
- The reagent, cation, medium and colour must all agree with a recognised test; an attractive precipitate colour alone does not establish a correct observation.

Step 1: Check the three characteristic positive tests Test A is correct: copper(II) ions give chocolate-

brown copper(II) ferrocyanide in the stated weakly acidic medium. The net precipitation can be represented as

$$2\,Cu^{2+} + [Fe(CN)_6]^{4-} \longrightarrow Cu_2[Fe(CN)_6](s).$$

Test B is also correct: ferric ions react with ferrocyanide to give Prussian blue, commonly represented by

$$4\,Fe^{3+} + 3\,[Fe(CN)_6]^{4-} \longrightarrow Fe_4[Fe(CN)_6]_3(s).$$

Test C is the recognised zinc test: in the appropriately neutralised solution zinc gives a white or bluish-white ferrocyanide precipitate. Therefore A, B and C all have the correct metal ion and observation.

Step 2: Reject the remaining observations and select the combination Test D is incorrect: magnesium(II) does not give the characteristic blue ferrocyanide precipitate stated here. The blue iron-ferrocyanide test must not be transferred to magnesium chloride. Test E, with barium chloride, ferrocyanide and sodium hydroxide as stated, is not the valid white-precipitate test in this set. The usual confirmatory white precipitate for barium is barium sulphate with a sulphate reagent; its reagent must not be confused with ferrocyanide. Hence the valid statements are precisely A, B and C. Option A includes the invalid D and E and misses B and C; option C includes D and E and misses A and C; option D includes D and E and misses A and B.

Step 3: Conclusion Tests A, B and C only have the stated correct observations. Hence, the correct option is (B).

Quick Tip to Solve: Link ferrocyanide to the characteristic copper-brown, ferric-blue and zinc-white tests, and check the stated medium before choosing a combination.

57. A solution is made by mixing one mole of volatile liquid A with 3 moles of volatile liquid B. The vapour pressure of pure A is 200 mm Hg and that of the solution is 500 mm Hg . The vapour pressure of pure B and the least volatile component of the solution, respectively, are: [4 marks]

(A) 1400 mm Hg, A

(B) 1400 mm Hg, B

(C) 600 mm Hg, A

(D) 600 mm Hg, B

Correct Answer: (C) 600 mm Hg, A

Solution:

Concept:

- For an ideal liquid mixture, Raoult's law gives $p = x_A p_A^\circ + x_B p_B^\circ$. Here x_i is the liquid mole fraction and p_i° is the pure component's vapour pressure at the same temperature.
- Volatility is compared using pure-liquid vapour pressures: the component with lower p° is less volatile, regardless of how many moles were mixed.

Step 1: Calculate the liquid mole fractions The mixture contains one mole of A and three moles of B,

so the total is four moles. Therefore

$$x_A = \frac{1}{1+3} = \frac{1}{4}, \qquad x_B = \frac{3}{1+3} = \frac{3}{4}.$$

These are the fractions in the liquid and they add to one. It would be incorrect to insert the mole numbers 1 and 3 directly as the coefficients in Raoult's law. A's partial vapour pressure is $p_A = x_A p_A^\circ = (1/4)(200) = 50\,\text{mm Hg}$.

Step 2: Determine the unknown pure vapour pressure and volatility Apply the total-pressure relation using the measured solution pressure:

$$500 = \frac{1}{4}(200) + \frac{3}{4}p_B^\circ.$$

Thus $(3/4)p_B^\circ = 450$, which gives

$$p_B^\circ = 450 \times \frac{4}{3} = 600\,\text{mm Hg}.$$

As a check, B contributes $(3/4)(600) = 450\,\text{mm Hg}$, and adding A's 50 reproduces the given 500. Since $200 < 600$, A has the lower pure vapour pressure and is the least volatile component. Options A and B give 1400, which would predict $50 + 1050 = 1100\,\text{mm Hg}$, contradicting the measurement. Option D has the right pressure but incorrectly identifies B as less volatile.

Step 3: Conclusion The vapour pressure of pure B is $600\,\text{mm Hg}$, and A is the least volatile component. Hence, the correct option is (C).

Quick Tip to Solve: First subtract the known component's partial pressure from the total; divide the remainder by the other mole fraction, then compare the two pure vapour pressures.

58. A molecule with the formula AX_4Y has all it's elements from p-block. Element A is rarest, monoatomic, non-radioactive from its group and has the lowest ionization enthalpy value among A, X and Y . Elements X and Y have first and second highest electronegativity values respectively among all the known elements. The shape of the molecule is: [4 marks]

(A) Pentagonal planar

(B) Square pyramidal

(C) Trigonal bipyramidal

(D) Octahedral

Correct Answer: (B) Square pyramidal

Solution:

Concept:

- Fluorine has the highest electronegativity and oxygen the second highest, identifying X as F and Y as O. Xenon is a rare, monatomic, stable noble gas with lower ionisation enthalpy than fluorine and oxygen.

- VSEPR counts regions of electron density at the central atom. A multiple bond counts as one region. Six regions give an octahedral electron-pair arrangement, while the molecular shape describes only the bonded atoms.

Step 1: Identify the formula and count the lone pairs The given clues identify the molecule as XeF_4O, conventionally written $XeOF_4$. Xenon has eight valence electrons. Four Xe–F single bonds and one Xe=O double bond use six of xenon's electrons in bonding, leaving one lone pair on xenon. The same result follows from the valence-electron total: $8 + 4(7) + 6 = 42$. Four Xe–F bonds and one Xe=O bond use 12 electrons, the four fluorine atoms use 24 nonbonding electrons, and oxygen uses four nonbonding electrons. The remaining two electrons constitute xenon's lone pair.

Step 2: Distinguish electron geometry from molecular shape Xenon consequently has five bonded-atom regions and one lone-pair region, six regions altogether. Their electron-pair arrangement is octahedral. Place the lone pair at one octahedral vertex; the oxygen occupies the opposite axial position and the four fluorines form the square base. Omitting the invisible lone pair gives a square-pyramidal molecular shape. Pentagonal planar would require all five bonded atoms to lie in one plane, which is not this arrangement. Trigonal bipyramidal describes five electron regions without the extra lone pair. Octahedral is the electron-pair geometry here, not the molecular shape of a species with only five surrounding atoms.

Step 3: Conclusion The molecule is $XeOF_4$, with a square-pyramidal shape. Hence, the correct option is (B).

Quick Tip to Solve: For xenon oxyfluorides, count a Xe=O double bond as one VSEPR region; five bonded atoms plus one lone pair gives a square pyramid.

59. Given below are two statements :

Statement (I) : In octahedral complexes, when $\Delta_0 < P$ high spin complexes are formed. When $\Delta_0 > P$ low spin complexes are formed.

Statement (II) : In tetrahedral complexes because of $\Delta_t < P$, low spin complexes are rarely formed.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are incorrect

(B) Statement I is correct but Statement II is incorrect

(C) Statement I is incorrect but Statement II is correct

(D) Both Statement I and Statement II are correct

Correct Answer: (D) Both Statement I and Statement II are correct

Solution:

Concept:

- Crystal-field splitting energy Δ is the energy cost of putting an electron into the higher d-orbital set. Pairing energy P is the cost of pairing two electrons in one orbital.
- When an unpaired higher-level arrangement is cheaper than pairing, a high-spin configuration

results. When pairing is cheaper, electrons preferentially occupy the lower set and low-spin configurations can result.

Step 1: Evaluate the octahedral statement In an octahedral field the lower set is t_{2g} and the higher set is e_g, separated by Δ_o. If $\Delta_o < P$, an electron can enter the higher set at lower energy cost than pairing in the lower set; this gives the high-spin arrangement for configurations admitting a spin choice. If $\Delta_o > P$, pairing in the lower orbitals is energetically preferred, giving low spin. For example, octahedral d^6 is $t_{2g}^4 e_g^2$ in high spin but $t_{2g}^6 e_g^0$ in low spin. Thus Statement I correctly gives the standard energy criterion.

Step 2: Evaluate the tetrahedral statement and compare the options Tetrahedral splitting is smaller, approximately $\Delta_t = (4/9)\Delta_o$ for comparable metal-ligand conditions. In the tetrahedral arrangement the lower set is e and the upper set is t_2. Ordinarily $\Delta_t < P$, so electrons occupy separate orbitals rather than paying the larger pairing cost. Tetrahedral complexes are therefore generally high spin, and low-spin tetrahedral complexes are rare. This is exactly what Statement II asserts; it does not claim that the small splitting favours low spin. Both statements are consequently correct. Option A wrongly rejects both energy arguments, B wrongly rejects the tetrahedral observation, and C wrongly rejects the octahedral comparison.

Step 3: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (D).

Quick Tip to Solve: Compare splitting with pairing energy: a small splitting favours promotion and high spin. Tetrahedral splitting is usually too small to make pairing favourable.

60. On complete combustion 1.0 g of an organic compound (X) gave 1.46 g of $\mathrm{CO_2}$ and 0.567 g of $\mathrm{H_2O}$. The empirical formula mass of compound (X) is _____ g. (Given molar mass in $\mathrm{gmol^{-1}C:12, H:1, O:16}$) [4 marks]

(A) 60

(B) 45

(C) 30

(D) 15

Correct Answer: (C) 30

Solution:

Concept:

- Complete combustion converts every carbon atom into $\mathrm{CO_2}$ and every pair of hydrogen atoms into $\mathrm{H_2O}$. Hence the product masses determine the amounts of carbon and hydrogen in the sample.
- For the carbon-hydrogen-oxygen compound used in this combustion calculation, obtain oxygen by subtracting the masses of carbon and hydrogen from the sample mass, then compare atom-mole ratios.

Step 1: Calculate the masses and amounts of the elements The molar masses of $\mathrm{CO_2}$ and $\mathrm{H_2O}$ are 44

and $18\,\mathrm{g\,mol^{-1}}$, respectively. Thus

$$m_C = 1.46\frac{12}{44} = 0.39818\,\mathrm{g}, \qquad n_C = \frac{1.46}{44} = 0.03318\,\mathrm{mol}.$$

Similarly,

$$m_H = 0.567\frac{2}{18} = 0.06300\,\mathrm{g}, \qquad n_H = 0.06300\,\mathrm{mol}.$$

Oxygen accounts for the remaining sample mass:

$$m_O = 1.000 - 0.39818 - 0.06300 = 0.53882\,\mathrm{g}, \qquad n_O = \frac{0.53882}{16} = 0.03368\,\mathrm{mol}.$$

Step 2: Obtain the simplest ratio and its formula mass Dividing by the smallest amount, 0.03318 mol, gives

$$C : H : O = 1 : 1.899 : 1.015.$$

The printed masses do not give an exact small-integer ratio; the simple ratio indicated by the data and the choices is approximately $1 : 2 : 1$, corresponding to $\mathrm{CH_2O}$. Its empirical formula mass is

$$12 + 2(1) + 16 = 30.$$

Option A, 60, doubles the simplest formula mass. Options B and D, 45 and 15, do not match the indicated atom ratio.

Step 3: Conclusion The approximate empirical formula is $\mathrm{CH_2O}$, with formula mass $30\,\mathrm{u}$ and empirical-formula molar mass $30\,\mathrm{g\,mol^{-1}}$. Hence, the correct option is (C).

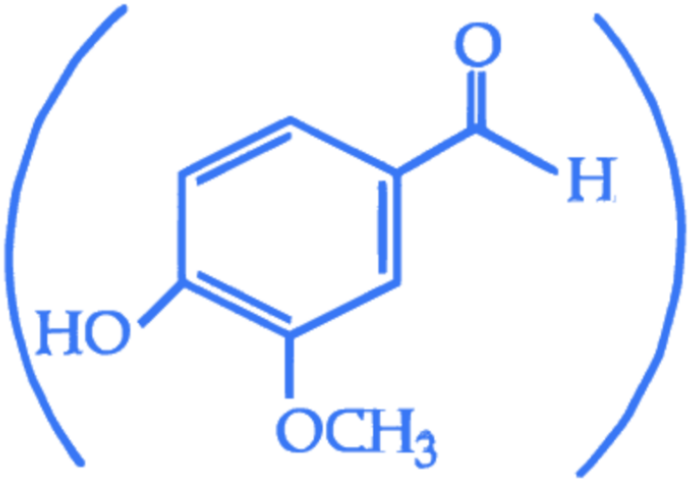
Quick Tip to Solve: Use the oxygen mass left in the original sample, not the oxygen in the combustion products; divide element moles before assigning the simplest whole-number ratio.

61. Given below are two statements:

Statement I : Vanilin will react with NaOH and also with Tollen's reagent.

Statement II : Vanilin will undergo self aldol condensation very easily.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]



(A) Statement I is correct but Statement II is incorrect

(B) Both Statement I and Statement II are correct

(C) Both Statement I and Statement II are incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (A) Statement I is correct but Statement II is incorrect

Solution:

Concept:

- Vanillin contains a phenolic $-OH$ group, a methoxy $-OCH_3$ group and an aromatic aldehyde $-CHO$ group. Each functional group must be tested separately.
- Phenols form phenoxide salts with NaOH; aldehydes reduce Tollens' reagent to metallic silver. Self aldol condensation requires an enolisable carbonyl compound with an alpha hydrogen.

Step 1: Test Statement I The phenolic hydrogen is sufficiently acidic to react with aqueous NaOH, producing the sodium phenoxide derivative and water. Writing Ar for the substituted aromatic group, the acid-base change is

$$\mathrm{ArOH} + \mathrm{NaOH} \longrightarrow \mathrm{ArONa} + \mathrm{H_2O}.$$

The aldehyde group is oxidised by Tollens' reagent to a carboxylate, while silver(I) is reduced to silver. Aromatic aldehydes give this test even though they do not possess an alpha hydrogen. Therefore vanillin reacts with both reagents, so Statement I is correct.

Step 2: Test Statement II and select the combination For self aldol condensation, base must remove an alpha hydrogen from a carbon adjacent to the carbonyl carbon and generate an enolate. In vanillin the $-CHO$ carbon is attached directly to the aromatic ring. There is no alpha carbon bearing an enolisable hydrogen. The hydrogen attached to the aldehyde carbon is not an alpha hydrogen; the methyl

hydrogens of the methoxy group are not alpha to the aldehyde either. Consequently vanillin cannot readily undergo self aldol condensation. Statement II is incorrect. Option B incorrectly accepts self aldol condensation, C incorrectly rejects the reactions in Statement I, and D reverses both judgements.

Step 3: Conclusion Statement I is correct but Statement II is incorrect. Hence, the correct option is (A).

Quick Tip to Solve: For an aromatic aldehyde, distinguish its Tollens test from its aldol behaviour: the first needs the aldehyde group, whereas the second needs an alpha hydrogen.

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62. An optically active alkyl halide $C_4H_9Br[A]$ reacts with hot KOH dissolved in ethanol and forms alkene $[B]$ as major product which reacts with bromine to give dibromide $[C]$. The compound [C] is converted into a gas [D] upon reacting with alcoholic $NaNH_2$. During hydration 18 gram of water is added to 1 mole of gas [D] on warming with mercuric sulphate and dilute acid at 333 K to form compound [E]. The IUPAC name of compound [E] is : [4 marks]

(A) Butan-2-ol

(B) But-2-yne

(C) Butan-2-one

(D) Butan-1-al

Correct Answer: (C) Butan-2-one

Solution:

Concept:

- Optical activity requires a stereogenic carbon with four different substituents. Among the C_4H_9Br isomers, 2-bromobutane has this feature.
- Hot alcoholic KOH favours elimination; bromine adds to an alkene, and strong base removes two hydrogen halide molecules from a vicinal dibromide. Acidic mercury(II)-catalysed hydration of an alkyne gives an enol that tautomerises to a carbonyl compound.

Step 1: Identify the alkene and dibromide The starting compound A is 2-bromobutane, $CH_3CHBrCH_2CH_3$. Its bromine-bearing carbon has H, Br, methyl and ethyl groups. The primary bromides and tert-butyl bromide do not satisfy this optical-activity condition. With hot ethanolic KOH, the major alkene is the more substituted but-2-ene:

$$CH_3CHBrCH_2CH_3 \longrightarrow CH_3CH{=}CHCH_3.$$

Addition of Br_2 across its double bond gives C, 2,3-dibromobutane, $CH_3CHBrCHBrCH_3$. The four-carbon skeleton is preserved throughout these reactions.

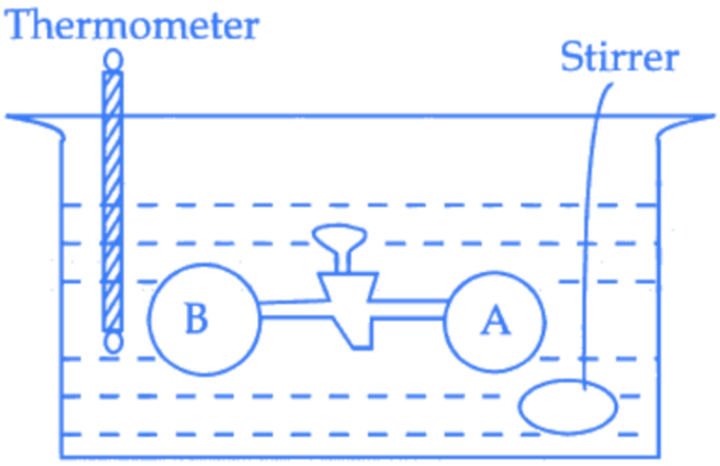
Step 2: Form the alkyne and hydrate it Two successive dehydrohalogenations of C with the strong base $NaNH_2$ give but-2-yne, D, $CH_3C{\equiv}CCH_3$. One mole of this alkyne adds one mole of water, whose mass is $18\,g$, under the stated $HgSO_4$/dilute-acid conditions. Initial addition forms the enol $CH_3C(OH){=}CHCH_3$, which rearranges by keto-enol tautomerism to $CH_3COCH_2CH_3$. The carbonyl carbon receives the lowest locant, 2. Butan-2-ol is an alcohol, not the keto tautomer; but-2-yne is the

intermediate D before hydration. Butan-1-al would require a terminal aldehyde and does not arise by hydration of this symmetrical internal alkyne.

Step 3: Conclusion The final compound E is $CH_3COCH_2CH_3$, whose IUPAC name is butan-2-one. Hence, the correct option is (C).

Quick Tip to Solve: Track the carbon skeleton first: 2-bromobutane gives but-2-ene, then but-2-yne, and hydration of the internal alkyne ends at a ketone.

63. Two vessels A and B are connected via stopcock. The vessel A is filled with a gas at a certain pressure. The entire assembly is immersed in water and is allowed to come to thermal equilibrium with water. After opening the stopcock the gas from vessel A expands into vessel B and no change in temperature is observed in the thermometer. Which of the following statement is true ? [4 marks]



(A) $\mathrm{dq} \neq 0$

(B) The pressure in the vessel B before opening the stopcock is zero

(C) $\mathrm{dw} \neq 0$

(D) $\mathrm{dU} \neq 0$

Correct Answer: (B) The pressure in the vessel B before opening the stopcock is zero

Solution:

Concept:

- For a closed system the first law is $\Delta U = q + w$, using the chemistry sign convention that work done on the system is positive.
- Expansion work is $w = -\int P_{\text{ext}}\, dV$. Free expansion into an evacuated chamber has zero opposing pressure and hence zero expansion work. For an ideal gas, internal energy depends only on temperature.

Step 1: Recognise the experimental arrangement The two rigid vessels and the water bath represent Joule's free-expansion experiment. Initially gas occupies A and B is evacuated. Opening the stopcock allows the gas to spread into the available volume without pushing against gas in B. Thus the initial

pressure of B is zero. The assembly's exterior is rigid, so it also performs no displacement work on its surroundings. Internal spreading is not by itself evidence of nonzero mechanical work: work requires an opposing pressure acting through a boundary displacement.

Step 2: Apply the energy balance and eliminate alternatives For the free expansion, $P_{\text{ext}} = 0$, and hence

$$w = -\int_{V_i}^{V_f} 0\, dV = 0.$$

The thermometer detects no water-bath temperature change, indicating no measurable heat transfer between the gas assembly and bath in this experiment, so $q = 0$. The first law then gives

$$\Delta U = q + w = 0 + 0 = 0.$$

This is also consistent with no temperature change for the ideal gas considered in Joule's experiment. Therefore the propositions $dq \neq 0$, $dw \neq 0$ and $dU \neq 0$ are not the description of this expansion. A final gas pressure in B after opening must not be confused with its zero initial pressure.

Step 3: Conclusion Vessel B is initially evacuated, so its pressure before the stopcock is opened is zero. Hence, the correct option is (B).

Quick Tip to Solve: In a free-expansion question, use the external opposing pressure to calculate work; the gas can expand even though the work is zero.

64. Among $SO_2, NF_3, NH_3, XeF_2, ClF_3$ and SF_4, the hybridization of the molecule with nonzero dipole moment and highest number of lone-pairs of electrons on the central atom is [4 marks]

(A) sp^3

(B) dsp^2

(C) $sp^3\, d^2$

(D) $sp^3\, d$

Correct Answer: (D) $sp^3\, d$

Solution:

Concept:

- Count sigma bonds and lone pairs on the central atom to find the steric number. A double bond counts as one electron domain for molecular geometry.
- In the conventional hybridisation description, steric numbers 3, 4 and 5 correspond to sp^2, sp^3 and sp^3d, respectively. A molecule is polar only if its bond-dipole vectors do not cancel.

Step 1: Count lone pairs for every candidate In SO_2, sulfur has two bonding domains and one lone pair; the bent molecule is polar. The central nitrogen in each of NF_3 and NH_3 has three sigma bonds and one lone pair, and both pyramidal molecules have nonzero dipole moments. XeF_2 has two sigma bonds and three lone pairs on xenon. ClF_3 has three sigma bonds and two lone pairs on chlorine. Finally, SF_4 has four sigma bonds and one lone pair on sulfur. Thus the greatest lone-pair count is initially that of XeF_2, but the polarity condition must still be applied.

Step 2: Apply polarity and obtain the hybridisation The three lone pairs of XeF_2 occupy equatorial positions of a trigonal-bipyramidal electron-domain arrangement. Its two axial Xe–F bonds are opposite, giving a linear molecule with zero net dipole, so it is excluded. ClF_3, with two equatorial lone pairs, is T-shaped and its bond dipoles do not all cancel. It therefore has the highest lone-pair count, two, among the allowed polar molecules. Its steric number is

$$3 \text{ sigma bonds} + 2 \text{ lone pairs} = 5.$$

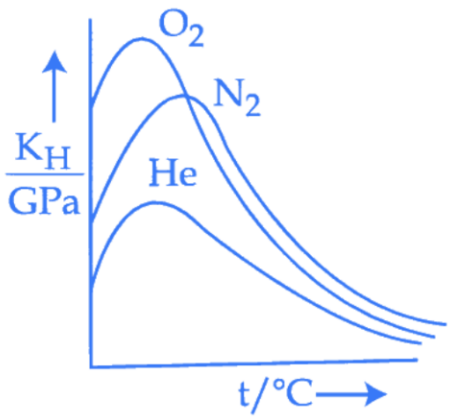
The corresponding hybridisation is $\mathrm{sp^3d}$. Option A describes four electron domains; B describes square-planar $\mathrm{dsp^2}$ hybridisation; C requires six domains. None describes chlorine's five-domain arrangement here.

Step 3: Conclusion The molecule is ClF_3, with two lone pairs and a nonzero dipole moment; its hybridisation is $\mathrm{sp^3d}$. Hence, the correct option is (D).

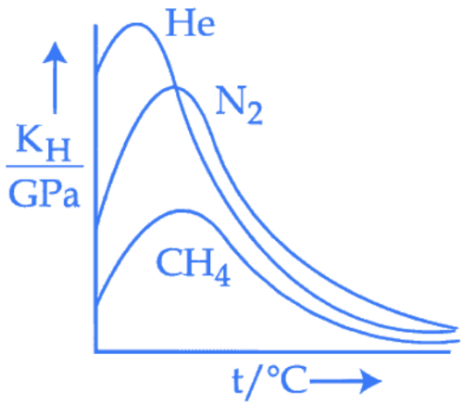
Quick Tip to Solve: Apply both conditions in order: count lone pairs, then discard symmetric molecules whose bond dipoles cancel before choosing the largest remaining count.

65. Which of the following graph correctly represents the plots of K_H at 1 bar for gases in water versus temperature? [4 marks]

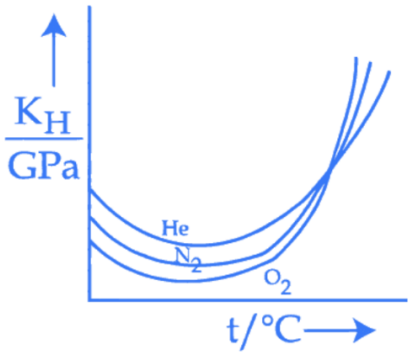
(A)



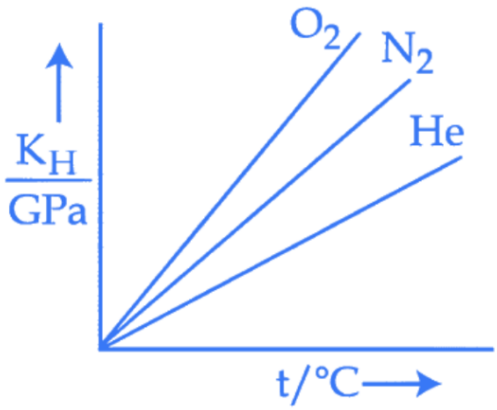
(B)



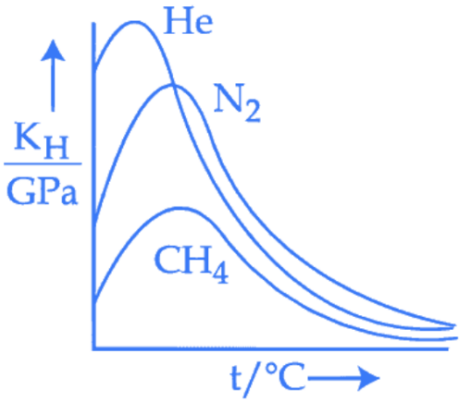
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- Henry's law in the convention used here is $p = K_{\mathrm{H}}x$, where p is the gas partial pressure and x its mole fraction in water. At fixed pressure, larger K_{H} means smaller solubility.
- The characteristic temperature plots for gases in water are not straight lines: over a sufficiently wide temperature interval they exhibit a maximum. Gas identity controls both the height and location of that maximum.

Step 1: Compare the gas identities on the curves At a fixed pressure, rearranging Henry's law gives $x = p/K_{\mathrm{H}}$, so lower dissolved mole fraction corresponds to a larger constant. Helium is only sparingly

soluble in water and has a large Henry constant. Around ordinary temperatures the order for the gases appearing in option B is

$$K_{\mathrm{H}}(\mathrm{He}) > K_{\mathrm{H}}(\mathrm{N_2}) > K_{\mathrm{H}}(\mathrm{CH_4}).$$

For scale, near 298 K the tabulated constants are about 145 kbar for He, 76 kbar for nitrogen and 41 kbar for methane; the same relative order holds if all values are converted to GPa. Thus the upper low-temperature curve should represent He, the intermediate one nitrogen, and the lower one methane, as drawn in B. A incorrectly puts oxygen above helium, even though oxygen is appreciably more soluble.

Step 2: Check the temperature dependence and reject the other shapes As temperature initially rises, gas solubility in water falls and the Henry constant in the p/x convention rises. The full characteristic curves reach maxima and subsequently decline; the question's wide-range plots must reproduce this behaviour rather than extend a local trend indefinitely. B has maxima and the correct gas ordering. C instead displays minima, reversing the initial temperature trend. D makes the curves linear through the Celsius origin, which would require zero Henry constant at $0^{\circ}\mathrm{C}$, and also places helium below the other gases. Neither requirement is physical. A has the maximum-type shapes but assigns the relative gas constants incorrectly.

Step 3: Conclusion The graph with maximum-type curves labelled He, N_2 and CH_4 in the appropriate order is (B). Hence, the correct option is (B).

Quick Tip to Solve: Check the definition first: for $p = K_{\mathrm{H}}x$, the least soluble gas has the largest Henry constant. Then compare the temperature trend and curve labels together.

66. According to Bohr's model of hydrogen atom, which of the following statement is incorrect?

[4 marks]

(A) Radius of 4^{th} orbit is four times larger than that of 2^{nd} orbit
(B) Radius of 8^{th} orbit is four times larger than that of 4^{th} orbit
(C) Radius of 6^{th} orbit is three times larger than that of 4^{th} orbit
(D) Radius of 3^{rd} orbit is nine times larger than that of 1^{st} orbit

Correct Answer: (C) Radius of 6^{th} orbit is three times larger than that of 4^{th} orbit

Solution:

Concept:

- For a hydrogen-like atom the Bohr radius of the nth orbit is $r_n = a_0 n^2/Z$, where a_0 is the Bohr radius and Z the atomic number. Hydrogen has $Z = 1$.
- For two orbits of the same atom, $r_m/r_n = (m/n)^2$. The dependence is quadratic in the principal quantum number, not linear; all dimensional constants cancel from the ratio.

Step 1: Check the first two comparisons For the fourth and second orbits,

$$\frac{r_4}{r_2} = \frac{4^2}{2^2} = \frac{16}{4} = 4.$$

Thus A correctly gives a factor of four. For the eighth and fourth orbits,

$$\frac{r_8}{r_4} = \frac{8^2}{4^2} = \frac{64}{16} = 4.$$

Thus B is also correct. In both cases the principal quantum number doubles, so the radius becomes four times its previous value. These two checks prevent confusing a change in the orbit label with a proportional change of the same size in radius.

Step 2: Check the sixth and third orbit statements For the sixth orbit relative to the fourth,

$$\frac{r_6}{r_4} = \frac{6^2}{4^2} = \frac{36}{16} = \frac{9}{4} = 2.25.$$

The factor is not three, so C is the incorrect statement demanded by the question. For completeness, the third orbit relative to the first gives

$$\frac{r_3}{r_1} = \frac{3^2}{1^2} = 9,$$

which agrees with D. Therefore A, B and D all obey the Bohr formula, whereas only C does not. No numerical value of a_0 or length-unit conversion is needed because this is a comparison of radii within the same hydrogen atom.

Step 3: Conclusion The sixth-orbit radius is $9/4$, not three, times the fourth-orbit radius. Hence, the incorrect statement, and the correct option to select, is (C).

Quick Tip to Solve: For orbit-radius comparisons, square the ratio of the two quantum numbers before comparing it with the claimed factor.

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67. Identify the correct statement among the following : [4 marks]

(A) All naturally occurring amino acids except glycine contain one chiral centre.

(B) Amino acid, cysteine can easily undergo dimerisation due to the presence of free SH group.

(C) Glutamic acid is the only amino acid that contains a $-$COOH group at the side chain.

(D) All naturally occurring amino acids are optically active.

Correct Answer: (B) Amino acid, cysteine can easily undergo dimerisation due to the presence of free SH group.

Solution:

Concept:

- An amino acid is chiral when a carbon bears four different substituents. Glycine has two hydrogen atoms at its alpha carbon and is achiral.
- Cysteine contains a thiol side chain, $-CH_2SH$. Oxidation of two thiol groups produces a disulphide bond, linking two cysteine units as cystine.

Step 1: Test the statement about cysteine Two cysteine molecules can combine through their side-chain sulphur atoms. The free thiol groups make this oxidation and dimer formation possible. Writing

R for the rest of each amino-acid unit, the change is

$$2\,\mathrm{RSH} + [\mathrm{O}] \longrightarrow \mathrm{RSSR} + \mathrm{H_2O}.$$

The product contains an $\mathrm{S{-}S}$ linkage and is called cystine. Thus statement (B) is correct; this is also the chemical basis of disulphide bridges in proteins.

Step 2: Check the other three claims Statement (A) overstates the number of chiral centres. Threonine and isoleucine are naturally occurring amino acids with two chiral centres, so it is false that every amino acid other than glycine has exactly one. Statement (C) is false because aspartic acid, as well as glutamic acid, carries a carboxyl group in its side chain: their side chains are $-\mathrm{CH_2COOH}$ and $-\mathrm{CH_2CH_2COOH}$, respectively. Statement (D) is false because glycine is achiral and therefore does not show optical activity. These counterexamples eliminate the remaining options without altering the meaning of the word all.

Step 3: Conclusion Cysteine can undergo dimerisation to cystine through its free thiol groups. Hence, the correct option is (B).

Quick Tip to Solve: For absolute claims about amino acids, check glycine for optical activity, threonine/isoleucine for extra chiral centres, and aspartic acid for a second acidic side chain.

68. The correct order of basic nature in aqueous solution for the bases $\mathrm{NH_3}, \mathrm{H_2\,N-NH_2}, \mathrm{CH_3CH_2NH_2}, (\mathrm{CH_3CH_2})_2\,\mathrm{NH}$ **and** $(\mathrm{CH_3CH_2})_3\ \mathrm{N}$ **is :** [4 marks]

(A) $\mathrm{NH_3} < \mathrm{H_2\,N-NH_2} < (\mathrm{CH_3CH_2})_3\ \mathrm{N} < \mathrm{CH_3CH_2NH_2} < (\mathrm{CH_3CH_2})_2\,\mathrm{NH}$

(B) $\mathrm{H_2\,N-NH_2} < \mathrm{NH_3} < (\mathrm{CH_3CH_2})_3\ \mathrm{N} < \mathrm{CH_3CH_2NH_2} < (\mathrm{CH_3CH_2})_2\,\mathrm{NH}$

(C) $\mathrm{NH_2-NH_2} < \mathrm{NH_3} < \mathrm{CH_3CH_2NH_2} < (\mathrm{CH_3CH_2})_3\ \mathrm{N} < (\mathrm{CH_3CH_2})_2\,\mathrm{NH}$

(D) $\mathrm{NH_3} < \mathrm{H_2\,N-NH_2} < \mathrm{CH_3CH_2NH_2} < (\mathrm{CH_3CH_2})_2\,\mathrm{NH} < (\mathrm{CH_3CH_2})_3\ \mathrm{N}$

Correct Answer: (C) $\mathrm{NH_2-NH_2} < \mathrm{NH_3} < \mathrm{CH_3CH_2NH_2} < (\mathrm{CH_3CH_2})_3\ \mathrm{N} < (\mathrm{CH_3CH_2})_2\,\mathrm{NH}$

Solution:

Concept:

- Aqueous basicity depends on both the availability of the nitrogen lone pair and the stabilization of the protonated base by water. A smaller pK_b means a stronger base.
- Ethyl groups donate electron density inductively. Hydration and steric effects must also be included; counting alkyl groups alone does not determine aqueous basicity.

Step 1: Compare hydrazine and ammonia In hydrazine, $\mathrm{H_2N{-}NH_2}$, the neighbouring nitrogen exerts an electron-withdrawing inductive influence. Hydrazine is a weaker aqueous base than ammonia: their approximate pK_b values are 5.9 and 4.75, respectively. Thus the beginning of the increasing basicity order is

$$\mathrm{H_2N{-}NH_2} < \mathrm{NH_3}.$$

This immediately rejects options (A) and (D), which reverse that pair.

Step 2: Order the three ethylamines For the ethyl-substituted amines, representative aqueous pK_b

values are about 3.29 for ethylamine, 3.25 for triethylamine, and 3.00 for diethylamine. The secondary amine provides the most favourable balance of electron donation and hydration of its conjugate acid. With ethyl groups, triethylamine is slightly more basic than ethylamine despite its less effective hydration. Hence

$$C_2H_5NH_2 < (C_2H_5)_3N < (C_2H_5)_2NH.$$

Option (B) interchanges the primary and tertiary ethylamines and is therefore incorrect. The purely inductive sequence ending with the tertiary amine in (D) also neglects hydration. Combining both comparisons gives exactly option (C).

Step 3: Conclusion The increasing aqueous basicity order is

$$H_2N{-}NH_2 < NH_3 < C_2H_5NH_2 < (C_2H_5)_3N < (C_2H_5)_2NH.$$

Hence, the correct option is (C).

Quick Tip to Solve: Distinguish methylamines from ethylamines in water. For ethylamines the increasing order is primary, tertiary, secondary; include hydration before ranking the bases.

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69. The property/properties that show irregularity in first four elements of group-17 is/are

(A) Covalent radius

(B) Electron affinity

(C) Ionic radius

(D) First ionization energy

Choose the correct answer from the options given below : [4 marks]

(A) A, B, C and D

(B) A and C only

(C) B only

(D) B and D only

Correct Answer: (C) B only

Solution:

Concept:

- The first four group-17 elements are fluorine, chlorine, bromine and iodine. Additional occupied shells generally increase atomic and ionic size down this group.
- Electron affinity describes the energy released when a gaseous neutral atom gains an electron. Electron-electron repulsion in a very compact orbital can produce an exception to a simple down-group trend.

Step 1: Check the two radius trends and ionization energy The covalent radii increase from fluorine to iodine as additional shells are added. The radii of their singly charged anions also increase in the sequence

$$F^- < Cl^- < Br^- < I^-.$$

Both radius properties therefore show the regular down-group increase, not an anomalous reversal. First ionization energy decreases down these four elements: increasing shielding and the larger distance of the outer electron from the nucleus make its removal progressively easier. Thus properties (A), (C) and (D) are regular.

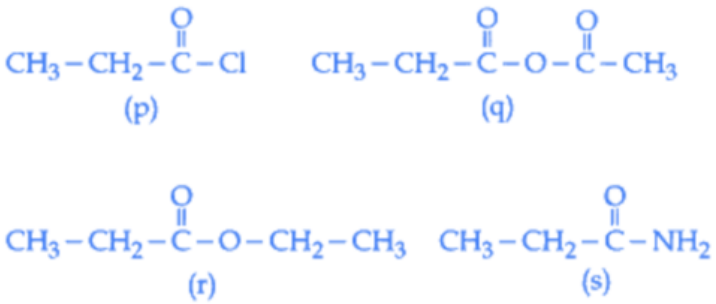
Step 2: Identify the electron-affinity exception Chlorine releases more energy on gaining an electron than fluorine does. The incoming electron in fluorine enters a compact $2p$ shell and experiences appreciable repulsion from the electrons already present. Chlorine has a more spacious $3p$ shell, so electron attachment is more favourable. Representative energy-release magnitudes, in $\mathrm{kJ\,mol^{-1}}$, are approximately 328, 349, 325 and 295 for F, Cl, Br and I. The rise from F to Cl followed by a fall is the irregularity. Thus only property (B) qualifies. Options (A) and (B) incorrectly include regular radius trends, while option (D) incorrectly adds first ionization energy.

Step 3: Conclusion Only electron affinity shows the stated irregularity. Hence, the correct option is (C).

Quick Tip to Solve: Remember that chlorine has a larger electron-affinity magnitude than fluorine; the compact fluorine shell is the exception, while halogen radii and first ionization energies change regularly.

70. Consider the following molecules:

The correct order of rate of hydrolysis is : [4 marks]



(A) $\mathrm{p} > \mathrm{q} > \mathrm{r} > \mathrm{s}$

(B) $r > q > p > s$

(C) $\mathrm{p} > \mathrm{r} > \mathrm{q} > \mathrm{s}$

(D) $q > p > r > s$

Correct Answer: (A) $\mathrm{p} > \mathrm{q} > \mathrm{r} > \mathrm{s}$

Solution:

Concept:

- Hydrolysis of an acyl derivative proceeds by nucleophilic attack at its carbonyl carbon followed by elimination. Reactivity depends on carbonyl electrophilicity and how readily the attached group leaves.
- The standard reactivity order under comparable hydrolysis conditions is acyl chloride, acid anhydride, ester, then amide. Strong resonance donation to the carbonyl makes amides especially

resistant.

Step 1: Identify the four functional groups In the displayed structures, p is propanoyl chloride, CH_3CH_2COCl; q is a mixed acid anhydride, $CH_3CH_2COOCOCH_3$; r is the ester ethyl propanoate, $CH_3CH_2COOCH_2CH_3$; and s is propanamide, $CH_3CH_2CONH_2$. Their similar acyl frameworks allow the attached groups to decide the main reactivity difference. The chloride in p is an excellent leaving group, making the acyl chloride the fastest to hydrolyse.

Step 2: Compare leaving ability and resonance stabilization Anhydride q can expel a resonance-stabilized carboxylate, so it hydrolyses more readily than ester r, whose alkoxy group is a poorer leaving group. Amide s is least reactive: the nitrogen lone pair donates strongly into the carbonyl, lowering the electrophilicity of the acyl carbon, and an unprotonated amino group is a very poor leaving group. Consequently

$$p > q > r > s.$$

Option (B) wrongly places the ester above both the anhydride and the acyl chloride. Option (C) wrongly puts the ester above the anhydride. Option (D) wrongly puts the anhydride above the acyl chloride. Only (A) preserves all three pairwise comparisons.

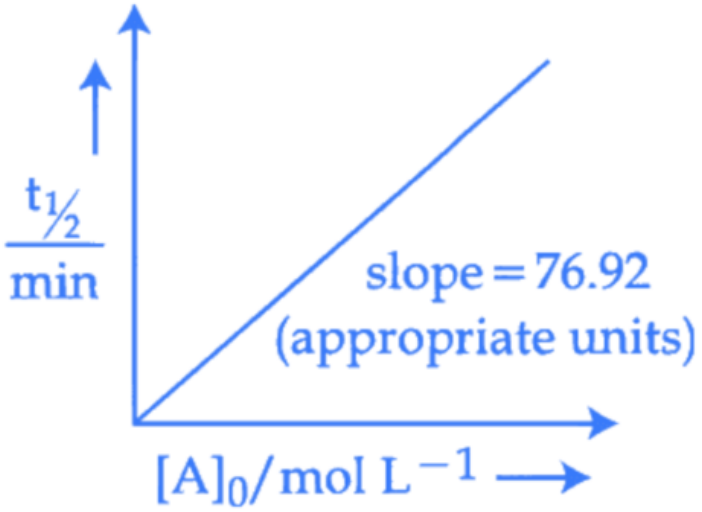
Step 3: Conclusion The rate of hydrolysis decreases as $p > q > r > s$. Hence, the correct option is (A).

Quick Tip to Solve: Identify the acyl derivative first: chloride hydrolyses fastest, then anhydride, ester and amide. The alkyl-chain length here is not the deciding comparison.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. For the reaction A $\rightarrow$ products.

The concentration of A at 10 minutes is _____ $\times 10^{-3}$ mol L^{-1} (nearest integer). The reaction was started with 2.5 mol L^{-1} of A . [4 marks]



Solution:

Concept:

- For a zero-order reaction, concentration decreases linearly with time: $[A]_t = [A]_0 - kt$, where k has units of concentration per unit time.
- Putting $[A]_t = [A]_0/2$ in this equation gives $t_{1/2} = [A]_0/(2k)$. A straight line through the origin for half-life against initial concentration therefore identifies zero-order kinetics.

Step 1: Read the half-life graph The vertical axis is half-life in minutes and the horizontal axis is initial concentration in $\text{mol}\,\text{L}^{-1}$. The line passes through the origin and has slope 76.92. Comparing with

$$t_{1/2} = \frac{1}{2k}[A]_0,$$

its slope is $1/(2k)$, not k itself. A first-order reaction would instead have a concentration-independent half-life and give a horizontal line. Hence the graph supports the zero-order integrated rate law.

Step 2: Calculate the rate constant With time measured in minutes,

$$k = \frac{1}{2(76.92)} = 0.00650026\,\text{mol}\,\text{L}^{-1}\,\text{min}^{-1}.$$

The units follow directly because the slope has units $\text{min}\,\text{L}\,\text{mol}^{-1}$. Keep these time units when substituting ten minutes; converting only one of the two quantities to seconds would give a wrong change in concentration.

Step 3: Find the remaining concentration and the requested integer Initially $[A]_0 = 2.5\,\text{mol}\,\text{L}^{-1}$. After

10 minutes,

$$\begin{aligned}[A]_{10} &= 2.5 - (0.00650026)(10) \\ &= 2.4349974\,\text{mol}\,\text{L}^{-1} \\ &= 2434.9974 \times 10^{-3}\,\text{mol}\,\text{L}^{-1}.\end{aligned}$$

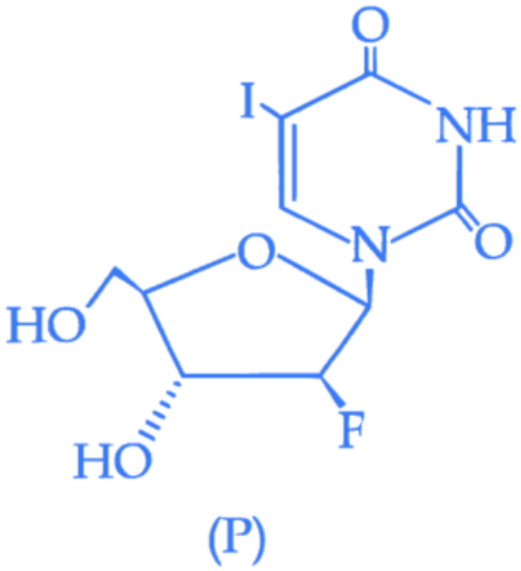
The blank asks for the coefficient of 10^{-3}, so round 2434.9974 to the nearest integer, rather than rounding the concentration in unscaled units.

Step 4: Conclusion The concentration is approximately $2.435\,\text{mol}\,\text{L}^{-1} = 2435 \times 10^{-3}\,\text{mol}\,\text{L}^{-1}$. The required integer is $\boxed{2435}$.

Quick Tip to Solve: For a half-life versus initial-concentration graph, a line through the origin means zero order. Its slope equals $1/(2k)$, so invert twice the slope before applying the rate law.

72. 0.1 mol of the following given antiviral compound (P) will weigh _____ $\times 10^{-1}$ g (nearest integer).

(Given : molar mass in $\text{gmol}^{-1}\text{H} : 1, \text{C} : 12,\ \text{N} : 14, \text{O} : 16,\ \text{F} : 19, \text{I} : 127$ **)** [4 marks]



(P)

Solution:

Concept:

- The mass of a sample is $m = nM$, where n is the amount in moles and M is molar mass in $\text{g}\,\text{mol}^{-1}$.
- In a skeletal formula, unlabelled vertices and line ends represent carbon. Add implicit hydrogen atoms to complete carbon valence four, retaining all explicitly shown heteroatoms and hydrogens.

Step 1: Count the atoms in the two ring fragments The iodinated heterocyclic base contains four carbon atoms, two nitrogen atoms, two carbonyl oxygens and one iodine atom. Its visible NH provides one hydrogen, and the unsubstituted alkene carbon provides another; the other ring nitrogen is attached to the sugar fragment and bears no hydrogen. The sugar portion contains four ring carbons and one exocyclic CH_2OH carbon, giving five more carbons. It contains one ring oxygen, two hydroxyl

oxygens and one fluorine atom.

Step 2: Complete the hydrogen count and obtain molar mass Each of the four sugar-ring carbons bears one hydrogen. The exocyclic carbon bears two hydrogens, and the two hydroxyl groups supply two more, making eight hydrogens in the sugar fragment. Together with the two in the base, the molecular formula is $C_9H_{10}FIN_2O_5$. Therefore, using the supplied atomic molar masses,

$$\begin{aligned} M &= 9(12) + 10(1) + 19 + 127 + 2(14) + 5(16) \\ &= 108 + 10 + 19 + 127 + 28 + 80 \\ &= 372\,\text{g mol}^{-1}. \end{aligned}$$

The iodine atom must be included; its contribution alone is $127\,\text{g mol}^{-1}$.

Step 3: Convert the amount to mass in the requested scale For 0.1 mole,

$$m = (0.1)(372) = 37.2\,\text{g} = 372 \times 10^{-1}\,\text{g}.$$

Thus the number required before 10^{-1} g is 372. The factor 0.1 from the mole amount has already been applied and must not be applied a second time.

Step 4: Conclusion The sample weighs $37.2\,\text{g} = 372 \times 10^{-1}\,\text{g}$. The required nearest integer is $\boxed{372}$.

Quick Tip to Solve: Count the base and sugar separately, then add them. Include hydrogens on hydroxyl and NH groups; the glycosidic nitrogen has no additional hydrogen.

73. Consider the following electrochemical cell at standard condition.

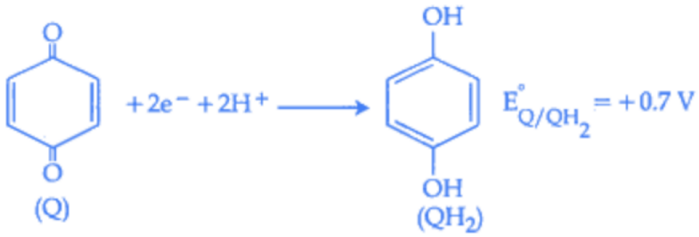
$$\text{Au(s) }|\text{QH}_2, \text{Q}|\,\text{NH}_4\text{X}(0.01\text{M})||\text{Ag}^+(1\text{M})\mid \text{Ag(s)}\,\text{E}_{\text{cell}} = +0.4\text{ V}$$

The couple QH_2/Q represents quinhydrone electrode, the half cell reaction is given below:

$$\left[\textbf{Given : } \text{E}^0_{\text{Ag}^+/\text{Ag}} = +0.8\text{ V } \textbf{and } \frac{2.303\text{RT}}{\text{F}} = 0.06\text{ V}\right]$$

The pK_b value of the ammonium halide salt (NH_4X) used here is _____ . (nearest integer)

[4 marks]



Solution:

Concept:

- A galvanic cell has $E_{\text{cell}} = E_{\text{right}} - E_{\text{left}}$, with both electrode potentials expressed as reduction potentials. The gold electrode is inert; the left-hand potential is determined by the quinone–hydroquinone couple.
- For $\text{Q} + 2\,\text{H}^+ + 2\,\text{e}^- \longrightarrow \text{QH}_2$, equal quinone and hydroquinone activities give $E_{\text{Q/QH}_2} = E^\circ_{\text{Q/QH}_2} - 0.06\,\text{pH}$. The factor two from hydrogen-ion activity cancels the two transferred electrons.
- An ammonium halide is a salt of the weak base ammonia and a strong acid. Its cation hydrolyses with $K_a = K_w/K_b$. For concentration C, $[\text{H}^+] \simeq \sqrt{K_a C}$, so $\text{pH} = \frac{1}{2}(\text{p}K_w - \text{p}K_b - \log C)$.

Step 1: Find the left-hand electrode potential The silver-ion concentration is $1\,\text{M}$, so the right-hand silver electrode has its standard reduction potential, $E_{\text{right}} = +0.80\,\text{V}$. Substitute the measured cell emf with the stated left-to-right cell orientation:

$$0.40 = 0.80 - E_{\text{left}}, \qquad E_{\text{left}} = +0.40\,\text{V}.$$

Subtracting the cell emf from the silver potential is essential; adding it would give the wrong acidity for the solution.

Step 2: Apply the quinhydrone Nernst equation The figure gives $E^\circ_{\text{Q/QH}_2} = +0.70\,\text{V}$. With unit activity ratio for the two quinhydrone components, the reaction quotient is $1/[\text{H}^+]^2$. Hence

$$E_{\text{left}} = 0.70 - \frac{0.06}{2}\log\frac{1}{[\text{H}^+]^2} = 0.70 - 0.06\,\text{pH}.$$

Therefore $0.40 = 0.70 - 0.06\,\text{pH}$, which gives $\text{pH} = 5$ and $[\text{H}^+] = 10^{-5}\,\text{M}$.

Step 3: Relate the acidity to the base constant The salt concentration is $C = 0.01 = 10^{-2}\,\text{M}$. For $\text{NH}_4{}^+ + \text{H}_2\text{O} \rightleftharpoons \text{NH}_3 + \text{H}_3\text{O}^+$, the weak-hydrolysis approximation gives

$$K_a \simeq \frac{(10^{-5})^2}{10^{-2}} = 10^{-8}.$$

At the standard temperature, $K_w = 10^{-14}$, so

$$K_b = \frac{K_w}{K_a} = \frac{10^{-14}}{10^{-8}} = 10^{-6}, \qquad \text{p}K_b = 6.$$

The hydrolysed fraction is only $10^{-5}/10^{-2} = 10^{-3}$, validating the approximation. Here the salt's $\text{p}K_b$ refers to its parent weak base.

Step 4: Conclusion The requested nearest-integer value is $\boxed{\text{p}K_b = 6}$. This logarithmic equilibrium-constant value is dimensionless.

Quick Tip to Solve: First obtain the quinhydrone potential by subtracting the cell emf from the silver potential; then convert its 0.30 V shift into pH before applying salt hydrolysis.

74. Consider the following equilibrium,

$$CO(g) + 2H_2(g) \rightleftharpoons CH_3OH(g)$$

0.1 mol of CO along with a catalyst is present in a $2dm^3$ flask maintained at 500 K . Hydrogen is introduced into the flask until the pressure is 5 bar and 0.04 mol of CH_3OH is formed. The K_p^θ is _____ $\times 10^{-3}$ (nearest integer).

Given : $R = 0.08\ dm^3$ **bar** $K^{-1}\ mol^{-1}$

Assume only methanol is formed as the product and the system follows ideal gas behaviour. [4 marks]

Solution:

Concept:

- For an ideal gas mixture, total pressure is $P = n_{tot}RT/V$, and a component's partial pressure is $p_i = n_iRT/V$. All amounts used to calculate the equilibrium constant must describe the equilibrium mixture.
- Stoichiometry for $CO + 2\,H_2 \rightleftharpoons CH_3OH$ consumes one mole of carbon monoxide and two moles of hydrogen for every mole of methanol formed. A catalyst changes the rate of reaching equilibrium, not the equilibrium constant.
- The standard thermodynamic constant is $K_p^\circ = (p_{CH_3OH}/p^\circ)/[(p_{CO}/p^\circ)(p_{H_2}/p^\circ)^2]$. Here $p^\circ = 1\,bar$, so numerical pressures in bar can be substituted and the resulting constant is dimensionless.

Step 1: Determine the total equilibrium amount The flask has volume $2\,dm^3$, temperature $500\,K$ and equilibrium pressure $5\,bar$. Using the supplied value of the gas constant,

$$n_{tot} = \frac{PV}{RT} = \frac{5 \times 2}{0.08 \times 500} = 0.25\,mol.$$

This includes unreacted carbon monoxide, unreacted hydrogen and the methanol product. The stated pressure is used for the mixture after the specified methanol has formed, rather than for a hypothetical unreacted mixture.

Step 2: Obtain all equilibrium mole amounts Initially there is no methanol, so formation of $0.04\,mol$ methanol consumes $0.04\,mol$ carbon monoxide. Thus

$$n_{CO} = 0.10 - 0.04 = 0.06\,mol.$$

Subtract the two known equilibrium amounts from the total:

$$n_{H_2} = 0.25 - 0.06 - 0.04 = 0.15\,mol.$$

As a stoichiometric check, hydrogen introduced before its consumption would total $0.15 + 2(0.04) = 0.23\,mol$; all final mole amounts are positive.

Step 3: Calculate partial pressures and the equilibrium constant For this flask, $RT/V = 0.08(500)/2 = 20\,bar\,mol^{-1}$. Consequently,

$$p_{CO} = 0.06(20) = 1.2\,bar, \quad p_{H_2} = 0.15(20) = 3.0\,bar, \quad p_{CH_3OH} = 0.04(20) = 0.8\,bar.$$

Their sum is $5.0\,\text{bar}$, as required. Substitute their ratios to standard pressure:

$$K_p^\circ = \frac{0.8}{(1.2)(3.0)^2} = 0.074074\ldots = 74.074\ldots \times 10^{-3}.$$

Round the coefficient of 10^{-3}, rather than rounding the small equilibrium constant itself to an integer.

Step 4: Conclusion $K_p^\circ \simeq 74 \times 10^{-3}$, so the nearest-integer value to enter in the blank is $\boxed{74}$. The standard equilibrium constant is dimensionless.

Quick Tip to Solve: Use total equilibrium pressure to find total moles, subtract CO and methanol to obtain hydrogen, and only then form the partial-pressure quotient.

75. A transition metal (M) among $\text{Mn}, \text{Cr}, \text{Co}$ and Fe has the highest standard electrode potential $(\text{M}^{3+}/\text{M}^{2+})$. It forms a metal complex of the type $[\text{M}(\text{CN})_6]^{4-}$. The number of electrons present in the e_g orbital of the complex is _____. [4 marks]

Solution:

Concept:

- A more positive standard reduction potential $E^\circ(\text{M}^{3+}/\text{M}^{2+})$ means a greater tendency of the trivalent ion to gain an electron and become divalent. The comparison is for this particular couple, not for reduction of the divalent ion to the metal.
- In an octahedral field the five d orbitals split into lower-energy t_{2g} orbitals and higher-energy e_g orbitals. The lower set holds at most six electrons, while the upper set holds at most four.
- Cyanide, CN^-, is a strong-field ligand. Its large octahedral splitting favours a low-spin electron arrangement, with pairing in t_{2g} before occupation of the higher-energy e_g set.

Step 1: Identify the metal from its reduction potential The approximate standard reduction potentials for the specified couples are

$$E^\circ(\text{Cr}^{3+}/\text{Cr}^{2+}) \simeq -0.41\,\text{V},$$
$$E^\circ(\text{Fe}^{3+}/\text{Fe}^{2+}) \simeq +0.77\,\text{V},$$
$$E^\circ(\text{Mn}^{3+}/\text{Mn}^{2+}) \simeq +1.51\,\text{V},$$
$$E^\circ(\text{Co}^{3+}/\text{Co}^{2+}) \simeq +1.82\,\text{V}.$$

Cobalt therefore has the highest value among the four candidates. Although reduction of manganese(III) gives stable half-filled manganese(II), its potential is still below that of the cobalt couple; that qualitative argument alone must not override the actual ordering.

Step 2: Find the metal oxidation state and d-electron count The complex is $[\text{Co}(\text{CN})_6]^{4-}$. Each of the six cyanide ligands has charge -1. If the cobalt oxidation state is x, charge balance gives

$$x + 6(-1) = -4, \qquad x = +2.$$

Neutral cobalt has configuration $[\text{Ar}]\,3d^7 4s^2$. Removing the two outer 4s electrons yields Co^{2+} with a $3d^7$ configuration. The $+3/+2$ couple identifies the metal but does not imply that its complex contains cobalt(III).

Step 3: Distribute the seven d electrons in the strong field The six ligands produce octahedral splitting. Because cyanide is strong-field, the three lower t_{2g} orbitals are completely paired before the remaining electron enters the upper set:

$$d^7 \text{ (low spin)} : \qquad t_{2g}^6 e_g^1.$$

The lower set cannot accommodate a seventh electron, so one electron necessarily occupies an e_g orbital. A high-spin distribution $t_{2g}^5 e_g^2$ would give two upper-set electrons, but it does not apply to the strong-field complex specified here.

Step 4: Conclusion The metal is cobalt and the complex has the low-spin configuration $t_{2g}^6 e_g^1$. The requested number of electrons in the e_g set is therefore $\boxed{1}$, a dimensionless count.

Quick Tip to Solve: Identify the metal using the specified redox couple, then independently obtain its complex oxidation state from ligand charges before counting d electrons.