JEE Main 2025 Paper 1 (B.E./B.Tech) Apr 7 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 7 April 2025, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. From a group of 7 batsmen and 6 bowlers, 10 players are to be chosen for a team, which should include atleast 4 batsmen and atleast 4 bowlers. One batsmen and one bowler who are captain and vice-captain respectively of the team should be included. Then the total number of ways such a selection can be made, is [4 marks]

(A) 145

(B) 165

(C) 155

(D) 135

Correct Answer: (C) 155

Solution:

Concept:

- A team is an unordered selection, so use combinations: $\binom{n}{r}$ counts the ways to select r people from n distinct people.
- The specified captain and vice-captain are compulsory members. Select the remaining eight players from six batsmen and five bowlers; do not select the officers again.

Step 1: List the allowed team compositions The team has ten players and must contain at least four of each kind. Therefore its possible numbers of batsmen and bowlers are $(6,4),(5,5),(4,6)$. These cases are mutually exclusive. Since one member of each kind is already fixed, the corresponding remaining selections are $(5,3),(4,4),(3,5)$. A composition such as $(7,3)$ is forbidden by the minimum number of bowlers.

Step 2: Count each case and add Choose independently within the two groups and multiply within a case. Then add across the three disjoint cases:

$$\begin{aligned} N &= \binom{6}{5}\binom{5}{3} + \binom{6}{4}\binom{5}{4} + \binom{6}{3}\binom{5}{5} \\ &= 6(10) + 15(5) + 20(1) \\ &= 60 + 75 + 20 = 155. \end{aligned}$$

There is no extra factor for assigning captain or vice-captain: the problem has already specified those two players and their roles. Nor is there a factorial for arranging the ten selected players, because only team membership is being selected. The other offered counts 145, 165 and 135 do not equal this exhaustive count.

Step 3: Conclusion The total number of admissible teams is 155. Hence, the correct option is (C).

Quick Tip to Solve: Include the two fixed players first, then apply the minimum requirements to the remaining selection; this prevents counting the officers twice.

2. Let C_1 be the circle in the third quadrant of radius 3 , that touches both coordinate axes. Let C_2 be the circle with centre $(1,3)$ that touches C_1 externally at the point (α,β). If $(\beta-\alpha)^2=\frac{m}{n}$, $\gcd(m,n)=1$, then $m+n$ is equal to [4 marks]

(A) 22

(B) 13

(C) 9

(D) 31

Correct Answer: (A) 22

Solution:

Concept:

- A circle of radius 3 touching both coordinate axes has centre at distance 3 from each axis. The third quadrant fixes both centre coordinates as negative.
- The point of external contact lies on the segment joining the centres. Starting at the first centre, travel one first-circle radius toward the second centre.

Step 1: Find the centres and their separation Let the centres be O_1 and O_2. The quadrant and the axis tangencies give $O_1=(-3,-3)$, while $O_2=(1,3)$. Thus

$$\overrightarrow{O_1O_2}=(4,6), \qquad d=|O_1O_2|=\sqrt{4^2+6^2}=2\sqrt{13}.$$

External tangency would give the second radius as $d-3>0$. Its exact value is unnecessary, because the contact point is exactly three units from O_1 in the direction of O_2.

Step 2: Locate the contact point and reduce the fraction Multiply the unit vector along the line of centres by the first radius:

$$(\alpha,\beta)=(-3,-3)+\frac{3}{2\sqrt{13}}(4,6).$$

Consequently $\alpha=-3+6/\sqrt{13}$ and $\beta=-3+9/\sqrt{13}$, so

$$\beta-\alpha=\frac{3}{\sqrt{13}}, \qquad (\beta-\alpha)^2=\frac{9}{13}.$$

The fraction is already in lowest terms, as 9 and 13 are coprime. Hence $m=9$, $n=13$. Option C gives only the numerator, and options B and D do not give the required numerator-plus-denominator of the reduced fraction.

Step 3: Conclusion We obtain $m+n=9+13=22$. Hence, the correct option is (A).

Quick Tip to Solve: For externally tangent circles, locate the contact point by moving one radius from a centre along the unit vector toward the other centre.

3. Let P be the parabola, whose focus is $(-2, 1)$ and directrix is $2x + y + 2 = 0$. Then the sum of the ordinates of the points on P, whose abscissa is -2, is [4 marks]

(A) $\frac{5}{2}$
(B) $\frac{3}{2}$
(C) $\frac{3}{4}$
(D) $\frac{1}{4}$

Correct Answer: (B) $\frac{3}{2}$

Solution:

Concept:

- Every point on a parabola is equidistant from its focus and its directrix. The distance of (x, y) from $ax + by + c = 0$ is $|ax + by + c|/\sqrt{a^2 + b^2}$.
- After fixing the abscissa, the distance equality becomes a quadratic in the ordinate. For $ay^2+by+c = 0$, the sum of roots is $-b/a$.

Step 1: Apply the distance definition at the fixed abscissa Write an intersection point as $(-2, y)$. Its distance from the focus $(-2, 1)$ is $|y - 1|$. Its perpendicular distance from the directrix is

$$\frac{|2(-2) + y + 2|}{\sqrt{2^2 + 1^2}} = \frac{|y - 2|}{\sqrt{5}}.$$

Equate these nonnegative distances. Squaring is legitimate here, and it gives $5(y - 1)^2 = (y - 2)^2$. The absolute values matter in the distance formula, although squaring removes them without introducing a sign restriction.

Step 2: Extract the sum of the ordinates Expand and collect the terms in the squared relation:

$$5y^2 - 10y + 5 = y^2 - 4y + 4,$$
$$4y^2 - 6y + 1 = 0.$$

The discriminant is $36 - 16 = 20 > 0$, so two real points occur at this abscissa. If their ordinates are y_1, y_2, Vieta's relation gives

$$y_1 + y_2 = -\frac{-6}{4} = \frac{3}{2}.$$

The coefficient of y must be taken after moving both sides to the same side; simply adding the focus and directrix ordinates would not use the parabola definition. The alternatives $5/2$, $3/4$ and $1/4$ are not this root sum.

Step 3: Conclusion The sum of the two ordinates is $\frac{3}{2}$. Hence, the correct option is (B).

Quick Tip to Solve: When only the sum of the intersection ordinates is asked, form the quadratic and use Vieta; solving both roots individually is unnecessary.

4. The mean and standard deviation of 100 observations are 40 and 5.1 , respectively. By mistake one observation is taken as 50 instead of 40 . If the correct mean and the correct standard deviation are μ and σ respectively, then $10(\mu+\sigma)$ is equal to [4 marks]

(A) 447

(B) 445

(C) 449

(D) 451

Correct Answer: (C) 449

Solution:

Concept:

- For N observations, the population mean is $\bar{x} = \sum x/N$, and the variance is $\sigma^2 = \sum x^2/N - \bar{x}^2$.
- Correct the sum and the sum of squares separately. Replacing an erroneous value a by b changes them by $b-a$ and b^2-a^2, respectively.

Step 1: Recover the sums and correct the mean The given mean and standard deviation describe the list containing the mistaken value 50. Therefore its sum is $100(40) = 4000$. Replacing 50 by 40 gives

$$S = 4000 - 50 + 40 = 3990, \qquad \mu = \frac{3990}{100} = 39.9.$$

The number of observations remains 100: this is replacement of a value, not removal of an observation. In particular the mean decreases by $10/100 = 0.1$, which is a useful sign check.

Step 2: Correct the second moment and calculate the required value Using the erroneous standard deviation 5.1, the original sum of squares is

$$S_{2,\text{old}} = 100\left(5.1^2 + 40^2\right) = 100(26.01 + 1600) = 162601.$$

Now replace the contribution 50^2 by 40^2:

$$S_2 = 162601 - 2500 + 1600 = 161701.$$

Hence the corrected variance and standard deviation are

$$\sigma^2 = \frac{161701}{100} - (39.9)^2 = 1617.01 - 1592.01 = 25, \qquad \sigma = 5.$$

Finally $10(\mu+\sigma) = 10(39.9+5) = 449$. Using the original mean in the variance formula would be inconsistent; subtracting only 10 from the sum of squares would also be wrong. The other integers offered do not satisfy both corrected moments.

Step 3: Conclusion The required value is 449. Hence, the correct option is (C).

Quick Tip to Solve: For a corrected observation, update both sums first and only then recompute the mean and variance; a standard deviation cannot be corrected by direct subtraction.

5. Let ABC be the triangle such that the equations of lines AB and AC be $3y - x = 2$ and $x + y = 2$, respectively, and the points B and C lie on x-axis. If P is the orthocentre of the triangle ABC , then the area of the triangle PBC is equal to [4 marks]

(A) 8

(B) 4

(C) 10

(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- The orthocentre is the intersection of two altitudes. An altitude is perpendicular to the opposite side and passes through the corresponding vertex.
- If a triangle has its base on the x-axis, its area is one-half the base length times the absolute ordinate of its third vertex.

Step 1: Find the vertices and the vertical altitude Since B lies on AB and on the x-axis, put $y = 0$ in $3y - x = 2$ to obtain $B = (-2, 0)$. Similarly $x + y = 2$ gives $C = (2, 0)$. Solve the two side equations for A:

$$x = 3y - 2, \quad x + y = 2 \implies 4y = 4 \implies A = (1, 1).$$

The side BC is horizontal and has length $2 - (-2) = 4$. Its perpendicular altitude through A is therefore the vertical line $x = 1$. This altitude contains the required orthocentre P.

Step 2: Intersect a second altitude and evaluate the area The line AC has slope -1, so an altitude perpendicular to AC has slope 1. Through $B = (-2, 0)$ its equation is $y = x + 2$. Its intersection with $x = 1$ is

$$P = (1, 3).$$

As a check, AB has slope $1/3$; the altitude from C has slope -3 and equation $y = -3(x - 2)$, which also contains $(1, 3)$. The height of triangle PBC above the x-axis is thus 3, not the height 1 of A. Consequently

$$[\triangle PBC] = \frac{1}{2} \times 4 \times 3 = 6.$$

The orthocentre lies outside this obtuse triangle, which is allowed. The alternative areas 8, 4 and 10 use neither the computed base-height pair nor the correct orthocentre.

Step 3: Conclusion The area of $\triangle PBC$ is 6 square units. Hence, the correct option is (D).

Quick Tip to Solve: With a horizontal base, first use the vertical altitude to fix the orthocentre abscissa; one more perpendicular line then gives its height immediately.

6. $\lim_{x\to 0^+} \dfrac{\tan\left(5x^{\frac{1}{3}}\right)\log_e\left(1+3x^2\right)}{\left(\tan^{-1}(3\sqrt{x})\right)^2\left(e^{5x^{\frac{4}{3}}}-1\right)}$ **is equal to** [4 marks]

(A) $\frac{5}{3}$

(B) 1

(C) $\frac{1}{3}$

(D) $\frac{1}{15}$

Correct Answer: (C) $\frac{1}{3}$

Solution:

Concept:

- As $t \to 0$, the standard limits give $\tan t/t \to 1$, $\log(1+t)/t \to 1$, $\tan^{-1} t/t \to 1$, and $(e^t-1)/t \to 1$.
- For products and quotients of these factors, isolate each standard-limit ratio and keep all numerical coefficients and powers of x. The one-sided limit ensures $\sqrt{x}$ is real.

Step 1: Identify the leading factors For $x \to 0^+$, the arguments $5x^{1/3}$, $3x^2$, $3\sqrt{x}$ and $5x^{4/3}$ all tend to zero. The corresponding leading expressions are

$$\tan(5x^{1/3}) \sim 5x^{1/3}, \quad \log_e(1+3x^2) \sim 3x^2,$$
$$\left(\tan^{-1}(3\sqrt{x})\right)^2 \sim 9x, \qquad e^{5x^{4/3}} - 1 \sim 5x^{4/3}.$$

The inverse tangent is squared as a whole. Thus its leading coefficient is $3^2 = 9$, and its leading power is $(\sqrt{x})^2 = x$. This distinction determines the numerical answer.

Step 2: Cancel equal powers and take the limit Equivalently, write the original quotient as the product of the standard-limit ratios for the four functions, multiplying by the quotient of their leading expressions. Each standard-limit ratio tends to 1, and the remaining quotient is

$$\frac{(5x^{1/3})(3x^2)}{(9x)(5x^{4/3})} = \frac{15x^{7/3}}{45x^{7/3}} = \frac{1}{3}.$$

Both numerator and denominator have the same total exponent $7/3$, so no residual power of x remains. All the factors used are nonzero for sufficiently small positive x, making the quotient-limit step valid. Options $5/3$ and 1 omit coefficients from the denominator; $1/15$ also does not equal the full coefficient ratio.

Step 3: Conclusion The given limit equals $\frac{1}{3}$. Hence, the correct option is (C).

Quick Tip to Solve: For a product of small-angle and exponential factors, match the powers first, then compute the coefficient ratio; remember to square both coefficient and power in an inverse-tangent square.

7. Let $y = y(x)$ be the solution curve of the differential equation $x\left(x^2+e^x\right)dy + \left(e^x(x-2)y - x^3\right)dx = 0, x > 0$, passing through the point $(1,0)$. Then $y(2)$ is equal to : [4 marks]

(A) $\frac{2}{2+e^2}$
(B) $\frac{4}{4-e^2}$
(C) $\frac{4}{4+e^2}$
(D) $\frac{2}{2-e^2}$

Correct Answer: (C) $\frac{4}{4+e^2}$

Solution:

Concept:

- A first-order linear equation has the form $y' + P(x)y = Q(x)$. Multiplication by an integrating factor $I(x)$, satisfying $I'/I = P$, turns its left side into the derivative of Iy.
- The initial point determines the integration constant. Here $x > 0$, so division by x and x^2 is valid, and $x^2 + e^x$ is always positive.

Step 1: Put the equation in linear form Divide the printed equation by $x(x^2+e^x)\,dx$. This gives

$$y' + \frac{e^x(x-2)}{x(x^2+e^x)}y = \frac{x^2}{x^2+e^x}.$$

To find an integrating factor without a long integration, compare the coefficient of y with the logarithmic derivative of $(x^2+e^x)/x^2$:

$$\frac{d}{dx}\log\frac{x^2+e^x}{x^2} = \frac{2x+e^x}{x^2+e^x} - \frac{2}{x} = \frac{e^x(x-2)}{x(x^2+e^x)}.$$

Step 2: Integrate and impose the initial value Thus take $I = (x^2+e^x)/x^2$. On multiplying the linear equation by this factor, its right side becomes exactly 1:

$$\frac{d}{dx}\left(\frac{x^2+e^x}{x^2}y\right) = 1.$$

Integration gives $(x^2+e^x)y/x^2 = x + C$. At $x = 1$, the specified value $y = 0$ gives $C = -1$. Therefore

$$y(x) = \frac{x^2(x-1)}{x^2+e^x}, \qquad y(2) = \frac{4}{4+e^2}.$$

The minus-denominator expressions in (B) and (D) have the wrong sign; (A) also has the wrong numerator and denominator after substitution.

Step 3: Conclusion The required value is $y(2) = \frac{4}{4+e^2}$. Hence, the correct option is (C).

Quick Tip to Solve: Look for a logarithmic derivative involving the coefficient denominator; here dividing x^2+e^x by x^2 immediately exposes the integrating factor.

8. Let A be a 3×3 matrix such that $|\operatorname{adj}(\operatorname{adj}(\operatorname{adj} A))| = 81$.

If $S = \left\{ n \in \mathbb{Z} : (|\operatorname{adj}(\operatorname{adj} A)|)^{\frac{(n-1)^2}{2}} = |A|^{(3n^2-5n-4)} \right\}$, then $\sum_{n \in S} \left| A^{(n^2+n)} \right|$ is equal to : [4 marks]

(A) 820

(B) 866

(C) 750

(D) 732

Correct Answer: (D) 732

Solution:

Concept:

- For a 3×3 matrix B, $\det(\operatorname{adj} B) = (\det B)^2$. Each application of adjoint therefore doubles the exponent on the determinant.
- For an invertible matrix, $\det(A^k) = (\det A)^k$ for every integer k. A determinant may be negative, so its sign must be checked before equating powers.

Step 1: Determine the possible determinant values Write $d = \det A$. Successive adjoints have determinants d^2, d^4, d^8. Consequently,

$$d^8 = 81 = 3^4, \qquad |d| = \sqrt{3}, \qquad d = \pm\sqrt{3}.$$

In particular $d \neq 0$, so all integer matrix powers in the question exist. The determinant of the double adjoint is $d^4 = 9$; this value is positive for either possible sign of d.

Step 2: Find the integer set and evaluate the sum The condition defining S becomes

$$9^{(n-1)^2/2} = d^{3n^2-5n-4}.$$

Taking absolute values gives a necessary equality of positive powers of 3. Since 3 is not 1, their exponents must be equal:

$$(n-1)^2 = \frac{3n^2-5n-4}{2} \quad \Longrightarrow \quad n^2 - n - 6 = 0.$$

Thus the only candidates are $n = 3, -2$. Their exponents on the right are respectively 8 and 18, both even, so they satisfy the original equality even when $d < 0$. Hence $S = \{-2, 3\}$, and

$$\sum_{n \in S} \det(A^{n^2+n}) = d^2 + d^{12} = 3 + 3^6 = 732.$$

The other numerical options do not equal the sum obtained from these two admissible integers.

Step 3: Conclusion The required determinant sum is 732. Hence, the correct option is (D).

Quick Tip to Solve: With repeated adjoints, track determinant exponents as $1, 2, 4, 8$; if an even power leaves a sign choice, test the candidate integers in the original equation.

9. The remainder when $\left((64)^{(64)}\right)^{(64)}$ is divided by 7 is equal to [4 marks]

(A) 4

(B) 6

(C) 3

(D) 1

Correct Answer: (D) 1

Solution:

Concept:

- Integers that differ by a multiple of 7 have the same remainder modulo 7. Congruences remain valid when both sides are raised to the same positive integer power.
- If a base is one more than a multiple of the divisor, every positive integer power of that base has remainder 1. The size of the exponent does not affect this result.

Step 1: Reduce the base before handling the powers The innermost base is 64. Division by 7 gives

$$64 = 7 \cdot 9 + 1, \qquad 64 \equiv 1 \pmod 7.$$

The printed expression has parentheses $(64^{64})^{64}$, so the power-of-a-power rule also writes it as $64^{64 \cdot 64} = 64^{4096}$. This identifies a positive integer exponent; no expansion of the huge integer is needed. In particular the exponent is a product, rather than the tower 64^{64}.

Step 2: Raise the reduced base and identify the remainder Using multiplication of congruences, any product of 4096 copies of 64 has the same remainder as a product of 4096 copies of 1. Thus

$$(64^{64})^{64} \equiv (1^{64})^{64} = 1 \pmod 7.$$

Equivalently, the binomial form $(63 + 1)^{4096}$ has constant term 1, while every other term contains a factor 63 and hence a factor 7. The remainder must be the representative between 0 and 6, so the congruence already provides the actual remainder. The proposed values 4, 6 and 3 are different residue classes and cannot be correct. This direct reduction avoids both computing the large power and unnecessarily searching for a cycle of remainders.

Step 3: Conclusion The remainder on division by 7 is 1. Hence, the correct option is (D).

Quick Tip to Solve: Always reduce a large-power base modulo the divisor first: a base congruent to 1 ends the calculation immediately, regardless of how large the positive exponent is.

10. If for $\theta \in \left[-\frac{\pi}{3}, 0\right]$, the points $(x, y) = \left(3\tan\left(\theta + \frac{\pi}{3}\right), 2\tan\left(\theta + \frac{\pi}{6}\right)\right)$ lie on $xy + \alpha x + \beta y + \gamma = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to : [4 marks]

(A) 75

(B) 96

(C) 80

(D) 72

Correct Answer: (A) 75

Solution:

Concept:

- The tangent subtraction identity is $\tan(u - v) = (\tan u - \tan v)/(1 + \tan u \tan v)$, whenever the tangents and the displayed quotient are defined.
- For this parametrisation, put $u = \theta + \pi/3$ and $v = \theta + \pi/6$; their difference is the fixed angle $\pi/6$. The scaling factors give $\tan u = x/3$ and $\tan v = y/2$.

Step 1: Eliminate the parameter Throughout the given interval, $u \in [0, \pi/3]$ and $v \in [-\pi/6, \pi/6]$, so both tangents are finite. Their difference is always $\pi/6$. Substitution in the subtraction identity yields

$$\frac{x/3 - y/2}{1 + xy/6} = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}.$$

The denominator is nonzero: equivalently it is $\cos(u - v)/(\cos u \cos v)$, whose numerator and denominator are positive in these intervals. We can therefore multiply through without losing any of the given points.

Step 2: Compare the polynomial coefficients Clear the fractions and rearrange so that the coefficient of xy is the specified 1:

$$\sqrt{3}(2x - 3y) = 6 + xy,$$

$$xy - 2\sqrt{3}\,x + 3\sqrt{3}\,y + 6 = 0.$$

Comparison with $xy + \alpha x + \beta y + \gamma = 0$ gives $\alpha = -2\sqrt{3}$, $\beta = 3\sqrt{3}$, $\gamma = 6$. Therefore

$$\alpha^2 + \beta^2 + \gamma^2 = 12 + 27 + 36 = 75.$$

Keeping the scaling factors 3 and 2 is essential; omitting either changes the linear coefficients. The other proposed totals 96, 80 and 72 disagree with the three squared coefficients.

Step 3: Conclusion The required value is 75. Hence, the correct option is (A).

Quick Tip to Solve: For coordinates involving tangents of two shifted angles, use their fixed angle difference first, then normalise the coefficient of xy before comparing coefficients.

11. Let the system of equations :

$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$12x + 3y - (4 + \lambda)z = 16 - \mu$$

have infinitely many solutions. Then the radius of the circle centred at (λ, μ) and touching the line $4x = 3y$ is : [4 marks]

(A) $\frac{7}{5}$
(B) $\frac{21}{5}$
(C) 7
(D) $\frac{17}{5}$

Correct Answer: (A) $\frac{7}{5}$

Solution:

Concept:

- A system has infinitely many solutions when it is consistent and its coefficient rank is smaller than the number of unknowns. A dependent coefficient row must have the same dependence in the right-hand constants.
- The radius of a circle tangent to a line is the perpendicular distance from its centre to the line. For $ax + by + c = 0$, that distance from (h, k) is $|ah + bk + c|/\sqrt{a^2 + b^2}$.

Step 1: Use row dependence to find the parameters Denote the first two equations by E_1, E_2. Their coefficient rows are independent: for example, the determinant from the x, y columns is $2 \cdot 3 - 7 \cdot 3 = -15 \neq 0$. To express the third coefficient row in their span, write it as $aE_1 + bE_2$. The first two coefficients require

$$a + b = 1, \qquad 2a + 7b = 12,$$

so $a = -1, b = 2$. Thus the only possible dependence is $E_3 = 2E_2 - E_1$. Its z-coefficient and constant term are

$$2(-2) - 5 = -9, \qquad 2(8) - 9 = 7.$$

Hence $-(4 + \lambda) = -9$ and $16 - \mu = 7$, giving $\lambda = 5, \mu = 9$. The augmented rank is then 2, leaving one free variable and indeed infinitely many solutions.

Step 2: Calculate the tangent-circle radius The centre is therefore $(5, 9)$. Write the given line as $4x - 3y = 0$; its normal length is $\sqrt{4^2 + (-3)^2} = 5$. Its distance from the centre is

$$r = \frac{|4(5) - 3(9)|}{5} = \frac{|20 - 27|}{5} = \frac{7}{5}.$$

The modulus makes the radius positive. Option (C) omits the normal-length divisor, while (B) and (D) do not equal the perpendicular distance for this centre.

Step 3: Conclusion The circle has radius $\frac{7}{5}$ length units. Hence, the correct option is (A).

Quick Tip to Solve: When two coefficient columns already determine a row dependence, use the remaining coefficient and the constant term to get both parameters before applying the distance formula.

12. Let the set of all values of $p \in \mathbb{R}$, for which both the roots of the equation $x^2-(p+2)x+(2p+9)=0$ are negative real numbers, be the interval $(\alpha, \beta]$. Then $\beta - 2\alpha$ is equal to [4 marks]

(A) 5

(B) 0

(C) 20

(D) 9

Correct Answer: (A) 5

Solution:

Concept:

- For a real monic quadratic with roots r, s, both roots are negative precisely when the roots are real, $r+s<0$, and $rs>0$. A repeated negative root is permitted.
- Vieta's formulas give the sum and product directly from the coefficients, while the discriminant tests reality. All three conditions must be intersected; none alone is sufficient.

Step 1: Translate the root conditions into inequalities For $x^2-(p+2)x+(2p+9)=0$, Vieta's formulas give

$$r+s=p+2, \qquad rs=2p+9.$$

Strict negativity of both roots therefore requires $p<-2$ and $p>-9/2$. The strict product inequality excludes a zero root. The discriminant is

$$\Delta=(p+2)^2-4(2p+9)=p^2-4p-32=(p-8)(p+4).$$

For real roots, $\Delta \geq 0$, so $p \leq -4$ or $p \geq 8$. The latter branch contradicts the negative-sum condition and must be discarded.

Step 2: Intersect the conditions and read the endpoints Combining the surviving reality condition with the sum and product conditions gives

$$-\frac{9}{2}<p\leq -4.$$

The left endpoint is excluded because at $p=-9/2$ the product vanishes and one root is zero. The right endpoint is included because $p=-4$ gives $x^2+2x+1=(x+1)^2$, with both roots equal to -1. Thus the interval has $\alpha=-9/2$ and $\beta=-4$, and

$$\beta-2\alpha=-4-2\left(-\frac{9}{2}\right)=5.$$

The other options 0, 20 and 9 do not result from these correctly included and excluded endpoints. In particular, using only the discriminant would admit positive-root cases that the question forbids.

Step 3: Conclusion The requested value is $\beta-2\alpha=5$. Hence, the correct option is (A).

Quick Tip to Solve: For two strictly negative real roots, intersect discriminant nonnegativity with negative sum and positive product; check zero-root and repeated-root endpoints separately.

13. Among the statements

(S1) : The set $\{z \in \mathbb{C} - \{-i\} : |z| = 1$ and $\frac{z-i}{z+i}$ is purely real $\}$ contains exactly two elements, and

(S2) : The set $\{z \in \mathbb{C} - \{-1\} : |z| = 1$ and $\frac{z-1}{z+1}$ is purely imaginary $\}$ contains infinitely many elements.

[4 marks]

(A) both are incorrect

(B) both are correct

(C) only (S2) is correct

(D) only (S1) is correct

Correct Answer: (C) only (S2) is correct

Solution:

Concept:

- A complex number $z = x + iy$ has unit modulus precisely when $x^2 + y^2 = 1$. A quotient is real when its imaginary part vanishes, and purely imaginary when its real part vanishes.
- Multiply numerator and denominator by the conjugate of the denominator. Excluded denominator zeros must be removed before counting points on the unit circle.

Step 1: Test the first statement Write $z = x + iy$. For $z \neq -i$, rationalising gives

$$\frac{z-i}{z+i} = \frac{x^2 + y^2 - 1 - 2ix}{x^2 + (y+1)^2}.$$

On the unit circle the real part is zero. For the quotient also to be real, its imaginary part must vanish, so $x = 0$. The circle then gives $y = 1$ or $y = -1$. However, $y = -1$ gives the excluded point $-i$. Only $z = i$ remains, where the quotient is zero. Thus the set has one element, rather than two, and (S1) is incorrect.

Step 2: Test the second statement For $z \neq -1$, similarly

$$\frac{z-1}{z+1} = \frac{x^2 + y^2 - 1 + 2iy}{(x+1)^2 + y^2}.$$

The real part vanishes at every point of the unit circle except the excluded denominator zero -1. Therefore every other point of that circle satisfies the purely imaginary condition. There are infinitely many such points, so (S2) is correct. Even if zero is excluded from the phrase purely imaginary, removing the additional point $z = 1$ still leaves infinitely many points.

Step 3: Compare the statement combinations Option (A) incorrectly rejects (S2), option (B) incorrectly accepts (S1), and option (D) reverses the two truth values. The combination in option (C) alone agrees with the direct calculation and the excluded points.

Step 4: Conclusion Only (S2) is correct. Hence, the correct option is (C).

Quick Tip to Solve: For unit-circle quotient questions, rationalise first and apply the circle equation before counting points; remember every excluded denominator zero.

14. The integral $\int_0^{\pi} \frac{(x+3)\sin x}{1+3\cos^2 x}dx$ is equal to [4 marks]

(A) $\frac{\pi}{\sqrt{3}}(\pi+1)$
(B) $\frac{\pi}{3\sqrt{3}}(\pi+6)$
(C) $\frac{\pi}{\sqrt{3}}(\pi+2)$
(D) $\frac{\pi}{2\sqrt{3}}(\pi+4)$

Correct Answer: (B) $\frac{\pi}{3\sqrt{3}}(\pi+6)$

Solution:

Concept:

- If $g(\pi-x)=g(x)$, reflection of a definite integral gives $\int_0^{\pi} xg(x)\,dx = \frac{\pi}{2}\int_0^{\pi} g(x)\,dx$.
- The substitution $u=\cos x$, with $du=-\sin x\,dx$, reduces an integrand involving $\sin x$ and $\cos^2 x$ to a rational function.

Step 1: Use reflection to eliminate the factor x Let $g(x)=\sin x/(1+3\cos^2 x)$, and denote the required integral by I. Since $\sin(\pi-x)=\sin x$ and $\cos^2(\pi-x)=\cos^2 x$, the denominator and g are unchanged under reflection. Consequently,

$$I=\int_0^{\pi}(\pi-x+3)g(x)\,dx.$$

Adding this identity to the original expression cancels the variable factors:

$$2I=(\pi+6)J, \qquad J=\int_0^{\pi}\frac{\sin x}{1+3\cos^2 x}\,dx.$$

The endpoints cause no singularity because the denominator is at least one.

Step 2: Evaluate the remaining integral Set $u=\cos x$. The limits $x=0,\pi$ become $u=1,-1$, respectively. The minus sign in the differential reverses the order:

$$J=\int_{-1}^{1}\frac{du}{1+3u^2}=\frac{1}{\sqrt{3}}\left[\tan^{-1}(\sqrt{3}u)\right]_{-1}^{1}.$$

Using $\tan^{-1}\sqrt{3}=\pi/3$, obtain

$$J=\frac{2\pi}{3\sqrt{3}}, \qquad I=\frac{\pi+6}{2}\frac{2\pi}{3\sqrt{3}}=\frac{\pi(\pi+6)}{3\sqrt{3}}.$$

Step 3: Check the listed expressions The coefficient of the linear factor must come from both the reflection factor $(\pi+6)/2$ and the full range of the inverse tangent. Option (B) has both factors correctly. Options (A), (C), and (D) have different constants or prefactors and do not equal the evaluated integral.

Step 4: Conclusion The integral equals $\frac{\pi}{3\sqrt{3}}(\pi+6)$. Hence, the correct option is (B).

Quick Tip to Solve: When the non-polynomial factor is symmetric about the midpoint, replace x by its midpoint value inside its integral before making a substitution.

15. Let the line L pass through $(1,1,1)$ and intersect the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z}{1}$. Then, which of the following points lies on the line L ? [4 marks]

(A) $(7,15,13)$

(B) $(4,22,7)$

(C) $(10,-29,-50)$

(D) $(5,4,3)$

Correct Answer: (A) $(7,15,13)$

Solution:

Concept:

- A line passing through a point and intersecting another line lies in the plane containing that point and the other line, provided the point is not on that line.
- Once two points on a line are known, their difference gives a direction vector. A proposed point lies on the line only when all three displacement ratios agree.

Step 1: Find the plane containing L and the first line Let $P=(1,1,1)$. The first given line has a point $A=(1,-1,1)$ and direction $\mathbf{d}=(2,3,4)$. The vectors $\overrightarrow{PA}=(0,-2,0)$ and $\mathbf{d}$ span their plane. A normal vector is $(-8,0,4)$, so its equation is

$$-8(x-1)+4(z-1)=0, \qquad z=2x-1.$$

The desired transversal lies in this plane because it passes through P and meets the first line.

Step 2: Intersect the second line with that plane Parameterise the second line as $(x,y,z)=(3+s,4+2s,s)$. Substitution in the plane gives

$$s=2(3+s)-1, \qquad s=-5.$$

Thus its intersection is $Q=(-2,-6,-5)$. The vector $\overrightarrow{PQ}=(-3,-7,-6)$ supplies the direction of L; reversing the direction is harmless. Hence

$$(x,y,z)=(1,1,1)+t(3,7,6).$$

For completeness, at $t=-4/5$, this line also meets the first line with its parameter $-6/5$, confirming the required intersection.

Step 3: Test all four candidate points For $(7,15,13)$, the displacement from P is $(6,14,12)=2(3,7,6)$, so it lies on L. For option (B), the ratios are $3/3, 21/7, 6/6=1,3,1$, which disagree. For option (C) they are $9/3,-30/7,-51/6$, and for option (D) they are $4/3,3/7,2/6$; neither triple is equal. Thus only the first candidate satisfies all coordinates simultaneously.

Step 4: Conclusion The point $(7,15,13)$ lies on L. Hence, the correct option is (A).

Quick Tip to Solve: Construct the plane through the known point and one line, intersect the other line with it, then test candidate points using displacement ratios.

16. If the area of the region bounded by the curves $y = 4 - \frac{x^2}{4}$ and $y = \frac{x-4}{2}$ is equal to α, then 6α. equals [4 marks]

(A) 210

(B) 250

(C) 240

(D) 220

Correct Answer: (B) 250

Solution:

Concept:

- The area between two curves is the integral of upper ordinate minus lower ordinate between their intersection abscissae.
- Factorising the difference identifies both intersection points and its sign, preventing a signed integral from being mistaken for geometric area.

Step 1: Find the intersections and identify the upper curve Equate the two ordinates:

$$4 - \frac{x^2}{4} = \frac{x-4}{2}.$$

Multiplying by four and collecting terms gives

$$x^2 + 2x - 24 = 0, \qquad (x+6)(x-4) = 0.$$

Thus the bounded region extends from $x = -6$ to $x = 4$. The difference of the parabola and the line is

$$\left(4 - \frac{x^2}{4}\right) - \frac{x-4}{2} = 6 - \frac{x}{2} - \frac{x^2}{4} = \frac{(4-x)(x+6)}{4}.$$

Both factors are nonnegative on this interval, so the parabola is the upper curve throughout the interior.

Step 2: Integrate the vertical separation Therefore the geometric area is

$$\alpha = \int_{-6}^{4} \left(6 - \frac{x}{2} - \frac{x^2}{4}\right) dx = \left[6x - \frac{x^2}{4} - \frac{x^3}{12}\right]_{-6}^{4}.$$

At the upper limit the expression is $24 - 4 - 16/3 = 44/3$. At the lower limit it is $-36 - 9 + 18 = -27$. Subtracting the lower value gives

$$\alpha = \frac{44}{3} + 27 = \frac{125}{3} \quad \text{square units.}$$

There is no need to split this integral because the curves do not exchange their upper and lower positions between the roots.

Step 3: Evaluate the requested multiple The question asks for six times the area, rather than the area

itself. Hence

$$6\alpha = 6\left(\frac{125}{3}\right) = 250.$$

This matches option (B). The values 210, 240, and 220 in the remaining options differ from the exact integral. As a check, shifting the midpoint to $x = -1$ makes the separation $(25 - (x+1)^2)/4$, whose symmetric integral gives the same area.

Step 4: Conclusion $\alpha = 125/3$ square units, and $6\alpha = 250$. Hence, the correct option is (B).

Quick Tip to Solve: Factor upper minus lower first: its roots provide the limits and its sign confirms the correct order of subtraction.

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17. If the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x}{1} = \frac{y}{\alpha} = \frac{z-5}{1}$ is $\frac{5}{\sqrt{6}}$, then the sum of all possible values of α is [4 marks]

(A) $\frac{3}{2}$

(B) 3

(C) -3

(D) $-\frac{3}{2}$

Correct Answer: (C) -3

Solution:

Concept:

- For lines through $\mathbf{a}_1, \mathbf{a}_2$ with nonparallel direction vectors $\mathbf{d}_1, \mathbf{d}_2$, their shortest distance is $|(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{d}_1 \times \mathbf{d}_2)| \,/ |\mathbf{d}_1 \times \mathbf{d}_2|$.
- The absolute value makes distance nonnegative. Squaring a distance equation is valid when both sides are nonnegative, but the resulting candidates should still be checked.

Step 1: Form the scalar triple product Choose $\mathbf{a}_1 = (1, 2, 3)$, $\mathbf{a}_2 = (0, 0, 5)$, $\mathbf{d}_1 = (2, 3, 4)$, and $\mathbf{d}_2 = (1, \alpha, 1)$. Then

$$\mathbf{d}_1 \times \mathbf{d}_2 = (3 - 4\alpha, 2, 2\alpha - 3).$$

Its middle component is always 2, so the lines are never parallel for any real α. Using $\mathbf{a}_2 - \mathbf{a}_1 = (-1, -2, 2)$, the numerator is

$$|-(3 - 4\alpha) - 4 + 2(2\alpha - 3)| = |8\alpha - 13|.$$

The squared denominator is

$$(3 - 4\alpha)^2 + 4 + (2\alpha - 3)^2 = 20\alpha^2 - 36\alpha + 22.$$

Step 2: Solve the distance equation The prescribed distance gives

$$\frac{|8\alpha - 13|}{\sqrt{20\alpha^2 - 36\alpha + 22}} = \frac{5}{\sqrt{6}}.$$

Square and cross-multiply to obtain

$$6(8\alpha - 13)^2 = 25(20\alpha^2 - 36\alpha + 22).$$

Expanding both sides and collecting terms gives

$$116\alpha^2 + 348\alpha - 464 = 0, \qquad \alpha^2 + 3\alpha - 4 = 0.$$

Thus $(\alpha - 1)(\alpha + 4) = 0$, giving $\alpha = 1, -4$.

Step 3: Check the candidates and add them At $\alpha = 1$, the distance is $5/\sqrt{6}$. At $\alpha = -4$, it is $45/\sqrt{486} = 5/\sqrt{6}$, so neither candidate was introduced spuriously by squaring. Their sum is $1 - 4 = -3$. Option (C) matches this sum; the positive sums in (A) and (B), and the value $-3/2$ in (D), do not. The same sum follows from the coefficient ratio of the quadratic after the distance formula is established.

Step 4: Conclusion The possible values are 1 and -4, whose sum is -3. Hence, the correct option is (C).

Quick Tip to Solve: After deriving the quadratic from the shortest-distance formula, use the sum of roots, but first confirm that the direction vectors are never parallel.

18. Let $x = -1$ and $x = 2$ be the critical points of the function $f(x) = x^3 + ax^2 + b\log_e|x| + 1, x \neq 0$. Let m and M respectively be the absolute minimum and the absolute maximum values of f in the interval $\left[-2, -\frac{1}{2}\right]$. Then $|M + m|$ is equal to (Take $\log_e 2 = 0.7$) : [4 marks]

(A) 21.1

(B) 19.8

(C) 22.1

(D) 20.9

Correct Answer: (A) 21.1

Solution:

Concept:

- For $x \neq 0$, $d(\log_e|x|)/dx = 1/x$. Critical points satisfy $f'(x) = 0$, so the supplied critical points determine the unknown coefficients.
- A continuous function on a closed interval attains absolute extrema at endpoints or interior critical points. Comparing only local extrema can miss the absolute minimum.

Step 1: Determine the coefficients Differentiation gives

$$f'(x) = 3x^2 + 2ax + \frac{b}{x}.$$

At $x = -1$ and $x = 2$, respectively,

$$3 - 2a - b = 0, \qquad 12 + 4a + \frac{b}{2} = 0.$$

The first equation gives $b = 3 - 2a$. Substituting this into twice the second equation yields $24 + 8a + 3 - 2a = 0$, so

$$a = -\frac{9}{2}, \qquad b = 12.$$

Thus $f(x) = x^3 - \frac{9}{2}x^2 + 12\log_e|x| + 1$.

Step 2: Locate all candidates in the interval The derivative can now be written

$$f\prime(x) = \frac{3x^3 - 9x^2 + 12}{x} = \frac{3(x+1)(x-2)^2}{x}.$$

In $[-2, -1/2]$, only -1 is an interior critical point; 2 is outside. The function is continuous throughout the interval because zero is excluded from it. The derivative is positive to the left of -1 and negative to its right, so -1 is a local maximum. We still compare both endpoints to determine the absolute minimum.

Step 3: Evaluate and compare the function values Using the stipulated $\log_e 2 = 0.7$ and $\log_e(1/2) = -0.7$, compute

$$f(-2) = -8 - 18 + 8.4 + 1 = -16.6,$$
$$f(-1) = -1 - 4.5 + 0 + 1 = -4.5,$$
$$f(-1/2) = -\frac{1}{8} - \frac{9}{8} - 8.4 + 1 = -8.65.$$

Consequently $m = -16.6$ and $M = -4.5$. Therefore

$$|M + m| = |-4.5 - 16.6| = 21.1.$$

This is option (A); none of 19.8, 22.1, or 20.9 equals the absolute value of the correctly identified extreme values.

Step 4: Conclusion The absolute minimum is $m = -16.6$, the absolute maximum is $M = -4.5$, and $|M + m| = 21.1$. Hence, the correct option is (A).

Quick Tip to Solve: For absolute extrema, list every endpoint and each critical point inside the stated interval; a critical point outside the interval does not participate.

19. Let the angle $\theta, 0 < \theta < \frac{\pi}{2}$ between two unit vectors $\hat{a}$ and $\hat{b}$ be $\sin^{-1}\left(\frac{\sqrt{65}}{9}\right)$. If the vector $\vec{c} = 3\hat{a} + 6\hat{b} + 9(\hat{a} \times \hat{b})$, then the value of $9(\vec{c} \cdot \hat{a}) - 3(\vec{c} \cdot \hat{b})$ is [4 marks]

(A) 31

(B) 29

(C) 24

(D) 27

Correct Answer: (B) 29

Solution:

Concept:

- For unit vectors, $\hat{a} \cdot \hat{a} = \hat{b} \cdot \hat{b} = 1$ and $\hat{a} \cdot \hat{b} = \cos\theta$. A scalar product measures the component along the other vector.
- The cross product $\hat{a} \times \hat{b}$ is perpendicular to both original vectors. Its scalar product with either one is zero, regardless of its magnitude.

Step 1: Find the positive cosine The angle lies in the first quadrant, so its cosine is positive. Use the given sine and the identity $\sin^2\theta + \cos^2\theta = 1$:

$$\cos\theta = \sqrt{1 - \frac{65}{81}} = \sqrt{\frac{16}{81}} = \frac{4}{9}.$$

The negative square root is excluded by the stated interval for the angle.

Step 2: Take the two scalar products Distribute the scalar product over the three terms defining $\vec{c}$. The cross-product term vanishes in each calculation, while the same-vector products equal one:

$$\vec{c} \cdot \hat{a} = 3 + 6\left(\frac{4}{9}\right) + 0 = \frac{17}{3},$$
$$\vec{c} \cdot \hat{b} = 3\left(\frac{4}{9}\right) + 6 + 0 = \frac{22}{3}.$$

The coefficient 9 of the perpendicular vector therefore makes no contribution to either projection.

Step 3: Evaluate the requested combination Substitute these two scalar products in the order printed in the question:

$$9(\vec{c} \cdot \hat{a}) - 3(\vec{c} \cdot \hat{b}) = 9\left(\frac{17}{3}\right) - 3\left(\frac{22}{3}\right) = 51 - 22 = 29.$$

Thus 31, 24 and 27 do not equal the required linear combination; only the second option matches.

Step 4: Conclusion The required value is $\boxed{29}$. Hence, the correct option is (B).

Quick Tip to Solve: Take the scalar products before expanding any cross product: perpendicular terms disappear immediately. Use the angle interval to choose the sign of the cosine.

20. Let x_1, x_2, x_3, x_4 be in a geometric progression. If $2, 7, 9, 5$ are subtracted respectively from x_1, x_2, x_3, x_4, then the resulting numbers are in an arithmetic progression. Then the value of $\frac{1}{24}(x_1x_2x_3x_4)$ is: [4 marks]

(A) 18

(B) 216

(C) 36

(D) 72

Correct Answer: (B) 216

Solution:

Concept:

- Write four consecutive GP terms as a, ar, ar^2, ar^3, where a is the first term and r is the common ratio.
- Three consecutive terms u, v, w form an AP precisely when $u - 2v + w = 0$. Applying this condition to both overlapping triples supplies two equations.

Step 1: Use the first AP condition After the specified subtractions, the first three terms are $a - 2, ar - 7, ar^2 - 9$. Their second difference must vanish:

$$\begin{aligned}(a-2) - 2(ar-7) + (ar^2-9) &= 0,\\ a(1-2r+r^2) + 3 &= 0,\\ a(r-1)^2 &= -3.\end{aligned}$$

In particular, neither $a = 0$ nor $r = 1$ is possible, because the right-hand side is nonzero.

Step 2: Use the second AP condition The last three adjusted terms also have zero second difference:

$$\begin{aligned}(ar-7) - 2(ar^2-9) + (ar^3-5) &= 0,\\ ar(1-2r+r^2) + 6 &= 0,\\ ar(r-1)^2 &= -6.\end{aligned}$$

Divide this equation by $a(r-1)^2 = -3$, which is nonzero, to obtain $r = 2$. Substituting in the first equation gives $a = -3$.

Step 3: Calculate and verify the product The original GP is $-3, -6, -12, -24$. Subtracting $2, 7, 9, 5$ respectively gives $-5, -13, -21, -29$, whose common difference is indeed -8. Now evaluate

$$\frac{x_1x_2x_3x_4}{24} = \frac{(-3)(-6)(-12)(-24)}{24} = \frac{5184}{24} = 216.$$

Four negative factors give a positive product. The alternatives 18, 36 and 72 fail this direct product check.

Step 4: Conclusion The required value is $\boxed{216}$. Hence, the correct option is (B).

Quick Tip to Solve: For a GP changed into an AP by known subtractions, take second differences of the adjusted terms. The two resulting equations often reveal the common ratio by division.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. The number of singular matrices of order 2 , whose elements are from the set $\{2, 3, 6, 9\}$, is _____. [4 marks]

Solution:

Concept:

- A matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is singular exactly when its determinant is zero, that is, $ad = bc$. All four entries must belong to $S = \{2, 3, 6, 9\}$.
- Count ordered pairs of entries producing each possible product. If a product occurs for m ordered pairs, independently choosing (a, d) and (b, c) with that product gives m^2 matrices.

Step 1: List the product frequencies There are $4^2 = 16$ ordered pairs from $S \times S$. Their product frequencies are

t	4	6	9	12	18	27	36	54	81
$m(t)$	1	2	1	2	4	2	1	2	1

For example, 18 is obtained from $(2, 9), (9, 2), (3, 6), (6, 3)$, so its frequency is four. The products 4, 9, 36 and 81 each come from a single equal-entry pair. Each remaining product comes from two differently ordered entries. The frequencies sum to 16, confirming that every ordered pair was counted.

Step 2: Match the two diagonal products Fix a product t. There are $m(t)$ choices for the ordered pair (a, d), and independently $m(t)$ choices for (b, c). These choices uniquely determine all four matrix positions, so they give $m(t)^2$ distinct singular matrices. Conversely, every singular matrix has one common value $t = ad = bc$, so it appears in exactly one of these groups. Thus neither a division for repeated entries nor an extra permutation factor is needed.

Step 3: Add the disjoint cases The total is

$$\begin{aligned} N &= 1^2 + 2^2 + 1^2 + 2^2 + 4^2 + 2^2 + 1^2 + 2^2 + 1^2 \\ &= 4(1) + 4(4) + 16 = 36. \end{aligned}$$

Ordered positions matter: interchanging two entries can produce a different matrix even though the product remains unchanged.

Step 4: Conclusion The number of singular matrices is $\boxed{36}$. This is a dimensionless count.

Quick Tip to Solve: For a two-by-two determinant with entries in a small finite set, tabulate ordered-pair product frequencies and add their squares.

22. The number of relations on the set $A = \{1, 2, 3\}$, containing at most 6 elements including $(1, 2)$, which are reflexive and transitive but not symmetric, is _____. [4 marks]

Solution:

Concept:

- Reflexivity on $A = \{1, 2, 3\}$ forces $D = \{(1, 1), (2, 2), (3, 3)\}$. Write every candidate relation as $R = D \cup E$, with E containing only off-diagonal pairs.
- The required pair $(1, 2)$ belongs to E. Since R has at most six elements, $1 \leq |E| \leq 3$. Transitivity requires (i, k) whenever both (i, j) and (j, k) occur.

Step 1: Count one and two additional pairs Use ij as an abbreviation for the ordered pair (i, j). If $|E| = 1$, only $E = \{12\}$ is possible; it is transitive and not symmetric. For $|E| = 2$, the additional pair can be 21, 13, 31, 23 or 32. The choice 21 makes the relation symmetric. The choices 31 and 23 respectively require the absent pairs 32 and 13, so they fail transitivity. The valid sets are therefore $\{12, 13\}$ and $\{12, 32\}$, giving two relations.

Step 2: Count the sets with three off-diagonal pairs Choose two pairs from $\{21, 13, 31, 23, 32\}$ along with 12. There are ten possibilities. The only transitive choices are

$$\{12, 13, 23\}, \qquad \{12, 13, 32\}, \qquad \{12, 31, 32\}.$$

These describe the three possible strict total orders compatible with 1 preceding 2. To check exhaustiveness, if 21 is chosen, connecting either vertex 1 or 2 to vertex 3 forces a fourth off-diagonal pair, so all four such choices fail. Without 21, the other rejected sets are $\{12, 13, 31\}$, $\{12, 31, 23\}$ and $\{12, 23, 32\}$, each missing a pair forced by a two-step chain.

Step 3: Verify the conditions and add Each displayed valid set becomes reflexive after adjoining D, and every two-step off-diagonal chain has its required direct pair. None contains 21, whereas all contain 12, so none is symmetric. The three sizes contribute

$$1 + 2 + 3 = 6.$$

The compulsory diagonal pairs do not create further constraints: composing with an identity pair only reproduces a pair already present.

Step 4: Conclusion The required number of relations is $\boxed{6}$, a dimensionless count.

Quick Tip to Solve: Remove the compulsory diagonal pairs first, then enumerate the few remaining arrows. Check every two-arrow chain before testing symmetry.

23. The number of points of discontinuity of the function $f(x) = \left[\frac{x^2}{2}\right] - [\sqrt{x}], x \in [0,4]$, where $[\cdot]$ denotes the greatest integer function, is _____. [4 marks]

Solution:

Concept:

- For an increasing continuous argument, its greatest-integer function jumps when the argument reaches a positive integer. A jump in a difference can disappear if both terms jump equally at the same point.
- Continuity at the endpoints of $[0,4]$ is tested using limits taken within this domain: a right limit at 0 and a left limit at 4.

Step 1: Locate jumps of the first term The argument $x^2/2$ increases from 0 to 8 on the domain. It reaches the positive integer k at $x = \sqrt{2k}$, for $k = 1, \ldots, 8$. Thus the candidates are

$$\sqrt{2},\ 2,\ \sqrt{6},\ \sqrt{8},\ \sqrt{10},\ \sqrt{12},\ \sqrt{14},\ 4.$$

At each interior candidate, $[x^2/2]$ increases by one; away from these points it is locally constant.

Step 2: Locate jumps of the second term The argument $\sqrt{x}$ increases from 0 to 2. Hence $[\sqrt{x}]$ jumps at $x = 1$ and $x = 4$. The point 1 is not in the first list, so f jumps downward there. The seven first-term candidates strictly below 4 are distinct from 1, and the second term is locally constant at them. Consequently all seven yield upward jumps in f.

Step 3: Check cancellation and endpoints At the shared endpoint 4 the two jumps cancel. For $x < 4$ sufficiently close to 4, $[x^2/2] = 7$ and $[\sqrt{x}] = 1$, so the left limit is 6. At the endpoint itself, $f(4) = 8 - 2 = 6$, hence 4 is continuous relative to the domain. At 0, both greatest-integer terms equal zero throughout a sufficiently small right neighbourhood, so 0 is also continuous. On all remaining open intervals both terms are constant, giving no additional discontinuities. Therefore the count is

$$7 + 1 = 8.$$

Step 4: Conclusion There are $\boxed{8}$ points of discontinuity. This is a dimensionless count.

Quick Tip to Solve: List the integer-crossing points of both floor arguments, then compare jump sizes at common points. Test boundary points using one-sided limits within the domain.

24. For $n \geq 2$, let S_n denote the set of all subsets of $\{1, 2, \ldots, n\}$ with no two consecutive numbers. For example $\{1, 3, 5\} \in S_6$, but $\{1, 2, 4\} \notin S_6$. Then $n(S_5)$ is equal to _____ [4 marks]

Solution:

Concept:

- Partition the admissible subsets according to their size. The empty set is included because it contains no consecutive numbers.
- If an admissible subset of $\{1, \ldots, n\}$ has sorted elements $a_1 < \cdots < a_k$, then $a_{j+1} \geq a_j + 2$. Removing these mandatory gaps converts the count into an ordinary binomial coefficient.

Step 1: Transform the spacing restriction Define $b_j = a_j - (j-1)$ for $j = 1, \ldots, k$. The gap condition implies

$$1 \le b_1 < b_2 < \cdots < b_k \le n - k + 1.$$

Conversely, any such increasing sequence gives $a_j = b_j + j - 1$, whose consecutive selected elements differ by at least two. Thus this is a bijection, and the number of admissible subsets of size k is $\binom{n-k+1}{k}$. The smallest possible last element is $2k-1$, so for $n = 5$ only $k = 0, 1, 2, 3$ can occur.

Step 2: Count each possible subset size For $n = 5$, the empty subset contributes one, the singleton subsets contribute five, the pairs contribute $\binom{4}{2} = 6$, and the triples contribute $\binom{3}{3} = 1$. Explicitly, the allowed pairs are

$$\{1,3\}, \{1,4\}, \{1,5\}, \{2,4\}, \{2,5\}, \{3,5\}.$$

The only allowed triple is $\{1,3,5\}$. A subset of four elements is impossible because its smallest possible last element would be 7, outside the ground set.

Step 3: Add and cross-check the total The classes by subset size are disjoint and cover every admissible subset. Therefore

$$n(S_5) = 1 + 5 + 6 + 1 = 13.$$

As an independent check, let T_n count admissible subsets of $\{1, \ldots, n\}$. Those omitting n contribute T_{n-1}; those containing it must omit $n-1$, contributing T_{n-2}. From $T_0 = 1, T_1 = 2$, this gives $T_2 = 3, T_3 = 5, T_4 = 8, T_5 = 13$, agreeing with the direct count.

Step 4: Conclusion The required cardinality is $n(S_5) = \boxed{13}$, a dimensionless count.

Quick Tip to Solve: For subsets with no adjacent integers, either use the recurrence based on including the last element or count size-k subsets by $\binom{n-k+1}{k}$.

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25. Consider the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having one of its focus at $\mathrm{P}(-3, 0)$. If the latus ractum through its other focus subtends a right angle at P and $a^2b^2 = \alpha\sqrt{2} - \beta, \alpha, \beta \in \mathbb{N}$, then $\alpha + \beta$ is _____ .

[4 marks]

Solution:

Concept:

- For the hyperbola $x^2/a^2 - y^2/b^2 = 1$, the focal distance satisfies $c^2 = a^2 + b^2$. The given focus fixes $c = 3$.
- The latus rectum through the positive focus has endpoints $(c, \pm b^2/a)$. Perpendicular vectors have zero scalar product.

Step 1: Use the angle condition Let the other focus be $Q = (3, 0)$, and let the latus-rectum endpoints be $R = (3, b^2/a)$ and $S = (3, -b^2/a)$. Relative to $P = (-3, 0)$, the two vectors are $\overrightarrow{PR} = (6, b^2/a)$ and $\overrightarrow{PS} = (6, -b^2/a)$. Hence the prescribed right angle gives

$$36 - \frac{b^4}{a^2} = 0 \quad \Longrightarrow \quad \frac{b^2}{a} = 6.$$

The positive sign is required because both semiaxis parameters are positive.

Step 2: Find the parameters Substitute $b^2 = 6a$ in $a^2 + b^2 = 9$:

$$a^2 + 6a - 9 = 0, \qquad a = -3 + 3\sqrt{2}.$$

The other root is negative and cannot be a semitransverse axis. Now expand the product without rounding:

$$\begin{aligned} a^2b^2 = 6a^3 &= 162(\sqrt{2} - 1)^3 \\ &= 162(5\sqrt{2} - 7) = 810\sqrt{2} - 1134. \end{aligned}$$

Comparing rational and irrational coefficients identifies the two natural numbers uniquely.

Step 3: Conclusion We obtain $\alpha = 810$, $\beta = 1134$, and therefore $\boxed{\alpha + \beta = 1944}$. This is a dimensionless integer.

Quick Tip to Solve: Use the dot product of the two endpoint vectors to convert the angle condition directly into the semilatus rectum.

Physics – Section A: Multiple Choice Questions (80 marks)

26. If ϵ_0 denotes the permittivity of free space and Φ_E is the flux of the electric field through the area bounded by the closed surface, then dimensions of $\left(\epsilon_0 \frac{d\phi_E}{dt}\right)$ are that of : [4 marks]

(A) electric charge

(B) electric field

(C) electric current

(D) electric potential

Correct Answer: (C) electric current

Solution:

Concept:

- Gauss's law gives $\epsilon_0 \Phi_E = Q_{\text{enc}}$, where Q_{enc} is the enclosed charge.
- Differentiating a quantity with respect to time divides its dimensions by T. Electric current has dimension charge per time.

Step 1: Relate flux to charge The flux in the question is through a closed surface, so it is precisely the flux appearing in Gauss's law. The factor ϵ_0 converts its dimensions into those of charge:

$$[\epsilon_0 \Phi_E] = [Q] = [IT].$$

The value of the enclosed charge, and the geometry of the surface, do not affect this dimensional statement.

Step 2: Include the time derivative Permittivity is a constant, so

$$\left[\epsilon_0 \frac{d\Phi_E}{dt}\right] = \frac{[\epsilon_0 \Phi_E]}{[t]} = \frac{IT}{T} = I.$$

This is also the dimensional form of displacement current. Charge itself retains a factor of time,

so option (A) misses the derivative. Electric field and electric potential contain mass and length dimensions, unlike the result, ruling out (B) and (D).

Step 3: Conclusion The expression has the dimensions of $\boxed{\text{electric current}}$, measured in amperes. Hence, the correct option is (C).

Quick Tip to Solve: For a closed-surface flux, replace $\epsilon_0\Phi_E$ by charge before taking dimensions of its derivative.

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27. The percentage increase in magnetic field (B) when space within a current carrying solenoid is filled with magnesium (magnetic susceptibility $\chi_{Mg} = 1.2 \times 10^{-5}$) is : [4 marks]

(A) $\frac{5}{3} \times 10^{-5}\%$

(B) $\frac{5}{6} \times 10^{-4}\%$

(C) $\frac{5}{6} \times 10^{-5}\%$

(D) $\frac{6}{5} \times 10^{-3}\%$

Correct Answer: (D) $\frac{6}{5} \times 10^{-3}\%$

Solution:

Concept:

- A linear magnetic medium has $M = \chi H$ and $B = \mu_0(H + M) = \mu_0(1 + \chi)H$.
- With the solenoid current unchanged, its magnetising field H is unchanged; compare the new field with the original vacuum field $B_0 = \mu_0 H$.

Step 1: Calculate the fractional change Before filling the solenoid, the field is $B_0 = \mu_0 nI$. After inserting magnesium, it is $B = (1 + \chi_{Mg})B_0$. Consequently,

$$\frac{B - B_0}{B_0} = \chi_{Mg} = 1.2 \times 10^{-5}.$$

The susceptibility is positive, so the field increases: magnesium is weakly paramagnetic. No inverse susceptibility is needed.

Step 2: Convert the fraction to a percentage A percentage is one hundred times the fractional increase. Thus

$$\frac{B - B_0}{B_0} \times 100\% = 1.2 \times 10^{-3}\% = \frac{6}{5} \times 10^{-3}\%.$$

Options (A) and (C) retain an exponent of -5, so they do not make the required conversion to percent. Option (B) has both a different coefficient and exponent and does not equal 0.0012%. Only the fourth expression agrees with the calculated increase.

Step 3: Conclusion The percentage increase is $\boxed{\frac{6}{5} \times 10^{-3}\%}$. Hence, the correct option is (D).

Quick Tip to Solve: At fixed solenoid current, the fractional field increase equals χ; multiply it by 100 to express it as a percentage.

---

28. An object of mass 1000 g experiences a time dependent force $\vec{F} = \left(2t\hat{i} + 3t^2\hat{j}\right) N$. The power generated by the force at time t is: [4 marks]

(A) $\left(2t^2 + 18t^3\right) W$

(B) $\left(3t^3 + 5t^5\right) w$

(C) $\left(2t^3 + 3t^5\right) w$

(D) $\left(2t^2 + 3t^3\right) w$

Correct Answer: (C) $\left(2t^3 + 3t^5\right) w$

Solution:

Concept:

- Instantaneous mechanical power is the scalar product $P = \vec{F} \cdot \vec{v}$, rather than the product of the magnitudes of unrelated components.
- Newton's second law gives $d\vec{v}/dt = \vec{F}/m$; integrate the acceleration component by component. Here $1000\,\text{g} = 1\,\text{kg}$.

Step 1: Integrate the acceleration For $m = 1\,\text{kg}$,

$$\frac{dv_x}{dt} = 2t, \qquad \frac{dv_y}{dt} = 3t^2.$$

The velocity is therefore

$$\vec{v} = (t^2 + v_{0x})\hat{i} + (t^3 + v_{0y})\hat{j}.$$

The listed options use the conventional zero initial velocity, giving $\vec{v} = t^2\hat{i} + t^3\hat{j}$. The printed statement does not explicitly specify this initial condition; keeping the constants shows exactly where it enters.

Step 2: Form the scalar product For that zero-initial-velocity motion, multiply corresponding components:

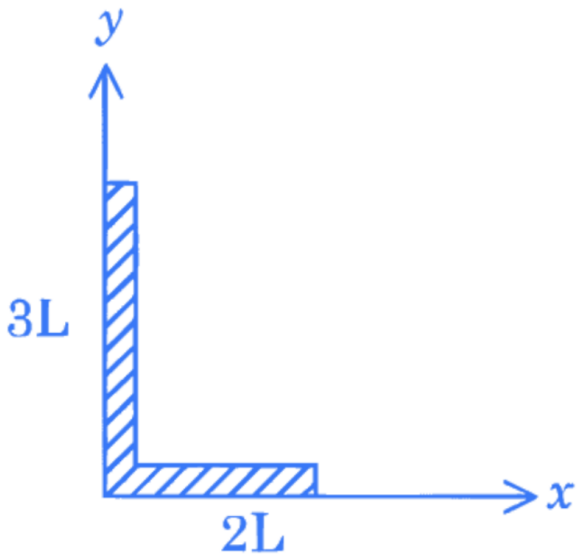
$$P = (2t)(t^2) + (3t^2)(t^3) = 2t^3 + 3t^5.$$

Thus option (C) has the correct exponents and coefficients. Options (A) and (D) do not use the integrated velocity, while (B) changes both coefficients. In general, a nonzero initial velocity would add $2tv_{0x} + 3t^2v_{0y}$ to this power, so the force alone does not determine those extra terms.

Step 3: Conclusion With the zero initial velocity implicit in the options, $\boxed{P = (2t^3 + 3t^5)\,\text{W}}$. Hence, the correct option is (C). Without that initial condition the general result contains the additional velocity-dependent terms shown above.

Quick Tip to Solve: First integrate force divided by mass to obtain velocity, retaining initial-velocity constants when none are explicitly stated.

29. A rod of length 5 L is bent right angle keeping one side length as 2 L .
The position of the centre of mass of the system :
(Consider $\mathrm{L} = 10\ \mathrm{cm}$**)** [4 marks]



(A) $4\hat{i} + 9\hat{j}$
(B) $2\hat{i} + 3\hat{j}$
(C) $5\hat{i} + 8\hat{j}$
(D) $3\hat{i} + 7\hat{j}$
Correct Answer: (A) $4\hat{i} + 9\hat{j}$

Solution:

Concept:

- A uniform rod has mass proportional to length. Treat each straight portion as a separate uniform rod with its mass concentrated at its midpoint.
- For components of masses m_1, m_2, the centre of mass is $\vec{r}_{\mathrm{CM}} = (m_1\vec{r}_1 + m_2\vec{r}_2)/(m_1 + m_2)$.

Step 1: Locate the component centres The figure places the bend at the origin, with a horizontal arm of length $2L$ along positive x and a vertical arm of length $3L$ along positive y. For linear mass density λ, their masses are $2\lambda L$ and $3\lambda L$, respectively. Their midpoints are

$$\vec{r}_1 = L\hat{i}, \qquad \vec{r}_2 = \frac{3L}{2}\hat{j}.$$

The bend carries no additional point mass.

Step 2: Take the mass-weighted average The total mass is $5\lambda L$, so

$$x_{\mathrm{CM}} = \frac{(2\lambda L)L}{5\lambda L} = \frac{2L}{5},$$
$$y_{\mathrm{CM}} = \frac{(3\lambda L)(3L/2)}{5\lambda L} = \frac{9L}{10}.$$

For $L = 10\,\text{cm}$, these coordinates are $(4, 9)\,\text{cm}$. The density cancels, as expected. Equal weighting of the two midpoints would be incorrect because the arms have different lengths and masses. Options (B), (C) and (D) do not reproduce both coordinates.

Step 3: Conclusion The position is $\boxed{\vec{r}_{\text{CM}} = (4\hat{i} + 9\hat{j})\,\text{cm}}$, measured from the bend. Hence, the correct option is (A).

Quick Tip to Solve: For a bent uniform wire, weight each arm's midpoint by its length, not equally by the number of arms.

30. An ac current is represented as

$$i = 5\sqrt{2} + 10\cos\left(650\pi t + \frac{\pi}{6}\right) Amp$$

The r.m.s value of the current is [4 marks]

(A) 10 Amp

(B) $5\sqrt{2}$ Amp

(C) 100 Amp

(D) 50 Amp

Correct Answer: (A) 10 Amp

Solution:

Concept:

- The rms current is $I_{\text{rms}} = \sqrt{\langle i^2 \rangle}$, where the average is taken over a complete cycle.
- Over a cycle, $\langle \cos(\omega t + \phi) \rangle = 0$ and $\langle \cos^2(\omega t + \phi) \rangle = 1/2$, independently of phase.

Step 1: Separate the constant and alternating parts Write the given current as $i = I_0 + I_m \cos\theta$, with $I_0 = 5\sqrt{2}\,\text{A}$, $I_m = 10\,\text{A}$, and $\theta = 650\pi t + \pi/6$. Squaring before averaging gives

$$i^2 = I_0^2 + 2I_0 I_m \cos\theta + I_m^2 \cos^2\theta.$$

The constant term must not be divided by $\sqrt{2}$, since it is present throughout the cycle.

Step 2: Average the square and take its root The cross term averages to zero, while the squared cosine averages to one half:

$$I_{\text{rms}}^2 = I_0^2 + \frac{I_m^2}{2}$$
$$= (5\sqrt{2})^2 + \frac{10^2}{2} = 50 + 50 = 100\,\text{A}^2.$$

Therefore $I_{\text{rms}} = 10\,\text{A}$. Option (B) is only the dc offset or the rms of the alternating component alone. Option (C) gives the mean square as though it were the rms, and (D) is one contribution to that mean square, so neither is the required current.

Step 3: Conclusion The rms current is $\boxed{10\,\text{A}}$. Hence, the correct option is (A).

Quick Tip to Solve: Add the squares of the dc current and the sinusoidal rms current, then take a square root; do not add their rms values directly.

31. Two harmonic waves moving in the same direction superimpose to form a wave $x = \mathrm{a}\cos(1.5\mathrm{t})\cos(50.5\mathrm{t})$ where t is in seconds. Find the period with which they beat. (close to nearest integer) [4 marks]

(A) 1 s

(B) 4 s

(C) 2 s

(D) 6 s

Correct Answer: (C) 2 s

Solution:

Concept:

- The superposition of two nearby angular frequencies obeys $\cos\alpha + \cos\beta = 2\cos[(\alpha-\beta)/2]\cos[(\alpha+\beta)/2]$. The slow factor controls the amplitude envelope.
- A beat is one complete variation of loudness from a maximum through a minimum to the next maximum. Intensity depends on the square of displacement amplitude, so both signs of the envelope give maxima.

Step 1: Identify the two angular frequencies Write the given displacement in its sum form:

$$x = \frac{a}{2}\{\cos(52t) + \cos(49t)\}.$$

Thus the waves have angular frequencies $\omega_1 = 52\,\mathrm{rad\,s^{-1}}$ and $\omega_2 = 49\,\mathrm{rad\,s^{-1}}$. Their mean is $50.5\,\mathrm{rad\,s^{-1}}$, while half their difference is $1.5\,\mathrm{rad\,s^{-1}}$, consistent with both factors in the question.

Step 2: Find the interval between successive beat maxima The frequency difference, rather than the angular-frequency difference itself, is the beat frequency:

$$f_b = \frac{|\omega_1 - \omega_2|}{2\pi} = \frac{3}{2\pi}\,\mathrm{Hz}.$$

Therefore

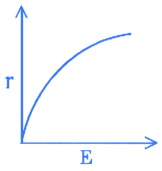
$$T_b = \frac{1}{f_b} = \frac{2\pi}{3}\,\mathrm{s} \approx 2.094\,\mathrm{s}.$$

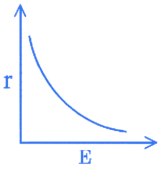
Equivalently, consecutive maxima of $a^2\cos^2(1.5t)$ occur when $1.5t$ changes by π. Using the full signed-envelope period would incorrectly double the beat period. The options 1 s, 4 s and 6 s do not equal the required rounded interval.

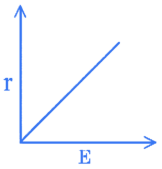
Step 3: Conclusion The beat period is approximately $2.094\,\mathrm{s}$, or $2\,\mathrm{s}$ to the nearest integer. Hence, the correct option is (C).

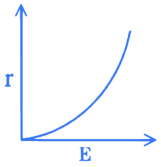
Quick Tip to Solve: For a product envelope $\cos(kt)$, successive loudness maxima are separated by π/k, since intensity involves $\cos^2(kt)$.

32. A particle of charge q, mass m and kinetic energy E enters in magnetic field perpendicular to its velocity and undergoes a circular arc of radius (r). Which of the following curves represents the variation of r with E ? [4 marks]

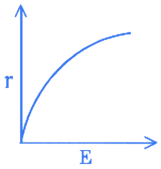
(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- A magnetic force perpendicular to the velocity supplies centripetal force: $|q|vB = mv^2/r$. Here m is the particle mass, $|q|$ its charge magnitude and B the fixed field strength.
- The nonrelativistic kinetic energy is $E = mv^2/2$. A magnetic field changes the direction of velocity without changing the speed or kinetic energy.

Step 1: Eliminate the speed from the radius formula For a nonzero speed, cancelling one factor of v in the force equation gives

$$r = \frac{mv}{|q|B}.$$

From the given kinetic energy,

$$v = \sqrt{\frac{2E}{m}}.$$

Substituting this into the radius expression yields

$$r = \frac{\sqrt{2mE}}{|q|B} = C\sqrt{E}, \qquad C = \frac{\sqrt{2m}}{|q|B} > 0.$$

All quantities in C are constant for the comparison. The sign of the charge determines the turning

direction, not the radius.

Step 2: Compare the shapes of the four curves The radius tends to zero as the energy tends to zero and increases as energy increases. However, it does not increase linearly: quadrupling the energy only doubles the radius. For positive energy,

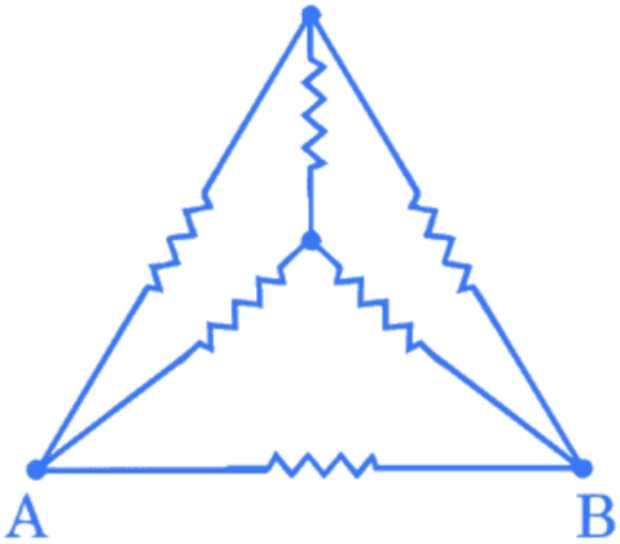
$$\frac{dr}{dE} = \frac{C}{2\sqrt{E}} > 0, \qquad \frac{d^2r}{dE^2} = -\frac{C}{4E^{3/2}} < 0.$$

Thus the graph rises from the origin with a progressively smaller slope, as in curve A. Curve B decreases, curve C has constant slope, and curve D has increasing slope; all three contradict the square-root dependence.

Step 3: Conclusion The required curve is the increasing, concave-down graph through the origin, because $r \propto \sqrt{E}$. Hence, the correct option is (A).

Quick Tip to Solve: First use $r = p/(|q|B)$, then $p = \sqrt{2mE}$. A plot of r^2 against energy would be a straight line, but a plot of r against energy is not.

33. A wire of resistance R is bent into a triangular pyramid as shown in figure with each segment having same length. The resistance between points A and B is R/n. The value of n is : [4 marks]



(A) 16

(B) 10

(C) 12

(D) 14

Correct Answer: (C) 12

Solution:

Concept:

- A triangular pyramid has four vertices and six equal-length edges. For a uniform wire of total resistance R, each of the six segments therefore has resistance $r = R/6$.
- In a symmetric resistor network, vertices that exchange places without changing the terminal connections have equal potential. A resistor joining equipotential points carries no current.

Step 1: Use symmetry to remove the inactive edge Call the other two vertices of the triangular pyramid C and D, keeping A and B as the terminals. Interchanging C and D leaves all six equal edge resistances and both terminal potentials unchanged. Hence $V_C = V_D$. The resistor on edge CD therefore carries zero current, since its potential difference is zero. It can be ignored for calculating the current drawn between A and B; this is a consequence of symmetry, not a short circuit of the whole network.

Step 2: Combine the three conducting branches There are now three independent routes between the terminals: the direct edge AB, of resistance r; the route $A \to C \to B$, of resistance $2r$; and the route $A \to D \to B$, also of resistance $2r$. These routes are in parallel, giving

$$\frac{1}{R_{AB}} = \frac{1}{r} + \frac{1}{2r} + \frac{1}{2r} = \frac{2}{r}.$$

Thus

$$R_{AB} = \frac{r}{2} = \frac{R/6}{2} = \frac{R}{12}.$$

Comparing this result with R/n fixes n uniquely. The values 16, 10 and 14 do not satisfy the parallel conductance sum and cannot represent this six-edge symmetric network.

Step 3: Conclusion The resistance between A and B is $R/12$, so $n = 12$. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish total wire resistance from resistance per edge: divide by six first, then exploit equal-potential vertices before combining branches.

34. Two wires A and B are made of same material having ratio of lengths $\frac{L_A}{L_B} = \frac{1}{3}$ and their diameters ratio $\frac{d_A}{d_B} = 2$. If both the wires are stretched using same force, what would be the ratio of their respective elongations? [4 marks]

(A) $3 : 4$

(B) $1 : 12$

(C) $1 : 3$

(D) $1 : 6$

Correct Answer: (B) $1 : 12$

Solution:

Concept:

- Young's modulus is the ratio of tensile stress to longitudinal strain: $Y = (F/A)/(\Delta L/L)$. Consequently the extension of a uniform wire is $\Delta L = FL/(AY)$.
- The cross-sectional area of a circular wire is $A = \pi d^2/4$. For wires under the same force and made of the same material, the extension is proportional to L/d^2.

Step 1: Form the ratio before substituting Let ΔL_A and ΔL_B denote the elongations. Because the material is the same, the two Young moduli are equal. The common tensile force also cancels:

$$\frac{\Delta L_A}{\Delta L_B} = \frac{FL_A/(A_A Y)}{FL_B/(A_B Y)} = \frac{L_A}{L_B}\frac{A_B}{A_A}.$$

This relation compares actual elongations, not strains: omitting the length ratio would compare a different quantity. The wires are understood to remain within the elastic range where Young's law applies.

Step 2: Convert diameters to areas and evaluate The given diameter ratio is $d_A/d_B = 2$, so

$$\frac{A_A}{A_B} = \left(\frac{d_A}{d_B}\right)^2 = 4, \qquad \frac{A_B}{A_A} = \frac{1}{4}.$$

Using the stated length ratio,

$$\frac{\Delta L_A}{\Delta L_B} = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}.$$

Wire A is both shorter and thicker, and each feature reduces its elongation under the same load. The ratio 1:3 ignores the area difference; 1:6 uses diameter rather than diameter squared. The remaining ratio 3:4 also fails the required product of the two factors.

Step 3: Conclusion The respective elongations are in the ratio $\Delta L_A : \Delta L_B = 1 : 12$. Hence, the correct option is (B).

Quick Tip to Solve: Under equal load and equal material, write $\Delta L \propto L/d^2$. Square a diameter ratio before using it as an area ratio.

35. Two plane polarized light waves combine at a certain point whose electric field components are

$$E_1 = E_0 \,\mathrm{Sin}\,\omega t$$
$$E_2 = E_0 \,\mathrm{Sin}\left(\omega t + \frac{\pi}{3}\right)$$

Find the amplitude of the resultant wave. [4 marks]

(A) 1.7 E_0

(B) E_0

(C) 0.9 E_0

(D) 3.4 E_0

Correct Answer: (A) 1.7 E_0

Solution:

Concept:

- For two harmonic electric fields of the same frequency, their relative phase is constant. The resultant amplitude follows phasor addition: $A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$.
- Here both component amplitudes are E_0, and the relative phase is $\phi = \pi/3$. The field amplitudes add algebraically as functions of time; intensities cannot simply be added without the interference term.

Step 1: Combine the fields using the phase difference The resultant field is

$$E = E_0 \sin\omega t + E_0 \sin(\omega t + \pi/3).$$

Using $\sin u + \sin v = 2\sin[(u+v)/2]\cos[(u-v)/2]$,

$$E = 2E_0 \cos(\pi/6)\sin(\omega t + \pi/6).$$

The time-dependent sine has maximum absolute value one, so the positive coefficient multiplying it is the resultant amplitude. The extra phase $\pi/6$ shifts the oscillation in time but does not alter its maximum magnitude.

Step 2: Evaluate the amplitude and check the options Since $\cos(\pi/6) = \sqrt{3}/2$,

$$A = 2E_0\frac{\sqrt{3}}{2} = \sqrt{3}\,E_0 \approx 1.732E_0.$$

As a check, the phasor formula gives

$$A^2 = E_0^2 + E_0^2 + 2E_0^2\cos(\pi/3) = 3E_0^2.$$

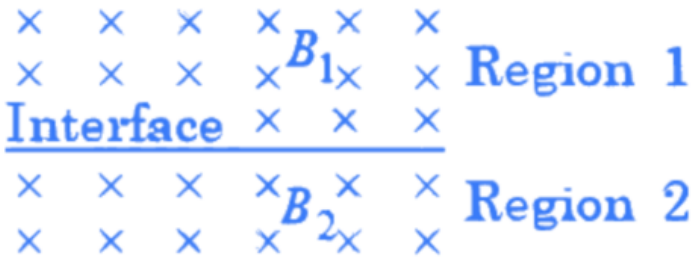
The listed value $1.7E_0$ matches this result to the indicated precision. The values E_0 and $0.9E_0$ do not give the required squared amplitude. The value $3.4E_0$ even exceeds the upper limit $2E_0$ for two equal component amplitudes, so it is impossible.

Step 3: Conclusion The resultant amplitude is $\sqrt{3}\,E_0 \approx 1.7E_0$. Hence, the correct option is (A).

Quick Tip to Solve: For two equal amplitudes, use $A = 2E_0|\cos(\phi/2)|$. Check that the result lies between zero and $2E_0$.

36. Uniform magnetic fields of different strengths (B_1 and B_2), both normal to the plane of the paper exist as shown in the figure. A charged particle of mass m and charge q, at the interface at an instant, moves into the region 2 with velocity v and returns to the interface. It continues to move into region 1 and finally reaches the interface. What is the displacement of the particle during this movement along the interface?

(Consider the velocity of the particle to be normal to the magnetic field and $B_2 > B_1$) [4 marks]



(A) $\frac{mv}{qB_1}\left(1-\frac{B_1}{B_2}\right)\times 2$

(B) $\frac{mv}{qB_1}\left(1-\frac{B_2}{B_1}\right)\times 2$

(C) $\frac{mv}{qB_1}\left(1-\frac{B_2}{B_1}\right)$

(D) $\frac{mv}{qB_1}\left(1-\frac{B_1}{B_2}\right)$

Correct Answer: (A) $\frac{mv}{qB_1}\left(1-\frac{B_1}{B_2}\right)\times 2$

Solution:

Concept:

- A charge moving perpendicular to a uniform magnetic field follows a circular path of radius $r = mv/(|q|B)$. The magnetic force does no work, so the speed remains v in both regions.
- For entry normal to the straight interface, each return to that interface completes a semicircle and shifts the particle along the interface by one diameter. The two semicircles lie on opposite sides of the boundary and produce opposite shifts.

Step 1: Calculate the radii in the two regions For the two field magnitudes,

$$r_1 = \frac{mv}{|q|B_1}, \qquad r_2 = \frac{mv}{|q|B_2}.$$

Since $B_2 > B_1$, the stronger-field semicircle in region 2 has the smaller radius: $r_2 < r_1$. On returning to the interface after the first semicircle, the velocity points into region 1. Because both fields have the same into-the-page direction, the next semicircle curves in the same rotational sense, giving a shift along the interface opposite to the first shift.

Step 2: Subtract the two diameters The first displacement along the interface has magnitude $2r_2$; the second has magnitude $2r_1$ in the opposite direction. Thus the magnitude of the net displacement is

$$d = 2(r_1 - r_2) = \frac{2mv}{|q|}\left(\frac{1}{B_1} - \frac{1}{B_2}\right).$$

Factoring the larger radius gives

$$d = \frac{2mv}{|q|B_1}\left(1 - \frac{B_1}{B_2}\right).$$

With the charge magnitude denoted by q as in the options, this is option A. Option D omits the diameter factor two, while B and C have the reversed field ratio and do not give this positive distance for $B_2 > B_1$. Reversing the charge reverses the direction of the net shift but preserves its magnitude.

Step 3: Conclusion The displacement magnitude is $\frac{mv}{qB_1}\left(1 - \frac{B_1}{B_2}\right) \times 2$, using q for charge magnitude. Hence, the correct option is (A).

Quick Tip to Solve: Across a straight boundary, track each semicircle as a signed diameter. Subtract the opposite shifts rather than adding the arc lengths.

37. Two projectiles are fired from ground with same initial speeds from same point at angles $(45° + \alpha)$ and $(45° - \alpha)$ with horizontal direction. The ratio of their times of flights is [4 marks]

(A) $\frac{1+\tan\alpha}{1-\tan\alpha}$
(B) $\frac{1+\sin 2\alpha}{1-\sin 2\alpha}$
(C) $\frac{1-\tan\alpha}{1+\tan\alpha}$
(D) 1

Correct Answer: (A) $\frac{1+\tan\alpha}{1-\tan\alpha}$

Solution:

Concept:

- For a projectile launched with speed u at angle θ, the vertical component is $u\sin\theta$. With launch and landing at the same height, its flight time is $T = 2u\sin\theta/g$.
- The speeds and gravitational acceleration are identical for the two projectiles. Their ratio of flight times therefore depends only on the sine of their respective launch angles.

Step 1: Write the two flight times Let the first projectile have angle $45° + \alpha$ and the second have angle $45° - \alpha$, preserving the order in the question. Vertical displacement is zero at landing, so $0 = u\sin\theta\, T - gT^2/2$. Taking the nonzero root gives

$$T_1 = \frac{2u}{g}\sin(45° + \alpha), \qquad T_2 = \frac{2u}{g}\sin(45° - \alpha).$$

Step 2: Expand the sines and simplify Using $\sin(45° \pm \alpha) = (\cos\alpha \pm \sin\alpha)/\sqrt{2}$, we obtain

$$\frac{T_1}{T_2} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{1 + \tan\alpha}{1 - \tan\alpha}.$$

For physically upward launches both angles lie between zero and $90°$. Option (C) reverses the requested order. Option (B) is the square of the required ratio, since $\sin^2(45° \pm \alpha) = (1 \pm \sin 2\alpha)/2$. Option (D) holds only for the special case $\alpha = 0$, not generally.

Step 3: Conclusion The required ratio is $\frac{1+\tan\alpha}{1-\tan\alpha}$. Hence, the correct option is (A).

Quick Tip to Solve: Complementary projectile angles give equal ranges, but flight times follow the sine of the angle; do not infer equal times from equal ranges.

38. In a hydrogen like ion, the energy difference between the 2^{nd} excitation energy state and ground is 108.8 eV . The atomic number of the ion is: [4 marks]

(A) 1

(B) 4

(C) 3

(D) 2

Correct Answer: (C) 3

Solution:

Concept:

- The Bohr energy of a one-electron ion is $E_n = -13.6Z^2/n^2\,\text{eV}$, where Z is its atomic number and n is the principal quantum number.
- The ground state is $n = 1$, the first excited state is $n = 2$, and the second excited state is $n = 3$. Excitation energy is the positive difference between the higher level and the ground level.

Step 1: Identify the correct excited level The wording specifies the second excitation state, which is the second excited level, $n = 3$. It does not mean $n = 2$; that is the first excited state. The relevant energies are

$$E_1 = -13.6Z^2\,\text{eV}, \qquad E_3 = -\frac{13.6Z^2}{9}\,\text{eV}.$$

The energy supplied to excite the electron is positive because the bound electron becomes less tightly bound.

Step 2: Use the energy difference to find the atomic number Subtract the ground energy from the excited energy:

$$108.8 = 13.6Z^2\left(1-\frac{1}{9}\right) = 13.6Z^2\frac{8}{9}.$$

Thus

$$Z^2 = \frac{108.8}{13.6}\frac{9}{8} = 8\frac{9}{8} = 9, \qquad Z = 3.$$

Only the positive root is admissible because atomic number counts protons. As a check, $E_1 = -122.4\,\text{eV}$ and $E_3 = -13.6\,\text{eV}$, whose difference is $108.8\,\text{eV}$. The alternatives $Z = 1, 2, 4$ give approximately $12.09, 48.36, 193.42\,\text{eV}$, respectively, and do not fit.

Step 3: Conclusion The atomic number is $Z = 3$, corresponding to the hydrogen-like ion Li^{2+}. Hence, the correct option is (C).

Quick Tip to Solve: Translate excited-state language into the principal quantum number first: second excited means the third energy level.

39. Two thin convex lenses of focal lengths 30 cm and 10 cm are placed coaxially, 10 cm apart. The power of this combination is: [4 marks]

(A) 5 D

(B) 1 D

(C) 20 D

(D) 10 D

Correct Answer: (D) 10 D

Solution:

Concept:

- For two thin lenses of powers P_1 and P_2 separated by distance d in air, the equivalent power is $P = P_1 + P_2 - dP_1P_2$. Use metres for d and dioptres for all powers.
- A convex lens has positive focal length and positive power. Power is the reciprocal of focal length in metres; the separation term accounts for ray propagation between the lenses.

Step 1: Convert the focal lengths and separation The focal lengths are $f_1 = 0.30\,\mathrm{m}$ and $f_2 = 0.10\,\mathrm{m}$, and the separation is $d = 0.10\,\mathrm{m}$. Therefore

$$P_1 = \frac{1}{0.30} = \frac{10}{3}\,\mathrm{D}, \qquad P_2 = \frac{1}{0.10} = 10\,\mathrm{D}.$$

Both signs are positive since both lenses are converging. Simply adding these powers would apply only if the lenses were in contact.

Step 2: Include the separation correction Substitution gives

$$\begin{aligned} P &= \frac{10}{3} + 10 - (0.10)\left(\frac{10}{3}\right)(10) \\ &= \frac{10}{3} + 10 - \frac{10}{3} = 10\,\mathrm{D}. \end{aligned}$$

Equivalently,

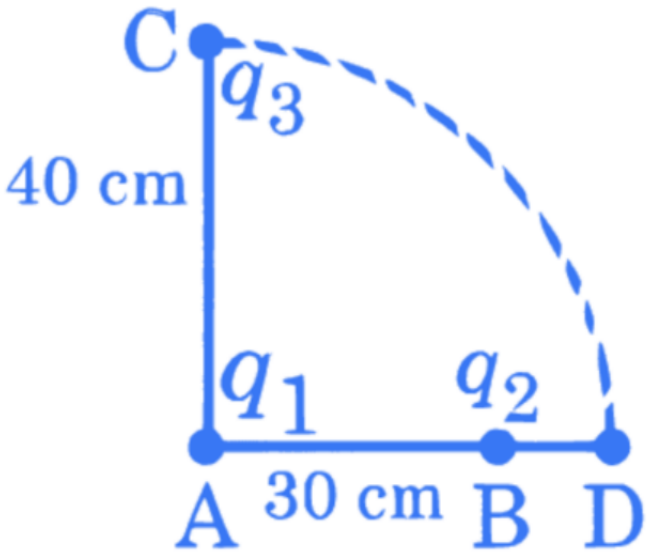
$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = 10\,\mathrm{m}^{-1},$$

so $F = 0.10\,\mathrm{m}$. Here the distance equals the shorter focal length, making the correction cancel the power of the longer lens. The proposed powers 5, 1, and $20\,\mathrm{D}$ fail this separated-lens relation; no change of sign is justified for these two convex lenses.

Step 3: Conclusion The equivalent power of the combination is $+10\,\mathrm{D}$. Hence, the correct option is (D).

Quick Tip to Solve: For separated lenses, use $P_1 + P_2 - dP_1P_2$, with the separation in metres; the contact-lens sum alone is insufficient.

40. Two charges q_1 and q_2 are separated by a distance of 30 cm . A third charge q_3 initially at ' C ' as shown in the figure, is moved along the circular path of radius 40 cm from C to D . If the difference in potential energy due to movement of q_3 from C to D is given by $\frac{q_3\ K}{4\pi\epsilon_0}$, the value of K is : [4 marks]



(A) $6q_2$

(B) $6q_1$

(C) $8q_1$

(D) $8q_2$

Correct Answer: (D) $8q_2$

Solution:

Concept:

- The interaction energy of two point charges is $U = q_i q_j/(4\pi\epsilon_0 r_{ij})$. The change in energy is final minus initial; unchanged pair separations make no contribution.
- Electrostatic forces are conservative, so only the initial and final separations matter. A charge moving on a circle centred at another charge has unchanged interaction energy with that central charge.

Step 1: Calculate the endpoint distances The figure places q_1 at A, q_2 at B, and q_3 initially at C. Since C and D lie on the circle centred at A,

$$AC = AD = 0.40\,\text{m}, \qquad AB = 0.30\,\text{m}.$$

The right triangle ABC gives

$$BC = \sqrt{0.30^2 + 0.40^2} = 0.50\,\text{m}.$$

At D, the distance to B is $BD = AD - AB = 0.10\,\text{m}$, not $0.30\,\text{m}$.

Step 2: Subtract the initial potential energy The $q_1 q_2$ interaction stays constant because neither

charge moves. The q_1q_3 interaction also stays constant because $AC = AD$. Hence

$$\Delta U = \frac{q_3}{4\pi\epsilon_0}\left[q_1\left(\frac{1}{0.40}-\frac{1}{0.40}\right)+q_2\left(\frac{1}{0.10}-\frac{1}{0.50}\right)\right]$$
$$= \frac{q_3}{4\pi\epsilon_0}\left(8q_2\,\mathrm{m}^{-1}\right).$$

Comparing with the given expression identifies the coefficient. Options (B) and (C) incorrectly retain a contribution from q_1. Option (A) gives the wrong reciprocal-distance difference: it is $10-2=8$, not 6.

Step 3: Conclusion In the SI convention used by the options, $K = 8q_2$ (the numerical coefficient carries m^{-1}). Hence, the correct option is (D).

Quick Tip to Solve: Cancel interactions whose separations do not change before calculating; the circular path removes the entire q_1 contribution.

41. A lens having refractive index 1.6 has focal length of 12 cm , when it is in air. Find the focal length of the lens when it is placed in water. (Take refractive index of water as 1.28) [4 marks]

(A) 355 mm

(B) 655 mm

(C) 288 mm

(D) 555 mm

Correct Answer: (C) 288 mm

Solution:

Concept:

- For a thin lens immersed in a medium, lens maker's formula is $1/f = (n_\ell/n_m - 1)(1/R_1 - 1/R_2)$, where n_ℓ and n_m are the absolute refractive indices of lens and medium.
- The lens surfaces keep the same radii when the surrounding medium changes. Consequently its power changes only through the relative refractive-index factor, and its focal length changes inversely with that factor.

Step 1: Find the relative-index factors In air, with refractive index approximately one, the factor multiplying the curvature is

$$\frac{n_\ell}{n_{\text{air}}} - 1 = 1.6 - 1 = 0.6.$$

In water the corresponding factor is

$$\frac{n_\ell}{n_w} - 1 = \frac{1.6}{1.28} - 1 = 1.25 - 1 = 0.25.$$

The lens still converges because its index exceeds that of water, but it is weaker than in air. Its focal length must therefore increase.

Step 2: Take the ratio of lens-maker equations Writing $S = 1/R_1 - 1/R_2$, the two equations are

$1/f_a = 0.6S$ and $1/f_w = 0.25S$. Eliminating the unknown curvature gives

$$\frac{f_w}{f_a} = \frac{0.6}{0.25} = 2.4.$$

Thus

$$f_w = 12 \times 2.4 = 28.8\,\text{cm} = 288\,\text{mm}.$$

The conversion uses $1\,\text{cm} = 10\,\text{mm}$. Values 355, 655, and $555\,\text{mm}$ do not satisfy the required ratio $f_w/f_a = 2.4$. Using only the index difference $1.6 - 1.28$ would omit the division by the surrounding medium's index.

Step 3: Conclusion The focal length in water is $28.8\,\text{cm} = 288\,\text{mm}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the refractive index relative to the surrounding medium, n_ℓ/n_m, rather than replacing it by the difference of absolute indices.

42. For a hydrogen atom, the ratio of the largest wavelength of Lyman series to that of the Balmer series is [4 marks]

(A) 5 : 27

(B) 27 : 5

(C) 3 : 4

(D) 5 : 36

Correct Answer: (A) 5 : 27

Solution:

Concept:

- The Rydberg formula is $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$, with $n_i > n_f$. The Lyman series terminates at $n_f = 1$, and the Balmer series terminates at $n_f = 2$.
- Within a given series, the largest wavelength corresponds to its smallest photon energy. This occurs for the nearest initial level, $n_i = n_f + 1$, not for the series limit $n_i \to \infty$.

Step 1: Identify each longest-wavelength transition For Lyman, the smallest energy transition is $2 \to 1$; for Balmer it is $3 \to 2$. Therefore

$$\frac{1}{\lambda_L} = R_H\left(1 - \frac{1}{4}\right) = \frac{3R_H}{4}, \qquad \lambda_L = \frac{4}{3R_H},$$

$$\frac{1}{\lambda_B} = R_H\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R_H}{36}, \qquad \lambda_B = \frac{36}{5R_H}.$$

The same Rydberg constant is used for both because both emissions are from hydrogen.

Step 2: Form the wavelength ratio in the requested order Dividing Lyman by Balmer, the common constant cancels:

$$\frac{\lambda_L}{\lambda_B} = \frac{4}{3R_H}\frac{5R_H}{36} = \frac{5}{27}.$$

This is less than one, as expected: the Lyman photon has the larger energy and therefore the shorter wavelength. Option (B) reverses the ratio. Options (C) and (D) do not follow from these two reciprocal-wavelength expressions. In particular, $5/36$ is the Balmer inverse-wavelength factor in units of R_H, not the ratio of the wavelengths.

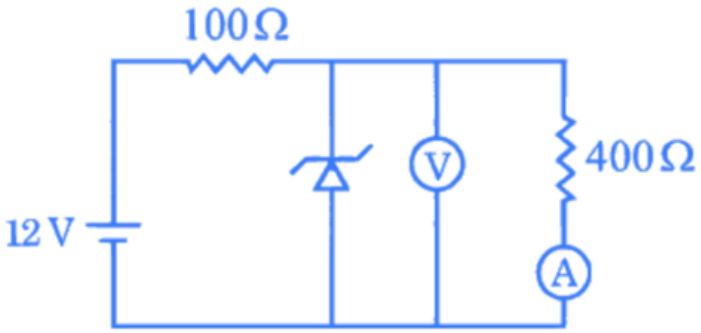
Step 3: Conclusion The ratio of the largest Lyman wavelength to the largest Balmer wavelength is $5 : 27$. Hence, the correct option is (A).

Quick Tip to Solve: Largest wavelength means the first line of a series. Find that transition before applying the Rydberg formula and remember to invert the energy ratio.

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43. In the following circuit, the reading of the ammeter will be

(**Take Zener breakdown voltage** $= 4$ V) [4 marks]



(A) 60 mA

(B) 80 mA

(C) 10 mA

(D) 24 mA

Correct Answer: (C) 10 mA

Solution:

Concept:

- A reverse-biased Zener diode in breakdown maintains its terminal voltage at the breakdown value, provided a positive Zener current is available.
- The load resistor and Zener diode are parallel, so their voltages are equal. Kirchhoff's current law separates the series current into the Zener and load currents.

Step 1: Identify the regulated voltage The cathode of the Zener is connected to the positive supply through the $100\,\Omega$ series resistor. It is therefore reverse biased. Without breakdown the load voltage would be $12(400)/(100+400) = 9.6\,\text{V}$, exceeding $4\,\text{V}$. Thus the diode reaches breakdown and clamps the parallel load voltage to $V_L = 4\,\text{V}$. The ideal voltmeter draws no current.

Step 2: Calculate the current measured by the ammeter The ammeter is in series with the $400\,\Omega$ load,

not with the supply resistor. Hence it measures only the load current:

$$I_A = I_L = \frac{V_L}{400} = \frac{4}{400} = 0.010\,\text{A} = 10\,\text{mA}.$$

The supply current is $I_s = (12-4)/100 = 80\,\text{mA}$, leaving $I_Z = 80 - 10 = 70\,\text{mA} > 0$. This verifies that the assumed breakdown operation is self-consistent.

Step 3: Distinguish the other readings The $80\,\text{mA}$ option is the total supply current, so it is not the current in the load ammeter. The $24\,\text{mA}$ result would follow from ignoring the Zener and treating the two resistors as a simple series circuit. The $60\,\text{mA}$ option does not satisfy Ohm's law for the regulated load voltage.

Step 4: Conclusion The ammeter reads $\boxed{10\,\text{mA}}$. Hence, the correct option is (C).

Quick Tip to Solve: First locate the ammeter branch: a load ammeter measures the load current, whereas the series resistor carries load current plus Zener current.

44. A cubic block of mass m is sliding down on an inclined plane at 60° with an acceleration of $\frac{g}{2}$, the value of coefficient of kinetic friction is [4 marks]

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{\sqrt{2}}{3}$

(C) $1 - \frac{\sqrt{3}}{2}$

(D) $\sqrt{3} - 1$

Correct Answer: (D) $\sqrt{3} - 1$

Solution:

Concept:

- Resolve gravity into $mg\sin\theta$ along the slope and $mg\cos\theta$ perpendicular to it. There is no acceleration normal to the plane.
- Kinetic friction has magnitude $f_k = \mu_k N$ and opposes the sliding motion. Use Newton's second law along the slope with downhill positive.

Step 1: Resolve the forces on the block The block slides downhill, so kinetic friction acts uphill. The normal reaction is $N = mg\cos 60^\circ = mg/2$. The downhill gravitational component is $mg\sin 60^\circ = \sqrt{3}mg/2$. The cubic shape does not change these force components in the stated sliding motion; no rolling equation is needed.

Step 2: Apply the given acceleration With downhill chosen positive, the net force equation is

$$mg\sin 60^\circ - \mu_k mg\cos 60^\circ = ma.$$

Substitute $a = g/2$ and the trigonometric values:

$$\frac{\sqrt{3}}{2}mg - \frac{\mu_k}{2}mg = \frac{1}{2}mg.$$

Cancelling the common nonzero factor $mg/2$ gives $\sqrt{3} - \mu_k = 1$, and therefore $\mu_k = \sqrt{3} - 1$.

Step 3: Check the sign and the alternatives This value is positive and less than $\tan 60^\circ = \sqrt{3}$, so the resultant acceleration is downhill, as required. Inserting it back gives $a/g = (\sqrt{3} - (\sqrt{3} - 1))/2 = 1/2$. Options (A), (B), and (C) give respectively $\sqrt{3}/4$, $(\sqrt{3} - \sqrt{2}/3)/2$, and $3\sqrt{3}/4 - 1/2$ for a/g; none equals $1/2$.

Step 4: Conclusion $\boxed{\mu_k = \sqrt{3} - 1}$, a dimensionless coefficient. Hence, the correct option is (D).

Quick Tip to Solve: For downhill sliding, write $a/g = \sin\theta - \mu_k \cos\theta$; solving this directly avoids reversing the friction sign.

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45. Match the List I with List II

	List - I		List - II
(A)	**Triatomic rigid gas**	**(I)**	$\frac{C_p}{C_v} = \frac{5}{3}$
(B)	**Diatomic non-rigid gas**	**(II)**	$\frac{C_p}{C_v} = \frac{7}{5}$
(C)	**Monoatomic gas**	**(III)**	$\frac{C_p}{C_v} = \frac{4}{3}$
(D)	**Diatomic rigid gas**	**(IV)**	$\frac{C_p}{C_v} = \frac{9}{7}$

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-IV, C-I, D-II

(B) A-II, B-IV, C-I, D-III

(C) A-IV, B-II, C-III, D-I

(D) A-III, B-II, C-IV, D-I

Correct Answer: (A) A-III, B-IV, C-I, D-II

Solution:

Concept:

- For an ideal gas with f active quadratic degrees of freedom, $C_v = fR/2$ and $C_p = C_v + R$, so $\gamma = C_p/C_v = (f+2)/f$.
- A monatomic gas has three translational degrees of freedom. Rotations and active vibrations add degrees of freedom; each vibrational mode contributes two quadratic terms.

Step 1: Assign the degrees of freedom For the rigid triatomic gas in the matching list, use the standard non-linear rigid-molecule model: three translational and three rotational degrees of freedom, giving $f = 6$. A diatomic non-rigid molecule has three translational, two rotational, and two vibrational contributions, giving $f = 7$. A monatomic molecule has $f = 3$. A rigid diatomic molecule has three translational and two rotational contributions, giving $f = 5$.

Step 2: Evaluate each heat-capacity ratio Apply the same formula separately to all four entries:

$$\begin{aligned} A: \quad \gamma &= \frac{6+2}{6} = \frac{4}{3} \quad \longrightarrow \text{III}, \\ B: \quad \gamma &= \frac{7+2}{7} = \frac{9}{7} \quad \longrightarrow \text{IV}, \\ C: \quad \gamma &= \frac{3+2}{3} = \frac{5}{3} \quad \longrightarrow \text{I}, \\ D: \quad \gamma &= \frac{5+2}{5} = \frac{7}{5} \quad \longrightarrow \text{II}. \end{aligned}$$

The ratios are dimensionless; the common molar gas constant cancels.

Step 3: Compare the complete matching codes The complete pairing is A-III, B-IV, C-I, D-II. Only option (A) preserves every one of these matches. Option (B) incorrectly gives the rigid triatomic gas the diatomic rigid ratio. Options (C) and (D) fail even the monatomic match, which must be $5/3$. Increasing the number of active degrees of freedom also lowers γ, consistent with the calculated order.

Step 4: Conclusion The matching is $\boxed{\text{A}-\text{III},\ \text{B}-\text{IV},\ \text{C}-\text{I},\ \text{D}-\text{II}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use $\gamma = 1 + 2/f$ and anchor the monatomic entry at $5/3$; count one active vibration as two degrees of freedom.

Physics – Section B: Numerical Value Questions (20 marks)

46. A wire of length 10 cm and diameter 0.5 mm is used in a bulb. The temperature of the wire is 1727°C and power radiated by the wire is 94.2 W . Its emissivity is $\frac{x}{8}$ where $x =$ _____.
(Given $\sigma = 6.0 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4}, \pi = 3.14$ and assume that the emissivity of wire material is same at all wavelength.) [4 marks]

Solution:

Concept:

- Stefan–Boltzmann radiation from an emitting surface is $P = e\sigma AT^4$, with emissivity e, emitting area A, and absolute temperature T.
- A long thin wire has cylindrical lateral area $A = \pi dL$. Its small end areas are neglected in the filament model. Emissivity and the parameter x are dimensionless.

Step 1: Convert temperature and dimensions The absolute temperature, not the Celsius reading, enters the fourth power:

$$T = 1727 + 273 = 2000\,\text{K}.$$

Convert the length and diameter into SI units: $L = 10\,\text{cm} = 0.10\,\text{m}$ and $d = 0.5\,\text{mm} = 5.0 \times 10^{-4}\,\text{m}$. The lateral emitting surface is therefore

$$A = \pi dL = 3.14(5.0 \times 10^{-4})(0.10) = 1.57 \times 10^{-4}\,\text{m}^2.$$

Step 2: Calculate the black-body power for this area Using the stated Stefan–Boltzmann constant

and temperature,

$$\begin{aligned} P_{\text{black}} &= \sigma A T^4 \\ &= (6.0 \times 10^{-8})(1.57 \times 10^{-4})(2000)^4 \\ &= (6.0 \times 10^{-8})(1.57 \times 10^{-4})(1.6 \times 10^{13}) \\ &= 150.72\,\text{W}. \end{aligned}$$

The given radiated power is lower because the wire is not a perfect black body. No ambient-temperature correction is specified in the radiation data.

Step 3: Extract emissivity and the requested parameter Now divide the measured power by the black-body value:

$$e = \frac{94.2}{150.72} = 0.625 = \frac{5}{8}.$$

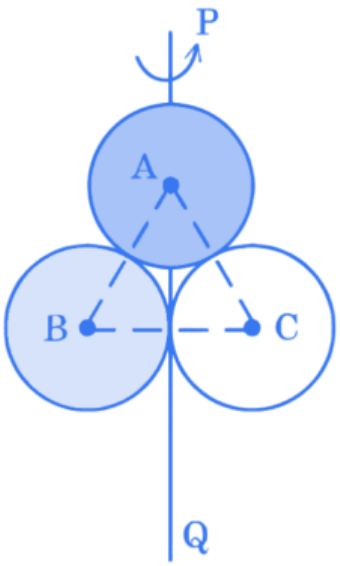
Since the question defines $e = x/8$, comparison of numerators gives $x = 5$. The result obeys the required physical range $0 < e < 1$. Using a cross-sectional area instead of the lateral area would not describe the radiating wire surface.

Step 4: Conclusion The emissivity is $5/8$, so $\boxed{x = 5}$ (dimensionless).

Quick Tip to Solve: For a hot thin filament use πdL, not $\pi d^2/4$, and convert Celsius to kelvin before raising the temperature to the fourth power.

47. A, B **and C are disc, solid sphere and spherical shell respectively with same radii and masses. These masses are placed as shown in figure.**

The moment of inertia of the given system about PQ axis is $\frac{x}{15}\text{I}$, where I is the moment of inertia of the disc about its diameter. The value of x is _____. [4 marks]



Solution:

Concept:

- Moments of inertia of rigid bodies about the same axis add. The parallel-axis theorem is $I = I_{\text{CM}} + Md^2$, where d is the perpendicular axis separation.
- For a disc about a diameter, $I_d = MR^2/4$. For a solid sphere and a thin spherical shell about central diameters, the values are $2MR^2/5$ and $2MR^2/3$, respectively.

Step 1: Locate each centre relative to the axis Let each object have mass M and radius R. In the figure, PQ passes through the centre of disc A and the contact point of the two lower spheres. The centres of B and C are horizontally one radius to the left and right of PQ. Their perpendicular distances from the rotation axis are consequently both R. The tilted dashed centre-to-centre lines do not represent the distances to the vertical axis.

Step 2: Calculate all three contributions The axis is a diameter of disc A, so its contribution is $MR^2/4$. For solid sphere B, shift a parallel central diameter by R:

$$I_B = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2.$$

The same shift for the spherical shell C gives

$$I_C = \frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2.$$

Each shift is positive, as moving an axis away from a centre must increase its moment of inertia.

Step 3: Express the sum using the stated reference inertia Adding the three bodies and taking a common denominator,

$$I_{PQ} = \left(\frac{1}{4} + \frac{7}{5} + \frac{5}{3}\right) MR^2 = \frac{15 + 84 + 100}{60} MR^2 = \frac{199}{60} MR^2.$$

The question denotes the disc diameter inertia by $I = MR^2/4$, hence $MR^2 = 4I$. Thus

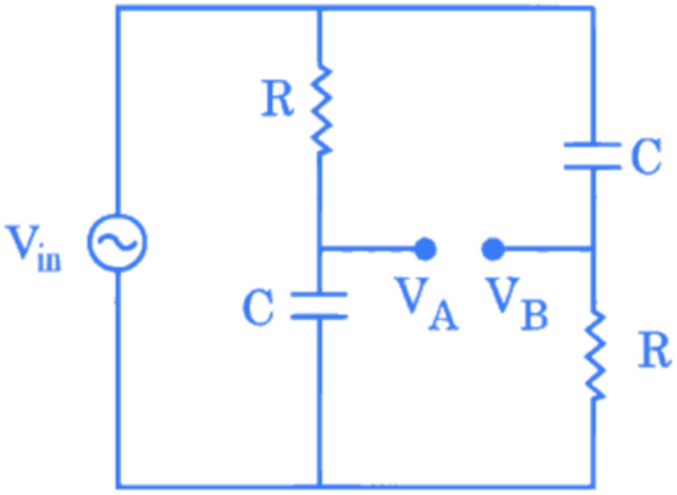
$$I_{PQ} = \frac{199}{60}(4I) = \frac{199}{15} I.$$

Comparing this with $(x/15)I$ yields the requested dimensionless numerator.

Step 4: Conclusion $\boxed{x = 199}$ (dimensionless), with $I_{PQ} = (199/15)I$.

Quick Tip to Solve: In the parallel-axis term use the perpendicular centre-to-axis distance, which is R for each lower sphere here, rather than the full separation between centres.

48. For ac circuit shown in figure, $\mathrm{R} = 100\mathrm{k}\Omega$ and $\mathrm{C} = 100\mathrm{pF}$ and the phase difference between $\mathrm{V_{in}}$ and $(\mathrm{V_B} - \mathrm{V_A})$ is $90°$. The input signal frequency is $10^x \mathrm{rad/sec}$, where ' x ' is _____ . [4 marks]



Solution:

Concept:

- For sinusoidal steady state, use complex impedances $Z_R = R$ and $Z_C = 1/(j\omega C)$, with $j^2 = -1$ and angular frequency ω.
- Each arm is a voltage divider across the same input. A phase difference of $90°$ means that the output-to-input transfer ratio is purely imaginary and nonzero.

Step 1: Write the two midpoint voltages Take the lower rail as zero potential and the upper rail as V_{in}. The left midpoint A is across the lower capacitor; the right midpoint B is across the lower resistor. With $u = \omega RC > 0$, their phasors are

$$V_A = \frac{Z_C}{R + Z_C} V_{\text{in}} = \frac{1}{1 + ju} V_{\text{in}}, \qquad V_B = \frac{R}{R + Z_C} V_{\text{in}} = \frac{ju}{1 + ju} V_{\text{in}}.$$

The common series impedance in the two arms makes the subtraction direct.

Step 2: Apply the quadrature condition The requested output is $V_B - V_A$, so

$$H = \frac{V_B - V_A}{V_{\text{in}}} = \frac{ju - 1}{1 + ju} = \frac{u^2 - 1 + 2ju}{1 + u^2}.$$

For H to have phase $90°$, its real part must vanish:

$$\frac{u^2 - 1}{1 + u^2} = 0 \quad \Rightarrow \quad u = 1.$$

The negative root is excluded since frequency, resistance, and capacitance are positive. At $u = 1$, $H = j$, explicitly confirming a positive right-angle phase difference.

Step 3: Substitute resistance and capacitance Use $R = 100\,\mathrm{k}\Omega = 10^5\,\Omega$ and $C = 100\,\mathrm{pF} = 10^{-10}\,\mathrm{F}$.

Therefore

$$RC = 10^{-5}\,\mathrm{s}, \qquad \omega = \frac{1}{RC} = 10^{5}\,\mathrm{rad\,s^{-1}}.$$

The question expresses the angular frequency as $10^{x}\,\mathrm{rad\,s^{-1}}$, so comparing powers of ten gives $x = 5$. No factor of 2π is needed because the supplied unit is radians per second, not hertz.

Step 4: Conclusion The input angular frequency is $10^{5}\,\mathrm{rad\,s^{-1}}$, so $\boxed{x = 5}$ (dimensionless).

Quick Tip to Solve: For this swapped RC divider bridge, subtract the midpoint phasors first; quadrature occurs at $\omega RC = 1$.

---

49. A container contains a liquid with refractive index of 1.2 up to a height of 60 cm and another liquid having refractive index 1.6 is added to height H above first liquid. If viewed from above, the apparent shift in the position of bottom of container is 40 cm . The value of H is _____ cm . (Consider liquids are immisible) [4 marks]

Solution:

Concept:

- For near-normal viewing from air through plane horizontal interfaces, each transparent layer of thickness t and refractive index μ contributes an apparent thickness t/μ. The observer sees the bottom closer to the surface than its real position.
- The apparent upward shift is real depth minus apparent depth. For several layers, their separate shifts add: $s = \sum t_i(1 - 1/\mu_i)$. All thicknesses and the shift must use the same length unit.

Step 1: Write the real and apparent depths The two liquids remain in separate layers, so the actual depth of the bottom below the upper surface is $60 + H$ cm. The apparent depth measured from that same surface is

$$d_{\mathrm{app}} = \frac{60}{1.2} + \frac{H}{1.6} = 50 + \frac{5H}{8} \quad \text{cm}.$$

The lower layer is viewed through the upper one, but successive refraction still gives this sum for near-normal rays. One must not apply a single average refractive index to the total depth.

Step 2: Use the specified shift to determine the upper thickness The stated upward displacement of the bottom is 40 cm. Subtract the apparent depth from the real depth, keeping the reference surface unchanged:

$$\begin{aligned} 40 &= (60 + H) - \left(50 + \frac{5H}{8}\right) \\ &= 10 + \frac{3H}{8}. \end{aligned}$$

The first liquid alone contributes a shift of 10 cm. The remaining 30 cm must therefore be due to the second liquid:

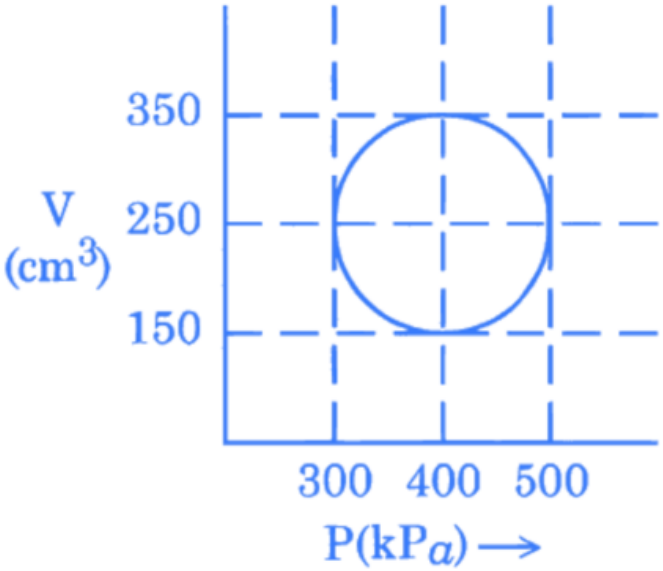
$$\frac{3H}{8} = 30, \qquad H = 80\ \mathrm{cm}.$$

As a direct check, the real depth is 140 cm and the apparent depth is $50 + 80/1.6 = 100$ cm, giving the required difference of 40 cm.

Step 3: Conclusion The height of the second liquid is $\boxed{H = 80\ \mathrm{cm}}$. Thus the numerical entry is 80.

Quick Tip to Solve: Compute each layer's shift separately: here the first contributes 10 cm, leaving 30 cm for the second layer.

50. An ideal gas has undergone through the cyclic process as shown in the figure. Work done by the gas in the entire cycle is _____ $\times 10^{-1}$ J. (Take $\pi = 3.14$) [4 marks]



Solution:

Concept:

- For a quasi-static gas process, the work is $W = \int P\,dV$. For a complete cycle, its magnitude is the area enclosed by the pressure-volume curve; a closed loop need not have zero work even though its endpoints coincide.
- The plotted horizontal coordinate is pressure P in kPa and the vertical coordinate is volume V in cm^3. For an ellipse, the area in these physical coordinates is πab, where a and b are the two semi-axis increments.

Step 1: Read the pressure and volume semi-axes The pressure extrema are 300 and 500 kPa, with centre 400 kPa. The volume extrema are 150 and 350 cm^3, with centre 250 cm^3. Consequently,

$$a = \frac{500 - 300}{2} = 100\ \mathrm{kPa},$$

$$b = \frac{350 - 150}{2} = 100\ \mathrm{cm}^3.$$

These are semi-axes, not full widths. Although the loop looks circular on the printed graph, pressure and volume have different units, so both coordinate increments must be retained.

Step 2: Convert the enclosed area to work in joules Using the specified value of π, the enclosed area gives the required work magnitude:

$$|W| = \pi ab = 3.14(100)(100) = 31400\ \mathrm{kPa\,cm}^3.$$

Convert both units before interpreting this as energy:

$$1\ \mathrm{kPa\,cm}^3 = 10^3\ \mathrm{Pa} \times 10^{-6}\ \mathrm{m}^3 = 10^{-3}\ \mathrm{J}.$$

Hence

$$|W| = 31400 \times 10^{-3} = 31.4 \text{ J} = 314 \times 10^{-1} \text{ J}.$$

The constant offsets of 400 kPa and 250 cm^3 locate the centre but do not change the enclosed area; using the coordinates of the centre as semi-axes would overestimate the work.

Step 3: Conclusion The enclosed-area work is $\boxed{31.4 \text{ J} = 314 \times 10^{-1} \text{ J}}$. The requested numerical entry is 314.

Quick Tip to Solve: Read half the range on each axis, then multiply by pi; remember that one kPa times one cubic centimetre is only one millijoule.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. An aqueous solution of HCl with pH 1.0 is diluted by adding equal volume of water (ignoring dissociation of water). The pH of HCl solution would

(**Given** $\log 2 = 0.30$) [4 marks]

(A) increase to 1.3

(B) reduce to 0.5

(C) increase to 2

(D) remain same

Correct Answer: (A) increase to 1.3

Solution:

Concept:

- Hydrochloric acid is a strong acid and is treated as fully ionized in this question. Ignoring the ionization of water, its hydrogen-ion concentration is therefore the analytical acid concentration.
- The pH is $-\log_{10}[\text{H}^+]$. Dilution conserves the number of moles of acid while increasing solution volume, so $c_1V_1 = c_2V_2$; pH itself does not obey that inverse-volume relation.

Step 1: Determine the new hydrogen-ion concentration The initial pH is 1.0. Therefore

$$[\text{H}^+]_1 = 10^{-1} \text{ mol L}^{-1}.$$

Let the starting volume be V. Adding an equal volume of water makes the final volume $2V$, while the moles of hydrogen ions contributed by the acid are unchanged. Thus

$$[\text{H}^+]_2 = \frac{10^{-1}V}{2V} = \frac{10^{-1}}{2} = 5.0 \times 10^{-2} \text{ mol L}^{-1}.$$

This remains sufficiently acidic for the stated neglect of water ionization.

Step 2: Calculate the logarithm and compare all options Take the negative common logarithm of the

concentration, rather than halving the pH:

$$\mathrm{pH}_2 = -\log_{10}\left(\frac{10^{-1}}{2}\right)$$
$$= 1 + \log_{10} 2 = 1 + 0.30 = 1.30.$$

Thus dilution raises the pH by 0.30. Option (B) incorrectly lowers the pH to 0.5, which would imply a more concentrated acid. Option (C), pH 2, requires a tenfold decrease of hydrogen-ion concentration, whereas the actual decrease is only twofold. Option (D) ignores that the volume, and hence concentration, has changed. Only option (A) gives the correct direction and numerical change.

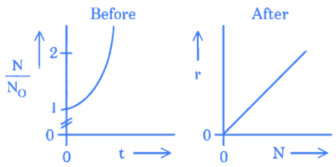
Step 3: Conclusion The pH increases to $\boxed{1.3}$. Hence, the correct option is (A).

Quick Tip to Solve: For an acid diluted by a factor D, add log D to its pH; doubling the volume adds 0.30, rather than doubling or halving the pH.

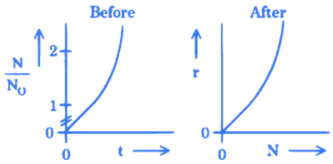
52. A person's wound was exposed to some bacteria and then bacterial growth started to happen at the same place. The wound was later treated with some antibacterial medicine and the rate of bacterial decay(r) was found to be proportional with the square of the existing number of bacteria at any instance. Which of the following set of graphs correctly represents the 'before' and 'after' situation of the application of the medicine?

[Given: N = No. of bacteria, t = time, bacterial growth follows 1^{st} order kinetics.] [4 marks]

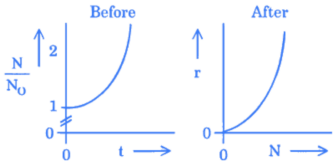
(A)



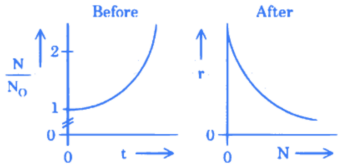
(B)



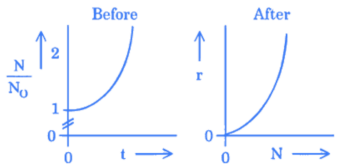
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- First-order bacterial growth means $dN/dt = k_g N$, where N is the bacterial population and $k_g > 0$. Integrating gives $N = N_0 e^{k_g t}$, so the normalized population starts at one, not zero.
- The positive rate of bacterial disappearance after treatment is $r = -dN/dt = k_d N^2$, with $k_d > 0$. A graph of this rate against the current population is an upward-opening parabola through the origin.

Step 1: Identify the correct graph before treatment Before medicine is applied, separate variables in the growth law:

$$\frac{dN}{N} = k_g\, dt, \qquad \ln \frac{N}{N_0} = k_g t.$$

Hence

$$\frac{N}{N_0} = e^{k_g t}.$$

At $t = 0$, this ratio is exactly 1, and it increases with an increasing slope for positive time. The graph must therefore be exponential and pass through $(0, 1)$. The left-hand graph in option (B) starts at zero and is inconsistent with the normalization, even though it rises with time.

Step 2: Match the rate-population relation after treatment After the antibacterial medicine is administered, the question gives $r \propto N^2$, or

$$r = k_d N^2.$$

The rate is zero when no bacteria remain, increases as the population increases, and has slope $dr/dN = 2k_d N$, which is zero at the origin and grows thereafter. Option (C) combines this curve with the correct earlier exponential growth. Option (A) uses a straight line after treatment, corresponding to a first-order decay law instead. Option (D) makes the decay rate decrease with increasing population, contrary to the square dependence. Do not confuse a graph of rate versus population with population versus elapsed time: the latter decreases during treatment, but the former increases.

Step 3: Conclusion The correct pair is exponential growth from $N/N_0 = 1$, followed by $r \propto N^2$. Hence, the correct option is (C).

Quick Tip to Solve: Check axis labels before interpreting a decay graph: rate versus population rises for r proportional to N squared, even while population falls with time.

53. Total enthalpy change for freezing of 1 mol of water at $10^\circ C$ to ice at $-10^\circ C$ is _____

(Given: $\Delta_{fus}\, H = x$ kJ/mol

$$C_p\,[H_2O(l)] = y \text{ J mol}^{-1} \text{ K}^{-1}$$
$$C_p\,[H_2O(s)] = z \text{ J mol}^{-1} \text{ K}^{-1}$$

[4 marks]

(A) $-x - 10y - 10z$

(B) $x - 10y - 10z$

(C) $-10(100x + y + z)$

(D) $10(100x + y + z)$

Correct Answer: (C) $-10(100x + y + z)$

Solution:

Concept:

- Enthalpy is a state function. Evaluate the change using three steps: cool liquid water to its freezing point, freeze it there, and cool the resulting ice to its final temperature.
- For a temperature change at constant pressure, $\Delta H = nC_p\Delta T$. Freezing is the reverse of fusion, so its molar enthalpy change is negative. Convert the given fusion enthalpy from kJ to J before adding it to the sensible-heat terms.

Step 1: Cool the liquid and reverse the fusion process For one mole of liquid water cooling from $10^\circ C$ to $0^\circ C$, the temperature change is -10 K. Therefore

$$\Delta H_1 = (1)y(-10) = -10y \text{ J}.$$

At $0^\circ C$, freezing releases the energy that fusion would absorb. Since the given molar fusion enthalpy is x kJ mol^{-1},

$$\Delta H_2 = -x \text{ kJ} = -1000x \text{ J}.$$

The minus sign reflects heat release by the sample, not heat absorbed by its surroundings.

Step 2: Cool the ice and add the three contributions The ice then cools from $0^\circ C$ to $-10^\circ C$. Its heat capacity is z J mol^{-1} K^{-1}, giving

$$\Delta H_3 = (1)z(-10) = -10z \text{ J}.$$

Add all terms in joules:

$$\Delta H = -10y - 1000x - 10z$$
$$= -10(100x + y + z) \text{ J}.$$

Option (A) fails to convert x from kilojoules to joules. Option (B) additionally assigns the wrong sign to freezing. Option (D) makes the total positive, although cooling and freezing both release heat for positive heat capacities and fusion enthalpy. Thus only option (C) respects units and signs throughout.

Step 3: Conclusion The total enthalpy change for the one-mole sample is $\boxed{-10(100x + y + z) \text{ J}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use one energy unit throughout: the fusion term contains 1000x joules, whereas each ten-degree cooling term contains only 10 times its heat capacity.

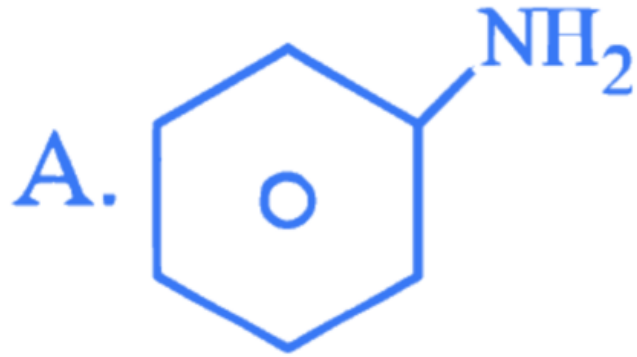
54. Which of the following amine (s) show (s) positive carbylamine test?

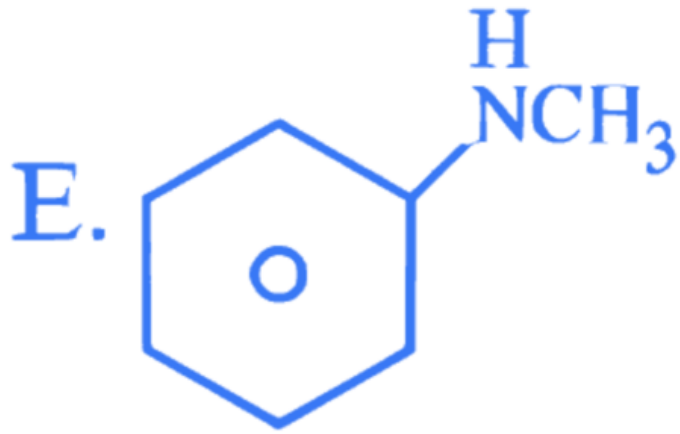
B. $(CH_3)_2\,NH$

C. CH_3NH_2

D. $(CH_3)_3\ N$

Choose the correct answer from the options given below: [4 marks]

A. 

E. 

(A) C Only

(B) B, C and D Only

(C) A and E Only

(D) A and C Only

Correct Answer: (D) A and C Only

Solution:

Concept:

- The carbylamine test is given by primary amines, whether aliphatic or aromatic. Heating a primary amine with chloroform and alcoholic potassium hydroxide produces a foul-smelling isocyanide.
- Classify an amine by the number of carbon-containing groups directly attached to nitrogen. A primary amine has $-NH_2$; a secondary amine has $-NH-$ between two such groups; a tertiary amine has no nitrogen-hydrogen bond.

Step 1: Classify the five listed compounds Compound A is aniline, $C_6H_5NH_2$. Its nitrogen is attached

to one phenyl group and two hydrogen atoms, so it is a primary aromatic amine. Compound B, $(CH_3)_2NH$, is secondary because two methyl groups are directly bonded to nitrogen. Compound C, CH_3NH_2, is a primary aliphatic amine. Compound D, $(CH_3)_3N$, is tertiary. Compound E is N-methylaniline, $C_6H_5NHCH_3$; its nitrogen bears both phenyl and methyl groups and is therefore secondary despite retaining one hydrogen.

Step 2: Apply the test and eliminate the alternatives The characteristic reaction for a primary amine is

$$RNH_2 + CHCl_3 + 3\,KOH \longrightarrow RNC + 3\,KCl + 3\,H_2O.$$

Here R may be an alkyl group or a phenyl group. Consequently, both A and C form isocyanides and give a positive test, while B, D and E do not. Option (A) omits aniline, which is a primary amine and does respond. Option (B) incorrectly includes secondary B and tertiary D, and also omits A. Option (C) incorrectly includes secondary E and omits methylamine C. The aromatic ring does not prevent a primary amino group from undergoing this test; the deciding feature is the substitution at nitrogen.

Step 3: Conclusion Only A (aniline) and C (methylamine) give a positive carbylamine test. Hence, the correct option is (D).

Quick Tip to Solve: Count groups bonded directly to nitrogen: both alkyl and phenyl count. Having one N-H bond alone does not make an amine primary.

55. Which of the following compounds is least likely to give effervescence of CO_2 in presence of aq. $NaHCO_3$? [4 marks]

(A) Anilinium chloride, $C_6H_5NH_3^+\,Cl^-$

(B) 4-Nitrobenzoic acid, $4\text{-}NO_2{-}C_6H_4{-}COOH$

(C) 2,4,6-Trinitrophenol (picric acid)

(D) 3-Nitrophenol (m-nitrophenol)

Correct Answer: (D) 3-Nitrophenol (m-nitrophenol)

Solution:

Concept:

- An acid liberates carbon dioxide from aqueous bicarbonate when proton transfer to HCO_3^- is sufficiently favourable. The products are the conjugate base, water and CO_2.
- Compare acid strengths using pK_a: a smaller value means a stronger acid. Carbonic acid has a first pK_a near 6.35, so ordinary weak phenols generally fail the bicarbonate test.

Step 1: Identify the acidic species Option (A) is anilinium chloride. Its acidic species is $C_6H_5NH_3^+$, not neutral aniline. Its pK_a is about 4.6. Option (B) is a nitro-substituted carboxylic acid; its carboxyl group is sufficiently acidic to react readily with bicarbonate.

Step 2: Compare the two nitrophenols Option (C), picric acid, contains three electron-withdrawing nitro groups at the ortho and para positions relative to the hydroxyl group. They strongly stabilise its conjugate base, so picric acid also liberates carbon dioxide. In option (D), the single nitro group is meta to the hydroxyl group. It exerts an inductive effect but cannot provide the same direct resonance

stabilisation of phenoxide as an ortho or para nitro group. Its pK_a is around 8.4, making it weaker than carbonic acid.

Step 3: Apply the bicarbonate test The relevant proton-transfer equilibrium is

$$\mathrm{ArOH} + \mathrm{HCO_3^-} \rightleftharpoons \mathrm{ArO^-} + \mathrm{H_2CO_3}.$$

For 3-nitrophenol it is unfavourable because the acid on the left is weaker. The alternatives (A), (B) and (C) are appreciably stronger acids and therefore do not fit the requested least-likely behaviour.

Step 4: Conclusion 3-Nitrophenol is least likely to give effervescence with aqueous $\mathrm{NaHCO_3}$. Hence, the correct option is (D).

Quick Tip to Solve: Do not treat an ammonium salt as its neutral amine: first identify the proton-donating species, then compare its acidity with carbonic acid.

56. Match the List I with List II.

	List I: Molecule/Ion		List II: Bond pair : lone pair (on the central atom)
(A)	$\mathrm{ICl_2^-}$	**(I)**	4 : 2
(B)	$\mathrm{H_2O}$	**(II)**	4 : 1
(C)	$\mathrm{SO_2}$	**(III)**	2 : 3
(D)	$\mathrm{XeF_4}$	**(IV)**	2 : 2

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-III, C-II, D-I

(B) A-III, B-IV, C-I, D-II

(C) A-II, B-I, C-IV, D-III

(D) A-III, B-IV, C-II, D-I

Correct Answer: (D) A-III, B-IV, C-II, D-I

Solution:

Concept:

- A Lewis structure distributes valence electrons into bonding pairs and lone pairs. For this matching question, a double bond contains two shared electron pairs; bond-pair counting is distinct from counting VSEPR electron domains.
- Count lone pairs on the central atom only. Terminal-atom lone pairs do not enter the listed ratio.

Step 1: Count the pairs for iodine and oxygen In $\mathrm{ICl_2^-}$, iodine has seven valence electrons and the negative charge supplies one more. Two single bonds use two of the central atom's electrons; the remaining six form three lone pairs. Therefore its bond-pair : lone-pair ratio is 2 : 3, so A matches III. In $\mathrm{H_2O}$, oxygen forms two single bonds and retains two lone pairs, giving 2 : 2, so B matches IV.

Step 2: Count the pairs for sulfur and xenon The expanded-octet Lewis representation of $\mathrm{SO_2}$ has two sulfur–oxygen double bonds and one lone pair on sulfur. Each double bond contains two bond pairs,

hence the required ratio is $4:1$: C matches II. In XeF_4, xenon has eight valence electrons, makes four single bonds and retains four nonbonding electrons as two lone pairs. Its ratio is $4:2$, so D matches I.

Step 3: Select the complete matching code The resulting assignment is

$$\mathrm{A}-\mathrm{III},\ \mathrm{B}-\mathrm{IV},\ \mathrm{C}-\mathrm{II},\ \mathrm{D}-\mathrm{I}.$$

Option (A) gives an incorrect iodine assignment. Option (B) interchanges the sulfur and xenon counts. Option (C) gives incorrect counts for iodine and water. Only option (D) agrees with all four Lewis structures. In particular, treating a double bond as one VSEPR domain would answer a different question from the printed bond-pair ratio.

Step 4: Conclusion The correct matching is A-III, B-IV, C-II, D-I. Hence, the correct option is (D).

Quick Tip to Solve: For bond pairs count both shared pairs in a double bond; for VSEPR domains count that same double bond as one domain. Read which count the question asks for.

57. The first transition series metal ' M ' has the highest enthalpy of atomisation in its series. One of its aquated ion (M^{n+}) exists in green colour. The nature of the oxide formed by the above M^{n+} ion is: [4 marks]

(A) basic

(B) neutral

(C) amphoteric

(D) acidic

Correct Answer: (A) basic

Solution:

Concept:

- Enthalpy of atomisation measures the energy needed to form gaseous atoms from a metal in its standard state. Stronger metallic bonding corresponds to a larger value.
- The acid-base character of a transition-metal oxide depends on the metal oxidation state. Lower oxidation states tend to form basic oxides; very high oxidation states tend to form acidic oxides.

Step 1: Identify the first-row metal The first transition series has its highest standard enthalpy of atomisation for vanadium. Thus $M = V$. This identification must use the actual atomisation trend, which is irregular; simply selecting the middle element or the atom with the largest free-atom count of unpaired electrons is not reliable.

Step 2: Use the colour to identify the ion The characteristic colours of aqueous vanadium species distinguish the oxidation states. Aqueous V^{2+} is violet, V^{3+} is green, and the vanadyl ion containing vanadium(IV) is blue. The green aquated ion in the question therefore identifies $n = 3$. For an oxide containing vanadium in this oxidation state, charge balance gives

$$2(+3) + 3(-2) = 0, \qquad V_2O_3.$$

Step 3: Classify the corresponding oxide Vanadium(III) oxide, V_2O_3, is a basic oxide. It reacts with acids to form vanadium(III) salts and water; the acid-base stoichiometry can be represented as

$$V_2O_3 + 6\,H^+ \longrightarrow 2\,V^{3+} + 3\,H_2O.$$

It is therefore not neutral. The amphoteric and acidic alternatives belong to different oxide behaviour and cannot be assigned solely because vanadium also forms higher oxidation states. In particular, the acidic character of its high-oxidation-state oxide does not describe the oxide of the green V^{3+} ion.

Step 4: Conclusion The metal is vanadium, the green ion is V^{3+}, and its oxide V_2O_3 is basic. Hence, the correct option is (A).

Quick Tip to Solve: Use the colour to fix the oxidation state before classifying an oxide; one transition metal can form oxides with very different acid-base character.

58. The number of valence electrons present in the metal among Cr, Co, Fe and Ni which has the lowest enthalpy of atomisation is : [4 marks]

(A) 10

(B) 6

(C) 9

(D) 8

Correct Answer: (B) 6

Solution:

Concept:

- For first-row transition metals, the valence-electron count includes both the $3d$ and $4s$ electrons outside the argon core.
- Enthalpies of atomisation vary irregularly across the transition series. Use the observed metallic-bonding trend, rather than assuming a direct monotonic dependence on atomic number.

Step 1: Compare the four atomisation enthalpies Among the specified metals, the approximate standard enthalpies of atomisation are

$$Cr : 397\ \text{kJ mol}^{-1}, \quad Fe : 416\ \text{kJ mol}^{-1},$$
$$Co : 425\ \text{kJ mol}^{-1}, \quad Ni : 430\ \text{kJ mol}^{-1}.$$

Chromium therefore has the lowest enthalpy of atomisation in this particular list. These relative values identify the metal before any electron counting is done.

Step 2: Write chromium's ground-state configuration Chromium has atomic number 24. Its stable ground-state configuration is

$$Cr : \ [Ar]\,3d^5 4s^1.$$

The half-filled $3d$ subshell accounts for the familiar exception to the simple filling expectation $3d^4 4s^2$. Both configurations would contain six electrons outside the argon core, but the actual ground-state

configuration should be stated correctly.

Step 3: Count and reject the other options The total number of valence electrons is

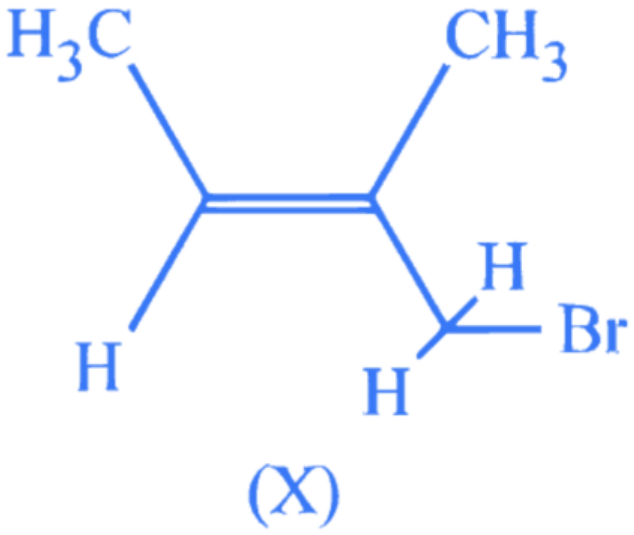
$$N_{\text{valence}} = 5 + 1 = 6.$$

Counting only the outermost $4s$ electron would miss the transition-metal $3d$ valence electrons. The values 8, 9 and 10 correspond to the total $3d + 4s$ valence counts of iron, cobalt and nickel respectively; none corresponds to the metal selected by the lowest-atomisation condition. Thus the electron-counting and property-selection steps point consistently to chromium and six electrons.

Step 4: Conclusion Chromium has the lowest enthalpy of atomisation among the four and has 6 valence electrons. Hence, the correct option is (B).

Quick Tip to Solve: First select the metal from the property trend, then add its $3d$ and $4s$ populations; the outermost-shell count alone is insufficient for transition metals.

59. Which of the following is the correct IUPAC name of given organic compound (X)? [4 marks]



(X)

(A) 4-Bromo-3-methylbut-2-ene

(B) 1-Bromo-2-methylbut-2-ene

(C) 3-Bromo-3-methylprop-2-ene

(D) 2-Bromo-2-methylbut-2-ene

Correct Answer: (B) 1-Bromo-2-methylbut-2-ene

Solution:

Concept:

- Choose the longest carbon chain containing the carbon–carbon double bond. Number it to give the double bond the lowest possible locant.
- If either numbering direction gives the same double-bond locant, compare substituent locants at

the first point of difference. Write different substituent prefixes in alphabetical order.

Step 1: Choose the alkene parent chain The displayed structure can be followed along the chain

$$\mathrm{BrCH_2{-}C(CH_3){=}CH{-}CH_3}.$$

This chain contains four carbon atoms and includes the double bond, so the parent is but-2-ene. A three-carbon parent would omit a carbon that can be included without losing the double bond and is therefore incorrect. The remaining methyl group is a substituent attached to a double-bonded carbon.

Step 2: Compare the two numbering directions Numbering from the $\mathrm{CH_2Br}$ end gives bromine at carbon 1, methyl at carbon 2, and the double bond at carbon 2. Numbering from the opposite methyl end also gives the double bond at carbon 2, but produces bromine at carbon 4 and methyl at carbon 3. Since the principal double-bond locant is tied, compare the substituent locant sets:

$$(1,2) \quad \text{versus} \quad (3,4).$$

The first set is lower, so numbering must begin at the bromomethyl end.

Step 3: Assemble and compare the name Place bromo before methyl alphabetically and retain the parent suffix but-2-ene. This yields 1-bromo-2-methylbut-2-ene. Option (A) uses the opposite numbering direction despite the substituent tie-break. Option (C) selects the wrong parent-chain length. Option (D) puts bromine on carbon 2, whereas the drawing clearly shows it bonded to the terminal $\mathrm{CH_2}$ carbon. The name in (B) alone reproduces the full carbon connectivity and the chosen locants.

Step 4: Conclusion The correct listed IUPAC name is 1-bromo-2-methylbut-2-ene. Hence, the correct option is (B).

Quick Tip to Solve: When an alkene double bond has the same locant from both ends, compare substituent locants before writing the name; here the terminal $\mathrm{CH_2Br}$ end must be carbon 1.

60. At the sea level, the dry air mass percentage composition is given as nitrogen gas: 70.0 , oxygen gas: 27.0 and argon gas: 3.0 . If total pressure is 1.15 atm , then calculate the ratio of following respectively:

(i) partial pressure of nitrogen gas to partial pressure of oxygen gas

(ii) partial pressure of oxygen gas to partial pressure of argon gas

(Given: Molar mass of N, O and Ar are 14, 16 and $40\ \text{g mol}^{-1}$ respectively.) [4 marks]

(A) $5.46, 17.8$

(B) $2.96, 11.2$

(C) $4.26, 19.3$

(D) $2.59, 11.85$

Correct Answer: (B) $2.96, 11.2$

Solution:

Concept:

- Dalton's law gives the partial pressure $p_i = x_i P$, where x_i is the mole fraction and P is total pressure. Hence $p_i/p_j = n_i/n_j$ for gases in the same mixture.
- A mass percentage is not a mole percentage. Convert each component mass to moles using its molecular or atomic molar mass before comparing partial pressures.

Step 1: Convert a convenient air sample to moles Take $100\ \text{g}$ of dry air, containing $70.0\ \text{g}$ nitrogen, $27.0\ \text{g}$ oxygen and $3.0\ \text{g}$ argon. Nitrogen and oxygen are diatomic, so their gas molar masses are 28 and $32\ \text{g mol}^{-1}$, while monatomic argon has molar mass $40\ \text{g mol}^{-1}$. Therefore

$$n_{\text{N}_2} = 70.0/28 = 2.500\ \text{mol},$$
$$n_{\text{O}_2} = 27.0/32 = 0.84375\ \text{mol},$$
$$n_{\text{Ar}} = 3.0/40 = 0.0750\ \text{mol}.$$

Step 2: Form the requested pressure ratios The common total pressure and common total mole number cancel in each ratio. Thus

$$\frac{p_{\text{N}_2}}{p_{\text{O}_2}} = \frac{2.500}{0.84375} = 2.96296\ldots \approx 2.96,$$

and

$$\frac{p_{\text{O}_2}}{p_{\text{Ar}}} = \frac{0.84375}{0.0750} = 11.25.$$

The second ratio is represented by 11.2 in the listed answers; this is the option consistent with the calculated value to the displayed precision.

Step 3: Check the interpretation of the data The given total pressure $1.15\ \text{atm}$ would be necessary for individual partial pressures, but not their ratios. Directly dividing mass percentages instead of mole amounts would give incorrect ratios because the three molar masses differ. Similarly, using the atomic masses 14 and 16 for the diatomic gases would distort the oxygen-to-argon ratio. Options (A), (C) and (D) fail the first calculated ratio; only (B) gives 2.96 together with the matching second value.

Step 4: Conclusion The requested ratios are approximately 2.96 and 11.25, corresponding to the

listed pair $(2.96, 11.2)$. Both ratios are dimensionless. Hence, the correct option is (B).

Quick Tip to Solve: For partial-pressure ratios from mass percentages, divide each mass percentage by the gas molar mass first; the total pressure cancels automatically.

61. Given below are two statements :

Statement I : Dimethyl ether is completely soluble in water. However, diethyl ether is soluble in water to a very small extent.

Statement II : Sodium metal can be used to dry diethyl ether and not ethyl alcohol.

In the light of given statements. choose the correct answer from the options given below [4 marks]

(A) Statement I is false but Statement II are true

(B) Statement I is true but Statement II is false

(C) Both Statement I and Statement II are true

(D) Both Statement I and Statement II are False

Correct Answer: (A) Statement I is false but Statement II are true

Solution:

Concept:

- Ether oxygen accepts hydrogen bonds from water, but hydrogen bonding does not imply unlimited solubility. A finite saturation concentration differs from complete miscibility.
- Sodium consumes water by forming sodium hydroxide and hydrogen. It also reacts with the acidic O–H hydrogen of an alcohol; an ether has no O–H bond.

Step 1: Evaluate the complete-solubility claim Dimethyl ether is more water-soluble than larger ethers because its nonpolar alkyl portion is smaller. However, the word "completely makes Statement I false when read literally: greater solubility is not solubility in all proportions. The ILO/WHO International Chemical Safety Card 0454 lists a finite water solubility of $2.4\,\mathrm{g}/100\,\mathrm{mL}$, demonstrating saturation rather than unlimited mixing. See ICSC 0454, physical and chemical information. Diethyl ether has limited solubility too; that true second clause cannot make the whole first statement true.

Step 2: Check the suitability of sodium as a drying agent In diethyl ether, sodium removes traces of water through

$$2\,\mathrm{Na} + 2\,\mathrm{H_2O} \longrightarrow 2\,\mathrm{NaOH} + \mathrm{H_2}.$$

The ether itself has no O–H hydrogen and does not undergo the corresponding alcohol reaction under the drying conditions. Ethanol instead consumes sodium even when no water is present:

$$2\,\mathrm{C_2H_5OH} + 2\,\mathrm{Na} \longrightarrow 2\,\mathrm{C_2H_5ONa} + \mathrm{H_2}.$$

Thus sodium cannot selectively remove water from ethanol without reacting with the solvent, and Statement II is true. Options (B) and (D) incorrectly deny Statement II; option (C) incorrectly accepts complete water solubility.

Step 3: Conclusion For the literal word "completely, Statement I is false and Statement II is true. Hence, the correct option is (A). The supplied final key treats both statements as true, but the finite

solubility documented above contradicts the complete-solubility claim; the literal statement supports (A).

Quick Tip to Solve: Distinguish "more soluble from "completely soluble. For sodium drying, check whether the solvent itself contains an O–H bond.

62. Given below are two statements :

Statement I : Mohr's salt is composed of only three types of ions-ferrous, ammonium and sulfate.

Statement II : If the molar conductance at infinite dilution for ferrous, ammonium and sulfate ions are x_1, x_2 and x_3 S cm^2 mol^{-1}, respectively then the molar conductance for Mohr's salt solution at infinite dilution would be given by $x_1 + x_2 + 2x_3$

In the light of the given statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statements I and Statement II are true

(B) Statement I is true but Statement II are false

(C) Both Statements I and Statement II are false

(D) Statement I is false but Statement II is true

Correct Answer: (B) Statement I is true but Statement II are false

Solution:

Concept:

- Mohr's salt is the double salt $(NH_4)_2Fe(SO_4)_2 \cdot 6\,H_2O$; dissolution releases the constituent simple ions rather than a persistent complex ion.
- Kohlrausch's law gives the limiting molar conductivity as the sum of ionic limiting conductivities, each multiplied by its stoichiometric coefficient.

Step 1: List the ions released by one formula unit The dissolved salt separates as

$$(NH_4)_2Fe(SO_4)_2 \cdot 6\,H_2O \longrightarrow Fe^{2+} + 2\,NH_4{}^{+} + 2\,SO_4{}^{2-} + 6\,H_2O.$$

There are three chemically distinct ionic species: ferrous, ammonium and sulfate. The six waters are neutral water molecules, so they do not constitute a fourth type of ion in the salt formula. The number of ion types is three although the total number of ions released per formula unit is five. Statement I therefore correctly describes the composition.

Step 2: Apply independent migration of ions At infinite dilution, the contributions of these ions are independent. If the respective ionic conductivities are x_1, x_2, x_3, the coefficients must follow the dissolution equation:

$$\Lambda_m^\circ = \lambda^\circ_{Fe^{2+}} + 2\lambda^\circ_{NH_4{}^{+}} + 2\lambda^\circ_{SO_4{}^{2-}} = x_1 + 2x_2 + 2x_3.$$

The unit is S cm^2 mol^{-1}. Statement II misses one ammonium contribution: it uses x_2 instead of $2x_2$. Ionic charge does not replace the formula-unit coefficient in this sum. Statement II is false; options (A) and (D) wrongly accept it, whereas option (C) wrongly rejects Statement I.

Step 3: Conclusion Statement I is true and Statement II is false. The limiting molar conductivity is $x_1 + 2x_2 + 2x_3\,\mathrm{S\,cm^2\,mol^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the complete ionic dissociation before summing conductivities. Count ammonium twice in one formula unit of Mohr's salt.

63. Given below are two statements:

Statement I: D-(+)-glucose + D-(+) fructose $\xrightarrow{-H_2O}$ **Sucrose**

$$\textbf{sucrose} \xrightarrow{\textbf{hydrolysis}} \mathrm{D-(+)}\ \textbf{glucose} + \mathrm{D-(+)}\ \textbf{fructose}$$

Statement II: Invert sugar is formed during sucrose hydrolysis

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are False

(B) Statement I is false but Statement II are true

(C) Both Statement I and Statement II are true

(D) Statement I is true but Statement II is false

Correct Answer: (B) Statement I is false but Statement II are true

Solution:

Concept:

- The D/L designation describes configuration relative to glyceraldehyde, whereas $(+)$ and $(-)$ describe the measured direction of optical rotation. D does not imply dextrorotatory.
- Sucrose hydrolysis gives equimolar D-(+)-glucose and D-(-)-fructose. The hydrolysate is called invert sugar because its net rotation has the opposite sign to sucrose.

Step 1: Check the signs assigned to the monosaccharides Glucose is correctly represented here as D-(+)-glucose, but naturally occurring D-fructose is levorotatory, not dextrorotatory. Accordingly, both appearances of D-(+)-fructose in Statement I have the wrong rotation sign. Formation of sucrose involves condensation through the anomeric positions of glucose and fructose, and hydrolysis cleaves that glycosidic bond. Neither process changes D-fructose into a dextrorotatory form. The correct product description is

$$\text{sucrose} + H_2O \longrightarrow \text{D-(+)-glucose} + \text{D-(-)-fructose}.$$

Thus Statement I is false even though its general condensation/hydrolysis pattern is recognizable.

Step 2: Explain the formation of invert sugar Before hydrolysis, sucrose rotates plane-polarized light to the right. Hydrolysis produces equal amounts of glucose and fructose; the left rotation due to fructose is greater in magnitude than the right rotation due to glucose, so the mixture rotates light to the left overall. This reversal of sign is called inversion, and the mixture is invert sugar. It is not a separate third monosaccharide. Statement II is therefore true. Option (A) incorrectly rejects this true fact, while options (C) and (D) incorrectly accept the positive rotation sign printed for fructose.

Step 3: Conclusion Statement I is false because D-fructose is levorotatory, and Statement II is true because sucrose hydrolysis forms invert sugar. Hence, the correct option is (B).

Quick Tip to Solve: Keep configuration and rotation separate: D-fructose is D-(-), despite having the same D configuration family as D-(+)-glucose.

64. Which of the following statements are correct, if the threshold frequency of caesium is 5.16×10^{14} Hz **?**

A. When Cs is placed inside a vacuum chamber with an ammeter connected to it and yellow light is focused on Cs , the ammeter shows the presence of current.

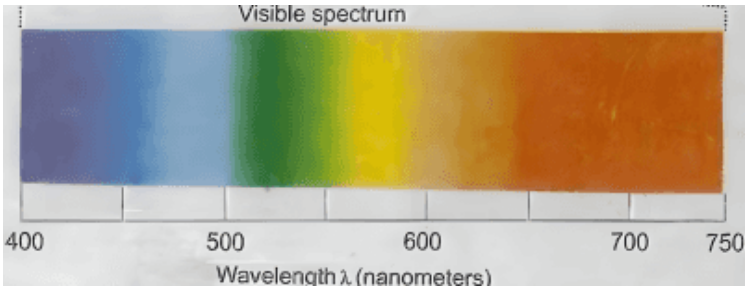
B. When the brightness of the yellow light is dimmed, the value of the current in the ammeter is reduced.

C. When a red light is used instead of the yellow light, the current produced is higher with respect to the yellow light.

D. When a blue light is used, the ammeter shows the formation of current.

E. When a white light is used. the ammeter shows formation of current.

Choose the correct answer from the options given below: [4 marks]



(A) B, C and D Only

(B) A, C, D and E Only

(C) A, B, D and E Only

(D) A, D and E Only

Correct Answer: (C) A, B, D and E Only

Solution:

Concept:

- Photoemission requires $h\nu \geq h\nu_0$. The corresponding longest effective wavelength is $\lambda_0 = c/\nu_0$, with c the speed of light.
- For light above threshold, photon energy controls maximum kinetic energy, while intensity controls photon flux and hence the photoelectric current under the same collection conditions.

Step 1: Calculate the threshold wavelength and test the colours Using the stated threshold frequency,

$$\lambda_0 = \frac{3.00 \times 10^8}{5.16 \times 10^{14}}\,\mathrm{m} = 5.81 \times 10^{-7}\,\mathrm{m} \simeq 581\,\mathrm{nm}.$$

The yellow light represented near the short-wavelength side of the yellow region in the supplied spectrum is capable of reaching the threshold, so the intended yellow illumination produces photoelectrons and A is true. Red light has a longer wavelength and lower photon frequency than the threshold and cannot produce a greater photoelectric current: C is false. Blue light has a shorter wavelength and a frequency safely above threshold, so D is true.

Step 2: Assess dimming and white illumination Dimming the effective yellow illumination decreases the number of incident photons per second without changing the energy of each photon. Fewer electrons are emitted per second, so the current decreases; B is true. This conclusion concerns current and does not mean that the stopping potential decreases with brightness. White light contains blue and other sufficiently high-frequency components, which can cause emission even though its red components cannot. Hence E is true. The correct set is A, B, D and E. Options (A) and (B) contain the false statement C; option (D) omits the true intensity statement B.

Step 3: Conclusion The correct statements are A, B, D and E for the yellow illumination represented in the question. Hence, the correct option is (C).

Quick Tip to Solve: Convert a threshold frequency into a wavelength first. Compare colour with the threshold for emission, then use brightness only to compare the current.

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65. The group 14 elements A and B have the first ionisation enthalpy values of 708 and $715\ \text{kJ mol}^{-1}$ respectively. The above values are lowest among their group members. The nature of their ions A^{2+} and B^{4+} respectively is [4 marks]

(A) both reducing

(B) oxidising and reducing

(C) both oxidising

(D) reducing and oxidising

Correct Answer: (D) reducing and oxidising

Solution:

Concept:

- In group 14, the stability of the +2 oxidation state increases down the group because the outer ns^2 pair participates less readily in bonding: the inert-pair effect.
- A species that is readily oxidised acts as a reducing agent; one that is readily reduced acts as an oxidising agent. The direction of electron transfer determines the label.

Step 1: Identify the two elements The first ionisation enthalpies given, approximately 708 and $715\,\text{kJ mol}^{-1}$, identify tin and lead, respectively. The small increase from tin to lead is an important exception to a simple monotonic decrease down the group; it should not be used to interchange their identities. Hence

$$A = \text{Sn}, \qquad B = \text{Pb}.$$

The ions requested are therefore Sn^{2+} and Pb^{4+}, not the corresponding neutral atoms. Their redox behaviour must be judged by which oxidation state each element favours.

Step 2: Determine the favourable electron-transfer directions Tin(II) is readily oxidised to tin(IV), losing two electrons:

$$\mathrm{Sn^{2+} \longrightarrow Sn^{4+} + 2\,e^{-}}.$$

By donating these electrons to another substance, $\mathrm{Sn^{2+}}$ reduces that substance and therefore acts as a reducing agent. For lead, the inert-pair effect strongly stabilises the +2 state relative to +4. Lead(IV) consequently tends to gain electrons:

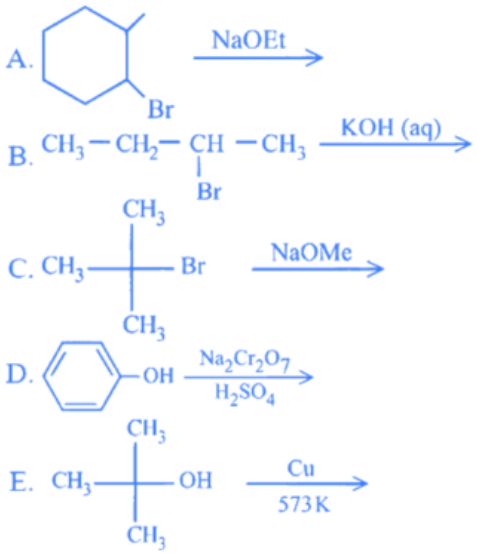
$$\mathrm{Pb^{4+} + 2\,e^{-} \longrightarrow Pb^{2+}}.$$

It oxidises the other reactant and is an oxidising agent. Option (A) wrongly makes lead(IV) reducing; option (C) wrongly makes tin(II) oxidising; option (B) reverses both required roles.

Step 3: Conclusion $\mathrm{A^{2+} = Sn^{2+}}$ is reducing and $\mathrm{B^{4+} = Pb^{4+}}$ is oxidising. Hence, the correct option is (D).

Quick Tip to Solve: Write the half-reaction: electron loss means the ion is a reducing agent. Remember that lead favours +2 more strongly than tin does.

66. The reactions which cannot be applied to prepare an alkene by elimination, are
Choose the correct answer from the options given below: [4 marks]



(A) B & E Only
(B) B, C & D Only
(C) A, C & D Only
(D) B & D Only
Correct Answer: (D) B & D Only

Solution:

Concept:

- Base-induced dehydrohalogenation eliminates a beta hydrogen and a halide from neighbouring carbon atoms. Alkoxide bases favour elimination when the substrate and conditions permit it.
- Aqueous hydroxide normally favours nucleophilic substitution, while oxidation is a different transformation from elimination. Tertiary alcohols on hot copper can undergo dehydration.

Step 1: Classify reactions A, B and C In A, the brominated cyclohexane derivative has beta hydrogens and is treated with sodium ethoxide. Loss of a beta hydrogen and bromide can produce a cyclohexene derivative; A can therefore prepare an alkene. In B, 2-bromobutane is treated with aqueous KOH. The stated aqueous conditions favour replacement of bromide by hydroxide to form butan-2-ol, rather than the standard alcoholic-KOH elimination used to prepare butenes. Thus B is unsuitable as the indicated elimination preparation. In C, the tertiary alkyl bromide is treated with sodium methoxide. Steric hindrance disfavors substitution at the tertiary centre, while a strong base removes a beta hydrogen, producing 2-methylpropene. C is a valid elimination reaction.

Step 2: Classify reactions D and E and select the set D treats phenol with acidified sodium dichromate. This is an oxidising system, not an alcohol-dehydration or alkyl-halide dehydrohalogenation system. It therefore cannot be used for the specified alkene preparation by elimination. E is tert-butyl alcohol over copper at $573\,\mathrm{K}$. A tertiary alcohol lacks a hydrogen on its hydroxyl-bearing carbon, so it cannot undergo the usual copper-catalysed dehydrogenation to a carbonyl compound. Instead it dehydrates to 2-methylpropene:

$$(CH_3)_3COH \longrightarrow (CH_3)_2C{=}CH_2 + H_2O.$$

Thus E is suitable. Only B and D are unsuitable. Option (A) wrongly excludes E, and options (B) and (C) wrongly exclude the valid alkoxide-base reactions.

Step 3: Conclusion The reactions that cannot be applied as shown to prepare an alkene by elimination are B and D only. Hence, the correct option is (D).

Quick Tip to Solve: Read the solvent and reagent before classifying the substrate: aqueous KOH suggests substitution, while alkoxide bases and hot tertiary alcohols can give elimination.

67. When a salt is treated with sodium hydroxide solution it gives gas X . On passing gas X through reagent Y a brown coloured precipitate is formed. X and Y respectively, are [4 marks]

(A) $X = NH_3$ and $Y = HgO$

(B) $X = NH_4Cl$ and $Y = KOH$

(C) $X = NH_3$ and $Y = K_2HgI_4 + KOH$

(D) $X = HCl$ and $Y = NH_4Cl$

Correct Answer: (C) $X = NH_3$ and $Y = K_2HgI_4 + KOH$

Solution:

Concept:

- Ammonium salts release ammonia on warming with a strong base: $NH_4{}^+ + OH^- \longrightarrow NH_3 + H_2O$. The gas is identified by its characteristic chemical tests.
- Nessler's reagent is alkaline potassium tetraiodomercurate(II), $K_2[HgI_4]$ in KOH. It gives a brown precipitate with ammonia.

Step 1: Identify the gas liberated by sodium hydroxide The hydroxide ion removes a proton from an ammonium ion in the salt. For example, the reaction of ammonium chloride is

$$NH_4Cl + NaOH \longrightarrow NH_3\uparrow + NaCl + H_2O.$$

Thus gas X is ammonia, not ammonium chloride. The latter is an ionic solid and is the starting salt in this example. Hydrogen chloride is not liberated by treating its salt with a strong aqueous base; the basic medium instead favours formation of ammonia.

Step 2: Match the brown precipitate test Pass the liberated ammonia into Nessler's reagent, an alkaline solution of $K_2[HgI_4]$. The brown basic mercury amidoiodide precipitate can be represented as

$$NH_3 + 2\,K_2[HgI_4] + 3\,KOH \longrightarrow HgO\cdot Hg(NH_2)I\downarrow + 7\,KI + 2\,H_2O.$$

The combination of ammonia and alkaline tetraiodomercurate is exactly option (C). Option (A) gives mercury(II) oxide rather than the specified Nessler reagent. Option (B) incorrectly calls ammonium chloride the gas and gives only potassium hydroxide as the detecting reagent. Option (D) gives neither the gas released by the alkali nor the reagent responsible for this brown precipitate.

Step 3: Conclusion Gas X is NH_3, and reagent Y is $K_2[HgI_4] + KOH$, Nessler's reagent. Hence, the correct option is (C).

Quick Tip to Solve: Link an ammonium salt plus alkali to ammonia; a brown ammonia test points specifically to Nessler's reagent.

68. Reaction $\mathrm{A(g)} \rightarrow 2\ \mathrm{B(\ g)} + \mathrm{C(g)}$ is a first order reaction. It was started with pure A

t/min	Pressure of system at time t/mm Hg
10	160
∞	240

Which of the following option is incorrect? [4 marks]

(A) Initial pressure of A is 80 mm Hg

(B) The reaction never goes to completion

(C) Partial pressure of A after 10 minute is 40 mm Hg

(D) Rate constant of the reaction is $1.693\ \mathrm{min}^{-1}$

Correct Answer: (D) Rate constant of the reaction is $1.693\ \mathrm{min}^{-1}$

Solution:

Concept:

- At constant temperature and volume, partial pressures are proportional to amounts. For $\mathrm{A\,(g)} \longrightarrow 2\,\mathrm{B\,(g)} + \mathrm{C\,(g)}$, one mole of reactant ultimately produces three moles of gaseous products.
- For a first-order reaction, $k = t^{-1} \ln(p_{A,0}/p_{A,t})$. Reactant pressure tends to zero asymptotically, rather than reaching zero at a finite time.

Step 1: Calculate the initial and remaining reactant pressures Let the initial pressure of pure A be p_0, and let x denote the pressure equivalent of A decomposed by time t. Then

$$p_A = p_0 - x, \qquad p_B = 2x, \qquad p_C = x,$$

so the total pressure is $P_t = p_0 + 2x$. At the limiting completion of the reaction, $x = p_0$; hence $P_\infty = 3p_0 = 240\ \mathrm{mm\,Hg}$, giving $p_0 = 80\ \mathrm{mm\,Hg}$. At 10 min, $160 = 80 + 2x$, so $x = 40\ \mathrm{mm\,Hg}$ and $p_{A,10} = 80 - 40 = 40\ \mathrm{mm\,Hg}$. Options (A) and (C) therefore state correct pressures.

Step 2: Evaluate the rate constant and test each statement Substitute the initial and remaining pressures in the integrated first-order equation:

$$k = \frac{1}{10} \ln\left(\frac{80}{40}\right) = \frac{\ln 2}{10} = 0.0693\ \mathrm{min}^{-1}.$$

Thus the first 10 minutes constitute one half-life. Moreover, $p_A = p_0 e^{-kt}$ remains positive at every finite time and vanishes only in the limit $t \to \infty$. This is the first-order idealisation meant by option (B), which is therefore correct. The limiting pressure in the table is consistent with this asymptotic statement. Option (D), which gives $1.693\ \mathrm{min}^{-1}$, does not equal the calculated rate constant and is the only incorrect option.

Step 3: Conclusion The rate constant is $0.0693\ \mathrm{min}^{-1}$, not $1.693\ \mathrm{min}^{-1}$. Since the question asks for the incorrect statement, the correct option is (D).

Quick Tip to Solve: First obtain the initial pressure from the final stoichiometric mole ratio; total pressure at time t is not the pressure of A.

69. An octahedral complex having molecular composition $Co \cdot 5NH_3 \cdot Cl^2.SO_4$ has two isomers A and B. The solution of A gives a white precipitate with $AgNO_3$ solution and the solution of B gives white precipitate with $BaCl_2$ solution. The type of isomerism exhibited by the complex is, [4 marks]

(A) Co-ordinate isomerism

(B) Ionisation isomerism

(C) Geometrical isomerism

(D) Linkage isomerism

Correct Answer: (B) Ionisation isomerism

Solution:

Concept:

- Ionisation isomers interchange a coordinated anion and an anion outside the coordination sphere. Their aqueous solutions consequently supply different free ions.
- Free chloride forms white $AgCl$ with $AgNO_3$, while free sulfate forms white $BaSO_4$ with $BaCl_2$. These tests identify the counterion rather than a ligand bound to cobalt.

Step 1: Use the precipitation tests to locate each anion The white precipitate obtained from A with silver nitrate shows that A releases chloride ions in water:

$$Ag^+ + Cl^- \longrightarrow AgCl\downarrow.$$

Accordingly, the chloride is outside the coordination sphere in A; sulfate occupies the sixth coordination position alongside the five ammonia ligands. Conversely, B releases sulfate ions, as established by

$$Ba^{2+} + {SO_4}^{2-} \longrightarrow BaSO_4\downarrow.$$

Its sulfate is therefore outside the coordination sphere, while chloride is coordinated. Both arrangements retain an octahedral coordination number of six.

Step 2: Identify the exchanged groups and exclude the alternatives The two coordination arrangements are represented by $[Co(NH_3)_5(SO_4)]Cl$ and $[Co(NH_3)_5Cl]SO_4$. They exchange sulfate and chloride between the coordination sphere and the ionisable part of the compound. This is the defining feature of ionisation isomerism, option (B). Coordination isomerism, option (A), requires redistribution of ligands between complex cations and complex anions, which is not the situation here. Geometrical isomerism, option (C), changes relative ligand positions without changing the counterion released. Linkage isomerism, option (D), changes the donor atom through which the same ambidentate ligand binds, rather than exchanging chloride and sulfate between inner and outer spheres.

Step 3: Conclusion The different precipitating ions arise from interchange of coordinated and ionisable anions. The complex exhibits ionisation isomerism. Hence, the correct option is (B).

Quick Tip to Solve: Use the precipitate to identify the free counterion first: silver chloride indicates outer-sphere chloride, and barium sulfate outer-sphere sulfate.

70. Given below are two statements:

Statement I : Ozonolysis followed by treatment with Zn, H_2O of cis-2-butene gives ethanal.

Statement II : The product obtained by ozonolysis followed by treatment with Zn, H_2O of 3, 6-dimethyloct-4-ene has no chiral carbon atom.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Statement I is false but Statement II are true

(B) Both Statement I and Statement II are False

(C) Statement I is true but Statement II is false

(D) Both Statement I and Statement II are true

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- Reductive ozonolysis cleaves a carbon-carbon double bond and converts each alkene carbon into a carbonyl carbon. A double-bond carbon bearing hydrogen gives an aldehyde with Zn/H_2O work-up.
- A tetrahedral carbon is chiral when it is attached to four different groups. Cutting a remote double bond does not automatically remove an existing stereogenic carbon.

Step 1: Check Statement I by cleaving cis-2-butene The carbon skeleton of cis-2-butene is $CH_3CH{=}CHCH_3$. Each double-bond carbon has one hydrogen and one methyl group. On cleavage, each becomes the carbonyl carbon of ethanal:

$$CH_3CH{=}CHCH_3 \xrightarrow[2.\ Zn/H_2O]{1.\ O_3} 2\,CH_3CHO.$$

The cis arrangement does not alter the identity of the products: the bond is broken, and each half has the same substituents. The reductive work-up preserves the aldehyde rather than oxidising it to the carboxylic acid. Statement I is therefore true.

Step 2: Identify the product and stereogenic carbon in Statement II Write the second alkene as

$$CH_3CH_2CH(CH_3)CH{=}CHCH(CH_3)CH_2CH_3.$$

Cleavage of the central double bond gives two molecules of 2-methylbutanal. Its condensed structure is

$$OHC{-}CH(CH_3){-}CH_2CH_3.$$

The carbon next to the aldehyde group is tetrahedral and carries four different substituents: $-CHO$, $-CH_3$, $-CH_2CH_3$, and hydrogen. It is therefore a chiral carbon atom. Whether a sample contains one stereoisomer or a mixture does not change the presence of this stereogenic centre. The claim that the product has no chiral carbon is false. Thus Statement II is false. Options (A) and (B) incorrectly reject Statement I; option (D) incorrectly accepts Statement II.

Step 3: Conclusion Statement I is true because ethanal is formed; Statement II is false because 2-methylbutanal contains a chiral carbon. Hence, the correct option is (C).

Quick Tip to Solve: After cutting the double bond, redraw the product and list the four groups on each tetrahedral carbon; do not infer chirality from alkene symmetry.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. An organic compound weighing 500 mg , produced 220 mg of CO_2, on complete combustion. The percentage composition of carbon in the compound is _____ %. (nearest integer)
(Given molar mass in $gmol^{-1}$ of $C : 12, O : 16$) [4 marks]

Solution:

Concept:

- Complete combustion transfers every carbon atom in the organic sample to CO_2. Thus one mole of collected carbon dioxide corresponds to one mole of carbon originally present.
- The mass fraction of carbon in CO_2 is $12/(12 + 2 \times 16) = 12/44$. Carbon percentage is carbon mass divided by the original sample mass, multiplied by 100.

Step 1: Convert the carbon dioxide mass into carbon mass The measured 220 mg is the mass of the carbon dioxide produced, not the mass of carbon alone. Its oxygen also contributes to its mass. With the given atomic masses,

$$M(CO_2) = 12 + 2(16) = 44\ \text{g mol}^{-1}.$$

Therefore the carbon mass represented by the product is

$$m_C = 220 \times \frac{12}{44} = 60\ \text{mg}.$$

As an independent unit check, 220 mg is 0.220 g, so the amount of carbon dioxide is $0.220/44 = 0.005$ mol. It contains 0.005 mol of carbon, whose mass is $0.005 \times 12 = 0.060$ g, again 60 mg.

Step 2: Calculate the sample percentage and round the result Use the original 500 mg sample as the denominator, because percentage composition describes the material before combustion:

$$\%C = \frac{60\ \text{mg}}{500\ \text{mg}} \times 100 = 12\%.$$

The same mass units in numerator and denominator cancel. Dividing 220 mg directly by 500 mg would wrongly count the oxygen in carbon dioxide as carbon from the compound. No molecular formula or amount of hydrogen is needed, because the carbon-containing combustion product alone fixes the total carbon mass. The result is exactly an integer, so rounding to the nearest integer makes no change.

Step 3: Conclusion The percentage composition of carbon is 12%. The required nearest-integer numerical entry is $\boxed{12}$.

Quick Tip to Solve: For carbon estimation, multiply the collected carbon dioxide mass by 12/44 before dividing by the original sample mass.

72. The number of paramagnetic complexes among $[FeF_6]^{3-}$, $[Fe(CN)_6]^{3-}$, $[Mn(CN)_6]^{3-}$, $[Co\,(C_2O_4)_3]^{3-}$, $[MnCl_6]^{3-}$, and $[CoF_6]^{3-}$, which involved d^2sp^3 hybridization is _____. [4 marks]

Solution:

Concept:

- An octahedral inner-orbital complex uses two 3d orbitals with one 4s and three 4p orbitals, giving d^2sp^3 hybridisation. Strong-field ligands can produce low-spin configurations by pairing 3d electrons.
- Paramagnetism requires at least one unpaired electron. A low-spin complex may still be paramagnetic: low-spin d^5 has one unpaired electron and low-spin d^4 has two.

Step 1: Determine oxidation states and d-electron counts Each metal is in oxidation state $+3$. For the fluoride, chloride and cyanide complexes, the six singly negative ligands and overall charge -3 give $x - 6 = -3$. For the oxalate complex, three doubly negative ligands give the same result. Thus Fe^{3+} is d^5, Mn^{3+} is d^4, and Co^{3+} is d^6. Both conditions in the question must be satisfied together: an unpaired electron alone is insufficient if the complex uses outer-orbital hybridisation.

Step 2: Apply the field strength and count the qualifying complexes The cyanide complexes are low spin. $[Fe(CN)_6]^{3-}$ has t_{2g}^5, one unpaired electron, and two vacant 3d orbitals for d^2sp^3 hybridisation. $[Mn(CN)_6]^{3-}$ has t_{2g}^4, two unpaired electrons, and likewise uses d^2sp^3. Both qualify. The cobalt(III) oxalate complex is low-spin d^6, t_{2g}^6: it is inner orbital but diamagnetic, so does not qualify. The weak-field complexes $[FeF_6]^{3-}$, $[MnCl_6]^{3-}$, and $[CoF_6]^{3-}$ are high spin and paramagnetic, but use outer-orbital sp^3d^2 hybridisation in the valence-bond description. They fail the hybridisation condition and must not be included.

Step 3: Conclusion The qualifying complexes are $[Fe(CN)_6]^{3-}$ and $[Mn(CN)_6]^{3-}$. Hence the required number is $\boxed{2}$, a dimensionless count.

Quick Tip to Solve: Make two checks for each complex: first inner versus outer orbital, then unpaired electrons. Neither condition by itself decides the count.

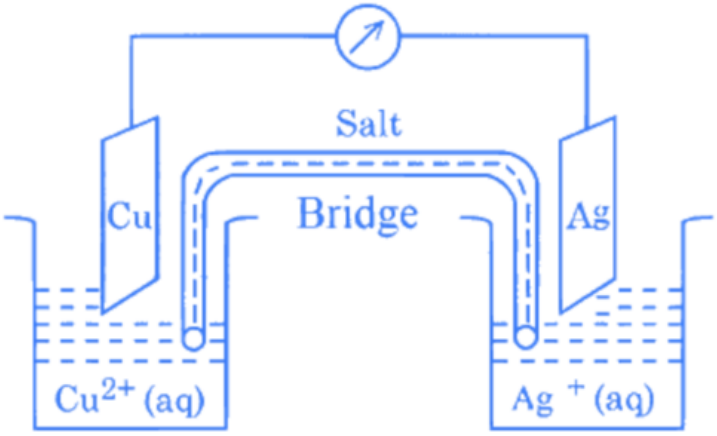
73. 1 Faraday electricity was passed through $Cu^{2+}(1.5M, 1\ L)/Cu$ and 0.1 Faraday was passed through $Ag^{+}(0.2M, 1\ L)/Ag$ electrolytic cells. After this the two cells were connected as shown below to make an electrochemical cell. The emf of the cell thus formed at 298 K is _____ mV (nearest integer)

Given : $E°Cu^{2+}/Cu = 0.34\ V$

$$E°Ag^{+}/Ag = 0.8\ V$$

$$\frac{2 \cdot 303RT}{F} = 0.06\ V$$

[4 marks]



Solution:

Concept:

- One faraday supplies one mole of electrons. In electrolysis, the amount of a metal ion consumed equals the moles of electrons divided by the number of electrons required per ion.
- For a galvanic cell, $E = E° - (0.06/n)\log_{10} Q$ at the stated temperature. Pure metal activities are unity; the concentrations used in Q are those remaining after electrolysis.

Step 1: Calculate the remaining ion concentrations Copper deposition follows $Cu^{2+} + 2\,e^{-} \longrightarrow Cu$. Passing $1\,F$ therefore consumes $1/2 = 0.5$ mol of copper ions from the initial $1.5 \times 1 = 1.5$ mol. In the specified one-litre solution, the remaining concentration is

$$[Cu^{2+}] = \frac{1.5 - 0.5}{1} = 1.0\,mol\,L^{-1}.$$

Silver deposition follows $Ag^{+} + e^{-} \longrightarrow Ag$. The charge $0.1\,F$ consumes 0.1 mol of silver ions from the initial 0.2 mol. Thus

$$[Ag^{+}] = \frac{0.2 - 0.1}{1} = 0.1\,mol\,L^{-1}.$$

Step 2: Write the spontaneous cell reaction Silver has the higher reduction potential and forms the

cathode; copper is oxidised at the anode. Combining the half-reactions gives

$$\mathrm{Cu} + 2\,\mathrm{Ag^+} \longrightarrow \mathrm{Cu^{2+}} + 2\,\mathrm{Ag}.$$

Two electrons are transferred, so $n = 2$, and

$$E^\circ = 0.80 - 0.34 = 0.46\,\mathrm{V}, \qquad Q = \frac{[\mathrm{Cu^{2+}}]}{[\mathrm{Ag^+}]^2} = \frac{1}{(0.1)^2} = 100.$$

Step 3: Apply the Nernst equation Use the supplied coefficient $0.06\,\mathrm{V}$, rather than introducing a different rounded temperature constant:

$$E = 0.46 - \frac{0.06}{2}\log_{10}(100) = 0.46 - 0.03(2) = 0.40\,\mathrm{V}.$$

The potential is positive, confirming the electrode directions used. Convert volts to millivolts only after evaluating the logarithmic correction.

Step 4: Conclusion The cell emf is $0.400\,\mathrm{V} = \boxed{400\,\mathrm{mV}}$. The nearest-integer entry is $\boxed{400}$.

Quick Tip to Solve: First subtract the ions discharged during electrolysis; then use the remaining concentrations in the galvanic-cell quotient. Silver concentration is squared because its reaction coefficient is two.

74. The percentage dissociation of a salt $(\mathrm{MX_3})$ solution at given temperature (van't Hoff factor $\mathrm{i} = 2$) is _____ %(Nearest integer) [4 marks]

Solution:

Concept:

- The van't Hoff factor i is the ratio of the actual number of solute particles in solution to the number that would be present without dissociation or association.
- If one formula unit dissociates into ν ions and the fraction dissociated is α, then $i = 1 + (\nu - 1)\alpha$. The percentage dissociation is 100α, not α itself.

Step 1: Count the particles produced by one formula unit The salt has one metal ion and three counterions per formula unit. Its idealised dissociation is

$$\mathrm{MX_3} \longrightarrow \mathrm{M^{3+}} + 3\,\mathrm{X^-}.$$

Thus one dissociated formula unit gives four ions in total, so $\nu = 4$. The subscript three counts the counterions only; it does not count the metal ion. Complete dissociation would therefore give $i = 4$, whereas an undissociated solution would give $i = 1$.

Step 2: Express the particle count using the dissociated fraction Begin with one mole of salt and let α be the fraction dissociated. The undissociated salt contributes $1 - \alpha$ mol of particles. The metal ions contribute α mol, and the counterions contribute 3α mol. Adding all three contributions gives

$$n_{\text{particles}} = (1 - \alpha) + \alpha + 3\alpha = 1 + 3\alpha.$$

Relative to the original one mole of formula units, this particle count is the van't Hoff factor. This accounting also explains why only the increase of three particles per dissociated formula unit enters the formula.

Step 3: Substitute the observed factor and round the percentage Insert $i = 2$ into the expression just obtained:

$$2 = 1 + 3\alpha, \qquad \alpha = \frac{2-1}{3} = \frac{1}{3}.$$

Convert the fraction into a percentage before rounding:

$$100\alpha = \frac{100}{3}\% = 33.333\ldots\%.$$

The fraction lies between zero and one, consistent with partial rather than complete dissociation.

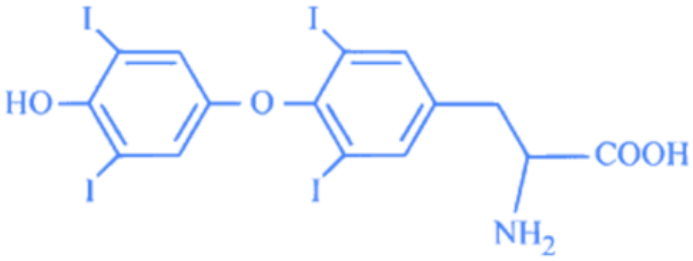
Step 4: Conclusion The percentage dissociation is $33.333\ldots\%$, which to the nearest integer is $\boxed{33\%}$. The numerical entry is $\boxed{33}$.

Quick Tip to Solve: Count all ions from one formula unit, including the metal ion. For a four-ion salt, use $\alpha = (i-1)/3$, and multiply by 100 only at the end.

75. Thyroxine, the hormone has given below structure

The percentage of iodine in thyroxine is _____ %. (nearest integer)

(Given molar mass in gmol^{-1} $\text{C}: 12, \text{H}: 1, \text{O}: 16,\ \text{N}: 14, \text{I}: 127$ **)** [4 marks]



Solution:

Concept:

- The mass percentage of an element is its mass contribution in one mole of the compound divided by the molar mass of the compound, multiplied by 100.
- In a skeletal structure, every unlabelled vertex or line end is carbon. Add enough implicit hydrogens to satisfy carbon valency, and count hydrogens explicitly attached to oxygen and nitrogen as well.

Step 1: Count the atoms in the displayed thyroxine structure The two benzene rings provide twelve carbon atoms. The side chain $-\text{CH}_2-\text{CH}(\text{NH}_2)-\text{COOH}$ provides three more, giving fifteen carbons. Four iodine atoms are explicitly shown, two on each ring. There is one nitrogen in the amino group. The oxygen count is four: one phenolic oxygen, one ether oxygen joining the rings, and two oxygens in the carboxyl group. For hydrogen, the phenolic ring has two carbon-bound hydrogens, and the other ring also has two. The side-chain carbon groups supply $2 + 1 = 3$ hydrogens, the amino group

supplies two, and the phenolic and carboxyl hydroxyl groups supply one each. Hence the total is $2+2+3+2+1+1=11$, and the molecular formula is $C_{15}H_{11}I_4NO_4$.

Step 2: Calculate the molar mass with the given atomic masses Multiply each atomic mass by its atom count and add the contributions:

$$\begin{aligned} M &= 15(12)+11(1)+4(127)+1(14)+4(16) \\ &= 180+11+508+14+64 \\ &= 777\,\text{g mol}^{-1}. \end{aligned}$$

In particular, iodine contributes $4(127)=508\,\text{g}$ per mole of thyroxine. The molecule contains iodine atoms rather than a separate molecule of iodine, so no additional factor of two is introduced.

Step 3: Form the iodine mass fraction and convert it to a percentage Divide the iodine contribution by the complete molar mass, including all the other elements:

$$\%\text{I} = \frac{508}{777} \times 100 = 65.379\ldots\,\%.$$

This exceeds half the molecular mass because iodine is much heavier than carbon, hydrogen, nitrogen and oxygen. Keeping the unrounded molar mass and ratio until this step prevents a premature rounding error.

Step 4: Conclusion The iodine content of thyroxine is $65.379\ldots\,\%$. To the nearest integer, the required percentage is $\boxed{65\%}$, so the numerical entry is $\boxed{65}$.

Quick Tip to Solve: Count the four visible iodine atoms first, but include both hydroxyl hydrogens and the amino hydrogens when finding the molecular mass from the skeletal formula.