JEE Main 2025 Paper 1 (B.E./B.Tech) Apr 3 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 3 April 2025, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1.

If the domain of the function $f(x) = \log_e\left(\frac{2x-3}{5+4x}\right) + \sin^{-1}\left(\frac{4+3x}{2-x}\right)$ **is** $[\alpha, \beta)$**, then** $\alpha^2+4\beta$ **is equal to**

[4 marks]

(A) 4

(B) 3

(C) 7

(D) 5

Correct Answer: (A) 4

Solution:

Concept:

- The real logarithm requires its argument to be strictly positive; its argument cannot be zero and its denominator cannot vanish.
- The principal inverse sine is real precisely when its argument lies in $[-1, 1]$. The domain of a sum is the intersection of the two domains.

Step 1: Solve the logarithmic restriction The critical values of the quotient are $-5/4$ and $3/2$. The numerator and denominator must have the same sign. A sign chart gives

$$\frac{2x-3}{5+4x} > 0 \quad \Longleftrightarrow \quad x \in (-\infty, -5/4) \cup (3/2, \infty).$$

Both critical endpoints are excluded: one makes the denominator zero, and the other gives the forbidden logarithm of zero.

Step 2: Solve the inverse-sine restriction Require $x \neq 2$ and square the inequality $|(4+3x)/(2-x)| \leq 1$. Since $(2-x)^2 > 0$, this is equivalent to

$$(4+3x)^2 - (2-x)^2 \leq 0 \quad \Longleftrightarrow \quad (4x+2)(2x+6) \leq 0.$$

Hence $-3 \leq x \leq -1/2$. The excluded denominator value 2 is outside this interval, so it introduces no further deletion.

Step 3: Intersect the intervals and evaluate Intersecting with the logarithmic domain leaves $[-3, -5/4)$. At -3, the inverse-sine argument is -1 and the logarithmic argument is positive, so the left endpoint is included. Thus $\alpha = -3$ and $\beta = -5/4$, giving

$$\alpha^2 + 4\beta = 9 - 5 = 4.$$

The other three offered numbers do not result from this intersection.

Step 4: Conclusion The domain is $[-3, -5/4)$ and the required value is $\boxed{4}$. Hence, the correct option is (A).

Quick Tip to Solve: Find the two domains separately, then intersect them; preserve the strict endpoint exclusion imposed by the logarithm.

2. Let $a_1, a_2, a_3, \ldots$. be a G.P. of increasing positive numbers. If $a_3 a_5 = 729$ and $a_2 + a_4 = \frac{111}{4}$, then $24\,(a_1 + a_2 + a_3)$ is equal to [4 marks]

(A) 128

(B) 129

(C) 131

(D) 130

Correct Answer: (B) 129

Solution:

Concept:

- For a G.P. with first term $a > 0$ and common ratio r, the term $a_n = ar^{n-1}$. Increasing positive terms require $r > 1$.
- The product of terms equally spaced about a middle term equals the square of that middle term; positivity removes the negative square root.

Step 1: Find the fourth and second terms The third and fifth terms are equally far from the fourth term. Therefore

$$a_3 a_5 = (ar^2)(ar^4) = (ar^3)^2 = a_4^2 = 729.$$

Every term is positive, so $a_4 = 27$, rather than -27. Now use the sum supplied in the question:

$$a_2 = \frac{111}{4} - 27 = \frac{3}{4}.$$

This extracts individual terms without solving simultaneously for two unknowns at the outset.

Step 2: Determine the common ratio The indices 2 and 4 differ by two, so their quotient is r^2, not r:

$$r^2 = \frac{a_4}{a_2} = \frac{27}{3/4} = 36.$$

Positivity of adjacent terms gives $r > 0$, and the increasing condition selects $r = 6 > 1$. Thus

$$a_1 = \frac{a_2}{r} = \frac{1}{8}, \qquad a_3 = a_2 r = \frac{9}{2}.$$

These values also reproduce $a_5 = 162$, with $a_3 a_5 = 729$.

Step 3: Evaluate the requested expression Add the first three terms with a common denominator:

$$a_1 + a_2 + a_3 = \frac{1}{8} + \frac{3}{4} + \frac{9}{2} = \frac{43}{8}.$$

Consequently $24(a_1 + a_2 + a_3) = 24(43/8) = 129$. The options 128, 131 and 130 fail this exact sum; rounding is unnecessary.

Step 4: Conclusion The required value is $\boxed{129}$. Hence, the correct option is (B).

Quick Tip to Solve: For a positive G.P., identify the middle term from the product first; remember that a two-index gap produces the square of the ratio.

3. The sum $1+3+11+25+45+71+\ldots$ upto 20 terms, is equal to [4 marks]

(A) 7240

(B) 8124

(C) 7130

(D) 6982

Correct Answer: (A) 7240

Solution:

Concept:

- A sequence with constant second difference is quadratic in its index: $a_n = An^2 + Bn + C$. Its second difference equals $2A$.
- The identities $\sum_{n=1}^{N} n = N(N+1)/2$ and $\sum_{n=1}^{N} n^2 = N(N+1)(2N+1)/6$ sum the resulting polynomial.

Step 1: Identify the pattern of differences The first differences of the displayed terms are

$$3-1=2, \quad 11-3=8, \quad 25-11=14, \quad 45-25=20, \quad 71-45=26.$$

They form an arithmetic progression of common difference 6. Thus the displayed sequence is the standard quadratic sequence with $A=3$. Put $a_n = 3n^2+Bn+C$. The first two terms give $3+B+C=1$ and $12+2B+C=3$, whence $B=-7$, $C=5$.

Step 2: Verify and sum the general term Therefore $a_n = 3n^2-7n+5$. For example, $a_3 = 27-21+5 = 11$ and $a_6 = 108-42+5 = 71$, agreeing with the printed sequence. The sum through the twentieth term is

$$S_{20} = 3\sum_{n=1}^{20} n^2 - 7\sum_{n=1}^{20} n + 5(20).$$

Here $\sum n = 210$ and $\sum n^2 = 20 \cdot 21 \cdot 41/6 = 2870$. Substituting gives

$$S_{20} = 3(2870) - 7(210) + 100 = 8610 - 1470 + 100 = 7240.$$

This is a sum of twenty terms, not merely the twentieth term, which would be 1065. None of the other listed totals equals this exact calculation.

Step 3: Conclusion The required sum is $\boxed{7240}$. Hence, the correct option is (A).

Quick Tip to Solve: When successive differences form an A.P., write a quadratic general term and sum it with the standard sums of indices and squared indices.

4.

The number of solutions of the equation $2x + 3\tan x = \pi, x \in [-2\pi, 2\pi] - \left\{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}\right\}$ **is:**

[4 marks]

(A) 4

(B) 5

(C) 3

(D) 6

Correct Answer: (B) 5

Solution:

Concept:

- On each interval where tangent is defined, a continuous strictly increasing function can have at most one zero.
- The derivative of tangent is $\sec^2 x$, which is positive. Its limits on either side of each vertical asymptote have opposite signs.

Step 1: Introduce a strictly increasing function Put $g(x) = 2x + 3\tan x - \pi$. The required solutions are precisely the zeros of g. Wherever it is defined,

$$g'(x) = 2 + 3\sec^2 x > 0.$$

Thus there cannot be two roots in the same continuous branch: the function increases strictly throughout that branch. The four excluded points divide the given domain into five branches, but each branch must still be checked for existence of a root.

Step 2: Count roots on the three interior branches The intervals $(-3\pi/2, -\pi/2)$, $(-\pi/2, \pi/2)$ and $(\pi/2, 3\pi/2)$ are complete branches of tangent. At the left endpoint of each branch, g tends to $-\infty$; at its right endpoint, g tends to $+\infty$. The finite term $2x - \pi$ cannot change these infinite limits. Continuity and strict increase therefore give exactly one root on each of these intervals, contributing three roots.

Step 3: Check the two end branches On $[-2\pi, -3\pi/2)$, the initial value is $g(-2\pi) = -5\pi < 0$, and the right limit is $+\infty$. There is exactly one root. On $(3\pi/2, 2\pi]$, the left limit is $-\infty$, while $g(2\pi) = 3\pi > 0$, again giving one root. Neither included endpoint is a root. Consequently the total is $1 + 3 + 1 = 5$. Counts 3 or 4 omit end branches; 6 is impossible since five increasing branches allow at most five roots.

Step 4: Conclusion The number of solutions is $\boxed{5}$. Hence, the correct option is (B). The printed equation has five roots, despite the supplied key indicating four; that count omits a valid branch.

Quick Tip to Solve: Count continuous tangent branches first, then check the endpoint signs on the two truncated branches; monotonicity alone proves uniqueness, not existence.

5. Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$. Let R be a relation on A defined by xRy if and only if $0 \le x^2 + 2y \le 4$. Let l be the number of elements in R and m be the minimum number of elements required to be added in R to make it a reflexive relation. Then $l + m$ is equal to [4 marks]

(A) 18

(B) 20

(C) 17

(D) 19

Correct Answer: (A) 18

Solution:

Concept:

- A relation on A is a set of ordered pairs (x, y). Count admissible values of y separately for each fixed x.
- Reflexivity requires precisely the seven diagonal pairs (x, x), one for every $x \in A$; add only those diagonal pairs currently absent.

Step 1: Count all ordered pairs in the relation The defining inequality is equivalent to

$$-\frac{x^2}{2} \le y \le \frac{4 - x^2}{2}, \qquad y \in A.$$

For $x = \pm 3$, only $y = -3$ is allowed, contributing 2 pairs. For $x = \pm 2$, the choices are $-2, -1, 0$, contributing 6 pairs. For $x = \pm 1$, only $0, 1$ are allowed, contributing 4 pairs. For $x = 0$, the choices are $0, 1, 2$, contributing 3 pairs. Thus

$$l = 2 + 6 + 4 + 3 = 15.$$

The signs of x give distinct ordered pairs even when they share the same admissible y values.

Step 2: Check the diagonal pairs for reflexivity A diagonal pair belongs to R when $0 \le x^2 + 2x \le 4$. For $x = -3, -2, -1, 0, 1, 2, 3$, respectively, the expression equals

$$3, \quad 0, \quad -1, \quad 0, \quad 3, \quad 8, \quad 15.$$

Thus the diagonal pairs at $-3, -2, 0, 1$ already occur, whereas $(-1, -1), (2, 2), (3, 3)$ are missing. All three must be added and no other addition is required, so $m = 3$.

Step 3: Combine the two counts The minimum reflexive completion has $15 + 3$ elements, and the requested number is $l + m = 18$. The options 20, 17 and 19 reflect incorrect counting of the diagonal pairs or of the signs of x; none equals the direct enumeration.

Step 4: Conclusion We obtain $l = 15$, $m = 3$, and $\boxed{l + m = 18}$. Hence, the correct option is (A).

Quick Tip to Solve: First count the whole relation by fixing x; then test only the diagonal pairs to determine the smallest reflexive completion.

6. The radius of the smallest circle which touches the parabolas $y = x^2 + 2$ and $x = y^2 + 2$ is [4 marks]

(A) $\frac{7\sqrt{2}}{16}$

(B) $\frac{7\sqrt{2}}{8}$

(C) $\frac{7\sqrt{2}}{2}$

(D) $\frac{7\sqrt{2}}{4}$

Correct Answer: (B) $\frac{7\sqrt{2}}{8}$

Solution:

Concept:

- A circle touching two disjoint curves at points P and Q has diameter at least PQ, because the distance between any two points on a circle is at most its diameter.
- Parallel supporting lines give a lower bound for the separation of curves. If points on the two curves attain this bound, a circle with their joining segment as diameter attains the smallest possible radius.

Step 1: Find parallel lines separating the parabolas On the first parabola,

$$y - x = x^2 - x + 2 = \left(x - \frac{1}{2}\right)^2 + \frac{7}{4} \geq \frac{7}{4}.$$

On the second parabola,

$$x - y = y^2 - y + 2 = \left(y - \frac{1}{2}\right)^2 + \frac{7}{4} \geq \frac{7}{4}.$$

Thus the curves lie on opposite outer sides of the parallel lines $y - x = 7/4$ and $y - x = -7/4$. Their separation is

$$d = \frac{7/4 - (-7/4)}{\sqrt{(-1)^2 + 1^2}} = \frac{7\sqrt{2}}{4}.$$

Every segment joining the curves has length at least d, by its perpendicular projection onto these lines.

Step 2: Exhibit contact points attaining the bound Equality occurs at $P = (1/2, 9/4)$ on the first parabola and $Q = (9/4, 1/2)$ on the second. Their joining line has slope -1 and is perpendicular to the supporting lines. Also $PQ = 7\sqrt{2}/4 = d$. The midpoint is $C = (11/8, 11/8)$. The circle with centre C and radius $d/2$ touches the first parabola at P and the second at Q: each parabola has tangent slope 1 there, perpendicular to its radius.

Step 3: Apply the lower bound for the radius Any circle touching both curves has two contact points whose separation is at least d, and therefore its radius is at least $d/2$. The circle just constructed attains that bound. Consequently

$$r_{\min} = \frac{d}{2} = \frac{7\sqrt{2}}{8}.$$

Option A is too small to bridge the supporting lines. Options C and D exceed this attained minimum.

Step 4: Conclusion The smallest radius is $\boxed{7\sqrt{2}/8}$. Hence, the correct option is (B).

Quick Tip to Solve: For reflected parabolas, complete squares in y minus x and x minus y; the parallel supporting-line gap is the minimum distance, then halve it.

7.

Let $f(x) = \int x^3\sqrt{3-x^2}dx$. If $5f(\sqrt{2}) = -4$, then $f(1)$ is equal to

[4 marks]

(A) $-\frac{6\sqrt{2}}{5}$

(B) $-\frac{8\sqrt{2}}{5}$

(C) $-\frac{2\sqrt{2}}{5}$

(D) $-\frac{4\sqrt{2}}{5}$

Correct Answer: (A) $-\frac{6\sqrt{2}}{5}$

Solution:

Concept:

- For $\int x^3\sqrt{3-x^2}\,dx$, set $u = 3 - x^2$; then $du = -2x\,dx$ and the remaining factor x^2 becomes $3 - u$.
- An indefinite integral has an arbitrary additive constant, which must be determined using the stated value before evaluating it elsewhere.

Step 1: Integrate after changing the variable Write $x^3dx = x^2(xdx)$. With $u = 3 - x^2$,

$$\begin{aligned} f(x) &= -\frac{1}{2}\int (3-u)u^{1/2}\,du \\ &= -u^{3/2} + \frac{1}{5}u^{5/2} + C \\ &= -(3-x^2)^{3/2} + \frac{1}{5}(3-x^2)^{5/2} + C. \end{aligned}$$

The substitution retains the minus sign from du. Both evaluation points lie inside $|x| < \sqrt{3}$, so the real square root and this antiderivative are valid there.

Step 2: Use the given value to fix the constant At $x = \sqrt{2}$, the quantity $3 - x^2$ is 1. Hence

$$f(\sqrt{2}) = -1 + \frac{1}{5} + C = -\frac{4}{5} + C.$$

The condition $5f(\sqrt{2}) = -4$ states $f(\sqrt{2}) = -4/5$, so $C = 0$. One must use this condition rather than assume that the integration constant vanishes.

Step 3: Evaluate and check the result At $x = 1$, the substituted variable is 2. Therefore

$$f(1) = -2^{3/2} + \frac{1}{5}2^{5/2} = -2\sqrt{2} + \frac{4}{5}\sqrt{2} = -\frac{6}{5}\sqrt{2}.$$

For a check, differentiating the antiderivative gives $3x\sqrt{3-x^2} - x(3-x^2)^{3/2} = x^3\sqrt{3-x^2}$, recovering the integrand. The remaining options have incorrect coefficients.

Step 4: Conclusion The required value is $\boxed{-6\sqrt{2}/5}$. Hence, the correct option is (A). The printed integral gives this value directly, despite the supplied key indicating a different coefficient.

Quick Tip to Solve: With an odd power of x next to a function of x squared, retain one x dx for substitution and express the remaining x squared in the new variable.

8. Let the domain of the function $f(x) = \log_2 \log_4 \log_6 \left(3 + 4x - x^2\right)$ be (a, b). If $\int_0^{b-a} \left[x^2\right] dx = p - \sqrt{q} - \sqrt{r}, p, q, r \in \mathbb{N}, \gcd(p, q, r) = 1$, where $[\cdot]$ is the greatest integer function, then $p + q + r$ is equal to [4 marks]

(A) 10

(B) 11

(C) 9

(D) 8

Correct Answer: (A) 10

Solution:

Concept:

- A real logarithm requires a strictly positive argument. For a base greater than one, the logarithm is increasing.
- The greatest-integer function $[x^2]$ is constant on the intervals between consecutive values $\sqrt{k}$. Integrate it by adding rectangle areas.

Step 1: Work inward through the logarithms Write $S = 3 + 4x - x^2$. The outer logarithm requires $\log_4(\log_6 S) > 0$. Since the base 4 exceeds one, this is equivalent to $\log_6 S > 1$, and hence to $S > 6$. This condition automatically makes all three logarithms defined. Therefore

$$3 + 4x - x^2 > 6 \iff (x-2)^2 < 1 \iff 1 < x < 3.$$

So $a = 1$, $b = 3$, and the upper integration limit $b - a$ is 2. Merely requiring $S > 0$ would overlook the stricter positivity conditions imposed by the outer logarithms.

Step 2: Split the integral at the integer jumps As x increases from 0 to 2, the value of $[x^2]$ is 0 on $[0, 1)$, 1 on $[1, \sqrt{2})$, 2 on $[\sqrt{2}, \sqrt{3})$, and 3 on $[\sqrt{3}, 2)$. Its values at the isolated endpoints do not affect the definite integral. Thus

$$\begin{aligned}\int_0^2 [x^2]\, dx &= (\sqrt{2} - 1) + 2(\sqrt{3} - \sqrt{2}) + 3(2 - \sqrt{3}) \\ &= 5 - \sqrt{2} - \sqrt{3}.\end{aligned}$$

The integer part cannot be replaced by x^2; that would give the area beneath a different function.

Step 3: Identify the integers Comparing with $p - \sqrt{q} - \sqrt{r}$ gives $p = 5$, $q = 2$, $r = 3$, with the last two interchangeable. These are positive integers and $\gcd(5, 2, 3) = 1$, as required. Their sum is $5 + 2 + 3 = 10$, ruling out 11, 9 and 8.

Step 4: Conclusion $p + q + r = \boxed{10}$. Hence, the correct option is (A).

Quick Tip to Solve: For nested logarithms with bases above one, start with positivity of the outermost argument; for floor integrals, split at square roots of integers.

9. Let α and β be the roots of $x^2+\sqrt{3}x-16=0$, and γ and δ be the roots of $x^2+3x-1=0$. If $P_n=\alpha^n+\beta^n$ and $Q_n=\gamma^n+\delta^n$, then $\frac{P_{25}+\sqrt{3}P_{24}}{2P_{23}}+\frac{Q_{25}-Q_{23}}{Q_{24}}$ is equal to [4 marks]

(A) 4

(B) 3

(C) 5

(D) 7

Correct Answer: (C) 5

Solution:

Concept:

- If a root satisfies a quadratic equation, multiplying that equation by a power of the root yields a recurrence for higher powers.
- Adding the recurrences for both roots gives an identity for the power sums. This avoids computing the roots or expanding large powers.

Step 1: Obtain the recurrence for the first quadratic Each of α,β satisfies $t^2+\sqrt{3}t-16=0$. Multiply by t^{23}, and then add the resulting equations for the two roots:

$$P_{25}+\sqrt{3}P_{24}-16P_{23}=0.$$

Consequently,

$$\frac{P_{25}+\sqrt{3}P_{24}}{2P_{23}}=\frac{16P_{23}}{2P_{23}}=8.$$

The denominator is nonzero: the roots have opposite signs, negative sum and unequal magnitudes, so the negative root has the larger absolute value and their odd power sum P_{23} is strictly negative. The coefficient 16, rather than the individual root values, is all that is needed.

Step 2: Obtain the recurrence for the second quadratic For $t=\gamma$ or δ, use $t^2+3t-1=0$. Multiplying by t^{23} and adding gives

$$Q_{25}+3Q_{24}-Q_{23}=0.$$

It follows that

$$\frac{Q_{25}-Q_{23}}{Q_{24}}=-3.$$

Both roots are real and nonzero, and $Q_{24}=\gamma^{24}+\delta^{24}>0$, so this division is valid. The minus sign comes from moving $3Q_{24}$ to the other side; changing it would produce an incorrect option.

Step 3: Combine the two values The requested sum is $8+(-3)=5$. All exponents have disappeared through the quadratic identities, so direct evaluation of 25th powers is unnecessary. The other listed values 4, 3 and 7 do not satisfy the exact recurrences, whereas 5 follows from both of them simultaneously.

Step 4: Conclusion The expression equals $\boxed{5}$. Hence, the correct option is (C).

Quick Tip to Solve: When adjacent high powers of quadratic roots appear, multiply the defining quadratic by the lowest needed power before doing any numerical calculation.

10. Let a line passing through the point $(4,1,0)$ intersect the line $\mathrm{L}_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ at the point $A(\alpha,\beta,\gamma)$ and the line $\mathrm{L}_2 : x-6 = y = -z+4$ at the point $B(a,b,c)$. Then $\begin{vmatrix} 1 & 0 & 1 \\ \alpha & \beta & \gamma \\ a & b & c \end{vmatrix}$ is equal to [4 marks]

(A) 16

(B) 6

(C) 8

(D) 12

Correct Answer: (C) 8

Solution:

Concept:

- Parameterise each line to describe its intersection point. Three points are collinear exactly when their displacement vectors are proportional.
- After locating both intersection points, expand the determinant along the row containing zero to minimise arithmetic.

Step 1: Parameterise the intersection points Let $P = (4,1,0)$. Taking the common ratio on L_1 as t, and the common value on L_2 as u, gives

$$A = (1+2t, 2+3t, 3+4t), \qquad B = (6+u, u, 4-u).$$

Thus

$$A - P = (-3+2t, 1+3t, 3+4t), \qquad B - P = (2+u, u-1, 4-u).$$

Since the desired line passes through all three points, write $A - P = \lambda(B-P)$. This uses one proportionality constant for all coordinates, rather than equating the separate line parameters.

Step 2: Use coordinate combinations to determine the parameters The sum of the second and third coordinates of $B-P$ is 3, and the difference of its first and second coordinates is also 3. Therefore the corresponding combinations for $A-P$ give

$$4+7t = 3\lambda, \qquad -4-t = 3\lambda.$$

Equating these expressions yields $8t = -8$, so $t = -1$ and $\lambda = -1$. Hence $A = (-1,-1,-1)$, and $A - P = -(B-P)$ gives $B = (9,3,1)$. As a check, the latter point satisfies $x-6 = y = -z+4 = 3$, exactly as required by L_2.

Step 3: Evaluate the determinant Substitute the coordinates in the given order and expand along the first row:

$$\begin{vmatrix} 1 & 0 & 1 \\ -1 & -1 & -1 \\ 9 & 3 & 1 \end{vmatrix} = 1\big((-1)(1) - (-1)(3)\big) + 1\big((-1)(3) - (-1)(9)\big)$$

$$= 2 + 6 = 8.$$

The last cofactor has a positive sign. Thus neither 16, 6 nor 12 is the determinant of these correctly

located points.

Step 4: Conclusion The determinant is $\boxed{8}$. Hence, the correct option is (C).

Quick Tip to Solve: For a line meeting two parameterised lines through a known point, combine coordinates that eliminate one parameter before solving all three proportionality equations.

11. Let $f(x) = \begin{cases} (1+ax)^{1/x}, & x < 0, \\ 1+b, & x = 0, \\ \dfrac{(x+4)^{1/2}-2}{(x+c)^{1/3}-2}, & x > 0 \end{cases}$ **be continuous at** $x = 0$**. Then** $e^a bc$ **is equal to:** [4 marks]

(A) 64

(B) 48

(C) 36

(D) 72

Correct Answer: (B) 48

Solution:

Concept:

- Continuity at zero requires the left-hand limit, the function value and the right-hand limit to be equal.
- The standard exponential limit is $\lim_{x\to 0}(1+ax)^{1/x} = e^a$. First-order expansions or derivatives evaluate a ratio whose numerator and denominator both vanish.

Step 1: Match the left-hand limit to the value at zero For $x < 0$, the given branch has the familiar exponential limit. Near zero its base is positive for every fixed real a, and taking logarithms gives $\log(1+ax)/x \to a$. Therefore

$$\lim_{x\to 0^-} f(x) = e^a = 1 + b.$$

In particular, the common value is strictly positive, so $1 + b$ cannot be zero.

Step 2: Determine the parameter in the denominator As $x \to 0^+$, the numerator $\sqrt{x+4} - 2$ tends to zero. If $\sqrt[3]{c} - 2 \neq 0$, the denominator would tend to a nonzero number and the right-hand limit would be zero. This contradicts the positive left-hand limit. Consequently

$$\sqrt[3]{c} - 2 = 0 \quad \Longrightarrow \quad c = 8.$$

This necessity follows from continuity; it should be established before applying a rule for a zero-over-zero limit.

Step 3: Evaluate the right-hand limit and the product With $c = 8$, use the derivatives of the numerator and denominator at zero, or their first-order expansions:

$$\sqrt{4+x} - 2 = \frac{x}{4} + o(x), \qquad \sqrt[3]{8+x} - 2 = \frac{x}{12} + o(x).$$

Thus the right-hand limit is $(1/4)/(1/12) = 3$. Continuity now gives $e^a = 3$ and $1 + b = 3$, so $b = 2$. Hence

$$e^a bc = 3 \times 2 \times 8 = 48.$$

The listed values 64, 36 and 72 fail this product after both one-sided limits have been matched.

Step 4: Conclusion $e^a bc = \boxed{48}$. Hence, the correct option is (B).

Quick Tip to Solve: In a continuous piecewise expression, first determine whether the common limit is nonzero; that often forces the denominator parameter before the limit is evaluated.

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12. A line passing through the point $P(\sqrt{5}, \sqrt{5})$ intersects the ellipse $\frac{x^2}{36} + \frac{y^2}{25} = 1$ at A and B such that $(PA) \cdot (PB)$ is maximum. Then $5\left(PA^2 + PB^2\right)$ is equal to : [4 marks]

(A) 290

(B) 377

(C) 338

(D) 218

Correct Answer: (C) 338

Solution:

Concept:

- Parameterise a line through an interior point by a unit direction vector, so its signed parameter is the actual directed distance.
- For the resulting quadratic, the product and sum of roots follow from Vieta. Since the point is inside the ellipse, the two roots have opposite signs.

Step 1: Express the chord product in terms of direction Let a point on the line be $(x, y) = (\sqrt{5}, \sqrt{5}) + t(u, v)$, with $u^2 + v^2 = 1$. Substitution in the ellipse equation gives

$$\left(\frac{u^2}{36} + \frac{v^2}{25}\right) t^2 + 2\sqrt{5}\left(\frac{u}{36} + \frac{v}{25}\right) t - \frac{119}{180} = 0.$$

The constant is negative, confirming that P lies inside the ellipse and that the two intersection parameters have opposite signs. Because the direction vector is a unit vector, the positive distance product is

$$PA \cdot PB = \frac{119/180}{u^2/36 + v^2/25}.$$

Step 2: Choose the direction giving the largest product The numerator is fixed and positive. The denominator can be written as

$$\frac{1}{36} + v^2\left(\frac{1}{25} - \frac{1}{36}\right).$$

The coefficient of v^2 is positive, so its minimum occurs at $v = 0$. Therefore the line that maximises the distance product is horizontal, namely $y = \sqrt{5}$. The criterion is maximising $PA \cdot PB$, not separately maximising either distance, so a generic sloping chord need not be considered further.

Step 3: Find the two distances on this horizontal line Putting $y = \sqrt{5}$ in the ellipse gives

$$\frac{x^2}{36} = 1 - \frac{5}{25} = \frac{4}{5}, \qquad x = \pm\frac{12}{\sqrt{5}}.$$

The point P has x-coordinate $\sqrt{5} = 5/\sqrt{5}$, between these endpoints. Thus the distances, in either order, are $7/\sqrt{5}$ and $17/\sqrt{5}$. It follows that

$$5(PA^2 + PB^2) = 5\left(\frac{49}{5} + \frac{289}{5}\right) = 338.$$

The exact chord computation rules out 290, 377 and 218.

Step 4: Conclusion $5(PA^2 + PB^2) = \boxed{338}$. Hence, the correct option is (C).

Quick Tip to Solve: Use a unit direction vector when turning chord intersections into a quadratic; then Vieta gives actual distance products without a missing scale factor.

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13. Line L_1 passes through the point $(1, 2, 3)$ and is parallel to z-axis. Line L_2 passes through the point $(\lambda, 5, 6)$ and is parallel to y-axis. Let for $\lambda = \lambda_1, \lambda_2, \lambda_2 < \lambda_1$, the shortest distance between the two lines be 3 . Then the square of the distance of the point $(\lambda_1, \lambda_2, 7)$ from the line L_1 is [4 marks]

(A) 25

(B) 32

(C) 40

(D) 37

Correct Answer: (A) 25

Solution:

Concept:

- A line parallel to the z-axis has fixed x, y coordinates; its z coordinate is unrestricted.
- For two perpendicular axis-parallel lines, their common perpendicular is parallel to the remaining coordinate axis. The shortest distance is the absolute difference of that fixed coordinate.

Step 1: Parametrize the two lines The given lines can be written as

$$L_1 : (x, y, z) = (1, 2, 3 + t), \qquad L_2 : (x, y, z) = (\lambda, 5 + s, 6).$$

Here $t, s \in \mathbb{R}$. For any pair of points, the squared separation is

$$D^2 = (\lambda - 1)^2 + (3 + s)^2 + (3 - t)^2.$$

The last two terms are nonnegative. They can simultaneously vanish by choosing $s = -3$ and $t = 3$, so this really gives the minimum distance, not merely a particular separation.

Step 2: Find the ordered values of the parameter The minimum separation is therefore $|\lambda - 1|$. The

prescribed distance gives

$$|\lambda - 1| = 3 \quad \Longrightarrow \quad \lambda = 4 \text{ or } -2.$$

Since $\lambda_2 < \lambda_1$, we must use $\lambda_1 = 4$ and $\lambda_2 = -2$. Thus the point specified in the question is $P = (4, -2, 7)$; interchanging the two roots would change this point.

Step 3: Project the point onto the line The closest point on L_1 has the same height as P, namely $H = (1, 2, 7)$. This point belongs to L_1 because its z coordinate is free. Hence

$$PH^2 = (4-1)^2 + (-2-2)^2 + (7-7)^2 = 9 + 16 = 25.$$

No height difference should be included. The other three listed values do not equal this squared perpendicular distance.

Step 4: Conclusion The required square of the distance is $\boxed{25}$. Hence, the correct option is (A).

Quick Tip to Solve: For a line parallel to a coordinate axis, ignore displacement along that axis when finding point-to-line distance.

---

14. Let g be a differentiable function such that $\int_0^x g(t)dt = x - \int_0^x \text{tg}(t)dt, x \geq 0$ and let $y = y(x)$ satisfy the differential equation $\frac{dy}{dx} - y\tan x = 2(x+1)\sec x g(x), x \in \left[0, \frac{\pi}{2}\right)$. If $y(0) = 0$, then $y\left(\frac{\pi}{3}\right)$ is equal to [4 marks]

(A) $\frac{4\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{2\pi}{3\sqrt{3}}$

(D) $\frac{4\pi}{3\sqrt{3}}$

Correct Answer: (A) $\frac{4\pi}{3}$

Solution:

Concept:

- The fundamental theorem of calculus gives $\frac{d}{dx}\int_0^x f(t)\,dt = f(x)$ for continuous f. In the given integral, $tg(t)$ denotes a product.
- For $y' + P(x)y = Q(x)$, an integrating factor is $e^{\int P(x)\,dx}$; multiplication converts the left side into a product derivative.

Step 1: Determine the function in the integral relation Differentiate both sides of the relation with respect to x. The differentiability of g ensures that the fundamental theorem applies. We obtain

$$g(x) = 1 - xg(x), \qquad (1+x)g(x) = 1.$$

Consequently

$$g(x) = \frac{1}{1+x}, \quad x \geq 0.$$

The denominator never vanishes on this domain. This function also satisfies the original integral relation: its two integrands add to $(1+t)g(t) = 1$, whose integral from zero to x is x.

Step 2: Use the integrating factor Substituting $g(x)$ simplifies the differential equation to

$$y' - y\tan x = 2\sec x.$$

Since $\cos x > 0$ throughout the stated interval, an integrating factor is

$$e^{\int -\tan x\,dx} = \cos x.$$

Multiplying through gives

$$y'\cos x - y\sin x = 2, \qquad \frac{d}{dx}(y\cos x) = 2.$$

The product derivative includes the minus sign from differentiating $\cos x$.

Step 3: Apply the initial condition and evaluate Integration yields $y(x)\cos x = 2x + C$. At $x = 0$, the initial condition gives $C = 0$, so

$$y(x) = 2x\sec x.$$

Therefore

$$y\left(\frac{\pi}{3}\right) = 2\cdot\frac{\pi}{3}\cdot 2 = \frac{4\pi}{3}.$$

The options involving $\sqrt{3}$ would arise from using a tangent or sine value rather than $\sec(\pi/3) = 2$; the smaller rational multiple misses a factor of two.

Step 4: Conclusion The required value is $\boxed{\frac{4\pi}{3}}$. Hence, the correct option is (A).

Quick Tip to Solve: Differentiate the integral relation before solving the differential equation; the factor $(x+1)g(x)$ then cancels immediately.

15. The sum of all rational terms in the expansion of $(2+\sqrt{3})^8$ is : [4 marks]

(A) 16923

(B) 18817

(C) 3763

(D) 33845

Correct Answer: (B) 18817

Solution:

Concept:

- The binomial term indexed by r in $(a+b)^n$ is $\binom{n}{r}a^{n-r}b^r$, where $r = 0, 1, \ldots, n$.
- An even power of $\sqrt{3}$ is an integer, while an odd power is an integer multiple of the irrational number $\sqrt{3}$. Thus precisely the even-indexed terms are rational.

Step 1: Select the rational terms Here the general term is

$$T_{r+1} = \binom{8}{r} 2^{8-r} (\sqrt{3})^r.$$

The factors $\binom{8}{r}$ and 2^{8-r} are nonzero integers. Hence the term is rational exactly for $r = 0, 2, 4, 6, 8$. The requested sum is a sum of five individual terms; it is neither the entire value of the expansion nor just its final constant term.

Step 2: Evaluate each selected term Using the even binomial coefficients $1, 28, 70, 28, 1$, we get

$$\begin{aligned} S &= 2^8 + \binom{8}{2} 2^6 \cdot 3 + \binom{8}{4} 2^4 \cdot 3^2 \\ &\quad + \binom{8}{6} 2^2 \cdot 3^3 + 3^4 \\ &= 256 + 28 \cdot 64 \cdot 3 + 70 \cdot 16 \cdot 9 \\ &\quad + 28 \cdot 4 \cdot 27 + 81 \\ &= 256 + 5376 + 10080 + 3024 + 81 \\ &= 18817. \end{aligned}$$

In particular, the middle term uses $\binom{8}{4} = 70$, and the last term is $(\sqrt{3})^8 = 81$, not 3^8.

Step 3: Check the result by conjugation Replacing $\sqrt{3}$ by $-\sqrt{3}$ leaves every even term unchanged and reverses every odd term. Consequently

$$S = \frac{(2+\sqrt{3})^8 + (2-\sqrt{3})^8}{2}.$$

This identity confirms that no irrational terms belong in the sum and that a factor of two is needed when using the two conjugate expansions. Of the listed values, only 18817 agrees with the direct five-term calculation.

Step 4: Conclusion The sum of all rational terms is $\boxed{18817}$. Hence, the correct option is (B).

Quick Tip to Solve: Add the conjugate binomial expansions and divide by two to isolate all even powers of the radical at once.

16. If $\sum_{r=1}^{9} \left(\frac{r+3}{2^r}\right) \cdot {}^9C_r = \alpha\left(\frac{3}{2}\right)^9 - \beta, \alpha, \beta \in \mathbb{N}$**, then** $(\alpha+\beta)^2$ **is equal to** [4 marks]

(A) 27

(B) 81

(C) 18

(D) 9

Correct Answer: (B) 81

Solution:

Concept:

- The binomial theorem gives $F(x) = \sum_{r=0}^{9} \binom{9}{r} x^r = (1+x)^9$.
- Differentiating and multiplying by x introduces the index factor: $xF\prime(x) = \sum_{r=0}^{9} r\binom{9}{r} x^r = 9x(1+x)^8$.

Step 1: Include the missing zero-index term temporarily The denominator 2^r suggests evaluating a binomial generating expression at $x = 1/2$. First sum from $r = 0$, separating the weight $r + 3$:

$$\sum_{r=0}^{9}(r+3)\binom{9}{r}x^r = xF\prime(x) + 3F(x)$$
$$= 9x(1+x)^8 + 3(1+x)^9.$$

Step 2: Substitute the value of the variable At $x = 1/2$, the weighted-index term becomes

$$\frac{9}{2}\left(\frac{3}{2}\right)^8 = 3\left(\frac{3}{2}\right)^9.$$

Adding the unweighted part therefore gives

$$\sum_{r=0}^{9} \frac{r+3}{2^r}\binom{9}{r} = 6\left(\frac{3}{2}\right)^9.$$

The actual question starts at $r = 1$. Its omitted term is $(0+3)\binom{9}{0}/2^0 = 3$, so the required sum is

$$6\left(\frac{3}{2}\right)^9 - 3.$$

Step 3: Identify the natural-number parameters The binomial simplification gives $\alpha = 6$ and $\beta = 3$. Finally

$$(\alpha+\beta)^2 = (6+3)^2 = 81.$$

The equality also permits $\alpha = 6 + 512k$, $\beta = 3 + 19683k$, with integer $k \geq 0$. Their sum is $9 + 20195k$. For $k \geq 1$, the square exceeds every listed option, leaving only 81 among the choices.

Step 4: Conclusion The intended coefficients are $\alpha = 6$, $\beta = 3$, giving $\boxed{81}$. Hence, the correct option is (B).

Quick Tip to Solve: Use $x\frac{d}{dx}(1+x)^n$ for a factor r, and check whether the summation excludes the $r = 0$ term.

17. Let $z \in C$ be such that $\frac{z^2+3i}{z-2+i} = 2+3i$. Then the sum of all possible values of z^2 is : [4 marks]

(A) $-19+2i$

(B) $-19-2i$

(C) $19-2i$

(D) $19+2i$

Correct Answer: (B) $-19-2i$

Solution:

Concept:

- After multiplying a rational equation by its denominator, check separately that the excluded denominator-zero value is not admitted as a solution.
- For roots z_1, z_2 of a monic quadratic $z^2 - Sz + P = 0$, Vieta gives $z_1 + z_2 = S$, $z_1 z_2 = P$, and $z_1^2 + z_2^2 = S^2 - 2P$.

Step 1: Convert the rational equation to a quadratic The original denominator requires $z \neq 2-i$. Multiplication gives

$$z^2 + 3i = (2+3i)(z-2+i).$$

Now

$$(2+3i)(2-i) = 4 - 2i + 6i - 3i^2 = 7 + 4i.$$

Therefore the quadratic equation is

$$z^2 - (2+3i)z + (7+7i) = 0.$$

The extra $3i$ on the left must be included when collecting its constant term.

Step 2: Check that both quadratic roots are admissible At the excluded value $z = 2-i$, the original numerator is

$$(2-i)^2 + 3i = 3 - i \neq 0.$$

Equivalently, the quadratic polynomial at that value is nonzero, so cross multiplication has not added this forbidden value. Its discriminant is

$$(2+3i)^2 - 4(7+7i) = -33 - 16i \neq 0,$$

so the quadratic has two distinct complex roots. Both are consequently among the possible values of z in the original equation.

Step 3: Find the sum of the squares without solving for roots Apply Vieta directly:

$$z_1 + z_2 = 2+3i, \qquad z_1 z_2 = 7+7i.$$

Hence

$$\begin{aligned} z_1^2 + z_2^2 &= (2+3i)^2 - 2(7+7i) \\ &= (-5+12i) - (14+14i) \\ &= -19 - 2i. \end{aligned}$$

The sign of $i^2 = -1$ makes the real part negative, and subtracting the product term makes the imaginary part negative too. This eliminates the three alternatives with an incorrect real or imaginary sign.

Step 4: Conclusion The sum of all possible values of z^2 is $\boxed{-19 - 2i}$. Hence, the correct option is (B).

Quick Tip to Solve: For a sum of squares of quadratic roots, use $S^2 - 2P$; explicit complex square roots are unnecessary.

18.

If $y(x) = \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{vmatrix}, x \in \mathbb{R}$**, then** $\frac{d^2y}{dx^2} + y$ **is equal to**

[4 marks]

(A) 28

(B) 27

(C) -1

(D) 1

Correct Answer: (C) -1

Solution:

Concept:

- Expanding a determinant along one row expresses it as a linear combination of that row's entries, with alternating cofactor signs.
- Every combination $a\sin x + b\cos x$ satisfies $f'' + f = 0$. A constant term survives unchanged when forming $y'' + y$.

Step 1: Simplify the determinant first The second and third rows are constant, so expansion along the first row is immediate:

$$\begin{aligned} y(x) &= \sin x(28 \cdot 1 - 27 \cdot 1) \\ &\quad - \cos x(27 \cdot 1 - 27 \cdot 1) \\ &\quad + (\sin x + \cos x + 1)(27 \cdot 1 - 28 \cdot 1) \\ &= \sin x - 0 - (\sin x + \cos x + 1) \\ &= -\cos x - 1. \end{aligned}$$

The cofactor of the middle entry is zero. The final minor is -1, which is essential for the sign of the constant.

Step 2: Differentiate the simplified expression twice Since $y(x) = -\cos x - 1$, ordinary differentiation gives

$$y'(x) = \sin x, \qquad y''(x) = \cos x.$$

Therefore

$$y''(x) + y(x) = \cos x - \cos x - 1 = -1.$$

The result is independent of x, as expected: differentiation twice cancels the trigonometric part after it is added back to the original function.

Step 3: Check the sign using a convenient value At $x = 0$, the simplified determinant gives $y(0) = -2$, while $y''(0) = 1$. Their sum is again -1. Thus the positive option 1 has the wrong sign, and 27 and 28 are entries of the determinant rather than its surviving constant contribution. Expanding before differentiation prevents lengthy product-rule work on an expression that reduces to a single cosine and a constant.

Step 4: Conclusion For every real x, $\boxed{\frac{d^2y}{dx^2} + y = -1}$. Hence, the correct option is (C).

Quick Tip to Solve: Simplify a determinant with only one variable row before differentiating; then $y'' + y$ isolates its constant term.

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19. A line passes through the origin and makes equal angles with the positive coordinate axes. It intersects the lines $L_1 : 2x + y + 6 = 0$ and $L_2 : 4x + 2y - p = 0, p > 0$, at the points A and B , respectively. If $AB = \frac{9}{\sqrt{2}}$ and the foot of the perpendicular from the point A on the line L_2 is M, then $\frac{AM}{BM}$ is equal to [4 marks]

(A) 5

(B) 3

(C) 2

(D) 4

Correct Answer: (B) 3

Solution:

Concept:

- A line making equal angles with the positive coordinate axes in the plane has equation $y = x$. Substitution gives its intersections with the two given lines.
- The perpendicular distance of (x_0, y_0) from $ax + by + c = 0$ is $|ax_0 + by_0 + c|/\sqrt{a^2 + b^2}$. Since $AM \perp BM$, triangle AMB is right-angled at M.

Step 1: Locate the two intersections Substitute $y = x$ into each equation. For the first line, $3x + 6 = 0$, so $A = (-2, -2)$. For the second line, $6x - p = 0$, so $B = (p/6, p/6)$. Because $p > 0$, the coordinate difference $p/6 + 2$ is positive. Hence no extra sign choice is needed when using the distance formula.

Step 2: Determine the parameter and perpendicular distance Using the given separation,

$$\sqrt{2}\left(\frac{p}{6} + 2\right) = \frac{9}{\sqrt{2}} \quad \Longrightarrow \quad \frac{p}{6} + 2 = \frac{9}{2}, \qquad p = 15.$$

Thus $B = (5/2, 5/2)$, and the second line is $4x + 2y - 15 = 0$. Its distance from A is

$$AM = \frac{|4(-2) + 2(-2) - 15|}{\sqrt{4^2 + 2^2}} = \frac{27}{2\sqrt{5}}.$$

Step 3: Use the right triangle The hypotenuse is AB, not AM, because both B and M lie on the second line. Therefore

$$BM^2 = AB^2 - AM^2 = \frac{81}{2} - \frac{729}{20} = \frac{81}{20}.$$

Taking the positive length gives $BM = 9/(2\sqrt{5})$. Consequently $AM/BM = 27/9 = 3$. The values 5, 2 and 4 do not satisfy these computed lengths.

Step 4: Conclusion The required ratio is $\boxed{3}$. Hence, the correct option is (B).

Quick Tip to Solve: After finding the perpendicular distance, use Pythagoras with AB as the hypotenuse; this avoids calculating the coordinates of M.

20. Let A be a matrix of order 3×3 and $|A| = 5$. If $|2\,\mathrm{adj}(3A\,\mathrm{adj}(2A))| = 2^\alpha \cdot 3^\beta \cdot 5^\gamma, \alpha, \beta, \gamma \in N$, then $\alpha + \beta + \gamma$ is equal to [4 marks]

(A) 26

(B) 27

(C) 25

(D) 28

Correct Answer: (B) 27

Solution:

Concept:

- For a nonsingular matrix of order n, $A\,\mathrm{adj}\,A = |A|I$. Here $|A| = 5 \neq 0$, so these adjugate identities apply without qualification.
- For order three, $\mathrm{adj}(kA) = k^2\,\mathrm{adj}\,A$, and $|kM| = k^3|M|$. Also, the adjugate of tI_3 is $t^2 I_3$.

Step 1: Simplify the matrix inside the adjugate The scalar 2 multiplies every entry of A. Each entry of its adjugate is a second-order cofactor, so it receives the factor 2^2. Thus

$$\mathrm{adj}(2A) = 4\,\mathrm{adj}\,A.$$

Multiply by the other factors in the order given:

$$3A\,\mathrm{adj}(2A) = 12A\,\mathrm{adj}\,A = 12|A|I_3 = 60I_3.$$

The unknown entries of A have disappeared; only its determinant matters.

Step 2: Evaluate the outer adjugate and determinant Each diagonal cofactor of $60I_3$ equals 60^2, while

the off-diagonal cofactors vanish. Therefore

$$2\,\mathrm{adj}(60I_3) = 2 \cdot 60^2 I_3 = 7200 I_3.$$

The bars in the question denote a determinant, so the value required before prime factorisation is

$$|7200 I_3| = 7200^3.$$

In particular, multiplying a matrix by 2 does not merely double its determinant in order three.

Step 3: Collect the prime powers Factor the scalar before taking its cube:

$$7200 = 72 \cdot 100 = 2^5 \cdot 3^2 \cdot 5^2.$$

Hence

$$7200^3 = 2^{15} \cdot 3^6 \cdot 5^6.$$

By uniqueness of prime factorisation, $\alpha = 15$, $\beta = 6$, and $\gamma = 6$. Their sum is $15 + 6 + 6 = 27$, excluding the other listed sums 26, 25 and 28.

Step 4: Conclusion $\boxed{\alpha + \beta + \gamma = 27}$. Hence, the correct option is (B).

Quick Tip to Solve: Collapse A times adj(A) to its determinant times the identity before expanding any determinant; keep the matrix order in every scalar power.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. All five letter words are made using all the letters A, B, C, D, E and arranged as in an English dictionary with serial numbers. Let the word at serial number n be denoted by $\mathrm{W_n}$. Let the probability $\mathrm{P}(\mathrm{W_n})$ of choosing the word $\mathrm{W_n}$ satisfy $\mathrm{P}(\mathrm{W_n}) = 2\mathrm{P}(\mathrm{W_{n-1}}), \mathrm{n} > 1$.
If $\mathrm{P(CDBEA)} = \frac{2^\alpha}{2^\beta - 1}, \alpha, \beta \in \mathbb{N}$, then $\alpha + \beta$ is equal to :_____ [4 marks]

Solution:

Concept:

- The five distinct letters have $5! = 120$ permutations. Dictionary order can be counted by fixing the first differing position and permuting the remaining letters.
- The recurrence makes the probabilities a geometric progression: if the first probability is c, then $P(W_n) = c\,2^{n-1}$. Their sum over all 120 words is 1.

Step 1: Count words preceding CDBEA Words starting with A or B precede the target. Their number is $2 \cdot 4! = 48$. Fix C: the second letter can be A or B before reaching D, contributing $2 \cdot 3! = 12$. Fix CD: among A, B, E, only A precedes the third letter B, contributing $1 \cdot 2! = 2$. Finally fix CDB: of A and E, A precedes the fourth letter E, contributing $1 \cdot 1! = 1$. Once CDBE is fixed, the final A introduces no further preceding words.

Step 2: Convert the count to the serial number There are $48 + 12 + 2 + 1 = 63$ earlier words. Serial

numbers begin at 1, so

$$\text{rank}(\text{CDBEA}) = 63 + 1 = 64.$$

This distinction matters: the exponent in the geometric progression is one less than the serial number, so the target probability contains the power 2^{63}, not 2^{64}.

Step 3: Normalise the probabilities Summing the geometric progression over all possible words gives

$$1 = c(1 + 2 + 2^2 + \cdots + 2^{119}) = c(2^{120} - 1).$$

Therefore $c = 1/(2^{120} - 1)$, and

$$P(\text{CDBEA}) = P(W_{64}) = \frac{2^{63}}{2^{120} - 1}.$$

This expression is already in the requested form, with $\alpha = 63$ and $\beta = 120$. Its denominator is odd, so there is no cancellation with the numerator.

Step 4: Conclusion The required numerical answer is $\boxed{\alpha + \beta = 63 + 120 = 183}$.

Quick Tip to Solve: Count preceding permutations first. The dictionary rank is that count plus one, but the geometric-probability exponent is the preceding count itself.

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22. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}, \vec{c} = \lambda\hat{j} + \mu\hat{k}$ and $\hat{d}$ be a unit vector such that $\vec{a} \times \hat{d} = \vec{b} \times \hat{d}$ and $\vec{c} \cdot \hat{d} = 1$. If $\vec{c}$ is perpendicular to $\vec{a}$, then $|3\lambda\hat{d} + \mu\vec{c}|^2$ is equal to_____ [4 marks]

Solution:

Concept:

- Equality of the two cross products implies $(\vec{a} - \vec{b}) \times \hat{d} = \vec{0}$. Thus $\hat{d}$ is parallel or antiparallel to $\vec{a} - \vec{b}$.
- Use $|\mathbf{u} + \mathbf{v}|^2 = |\mathbf{u}|^2 + |\mathbf{v}|^2 + 2\mathbf{u} \cdot \mathbf{v}$. The condition $\vec{c} \cdot \hat{d} = 1$ directly supplies the mixed term.

Step 1: Parameterise both possible unit directions The difference of the given vectors is

$$\vec{a} - \vec{b} = (-2, -1, 2), \qquad |\vec{a} - \vec{b}| = 3.$$

Write

$$\hat{d} = \frac{s}{3}(-2, -1, 2), \qquad s \in \{1, -1\}.$$

Keeping the sign s avoids prematurely discarding one of the valid unit vectors. The vector $\vec{c}$ has coordinates $(0, \lambda, \mu)$.

Step 2: Apply the two dot-product constraints Perpendicularity of $\vec{c}$ and $\vec{a}$ gives

$$\lambda + \mu = 0, \qquad \mu = -\lambda.$$

Next use the specified dot product with $\hat{d}$:

$$1 = \frac{s}{3}(-\lambda + 2\mu) = -s\lambda.$$

Since $s^2 = 1$, this implies $\lambda = -s$ and $\mu = s$. Consequently $\lambda^2 = \mu^2 = 1$, $\lambda\mu = -1$, and $|\vec{c}|^2 = \lambda^2 + \mu^2 = 2$.

Step 3: Expand the requested squared norm Use the unit length of $\hat{d}$ and the known dot product rather than expanding coordinates again:

$$\begin{aligned} |3\lambda\hat{d} + \mu\vec{c}|^2 &= 9\lambda^2|\hat{d}|^2 + \mu^2|\vec{c}|^2 + 6\lambda\mu(\hat{d}\cdot\vec{c}) \\ &= 9\cdot 1 + 1\cdot 2 + 6(-1)(1) \\ &= 5. \end{aligned}$$

Both choices of s give the same squared magnitude, so the answer is independent of the direction chosen for the parallel unit vector.

Step 4: Conclusion The required value is $\boxed{5}$.

Quick Tip to Solve: Keep a plus-or-minus sign for an unspecified parallel unit vector; use dot products to evaluate the final norm without choosing a direction.

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23. Let the product of the focal distances of the point $\mathbf{P}(4, 2\sqrt{3})$ on the hyperbola $\mathrm{H} : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be 32 . Let the length of the conjugate axis of H be p and the length of its latus rectum be q. Then $p^2 + q^2$ is equal to_____ [4 marks]

Solution:

Concept:

- For $x^2/a^2 - y^2/b^2 = 1$, the eccentricity satisfies $e^2 = 1 + b^2/a^2$. On its right branch the focal distances are $ex - a$ and $ex + a$.
- The conjugate axis has length $2b$, while the latus rectum has length $2b^2/a$. The given point must also satisfy the hyperbola equation.

Step 1: Express the two conditions algebraically Set $u = a^2 > 0$ and $v = b^2 > 0$. Substitution of $P = (4, 2\sqrt{3})$ yields

$$\frac{16}{u} - \frac{12}{v} = 1, \qquad v = \frac{12u}{16 - u}.$$

Positivity of v also shows $u < 16$. The product of the two focal distances is

$$(4e - a)(4e + a) = 16e^2 - a^2 = 32.$$

Thus the second independent equation is $16(1 + v/u) - u = 32$.

Step 2: Solve for the squared semiaxes From the focal-distance equation,

$$\frac{v}{u} = 1 + \frac{u}{16}.$$

From the point equation, $v/u = 12/(16-u)$. Equating these gives

$$\frac{12}{16-u} = 1 + \frac{u}{16}.$$

Multiply by $16(16-u)$, which is nonzero because $0 < u < 16$:

$$192 = (16+u)(16-u) = 256 - u^2.$$

Hence $u^2 = 64$, and positivity selects $u = 8$. Then $v = 12u/(16-u) = 12$. The negative root cannot represent a^2.

Step 3: Calculate the requested axis and chord lengths The full conjugate axis is $p = 2b$, so $p^2 = 4b^2 = 48$. The full latus rectum is $q = 2b^2/a$, so

$$q^2 = \frac{4b^4}{a^2} = \frac{4 \cdot 12^2}{8} = 72.$$

Therefore $p^2 + q^2 = 48 + 72 = 120$. As a check, the point satisfies $16/8 - 12/12 = 1$, and $16(1 + 12/8) - 8 = 32$ reproduces the focal-distance product.

Step 4: Conclusion The numerical answer is $\boxed{120}$.

Quick Tip to Solve: Distinguish a semiaxis from the full axis and the semilatus rectum from the full latus rectum before squaring their lengths.

24. The area of the region bounded by the curve $y = \max\{|x|, x|x-2|\}$, the x-axis and the lines $x = -2$ and $x = 4$ is equal to_____ [4 marks]

Solution:

Concept:

- The maximum function must be compared on intervals where the absolute-value expressions change. The breakpoints of the absolute values are $x = 0$ and $x = 2$.
- Since $\max\{|x|, x|x-2|\} \geq |x| \geq 0$, the bounded area is the ordinary integral of the upper function from -2 to 4; no signed-area cancellation occurs.

Step 1: Find which expression forms the upper boundary For $-2 \leq x \leq 0$, the term $x|x-2|$ is nonpositive, so the maximum is $-x$. For $0 \leq x \leq 2$, compare x and $x(2-x)$: their difference is $x(1-x)$. Hence the quadratic dominates on $[0,1]$ and x dominates on $[1,2]$. For $2 \leq x \leq 4$, compare x and $x(x-2)$; their difference is $x(x-3)$. The line dominates on $[2,3]$, and the quadratic dominates on $[3,4]$.

Step 2: Set up the piecewise area integral The neighbouring line intervals combine into $[1,3]$. Thus

$$\mathcal{A} = \int_{-2}^{0} (-x)\,dx + \int_{0}^{1} (2x - x^2)\,dx + \int_{1}^{3} x\,dx + \int_{3}^{4} (x^2 - 2x)\,dx.$$

The two expressions agree at each switch, so the boundary is continuous. The value $x = 2$ changes

the formula inside the second candidate but does not change the actual maximum there.

Step 3: Evaluate and add the four contributions The four positive areas are

$$\int_{-2}^{0}(-x)\,dx = 2,$$
$$\int_{0}^{1}(2x - x^2)\,dx = 1 - \frac{1}{3} = \frac{2}{3},$$
$$\int_{1}^{3} x\,dx = \frac{9-1}{2} = 4,$$
$$\int_{3}^{4}(x^2 - 2x)\,dx = \left[\frac{x^3}{3} - x^2\right]_3^4 = \frac{16}{3}.$$

Their sum is $2 + 2/3 + 4 + 16/3 = 12$. Integrating only one of the two candidates over the whole interval would omit the changes of the upper boundary.

Step 4: Conclusion The required area is $\boxed{12}$ square units.

Quick Tip to Solve: For a maximum, first solve equality of the candidates separately on the absolute-value intervals; integrate only the candidate that is larger.

25. If the number of seven-digit numbers, such that the sum of their digits is even, is $m \cdot n \cdot 10^n; m, n \in \{1, 2, 3, \ldots, 9\}$, then $m + n$ is equal to_____ [4 marks]

Solution:

Concept:

- A seven-digit number has a nonzero first digit, while each remaining digit can be any of the ten digits from 0 to 9.
- An integer sum is even when its last summand has the same parity as the sum of the preceding summands. There are five digits of each parity among 0 through 9.

Step 1: Count the allowed prefixes The leading digit has 9 choices, namely 1 through 9. Each of the next five digits has 10 choices. Thus the first six digits can be chosen in 9×10^5 ways. Keeping the leading digit nonzero is essential: counting all seven-place strings would also include numbers with fewer than seven digits.

Step 2: Choose the final digit by parity Fix any one of these six-digit prefixes, and let its digit sum be s. If s is even, the final digit must be one of $0, 2, 4, 6, 8$; if s is odd, it must be one of $1, 3, 5, 7, 9$. In either case exactly five final digits work. Consequently the required number is

$$N = 9 \times 10^5 \times 5 = 45 \times 10^5 = 4,500,000.$$

This argument pairs the even-sum and odd-sum numbers without assuming that the leading digit has equally many even and odd choices.

Step 3: Identify the permitted parameters Compare $N = 45 \times 10^5$ with $mn10^n$. Taking $n = 5$ gives $5m = 45$, so $m = 9$. Both parameters belong to the prescribed set. For $n \leq 4$, even $m = 9$ gives at

most $36 \times 10^4 < N$; for $n \geq 6$, even $m = 1$ gives at least $6 \times 10^6 > N$. Hence the admissible pair is unique.

Step 4: Conclusion The parameters are $m = 9$ and $n = 5$. Therefore $\boxed{m + n = 14}$.

Quick Tip to Solve: Fix all but the last digit first: exactly five choices of the last digit make the total digit sum even.

Physics – Section A: Multiple Choice Questions (80 marks)

26. A gas is kept in a container having walls which are thermally non-conducting. Initially the gas has a volume of $800\ \text{cm}^3$ and temperature 27°C. The change in temperature when the gas is adiabatically compressed to $200\ \text{cm}^3$ is:

(Take $\gamma = 1.5$; γ is the ratio of specific heats at constant pressure and at constant volume) [4 marks]

(A) 300 K

(B) 600 K

(C) 327 K

(D) 522 K

Correct Answer: (A) 300 K

Solution:

Concept:

- For a reversible adiabatic change of an ideal gas, $TV^{\gamma-1} =$ constant, with temperature measured in kelvin.
- The required temperature change is $\Delta T = T_2 - T_1$, rather than the final temperature itself. Compression does work on an insulated gas and raises its temperature.

Step 1: Express the initial state in absolute units The starting temperature is 27°C, so $T_1 = 27 + 273 = 300\,\text{K}$. The initial and final volumes are $800\,\text{cm}^3$ and $200\,\text{cm}^3$. Only their ratio enters the adiabatic relation, so it is unnecessary to convert both volumes to cubic metres. Their common unit cancels.

Step 2: Apply the adiabatic temperature relation With $\gamma = 1.5$, the exponent is $\gamma - 1 = 0.5$. Hence

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}, \qquad T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}.$$

Substituting the values gives

$$T_2 = 300 \left(\frac{800}{200}\right)^{1/2} = 300\sqrt{4} = 600\,\text{K}.$$

The rise agrees with the first law: no heat is supplied through the nonconducting walls, and the work done on the gas increases its internal energy.

Step 3: Calculate the rise and compare the options The question asks for the change in temperature,

so subtract the initial value:

$$\Delta T = 600 - 300 = 300\,\mathrm{K}.$$

The value $600\,\mathrm{K}$ is the final absolute temperature and therefore is not the requested quantity. The final Celsius reading is $327^{\circ}\mathrm{C}$, but 327 K is neither the final temperature nor the temperature rise. The remaining value 522 K does not follow from the given volume ratio.

Step 4: Conclusion The temperature increases by $\boxed{300\,\mathrm{K}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use kelvin in the adiabatic power law, and distinguish the final temperature from the temperature change.

---

27. A person measures mass of 3 different particles as $435.42\ \mathrm{g}, 226.3\ \mathrm{g}$ and 0.125 g . According to the rules for arithmetic operations with significant figures, the addition of the masses of 3 particles will be. [4 marks]

(A) 661.845 g

(B) 661.84 g

(C) 662 g

(D) 661.8 g

Correct Answer: (D) 661.8 g

Solution:

Concept:

- In addition and subtraction of measured quantities, the reported decimal place is set by the least precise decimal place among the inputs.
- The significant-figure count rule for multiplication is different; it must not replace the decimal-place rule when adding measured masses.

Step 1: Identify the limiting decimal precision The measurements are $435.42\,\mathrm{g}$, $226.3\,\mathrm{g}$, and $0.125\,\mathrm{g}$. They are reported to the hundredths, tenths, and thousandths places, respectively. The measurement $226.3\,\mathrm{g}$ is the least precise in absolute decimal resolution. Therefore the total must be reported to the tenths place, meaning one digit after the decimal point.

Step 2: Add without prematurely rounding Keep all supplied digits during the arithmetic, using zeros only to align the decimal points:

$$\begin{aligned} M &= 435.420 + 226.300 + 0.125 \\ &= 661.845\,\mathrm{g}. \end{aligned}$$

These extra displayed zeros help with addition; they do not improve the experimental precision of any measurement. Rounding the individual inputs first can unnecessarily lose information, so perform the addition before the final rounding operation.

Step 3: Round the result once In 661.845, the tenths digit is 8 and the following hundredths digit is 4. Since 4 is less than 5, retain the tenths digit. Thus the properly reported sum is $661.8\,\mathrm{g}$. Option (A)

retains unjustified thousandths, while option (B) retains unjustified hundredths. Option (C), 662 g, rounds to the units place and discards more precision than the measurements require. Only option (D) retains the correct decimal place.

Step 4: Conclusion The sum with the correct measurement precision is $\boxed{661.8\,\text{g}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a sum, line up decimal points and round the final total to the least precise decimal place of the measurements.

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28. A wire of length 25 m and cross-sectional area $5\ \text{mm}^2$ having resistivity of $2 \times 10^{-6}\Omega\ \text{m}$ is bent into a complete circle. The resistance between diametrically opposite points will be [4 marks]

(A) 100Ω

(B) 2.5Ω

(C) 12.5Ω

(D) 50Ω

Correct Answer: (B) 2.5Ω

Solution:

Concept:

- A uniform wire has resistance $R = \rho L/A$, where ρ is resistivity, L is wire length and A is cross-sectional area.
- Diametrically opposite terminals divide a circular wire into equal semicircular paths. These paths connect the same pair of terminals and are therefore in parallel.

Step 1: Calculate the resistance of the complete wire Convert the area carefully: $5\,\text{mm}^2 = 5\times10^{-6}\,\text{m}^2$, since the linear conversion factor must be squared. The resistance before bending is

$$R = \frac{(2 \times 10^{-6})(25)}{5 \times 10^{-6}} = 10\,\Omega.$$

Bending does not change the wire length, cross-sectional area or resistivity, so this is also the resistance obtained by traversing the entire circumference once.

Step 2: Find the resistance of each semicircle The two terminals lie at opposite ends of a diameter. Each arc between them has length $25/2 = 12.5\,\text{m}$. Since resistance is proportional to length for a uniform wire, each semicircle has resistance $10/2 = 5\,\Omega$. Current supplied at one terminal can reach the other through either arc; it need not pass through both arcs in succession.

Step 3: Combine the two parallel branches The equivalent resistance between the specified points is

$$\frac{1}{R_{\text{eq}}} = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}, \qquad R_{\text{eq}} = 2.5\,\Omega.$$

Equivalently, two equal resistances in parallel give half the resistance of either branch. The result is one quarter of the full-wire resistance. The options 12.5, 50 and 100 ohms cannot be correct:

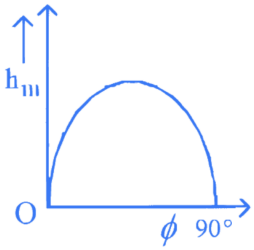
parallel connection must give a resistance smaller than either 5-ohm semicircle, and 12.5 is merely the numerical arc length, not its resistance.

Step 4: Conclusion The resistance between diametrically opposite points is $\boxed{2.5\,\Omega}$. Hence, the correct option is (B).

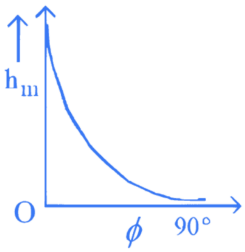
Quick Tip to Solve: For a uniform circular wire measured across a diameter, divide the full-wire resistance by four: two half-wire resistances are in parallel.

29. The angle of projection of a particle is measured from the vertical axis as ϕ and the maximum height reached by the particle is h_m. Here h_m as function of ϕ can be presented as [4 marks]

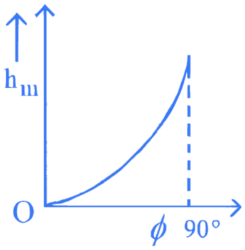
(A)



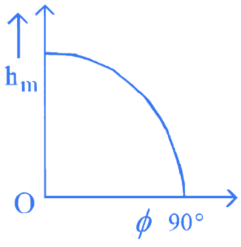
(B)



(C)



(D)



Correct Answer: Neither decreasing graphical option has both horizontal endpoint tangents and the inflection required by the exact cos-squared dependence.

Solution:

Concept:

- When the projection angle ϕ is measured from the vertical, the initial vertical velocity is $u_y = u\cos\phi$.
- For fixed launch speed u under uniform gravity, the maximum height above the launch point is $h_m = u_y^2/(2g)$.

Step 1: Derive the height function At the highest point, the vertical velocity is zero. Applying $v_y^2 = u_y^2 - 2gh_m$ therefore gives

$$h_m(\phi) = \frac{u^2}{2g}\cos^2\phi = H\cos^2\phi, \qquad H = \frac{u^2}{2g}.$$

Here u and g remain fixed as the projection angle varies. In particular, $h_m(0) = H$, $h_m(45^\circ) = H/2$, and $h_m(90^\circ) = 0$. The graph decreases throughout the interval, so the hump in (A) and increasing curve in (C) are excluded.

Step 2: Check the slopes and curvature Differentiate with respect to the angle in radians:

$$\frac{dh_m}{d\phi} = -H\sin 2\phi, \qquad \frac{d^2h_m}{d\phi^2} = -2H\cos 2\phi.$$

Both endpoint tangents are horizontal. The curve is initially concave downward, changes curvature at 45°, and is concave upward near 90°. Thus it must flatten as it approaches zero as well as start flat at its maximum.

Step 3: Assess the supplied graphical choices Of the decreasing sketches, (B) falls steeply from the vertical axis and is convex throughout, missing the initially horizontal tangent and downward concavity. Sketch (D) starts flat but becomes steeper toward its zero, missing the final flattening and change of curvature. These are not exact graphs of $\cos^2\phi$. Endpoint values and monotonic decrease alone do not distinguish the two decreasing sketches; the derivative test exposes the problem with both.

Step 4: Conclusion The exact relation is $\boxed{h_m = \frac{u^2}{2g}\cos^2\phi}$. None of the four supplied sketches represents all its required features; (D) resembles its initial behaviour, but neither decreasing sketch is mathematically exact.

Quick Tip to Solve: For an angle measured from the vertical, use cosine for the vertical component; check both endpoint slopes when comparing graphs.

30. The work function of a metal is 3 eV . The color of the visible light that is required to cause emission of photoelectrons is [4 marks]

(A) Red

(B) Green

(C) Blue

(D) Yellow

Correct Answer: (C) Blue

Solution:

Concept:

- Photoelectric emission requires a photon energy $E = h\nu$ at least equal to the work function W; increasing intensity alone cannot overcome the threshold.
- Photon energy increases as wavelength decreases: $E = hc/\lambda$, with $hc \simeq 1240\,\text{eV nm}$.

Step 1: Compute the threshold wavelength The work function is $W = 3\,\text{eV}$. At threshold, the emitted electrons have zero maximum kinetic energy, so

$$\lambda_0 = \frac{hc}{W} = \frac{1240}{3}\,\text{nm} \simeq 413\,\text{nm}.$$

Emission requires $\lambda \leq 413\,\text{nm}$, or equivalently $\nu \geq W/h$. This numerical threshold is the precise physical condition and is more definite than a broad colour name.

Step 2: Compare the listed colour regions Red, yellow and green light have longer wavelengths and lower photon energies than the short-wavelength blue/violet end of the visible spectrum. Their photons cannot supply the stated 3 eV work function. Blue is the shortest-wavelength and highest-frequency choice in the list, and it is the supplied keyed answer. Merely making red, yellow or green light brighter increases the number of photons without increasing the energy of each photon.

Step 3: State the wavelength qualification The threshold falls at about 413 nm, conventionally in the violet part of the visible range. Ordinary blue light around 450 nm has energy $1240/450 \simeq 2.76\,\text{eV}$, so it would not actually emit electrons from this metal. Thus the choice Blue must be read as the short-wavelength blue/violet end intended by the options; it does not mean that every wavelength called blue suffices. The decisive experimental requirement remains the computed wavelength cutoff, independently of colour naming.

Step 4: Conclusion Among the listed colours, the keyed choice is Blue. Hence, the correct option is (C). Precisely, emission requires $\lambda \leq 413\,\text{nm}$, at the violet end of the visible spectrum; ordinary longer-wavelength blue light is insufficient.

Quick Tip to Solve: Calculate the threshold wavelength before choosing a colour; broad colour names can obscure a cutoff near the blue-violet boundary.

31.

Match the LIST-I with LIST-II

List - I		List - II	
A.	**Gravitational constant**	**I.**	$[\mathrm{LT}^{-2}]$
B.	**Gravitational potential energy**	**II.**	$[\mathrm{L}^2\ \mathrm{T}^{-2}]$
C.	**Gravitational potential**	**III.**	$[\mathrm{ML}^2\ \mathrm{T}^{-2}]$
D.	**Acceleration due to gravity**	**IV.**	$[\mathrm{M}^{-1}\ \mathrm{L}^3\ \mathrm{T}^{-2}]$

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-III, C-II, D-I

(B) A-II, B-IV, C-III, D-I

(C) A-I, B-III, C-IV, D-II

(D) A-III, B-II, C-I, D-IV

Correct Answer: (A) A-IV, B-III, C-II, D-I

Solution:

Concept:

- Dimensions express a physical quantity in powers of mass M, length L, and time T; numerical factors have no dimensions.
- Use Newton's law $F = Gm_1m_2/r^2$, energy $U = Fr$, potential $V = U/m$, and acceleration $g = F/m$ to identify the four entries.

Step 1: Find the dimensions of the gravitational constant Force has dimensions $[F] = \mathrm{MLT}^{-2}$. Rearranging Newton's gravitational law isolates the constant, rather than treating it as an acceleration:

$$G = \frac{Fr^2}{m_1m_2}, \qquad [G] = \frac{(\mathrm{MLT}^{-2})\mathrm{L}^2}{\mathrm{M}^2} = \mathrm{M}^{-1}\mathrm{L}^3\mathrm{T}^{-2}.$$

Thus A matches IV.

Step 2: Distinguish gravitational energy from potential Gravitational potential energy is work, so its dimensions are force times displacement:

$$[U] = [F][r] = \mathrm{ML}^2\mathrm{T}^{-2}.$$

Hence B matches III. Gravitational potential is potential energy per unit test mass, not total energy:

$$[V] = \frac{[U]}{\mathrm{M}} = \mathrm{L}^2\mathrm{T}^{-2}.$$

Hence C matches II. Dividing by mass is essential: it removes the mass power from the dimensions.

Step 3: Match acceleration and eliminate the other combinations Acceleration is change in velocity per unit time, hence $[g] = \mathrm{LT}^{-2}$, which gives D-I. Options B, C and D all assign an incorrect dimension to G; they also confuse energy, potential, or acceleration. The four independently derived matches therefore select a unique combination.

Step 4: Conclusion The matching is A-IV, B-III, C-II, D-I. Hence, the correct option is (A).

Quick Tip to Solve: Potential is energy per unit mass: remove one mass power from the energy dimensions before matching it.

32. Consider a completely full cylindrical water tank of height 1.6 m and of cross-sectional area $0.5\ \mathrm{m}^2$. It has a small hole in its side at a height 90 cm from the bottom. Assume, the crosssectional area of the hole to be negligibly small as compared to that of the water tank. If a load 50 kg is applied at the top surface of the water in the tank then the velocity of the water coming out at the instant when the hole is opened is:

$$\left(\mathrm{g} = 10\ \mathrm{m/s}^2\right)$$

[4 marks]

(A) $2\ \mathrm{m/s}$

(B) $5\ \mathrm{m/s}$

(C) $3\ \mathrm{m/s}$

(D) $4\ \mathrm{m/s}$

Correct Answer: (D) $4\ \mathrm{m/s}$

Solution:

Concept:

- For steady, incompressible flow with negligible viscous loss, Bernoulli's equation conserves $p + \rho g z + \frac{1}{2}\rho v^2$ along a streamline.
- The surface load supplies gauge pressure $p_L = Mg/A$. Since the hole is very small compared with the tank area, the initial speed of the upper surface is negligible.

Step 1: Find the driving head and load pressure The water surface is at $1.6\,\mathrm{m}$ above the bottom, while the hole is at $0.90\,\mathrm{m}$. The depth of the hole below the water surface is therefore

$$h = 1.6 - 0.90 = 0.70\,\mathrm{m}.$$

The applied load acts over the tank cross-sectional area $A = 0.5\,\mathrm{m}^2$, giving

$$p_L = \frac{50 \times 10}{0.5} = 1000\,\mathrm{Pa}.$$

Use the density of water, $\rho = 1000\,\mathrm{kg\,m^{-3}}$.

Step 2: Apply Bernoulli between the surface and the outlet The water surface has pressure $p_{\mathrm{atm}} + p_L$, whereas the emerging jet is at atmospheric pressure. The atmospheric terms cancel; the load term does not. Taking the hole as the zero of height,

$$p_L + \rho g h = \frac{1}{2}\rho v^2.$$

Thus

$$v^2 = 2gh + \frac{2p_L}{\rho} = 2(10)(0.70) + \frac{2(1000)}{1000} = 16.$$

The additional pressure corresponds to a further $0.10\,\mathrm{m}$ of water head.

Step 3: Select the physical speed Taking the positive square root gives $v = 4\,\mathrm{m\,s^{-1}}$. Speeds 2, 5 and 3

would require $v^2 = 4, 25, 9$, respectively, and do not satisfy the energy balance. In particular, using the full tank height instead of the hole depth would incorrectly overestimate the available gravitational energy.

Step 4: Conclusion The initial efflux speed is $4\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a loaded surface, add the pressure head $M/(\rho A)$ to the water depth above the hole before using Torricelli's formula.

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33. Consider following statements for refraction of light through prism, when angle of deviation is minimum.

A. The refracted ray inside prism becomes parallel to the base.

B. Larger angle prisms provide smaller angle of minimum deviation.

C. Angle of incidence and angle of emergence becomes equal.

D. There are always two sets of angle of incidence for which deviation will be same except at minimum deviation setting.

E. Angle of refraction becomes double of prism angle.

Choose the correct answer from the options given below : [4 marks]

(A) A, B and E Only

(B) B, C and D Only

(C) B, D and E Only

(D) A, C and D Only

Correct Answer: (D) A, C and D Only

Solution:

Concept:

- For refraction through a prism, $r_1 + r_2 = A$ and $\delta = i + e - A$, where A is its refracting angle.
- At minimum deviation the light path is symmetric: $i = e$, $r_1 = r_2 = A/2$, and $\mu = \sin[(A + \delta_m)/2]/\sin(A/2)$.

Step 1: Check the symmetric-path statements At the minimum of the deviation-versus-incidence curve, symmetry gives equal incidence and emergence angles, so statement C is correct. For the usual prism geometry represented in this question, the symmetric refracted ray is parallel to its base, so statement A is also correct. Both internal refraction angles are $A/2$, since they are equal and sum to A. They are not $2A$; therefore statement E is incorrect.

Step 2: Test the effect of the prism angle For the same refractive index in air,

$$\delta_m = 2\sin^{-1}\left(\mu \sin\frac{A}{2}\right) - A.$$

In the transmission range and for $\mu > 1$, this increases with A. For a thin prism the same result is especially transparent: $\delta_m \simeq (\mu - 1)A$. A larger refracting angle therefore does not give a smaller minimum deviation, and statement B is incorrect.

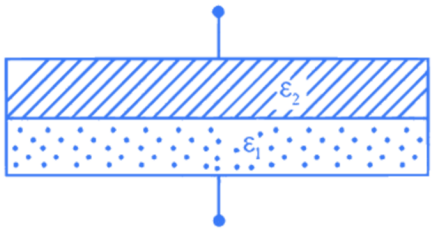
Step 3: Use reversibility to check the paired settings A ray path with incidence i and emergence e can be reversed, interchanging these angles without changing $i + e - A$. Thus an accessible nonminimum deviation corresponds to two incidence settings on the two branches of the deviation curve. At the minimum they coincide because $i = e$. This is statement D. Every other offered option contains false statement B or E, whereas option D contains exactly A, C and D.

Step 4: Conclusion The correct statements are A, C and D only. Hence, the correct option is (D).

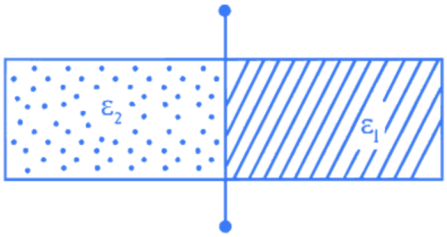
Quick Tip to Solve: At minimum deviation remember the pair $i = e$ and $r_1 = r_2 = A/2$; reversibility explains the two equal-deviation settings away from the minimum.

34. A parallel plate capacitor is filled equally(half) with two dielectrics of dielectric constants ε_1 and ε_2, as shown in figures. The distance between the plates is d and area of each plate is A. If capacitance in first configuration and second configuration are C_1 and C_2 respectively, then $\frac{C_1}{C_2}$ is: [4 marks]

First Configuration



Second Configuration



(A) $\frac{\varepsilon_0(\varepsilon_1+\varepsilon_2)}{2}$

(B) $\frac{\varepsilon_1\varepsilon_2^2}{(\varepsilon_1+\varepsilon_2)^2}$

(C) $\frac{4\varepsilon_1\varepsilon_2}{(\varepsilon_1+\varepsilon_2)^2}$

(D) $\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}$

Correct Answer: (C) $\frac{4\varepsilon_1\varepsilon_2}{(\varepsilon_1+\varepsilon_2)^2}$

Solution:

Concept:

- A dielectric-filled parallel-plate capacitor has capacitance $C = \varepsilon_0\varepsilon_r A/d$, where ε_r is its relative dielectric constant.
- Layers stacked along the electric field behave as capacitors in series; regions occupying separate parts of the plate area behave as capacitors in parallel.

Step 1: Reduce the first configuration to series capacitors In the first figure the dielectric interface is parallel to the plates, so the field traverses both materials. Each layer has thickness $d/2$ and area A. Their individual capacitances are

$$C_a = \frac{2\varepsilon_0\varepsilon_1 A}{d}, \qquad C_b = \frac{2\varepsilon_0\varepsilon_2 A}{d}.$$

The same free charge passes through the series arrangement, and the voltage drops add. Therefore

$$C_1 = \frac{C_a C_b}{C_a + C_b} = \frac{2\varepsilon_0 A\varepsilon_1\varepsilon_2}{d(\varepsilon_1 + \varepsilon_2)}.$$

Step 2: Reduce the second configuration to parallel capacitors In the second figure the materials are side by side, each covering plate area $A/2$ across the full gap d. Both regions have the same potential difference, and their charges add. Thus

$$C_2 = \frac{\varepsilon_0\varepsilon_1(A/2)}{d} + \frac{\varepsilon_0\varepsilon_2(A/2)}{d} = \frac{\varepsilon_0 A(\varepsilon_1 + \varepsilon_2)}{2d}.$$

Step 3: Divide and check the equal-dielectric limit The geometric factors $\varepsilon_0 A/d$ cancel in the ratio:

$$\frac{C_1}{C_2} = \frac{4\varepsilon_1\varepsilon_2}{(\varepsilon_1 + \varepsilon_2)^2}.$$

If both dielectric constants equal k, both arrangements become the same homogeneous capacitor and the ratio must be one. Option C gives one; options B and D give $k/4$ and $k/2$, and option A incorrectly retains ε_0 in a dimensionless ratio.

Step 4: Conclusion $C_1/C_2 = 4\varepsilon_1\varepsilon_2/(\varepsilon_1 + \varepsilon_2)^2$. Hence, the correct option is (C).

Quick Tip to Solve: Decide series or parallel by the field direction: successive layers share charge, while side-by-side plate regions share voltage.

35. During the melting of a slab of ice at 273 K at atmospheric pressure : [4 marks]

(A) Internal energy of ice-water system remains unchanged.

(B) Positive work is done by the ice-water system on the atmosphere.

(C) Positive work is done on the ice-water system by the atmosphere.

(D) Internal energy of the ice-water system decreases.

Correct Answer: (C) Positive work is done on the ice-water system by the atmosphere.

Solution:

Concept:

- At constant external pressure, work done by the system is $W_{\text{by}} = p\Delta V$; work done on it is $W_{\text{on}} = -p\Delta V$.
- Ice is less dense than liquid water at the melting point. Melting therefore decreases the volume for a fixed mass, while latent heat is absorbed at constant temperature.

Step 1: Determine the sign of the volume change Let the fixed mass of ice be m. Initially its volume is $V_i = m/\rho_i$, and after melting its volume is $V_f = m/\rho_w$. Since $\rho_w > \rho_i$,

$$V_f < V_i, \qquad \Delta V = V_f - V_i < 0.$$

The ice-water system contracts as the solid melts. The surrounding atmosphere consequently pushes inward through a net displacement; it is not pushed outward by an expanding system.

Step 2: Identify who performs positive work The atmospheric pressure is positive, so

$$W_{\text{by}} = p\Delta V < 0, \qquad W_{\text{on}} = -p\Delta V > 0.$$

Thus the atmosphere does positive work on the ice-water system, as stated in option C. Option B reverses the direction of energy transfer by treating the melting ice as if it expanded.

Step 3: Check the internal-energy alternatives Using the first law with work done by the system,

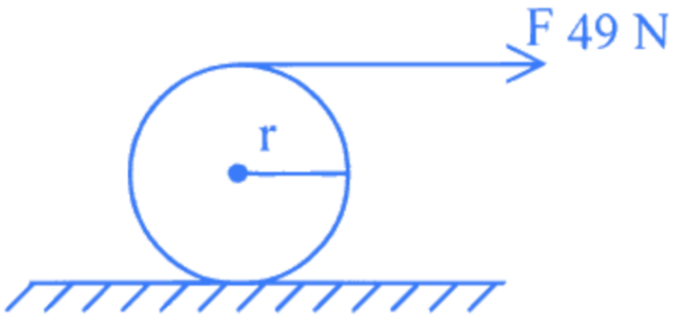
$$\Delta U = Q - W_{\text{by}}.$$

Melting requires positive latent heat input $Q = mL_f > 0$, and $W_{\text{by}} < 0$, so $\Delta U > 0$. The constant temperature does not imply constant internal energy: intermolecular potential energy changes during a phase transition. Therefore option A, which claims no change, and option D, which claims a decrease, are both incorrect. No temperature rise is required for this energy increase.

Step 4: Conclusion Positive work is done on the ice-water system by the atmosphere during melting. Hence, the correct option is (C).

Quick Tip to Solve: For ice melting into water, first remember the contraction; then use $p\Delta V$ to determine the work sign before applying the first law.

36. A force of 49 N acts tangentially at the highest point of a sphere (solid) of mass 20 kg , kept on a rough horizontal plane. If the sphere rolls without slipping, then the acceleration of the center of the sphere is [4 marks]



(A) $2.5\ \text{m/s}^2$

(B) $3.5\ \text{m/s}^2$

(C) $0.25\ \text{m/s}^2$

(D) $0.35\ \text{m/s}^2$

Correct Answer: (B) $3.5\ \text{m/s}^2$

Solution:

Concept:

- For a uniform solid sphere, $I = \frac{2}{5}mR^2$ about its centre. Pure rolling on a stationary surface imposes $a = \alpha R$.
- The applied tangential force at the top supplies both a horizontal force and a torque FR; static friction must be found from simultaneous translation and rotation.

Step 1: Write the translational and rotational equations Take rightward acceleration and clockwise angular acceleration as positive. Let f denote signed friction, positive to the right. Translation gives

$$F + f = ma.$$

The applied top force gives clockwise torque FR. A rightward friction force at the bottom gives an anticlockwise torque, so

$$(F - f)R = I\alpha.$$

This sign convention allows friction to emerge from the equations instead of assuming it always opposes the motion of the centre.

Step 2: Use the rolling constraint to eliminate friction Substitute $I = \frac{2}{5}mR^2$ and $\alpha = a/R$:

$$F - f = \frac{2}{5}ma.$$

Adding this to $F + f = ma$ eliminates f, giving

$$2F = \frac{7}{5}ma, \qquad a = \frac{10F}{7m}.$$

With $F = 49\,\text{N}$ and $m = 20\,\text{kg}$,

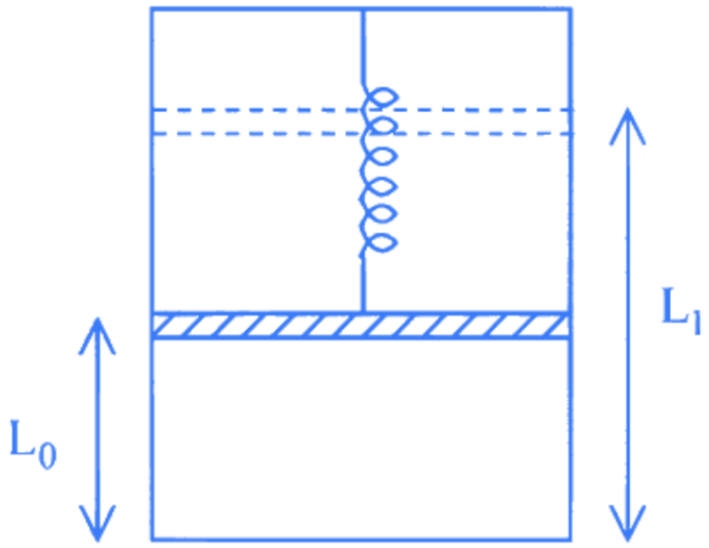
$$a = \frac{10 \times 49}{7 \times 20} = 3.5\,\text{m s}^{-2}.$$

Step 3: Check the required friction and reject the alternatives The friction obtained from translation is $f = ma - F = 70 - 49 = 21\,\text{N}$, directed rightward. Without friction the top force would spin the sphere too rapidly for its translational motion; rightward friction adds translation and reduces the clockwise torque. The assumed rough surface supplies this static friction for pure rolling. A value near $F/m = 2.45\,\text{m s}^{-2}$ would ignore the coupling between translation and rotation; the smaller offered accelerations also fail the combined equations.

Step 4: Conclusion The centre accelerates at $3.5\,\text{m s}^{-2}$ in the direction of the applied force. Hence, the correct option is (B).

Quick Tip to Solve: For a force at the top of a rolling sphere, combine $F + f = ma$ with $F - f = Ia/R^2$; adding eliminates the unknown friction immediately.

37. A piston of mass M is hung from a massless spring whose restoring force law goes as $F = -kx^3$, where k is the spring constant of appropriate dimension. The piston separates the vertical chamber into two parts, where the bottom part is filled with ' n ' moles of an ideal gas. An external work is done on the gas isothermally (at a constant temperature T) with the help of a heating filament (with negligible volume) mounted in lower part of the chamber, so that the piston goes up from a height L_0 to L_1, the total energy delivered by the filament is:(Assume spring to be in its natural length before heating) [4 marks]



(A) $nRT \ln\left(\frac{L_1}{L_0}\right) + Mg\left(L_1 - L_0\right) + \frac{3k}{4}\left({L_1}^4 - {L_0}^4\right)$

(B) $nRT \ln\left(\frac{L_1}{L_0}\right) + Mg\left(L_1 - L_0\right) + \frac{k}{4}\left(L_1^4 - {L_0}^4\right)$

(C) $nRT \ln\left(\frac{L_1^2}{L_0^2}\right) + \frac{Mg}{2}\left(L_1 - L_0\right) + \frac{k}{4}\left(L_1^4 - {L_0}^4\right)$

(D) $3nRT \ln\left(\frac{L_1}{L_0}\right) + 2Mg\left(L_1 - L_0\right) + \frac{k}{3}\left({L_1}^3 - {L_0}^3\right)$

Correct Answer: No physically consistent option follows from the stated isothermal process and initially natural spring.

Solution:

Concept:

- For an ideal gas, internal energy depends only on temperature. In an isothermal process, $\Delta U = 0$, so the net heat received by the gas equals the work it does: $Q = W_{\text{gas}}$.
- For a quasistatic isothermal expansion, $W_{\text{gas}} = nRT \ln(V_1/V_0)$. A nonlinear spring with force $F = -kx^3$ stores energy $U_s = kx^4/4$, where x is deformation measured from its natural length.

Step 1: Apply the first law to the gas Let the cylinder have constant cross-sectional area A. Its gas volumes are $V_0 = AL_0$ and $V_1 = AL_1$. Thus, if the filament is the only heat source and the expansion is quasistatic, its heat supplied to the gas is

$$Q = nRT \ln\left(\frac{V_1}{V_0}\right) = nRT \ln\left(\frac{L_1}{L_0}\right).$$

The work that lifts the piston and changes the spring energy is part of the gas work, rather than an additional heat term to be added to that same work. Isothermal heating alone cannot prescribe an arbitrary expansion between two equilibrium states against an unchanged external loading.

Step 2: Check the spring coordinate and the offered expressions The question says that the spring is initially at its natural length. On raising the piston by $L_1 - L_0$, the magnitude of its deformation is therefore $L_1 - L_0$, and its energy increase is

$$\Delta U_s = \frac{k}{4}(L_1 - L_0)^4.$$

The rise in gravitational potential energy is $Mg(L_1 - L_0)$. In contrast, every offered expression uses a spring-energy term based on absolute piston heights; even option (B) uses $k(L_1^4 - L_0^4)/4$. That expression would require L itself to be the spring deformation, contrary to the initial-natural-length condition. Moreover, adding $nRT\ln(L_1/L_0)$ to all the mechanical-energy changes counts overlapping work contributions.

Step 3: Conclusion As written, no offered option gives a physically consistent unique answer. The usual intended selection is (B) if one adds the three formal terms $nRT\ln(L_1/L_0)$, $Mg(L_1 - L_0)$ and $\int_{L_0}^{L_1} kx^3\,dx$, but that interpretation contradicts the stated spring reference and the isothermal first-law accounting. Hence the printed question is ambiguous.

Quick Tip to Solve: Choose the thermodynamic system first, and measure spring deformation from its natural length; never add gas work again as a separate energy cost.

38.

Match the LIST-I with LIST-II

List - I		List - II	
A.	${}_0^1\text{n} + {}_{92}^{235}\text{U} \rightarrow {}_{54}^{140}\text{Xe} + {}_{38}^{94}\text{Sr} + 2{}_0^1\text{n}$	**I.**	**Chemical reaction**
B.	$2\text{H}_2 + \text{O}_2 \rightarrow 2\text{H}_2\text{O}$	**II.**	**Fusion with +ve Q value**
C.	${}_1^2\text{H} + {}_1^2\text{H} \rightarrow {}_2^3\text{He} + {}_0^1\text{n}$	**III.**	**Fission**
D.	${}_1^1\text{H} + {}_1^3\text{H} \rightarrow {}_1^2\text{H} + {}_1^2\text{H}$	**IV.**	**Fusion with -ve Q value**

Choose the correct answer from the options given below:

[4 marks]

(A) A-II, B-I, C-III, D-IV

(B) A-III, B-I, C-II, D-IV

(C) A-III, B-I, C-IV, D-II

(D) A-II, B-I, C-IV, D-III

Correct Answer: (B) A-III, B-I, C-II, D-IV

Solution:

Concept:

- Fission splits a heavy nucleus into lighter nuclei. Fusion combines light nuclei or rearranges them into a more tightly bound nuclear system.
- The reaction energy is $Q = (m_{\text{initial}} - m_{\text{final}})c^2$. Positive Q means energy release; negative Q means energy must be supplied. Chemical reactions rearrange electrons and chemical bonds without changing the nuclei.

Step 1: Identify the heavy-nucleus and chemical reactions In reaction A, a neutron is absorbed by uranium-235 and the heavy nucleus splits into xenon and strontium, with additional neutrons emitted. This is nuclear fission, so A matches III. In reaction B, hydrogen and oxygen form water. The hydrogen and oxygen nuclei remain unchanged; only bonding changes. It is therefore a chemical reaction, and B matches I. These two identifications already reject options (A) and (D), which incorrectly describe the uranium reaction as fusion.

Step 2: Compare the energy balances of the light-nucleus reactions For reaction C, two deuterium nuclei give helium-3 and a neutron. The products are more tightly bound overall; using approximate binding energies gives $Q \simeq 7.72 - 2(2.22) = +3.28\,\text{MeV}$. Hence C is fusion with positive Q, matching II. For D, a proton and tritium yield two deuterons. Its binding-energy change is $Q \simeq 2(2.22) - 8.48 = -4.04\,\text{MeV}$, so energy is absorbed and D matches IV. Although it involves light nuclei, its negative energy balance distinguishes it from the exothermic deuterium reaction. Option (C) reverses these two energy signs.

Step 3: Conclusion The matching is A-III, B-I, C-II, D-IV. Hence, the correct option is (B).

Quick Tip to Solve: Use total binding energy of products minus reactants to find the sign of Q; a light-nucleus reaction need not release energy.

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39. A particle is released from height S above the surface of the earth. At certain height its kinetic energy is three times its potential energy. The height from the surface of the earth and the speed of the particle at that instant are respectively. [4 marks]

(A) $\frac{S}{4}, \frac{3gS}{2}$

(B) $\frac{S}{2}, \frac{3gS}{2}$

(C) $\frac{S}{4}, \sqrt{\frac{3gS}{2}}$

(D) $\frac{S}{2}, \sqrt{\frac{3gS}{2}}$

Correct Answer: (C) $\frac{S}{4}, \sqrt{\frac{3gS}{2}}$

Solution:

Concept:

- With air resistance neglected and g constant, gravitational potential energy relative to the ground is $U = mgh$. Mechanical energy $K + U$ is conserved.
- Released from rest means initial kinetic energy is zero. At the required point the condition is

$K = 3U$, while kinetic energy also equals $\frac{1}{2}mv^2$.

Step 1: Use the energy ratio to find the height Let the particle have mass m and let h be its height above the ground at the required instant. Initially its total mechanical energy is mgS. At height h, energy conservation gives

$$K + mgh = mgS.$$

Substitute the stated ratio $K = 3mgh$ into this equation:

$$3mgh + mgh = mgS \quad \Longrightarrow \quad 4h = S.$$

Thus the particle is at $h = S/4$. It has fallen through $S - h = 3S/4$, rather than through half the original height. The mass cancels, as expected for free fall under gravity.

Step 2: Calculate the speed and check the options The kinetic energy equals the loss in gravitational potential energy:

$$\frac{1}{2}mv^2 = mg(S - h) = \frac{3}{4}mgS.$$

Consequently,

$$v^2 = \frac{3gS}{2}, \qquad v = \sqrt{\frac{3gS}{2}}.$$

The positive square root is used because speed is a nonnegative magnitude. This is also obtained from $v^2 = u^2 + 2g(3S/4)$, with initial speed $u = 0$. Options (A) and (B) omit the square root, so their second quantities have the dimensions of speed squared rather than speed. Option (D) gives the correct speed but the wrong height; at $S/2$, kinetic and potential energies would be equal.

Step 3: Conclusion The height and speed are $\frac{S}{4}$ and $\sqrt{\frac{3gS}{2}}$, respectively. Hence, the correct option is (C).

Quick Tip to Solve: If K is r times U during a fall from rest, the remaining height is S/(r+1); use the distance fallen to calculate speed.

40. Two blocks of masses m and $M, (M > m)$, are placed on a frictionless table as shown in figure. A massless spring with spring constant k is attached with the lower block. If the system is slightly displaced and released, then (μ = coefficient of friction between the two blocks)

A. The time period of small oscillation of the two blocks is $T = 2\pi\sqrt{\frac{(m+M)}{k}}$

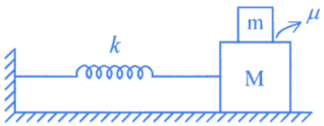
B. The acceleration of the blocks is $a = -\frac{kx}{M+m}$ (x = displacement of the blocks from the mean position)

C. The magnitude of the frictional force on the upper block is $\frac{m\mu|x|}{M+m}$

D. The maximum amplitude of the upper block, if it does not slip, is $\frac{\mu(M+m)g}{k}$

E. Maximum frictional force can be $\mu(\mathrm{M}+\mathrm{m})\mathrm{g}$.

Choose the correct answer from the options given below : [4 marks]



(A) B, C, D Only

(B) C, D, E Only

(C) A, B, D Only

(D) A, B, C Only

Correct Answer: (C) A, B, D Only

Solution:

Concept:

- When the blocks do not slip, they move as a single mass $M+m$, acted on horizontally by the spring restoring force $-kx$.
- Static friction supplies the upper block's horizontal acceleration. Its required value is $f = m|a|$, and its maximum available magnitude is $\mu N = \mu mg$.

Step 1: Find the common acceleration and oscillation period For the two-block system, internal friction forces cancel, leaving

$$(M+m)a = -kx \quad \Longrightarrow \quad a = -\frac{k}{M+m}x.$$

This is simple harmonic motion with $\omega^2 = k/(M+m)$. Therefore

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M+m}{k}}.$$

Thus statements A and B are correct. The larger block mass and the smaller block mass both contribute to inertia while sticking; using only the lower block mass would incorrectly ignore the acceleration of the upper block.

Step 2: Apply the no-slip condition to the upper block The upper block has no spring directly attached

to it, so static friction is its only horizontal force. Its required magnitude is

$$f = m|a| = \frac{mk|x|}{M+m}.$$

This is not the expression in statement C, which contains μ instead of the spring constant k; C is false. At maximum displacement A, the required friction is greatest. No slipping requires

$$\frac{mkA}{M+m} \le \mu mg \quad \Longrightarrow \quad A \le \frac{\mu(M+m)g}{k}.$$

Hence D is correct. The normal reaction between the blocks is mg, so the limiting friction is μmg, not $\mu(M+m)g$. Statement E is therefore false. Every alternative containing C or E must be rejected.

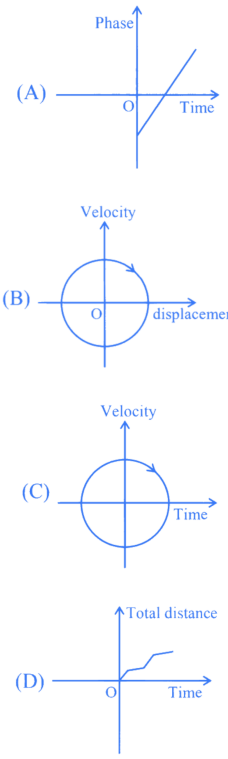
Step 3: Conclusion Only statements A, B and D are correct. Hence, the correct option is (C).

Quick Tip to Solve: Find the common SHM acceleration first, then use f = m|a| for the upper block and compare its maximum with mu m g.

41.

Which of the following curves possibly represent one-dimensional motion of a particle?

Choose the correct answer from the options given below : [4 marks]



(A) A, B and C only

(B) A, C and D only

(C) A, B and D only

(D) A and B only

Correct Answer: (C) A, B and D only

Solution:

Concept:

- At each instant of time a particle has a single velocity, position and phase. A graph of any of these quantities against time must therefore be single-valued.
- A phase-space graph plots velocity against displacement rather than time; it can be a closed curve because the particle can revisit the same displacement with a different velocity. Total distance travelled can increase or remain constant but cannot decrease.

Step 1: Test graphs A and B against possible motions Graph A plots phase against time as an increasing straight line. This is possible for simple harmonic motion, whose phase is $\phi = \omega t + \phi_0$, with constant positive angular frequency ω. The phase may begin at a negative value; it is defined relative to a chosen origin. Graph B plots velocity against displacement as a closed circular curve. For SHM,

$$v^2 + \omega^2 x^2 = \omega^2 A^2.$$

This is an ellipse in a velocity-displacement plot, and it appears circular for a suitable choice of numerical scales on the two axes. At a given displacement the particle passes once with positive and once with negative velocity, at different times. Thus the two branches cause no physical contradiction.

Step 2: Check the time-axis restriction and accumulated distance Graph C is a circle in the velocity-time plane. A vertical line at a time within its horizontal span intersects both upper and lower branches, assigning two distinct velocities to one particle at the same instant. The indicated direction also requires time to run backwards on part of the curve. It cannot represent one-dimensional motion. Graph D gives total distance travelled as a piecewise increasing function of time. Its slope is the nonnegative speed; changes of slope represent changes of speed. It never decreases, so it is possible. Options (A) and (B) include the impossible graph C, while option (D) unnecessarily excludes the possible distance-time graph D.

Step 3: Conclusion Graphs A, B and D can represent one-dimensional motion, whereas C cannot. Hence, the correct option is (C).

Quick Tip to Solve: Check which quantity labels the horizontal axis: a loop is allowed in phase space, but a velocity-time loop gives two velocities at one instant.

42. The radii of curvature for a thin convex lens are 10 cm and 15 cm respectively. The focal length of the lens is 12 cm . The refractive index of the lens material is [4 marks]

(A) 1.4

(B) 1.8

(C) 1.5

(D) 1.2

Correct Answer: (C) 1.5

Solution:

Concept:

- For a thin lens in air, the lens maker formula is $1/f = (\mu - 1)(1/R_1 - 1/R_2)$, where μ is the lens material's refractive index.
- With light travelling from left to right, a biconvex lens has $R_1 > 0$ and $R_2 < 0$. The stated radii are magnitudes; their signs are assigned from the surface geometry.

Step 1: Assign the signs of the two curvatures Take the first surface radius as $R_1 = +10\,\text{cm}$ and the second as $R_2 = -15\,\text{cm}$. The first centre of curvature lies to the right of its surface, while the second lies to its left. The lens is converging, so $f = +12\,\text{cm}$. The curvature factor is therefore

$$\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{10} - \frac{1}{-15} = \frac{1}{10} + \frac{1}{15} = \frac{1}{6}\,\text{cm}^{-1}.$$

Reversing the lens exchanges the two magnitudes but leaves their positive sum, and thus the focal length, unchanged.

Step 2: Substitute the focal length and solve for the index Insert the curvature factor into the lens maker formula:

$$\frac{1}{12} = (\mu - 1)\frac{1}{6}.$$

Multiplication by $6\,\text{cm}$ gives $\mu - 1 = 1/2$, hence $\mu = 1.5$. The refractive index is dimensionless, so no length unit accompanies the result. As a check, the lens power predicted by this index is $(1.5 - 1)/6 = 1/12\,\text{cm}^{-1}$, reproducing the given focal length. The other indices 1.4, 1.8 and 1.2 would instead give focal lengths 15 cm, 7.5 cm and 30 cm respectively, so none matches the given 12 cm.

Step 3: Conclusion The refractive index of the lens material is $\boxed{1.5}$. Hence, the correct option is (C).

Quick Tip to Solve: For a biconvex lens, use the sum of the reciprocal radius magnitudes in the lens maker formula, because the signed second radius is negative.

43. The electrostatic potential on the surface of uniformly charged spherical shell of radius $\mathrm{R} = 10$ cm is 120 V . The potential at the centre of shell, at a distance $\mathrm{r} = 5$ cm from centre, and at a distance $\mathrm{r} = 15$ cm from the centre of the shell respectively, are: [4 marks]

(A) 0 V, 120 V, 40 V

(B) 120 V, 120 V, 80 V

(C) 40 V, 40 V, 80 V

(D) 0 V, 0 V, 80 V

Correct Answer: (B) 120 V, 120 V, 80 V

Solution:

Concept:

- The electric potential is measured relative to infinity. Outside a uniformly charged spherical shell, its potential is the same as that of a point charge at its centre: $V(r) = kQ/r$, where $k = 1/(4\pi\varepsilon_0)$.
- The electric field everywhere inside the shell is zero. Hence the potential has no spatial variation there and equals its surface value, rather than necessarily being zero.

Step 1: Find the potential throughout the interior The radius of the shell is $R = 10\,\mathrm{cm}$, and its surface potential is $V(R) = 120\,\mathrm{V}$. Since $E = -dV/dr = 0$ at every interior point, moving from the surface to any point inside requires no work per unit test charge. Therefore the centre and the point at $r = 5\,\mathrm{cm} < R$ both have the surface potential:

$$V(0) = V(5\,\mathrm{cm}) = 120\,\mathrm{V}.$$

The zero interior field implies a constant potential, not a zero potential: the reference was already fixed at infinity.

Step 2: Use the inverse-distance law outside The point at $r = 15\,\mathrm{cm}$ is outside the shell. Dividing its point-charge potential by the known surface potential eliminates the unknown total charge:

$$\frac{V(15\,\mathrm{cm})}{120} = \frac{R}{15\,\mathrm{cm}} = \frac{10}{15}.$$

$$V(15\,\mathrm{cm}) = 80\,\mathrm{V}.$$

Option (A) incorrectly makes the central potential zero and also gives the wrong exterior value. Option (C) incorrectly lets the potential decrease within the shell, while option (D) confuses zero electric field with zero potential.

Step 3: Conclusion The three potentials, in the stated order, are $120\,\mathrm{V},\ 120\,\mathrm{V},\ 80\,\mathrm{V}$. Hence, the correct option is (B).

Quick Tip to Solve: For a charged spherical shell, keep V constant for all $r \le R$; use $V(r) = V(R)R/r$ only outside.

44. The radiation pressure exerted by a 450 W light source on a perfectly reflecting surface placed at 2 m away from it, is [4 marks]

(A) 3×10^{-8} Pascals

(B) 0

(C) 1.5×10^{-8} Pascals

(D) 6×10^{-8} Pascals

Correct Answer: (D) 6×10^{-8} Pascals

Solution:

Concept:

- For an isotropic source of power P, the intensity at distance r is $I = P/(4\pi r^2)$, since the power spreads over a spherical surface.
- At normal incidence on a perfect reflector, the momentum of each photon reverses. Thus the pressure is $p = 2I/c$, where c is the speed of light. A perfectly absorbing surface would instead have pressure I/c.

Step 1: Calculate the intensity at the surface The source radiates $P = 450\,\mathrm{W}$, and the reflecting surface is $r = 2\,\mathrm{m}$ away. The area over which the source power is distributed at that distance is $4\pi(2)^2 = 16\pi\,\mathrm{m}^2$. Consequently,

$$I = \frac{450}{16\pi}\,\mathrm{W\,m^{-2}} \approx 8.95\,\mathrm{W\,m^{-2}}.$$

The source power itself is not the intensity: dividing by the area is essential before applying a radiation-pressure formula.

Step 2: Account for reversal of photon momentum The incident light carries momentum towards the surface. Reflection sends the light back with equal momentum magnitude, so the momentum transfer is twice that for absorption. Taking $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$,

$$p = \frac{2I}{c} = \frac{2(450)}{16\pi(3 \times 10^8)}\,\mathrm{Pa} \approx 5.97 \times 10^{-8}\,\mathrm{Pa}.$$

This rounds to $6 \times 10^{-8}\,\mathrm{Pa}$. Option (A) is the absorbing-surface result and misses the factor of two; option (B) incorrectly assumes reflection gives no momentum transfer. Option (C) is too small by a factor of four relative to the reflecting value.

Step 3: Conclusion The radiation pressure is approximately $6 \times 10^{-8}\,\mathrm{Pa}$. Hence, the correct option is (D).

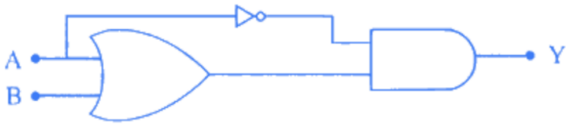
Quick Tip to Solve: First turn source power into intensity using $4\pi r^2$, then double I/c for perfect reflection.

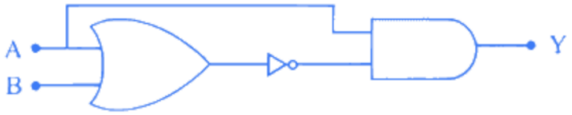
45. Choose the correct logic circuit for the given truth table having inputs A and B.

Inputs		Output
A	B	Y
0	**0**	**0**
0	**1**	**0**
1	**0**	**1**
1	**1**	**1**

[4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- An OR gate gives the Boolean sum $A + B$, while an AND gate gives the Boolean product of its inputs. An inverter changes A to $\overline{A}$.
- The absorption identity is $A(A+B) = A$. It follows either by distributivity, $A^2+AB = A+AB = A$, or by checking separately the cases $A = 0$ and $A = 1$.

Step 1: Identify the required Boolean function Read the output column in the order $(A, B) = (0, 0), (0, 1), (1, 0), (1, 1)$. The required outputs are $0, 0, 1, 1$. Changing B while keeping A fixed never changes the output: when $A = 0$, the output is always zero, and when $A = 1$, it is always one. Therefore the required circuit must implement

$$Y = A.$$

This is stronger than checking a single row; every row must agree.

Step 2: Trace the gates in all four alternatives In option (A), the OR output $A + B$ and the direct

input A enter an AND gate. Hence

$$Y_A = A(A + B) = A,$$

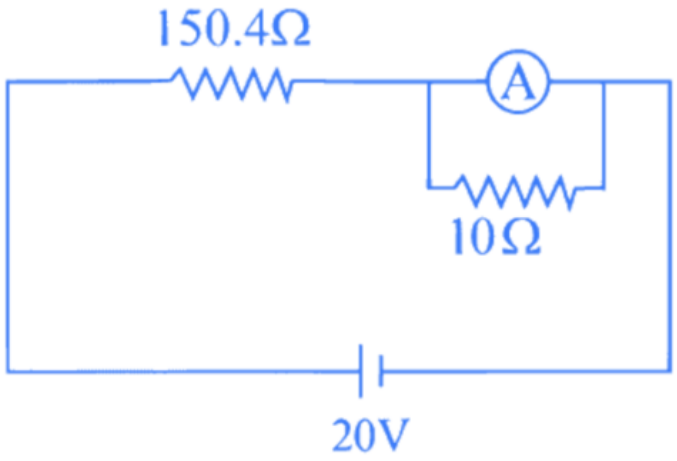
which reproduces the complete table. In option (B), the direct branch is inverted, so $Y_B = \overline{A}(A+B) = \overline{A}B$; it incorrectly gives one for $(0, 1)$ and zero for $(1, 0)$. In option (C), the OR output is inverted before the AND gate, giving $Y_C = A\overline{(A + B)} = 0$, so the last two rows fail. In option (D), the direct AND input is B, giving $Y_D = B(A + B) = B$, so the middle two rows are interchanged. Only the first circuit has the required independence from B.

Step 3: Conclusion The required output is $Y = A$, implemented by $A(A + B)$. Hence, the correct option is (A).

Quick Tip to Solve: If a truth-table output duplicates one input column, simplify each proposed circuit with the absorption law before evaluating all rows.

Physics – Section B: Numerical Value Questions (20 marks)

46. In the figure shown below, a resistance of 150.4Ω is connected in series to an ammeter A of resistance 240Ω. A shunt resistance of 10Ω is connected in parallel with the ammeter. The reading of the ammeter is_____mA . [4 marks]



Solution:

Concept:

- A shunt is a low resistance connected in parallel with the meter. The meter and shunt have equal potential differences, but generally carry different currents.
- For parallel resistances R_A and R_s, their equivalent resistance is $R_p = R_A R_s/(R_A + R_s)$. The source current is found from the total series resistance; current division then gives the meter current.

Step 1: Reduce the meter and shunt combination The ammeter has resistance $R_A = 240\,\Omega$, and the

shunt has resistance $R_s = 10\,\Omega$. Their parallel equivalent is

$$R_p = \frac{240 \times 10}{240 + 10} = 9.6\,\Omega.$$

This combination is in series with the $150.4\,\Omega$ resistor shown in the circuit. Thus

$$R_{\text{total}} = 150.4 + 9.6 = 160\,\Omega.$$

With the $20\,\text{V}$ battery in the figure, the current entering the parallel combination is

$$I_{\text{total}} = \frac{20}{160} = 0.125\,\text{A}.$$

This total current is not the ammeter reading because part of it bypasses the meter through the shunt.

Step 2: Find the current through the ammeter branch The potential difference across the parallel branches is

$$V_p = I_{\text{total}} R_p = 0.125(9.6) = 1.2\,\text{V}.$$

Therefore the ammeter current is

$$I_A = \frac{V_p}{R_A} = \frac{1.2}{240} = 0.005\,\text{A} = 5\,\text{mA}.$$

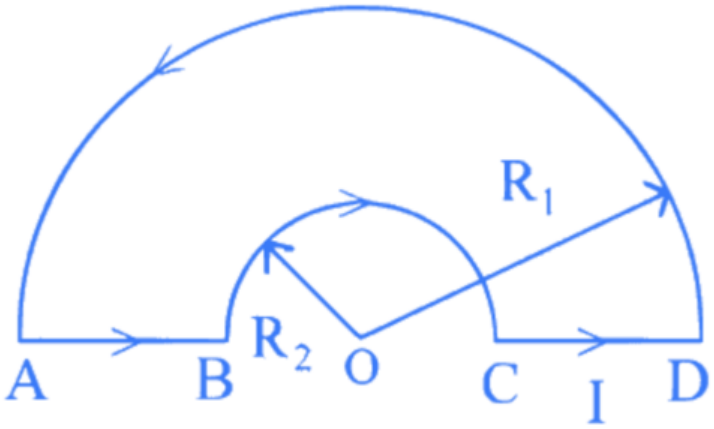
As a check, the shunt current is $1.2/10 = 0.12\,\text{A}$, and $0.12 + 0.005 = 0.125\,\text{A}$, consistent with the source current. The large meter resistance explains why nearly all the current chooses the low-resistance shunt.

Step 3: Conclusion The reading of the ammeter is $5\,\text{mA}$. The numerical entry is 5.

Quick Tip to Solve: Calculate the parallel equivalent before the series current, then use the common branch voltage to find the actual meter reading.

47. A loop ABCDA , carrying current $\mathrm{I} = 12\ \mathrm{A}$, is placed in a plane, consists of two semi-circular segments of radius $R_1 = 6\pi\ \mathrm{m}$ and $\mathrm{R}_2 = 4\pi\ \mathrm{m}$. The magnitude of the resultant magnetic field at center O is $\mathrm{k} \times 10^{-7}\ \mathrm{T}$. The value of k is_____.

(Given $\mu_0 = 4\pi \times 10^{-7}\mathrm{TmA}^{-1}$) [4 marks]



Solution:

Concept:

- The magnetic field at the centre of a circular arc of radius R, current I, and angle θ is $B = \mu_0 I\theta/(4\pi R)$. For a semicircle, $\theta = \pi$, so $B = \mu_0 I/(4R)$.
- A straight radial segment gives zero field at the centre because its current element and the vector towards the centre are parallel. The right-hand rule determines whether the fields from different arcs add or subtract.

Step 1: Determine the directions of the arc fields The current follows the loop through the inner semicircle from B to C and returns through the outer semicircle from D to A, as shown. These two circulations about O are opposite. Their fields at O therefore oppose. The connecting segments AB and CD lie along radii through O, so their Biot–Savart cross products vanish. Because the inner radius $R_2 = 4\pi\ \mathrm{m}$ is smaller than $R_1 = 6\pi\ \mathrm{m}$, the inner arc produces the larger field.

Step 2: Subtract the semicircular contributions For the magnitude of the resultant, subtract the outer-arc contribution from the inner one:

$$B = \frac{\mu_0 I}{4}\left(\frac{1}{R_2} - \frac{1}{R_1}\right).$$

Substituting the stated values gives

$$B = \frac{(4\pi \times 10^{-7})(12)}{4}\left(\frac{1}{4\pi} - \frac{1}{6\pi}\right).$$

$$B = (12\pi \times 10^{-7})\frac{1}{12\pi} = 1 \times 10^{-7}\ \mathrm{T}.$$

Equivalently, the inner arc gives $3\times10^{-7}\ \mathrm{T}$ and the outer arc gives $2\times10^{-7}\ \mathrm{T}$. Adding those magnitudes would ignore their opposite directions and produce the wrong coefficient.

Step 3: Conclusion The resultant magnitude is 1×10^{-7} T. Comparing with $k \times 10^{-7}$ T, the required value is $k = 1$.

Quick Tip to Solve: For concentric arcs, decide the field directions before combining $1/R$ terms; radial connectors contribute zero at the common centre.

48. Two coherent monochromatic light beams of intensities 4I and 9I are superimposed. The difference between the maximum and minimum intensities in the resulting interference pattern is xI. The value of x is_____ . [4 marks]

Solution:

Concept:

- For coherent beams, the resultant intensity is $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$, where δ is their phase difference. The square roots of intensities are proportional to wave amplitudes.
- The maximum occurs at $\cos\delta = 1$, and the minimum occurs at $\cos\delta = -1$. Thus $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$ and $I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$.

Step 1: Calculate the maximum intensity Here $I_1 = 4I$ and $I_2 = 9I$, where I is the common intensity unit used in the question. Their amplitude factors are proportional to $2\sqrt{I}$ and $3\sqrt{I}$. At constructive interference these amplitudes add, giving

$$I_{\max} = (2\sqrt{I} + 3\sqrt{I})^2 = 25I.$$

Simply adding the separate intensities would give $13I$, which omits the interference term and cannot describe a bright fringe of coherent beams.

Step 2: Calculate the minimum and take the difference At destructive interference the amplitudes subtract, so

$$I_{\min} = (3\sqrt{I} - 2\sqrt{I})^2 = I.$$

The minimum is not zero because the two amplitudes are unequal: complete cancellation requires equal incident intensities. Therefore

$$I_{\max} - I_{\min} = 25I - I = 24I.$$

As a separate algebraic check, subtracting the general maximum and minimum expressions directly gives

$$I_{\max} - I_{\min} = 4\sqrt{I_1 I_2} = 4\sqrt{(4I)(9I)} = 24I.$$

Both calculations retain the same common intensity factor, so comparison with the stated xI identifies a dimensionless coefficient.

Step 3: Conclusion The difference between the maximum and minimum intensities is $24I$. Therefore the required numerical value is $x = 24$.

Quick Tip to Solve: For unequal coherent beams, use amplitude factors $\sqrt{I_1}$ and $\sqrt{I_2}$; the fringe-intensity difference is $4\sqrt{I_1 I_2}$.

49. Three identical spheres of mass m , are placed at the vertices of an equilateral triangle of length a. When released, they interact only through gravitational force and collide after a time $\mathrm{T} = 4$ seconds. If the sides of the triangle are increased to length $2a$ and also the masses of the spheres are made 2 m , then they will collide after_____seconds. [4 marks]

Solution:

Concept:

- For initially stationary particles interacting only through Newtonian gravitation, acceleration scales as Gm/a^2, where m is the particle mass and a is the initial separation.
- For geometrically similar configurations, the time scale is $t_0 = \sqrt{a^3/(Gm)}$. The dimensionless collision time is the same for both equilateral triangles.

Step 1: Identify the common motion by symmetry At every vertex the forces from the other two spheres are equal and make an angle of $60°$. Their resultant points towards the centroid. Thus the triangle contracts symmetrically, remaining equilateral until the three spheres meet. The changed initial triangle follows the same dimensionless contraction; only its length, mass and time scales change. No initial velocity introduces an additional independent time scale.

Step 2: Derive the gravitational time scaling Write each position as $\mathbf{r} = a\mathbf{x}$ and time as $t = t_0\tau$. Newton's equation has the form $\ddot{\mathbf{r}} = Gm\sum(\mathbf{r}_j - \mathbf{r}_i)/|\mathbf{r}_j - \mathbf{r}_i|^3$. Its two sides have scales a/t_0^2 and Gm/a^2. Hence choose

$$t_0^2 = \frac{a^3}{Gm}, \qquad T \propto \sqrt{\frac{a^3}{m}}.$$

The same dimensionless equation and initial shape guarantee the same proportionality constant.

Step 3: Apply both changes together The new side is $a' = 2a$, and each new mass is $m' = 2m$. Therefore

$$\frac{T'}{T} = \sqrt{\frac{(2a)^3/(2m)}{a^3/m}} = \sqrt{\frac{8}{2}} = 2.$$

With the given collision time $T = 4\,\mathrm{s}$, this gives $T' = 2 \times 4 = 8\,\mathrm{s}$. Increasing separation alone would give a different factor; doubling mass partly offsets that increase.

Step 4: Conclusion The spheres collide after $\boxed{8\,\mathrm{s}}$. The numerical entry is 8.

Quick Tip to Solve: For gravitationally similar systems released from rest, take the ratio $T_2/T_1 = \sqrt{(a_2/a_1)^3/(m_2/m_1)}$; include the mass change as well as the separation change.

50. A 4.0 cm long straight wire carrying a current of 8 A is placed perpendicular to a uniform magnetic field of strength 0.15 T . The magnetic force on the wire is _____mN . [4 marks]

Solution:

Concept:

- A straight wire of length L carrying current I in a uniform magnetic field B experiences force $F = BIL\sin\theta$, where θ is the angle between current and field.
- Use SI units before substituting: $1\,\mathrm{cm} = 10^{-2}\,\mathrm{m}$ and $1\,\mathrm{mN} = 10^{-3}\,\mathrm{N}$. The perpendicular orientation

gives the maximum force.

Step 1: Convert the wire length and identify the angle The current is $I = 8\,\text{A}$, the uniform field is $B = 0.15\,\text{T}$, and the wire length in the field is

$$L = 4.0\,\text{cm} = 4.0 \times 10^{-2}\,\text{m} = 0.040\,\text{m}.$$

The wire is perpendicular to the field, so $\theta = 90^\circ$ and $\sin\theta = 1$. It is the wire direction, equivalently the conventional-current direction, that determines this angle. Using cosine would incorrectly make the force vanish for the stated arrangement.

Step 2: Calculate the force in newtons For the whole straight segment in the uniform field, the magnetic force has magnitude

$$\begin{aligned} F &= BIL\sin 90^\circ \\ &= 0.15 \times 8 \times 0.040 \\ &= 0.048\,\text{N}. \end{aligned}$$

The product of tesla, ampere and metre is a newton, so the units are consistent. The magnetic force is transverse to both the current and field; its direction is unnecessary because the question asks only for its magnitude.

Step 3: Express the result in millinewtons The requested unit is millinewton rather than newton. Since one newton equals one thousand millinewtons,

$$F = 0.048 \times 10^3\,\text{mN} = 48\,\text{mN}.$$

This conversion must be made after the length has been converted to metres; keeping the centimetre value in the SI force formula would make the answer too large by a factor of one hundred.

Step 4: Conclusion The magnetic force is $\boxed{48\,\text{mN}}$. The numerical entry is 48.

Quick Tip to Solve: For a perpendicular straight wire use $F = BIL$; convert centimetres to metres first, then multiply the force in newtons by 1000 to obtain millinewtons.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. The correct order of the complexes $[\mathrm{Co}\,(\mathrm{NH_3})_5\,(\mathrm{H_2O})]^{3+}$ (A), $[\mathrm{Co}\,(\mathrm{NH_3})_6]^{3+}$ (B), $[\mathrm{Co(CN)_6}]^{3-}$ (C) and $[\mathrm{CoCl}\,(\mathrm{NH_3})_5]^{2+}$ (D) in terms of wavelength of light absorbed is [4 marks]

(A) $\mathrm{D} > \mathrm{C} > \mathrm{B} > \mathrm{A}$

(B) $\mathrm{C} > \mathrm{B} > \mathrm{A} > \mathrm{D}$

(C) $\mathrm{D} > \mathrm{A} > \mathrm{B} > \mathrm{C}$

(D) $\mathrm{C} > \mathrm{B} > \mathrm{D} > \mathrm{A}$

Correct Answer: (C) $\mathrm{D} > \mathrm{A} > \mathrm{B} > \mathrm{C}$

Solution:

Concept:

- The energy of light absorbed in a crystal-field transition is $\Delta_o = hc/\lambda$. A larger octahedral splitting therefore corresponds to a shorter absorbed wavelength.
- For the relevant ligands, the spectrochemical series gives $\mathrm{Cl^-} < \mathrm{H_2O} < \mathrm{NH_3} < \mathrm{CN^-}$ in increasing ligand-field strength.

Step 1: Check the oxidation state and geometry All four complexes contain cobalt in oxidation state $+3$. In A and B all ligands are neutral, so the metal charge equals the complex charge. In C six cyanide ions contribute -6, giving $x - 6 = -3$, hence $x = +3$. In D the chloride ligand contributes -1, giving $x - 1 = +2$, again $x = +3$. Each complex has six donor atoms and is compared in an octahedral ligand field. Thus the relevant trend is mainly ligand strength, rather than a change in cobalt oxidation state.

Step 2: Order the crystal-field splittings D has five ammine ligands and one weak-field chloride ligand. Replacing chloride by water gives A and increases the splitting; replacing water by a sixth ammine ligand gives B and increases it further. C contains six strong-field cyanide ligands and has the largest splitting. Therefore

$$\Delta_o(\mathrm{D}) < \Delta_o(\mathrm{A}) < \Delta_o(\mathrm{B}) < \Delta_o(\mathrm{C}).$$

The complex labels here refer to the labels in the question, not to the answer-option letters.

Step 3: Reverse the order for absorbed wavelength Because $\lambda = hc/\Delta_o$, the wavelength order is the reverse of the splitting-energy order:

$$\lambda_{\mathrm{D}} > \lambda_{\mathrm{A}} > \lambda_{\mathrm{B}} > \lambda_{\mathrm{C}}.$$

Options placing C first confuse strongest absorption energy with longest wavelength. The alternative D, C, B, A also incorrectly puts the strongest-field cyanide complex ahead of the ammine complexes in wavelength.

Step 4: Conclusion The correct order of wavelength absorbed is $\boxed{\mathrm{D} > \mathrm{A} > \mathrm{B} > \mathrm{C}}$. Hence, the correct option is (C).

Quick Tip to Solve: Order ligand-field strength first and then reverse that order for wavelength, since $\Delta_o\lambda = hc$. Check that the metal oxidation state is the same before using this shortcut.

52. Which of the following statements are correct?

A. The process of adding an electron to a neutral gaseous atom is always exothermic.

B. The process of removing an electron from an isolated gaseous atom is always endothermic.

C. The 1^{st} ionization energy of boron is less than that of beryllium.

D. The electronegativity of C is 2.5 in CH_4 and CCl_4

E. Li is the most electropositive among elements of group I.

Choose the correct answer from the options given below: [4 marks]

(A) A, C and D Only

(B) B and C Only

(C) B and D Only

(D) B, C and E Only

Correct Answer: (B) B and C Only

Solution:

Concept:

- Ionization of a neutral isolated gaseous atom requires energy, whereas addition of an electron can release or require energy depending on the electronic configuration.
- Electronegativity describes attraction for shared electrons in a bonded state. It is not an invariant property of an isolated atom; its effective value depends on the chemical environment.

Step 1: Test statements A and B A is false: adding an electron is not always exothermic. A noble-gas atom, for example, has a stable filled shell, and adding an electron requires occupation of a higher shell, so the electron-gain process is endothermic. B is true: removing a bound electron from a neutral isolated gaseous atom requires work against the attraction of the nucleus. Hence the first ionization energy is positive, and the ionization process is endothermic.

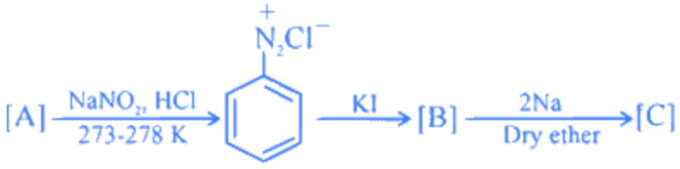
Step 2: Explain the beryllium-boron exception Beryllium has configuration $\mathrm{Be}: 1s^2 2s^2$, while boron has $\mathrm{B}: 1s^2 2s^2 2p^1$. The electron removed from B occupies the higher-energy, less penetrating $2p$ subshell, whereas Be loses a $2s$ electron from a filled subshell. Consequently the first ionization energy of B is lower than that of Be, despite the general increase across the period. Statement C is true.

Step 3: Test statements D and E and select the combination D is false as an absolute statement: the quoted tabulated value 2.5 does not make carbon's electronegativity an invariant value in every compound. Carbon bonded to H and carbon bonded to Cl have different electronic environments. E is also false: electropositive character generally increases down group 1 as atomic size increases and electron loss becomes easier; Li is not its most electropositive element. Thus only B and C are correct. Option A includes false A and D, option C includes false D and omits true C, and option D includes false E.

Step 4: Conclusion Only statements $\boxed{\mathrm{B \text{ and } C}}$ are correct. Hence, the correct option is (B).

Quick Tip to Solve: Remember both exceptions: electron gain need not release energy, and B loses a higher-energy $2p$ electron more easily than Be loses a $2s$ electron.

53. Identify [A], [B] and [C], respectively, in the following reaction sequence: [4 marks]



(A) Aniline; chlorobenzene; benzene.

(B) Nitrobenzene; iodobenzene; biphenyl.

(C) Aniline; iodobenzene; biphenyl.

(D) Aniline; chlorobenzene; iodobenzene.

Correct Answer: (C) Aniline; iodobenzene; biphenyl.

Solution:

Concept:

- A primary aromatic amine undergoes diazotization with $NaNO_2/HCl$ at 273–278 K to give an arenediazonium salt.
- An arenediazonium group is replaced by iodine on reaction with KI; sodium in dry ether couples aryl halides to form a biaryl compound in the Fittig reaction.

Step 1: Identify the precursor of the diazonium salt The intermediate shown is benzenediazonium chloride, $C_6H_5N_2{}^+Cl^-$. Its formation with nitrous acid generated in situ from sodium nitrite and hydrochloric acid requires the primary aromatic amine aniline. Hence $[A] = C_6H_5NH_2$. The cold conditions stabilize the diazonium salt and prevent its premature decomposition. Nitrobenzene cannot undergo this direct diazotization, so the option starting from a nitro group is excluded.

Step 2: Replace the diazonium group with iodine Potassium iodide supplies iodide, which replaces the diazonium group with liberation of nitrogen:

$$C_6H_5N_2{}^+Cl^- + KI \longrightarrow C_6H_5I + N_2 + KCl.$$

Thus [B] is iodobenzene, C_6H_5I. The presence of chloride as the counterion of the original salt does not make chlorobenzene the product of reaction with KI. This excludes the alternatives that assign chlorobenzene to B.

Step 3: Couple the aryl halide with sodium Treatment of iodobenzene with sodium in dry ether forms a carbon-carbon bond between two phenyl groups:

$$2\,C_6H_5I + 2\,Na \longrightarrow C_6H_5{-}C_6H_5 + 2\,NaI.$$

This Fittig coupling gives biphenyl, not benzene or a further halogenated benzene. Consequently $[C] = C_6H_5{-}C_6H_5$. Comparing the complete triple, the only option with aniline, iodobenzene and biphenyl in that order is option C.

Step 4: Conclusion [A] is aniline, [B] is iodobenzene and [C] is biphenyl. Hence, the correct option is (C).

Quick Tip to Solve: Trace the functional group through the sequence: $-NH_2 \longrightarrow -N_2{}^+ \longrightarrow -I$, then couple two phenyl groups with sodium in dry ether.

54. Match LIST-I with LIST-II.

LIST-I (Molecules/ion)		LIST-II (Hybridisation)	
A	PF_5	**I**	dsp^2
B	SF_6	**II**	sp^3d
C	$Ni(CO)_4$	**III**	sp^3d^2
D	$[PtCl_4]^{2-}$	**IV**	sp^3

Choose the correct answer from the options given below. [4 marks]

(A) A-IV, B-I, C-II, D-III

(B) A-III, B-I, C-IV, D-II

(C) A-II, B-III, C-IV, D-I

(D) A-I, B-II, C-III, D-IV

Correct Answer: (C) A-II, B-III, C-IV, D-I

Solution:

Concept:

- In the conventional valence-bond description, five and six bonding directions correspond to sp^3d and sp^3d^2 hybridisation, respectively.
- Four-coordinate complexes can be tetrahedral sp^3 or square planar dsp^2; the metal electronic configuration and geometry distinguish these possibilities.

Step 1: Match the phosphorus and sulfur compounds In PF_5, phosphorus has five P–F sigma bonds and no lone pair, giving a trigonal-bipyramidal arrangement. Its conventional hybridisation is sp^3d, so A matches II. In SF_6, sulfur forms six S–F sigma bonds with no lone pair, giving octahedral geometry. Its conventional hybridisation is sp^3d^2, so B matches III. These are the hybridisation labels asked for by the table.

Step 2: Distinguish the two four-coordinate complexes $Ni(CO)_4$ contains nickel in oxidation state zero. Pairing in the metal d orbitals gives a filled $3d$ set, and the four equivalent bonding directions use $4s$ and $4p$ orbitals in the conventional description. The molecule is tetrahedral and sp^3-hybridised, so C matches IV. In $[PtCl_4]^{2-}$, platinum is $+2$, with a $5d^8$ configuration. This platinum(II) complex is square planar and is described by dsp^2 hybridisation, so D matches I.

Step 3: Assemble the complete mapping The individual matches give

$$\mathrm{A:II}, \qquad \mathrm{B:III}, \qquad \mathrm{C:IV}, \qquad \mathrm{D:I}.$$

Option C contains all four matches. Options A and D already fail for PF_5, and option B assigns the octahedral six-direction label to PF_5 instead of SF_6. Equal coordination number four does not imply equal hybridisation for the nickel and platinum complexes.

Step 4: Conclusion The matching is $\boxed{\mathrm{A:II,\ B:III,\ C:IV,\ D:I}}$. Hence, the correct option is (C).

Quick Tip to Solve: Fix the easy five- and six-coordinate matches first; then remember tetrahedral $Ni(CO)_4$ versus square-planar $[PtCl_4]^{2-}$.

55. Which of the following postulate of Bohr's model of hydrogen atom is not in agreement with quantum mechanical model of an atom? [4 marks]

(A) The electron in a H atom's stationary state moves in a circle around the nucleus.

(B) An atom can take only certain distinct energies E_1, E_2, E_3, etc. These allowed states of constant energy are called the stationary states of atom.

(C) An atom in a stationary state does not emit electromagnetic radiation as long as it stays in the same state.

(D) When an electron makes a transition from a higher energy stationary state to a lower energy stationary state, then it emits a photon of light.

Correct Answer: (A) The electron in a H atom's stationary state moves in a circle around the nucleus.

Solution:

Concept:

- Bohr assigns an electron a definite circular orbit. Quantum mechanics instead describes it by a wavefunction ψ, whose squared magnitude gives a probability density.
- A stationary quantum state has a definite allowed energy, but this does not assign the electron a classical trajectory. A radiative transition between two levels satisfies $h\nu = E_i - E_f$.

Step 1: Identify the assumption about motion Option (A) asserts that the electron moves along a circle around the nucleus in its stationary state. This specifies a definite path, including a definite radius and tangential momentum. Such an orbit is a feature of the Bohr picture, not the quantum mechanical description. An orbital is a probability distribution in space, not the track followed by an electron. The uncertainty principle also prevents assigning the simultaneously precise position and momentum required for a classical trajectory.

Step 2: Check the remaining postulates Option (B) concerns discrete allowed energies, which the bound-state solutions of the hydrogen Schrodinger equation also produce. Option (C) correctly says that an atom remaining in one stationary state does not continually radiate away energy. Option (D) describes photon emission on a transition to a lower energy level, with the photon carrying the energy difference. Both models retain these energy and transition statements. They therefore do not identify the disagreement asked for here.

Step 3: Conclusion The definite circular electron trajectory is inconsistent with the quantum mechanical model. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish an orbit, which is a path, from an orbital, which describes the probability of finding the electron. Quantised energy alone is common to both models.

56. In the following system, $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$ at equilibrium, upon addition of xenon gas at constant T and p, the concentration of [4 marks]

(A) $PCl_5, PCl_3 \& Cl_2$ remain constant

(B) PCl_3 will increase

(C) Cl_2 will decrease

(D) PCl_5 will increase

Correct Answer: (C) Cl_2 will decrease

Solution:

Concept:

- For $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$, $K_c = [PCl_3][Cl_2]/[PCl_5]$ remains fixed at constant temperature.
- Adding inert xenon at constant total pressure expands the vessel. A rightward shift increases the product mole amounts, but it does not imply that their concentrations increase.

Step 1: Separate dilution from the equilibrium shift Let the volume initially increase by a factor $f > 1$. Before any reaction occurs, every reactive concentration is divided by f, and the reaction quotient becomes $Q_c = K_c/f < K_c$. Dissociation therefore proceeds to the right. Nevertheless, the additional product formed only partly offsets its dilution; concentration is amount divided by the expanded volume.

Step 2: Verify the final concentration change For an infinitesimal expansion, write n_A, n_B, n_C for the mole amounts of PCl_5, PCl_3, Cl_2, and $d\xi$ for the forward extent. Differentiating the equilibrium relation $K_c = n_B n_C/(n_A V)$ gives

$$\left(\frac{1}{n_A} + \frac{1}{n_B} + \frac{1}{n_C}\right) d\xi = d\ln V.$$

Thus $d\xi/d\ln V < n_B$ and also $< n_C$. Consequently

$$\frac{d[Cl_2]}{d\ln V} = \frac{1}{V}\left(\frac{d\xi}{d\ln V} - n_C\right) < 0.$$

The same argument makes the trichloride concentration decrease, while pentachloride decreases through dilution and dissociation. Repeating small expansions establishes the finite-change result. Option (A) would apply to an inert gas added at constant volume, not constant pressure; (B) confuses product mole amount with concentration, and (D) contradicts both dilution and dissociation.

Step 3: Conclusion The chlorine concentration decreases. Hence, the correct option is (C). The supplied answer key instead indicates increasing trichloride concentration, but that confuses the increase in product amount with its concentration under the printed constant-pressure condition.

Quick Tip to Solve: For inert-gas additions, first identify whether volume or pressure is fixed. At fixed pressure, account for dilution before interpreting the equilibrium shift.

57.

In the following reactions, which one is NOT correct?

[4 marks]

(A) $C_6H_5N_2Cl \xrightarrow{H_3PO_2/H_2O} C_6H_6$

(B) $C_6H_5N_2Cl \xrightarrow{CuCN/KCN} C_6H_5CN$

(C) $C_6H_5N_2Cl \xrightarrow{C_2H_5OH} C_6H_5OC_2H_5$

(D) $C_6H_5N_2Cl \xrightarrow{KI} C_6H_5I$

Correct Answer: (C) $C_6H_5N_2Cl \xrightarrow{C_2H_5OH} C_6H_5OC_2H_5$

Solution:

Concept:

- The diazonium group in benzenediazonium chloride, $C_6H_5N_2^+Cl^-$, can be replaced with loss of stable nitrogen gas. The reagent determines the group replacing it.
- Hypophosphorous acid and ethanol reduce an arenediazonium salt to the corresponding arene; ethanol does not directly replace the diazonium group with an ethoxy group.

Step 1: Check the reduction reactions In (A), H_3PO_2/H_2O replaces the diazonium group by hydrogen, giving benzene. This is a standard reductive deamination and the displayed product is correct. In (C), the displayed product is ethoxybenzene, $C_6H_5OC_2H_5$, whereas the stated ethanol reagent reduces the diazonium salt to benzene. Ethanol is oxidised to ethanal while the diazonium group is lost as nitrogen:

$$C_6H_5N_2Cl + C_2H_5OH \longrightarrow C_6H_6 + N_2 + CH_3CHO + HCl.$$

The aromatic product therefore has hydrogen, rather than an ethoxy group, at the original attachment site.

Step 2: Check cyanide and iodide replacement In (B), cuprous cyanide in the cyanide medium replaces the diazonium group by $-CN$, forming benzonitrile. This is the cyanation associated with the Sandmeyer reaction. In (D), potassium iodide supplies iodide, producing iodobenzene with elimination of nitrogen; a copper catalyst is not needed for this iodide replacement. The products shown in (B) and (D) therefore agree with their reagents. Only the ethanol reaction assigns the wrong aromatic product.

Step 3: Conclusion Ethanol gives benzene rather than ethoxybenzene, so reaction (C) is NOT correct. Hence, the correct option is (C).

Quick Tip to Solve: Remember the pair of reducing reagents for diazonium salts: hypophosphorous acid and ethanol both replace the diazonium group by hydrogen.

58. Given below are two statements :

Statement I : The N - N single bond is weaker and longer than that of P - P single bond.

Statement II : Compounds of group 15 elements in +3 oxidation states readily undergo disproportionation reactions.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are true

(C) Both Statement I and Statement II are false

(D) Statement I is true but Statement II is false

Correct Answer: (C) Both Statement I and Statement II are false

Solution:

Concept:

- Bond strength and bond length are separate properties. The small nitrogen atom gives a short N–N bond, but strong lone-pair repulsion makes that single bond unusually weak.
- Disproportionation means simultaneous oxidation and reduction of the same element. Whether it occurs depends on the relative stability of the available oxidation states.

Step 1: Evaluate Statement I The N–N single bond is indeed weaker than the P–P single bond, because the lone pairs on the small adjacent nitrogen atoms repel strongly. However, it is shorter, not longer: representative single-bond lengths are about $145\,\text{pm}$ for N–N and $213\,\text{pm}$ for P–P. The larger size of phosphorus explains the longer P–P bond. Since Statement I combines a correct strength comparison with an incorrect length comparison, the statement as a whole is false.

Step 2: Evaluate Statement II The claim is too broad. Nitrogen in the +3 state can disproportionate, as illustrated by nitrous acid:

$$3\,\text{HNO}_2 \longrightarrow \text{HNO}_3 + 2\,\text{NO} + \text{H}_2\text{O}.$$

Here nitrogen changes from +3 to +5 in nitric acid and to +2 in nitric oxide. This behaviour cannot be assigned generally to all group 15 compounds in the +3 state. Down the group the +3 state becomes more stable relative to +5 because of the inert-pair effect; bismuth(III), for example, is the stable common oxidation state and does not readily undergo the asserted general disproportionation. Statement II is therefore false. Options (A), (B), and (D) each require at least one false statement to be true.

Step 3: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (C).

Quick Tip to Solve: Check every clause of a combined statement. A weak bond need not be long, and a reaction typical of nitrogen must not automatically be extended to its entire group.

59.

Which compound would give 3-methyl-6-oxoheptanal upon ozonolysis?

[4 marks]

(A) 1,4-Dimethylcyclohex-1-ene

(B) 1,6-Dimethylcyclohex-1-ene

(C) 1,3-Dimethylcyclohex-1-ene

(D) 1,5-Dimethylcyclohex-1-ene

Correct Answer: (A) 1,4-Dimethylcyclohex-1-ene

Solution:

Concept:

- Ozonolysis cleaves the alkene $C=C$ bond and converts its two carbon atoms into carbonyl carbons. An alkene carbon bearing hydrogen gives an aldehyde; one bearing two carbon groups gives a ketone.
- Cleaving one double bond in a cycloalkene opens the ring to a single chain. Preserve every original single bond and substituent while tracing the product.

Step 1: Reconstruct the required carbonyl chain The named product is

$$OHC-CH_2-CH(CH_3)-CH_2-CH_2-CO-CH_3.$$

Numbering begins at the aldehyde carbon, so the methyl substituent is at C3 and the ketone carbonyl is at C6. Rejoining the two carbonyl carbons by a double bond closes a six-membered ring. The carbon which becomes the ketone must carry a methyl group, while the carbon which becomes the aldehyde must carry hydrogen.

Step 2: Match the ring substituent positions In option (A), the double bond is between ring carbons 1 and 2; carbon 1 bears methyl, and a second methyl group is on ring carbon 4. Cut the 1–2 double bond and trace from the hydrogen-bearing end: old ring carbon 2 becomes product C1, old carbon 3 becomes C2, old carbon 4 becomes C3, old carbon 5 becomes C4, old carbon 6 becomes C5, and old carbon 1 becomes C6. The methyl on old carbon 4 is therefore exactly the required C3 methyl group, while the methyl on old carbon 1 supplies product C7. The other pictured isomers place the second methyl at product C5 in (B), C2 in (C), or C4 in (D), so none gives the specified 3-methyl product.

Step 3: Conclusion Option (A), 1,4-dimethylcyclohex-1-ene, opens to 3-methyl-6-oxoheptanal. Hence, the correct option is (A).

Quick Tip to Solve: For ozonolysis of a cyclic alkene, cut only the double bond, turn its ends into carbonyl groups, and number the opened chain from the aldehyde end.

60. 2 moles each of ethylene glycol and glucose are dissolved in 500 g of water. The boiling point of the resulting solution is: (Given : Ebullioscopic constant of water $= 0.52\ \text{K kg mol}^{-1}$) [4 marks]

(A) 377.3 K

(B) 379.2 K

(C) 375.3 K

(D) 277.3 K

Correct Answer: (A) 377.3 K

Solution:

Concept:

- For nonvolatile, non-electrolyte solutes, the boiling-point elevation is $\Delta T_b = K_b m_{\text{total}}$. Both glucose and ethylene glycol remain molecular in water, so each has van't Hoff factor $i = 1$.
- Molality is total moles of dissolved solute particles per kilogram of solvent, not per kilogram of solution. The boiling point of pure water is approximately $373.15\,\text{K}$.

Step 1: Add the solute amounts and find total molality There are 2 moles of ethylene glycol and 2 moles of glucose. Since both contribute independently to this colligative property, the effective amount is their sum:

$$n_{\text{solute}} = 2 + 2 = 4\,\text{mol}.$$

The mass of water is $500\,\text{g} = 0.500\,\text{kg}$. Therefore

$$m_{\text{total}} = \frac{4}{0.500} = 8\,\text{mol kg}^{-1}.$$

Using just one solute would omit half the particles; using grams directly would also introduce an incorrect factor of one thousand.

Step 2: Calculate the elevation and add it to the solvent boiling point Substitute the given ebullioscopic constant:

$$\Delta T_b = (0.52\,\text{K kg mol}^{-1})(8\,\text{mol kg}^{-1}) = 4.16\,\text{K}.$$

Hence the absolute boiling temperature is

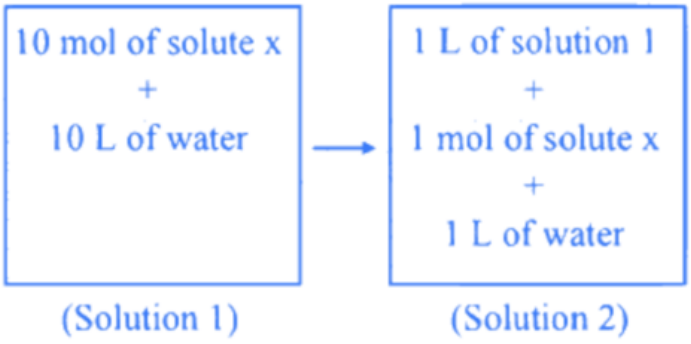
$$T_b = 373.15 + 4.16 = 377.31\,\text{K} \simeq 377.3\,\text{K}.$$

Option (B) has too large an elevation and (C) too small an elevation. Option (D) lies below the boiling point of pure water, which contradicts the positive elevation caused by nonvolatile solutes. The calculation also distinguishes a temperature rise from the final absolute temperature.

Step 3: Conclusion The solution boils at approximately $377.3\,\text{K}$. Hence, the correct option is (A).

Quick Tip to Solve: With several nonvolatile solutes, add their effective particle mole amounts first, divide by the solvent mass in kilograms, and then calculate the common boiling-point elevation.

61. Which of the following properties will change when system containing solution 1 will become solution 2 ? [4 marks]



(A) Molar heat capacity

(B) Concentration

(C) Gibbs free energy

(D) Density

Correct Answer: (C) Gibbs free energy

Solution:

Concept:

- An intensive property depends on the nature and composition of a homogeneous system, rather than the amount present. Density, concentration and molar heat capacity are intensive properties.
- Gibbs free energy is extensive: at fixed temperature, pressure and composition, $G = \sum_i n_i \mu_i$, where n_i is the amount of component i and μ_i is its chemical potential.

Step 1: Compare the compositions Solution 1 contains 10 mol of solute with 10 L of water, so the solute-to-solvent ratio is 1 mol per litre of water. The added mixture contains 1 mol of the same solute and 1 L of water, in the same ratio. Adding it to 1 L of solution 1 therefore increases the amount of solution without changing its composition. The labels do not require treating solvent volume as solution volume; it is the unchanged component ratio that is decisive.

Step 2: Classify the four properties Because the composition is unchanged, the concentration and density remain the same under the same conditions. The molar heat capacity is heat capacity per mole, so it also remains unchanged. By contrast, total Gibbs free energy refers to the entire system. It changes when the amount of solution changes, even though the chemical potentials at the given composition remain the same. Thus the extensive quantity is selected; the three intensive alternatives do not describe the change in system size.

Step 3: Conclusion Gibbs free energy changes. Hence, the correct option is (C).

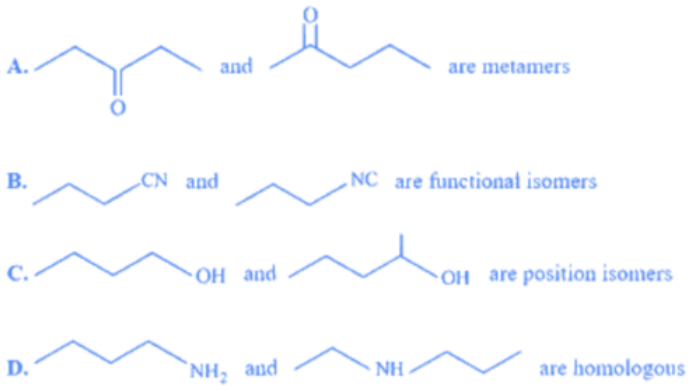
Quick Tip to Solve: When two portions have the same composition, distinguish total properties such as G from properties expressed per mole or per unit volume.

62.

Identify the correct statements from the following.

Choose the correct answer from the options given below:

[4 marks]



(A) B & C Only

(B) C & D Only

(C) A,B & C Only

(D) A & B Only

Correct Answer: (D) A & B Only

Solution:

Concept:

- Structural isomers have the same molecular formula but different connectivity. Metamers differ in the alkyl groups on opposite sides of a polyvalent functional group.
- Position isomerism requires the same carbon framework with a functional group at different positions. Homologues belong to the same homologous series and successive members differ by CH_2.

Step 1: Check statements A and B In A, the ketones are pentan-3-one, $C_2H_5COC_2H_5$, and pentan-2-one, $CH_3COC_3H_7$. Both have formula $C_5H_{10}O$, but their carbonyl groups have different distributions of alkyl groups on the two sides: ethyl/ethyl versus methyl/propyl. They are metamers, so A is correct. In B, the structures are an alkyl cyanide and the corresponding alkyl isocyanide. Bonding through carbon in $R-CN$ versus nitrogen in $R-NC$ gives different functional groups at the same formula. Therefore B is also correct.

Step 2: Check statements C and D In C, count the carbon vertices and endpoints before comparing the hydroxyl positions. The first alcohol has four carbons, whereas the second has five: the upward terminal line on the right-hand structure contributes an additional carbon. Their molecular formulae differ, so they cannot be position isomers. In D, the first amine is primary, with an NH_2 group, whereas the second is secondary, with two carbon groups bonded to NH. They do not form the stated homologous pair within one amine series. Consequently C and D are false; each other answer choice

incorrectly includes C.

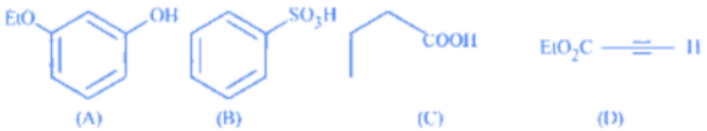
Step 3: Conclusion Only statements A and B are correct. Hence, the correct option is (D).

Quick Tip to Solve: Count every skeletal endpoint and vertex first; an unnoticed methyl branch can invalidate a proposed isomeric relationship.

63.

The least acidic compound, among the following is:

[4 marks]



(A) B

(B) D

(C) A

(D) C

Correct Answer: (B) D

Solution:

Concept:

- An acid is stronger when the conjugate base produced by loss of H^+ is more stable. Compare both the acidic group and delocalisation of its conjugate base.
- Sulphonic acids and carboxylic acids are appreciably stronger acids than phenols. A terminal alkyne proton is generally less acidic than a phenolic proton, even when an adjacent electron-withdrawing group increases its acidity.

Step 1: Identify the acidic sites Compound A is an ethoxy-substituted phenol, so its acidic hydrogen belongs to the phenolic $O-H$ bond. Compound B is benzenesulphonic acid, with an acidic SO_3H group. Compound C is an aliphatic carboxylic acid, containing $COOH$. Compound D is the terminal alkyne $EtO_2C-C\equiv C-H$; its acidic hydrogen is bonded to an sp-hybridised carbon. These are different functional classes, so choosing from only the substituent on the benzene ring would miss the main comparison.

Step 2: Compare conjugate-base stabilisation Deprotonation of B gives a strongly stabilised sulphonate ion, while C gives a carboxylate ion with charge shared over two oxygens. A gives a resonance-stabilised phenoxide ion. D gives an acetylide-type carbanion: the ester group increases the acidity relative to an ordinary terminal alkyne, but it remains a weaker acid than the phenol and the oxygen acids listed. Therefore the least acidic compound is D. The alternatives naming B, A or C respectively select a sulphonic acid, a phenol or a carboxylic acid, all more acidic than D.

Step 3: Conclusion Compound D is the least acidic; it is listed in option (B). Hence, the correct option is (B).

Quick Tip to Solve: Identify the bond that loses hydrogen first: a terminal alkyne $C-H$ is much less acidic than the $O-H$ of the oxygen acids here.

64. Among 10^{-9} g (each) of the following elements, which one will have the highest number of atoms?

Element: $\mathrm{Pb}, \mathrm{Po}, \mathrm{Pr}$ **and Pt** [4 marks]

(A) Pr

(B) Pt

(C) Po

(D) Pb

Correct Answer: (A) Pr

Solution:

Concept:

- For an elemental sample of mass m and atomic molar mass M, the amount is $n = m/M$ and the number of atoms is $N = (m/M)N_{\mathrm{A}}$.
- When all sample masses are identical, mN_{A} is common. Thus atom count is inversely proportional to atomic molar mass, not directly proportional to atomic number or density.

Step 1: Compare the atomic molar masses The approximate molar masses in $\mathrm{g\,mol^{-1}}$ are Pr: 140.9, Pt: 195.1, Pb: 207.2 and Po: about 209. Praseodymium therefore has the smallest molar mass. The order by increasing molar mass is $\mathrm{Pr} < \mathrm{Pt} < \mathrm{Pb} < \mathrm{Po}$. Each given sample weighs 10^{-9} g; the small absolute mass does not change this comparison because the same mass is used for every element.

Step 2: Calculate and rank the atom counts For praseodymium,

$$N_{\mathrm{Pr}} = \frac{10^{-9}}{140.9}(6.022 \times 10^{23}) \simeq 4.27 \times 10^{12} \text{ atoms.}$$

The corresponding values are approximately 3.09×10^{12} for Pt, 2.91×10^{12} for Pb and 2.88×10^{12} for Po. Thus $N_{\mathrm{Pr}} > N_{\mathrm{Pt}} > N_{\mathrm{Pb}} > N_{\mathrm{Po}}$. The alternatives Pt, Po and Pb all have a larger mass per mole and hence fewer atoms in the same sample mass. No information about sample volume is needed.

Step 3: Conclusion Pr has the highest number of atoms in 10^{-9} g. Hence, the correct option is (A).

Quick Tip to Solve: For equal masses, choose the element with the lowest atomic molar mass; Avogadro's constant cancels in the comparison.

65. The metal ions that have the calculated spin-only magnetic moment value of 4.9 B.M. are :

A. Cr^{2+}

B. Fe^{2+}

C. Fe^{3+}

D. Co^{2+}

E. Mn^{3+}

Choose the correct answer from the options given below: [4 marks]

(A) A, D and E Only

(B) A, B and E Only

(C) A, C and E Only

(D) B and E Only

Correct Answer: (B) A, B and E Only

Solution:

Concept:

- The spin-only magnetic moment of an ion with n unpaired electrons is $\mu = \sqrt{n(n+2)}\,\text{B.M.}$. A value close to $4.9\,\text{B.M.}$ corresponds to four unpaired electrons.
- For transition-metal cations, remove the $4s$ electrons before the $3d$ electrons. In the spin-only free-ion comparison, fill the five d orbitals singly before pairing, according to Hund's rule.

Step 1: Find the required unpaired-electron count Substituting $n = 4$ gives

$$\mu = \sqrt{4(4+2)} = \sqrt{24} = 4.899\ldots\ \text{B.M.}$$

Thus each correct ion must have four unpaired electrons. In a d subshell this is possible for high-spin d^4 and d^6 populations. The question asks for the calculated spin-only value, so orbital contributions to experimentally measured moments are not included.

Step 2: Test all five ions Chromium is $[\text{Ar}]3d^5 4s^1$, so Cr^{2+} is $3d^4$ with four unpaired electrons: A qualifies. Iron gives Fe^{2+} : $3d^6$, with four unpaired electrons, so B qualifies; Fe^{3+} : $3d^5$ has five and moment $\sqrt{35} \simeq 5.92\,\text{B.M.}$, so C does not. Co^{2+} : $3d^7$ has three and moment $\sqrt{15} \simeq 3.87\,\text{B.M.}$, excluding D. Finally Mn^{3+} : $3d^4$ has four, so E qualifies. Options containing D or C, or omitting A, fail these electron counts.

Step 3: Conclusion The required ions are A, B and E only: Cr^{2+}, Fe^{2+} and Mn^{3+}. Hence, the correct option is (B).

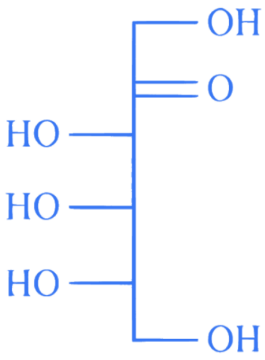
Quick Tip to Solve: Memorise the spin-only pairing: four unpaired electrons occur in high-spin d^4 and d^6, both giving $\sqrt{24}\,\text{B.M.}$.

66.

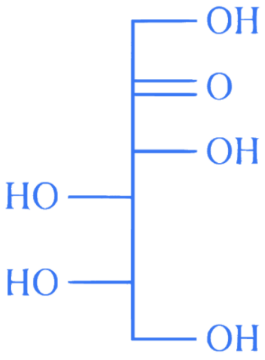
Which of the following is the correct structure of L-Fructose?

[4 marks]

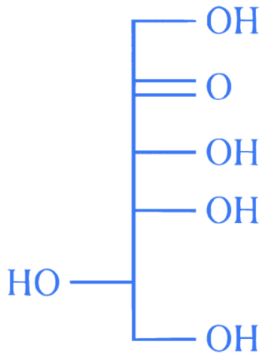
(A)



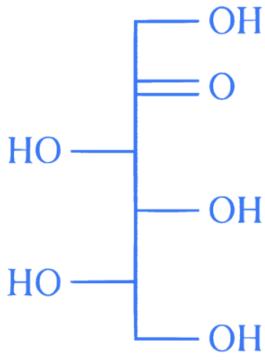
(B)



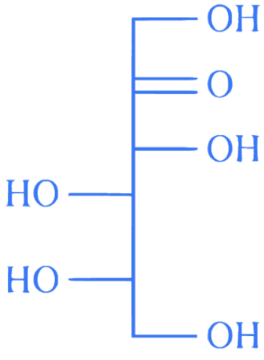
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- Fructose is a ketohexose: its open-chain Fischer projection has six carbons, a ketone at C2, and CH_2OH groups at C1 and C6.
- The D/L designation is fixed by the stereocentre farthest from the carbonyl group, C5 in fructose. The L form has the C5 hydroxyl group on the left; the full stereochemistry must also be that of fructose.

Step 1: Determine the hydroxyl arrangement of L-fructose With C1 at the top and C6 at the bottom, standard D-fructose has hydroxyl groups at C3, C4 and C5 on the left, right and right, respectively. L-fructose is its mirror image, so all three stereocentres invert. The required sequence from C3 through C5 is therefore right, left, left. The C2 carbonyl and the terminal CH_2OH groups remain in the same positions when comparing these Fischer projections.

Step 2: Compare each printed projection Option (A) places the C3, C4 and C5 hydroxyl groups left, left and left, so its C3 configuration is incorrect for L-fructose. Option (B) places them right, left and left and matches all three required centres. Option (C) gives right, right and left, with the wrong C4 configuration. Option (D) gives left, right and left, with incorrect C3 and C4 configurations. Although all four show a left-sided hydroxyl at C5 and hence an L configuration, that feature alone does not identify the sugar as fructose; its other stereocentres must also match.

Step 3: Conclusion The second Fischer projection is L-fructose, with the C3–C5 hydroxyl sequence right–left–left. Hence, the correct option is (B).

Quick Tip to Solve: Use C5 to decide D or L, then check C3 and C4 as well; L does not uniquely identify which ketohexose is shown.

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67. Given below are two statements :

Statement I : A catalyst cannot alter the equilibrium constant (K_c) of the reaction, temperature remaining constant.

Statement II : A homogenous catalyst can change the equilibrium composition of a system, temperature remaining constant.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Statement I is true but Statement II is false

(B) Statement I is false but Statement II is true

(C) Both Statement I and Statement II are true

(D) Both Statement I and Statement II are false

Correct Answer: (A) Statement I is true but Statement II is false

Solution:

Concept:

- At a fixed temperature, the equilibrium constant is determined by the standard Gibbs energy change: $\Delta_r G^\circ = -RT \ln K$. A catalyst changes the rate of approach to equilibrium.

- A catalyst supplies an alternative reaction pathway with a lower activation barrier for both forward and reverse reactions; it does not change the energies of reactants or products.

Step 1: Evaluate Statement I The equilibrium constant measures the equilibrium ratio of product and reactant activities (or the corresponding concentration expression for K_c). At unchanged temperature, introducing a catalyst does not change $\Delta_r G^\circ$, so it cannot change the equilibrium constant. Thus Statement I is true. Faster establishment of equilibrium must not be confused with a different equilibrium constant.

Step 2: Evaluate Statement II A homogeneous catalyst is in the same phase as the reacting mixture. This affects how it participates in the catalytic pathway, but does not give it any ability to shift the thermodynamic equilibrium. Starting from a nonequilibrium mixture, the composition changes more rapidly towards the same equilibrium composition. Starting at equilibrium, there is no net conversion merely because catalyst is added. Hence Statement II, which claims a change in equilibrium composition at fixed temperature, is false.

Step 3: Select the matching combination The required combination is true for Statement I and false for Statement II. Option (B) reverses these truth values; (C) incorrectly accepts a composition shift, and (D) incorrectly rejects the temperature dependence criterion for the equilibrium constant.

Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (A).

Quick Tip to Solve: A catalyst changes how quickly equilibrium is reached; temperature changes the value of the equilibrium constant.

68. Correct order of limiting molar conductivity for cations in water at 298 K is : [4 marks]

(A) $H^+ > Na^+ > K^+ > Ca^{2+} > Mg^{2+}$

(B) $H^+ > Ca^{2+} > Mg^{2+} > K^+ > Na^+$

(C) $Mg^{2+} > H^+ > Ca^{2+} > K^+ > Na^+$

(D) $H^+ > Na^+ > Ca^{2+} > Mg^{2+} > K^+$

Correct Answer: (B) $H^+ > Ca^{2+} > Mg^{2+} > K^+ > Na^+$

Solution:

Concept:

- Limiting ionic molar conductivity λ_i° is the contribution of one mole of an ion to conductivity at infinite dilution. Both ionic charge and mobility matter.
- In water, hydration controls effective ionic size and mobility. The proton has unusually high conductivity because charge moves through the hydrogen-bond network as well as by ordinary ionic motion.

Step 1: Place the proton first The limiting ionic molar conductivity of H^+ at 298 K is approximately $349.6\,S\,cm^2\,mol^{-1}$, much larger than those of the listed metal cations. Its exceptional value results from the proton-transfer mechanism in water. Therefore any ordering with magnesium above hydrogen, such as option (C), is excluded immediately.

Step 2: Compare the metal-ion contributions The approximate limiting molar ionic conductivities, on

a per-mole-of-ion basis, are

$$\lambda^\circ(\mathrm{Ca}^{2+}) \simeq 119, \quad \lambda^\circ(\mathrm{Mg}^{2+}) \simeq 106,$$
$$\lambda^\circ(\mathrm{K}^{+}) \simeq 73.5, \quad \lambda^\circ(\mathrm{Na}^{+}) \simeq 50.1$$

in $\mathrm{S\,cm^2\,mol^{-1}}$. Thus calcium exceeds magnesium, and potassium exceeds sodium. The divalent ions have larger molar contributions than either alkali ion. Magnesium is more strongly hydrated than calcium, and sodium more strongly hydrated than potassium, reducing their respective mobilities.

Step 3: Reject the remaining orders Combining the comparisons gives hydrogen, calcium, magnesium, potassium, sodium in descending order. Options (A) and (D) put sodium too high and reverse its order relative to potassium. Be careful not to substitute equivalent ionic conductivity values for molar ionic conductivity of the divalent ions.

Step 4: Conclusion The correct order is $\mathrm{H}^+ > \mathrm{Ca}^{2+} > \mathrm{Mg}^{2+} > \mathrm{K}^+ > \mathrm{Na}^+$. Hence, the correct option is (B).

Quick Tip to Solve: For molar ionic conductivity, retain the ion charge factor; then remember the exceptional proton mobility and the hydration order within each pair.

69. In a reaction $A + B \rightarrow C$, initial concentrations of A and B are related as $[A]_0 = 8[B]_0$. The half lives of A and B are 10 min and 40 min , respectively. If they start to disappear at the same time, both following first order kinetics, after how much time will the concentration of both the reactants be same? [4 marks]

(A) 20 min

(B) 40 min

(C) 80 min

(D) 60 min

Correct Answer: (B) 40 min

Solution:

Concept:

- For the specified first-order disappearance of a reactant, $[R]_t = [R]_0 e^{-kt} = [R]_0\, 2^{-t/t_{1/2}}$, where $k = \ln 2/t_{1/2}$.
- To find when two independently specified exponential concentration laws cross, form their ratio and set it equal to one; the common initial concentration scale cancels.

Step 1: Write the two concentration laws Let $[B]_0 = b$. The initial concentration of A is then $8b$. Using the two half-lives given in the question, the stipulated first-order laws give

$$[A]_t = 8b\, 2^{-t/10}, \qquad [B]_t = b\, 2^{-t/40},$$

with t in minutes. A disappears more rapidly because its half-life is smaller. Therefore its initially larger concentration can eventually become equal to that of B.

Step 2: Equate the concentrations Setting $[A]_t = [B]_t$ and cancelling b gives

$$8\,2^{-t/10} = 2^{-t/40}.$$

Since $8 = 2^3$, equality of exponents yields

$$3 - \frac{t}{10} = -\frac{t}{40}.$$

Hence

$$3 = t\left(\frac{1}{10} - \frac{1}{40}\right) = \frac{3t}{40}, \qquad t = 40\,\text{min}.$$

The different first-order rates, rather than the initial ratio alone, determine this crossing time.

Step 3: Check the result and alternatives At 40 min, A has completed four half-lives and B one half-life: $[A]_{40} = 8b/16 = b/2$ and $[B]_{40} = b/2$. More generally their ratio is $8\,2^{-3t/40}$. It is greater than one at 20 min and less than one at 60 and 80 min, so those alternatives cannot be the first equality time under the stated kinetic laws.

Step 4: Conclusion The two concentrations become equal after $40\,\text{min}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the concentration ratio directly as $8\,2^{-t(1/10-1/40)}$; equality means its exponent must remove the initial factor of eight.

70. Number of molecules from below which cannot give iodoform reaction is :

Ethanol, Isopropyl alcohol, Bromoacetone, 2-Butanol, 2-Butanone, Butanal, 2-Pentanone, 3-Pentanone, Pentanal and 3-Pentanol. [4 marks]

(A) 4

(B) 3

(C) 5

(D) 2

Correct Answer: (A) 4

Solution:

Concept:

- The iodoform test is given by methyl ketones containing CH_3CO-, which undergo exhaustive iodination followed by alkaline cleavage to yellow CHI_3.
- Alcohols oxidisable to methyl ketones, containing $CH_3CH(OH)-$, also respond. Ethanol is a special positive case because it forms ethanal, which undergoes the haloform reaction.

Step 1: Identify the positive alcohols Ethanol is oxidised to ethanal, so it gives the iodoform test. Isopropyl alcohol is oxidised to acetone, a methyl ketone, and is also positive. 2-Butanol contains the $CH_3CH(OH)-$ unit and produces 2-butanone, so it is positive. By contrast, 3-pentanol produces 3-pentanone, $CH_3CH_2COCH_2CH_3$, which lacks a carbonyl-bound methyl group; therefore 3-pentanol

is negative.

Step 2: Classify the carbonyl compounds Bromoacetone, CH_3COCH_2Br, still has a CH_3CO- group and is positive. Both 2-butanone, $CH_3COCH_2CH_3$, and 2-pentanone, $CH_3COCH_2CH_2CH_3$, are methyl ketones and are positive. 3-Pentanone has ethyl groups on both sides of the carbonyl and is negative. Butanal and pentanal are aldehydes other than ethanal, so neither has the structural arrangement required for iodoform formation.

Step 3: Count the negative compounds The negative compounds are butanal, 3-pentanone, pentanal and 3-pentanol. Thus there are four negatives and six positives among the ten compounds. Counting only the negative ketone would miss the two aldehydes and the secondary alcohol that oxidises to a non-methyl ketone. The counts three, five and two do not match the complete classification.

Step 4: Conclusion Four compounds cannot give the iodoform reaction: butanal, 3-pentanone, pentanal and 3-pentanol. Hence, the correct option is (A).

Quick Tip to Solve: For every alcohol, first write its oxidation product; then look specifically for CH_3CO-, with ethanol/ethanal as the familiar exception.

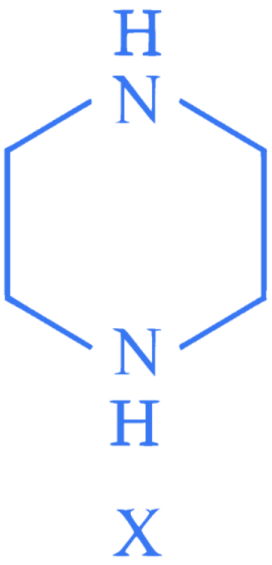
Chemistry – Section B: Numerical Value Questions (20 marks)

71.

During estimation of nitrogen by Dumas' method of compound $\mathrm{X}(0.42\ \mathrm{g})$

_____mL of $\mathrm{N_2}$ **gas will be liberated at STP. (nearest integer)**

(Given molar mass in $\mathrm{g\ mol^{-1}}$ **:** $\mathrm{C:12, H:1,\ N:14}$ **)** [4 marks]



Solution:

Concept:

- In Dumas' method, the nitrogen atoms of an organic sample are converted to molecular nitrogen. Two nitrogen atoms in one molecule give one molecule of $\mathrm{N_2}$.
- At STP, one mole of ideal gas occupies $22.4\,\mathrm{L} = 22400\,\mathrm{mL}$. Obtain the sample molar mass from the molecular structure before applying this volume.

Step 1: Read the molecular formula from the ring The displayed compound is a saturated six-membered ring containing two nitrogen atoms opposite one another and four carbon atoms. Each carbon is a $\mathrm{CH_2}$ group and each nitrogen bears one hydrogen, so the molecular formula is $\mathrm{C_4H_{10}N_2}$. Its molar mass is

$$M = 4(12) + 10(1) + 2(14) = 86\,\mathrm{g\,mol^{-1}}.$$

Counting the hydrogen atoms on nitrogen is necessary: the four carbon groups contribute eight hydrogens and the two NH groups contribute two more.

Step 2: Convert sample amount into nitrogen amount The amount of compound in the 0.42 g sample is

$$n_X = \frac{0.42}{86}\,\mathrm{mol}.$$

Each molecule has two nitrogen atoms, which together produce one $\mathrm{N_2}$ molecule during the estimation.

Consequently

$$n_{N_2} = \frac{2}{2} n_X = \frac{0.42}{86}\,\text{mol}.$$

Multiplying by two at this stage would count nitrogen atoms as if they were nitrogen molecules.

Step 3: Calculate the gas volume and round The volume liberated at STP is

$$V_{N_2} = \frac{0.42}{86} \times 22400 = 109.395\ldots\,\text{mL}.$$

Equivalently, the nitrogen mass is $(0.42/86) \times 28$ g, and dividing by the molar mass 28 of nitrogen gas gives the same amount. Rounding only the final volume to the nearest integer gives 109 mL.

Step 4: Conclusion The volume of nitrogen gas liberated at STP, to the nearest integer, is $\boxed{109\,\text{mL}}$.

Quick Tip to Solve: For a molecule containing two nitrogen atoms, one mole of compound yields one mole of N_2; distinguish atoms from gas molecules before using 22.4 L.

72. Consider the following reactions

$$\text{A} + \underset{\substack{\text{Little}\\ \text{amount}}}{\text{NaCl}} + \text{H}_2\text{SO}_4 \rightarrow \text{CrO}_2\text{Cl}_2 + \textbf{ Side Products}$$

$$\text{CrO}_2\text{Cl}_2\ \textbf{(Vapour)} + \text{NaOH} \rightarrow \text{B} + \text{NaCl} + \text{H}_2\text{O}$$

$$\text{B} + \text{H}^+ \rightarrow \text{C} + \text{H}_2\text{O}$$

The number of terminal ' O ' present in the compound ' C ' is_____ [4 marks]

Solution:

Concept:

- Acidified dichromate reacts with a small quantity of chloride to form volatile chromyl chloride, CrO_2Cl_2. Alkaline absorption converts chromyl chloride into chromate.
- Acidification of chromate forms dichromate: $2\,{\text{CrO}_4}^{2-} + 2\,\text{H}^+ \longrightarrow {\text{Cr}_2\text{O}_7}^{2-} + \text{H}_2\text{O}$. A terminal oxygen is attached to just one chromium atom, while a bridging oxygen joins both chromium atoms.

Step 1: Identify the chromate formed in alkali The first reaction is the chromyl chloride test, in which the chromium(VI) reagent A supplies the chromium for CrO_2Cl_2. On absorption into sodium hydroxide the balanced equation is

$$\text{CrO}_2\text{Cl}_2 + 4\,\text{NaOH} \longrightarrow \text{Na}_2\text{CrO}_4 + 2\,\text{NaCl} + 2\,\text{H}_2\text{O}.$$

Thus B is sodium chromate, Na_2CrO_4, whose anion is ${\text{CrO}_4}^{2-}$. The chloride ions become sodium chloride and do not remain in the chromium oxyanion.

Step 2: Identify C after acidification Adding acid changes chromate into dichromate through the equilibrium

$$2\,{\text{CrO}_4}^{2-} + 2\,\text{H}^+ \longrightarrow {\text{Cr}_2\text{O}_7}^{2-} + \text{H}_2\text{O}.$$

Therefore the chromium-containing species in C is the dichromate ion, $Cr_2O_7^{2-}$, present as the corresponding dichromate compound. The counterion does not affect the oxygen connectivity being counted.

Step 3: Count terminal and bridging oxygens The dichromate ion consists of two tetrahedral chromium-oxygen units sharing one oxygen vertex. Of the seven oxygen atoms in $Cr_2O_7^{2-}$, one is the shared bridge in a $Cr-O-Cr$ linkage. The other six are each bonded to only one chromium centre and are terminal oxygens. Equivalently, each chromium has three terminal oxygens and the common bridging oxygen, so the terminal count is $3 + 3 = 6$. Count connectivity, rather than only formally double-bonded oxygens in a particular resonance drawing.

Step 4: Conclusion The number of terminal oxygen atoms in C is $\boxed{6}$.

Quick Tip to Solve: Dichromate has two tetrahedra sharing one oxygen: subtract the single bridging oxygen from the total seven to obtain six terminal oxygens.

73. 0.5 g of an organic compound on combustion gave 1.46 g of CO_2 and 0.9 g of H_2O. The percentage of carbon in the compound is _____. (Nearest integer)

[Given : Molar mass (in $gmol^{-1}$) C : 12, H : 1, O : 16] [4 marks]

Solution:

Concept:

- In complete combustion, every carbon atom in the organic sample is converted into CO_2. Carbon can therefore be estimated from the mass of carbon dioxide collected.
- The carbon mass fraction in CO_2 is $12/(12 + 2 \times 16) = 12/44$. Percentage composition uses the mass of the original compound as its denominator.

Step 1: Convert carbon dioxide to carbon mass The molar mass of CO_2 is $44\,g\,mol^{-1}$. One mole of carbon dioxide contains one mole, or $12\,g$, of carbon. Thus the $1.46\,g$ of carbon dioxide contains

$$m_C = 1.46 \times \frac{12}{44} = 0.39818\ldots\ g.$$

This is the mass of carbon originally present in the $0.5\,g$ sample. Oxygen added during combustion contributes to the remaining mass of carbon dioxide, so the full $1.46\,g$ must not be treated as carbon.

Step 2: Calculate the percentage and round once Divide the carbon mass by the original sample mass and multiply by 100:

$$\%C = \frac{0.39818\ldots}{0.5} \times 100 = 79.63636\ldots\%.$$

The water yield is used for hydrogen analysis and is not needed to obtain carbon percentage. Keeping the unrounded carbon mass until this final calculation avoids an unnecessary rounding error. The requested nearest integer is obtained by rounding $79.63636\ldots$ upward.

Step 3: Conclusion The carbon percentage is $79.63636\ldots\%$, giving the required nearest-integer answer $\boxed{80}$.

Quick Tip to Solve: For combustion analysis, multiply the carbon dioxide mass by $12/44$, then divide by the original sample mass; water is irrelevant to carbon percentage.

74. The number of optical isomers exhibited by the iron complex (A) obtained from the following reaction is_____.

$$FeCl_3 + KOH + H_2C_2O_4 \rightarrow A$$

[4 marks]

Solution:

Concept:

- Oxalic acid forms the oxalate ion, $C_2O_4^{2-}$, in alkaline medium. Oxalate is a bidentate ligand: its two donor oxygen atoms bind to one metal ion and form a chelate ring.
- An octahedral complex with three identical bidentate ligands can have two nonsuperimposable mirror-image arrangements, designated Δ and Λ. These are an enantiomeric pair.

Step 1: Identify the iron complex Iron is supplied as $FeCl_3$, with iron in the $+3$ oxidation state. In the presence of KOH, oxalic acid supplies oxalate ligands. Three oxalate ions provide six oxygen donor atoms around iron, producing the octahedral tris(oxalato)ferrate(III) ion:

$$Fe^{3+} + 3\,C_2O_4^{2-} \longrightarrow [Fe(C_2O_4)_3]^{3-}.$$

Charge balance gives $+3 + 3(-2) = -3$, so its potassium salt is $K_3[Fe(C_2O_4)_3]$. The potassium ions are outside the coordination sphere and do not determine its optical isomerism.

Step 2: Count the mirror-image arrangements In $[Fe(C_2O_4)_3]^{3-}$, each oxalate joins a pair of adjacent positions of the octahedron. The three chelate rings can twist around the metal in either a right-handed or a left-handed sense. The two arrangements are denoted Δ and Λ, respectively. Reflection interchanges them, but rotation cannot superimpose them, so both are optically active stereoisomers. There is just one such mirror-image pair for this tris-chelate complex. The six donor atoms are not six independent monodentate ligands, and the two donor atoms within any one oxalate must remain connected when counting configurations.

Step 3: Conclusion The iron complex exhibits two optical isomers, Δ and Λ. The required number is $\boxed{2}$.

Quick Tip to Solve: Recognise an octahedral tris-chelate complex $[M(AA)_3]$: three identical bidentate ligands give one Δ/Λ enantiomeric pair.

75. Given :

$$\Delta H^{\ominus}{}_{\textbf{sub}}\left[\mathrm{C}\ \textbf{(graphite}\ )\right] = 710\ \mathrm{kJ\ mol^{-1}}$$
$$\Delta_{\mathrm{C-H}}H^{\ominus} = 414\ \mathrm{kJ\ mol^{-1}}$$
$$\Delta_{\mathrm{H-H}}H^{\ominus} = 436\ \mathrm{kJ\ mol^{-1}}$$
$$\Delta_{\mathrm{C}} = \mathrm{CH}^{\ominus} = 611\ \mathrm{kJ\ mol^{-1}}$$

The $\Delta H_f\ominus$ for $CH_2 = CH_2$ is_____ $kJmol^{-1}$ (nearest integer value) [4 marks]

Solution:

Concept:

- Standard enthalpy of formation is the enthalpy change for making one mole of a compound from its constituent elements in their standard states. For ethene, the elements are graphite and molecular hydrogen.
- Hess's law allows a path through gaseous atoms. Atomisation and bond breaking absorb energy; formation of bonds from gaseous atoms releases energy. Thus the formation enthalpy equals atomisation energy minus the energy released on bond formation.

Step 1: Write the formation reaction and atomise the elements For one mole of gaseous ethene the formation reaction is

$$2\,\mathrm{C(graphite)} + 2\,\mathrm{H_2(g)} \longrightarrow \mathrm{C_2H_4(g)}.$$

Two moles of carbon atoms must first be obtained by subliming graphite, and two moles of hydrogen molecules must be dissociated to give four moles of hydrogen atoms. Using the supplied data, the energy absorbed is

$$E_{\text{atoms}} = 2(710) + 2(436) = 1420 + 872 = 2292\,\mathrm{kJ\,mol^{-1}}.$$

The factors of two follow from the balanced formation reaction; only one mole of ethene is formed.

Step 2: Form the ethene bonds and apply Hess's law Ethene, $CH_2{=}CH_2$, contains four carbon–hydrogen bonds and one carbon–carbon double bond. Forming these bonds from the separated gaseous atoms releases

$$E_{\text{released}} = 4(414) + 611 = 1656 + 611 = 2267\,\mathrm{kJ\,mol^{-1}}.$$

There is no additional carbon–carbon single-bond term: the supplied double-bond enthalpy already corresponds to the complete double bond. The net enthalpy change is therefore

$$\Delta_f H^{\ominus} = 2292 - 2267 = +25\,\mathrm{kJ\,mol^{-1}}.$$

The positive sign means that the formation reaction is slightly endothermic with the given bond-enthalpy data.

Step 3: Conclusion The standard enthalpy of formation is $\boxed{+25\,\mathrm{kJ\,mol^{-1}}}$. The requested nearest-integer value is $\boxed{25}$.

Quick Tip to Solve: For ethene count four C–H bonds and one C=C bond. Atomise the elements first, then subtract the energy released by forming those five bonds.