JEE Main 2025 Paper 1 (B.E./B.Tech) Apr 7 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 7 April 2025, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let f : $\mathbb{R} \to \mathbb{R}$ be a polynomial function of degree four having extreme values at x = 4 and x = 5. If $\lim_{x\to 0} \frac{f(x)}{x^2} = 5$, then f(2) is equal to : [4 marks]

(A) 8

(B) 10

(C) 12

(D) 14

Correct Answer: (B) 10

Solution:

Concept:

- A finite limit of $f(x)/x^2$ at zero forces the constant and linear coefficients of the polynomial to vanish. The value of that limit is its quadratic coefficient.
- An interior extremum of a differentiable function satisfies $f\prime(x) = 0$. For a quartic polynomial, its derivative is cubic, so its three known roots determine it up to a constant.

Step 1: Use the limit at zero Write the quartic as $f(x) = ax^4 + bx^3 + cx^2 + dx + e$, where $a \neq 0$. Dividing by x^2, the terms d/x and e/x^2 must disappear for the given two-sided finite limit to exist. Thus $d = e = 0$, while $c = 5$. Consequently,

$$f(x) = ax^4 + bx^3 + 5x^2, \qquad f\prime(x) = 4ax^3 + 3bx^2 + 10x.$$

Step 2: Determine the derivative from its roots The derivative vanishes at zero automatically and at 4 and 5 by the extremum condition. These are three distinct roots of a cubic, giving

$$f\prime(x) = kx(x-4)(x-5) = k(x^3 - 9x^2 + 20x).$$

Comparing the coefficient of x gives $20k = 10$, hence $k = 1/2$. Comparing the other coefficients now yields $a = 1/8$ and $b = -3/2$. The simple roots at 4 and 5 do change the derivative sign, consistent with actual extrema.

Step 3: Evaluate the required value Substitute $x = 2$ into the determined polynomial:

$$f(2) = \frac{2^4}{8} - \frac{3(2^3)}{2} + 5(2^2) = 2 - 12 + 20 = 10.$$

Thus 8, 12 and 14 do not equal the value forced simultaneously by the limit and both extrema.

Step 4: Conclusion $f(2) = 10$. Hence, the correct option is (B).

Quick Tip to Solve: Use the limit to fix the lowest powers first; then factor the derivative using the extremum locations instead of solving three simultaneous equations.

2. Let y = y(x) be the solution of the differential equation $(x^2+1)y' - 2xy = (x^4+2x^2+1)\cos x$, $y(0) = 1$. **Then** $\int_{-3}^{3} y(x)\,dx$ **is :** [4 marks]

(A) 36

(B) 24

(C) 18

(D) 30

Correct Answer: (B) 24

Solution:

Concept:

- For $y' + P(x)y = Q(x)$, multiplication by the integrating factor $e^{\int P(x)\,dx}$ turns the left-hand side into a product derivative.
- An odd integrable function contributes zero over $[-a, a]$. The factor $1 + x^2$ is even, so its product with $\sin x$ is odd.

Step 1: Reduce to a linear equation Since $1 + x^2 > 0$ for every real x, divide the given equation throughout by this factor. Note that $x^4 + 2x^2 + 1 = (1 + x^2)^2$. We obtain

$$y' - \frac{2x}{1+x^2}y = (1+x^2)\cos x.$$

Its integrating factor is

$$\exp\left(\int \frac{-2x}{1+x^2}\,dx\right) = \frac{1}{1+x^2}.$$

No absolute-value ambiguity occurs because the denominator is positive.

Step 2: Apply the initial condition After multiplication by the integrating factor, the equation becomes

$$\frac{d}{dx}\left(\frac{y}{1+x^2}\right) = \cos x.$$

Integrating gives $y/(1 + x^2) = \sin x + C$. At $x = 0$, the condition $y(0) = 1$ gives $C = 1$. Hence

$$y(x) = (1+x^2)(1+\sin x).$$

This expression also verifies the initial value directly.

Step 3: Use symmetry before integrating The part $(1 + x^2)\sin x$ is odd and therefore integrates to zero between -3 and 3. Only the even polynomial remains:

$$\int_{-3}^{3} y(x)\,dx = 2\int_{0}^{3}(1+x^2)\,dx = 2\left[x + \frac{x^3}{3}\right]_0^3 = 2(3+9) = 24.$$

Careful evaluation therefore selects 24; the other three listed numbers do not equal this integral.

Step 4: Conclusion The required integral equals 24. Hence, the correct option is (B).

Quick Tip to Solve: After finding the solution, separate its even and odd parts before integrating across symmetric limits.

3. If the locus of z $\in \mathbb{C}$, such that Re$\left(\frac{z-1}{2z+i}\right)$ + Re $\left(\frac{\bar{z}-1}{2\bar{z}-i}\right) = 2$, is a circle of radius r and center (a, b), then $\frac{15ab}{r^2}$ is equal to : [4 marks]

(A) 16

(B) 24

(C) 12

(D) 18

Correct Answer: (D) 18

Solution:

Concept:

- For any complex number w, $\mathrm{Re}(\overline{w}) = \mathrm{Re}(w)$. Conjugating a quotient conjugates both its numerator and denominator.
- Put $z = x + iy$ and multiply a complex quotient by the conjugate of its denominator to extract its real part. Complete squares to identify a circle.

Step 1: Recognise the conjugate pair The second quotient is the complex conjugate of the first, because $\overline{2z + i} = 2\bar{z} - i$. Thus the given equality is equivalent, wherever the denominator is nonzero, to

$$\mathrm{Re}\left(\frac{z-1}{2z+i}\right) = 1.$$

Write $z = x + iy$; the forbidden point is $z = -i/2$, which must remain excluded throughout.

Step 2: Convert the locus to Cartesian form The numerator is $(x - 1) + iy$ and the denominator is $2x + i(2y + 1)$. Multiply by the conjugate of the denominator. Its real part equals 1 precisely when

$$\frac{2x(x-1) + y(2y+1)}{4x^2 + (2y+1)^2} = 1.$$

Expanding gives $2x^2 + 2y^2 + 2x + 3y + 1 = 0$. Divide by 2 and complete the squares:

$$\left(x + \frac{1}{2}\right)^2 + \left(y + \frac{3}{4}\right)^2 = \frac{5}{16}.$$

The undefined point $-i/2$ lies on this circle and is excluded from the original locus; it does not change the centre or radius.

Step 3: Calculate the requested ratio The circle parameters are $a = -1/2$, $b = -3/4$, and $r^2 = 5/16$. In particular, the two negative coordinates have a positive product. Therefore

$$\frac{15ab}{r^2} = \frac{15(3/8)}{5/16} = 18.$$

None of 16, 24 or 12 equals this ratio; using the radius rather than its square would not answer the printed expression.

Step 4: Conclusion The requested value is 18. Hence, the correct option is (D).

Quick Tip to Solve: First identify conjugate quotients: their real parts are equal, so the sum immediately halves the algebra.

4. Let a_n be the n^{th} term of an A.P. If $S_n = a_1 + a_2 + a_3 + \ldots + a_n = 700$, $a_6 = 7$ and $S_7 = 7$, then a_n is equal to : [4 marks]

(A) 65

(B) 56

(C) 70

(D) 64

Correct Answer: (D) 64

Solution:

Concept:

- For an arithmetic progression with first term a and common difference d, $a_k = a + (k-1)d$ and $S_n = n[2a + (n-1)d]/2$.
- The average of seven consecutive terms of an A.P. is the middle term, so $S_7 = 7a_4$. A term index must be a positive integer.

Step 1: Find the first term and difference Using the sum of the first seven terms gives

$$7 = \frac{7}{2}(2a + 6d), \qquad a + 3d = 1.$$

The sixth-term condition is $a + 5d = 7$. Subtracting these equations gives $2d = 6$, so $d = 3$ and $a = 1 - 9 = -8$. Check: the first seven terms are $-8, -5, -2, 1, 4, 7, 10$, whose sum is indeed 7 and whose sixth term is 7.

Step 2: Determine the index from the given sum Insert the obtained values into the general sum:

$$700 = \frac{n}{2}[-16 + 3(n-1)] = \frac{n}{2}(3n - 19).$$

Consequently,

$$3n^2 - 19n - 1400 = 0, \qquad (n - 25)(3n + 56) = 0.$$

The algebraic roots are 25 and $-56/3$. The second cannot be a number of terms, so the required index is $n = 25$. It would be incorrect to treat the supplied 700 as the value of the last term.

Step 3: Find the last term Use the term formula, remembering that the common difference is added $n - 1$ times:

$$a_n = a_{25} = -8 + 24(3) = 64.$$

As an independent check, $S_{25} = 25(-8 + 64)/2 = 700$. Thus 65, 56 and 70 fail the term or sum condition, while 64 satisfies both.

Step 4: Conclusion $a_n = 64$. Hence, the correct option is (D).

Quick Tip to Solve: For an odd number of A.P. terms, divide the sum by the number of terms to read off the middle term immediately.

5. A bag contains 19 unbiased coins and one coin with head on both sides. One coin drawn at random is tossed and head turns up. If the probability that the drawn coin was unbiased, is $\frac{m}{n}$, $\gcd(m,n)=1$, then n^2-m^2 is equal to : [4 marks]

(A) 64

(B) 80

(C) 60

(D) 72

Correct Answer: (B) 80

Solution:

Concept:

- Bayes' theorem updates the probability of the selected coin type after observing a head: $P(U \mid H) = P(U)P(H \mid U)/P(H)$.
- The unbiased and double-headed coin events form a partition. Their head probabilities are respectively $1/2$ and 1, while drawing any individual coin is equally likely.

Step 1: Define the events and prior probabilities Let U mean that an unbiased coin was drawn, D that the double-headed coin was drawn, and H that the toss shows head. The bag contains 20 coins in total, so

$$P(U)=\frac{19}{20},\quad P(D)=\frac{1}{20},\quad P(H\mid U)=\frac{1}{2},\quad P(H\mid D)=1.$$

Although unbiased coins are much more numerous, the double-headed coin is twice as likely as any particular unbiased coin to produce the observation.

Step 2: Condition on the observed head By the law of total probability,

$$P(H)=\frac{19}{20}\cdot\frac{1}{2}+\frac{1}{20}\cdot 1=\frac{21}{40}.$$

The joint probability that an unbiased coin is selected and produces a head is $19/40$. Therefore Bayes' theorem gives

$$P(U\mid H)=\frac{19/40}{21/40}=\frac{19}{21}.$$

The prior value $19/20$ cannot be retained after the observation; conditioning increases the relative chance of the double-headed coin.

Step 3: Reduce the fraction and evaluate The integers 19 and 21 have no common divisor greater than 1, so the required reduced fraction has $m=19$ and $n=21$. Use the difference of squares:

$$n^2-m^2=21^2-19^2=(21-19)(21+19)=2\cdot 40=80.$$

The other proposed values 64, 60 and 72 do not result from the reduced conditional probability.

Step 4: Conclusion $n^2-m^2=80$. Hence, the correct option is (B).

Quick Tip to Solve: After one observed head, assign weight 1 to each unbiased coin and weight 2 to the double-headed coin; the desired probability is then 19 out of 21.

6. If the range of the function $f(x) = \frac{5-x}{x^2-3x+2}$, $x \neq 1, 2$, is $(-\infty, \alpha] \cup [\beta, \infty)$, then $\alpha^2 + \beta^2$ is equal to : [4 marks]

(A) 188

(B) 192

(C) 190

(D) 194

Correct Answer: (D) 194

Solution:

Concept:

- A real value y lies in the range when the equation $y = f(x)$ has at least one real solution in the original domain. For a genuine quadratic, this requires its discriminant to be nonnegative.
- When clearing a denominator, check both excluded denominator zeros and any value of y that reduces the quadratic to a linear equation.

Step 1: Express the inverse condition as a quadratic Set $y = (5 - x)/(x^2 - 3x + 2)$. Multiplication by the denominator gives

$$yx^2 + (1 - 3y)x + (2y - 5) = 0.$$

For $y \neq 0$, this is quadratic in x. Its discriminant must satisfy

$$\Delta = (1 - 3y)^2 - 4y(2y - 5) = y^2 + 14y + 1 \geq 0.$$

Solving this inequality gives

$$y \leq -7 - 4\sqrt{3} \quad \text{or} \quad y \geq -7 + 4\sqrt{3}.$$

Equality is permitted because a repeated real root still supplies a value of the function.

Step 2: Check the original domain and the linear case Neither excluded input is an accidental root of the cleared equation: substituting $x = 1$ gives -4, and substituting $x = 2$ gives -3, independently of y. Thus every real root obtained above is admissible. For $y = 0$, the equation becomes $x - 5 = 0$, giving the allowed input $x = 5$. Zero is already inside the upper interval, so no value needs to be removed or added. This check establishes sufficiency as well as necessity of the range condition.

Step 3: Use the endpoint sum and product We identify $\alpha = -7 - 4\sqrt{3}$ and $\beta = -7 + 4\sqrt{3}$. Their sum is -14 and their product is $49 - 48 = 1$. Hence

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 196 - 2 = 194.$$

Therefore the other choices 188, 192 and 190 do not match the endpoint squares.

Step 4: Conclusion $\alpha^2 + \beta^2 = 194$. Hence, the correct option is (D).

Quick Tip to Solve: Find a rational function's range by treating its value as a parameter and demanding real admissible roots for the input.

7. The number of real roots of the equation $x|x-2|+3|x-3|+1=0$ is : [4 marks]

(A) 4

(B) 3

(C) 2

(D) 1

Correct Answer: (D) 1

Solution:

Concept:

- The expressions inside the absolute values change sign at $x=2$ and $x=3$. Split the real line at these points before removing the modulus signs.
- A root of a quadratic obtained on one interval counts only if it belongs to that interval. Squaring or solving a polynomial without checking its interval can introduce inadmissible answers.

Step 1: Examine the interval below 2 For $x<2$, both $x-2$ and $x-3$ are negative. Consequently the given equation becomes

$$x(2-x)+3(3-x)+1=0,$$

or

$$x^2+x-10=0.$$

Its two candidates are

$$x=\frac{-1\pm\sqrt{41}}{2}.$$

Since $\sqrt{41}>5$, the positive candidate is greater than 2 and must be rejected on this interval. The negative candidate is less than 2, so it gives exactly one valid root.

Step 2: Examine the interval from 2 to 3 For $2\le x<3$, use $|x-2|=x-2$ and $|x-3|=3-x$. The equation is

$$x(x-2)+3(3-x)+1=x^2-5x+10=0.$$

Its discriminant is

$$(-5)^2-4(1)(10)=-15<0.$$

There are no real roots, hence no roots on this interval. In particular, the boundary value $x=2$ does not solve the original equation.

Step 3: Examine the interval from 3 onwards For $x\ge 3$, both absolute values open with positive signs. We get

$$x(x-2)+3(x-3)+1=x^2+x-8=0.$$

Its roots are $(-1\pm\sqrt{33})/2$. The positive root is less than 3 because $\sqrt{33}<7$, while the other root is negative. Neither belongs to this interval, so this interval contributes no root. The three disjoint intervals exhaust the real line.

Step 4: Conclusion The equation has exactly 1 real root, namely $(-1-\sqrt{41})/2$. Counts 4, 3 and 2 include inadmissible quadratic roots. Hence, the correct option is (D).

Quick Tip to Solve: After splitting at every modulus breakpoint, write the interval beside each quadratic and test each candidate against it immediately.

8. Let the length of a latus rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be 10. If its eccentricity is the minimum value of the function $f(t)=t^2+t+\frac{11}{12}$, $t\in\mathbb{R}$, then a^2+b^2 is equal to : [4 marks]

(A) 115

(B) 120

(C) 125

(D) 126

Correct Answer: (D) 126

Solution:

Concept:

- For an ellipse with major semiaxis a and minor semiaxis b, $e^2=1-b^2/a^2$. The length of a latus rectum is $2b^2/a$.
- The minimum of a quadratic with positive leading coefficient can be read by completing the square. The eccentricity itself, rather than its square, equals the stated minimum.

Step 1: Find the eccentricity from the quadratic Complete the square in the expression provided:

$$f(t)=t^2+t+\frac{11}{12}=\left(t+\frac{1}{2}\right)^2+\frac{2}{3}.$$

The squared term is nonnegative for every real t, and is zero at $t=-1/2$. Thus its minimum is attained and equals $2/3$. The ellipse therefore has

$$e=\frac{2}{3},\qquad e^2=\frac{4}{9}.$$

Using $e=4/9$ would incorrectly confuse eccentricity with its square.

Step 2: Relate the two semiaxes Apply the eccentricity formula:

$$\frac{b^2}{a^2}=1-e^2=1-\frac{4}{9}=\frac{5}{9}.$$

Thus $b^2=5a^2/9$. This is the ratio of the squares of the minor and major semiaxes. Substituting it into the latus-rectum condition gives

$$\frac{2b^2}{a}=\frac{10a}{9}=10.$$

Since a semiaxis is a positive length, it follows that $a=9$.

Step 3: Calculate the requested sum and check Now

$$a^2=81,\qquad b^2=\frac{5}{9}(81)=45,$$

and hence

$$a^2+b^2=81+45=126.$$

The values satisfy both original conditions: $2b^2/a=90/9=10$, and $\sqrt{1-45/81}=2/3$. If the names of the major and minor axes are exchanged, their squared sum is unchanged. The other listed sums 115, 120 and 125 do not satisfy the two simultaneous conditions.

Step 4: Conclusion The required value is $\boxed{126}$. Hence, the correct option is (D).

Quick Tip to Solve: First find the eccentricity, then the ratio of squared semiaxes; the latus rectum fixes their absolute size.

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9. Let e_1 and e_2 be the eccentricities of the ellipse $\frac{x^2}{b^2}+\frac{y^2}{25}=1$ and the hyperbola $\frac{x^2}{16}-\frac{y^2}{b^2}=1$, respectively. If $b<5$ and $e_1e_2=1$, then the eccentricity of the ellipse having its axes along the coordinate axes and passing through all four foci (two of the ellipse and two of the hyperbola) is : [4 marks]

(A) $\frac{4}{5}$

(B) $\frac{3}{5}$

(C) $\frac{\sqrt{7}}{4}$

(D) $\frac{\sqrt{3}}{2}$

Correct Answer: (B) $\frac{3}{5}$

Solution:

Concept:

- An ellipse with squared major and minor semiaxes A^2, B^2 has focal distance $c=\sqrt{A^2-B^2}$ and eccentricity c/A. Its foci lie along its major axis.
- For $x^2/a^2-y^2/b^2=1$, the hyperbola has foci $(\pm\sqrt{a^2+b^2},0)$ and eccentricity $\sqrt{1+b^2/a^2}$.

Step 1: Use the product of eccentricities Write $u=b^2$, so the nondegenerate given ellipse has $0<u<25$. Its major axis is vertical, and

$$e_1^2=\frac{25-u}{25}.$$

The hyperbola has $a^2=16$, so

$$e_2^2=\frac{16+u}{16}.$$

Both eccentricities are positive. Squaring the given product $e_1e_2=1$ is therefore valid and gives

$$(25-u)(16+u)=400.$$

Step 2: Determine the four foci Expand and simplify the last relation:

$$400+9u-u^2=400, \qquad u(9-u)=0.$$

The value $u=0$ is excluded because it makes the given conics degenerate and their denominators invalid. Hence $u=9$. The original ellipse has focal distance $\sqrt{25-9}=4$, giving foci $(0,\pm4)$. The hyperbola has focal distance $\sqrt{16+9}=5$, giving foci $(\pm5,0)$.

Step 3: Form the ellipse through the four foci An ellipse with axes along the coordinate axes has equation $x^2/A^2+y^2/B^2=1$. Passing through $(\pm5,0)$ forces $A^2=25$, and passing through $(0,\pm4)$

forces $B^2 = 16$. Its major axis is therefore horizontal. The required eccentricity is

$$e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}.$$

The ratio $4/5$ is the minor-to-major semiaxis ratio, not the eccentricity; the remaining two options have different squared ratios.

Step 4: Conclusion The eccentricity of the required ellipse is $\boxed{3/5}$. Hence, the correct option is (B).

Quick Tip to Solve: Locate the foci of the original conics first; their coordinates become the axis intercepts of the new ellipse.

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10. If the equation of the line passing through the point $\left(0, -\frac{1}{2}, 0\right)$ and perpendicular to the lines $\vec{r} = \lambda\left(\hat{i} + a\hat{j} + b\hat{k}\right)$ and $\vec{r} = \left(\hat{i} - \hat{j} - 6\hat{k}\right) + \mu\left(-b\hat{i} + a\hat{j} + 5\hat{k}\right)$ is $\frac{x-1}{-2} = \frac{y+4}{d} = \frac{z-c}{-4}$, then $a + b + c + d$ is equal to : [4 marks]

(A) 13

(B) 14

(C) 12

(D) 10

Correct Answer: (B) 14

Solution:

Concept:

- The line $(x-1)/(-2) = (y+4)/d = (z-c)/(-4)$ has direction vector $(-2, d, -4)$ and passes through $(1, -4, c)$.
- Two lines have perpendicular directions when the dot product of their direction vectors is zero. To impose passage through a point, substitute the same parameter into all three coordinate equations.

Step 1: Apply the condition that the line passes through the point Introduce a parameter s in the required line:

$$x = 1 - 2s, \qquad y = -4 + ds, \qquad z = c - 4s.$$

At the given point $(0, -1/2, 0)$, the first equation gives $s = 1/2$. Substituting this same parameter in the other two equations yields

$$-\frac{1}{2} = -4 + \frac{d}{2}, \qquad 0 = c - 2.$$

Therefore $d = 7$ and $c = 2$. A separate parameter for each coordinate would fail to describe one point on the line.

Step 2: Use perpendicularity to the first line The first given line has direction vector $(1, a, b)$. The direction of the required line is now $(-2, 7, -4)$. Their dot product must vanish:

$$(-2)(1) + 7a - 4b = 0.$$

Thus

$$7a - 4b = 2.$$

Only the direction vector enters this condition; the position vector in the second given line will likewise not affect its direction.

Step 3: Use perpendicularity to the second line and sum The second line has direction vector $(-b, a, 5)$. Thus

$$(-2)(-b) + 7a + (-4)(5) = 0,$$

giving

$$7a + 2b = 20.$$

Subtracting $7a - 4b = 2$ gives $6b = 18$, so $b = 3$. Then $7a = 14$, so $a = 2$. Both dot products check to zero: $-2 + 14 - 12 = 0$ and $6 + 14 - 20 = 0$. Finally,

$$a + b + c + d = 2 + 3 + 2 + 7 = 14.$$

The other proposed sums cannot arise from these uniquely determined values.

Step 4: Conclusion The required sum is $\boxed{14}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the known point to determine the direction parameter first, then take two dot products to find the remaining constants.

11. If the sum of the second, fourth and sixth terms of a G.P. of positive terms is 21 and the sum of its eighth, tenth and twelfth terms is 15309, then the sum of its first nine terms is : [4 marks]

(A) 757

(B) 755

(C) 750

(D) 760

Correct Answer: (A) 757

Solution:

Concept:

- For a geometric progression with first term a and common ratio r, its nth term is ar^{n-1}. Positive terms imply $a > 0$ and $r > 0$.
- If corresponding terms in two sums are all shifted forward by six positions, the second sum is r^6 times the first. For $r \neq 1$, $S_n = a(r^n - 1)/(r - 1)$.

Step 1: Compare the two given sums The sum of the second, fourth and sixth terms is

$$ar + ar^3 + ar^5 = ar(1 + r^2 + r^4) = 21.$$

The sum of the eighth, tenth and twelfth terms is

$$ar^7 + ar^9 + ar^{11} = ar^7(1 + r^2 + r^4) = 15309.$$

Each selected term in the second sum occurs six positions after its counterpart in the first. Divide the second equation by the first; division is valid since the first sum is 21, not zero.

Step 2: Find the common ratio and the first term The division gives

$$r^6 = \frac{15309}{21} = 729 = 3^6.$$

Since all terms of the progression are positive, its common ratio is positive, so $r = 3$. The negative sixth root is not admissible here. Substitute in the first sum:

$$a(3 + 27 + 243) = 273a = 21.$$

Therefore

$$a = \frac{21}{273} = \frac{1}{13}.$$

The common ratio cannot be 1 because the two given sums differ.

Step 3: Compute the first-nine-term sum Use the ordinary sum formula for nine consecutive terms, including the first term:

$$S_9 = \frac{1}{13}\frac{3^9 - 1}{3 - 1} = \frac{19683 - 1}{26} = \frac{19682}{26} = 757.$$

As a check, the initial sum is $(3 + 27 + 243)/13 = 21$, and shifting its indices by six multiplies it by 729 to give 15309. The requested sum is of consecutive terms, so summing only odd or only even terms would be incorrect. The alternative values 755, 750 and 760 do not equal the exact geometric sum.

Step 4: Conclusion The sum of the first nine terms is $\boxed{757}$. Hence, the correct option is (A).

Quick Tip to Solve: When two sums contain the same index pattern shifted by a fixed amount, divide them before solving separately for the first term.

12. If the area of the region $\{(x, y) : 1 + x^2 \leq y \leq \min\{x + 7, 11 - 3x\}\}$ is A, then $3A$ is equal to :

[4 marks]

(A) 50

(B) 46

(C) 49

(D) 47

Correct Answer: (A) 50

Solution:

Concept:

- The lower boundary is $y = 1 + x^2$. The upper boundary is the lower of the two lines $y = x + 7$ and $y = 11 - 3x$, and changes at their intersection.
- Area is the integral of upper minus lower over the interval where this difference is nonnegative. Both line inequalities must hold because the upper bound is their minimum.

Step 1: Find the interval supporting the region The parabola lies below the first line when

$$1 + x^2 \leq x + 7 \iff (x - 3)(x + 2) \leq 0,$$

giving $-2 \leq x \leq 3$. It lies below the second line when

$$1 + x^2 \leq 11 - 3x \iff (x + 5)(x - 2) \leq 0,$$

giving $-5 \leq x \leq 2$. Both restrictions are required, so the actual horizontal extent is their intersection $[-2, 2]$. Taking their union would include points for which one upper bound lies below the parabola.

Step 2: Identify the upper boundary on each subinterval The lines meet when

$$x + 7 = 11 - 3x \iff x = 1.$$

Their difference is $(x + 7) - (11 - 3x) = 4(x - 1)$. Thus $x + 7$ is the lower line for $x \leq 1$, and $11 - 3x$ is the lower line for $x \geq 1$. The area consequently splits as

$$A = \int_{-2}^{1} (6 + x - x^2)\, dx + \int_{1}^{2} (10 - 3x - x^2)\, dx.$$

Both integrands are nonnegative on their respective intervals.

Step 3: Evaluate the two areas and the requested multiple The first integral is

$$\left[6x + \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^{1} = \frac{37}{6} + \frac{22}{3} = \frac{27}{2}.$$

The second integral is

$$\left[10x - \frac{3x^2}{2} - \frac{x^3}{3}\right]_{1}^{2} = \frac{34}{3} - \frac{49}{6} = \frac{19}{6}.$$

Therefore

$$A = \frac{27}{2} + \frac{19}{6} = \frac{50}{3}, \qquad 3A = 50.$$

The remaining options differ from this exact integral and are rejected.

Step 4: Conclusion The required value is $\boxed{3A = 50}$. Hence, the correct option is (A).

Quick Tip to Solve: For a minimum of two upper boundaries, intersect the allowable x-intervals and split the area integral where the upper curves cross.

13. The number of solutions of the equation

$\cos 2\theta \cos \frac{\theta}{2} + \cos \frac{5\theta}{2} = 2\cos^3 \frac{5\theta}{2}$ **in** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ **is :** [4 marks]

(A) 5

(B) 7

(C) 6

(D) 9

Correct Answer: (B) 7

Solution:

Concept:

- Use the product-to-sum identity $2\cos A \cos B = \cos(A+B) + \cos(A-B)$ to express both products in terms of single cosine functions.
- The identity $2\cos^3 u - \cos u = \cos u \cos 2u$ reduces the cubic expression. Count distinct angles in the specified closed interval, counting common roots only once.

Step 1: Remove the cubic cosine Put $u = 5\theta/2$. Moving the single cosine to the right gives

$$\cos 2\theta \cos \frac{\theta}{2} = 2\cos^3 \frac{5\theta}{2} - \cos \frac{5\theta}{2} = \cos \frac{5\theta}{2} \cos 5\theta.$$

This is an identity-based transformation and involves no division, so zeros of any cosine have not been discarded.

Step 2: Factor the resulting equation Apply product-to-sum on both sides and cancel their common term:

$$\cos \frac{5\theta}{2} + \cos \frac{3\theta}{2} = \cos \frac{15\theta}{2} + \cos \frac{5\theta}{2}.$$

Consequently,

$$\cos \frac{3\theta}{2} - \cos \frac{15\theta}{2} = 2\sin \frac{9\theta}{2} \sin 3\theta = 0.$$

Thus either sine factor must vanish.

Step 3: Count both root families The first family is $\theta = 2k\pi/9$. The interval condition gives $-9/4 \le k \le 9/4$, hence $k = -2, -1, 0, 1, 2$: five roots. The second family is $\theta = k\pi/3$, with $k = -1, 0, 1$: three roots. A common root requires $2k = 3j$; within these ranges only zero is common. Therefore the distinct-root count is $5 + 3 - 1 = 7$. Neither endpoint satisfies either family. The other offered counts omit or duplicate roots.

Step 4: Conclusion There are 7 distinct solutions in the given interval. Hence, the correct option is (B).

Quick Tip to Solve: Convert the cubic cosine into a product, then count the two sine-zero families using inclusion-exclusion.

14. Consider the lines L_1: x - 1 = y - 2 = z and L_2: x - 2 = y = z - 1. Let the feet of the perpendiculars from the point P(5, 1, -3) on the lines L_1 and L_2 be Q and R respectively. If the area of the triangle PQR is A, then $4A^2$ is equal to : [4 marks]

(A) 151

(B) 147

(C) 139

(D) 143

Correct Answer: (B) 147

Solution:

Concept:

- For a line $\mathbf{r} = \mathbf{a} + t\mathbf{d}$, the perpendicular foot from P has $t = (\mathbf{P} - \mathbf{a}) \cdot \mathbf{d}/(\mathbf{d} \cdot \mathbf{d})$. The connecting vector is orthogonal to the line.
- If two sides of a triangle are vectors $\mathbf{u}, \mathbf{v}$, its area is $A = \frac{1}{2}|\mathbf{u} \times \mathbf{v}|$, so $4A^2 = |\mathbf{u} \times \mathbf{v}|^2$.

Step 1: Find the first perpendicular foot The equal parameters in the first line give

$$L_1 : (x, y, z) = (1, 2, 0) + t(1, 1, 1).$$

For $P = (5, 1, -3)$, the displacement from its base point is $(4, -1, -3)$. Its dot product with $(1, 1, 1)$ is $4 - 1 - 3 = 0$, so the base point already is the foot. Therefore $Q = (1, 2, 0)$.

Step 2: Find the second perpendicular foot Similarly,

$$L_2 : (x, y, z) = (2, 0, 1) + s(1, 1, 1).$$

Here $P - (2, 0, 1) = (3, 1, -4)$, whose dot product with the line direction is $3 + 1 - 4 = 0$. Thus $s = 0$ and $R = (2, 0, 1)$. Both feet lie on the specified lines and both perpendicularity checks are exact.

Step 3: Calculate the squared cross product Take the two sides from the same vertex:

$$\overrightarrow{PQ} = (-4, 1, 3), \qquad \overrightarrow{PR} = (-3, -1, 4).$$

Their cross product is

$$\overrightarrow{PQ} \times \overrightarrow{PR} = (4 + 3,\ -9 + 16,\ 4 + 3) = (7, 7, 7).$$

In the middle component the correct calculation is $3(-3) - (-4)(4) = -9 + 16 = 7$; thus all three components equal 7. Hence $4A^2 = 7^2 + 7^2 + 7^2 = 147$. The other numerical options do not equal this squared norm.

Step 4: Conclusion $4A^2 = 147$. Hence, the correct option is (B).

Quick Tip to Solve: Check the base-point displacement dot the line direction first: a zero dot product identifies the foot immediately.

15. Let A = { $(\alpha, \beta) \in \mathbb{R} \times \mathbb{R}$: $|\alpha - 1| \leq 4$ and $|\beta - 5| \leq 6$ }
and B = { $(\alpha, \beta) \in \mathbb{R} \times \mathbb{R}$: $16(\alpha - 2)^2 + 9(\beta - 6)^2 \leq 144$ }.
Then [4 marks]

(A) A $\subset$ B

(B) B $\subset$ A

(C) neither A $\subset$ B nor B $\subset$ A

(D) $A \cup B = \{(x, y) : -4 \leqslant x \leqslant 4, -1 \leqslant y \leqslant 11\}$

Correct Answer: (B) B $\subset$ A

Solution:

Concept:

- An inequality $|x - a| \leq r$ describes the closed interval $[a - r, a + r]$. Applying this to each coordinate gives the rectangular set A.
- The ellipse $(x - h)^2/a^2 + (y - k)^2/b^2 \leq 1$ lies within $h - a \leq x \leq h + a$ and $k - b \leq y \leq k + b$. Coordinate bounds can establish containment in a rectangle.

Step 1: Describe the rectangle The first pair of inequalities gives

$$|\alpha - 1| \leq 4 \Longrightarrow -3 \leq \alpha \leq 5,$$

$$|\beta - 5| \leq 6 \Longrightarrow -1 \leq \beta \leq 11.$$

Thus $A = [-3, 5] \times [-1, 11]$, including its boundary. The lower horizontal bound is -3, not the -4 appearing in option (D).

Step 2: Bound every point of the ellipse Divide the defining inequality for B by 144:

$$\frac{(\alpha - 2)^2}{9} + \frac{(\beta - 6)^2}{16} \leq 1.$$

Both terms are nonnegative, so each is at most 1. It follows that $|\alpha - 2| \leq 3$ and $|\beta - 6| \leq 4$. Consequently every point of B satisfies $-1 \leq \alpha \leq 5$ and $2 \leq \beta \leq 10$. These coordinate intervals are contained in the corresponding intervals defining A, proving $B \subseteq A$.

Step 3: Check that the containment is proper The point $(-3, -1)$ belongs to A. In the inequality for B, however, its left side is $16(-5)^2 + 9(-7)^2 = 400 + 441 = 841 > 144$, so it does not belong to B. Thus A is not contained in B, and B is a proper subset of A. This rejects (A) and (C). Also $A \cup B = A$, whose bounds differ from (D); for example $(5, 5) \in A$ lies outside its proposed set.

Step 4: Conclusion $B \subset A$, with proper containment. Hence, the correct option is (B).

Quick Tip to Solve: To place an axis-aligned ellipse inside a rectangle, compare its extreme coordinate bounds and then check properness with one point.

16. Let $\vec{a}$ and $\vec{b}$ be the vectors of the same magnitude such that $\frac{|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|-|\vec{a}-\vec{b}|} = \sqrt{2}+1$. Then $\frac{|\vec{a}+\vec{b}|^2}{|\vec{a}|^2}$ is : [4 marks]

(A) $2 + \sqrt{2}$

(B) $2 + 4\sqrt{2}$

(C) $4 + 2\sqrt{2}$

(D) $1 + \sqrt{2}$

Correct Answer: (A) $2 + \sqrt{2}$

Solution:

Concept:

- The parallelogram identity is $|\vec{a}+\vec{b}|^2 + |\vec{a}-\vec{b}|^2 = 2(|\vec{a}|^2 + |\vec{b}|^2)$. For equal magnitudes m, the right side is $4m^2$.
- A ratio involving the sum and difference of two nonnegative magnitudes can first be solved for their quotient. The original denominator must remain nonzero.

Step 1: Name the two magnitudes Let $X = |\vec{a}+\vec{b}|$, $Y = |\vec{a}-\vec{b}|$, and $m = |\vec{a}| = |\vec{b}|$. The given finite ratio requires $X - Y \neq 0$. Since its value is positive and $X + Y > 0$, it also follows that $X > Y \geq 0$, so dividing by X is legitimate. Write

$$\frac{1 + Y/X}{1 - Y/X} = 1 + \sqrt{2}.$$

Step 2: Solve the magnitude ratio Cross-multiplication gives

$$1 + Y/X = (1 + \sqrt{2})(1 - Y/X),$$

$$\frac{Y}{X} = \frac{\sqrt{2}}{2 + \sqrt{2}} = \sqrt{2} - 1.$$

This value lies between zero and one, consistently with the sign of the original denominator. Squaring now yields $Y^2 = (3 - 2\sqrt{2})X^2$.

Step 3: Use equal vector lengths By the parallelogram identity,

$$X^2 + Y^2 = 4m^2.$$

Substituting the ratio therefore gives

$$\frac{X^2}{m^2} = \frac{4}{1 + (\sqrt{2} - 1)^2} = \frac{4}{4 - 2\sqrt{2}} = 2 + \sqrt{2}.$$

As a check, the squared sum magnitude cannot exceed $(2m)^2$, so the requested ratio is at most 4. Options (B) and (C) exceed this bound. Option (D) is below 4 but does not satisfy the exact ratio equation, leaving the calculated value uniquely.

Step 4: Conclusion $\frac{|\vec{a}+\vec{b}|^2}{|\vec{a}|^2} = 2 + \sqrt{2}$. Hence, the correct option is (A).

Quick Tip to Solve: Solve for the ratio of the two lengths first, then use the parallelogram identity to avoid introducing the angle between vectors.

17. Let p be the number of all triangles that can be formed by joining the vertices of a regular polygon P of n sides and q be the number of all quadrilaterals that can be formed by joining the vertices of P. If p + q = 126, then the eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{n} = 1$ is : [4 marks]

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{7}}{4}$

(D) $\frac{3}{4}$

Correct Answer: (A) $\frac{1}{\sqrt{2}}$

Solution:

Concept:

- Any three distinct vertices of a regular polygon form a triangle, and any four form a quadrilateral, since the vertices lie on a circle and no three are collinear.
- Pascal's identity gives $\binom{n}{3} + \binom{n}{4} = \binom{n+1}{4}$. For an ellipse with major and minor squared semiaxes $a^2 > b^2$, its eccentricity is $e = \sqrt{1 - b^2/a^2}$.

Step 1: Count the polygons formed from vertices Choosing three vertices completely determines a triangle, without regard to the order of selection. Similarly, four chosen vertices determine a convex quadrilateral when joined in their circular order. Therefore

$$p = \binom{n}{3}, \qquad q = \binom{n}{4}.$$

There is no additional factor for ordering or for rotations: the vertices are fixed and we count the distinct selections.

Step 2: Determine the number of sides The given total and Pascal's identity imply

$$126 = \binom{n}{3} + \binom{n}{4} = \binom{n+1}{4}.$$

Now $\binom{9}{4} = 126$, so $n + 1 = 9$, hence $n = 8$. This is unique because $\binom{m}{4}$ increases strictly for integral $m \geq 4$. Direct substitution confirms $\binom{8}{3} + \binom{8}{4} = 56 + 70 = 126$.

Step 3: Identify the major axis and eccentricity The ellipse becomes

$$\frac{x^2}{16} + \frac{y^2}{8} = 1.$$

The larger denominator is 16, so $a^2 = 16$ and $b^2 = 8$. Hence

$$e = \sqrt{1 - \frac{8}{16}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}.$$

In particular $e^2 = 1/2$; the squares of the other proposed values are $1/4, 7/16, 9/16$, respectively. None equals $1/2$, so those options are excluded.

Step 4: Conclusion The eccentricity is $e = 1/\sqrt{2}$. Hence, the correct option is (A).

Quick Tip to Solve: Combine consecutive binomial coefficients with Pascal's identity before finding the ellipse parameters; compare squared eccentricities to check options.

18. Let a random variable X take values 0, 1, 2, 3 with P(X=0)=P(X=1)=p, P(X=2)=P(X=3) and $E(X^2)=2E(X)$. Then the value of 8p−1 is : [4 marks]

(A) 2

(B) 0

(C) 3

(D) 1

Correct Answer: (A) 2

Solution:

Concept:

- For a discrete distribution, all probabilities are nonnegative and their sum is 1. If the probabilities at 0 and 1 are p, and those at 2 and 3 are q, normalization gives $2p + 2q = 1$.
- The moments are $E(X) = \sum xP(X = x)$ and $E(X^2) = \sum x^2P(X = x)$. In general $E(X^2)$ is not the square of $E(X)$.

Step 1: Write the normalization equation Let $P(X = 2) = P(X = 3) = q$. The four probabilities are thus p, p, q, q. Adding them gives

$$2p + 2q = 1, \qquad p + q = \frac{1}{2}.$$

This condition must be used together with the given relation between the moments; the latter alone does not determine the scale of the probabilities.

Step 2: Calculate both moments explicitly Multiplying each possible value by its probability gives

$$E(X) = 0p + 1p + 2q + 3q = p + 5q.$$

For the second moment, square the values before weighting:

$$E(X^2) = 0^2p + 1^2p + 2^2q + 3^2q = p + 13q.$$

The supplied relation therefore reads

$$p + 13q = 2(p + 5q),$$

which simplifies to $p = 3q$.

Step 3: Solve and evaluate the requested expression Substitute $p = 3q$ into normalization:

$$3q + q = \frac{1}{2} \Longrightarrow q = \frac{1}{8}, \qquad p = \frac{3}{8}.$$

All four probabilities lie between zero and one and sum to one. Checking the moments, $E(X) = 3/8 + 5/8 = 1$ and $E(X^2) = 3/8 + 13/8 = 2$, exactly as required. Finally,

$$8p - 1 = 8\left(\frac{3}{8}\right) - 1 = 2.$$

The alternatives 0, 3 and 1 correspond to other values of p and cannot meet both normalization and the stated moment equation.

Step 4: Conclusion $8p - 1 = 2$. Hence, the correct option is (A).

Quick Tip to Solve: Assign one symbol to each repeated probability, normalize first, and calculate the two moments separately to avoid confusing them.

19. Let the system of equations

x + 5y - z = 1

4x + 3y - 3z = 7

24x + y + λz = μ

λ, $\mu \in \mathbb{R}$, have infinitely many solutions. Then the number of the solutions of this system, if x, y, z are integers and satisfy 7 $\leq$ x + y + z $\leq$ 77, is : [4 marks]

(A) 4

(B) 5

(C) 3

(D) 6

Correct Answer: (C) 3

Solution:

Concept:

- Infinitely many solutions require the third equation to be dependent on the first two, including its constant term.
- For integer solutions, eliminate a variable first and use one integer parameter; then impose the bounds on the sum.

Step 1: Parametrise the first two equations The first equation gives $z = x + 5y - 1$. Substitution into the second equation yields

$$4x + 3y - 3(x + 5y - 1) = 7 \quad \Longrightarrow \quad x - 12y = 4.$$

Thus, writing $y = t$, we obtain

$$x = 12t + 4, \qquad y = t, \qquad z = 17t + 3.$$

Every integer solution has integer t, and every integer t makes all three coordinates integers.

Step 2: Use the dependence condition Substitute the parametrisation into the last equation:

$$24x + y + \lambda z = (289 + 17\lambda)t + 96 + 3\lambda = \mu.$$

For infinitely many solutions, the coefficient of t must vanish. Consequently $\lambda = -17$ and $\mu = 45$. With these values, the third equation adds no further restriction. This confirms that counting the

parameter values counts the solutions of the entire system.

Step 3: Apply the given bounds The sum of the three coordinates is

$$x + y + z = 30t + 7.$$

Therefore

$$7 \le 30t + 7 \le 77 \quad \Longrightarrow \quad 0 \le t \le \frac{7}{3}.$$

The admissible integers are exactly $0, 1, 2$, producing $(4, 0, 3)$, $(16, 1, 20)$, and $(28, 2, 37)$. Their sums are respectively $7, 37, 67$; the next integer parameter gives sum 97, outside the interval. Hence the counts 4, 5 and 6 cannot satisfy the upper bound.

Step 4: Conclusion There are $\boxed{3}$ integer solutions. Hence, the correct option is (C).

Quick Tip to Solve: After parametrising the integer solutions, count integer parameter values in the resulting closed interval.

20. If the orthocenter of the triangle formed by the lines y = x + 1, y = 4x - 8 and y = mx + c is at (3, -1), then m - c is : [4 marks]

(A) 0

(B) 2

(C) -2

(D) 4

Correct Answer: (A) 0

Solution:

Concept:

- The orthocentre lies on every altitude of a triangle. An altitude is perpendicular to the opposite side.
- Two nonvertical perpendicular lines have slopes whose product is -1; a vertical altitude has a horizontal opposite side.

Step 1: Find the known vertex and its altitude Let A be the intersection of the first two lines. Equating their ordinates gives

$$x + 1 = 4x - 8 \quad \Longrightarrow \quad A = (3, 4).$$

The orthocentre is $H = (3, -1)$. Since A and H have the same abscissa and different ordinates, AH is the vertical line $x = 3$. This line is the altitude from A, so its opposite side, the third given line, must be horizontal. Therefore $m = 0$, and that side is $y = c$.

Step 2: Use an altitude from another vertex Let B be the intersection of $y = c$ with $y = x + 1$. Then $B = (c - 1, c)$. The side opposite B lies on $y = 4x - 8$, whose slope is 4. Thus the altitude BH has slope $-1/4$ and equation

$$y + 1 = -\frac{1}{4}(x - 3).$$

Substituting the coordinates of B gives

$$c+1=-\frac{1}{4}(c-4), \qquad 4c+4=-c+4,$$

so $c=0$. Both altitude conditions have now been used; merely finding the horizontal direction would not determine its intercept.

Step 3: Check the resulting triangle The third side is $y=0$, meeting the other two lines at $B=(-1,0)$ and $C=(2,0)$. These points are distinct from $A=(3,4)$, so they form a nondegenerate triangle. The slope of BH is $-1/4$, perpendicular to AC, while AH is perpendicular to BC. Hence the given point is indeed the orthocentre. We have $m-c=0-0=0$, ruling out the other listed values.

Step 4: Conclusion $\boxed{m-c=0}$. Hence, the correct option is (A).

Quick Tip to Solve: Start with the vertex obtained from the two known sides; its altitude immediately fixes the direction of the unknown side.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. The sum of the series $2\times1\times{}^{20}C_4-3\times2\times{}^{20}C_5+4\times3\times{}^{20}C_6-5\times4\times{}^{20}C_7+\cdots\cdots+18\times17\times{}^{20}C_{20}$, is equal to _____. [4 marks]

Solution:

Concept:

- The binomial theorem gives $F(t)=(1+t)^{20}=\sum_{k=0}^{20}\binom{20}{k}t^k$.
- Derivatives generate factors k and $k(k-1)$. Evaluating at $t=-1$ produces alternating sums.

Step 1: Write the general term The coefficient multiplying $\binom{20}{k}$ in the stated series is $(k-2)(k-3)$, and its sign is $(-1)^k$. Thus the required sum is

$$S=\sum_{k=4}^{20}(-1)^k(k-2)(k-3)\binom{20}{k}.$$

This expression reproduces the first coefficient $2\cdot1$, the next coefficient $-3\cdot2$, and the positive last coefficient $18\cdot17$. Expand the quadratic as $(k-2)(k-3)=k(k-1)-4k+6$.

Step 2: Evaluate the complete alternating sum Differentiating the finite polynomial introduces no convergence issue:

$$\sum k\binom{20}{k}t^k=tF'(t), \qquad \sum k(k-1)\binom{20}{k}t^k=t^2F''(t).$$

All sums here run from 0 to 20. Hence

$$\sum_{k=0}^{20}(k-2)(k-3)\binom{20}{k}t^k=t^2F''(t)-4tF'(t)+6F(t).$$

At $t = -1$, each term on the right vanishes: F, F', and F'' contain respectively the factors $(1+t)^{20}$, $(1+t)^{19}$, and $(1+t)^{18}$. The complete alternating sum is therefore zero.

Step 3: Remove the initial terms The terms omitted from the requested series correspond to $k = 0, 1, 2, 3$. Their total is

$$6\binom{20}{0} - 2\binom{20}{1} + 0 + 0 = 6 - 40 = -34.$$

The two zero terms arise from the factors $k-2$ and $k-3$, not from vanishing binomial coefficients. Since the complete sum is zero, we have $0 = -34 + S$, so $S = 34$.

Step 4: Conclusion The required sum is $\boxed{34}$.

Quick Tip to Solve: Complete an alternating binomial sum from index 0, use derivatives for polynomial factors, then subtract the extra terms.

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22. If the function $f(x) = \frac{\tan(\tan x) - \sin(\sin x)}{\tan x - \sin x}$ is continuous at $x = 0$, then $f(0)$ is equal to _____.

[4 marks]

Solution:

Concept:

- Continuity at the origin requires $f(0) = \lim_{x \to 0} f(x)$, because the displayed quotient is initially undefined there.
- Near zero, $\tan u = u + u^3/3 + O(u^5)$ and $\sin u = u - u^3/6 + O(u^5)$.

Step 1: Expand the denominator to its first nonzero term The linear terms cancel when the two elementary functions are subtracted:

$$\tan x - \sin x = \left(x + \frac{x^3}{3} + O(x^5)\right) - \left(x - \frac{x^3}{6} + O(x^5)\right) = \frac{x^3}{2} + O(x^5).$$

Therefore a first-order approximation alone cannot evaluate this limit. We must also retain cubic terms in the composed functions in the numerator.

Step 2: Expand the two compositions For the tangent composition, use $u = \tan x = x + x^3/3 + O(x^5)$. Since $u^3 = x^3 + O(x^5)$,

$$\tan(\tan x) = u + \frac{u^3}{3} + O(u^5) = x + \frac{2x^3}{3} + O(x^5).$$

Likewise, for $v = \sin x = x - x^3/6 + O(x^5)$,

$$\sin(\sin x) = v - \frac{v^3}{6} + O(v^5) = x - \frac{x^3}{3} + O(x^5).$$

Both the inner and outer functions contribute to the cubic coefficient. Subtracting these expressions makes the numerator $x^3 + O(x^5)$.

Step 3: Take the two-sided limit For nonzero x sufficiently close to zero, divide numerator and denominator by x^3:

$$f(x) = \frac{1 + O(x^2)}{\frac{1}{2} + O(x^2)} \longrightarrow 2.$$

The denominator after division tends to the nonzero number $1/2$, so the quotient limit exists from both sides. Assigning the value 2 at the origin removes the discontinuity and is the unique assignment that makes the function continuous there.

Step 4: Conclusion Continuity requires $\boxed{f(0) = 2}$.

Quick Tip to Solve: When linear terms cancel in a quotient of odd functions, retain cubic terms and include the contribution from each composition.

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23. If $\int \left(\frac{1}{x} + \frac{1}{x^3}\right) \left(\sqrt[23]{3x^{-24} + x^{-26}}\right) \mathrm{d}x = -\frac{\alpha}{3(\alpha+1)} \left(3x^{\beta} + x^{\gamma}\right)^{\frac{\alpha+1}{\alpha}} + C, x > 0, (\alpha, \beta, \gamma \in \mathbf{Z})$, where C is the constant of integration, then $\alpha + \beta + \gamma$ is equal to _____. [4 marks]

Solution:

Concept:

- A substitution is effective when the integrand can be written as a constant multiple of $u^r\,\mathrm{d}u$.
- For $x > 0$, fractional powers of positive expressions obey the usual exponent laws without absolute-value ambiguity.

Step 1: Factor the expression under the root Rewrite the radical to expose a useful substitution:

$$3x^{-24} + x^{-26} = x^{-23}(3x^{-1} + x^{-3}).$$

Because $x > 0$, its twenty-third root is

$$x^{-1}(3x^{-1} + x^{-3})^{1/23}.$$

Multiplication by the remaining factor transforms the integral into

$$I = \int (x^{-2} + x^{-4})(3x^{-1} + x^{-3})^{1/23}\,\mathrm{d}x.$$

This factoring is chosen so that differentiating the bracket produces precisely the two powers outside it.

Step 2: Integrate using the bracket as a variable Set $u = 3x^{-1} + x^{-3}$. Then

$$\mathrm{d}u = -3(x^{-2} + x^{-4})\,\mathrm{d}x.$$

The integral becomes

$$I = -\frac{1}{3}\int u^{1/23}\,\mathrm{d}u = -\frac{23}{72}u^{24/23} + C.$$

Restoring x,

$$I = -\frac{23}{3(23+1)}(3x^{-1} + x^{-3})^{(23+1)/23} + C.$$

Comparing the coefficient, bracket and exponent with the given form gives $\alpha = 23$, $\beta = -1$, and $\gamma = -3$.

Step 3: Check the result by differentiation Differentiating the antiderivative gives

$$-\frac{23}{72}\frac{24}{23}u^{1/23}\big[-3(x^{-2} + x^{-4})\big] = (x^{-2} + x^{-4})u^{1/23}.$$

Since $u^{1/23} = x(3x^{-24} + x^{-26})^{1/23}$, this equals the original integrand exactly. Thus the negative exponents have been retained correctly. Finally, $\alpha + \beta + \gamma = 23 - 1 - 3 = 19$.

Step 4: Conclusion $\boxed{\alpha + \beta + \gamma = 19}$.

Quick Tip to Solve: For a radical containing large negative exponents, factor a power divisible by the root index before choosing the substitution.

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24. Let the lengths of the transverse and conjugate axes of a hyperbola in standard form be $2a$ and $2b$, respectively, and one focus and the corresponding directrix of this hyperbola be $(-5, 0)$ and $5x + 9 = 0$, respectively. If the product of the focal distances of a point $(\alpha, 2\sqrt{5})$ on the hyperbola is p, then $4p$ is equal to _____. [4 marks]

Solution:

Concept:

- For $x^2/a^2 - y^2/b^2 = 1$, the foci are $(\pm ae, 0)$, directrices are $x = \pm a/e$, and $b^2 = a^2(e^2 - 1)$.
- The distance from a point on the hyperbola to a focus is e times its perpendicular distance to the corresponding directrix.

Step 1: Determine the hyperbola The negative focus is $(-5, 0)$, so $ae = 5$. Its corresponding directrix is $x = -9/5$, giving $a/e = 9/5$. Multiplying these equations yields

$$a^2 = 9, \qquad a = 3, \qquad e = \frac{5}{3}.$$

Consequently

$$b^2 = a^2(e^2 - 1) = 9\left(\frac{25}{9} - 1\right) = 16.$$

The hyperbola therefore has equation $x^2/9 - y^2/16 = 1$. The focus lies on the horizontal axis, which fixes the orientation of the standard equation.

Step 2: Find the square of the point abscissa Substitute the ordinate $2\sqrt{5}$, whose square is 20:

$$\frac{\alpha^2}{9} - \frac{20}{16} = 1 \quad \Longrightarrow \quad \frac{\alpha^2}{9} = \frac{9}{4} \quad \Longrightarrow \quad \alpha^2 = \frac{81}{4}.$$

Thus $\alpha = \pm 9/2$. Both branches of the hyperbola are possible; the requested product must be evaluated

in a way that covers either sign.

Step 3: Multiply the two focal distances The two directrices are $x = \pm 9/5$. By the focus-directrix property, the focal distances have product

$$p = e^2 \left|\alpha - \frac{9}{5}\right| \left|\alpha + \frac{9}{5}\right| = \frac{25}{9} \left|\alpha^2 - \frac{81}{25}\right|.$$

As $\alpha^2 = 81/4 > 81/25$, the quantity inside the absolute value is positive. Therefore

$$p = \frac{25}{9}\frac{81}{4} - 9 = \frac{225}{4} - \frac{36}{4} = \frac{189}{4}.$$

The product is the same on either branch, as expected from symmetry, and multiplying it by four gives 189.

Step 4: Conclusion The required value is $\boxed{4p = 189}$.

Quick Tip to Solve: Multiply the focus coordinate magnitude ae by the directrix distance a/e to obtain a^2 immediately.

25. For $\mathrm{t} > -1$, let α_t and β_t be the roots of the equation

$$\left((\mathrm{t}+2)^{1/7} - 1\right)x^2 + \left((\mathrm{t}+2)^{1/6} - 1\right)x + \left((\mathrm{t}+2)^{1/21} - 1\right) = 0.$$ **If** $\lim\limits_{\mathrm{t}\to -1^+} \alpha_\mathrm{t} = \mathrm{a}$ **and** $\lim\limits_{\mathrm{t}\to -1^+} \beta_\mathrm{t} = \mathrm{b}$,

then $72(a+b)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- For a quadratic $Ax^2 + Bx + C = 0$ with $A \neq 0$, Vieta's relation gives the sum of roots as $-B/A$. The individual roots need not be evaluated to find their sum.
- The standard limit $\lim_{u\to 0}((1+u)^r - 1)/u = r$ follows from the derivative of $(1+u)^r$ at zero. It applies to each of the three rational exponents here.

Step 1: Express the sum before taking the limit For $t > -1$, we have $t + 2 > 1$, so the coefficient $(t+2)^{1/7} - 1$ is positive and the equation remains quadratic. Thus

$$\alpha_t + \beta_t = -\frac{(t+2)^{1/6} - 1}{(t+2)^{1/7} - 1}.$$

Since both root limits are specified, the limit of this sum is $a + b$. This avoids separately solving two varying quadratic roots.

Step 2: Evaluate the ratio using the standard limit Put $u = t + 1$, so $u \to 0^+$ and $t + 2 = 1 + u$. Divide

numerator and denominator by u:

$$a+b=-\frac{\lim\limits_{u\to0^+}\dfrac{(1+u)^{1/6}-1}{u}}{\lim\limits_{u\to0^+}\dfrac{(1+u)^{1/7}-1}{u}}=-\frac{1/6}{1/7}=-\frac{7}{6}.$$

The denominator limit is nonzero, so the quotient limit is valid. The constant coefficient affects the individual roots but not their sum.

Step 3: Calculate the requested quantity Square the sum only after retaining its sign in the preceding calculation:

$$72(a+b)^2=72\left(-\frac{7}{6}\right)^2=72\frac{49}{36}=98.$$

The two roots of the limiting scaled quadratic are real and distinct, so there is no coalescence issue in the given root limits.

Step 4: Conclusion The required numerical value is $\boxed{98}$.

Quick Tip to Solve: When a root-limit problem asks only for the sum, apply Vieta first and take one coefficient-ratio limit instead of using the quadratic formula.

Physics – Section A: Multiple Choice Questions (80 marks)

26. The dimension of $\sqrt{\frac{\mu_0}{\epsilon_0}}$ is equal to that of:

(μ_0 = Vacuum permeability and ϵ_0 = Vacuum permittivity) [4 marks]

(A) Voltage

(B) Inductance

(C) Resistance

(D) Capacitance

Correct Answer: (C) Resistance

Solution:

Concept:

- A dimension is expressed in fundamental mass M, length L, time T and current I. Quantities can be compared dimensionally without substituting their numerical SI values.
- Vacuum permeability has dimension $[\mu_0]=MLT^{-2}I^{-2}$, while vacuum permittivity has dimension $[\epsilon_0]=M^{-1}L^{-3}T^4I^2$.

Step 1: Find the dimension of the square root Using the magnetic force per unit length between parallel currents gives the stated permeability dimension. Coulomb's law similarly gives the permittivity dimension. Therefore

$$\left[\sqrt{\frac{\mu_0}{\epsilon_0}}\right]=\left[\frac{MLT^{-2}I^{-2}}{M^{-1}L^{-3}T^4I^2}\right]^{1/2}=ML^2T^{-3}I^{-2}.$$

Taking the square root halves every exponent, including the negative powers of time and current.

Step 2: Compare with electrical resistance Voltage is energy per charge, so $[V] = ML^2T^{-3}I^{-1}$. Resistance is voltage divided by current:

$$[R] = \frac{[V]}{I} = ML^2T^{-3}I^{-2}.$$

This matches the computed dimension exactly. Physically, $\sqrt{\mu_0/\epsilon_0}$ is the characteristic wave impedance of free space, whose SI unit is the ohm.

Step 3: Eliminate the remaining choices Voltage has only I^{-1}, so option (A) fails. Inductance satisfies $V = L_{\text{ind}}\, dI/dt$, hence its dimension is $ML^2T^{-2}I^{-2}$, different in its time power. Capacitance is charge per voltage and has dimension $M^{-1}L^{-2}T^4I^2$. Neither (B) nor (D) matches.

Step 4: Conclusion The dimension is that of resistance, $ML^2T^{-3}I^{-2}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $c = 1/\sqrt{\mu_0\epsilon_0}$ to rewrite the expression as $\mu_0 c$; its unit is henry per second, which equals ohm.

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27. A photoemissive substance is illuminated with a radiation of wavelength λ_i so that it releases electrons with de-Broglie wavelength λ_e. The longest wavelength of radiation that can emit photoelectron is λ_o. Expression for de-Broglie wavelength is given by:

(m: mass of the electron, h: Planck's constant and c: speed of light) [4 marks]

(A) $\lambda_e = \frac{\sqrt{h\lambda_i}}{\sqrt{2mc}}$

(B) $\lambda_e = \frac{h}{\sqrt{2mc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_o}\right)}}$

(C) $\lambda_e = \sqrt{\frac{h}{2mc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_o}\right)}}$

(D) $\lambda_e = \sqrt{\frac{h\lambda_0}{2mc}}$

Correct Answer: (C) $\lambda_e = \sqrt{\frac{h}{2mc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_o}\right)}}$

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where ϕ is the work function. The threshold wavelength λ_o gives $\phi = hc/\lambda_o$.
- For a nonrelativistic photoelectron, $K_{\max} = p^2/(2m)$ and its de Broglie wavelength is $\lambda_e = h/p$. The incident photon energy must exceed the work function.

Step 1: Write the available kinetic energy The incident wavelength λ_i corresponds to photon energy

hc/λ_i. Subtract the threshold energy needed to release the electron:

$$K_{\max} = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_o} = hc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_o}\right).$$

Emission with nonzero kinetic energy requires $\lambda_i < \lambda_o$, which makes the expression inside parentheses positive.

Step 2: Convert energy into the electron wavelength Using $p = \sqrt{2mK_{\max}}$, we obtain

$$\lambda_e = \frac{h}{\sqrt{2mhc(1/\lambda_i - 1/\lambda_o)}} = \sqrt{\frac{h}{2mc(1/\lambda_i - 1/\lambda_o)}}.$$

One factor of h remains inside the square root after simplifying the numerator h^2 against the denominator factor h.

Step 3: Check the alternative expressions Option (C) is exactly this expression. Option (B) leaves h, rather than $\sqrt{h}$, outside a denominator from which h has already been removed and is dimensionally incorrect. Options (A) and (D) omit the subtraction of the work function. In particular, as $\lambda_i \to \lambda_o^-$, the electron momentum tends to zero and λ_e must diverge; these two finite expressions fail that threshold check.

Step 4: Conclusion $\lambda_e = \sqrt{\frac{h}{2mc(1/\lambda_i - 1/\lambda_o)}}$. Hence, the correct option is (C).

Quick Tip to Solve: First subtract the threshold photon energy, then use $\lambda = h/\sqrt{2mK}$; a valid expression must diverge as the incident wavelength approaches threshold.

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28. A helicopter flying horizontally with a speed of 360 km/h at an altitude of 2 km, drops an object at an instant. The object hits the ground at a point O, 20 s after it is dropped. Displacement of 'O' from the position of helicopter where the object was released is :
(use acceleration due to gravity g = 10 m/s^2 and neglect air resistance) [4 marks]

(A) 7.2 km
(B) $2\sqrt{5}$ km
(C) $2\sqrt{2}$ km
(D) 4 km

Correct Answer: (C) $2\sqrt{2}$ km

Solution:

Concept:

- After release, the object retains the helicopter's horizontal velocity. With air resistance neglected, horizontal acceleration is zero, while vertical acceleration is g downward.
- Displacement is the straight-line vector from the release point to the impact point. Its magnitude is $\sqrt{(\Delta x)^2 + (\Delta y)^2}$, rather than the distance travelled along the curved trajectory.

Step 1: Calculate the horizontal displacement Convert the helicopter speed to SI units before multiplying by time:

$$v_x = 360\frac{1000}{3600} = 100\,\mathrm{m\,s^{-1}}.$$

No horizontal force acts on the dropped object in this model. During the stated flight time $t = 20\,\mathrm{s}$, its horizontal displacement is

$$\Delta x = v_x t = 100(20) = 2000\,\mathrm{m} = 2\,\mathrm{km}.$$

Step 2: Determine the vertical displacement The object starts with zero vertical velocity, since the helicopter flies horizontally. Taking upward as positive gives

$$\Delta y = -\frac{1}{2}gt^2 = -\frac{1}{2}(10)(20)^2 = -2000\,\mathrm{m}.$$

The downward displacement of $2\,\mathrm{km}$ agrees with the given altitude. Thus the stated fall time and the height are mutually consistent.

Step 3: Find the resultant and compare the choices The perpendicular components have equal magnitudes, so

$$|\Delta\mathbf{r}| = \sqrt{(2)^2 + (-2)^2} = 2\sqrt{2}\,\mathrm{km}.$$

This is the release-to-impact displacement even though the helicopter continues moving after release. Option (D), $4\,\mathrm{km}$, adds perpendicular magnitudes as scalars. Options (A) and (B) do not satisfy the computed components or their Pythagorean resultant.

Step 4: Conclusion The displacement magnitude is $\boxed{2\sqrt{2}\,\mathrm{km}}$, directed downward at 45° to the horizontal. Hence, the correct option is (C).

Quick Tip to Solve: For a horizontally dropped projectile, calculate horizontal travel and vertical drop separately; the required displacement is their vector resultant.

29. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): Refractive index of glass is higher than that of air.

Reason (R): Optical density of a medium is directly proportionate to its mass density which results in a proportionate refractive index.

In the light of the above statements, choose the most appropriate answer from the options given below: [4 marks]

(A) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(B) (A) is correct but (R) is not correct

(C) (A) is not correct but (R) is correct

(D) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Correct Answer: (B) (A) is correct but (R) is not correct

Solution:

Concept:

- The absolute refractive index is $n = c/v$, with c the vacuum speed of light and v its speed in the medium. A larger refractive index means a smaller light speed.
- Optical density describes the slowing of light and is governed by electromagnetic response. Mass density is mass per unit volume; it does not determine refractive index by a universal direct proportionality.

Step 1: Evaluate the assertion For ordinary visible light, air has refractive index close to one, while common glass has refractive index around 1.5. Light therefore travels more slowly in glass than in air. Thus the statement that glass has a higher refractive index than air is correct. The comparison refers to refractive indices at the same optical wavelength, since refractive index can vary with wavelength.

Step 2: Evaluate the proposed reason The reason claims that optical density is directly proportional to mass density and therefore that refractive index scales proportionately with mass density. This is not a general physical law. Refractive index depends on how the atoms or molecules respond to an electric field, including their polarizability and the frequency of the light. Mass density alone cannot specify that response. For example, a liquid such as kerosene can have a greater refractive index than water despite having a smaller mass density.

Step 3: Choose the assertion-reason combination The assertion is true and the reason is false, so the reason cannot explain the glass-air comparison. Both choices that require a correct reason are excluded, and the choice asserting that the assertion is incorrect is also excluded. Select the option that explicitly states the assertion is correct but the reason is not correct; this is option (B).

Step 4: Conclusion The assertion is correct, but the reason is not correct. Hence, the correct option is (B).

Quick Tip to Solve: Keep optical density separate from mass density: compare light speeds or refractive indices, and reject a claimed universal proportionality to mass per volume.

30. Which one of the following forces cannot be expressed in terms of potential energy? [4 marks]

(A) Frictional force

(B) Coulomb's force

(C) Restoring force

(D) Gravitational force

Correct Answer: (A) Frictional force

Solution:

Concept:

- A position-dependent potential energy U represents a conservative force through $\mathbf{F} = -\nabla U$. The work between two points is then $-\Delta U$ and is independent of the path.
- For a conservative force, work around a closed path is zero. Ordinary friction dissipates mechanical energy and its work depends on the distance travelled and the contact conditions.

Step 1: Apply the closed-path criterion to friction Consider a block sliding from one point to another and then returning along the same track. Kinetic friction opposes the motion on each leg, so its work is negative on both legs. For constant friction magnitude f and one-way distance ℓ,

$$W_{\text{round trip}} = -f\ell - f\ell = -2f\ell \neq 0.$$

A position-only potential would return to its initial value after this closed trip, requiring zero net work. Friction therefore cannot be represented by such a potential energy.

Step 2: Check the electrostatic and gravitational choices Coulomb's force between fixed charges is conservative, with potential energy $U = kq_1q_2/r$. Newtonian gravitational attraction has potential energy $U = -GMm/r$, or mgh in a uniform field near Earth. In both cases the work depends only on initial and final positions, not on the route. These facts exclude options (B) and (D).

Step 3: Check the restoring-force choice For the standard elastic restoring force in this question, $F = -kx$, integration gives $U = \frac{1}{2}kx^2$, taking zero potential at equilibrium. Its work over a complete return to the same extension is zero. Thus the usual restoring force is conservative, excluding option (C). Friction is the dissipative force among the four choices; mechanical energy lost to it appears as internal energy rather than a position potential.

Step 4: Conclusion Frictional force cannot be expressed in terms of a position-dependent potential energy. Hence, the correct option is (A).

Quick Tip to Solve: Test a force over an outward-and-return path: if its work stays negative on both legs, it is nonconservative and has no position-only potential.

31. Match List - I with List - II.

List - I	List - II
(A) Mass density	**(I)** $[\mathrm{ML^2T^{-3}}]$
(B) Impulse	**(II)** $[\mathrm{MLT^{-1}}]$
(C) Power	**(III)** $[\mathrm{ML^2T^0}]$
(D) Moment of inertia	**(IV)** $[\mathrm{ML^{-3}T^0}]$

Choose the correct answer from the options given below: [4 marks]

(A) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(B) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(C) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(D) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

Correct Answer: (D) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

Solution:

Concept:

- Write dimensions in terms of mass M, length L and time T, using each quantity's defining equation rather than its name.
- Density is mass per volume, impulse equals change in momentum, power is work per time, and moment of inertia is a sum of mass times squared distance.

Step 1: Match density and impulse Mass density is $\rho = m/V$. Since volume has dimension L^3,

$$[\rho] = \frac{M}{L^3} = ML^{-3}T^0.$$

This is entry (IV), so (A) matches (IV). Impulse is force multiplied by its duration, or equivalently the change in momentum. Therefore

$$[J] = [F][t] = (MLT^{-2})T = MLT^{-1}.$$

This is entry (II), so (B) matches (II). The time power is negative one, not zero or negative three.

Step 2: Match power and moment of inertia Work is force times displacement and has dimension ML^2T^{-2}. Dividing by time gives

$$[P] = \frac{[W]}{[t]} = ML^2T^{-3},$$

which is entry (I), so (C) matches (I). For moment of inertia about a specified axis, $I = \sum m_i r_i^2$; consequently

$$[I] = ML^2T^0.$$

This is entry (III), so (D) matches (III). Moment of inertia has no time factor, even though it occurs in equations of rotational dynamics.

Step 3: Select the complete matching Combining all four independent results gives (A)-(IV), (B)-(II), (C)-(I), (D)-(III). The first option interchanges the power and inertia dimensions. The second already assigns density a momentum dimension. The third assigns impulse a moment-of-inertia dimension.

Only the fourth option preserves every match.

Step 4: Conclusion The matching is $\boxed{\text{A} - \text{IV},\ \text{B} - \text{II},\ \text{C} - \text{I},\ \text{D} - \text{III}}$. Hence, the correct option is (D).

Quick Tip to Solve: Check the time exponent first: inertia and density have zero, impulse has minus one, and power has minus three.

32. The helium and argon are put in the flask at the same room temperature (300 K). The ratio of average kinetic energies (per molecule) of helium and argon is:

(Give: Molar mass of helium = 4 g/mol, Molar mass of argon = 40 g/mol) [4 marks]

(A) $1 : \sqrt{10}$

(B) $10 : 1$

(C) $1 : 10$

(D) $1 : 1$

Correct Answer: (D) $1 : 1$

Solution:

Concept:

- The mean translational kinetic energy of one ideal-gas molecule is $\langle K \rangle = \frac{3}{2} k_B T$, where k_B is the Boltzmann constant and T is absolute temperature.
- Molecular mass affects the root-mean-square speed through $v_{\text{rms}} = \sqrt{3k_BT/m}$, but the energy contains the compensating mass factor $\frac{1}{2}m$.

Step 1: Apply the common temperature Both helium and argon are in the flask at $T = 300\,\text{K}$. Each is monatomic, so its molecular kinetic energy is translational. For either gas,

$$\langle K \rangle = \frac{3}{2} k_B (300).$$

The same temperature and the same universal Boltzmann constant occur for the two species; their different molar masses do not enter this expression.

Step 2: Form the energy ratio Taking the ratio cancels every common factor:

$$\frac{\langle K_{\text{He}} \rangle}{\langle K_{\text{Ar}} \rangle} = \frac{(3/2) k_B (300)}{(3/2) k_B (300)} = 1.$$

Thus helium atoms move faster on average, but they do not have a greater mean kinetic energy. In fact, $m_{\text{Ar}} = 10 m_{\text{He}}$ makes $v^2_{\text{rms,He}} = 10 v^2_{\text{rms,Ar}}$; multiplying by the smaller helium mass gives equal energies.

Step 3: Exclude ratios based on mass or speed The ratios $10 : 1$ and $1 : 10$ use the mass ratio without accounting for speed. The ratio involving $\sqrt{10}$ describes a possible speed comparison rather than an energy comparison. Since the question explicitly asks for energy per molecule, neither the total number of atoms nor the total gas mass is relevant.

Step 4: Conclusion The average kinetic energies per molecule are equal, so their ratio is $1:1$. Hence, the correct option is (D).

Quick Tip to Solve: For kinetic energy per molecule at equilibrium, compare temperatures; reserve the inverse square-root mass rule for molecular speeds.

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33. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): The radius vector from the Sun to a planet sweeps out equal areas in equal intervals of time and thus areal velocity of planet is constant.

Reason (R): For a central force field the angular momentum is a constant.

In the light of the above statements, choose the most appropriate answer from the options given below: [4 marks]

(A) (A) is not correct but (R) is correct

(B) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(C) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(D) (A) is correct but (R) is not correct

Correct Answer: (C) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Solution:

Concept:

- A central force acts along the radius vector, so its torque about the force centre is $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \mathbf{0}$. Therefore angular momentum about that centre is conserved.
- The instantaneous areal velocity is $dA/dt = \frac{1}{2}r^2\dot{\theta} = L/(2m)$, with m the planet mass and L its orbital angular momentum magnitude.

Step 1: Evaluate the assertion Kepler's second law states that the Sun-to-planet radius vector sweeps equal areas in equal time intervals. Equivalently, the area swept per unit time is constant throughout the orbit. Thus the assertion correctly states both the law and its mathematical meaning. A planet need not have constant linear speed: it moves faster nearer the Sun while sweeping the same area per unit time.

Step 2: Evaluate the reason The gravitational force on the planet is directed along the line joining it to the Sun. It is a central force and produces zero torque about the Sun:

$$\frac{d\mathbf{L}}{dt} = \boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \mathbf{0}.$$

Hence $\mathbf{L}$ is constant, confirming the reason. This conclusion does not require a circular orbit or constant force magnitude.

Step 3: Establish the explanatory link In a small time dt, the radius vector sweeps an area $dA = \frac{1}{2}r^2 d\theta$. Since $L = mr^2\dot{\theta}$,

$$\frac{dA}{dt} = \frac{1}{2}r^2\dot{\theta} = \frac{L}{2m} = \text{constant}.$$

Conservation of angular momentum therefore directly proves the assertion. Options denying either statement are false, and the option claiming that the reason does not explain the assertion misses this relation.

Step 4: Conclusion Both the assertion and reason are correct, and the reason correctly explains the assertion. Hence, the correct option is (C).

Quick Tip to Solve: For an areal-velocity assertion, write $dA/dt = L/(2m)$; it immediately connects the statement to zero torque.

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34. The unit of $\sqrt{\frac{2I}{\varepsilon_0 c}}$ is :
(I = intensity of an electromagnetic wave, c = speed of light) [4 marks]

(A) Vm

(B) NC^{-1}

(C) NC

(D) Nm

Correct Answer: (B) NC^{-1}

Solution:

Concept:

- The average intensity of a sinusoidal electromagnetic wave in vacuum is $I = \frac{1}{2}\varepsilon_0 c E_0^2$, where E_0 is the electric-field amplitude.
- Electric field is force per unit test charge, $E = F/q$, so its SI unit is $\mathrm{N\,C^{-1}}$, equivalently $\mathrm{V\,m^{-1}}$.

Step 1: Identify the physical quantity Rearranging the wave-intensity relation gives

$$E_0^2 = \frac{2I}{\varepsilon_0 c}, \qquad E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}}.$$

The expression asked about is therefore the amplitude of the electric field of the electromagnetic wave. The numerical factor 2 changes the amplitude convention but cannot change the dimensions or unit.

Step 2: Check the unit directly Intensity has unit $\mathrm{W\,m^{-2}} = \mathrm{kg\,s^{-3}}$. Also ε_0 has unit $\mathrm{C^2\,N^{-1}\,m^{-2}}$ and c has unit $\mathrm{m\,s^{-1}}$. Consequently

$$\left[\frac{I}{\varepsilon_0 c}\right] = \mathrm{N^2\,C^{-2}},$$

whose square root is $\mathrm{N\,C^{-1}}$. This independently verifies the identification as electric field.

Step 3: Compare with the options The option $\mathrm{N\,C^{-1}}$ has precisely the required force-per-charge form. The printed $\mathrm{V\,m}$ has a product of volt and metre, whereas electric field requires volt divided by metre. The unit $\mathrm{N\,C}$ multiplies force by charge, and $\mathrm{N\,m}$ represents work or torque; neither can measure electric-field amplitude.

Step 4: Conclusion The required SI unit is $\mathrm{N\,C^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: Recognize $I = \frac{1}{2}\varepsilon_0 c E_0^2$ before expanding dimensions; the square root then gives an electric field.

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35. A dipole with two electric charges of 2 μC magnitude each, with separation distance 0.5 μm, is placed between the plates of a capacitor such that its axis is parallel to an electric field established between the plates when a potential difference of 5 V is applied. Separation between the plates is 0.5 mm. If the dipole is rotated by 30° from the axis, it tends to realign in the direction due to a torque. The value of torque is: [4 marks]

(A) 2.5×10^{-9} Nm

(B) 2.5×10^{-12} Nm

(C) 5×10^{-3} Nm

(D) 5×10^{-9} Nm

Correct Answer: (D) 5×10^{-9} Nm

Solution:

Concept:

- The electric dipole moment has magnitude $p = q\ell$, where q is the magnitude of either charge and ℓ is the full separation between the charges.
- For parallel capacitor plates, $E = V/d$. The torque magnitude on a dipole at angle θ to the uniform field is $\tau = pE\sin\theta$.

Step 1: Calculate the dipole moment The given charge magnitude is $q = 2 \times 10^{-6}\,\mathrm{C}$, and the separation of the charges is $\ell = 0.5 \times 10^{-6}\,\mathrm{m}$. Therefore

$$p = q\ell = (2 \times 10^{-6})(0.5 \times 10^{-6}) = 1.0 \times 10^{-12}\,\mathrm{C\,m}.$$

The separation is already the full distance between the charges; it must not be doubled.

Step 2: Find the field between the plates The plate separation is $d = 0.5\,\mathrm{mm} = 5 \times 10^{-4}\,\mathrm{m}$, while the potential difference is $5\,\mathrm{V}$. Thus

$$E = \frac{V}{d} = \frac{5}{5 \times 10^{-4}} = 10^{4}\,\mathrm{V\,m^{-1}}.$$

The plate spacing and the charge spacing describe different distances and must be converted independently.

Step 3: Apply the angular factor After the dipole is turned through 30° from its original field-parallel orientation, the angle between its axis and the field is 30°. Hence

$$\tau = pE\sin 30^\circ = (10^{-12})(10^{4})\left(\frac{1}{2}\right) = 5 \times 10^{-9}\,\mathrm{N\,m}.$$

The torque acts to decrease the angle and restore alignment. The other listed magnitudes either introduce an extra factor of two or use incorrect powers of ten in the length conversions.

Step 4: Conclusion The restoring torque has magnitude $5 \times 10^{-9}\,\mathrm{N\,m}$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the two separations distinct: use the microscopic charge spacing in $p = q\ell$ and the plate spacing in $E = V/d$.

36. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : The outer body of an aircraft is made of metal which protects persons sitting inside from lightning strikes.

Reason (R) : The electric field inside the cavity enclosed by a conductor is zero.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(B) (A) is correct but (R) is not correct

(C) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(D) (A) is not correct but (R) is correct

Correct Answer: (C) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Solution:

Concept:

- A closed conducting shell redistributes its free charges in response to an external electric field. For a charge-free cavity under electrostatic conditions, the field inside the cavity is zero.
- This shielding is the Faraday-cage principle: the conducting enclosure carries induced charge and current on its exterior while shielding the enclosed region from the external field.

Step 1: Evaluate the assertion A metal aircraft body provides a conducting path for lightning current around its exterior. The passengers occupy the enclosed interior rather than this conducting exterior path. The metal enclosure consequently offers protection against lightning strikes by shielding the cabin. Within the ideal conducting-enclosure model used in this question, the assertion is correct; the protection does not require the aircraft to be connected to the ground.

Step 2: Evaluate the reason In electrostatic equilibrium a conductor is an equipotential. The inner surface bounding an empty cavity is therefore at one constant potential. With no charge inside the cavity, the potential throughout it is constant, and $\mathbf{E} = -\nabla V = \mathbf{0}$. Thus the reason correctly states the shielding property for the charge-free cavity relevant here. The applied external field rearranges charges outside without producing a field in that interior region.

Step 3: Connect the shielding to passenger protection The aircraft body acts as the conducting enclosure and its cabin corresponds to the protected cavity. The zero interior field in the ideal model explains why an external electrical disturbance does not establish the same dangerous field through the passengers. Hence the reason explains the assertion, rather than merely being an unrelated true fact. Options claiming either statement is false are inconsistent with electrostatic shielding, and the option denying the explanatory link overlooks the Faraday-cage mechanism.

Step 4: Conclusion Both the assertion and reason are correct, and the reason correctly explains the assertion. Hence, the correct option is (C).

Quick Tip to Solve: For a metal-enclosure question, identify the conducting exterior and the charge-free cavity; grounding is not needed for shielding.

37. A mirror is used to produce an image with magnification of $\frac{1}{4}$. If the distance between object and its image is 40 cm, then the focal length of the mirror is _____. [4 marks]

(A) 10 cm

(B) 12.7 cm

(C) 10.7 cm

(D) 15 cm

Correct Answer: (C) 10.7 cm

Solution:

Concept:

- The signed mirror magnification is $m = -v/u$, where u and v are the object and image distances from the pole.
- The mirror formula is $1/f = 1/u + 1/v$. A positive diminished magnification for a real object describes an erect virtual image behind a convex mirror.

Step 1: Interpret the magnification The given signed magnification is positive, $m = +1/4$. Thus the object and image are on opposite sides of the mirror. Write $u = -a$, with $a > 0$. Then

$$\frac{1}{4} = -\frac{v}{-a} \quad \Longrightarrow \quad v = \frac{a}{4}.$$

The virtual image is a quarter as far behind the pole as the object is in front; the object-image separation is the sum of these two positive distances.

Step 2: Use the given separation The separation is $40\,\text{cm}$, so

$$a + \frac{a}{4} = 40 \quad \Longrightarrow \quad a = 32\,\text{cm}.$$

Consequently $u = -32\,\text{cm}$ and $v = +8\,\text{cm}$. Substituting the signed values gives

$$\frac{1}{f} = -\frac{1}{32} + \frac{1}{8} = \frac{3}{32}, \qquad f = \frac{32}{3}\,\text{cm} = 10.67\,\text{cm}.$$

The positive focal length is consistent with a convex mirror. The other listed focal lengths do not satisfy both the positive magnification and the specified separation.

Step 3: Conclusion The focal length is $f = +10.7\,\text{cm}$, to the precision of the options. Hence, the correct option is (C).

Quick Tip to Solve: Use the sign of magnification first: an erect virtual image lies opposite the object, so their separation is the sum of the distance magnitudes.

38. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : The density of the copper $(^{64}_{29}\text{Cu})$ nucleus is greater than that of the carbon $(^{12}_{6}\text{C})$ nucleus.

Reason (R) : The nucleus of mass number A has a radius proportional to $A^{1/3}$.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) (A) is correct but (R) is not correct

(B) Both (A) and (R) are correct but (R) is **not** the correct explanation of (A)

(C) (A) is not correct but (R) is correct

(D) Both (A) and (R) are correct and (R) **is** the correct explanation of (A)

Correct Answer: (C) (A) is not correct but (R) is correct

Solution:

Concept:

- The empirical nuclear radius law is $R = r_0 A^{1/3}$, with approximately the same constant r_0 for different nuclei.
- The nuclear mass is approximately Am_n, where m_n is the mass per nucleon; nuclear volume is $4\pi R^3/3$.

Step 1: Evaluate the assertion For a nucleus of mass number A, its average density is

$$\rho = \frac{Am_n}{(4\pi/3)R^3} = \frac{Am_n}{(4\pi/3)r_0^3 A} = \frac{3m_n}{4\pi r_0^3}.$$

The factor A cancels. Copper has more nucleons than carbon, but its volume increases in the same proportion as its mass. Thus the standard nuclear-size model gives approximately equal nuclear densities, rather than a greater density for copper. Assertion (A) is false.

Step 2: Evaluate the reason and select the pair The relation stated in Reason (R), $R \propto A^{1/3}$, is exactly the nuclear-radius law used above. For these nuclei,

$$\frac{R_{\text{Cu}}}{R_{\text{C}}} = \left(\frac{64}{12}\right)^{1/3}, \qquad \frac{V_{\text{Cu}}}{V_{\text{C}}} = \frac{64}{12}.$$

Their mass ratio is also approximately $64/12$, so the density ratio is approximately one. Reason (R) is true. Options claiming the assertion is true are therefore rejected; the option claiming the radius law is false is also rejected. Small binding-energy differences are outside the approximate model invoked by the question.

Step 3: Conclusion The assertion is incorrect and the reason is correct. Hence, the correct option is (C).

Quick Tip to Solve: Cube the nuclear-radius law before comparing densities: volume scales as mass number, cancelling the same factor in nuclear mass.

39. A capillary tube of radius 0.1 mm is partly dipped in water (surface tension 70 dyn/cm and glass water contact angle $\approx 0^\circ$) with 30° inclined with the vertical. The length of water risen in the capillary is _____ cm. (Take $g = 9.8$ m/s^2) [4 marks]

(A) $\frac{82}{5}$

(B) $\frac{68}{5}$

(C) $\frac{57}{2}$

(D) $\frac{71}{5}$

Correct Answer: (A) $\frac{82}{5}$

Solution:

Concept:

- Capillary rise has vertical height $h = 2T\cos\theta/(\rho g r)$, where T is surface tension and θ is the liquid-glass contact angle.
- If the capillary axis makes angle α with the vertical, the length of the raised liquid column is $L = h/\cos\alpha$.

Step 1: Convert the data and calculate the vertical rise The contact angle is approximately zero, so $\cos\theta \simeq 1$. In SI units,

$$T = 70\,\text{dyn/cm} = 0.070\,\text{N/m}, \qquad r = 0.1\,\text{mm} = 10^{-4}\,\text{m}.$$

For water take $\rho = 1000\,\text{kg/m}^3$. Therefore

$$h = \frac{2(0.070)}{1000(9.8)(10^{-4})} = \frac{1}{7}\,\text{m} = 14.286\,\text{cm}.$$

This is the vertical height above the external water surface, not the distance measured along the inclined glass tube.

Step 2: Project the column along the tube The tube is inclined at 30° to the vertical. Its vertical projection satisfies $L\cos 30^\circ = h$, hence

$$L = \frac{14.286}{\sqrt{3}/2} = 16.50\,\text{cm approximately}.$$

Among the listed values, $82/5 = 16.4\,\text{cm}$ is the closest. The alternatives are 13.6, 28.5, and $14.2\,\text{cm}$. In particular $14.2\,\text{cm}$ is close to the vertical rise and misses the inclination correction. The computed value and the closest listed value differ slightly because of the numerical precision of the options.

Step 3: Conclusion The liquid rises about $16.5\,\text{cm}$ along the tube; the matching listed approximation is $82/5\,\text{cm}$. Hence, the correct option is (A).

Quick Tip to Solve: Capillary rise determines a vertical height. Divide by the cosine of the angle to the vertical to obtain the liquid length in an inclined tube.

40. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): Magnetic monopoles do not exist.

Reason (R): Magnetic field lines are continuous and form closed loops.

In the light of the above statements, choose the most appropriate answer from the options given below: [4 marks]

(A) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(B) (A) is not correct but (R) is correct

(C) (A) is correct but (R) is not correct

(D) Both (A) and (R) are correct but (R) is **not** the correct explanation of (A)

Correct Answer: (A) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Solution:

Concept:

- For ordinary magnets, isolated north or south magnetic poles are not observed; cutting a magnet produces smaller dipoles.
- Gauss's law for magnetism is $\oint \mathbf{B} \cdot d\mathbf{A} = 0$: magnetic field lines have no isolated source or sink.

Step 1: Evaluate the assertion A bar magnet always has both a north pole and a south pole. If it is divided, each piece again has both poles, rather than producing a separate north pole in one piece and a separate south pole in the other. In the classical magnetism framework of this question, the statement that magnetic monopoles do not exist is therefore accepted as true. It expresses the absence of isolated magnetic poles in ordinary magnetic phenomena.

Step 2: Evaluate the reason Magnetic field lines form continuous closed loops. For a bar magnet they go from north to south outside the magnet and return from south to north inside it. They do not begin at a north pole or end at a south pole as independent endpoints. Thus the reason is also true. This differs from electric field lines, which can begin or end on isolated electric charges.

Step 3: Connect the reason to the assertion An isolated magnetic pole would act as a source or sink of magnetic field lines and would give a nonzero net flux through a closed surface enclosing it. The closed-loop property rules out such a beginning or end:

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0.$$

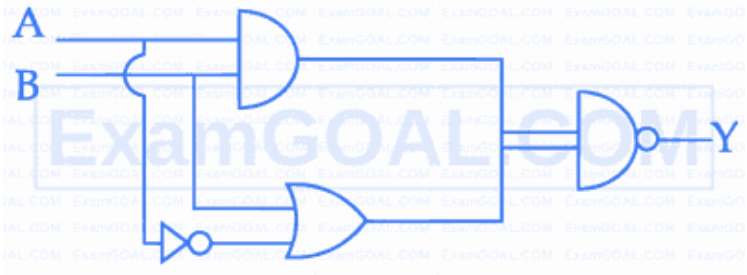
Thus the reason directly explains the absence of isolated poles in this model. The alternatives making either statement false are excluded; the alternative denying their explanatory connection is also excluded.

Step 4: Conclusion Both Assertion (A) and Reason (R) are correct, and (R) correctly explains (A). Hence, the correct option is (A).

Quick Tip to Solve: For magnetic assertion-reason questions, connect closed field loops with zero net magnetic flux and the absence of isolated poles.

41. Consider the following logic circuit.

The output is Y = 0 when : [4 marks]



(A) A = 1 and B = 0

(B) A = 0 and B = 0

(C) A = 1 and B = 1

(D) A = 0 and B = 1

Correct Answer: (C) A = 1 and B = 1

Solution:

Concept:

- An AND gate gives output 1 only when both its inputs are 1; an OR gate gives output 1 when at least one input is 1.
- A NOT gate complements its input, and a NAND gate complements the AND of its inputs. Boolean notation uses multiplication for AND and $+$ for OR.

Step 1: Write the two intermediate outputs The upper gate is an AND gate with inputs A and B, so its output is $P = AB$. The lower branch inverts A and ORs it with B, giving

$$Q = \overline{A} + B.$$

The small bridge where wires cross indicates that the crossing wires are not joined. The final gate has an output bubble, so it is a NAND gate acting on P and Q, not an ordinary AND gate.

Step 2: Simplify the final output The final output is

$$Y = \overline{PQ} = \overline{AB(\overline{A} + B)}.$$

Using $A\overline{A} = 0$ and $BB = B$, the expression inside the complement becomes

$$AB\overline{A} + ABB = 0 + AB = AB.$$

Thus $Y = \overline{AB}$. The output is zero only when $AB = 1$, requiring both $A = 1$ and $B = 1$.

Step 3: Check all four listed inputs For $(A, B) = (1, 0)$, the upper AND output is zero, so the final NAND output is one. The same applies for $(0, 0)$ and $(0, 1)$. For $(1, 1)$, both intermediate outputs are one: $P = 1$ and $Q = 0 + 1 = 1$. Only this input pair makes the final output zero. This confirms the simplified Boolean expression directly against the circuit.

Step 4: Conclusion The output is $Y = 0$ for $A = 1$ and $B = 1$. Hence, the correct option is (C).

Quick Tip to Solve: A final NAND produces zero only if every input to it is one. Start with that condition, then work backwards through the circuit.

42. The equation of a wave travelling on a string is $y = \sin[20\pi x + 10\pi t]$, where x and t are distance and time in SI units. The minimum distance between two points having the same oscillating speed is : [4 marks]

(A) 10 cm

(B) 2.5 cm

(C) 20 cm

(D) 5.0 cm

Correct Answer: (D) 5.0 cm

Solution:

Concept:

- For a harmonic wave $y = a\sin(kx+\omega t)$, the transverse particle velocity is $v_y = \partial y/\partial t = a\omega\cos(kx+\omega t)$.
- Oscillating speed is the magnitude $|v_y|$, so the spatial speed pattern repeats after a phase change π, half the displacement wavelength.

Step 1: Identify the wave number and particle speed Comparing the equation with $y = a\sin(kx+\omega t)$ gives $k = 20\pi\,\mathrm{rad/m}$ and $\omega = 10\pi\,\mathrm{rad/s}$. At a fixed point of the string,

$$v_y = 10\pi\cos(20\pi x + 10\pi t), \qquad s = 10\pi\,|\cos(20\pi x + 10\pi t)|\,.$$

This derivative describes the motion of a string particle, rather than the speed ω/k of propagation of the wave.

Step 2: Find the smallest repeat separation Two positions separated by Δx have the same oscillating-speed history when their phases differ by the smallest positive period of $|\cos\phi|$, namely π. Thus

$$k\Delta x = \pi \quad\Longrightarrow\quad \Delta x = \frac{\pi}{20\pi} = 0.050\,\mathrm{m} = 5.0\,\mathrm{cm}.$$

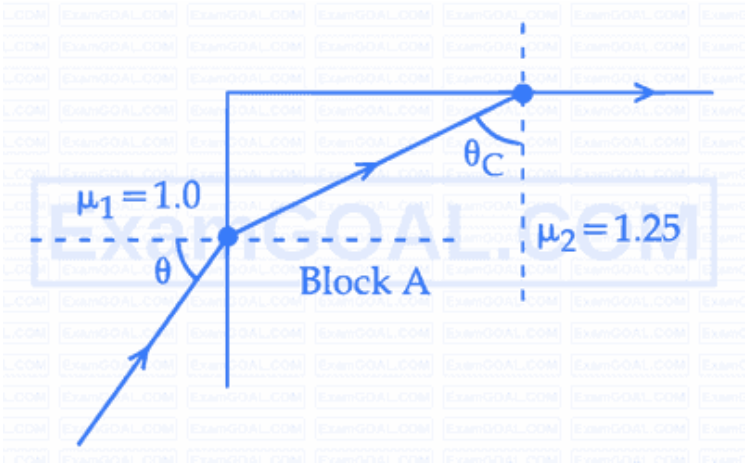
Their signed velocities are opposite, but their speed magnitudes agree at all times. The wavelength is $2\pi/k = 10\,\mathrm{cm}$, so this distance is $\lambda/2$. A full wavelength also works but is not the smallest repeat separation.

Step 3: Distinguish speed from signed velocity The $2.5\,\mathrm{cm}$ option corresponds to a phase shift $\pi/2$, which interchanges cosine and sine magnitudes and does not preserve the speed throughout the oscillation. The $10\,\mathrm{cm}$ and $20\,\mathrm{cm}$ options are larger repeats. Here equal oscillating speed means equal speed as functions of time; a coincidental equality at one instant does not define a fixed minimum spatial period.

Step 4: Conclusion The minimum positive repeat separation of the oscillating-speed pattern is $5.0\,\mathrm{cm}$. Hence, the correct option is (D).

Quick Tip to Solve: Speed ignores the sign of velocity: $|\cos\phi|$ repeats after π, so equal speed histories recur at half a wavelength.

43. A transparent block A having refractive index $\mu = 1.25$ is surrounded by another medium of refractive index $\mu = 1.0$ as shown in the figure. A light ray is incident on the flat face of the block with incident angle θ as shown in the figure. What is the maximum value of θ for which light suffers total internal reflection at the top surface of the block? [4 marks]



(A) $\tan^{-1}(4/3)$

(B) $\sin^{-1}(3/4)$

(C) $\tan^{-1}(3/4)$

(D) $\cos^{-1}(3/4)$

Correct Answer: (B) $\sin^{-1}(3/4)$

Solution:

Concept:

- Total internal reflection from the block to the surrounding medium requires an internal incidence angle greater than the critical angle C, with $\sin C = n_{\text{out}}/n_{\text{block}}$.
- Snell's law at the vertical face is $n_{\text{out}} \sin\theta = n_{\text{block}} \sin r$, where r is measured from the horizontal normal. The incidence angle at the horizontal top face is $90^\circ - r$.

Step 1: Find the critical angle The ray travels from refractive index $1.25 = 5/4$ to index 1 at the top face. Therefore

$$\sin C = \frac{1}{1.25} = \frac{4}{5}, \qquad \cos C = \frac{3}{5}.$$

At the limiting condition the transmitted ray grazes the top surface. Increasing the external incidence angle increases r, hence decreases the top-face incidence angle; the limiting external angle is therefore the upper boundary for total reflection.

Step 2: Apply the complementary-angle geometry At this boundary, $90^\circ - r = C$. Thus

$$\sin r = \sin(90^\circ - C) = \cos C = \frac{3}{5}.$$

Using Snell's law at entry gives

$$\sin\theta_{\text{lim}} = \frac{5}{4} \times \frac{3}{5} = \frac{3}{4}.$$

This uses the entry-face normal, rather than confusing the external angle with the critical angle at the

top face.

Step 3: Check the option functions The required boundary is $\sin^{-1}(3/4)$, approximately $48.6°$. The tangent alternatives do not satisfy the entry relation $\sin\theta = 3/4$; the cosine alternative is the complementary angle and also gives the wrong sine. Strictly, total internal reflection occurs for angles below this boundary, while equality is the critical, grazing limit.

Step 4: Conclusion The limiting maximum angle is $\boxed{\theta_{\lim} = \sin^{-1}(3/4)}$. Hence, the correct option is (B).

Quick Tip to Solve: For perpendicular entry and reflecting faces, the two internal angles are complementary; apply the critical-angle condition first and Snell's law second.

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44. Match List - I with List - II.

List - I	List - II
(A) Isothermal	**(I)** ΔW **(work done)** $= 0$
(B) Adiabatic	**(II)** ΔQ **(supplied heat)** $= 0$
(C) Isobaric	**(III)** ΔU **(change in internal energy)** $\neq 0$
(D) Isochoric	**(IV)** $\Delta U = 0$

Choose the correct answer from the options given below: [4 marks]

(A) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(B) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(C) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(D) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

Correct Answer: (C) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

Solution:

Concept:

- For a fixed amount of ideal gas, internal energy depends only on temperature: $\Delta U = nC_V\Delta T$. A constant-temperature process therefore has no change in internal energy.
- The work done by a gas is $W = \int P\,dV$, while the first law is $Q = \Delta U + W$. Constant volume forces zero boundary work; an adiabatic wall prevents heat transfer.

Step 1: Match the constant-temperature and insulated processes In an isothermal process the temperature remains constant. Thus $\Delta T = 0$, giving $\Delta U = 0$, so (A) matches (IV). In an adiabatic process there is no heat supplied or removed: $Q = 0$. Thus (B) matches (II). Adiabatic does not mean zero work: the work can be supplied by a decrease of internal energy.

Step 2: Match the constant-pressure and constant-volume processes For an isobaric process, pressure remains constant while the gas expands or contracts. The ideal-gas equation $PV = nRT$ then implies a temperature change when volume changes. Consequently $\Delta U \neq 0$, giving (C)-(III). For an isochoric process, $dV = 0$ throughout, so $W = \int P\,dV = 0$, giving (D)-(I). Constant volume does not require zero heat or zero internal-energy change.

Step 3: Select the complete matching The sequence is (A)-(IV), (B)-(II), (C)-(III), (D)-(I). Only the third listed option contains all four of these associations. The first and second options already fail for the isothermal process, and the fourth incorrectly assigns zero work to an adiabatic process and zero heat to an isochoric one.

Step 4: Conclusion The processes match, in their listed order, with IV, II, III and I respectively. Hence, the correct option is (C).

Quick Tip to Solve: Identify the fixed variable first: isothermal fixes temperature, adiabatic fixes heat transfer, and isochoric fixes volume.

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45. An object with mass 500 g moves along x-axis with speed $v = 4\sqrt{x}$ m/s. The force acting on the object is : [4 marks]

(A) 8 N

(B) 4 N

(C) 5 N

(D) 6 N

Correct Answer: (B) 4 N

Solution:

Concept:

- When speed is supplied as a function of position, use the chain rule $a = dv/dt = (dv/dx)(dx/dt) = v\,dv/dx$.
- Newton's second law gives the net force $F = ma$, with mass converted into kilograms before substitution.

Step 1: Differentiate the speed with respect to position The speed law is $v = 4\sqrt{x} = 4x^{1/2}$. Its position derivative is

$$\frac{dv}{dx} = 2x^{-1/2} = \frac{2}{\sqrt{x}}.$$

This derivative alone is not the acceleration: it describes how speed changes per unit distance. To obtain change per unit time, multiply it by the speed. That distinction is essential because the question specifies $v(x)$, rather than $v(t)$.

Step 2: Evaluate the acceleration For motion along the positive x direction, the chain rule gives

$$a = v\frac{dv}{dx} = 4\sqrt{x}\frac{2}{\sqrt{x}} = 8\,\mathrm{m\,s^{-2}}.$$

The position factors cancel, so the acceleration is constant even though speed is not. The same result follows without the intermediate singular derivative by squaring the speed: $v^2 = 16x$, hence $a = \frac{1}{2}d(v^2)/dx = 8\,\mathrm{m\,s^{-2}}$.

Step 3: Convert the mass and calculate force The given mass is $500\,\mathrm{g} = 0.500\,\mathrm{kg}$. Therefore

$$F = ma = 0.500 \times 8 = 4\,\mathrm{N}.$$

The value 8 N confuses acceleration with force and omits the half-kilogram mass. Neither 5 N nor 6 N satisfies Newton's law with this acceleration. Thus only the second option has the required force magnitude.

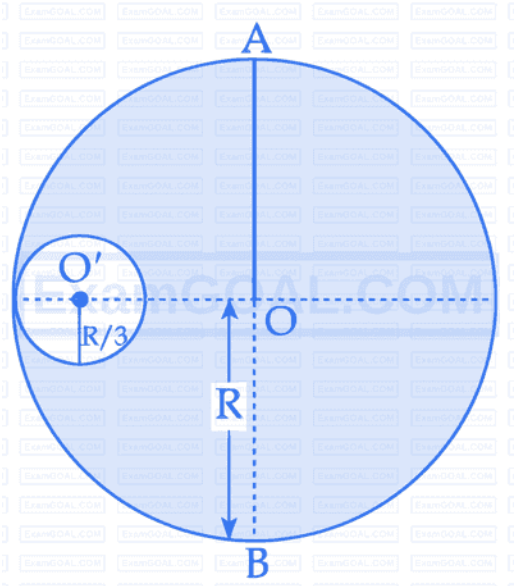
Step 4: Conclusion The net force has magnitude $\boxed{4\,\mathrm{N}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a speed law involving a square root of position, differentiate v^2 and use $a = \frac{1}{2}d(v^2)/dx$ to avoid unnecessary radicals.

Physics – Section B: Numerical Value Questions (20 marks)

46. M and R be the mass and radius of a disc. A small disc of radius $\mathrm{R}/3$ is removed from the bigger disc as shown in figure. The moment of inertia of remaining part of bigger disc about an axis AB passing through the centre O and perpendicular to the plane of disc is $\frac{4}{x}\mathrm{MR}^2$. The value of x is _____.

[4 marks]



Solution:

Concept:

- For a uniform disc about its central axis perpendicular to its plane, $I = \frac{1}{2}MR^2$. Removing material subtracts that material's moment of inertia about the same axis.
- The mass of a cut-out is proportional to its area, and the parallel-axis theorem gives $I_O = I_{O'} + md^2$ for a displaced circular disc.

Step 1: Find the removed mass and its centre distance The removed circle has radius $r = R/3$. Uniform surface density means

$$m = M\frac{\pi(R/3)^2}{\pi R^2} = \frac{M}{9}.$$

The diagram shows the smaller circle internally tangent to the large circle, so the separation of centres

is

$$d = OO\prime = R - r = \frac{2R}{3}.$$

Use the original full-disc mass M in this area ratio; it is not the mass of the remaining piece.

Step 2: Calculate the inertia removed about the original centre The required axis passes through O perpendicular to the disc, as stated. The small disc's own central perpendicular inertia is $\frac{1}{2}mr^2$. Shift it to O:

$$I_{\text{removed}} = \frac{1}{2}\frac{M}{9}\left(\frac{R}{3}\right)^2 + \frac{M}{9}\left(\frac{2R}{3}\right)^2$$
$$= \frac{MR^2}{162} + \frac{4MR^2}{81} = \frac{MR^2}{18}.$$

The displacement contribution cannot be dropped, since the cut-out is not concentric.

Step 3: Subtract and compare coefficients The full disc has $I_{\text{full}} = MR^2/2$. Hence

$$I_{\text{remaining}} = \frac{MR^2}{2} - \frac{MR^2}{18} = \frac{4}{9}MR^2.$$

Comparing this with the specified form $(4/x)MR^2$ gives $4/x = 4/9$. The positive reduction from one half to four ninths is consistent with removing a small off-centre portion of the disc.

Step 4: Conclusion The remaining moment of inertia is $\boxed{\frac{4}{9}MR^2}$, giving $\boxed{x = 9}$.

Quick Tip to Solve: For an off-centre hole, subtract its inertia about the original axis using both its own central inertia and the parallel-axis term.

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47. Two cylindrical rods A and B made of different materials, are joined in a straight line. The ratios of lengths, radii and thermal conductivites of these rods are: $\frac{L_A}{L_B} = \frac{1}{2}, \frac{r_A}{r_B} = 2$ and $\frac{K_A}{K_B} = \frac{1}{2}$. The free ends of rods A and B are maintained at 400 K , 200 K , respectively. The temperature of rods interface is _____ K, when equilibrium is established. [4 marks]

Solution:

Concept:

- The thermal resistance of a uniform rod is $\mathcal{R} = L/(KA)$, where $A = \pi r^2$ is its cross-sectional area.
- In the steady state, no heat accumulates at the interface. The same heat current passes through both rods, and their temperature drops are proportional to their thermal resistances.

Step 1: Calculate the resistance ratio The supplied ratios are $L_A/L_B = 1/2$, $r_A/r_B = 2$ and $K_A/K_B = 1/2$. Since cross-sectional area depends on the square of the radius,

$$\frac{A_A}{A_B} = \left(\frac{r_A}{r_B}\right)^2 = 4.$$

Therefore

$$\frac{\mathcal{R}_A}{\mathcal{R}_B} = \frac{L_A}{L_B}\frac{K_B}{K_A}\frac{A_B}{A_A} = \frac{1}{2} \times 2 \times \frac{1}{4} = \frac{1}{4}.$$

Although A has lower conductivity, its shorter length and larger area give it a lower thermal resistance.

Step 2: Equate the heat currents at the interface Let the interface temperature be T. Heat flows from the 400 K end through A to the interface, then through B to the 200 K end. At equilibrium,

$$\frac{400 - T}{\mathcal{R}_A} = \frac{T - 200}{\mathcal{R}_B}.$$

Consequently

$$\frac{400 - T}{T - 200} = \frac{1}{4}, \qquad 4(400 - T) = T - 200.$$

Using equal temperature drops would be incorrect because the rods have different resistances.

Step 3: Solve and check the temperature Rearranging gives $1600 - 4T = T - 200$, hence $5T = 1800$ and $T = 360\,\text{K}$. The drops are 40 K across A and 160 K across B, in the required ratio 1:4. The interface lies between the two reservoir temperatures and is closer to the hotter end's temperature because the hot-side rod offers the smaller thermal resistance.

Step 4: Conclusion The equilibrium interface temperature is $\boxed{360\,\text{K}}$.

Quick Tip to Solve: Treat joined rods as thermal resistances in series and square the radius ratio before comparing their temperature drops.

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48. The electric field in a region is given by $\vec{E} = (2\hat{i} + 4\hat{j} + 6\hat{k}) \times 10^3$ N/C. The flux of the field through a rectangular surface parallel to $x - z$ plane is $6.0\,\text{Nm}^2\text{C}^{-1}$. The area of the surface is _____ cm^2. [4 marks]

Solution:

Concept:

- For a uniform field and a flat surface, electric flux is $\Phi = \vec{E} \cdot \vec{A} = E_n A$, where E_n is the component normal to the surface.
- A surface parallel to the x-z plane has its area vector along the y direction, so only the y component of the field contributes to its flux.

Step 1: Identify the field component perpendicular to the surface The given field is

$$\vec{E} = (2\hat{i} + 4\hat{j} + 6\hat{k}) \times 10^3\,\text{N C}^{-1}.$$

The rectangle is parallel to the x-z plane. Its unit normal is therefore $\hat{j}$ or $-\hat{j}$, depending on the orientation chosen. For the stated positive flux choose $+\hat{j}$, giving

$$E_n = \vec{E} \cdot \hat{j} = 4 \times 10^3\,\text{N C}^{-1}.$$

The x and z components are tangential and have zero dot product with the area vector.

Step 2: Calculate the area in SI units Because the field is uniform over the rectangle, the surface

integral reduces to the normal field multiplied by its area. Thus

$$A = \frac{\Phi}{E_n} = \frac{6.0}{4 \times 10^3} = 1.5 \times 10^{-3}\,\mathrm{m}^2.$$

Using the magnitude of the whole field would incorrectly include the tangential components. The dimensions also check: dividing $\mathrm{N\,m^2\,C^{-1}}$ by $\mathrm{N\,C^{-1}}$ leaves square metres.

Step 3: Convert square metres into square centimetres The question asks for area in square centimetres. Since $1\,\mathrm{m} = 100\,\mathrm{cm}$, the area conversion factor is squared:

$$1\,\mathrm{m}^2 = 10^4\,\mathrm{cm}^2.$$

It follows that

$$A = 1.5 \times 10^{-3} \times 10^4 = 15\,\mathrm{cm}^2.$$

Substitution back gives $(4 \times 10^3)(15 \times 10^{-4}) = 6.0\,\mathrm{N\,m^2\,C^{-1}}$, reproducing the supplied flux.

Step 4: Conclusion The rectangular surface has area $\boxed{15\,\mathrm{cm}^2}$.

Quick Tip to Solve: Name the plane's missing coordinate to identify its normal field component; then remember that converting an area squares the length conversion factor.

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49. An inductor of reactance 100Ω, a capacitor of reactance 50Ω, and a resistor of resistance 50Ω are connected in series with an AC source of $10\ \mathrm{V}, 50\ \mathrm{Hz}$. Average power dissipated by the circuit is _____ W. [4 marks]

Solution:

Concept:

- In a series AC circuit the same root-mean-square current flows through the resistor, inductor and capacitor. The impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$.
- Only the resistance dissipates average power in ideal components: $P = I_{\mathrm{rms}}^2 R = V_{\mathrm{rms}}^2 R/Z^2$. Inductive and capacitive reactances affect current but return their stored energy each cycle.

Step 1: Find the net reactance The inductive and capacitive reactances have opposite signs, so subtract them rather than adding their magnitudes. Here $X_L = 100\,\Omega$, $X_C = 50\,\Omega$, and $R = 50\,\Omega$. Thus the circuit is net inductive, with

$$X = X_L - X_C = 100 - 50 = 50\,\Omega.$$

The supply frequency is already incorporated in the specified reactances; there is no need to calculate the inductance or capacitance separately.

Step 2: Calculate the current and real power The quoted AC source voltage is its rms value. The impedance and current are

$$Z = \sqrt{50^2 + 50^2} = 50\sqrt{2}\,\Omega, \qquad I_{\mathrm{rms}} = \frac{10}{50\sqrt{2}} = \frac{\sqrt{2}}{10}\,\mathrm{A}.$$

Therefore

$$P = I_{\text{rms}}^2 R = \left(\frac{\sqrt{2}}{10}\right)^2 (50) = 1\,\text{W}.$$

As a check, the power factor is $R/Z = 1/\sqrt{2}$, so $VI\cos\phi$ gives the same result. Using V^2/R directly would incorrectly ignore the reactance.

Step 3: Conclusion The average power dissipated by the circuit is $\boxed{1\,\text{W}}$. The required numerical entry is $\boxed{1}$.

Quick Tip to Solve: For a series LCR circuit, use the net reactance in the impedance, then apply $P = V_{\text{rms}}^2 R/[R^2 + (X_L - X_C)^2]$.

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50. A parallel plate capacitor has charge 5×10^{-6}C. A dielectric slab is inserted between the plates and almost fills the space between the plates. If the induced charge on one face of the slab is 4×10^{-6}C then the dielectric constant of the slab is _____. [4 marks]

Solution:

Concept:

- Polarisation produces equal and opposite bound charges on the two faces of a dielectric slab. Their magnitude is less than the free charge on the capacitor plate for a finite dielectric constant.
- For a slab filling the plate separation, the induced charge magnitude is $Q_b = Q(1 - 1/K)$, where Q is the free plate charge and K is the relative permittivity.

Step 1: Relate free charge to bound charge Let the plate area be A. The free surface charge density is $\sigma = Q/A$. In the dielectric the electric field is $E = \sigma/(K\varepsilon_0)$, and the polarisation magnitude is $P = \varepsilon_0(K-1)E$. The bound surface charge density equals this polarisation. Consequently,

$$Q_b = PA = \varepsilon_0(K-1)\frac{Q}{AK\varepsilon_0}A = Q\frac{K-1}{K}.$$

The bound charge on either individual face is used here; adding the magnitudes on both faces would count the same polarisation twice.

Step 2: Substitute the two charge magnitudes Using the stated free charge $Q = 5 \times 10^{-6}\,\text{C}$ and induced charge $Q_b = 4 \times 10^{-6}\,\text{C}$,

$$\frac{Q_b}{Q} = \frac{4}{5} = 1 - \frac{1}{K}.$$

Hence

$$\frac{1}{K} = \frac{1}{5}, \qquad K = 5.$$

Equivalently, the residual charge responsible for the internal field is $Q - Q_b = 1 \times 10^{-6}\,\text{C}$, giving $K = Q/(Q - Q_b) = 5$. The answer is dimensionless, as a dielectric constant must be.

Step 3: Conclusion The dielectric constant of the slab is $\boxed{K = 5}$. The required numerical entry is $\boxed{5}$.

Quick Tip to Solve: Use the induced charge on one face: $K = Q/(Q - Q_b)$. Never add the opposite bound charges on the two faces.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Match List - I with List - II.

List - I (Complex)	List - II (Primary valency and Secondary valency)
(A) $[Co(en)_2Cl_2]Cl$	**(I) 3, 6**
(B) $[Pt(NH_3)_2Cl(NO_2)]$	**(II) 3, 4**
(C) $Hg[Co(SCN)_4]$	**(III) 2, 6**
(D) $[Mg(EDTA)]^{2-}$	**(IV) 2, 4**

Choose the correct answer from the options given below: [4 marks]

(A) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

(B) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(C) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

(D) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

Correct Answer: The first, second and fourth complexes point to option (C), but the printed mercury salt gives an inconsistent third match under the usual Hg(II) assignment.

Solution:

Concept:

- Primary valency is the oxidation state of the central metal. Secondary valency is its coordination number, counting donor atoms rather than ligand molecules.
- Ethylenediamine (en) is neutral and bidentate, EDTA is a hexadentate tetravalent anion, and each chloride, nitro and thiocyanato ligand carries charge minus one and normally supplies one donor atom here.

Step 1: Match the cobalt and platinum complexes In $[Co(en)_2Cl_2]Cl$, the coordination entity has charge plus one. Thus $x - 2 = 1$, so cobalt has oxidation state $+3$. Two en ligands supply four donor atoms and two coordinated chlorides supply two, giving coordination number six. Hence A matches I, namely $(3, 6)$. In neutral $[Pt(NH_3)_2Cl(NO_2)]$, the two ammonia ligands are neutral and the other ligands together carry charge minus two. Platinum is therefore $+2$, with four donor atoms, so B matches IV, namely $(2, 4)$.

Step 2: Match the magnesium complex Let the oxidation state of magnesium in $[Mg(EDTA)]^{2-}$ be x. Charge balance gives $x - 4 = -2$, so $x = +2$. One EDTA ligand supplies six donor atoms, giving secondary valency six. Therefore D matches III, namely $(2, 6)$. These three secure matches occur together only in option (C); the other options already contradict at least one of them.

Step 3: Check the printed mercury salt before accepting the full matching The remaining printed formula is $Hg[Co(SCN)_4]$. With mercury in its usual $+2$ state in this salt, the complex anion has charge minus two, so $x - 4 = -2$ and cobalt is $+2$. Four thiocyanato donor atoms give secondary valency four, so this gives $(2, 4)$, entry IV, rather than entry II $(3, 4)$ required by option (C). Forcing Co(III) would instead require a monatomic Hg(I) counterion; ordinary mercury(I) salts contain the dimeric Hg_2^{2+} cation, and that composition is not printed. Consequently the third match is inconsistent with the conventional reading of the given salt; it must not be silently assigned a different formula.

Step 4: Conclusion A-I, B-IV and D-III identify option (C) as the intended matching. However, the printed $Hg[Co(SCN)_4]$ gives C-IV for the usual Hg(II) salt, while option (C) requires C-II. Thus the literal question has no fully consistent listed option under the standard oxidation-state assignment.

Quick Tip to Solve: Count donor atoms separately from ligand charge, then check the counterion before assigning the metal oxidation state; chelating ligands change the coordination count.

52. The correct statements from the following are :

(A) Tl^{3+} is a powerful oxidising agent

(B) Al^{3+} does not get reduced easily

(C) Both Al^{3+} and Tl^{3+} are very stable in solution

(D) Tl^{1+} is more stable than Tl^{3+}

(E) Al^{3+} and Tl^{1+} are highly stable

Choose the correct answer from the options given below : [4 marks]

(A) (A), (B), (D) and (E) only

(B) (A), (C) and (D) only

(C) (A), (B), (C) and (E)

(D) (B), (D) and (E) only

Correct Answer: (A) (A), (B), (D) and (E) only

Solution:

Concept:

- The inert-pair effect stabilises the lower oxidation state down Group 13. Thallium therefore favours $+1$, whereas aluminium strongly favours $+3$.
- An ion that readily accepts electrons and is reduced acts as an oxidising agent. The relative stability of oxidation states determines which reduction is favourable.

Step 1: Evaluate the aluminium statements Aluminium has the valence configuration $3s^2 3p^1$, and its common stable oxidation state is $+3$. Aluminium(III) ions are not readily reduced in ordinary aqueous chemical conditions; the strongly negative standard reduction potential of the Al^{3+}/Al couple is consistent with this. Statement (B) is therefore correct. Aluminium(III) is also the stable aluminium state described in (E). The behaviour of aluminium alone cannot establish (C), because that statement also claims high stability for thallium(III).

Step 2: Evaluate thallium and select the set For the heavy element thallium, the $6s^2$ electron pair tends to remain uninvolved in bonding. The $+1$ state is thus more stable than $+3$, making statement (D) correct. Thallium(III) readily undergoes reduction,

$$Tl^{3+} + 2\,e^- \longrightarrow Tl^+,$$

and hence acts as a powerful oxidising agent: (A) is correct. This instability of thallium(III) makes (C) false. Statement (E), which pairs stable Al^{3+} with stable Tl^+, is correct. Thus the complete true set is (A), (B), (D), and (E). Options containing (C) fail, while the option listing only (B), (D), and (E)

omits the true oxidising-agent statement (A).

Step 3: Conclusion The four correct statements are the oxidising-agent statement, the resistance of aluminium(III) to reduction, the preference for thallium(I), and the stability comparison. Hence, the correct option is (A). This differs from the supplied answer key because the printed first statement about thallium(III) being a powerful oxidising agent is also true.

Quick Tip to Solve: In Group 13, compare aluminium(III) with thallium(I): the inert-pair effect makes thallium(III) an oxidising agent.

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53. Given below are two statements :

Statement (I) : On hydrolysis, oligo peptides give rise to fewer number of α-amino acids while proteins give rise to a large number of β-amino acids.

Statement (II) : Natural proteins are denatured by acids which convert the water soluble form of fibrous proteins to their water insoluble form.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) **Statement I** is incorrect but **Statement II** is correct

(B) Both **Statement I** and **Statement II** are incorrect

(C) Both **Statement I** and **Statement II** are correct

(D) **Statement I** is correct but **Statement II** is incorrect

Correct Answer: (B) Both **Statement I** and **Statement II** are incorrect

Solution:

Concept:

- Peptides and proteins are built from α-amino acids linked through peptide bonds. Hydrolysis cleaves these bonds to release their constituent amino acids.
- Denaturation disrupts the higher-order structure of a protein without normally destroying its primary peptide-bond sequence. Globular and fibrous proteins have different typical solubilities.

Step 1: Test statement I against protein composition Oligopeptides contain relatively few amino-acid residues and therefore give a relatively small number of amino-acid molecules on complete hydrolysis. Proteins contain many residues and yield many amino-acid molecules. However, the amino acids released from natural proteins are α-amino acids, not the β-amino acids stated in the question. The carbon bearing the amino group is adjacent to the carboxyl carbon in an α-amino acid. Thus the first clause about oligopeptides does not rescue the incorrect second clause: Statement I as a whole is false.

Step 2: Test statement II and compare the options Acids can indeed denature a natural protein by altering the interactions responsible for its secondary and tertiary structures. The familiar description is that water-soluble globular proteins lose their native structure and can become insoluble or coagulated. Fibrous proteins are generally already water-insoluble; describing a water-soluble fibrous form as the starting material confuses the two classes. Therefore Statement II is also false as written. Options (A)

and (D) each require one false statement to be true, while (C) requires both to be true. The option asserting that both statements are incorrect is the only consistent choice.

Step 3: Conclusion Both Statement I and Statement II are incorrect. Hence, the correct option is (B).

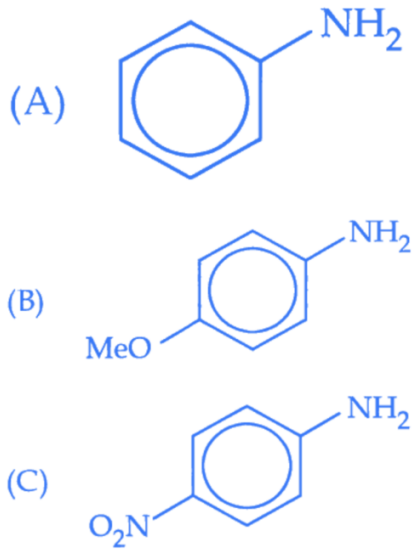
Quick Tip to Solve: Check both the amino-acid type and the protein class: protein hydrolysis gives alpha-amino acids, and soluble proteins are typically globular.

54. The descending order of basicity of the following amines is :

(D) CH_3NH_2

(E) $(CH_3)_2\,NH$

Choose the correct answer from the options given below : [4 marks]



(A) E > A > D > C > B

(B) E > D > A > B > C

(C) E > D > B > A > C

(D) B > E > D > A > C

Correct Answer: (C) E > D > B > A > C

Solution:

Concept:

- An amine is basic because its nitrogen lone pair accepts a proton. Electron donation increases availability of the lone pair, while delocalisation or electron withdrawal decreases it.
- For the methylamines in aqueous solution, dimethylamine is more basic than methylamine. Aniline derivatives are weaker because their nitrogen lone pair is delocalised into the benzene ring.

Step 1: Identify and compare the aliphatic amines The structures are (A) aniline, (B) para-methoxyaniline, (C) para-nitroaniline, (D) methylamine and (E) dimethylamine. In the aliphatic pair, methyl groups donate electron density through their positive inductive effect. Together with the

favourable solvation of the conjugate acids in the usual aqueous comparison, this gives

$$(E) > (D).$$

Both are stronger bases than the aromatic amines because their nitrogen lone pair is not shared with a benzene pi system. A direct comparison based only on the number of atoms or molecular mass would not explain proton acceptance.

Step 2: Order the aromatic amines and combine the groups In aniline (A), delocalisation of the nitrogen lone pair lowers its availability for protonation. The para-methoxy group in (B) donates electron density to the aromatic system by resonance; its resonance donation outweighs its inductive withdrawal in this comparison. Thus (B) is more basic than (A). The para-nitro group in (C) withdraws electron density strongly by both resonance and induction, making (C) the weakest base. Consequently,

$$(B) > (A) > (C).$$

Combining the aliphatic and aromatic comparisons gives $(E) > (D) > (B) > (A) > (C)$. The other options incorrectly place aniline above its para-methoxy derivative, elevate the nitro compound, or put the aromatic methoxy compound above the methylamines.

Step 3: Conclusion The descending order is $E > D > B > A > C$. Hence, the correct option is (C).

Quick Tip to Solve: First separate aliphatic from aromatic amines, then compare ring substituents: para-methoxy increases and para-nitro decreases aniline basicity.

55. In SO_2, NO_2^- and N_3^-, the hybridizations at the central atom are respectively : [4 marks]

(A) sp^2, sp^2 and sp^2

(B) sp, sp^2 and sp

(C) sp^2, sp and sp

(D) sp^2, sp^2 and sp

Correct Answer: (D) sp^2, sp^2 and sp

Solution:

Concept:

- Hybridisation is assigned from the steric number: the number of sigma-bond directions plus lone pairs around the atom. A multiple bond counts as one direction.
- Three electron domains correspond to sp^2 hybridisation; two electron domains correspond to sp. Pi bonding uses the remaining unhybridised orbitals.

Step 1: Count the domains around sulphur In SO_2, sulphur has two S–O sigma-bond directions and one lone pair. Resonance changes the description of the pi bonds but does not add another sigma direction. Thus its steric number is $2 + 1 = 3$, giving sp^2 hybridisation and a bent molecular shape.

Step 2: Count the domains around nitrogen In NO_2^-, the central nitrogen has two N–O sigma bonds and one lone pair. Its steric number is also three, so it is sp^2-hybridised. In the azide ion, N_3^-, the

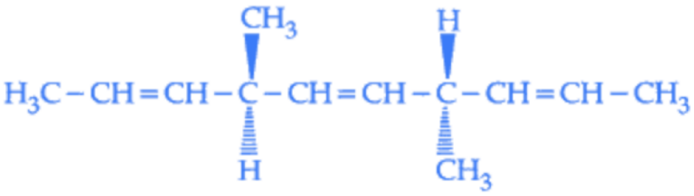
central nitrogen is connected to the two terminal nitrogens along a straight line. The representative resonance structure $^{-}N{=}N^{+}{=}N^{-}$ shows two sigma directions and no central lone pair. Its steric number is two and its hybridisation is sp.

Step 3: Compare the proposed sequences The required sequence is therefore sp^2, sp^2, sp. Option (A) wrongly assigns sp^2 to the linear azide centre. Option (B) incorrectly makes sulphur sp, while option (C) incorrectly assigns sp to the bent nitrite centre. Only (D) has all three assignments correct.

Step 4: Conclusion The central-atom hybridisations are $\boxed{sp^2,\ sp^2,\ sp}$, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Count sigma directions and lone pairs, not the total number of bond lines: a double bond contributes only one electron domain.

56. The number of optically active products obtained from the complete ozonolysis of the given compound is : [4 marks]



(A) 4

(B) 0

(C) 2

(D) 1

Correct Answer: (B) 0

Solution:

Concept:

- Complete ozonolysis cleaves every carbon–carbon double bond and replaces each alkene carbon by a carbonyl group.
- A tetrahedral carbon is a stereogenic centre only when it has four different substituents. Identical substituents make it achiral, even if wedge-and-dash bonds were present in the starting molecule.

Step 1: Cleave all three double bonds The displayed chain has three double bonds, with a methyl-substituted tetrahedral carbon between successive double bonds. Its two tetrahedral centres are shown stereochemically. Cutting each of its three double bonds gives two terminal molecules of ethanal, CH_3CHO, and two central fragments of the form $OHC{-}CH(CH_3){-}CHO$. Each carbon of an original double bond bears hydrogen, so the corresponding carbonyl end is an aldehyde.

Step 2: Inspect each product for chirality Ethanal has no tetrahedral carbon bearing four different groups, and hence is optically inactive. In each central fragment, the carbon that originally carried a wedge and a dashed bond is now bonded to H, CH_3, CHO and CHO. The two aldehyde substituents are identical. Consequently, this carbon is no longer a stereogenic centre. Both central fragments are

achiral despite the stereochemistry drawn in the starting material.

Step 3: Count optically active products No fragment retains a stereogenic centre or another source of molecular chirality. Thus none of the ozonolysis products is optically active. Counting the four carbonyl-containing fragments as four active products would confuse the number of fragments with the number of chiral products; counting the two original stereocentres would ignore their changed substituents.

Step 4: Conclusion The number of optically active products is $\boxed{0}$. Hence, the correct option is (B).

Quick Tip to Solve: After cleavage, relabel all four groups on each former stereocentre. Here the two new CHO groups become identical and destroy chirality.

57. A mixture of 1 g each of chlorobenzene, aniline, and benzoic acid is dissolved in 50 mL ethyl acetate and placed in a separating funnel. 5 M NaOH (30 mL) was added in the same funnel. The funnel was shaken vigorously and then kept aside. The ethyl acetate layer in the funnel contains :

[4 marks]

(A) chlorobenzene and aniline

(B) benzoic acid and aniline

(C) benzoic acid

(D) benzoic acid and chlorobenzene

Correct Answer: (A) chlorobenzene and aniline

Solution:

Concept:

- Acid-base extraction separates a neutral organic solute from an ion formed by reaction with aqueous acid or base.
- A carboxylic acid is deprotonated by aqueous NaOH, producing an ionic sodium carboxylate that preferentially dissolves in the aqueous layer.

Step 1: Identify the compound that reacts with base Benzoic acid reacts with hydroxide to form sodium benzoate:

$$\mathrm{C_6H_5COOH + NaOH \longrightarrow C_6H_5COONa + H_2O.}$$

One gram of benzoic acid corresponds to approximately $1/122 = 0.0082\ \mathrm{mol}$. The added hydroxide is $5 \times 0.030 = 0.150\ \mathrm{mol}$, so it is in large excess and is sufficient to convert the acid into its salt.

Step 2: Assign the compounds to the layers Sodium benzoate is ionic and moves into the aqueous phase. Chlorobenzene does not undergo an acid-base reaction with aqueous hydroxide under these extraction conditions; it remains a neutral organic solute. Aniline is a weak base, not a carboxylic acid. Adding another base does not convert it into a water-soluble anilinium salt. It therefore also remains in the ethyl acetate phase in the intended extraction.

Step 3: Eliminate the alternatives The ethyl acetate layer thus retains chlorobenzene and aniline. All the other alternatives include benzoic acid as a neutral organic component; they overlook its conversion into sodium benzoate. Acidifying the separated aqueous layer would regenerate benzoic

acid, but that later recovery step has not been performed here.

Step 4: Conclusion The ethyl acetate layer contains $\boxed{\text{chlorobenzene and aniline}}$. Hence, the correct option is (A).

Quick Tip to Solve: For aqueous NaOH extraction, move carboxylic acids to water as salts; to extract aniline as a salt, use aqueous acid instead.

58. " P " is an optically active compound with molecular formula $C_6H_{12}O$. When ' P ' is treated with 2, 4-dinitrophenylhydrazine, it gives a positive test. However, in presence of Tollens reagent, " P " gives a negative test. Predict the structure of " P ". [4 marks]

(A) $OHCCH_2CH(CH_3)CH_2CH_3$

(B) $CH_3COCH(CH_3)CH_2CH_3$

(C) $CH_3COCH_2CH_2CH_2CH_3$

(D) $CH_3COCH_2CH(CH_3)_2$

Correct Answer: (B) $CH_3COCH(CH_3)CH_2CH_3$

Solution:

Concept:

- 2,4-Dinitrophenylhydrazine gives a positive condensation test with aldehydes and ketones, indicating a carbonyl group.
- An ordinary aldehyde reduces Tollens reagent, whereas an ordinary ketone does not. Optical activity additionally requires a chiral structure.

Step 1: Use the two functional-group tests The positive 2,4-DNPH test identifies a carbonyl compound. The negative Tollens test excludes the aldehyde option and requires a ketone among the displayed ordinary carbonyl compounds. For $C_6H_{12}O$, the degree of unsaturation is $(2 \times 6 + 2 - 12)/2 = 1$, consistent with one carbonyl double bond and an otherwise saturated, open-chain structure.

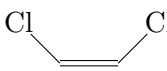
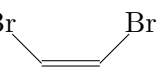
Step 2: Test the ketone options for chirality Option (B) is $CH_3COCH(CH_3)CH_2CH_3$, or 3-methylpentan-2-one. Its carbon at position 3 is bonded to H, methyl, ethyl and acetyl groups. These four groups are different, so this carbon is stereogenic. An individual enantiomer can therefore be optically active. The carbonyl group gives the DNPH test, while its ketone character accounts for the negative Tollens test.

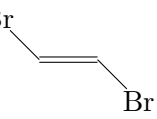
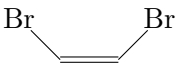
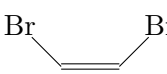
Step 3: Reject the remaining structures Option (A), $OHCCH_2CH(CH_3)CH_2CH_3$, is an aldehyde: although it has a stereogenic centre, it would give a positive Tollens test. Option (C), hexan-2-one, has no stereogenic carbon. Option (D), 4-methylpentan-2-one, has a methine carbon carrying two identical methyl groups, so it is also achiral. Thus only the second structure satisfies all three observations together.

Step 4: Conclusion P is $\boxed{CH_3COCH(CH_3)CH_2CH_3}$, 3-methylpentan-2-one. Hence, the correct option is (B).

Quick Tip to Solve: Apply the chemical tests first to select a ketone, then check whether any tetrahedral carbon has four different substituents.

59. Given below are two statements :

Statement (I) :  **is more polar than** 

Statement (II) : Boiling point of  **is lower than**  **but it is more polar than** .

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) **Both Statement I and Statement II are correct**

(B) **Statement I is incorrect but Statement II is correct**

(C) **Both Statement I and Statement II are incorrect**

(D) **Statement I is correct but Statement II is incorrect**

Correct Answer: (D) **Statement I is correct but Statement II is incorrect**

Solution:

Concept:

- Molecular polarity depends on the vector sum of bond dipoles. In a symmetric trans-1,2-dihaloethene these contributions cancel, whereas the cis isomer has a nonzero resultant.
- For comparable cis compounds, the more polar C–Cl bonds give a larger dipole resultant than C–Br bonds. Boiling point depends on intermolecular attraction as well as other molecular properties.

Step 1: Evaluate Statement I The first displayed molecule is cis-1,2-dichloroethene, and the second is cis-1,2-dibromoethene. Both have the two halogens on the same side of the carbon–carbon double bond, so the relevant bond dipoles reinforce rather than cancel. Chlorine is more electronegative than bromine, and cis-1,2-dichloroethene has the larger net dipole moment. Thus the polarity comparison made in Statement I is correct.

Step 2: Evaluate Statement II The first molecule in Statement II is trans-1,2-dibromoethene; the following molecule is its cis isomer. The trans isomer has opposite, equal bond-dipole contributions and is nonpolar by symmetry. The cis isomer has a nonzero dipole moment and is more polar. The trans isomer has a lower boiling point than the cis isomer, so the boiling-point clause is correct. However, the added assertion that the trans isomer is more polar than the cis isomer is false.

Step 3: Select the combined verdict A statement containing a correct clause and an incorrect clause is incorrect as a whole. Therefore Statement I is correct but Statement II is incorrect. The alternatives asserting that Statement II is correct fail the dipole-cancellation test, and the alternative declaring both incorrect wrongly rejects the first comparison.

Step 4: Conclusion Statement I is correct and Statement II is incorrect. Hence, the correct option is (D).

Quick Tip to Solve: Check each clause separately: the trans dibromoethene can boil lower while still being less polar because its bond dipoles cancel.

60. Choose the incorrect trend in the atomic radii (r) of the elements. [4 marks]

(A) $r_{Rb} < r_{Cs}$

(B) $r_{At} < r_{Cs}$

(C) $r_{Br} < r_{K}$

(D) $r_{Mg} < r_{Al}$

Correct Answer: (D) $r_{Mg} < r_{Al}$

Solution:

Concept:

- Atomic radius generally increases down a group because an additional principal shell is occupied.
- Across a period, increasing effective nuclear charge attracts electrons in the same principal shell more strongly, so atomic radius generally decreases from left to right.

Step 1: Check the down-group and same-period comparisons Rubidium and caesium belong to Group 1. Caesium lies below rubidium and has an additional occupied shell, so $r_{Rb} < r_{Cs}$, as in (A). Astatine and caesium lie in Period 6, with caesium at the left and astatine toward the right. Contraction across the period gives $r_{At} < r_{Cs}$, as stated in (B).

Step 2: Check potassium and bromine Potassium and bromine lie in Period 4. Moving from potassium toward bromine increases nuclear charge while the added outer electrons occupy the same principal shell. Shielding does not fully offset the increased nuclear attraction, and the atomic radius decreases. Hence $r_{Br} < r_{K}$, so option (C) is also a correct trend.

Step 3: Identify the reversed inequality Magnesium and aluminium are adjacent Period-3 elements, with aluminium to the right of magnesium. The increased effective nuclear charge in aluminium contracts the atom. Therefore

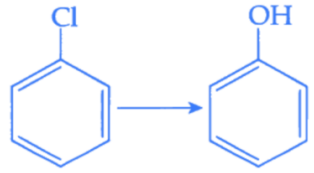
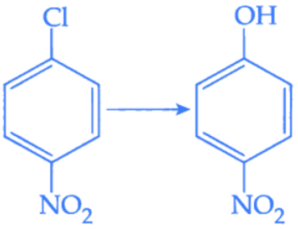
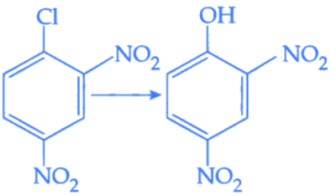
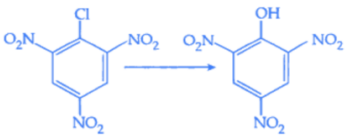
$$r_{Mg} > r_{Al}.$$

Option (D) prints the opposite inequality. Since the question asks for the incorrect trend, the reversed Mg–Al comparison is the required selection; the other three inequalities agree with the periodic pattern.

Step 4: Conclusion The incorrect trend is $\boxed{r_{Mg} < r_{Al}}$; the actual trend is $r_{Mg} > r_{Al}$. Hence, the correct option is (D).

Quick Tip to Solve: For adjacent elements in one period, compare their left-to-right positions before using group trends; Mg lies to the left of Al.

61. **Match List - I with List - II. Choose the correct answer from the options given below:** [4 marks]

List - I (Conversion)	List - II (Reagents, Conditions used)
(A) 	(I) Warm H_2O
(B) 	(II) (a) NaOH, 368K; (b) H_3O^+
(C) 	(III) (a) NaOH, 443K; (b) H_3O^+
(D) 	(IV) (a) NaOH, 623K, 300 atm; (b) H_3O^+

(A) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(B) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(C) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(D) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

Correct Answer: (D) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

Solution:

Concept:

- Electron-withdrawing nitro groups at ortho or para positions activate an aryl chloride towards nucleophilic aromatic substitution by stabilising its anionic addition intermediate.
- Increasing the number of activating ortho/para nitro groups permits hydrolysis under progressively milder conditions. Alkaline hydrolysis first gives a phenoxide ion, and acidification gives the corresponding phenol.

Step 1: Match unsubstituted chlorobenzene Conversion (A) replaces chlorine on chlorobenzene by hydroxyl. The carbon–chlorine bond has partial double-bond character through resonance and cannot undergo ordinary easy hydrolysis. Consequently this conversion requires the harshest listed conditions: aqueous NaOH at $623\,\mathrm{K}$ and $300\,\mathrm{atm}$, followed by acidification. Therefore (A) matches

(IV).

Step 2: Compare the nitro-substituted substrates Conversion (B) starts from para-nitrochlorobenzene. Its single para nitro group stabilises the intermediate, so hydrolysis occurs at $443\,\mathrm{K}$: (B) matches (III). Conversion (C) has two nitro groups, one ortho and one para to chlorine. Their combined activation allows hydrolysis with NaOH at $368\,\mathrm{K}$, followed by H_3O^+: (C) matches (II).

Step 3: Identify the most activated substrate Conversion (D) starts from 2,4,6-trinitrochlorobenzene. Three suitably placed nitro groups strongly stabilise the substitution intermediate, and even warm water converts it into 2,4,6-trinitrophenol. Thus (D) matches (I). Options (A) and (B) incorrectly give chlorobenzene mild conditions; option (C) interchanges the conditions for the di- and trinitro substrates.

Step 4: Conclusion The required matching assigns the four conversions to IV, III, II and I, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Rank the aryl chlorides by the number of ortho/para nitro groups: more activation means a lower hydrolysis temperature.

62. Liquid A and B form an ideal solution. The vapour pressures of pure liquids A and B are 350 and 750 mm Hg respectively at the same temperature. If x_A and x_B are the mole fraction of A and B in solution while y_A and y_B are the mole fraction of A and B in vapour phase, then, [4 marks]

(A) $(x_A - y_A) < (x_B - y_B)$
(B) $\frac{x_A}{x_B} = \frac{y_A}{y_B}$
(C) $\frac{x_A}{x_B} < \frac{y_A}{y_B}$
(D) $\frac{x_A}{x_B} > \frac{y_A}{y_B}$

Correct Answer: (D) $\frac{x_A}{x_B} > \frac{y_A}{y_B}$

Solution:

Concept:

- For an ideal solution, The ideal-solution form of Raoult's law gives partial pressures $p_A = x_A p_A^0$ and $p_B = x_B p_B^0$. Dalton's law gives $y_i = p_i/P$, where $P = p_A + p_B$.
- The vapour is enriched in the more volatile component, which has the higher pure-liquid vapour pressure.

Step 1: Form the vapour-composition ratio At the stated temperature, $p_A^0 = 350\,\mathrm{mm\,Hg}$ and $p_B^0 = 750\,\mathrm{mm\,Hg}$. Applying Raoult's law and then taking a ratio removes the unknown total pressure:

$$\frac{y_A}{y_B} = \frac{p_A/P}{p_B/P} = \frac{x_A p_A^0}{x_B p_B^0} = \frac{350}{750}\frac{x_A}{x_B} = \frac{7}{15}\frac{x_A}{x_B}.$$

For a mixture containing both liquids, this is strictly smaller than x_A/x_B.

Step 2: Check the remaining relations The total pressure is $P = 350x_A + 750x_B$, which lies between 350 and 750 mm Hg. Consequently $y_A = x_A(350/P) < x_A$, whereas $y_B = x_B(750/P) > x_B$. Thus $x_A - y_A$ is positive and $x_B - y_B$ is negative, ruling out option (A). The mole fractions in each phase

separately sum to one, so these two differences have equal magnitude and opposite signs. Option (B) would require equal volatilities, contrary to the stated pressures. Option (C) reverses the ratio relation. The higher vapour pressure of B means its relative abundance increases upon vaporisation; this also supplies a physical check on the algebra.

Step 3: Conclusion

$$\frac{x_A}{x_B} > \frac{y_A}{y_B}.$$

Hence, the correct option is (D).

Quick Tip to Solve: For an ideal binary mixture, divide the vapour mole-fraction ratio by the liquid ratio; the result is simply the ratio of pure-component vapour pressures.

63. Match List - I with List - II.

List - I	List - II
(A) Solution of chloroform and acetone	(I) Minimum boiling azeotrope
(B) Solution of ethanol and water	(II) Dimerizes
(C) Solution of benzene and toluene	(III) Maximum boiling azeotrope
(D) Solution of acetic acid in benzene	(IV) $\Delta V_{\text{mix}} = 0$

Choose the correct answer from the options given below: [4 marks]

(A) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(B) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(C) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(D) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

Correct Answer: (A) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Solution:

Concept:

- Positive deviations from Raoult's law give a higher vapour pressure and can form minimum-boiling azeotropes. Negative deviations give a lower vapour pressure and can form maximum-boiling azeotropes.
- An ideal solution has negligible enthalpy and volume changes on mixing. Association of a solute, such as carboxylic-acid dimer formation, is a distinct molecular effect.

Step 1: Match the two azeotropic mixtures Chloroform and acetone develop stronger unlike-molecule interactions through hydrogen bonding between the acidic hydrogen of chloroform and the carbonyl oxygen of acetone. The escaping tendency decreases, giving negative deviation and a maximum-boiling azeotrope. Thus (A) matches (III). Ethanol and water show positive deviation because mixing disrupts the original molecular interactions sufficiently to increase the total escaping tendency. Their azeotrope boils below either pure component, so (B) matches (I).

Step 2: Match the ideal and associating systems Benzene and toluene have similar molecular sizes and intermolecular attractions, so their solution is approximately ideal. Its volume change on mixing

is zero in the ideal-solution treatment: (C) matches (IV). Acetic acid in non-polar benzene forms hydrogen-bonded dimers, so (D) matches (II). Collecting these independent matches gives

$$(A)\to(III),\quad (B)\to(I),\quad (C)\to(IV),\quad (D)\to(II).$$

Only the first listed option has all four assignments. The other combinations interchange the azeotrope type, ideal behaviour or acid association and therefore fail at least one of these molecular checks.

Step 3: Conclusion The four entries match III, I, IV and II respectively. Hence, the correct option is (A).

Quick Tip to Solve: Remember the pair: stronger unlike attractions mean negative deviation and a maximum-boiling azeotrope; acetic acid associates in benzene.

64. The hydration energies of K^+ and Cl^- are $-x$ and $-y$ kJ/mol respectively. If the lattice energy of KCl is $-z$ kJ/mol, then the heat of solution of KCl is : [4 marks]

(A) $x+y+z$

(B) $z-(x+y)$

(C) $-z-(x+y)$

(D) $x-y-z$

Correct Answer: (B) $z-(x+y)$

Solution:

Concept:

- By Hess's law, the net enthalpy change depends only on the initial solid and the final aqueous ions. Dissolving an ionic solid can be resolved into lattice dissociation and hydration of the separated gaseous ions.
- Reversing a thermochemical process reverses its enthalpy sign. Hydration is exothermic, while separating the crystal into gaseous ions requires energy.

Step 1: Reverse the lattice-formation process The stated lattice energy is negative, $-z\,\mathrm{kJ\,mol^{-1}}$, so it describes formation of crystalline KCl from gaseous ions. Dissolution requires the opposite step:

$$\mathrm{KCl(s)} \longrightarrow \mathrm{K^+(g)} + \mathrm{Cl^-(g)}, \qquad \Delta H_1 = +z\,\mathrm{kJ\,mol^{-1}}.$$

Using $-z$ for this separation would incorrectly count lattice formation rather than lattice breaking.

Step 2: Hydrate the ions and add the changes The two hydration processes have the given enthalpies:

$$\mathrm{K^+(g)} \longrightarrow \mathrm{K^+(aq)}, \qquad \Delta H_2 = -x,$$

$$\mathrm{Cl^-(g)} \longrightarrow \mathrm{Cl^-(aq)}, \qquad \Delta H_3 = -y,$$

where both values are in $\mathrm{kJ\,mol^{-1}}$. Adding all three reactions cancels the gaseous ions and leaves

exactly $\mathrm{KCl(s)} \longrightarrow \mathrm{K^+(aq)} + \mathrm{Cl^-(aq)}$. Therefore

$$\Delta H_{\text{sol}} = z - x - y = z - (x + y).$$

Option (A) wrongly makes hydration endothermic. Option (C) fails to reverse the negative lattice energy. Option (D) assigns inconsistent signs to the two hydration terms. The net dissolution can be endothermic or exothermic, depending on whether lattice dissociation or total hydration dominates.

Step 3: Conclusion The heat of solution is $\boxed{z - (x + y)\,\mathrm{kJ\,mol^{-1}}}$. Hence, the correct option is (B).

Quick Tip to Solve: When lattice energy is supplied as a negative formation value, change its sign before adding the negative hydration enthalpies.

65. The number of unpaired electrons responsible for the paramagnetic nature of the following complex species are respectively :

$[Fe(CN)_6]^{3-}$, $[FeF_6]^{3-}$, $[CoF_6]^{3-}$, $[Mn(CN)_6]^{3-}$ [4 marks]

(A) 1, 5, 4, 2

(B) 1, 4, 4, 2

(C) 1, 5, 5, 2

(D) 1, 1, 4, 2

Correct Answer: (A) 1, 5, 4, 2

Solution:

Concept:

- For these octahedral complexes, the six ligands split the metal d orbitals into lower t_{2g} and higher e_g sets.
- Cyanide is a strong-field ligand and favours low-spin arrangements here; fluoride is weak-field and gives high-spin arrangements. The oxidation state determines the metal d-electron count.

Step 1: Determine the oxidation states and d counts Each complex has six singly negative ligands and an overall charge of -3, so every central metal has oxidation state $+3$. Neutral iron, cobalt and manganese belong respectively to groups 8, 9 and 7. Their trivalent ions are therefore $\mathrm{Fe^{3+}} : d^5$, $\mathrm{Co^{3+}} : d^6$ and $\mathrm{Mn^{3+}} : d^4$. Ligand strength changes how these electrons occupy the split orbitals; it does not change their total number.

Step 2: Fill the four complexes in the printed order In $[\mathrm{Fe(CN)_6}]^{3-}$, strong-field cyanide gives low-spin d^5, namely $t_{2g}^5 e_g^0$, with one unpaired electron. In $[\mathrm{FeF_6}]^{3-}$, weak-field fluoride gives high-spin d^5, namely $t_{2g}^3 e_g^2$, with five unpaired electrons. In $[\mathrm{CoF_6}]^{3-}$, high-spin d^6 has configuration $t_{2g}^4 e_g^2$: one pair and four singly occupied orbitals give four unpaired electrons. Finally, low-spin d^4 in $[\mathrm{Mn(CN)_6}]^{3-}$ is $t_{2g}^4 e_g^0$, with two unpaired electrons. Option (B) undercounts high-spin iron, (C) assigns five unpaired electrons to a six-electron ion, and (D) treats fluoride as strong-field for iron.

Step 3: Conclusion The numbers of unpaired electrons are $\boxed{1,\ 5,\ 4,\ 2}$, respectively. Hence, the correct option is (A).

Quick Tip to Solve: First write the metal oxidation state and d count; then choose high or low spin from the ligand before counting singly occupied orbitals.

66. A(g) → B(g) + C(g) is a first order reaction.

Time	t	∞
P_{system}	P_t	P_∞

The reaction was started with reactant A only. Which of the following expressions is correct for rate constant k? [4 marks]

(A) $k = \frac{1}{t} \ln \frac{p_\infty}{p_t}$

(B) $k = \frac{1}{t} \ln \frac{p_\infty}{2(p_\infty - p_t)}$

(C) $k = \frac{1}{t} \ln \frac{2(p_\infty - p_t)}{p_t}$

(D) $k = \frac{1}{t} \ln \frac{p_\infty}{(p_\infty - p_t)}$

Correct Answer: (B) $k = \frac{1}{t} \ln \frac{p_\infty}{2(p_\infty - p_t)}$

Solution:

Concept:

- At fixed temperature and volume, a gas partial pressure is proportional to its amount. For first-order disappearance of A, $k = t^{-1} \ln(p_{A,0}/p_{A,t})$.
- In $\mathrm{A(g)} \longrightarrow \mathrm{B(g)} + \mathrm{C(g)}$, each mole of A consumed increases the total number of gas moles by one. Total pressure is therefore not the remaining pressure of A.

Step 1: Express the total pressures using the extent Let the initial pressure of pure A be P_0, and let x be the pressure-equivalent of A decomposed by time t. The partial pressures are then $p_A = P_0 - x$, $p_B = x$ and $p_C = x$. Thus

$$P_t = (P_0 - x) + x + x = P_0 + x.$$

At complete decomposition, $x = P_0$, so $P_\infty = 2P_0$, or $P_0 = P_\infty/2$. No initial products are present, which is essential to this relation.

Step 2: Substitute the remaining reactant pressure From $x = P_t - P_0$, the pressure of undecomposed A is

$$p_{A,t} = P_0 - (P_t - P_0) = 2P_0 - P_t = P_\infty - P_t.$$

The first-order integrated law therefore gives

$$k = \frac{1}{t} \ln \frac{P_0}{p_{A,t}} = \frac{1}{t} \ln \frac{P_\infty}{2(P_\infty - P_t)}.$$

Option (A) uses total pressure as the reactant pressure and would produce a decreasing or negative logarithm. Option (C) has the wrong pressure ratio. Option (D) misses the factor of two relating the initial and final total pressures. As a consistency check, at $t = 0$, $P_t = P_\infty/2$, so the logarithm in the correct integrated expression is zero.

Step 3: Conclusion

$$\boxed{k = \frac{1}{t} \ln \frac{P_\infty}{2(P_\infty - P_t)}}.$$

Hence, the correct option is (B).

Quick Tip to Solve: Build a pressure table first: for A forming two gaseous products, the remaining reactant pressure is final total pressure minus current total pressure.

67. Given below are two statements :

1 M aqueous solutions of each of $Cu(NO_3)_2$, $AgNO_3$, $Hg_2(NO_3)_2$, $Mg(NO_3)_2$ are electrolysed using inert electrodes. Given: $E^0_{Ag^+/Ag}$ = 0.80 V, $E^0_{Hg_2^{2+}/Hg}$ = 0.79 V, $E^0_{Cu^{2+}/Cu}$ = 0.24 V and $E^0_{Mg^{2+}/Mg}$ = -2.37 V.

Statement (I) : With increasing voltage, the sequence of deposition of metals on the cathode will be Ag, Hg and Cu.

Statement (II) : Magnesium will not be deposited at the cathode instead oxygen gas will be evolved at the cathode.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both **Statement I** and **Statement II** are incorrect

(B) **Statement I** is incorrect but **Statement II** is correct

(C) **Statement I** is correct but **Statement II** is incorrect

(D) Both **Statement I** and **Statement II** are correct

Correct Answer: (C) **Statement I** is correct but **Statement II** is incorrect

Solution:

Concept:

- At an electrolytic cathode reduction takes place. Under comparable conditions, a more positive reduction potential favours discharge of the corresponding cation.
- In an aqueous electrolyte, reduction of water competes with reduction of the metal ion. Very electropositive metals such as magnesium are not normally obtained by electrolysis of their aqueous salts.

Step 1: Compare the given reduction potentials The solutions have the same stated concentration and inert electrodes are used. Arrange the metal couples by decreasing standard reduction potential:

$$E^\circ_{Ag^+/Ag} = 0.80 > E^\circ_{Hg_2^{2+}/Hg} = 0.79 > E^\circ_{Cu^{2+}/Cu} = 0.24\ \text{V}.$$

Consequently the order of metal deposition, as the applied voltage is increased sufficiently, is silver, mercury and copper. The very small difference between silver and mercury does not reverse their given order. Statement I is correct.

Step 2: Identify the gas produced instead of magnesium The reduction potential for Mg^{2+}/Mg is very negative, $-2.37\,\text{V}$. In aqueous solution, water is reduced preferentially:

$$2\,H_2O + 2\,e^- \longrightarrow H_2 + 2\,OH^-.$$

Thus magnesium is not deposited, but the cathodic gas is hydrogen, not oxygen. Oxygen evolution is an oxidation process and belongs at the anode. Statement II joins a correct observation to an incorrect gas assignment, so the statement as a whole is incorrect. Options A and B wrongly reject I, while D wrongly accepts II.

Step 3: Conclusion Statement I is correct and Statement II is incorrect. Hence, the correct option is (C).

Quick Tip to Solve: First check the electrode process: a cathode reduces species and can evolve hydrogen; oxygen evolution requires oxidation at an anode.

68. The extra stability of half-filled subshell is due to :

(A) Symmetrical distribution of electrons

(B) Smaller coulombic repulsion energy

(C) The presence of electrons with the same spin in non-degenerate orbitals

(D) Larger exchange energy

(E) Relatively smaller shielding of electrons by one another

Indentify the correct statements : [4 marks]

(A) (B), (D) and (E) only

(B) (A), (B), (D) and (E) only

(C) (B), (C) and (D) only

(D) (A), (B) and (D) only

Correct Answer: (B) (A), (B), (D) and (E) only

Solution:

Concept:

- By Hund's rule, electrons occupy the degenerate orbitals of a subshell singly with parallel spins before pairing. A half-filled subshell therefore has a particularly symmetrical distribution.
- Exchange stabilization arises between electrons with parallel spins in degenerate orbitals. Singly occupied orbitals also avoid the additional repulsion associated with pairing two electrons in the same orbital.

Step 1: Assess the proposed stabilizing features In a half-filled subshell every orbital has one electron. The equivalent occupancy gives symmetry, so statement (A) is correct. Keeping electrons in separate orbitals reduces their close mutual repulsion compared with paired occupancy, supporting (B). The parallel-spin arrangement provides the maximum exchange stabilization available for that half-filled configuration, so (D) is correct. The spread of electrons among the orbitals also gives relatively less mutual shielding; the electrons experience the nuclear attraction more effectively. Thus (E) is included among the stabilizing features.

Step 2: Reject the incorrect orbital description Statement (C) refers to electrons in non-degenerate orbitals. The exchange explanation used here concerns equivalent, degenerate orbitals of the same subshell, such as the three p orbitals or the five d orbitals. Having parallel spins in arbitrary non-degenerate orbitals is not the stated cause of half-filled-subshell stability. Therefore (C) is incorrect. The valid set is (A), (B), (D) and (E). Option A omits symmetry, option C includes the incorrect statement (C), and option D omits reduced mutual shielding.

Step 3: Conclusion Symmetry, reduced repulsion, exchange stabilization and reduced mutual shielding are the correct features. Hence, the correct option is (B).

Quick Tip to Solve: For a half-filled subshell picture one parallel-spin electron in every degenerate orbital; check symmetry, pairing repulsion and exchange stabilization together.

69. 'X' is the number of acidic oxides among VO_2, V_2O_3, CrO_3, V_2O_5 and Mn_2O_7. The primary valency of cobalt in

$[Co(H_2NCH_2CH_2NH_2)_3]_2(SO_4)_3$ is Y. The value of X + Y is _____. [4 marks]

(A) 3

(B) 4

(C) 2

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- Transition-metal oxides generally become more acidic as the oxidation state of the metal increases. Some intermediate oxides are amphoteric and must not be counted as purely acidic oxides.
- In Werner's terminology the primary valency is the oxidation state of the central metal. It is distinct from the secondary valency, which is the coordination number.

Step 1: Count the acidic oxides The listed oxides have the following conventional acid-base characters: V_2O_3, with vanadium in the $+3$ state, is basic; VO_2, with vanadium in the $+4$ state, is amphoteric; and V_2O_5, with vanadium in the $+5$ state, is also amphoteric. The remaining two, CrO_3 and Mn_2O_7, are acidic oxides. They are associated with chromic acid and permanganic acid, respectively. The count is therefore $X = 2$. Counting every high-oxidation-state oxide as acidic would incorrectly include amphoteric vanadium(V) oxide.

Step 2: Find cobalt oxidation state and add The ligand $H_2NCH_2CH_2NH_2$, ethane-1,2-diamine, is neutral even though it occupies two coordination positions. Three sulfate counterions carry total charge -6, so the two complex cations together carry $+6$. If the oxidation state of each cobalt is y, charge neutrality requires

$$2y + 3(-2) = 0, \qquad y = +3.$$

Thus the primary valency is $Y = 3$, whereas the coordination number is six and is not the quantity requested. Adding gives $X + Y = 2 + 3 = 5$. The other offered numbers do not equal this sum.

Step 3: Conclusion $X = 2$, $Y = 3$, and $X + Y = 5$. Hence, the correct option is (D).

Quick Tip to Solve: Do not replace primary valency by coordination number: neutral bidentate en raises the coordination number without changing the charge-balance calculation.

70. The correct statement amongst the following is : [4 marks]

(A) The standard state of a pure gas is the pure gas at a pressure of 1 bar and temperature 273 K.

(B) $\Delta_f H^\circ_{500}$ is zero for $O_2(g)$.

(C) $\Delta_f H^\circ_{298}$ is zero for $O(g)$.

(D) The term 'standard state' implies that the temperature is 0°C.

Correct Answer: (B) $\Delta_f H^\circ_{500}$ is zero for $O_2(g)$.

Solution:

Concept:

- A standard thermodynamic state specifies a reference pressure, conventionally 1 bar, and the physical state at a stated temperature. It does not itself fix the temperature to 273 K or 298 K.
- The standard enthalpy of formation of an element in its reference form is zero by convention at the temperature considered. An isolated gaseous atom need not be that reference form.

Step 1: Check the elemental reference state at 500 K At $500\,\mathrm{K}$, oxygen in its elemental reference form is diatomic oxygen gas, $\mathrm{O_2(g)}$. Formation of this reference substance from the element in its reference state involves no chemical change. Therefore

$$\Delta_\mathrm{f} H^\circ_{500}[\mathrm{O_2(g)}] = 0.$$

This zero is a convention for formation enthalpy, not a claim that all enthalpy changes involving oxygen vanish. Heating oxygen from one temperature to another still requires an enthalpy change.

Step 2: Eliminate the remaining definitions Option A combines the correct standard pressure of 1 bar with an unjustified compulsory temperature of $273\,\mathrm{K}$. A standard state can be specified at other temperatures, including the 500 K explicitly used in option B. Option D makes the same error by insisting on $0^\circ\mathrm{C}$. Option C refers to monatomic $\mathrm{O\,(g)}$, not the stable diatomic reference form. Producing gaseous oxygen atoms from $\mathrm{O_2(g)}$ requires dissociation, so its standard formation enthalpy at 298 K is not zero. Only option B obeys both the reference-state convention and the temperature distinction.

Step 3: Conclusion $\Delta_\mathrm{f} H^\circ_{500}$ is zero for $\mathrm{O_2(g)}$. Hence, the correct option is (B).

Quick Tip to Solve: A superscript standard-state symbol fixes the reference conditions; it does not force 0 degrees Celsius. Always check the element's actual reference form.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. In Dumas' method 292 mg of an organic compound released 50 mL of nitrogen gas (N_2) at 300 K temperature and 715 mm Hg pressure. The percentage composition of ' N ' in the organic compound is _____ % (Nearest integer) (Aqueous tension at $300\ K = 15\ mmHg$) [4 marks]

Solution:

Concept:

- Gas collected over water contains water vapour. Dalton's law gives the dry nitrogen pressure as the observed pressure minus the aqueous tension at the collection temperature.
- The ideal-gas equation gives $n = PV/(RT)$. Multiplying moles of N_2 by $28\,g\,mol^{-1}$ gives the mass of nitrogen in the sample; divide by sample mass for its percentage.

Step 1: Correct the measured gas pressure The measured pressure includes both nitrogen and water vapour. Use the given aqueous tension, rather than treating 715 mm Hg as the dry gas pressure:

$$P_{N_2} = 715 - 15 = 700\,mm\,Hg = \frac{700}{760}\,atm.$$

The gas volume is $50\,mL = 0.050\,L$, its temperature is $300\,K$, and the compound mass is $292\,mg = 0.292\,g$. All quantities are now consistent with $R = 0.082057\,L\,atm\,mol^{-1}\,K^{-1}$.

Step 2: Calculate nitrogen mass and percentage Substitute into the ideal-gas equation, retaining unrounded values until the final percentage:

$$n(N_2) = \frac{(700/760)(0.050)}{0.082057(300)} = 1.8707 \times 10^{-3}\,mol.$$

$$m(N) = 28n(N_2) = 0.05238\,g.$$

The molar mass 28 already accounts for both nitrogen atoms in each molecule; an additional factor of two would double-count nitrogen. Therefore

$$\%N = \frac{0.05238}{0.292} \times 100 = 17.94\%.$$

The nearest-integer instruction applies to this final result, giving 18, in agreement with the stated numerical key.

Step 3: Conclusion The nitrogen content is approximately 17.94%, so the required nearest-integer answer is $\boxed{18}$.

Quick Tip to Solve: Subtract aqueous tension before applying the gas law, and use 28 g per mole of collected nitrogen gas without multiplying by two again.

72. Butane reacts with oxygen to produce carbon dioxide and water following the equation given below.

$$C_4H_{10}(g) + \frac{13}{2}O_2(g) \rightarrow 4CO_2(g) + 5H_2O(l)$$

If 174.0 kg of butane is mixed with 320.0 kg of O_2, the volume of water formed in liters is _____. (Nearest integer)

[Given : (a) Molar mass of C, H, O are $12, 1, 16$ g mol^{-1} respectively, (b) Density of water $= 1$ g mL^{-1}]

[4 marks]

Solution:

Concept:

- The limiting reactant is found by comparing each available amount divided by its stoichiometric coefficient. It determines the maximum amount of product even if the other reactant is in excess.
- For the given reaction, 6.5 mol of O_2 produces 5 mol of liquid water. Water volume follows from $V = m/\rho$, not from a molar gas volume.

Step 1: Identify the limiting reactant The molar masses are $M(C_4H_{10}) = 4(12) + 10(1) = 58\,\text{g mol}^{-1}$ and $M(O_2) = 32\,\text{g mol}^{-1}$. Consequently

$$n(C_4H_{10}) = \frac{174000}{58} = 3000\,\text{mol}, \qquad n(O_2) = \frac{320000}{32} = 10000\,\text{mol}.$$

Complete combustion of all 3000 mol of butane would need $3000(13/2) = 19500$ mol of oxygen. Only 10000 mol is available, so oxygen is limiting. The excess butane cannot increase the water yield after oxygen is consumed.

Step 2: Calculate liquid-water volume Use the oxygen-to-water mole ratio from the balanced equation:

$$n(H_2O) = 10000\frac{5}{13/2} = \frac{100000}{13}\,\text{mol}.$$

With $M(H_2O) = 18\,\text{g mol}^{-1}$,

$$m(H_2O) = \frac{1800000}{13}\,\text{g} = 138461.54\,\text{g}.$$

The density is $1\,\text{g mL}^{-1}$, so the numerical volume in millilitres equals this mass in grams. Converting millilitres to litres gives

$$V = 138461.54\,\text{mL} = 138.46154\,\text{L}.$$

Round only now to obtain the required integer 138. Using butane alone as the yield basis would overestimate the product because it ignores the oxygen shortage.

Step 3: Conclusion The water volume is $138.46\,\text{L}$, giving the nearest-integer answer $\boxed{138}$.

Quick Tip to Solve: Check oxygen demand before calculating combustion products. Because the product is liquid water, use its density rather than the molar volume of a gas.

73. The number of paramagnetic metal complex species among $[Co(NH_3)_6]^{3+}$, $[Co(C_2O_4)_3]^{3-}$, $[MnCl_6]^{3-}$, $[Mn(CN)_6]^{3-}$, $[CoF_6]^{3-}$, $[Fe(CN)_6]^{3-}$ and $[FeF_6]^{3-}$ with same number of unpaired electrons is _____. [4 marks]

Solution:

Concept:

- The oxidation state gives the metal-ion electron count: Co^{3+} is d^6, Mn^{3+} is d^4, and Fe^{3+} is d^5.
- In an octahedral complex, electrons occupy the lower t_{2g} and upper e_g orbitals. Weak fields favour high spin; a sufficiently strong field favours pairing. A species is paramagnetic when it has unpaired electrons.

Step 1: Assign the oxidation states Each ammine ligand is neutral, whereas chloride, cyanide and fluoride carry charge -1, and oxalate carries charge -2. Charge balance therefore gives the metal oxidation state $+3$ in all seven complexes. We must compare the unpaired-electron counts, rather than simply count all the paramagnetic species.

Step 2: Count unpaired electrons in each complex For the cobalt(III) ammine and oxalate complexes, the octahedral field gives the low-spin d^6 configuration $t_{2g}^6 e_g^0$, with no unpaired electrons. The fluoride complex is high-spin d^6, with $t_{2g}^4 e_g^2$, giving four unpaired electrons.

For manganese(III), chloride gives high-spin d^4: $t_{2g}^3 e_g^1$, with four unpaired electrons. Cyanide gives low-spin d^4: $t_{2g}^4 e_g^0$, with two unpaired electrons.

For iron(III), cyanide gives low-spin d^5: $t_{2g}^5 e_g^0$, with one unpaired electron. Fluoride gives high-spin d^5: $t_{2g}^3 e_g^2$, with five unpaired electrons.

Step 3: Select the repeated nonzero count The counts in the listed order are $0, 0, 4, 2, 4, 1, 5$. Although the first two complexes have the same zero count, they are diamagnetic and cannot enter the requested selection. The repeated nonzero count is four, occurring only for $[MnCl_6]^{3-}$ and $[CoF_6]^{3-}$.

Step 4: Conclusion Two paramagnetic complexes have the same number of unpaired electrons: $[MnCl_6]^{3-}$ and $[CoF_6]^{3-}$, each with four. The numerical answer is $\boxed{2}$.

Quick Tip to Solve: First list the unpaired-electron counts in the given order, then look for repeated nonzero values; a repeated zero describes diamagnetic complexes.

74. One litre buffer solution was prepared by adding 0.10 mol each of NH_3 and NH_4Cl in deionised water. The change in pH on addition of 0.05 mol of HCl to the above solution is _____ $\times 10^{-2}$. (Nearest integer)

Given : pK_b **of** $NH_3 = 4.745$ **and** $\log_{10} 3 = 0.477$ [4 marks]

Solution:

Concept:

- An ammonia/ammonium buffer obeys $pOH = pK_b + \log_{10}([NH_4^+]/[NH_3])$. At a common solution volume the concentration ratio equals the mole ratio.
- Added strong acid first reacts quantitatively with the weak base: $NH_3 + H^+ \longrightarrow NH_4^+$. Apply the buffer equation only after this neutralisation.

Step 1: Calculate the initial pH The solution initially contains 0.10 mol each of ammonia and ammonium chloride. Their ratio is unity, so the logarithmic term vanishes. Thus $\mathrm{pOH}_i = 4.745$. Using $\mathrm{pH} + \mathrm{pOH} = 14$, the initial pH is $\mathrm{pH}_i = 14 - 4.745 = 9.255$. The equal amounts do not make the solution neutral; its basicity is fixed by the ammonia equilibrium.

Step 2: Update the buffer composition after adding acid The 0.05 mol of hydrogen chloride supplies 0.05 mol of hydrogen ions, which consume the same amount of ammonia and form the same amount of ammonium ions:

$$n_f(\mathrm{NH_3}) = 0.10 - 0.05 = 0.05 \text{ mol}, \qquad n_f(\mathrm{NH_4^+}) = 0.10 + 0.05 = 0.15 \text{ mol}.$$

Both buffer components remain present, so the buffer equation still applies. There is no excess strong acid to use directly in a hydrogen-ion concentration calculation. Any common final volume cancels in their ratio.

Step 3: Calculate the pH decrease and the requested coefficient The final salt-to-base ratio is $0.15/0.05 = 3$. Therefore,

$$\mathrm{pOH}_f = 4.745 + \log_{10} 3 = 4.745 + 0.477 = 5.222,$$

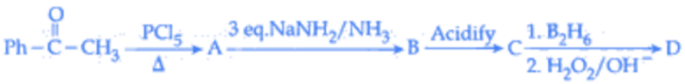
$$\mathrm{pH}_f = 14 - 5.222 = 8.778.$$

Thus $\mathrm{pH}_f - \mathrm{pH}_i = -0.477$: the pH decreases. Its decrease in the question's numerical format is $0.477 = 47.7 \times 10^{-2}$, and 47.7 rounds to 48.

Step 4: Conclusion The pH decreases from 9.255 to 8.778, a decrease of 0.477. The nearest-integer coefficient of 10^{-2} requested is $\boxed{48}$.

Quick Tip to Solve: For an initially equimolar buffer, the magnitude of its pH change is simply the logarithm of the final salt-to-base mole ratio after neutralisation.

75. Identify the structure of the final product (D) in the following sequence of the reactions :
Total number of $\mathrm{sp^2}$ hybridised carbon atoms in product D is _____. [4 marks]



Solution:

Concept:

- Phosphorus pentachloride replaces a ketone carbonyl oxygen by two chlorine atoms, producing a geminal dichloride. Strong base then removes two molecules of hydrogen chloride to form an alkyne.
- A terminal alkyne is deprotonated by excess sodium amide. Acidification restores its terminal hydrogen. Hydroboration followed by alkaline peroxide oxidation gives the anti-Markovnikov carbonyl product after enol tautomerisation.

Step 1: Identify the geminal dichloride A The starting compound is acetophenone, $C_6H_5COCH_3$. Treatment with PCl_5 replaces $C{=}O$ by CCl_2, while retaining the carbon skeleton:

$$C_6H_5COCH_3 \xrightarrow{PCl_5} C_6H_5CCl_2CH_3.$$

Thus A is $C_6H_5CCl_2CH_3$, not a ring-chlorinated compound.

Step 2: Follow the three equivalents of sodium amide Two equivalents of $NaNH_2$ remove two equivalents of HCl from the geminal dichloride, generating phenylacetylene. The third equivalent deprotonates that terminal alkyne, so B is the sodium acetylide, $C_6H_5C{\equiv}C^-\ Na^+$. Acidification then protonates the negatively charged terminal carbon, giving C as phenylacetylene, $C_6H_5C{\equiv}CH$. No carbon atom is added or removed in these steps; the two-carbon side chain remains attached to the phenyl ring.

Step 3: Identify D and count its trigonal carbon atoms Hydroboration places boron at the less substituted terminal carbon. Oxidation replaces the carbon-boron bond by a carbon-oxygen bond, giving the enol $C_6H_5CH{=}CHOH$. Keto-enol tautomerisation produces phenylacetaldehyde:

$$C_6H_5CH{=}CHOH \longrightarrow C_6H_5CH_2CHO.$$

In D, all six carbons of the benzene ring are sp^2-hybridised. The aldehyde carbon is also sp^2, because it has a carbonyl double bond and trigonal planar bonding. The intervening CH_2 carbon has four sigma bonds and is sp^3; it is not counted. The total is therefore $6 + 1 = 7$.

Step 4: Conclusion The final product D is phenylacetaldehyde, $C_6H_5CH_2CHO$. It contains $\boxed{7}$ sp^2-hybridised carbon atoms.

Quick Tip to Solve: Complete the reaction sequence before counting hybridisation: the terminal alkyne becomes an aldehyde, while the six aromatic ring carbons retain their hybridisation.