JEE Main 2025 Paper 1 (B.E./B.Tech) Apr 2 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 2 April 2025, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If the mean and the variance of $6, 4, a, 8, b, 12, 10, 13$ are 9 and 9.25 respectively, then $a + b + ab$ is equal to : [4 marks]

(A) 103

(B) 106

(C) 100

(D) 105

Correct Answer: (A) 103

Solution:

Concept:

- The arithmetic mean of eight observations is their sum divided by eight: $\bar{x} = \frac{1}{8}\sum x_i$.
- The population variance is $\sigma^2 = \frac{1}{8}\sum x_i^2 - \bar{x}^2$. Thus the mean determines the sum of the unknowns, while the variance determines their sum of squares.

Step 1: Use the mean to obtain the sum The six known observations have sum $6+4+8+12+10+13 = 53$. Since all eight observations have mean 9, their total is $8 \times 9 = 72$. Therefore,

$$a + b = 72 - 53 = 19.$$

We need the symmetric expression involving both unknowns, so there is no need to decide which unknown occurs at which position.

Step 2: Use the variance to obtain the squares Apply the variance formula with $\sigma^2 = 9.25$ and $\bar{x} = 9$:

$$\sum x_i^2 = 8(9.25 + 81) = 722.$$

The sum of the six known squares is

$$6^2 + 4^2 + 8^2 + 12^2 + 10^2 + 13^2 = 529.$$

Consequently, $a^2 + b^2 = 722 - 529 = 193$. The variance here uses all eight observations, rather than a sample divisor of seven.

Step 3: Recover the product and compare the options Expand the square of the sum:

$$(a + b)^2 = a^2 + b^2 + 2ab.$$

Substitution gives $361 = 193 + 2ab$, hence $ab = 84$. Thus $a + b + ab = 19 + 84 = 103$. As a check, the unknowns are 7 and 12 in either order; these give both the required mean and variance. The other listed values 106, 100 and 105 do not equal this symmetric expression.

Step 4: Conclusion Therefore, $a + b + ab = \boxed{103}$. Hence, the correct option is (A).

Quick Tip to Solve: When both mean and variance are given, find the sum and sum of squares first; use their difference to obtain the product.

2. Let $f : [1, \infty) \to [2, \infty)$ be a differentiable function. If $10 \int_1^x f(\mathrm{t})\mathrm{dt} = 5xf(x) - x^5 - 9$ for all $x \geqslant 1$, then the value of $f(3)$ is : [4 marks]

(A) 22

(B) 26

(C) 32

(D) 18

Correct Answer: (C) 32

Solution:

Concept:

- The fundamental theorem of calculus gives $\frac{d}{dx} \int_1^x f(t)\, dt = f(x)$ for this differentiable function.
- A first-order linear differential equation can be solved using an integrating factor. The original integral equation also supplies its initial value at the lower limit.

Step 1: Find the initial value Set $x = 1$ in the original equation. The integral over an interval of zero length vanishes, so

$$0 = 5f(1) - 1 - 9.$$

Thus $f(1) = 2$. This condition must be retained when differentiating, since differentiation alone introduces an arbitrary integration constant.

Step 2: Differentiate and solve the equation Differentiate both sides, applying the product rule to $xf(x)$:

$$10f(x) = 5f(x) + 5xf\prime(x) - 5x^4.$$

After rearrangement and division by x, which is nonzero on the given domain,

$$f\prime(x) - \frac{f(x)}{x} = x^3.$$

The integrating factor is $e^{-\int dx/x} = 1/x$. Therefore,

$$\frac{d}{dx}\left(\frac{f(x)}{x}\right) = x^2, \qquad \frac{f(x)}{x} = \frac{x^3}{3} + C.$$

Hence $f(x) = x^4/3 + Cx$.

Step 3: Apply the condition and evaluate Using $f(1) = 2$, we obtain $C = 5/3$. Thus

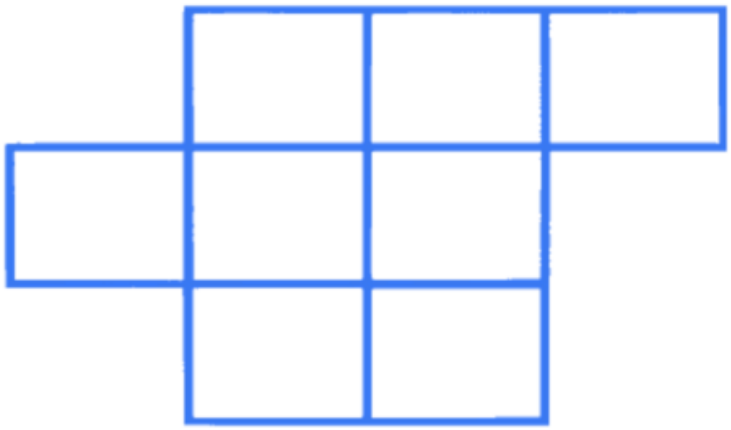
$$f(x) = \frac{x^4 + 5x}{3}, \qquad f(3) = \frac{81 + 15}{3} = 32.$$

This function is increasing for $x \geq 1$ and starts at 2, consistent with the stated codomain. Substitution also gives $10 \int_1^x f(t)dt = \frac{2}{3}x^5 + \frac{25}{3}x^2 - 9$, equal to the original right-hand side. Consequently the alternatives 22, 26 and 18 fail the given condition.

Step 4: Conclusion The required value is $\boxed{f(3) = 32}$. Hence, the correct option is (C).

Quick Tip to Solve: Before differentiating an integral equation, substitute its lower limit to preserve the initial condition needed for the constant.

3. The number of ways, in which the letters A, B, C, D, E can be placed in the 8 boxes of the figure below so that no row remains empty and at most one letter can be placed in a box, is : [4 marks]



(A) 5880

(B) 840

(C) 960

(D) 5760

Correct Answer: (D) 5760

Solution:

Concept:

- The five letters are distinct, and at most one letter occupies each box. Select the five occupied boxes, then permute the five letters among them.
- The displayed rows contain 3, 3 and 2 boxes. Inclusion-exclusion removes selections that leave any entire row empty.

Step 1: Count all selections of occupied boxes Ignoring the restriction on rows for a moment, any five of the eight boxes can be occupied. The number of such selections is

$$\binom{8}{5} = 56.$$

Once the occupied positions are fixed, the five distinct letters A, B, C, D and E can be assigned bijectively to them in $5! = 120$ ways. The shape does not introduce any further constraint between adjacent boxes.

Step 2: Exclude selections with an empty row If the top row, which contains three boxes, is empty, all five boxes of the other two rows must be occupied. This gives $\binom{5}{5} = 1$ selection. The middle row also has three boxes, so leaving it empty gives another one selection. If the bottom row with two boxes is empty, five occupied boxes must be selected from the other six, giving $\binom{6}{5} = 6$ selections. No selection can leave two rows empty: each remaining single row has fewer than five boxes. Thus no overlap correction is required.

Step 3: Assign the letters and identify the result The number of valid occupied-box selections is

$$56 - 1 - 1 - 6 = 48.$$

Every valid selection allows all 120 assignments of the distinct letters, so the total is

$$48 \times 120 = 5760.$$

The value 5880 would subtract only seven invalid selections; 840 and 960 also fail to include all valid choices and letter permutations. An empty box is permitted, but an empty row is not.

Step 4: Conclusion The required number of arrangements is $\boxed{5760}$. Hence, the correct option is (D).

Quick Tip to Solve: For distinct objects in irregular rows, choose occupied boxes first, exclude empty rows, and only then multiply by the factorial.

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4. Let the point P of the focal chord PQ of the parabola $y^2 = 16x$ be $(1, -4)$. If the focus of the parabola divides the chord PQ in the ratio $m : n, \gcd(m, n) = 1$, then $m^2 + n^2$ is equal to : [4 marks]

(A) 17

(B) 37

(C) 10

(D) 26

Correct Answer: (A) 17

Solution:

Concept:

- For the standard parabola $y^2 = 4ax$, the focus is $(a, 0)$. A focal chord is a chord whose line passes through this focus.
- Points on a straight line can be parametrised by a direction vector; distances along that line are proportional to changes in the parameter.

Step 1: Locate the focus and the chord line Compare $y^2 = 16x$ with $y^2 = 4ax$. This gives $a = 4$, so the focus is $S = (4, 0)$. The given endpoint $P = (1, -4)$ lies on the parabola since $(-4)^2 = 16 \cdot 1$. The direction from P to S is $(3, 4)$, and the focal chord therefore has parametrisation

$$(x, y) = (1 + 3t, -4 + 4t).$$

Here P corresponds to $t = 0$, and S to $t = 1$.

Step 2: Find the second intersection Substitute the line into the parabola:

$$(-4 + 4t)^2 = 16(1 + 3t).$$

Dividing by 16 and expanding gives

$$(t - 1)^2 = 1 + 3t, \qquad t^2 - 5t = 0.$$

One root is $t = 0$, representing P, and the second is $t = 5$. Hence the other endpoint is $Q = (16, 16)$. It indeed satisfies $16^2 = 16 \cdot 16$.

Step 3: Determine the reduced ratio The focus lies between P and Q because its parameter 1 lies between 0 and 5. Along this line,

$$PS : SQ = (1 - 0) : (5 - 1) = 1 : 4.$$

Equivalently, the distances are 5 and 20 because the direction vector has length 5. Thus the relatively prime integers are 1 and 4 (or their reverse if the ratio is read oppositely). Their squares have the same sum, $1 + 16 = 17$. The other listed sums do not match this reduced ratio.

Step 4: Conclusion We obtain $m^2 + n^2 = \boxed{17}$. Hence, the correct option is (A).

Quick Tip to Solve: Parameterise a focal chord from its given endpoint toward the focus; its intersection parameters directly give the division ratio.

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5. If the system of equations

$$\begin{aligned} 2x + \lambda y + 3z &= 5 \\ 3x + 2y - z &= 7 \\ 4x + 5y + \mu z &= 9 \end{aligned}$$

has infinitely many solutions, then $(\lambda^2 + \mu^2)$ is equal to : [4 marks]

(A) 30

(B) 26

(C) 22

(D) 18

Correct Answer: (B) 26

Solution:

Concept:

- An underdetermined consistent system in three unknowns has infinitely many solutions when its coefficient and augmented matrices have the same rank below three.
- If two augmented rows are independent, the third must be a linear combination of them. The constants must satisfy the same combination as every coefficient.

Step 1: Express the third augmented row using the first two The first two augmented rows are $(2, \lambda, 3 \mid 5)$ and $(3, 2, -1 \mid 7)$. They cannot be proportional because the ratios of their x coefficients and their constants are different. For infinite solutions the third row must therefore equal $uR_1 + vR_2$. Comparing its x coefficient and constant gives

$$2u + 3v = 4, \qquad 5u + 7v = 9.$$

Multiply the first relation by 5 and the second by 2 to eliminate u; the result is $v = 2$, followed by $u = -1$.

Step 2: Determine both parameters The required dependence is thus $R_3 = -R_1 + 2R_2$. Comparing

the y coefficients gives

$$5 = -\lambda + 2 \cdot 2, \qquad \lambda = -1.$$

Comparing the z coefficients gives

$$\mu = -3 + 2(-1) = -5.$$

The same combination already produces the correct x coefficient $-2+6 = 4$ and constant $-5+14 = 9$, so the system is consistent.

Step 3: Verify the rank condition and evaluate At these parameter values, the first two coefficient rows remain independent; for example, their x-z minor is $2(-1) - 3(3) = -11 \neq 0$. The third is dependent, so both ranks are exactly two. There are three unknowns, leaving one free parameter and hence infinitely many solutions. Finally,

$$\lambda^2 + \mu^2 = (-1)^2 + (-5)^2 = 26.$$

Thus 30, 22 and 18 do not satisfy the necessary consistent row dependence.

Step 4: Conclusion The required value is $\boxed{26}$. Hence, the correct option is (B).

Quick Tip to Solve: For infinite solutions, compare augmented rows: a zero coefficient determinant alone does not guarantee consistency.

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6. Let A be a 3×3 real matrix such that $A^2(A - 2I) - 4(A - I) = O$, where I and O are the identity and null matrices, respectively. If $A^5 = \alpha A^2 + \beta A + \gamma I$, where α, β, and γ are real constants, then $\alpha + \beta + \gamma$ is equal to : [4 marks]

(A) 76

(B) 12

(C) 4

(D) 20

Correct Answer: (B) 12

Solution:

Concept:

- The given relation is a polynomial identity in A: $A^3 - 2A^2 - 4A + 4I = O$. It allows every higher power to be reduced to a combination of A^2, A, I.
- Matrix multiplication distributes over sums, and powers of the same matrix commute. Keep the identity matrix in the constant term throughout the reduction.

Step 1: Isolate the cubic power Expand the given expression without changing the sign of its constant term:

$$A^2(A - 2I) - 4(A - I) = A^3 - 2A^2 - 4A + 4I = O.$$

Hence,

$$A^3 = 2A^2 + 4A - 4I.$$

This is the recurrence we will use. In particular, replacing the matrix by the scalar 1 in the original relation is not legitimate; we must first reduce the desired matrix power.

Step 2: Reduce the fourth power Multiply the cubic relation by A and then replace the resulting cubic term:

$$\begin{aligned} A^4 &= 2A^3 + 4A^2 - 4A \\ &= 2(2A^2 + 4A - 4I) + 4A^2 - 4A \\ &= 8A^2 + 4A - 8I. \end{aligned}$$

All multiplications here involve powers of A and I, so no ordering issue arises despite general matrix multiplication being noncommutative.

Step 3: Reduce the fifth power and compare coefficients Multiply once more by A and use the same cubic identity:

$$\begin{aligned} A^5 &= 8A^3 + 4A^2 - 8A \\ &= 8(2A^2 + 4A - 4I) + 4A^2 - 8A \\ &= 20A^2 + 24A - 32I. \end{aligned}$$

Thus the polynomial reduction gives $\alpha = 20$, $\beta = 24$ and $\gamma = -32$. Their sum is $20 + 24 - 32 = 12$. The value 20 is only the coefficient of A^2, while 76 incorrectly changes the sign of the constant coefficient; neither is the requested sum. The remaining value 4 also fails this reduction.

Step 4: Conclusion Therefore, $\alpha + \beta + \gamma = \boxed{12}$. Hence, the correct option is (B).

Quick Tip to Solve: Reduce successive powers using the given polynomial relation, keeping the sign of the identity-matrix term at every substitution.

7.

$$If \lim_{x \to 0} \frac{\cos(2x) + a\cos(4x) - b}{x^4} is\ finite,\ then\ (a+b)\ is\ equal\ to:$$

[4 marks]

(A) 0

(B) $\frac{3}{4}$

(C) -1

(D) $\frac{1}{2}$

Correct Answer: (D) $\frac{1}{2}$

Solution:

Concept:

- A finite quotient with denominator x^4 requires the numerator to vanish through orders below four. In particular, its constant and quadratic coefficients must both be zero.
- For a fixed real number t, the expansion near zero is $\cos(tx) = 1 - \frac{t^2x^2}{2} + \frac{t^4x^4}{24} + O(x^6)$.

Step 1: Expand the numerator Expand both cosine terms to the order required by the denominator:

$$\cos(2x) = 1 - 2x^2 + \frac{2}{3}x^4 + O(x^6),$$
$$\cos(4x) = 1 - 8x^2 + \frac{32}{3}x^4 + O(x^6).$$

Thus the numerator equals $(1+a-b) + (-2-8a)x^2 + (\frac{2}{3} + \frac{32a}{3})x^4 + O(x^6)$. There are no odd powers because cosine is even.

Step 2: Cancel the divergent terms The constant term must vanish; otherwise division by x^4 diverges. After that cancellation, the coefficient of x^2 must also vanish, since it would produce an unbounded $1/x^2$ term. Hence

$$1 + a - b = 0, \qquad -2 - 8a = 0.$$

The second equation gives $a = -\frac{1}{4}$, and the first then gives $b = \frac{3}{4}$. These conditions are sufficient: the remaining quotient tends to $\frac{2}{3} - \frac{8}{3} = -2$, a finite real number.

Step 3: Evaluate the requested sum Use the two coefficients, rather than confusing their sum with the value of the limit:

$$a + b = -\frac{1}{4} + \frac{3}{4} = \frac{1}{2}.$$

The values 0, $\frac{3}{4}$, and -1 do not equal this sum; $\frac{3}{4}$ is only the value of b.

Step 4: Conclusion The required value is $a + b = \frac{1}{2}$. Hence, the correct option is (D).

Quick Tip to Solve: For a finite cosine-expression limit over x^4, cancel the constant and x^2 coefficients before evaluating any remaining term.

8. If the image of the point $\mathrm{P}(1,0,3)$ in the line joining the points $\mathrm{A}(4,7,1)$ and $\mathrm{B}(3,5,3)$ is $Q(\alpha,\beta,\gamma)$, then $\alpha+\beta+\gamma$ is equal to : [4 marks]

(A) $\frac{46}{3}$

(B) 18

(C) 13

(D) $\frac{47}{3}$

Correct Answer: (A) $\frac{46}{3}$

Solution:

Concept:

- The reflection of a point P in a line has the foot H of its perpendicular as the midpoint of PQ. Therefore $Q = 2H - P$.
- A point on the line through A in direction $\mathbf{d} = B - A$ is $H = A + t\mathbf{d}$. Perpendicularity gives $(P - H) \cdot \mathbf{d} = 0$.

Step 1: Find the direction and projection parameter For the given points,

$$\mathbf{d} = B - A = (-1, -2, 2), \qquad P - A = (-3, -7, 2).$$

The direction has squared length $1+4+4=9$. Write the perpendicular foot as $H = A + t\mathbf{d}$. Taking the dot product of $P - A - t\mathbf{d}$ with $\mathbf{d}$ yields

$$t = \frac{(P-A)\cdot\mathbf{d}}{\mathbf{d}\cdot\mathbf{d}} = \frac{3+14+4}{9} = \frac{7}{3}.$$

The parameter need not lie between zero and one: the reflection is in the entire line, rather than only in the segment AB.

Step 2: Construct the reflected point Substitute the parameter into the line:

$$H = (4,7,1) + \frac{7}{3}(-1,-2,2) = \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right).$$

Because this is the midpoint of the original point and its image, the reflected point is

$$Q = 2H - P = \left(\frac{7}{3}, \frac{14}{3}, \frac{25}{3}\right).$$

As a check, $P - H = (-\frac{2}{3}, -\frac{7}{3}, -\frac{8}{3})$ has zero dot product with $(-1,-2,2)$, so the midpoint really lies on the required perpendicular.

Step 3: Add the coordinates The required sum is

$$\alpha + \beta + \gamma = \frac{7+14+25}{3} = \frac{46}{3}.$$

The other three listed numbers do not equal this coordinate sum. Using the foot itself as the image would miss the necessary doubling about the line.

Step 4: Conclusion The coordinate sum is $\frac{46}{3}$. Hence, the correct option is (A).

Quick Tip to Solve: Find the perpendicular foot first, then use image = twice the foot minus the original point; a projection alone is not a reflection.

9. If $\theta \epsilon \left[-\frac{7\pi}{6}, \frac{4\pi}{3}\right]$, then the number of solutions of $\sqrt{3}\,\mathrm{cosec}^2\theta - 2(\sqrt{3}-1)\,\mathrm{cosec}\,\theta - 4 = 0$, is equal to : [4 marks]

(A) 7

(B) 10

(C) 6

(D) 8

Correct Answer: (C) 6

Solution:

Concept:

- Treat a polynomial in $\mathrm{cosec}\,\theta$ as a quadratic, while retaining $\sin\theta \neq 0$. Convert its real roots to sine equations before counting angles.

- The interval is closed: endpoint angles count if they satisfy the equation. General solutions must be filtered to the given interval, rather than counted over a single period.

Step 1: Factor the equation Put $u = \operatorname{cosec}\theta$. Then the equation factors as

$$\sqrt{3}u^2 - 2(\sqrt{3}-1)u - 4 = (u-2)(\sqrt{3}u+2) = 0.$$

Thus $u = 2$ or $u = -2/\sqrt{3}$, giving respectively

$$\sin\theta = \frac{1}{2} \quad \text{or} \quad \sin\theta = -\frac{\sqrt{3}}{2}.$$

Both values are within the range of sine, and neither is zero, so neither branch introduces a point at which cosecant is undefined.

Step 2: Count the first sine branch For $\sin\theta = 1/2$, the two families are

$$\theta = \frac{\pi}{6} + 2n\pi \quad \text{or} \quad \theta = \frac{5\pi}{6} + 2n\pi, \qquad n \in \mathbb{Z}.$$

Within $[-7\pi/6, 4\pi/3]$, the surviving angles are $-7\pi/6, \pi/6, 5\pi/6$. In particular, the left endpoint belongs to the second family with $n = -1$, and must be included. This branch contributes three solutions.

Step 3: Count the second sine branch For $\sin\theta = -\sqrt{3}/2$, use the families $\theta = -\pi/3 + 2n\pi$ and $\theta = -2\pi/3 + 2n\pi$. Filtering to the same closed interval leaves

$$-\frac{2\pi}{3}, \quad -\frac{\pi}{3}, \quad \frac{4\pi}{3}.$$

The last angle is the right endpoint and is included. These three are distinct from the first branch because their sine value is different. The total is $3 + 3 = 6$, ruling out the listed totals 7, 10 and 8.

Step 4: Conclusion There are 6 solutions in the specified interval. Hence, the correct option is (C).

Quick Tip to Solve: List both sine families and check the two endpoints explicitly; an interval longer than one period cannot be counted by a one-cycle shortcut.

10. Let $A = \{1, 2, 3, \ldots, 100\}$ and R be a relation on A such that $R = \{(a, b) : a = 2b + 1\}$. Let $(a_1, a_2), (a_2, a_3), (a_3, a_4), \ldots, (a_k, a_{k+1})$ be a sequence of k elements of R such that the second entry of an ordered pair is equal to the first entry of the next ordered pair. Then the largest integer k , for which such a sequence exists, is equal to : [4 marks]

(A) 6

(B) 8

(C) 7

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- Each relation pair satisfies $a_i = 2a_{i+1} + 1$. Thus a chain of k ordered pairs contains $k + 1$ entries.
- Iterating an affine recurrence gives an exponential bound. Since all entries lie in $\{1, \ldots, 100\}$, the terminal entry is at least one and the initial entry is at most one hundred.

Step 1: Express the first entry using the last For consecutive pairs, the relation requires

$$a_1 = 2a_2 + 1, \quad a_2 = 2a_3 + 1, \quad \ldots, \quad a_k = 2a_{k+1} + 1.$$

Substituting successively gives $a_1 = 4a_3 + 3$, then $a_1 = 8a_4 + 7$, and in general

$$a_1 = 2^k a_{k+1} + (2^k - 1).$$

The added constants form the geometric sum $1 + 2 + \cdots + 2^{k-1}$. Equivalently, adding one to every entry makes each preceding entry exactly twice the next.

Step 2: Obtain the maximum possible length The smallest admissible terminal value is $a_{k+1} = 1$. Therefore every possible chain obeys

$$a_1 \geq 2^k + (2^k - 1) = 2^{k+1} - 1.$$

Since $a_1 \leq 100$, we need $2^{k+1} \leq 101$. Now $2^6 = 64 \leq 101$, whereas $2^7 = 128 > 101$. Hence $k \leq 5$. Choosing a larger terminal entry can only increase a_1, so it cannot allow a longer chain.

Step 3: Exhibit a chain attaining the bound Take the entries

$$63,\ 31,\ 15,\ 7,\ 3,\ 1.$$

They all belong to A, and each entry is twice the next plus one. The ordered pairs $(63, 31)$, $(31, 15)$, $(15, 7)$, $(7, 3)$, and $(3, 1)$ therefore form a valid sequence of five elements of R. A sixth pair would require a first entry of at least 127, outside A. Thus the options 6, 8 and 7 are impossible.

Step 4: Conclusion The largest possible integer is $k = 5$. Hence, the correct option is (D).

Quick Tip to Solve: Count ordered pairs, not entries: the extremal chain has six entries but only five links. Start backward from the smallest allowed terminal entry.

11. If the domain of the function $f(x) = \frac{1}{\sqrt{10+3x-x^2}} + \frac{1}{\sqrt{x+|x|}}$ is (a, b), then $(1+a)^2 + b^2$ is equal to :

[4 marks]

(A) 29

(B) 30

(C) 25

(D) 26

Correct Answer: (D) 26

Solution:

Concept:

- A square root in a denominator must have a strictly positive radicand: zero makes the denominator vanish, while a negative radicand is not real.
- The domain of a sum is the intersection of the domains of its terms. An absolute value is evaluated separately for nonnegative and negative inputs.

Step 1: Solve the first radicand inequality For the first denominator, require

$$10 + 3x - x^2 > 0.$$

Factor the expression as $(5-x)(x+2)$. Its roots are -2 and 5; because the coefficient of x^2 is negative, the expression is positive between these roots. Thus the first term is defined precisely when

$$-2 < x < 5.$$

The endpoints cannot be admitted: each gives a zero denominator, even though its square root exists.

Step 2: Solve the absolute-value restriction The second denominator requires $x + |x| > 0$. If $x < 0$, then $|x| = -x$, so the radicand is zero and the term is undefined. If $x \geq 0$, then $|x| = x$, so the radicand is $2x$, which is strictly positive exactly when $x > 0$. Combining the restrictions for both terms gives

$$(-2, 5) \cap (0, \infty) = (0, 5).$$

This intersection excludes all negative numbers and zero, while keeping every positive number below five.

Step 3: Identify the interval endpoints and evaluate Comparison with the specified notation (a, b) gives $a = 0$ and $b = 5$. Hence

$$(1+a)^2 + b^2 = (1+0)^2 + 5^2 = 1 + 25 = 26.$$

The value 25 would omit the first squared term. The options 29 and 30 do not follow from the actual interval endpoints; using the first radicand alone would fail the second denominator condition.

Step 4: Conclusion The requested value is 26. Hence, the correct option is (D).

Quick Tip to Solve: For reciprocal square roots use strict inequalities, then intersect the restrictions; $x + |x|$ vanishes for every nonpositive input.

12. Let the area of the triangle formed by a straight line $\mathrm{L}: x+\mathrm{b}y+\mathrm{c}=0$ with co-ordinate axes be 48 square units. If the perpendicular drawn from the origin to the line L makes an angle of 45° with the positive x-axis, then the value of $\mathrm{b}^2+\mathrm{c}^2$ is : [4 marks]

(A) 90

(B) 83

(C) 93

(D) 97

Correct Answer: (D) 97

Solution:

Concept:

- The coefficients $(1, b)$ of the line $x+by+c=0$ form a normal vector. The perpendicular from the origin to the line is parallel to this vector.
- The area of the triangle cut off by nonzero axis intercepts p, q is $\frac{1}{2}|pq|$. The absolute value is essential because the intercepts can have either sign.

Step 1: Use the direction of the perpendicular The perpendicular to the line makes angle 45° with the positive x-axis, so its slope is $\tan 45^\circ = 1$. The normal vector $(1, b)$ has slope b; consequently $b = 1$. This also follows from the line's slope $-1/b$, whose product with the perpendicular slope must be -1. The direction condition therefore fixes $b^2 = 1$, independent of how the line's constant is signed.

Step 2: Apply the axis-intercept area formula On the x-axis, setting $y = 0$ gives intercept $(-c, 0)$. On the y-axis, setting $x = 0$ gives $(0, -c/b)$. Their triangle with the origin has area

$$\frac{1}{2}\left|(-c)\left(-\frac{c}{b}\right)\right| = \frac{c^2}{2|b|}.$$

The given area is 48 square units, so with $|b| = 1$,

$$\frac{c^2}{2} = 48 \quad \Longrightarrow \quad c^2 = 96.$$

The line cannot pass through the origin, since that would make this area zero. The two possible signs of c do not affect its square.

Step 3: Calculate the required sum Add the two squared coefficients:

$$b^2 + c^2 = 1 + 96 = 97.$$

The values 90, 83 and 93 are inconsistent with the product of intercept lengths required by area 48. In particular, the slope condition fixes the magnitude of the two intercepts to be equal, so the area must be half their common squared magnitude.

Step 4: Conclusion The required value is $b^2 + c^2 = 97$. Hence, the correct option is (D).

Quick Tip to Solve: The normal vector reads directly from the coefficients of a line. Use its direction before applying the absolute-value intercept area formula.

13. The line L_1 is parallel to the vector $\vec{a} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ and passes through the point $(7, 6, 2)$ and the line L_2 is parallel to the vector $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through the point $(5, 3, 4)$. The shortest distance between the lines L_1 and L_2 is : [4 marks]

(A) $\frac{23}{\sqrt{38}}$
(B) $\frac{21}{\sqrt{38}}$
(C) $\frac{23}{\sqrt{57}}$
(D) $\frac{21}{\sqrt{57}}$

Correct Answer: (A) $\frac{23}{\sqrt{38}}$

Solution:

Concept:

- For lines through points P, Q, with nonparallel directions $\mathbf{a}, \mathbf{b}$, the shortest distance is $d = |\overrightarrow{PQ} \cdot (\mathbf{a} \times \mathbf{b})|/|\mathbf{a} \times \mathbf{b}|$.
- The cross product is perpendicular to both lines. Projecting a vector joining the two lines onto its unit direction gives the common perpendicular separation.

Step 1: Find a common normal Use the given direction vectors, without normalising either one. Their cross product is

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 4 \\ 2 & 1 & 3 \end{vmatrix} = 2\hat{i} + 17\hat{j} - 7\hat{k}.$$

It is nonzero, so the parallel-lines formula does not apply. As a check, its dot product with each direction vector is zero: $-6 + 34 - 28 = 0$ and $4 + 17 - 21 = 0$.

Step 2: Project the displacement onto the normal Take $P = (7, 6, 2)$ and $Q = (5, 3, 4)$. Then

$$\overrightarrow{PQ} = (-2, -3, 2), \qquad |\mathbf{a} \times \mathbf{b}| = \sqrt{4 + 289 + 49} = 3\sqrt{38}.$$

The scalar triple product is $(-2)(2) + (-3)(17) + 2(-7) = -69$. Its sign depends on the direction chosen for the joining vector; distance uses its absolute value. Hence

$$d = \frac{|-69|}{3\sqrt{38}} = \frac{23}{\sqrt{38}}.$$

Using the unprojected distance between the stated points would not give the shortest distance. The other options have either the wrong numerator or the wrong normal-vector magnitude.

Step 3: Conclusion The shortest distance is $\frac{23}{\sqrt{38}}$. Hence, the correct option is (A).

Quick Tip to Solve: Compute the cross product first and check it is perpendicular to both direction vectors; take the absolute value only after the scalar triple product.

14. The number of terms of an A.P. is even; the sum of all the odd terms is 24 , the sum of all the even terms is 30 and the last term exceeds the first by $\frac{21}{2}$. Then the number of terms which are integers in the A.P. is : [4 marks]

(A) 6

(B) 4

(C) 8

(D) 10

Correct Answer: (B) 4

Solution:

Concept:

- An A.P. with first term a, difference d, and $2m$ terms has exactly m odd-position terms and m even-position terms.
- Each even-position term exceeds the preceding odd-position term by d. Thus the difference of those two sums is md, while last minus first is $(2m-1)d$.

Step 1: Determine the number of terms and common difference The words odd and even refer to positions in the progression. Pair term 2 with term 1, term 4 with term 3, and so on. The sum of all pairwise differences is

$$md = 30 - 24 = 6.$$

The last term exceeds the first by the given amount, so

$$(2m-1)d = \frac{21}{2}.$$

Substitute $d = 6/m$ to obtain $12 - 6/m = 21/2$. Hence $6/m = 3/2$, so $m = 4$, $2m = 8$, and $d = 3/2$. Eight is the total number of terms, not the requested number of integer terms.

Step 2: Find the progression and count its integer members The total sum is $24 + 30 = 54$. Apply the sum formula to all eight terms:

$$54 = \frac{8}{2}\left(2a + 7 \cdot \frac{3}{2}\right), \qquad 2a + \frac{21}{2} = \frac{27}{2}.$$

Thus $a = 3/2$, and the progression is

$$\frac{3}{2},\ 3,\ \frac{9}{2},\ 6,\ \frac{15}{2},\ 9,\ \frac{21}{2},\ 12.$$

The integers are $3, 6, 9, 12$. The odd-position sum is $(3 + 9 + 15 + 21)/2 = 24$, and the even-position sum is $3 + 6 + 9 + 12 = 30$, confirming both supplied conditions. Counts 6 and 10 do not agree with this explicit list.

Step 3: Conclusion Exactly 4 terms are integers. Hence, the correct option is (B).

Quick Tip to Solve: For an even-length A.P., subtract the sums of the even and odd positions first: each adjacent pair contributes exactly one common difference.

15. Let (a, b) be the point of intersection of the curve $x^2 = 2y$ and the straight line $y - 2x - 6 = 0$ in the second quadrant. Then the integral $\mathrm{I} = \int_{\mathrm{a}}^{\mathrm{b}} \frac{9x^2}{1+5^x}\,\mathrm{d}x$ is equal to : [4 marks]

(A) 27

(B) 18

(C) 24

(D) 21

Correct Answer: (C) 24

Solution:

Concept:

- Intersection coordinates satisfy both equations. The second quadrant requires $x < 0$ and $y > 0$, which selects the appropriate intersection.
- For symmetric limits, substitution $x \mapsto -x$ pairs the integrands. The identity $1/(1 + 5^x) + 1/(1 + 5^{-x}) = 1$ removes the exponential denominator.

Step 1: Find the integration limits From the line, $y = 2x + 6$. Substituting in the parabola gives

$$x^2 = 4x + 12, \qquad (x - 6)(x + 2) = 0.$$

The two possible abscissae are 6 and -2. The second-quadrant point has $a = -2$, and its ordinate is $b = 2(-2) + 6 = 2$. Notice that the upper integration limit is the ordinate of that selected point, not the abscissa of the other intersection. Therefore

$$I = \int_{-2}^{2} \frac{9x^2}{1 + 5^x}\,dx.$$

Step 2: Pair the reflected integrands Changing variable to $-x$ leaves the limits symmetric and gives

$$I = \int_{-2}^{2} \frac{9x^2}{1 + 5^{-x}}\,dx.$$

Add this equation to the original integral. Since $5^{-x} = 1/5^x$, the two reciprocal denominators sum to one:

$$2I = \int_{-2}^{2} 9x^2\,dx = 9\left[\frac{x^3}{3}\right]_{-2}^{2} = 48.$$

Consequently $I = 24$. The factor of two on the left is essential: symmetry evaluates the sum of two equal integrals, rather than the original integral directly. Values 27, 18 and 21 do not satisfy this identity. No approximation to the exponential or numerical integration is needed.

Step 3: Conclusion $I = 24$. Hence, the correct option is (C).

Quick Tip to Solve: When an even numerator is divided by $1 + a^x$ over symmetric limits, add the integral after replacing x by $-x$.

16. If the length of the minor axis of an ellipse is equal to one fourth of the distance between the foci, then the eccentricity of the ellipse is : [4 marks]

(A) $\frac{3}{\sqrt{19}}$

(B) $\frac{\sqrt{3}}{16}$

(C) $\frac{4}{\sqrt{17}}$

(D) $\frac{\sqrt{5}}{7}$

Correct Answer: (C) $\frac{4}{\sqrt{17}}$

Solution:

Concept:

- For an ellipse with semiaxes $a > b > 0$, the focal distance from the centre is c, where $c^2 = a^2 - b^2$, and eccentricity is $e = c/a$.
- The full minor-axis length is $2b$, and the distance between the two foci is $2c$. Both factors of two must be retained when translating the wording.

Step 1: Translate the given ratio into semiaxes The condition concerns the entire minor axis and the entire distance between the foci. Thus

$$2b = \frac{1}{4}(2c), \qquad b = \frac{c}{4}, \qquad c = 4b.$$

It would be incorrect to equate b directly to a quarter of $2c$, because that would compare a semiaxis with the stated full axis. The ratio is independent of whether the major axis is horizontal or vertical; only the lengths enter the eccentricity.

Step 2: Use the focal relation and identify the option Insert $c = 4b$ into $a^2 = b^2 + c^2$:

$$a^2 = b^2 + 16b^2 = 17b^2, \qquad a = \sqrt{17}\, b.$$

All lengths are positive, so the positive root is appropriate. Hence

$$e = \frac{c}{a} = \frac{4b}{\sqrt{17}\, b} = \frac{4}{\sqrt{17}}.$$

As a check, $e^2 = 16/17$, so $1 - e^2 = 1/17 = b^2/a^2$. The result is between zero and one, as an ellipse requires, and its focal-to-minor-semiaxis ratio is exactly four. The other three options fail the stronger relation $e^2/(1 - e^2) = c^2/b^2 = 16$, even though they may individually lie in the allowed eccentricity range.

Step 3: Conclusion The eccentricity is $\frac{4}{\sqrt{17}}$. Hence, the correct option is (C).

Quick Tip to Solve: Write full axis lengths as $2a, 2b$ and interfocal distance as $2c$ before cancelling factors; then use $a^2 = b^2 + c^2$.

17.

$$If \sum_{r=0}^{10} (\frac{10^{r+1}-1}{10^r}).{}^{11}C_{r+1} = \frac{\alpha 11 - 11^{11}}{10^{10}}, \text{ then } \alpha \text{ is equal to :}$$

[4 marks]

(A) 11

(B) 20

(C) 24

(D) 15

Correct Answer: Printed power placement is ambiguous; the intended α^{11} interpretation gives option (B), 20.

Solution:

Concept:

- The binomial theorem gives $\sum_{k=0}^{11} \binom{11}{k} t^k = (1+t)^{11}$. Subtract the $k=0$ term when the sum starts at one.
- Reindex with $k = r+1$, so that $(10^{r+1}-1)/10^r = 10(1-10^{-k})$. This separates the expression into two standard binomial sums.

Step 1: Reindex and separate the two sums Let the left-hand side be S. As r runs from 0 to 10, the new index $k = r+1$ runs from 1 to 11. Therefore

$$S = 10\sum_{k=1}^{11}\binom{11}{k} - 10\sum_{k=1}^{11}\binom{11}{k}\left(\frac{1}{10}\right)^k.$$

Step 2: Apply the binomial theorem and compare powers At $t = 1$, the first sum is $2^{11} - 1$. At $t = 1/10$, the second sum is $(11/10)^{11} - 1$. Thus

$$\begin{aligned} S &= 10(2^{11}-1) - 10\left[\left(\frac{11}{10}\right)^{11} - 1\right] \\ &= 10 \cdot 2^{11} - \frac{11^{11}}{10^{10}} \\ &= \frac{20^{11} - 11^{11}}{10^{10}}. \end{aligned}$$

In the last equality, $10 \cdot 2^{11} \cdot 10^{10} = 2^{11}10^{11} = 20^{11}$. The unclear printed numerator must equal 20^{11}. If it denotes α^{11}, then the real value is $\alpha = 20$, corresponding to option (B). If the juxtaposition denotes 11α, instead it gives $\alpha = 20^{11}/11$, which is none of the options. Thus the intended power interpretation is consistent with (B), but the literal typography does not determine a unique listed answer.

Step 3: Conclusion The exact sum is $(20^{11} - 11^{11})/10^{10}$. The printed typography is ambiguous; interpreting its numerator as α^{11} gives $\alpha = 20$, option (B).

Quick Tip to Solve: Shift $r+1$ to one index before using the binomial theorem, and subtract the missing constant terms in both sums; here they cancel.

18. $4\int_0^1 \left(\frac{1}{\sqrt{3+x^2}+\sqrt{1+x^2}}\right) dx - 3\log_e(\sqrt{3})$ **is equal to :** [4 marks]

(A) $2-\sqrt{2}-\log_e(1+\sqrt{2})$

(B) $2+\sqrt{2}+\log_e(1+\sqrt{2})$

(C) $2+\sqrt{2}-\log_e(1+\sqrt{2})$

(D) $2-\sqrt{2}+\log_e(1+\sqrt{2})$

Correct Answer: (A) $2-\sqrt{2}-\log_e(1+\sqrt{2})$

Solution:

Concept:

- Rationalise a reciprocal sum of square roots using the difference of their squares. Here $(3+x^2)-(1+x^2)=2$, a constant.
- Use $\int \sqrt{x^2+a^2}\,dx = \frac{1}{2}[x\sqrt{x^2+a^2}+a^2\ln(x+\sqrt{x^2+a^2})]$. The logarithm arguments are positive on the interval $[0,1]$.

Step 1: Rationalise before integrating Denote the requested expression by E. Multiplying numerator and denominator by the difference of the roots gives

$$\frac{1}{\sqrt{3+x^2}+\sqrt{1+x^2}} = \frac{\sqrt{3+x^2}-\sqrt{1+x^2}}{2}.$$

Hence, if $J_a = \int_0^1 \sqrt{x^2+a^2}\,dx$, then

$$E = 2(J_{\sqrt{3}} - J_1) - 3\ln\sqrt{3}.$$

The factor four in the question becomes two after rationalisation. Keeping this factor prevents an erroneous doubling of the final logarithms.

Step 2: Evaluate the two definite integrals Applying the square-root antiderivative at both limits yields

$$2J_{\sqrt{3}} = 2+3\ln 3-3\ln\sqrt{3}, \qquad 2J_1 = \sqrt{2}+\ln(1+\sqrt{2}).$$

For the first integral, the upper root is $\sqrt{1+3}=2$, so its upper logarithm is $\ln(1+2)=\ln 3$; the lower logarithm is $\ln\sqrt{3}$, not zero. Substitute these results:

$$\begin{aligned} E &= 2-\sqrt{2}-\ln(1+\sqrt{2}) \\ &\quad +3\ln 3-6\ln\sqrt{3} \\ &= 2-\sqrt{2}-\ln(1+\sqrt{2}). \end{aligned}$$

The last cancellation follows from $2\ln\sqrt{3}=\ln 3$. The signs of both subtracted terms are negative; the three other options reverse one or both of those signs and are inconsistent with the definite evaluation.

Step 3: Conclusion The value is $2-\sqrt{2}-\ln(1+\sqrt{2})$. Hence, the correct option is (A).

Quick Tip to Solve: When two square roots differ by a constant after squaring, rationalise first; keep the lower-limit logarithm until all logarithmic terms are combined.

19. Let $\overrightarrow{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\overrightarrow{b} = 3\hat{i} + 2\hat{j} + 5\hat{k}$ and a vector $\overrightarrow{c}$ be such that $(\vec{a} - \vec{c}) \times \vec{b} = -18\hat{i} - 3\hat{j} + 12\hat{k}$ and $\vec{a} \cdot \vec{c} = 3$. If $\vec{b} \times \vec{c} = \vec{d}$, then $|\vec{a} \cdot \vec{d}|$ is equal to : [4 marks]

(A) 15

(B) 18

(C) 12

(D) 9

Correct Answer: (A) 15

Solution:

Concept:

- The cross product is distributive and antisymmetric: $(\vec{a} - \vec{c}) \times \vec{b} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c}$.
- A vector is perpendicular to its cross product with another vector, so $\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$. The required quantity is the absolute value of a scalar triple product.

Step 1: Express the given cross product using the required vector Let $\vec{r} = -18\hat{i} - 3\hat{j} + 12\hat{k}$. Since $\vec{d} = \vec{b} \times \vec{c}$, distributivity gives

$$\vec{r} = \vec{a} \times \vec{b} - \vec{c} \times \vec{b} = \vec{a} \times \vec{b} + \vec{d}.$$

The plus sign before $\vec{d}$ is essential: reversing the order in a cross product changes its sign. There is no need to determine all three components of $\vec{c}$.

Step 2: Take the scalar product with the given vector Taking the scalar product with $\vec{a}$, the term containing $\vec{a} \times \vec{b}$ vanishes by perpendicularity. Therefore

$$\vec{a} \cdot \vec{d} = \vec{a} \cdot \vec{r} = 2(-18) + (-3)(-3) + 1(12) = -15.$$

The additional condition $\vec{a} \cdot \vec{c} = 3$ can determine the otherwise free component of $\vec{c}$ parallel to $\vec{b}$; it does not affect the triple product. As a consistency check, $\vec{r} \cdot \vec{b} = -54 - 6 + 60 = 0$, as a cross product with $\vec{b}$ requires. Taking the modulus changes the negative scalar to a positive number. Options 18, 12 and 9 do not equal the computed modulus.

Step 3: Conclusion $|\vec{a} \cdot \vec{d}| = |-15| = 15$. Hence, the correct option is (A).

Quick Tip to Solve: Dot the given cross-product identity with the repeated vector first; its own cross-product term vanishes.

20.

Given three indentical bags each containing 10 balls, whose colours are as follows :

	Red	Blue	Green
Bag I	3	2	5
Bag II	4	3	3
Bag III	5	1	4

A person chooses a bag at random and takes out a ball. If the ball is Red, the probability that it is from bag I is p and if the ball is Green, the probability that it is from bag III is q, then the value of $\left(\frac{1}{p}+\frac{1}{q}\right)$ is: [4 marks]

(A) 6

(B) 9

(C) 7

(D) 8

Correct Answer: (C) 7

Solution:

Concept:

- For equally likely bags, $P(B_i) = 1/3$. Bayes theorem gives $P(B_i \mid C) = P(B_i)P(C \mid B_i)/P(C)$, where C denotes an observed colour.
- Every bag contains ten balls, so a colour count divided by ten is its conditional probability. Equal prior probabilities and equal bag sizes cancel when computing a posterior ratio.

Step 1: Condition on the red ball Write R for the event that the selected ball is red. The probabilities of red given bags I, II and III are $3/10, 4/10, 5/10$, respectively. By total probability,

$$P(R) = \frac{1}{3}\left(\frac{3}{10}+\frac{4}{10}+\frac{5}{10}\right) = \frac{2}{5}.$$

Consequently,

$$p = P(B_1 \mid R) = \frac{(1/3)(3/10)}{2/5} = \frac{1}{4}.$$

The denominator includes red outcomes from every bag, rather than all colours from bag I.

Step 2: Condition separately on the green ball Let G denote green. Its counts in the three bags are 5, 3 and 4. Thus

$$P(G) = \frac{1}{3}\left(\frac{5}{10}+\frac{3}{10}+\frac{4}{10}\right) = \frac{2}{5}, \qquad q = P(B_3 \mid G) = \frac{(1/3)(4/10)}{2/5} = \frac{1}{3}.$$

Although the total red and green counts happen to coincide, their favourable bag counts differ. The requested expression is the sum of the reciprocals, not the reciprocal of their sum:

$$\frac{1}{p}+\frac{1}{q} = 4+3 = 7.$$

Options 6, 9 and 8 do not match these two posterior probabilities.

Step 3: Conclusion The requested value is 7. Hence, the correct option is (C).

Quick Tip to Solve: With equal bag priors and equal bag sizes, divide the chosen bag colour count by that colour total across all bags.

Mathematics – Section B: Numerical Value Questions (20 marks)

21.

If $y = \cos\left(\frac{\pi}{3} + \cos^{-1}\frac{x}{2}\right)$, then $(x-y)^2 + 3y^2$ is equal to

[4 marks]

Solution:

Concept:

- Put $\theta = \cos^{-1}(x/2)$. The principal inverse cosine has $0 \le \theta \le \pi$, so $-2 \le x \le 2$ and $\sin\theta = \sqrt{1 - x^2/4} \ge 0$.
- The cosine addition formula is $\cos(A+B) = \cos A\cos B - \sin A\sin B$. Squaring an identity satisfied by the given variables is valid for simplifying the requested expression.

Step 1: Expand the cosine while retaining the principal square root Use $\cos(\pi/3) = 1/2$ and $\sin(\pi/3) = \sqrt{3}/2$. Substitution into the addition formula gives

$$y = \frac{x}{4} - \frac{\sqrt{3}}{4}\sqrt{4 - x^2}.$$

The square root is real throughout the domain of the original inverse cosine. Its nonnegative sign follows from the principal range of θ, so no arbitrary sign is inserted. Rearranging gives

$$x - 4y = \sqrt{3}\sqrt{4 - x^2}.$$

Step 2: Eliminate the radical and recognise the requested form Square the last equality and expand both sides:

$$(x - 4y)^2 = 3(4 - x^2),$$

$$x^2 - 8xy + 16y^2 = 12 - 3x^2.$$

Collect the terms and divide by four to obtain

$$x^2 - 2xy + 4y^2 = 3.$$

But the expression asked for is

$$(x-y)^2 + 3y^2 = x^2 - 2xy + y^2 + 3y^2 = x^2 - 2xy + 4y^2.$$

Thus it is constant, independent of the permitted value of x. Squaring here is used only as an implication from the original definition; no extraneous values are introduced into the answer. For

instance, at $x = 2$, $y = 1/2$, and the expression is $9/4 + 3/4 = 3$, confirming the calculation.

Step 3: Conclusion For all $x \in [-2, 2]$, $(x - y)^2 + 3y^2 = 3$. The required numerical value is 3.

Quick Tip to Solve: Expand the cosine, isolate the radical, and square once; the resulting quadratic identity is exactly the required expression.

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22. If the set of all $\mathrm{a} \in \mathbf{R} - \{1\}$, for which the roots of the equation $(1 - \mathrm{a})x^2 + 2(\mathrm{a} - 3)x + 9 = 0$ are positive is $(-\infty, -\alpha] \cup [\beta, \gamma)$, then $2\alpha + \beta + \gamma$ is equal to . [4 marks]

Solution:

Concept:

- A real quadratic $Ax^2 + Bx + C = 0$, with $A \neq 0$, has both roots positive when its discriminant is nonnegative, its root sum $-B/A$ is positive, and its root product C/A is positive.
- A zero discriminant is allowed because two equal positive roots satisfy the condition. Vieta relations give the sum and product without explicitly solving the quadratic.

Step 1: Apply the product and sum conditions Here $A = 1 - a$, $B = 2(a - 3)$ and $C = 9$, with $a \neq 1$. The product condition is

$$\frac{9}{1 - a} > 0 \quad \Longrightarrow \quad a < 1.$$

For this range, the sum of the roots is

$$\frac{2(3 - a)}{1 - a} > 0,$$

since both numerator and denominator are positive. Thus the sum places no further restriction after the product condition. In particular, $a > 1$ cannot give both positive roots because their product would be negative.

Step 2: Require real roots and identify the interval endpoints The discriminant simplifies to

$$\Delta = 4(a - 3)^2 - 36(1 - a) = 4(a^2 + 3a) = 4a(a + 3).$$

Its nonnegative range is $a \leq -3$ or $a \geq 0$. Intersecting with $a < 1$ gives

$$(-\infty, -3] \cup [0, 1).$$

Both finite included endpoints are valid: at $a = -3$, the equation is $(2x - 3)^2 = 0$, and at $a = 0$, it is $(x - 3)^2 = 0$. Their repeated roots are positive. The endpoint 1 is excluded as required because the leading coefficient vanishes there. Comparing with $(-\infty, -\alpha] \cup [\beta, \gamma)$, we get $\alpha = 3$, $\beta = 0$, $\gamma = 1$.

Step 3: Conclusion Therefore $2\alpha + \beta + \gamma = 2(3) + 0 + 1 = 7$. The required value is 7.

Quick Tip to Solve: For two positive real roots, combine product, sum and discriminant conditions; include repeated positive roots at discriminant zero.

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23. If the sum of the first 10 terms of the series $\frac{4\cdot1}{1+4\cdot1^4}+\frac{4\cdot2}{1+4\cdot2^4}+\frac{4\cdot3}{1+4\cdot3^4}+\ldots.$ is $\frac{m}{n}$, where $\gcd(m,n)=1$, then $m+n$ is equal to _____ [4 marks]

Solution:

Concept:

- A telescoping sum is simplified by writing each term as the difference of two consecutive terms of an auxiliary sequence; all interior contributions then cancel.
- The denominator factors as $4k^4+1=(2k^2-2k+1)(2k^2+2k+1)$. The difference of these two quadratic factors is $4k$, matching the numerator.

Step 1: Rewrite the general term as a difference For a positive integer k, use the factorisation

$$4k^4+1=(2k^2+1)^2-(2k)^2.$$

Hence

$$\frac{4k}{1+4k^4}=\frac{1}{2k^2-2k+1}-\frac{1}{2k^2+2k+1}.$$

This identity follows directly by taking the common denominator: the numerator is $(2k^2+2k+1)-(2k^2-2k+1)=4k$. Define $f(k)=1/(2k^2-2k+1)$. Then the second fraction is $f(k+1)$, since $2(k+1)^2-2(k+1)+1=2k^2+2k+1$.

Step 2: Cancel the intermediate fractions and reduce the result The first ten terms therefore sum to

$$S_{10}=\sum_{k=1}^{10}[f(k)-f(k+1)]=f(1)-f(11).$$

In expanded form the first few differences are $(1-1/5)+(1/5-1/13)+(1/13-1/25)+\cdots$. Every negative interior fraction cancels the next positive one. The two surviving boundary terms give

$$S_{10}=1-\frac{1}{2(11)^2-2(11)+1}=1-\frac{1}{221}=\frac{220}{221}.$$

The integers 220 and 221 are consecutive, so their greatest common divisor is 1. Thus the fraction already has the required coprime numerator and denominator; neither cancellation nor a further common factor changes their values.

Step 3: Conclusion We have $m=220$, $n=221$, so $m+n=441$. The required numerical value is 441.

Quick Tip to Solve: For a quartic denominator $4k^4+1$, factor it into quadratics and look for neighbouring reciprocal denominators.

24. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + 2y\sec^2 x = 2\sec^2 x + 3\tan x \cdot \sec^2 x$ such that $y(0) = \frac{5}{4}$. Then $12\left(y\left(\frac{\pi}{4}\right) - e^{-2}\right)$ is equal to_____ [4 marks]

Solution:

Concept:

- For $y' + P(x)y = Q(x)$, an integrating factor is $\exp(\int P(x)\,dx)$. Equivalently, a substitution can first remove a common derivative factor.
- Here $t = \tan x$ gives $dt/dx = \sec^2 x$. On the interval containing 0 and $\pi/4$, this substitution is valid and transforms the equation to one with constant coefficient.

Step 1: Transform the equation using the tangent substitution Write $Y(t) = y(x)$ with $t = \tan x$. The chain rule gives $dy/dx = (dY/dt)\sec^2 x$. Divide the differential equation by $\sec^2 x$ to obtain

$$\frac{dY}{dt} + 2Y = 2 + 3t.$$

The initial point becomes $t = 0$, with $Y(0) = 5/4$. A linear particular solution $Y_p = At + B$ satisfies $A + 2At + 2B = 2 + 3t$. Comparing coefficients gives $A = 3/2$ and $B = 1/4$.

Step 2: Apply the initial condition and evaluate the requested combination The homogeneous equation has solution Ce^{-2t}, so the full solution is

$$Y(t) = \frac{3}{2}t + \frac{1}{4} + Ce^{-2t}.$$

At $t = 0$, the initial condition yields $5/4 = 1/4 + C$, hence $C = 1$. Therefore

$$y(x) = \frac{3}{2}\tan x + \frac{1}{4} + e^{-2\tan x}.$$

As a check, differentiating in t gives $Y' = 3/2 - 2e^{-2t}$, so $Y' + 2Y = 2 + 3t$ exactly. At $x = \pi/4$, $t = 1$, and

$$y(\pi/4) = \frac{7}{4} + e^{-2}.$$

The exponential contribution is precisely the term subtracted in the problem, leaving $7/4$. Multiplication by 12 gives an integer without any rounding.

Step 3: Conclusion

$$12\left(y(\pi/4) - e^{-2}\right) = 12 \cdot \frac{7}{4} = 21.$$

The required numerical value is 21.

Quick Tip to Solve: When every term contains $\sec^2 x$ except the derivative, try $t = \tan x$ before computing an integrating factor.

25. Let $\mathrm{A}(4,-2), \mathrm{B}(1,1)$ and $\mathrm{C}(9,-3)$ be the vertices of a triangle ABC . Then the maximum area of the parallelogram AFDE, formed with vertices D, E and F on the sides BC, CA and AB of the triangle ABC respectively, is_____ [4 marks]

Solution:

Concept:

- Use vectors along the two sides meeting at A. The area of a parallelogram generated by vectors $\mathbf{u}, \mathbf{v}$ is $|\det(\mathbf{u}, \mathbf{v})|$.
- A point on BC divides the side into fractions whose sum is one. The nonnegative product $t(1-t)$ is at most $1/4$.

Step 1: Parameterise the adjacent sides Let $\mathbf{b} = \overrightarrow{AB} = (-3, 3)$ and $\mathbf{c} = \overrightarrow{AC} = (5, -1)$. Write $\overrightarrow{AF} = t\mathbf{b}$ and $\overrightarrow{AE} = s\mathbf{c}$, where $0 \le t, s \le 1$. Since AFDE is a parallelogram in this cyclic order, its opposite vertex satisfies $\overrightarrow{AD} = t\mathbf{b} + s\mathbf{c}$. This relation follows by adding its two adjacent side vectors; the point D is not chosen independently.

Step 2: Apply the condition that D lies on BC Every point of BC has the form $A + (1-u)\mathbf{b} + u\mathbf{c}$, for $0 \le u \le 1$. The vectors $\mathbf{b}$ and $\mathbf{c}$ are independent, so comparison of coefficients gives $t = 1-u$, $s = u$, and therefore $s + t = 1$. Thus increasing one adjacent side fraction necessarily decreases the other.

Step 3: Maximise the area The fixed determinant is $\det(\mathbf{b}, \mathbf{c}) = (-3)(-1) - 3(5) = -12$. Hence the parallelogram area is

$$K = 12st = 12t(1-t) = 3 - 12\left(t - \frac{1}{2}\right)^2 \le 3.$$

Equality is attained when $s = t = 1/2$, so F and E are the midpoints of AB and AC and D is the midpoint of BC. These points all lie on the required sides, proving that the bound is attainable rather than merely an algebraic upper bound.

Step 4: Conclusion The maximum area of AFDE is $\boxed{3}$ square units. The required numerical value is 3.

Quick Tip to Solve: For a parallelogram with one vertex at a triangle vertex and its opposite vertex on the opposite side, the maximum area is half the triangle area; equality uses the three side midpoints.

26. The moment of inertia of a circular ring of mass M and diameter r about a tangential axis lying in the plane of the ring is : [4 marks]

(A) $\frac{3}{8}\text{Mr}^2$

(B) 2Mr^2

(C) $\frac{1}{2}\text{Mr}^2$

(D) $\frac{3}{2}\text{Mr}^2$

Correct Answer: (A) $\frac{3}{8}\text{Mr}^2$

Solution:

Concept:

- The perpendicular-axis theorem for a planar ring gives $I_z = I_x + I_y$, with the axes passing through its centre.
- The parallel-axis theorem is $I = I_{\text{CM}} + Md^2$. The given r is a diameter, so the radius is $a = r/2$.

Step 1: Find the moment about a central diameter For a uniform thin circular ring of radius a, every mass element is at distance a from the central axis perpendicular to its plane. Thus $I_z = Ma^2$. Rotational symmetry makes the two mutually perpendicular in-plane diameters equivalent: $I_x = I_y$. The perpendicular-axis theorem then gives

$$I_x = I_y = \frac{Ma^2}{2}.$$

The tangent requested is in the plane, so its parallel central axis is a diameter, not the perpendicular symmetry axis.

Step 2: Shift the axis to the tangent The perpendicular distance from the centre to an in-plane tangent is exactly a. Applying the parallel-axis theorem to the central diameter parallel to that tangent,

$$I_{\text{tangent}} = \frac{Ma^2}{2} + Ma^2 = \frac{3}{2}Ma^2.$$

Substituting the stated diameter rather than treating it as a radius,

$$I_{\text{tangent}} = \frac{3}{2}M\left(\frac{r}{2}\right)^2 = \frac{3}{8}Mr^2.$$

The shift term is positive, as a tangent must have a larger moment than the parallel diameter.

Step 3: Check the alternatives Option (A) has the derived coefficient $3/8$. Option (D), $3Mr^2/2$, results from incorrectly using the diameter as the radius. The coefficient in (B) corresponds to neither the correct in-plane central moment nor the correct radius. Option (C) also fails the perpendicular-axis and parallel-axis combination. All options have the same physical dimensions, so dimensions alone cannot select the answer here.

Step 4: Conclusion The required moment of inertia is $\boxed{\frac{3}{8}Mr^2}$. Hence, the correct option is (A).

Quick Tip to Solve: Write radius = diameter/2 before using a ring formula, and identify whether the tangent lies in the plane or is perpendicular to it.

27. In a moving coil galvanometer, two moving coils $\mathrm{M_1}$ and $\mathrm{M_2}$ have the following particulars :

$$\mathrm{R_1 = 5\Omega,\ N_1 = 15,\ A_1 = 3.6 \times 10^{-3}\ m^2,\ B_1 = 0.25\ T}$$
$$\mathrm{R_2 = 7\Omega,\ N_2 = 21,\ A_2 = 1.8 \times 10^{-3}\ m^2,\ B_2 = 0.50\ T}$$

Assuming that torsional constant of the springs are same for both coils, what will be the ratio of voltage sensitivity of M_1 and M_2 ? [4 marks]

(A) $1 : 1$

(B) $1 : 3$

(C) $1 : 2$

(D) $1 : 4$

Correct Answer: (A) $1 : 1$

Solution:

Concept:

- At equilibrium, magnetic torque $NABI$ balances restoring torque $\kappa\theta$, where κ is the torsional constant.
- Voltage sensitivity is $S_V = \theta/V = NAB/(\kappa R)$, because the applied voltage across a coil is $V = IR$.

Step 1: Obtain the appropriate sensitivity The current sensitivity $\theta/I = NAB/\kappa$ is not the quantity requested. Dividing the deflection by the voltage IR introduces the additional factor $1/R$. Thus a coil with more turns or greater field need not be more voltage-sensitive if its resistance increases by a compensating factor. Here the two springs have the same κ, so this common constant cancels only after the voltage-sensitivity ratio is written correctly.

Step 2: Substitute the data into the ratio Keeping each coil's resistance with its own magnetic parameters gives

$$\frac{S_{V1}}{S_{V2}} = \frac{N_1 A_1 B_1 R_2}{N_2 A_2 B_2 R_1} = \frac{15(3.6 \times 10^{-3})(0.25)(7)}{21(1.8 \times 10^{-3})(0.50)(5)}.$$

Cancel the common powers of ten. The area ratio is 2 and the field ratio is $1/2$, so their product is 1. The remaining factor is

$$\frac{15}{21}\frac{7}{5} = \frac{5}{7}\frac{7}{5} = 1.$$

Step 3: Check the balance and eliminate the other ratios An independent check is to calculate NAB/R for each coil. The first gives $0.0135/5 = 0.0027$; the second gives $0.0189/7 = 0.0027$. Thus both give the same deflection for the same applied voltage. Ratios $1 : 3$, $1 : 2$, and $1 : 4$ would require unequal values of this quantity and are inconsistent with the supplied data. Ignoring resistance would instead give $5 : 7$, also not the required voltage ratio.

Step 4: Conclusion The voltage sensitivities are equal: $\boxed{S_{V1} : S_{V2} = 1 : 1}$. Hence, the correct option is (A).

Quick Tip to Solve: For voltage sensitivity compare NAB/R, whereas for current sensitivity compare NAB; cancelling the area and field ratios first makes this example immediate.

28. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Net dipole moment of a polar linear isotropic dielectric substance is not zero even in the absence of an external electric field.

Reason (R) : In absence of an external electric field, the different permanent dipoles of a polar dielectric substance are oriented in random directions.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both (A) and (R) are correct and (R) is the correct explanation of (A) 2.

(B) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(C) (A) is not correct but (R) is correct

(D) (A) is correct but (R) is not correct

Correct Answer: (C) (A) is not correct but (R) is correct

Solution:

Concept:

- A polar molecule has a permanent electric dipole moment even without an applied field. This is a microscopic molecular property.
- The net dipole moment of a specimen is the vector sum of all molecular moments. For an isotropic polar dielectric without an external field, random orientations give zero macroscopic polarization.

Step 1: Evaluate the assertion The assertion concerns the net dipole moment of the dielectric substance, not the permanent moment of one molecule. In a linear isotropic polar dielectric, there is no preferred direction when the applied field vanishes. The average vector sum of the permanent moments therefore vanishes. In macroscopic notation,

$$\mathbf{P} = \frac{1}{\mathcal{V}} \sum_i \mathbf{p}_i = \mathbf{0}$$

in the unpolarized state. Equivalently, the linear response $\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$ is zero at $\mathbf{E} = 0$. Hence the assertion that the net moment is nonzero is false.

Step 2: Evaluate the reason independently Thermal motion leaves the permanent molecular dipoles randomly oriented in the absence of an aligning external field. Individual dipoles remain present, but their directions are distributed symmetrically, and their vector contributions cancel in the macroscopic average. This is precisely the situation described by the reason, so the reason is true. A random ensemble must not be confused with an ensemble of molecules having no permanent moment.

Step 3: Select the truth combination We have a false assertion and a true reason. Thus the two alternatives claiming both statements are correct are excluded immediately. The alternative claiming a true assertion and a false reason reverses both truth values. The correct combination is the option stating that (A) is not correct but (R) is correct. Random orientation explains the vanishing bulk moment; it cannot explain a nonzero bulk moment as claimed in the assertion.

Step 4: Conclusion The assertion is false, while the reason is true. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish a permanent dipole moment of each polar molecule from the net dipole moment of the whole sample; the latter depends on orientational order.

29. If μ_0 and ϵ_0 are the permeability and permittivity of free space, respectively, then the dimension of $\left(\frac{1}{\mu_0\epsilon_0}\right)$ is : [4 marks]

(A) T^2/L

(B) $\mathrm{L}^2/\mathrm{T}^2$

(C) $\mathrm{T}^2/\mathrm{L}^2$

(D) L/T^2

Correct Answer: (B) $\mathrm{L}^2/\mathrm{T}^2$

Solution:

Concept:

- The speed of electromagnetic waves in vacuum obeys $c = 1/\sqrt{\mu_0\epsilon_0}$, where c has dimensions LT^{-1}.
- Dimensional equations follow algebraic powers: squaring a speed gives L^2T^{-2}, and taking a reciprocal reverses every exponent.

Step 1: Relate the constants to the wave speed The permeability and permittivity of free space appear together in the electromagnetic wave equation. Its propagation speed is

$$c^2 = \frac{1}{\mu_0\epsilon_0}.$$

Therefore the expression asked in the question is a speed squared, not a speed or the reciprocal of a speed. No numerical values of the two constants are needed, since only their combined dimensions are requested.

Step 2: Compute the dimensions Using $[c] = LT^{-1}$, square both sides of the dimensional relation:

$$\left[\frac{1}{\mu_0\epsilon_0}\right] = [c]^2 = (LT^{-1})^2 = L^2T^{-2}.$$

As an independent units check, $[\mu_0] = MLT^{-2}I^{-2}$ and $[\epsilon_0] = M^{-1}L^{-3}T^4I^2$. Their product is $L^{-2}T^2$, whose reciprocal is again L^2T^{-2}. Both mass and electric-current dimensions cancel, as expected for a kinematic quantity.

Step 3: Compare with all the choices Option (B) is L^2/T^2, exactly the obtained result. Option (C), T^2/L^2, is instead the dimension of the product $\mu_0\epsilon_0$, before taking its reciprocal. Option (D), L/T^2, is an acceleration and lacks one power of length. Option (A), T^2/L, is the reciprocal of acceleration and is also inconsistent with a squared propagation speed. Thus only one option has both the required length and time powers.

Step 4: Conclusion The required dimension is $\boxed{L^2T^{-2}}$, or L^2/T^2. Hence, the correct option is (B).

Quick Tip to Solve: Remember c squared = 1/(permeability times permittivity); this avoids carrying mass and current dimensions unless an independent check is needed.

30. A sinusoidal wave of wavelength 7.5 cm travels a distance of 1.2 cm along the x-direction in 0.3 sec . The crest P is at $x = 0$ at $\mathrm{t} = 0\mathrm{sec}$ and maximum displacement of the wave is 2 cm . Which equation correctly represents this wave? [4 marks]

(A) $y = 2\cos(0.83x - 3.35t)\mathrm{cm}$

(B) $y = 2\sin(0.83x - 3.5\mathrm{t})\mathrm{cm}$

(C) $y = 2\cos(0.13x - 0.5t)\mathrm{cm}$

(D) $y = 2\cos(3.35x - 0.83\mathrm{t})\mathrm{cm}$

Correct Answer: (A) $y = 2\cos(0.83x - 3.35t)\mathrm{cm}$

Solution:

Concept:

- A sinusoidal wave travelling along positive x can be written $y = A\cos(kx - \omega t + \phi)$, with $k = 2\pi/\lambda$ and $\omega = kv$.
- The amplitude is the maximum displacement. A crest at $x = 0, t = 0$ requires $y = A$ there, which fixes $\phi = 0$ modulo 2π.

Step 1: Find the wave speed and wave number Use centimetres consistently because the wavelengths, distances and option amplitudes are given in centimetres. The propagation speed is

$$v = \frac{1.2}{0.3} = 4\,\mathrm{cm\,s^{-1}}.$$

The angular spatial frequency is

$$k = \frac{2\pi}{7.5} = 0.8378\,\mathrm{rad\,cm^{-1}} \simeq 0.83\,\mathrm{rad\,cm^{-1}}.$$

The factor 2π is essential: $1/\lambda$ alone is the number of cycles per centimetre, not the coefficient of x in a trigonometric phase measured in radians.

Step 2: Find the angular frequency and initial phase The angular frequency follows from $\omega = kv$:

$$\omega = \frac{2\pi}{7.5}(4) = 3.351\,\mathrm{rad\,s^{-1}} \simeq 3.35\,\mathrm{rad\,s^{-1}}.$$

The amplitude is $A = 2\,\mathrm{cm}$. At the origin initially the crest has maximum positive displacement, so cosine with zero phase satisfies the condition. A negative sign before ωt makes a constant phase move toward increasing x.

Step 3: Check each proposed wave Option (A) has the correct amplitude, wave number, angular frequency and initial crest. Option (B) has zero displacement at the origin initially because it uses sine with no phase shift. Option (C) uses approximately $1/\lambda$ rather than $2\pi/\lambda$, so it gives the wrong wavelength and period. Option (D) exchanges the spatial and temporal coefficients, producing neither the specified wavelength nor the correct speed. The small rounding of k in (A) is consistent with the quoted coefficients.

Step 4: Conclusion The wave is $\boxed{y = 2\cos(0.83x - 3.35t)\,\mathrm{cm}}$, with x in centimetres and t in seconds. Hence, the correct option is (A).

Quick Tip to Solve: Check the initial crest first to select cosine, then calculate k from the wavelength and omega from kv; keep length units consistent with the options.

31. A solenoid having area A and length ' l ' is filled with a material having relative permeability 2. The magnetic energy stored in the solenoid is : [4 marks]

(A) $\frac{\mathrm{B}^2\ \mathrm{Al}}{4\mu_0}$

(B) $\frac{\mathrm{B}^2\ \mathrm{Al}}{2\mu_0}$

(C) $\frac{\mathrm{B}^2\ \mathrm{Al}}{\mu_0}$

(D) $\mathrm{B}^2\mathrm{Al}$

Correct Answer: (A) $\frac{\mathrm{B}^2\ \mathrm{Al}}{4\mu_0}$

Solution:

Concept:

- For a linear magnetic medium, the magnetic energy per unit volume is $u = \frac{1}{2}BH = B^2/(2\mu)$. Here B is the magnetic flux density inside the solenoid and μ is its absolute permeability.
- The relative permeability is defined by $\mu_r = \mu/\mu_0$, where μ_0 is the permeability of free space. The approximately uniform field occupies volume $V = Al$.

Step 1: Identify the permeability and field volume The filling material has $\mu_r = 2$, so its absolute permeability is $\mu = 2\mu_0$. The cross-sectional area is A and the solenoid length is l; hence the volume over which the stated field stores energy is Al. The relevant permeability is that of the material, rather than that of empty space.

Step 2: Multiply the energy density by the volume Using the field magnitude B appearing in the options,

$$U = uV = \frac{B^2}{2\mu}\,Al = \frac{B^2 Al}{2(2\mu_0)} = \frac{B^2 Al}{4\mu_0}.$$

The factor of two from energy density and the factor of two from relative permeability both occur in the denominator. Option (B) uses the vacuum value of permeability; option (C) misses both factors, and option (D) lacks the permeability factor needed for energy dimensions.

Step 3: Conclusion The magnetic energy stored is $\boxed{U = B^2 Al/(4\mu_0)}$. Hence, the correct option is (A).

Quick Tip to Solve: When the answer is expressed in terms of the actual field B, use $B^2/(2\mu)$, then multiply by the field-filled volume.

32. Two identical objects are placed in front of convex mirror and concave mirror having same radii of curvature of 12 cm , at same distance of 18 cm from the respective mirrors. The ratio of sizes of the images formed by convex mirror and by concave mirror is : [4 marks]

(A) 2

(B) $1/2$

(C) $1/3$

(D) 3

Correct Answer: (B) $1/2$

Solution:

Concept:

- For a spherical mirror, $1/f = 1/v + 1/u$ and the transverse magnification is $m = -v/u$. Image size is proportional to $|m|$, so an inverted image contributes a positive size magnitude.
- The focal length magnitude is half the radius of curvature: $|f| = R/2$. In the Cartesian convention a real object in front of either mirror has negative object distance.

Step 1: Find the image formed by the convex mirror Both objects are $18\,\text{cm}$ in front of their mirrors, so $u = -18\,\text{cm}$. With $R = 12\,\text{cm}$, the convex focal length is $f = +6\,\text{cm}$. Thus

$$\frac{1}{v_{\text{convex}}} = \frac{1}{6} + \frac{1}{18} = \frac{2}{9}, \qquad v_{\text{convex}} = \frac{9}{2}\,\text{cm}.$$

Its magnification is $m_{\text{convex}} = -(9/2)/(-18) = 1/4$, giving an erect image one quarter of the object size.

Step 2: Find the concave image and compare sizes The concave mirror has $f = -6\,\text{cm}$. Consequently

$$\frac{1}{v_{\text{concave}}} = -\frac{1}{6} + \frac{1}{18} = -\frac{1}{9}, \qquad v_{\text{concave}} = -9\,\text{cm}.$$

Its magnification is $m_{\text{concave}} = -(-9)/(-18) = -1/2$. Since the objects are identical, their common height cancels, leaving the size ratio

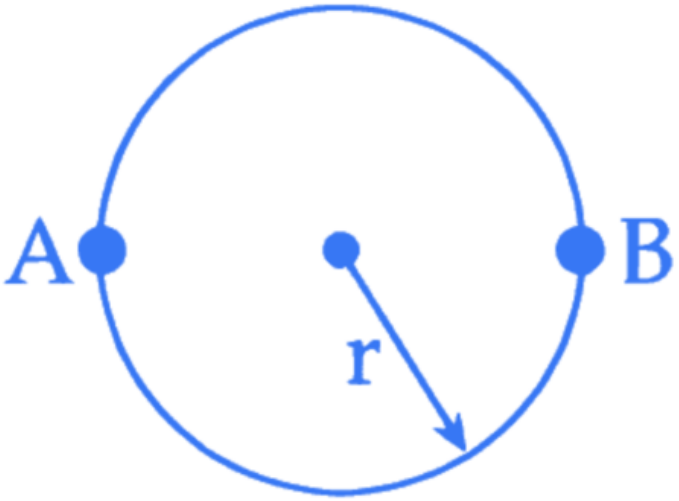
$$\frac{h_{\text{convex}}}{h_{\text{concave}}} = \frac{|1/4|}{|-1/2|} = \frac{1}{2}.$$

The value 2 reverses the requested order; neither 3 nor $1/3$ follows from the mirror distances.

Step 3: Conclusion The ratio of convex-mirror image size to concave-mirror image size is $\boxed{1/2}$. Hence, the correct option is (B).

Quick Tip to Solve: Use signed distances to find magnifications, but take their absolute values when comparing image sizes.

33. A sportsman runs around a circular track of radius r such that he traverses the path $ABAB$. The distance travelled and displacement, respectively, are [4 marks]



(A) $\pi r, 3r$

(B) $2\mathrm{r}, 3\pi\mathrm{r}$

(C) $3\pi\mathrm{r}, 2\mathrm{r}$

(D) $3\pi r, \pi r$

Correct Answer: (C) $3\pi\mathrm{r}, 2\mathrm{r}$

Solution:

Concept:

- Distance is the total length of the actual path travelled, whereas displacement is the vector from the initial position to the final position. Intermediate returns affect distance but cancel in displacement.
- The marked points A and B are diametrically opposite on the circle of radius r. A semicircular arc has length πr, while the straight diameter has length $2r$.

Step 1: Count the semicircular traversals The path $ABAB$ begins at A, reaches B, returns to A, and finally reaches B again. It therefore consists of three successive traversals between opposite ends of a diameter. Each traversal along the circular track covers a semicircle, irrespective of whether the runner uses the upper or lower half. Thus

$$s = \pi r + \pi r + \pi r = 3\pi r.$$

There are three arcs, not just the single arc joining the starting and finishing positions.

Step 2: Compare the initial and final positions The initial point is A and the final point is B. The displacement therefore points directly from A to B, across the diameter, and its magnitude is

$$|\Delta\vec{r}| = AB = 2r.$$

A displacement magnitude of $3r$ exceeds the diameter and cannot join any two points of this circle.

The value πr measures a semicircular path, not the straight separation. Option (B) also interchanges distance and displacement, contrary to the order asked.

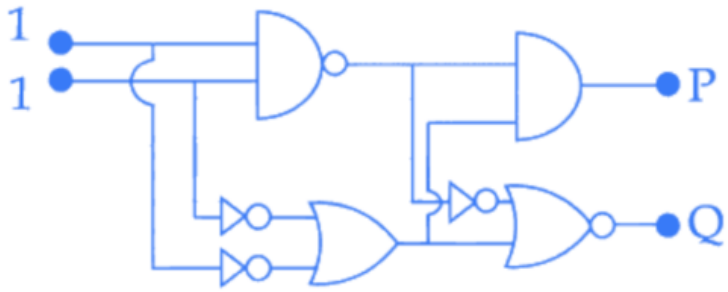
Step 3: Conclusion The distance travelled and displacement magnitude are respectively $\boxed{3\pi r,\ 2r}$. Hence, the correct option is (C).

Quick Tip to Solve: For repeated motion, write the first and last positions separately; only these determine displacement, while every arc contributes to distance.

34.

In the digital circuit shown in the figure, for the given inputs the P and Q values are :

[4 marks]



(A) $P = 0, Q = 1$

(B) $P = 1, Q = 0$

(C) $\mathrm{P} = 0, \mathrm{Q} = 0$

(D) $P = 1, Q = 1$

Correct Answer: (C) $\mathrm{P} = 0, \mathrm{Q} = 0$

Solution:

Concept:

- An AND gate outputs 1 only when both inputs are 1. A NAND gate is an AND gate followed by inversion; hence its output is 0 for inputs $(1, 1)$.
- A NOT gate changes 1 to 0 and 0 to 1. An OR gate outputs 1 when at least one input is 1, while a NOR gate inverts the OR output. Crossing wires with a bridge are not joined.

Step 1: Evaluate the two left-hand branches Let the output of the upper NAND gate be X, and let the output of the lower OR gate be Y. Both external inputs are 1. The upper gate has the AND shape with an output bubble, so

$$X = \overline{1 \cdot 1} = 0.$$

Each external input also passes through its own NOT gate before reaching the lower OR gate. Both inverted inputs are therefore 0, giving

$$Y = \overline{1} + \overline{1} = 0 + 0 = 0.$$

Here the plus sign denotes the logical OR operation, not ordinary addition.

Step 2: Propagate the intermediate values to the outputs The final upper AND gate receives X and Y, so $P = X \cdot Y = 0$. In the lower branch, X passes through a NOT gate, giving $\overline{X} = 1$; the final NOR gate receives this signal and Y. Hence

$$Q = \overline{\overline{X} + Y} = \overline{1 + 0} = 0.$$

The bubble on the final lower gate is essential: without applying this inversion one would incorrectly obtain $Q = 1$. Options (B) and (D) require $P = 1$, impossible because the upper NAND output is already zero.

Step 3: Conclusion The two outputs are $\boxed{P = 0,\ Q = 0}$. Hence, the correct option is (C).

Quick Tip to Solve: Label each intermediate wire and evaluate gates from left to right; apply every output bubble immediately before following the next wire.

---

35. An electron with mass ' m ' with an initial velocity $(\mathrm{t} = 0)\overrightarrow{\mathrm{v}} = \mathrm{v}_0\hat{i}\,(\mathrm{v}_0 > 0)$ enters a magnetic field $\overrightarrow{\mathrm{B}} = \mathrm{B}_0\hat{j}$. If the initial de-Broglie wavelength at $\mathrm{t} = 0$ is λ_0 then its value after time ' t ' would be : [4 marks]

(A) $\frac{\lambda_0}{\sqrt{1-\frac{\mathrm{e}^2\ \mathrm{B}_0^2\mathrm{t}^2}{\mathrm{m}^2}}}$

(B) λ_0

(C) $\lambda_0\sqrt{1+\frac{\mathrm{e}^2\ \mathrm{B}_0^2\mathrm{t}^2}{\mathrm{m}^2}}$

(D) $\frac{\lambda_0}{\sqrt{1+\frac{\mathrm{e}^2\ \mathrm{B}_0^2\mathrm{t}^2}{\mathrm{m}^2}}}$

Correct Answer: (B) λ_0

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, where p is the magnitude of the particle momentum. A changing direction of momentum alone does not change its wavelength.
- The magnetic Lorentz force is $\vec{F} = q\vec{v} \times \vec{B}$. It is perpendicular to the instantaneous velocity and therefore does no work: $\vec{F} \cdot \vec{v} = 0$.

Step 1: Determine whether the magnetic field changes the speed The electron initially moves along $\hat{i}$ while the magnetic field points along $\hat{j}$. Its velocity is perpendicular to the field, so the magnetic force bends its trajectory. However, at every later instant that force remains perpendicular to its instantaneous velocity. Therefore

$$\frac{dK}{dt} = \vec{F} \cdot \vec{v} = q(\vec{v} \times \vec{B}) \cdot \vec{v} = 0.$$

The kinetic energy is constant. Consequently the speed remains v_0; the force changes direction of motion rather than the magnitude of velocity.

Step 2: Use the constant momentum magnitude For the stated mass and initial speed, the momentum magnitude is $p_0 = mv_0$, and it remains unchanged throughout the motion. Thus

$$\lambda(t) = \frac{h}{p(t)} = \frac{h}{mv_0} = \lambda_0.$$

The electron charge fixes the sense of curvature but does not affect this energy argument. Each of options (A), (C), and (D) introduces a time-dependent momentum magnitude and hence a change in kinetic energy. That would require work, which a purely magnetic field cannot supply. In particular, treating the initial magnetic acceleration as a constant vector for all time is invalid because its direction turns with the velocity.

Step 3: Conclusion The wavelength remains $\boxed{\lambda(t) = \lambda_0}$ at every time. Hence, the correct option is (B).

Quick Tip to Solve: Before resolving magnetic motion into components, check work: a purely magnetic force preserves speed and therefore preserves de Broglie wavelength.

36. A bi-convex lens has radius of curvature of both the surfaces same as $1/6$ cm. If this lens is required to be replaced by another convex lens having different radii of curvatures on both sides $(R_1 \neq R_2)$, without any change in lens power then possible combination of R_1 and R_2 is : [4 marks]

(A) $\frac{1}{3}$ cm and $\frac{1}{7}$ cm
(B) $\frac{1}{5}$ cm and $\frac{1}{7}$ cm
(C) $\frac{1}{3}$ cm and $\frac{1}{3}$ cm
(D) $\frac{1}{6}$ cm and $\frac{1}{9}$ cm

Correct Answer: (B) $\frac{1}{5}$ cm and $\frac{1}{7}$ cm

Solution:

Concept:

- For a thin lens of refractive index n in air, lens power is $P = 1/f = (n-1)(1/R_1 - 1/R_2)$, using signed radii. Replacing the lens without changing its material leaves the factor $n-1$ unchanged.
- For a biconvex lens, the two signed radii have opposite signs. If their positive magnitudes are r_1, r_2, the curvature factor is $1/r_1 + 1/r_2$. Equal power requires equal sums of reciprocal magnitudes.

Step 1: Find the original curvature factor Take light incident from the left. The original symmetric biconvex lens then has signed radii $R_1 = +1/6\,\text{cm}$ and $R_2 = -1/6\,\text{cm}$. Its power is proportional to

$$\frac{1}{R_1} - \frac{1}{R_2} = 6 - (-6) = 12\,\text{cm}^{-1}.$$

Equivalently, the sum of the reciprocals of the two positive curvature magnitudes must be 12. The numerical power itself need not be calculated because the common refractive-index factor cancels.

Step 2: Test the proposed pairs of curvature magnitudes For option (A), the reciprocal sum is

$3 + 7 = 10\,\text{cm}^{-1}$, so the power is too small. For option (B),

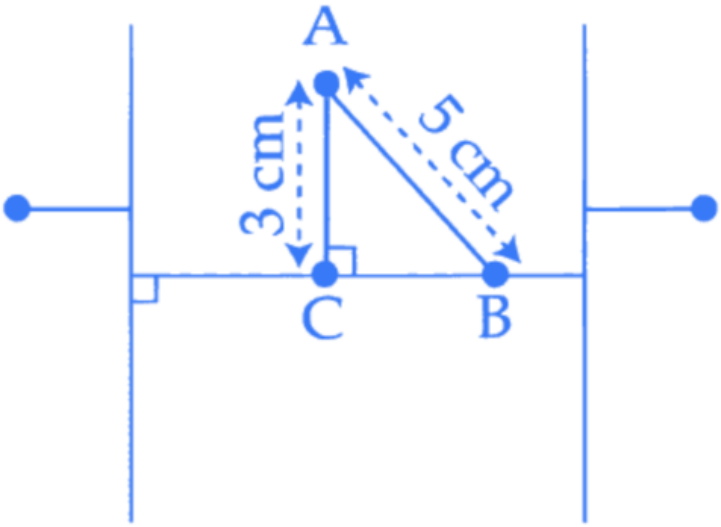
$$\frac{1}{1/5} + \frac{1}{1/7} = 5 + 7 = 12\,\text{cm}^{-1},$$

which reproduces the original power and uses unequal radii as required. Option (C) gives $3 + 3 = 6\,\text{cm}^{-1}$ and also has equal radii, while option (D) gives $6 + 9 = 15\,\text{cm}^{-1}$. Thus only the second pair retains the required curvature factor. The displayed radii in the choices are magnitudes; the rear surface receives the negative sign in the lens-maker convention.

Step 3: Conclusion The required radii have magnitudes $\boxed{1/5\,\text{cm and } 1/7\,\text{cm}}$. Hence, the correct option is (B).

Quick Tip to Solve: For the same material, compare the sums of reciprocal curvature magnitudes for biconvex lenses; adding the radii themselves does not determine power.

37. Two large plane parallel conducting plates are kept 10 cm apart as shown in figure. The potential difference between them is V . The potential difference between the points A and B (shown in the figure) is : [4 marks]



(A) 1 V
(B) $\frac{1}{4}$ V
(C) $\frac{3}{4}$ V
(D) $\frac{2}{5}$ V

Correct Answer: (D) $\frac{2}{5}$ V

Solution:

Concept:

- Between large parallel plates, away from edges, the electric field is uniform and perpendicular to the plates. If their separation is d and potential difference is V, its magnitude is $E = V/d$.

- The potential change depends only on displacement parallel to the electric field: $|\Delta V| = E|\Delta x|$. Displacement along an equipotential surface contributes no potential change.

Step 1: Read the displacement along the field The plates are vertical, so the electric field is horizontal. The diagram gives $AC = 3\,\text{cm}$, $AB = 5\,\text{cm}$, and a right angle at C. Thus A and C have the same horizontal coordinate and lie on the same equipotential surface. The effective separation of A and B is CB, rather than the sloping distance AB. By Pythagoras,

$$CB = \sqrt{AB^2 - AC^2} = \sqrt{5^2 - 3^2} = 4\,\text{cm}.$$

Step 2: Apply the linear potential variation The full plate separation is $10\,\text{cm}$. Consequently, the potential difference across the horizontal separation CB is

$$|V_A - V_B| = |V_C - V_B| = \frac{V}{10\,\text{cm}}(4\,\text{cm}) = \frac{2V}{5}.$$

The question asks for the magnitude; no plate polarity is needed. Using AB would give an incorrect fraction because its vertical component produces no potential change. The fractions 1, $1/4$, and $3/4$ do not correspond to the horizontal separation shown.

Step 3: Conclusion The potential difference between A and B is $\frac{2}{5}V$. Hence, the correct option is (D).

Quick Tip to Solve: First project the separation onto the direction perpendicular to the plates; points separated parallel to a plate have equal potential.

38. Two water drops each of radius ' r ' coalesce to form a bigger drop. If ' T ' is the surface tension, the surface energy released in this process is : [4 marks]

(A) $4\pi r^2\ T\left[2 - 2^{1/3}\right]$

(B) $4\pi r^2\ T[1 + \sqrt{2}]$

(C) $4\pi r^2\ T\left[2 - 2^{2/3}\right]$

(D) $4\pi r^2\ T[\sqrt{2} - 1]$

Correct Answer: (C) $4\pi r^2\ T\left[2 - 2^{2/3}\right]$

Solution:

Concept:

- A liquid drop has one liquid–air surface, so its surface energy is $U = TA$, where T is surface tension and A is surface area.
- On coalescence, the volume of the incompressible liquid is conserved. The energy released is the decrease in total surface energy, $U_i - U_f$.

Step 1: Find the radius after coalescence Let R be the radius of the single larger spherical drop. Each original drop has volume $4\pi r^3/3$. Adding both volumes and equating to the final volume gives

$$\frac{4\pi R^3}{3} = 2\frac{4\pi r^3}{3}, \qquad R = 2^{1/3}r.$$

The radius does not double: it is the cube of radius that is proportional to volume. This distinction determines the power of 2 in the surface area.

Step 2: Subtract the final surface energy from the initial energy The two initial surfaces have total area $A_i = 8\pi r^2$. The final area is

$$A_f = 4\pi R^2 = 4\pi\, 2^{2/3} r^2.$$

Therefore the energy released is

$$U_i - U_f = T(A_i - A_f) = 4\pi r^2 T\left(2 - 2^{2/3}\right).$$

The bracket is positive because $2^{2/3} < 2$, consistent with coalescence reducing surface area. The exponent $1/3$ belongs to radius, not area, so option (A) fails. The square-root expressions in (B) and (D) do not satisfy the three-dimensional volume relation. There is no extra factor of two for a second surface, as these are water drops rather than soap bubbles.

Step 3: Conclusion The released surface energy is $4\pi r^2 T\left(2 - 2^{2/3}\right)$. Hence, the correct option is (C).

Quick Tip to Solve: For n identical drops merging, conserve volume first: the radius scales as $n^{1/3}$, but the final area scales as $n^{2/3}$.

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39. Energy released when two deuterons $\left({}_1H^2\right)$ fuse to form a helium nucleus $\left({}_2He^4\right)$ is : (Given : Binding energy per nucleon of ${}_1H^2 = 1.1\text{MeV}$ and binding energy per nucleon of ${}_2He^4 = 7.0\text{MeV}$) [4 marks]

(A) 26.8 MeV

(B) 8.1 MeV

(C) 23.6 MeV

(D) 5.9 MeV

Correct Answer: (C) 23.6 MeV

Solution:

Concept:

- Total binding energy is the energy needed to separate a nucleus into its nucleons. It equals mass number times binding energy per nucleon: $B = A\bar{B}$.
- When the same nucleons form a more tightly bound final nucleus, their rest energy decreases. The released energy is the increase in total binding energy, $Q = B_f - B_i$.

Step 1: Calculate the initial total binding energy Each deuteron contains two nucleons. The given binding energy per nucleon is $1.1\,\text{MeV}$, so the binding energy of one deuteron is

$$B_d = 2(1.1) = 2.2\,\text{MeV}.$$

There are two deuterons initially. Their combined binding energy is therefore

$$B_i = 2B_d = 4.4\,\text{MeV}.$$

The factor of two for nucleons in each deuteron and the factor of two for the number of deuterons must both be included.

Step 2: Calculate the increase on forming helium The helium nucleus contains four nucleons, each contributing the given average binding energy of $7.0\,\mathrm{MeV}$. Thus

$$B_f = 4(7.0) = 28.0\,\mathrm{MeV}, \qquad Q = 28.0 - 4.4 = 23.6\,\mathrm{MeV}.$$

The positive result confirms energy release. The nucleon numbers balance: the two initial deuterons together contain the same four nucleons as the helium nucleus. The value $5.9\,\mathrm{MeV}$ is only the increase per nucleon; multiplying it by four gives the total. Adding the given per-nucleon values gives $8.1\,\mathrm{MeV}$, which is not the reaction energy. Option (A) also does not follow from the total binding energies.

Step 3: Conclusion The energy released per fusion event is $23.6\,\mathrm{MeV}$. Hence, the correct option is (C).

Quick Tip to Solve: Convert every binding energy per nucleon into total binding energy before subtracting final and initial values.

40.

Match List - I with List - II.

	List - I	$List - II$
(A)	**Heat capacity of body**	**(I)** $\mathrm{J\ kg^{-1}}$
(B)	**Specific heat capacity of body**	**(II)** $\mathrm{J\ K^{-1}}$
(C)	**Latent heat**	**(III)** $\mathrm{J\ kg^{-1}\ K^{-1}}$
(D)	**Thermal conductivity**	**(IV)** $\mathrm{J\ m^{-1}\ K^{-1}\ s^{-1}}$

Choose the correct answer from the options given below :

[4 marks]

(A) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(B) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

(C) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(D) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Correct Answer: (D) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Solution:

Concept:

- Heat capacity C refers to a complete body and is defined by $C = Q/\Delta T$. Specific heat capacity c refers to unit mass: $c = Q/(m\Delta T)$.
- Specific latent heat is $L = Q/m$. Thermal conductivity k occurs in the conduction equation $Q/t = kA\Delta T/\ell$, with area A and thickness ℓ.

Step 1: Match the heat capacity and specific heat capacity Heat Q is measured in joules and a temperature change in kelvin. Dividing the supplied energy by the temperature rise gives

$$[C] = \mathrm{J\,K^{-1}}, \qquad [c] = \mathrm{J\,kg^{-1}\,K^{-1}}.$$

Hence heat capacity of a body, entry (A), matches (II), while specific heat capacity, entry (B), matches (III). The mass factor distinguishes the whole-body property from the material property and cannot be interchanged.

Step 2: Match latent heat and thermal conductivity In the list, latent heat denotes the specific latent heat used in $Q = mL$; its unit is therefore $\mathrm{J\,kg^{-1}}$, so (C) matches (I). For conductivity, rearrange the rate of heat flow:

$$k = \frac{(Q/t)\ell}{A\Delta T}.$$

Its unit is

$$[k] = \frac{\mathrm{J\,s^{-1}\,m}}{\mathrm{m^2\,K}} = \mathrm{J\,m^{-1}\,K^{-1}\,s^{-1}},$$

so (D) matches (IV). Only the fourth option contains all four of these pairings. The other choices already fail the whole-body heat capacity test, since they assign it a unit containing inverse mass or inverse length rather than energy per temperature change.

Step 3: Conclusion Heat capacity matches II, specific heat capacity III, latent heat I, and thermal conductivity IV. Hence, the correct option is (D).

Quick Tip to Solve: Write each defining equation before matching units; the word specific introduces a division by mass.

41. Consider a circular loop that is uniformly charged and has a radius $\mathrm{a}\sqrt{2}$. Find the position along the positive z-axis of the cartesian coordinate system where the electric field is maximum if the ring was assumed to be placed in xy plane at the origin : [4 marks]

(A) a

(B) $a/2$

(C) 0

(D) $a/\sqrt{2}$

Correct Answer: (A) a

Solution:

Concept:

- For a uniformly charged ring of radius R and total charge Q, transverse field components cancel by symmetry. The axial field magnitude is $E(z) = k|Q|z/(R^2 + z^2)^{3/2}$ for $z > 0$.
- A stationary point of the field satisfies $dE/dz = 0$. The sign of this derivative on either side identifies whether the point is a maximum.

Step 1: Differentiate the axial field Treat R and Q as constants. Differentiating the product of z and

$(R^2 + z^2)^{-3/2}$ gives

$$\frac{dE}{dz} = k|Q| \left[(R^2 + z^2)^{-3/2} - 3z^2(R^2 + z^2)^{-5/2}\right].$$

Combining the terms over a common denominator yields

$$\frac{dE}{dz} = k|Q| \frac{R^2 - 2z^2}{(R^2 + z^2)^{5/2}}.$$

The denominator is always positive, so the stationary point comes entirely from the numerator.

Step 2: Locate and verify the maximum Setting the numerator to zero gives $R^2 = 2z^2$. Since the position is on the positive axis,

$$z = \frac{R}{\sqrt{2}} = \frac{a\sqrt{2}}{\sqrt{2}} = a.$$

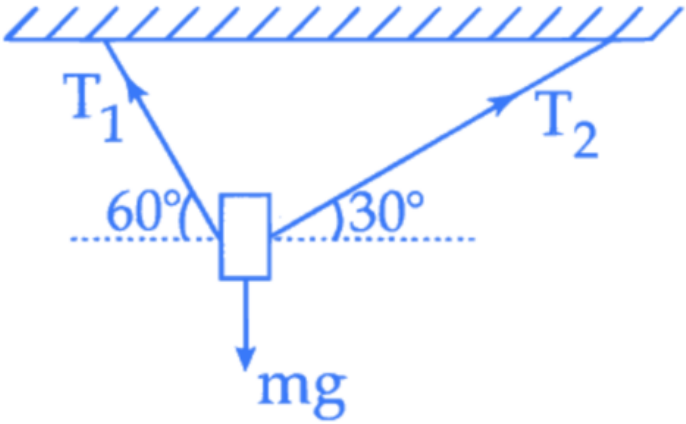
For $0 < z < R/\sqrt{2}$, the derivative is positive; beyond this point it is negative. Thus the field increases and then decreases, establishing a maximum. Also $E(0) = 0$ and $E(z) \to 0$ as $z \to \infty$, consistent with this being the global maximum on the positive axis. The positions $a/2$ and $a/\sqrt{2}$ lie before the maximum for the stated radius; zero gives a vanishing field, so none of the other options qualifies.

Step 3: Conclusion The electric field is maximum at $z = a$ on the positive z-axis. Hence, the correct option is (A).

Quick Tip to Solve: The axial maximum for any uniformly charged ring is at radius divided by $\sqrt{2}$; substitute the given radius only at the end.

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42. A body of mass 1 kg is suspended with the help of two strings making angles as shown in figure. Magnitudes of tensions T_1 and T_2, respectively, are (in N) :
(Take acceleration due to gravity $10\ m/s^2$) [4 marks]



(A) $5\sqrt{3}, 5$
(B) $5\sqrt{3}, 5\sqrt{3}$
(C) 5,5
(D) $5, 5\sqrt{3}$
Correct Answer: (A) $5\sqrt{3}, 5$

Solution:

Concept:

- A body at rest has zero resultant force along both horizontal and vertical directions. Its weight here is $mg = 1 \times 10 = 10\,\text{N}$ downward.
- Tension acts along a string away from the suspended body. For an angle measured from the horizontal, its horizontal component is $T\cos\theta$ and vertical component is $T\sin\theta$.

Step 1: Balance the horizontal components The figure shows the left string with tension T_1 at 60° to the horizontal and the right string with tension T_2 at 30°. Their horizontal components act in opposite directions, hence

$$T_1\cos 60^\circ = T_2\cos 30^\circ.$$

Using $\cos 60^\circ = 1/2$ and $\cos 30^\circ = \sqrt{3}/2$, this becomes

$$T_1 = \sqrt{3}\,T_2.$$

The steeper string therefore has the larger tension; equal tensions would leave a horizontal resultant.

Step 2: Balance the vertical components and solve Both vertical components act upward and must support the weight:

$$T_1\sin 60^\circ + T_2\sin 30^\circ = 10.$$

Substitute $T_1 = \sqrt{3}T_2$:

$$\frac{\sqrt{3}}{2}(\sqrt{3}T_2) + \frac{1}{2}T_2 = 10, \qquad 2T_2 = 10.$$

Thus $T_2 = 5\,\text{N}$ and $T_1 = 5\sqrt{3}\,\text{N}$. Their vertical components are $7.5\,\text{N}$ and $2.5\,\text{N}$, which add to the required $10\,\text{N}$. Options (B) and (C) violate horizontal balance by making tensions equal. Option (D) reverses the tensions and also fails horizontal balance for the angles shown.

Step 3: Conclusion The tensions are $T_1 = 5\sqrt{3}\,\text{N}$ and $T_2 = 5\,\text{N}$, respectively. Hence, the correct option is (A).

Quick Tip to Solve: Check whether angles are measured from the horizontal or vertical before choosing sine and cosine for the force components.

43. Identify the characteristics of an adiabatic process in a monoatomic gas. (A) Internal energy is constant. (B) Work done in the process is equal to the change in internal energy. (C) The product of temperature and volume is a constant. (D) The product of pressure and volume is a constant. (E) The work done to change the temperature from T_1 to T_2 is proportional to $(T_2 - T_1)$. Choose the correct answer from the options given below : [4 marks]

(A) (B), (D) only

(B) (B), (E) only

(C) (A), (C), (E) only

(D) (A), (C), (D) only

Correct Answer: (B) (B), (E) only

Solution:

Concept:

- In an adiabatic process no heat crosses the boundary: $Q = 0$. The first law is $\Delta U = Q + W_{\text{on}}$, where W_{on} is the work done on the gas. Equivalently, $\Delta U = Q - W_{\text{by}}$.
- For n moles of an ideal monoatomic gas, $U = \frac{3}{2}nRT$. Its heat-capacity ratio is $\gamma = C_P/C_V = 5/3$; a reversible adiabatic change obeys $PV^{\gamma} =$ constant and $TV^{\gamma-1} =$ constant.

Step 1: Apply the first law with no heat transfer Adiabatic does not mean constant temperature. During compression work increases the internal energy; during expansion the gas can do work at the expense of its internal energy. Setting $Q = 0$ gives

$$\Delta U = W_{\text{on}} = -W_{\text{by}}.$$

Thus statement B expresses the work–internal-energy equality using the work-on convention. If work is instead defined as work done by the gas, the minus sign must be retained. Statement A is false in general: zero heat transfer does not require zero change in internal energy.

Step 2: Test the proposed constant products For a reversible adiabatic change of the monoatomic ideal gas, the correct relations are

$$PV^{5/3} = \text{constant}, \qquad TV^{2/3} = \text{constant}.$$

Neither TV nor PV is the corresponding adiabatic invariant. In particular, $PV = nRT$ changes whenever the temperature changes. Statements C and D are therefore false. The relation $PV =$ constant would instead describe an isothermal ideal-gas process.

Step 3: Relate the work to the temperature change With fixed amount of gas, its change in internal energy is

$$\Delta U = \frac{3}{2}nR(T_2 - T_1).$$

Hence the work done on the gas in the adiabatic process is $\frac{3}{2}nR(T_2 - T_1)$, whereas the work done by the gas is its negative. Under either sign convention the work is proportional to $T_2 - T_1$, with a constant proportionality factor. Statement E is true. The options containing A, C or D cannot be selected.

Step 4: Conclusion The valid statements are B and E, with statement B using work done on the gas.

Hence, the correct option is (B).

Quick Tip to Solve: Write the first law with an explicit work sign convention before setting heat transfer to zero. Adiabatic means no heat exchange, whereas isothermal means constant temperature.

44. Given a charge q , current I and permeability of vacuum μ_{o^*}. Which of the following quantity has the dimension of momentum ? [4 marks]

(A) qI/μ_o

(B) $q^2\mu_o I$

(C) $q\mu_0/I$

(D) $\mathrm{q}\mu_{\mathrm{o}}\mathrm{I}$

Correct Answer: (D) $\mathrm{q}\mu_{\mathrm{o}}\mathrm{I}$

Solution:

Concept:

- The dimensions of charge are $[q] = [I]T$, since charge is current multiplied by time.
- The permeability of vacuum has dimensions $[\mu_0] = MLT^{-2}[I]^{-2}$, obtained from force per unit length between parallel currents. Momentum has dimensions MLT^{-1}.

Step 1: Express the permeability in base dimensions For two parallel conductors, force per unit length is proportional to $\mu_0 I_1 I_2/r$, where r is their separation. Thus $[\mu_0] = [F]/[I]^2 = MLT^{-2}[I]^{-2}$. Here the symbol $[I]$ denotes the independent dimension of electric current; it must not be confused with a dimension of charge. We compare dimensions, so numerical constants in the magnetic-force relation do not affect the result.

Step 2: Test the product in option D Multiply the dimensions of the three quantities appearing in this option:

$$[q\mu_0 I] = ([I]T)(MLT^{-2}[I]^{-2})([I]) = MLT^{-1}.$$

The two powers of current in the numerator cancel the inverse square current dimension of permeability. One power of time then cancels, leaving precisely mass times velocity, the dimensions required for momentum.

Step 3: Eliminate the other expressions For option A, $[qI/\mu_0] = M^{-1}L^{-1}T^3[I]^4$, which is not momentum. Option B contains one extra charge relative to option D, so its dimensions are $ML[I]$. Option C divides the correct product by I^2, leaving $MLT^{-1}[I]^{-2}$. Each has an unwanted electric-current dimension, whereas mechanical momentum has none.

Step 4: Conclusion The expression with dimensions of momentum is $q\mu_0 I$. Hence, the correct option is (D).

Quick Tip to Solve: For an electromagnetic dimensional test, first use $[q] = [I]T$; cancellation of all current dimensions is essential for a purely mechanical quantity.

45. Assuming the validity of Bohr's atomic model for hydrogen like ions the radius of Li^{++} ion in its ground state is given by $\frac{1}{X}a_0$, where $X =$ _____ (Where a_0 is the first Bohr's radius.) [4 marks]

(A) 2

(B) 9

(C) 1

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- A hydrogen-like ion has one electron moving in the Coulomb field of a nucleus of charge $+Ze$, where Z is its atomic number.
- In the Bohr model its orbital radius is $r_n = a_0 n^2/Z$, where n is the principal quantum number and a_0 is the ground-state radius of hydrogen.

Step 1: Identify the nuclear charge and orbit Lithium has atomic number $Z = 3$. The ion Li^{++} has lost two of its three electrons and therefore retains one electron, so the hydrogen-like Bohr formula applies. Ionisation changes the number of electrons but does not change the nuclear charge. The phrase ground state means that the remaining electron occupies the lowest allowed Bohr orbit, so $n = 1$.

Step 2: Apply the radius formula Substitute these values into the hydrogen-like radius relation:

$$r_1 = a_0 \frac{1^2}{3} = \frac{a_0}{3}.$$

The factor $1/Z$ reflects the stronger attraction of the lithium nucleus: at the same quantum number, its electron orbit is smaller than the corresponding hydrogen orbit. The comparison is between ground states, so there is no additional factor from an excited orbit.

Step 3: Compare with the requested form The question defines the radius as a_0/X. Equating the two expressions gives $a_0/X = a_0/3$, hence $X = 3$. The value 9 would result from using the Z^2 dependence of energy incorrectly for radius. The value 1 is the hydrogen result and ignores the higher nuclear charge. The value 2 is the ionic charge number, not the atomic number that appears in the radius formula.

Step 4: Conclusion The ground-state radius is $a_0/3$, so $X = 3$. Hence, the correct option is (D).

Quick Tip to Solve: For a one-electron ion, radius varies as n^2/Z, whereas binding-energy magnitude varies as Z^2/n^2; keep these two dependences distinct.

Physics – Section B: Numerical Value Questions (20 marks)

46. A ray of light suffers minimum deviation when incident on a prism having angle of the prism equal to 60°. The refractive index of the prism material is $\sqrt{2}$. The angle of incidence (in degrees) is_____ [4 marks]

Solution:

Concept:

- At minimum deviation a ray has a symmetric path through a prism: the angle of incidence equals the angle of emergence, $i = e$.
- The internal refraction angles obey $r_1 + r_2 = A$. For a symmetric path $r_1 = r_2 = A/2$; at the air-prism interface Snell's law is $\sin i = \mu \sin r_1$.

Step 1: Use the minimum-deviation condition The prism angle is $A = 60^\circ$. Since the ray suffers minimum deviation, the two refraction angles inside the prism are equal. Combining this equality with the prism geometry gives

$$r_1 = r_2 = \frac{A}{2} = 30^\circ.$$

It is the internal refraction angle that equals half the prism angle; the external angle of incidence need not equal 30°. Both angles are measured from their respective face normals.

Step 2: Apply refraction at the first face Taking the surrounding air to have refractive index 1, Snell's law gives

$$1 \sin i = \sqrt{2} \sin 30^\circ = \frac{\sqrt{2}}{2}.$$

The physically incident ray has an angle between 0° and 90° with the inward normal. Therefore the appropriate solution is

$$i = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ.$$

The obtuse supplementary angle is not an angle of incidence for the entering ray.

Step 3: Check the symmetric result The angle of emergence also equals 45°, so the minimum deviation would be $i + e - A = 45^\circ + 45^\circ - 60^\circ = 30^\circ$. Equivalently, the prism relation $\mu = \sin[(A + \delta_{\min})/2] / \sin(A/2)$ gives $\sin 45^\circ / \sin 30^\circ = \sqrt{2}$, agreeing with the given material. This verifies the selected incidence angle without confusing it with the deviation.

Step 4: Conclusion The angle of incidence is 45°. The required numerical entry is 45.

Quick Tip to Solve: At minimum deviation, set each internal angle equal to half the prism angle before using Snell's law; do not halve the external incidence angle.

47. The length of a light string is 1.4 m when the tension on it is 5 N . If the tension increases to 7 N , the length of the string is 1.56 m . The original length of the string is_____m. [4 marks]

Solution:

Concept:

- Within the elastic limit, Hooke's law gives $F = k(L - L_0)$, where L_0 is the original length and k is the effective force constant.
- For the same string under two tensions, $\Delta F = k\Delta L$. The difference of loaded lengths is the extra extension, not the original length.

Step 1: Find the change in extension Let the natural length be L_0. The two stated tensions give

$$5 = k(1.40 - L_0), \qquad 7 = k(1.56 - L_0).$$

Lengths are in metres and forces are in newtons. Subtracting these equations removes the unknown natural length. This is valid because the two measurements refer to the same string, so the material, cross-section and elastic force constant are unchanged.

Step 2: Calculate the force constant The tension increase is $7 - 5 = 2\,\mathrm{N}$, and the corresponding increase in length is $1.56 - 1.40 = 0.16\,\mathrm{m}$. Therefore

$$k = \frac{2}{0.16} = 12.5\,\mathrm{N\,m^{-1}}.$$

The measured increase of $0.16\,\mathrm{m}$ is produced only by the additional tension of $2\,\mathrm{N}$; it is not the full extension at either stated load.

Step 3: Recover and verify the original length At the first load, the extension is

$$x_1 = \frac{5}{12.5} = 0.40\,\mathrm{m}.$$

Subtract this extension from the loaded length:

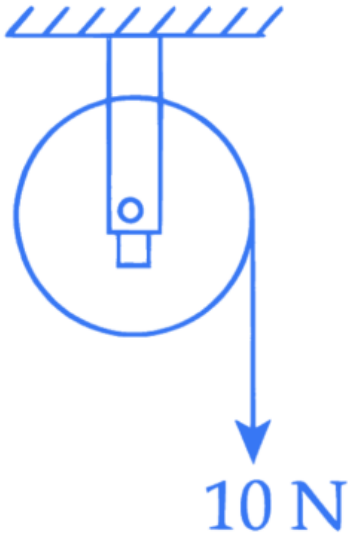
$$L_0 = 1.40 - 0.40 = 1.00\,\mathrm{m}.$$

As a check, the extension at $7\,\mathrm{N}$ is $7/12.5 = 0.56\,\mathrm{m}$, and the predicted loaded length is $1.00 + 0.56 = 1.56\,\mathrm{m}$, exactly the second observation. A natural length longer than either loaded length would give a negative extension and is therefore inconsistent with a stretched string.

Step 4: Conclusion The original length of the string is $1.00\,\mathrm{m}$. The required numerical entry is 1.

Quick Tip to Solve: With two force-length measurements, subtract the Hooke-law equations first to find stiffness, then subtract the full extension from one loaded length.

48. A wheel of radius 0.2 m rotates freely about its center when a string that is wrapped over its rim is pulled by force of 10 N as shown in figure. The established torque produces an angular acceleration of 2rad/s^2. Moment of intertia of the wheel is_____ kg m^2. (Acceleration due to gravity $= 10\ \text{m/s}^2$) [4 marks]



Solution:

Concept:

- For rotation about a fixed axis, the net torque is $\tau = I\alpha$, where I is moment of inertia and α is angular acceleration.
- A tangential force at the rim has perpendicular lever arm equal to the wheel radius, so its torque magnitude is $\tau = FR$.

Step 1: Identify the force producing rotation The string is wrapped around the rim, and its pulled segment is tangent to the wheel as shown. Thus the tension of $10\,\text{N}$ acts with a lever arm of $0.2\,\text{m}$ about the fixed central axis. The force is perpendicular to the radius at its point of application, so the sine factor in $\tau = RF\sin\theta$ is 1. The force at the axle acts through the axis and contributes no torque about it.

Step 2: Calculate the applied torque The wheel rotates freely, so a resisting frictional torque is not included. Its weight acts through its centre and also gives zero torque about this axis. The stated pulling force therefore supplies the net torque:

$$\tau = 10 \times 0.2 = 2.0\,\text{N m}.$$

The downward direction shown would make the wheel rotate clockwise, but only the magnitudes of torque and angular acceleration are needed to find the positive moment of inertia.

Step 3: Use the rotational equation of motion The angular acceleration is $\alpha = 2\,\text{rad s}^{-2}$. Hence

$$I = \frac{\tau}{\alpha} = \frac{2.0}{2} = 1.0\,\text{kg m}^2.$$

Radians are dimensionless, so torque divided by angular acceleration has the required dimensions of mass times length squared. The supplied value of gravitational acceleration is unnecessary: no mass is falling here, and the applied force is already given directly in newtons. Using $10g$ as the force would incorrectly treat the stated force as a mass.

Step 4: Conclusion The moment of inertia of the wheel is $1.0\,\mathrm{kg\,m^2}$. The required numerical entry is 1.

Quick Tip to Solve: For a fixed wheel pulled tangentially at its rim, use $I = FR/\alpha$; a force already expressed in newtons must not be multiplied by g.

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49. The internal energy of air in $4\text{ m} \times 4\text{ m} \times 3\text{ m}$ sized room at 1 atmospheric pressure will be_____$\times 10^6$ J

(Consider air as diatomic molecule) [4 marks]

Solution:

Concept:

- For an ideal diatomic gas at ordinary temperature, three translational and two rotational modes are active, so the number of degrees of freedom is $f = 5$. Vibrational modes are neglected in the usual diatomic-gas model.
- Equipartition gives $U = (f/2)nRT$, where n is the amount of gas, R is the gas constant and T is its absolute temperature. The ideal-gas equation $nRT = PV$ eliminates the unknown temperature and amount.

Step 1: Find the volume of air The gas occupies the entire room. Multiply its three internal dimensions, all already in metres:

$$V = 4 \times 4 \times 3 = 48\,\mathrm{m^3}.$$

This is the gas volume, rather than the floor area; omitting the height would produce an incorrect energy and incorrect units.

Step 2: Apply equipartition to the diatomic gas With five active degrees of freedom, the internal energy is

$$U = \frac{5}{2}nRT = \frac{5}{2}PV.$$

Use the room pressure, which is one atmosphere. In the standard approximation $1\,\mathrm{atm} \simeq 10^5\,\mathrm{Pa}$,

$$U = \frac{5}{2} \times 10^5 \times 48 = 120 \times 10^5\,\mathrm{J} = 12 \times 10^6\,\mathrm{J}.$$

Pressure times volume has units $\mathrm{Pa\,m^3} = \mathrm{J}$. Using $1.013 \times 10^5\,\mathrm{Pa}$ gives $12.156 \times 10^6\,\mathrm{J}$, whose integer coefficient is still 12. The monatomic factor $3/2$ would incorrectly omit the two rotational modes.

Step 3: Conclusion The internal energy is approximately $12 \times 10^6\,\mathrm{J}$. The numerical value to enter is $\boxed{12}$.

Quick Tip to Solve: When pressure and volume are given, replace nRT by PV; temperature is unnecessary. Use $5/2$ for an ordinary diatomic gas.

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50. A satellite of mass 1000 kg is launched to revolve around the earth in an orbit at a height of 270 km from the earth's surface. Kinetic energy of the satellite in this orbit is_____ $\times 10^{10}$ J.
(Mass of earth $= 6 \times 10^{24}$ kg**, Radius of earth** $= 6.4 \times 10^{6}$ m**, Gravitational constant** $= 6.67 \times 10^{-11} \mathrm{Nm}^2\ \mathrm{kg}^{-2}$ **)** [4 marks]

Solution:

Concept:

- In a circular orbit, gravitational attraction supplies centripetal force: $GMm/r^2 = mv^2/r$. Here M is Earth's mass, m is the satellite mass, and r is the distance from Earth's centre.
- The orbital kinetic energy is $K = mv^2/2 = GMm/(2r)$. The relevant radius is Earth's radius plus altitude, not the height above the surface alone.

Step 1: Calculate the orbital radius Convert the altitude to metres before adding it to Earth's radius:

$$h = 270\,\mathrm{km} = 2.70 \times 10^5\,\mathrm{m}, \qquad r = 6.4 \times 10^6 + 0.27 \times 10^6 = 6.67 \times 10^6\,\mathrm{m}.$$

A radius of only 270 km would put the assumed orbit deep inside Earth, so it cannot be used in the external gravitational formula.

Step 2: Obtain speed from circular-orbit balance Cancel the satellite mass from the force equation and solve for the square of its speed:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \quad \Rightarrow \quad v^2 = \frac{GM}{r}.$$

Substituting this into kinetic energy gives $K = GMm/(2r)$. Only the kinetic energy while in orbit is requested; the work required to launch from the surface or any change in gravitational potential energy is a different quantity.

Step 3: Substitute the given constants Use all the supplied values in SI units:

$$K = \frac{(6.67 \times 10^{-11})(6 \times 10^{24})(1000)}{2(6.67 \times 10^6)}$$
$$= \frac{6}{2} \times 10^{10} = 3 \times 10^{10}\,\mathrm{J}.$$

The factor of two follows from the definition of kinetic energy. Using GMm/r instead would double the result.

Step 4: Conclusion The satellite's orbital kinetic energy is $\boxed{3 \times 10^{10}\,\mathrm{J}}$, so the required numerical coefficient is $\boxed{3}$.

Quick Tip to Solve: For a circular satellite orbit, use $K = GMm/[2(R+h)]$; first express both radius and altitude in the same unit.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Given below are two statements :

Statement (I) : Neopentane forms only one monosubstituted derivative.

Statement (II) : Melting point of neopentane is higher than n-pentane.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is incorrect but Statement II is correct

(B) Both Statement I and Statement II are correct

(C) Statement I is correct but Statement II is incorrect

(D) Both Statement I and Statement II are incorrect

Correct Answer: (B) Both Statement I and Statement II are correct

Solution:

Concept:

- Neopentane is 2,2-dimethylpropane, $C(CH_3)_4$. Its four methyl groups are symmetry-equivalent, and replacement of any one hydrogen therefore produces the same constitutional monosubstitution product.
- Melting point depends strongly on packing in a solid crystal. A compact, highly symmetric molecule can pack efficiently even when branching lowers its boiling point relative to a less branched isomer.

Step 1: Evaluate Statement I using equivalent hydrogens The central carbon of neopentane has no hydrogen. All twelve hydrogens belong to four identical methyl groups arranged symmetrically about that carbon. Replacing one hydrogen by a substituent gives the same connectivity whichever group is selected. Thus there is only one distinct monosubstituted derivative, and Statement I is correct. Counting the four methyl groups as four different positions would ignore the molecular symmetry.

Step 2: Evaluate Statement II using solid-state packing Neopentane has a substantially higher melting point than normal pentane because its greater symmetry favours efficient crystal packing. Approximately, neopentane melts near $-16.6\,^{\circ}C$, whereas n-pentane melts near $-130\,^{\circ}C$. The less negative temperature is the higher melting point, so Statement II is also correct. The usual decrease of boiling point on branching is not a rule for melting points.

Step 3: Select the pair of truth values Both statements have independently been established as true. Option (A) rejects the correct first statement, option (C) rejects the correct second statement, and option (D) rejects both. Only the option asserting that both statements are correct agrees with symmetry and crystal packing.

Step 4: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (B).

Quick Tip to Solve: Use molecular symmetry to count substitution products; use crystal packing, rather than the boiling-point branching trend, to compare melting points.

52. **Match List - I with List - II. Choose the correct answer from the options given below:** [4 marks]

List - I (Reaction)		List - II (Name of reaction)	
(A)		(I)	Lucas reaction
(B)	$ArN_2^+X^- \xrightarrow[HCl]{Cu} ArCl + N_2 \uparrow + CuX$	(II)	Finkelstein reaction
(C)	$C_2H_5Br + NaI \xrightarrow[Acetone]{Dry} C_2H_5I + NaBr$	(III)	Fittig reaction
(D)	$CH_3C(OH)(CH_3)CH_3 \xrightarrow[ZnCl_2]{HCl} CH_3C(Cl)(CH_3)CH_3$	(IV)	Gatterman reaction

(A) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(B) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(C) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(D) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Correct Answer: (D) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Solution:

Concept:

- Identify named reactions from their substrate, reagent and characteristic bond change. Sodium in dry ether couples aryl halides in the Fittig reaction, whereas copper and hydrochloric acid replace an aryl diazonium group in the Gattermann reaction.
- Finkelstein exchange uses sodium iodide in dry acetone to replace chlorine or bromine by iodine. Lucas reagent, concentrated hydrochloric acid with anhydrous zinc chloride, converts an alcohol into an alkyl chloride.

Step 1: Match the aryl reactions In (A), two aryl halide molecules couple in the presence of sodium and dry ether to form biphenyl:

$$2\,C_6H_5X + 2\,Na \longrightarrow C_6H_5{-}C_6H_5 + 2\,NaX.$$

This is the Fittig reaction, so (A) matches (III). In (B), an aryl diazonium salt reacts with copper and hydrochloric acid, eliminating nitrogen and giving an aryl chloride. This is the Gattermann reaction, so (B) matches (IV). It is not Finkelstein exchange, which uses an alkyl halide and iodide.

Step 2: Match the alkyl reactions In (C), ethyl bromide reacts with sodium iodide in dry acetone to give ethyl iodide and sodium bromide. This halogen exchange is the Finkelstein reaction, giving (C)-(II); precipitation of sodium bromide in acetone helps drive the exchange. In (D), tert-butyl alcohol is converted to tert-butyl chloride using hydrochloric acid and zinc chloride. These are the Lucas reagents, giving (D)-(I).

Step 3: Compare the complete code The four independent matches are (A)-(III), (B)-(IV), (C)-(II), (D)-(I). Option (A) interchanges the assignments for (B) and (C). Options (B) and (C) incorrectly assign the aryl coupling to Gattermann chemistry. Only option (D) contains all four correct matches, so a single familiar reaction need not be used to guess the code.

Step 4: Conclusion The matching is $\boxed{\text{A-III, B-IV, C-II, D-I}}$. Hence, the correct option is (D).

Quick Tip to Solve: Anchor the match with NaI in acetone (Finkelstein) and HCl/ZnCl2 (Lucas); then distinguish aryl coupling from diazonium replacement.

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53. Electronic configuration of four elements A, B, C and D are given below :

(A) $1s^2 2s^2 2p^3$

(B) $1s^2 2s^2 2p^4$

(C) $1s^2 2s^2 2p^5$

(D) $1s^2 2s^2 2p^2$

Which of the following is the correct order of increasing electronegativity (Pauling's scale)? [4 marks]

(A) $\mathrm{D} < \mathrm{A} < \mathrm{B} < \mathrm{C}$

(B) $\mathrm{A} < \mathrm{B} < \mathrm{C} < \mathrm{D}$

(C) $\mathrm{A} < \mathrm{C} < \mathrm{B} < \mathrm{D}$

(D) $\mathrm{A} < \mathrm{D} < \mathrm{B} < \mathrm{C}$

Correct Answer: (A) $\mathrm{D} < \mathrm{A} < \mathrm{B} < \mathrm{C}$

Solution:

Concept:

- Electronegativity is an atom's tendency to attract a shared electron pair in a chemical bond. Across a period from left to right, increasing effective nuclear charge and decreasing atomic size generally increase this tendency.
- For the second-period elements carbon, nitrogen, oxygen and fluorine, the Pauling electronegativity values increase in the sequence $\mathrm{C} < \mathrm{N} < \mathrm{O} < \mathrm{F}$. Configurations must first be mapped to elements before the labels are ordered.

Step 1: Identify each element from its configuration Count all electrons in each neutral-atom configuration. Element A has $2 + 2 + 3 = 7$ electrons and is nitrogen. Element B has $2 + 2 + 4 = 8$ electrons and is oxygen. Element C has $2 + 2 + 5 = 9$ electrons and is fluorine. Element D has $2 + 2 + 2 = 6$ electrons and is carbon. All four lie in the second period, with valence electrons in the same principal shell.

Step 2: Apply the left-to-right trend As nuclear charge increases from carbon to fluorine, the additional electrons enter the same shell, so shielding does not offset the increased nuclear attraction. Atomic size decreases and the attraction for bonding electrons increases. The approximate Pauling values support the order: carbon 2.55, nitrogen 3.04, oxygen 3.44 and fluorine 3.98. Therefore the increasing sequence is carbon, nitrogen, oxygen, fluorine.

Step 3: Translate the order back into labels Replacing element names by the assigned labels gives

$$\mathrm{D} < \mathrm{A} < \mathrm{B} < \mathrm{C}.$$

Option (B) puts carbon above fluorine, while options (C) and (D) also place carbon above elements to its right in the period. These contradict the Pauling trend. Nitrogen's half-filled subshell causes an ionisation-energy anomaly relative to oxygen, but it does not reverse their electronegativity order.

Step 4: Conclusion The increasing electronegativity order is $\boxed{\mathrm{D} < \mathrm{A} < \mathrm{B} < \mathrm{C}}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert configurations to atomic numbers before comparing properties; do not transfer the nitrogen-oxygen ionisation-energy exception to electronegativity.

54. A tetrapeptide, " x " on complete hydrolysis produced glycine (Gly), alanine (Ala), valine (Val), leucine (Leu) in equimolar proportion each. The number of tetrapeptides (sequences) possible involving each of these amino acids is : [4 marks]

(A) 32

(B) 24

(C) 8

(D) 16

Correct Answer: (B) 24

Solution:

Concept:

- A tetrapeptide contains four amino-acid residues joined in a definite sequence from the N-terminus to the C-terminus. Complete hydrolysis breaks its peptide bonds and reveals the identities and proportions of the constituent residues.
- When all four residues are different and each is used once, the number of linear sequences is the number of permutations of four distinguishable objects: $4! = 4 \times 3 \times 2 \times 1$. A sequence and its reverse generally represent different peptides.

Step 1: Deduce the residue composition Complete hydrolysis gives glycine, alanine, valine and leucine in equal molar amounts. Since there are exactly four residues in a tetrapeptide and four different amino acids appear, each peptide must contain one Gly, one Ala, one Val and one Leu. No amino acid can be repeated without either exceeding four residues or leaving another reported amino acid absent.

Step 2: Count positions from the N-terminus There are four possible choices for the first residue. After choosing it, three amino acids remain for the second position, two for the third, and the last amino acid is fixed at the fourth position. By the multiplication principle,

$$N = 4 \times 3 \times 2 \times 1 = 4! = 24.$$

This counts every possible placement once. A sequence such as Gly-Ala-Val-Leu differs from Leu-Val-

Ala-Gly because the peptide backbone has distinct N- and C-termini.

Step 3: Exclude alternative counting rules No division by repetition factorials is needed because all four amino acids are different. No division by two for reversal or by four for cyclic rotations is justified: the question concerns ordinary linear tetrapeptide sequences, not an undirected chain or a cyclic peptide. Choices 32, 8 and 16 do not equal the permutation count for these four distinct residues.

Step 4: Conclusion The number of possible tetrapeptide sequences is $\boxed{24}$. Hence, the correct option is (B).

Quick Tip to Solve: For a linear peptide containing distinct residues exactly once, count permutations from N to C; equal hydrolysis proportions fix composition, not order.

55. Which among the following molecules is (a) involved in $\mathrm{sp^3\ d}$ hybridization, (b) has different bond lengths and (c) has lone pair of electrons on the central atom? [4 marks]

(A) $\mathrm{XeF_4}$

(B) $\mathrm{XeF_2}$

(C) $\mathrm{PF_5}$

(D) $\mathrm{SF_4}$

Correct Answer: (D) $\mathrm{SF_4}$

Solution:

Concept:

- The steric number is the number of sigma bonds plus lone pairs on the central atom. Steric number five corresponds to the conventional sp^3d description and a trigonal-bipyramidal electron-pair arrangement.
- A lone pair prefers an equatorial site in this arrangement because that site has fewer interactions at $90°$. Axial and equatorial bonds can consequently have different lengths.

Step 1: Identify the electron pairs in sulphur tetrafluoride Sulphur has six valence electrons. In $\mathrm{SF_4}$, four electrons form four S–F sigma bonds and the remaining two form one lone pair. Thus the central atom has four bond pairs and one lone pair: its steric number is five, giving the conventional sp^3d hybridization. The molecular shape is see-saw because one equatorial position is occupied by the lone pair.

Step 2: Check the bond lengths and eliminate the other molecules The two axial S–F bonds experience more repulsive interactions than the two equatorial bonds, so the axial bonds are longer. Thus $\mathrm{SF_4}$ satisfies all three requirements. $\mathrm{XeF_4}$ has six electron domains and the conventional sp^3d^2 description. $\mathrm{XeF_2}$ has five domains and lone pairs, but its two axial Xe–F bonds are equivalent. $\mathrm{PF_5}$ has unequal axial and equatorial bonds, but phosphorus has no lone pair. Each of these alternatives therefore fails at least one condition.

Step 3: Conclusion $\mathrm{SF_4}$ has sp^3d hybridization, unequal axial and equatorial bond lengths, and one lone pair on sulphur. Hence, the correct option is (D).

Quick Tip to Solve: Apply every condition in the stem: unequal bonds alone would also suggest $\mathrm{PF_5}$, but the lone-pair condition excludes it.

56. In 3,3-dimethylhex-1-en-4-yne, there are _____sp^3,_____sp^2 and_____ sp hybridised carbon atoms respectively. [4 marks]

(A) $4, 2, 2$

(B) $2, 4, 2$

(C) $3, 3, 2$

(D) $2, 2, 4$

Correct Answer: (A) $4, 2, 2$

Solution:

Concept:

- A carbon in a C=C bond is sp^2-hybridized, with three sigma-bond directions and one unhybridized p orbital. A carbon in a C≡C bond is sp-hybridized, with two sigma-bond directions.
- A carbon bonded only through four single bonds is sp^3-hybridized. Count the carbon atoms in substituents as well as those in the named parent chain.

Step 1: Write the carbon skeleton from the name The parent is a six-carbon chain with a double bond between C1 and C2 and a triple bond between C4 and C5. Two methyl groups are attached to C3. Its condensed structure is

$$\mathrm{CH_2{=}CH{-}C(CH_3)_2{-}C{\equiv}C{-}CH_3}.$$

There are eight carbon atoms in total: six in the parent chain and two in the methyl substituents. The locant 3,3 identifies the positions of these substituents; it does not introduce any additional multiple bond.

Step 2: Classify each carbon and check the total C1 and C2 are the two carbons of the double bond, so both are sp^2. C4 and C5 are the two carbons of the triple bond, so both are sp. C3 has four single bonds, C6 is a terminal methyl carbon, and each of the two methyl substituents has only single bonds: these four carbons are sp^3. Thus the count is

$$N(sp^3) = 4, \qquad N(sp^2) = 2, \qquad N(sp) = 2.$$

The sum $4 + 2 + 2 = 8$ agrees with the molecular skeleton. Options (B), (C) and (D) incorrectly assign additional carbons to double or triple bonds that the name does not contain.

Step 3: Conclusion The numbers of sp^3, sp^2 and sp carbon atoms are respectively $\boxed{4, 2, 2}$. Hence, the correct option is (A).

Quick Tip to Solve: Mark the two ends of each multiple bond first, then classify all remaining carbon atoms, including methyl branches, as sp^3.

57. In Dumas' method for estimation of nitrogen, 0.5 gram of an organic compound gave 60 mL of nitrogen collected at 300 K temperature and 715 mm Hg pressure. The percentage composition of nitrogen in the compound (Aqueous tension at $300\ \text{K} = 15\ \text{mmHg}$) is_____%. [4 marks]

(A) 20.87

(B) 12.57

(C) 1.257

(D) 18.67

Correct Answer: (B) 12.57

Solution:

Concept:

- Gas collected over water contains nitrogen and water vapour. Dalton's law gives the dry nitrogen pressure as $P_{\text{N}_2} = P_{\text{total}} - P_{\text{water}}$.
- Use $n = PV/(RT)$, where pressure is in atm, volume in litres and $R = 0.0821\,\text{L atm mol}^{-1}\,\text{K}^{-1}$. The mass of nitrogen is $28n$ grams because the gas is N_2.

Step 1: Correct the measured gas pressure The total pressure includes the aqueous tension, so subtract it before using the gas equation:

$$P_{\text{N}_2} = 715 - 15 = 700\,\text{mm Hg} = \frac{700}{760}\,\text{atm}.$$

The measured gas volume is $60\,\text{mL} = 0.060\,\text{L}$, and its temperature is already given on the absolute scale as $300\,\text{K}$. There is no need to add 273 to this temperature.

Step 2: Find the nitrogen mass and its percentage The amount of dry nitrogen is

$$n(\text{N}_2) = \frac{(700/760)(0.060)}{(0.0821)(300)} \simeq 2.244 \times 10^{-3}\,\text{mol}.$$

Hence its mass is $m_N = 28n \simeq 0.06283\,\text{g}$. All the mass of this nitrogen gas came from the nitrogen in the original compound. Therefore

$$\%N = \frac{0.06283}{0.500} \times 100 \simeq 12.57\%.$$

Option (C) is smaller by a factor of ten, while (A) and (D) do not agree with the pressure-corrected gas calculation. Using atomic mass 14 for a mole of nitrogen gas would incorrectly halve the result.

Step 3: Conclusion The percentage composition of nitrogen is $\boxed{12.57\%}$. Hence, the correct option is (B).

Quick Tip to Solve: For moist nitrogen, subtract aqueous tension before calculating moles, then use molar mass 28 for N_2, not 14.

58. Which of the following statements are true?

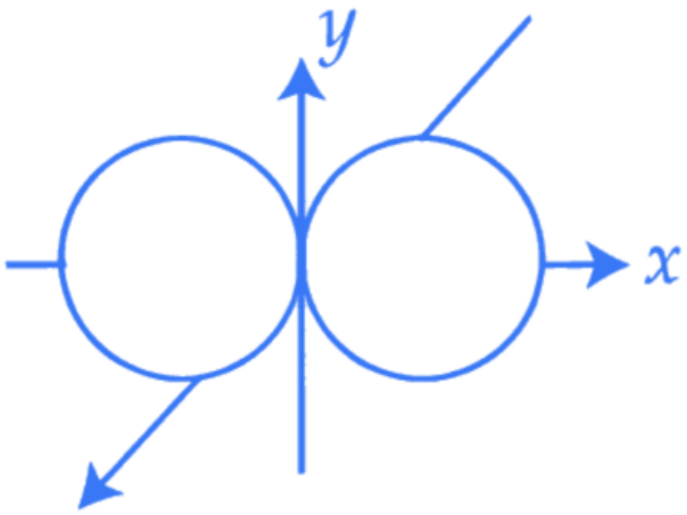
(A) The subsidiary quantum number l describes the shape of the orbital occupied by the electron.

(B) is the boundary surface diagram of the $2\mathrm{p}_x$ orbital.

(C) The + and - signs in the wave function of the $2p_x$ orbital refer to charge.

(D) The wave function of $2\mathrm{p}_x$ orbital is zero everywhere in the xy plane.

Choose the correct answer from the options given below : [4 marks]



(A) (A) and (B) only

(B) (B) and (D) only

(C) (C) and (D) only

(D) (A), (B) and (C) only

Correct Answer: (A) (A) and (B) only

Solution:

Concept:

- The azimuthal or subsidiary quantum number l identifies the orbital type and hence its shape: $l = 0, 1, 2$ correspond to s, p and d orbitals respectively.
- The signs of the wave function indicate its phase, whereas probability density is proportional to $|\psi|^2$. A nodal plane is a plane on which the wave function vanishes.

Step 1: Evaluate the shape and orientation statements Statement (A) is true: the subsidiary quantum number distinguishes orbital shapes, such as the spherical s orbital and the dumb-bell-shaped p orbital. Statement (B) is also true. The displayed diagram has two lobes directed along the x axis, one on each side of the nucleus, which is the orientation of a $2p_x$ orbital. The lobe direction, rather than the direction of an unrelated axis drawn through the origin, identifies the orbital.

Step 2: Distinguish phase from charge and locate the node Statement (C) is false. A plus or minus sign on a wave-function lobe does not denote positive or negative electric charge. The electron has the same negative charge in either lobe, and squaring the wave function gives nonnegative probability

density. Statement (D) is false because a $2p_x$ wave function contains an angular factor proportional to x. It therefore vanishes on $x = 0$, the yz plane, rather than throughout the xy plane. For example, a point on the positive x axis lies in the xy plane yet belongs to a lobe, so its wave function need not be zero. Only statements (A) and (B) survive; every other listed combination includes at least one false statement.

Step 3: Conclusion Only the orbital-shape and boundary-surface statements are true. Hence, the correct option is (A).

Quick Tip to Solve: For p_x, the lobes follow x and the nodal plane is perpendicular to x: set $x = 0$ to obtain the yz plane.

---

59. Arrange the following in order of magnitude of work done by the system/on the system at constant temperature.

(a) $|w_{\text{reversible}}|$ **for expansion in infinite stages.**

(b) $|w_{\text{irreversible}}|$ **for expansion in single stage.**

(c) $|\text{w}_{\text{reversible}}|$ **for compression in infinite stages.**

(d) $|w_{\text{irreversible}}|$ **for compression in single stage.**

Choose the correct answer from the options given below : [4 marks]

(A) $\text{d} > \text{c} = \text{a} > \text{b}$

(B) $\text{c} = \text{a} > \text{d} > \text{b}$

(C) $a > c > b > d$

(D) $a > b > c > d$

Correct Answer: (A) $\text{d} > \text{c} = \text{a} > \text{b}$

Solution:

Concept:

- For the same initial and final states, reversible isothermal expansion yields the maximum work by the system. Reversible compression requires the minimum work on the system.
- For an ideal gas at fixed temperature, the reversible work magnitude is $nRT\,|\ln(V_f/V_i)|$. Reversing the path changes its sign but leaves its magnitude unchanged.

Step 1: Compare the two expansion paths Take the expansion between volumes V_1 and V_2, with $V_2 > V_1$. In the reversible path the opposing pressure tracks the gas pressure through infinitely small changes. Its work magnitude is

$$a = nRT \ln \frac{V_2}{V_1}.$$

In a one-stage expansion the gas pushes against a smaller constant external pressure. The area under this horizontal pressure line is smaller than the area under the reversible isotherm, so the work produced is smaller: $a > b$. This comparison concerns magnitudes, so no negative expansion sign enters the ranking.

Step 2: Compare the compression paths and combine the inequalities Reversibly returning between

the same endpoint states retraces the isotherm. Consequently its compression work magnitude equals that of reversible expansion: $c = a$. Single-stage compression requires an external pressure greater than the instantaneous equilibrium gas pressure over the path, so more work must be supplied than for reversible compression. Thus $d > c$. Combining these facts gives

$$d > c = a > b.$$

Option (B) puts irreversible compression below reversible compression, while (C) and (D) incorrectly separate the equal reversible magnitudes. Their rankings therefore contradict the endpoint and maximum/minimum work arguments.

Step 3: Conclusion The correct order of magnitudes is $\boxed{d > c = a > b}$. Hence, the correct option is (A).

Quick Tip to Solve: Reversible expansion gives the most work; reversible compression needs the least. The reversible forward and reverse paths have equal work magnitudes.

60. When a concentrated solution of sulphanilic acid and 1-naphthylamine is treated with nitrous acid $(273\ \mathrm{K})$ **and acidified with acetic acid, the mass** (g) **of 0.1 mole of product formed is :**

(Given molar mass in $\mathrm{gmol}^{-1}\mathrm{H} : 1, \mathrm{C} : 12,\ \mathrm{N} : 14, \mathrm{O} : 16,\ \mathrm{S} : 32$ **)** [4 marks]

(A) 330

(B) 33

(C) 343

(D) 66

Correct Answer: (B) 33

Solution:

Concept:

- An aromatic primary amine reacts with nitrous acid at low temperature to form a diazonium species. The diazonium group can couple with an activated aromatic ring to form an azo linkage, $-\mathrm{N}{=}\mathrm{N}-$.
- The coloured coupling product from diazotized sulphanilic acid and 1-naphthylamine retains the sulfonic acid group and the amino group on the naphthalene ring. Its molecular mass follows from its elemental formula.

Step 1: Identify the formula of the azo dye Sulphanilic acid provides the para-sulfophenyl part, $\mathrm{HO_3S{-}C_6H_4^-}$, whose amino group is converted into the diazonium group. Coupling with the activated naphthylamine ring joins the rings through $-\mathrm{N}{=}\mathrm{N}-$. The naphthalene unit retains its $\mathrm{NH_2}$ group and loses one ring hydrogen at the coupling position. The product may therefore be represented as $\mathrm{HO_3S{-}C_6H_4{-}N{=}N{-}C_{10}H_6{-}NH_2}$. Counting atoms gives

$$\mathrm{C_{16}H_{13}N_3O_3S}.$$

The low temperature supports diazotization, and the acidified medium is part of the stated coupling

conditions.

Step 2: Calculate the molar mass and the requested mass Using the atomic masses supplied in the question,

$$M = 16(12) + 13(1) + 3(14) + 3(16) + 1(32)$$
$$= 192 + 13 + 42 + 48 + 32 = 327\,\text{g mol}^{-1}.$$

For 0.1 mole, the mass is

$$m = nM = 0.1(327) = 32.7\,\text{g}.$$

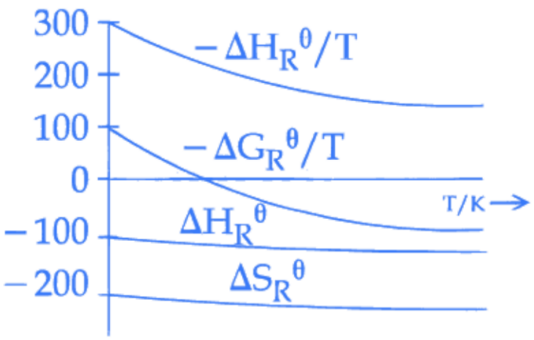
Among the listed whole-gram choices this corresponds to $33\,\text{g}$. The choice 330 is of the order of a molar mass rather than the mass of 0.1 mole; 343 does not match the atom count, and 66 would approximately double the required mass.

Step 3: Conclusion The azo product has formula $C_{16}H_{13}N_3O_3S$, giving $32.7\,\text{g}$ for 0.1 mole, or $\boxed{33\,\text{g}}$ to the listed precision. Hence, the correct option is (B).

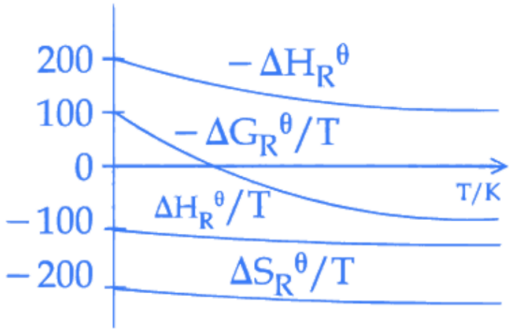
Quick Tip to Solve: Count the atoms in the coupled dye before multiplying by 0.1; the nitrogen in the azo bridge is additional to the retained amino nitrogen.

61. Which of the following graphs correctly represents the variation of thermodynamic properties of Haber's process? [4 marks]

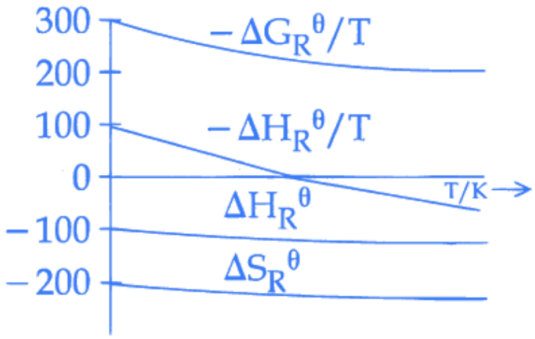
(A)



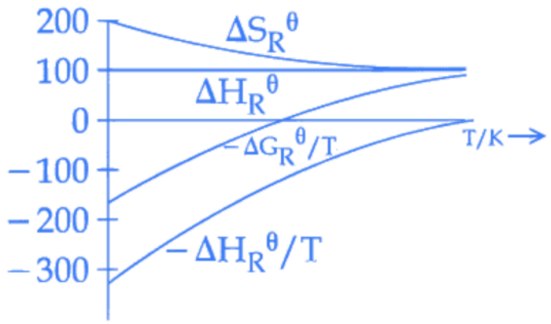
(B)



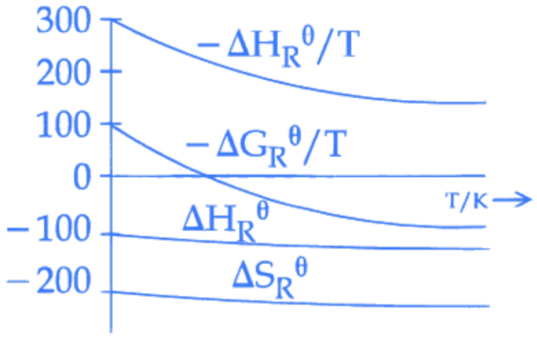
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- Haber's process is $N_2(g) + 3\,H_2(g) \rightleftharpoons 2\,NH_3(g)$. It is exothermic, so its standard reaction enthalpy ΔH_R° is negative.
- The number of gaseous moles decreases from four to two; therefore the standard reaction entropy ΔS_R° is negative. Gibbs energy satisfies $\Delta G_R^\circ = \Delta H_R^\circ - T\Delta S_R^\circ$.

Step 1: Determine the signs of the plotted quantities The enthalpy and entropy curves must both lie below zero for ammonia formation. Since the absolute temperature is positive, the quantity $-\Delta H_R^\circ/T$ must lie above zero. Its magnitude generally decreases as temperature rises in the qualitative representation used in the options. These sign tests immediately exclude the graph showing positive reaction enthalpy and positive reaction entropy.

Step 2: Compare the Gibbs-energy curve with the enthalpy curve Divide the Gibbs equation by $-T$ to

obtain

$$-\frac{\Delta G^\circ_R}{T} = -\frac{\Delta H^\circ_R}{T} + \Delta S^\circ_R.$$

The entropy term is negative, so the Gibbs-energy curve must lie below the positive $-\Delta H^\circ_R/T$ curve. At lower temperature the favourable enthalpy contribution dominates; at higher temperature the negative entropy contribution can make $-\Delta G^\circ_R/T$ negative. Graph A displays precisely this arrangement. Graph B assigns the ratios to the wrong curves, while graph C incorrectly makes the enthalpy ratio cross zero and places the Gibbs ratio above it.

Step 3: Conclusion The graph with negative ΔH°_R, negative ΔS°_R, positive $-\Delta H^\circ_R/T$, and the lower Gibbs-energy ratio is graph A. Hence, the correct option is (A).

Quick Tip to Solve: For a thermodynamic graph, establish the signs first and then use $-\Delta G/T = -\Delta H/T + \Delta S$ to compare the heights of the two ratio curves.

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62. Reactant A converts to product D through the given mechanism (with the net evolution of heat):

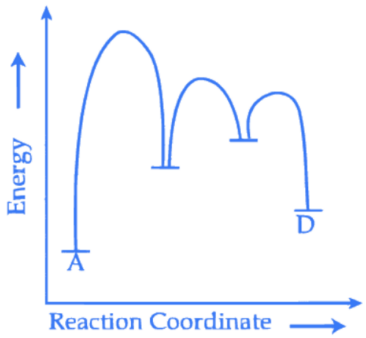
A $\rightarrow$ B slow; ΔH = +ve

B $\rightarrow$ C fast; ΔH = -ve

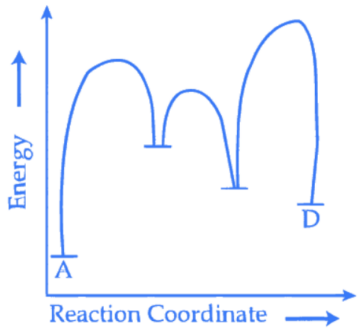
C $\rightarrow$ D fast; ΔH = -ve

Which of the following represents the above reaction mechanism? [4 marks]

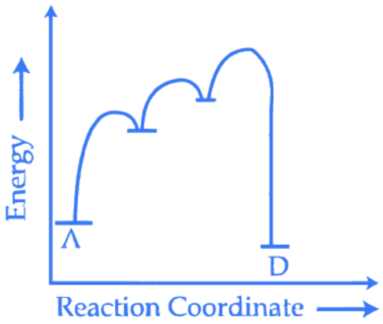
(A)



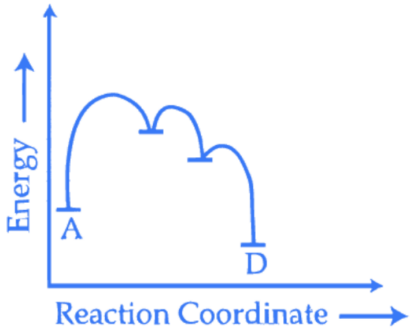
(B)



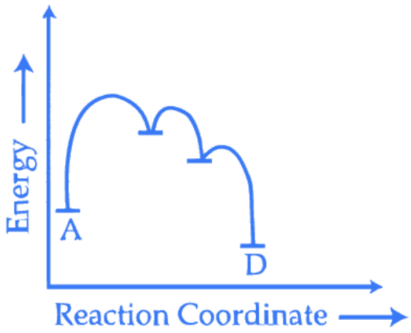
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- In an energy profile, the vertical difference between consecutive minima gives the enthalpy change of a step. A higher final minimum denotes an endothermic step; a lower one denotes an exothermic step.
- The activation energy of a forward step is measured from its own starting minimum to the following maximum. The slow step has the dominant forward barrier, and overall heat evolution requires $E_D < E_A$.

Step 1: Translate the three steps into energy levels The first step, A to B, has positive enthalpy change, so B must be above A. The second step releases heat, so C must lie below B. The third step also releases heat, so D must lie below C. In addition, the stated net evolution of heat requires the final product D to lie below the original reactant A. There must be three maxima because the mechanism contains three elementary steps and two intermediate minima.

Step 2: Check the barriers and reject the other profiles In graph D, the first rise from A to the first maximum is the largest forward activation barrier. Its first intermediate lies above A, its next intermediate lies below the first, and its product D lies below both C and A. Thus it satisfies both the rate information and all enthalpy signs. Graph A has its second intermediate above the first and its product above A. Graph B also finishes above A and has an unsuitable final barrier. Graph C has successive intermediate levels rising, so its second step is endothermic rather than exothermic. Comparing peak heights alone would miss the fact that each barrier starts at a different minimum.

Step 3: Conclusion Graph D represents a slow endothermic first step, two subsequent fast exothermic steps, and an overall exothermic reaction. Hence, the correct option is (D).

Quick Tip to Solve: Read reaction enthalpy from minimum to minimum, but activation energy from a minimum to the next peak; these are different vertical distances.

---

63. Formation of $Na_4\,[Fe(CN)_5NOS]$, a purple coloured complex formed by addition of sodium nitroprusside in sodium carbonate extract of salt indicates the presence of :

[4 marks]

(A) Sulphide ion

(B) Sulphite ion

(C) Sulphate ion

(D) Sodium ion

Correct Answer: (A) Sulphide ion

Solution:

Concept:

- Sodium nitroprusside, $Na_2[Fe(CN)_5NO]$, is a qualitative reagent for sulphide ions in an alkaline medium.
- The sulphide ion reacts with the nitroprusside complex to produce the characteristic purple thionitroprusside complex $[Fe(CN)_5NOS]^{4-}$. The sodium ions are counterions.

Step 1: Identify the ion incorporated into the coloured complex The product differs from the nitroprusside reagent by the incorporation of sulphur into the nitrosyl-containing complex. The diagnostic reaction is

$$[Fe(CN)_5NO]^{2-} + S^{2-} \longrightarrow [Fe(CN)_5NOS]^{4-}.$$

The charges balance: two negative charges on the original complex and two on sulphide give the four negative charges on the coloured product. Four sodium counterions therefore give the printed formula $Na_4[Fe(CN)_5NOS]$.

Step 2: Distinguish sulphide from the other listed ions The characteristic purple colour is the sodium nitroprusside test for sulphide, not a general test for every sulphur-containing anion. Sulphite contains oxygen and has sulphur in a different oxidation state; it is commonly recognised through sulphur dioxide formation and reducing behaviour. Sulphate is a stable oxoanion and is tested by formation of an insoluble barium salt under appropriate conditions. Sodium cannot be inferred from this colour

because sodium is already present in the reagent and in the sodium carbonate extract; it acts as a spectator counterion rather than the species responsible for the diagnostic reaction.

Step 3: Conclusion The purple thionitroprusside complex indicates the presence of S^{2-}, the sulphide ion. Hence, the correct option is (A).

Quick Tip to Solve: Associate the purple nitroprusside test specifically with sulphide; the sodium in the product formula is only a counterion and is not the detected species.

64. Consider the following reactions. From these reactions which reaction will give carboxylic acid as a major product ?

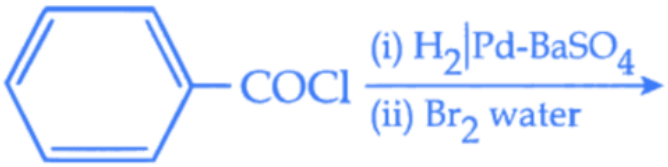
(A) $R-C\equiv N \xrightarrow[\text{mild condition}]{\text{(i) } \overset{+}{H}/H_2O}$

(B) $R-MgX \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } CO_2}$

(C) $R-C\equiv N \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } SnCl_2/HCl}$

(D) $R\cdot CH_2\cdot OH \xrightarrow{PCC}$

(E)



Choose the correct answer from the options given below : [4 marks]

(A) A, B and E only

(B) A and D only

(C) B, C and E only

(D) B and E only

Correct Answer: (D) B and E only

Solution:

Concept:

- Carboxylation of a Grignard reagent followed by acidification gives a carboxylic acid with one more carbon atom than the original alkyl group.
- Mild nitrile hydrolysis can stop at the amide; Stephen reduction and PCC oxidation produce aldehydes. Rosenmund reduction gives an aldehyde, which bromine water can oxidise to a carboxylic acid.

Step 1: Evaluate reactions A, B, C and D In A, the specified mild acidic hydrolysis of RCN leads primarily to the amide $RCONH_2$; complete conversion to the acid requires further hydrolysis under

stronger conditions. In B, carbon dioxide inserts into the carbon–magnesium bond:

$$\mathrm{RMgX} + \mathrm{CO_2} \longrightarrow \mathrm{RCOOMgX} \xrightarrow{\mathrm{H_3O^+}} \mathrm{RCOOH}.$$

Therefore B gives the acid. Reaction C is Stephen reduction: $\mathrm{SnCl_2/HCl}$ reduces the nitrile to an iminium salt, whose hydrolysis gives RCHO. In D, PCC oxidises the primary alcohol to RCHO and normally stops before the acid. Hence neither C nor D belongs in the required set.

Step 2: Follow both stages of reaction E The displayed starting material in E is benzoyl chloride. Hydrogen over poisoned palladium on barium sulphate carries out Rosenmund reduction to benzaldehyde, rather than reducing the acyl chloride directly to an alcohol. The second reagent is bromine water, an oxidising agent for aldehydes:

$$\mathrm{C_6H_5COCl} \xrightarrow{\mathrm{H_2/Pd-BaSO_4}} \mathrm{C_6H_5CHO} \xrightarrow{\mathrm{Br_2/H_2O}} \mathrm{C_6H_5COOH}.$$

Thus E ultimately gives benzoic acid. Only B and E meet the requested outcome; the choices containing A, C or D include reactions whose major products stop at an amide or aldehyde.

Step 3: Conclusion Reactions B and E give carboxylic acids as their major final products. Hence, the correct option is (D).

Quick Tip to Solve: For a sequence of reagents, follow the product through every stage; an aldehyde formed in the first stage may be oxidised to an acid in the second.

65. The nature of oxide $(\mathrm{TeO_2})$ and hydride $(\mathrm{TeH_2})$ formed by Te , respectively are : [4 marks]

(A) Reducing and basic

(B) Reducing and acidic

(C) Oxidising and acidic

(D) Oxidising and basic

Correct Answer: (C) Oxidising and acidic

Solution:

Concept:

- In group 16, the +6 oxidation state becomes less stable relative to +4 down the group. The reducing character of the dioxides decreases from sulphur dioxide to tellurium dioxide; tellurium dioxide is oxidising.
- The acidity of group 16 hydrides increases down the group because the element–hydrogen bond weakens. Thus $\mathrm{H_2Te}$, written as $\mathrm{TeH_2}$ in the question, is acidic.

Step 1: Identify the redox nature of tellurium dioxide Each oxygen in $\mathrm{TeO_2}$ has oxidation number -2, so tellurium has oxidation number $+4$. The group trend must be applied rather than assuming that every +4 oxide behaves like sulphur dioxide. Sulphur dioxide is reducing, because sulphur(IV) is readily oxidised to sulphur(VI). Down the group, the +6 state becomes less stable and this reducing tendency diminishes. Selenium dioxide and tellurium dioxide instead show oxidising character: in

an oxidation reaction they can accept electrons and the central atom is reduced. Thus the required description of the tellurium oxide is oxidising. This redox classification is separate from the acid–base description of the oxide.

Step 2: Determine the hydride character and compare the pairs Tellurium hydride belongs to the sequence H_2O, H_2S, H_2Se, H_2Te. Down this sequence, the longer and weaker E–H bond makes release of a proton easier, so acidic strength increases. Consequently H_2Te is acidic, not basic. The options pairing a basic hydride with either oxide description are ruled out. Of the two remaining acidic choices, only the pair oxidising and acidic follows both group trends. Choosing reducing and acidic incorrectly transfers the familiar reducing property of sulphur dioxide to the much heavier tellurium dioxide. Hydride acidity is governed chiefly by the weakening bond, not just the central atom electronegativity.

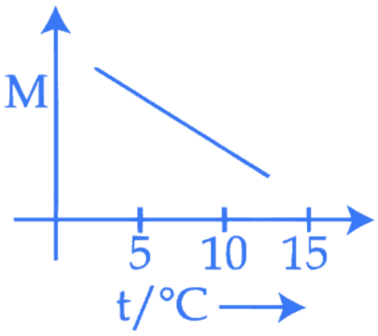
Step 3: Conclusion The oxide TeO_2 is oxidising, and the hydride H_2Te is acidic. Hence, the correct option is (C).

Quick Tip to Solve: Keep the trends separate: reducing character of group 16 dioxides decreases down the group, while acidity of their hydrides increases.

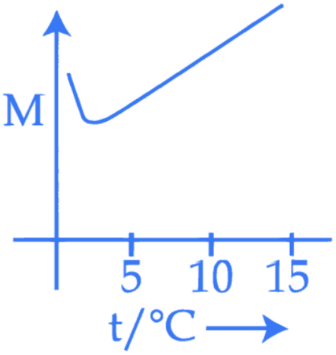
66. ' x ' g of NaCl is added to water in a beaker with a lid. The temperature of the system is raised from $1^\circ C$ to $25^\circ C$. Which out of the following plots, is best suited for the change in the molarity (M) of the solution with respect to temperature ?

[Consider the solubility of NaCl remains unchanged over the temperature range] [4 marks]

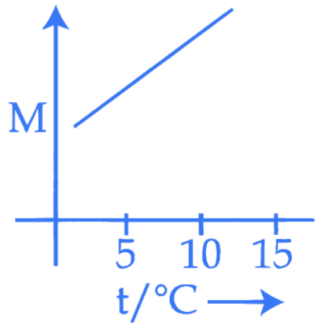
(A)



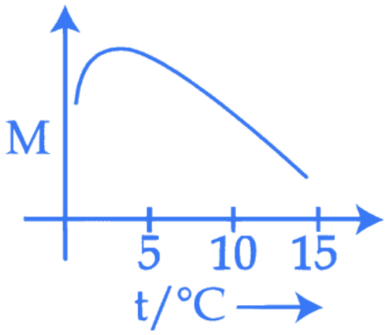
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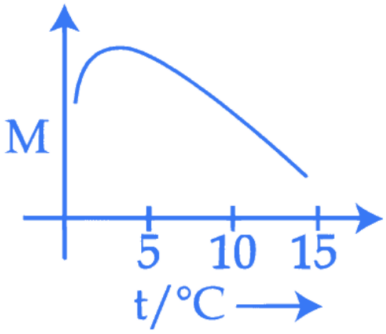
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- Molarity is the amount of solute per litre of solution: $M = n/V$. With fixed dissolved solute, it varies inversely with solution volume.
- Water shows anomalous expansion near 4°C: on warming from 1°C to about 4°C, its volume decreases, and on further warming its volume increases.

Step 1: Hold the amount of dissolved salt fixed The mass of added salt is x grams, giving a fixed amount $n = x/M_{\text{NaCl}}$, where M_{NaCl} is its molar mass. The lid prevents loss of water by evaporation, and the stated unchanged solubility eliminates a temperature-driven change in the amount dissolved. Thus the qualitative concentration change in the options is controlled by the volume of the aqueous medium, not by formation or destruction of salt. Molarity must therefore increase when the liquid contracts and decrease when it expands.

Step 2: Apply the volume anomaly over the full temperature interval Between 1°C and approximately

4°C, the water contracts on heating. The denominator V decreases, so molarity initially rises. Above this region, ordinary thermal expansion increases V, causing molarity to fall. The required curve consequently has a maximum near the anomalous-density region and is represented by graph D. Graph A omits the initial rise. Graph B gives a minimum instead of a maximum, reversing the inverse-volume relationship. Graph C predicts a continuous increase and therefore ignores the expansion at higher temperatures. The plots are qualitative; a straight-line law for the entire interval is not required.

Step 3: Conclusion The molarity first increases and then decreases, giving the curve with a maximum shown in graph D. Hence, the correct option is (D).

Quick Tip to Solve: For fixed solute, draw the molarity trend opposite to the volume trend; near water's density maximum, molarity has a corresponding maximum.

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67. The type of hybridization and the magnetic property of $[MnCl_6]^{3-}$ are, [4 marks]

(A) sp^3d^2, paramagnetic with four unpaired electrons.

(B) d^2sp^3, paramagnetic with four unpaired electrons.

(C) $sp^3\ d^2$, paramagnetic with two unpaired electrons.

(D) d^2sp^3, paramagnetic with two unpaired electrons.

Correct Answer: (A) sp^3d^2, paramagnetic with four unpaired electrons.

Solution:

Concept:

- The oxidation state determines the metal ion configuration. Manganese has atomic number 25 and the neutral configuration $[Ar]3d^5 4s^2$.
- Chloride is a weak-field ligand. An octahedral high-spin complex uses outer-orbital sp^3d^2 hybridization; unpaired electrons make it paramagnetic.

Step 1: Determine the oxidation state and d count Let the oxidation state of manganese be x. Each of the six chloride ligands contributes charge -1, so $x - 6 = -3$, giving $x = +3$. Remove the two 4s electrons and then one 3d electron from neutral Mn. Thus Mn^{3+} is a $3d^4$ ion. The number of ligands fixes an octahedral coordination environment, but does not alone decide whether the complex is high spin.

Step 2: Fill the orbitals and identify the hybridization Because Cl^- is a weak-field ligand, the octahedral splitting is insufficient to force pairing before occupancy of the upper set. The configuration is $t_{2g}^3 e_g^1$, with one electron in each of four orbitals and hence four unpaired electrons. There are not two vacant inner 3d orbitals available for d^2sp^3 bonding in this high-spin arrangement. The valence-bond description therefore uses 4s, three 4p and two 4d orbitals, giving outer-orbital sp^3d^2 hybridization. Options with two unpaired electrons or inner-orbital hybridization do not describe this chloride complex.

Step 3: Conclusion The complex has sp^3d^2 hybridization and is paramagnetic with four unpaired electrons. Hence, the correct option is (A).

Quick Tip to Solve: Find the metal d count before considering hybridization: weak-field chloride keeps the octahedral d4 ion high spin.

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68. Match List-I with List-II.

List-I: Purification technique		List-II: Mixture	
(A)	Distillation (simple)	(I)	Diesel + Petrol
(B)	Fractional distillation	(II)	Aniline + Water
(C)	Distillation under reduced pressure	(III)	Chloroform + Aniline
(D)	Steam distillation	(IV)	Glycerol + Spent-lye

Choose the correct answer from the options given below: [4 marks]

(A) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(B) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(C) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(D) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

Correct Answer: (A) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Solution:

Concept:

- Distillation separates liquids by their volatility. Simple distillation is suitable for a large boiling-point difference, whereas fractional distillation uses repeated vaporization and condensation.
- Reduced pressure lowers the boiling temperature of a high-boiling liquid. Steam distillation is suitable for a steam-volatile organic liquid that is substantially immiscible with water.

Step 1: Match the ordinary distillation methods Chloroform boils near $61\,^{\circ}\mathrm{C}$, whereas aniline boils near $184\,^{\circ}\mathrm{C}$. Their large boiling-point separation permits simple distillation, so (A) matches (III). Petrol and diesel are petroleum fractions containing hydrocarbons with different boiling ranges. A fractionating column supplies repeated equilibration between vapour and liquid, making fractional distillation the suitable listed technique for their separation. Thus (B) matches (I).

Step 2: Match the pressure-assisted methods and check the options Glycerol has a very high atmospheric boiling point and may decompose when strongly heated. Distillation under reduced pressure permits its recovery from spent-lye at a lower temperature, so (C) matches (IV). Aniline is an organic liquid that is only sparingly soluble in water and can be carried over with steam; the sum of the vapour pressures enables distillation below its ordinary boiling point. Hence steam distillation matches aniline and water: (D) matches (II). Only the first option has all four assignments. The remaining options incorrectly assign reduced-pressure distillation to a different mixture or steam distillation to the petroleum fractions.

Step 3: Conclusion Simple, fractional, reduced-pressure and steam distillation match III, I, IV and II respectively. Hence, the correct option is (A).

Quick Tip to Solve: Use glycerol as the reduced-pressure clue and aniline with water as the steam-distillation clue before matching the other two.

69. The d-orbital electronic configuration of the complex among $[Co(en)_3]^{3+}$, $[CoF_6]^{3-}$, $[Mn(H_2O)_6]^{2+}$ and $[Zn(H_2O)_6]^{2+}$ that has the highest CFSE is : [4 marks]

(A) $t_{2g}{}^6e_g^0$

(B) $t_{2g}{}^3e_g^2$

(C) $t_{2\,g}{}^4e_g{}^2$

(D) $t_{2g}{}^6e_g^4$

Correct Answer: (A) $t_{2g}{}^6e_g^0$

Solution:

Concept:

- For an octahedral complex, each t_{2g} electron contributes $-0.4\Delta_o$ and each e_g electron contributes $+0.6\Delta_o$ to the orbital crystal-field energy.
- A larger lowering of energy means greater crystal-field stabilization. Ligand strength determines whether a d6 ion adopts the low-spin or high-spin configuration.

Step 1: Assign the configurations of the four complexes In $[Co(en)_3]^{3+}$, ethylenediamine is neutral, so cobalt is +3 and has a d6 configuration. The strong ligand field produces the low-spin arrangement $t_{2g}^6e_g^0$. In $[CoF_6]^{3-}$, cobalt is also +3, but weak-field fluoride gives high-spin $t_{2g}^4e_g^2$. Water ligands in $[Mn(H_2O)_6]^{2+}$ give high-spin Mn(II), d5: $t_{2g}^3e_g^2$. Zinc(II) is d10, so its aqua complex has $t_{2g}^6e_g^4$.

Step 2: Compare the orbital stabilization energies Applying the octahedral contributions to the four candidate arrangements gives

$$\begin{aligned} t_{2g}^6e_g^0 &: \quad -2.4\Delta_o, \\ t_{2g}^3e_g^2 &: \quad 0, \\ t_{2g}^4e_g^2 &: \quad -0.4\Delta_o, \\ t_{2g}^6e_g^4 &: \quad 0. \end{aligned}$$

Thus the completely occupied lower set with an empty upper set provides the greatest crystal-field stabilization. The high-spin d5 and d10 configurations have zero net orbital CFSE, and high-spin d6 has a much smaller lowering. Pairing energy is a separate contribution in deciding spin states; the comparison asked here is the CFSE.

Step 3: Conclusion The configuration with the highest CFSE is $t_{2g}^6e_g^0$, belonging to $[Co(en)_3]^{3+}$. Hence, the correct option is (A).

Quick Tip to Solve: For CFSE, count lower and upper electrons separately: a filled t2g set and empty eg set gives the largest octahedral lowering here.

70. Consider the following chemical equilibrium of the gas phase reaction at a constant temperature :
$A(g) \rightleftharpoons B(g) + C(g)$

If p being the total pressure, K_p is the pressure equilibrium constant and α is the degree of dissociation, then which of the following is true at equilibrium? [4 marks]

(A) If K_p value is extremely high compared to p, α becomes much less than unity

(B) When p increases α increases

(C) If p value is extremely high compared to K_p, $\alpha \approx 1$

(D) When p increases α decreases

Correct Answer: (D) When p increases α decreases

Solution:

Concept:

- At fixed temperature K_p is constant. For $A(g) \rightleftharpoons B(g) + C(g)$, $K_p = p_B p_C / p_A$.
- For dissociation of initially pure A, equilibrium mole numbers proportional to $1-\alpha, \alpha, \alpha$ give a total proportional to $1+\alpha$.

Step 1: Express the equilibrium constant in terms of dissociation Start with one mole of A. At equilibrium the amounts of A, B and C are $1-\alpha$, α and α mole respectively, and the total is $1+\alpha$. Their partial pressures are the corresponding mole fractions multiplied by total pressure p. Therefore

$$K_p = \frac{[\alpha p/(1+\alpha)]^2}{(1-\alpha)p/(1+\alpha)} = \frac{\alpha^2 p}{1-\alpha^2}.$$

This expression includes the change in total moles; omitting that change gives an incorrect pressure dependence.

Step 2: Determine the pressure dependence and test the limiting claims Rearranging gives

$$\alpha^2 = \frac{K_p}{K_p + p}, \qquad \alpha = \sqrt{\frac{K_p}{K_p + p}}.$$

Since K_p stays fixed at constant temperature, increasing p enlarges the denominator and decreases α. When $K_p \gg p$, the fraction approaches one and dissociation approaches completion, so option (A) reverses the limit. When $p \gg K_p$, $\alpha \approx \sqrt{K_p/p} \ll 1$, contradicting (C). Option (B) also reverses the dependence. The result agrees with compression favouring the side with fewer gas molecules, namely A.

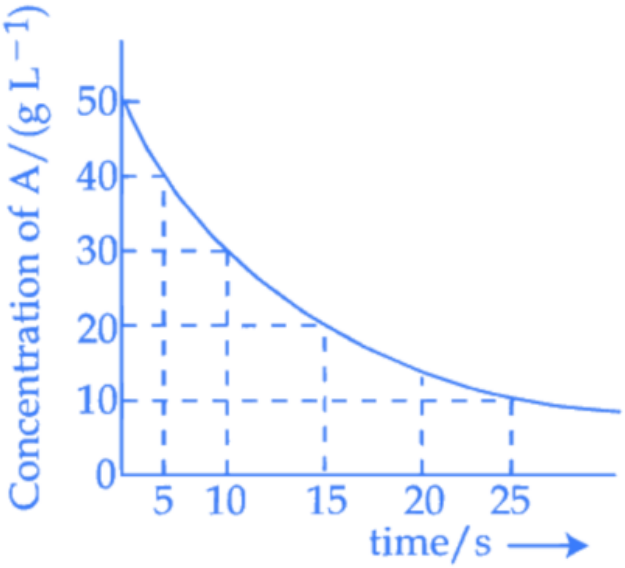
Step 3: Conclusion When total pressure p increases at constant temperature, the degree of dissociation α decreases. Hence, the correct option is (D).

Quick Tip to Solve: Derive alpha squared as Kp/(Kp+p); both the pressure trend and the two extreme limits then follow immediately.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. For the reaction $\mathrm{A} \rightarrow \mathrm{B}$ the following graph was obtained. The time required (in seconds) for the concentration of A to reduce to $2.5\ \mathrm{g\ L^{-1}}$ (if the initial concentration of A was $50\ \mathrm{g\ L^{-1}}$) is . (Nearest integer)

Given : $\log 2 = 0.3010$ [4 marks]



Solution:

Concept:

- A first-order reaction has concentration $[A]_t = [A]_0 e^{-kt}$, and its half-life $t_{1/2} = 0.693/k$ is independent of the starting concentration.
- The integrated rate law is $t = (2.303/k)\log([A]_0/[A]_t)$, where k is the first-order rate constant and time is in seconds.

Step 1: Read a repeated halving interval from the graph The graph shows the concentration falling from $40\,\mathrm{g\,L^{-1}}$ at $5\,\mathrm{s}$ to $20\,\mathrm{g\,L^{-1}}$ at $15\,\mathrm{s}$, and then to $10\,\mathrm{g\,L^{-1}}$ at $25\,\mathrm{s}$. Thus successive halvings each take $10\,\mathrm{s}$, identifying the first-order half-life rather than a constant rate of concentration loss. Hence

$$t_{1/2} = 10\,\mathrm{s}, \qquad k = \frac{0.693}{10} = 0.0693\,\mathrm{s^{-1}}.$$

The initial value 50 is used in the requested overall concentration ratio; it is not itself one of these plotted half-life pairs.

Step 2: Calculate the time for the specified concentration ratio The required decrease is from 50 to $2.5\,\mathrm{g\,L^{-1}}$, a factor of twenty. Using the given logarithm,

$$\log 20 = \log(2 \times 10) = 0.3010 + 1 = 1.3010.$$

The required number of half-lives is $\log 20/\log 2$, so

$$t = 10\frac{1.3010}{0.3010} = 43.22\,\text{s}.$$

This also agrees with the integrated first-order rate law. After four half-lives, the concentration is $50/16 = 3.125\,\text{g L}^{-1}$; the target is slightly smaller, so a time slightly greater than forty seconds is physically consistent. Round only the final time, as requested.

Step 3: Conclusion The time is approximately $43.22\,\text{s}$, giving the nearest-integer answer $\boxed{43}$.

Quick Tip to Solve: Read equal halving intervals from the graph, then multiply that half-life by log(initial/final)/log 2.

72. When 1 g each of compounds AB and AB_2 are dissolved in 15 g of water separately, they increased the boiling point of water by 2.7 K and 1.5 K respectively. The atomic mass of A (in amu) is_____ $\times 10^{-1}$ (Nearest integer)

(Given : Molal boiling point elevation constant is $0.5\ \text{K kg mol}^{-1}$) [4 marks]

Solution:

Concept:

- For a non-dissociating solute, boiling-point elevation is $\Delta T_b = K_b m$, with molality in moles per kilogram of solvent.
- If the atomic masses of A and B are a and b, the formula masses are $M_{AB} = a+b$ and $M_{AB_2} = a+2b$. Two measured formula masses determine both atomic masses.

Step 1: Calculate the two molar masses from their boiling elevations Each solution contains $1\,\text{g}$ solute in $15\,\text{g} = 0.015\,\text{kg}$ water. With molar mass M, the molality is $(1/M)/0.015$. Consequently

$$M = \frac{K_b \times 1}{0.015\Delta T_b}.$$

For AB and AB_2, respectively, this gives

$$M_{AB} = \frac{0.5}{0.015 \times 2.7} = \frac{1000}{81}\,\text{g mol}^{-1},$$

$$M_{AB_2} = \frac{0.5}{0.015 \times 1.5} = \frac{200}{9}\,\text{g mol}^{-1}.$$

The larger elevation corresponds to the lower molar mass because the same mass then supplies more solute particles.

Step 2: Eliminate the atomic mass of B and apply the requested scale Write $a + b = 1000/81$ and $a + 2b = 200/9$. Twice the first equation minus the second eliminates b:

$$a = 2\left(\frac{1000}{81}\right) - \frac{200}{9} = \frac{200}{81} = 2.469135\ldots\ \text{amu}.$$

As a check, $b = 800/81$ is positive and reproduces both formula masses. The question expresses a as an integer coefficient times 10^{-1} amu. Thus the coefficient before rounding is

$$\frac{2.469135\ldots}{10^{-1}} = 24.69135\ldots.$$

Round this coefficient rather than first rounding the atomic mass itself; premature rounding would lose the scale specified in the blank.

Step 3: Conclusion The atomic mass is $2.4691\ldots$ amu $= 24.691\ldots \times 10^{-1}$ amu. The required nearest-integer coefficient is $\boxed{25}$.

Quick Tip to Solve: With equal solute and solvent masses, first compute each formula mass, then use a = 2M(AB) - M(AB2).

73. The spin-only magnetic moment value of M^{n+} ion formed among Ni, Zn, Mn and Cu that has the least enthalpy of atomisation is_____ . (in nearest integer) Here n is equal to the number of diamagnetic complexes among $K_2[NiCl_4]$, $[Zn(H_2O)_6]Cl_2$, $K_3[Mn(CN)_6]$ and $[Cu(PPh_3)_3I]$ [4 marks]

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{u(u+2)}\,\text{BM}$, where u is the number of unpaired electrons.
- Find oxidation states and d-electron counts before assigning the magnetic character. A diamagnetic complex has no unpaired electrons.

Step 1: Examine the nickel and zinc complexes In $[NiCl_4]^{2-}$, nickel is +2 and has configuration $3d^8$. Chloride is a weak-field ligand and the complex is tetrahedral, with two unpaired electrons; it is paramagnetic. In $[Zn(H_2O)_6]^{2+}$, zinc is +2 with $3d^{10}$. All d orbitals are filled, so this complex is diamagnetic regardless of the weak field of water.

Step 2: Examine the manganese and copper complexes In $[Mn(CN)_6]^{3-}$, manganese is +3, hence $3d^4$. Strong-field cyanide gives a low-spin octahedral arrangement t_{2g}^4, which still has two unpaired electrons. This complex is therefore paramagnetic, not diamagnetic. In $[Cu(PPh_3)_3I]$, the phosphine ligands are neutral and iodide is -1. Copper is consequently +1, with $3d^{10}$, and this complex is diamagnetic. Exactly two listed complexes are diamagnetic, giving $n = 2$.

Step 3: Select the metal and calculate its moment Among Ni, Zn, Mn and Cu, zinc has the least enthalpy of atomisation. Its filled $3d^{10}$ shell does not provide the partly filled d orbitals that strengthen metallic bonding in the other listed transition metals. The ion required is therefore Zn^{2+}, whose configuration is $[Ar]3d^{10}$. Thus $u = 0$, and

$$\mu = \sqrt{0(0+2)} = 0\,\text{BM}.$$

Step 4: Conclusion The required ion is Zn^{2+}, and its spin-only magnetic moment is $0\,\text{BM}$. The nearest-integer answer is $\boxed{0}$.

Quick Tip to Solve: Do not call every strong-field complex diamagnetic: low-spin d^4 still has two unpaired electrons. Determine n before choosing the metal ion.

74. $0.2\%(\mathrm{w/v})$ **solution of NaOH is measured to have resistivity** $870.0\ \mathrm{m\Omega\ m}$**. The molar conductivity of the solution will be_____**$\times 10^2 \mathrm{mS dm^2\ mol^{-1}}$**. (Nearest integer)** [4 marks]

Solution:

Concept:

- Conductivity is the reciprocal of resistivity: $\kappa = 1/\rho$. Molar conductivity is $\Lambda_m = \kappa/c$ when concentration c is expressed in $\mathrm{mol\,m^{-3}}$.
- A percentage expressed as w/v gives grams of solute in 100 mL of solution. Convert every quantity to consistent units before evaluating the requested numerical coefficient.

Step 1: Convert the given solution strength The solution contains 0.2 g of NaOH per 100 mL, or 2.0 g per litre. Using its molar mass $40\,\mathrm{g\,mol^{-1}}$, the concentration is

$$c = \frac{2.0}{40} = 0.050\,\mathrm{mol\,L^{-1}} = 50\,\mathrm{mol\,m^{-3}}.$$

The volume is the volume of solution, so no density conversion is needed.

Step 2: Take the reciprocal of resistivity The prefix milli belongs to ohm:

$$\rho = 870.0\,\mathrm{m\Omega\,m} = 0.8700\,\Omega\,\mathrm{m}.$$

Therefore the conductivity is

$$\kappa = \frac{1}{0.8700} = 1.149425\,\mathrm{S\,m^{-1}}.$$

Keeping this value unrounded avoids premature error in the final coefficient.

Step 3: Convert the molar conductivity to the requested units Using the SI concentration gives

$$\Lambda_m = \frac{1.149425}{50} = 0.0229885\,\mathrm{S\,m^2\,mol^{-1}}.$$

Since $1\,\mathrm{S} = 10^3\,\mathrm{mS}$ and $1\,\mathrm{m^2} = 10^2\,\mathrm{dm^2}$, the combined conversion factor is 10^5. Hence

$$\Lambda_m = 2298.85\,\mathrm{mS\,dm^2\,mol^{-1}} = 22.9885 \times 10^2\,\mathrm{mS\,dm^2\,mol^{-1}}.$$

It is the coefficient of the printed power of ten that must be rounded, rather than the full conductivity value.

Step 4: Conclusion The coefficient is 22.9885, which rounds to $\boxed{23}$. Thus the molar conductivity is approximately $23 \times 10^2\,\mathrm{mS\,dm^2\,mol^{-1}}$.

Quick Tip to Solve: For this unit conversion, multiply $\mathrm{S\,m^2}$ by 10^5 to obtain $\mathrm{mS\,dm^2}$; both the conductance and area units change.

75. Consider the above sequence of reactions. 151 g of 2-bromopentane is made to react. Yield of major product P is 80% whereas Q is 100%.

Mass of product Q obtained is_____ g.

(Given molar mass in gmol^{-1} $\mathrm{H}:1, \mathrm{C}:12, \mathrm{O}:16, \mathrm{Br}:80$) [4 marks]



Solution:

Concept:

- Alcoholic KOH favours beta-elimination of hydrogen halide from a haloalkane. The more substituted alkene is the major product by the Saytzeff rule.
- Bromine adds across an alkene double bond in a 1:1 mole ratio. Apply each percentage yield to moles before converting the final product into mass.

Step 1: Identify the major elimination product In 2-bromopentane, loss of a beta hydrogen and bromide can give pent-1-ene or pent-2-ene. Pent-2-ene has the more substituted double bond, so it is the major product P under alcoholic KOH. Its formation is represented by

$$\mathrm{CH_3CHBrCH_2CH_2CH_3} \longrightarrow \mathrm{CH_3CH{=}CHCH_2CH_3} + \mathrm{HBr}.$$

This equation indicates the organic elimination; the base consumes the eliminated hydrogen halide. Any E/Z forms of pent-2-ene have the same molar mass and give the same final mass calculation.

Step 2: Calculate the moles of P actually obtained The starting compound is $\mathrm{C_5H_{11}Br}$, with molar mass

$$M = 5(12) + 11(1) + 80 = 151\,\mathrm{g\,mol^{-1}}.$$

Thus 151 g corresponds to 1.00 mol of 2-bromopentane. Elimination has a 1:1 stoichiometric ratio, but P is obtained in 80 percent yield. Consequently,

$$n(P) = 1.00 \times 0.80 = 0.80\,\mathrm{mol}.$$

The given yield already describes the isolated major product, so no further product-distribution factor is required.

Step 3: Add bromine and calculate the mass of Q Pent-2-ene adds one molecule of $\mathrm{Br_2}$ to give 2,3-dibromopentane, Q:

$$\mathrm{CH_3CH{=}CHCH_2CH_3} + \mathrm{Br_2} \longrightarrow \mathrm{CH_3CHBrCHBrCH_2CH_3}.$$

Its formula is $\mathrm{C_5H_{10}Br_2}$ and its molar mass is

$$M(Q) = 5(12) + 10(1) + 2(80) = 230\,\mathrm{g\,mol^{-1}}.$$

The second step has 100 percent yield, so $n(Q) = 0.80\,\text{mol}$. Therefore

$$m(Q) = 0.80 \times 230 = 184\,\text{g}.$$

Step 4: Conclusion The mass of product Q, 2,3-dibromopentane, is $\boxed{184\,\text{g}}$. The numerical answer is 184.

Quick Tip to Solve: Track moles through the two reactions: the 80 percent yield reduces the amount to 0.80 mol, while bromine addition raises the molar mass to 230 without changing that mole count.