JEE Main 2025 Paper 1 (B.E./B.Tech) Apr 4 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 4 April 2025, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function satisfying $f(0) = 1$ and $f(2x) - f(x) = x$ for all $x \in \mathbb{R}$. If $\lim_{n\to\infty} \left\{f(x) - f\left(\frac{x}{2^n}\right)\right\} = G(x)$, then $\sum_{r=1}^{10} G\left(r^2\right)$ is equal to [4 marks]

(A) 215

(B) 420

(C) 385

(D) 540

Correct Answer: (C) 385

Solution:

Concept:

- The relation $f(2u) - f(u) = u$ can be applied at successively halved arguments, so that intermediate function values cancel in a telescoping sum.
- Continuity at zero gives $f(x/2^n) \to f(0) = 1$. The required finite sum is evaluated using $\sum_{r=1}^{N} r^2 = N(N+1)(2N+1)/6$.

Step 1: Apply the functional relation at halved arguments Fix any real x. Substituting $u = x/2^k$, for $k = 1, \ldots, n$, gives $f(x/2^{k-1}) - f(x/2^k) = x/2^k$. Add these identities; every intermediate function value appears once with each sign. Consequently,

$$f(x) - f(x/2^n) = x \sum_{k=1}^{n} 2^{-k} = x(1 - 2^{-n}).$$

This computation is valid for negative x as well as positive x, and for zero both sides vanish.

Step 2: Take the limit and evaluate the requested sum As $n \to \infty$, the right-hand side tends to x. Hence the function defined in the question is $G(x) = x$. Continuity also confirms $f(x) - 1 = x$, so $f(x) = x + 1$, which satisfies both original conditions. In particular, $G(r^2) = r^2$, not r or $2r^2$. Thus

$$\sum_{r=1}^{10} G(r^2) = \sum_{r=1}^{10} r^2 = \frac{10 \cdot 11 \cdot 21}{6} = 385.$$

The other listed values, 215, 420 and 540, do not equal this exact sum; no approximation or rounding is involved.

Step 3: Conclusion The required sum is $\boxed{385}$. Hence, the correct option is (C).

Quick Tip to Solve: In a functional relation involving $f(2x) - f(x)$, substitute halved arguments and add before taking a limit; the cancellation determines the expression directly.

2. Let the shortest distance between the lines $\frac{x-3}{3} = \frac{y-\alpha}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-\beta}{4}$ be $3\sqrt{30}$. Then the positive value of $5\alpha + \beta$ is [4 marks]

(A) 42

(B) 40

(C) 48

(D) 46

Correct Answer: (D) 46

Solution:

Concept:

- For lines through points $\mathbf{a}_1, \mathbf{a}_2$ with nonparallel direction vectors $\mathbf{b}_1, \mathbf{b}_2$, their shortest distance is $d = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|/|\mathbf{b}_1 \times \mathbf{b}_2|$.
- The absolute value in the scalar triple product gives two algebraic possibilities. The question asks for the positive value of the specified combination, not separate values of the parameters.

Step 1: Extract the points and direction vectors The first line passes through $\mathbf{a}_1 = (3, \alpha, 3)$ and has direction $\mathbf{b}_1 = (3, -1, 1)$. The second passes through $\mathbf{a}_2 = (-3, -7, \beta)$ and has direction $\mathbf{b}_2 = (-3, 2, 4)$. Their common normal is

$$\mathbf{b}_1 \times \mathbf{b}_2 = (-6, -15, 3), \qquad |\mathbf{b}_1 \times \mathbf{b}_2| = \sqrt{36 + 225 + 9} = 3\sqrt{30}.$$

It is nonzero, so the shortest-distance formula applies.

Step 2: Use the given distance and select the positive value The joining displacement is $\mathbf{a}_2 - \mathbf{a}_1 = (-6, -7 - \alpha, \beta - 3)$. Its scalar product with the normal equals

$$36 + 105 + 15\alpha + 3\beta - 9 = 3(44 + 5\alpha + \beta).$$

Writing $s = 5\alpha + \beta$, the distance condition becomes

$$\frac{|s + 44|}{\sqrt{30}} = 3\sqrt{30}, \qquad |s + 44| = 90.$$

Thus $s = 46$ or $s = -134$. Only 46 is positive. As a check, the other offered positive values 42, 40 and 48 give $|s + 44| = 86, 84, 92$, respectively, so none gives the stated distance.

Step 3: Conclusion The positive value is $\boxed{5\alpha + \beta = 46}$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the absolute value until the final step in a skew-line distance problem; dropping it can hide the second parameter value.

3. Let $A = \{1, 6, 11, 16, \ldots\}$ and $B = \{9, 16, 23, 30, \ldots\}$ be the sets consisting of the first 2025 terms of two arithmetic progressions. Then $n(A \cup B)$ is [4 marks]

(A) 3814

(B) 4003

(C) 4027

(D) 3761

Correct Answer: (D) 3761

Solution:

Concept:

- For finite sets, inclusion and exclusion gives $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. Each progression has 2025 distinct terms.
- An integer common to progressions of differences 5 and 7 recurs every $\text{lcm}(5, 7) = 35$. Count common terms only up to the smaller of the two last terms.

Step 1: Find the common progression and its upper bound The final term of A is $1 + 2024 \cdot 5 = 10121$, while the final term of B is $9 + 2024 \cdot 7 = 14177$. Any common term must therefore be at most 10121. It must satisfy $t \equiv 1 \pmod 5$ and $t \equiv 2 \pmod 7$. The first such term belonging to both sets is 16: it is the fourth term of A and the second term of B. Every subsequent common term differs by 35. Hence the intersection is formed by $16 + 35k$, with integral $k \geq 0$, subject to the upper bound.

Step 2: Count the intersection and subtract its duplicate count The bound gives

$$16 + 35k \leq 10121 \quad \Longrightarrow \quad k \leq \frac{10105}{35} = 288 + \frac{5}{7}.$$

Thus $k = 0, 1, \ldots, 288$, giving 289 common terms. The last common term is $16 + 35 \cdot 288 = 10096$, and the next one, 10131, exceeds the last term of A. Therefore

$$n(A \cup B) = 2025 + 2025 - 289 = 3761.$$

Adding the two cardinalities alone would count every common term twice. Values 3814, 4003 and 4027 would require intersection counts 236, 47 and 23, contradicting the explicit count of 289.

Step 3: Conclusion The union contains $\boxed{3761}$ elements. Hence, the correct option is (D).

Quick Tip to Solve: For two finite progressions, first find their first common term and common difference; the smaller last term sets the cutoff for their intersection.

4. The length of the latus-rectum of the ellipse, whose foci are $(2, 5)$ and $(2, -3)$ and eccentricity is $\frac{4}{5}$, is [4 marks]

(A) $\frac{50}{3}$
(B) $\frac{18}{5}$
(C) $\frac{6}{5}$
(D) $\frac{10}{3}$

Correct Answer: (B) $\frac{18}{5}$

Solution:

Concept:

- For an ellipse with semi-major axis a, semi-minor axis b, and centre-to-focus distance c, the relations are $e = c/a$ and $b^2 = a^2 - c^2$.
- The full length of a latus rectum is $2b^2/a$, irrespective of whether the major axis is horizontal or vertical. The foci are separated by $2c$, not c.

Step 1: Locate the centre and determine the semi-axes The midpoint of the foci $(2, 5)$ and $(2, -3)$ is $(2, 1)$. Their separation is $5 - (-3) = 8$, so each focus lies a distance $c = 4$ from the centre. Since $e = 4/5$,

$$a = \frac{c}{e} = \frac{4}{4/5} = 5.$$

The semi-minor axis follows from $b^2 = a^2 - c^2 = 25 - 16 = 9$, so $b = 3$. In particular, the focal separation 8 must not be substituted for c.

Step 2: Calculate the complete focal chord The length of the latus rectum is

$$L = \frac{2b^2}{a} = \frac{2 \cdot 9}{5} = \frac{18}{5}.$$

One can verify the factor of two directly. Because the major axis is vertical, the ellipse is $(x-2)^2/9 + (y-1)^2/25 = 1$. At the focus $(2, 5)$, put $y = 5$; this gives $(x-2)^2/9 = 9/25$, hence $x - 2 = \pm 9/5$. The distance between those two endpoints is $18/5$. Therefore the values $50/3$, $6/5$ and $10/3$ cannot represent this full chord; they do not satisfy the focal-section calculation.

Step 3: Conclusion The latus-rectum length is $\boxed{\frac{18}{5}}$ units. Hence, the correct option is (B).

Quick Tip to Solve: Take half the distance between the foci to find c, then obtain $a = c/e$; use $2b^2/a$ for the full latus rectum.

5. Consider the equation $x^2 + 4x - n = 0$, where $n \in [20, 100]$ is a natural number. Then the number of all distinct values of n, for which the given equation has integral roots, is equal to [4 marks]

(A) 6

(B) 5

(C) 8

(D) 7

Correct Answer: (A) 6

Solution:

Concept:

- Completing the square in $x^2 + 4x - n = 0$ gives $(x + 2)^2 = n + 4$. Integral roots therefore require $n + 4$ to be a perfect square.
- The two roots are $-2 \pm \sqrt{n + 4}$. Since the centre -2 is an integer, an integral square root is both necessary and sufficient for both roots to be integers.

Step 1: Translate the allowed interval into a square condition The condition $20 \leq n \leq 100$ gives

$$24 \leq n + 4 \leq 104.$$

Let $n + 4 = m^2$. We may take $m \geq 0$, since changing its sign exchanges the two roots and produces the same n. Because $4^2 = 16 < 24$, $5^2 = 25 \geq 24$, $10^2 = 100 \leq 104$, and $11^2 = 121 > 104$, the only possible nonnegative integers are $m = 5, 6, 7, 8, 9, 10$.

Step 2: List the parameter values and check sufficiency The corresponding values $n = m^2 - 4$ are

$$21,\ 32,\ 45,\ 60,\ 77,\ 96.$$

All six are distinct natural numbers in the required closed interval. For each, the roots $-2 + m$ and $-2 - m$ are integers; for example, $n = 21$ gives roots 3 and -7, and $n = 96$ gives roots 8 and -12. No endpoint adds another solution, since 24 and 104 are not squares. Negative values of m cannot be counted again because the question counts distinct values of n, not ordered root pairs. Thus 5 undercounts the six listed possibilities, while 7 and 8 require additional squares outside the permitted interval.

Step 3: Conclusion There are $\boxed{6}$ distinct allowed values of n. Hence, the correct option is (A).

Quick Tip to Solve: When the linear coefficient is even, completing the square often turns integral-root counting into counting perfect squares in an interval.

6. Consider the sets $A = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 = 25\}$, $B = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + 9y^2 = 144\}$, $C = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \leq 4\}$ and $D = A \cap B$. The total number of one-one functions from the set D to the set C is: [4 marks]

(A) 15120

(B) 18290

(C) 17160

(D) 19320

Correct Answer: (C) 17160

Solution:

Concept:

- If a domain has r distinct elements and a codomain has $s \geq r$ elements, the number of one-one functions is ${}^{s}P_r = s!/(s-r)!$: each new domain element needs a different image.
- The set D is obtained by solving both curve equations simultaneously over the reals. The set C is counted separately using integer lattice points in the disk $x^2 + y^2 \leq 4$.

Step 1: Count the real intersection points Subtract $x^2 + y^2 = 25$ from $x^2 + 9y^2 = 144$. This yields $8y^2 = 119$, so $y^2 = 119/8$. Substitution into the circle gives

$$x^2 = 25 - \frac{119}{8} = \frac{81}{8}.$$

Both squared coordinates are strictly positive. Each coordinate therefore has two signs, and the signs can be chosen independently. Consequently, D contains exactly four distinct real ordered pairs. They need not be integer pairs, because the integrality restriction applies to C, not to A or B.

Step 2: Count the lattice points and assign distinct images For $x = 0$, the possible integral values $y = -2, -1, 0, 1, 2$ give five points. For each of $x = 1, -1$, the values $y = -1, 0, 1$ give three points, hence six in all. For each of $x = 2, -2$, only $y = 0$ works, giving two more. Thus $|C| = 5 + 6 + 2 = 13$. Assign images successively to the four domain elements:

$$^{13}P_4 = 13 \cdot 12 \cdot 11 \cdot 10 = 17160.$$

The product counts functions, so the domain points are distinguished. A combination $\binom{13}{4}$ would select images without assigning them. The other offered totals, 15120, 18290 and 19320, disagree with the required permutation product.

Step 3: Conclusion The number of one-one functions is $\boxed{17160}$. Hence, the correct option is (C).

Quick Tip to Solve: Keep real intersection counting separate from integer lattice-point counting; then use permutations, since each domain element needs its own assigned image.

7. $1+3+5^2+7+9^2+\ldots$ **upto 40 terms is equal to** [4 marks]

(A) 40870

(B) 41880

(C) 43890

(D) 33980

Correct Answer: (B) 41880

Solution:

Concept:

- The odd numbers form the arithmetic progression $1, 3, 5, \ldots, 79$. In the displayed series the terms in odd positions are squared, while those in even positions are unchanged; $1^2 = 1$.
- For m consecutive nonnegative integers, $\sum_{k=0}^{m-1} k = m(m-1)/2$ and $\sum_{k=0}^{m-1} k^2 = m(m-1)(2m-1)/6$. Use these identities after separating the two progressions.

Step 1: Separate the forty terms into twenty pairs The first pair is 1^2+3, the second is 5^2+7, and the third is 9^2+11. Thus, with $k = 0, 1, \ldots, 19$, the terms of a pair are $(4k+1)^2$ and $4k+3$. There are twenty pairs, because each consumes two of the forty displayed positions. The final pair is 77^2+79. Consequently,

$$S = \sum_{k=0}^{19} \left[(4k+1)^2 + (4k+3)\right].$$

Step 2: Expand and evaluate the finite sums Expanding before summing avoids treating the expression as an ordinary sum of odd numbers:

$$(4k+1)^2 + (4k+3) = 16k^2 + 12k + 4.$$

For twenty values of k,

$$\sum_{k=0}^{19} k = 190, \qquad \sum_{k=0}^{19} k^2 = 2470.$$

Therefore,

$$\begin{aligned} S &= 16(2470) + 12(190) + 4(20) \\ &= 39520 + 2280 + 80 = 41880. \end{aligned}$$

The two contributions can also be checked separately: the squared terms total 41060, and the unchanged terms total 820. The values in (A), (C) and (D) disagree with this exact total.

Step 3: Conclusion The sum of the first forty terms is $\boxed{41880}$. Hence, the correct option is (B).

Quick Tip to Solve: Pair each squared odd number with the following unsquared odd number. Count twenty pairs, and remember that the first term is also 1^2.

8. If $10\sin^4\theta + 15\cos^4\theta = 6$, then the value of $\frac{27\,\mathrm{cosec}^6\theta + 8\sec^6\theta}{16\sec^8\theta}$ is [4 marks]

(A) $\frac{2}{5}$

(B) $\frac{3}{5}$

(C) $\frac{1}{5}$

(D) $\frac{3}{4}$

Correct Answer: (A) $\frac{2}{5}$

Solution:

Concept:

- Use $\sin^2\theta + \cos^2\theta = 1$ to express the fourth-power equation as a quadratic in $t = \sin^2\theta$, with $0 \le t \le 1$.
- Reciprocal powers can then be evaluated from $\mathrm{cosec}^6\theta = (\sin^2\theta)^{-3}$, $\sec^6\theta = (\cos^2\theta)^{-3}$ and $\sec^8\theta = (\cos^2\theta)^{-4}$.

Step 1: Determine the squared sine and cosine Substitute $\cos^2\theta = 1 - t$ into the given relation. This produces

$$10t^2 + 15(1-t)^2 = 6,$$
$$25t^2 - 30t + 9 = 0,$$
$$(5t-3)^2 = 0.$$

The repeated root is $t = 3/5$, so $\sin^2\theta = 3/5$ and $\cos^2\theta = 2/5$. Both are strictly positive; therefore all reciprocal powers in the requested expression are defined. The signs of sine and cosine do not affect any of the even powers.

Step 2: Substitute the reciprocal powers and simplify The required quantities are

$$\mathrm{cosec}^6\theta = \left(\frac{5}{3}\right)^3 = \frac{125}{27}, \qquad \sec^6\theta = \left(\frac{5}{2}\right)^3 = \frac{125}{8},$$

$$\sec^8\theta = \left(\frac{5}{2}\right)^4 = \frac{625}{16}.$$

Hence the numerator is $27(125/27) + 8(125/8) = 250$, while the entire denominator is $16(625/16) = 625$. Their quotient is $250/625 = 2/5$. In particular, the denominator uses the eighth power, not the sixth power; replacing it would give a different value. The alternatives $3/5$, $1/5$ and $3/4$ do not equal the computed quotient.

Step 3: Conclusion The value is $\boxed{\frac{2}{5}}$. Hence, the correct option is (A).

Quick Tip to Solve: When a weighted fourth-power identity equals its minimum, substitute the squared sine: the resulting quadratic often becomes a perfect square.

9. The value of $\int_{-1}^{1} \frac{(1+\sqrt{|x|-x})e^x+(\sqrt{|x|-x})e^{-x}}{e^x+e^{-x}}dx$ is equal to [4 marks]

(A) $1+\frac{2\sqrt{2}}{3}$

(B) $1-\frac{2\sqrt{2}}{3}$

(C) $2+\frac{2\sqrt{2}}{3}$

(D) $3-\frac{2\sqrt{2}}{3}$

Correct Answer: (A) $1+\frac{2\sqrt{2}}{3}$

Solution:

Concept:

- For a symmetric interval, the substitution $x \mapsto -x$ gives $\int_{-a}^{a} g(x)\,dx = \int_{-a}^{a} g(-x)\,dx$. Adding the two forms can simplify exponential quotients.
- The absolute value must be resolved by sign: $|x| - x = -2x$ when $x < 0$, and $|x| - x = 0$ when $x \geq 0$. The square root is real throughout the given interval.

Step 1: Separate the radical from the exponential quotient Write $h(x) = \sqrt{|x| - x}$. The numerator equals $e^x + h(x)(e^x + e^{-x})$. Since the denominator is positive, division gives

$$I = \int_{-1}^{1} \frac{e^x}{e^x + e^{-x}}\,dx + \int_{-1}^{1} \sqrt{|x| - x}\,dx.$$

For $g(x) = e^x/(e^x + e^{-x})$, reflection gives $g(-x) = e^{-x}/(e^x + e^{-x})$; hence $g(x) + g(-x) = 1$. If the first integral is J, then

$$2J = \int_{-1}^{1} 1\,dx = 2, \qquad J = 1.$$

Step 2: Integrate the radical on its nonzero half-interval For $0 \leq x \leq 1$, the radical vanishes. For $-1 \leq x < 0$, it is $\sqrt{-2x}$. Put $u = -x$, changing the limits from $(-1, 0)$ to $(1, 0)$, to get

$$\begin{aligned}\int_{-1}^{0} \sqrt{-2x}\,dx &= \sqrt{2}\int_{0}^{1} u^{1/2}\,du \\ &= \sqrt{2}\left[\frac{2}{3}u^{3/2}\right]_0^1 = \frac{2\sqrt{2}}{3}.\end{aligned}$$

The radical contribution is positive and the exponential contribution is exactly one, so the result is $1 + 2\sqrt{2}/3$. Option (B) subtracts a positive contribution, (C) counts the symmetry integral as two, and (D) also disagrees with the evaluated sum.

Step 3: Conclusion The integral equals $\boxed{1+\frac{2\sqrt{2}}{3}}$. Hence, the correct option is (A).

Quick Tip to Solve: First collect the common radical in the numerator. Then use reflection for the exponential fraction and split the absolute-value term at zero.

10. Let $f : [0, \infty) \to \mathbb{R}$ be a differentiable function such that $f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)dt$ for all $x \in [0, \infty)$.

Then the area of the region bounded by $y = f(x)$ and the coordinate axes is [4 marks]

(A) $\sqrt{5}$

(B) 2

(C) $\sqrt{2}$

(D) $\frac{1}{2}$

Correct Answer: (D) $\frac{1}{2}$

Solution:

Concept:

- For $H(x) = \int_0^x e^{x-t} f(t)\, dt$, differentiating gives $H\prime(x) = f(x) + H(x)$: the upper-limit term is $f(x)$, and differentiating the kernel reproduces the integral.
- The area bounded by a nonnegative graph and the coordinate axes is $\int_0^a f(x)\, dx$, where a is the first positive intercept. Find the function before computing this area.

Step 1: Differentiate the defining relation At $x = 0$, the integral is zero, so $f(0) = 1$. Differentiating the relation and using the kernel rule yields

$$f\prime(x) = -2 + f(x) + \int_0^x e^{x-t} f(t)\, dt.$$

The original equation also gives the integral as $f(x) - 1 + 2x$. Substitution therefore eliminates the integral:

$$f\prime(x) = 2f(x) + 2x - 3, \qquad f(0) = 1.$$

Keeping the kernel derivative is essential; omitting it would produce an incorrect differential equation.

Step 2: Solve the initial-value equation and locate the intercept A particular solution is $f_p(x) = 1 - x$, since its derivative is -1 and $2(1 - x) + 2x - 3 = -1$. The associated homogeneous equation has solution Ce^{2x}, giving

$$f(x) = 1 - x + Ce^{2x}.$$

Using $f(0) = 1$ forces $C = 0$. Thus the graph is the straight line $y = 1 - x$. It meets the axes at $(0, 1)$ and $(1, 0)$, and is nonnegative between these intercepts.

Step 3: Evaluate the bounded area The enclosed region lies in the first quadrant, between $x = 0$, $y = 0$ and $y = 1 - x$. Therefore,

$$A = \int_0^1 (1 - x)\, dx = \left[x - \frac{x^2}{2}\right]_0^1 = \frac{1}{2}.$$

This also equals the triangular area $\frac{1}{2}(1)(1)$. None of $\sqrt{5}$, 2 or $\sqrt{2}$ matches this area.

Step 4: Conclusion The bounded area is $\boxed{\frac{1}{2}}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: For the kernel e^{x-t}, differentiation reproduces the same integral. Replace that integral using the original equation before solving for f.

11. The probability, of forming a 12 persons committee from 4 engineers, 2 doctors and 10 professors containing at least 3 engineers and at least 1 doctor, is [4 marks]

(A) $\frac{129}{182}$
(B) $\frac{17}{26}$
(C) $\frac{19}{26}$
(D) $\frac{103}{182}$

Correct Answer: (A) $\frac{129}{182}$

Solution:

Concept:

- Every unordered twelve-person committee selected from sixteen distinct people is equally likely. The total number of committees is $\binom{16}{12}$; order of selection is irrelevant.
- The engineer count can only be three or four, while the doctor count can only be one or two. These four disjoint possibilities enforce both lower bounds without double counting.

Step 1: List the permitted compositions Let e, d, and p be the numbers of engineers, doctors and professors. They satisfy $e + d + p = 12$, with $e \in \{3, 4\}$ and $d \in \{1, 2\}$. Thus the triples are $(3, 1, 8)$, $(3, 2, 7)$, $(4, 1, 7)$, and $(4, 2, 6)$. Every professor count lies between zero and ten, so all four possibilities are feasible. For each triple, independent choices from the three professional groups multiply.

Step 2: Count favourable committees The favourable count is

$$\begin{aligned} N &= \binom{4}{3}\binom{2}{1}\binom{10}{8} + \binom{4}{3}\binom{2}{2}\binom{10}{7} \\ &\quad + \binom{4}{4}\binom{2}{1}\binom{10}{7} + \binom{4}{4}\binom{2}{2}\binom{10}{6} \\ &= 4(2)(45) + 4(1)(120) \\ &\quad + 1(2)(120) + 1(1)(210) \\ &= 360 + 480 + 240 + 210 = 1290. \end{aligned}$$

The cases are disjoint because a committee has only one engineer count and one doctor count.

Step 3: Divide by all committees The total is $\binom{16}{12} = \binom{16}{4} = 1820$. Hence

$$P = \frac{1290}{1820} = \frac{129}{182}.$$

For comparison, the other options have denominator 182 values $119/182$, $133/182$ and $103/182$. None agrees with the exhaustive favourable count. Using $\binom{16}{4}$ counts the total via omitted people; it does not change which committees satisfy the given restrictions.

Step 4: Conclusion The required probability is $\boxed{\frac{129}{182}}$. Hence, the correct option is (A).

Quick Tip to Solve: When selecting almost everyone, count all committees by their small complement, but enforce profession restrictions with explicit disjoint compositions.

12. A box contains 10 pens of which 3 are defective. A sample of 2 pens is drawn at random and let X denote the number of defective pens. Then the variance of X is [4 marks]

(A) $\frac{11}{15}$
(B) $\frac{2}{15}$
(C) $\frac{3}{5}$
(D) $\frac{28}{75}$

Correct Answer: (D) $\frac{28}{75}$

Solution:

Concept:

- Sampling two pens without replacement gives a hypergeometric distribution. For $X = x$, choose x defective pens from three and $2 - x$ good pens from seven.
- The variance is $\text{Var}(X) = E(X^2) - [E(X)]^2$. Constructing the full distribution keeps the distinction between the first and second moments clear.

Step 1: Construct the distribution of defective pens The possible values of X are zero, one and two. There are $\binom{10}{2} = 45$ equally likely unordered samples. Therefore,

$$P(X = 0) = \frac{\binom{3}{0}\binom{7}{2}}{45} = \frac{21}{45} = \frac{7}{15},$$
$$P(X = 1) = \frac{\binom{3}{1}\binom{7}{1}}{45} = \frac{21}{45} = \frac{7}{15},$$
$$P(X = 2) = \frac{\binom{3}{2}\binom{7}{0}}{45} = \frac{3}{45} = \frac{1}{15}.$$

The probabilities sum to $(7 + 7 + 1)/15 = 1$, checking that the distribution includes every possible sample.

Step 2: Calculate the two moments The first moment weights each outcome by its value:

$$E(X) = 0\left(\frac{7}{15}\right) + 1\left(\frac{7}{15}\right) + 2\left(\frac{1}{15}\right) = \frac{3}{5}.$$

For the second moment the outcome is squared before multiplication:

$$E(X^2) = 0^2\left(\frac{7}{15}\right) + 1^2\left(\frac{7}{15}\right) + 2^2\left(\frac{1}{15}\right) = \frac{11}{15}.$$

Thus $11/15$ is the second moment and $3/5$ is the mean, rather than the variance.

Step 3: Subtract the square of the mean Apply the variance identity:

$$\text{Var}(X) = \frac{11}{15} - \left(\frac{3}{5}\right)^2 = \frac{55 - 27}{75} = \frac{28}{75}.$$

As an independent check, the finite-population formula gives $2(3/10)(7/10)(8/9) = 28/75$. Options (A) and (C) confuse a moment with the variance; (B), $2/15$, also fails the subtraction above.

Step 4: Conclusion The variance is $\boxed{\frac{28}{75}}$. Hence, the correct option is (D).

Quick Tip to Solve: For sampling without replacement, use the finite-population correction. If listing outcomes, compute $E(X^2)$ separately before subtracting $[E(X)]^2$.

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13. Consider two vectors $\vec{u} = 3\hat{i} - \hat{j}$ and $\vec{v} = 2\hat{i} + \hat{j} - \lambda\hat{k}, \lambda > 0$. The angle between them is given by $\cos^{-1}\left(\frac{\sqrt{5}}{2\sqrt{7}}\right)$. Let $\vec{v} = \vec{v_1} + \vec{v_2}$, where $\vec{v_1}$ is parallel to $\vec{u}$ and $\vec{v_2}$ is perpendicular to $\vec{u}$. Then the value $|\vec{v_1}|^2 + |\vec{v_2}|^2$ is equal to [4 marks]

(A) $\frac{23}{2}$

(B) $\frac{25}{2}$

(C) 10

(D) 14

Correct Answer: (D) 14

Solution:

Concept:

- The scalar product determines the angle: $\cos\theta = (\vec{u}\cdot\vec{v})/(|\vec{u}||\vec{v}|)$. Here the third component of $\vec{u}$ is zero, so the dot product is independent of λ.
- If $\vec{v}_1 \parallel \vec{u}$ and $\vec{v}_2 \perp \vec{u}$, the two components are perpendicular. Thus $|\vec{v}|^2 = |\vec{v}_1|^2 + |\vec{v}_2|^2$, the vector form of the Pythagorean theorem.

Step 1: Calculate the dot product and magnitudes Use the Cartesian components directly, preserving the negative sign in the second component of $\vec{u}$:

$$\vec{u}\cdot\vec{v} = 3(2) + (-1)(1) + 0(-\lambda) = 5.$$

$$|\vec{u}| = \sqrt{10}, \qquad |\vec{v}| = \sqrt{5+\lambda^2}.$$

The dot product is positive, consistently giving the acute angle stated in the question.

Step 2: Use the specified angle Equate the cosine formula to the given value and square both positive sides:

$$\frac{5}{\sqrt{10}\sqrt{5+\lambda^2}} = \frac{\sqrt{5}}{2\sqrt{7}}.$$

$$\frac{25}{10(5+\lambda^2)} = \frac{5}{28} \quad \Longrightarrow \quad 5+\lambda^2 = 14.$$

Thus $\lambda^2 = 9$, and the condition $\lambda > 0$ selects $\lambda = 3$.

Step 3: Add the squared orthogonal components The requested sum is the squared magnitude of the original vector, rather than the sum of its component magnitudes:

$$|\vec{v}_1|^2 + |\vec{v}_2|^2 = |\vec{v}|^2 = 5 + 9 = 14.$$

As a check, the parallel projection has squared magnitude $(\vec{u}\cdot\vec{v})^2/|\vec{u}|^2 = 25/10 = 5/2$, leaving $23/2$ for the perpendicular component. Option (A) gives just that latter contribution; (B) and (C) give neither the required sum nor the angle-compatible norm.

Step 4: Conclusion The required value is $\boxed{14}$. Hence, the correct option is (D).

Quick Tip to Solve: For perpendicular projections, find the original vector norm first: their squared lengths add to that norm squared, so separate projection vectors are unnecessary.

---

14. For an integer $n \geq 2$, if the arithmetic mean of all coefficients in the binomial expansion of $(x+y)^{2n-3}$ is 16 , then the distance of the point $\mathrm{P}\left(2n-1, n^2-4n\right)$ from the line $x+y=8$ is [4 marks]

(A) $\sqrt{2}$

(B) $2\sqrt{2}$

(C) $5\sqrt{2}$

(D) $3\sqrt{2}$

Correct Answer: (D) $3\sqrt{2}$

Solution:

Concept:

- For a nonnegative integer exponent m, the expansion of $(x+y)^m$ has $m+1$ coefficients, whose sum is $(1+1)^m = 2^m$. Their arithmetic mean is therefore $2^m/(m+1)$.
- The perpendicular distance of (x_0, y_0) from $ax+by+c=0$ is $|ax_0+by_0+c|/\sqrt{a^2+b^2}$. Use the absolute value only after substituting the point.

Step 1: Convert the mean into an equation Put $m = 2n-3$. Since $n \geq 2$, this is a positive odd integer. There are $2n-2$ coefficients, including both endpoint coefficients, and the condition becomes

$$\frac{2^{2n-3}}{2n-2} = 16.$$

Equivalently, $2^m = 16(m+1)$. At $m=7$, the two sides are 128 and $16\cdot 8 = 128$, respectively.

Step 2: Determine the integer uniquely To ensure that no other admissible exponent was missed, let $a_m = 2^m/(m+1)$. For every integer $m \geq 1$,

$$\frac{a_{m+1}}{a_m} = \frac{2(m+1)}{m+2} > 1.$$

The mean increases strictly with the exponent, so the solution $m=7$ is unique. Hence $2n-3=7$, giving $n=5$.

Step 3: Calculate the point and its distance Substitute $n=5$ in both coordinates:

$$P = (2\cdot 5 - 1,\ 5^2 - 4\cdot 5) = (9, 5).$$

The given line is $x + y - 8 = 0$, so

$$d = \frac{|9 + 5 - 8|}{\sqrt{1^2 + 1^2}} = \frac{6}{\sqrt{2}} = 3\sqrt{2}.$$

The value 6 is the unnormalised line expression, not the distance. Options (A), (B) and (C) would require numerators $2, 4, 10$, respectively, whereas the actual numerator is 6.

Step 4: Conclusion The distance is $\boxed{3\sqrt{2}}$ units. Hence, the correct option is (D).

Quick Tip to Solve: For the mean of all binomial coefficients, divide the sum by exponent plus one. Do not count only the interior terms before using the point-to-line distance formula.

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15. Let A and B be two distinct points on the line $L : \frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Both A and B are at a distance $2\sqrt{17}$ from the foot of perpendicular drawn from the point $(1, 2, 3)$ on the line L. If O is the origin, then $\overrightarrow{OA} \cdot \overrightarrow{OB}$ is equal to [4 marks]

(A) 49

(B) 21

(C) 47

(D) 62

Correct Answer: (C) 47

Solution:

Concept:

- A point on the given line is $\vec{r}(t) = (6, 7, 7) + t(3, 2, -2)$. The direction vector $\vec{d} = (3, 2, -2)$ has squared magnitude 17.
- At the perpendicular foot H from Q, the vector $\overrightarrow{QH}$ has zero scalar product with the direction vector. Points equally distant on either side of H can then be written as $\vec{h} \pm \vec{a}$.

Step 1: Find the perpendicular foot Let $Q = (1, 2, 3)$ and write $H = (6 + 3t, 7 + 2t, 7 - 2t)$. Perpendicularity gives

$$(H - Q) \cdot (3, 2, -2) = 0.$$

$$3(5 + 3t) + 2(5 + 2t) - 2(4 - 2t) = 0.$$

This simplifies to $17 + 17t = 0$, hence $t = -1$ and $H = (3, 5, 9)$. Direct substitution checks that $(2, 3, 6) \cdot (3, 2, -2) = 0$.

Step 2: Locate the two distinct points A parameter change of s along the line moves a distance $|s|\sqrt{17}$. Therefore distance $2\sqrt{17}$ from H corresponds to $s = \pm 2$. Since the points are distinct, they occupy the two opposite positions:

$$A = H + 2\vec{d} = (9, 9, 5), \qquad B = H - 2\vec{d} = (-3, 1, 13).$$

Interchanging their names cannot change the scalar product.

Step 3: Evaluate the scalar product Using the coordinates relative to the origin,

$$\overrightarrow{OA} \cdot \overrightarrow{OB} = 9(-3) + 9(1) + 5(13) = -27 + 9 + 65 = 47.$$

Alternatively, the symmetric form gives $(\vec{h}+2\vec{d}) \cdot (\vec{h}-2\vec{d}) = |\vec{h}|^2 - 4|\vec{d}|^2$. Here $|\vec{h}|^2 = 9+25+81 = 115$, so the same result is $115 - 68 = 47$. The other proposed values $49, 21, 62$ disagree with this exact product; no square root is needed because a scalar product is requested.

Step 4: Conclusion $\overrightarrow{OA} \cdot \overrightarrow{OB} = \boxed{47}$. Hence, the correct option is (C).

Quick Tip to Solve: For two points symmetric about a known midpoint vector, use $(\vec{h}+\vec{a}) \cdot (\vec{h}-\vec{a}) = |\vec{h}|^2 - |\vec{a}|^2$; the cross terms cancel automatically.

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16. Let $f, g : (1, \infty) \to \mathbb{R}$ be defined as $f(x) = \frac{2x+3}{5x+2}$ and $g(x) = \frac{2-3x}{1-x}$. If the range of the function fog: $[2, 4] \to \mathbb{R}$ is $[\alpha, \beta]$, then $\frac{1}{\beta-\alpha}$ is equal to [4 marks]

(A) 56

(B) 2

(C) 29

(D) 68

Correct Answer: (A) 56

Solution:

Concept:

- The composition $(f \circ g)(x)$ means substituting the complete value $g(x)$ for the input of f, not multiplying the two functions.
- A continuous monotone function on a closed interval takes its minimum and maximum at the endpoints. Check that its denominator is nonzero throughout that interval before using this fact.

Step 1: Simplify the composition and check its domain For $2 \le x \le 4$, write

$$g(x) = \frac{3x-2}{x-1} = 3 + \frac{1}{x-1} > 1.$$

Thus every inner value lies in the stated domain of f. Substitution gives

$$(f \circ g)(x) = \frac{2(3x-2)+3(x-1)}{5(3x-2)+2(x-1)} = \frac{9x-7}{17x-12}.$$

The common denominator $x-1$ cancels legally because $x \ge 2$; also $17x - 12 \ge 22 > 0$.

Step 2: Establish the order of the endpoint values Differentiate the simplified expression using the quotient rule:

$$(f \circ g)'(x) = \frac{9(17x-12) - 17(9x-7)}{(17x-12)^2} = \frac{11}{(17x-12)^2} > 0.$$

The function is continuous and strictly increasing over the whole interval. Consequently the lower endpoint of the range is the value at $x = 2$, and the upper endpoint is the value at $x = 4$.

Step 3: Find the width of the range Evaluate exactly rather than rounding the two close fractions:

$$\alpha = \frac{18-7}{34-12} = \frac{1}{2}, \qquad \beta = \frac{36-7}{68-12} = \frac{29}{56}.$$

$$\beta - \alpha = \frac{29}{56} - \frac{28}{56} = \frac{1}{56}, \qquad \frac{1}{\beta-\alpha} = 56.$$

Option (B) is the reciprocal of the lower endpoint alone; (C) is just the numerator of the upper endpoint. Option (D), like those, does not equal the reciprocal of the difference.

Step 4: Conclusion The range is $[1/2, 29/56]$, and the required value is $\boxed{56}$. Hence, the correct option is (A).

Quick Tip to Solve: For a fractional linear composition, simplify first and use the derivative sign to order the endpoints. Keep exact fractions when the range width is small.

17. If $\lim_{x\to 1^+} \frac{(x-1)(6+\lambda\cos(x-1))+\mu\sin(1-x)}{(x-1)^3} = -1$**, where** $\lambda, \mu \in \mathbb{R}$**, then** $\lambda+\mu$ **is equal to** [4 marks]

(A) 20

(B) 19

(C) 18

(D) 17

Correct Answer: (C) 18

Solution:

Concept:

- Write $h = x - 1$, so that the one-sided limit becomes $h \to 0^+$ and $\sin(1-x) = -\sin h$. This sign is essential for the cancellation condition.
- Near zero, $\cos h = 1 - h^2/2 + O(h^4)$ and $\sin h = h - h^3/6 + O(h^5)$. After division by h^3, a nonzero linear numerator term would diverge as $1/h^2$.

Step 1: Identify the necessary cancellation The numerator becomes $h(6+\lambda\cos h) - \mu\sin h$. Expanding to the order required by the denominator gives

$$h(6+\lambda\cos h) - \mu\sin h = (6+\lambda-\mu)h + \left(-\frac{\lambda}{2} + \frac{\mu}{6}\right)h^3 + O(h^5).$$

Because the prescribed limit is the finite number -1, the coefficient of h must vanish. Thus

$$\mu = 6 + \lambda.$$

Step 2: Match the finite cubic coefficient With that cancellation imposed, division by h^3 and passage to the limit leave

$$-\frac{\lambda}{2} + \frac{\mu}{6} = -1.$$

Multiplying by six gives $-3\lambda + \mu = -6$. Insert $\mu = 6 + \lambda$:

$$-3\lambda + 6 + \lambda = -6 \quad \Longrightarrow \quad \lambda = 6, \qquad \mu = 12.$$

The discarded remainder contributes $O(h^2)$, which tends to zero, so no further conditions arise.

Step 3: Verify the parameters and compare the choices For these values the linear coefficient is $6 + 6 - 12 = 0$, while the cubic coefficient is $-6/2 + 12/6 = -3 + 2 = -1$, exactly the specified limit. Their sum is

$$\lambda + \mu = 6 + 12 = 18.$$

The two independent coefficient conditions fix both parameters uniquely. Hence sums $20, 19, 17$, appearing in the other options, cannot satisfy the same finite limit.

Step 4: Conclusion $\lambda = 6$, $\mu = 12$, and $\lambda + \mu = \boxed{18}$. Hence, the correct option is (C).

Quick Tip to Solve: In a limit divided by a cube, cancel the linear term before matching the cubic term. Replacing $\sin(1 - x)$ by $-\sin(x - 1)$ prevents the common sign error.

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18. Let the three sides of a triangle are on the lines $4x - 7y + 10 = 0, x + y = 5$ and $7x + 4y = 15$. Then the distance of its orthocentre from the orthocentre of the tringle formed by the lines $x = 0, y = 0$ and $x + y = 1$ is [4 marks]

(A) $\sqrt{20}$

(B) 20

(C) $\sqrt{5}$

(D) 5

Correct Answer: (C) $\sqrt{5}$

Solution:

Concept:

- When two sides of a triangle are perpendicular, their common vertex is the right-angle vertex and also the orthocentre: the two perpendicular sides themselves form two altitudes.
- For lines of finite slopes m_1, m_2, perpendicularity means $m_1 m_2 = -1$. Once both orthocentres are known, use the ordinary Cartesian distance formula.

Step 1: Recognise the first right triangle The lines $4x - 7y + 10 = 0$ and $7x + 4y = 15$ have slopes $4/7$ and $-7/4$, respectively. Their product is -1, so these sides meet at a right angle. Their intersection is the first orthocentre H_1, regardless of the position of the third side, provided the three lines form a nondegenerate triangle as stated.

Step 2: Locate the first orthocentre exactly Solve the two perpendicular side equations:

$$4x - 7y = -10, \qquad 7x + 4y = 15.$$

Multiply the first by four and the second by seven, then add:

$$16x - 28y = -40, \qquad 49x + 28y = 105 \quad \Longrightarrow \quad 65x = 65.$$

Therefore $x = 1$, and $7 + 4y = 15$ gives $y = 2$. Thus $H_1 = (1, 2)$. It is not on the remaining side $x + y = 5$, since $1 + 2 = 3$, confirming that the triangle is nondegenerate.

Step 3: Find the second orthocentre and the distance The lines $x = 0$ and $y = 0$ are perpendicular coordinate axes. With $x + y = 1$, they bound a right triangle whose right-angle vertex and orthocentre are $H_2 = (0, 0)$. Therefore

$$H_1H_2 = \sqrt{(1-0)^2 + (2-0)^2} = \sqrt{5}.$$

Option (D), 5, is the squared distance rather than the distance. Options (A) and (B) also fail the exact squared-distance check: the coordinate differences give $1^2 + 2^2 = 5$, not 20 or 400.

Step 4: Conclusion The distance between the orthocentres is $\boxed{\sqrt{5}}$ units. Hence, the correct option is (C).

Quick Tip to Solve: Before constructing altitudes, test the side slopes. In a right triangle the orthocentre is already the intersection of the perpendicular sides.

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19. In the expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n, n \in \mathrm{N}$, if the ratio of 15^{th} term from the beginning to the 15^{th} term from the end is $\frac{1}{6}$, then the value of ${}^n\mathrm{C}_3$ is [4 marks]

(A) 4960

(B) 2300

(C) 1040

(D) 4060

Correct Answer: (B) 2300

Solution:

Concept:

- The term in position $r + 1$ in $(a + b)^n$ is $T_{r+1} = \binom{n}{r} a^{n-r} b^r$. Position and exponent are different: position 15 corresponds to $r = 14$.
- The fifteenth term from the end has position $n - 13$ from the beginning. Its binomial coefficient is $\binom{n}{n-14} = \binom{n}{14}$, so the two coefficients cancel in their ratio.

Step 1: Write the two specified terms Put $a = \sqrt[3]{2}$ and $b = 1/\sqrt[3]{3}$. Counting from either end requires at least fifteen terms. The terms are

$$T_{15} = \binom{n}{14} a^{n-14} b^{14}, \qquad T_{15 \text{ from end}} = \binom{n}{14} a^{14} b^{n-14}.$$

Both quantities are positive, so taking their ratio and comparing positive powers is legitimate.

Step 2: Determine the exponent and evaluate the combination After cancelling the equal coefficients,

the given ratio becomes

$$\left(\frac{a}{b}\right)^{n-28} = \frac{1}{6}.$$

Since $a/b = \sqrt[3]{6}$, this is

$$6^{(n-28)/3} = 6^{-1} \quad \Longrightarrow \quad n - 28 = -3 \quad \Longrightarrow \quad n = 25.$$

This value provides 26 terms, so both specified positions exist. Finally,

$$\binom{25}{3} = \frac{25 \cdot 24 \cdot 23}{3 \cdot 2 \cdot 1} = 2300.$$

The other printed values, 4960, 1040 and 4060, do not equal this combination; they cannot satisfy the derived value of n.

Step 3: Conclusion The required value is $\boxed{2300}$. Hence, the correct option is (B).

Quick Tip to Solve: For equally placed terms counted from opposite ends, cancel the symmetric binomial coefficients before solving for the exponent.

20. Considering the principal values of the inverse trigonometric functions,

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{1-x^2}\right), \qquad -\frac{1}{2} < x < \frac{1}{\sqrt{2}},$$

is equal to [4 marks]

(A) $\frac{-5\pi}{6} - \sin^{-1} x$

(B) $\frac{5\pi}{6} - \sin^{-1} x$

(C) $\frac{\pi}{6} + \sin^{-1} x$

(D) $\frac{\pi}{4} + \sin^{-1} x$

Correct Answer: (C) $\frac{\pi}{6} + \sin^{-1} x$

Solution:

Concept:

- The principal value $t = \sin^{-1} x$ lies in $[-\pi/2, \pi/2]$. On this interval $\cos t \geq 0$, hence $\sqrt{1-x^2} = \cos t$.
- The addition identity $\sin(t+\alpha) = \sin t \cos\alpha + \cos t \sin\alpha$ converts the argument to one sine. The identity $\sin^{-1}(\sin u) = u$ requires $u \in [-\pi/2, \pi/2]$.

Step 1: Substitute a principal angle Let $t = \sin^{-1} x$. Since inverse sine increases throughout its domain, the given strict bounds imply

$$-\frac{\pi}{6} < t < \frac{\pi}{4}.$$

In particular, $\cos t$ is positive. Therefore the positive square root in the question is exactly $\cos t$, rather

than its negative. The argument of the outer inverse sine becomes

$$\frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t = \sin\left(t + \frac{\pi}{6}\right).$$

Step 2: Check the branch and compare the options The angle to be recovered satisfies

$$0 < t + \frac{\pi}{6} < \frac{5\pi}{12} < \frac{\pi}{2}.$$

It stays inside the principal interval of inverse sine for every permitted x. Thus no reflection or periodic adjustment is needed, and

$$\sin^{-1}\left(\sin\left(t + \frac{\pi}{6}\right)\right) = t + \frac{\pi}{6}.$$

As a direct check on the alternatives, $x = 0$ is allowed and the original expression is $\sin^{-1}(1/2) = \pi/6$. Options (A), (B) and (D) instead give $-5\pi/6$, $5\pi/6$ and $\pi/4$, respectively. They therefore fail even this admissible test value.

Step 3: Conclusion The expression is $\boxed{\frac{\pi}{6} + \sin^{-1} x}$. Hence, the correct option is (C).

Quick Tip to Solve: Always bound the angle inside the sine before cancelling inverse sine with sine; checking only the algebra can select the wrong principal branch.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let C be the circle $x^2 + (y-1)^2 = 2$, E_1 and E_2 be two ellipses whose centres lie at the origin and major axes lie on x -axis and y -axis respectively. Let the straight line $x + y = 3$ touch the curves C, E_1 and E_2 at $P(x_1, y_1), Q(x_2, y_2)$ and $R(x_3, y_3)$ respectively. Given that P is the mid point of the line segment QR and $PQ = \frac{2\sqrt{2}}{3}$, the value of $9(x_1y_1 + x_2y_2 + x_3y_3)$ is equal to _____. [4 marks]

Solution:

Concept:

- A circle touches a straight line at the foot of the perpendicular from its centre. The line joining the centre to the point of tangency is normal to the tangent.
- A displacement along $x + y = 3$ has components $(h, -h)$ and length $\sqrt{2}|h|$. If P is the midpoint of QR, the two endpoint displacements from P are opposites.

Step 1: Find the circle tangency point The centre of C is $(0, 1)$ and its radius is $\sqrt{2}$. The line $x + y = 3$ has normal vector $(1, 1)$. A perpendicular through the centre can be written $(x, y) = (t, 1 + t)$. Substitution into the tangent equation gives

$$t + 1 + t = 3 \quad \Longrightarrow \quad t = 1.$$

Consequently $P = (1, 2)$, and its distance from the centre is indeed $\sqrt{1^2 + 1^2} = \sqrt{2}$, confirming tangency.

Step 2: Locate the other two contact points Write the endpoints on the same line as

$$Q = (1 + h, 2 - h), \qquad R = (1 - h, 2 + h).$$

This automatically incorporates the midpoint condition. The stated distance gives

$$PQ = \sqrt{h^2 + (-h)^2} = \sqrt{2}|h| = \frac{2\sqrt{2}}{3}, \qquad |h| = \frac{2}{3}.$$

Thus the unordered pair of endpoints is $(5/3, 4/3)$ and $(1/3, 8/3)$. Their assignment to the two ellipses need not be determined, because the required sum is symmetric in Q, R.

Step 3: Evaluate the symmetric sum The three coordinate products are

$$x_1y_1 = 2, \qquad \{x_2y_2, x_3y_3\} = \left\{\frac{20}{9}, \frac{8}{9}\right\}.$$

Hence

$$9(x_1y_1 + x_2y_2 + x_3y_3) = 9\left(2 + \frac{20}{9} + \frac{8}{9}\right) = 18 + 20 + 8 = 46.$$

The midpoint and distance data already fix this quantity; finding either ellipse equation would add unnecessary unknowns.

Step 4: Conclusion The required numerical value is $\boxed{46}$. It is a dimensionless coordinate expression.

Quick Tip to Solve: When two points on a fixed line have a known midpoint and distance from it, determine the endpoints directly before introducing conic equations.

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22. Let m and n be the number of points at which the function $f(x) = \max\left\{x, x^3, x^5, \ldots x^{21}\right\}, x \in \mathbb{R}$, is not differentiable and not continuous, respectively. Then $m + n$ is equal to _____. [4 marks]

Solution:

Concept:

- The maximum of finitely many continuous real functions is continuous. Every function $x, x^3, \ldots, x^{21}$ is a polynomial, so discontinuities cannot arise here.
- To check differentiability of a maximum, first identify its active polynomial on each interval. A change in the maximizing polynomial is a possible corner; compare its one-sided derivatives.

Step 1: Identify the maximizing power on each interval For $x > 1$, larger positive powers are larger, so x^{21} is the maximum. For $0 < x < 1$, larger powers are smaller and the maximum is x. If $-1 < x < 0$, larger odd powers are less negative, so the maximum is x^{21}. Finally, for $x < -1$, larger odd powers

are more negative, and the maximum is x. Thus

$$f(x) = \begin{cases} x, & x < -1, \\ x^{21}, & -1 < x < 0, \\ x, & 0 < x < 1, \\ x^{21}, & x > 1. \end{cases}$$

At $-1, 0, 1$, all the candidate powers agree, with values $-1, 0, 1$, respectively.

Step 2: Compare derivatives at every switching point Each open interval has a polynomial formula, so there are no other candidates for non-differentiability. At the boundaries the left and right derivatives are

x	$f'_-(x)$	$f'_+(x)$
-1	1	21
0	0	1
1	1	21

At $x = -1$, use $21(-1)^{20} = 21$; at zero the derivative of x^{21} is zero. Each pair is unequal, so all three points are corners. Nevertheless, the one-sided function limits equal the common values at each boundary, consistent with continuity of a finite maximum. Therefore $m = 3$ and $n = 0$.

Step 3: Conclusion The function is non-differentiable at exactly $-1, 0, 1$ and continuous everywhere. Hence $\boxed{m + n = 3}$.

Quick Tip to Solve: For odd powers, treat the two negative intervals separately: multiplication by a negative sign reverses the ordering of their magnitudes.

23. Let $A = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$**. If for some** $\theta \in (0, \pi)$, $A^2 = A^T$**, then the sum of the diagonal elements of the matrix** $(\mathrm{A} + \mathrm{I})^3 + (\mathrm{A} - \mathrm{I})^3 - 6\ \mathrm{A}$ **is equal to _____ .** [4 marks]

Solution:

Concept:

- The displayed matrix is orthogonal: its transpose is its inverse. This follows from $\cos^2\theta + \sin^2\theta = 1$ and the cancellation of cross terms.
- The sum of diagonal entries is the trace. Since I commutes with A, the usual binomial expansion applies to powers of $A + I$ and $A - I$.

Step 1: Use orthogonality to find the cube Put $c = \cos\theta$, $s = \sin\theta$. Direct multiplication of the first and third rows against the corresponding transpose columns gives diagonal terms $c^2 + s^2 = 1$ and off-diagonal terms $cs - sc = 0$. The middle entry remains one, so

$$AA^T = I.$$

The given condition $A^2 = A^T$ can therefore be multiplied on the left by A, giving

$$A^3 = AA^T = I.$$

There is no need to solve separately for the permitted angle to evaluate the requested expression.

Step 2: Expand and cancel the matrix polynomial Expand both cubes with the identity matrix in its correct role:

$$(A+I)^3 = A^3 + 3A^2 + 3A + I,$$
$$(A-I)^3 = A^3 - 3A^2 + 3A - I.$$

Adding these equations cancels the quadratic and constant terms. Subtracting the additional $6A$ specified in the question then cancels the linear terms:

$$(A+I)^3 + (A-I)^3 - 6A = 2A^3 = 2I.$$

Since these are three-by-three matrices, the final matrix has diagonal entries $2, 2, 2$. Its trace is therefore $2+2+2=6$. This computation concerns the trace of the complete polynomial, rather than the trace of A alone.

Step 3: Conclusion The sum of the diagonal elements of the given matrix expression is $\boxed{6}$.

Quick Tip to Solve: Combine symmetric cubes before calculating entries: their even-power terms cancel, and orthogonality can turn a transpose condition into a simple power identity.

24. If the area of the region $\{(x, y) : |x-5| \le y \le 4\sqrt{x}\}$ is A, then $3A$ is equal to _____. [4 marks]

Solution:

Concept:

- The region exists where its upper boundary $4\sqrt{x}$ is at least the lower boundary $|x-5|$, with $x \ge 0$ required by the square root.
- Area is the integral of upper minus lower boundary. The absolute value changes formula at $x = 5$, so its integral must be split there even though the upper boundary is unchanged.

Step 1: Find both endpoints of the region For $0 \le x \le 5$, the intersection equation is $5 - x = 4\sqrt{x}$. Set $t = \sqrt{x} \ge 0$; then

$$t^2 + 4t - 5 = 0 \quad \Longrightarrow \quad (t-1)(t+5) = 0.$$

The acceptable root is $t = 1$, giving $x = 1$. For $x \ge 5$, use $x - 5 = 4\sqrt{x}$, hence

$$t^2 - 4t - 5 = 0 \quad \Longrightarrow \quad (t-5)(t+1) = 0,$$

which gives $x = 25$. The inequalities hold between these intersections: $1 \le x \le 25$. The rejected negative roots cannot represent $\sqrt{x}$.

Step 2: Integrate the upper boundary and subtract the V-shaped boundary The area can be calculated

as

$$A = \int_1^{25} 4\sqrt{x}\,dx - \left(\int_1^{5} (5-x)\,dx + \int_5^{25} (x-5)\,dx \right).$$

The first integral is

$$\left[\frac{8}{3}x^{3/2}\right]_1^{25} = \frac{8}{3}(125-1) = \frac{992}{3}.$$

The two lower-boundary integrals are triangles of base and height 4 and 20, respectively, so their sum is

$$\frac{1}{2}(4)(4) + \frac{1}{2}(20)(20) = 8 + 200 = 208.$$

Therefore $A = 992/3 - 208 = 368/3$. Both contributions were subtracted from the upper-boundary integral; adding a signed integral across the absolute-value corner would give an incorrect area.

Step 3: Conclusion The area is $A = \dfrac{368}{3}$ square units. Consequently, the requested numerical value is $\boxed{3A = 368}$.

Quick Tip to Solve: Find intersections separately on either side of the absolute-value vertex, then subtract the two simple triangular areas from the upper-curve integral.

---

25. Let $\mathrm{A} = \{z \in \mathrm{C} : |z-2-i| = 3\}, \mathrm{B} = \{z \in \mathrm{C} : \mathrm{Re}(z - iz) = 2\}$ **and** $\mathrm{S} = \mathrm{A} \cap \mathrm{B}$**. Then** $\sum_{z \in S} |z|^2$ **is equal to _____.** [4 marks]

Solution:

Concept:

- Write a complex number as $z = x + iy$, where x, y are real. Then $|z|^2 = x^2 + y^2$, and a modulus equation describes a circle in the Argand plane.
- The real-part condition gives a straight line. Substitution into the circle gives a quadratic whose two roots correspond to the intersection points; Vieta's formula supplies their sum without requiring radicals.

Step 1: Convert both conditions to Cartesian equations The first locus has centre $(2, 1)$ and radius 3, so its equation is

$$(x-2)^2 + (y-1)^2 = 9.$$

For the second locus, expand the product carefully:

$$z - iz = (x+y) + i(y-x).$$

Thus its real part is $x + y$, and the second condition is $x + y = 2$. In particular, $y = 2 - x$; using $x - y = 2$ would describe the wrong line.

Step 2: Obtain the quadratic and sum the squared moduli Substitute the line equation into the circle equation:

$$(x-2)^2 + (1-x)^2 = 9.$$

On expansion this becomes $2x^2 - 6x - 4 = 0$, or

$$x^2 - 3x - 2 = 0.$$

Its discriminant is $9 + 8 = 17 > 0$, so there are two distinct real intersections. If their abscissae are x_1, x_2, Vieta's relation gives $x_1 + x_2 = 3$. At either root, $x^2 = 3x + 2$, and hence

$$|z|^2 = x^2 + (2 - x)^2 = 2x^2 - 4x + 4 = 2x + 8.$$

Therefore the requested sum over both elements of S is

$$(2x_1 + 8) + (2x_2 + 8) = 2(3) + 16 = 22.$$

Each line-circle intersection contributes once; there is no need to calculate either root explicitly.

Step 3: Conclusion The required dimensionless sum is $\sum_{z \in S} |z|^2 = \boxed{22}$.

Quick Tip to Solve: After finding a quadratic for the intersections, reduce the expression to a linear function of a root and use Vieta's sum instead of solving for the roots.

Physics – Section A: Multiple Choice Questions (80 marks)

26. Consider the sound wave travelling in ideal gases of $\mathrm{He}, \mathrm{CH_4}$, and $\mathrm{CO_2}$. All the gases have the same ratio $\frac{P}{\rho}$, where P is the pressure and ρ is the density. The ratio of the speed of sound through the gases $\mathrm{V_{He}} : \mathrm{V_{CH_4}} : \mathrm{V_{CO_2}}$ is given by [4 marks]

(A) $\sqrt{\frac{7}{5}} : \sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}}$

(B) $\sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}} : \sqrt{\frac{4}{3}}$

(C) $\sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}} : \sqrt{\frac{7}{5}}$

(D) $\sqrt{\frac{4}{3}} : \sqrt{\frac{5}{3}} : \sqrt{\frac{7}{5}}$

Correct Answer: (C) $\sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}} : \sqrt{\frac{7}{5}}$

Solution:

Concept:

- The adiabatic speed of sound in an ideal gas is $v = \sqrt{\gamma P/\rho}$, where $\gamma = C_P/C_V$. Sound propagates through rapid compressions and rarefactions, so the adiabatic rather than isothermal modulus applies.
- For a gas with f active translational and rotational degrees of freedom, $C_V = fR/2$, $C_P = C_V + R$, and $\gamma = (f + 2)/f$. Vibrational modes are neglected in the usual ideal-gas treatment here.

Step 1: Identify the heat-capacity ratio of each gas Helium is monatomic and has three translational degrees of freedom. Methane is a nonlinear polyatomic molecule, with three translational and three rotational degrees of freedom. Carbon dioxide is linear, with three translational and two rotational

degrees of freedom. Therefore,

$$\gamma_{\mathrm{He}} = \frac{5}{3}, \qquad \gamma_{\mathrm{CH_4}} = \frac{4}{3}, \qquad \gamma_{\mathrm{CO_2}} = \frac{7}{5}.$$

The distinction between linear and nonlinear molecules is essential: merely calling both molecules polyatomic would miss their different rotational contributions.

Step 2: Cancel the common pressure-to-density factor The problem states that P/ρ has the same value for all three gases. Its square root is consequently a common multiplier and cancels from the speed ratio:

$$v_{\mathrm{He}} : v_{\mathrm{CH_4}} : v_{\mathrm{CO_2}} = \sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}} : \sqrt{\frac{7}{5}}.$$

Option (A) incorrectly assigns $7/5$ to helium, option (B) treats carbon dioxide as nonlinear, and option (D) interchanges the helium and methane ratios. None has the required ordered set of heat-capacity ratios.

Step 3: Conclusion The ratio is $\boxed{\sqrt{5/3} : \sqrt{4/3} : \sqrt{7/5}}$. Hence, the correct option is (C).

Quick Tip to Solve: When P/ρ is common, compare only $\sqrt{\gamma}$; classify each molecule as monatomic, linear or nonlinear before assigning γ.

27. In an electromagnetic system, the quantity representing the ratio of electric flux and magnetic flux has dimension of $M^P L^Q T^R A^S$, where value of ' Q ' and ' R ' are [4 marks]

(A) $(3, -5)$

(B) $(-2, 1)$

(C) $(-2, 2)$

(D) $(1, -1)$

Correct Answer: (D) $(1, -1)$

Solution:

Concept:

- Electric flux is $\Phi_E = \int \vec{E} \cdot d\vec{A}$, and magnetic flux is $\Phi_B = \int \vec{B} \cdot d\vec{A}$. Both contain an area factor, so the dimensions of their ratio equal those of E/B.
- Use force definitions to derive field dimensions: $F = qE$ and the magnetic-force magnitude $F = qvB$ for perpendicular motion. Charge has dimension $[q] = AT$.

Step 1: Find the dimensions of the electric and magnetic fields Force has dimensions MLT^{-2}. Dividing by charge gives

$$[E] = \frac{MLT^{-2}}{AT} = MLT^{-3}A^{-1}.$$

For the magnetic field, divide force by charge and velocity:

$$[B] = \frac{MLT^{-2}}{(AT)(LT^{-1})} = MT^{-2}A^{-1}.$$

The velocity in the Lorentz-force expression is needed; omitting it would incorrectly give the same dimensions for the two fields.

Step 2: Form the flux ratio and read the exponents Multiplying each field dimension by area gives

$$[\Phi_E] = ML^3T^{-3}A^{-1}, \qquad [\Phi_B] = ML^2T^{-2}A^{-1}.$$

Consequently,

$$\left[\frac{\Phi_E}{\Phi_B}\right] = \frac{ML^3T^{-3}A^{-1}}{ML^2T^{-2}A^{-1}} = LT^{-1}.$$

Comparing with $M^PL^QT^RA^S$, we obtain $P = 0$, $Q = 1$, $R = -1$, and $S = 0$. The requested pair is therefore $(1, -1)$. Options (A), (B) and (C) have incorrect length and time powers; in particular, this ratio must have the dimensions of a speed, not inverse area or an acceleration.

Step 3: Conclusion The exponents are $\boxed{Q = 1,\ R = -1}$. Hence, the correct option is (D).

Quick Tip to Solve: The common area dimensions cancel between the two fluxes, leaving E/B, which has the dimensions of velocity.

28. Two simple pendulums having lengths l_1 and l_2 with negligible string mass undergo angular displacements θ_1 and θ_2, from their mean positions, respectively. If the angular accelerations of both pendulums are same, then which expression is correct? [4 marks]

(A) $\theta_1 l_2 = \theta_2 l_1$

(B) $\theta_1 l_1 = \theta_2 l_2$

(C) $\theta_1 l_2^2 = \theta_2 l_1^2$

(D) $\theta_1 l_1^2 = \theta_2 l_2^2$

Correct Answer: (A) $\theta_1 l_2 = \theta_2 l_1$

Solution:

Concept:

- For a simple pendulum with a massless string, the restoring torque about the support is $\tau = -mgl\sin\theta$, and its moment of inertia is $I = ml^2$.
- In the small-angle approximation used for simple-pendulum oscillations, $\sin\theta \simeq \theta$ with angles in radians. The angular acceleration is consequently $\alpha = -g\theta/l$, directed towards the mean position.

Step 1: Relate angular acceleration to displacement and length Apply the rotational equation of motion, $\tau = I\alpha$, to a pendulum of length l:

$$-mgl\sin\theta = ml^2\alpha.$$

Cancelling the bob mass and one factor of the length gives $\alpha = -(g/l)\sin\theta$. For small oscillations this becomes

$$\alpha = -\frac{g}{l}\theta.$$

Thus angular acceleration is proportional to angular displacement but inversely proportional to length.

It is not proportional to l or to l^2.

Step 2: Equate the accelerations and compare the options For the two pendulums at the same gravitational acceleration,

$$\alpha_1 = -\frac{g\theta_1}{l_1}, \qquad \alpha_2 = -\frac{g\theta_2}{l_2}.$$

The given equality $\alpha_1 = \alpha_2$ implies

$$\frac{\theta_1}{l_1} = \frac{\theta_2}{l_2} \quad \Longrightarrow \quad \theta_1 l_2 = \theta_2 l_1.$$

This is option (A). Option (B) would instead require equal products θl; options (C) and (D) incorrectly introduce squared lengths. For example, doubling the length requires doubling the small angular displacement to retain the same angular acceleration. Outside the small-angle regime the exact relation would involve $\sin\theta_1/l_1 = \sin\theta_2/l_2$.

Step 3: Conclusion For simple-pendulum small oscillations, $\boxed{\theta_1 l_2 = \theta_2 l_1}$. Hence, the correct option is (A).

Quick Tip to Solve: Derive angular acceleration from torque divided by ml^2; its length dependence is $1/l$, whereas the angular frequency varies as $1/\sqrt{l}$.

29. In a Young's double slit experiment, the slits are separated by 0.2 mm . If the slits separation is increased to 0.4 mm , the percentage change of the fringe width is : [4 marks]

(A) 25%

(B) 50%

(C) 0%

(D) 100%

Correct Answer: (B) 50%

Solution:

Concept:

- In Young's double-slit experiment, the separation between successive bright fringes is $\beta = \lambda D/d$, where λ is the wavelength, D is the screen distance and d is the slit separation.
- With wavelength and screen distance unchanged, β is inversely proportional to d. Percentage change must be measured relative to the original fringe width, not relative to the new value.

Step 1: Calculate the ratio of the fringe widths Let the initial fringe width be β_1 for $d_1 = 0.2\,\text{mm}$, and let the final width be β_2 for $d_2 = 0.4\,\text{mm}$. The common factors cancel in their ratio:

$$\frac{\beta_2}{\beta_1} = \frac{\lambda D/d_2}{\lambda D/d_1} = \frac{d_1}{d_2} = \frac{0.2}{0.4} = \frac{1}{2}.$$

The slit separation has doubled, so the fringes become half as wide. Both slit separations are in the same unit, so converting millimetres to metres is unnecessary in this ratio.

Step 2: Express the change as a percentage of the original width The signed percentage change is

$$\frac{\beta_2 - \beta_1}{\beta_1} \times 100 = \left(\frac{1}{2} - 1\right) 100 = -50\%.$$

The negative sign means a decrease. Since the options give positive magnitudes of change, choose 50%. The 25% choice would require a final width of three quarters of the original, 0% would require no change in separation, and 100% confuses the doubling of slit separation with the percentage change of fringe width.

Step 3: Conclusion The fringe width decreases by $\boxed{50\%}$, a signed change of -50%. Hence, the correct option is (B).

Quick Tip to Solve: For a finite change, first use $\beta_2/\beta_1 = d_1/d_2$, then subtract one; do not apply the small-change approximation to a doubling.

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30. Current passing through a wire as function of time is given as $I(t) = 0.02t + 0.01$ A. The charge that will flow through the wire from $t = 1$ s to $t = 2$ s is [4 marks]

(A) 0.02 C

(B) 0.07 C

(C) 0.06 C

(D) 0.04 C

Correct Answer: (D) 0.04 C

Solution:

Concept:

- Instantaneous current is the rate of charge flow: $I = dq/dt$. The charge passing a cross-section between two times is the definite integral $\Delta q = \int_{t_1}^{t_2} I(t)\, dt$.
- For current varying linearly with time, its average over an interval equals the arithmetic mean of the endpoint currents. Multiplying that average by the interval duration provides an independent check on integration.

Step 1: Integrate the given current over the stated interval Here $I(t) = 0.02t + 0.01$ in amperes when t is in seconds. Use the limits $1\,\text{s}$ and $2\,\text{s}$, rather than integrating from zero:

$$\Delta q = \int_1^2 (0.02t + 0.01)\, dt = \left[0.01t^2 + 0.01t\right]_1^2.$$

The antiderivative at the upper limit is $0.01(4) + 0.01(2) = 0.06$, while at the lower limit it is $0.01(1) + 0.01(1) = 0.02$. Thus,

$$\Delta q = 0.06 - 0.02 = 0.04\,\text{C}.$$

The unit is ampere-second, equivalent to coulomb.

Step 2: Check using the average current and eliminate alternatives At the endpoints, $I(1) = 0.03\,\text{A}$

and $I(2) = 0.05\,\text{A}$. Since the graph is a straight line, its average height is

$$I_{\text{avg}} = \frac{0.03 + 0.05}{2} = 0.04\,\text{A}.$$

Over $2 - 1 = 1\,\text{s}$, this gives $\Delta q = 0.04(1) = 0.04\,\text{C}$, confirming the integral. Option (C), $0.06\,\text{C}$, is merely the upper-limit antiderivative value before subtracting the lower limit. Option (A) is the lower-limit value, and option (B) exceeds even $I(2)\Delta t = 0.05\,\text{C}$, so cannot be correct.

Step 3: Conclusion The charge flowing from $1\,\text{s}$ to $2\,\text{s}$ is $\boxed{0.04\,\text{C}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a linear current, use the area of a trapezium: endpoint-current average times the time interval; always subtract the lower integration limit.

---

31. An alternating current is represented by the equation, $i = 100\sqrt{2}\sin(100\pi t)$ ampere. The RMS value of current and the frequency of the given alternating current are [4 marks]

(A) $\frac{100}{\sqrt{2}}$ A, 100 Hz

(B) $50\sqrt{2}$ A, 50 Hz

(C) $100\sqrt{2}$ A, 100 Hz

(D) 100 A, 50 Hz

Correct Answer: (D) 100 A, 50 Hz

Solution:

Concept:

- For a sinusoidal current $i = I_0 \sin(\omega t)$, I_0 is its peak value and ω is its angular frequency in radians per second.
- The root mean square current is $I_{\text{rms}} = I_0/\sqrt{2}$, while the frequency in cycles per second is $f = \omega/(2\pi)$. These are different conversions.

Step 1: Identify the peak current and angular frequency Compare the given expression $i = 100\sqrt{2}\sin(100\pi t)\,\text{A}$ with the standard sinusoidal form. The coefficient outside the sine gives the peak current, whereas the coefficient of time inside the sine gives the angular frequency. Thus

$$I_0 = 100\sqrt{2}\,\text{A}, \qquad \omega = 100\pi\,\text{rad}\,\text{s}^{-1}.$$

The peak current is not itself the RMS current, and angular frequency is not the frequency in hertz.

Step 2: Convert to RMS current and ordinary frequency Taking the square root of the mean of i^2 over one complete cycle uses $\langle \sin^2(\omega t) \rangle = 1/2$. Therefore

$$I_{\text{rms}} = \frac{100\sqrt{2}}{\sqrt{2}} = 100\,\text{A}.$$

One cycle corresponds to a phase change of 2π, so

$$f = \frac{100\pi}{2\pi} = 50\,\text{Hz}.$$

Option (A) has both an incorrect current and frequency. Option (B) has the correct frequency but an incorrect RMS current. Option (C) retains the peak current and also gives the wrong frequency.

Step 3: Conclusion The RMS current is 100 A and the frequency is 50 Hz. Hence, the correct option is (D).

Quick Tip to Solve: Read the amplitude and coefficient of time separately: divide the former by $\sqrt{2}$, and the latter by 2π.

32. Two liquids A and B have θ_A and θ_B as contact angles in a capillary tube. If $K = \cos\theta_A/\cos\theta_B$, then identify the correct statement: [4 marks]

(A) K is negative, then liquid A and liquid B have convex meniscus.

(B) K is negative, then liquid A and liquid B have concave meniscus.

(C) K is zero, then liquid A has convex meniscus and liquid B has concave meniscus.

(D) K is negative, then liquid A has concave meniscus and liquid B has convex meniscus.

Correct Answer: (D) K is negative, then liquid A has concave meniscus and liquid B has convex meniscus.

Solution:

Concept:

- The contact angle is measured through the liquid. A wetting liquid has $\theta < 90^\circ$, $\cos\theta > 0$, and a concave meniscus; a non-wetting liquid has $\theta > 90^\circ$, $\cos\theta < 0$, and a convex meniscus.
- The sign of $K = \cos\theta_A/\cos\theta_B$ compares the two wetting behaviours. A negative quotient requires opposite signs of the two nonzero cosines.

Step 1: Determine the signs for the proposed shapes If liquid A has a concave meniscus, its contact angle is acute and $\cos\theta_A > 0$. If liquid B has a convex meniscus, its contact angle is obtuse and $\cos\theta_B < 0$. For this configuration,

$$K = \frac{\text{positive}}{\text{negative}} < 0.$$

Thus the configuration described in option (D) is the admissible concave–convex pairing for a negative ratio.

Step 2: Reject the other proposed pairings If both menisci are convex, both cosines are negative, so their quotient is positive; option (A) cannot represent $K < 0$. If both are concave, both cosines are positive and the quotient is again positive; option (B) is also excluded. A zero value of K, with a nonzero denominator, requires $\cos\theta_A = 0$, hence $\theta_A = 90^\circ$. Liquid A then has a plane meniscus rather than the convex one stated in option (C). Strictly, $K < 0$ alone does not specify which named liquid is wetting: the reversed convex–concave pairing also has a negative ratio. Among the listed configurations, (D) is the compatible one.

Step 3: Conclusion A concave meniscus for A and a convex meniscus for B gives $K < 0$, while the

other listed pairings fail their stated signs. Hence, the correct option is (D).

Quick Tip to Solve: Map concave to a positive cosine and convex to a negative cosine before taking the ratio; a right contact angle gives a plane meniscus.

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33. Which of the following are correct expression for torque acting on a body?

A. $\vec{\tau} = \vec{r} \times \vec{L}$

B. $\vec{\tau} = \frac{d}{dt}(\vec{r} \times \vec{p})$

C. $\vec{\tau} = \vec{r} \times \frac{d\vec{p}}{dt}$

D. $\vec{\tau} = I\vec{\alpha}$

E. $\vec{\tau} = \vec{r} \times \vec{F}$

($\vec{r}$ = position vector; $\vec{p}$ = linear momentum; $\vec{L}$ = angular momentum; $\vec{\alpha}$ = angular acceleration; I = moment of inertia; $\vec{F}$ = force; t = time)

Choose the correct answer from the options given below: [4 marks]

(A) A, B, D and E Only

(B) C and D Only

(C) B, C, D and E Only

(D) B, D and E Only

Correct Answer: (C) B, C, D and E Only

Solution:

Concept:

- About a fixed origin, the torque of a force is $\vec{\tau} = \vec{r} \times \vec{F}$, and angular momentum is $\vec{L} = \vec{r} \times \vec{p}$. Newton's second law is $d\vec{p}/dt = \vec{F}$.
- The net torque equals $d\vec{L}/dt$. For rigid-body rotation about a fixed axis of constant moment of inertia, the rotational equation reduces to $\tau = I\alpha$.

Step 1: Check the angular-momentum and force expressions Differentiate the angular momentum using the product rule:

$$\frac{d}{dt}(\vec{r} \times \vec{p}) = \vec{v} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}.$$

For a particle of fixed mass, $\vec{p} = m\vec{v}$, so $\vec{v} \times \vec{p} = 0$. Therefore

$$\frac{d}{dt}(\vec{r} \times \vec{p}) = \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \vec{F} = \vec{\tau}.$$

Consequently statements B, C and E are correct. For a collection of particles these expressions are summed over the particles to obtain the net external torque.

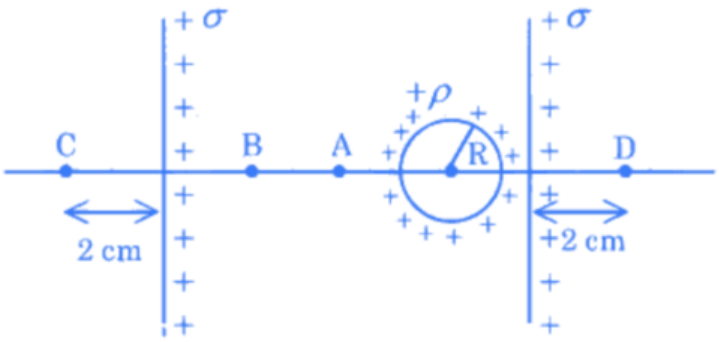
Step 2: Check the rotational equation and eliminate the false expression Under the usual fixed-axis, constant-I rotational model, $L = I\omega$, and differentiation gives $\tau = I\alpha$, which is statement D. Statement A, $\vec{r} \times \vec{L}$, is not torque: it does not take a time derivative of angular momentum and has dimensions $\mathrm{kg\,m^3\,s^{-1}}$, rather than $\mathrm{kg\,m^2\,s^{-2}}$. Option (A) includes this false statement; option (B)

omits B and E; option (D) omits the valid statement C. Only option (C) includes exactly B, C, D and E.

Step 3: Conclusion The correct expressions are B, C, D and E only, with D understood in the standard fixed-axis rotational setting. Hence, the correct option is (C).

Quick Tip to Solve: For torque questions start from $\vec{r}\times\vec{F}$ or $d\vec{L}/dt$; a dimensional check immediately eliminates $\vec{r}\times\vec{L}$.

34. Two infinite identical charged sheets and a charged spherical body of charge density ' ρ ' are arranged as shown in figure. Then the correct relation between the electrical fields at $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D points is: [4 marks]



(A) $\left|\vec{E}_A\right| = \left|\vec{E}_B\right|; \vec{E}_C > \vec{E}_D$
(B) $\vec{E}_A = \vec{E}_B; \vec{E}_C = \vec{E}_D$
(C) $\vec{E}_C \neq \vec{E}_D; \vec{E}_A > \vec{E}_B$
(D) $\vec{E}_A > \vec{E}_B; \vec{E}_C = \vec{E}_D$

Correct Answer: (C) $\vec{E}_C \neq \vec{E}_D; \vec{E}_A > \vec{E}_B$

Solution:

Concept:

- An infinite sheet of positive surface charge density σ produces a uniform field of magnitude $\sigma/(2\varepsilon_0)$, directed away from the sheet on either side.
- For a spherically symmetric charged body, the exterior field is that of its total charge Q concentrated at its centre: $E_s = Q/(4\pi\varepsilon_0 r^2)$. Fields add vectorially.

Step 1: Compare the fields between the two identical sheets At both A and B, the field of the left positive sheet points right and that of the right positive sheet points left. Their equal magnitudes cancel exactly. The resultant field at each point is therefore the sphere's field. Both points lie to the left of the positively charged sphere, so those fields point left. Since A is closer to the centre than B,

$$E_A = \frac{Q}{4\pi\varepsilon_0 r_A^2} > \frac{Q}{4\pi\varepsilon_0 r_B^2} = E_B.$$

The comparisons in the answer choices are understood as comparisons of field magnitudes.

Step 2: Compare the exterior fields and select the relation Outside the pair of sheets their fields reinforce. Their combined magnitude is the same, σ/ε_0, at C and at D, directed left at C and right at D. The sphere's field points in the same respective directions. The diagram places C and D equally far from their adjacent sheets, but the sphere is closer to the right sheet; therefore $r_D < r_C$. Hence

$$E_C = \frac{\sigma}{\varepsilon_0} + \frac{Q}{4\pi\varepsilon_0 r_C^2}, \qquad E_D = \frac{\sigma}{\varepsilon_0} + \frac{Q}{4\pi\varepsilon_0 r_D^2},$$

giving $E_D > E_C$. Their vectors are also unequal because they point oppositely. Options (A) and (B) incorrectly equate the interior magnitudes, while (D) incorrectly equates the exterior fields.

Step 3: Conclusion The field magnitudes obey $E_A > E_B$, and $\vec{E}_C \neq \vec{E}_D$. Hence, the correct option is (C).

Quick Tip to Solve: Cancel the identical sheet fields only between the sheets. Outside, add them first and then compare distances from the sphere's centre.

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35. Considering the Bohr model of hydrogen like atoms, the ratio of the radius of 5^{th} orbit of the electron in Li^{2+} and $\text{He}^{+}$is [4 marks]

(A) $\frac{3}{2}$
(B) $\frac{2}{3}$
(C) $\frac{4}{9}$
(D) $\frac{9}{4}$

Correct Answer: (B) $\frac{2}{3}$

Solution:

Concept:

- For a one-electron ion of nuclear charge $+Ze$, the Bohr radius of orbit n is $r_n = a_0 n^2/Z$, with a_0 the hydrogen ground-state radius.
- For two ions in the same principal quantum state, n^2 cancels in a radius ratio. The radius is inversely proportional to nuclear charge, rather than proportional to it.

Step 1: Identify the nuclear charges and the common orbit Lithium has atomic number 3, so Li^{2+} has nuclear charge $+3e$ and one remaining electron. Helium has atomic number 2, so He^{+} has nuclear charge $+2e$ and is also a one-electron ion. Both electrons are specified to occupy the fifth orbit, so $n = 5$ in each case. Thus

$$r_5(\text{Li}^{2+}) = \frac{25a_0}{3}, \qquad r_5(\text{He}^{+}) = \frac{25a_0}{2}.$$

The ionic superscript gives the net ionic charge; it is not the value of Z.

Step 2: Take the ratio in the requested order The requested ratio has lithium in the numerator and helium in the denominator. Therefore

$$\frac{r_5(\text{Li}^{2+})}{r_5(\text{He}^{+})} = \frac{25a_0/3}{25a_0/2} = \frac{2}{3}.$$

Lithium's greater nuclear attraction makes its fifth orbit smaller, providing a physical check that the ratio must be below one. Option (A) is the reciprocal of the requested ratio. Options (C) and (D) square a nuclear-charge ratio unnecessarily: the radius depends on $1/Z$, not on $1/Z^2$. The common fifth-orbit factor does not alter the result.

Step 3: Conclusion The ratio of the fifth-orbit radius in Li^{2+} to that in He^{+} is $2/3$. Hence, the correct option is (B).

Quick Tip to Solve: For equal orbit numbers in hydrogen-like ions, write the inverse atomic-number ratio directly; use nuclear charge, not net ionic charge.

36. When an object is placed 40 cm away from a spherical mirror an image of magnification $\frac{1}{2}$ is produced. To obtain an image with magnification of $\frac{1}{3}$, the object is to be moved : [4 marks]

(A) 20 cm away from the mirror.

(B) 20 cm towards the mirror.

(C) 80 cm away from the mirror.

(D) 40 cm away from the mirror.

Correct Answer: (D) 40 cm away from the mirror.

Solution:

Concept:

- Using the Cartesian sign convention, a real object in front of a mirror has $u < 0$, and the signed transverse magnification is $m = -v/u$.
- The spherical mirror equation is $1/f = 1/v + 1/u$. The stated positive magnifications describe erect diminished virtual images, consistent with a convex mirror.

Step 1: Find the focal length from the first position The object is initially $40\,\text{cm}$ in front of the mirror, so $u_1 = -40\,\text{cm}$. Taking the printed magnification $m_1 = +1/2$,

$$v_1 = -m_1 u_1 = -\frac{1}{2}(-40) = +20\,\text{cm}.$$

The positive image distance places the virtual image behind the mirror. Substitute in the mirror equation:

$$\frac{1}{f} = \frac{1}{20} + \frac{1}{-40} = \frac{1}{40}, \qquad f = +40\,\text{cm}.$$

This positive focal length confirms a convex mirror, which gives diminished erect images for real objects.

Step 2: Find the position for the new magnification For $m_2 = +1/3$, the magnification relation gives $v_2 = -u_2/3$. The focal length remains unchanged when the object is moved. Hence

$$\frac{1}{40} = \frac{1}{-u_2/3} + \frac{1}{u_2} = -\frac{2}{u_2}, \qquad u_2 = -80\,\text{cm}.$$

The object must move from a distance of $40\,\text{cm}$ to $80\,\text{cm}$, an increase of $40\,\text{cm}$. A move towards the

mirror increases, rather than decreases, the erect image magnification. Moving away by $20\,\text{cm}$ or $80\,\text{cm}$ would give object distances $60\,\text{cm}$ or $120\,\text{cm}$, neither of which gives $m = 1/3$.

Step 3: Conclusion The object is to be moved $40\,\text{cm}$ away from the mirror. Hence, the correct option is (D).

Quick Tip to Solve: Preserve the sign of the stated magnification. For a convex mirror and object distance $d > 0$, $m = f/(f + d)$, so a smaller image requires a larger d.

37. In an experiment with a closed organ pipe, it is filled with water by $\left(\frac{1}{5}\right)$ th of its volume. The frequency of the fundamental note will change by [4 marks]

(A) 20%

(B) 25%

(C) -20%

(D) -25%

Correct Answer: (B) 25%

Solution:

Concept:

- A closed organ pipe has a displacement node at its closed end and an antinode at its open end. Its fundamental wavelength is $4L$, where L is the length of the air column.
- For sound speed v, the fundamental frequency is $f = v/(4L)$. At unchanged temperature, shortening the air column increases its frequency inversely with length.

Step 1: Find the new vibrating air length Let the original air-column length be L. The pipe has uniform cross-section, so filling one-fifth of its volume with water occupies one-fifth of its length. The water surface becomes the closed boundary for the air above it. Thus the remaining air length is

$$L\prime = L - \frac{L}{5} = \frac{4L}{5}.$$

The vibrating medium is the air column, not the water-filled part; the pipe remains closed at one end and open at the other.

Step 2: Compare the two fundamental frequencies Initially $f = v/(4L)$, whereas after filling,

$$f\prime = \frac{v}{4(4L/5)} = \frac{5}{4}f.$$

Therefore the fractional increase is

$$(f\prime - f)/f = \frac{5}{4} - 1 = \frac{1}{4}.$$

Multiplying by 100 gives a positive change of 25%. Option (A) confuses the 20 percent reduction in length with the percentage increase in its reciprocal. Options (C) and (D) predict a decrease, contrary to the shorter air column and its higher frequency.

Step 3: Conclusion The fundamental frequency increases by 25%. Hence, the correct option is (B).

Quick Tip to Solve: When a closed pipe is filled through a fraction x of its length, use $f\prime/f = 1/(1-x)$, rather than assigning the same percentage change to frequency.

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38. A body of mass m is suspended by two strings making angles θ_1 and θ_2 with the horizontal ceiling with tensions T_1 and T_2 simultaneously. T_1 and T_2 are related by $T_1 = \sqrt{3}T_2$, the angles θ_1 and θ_2 are [4 marks]

(A) $\theta_1 = 30°\theta_2 = 60°$ with $T_2 = \frac{3\text{mg}}{4}$
(B) $\theta_1 = 45°\theta_2 = 45°$ with $T_2 = \frac{3mg}{4}$
(C) $\theta_1 = 30°\theta_2 = 60°$ with $T_2 = \frac{4mg}{5}$
(D) $\theta_1 = 60°\theta_2 = 30°$ with $T_2 = \frac{mg}{2}$

Correct Answer: (D) $\theta_1 = 60°\theta_2 = 30°$ with $T_2 = \frac{mg}{2}$

Solution:

Concept:

- For a suspended body at rest, the resultant force vanishes separately in the horizontal and vertical directions.
- When a string makes an angle θ with the horizontal, its horizontal and upward components are $T\cos\theta$ and $T\sin\theta$, respectively.

Step 1: Use horizontal force balance The two strings pull toward opposite sides of the ceiling. Their horizontal components must cancel:

$$T_1\cos\theta_1 = T_2\cos\theta_2.$$

Substituting the given tension relation $T_1 = \sqrt{3}T_2$ and cancelling the nonzero tension gives

$$\sqrt{3}\cos\theta_1 = \cos\theta_2.$$

For $\theta_1 = 60°$ and $\theta_2 = 30°$, both sides equal $\sqrt{3}/2$. Equal angles would require equal tensions, so the 45-degree pair in (B) fails. The 30-degree, 60-degree ordering in (A) and (C) gives left and right sides $3/2$ and $1/2$, so these also fail.

Step 2: Check the vertical balance and tension The upward components together support the weight:

$$T_1\sin\theta_1 + T_2\sin\theta_2 = mg.$$

For the angle pair that passes horizontal balance,

$$\sqrt{3}T_2\frac{\sqrt{3}}{2} + T_2\frac{1}{2} = mg.$$

Hence $2T_2 = mg$, giving

$$T_2 = \frac{mg}{2}, \qquad T_1 = \frac{\sqrt{3}mg}{2}.$$

These tensions also confirm the specified ratio. In particular, the more steeply inclined string carries the larger tension here; using sine for the horizontal component would reverse the angles incorrectly.

Step 3: Conclusion The angles are $\theta_1 = 60°$, $\theta_2 = 30°$, and $T_2 = mg/2$. Hence, the correct option is (D).

Quick Tip to Solve: Check horizontal equilibrium first to select the angle pair, then use vertical equilibrium to verify the quoted tension. Measure the components from the stated horizontal ceiling.

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39. A small mirror of mass m is suspended by a massless thread of length l. Then the small angle through which the thread will be deflected when a short pulse of laser of energy E falls normal on the mirror

(c = speed of light in vacuum and g = acceleration due to gravity) [4 marks]

(A) $\theta = \frac{E}{mc\sqrt{gl}}$

(B) $\theta = \frac{E}{2mc\sqrt{gl}}$

(C) $\theta = \frac{3E}{4mc\sqrt{gl}}$

(D) $\theta = \frac{2E}{mc\sqrt{gl}}$

Correct Answer: (D) $\theta = \frac{2E}{mc\sqrt{gl}}$

Solution:

Concept:

- A light pulse of energy E carries momentum E/c. Normal reflection reverses its momentum and transfers an impulse $2E/c$ to the mirror.
- After the short impulse, the mirror swings as a pendulum. Its kinetic energy changes to gravitational potential energy, and $1 - \cos\theta \simeq \theta^2/2$ for a small angle in radians.

Step 1: Calculate the initial speed of the mirror During the very short pulse the mirror acquires horizontal momentum before it moves through an appreciable height. The incoming light momentum is E/c, while the reflected light has momentum $-E/c$ along the initial direction. The mirror consequently receives

$$J = \frac{E}{c} - \left(-\frac{E}{c}\right) = \frac{2E}{c}.$$

Thus its speed immediately after the pulse is

$$u = \frac{J}{m} = \frac{2E}{mc}.$$

Using only E/c would describe absorption and would miss the factor of two for a reflecting mirror.

Step 2: Convert the recoil energy into a swing At the maximum deflection the mirror momentarily stops. The thread tension does no work because it is perpendicular to the instantaneous motion. If

the thread length is l, the height gained is $l(1-\cos\theta)$. Energy conservation therefore gives

$$\frac{1}{2}mu^2 = mgl(1-\cos\theta) \simeq \frac{1}{2}mgl\theta^2.$$

Cancelling common factors gives $\theta = u/\sqrt{gl}$. Substituting the recoil speed yields

$$\theta = \frac{2E}{mc\sqrt{gl}}.$$

Option (A) is smaller by two, and (B) and (C) also have incorrect impulse factors; none follows from normal reflection.

Step 3: Conclusion The maximum small deflection is $\theta = 2E/(mc\sqrt{gl})$ radians. Hence, the correct option is (D).

Quick Tip to Solve: Use momentum conservation during the short light pulse and mechanical energy conservation during the later swing. The reflecting surface receives twice the incident light momentum.

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40. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : The kinetic energy needed to project a body of mass m from earth surface to infinity is $\frac{1}{2}$mgR, where R is the radius of earth.

Reason R : The maximum potential energy of a body is zero when it is projected to infinity from earth surface.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) **A** is false but **R** is true

(B) Both **A** and **R** are true and **R** is the correct explanation of **A**

(C) **A** is true but **R** is false

(D) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

Correct Answer: (A) **A** is false but **R** is true

Solution:

Concept:

- With gravitational potential energy defined as zero at infinity, a mass m outside Earth has $U(r) = -GMm/r$, where M is Earth's mass.
- The minimum escape kinetic energy raises the total energy from the surface value to zero, with zero limiting speed at infinity. Surface gravity satisfies $g = GM/R^2$.

Step 1: Evaluate Assertion A At the Earth's surface,

$$U(R) = -\frac{GMm}{R} = -mgR.$$

For the minimum projection energy needed to reach infinity, the final kinetic and potential energies

both tend to zero. Conservation of mechanical energy gives

$$K_{\min} - \frac{GMm}{R} = 0, \qquad K_{\min} = \frac{GMm}{R} = mgR.$$

Thus the required kinetic energy is mgR, not $mgR/2$. Assertion A is false. The familiar factor one-half in kinetic energy does not halve this result: escape speed squared is $2gR$, so $\frac{1}{2}mv_e^2 = mgR$.

Step 2: Evaluate Reason R At every finite positive distance from Earth's centre the potential energy $-GMm/r$ is negative. As the distance increases, it increases toward zero. Its limiting maximum in this convention is therefore

$$\lim_{r\to\infty} U(r) = 0.$$

Reason R correctly states this reference and limiting value, so it is true. It supports the escape-energy calculation, but it cannot make the incorrect numerical value in the assertion true.

Step 3: Match the truth values to the options We have independently obtained a false assertion and a true reason. Option (A) states precisely this combination. Options (B) and (D) require the assertion to be true, while option (C) also incorrectly labels the reason false. The distinction concerns the minimum launch energy; any additional initial energy would remain as kinetic energy at infinity.

Step 4: Conclusion Assertion A is false, whereas Reason R is true. Hence, the correct option is (A).

Quick Tip to Solve: Remember that $mgR/2$ is the magnitude of the total energy of a circular orbit at radius R; the minimum energy to escape from rest at the surface is mgR.

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41. The Boolean expression $Y = A\bar{B}C + \bar{A}\bar{C}$ can be realised with which of the following gate configurations.

A. One 3-input AND gate, 3 NOT gates and one 2-input OR gate, One 2-input AND gate,

B. One 3 -input AND gate, 1 NOT gate, One 2 -input NOR gate and one 2 -input OR gate

C. 3 -input OR gate, 3 NOT gates and one 2 -input AND gate

Choose the correct answer from the options given below: [4 marks]

(A) B, C Only

(B) A, B, C Only

(C) A, B Only

(D) A, C Only

Correct Answer: (C) A, B Only

Solution:

Concept:

- An AND gate forms a product, an OR gate forms a sum, and a NOT gate complements a Boolean variable. Thus the two product terms of $Y = A\bar{B}C + \bar{A}\bar{C}$ can be built separately and then added.
- De Morgan's identity gives $\bar{A}\bar{C} = \overline{A + C}$. A two-input NOR gate therefore directly produces the second term.

Step 1: Check configuration A by direct construction Use the three NOT gates to obtain $\bar{A}$, $\bar{B}$, and $\bar{C}$ from the original variables. Feed $A, \bar{B}, C$ into the three-input AND gate to form $P = A\bar{B}C$. Feed $\bar{A}, \bar{C}$ into the two-input AND gate to form $Q = \bar{A}\bar{C}$. Finally, the two-input OR gate combines them as

$$Y = P + Q = A\bar{B}C + \bar{A}\bar{C}.$$

The stated gates supply every required intermediate signal, so configuration A works.

Step 2: Check configuration B using De Morgan's theorem The single NOT gate supplies $\bar{B}$, and the three-input AND gate then produces $A\bar{B}C$. The NOR gate takes A, C as inputs and produces

$$Q = \overline{A + C} = \bar{A}\bar{C}.$$

The final OR gate combines this with the first product term. Thus configuration B also gives the required output with exactly the available gates.

Step 3: Reject configuration C and select the combination Configuration C contains only one three-input OR gate and one two-input AND gate, besides the three inverters. These do not provide either the two product gates followed by their sum or the AND–NOR–OR construction established above. For example, complementing three literals around an OR gate supplies one product by De Morgan's theorem, leaving no OR stage to add the other independent product. Hence C does not realise the specified sum. Every option except (C) includes configuration C or omits valid configuration A.

Step 4: Conclusion Configurations A and B realise the expression, while C does not. Hence, the correct option is (C).

Quick Tip to Solve: Before counting gates, simplify complemented products with De Morgan's theorem: a NOR gate can replace two input inverters followed by an AND gate.

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42. The mean free path and the average speed of oxygen molecules at 300 K and 1 atm are 3×10^{-7} m and 600 m/s, respectively. Find the frequency of its collisions. [4 marks]

(A) $5 \times 10^{8}/\text{s}$

(B) $9 \times 10^{5}/\text{s}$

(C) $2 \times 10^{10}/\text{s}$

(D) $2 \times 10^{9}/\text{s}$

Correct Answer: (D) $2 \times 10^{9}/\text{s}$

Solution:

Concept:

- The mean free path λ is the average distance travelled by a gas molecule between successive collisions. The average speed $\bar{v}$ is distance travelled per unit time.
- The mean collision time is $\tau = \lambda/\bar{v}$, so the collision frequency for one molecule is $z = 1/\tau = \bar{v}/\lambda$, measured in s^{-1}.

Step 1: Find the time between successive collisions The given quantities are $\lambda = 3 \times 10^{-7}$ m and

$\bar{v} = 600\,\mathrm{m\,s^{-1}}$. The molecule takes, on average,

$$\tau = \frac{\lambda}{\bar{v}} = \frac{3 \times 10^{-7}}{600}\,\mathrm{s} = 5 \times 10^{-10}\,\mathrm{s}$$

to traverse one mean free path. This very short time is consistent with frequent collisions in a gas at atmospheric pressure. Temperature and pressure describe the conditions under which the stated mean free path and speed apply; they need not be inserted again into a separate gas-law calculation.

Step 2: Take the reciprocal and compare the choices The number of collisions per second for a molecule is the reciprocal of this average interval:

$$z = \frac{1}{5 \times 10^{-10}} = 2 \times 10^{9}\,\mathrm{s^{-1}}.$$

Equivalently,

$$z = \frac{600}{3 \times 10^{-7}} = 200 \times 10^{7} = 2 \times 10^{9}\,\mathrm{s^{-1}}.$$

The units follow directly from speed divided by length: $(\mathrm{m\,s^{-1}})/\mathrm{m} = \mathrm{s^{-1}}$. Options (A) and (B) underestimate the resulting rate, and (C) is too large by a factor of ten. An extra factor of $\sqrt{2}$ should not be inserted, since it is already included when mean free path is derived from the molecular collision model.

Step 3: Conclusion The collision frequency is $2 \times 10^{9}\,\mathrm{s^{-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: Once both mean speed and mean free path are given, use $z = \bar{v}/\lambda$ directly and check that the result has units of inverse seconds.

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43. If $\vec{L}$ and $\vec{P}$ represent the angular momentum and linear momentum respectively of a particle of mass ' m ' having position vector as $\vec{r} = a(\hat{i}\cos\omega t + \hat{j}\sin\omega t)$. The direction of force is [4 marks]

(A) Opposite to the direction of $\vec{L}$

(B) Opposite to the direction of $\vec{L} \times \vec{P}$

(C) Opposite to the direction of $\vec{r}$

(D) Opposite to the direction of $\vec{P}$

Correct Answer: (C) Opposite to the direction of $\vec{r}$

Solution:

Concept:

- The force follows Newton's second law: $\vec{F} = m\vec{a}$, where $\vec{a} = d^2\vec{r}/dt^2$. The direction is obtained from the acceleration, without assuming that force follows momentum.
- For planar circular motion, $\vec{P} = m\vec{v}$ is tangential and $\vec{L} = \vec{r} \times \vec{P}$ is perpendicular to the plane. Centripetal acceleration points toward the centre.

Step 1: Differentiate the position vector The given position has constant magnitude a, so the trajectory

is a circle centred at the origin. Differentiating each component gives

$$\vec{v} = a\omega(-\hat{i}\sin\omega t + \hat{j}\cos\omega t).$$

Differentiating once more gives

$$\vec{a} = -a\omega^2(\hat{i}\cos\omega t + \hat{j}\sin\omega t) = -\omega^2\vec{r}.$$

Thus $\vec{F} = -m\omega^2\vec{r}$: its radial direction is inward at every instant.

Step 2: Check the listed vector directions The angular momentum is $\vec{L} = ma^2\omega\hat{k}$, perpendicular to the orbit, whereas $\vec{P}$ is tangent to it. Neither opposite $\vec{L}$ nor opposite $\vec{P}$ is the radial inward direction. Also,

$$\vec{L} \times \vec{P} = -m^2a^2\omega^2\vec{r}.$$

This cross product already points inward, in the direction of force; its opposite points outward. Hence options (A), (B) and (D) do not describe the force. The acceleration calculation directly selects option (C).

Step 3: Conclusion The force is opposite to the position vector $\vec{r}$, toward the centre of the circle. Hence, the correct option is (C).

Quick Tip to Solve: For a position written with sine and cosine at the same angular frequency, differentiate twice: $\vec{a} = -\omega^2\vec{r}$ reveals the force direction immediately.

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44. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : In photoelectric effect, on increasing the intensity of incident light the stopping potential increases.

Reason R : Increase in intensity of light increases the rate of photoelectrons emitted, provided the frequency of incident light is greater than threshold frequency.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**

(B) **A** is false but **R** is true

(C) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(D) **A** is true but **R** is false

Correct Answer: (B) **A** is false but **R** is true

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi = eV_s$, where ϕ is the metal's work function and V_s is the stopping potential.
- At fixed frequency, intensity measures the incident photon flux. Each absorbed photon supplies energy $h\nu$, so changing the number of photons changes the emission rate rather than the maximum

energy per electron.

Step 1: Evaluate Assertion A Keep the frequency and emitting material fixed while increasing the intensity, as required when testing intensity dependence. The equation

$$V_s = \frac{h\nu - \phi}{e}$$

contains frequency and work function, but no intensity. A brighter beam supplies more photons per second, each with unchanged energy. The fastest emitted electrons therefore retain the same maximum kinetic energy, and the retarding potential needed to stop them is unchanged. Assertion A, which says that stopping potential increases with intensity, is false.

Step 2: Evaluate Reason R When $\nu > \nu_0 = \phi/h$, incident photons can eject electrons. Increasing intensity at that frequency increases the number of incident photons per unit time, and consequently increases the rate of photoelectron emission and the saturation photocurrent. Reason R is therefore true. This statement concerns the number of electrons emitted, whereas stopping potential concerns their maximum energy.

Step 3: Match the truth values to an option The combination is A false and R true. Options (A) and (C) both require a true assertion, and option (D) requires a true assertion and false reason. All three conflict with the photoelectric equation and the photon-flux interpretation. Only option (B) has the required truth values.

Step 4: Conclusion Assertion A is false, while Reason R is true. Hence, the correct option is (B).

Quick Tip to Solve: Separate photon energy from photon number: frequency determines stopping potential, while intensity determines the emission rate above threshold.

45. Two small spherical balls of mass 10 g each with charges $-2\mu\text{C}$ and $2\mu\text{C}$, are attached to two ends of very light rigid rod of length 20 cm . The arrangement is now placed near an infinite nonconducting charge sheet with uniform charge density of $100\mu\text{C/m}^2$ such that length of rod makes an angle of 30° with electric field generated by charge sheet. Net torque acting on the rod is: (Take $\varepsilon_o : 8.85 \times 10^{-12}\text{C}^2/\text{Nm}^2$) [4 marks]

(A) 1.12 Nm

(B) 2.24 Nm

(C) 11.2 Nm

(D) 112 Nm

Correct Answer: (A) 1.12 Nm

Solution:

Concept:

- An infinite nonconducting sheet of surface charge density σ produces a uniform electric field $E = \sigma/(2\varepsilon_0)$ on either side.
- Two opposite charges separated by distance d form a dipole with moment magnitude $p = qd$. In a

uniform field the equal and opposite forces form a couple of magnitude $\tau = pE\sin\theta$.

Step 1: Calculate the electric field and dipole moment Convert the supplied quantities to SI units:

$$\sigma = 100 \times 10^{-6}\,\mathrm{C\,m^{-2}}, \quad q = 2 \times 10^{-6}\,\mathrm{C}, \quad d = 0.20\,\mathrm{m}.$$

For a nonconducting sheet the field is

$$E = \frac{100 \times 10^{-6}}{2(8.85 \times 10^{-12})} = 5.65 \times 10^{6}\,\mathrm{N\,C^{-1}}.$$

The separation is the entire rod length, so

$$p = (2 \times 10^{-6})(0.20) = 4.0 \times 10^{-7}\,\mathrm{C\,m}.$$

Step 2: Apply the torque formula The rod and the field make the given angle 30°. Taking the magnitude avoids any dependence on which end of the rod carries the positive charge:

$$\begin{aligned}\tau &= pE\sin 30^\circ \\ &= (4.0 \times 10^{-7})(5.65 \times 10^{6})\left(\frac{1}{2}\right) \\ &\simeq 1.13\,\mathrm{N\,m}.\end{aligned}$$

The closest listed value is $1.12\,\mathrm{N\,m}$. The equal masses do not enter the electric couple; their weights produce no resultant torque about the rod's centre.

Step 3: Rule out the other magnitudes Option (B), $2.24\,\mathrm{N\,m}$, misses the factor $\sin 30^\circ = 1/2$, or uses the conducting-sheet field instead of the nonconducting-sheet field. Options (C) and (D) are too large by factors near ten and one hundred; they are inconsistent with the SI conversions and the calculated dipole moment.

Step 4: Conclusion The electric torque is approximately $1.13\,\mathrm{N\,m}$, represented by $1.12\,\mathrm{N\,m}$ in the choices. Hence, the correct option is (A).

Quick Tip to Solve: Use $E = \sigma/(2\varepsilon_0)$ for a nonconducting sheet and use the full charge separation in $p = qd$; then include the sine of the rod-field angle.

Physics – Section B: Numerical Value Questions (20 marks)

46. A circular ring and a solid sphere having same radius roll down on an inclined plane from rest without slipping. The ratio of their velocities when reached at the bottom of the plane is $\sqrt{\frac{x}{5}}$ where $x =$ _____. [4 marks]

Solution:

Concept:

- For rolling without slipping, $v = R\omega$. Static friction does no work on a fixed incline, so the loss of gravitational potential energy becomes translational plus rotational kinetic energy.
- Writing $I = kmR^2$, conservation of energy gives $mgh = \frac{1}{2}mv^2(1+k)$, and hence $v^2 = 2gh/(1+k)$. A ring has $k = 1$, and a solid sphere has $k = 2/5$.

Step 1: Find the speed of the ring Both bodies start from rest and descend through the same vertical height h. For the ring, $I_r = m_r R^2$. Therefore

$$m_r gh = \frac{1}{2}m_r v_r^2 + \frac{1}{2}m_r R^2 \frac{v_r^2}{R^2} = m_r v_r^2.$$

Thus $v_r^2 = gh$. The mass cancels, so equality of the two bodies' masses is not required. This cancellation is why the unspecified masses do not prevent calculation of the ratio.

Step 2: Find the speed of the solid sphere For a uniform solid sphere, $I_s = \frac{2}{5}m_s R^2$. Its energy equation becomes

$$m_s gh = \frac{1}{2}m_s v_s^2 + \frac{1}{2}\left(\frac{2}{5}m_s R^2\right)\frac{v_s^2}{R^2} = \frac{7}{10}m_s v_s^2.$$

Consequently $v_s^2 = 10gh/7$. The sphere moves faster because a smaller fraction of its energy is tied up in rotation at any given translational speed.

Step 3: Form the stated ratio and determine x Take the ratio in the order in which the bodies are named: ring to solid sphere. Then

$$\frac{v_r}{v_s} = \sqrt{\frac{gh}{10gh/7}} = \sqrt{\frac{7}{10}} = \sqrt{\frac{3.5}{5}}.$$

Comparing with $\sqrt{x/5}$ gives the exact value $x = 3.5$. If an integer entry is required, this rounds to 4; the accepted answer key lists both 3.5 and 4. The ratio, and therefore x, is dimensionless.

Step 4: Conclusion The exact dimensionless value is $\boxed{x = 3.5}$. Its nearest-integer entry is $\boxed{4}$; the final key accepts both forms.

Quick Tip to Solve: Use $v^2 = 2gh/(1 + I/mR^2)$ for each rolling body before taking the speed ratio, and keep the bodies in the order named.

47. Distance between object and its image (magnified by $-\frac{1}{3}$) is 30 cm . The focal length of the mirror used is $\left(\frac{x}{4}\right)$ cm, where magnitude of value of x is _____. [4 marks]

Solution:

Concept:

- With Cartesian mirror signs, transverse magnification is $m = -v/u$, and the mirror formula is $1/f = 1/v + 1/u$. Distances are measured from the pole.
- A negative magnification means an inverted real image. For a real object and real image formed by a concave mirror, both u and v are negative; their separation is $|u - v|$, not the sum of their magnitudes.

Step 1: Relate object and image distances The given magnification is $-1/3$. Therefore

$$-\frac{v}{u} = -\frac{1}{3} \quad \Longrightarrow \quad v = \frac{u}{3}.$$

Write $u = -U$, where $U > 0$. Then $v = -U/3$. The object and real image lie on the same side of the mirror, and the diminished image lies nearer its pole. Their stated separation gives

$$\left|-U + \frac{U}{3}\right| = \frac{2U}{3} = 30\,\text{cm}.$$

It follows that $U = 45\,\text{cm}$, so $u = -45\,\text{cm}$ and $v = -15\,\text{cm}$.

Step 2: Calculate the focal length Substitute these signed distances into the mirror formula:

$$\begin{aligned}\frac{1}{f} &= \frac{1}{-45} + \frac{1}{-15} \\ &= -\frac{1}{45} - \frac{3}{45} = -\frac{4}{45}\,\text{cm}^{-1}.\end{aligned}$$

Hence $f = -45/4\,\text{cm} = -11.25\,\text{cm}$. The negative focal length confirms a concave mirror. The image distance, $15\,\text{cm}$ in magnitude, lies beyond its focal point but inside its centre of curvature, consistent with a diminished real image.

Step 3: Extract the requested magnitude The question writes the focal length as $(x/4)\,\text{cm}$. Thus the signed value is $x = -45$, and the requested magnitude is $|x| = 45$. Since x is the numerical coefficient multiplying the stated centimetre unit, the response to the blank is dimensionless; the physical focal length has magnitude $11.25\,\text{cm}$.

Step 4: Conclusion The mirror has $f = -11.25\,\text{cm}$, so the magnitude of the requested coefficient is $\boxed{|x| = 45}$.

Quick Tip to Solve: For an inverted mirror image, first use $m = -v/u$. Object-image separation is the difference of distances when both lie in front of the mirror.

48. Two slabs with square cross section of different materials $(1, 2)$ with equal sides (l) and thickness d_1 and d_2 such that $d_2 = 2d_1$ and $l > d_2$. Considering lower edges of these slabs are fixed to the floor, we apply equal shearing force on the narrow faces. The angle of deformation is $\theta_2 = 2\theta_1$. If the shear moduli of material 1 is 4×10^9 N/m^2, then shear moduli of material 2 is $x \times 10^9$ N/m^2, where value of x is _____. [4 marks]

Solution:

Concept:

- Shear modulus is the ratio of shearing stress to shear strain: $G = (F/A)/\tan\theta$. For small deformation, $\tan\theta \simeq \theta$, with the angle measured in radians.
- Each upright slab has square face $l \times l$ and thickness d. A narrow face has area $A = ld$; therefore the two equal forces act over different areas when the slab thicknesses differ.

Step 1: Identify the stressed areas The lower edges are fixed and the same shearing force F is applied to each narrow face. The relevant areas are

$$A_1 = ld_1, \qquad A_2 = ld_2 = 2ld_1 = 2A_1.$$

Thus the shearing stress in slab 2 is half the stress in slab 1:

$$\frac{F/A_2}{F/A_1} = \frac{A_1}{A_2} = \frac{1}{2}.$$

Using l^2 as the stressed area would describe the broad square face, whereas the question explicitly applies force on the narrow faces.

Step 2: Compare stress-to-strain ratios The given deformation angles satisfy $\theta_2 = 2\theta_1$. In the small elastic-deformation regime, the corresponding shear strains have the same ratio. Consequently

$$\begin{aligned}\frac{G_2}{G_1} &= \frac{F/(ld_2\theta_2)}{F/(ld_1\theta_1)} \\ &= \frac{d_1\theta_1}{d_2\theta_2} = \frac{d_1\theta_1}{(2d_1)(2\theta_1)} = \frac{1}{4}.\end{aligned}$$

The second slab has lower stress but a greater strain, so its modulus must be smaller, providing an independent direction check on the ratio.

Step 3: Substitute the supplied modulus With $G_1 = 4 \times 10^9\,\mathrm{N\,m^{-2}}$,

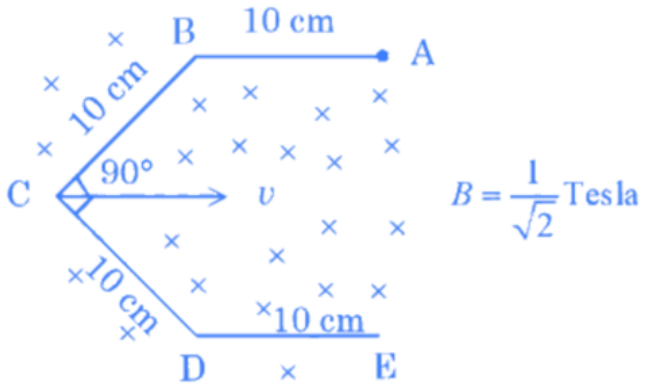
$$G_2 = \frac{1}{4}(4 \times 10^9) = 1 \times 10^9\,\mathrm{N\,m^{-2}}.$$

Comparison with $G_2 = x \times 10^9\,\mathrm{N\,m^{-2}}$ gives $x = 1$. The common force and common side length cancel; no numerical value of either is needed. The modulus carries pressure units, while x itself is a dimensionless numerical coefficient.

Step 4: Conclusion The shear modulus is $G_2 = 1.0 \times 10^9\,\mathrm{N\,m^{-2}}$. Hence the requested value is $\boxed{x = 1}$.

Quick Tip to Solve: For equal shear forces, compare $A\theta$: modulus is inversely proportional to this product. Here both the narrow-face area and the strain double.

49. Conductor wire ABCDE with each arm 10 cm in length is placed in magnetic field of $\frac{1}{\sqrt{2}}$ Tesla, perpendicular to its plane. When conductor is pulled towards right with constant velocity of $10\ \text{cm/s}$, induced emf between points A and E is _____ mV . [4 marks]



Solution:

Concept:

- For a rigid conductor translating with uniform velocity in a uniform magnetic field, the motional emf is $\mathcal{E}_{AE} = \int_A^E (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$. The cross product is constant along the wire.
- Consequently, the emf depends on the separation of the endpoints along $\mathbf{v} \times \mathbf{B}$, rather than on the total bent length of the conductor. Here this is the vertical separation of A and E.

Step 1: Find the effective transverse length The arms AB and DE are horizontal, hence parallel to the velocity, and have zero projection along the magnetic force. The two oblique arms BC and CD are each $10\,\text{cm}$ long and enclose $90°$ at C. Their symmetric inclinations to the horizontal are $45°$. Thus the vertical endpoint separation is

$$h = 2(10\,\text{cm}) \sin 45° = 10\sqrt{2}\,\text{cm} = 0.10\sqrt{2}\,\text{m}.$$

Adding all four arm lengths would incorrectly count segments which contribute no emf.

Step 2: Calculate the emf between the endpoints The velocity is $v = 10\,\text{cm}\,\text{s}^{-1} = 0.10\,\text{m}\,\text{s}^{-1}$, and the field magnitude is $B = 1/\sqrt{2}\,\text{T}$. The magnitude of the emf therefore is

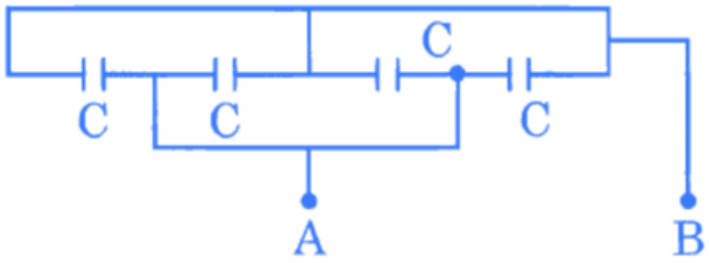
$$|\mathcal{E}| = Bvh = \frac{1}{\sqrt{2}}(0.10)(0.10\sqrt{2}) = 0.010\,\text{V}.$$

The field is into the page and the velocity is to the right, so the magnetic force on positive charges is upwards. A is therefore at a higher potential than E; the requested value is the positive magnitude in millivolts.

Step 3: Conclusion The induced emf between A and E is $0.010\,\text{V} = \boxed{10\,\text{mV}}$. The numerical entry is 10.

Quick Tip to Solve: For a bent wire moving rigidly in a uniform field, project the endpoint separation onto $\mathbf{v} \times \mathbf{B}$; do not add the lengths of all its arms.

50. Four capacitors each of capacitance $16\mu F$ are connected as shown in the figure. The capacitance between points A and B is : _____ (in μF). [4 marks]



Solution:

Concept:

- Points joined by ideal conducting wires belong to the same electrical node and have the same potential. The appearance of capacitors in a straight row does not, by itself, imply a series connection.
- Capacitors are in parallel when each is connected between the same two nodes. Their potential differences are equal, their charges add, and the equivalent capacitance is $C_{\text{eq}} = \sum_i C_i$.

Step 1: Identify the two conducting nodes Number the four capacitors from left to right. The upper connecting wire joins the left end of the first capacitor, the junction between the second and third capacitors, and the right end of the fourth capacitor. This entire upper conductor is connected to B. The lower wire joins the junction between the first and second capacitors to the junction between the third and fourth capacitors, and is connected to A. Thus the nodes encountered along the horizontal row alternate as B, A, B, A, B. No pair of successive capacitors has an isolated middle junction, so the series formula cannot be applied.

Step 2: Add the four parallel capacitances Each capacitor has one plate on node A and its other plate on node B. If the terminal potential difference is V, each capacitor stores charge of magnitude $Q_i = CV$ on its A-side plate. The terminal charge is consequently

$$Q = Q_1 + Q_2 + Q_3 + Q_4 = 4CV.$$

Dividing by the common voltage gives

$$C_{AB} = \frac{Q}{V} = 4C = 4(16\,\mu\text{F}) = 64\,\mu\text{F}.$$

This result is greater than the individual capacitance, as expected for four parallel branches, and not smaller as a mistaken series reading would suggest.

Step 3: Conclusion The equivalent capacitance between A and B is $\boxed{64\,\mu\text{F}}$. The numerical entry is 64.

Quick Tip to Solve: Colour or label each uninterrupted wire as one node first. Here the row alternates B–A–B–A–B, revealing four parallel capacitors.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. For $A_2 + B_2 \rightleftharpoons 2AB$

E_a **for forward and backward reaction are 180 and** $200\ \text{kJ mol}^{-1}$ **respectively**

If catalyst lowers E_a **for both reaction by** $100\ \text{kJ mol}^{-1}$**.**

Which of the following statement is correct? [4 marks]

(A) Catalyst does not alter the Gibbs energy change of a reaction.

(B) The enthalpy change for the reaction is $+20\ \text{kJ mol}^{-1}$.

(C) Catalyst can cause non-spontaneous reactions to occur.

(D) The enthalpy change for the catalysed reaction is different from that of uncatalysed reaction.

Correct Answer: (A) Catalyst does not alter the Gibbs energy change of a reaction.

Solution:

Concept:

- Forward and reverse activation energies measured from the same transition state satisfy $\Delta H = E_{a,\text{f}} - E_{a,\text{b}}$. The reaction enthalpy depends only on the reactant and product states.
- A catalyst supplies a pathway with lower activation energy, increasing the rates of both directions. It changes neither the Gibbs energy change ΔG nor the equilibrium constant at a fixed temperature.

Step 1: Calculate the reaction enthalpy before and after catalysis For the uncatalysed reaction, subtract the reverse activation energy from the forward one:

$$\Delta H = 180 - 200 = -20\,\text{kJ mol}^{-1}.$$

The forward process is therefore exothermic, so the positive enthalpy stated in option (B) has the wrong sign. The catalyst lowers both barriers by $100\,\text{kJ mol}^{-1}$, giving

$$E_{a,\text{f}}^{\text{cat}} = 80, \qquad E_{a,\text{b}}^{\text{cat}} = 100 \quad (\text{kJ mol}^{-1}).$$

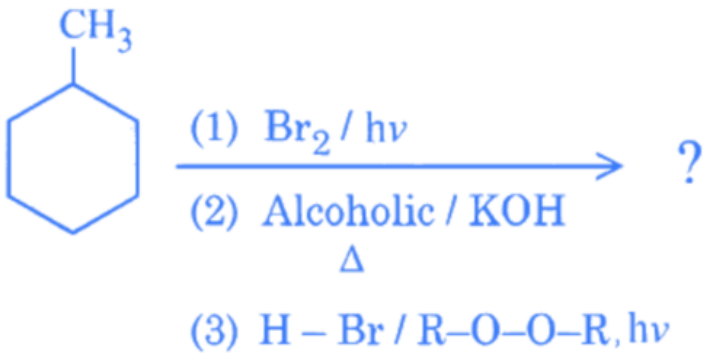
Their difference remains $80 - 100 = -20\,\text{kJ mol}^{-1}$, which also rules out option (D).

Step 2: Distinguish kinetic acceleration from thermodynamic change The initial and final chemical states are identical whether or not a catalyst is present. Since Gibbs energy is a state function, its change cannot be altered by choosing another reaction pathway. Option (A) expresses this correctly. Option (C) incorrectly assigns a thermodynamic role to a catalyst: lowering a barrier does not reverse the sign of ΔG or make a non-spontaneous forward reaction spontaneous under the same conditions. Catalysis speeds the approach to the existing equilibrium rather than changing which equilibrium is favoured.

Step 3: Conclusion A catalyst does not alter the Gibbs energy change of the reaction. Hence, the correct option is (A).

Quick Tip to Solve: Subtract reverse activation energy from forward activation energy to obtain ΔH. Equal lowering of the two barriers leaves this difference unchanged.

52. Predict the major product of the following reaction sequence:- [4 marks]



(A) 1-bromo-1-methylcyclohexane

(B) 1-bromo-2-methylcyclohexane

(C) (bromomethyl)cyclohexane

(D) Bromocycloheptane

Correct Answer: (B) 1-bromo-2-methylcyclohexane

Solution:

Concept:

- Photochemical bromination of an alkane is a free-radical substitution. Bromination is selective: abstraction of a tertiary hydrogen produces the most stable available carbon radical and gives the major monobrominated product.
- Hot alcoholic KOH promotes elimination of HBr, generally favouring the more substituted alkene. Subsequent HBr addition in the presence of peroxide follows the radical, anti-Markovnikov orientation.

Step 1: Apply bromination and elimination in order The starting compound is methylcyclohexane. Its ring carbon bearing the methyl group has one tertiary hydrogen. Replacement of that hydrogen by bromine under $Br_2/h\nu$ gives predominantly 1-bromo-1-methylcyclohexane. On heating this bromide with alcoholic KOH, a beta hydrogen is removed from a neighbouring ring carbon. Loss of bromide then gives the more substituted endocyclic alkene, 1-methylcyclohexene. The alternative exocyclic alkene is less substituted and is not the major elimination product.

Step 2: Orient the final peroxide-assisted addition In the final stage, a bromine radical adds to the less substituted carbon of the double bond. This places the resulting carbon radical on the methyl-bearing ring carbon, where it is tertiary and more stable. That radical abstracts hydrogen from HBr. The bromine and methyl groups thus occupy adjacent ring carbons, giving 1-bromo-2-methylcyclohexane. Option (A), with both substituents on the same carbon, corresponds to the opposite addition orientation. Option (C) puts bromine on the side chain, whereas the alkene being attacked is within the ring. Option (D) changes the ring size; none of these steps requires ring expansion.

Step 3: Conclusion The major product is 1-bromo-2-methylcyclohexane, with bromine adjacent to the methyl-bearing ring carbon. Hence, the correct option is (B).

Quick Tip to Solve: Track the intermediate after every reagent: tertiary bromide, then the Zaitsev alkene, then anti-Markovnikov HBr addition. The last reagent decides the final bromine position.

53. Which one of the following about an electron occupying the 1 s orbital in a hydrogen atom is incorrect?

(Bohr's radius is represented by a_0) [4 marks]

(A) The probability density of finding the electron is maximum at the nucleus

(B) The total energy of the electron is maximum when it is at a distance a_0 from the nucleus

(C) The electron can be found at a distance $2a_0$ from the nucleus

(D) The 1 s orbital is spherically symmetrical

Correct Answer: (B) The total energy of the electron is maximum when it is at a distance a_0 from the nucleus

Solution:

Concept:

- The hydrogen $1s$ state is a stationary energy eigenstate with energy $E_1 = -13.6\,\text{eV}$. Its total energy is a property of the state and is not a function that becomes maximal at one electron position.
- The $1s$ probability density is $|\psi_{1s}|^2 = (\pi a_0^3)^{-1} e^{-2r/a_0}$, where r is the distance from the nucleus. This density must be distinguished from the radial probability distribution $4\pi r^2 |\psi_{1s}|^2$.

Step 1: Check the spatial statements about the orbital The exponential probability density decreases continuously with increasing r. It is therefore largest at $r = 0$, so option (A) is correct. At $r = 2a_0$, its value is $(\pi a_0^3)^{-1} e^{-4}$, which is positive. Hence finding the electron in a small region at that distance is possible, and option (C) is correct. The wavefunction depends only on r, with no angular dependence, so the probability distribution is spherically symmetrical as stated in option (D).

Step 2: Identify what is special about the Bohr radius The radial probability distribution includes the increasing volume of a spherical shell:

$$P(r) = \frac{4r^2}{a_0^3} e^{-2r/a_0}.$$

Its derivative vanishes at the nonzero maximum $r = a_0$. This makes a_0 the most probable radial distance; it does not make the total energy maximum there. A position measurement can find the electron at many radii, while the energy of the unmeasured stationary $1s$ state remains the fixed eigenvalue $-13.6\,\text{eV}$. Option (B) confuses a radial probability maximum with an energy maximum.

Step 3: Conclusion The incorrect statement is that the total electron energy is maximum at $r = a_0$. Hence, the correct option is (B).

Quick Tip to Solve: Keep three ideas separate: $1s$ probability density peaks at the nucleus, radial probability peaks at a_0, and the state has one fixed total energy.

54. Rate law for a reaction between A and B is given by

$$\mathrm{r = k[\ A]^n[\ B]^m}$$

If concentration of A is doubled and concentration of B is halved from their initial value, the ratio of new rate of reaction to the initial rate of reaction $\left(\frac{r_2}{r_1}\right)$ is [4 marks]

(A) $(\mathrm{n - m})$

(B) $2^{(\mathrm{n}-m)}$

(C) $\frac{1}{2^{m+n}}$

(D) $(\mathrm{m + n})$

Correct Answer: (B) $2^{(\mathrm{n}-m)}$

Solution:

Concept:

- For the empirical rate law $r = k[A]^n[B]^m$, the exponents n and m are the reaction orders with respect to A and B. A multiplicative concentration change must itself be raised to the corresponding order.
- At the same temperature, the rate constant k is unchanged. Taking a ratio of rates eliminates both k and the initial concentrations, leaving only the concentration scaling factors.

Step 1: Write the initial and final rates Let the initial concentrations be $[A]_1 = a$ and $[B]_1 = b$. Their initial rate is

$$r_1 = ka^n b^m.$$

After the specified changes, the concentrations become $[A]_2 = 2a$ and $[B]_2 = b/2$. Substituting both changes into the same rate law gives

$$r_2 = k(2a)^n \left(\frac{b}{2}\right)^m.$$

The first factor reflects doubling A; the second reflects halving B. Neither change permits adding or subtracting the orders directly as ordinary concentration factors.

Step 2: Divide and simplify the powers of two Dividing by the initial rate and cancelling the common factors yields

$$\frac{r_2}{r_1} = \frac{k(2a)^n (b/2)^m}{ka^n b^m} = 2^n 2^{-m} = 2^{n-m}.$$

The minus sign in the exponent is due to halving B. Options (A) and (D) give sums or differences of orders rather than the required exponential scaling. Option (C) would halve both reactant concentrations, whereas A was doubled. As a check, setting $n = m = 1$ makes the doubling and halving cancel and gives a rate ratio of one; the expression 2^{n-m} has precisely this behaviour.

Step 3: Conclusion The ratio of the new rate to the initial rate is $\boxed{2^{n-m}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a rate ratio, replace each concentration by its scale factor: doubling gives 2^n, halving gives 2^{-m}, and the factors multiply.

55. Pair of transition metal ions having the same number of unpaired electrons is [4 marks]

(A) Ti^{3+}, Mn^{2+}

(B) Ti^{2+}, Co^{2+}

(C) Fe^{3+}, Cr^{2+}

(D) V^{2+}, Co^{2+}

Correct Answer: (D) V^{2+}, Co^{2+}

Solution:

Concept:

- Transition-metal cations lose the outer $4s$ electrons before the $3d$ electrons. The ionic configuration, rather than the neutral configuration, determines the unpaired-electron count.
- For these free ions, Hund's rule fills the five d orbitals singly before pairing. Thus $d^1, d^2, d^3, d^4, d^5, d^7$ have respectively $1, 2, 3, 4, 5, 3$ unpaired electrons.

Step 1: Determine the ionic configurations Titanium has atomic number 22, vanadium 23, chromium 24, manganese 25, iron 26 and cobalt 27. Remove the $4s$ electrons first; chromium starts from $[Ar]3d^5 4s^1$. The required configurations are

$$\begin{aligned} Ti^{3+} &: [Ar]3d^1, \quad Ti^{2+} : [Ar]3d^2, \\ V^{2+} &: [Ar]3d^3, \quad Mn^{2+} : [Ar]3d^5, \\ Fe^{3+} &: [Ar]3d^5, \quad Cr^{2+} : [Ar]3d^4, \\ Co^{2+} &: [Ar]3d^7. \end{aligned}$$

Step 2: Compare the counts in every option Option (A) compares d^1 with d^5, giving 1 and 5 unpaired electrons, so it fails. Option (B) compares d^2 with d^7, giving 2 and 3, so it fails. Option (C) compares d^5 with d^4, giving 5 and 4, so it also fails. In option (D), V^{2+} is d^3: three orbitals are singly occupied. Co^{2+} is d^7: after five single occupations, two electrons pair, leaving three singly occupied orbitals. No ligand field or low-spin complex is specified here.

Step 3: Conclusion V^{2+} and Co^{2+} each have three unpaired electrons. Hence, the correct option is (D).

Quick Tip to Solve: Write the ionic d^n count first, then count singles: for a free d shell the count is n up to d^5, and $10 - n$ thereafter.

56. On charging the lead storage battery, the oxidation state of lead changes from x_1 to y_1 at the anode and from x_2 to y_2 at the cathode. The values of x_1, y_1, x_2, y_2 are respectively : [4 marks]

(A) $0, +2, +4, +2$

(B) $+2, 0, 0, +4$

(C) $+4, +2, 0, +2$

(D) $+2, 0, +2, +4$

Correct Answer: (D) $+2, 0, +2, +4$

Solution:

Concept:

- The discharged lead storage battery has $\mathrm{PbSO_4}$ on both plates, with lead in oxidation state $+2$. Charging restores metallic lead and lead dioxide.
- The battery plates are commonly named by their discharge functions: the lead plate is called the anode and the lead-dioxide plate the cathode. During charging their reaction functions reverse.

Step 1: Restore the lead plate during charging At the plate normally called the battery anode, discharged $\mathrm{PbSO_4}$ accepts electrons and regenerates Pb:

$$\mathrm{PbSO_4 + 2\,e^- \longrightarrow Pb + {SO_4}^{2-}}.$$

Sulphate has charge -2, so lead in $\mathrm{PbSO_4}$ is $+2$; elemental lead is 0. Hence, for the named lead plate, $x_1 = +2$ and $y_1 = 0$. This process is reduction. It is the reverse of the oxidation occurring at this plate when the battery supplies current.

Step 2: Restore the lead-dioxide plate and identify the convention At the plate normally called the battery cathode, charging converts $\mathrm{PbSO_4}$ back into $\mathrm{PbO_2}$:

$$\mathrm{PbSO_4 + 2\,H_2O \longrightarrow PbO_2 + {SO_4}^{2-} + 4\,H^+ + 2\,e^-}.$$

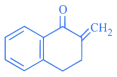
Lead changes from $+2$ to $+4$, because the two oxygen atoms in $\mathrm{PbO_2}$ contribute -4. Thus $x_2 = +2$, $y_2 = +4$. Option (A) gives the discharge changes, (B) incorrectly begins the second change with elemental lead, and (C) also describes the reverse direction. Strictly naming electrodes by their instantaneous charging reactions would call the oxidation plate the electrolytic anode and the reduction plate the electrolytic cathode; that convention reverses the two pairs and is absent from the choices. The listed answer uses the conventional battery-plate names.

Step 3: Conclusion With the battery plates named by their discharge roles, the requested tuple is $(+2, 0, +2, +4)$. Hence, the correct option is (D).

Quick Tip to Solve: For charging, start from $\mathrm{PbSO_4}$ on both plates: one $\mathrm{Pb^{2+}}$ becomes Pb, and the other becomes $\mathrm{PbO_2}$. Check the electrode-naming convention.

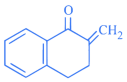
57. Aldol condensation is a popular and classical method to prepare α,β-unsaturated carbonyl compounds. This reaction can be both intermolecular and intramolecular. Predict which one of the following is not a product of intramolecular aldol condensation? [4 marks]

(A) Fused bicyclic enone (ring-junction double bond). 

(B) Cyclic ketone with exocyclic alpha-methylene group. 

(C) Cyclohex-2-en-1-one. 

(D) Benzene-fused five-membered enone. 

Correct Answer: (B) Cyclic ketone with exocyclic alpha-methylene group. 

Solution:

Concept:

- An intramolecular aldol reaction forms a carbon–carbon bond when an enolate attacks a second carbonyl group in the same molecule. Dehydration produces an α,β-unsaturated carbonyl group.
- A useful reverse test cleaves the newly formed α–β bond and restores a carbonyl at the β carbon. The resulting two carbonyl groups must still belong to one connected precursor for the reaction to be intramolecular.

Step 1: Apply the reverse aldol test to the exocyclic product In option (B), the cyclic ketone bears an exocyclic $=CH_2$ group at its alpha carbon. This terminal methylene carbon is the beta carbon of the enone. Cutting the alpha–beta bond and restoring its oxygen separates a molecule of formaldehyde, $HCHO$, from the benzene-fused cyclic ketone. The methylene carbon has no second carbon connection to keep it tethered to the ketone. Such a product can be obtained by a crossed aldol reaction with formaldehyde, but that is intermolecular. The fact that the ketone itself contains a ring does not make the aldol reaction intramolecular.

Step 2: Check why the remaining structures can form internally Option (C), cyclohex-2-en-1-one, follows by cyclising an appropriately spaced keto-aldehyde, for example $CH_3COCH_2CH_2CH_2CHO$, and dehydrating the aldol. The new ring bond connects two positions of the same chain. In option (D), breaking the enone alpha–beta bond opens the five-membered ring while the benzene framework still connects the two carbonyl-containing side chains. In option (A), breaking the shared fusion alpha–beta bond leaves the outer carbon skeleton connected, giving a cyclic dicarbonyl precursor capable of internal closure. Thus these three pass the connectivity test, whereas the isolated formaldehyde fragment in (B) does not.

Step 3: Conclusion The exocyclic alpha-methylene ketone in option (B) requires an intermolecular aldol step with formaldehyde; it is not an intramolecular aldol condensation product. Hence, the correct option is (B).

Quick Tip to Solve: Disconnect the enone alpha–beta bond mentally. If this releases $HCHO$ as a separate molecule, the methylene group came from an intermolecular aldol reaction.

58. Benzene is treated with oleum to produce compound (X) which when further heated with molten sodium hydroxide followed by acidification produces compound (Y). The compound Y is treated with zinc metal to produce compound (Z). Identify the structure of compound (Z) from the following option. [4 marks]

(A) Catechol (benzene-1,2-diol). 

(B) Hydroquinone (benzene-1,4-diol). 

(C) Benzene. 

(D) Phenol. 

Correct Answer: (C) Benzene. 

Solution:

Concept:

- Oleum sulphonates benzene by electrophilic aromatic substitution, introducing a $-SO_3H$ group without changing the six-carbon aromatic ring.
- Fusion of a benzene sulphonate with molten $NaOH$, followed by acidification, gives phenol. Heating phenol with zinc dust removes its oxygen as ZnO and regenerates benzene.

Step 1: Identify the sulphonation and fusion products The first product X is benzenesulphonic acid:

$$C_6H_6 + SO_3 \longrightarrow C_6H_5SO_3H.$$

Under the strongly basic fusion conditions, its sodium salt reacts with molten sodium hydroxide. The sulphonic acid group is replaced by an oxygen substituent:

$$C_6H_5SO_3Na + 2\,NaOH \longrightarrow C_6H_5ONa + Na_2SO_3 + H_2O.$$

The immediate organic product is sodium phenoxide, rather than free phenol, because the medium is basic. Acidification protonates it:

$$C_6H_5ONa + HCl \longrightarrow C_6H_5OH + NaCl.$$

Thus Y is phenol.

Step 2: Reduce phenol with zinc and eliminate the other options Zinc removes oxygen from phenol on heating, giving zinc oxide and benzene:

$$C_6H_5OH + Zn \longrightarrow C_6H_6 + ZnO.$$

The ring is retained, while its hydroxyl group is replaced by hydrogen. Option (A), catechol, and option (B), hydroquinone, each have two hydroxyl groups. Neither can result from this final deoxygenation step; no reagent here introduces a second hydroxyl group. Option (D), phenol, is the intermediate Y, before treatment with zinc. Option (C) is the unsubstituted aromatic six-membered ring with alternating double bonds, namely benzene, and is therefore Z.

Step 3: Conclusion X is benzenesulphonic acid, Y is phenol and Z is benzene, C_6H_6. Hence, the

correct option is (C).

Quick Tip to Solve: Anchor the sequence at its last step: phenol heated with zinc dust gives benzene. Molten NaOH fusion first supplies the phenol intermediate.

59. One mole of an ideal gas expands isothermally and reversibly from 10dm^3 to 20dm^3 at 300 K . $\Delta\text{U}, \text{q}$ and work done in the process respectively are

Given: $\text{R} = 8.3\ \text{J K}^{-1}\ \text{mol}^{-1}$

$$\ln 10 = 2.3$$

$$\log 2 = 0.30$$

$$\log 3 = 0.48$$

[4 marks]

(A) $0, 21.84\ \text{kJ}, -1.726\ \text{J}$

(B) $0, 21.84\ \text{kJ}, 21.84\ \text{kJ}$

(C) $0, 1.718\ \text{kJ}, -1.718\ \text{kJ}$

(D) $0, -17.18\ \text{kJ}, 1.718\ \text{J}$

Correct Answer: (C) $0, 1.718\ \text{kJ}, -1.718\ \text{kJ}$

Solution:

Concept:

- For an ideal gas, internal energy depends only on temperature. An isothermal process has $\Delta T = 0$, so $\Delta U = 0$.
- With the chemistry sign convention, $\Delta U = q + w$, where work done on the gas is positive. Reversible expansion gives $w = -nRT\ln(V_2/V_1)$, and the gas absorbs heat to keep its temperature fixed.

Step 1: Calculate the reversible work with the supplied logarithm Here $n = 1\,\text{mol}$, $T = 300\,\text{K}$, and $V_2/V_1 = 20/10 = 2$. The volume ratio is dimensionless, so conversion from cubic decimetres to cubic metres is unnecessary. From the given logarithms,

$$\ln 2 = (\ln 10)\log 2 = 2.3(0.30) = 0.69.$$

Therefore,

$$\begin{aligned} w &= -(1)(8.3)(300)(0.69)\,\text{J} \\ &= -1718.1\,\text{J} = -1.7181\,\text{kJ}. \end{aligned}$$

The negative sign expresses that the gas does work on its surroundings. The supplied value of $\log 3$ is not needed because the volume doubles.

Step 2: Use the first law and check all three entries Because the gas is ideal and its temperature stays

at 300 K, $\Delta U = 0$. Consequently,

$$q = \Delta U - w = 0 - (-1.7181) = +1.7181\,\text{kJ}.$$

Heat enters the gas to compensate for the energy transferred as expansion work. Option (C) has both the correct magnitude and opposite signs for heat and work. Option (A) has an incorrect heat magnitude and an inconsistent work unit. Option (B) assigns equal positive signs, violating $q + w = 0$. Option (D) assigns heat a negative sign during this reversible isothermal expansion and gives an inconsistent magnitude and unit. The choices thus confirm the chemistry convention for the work entry.

Step 3: Conclusion Using the supplied logarithms, $(\Delta U, q, w) = (0, +1.718\,\text{kJ}, -1.718\,\text{kJ})$. Hence, the correct option is (C).

Quick Tip to Solve: For ideal-gas isothermal expansion, immediately set $\Delta U = 0$ and $q = -w$. Then use the dimensionless volume ratio inside the logarithm.

---

60. Given below are two statements.

Statement I: The dipole moment of $CH_3{-}CH{=}CH{-}CH{=}O$

is greater than that of $CH_3{-}CH_2{-}CH_2{-}CH{=}O$.

Statement II: The $C_1 - C_2$ **bond length of** $CH_3{-}CH{=}CH{-}CH{=}O$ **is greater than the** $C_1 - C_2$ **bond length of** $CH_3{-}CH_2{-}CH_2{-}CH{=}O$. **In both compounds the aldehyde carbon is** C_1**, and the remaining chain carbons are numbered consecutively to** C_4.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) Both Statement I and Statement II are true

(B) Statement I is true but Statement II is false

(C) Statement I is false but Statement II is true

(D) Both Statement I and Statement II are false

Correct Answer: (B) Statement I is true but Statement II is false

Solution:

Concept:

- In an α,β-unsaturated aldehyde, the alkene and carbonyl groups form a conjugated π system. Resonance redistributes electron density along this system.
- Partial double-bond character shortens a formally single carbon–carbon bond. Dipole moment depends on the extent and separation of charge, rather than solely on the presence of an oxygen atom.

Step 1: Evaluate Statement I using conjugation In $CH_3{-}CH{=}CH{-}CHO$, the C=C bond is adjacent to C=O, allowing resonance donation towards oxygen. The usual polar carbonyl contributor is supplemented by a contributor with negative charge on oxygen and positive charge at the beta carbon. This extends the charge separation across the conjugated system and increases its molecular dipole

moment. In $CH_3-CH_2-CH_2-CHO$, the intervening saturated carbons prevent such conjugation. Thus the stated comparison, a greater dipole moment for the unsaturated aldehyde, is true.

Step 2: Evaluate Statement II from the numbered bond Numbering begins at the aldehyde carbon: C_1 is the carbonyl carbon and C_2 is its neighbouring carbon. In the conjugated aldehyde, resonance gives the $C_1 - C_2$ linkage partial double-bond character; both participating carbons are also sp^2-hybridised. In the saturated aldehyde, the corresponding linkage joins an sp^2 carbonyl carbon to an sp^3 carbon and lacks this conjugative double-bond contribution. The conjugated linkage is therefore shorter, rather than longer. Statement II is false. Option (A) incorrectly treats it as true; (C) reverses both judgements; (D) incorrectly rejects Statement I. Only (B) gives the required true–false combination.

Step 3: Conclusion Statement I is true, while Statement II is false: conjugation increases the dipole moment and shortens the $C_1 - C_2$ bond. Hence, the correct option is (B).

Quick Tip to Solve: For a conjugated $C=C-C=O$ unit, the formally single central bond acquires partial double-bond character and becomes shorter, not longer.

61. Which one of the following complexes will have $\Delta_o = 0$ and $\mu = 5.96$ B.M? [4 marks]

(A) $[FeF_6]^{4-}$

(B) $[Mn(SCN)_6]^{4-}$

(C) $[Co\,(NH_3)_6]^{3+}$

(D) $[Fe(CN)_6]^{4-}$

Correct Answer: (B) $[Mn(SCN)_6]^{4-}$

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}$ B.M., where n is the number of unpaired electrons. Five unpaired electrons give $\sqrt{35} = 5.92$ B.M., close to the stated 5.96 B.M.
- For an octahedral complex, $\text{CFSE} = (-0.4n_t + 0.6n_e)\Delta_o$. A high-spin d^5 ion has $t_{2g}^3 e_g^2$, so its CFSE vanishes although its orbital splitting Δ_o is not zero.

Step 1: Identify the required electron configuration The given magnetic moment points to five unpaired electrons, hence a high-spin d^5 metal ion. In option (B), each thiocyanate ligand has charge -1, so the manganese oxidation state is $x - 6 = -4$, giving $x = +2$. Neutral manganese is $[Ar]3d^5 4s^2$; removing its two outer electrons gives Mn^{2+}, a d^5 ion. With the weak-field ligand thiocyanate, all five d electrons remain unpaired.

Step 2: Check stabilisation and eliminate the other ions For this high-spin state, the lower three orbitals and upper two orbitals are singly occupied. Thus

$$\text{CFSE} = 3(-0.4\Delta_o) + 2(0.6\Delta_o) = 0.$$

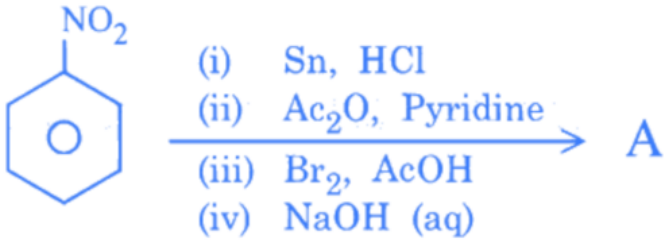
Option (A) contains high-spin Fe^{2+}, which is d^6 with four unpaired electrons, not five. Option (C) contains low-spin d^6 Co^{3+} with paired electrons. Option (D) contains low-spin d^6 Fe^{2+} because cyanide is strong-field. Neither has the required moment.

Step 3: Distinguish splitting from stabilisation The printed condition $\Delta_o = 0$ is not the same as zero CFSE: an actual octahedral ligand field splits the d levels. Therefore none of the listed complexes literally satisfies zero splitting. The chemically consistent interpretation of the stated condition is zero crystal-field stabilisation, for which (B) is the unique match.

Step 4: Conclusion With zero CFSE as intended by the stated zero-energy condition, the complex is $[Mn(SCN)_6]^{4-}$, option (B), with five unpaired electrons. Strictly, the printed $\Delta_o = 0$ condition has no matching option; the spin-only moment is 5.92 B.M. rather than exactly 5.96 B.M.

Quick Tip to Solve: For a weak-field octahedral ion, count unpaired electrons first; high-spin d^5 uniquely combines five unpaired electrons with zero CFSE, not zero splitting.

62. The major product (A) formed in the following reaction sequence is [4 marks]



(A) 2,4,6-Tribromoaniline

(B) 4-Bromoaniline

(C) 2,4-Dibromoaniline

(D) 3-Bromoaniline

Correct Answer: (B) 4-Bromoaniline

Solution:

Concept:

- Tin and hydrochloric acid reduce an aromatic nitro group to an amino group. Acetic anhydride converts aniline to acetanilide, protecting the strongly activating $-NH_2$ group as $-NHCOCH_3$.
- The acetamido group directs electrophilic substitution to ortho and para positions, but its reduced activation and steric bulk favour para monobromination. Hydrolysis of the amide restores the amino group.

Step 1: Reduce and protect the amino group The starting compound is nitrobenzene. Treatment with Sn/HCl reduces $-NO_2$ to $-NH_2$, giving aniline after work-up. Acetic anhydride in pyridine then acetylates nitrogen to form acetanilide:

$$C_6H_5NH_2 + (CH_3CO)_2O \longrightarrow C_6H_5NHCOCH_3 + CH_3COOH.$$

The protecting step is essential because direct bromination of unprotected aniline readily produces substitution at both ortho positions and the para position.

Step 2: Brominate the protected ring Bromine in acetic acid attacks the aromatic ring of acetanilide.

The $-NHCOCH_3$ group is ortho/para directing; the para position is less crowded than the ortho positions. The major intermediate is therefore 4-bromoacetanilide, with bromine opposite the acetamido group. This gives one bromine atom rather than the three bromines in option (A), or the two bromines in option (C).

Step 3: Hydrolyse the protecting group and compare products Aqueous sodium hydroxide hydrolyses the acetyl group, converting the para-bromoacetanilide into 4-bromoaniline and acetate. The bromine attached to the benzene ring remains in place during amide hydrolysis. Option (D), 3-bromoaniline, would require meta substitution, inconsistent with the directing effect of the acetamido group. Option (B) alone represents the para monobrominated amine.

Step 4: Conclusion The major product of the sequence is 4-bromoaniline (p-bromoaniline), $p\text{-}BrC_6H_4NH_2$. Hence, the correct option is (B).

Quick Tip to Solve: An acetylation step before bromination signals controlled para monobromination; remove the acetyl group only after locating bromine on the ring.

63. Given below are two statements:

Statement I: Nitrogen forms oxides with +1 to +5 oxidation states due to the formation of $p\pi - p\pi$ bond with oxygen.

Statement II: Nitrogen does not form halides with +5 oxidation state due to the absence of d-orbital in it.

In the light of given statements, choose the correct answer from the options given below. [4 marks]

(A) Both Statement I and Statement II are true

(B) Statement I is true but Statement II is false

(C) Statement I is false but Statement II are true

(D) Both Statement I and Statement II are False

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- Nitrogen has valence configuration $2s^2 2p^3$. Its small size permits effective overlap of its 2p orbitals with oxygen 2p orbitals, producing strong $p\pi - p\pi$ multiple bonding in nitrogen oxides.
- Oxidation state is a formal electron-counting assignment and does not equal covalency. Nitrogen has no d orbital in its valence shell and cannot expand its second-shell octet to make five ordinary nitrogen–halogen bonds.

Step 1: Evaluate the range of nitrogen oxides Nitrogen forms oxides spanning the positive oxidation states from $+1$ to $+5$. Using oxygen as -2, the familiar series is N_2O ($+1$ average), NO ($+2$), N_2O_3 ($+3$), NO_2 ($+4$), and N_2O_5 ($+5$). Efficient nitrogen–oxygen $p\pi - p\pi$ bonding stabilises these oxides. Therefore Statement I is true. The $+5$ value in an oxide does not imply that nitrogen must have five separate single bonds.

Step 2: Evaluate the pentahalide statement A neutral nitrogen pentahalide would require five

nitrogen–halogen bonds and ten electrons around nitrogen in the usual covalent description. Nitrogen is a second-period element whose valence shell contains only the 2s and 2p orbitals, so it cannot accommodate this expanded octet. It consequently forms trihalides such as NF_3, but no ordinary pentahalides such as NF_5. This is the standard bonding reason expressed by Statement II, so Statement II is also true.

Step 3: Select the statement combination The two statements concern different kinds of bonding: multiple bonding with oxygen can give high formal oxidation states without an expanded octet, whereas five single bonds to halogens would exceed the octet. Thus both statements can hold together. Options (B) and (C) incorrectly reject one true statement, and option (D) incorrectly rejects both.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: Do not equate oxidation state with the number of bonds: nitrogen can be +5 in an oxide while still obeying the octet rule.

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64. Number of stereoisomers possible for the complexes, $[CrCl_3(py)_3]$ and $[CrCl_2(ox)_2]^{3-}$ are respectively (py = pyridine, ox = oxalate)

[4 marks]

(A) 3 & 3

(B) 2 & 3

(C) 1 & 2

(D) 2 & 2

Correct Answer: (B) 2 & 3

Solution:

Concept:

- An octahedral complex of type MA_3B_3 has facial (fac) and meridional (mer) arrangements. Identical monodentate ligands do not create an additional optical pair in either arrangement.
- In an octahedral complex $M(AA)_2B_2$ with two identical symmetrical bidentate ligands, the cis arrangement forms an enantiomeric pair, while the trans arrangement is optically inactive. Stereoisomer counting includes both geometrical and optical forms.

Step 1: Count the pyridine complex Pyridine is a monodentate neutral ligand, so $[CrCl_3(py)_3]$ has three chloride and three pyridine donor sites in an octahedron. In the fac form all three chlorides occupy one triangular face and are pairwise cis. In the mer form the three chlorides lie in a plane through the metal, with one chloride pair trans. These two geometrical arrangements are distinct, and each is achiral. Consequently the first complex has exactly two stereoisomers.

Step 2: Count the bis-oxalate complex Each oxalate binds through two oxygen atoms and occupies adjacent coordination sites. The two chloride ligands of $[CrCl_2(ox)_2]^{3-}$ can be cis or trans. When the chlorides are cis, the chelate arrangement has two non-superimposable mirror images, conventionally denoted Δ and Λ. The trans form possesses symmetry and gives no separate optical pair. The count is therefore

$$2 \text{ cis enantiomers} + 1 \text{ trans form} = 3.$$

Step 3: Compare the ordered counts The requested order is the pyridine complex first and the oxalate complex second, giving 2 and 3. Option (A) overcounts the first complex by adding a nonexistent optical isomer; option (C) misses geometrical isomerism; option (D) counts only cis/trans for the chelate and omits its optical pair.

Step 4: Conclusion The numbers of stereoisomers are respectively 2 and 3. Hence, the correct option is (B).

Quick Tip to Solve: For $M(AA)_2B_2$, count the cis mirror-image pair separately: cis contributes two and trans contributes one.

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65. An organic compound (X) with molecular formula C_3H_6O is not readily oxidised. On reduction it gives $C_3H_8O(Y)$ which reacts with HBr to give a bromide (Z) which is converted to Grignard reagent. This Grignard reagent on reaction with (X) followed by hydrolysis give 2,3-dimethylbutan-2-ol. Compounds $(X), (Y)$ and (Z) respectively are: [4 marks]

(A) $CH_3CH_2CHO, CH_3CH_2CH_2OH, CH_3CH_2CH_2Br$

(B) $CH_3COCH_3, CH_3CH(OH)CH_3, CH_3CH(Br)CH_3$

(C) $CH_3COCH_3, CH_3CH_2CH_2OH, CH_3CH(Br)CH_3$

(D) $CH_3CH_2CHO, CH_3CH=CH_2, CH_3CH(Br)CH_3$

Correct Answer: (B) $CH_3COCH_3, CH_3CH(OH)CH_3, CH_3CH(Br)CH_3$

Solution:

Concept:

- An aldehyde is readily oxidised to a carboxylic acid, whereas an ordinary ketone resists mild oxidation. Reduction of a ketone converts its carbonyl group into a secondary alcohol without changing the carbon skeleton.
- An alkyl bromide reacts with magnesium in dry ether to form a Grignard reagent $RMgBr$. Its alkyl group adds to the carbonyl carbon of a ketone; hydrolysis of the magnesium alkoxide gives a tertiary alcohol.

Step 1: Identify the carbonyl compound and its reduction product The formula C_3H_6O together with resistance to ready oxidation identifies propanone, CH_3COCH_3, rather than propanal. Reducing its carbonyl group gives propan-2-ol:

$$CH_3COCH_3 \xrightarrow{\text{reduction}} CH_3CH(OH)CH_3.$$

Thus X is propanone and Y is the secondary alcohol. A primary alcohol, propan-1-ol, would instead come from reducing propanal.

Step 2: Form the bromide and Grignard reagent With hydrogen bromide, the hydroxyl group of propan-2-ol is replaced by bromine to give 2-bromopropane, $CH_3CH(Br)CH_3$, which is Z. Magnesium in dry ether then inserts into the carbon–bromine bond to form isopropylmagnesium bromide, $(CH_3)_2CHMgBr$. The carbon skeleton remains the three-carbon isopropyl group.

Step 3: Verify the final tertiary alcohol Addition of that Grignard reagent to propanone, followed by

hydrolysis, gives

$$(CH_3)_2CO + (CH_3)_2CHMgBr \longrightarrow (CH_3)_2C(OMgBr)CH(CH_3)_2.$$

$$(CH_3)_2C(OMgBr)CH(CH_3)_2 \xrightarrow{H_2O} (CH_3)_2C(OH)CH(CH_3)_2.$$

The product is 2,3-dimethylbutan-2-ol as required. Options (A) and (D) use the readily oxidised aldehyde; option (C) wrongly makes a primary alcohol by reducing propanone. Option (D) also lists an alkene for Y, contradicting its oxygen-containing formula.

Step 4: Conclusion $X = CH_3COCH_3$, $Y = CH_3CH(OH)CH_3$, and $Z = CH_3CH(Br)CH_3$. Hence, the correct option is (B).

Quick Tip to Solve: Trace the carbon attached to OH backwards: a tertiary alcohol from Grignard addition retains both original ketone substituents and gains the reagent alkyl group.

66. Let us consider a reversible reaction at temperature, T. In this reaction, both ΔH and ΔS were observed to have positive values. If the equilibrium temperature is Te , then the reaction becomes spontaneous at: [4 marks]

(A) $\mathrm{Te} > \mathrm{T}$

(B) $\mathrm{T} > \mathrm{Te}$

(C) $\mathrm{T} = \mathrm{Te}$

(D) $\mathrm{Te} = 5\ \mathrm{T}$

Correct Answer: (B) $\mathrm{T} > \mathrm{Te}$

Solution:

Concept:

- At constant temperature and pressure, the Gibbs-energy change of the forward reaction is $\Delta G = \Delta H - T\Delta S$. The forward direction is spontaneous when $\Delta G < 0$, while equilibrium requires $\Delta G = 0$.
- When both ΔH and ΔS are positive, the enthalpy term opposes the forward process and the entropy term favours it. Raising temperature increases the magnitude of the favourable $-T\Delta S$ contribution.

Step 1: Express the equilibrium temperature At the stated equilibrium temperature T_e, the competing enthalpy and entropy contributions balance:

$$0 = \Delta H - T_e\Delta S.$$

Since $\Delta S > 0$, rearrangement gives $T_e = \Delta H/\Delta S$. This ratio is positive and has units of kelvin when the enthalpy and entropy units are consistent. The question uses this temperature as the threshold between the two directions.

Step 2: Apply the condition for spontaneity The forward reaction becomes spontaneous when

$$\Delta H - T\Delta S < 0.$$

Dividing by the positive quantity ΔS preserves the direction of the inequality:

$$T > \frac{\Delta H}{\Delta S} = T_e.$$

Equivalently, inserting $\Delta H = T_e \Delta S$ gives $\Delta G = (T_e - T)\Delta S$. Its sign is negative only above the equilibrium temperature, under the stated thermodynamic comparison.

Step 3: Reject the other temperature relations Option (A), $T_e > T$, makes $\Delta G > 0$, so the forward process is nonspontaneous. Option (C), $T = T_e$, gives equilibrium rather than a spontaneous net forward reaction. Option (D), $T_e = 5T$, also puts the system below the equilibrium temperature and gives $\Delta G = 4T\Delta S > 0$. Only option (B) has the necessary high-temperature condition.

Step 4: Conclusion The forward reaction is spontaneous for $T > T_e$, when $T\Delta S$ exceeds ΔH. Hence, the correct option is (B).

Quick Tip to Solve: For positive enthalpy and entropy changes, divide ΔH by ΔS to find the threshold; spontaneity lies on its higher-temperature side.

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67. Which of the following molecule(s) show/s paramagnetic behavior?

A. O_2

B. N_2

C. F_2

D. S_2

E. Cl_2

Choose the correct answer from the options given below: [4 marks]

(A) A & C only

(B) A & E Only

(C) A & D Only

(D) B Only

Correct Answer: (C) A & D Only

Solution:

Concept:

- Paramagnetism requires at least one unpaired electron in the molecular ground state. A molecule with all occupied orbitals containing electron pairs is diamagnetic.
- Degenerate molecular orbitals are occupied singly with parallel spins before pairing, according to Hund's rule. Count molecular electrons rather than using the isolated atoms.

Step 1: Identify the oxygen-family molecules Oxygen contributes six valence electrons per atom, so O_2

has twelve valence electrons. Filling its molecular orbitals leaves two electrons in the two degenerate antibonding π^*_{2p} orbitals. These occupy separate orbitals and remain unpaired. Sulfur belongs to the same group: S_2 also has twelve valence electrons and the analogous $3p$-derived occupation, including two unpaired antibonding electrons. Thus both oxygen and sulfur molecules are paramagnetic.

Step 2: Check the remaining molecules N_2 has ten valence electrons. Its occupied molecular orbitals are filled with pairs, so it is diamagnetic. F_2 has fourteen valence electrons: the two antibonding π^* orbitals now each contain a pair, leaving no unpaired electrons. Chlorine has the corresponding closed-shell molecular configuration in Cl_2, which is also diamagnetic. These classifications depend on the molecular ground states, not the magnetic properties of the individual free atoms.

Step 3: Test the answer combinations The paramagnetic entries are A and D. A and C incorrectly includes diamagnetic fluorine; A and E incorrectly includes diamagnetic chlorine. The B-only choice selects nitrogen, whose electrons are all paired. Therefore the only correct combination is A and D, corresponding to oxygen and sulfur molecules.

Step 4: Conclusion O_2 and S_2, statements A and D only, are paramagnetic. Hence, the correct option is (C).

Quick Tip to Solve: For a diatomic molecule, count the unpaired electrons after filling the molecular orbitals; isolated-atom configurations do not decide molecular magnetism.

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68. Identify the pair of reactants that upon reaction, with elimination of HCl will give rise to the dipeptide Gly-Ala. [4 marks]

(A) NH_2CH_2COOH and $NH_2CH(CH_3)COOH$

(B) NH_2CH_2COCl and $H_3N^+CH(CH_3)COCl$

(C) NH_2CH_2COCl and $NH_2CH(CH_3)COOH$

(D) NH_2CH_2COOH and $NH_2CH(CH_3)COCl$

Correct Answer: (C) NH_2CH_2COCl and $NH_2CH(CH_3)COOH$

Solution:

Concept:

- Gly-Ala specifies glycine at the N terminus and alanine at the C terminus. The peptide linkage is $-CO-NH-$.
- An acid chloride reacts with a free amino group by nucleophilic acyl substitution, forming an amide and eliminating HCl. The activated carbonyl determines the first residue.

Step 1: Write the required peptide sequence Glycine contributes the amino-terminal fragment $NH_2CH_2CO^-$, while alanine contributes $-NHCH(CH_3)COOH$. Joining these gives $NH_2CH_2CONHCH(CH_3)COOH$. Reading from the free amino end toward the carboxyl end gives Gly-Ala. The alanine carboxyl group must remain at the free C terminus; activation of alanine instead would reverse the residue order.

Step 2: Select the reacting functional groups The glycine derivative must therefore be NH_2CH_2COCl, with its carboxyl group activated as an acid chloride. It reacts with the free amino group of alanine,

$NH_2CH(CH_3)COOH$. Nitrogen attacks the glycine carbonyl carbon, and chloride is displaced. Loss of one nitrogen-bound hydrogen together with chloride accounts for the elimination of HCl. The resulting amide bond is between the glycine carbonyl and the alanine nitrogen, exactly as the specified sequence requires.

Step 3: Eliminate the other options Option A contains two unactivated carboxylic acids and does not describe the acid-chloride reaction with elimination of hydrogen chloride. Option B has the alanine amino group protonated, so it lacks the free nitrogen lone pair for this coupling, and its carboxyl group is also an acid chloride rather than the required free acid. Option D activates the alanine carboxyl group: reaction with glycine would produce Ala-Gly, not Gly-Ala. Only option C has glycine acid chloride and free alanine in the required roles.

Step 4: Conclusion The required pair is NH_2CH_2COCl and $NH_2CH(CH_3)COOH$. Hence, the correct option is (C).

Quick Tip to Solve: For Gly-Ala, activate glycine's carboxyl group and retain alanine's free amino group. Reversing these roles reverses the peptide sequence.

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69. Given below are the pairs of group 13 elements showing their relation in terms of atomic radius. $(B < Al), (Al < Ga), (Ga < In)$ and $(In < Tl)$ Identify the elements present in the incorrect pair and in that pair find out the element (X) that has higher ionic radius (M^{3+}) than the other one. The atomic number of the element (X) is [4 marks]

(A) 49

(B) 81

(C) 31

(D) 13

Correct Answer: (C) 31

Solution:

Concept:

- Atomic radius generally increases down a group, but poor shielding by filled d electrons causes the gallium contraction: neutral gallium is slightly smaller than aluminium.
- The radius of a neutral atom and that of its trivalent cation follow different comparisons. First identify the incorrect atomic-radius pair, then compare only those two M^{3+} ions.

Step 1: Identify the incorrect atomic-radius relation The increase from boron to aluminium is normal because an additional occupied principal shell is introduced. From aluminium to gallium, however, the filled $3d^{10}$ electrons shield the nuclear charge relatively poorly. The stronger effective attraction contracts the gallium atom, making $r(Ga) < r(Al)$. Thus the printed relation $Al < Ga$ is incorrect. The stated increases from gallium to indium and from indium to thallium do not identify the exceptional pair requested.

Step 2: Compare the trivalent cations Aluminium forms Al^{3+} by losing three valence electrons, leaving the configuration $[Ne]$. Gallium forms Ga^{3+}, leaving $[Ar]3d^{10}$. The aluminium ion has its outermost occupied electrons in the second shell, whereas the gallium ion retains occupied third-shell

orbitals. Consequently gallium has the larger trivalent ion: $r(\mathrm{Ga}^{3+}) > r(\mathrm{Al}^{3+})$. The contraction of the neutral gallium atom must not be incorrectly extended to reverse this ionic comparison.

Step 3: Select the atomic number The element X is gallium, atomic number 31. Aluminium has atomic number 13 but is the smaller trivalent ion in the relevant pair. The other proposed numbers, 49 and 81, belong to indium and thallium; neither belongs to the incorrect aluminium-gallium pair. Therefore the answer follows from two distinct size comparisons rather than a single down-group rule.

Step 4: Conclusion $X = \mathrm{Ga}$, so its atomic number is $\boxed{31}$. Hence, the correct option is (C).

Quick Tip to Solve: Keep the two tests separate: Al/Ga is the anomalous neutral-atom pair, while Ga has the larger trivalent ion.

70. XY is the membrane/partition between two chambers 1 and 2 containing sugar solutions of concentration c_1 and $c_2\,(c_1 > c_2)\,\mathrm{molL}^{-1}$. For the reverse osmosis to take place identify the correct condition.

(Here p_1 and p_2 are pressures applied on chamber 1 and 2).

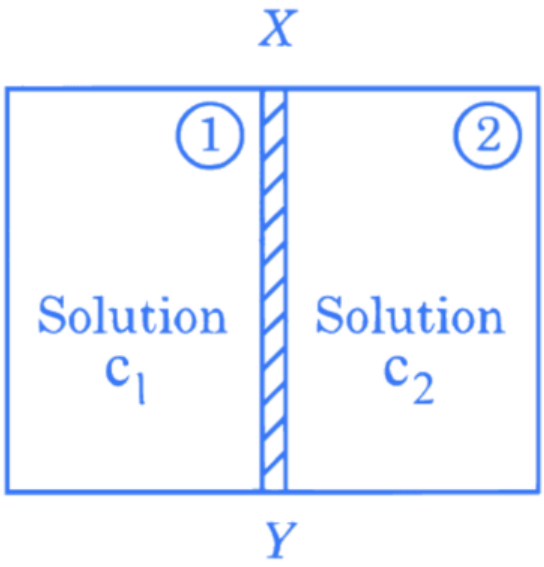
A. Membrane/Partition : Cellophane, $\mathrm{p_1} > \pi$

B. Membrane/Partition : Porous, $\mathrm{p_2} > \pi$

C. Membrane/Partition : Parchment paper, $p_1 > \pi$

D. Membrane/Partition : Cellophane, $\mathrm{p_2} > \pi$

Choose the correct answer from the option given below: [4 marks]



(A) A and C Only

(B) A and D Only

(C) B and D Only

(D) C Only

Correct Answer: (A) A and C Only

Solution:

Concept:

- A semipermeable membrane permits solvent to pass but restricts the solute. Ordinary osmosis moves solvent from a dilute solution toward the concentrated solution.
- Reverse osmosis requires a pressure difference on the concentrated side exceeding the osmotic-pressure difference. For ideal sugar solutions, $\Delta\pi = (c_1 - c_2)RT$.

Step 1: Find the direction to be reversed The concentration inequality is $c_1 > c_2$, so chamber 1 contains the more concentrated sugar solution. In ordinary osmosis, water moves from chamber 2 through the membrane into chamber 1. To reverse this natural flow, the applied pressure must drive water from chamber 1 toward chamber 2. Pressure on chamber 2 would instead reinforce the ordinary direction.

Step 2: Choose the membrane and pressure condition Cellophane and parchment paper can serve as semipermeable membranes in the stated experiment. A merely porous partition is unsuitable because it does not selectively restrict sugar while allowing solvent through. The precise reverse-osmosis criterion is $p_1 - p_2 > \pi_1 - \pi_2$. In the question, π denotes the osmotic pressure opposing reversal and the condition is written as the applied pressure on chamber 1 exceeding it, $p_1 > \pi$. Thus the concentrated side is the side to pressurise.

Step 3: Evaluate all the listed statements Statement A combines cellophane with pressure on chamber 1 and is correct. Statement C combines parchment paper with pressure on chamber 1 and is also correct. Statement B specifies both an unsuitable porous partition and pressure on the wrong side. Statement D has a suitable cellophane membrane but pressurises chamber 2, which cannot reverse osmosis in the stated direction. Therefore combinations containing B or D fail. The C-only choice omits valid statement A, leaving only the A-and-C combination.

Step 4: Conclusion A and C only satisfy the reverse-osmosis requirements. Hence, the correct option is (A).

Quick Tip to Solve: Locate the concentrated solution first, then apply sufficient extra pressure there; a solvent-selective membrane is essential.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The pH of a 0.01 M weak acid $\mathrm{HX}\left(\mathrm{K}_a = 4 \times 10^{-10}\right)$ is found to be 5 . Now the acid solution is diluted with excess of water so that the pH of the solution changes to 6 . The new concentration of the diluted weak acid is given as $x \times 10^{-4}$M. The value of x is _____ (nearest integer) [4 marks]

Solution:

Concept:

- For $\mathrm{HX} \rightleftharpoons \mathrm{H}^+ + \mathrm{X}^-$, $K_a = [\mathrm{H}^+][\mathrm{X}^-]/[\mathrm{HX}]$. The final pH gives the final hydrogen-ion concentration directly.
- For small dissociation, $K_a \simeq h^2/C$, where $h = [\mathrm{H}^+]$ and C is the analytical acid concentration. Small water-ionisation corrections can be checked after calculating the leading value.

Step 1: Use the final equilibrium condition The final pH is 6, so $h = 10^{-6}\,\mathrm{mol\,L^{-1}}$. The supplied initial data are inconsistent: $K_a = 4 \times 10^{-10}$ and $C_0 = 0.01\,\mathrm{M}$ predict $h_0 \simeq 2 \times 10^{-6}\,\mathrm{M}$, or pH 5.70, rather than pH 5. Therefore a dilution ratio based on the two stated pH values would not be consistent with the stated acid constant. Calculate the final concentration using the supplied K_a and final pH, which independently determine the requested result.

Step 2: Calculate the concentration from the acid constant Neglecting water autoionisation initially, $[\mathrm{X^-}] \simeq h$ and $[\mathrm{HX}] \simeq C$. Therefore

$$C \simeq \frac{h^2}{K_a} = \frac{10^{-12}}{4 \times 10^{-10}} = 2.5 \times 10^{-3}\,\mathrm{M}.$$

The dissociated fraction is only $h/C = 4 \times 10^{-4}$, so replacing the undissociated acid concentration by the analytical concentration is justified.

Step 3: Check the rounding The leading result is $2.5 \times 10^{-3} = 25 \times 10^{-4}\,\mathrm{M}$. If water ionisation is retained with $K_w = 10^{-14}$, then $[\mathrm{OH^-}] = 10^{-8}\,\mathrm{M}$, and charge balance gives $[\mathrm{X^-}] = 9.9 \times 10^{-7}\,\mathrm{M}$. The exact acid-balance expression gives

$$C = [\mathrm{X^-}]\left(1 + \frac{h}{K_a}\right) = 2.47599 \times 10^{-3}\,\mathrm{M}.$$

Its coefficient 24.7599 still rounds to 25. Hence the small correction does not alter the requested nearest integer.

Step 4: Conclusion The diluted concentration is approximately $25 \times 10^{-4}\,\mathrm{M}$, giving the dimensionless coefficient $\boxed{x = 25}$.

Quick Tip to Solve: Use the final pH and acid constant directly. Check consistency before using a ratio of initial and final hydrogen-ion concentrations.

72. In Dumas' method for estimation of nitrogen 1 g of an organic compound gave 150 mL of nitrogen collected at 300 K temperature and 900 mm Hg pressure. The percentage composition of nitrogen in the compound is _____ % (nearest integer)

(Aqueous tension at $300\ \mathrm{K} = 15\ \mathrm{mm\ Hg}$ **)** [4 marks]

Solution:

Concept:

- Gas collected over water contains nitrogen plus water vapour. Subtract aqueous tension from the total pressure to obtain dry nitrogen pressure before using $pV = nRT$.
- The molar mass of collected nitrogen gas, $\mathrm{N_2}$, is $28\,\mathrm{g\,mol^{-1}}$. Its mass divided by the sample mass and multiplied by 100 gives nitrogen percentage.

Step 1: Correct the pressure and volume The measured total pressure is $900\,\mathrm{mm\,Hg}$, including

$15\,\text{mm Hg}$ of water vapour. By Dalton's law,

$$p_{N_2} = 900 - 15 = 885\,\text{mm Hg} = \frac{885}{760}\,\text{atm}.$$

The volume is $150\,\text{mL} = 0.150\,\text{L}$, and the temperature is $300\,\text{K}$. These conversions permit use of the litre-atmosphere value of the gas constant.

Step 2: Calculate nitrogen moles and mass Using $R = 0.0821\,\text{L atm mol}^{-1}\,\text{K}^{-1}$,

$$n = \frac{pV}{RT} = \frac{(885/760)(0.150)}{0.0821(300)} \simeq 7.09 \times 10^{-3}\,\text{mol}.$$

The collected gas consists of nitrogen molecules, each containing two atoms, so multiply by 28 rather than 14:

$$m(N_2) = (7.09 \times 10^{-3})(28) \simeq 0.1986\,\text{g}.$$

All of this gas mass is nitrogen derived from the original compound.

Step 3: Express the result as a percentage The organic sample has mass 1 g. Consequently,

$$\%\text{N} = \frac{0.1986}{1.000} \times 100 \simeq 19.86\%.$$

The nearest integer is 20. Omitting the aqueous-tension correction would count water vapour as nitrogen, while using the atomic mass 14 would incorrectly halve the nitrogen mass. Keeping the dry-gas pressure and molecular molar mass separate prevents both errors. The unrounded percentage is slightly below 20, but rounds upward as requested.

Step 4: Conclusion The nitrogen content is approximately 19.86%, giving the nearest-integer answer $\boxed{20\%}$.

Quick Tip to Solve: In Dumas' method, subtract water-vapour pressure first, use compatible gas-law units, and multiply the moles of collected gas by 28.

73. Fortification of food with iron is done using $FeSO_4 \cdot 7H_2O$. The mass in grams of the $FeSO_4 \cdot 7H_2O$ required to achieve 12 ppm of iron in 150 kg of wheat is _____ (Nearest integer)

[Given: Molar mass of Fe, S and and O respectively are 56, 32 and $16\ \text{g mol}^{-1}$] [4 marks]

Solution:

Concept:

- For a dilute solid mixture, concentration in parts per million by mass is $10^6 m_{\text{solute}}/m_{\text{mixture}}$, with both masses in the same units. Thus 12 ppm of iron means 12 mg of elemental iron per kilogram of wheat.
- Each mole of $FeSO_4 \cdot 7\,H_2O$ supplies one mole of iron. Its water of crystallisation contributes to the salt's mass but supplies no additional iron. The iron mass fraction is the molar mass of iron divided by the molar mass of the hydrated salt.

Step 1: Calculate the required mass of elemental iron The concentration specified is that of iron,

rather than of ferrous sulphate. For 150 kg of wheat the required iron mass is

$$m_{\mathrm{Fe}} = 12\,\mathrm{mg\,kg^{-1}} \times 150\,\mathrm{kg} = 1800\,\mathrm{mg} = 1.8\,\mathrm{g}.$$

The ppm calculation uses the wheat mass as stated in the question; the tiny mass added during fortification does not affect the required nearest-integer result.

Step 2: Convert iron mass into hydrated-salt mass Using the given atomic masses and $M_{\mathrm{H}} = 1\,\mathrm{g\,mol^{-1}}$, the molar mass of the heptahydrate is

$$M_{\mathrm{FeSO_4\cdot 7\,H_2O}} = 56 + 32 + 4(16) + 7\{2(1) + 16\} = 278\,\mathrm{g\,mol^{-1}}.$$

Since 278 g of this salt contains 56 g of iron, the mass required to supply 1.8 g of iron is

$$m_{\mathrm{salt}} = 1.8 \times \frac{278}{56} = 8.9357\ldots\ \mathrm{g}.$$

Using the anhydrous molar mass would underestimate this mass because seven water molecules are present in every formula unit.

Step 3: Conclusion The required mass of $\mathrm{FeSO_4\cdot 7\,H_2O}$ is approximately $8.94\,\mathrm{g}$. Rounded to the nearest integer, the answer is $\boxed{9\,\mathrm{g}}$.

Quick Tip to Solve: Convert ppm to mg per kg to find elemental iron first, then multiply by the hydrated-salt molar mass divided by 56. Include all seven waters of crystallisation.

74. $\mathrm{KMnO_4}$ **acts as an oxidising agent in acidic medium. ' X ' is the difference between the oxidation states of Mn in reactant and product. ' Y ' is the number of ' d ' electrons present in the brown red precipitate formed at the end of the acetate ion test with neutral ferric chloride. The value of** $\mathrm{X + Y}$ **is _____.**

[4 marks]

Solution:

Concept:

- Oxidation numbers add to the charge on a species. In acidic solution permanganate is reduced from manganese(VII) to manganese(II), so its oxidising action involves a five-electron change per manganese atom.
- The acetate test with neutral ferric chloride involves iron(III). The brown-red basic ferric acetate precipitate formed on heating retains iron in the +3 oxidation state. Its colour or precipitation does not imply reduction to iron(II).
- The ground-state configuration of iron is $[\mathrm{Ar}]3d^6 4s^2$. In forming cations, the 4s electrons are removed before the 3d electrons; consequently iron(III) has a $3d^5$ configuration.

Step 1: Find the change in manganese oxidation state Let the oxidation number of Mn in $\mathrm{KMnO_4}$ be n. Potassium is +1 and oxygen is -2, hence

$$1 + n + 4(-2) = 0 \quad \Rightarrow \quad n = +7.$$

The product in acidic medium is Mn^{2+}, as shown by the reduction half-reaction

$$MnO_4^- + 8\,H^+ + 5\,e^- \longrightarrow Mn^{2+} + 4\,H_2O.$$

Therefore the magnitude of the difference between the reactant and product oxidation states is $X = 7 - 2 = 5$. The product would be different in neutral or alkaline medium, which is why the specified acidity matters.

Step 2: Count the d electrons in the acetate-test precipitate Ferric ions give the acetate test and form brown-red basic ferric acetate on heating. Acetate and hydroxide ligands do not change the metal's ferric oxidation state during this process. Thus the metal ion to consider is Fe^{3+}, rather than Fe^{2+}. Starting from neutral iron, remove two 4s electrons and one 3d electron:

$$Fe : [Ar]3d^6 4s^2, \qquad Fe^{3+} : [Ar]3d^5.$$

Hence $Y = 5$. This is the total number of d electrons, not a request to count ligands or infer the number from the colour alone.

Step 3: Conclusion The two quantities are $X = 5$ and $Y = 5$. Their dimensionless sum is $\boxed{X + Y = 10}$.

Quick Tip to Solve: Identify both metal oxidation states before counting electrons: acidic permanganate gives manganese(II), while the acetate test retains ferric iron with five d electrons.

75. The total number of hydrogen bonds of a DNA-double Helix strand whose one strand has the following sequence of bases is _____.

$$5' - G - G - C - A - A - A - T - C - G - G - C - T - A - 3'$$

[4 marks]

Solution:

Concept:

- The two strands of a DNA double helix are complementary: adenine pairs with thymine, and guanine pairs with cytosine. An A-T pair contains two hydrogen bonds, whereas a G-C pair contains three.
- A base written on one strand corresponds to exactly one base pair in the double helix. Therefore count the G and C positions together, and the A and T positions together, then weight the two counts by three and two respectively.

Step 1: Count the bases on the given strand Read the entire sequence from its 5-prime end to its 3-prime end:

$$G\ G\ C\ A\ A\ A\ T\ C\ G\ G\ C\ T\ A.$$

There are four guanines (positions 1, 2, 9 and 10), three cytosines (positions 3, 8 and 11), four

adenines (positions 4, 5, 6 and 13), and two thymines (positions 7 and 12). Thus

$$N_G + N_C = 4 + 3 = 7, \qquad N_A + N_T = 4 + 2 = 6.$$

As a check, these counts total $7 + 6 = 13$, equal to the number of bases shown. The 5-prime and 3-prime end labels indicate direction and are not additional bases.

Step 2: Assign hydrogen bonds to each complementary pair The seven G or C positions each form a G-C pair with the complementary strand. The six A or T positions each form an A-T pair. Hence

$$N_{\text{H bonds}} = 3(N_G + N_C) + 2(N_A + N_T) = 3(7) + 2(6) = 21 + 12 = 33.$$

Do not double this number after imagining the complementary strand: a hydrogen bond is shared by both bases of a pair and has already been counted once. Phosphodiester bonds within an individual strand are covalent backbone bonds and are not part of this hydrogen-bond total.

Step 3: Conclusion The DNA double helix contains seven G-C pairs and six A-T pairs. The required total number of hydrogen bonds is $\boxed{33}$, a dimensionless count.

Quick Tip to Solve: Start with two bonds for each of the 13 base pairs, then add one extra for every G or C on the stated strand: $2(13) + 7 = 33$.