JEE Main 2025 Paper 1 (B.E./B.Tech) Apr 4 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 4 April 2025, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $\mathrm{A} = \{-3, -2, -1, 0, 1, 2, 3\}$ and R be a relation on A defined by $x\mathrm{R}y$ if and only if $2x - y \in \{0, 1\}$. Let l be the number of elements in R. Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations, respectively. Then $l + \mathrm{m} + \mathrm{n}$ is equal to: [4 marks]

(A) 17

(B) 18

(C) 15

(D) 16

Correct Answer: (A) 17

Solution:

Concept:

- A relation on A is a subset of $A \times A$. Here $2x - y \in \{0, 1\}$ means that either $y = 2x$ or $y = 2x - 1$, with both coordinates belonging to A.
- Reflexivity requires every diagonal pair (x, x). Symmetry requires (y, x) whenever (x, y) is present; the two requested additions are calculated independently for the original relation.

Step 1: List all admissible ordered pairs Test the seven possible first coordinates and retain only second coordinates in A. For $x = -3, -2$, both candidates are outside A; for $x = -1$, they are $-2, -3$; for $x = 0$, they are $0, -1$; for $x = 1$, they are $2, 1$; for $x = 2$, only 3 is allowed; and for $x = 3$, neither is allowed. Thus

$$R = \{(-1, -2), (-1, -3), (0, 0), (0, -1), (1, 2), (1, 1), (2, 3)\},$$

so $l = 7$.

Step 2: Count missing reflexive and symmetric pairs Of the seven required diagonal pairs, only $(0, 0)$ and $(1, 1)$ already occur. Hence $m = 7 - 2 = 5$. The five non-diagonal pairs require their reverses:

$$(-2, -1),\ (-3, -1),\ (-1, 0),\ (2, 1),\ (3, 2).$$

None of these is already in R, while the two diagonal pairs are their own reverses. Therefore $n = 5$. Do not count a diagonal pair again for symmetry or require transitivity, which is not asked for.

Step 3: Conclusion The required sum is

$$l + m + n = 7 + 5 + 5 = 17.$$

Hence, the correct option is (A). The other values arise from omitting or double-counting required ordered pairs.

Quick Tip to Solve: List the relation first, then count absent diagonal pairs and absent reverse pairs separately; symmetry does not demand transitivity.

2. Let the domains of the functions $f(x) = \log_4 \log_3 \log_7 \left(8 - \log_2 \left(x^2 + 4x + 5\right)\right)$ and $g(x) = \sin^{-1}\left(\frac{7x+10}{x-2}\right)$ be (α, β) and $[\gamma, \delta]$, respectively. Then $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ is equal to : [4 marks]

(A) 15

(B) 13

(C) 16

(D) 14

Correct Answer: (A) 15

Solution:

Concept:

- Every real logarithm must have a positive argument. Since the bases $2, 3, 4, 7$ exceed one, their logarithmic functions are strictly increasing.
- The real domain of $\sin^{-1} u$ requires $-1 \le u \le 1$, including the endpoints; its rational argument must also have a nonzero denominator.

Step 1: Work inward through the nested logarithms For the outermost logarithm we need $\log_3 \log_7(8 - \log_2(x^2 + 4x + 5)) > 0$. Successive monotonicity gives

$$\begin{aligned} \log_7(8 - \log_2(x^2 + 4x + 5)) &> 1, \\ 8 - \log_2(x^2 + 4x + 5) &> 7, \\ \log_2(x^2 + 4x + 5) &< 1. \end{aligned}$$

Also $x^2 + 4x + 5 = (x + 2)^2 + 1 > 0$ for every real x. Thus the remaining condition is $(x + 2)^2 + 1 < 2$, or $-3 < x < -1$. This strict inequality already guarantees positivity at every inner logarithm. Hence $\alpha = -3$ and $\beta = -1$.

Step 2: Find the inverse-sine domain and add the endpoint squares For $x \neq 2$, the argument condition is equivalent to

$$\begin{aligned} (7x + 10)^2 &\le (x - 2)^2, \\ 48x^2 + 144x + 96 &\le 0, \\ (x + 1)(x + 2) &\le 0. \end{aligned}$$

Therefore $x \in [-2, -1]$; the excluded point 2 does not lie in this interval. Hence $\gamma = -2$, $\delta = -1$. The logarithmic endpoints are excluded, but the inverse-sine endpoints are allowed; confusing these conditions changes the domains.

Step 3: Conclusion Consequently,

$$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 9 + 1 + 4 + 1 = 15.$$

Hence, the correct option is (A); 13, 16 and 14 do not equal this sum.

Quick Tip to Solve: For nested logarithms, impose the outermost positivity first and move inward; do not stop at positivity of the innermost expression.

3. Let the mean and the standard deviation of the observation $2, 3, 3, 4, 5, 7, a, b$ be 4 and $\sqrt{2}$ respectively. Then the mean deviation about the mode of these observations is : [4 marks]

(A) $\frac{1}{2}$

(B) $\frac{3}{4}$

(C) 1

(D) 2

Correct Answer: (C) 1

Solution:

Concept:

- For N observations, $\overline{x} = \sum x_i/N$ and $\sigma^2 = \sum x_i^2/N - \overline{x}^2$. The stated standard deviation must be squared before using the variance formula.
- The mode is the most frequently occurring observation. Mean deviation about a number M is $N^{-1}\sum |x_i - M|$, so deviations must be taken in absolute value.

Step 1: Determine the two unknown observations There are eight observations and their mean is 4, so their total is $8 \times 4 = 32$. The six known values add to 24. Therefore $a + b = 8$. Since $\sigma = \sqrt{2}$, the variance is 2 and

$$\sum x_i^2 = 8(2 + 4^2) = 144.$$

The known square sum is $4 + 9 + 9 + 16 + 25 + 49 = 112$, giving $a^2 + b^2 = 32$. Now

$$2ab = (a + b)^2 - (a^2 + b^2) = 64 - 32 = 32.$$

Thus $ab = 16$, and a, b are roots of $t^2 - 8t + 16 = (t - 4)^2 = 0$. Both unknown observations equal 4.

Step 2: Identify the mode and calculate absolute deviations The complete data are $2, 3, 3, 4, 5, 7, 4, 4$. The value 4 occurs three times, whereas 3 occurs twice and each other value occurs once. The unique mode is therefore 4, not 3. The absolute deviations from it are $2, 1, 1, 0, 1, 3, 0, 0$, whose sum is 8. Hence

$$\text{Mean deviation about mode} = \frac{2 + 1 + 1 + 0 + 1 + 3 + 0 + 0}{8}.$$

The divisor remains eight, including the zero deviations; neither the number of distinct observations nor the frequency of the mode is the divisor.

Step 3: Conclusion The required mean deviation is $8/8 = 1$. Hence, the correct option is (C). The values $1/2$, $3/4$ and 2 do not equal the average absolute deviation.

Quick Tip to Solve: Use the mean and variance to recover the missing data before identifying the mode; the known observations alone may suggest the wrong mode.

4. Let the sum of the focal distances of the point $\mathrm{P}(4,3)$ on the hyperbola $\mathrm{H}: \frac{x^2}{\mathrm{a}^2}-\frac{y^2}{\mathrm{b}^2}=1$ be $8\sqrt{\frac{5}{3}}$. If for H , the length of the latus rectum is l and the product of the focal distances of the point P is m , then $9l^2+6\,\mathrm{m}$ is equal to : [4 marks]

(A) 187

(B) 184

(C) 186

(D) 185

Correct Answer: (D) 185

Solution:

Concept:

- For $x^2/a^2-y^2/b^2=1$, the eccentricity satisfies $e^2=1+b^2/a^2$ and the latus rectum has length $l=2b^2/a$, where $a,b>0$.
- At a point of the right branch with abscissa x, the two focal distances are $ex-a$ and $ex+a$. Their sum is $2ex$, their difference is $2a$, and their product is $e^2x^2-a^2$.

Step 1: Recover the hyperbola parameters from the given point At $P(4,3)$, the focal-distance sum is $8e$. Comparing with $8\sqrt{5/3}$ gives $e=\sqrt{5/3}$, so $b^2/a^2=e^2-1=2/3$. Substitute P in the hyperbola and use $b^2=2a^2/3$:

$$\frac{16}{a^2}-\frac{9}{2a^2/3}=1.$$

The left side simplifies to $(16-27/2)/a^2$. Consequently $a^2=5/2$ and $b^2=5/3$. It is the difference of focal distances that is constant on a hyperbola; treating their given sum as $2a$ would be incorrect.

Step 2: Evaluate the requested squared length and distance product The square of the latus-rectum length is

$$l^2=\frac{4b^4}{a^2}=\frac{4(25/9)}{5/2}=\frac{40}{9}.$$

The focal-distance product at the same point is

$$m=(4e-a)(4e+a)=16e^2-a^2=\frac{80}{3}-\frac{5}{2}=\frac{145}{6}.$$

Both distances are positive because $4e>a$. Squaring the latus-rectum length first avoids unnecessary radicals and gives exactly the quantity needed in the final expression.

Step 3: Conclusion Therefore

$$9l^2+6m=9\left(\frac{40}{9}\right)+6\left(\frac{145}{6}\right)=40+145=185.$$

Hence, the correct option is (D); the other three nearby integers fail this exact calculation.

Quick Tip to Solve: For a right-branch hyperbola point, use focal-distance sum $2ex$ and product $e^2x^2-a^2$; only the difference equals $2a$.

5. Let $a > 0$. If the function $f(x) = 6x^3 - 45ax^2 + 108a^2x + 1$ attains its local maximum and minimum values at the points x_1 and x_2 respectively such that $x_1x_2 = 54$, then $a + x_1 + x_2$ is equal to : [4 marks]

(A) 15

(B) 13

(C) 24

(D) 18

Correct Answer: (D) 18

Solution:

Concept:

- A differentiable function has an interior local extremum only at a stationary point where $f'(x) = 0$. A sign change of the derivative or the second-derivative test identifies its type.
- For a factored quadratic derivative, the stationary abscissae can be obtained directly; the given positivity of the parameter fixes both their order and the sign of its square root.

Step 1: Find and classify the stationary points Differentiate with respect to x, treating a as a constant:

$$\begin{aligned} f'(x) &= 18x^2 - 90ax + 108a^2 \\ &= 18(x^2 - 5ax + 6a^2) \\ &= 18(x - 2a)(x - 3a). \end{aligned}$$

Thus the stationary points are $2a$ and $3a$. Since $a > 0$, they are distinct and $2a < 3a$. Also $f''(x) = 36x - 90a$, so $f''(2a) = -18a < 0$ while $f''(3a) = 18a > 0$. Therefore the local maximum occurs at $x_1 = 2a$ and the local minimum at $x_2 = 3a$, in the order required.

Step 2: Use the prescribed product to determine the parameter The condition is on the abscissae of the extrema, rather than their function values. Substitute the stationary abscissae:

$$x_1x_2 = (2a)(3a) = 6a^2 = 54.$$

It follows that $a^2 = 9$. Although the equation for the square has two algebraic roots, the given $a > 0$ forces $a = 3$. Consequently $x_1 = 6$ and $x_2 = 9$. There is no need to evaluate the cubic at these points: its constant term does not affect either stationary abscissa or their product.

Step 3: Conclusion The requested value is

$$a + x_1 + x_2 = 3 + 6 + 9 = 18.$$

Hence, the correct option is (D). In particular 15 is only $x_1 + x_2$, omitting the required parameter; 13 and 24 also fail the condition.

Quick Tip to Solve: Factor the derivative before using the extremum condition, and check whether the question constrains the abscissae or the extremum values.

6. Let the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ satisfy $A^n = A^{n-2} + A^2 - I$ for $n \geqslant 3$. Then the sum of all the elements of A^{50} is : [4 marks]

(A) 44

(B) 39

(C) 52

(D) 53

Correct Answer: (D) 53

Solution:

Concept:

- The given recurrence decreases the exponent by two, so even powers form a separate sequence beginning with A^2. Its increment at each step is the fixed matrix $A^2 - I$.
- Matrix addition and scalar multiplication act entry by entry. The sum of all entries is therefore a linear operation, but the identity matrix contributes three, rather than one, in order three.

Step 1: Iterate the recurrence for even powers Write $B = A^2 - I$. For even exponents the recurrence gives $A^4 = A^2 + B$, $A^6 = A^2 + 2B$, and generally

$$A^{2k} = A^2 + (k-1)(A^2 - I).$$

This follows by induction: replacing k by $k+1$ adds one more copy of B. The exponent 50 corresponds to $k = 25$, hence

$$A^{50} = 25A^2 - 24I.$$

There are 24 recurrence steps from exponent 2 to exponent 50; counting 25 steps would give an erroneous coefficient.

Step 2: Square the given matrix and sum its entries Multiply rows of A by columns of A to obtain

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

Its entry sum is $1 + 2 + 2 = 5$, while the entry sum of I is 3. Thus the linearity of the entry-sum operation yields $25 \times 5 - 24 \times 3$. Equivalently, substitution gives

$$A^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix},$$

whose entries can be added directly as a check. No diagonalisation or repeated multiplication through all fifty powers is necessary; the supplied recurrence already reduces the calculation to one square.

Step 3: Conclusion The sum of all entries is

$$25(5) - 24(3) = 125 - 72 = 53.$$

Hence, the correct option is (D). The alternative totals 44, 39 and 52 do not match either the recurrence calculation or the explicit matrix.

Quick Tip to Solve: When a matrix-power recurrence jumps by two, iterate within the correct parity and count the steps from its base power carefully.

7. If a curve $y = y(x)$ passes through the point $\left(1, \frac{\pi}{2}\right)$ and satisfies the differential equation $\left(7x^4 \cot y - e^x \operatorname{cosec} y\right) \frac{dx}{dy} = x^5, x \geq 1$, then at $x = 2$, the value of $\cos y$ is : [4 marks]

(A) $\frac{2e^2+e}{64}$
(B) $\frac{2e^2-e}{64}$
(C) $\frac{2e^2-e}{128}$
(D) $\frac{2e^2+e}{128}$

Correct Answer: (C) $\frac{2e^2-e}{128}$

Solution:

Concept:

- The derivative of $z = \cos y$, with respect to x, is $z' = -\sin y\, y'$. Multiplying by $\sin y$ converts the trigonometric differential equation into a linear equation in z.
- For $z' + P(x)z = Q(x)$, the integrating factor is $\exp(\int P(x)\, dx)$. Here $x \geq 1$, so division by powers of x is valid.

Step 1: Convert the dependent variable Multiply the given equation by dy/dx, and then by $\sin y$, to obtain

$$7x^4 \cos y - e^x = x^5 \sin y\, y'.$$

Since $\sin y\, y' = -z'$, rearranging and dividing by x^5 gives

$$z' + \frac{7}{x} z = \frac{e^x}{x^5}.$$

The negative sign in the derivative of cosine is essential; changing it would give a different integrating factor.

Step 2: Integrate and impose the initial point The integrating factor is x^7. Thus

$$\frac{d}{dx}(x^7 z) = x^2 e^x.$$

Integrating twice by parts, or differentiating the expression on the right to check it, gives

$$x^7 z = e^x(x^2 - 2x + 2) + K.$$

At the prescribed point, $x = 1$ and $z = \cos(\pi/2) = 0$. Hence $0 = e + K$, so $K = -e$. This determines the solution through the initial point, rather than merely a family of curves.

Step 3: Evaluate at the requested abscissa Substitute $x = 2$ into the initial-value solution:

$$2^7 \cos y = e^2(4 - 4 + 2) - e = 2e^2 - e.$$

The denominator is $2^7 = 128$, not 64, and the integration constant contributes a minus sign. These two checks exclude all the other listed expressions.

Step 4: Conclusion At $x = 2$, $\cos y = \dfrac{2e^2 - e}{128}$. Hence, the correct option is (C).

Quick Tip to Solve: After setting $z = \cos y$, carry its negative derivative explicitly; then use the initial point before substituting the final value of x.

8. If the sum of the first 20 terms of the series $\frac{4\cdot1}{4+3\cdot1^2+1^4} + \frac{4\cdot2}{4+3\cdot2^2+2^4} + \frac{4\cdot3}{4+3\cdot3^2+3^4} + \frac{4\cdot4}{4+3\cdot4^2+4^4} + \ldots\cdot$ is $\frac{\mathrm{m}}{\mathrm{n}}$, where m and n are coprime, then $\mathrm{m} + \mathrm{n}$ is equal to : [4 marks]

(A) 423

(B) 421

(C) 422

(D) 420

Correct Answer: (B) 421

Solution:

Concept:

- A quartic denominator can sometimes be factored as a difference of squares. Here $k^4 + 3k^2 + 4 = (k^2 + 2)^2 - k^2$.
- A telescoping sum cancels adjacent terms when the second denominator of one summand equals the first denominator of the next summand. Only the two endpoint contributions remain.

Step 1: Factor and split the general summand For the k-th term, factor the denominator as

$$k^4 + 3k^2 + 4 = (k^2 - k + 2)(k^2 + k + 2).$$

The difference of the two quadratic factors is $2k$. Therefore the required numerator $4k$ is obtained by

$$\frac{4k}{k^4 + 3k^2 + 4} = \frac{2}{k^2 - k + 2} - \frac{2}{k^2 + k + 2}.$$

This identity follows directly by combining the two fractions, so no approximation to the finite sum is involved.

Step 2: Cancel consecutive terms Write $f(k) = 2/(k^2 - k + 2)$. Since

$$(k + 1)^2 - (k + 1) + 2 = k^2 + k + 2,$$

our summand is $f(k) - f(k+1)$. Consequently

$$S_{20} = f(1) - f(21) = \frac{2}{2} - \frac{2}{441 - 21 + 2}.$$

Every denominator between the endpoints appears once with each sign. The upper endpoint is 21, because the last negative term comes from $k = 20$, not from $k = 19$.

Step 3: Reduce the fraction and compare the options The remaining denominator is 422, so

$$S_{20} = 1 - \frac{2}{422} = 1 - \frac{1}{211} = \frac{210}{211}.$$

Consecutive integers have greatest common divisor 1, so this is already the required coprime representation. Hence $m = 210$ and $n = 211$; the nearby alternatives arise from an endpoint or arithmetic error.

Step 4: Conclusion $m + n = 210 + 211 = 421$. Hence, the correct option is (B).

Quick Tip to Solve: Try $(k^2+2)^2 - k^2$ when a quartic has the form $k^4 + 3k^2 + 4$; the resulting quadratic factors differ by $2k$.

9. Let for two distinct values of p the lines $y = x + \mathrm{p}$ touch the ellipse $\mathrm{E} : \frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$ at the points A and B . Let the line $y = x$ intersect E at the points C and D . Then the area of the quadrilateral $ABCD$ is equal to : [4 marks]

(A) 48

(B) 20

(C) 24

(D) 36

Correct Answer: (C) 24

Solution:

Concept:

- For the ellipse $x^2/a^2 + y^2/b^2 = 1$, the tangent at (x_1, y_1) is $xx_1/a^2 + yy_1/b^2 = 1$. Parallel tangents touch at diametrically opposite points.
- A quadrilateral with opposite vertex vectors $\pm\mathbf{u}$ and $\pm\mathbf{v}$ has area $2|\det(\mathbf{u}, \mathbf{v})|$. Equivalently, it is composed of four triangles with the centre as a common vertex.

Step 1: Find the two points of tangency For the tangent to have slope 1, its slope from the tangent equation must satisfy

$$-\frac{9x_1}{16y_1} = 1, \qquad y_1 = -\frac{9}{16}x_1.$$

Substituting this relation in the ellipse equation yields $x_1^2 = 256/25$. We may therefore choose

$$A = \left(-\frac{16}{5}, \frac{9}{5}\right), \qquad B = -A.$$

Indeed, the tangent at this chosen point is $y = x + 5$, and the opposite point has tangent $y = x - 5$. Thus both given parallel touching lines are accounted for.

Step 2: Locate the endpoints on the line through the centre Put $y = x$ in the ellipse:

$$\left(\frac{1}{16} + \frac{1}{9}\right) x^2 = 1 \quad \Longrightarrow \quad x^2 = \frac{144}{25}.$$

Thus choose

$$C = \left(\frac{12}{5}, \frac{12}{5}\right), \qquad D = -C.$$

The segments AB and CD are the two diagonals of the quadrilateral formed by these four points. In taking its area, traverse the boundary through alternating diagonal endpoints.

Step 3: Add the four central triangles The determinant of the vectors to A and C is

$$\det(A, C) = \frac{-16 \cdot 12 - 9 \cdot 12}{25} = -12.$$

Each of the four triangles determined by the two diagonals has area $|\det(A, C)|/2 = 6$. Their sum is 24. This calculation does not assume that the two diagonals are perpendicular; that assumption would give an incorrect area.

Step 4: Conclusion The quadrilateral has area 24 square units. Hence, the correct option is (C).

Quick Tip to Solve: When both pairs of points are opposite about the centre, use twice the determinant of one point from each pair; perpendicular diagonals are unnecessary.

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10. The centre of a circle C is at the centre of the ellipse $\mathrm{E} : \frac{x^2}{\mathrm{a}^2} + \frac{y^2}{\mathrm{b}^2} = 1, \mathrm{a} > \mathrm{b}$**. Let C pass through the foci** F_1 **and** F_2 **of E such that the circle** C **and the ellipse** E **intersect at four points. Let P be one of these four points. If the area of the triangle** $\mathrm{PF_1\ F_2}$ **is 30 and the length of the major axis of** E **is 17 , then the distance between the foci of** E **is :** [4 marks]

(A) 12

(B) 26

(C) 13

(D) $\frac{13}{2}$

Correct Answer: (C) 13

Solution:

Concept:

- For an ellipse with semiaxes $a > b$, the foci are $(\pm c, 0)$, where $c^2 = a^2 - b^2$. A circle centred at the origin through either focus has equation $x^2 + y^2 = c^2$.
- The area of a triangle with the two foci as its base and third vertex (x, y) is $\frac{1}{2}(2c)|y| = c|y|$. Its altitude is the perpendicular distance from the vertex to the x-axis.

Step 1: Relate an intersection coordinate to the semiaxes Let $P = (x, y)$ be one of the four intersection

points. Since P lies on both curves,

$$x^2 + y^2 = c^2, \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Substitute $x^2 = c^2 - y^2$ in the second equation. Using $a^2 - b^2 = c^2$, this becomes

$$\frac{c^2}{a^2} + \frac{c^2 y^2}{a^2 b^2} = 1.$$

Multiplication by a^2 gives $c^2 y^2 / b^2 = b^2$, and hence $|y| = b^2/c$, since lengths b and c are positive.

Step 2: Use the specified triangle area The base F_1F_2 has length $2c$, so

$$[PF_1F_2] = c|y| = c\frac{b^2}{c} = b^2.$$

The given area 30 therefore means $b^2 = 30$. The major-axis length is $2a = 17$, not $a = 17$, giving $a = 17/2$. We now have both squared semiaxes without needing the abscissa of P.

Step 3: Calculate the focal separation Apply the focal relation:

$$c^2 = \frac{289}{4} - 30 = \frac{169}{4}, \qquad c = \frac{13}{2}.$$

Thus the distance between the foci is twice this value. As a consistency check, $y^2 = b^4/c^2 = 3600/169 < c^2$, so the circle really does meet the ellipse in four points. Option (D) is only the centre-to-focus distance; the other numerical options do not satisfy the focal relation.

Step 4: Conclusion $F_1F_2 = 2c = 13$. Hence, the correct option is (C).

Quick Tip to Solve: At the intersections with the circle through the foci, the triangle area simplifies to b^2; distinguish the semimajor axis from the full major axis.

11. A line passing through the point $\mathrm{A}(-2, 0)$, touches the parabola $\mathrm{P} : y^2 = x - 2$ at the point B in the first quadrant. The area, of the region bounded by the line AB, parabola P and the x-axis, is : [4 marks]

(A) 3

(B) $\frac{7}{3}$

(C) $\frac{8}{3}$

(D) 2

Correct Answer: (C) $\frac{8}{3}$

Solution:

Concept:

- It is convenient to write the parabola as $x = y^2 + 2$ and integrate horizontal strips. Their width is the right-hand x-coordinate minus the left-hand x-coordinate.

- At a point (t^2+2, t) on this parabola, differentiation gives $dx/dy = 2t$. Thus its tangent is $x = 2ty - t^2 + 2$. The first-quadrant condition selects $t > 0$.

Step 1: Determine the point where the line touches Use $B = (t^2+2, t)$. Its tangent equation can be written

$$x - (t^2+2) = 2t(y-t), \quad \text{or} \quad x = 2ty - t^2 + 2.$$

Because this tangent passes through $A = (-2, 0)$, substitution gives

$$-2 = 2 - t^2 \quad \Longrightarrow \quad t^2 = 4.$$

The point of contact is in the first quadrant, so $t = 2$. Consequently $B = (6, 2)$ and the line has equation $x = 4y - 2$. The negative root would correspond to the lower tangent, which is not requested.

Step 2: Identify the region with horizontal strips The boundary on the x-axis runs from $A = (-2, 0)$ to the vertex $(2, 0)$. For $0 \le y \le 2$, the line is the left boundary and the parabola is the right boundary. Their horizontal separation is

$$x_{\text{parabola}} - x_{\text{line}} = y^2 + 2 - (4y - 2) = (y-2)^2.$$

It is nonnegative throughout the interval and becomes zero exactly at the point of tangency. This also checks that the subtraction order is correct.

Step 3: Integrate the width The enclosed area is therefore

$$\begin{aligned} \mathcal{A} &= \int_0^2 (y^2 - 4y + 4)\, dy \\ &= \left[\frac{y^3}{3} - 2y^2 + 4y\right]_0^2 \\ &= \frac{8}{3} - 8 + 8 = \frac{8}{3}. \end{aligned}$$

Integrating with respect to y avoids splitting the region at the vertex in a vertical-strip calculation. None of the other listed values equals this exact integral.

Step 4: Conclusion The required area is $\frac{8}{3}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: For a sideways parabola and an oblique tangent, horizontal strips often give one polynomial integral instead of two separate vertical-strip integrals.

12. Let the product of $\omega_1 = (8+i)\sin\theta + (7+4i)\cos\theta$ and $\omega_2 = (1+8i)\sin\theta + (4+7i)\cos\theta$ be $\alpha + i\beta$, $i = \sqrt{-1}$. Let p and q be the maximum and the minimum values of $\alpha + \beta$ respectively. Then $\mathrm{p} + \mathrm{q}$ is equal to : [4 marks]

(A) 130

(B) 150

(C) 160

(D) 140

Correct Answer: (A) 130

Solution:

Concept:

- If u, v are real, then $(u+iv)(v+iu) = i(u^2+v^2)$: the two real cross-products cancel because $i^2 = -1$.
- Use $\sin^2\theta + \cos^2\theta = 1$ and $2\sin\theta\cos\theta = \sin 2\theta$. A real expression $A + B\sin 2\theta$ ranges from $A - |B|$ to $A + |B|$.

Step 1: Expose the symmetry of the two factors Put $s = \sin\theta$, $c = \cos\theta$, and define the real quantities

$$u = 8s + 7c, \qquad v = s + 4c.$$

The first complex number is $u + iv$, while the second is $v + iu$. Multiplication gives

$$(u+iv)(v+iu) = uv + iu^2 + iv^2 - uv = i(u^2+v^2).$$

Thus $\alpha = 0$, so it is sufficient to find the range of the imaginary part. This cancellation prevents unnecessary expansion of all sixteen individual terms.

Step 2: Simplify the imaginary part Expand the two real squares and collect coefficients:

$$\begin{aligned}\alpha + \beta &= (8s+7c)^2 + (s+4c)^2 \\ &= (64+1)s^2 + (49+16)c^2 + (112+8)sc \\ &= 65(s^2+c^2) + 120sc \\ &= 65 + 60\sin 2\theta.\end{aligned}$$

Both squared coefficients are 65, which is why the expression reduces to one sine term rather than a general sine-and-cosine combination.

Step 3: Find the two extrema and add them As θ varies over the reals, $\sin 2\theta$ takes every value in $[-1, 1]$. The maximum and minimum are therefore

$$p = 65 + 60 = 125, \qquad q = 65 - 60 = 5.$$

They are attained, for example, at $\theta = \pi/4$ and $\theta = -\pi/4$, respectively. Their sum is twice the central value 65. The oscillating term cancels in the sum of the extrema, so the other three proposed totals cannot occur.

Step 4: Conclusion $p + q = 125 + 5 = 130$. Hence, the correct option is (A).

Quick Tip to Solve: When the real and imaginary coefficients swap between two factors, use $(u+iv)(v+iu) = i(u^2+v^2)$ before expanding.

13. If $1^2 \cdot \binom{15}{1} + 2^2 \cdot \binom{15}{2} + 3^2 \cdot \binom{15}{3} + \ldots + 15^2 \cdot \binom{15}{15} = 2^m \cdot 3^n \cdot 5^k$**, where** $m, n, k \in$ **N, then** $\mathrm{m} + \mathrm{n} + \mathrm{k}$ **is equal to :** [4 marks]

(A) 20

(B) 19

(C) 18

(D) 21

Correct Answer: (B) 19

Solution:

Concept:

- The binomial theorem gives $(1+t)^n = \sum_{r=0}^{n} \binom{n}{r} t^r$. Differentiating generates sums with factors r and $r(r-1)$.
- Use $r^2 = r(r-1) + r$, then express the resulting positive integer as a product of prime powers to identify the exponents.

Step 1: Generate the two weighted sums Differentiate the binomial expansion once and twice, and then put $t = 1$. This gives

$$\sum_{r=1}^{15} r\binom{15}{r} = 15\,2^{14}, \qquad \sum_{r=2}^{15} r(r-1)\binom{15}{r} = 15 \cdot 14\,2^{13}.$$

The second derivative counts ordered selections of two distinguished positions, explaining the factor $r(r-1)$. The omitted terms are zero, so both expressions may be summed over the full range.

Step 2: Combine and factor the sum Writing each square as $r(r-1) + r$, the required sum is

$$S = 15 \cdot 14\,2^{13} + 15\,2^{14} = 15\,2^{13}(14+2).$$

Thus

$$S = 15 \cdot 16 \cdot 2^{13} = 2^{17} \cdot 3 \cdot 5.$$

The prime factorisation is unique, so $m = 17$, $n = 1$, and $k = 1$. In particular, the factor 16 contributes four further powers of 2; it must not be treated as an additional prime factor.

Step 3: Check the requested sum Adding the exponents, rather than the prime factors themselves, gives

$$m + n + k = 17 + 1 + 1 = 19.$$

The other listed totals 20, 18 and 21 are inconsistent with these uniquely determined exponents. A direct numerical check is also available: the sum is $15 \cdot 16 \cdot 8192 = 1966080$, which equals $2^{17} \cdot 15$.

Step 4: Conclusion Therefore $m + n + k = \boxed{19}$. Hence, the correct option is (B).

Quick Tip to Solve: For a squared binomial weight, replace r^2 by $r(r-1) + r$ before differentiating the binomial expansion.

14. Let A be the point of intersection of the lines $L_1 : \frac{x-7}{1} = \frac{y-5}{0} = \frac{z-3}{-1}$ and $L_2 : \frac{x-1}{3} = \frac{y+3}{4} = \frac{z+7}{5}$. Let B and C be the points on the lines L_1 and L_2 respectively such that $AB = AC = \sqrt{15}$. Then the square of the area of the triangle ABC is : [4 marks]

(A) 63

(B) 57

(C) 60

(D) 54

Correct Answer: (D) 54

Solution:

Concept:

- A line in symmetric form supplies a direction vector. The zero denominator in the first line means $y = 5$, so its direction vector has zero second component.
- For two sides of lengths a, b enclosing angle θ, the squared triangle area is $\Delta^2 = \frac{1}{4}a^2b^2\sin^2\theta$.

Step 1: Read the line directions From the denominators, take

$$\mathbf{u} = (1, 0, -1), \qquad \mathbf{v} = (3, 4, 5).$$

The vectors along AB and AC are parallel to these respectively. It is unnecessary to find the coordinates of the intersection because the side lengths from that intersection are already specified. Reversing either vector changes the sign of the angle cosine but not its square.

Step 2: Find the squared sine of the included angle Compute the scalar product and lengths:

$$\mathbf{u}\cdot\mathbf{v} = 3 - 5 = -2, \qquad |\mathbf{u}|^2 = 2, \qquad |\mathbf{v}|^2 = 50.$$

Consequently

$$\cos^2\theta = \frac{(-2)^2}{2\cdot 50} = \frac{1}{25}, \qquad \sin^2\theta = 1 - \frac{1}{25} = \frac{24}{25}.$$

This calculation covers either possible choice of B and C on their lines. Those choices replace the included angle by its supplement without changing the triangle area.

Step 3: Insert the two side lengths The given lengths satisfy $AB^2 = AC^2 = 15$. Therefore

$$\Delta^2 = \frac{1}{4}(15)(15)\frac{24}{25} = \frac{225\cdot 24}{100} = 54.$$

As a check, a right angle would give the larger value $225/4 = 56.25$; our lines are not exactly perpendicular, so 54 is reasonable. The alternatives 63, 57 and 60 even exceed this maximum possible squared area for the fixed side lengths.

Step 4: Conclusion The square of the area is $\boxed{54}$ units^4. Hence, the correct option is (D).

Quick Tip to Solve: When the distances from the intersection are given, use direction vectors and the area formula; calculating the intersection adds no useful information.

15. The sum of the infinite series $\cot^{-1}\left(\frac{7}{4}\right)+\cot^{-1}\left(\frac{19}{4}\right)+\cot^{-1}\left(\frac{39}{4}\right)+\cot^{-1}\left(\frac{67}{4}\right)+\ldots$ **. is :** [4 marks]

(A) $\frac{\pi}{2}+\cot^{-1}\left(\frac{1}{2}\right)$

(B) $\frac{\pi}{2}-\cot^{-1}\left(\frac{1}{2}\right)$

(C) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\right)$

(D) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{2}\right)$

Correct Answer: (C) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\right)$

Solution:

Concept:

- For positive u, the principal value satisfies $\cot^{-1}u=\tan^{-1}(1/u)$, with both sides between 0 and $\pi/2$.
- The subtraction identity $\tan(A-B)=(\tan A-\tan B)/(1+\tan A\tan B)$ can turn each positive term into a difference that telescopes.

Step 1: Identify the general term The numerators 7, 19, 39 and 67 have successive differences 12, 20 and 28, so the displayed pattern is $4r^2+3$, starting at $r=1$. Thus

$$a_r=\cot^{-1}\frac{4r^2+3}{4}=\tan^{-1}\frac{4}{4r^2+3}.$$

All denominators are positive. The use of the acute principal arctangent is therefore consistent throughout the series.

Step 2: Express each term as an angle difference Put $A_r=\tan^{-1}((2r+1)/2)$ and $B_r=\tan^{-1}((2r-1)/2)$. Their difference is positive and less than $\pi/2$. Its tangent is

$$\frac{(2r+1)/2-(2r-1)/2}{1+(2r+1)(2r-1)/4}=\frac{4}{4r^2+3}.$$

Hence $a_r=A_r-B_r$, with no added multiple of π. This branch check is essential when using inverse trigonometric identities.

Step 3: Sum and take the limit Adjacent terms cancel, leaving the partial sum

$$S_N=\tan^{-1}\frac{2N+1}{2}-\tan^{-1}\frac{1}{2}.$$

As N tends to infinity, the first term tends to $\pi/2$. Therefore

$$S=\frac{\pi}{2}-\tan^{-1}\frac{1}{2}.$$

The sum is positive and below $\pi/2$, ruling out (A) and (D). Option (B) instead equals $\tan^{-1}(1/2)$, which is the complementary angle, not this telescoped limit.

Step 4: Conclusion The series sums to $\boxed{\frac{\pi}{2}-\tan^{-1}\frac{1}{2}}$. Hence, the correct option is (C).

Quick Tip to Solve: For quadratic denominators in an inverse tangent series, test neighbouring linear arguments whose product creates the quadratic denominator.

16. The axis of a parabola is the line $y = x$ and its vertex and focus are in the first quadrant at distances $\sqrt{2}$ and $2\sqrt{2}$ units from the origin, respectively. If the point $(1, k)$ lies on the parabola, then a possible value of k is : [4 marks]

(A) 8

(B) 3

(C) 9

(D) 4

Correct Answer: (C) 9

Solution:

Concept:

- A parabola consists of points whose distance from its focus equals their perpendicular distance from its directrix.
- The vertex lies halfway between the focus and directrix along the axis. First locate these geometrical objects from the given distances and quadrant.

Step 1: Locate the vertex, focus and directrix A point in the first quadrant on $y = x$ is (t, t), where $t > 0$, and its distance from the origin is $t\sqrt{2}$. Therefore the vertex and focus are

$$V = (1, 1), \qquad F = (2, 2).$$

Their separation is $\sqrt{2}$, and the parabola opens from V towards F. Moving the same distance from V in the opposite direction reaches the origin. The directrix passes through that point and is perpendicular to $y = x$, giving $x + y = 0$.

Step 2: Apply the focus–directrix condition For a general point (x, y), equality of the squared distances yields

$$(x - 2)^2 + (y - 2)^2 = \frac{(x + y)^2}{2}.$$

Using the squared condition loses no sign information because both original distances are nonnegative. With $x = 1$ and $y = k$, this becomes

$$1 + (k - 2)^2 = \frac{(1 + k)^2}{2}.$$

Step 3: Solve and compare the possible values Multiplying by 2 and collecting terms,

$$2 + 2(k^2 - 4k + 4) = k^2 + 2k + 1,$$

so

$$k^2 - 10k + 9 = (k - 1)(k - 9) = 0.$$

Thus k can be 1 or 9. The first is the vertex itself; the second gives another point on the parabola. Among the options, only 9 occurs. The values 8, 3 and 4 do not satisfy the quadratic and therefore fail the focus–directrix condition.

Step 4: Conclusion A possible listed value is $\boxed{k = 9}$. Hence, the correct option is (C).

Quick Tip to Solve: For a parabola with a tilted axis, the focus–directrix distance equation often avoids introducing rotated coordinates.

17. Let $f(x)+2f\left(\frac{1}{x}\right)=x^2+5$ and $2g(x)-3g\left(\frac{1}{x}\right)=x, x>0$. If $\alpha=\int_1^2 f(x)\mathrm{d}x$, and $\beta=\int_1^2 g(x)\mathrm{d}x$, then the value of $9\alpha+\beta$ is : [4 marks]

(A) 0

(B) 10

(C) 1

(D) 11

Correct Answer: None of the listed options equals the exact result for the printed functional equations.

Solution:

Concept:

- For positive x, replacing x by $1/x$ produces a second linear equation for the two unknown function values. Eliminate the reciprocal value before integrating.
- Use $\int x^{-2}\,dx=-x^{-1}$, $\int x^2\,dx=x^3/3$, and $\int x^{-1}\,dx=\ln x$ on the positive interval $[1,2]$.

Step 1: Determine f and its integral The reciprocal substitution gives $f(1/x)+2f(x)=x^{-2}+5$. Twice this equation minus the original gives

$$3f(x)=2x^{-2}-x^2+5.$$

Therefore

$$\alpha=\frac{1}{3}\left[-\frac{2}{x}-\frac{x^3}{3}+5x\right]_1^2=\frac{1}{3}\left(1-\frac{7}{3}+5\right)=\frac{11}{9}.$$

Substitution of this f verifies the first printed equation.

Step 2: Determine g and retain the logarithm The reciprocal equation is $2g(1/x)-3g(x)=1/x$. Twice the original plus three times this equation eliminates $g(1/x)$, leaving

$$-5g(x)=2x+\frac{3}{x}.$$

Hence

$$\beta=-\frac{1}{5}\left[x^2+3\ln x\right]_1^2=-\frac{3+3\ln 2}{5}.$$

Retain the logarithm from integrating $1/x$. Substitution of this g into the original equation gives exactly x.

Step 3: Evaluate the literal expression and test the choices Combining the two exact integrals gives

$$9\alpha+\beta=11-\frac{3+3\ln 2}{5}=\frac{52-3\ln 2}{5}\approx 9.9841117.$$

This is not 10: equality would require $\ln 2=2/3$, which is false. It is also neither 0, 1 nor 11. No nearest-integer rounding is requested. Thus no listed option equals the exact result for these functional equations.

Step 4: Conclusion For the equations as printed, $\boxed{9\alpha+\beta=(52-3\ln 2)/5}$. None of the listed options is mathematically correct. The supplied answer key selects (D), 11, but the nonzero integral of g makes that value inconsistent with the printed equations.

Quick Tip to Solve: Solve reciprocal functional equations algebraically first. Keep logarithmic terms exact and do not round an exact-value MCQ to force a listed option.

18. Consider two sets A and B, each containing three numbers in A.P. Let the sum and the product of the elements of A be 36 and p respectively and the sum and the product of the elements of B be 36 and q respectively. Let d and D be the common differences of $\mathrm{AP's}$ in A and B respectively such that $D = d + 3, d > 0$. If $\frac{p+q}{p-q} = \frac{19}{5}$, then $\mathrm{p - q}$ is equal to [4 marks]

(A) 540

(B) 450

(C) 600

(D) 630

Correct Answer: (A) 540

Solution:

Concept:

- Three numbers in A.P. with sum 36 have middle term 12, so write them symmetrically as $12 - d, 12, 12 + d$.
- The product of the symmetric triple is $12(144 - d^2)$. For the other triple, replace d by its common difference $D = d + 3$.

Step 1: Express both products using the differences The equal sums fix the middle term of both progressions. Thus

$$p = 12(144 - d^2), \qquad q = 12\{144 - (d + 3)^2\}.$$

This symmetric parametrisation automatically imposes the A.P. condition. The given ratio also ensures $p - q \neq 0$.

Step 2: Use the ratio to find d Cross-multiplication gives

$$5(p + q) = 19(p - q), \qquad 14p = 24q, \qquad 7p = 12q.$$

Cancelling the common factor 12 from the product expressions,

$$7(144 - d^2) = 12\{144 - (d + 3)^2\}.$$

After expansion this is

$$5d^2 + 72d - 612 = 0 = (d - 6)(5d + 102).$$

The two roots are 6 and $-102/5$. Since the question explicitly states $d > 0$, only $d = 6$ is allowed, and consequently $D = 9$.

Step 3: Calculate and verify the product difference The first triple is $(6, 12, 18)$ and the second is $(3, 12, 21)$. Their products are

$$p = 1296, \qquad q = 756, \qquad p - q = 540.$$

A useful independent check is $(p + q)/(p - q) = 2052/540 = 19/5$, agreeing with the given condition. The differences 450, 600 and 630 fail this calculation.

Step 4: Conclusion Therefore $\boxed{p - q = 540}$. Hence, the correct option is (A).

Quick Tip to Solve: Write a three-term A.P. around its middle term; a known sum then fixes the middle immediately, and its product becomes a difference of squares.

19. Let the values of p , for which the shortest distance between the lines $\frac{x+1}{3} = \frac{y}{4} = \frac{z}{5}$ and $\vec{r} = (p\hat{i} + 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ is $\frac{1}{\sqrt{6}}$, be $a, b, (a < b)$. Then the length of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is : [4 marks]

(A) $\frac{3}{2}$

(B) 9

(C) 18

(D) $\frac{2}{3}$

Correct Answer: (D) $\frac{2}{3}$

Solution:

Concept:

- For nonparallel lines through position vectors $\mathbf{a}_1, \mathbf{a}_2$, with directions $\mathbf{d}_1, \mathbf{d}_2$, the shortest distance is $D = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{d}_1 \times \mathbf{d}_2)|/|\mathbf{d}_1 \times \mathbf{d}_2|$.
- An ellipse with semimajor axis A and semiminor axis B has full latus rectum length $2B^2/A$. The larger squared denominator identifies the major axis, irrespective of the names of the parameters.

Step 1: Identify the two lines The symmetric equation gives $\mathbf{a}_1 = (-1, 0, 0)$ and $\mathbf{d}_1 = (3, 4, 5)$. The vector equation gives $\mathbf{a}_2 = (p, 2, 1)$ and $\mathbf{d}_2 = (2, 3, 4)$. Their direction vectors are not parallel. Calculate the perpendicular direction:

$$\mathbf{d}_1 \times \mathbf{d}_2 = (16 - 15, 10 - 12, 9 - 8) = (1, -2, 1), \qquad |\mathbf{d}_1 \times \mathbf{d}_2| = \sqrt{6}.$$

Step 2: Determine the two parameter values The displacement between the selected points is $(p + 1, 2, 1)$, whose scalar product with $(1, -2, 1)$ is $p + 1 - 4 + 1 = p - 2$. Thus

$$\frac{|p - 2|}{\sqrt{6}} = \frac{1}{\sqrt{6}} \quad \Longrightarrow \quad |p - 2| = 1.$$

The two possibilities are $p = 1$ and $p = 3$. Since $a < b$, we must take $a = 1$ and $b = 3$; these are not interchangeable.

Step 3: Calculate the latus rectum The ellipse becomes $x^2 + y^2/9 = 1$, with major axis along the y-direction. Hence its semimajor axis is $A = 3$, and its semiminor axis is $B = 1$. Therefore

$$L = \frac{2B^2}{A} = \frac{2 \cdot 1^2}{3} = \frac{2}{3}.$$

Option (A) reverses this ratio, while (B) and (C) incorrectly use the squared major-axis denominator in the numerator.

Step 4: Conclusion The length of the latus rectum is $\boxed{2/3}$. Hence, the correct option is (D).

Quick Tip to Solve: After solving the distance equation, identify the larger semiaxis before using the ellipse formula; the symbol b is the major semiaxis here.

20. Let f be a differentiable function on R such that $f(2) = 1, f'(2) = 4$. Let $\lim_{x\to 0}(f(2+x))^{3/x} = e^{\alpha}$. Then the number of times the curve $y = 4x^3 - 4x^2 - 4(\alpha - 7)x - \alpha$ meets x-axis is : [4 marks]

(A) 3

(B) 1

(C) 2

(D) 0

Correct Answer: (C) 2

Solution:

Concept:

- Differentiability gives $f(2+x) = f(2) + xf'(2) + o(x) = 1 + 4x + o(x)$. Since $f(2) = 1$, continuity also ensures $f(2+x) > 0$ for sufficiently small $|x|$, so logarithms are valid.
- The standard limit $\log(1+u)/u \to 1$ converts a power of the form 1^{∞} into an exponential. Intersections with the x-axis correspond to distinct real roots; a repeated root gives one point.

Step 1: Evaluate the exponent Let the stated positive limit be $L = e^{\alpha}$. Taking logarithms in a sufficiently small neighbourhood of zero yields

$$\log L = \lim_{x\to 0} \frac{3\log f(2+x)}{x}.$$

With $u = f(2+x) - 1$, differentiability gives $u/x \to 4$, whereas $\log(1+u)/u \to 1$. Thus

$$\alpha = 3\lim_{x\to 0} \frac{\log(1+u)}{u}\frac{u}{x} = 3 \cdot 1 \cdot 4 = 12.$$

This uses the local behaviour of f, so no formula for f away from 2 is needed.

Step 2: Factor the resulting polynomial Substitute $\alpha = 12$ into the curve:

$$y = 4x^3 - 4x^2 - 20x - 12 = 4(x^3 - x^2 - 5x - 3).$$

At $x = 3$, the bracket is $27 - 9 - 15 - 3 = 0$. Division by $x - 3$ gives $x^2 + 2x + 1 = (x+1)^2$. Consequently

$$y = 4(x-3)(x+1)^2.$$

Step 3: Count distinct meeting points The real zeros are $x = 3$ and $x = -1$. The latter has multiplicity two, but its two factors describe the same point $(-1, 0)$; the curve touches the axis there. At $(3, 0)$ the simple root gives a crossing. Both count as meeting the axis. Hence three counts multiplicities rather than distinct points, while one and zero miss real meeting points.

Step 4: Conclusion The curve meets the x-axis at exactly $\boxed{2}$ distinct points. Hence, the correct option is (C).

Quick Tip to Solve: For a power limit based at 1, take logarithms first; when counting axis meetings, count distinct real zeros rather than their multiplicities.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. If $\int \frac{\left(\sqrt{1+x^2}+x\right)^{10}}{\left(\sqrt{1+x^2}-x\right)^9}\,\mathrm{d}x = \frac{1}{\mathrm{m}}\left(\left(\sqrt{1+x^2}+x\right)^{\mathrm{n}}\left(\mathrm{n}\sqrt{1+x^2}-x\right)\right)+\mathrm{C}$ **where C is the constant of integration and** $\mathrm{m},\mathrm{n}\in\mathbf{N}$**, then** $\mathrm{m}+\mathrm{n}$ **is equal to _____ .**

[4 marks]

Solution:

Concept:

- Put $s=\sqrt{1+x^2}$. The identity $(s+x)(s-x)=1$ allows every denominator power of $s-x$ to become a numerator power of $s+x$.
- The substitution $t=s+x>0$ gives $s-x=t^{-1}$, and therefore $x=(t-t^{-1})/2$, $s=(t+t^{-1})/2$ and $dx=(1+t^{-2})dt/2$. These formulas hold for every real x.

Step 1: Simplify the integrand The conjugate factors have product one, so division by $(s-x)^9$ multiplies by $(s+x)^9$. Thus the integral is

$$I=\int (s+x)^{19}\,dx=\int t^{19}\,dx.$$

The exponent is $10+9=19$, rather than $10-9$, because the denominator is the reciprocal factor. The positivity of t also prevents a substitution singularity at any finite real x.

Step 2: Integrate the powers of the new variable Using the differential from the concept block,

$$I=\frac{1}{2}\int (t^{19}+t^{17})\,dt=\frac{t^{20}}{40}+\frac{t^{18}}{36}+C.$$

Bring the terms to a common denominator and factor a power of t:

$$I=\frac{9t^{20}+10t^{18}}{360}+C=\frac{t^{19}}{360}\left(9t+\frac{10}{t}\right)+C.$$

Step 3: Match the prescribed expression and check Substituting $t=s+x$ and $t^{-1}=s-x$ gives

$$9t+10t^{-1}=9(s+x)+10(s-x)=19s-x.$$

Therefore $n=19$ and $m=360$, both natural numbers as required. As a check, differentiating the intermediate antiderivative with respect to t gives $(t^{19}+t^{17})/2$; multiplying by $dt/dx=2/(1+t^{-2})$ recovers exactly t^{19}, the original integrand.

Step 4: Conclusion Here $m=360$ and $n=19$, so $\boxed{m+n=379}$.

Quick Tip to Solve: For conjugate radicals whose product is 1, use $t=\sqrt{1+x^2}+x$; write both x and dx in terms of t before integrating.

22. A card from a pack of 52 cards is lost. From the remaining 51 cards, n cards are drawn and are found to be spades. If the probability of the lost card to be a spade is $\frac{11}{50}$, then n is equal to _____ .

[4 marks]

Solution:

Concept:

- Condition on the observed event E: the n drawn cards are all spades. For a uniformly shuffled pack, the lost card and the drawn cards are distinct random positions in one permutation.
- After n spades have been observed, the lost card is equally likely to be any of the $52-n$ unobserved cards. Among those, exactly $13-n$ are spades. This is a conditional probability, rather than the initial probability $13/52$.

Step 1: Express the conditional probability Initially the pack has 13 spades and 39 other cards. The observed draw removes n known spades from consideration. There are $52-n$ unobserved cards: one is the lost card and the others are still in the pack. By exchangeability, no one of these unobserved positions is favoured for the lost card. Hence

$$P(\text{lost card is spade} \mid E) = \frac{13-n}{52-n}.$$

The denominator must include the lost card; using $51-n$ would omit one possible unobserved card.

Step 2: Use the specified probability Equate this expression to the stated value:

$$\frac{13-n}{52-n} = \frac{11}{50}.$$

Cross-multiplication is valid since a possible all-spade draw has $n \le 13 < 52$. It gives

$$650 - 50n = 572 - 11n, \qquad 78 = 39n, \qquad n = 2.$$

Step 3: Verify the draw is feasible For $n=2$, the two observed cards are spades, leaving 11 unobserved spades among 50 unobserved cards. The conditional chance for the lost card is indeed $11/50$. This is consistent with the original pack, and the positive integer answer does not exceed the number of spades. Observing spades reduces the chance that the lost card was a spade, as the result also confirms.

Step 4: Conclusion The number of spades drawn is $\boxed{n=2}$.

Quick Tip to Solve: Include the lost card among the unseen cards: after observing n spades, count $13-n$ spades in $52-n$ unseen positions.

23. Let the three sides of a triangle ABC be given by the vectors $2\hat{i}-\hat{j}+\hat{k}, \hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$. Let G be the centroid of the triangle ABC. Then $6\left(|\overrightarrow{AG}|^2+|\overrightarrow{BG}|^2+|\overrightarrow{CG}|^2\right)$ is equal to _____.

[4 marks]

Solution:

Concept:

- If G is the centroid of a triangle, then $\overrightarrow{AG}+\overrightarrow{BG}+\overrightarrow{CG}=\mathbf{0}$. The sum of their squared lengths is one third of the sum of the squared side lengths.
- For a vector $\mathbf{v}=(v_1,v_2,v_3)$, its squared magnitude is $|\mathbf{v}|^2=v_1^2+v_2^2+v_3^2$. The signs of components therefore do not affect the side-length squares.

Step 1: Establish the centroid identity Take A as the origin, and write $\overrightarrow{AB}=\mathbf{u}$, $\overrightarrow{AC}=\mathbf{v}$. Then G has position vector $(\mathbf{u}+\mathbf{v})/3$, so

$$\overrightarrow{AG}=\frac{\mathbf{u}+\mathbf{v}}{3},\quad \overrightarrow{BG}=\frac{\mathbf{v}-2\mathbf{u}}{3},\quad \overrightarrow{CG}=\frac{\mathbf{u}-2\mathbf{v}}{3}.$$

Squaring and adding gives $\frac{2}{3}(|\mathbf{u}|^2+|\mathbf{v}|^2-\mathbf{u}\cdot\mathbf{v})$. Since $|\mathbf{u}-\mathbf{v}|^2=|\mathbf{u}|^2+|\mathbf{v}|^2-2\mathbf{u}\cdot\mathbf{v}$, this equals $(AB^2+AC^2+BC^2)/3$.

Step 2: Find the squared side lengths The three supplied vectors determine the side lengths regardless of their orientation around the triangle. Their squared magnitudes are

$$2^2+(-1)^2+1^2=6,$$

$$1^2+(-3)^2+(-5)^2=35,$$

$$3^2+(-4)^2+(-4)^2=41.$$

They are geometrically consistent: the first two vectors add to the third. We need their squared magnitudes, not the sum of their individual components.

Step 3: Evaluate the requested multiple The sum of squared distances from the vertices to the centroid is therefore $(6+35+41)/3=82/3$. Multiply only after applying the centroid identity:

$$6\left(|\overrightarrow{AG}|^2+|\overrightarrow{BG}|^2+|\overrightarrow{CG}|^2\right)=6\cdot\frac{82}{3}=164.$$

Step 4: Conclusion The required value is $\boxed{164}$.

Quick Tip to Solve: For a centroid question involving squared vertex distances, replace their sum by one third of the sum of squared side lengths before doing coordinates.

24. If α is a root of the equation $x^2 + x + 1 = 0$ and $\sum_{k=1}^{n} \left(\alpha^k + \frac{1}{\alpha^k}\right)^2 = 20$, then n is equal to _____.

[4 marks]

Solution:

Concept:

- From $\alpha^2 + \alpha + 1 = 0$, we obtain $\alpha^3 = 1$ and $\alpha \neq 1$. Thus α is a nonreal cube root of unity, with $\alpha^{-1} = \alpha^2$ and $\alpha + \alpha^{-1} = -1$.
- Powers of α repeat every three exponents. The summands can therefore be grouped into complete blocks of length three, plus at most two remaining terms.

Step 1: Evaluate the three types of summands When k is divisible by 3, $\alpha^k = \alpha^{-k} = 1$, and hence the squared sum is $(1 + 1)^2 = 4$. When $k \equiv 1 \pmod 3$, the two terms are α and α^2, whose sum is -1, giving square 1. When $k \equiv 2 \pmod 3$, the same two roots occur in reversed order and again give square 1. Thus the summands, beginning with $k = 1$, are

$$1,\ 1,\ 4,\ 1,\ 1,\ 4, \ldots$$

Step 2: Express the partial sum in blocks Write $n = 3q + r$, where q is a nonnegative integer and $r \in \{0, 1, 2\}$. Each complete block contributes $1 + 1 + 4 = 6$; each of the first two terms of the next block contributes 1. Therefore

$$\sum_{k=1}^{n} (\alpha^k + \alpha^{-k})^2 = 6q + r.$$

This formula also equals $n + 3\lfloor n/3 \rfloor$, since each multiple of three contributes three more than the baseline value 1.

Step 3: Solve the integer condition and check The equation is $6q + r = 20$. Of the possible remainders 0, 1 and 2, only $r = 2$ makes $20 - r$ divisible by 6. Thus $q = 3$, and

$$n = 3q + r = 9 + 2 = 11.$$

Checking directly, the first nine terms contribute 18 and terms 10 and 11 contribute 1 each, yielding 20. Since every summand is positive, the partial sums strictly increase and there is no second possible value of n.

Step 4: Conclusion The required upper summation limit is $\boxed{n = 11}$.

Quick Tip to Solve: For cube roots of unity, list a single period before summing: here every block of three squared sums contributes $1 + 1 + 4 = 6$.

25. Let m and $\mathrm{n}, (\mathrm{m} < \mathrm{n})$, be two 2-digit numbers. Then the total numbers of pairs (m, n), such that $\gcd(m, n) = 6$, is _____ . [4 marks]

Solution:

Concept:

- Write the required numbers as $m = 6a$ and $n = 6b$. Then $\gcd(m, n) = 6\gcd(a, b)$, so we must count pairs with $\gcd(a, b) = 1$.
- For a fixed integer $b > 1$, Euler's totient $\phi(b)$ counts the integers from 1 to $b - 1$ coprime to b. Here the lower bound on a excludes 1.

Step 1: Convert the two-digit restriction Both original numbers lie between 10 and 99. Their possible multiples of 6 run from 12 to 96, inclusive. Consequently

$$2 \le a < b \le 16.$$

This conversion is one-to-one: each admissible coprime pair gives exactly one original pair, and division of any required pair by 6 gives an admissible coprime pair. The condition $a < b$ also ensures that we do not count reversed pairs or equal numbers.

Step 2: Count the smaller partner for each larger partner Fix b between 3 and 16. Among the $\phi(b)$ positive integers below b coprime to it, precisely one, namely 1, lies outside the permitted interval. Thus the contribution is $\phi(b) - 1$. The complete enumeration is

b	3	4	5	6	7	8	9
$\phi(b) - 1$	1	1	3	1	5	3	5

b	10	11	12	13	14	15	16
$\phi(b) - 1$	3	9	3	11	5	7	7

For example, at $b = 12$ the eligible smaller integers are 5, 7 and 11, and at $b = 13$ every integer from 2 to 12 works because 13 is prime. Adding the contributions gives

$$1 + 1 + 3 + 1 + 5 + 3 + 5 + 3 + 9 + 3 + 11 + 5 + 7 + 7 = 64.$$

There is no contribution from $b = 2$, because no integer $a \ge 2$ is smaller than it.

Step 3: Conclusion The total number of ordered-by-size pairs (m, n) satisfying the conditions is $\boxed{64}$.

Quick Tip to Solve: First divide both numbers by their specified gcd. Then count coprime pairs in the resulting interval, remembering to exclude 1 if the original numbers must be two-digit.

26. A finite size object is placed normal to the principal axis at a distance of 30 cm from a convex mirror of focal length 30 cm . A plane mirror is now placed in such a way that the image produced by both the mirrors coincide with each other. The distance between the two mirrors is : [4 marks]

(A) 45 cm

(B) 15 cm

(C) 22.5 cm

(D) 7.5 cm

Correct Answer: (D) 7.5 cm

Solution:

Concept:

- Use the Cartesian mirror formula $1/f = 1/v + 1/u$. A convex mirror has positive focal length and forms its virtual image behind the mirror.
- A plane mirror places the image as far behind its plane as the object is in front. Its plane therefore bisects the object-image separation.

Step 1: Locate the convex mirror image Place the pole of the convex mirror at coordinate $x = 0$, with the object at $x = -30\,\text{cm}$. With incident light travelling towards increasing x, the given quantities are $u = -30\,\text{cm}$ and $f = +30\,\text{cm}$. The mirror formula gives

$$\frac{1}{30} = \frac{1}{v} - \frac{1}{30}, \qquad \frac{1}{v} = \frac{2}{30}, \qquad v = +15\,\text{cm}.$$

Thus the convex mirror image is virtual and lies at coordinate $+15\,\text{cm}$. Its positive image distance is essential: treating it as a real image would give an incorrect location for the second mirror.

Step 2: Place the plane mirror at the midpoint Let the plane mirror be parallel to the convex mirror's transverse plane at coordinate $x = p$. Reflection in a plane sends the object's axial coordinate x_o to $x_i = 2p - x_o$. To put its image at the same axial position as the convex mirror image, require

$$15 = 2p - (-30) = 2p + 30, \qquad p = -7.5\,\text{cm}.$$

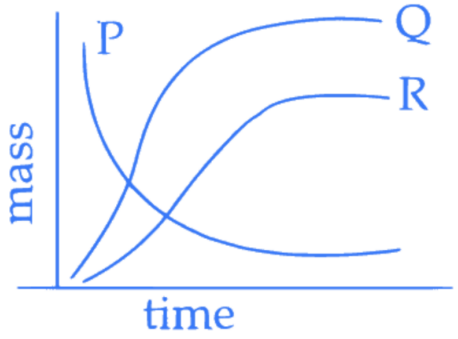
Equivalently, the object and common image are separated by $30 + 15 = 45\,\text{cm}$, so the plane mirror is 22.5 cm from each. Since the object is 30 cm from the convex mirror, the separation of the mirrors is $30 - 22.5 = 7.5\,\text{cm}$. The value 22.5 cm is the object-plane-mirror distance, rather than the requested mirror separation; 15 cm is the convex image distance, and 45 cm is the full object-image distance.

Step 3: Conclusion The distance between the mirrors is $\boxed{7.5\,\text{cm}}$. Hence, the correct option is (D).

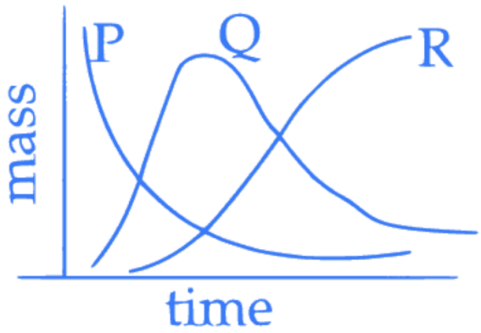
Quick Tip to Solve: Find the curved mirror image first. The plane mirror must lie at the midpoint between the object and that image; only then measure the distance between the mirrors.

27. A radioactive material P first decays into Q and then Q decays to non-radioactive material R. Which of the following figure represents time dependent mass of P, Q and R ? [4 marks]

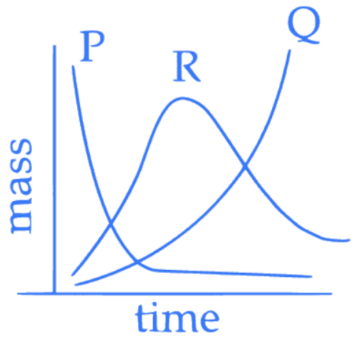
(A)



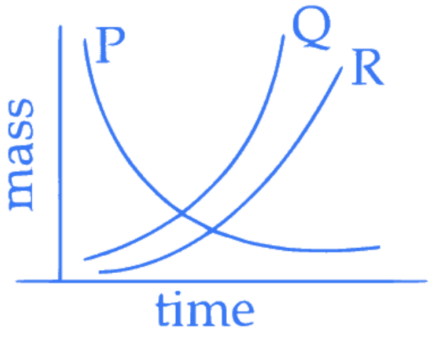
(B)



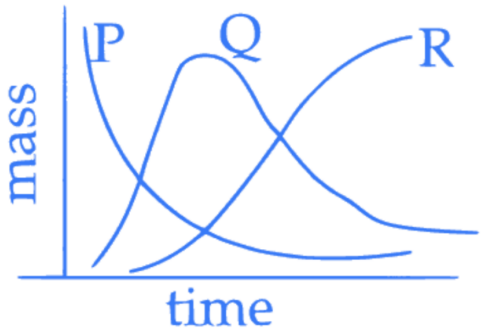
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- For the chain $P \longrightarrow Q \longrightarrow R$, the parent is consumed continuously, the intermediate is both produced and consumed, and the final stable product accumulates.
- With initially pure P, the rate equations are $\dot{N}_P = -\lambda_P N_P$, $\dot{N}_Q = \lambda_P N_P - \lambda_Q N_Q$, and $\dot{N}_R = \lambda_Q N_Q$, where both decay constants are positive.

Step 1: Determine the parent and intermediate curves Initially, the sample consists of the parent material, so $N_P(0) = N_0$ and $N_Q(0) = N_R(0) = 0$. The parent follows

$$N_P(t) = N_0 e^{-\lambda_P t}.$$

It therefore decreases monotonically towards zero. The intermediate Q initially increases because its production term is positive while its initial decay term is zero. As the parent is exhausted, production weakens; the remaining intermediate continues to decay. For unequal decay constants,

$$N_Q(t) = \frac{\lambda_P N_0}{\lambda_Q - \lambda_P}\left(e^{-\lambda_P t} - e^{-\lambda_Q t}\right).$$

It starts at zero, reaches a maximum, and ultimately returns towards zero. If the constants are equal, the corresponding expression is $N_Q = N_0 \lambda t e^{-\lambda t}$, with the same qualitative rise and fall.

Step 2: Identify the stable-product curve and compare the options The final material R is non-radioactive, so it has no decay loss term. Since $\dot{N}_R = \lambda_Q N_Q \geq 0$, its amount increases monotonically, approaching a final limiting value when both radioactive species have decayed. Its initial slope is zero because there is initially no Q to feed it. Multiplying each number of nuclei by its fixed nuclear mass does not change these qualitative time dependences.

Only the second graph has all three required features: a falling P, a peaked intermediate Q, and a rising stable R. The first graph incorrectly leaves Q at a nonzero plateau. The third makes the stable product decrease after a peak while Q grows. The fourth makes both products grow without the intermediate turning over, contrary to its radioactive decay.

Step 3: Conclusion The second graph correctly represents the sequential decay: P decreases, Q rises and falls, and stable R accumulates. Hence, the correct option is (B).

Quick Tip to Solve: In a decay chain, look first for the intermediate: it must rise from zero and then fall. The stable final product can accumulate but cannot decay away.

28. Displacement of a wave is expressed as $x(t) = 5\cos\left(628t + \frac{\pi}{2}\right)$ m. The wavelength of the wave when its velocity is 300 m/s is :

$$(\pi = 3.14)$$

[4 marks]

(A) 0.33 m

(B) 0.5 m

(C) 3 m

(D) 5 m

Correct Answer: (C) 3 m

Solution:

Concept:

- Compare the displacement with $x = A\cos(\omega t + \varphi)$, where A is amplitude, ω angular frequency and φ initial phase.
- The frequency is $f = \omega/(2\pi)$, and the propagation speed satisfies $v = f\lambda$. Therefore $\lambda = 2\pi v/\omega$.

Step 1: Extract the frequency from the time dependence The given displacement is

$$x(t) = 5\cos\left(628t + \frac{\pi}{2}\right)\ \text{m}.$$

The coefficient multiplying time in the cosine is the angular frequency, so $\omega = 628\,\text{rad}\,\text{s}^{-1}$. It is not the frequency in cycles per second. One cycle corresponds to a phase increase of 2π radians, which gives

$$f = \frac{\omega}{2\pi} = \frac{628}{2(3.14)} = 100\,\text{Hz}.$$

The amplitude 5 m describes how far a particle moves from its mean position, while the phase $\pi/2$ specifies its state at $t = 0$. Neither quantity changes the time for a complete oscillation. In particular, the phase constant must not be added to the coefficient of time.

Step 2: Use the propagation speed to calculate the wavelength In one period $T = 1/f = 0.01\,\text{s}$, a wave disturbance travels exactly one wavelength. With the stated propagation speed $v = 300\,\text{m}\,\text{s}^{-1}$,

$$\lambda = vT = \frac{v}{f} = \frac{300}{100} = 3\,\text{m}.$$

The units are metres because metres per second divided by cycles per second gives the distance travelled per cycle. A direct check is

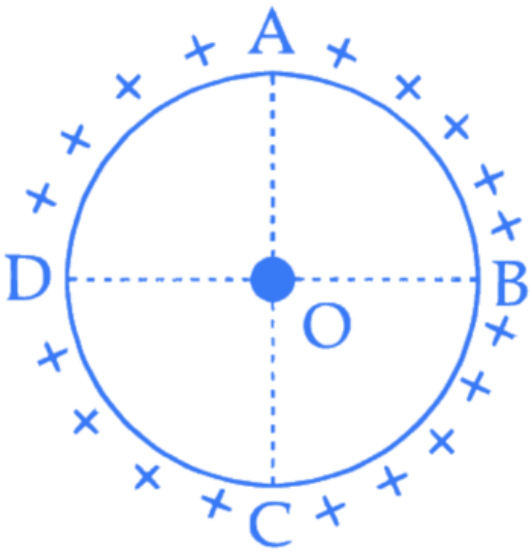
$$f\lambda = (100\,\text{s}^{-1})(3\,\text{m}) = 300\,\text{m}\,\text{s}^{-1},$$

which reproduces the specified wave speed. The other proposed wavelengths, 0.33 m, 0.5 m and 5 m, would respectively require different frequencies at the same wave speed. In particular, choosing 5 m would confuse displacement amplitude with wavelength.

Step 3: Conclusion The wavelength is $\boxed{3\,\text{m}}$. Hence, the correct option is (C).

Quick Tip to Solve: The coefficient of time inside a sine or cosine is angular frequency. Divide it by 2π before using $v = f\lambda$; amplitude and initial phase do not enter.

29. A metallic ring is uniformly charged as shown in figure. AC and BD are two mutually perpendicular diameters. Electric field due to arc AB at ' O ' is ' E ' in magnitude. What would be the magnitude of electric field at ' O ' due to arc ABC ? [4 marks]



(A) 2E

(B) Zero

(C) E/2

(D) $\sqrt{2}$E

Correct Answer: (D) $\sqrt{2}$E

Solution:

Concept:

- The field at the centre due to a uniformly charged circular arc lies along its symmetry bisector. For positive charge it points away from the arc.
- Fields add as vectors. The two adjacent quarter-arcs AB and BC have equal field magnitudes because they have equal radius, angle and uniform linear charge density.

Step 1: Resolve the first quadrant field Take OB along the positive horizontal axis and OA along the positive vertical axis. Arc AB occupies the upper-right quarter of the ring. Its positive charges produce a resultant at O directed downwards and leftwards, at 45° to either axis. The magnitude is given as E, so

$$\boldsymbol{E}_{AB} = -\frac{E}{\sqrt{2}}\,\hat{\boldsymbol{i}} - \frac{E}{\sqrt{2}}\,\hat{\boldsymbol{j}}.$$

The equality of these components follows from reflection symmetry about the quadrant's bisector. No numerical value of the ring's radius or total charge is needed because the specified quarter-arc field already fixes the relevant scale.

Step 2: Add the reflected quarter-arc contribution Arc BC is the reflection of AB in the horizontal

diameter BD. Its field has the same leftward component and an opposite vertical component:

$$\boldsymbol{E}_{BC} = -\frac{E}{\sqrt{2}}\,\hat{\boldsymbol{i}} + \frac{E}{\sqrt{2}}\,\hat{\boldsymbol{j}}.$$

The requested arc ABC is the right semicircle, comprising these two quarters. Thus

$$\boldsymbol{E}_{ABC} = \boldsymbol{E}_{AB} + \boldsymbol{E}_{BC} = -\sqrt{2}E\,\hat{\boldsymbol{i}}, \qquad |\boldsymbol{E}_{ABC}| = \sqrt{2}E.$$

Equivalently, the equal vectors of magnitude E are perpendicular, giving resultant $\sqrt{E^2 + E^2}$. They do not add to $2E$ because they are not parallel. Zero field would require the whole uniformly charged ring or diametrically opposite arcs, whereas here only the right semicircle is included. The value $E/2$ is also inconsistent with the two leftward components reinforcing each other.

Step 3: Conclusion The electric field magnitude due to arc ABC is $\boxed{\sqrt{2}E}$, directed towards D. Hence, the correct option is (D).

Quick Tip to Solve: For adjacent equal quarter-arcs, reflect one field vector across the common diameter: one component cancels and the other doubles. Do not add field magnitudes directly.

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30. A wheel is rolling on a plane surface. The speed of a particle on the highest point of the rim is $8\ \mathrm{m/s}$. The speed of the particle on the rim of the wheel at the same level as the centre of wheel, will be : [4 marks]

(A) $4\sqrt{2}\ \mathrm{m/s}$

(B) $8\ \mathrm{m/s}$

(C) $4\ \mathrm{m/s}$

(D) $8\sqrt{2}\ \mathrm{m/s}$

Correct Answer: (A) $4\sqrt{2}\ \mathrm{m/s}$

Solution:

Concept:

- For pure rolling without slipping, the centre speed V and angular speed ω satisfy $V = \omega R$, where R is the wheel radius.
- The ground-frame velocity of any rim point is the vector sum of the centre translation and the point's rotational velocity relative to the centre.

Step 1: Find the translation speed from the top point At the highest point of a wheel rolling forwards, the rotational velocity relative to the centre points forwards, in the same direction as the centre's translation. Both contributions have magnitude V, because $\omega R = V$. Therefore

$$v_{\text{top}} = V + \omega R = 2V.$$

The given top speed is $8\,\mathrm{m\,s^{-1}}$, so

$$2V = 8, \qquad V = 4\,\mathrm{m\,s^{-1}}, \qquad \omega R = 4\,\mathrm{m\,s^{-1}}.$$

This uses rolling without slipping: the contact point is instantaneously at rest relative to the plane, while the top point has twice the centre speed. The top speed must not itself be used as the centre speed.

Step 2: Add perpendicular velocities at centre height A rim point at the same height as the centre lies at the front or rear end of the horizontal diameter. Its rotational velocity is vertical, whereas its translational velocity remains horizontal. The two contributions are therefore perpendicular. The sign of the vertical contribution is different at the two ends, but their speeds are identical:

$$v = \sqrt{V^2 + (\omega R)^2} = \sqrt{4^2 + 4^2} = 4\sqrt{2}\,\mathrm{m\,s^{-1}}.$$

The value $4\,\mathrm{m\,s^{-1}}$ includes only one velocity contribution. The value $8\,\mathrm{m\,s^{-1}}$ adds the two magnitudes as though the vectors were parallel, which is true at the top but not at centre height. The value $8\sqrt{2}\,\mathrm{m\,s^{-1}}$ results from incorrectly treating the given top speed as the translation speed. The required speed lies between the centre speed and the top speed, as this vector calculation predicts.

Step 3: Conclusion The speed of either rim point at centre height is $\boxed{4\sqrt{2}\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (A).

Quick Tip to Solve: In pure rolling, first halve the top speed to obtain the centre speed. At centre height the translation and rotation velocities are perpendicular, so multiply the centre speed by $\sqrt{2}$.

31. In an electromagnetic system, a quantity defined as the ratio of electric dipole moment and magnetic dipole moment has dimension of $[\mathrm{M^P L^Q T^R A^S}]$. The value of P and Q are : [4 marks]

(A) $-1, 0$

(B) $0, -1$

(C) $-1, 1$

(D) $1, -1$

Correct Answer: (B) $0, -1$

Solution:

Concept:

- Electric dipole moment is charge multiplied by separation, $p = q\ell$, where $[q] = [\mathrm{AT}]$. Thus its dimension is obtained without introducing force or energy.
- Magnetic dipole moment of a current loop is $\mu = IA$, where A here denotes geometrical area. Divide the two dimensional expressions and compare powers of the fundamental quantities.

Step 1: Write the dimensions of each moment For a pair of opposite charges separated by a distance, the electric dipole moment has magnitude $p = q\ell$. Since charge equals current multiplied by time,

$$[p] = [\mathrm{AT}][\mathrm{L}] = [\mathrm{LTA}].$$

For a current loop, the magnetic dipole moment equals the current multiplied by the area enclosed by

the loop:

$$[\mu] = [\mathrm{A}][\mathrm{L}^2] = [\mathrm{L}^2\mathrm{A}].$$

The area in this relation is not another fundamental quantity; its dimensions are length squared.

Step 2: Divide and compare exponents The requested quantity is the ratio of electric dipole moment to magnetic dipole moment, so the order of division matters.

$$\left[\frac{p}{\mu}\right] = \frac{[\mathrm{LTA}]}{[\mathrm{L}^2\mathrm{A}]} = [\mathrm{M}^0\mathrm{L}^{-1}\mathrm{T}^1\mathrm{A}^0].$$

The current dimensions cancel, and neither dipole moment introduces mass. Comparing this expression with $[\mathrm{M}^P\mathrm{L}^Q\mathrm{T}^R\mathrm{A}^S]$ gives $P = 0$, $Q = -1$, $R = 1$ and $S = 0$. Options (A), (C) and (D) all introduce a nonzero mass exponent and therefore cannot represent this ratio.

Step 3: Conclusion The required exponents are $\boxed{P = 0, \quad Q = -1}$. Hence, the correct option is (B).

Quick Tip to Solve: Use electric moment = charge times length and magnetic moment = current times area; the current cancels immediately in their ratio.

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32. The displacement x versus time graph is shown below.

(A) The average velocity during 0 to 3 s is $10\ \mathrm{m/s}$

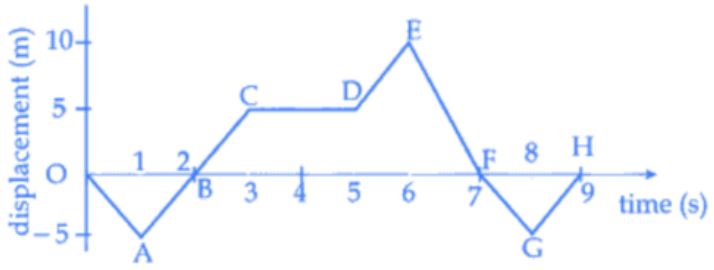
(B) The average velocity during 3 to 5 s is $0\ \mathrm{m/s}$

(C) The instantaneous velocity at $\mathrm{t} = 2\ \mathrm{s}$ **is** $5\ \mathrm{m/s}$

(D) The average velocity during 5 to 7 s and instantaneous velocity at $\mathrm{t} = 6.5\ \mathrm{s}$ **are equal**

(E) The average velocity from $t = 0$ **to** $t = 9\ \mathrm{s}$ **is zero**

Choose the correct answer from the options given below : [4 marks]



(A) (B), (C), (D) only

(B) (B), (D), (E) only

(C) (A), (D), (E) only

(D) (B), (C), (E) only

Correct Answer: (D) (B), (C), (E) only

Solution:

Concept:

- Average velocity is net displacement divided by elapsed time: $v_{\text{av}} = [x(t_2) - x(t_1)]/(t_2 - t_1)$. It does not use the total distance travelled.
- Instantaneous velocity is the slope of the displacement-time graph. On a straight segment this slope is constant, but an average across two differently sloped segments must be calculated separately.

Step 1: Test statements A, B and C The graph gives $x(0) = 0$, $x(1) = -5\,\text{m}$, $x(3) = 5\,\text{m}$ and $x(5) = 5\,\text{m}$. Therefore,

$$v_{0\to3} = \frac{5-0}{3} = \frac{5}{3}\,\text{m s}^{-1},$$

so (A) is false. The graph is horizontal from 3 s to 5 s, giving $v_{3\to5} = 0$; hence (B) is true. At 2 s the particle lies on the straight segment from $(1, -5)$ to $(3, 5)$:

$$v(2) = \frac{5-(-5)}{3-1} = 5\,\text{m s}^{-1}.$$

Thus (C) is true.

Step 2: Test statements D and E From 5 s to 7 s, the displacement changes from 5 m to 0, so

$$v_{5\to7} = \frac{0-5}{7-5} = -2.5\,\text{m s}^{-1}.$$

However, 6.5 s lies on the descending segment from $(6, 10)$ to $(7, 0)$, whose slope is

$$v(6.5) = \frac{0-10}{7-6} = -10\,\text{m s}^{-1}.$$

The two velocities differ, making (D) false. Finally, $x(9) = x(0) = 0$, so the average velocity over the complete 9 s is zero and (E) is true. Every alternative containing (A) or (D) is excluded.

Step 3: Conclusion The three true statements are B, C and E; both of the others are false. Hence, the correct option is (D).

Quick Tip to Solve: Read endpoint displacements for average velocity; use only the local straight-line slope for instantaneous velocity.

33. Consider a rectangular sheet of solid material of length $l = 9\ \text{cm}$ and width $\text{d} = 4\ \text{cm}$. The coefficient of linear expansion is $\alpha = 3.1 \times 10^{-5}\ \text{K}^{-1}$ at room temperature and one atmospheric pressure. The mass of sheet $m = 0.1\ \text{kg}$ and the specific heat capacity $C_\text{v} = 900\ \text{J kg}^{-1}\ \text{K}^{-1}$. If the amount of heat supplied to the material is $8.1 \times 10^2\ \text{J}$ then change in area of the rectangular sheet is : [4 marks]

(A) $2.0 \times 10^{-6}\ \text{m}^2$

(B) $6.0 \times 10^{-7}\ \text{m}^2$

(C) $3.0 \times 10^{-7}\ \text{m}^2$

(D) $4.0 \times 10^{-7}\ \text{m}^2$

Correct Answer: (A) $2.0 \times 10^{-6}\ \text{m}^2$

Solution:

Concept:

- The heat supplied raises the temperature by $\Delta T = Q/(mC_v)$, using the specific heat supplied in the question and neglecting heat loss.
- For an isotropic solid with small thermal expansion, the areal expansion coefficient is $\beta = 2\alpha$. Thus $\Delta A = 2\alpha A_0 \Delta T$, with $A_0 = ld$.

Step 1: Calculate the temperature rise and initial area Convert the sides into metres before calculating area:

$$A_0 = (9 \times 10^{-2})(4 \times 10^{-2}) = 3.6 \times 10^{-3}\,\text{m}^2.$$

The sheet has heat capacity $mC_v = (0.1)(900) = 90\,\text{J K}^{-1}$. Hence,

$$\Delta T = \frac{8.1 \times 10^2}{90} = 9\,\text{K}.$$

The rise of temperature is positive, so both dimensions of the sheet increase. The initial area is 36 square centimetres, which is 36×10^{-4} square metres, providing a check on the conversion.

Step 2: Apply expansion in both directions Each side becomes its initial value multiplied by $1 + \alpha\Delta T$. Therefore,

$$A = A_0(1 + \alpha\Delta T)^2.$$

Since $\alpha\Delta T = 2.79 \times 10^{-4}$, its square is negligible at the precision of the options. Keeping the first-order term gives

$$\begin{aligned}\Delta A &= 2\alpha A_0 \Delta T \\ &= 2(3.1 \times 10^{-5})(3.6 \times 10^{-3})(9) \\ &= 2.0088 \times 10^{-6}\,\text{m}^2.\end{aligned}$$

Using only one linear expansion factor would miss the expansion of the other side. The values in (B), (C) and (D) do not agree with the two-direction area change.

Step 3: Conclusion The increase in area is $\boxed{2.0 \times 10^{-6}\,\text{m}^2}$, to the precision shown. Hence, the correct option is (A).

Quick Tip to Solve: Find the temperature rise from heat first, and use twice the linear expansion coefficient for the change in area.

34. An object is kept at rest at a distance of $3R$ above the earth's surface where R is earth's radius. The minimum speed with which it must be projected so that it does not return to earth is : (Assume M = mass of earth, G = Universal gravitational constant) [4 marks]

(A) $\sqrt{\frac{3\mathrm{GM}}{\mathrm{R}}}$

(B) $\sqrt{\frac{2\mathrm{GM}}{\mathrm{R}}}$

(C) $\sqrt{\frac{\mathrm{GM}}{2\mathrm{R}}}$

(D) $\sqrt{\frac{\mathrm{GM}}{\mathrm{R}}}$

Correct Answer: (C) $\sqrt{\frac{\mathrm{GM}}{2\mathrm{R}}}$

Solution:

Concept:

- With gravitational potential energy zero at infinity, an object of mass m at distance r from Earth's centre has $U = -GMm/r$.
- Minimum escape speed corresponds to zero final speed at infinity. Conservation of mechanical energy gives $v_e = \sqrt{2GM/r}$, where r is measured from the centre, not the surface.

Step 1: Convert the altitude to distance from the centre The object is at a height $h = 3R$ above the surface. The surface itself is a distance R from the centre, so

$$r = R + h = R + 3R = 4R.$$

Consequently, the initial gravitational potential energy is

$$U_i = -\frac{GMm}{4R}.$$

Putting $3R$ directly into the escape-speed formula would confuse altitude with the radial distance that appears in Newton's gravitational potential. We treat Earth as spherical and neglect atmospheric resistance.

Step 2: Set the threshold total energy to zero At the minimum projection speed that permits escape, the object can reach infinity with speed tending to zero. Its final kinetic and potential energies then both tend to zero. Energy conservation yields

$$\frac{1}{2}mv_e^2 - \frac{GMm}{4R} = 0.$$

Solving and cancelling the object's mass gives

$$v_e^2 = \frac{GM}{2R}, \qquad v_e = \sqrt{\frac{GM}{2R}}.$$

This is half the escape speed at Earth's surface because the starting radius is four times as large. Option (B) is the surface escape speed, while (A) and (D) also require excessive energy for the minimum threshold at this altitude.

Step 3: Conclusion The minimum projection speed is $\boxed{\sqrt{GM/(2R)}}$. Hence, the correct option is (C).

Quick Tip to Solve: Always convert height above the surface into radius from the centre: escape speed varies inversely as the square root of that radius.

---

35. A cylindrical rod of length 1 m and radius 4 cm is mounted vertically. It is subjected to a shear force of 10^5 N at the top. Considering infinitesimally small displacement in the upper edge, the angular displacement θ of the rod axis from its original position would be : (shear moduli, $G = 10^{10}$ N/m^2) [4 marks]

(A) $1/160\pi$

(B) $1/2\pi$

(C) $1/4\pi$

(D) $1/40\pi$

Correct Answer: (A) $1/160\pi$

Solution:

Concept:

- Shear stress is tangential force per cross-sectional area, $\tau = F/A$. For a circular rod, $A = \pi r^2$.
- Shear modulus is shear stress divided by shear strain: $G = \tau/\tan\theta$. At an infinitesimally small angular displacement, $\tan\theta \simeq \theta$ in radians.

Step 1: Calculate the cross-sectional area and shear stress The radius is $r = 4\,\text{cm} = 4 \times 10^{-2}\,\text{m}$, so the area supporting the tangential force is

$$A = \pi(4 \times 10^{-2})^2 = 1.6 \times 10^{-3}\pi\,\text{m}^2.$$

The stress produced by the force is therefore

$$\tau = \frac{10^5}{1.6 \times 10^{-3}\pi}\,\text{N m}^{-2}.$$

The length of the rod would be required to find the sideways displacement at the top; it will cancel when finding the angle because angular strain is that displacement divided by length.

Step 2: Divide the shear stress by the modulus Using the small-angle form of the shear-modulus relation,

$$\begin{aligned}\theta \simeq \frac{\tau}{G} &= \frac{F}{\pi r^2 G} \\ &= \frac{10^5}{\pi(1.6 \times 10^{-3})(10^{10})} \\ &= \frac{10^5}{1.6 \times 10^7\pi} = \frac{1}{160\pi}\,\text{rad}.\end{aligned}$$

Numerically this is about $1.99 \times 10^{-3}\,\text{rad}$, justifying the small-angle approximation. A dimensional check also works: force divided by area and modulus is dimensionless, as an angle in radians must be.

The other three options exceed this value and do not satisfy the stated shear-modulus relation.

Step 3: Conclusion The angular displacement is $\boxed{\theta = 1/(160\pi)\,\text{rad}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a small shear angle use force divided by area and shear modulus; rod length is needed only if the top displacement is requested.

36. For the determination of refractive index of glass slab, a travelling microscope is used whose main scale contains 300 equal divisions equals to 15 cm . The vernier scale attached to the microscope has 25 divisions equals to 24 divisions of main scale. The least count (LC) of the travelling microscope is (in cm) : [4 marks]

(A) 0.002

(B) 0.0025

(C) 0.0005

(D) 0.001

Correct Answer: (A) 0.002

Solution:

Concept:

- For a direct vernier, the least count is the difference between one main-scale division and one vernier-scale division: $LC = 1\,\text{MSD} - 1\,\text{VSD}$.
- When n vernier divisions equal $(n-1)$ main-scale divisions, $1\,\text{VSD} = (n-1)\,\text{MSD}/n$, so the least count is one main-scale division divided by n.

Step 1: Determine the size of one main-scale division The complete length of 300 equal main-scale divisions is 15 cm. Dividing the length by the number of divisions gives

$$1\,\text{MSD} = \frac{15}{300} = 0.05\,\text{cm}.$$

This is 0.5 mm, not 0.05 mm. Maintaining centimetres throughout is useful because the answer options ask for the least count in centimetres. This main-scale spacing is the starting length to which the vernier comparison must be applied.

Step 2: Compare one vernier division with one main division The condition on the vernier is

$$25\,\text{VSD} = 24\,\text{MSD}.$$

Consequently,

$$1\,\text{VSD} = \frac{24}{25}(0.05) = 0.048\,\text{cm}.$$

Subtracting these nearly equal spacings yields

$$LC = 0.05 - 0.048 = 0.002\,\text{cm}.$$

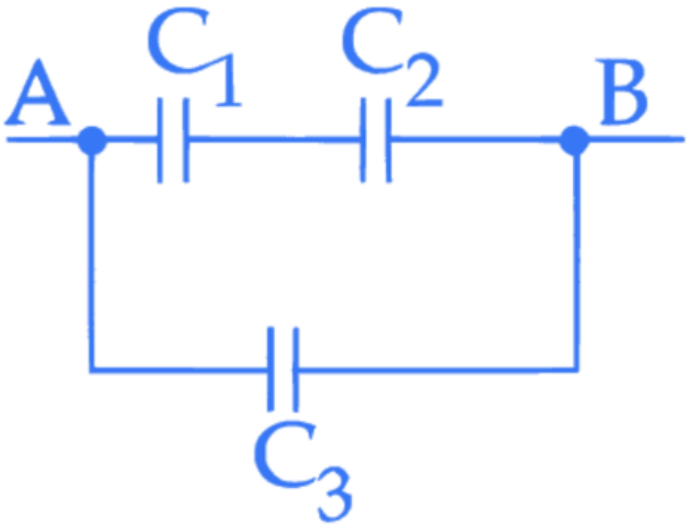
Equivalently, $LC = (0.05)/25$, giving the same value directly. This corresponds to 0.02 mm. The

refractive-index experiment does not change the instrument's least count: it is fixed by the scale geometry. The alternatives 0.0025, 0.0005 and 0.001 cm do not equal the difference of the stated scale divisions.

Step 3: Conclusion The least count is $\boxed{0.002\,\text{cm}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a vernier with 25 divisions equal to 24 main divisions, divide the main-scale spacing by 25, keeping the requested unit throughout.

37. Three parallel plate capacitors C_1, C_2 and C_3 each of capacitance 5μ F are connected as shown in figure. The effective capacitance between points A and B, when the space between the parallel plates of C_1 capacitor is filled with a dielectric medium having dielectric constant of 4, is : [4 marks]



(A) 9μ F

(B) 30μ F

(C) 7.5μ F

(D) 22.5μ F

Correct Answer: (A) 9μ F

Solution:

Concept:

- For a parallel-plate capacitor completely filled by a dielectric of relative permittivity K, the new capacitance is $C = KC_0$, with the plate geometry unchanged.
- Capacitors in series carry equal charge and obey $C_s = C_aC_b/(C_a + C_b)$. Parallel branches have the same potential difference, so their capacitances add.

Step 1: Update the dielectric-filled capacitor Only the space inside C_1 is filled. Its capacitance is therefore multiplied by four; neither C_2 nor C_3 changes. Thus the three values to use in the network

are

$$C_1 = 4(5) = 20\,\mu\text{F}, \qquad C_2 = C_3 = 5\,\mu\text{F}.$$

The factor four applies to the individual capacitor, rather than to the equivalent capacitance of the entire circuit.

Step 2: Reduce the upper branch first The junction between C_1 and C_2 has no other connection. These two capacitors are in series along the upper branch from A to B. Therefore

$$C_{12} = \frac{20 \times 5}{20 + 5} = 4\,\mu\text{F}.$$

The result is smaller than either member of the series pair, as required. Adding 20 and 5 would incorrectly treat their different potential differences as equal.

Step 3: Add the branch across the same terminals The lower branch contains C_3 directly between A and B, in parallel with the equivalent upper branch. Hence

$$C_{AB} = C_{12} + C_3 = 4 + 5 = 9\,\mu\text{F}.$$

The value $30\,\mu\text{F}$ would require all three to be parallel. The value $7.5\,\mu\text{F}$ is the network value before insertion of the dielectric, while $22.5\,\mu\text{F}$ also uses an incorrect grouping.

Step 4: Conclusion The effective capacitance between A and B is $9\,\mu\text{F}$. Hence, the correct option is (A).

Quick Tip to Solve: Mark the two common terminals first: reduce the series pair in the upper branch before adding the capacitor in the lower branch.

---

38. A block of mass 25 kg is pulled along a horizontal surface by a force at an angle 45° with the horizontal. The friction coefficient between the block and the surface is 0.25 . The block travels at a uniform velocity. The work done by the applied force during a displacement of 5 m of the block is : [4 marks]

(A) 490 J

(B) 970 J

(C) 735 J

(D) 245 J

Correct Answer: (D) 245 J

Solution:

Concept:

- Uniform velocity means zero acceleration, so the horizontal and vertical components of the resultant force both vanish.
- For horizontal displacement s, the applied force does work $W = Fs\cos\theta$. Sliding friction has magnitude $f = \mu N$, where N is the normal reaction.

Step 1: Balance the vertical forces Let the magnitude of the pull be F. Its upward component reduces

the load carried by the surface. Since the block has no vertical acceleration,

$$N + F\sin 45^\circ = mg, \qquad N = mg - \frac{F}{\sqrt{2}}.$$

It would be incorrect to put $N = mg$, because the applied force is inclined upwards.

Step 2: Balance the horizontal forces At constant speed the horizontal pull equals the frictional resistance:

$$F\cos 45^\circ = \mu N = 0.25\,(mg - F\sin 45^\circ)\,.$$

Both trigonometric components have the same magnitude at 45°. Put $X = F/\sqrt{2}$. Then

$$X = 0.25(mg - X), \qquad 1.25X = 0.25mg.$$

Using $m = 25\,\text{kg}$ and $g = 9.8\,\text{m}\,\text{s}^{-2}$, we obtain

$$X = \frac{0.25}{1.25}(25)(9.8) = 49\,\text{N}.$$

Step 3: Calculate the work along the displacement The displacement is horizontal, so only $X = F\cos 45^\circ$ contributes to the scalar product. Consequently

$$W = Xs = 49 \times 5 = 245\,\text{J}.$$

Gravity and the normal reaction do no work because they are perpendicular to the displacement. Friction does $-245\,\text{J}$, leaving zero net work and no change in kinetic energy, consistent with uniform velocity. The larger proposed values do not satisfy the horizontal force balance with the reduced normal reaction.

Step 4: Conclusion The applied force does positive work $245\,\text{J}$. Hence, the correct option is (D).

Quick Tip to Solve: For an upward inclined pull, calculate the reduced normal force before calculating friction; use only the horizontal component in the work.

39. There are two vessels filled with an ideal gas where volume of one is double the volume of other. The large vessel contains the gas at 8 kPa at 1000 K while the smaller vessel contains the gas at 7 kPa at 500 K . If the vessels are connected to each other by a thin tube allowing the gas to flow and the temperature of both vessels is maintained at 600 K , at steady state the pressure in the vessels will be (in kPa). [4 marks]

(A) 24

(B) 4.4

(C) 18

(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- The number of moles in an ideal gas is $n = PV/(RT)$. In a closed pair of connected vessels, the total number of moles is conserved.
- After equilibrium is established, the pressure and temperature are common to both vessels. Their fixed volumes add to give the final total volume.

Step 1: Express the initial mole numbers Let the volume of the smaller vessel be V, so that the larger one has volume $2V$. Use pressures in kPa consistently throughout; the conversion to pascals cancels between the initial and final ideal-gas equations. The initial mole numbers satisfy

$$n_L R = \frac{8(2V)}{1000}, \qquad n_S R = \frac{7V}{500}.$$

The temperatures are already absolute temperatures. The larger vessel does not contain simply twice the gas, since its initial pressure and temperature are different.

Step 2: Apply conservation of gas at the final temperature The thin connecting tube allows redistribution of gas without changing its total amount; its volume is negligible compared with the vessels. Adding the two initial quantities gives

$$(n_L + n_S)R = \left(\frac{16}{1000} + \frac{7}{500}\right) V = 0.030V.$$

At the final common temperature $600\,\mathrm{K}$, the gas occupies $3V$. Therefore the final pressure P_f obeys

$$P_f(3V) = (n_L + n_S)R(600) = 0.030V(600).$$

Step 3: Evaluate the common pressure Cancel the unknown vessel volume to obtain

$$P_f = \frac{600}{3}(0.030) = 6\,\mathrm{kPa}.$$

A direct arithmetic average of the two initial pressures is inapplicable because both the volumes and temperatures differ. The options 24 and 18 overestimate the pressure, and 4.4 does not conserve the total PV/T of the gas. As a check, the initial pressure-volume sum divided by temperature is $0.030V$,

exactly equal to $6(3V)/600$ in the final state.

Step 4: Conclusion The steady common pressure in both vessels is $6\,\text{kPa}$. Hence, the correct option is (D).

Quick Tip to Solve: For connected rigid vessels at a final temperature, add the initial quantities PV/T, then multiply by $T_f/(V_1+V_2)$.

40. Two polarisers P_1 and P_2 are placed in such a way that the intensity of the transmitted light will be zero. A third polariser P_3 is inserted in between P_1 and P_2, at particular angle between P_2 and P_3. The transmitted intensity of the light passing the through all three polarisers is maximum. The angle between the polarisers P_2 and P_3 is : [4 marks]

(A) $\pi/6$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) $\pi/8$

Correct Answer: (C) $\frac{\pi}{4}$

Solution:

Concept:

- Malus law states that a plane-polarised beam of intensity I transmits intensity $I\cos^2\theta$ through an ideal polariser whose axis makes angle θ with its polarisation.
- Crossed transmission axes differ by $\pi/2$. The intermediate polariser changes the polarisation direction, permitting light to pass through the final polariser.

Step 1: Relate the two successive angles The zero transmission through P_1 and P_2 means their axes are perpendicular. Let the angle between P_2 and the inserted P_3 be θ, with $0 \le \theta \le \pi/2$. The angle between P_1 and P_3 is then $\pi/2-\theta$. Let I_1 denote the intensity just after P_1; any reduction at the first polariser is independent of θ.

Step 2: Apply Malus law twice After the intermediate polariser, the intensity is

$$I_3 = I_1\cos^2(\pi/2-\theta) = I_1\sin^2\theta.$$

The light is now polarised along the axis of P_3. Transmission through P_2 gives

$$I_{\text{out}} = I_3\cos^2\theta = I_1\sin^2\theta\cos^2\theta = \frac{I_1}{4}\sin^2(2\theta).$$

It is essential to include both transmission factors; maximising just one of them would leave the other polariser blocking the light.

Step 3: Maximise the transmission The largest possible value of $\sin^2(2\theta)$ is one. In the allowed interval it occurs at

$$2\theta = \frac{\pi}{2}, \qquad \theta = \frac{\pi}{4}.$$

The options $\pi/6$ and $\pi/3$ each give $\sin^2(2\theta) = 3/4$, below the maximum. The option $\pi/8$ gives $1/2$. Thus none of those orientations can give the greatest transmitted intensity. The optimal intermediate axis bisects the right angle between the crossed axes.

Step 4: Conclusion The required angle between P_2 and P_3 is $\pi/4$, or $45°$. Hence, the correct option is (C).

Quick Tip to Solve: Between crossed polarisers, write the product $\sin^2\theta\cos^2\theta$; its maximum occurs when the intermediate axis bisects their angle.

41. Match List - I with List - II.

	List - I		List - II
(A)	**Isobaric**	**(I)**	$\Delta Q = \Delta W$
(B)	**Isochoric**	**(II)**	$\Delta Q = \Delta U$
(C)	**Adiabatic**	**(III)**	$\Delta Q =$ **zero**
(D)	**Isothermal**	**(IV)**	$\Delta Q = \Delta U + P\Delta V$

$\Delta Q =$ **Heat supplied**

$\Delta W =$ **Work done by the system**

ΔU $=$ **Change in internal energy**

P $=$ **Pressure of the system**

ΔV $=$ **Change in volume of the system**

Choose the correct answer from the options given below : [4 marks]

(A) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

(B) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(C) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(D) (A)-(II), (B)-(IV), (C)-(III), (D)-(I)

Correct Answer: (B) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

Solution:

Concept:

- With work taken as work done by the system, the first law is $\Delta Q = \Delta U + \Delta W$. For a quasistatic expansion, $\Delta W = \int P\,dV$.
- For an ideal gas the internal energy depends only on temperature. Each named process imposes its own constraint on pressure, volume, heat transfer or temperature.

Step 1: Match isobaric and isochoric changes For an isobaric process the pressure is constant, so the work integral becomes $P\Delta V$. The first law is therefore

$$\Delta Q = \Delta U + P\Delta V,$$

which pairs A with IV. For an isochoric process the volume is fixed: $dV = 0$, so $\Delta W = 0$. All heat

supplied changes the internal energy, giving

$$\Delta Q = \Delta U.$$

Thus B pairs with II. These two constraints refer to different state variables and must not be interchanged.

Step 2: Match adiabatic and isothermal changes An adiabatic process has no heat exchange with the surroundings, by definition:

$$\Delta Q = 0.$$

Hence C pairs with III. In an isothermal process of an ideal gas, the temperature and consequently internal energy remain constant. Setting $\Delta U = 0$ in the first law gives

$$\Delta Q = \Delta W,$$

so D pairs with I. Isothermal does not mean zero heat transfer: heat can enter while the gas does an equal amount of work.

Step 3: Compare the complete mapping with the options The four independent matches combine into

$$\mathrm{A} \to \mathrm{IV}, \quad \mathrm{B} \to \mathrm{II}, \quad \mathrm{C} \to \mathrm{III}, \quad \mathrm{D} \to \mathrm{I}.$$

The first option wrongly pairs constant volume with heat equal to work and constant temperature with heat equal to internal-energy change. The third option interchanges the isochoric and adiabatic conditions. The fourth misidentifies both isobaric and isochoric relations. Only the second option satisfies all four process constraints together.

Step 4: Conclusion The correct matching is A-IV, B-II, C-III and D-I. Hence, the correct option is (B).

Quick Tip to Solve: Start with the first law and set the quantity constrained by the process to zero; for constant pressure replace the work integral by $P\Delta V$.

42. There are ' n ' number of identical electric bulbs, each is designed to draw a power p independently from the mains supply. They are now joined in series across the mains supply. The total power drawn by the combination is : [4 marks]

(A) np

(B) p

(C) $\frac{\mathrm{p}}{\mathrm{n}^2}$

(D) $\frac{p}{n}$

Correct Answer: (D) $\frac{p}{n}$

Solution:

Concept:

- At a fixed mains voltage V, a bulb of resistance R has rated power $p = V^2/R$. Treat the identical bulbs as equal resistances in the standard circuit model.

- Series resistances add, and every member carries the same current. The total electrical power taken from a supply is $P = VI = V^2/R_{\text{eq}}$.

Step 1: Infer the resistance of one bulb Each bulb individually is intended to be connected across the same mains voltage V and then draw power p. Its resistance in the given model is therefore

$$R = \frac{V^2}{p}.$$

The given p is the rating when a bulb receives the full mains voltage; it is not automatically the power dissipated by that bulb after the connections are changed.

Step 2: Find the series current and power per bulb For n identical bulbs in series, the equivalent resistance is

$$R_{\text{eq}} = nR.$$

Across the same mains, their common current is

$$I = \frac{V}{nR}.$$

Each bulb receives voltage V/n, since the resistances are equal. Thus the power of a single bulb in this series combination is

$$P_{\text{one}} = I^2R = \frac{V^2}{n^2R} = \frac{p}{n^2}.$$

This is the third option, but it describes one bulb rather than the whole combination.

Step 3: Sum over all the bulbs There are n bulbs, so their total power is

$$P_{\text{total}} = nP_{\text{one}} = n\frac{p}{n^2} = \frac{p}{n}.$$

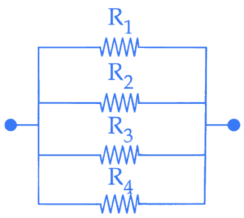
Equivalently the supply directly delivers $V^2/(nR) = p/n$, confirming the result. The value np would apply to n bulbs connected in parallel at full voltage. The value p applies to just one bulb alone, not generally to a series string. The total decreases with increasing n, consistent with the larger equivalent resistance.

Step 4: Conclusion The series combination draws total power p/n from the mains. Hence, the correct option is (D).

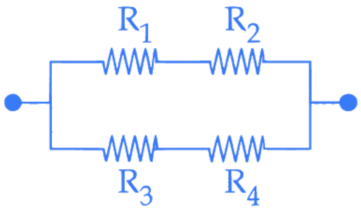
Quick Tip to Solve: Distinguish the power of each series bulb, p/n^2, from the power of the entire string, n times that value.

43. From the combination of resistors with resistances values $R_1 = R_2 = R_3 = 5\Omega$ and $R_4 = 10\Omega$, which of the following combination is the best circuit to get an equivalent resistance of 6Ω ? [4 marks]

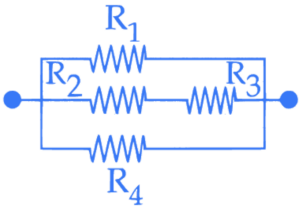
(A)



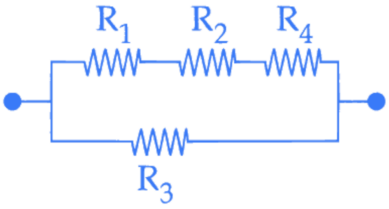
(B)



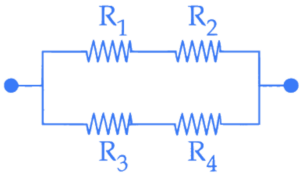
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- Series resistors carry the same current, so their resistances add: $R_s = R_a + R_b$. Parallel branches share a potential difference, so their conductances add: $1/R_p = 1/R_a + 1/R_b$.
- Identify the two terminal nodes first. A branch containing several resistors in series must be reduced before combining it in parallel with another branch.

Step 1: Reduce the branches in option B The upper branch contains R_1 and R_2 in series, while the lower branch contains R_3 and R_4 in series. Their resistances are therefore

$$R_u = 5 + 5 = 10\,\Omega, \qquad R_l = 5 + 10 = 15\,\Omega.$$

These branches connect the same pair of terminals, so they are in parallel rather than in series with each other.

Step 2: Calculate the equivalent resistance The current divides between the two branches. Using the product-over-sum formula for two parallel resistances gives

$$R_B = \frac{R_u R_l}{R_u + R_l} = \frac{10 \times 15}{10 + 15} = \frac{150}{25} = 6\,\Omega.$$

This exactly matches the required value; all four given resistors are used.

Step 3: Check the other circuit choices In A all four resistors are in parallel, giving

$$R_A = \left(\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{10}\right)^{-1} = \frac{10}{7}\,\Omega.$$

In C the three parallel branches have resistances $5, 10, 10\,\Omega$, so $R_C = 2.5\,\Omega$. In D a $20\,\Omega$ upper branch is parallel to $5\,\Omega$, giving $R_D = 4\,\Omega$. None of these equals the required resistance.

Step 4: Conclusion The correct network is $(R_1 + R_2) \parallel (R_3 + R_4)$, whose equivalent resistance is $6\,\Omega$. Hence, the correct option is (B).

Quick Tip to Solve: Mark the two terminal nodes and sum within each uninterrupted branch before applying a parallel formula.

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44. Consider a n-type semiconductor in which $\mathrm{n_e}$ and $\mathrm{n_h}$ are number of electrons and holes, respectively.

(A) Holes are minority carriers

(B) The dopant is a pentavalent atom

(C) $\mathrm{n_e n_h} \neq \mathrm{n_i^2}$

(where $\mathrm{n_i}$ is number of electrons or holes in semiconductor when it is intrinsic form)

(D) $\mathrm{n_e n_h} \geqslant \mathrm{n_i^2}$

(E) The holes are not generated due to the donors

Choose the correct answer from the options given below : [4 marks]

(A) (A), (B), (E) only

(B) (A), (C), (E) only

(C) (A), (B), (C) only

(D) (A), (C), (D) only

Correct Answer: No option lists all true statements; printed statement (D) allows equality. The key intends option (A).

Solution:

Concept:

- An n-type semiconductor is obtained by doping a tetravalent semiconductor with pentavalent donor atoms. Electrons are majority carriers and holes are minority carriers.
- At thermal equilibrium the mass-action law is $n_e n_h = n_i^2$, where n_i is the intrinsic carrier concentration at the same temperature. Equality satisfies a greater-than-or-equal inequality as well.

Step 1: Evaluate the carrier statements A donor contributes an extra electron to the conduction band without leaving a corresponding mobile valence-band hole. Hence electrons are the majority carriers, holes are the minority carriers, and the dopant is pentavalent. Statements (A), (B) and (E) are therefore correct. The minority holes arise from thermal generation, rather than from ionisation of the donor atoms.

Step 2: Apply the equilibrium relation carefully The product of the equilibrium electron and hole concentrations obeys

$$n_e n_h = n_i^2.$$

Consequently statement (C), which asserts inequality $n_e n_h \neq n_i^2$, is false. However, statement (D) is printed with $\geqslant$, not with a strict greater-than sign. Because equality is allowed by that symbol, statement (D) is also mathematically true. Doping changes the separate carrier populations but does not invalidate the mass-action equality at a fixed temperature.

Step 3: Compare the full true set with the options The literal set of true statements is (A), (B), (D) and (E). Option A lists (A), (B), (E) only and thus omits the true inclusive inequality (D). Options B, C and D all include false statement (C). The intended keyed selection is evidently option A if statement (D) was meant to assert a strict inequality, but that is not what its printed symbol says.

Step 4: Conclusion As printed, there is no fully correct option: (A), (B), (D) and (E) are true. The supplied answer key selects (A), corresponding to (A), (B), (E) only, but the inclusive $\geqslant$ in statement (D) also holds at equality.

Quick Tip to Solve: Check whether an inequality is strict or inclusive: an exact equality makes both the equality and its inclusive inequality true.

45. Given below are two statements :

Statement (I) : The dimensions of Planck's constant and angular momentum are same.

Statement (II) : In Bohr's model electron revolve around the nucleus only in those orbits for which angular momentum is integral multiple of Planck's constant.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are correct

(B) Statement I is correct but Statement II is incorrect

(C) Both Statement I and Statement II are incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (B) Statement I is correct but Statement II is incorrect

Solution:

Concept:

- Planck's relation $E = h\nu$ gives the dimensions of h as energy divided by frequency.
- Bohr quantises orbital angular momentum in units of the reduced Planck constant $\hbar = h/(2\pi)$: $L = n\hbar$, for positive integers n.

Step 1: Check Statement I using dimensions Energy has dimensions $[E] = ML^2T^{-2}$, and frequency has dimensions $[\nu] = T^{-1}$. Therefore

$$[h] = \frac{[E]}{[\nu]} = ML^2T^{-1}.$$

Angular momentum is $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, so its dimensions are length times linear momentum:

$$[L] = L\,(MLT^{-1}) = ML^2T^{-1}.$$

The two dimensional expressions agree. Thus Statement I is correct; dimensions alone do not imply that the two quantities are numerically equal.

Step 2: Test Statement II against the quantisation postulate For a stationary circular orbit in Bohr's model, the allowed angular momentum is

$$mvr = \frac{nh}{2\pi} = n\hbar, \qquad n = 1, 2, 3, \ldots.$$

The integer multiplies $h/(2\pi)$, not h. The factor 2π cannot be dropped, even though it is dimensionless. Hence the wording of Statement II is incorrect. For instance, the first allowed orbit has $L = h/(2\pi)$, which is not an integral multiple of h.

Step 3: Choose the matching truth combination Option A incorrectly accepts Statement II. Option C incorrectly rejects the dimension equality established above. Option D reverses both truth values. Only the choice stating that Statement I is correct and Statement II is incorrect matches both independent checks.

Step 4: Conclusion Statement I is correct, while Statement II is incorrect because Bohr's angular momentum is an integral multiple of $\hbar$, not h. Hence, the correct option is (B).

Quick Tip to Solve: Remember Bohr's rule as $L = n\hbar$; expand $\hbar = h/(2\pi)$ before judging a statement mentioning Planck's constant.

Physics – Section B: Numerical Value Questions (20 marks)

46. A solid sphere with uniform density and radius R is rotating initially with constant angular velocity (ω_1) about its diameter. After some time during the rotation its starts loosing mass at a uniform rate, with no change in its shape. The angular velocity of the sphere when its radius become $\mathrm{R}/2$ is $x\omega_1$. The value of x is _____. [4 marks]

Solution:

Concept:

- The moment of inertia of a uniform solid sphere about its diameter is $I = \frac{2}{5}MR^2$. Its angular momentum about that axis is $L = I\omega$.
- For unchanged density, mass is proportional to volume: $M = \frac{4}{3}\pi\rho R^3$. Consequently the sphere's moment of inertia is proportional to the fifth power of its radius.

Step 1: Find the mass at half the radius Let the initial mass be M and the final mass be M_f. With uniform unchanged density and the same spherical shape,

$$\frac{M_f}{M} = \frac{(R/2)^3}{R^3} = \frac{1}{8}.$$

Thus halving the radius reduces the mass to one-eighth; retaining the initial mass in the final moment of inertia would miss this important factor.

Step 2: Compare the moments of inertia The initial and final moments of inertia about the diameter are

$$I_i = \frac{2}{5}MR^2, \qquad I_f = \frac{2}{5}\frac{M}{8}\left(\frac{R}{2}\right)^2 = \frac{I_i}{32}.$$

Equivalently, using $I \propto R^5$ gives $I_f/I_i = (1/2)^5$. The specified uniform rate of mass loss determines how rapidly this state is reached, but does not enter this ratio.

Step 3: Use the angular-momentum conservation model In the angular-momentum conservation model used for this question, take the shrinking sphere's axial angular momentum to remain unchanged. Then

$$I_i\omega_1 = I_f\omega_f, \qquad \omega_f = \frac{I_i}{I_f}\omega_1 = 32\omega_1.$$

This is the model underlying the stated answer: an additional specification that shed material carries away angular momentum would require an angular-momentum flux calculation and would not follow from this closed-system equation alone.

Step 4: Conclusion Using conservation of the sphere's angular momentum with $I \propto R^5$, $\omega_f = 32\omega_1$. Therefore the required dimensionless value is $\boxed{x = 32}$.

Quick Tip to Solve: For a shrinking uniform-density sphere, combine $M \propto R^3$ with $I \propto MR^2$ before conserving angular momentum.

47. An inductor of self inductance 1 H is connected in series with a resistor of 100π ohm and an ac supply of 100π volt, 50 Hz . Maximum current flowing in the circuit is _____ A. [4 marks]

Solution:

Concept:

- For a sinusoidal supply of frequency f, an ideal inductor has reactance $X_L = \omega L = 2\pi f L$. Resistance and inductive reactance are perpendicular components of impedance.
- A series RL circuit has impedance magnitude $Z = \sqrt{R^2 + X_L^2}$. An ac voltage specified without a peak qualifier is its rms value, so $I_{\max} = \sqrt{2}\,V_{\text{rms}}/Z$.

Step 1: Calculate the inductive reactance The inductance is $L = 1\,\text{H}$ and the supply frequency is $f = 50\,\text{Hz}$. Hence

$$X_L = 2\pi \times 50 \times 1 = 100\pi\,\Omega.$$

The inductive reactance equals the resistance $R = 100\pi\,\Omega$. It must still be combined vectorially with the resistance because the inductor voltage is a quarter-cycle ahead of the current.

Step 2: Find the impedance and rms current The impedance magnitude is

$$Z = \sqrt{(100\pi)^2 + (100\pi)^2} = 100\pi\sqrt{2}\,\Omega.$$

Using the supply voltage $V_{\text{rms}} = 100\pi$ V, the rms current is

$$I_{\text{rms}} = \frac{100\pi}{100\pi\sqrt{2}} = \frac{1}{\sqrt{2}}\,\text{A}.$$

Adding R and X_L directly would wrongly treat their voltage drops as being in phase.

Step 3: Convert from rms to maximum current For a sine wave $i(t) = I_{\max}\sin(\omega t - \phi)$, the rms value is $I_{\max}/\sqrt{2}$. Therefore

$$I_{\max} = \sqrt{2}\,I_{\text{rms}} = \sqrt{2}\left(\frac{1}{\sqrt{2}}\right) = 1\,\text{A}.$$

The intermediate $1/\sqrt{2}\,\text{A}$ is the rms current and is not the maximum requested in the question.

Step 4: Conclusion The maximum current flowing in the series RL circuit is $\boxed{1\,\text{A}}$. The numerical value to enter is 1.

Quick Tip to Solve: Compute the impedance first; convert rms current to peak current only at the final step to avoid losing the factor $\sqrt{2}$.

---

48. If an optical medium possesses a relative permeability of $\frac{10}{\pi}$ and relative permittivity of $\frac{1}{0.0885}$, then the velocity of light is greater in vacuum than that in this medium by _____ times.

$$\left(\mu_0 = 4\pi \times 10^{-7}\text{H/m}, \epsilon_0 = 8.85 \times 10^{-12}\ \text{F/m}, \text{c} = 3 \times 10^8\ \text{m/s}\right)$$

[4 marks]

Solution:

Concept:

- The speed of an electromagnetic wave in a medium is $v = 1/\sqrt{\mu\epsilon}$. Write $\mu = \mu_r\mu_0$ and $\epsilon = \epsilon_r\epsilon_0$.
- The requested factor is the ratio of speeds c/v, not the difference $c - v$. Relative permeability and relative permittivity are dimensionless.

Step 1: Calculate the absolute material constants The relative permeability is $\mu_r = 10/\pi$, so

$$\mu = \frac{10}{\pi}(4\pi \times 10^{-7}) = 4 \times 10^{-6}\,\text{H m}^{-1}.$$

The relative permittivity is $\epsilon_r = 1/0.0885$. Thus

$$\epsilon = \frac{8.85 \times 10^{-12}}{0.0885} = 1.00 \times 10^{-10}\,\text{F m}^{-1}.$$

The cancellation in the second calculation is useful: $0.0885 = 8.85 \times 10^{-2}$.

Step 2: Determine the light speed in the medium Substitute these absolute constants into the wave-speed formula:

$$\mu\epsilon = (4 \times 10^{-6})(10^{-10}) = 4 \times 10^{-16},$$

$$v = \frac{1}{\sqrt{4 \times 10^{-16}}} = \frac{1}{2 \times 10^{-8}} = 5 \times 10^{7}\,\mathrm{m\,s^{-1}}.$$

The medium therefore reduces the propagation speed relative to vacuum. No frequency is needed because the material constants needed for this calculation are already supplied.

Step 3: Form the requested vacuum-to-medium ratio Using the stated vacuum speed $c = 3\times10^{8}\,\mathrm{m\,s^{-1}}$,

$$\frac{c}{v} = \frac{3 \times 10^{8}}{5 \times 10^{7}} = 6.$$

Both speeds have the same units, which cancel. Directly using $\sqrt{\mu_r \epsilon_r}$ gives the same factor to the precision of the supplied vacuum constants; the absolute-constant route makes the numerical cancellations exact here.

Step 4: Conclusion The vacuum light speed is $\boxed{6}$ times the speed in this optical medium. The required dimensionless numerical value is 6.

Quick Tip to Solve: Convert the relative constants to μ and ϵ when the supplied decimal values cancel neatly, then evaluate $1/\sqrt{\mu\epsilon}$.

---

49. A particle of charge 1.6μC and mass 16μ g is present in a strong magnetic field of 6.28 T . The particle is then fired perpendicular to magnetic field. The time required for the particle to return to original location for the first time is _____ s. $(\pi = 3.14)$ [4 marks]

Solution:

Concept:

- A charge moving perpendicular to a uniform magnetic field follows a circular path because the magnetic force is always perpendicular to its velocity and does no work.
- The first return to the starting point occurs after one complete revolution, with period $T = 2\pi m/(|q|B)$, where m is mass, q is charge and B is magnetic flux density.

Step 1: Convert the mass and charge to SI units The prefix micro means 10^{-6}, but grams must also be converted to kilograms. Thus

$$m = 16 \times 10^{-6}\,\mathrm{g} = 16 \times 10^{-9}\,\mathrm{kg}, \qquad |q| = 1.6 \times 10^{-6}\,\mathrm{C}.$$

The field is $B = 6.28\,\mathrm{T}$. Using $16 \times 10^{-6}\,\mathrm{kg}$ would make the period too large by a factor of one thousand.

Step 2: Obtain the time for a full revolution Equate the magnetic force to the required centripetal force:

$$|q|vB = \frac{mv^2}{r}, \qquad r = \frac{mv}{|q|B}.$$

Dividing the circumference by the speed gives

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{|q|B}.$$

Therefore the unspecified firing speed cancels; no value of the speed or radius is needed.

Step 3: Substitute the given values Using the supplied value $\pi = 3.14$,

$$T = \frac{2(3.14)(16 \times 10^{-9})}{(1.6 \times 10^{-6})(6.28)} = \frac{16 \times 10^{-9}}{1.6 \times 10^{-6}} = 1.0 \times 10^{-2}\,\text{s}.$$

Half a revolution reaches the opposite end of the diameter and does not return the particle to its original point.

Step 4: Conclusion The first-return time is $\boxed{0.01\,\text{s}}$. When the numerical response is rounded to the nearest integer, the entry is $\boxed{0}$, consistent with the accepted numerical key.

Quick Tip to Solve: For a cyclotron period, cancel the speed first and convert micrograms to kilograms before substitution.

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50. In a Young's double slit experiment, two slits are located 1.5 mm apart. The distance of screen from slits is 2 m and the wavelength of the source is 400 nm . If the 20 maxima of the double slit pattern are contained within the central maximum of the single slit diffraction pattern, then the width of each slit is $x \times 10^{-3}$ cm, where x-value is _____ . [4 marks]

Solution:

Concept:

- The double-slit interference fringe spacing is $\beta = \lambda D/d$, where d is the slit separation and D is the screen distance.
- For slits of width a, the central single-slit diffraction maximum has width $W = 2\lambda D/a$, between its first minima on either side.

Step 1: Relate the two widths The interference maxima are spaced by β, while the much broader diffraction envelope determines how many of them are visible in its central region. For the stated count $N = 20$, use

$$N = \frac{W}{\beta} = \frac{2\lambda D/a}{\lambda D/d} = \frac{2d}{a}.$$

Both the wavelength and the screen distance cancel because they scale the interference pattern and diffraction envelope in the same way.

Step 2: Calculate the width of one slit Rearrange the count relation before inserting the separation:

$$a = \frac{2d}{N} = \frac{2(1.5\,\text{mm})}{20} = 0.15\,\text{mm}.$$

As a direct check, the given wavelength and screen distance give

$$\beta = \frac{400 \times 10^{-9} \times 2}{1.5 \times 10^{-3}} = 5.33 \times 10^{-4}\,\text{m}.$$

With $a = 1.5 \times 10^{-4}\,\text{m}$, the central-envelope width is $1.067 \times 10^{-2}\,\text{m}$, twenty times this spacing.

Step 3: Express the result in the requested unit The unit conversion is $1\,\text{mm} = 0.1\,\text{cm}$, so

$$a = 0.15\,\text{mm} = 0.015\,\text{cm} = 15 \times 10^{-3}\,\text{cm}.$$

Compare this directly with the printed form $x \times 10^{-3}\,\text{cm}$. The requested number is the coefficient, not the slit width expressed in metres.

Step 4: Conclusion The width of each slit is $\boxed{15 \times 10^{-3}\,\text{cm}}$, and hence $\boxed{x = 15}$.

Quick Tip to Solve: Take the ratio of diffraction width to fringe spacing first: $N = 2d/a$ removes both D and λ.

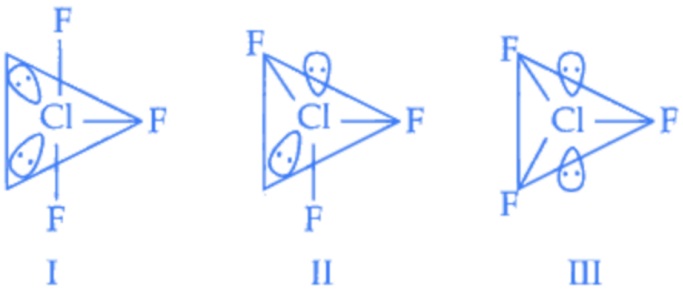
Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Given below are two statements:

Statement (I): For ClF_3, all three possible structures may be drawn as follows.

Statement (II): Structure III is most stable, as the orbitals having the lone pairs are axial, where the lp – bp repulsion is minimum.

In the light of the above statements, choose the most appropriate answer from the options given below: [4 marks]



(A) Both Statement I and Statement II are correct

(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (C) Statement I is correct but Statement II is incorrect

Solution:

Concept:

- In ClF_3, chlorine has three bonding pairs and two lone pairs, giving five electron domains and a trigonal-bipyramidal electron-pair arrangement.
- Lone pairs repel more strongly than bonding pairs. In a trigonal bipyramid an equatorial site has fewer 90° interactions than an axial site, so lone pairs preferentially occupy equatorial positions.

Step 1: Identify the possible lone-pair placements The diagram compares three arrangements of two lone pairs among five sites. They can both be equatorial (structure I), one equatorial and one axial

(structure II), or both axial (structure III). These are the three distinct placements before comparing their energies. Thus the drawings represent possible arrangements to consider, even though they are not equally stable. Statement I is correct.

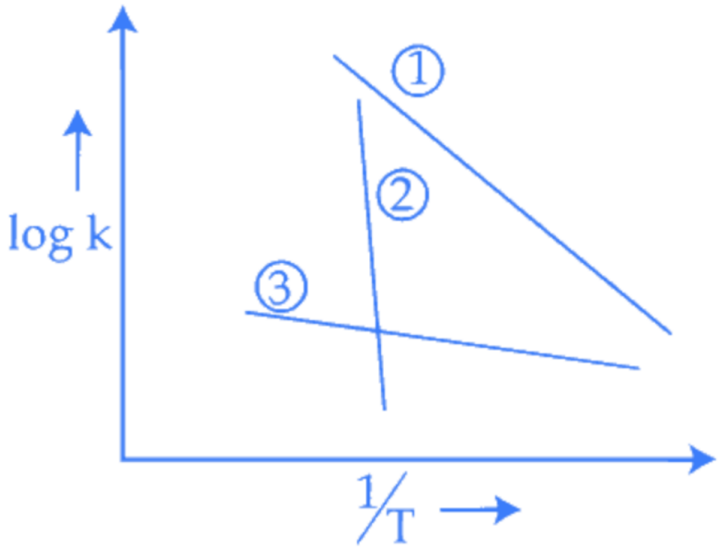
Step 2: Compare repulsions in the three structures An axial lone pair has three neighbouring equatorial domains at $90°$. An equatorial lone pair has only two axial domains at $90°$, while its other equatorial neighbours are at $120°$. Placing both lone pairs equatorially therefore reduces the unfavourable close-angle interactions. In structure I the two lone pairs are separated by $120°$, and the remaining three bonds form the familiar T-shaped molecule. Structure III puts both lone pairs axially and increases their $90°$ interactions with bonding pairs.

Step 3: Judge the combined statements Statement II claims that axial lone pairs minimise lone-pair–bond-pair repulsion and make structure III most stable. Its site preference and its stability conclusion are both wrong: structure I is preferred. Consequently Statement II is incorrect. Options (A) and (D) incorrectly accept Statement II, while option (B) incorrectly rejects Statement I. Only option (C) matches the two independent evaluations.

Step 4: Conclusion Statement I is correct and Statement II is incorrect; the most stable arrangement is structure I with equatorial lone pairs. Hence, the correct option is (C).

Quick Tip to Solve: In a trigonal bipyramid, count the $90°$ neighbours: an equatorial lone pair encounters fewer than an axial one.

52. Consider the following plots of $\log$ of rate constant $k(\log k)$ vs $\frac{1}{T}$ for three different reactions. The correct order of activation energies of these reactions is : [4 marks]



(A) $Ea_2 > Ea_1 > Ea_3$

(B) $Ea_1 > Ea_3 > Ea_2$

(C) $Ea_3 > Ea_2 > Ea_1$

(D) $Ea_1 > Ea_2 > Ea_3$

Correct Answer: (A) $Ea_2 > Ea_1 > Ea_3$

Solution:

Concept:

- The Arrhenius equation is $k = A\exp[-E_a/(RT)]$, where A is the pre-exponential factor, E_a is activation energy and R is the gas constant.
- Taking common logarithms gives $\log k = \log A - [E_a/(2.303R)](1/T)$. Thus the slope of a plot of $\log k$ against $1/T$ is $-E_a/(2.303R)$.

Step 1: Read the magnitudes of the slopes All three plotted straight lines slope downward as $1/T$ increases. Line 2 is nearly vertical and has the largest magnitude of negative slope. Line 1 has an intermediate downward inclination. Line 3 is close to horizontal and has the smallest magnitude. In symbols,

$$|m_2| > |m_1| > |m_3|.$$

Compare the steepness against the same pair of axes, rather than comparing the heights or crossing points of the lines.

Step 2: Convert slope magnitudes into activation energies The activation energy is obtained by multiplying the magnitude of the slope by the same positive constant for every reaction:

$$E_{a,i} = 2.303R\,|m_i|.$$

Therefore the ordering of activation energies is exactly the ordering of slope magnitudes:

$$E_{a,2} > E_{a,1} > E_{a,3}.$$

The negative sign in the slope formula must not reverse this result: it is the magnitude of a downward slope that measures a positive barrier. The intercept depends on A, not directly on the barrier.

Step 3: Select the matching order Option (A) places reaction 2 first, reaction 1 second and reaction 3 last, as the graph requires. Options (B) and (D) wrongly put reaction 1 above the much steeper reaction 2. Option (C) puts the shallowest line, reaction 3, at the largest activation energy and so reverses the relevant visual comparison.

Step 4: Conclusion The correct order is $\boxed{E_{a,2} > E_{a,1} > E_{a,3}}$. Hence, the correct option is (A).

Quick Tip to Solve: For $\log k$ versus $1/T$, the steepest downward line has the largest activation energy; ignore its intercept.

53. Match List I with List II.

	List I (Separation of)		List II (Technique)
(A)	Aniline from aniline-water mixture	(I)	Simple distillation
(B)	Glycerol from spent-lye in soap industry	(II)	Fractional distillation
(C)	Different fractions of crude oil in petroleum industry	(III)	Distillation at reduced pressure
(D)	Chloroform-Aniline mixture	(IV)	Steam distillation

Choose the correct answer from the options given below: [4 marks]

(A) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(B) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(C) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(D) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

Correct Answer: (A) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

Solution:

Concept:

- Distillation separates liquids by volatility. Simple distillation is suitable when boiling points are well separated, while fractional distillation repeatedly equilibrates vapour and liquid to separate closer-boiling components.
- Steam distillation isolates steam-volatile water-immiscible substances below their normal boiling point. Reduced-pressure distillation lowers the boiling temperature of high-boiling substances that can decompose on strong heating.

Step 1: Assign the specialised methods Aniline can be separated from its mixture with water by steam distillation. Their combined vapour pressures allow the mixture to distil below the normal boiling point of aniline, so (A) matches (IV). Glycerol has a high boiling point and can decompose when heated at ordinary pressure; distillation at reduced pressure lowers the required temperature. Therefore glycerol from spent-lye, (B), matches (III).

Step 2: Assign the petroleum and binary-liquid methods Crude oil contains many hydrocarbons with different but overlapping boiling ranges. A fractionating column produces repeated vaporisation and condensation, giving different petroleum fractions. Hence (C) matches fractional distillation, (II). Chloroform and aniline have widely separated boiling points, approximately $61\,^{\circ}\mathrm{C}$ and $184\,^{\circ}\mathrm{C}$, respectively. Their mixture can therefore be separated by simple distillation, so (D) matches (I).

Step 3: Compare the complete matching The four assignments combine to give

$$(A)-(IV), \quad (B)-(III), \quad (C)-(II), \quad (D)-(I).$$

This is option (A). Every other option assigns the wrong method to aniline and fails to place petroleum fractions under fractional distillation; a single correct-looking pair would not suffice to select an entire matching.

Step 4: Conclusion Aniline requires steam distillation, glycerol reduced pressure, petroleum fractional distillation, and chloroform–aniline simple distillation. Hence, the correct option is (A).

Quick Tip to Solve: Associate steam with aniline, vacuum with heat-sensitive glycerol, and a fractionating column with petroleum.

54. Given below are two statements :

Statement (I) : The first ionisation enthalpy of group 14 elements is higher than the corresponding elements of group 13.

Statement (II) : Melting points and boiling points of group 13 elements are in general much higher than those of corresponding elements of group 14.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is correct but Statement II is incorrect

(B) Both Statement I and Statement II are incorrect

(C) Statement I is incorrect but Statement II is correct

(D) Both Statement I and Statement II are correct

Correct Answer: (A) Statement I is correct but Statement II is incorrect

Solution:

Concept:

- Across a period, increasing effective nuclear attraction generally raises the first ionisation enthalpy. Group 13 has outer configuration ns^2np^1, whereas group 14 has ns^2np^2.
- Melting and boiling points depend strongly on bonding and solid structure, so they should be compared using the actual group trends rather than inferred directly from atomic number.

Step 1: Evaluate the first ionisation enthalpy statement The corresponding group 14 atom lies to the right of the group 13 atom in the same period. The greater effective nuclear attraction generally binds its valence electrons more strongly, giving a higher first ionisation enthalpy. For example, carbon exceeds boron and silicon exceeds aluminium. The stated comparison is the accepted group trend for these corresponding elements, so Statement I is correct. The comparison concerns removal of the first electron, not removal of every valence electron.

Step 2: Evaluate the melting and boiling point statement Statement II asserts that group 13 generally has much higher melting and boiling points than group 14. This reverses the usual trend discussed for these groups. The strong extended bonding of carbon and silicon, for example, gives very high melting and boiling temperatures compared with boron and aluminium, respectively. The group 14 elements generally have higher values than their corresponding group 13 elements; structures and metallic character account for variations down the groups. Thus the broad claim made in Statement II is incorrect.

Step 3: Match the truth values to an option The independent evaluations give Statement I true and Statement II false. Option (A) states exactly this combination. Option (B) incorrectly rejects the ionisation-enthalpy trend, option (C) reverses both truth values, and option (D) incorrectly accepts the melting/boiling-point claim. Neither statement follows merely from the number of valence electrons; the relevant property and bonding must be considered separately.

Step 4: Conclusion Statement I is correct but Statement II is incorrect. Hence, the correct option is (A).

Quick Tip to Solve: Keep the two comparisons separate: ionisation reflects electron binding, while melting and boiling reflect the bonding of the material.

55. Which among the following compounds give yellow solid when reacted with $NaOI/NaOH$?

(A) $CH_3CH(OH)C_2H_5$
(B) $CH_3CH_2CH_2OH$
(C) $CH_3COC_2H_5$
(D) CH_3COOH
(E) CH_3CH_2CHO

Choose the correct answer from the options given below: [4 marks]

(A) (A) and (C) Only
(B) (C) and (D) Only
(C) (B), (C) and (E) Only
(D) (A), (C) and (D) Only

Correct Answer: (A) (A) and (C) Only

Solution:

Concept:

- The iodoform test gives a yellow precipitate of CHI_3 with alkaline hypoiodite. It is characteristic of methyl ketones, CH_3COR.
- Secondary alcohols containing the group $CH_3CH(OH)^-$ also respond because oxidation first gives a methyl ketone. A carbonyl group alone is not sufficient.

Step 1: Identify the responsive alcohol Compound (A) is butan-2-ol, $CH_3CH(OH)CH_2CH_3$. Its hydroxyl-bearing carbon carries one hydrogen and a methyl group. Oxidation by the reagent converts this group into the methyl-ketone group of butan-2-one. Therefore (A) gives the yellow solid. Compound (B), propan-1-ol, is a primary alcohol; its initial oxidation product is propanal, not a methyl ketone. Thus (B) is excluded.

Step 2: Identify the responsive carbonyl compound Compound (C) is butan-2-one, $CH_3COCH_2CH_3$, which already contains the required methyl-ketone unit. Successive iodination of its carbonyl-adjacent methyl group, followed by cleavage in base, produces iodoform. The net methyl-ketone reaction can be written as

$$RCOCH_3 + 3\,I_2 + 4\,OH^- \longrightarrow RCOO^- + CHI_3 + 3\,I^- + 3\,H_2O.$$

Compound (D) is ethanoic acid; a carboxyl group is not a ketone group and does not give this reaction. Compound (E) is propanal, CH_3CH_2CHO, which lacks the required methyl carbonyl unit. Among aldehydes, ethanal is the familiar positive case, whereas propanal is negative. Thus combinations including (B), (D) or (E) fail.

Step 3: Conclusion Only butan-2-ol and butan-2-one yield yellow CHI_3. Hence, the correct option is (A).

Quick Tip to Solve: Circle each CH_3CO^- group and each oxidisable $CH_3CH(OH)^-$ group; do not count a carboxylic acid as a methyl ketone.

56. Consider the ground state of chromium atom $(Z = 24)$. How many electrons are with Azimuthal quantum number $l = 1$ and $l = 2$ respectively ? [4 marks]

(A) 16 and 5

(B) 12 and 5

(C) 12 and 4

(D) 16 and 4

Correct Answer: (B) 12 and 5

Solution:

Concept:

- The azimuthal quantum number identifies the subshell: $l = 0, 1, 2, 3$ correspond to s, p, d and f respectively. Count electrons in every occupied subshell of the requested type.
- Chromium has the exceptional ground-state configuration $[\mathrm{Ar}]\, 3d^5 4s^1$, rather than $[\mathrm{Ar}]\, 3d^4 4s^2$. The half-filled d subshell is particularly stable.

Step 1: Write the complete ground-state configuration The 24 electrons of chromium are distributed as

$$1s^2\, 2s^2\, 2p^6\, 3s^2\, 3p^6\, 3d^5\, 4s^1.$$

The closed argon core contains 18 electrons, while the five d electrons and one outer s electron supply the remaining six. This also checks the total atomic number. It is essential to use the actual ground state: blindly following the simple filling sequence would give four d electrons and would select a wrong option.

Step 2: Count the p and d electrons separately For $l = 1$, only p electrons contribute. Both $2p$ and $3p$ are filled, whereas no $4p$ electrons are present:

$$N(l = 1) = 6 + 6 = 12.$$

For $l = 2$, only the $3d$ subshell is occupied:

$$N(l = 2) = 5.$$

The s electrons have $l = 0$ and must not be added to either count. In particular, the two p subshells together contain twelve electrons, not sixteen; sixteen would incorrectly include s electrons. Options with four d electrons correspond to the uncorrected Aufbau prediction, not chromium's observed ground state. Therefore only the pair twelve and five agrees with both the quantum-number definition and the exceptional electronic configuration.

Step 3: Conclusion There are 12 electrons with $l = 1$ and 5 electrons with $l = 2$, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Expand the noble-gas core when counting a quantum number: the p electrons of chromium lie inside the argon core, while its five d electrons lie outside it.

57. Given below are two statements:

Statement (I) : Molal depression constant K_f is given by $\frac{\mathrm{M_1RT}_f}{\Delta\ \mathrm{S_{fus}}}$, where symbols have their usual meaning.

Statement (II) : K_f for benzene is less than the K_f for water.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is correct but Statement II is incorrect

(B) Both Statement I and Statement II are incorrect

(C) Statement I is incorrect but Statement II is correct

(D) Both Statement I and Statement II are correct

Correct Answer: (A) Statement I is correct but Statement II is incorrect

Solution:

Concept:

- For a dilute ideal solution, the molal freezing-point depression constant is $K_f = RT_f^2 M_1/\Delta H_{\text{fus}}$, with solvent molar mass M_1 expressed in kg mol^{-1}.
- At the melting/freezing equilibrium, $\Delta H_{\text{fus}} = T_f \Delta S_{\text{fus}}$. The constant depends on the solvent, not on which dilute non-electrolyte is dissolved.

Step 1: Test Statement I using the fusion relation Substitute the entropy relation into the usual cryoscopic expression:

$$K_f = \frac{RT_f^2 M_1}{T_f \Delta S_{\text{fus}}} = \frac{M_1 R T_f}{\Delta S_{\text{fus}}}.$$

This is exactly the expression in Statement I, with the conventional SI molar-mass units. If molar mass is instead quoted in grams per mole, its numerical value must first be divided by 1000; this is a unit conversion, not a change in the physical expression. Thus Statement I is correct. The surviving single power of temperature results from cancelling the temperature in the latent-heat relation.

Step 2: Compare the constants of the two solvents The standard cryoscopic constants are approximately

$$K_f(\text{benzene}) = 5.12\ \text{K kg mol}^{-1}, \qquad K_f(\text{water}) = 1.86\ \text{K kg mol}^{-1}.$$

Consequently benzene has the larger, rather than the smaller, value. At the same dilute molality a non-electrolyte therefore produces a greater freezing-point depression in benzene than in water. Statement II reverses this known comparison and is incorrect. Options claiming that both statements are true, or that Statement I is false, cannot match these two independent checks.

Step 3: Conclusion Statement I is correct, but Statement II is incorrect. Hence, the correct option is (A).

Quick Tip to Solve: Remember the approximate pair benzene 5.12 and water 1.86, and derive the entropy form of K_f by using $\Delta H = T\Delta S$.

58. The IUPAC name of the compound $HC{\equiv}C{-}CH_2{-}CH(OH){-}CH_2{-}CH{=}CH_2$ is: [4 marks]

(A) 4-Hydroxyhept-6-en-1-yne

(B) Hept-1-en-6-yn-4-ol

(C) Hept-6-en-1-yn-4-ol

(D) 4-Hydroxyhept-1-en-6-yne

Correct Answer: (B) Hept-1-en-6-yn-4-ol

Solution:

Concept:

- The parent chain must contain the principal hydroxyl group and both multiple bonds. An alcohol is named with the suffix -ol, which has priority over a hydroxy prefix here.
- Number first to minimise the principal functional-group locant. If that locant ties, minimise the multiple-bond locants; a remaining double/triple-bond tie is resolved in favour of the double bond.

Step 1: Choose the chain and compare both numberings The displayed structure contains a continuous seven-carbon chain,

$$HC{\equiv}C{-}CH_2{-}CH(OH){-}CH_2{-}CH{=}CH_2.$$

Its parent is therefore hept-. Numbering from the alkyne end places the triple bond at 1, the hydroxyl group at 4 and the double bond at 6. Numbering from the alkene end places the double bond at 1, the hydroxyl group at 4 and the triple bond at 6. Both directions give the principal -ol group the same locant, 4, so the hydroxyl group does not decide the direction.

Step 2: Resolve the tie and assemble the name The multiple-bond locant set is $\{1, 6\}$ in either direction. At this final tie, the double bond receives the lower locant. Choose numbering from the alkene end: ene is at 1 and yne is at 6. In the combined unsaturation suffix, en is written before yn. Add the alcohol suffix at carbon 4 to obtain hept-1-en-6-yn-4-ol. Options (A) and (D) improperly demote the principal alcohol to the prefix hydroxy. Option (C) gives the alkyne the lower locant despite the unresolved equal locant set. It therefore violates the tie-breaking preference for the double bond. The correct name retains all three features: the seven-carbon chain, both unsaturations and the central hydroxyl group.

Step 3: Conclusion The compound is hept-1-en-6-yn-4-ol. Hence, the correct option is (B).

Quick Tip to Solve: For an enynol, give -ol the first numbering priority; use the double-bond preference only after the hydroxyl and multiple-bond locant sets tie.

59. Half life of zero order reaction $A \rightarrow$ product is 1 hour, when initial concentration of reactant is $2.0\ \text{mol L}^{-1}$. The time required to decrease concentration of A from 0.50 to $0.25\ \text{mol L}^{-1}$ is : [4 marks]

(A) 0.5 hour

(B) 15 min

(C) 60 min

(D) 4 hour

Correct Answer: (B) 15 min

Solution:

Concept:

- For a zero-order reaction, $[A]_t = [A]_0 - kt$; concentration falls by a fixed amount per unit time while the zero-order regime holds.
- The half-life is $t_{1/2} = [A]_0/(2k)$, so it depends on the starting concentration. The rate constant has concentration per time units.

Step 1: Determine the rate constant from the stated half-life Use the half-life measured with initial concentration $2.0\ \text{mol L}^{-1}$, not the later concentration interval:

$$k = \frac{[A]_0}{2t_{1/2}} = \frac{2.0}{2 \times 1} = 1.0\ \text{mol L}^{-1}\,\text{h}^{-1}.$$

During this one hour the concentration falls from 2.0 to 1.0, a decrease of one mole per litre. This is consistent with the derived constant rate and provides a direct check on its units and magnitude.

Step 2: Use the concentration change for the requested interval The time between any two concentrations is their difference divided by the same zero-order rate constant. Therefore

$$\Delta t = \frac{0.50 - 0.25}{1.0}\ \text{h} = 0.25\ \text{h} = 15\ \text{min}.$$

Although 0.25 is half of 0.50, this does not mean that the time equals the given one-hour half-life. That half-life belongs to a different initial concentration. A constant half-life is a first-order property, whereas this problem explicitly states zero order. Thirty minutes, sixty minutes and four hours would require removal of respectively 0.50, 1.0 and 4.0 mole per litre at the calculated rate; none equals the requested decrease of 0.25 mole per litre.

Step 3: Conclusion The required time is $15\ \text{min}$. Hence, the correct option is (B).

Quick Tip to Solve: In zero-order kinetics use concentration removed divided by k; never carry a half-life from one initial concentration to another unchanged.

60. The incorrect relationship in the following pairs in relation to ionisation enthalpies is : [4 marks]

(A) $Mn^{2+} < Fe^{2+}$

(B) $Mn^{+} < Mn^{2+}$

(C) $Mn^{+} < Cr^{+}$

(D) $Fe^{2+} < Fe^{3+}$

Correct Answer: (A) $Mn^{2+} < Fe^{2+}$

Solution:

Concept:

- Ionisation enthalpy of a specified ion is the energy required to remove one further electron from that gaseous ion. A stable half-filled $3d^5$ configuration resists removal.
- For transition-metal cations, 4s electrons are removed before 3d electrons. Successive removal from an increasingly positive ion generally requires more energy.

Step 1: Compare the doubly charged manganese and iron ions The configurations are $Mn^{2+} : [Ar]3d^5$ and $Fe^{2+} : [Ar]3d^6$. Removing an electron from manganese(II) destroys the especially stable half-filled d subshell. Removing one from iron(II) produces the stable Fe^{3+} configuration $3d^5$. Hence the next ionisation requires more energy for Mn^{2+} than for Fe^{2+}. The actual ordering is

$$IE(Mn^{2+}) > IE(Fe^{2+}),$$

which is the reverse of the inequality printed in option (A).

Step 2: Check why the remaining comparisons hold For manganese, Mn^{+} has $[Ar]3d^5 4s^1$. Its ionisation removes the remaining 4s electron and produces stable $3d^5$, whereas ionising Mn^{2+} breaks that shell and starts from a higher positive charge. Thus (B) is correct. Chromium(I) is $[Ar]3d^5$, so its next ionisation breaks the half-filled d subshell; manganese(I) instead loses its 4s electron. This supports (C). Finally Fe^{3+} is both more highly charged than Fe^{2+} and half-filled. Its next electron is harder to remove, supporting (D). Only (A) is incorrect.

Step 3: Conclusion The incorrect relationship is $Mn^{2+} < Fe^{2+}$ in option (A). Hence, the correct option is (A).

Quick Tip to Solve: Write the configuration of the ion before and after electron removal: forming d^5 is favoured, while destroying d^5 raises the required ionisation energy.

61. A toxic compound " A " when reacted with NaCN in aqueous acidic medium yields an edible cooking component and food preservative " B ". " B " is converted to " C " by diborane and can be used as an additive to petrol to reduce emission. "C" upon reaction with oleum at $140^\circ C$ yields an inhalable anesthetic " D ". Identify " A ", " B ", " C " & " D ", respectively : [4 marks]

(A) Methanol; acetic acid; ethanol; diethyl ether

(B) Ethanol; acetonitrile; ethylamine; ethylene

(C) Methanol; formaldehyde; methyl chloride; chloroform

(D) Acetaldehyde; 2-hydroxypropanoic acid; propanoic acid; dipropyl ether

Correct Answer: (A) Methanol; acetic acid; ethanol; diethyl ether

Solution:

Concept:

- Acidic cyanide conditions can introduce one carbon through a nitrile; hydrolysis of the nitrile gives a carboxylic acid.
- Diborane reduces a carboxylic acid to its primary alcohol, whereas acid-catalysed intermolecular dehydration of ethanol at $140^\circ C$ gives diethyl ether.

Step 1: Identify the acid and its alcohol The edible cooking ingredient and preservative is acetic acid, CH_3COOH, the acid in vinegar. Reduction by diborane replaces the carboxyl group with a primary alcohol group without changing the carbon skeleton. Therefore B is acetic acid and C is ethanol, CH_3CH_2OH. Ethanol is blended with petrol as an oxygenated fuel component. This immediately distinguishes the proposed sequence from options containing amines or chlorinated compounds.

Step 2: Trace the precursor and final ether A is methanol, a toxic alcohol. The cyanide route introduces an additional carbon through acetonitrile, CH_3CN; acidic hydrolysis gives acetic acid. Thus the relevant carbon sequence is methanol to acetonitrile to acetic acid to ethanol. At the stated temperature, two ethanol molecules undergo acid-catalysed dehydration:

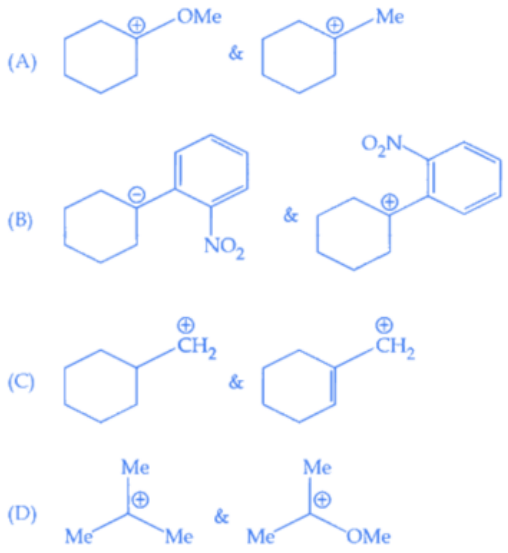
$$2\,CH_3CH_2OH \xrightarrow[140\ ^\circ C]{H^+} CH_3CH_2OCH_2CH_3 + H_2O.$$

The product is diethyl ether, historically used as an inhalation anaesthetic. The acetaldehyde option would initially give a cyanohydrin and then lactic acid, whose reduction does not give propanoic acid; the other sequences likewise do not fit the stated transformations.

Step 3: Conclusion A, B, C and D are methanol, acetic acid, ethanol and diethyl ether, respectively. Hence, the correct option is (A).

Quick Tip to Solve: Identify the familiar ethanol-to-diethyl-ether transformation at $140^\circ C$, then work backwards through carboxylic-acid reduction.

62. In which pairs, the first ion is more stable than the second? [4 marks]



(A) (A) & (B) only

(B) (B) & (D) only

(C) (A) & (C) only

(D) (B) & (C) only

Correct Answer: (A) (A) & (B) only

Solution:

Concept:

- Carbocations are stabilised by electron donation through resonance and hyperconjugation; a directly attached oxygen lone pair is especially effective.
- An electron-withdrawing nitro group stabilises a nearby conjugated carbanion but destabilises a carbocation. Allylic resonance must be checked before comparing simple substitution.

Step 1: Compare pairs A and B In pair (A), the methoxy oxygen is bonded directly to the positively charged carbon. Donation of an oxygen lone pair gives an additional resonance contributor with a carbon–oxygen double bond and the positive charge on oxygen. This provides octet completion at carbon and is more effective than the methyl group in the second ion, which mainly supplies hyperconjugative and inductive donation. Hence the first ion in (A) is more stable. In pair (B), the first ion is a benzylic carbanion adjacent to a nitro-substituted phenyl ring. The nitro group withdraws electron density and helps stabilise the negative charge. The second ion is a carbocation in the corresponding electron-withdrawing environment, so it is less stable. Thus (B) also satisfies the condition.

Step 2: Reject pairs C and D In pair (C), the first exocyclic primary carbocation lacks allylic resonance. The second carbocation is adjacent to the ring double bond and its positive charge is delocalised over an allylic system. The second is therefore more stable, contrary to the required comparison. In pair (D), the first ion is stabilised by methyl groups, but the second has a directly attached methoxy oxygen and gains strong lone-pair resonance stabilisation. Again the second is more stable. Therefore neither

(C) nor (D) can be included; options containing these pairs fail the individual stability tests.

Step 3: Conclusion The first ion is more stable only in pairs A and B. Hence, the correct option is (A).

Quick Tip to Solve: Check charge sign first: a nitro group helps a carbanion, while an attached methoxy oxygen strongly stabilises a carbocation by lone-pair donation.

63. ' X ' is the number of electrons in t_{2g} orbitals of the most stable complex ion among $[\mathrm{Fe}(\mathrm{NH_3})_6]^{3+}$, $[\mathrm{FeCl_6}]^{3-}$, $[\mathrm{Fe}(\mathrm{C_2O_4})_3]^{3-}$ and $[\mathrm{Fe}(\mathrm{H_2O})_6]^{3+}$. The nature of oxide of vanadium of the type $\mathrm{V_2O_X}$ is : [4 marks]

(A) Amphoteric

(B) Acidic

(C) Basic

(D) Neutral

Correct Answer: (A) Amphoteric

Solution:

Concept:

- Iron has atomic number 26; removal of three electrons gives $\mathrm{Fe^{3+}}$ a $3d^5$ configuration. All four listed complexes contain iron in oxidation state +3.
- For an octahedral low-spin d^5 configuration, the five electrons occupy the lower t_{2g} orbitals: $t_{2g}^5 e_g^0$. Vanadium(V) oxide is amphoteric, although its acidic character predominates.

Step 1: Determine the lower-orbital electron count In the crystal-field comparison intended here, ammonia is the strongest-field ligand among the listed ligands: chloride, oxalate, water and ammonia. The larger octahedral splitting favours occupation of the lower-energy set with pairing rather than promotion into the upper set. For the low-spin ferric ammine configuration the occupation is

$$\mathrm{Fe^{3+}} : d^5, \qquad t_{2g}^5 e_g^0.$$

Thus the number of electrons specified in the question is $X = 5$. The important count is the total population of the three lower orbitals, not the number of unpaired electrons; a low-spin d^5 ion has one unpaired electron but five electrons in t_{2g}.

Step 2: Classify the corresponding oxide Substituting $X = 5$ gives $\mathrm{V_2O_5}$. Its vanadium oxidation state follows from $2z + 5(-2) = 0$, so $z = +5$. The NCERT classification is amphoteric, with predominantly acidic behaviour: it reacts with bases to form vanadates, and with acids to give vanadium(V) oxocation salts. For example, its reaction with alkali can be represented as

$$\mathrm{V_2O_5 + 6\,OH^- \longrightarrow 2\,{VO_4}^{3-} + 3\,H_2O}.$$

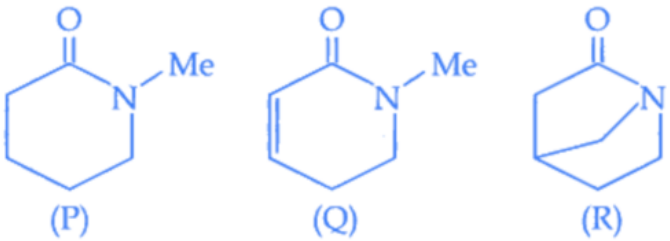
Merely calling every high-oxidation-state transition-metal oxide acidic would miss the specified amphoteric classification. It is neither a purely basic oxide nor a neutral oxide.

Step 3: Conclusion $X = 5$, so the oxide is $\mathrm{V_2O_5}$, which is amphoteric. Hence, the correct option is

(A).

Quick Tip to Solve: Count all electrons in t_{2g}, then identify the oxide; remember that V_2O_5 is classified as amphoteric despite predominantly acidic behaviour.

64. The correct order of basicity for the following molecules is : [4 marks]



(A) R > P > Q

(B) P > Q > R

(C) R > Q > P

(D) Q > P > R

Correct Answer: (C) R > Q > P

Solution:

Concept:

- Basicity increases when the nitrogen lone pair is more available for protonation. Delocalisation of that lone pair into a carbonyl group reduces nitrogen basicity.
- Amide resonance needs favourable orbital overlap and near-planarity. A bridged framework can inhibit this overlap, while additional conjugation elsewhere can compete with nitrogen donation.

Step 1: Compare the bridged compound with the lactams P is a normal saturated cyclic amide. Its nitrogen lone pair can overlap with the carbonyl system, giving the familiar amide resonance contributor with positive charge on nitrogen and negative charge on oxygen. This makes the lone pair less available for protonation. In R the nitrogen is constrained by the additional bridge. The framework prevents the nitrogen from adopting the optimal planar arrangement for amide conjugation. Inhibition of resonance localises its lone pair, so R is more basic than the ordinary lactams P and Q. Therefore any option placing R last is inconsistent with the structural constraint.

Step 2: Distinguish Q from P Q contains a carbon–carbon double bond conjugated with the carbonyl group. This additional conjugated pathway competes with donation from nitrogen into the same carbonyl system. Relative to P, the nitrogen lone pair in Q is therefore less strongly tied up in the simple amide resonance interaction and is more available for protonation. Consequently Q is more basic than P, while the strong geometric inhibition of resonance makes R the most basic member. The comparison concerns lone-pair availability, rather than merely counting methyl substituents: P and Q both have an N-methyl group, yet their conjugation differs. The order R greater than P greater than Q reverses this second comparison; the other two orders incorrectly put the bridged nitrogen last.

Step 3: Conclusion The order of basicity is R > Q > P. Hence, the correct option is (C).

Quick Tip to Solve: For cyclic amides, inspect both the geometry needed for nitrogen–carbonyl overlap and any competing conjugation before applying an alkyl-group rule.

65. Consider the given data :

(a) $\mathrm{HCl(g) + 10H_2O(l) \rightarrow HCl.10H_2O \Delta H = -69.01\ kJ\ mol^{-1}}$

(b) $\mathrm{HCl(g) + 40H_2O(l) \rightarrow HCl.40H_2O \Delta H = -72.79\ kJ\ mol^{-1}}$

Choose the correct statement : [4 marks]

(A) The heat of dilution for the $\mathrm{HCl\,(HCl.10H_2O\ to\ HCl.40H_2O)}$ is $3.78\ \mathrm{kJ\ mol^{-1}}$.

(B) Dissolution of gas in water is an endothermic process.

(C) The heat of solution depends on the amount of solvent.

(D) The heat of formation of HCl solution is represented by both (a) and (b).

Correct Answer: (C) The heat of solution depends on the amount of solvent.

Solution:

Concept:

- Hess's law allows subtraction of solution enthalpies to obtain the enthalpy change on dilution between the specified concentrations.
- Enthalpy of solution refers to dissolving solute in a specified amount of solvent; it need not be independent of the final concentration.

Step 1: Calculate the dilution enthalpy Reverse reaction (a) and add it to reaction (b). Gaseous hydrogen chloride cancels, leaving the solution containing ten moles of water plus thirty additional moles of water to produce the solution containing forty moles of water. Hence

$$\mathrm{HCl \cdot 10\,H_2O + 30\,H_2O \longrightarrow HCl \cdot 40\,H_2O.}$$

The corresponding enthalpy change is

$$\Delta H_{\mathrm{dil}} = (-72.79) - (-69.01) = -3.78\ \mathrm{kJ\,mol^{-1}}.$$

The negative sign means heat is released on dilution. Option (A) gives a positive value for the thermodynamic heat of dilution and therefore has the wrong sign.

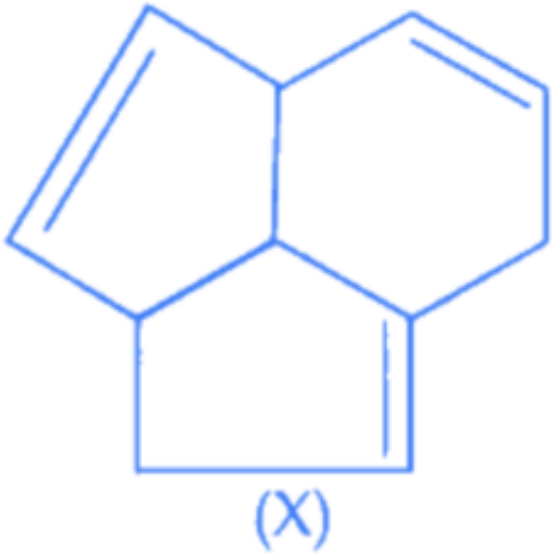
Step 2: Test the remaining statements Both given solution processes have negative enthalpy changes, so they describe exothermic dissolution, not the endothermic process asserted in (B). The two different negative values are obtained for different amounts of water, which directly demonstrates that the enthalpy of solution depends on the amount of solvent, as stated in (C). Finally, a standard enthalpy of formation describes formation of a substance from its constituent elements in their standard states. Here the reactants already include the compounds hydrogen chloride and water, so these are enthalpies of solution, not the formation enthalpies described in (D). All alternatives can thus be decided from the supplied data and the definitions without assuming any extra experimental values.

Step 3: Conclusion The heat of solution depends on the amount of solvent; dilution here has $\Delta H = -3.78\ \mathrm{kJ\,mol^{-1}}$. Hence, the correct option is (C).

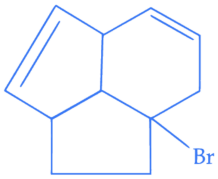
Quick Tip to Solve: For dilution, subtract the initial solution enthalpy from the final one and retain its sign; a positive heat-release magnitude is not a positive enthalpy change.

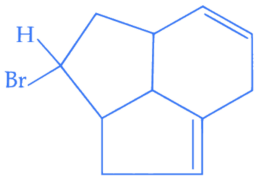
66. Consider the following molecule(X).

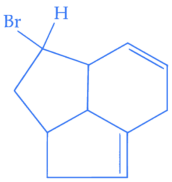
The major product formed when the given molecule (X) is treated with $\mathrm{HBr(1eq)}$ is : [4 marks]

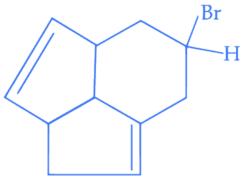


(X)

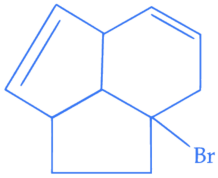
(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- Without a peroxide condition, addition of HBr to an alkene follows the ionic electrophilic-addition pathway, through a carbocation.
- With one equivalent and several double bonds, compare the carbocations obtainable by protonation at each site. The preferred pathway forms the more substituted, more stable carbocation.

Step 1: Locate the preferred double bond The molecule contains three distinct double bonds. The lower double bond includes a ring-junction carbon already bonded to three carbon atoms. Protonation at the other, less substituted end of that double bond places the positive charge on this junction carbon. The resulting intermediate is a tertiary carbocation, stabilised by donation from its neighbouring carbon framework. In contrast, addition at the double bond in the left ring or at the upper double bond in the right ring initially gives less substituted carbocations. These pathways do not offer the same tertiary-centre stabilisation. Thus the lower, more substituted alkene is the preferred site for reaction with the limited amount of reagent.

Step 2: Capture the carbocation with bromide After protonation, bromide attacks the positively charged ring-junction carbon. The lower double bond becomes a single bond, hydrogen has been added to its less substituted carbon, and bromine is attached to the junction carbon. The other two double bonds are retained because only one equivalent of hydrogen bromide is supplied. This is precisely the structure drawn in option (A). Options (B) and (C) instead consume the double bond in the left ring and put bromine on one of its two alkene carbons; option (D) consumes the upper double bond in the right ring. Those structures correspond to the less favourable initial protonation pathways rather than formation of the tertiary carbocation at the lower junction.

Step 3: Conclusion The major product has bromine on the junction carbon of the lower double bond, with the other two double bonds unchanged. Hence, the correct option is (A).

Quick Tip to Solve: When one equivalent of HBr meets several isolated alkenes, draw the carbocation from each possible protonation and compare substitution before choosing the product.

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67. The correct order of $[FeF_6]^{3-}$, $[CoF_6]^{3-}$, $[Ni(CO)_4]$ and $[Ni(CN)_4]^{2-}$ complex species based on the number of unpaired electrons present is: [4 marks]

(A) $[CoF_6]^{3-} > [FeF_6]^{3-} > [Ni(CO)_4] > [Ni(CN)_4]^{2-}$

(B) $[FeF_6]^{3-} > [CoF_6]^{3-} > [Ni(CN)_4]^{2-} > [Ni(CO)_4]$

(C) $[FeF_6]^{3-} > [CoF_6]^{3-} > [Ni(CN)_4]^{2-} = [Ni(CO)_4]$

(D) $[Ni(CN)_4]^{2-} > [FeF_6]^{3-} > [CoF_6]^{3-} > [Ni(CO)_4]$

Correct Answer: (C) $[FeF_6]^{3-} > [CoF_6]^{3-} > [Ni(CN)_4]^{2-} = [Ni(CO)_4]$

Solution:

Concept:

- The oxidation state of the metal fixes its d-electron count. Each fluoride or cyanide ligand has charge -1, whereas carbon monoxide is neutral.

- The number of unpaired electrons depends on both the d configuration and the ligand field. Fluoride is a weak-field ligand; cyanide and carbon monoxide favour paired arrangements.

Step 1: Determine the metal configurations In $[FeF_6]^{3-}$, the metal is Fe^{3+}, which has a $3d^5$ configuration. In $[CoF_6]^{3-}$, cobalt is Co^{3+}, with $3d^6$. The nickel centres are different: neutral $Ni(CO)_4$ contains nickel in oxidation state zero, whereas $[Ni(CN)_4]^{2-}$ contains Ni^{2+}, with a $3d^8$ configuration. Oxidation states must therefore be established before comparing the species.

Step 2: Count the unpaired electrons With weak-field fluoride, the octahedral iron(III) species is high spin: $t_{2g}^3 e_g^2$, giving five unpaired electrons. The cobalt(III) fluoride species is also high spin: $t_{2g}^4 e_g^2$, giving four unpaired electrons. Nickel tetracarbonyl is tetrahedral and diamagnetic, with all electrons paired. Strong-field cyanide produces square-planar $[Ni(CN)_4]^{2-}$, which is also diamagnetic. Thus their unpaired-electron counts, in the order named in the question, are $5, 4, 0, 0$.

Step 3: Compare all four options The iron complex must precede the cobalt complex, eliminating (A). Both nickel species have zero unpaired electrons and must be equal, eliminating the strict inequality in (B). Option (D) incorrectly puts diamagnetic nickel cyanide above the iron complex. Only (C) gives the sequence $5 > 4 > 0 = 0$.

Step 4: Conclusion The order is $[FeF_6]^{3-} > [CoF_6]^{3-} > [Ni(CN)_4]^{2-} = Ni(CO)_4$, based on $5 > 4 > 0 = 0$ unpaired electrons. Hence, the correct option is (C).

Quick Tip to Solve: Find oxidation state and d count first; then use ligand strength and geometry. Two diamagnetic species must tie in an unpaired-electron ranking.

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68. Given below are two statements :

Statement (I) : Alcohols are formed when alkyl chlorides are treated with aqueous potassium hydroxide by elimination reaction.

Statement (II) : In alcoholic potassium hydroxide, alkyl chlorides form alkenes by abstracting the hydrogen from the β-carbon.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are incorrect

(B) Both Statement I and Statement II are correct

(C) Statement I is incorrect but Statement II is correct

(D) Statement I is correct but Statement II is incorrect

Correct Answer: (C) Statement I is incorrect but Statement II is correct

Solution:

Concept:

- Aqueous potassium hydroxide supplies hydroxide ions that can replace the halide group by nucleophilic substitution, producing an alcohol.
- Alcoholic potassium hydroxide favours elimination of a hydrogen from the beta-carbon along with halide loss from the alpha-carbon, forming a carbon-carbon double bond.

Step 1: Test Statement I Formation of an alcohol from an alkyl chloride and aqueous potassium hydroxide involves replacement of Cl by OH. For example,

$$CH_3CH_2Cl + KOH\,(aq) \longrightarrow CH_3CH_2OH + KCl.$$

The carbon skeleton remains saturated in this substitution. Statement I names the usual product correctly, but incorrectly calls the reaction elimination. Since the mechanism or reaction type is part of the statement, the statement as a whole is incorrect.

Step 2: Test Statement II In alcoholic potassium hydroxide, the reagent acts predominantly as a base and removes a beta-hydrogen. A double bond forms between the alpha-carbon bearing chlorine and the adjacent beta-carbon. For chloroethane, the transformation is

$$CH_3CH_2Cl + KOH\,(alc) \longrightarrow CH_2{=}CH_2 + KCl + H_2O.$$

This is beta-elimination or dehydrohalogenation: the organic compound loses the elements of hydrogen chloride. Thus Statement II describes the expected elimination correctly.

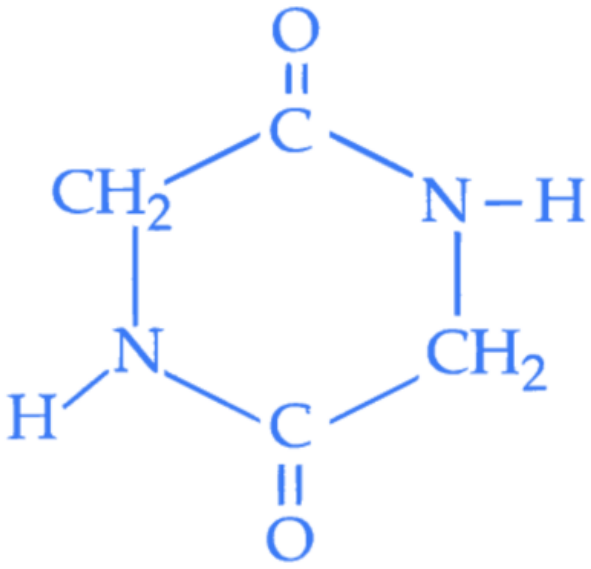
Step 3: Select the combination of truth values The required pair of truth values is false for Statement I and true for Statement II. Options (A) and (D) incorrectly reject Statement II. Option (B) incorrectly accepts the elimination description of alcohol formation. The distinction is the role of hydroxide as a nucleophile in aqueous medium and as a base under the usual alcoholic elimination conditions.

Step 4: Conclusion Statement I is incorrect, whereas Statement II is correct. Hence, the correct option is (C).

Quick Tip to Solve: Look at what changes: replacement of Cl by OH is substitution; loss of beta-H and Cl with formation of a double bond is elimination.

69. A dipeptide, " x " on complete hydrolysis gives " y " and " z ". " y " on treatment with aq. HNO_2 produces lactic acid. On the other hand " z " on heating gives the following cyclic molecule.

Based on the information given, the dipeptide X is : [4 marks]



(A) alanine-alanine

(B) valine-glycine

(C) valine-leucine

(D) alanine-glycine

Correct Answer: (D) alanine-glycine

Solution:

Concept:

- Complete hydrolysis of a dipeptide cleaves its peptide linkage and yields the two constituent amino acids.
- Nitrous acid replaces the amino group of a simple alpha-amino acid by a hydroxyl group. Heating two amino-acid molecules can form a cyclic dipeptide, a diketopiperazine.

Step 1: Identify the amino acid giving lactic acid Lactic acid is $CH_3CH(OH)COOH$. Replacing its hydroxyl group at the alpha-carbon by an amino group gives $CH_3CH(NH_2)COOH$, which is alanine. Its reaction with nitrous acid may be written

$$CH_3CH(NH_2)COOH + HNO_2 \longrightarrow CH_3CH(OH)COOH + N_2 + H_2O.$$

Thus the amino acid labelled y is alanine; its methyl side chain is retained during deamination.

Step 2: Read the cyclic product to identify the second amino acid The displayed six-membered ring contains two carbonyl carbons, two nitrogen atoms carrying hydrogen, and two methylene groups. These are the repeating units $-NH-CH_2-CO-$ of glycine. Two glycine molecules, NH_2CH_2COOH, condense on heating with loss of two water molecules to produce this cyclic dipeptide. If the precursor were alanine, the corresponding ring carbons would each carry a methyl substituent rather than being

the shown CH_2 groups. Hence z is glycine.

Step 3: Match the hydrolysis products to the options The constituents must be alanine and glycine. Alanine-alanine lacks glycine and would not give the unsubstituted cyclic product shown. Both valine-containing options fail the lactic-acid test, since valine has a larger side chain that would be retained in its hydroxy-acid product. Among the listed dipeptides, only alanine-glycine contains both identified amino acids. Hydrolysis establishes the constituents; the supplied options select the named dipeptide.

Step 4: Conclusion The amino acids are y = alanine and z = glycine, so the listed dipeptide is alanine-glycine. Hence, the correct option is (D).

Quick Tip to Solve: Keep the side chain unchanged in nitrous-acid deamination, and inspect whether cyclic-dipeptide ring carbons are CH2 or substituted CH groups.

70. The elements of Group 13 with highest and lowest first ionisation enthalpies are respectively :

[4 marks]

(A) B & Tl

(B) Tl & B

(C) B & In

(D) B & Ga

Correct Answer: (C) B & In

Solution:

Concept:

- First ionisation enthalpy is the energy required to remove the most loosely held electron from an isolated gaseous atom.
- Atomic size generally lowers ionisation enthalpy down a group, but incomplete shielding by intervening d and f electrons produces important irregularities in Group 13.

Step 1: Locate the highest first ionisation enthalpy Boron is the smallest of the listed Group 13 atoms, with its outer electron in the second shell. This electron experiences relatively strong nuclear attraction and is more difficult to remove than the outer electron of aluminium, gallium, indium or thallium. Boron therefore has the highest first ionisation enthalpy among the ordinary Group 13 elements considered here. This immediately excludes option (B), which assigns the highest value to thallium.

Step 2: Account for the irregular trend down the group The trend is not a smooth decrease from boron to thallium. Gallium has a slightly higher first ionisation enthalpy than aluminium because its intervening d electrons shield the nuclear charge poorly. Indium has a larger valence shell and its first ionisation enthalpy is lower than those of aluminium and gallium. Thallium rises again relative to indium; poor shielding by intervening d and f electrons and contraction strengthen the attraction on the outer electron. Hence one cannot choose thallium merely because it is lowest in the group.

Step 3: Choose the correct extreme pair The approximate first ionisation enthalpies, in $\mathrm{kJ\,mol^{-1}}$, are

boron 801, aluminium 578, gallium 579, indium 558 and thallium 589. They confirm that boron is the maximum and indium the minimum. Options (A) and (D) use thallium and gallium respectively as the minimum, contrary to the irregular trend. The requested order is highest first, then lowest.

Step 4: Conclusion Boron has the highest and indium the lowest first ionisation enthalpy in Group 13. Hence, the correct option is (C).

Quick Tip to Solve: Do not extrapolate a monotonic decrease in Group 13: remember the small Ga rise and the larger Tl rise, leaving In as the minimum.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. Sea water, which can be considered as a 6 molar $(6M)$ solution of NaCl , has a density of $2\ \text{g mL}^{-1}$. The concentration of dissolved oxygen (O_2) in sea water is 5.8 ppm . Then the concentration of dissolved oxygen (O_2) in sea water, is $x \times 10^{-4}$ m.

$x =$ _____. (Nearest integer)

Given: Molar mass of NaCl is $58.5\ \text{g mol}^{-1}$

Molar mass of O_2 is $32\ \text{g mol}^{-1}$ [4 marks]

Solution:

Concept:

- Mass-based parts per million is the mass of a solute divided by the mass of solution, multiplied by 10^6. Thus 5.8 ppm means 5.8 mg of oxygen per kg of sea-water solution.
- Molality is moles of the chosen solute per kilogram of solvent, not per kilogram of solution. The dissolved sodium chloride must therefore be removed from the solution mass to find the water mass.

Step 1: Choose one litre of sea water A one-litre basis makes the given molarity directly useful. From the density, the total mass is

$$m_{\text{solution}} = 1000\,\text{mL} \times 2\,\text{g mL}^{-1} = 2000\,\text{g}.$$

A 6 molar sodium chloride solution contains 6 moles of salt in this litre, so

$$m_{\text{NaCl}} = 6 \times 58.5 = 351\,\text{g}.$$

The solvent mass is therefore approximately $2000 - 351 = 1649\,\text{g} = 1.649\,\text{kg}$. The oxygen mass is tiny on this scale.

Step 2: Convert the oxygen concentration to moles Apply ppm to the total solution mass, not to the water mass:

$$m_{O_2} = \frac{5.8}{10^6} \times 2000 = 0.0116\,\text{g}.$$

Its number of moles is

$$n_{O_2} = \frac{0.0116}{32} = 3.625 \times 10^{-4}\,\text{mol}.$$

The factor of 32 is the molar mass of molecular oxygen; using 16 would double the calculated concentration.

Step 3: Calculate molality and round the coefficient Using the solvent mass in kilograms,

$$m_{O_2}\ (\text{molality}) = \frac{3.625 \times 10^{-4}}{1.649} = 2.1983 \times 10^{-4}\,\text{mol kg}^{-1}.$$

Subtracting the additional $0.0116\,\text{g}$ of oxygen from the solvent mass changes no reported integer. Comparing with $x \times 10^{-4}\,\text{m}$ gives $x \approx 2.1983$. Round this coefficient, rather than rounding the small molality itself to an integer.

Step 4: Conclusion The oxygen molality is approximately $2.1983 \times 10^{-4}\,\text{m}$. Therefore the requested nearest-integer value is $\boxed{x = 2}$.

Quick Tip to Solve: For ppm-to-molality conversion in salt water, use ppm with the solution mass but divide moles by the mass of water after subtracting dissolved salt.

72. The amount of calcium oxide produced on heating 150 kg limestone (75% pure) is _____ kg. (Nearest integer)

Given: Molar mass (in gmol^{-1}) of Ca-40, O-16, C-12 [4 marks]

Solution:

Concept:

- On heating, calcium carbonate decomposes to calcium oxide and carbon dioxide: $CaCO_3 \longrightarrow CaO + CO_2$. The mole ratio of calcium carbonate to calcium oxide is 1:1.
- Percentage purity gives the mass fraction of the reacting compound in an impure sample. Apply this correction before using the stoichiometric molar-mass ratio.

Step 1: Find the mass of pure calcium carbonate The 150 kg sample is only 75 percent pure limestone, so not all of its mass can produce calcium oxide. Taking the stated purity as the calcium carbonate content,

$$m_{CaCO_3} = 150 \times \frac{75}{100} = 112.5\,\text{kg}.$$

The remaining $37.5\,\text{kg}$ is impurity and does not contribute to the calcium oxide yield in the stated reaction. This avoids treating the full sample as calcium carbonate.

Step 2: Establish the mass ratio from the balanced reaction Using the given atomic masses,

$$M_{CaCO_3} = 40 + 12 + 3(16) = 100\,\text{g mol}^{-1},$$

whereas

$$M_{CaO} = 40 + 16 = 56\,\text{g mol}^{-1}.$$

Since one mole of carbonate produces one mole of oxide, every 100 mass units of pure carbonate gives 56 mass units of calcium oxide. The remaining 44 mass units leave as carbon dioxide. The ratio is dimensionless, so it may be applied directly to masses in kilograms.

Step 3: Calculate and check the product mass The calcium oxide yield is

$$m_{\mathrm{CaO}} = 112.5 \times \frac{56}{100} = 63.0\,\mathrm{kg}.$$

As a mass-balance check, the evolved carbon dioxide has mass $112.5 \times 44/100 = 49.5\,\mathrm{kg}$, and $63.0 + 49.5 = 112.5\,\mathrm{kg}$, equal to the pure carbonate consumed. The product mass is less than the carbonate mass because a gaseous product escapes. No extra yield factor is specified, so complete decomposition is used.

Step 4: Conclusion The amount of calcium oxide produced is $\boxed{63\,\mathrm{kg}}$, so the nearest-integer entry is 63.

Quick Tip to Solve: Multiply sample mass by the purity fraction first, then by $M_{\mathrm{CaO}}/M_{\mathrm{CaCO_3}} = 56/100$; do not apply purity twice.

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73. A metal complex with a formula $\mathrm{MCl_4 \cdot 3NH_3}$ is involved in $\mathrm{sp^3\ d^2}$ hybridisation. It upon reaction with excess of $\mathrm{AgNO_3}$ solution gives ' x ' moles of AgCl . Consider ' x ' is equal to the number of lone pairs of electron present in central atom of $\mathrm{BrF_5}$. Then the number of geometrical isomers exhibited by the complex is _____. [4 marks]

Solution:

Concept:

- The $\mathrm{sp^3d^2}$ hybridisation corresponds to coordination number six and an octahedral complex. Each ammonia molecule and each coordinated chloride occupies one coordination position.
- Chloride ions outside the coordination sphere precipitate as AgCl with excess $\mathrm{AgNO_3}$. Coordinated chloride ligands are not counted as free chloride ions in this test.
- In an octahedral complex of the form $[\mathrm{MA_3B_3}]$, geometrical isomerism gives facial and meridional arrangements, distinguished by the relative positions of the three identical ligands.

Step 1: Count the lone pairs on bromine Bromine has seven valence electrons. In $\mathrm{BrF_5}$, it forms five single bonds with fluorine, leaving two electrons as one lone pair on the central atom. Thus the specified value is

$$x = \frac{7-5}{2} = 1.$$

Although fluorine atoms also have lone pairs, the question explicitly asks for those on the central atom, so they are not included.

Step 2: Determine the coordination sphere Formation of one mole of AgCl per mole of complex means that one of the four chlorides lies outside the coordination sphere. The remaining three chlorides, together with the three ammonia ligands, give the required six-coordinate species:

$$[\mathrm{MCl_3(NH_3)_3}]\mathrm{Cl}.$$

The coordination entity is therefore of the octahedral $\mathrm{MA_3B_3}$ type, with $\mathrm{Cl^-}$ and $\mathrm{NH_3}$ as the two ligand types.

Step 3: Enumerate the distinct arrangements In the facial, or fac, isomer, the three chloride ligands occupy the vertices of one triangular face; each chloride pair is cis. In the meridional, or mer, isomer, the three chlorides lie along a meridian of the octahedron, with one pair trans and the other pairs cis. These exhaust the distinct geometrical arrangements: rotation cannot turn an all-cis chloride trio into one containing a trans pair.

Step 4: Conclusion The complex is $[MCl_3(NH_3)_3]Cl$, which exhibits fac and mer geometrical isomers. Hence the required number is $\boxed{2}$.

Quick Tip to Solve: First use the silver nitrate precipitate to locate outer-sphere chloride; an octahedral three-plus-three ligand distribution then immediately suggests fac and mer isomers.

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74. The molar conductance of an infinitely dilute solution of ammonium chloride was found to be $185\ \text{S cm}^2\ \text{mol}^{-1}$ and the ionic conductance of hydroxyl and chloride ions are 170 and $70\ \text{S cm}^2\ \text{mol}^{-1}$, respectively. If molar conductance of 0.02 M solution of ammonium hydroxide is $85.5\ \text{S cm}^2\ \text{mol}^{-1}$, its degree of dissociation is given by $x \times 10^{-1}$. The value of x is _____ . (Nearest integer) [4 marks]

Solution:

Concept:

- Kohlrausch's law states that the limiting molar conductivity of an electrolyte is the sum of the limiting ionic conductivities of its constituent ions, with stoichiometric coefficients included.
- For a weak electrolyte, the degree of dissociation is $\alpha = \Lambda_m/\Lambda_m^\circ$, where Λ_m is its molar conductivity at the given concentration and Λ_m° is its limiting molar conductivity.

Step 1: Extract the ammonium ionic contribution At infinite dilution, ammonium chloride contributes one ammonium ion and one chloride ion per formula unit. Applying independent ionic migration gives

$$\Lambda_m^\circ(NH_4Cl) = \lambda^\circ(NH_4{}^+) + \lambda^\circ(Cl^-).$$

Thus, using the supplied values in $\text{S cm}^2\,\text{mol}^{-1}$,

$$\lambda^\circ(NH_4{}^+) = 185 - 70 = 115.$$

Subtracting chloride removes only its contribution; the hydroxide value is used in the next step.

Step 2: Construct the limiting conductivity of the weak base Ammonium hydroxide gives ammonium and hydroxide ions. Their limiting ionic contributions add, so

$$\Lambda_m^\circ(NH_4OH) = 115 + 170 = 285\ \text{S cm}^2\,\text{mol}^{-1}.$$

The limiting value cannot be replaced by the ammonium chloride value, because chloride and hydroxide have different ionic conductivities. This is why both ionic data are needed.

Step 3: Calculate the degree of dissociation and identify the coefficient The measured molar conductivity at $0.02\ \text{M}$ already includes the concentration dependence. Divide it directly by the

limiting value:

$$\alpha = \frac{85.5}{285} = 0.300 = 3.00 \times 10^{-1}.$$

This is a dimensionless fraction, equivalent to thirty percent dissociation. An additional multiplication by the concentration would be inappropriate, because the question gives molar conductivity rather than specific conductivity.

Step 4: Conclusion The degree of dissociation is $0.300 = 3 \times 10^{-1}$. Therefore the requested coefficient, to the nearest integer, is $\boxed{x = 3}$.

Quick Tip to Solve: Replace the unwanted anion contribution in the known strong electrolyte to obtain the weak electrolyte's limiting conductivity, then take the measured-to-limiting ratio.

75. $x$mg **of** $\mathrm{Mg(OH)_2}$(**molar mass** $= 58$) **is required to be dissolved in 1.0 L of water to produce a pH of 10.0 at 298 K . The value of** x **is _____ mg. (Nearest integer)**

(Given : $\mathrm{Mg(OH)_2}$ **is assumed to dissociate completely in** $\mathrm{H_2O}$ **]** [4 marks]

Solution:

Concept:

- At 298 K, $\mathrm{pH} + \mathrm{pOH} = 14.00$, and the hydroxide concentration follows $[\mathrm{OH^-}] = 10^{-\mathrm{pOH}}\ \mathrm{mol\,L^{-1}}$.
- Under the stated complete-dissociation assumption, one mole of $\mathrm{Mg(OH)_2}$ yields two moles of hydroxide ions. The required amount of solute therefore equals half the required amount of hydroxide.
- Convert concentration to moles using $n = cV$, then convert moles to mass using the supplied molar mass $58\ \mathrm{g\,mol^{-1}}$. Finally use $1\ \mathrm{g} = 1000\ \mathrm{mg}$.

Step 1: Determine the required hydroxide concentration The target pH is 10.0 at 298 K. Hence

$$\mathrm{pOH} = 14.0 - 10.0 = 4.0, \qquad [\mathrm{OH^-}] = 10^{-4}\ \mathrm{mol\,L^{-1}}.$$

This is the hydroxide concentration, not the concentration of magnesium hydroxide formula units. The hydrogen-ion concentration is only $10^{-10}\ \mathrm{mol\,L^{-1}}$, so the contribution from water is negligible at the requested integer precision.

Step 2: Use the dissociation stoichiometry The question explicitly assumes complete dissociation:

$$\mathrm{Mg(OH)_2} \longrightarrow \mathrm{Mg^{2+}} + 2\,\mathrm{OH^-}.$$

Therefore the concentration of dissolved solute required is

$$c(\mathrm{Mg(OH)_2}) = \frac{10^{-4}}{2} = 5.0 \times 10^{-5}\ \mathrm{mol\,L^{-1}}.$$

In 1.0 L of solution this corresponds to 5.0×10^{-5} mol. No solubility-product calculation is needed under the assumption given in the question.

Step 3: Convert the amount to milligrams Multiplying by the given molar mass gives

$$m = (5.0 \times 10^{-5})(58) = 2.9 \times 10^{-3}\ \text{g} = 2.9\ \text{mg}.$$

The requested answer is the nearest integer in milligrams, so rounding is performed only after the mass conversion. Rounding the small molar amount at an earlier stage would destroy the required precision.

Step 4: Conclusion The required mass is $2.9\ \text{mg}$, which rounds to $\boxed{x = 3}$, corresponding to $3\ \text{mg}$ to the nearest integer.

Quick Tip to Solve: After converting pH to hydroxide concentration, divide by two for magnesium hydroxide before multiplying by the volume and molar mass.