JEE Main 2025 Paper 1 (B.E./B.Tech) Apr 3 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 3 April 2025, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The area of the region $\{(x, y) : |x - y| \le y \le 4\sqrt{x}\}$ is [4 marks]

(A) $\frac{512}{3}$

(B) $\frac{2048}{3}$

(C) 512

(D) $\frac{1024}{3}$

Correct Answer: (D) $\frac{1024}{3}$

Solution:

Concept:

- For vertical strips, the area between an upper curve and a lower curve is $\int (y_{\text{upper}} - y_{\text{lower}})\, dx$, over the interval where the upper curve is above the lower one.
- The inequality $|x - y| \le y$ is equivalent to $-y \le x - y \le y$. It must be combined with the domain $x \ge 0$ of $\sqrt{x}$, rather than treated as only one boundary line.

Step 1: Simplify the absolute-value condition From $-y \le x - y \le y$, adding y gives $0 \le x \le 2y$. Therefore the allowed points satisfy

$$x \ge 0, \qquad \frac{x}{2} \le y \le 4\sqrt{x}.$$

This also ensures $y \ge 0$. Thus the lower boundary for a vertical strip is the straight line $y = x/2$, and the upper boundary is $y = 4\sqrt{x}$. The absolute value does not produce an additional disjoint region.

Step 2: Find the intersection interval At an intersection, $x/2 = 4\sqrt{x}$. Since $x \ge 0$, putting $t = \sqrt{x}$ gives $t^2 = 8t$, so $t = 0$ or 8. Hence the intersection abscissae are 0 and 64. Between these values the upper curve exceeds the lower line; beyond 64 no allowed vertical strip remains.

Step 3: Integrate the strip height The total area is therefore

$$\begin{aligned} A &= \int_0^{64} \left(4\sqrt{x} - \frac{x}{2}\right) dx \\ &= \left[\frac{8}{3}x^{3/2} - \frac{x^2}{4}\right]_0^{64} \\ &= \frac{8}{3}(512) - 1024 = \frac{1024}{3}. \end{aligned}$$

There is no symmetry factor to insert: this integral already covers every point of the stated region. The other three listed values do not equal this upper-minus-lower integral.

Step 4: Conclusion The area is $\boxed{\frac{1024}{3}}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: Convert the absolute-value inequality into a double inequality first; then identify the actual upper and lower curves before integrating.

2. Line L_1 of slope 2 and line L_2 of slope $\frac{1}{2}$ intersect at the origin O . In the first quadrant, $\mathrm{P}_1, P_2, \ldots, P_{12}$ are 12 points on line L_1 and $Q_1, Q_2, \ldots, Q_9$ are 9 points on line L_2. Then the total number of triangles, that can be formed having vertices at three of the 22 points $\mathrm{O}, \mathrm{P}_1, \mathrm{P}_2, \ldots, \mathrm{P}_{12}, \mathrm{Q}_1, \mathrm{Q}_2, \ldots, \mathrm{Q}_9$, is:

[4 marks]

(A) 1026

(B) 1188

(C) 1134

(D) 1080

Correct Answer: (C) 1134

Solution:

Concept:

- Three distinct points determine a triangle exactly when they are not collinear. The number of unordered triples from n points is $\binom{n}{3}$.
- When all available points lie on two distinct intersecting lines, a degenerate triple must lie entirely on one of those lines. A line different from both given lines meets each of them in at most one point.

Step 1: Count all possible triples There are $1 + 12 + 9 = 22$ distinct available points: the common origin, twelve nonzero points on L_1, and nine nonzero points on L_2. Ignoring collinearity for the moment gives

$$\binom{22}{3} = \frac{22 \cdot 21 \cdot 20}{6} = 1540.$$

This selection counts each possible set of three vertices once; ordering the vertices is unnecessary.

Step 2: Remove the collinear selections Line L_1 contains thirteen available points when the origin is included, so it contributes $\binom{13}{3} = 286$ degenerate triples. Line L_2 contains ten points including the origin, giving $\binom{10}{3} = 120$ degenerate triples. No triple belongs to both sets because the lines share only one point. Moreover, there are no further collinear triples: any other line intersects each given line in at most one point and therefore cannot contain three available points.

Step 3: Evaluate and check the count Subtracting exactly these two disjoint sets gives

$$N = 1540 - 286 - 120 = 1134.$$

An independent case count gives the same result. With the origin as a vertex, choose one point from each line in $12 \cdot 9 = 108$ ways. Without the origin, choose either two P-points and one Q-point, or one P-point and two Q-points:

$$N = 108 + \binom{12}{2}9 + 12\binom{9}{2} = 108 + 594 + 432 = 1134.$$

Thus 1026 omits the origin-containing triangles, while 1188 and 1080 do not match the complete case count.

Step 4: Conclusion The total number of triangles is $\boxed{1134}$. Hence, the correct option is (C).

Quick Tip to Solve: Include the common origin in each line when subtracting collinear triples, but count it only once in the total set of points.

3. Each of the angles β and γ that a given line makes with the positive y - and z-axes, respectively, is half of the angle that this line makes with the positive x-axes. Then the sum of all possible values of the angle β is [4 marks]

(A) $\frac{\pi}{2}$

(B) π

(C) $\frac{3\pi}{4}$

(D) $\frac{3\pi}{2}$

Correct Answer: (C) $\frac{3\pi}{4}$

Solution:

Concept:

- If a line has direction angles α, β, γ, measured from the positive coordinate axes, its direction cosines satisfy $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. Each direction angle lies in $[0, \pi]$.
- The double-angle identity $\cos 2\beta = 2\cos^2\beta - 1$ reduces the prescribed relationship among the angles to a quadratic in $\cos^2\beta$.

Step 1: Apply the given relationships The question gives $\beta = \gamma = \alpha/2$, so $\alpha = 2\beta$. Because $0 \le \alpha \le \pi$, the admissible interval is $0 \le \beta \le \pi/2$. Substituting the angle relationships into the direction-cosine identity gives

$$\cos^2 2\beta + 2\cos^2\beta = 1.$$

Keeping the interval restriction is essential, since arbitrary trigonometric solutions outside the range of direction angles are not allowed.

Step 2: Solve for the admissible direction angles Put $u = \cos^2\beta$. The double-angle identity gives

$$\begin{aligned}(2u-1)^2 + 2u &= 1,\\ 4u^2 - 2u &= 0,\\ 2u(2u-1) &= 0.\end{aligned}$$

Hence $u = 0$ or $u = 1/2$. On $[0, \pi/2]$, these give $\beta = \pi/2$ and $\beta = \pi/4$, respectively. Both are legitimate: the first has direction cosines $(-1, 0, 0)$, while the second has $(0, 1/\sqrt{2}, 1/\sqrt{2})$. Each triple has sum of squares one and satisfies the stated angle relationships.

Step 3: Add all possible values The question asks for the sum of the possible values, not just one admissible angle. Thus

$$\sum \beta = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}.$$

Option (A) keeps only the boundary solution and misses $\pi/4$. Options (B) and (D) cannot be obtained from the two admissible values; they would require extra angles outside the derived interval or double counting.

Step 4: Conclusion The sum of all possible values of β is $\boxed{3\pi/4}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the range of direction angles before solving the trigonometric equation; the boundary value $\beta = \pi/2$ is valid and must not be discarded.

4. If z_1, z_2, z_3 are the vertices of an equilateral triangle, whose centroid is z_0, then $\sum_{k=1}^{3}(z_k - z_0)^2$ is equal to [4 marks]

(A) 0

(B) 1

(C) i

(D) -i

Correct Answer: (A) 0

Solution:

Concept:

- Translating all vertices by the centroid does not change their relative geometry. The numbers $w_k = z_k - z_0$ are the position vectors of the vertices relative to the centre of the equilateral triangle.
- Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$. Multiplication by ω rotates a complex vector through $120°$.

Step 1: Centre the triangle at the origin In an equilateral triangle, the centroid is also the circumcentre, so its three vertex vectors have equal lengths and successive directions separated by $120°$. Therefore, after a possible relabelling, there is a nonzero complex number u such that

$$w_1 = u, \qquad w_2 = u\omega, \qquad w_3 = u\omega^2.$$

The value of u accounts for the size and orientation of the triangle. Reversing the orientation only exchanges the last two vertices and does not alter their sum of squares.

Step 2: Sum the squared complex displacements Squaring these complex numbers and adding gives

$$\sum_{k=1}^{3}(z_k - z_0)^2 = u^2\left(1 + \omega^2 + \omega^4\right)$$
$$= u^2\left(1 + \omega^2 + \omega\right) = 0,$$

since $\omega^4 = \omega$. These are complex squares, not squared absolute values: $\sum |z_k - z_0|^2$ would instead be three times the square of the circumradius and would generally be positive.

Step 3: Check the alternatives against rotation symmetry The result is zero for every possible size, orientation and centroid of the equilateral triangle. A nonzero fixed value such as 1, i, or $-i$ cannot remain unchanged when all vertex displacements are multiplied by an arbitrary scale and rotation factor: the requested sum would multiply by its square. The exact root-of-unity calculation establishes the cancellation, and this symmetry check independently rules out every nonzero listed alternative.

Step 4: Conclusion For an equilateral triangle, $\boxed{\sum_{k=1}^{3}(z_k - z_0)^2 = 0}$. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish $(z - z_0)^2$ from $|z - z_0|^2$: complex squares cancel under the threefold rotation of an equilateral triangle.

5. Let C be the circle of minimum area enclosing the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with eccentricity $\frac{1}{2}$ and foci $(\pm 2, 0)$. Let PQR be a variable triangle, whose vertex P is on the circle C and the side QR of length $2a$ is parallel to the major axis of E and contains the point of intersection of E with the negative y-axis. Then the maximum area of the triangle PQR is : [4 marks]

(A) $8(3 + \sqrt{2})$

(B) $8(2 + \sqrt{3})$

(C) $6(3 + \sqrt{2})$

(D) $6(2 + \sqrt{3})$

Correct Answer: (B) $8(2 + \sqrt{3})$

Solution:

Concept:

- For an ellipse with horizontal major axis, the focal distance is $c = ae$ and $b^2 = a^2 - c^2$, where a and b are its semimajor and semiminor axes.
- For a triangle with fixed base length, maximizing its area $\frac{1}{2} \times$ base $\times$ height is equivalent to maximizing the perpendicular distance of its third vertex from the line containing the base.

Step 1: Determine the ellipse dimensions The foci $(\pm 2, 0)$ give $c = 2$. Using the eccentricity $e = 1/2$,

$$a = \frac{c}{e} = 4, \qquad b^2 = a^2 - c^2 = 16 - 4 = 12,$$

so $b = 2\sqrt{3}$. The ellipse meets the negative y-axis at $(0, -2\sqrt{3})$, and its major axis is horizontal.

Step 2: Identify the smallest enclosing circle The two major-axis endpoints $(4, 0)$ and $(-4, 0)$ are eight units apart. Any circle containing both must have radius at least four. The circle centred at the origin with radius four contains the entire ellipse because every point of the ellipse satisfies

$$x^2 + y^2 \leq 16\left(\frac{x^2}{16} + \frac{y^2}{12}\right) = 16.$$

It therefore attains the lower bound and is precisely the minimum-area enclosing circle C. Its highest point is $(0, 4)$.

Step 3: Maximize the triangle height and area The side QR has length $2a = 8$, is parallel to the major axis and contains $(0, -2\sqrt{3})$; consequently its supporting line is $y = -2\sqrt{3}$. For $P = (x, y)$ on C, the height is $|y + 2\sqrt{3}|$. Since $-4 \leq y \leq 4$, its maximum is $4 + 2\sqrt{3}$, attained at $P = (0, 4)$. Hence

$$A_{\max} = \frac{1}{2}(8)(4 + 2\sqrt{3}) = 16 + 8\sqrt{3} = 8(2 + \sqrt{3}).$$

The horizontal location of the base segment does not affect this height. Options involving $\sqrt{2}$ use an incorrect semiminor axis, while the coefficient six in option (D) is incompatible with the actual base length eight.

Step 4: Conclusion The maximum area is $\boxed{8(2 + \sqrt{3})}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: First find the smallest enclosing circle from the major-axis endpoints; then maximize distance from the base line, rather than tracking both base vertices.

6. The shortest distance between the curves $y^2 = 8x$ and $x^2 + y^2 + 12y + 35 = 0$ is: [4 marks]

(A) $2\sqrt{3} - 1$

(B) $2\sqrt{2} - 1$

(C) $3\sqrt{2} - 1$

(D) $\sqrt{2}$

Correct Answer: (B) $2\sqrt{2} - 1$

Solution:

Concept:

- The equation $x^2 + (y + 6)^2 = 1$ represents a circle with centre $S = (0, -6)$ and radius one. For an exterior point P, its distance from the circle is $SP - 1$.
- Every point of $y^2 = 8x$ has the parametrization $P = (t^2/8, t)$ for a real number t. Minimizing SP^2 is equivalent to minimizing SP, since distance is nonnegative.

Step 1: Reduce the problem to distance from the centre Complete the square in the second equation:

$$x^2 + y^2 + 12y + 35 = 0 \quad \Longrightarrow \quad x^2 + (y + 6)^2 = 1.$$

For $P = (t^2/8, t)$ on the parabola, the squared distance from S is

$$f(t) = SP^2 = \frac{t^4}{64} + (t + 6)^2.$$

We first find its global minimum, then check that it exceeds the squared radius so that subtracting one is legitimate.

Step 2: Find the global minimum of the squared distance Differentiate with respect to the real parameter:

$$\begin{aligned} f'(t) &= \frac{t^3}{16} + 2t + 12 \\ &= \frac{(t + 4)(t^2 - 4t + 48)}{16}. \end{aligned}$$

The second factor is $(t - 2)^2 + 44 > 0$, so the only stationary point is $t = -4$. The derivative is negative before -4 and positive afterwards, proving this is the global minimum. Equivalently, $f''(t) = 3t^2/16 + 2 > 0$ everywhere.

Step 3: Subtract the circle radius At $t = -4$, the nearest parabola point to the centre is $P = (2, -4)$, with

$$SP_{\min} = \sqrt{2^2 + (-4 + 6)^2} = \sqrt{8} = 2\sqrt{2} > 1.$$

Every parabola point is therefore outside the circle. For each such point the nearest circle point lies along the radius towards it, so minimizing the distance between the curves gives

$$d_{\min} = SP_{\min} - 1 = 2\sqrt{2} - 1.$$

The alternatives $2\sqrt{3} - 1$ and $3\sqrt{2} - 1$ use incorrect centre distances; $\sqrt{2}$ does not subtract the unit radius from the actual minimum centre distance.

Step 4: Conclusion The shortest distance is $\boxed{2\sqrt{2}-1}$ units. Hence, the correct option is (B).

Quick Tip to Solve: For distance from a curve to a circle, minimize distance to the centre first, verify the curve stays outside, and only then subtract the radius.

7. Consider the lines $x(3\lambda+1)+y(7\lambda+2)=17\lambda+5, \lambda$ being a parameter, all passing through a point P. One of these lines (say L) is farthest from the origin. If the distance of L from the point $(3,6)$ is d, then the value of d^2 is [4 marks]

(A) 10

(B) 20

(C) 15

(D) 30

Correct Answer: (B) 20

Solution:

Concept:

- A line in the family can be written as $(x+2y-5)+\lambda(3x+7y-17)=0$. Its fixed point is the intersection of the two lines obtained by equating the constant and parameter coefficients to zero.
- For every line through a fixed point P, the perpendicular distance from the origin cannot exceed OP. Equality holds when the line is perpendicular to OP. The distance from (x_0, y_0) to $ax+by+c=0$ is $|ax_0+by_0+c|/\sqrt{a^2+b^2}$.

Step 1: Find the point common to the family For the equation to hold for every value of λ at P, both coefficients must vanish. Thus

$$x+2y=5, \qquad 3x+7y=17.$$

Subtracting three times the first equation from the second gives $y=2$, and then $x=1$. Therefore $P=(1,2)$.

Step 2: Identify the line farthest from the origin The vector from the origin to P is $(1,2)$. A line perpendicular to this vector and passing through P has equation

$$x+2y=1+2(2)=5.$$

This is indeed a member of the given family: take $\lambda=0$. If H is the perpendicular foot from the origin to any line through P, then $OH^2+HP^2=OP^2$, so $OH \le OP$. For the chosen line, $H=P$; hence its distance is the maximum.

Step 3: Calculate the requested squared distance Use the point $(3,6)$, rather than the origin, in the distance formula for this identified line:

$$d=\frac{|3+2(6)-5|}{\sqrt{1^2+2^2}}=\frac{10}{\sqrt{5}}.$$

Consequently $d^2=100/5=20$. The alternatives 10, 15 and 30 are inconsistent with this exact

squared distance; in particular, the numerator 10 is not itself the squared distance.

Step 4: Conclusion The required value is $\boxed{d^2 = 20}$. Hence, the correct option is (B).

Quick Tip to Solve: First find the fixed point of a pencil of lines. Its farthest member from the origin is perpendicular to the radius joining the origin to that point.

8. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + 3\left(\tan^2 x\right) y + 3y = \sec^2 x, y(0) = \frac{1}{3} + e^3$. Then $y\left(\frac{\pi}{4}\right)$ is equal to : [4 marks]

(A) $\frac{4}{3}$

(B) $\frac{2}{3} + e^3$

(C) $\frac{4}{3} + e^3$

(D) $\frac{2}{3}$

Correct Answer: (A) $\frac{4}{3}$

Solution:

Concept:

- The identity $1 + \tan^2 x = \sec^2 x$ combines the two terms multiplying y. A linear equation $y\prime + P(x)y = Q(x)$ has integrating factor $e^{\int P(x)\,dx}$.
- Here $P(x) = 3\sec^2 x$, and an integrating factor is $e^{3\tan x}$. The interval from 0 to $\pi/4$ contains no pole of $\tan x$, so the initial condition and the requested value lie on the same solution interval.

Step 1: Put the equation in linear form Combine $3\tan^2 x\, y + 3y$ before choosing an integrating factor:

$$\frac{dy}{dx} + 3\sec^2 x\, y = \sec^2 x.$$

Since $\int 3\sec^2 x\, dx = 3\tan x$, multiplying the equation by $e^{3\tan x}$ makes its left side a product derivative.

Step 2: Integrate and impose the initial value The resulting equation and its integral are

$$\frac{d}{dx}\left(ye^{3\tan x}\right) = \sec^2 x\, e^{3\tan x},$$

$$ye^{3\tan x} = \frac{1}{3}e^{3\tan x} + C.$$

In the integral, the substitution $u = 3\tan x$ gives $du = 3\sec^2 x\, dx$. Therefore

$$y = \frac{1}{3} + Ce^{-3\tan x}.$$

At $x = 0$, this gives $1/3 + C = 1/3 + e^3$, so $C = e^3$.

Step 3: Evaluate at the required argument As $\tan(\pi/4) = 1$, the exponential term becomes $e^3 e^{-3} = 1$, not e^3. Hence

$$y\left(\frac{\pi}{4}\right) = \frac{1}{3} + 1 = \frac{4}{3}.$$

This rules out both options retaining an e^3 term, as well as $2/3$. As a check, differentiating the solution

gives $y' = -3\sec^2 x\,(y - 1/3)$, which rearranges to the original linear equation.

Step 4: Conclusion $\boxed{y(\pi/4) = 4/3}$. Hence, the correct option is (A).

Quick Tip to Solve: Use $1 + \tan^2 x = \sec^2 x$ first; then $3\tan x$ is the exponent in the integrating factor, and the initial exponential cancels at $x = \pi/4$.

9. Let f be a function such that $f(x) + 3f\left(\frac{24}{x}\right) = 4x, x \neq 0$. Then $f(3) + f(8)$ is equal to [4 marks]

(A) 13

(B) 11

(C) 10

(D) 12

Correct Answer: (B) 11

Solution:

Concept:

- The transformation $x \mapsto 24/x$ interchanges 3 and 8. Evaluating the functional equation at these two inputs creates a pair of simultaneous linear equations.
- When only the sum of two function values is required, adding the simultaneous equations can determine that sum directly; finding a complete formula for the function is unnecessary.

Step 1: Evaluate the equation at the first input Put $x = 3$. Since $24/3 = 8$, the given identity becomes

$$f(3) + 3f(8) = 4(3) = 12.$$

Both arguments are permitted because the excluded input is only zero. Denote $f(3) = a$ and $f(8) = b$, so the first equation is $a + 3b = 12$.

Step 2: Use the paired input and add Now put $x = 8$. The reciprocal argument returns to 3, giving

$$f(8) + 3f(3) = 4(8) = 32.$$

Thus $3a + b = 32$. Add the two equations so that the coefficients of a and b become equal:

$$4a + 4b = 44 \quad \Longrightarrow \quad a + b = 11.$$

This directly yields the requested sum without assuming any particular form such as a linear or polynomial function.

Step 3: Check the individual values and the options Subtracting the first equation from three times the second gives $8a = 84$, so $a = 21/2$. Then $b = 1/2$. The original equations check as

$$\frac{21}{2} + 3\left(\frac{1}{2}\right) = 12, \qquad \frac{1}{2} + 3\left(\frac{21}{2}\right) = 32.$$

Their sum is exactly 11. Therefore 13, 10 and 12 are each ruled out by the simultaneous equations.

In particular, 12 is the right side at $x = 3$, which is a weighted sum rather than the unweighted sum asked for.

Step 4: Conclusion $\boxed{f(3) + f(8) = 11}$. Hence, the correct option is (B).

Quick Tip to Solve: Look for paired inputs whose product is 24. Substituting both and adding cancels the unequal weights and gives the sum immediately.

10. Let the equation $x(x+2)(12-k) = 2$ have equal roots. Then the distance of the point $\left(k, \frac{k}{2}\right)$ from the line $3x + 4y + 5 = 0$ is [4 marks]

(A) 15

(B) 12

(C) $5\sqrt{3}$

(D) $15\sqrt{5}$

Correct Answer: (A) 15

Solution:

Concept:

- A genuine quadratic $Ax^2 + Bx + C = 0$, with $A \neq 0$, has equal roots precisely when $B^2 - 4AC = 0$. Any parameter value making the leading coefficient zero must be checked in the original equation.
- The perpendicular distance of (x_0, y_0) from $ax + by + c = 0$ is $|ax_0 + by_0 + c|/\sqrt{a^2 + b^2}$; both the absolute value and the normalising denominator are essential.

Step 1: Apply the repeated-root condition Write $t = 12 - k$. The equation becomes

$$tx^2 + 2tx - 2 = 0.$$

Its discriminant is

$$D = (2t)^2 - 4(t)(-2) = 4t(t+2).$$

Equal roots require $D = 0$, giving the candidates $t = 0$ and $t = -2$.

Step 2: Reject the degenerate candidate and find the point If $t = 0$, the original equation reduces to $0 = 2$, which has no root at all. Thus this candidate cannot describe equal roots. The valid value is $t = -2$, so

$$12 - k = -2 \quad \Longrightarrow \quad k = 14.$$

Indeed, with this value the equation reduces to $(x+1)^2 = 0$, confirming the repeated root. The required point is therefore $(k, k/2) = (14, 7)$.

Step 3: Find the perpendicular distance For the line $3x + 4y + 5 = 0$, substitute the point into the numerator and divide by the length of its normal vector:

$$d = \frac{|3(14) + 4(7) + 5|}{\sqrt{3^2 + 4^2}} = \frac{42 + 28 + 5}{5} = 15.$$

The options 12 and $5\sqrt{3}$ do not equal this exact value. The value $15\sqrt{5}$ also fails the distance formula; no extra factor arises from the coordinates of the point.

Step 4: Conclusion The required distance is $\boxed{15}$ units. Hence, the correct option is (A).

Quick Tip to Solve: After setting a parameter-dependent discriminant to zero, reject values that destroy the quadratic before using the remaining parameter in the geometry.

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11. Let $A = \{-2, -1, 0, 1, 2, 3\}$. Let R be a relation on A defined by $x\mathrm{R}y$ if and only if $y = \max\{x, 1\}$. Let l be the number of elements in R . Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations, respectively. Then $l + m + n$ is equal to [4 marks]

(A) 11

(B) 12

(C) 14

(D) 13

Correct Answer: (B) 12

Solution:

Concept:

- A relation on A is a set of ordered pairs in $A \times A$. It is reflexive when every diagonal pair (a, a) is present, and symmetric when the reverse (y, x) accompanies each pair (x, y).
- To find a minimum number of additions, count only pairs missing from the original relation. The reflexive and symmetric requirements in this question are assessed separately, so their added pairs need not be combined into one new relation.

Step 1: List and count the original relation For $x = -2, -1, 0, 1$, the maximum of x and 1 is 1. For $x = 2, 3$, it is x itself. Hence

$$R = \{(-2, 1), (-1, 1), (0, 1), (1, 1), (2, 2), (3, 3)\}.$$

Every element of A supplies exactly one distinct ordered pair, and every second coordinate belongs to A. Thus $l = 6$.

Step 2: Count the missing diagonal pairs Reflexivity requires all six diagonal pairs. The relation already contains $(1, 1), (2, 2), (3, 3)$, but lacks

$$(-2, -2), \quad (-1, -1), \quad (0, 0).$$

Each missing diagonal pair is indispensable; adding just these three also suffices. Therefore the minimum is $m = 3$.

Step 3: Count the missing reverses and form the sum The diagonal pairs are their own reverses. The

remaining three pairs require the missing reverse pairs

$$(1,-2), \quad (1,-1), \quad (1,0).$$

Adding these makes the relation symmetric, since reversing them recovers pairs already present. Consequently $n = 3$, and

$$l + m + n = 6 + 3 + 3 = 12.$$

The other proposed totals 11, 14 and 13 cannot follow from the exact pair counts. Counting diagonal pairs again while checking symmetry would incorrectly add pairs that are already self-reversing.

Step 4: Conclusion $\boxed{l + m + n = 12}$. Hence, the correct option is (B).

Quick Tip to Solve: For reflexivity count missing diagonal pairs; for symmetry count missing reverse pairs. A diagonal pair already satisfies symmetry by itself.

12. Let the Mean and Variance of five observations $x_1 = 1, x_2 = 3, x_3 = a, x_4 = 7$ and $x_5 = \mathrm{b}, a > \mathrm{b}$, be 5 and 10 respectively. Then the Variance of the observations $n + x_n, n = 1, 2, \ldots, 5$ is [4 marks]

(A) 17

(B) 16

(C) 16.4

(D) 17.4

Correct Answer: (B) 16

Solution:

Concept:

- For N observations with mean $\bar{x}$, the population variance is $\sigma^2 = \frac{1}{N}\sum x_i^2 - \bar{x}^2$. The given mean and variance determine the sum and sum of squares of the original observations.
- The new values are $z_n = n + x_n$, so the increment varies with the position n. This is not addition of one common constant; the new variance must be computed using the new observations.

Step 1: Recover the two missing observations The mean is 5, so

$$1 + 3 + a + 7 + b = 5(5) \quad \Longrightarrow \quad a + b = 14.$$

Using the variance 10 gives

$$\frac{1^2 + 3^2 + a^2 + 7^2 + b^2}{5} - 25 = 10.$$

Thus the total sum of squares is 175 and $a^2 + b^2 = 175 - 59 = 116$. It follows that

$$2ab = (a + b)^2 - (a^2 + b^2) = 196 - 116 = 80.$$

Therefore a, b are the roots of $t^2 - 14t + 40 = (t - 10)(t - 4) = 0$. The condition $a > b$ fixes $a = 10$ and $b = 4$.

Step 2: Construct the new data and its mean Keep the observations in their stated order, because the added index depends on position. The original sequence is $1, 3, 10, 7, 4$, and the new sequence is

$$2, \quad 5, \quad 13, \quad 11, \quad 9.$$

Its total is 40, so the new mean is $\bar{z} = 40/5 = 8$. Interchanging a and b would change this position-dependent transformation and is forbidden by $a > b$.

Step 3: Compute and check the new variance The new sum of squares is

$$2^2 + 5^2 + 13^2 + 11^2 + 9^2 = 4 + 25 + 169 + 121 + 81 = 400.$$

Hence

$$\text{Var}(z) = \frac{400}{5} - 8^2 = 80 - 64 = 16.$$

Equivalently, the squared deviations from 8 sum to $36 + 9 + 25 + 9 + 1 = 80$, again yielding 16. Thus 17, 16.4 and 17.4 are each excluded. Using a sample-variance denominator would not match the population variance specified here.

Step 4: Conclusion The variance of $n + x_n$ is $\boxed{16}$. Hence, the correct option is (B).

Quick Tip to Solve: When a different index is added to each observation, rebuild the data explicitly; variance stays unchanged only when the same constant is added to every value.

13. The integral $\int_0^{\pi} \frac{8x\,dx}{4\cos^2 x + \sin^2 x}$ is equal to [4 marks]

(A) $2\pi^2$

(B) $4\pi^2$

(C) π^2

(D) $\frac{3\pi^2}{2}$

Correct Answer: (A) $2\pi^2$

Solution:

Concept:

- When $g(\pi - x) = g(x)$, substituting $x \mapsto \pi - x$ gives $\int_0^{\pi} x g(x)\,dx = \frac{\pi}{2}\int_0^{\pi} g(x)\,dx$.
- For a rational expression in $\sin^2 x$ and $\cos^2 x$, first use symmetry about $\pi/2$, then substitute $t = \tan x$ on $[0, \pi/2)$.

Step 1: Remove the factor of x by reflection Let $D(x) = 4\cos^2 x + \sin^2 x$ and let the required integral be I. Since $D(\pi - x) = D(x)$, reflection gives a second expression for the same integral:

$$I = \int_0^{\pi} \frac{8(\pi - x)}{D(x)}\,dx.$$

Add this to the original expression. The terms containing x cancel in the numerator, so

$$2I = 8\pi \int_0^{\pi} \frac{dx}{D(x)}, \qquad I = 4\pi J, \quad J = \int_0^{\pi} \frac{dx}{D(x)}.$$

This reflection is valid because the denominator is strictly positive throughout the closed interval.

Step 2: Evaluate the remaining trigonometric integral The two halves of J are equal. On the first half, dividing numerator and denominator by $\cos^2 x$, and setting $t = \tan x$, gives

$$J = 2\int_0^{\pi/2} \frac{\sec^2 x\, dx}{4 + \tan^2 x} = 2\int_0^{\infty} \frac{dt}{4 + t^2}.$$

Use the inverse-tangent antiderivative with its scale factor:

$$J = 2\left[\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)\right]_0^{\infty} = \frac{\pi}{2}.$$

Consequently $I = 4\pi(\pi/2) = 2\pi^2$. The values $4\pi^2$, π^2 and $3\pi^2/2$ do not equal this exact result; the first two can arise by losing a factor in the reflection or in the half-interval reduction.

Step 3: Conclusion The integral equals $\boxed{2\pi^2}$. Hence, the correct option is (A).

Quick Tip to Solve: For an integral containing x times a function of squared sine and cosine on zero to pi, reflect first; this avoids integration by parts.

14. The distance of the point $(7, 10, 11)$ from the line $\frac{x-4}{1} = \frac{y-4}{0} = \frac{z-2}{3}$ along the line $\frac{x-9}{2} = \frac{y-13}{3} = \frac{z-17}{6}$ is [4 marks]

(A) 16

(B) 12

(C) 18

(D) 14

Correct Answer: (D) 14

Solution:

Concept:

- Distance measured along a specified line direction means intersecting the target line with the line through the given point parallel to that direction.
- A direction vector $\mathbf{d}$ gives points $P + t\mathbf{d}$; the distance from P to such a point is $|t|\,|\mathbf{d}|$. A zero direction ratio fixes the corresponding coordinate.

Step 1: Parameterise the target line and the required direction Write the first line using a parameter s. Its middle direction ratio is zero, which means $y = 4$, rather than requiring division by zero:

$$L : (x, y, z) = (4 + s, 4, 2 + 3s).$$

The second displayed line has direction vector $(2, 3, 6)$. The given point $P = (7, 10, 11)$ lies on it: setting its parameter to -1 in $(9 + 2u, 13 + 3u, 17 + 6u)$ produces P. Hence the line through P in the required direction is

$$(x, y, z) = (7 + 2t, 10 + 3t, 11 + 6t).$$

This asks for a directional distance, so the usual shortest perpendicular-distance formula would answer a different question.

Step 2: Find the intersection and calculate its distance At an intersection with L, the y-coordinate must be 4. Therefore

$$10 + 3t = 4 \quad \Rightarrow \quad t = -2.$$

The corresponding point is $Q = (3, 4, -1)$. It is on the target line as well: $s = -1$ gives $4 + s = 3$ and $2 + 3s = -1$. This checks all three coordinates, so the two parameterisations are consistent. The displacement and its magnitude are

$$\overrightarrow{PQ} = (-4, -6, -12), \qquad PQ = \sqrt{16 + 36 + 144} = \sqrt{196} = 14.$$

Equivalently $|t|\sqrt{2^2 + 3^2 + 6^2} = 2\sqrt{49} = 14$. The other numerical choices, 16, 12 and 18, do not have the required displacement magnitude.

Step 3: Conclusion The distance along the specified line is $\boxed{14}$ units. Hence, the correct option is (D).

Quick Tip to Solve: Use the coordinate with zero direction ratio first: it immediately determines the parameter on the other line.

15. If the probability that the random variable X takes the value x is given by $P(X = x) = k(x + 1)3^{-x}, x = 0, 1, 2, 3 \ldots$, where k is a constant, then $P(X \geq 3)$ is equal to [4 marks]

(A) $\frac{1}{9}$
(B) $\frac{8}{27}$
(C) $\frac{7}{27}$
(D) $\frac{4}{9}$

Correct Answer: (A) $\frac{1}{9}$

Solution:

Concept:

- The probabilities of all values of a discrete random variable sum to one. This determines the normalising constant k.
- For $|r| < 1$, the geometric-series identities give $\sum_{x=0}^{\infty}(x + 1)r^x = (1 - r)^{-2}$. A tail probability can then be evaluated using its finite complement.

Step 1: Determine the normalising constant Since the possible values of X are the nonnegative

integers, normalisation requires

$$1 = k\sum_{x=0}^{\infty}(x+1)\left(\frac{1}{3}\right)^{x}.$$

To evaluate this sum, differentiate $\sum_{x=0}^{\infty} r^{x+1} = r/(1-r)$ with respect to r, valid for $|r| < 1$. This gives the stated identity. At $r = 1/3$, its value is

$$\sum_{x=0}^{\infty}(x+1)3^{-x} = \frac{1}{(1-1/3)^2} = \frac{9}{4}.$$

Thus $k = 4/9$. It is positive, as required, and all terms in the probability distribution are nonnegative.

Step 2: Subtract the first three probabilities from one The event $X \geq 3$ excludes precisely $X = 0, 1, 2$. Their probabilities are

$$P(X=0) = \frac{4}{9}, \qquad P(X=1) = \frac{4}{9}\frac{2}{3} = \frac{8}{27}, \qquad P(X=2) = \frac{4}{9}\frac{3}{9} = \frac{4}{27}.$$

Therefore

$$P(X \geq 3) = 1 - \left(\frac{12}{27} + \frac{8}{27} + \frac{4}{27}\right) = 1 - \frac{24}{27} = \frac{1}{9}.$$

The choice $8/27$ is the probability of $X = 1$, and $4/9$ is the probability of $X = 0$, rather than the requested tail. The remaining choice $7/27$ also fails the complement calculation. Including $X = 3$ in the subtracted sum would incorrectly compute the stricter tail $X > 3$.

Step 3: Conclusion The probability is $\boxed{P(X \geq 3) = \frac{1}{9}}$. Hence, the correct option is (A).

Quick Tip to Solve: Normalise before computing a tail; for a tail starting at 3, subtract only the probabilities at 0, 1 and 2.

16. The number of solutions of the equation

$(4-\sqrt{3})\sin x - 2\sqrt{3}\cos^2 x = -\frac{4}{1+\sqrt{3}}, x \in \left[-2\pi, \frac{5\pi}{2}\right]$ **is** [4 marks]

(A) 4

(B) 3

(C) 6

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- Replacing $\cos^2 x$ by $1 - \sin^2 x$ converts this equation into a quadratic in $\sin x$. Only roots in $[-1, 1]$ can yield real angles.
- For $\sin x = \sin\theta$, the two families are $x = \theta + 2k\pi$ and $x = \pi - \theta + 2k\pi$, with integer k. Both must be counted on the given closed interval.

Step 1: Reduce the equation to a quadratic Put $s = \sin x$ and rationalise the right-hand side:

$$-\frac{4}{1+\sqrt{3}} = 2 - 2\sqrt{3}.$$

Substitution of $\cos^2 x = 1 - s^2$ into the given equation gives

$$(4-\sqrt{3})s - 2\sqrt{3}(1-s^2) = 2 - 2\sqrt{3}.$$

After cancelling the constant terms involving $\sqrt{3}$, factorise:

$$2\sqrt{3}s^2 + (4-\sqrt{3})s - 2 = 0, \qquad (2s-1)(\sqrt{3}s+2) = 0.$$

Hence $s = 1/2$ or $s = -2/\sqrt{3}$. The second root is smaller than -1, so it cannot be a sine value of a real angle and contributes no solution.

Step 2: Count both valid angle families The remaining equation is $\sin x = 1/2$, with solutions

$$x = \frac{\pi}{6} + 2k\pi \quad \text{or} \quad x = \frac{5\pi}{6} + 2k\pi.$$

In the first family, the condition $-2\pi \le x \le 5\pi/2$ allows $k = -1, 0, 1$, giving $-11\pi/6, \pi/6, 13\pi/6$. In the second family it allows only $k = -1, 0$, giving $-7\pi/6, 5\pi/6$. These five angles are distinct, and neither endpoint adds another solution because its sine is 0 or 1. Thus there are $3 + 2 = 5$ solutions. Counts 3 or 4 omit valid angles, while 6 includes an angle outside the interval or an inadmissible quadratic root.

Step 3: Conclusion The number of solutions is $\boxed{5}$. Hence, the correct option is (D).

Quick Tip to Solve: Reject impossible sine roots before counting; then list both periodic families and check each interval endpoint.

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17. If the domain of the function $f(x) = \log_7\left(1 - \log_4\left(x^2 - 9x + 18\right)\right)$ is $(\alpha, \beta) \cup (\gamma, o)$, then $\alpha + \beta + \gamma + \hat{o}$ is equal to [4 marks]

(A) 17

(B) 15

(C) 16

(D) 18

Correct Answer: (D) 18

Solution:

Concept:

- A real logarithm requires a strictly positive argument. Both the inner and the outer logarithm impose conditions, so their domain restrictions must be intersected.
- As the base 4 is greater than one, $\log_4 u < 1$ is equivalent to $u < 4$ when $u > 0$. Quadratic inequalities then determine the open domain intervals.

Step 1: Apply the restrictions of both logarithms Let $u = x^2 - 9x + 18$. The inner logarithm exists only for $u > 0$. The outer logarithm, with base 7, requires

$$1 - \log_4 u > 0 \quad \Rightarrow \quad \log_4 u < 1.$$

Since base 4 is increasing, the latter inequality is $u < 4$. Combining them, rather than keeping only the outer restriction, gives

$$0 < x^2 - 9x + 18 < 4.$$

The inequalities are strict: $u = 0$ makes the inner logarithm undefined, and $u = 4$ makes the outer logarithm's argument zero.

Step 2: Solve and intersect the quadratic inequalities The positivity condition factors as

$$(x - 3)(x - 6) > 0 \quad \Rightarrow \quad x < 3 \text{ or } x > 6.$$

The upper restriction similarly gives

$$x^2 - 9x + 14 < 0 \quad \Rightarrow \quad (x - 2)(x - 7) < 0 \quad \Rightarrow \quad 2 < x < 7.$$

Their intersection is $(2, 3) \cup (6, 7)$. Thus the four successive endpoints named in the question are 2, 3, 6 and 7, whose sum is

$$2 + 3 + 6 + 7 = 18.$$

As a quick independent check, both quadratic polynomials have root sum 9, so the sum of their four roots is $9 + 9 = 18$. The choices 17, 15 and 16 cannot be the sum of these boundary values. Including any boundary point would also violate a logarithm's strict domain condition.

Step 3: Conclusion The domain is $(2, 3) \cup (6, 7)$, and the required endpoint sum is $\boxed{18}$. Hence, the correct option is (D).

Quick Tip to Solve: For nested logarithms, work outward from the inner argument and combine all strict inequalities before solving for x.

18. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \big|\,|x+2| - 2|x|\,\big|$. If m is the number of points of local minima and n is the number of points of local maxima of f, then $m+n$ is [4 marks]

(A) 3

(B) 4

(C) 2

(D) 5

Correct Answer: (A) 3

Solution:

Concept:

- Resolve inner absolute values on the intervals where their signs are fixed, then resolve the outer absolute value at its zeros. This produces a continuous piecewise linear function.
- A local minimum occurs when the slope changes from negative to positive, and a local maximum when it changes from positive to negative. A corner without such a sign change is not an extremum.

Step 1: Resolve the nested absolute values First put $g(x) = |x+2| - 2|x|$, so that $f(x) = |g(x)|$. The sign boundaries for the inner absolute values are -2 and 0. On the corresponding intervals,

$$g(x) = \begin{cases} x - 2, & x < -2, \\ 3x + 2, & -2 \le x < 0, \\ 2 - x, & x \ge 0. \end{cases}$$

In the first region g is always negative. Its zeros in the other two regions are $-2/3$ and 2. Consequently the full expression is

$$f(x) = \begin{cases} 2 - x, & x < -2, \\ -3x - 2, & -2 \le x < -2/3, \\ 3x + 2, & -2/3 \le x < 0, \\ 2 - x, & 0 \le x < 2, \\ x - 2, & x \ge 2. \end{cases}$$

Adjacent formulae give the same values at their shared boundaries, which also verifies continuity there.

Step 2: Inspect the slope changes at every boundary The successive slopes are $-1, -3, +3, -1, +1$. At $x = -2$ the function decreases on both sides, so that corner is not a local extremum. At $x = -2/3$, its slope changes from -3 to $+3$, giving a local minimum with value zero. At $x = 0$, the slope changes from $+3$ to -1, giving a local maximum with value 2. At $x = 2$, the slope changes from -1 to $+1$, giving the other local minimum, again with value zero. Within each open linear piece the slope is nonzero, so there are no further extrema. Thus $m = 2$, $n = 1$. Counting every absolute-value corner would give an incorrect extra extremum at -2; counting only zeros would miss the maximum at 0.

Step 3: Conclusion There are two local minima and one local maximum, so $\boxed{m+n=3}$. Hence, the correct option is (A).

Quick Tip to Solve: Count changes in the sign of the slope, not merely changes in its value: the corner at -2 is not a turning point.

19. The sum $1+\frac{1+3}{2!}+\frac{1+3+5}{3!}+\frac{1+3+5+7}{4!}+\ldots$ upto ∞ terms, is equal to [4 marks]

(A) $3e$

(B) $2e$

(C) $4e$

(D) $6e$

Correct Answer: (B) $2e$

Solution:

Concept:

- The sum of the first n odd positive integers is $1+3+\cdots+(2n-1)=n^2$. Thus the numerator of the term with denominator $n!$ is n^2.
- The exponential series gives $e=\sum_{m=0}^{\infty}1/m!$. Splitting a polynomial in n into falling factorials allows cancellation with $n!$.

Step 1: Express the general term Let the required sum be S. The first term is also $1^2/1!$, so the same pattern applies starting from $n=1$. Therefore

$$S=\sum_{n=1}^{\infty}\frac{n^2}{n!}.$$

These terms are positive, and their ratio tends to zero, so the series converges absolutely. We may split it into the two convergent sums below. The useful decomposition is $n^2=n(n-1)+n$, which matches the factors in the factorial denominator.

Step 2: Cancel factorial factors and shift indices The factor $n(n-1)$ vanishes when $n=1$, so that part starts at $n=2$. Cancelling gives

$$\begin{aligned}S&=\sum_{n=2}^{\infty}\frac{n(n-1)}{n!}+\sum_{n=1}^{\infty}\frac{n}{n!}\\&=\sum_{n=2}^{\infty}\frac{1}{(n-2)!}+\sum_{n=1}^{\infty}\frac{1}{(n-1)!}.\end{aligned}$$

In the first sum put $m=n-2$, and in the second put $m=n-1$. Both now start at $m=0$, so each is exactly e, with no missing constant term.

Step 3: Compare the printed options Thus $S=e+e=2e$. Options (A), (C) and (D) would require respectively three, four or six copies of the exponential series. The decomposition produces exactly two copies, so those values cannot equal the given sum.

Step 4: Conclusion The required infinite sum is $\boxed{2e}$. Hence, the correct option is (B).

Quick Tip to Solve: For a series containing $n^2/n!$, write $n^2=n(n-1)+n$; each resulting factorial sum becomes an exponential series.

20. If the four distinct points $(4,6),(-1,5),(0,0)$ and $(k,3k)$ lie on a circle of radius r, then $10k+r^2$ is equal to [4 marks]

(A) 34

(B) 32

(C) 35

(D) 33

Correct Answer: (C) 35

Solution:

Concept:

- A circle through the origin has equation $x^2+y^2+Dx+Ey=0$, since its constant term is zero.
- Its centre is $(-D/2,-E/2)$ and its radius satisfies $r^2=(D^2+E^2)/4$. A point lying on the circle satisfies its equation.

Step 1: Determine the circle from the first three points Substitute $(4,6)$ and $(-1,5)$ into the equation of a circle through $(0,0)$:

$$52+4D+6E=0,$$
$$26-D+5E=0.$$

The second equation gives $D=26+5E$. Using this in the first gives $104+26E=-52$, so $E=-6$ and $D=-4$. Hence the circle is

$$x^2+y^2-4x-6y=0.$$

Completing squares checks that it is a real circle: $(x-2)^2+(y-3)^2=13$.

Step 2: Use the fourth point and distinctness Put $x=k$ and $y=3k$ in this circle:

$$k^2+9k^2-4k-18k=0,$$
$$2k(5k-11)=0.$$

Thus $k=0$ or $k=11/5$. The root $k=0$ repeats the already listed point $(0,0)$, contradicting the requirement that the four points be distinct. Therefore $k=11/5$. This gives a point different from each of the other three.

Step 3: Evaluate the requested combination The completed-square equation gives $r^2=13$, rather than $r=13$. Consequently

$$10k+r^2=10\left(\frac{11}{5}\right)+13=22+13=35.$$

Only option (C) has this value. The alternatives 34, 32 and 33 do not satisfy the calculated expression with the admissible value of k.

Step 4: Conclusion The required value is $\boxed{35}$. Hence, the correct option is (C).

Quick Tip to Solve: When a circle includes the origin, set its constant term to zero immediately. Reject any intersection root that repeats a point required to be distinct.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. If $\lim\limits_{x\to 0}\left(\dfrac{\tan x}{x}\right)^{1/x^2} = p$, **then** $96\log_e p$ **is equal to** _____. [4 marks]

Solution:

Concept:

- The given expression has the indeterminate form 1^∞. Taking a natural logarithm converts the variable power into a quotient suitable for evaluating a limit.
- Near zero, $\sin x = x - x^3/6 + O(x^5)$ and $\cos x = 1 - x^2/2 + O(x^4)$. Also $\log(1+u) = u + O(u^2)$ as $u \to 0$.

Step 1: Take the logarithm of the limiting expression For all sufficiently small nonzero real x, $\tan x/x$ is positive and tends to 1. Let L be the limit of the logarithm. Then

$$L = \lim_{x\to 0}\frac{\log(\tan x/x)}{x^2}.$$

If this finite limit exists, continuity of the exponential function gives $p = e^L$, and hence $\log_e p = L$. The quantity requested in the question is therefore $96L$, so it is unnecessary to approximate p numerically.

Step 2: Find the first nonzero term in the base Using the small-angle expansions and dividing the two series,

$$\begin{aligned}\frac{\tan x}{x} &= \frac{\sin x/x}{\cos x}\\ &= \left(1 - \frac{x^2}{6} + O(x^4)\right)\left(1 + \frac{x^2}{2} + O(x^4)\right)\\ &= 1 + \frac{x^2}{3} + O(x^4).\end{aligned}$$

Only the quadratic coefficient matters because the logarithm will subsequently be divided by x^2. Terms of order x^4 contribute zero in that limit.

Step 3: Evaluate the logarithmic limit Apply $\log(1+u) = u + O(u^2)$ with $u = x^2/3 + O(x^4)$:

$$\log(\tan x/x) = \frac{x^2}{3} + O(x^4).$$

Therefore $L = 1/3$, so $p = e^{1/3}$. Substitution in the requested expression gives

$$96\log_e p = 96\left(\frac{1}{3}\right) = 32.$$

Step 4: Conclusion The required numerical value is $\boxed{32}$.

Quick Tip to Solve: For $(1 + ax^2 + O(x^4))^{1/x^2}$, the limit is e^a. Identify the quadratic coefficient before doing any numerical calculation.

22. If the equation of the hyperbola with foci $(4,2)$ and $(8,2)$ is $3x^2 - y^2 - \alpha x + \beta y + \gamma = 0$, then $\alpha + \beta + \gamma$ is equal to_____. [4 marks]

Solution:

Concept:

- The centre of a hyperbola is the midpoint of its foci. For a horizontal transverse axis, its standard equation is $(x-h)^2/a^2 - (y-k)^2/b^2 = 1$.
- The focal distance satisfies $c^2 = a^2 + b^2$, where c is the distance from the centre to either focus. Completing squares connects the general and standard equations.

Step 1: Find the centre and the linear coefficients The foci $(4,2)$ and $(8,2)$ have midpoint $(6,2)$. Thus the centre is $(h,k) = (6,2)$, the transverse axis is horizontal, and the focal distance is $c = 2$. Complete squares in the given equation, or compare the linear coefficients with $3(x-6)^2 - (y-2)^2$. Its linear terms are $-36x + 4y$, so

$$\alpha = 36, \qquad \beta = 4.$$

The signs are fixed by the given form $-\alpha x + \beta y$, which is essential when comparing coefficients.

Step 2: Relate the constant to the focal distance With these coefficients the equation becomes

$$3(x-6)^2 - (y-2)^2 = 104 - \gamma.$$

Write $K = 104 - \gamma$. Because the foci lie on a horizontal line, the coefficient of the positive squared term must correspond to the transverse axis, so $K > 0$. Dividing by K yields

$$\frac{(x-6)^2}{K/3} - \frac{(y-2)^2}{K} = 1.$$

Thus $a^2 = K/3$, $b^2 = K$, and $c^2 = a^2 + b^2 = 4K/3$.

Step 3: Determine the constant and add the coefficients Since $c = 2$, we have $4 = 4K/3$, giving $K = 3$. Hence $104 - \gamma = 3$ and $\gamma = 101$. As a check, the hyperbola is $(x-6)^2 - (y-2)^2/3 = 1$, with $c^2 = 1 + 3 = 4$ as required. Finally,

$$\alpha + \beta + \gamma = 36 + 4 + 101 = 141.$$

Step 4: Conclusion The required numerical value is $\boxed{141}$.

Quick Tip to Solve: Use the midpoint of the foci to obtain the linear coefficients first. Then use $c^2 = a^2 + b^2$ to fix the remaining constant.

23. Let $\left(1+x+x^2\right)^{10} = a_0 + a_1x + a_2x^2 + \ldots + a_{20}x^{20}$. If $(a_1 + a_3 + a_5 + \ldots + a_{19}) - 11a_2 = 121k$, then k is equal to_____ . [4 marks]

Solution:

Concept:

- For a polynomial $P(x) = \sum a_j x^j$, substitution of $x = 1$ and $x = -1$ separates even-indexed and odd-indexed coefficients: their odd-indexed sum is $(P(1) - P(-1))/2$.
- The coefficient of a chosen power in a product is obtained by selecting terms whose exponents add to that power, including the number of ways to make each selection.

Step 1: Calculate the sum of the odd-indexed coefficients Let $P(x) = (1 + x + x^2)^{10}$. At $x = 1$, all coefficients enter with positive sign, giving $P(1) = 3^{10} = 59049$. At $x = -1$, the even-indexed terms enter positively and the odd-indexed terms negatively, giving $P(-1) = (1 - 1 + 1)^{10} = 1$. Subtracting cancels every even-indexed coefficient, so

$$a_1 + a_3 + \cdots + a_{19} = \frac{59049 - 1}{2} = 29524.$$

There are no odd exponents beyond 19 because the degree of this polynomial is 20.

Step 2: Obtain the coefficient of the quadratic term To obtain x^2 from the ten identical factors, either choose x^2 from one factor and 1 from the others, or choose x from two distinct factors and 1 from the others. The first selection has 10 possibilities; the second has $\binom{10}{2} = 45$ possibilities. These are disjoint and exhaust the possible exponent sums of 2. Hence

$$a_2 = 10 + 45 = 55.$$

Step 3: Substitute and solve for the requested number The relation in the question now gives

$$\begin{aligned} 121k &= 29524 - 11(55) \\ &= 29524 - 605 = 28919. \end{aligned}$$

Dividing by 121 gives $k = 239$. The exact multiplication $121 \times 239 = 28919$ checks the arithmetic and confirms that the result is an integer as required by the numerical response.

Step 4: Conclusion The required value is $\boxed{239}$.

Quick Tip to Solve: Use $P(1) - P(-1)$ for the sum of odd coefficients; find an individual low-degree coefficient by counting selections rather than expanding the polynomial.

24. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = 3\hat{i} - 3\hat{j} + 3\hat{k}, \vec{c} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{d}$ be a vector such that $\vec{b} \times \vec{d} = \vec{c} \times \vec{d}$ and $\vec{a} \cdot \vec{d} = 4$. Then $|(\vec{a} \times \vec{d})|^2$ is equal to_____. [4 marks]

Solution:

Concept:

- Bilinearity of the cross product gives $\vec{b} \times \vec{d} - \vec{c} \times \vec{d} = (\vec{b} - \vec{c}) \times \vec{d}$. If a nonzero vector has zero cross product with another vector, the two are parallel.
- The identity $|\vec{a} \times \vec{d}|^2 = |\vec{a}|^2|\vec{d}|^2 - (\vec{a} \cdot \vec{d})^2$ evaluates the required squared magnitude without calculating a determinant.

Step 1: Use the cross-product condition to determine the direction The equality of the two cross products implies

$$(\vec{b} - \vec{c}) \times \vec{d} = \vec{0}.$$

Subtracting the given vectors gives

$$\vec{b} - \vec{c} = (3-2)\hat{i} + (-3+1)\hat{j} + (3-2)\hat{k} = \hat{i} - 2\hat{j} + \hat{k}.$$

This difference is nonzero. Therefore $\vec{d} = t(\hat{i} - 2\hat{j} + \hat{k})$ for a real scalar t. The zero vector is already excluded by the separate condition $\vec{a} \cdot \vec{d} = 4$.

Step 2: Use the dot-product condition to determine the scale Take the dot product with $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$:

$$\vec{a} \cdot \vec{d} = t(1 - 4 + 1) = -2t = 4,$$
$$t = -2.$$

Thus $\vec{d} = -2\hat{i} + 4\hat{j} - 2\hat{k}$. Substitution checks the dot product directly: $-2 + 8 - 2 = 4$, so the sign of the scalar is correct. Also $\vec{d}$ is a multiple of $\vec{b} - \vec{c}$, as required by the cross-product condition.

Step 3: Evaluate the squared cross-product magnitude The squared lengths are

$$|\vec{a}|^2 = 1 + 4 + 1 = 6, \qquad |\vec{d}|^2 = 4 + 16 + 4 = 24.$$

Using the identity with the given dot product,

$$|\vec{a} \times \vec{d}|^2 = 6(24) - 4^2 = 144 - 16 = 128.$$

As an independent arithmetic check, direct expansion gives $\vec{a} \times \vec{d} = -8\hat{i} + 8\hat{k}$, whose squared magnitude is $64 + 64 = 128$.

Step 4: Conclusion The required numerical value is $\boxed{128}$.

Quick Tip to Solve: First combine equal cross products to obtain a direction for the unknown vector. Use the dot product to find its scale, then apply the squared-magnitude identity.

25. Let I be the identity matrix of order 3×3 and for the matrix $A = \begin{bmatrix} \lambda & 2 & 3 \\ 4 & 5 & 6 \\ 7 & -1 & 2 \end{bmatrix}, |A| = -1$. Let B be the inverse of the matrix $\text{adj}\left(\text{Aadj}\left(A^2\right)\right)$. Then $|(\lambda B + I)|$ is equal to_____ [4 marks]

Solution:

Concept:

- For an invertible matrix M, $\text{adj}\, M = |M|M^{-1}$. Since $|A| = -1$, every inverse used here exists.
- Determinants are multiplicative: $|XY| = |X||Y|$. Thus $|I - \lambda A^{-1}| = |A - \lambda I|/|A|$, which avoids computing all entries of the inverse.

Step 1: Determine the parameter from the given determinant Expand $|A|$ along its first row, retaining the alternating cofactor signs:

$$\begin{aligned} |A| &= \lambda(10 + 6) - 2(8 - 42) + 3(-4 - 35) \\ &= 16\lambda + 68 - 117 = 16\lambda - 49. \end{aligned}$$

Equating this to -1 gives $16\lambda = 48$, so $\lambda = 3$. This value also confirms that the specified matrix is nonsingular.

Step 2: Simplify the nested adjugates before taking the inverse The expression inside the outer adjugate is $A\,\text{adj}(A^2)$. First, $|A^2| = |A|^2 = 1$, hence

$$\text{adj}(A^2) = A^{-2}, \qquad A\,\text{adj}(A^2) = A^{-1}.$$

Now $|A^{-1}| = -1$, so

$$\text{adj}(A^{-1}) = |A^{-1}|A = -A.$$

As B is the inverse of this entire matrix, $B = (-A)^{-1} = -A^{-1}$. The minus sign must be retained.

Step 3: Evaluate the required determinant Factor $I - 3A^{-1} = A^{-1}(A - 3I)$. Substituting the determined value of λ,

$$A - 3I = \begin{pmatrix} 0 & 2 & 3 \\ 4 & 2 & 6 \\ 7 & -1 & -1 \end{pmatrix}.$$

Expansion along its first row gives

$$|A - 3I| = -2(-4 - 42) + 3(-4 - 14) = 92 - 54 = 38.$$

Therefore $|\lambda B + I| = |A^{-1}||A - 3I| = (-1)(38) = -38$. A positive result would incorrectly discard the determinant of A^{-1}.

Step 4: Conclusion The required dimensionless determinant is $\boxed{-38}$. The accepted key lists both signs, but the literal nested-adjugate expression gives -38, as the factor $|A^{-1}| = -1$ shows.

Quick Tip to Solve: Replace each adjugate by determinant times inverse from the innermost bracket outward; simplify the matrix product before expanding the final determinant.

26. Two cylindrical vessels of equal cross sectional area of $2\ \mathrm{m}^2$ contain water upto heights 10 m and 6 m , respectively. If the vessels are connected at their bottom then the work done by the force of gravity is (Density of water is $10^3\ \mathrm{kg/m}^3$ and $\mathrm{g} = 10\ \mathrm{m/s}^2$) [4 marks]

(A) 1×10^5 J

(B) 4×10^4 J

(C) 8×10^4 J

(D) 6×10^4 J

Correct Answer: (C) 8×10^4 J

Solution:

Concept:

- A uniform liquid column of height h and cross-sectional area S has mass ρSh, with its centre of mass at $h/2$ above the bottom. Its gravitational potential energy is $U = \rho Sgh^2/2$.
- Gravity is conservative, so its work equals the decrease in gravitational potential energy: $W_g = U_i - U_f$. Equal connected vessels reach the same final water level.

Step 1: Find the final common height Choose the common bottom as zero gravitational potential. Water flows from the taller column into the shorter one until the pressures at their connected bottoms are equal. Since both exposed surfaces are at atmospheric pressure, their final heights are equal. Conservation of water volume gives

$$S(10) + S(6) = 2Sh_f, \qquad h_f = 8\,\mathrm{m}.$$

Here $S = 2\,\mathrm{m}^2$ is the area of each vessel, not their combined area.

Step 2: Calculate the initial and final potential energies Add the energy of both columns separately. Using $\rho = 10^3\,\mathrm{kg\,m^{-3}}$ and $g = 10\,\mathrm{m\,s^{-2}}$,

$$\begin{aligned} U_i &= \frac{\rho Sg}{2}(10^2 + 6^2) \\ &= 10^4(136) = 1.36 \times 10^6\,\mathrm{J}, \\ U_f &= \frac{\rho Sg}{2}(8^2 + 8^2) \\ &= 10^4(128) = 1.28 \times 10^6\,\mathrm{J}. \end{aligned}$$

The system's centre of mass falls, so the potential-energy change is negative and gravity does positive work.

Step 3: Take the energy decrease and compare the options The required work is

$$W_g = 1.36 \times 10^6 - 1.28 \times 10^6 = 8.0 \times 10^4\,\mathrm{J}.$$

As a check, equal-area vessels give $W_g = \rho Sg(h_1 - h_2)^2/4$, which here is $10^3(2)(10)(4)^2/4 = 8.0 \times 10^4$ J. Options (A), (B) and (D) therefore do not equal the computed energy decrease. In particular, option

(B) misses a factor of two in the total column-energy change.

Step 4: Conclusion Gravity does positive work $\boxed{8 \times 10^4\ \text{J}}$ while the levels become equal. Hence, the correct option is (C).

Quick Tip to Solve: For two equal-area connected columns, first average their heights, then subtract their total gravitational energies; do not multiply all the transferred water by a fixed height drop.

27. Two monochromatic light beams have intensities in the ratio 1:9. An interference pattern is obtained by these beams. The ratio of the intensities of maximum to minimum is [4 marks]

(A) $8:1$

(B) $4:1$

(C) $3:1$

(D) $9:1$

Correct Answer: (B) $4:1$

Solution:

Concept:

- Light intensity is proportional to the square of electric-field amplitude: $I \propto a^2$. Interfering amplitudes, rather than the individual intensities, add or subtract.
- For coherent beams, $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$, where δ is their phase difference. Maxima have $\cos\delta = 1$; minima have $\cos\delta = -1$.

Step 1: Convert the intensity ratio into an amplitude ratio Let the individual intensities be $I_1 = I_0$ and $I_2 = 9I_0$, with $I_0 > 0$. Because intensity is quadratic in amplitude, the corresponding amplitude ratio is

$$a_1 : a_2 = \sqrt{I_1} : \sqrt{I_2} = 1 : 3.$$

Using $1:9$ as the amplitude ratio would confuse the given energy-flow ratio with the oscillating electric-field strengths.

Step 2: Evaluate the constructive and destructive intensities At a bright fringe the two amplitudes add, whereas at a dark fringe they subtract. Thus

$$\begin{aligned} I_{\max} &= (\sqrt{I_1} + \sqrt{I_2})^2 \\ &= (\sqrt{I_0} + 3\sqrt{I_0})^2 = 16I_0, \\ I_{\min} &= (\sqrt{I_2} - \sqrt{I_1})^2 \\ &= (3\sqrt{I_0} - \sqrt{I_0})^2 = 4I_0. \end{aligned}$$

The minimum is nonzero because the unequal amplitudes cannot cancel completely. Complete darkness would require equal beam intensities.

Step 3: Form the requested maximum-to-minimum ratio Divide the two fringe intensities in the order

asked:

$$\frac{I_{\max}}{I_{\min}} = \frac{16I_0}{4I_0} = 4.$$

Hence the ratio is $4:1$. Option (D), $9:1$, merely repeats the incident-beam intensity ratio. Option (C), $3:1$, is the amplitude ratio, not the fringe-intensity ratio. Option (A), $8:1$, also disagrees with the squared sum-to-difference calculation. The unknown absolute intensity scale cancels, as a ratio must.

Step 4: Conclusion The ratio of maximum intensity to minimum intensity is $\boxed{4:1}$. Hence, the correct option is (B).

Quick Tip to Solve: Take square roots of the two incident intensities first, then square the ratio of their sum to their difference to obtain the fringe contrast ratio.

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28. A particle moves along the x-axis and has its displacement x varying with time t according to the equation:

$$x = \mathrm{c}_0\left(\mathrm{t}^2 - 2\right) + \mathrm{c}(\mathrm{t}-2)^2$$

where c_0 and c are constants of appropriate dimensions.

Then, which of the following statements is correct? [4 marks]

(A) the acceleration of the particle is $2\,(c + c_0)$

(B) the acceleration of the particle is $2c_0$

(C) the acceleration of the particle is 2 c

(D) the initial velocity of the particle is $4c$

Correct Answer: (A) the acceleration of the particle is $2\,(c + c_0)$

Solution:

Concept:

- For one-dimensional motion, instantaneous velocity is $v = dx/dt$ and acceleration is $a = dv/dt = d^2x/dt^2$. Constant displacement terms vanish on differentiation.
- A quadratic position-time expression describes constant acceleration. Its linear coefficient gives the initial velocity, while twice its quadratic coefficient gives the acceleration.

Step 1: Expand the position expression to identify each term Starting with the given displacement, expand only the shifted square and collect powers of time:

$$\begin{aligned} x &= c_0(t^2 - 2) + c(t-2)^2 \\ &= c_0t^2 - 2c_0 + c(t^2 - 4t + 4) \\ &= (c_0 + c)t^2 - 4ct + (4c - 2c_0). \end{aligned}$$

Both c_0t^2 and the ct^2 contribution from the shifted square affect acceleration. The last bracket is independent of time and only fixes the initial position.

Step 2: Differentiate to obtain velocity and acceleration Differentiate the collected expression once

and then again:

$$v(t) = 2(c_0 + c)t - 4c,$$

$$a(t) = 2(c_0 + c).$$

The acceleration has no time dependence. Direct differentiation of the original expression provides the same check: $v = 2c_0t + 2c(t-2)$, whose derivative is $2c_0 + 2c$. Thus the shift in the square changes the initial velocity but does not remove that term's acceleration contribution.

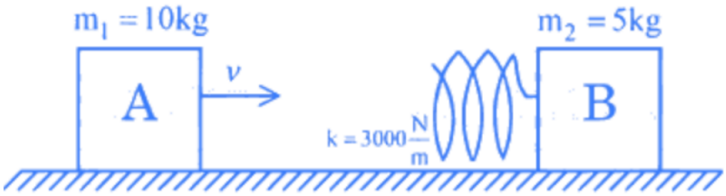
Step 3: Check the initial velocity and eliminate the other statements At $t = 0$, the velocity is $v(0) = -4c$, with the sign measured along the stated x-axis. Therefore option (D), which gives $+4c$, has the wrong sign. Option (B) omits the acceleration contribution $2c$, and option (C) omits $2c_0$. With arbitrary constants neither omission is justified. Only option (A) contains the full second derivative. The same result follows from comparison with $x = x_0 + ut + \frac{1}{2}at^2$.

Step 4: Conclusion The particle has constant acceleration $\boxed{a = 2(c + c_0)}$ and initial velocity $-4c$. Hence, the correct option is (A).

Quick Tip to Solve: Expand a shifted time square before identifying the initial velocity; its quadratic coefficient still contributes twice that coefficient to the acceleration.

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29. Consider two blocks A and B of masses $m_1 = 10$ kg and $m_2 = 5$ kg that are placed on a frictionless table. The block A moves with a constant speed $v = 3$ m/s towards the block B kept at rest. A spring with spring constant $k = 3000$ N/m is attached with the block B as shown in the figure. After the collision, suppose that the blocks A and B, along with the spring in constant compression state, move together, then the compression in the spring is, (Neglect the mass of the spring) [4 marks]



(A) 0 .3 m

(B) 0.1 m

(C) 0.4 m

(D) 0.2 m

Correct Answer: (B) 0.1 m

Solution:

Concept:

- The frictionless table exerts no external horizontal force, so total horizontal momentum of the two blocks is conserved. Spring forces redistribute momentum internally.
- At the common-velocity compression state, relative velocity is zero. For an ideal massless spring, the reduction in the blocks' kinetic energy is stored as elastic potential energy $kx^2/2$, where x is

compression.

Step 1: Determine the common velocity from momentum conservation Initially block A has mass $m_1 = 10\,\text{kg}$ and speed $v = 3\,\text{m s}^{-1}$; block B has mass $m_2 = 5\,\text{kg}$ and is at rest. Let V be their velocity when they move together at the specified compression state. Then

$$m_1 v = (m_1 + m_2)V,$$

$$V = \frac{10 \times 3}{10 + 5} = 2\,\text{m s}^{-1}.$$

The common velocity is also the conserved centre-of-mass velocity. At maximum compression there is no relative motion of the two blocks.

Step 2: Calculate the kinetic energy converted into spring energy The initial and common-velocity kinetic energies are

$$K_i = \frac{1}{2}(10)(3)^2 = 45\,\text{J},$$

$$K_f = \frac{1}{2}(10 + 5)(2)^2 = 30\,\text{J}.$$

Thus $45 - 30 = 15\,\text{J}$ is stored in the spring. The common translational energy cannot be assigned to spring compression because the centre of mass continues to move. The massless spring has no separate translational kinetic energy.

Step 3: Solve for the compression and test the choices With $k = 3000\,\text{N m}^{-1}$, mechanical-energy conservation gives

$$\frac{1}{2}kx^2 = 15, \qquad x^2 = \frac{30}{3000} = 0.0100\,\text{m}^2.$$

Compression is a nonnegative length, so $x = 0.100\,\text{m}$. As a check, compressions 0.2, 0.3 and $0.4\,\text{m}$ store 60, 135 and $240\,\text{J}$, respectively. Each exceeds even the initial $45\,\text{J}$, ruling out options (D), (A) and (C). The remaining value stores precisely $15\,\text{J}$.

Step 4: Conclusion The spring compression at the common-velocity state is $\boxed{0.1\,\text{m}}$. Hence, the correct option is (B).

Quick Tip to Solve: Use momentum to find the shared velocity first; only the kinetic energy lost from relative motion, not the centre-of-mass kinetic energy, can become spring energy.

30. A magnetic dipole experiences a torque of $80\sqrt{3}$ N m when placed in uniform magnetic field in such a way that dipole moment makes angle of 60° with magnetic field. The potential energy of the dipole is : [4 marks]

(A) $-40\sqrt{3}$ J

(B) -80 J

(C) 80 J

(D) -60 J

Correct Answer: (B) -80 J

Solution:

Concept:

- A magnetic dipole of moment magnitude m in a uniform field of magnitude B experiences torque magnitude $\tau = mB\sin\theta$, where θ is the angle between moment and field.
- Its magnetic potential energy is $U = -\boldsymbol{m}\cdot\boldsymbol{B} = -mB\cos\theta$, taking zero energy for a perpendicular dipole. Torque and energy therefore involve different trigonometric factors.

Step 1: Extract the product of magnetic moment and field The given torque is $80\sqrt{3}\,\mathrm{N\,m}$ at $\theta = 60^\circ$. Substitute into the torque law with $\sin 60^\circ = \sqrt{3}/2$:

$$80\sqrt{3} = mB\frac{\sqrt{3}}{2}.$$

Cancelling the positive common factor gives

$$mB = 160\,\mathrm{N\,m} = 160\,\mathrm{J}.$$

The torque determines only the product mB; separate values of dipole moment and magnetic field are not needed to determine the potential energy.

Step 2: Apply the potential-energy formula with its sign Use the same orientation in $U = -mB\cos\theta$. Since $\cos 60^\circ = 1/2$,

$$U = -(160)\frac{1}{2} = -80\,\mathrm{J}.$$

The negative sign is physically consistent: the moment makes an acute angle with the field, so their scalar product is positive and its negative is below the perpendicular-orientation energy. Alignment would give the still lower value -160 J, while anti-alignment would give $+160$ J.

Step 3: Cross-check directly and eliminate the other options Eliminate mB between the torque and energy relations to obtain

$$U = -\tau\cot\theta = -80\sqrt{3}\,\frac{1}{\sqrt{3}} = -80\,\mathrm{J}.$$

Option (A) incorrectly introduces a remaining $\sqrt{3}$ factor, and option (D) does not satisfy either relation for the stated angle. Option (C), $+80$ J, has the right magnitude but the wrong sign for an acute orientation. Only option (B) agrees with both the torque magnitude and the magnetic energy convention.

Step 4: Conclusion The magnetic dipole has potential energy $\boxed{-80\,\mathrm{J}}$ in the given orientation. Hence,

the correct option is (B).

Quick Tip to Solve: When torque and angle are given, combine the two dipole laws as $U = -\tau \cot\theta$; this retains the energy sign and avoids finding moment and field separately.

31. An ideal gas exists in a state with pressure P_0, volume V_0. It is isothermally expanded to 4 times of its initial volume (V_0), then isobarically compressed to its original volume. Finally the system is heated isochorically to bring it to its initial state. The amount of heat exchanged in this process is [4 marks]

(A) $\mathrm{P}_0\ \mathrm{V}_0(\ln 2 - 0.75)$

(B) $\mathrm{P}_0\ \mathrm{V}_0(2\ln 2 - 0.75)$

(C) $\mathrm{P}_0\ \mathrm{V}_0(2\ln 2 - 0.25)$

(D) $\mathrm{P}_0\ \mathrm{V}_0(\ln 2 - 0.25)$

Correct Answer: (B) $\mathrm{P}_0\ \mathrm{V}_0(2\ln 2 - 0.75)$

Solution:

Concept:

- In a complete thermodynamic cycle the internal energy returns to its initial value, so the first law gives net heat supplied equal to net work done by the gas: $Q_{\text{net}} = W_{\text{net}}$.
- For an ideal gas, isothermal work is $nRT\ln(V_f/V_i)$, isobaric work is $P(V_f - V_i)$, and isochoric work is zero. Work by the gas is positive for expansion.

Step 1: Find the pressure after isothermal expansion Call the initial state A and the expanded state B. Since PV remains constant on an isotherm, the temperature stays fixed and

$$P_B(4V_0) = P_0V_0, \qquad P_B = \frac{P_0}{4}.$$

The work during A to B is therefore

$$W_{AB} = P_0V_0\ln\frac{4V_0}{V_0} = P_0V_0\ln 4 = 2P_0V_0\ln 2.$$

Step 2: Calculate the compression and constant-volume work Compression from B to C occurs at the fixed pressure $P_0/4$, taking the volume from $4V_0$ back to V_0. Its negative work is

$$W_{BC} = \frac{P_0}{4}(V_0 - 4V_0) = -\frac{3}{4}P_0V_0.$$

In the final heating C to A the volume is fixed, hence $W_{CA} = 0$. The return to the original pressure and volume also restores the original temperature and internal energy.

Step 3: Apply the first law over the full cycle Adding the three contributions gives

$$Q_{\text{net}} = W_{AB} + W_{BC} + W_{CA} = P_0V_0\left(2\ln 2 - \frac{3}{4}\right).$$

The coefficient is positive, so this is net heat absorbed. Options A and D omit the factor 2 from $\ln 4 = 2\ln 2$; option C uses an incorrect compression term of $1/4$ instead of $3/4$.

Step 4: Conclusion The net heat exchanged is $P_0V_0(2\ln 2 - 0.75)$, absorbed by the gas. Hence, the correct option is (B).

Quick Tip to Solve: For a closed ideal-gas cycle, calculate net work directly; there is no need to know the heat capacity because the net internal-energy change is zero.

32. Width of one of the two slits in a Young's double slit interference experiment is half of the other slit. The ratio of the maximum to the minimum intensity in the interference pattern is : [4 marks]

(A) $3 : 1$

(B) $(2\sqrt{2}+1) : (2\sqrt{2}-1)$

(C) $9 : 1$

(D) $(3+2\sqrt{2}) : (3-2\sqrt{2})$

Correct Answer: (D) $(3+2\sqrt{2}) : (3-2\sqrt{2})$

Solution:

Concept:

- For equally illuminated slits, the light intensity contributed by each slit is proportional to its width. Thus a slit of half the width supplies half the intensity of the wider slit.
- Interference adds field amplitudes, with amplitude proportional to the square root of intensity. Hence $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$ and $I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$.

Step 1: Relate the individual slit intensities Let the narrower slit have width w and the wider one width $2w$. With the same incident illumination, their intensities in the usual two-slit treatment are in the ratio

$$I_1 : I_2 = 1 : 2.$$

Write $I_1 = I$ and $I_2 = 2I$. The corresponding amplitudes are in the ratio $1 : \sqrt{2}$. The width ratio must therefore not be inserted directly as an amplitude ratio.

Step 2: Calculate constructive and destructive intensities At a bright fringe the phase difference is an integral multiple of 2π, so the two amplitudes add. At a dark fringe they oppose. Consequently,

$$I_{\max} = (\sqrt{I} + \sqrt{2I})^2 = I(3+2\sqrt{2}),$$
$$I_{\min} = (\sqrt{2I} - \sqrt{I})^2 = I(3-2\sqrt{2}).$$

The minimum is not zero because the individual slit intensities are unequal.

Step 3: Form the ratio and compare the options Cancelling the common intensity scale gives

$$I_{\max} : I_{\min} = (3+2\sqrt{2}) : (3-2\sqrt{2}).$$

Option A does not include the interference cross term. Option B has an incorrect constant term. Option C would result from wrongly treating the width ratio $1 : 2$ as the amplitude ratio, instead of

using its square root.

Step 4: Conclusion The maximum-to-minimum intensity ratio is $(3+2\sqrt{2}) : (3-2\sqrt{2})$. Hence, the correct option is (D).

Quick Tip to Solve: Convert the slit-width ratio to the individual intensity ratio first, then take square roots before using the sum-and-difference amplitude formula.

33. Using a battery, a 100 pF capacitor is charged to 60 V and then the battery is removed. After that, a second uncharged capacitor is connected to the first capacitor in parallel. If the final voltage across the second capacitor is 20 V , its capacitance is: (in pF) [4 marks]

(A) 600

(B) 100

(C) 400

(D) 200

Correct Answer: (D) 200

Solution:

Concept:

- A capacitor stores charge $Q = CV$. Once the battery is removed, the total charge of the connected isolated capacitor system is conserved.
- Capacitors connected in parallel have the same final potential difference. Their equivalent capacitance is the sum $C_1 + C_2$, so the final charge is $(C_1 + C_2)V_f$.

Step 1: Calculate the initially stored charge The first capacitor has $C_1 = 100\,\text{pF}$ and is initially at $V_i = 60\,\text{V}$. The second capacitor is uncharged, so the entire initial charge comes from the first:

$$Q_i = C_1 V_i = (100)(60)\,\text{pC} = 6000\,\text{pC}.$$

Using picofarads and volts directly is convenient because their product is picocoulombs.

Step 2: Conserve charge at the common final voltage After the parallel connection both capacitors are at $V_f = 20\,\text{V}$. If the unknown capacitance is C_2, charge conservation gives

$$C_1 V_i = (C_1 + C_2)V_f.$$

Thus the combined capacitance is

$$C_1 + C_2 = \frac{6000\,\text{pC}}{20\,\text{V}} = 300\,\text{pF}.$$

Subtracting the first capacitor gives $C_2 = 300 - 100 = 200\,\text{pF}$.

Step 3: Check the individual final charges The final charges are $Q_1 = 100 \times 20 = 2000\,\text{pC}$ and $Q_2 = 200 \times 20 = 4000\,\text{pC}$, adding to the initial $6000\,\text{pC}$. The alternatives 600, 100 and 400 pF would instead give final voltages $60/7$, 30 and 12 V, respectively, so none satisfies 20 V. Electrostatic energy

need not be conserved during redistribution; charge is the relevant conserved quantity.

Step 4: Conclusion The capacitance of the second capacitor is $200\,\mathrm{pF}$. Hence, the correct option is (D).

Quick Tip to Solve: For an initially uncharged parallel partner, use $C_2 = C_1(V_i/V_f - 1)$; conserve total charge rather than electrostatic energy.

34. A motor operating on 100 V draws a current of 1 A . If the efficiency of the motor is 91.6%, then the loss of power in units of $\mathrm{cal/s}$ is [4 marks]

(A) 6.2

(B) 2

(C) 8.4

(D) 4

Correct Answer: (B) 2

Solution:

Concept:

- Electrical input power is $P_{\text{in}} = VI$, where V is the supply voltage and I the current drawn. A watt is one joule per second.
- Motor efficiency is $\eta = P_{\text{useful}}/P_{\text{in}}$. The lost power is the unused fraction, $P_{\text{loss}} = (1-\eta)P_{\text{in}}$, and $1\,\text{cal} \simeq 4.2\,\text{J}$.

Step 1: Determine the electrical input power The motor operates at $100\,\text{V}$ and draws $1\,\text{A}$, so the rate at which it receives electrical energy is

$$P_{\text{in}} = VI = 100 \times 1 = 100\,\text{W}.$$

This is input power; the useful mechanical output is smaller because the efficiency is below unity.

Step 2: Separate useful output from the losses Express the stated percentage as a fraction: $\eta = 91.6/100 = 0.916$. The useful output and the difference from the input are

$$P_{\text{useful}} = 0.916 \times 100 = 91.6\,\text{W},$$
$$P_{\text{loss}} = 100 - 91.6 = 8.4\,\text{W}.$$

Equivalently, 8.4 percent of the input is lost. The value 8.4 is in joules per second at this stage, not calories per second.

Step 3: Convert the loss to calories per second Divide the energy rate in joules per second by the joules in one calorie:

$$P_{\text{loss}} = \frac{8.4}{4.2} = 2\,\text{cal}\,\text{s}^{-1}.$$

Using the more precise conversion 4.184 gives approximately 2.01, consistent with the listed value 2. Option C is the unconverted watt value; options A and D do not give the lost fraction in the requested units.

Step 4: Conclusion The power lost by the motor is $2\,\text{cal}\,\text{s}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: Multiply input power by the inefficiency $1-\eta$, then divide by 4.2 to change watts into calories per second.

35. In the resonance experiment, two air columns (closed at one end) of 100 cm and 120 cm long, give 15 beats per second when each one is sounding in the respective fundamental modes. The velocity of sound in the air column is: [4 marks]

(A) 370 m/s

(B) 340 m/s

(C) 335 m/s

(D) 360 m/s

Correct Answer: (D) 360 m/s

Solution:

Concept:

- A pipe closed at one end has a displacement node at the closed end and an antinode at the open end. In its fundamental mode, its length is one-quarter wavelength, so $f = v/(4L)$.
- Two simultaneous sounds of nearby frequencies produce beats at frequency $f_b = |f_1 - f_2|$. The shorter air column has the higher fundamental frequency.

Step 1: Express the two fundamental frequencies Convert the column lengths to metres: $L_1 = 100\,\text{cm} = 1.00\,\text{m}$ and $L_2 = 120\,\text{cm} = 1.20\,\text{m}$. If sound travels at speed v, their fundamental frequencies are

$$f_1 = \frac{v}{4(1.00)} = \frac{v}{4}, \qquad f_2 = \frac{v}{4(1.20)} = \frac{v}{4.8}.$$

Both columns contain the same air, so the speed is common even though the resonant frequencies differ.

Step 2: Use the measured beat frequency Since $f_1 > f_2$, the observed 15 beats per second means

$$\begin{aligned} 15 &= \frac{v}{4} - \frac{v}{4.8} \\ &= \frac{v}{4}\left(1 - \frac{5}{6}\right) = \frac{v}{24}. \end{aligned}$$

Therefore $v = 15 \times 24 = 360\,\text{m}\,\text{s}^{-1}$. The length difference alone is insufficient; the frequencies depend on reciprocal lengths.

Step 3: Verify the resonances and reject alternatives At $v = 360\,\text{m}\,\text{s}^{-1}$, the frequencies are $90\,\text{Hz}$ and $75\,\text{Hz}$, whose difference is exactly 15 Hz. The other suggested speeds 370, 340 and 335 m/s would give beat frequencies $370/24$, $340/24$ and $335/24$ Hz, respectively, none equal to 15. Using the open-pipe formula would introduce an incorrect factor of two for these closed-end columns.

Step 4: Conclusion The speed of sound in the air columns is $360\,\text{m}\,\text{s}^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: For two closed pipes in their fundamental modes, write $f_b = (v/4)|1/L_1 - 1/L_2|$ after converting both lengths to metres.

36. A block of mass 1 kg , moving along x with speed $v_i = 10\ \mathrm{m/s}$ enters a rough region ranging from $x = 0.1\ \mathrm{m}$ to $x = 1.9\ \mathrm{m}$. The retarding force acting on the block in this range is $\mathrm{F_r} = -\mathrm{krN}$, with k $= 10\ \mathrm{N/m}$. Then the final speed of the block as it crosses rough region is. [4 marks]

(A) $4\ \mathrm{m/s}$

(B) $10\ \mathrm{m/s}$

(C) $8\ \mathrm{m/s}$

(D) $6\ \mathrm{m/s}$

Correct Answer: (C) $8\ \mathrm{m/s}$

Solution:

Concept:

- The work-energy theorem states that the net work equals the change in kinetic energy: $W = \frac{1}{2}m(v_f^2 - v_i^2)$. A force opposite to the motion removes kinetic energy.
- For a force varying with position, its work is the definite integral $W = \int F_x\,dx$. Here the position r in the linear retarding-force law is the coordinate along the stated straight path, so $F_x = -kx$.

Step 1: Calculate the initial kinetic energy The mass is $m = 1\,\mathrm{kg}$, and the entry speed is $v_i = 10\,\mathrm{m\,s^{-1}}$. Therefore

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(1)(10)^2 = 50\,\mathrm{J}.$$

Only the rough interval from 0.1 m to 1.9 m contributes to the specified retarding work. The constant $k = 10\,\mathrm{N\,m^{-1}}$ multiplies the position, so the force is not constant over that interval.

Step 2: Integrate the retarding force over the rough interval With positive displacement along the direction of motion, the force has a negative sign. Its work is

$$W = \int_{0.1}^{1.9}(-kx)\,dx = -\frac{k}{2}\left[(1.9)^2 - (0.1)^2\right]$$
$$= -5(3.61 - 0.01) = -18\,\mathrm{J}.$$

The initial kinetic energy exceeds this loss, so the block does reach the end of the rough region.

Step 3: Determine and check the exit speed The final kinetic energy is $K_f = 50 - 18 = 32\,\mathrm{J}$, hence

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{64} = 8\,\mathrm{m\,s^{-1}}.$$

A final speed of 10 m/s would mean no energy loss, while 4 and 6 m/s would require losses of 42 and 32 J, respectively. These are inconsistent with the integrated work of 18 J. Taking one endpoint's force as constant would also give an incorrect loss.

Step 4: Conclusion The final speed as the block crosses the rough region is $8\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: A retarding force proportional to position gives work $-\frac{1}{2}k(x_f^2 - x_i^2)$; use the difference of squares, not the square of the travelled distance.

37.

The truth table corresponding to the circuit given below is:

[4 marks]



(A)

A	B	C
0	0	0
0	1	0
1	0	1
1	1	1

(B)

A	B	C
0	0	1
1	0	0
0	1	0
1	1	0

(C)

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

(D)

A	B	C
0	0	0
1	0	0
0	1	0
1	1	1

Correct Answer: (A)

A	B	C
0	0	0
0	1	0
1	0	1
1	1	1

Solution:

Concept:

- An OR gate gives output $X = A + B$, where $+$ denotes Boolean OR: the output is 1 if at least one input is 1.
- An AND gate gives the Boolean product of its inputs; its output is 1 only when both inputs are 1. The absorption identity is $A(A + B) = A$.

Step 1: Read the two stages of the circuit The first curved gate is an OR gate with inputs A and B.

Call its output X. The output wire from this gate enters the lower input of the second gate. A separate wire takes A directly to the upper input of the second gate, which is an AND gate. Thus the gates are connected as

$$X = A + B, \qquad C = A \cdot X = A(A + B).$$

Step 2: Evaluate the output for each input pair If $A = 0$, the AND gate has a zero input, so $C = 0$, irrespective of B. If $A = 1$, the OR output is necessarily 1, and hence $C = 1$. Equivalently, Boolean expansion gives

$$C = A^2 + AB = A + AB = A.$$

The four input cases therefore give

A	B	$X = A + B$	$C = AX$
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

Step 3: Compare the printed tables Option (A) gives precisely the sequence $0, 0, 1, 1$ for inputs $(0, 0), (0, 1), (1, 0), (1, 1)$. Options (B) and (C), although their middle rows are written in different orders, both describe a NOR output: 1 only for $(0, 0)$. Option (D) describes AND, giving 1 only for $(1, 1)$, and incorrectly gives 0 for $(1, 0)$.

Step 4: Conclusion The final output equals the input A, independently of B. Hence, the correct option is (A).

Quick Tip to Solve: Follow the bypass wire as well as the gate outputs. Whenever an AND gate receives A and $A + B$, apply absorption immediately: $A(A + B) = A$.

38. A monochromatic light of frequency 5×10^{14} Hz travelling through air, is incident on a medium of refractive index ' 2 '. Wavelength of the refracted light will be : [4 marks]

(A) 400 nm

(B) 300 nm

(C) 600 nm

(D) 500 nm

Correct Answer: (B) 300 nm

Solution:

Concept:

- The refractive index is $n = c/v$, where c is the speed of light in vacuum (approximately air) and v is its speed in the medium.
- The frequency f of light remains unchanged at a stationary interface. Its wavelength in a medium is $\lambda = v/f = c/(nf)$.

Step 1: Find the speed in the refracting medium The light has frequency $f = 5 \times 10^{14}\,\text{Hz}$, and the second medium has refractive index $n = 2$. Its optical speed is therefore half the speed in air:

$$v = \frac{c}{n} = \frac{3 \times 10^8}{2} = 1.5 \times 10^8\,\text{m s}^{-1}.$$

Refraction changes the propagation speed and wavelength, while the source fixes the frequency. We must therefore use the same value of f on both sides of the interface.

Step 2: Calculate and convert the wavelength Using the wave relation $v = f\lambda$ inside the medium gives

$$\lambda = \frac{1.5 \times 10^8}{5 \times 10^{14}} = 0.3 \times 10^{-6}\,\text{m} = 3 \times 10^{-7}\,\text{m}.$$

Since $1\,\text{nm} = 10^{-9}\,\text{m}$, this is

$$\lambda = 300\,\text{nm}.$$

As a cross-check, the air wavelength is $c/f = 600\,\text{nm}$; dividing this by the refractive index 2 gives the same 300 nm result.

Step 3: Reject wavelengths inconsistent with the index Option (C), 600 nm, is the wavelength in air and would neglect the reduction of speed. The values 400 nm and 500 nm in options (A) and (D) would correspond to refractive indices $600/400 = 1.5$ and $600/500 = 1.2$, respectively, rather than the given index 2. Only option (B) satisfies both the fixed frequency and the stated refractive index.

Step 4: Conclusion The wavelength of the refracted light is $300\,\text{nm}$. Hence, the correct option is (B).

Quick Tip to Solve: Across a stationary interface keep frequency fixed, then divide the air wavelength by the refractive index: $\lambda_{\text{medium}} = \lambda_{\text{air}}/n$.

39.

Match the LIST-I with LIST-II

LIST-I		LIST-II	
A.	**Boltzmann constant**	**I**	$\text{ML}^2\,\text{T}^{-1}$
B	**Coefficient of viscosity**	**II**	$\text{MLT}^{-3}\,\text{K}^{-1}$
C	**Planck's constant**	**III**	$\text{ML}^2\,\text{T}^{-2}\,\text{K}^{-1}$
D	**Thermal conductivity**	**IV**	$\text{ML}^{-1}\,\text{T}^{-1}$

Choose the correct answer from the options given below: [4 marks]

(A) A - III, B - IV, C - I, D - II

(B) A - III, B - IV, C - II, D - I

(C) A - III, B - II, C - I, D - IV

(D) A - II, B - III, C - IV, D - I

Correct Answer: (A) A - III, B - IV, C - I, D - II

Solution:

Concept:

- Dimensions follow from defining equations. Energy has dimensions $[E] = \mathrm{ML^2T^{-2}}$, force has $[F] = \mathrm{MLT^{-2}}$, and temperature is denoted by K in the lists.
- Use $E = k_{\mathrm{B}}T$, $E = h\nu$, the viscous-force law $F = \eta A\, dv/dy$, and the heat-conduction law $\dot{Q} = \kappa A\, \Delta T/\ell$.

Step 1: Match the two energy-related constants Boltzmann constant is energy per unit temperature. Consequently,

$$[k_{\mathrm{B}}] = \frac{[E]}{[T_{\text{temperature}}]} = \mathrm{ML^2T^{-2}K^{-1}}.$$

This matches entry III, so A maps to III. Planck constant is energy divided by frequency, where $[\nu] = \mathrm{T^{-1}}$. Thus

$$[h] = \frac{\mathrm{ML^2T^{-2}}}{\mathrm{T^{-1}}} = \mathrm{ML^2T^{-1}}.$$

This is entry I, giving C maps to I.

Step 2: Match viscosity and thermal conductivity For dynamic viscosity, $[dv/dy] = \mathrm{T^{-1}}$. Therefore the viscous-force law gives

$$[\eta] = \frac{[F]}{[A][dv/dy]} = \frac{\mathrm{MLT^{-2}}}{\mathrm{L^2T^{-1}}} = \mathrm{ML^{-1}T^{-1}}.$$

Hence B maps to IV. Heat flow per unit time has dimensions of power, $\mathrm{ML^2T^{-3}}$. Fourier's conduction law then gives

$$[\kappa] = \frac{[\dot{Q}][\ell]}{[A][\Delta T]} = \frac{\mathrm{ML^2T^{-3}\,L}}{\mathrm{L^2K}} = \mathrm{MLT^{-3}K^{-1}}.$$

This is entry II, so D maps to II.

Step 3: Select the complete correspondence The unique complete matching is A–III, B–IV, C–I, D–II. Option (B) interchanges the dimensions of Planck constant and thermal conductivity. Option (C) interchanges viscosity and thermal conductivity, while option (D) mismatches all four quantities. In particular, the temperature dimension must appear in Boltzmann constant and thermal conductivity but not in Planck constant or viscosity.

Step 4: Conclusion The matching is A–III, B–IV, C–I, D–II. Hence, the correct option is (A).

Quick Tip to Solve: Obtain each constant from its defining equation rather than memorising all four dimensions; the temperature dimension immediately separates the thermal quantities.

40. Pressure of an ideal gas, contained in a closed vessel, is increased by 0.4% when heated by $1^\circ\mathrm{C}$. Its initial temperature must be: [4 marks]

(A) 2500 K

(B) $25^\circ\mathrm{C}$

(C) $250^\circ\mathrm{C}$

(D) 250 K

Correct Answer: (D) 250 K

Solution:

Concept:

- For a fixed quantity of ideal gas in a rigid closed vessel, $PV = NTk_{\mathrm{B}}$. Therefore pressure is proportional to absolute temperature: $P/T = Nk_{\mathrm{B}}/V$.
- At constant volume, the exact finite-change relation is $\Delta P/P = \Delta T/T$, where P and T are the initial values and T is in kelvin.

Step 1: Translate the increase into a fractional change The gas is enclosed in the vessel, so its amount is fixed; the vessel volume is constant. Heating raises its temperature and therefore its pressure. The pressure increase is 0.4 percent of the initial pressure, giving

$$\frac{\Delta P}{P} = \frac{0.4}{100} = 0.004.$$

A temperature increase of $1^\circ\mathrm{C}$ equals an increase of $1\,\mathrm{K}$. This equality applies to temperature differences; it does not mean a Celsius temperature itself equals the kelvin temperature.

Step 2: Solve for the initial absolute temperature Write the initial and final gas equations and subtract them at unchanged volume:

$$P = \frac{Nk_{\mathrm{B}}}{V}T, \qquad P + \Delta P = \frac{Nk_{\mathrm{B}}}{V}(T + \Delta T).$$

Dividing the difference by the initial equation gives

$$0.004 = \frac{1\,\mathrm{K}}{T} \quad \Longrightarrow \quad T = \frac{1}{0.004}\,\mathrm{K} = 250\,\mathrm{K}.$$

Step 3: Check the units and alternatives At 250 K the increase from 250 K to 251 K produces a pressure ratio $251/250 = 1.004$, exactly the required increase. Option (A), 2500 K, gives only 0.04 percent. Option (B), $25^\circ\mathrm{C}$, is about 298 K and gives about 0.336 percent; option (C), $250^\circ\mathrm{C}$, is about 523 K and gives about 0.191 percent. The required initial value is an absolute temperature, so 250 K cannot be replaced by 250 degrees Celsius.

Step 4: Conclusion The initial temperature is $250\,\mathrm{K}$, approximately $-23^\circ\mathrm{C}$. Hence, the correct option is (D).

Quick Tip to Solve: For constant-volume heating, divide the kelvin temperature rise by the fractional pressure rise. Convert a percentage to a fraction before substituting.

41. A solid steel ball of diameter 3.6 mm acquired terminal velocity $2.45 \times 10^{-2}\ \mathrm{m/s}$ while falling under gravity through an oil of density $925\ \mathrm{kg\ m^{-3}}$. Take density of steel as $7825\ \mathrm{kg\ m^{-3}}$ and g as $9.8\ \mathrm{m/s^2}$. The viscosity of the oil in SI unit is [4 marks]

(A) 2.18

(B) 1.68

(C) 2.38

(D) 1.99

Correct Answer: (D) 1.99

Solution:

Concept:

- At terminal speed the net force on a falling sphere is zero: its weight equals buoyancy plus the viscous drag.
- For slow viscous flow, Stokes law gives $F_{\text{drag}} = 6\pi\eta r v_t$. Combining it with buoyancy gives $v_t = 2r^2(\rho_s - \rho_o)g/(9\eta)$.

Step 1: Convert the radius and find the density contrast The stated 3.6 mm is the diameter. The radius needed in Stokes law is

$$r = \frac{3.6}{2}\,\mathrm{mm} = 1.8 \times 10^{-3}\,\mathrm{m}.$$

The density contrast between steel and oil is

$$\rho_s - \rho_o = 7825 - 925 = 6900\,\mathrm{kg\,m^{-3}}.$$

The upward buoyant force must be included because the sphere displaces oil; using the steel density alone would overestimate the effective downward force.

Step 2: Balance forces at terminal speed The sphere has volume $4\pi r^3/3$. Since acceleration is zero at terminal speed, write

$$\frac{4\pi r^3}{3}\rho_s g = \frac{4\pi r^3}{3}\rho_o g + 6\pi\eta r v_t.$$

Rearranging and cancelling the common geometric factors gives

$$\eta = \frac{2r^2(\rho_s - \rho_o)g}{9v_t}.$$

Here $v_t = 2.45 \times 10^{-2}\,\mathrm{m\,s^{-1}}$ and $g = 9.8\,\mathrm{m\,s^{-2}}$.

Step 3: Substitute and compare with the options Using the SI values throughout,

$$\eta = \frac{2(1.8 \times 10^{-3})^2(6900)(9.8)}{9(2.45 \times 10^{-2})} = 1.9872\,\mathrm{Pa\,s}.$$

Thus the viscosity rounded to two decimal places is $1.99\,\mathrm{Pa\,s}$. Dimensional checking gives $\mathrm{kg\,m^{-1}\,s^{-1}}$, equivalent to $\mathrm{Pa\,s}$, as required for dynamic viscosity. Options (A), (B) and (C), namely 2.18, 1.68 and 2.38, do not equal the value obtained from this force balance; only option (D) does.

Step 4: Conclusion The viscosity of the oil is $1.99\,\mathrm{Pa\,s}$ ($\mathrm{kg\,m^{-1}\,s^{-1}}$). Hence, the correct option is (D).

Quick Tip to Solve: Use half the stated diameter in Stokes law and use the density difference, not the sphere density alone; both corrections are essential in terminal-speed problems.

42. A particle is projected with velocity u so that its horizontal range is three times the maximum height attained by it. The horizontal range of the projectile is given as $\frac{nu^2}{25g}$, where value of n is: (Given, ' g ' is the acceleration due to gravity.) [4 marks]

(A) 6

(B) 12

(C) 18

(D) 24

Correct Answer: (D) 24

Solution:

Concept:

- For a projectile launched at speed u and angle θ, returning to its launch level, the range is $R = u^2 \sin 2\theta/g$ and maximum height is $H = u^2 \sin^2\theta/(2g)$.
- The ratio eliminates the launch speed and gravity: $R/H = 4\cot\theta$. For a nonzero projectile range and height, $0 < \theta < 90°$.

Step 1: Use the stated ratio to determine the angle The condition in the question is $R = 3H$, so the ratio of the standard range and height expressions is

$$\frac{R}{H} = \frac{u^2(2\sin\theta\cos\theta)/g}{u^2\sin^2\theta/(2g)} = 4\cot\theta = 3.$$

Therefore $\cot\theta = 3/4$, or $\tan\theta = 4/3$. For the physical acute launch angle, the associated 3–4–5 triangle gives

$$\sin\theta = \frac{4}{5}, \qquad \cos\theta = \frac{3}{5}.$$

Step 2: Calculate the horizontal range Use these values to obtain the double-angle sine:

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) = \frac{24}{25}.$$

Substituting into the range formula yields

$$R = \frac{u^2}{g}\cdot\frac{24}{25} = \frac{24u^2}{25g}.$$

As a check, the maximum height is $H = (u^2/2g)(16/25) = 8u^2/(25g)$. The computed range is exactly three times this height.

Step 3: Identify the requested coefficient The question writes the same range as $nu^2/(25g)$, so the dimensionless numerator coefficient is $n = 24$. Options (A), (B) and (C), giving 6, 12 and 18, would instead make the range-to-height ratios $6/8$, $12/8$ and $18/8$ with the angle fixed by the stated condition, none of which is 3. Also u^2/g has dimensions of length, confirming the range expression is dimensionally consistent.

Step 4: Conclusion The horizontal range is $24u^2/(25g)$, giving $n = 24$. Hence, the correct option is (D).

Quick Tip to Solve: When range is specified as a multiple of maximum height, first use $R/H = 4\cot\theta$; this cancels u and g before any trigonometric calculation.

43. Given below are two statements: one is labelled as $\mathrm{Assertion A}$ and the other is labelled as Reason R

Assertion A : The Bohr model is applicable to hydrogen and hydrogen-like atoms only.

Reason R : The formulation of Bohr model does not include repulsive force between electrons.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) **A** is true but **R** is false

(B) Both **A** and **R** are true and **R** is the correct explanation of **A**

(C) **A** is false but **R** is true

(D) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

Correct Answer: (B) Both **A** and **R** are true and **R** is the correct explanation of **A**

Solution:

Concept:

- The Bohr model describes a single electron moving in the Coulomb attraction of a nucleus of charge $+Ze$. Its hydrogen-like energy levels are $E_n = -13.6Z^2/n^2\,\mathrm{eV}$.
- An atom or ion with more than one electron also has electron–electron repulsion. This interaction is absent from the elementary Bohr model and cannot be represented by the nuclear attraction alone.

Step 1: Evaluate Assertion A Hydrogen has one electron. Hydrogen-like ions, such as He^{+} and Li^{2+}, also have exactly one electron, although their nuclear charges differ. In each case the electron is subject to the central nuclear Coulomb force used in the Bohr calculation. Thus the basic model applies to hydrogen and hydrogen-like species, and Assertion A is true. Neutral helium, for example, is not hydrogen-like merely because it is a small atom: it contains two electrons.

Step 2: Evaluate Reason R The force used in the model is the attractive force $F = Ze^2/(4\pi\varepsilon_0 r^2)$. The model balances this force against the required centripetal force and imposes angular-momentum quantisation. It does not add a repulsive force arising from a second electron. Reason R is therefore true; it identifies an interaction that becomes essential for a many-electron atom.

Step 3: Connect the reason to the limitation With only one electron there is no electron pair, so no electron–electron repulsion has been omitted. With several electrons, omitting their mutual repulsion makes the simple one-electron treatment inadequate. This explains the restriction in A directly. Options (A) and (C) incorrectly make one statement false, while (D) incorrectly denies this explanatory connection.

Step 4: Conclusion Both A and R are true, and R is the correct explanation of A. Hence, the correct option is (B).

Quick Tip to Solve: Count electrons, not protons: any one-electron ion is hydrogen-like, so the elementary Bohr formula uses its nuclear charge Z.

44. An electric bulb rated as $100\ \text{W} - 220\ \text{V}$ is connected to an ac source of rms voltage 220 V. The peak value of current through the bulb is : [4 marks]

(A) 0.32 A

(B) 0.64 A

(C) 0.45 A

(D) 2.2 A

Correct Answer: (B) 0.64 A

Solution:

Concept:

- For a resistive bulb operated at its rated rms voltage, average electrical power is $P = V_{\text{rms}} I_{\text{rms}} = V_{\text{rms}}^2/R$. The rating uses rms values rather than peak values.
- For sinusoidal current, $I_{\text{rms}} = I_0/\sqrt{2}$, where I_0 is the peak amplitude. The bulb is treated as a resistive load at its operating resistance.

Step 1: Calculate the rms current The bulb is supplied at the same rms voltage as its rating, so its power is $100\,\text{W}$. Using the power relation gives

$$I_{\text{rms}} = \frac{P}{V_{\text{rms}}} = \frac{100}{220} = \frac{5}{11}\,\text{A} \approx 0.4545\,\text{A}.$$

This is the heating-equivalent current; it is not yet the requested peak current. Equivalently, the hot resistance is $R = 220^2/100 = 484\,\Omega$.

Step 2: Convert to the peak value Multiply the rms current by $\sqrt{2}$, because the mean square of a sinusoidal current is one half of its amplitude squared:

$$I_0 = \sqrt{2} I_{\text{rms}} = \frac{5\sqrt{2}}{11}\,\text{A} \approx 0.6428\,\text{A}.$$

As a check, the peak voltage is $220\sqrt{2}\,\text{V}$, and dividing it by $484\,\Omega$ gives the same current amplitude.

Step 3: Compare the options The value $0.64\,\text{A}$ matches the peak current. The option $0.45\,\text{A}$ is approximately the rms current and misses the conversion. The option $0.32\,\text{A}$ results from dividing, rather than multiplying, by $\sqrt{2}$. A current of $2.2\,\text{A}$ is inconsistent with the stated power at $220\,\text{V}$.

Step 4: Conclusion The peak current through the bulb is $I_0 \approx 0.64\,\text{A}$. Hence, the correct option is (B).

Quick Tip to Solve: Use P/V to obtain the rms current from a bulb rating; multiply by the square root of two only when the question asks for the peak current.

45. Given below are two statements: one is labelled as AssertionA and the other is labelled as Reason R

Assertion A : If Oxygen ion $\left(O^{-2}\right)$ and Hydrogen ion (H^{+})enter normal to the magnetic field with equal momentum, then the path of O^{-2} ion has a smaller curvature than that of H^{+}.

Reason R : A proton with same linear momentum as an electron will form a path of smaller radius of curvature on entering a uniform magnetic field perpendicularly.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(B) **A** is false but **R** is true

(C) **A** is true but **R** is false

(D) Both **A** and **R** are true and **R** is the correct explanation of **A**

Correct Answer: Ambiguous wording: (C) is intended for radius of curvature; literally both A and R are false.

Solution:

Concept:

- For motion perpendicular to a uniform magnetic field, $|q|vB = mv^2/r$, so the radius of curvature is $r = p/(|q|B)$. Equal momentum removes any separate dependence on mass.
- The sign of charge reverses the direction of bending, whereas its magnitude determines the radius. Geometrical curvature is $\kappa = 1/r$; it must be distinguished from the radius of curvature.

Step 1: Examine Assertion A and its terminology The charge magnitudes are $2e$ for O^{2-} and e for H^{+}. At equal momentum in the same field,

$$r_O = \frac{p}{2eB}, \qquad r_H = \frac{p}{eB}, \qquad r_O = \frac{r_H}{2}.$$

Thus oxygen has the smaller radius of curvature. The assertion uses the phrase "smaller curvature" in this intended radius-of-curvature sense. Strictly, its geometrical curvature is larger: $\kappa_O = 2\kappa_H$. The wording therefore needs this distinction.

Step 2: Evaluate Reason R A proton and an electron have equal charge magnitudes, even though their masses differ greatly. For the equal momentum specified in R,

$$r_p = \frac{p}{eB} = r_e.$$

Their circular paths bend in opposite senses because their charges have opposite signs, but the radii are equal. Reason R says the proton has a smaller radius, so it is false. Comparing masses at equal speed would be a different question.

Step 3: Identify the intended option Interpreting the assertion as a comparison of radii, A is true and R is false, which gives (C), consistent with the stated answer key. Both (A) and (D) incorrectly call R true, and (B) also calls R true. If "curvature" is read in its strict mathematical sense of inverse radius, A and R are both false and none of the offered combinations applies. The physical radii calculated above are unambiguous.

Step 4: Conclusion The intended answer is (C) if A compares radii of curvature. With geometric

curvature $\kappa = 1/r$, both A and R are false, and no listed option is correct.

Quick Tip to Solve: At equal momentum compare charge magnitudes using r=p/(|q|B). Do not compare masses, and distinguish a smaller radius from a larger inverse-radius curvature.

Physics – Section B: Numerical Value Questions (20 marks)

46. An electron in the hydrogen atom initially in the fourth excited state makes a transition to n^{th} energy state by emitting a photon of energy 2.86 eV . The integer value of n will be_____.

[4 marks]

Solution:

Concept:

- The energy of the hydrogen level with principal quantum number n is $E_n = -13.6/n^2\,\text{eV}$, measured relative to the separated electron and proton.
- An emitted photon carries the decrease in atomic energy: $E_\gamma = E_i - E_f = 13.6(1/n_f^2 - 1/n_i^2)\,\text{eV}$. The final level must be an integer below the initial level.

Step 1: Identify the initial principal quantum number The ground state is $n = 1$, the first excited state is $n = 2$, and each further excited state increases this number by one. The fourth excited state is therefore $n_i = 5$, rather than $n_i = 4$. Its energy is

$$E_i = -\frac{13.6}{25}\,\text{eV} = -0.544\,\text{eV}.$$

Emission places the electron in a more negative energy state, so the sign of the energy change is fixed.

Step 2: Apply conservation of energy The atom loses $2.86\,\text{eV}$, giving

$$E_f = E_i - E_\gamma = -0.544 - 2.86 = -3.404\,\text{eV}.$$

Equating this to the hydrogen level formula yields

$$\frac{13.6}{n^2} = 3.404, \qquad n^2 = \frac{13.6}{3.404} \approx 3.995.$$

Thus the allowed integer level is $n = 2$. The slight departure from exactly four in n^2 is due to the quoted photon energy being rounded.

Step 3: Verify the transition directly For $5 \to 2$, the photon energy is

$$13.6\left(\frac{1}{4} - \frac{1}{25}\right) = 13.6 \times 0.21 = 2.856\,\text{eV},$$

which rounds to $2.86\,\text{eV}$. This is an allowed downward transition and confirms the result. A principal quantum number carries no physical unit, so the requested entry is simply the integer.

Step 4: Conclusion The electron reaches the second energy level, so the required integer is $\boxed{n = 2}$.

Quick Tip to Solve: The kth excited state has principal quantum number k+1. For emission subtract the photon energy from the initial atomic energy before identifying the final level.

47. Light from a point source in air falls on a spherical glass surface (refractive index, $\mu = 1.5$ and radius of curvature $= 50$ cm). The image is formed at a distance of 200 cm from the glass surface inside the glass. The magnitude of distance of the light source from the glass surface is _____m. [4 marks]

Solution:

Concept:

- For refraction at one spherical surface, the Cartesian sign convention gives $\mu_2/v - \mu_1/u = (\mu_2 - \mu_1)/R$, where all distances are measured from the pole.
- Take the incident light direction, from air into glass, as positive. A real source in air has $u < 0$, while the real image inside glass has $v > 0$.

Step 1: Assign the data with their signs The incident medium is air, so $\mu_1 = 1$, while the glass has $\mu_2 = 1.5$. The image lies inside the glass at $v = +200\,\text{cm}$. The converging air-to-glass surface has its centre of curvature in the glass, giving $R = +50\,\text{cm}$. This orientation is also the one compatible with a real source in air and the stated real image. Keep every length in centimetres while applying the formula.

Step 2: Solve for the object distance Substituting the signed quantities gives

$$\frac{1.5}{200} - \frac{1}{u} = \frac{1.5 - 1}{50}.$$

Hence

$$0.0075 - \frac{1}{u} = 0.0100, \qquad -\frac{1}{u} = 0.0025\,\text{cm}^{-1},$$

and therefore

$$u = -400\,\text{cm}.$$

The negative result places the object on the incident-light side, as required for the point source in air.

Step 3: Convert the magnitude to metres The question requests the magnitude, so the negative sign is not included in the numerical response:

$$|u| = 400\,\text{cm} = 4\,\text{m}.$$

A check in metres gives $1.5/2 - 1/(-4) = 0.75 + 0.25 = 1\,\text{m}^{-1}$, equal to $(1.5 - 1)/0.5$. The formula applies to a single refracting surface, not a thin lens with two surfaces.

Step 4: Conclusion The magnitude of the source distance from the glass surface is $\boxed{4\,\text{m}}$. The required numerical entry is 4.

Quick Tip to Solve: Apply the spherical-surface refraction formula with signed distances, then take the magnitude only at the end; convert centimetres to metres after solving.

48. Two cells of emfs 1 V and 2 V and internal resistances 2Ω and 1Ω, respectively, are connected in series with an external resistance of 6Ω. The total current in the circuit is I_1. Now the same two cells in parallel configuration are connected to same external resistance. In this case, the total current drawn is I_2. The value of $\left(\frac{I_1}{I_2}\right)$ is $\frac{x}{3}$. The value of x is_____. [4 marks]

Solution:

Concept:

- For cells connected in series aiding, their emfs and internal resistances add: $E_s = E_1 + E_2$, $r_s = r_1 + r_2$, and load current is $I = E_s/(R + r_s)$.
- For like-polarity cells in parallel, $E_p = (E_1/r_1 + E_2/r_2)/(1/r_1 + 1/r_2)$ and $r_p = r_1 r_2/(r_1 + r_2)$. Unequal emfs must not be averaged without their conductance weights.

Step 1: Calculate the series current With $E_1 = 1\,\text{V}$, $E_2 = 2\,\text{V}$, $r_1 = 2\,\Omega$, $r_2 = 1\,\Omega$, and $R = 6\,\Omega$, the series-aiding arrangement has

$$E_s = 3\,\text{V}, \qquad r_s = 3\,\Omega.$$

Thus the current through the external resistor is

$$I_1 = \frac{3}{6+3} = \frac{1}{3}\,\text{A}.$$

The internal resistances remain part of the circuit; using only the external resistance would overestimate this current.

Step 2: Reduce the parallel combination The equivalent internal resistance and emf are

$$r_p = \frac{2 \times 1}{2+1} = \frac{2}{3}\,\Omega,$$

$$E_p = \frac{1/2 + 2/1}{1/2 + 1/1} = \frac{5}{3}\,\text{V}.$$

Therefore the current delivered to the same load is

$$I_2 = \frac{5/3}{6 + 2/3} = \frac{5/3}{20/3} = \frac{1}{4}\,\text{A}.$$

This is the total external current, rather than either individual cell current; unequal-emf cells can have a circulating component internally.

Step 3: Form the requested ratio Using the two load currents gives

$$\frac{I_1}{I_2} = \frac{1/3}{1/4} = \frac{4}{3}.$$

Comparing with the specified expression $x/3$ immediately gives $x = 4$. As a circuit check, the parallel load voltage is $I_2 R = 1.5\,\text{V}$; the higher-emf cell supplies current while the lower-emf cell receives some current, leaving the calculated net load current.

Step 4: Conclusion The ratio of total external currents is $I_1/I_2 = 4/3$, so the required dimensionless value is $\boxed{x = 4}$.

Quick Tip to Solve: For unequal cells in parallel use the conductance-weighted emf and parallel internal resistance, then calculate the external-load current before taking a ratio.

49. The excess pressure inside a soap bubble A in air is half the excess pressure inside another soap bubble B in air. If the volume of the bubble A is n times the volume of the bubble B, then, the value of n is_____. [4 marks]

Solution:

Concept:

- A soap bubble in air has two liquid surfaces. Its excess pressure is $\Delta P = 4T/R$, where T is the surface tension and R is its radius.
- For two bubbles of the same soap solution, surface tension cancels in the pressure ratio. Their spherical volumes satisfy $V = 4\pi R^3/3$.

Step 1: Write the pressure ratio Let the radii of bubbles A and B be R_A and R_B. The given statement concerns excess pressure, namely the pressure inside minus atmospheric pressure, and gives $\Delta P_A = \Delta P_B/2$. Apply the soap-bubble formula to both bubbles:

$$\frac{\Delta P_A}{\Delta P_B} = \frac{4T/R_A}{4T/R_B} = \frac{R_B}{R_A} = \frac{1}{2}.$$

The common factor $4T$ cancels, so no numerical surface tension or atmospheric pressure is required.

Step 2: Find the radius and volume ratios Rearranging the ratio gives $R_A = 2R_B$. This direction is physically sensible: the larger bubble has the smaller curvature and hence the smaller excess pressure. Now use the cube dependence of spherical volume:

$$n = \frac{V_A}{V_B} = \frac{(4\pi/3)R_A^3}{(4\pi/3)R_B^3} = \left(\frac{R_A}{R_B}\right)^3 = 2^3 = 8.$$

A pressure ratio is inversely a radius ratio, rather than directly a volume ratio; cubing is essential here.

Step 3: Conclusion Bubble A has twice the radius and eight times the volume of bubble B. Therefore, the required dimensionless value is $\boxed{n = 8}$.

Quick Tip to Solve: For equal-surface-tension soap bubbles, first invert the excess-pressure ratio to obtain the radius ratio, then cube it for the volume ratio.

50. A physical quantity C is related to four other quantities $\mathrm{p}, \mathrm{q}, \mathrm{r}$ and s as follows

$$C = \frac{pq^2}{r^3\sqrt{s}}$$

The percentage errors in the measurement of $\mathrm{p}, \mathrm{q}, \mathrm{r}$ and s are $1\%, 2\%, 3\%$ and 2%, respectively. The percentage error in the measurement of C will be_____% [4 marks]

Solution:

Concept:

- For a product of powers, the maximum fractional error is the sum of the absolute exponent multiplied by each fractional error.

- A denominator contributes positively to the maximum error. In $C = pq^2r^{-3}s^{-1/2}$, the error weights are $1, 2, 3, 1/2$, respectively.

Step 1: Express the error rule for this quantity Rewrite the square root and denominator as powers: $C = pq^2r^{-3}s^{-1/2}$. Logarithmic differentiation gives the signed small relative change

$$\frac{dC}{C} = \frac{dp}{p} + 2\frac{dq}{q} - 3\frac{dr}{r} - \frac{1}{2}\frac{ds}{s}.$$

Measurement errors may act in whichever direction increases the error in the result. Thus the maximum uncertainty uses the sum of magnitudes, not a subtraction of denominator errors.

Step 2: Substitute all four percentage errors Writing each fractional error as a percentage gives

$$\frac{\Delta C}{C}\,100 = 1(1) + 2(2) + 3(3) + \frac{1}{2}(2).$$

The contributions from p, q, r, s are therefore $1\%, 4\%, 9\%, 1\%$. Their sum is

$$1 + 4 + 9 + 1 = 15\%.$$

The square on q doubles its percentage error, the cube on r triples its error and the square root of s halves its error. A root-sum-square rule would address a different statistical uncertainty model; the standard requested percentage error here is the maximum propagated error.

Step 3: Conclusion The percentage error in C is $\boxed{15\%}$, so the numerical value to fill in the blank is 15.

Quick Tip to Solve: Write every factor as a power first. Multiply each percentage error by the magnitude of its exponent, including one-half for a square root, and add.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Given below are two statements:

Statement I : Wet cotton clothes made of cellulose based carbohydrate takes comparatively longer time to get dried than wet nylon polymer based clothes.

Statement II : Intermolecular hydrogen bonding with water molecule is more in nylon-based clothes than in the case of cotton clothes.

In the light of above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are false

(B) Both Statement I and Statement II are true

(C) Statement I is false but Statement II is true

(D) Statement I is true but Statement II is false

Correct Answer: (D) Statement I is true but Statement II is false

Solution:

Concept:

- Cotton consists mainly of cellulose, a carbohydrate polymer whose glucose units carry several hydroxyl groups capable of hydrogen bonding with water.
- Nylon contains polar amide groups and can hydrogen bond, but cellulose-based cotton retains more water in the comparison made here. Stronger water retention delays drying under comparable conditions.

Step 1: Evaluate Statement I Cellulose has many accessible hydroxyl groups along its polymer chains. These groups can donate and accept hydrogen bonds with water, so cotton absorbs and retains moisture. Removing this retained water during drying requires breaking its interactions with the fibre. Compared under similar drying conditions, cotton therefore takes longer to dry than nylon. The observation asserted in Statement I is true; it does not mean nylon has no affinity for water.

Step 2: Evaluate Statement II The amide links in nylon also participate in hydrogen bonding, including interactions with water. However, the statement says that intermolecular hydrogen bonding with water is greater for nylon than for cotton. That reverses the comparison responsible for the drying behaviour. Cotton cellulose has numerous hydroxyl groups and greater water retention in this comparison, so Statement II is false. One should distinguish hydrogen bonding between nylon chains from the relative binding and retention of water by the two fibres.

Step 3: Match the truth values to the options The required pair is true for Statement I and false for Statement II. Option A wrongly calls Statement I false; option B wrongly accepts Statement II; option C reverses both truth values. Only option D represents the derived combination.

Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (D).

Quick Tip to Solve: Do not infer that a polymer cannot hydrogen bond merely because it dries faster. Compare its water retention with cellulose, which carries many hydroxyl groups.

52. Fat soluble vitamins are:

A. Vitamin B_1

B. Vitamin C

C. Vitamin E

D. Vitamin B 12

E. Vitamin K

Choose the correct answer from the options given below: [4 marks]

(A) B & C Only

(B) C & D Only

(C) C & E Only

(D) A & B Only

Correct Answer: (C) C & E Only

Solution:

Concept:

- The fat-soluble vitamins are A, D, E and K. They are associated with dietary fats and can be stored in the body.
- Vitamin C and the B-group vitamins, including B_1 and B_{12}, are classified as water-soluble vitamins.

Step 1: Classify each listed vitamin Apply the solubility classification to the five entries rather than choosing by the entry letters alone. Entry A, vitamin B_1 (thiamine), belongs to the water-soluble B group. Entry B is vitamin C (ascorbic acid), which is also water soluble. Entry C is vitamin E, a fat-soluble vitamin. Entry D is vitamin B_{12} (cobalamin), another water-soluble B vitamin. Entry E is vitamin K, which is fat soluble.

Step 2: Select the fat-soluble entries The only entries satisfying the question are C and E. The list letters label the entries; for example, entry A is vitamin B_1, not vitamin A. Similarly, entry D is vitamin B_{12}, not vitamin D. This distinction matters because vitamins A and D are indeed fat soluble, but neither occurs as a vitamin in the supplied list.

Step 3: Eliminate the incorrect combinations Option A combines vitamin C with vitamin E, so it includes a water-soluble vitamin. Option B combines vitamin E with vitamin B_{12}, again including a water-soluble vitamin. Option D consists of B_1 and C, both water soluble. Option C alone selects vitamin E and vitamin K, namely entries C and E.

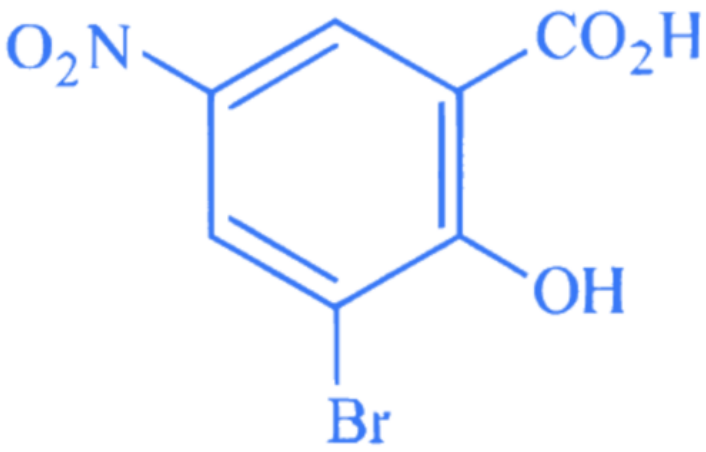
Step 4: Conclusion The fat-soluble vitamins in the list are vitamin E and vitamin K, corresponding to entries C and E only. Hence, the correct option is (C).

Quick Tip to Solve: Remember the fat-soluble set A, D, E, K, and distinguish vitamin names from the letters used to label the entries in the question.

53.

What is the correct IUPAC name of

? [4 marks]



(A) 3-Bromo-2-hydroxy-5-nitrobenzoic acid

(B) 2-Hydroxy-3-bromo-5-nitrobenzoic acid

(C) 3-Bromo-4-hydroxy-1-nitrobenzoic acid

(D) 5-Nitro-3-bromo-2-hydroxybenzoic acid

Correct Answer: (A) 3-Bromo-2-hydroxy-5-nitrobenzoic acid

Solution:

Concept:

- The carboxylic acid group has priority over the phenolic hydroxyl group, so the parent is benzoic acid and the ring carbon attached to CO_2H is carbon 1.
- Choose the direction giving the lowest set of substituent locants, then list substituent prefixes alphabetically: bromo, hydroxy, nitro.

Step 1: Identify the principal group and number the ring The structure contains a benzene ring bearing carboxylic acid, hydroxyl, bromo and nitro groups. Use benzoic acid as the parent; the carboxyl group is expressed by the parent acid name, while the hydroxyl group is expressed as the prefix hydroxy. Start ring numbering at the carbon bonded to CO_2H. Numbering toward the adjacent hydroxyl places hydroxy at 2, bromo at 3 and nitro at 5.

Step 2: Check the direction and arrange the prefixes The opposite numbering direction would place nitro at 3, bromo at 5 and hydroxy at 6, giving the sorted locant set $(3, 5, 6)$ instead of $(2, 3, 5)$. The latter wins at the first point of difference. Alphabetical prefix order is independent of numerical locant order: bromo precedes hydroxy, which precedes nitro. Combining these with the parent gives 3-bromo-2-hydroxy-5-nitrobenzoic acid.

Step 3: Compare the proposed names Option B has the correct locants but puts hydroxy before bromo. Option D also uses the correct locants but begins with nitro instead of arranging the prefixes alphabetically. Option C assigns incompatible substituent positions and even puts nitro at carbon 1, already fixed by the acid parent. Option A alone satisfies both numbering and prefix-order rules.

Step 4: Conclusion The correct IUPAC name is 3-bromo-2-hydroxy-5-nitrobenzoic acid. Hence, the correct option is (A).

Quick Tip to Solve: For substituted benzoic acids, fix the carboxyl-bearing ring carbon as 1, minimize the locant set, and only then alphabetize the prefixes.

54. Mass of magnesium required to produce 220 mL of hydrogen gas at STP on reaction with excess of dil. HCl is

Given: Molar mass of Mg is $24\ \text{g mol}^{-1}$**.** [4 marks]

(A) 0.24 mg

(B) 235.7 g

(C) 2.444 g

(D) 236 mg

Correct Answer: (D) 236 mg

Solution:

Concept:

- Magnesium reacts with dilute hydrochloric acid according to $\text{Mg} + 2\,\text{HCl} \longrightarrow \text{MgCl}_2 + \text{H}_2$. One mole of magnesium produces one mole of hydrogen.
- At STP use the standard molar gas volume $22.4\,\text{L mol}^{-1}$ in this calculation. Convert the gas volume to litres before calculating $n = V/V_m$.

Step 1: Convert the hydrogen volume into moles The acid is present in excess, so magnesium is the limiting reactant and the hydrogen yield determines the magnesium required. The stated volume is

$$220\,\text{mL} = 0.220\,\text{L}.$$

At STP the amount of hydrogen is

$$n(\text{H}_2) = \frac{0.220}{22.4} = 9.8214 \times 10^{-3}\,\text{mol}.$$

Using millilitres directly with a molar volume in litres would introduce an erroneous factor of one thousand.

Step 2: Apply the mole ratio and convert the mass The balanced equation gives $n(\text{Mg}) = n(\text{H}_2)$, not twice that amount. The coefficient 2 belongs to the acid, not to magnesium. Therefore

$$m(\text{Mg}) = 9.8214 \times 10^{-3} \times 24 = 0.235714\,\text{g}.$$

Converting grams to milligrams gives

$$m(\text{Mg}) = 235.714\,\text{mg} \simeq 236\,\text{mg}.$$

Step 3: Compare with the options Option A is about one thousand times too small because it gives

the mass in milligrams rather than approximately 0.24 grams. Option B is one thousand times too large. Option C does not follow the one-to-one mole ratio or the given gas volume. The rounded calculated mass agrees with option D.

Step 4: Conclusion The mass of magnesium needed is approximately $\boxed{236\,\text{mg}}$. Hence, the correct option is (D).

Quick Tip to Solve: Balance the reaction before using gas volume: one mole of Mg gives one mole of hydrogen. Convert 220 mL to 0.220 L, then convert the final grams to milligrams.

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55. Given below are two statements:

Statement I : When a system containing ice in equilibrium with water (liquid) is heated, heat is absorbed by the system and there is no change in the temperature of the system until whole ice gets melted.

Statement II : At melting point of ice, there is absorption of heat in order to overcome intermolecular forces of attraction within the molecules of water in ice and kinetic energy of molecules is not increased at melting point.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are false

(B) Statement I is true but Statement II is false

(C) Both Statement I and Statement II are true

(D) Statement I is false but Statement II is true

Correct Answer: (C) Both Statement I and Statement II are true

Solution:

Concept:

- For a pure substance melting at constant pressure, the heat supplied during coexistence of solid and liquid is latent heat: $Q = mL_f$, where m melts and L_f is the specific latent heat of fusion.
- Temperature measures the average kinetic energy of molecular motion. During melting the absorbed energy primarily increases intermolecular potential energy by disrupting the solid structure, rather than increasing temperature.

Step 1: Evaluate Statement I An ice-water mixture at its melting point contains both phases in equilibrium. On gentle heating at constant pressure, some ice changes into liquid water. The supplied energy is used for fusion, so the temperature remains at the melting point while both phases remain present. Thus heat can enter a system without producing a temperature rise. Only after the ice has completely melted does further heating raise the temperature of the liquid. Statement I is therefore true.

Step 2: Evaluate Statement II Ice has an ordered hydrogen-bonded arrangement. Melting loosens this intermolecular network and permits the molecules greater freedom of movement. The energy supplied over the melting interval increases the intermolecular potential energy; it does not raise the average kinetic energy because the temperature is constant. The attraction mentioned in the

statement is between water molecules: melting does not break the covalent O–H bonds within a water molecule. Statement II describes the latent-heat explanation and is true.

Step 3: Compare the offered combinations Both statements agree with the constant-temperature melting plateau. Option (A) incorrectly rejects both, option (B) incorrectly rejects the latent-heat explanation, and option (D) incorrectly rejects the observed temperature plateau. The combination declaring both statements true is the only consistent choice.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (C).

Quick Tip to Solve: During a phase change ask where the heat goes: it changes intermolecular potential energy, so a temperature rise is not required.

56. The correct orders among the following are

Atomic radius : $B < Al < Ga < In < Tl$

Electronegativity : $Al < Ga < In < Tl < B$

Density : $Tl < In < Ga < Al < B$

1st Ionisation Energy : $In < Al < Ga < Tl < B$

Choose the correct answer from the options given below: [4 marks]

(A) B and D Only

(B) A and B Only

(C) C and D Only

(D) A and C Only

Correct Answer: (A) B and D Only

Solution:

Concept:

- Group 13 trends are affected by incomplete shielding by filled d and f subshells. Atomic size and ionisation energy therefore do not follow a completely regular down-group trend.
- The electronegativity trend used for this group is a decrease from boron to aluminium followed by an increase towards thallium. Density generally increases down the group, rather than following atomic radius.

Step 1: Test the atomic-radius and electronegativity orders The first proposed order is incorrect. Gallium is slightly smaller than aluminium because the intervening 3d electrons shield the nuclear charge poorly; its outer electrons experience increased effective attraction. Thus the proposed Al < Ga step is reversed. Boron remains the smallest atom and indium and thallium are larger, but a single incorrect comparison invalidates the entire order. The second order is the textbook group-13 electronegativity trend: Al < Ga < In < Tl < B. Aluminium is the least electronegative member and boron the most; the rise after aluminium is associated with poor shielding and increasing effective nuclear attraction. This second order is correct in the trend convention tested here.

Step 2: Test the density and ionisation-energy orders The proposed density order places thallium at the low-density end and boron at the high-density end. This is reversed: density increases broadly

from B through Al, Ga and In to Tl. Hence the third order is false. The first ionisation energies, approximately 558, 578, 579, 589 and 801 kJ mol^{-1} for In, Al, Ga, Tl and B respectively, give the fourth printed order. The slight Ga-over-Al reversal reflects d-electron shielding, while the relatively high value for Tl includes contraction and poor shielding effects. The fourth order is true.

Step 3: Select the matching pair The four proposed orders, read from top to bottom as A, B, C and D, have truth values false, true, false and true. Therefore B and D alone survive. Every other offered pair includes either the incorrect atomic-radius order or the reversed density order.

Step 4: Conclusion Only the electronegativity order B and first-ionisation-energy order D are correct. Hence, the correct option is (A).

Quick Tip to Solve: For group 13, check the Al–Ga radius reversal first, and remember that the printed density order must put Tl near the high-density end.

57. 10 mL of 2 M NaOH solution is added to 20 mL of 1 M HCl solution kept in a beaker. Now, 10 mL of this mixture is poured into a volumetric flask of 100 mL containing 2 moles of HCl and made the volume upto the mark with distilled water. The solution in this flask is : [4 marks]

(A) 0.2 M NaCl solution

(B) 20 M HCl solution

(C) Neutral solution

(D) 10 M HCl solution

Correct Answer: (B) 20 M HCl solution

Solution:

Concept:

- The neutralisation reaction is $\mathrm{NaOH} + \mathrm{HCl} \longrightarrow \mathrm{NaCl} + \mathrm{H_2O}$. A mole of sodium hydroxide consumes one mole of hydrochloric acid.
- Molarity is $M = n/V$, with volume V in litres. When taking a homogeneous aliquot, every dissolved solute is transferred in the same volume fraction; subsequent dilution changes concentration, not moles.

Step 1: Neutralise the solutions in the beaker Calculate the initial amounts using the given volumes and molarities:

$$n(\mathrm{NaOH}) = 2(0.010) = 0.020 \text{ mol}, \qquad n(\mathrm{HCl}) = 1(0.020) = 0.020 \text{ mol}.$$

The reactants are present in exactly the 1:1 stoichiometric ratio. Both are consumed and 0.020 mol of sodium chloride forms in the 30 mL mixture. Thus the beaker mixture is neutral before the later acid is introduced; it contains no excess hydroxide that could consume the acid placed in the volumetric flask.

Step 2: Track the aliquot and the added acid The 10 mL aliquot is one third of the 30 mL beaker mixture. It transfers $0.020/3 = 0.006667$ mol of sodium chloride and no excess acid or base. The volumetric flask already contains 2 mol of hydrochloric acid. Adding the neutral aliquot does not

change these 2 mol. After making the solution up to the 100 mL mark, the final acid molarity is

$$M(\mathrm{HCl}) = \frac{2}{0.100} = 20 \text{ mol L}^{-1}.$$

The small amount of sodium chloride remains dissolved alongside the acid.

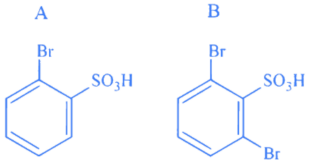
Step 3: Eliminate the distractors The solution cannot be neutral because 2 mol of strong acid have been added after the original neutralisation. Its salt concentration is only $0.006667/0.100 = 0.06667$ M, not 0.2 M, so option (A) is false. Option (D) would require 1 mol of acid in 100 mL, whereas the flask contains 2 mol. Only the 20 M acid description matches the mole balance.

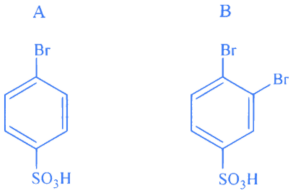
Step 4: Conclusion The final flask contains 20 M hydrochloric acid, together with dissolved sodium chloride. Hence, the correct option is (B).

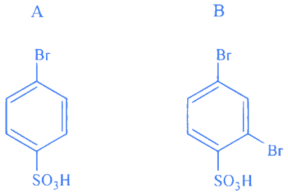
Quick Tip to Solve: Finish the beaker neutralisation before taking the aliquot; then divide the unchanged acid moles in the flask by its final 0.100 L volume.

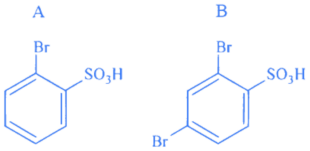
58. In the following series of reactions identify the major products A and B respectively. [4 marks]



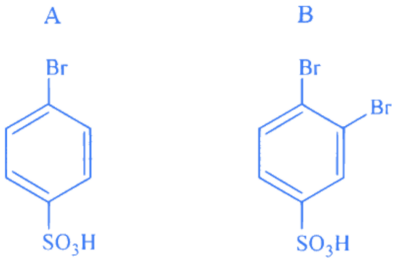
(A) 

(B) 

(C) 

(D) 

Correct Answer: (B)



Solution:

Concept:

- Bromine is deactivating by its inductive effect but ortho/para directing through lone-pair resonance. Sulfonation with SO_3/H_2SO_4 therefore favours substitution ortho or para to the bromine.
- The sulfonic acid group, $-SO_3H$, is strongly deactivating and meta directing. In a disubstituted ring, an incoming electrophile preferentially enters a position favoured by both existing directing effects.

Step 1: Identify the major sulfonation product A The starting compound is bromobenzene. Number the carbon bearing its original bromine as C1. Electrophilic sulfonation can place the sulfonic acid group at C2 or C4 because bromine directs to the ortho and para positions. The para product avoids the greater steric crowding associated with placing two bulky groups adjacent to one another. Hence the major product A has bromine at C1 and the sulfonic acid group at C4: it is p-bromobenzenesulfonic acid, also called 4-bromobenzenesulfonic acid. The first and fourth offered pairs show the ortho sulfonation product instead, so they cannot be correct.

Step 2: Locate the second bromine in product B For the para-disubstituted intermediate, the original bromine favours C2 and C6 for the next substitution; its para position is already occupied. The sulfonic acid group at C4 directs meta, again to C2 and C6. These two positions are equivalent by symmetry. Bromination with Br_2/Fe therefore introduces the new bromine adjacent to the original bromine and meta to the sulfonic acid group. Keeping the original bromobenzene numbering, the product has Br at C1 and C2 and the sulfonic acid group at C4. Numbering from the sulfonic acid carbon instead gives 3,4-dibromobenzenesulfonic acid.

Step 3: Match the structures to the options The second pair of structures shows para sulfonation followed by bromination next to the original bromine, precisely the mutually favoured site. The third pair starts with the correct para intermediate but places the next bromine ortho to the sulfonic acid group and meta to the original bromine, contradicting both directing effects. The first and fourth pairs already have the wrong major intermediate A.

Step 4: Conclusion A is 4-bromobenzenesulfonic acid and B is 3,4-dibromobenzenesulfonic acid, as shown in the second pair. Hence, the correct option is (B).

Quick Tip to Solve: After finding the para sulfonation product, mark the sites ortho to Br and meta to the sulfonic acid group; their overlap gives the second bromination site.

59. Identify the diamagnetic octahedral complex ions from below ;

A. $[Mn(CN)_6]^{3-}$

B. $[Co\,(NH_3)_6]^{3+}$

C. $[Fe(CN)_6]^{4-}$

D. $[Co\,(H_2O)_3\ F_3]$

Choose the correct answer from the options given below: [4 marks]

(A) B and C Only

(B) A and C Only

(C) B and D Only

(D) A and D Only

Correct Answer: (A) B and C Only

Solution:

Concept:

- A diamagnetic complex has no unpaired electrons. In an octahedral field the d orbitals split into lower t_{2g} and upper e_g sets; strong-field ligands can favour pairing in the lower set.
- Find the metal oxidation state before counting d electrons. Cyanide is strong field; fluoride and water are weak-field ligands. The cobalt(III) hexaammine complex is low spin because its octahedral splitting is sufficiently large.

Step 1: Count the electrons in complexes A and B In $[Mn(CN)_6]^{3-}$, six cyanide ligands contribute charge -6, so manganese is +3 and has a d^4 configuration. Even in the low-spin case the occupancy is $t_{2g}^4 e_g^0$, with two unpaired electrons; four electrons cannot fill three lower orbitals in pairs without first occupying all three. Complex A is therefore paramagnetic. In $[Co(NH_3)_6]^{3+}$, ammonia is neutral, so cobalt is +3 and is d^6. The low-spin octahedral occupancy is $t_{2g}^6 e_g^0$, in which all six electrons are paired. Complex B is diamagnetic.

Step 2: Count the electrons in complexes C and D For $[Fe(CN)_6]^{4-}$, the oxidation-state balance gives iron +2 and hence d^6. Strong-field cyanide produces the low-spin configuration $t_{2g}^6 e_g^0$; there are no unpaired electrons, so C is diamagnetic. In neutral $[Co(H_2O)_3F_3]$, the three fluoride ligands supply charge -3 and water is neutral. Cobalt is again +3, but the weak-field ligand environment gives the high-spin d^6 arrangement $t_{2g}^4 e_g^2$, with four unpaired electrons. D is paramagnetic.

Step 3: Choose the completely paired pair Only B and C have zero unpaired electrons. Option (B) includes the paramagnetic manganese complex A, option (C) includes the weak-field cobalt complex D, and option (D) includes both of those paramagnetic species. Thus none of those pairs is diamagnetic throughout.

Step 4: Conclusion Complexes B and C alone are diamagnetic. Hence, the correct option is (A).

Quick Tip to Solve: For each complex write oxidation state, d count, and then spin state; a strong-field d4 ion still has unpaired electrons, whereas low-spin d6 is fully paired.

60. Given below are two statements:

Statement I : CrO_3 **is a stronger oxidizing agent than** MoO_3

Statement II : $Cr(VI)$ **is more stable than** $Mo(VI)$

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Statement I is true but Statement II is false

(B) Both Statement I and Statement II are true

(C) Both Statement I and Statement II are false

(D) Statement I is false but Statement II is true

Correct Answer: (A) Statement I is true but Statement II is false

Solution:

Concept:

- An oxidising agent accepts electrons and is itself reduced. A high oxidation state that is less stable relative to a lower state generally gives a greater tendency towards reduction.
- In the heavier transition metals, high oxidation states are often stabilised more effectively than in the corresponding first-row element. Group-6 molybdenum(VI) is more stable than chromium(VI).

Step 1: Compare the oxidation states in the trioxides In both CrO_3 and MoO_3, each oxygen contributes an oxidation state of -2, so the metal has oxidation state +6. Chromium readily attains the comparatively stable +3 state after accepting electrons. Consequently chromium(VI) compounds can act as strong oxidising agents. Molybdenum(VI), by contrast, is more strongly stabilised in its oxide chemistry and has a smaller tendency to undergo the corresponding reduction. One must compare the stability of the oxidation states, rather than merely noting that both trioxides contain the same number of oxygen atoms.

Step 2: Evaluate the two statements separately Statement I says that chromium trioxide is the stronger oxidising agent. This agrees with the greater reduction tendency of chromium(VI), so Statement I is true. Statement II says chromium(VI) is more stable than molybdenum(VI). The relevant down-group trend is the opposite: the high +6 state is more stable for molybdenum than for chromium. Statement II is therefore false. Its proposed stability comparison would also run counter to the explanation for chromium(VI) being reduced more readily.

Step 3: Eliminate the remaining truth combinations Option (B) incorrectly accepts the reversed stability comparison. Option (C) incorrectly rejects the greater oxidising strength of chromium trioxide, and option (D) reverses both conclusions. The only option consistent with both the redox tendency and the oxidation-state stability trend is the one with Statement I true and Statement II false.

Step 4: Conclusion Statement I is true, but Statement II is false: Mo(VI) is more stable than Cr(VI). Hence, the correct option is (A).

Quick Tip to Solve: A less stable high oxidation state is usually reduced more readily; do not equate stronger oxidising power with greater stability of that high state.

61. Consider the following statements related to temperature dependence of rate constants. Identify the correct statements.

A. The Arrhenius equation holds true only for an elementary homogenous reaction.

B. The unit of A is same as that of k in Arrhenius equation.

C. At a given temperature, a low activation energy means a fast reaction.

D. A and Ea as used in Arrhenius equation depend on temperature.

E. When $\mathrm{Ea} \gg \mathrm{RT}, \mathrm{A}$ and Ea become interdependent.

Choose the correct answer from the options given below: [4 marks]

(A) A, C and D Only

(B) B, D and E Only

(C) B and C Only

(D) A and B Only

Correct Answer: (C) B and C Only

Solution:

Concept:

- The Arrhenius relation is $k = A\exp[-E_a/(RT)]$, where k is the rate constant, A the pre-exponential factor, E_a the activation energy, R the gas constant and T the absolute temperature.
- In the usual Arrhenius model over the temperature range considered, A and E_a are treated as independent, approximately temperature-independent parameters. The exponential is dimensionless.

Step 1: Test the scope and units Statement A is false: the Arrhenius relation is an empirical description of the temperature dependence of rate constants and is not restricted only to elementary homogeneous reactions. Many overall reactions also show Arrhenius behaviour over a suitable temperature interval. Since $E_a/(RT)$ is dimensionless, multiplication by its exponential cannot change units. Therefore $[A] = [k]$, making statement B true; these units depend on reaction order.

Step 2: Compare activation barriers For comparison at a fixed temperature with comparable frequency factors, lowering E_a increases the fraction of effective collisions. Indeed, if $E_{a,1} < E_{a,2}$ and the same A applies, then $k_1/k_2 = \exp[(E_{a,2} - E_{a,1})/(RT)] > 1$. This is the usual meaning of statement C, so it is true. The barrier must not be confused with the overall enthalpy change.

Step 3: Reject the remaining assertions Statement D is false in the Arrhenius model used here: temperature variation is represented by the exponential factor, with A and E_a treated as constants over the range. Statement E is also false: $E_a \gg RT$ merely makes the exponential small; it does not impose any relation making A and E_a interdependent. Option A contains false A and D, option B contains false D and E, and option D contains false A.

Step 4: Conclusion Only statements B and C are correct. Hence, the correct option is (C).

Quick Tip to Solve: In Arrhenius questions, first use the dimensionless exponential to check units, then vary the barrier while holding temperature and the comparison factor fixed.

62. Given below are two statements :

Statement I : Hyperconjugation is not a permanent effect.

Statement II : In general, greater the number of alkyl groups attached to a positively charged Cation, greater is the hyperconjugation interaction and stabilization of the cation.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Statement I is false but Statement II is true

(B) Statement I is true but Statement II is false

(C) Both Statement I and Statement II are false

(D) Both Statement I and Statement II are true

Correct Answer: (A) Statement I is false but Statement II is true

Solution:

Concept:

- Hyperconjugation is the delocalisation of electrons from a suitably oriented adjacent σ bond, commonly a C–H bond, into an empty p orbital or an adjacent π system.
- An ordinary alkyl carbocation has an electron-deficient, approximately planar carbon with a vacant p orbital. Electron donation by adjacent alkyl groups spreads its positive charge and stabilises it.

Step 1: Evaluate the permanence of the effect Hyperconjugation is a ground-state electronic effect of the molecule when the required adjacent orbitals are present. It is not a temporary polarisation that occurs only during the approach of a reagent. Its no-bond resonance forms represent delocalisation; they do not mean that a hydrogen atom is permanently detached in the actual molecule. Therefore the statement that hyperconjugation is not a permanent effect is false.

Step 2: Evaluate substitution and stabilisation In the usual comparison of simple alkyl carbocations, additional alkyl groups supply more adjacent C–H bonds capable of hyperconjugative donation into the empty orbital. For example, ethyl, isopropyl and tert-butyl cations have respectively three, six and nine adjacent hydrogens for the standard comparison. More contributing interactions spread the charge more effectively, giving the familiar order tertiary greater than secondary greater than primary, with methyl least stabilised. Thus statement II is true in its stated general sense.

Step 3: Select the matching combination The required truth values are false for I and true for II. Option B reverses both judgements, option C wrongly denies the general stabilising effect of additional alkyl substitution, and option D wrongly labels the first statement as true. No rearrangement or reagent-induced temporary effect needs to be invoked for this comparison.

Step 4: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish hyperconjugation, a permanent delocalisation effect, from a temporary electromeric shift; count adjacent donating bonds when comparing simple carbocations.

63. The sequence from the following that would result in giving predominantly $3,4,5-$ Tribromoaniline is : [4 marks]

(A) Nitrobenzene: (i) Br_2, acetic acid; (ii) Sn, HCl.

(B) Benzene: (i) Br_2, $AlBr_3$; (ii) NH_3.

(C) p-Nitroaniline: (i) excess Br_2, acetic acid; (ii) $NaNO_2$, HCl, CuBr; (iii) Sn, HCl.

(D) Aniline: Br_2, water.

Correct Answer: (C) p-Nitroaniline: (i) excess Br_2, acetic acid; (ii) $NaNO_2$, HCl, CuBr; (iii) Sn, HCl.

Solution:

Concept:

- The amino group is strongly activating and ortho/para directing in aromatic electrophilic substitution; the nitro group is strongly deactivating and meta directing.
- A primary aromatic amino group can be diazotised with $NaNO_2$/HCl, then replaced by bromine using CuBr (Sandmeyer reaction). Reduction with Sn/HCl converts NO_2 into NH_2.

Step 1: Follow bromination of para-nitroaniline In option C, label the initial amino-bearing ring carbon 1 and the nitro-bearing carbon 4. Positions 2 and 6 are ortho to NH_2 and meta to NO_2, so the directing effects favour both positions. The para position relative to the amino group is already occupied. Excess bromine therefore produces 2,6-dibromo-4-nitroaniline, with the amino and nitro groups still present.

Step 2: Replace the original amino group The second stage converts the original NH_2 group at carbon 1 into an aryl diazonium group, and CuBr substitutes it with bromine. The ring now bears bromine at the original positions 1, 2 and 6 and nitro at position 4. This step is essential: direct bromination alone cannot put bromine at the carbon initially occupied by the amino group.

Step 3: Reduce and renumber the ring Sn/HCl reduces the remaining nitro group to amino without changing those three ring substituent positions in the intended sequence. Renumber the product with its new amino carbon as position 1. The original positions 2, 1 and 6 are now positions 3, 4 and 5, respectively. The product is consequently 3,4,5-tribromoaniline.

Step 4: Eliminate the other sequences Option A starts from nitrobenzene; its meta-directing bromination followed by reduction does not supply the required three bromines. Option B gives electrophilic bromination of benzene followed by ammonia, without the directing and replacement sequence needed here. Option D gives the familiar 2,4,6-tribromoaniline with bromine water, a different substitution pattern from the required 3,4,5-isomer.

Step 5: Conclusion The para-nitroaniline sequence in option C yields predominantly 3,4,5-tribromoaniline. Hence, the correct option is (C).

Quick Tip to Solve: Track the original ring positions through diazotisation, then renumber only after the final nitro-to-amino reduction; the final amino group comes from the original nitro group.

64. Match LIST-I with LIST-II.

LIST-I (Family)		LIST-II (Symbol of Element)	
A	Pnictogen (group 15)	I	Ts
B	Chalcogen	II	Og
C	Halogen	III	Lv
D	Noble gas	IV	Mc

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-I, C-II, D-III

(B) A-IV, B-III, C-I, D-II

(C) A-III, B-I, C-IV, D-II

(D) A-II, B-III, C-IV, D-I

Correct Answer: (B) A-IV, B-III, C-I, D-II

Solution:

Concept:

- Pnictogens belong to group 15, chalcogens to group 16, halogens to group 17, and noble gases to group 18 of the periodic table.
- The final four named elements of period 7 are moscovium (Mc), livermorium (Lv), tennessine (Ts), and oganesson (Og), in order of atomic numbers 115 through 118.

Step 1: Identify the group-15 and group-16 elements Moscovium, atomic number 115, lies in group 15 beneath bismuth. It is therefore the pnictogen in the list, giving A-IV. Livermorium, atomic number 116, occupies group 16 beneath polonium. It is the chalcogen, giving B-III. These are family assignments from periodic position; the symbols must not be confused merely because the elements are all in the same period.

Step 2: Identify the group-17 and group-18 elements Tennessine, symbol Ts and atomic number 117, is in group 17 beneath astatine and therefore belongs to the halogen family: C-I. Oganesson, symbol Og and atomic number 118, completes period 7 in group 18 beneath radon. Its periodic-table family is the noble gases: D-II. The classification requested is by group, regardless of unusual predicted physical properties of these very heavy elements.

Step 3: Assemble and check the matching code The complete ordered match is A-IV, B-III, C-I, D-II. This is precisely option B. Option A has the pnictogen correct but exchanges the remaining family assignments. Option C puts livermorium among pnictogens and moscovium among halogens, so it fails. Option D puts oganesson in group 15 and tennessine among noble gases, and also fails. Checking both the family and the symbol prevents an accidental swap of adjacent columns.

Step 4: Conclusion The correct matching is A-IV, B-III, C-I, D-II. Hence, the correct option is (B).

Quick Tip to Solve: Write the period-7 tail in order as Mc, Lv, Ts, Og, then place the corresponding group numbers 15, 16, 17, 18 above it before matching.

65.

The major product (P) in the following reaction is :

[4 marks]



(A) $C_6H_5-C(=O)-COO^- K^+$

(B) $C_6H_5-CH(OH)-CH_2OH$

(C) $C_6H_5-CH(OH)-COO^- K^+$

(D) $C_6H_5-C(=O)-CH_2OH$

Correct Answer: (C) $C_6H_5-CH(OH)-COO^- K^+$

Solution:

Concept:

- The substrate is phenylglyoxal, $C_6H_5-CO-CHO$, an adjacent keto-aldehyde without an enolisable alpha hydrogen for an ordinary aldol reaction.
- Under strong basic conditions an intramolecular Cannizzaro process transfers hydride from the aldehyde centre to the adjacent ketone centre. The aldehyde is oxidised to carboxylate while the ketone is reduced to alcohol.

Step 1: Identify the two carbonyl roles The carbon directly attached to phenyl is a ketone carbonyl, whereas the terminal carbonyl bears a hydrogen and is an aldehyde. Hydroxide attacks the aldehyde carbonyl to generate a tetrahedral intermediate. Its aldehydic hydrogen can migrate as hydride to the neighbouring ketone carbonyl; the proximity of the two groups favours this intramolecular redox event.

Step 2: Follow the oxidation and reduction During hydride transfer, the aldehyde-derived carbon forms the carboxylic-acid oxidation level while the ketone-derived oxygen becomes an alkoxide. Proton transfer in the aqueous basic medium gives an alcohol at the phenyl-bearing carbon and a carboxylate at the terminal carbon. The strongly basic medium means the isolated form shown among the choices is the potassium carboxylate, not the free acid. The net transformation is

$$C_6H_5-CO-CHO + KOH \longrightarrow C_6H_5-CH(OH)-COOK.$$

Step 3: Compare every product choice Option A, potassium phenylglyoxylate, oxidises the aldehyde but leaves the ketone unreduced and misses the paired hydride-transfer change. Option B is a diol and would require reduction of both carbonyl groups, which potassium hydroxide does not supply. Option D reduces the aldehyde to a primary alcohol while leaving the ketone unchanged, also not the intramolecular Cannizzaro outcome. Only option C has the required secondary alcohol plus carboxylate combination. Carbon atoms remain connected throughout; no carbon-chain cleavage is required.

Step 4: Conclusion The major product is potassium mandelate, $C_6H_5-CH(OH)-COO^- K^+$. Hence,

the correct option is (C).

Quick Tip to Solve: For an adjacent keto-aldehyde under strong base, track hydride from the aldehydic carbon to the ketone carbon: the product has one alcohol and one carboxylate.

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66. The standard cell potential $\left(E^{\ominus}_{\text{cell}}\right)$ of a fuel cell based on the oxidation of methanol in air that has been used to power television relay station is measured as 1.21 V . The standard half cell reduction potential for O_2 $\left(E^{\circ}_{O_2/H_2O}\right)$ is 1.229 V .

Choose the correct statement : [4 marks]

(A) Reactants are fed at one go to each electrode.

(B) Oxygen is formed at the anode.

(C) The standard half cell reduction potential for the reduction of CO_2 $\left(E^{\circ}_{CO_2/CH_3OH}\right)$ is 19 mV

(D) Reduction of methanol takes place at the cathode.

Correct Answer: (C) The standard half cell reduction potential for the reduction of CO_2 $\left(E^{\circ}_{CO_2/CH_3OH}\right)$ is 19 mV

Solution:

Concept:

- For a galvanic cell written with standard reduction potentials, $E^{\circ}_{\text{cell}} = E^{\circ}_{\text{cathode}} - E^{\circ}_{\text{anode}}$. The anode potential in this expression is its reduction potential, even though oxidation actually occurs there.
- A fuel cell converts continuously supplied fuel and oxidant into electrical energy. Methanol is oxidised at the anode and oxygen is reduced at the cathode.

Step 1: Identify the electrode reactions The cathodic reduction is

$$O_2 + 4\,H^+ + 4\,e^- \longrightarrow 2\,H_2O.$$

Methanol oxidation at the anode can be written

$$CH_3OH + H_2O \longrightarrow CO_2 + 6\,H^+ + 6\,e^-.$$

The requested anode-couple reduction potential refers to the reverse reaction, conversion of carbon dioxide to methanol. Balancing electrons when combining these half-reactions does not multiply either electrode potential, because potential is an intensive quantity.

Step 2: Calculate the requested reduction potential Substitute the given standard cell potential and oxygen reduction potential:

$$1.210 = 1.229 - E^{\circ}_{CO_2/CH_3OH}.$$

Hence

$$E^{\circ}_{CO_2/CH_3OH} = 1.229 - 1.210 = 0.019\,\text{V} = 19\,\text{mV}.$$

The positive value belongs to the reduction convention for the carbon dioxide/methanol couple; changing to an oxidation potential would reverse its sign.

Step 3: Reject the alternative statements Option A is false because a working fuel cell receives

reactants continuously rather than charging both electrodes at one go as stated. Option B is false: oxygen is consumed by reduction at the cathode, not generated at the anode. Option D is false because methanol undergoes oxidation at the anode; oxygen is the substance reduced at the cathode. Thus option C alone correctly states both the reduction couple and its numerical standard potential.

Step 4: Conclusion The standard reduction potential of the carbon dioxide/methanol couple is $19\,\mathrm{mV}$. Hence, the correct option is (C).

Quick Tip to Solve: Always insert reduction potentials into cathode minus anode; do not change signs twice, and never multiply a potential when balancing electrons.

67. For electrons in ' 2 s ' and ' 2 p ' orbitals, the orbital angular momentum values, respectively are: [4 marks]

(A) 0 and $\sqrt{6}\frac{h}{2\pi}$
(B) 0 and $\sqrt{2}\frac{h}{2\pi}$
(C) $\sqrt{2}\frac{h}{2\pi}$ and 0
(D) $\frac{h}{2\pi}$ and $\sqrt{2}\frac{h}{2\pi}$
Correct Answer: (B) 0 and $\sqrt{2}\frac{h}{2\pi}$

Solution:

Concept:

- The magnitude of orbital angular momentum is $L = \sqrt{\ell(\ell+1)}\,\hbar$, where $\hbar = h/(2\pi)$ and ℓ is the azimuthal quantum number.
- The orbital labels s, p, d, f correspond to $\ell = 0, 1, 2, 3$, respectively. The principal quantum number n specifies the shell but does not directly replace ℓ in this formula.

Step 1: Identify the azimuthal quantum numbers Both given orbitals belong to the second shell, so $n = 2$ for each. Their orbital types are different, however: the $2s$ electron has $\ell = 0$, whereas the $2p$ electron has $\ell = 1$. Orbital angular momentum is determined by this orbital type, not just by the common shell number.

Step 2: Calculate the two magnitudes For the $2s$ electron, substitute $\ell = 0$:

$$L_{2s} = \sqrt{0(0+1)}\,\frac{h}{2\pi} = 0.$$

For the $2p$ electron, substitute $\ell = 1$:

$$L_{2p} = \sqrt{1(1+1)}\,\frac{h}{2\pi} = \sqrt{2}\,\frac{h}{2\pi}.$$

These are magnitudes, so neither is assigned a negative sign.

Step 3: Compare the options Option (A) assigns $\sqrt{6}\hbar$ to the p electron; that value belongs to $\ell = 2$, a d orbital. Option (C) reverses the two orbitals. Option (D) incorrectly assigns $\hbar$ to the s orbital. Only (B) keeps zero for s and $\sqrt{2}\hbar$ for p.

Step 4: Conclusion The orbital angular momenta of electrons in $2s$ and $2p$ are, respectively, 0 and

$\sqrt{2}\,h/(2\pi)$. Hence, the correct option is (B).

Quick Tip to Solve: Convert the orbital letter into ℓ before substituting: $s \to 0$, $p \to 1$, $d \to 2$. Do not substitute the shell number 2 for ℓ.

68. Compounds that should not be used as primary standards in titrimetric analysis are :

A. $Na_2Cr_2O_7$

B. Oxalic acid

C. NaOH

D. $FeSO_4 \cdot 6H_2O$

E. Sodium tetraborate

Choose the most appropriate answer from the options given below: [4 marks]

(A) C, D and E Only

(B) B and D Only

(C) D and E Only

(D) A, C and D Only

Correct Answer: (D) A, C and D Only

Solution:

Concept:

- A primary standard must be available in high purity, have a definite composition, remain stable during storage and weighing, and react quantitatively with a known stoichiometry.
- Hygroscopic substances, substances that absorb atmospheric gases, and compounds readily oxidised by air cannot provide a reliable concentration simply from their weighed mass.

Step 1: Exclude sodium dichromate and sodium hydroxide Compound A, $Na_2Cr_2O_7$, is hygroscopic, so its weighed mass can include a variable amount of absorbed water. Its composition during weighing is therefore unsuitable for direct preparation of a primary-standard solution. Compound C, NaOH, absorbs moisture and atmospheric CO_2; the latter forms carbonate. A weighed sample consequently need not contain the calculated amount of pure hydroxide.

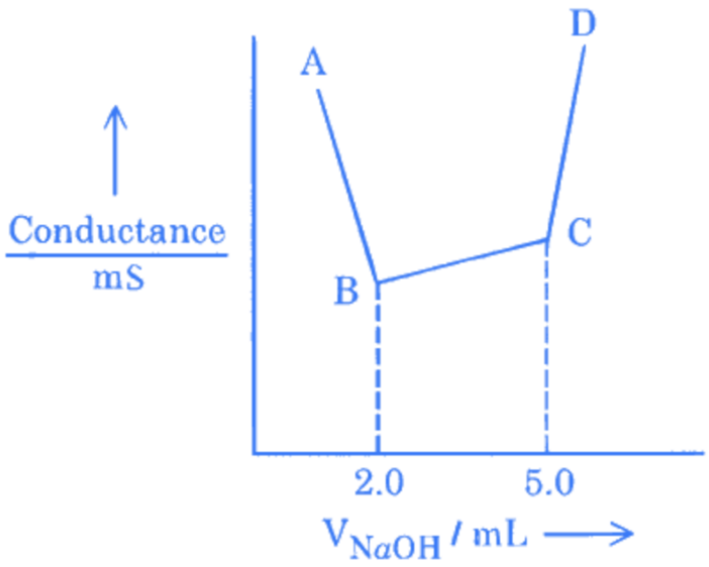
Step 2: Check the ferrous salt and the remaining compounds Compound D, $FeSO_4 \cdot 6\,H_2O$, contains ferrous ions, which are readily oxidised by atmospheric oxygen to ferric ions. Its reducing capacity changes on storage, making it unsuitable as a primary standard. Oxalic acid, B, is used as a primary standard in its well-defined crystalline form. Sodium tetraborate, E, is also a standard substance used for acid standardisation. They must not be included among the unsuitable choices.

Step 3: Select the complete set The compounds that fail the required stability or weighing criteria are A, C and D. Option (A) incorrectly includes E and omits A. Option (B) incorrectly includes oxalic acid and omits A and C. Option (C) incorrectly includes E while omitting A and C. Option (D) alone lists precisely the unsuitable three compounds.

Step 4: Conclusion $Na_2Cr_2O_7$, NaOH and $FeSO_4 \cdot 6\,H_2O$, namely A, C and D only, should not be used as primary standards. Hence, the correct option is (D).

Quick Tip to Solve: For a primary-standard question, check moisture absorption, reaction with air and oxidation-state stability before considering the titration reaction.

69. 40 mL of a mixture of CH_3COOH and HCl (aqueous solution) is titrated against 0.1 M NaOH solution conductometrically. Which of the following statement is correct? [4 marks]



(A) The concentration of CH_3COOH in the original mixture is 0.005 M

(B) The concentration of HCl in the original mixture is 0.005 M

(C) CH_3COOH is neutralised first followed by neutralisation of HCl

(D) Point ' C ' indicates the complete neutralisation of HCl

Correct Answer: (B) The concentration of HCl in the original mixture is 0.005 M

Solution:

Concept:

- In a mixture of hydrochloric acid and acetic acid, the strong acid is neutralised first. Replacing highly mobile hydrogen ions by sodium ions causes the initial conductance to decrease.
- The first break B gives the volume required for HCl; the second break C gives the total volume required for both acids. Each acid reacts with NaOH in a 1:1 mole ratio.

Step 1: Read the two equivalence volumes The graph reaches its minimum at B, corresponding to $2.0\,\text{mL}$ of base. This is the equivalence point for HCl. From B to C, acetic acid is converted into sodium acetate and the conductance rises gradually. C is at $5.0\,\text{mL}$, where both acids have been neutralised. Beyond C, excess hydroxide ions cause a steeper increase.

Step 2: Calculate the hydrochloric acid concentration The amount of base added at B is the initial amount of hydrochloric acid:

$$n(\text{HCl}) = 0.1 \times \frac{2.0}{1000} = 2.0 \times 10^{-4}\,\text{mol}.$$

The original mixture has volume $40\,\text{mL} = 0.040\,\text{L}$. Thus

$$[\text{HCl}] = \frac{2.0 \times 10^{-4}}{0.040} = 0.005\,\text{mol}\,\text{L}^{-1}.$$

Step 3: Check the other statements Acetic acid consumes only the additional $(5.0 - 2.0) = 3.0\,\text{mL}$ of base. Its original concentration is therefore $0.1 \times 3.0/40 = 0.0075\,\text{M}$, not $0.005\,\text{M}$; (A) is false. The strong acid reacts first, so (C) is false. C is the endpoint for total acid, whereas B is the endpoint for hydrochloric acid alone, so (D) is false.

Step 4: Conclusion The original hydrochloric acid concentration is $0.005\,\text{M}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the first break for strong acid, and the difference between the second and first break volumes for weak acid. Divide by the original mixture volume.

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70. In Dumas' method for estimation of nitrogen 0.4 g of an organic compound gave 60 mL of nitrogen collected at 300 K temperature and 715 mm Hg pressure. The percentage composition of nitrogen in the compound is (Given : Aqueous tension at $300\ \text{K} = 15\ \text{mmHg}$) [4 marks]

(A) 15.71%

(B) 17.46%

(C) 7.85%

(D) 20.95%

Correct Answer: (A) 15.71%

Solution:

Concept:

- Gas collected over water contains water vapour. The partial pressure of dry nitrogen is $P_{\text{N}_2} = P_{\text{total}} - P_{\text{water}}$.
- The ideal-gas equation gives $n = PV/(RT)$. The nitrogen mass is $28n$ grams, and its mass percentage is $100\, m_{\text{nitrogen}}/m_{\text{sample}}$.

Step 1: Correct the collected gas pressure The reported pressure is $715\,\text{mm Hg}$, and the aqueous tension is $15\,\text{mm Hg}$. Subtract the water-vapour contribution:

$$P_{\text{N}_2} = 715 - 15 = 700\,\text{mm Hg} = \frac{700}{760}\,\text{atm}.$$

The volume and temperature of this dry nitrogen are $0.060\,\text{L}$ and $300\,\text{K}$, respectively.

Step 2: Find the mass of nitrogen Use $R = 0.0821\,\text{L atm mol}^{-1}\,\text{K}^{-1}$:

$$n(\text{N}_2) = \frac{(700/760) \times 0.060}{0.0821 \times 300} \simeq 2.244 \times 10^{-3}\,\text{mol}.$$

Each mole of nitrogen gas has mass $28\,\text{g}$, because it is diatomic. Hence

$$m(\text{N}_2) = (2.244 \times 10^{-3}) \times 28 \simeq 0.06283\,\text{g}.$$

Step 3: Convert to mass percentage and compare The original sample mass is $0.4\,\mathrm{g}$, so

$$\%\mathrm{N} = \frac{0.06283}{0.4} \times 100 \simeq 15.71\%.$$

This agrees with (A). Option (C) is approximately half this result and would arise from treating nitrogen gas as monatomic. Options (B) and (D) do not satisfy the corrected gas-law calculation for the stated sample mass, volume and temperature.

Step 4: Conclusion The percentage composition of nitrogen is approximately 15.71%. Hence, the correct option is (A).

Quick Tip to Solve: First subtract aqueous tension, then use molar mass 28 for N_2. Mixing millilitres with a gas constant expressed per litre gives a factor-of-1000 error.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. X g of nitrobenzene on nitration gave 4.2 g of m-dinitrobenzene. X = _____ g. (nearest integer)

[Given: molar mass (in $\mathrm{g\,mol^{-1}}$): C: 12, H: 1, O: 16, N: 14] [4 marks]

Solution:

Concept:

- Nitration replaces an aromatic hydrogen atom by a nitro group. One mole of nitrobenzene produces one mole of dinitrobenzene; the nitro group directs further substitution mainly to the meta position.
- For a 1:1 conversion, $m_{\text{reactant}}/M_{\text{reactant}} = m_{\text{product}}/M_{\text{product}}$. Use the supplied atomic masses and round only the final mass.

Step 1: Write the formulas and molar masses Nitrobenzene is $C_6H_5NO_2$; adding a second nitro group in place of one ring hydrogen gives $C_6H_4N_2O_4$. Using the stated atomic masses,

$$M(\mathrm{C_6H_5NO_2}) = 6(12) + 5(1) + 14 + 2(16) = 123\,\mathrm{g\,mol^{-1}},$$

$$M(\mathrm{C_6H_4N_2O_4}) = 6(12) + 4(1) + 2(14) + 4(16) = 168\,\mathrm{g\,mol^{-1}}.$$

The product is heavier because a nitro group has replaced hydrogen.

Step 2: Apply the mole ratio The nitration equation is

$$\mathrm{C_6H_5NO_2 + HNO_3 \longrightarrow C_6H_4N_2O_4 + H_2O}.$$

Thus, one mole of nitrobenzene corresponds to one mole of the product. The $4.2\,\mathrm{g}$ of product represents

$$n = \frac{4.2}{168} = 0.025\,\mathrm{mol}.$$

Therefore the starting nitrobenzene mass is

$$X = 0.025 \times 123 = 3.075\,\mathrm{g}.$$

Step 3: Round as requested The question asks for the nearest integer. Since 3.075 is closer to 3 than to 4, the required entry is 3. The mole balance uses the 1:1 reaction ratio; equating the two masses directly would incorrectly ignore the mass supplied by the nitrating reagent.

Step 4: Conclusion The nitrobenzene mass is $3.075\,\mathrm{g}$, which rounds to $X = 3\,\mathrm{g}$ to the nearest integer.

Quick Tip to Solve: Calculate both molar masses first and use a 1:1 mole ratio. Nitration changes the mass per mole, so the reactant and product masses are not equal.

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72. The total number of structural isomers possible for the substituted benzene derivatives with the molecular formula C_9H_{12} is_____. [4 marks]

Solution:

Concept:

- A benzene ring accounts for six carbon atoms. For C_9H_{12}, the remaining three carbons form saturated alkyl substituents; partition them as 3, $2+1$, or $1+1+1$.
- Structural isomers differ in connectivity. Count distinct side-chain connectivities and distinct ring positions, identifying patterns related by rotation or reflection of the benzene ring.

Step 1: Count the single three-carbon substituents With all three extra carbons in one substituent, the side chain can be straight or branched. These give propylbenzene, $C_6H_5CH_2CH_2CH_3$, and isopropylbenzene, $C_6H_5CH(CH_3)_2$. A monosubstituted benzene has only one ring position type, so this partition contributes 2 structural isomers.

Step 2: Count ethyl-plus-methyl substitution The partition $2+1$ gives one ethyl group and one methyl group on the ring. Fix one substituent at carbon 1. The other can occupy position 2, 3 or 4, giving ortho-, meta- and para-ethyltoluene. Positions 6 and 5 are equivalent to 2 and 3 by reflection. This partition therefore contributes 3 isomers.

Step 3: Count the trimethylbenzenes The partition $1+1+1$ gives three identical methyl substituents. The distinct arrangements are 1,2,3-trimethylbenzene, 1,2,4-trimethylbenzene and 1,3,5-trimethylbenzene. Other apparent arrangements are related to these by renumbering, rotation or reflection, so they are not additional connectivities. This partition contributes another 3 isomers.

Step 4: Add the mutually exclusive cases These partitions cover every possible distribution of the three extra carbons while preserving a benzene nucleus. Their counts add without overlap:

$$N = 2 + 3 + 3 = 8.$$

Side-chain unsaturation or an additional ring would remove hydrogens and would not fit the stated alkylbenzene formula.

Step 5: Conclusion The total number of structural isomers of substituted benzene derivatives with formula C_9H_{12} is 8.

Quick Tip to Solve: Partition side-chain carbons before counting ring positions: $3 \to 2$, $2+1 \to 3$, $1+1+1 \to 3$. Benzene symmetry prevents double counting.

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73. Among, $\mathrm{Sc}, \mathrm{Mn}, \mathrm{Co}$ and Cu , identify the element with highest enthalpy of atomisation. The spin only magnetic moment value of that element in its +2 oxidation state is_____BM (in nearest integer). [4 marks]

Solution:

Concept:

- Enthalpy of atomisation is the energy required to form gaseous atoms from a mole of an element in its standard state. Its magnitude reflects the strength of metallic bonding in the solid.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mathrm{BM}$, where n is the number of unpaired electrons in the ion, not in the neutral atom.

Step 1: Compare the atomisation enthalpies The approximate atomisation enthalpies of the listed metals are Sc: 326, Mn: 281, Co: 425, and Cu: $339\,\mathrm{kJ\,mol^{-1}}$. Thus cobalt has the highest value among these four elements. This comparison is of the metals themselves, before forming their ions. Manganese does not have the largest value merely because its isolated atom has a half-filled $3d$ subshell; atomisation depends on bonding in the solid and the stability of the separated gaseous atoms.

Step 2: Write the electronic configuration of cobalt(II) Cobalt has atomic number 27 and neutral configuration $[\mathrm{Ar}]\,3d^7 4s^2$. Formation of its +2 ion removes the two $4s$ electrons first, so Co^{2+} has configuration $[\mathrm{Ar}]\,3d^7$. Under the usual free-ion spin-only treatment, the five d orbitals first receive one electron each, and the next two electrons pair in two of those orbitals. There are therefore two paired orbitals and three singly occupied orbitals, giving $n = 3$.

Step 3: Calculate and round the spin-only moment Substitute the unpaired-electron count of the ion into the spin-only expression:

$$\mu = \sqrt{3(3+2)} = \sqrt{15} = 3.873\,\mathrm{BM}.$$

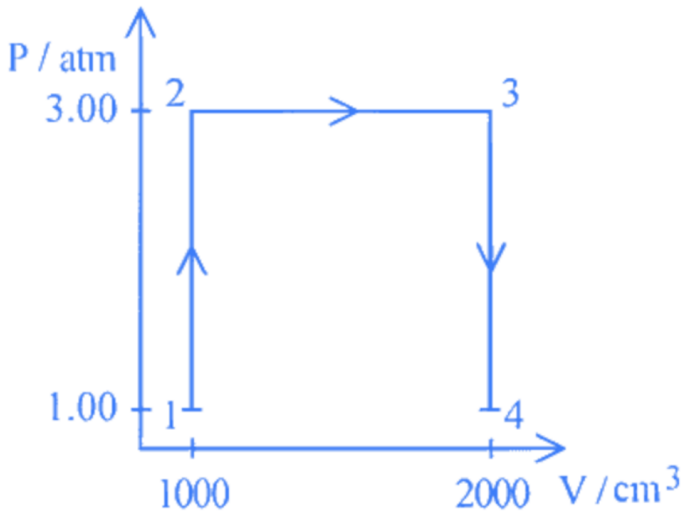
The question requests the nearest integer, so this value rounds upward to 4. No orbital contribution is added because the required quantity is explicitly the spin-only magnetic moment.

Step 4: Conclusion The selected element is cobalt, and its +2 ion has spin-only magnetic moment $3.87\,\mathrm{BM}$. The required nearest-integer entry is $\boxed{4}$.

Quick Tip to Solve: Select the metal using atomisation enthalpy first; then remove its 4s electrons before counting the unpaired d electrons for the +2 ion.

74. A perfect gas (0.1 mol) having $\overline{C}_v = 1.50R$ (independent of temperature) undergoes the above transformation from point 1 to point 4. If each step is reversible, the total work done (w) while going from point 1 to point 4 is $(-)$_____J (nearest integer)

[Given: $R = 0.082$ L atm K^{-1} mol^{-1} **]** [4 marks]



Solution:

Concept:

- With the chemistry sign convention, reversible pressure-volume work is $w = -\int P\,dV$. Expansion gives negative work because the gas transfers energy to the surroundings.
- A vertical segment on a pressure-volume plot has constant volume and hence zero pressure-volume work. A horizontal segment has constant pressure and contributes $-P\Delta V$.

Step 1: Read the path and convert the volumes The first segment takes the gas from state 1 to state 2 at $V = 1000\,\text{cm}^3$. The second segment is an expansion from 1000 to $2000\,\text{cm}^3$ at $P = 3.00\,\text{atm}$. The third segment lowers the pressure from state 3 to state 4 at constant $V = 2000\,\text{cm}^3$. Since $1000\,\text{cm}^3 = 1.000\,\text{L}$, the horizontal expansion has $\Delta V = 1.000\,\text{L}$.

Step 2: Add the work along the three segments For both vertical segments, $dV = 0$, irrespective of the pressure change. Only the horizontal expansion contributes to the integral:

$$\begin{aligned} w_{1\to2} &= 0, \\ w_{2\to3} &= -(3.00\,\text{atm})(2.000 - 1.000)\,\text{L} = -3.00\,\text{L atm}, \\ w_{3\to4} &= 0. \end{aligned}$$

Thus $w_{1\to4} = -3.00\,\text{L atm}$. This is an open path, so the relevant area is the area under its horizontal segment, not the area of a closed rectangle between the two pressure levels.

Step 3: Express the total in joules Use $1\,\text{L atm} = 101.325\,\text{J}$ to obtain

$$w = -3.00(101.325) = -303.975\,\text{J} \simeq -304\,\text{J}.$$

The amount of gas and the heat capacity would be needed for temperatures or internal-energy changes, but they are unnecessary for work once the complete pressure-volume path is specified. The sign agrees with the net expansion.

Step 4: Conclusion The total reversible work is $w \simeq \boxed{-304\,\text{J}}$. Since the minus sign is already printed before the blank, the required entry is $\boxed{304}$.

Quick Tip to Solve: On a pressure-volume path, discard vertical segments when calculating work; multiply the pressure by the volume change for each horizontal segment.

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75. A sample of n -octane $(1.14\ \text{g})$ was completely burnt in excess of oxygen in a bomb calorimeter, whose heat capacity is $5\ \text{kJ K}^{-1}$. As a result of combustion reaction, the temperature of the calorimeter is increased by 5 K . The magnitude of the heat of combustion of octane at constant volume is_____ kJmol^{-1} (nearest integer). [4 marks]

Solution:

Concept:

- A bomb calorimeter operates at constant volume. For the combustion reaction, the measured heat therefore equals the internal-energy change, $q_v = \Delta U$, rather than the constant-pressure enthalpy change.
- The calorimeter absorbs $q_{\text{cal}} = C_{\text{cal}}\Delta T$. Energy conservation gives $q_{v,\text{sample}} = -q_{\text{cal}}$, and division by the amount burned gives the molar heat of combustion.

Step 1: Calculate the heat absorbed by the calorimeter The given heat capacity is for the complete calorimeter, so no additional mass factor is required. Its temperature rises by $5\,\text{K}$, giving

$$q_{\text{cal}} = (5\,\text{kJ K}^{-1})(5\,\text{K}) = 25\,\text{kJ}.$$

The positive sign means that the calorimeter gains heat. Accordingly, combustion of the stated sample releases $25\,\text{kJ}$, so its constant-volume reaction heat is $-25\,\text{kJ}$.

Step 2: Convert the mass of octane into moles The molecular formula of n-octane is C_8H_{18}. With the usual atomic masses of carbon and hydrogen, its molar mass is

$$M = 8(12) + 18(1) = 114\,\text{g mol}^{-1}.$$

Thus the amount completely burned is

$$n = \frac{1.14\,\text{g}}{114\,\text{g mol}^{-1}} = 0.0100\,\text{mol}.$$

Excess oxygen ensures that all this octane undergoes complete combustion, so this amount is the proper denominator for the molar value.

Step 3: Obtain the molar constant-volume heat Divide the sample's reaction heat by its amount:

$$\Delta U_{\text{comb,m}} = \frac{-25\,\text{kJ}}{0.0100\,\text{mol}} = -2500\,\text{kJ mol}^{-1}.$$

This negative value describes an exothermic reaction. The question asks for its magnitude, which is positive. A gas-mole correction of the form $\Delta H = \Delta U + \Delta n_g RT$ is not applied: that would convert the answer to constant-pressure heat, whereas the required measurement is explicitly at constant volume.

Step 4: Conclusion The magnitude of the constant-volume heat of combustion is $\boxed{2500\,\text{kJ}\,\text{mol}^{-1}}$. The required nearest-integer entry is $\boxed{2500}$.

Quick Tip to Solve: A bomb-calorimeter heat capacity already includes the apparatus: calculate C times the temperature rise, then divide by the fuel's moles and take the requested magnitude.