# JEE Main 2025 Paper 1 (B.E./B.Tech) Apr 8 Shift 2 Question Paper with Solutions PDF

## Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

**Time Allowed: 3 Hours** **Maximum Marks: 300**

### General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 8 April 2025, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, $-1$ for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, $-1$ for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

## Mathematics – Section A: Multiple Choice Questions (80 marks)

**1. Let the values of $\lambda$ for which the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-\lambda}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ be $\lambda_1$ and $\lambda_2$. Then the radius of the circle passing through the points $(0,0), (\lambda_1, \lambda_2)$ and $(\lambda_2, \lambda_1)$ is** [4 marks]

(A) 3

(B) $\frac{5\sqrt{2}}{3}$

(C) $\frac{\sqrt{2}}{3}$

(D) 4

**Correct Answer:** (B) $\frac{5\sqrt{2}}{3}$

**Solution:**

**Concept:**

- The shortest distance between two nonparallel lines through position vectors $\mathbf{a}_1, \mathbf{a}_2$, with directions $\mathbf{b}_1, \mathbf{b}_2$, is $d = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|/|\mathbf{b}_1 \times \mathbf{b}_2|$.
- A circle centre is equidistant from its three points. Interchanging the coordinates of two points gives a useful symmetry for finding the centre.

**Step 1:** Find the normal to the two directions The direction vectors are $(2,3,4)$ and $(3,4,5)$. Their cross product and its length are

$$\mathbf{n} = (-1, 2, -1), \qquad |\mathbf{n}| = \sqrt{6}.$$

Because this vector is nonzero, the shortest-distance formula applies.

**Step 2:** Determine the two values of the parameter Take points $(1,2,3)$ and $(\lambda,4,5)$ on the respective lines. Their displacement is $(\lambda - 1, 2, 2)$, so

$$d = \frac{|-(\lambda-1)+4-2|}{\sqrt{6}} = \frac{|3-\lambda|}{\sqrt{6}}.$$

The specified distance gives $|3-\lambda| = 1$, hence the two values are 2 and 4, in either order.

**Step 3:** Locate the circle centre The required points are $O = (0,0), U = (2,4), V = (4,2)$. The perpendicular bisector of $UV$ is $y = x$, so write the centre as $(t,t)$. Equal distances to $O$ and $U$ give

$$2t^2 = (t-2)^2 + (t-4)^2 = 2t^2 - 12t + 20,$$

$$t = \frac{5}{3}.$$

**Step 4:** Calculate and compare the radius The radius is the distance from the centre to the origin:

$$R = \sqrt{(5/3)^2 + (5/3)^2} = \frac{5\sqrt{2}}{3}.$$

The alternatives 3, $\sqrt{2}/3$ and 4 do not have the squared radius $50/9$ required by this centre.

**Step 5:** Conclusion The radius is $\boxed{5\sqrt{2}/3}$. Hence, the correct option is (B).

**Quick Tip to Solve:** After finding the two parameter values, use the symmetry of the swapped coordinates to put the circle centre on $y = x$.

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**2. Let $f(x) = x - 1$ and $g(x) = e^x$ for $x \in \mathbb{R}$. If $\frac{dy}{dx} = \left(e^{-2\sqrt{x}} g\left(f(f(x))\right) - \frac{y}{\sqrt{x}}\right)$, $y(0) = 0$, then $y(1)$ is** [4 marks]

(A) $\frac{1-e^3}{e^4}$
(B) $\frac{e-1}{e^4}$
(C) $\frac{1-e^2}{e^4}$
(D) $\frac{2e-1}{e^3}$

**Correct Answer:** (B) $\frac{e-1}{e^4}$

**Solution:**

**Concept:**

- A first-order linear equation $y' + P(x)y = Q(x)$ is solved by multiplying by the integrating factor $I(x) = e^{\int P(x)\,dx}$. The left side then becomes $(Iy)'$.
- Function composition is evaluated from the inside out. The initial condition at a singular endpoint is imposed by the right-hand limit of the solution.

**Step 1:** Evaluate the composed functions Since $f(x) = x - 1$, applying $f$ twice gives $f(f(x)) = x - 2$. Therefore $g(f(f(x))) = e^{x-2}$. For $x > 0$, rearrange the stated equation as

$$y' + \frac{y}{\sqrt{x}} = e^{x-2-2\sqrt{x}}.$$

The coefficient is singular at zero, so we first solve on the positive half-line.

**Step 2:** Multiply by the integrating factor Integrating the coefficient of $y$ gives $\int x^{-1/2}\,dx = 2\sqrt{x}$. Thus

$$I(x) = e^{2\sqrt{x}}, \qquad \frac{d}{dx}\left(ye^{2\sqrt{x}}\right) = e^{x-2}.$$

The exponential involving $\sqrt{x}$ cancels exactly on the right; it must not be retained a second time.

**Step 3:** Apply the endpoint condition Integrating once gives

$$ye^{2\sqrt{x}} = e^{x-2} + C.$$

As $x \to 0^+$, the given value $y(0) = 0$ gives $0 = e^{-2} + C$. Hence

$$y = e^{-2\sqrt{x}}e^{-2}(e^x - 1).$$

This formula has the required continuous limit zero at the initial endpoint.

**Step 4:** Evaluate at one and test the choices Substitute $\sqrt{1} = 1$:

$$y(1) = e^{-2}e^{-2}(e-1) = \frac{e-1}{e^4}.$$

This is positive, excluding the first and third choices, whose numerators are negative. The fourth choice has a different numerator and exponential factor and does not follow from the initial condition.

**Step 5:** Conclusion $\boxed{y(1) = (e-1)/e^4}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Evaluate the composition first; the integrating factor $e^{2\sqrt{x}}$ then cancels the radical exponential in one step.

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**3. If** $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots \infty = \frac{\pi^4}{90}$,
$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots \infty = \alpha$,
$\frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + \ldots \infty = \beta$,
**then** $\frac{\alpha}{\beta}$ **is equal to :** [4 marks]

(A) 23

(B) 14

(C) 18

(D) 15

**Correct Answer:** (D) 15

**Solution:**

**Concept:**

- An absolutely convergent positive series can be partitioned into its even-index and odd-index terms without changing its sum.
- Factoring $(2n)^4 = 16n^4$ shows that the even-denominator fourth-power series is $1/16$ of the full series.

**Step 1:** Name the full series Put

$$S = \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}.$$

Every positive integer is either even or odd, and these sets do not overlap. Therefore the two series specified in the question satisfy $\alpha + \beta = S$. No evaluation of the odd terms individually is necessary.

**Step 2:** Extract the even terms Writing each even positive integer as $2n$ gives

$$\beta = \sum_{n=1}^{\infty} \frac{1}{(2n)^4} = \frac{1}{16}\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{S}{16}.$$

The factor is $2^4$, rather than 2 or 4, because the whole denominator is raised to the fourth power.

**Step 3:** Obtain the odd terms by subtraction Use the disjoint partition of the full sum:

$$\alpha = S - \beta = S - \frac{S}{16} = \frac{15S}{16}.$$

Equivalently, the given value of $S$ gives $\beta = \pi^4/1440$ and $\alpha = \pi^4/96$. Both quantities are positive, so taking their ratio is valid.

**Step 4:** Form the requested ratio The common full-series factor cancels:

$$\frac{\alpha}{\beta} = \frac{15S/16}{S/16} = 15.$$

As a check, the even terms account for one part out of sixteen of the total and the odd terms account for the other fifteen parts. Thus the ratio cannot be 23, 14 or 18; only 15 respects both the scaling relation and $\alpha + \beta = S$.

**Step 5:** Conclusion $\boxed{\alpha/\beta = 15}$. Hence, the correct option is (D).

**Quick Tip to Solve:** For reciprocal $p$-th powers, the even terms are $2^{-p}$ of the total; consequently odd sum divided by even sum is $2^p - 1$.

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**4. Let *a* be the length of a side of a square OABC with O being the origin. Its side OA makes an acute angle $\alpha$ with the positive x-axis and the equations of its diagonals are $(\sqrt{3}+1)x + (\sqrt{3}-1)y = 0$ and $(\sqrt{3}-1)x - (\sqrt{3}+1)y + 8\sqrt{3} = 0$. Then $a^2$ is equal to :** [4 marks]

(A) 48

(B) 16

(C) 24

(D) 32

**Correct Answer:** (A) 48

**Solution:**

**Concept:**

- The diagonals of a square bisect each other at right angles. If the side is $a$, each diagonal is $a\sqrt{2}$, so a vertex is $a/\sqrt{2}$ from the centre.
- The perpendicular distance from $(x_0, y_0)$ to $Ax + By + C = 0$ is $|Ax_0 + By_0 + C|/\sqrt{A^2 + B^2}$.

**Step 1:** Identify the diagonal through the origin The line

$$(\sqrt{3}+1)x + (\sqrt{3}-1)y = 0$$

passes through $O$, so it is the diagonal $OB$. The other diagonal is $AC$. Their normals have zero dot product, confirming the perpendicular relation expected in a square. Let their intersection, the square centre, be $M$.

**Step 2:** Relate the distance to the side Since $OM$ lies along $OB$ and $OB \perp AC$, the segment $OM$ is precisely the perpendicular from $O$ to the line containing $AC$. Also, the diagonals bisect each other,

giving

$$OM = \frac{OB}{2} = \frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}}.$$

Thus the point-line distance supplies half a diagonal, not a side.

**Step 3:** Calculate the perpendicular distance For the second diagonal,

$$(\sqrt{3}-1)x - (\sqrt{3}+1)y + 8\sqrt{3} = 0,$$

the distance from the origin is

$$OM = \frac{8\sqrt{3}}{\sqrt{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2}} = \frac{8\sqrt{3}}{\sqrt{8}} = 2\sqrt{6}.$$

The cross terms cancel in the sum of the two squares.

**Step 4:** Recover the squared side length Squaring $a/\sqrt{2} = 2\sqrt{6}$ gives

$$\frac{a^2}{2} = 24, \qquad a^2 = 48.$$

The value 24 is the squared half-diagonal, rather than the squared side. The alternatives 16 and 32 also fail the relation $OM^2 = a^2/2$. The acute-angle description selects the square orientation and does not change this distance calculation.

**Step 5:** Conclusion $\boxed{a^2 = 48}$. Hence, the correct option is (A).

**Quick Tip to Solve:** When one square diagonal passes through the origin, its distance to the other diagonal equals half the diagonal length.

**5. A line passing through the point *P*($a$, *0*) makes an acute angle $\alpha$ with the positive x-axis. Let this line be rotated about the point *P* through an angle $\frac{\alpha}{2}$ in the clockwise direction. If in the new position, the slope of the line is $2-\sqrt{3}$ and its distance from the origin is $\frac{1}{\sqrt{2}}$, then the value of $3a^2\tan^2\alpha - 2\sqrt{3}$ is :** [4 marks]

(A) 8

(B) 4

(C) 5

(D) 6

**Correct Answer:** (B) 4

**Solution:**

**Concept:**

- A line making inclination $\theta$ with the positive $x$-axis has slope $\tan\theta$. A clockwise rotation decreases that inclination.
- A line through $(a, 0)$ with inclination $\theta$ is at perpendicular distance $|a\sin\theta|$ from the origin.

**Step 1:** Find the original inclination The original angle is acute and is reduced by $\alpha/2$, so the new inclination is $\alpha/2$, between zero and $45^\circ$. Since

$$\tan(\alpha/2) = 2 - \sqrt{3} = \tan 15^\circ,$$

this interval fixes $\alpha/2 = 15^\circ$ and hence $\alpha = 30^\circ$. There is no additional periodic tangent solution in the permitted interval.

**Step 2:** Use the distance in the rotated position The new line still passes through $P = (a, 0)$ because the rotation is about $P$. If its slope is $m$, its equation is $mx - y - ma = 0$. Thus

$$d = \frac{|ma|}{\sqrt{1+m^2}} = |a| \sin 15^\circ = \frac{1}{\sqrt{2}}.$$

Squaring removes the irrelevant sign of the intercept $a$.

**Step 3:** Determine the squared intercept Using the half-angle identity gives

$$\sin^2 15^\circ = \frac{1 - \cos 30^\circ}{2} = \frac{2 - \sqrt{3}}{4}.$$

Consequently

$$a^2 \frac{2 - \sqrt{3}}{4} = \frac{1}{2}, \qquad a^2 = \frac{2}{2 - \sqrt{3}} = 4 + 2\sqrt{3}.$$

Rationalising uses $(2 - \sqrt{3})(2 + \sqrt{3}) = 1$.

**Step 4:** Evaluate the complete expression Since $\tan^2 30^\circ = 1/3$,

$$3a^2 \tan^2 \alpha - 2\sqrt{3} = a^2 - 2\sqrt{3} = (4 + 2\sqrt{3}) - 2\sqrt{3} = 4.$$

The radical terms cancel exactly. The options 8, 5 and 6 would require different values of $a^2$ and contradict the stated perpendicular distance.

**Step 5:** Conclusion The requested value is $\boxed{4}$. Hence, the correct option is (B).

**Quick Tip to Solve:** After a clockwise rotation by half the original acute inclination, the new inclination is simply its half; use the new angle in the distance formula.

**6. Let $\alpha$ be a solution of $x^2+x+1=0$, and for some $a$ and $b$ in $R$, $\begin{bmatrix}4 & a & b\end{bmatrix}\begin{bmatrix}1 & 16 & 13\\ -1 & -1 & 2\\ -2 & -14 & -8\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\end{bmatrix}$. If $\frac{4}{\alpha^4}+\frac{m}{\alpha^a}+\frac{n}{\alpha^b}=3$, then $m+n$ is equal to _____** [4 marks]

(A) 11

(B) 3

(C) 8

(D) 7

**Correct Answer:** (A) 11

**Solution:**

**Concept:**

- Row-by-matrix multiplication equates each resulting component to zero, producing simultaneous linear equations for the row entries.
- A root $\alpha$ of $x^2+x+1=0$ is a nonreal cube root of unity: $\alpha^3=1$, $\alpha\neq 1$, and $1+\alpha+\alpha^2=0$. Powers can therefore be reduced modulo three.

**Step 1:** Solve for the row entries The first two components of the given matrix equation give

$$4-a-2b=0, \qquad 64-a-14b=0.$$

Subtracting the equivalent equations $a+2b=4$ and $a+14b=64$ gives $12b=60$, so $b=5$ and $a=-6$. The third component checks these values: $52+2a-8b=52-12-40=0$.

**Step 2:** Reduce all powers of the root Because $\alpha^3=1$, we have $\alpha^4=\alpha$, $\alpha^{-6}=1$, and $\alpha^5=\alpha^2$. In particular, the denominator with the negative exponent must be handled carefully:

$$\frac{4}{\alpha^4}=4\alpha^2, \qquad \frac{m}{\alpha^a}=\frac{m}{\alpha^{-6}}=m, \qquad \frac{n}{\alpha^b}=\frac{n}{\alpha^5}=n\alpha.$$

**Step 3:** Compare the independent real coefficients The stated identity becomes

$$4\alpha^2+n\alpha+m=3.$$

Replace $\alpha^2$ by $-1-\alpha$ to obtain

$$(n-4)\alpha+(m-7)=0.$$

For real coefficients $m,n$, the imaginary part vanishes only if $n=4$, since $\alpha$ is nonreal. The real part then gives $m=7$.

**Step 4:** Add and verify the values We obtain $m+n=7+4=11$. Direct substitution also gives

$$4\alpha^2+7+4\alpha=4(\alpha^2+\alpha)+7=-4+7=3.$$

Thus both the matrix equation and the root identity hold. The alternatives 3, 8 and 7 do not equal the sum of the coefficients determined by these independent conditions.

**Step 5:** Conclusion $\boxed{m+n=11}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Solve for $a,b$ before reducing exponents modulo three; division by $\alpha^{-6}$ leaves the coefficient unchanged.

**7. Let the ellipse $3x^2 + py^2 = 4$ pass through the centre $C$ of the circle $x^2 + y^2 - 2x - 4y - 11 = 0$ of radius $r$. Let $f_1, f_2$ be the focal distances of the point $C$ on the ellipse. Then $6f_1f_2 - r$ is equal to** [4 marks]

(A) 78

(B) 68

(C) 70

(D) 74

**Correct Answer:** (C) 70

**Solution:**

**Concept:**

- Complete squares to read the circle centre and radius. For an ellipse with vertical major axis, write $x^2/b^2 + y^2/a^2 = 1$, where $a > b > 0$.
- The focal distances of a point with vertical coordinate $y$ are $a + ey$ and $a - ey$, where $e^2 = 1 - b^2/a^2$. Their product is $a^2 - e^2y^2$.

**Step 1:** Find the circle data and the ellipse parameter The circle equation becomes

$$(x - 1)^2 + (y - 2)^2 = 16.$$

Hence its centre is $C = (1, 2)$ and its radius is $r = 4$. Since this centre lies on the given ellipse, substitute its coordinates into that equation:

$$3(1)^2 + p(2)^2 = 4 \quad \Rightarrow \quad p = \frac{1}{4}.$$

The parameter is positive, as required for a real ellipse.

**Step 2:** Identify the major axis and the eccentricity The resulting equation is

$$3x^2 + \frac{y^2}{4} = 4 \quad \Rightarrow \quad \frac{x^2}{4/3} + \frac{y^2}{16} = 1.$$

The larger denominator belongs to the vertical coordinate, so $a^2 = 16$, $b^2 = 4/3$, and

$$e^2 = 1 - \frac{4/3}{16} = \frac{11}{12}.$$

Interchanging these axes would give an incorrect focal-distance calculation.

**Step 3:** Calculate the product and compare the choices At $C$, the vertical coordinate is $y = 2$. Thus

$$f_1f_2 = a^2 - e^2y^2 = 16 - \frac{11}{12} \cdot 4 = \frac{37}{3}.$$

Therefore

$$6f_1f_2 - r = 6 \cdot \frac{37}{3} - 4 = 74 - 4 = 70.$$

The value 74 omits the subtraction of the circle radius; 78 and 68 do not follow from the correct product and radius.

**Step 4:** Conclusion The required value is 70. Hence, the correct option is (C).

**Quick Tip to Solve:** First locate the larger denominator of the ellipse: the focal-distance formula uses the coordinate along that major axis.

**8. The sum of the squares of the roots of $|x-2|^2+|x-2|-2=0$ and the squares of the roots of $x^2-2|x-3|-5=0$, is** [4 marks]

(A) 24

(B) 26

(C) 36

(D) 30

**Correct Answer:** (C) 36

**Solution:**

**Concept:**

- For a real variable, $|x-2| \geq 0$, so substituting a nonnegative auxiliary variable can turn an absolute-value equation into a quadratic.
- A root obtained after replacing an absolute value by a signed expression must satisfy the interval used. For a quadratic with roots $u, v$, $u^2+v^2=(u+v)^2-2uv$.

**Step 1:** Solve the first equation Put $t=|x-2|$, with $t \geq 0$. The equation becomes

$$t^2+t-2=(t+2)(t-1)=0.$$

The root $t=-2$ is inadmissible because an absolute value cannot be negative. Hence $t=1$, so

$$|x-2|=1 \quad \Rightarrow \quad x=1,3.$$

Their squares contribute $1^2+3^2=10$ to the requested sum.

**Step 2:** Check both intervals for the second equation When $x \geq 3$, the equation reduces to

$$x^2-2(x-3)-5=(x-1)^2=0.$$

Its candidate $x=1$ violates $x \geq 3$, so this interval supplies no root. When $x<3$, use $|x-3|=3-x$, giving

$$x^2-2(3-x)-5=x^2+2x-11=0.$$

Its roots are $-1 \pm 2\sqrt{3}$. Both are below 3, since $2\sqrt{3}<4$, so both are valid.

**Step 3:** Add the squares of all valid roots For the second quadratic, the sum of roots is $-2$ and their product is $-11$. Their squared sum is therefore

$$(-2)^2-2(-11)=26.$$

Combining the two equations gives

$$10 + 26 = 36.$$

The option 26 counts only the second equation. The values 24 and 30 are incompatible with the complete valid-root list; an extraneous root from the wrong interval must not be added.

**Step 4:** Conclusion The sum of the squares of the roots is 36. Hence, the correct option is (C).

**Quick Tip to Solve:** After every absolute-value case split, check each candidate against that case before using Vieta formulas on the valid roots.

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**9. The value of** $\cot^{-1}\left(\frac{\sqrt{1+\tan^2(2)}-1}{\tan(2)}\right) - \cot^{-1}\left(\frac{\sqrt{1+\tan^2\left(\frac{1}{2}\right)}+1}{\tan\left(\frac{1}{2}\right)}\right)$ **is equal to** [4 marks]

(A) $\pi - \frac{3}{2}$
(B) $\pi + \frac{5}{2}$
(C) $\pi - \frac{5}{4}$
(D) $\pi + \frac{3}{2}$

**Correct Answer:** (C) $\pi - \frac{5}{4}$

**Solution:**

**Concept:**

- The principal range of $\cot^{-1}$ is $(0, \pi)$. Simplifying its argument does not remove the need to put the angle in this range.
- Use $\sqrt{1+\tan^2 t} = |\sec t|$, together with $(1+\cos t)/\sin t = \cot(t/2)$. All given angles are in radians.

**Step 1:** Simplify the first argument with its correct sign The angle 2 lies in the second quadrant because $\pi/2 < 2 < \pi$. Consequently $\cos 2 < 0$, so $\sqrt{1+\tan^2 2} = -\sec 2$, rather than $\sec 2$. Thus

$$\frac{\sqrt{1+\tan^2 2}-1}{\tan 2} = \frac{-\sec 2 - 1}{\tan 2} = -\frac{1+\cos 2}{\sin 2} = -\cot 1.$$

This sign is crucial for selecting the principal inverse value.

**Step 2:** Evaluate the two principal inverse values Since $0 < 1 < \pi$, the angle $\pi - 1$ belongs to the principal interval and satisfies $\cot(\pi - 1) = -\cot 1$. Therefore the first term equals $\pi - 1$. For the second term, $0 < 1/2 < \pi/2$, hence $\sec(1/2) > 0$. We obtain

$$\frac{\sqrt{1+\tan^2(1/2)}+1}{\tan(1/2)} = \frac{1+\cos(1/2)}{\sin(1/2)} = \cot\frac{1}{4}.$$

Because $1/4 \in (0, \pi)$, its inverse cotangent is exactly $1/4$.

**Step 3:** Subtract and test the alternatives The requested difference is

$$(\pi - 1) - \frac{1}{4} = \pi - \frac{5}{4}.$$

Options containing $\pi$ plus a positive number cannot be a difference of two numbers in $(0, \pi)$, since

such a difference is smaller than $\pi$. The remaining alternative $\pi - 3/2$ uses the wrong half-angle contribution; the second principal value is $1/4$, not $1/2$.

**Step 4:** Conclusion The value is $\pi - \frac{5}{4}$. Hence, the correct option is (C).

**Quick Tip to Solve:** Before applying a half-angle identity, replace the square root of secant squared by the absolute value of secant.

**10. Let** $A = \left\{\theta \in [0, 2\pi] \ : 1 + 10\,\mathrm{Re}\left(\frac{2\cos\theta + i\sin\theta}{\cos\theta - 3i\sin\theta}\right) = 0\right\}$**. Then** $\sum_{\theta \in A} \theta^2$ **is equal to** [4 marks]

(A) $\frac{21}{4}\pi^2$

(B) $6\pi^2$

(C) $\frac{27}{4}\pi^2$

(D) $8\pi^2$

**Correct Answer:** (A) $\frac{21}{4}\pi^2$

**Solution:**

**Concept:**

- Multiply a complex quotient by the conjugate of its denominator to isolate its real part. The real denominator is the squared modulus.
- After obtaining a trigonometric equation, list every solution in the specified closed interval before summing the required powers.

**Step 1:** Extract the real part Write $c = \cos\theta$ and $s = \sin\theta$. The denominator $c - 3is$ never vanishes for real $\theta$, because $c$ and $s$ cannot both be zero. Multiplication by its conjugate gives

$$\frac{2c + is}{c - 3is} = \frac{(2c + is)(c + 3is)}{c^2 + 9s^2} = \frac{2c^2 - 3s^2 + 7ics}{c^2 + 9s^2}.$$

Hence the real part is $(2c^2 - 3s^2)/(c^2 + 9s^2)$.

**Step 2:** Solve the defining equation for the set The denominator is strictly positive, so multiplying by it introduces no extraneous solutions. The condition becomes

$$c^2 + 9s^2 + 10(2c^2 - 3s^2) = 0,$$

or

$$21c^2 - 21s^2 = 0 \quad \Rightarrow \quad \cos 2\theta = 0.$$

Within $[0, 2\pi]$, the complete solution list is

$$A = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}.$$

Neither endpoint satisfies the equation, so no boundary term needs to be added.

**Step 3:** Sum the squared angles The angles themselves, rather than their sine or cosine values, must

be squared. Thus

$$\sum_{\theta\in A}\theta^2=\frac{\pi^2}{16}(1^2+3^2+5^2+7^2)=\frac{84\pi^2}{16}=\frac{21\pi^2}{4}.$$

All four permitted angles have been counted once. The other coefficients $6$, $27/4$, and $8$ fail this direct sum and can arise only from an incorrect root list or arithmetic.

**Step 4:** Conclusion The required sum is $\frac{21}{4}\pi^2$. Hence, the correct option is (A).

**Quick Tip to Solve:** Keep the squared modulus as a positive denominator; then the real-part condition reduces directly to a simple double-angle equation.

**11. Given below are two statements:**

**Statement I:** $\lim\limits_{x\to0}\left(\frac{\tan^{-1}x+\log_e\sqrt{\frac{1+x}{1-x}}-2x}{x^5}\right)=\frac{2}{5}$

**Statement II:** $\lim\limits_{x\to1}\left(x^{\frac{2}{1-x}}\right)=\frac{1}{e^2}$

**In the light of the above statements, choose the correct answer from the options given below:**

[4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are false

(C) Both Statement I and Statement II are true

(D) Statement I is true but Statement II is false

**Correct Answer:** (C) Both Statement I and Statement II are true

**Solution:**

**Concept:**

- Near zero, the Maclaurin expansions of inverse tangent and logarithm identify the first surviving power after cancellation.
- For an indeterminate power, take its logarithm first. The standard limit $\log x/(x-1)\to1$ as $x\to1$ then determines the exponent limit.

**Step 1:** Evaluate Statement I through the fifth power For sufficiently small $|x|<1$, both the logarithm and the square root are real and well defined. The needed expansions are

$$\tan^{-1}x=x-\frac{x^3}{3}+\frac{x^5}{5}+O(x^7),$$

and

$$\log\sqrt{\frac{1+x}{1-x}}=\frac{1}{2}[\log(1+x)-\log(1-x)]=x+\frac{x^3}{3}+\frac{x^5}{5}+O(x^7).$$

Adding them and subtracting $2x$ cancels the linear and cubic terms. The numerator is $2x^5/5+O(x^7)$, so division by $x^5$ gives the limit $2/5$. Statement I is true.

**Step 2:** Evaluate Statement II by taking logarithms Let $y = x^{2/(1-x)}$ for $x > 0$, $x \neq 1$. Then

$$\log y = \frac{2\log x}{1-x} = -2\frac{\log x}{x-1}.$$

As $x \to 1$, the ratio on the right tends to 1, so $\log y \to -2$. Continuity of the exponential function implies

$$y \to e^{-2} = \frac{1}{e^2}.$$

The negative sign comes from $1 - x = -(x-1)$; losing it would produce the reciprocal answer. Statement II is true.

**Step 3:** Compare the truth values with the options Both limits equal the values asserted. Options (A) and (D) each incorrectly declare one statement false, while option (B) declares both false. Neither statement requires an additional one-sided restriction beyond the local domains already used: the small interval around zero and the positive interval around one suffice.

**Step 4:** Conclusion Both Statement I and Statement II are true. Hence, the correct option is (C).

**Quick Tip to Solve:** In the first limit, retain terms through the fifth power: the cubic terms cancel and cannot determine the answer.

---

**12. The integral $\int_{-1}^{\frac{3}{2}} \left(|\pi^2 x \sin(\pi x)|\right) dx$ is equal to:** [4 marks]

(A) $2 + 3\pi$

(B) $4 + \pi$

(C) $1 + 3\pi$

(D) $3 + 2\pi$

**Correct Answer:** (C) $1 + 3\pi$

**Solution:**

**Concept:**

- An absolute-value integral must be split where the expression inside it changes sign. The product $x\sin(\pi x)$ is even.
- Integration by parts gives an antiderivative $F(x) = -\pi x\cos(\pi x) + \sin(\pi x)$ of $\pi^2 x\sin(\pi x)$.

**Step 1:** Locate the sign changes and exploit symmetry On $[0,1]$, both $x$ and $\sin(\pi x)$ are nonnegative. On $[1, 3/2]$, the sine is nonpositive while $x$ is positive, so their product is negative. On $[-1, 0]$, both factors are nonpositive and their product is nonnegative. Since the product is even, the contributions over $[-1, 0]$ and $[0, 1]$ are equal. Therefore

$$I = 2\int_0^1 \pi^2 x\sin(\pi x)\,dx - \int_1^{3/2} \pi^2 x\sin(\pi x)\,dx.$$

The minus sign in the last term implements the absolute value.

**Step 2:** Evaluate the antiderivative at the boundaries Integrating by parts with $u = x$ and $dv = \pi^2 \sin(\pi x)\, dx$ yields

$$F(x) = -\pi x \cos(\pi x) + \sin(\pi x).$$

Its derivative is $\pi^2 x \sin(\pi x)$, because the two terms involving $\pi \cos(\pi x)$ cancel. At the required endpoints,

$$F(0) = 0, \qquad F(1) = \pi, \qquad F(3/2) = -1.$$

The final value uses $\cos(3\pi/2) = 0$ and $\sin(3\pi/2) = -1$.

**Step 3:** Combine the positive contributions Substituting the endpoint values gives

$$I = 2(\pi - 0) - (-1 - \pi) = 1 + 3\pi.$$

Equivalently, the symmetric central interval contributes $2\pi$ and the last interval contributes $1 + \pi$. The alternatives $2 + 3\pi$, $4 + \pi$, and $3 + 2\pi$ disagree with these separate contributions; the endpoint sine term must be included exactly once.

**Step 4:** Conclusion The integral equals $1 + 3\pi$. Hence, the correct option is (C).

**Quick Tip to Solve:** Find zeros of the sine before integrating an absolute value, then use evenness only on the symmetric portion of the interval.

---

**13. Let** $A = \begin{bmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{bmatrix}$.

**If** $\det(\textbf{adj}(\textbf{adj}(3A))) = 2^m \cdot 3^n$, $m, n \in \mathbb{N}$, **then** $m + n$ **is equal to** [4 marks]

(A) 22

(B) 20

(C) 24

(D) 26

**Correct Answer:** (C) 24

**Solution:**

**Concept:**

- For a nonsingular matrix $M$ of order $r$, the adjugate identity gives $\det(\text{adj}\, M) = (\det M)^{r-1}$. Here the order is three, so each adjugate squares the determinant.
- Adding a multiple of another column does not change a determinant. Use column operations to remove the parameters before expanding; scalar multiplication satisfies $\det(kA) = k^3 \det A$ for a three by three matrix.

**Step 1:** Remove the parameters from the determinant First replace $C_2$ by $C_2 - \frac{p}{2}C_1$, giving the column

$(2, 6, 12)^T$. Using this updated second column, replace $C_3$ by $C_3 - \frac{p}{2}C_2 - \frac{q}{2}C_1$. Thus

$$\det A = \begin{vmatrix} 2 & 2 & 2 \\ 4 & 6 & 8 \\ 6 & 12 & 20 \end{vmatrix} = 2(120 - 96) - 2(80 - 48) + 2(48 - 36) = 8.$$

Both operations preserve the determinant, and the cancellation shows why the result is independent of the real parameters $p$ and $q$. In particular, the determinant is nonzero, so the adjugate formula applies without any singularity concern.

**Step 2:** Apply the adjugate identity twice Put $M = 3A$. Multiplying each of the three rows by three multiplies the determinant by $3^3$, not merely by three. Consequently,

$$\det M = 3^3 \cdot 8 = 2^3 \cdot 3^3, \qquad \det(\text{adj}(\text{adj } M)) = (\det M)^4.$$

Therefore the required determinant equals $2^{12}3^{12}$. The first adjugate doubles each prime exponent, and the second doubles it again. Hence $m = 12$ and $n = 12$; the other proposed sums, 22, 20 and 26, cannot follow from these exponents.

**Step 3:** Conclusion The exponent sum is $m + n = 12 + 12 = \boxed{24}$. Hence, the correct option is (C).

**Quick Tip to Solve:** For a three by three matrix, two successive adjugates raise its determinant to the fourth power. Simplify the original determinant before applying this power.

**14. Let A = {0, 1, 2, 3, 4, 5}. Let R be a relation on A defined by (x, y) $\in$ R if and only if max{x, y} $\in$ {3, 4}. Then among the statements**

**($S_1$): The number of elements in R is 18, and**

**($S_2$): The relation R is symmetric but neither reflexive nor transitive** [4 marks]

(A) both are false

(B) only ($S_1$) is true

(C) only ($S_2$) is true

(D) both are true

**Correct Answer:** (C) only ($S_2$) is true

**Solution:**

**Concept:**

- A relation consists of ordered pairs. Count pairs with a specified maximum by subtracting the pairs whose coordinates are both smaller: the number with maximum $k$ is $(k + 1)^2 - k^2$ in this set.
- Reflexivity requires every diagonal pair; symmetry requires the reverse of each pair; transitivity requires $(x, z)$ whenever both $(x, y)$ and $(y, z)$ belong to the relation. A single counterexample disproves a universal property.

**Step 1:** Count the ordered pairs If the maximum is three, both coordinates lie in $\{0, 1, 2, 3\}$, but they cannot both lie in $\{0, 1, 2\}$. This gives $4^2 - 3^2 = 7$ ordered pairs. Similarly, maximum four gives

$5^2 - 4^2 = 9$ pairs. These cases are disjoint, while any pair involving five has maximum five and is excluded. Thus

$$|R| = (16 - 9) + (25 - 16) = 7 + 9 = 16.$$

Equivalently, count all pairs from $\{0, 1, 2, 3, 4\}$ and remove the nine pairs from $\{0, 1, 2\}$. Both methods give sixteen, rather than eighteen, so statement $S_1$ is false.

**Step 2:** Test the three relation properties The maximum is unchanged on reversing coordinates: $\max\{x, y\} = \max\{y, x\}$. Thus every allowed pair has its reverse, and $R$ is symmetric. It is not reflexive on the given set because, for example, $(0, 0) \notin R$; only $(3, 3)$ and $(4, 4)$ satisfy the diagonal condition. For transitivity, take $(0, 3) \in R$ and $(3, 0) \in R$. Transitivity would then require $(0, 0) \in R$, which is false. Therefore $R$ is not transitive and $S_2$ is true. Options asserting both statements true, both false, or only the first true each contradict one of these checks.

**Step 3:** Conclusion Only statement $S_2$ is true: $R$ is symmetric, neither reflexive nor transitive, and has 16 elements. Hence, the correct option is (C).

**Quick Tip to Solve:** For a condition on the maximum, use square differences to count ordered pairs. To disprove transitivity here, reverse a pair whose first coordinate has no diagonal pair.

---

**15. Let *f(x)* be a positive function and $I_1 = \int_{-\frac{1}{2}}^{1} 2x\, f(2x(1-2x))\, dx$ and $I_2 = \int_{-1}^{2} f(x(1-x))\, dx$. Then the value of $\frac{I_2}{I_1}$ is equal to _____** [4 marks]

(A) 12

(B) 9

(C) 6

(D) 4

**Correct Answer:** (D) 4

**Solution:**

**Concept:**

- Under a linear substitution, transform both limits and the differential. The substitution $t = 2x$ makes the arguments of the two given integrals identical.
- For $g(t) = f(t(1-t))$, the equality $g(1-t) = g(t)$ follows from $(1-t)t = t(1-t)$. The interval $[-1, 2]$ is also unchanged by reflection about $t = 1/2$.

**Step 1:** Transform the first integral Let $t = 2x$, so $dx = dt/2$. The lower limit $x = -1/2$ becomes $t = -1$, and the upper limit $x = 1$ becomes $t = 2$. Therefore

$$I_1 = \frac{1}{2} \int_{-1}^{2} t f(t(1-t))\, dt.$$

Write $J = \int_{-1}^{2} t g(t)\, dt$, where $g(t) = f(t(1-t))$. Then $I_1 = J/2$, whereas $I_2 = \int_{-1}^{2} g(t)\, dt$. Keeping the factor one half from the differential is essential; the weight $2x$ becomes $t$, not $2t$.

**Step 2:** Reflect and combine the weighted integrals In $J$, replace $t$ by $1-u$. The reversed limits and the negative differential cancel. By the symmetry of $g$,

$$J = \int_{-1}^{2} (1-u)g(u)\,du.$$

Add this to the original expression for $J$, renaming the dummy variable consistently:

$$2J = \int_{-1}^{2} [t + (1-t)]g(t)\,dt = I_2.$$

Thus $J = I_2/2$ and $I_1 = I_2/4$. Positivity of $f$ ensures $I_2 > 0$ for the stated integrals, so division is legitimate and the ratio is four. The alternatives twelve, nine and six fail this exact symmetry identity; they cannot be obtained by a valid change of variables.

**Step 3:** Conclusion We obtain $I_1 = I_2/4$, hence $\boxed{I_2/I_1 = 4}$. Hence, the correct option is (D).

**Quick Tip to Solve:** When an integrand contains $t(1-t)$, reflect with $t \mapsto 1-t$. On an invariant interval, the weighted integral of $t$ equals half the unweighted integral.

**16. There are 12 points in a plane, no three of which are in the same straight line, except 5 points which are collinear. Then the total number of triangles that can be formed with the vertices at any three of these 12 points is** [4 marks]

(A) 230

(B) 210

(C) 200

(D) 220

**Correct Answer:** (B) 210

**Solution:**

**Concept:**

- Choosing the vertices of a triangle is an unordered selection of three distinct points. Thus the initial count from twelve points is the binomial coefficient $\binom{12}{3}$, without any additional factor for rearranging vertices.
- Three distinct points form a triangle exactly when they are not collinear. Subtract all selections made entirely from the exceptional five collinear points; the problem excludes every other collinear triple.

**Step 1:** Count all selections of three points Ignore collinearity temporarily and select any three of the twelve points. The number of unordered triples is

$$\binom{12}{3} = \frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1} = 220.$$

Each triple is counted once, since naming the same three vertices in another order does not create

another triangle. This total includes degenerate selections lying on the given straight line, so 220 is only an intermediate total. The proposed count 230 even exceeds the number of possible triples and is therefore impossible.

**Step 2:** Remove exactly the collinear selections Any choice of three of the five points on the exceptional line gives zero area rather than a triangle. The number of these invalid triples is

$$\binom{5}{3} = \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 10.$$

No additional invalid triple is possible because the statement explicitly excludes other sets of three collinear points. Hence the valid count is $220 - 10 = 210$. As an independent partition, the other seven points yield $\binom{7}{3} = 35$ triangles with no vertex on the line; one vertex on it gives $5\binom{7}{2} = 105$; two vertices on it give $\binom{5}{2} \cdot 7 = 70$. Their sum is again $35 + 105 + 70 = 210$. Subtracting twenty instead of ten would give 200, but would incorrectly remove additional nondegenerate triples.

**Step 3:** Conclusion The number of triangles is $\binom{12}{3} - \binom{5}{3} = \boxed{210}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Start with all unordered triples and subtract triples wholly on each exceptional line. Two vertices on a line and a third vertex off it still form a valid triangle.

---

**17. If *A* and *B* are two events such that $P(A) = 0.7$, $P(B) = 0.4$ and $P(A \cap \overline{B}) = 0.5$, where $\overline{B}$ denotes the complement of *B*, then $P\left(B \mid (A \cup \overline{B})\right)$ is equal to** [4 marks]

(A) $\frac{1}{3}$
(B) $\frac{1}{2}$
(C) $\frac{1}{4}$
(D) $\frac{1}{6}$

**Correct Answer:** (C) $\frac{1}{4}$

**Solution:**

**Concept:**

- For an event $E$ of positive probability, $P(B \mid E) = P(B \cap E)/P(E)$. The conditioning event changes the denominator and must be identified before substituting numbers.
- The disjoint events $A \cap B$ and $A \cap \overline{B}$ partition $A$. Also, intersecting $B$ with $A \cup \overline{B}$ leaves only $A \cap B$, since $B \cap \overline{B}$ is empty.

**Step 1:** Find the intersection probability Split $A$ according to whether $B$ occurs. From the supplied values,

$$P(A \cap B) = P(A) - P(A \cap \overline{B}) = 0.7 - 0.5 = 0.2.$$

Also $P(\overline{B}) = 1 - P(B) = 0.6$. For a consistency check, the four disjoint regions have probabilities $P(A \cap B) = 0.2$, $P(A \cap \overline{B}) = 0.5$, $P(\overline{A} \cap B) = 0.2$, and $P(\overline{A} \cap \overline{B}) = 0.1$. They are nonnegative and sum to one, confirming that the data are compatible.

**Step 2:** Evaluate the conditional probability Set $E = A \cup \overline{B}$. Its probability follows from inclusion

and exclusion:

$$P(E) = P(A) + P(\overline{B}) - P(A \cap \overline{B}) = 0.7 + 0.6 - 0.5 = 0.8.$$

By distributivity, $B \cap E = (B \cap A) \cup (B \cap \overline{B}) = A \cap B$. Thus

$$P(B \mid E) = \frac{0.2}{0.8} = \frac{1}{4}.$$

The denominator is neither $P(B) = 0.4$ nor $P(\overline{B}) = 0.6$: using those would produce the tempting alternatives one half and one third. The remaining alternative one sixth likewise fails the required numerator-to-conditioning-event ratio. The requested probability is dimensionless.

**Step 3:** Conclusion $P\big(B \mid (A \cup \overline{B})\big) = \boxed{\frac{1}{4}}$. Hence, the correct option is (C).

**Quick Tip to Solve:** For a conditional probability, simplify the intersection in the numerator using set identities first; then calculate the probability of the exact event written after the conditioning bar.

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**18. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$. Let $\hat{c}$ be a unit vector in the plane of the vectors $\vec{a}$ and $\vec{b}$ and be perpendicular to $\vec{a}$. Then such a vector $\hat{c}$ is:** [4 marks]

(A) $\frac{1}{\sqrt{2}}(-\hat{i} + \hat{k})$

(B) $\frac{1}{\sqrt{5}}(\hat{j} - 2\hat{k})$

(C) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$

(D) $\frac{1}{\sqrt{3}}(-\hat{i} + \hat{j} - \hat{k})$

**Correct Answer:** (A) $\frac{1}{\sqrt{2}}(-\hat{i} + \hat{k})$

**Solution:**

**Concept:**

- A vector in the plane spanned by $\vec{a}$ and $\vec{b}$, perpendicular to $\vec{a}$, is obtained by removing the projection of $\vec{b}$ on $\vec{a}$: $\vec{v} = \vec{b} - \frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\vec{a}$.
- Normalize a nonzero vector by dividing by its magnitude. Either sign of the resulting unit vector remains in the same plane and remains perpendicular to $\vec{a}$; select the sign present among the options.

**Step 1:** Remove the component along the first vector For $\vec{a} = (1, 2, 1)$ and $\vec{b} = (2, 1, -1)$, the required dot products are

$$\vec{a} \cdot \vec{a} = 1 + 4 + 1 = 6, \qquad \vec{a} \cdot \vec{b} = 2 + 2 - 1 = 3.$$

Therefore

$$\vec{v} = \vec{b} - \frac{1}{2}\vec{a} = \left(\frac{3}{2}, 0, -\frac{3}{2}\right) = \frac{3}{2}(\hat{i} - \hat{k}).$$

This construction is a linear combination of the two given vectors and hence lies in their plane. Its dot product with $\vec{a}$ is zero by construction, so the plane condition and perpendicularity have both been enforced, rather than only one.

**Step 2:** Normalize and distinguish the options The magnitude of $\hat{i} - \hat{k}$ is $\sqrt{2}$. Thus the two possible

unit directions are $\pm(\hat{i}-\hat{k})/\sqrt{2}$. Option (A) is the negative direction, $(-\hat{i}+\hat{k})/\sqrt{2}$, and therefore qualifies. Its squared magnitude is $(1+1)/2=1$. The other three displayed vectors also have zero dot product with $\vec{a}$, so perpendicularity alone cannot reject them. However every vector in the given plane perpendicular to $\vec{a}$ must be proportional to $\vec{v}$, because the spanning vectors are independent. Its middle coordinate must consequently vanish. Options (B), (C) and (D) all have a nonzero middle coordinate and fail the plane condition.

**Step 3:** Conclusion One such unit vector is $\boxed{\hat{c}=\dfrac{-\hat{i}+\hat{k}}{\sqrt{2}}}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Perpendicularity alone may leave several choices. Subtract the projection of the second vector onto the first, then normalize to enforce the plane condition as well.

**19. Let the function $f(x)=\frac{x}{3}+\frac{3}{x}+3, x\neq 0$ be strictly increasing in $(-\infty,\alpha_1)\cup(\alpha_2,\infty)$ and strictly decreasing in $(\alpha_3,\alpha_4)\cup(\alpha_4,\alpha_5)$. Then $\sum_{i=1}^{5}\alpha_i^2$ is equal to** [4 marks]

(A) 48

(B) 40

(C) 36

(D) 28

**Correct Answer:** (C) 36

**Solution:**

**Concept:**

- For a differentiable function, a positive derivative on an interval gives strict increase, while a negative derivative gives strict decrease. The domain must be checked before forming these intervals.
- The derivative of a reciprocal is obtained from the power rule: $\frac{d}{dx}(x^{-1})=-x^{-2}$. Points where the derivative vanishes and excluded domain points divide the sign chart.

**Step 1:** Differentiate the given function The constant term contributes zero. Differentiating the other two terms gives

$$f\prime(x)=\frac{1}{3}-\frac{3}{x^2}=\frac{x^2-9}{3x^2}, \qquad x\neq 0.$$

The numerator vanishes at $x=-3$ and $x=3$. The point $x=0$ is excluded from the original function as well as from its derivative.

**Step 2:** Determine the sign on each domain interval The denominator $3x^2$ is positive for every permitted value of $x$. Thus the sign depends entirely on $x^2-9$. It is positive when $|x|>3$, giving increase on $(-\infty,-3)\cup(3,\infty)$. It is negative when $0<|x|<3$, giving decrease on $(-3,0)\cup(0,3)$. The decrease cannot be described as one interval across zero because the function is undefined there.

**Step 3:** Identify the five endpoints and add their squares Comparing the intervals in order gives

$$\alpha_1=-3, \quad \alpha_2=3, \quad \alpha_3=-3, \quad \alpha_4=0, \quad \alpha_5=3.$$

Consequently,

$$\sum_{i=1}^{5} \alpha_i^2 = 9 + 9 + 9 + 0 + 9 = 36.$$

Each occurrence is counted because the sum is over the five indexed endpoints; equal numerical endpoints must not be discarded. The values 48, 40 and 28 therefore do not match the sign-chart result.

**Step 4:** Conclusion The required sum is $\boxed{36}$. Hence, the correct option is (C).

**Quick Tip to Solve:** Include excluded domain points in a derivative sign chart, even when they are not roots of the numerator.

---

**20. The number of integral terms in the expansion of $\left(5^{\frac{1}{2}} + 7^{\frac{1}{8}}\right)^{1016}$ is:** [4 marks]

(A) 127

(B) 128

(C) 130

(D) 129

**Correct Answer:** (B) 128

**Solution:**

**Concept:**

- The general term in $(a+b)^n$ is $T_{r+1} = \binom{n}{r} a^{n-r} b^r$, where the integer index satisfies $0 \le r \le n$.
- For a product of rational powers of distinct primes to be a nonzero integer, every prime exponent must be a nonnegative integer. A nonzero integer binomial coefficient cannot cancel a fractional prime exponent.

**Step 1:** Write the general term with prime exponents Choose $r$ factors of $7^{1/8}$ and $1016 - r$ factors of $5^{1/2}$. The resulting term is

$$T_{r+1} = \binom{1016}{r} 5^{(1016-r)/2}\, 7^{r/8}, \qquad 0 \le r \le 1016.$$

Here the coefficient is a positive integer and both exponents are nonnegative. Thus the only possible obstacle to an integral value is a fractional exponent of 5 or 7.

**Step 2:** Impose integrality of both exponents The exponent $r/8$ is integral precisely when $r$ is divisible by 8. The exponent $(1016 - r)/2$ is integral precisely when $r$ is even. Since every multiple of 8 is even and 1016 is even, the divisibility-by-8 condition already ensures the second requirement. Conversely, a nonmultiple of 8 has a fractional exponent of the prime 7, so its term cannot be an integer, regardless of the binomial coefficient.

**Step 3:** Count all admissible indices including the endpoints The permissible indices are

$$r = 0, 8, 16, \ldots, 1016.$$

Since $1016 = 8 \cdot 127$, write $r = 8k$ with $k = 0, 1, \ldots, 127$. There are $127 - 0 + 1 = 128$ choices. Both endpoint terms are integral: at $r = 0$ the term is $5^{508}$, while at $r = 1016$ it is $7^{127}$. Counting only the positive multiples would give 127 and miss the first term. The counts 129 and 130 include indices that do not meet the exponent conditions.

**Step 4:** Conclusion There are $\boxed{128}$ integral terms. Hence, the correct option is (B).

**Quick Tip to Solve:** Count indices from zero: after finding the required divisibility, check both endpoint terms before using the arithmetic-progression count.

## Mathematics – Section B: Numerical Value Questions (20 marks)

**21. The product of the last two digits of $(1919)^{1919}$ is** [4 marks]

**Solution:**

**Concept:**

- The last two digits of a positive integer are its residue modulo 100, written as a two-digit number when necessary.
- In a binomial expansion of $(a + 100b)^n$, every term except $a^n$ is divisible by 100. Powers can therefore be simplified modulo 100 before multiplication.

**Step 1:** Reduce the base to its last two digits Because $1919 \equiv 19 \pmod{100}$, we have

$$1919^{1919} \equiv 19^{1919} \pmod{100}.$$

This reduction preserves exactly the information needed about the last two digits and avoids attempting to evaluate the enormous integer itself. We now look for a short power cycle modulo 100.

**Step 2:** Find a convenient period of the powers Successive modular products give

$$19^2 \equiv 61, \qquad 19^4 \equiv 61^2 \equiv 21 \pmod{100}.$$

Multiplying the fourth power by 19 gives

$$19^5 \equiv 21 \cdot 19 = 399 \equiv 99 \equiv -1 \pmod{100}.$$

It follows that $19^{10} \equiv 1 \pmod{100}$. Since $1919 = 10 \cdot 191 + 9$, the required residue is that of $19^9$. Only a period dividing 10 is needed; there is no need to calculate every term of a full cycle.

**Step 3:** Calculate the remaining residue and multiply its digits Use the already obtained fourth and fifth powers:

$$19^9 = 19^5 \cdot 19^4 \equiv (-1) \cdot 21 = -21 \equiv 79 \pmod{100}.$$

Thus the final two digits are 7 and 9, in that order. The residue is 79 rather than 21 because the negative residue must be converted to its representative between 0 and 99. The question asks for the

product of these digits, not their sum or the two-digit number itself:

$$7 \times 9 = 63.$$

**Step 4:** Conclusion The last two digits are 79, so their product is $\boxed{63}$.

**Quick Tip to Solve:** When a power is congruent to -1 modulo 100, square it to obtain a short period, then reduce the exponent before multiplying.

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**22. Let the area of the bounded region $\{(x, y) : 0 \le 9x \le y^2, y \ge 3x - 6\}$ be $A$. Then $6A$ is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- For horizontal strips, the area between two boundaries is $\int (x_{\text{right}} - x_{\text{left}})\, dy$. Choose the right boundary by checking all inequalities, rather than integrating the two curves without checking the region.
- The inequalities imply $x \ge 0$, $x \le y^2/9$, and $x \le (y+6)/3$. The bounded lobe lies below the origin; the part extending to positive, arbitrarily large $y$ is unbounded.

**Step 1:** Locate the boundary intersections The parabola and line meet when

$$\frac{y^2}{9} = \frac{y+6}{3} \quad \Longrightarrow \quad y^2 - 3y - 18 = 0.$$

Hence $y = -3$ or $y = 6$, giving intersection points $(1, -3)$ and $(4, 6)$. On the axis $x = 0$, the line meets at $y = -6$ and the parabola meets at $y = 0$. The bounded lobe is enclosed by the axis segment from $(0, -6)$ to the origin, the line segment ending at $(1, -3)$, and the lower parabola arc returning to the origin.

**Step 2:** Select the active boundary on each interval For $-6 \le y \le -3$, the smaller upper bound for $x$ is $(y+6)/3$; for $-3 \le y \le 0$, it is $y^2/9$. In both strips the left boundary is $x = 0$. Thus the required finite area is

$$A = \int_{-6}^{-3} \frac{y+6}{3}\, dy + \int_{-3}^{0} \frac{y^2}{9}\, dy.$$

The positive intersection $y = 6$ belongs to the unbounded portion and does not close this bounded lobe.

**Step 3:** Evaluate the integrals and the requested multiple For the first part, put $u = y + 6$, so the limits become 0 and 3. Then

$$\int_{-6}^{-3} \frac{y+6}{3}\, dy = \left[\frac{(y+6)^2}{6}\right]_{-6}^{-3} = \frac{3}{2}.$$

The curved part contributes

$$\int_{-3}^{0} \frac{y^2}{9}\,dy = \left[\frac{y^3}{27}\right]_{-3}^{0} = 1.$$

Therefore $A = \frac{3}{2} + 1 = \frac{5}{2}$ square units, and $6A = 6 \cdot \frac{5}{2} = 15$.

**Step 4:** Conclusion The bounded area is $\frac{5}{2}$ square units, and the requested value is $\boxed{6A = 15}$.

**Quick Tip to Solve:** With inequalities, first express x as the smaller of its upper bounds; the bounded lobe may use an axis as well as the two named curves.

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**23. Let $r$ be the radius of the circle, which touches $x$ - axis at point $(a, 0), a < 0$ and the parabola $y^2 = 9x$ at the point $(4, 6)$. Then $r$ is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- A circle tangent to the x-axis at $(a, 0)$ has centre $(a, k)$ and radius $r = |k|$. Its centre lies on the vertical line through the point of contact.
- At a point of tangency between a circle and a smooth curve, the circle radius is normal to the curve. For $y^2 = 9x$, implicit differentiation gives $dy/dx = 9/(2y)$.

**Step 1:** Find the normal at the given parabola point At $(4, 6)$, the parabola tangent has slope $9/(2 \cdot 6) = 3/4$. Its normal therefore has slope $-4/3$. If $(a, k)$ is the circle centre, it lies on this normal:

$$k - 6 = -\frac{4}{3}(a - 4), \qquad k = \frac{34 - 4a}{3}.$$

The negative-reciprocal slope is essential: the radius is perpendicular to the common tangent, not parallel to it.

**Step 2:** Use the equal distances from the centre The radius equals the distance to the x-axis, so $r^2 = k^2$. Since the circle also passes through $(4, 6)$,

$$(a - 4)^2 + (k - 6)^2 = k^2.$$

Expanding and cancelling $k^2$ gives

$$(a - 4)^2 + 36 - 12k = 0.$$

Substitute the normal-line expression for $k$:

$$(a - 4)^2 + 36 - 4(34 - 4a) = 0.$$

This simplifies to $a^2 + 8a - 84 = 0$, or $(a - 6)(a + 14) = 0$.

**Step 3:** Apply the sign restriction and verify the radius The algebra permits $a = 6$ or $a = -14$, but

the question requires $a < 0$. We must therefore take $a = -14$. The centre ordinate is

$$k = \frac{34 - 4(-14)}{3} = \frac{90}{3} = 30.$$

Hence the circle centre is $(-14, 30)$ and $r = 30$. As a check, its distance to $(4, 6)$ is

$$\sqrt{18^2 + (-24)^2} = \sqrt{900} = 30.$$

The radius has slope $-24/18 = -4/3$, which confirms tangency to the parabola as well as passage through the required point.

**Step 4:** Conclusion The required circle has radius $\boxed{30}$ units.

**Quick Tip to Solve:** For a circle touching an axis and a curve, combine the curve normal with equality of the centre-to-axis and centre-to-contact distances.

**24. Let the area of the triangle formed by the lines $x+2 = y-1 = z$, $\frac{x-3}{5} = \frac{y}{-1} = \frac{z-1}{1}$ and $\frac{x}{-3} = \frac{y-3}{3} = \frac{z-2}{1}$ be $A$. Then $A^2$ is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- Two intersecting lines determine a vertex of the triangle. Write each line parametrically and equate coordinates to find the three pairwise intersections.
- For triangle vertices $P, Q, R$, the area is $A = \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}|$. Therefore $A^2 = \frac{1}{4}|\overrightarrow{PQ} \times \overrightarrow{PR}|^2$, avoiding a square root when only the square is requested.

**Step 1:** Parametrize all three lines Use independent real parameters $t, u, v$. The lines become

$$L_1 : (x, y, z) = (t - 2, t + 1, t),$$

$$L_2 : (x, y, z) = (3 + 5u, -u, 1 + u),$$

$$L_3 : (x, y, z) = (-3v, 3 + 3v, 2 + v).$$

The first line has direction $(1, 1, 1)$. In particular, the expression $x + 2 = y - 1 = z$ has common parameter $t$; no denominator has been omitted in this parametrization.

**Step 2:** Find the triangle vertices by pairwise intersection For $L_1$ and $L_2$, equating y and z gives $t + u = -1$ and $t - u = 1$, so $t = 0, u = -1$, yielding $P = (-2, 1, 0)$. These values also satisfy the x equation. For $L_2$ and $L_3$, equating z and y gives $u - v = 1$ and $-u - 3v = 3$. Thus $u = 0, v = -1$, yielding $Q = (3, 0, 1)$, again consistent with x. For $L_3$ and $L_1$, the x and y equations give $t = 2 - 3v$ and $t = 2 + 3v$. Hence $v = 0, t = 2$, yielding $R = (0, 3, 2)$, which also satisfies z.

**Step 3:** Compute the cross product and square the area Take the two edge vectors from the same vertex:

$$\overrightarrow{PQ} = (5, -1, 1), \qquad \overrightarrow{PR} = (2, 2, 2).$$

Their cross product is

$$\overrightarrow{PQ} \times \overrightarrow{PR} = (-4, -8, 12).$$

Its squared magnitude is $16 + 64 + 144 = 224$. This is the square of the corresponding parallelogram area, so the triangle requires the factor $1/4$:

$$A^2 = \frac{224}{4} = 56.$$

The nonzero cross product confirms that the three vertices form a nondegenerate triangle.

**Step 4:** Conclusion The triangle has area $\sqrt{56}$ square units, so $\boxed{A^2 = 56}$ in fourth-power coordinate units.

**Quick Tip to Solve:** When a problem asks for the square of a triangle area, compute one quarter of the squared cross-product magnitude directly.

---

**25. Let the domain of the function $f(x) = \cos^{-1}\left(\frac{4x+5}{3x-7}\right)$ be $[\alpha, \beta]$ and the domain of $g(x) = \log_2(2 - 6\log_{27}(2x+5))$ be $(\gamma, \delta)$.**

**Then $|7(\alpha+\beta) + 4(\gamma+\delta)|$ is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- For a real inverse cosine, its argument must lie in $[-1, 1]$. For a real logarithm, its argument must be strictly positive. Endpoints admitted by an inverse cosine can therefore be excluded by a logarithm.
- For the rational argument, impose $3x - 7 \neq 0$ before squaring the inequality $|(4x+5)/(3x-7)| \leq 1$. For the nested logarithm, both the inner and the outer arguments must satisfy their domain conditions.

**Step 1:** Find the closed interval for the inverse cosine The condition on the argument gives

$$(4x+5)^2 \leq (3x-7)^2.$$

Factor the difference of the two squares:

$$(x+12)(7x-2) \leq 0.$$

Since the leading coefficient is positive, the product is nonpositive between its roots. Thus

$$-12 \leq x \leq \frac{2}{7}.$$

The forbidden denominator value $7/3$ is outside this interval. Both endpoints give an argument of modulus one, so both are included. Hence $\alpha = -12$, $\beta = 2/7$.

**Step 2:** Find the open interval for the nested logarithm The inner logarithm requires $2x + 5 > 0$, so

$x > -5/2$. The outer logarithm requires

$$2 - 6\log_{27}(2x+5) > 0, \qquad \log_{27}(2x+5) < \frac{1}{3}.$$

Because the base 27 is greater than one, exponentiating preserves the inequality:

$$0 < 2x + 5 < 27^{1/3} = 3.$$

Therefore $-5/2 < x < -1$, giving $\gamma = -5/2$, $\delta = -1$. Equality at either end is forbidden: the inner argument vanishes at the left end and the outer argument vanishes at the right end.

**Step 3:** Evaluate the requested absolute value Use the endpoint sums rather than combining the two domains:

$$7(\alpha+\beta) = 7\left(-12 + \frac{2}{7}\right) = -82,$$

$$4(\gamma+\delta) = 4\left(-\frac{5}{2} - 1\right) = -14.$$

Their sum is $-96$, whose absolute value is positive.

**Step 4:** Conclusion The requested dimensionless value is $\boxed{96}$.

**Quick Tip to Solve:** For nested logarithms, first make the inner argument positive, then impose positivity of the entire outer argument; do not include their zero endpoints.

## Physics – Section A: Multiple Choice Questions (80 marks)

**26. A monoatomic gas having $\gamma = \frac{5}{3}$ is stored in a thermally insulated container and the gas is suddenly compressed to $\left(\frac{1}{8}\right)^{\text{th}}$ of its initial volume. The ratio of final pressure and initial pressure is: ($\gamma$ is the ratio of specific heats of the gas at constant pressure and at constant volume)** [4 marks]

(A) 16

(B) 32

(C) 28

(D) 40

**Correct Answer:** (B) 32

**Solution:**

**Concept:**

- Thermal insulation makes heat transfer zero, $Q = 0$. In the ideal adiabatic gas model, pressure and volume obey $PV^{\gamma} =$ constant, where $\gamma = C_P/C_V$.
- For a monoatomic ideal gas, $\gamma = 5/3$. Compression increases both temperature and pressure because the work done on the gas increases its internal energy.

**Step 1:** Identify the volume ratio and process relation Write the initial and final states as $(P_i, V_i)$ and $(P_f, V_f)$. The given final volume is $V_f = V_i/8$, so $V_i/V_f = 8$. Since the container is insulated, an

isothermal relation would not describe this compression: the gas cannot discard the work input as heat. In the adiabatic ideal-gas treatment used here,

$$P_i V_i^{5/3} = P_f V_f^{5/3}.$$

Keep the initial-to-final volume ratio in this order; its reciprocal would incorrectly predict a pressure decrease.

**Step 2:** Compute and check the pressure increase Rearranging the relation gives

$$\frac{P_f}{P_i} = \left(\frac{V_i}{V_f}\right)^{5/3} = 8^{5/3} = \left(2^3\right)^{5/3} = 2^5 = 32.$$

A useful consistency check uses $TV^{\gamma-1} =$ constant:

$$\frac{T_f}{T_i} = 8^{2/3} = 4, \qquad \frac{P_f}{P_i} = \frac{T_f}{T_i}\frac{V_i}{V_f} = 4 \times 8 = 32.$$

The ratio is dimensionless. Values 16, 28 and 40 do not satisfy the stated adiabatic power law with the given volume ratio; 32 is the only listed value that does.

**Step 3:** Conclusion The final pressure is 32 times the initial pressure, so $\boxed{P_f/P_i = 32}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Write 8 as $2^3$ before applying the fractional exponent; the cube root then cancels immediately.

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**27. A convex lens of focal length 30 cm is placed in contact with a concave lens of focal length 20 cm. An object is placed at 20 cm to the left of this lens system. The distance of the image from the lens in cm is _____.** [4 marks]

(A) $\frac{60}{7}$

(B) 15

(C) 45

(D) 30

**Correct Answer:** (B) 15

**Solution:**

**Concept:**

- Thin lenses in contact have additive optical powers: $1/F = 1/f_1 + 1/f_2$. A convex lens has positive focal length and a concave lens has negative focal length.
- Using the Cartesian sign convention with incident light travelling to the right, $1/v - 1/u = 1/F$. An object to the left has $u < 0$; a negative image distance denotes a virtual image on that same side.

**Step 1:** Combine the lens powers with their signs The given focal lengths are magnitudes. Insert

$f_1 = +30\,\text{cm}$ and $f_2 = -20\,\text{cm}$:

$$\frac{1}{F} = \frac{1}{30} - \frac{1}{20} = \frac{2-3}{60} = -\frac{1}{60}\,\text{cm}^{-1}.$$

Thus $F = -60\,\text{cm}$: the combination is diverging because the magnitude of the concave lens power exceeds that of the convex lens. Adding the two focal lengths directly would be incorrect, since powers, rather than focal lengths, add for contact lenses.

**Step 2:** Locate the image and compare the listed distances The object position is $u = -20\,\text{cm}$. Therefore

$$\frac{1}{v} = \frac{1}{F} + \frac{1}{u} = -\frac{1}{60} - \frac{1}{20} = -\frac{4}{60} = -\frac{1}{15}.$$

It follows that $v = -15\,\text{cm}$. The negative sign places the image 15 cm to the left of the system; the question asks for the distance, which is its positive magnitude. As a check, the magnification is $v/u = 3/4$, so the image is upright and diminished, as expected for this diverging combination. The other distances, $60/7$, 45 and 30 cm, fail the lens equation with $u = -20\,\text{cm}$ and $F = -60\,\text{cm}$.

**Step 3:** Conclusion The image is virtual and $15\,\text{cm}$ to the left of the lens system; its distance is $\boxed{15\,\text{cm}}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Assign focal-length signs before adding powers, and distinguish the signed image coordinate from the positive distance asked in the options.

**28. A concave-convex lens of refractive index 1.5 and the radii of curvature of its surfaces are 30 cm and 20 cm, respectively. The concave surface is upwards and is filled with a liquid of refractive index 1.3. The focal length of the liquid-glass combination will be** [4 marks]

(A) $\frac{700}{11}$ cm
(B) $\frac{600}{11}$ cm
(C) $\frac{800}{11}$ cm
(D) $\frac{500}{11}$ cm

**Correct Answer:** (B) $\frac{600}{11}$ cm

**Solution:**

**Concept:**

- For a thin lens in air, its optical power is $P = (\mu - 1)(1/R_1 - 1/R_2)$. Radii carry Cartesian signs, whereas the quoted radii of curvature are magnitudes.
- The liquid in the upward concavity acts as a plano-convex lens: its upper free surface is plane and its lower surface is curved. The thin liquid lens and the glass lens are in contact, so their powers add.

**Step 1:** Calculate the power of the glass meniscus Take light travelling downwards, first crossing the upward concave surface. Its centre of curvature lies above the first surface, so $R_1 = -30\,\text{cm}$. The

lower convex surface also has its centre above that surface, so $R_2 = -20\,\text{cm}$. Thus

$$P_g = (1.5-1)\left(-\frac{1}{30}+\frac{1}{20}\right) = \frac{1}{120}\,\text{cm}^{-1}.$$

The glass meniscus is converging: its more strongly curved convex face outweighs its concave face. Using opposite signs for the two radii would describe a different lens shape.

**Step 2:** Add the power of the liquid lens The liquid has a plane top surface, whose reciprocal radius is zero, and a convex lower surface with radius $-30\,\text{cm}$ in the chosen light direction. Therefore

$$P_\ell = (1.3-1)\left(0-\frac{1}{-30}\right) = \frac{1}{100}\,\text{cm}^{-1}.$$

Adding powers gives

$$\frac{1}{F} = P_g + P_\ell = \frac{1}{120}+\frac{1}{100} = \frac{5+6}{600} = \frac{11}{600}\,\text{cm}^{-1}.$$

This contact-lens calculation also accounts for the common liquid-glass interface, since the two contributions there combine to the refractive-index difference. Inverting the total power gives the focal length. Only $600/11\,\text{cm}$, among the four options, has reciprocal $11/600\,\text{cm}^{-1}$.

**Step 3:** Conclusion The combination is converging with focal length $\boxed{F = \frac{600}{11}\,\text{cm}}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Treat the liquid as a plano-convex lens in contact with the glass meniscus; add powers after assigning the signs of both curved surfaces.

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**29. A quantity Q is formulated as $X^{-2}Y^{+\frac{3}{2}}Z^{-\frac{2}{5}}$. X, Y, and Z are independent parameters which have fractional errors of 0.1, 0.2, and 0.5, respectively in measurement. The maximum fractional error of Q is** [4 marks]

(A) 0.6

(B) 0.8

(C) 0.7

(D) 0.1

**Correct Answer:** (C) 0.7

**Solution:**

**Concept:**

- For $Q = X^aY^bZ^c$, the maximum fractional error is $\Delta Q/|Q| = |a|\Delta X/|X| + |b|\Delta Y/|Y| + |c|\Delta Z/|Z|$, using the standard first-order error-propagation rule.
- Maximum errors are upper bounds and therefore add with the absolute values of the exponents. A negative power does not cancel an error contribution; independence alone does not change a worst-case bound to a statistical uncertainty.

**Step 1:** Identify the sensitivity to each measured parameter The powers are $a = -2$, $b = 3/2$, and

$c = -2/5$. Taking a logarithmic differential gives

$$\frac{dQ}{Q} = -2\frac{dX}{X} + \frac{3}{2}\frac{dY}{Y} - \frac{2}{5}\frac{dZ}{Z}.$$

For the greatest possible magnitude, each measurement error may have the sign that reinforces the others. Hence replace the coefficients by their magnitudes. In particular, the inverse dependence on $X$ and $Z$ changes the direction of response, but not the positive size of their error bounds.

**Step 2:** Add all three maximum contributions The supplied numbers are already fractional errors, not percentages. Their weighted contributions are

$$2(0.1) = 0.2, \qquad \frac{3}{2}(0.2) = 0.3, \qquad \frac{2}{5}(0.5) = 0.2.$$

Therefore

$$\left(\frac{\Delta Q}{|Q|}\right)_{\max} = 0.2 + 0.3 + 0.2 = 0.7.$$

The result is dimensionless and corresponds to a 70 percent maximum fractional error in the standard linear approximation. The value 0.1 would arise from inappropriate signed cancellation in the differential. Values 0.6 and 0.8 do not equal the sum of the three weighted absolute contributions. A root-sum-square calculation would address random standard uncertainties, which is a different quantity from the requested maximum error.

**Step 3:** Conclusion The maximum fractional error is $\boxed{0.7}$. Hence, the correct option is (C).

**Quick Tip to Solve:** For maximum fractional errors, write the absolute exponent beside each fractional error before adding; never retain the minus sign of an inverse power.

**30. For a nucleus of mass number A and radius R, the mass density of nucleus can be represented as** [4 marks]

(A) $A^{\frac{2}{3}}$

(B) Independent of A

(C) $A^3$

(D) $A^{\frac{1}{3}}$

**Correct Answer:** (B) Independent of A

**Solution:**

**Concept:**

- The empirical nuclear-radius relation is $R = R_0 A^{1/3}$, where $R_0$ is approximately constant and $A$ is the mass number. Nuclear mass is approximately $M = Am_N$, where $m_N$ is a representative nucleon mass.
- Mass density is mass divided by volume. Approximating the nucleus as a sphere gives $\rho = M/(4\pi R^3/3)$. The dependence on $A$ must be obtained from both mass and radius together.

**Step 1:** Express nuclear mass and volume in terms of mass number The nucleus contains $A$ nucleons,

so its mass is proportional to $A$, neglecting the small variation due to binding energy and the neutron-proton mass difference. Its radius grows only as the cube root of $A$. Cubing the radius relation gives

$$R^3 = R_0^3 A.$$

Consequently, the spherical volume is

$$V = \frac{4\pi}{3} R_0^3 A.$$

Thus volume, like mass, grows linearly with mass number. Confusing radius with volume would produce a spurious residual power of $A$.

**Step 2:** Cancel the common mass-number factor Substitute both expressions into the density:

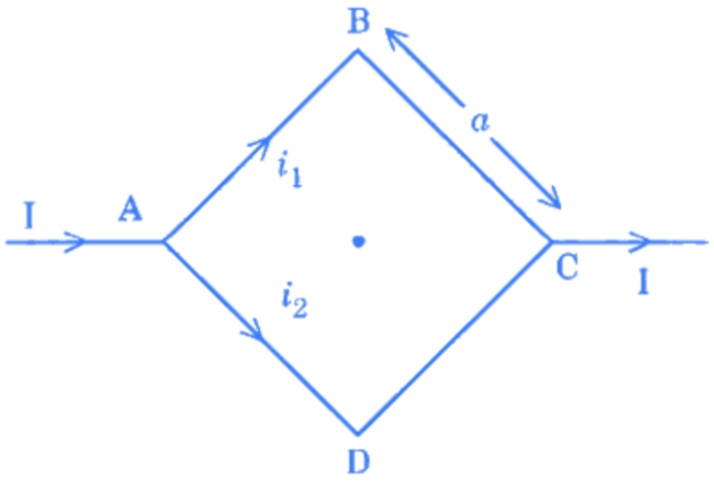
$$\rho = \frac{A m_N}{(4\pi/3) R_0^3 A} = \frac{3 m_N}{4\pi R_0^3}.$$

No mass-number factor remains. The dimensions are mass divided by length cubed, as required. For instance, increasing $A$ eightfold doubles the radius, multiplies the volume by eight, and multiplies the mass by eight, leaving the ratio unchanged. The options proportional to $A^{2/3}$, $A^3$, or $A^{1/3}$ all predict a systematic mass-number dependence absent from this expression. The approximate constancy reflects the nearly constant packing density of nucleons in ordinary nuclei.

**Step 3:** Conclusion The nuclear mass density is approximately $\boxed{\text{independent of } A}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Cube the nuclear-radius relation before comparing mass and volume: $R^3 \propto A$ cancels the mass factor $M \propto A$.

**31. Figure shows a current carrying square loop ABCD of edge length is 'a' lying in a plane. If the resistance of the ABC part is r and that of ADC part is 2r, then the magnitude of the resultant magnetic field at centre of the square loop is** [4 marks]




(A) $\frac{2\mu_0 I}{3\pi a}$

(B) $\frac{\sqrt{2}\mu_0 I}{3\pi a}$

(C) $\frac{3\pi\mu_0 I}{\sqrt{2}a}$

(D) $\frac{\mu_0 I}{2\pi a}$

**Correct Answer:** (B) $\frac{\sqrt{2}\mu_0 I}{3\pi a}$

**Solution:**

**Concept:**

- The two paths ABC and ADC are connected between the same junctions A and C, so their potential differences are equal. Current division therefore gives currents inversely proportional to their resistances.
- For a finite straight side carrying current $i$, the magnetic field at perpendicular distance $d$ is $B = \mu_0 i(\sin\theta_1 + \sin\theta_2)/(4\pi d)$. At the square centre, $d = a/2$ and both angles are $45°$.

**Step 1:** Divide the incoming current Let the currents along ABC and ADC be $i_1$ and $i_2$. Since their resistances are $r$ and $2r$, respectively,

$$i_1 r = i_2(2r), \qquad i_1 + i_2 = I.$$

Hence

$$i_1 = \frac{2I}{3}, \qquad i_2 = \frac{I}{3}.$$

The upper path carries the larger current. Both paths have two equal sides, so the geometry factors for their fields have equal magnitudes.

**Step 2:** Find each path field and its direction For one side, substitute the distance and endpoint

angles into the finite-wire result:

$$B_{\text{side}} = \frac{\mu_0 i}{4\pi(a/2)}(2\sin 45^\circ) = \frac{\sqrt{2}\mu_0 i}{2\pi a}.$$

Each path has two sides, giving $B_{\text{path}} = \sqrt{2}\mu_0 i/(\pi a)$. The current A to B to C circulates clockwise about the centre and produces an inward field; A to D to C produces an outward field. The two contributions must therefore be subtracted, rather than added.

**Step 3:** Combine and check the options The straight incoming and outgoing leads lie along the diagonal through the centre; their current elements are collinear with the displacement to the centre and add no field there. Thus

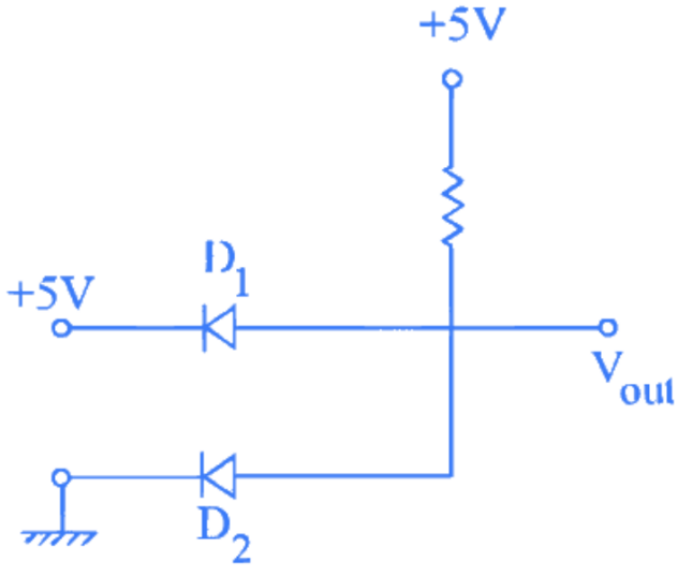
$$B_{\text{net}} = \frac{\sqrt{2}\mu_0}{\pi a}\left|\frac{2I}{3} - \frac{I}{3}\right| = \frac{\sqrt{2}\mu_0 I}{3\pi a}.$$

Option (A) has the wrong geometric coefficient, (C) places the factors of $\pi$ incorrectly, and (D) misses both the square geometry and current division.

**Step 4:** Conclusion The resultant field has magnitude $\boxed{\sqrt{2}\mu_0 I/(3\pi a)}$, directed into the plane of the figure. Hence, the correct option is (B).

**Quick Tip to Solve:** First divide current using branch resistance, then subtract the opposite fields; the two sides of each branch contribute equally.

**32. The output voltage in the following circuit is (Consider ideal diode case)** [4 marks]




(A) 0 V

(B) +5 V

(C) −5 V

(D) 10 V

**Correct Answer:** (A) 0 V

**Solution:**

**Concept:**

- An ideal diode conducts with zero voltage drop when its anode is at a higher potential than its cathode. In reverse bias it is an open circuit and carries no current.
- The resistor connects the output node to a $+5\,\mathrm{V}$ supply. Both diode anodes are at the output node, while their cathodes are connected to $+5\,\mathrm{V}$ and ground, respectively.

**Step 1:** Test the ground-connected diode Imagine first that neither diode conducts. No current flows through the resistor, so it would pull the output toward $+5\,\mathrm{V}$. But then the anode of $D_2$ would be positive relative to its grounded cathode. That forward-biases $D_2$, contradicting the assumption that it is off. Therefore the ground-connected diode must conduct. The current path is from the upper supply, through the resistor, through $D_2$, and into ground.

**Step 2:** Apply the ideal conducting condition Since $D_2$ is ideal and on, its anode and cathode have equal potentials. Its cathode is grounded, so

$$V_{\text{out}} = V_{K2} = 0\,\mathrm{V}.$$

The resistor has $5\,\mathrm{V}$ across it and limits the current to $5/R$, where $R$ is its resistance. The numerical resistance is unnecessary for finding the output because the ideal diode fixes the node voltage.

**Step 3:** Check the other diode and reject alternatives At this output potential, the anode of $D_1$ is at zero while its cathode is at $+5\,\mathrm{V}$. Thus $D_1$ is reverse-biased, consistent with no current through it. Option (B) would forward-bias $D_2$ and cannot persist. Option (C) requires a negative voltage source or another mechanism absent from this circuit, and option (D) exceeds either applied source potential. No nonzero forward diode drop is included because the question specifies ideal diodes.

**Step 4:** Conclusion The conducting diode $D_2$ clamps the output to ground: $\boxed{V_{\text{out}} = 0\,\mathrm{V}}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Locate the cathode bars first. With output-connected anodes, the diode tied to the lower reference voltage clamps the node.

**33. The amplitude and phase of a wave that is formed by the superposition of two harmonic travelling waves, $y_1(x,t) = 4\sin(kx - \omega t)$ and $y_2(x,t) = 2\sin(kx - \omega t + \frac{2\pi}{3})$, are:**

**(Take the angular frequency of initial waves same as $\omega$)** [4 marks]

(A) $\left[\sqrt{3}, \frac{\pi}{6}\right]$

(B) $\left[2\sqrt{3}, \frac{\pi}{6}\right]$

(C) $\left[6, \frac{2\pi}{3}\right]$

(D) $\left[6, \frac{\pi}{3}\right]$

**Correct Answer:** (B) $\left[2\sqrt{3}, \frac{\pi}{6}\right]$

**Solution:**

**Concept:**

- Waves with the same wave number and angular frequency add by combining their sine and cosine coefficients. Their phase difference is fixed, so the resultant is another harmonic travelling wave.
- Writing $y = A\sin(\theta + \phi)$ gives sine coefficient $A\cos\phi$ and cosine coefficient $A\sin\phi$, where $\theta = kx - \omega t$.

**Step 1:** Expand the shifted wave Put $\theta = kx - \omega t$. Using $\sin(\theta + \delta) = \sin\theta\cos\delta + \cos\theta\sin\delta$, together with $\cos(2\pi/3) = -1/2$ and $\sin(2\pi/3) = \sqrt{3}/2$, we obtain

$$y_2 = -\sin\theta + \sqrt{3}\cos\theta.$$

Adding $y_1 = 4\sin\theta$ then gives

$$y = y_1 + y_2 = 3\sin\theta + \sqrt{3}\cos\theta.$$

The amplitudes cannot simply be added because the initial phases differ.

**Step 2:** Determine the resultant amplitude and phase Compare the sum with $A\sin(\theta + \phi)$. The coefficients require

$$A\cos\phi = 3, \qquad A\sin\phi = \sqrt{3}.$$

Squaring and adding eliminates the phase:

$$A^2 = 3^2 + (\sqrt{3})^2 = 12, \qquad A = 2\sqrt{3}.$$

Dividing the coefficients gives

$$\tan\phi = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}.$$

Both coefficients are positive, so the phase belongs to the first quadrant, yielding $\phi = \pi/6$, measured relative to the phase of the first wave.

**Step 3:** Check the offered pairs The resultant can be written explicitly as

$$y = 2\sqrt{3}\sin\left(kx - \omega t + \frac{\pi}{6}\right).$$

This reproduces both original coefficients upon expansion. Option (A) has the right phase but an amplitude smaller by a factor of two. Options (C) and (D) both use amplitude 6, which would require

the waves to be in phase; their quoted phases also fail the coefficient comparison. The angular frequency remains $\omega$, because addition does not change the common time dependence.

**Step 4:** Conclusion The amplitude and phase are $\boxed{[2\sqrt{3},\ \pi/6]}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Expand the shifted sine before adding waves; use the signs of both resultant coefficients to select the phase quadrant.

---

**34. A 3 m long wire of radius 3 mm shows an extension of 0.1 mm when loaded vertically by a mass of 50 kg in an experiment to determine Young's modulus. The value of Young's modulus of the wire as per this experiment is $P \times 10^{11}$ Nm$^{-2}$, where the value of $P$ is: (Take $g = 3\pi$ m/s$^2$)** [4 marks]

(A) 2.5

(B) 25

(C) 10

(D) 5

**Correct Answer:** (D) 5

**Solution:**

**Concept:**

- Young's modulus is longitudinal stress divided by longitudinal strain: $Y = (F/A)/(\Delta L/L) = FL/(A\Delta L)$. Here $F$ is the tensile load and $A$ is the cross-sectional area.
- For a circular wire $A = \pi r^2$, and the hanging mass exerts force $F = mg$. The radius and extension must both be expressed in metres before substitution.

**Step 1:** Convert dimensions and calculate the load The original length is $L = 3\,\text{m}$. The given radius, rather than diameter, is

$$r = 3\,\text{mm} = 3 \times 10^{-3}\,\text{m},$$

and the extension is

$$\Delta L = 0.1\,\text{mm} = 10^{-4}\,\text{m}.$$

With the specified gravitational acceleration, the tensile force is

$$F = mg = 50(3\pi) = 150\pi\,\text{N}.$$

Using the stated value of $g$ keeps the calculation exact and avoids an unnecessary approximation.

**Step 2:** Evaluate stress and strain The circular cross-sectional area is

$$A = \pi(3 \times 10^{-3})^2 = 9\pi \times 10^{-6}\,\text{m}^2.$$

Thus the stress and strain are

$$\sigma = \frac{150\pi}{9\pi \times 10^{-6}} = \frac{50}{3} \times 10^6\,\text{Pa},$$

$$\epsilon = \frac{10^{-4}}{3}.$$

The strain is dimensionless, whereas stress has units of pressure. Consequently their quotient has the required units $\mathrm{N\,m^{-2}}$.

**Step 3:** Extract the requested coefficient Dividing stress by strain, or directly using the modulus formula, gives

$$Y = \frac{150\pi \times 3}{(9\pi \times 10^{-6})(10^{-4})} = 5 \times 10^{11}\,\mathrm{N\,m^{-2}}.$$

Comparison with $P \times 10^{11}\,\mathrm{N\,m^{-2}}$ gives $P = 5$. Values 2.5, 25 and 10 in options (A), (B) and (C) do not reproduce the measured extension when substituted into $\Delta L = FL/(AY)$; only 5 does.

**Step 4:** Conclusion The experimental modulus is $5 \times 10^{11}\,\mathrm{N\,m^{-2}}$, so $\boxed{P = 5}$. Hence, the correct option is (D).

**Quick Tip to Solve:** Convert the extension separately from the radius, then square the radius in the area; the given factor of pi in gravity cancels.

**35. Water falls from a height of 200 m into a pool. Calculate the rise in temperature of the water assuming no heat dissipation from the water in the pool.**

**(Take $g$ = 10 m/s$^2$, specific heat of water = 4200 J/(kg K))** [4 marks]

(A) 0.36 K

(B) 0.23 K

(C) 0.48 K

(D) 0.14 K

**Correct Answer:** (C) 0.48 K

**Solution:**

**Concept:**

- When falling water comes to rest in a pool without heat loss, its decrease in gravitational potential energy becomes thermal energy. For a falling mass $m$, the available energy is $mgh$.
- The heat needed to raise that same mass by temperature $\Delta T$ is $Q = mc\Delta T$, where $c$ is the specific heat capacity. A temperature difference has the same numerical value in kelvin and degrees Celsius.

**Step 1:** Write the energy balance The water loses gravitational potential energy during a fall through $h = 200\,\mathrm{m}$. On joining the pool and losing its organized motion, this energy becomes internal energy under the stated assumption of no heat dissipation. Equating the energy lost to the thermal energy gained gives

$$mgh = mc\Delta T.$$

Both sides refer to the falling water mass in the standard calorimetric balance; no separate water mass or pool heat capacity is specified.

**Step 2:** Cancel mass and substitute the values The water mass cancels, so the temperature rise follows from energy gained per unit mass:

$$\Delta T = \frac{gh}{c} = \frac{10 \times 200}{4200}\,\mathrm{K}.$$

The numerator is $2000\,\mathrm{J\,kg^{-1}}$, while the denominator is $4200\,\mathrm{J\,kg^{-1}\,K^{-1}}$, confirming that the quotient is a temperature change. Numerically,

$$\Delta T = \frac{10}{21}\,\mathrm{K} \approx 0.47619\,\mathrm{K}.$$

Rounding to the precision of the choices gives $0.48\,\mathrm{K}$.

**Step 3:** Compare the choices using the heat requirement Option (C) agrees with the calculated energy conversion. For a useful check, multiply each candidate temperature change by $c$. Values 0.36, 0.23 and 0.14 K would correspond to only 1512, 966 and 588 joules per kilogram, respectively, rather than the available 2000 joules per kilogram. They therefore violate the assumed complete conversion of gravitational energy into heat. No extra factor of two belongs in the balance: the kinetic energy just before impact already equals $mgh$.

**Step 4:** Conclusion The rise in temperature is $\boxed{\Delta T \approx 0.48\,\mathrm{K}}$. Hence, the correct option is (C).

**Quick Tip to Solve:** For complete conversion of a falling liquid's energy into heat, use temperature rise equal to gh divided by specific heat; the mass cancels.

**36. A rod of linear mass density '$\lambda$' and length 'L' is bent to form a ring of radius 'R'. Moment of inertia of ring about any of its diameter is.** [4 marks]

(A) $\frac{\lambda L^3}{8\pi^2}$
(B) $\frac{\lambda L^3}{16\pi^2}$
(C) $\frac{\lambda L^3}{4\pi^2}$
(D) $\frac{\lambda L^3}{12}$

**Correct Answer:** (A) $\frac{\lambda L^3}{8\pi^2}$

**Solution:**

**Concept:**

- The mass of a uniform rod with linear mass density $\lambda$ and length $L$ is $M = \lambda L$. Bending changes its shape but conserves this total mass.
- A thin circular ring has moment of inertia $MR^2$ about its central axis perpendicular to its plane. By the perpendicular-axis theorem and symmetry, either in-plane diameter has moment of inertia $MR^2/2$.

**Step 1:** Relate the ring radius to the original length The rod is bent into one complete circular ring without changing its length. Its original length therefore equals the circumference:

$$L = 2\pi R, \qquad R = \frac{L}{2\pi}.$$

The mass remains

$$M = \lambda L.$$

Keeping these relations separate prevents confusion between a mass density per unit length and

a total mass. The radius is not $L/\pi$, because that would correspond to using the diameter in the circumference relation.

**Step 2:** Choose the moment of inertia about a diameter Let $I_z$ be the moment of inertia about the axis normal to the ring plane through its centre, and $I_x, I_y$ about two perpendicular diameters in that plane. All the ring mass is at distance $R$ from the normal axis, so $I_z = MR^2$. The perpendicular-axis theorem gives

$$I_z = I_x + I_y.$$

Rotational symmetry of the circular ring makes the two diameter values equal. Hence

$$I_x = I_y = \frac{MR^2}{2}.$$

No parallel-axis shift is required because every stated axis passes through the centre.

**Step 3:** Substitute mass and radius and compare options Expressing the diameter result using the given rod quantities gives

$$I_{\text{diameter}} = \frac{1}{2}(\lambda L)\left(\frac{L}{2\pi}\right)^2 = \frac{\lambda L^3}{8\pi^2}.$$

Option (C) is instead the ring's perpendicular central-axis value and is twice as large. Option (B) introduces another unjustified factor of one-half. Option (D) is the straight-rod result about a central transverse axis; the rod is no longer straight. Dimensionally, $\lambda L^3$ has units $\mathrm{kg\,m^2}$, as a moment of inertia must.

**Step 4:** Conclusion The moment of inertia about any diameter is $\boxed{\lambda L^3/(8\pi^2)}$. Hence, the correct option is (A).

**Quick Tip to Solve:** After bending, replace the straight-rod formula by the ring formula and use circumference conservation to eliminate the radius.

**37. Two strings with circular cross section and made of same material, are stretched to have same amount of tension. A transverse wave is then made to pass through both the strings. The velocity of the wave in the first string having the radius of cross section R is $v_1$, and that in the other string having radius of cross section R/2 is $v_2$. Then $\frac{v_2}{v_1}$ =** [4 marks]

(A) 8

(B) 4

(C) 2

(D) $\sqrt{2}$

**Correct Answer:** (C) 2

**Solution:**

**Concept:**

- The transverse wave speed on a stretched string is $v = \sqrt{T/\mu}$, where $T$ is tension and $\mu$ is mass per unit length.

- For a uniform circular string of material density $\rho$ and radius $r$, $\mu = \rho\pi r^2$. Thus, at fixed tension and material density, wave speed is inversely proportional to radius.

**Step 1:** Compare the mass per unit length Both strings are made of the same material, so their volume mass density $\rho$ is equal. Their lengths need not be equal, because wave speed depends on mass per unit length rather than total mass. Using the given radii,

$$\mu_1 = \rho\pi R^2, \qquad \mu_2 = \rho\pi(R/2)^2 = \frac{\mu_1}{4}.$$

Halving the radius quarters the cross-sectional area, so the second string has only one quarter of the inertia per unit length.

**Step 2:** Apply the wave-speed relation The tension is the same in both strings. Therefore the common tension cancels on taking the ratio:

$$\frac{v_2}{v_1} = \sqrt{\frac{T/\mu_2}{T/\mu_1}} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{4} = 2.$$

The thinner string carries the wave faster, as expected from its smaller inertia. The value 4 is the inverse linear-density ratio, not the speed ratio; the square root must still be taken. The values 8 and $\sqrt{2}$ also fail the required inverse-radius dependence.

**Step 3:** Conclusion The required dimensionless ratio is $\boxed{v_2/v_1 = 2}$. Hence, the correct option is (C).

**Quick Tip to Solve:** For the same material and tension, use $vr =$ constant: halving radius doubles speed, while halving area increases speed only by $\sqrt{2}$.

**38. Two metal spheres of radius R and 3R have same surface charge density $\sigma$. If they are brought in contact and then separated, the surface charge density on smaller and bigger sphere becomes $\sigma_1$ and $\sigma_2$, respectively. The ratio $\frac{\sigma_1}{\sigma_2}$ is** [4 marks]

(A) $\frac{1}{3}$

(B) $\frac{1}{9}$

(C) 9

(D) 3

**Correct Answer:** (D) 3

**Solution:**

**Concept:**

- For an isolated conducting sphere of radius $r$, surface potential is $V = kQ/r$, with $k = 1/(4\pi\epsilon_0)$.
- Charge redistributes between spheres in electrical contact until their potentials are equal. After separation, $\sigma = Q/(4\pi r^2)$, so equal potential implies surface charge density inversely proportional to radius.

**Step 1:** Use equal potentials after charge sharing Let the final charges on the smaller and bigger

spheres be $Q_1$ and $Q_2$, respectively. The standard isolated-sphere charge-sharing condition gives

$$\frac{kQ_1}{R} = \frac{kQ_2}{3R} \quad \Longrightarrow \quad Q_2 = 3Q_1.$$

Thus equal potential does not mean equal charge: the bigger sphere retains three times as much charge. Its area, however, is nine times as large, which is essential when finding the surface densities.

**Step 2:** Convert the final charges to densities Divide each final charge by its own surface area:

$$\frac{\sigma_1}{\sigma_2} = \frac{Q_1/(4\pi R^2)}{Q_2/(4\pi(3R)^2)} = 9\frac{Q_1}{Q_2} = 9 \times \frac{1}{3} = 3.$$

As a conservation check, the initial charges are $4\pi R^2\sigma$ and $36\pi R^2\sigma$. Their total is $40\pi R^2\sigma$, giving $Q_1 = 10\pi R^2\sigma$ and $Q_2 = 30\pi R^2\sigma$. Hence $\sigma_1 = 5\sigma/2$ and $\sigma_2 = 5\sigma/6$, confirming the ratio. The values $1/3$ and $1/9$ reverse or confuse charge and area ratios; 9 ignores charge redistribution.

**Step 3:** Conclusion The smaller sphere has three times the final surface charge density of the bigger sphere: $\boxed{\sigma_1/\sigma_2 = 3}$. Hence, the correct option is (D).

**Quick Tip to Solve:** In the standard sphere charge-sharing model, equal potentials give $Q \propto r$, but dividing by area gives $\sigma \propto 1/r$.

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**39. Two balls with same mass and initial velocity, are projected at different angles in such a way that maximum height reached by first ball is 8 times higher than that of the second ball. $T_1$ and $T_2$ are the total flying times of first and second ball, respectively, then the ratio of $T_1$ and $T_2$ is** [4 marks]

(A) 2 : 1

(B) $\sqrt{2}$ : 1

(C) $2\sqrt{2}$ : 1

(D) 4 : 1

**Correct Answer:** (C) $2\sqrt{2}$ : 1

**Solution:**

**Concept:**

- For a projectile launched and landing at the same height, $H = u_y^2/(2g)$ and $T = 2u_y/g$, where $u_y = u\sin\theta$ is its initial vertical speed.
- Eliminating $u_y$ gives $H = gT^2/8$. Thus flight time is proportional to the square root of maximum height, independently of mass.

**Step 1:** Relate the vertical launch components Let the two launch angles be $\theta_1$ and $\theta_2$, and let their common initial speed be $u$. The maximum heights depend on the vertical components rather than the horizontal components:

$$H_1 = \frac{u^2\sin^2\theta_1}{2g}, \qquad H_2 = \frac{u^2\sin^2\theta_2}{2g}.$$

The given height ratio is $H_1/H_2 = 8$. Since both balls are projected upward, their vertical components

are positive, so taking the positive square root yields $\sin\theta_1/\sin\theta_2 = \sqrt{8}$.

**Step 2:** Calculate and check the flight-time ratio For return to the launch level, the total time is twice the time to the highest point. Therefore

$$\frac{T_1}{T_2} = \frac{2u\sin\theta_1/g}{2u\sin\theta_2/g} = \frac{\sin\theta_1}{\sin\theta_2} = \sqrt{8} = 2\sqrt{2}.$$

Equivalently, $H_1/H_2 = (T_1/T_2)^2$. The alternative ratios $2:1$, $\sqrt{2}:1$, and $4:1$ would imply height ratios 4, 2, and 16, respectively, rather than 8. The equality of masses is irrelevant because gravitational acceleration is mass independent; equal initial speed alone does not imply equal flight time when launch angles differ.

**Step 3:** Conclusion The ratio of total flying times is $\boxed{T_1 : T_2 = 2\sqrt{2} : 1}$. Hence, the correct option is (C).

**Quick Tip to Solve:** When the launch and landing levels agree, use $T^2 = 8H/g$; take the square root of a height ratio to obtain the time ratio.

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**40. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R**

**Assertion A : Work done in moving a test charge between two points inside a uniformly charged spherical shell is zero, no matter which path is chosen.**

**Reason R : Electrostatic potential inside a uniformly charged spherical shell is constant and is same as that on the surface of the shell.**

**In the light of the above statements, choose the correct answer from the options given below.**

[4 marks]

(A) A is true but R is false

(B) Both A and R are true and R is the correct explanation of A

(C) Both A and R are true but R is NOT the correct explanation of A

(D) A is false but R is true

**Correct Answer:** (B) Both A and R are true and R is the correct explanation of A

**Solution:**

**Concept:**

- Gauss's law gives zero electric field at every interior point of a uniformly charged spherical shell.
- Electrostatic work depends only on endpoint potentials: field work is $W = q(V_i - V_f)$, while the change in potential energy is $q(V_f - V_i)$. A constant potential therefore gives zero work for either convention.

**Step 1:** Evaluate the assertion using the interior field Choose a concentric Gaussian surface with radius smaller than the shell radius $a$. It encloses no charge; spherical symmetry makes the field radial

and constant in magnitude on this surface. Consequently,

$$E(4\pi r^2) = 0 \quad \Longrightarrow \quad E = 0 \qquad (r < a).$$

There is no electric force on a test charge in the interior, so moving it between any two interior points requires zero net electrostatic work. Electrostatic forces are conservative, so this endpoint result is independent of the path. Assertion A is true.

**Step 2:** Evaluate the reason and its explanatory link Since $\mathbf{E} = -\nabla V$, the zero interior field means the potential is spatially constant. Potential is continuous across the shell surface, so, taking zero at infinity,

$$V_{\text{inside}} = V_{\text{surface}} = \frac{kQ}{a}.$$

Reason R is therefore true. In particular the two endpoints have $V_i = V_f$; substituting into $W = q(V_i - V_f)$ directly proves the assertion. The reason is thus the correct explanation, not merely another true statement. Options claiming A or R is false fail, and the option denying the explanatory link fails because equal potential is precisely why the work vanishes.

**Step 3:** Conclusion Both A and R are true, and R is the correct explanation of A. Hence, the correct option is (B).

**Quick Tip to Solve:** Zero field inside a charged shell means constant potential, not zero potential. Work vanishes because the potential difference is zero.

**41. Electric charge is transferred to an irregular metallic disk as shown in the figure. If $\sigma_1$, $\sigma_2$, $\sigma_3$ and $\sigma_4$ are charge densities at given points then, choose the correct answer from the options given below:**

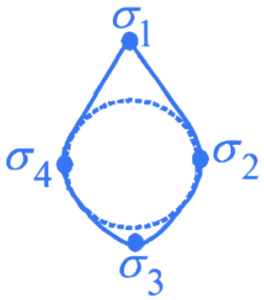
**A.** $\sigma_1 > \sigma_3; \sigma_2 = \sigma_4$

**B.** $\sigma_1 > \sigma_2; \sigma_3 > \sigma_4$

**C.** $\sigma_1 > \sigma_3 > \sigma_2 = \sigma_4$

**D.** $\sigma_1 < \sigma_3 < \sigma_2 = \sigma_4$

**E.** $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4$ [4 marks]




(A) B and C Only

(B) A and C Only

(C) D and E Only

(D) A, B and C Only

**Correct Answer:** (D) A, B and C Only

**Solution:**

**Concept:**

- At electrostatic equilibrium, a conductor is equipotential but its surface charge density need not be uniform. Charge concentrates more strongly at sharper convex regions, which have smaller local radius of curvature.
- The electric field just outside a conducting surface satisfies $E_n = \sigma/\epsilon_0$. Geometrically equivalent points related by reflection symmetry have equal surface charge densities.

**Step 1:** Read the curvature and symmetry of the disk In the given figure, point 1 is at the sharpest tip, point 3 is at the less sharp lower tip, and points 2 and 4 lie at the symmetric rounded side positions. The charge concentration is greatest at point 1 and next greatest at point 3. Reflection symmetry about the vertical axis interchanges points 2 and 4, so they have equal charge densities. For the charge-density comparison intended by the figure,

$$\sigma_1 > \sigma_3 > \sigma_2 = \sigma_4.$$

Being metallic establishes equal potential, not equal density; the differing local curvatures cause a nonuniform distribution.

**Step 2:** Test every listed statement against this order Statement A is true because $\sigma_1 > \sigma_3$ and the two side densities are equal. Statement B is also true: the full ordering implies both $\sigma_1 > \sigma_2$ and $\sigma_3 > \sigma_4$. Statement C gives exactly the full ordering and is true. Statement D reverses the curvature trend, so it is false; statement E incorrectly assumes uniform density and is false. Thus A, B and C must all be retained. The options containing only B and C or only A and C omit a true statement, while the option containing D and E contains two false statements.

**Step 3:** Conclusion The correct statements are A, B and C only, corresponding to $\boxed{\sigma_1 > \sigma_3 > \sigma_2 = \sigma_4}$. Hence, the correct option is (D).

**Quick Tip to Solve:** First write one complete density ordering from sharpness and symmetry; then test each statement as a logical consequence of that ordering.

**42. An infinitely long wire has uniform linear charge density $\lambda = 2$ nC/m. The net flux through a Gaussian cube of side length $\sqrt{3}$ cm, if the wire passes through any two corners of the cube, that are maximally displaced from each other, would be $x$ Nm$^2$C$^{-1}$, where $x$ is:**

**[Neglect any edge effects and use $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ SI units]** [4 marks]

(A) $6.48\pi$

(B) $0.72\pi$

(C) $1.44\pi$

(D) $2.16\pi$

**Correct Answer:** (D) $2.16\pi$

**Solution:**

**Concept:**

- Gauss's law relates net outward flux through a closed surface to enclosed charge: $\Phi = Q_{\text{enc}}/\epsilon_0$, regardless of the detailed field over its faces.
- A uniform line charge contributes $Q_{\text{enc}} = \lambda L$, where $L$ is the length of wire inside the surface. Opposite corners of a cube are separated by the body diagonal $a\sqrt{3}$.

**Step 1:** Find the wire length enclosed by the cube The maximally displaced corners are opposite vertices, so the wire traverses a body diagonal rather than an edge or a face diagonal. With cube side $a = \sqrt{3}\,\text{cm}$,

$$L = a\sqrt{3} = (\sqrt{3}\,\text{cm})\sqrt{3} = 3\,\text{cm} = 3 \times 10^{-2}\,\text{m}.$$

Only this segment contributes to the charge enclosed by the Gaussian cube. Charges on the rest of the infinite wire can affect local electric fields but contribute zero net flux through the closed surface.

**Step 2:** Apply Gauss law in SI units The enclosed charge is

$$Q_{\text{enc}} = (2 \times 10^{-9}\,\text{C m}^{-1})(3 \times 10^{-2}\,\text{m}) = 6 \times 10^{-11}\,\text{C}.$$

The supplied Coulomb constant gives $1/\epsilon_0 = 4\pi(9 \times 10^9)$. Therefore

$$\Phi = (6 \times 10^{-11})(36\pi \times 10^9) = 2.16\pi\,\text{N m}^2\,\text{C}^{-1}.$$

Thus $x = 2.16\pi$. The alternatives are excluded by the enclosed length: an edge or face diagonal is not the specified path between farthest corners. No extra factor for the number of cube faces belongs in Gauss's law, because this is already the total flux through all six faces.

**Step 3:** Conclusion The net outward flux is $\boxed{2.16\pi\,\text{N m}^2\,\text{C}^{-1}}$, so $x = 2.16\pi$. Hence, the correct option is (D).

**Quick Tip to Solve:** Find the length of charged wire inside the Gaussian surface first; then use $\Phi = \lambda L/\epsilon_0$ without calculating individual face fluxes.

**43. A body of mass 2 kg moving with velocity of $\vec{v}_{in} = 3\hat{i} + 4\hat{j}$ ms$^{-1}$ enters into a constant force field of 6N directed along positive z-axis. If the body remains in the field for a period of $\frac{5}{3}$ seconds, then velocity of the body when it emerges from force field is.** [4 marks]

(A) $3\hat{i} + 4\hat{j} + \sqrt{5}\hat{k}$

(B) $4\hat{i} + 3\hat{j} + 5\hat{k}$

(C) $3\hat{i} + 4\hat{j} - 5\hat{k}$

(D) $3\hat{i} + 4\hat{j} + 5\hat{k}$

**Correct Answer:** (D) $3\hat{i} + 4\hat{j} + 5\hat{k}$

**Solution:**

**Concept:**

- Newton's second law relates the constant net force to acceleration: $\vec{a} = \vec{F}/m$, where $m$ is the body's mass.
- For uniform acceleration, $\vec{v}_{\text{out}} = \vec{v}_{\text{in}} + \vec{a}t$. Cartesian components change independently; a force along the positive $z$-axis changes only the $z$-component.

**Step 1:** Calculate the acceleration vector The applied force is $\vec{F} = 6\hat{k}\,\text{N}$, and the mass is $m = 2\,\text{kg}$. Hence

$$\vec{a} = \frac{6\hat{k}}{2} = 3\hat{k}\,\text{m s}^{-2}.$$

There is no force along either the $x$- or the $y$-axis. Their acceleration components therefore vanish, and the corresponding velocity components remain 3 and $4\,\text{m s}^{-1}$, respectively.

**Step 2:** Apply the velocity change for the given time The body spends $t = 5/3\,\text{s}$ in the field. Its initial $z$-component of velocity is zero, so

$$\Delta\vec{v} = \vec{a}t = 3\hat{k}\left(\frac{5}{3}\right) = 5\hat{k}\,\text{m s}^{-1}.$$

Adding this change to the initial vector gives

$$\vec{v}_{\text{out}} = 3\hat{i} + 4\hat{j} + 5\hat{k} \quad \text{m s}^{-1}.$$

Option (A) uses an incorrect $z$-component, (B) interchanges the unchanged $x$- and $y$-components, and (C) gives a negative change despite the positive force.

**Step 3:** Conclusion The emerging velocity is $\boxed{3\hat{i} + 4\hat{j} + 5\hat{k}\ \text{m s}^{-1}}$. Hence, the correct option is (D).

**Quick Tip to Solve:** First resolve the force into components: components perpendicular to the force keep their original velocities, while the parallel component changes by $Ft/m$.

**44. A block of mass 2 kg is attached to one end of a massless spring whose other end is fixed at a wall. The spring-mass system moves on a frictionless horizontal table. The spring's natural length is 2 m and spring constant is 200 N/m. The block is pushed such that the length of the spring becomes 1 m and then released. At distance *x* m ($x < 2$) from the wall, the speed of the block will be** [4 marks]

(A) $10\left[1-(2-x)^2\right]^{\frac{1}{2}}\ m/s$

(B) $10\left[1-(2-x)^2\right]^{\frac{3}{2}}\ m/s$

(C) $10\left[1-(2-x)^2\right]\ m/s$

(D) $10\left[1-(2-x)^2\right]^{2}\ m/s$

**Correct Answer:** (A) $10\left[1-(2-x)^2\right]^{\frac{1}{2}}\ m/s$

**Solution:**

**Concept:**

- On a frictionless horizontal surface, kinetic energy plus spring potential energy is conserved. A spring of constant $k$ and deformation $u$ has elastic energy $ku^2/2$.
- The deformation is measured from the natural length, not the wall. If the block is at distance $x$ from the wall, the compression is $2-x$ metres for $x<2$.

**Step 1:** Find the energy at release The spring has natural length $2\,\mathrm{m}$ and is released from length $1\,\mathrm{m}$. Its initial compression and the amplitude are therefore $A=1\,\mathrm{m}$. Since the block is released from rest, all its initial mechanical energy is elastic:

$$E=\frac{1}{2}kA^2=\frac{1}{2}(200)(1)^2=100\,\mathrm{J}.$$

The wall does not move, gravity and the normal force do no work, and there is no frictional energy loss.

**Step 2:** Calculate the speed at length x At length $x$, the spring deformation has magnitude $2-x$, giving

$$\frac{1}{2}(2)v^2+\frac{1}{2}(200)(2-x)^2=100.$$

Thus

$$v^2=100\left[1-(2-x)^2\right],\qquad v=10\sqrt{1-(2-x)^2}\,\mathrm{m\,s^{-1}}.$$

Take the nonnegative root because the question asks for speed. The accessible interval with $x<2$ is $1\le x<2$. The other powers in (B), (C) and (D) fail the energy equation at intermediate positions; for example, at $x=1.5$, the squared speed must be $75\,\mathrm{m^2\,s^{-2}}$.

**Step 3:** Conclusion The speed is $\boxed{10[1-(2-x)^2]^{1/2}\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Use spring deformation $x-L_0$, not the absolute length $x$. Check that the result gives zero speed at the release point and maximum speed at the natural length.

**45. In a Young's double slit experiment, the source is white light. One of the slits is covered by red filter and another by a green filter. In this case:** [4 marks]

(A) there shall be alternate interference fringes of red and green.

(B) there shall be an interference pattern for red distinct from that for green.

(C) there shall be an interference pattern, where each fringe's pattern center is green and outer edges is red.

(D) there shall be no interference fringes.

**Correct Answer:** (D) there shall be no interference fringes.

**Solution:**

**Concept:**

- Sustained interference requires two waves of the same frequency with a fixed relative phase and overlapping polarization components.
- The observed intensity is averaged over many optical cycles. For different frequencies, the rapidly varying cross term averages to zero, so the time-averaged intensities simply add.

**Step 1:** Identify the light emerging from each slit White light contains a range of frequencies. The red filter transmits the red part through one slit, whereas the green filter transmits a different spectral band through the other. Thus the two slits do not provide a pair of coherent red beams or a pair of coherent green beams. Sharing the same original white-light source does not restore a fixed phase relation between these distinct frequencies.

**Step 2:** Apply the condition for a stationary fringe pattern For electric fields at angular frequencies $\omega_r$ and $\omega_g$, the interference cross term includes a factor of the form

$$\cos[(\omega_r - \omega_g)t + \delta].$$

Its average over the detector response time is zero because $\omega_r \neq \omega_g$. Consequently the measured intensity is $I = I_r + I_g$, without alternating bright and dark bands due to two-slit interference. Option (A) incorrectly treats different colours as a coherent pair. Option (B) would require both slits to transmit each colour, and (C) describes wavelength-dependent fringe locations for an arrangement in which each colour reaches both slits; neither condition holds here. A single-slit diffraction envelope may remain, but it is not a double-slit interference pattern.

**Step 3:** Conclusion There shall be no sustained interference fringes. Hence, the correct option is (D).

**Quick Tip to Solve:** Before using the fringe-width formula, check that both slits transmit the same wavelength. Separate nonoverlapping colour filters remove the coherent pair needed for interference.

## Physics – Section B: Numerical Value Questions (20 marks)

**46. A sample of a liquid is kept at 1 atm. It is compressed to 5 atm which leads to a change of volume of 0.8 cm$^3$. If the bulk modulus of the liquid is 2 GPa, the initial volume of the liquid was _____ litre.**

**(Take 1 atm = $10^5$ Pa)** [4 marks]

**Solution:**

**Concept:**

- The bulk modulus is $B = -V\,\Delta P/\Delta V$. For the magnitude of a small compression, $B = V\Delta P/|\Delta V|$, with $V$ the initial volume.
- The pressure entering the compression formula is the increase in pressure, not the final absolute pressure. Use pascals and cubic metres before converting the final volume into litres.

**Step 1:** Convert the pressure and volume changes The pressure rises from $1$ atmosphere to $5$ atmospheres. With the given conversion,

$$\Delta P = (5-1)\times 10^5 = 4\times 10^5\,\text{Pa}.$$

The reduction in volume is

$$|\Delta V| = 0.8\,\text{cm}^3 = 0.8\times 10^{-6}\,\text{m}^3.$$

The bulk modulus is $B = 2\,\text{GPa} = 2\times 10^9\,\text{Pa}$. The negative signed volume change expresses compression, while using its magnitude keeps the modulus positive.

**Step 2:** Rearrange the modulus formula for the original volume From $B = V\Delta P/|\Delta V|$,

$$V = \frac{B|\Delta V|}{\Delta P} = \frac{(2\times 10^9)(0.8\times 10^{-6})}{4\times 10^5} = 4\times 10^{-3}\,\text{m}^3.$$

Since $1\,\text{m}^3 = 10^3$ litres, the initial volume is $4$ litres. As a consistency check, the fractional compression is $\Delta P/B = 2\times 10^{-4}$. Multiplying this by $4000\,\text{cm}^3$ gives the stated reduction of $0.8\,\text{cm}^3$. Using $5\times 10^5\,\text{Pa}$ instead would incorrectly include the initial pressure as part of the increase.

**Step 3:** Conclusion The initial liquid volume was $\boxed{4\,\text{litres}}$. The required numerical entry is $4$.

**Quick Tip to Solve:** Always subtract the initial pressure from the final pressure before applying a bulk modulus. The small ratio $\Delta P/B$ is also a quick check on the fractional compression.

**47. A thin solid disk of 1 kg is rotating along its diameter axis at the speed of 1800 rpm . By applying an external torque of $25\pi$ Nm for 40 s , the speed increases to 2100 rpm . The diameter of the disk is _____ m.** [4 marks]

**Solution:**

**Concept:**

- A constant torque about a fixed axis gives $\tau = I\alpha$, with angular acceleration $\alpha = (\omega_f - \omega_i)/t$. Convert rotations per minute to radians per second.
- The moment of inertia of a thin uniform solid disc about an in-plane diameter is $I = MR^2/4$, where $R$ is its radius. This differs from $MR^2/2$ for its perpendicular central axis.

**Step 1:** Find the angular acceleration The increase in rotational speed is $2100 - 1800 = 300\,\text{rpm}$. Thus

$$\Delta\omega = 300\frac{2\pi}{60} = 10\pi\,\text{rad}\,\text{s}^{-1}, \qquad \alpha = \frac{10\pi}{40} = \frac{\pi}{4}\,\text{rad}\,\text{s}^{-2}.$$

Only the speed difference enters the acceleration; the initial speed itself is not divided by the torque duration. The torque acts in the direction that increases the speed, so the acceleration magnitude is positive.

**Step 2:** Calculate the inertia and then the diameter The applied torque gives

$$I = \frac{\tau}{\alpha} = \frac{25\pi}{\pi/4} = 100\,\text{kg}\,\text{m}^2.$$

For the stated diameter axis and $M = 1\,\text{kg}$,

$$100 = \frac{1 \cdot R^2}{4}, \qquad R^2 = 400\,\text{m}^2, \qquad R = 20\,\text{m}.$$

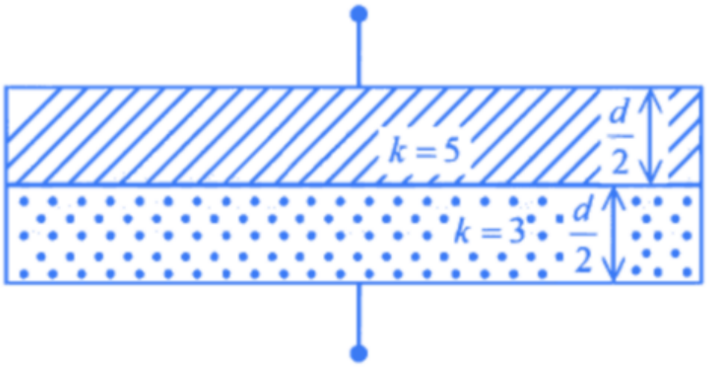
The required diameter is $D = 2R = 40\,\text{m}$. Checking the result, $I\alpha\, t = 100(\pi/4)(40) = 1000\pi\,\text{kg}\,\text{m}^2\,\text{s}^{-1}$, which equals the applied angular impulse $\tau t = 25\pi(40)$.

**Step 3:** Conclusion The diameter of the disc is $\boxed{40\,\text{m}}$. The required numerical entry is 40.

**Quick Tip to Solve:** Underline the rotation axis before selecting the inertia formula: for a thin disc, a diameter has half the inertia of the axis perpendicular to its plane.

**48. Space between the plates of a parallel plate capacitor of plate area 4 cm$^2$ and separation of 1.77 mm, is filled with uniform dielectric materials with dielectric constants (3 and 5) as shown in figure. Another capacitor of capacitance 7.5 pF is connected in parallel with it. The effective capacitance of this combination is _ pF.**

**(Given $\epsilon_0 = 8.85 \times 10^{-12}$ F/m)** [4 marks]




**Solution:**

**Concept:**

- Dielectric layers stacked along the electric-field direction behave as capacitors in series. For plate area $A$, layer thickness $d_i$, and dielectric constant $k_i$, the effective capacitance is $C = \epsilon_0 A / \sum_i (d_i/k_i)$.
- Capacitors connected across the same two terminals are in parallel, so their capacitances add: $C_{\text{total}} = C + C_p$. One picofarad is $10^{-12}$ F.

**Step 1:** Calculate the capacitance of the layered capacitor The figure divides the plate separation into two layers, each of thickness $d/2$, with dielectric constants 5 and 3. They span the full area and lie successively between the plates, so they are in series rather than side by side. With $A = 4 \times 10^{-4}\,\text{m}^2$ and $d = 1.77 \times 10^{-3}\,\text{m}$, the vacuum value is

$$C_0 = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12})(4 \times 10^{-4})}{1.77 \times 10^{-3}} = 2 \times 10^{-12}\,\text{F} = 2\,\text{pF}.$$

The individual layer capacitances are $C_5 = 10C_0 = 20\,\text{pF}$ and $C_3 = 6C_0 = 12\,\text{pF}$.

**Step 2:** Combine the layers and the additional capacitor The two layers have the same free charge, while their potential drops add. Their series equivalent is therefore

$$C = \frac{C_5 C_3}{C_5 + C_3} = \frac{20 \times 12}{20 + 12} = 7.5\,\text{pF}.$$

The additional $7.5\,\text{pF}$ capacitor is connected in parallel with this complete layered capacitor. Consequently,

$$C_{\text{total}} = 7.5 + 7.5 = 15\,\text{pF}.$$

The layer equivalent is smaller than either 12 or $20\,\text{pF}$, as required for a series combination; the final parallel combination increases it.

**Step 3:** Conclusion The effective capacitance is $\boxed{15\,\text{pF}}$. The required numerical entry is 15.

**Quick Tip to Solve:** Layers stacked through the plate separation are in series; regions sharing the separation but dividing the plate area are in parallel. Decide the geometry before combining capacitances.

**49. An electron is released from rest near an infinite non-conducting sheet of uniform charge density '$-\sigma$'. The rate of change of de-Broglie wave length associated with the electron varies inversely as n$^{th}$ power of time. The numerical value of n is _____.** [4 marks]

**Solution:**

**Concept:**

- The electric field of an infinite non-conducting sheet has constant magnitude $E = \sigma/(2\varepsilon_0)$, independent of distance from the sheet.
- The de Broglie wavelength is $\lambda = h/p$, where $h$ is Planck's constant and $p$ is the magnitude of momentum. A constant force gives $p = Ft$ for release from rest.

**Step 1:** Find the constant electric force The sheet is negatively charged, so its field is directed towards the sheet. The electron has charge $-e$, hence its force points away from the sheet. The direction does not affect the momentum magnitude used for wavelength. The constant force magnitude is

$$F = eE = \frac{e\sigma}{2\varepsilon_0}.$$

The sheet is non-conducting: use the factor $1/2$, rather than the field just outside a conductor.

**Step 2:** Express the wavelength as a function of time Newton's law in momentum form is $dp/dt = F$. Since the initial momentum is zero, integration gives

$$p(t) = Ft.$$

Therefore, for $t > 0$,

$$\lambda(t) = \frac{h}{Ft} = \frac{2h\varepsilon_0}{e\sigma}\, t^{-1}.$$

The wavelength itself is inversely proportional to time; this is not yet the rate requested in the question.

**Step 3:** Differentiate the wavelength Differentiating the power of time gives

$$\frac{d\lambda}{dt} = -\frac{2h\varepsilon_0}{e\sigma}\, t^{-2}.$$

The negative sign means the wavelength decreases as the electron gains momentum. Its rate of decrease has magnitude

$$\left|\frac{d\lambda}{dt}\right| = \frac{2h\varepsilon_0}{e\sigma t^2}.$$

Thus the inverse power for the rate is two, although the inverse power for wavelength is one.

**Step 4:** Conclusion The magnitude of the rate varies as $t^{-2}$. Hence the numerical value is $\boxed{n = 2}$ (dimensionless).

**Quick Tip to Solve:** For constant force and release from rest, write $p = Ft$ first. Wavelength scales as $t^{-1}$, so its time derivative scales as $t^{-2}$.

**50. A cube having a side of 10 cm with unknown mass and 200 gm mass were hung at two ends of a uniform rigid rod of 27 cm long. The rod along with masses was placed on a wedge keeping the distance between wedge point and 200 gm weight as 25 cm. Initially the masses were not at balance. A beaker is placed beneath the unknown mass and water is added slowly to it. At given point the masses were in balance and half volume of the unknown mass was inside the water.**

**(Take the density of unknown mass is more than that of the water, the mass did not absorb water and water density is 1 gm/cm$^3$.)**

**The unknown mass is _____ kg.** [4 marks]

**Solution:**

**Concept:**

- Archimedes' principle gives the upward buoyant force $F_B = \rho_w V_{\text{submerged}} g$. The cube therefore pulls down on the rod with its apparent weight $Mg - F_B$.
- For equilibrium about the wedge, clockwise and anticlockwise moments are equal. In the ideal lever calculation corresponding to the numerical answer, the rod's own weight is neglected.

**Step 1:** Locate the two lever arms The total rod length is $27\,\text{cm}$, and the wedge is $25\,\text{cm}$ from the $200\,\text{g}$ end. It is therefore

$$27 - 25 = 2\,\text{cm}$$

from the cube end. The cube acts through the shorter arm, whereas the known mass acts through the longer arm. The upward support force at the wedge has zero moment about the wedge.

**Step 2:** Calculate the displaced-water mass The side of the cube is $10\,\text{cm}$, so its volume is

$$V = (10)^3 = 1000\,\text{cm}^3.$$

Half is immersed at balance; hence

$$V_{\text{submerged}} = 500\,\text{cm}^3.$$

At water density $1\,\text{g}\,\text{cm}^{-3}$, the displaced water has mass

$$\rho_w V_{\text{submerged}} = 500\,\text{g} = 0.500\,\text{kg}.$$

Thus the buoyancy is $0.500g$ newtons, and the cube's downward force on the rod is $(M - 0.500)g$.

**Step 3:** Balance the moments about the wedge Using masses in kilograms and both lever arms in centimetres, the common factors cancel:

$$(M - 0.500)g(2) = 0.200g(25).$$

Therefore

$$M - 0.500 = 2.500, \qquad M = 3.000\,\text{kg}.$$

This is consistent with a cube denser than water: its density is $3\,\text{g}\,\text{cm}^{-3}$. If a non-negligible rod mass were included, its centre-of-mass moment would also be required; the stated numerical result uses the ideal lever model.

**Step 4:** Conclusion The unknown cube mass is $\boxed{3\,\text{kg}}$ in the ideal lever model.

**Quick Tip to Solve:** Use apparent weight for the immersed cube and actual weight for the dry mass. The short lever arm is $27 - 25 = 2\,\text{cm}$, not $25\,\text{cm}$.

## Chemistry – Section A: Multiple Choice Questions (80 marks)

**51. The atomic number of the element from the following with lowest 1st ionisation enthalpy is :**

[4 marks]

(A) 32

(B) 35

(C) 19

(D) 87

**Correct Answer:** (D) 87

**Solution:**

**Concept:**

- First ionisation enthalpy is the enthalpy required to remove the most loosely bound electron from an isolated gaseous atom: $\text{M}(\text{g}) \longrightarrow \text{M}^+(\text{g}) + \text{e}^-$.
- In the usual periodic trend, alkali metals have low first ionisation enthalpies because their single valence electron is shielded by filled inner shells. The value generally decreases down the group as atomic size increases.

**Step 1:** Identify the elements from their atomic numbers The four atomic numbers identify germanium ($Z = 32$), bromine ($Z = 35$), potassium ($Z = 19$), and francium ($Z = 87$). Germanium and bromine are p-block elements, whereas potassium and francium belong to Group 1. The Group 1 configuration contains one outer electron beyond a noble-gas core, so it is easier to remove that electron than an electron from the much more strongly bound p-block valence shells.

**Step 2:** Compare the two alkali metals Potassium has valence configuration $[\text{Ar}]4s^1$, and francium has $[\text{Rn}]7s^1$. The outer electron in francium is screened by many more inner shells and is at a much larger mean distance from the nucleus. Consequently its first ionisation enthalpy is lower than potassium's. Although subtle effects can affect trends among the very heaviest atoms, they do not reverse this comparison between potassium and francium.

**Step 3:** Eliminate the remaining choices Atomic number 32 represents germanium, which does not have the loosely bound solitary alkali-metal electron. Atomic number 35 represents bromine, whose relatively high effective nuclear attraction makes electron removal more difficult. Atomic number 19 is potassium, which has a low value but still a higher value than francium. Thus the lowest value among the four listed elements belongs to atomic number 87.

**Step 4:** Conclusion The element with the lowest first ionisation enthalpy among the given choices has atomic number $\boxed{87}$, francium. Hence, the correct option is (D).

**Quick Tip to Solve:** First identify the Group 1 choices, then compare their periods: the more distant and more shielded outer electron is easier to remove.

**52. Match the LIST-I with LIST-II**

| LIST-I(Complex/ Species) | LIST-II(Shape & magnetic moment) |
|---|---|
| A. $[Ni(CO)_4]$ | I. Tetrahedral, 2.8 BM |
| B. $[Ni(CN)_4]^{2-}$ | II. Square planar, 0 BM |
| C. $[NiCl_4]^{2-}$ | III. Tetrahedral, 0 BM |
| D. $[MnBr_4]^{2-}$ | IV. Tetrahedral, 5.9 BM |

**Choose the correct answer from the options given below:** [4 marks]

(A) A-I, B-II, C-III, D-IV

(B) A-IV, B-I, C-III, D-II

(C) A-III, B-II, C-I, D-IV

(D) A-III, B-IV, C-II, D-I

**Correct Answer:** (C) A-III, B-II, C-I, D-IV

**Solution:**

**Concept:**

- Find the metal oxidation state before counting d electrons. Nickel(0) has formal $d^{10}$ electron count in its carbonyl complex, $Ni^{2+}$ is $d^8$, and $Mn^{2+}$ is $d^5$.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, with $n$ unpaired electrons. Strong-field and weak-field ligands can give different geometries for four-coordinate nickel(II).

**Step 1:** Assign the two diamagnetic nickel species In $Ni(CO)_4$, neutral CO ligands leave nickel in oxidation state zero. The $d^{10}$ configuration has no unpaired electrons; the four-coordinate molecule is tetrahedral, with zero magnetic moment. Thus A matches III. In $[Ni(CN)_4]^{2-}$, nickel is in the $+2$ state. The strong-field cyanide ligands favour a square-planar $d^8$ arrangement in which all electrons are paired. Thus B matches II, also with zero moment.

**Step 2:** Assign the weak-field tetrahedral species In $[NiCl_4]^{2-}$, nickel(II) is $d^8$ and chloride is weak field. The complex is tetrahedral with two unpaired electrons. Its moment is

$$\mu = \sqrt{2(2+2)} = \sqrt{8} \simeq 2.8\,\text{BM}.$$

Therefore C matches I. In $[MnBr_4]^{2-}$, manganese is $+2$ and hence $d^5$. The weak-field tetrahedral bromide complex is high spin with five unpaired electrons:

$$\mu = \sqrt{5(5+2)} = \sqrt{35} \simeq 5.9\,\text{BM}.$$

Thus D matches IV.

**Step 3:** Compare the complete matching with the choices The matching is A–III, B–II, C–I, D–IV. Option (A) incorrectly makes the carbonyl paramagnetic and the chloride diamagnetic. Option (B) assigns the five-unpaired-electron moment to nickel carbonyl and gives incorrect nickel and manganese matches. Option (D) gives the square-planar nickel cyanide the manganese moment and incorrectly exchanges the chloride and manganese assignments. Only option (C) satisfies all four species simultaneously.

**Step 4:** Conclusion The correct matching is $\boxed{\text{A–III, B–II, C–I, D–IV}}$. Hence, the correct option is (C).

**Quick Tip to Solve:** For four-coordinate nickel, distinguish $d^{10}$ carbonyl from $d^8$ cyanide and chloride. Use the unpaired-electron count to check the geometry assignment.

**53. Match the LIST-I with LIST-II**

| LIST-I (Reagent) | LIST-II (Functional Group detected) |
|---|---|
| A. Sodium bicarbonate solution | I. double bond/unsaturation |
| B. Neutral ferric chloride | II. carboxylic acid |
| C. ceric ammonium nitrate | III. phenolic - OH |
| D. alkaline $KMnO_4$ | IV. alcoholic - OH |

**Choose the *correct* answer from the options given below:** [4 marks]

(A) A-II, B-III, C-I, D-IV

(B) A-II, B-IV, C-III, D-I

(C) A-III, B-II, C-IV, D-I

(D) A-II, B-III, C-IV, D-I

**Correct Answer:** (D) A-II, B-III, C-IV, D-I

**Solution:**

**Concept:**

- Functional-group tests rely on a characteristic observable reaction: gas evolution, colour formation, or oxidation. Match each reagent with its characteristic use in qualitative organic analysis.
- Carboxylic acids release carbon dioxide with bicarbonate; phenols give coloured complexes with neutral ferric chloride; alcohols respond to ceric ammonium nitrate; alkaline permanganate detects unsaturation.

**Step 1:** Match the acid and phenol reagents Sodium bicarbonate solution reacts with a carboxylic acid to produce brisk effervescence of carbon dioxide:

$$\text{RCOOH} + \text{NaHCO}_3 \longrightarrow \text{RCOONa} + \text{H}_2\text{O} + \text{CO}_2.$$

Ordinary phenols and alcohols do not liberate carbon dioxide in this test. Hence A matches II. Neutral ferric chloride gives a characteristic coloured complex with a phenolic hydroxyl group. This is the standard phenol test, so B matches III. The reagent is used neutral because strongly acidic or alkaline conditions interfere with the characteristic observation.

**Step 2:** Match the alcohol and unsaturation reagents Ceric ammonium nitrate gives a characteristic red colour with an alcoholic hydroxyl group under the test conditions. Thus C matches IV. Cold dilute alkaline potassium permanganate is Baeyer's reagent: it oxidises a carbon–carbon double bond, with loss of the purple permanganate colour and formation of manganese dioxide under the usual test conditions. Therefore D matches I. Although permanganate can oxidise other groups too, the intended characteristic match in this list is double bond/unsaturation.

**Step 3:** Check every option against the four matches Together the matches are A–II, B–III, C–IV, D–I. Option (A) interchanges the alcohol and unsaturation tests. Option (B) incorrectly assigns alcoholic hydroxyl to neutral ferric chloride and phenolic hydroxyl to ceric ammonium nitrate. Option (C) interchanges the bicarbonate and ferric chloride assignments. Option (D) alone retains all four characteristic reagent–functional-group pairings.

**Step 4:** Conclusion The correct matching is $\boxed{\text{A–II, B–III, C–IV, D–I}}$. Hence, the correct option is (D).

**Quick Tip to Solve:** Remember the observations: bicarbonate bubbles for carboxylic acid, ferric chloride colour for phenol, ceric nitrate red for alcohol, and permanganate decolourisation for unsaturation.

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**54. In a first order decomposition reaction, the time taken for the decomposition of reactant to one fourth and one eighth of its initial concentration are $t_1$ and $t_2$ (s), respectively. The ratio $t_1/t_2$ will be:** [4 marks]

(A) $\frac{4}{3}$
(B) $\frac{3}{2}$
(C) $\frac{3}{4}$
(D) $\frac{2}{3}$

**Correct Answer:** (D) $\frac{2}{3}$

**Solution:**

**Concept:**

- For a first-order reaction, the integrated rate law is $kt = \ln([A]_0/[A]_t)$, where $k$ is the rate constant and $[A]_0, [A]_t$ are the initial and remaining concentrations.
- The first-order half-life $t_{1/2} = \ln 2/k$ is independent of concentration. Reaching one fourth requires two half-lives; reaching one eighth requires three.

**Step 1:** Find the time to reach one fourth Here the concentration falls to one fourth of its initial value, so $[A]_{t_1} = [A]_0/4$. Substitute this remaining fraction in the integrated law:

$$kt_1 = \ln\left(\frac{[A]_0}{[A]_0/4}\right) = \ln 4.$$

Thus

$$t_1 = \frac{\ln 4}{k} = \frac{2\ln 2}{k}.$$

Equivalently, the successive remaining fractions are $1$, $1/2$, and $1/4$, reached after zero, one and two half-lives.

**Step 2:** Find the time to reach one eighth For the second time, $[A]_{t_2} = [A]_0/8$. Therefore

$$kt_2 = \ln\left(\frac{[A]_0}{[A]_0/8}\right) = \ln 8,$$

giving

$$t_2 = \frac{\ln 8}{k} = \frac{3\ln 2}{k}.$$

Since one eighth is a smaller remaining fraction, it must take longer to reach than one fourth; consequently $t_1/t_2$ must be less than unity.

**Step 3:** Take the ratio and test the choices The same rate constant applies to both measurements, so

$$\frac{t_1}{t_2} = \frac{2\ln 2/k}{3\ln 2/k} = \frac{2}{3}.$$

The rate constant and initial concentration cancel. Options (A) and (B) exceed unity and contradict the time ordering. Option (C), $3/4$, does not equal the logarithmic ratio. Option (D) is the required ratio of two half-lives to three half-lives. The ratio is dimensionless because both times are measured in seconds.

**Step 4:** Conclusion $\boxed{t_1/t_2 = 2/3}$. Hence, the correct option is (D).

**Quick Tip to Solve:** Convert the remaining fractions to powers of one half: $1/4 = (1/2)^2$ and $1/8 = (1/2)^3$. Their first-order times are in the ratio $2 : 3$.

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**55. Correct statements for an element with atomic number 9 are:**

**A. There can be 5 electrons for which $m_s = +\frac{1}{2}$ and 4 electrons for which $m_s = -\frac{1}{2}$.**

**B. There is only one electron in $p_z$ orbital.**

**C. The last electron goes to orbital with $n = 2$ and $l = 1$.**

**D. The sum of angular nodes of all the atomic orbitals is 1.**

**Choose the correct answer from the options given below:** [4 marks]

(A) A and B Only

(B) A, C and D Only

(C) A and C Only

(D) C and D Only

**Correct Answer:** (C) A and C Only

**Solution:**

**Concept:**

- Fluorine has atomic number 9, so its neutral atom contains nine electrons. The ground-state configuration follows the Aufbau principle: $1s^2\,2s^2\,2p^5$.
- Pauli exclusion allows two electrons of opposite spin in an orbital; Hund's rule places electrons singly with parallel spins before pairing. The angular-node count of an orbital is its azimuthal quantum number $l$.

**Step 1:** Write the orbital populations The filled $1s$ and $2s$ orbitals contribute two spin-up and two spin-down electrons in total. The five $2p$ electrons occupy the three degenerate orbitals with populations $2, 2, 1$. Taking the unpaired electron as spin-up, their spin populations are three up and

two down. Therefore the entire atom can have five electrons with $m_s = +\frac{1}{2}$ and four with $m_s = -\frac{1}{2}$. Statement A is correct.

**Step 2:** Check the orbital assignment The label $p_z$ denotes one choice of axis among three degenerate $p$ orbitals. The configuration $2p^5$ ensures one singly occupied orbital but does not require that orbital to be $p_z$. It is therefore not valid to assert that $p_z$ necessarily contains only one electron. Statement B is not a general property of the atom. The differentiating electron enters the second-shell $p$ subshell, whose quantum numbers are $n = 2$ and $l = 1$; hence C is correct.

**Step 3:** Count nodes and compare the combinations Each occupied $s$ orbital has $l = 0$ and therefore no angular nodes. Each of the three occupied $2p$ orbitals has one angular node. Summing over the occupied atomic orbitals gives

$$0 + 0 + 1 + 1 + 1 = 3.$$

Statement D confuses the node count of one $p$ orbital with the sum for all occupied orbitals. Thus the combinations containing D, namely (B) and (D), fail; option (A) incorrectly includes B.

**Step 4:** Conclusion Only statements A and C are correct. Hence, the correct option is (C).

**Quick Tip to Solve:** For a node sum, count the distinct occupied orbitals, not merely the occupied subshell names; the three p orbitals each contribute one angular node.

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**56.**

$$\mathrm{HA}(aq) \rightleftharpoons \mathrm{H}^+(aq) + \mathrm{A}^-(aq)$$

**The freezing point depression of a 0.1 m aqueous solution of a monobasic weak acid HA is 0.20 °C. The dissociation constant for the acid is**

**Given: $K_f$($H_2O$) = 1.8 K kg mol$^{-1}$, molality $\equiv$ molarity** [4 marks]

(A) $1.90 \times 10^{-3}$

(B) $1.38 \times 10^{-3}$

(C) $1.1 \times 10^{-2}$

(D) $1.89 \times 10^{-1}$

**Correct Answer:** (B) $1.38 \times 10^{-3}$

**Solution:**

**Concept:**

- Freezing-point depression counts dissolved particles: $\Delta T_f = iK_f m$, where $i$ is the van't Hoff factor, $K_f$ is the solvent constant and $m$ is molality.
- A monobasic acid dissociates as $\mathrm{HA} \rightleftharpoons \mathrm{H}^+ + \mathrm{A}^-$. If its degree of dissociation is $\alpha$, then $i = 1 + \alpha$, and $K_a = c\alpha^2/(1 - \alpha)$.

**Step 1:** Calculate the particle factor A temperature difference of $0.20\,^\circ\mathrm{C}$ is also $0.20\,\mathrm{K}$. An undissociated $0.1\,\mathrm{mol\,kg^{-1}}$ solution would give $K_f m = 1.8 \times 0.1 = 0.18\,\mathrm{K}$. The actual depression is larger

because dissociation creates additional particles. Thus

$$i = \frac{0.20}{1.8(0.1)} = \frac{10}{9}.$$

**Step 2:** Find the equilibrium concentrations Starting from one mole of acid, dissociation of a fraction $\alpha$ leaves $1 - \alpha$ mole of acid and creates $\alpha$ mole each of the two ions. The total is $1 + \alpha$, so

$$\alpha = i - 1 = \frac{1}{9}.$$

Using the given equivalence of molality and molarity, $c = 0.1\,\text{mol}\,\text{L}^{-1}$. At equilibrium, $[\text{H}^+] = [\text{A}^-] = c\alpha$ and $[\text{HA}] = c(1 - \alpha)$.

**Step 3:** Evaluate the acid constant and test the choices Substitute into the concentration quotient without dropping the undissociated fraction:

$$K_a = \frac{c\alpha^2}{1 - \alpha} = \frac{0.1(1/9)^2}{8/9} = \frac{0.1}{72} = 1.3889 \times 10^{-3}.$$

The reported value is $1.38 \times 10^{-3}$. Option (A) does not reproduce this quotient. Option (C) is approximately the ion concentration $c\alpha$, not the acid constant, and option (D) is much too large. The $1 - \alpha$ denominator should be retained since dissociation is about eleven percent.

**Step 4:** Conclusion The dissociation constant is approximately $K_a = 1.38 \times 10^{-3}$. Hence, the correct option is (B).

**Quick Tip to Solve:** First calculate i from the measured depression; for a one-to-two dissociation use alpha = i - 1, then retain the equilibrium denominator when evaluating the acid constant.

**57. What is the correct IUPAC name of the compound?** [4 marks]




(A) 1-Ethylcyclopent-2-en-3-ol

(B) 4-Ethylcyclopent-2-en-1-ol

(C) 1-Ethyl-3-hydroxycyclopent-2-ene

(D) 4-Ethyl-1-hydroxycyclopent-2-ene

**Correct Answer:** (B) 4-Ethylcyclopent-2-en-1-ol

**Solution:**

**Concept:**

- In a molecule containing an alcohol and a carbon-carbon double bond, the alcohol is the principal characteristic group and receives the suffix $-\mathrm{ol}$. Its locant takes priority over the double-bond locant.
- The ring containing the principal group is the parent. Number a monocyclic alcohol from the carbon bearing $-\mathrm{OH}$, then choose the direction giving the double bond the lower available locant.

**Step 1:** Choose the parent and principal group The figure contains a five-membered carbon ring, one double bond, one hydroxyl group attached directly to the ring and a two-carbon side chain. The ring is therefore a cyclopentene-derived parent, while the side chain is an ethyl substituent. Because the hydroxyl group is the principal functional group, its carbon is numbered 1 and the parent name must end in $-1$-ol. The hydroxyl group is not expressed merely as the prefix hydroxy when no higher-priority group is present.

**Step 2:** Number toward the double bond From the hydroxyl-bearing carbon, follow the ring toward the adjacent lower-right vertex in the figure. The double bond is then between carbons 2 and 3; the side chain is attached at carbon 4. Numbering in the opposite direction would put the double bond between carbons 3 and 4, which gives a larger double-bond locant. The numbering with the alkene beginning at carbon 2 is preferred, even though the opposite direction would lower the ethyl locant.

**Step 3:** Assemble the systematic name and exclude alternatives The parent is cyclopent-2-en-1-ol and the substituent is 4-ethyl, giving 4-ethylcyclopent-2-en-1-ol. Option (A) assigns the principal alcohol locant 3, so it violates functional-group numbering priority. Options (C) and (D) express the alcohol as hydroxy while using the alkene as the ending; that is inappropriate here because alcohol has higher suffix priority. Option (B) alone gives both the correct suffix and the correct ring numbering.

**Step 4:** Conclusion The compound is 4-ethylcyclopent-2-en-1-ol. Hence, the correct option is (B).

**Quick Tip to Solve:** For an unsaturated cyclic alcohol, fix the OH-bearing carbon as carbon 1 before considering the alkene or alkyl substituent locants.

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**58. 1,2-Dibromocyclooctane is treated successively with (i) alcoholic $\mathrm{KOH}$, (ii) $\mathrm{NaNH_2}$, (iii) $\mathrm{Hg^{2+}/H^+}$, and (iv) $\mathrm{Zn-Hg/H^+}$. The major final product $P$ is:** [4 marks]

(A) Cyclooctanol

(B) Cyclooctanone

(C) Cyclooctane

(D) Cyclooctyne

**Correct Answer:** (C) Cyclooctane

**Solution:**

**Concept:**

- A vicinal dibromide can lose two molecules of $\mathrm{HBr}$ through successive base-induced eliminations, producing an alkyne. Alcoholic $\mathrm{KOH}$ performs the first elimination and $\mathrm{NaNH_2}$ the second.

- Mercuric-ion-catalysed acidic hydration of an alkyne gives an enol that tautomerises to a ketone. Clemmensen reduction with $\mathrm{Zn{-}Hg/H^+}$ replaces the ketone carbonyl group by a methylene group.

**Step 1:** Form the cyclic alkyne The starting material is 1,2-dibromocyclooctane, with bromines on adjacent ring carbons. Alcoholic $\mathrm{KOH}$ eliminates one equivalent of $\mathrm{HBr}$, giving a bromoalkene. The stronger base $\mathrm{NaNH_2}$ removes another equivalent from the vinyl halide, introducing the second pi bond. The carbon skeleton remains an eight-membered ring; the intermediate is cyclooctyne. Neither step introduces oxygen or changes the number of carbon atoms.

**Step 2:** Hydrate and tautomerise Treatment with $\mathrm{Hg^{2+}/H^+}$ in the hydration step adds water across the triple bond. The immediate enol undergoes keto-enol tautomerism to give cyclooctanone. The two alkyne carbons are equivalent by ring symmetry, so there is no distinct regioisomeric ketone to choose between. At this stage the product has a carbonyl group on the ring, corresponding to the structure drawn in option (B), but the sequence has one further reagent.

**Step 3:** Apply the final reduction and select the product Zinc amalgam in acid performs Clemmensen reduction:

$$\mathrm{R_2C{=}O} \xrightarrow{\mathrm{Zn{-}Hg/H^+}} \mathrm{R_2CH_2}.$$

Consequently cyclooctanone becomes cyclooctane. The final product is the saturated eight-membered hydrocarbon shown in (C). The alcohol in (A) would require carbonyl reduction to an alcohol rather than Clemmensen deoxygenation. The ketone in (B) is only an intermediate, and the alkyne in (D) would remain only if the final hydration and reduction steps were omitted.

**Step 4:** Conclusion The major final product $P$ is cyclooctane. Hence, the correct option is (C).

**Quick Tip to Solve:** In a multistep conversion, carry the intermediate through every reagent: the ketone formed by alkyne hydration is deoxygenated by the final zinc-amalgam treatment.

**59. On combustion 0.210 g of an organic compound containing C, H and O gave 0.127 g $H_2O$ and 0.307 g $CO_2$. The percentages of hydrogen and oxygen in the given organic compound respectively are:** [4 marks]

(A) 7.55, 43.85

(B) 6.72, 53.41

(C) 6.72, 39.87

(D) 53.41, 39.6

**Correct Answer:** (B) 6.72, 53.41

**Solution:**

**Concept:**

- Complete combustion transfers all carbon in the sample to $\mathrm{CO_2}$ and all hydrogen to $\mathrm{H_2O}$. Their masses follow from the mass fractions $12/44$ and $2/18$, respectively.
- For a compound containing only carbon, hydrogen and oxygen, the oxygen mass in the original sample is found by difference: $m_{\mathrm{O}} = m_{\mathrm{sample}} - m_{\mathrm{C}} - m_{\mathrm{H}}$.

**Step 1:** Find the mass percentage of hydrogen The mass of water is $0.127\,\mathrm{g}$, but only two units out of

its molar mass 18 are hydrogen. Thus

$$m_{\mathrm{H}} = 0.127\frac{2}{18} = 0.014111\,\mathrm{g}.$$

Relative to the original $0.210\,\mathrm{g}$ sample,

$$\%\mathrm{H} = \frac{0.014111}{0.210} \times 100 = 6.7196\% \simeq 6.72\%.$$

Using the entire mass of water as hydrogen would incorrectly include oxygen supplied during combustion.

**Step 2:** Determine carbon before obtaining oxygen The carbon mass follows from the collected $0.307\,\mathrm{g}$ of carbon dioxide:

$$m_{\mathrm{C}} = 0.307\frac{12}{44} = 0.0837273\,\mathrm{g}.$$

Therefore

$$\%\mathrm{C} = \frac{0.0837273}{0.210} \times 100 = 39.8701\%.$$

The oxygen in the combustion gases comes partly from the oxidant, so their total oxygen cannot be assigned to the original compound. Instead use conservation of the sample mass and the fact that the compound contains no other elements.

**Step 3:** Subtract and compare the ordered percentages The required oxygen percentage is

$$\%\mathrm{O} = 100 - 39.8701 - 6.7196 = 53.4103\% \simeq 53.41\%.$$

The ordered pair requested is hydrogen followed by oxygen, namely $(6.72, 53.41)$. Option (A) has the wrong hydrogen value. Option (C) gives the carbon percentage as oxygen. Option (D) puts the oxygen percentage first, although the question asks for hydrogen first. This also checks that all three elemental percentages add to 100.

**Step 4:** Conclusion The hydrogen and oxygen percentages are $6.72\%$ and $53.41\%$, respectively. Hence, the correct option is (B).

**Quick Tip to Solve:** Use 2/18 of the water mass for hydrogen and 12/44 of the carbon-dioxide mass for carbon; find sample oxygen only by the remaining mass.

**60. The number of species from the following that are involved in $sp^3d^2$ hybridization is :**

**$[Co(NH_3)_6]^{3+}$, $SF_6$, $[CrF_6]^{3-}$, $[CoF_6]^{3-}$, $[Mn(CN)_6]^{3-}$, and $[MnCl_6]^{3-}$** [4 marks]

(A) 4

(B) 3

(C) 5

(D) 6

**Correct Answer:** (B) 3

**Solution:**

**Concept:**

- In the valence-bond description, an outer-orbital octahedral complex uses $sp^3d^2$ hybridisation, whereas an inner-orbital octahedral complex uses $d^2sp^3$. Coordination number six alone does not distinguish them.
- Find the metal oxidation state and d-electron count first. Strong-field ligands can pair electrons to create two vacant inner d orbitals; weak-field ligands usually preserve the high-spin arrangement.

**Step 1:** Classify the cobalt and chromium complexes In $[Co(NH_3)_6]^{3+}$, cobalt is $+3$, hence $3d^6$. The cobalt(III)-ammine complex is low spin; pairing leaves two vacant inner $3d$ orbitals, so its hybridisation is $d^2sp^3$, not the one being counted. In $[CrF_6]^{3-}$, chromium is $+3$, hence $3d^3$. Three singly occupied $3d$ orbitals already leave two inner orbitals empty, even with weak-field fluoride. It too uses $d^2sp^3$. By contrast, $[CoF_6]^{3-}$ is high-spin $d^6$; its inner shell has no pair of vacant d orbitals, so it uses outer-orbital $sp^3d^2$.

**Step 2:** Classify the manganese complexes and sulfur fluoride Both manganese complexes contain $Mn^{3+}$, with configuration $3d^4$. Strong-field cyanide in $[Mn(CN)_6]^{3-}$ produces a low-spin arrangement, leaving two inner d orbitals available; this is $d^2sp^3$. Weak-field chloride in $[MnCl_6]^{3-}$ gives high-spin $d^4$, which cannot supply two empty inner orbitals and therefore uses $sp^3d^2$. In the conventional hybridisation model used by the question, octahedral $SF_6$ is also designated $sp^3d^2$.

**Step 3:** Count only the outer-orbital species The counted species are $SF_6$, $[CoF_6]^{3-}$ and $[MnCl_6]^{3-}$, giving three. The other three use inner-orbital $d^2sp^3$ in this model. Therefore counts 4, 5 and 6 in the other options overcount the list. In particular, weak-field fluoride does not automatically force an outer-orbital description for a $d^3$ metal: chromium already has two vacant inner d orbitals.

**Step 4:** Conclusion There are 3 species with $sp^3d^2$ hybridisation. Hence, the correct option is (B).

**Quick Tip to Solve:** Check available inner d orbitals as well as ligand strength: a d3 ion already has two vacant inner orbitals, while high-spin d4 and d6 ions do not.

**61. Which of the following binary mixture does not show the behaviour of minimum boiling azeotropes?** [4 marks]

(A) $CH_3OH + CHCl_3$

(B) $C_6H_5OH + C_6H_5NH_2$

(C) $H_2O + CH_3COC_2H_5$

(D) $CS_2 + CH_3COCH_3$

**Correct Answer:** (B) $C_6H_5OH + C_6H_5NH_2$

**Solution:**

**Concept:**

- A minimum-boiling azeotrope boils below the boiling points of both pure components. It is associated with a maximum in total vapour pressure and a sufficiently large positive deviation from Raoult's law.
- The sign of the deviation depends on unlike-molecule attractions relative to the original like-molecule attractions. Stronger unlike attractions reduce the escaping tendency and favour negative deviation, rather than a minimum-boiling azeotrope.

**Step 1:** Connect boiling temperature with vapour pressure At a fixed external pressure, a liquid boils when its vapour pressure reaches that pressure. Weaker effective attractions in a mixture increase its vapour pressure, so that it reaches the boiling condition at a lower temperature. Conversely, stronger attractions lower vapour pressure and raise the boiling temperature. Thus the mixture sought must not be the positive-deviation case.

**Step 2:** Identify the strongly associating pair Phenol and aniline form strong intermolecular hydrogen bonds between the phenolic OH group and the basic nitrogen of aniline. These unlike interactions are stronger than the interactions that they replace. Their mixture therefore exhibits negative deviation and does not have the minimum-boiling behaviour asked about. Its tendency is toward maximum-boiling behaviour.

**Step 3:** Eliminate the remaining mixtures Methanol with chloroform, water with methyl ethyl ketone, and carbon disulphide with acetone are the positive-deviation mixtures in this comparison. Mixing disrupts the original liquid associations or dipole interactions sufficiently to increase the escaping tendency. They belong to the minimum-boiling class, whereas the phenol–aniline pair is distinguished by strong unlike association.

**Step 4:** Conclusion The required mixture is phenol with aniline, $C_6H_5OH + C_6H_5NH_2$. Hence, the correct option is (B).

**Quick Tip to Solve:** For azeotrope questions, first identify strong acid–base hydrogen bonding between unlike molecules: it points to negative deviation and higher, rather than lower, boiling behaviour.

**62. Given below are two statements:**

**Statement I: $H_2Se$ is more acidic than $H_2Te$**

**Statement II: $H_2Se$ has higher bond enthalpy for dissociation than $H_2Te$**

**In the light of the above statements, choose the correct answer from the options given below:**

[4 marks]

(A) Both Statement I and Statement II are true

(B) Statement I is true but Statement II is false

(C) Both Statement I and Statement II are false

(D) Statement I is false but Statement II is true

**Correct Answer:** (D) Statement I is false but Statement II is true

**Solution:**

**Concept:**

- Down Group 16, the central atom becomes larger, the H–E bond becomes longer and its bond enthalpy decreases. A weaker bond makes proton release easier in comparing the hydrides.
- For the heavier Group 16 hydrides, the acidity trend is $H_2S < H_2Se < H_2Te$. This trend follows the decrease in bond strength rather than an electronegativity-only argument.

**Step 1:** Evaluate the acidity statement Tellurium lies below selenium in Group 16. The H–Te bond is weaker than the H–Se bond, so hydrogen telluride releases a proton more readily. Thus $H_2Te$ is more acidic than $H_2Se$. Statement I asserts the reverse and is false. The increasing size and polarizability of the conjugate base also support the greater acidity down this group.

**Step 2:** Evaluate the bond-enthalpy statement The larger tellurium atom forms a longer bond to hydrogen with poorer orbital overlap than selenium does. More energy is therefore required to dissociate an H–Se bond than an H–Te bond. Consequently the bond enthalpy for dissociation of $H_2Se$ is higher, as Statement II says. Statement II is true.

**Step 3:** Match the truth values to the options The ordered pair of truth values is false for Statement I and true for Statement II. Option A incorrectly accepts the reversed acidity trend; option B assigns the opposite truth values to both statements; option C incorrectly rejects the bond-strength trend. Only option D gives the independently established pair. A stronger H–Se bond is consistent with lower acidity, so there is no contradiction between these conclusions.

**Step 4:** Conclusion Statement I is false and Statement II is true. Hence, the correct option is (D).

**Quick Tip to Solve:** For acidity of Group 16 hydrides, use the weakening H–E bond down the group; do not predict the trend from electronegativity alone.

**63.**

$$\mathrm{A} \xrightarrow[\text{(i) } H_3O^+]{\text{(i) NaH}} \mathrm{B} \xrightarrow[\text{(ii) } H_2SO_4,\Delta]{\text{(i) EOH}} \mathrm{C}$$

**' A ' shows positive Lassaign's test for N and its molar mass is 121 .**

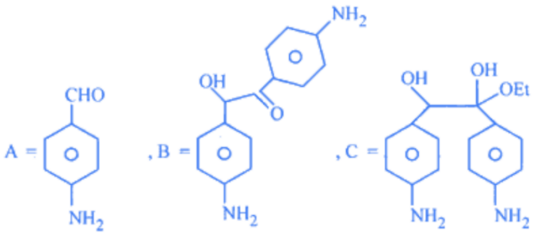
**' B ' gives effervescence with aq $NaHCO_3$.**

**'C' gives fruity smell.**

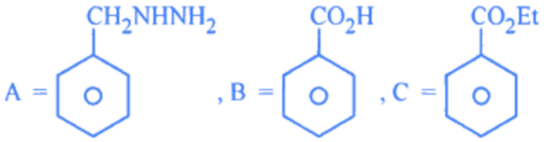
**Identify $A, B$ and C from the following.**

[4 marks]

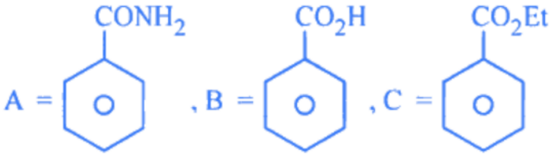
(A) A: 4-aminobenzaldehyde; B: the depicted amino-substituted hydroxy ketone; C: its depicted ethoxy hemiacetal.



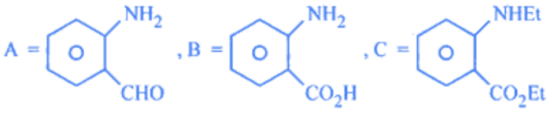

(B) A: $C_6H_5CH_2NHNH_2$; B: $C_6H_5COOH$; C: $C_6H_5COOC_2H_5$.




(C) A: $C_6H_5CONH_2$; B: $C_6H_5COOH$; C: $C_6H_5COOC_2H_5$.

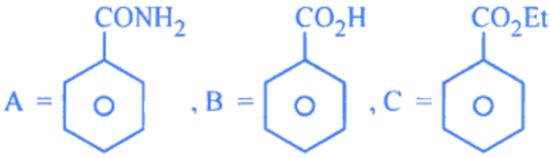



(D) A: 2-aminobenzaldehyde; B: 2-aminobenzoic acid; C: ethyl 2-(ethylamino)benzoate.




**Correct Answer:** (C)

A: $C_6H_5CONH_2$; B: $C_6H_5COOH$; C: $C_6H_5COOC_2H_5$.




**Solution:**

**Concept:**

- A positive Lassaigne's test for nitrogen establishes the presence of nitrogen in the organic compound. Molecular mass then helps distinguish candidate nitrogen-containing structures.
- Carboxylic acids liberate carbon dioxide from aqueous sodium hydrogen carbonate. Esterification of a carboxylic acid with ethanol in acidic conditions gives an ethyl ester, commonly with a fruity smell.

**Step 1:** Identify the acid and ester pair The effervescence of B with aqueous $NaHCO_3$ is the acid test:

$$C_6H_5COOH + NaHCO_3 \longrightarrow C_6H_5COONa + CO_2\uparrow + H_2O.$$

A benzoic-acid structure for B and an ethyl-benzoate structure for C are mutually consistent. Acid-

catalysed esterification is

$$C_6H_5COOH + C_2H_5OH \underset{\Delta}{\overset{H_2SO_4}{\rightleftharpoons}} C_6H_5COOC_2H_5 + H_2O.$$

The product is ethyl benzoate, a fruity-smelling ester.

**Step 2:** Check the nitrogen compound and its molecular mass Benzamide, $C_6H_5CONH_2$, contains nitrogen and has formula $C_7H_7NO$. Its molar mass is

$$7(12) + 7(1) + 14 + 16 = 121 \text{ g mol}^{-1}.$$

Hydrolysis of its amide group yields benzoic acid. These identities give the sequence benzamide, benzoic acid, ethyl benzoate appearing in option C. The printed reagent label NaH is not itself an amide-hydrolysis reagent; the identity is selected from the stated tests, mass and the acid–ester relationship rather than attributing hydrolysis to NaH.

**Step 3:** Check the other structural alternatives Option A has an aminoaldehyde followed by a hydroxy ketone rather than a carboxylic acid, so its B does not satisfy the bicarbonate test. Option B starts with $C_6H_5CH_2NHNH_2$, whose molar mass is $122 \text{ g mol}^{-1}$, not 121. In option D the amino group is ethylated in the proposed final product; simple acidic esterification does not produce that N-ethyl derivative. Thus the complete set of identity clues selects C.

**Step 4:** Conclusion A is benzamide $C_6H_5CONH_2$, B is benzoic acid $C_6H_5COOH$, and C is ethyl benzoate $C_6H_5COOC_2H_5$. Hence, the correct option is (C).

**Quick Tip to Solve:** Use the bicarbonate test to locate the carboxylic acid, then work backward with the molecular mass and forward with the alcohol used for esterification.

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**64. The correct decreasing order of spin only magnetic moment values (BM) of $Cu^+$, $Cu^{2+}$, $Cr^{2+}$ and $Cr^{3+}$ ions is :** [4 marks]

(A) $Cr^{3+} > Cr^{2+} > Cu^+ > Cu^{2+}$

(B) $Cu^+ > Cu^{2+} > Cr^{3+} > Cr^{2+}$

(C) $Cr^{2+} > Cr^{3+} > Cu^{2+} > Cu^+$

(D) $Cu^{2+} > Cu^+ > Cr^{2+} > Cr^{3+}$

**Correct Answer:** (C) $Cr^{2+} > Cr^{3+} > Cu^{2+} > Cu^+$

**Solution:**

**Concept:**

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}$ BM, where $n$ is the number of unpaired electrons. It increases monotonically with $n$.
- In forming transition-metal cations, remove the outer 4s electrons before 3d electrons. The neutral configurations are $Cu : [Ar]3d^{10}4s^1$ and $Cr : [Ar]3d^5 4s^1$.

**Step 1:** Obtain the ionic configurations Removing one electron from copper gives $Cu^+ : 3d^{10}$; removing one more gives $Cu^{2+} : 3d^9$. Chromium loses the 4s electron first and then a 3d electron,

giving $Cr^{2+}: 3d^4$, followed by $Cr^{3+}: 3d^3$. The question asks for the spin-only values of these ions, so count unpaired d electrons by Hund's rule.

**Step 2:** Count the unpaired electrons and calculate The filled $d^{10}$ shell has no unpaired electrons; $d^9$ has one; $d^4$ has four; and $d^3$ has three. Therefore

$$\mu(Cu^+) = 0, \qquad \mu(Cu^{2+}) = \sqrt{3} \approx 1.73 \text{ BM},$$
$$\mu(Cr^{2+}) = \sqrt{24} \approx 4.90 \text{ BM}, \quad \mu(Cr^{3+}) = \sqrt{15} \approx 3.87 \text{ BM}.$$

The largest moment is associated with four unpaired electrons.

**Step 3:** Arrange and eliminate the incorrect orders Sorting those numerical moments gives chromium(II), chromium(III), copper(II), then copper(I). Option A reverses the two chromium ions and incorrectly places diamagnetic copper(I) above copper(II). Option B begins with the ion of zero moment, and option D likewise places the copper ions ahead of the much more strongly paramagnetic chromium ions. Only C follows the unpaired-electron counts.

**Step 4:** Conclusion The decreasing order is $Cr^{2+} > Cr^{3+} > Cu^{2+} > Cu^+$. Hence, the correct option is (C).

**Quick Tip to Solve:** For a spin-only order, rank the number of unpaired electrons first; evaluating square roots is only a confirmation.

---

**65. Given below are two statements:**

**Statement I: A homoleptic octahedral complex, formed using monodentate ligands, will not show stereoisomerism.**

**Statement II: cis- and trans- platin are heteroleptic complexes of Pd.**

**In the light of the above statements, choose the correct answer from the options given below:**

[4 marks]

(A) Statement I is true but Statement II is false

(B) Both Statement I and Statement II are true

(C) Both Statement I and Statement II are false

(D) Statement I is false but Statement II is true

**Correct Answer:** (A) Statement I is true but Statement II is false

**Solution:**

**Concept:**

- A homoleptic complex contains one kind of ligand. With six identical monodentate ligands, an octahedral $[ML_6]$ coordination sphere has only one arrangement of ligand positions.
- Cisplatin and transplatin are square-planar platinum(II) complexes, $[Pt(NH_3)_2Cl_2]$. They are heteroleptic because they contain ammonia and chloride ligands.

**Step 1:** Assess the homoleptic octahedral complex All six coordination sites of the octahedron carry the same monodentate ligand. Interchanging any two sites cannot produce a distinguishable

arrangement, so there is no cis/trans or fac/mer pair. The symmetric coordination sphere is also not optically active. Thus the stated homoleptic octahedral complex does not show stereoisomerism, and Statement I is true. The restriction to monodentate ligands matters: homoleptic chelate complexes can have optical isomers.

**Step 2:** Identify the metal in the platin complexes In cisplatin the two chloride ligands are adjacent; in transplatin they are opposite. Both have platinum, symbol Pt, in the +2 oxidation state. Palladium has symbol Pd and is a different element. Although these complexes are correctly described as heteroleptic, Statement II specifies Pd and is therefore false. A statement containing both a correct ligand description and an incorrect metal identity is false as a whole.

**Step 3:** Select the corresponding combination The independent assessments are I true and II false, exactly the combination in option A. Option B incorrectly accepts Pd as the central metal of the platin drugs. Option C incorrectly rejects the symmetry argument for $[ML_6]$, while option D reverses both truth values. The familiar cis/trans distinction here belongs to the square-planar heteroleptic platinum complex, not to octahedral $[ML_6]$.

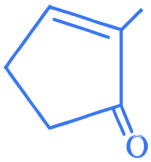
**Step 4:** Conclusion Statement I is true and Statement II is false: cisplatin and transplatin contain Pt, not Pd. Hence, the correct option is (A).

**Quick Tip to Solve:** Read the metal symbol as carefully as the ligand count: platin refers to platinum (Pt); Pd denotes palladium.

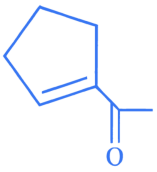
**66. When undergoes intramolecular aldol condensation, the major product formed is :** [4 marks]



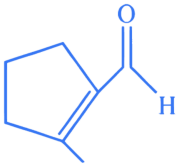

(A) 2-Methylcyclopent-2-en-1-one.



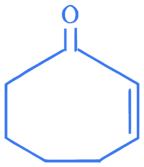

(B) 1-Acetylcyclopent-1-ene.



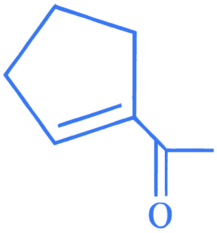

(C) 2-Methylcyclopent-1-ene-1-carbaldehyde.




(D) Cyclohept-2-en-1-one.




**Correct Answer:** (B)
1-Acetylcyclopent-1-ene.



**Solution:**

**Concept:**

- An enolate attacks a carbonyl carbon to form a carbon–carbon bond in an aldol addition. Intramolecular reactions strongly favour formation of five- or six-membered rings when the chain permits them.
- Aldol condensation includes dehydration of the initial beta-hydroxy carbonyl compound. The resulting carbon–carbon double bond is conjugated with the carbonyl group that was retained.

**Step 1:** Number the chain and choose the ring closure The substrate is $CH_3CO(CH_2)_4CHO$. Number the aldehyde carbon C1, the methylene atoms C2 through C5, the ketone carbon C6 and the terminal methyl C7. Enolate formation at C5, the methylene alpha to the ketone, allows attack on the more electrophilic aldehyde carbon C1. The new C5–C1 bond closes a five-membered ring containing C1 through C5; the acetyl group remains attached to C5.

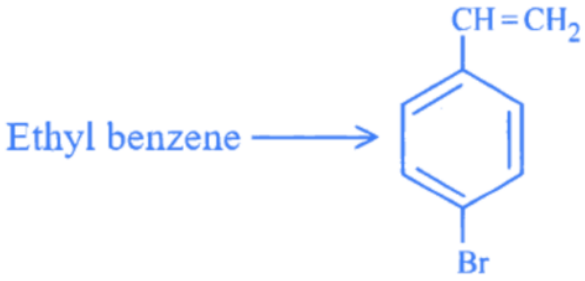
**Step 2:** Dehydrate the cyclic aldol intermediate After the C5 enolate attacks C1, protonation converts the aldehyde oxygen into an OH group on C1. The ketone carbonyl is retained outside the ring as $-COCH_3$. Loss of water between C1 and C5 then places the double bond between these two ring atoms. This produces a cyclopentene ring bearing an acetyl group on a double-bond carbon, namely 1-acetylcyclopent-1-ene. The double bond is conjugated with the retained ketone carbonyl.

**Step 3:** Compare all four products Option B shows precisely that five-membered cycloalkene with an exocyclic acetyl substituent. Option C instead retains the aldehyde, requiring an enolate of the less reactive aldehyde to attack the ketone; the preferred electrophile is the aldehyde. Option D would require a seven-membered closure from the terminal methyl enolate, disfavoured relative to a five-membered ring. Option A gives the wrong carbonyl connectivity: the retained ketone carbonyl is outside, rather than inside, the five-membered ring formed from C1–C5.

**Step 4:** Conclusion The major product is 1-acetylcyclopent-1-ene, the conjugated enone shown in option B. Hence, the correct option is (B).

**Quick Tip to Solve:** Before an intramolecular aldol mechanism, count the atoms in the proposed new ring and mark which carbonyl survives; a five-membered ring and aldehyde attack are favoured here.

**67. Choose the correct set of reagents for the following conversion.** [4 marks]




(A) $Cl_2$/ anhy ·$AlCl_3$; $Br_2/Fe$; alc. KOH

(B) $Cl_2/Fe$; $Br_2$/ anhy. $AlCl_3$; aq. KOH

(C) $Br_2/Fe$; $Cl_2, \Delta$; alc. KOH

(D) $Br_2$/ anhy. $AlCl_3$; $Cl_2, \Delta$; aq. KOH

**Correct Answer:** (C) $Br_2/Fe$; $Cl_2, \Delta$; alc. KOH

**Solution:**

**Concept:**

- Electrophilic aromatic bromination uses $Br_2$ with a Lewis-acid catalyst. An ethyl group activates its benzene ring and directs substitution mainly to the ortho and para positions.
- Chlorination with heat proceeds by a radical pathway at the benzylic carbon. Alcoholic KOH then favours elimination of hydrogen halide to form an alkene.

**Step 1:** Identify the changes in the target The target is para-bromostyrene: the ring bears bromine para to a vinyl group, $-CH{=}CH_2$. Starting ethylbenzene already supplies the two-carbon side chain, so we need ring bromination followed by conversion of $-CH_2CH_3$ into $-CH{=}CH_2$. Direct aromatic chlorination would put chlorine on the ring and would not create the required side-chain leaving group.

**Step 2:** Brominate the ring and chlorinate the side chain First use $Br_2/Fe$. The ethyl group directs the incoming bromine ortho/para, with the less crowded para product used for the stated conversion. Then $Cl_2$ with heat replaces a benzylic hydrogen:

$$\text{p-BrC}_6\text{H}_4\text{CH}_2\text{CH}_3 \xrightarrow{Cl_2,\ \Delta} \text{p-BrC}_6\text{H}_4\text{CHClCH}_3.$$

The chlorine at the benzylic carbon can now leave during elimination.

**Step 3:** Eliminate and compare all reagent sets Alcoholic KOH removes a beta-hydrogen while chloride leaves, giving p-$BrC_6H_4CH{=}CH_2$. Thus the required sequence is $Br_2/Fe;\ Cl_2, \Delta;$ alcoholic KOH. Sets (A) and (B) begin with aromatic chlorination and introduce an unwanted ring chlorine. Sets (B) and (D) use aqueous alkali, which favours substitution to an alcohol rather than the desired elimination. Only (C) contains all three required conditions.

**Step 4:** Conclusion The appropriate sequence is $Br_2/Fe$, then $Cl_2, \Delta$, then alcoholic KOH. Hence, the correct option is (C).

**Quick Tip to Solve:** Distinguish ring halogenation with a Lewis acid from benzylic halogenation under heat, then choose alcoholic alkali for elimination.

**68. Which one of the following reactions will not lead to the desired ether formation in major proportion?**

**(iso-** $\mathrm{Bu} \Rightarrow$ **isobutyl, sec** $-\mathrm{Bu} \Rightarrow$ **sec-butyl,** $\mathrm{nPr} \Rightarrow$ **n-propyl,** $^{t}\mathrm{Bu} \Rightarrow$ **tert-butyl,** $\mathrm{Et} \Rightarrow$ **ethyl)** [4 marks]

(A) $C_6H_5ONa + CH_3CH_2CH_2Br \longrightarrow C_6H_5OCH_2CH_2CH_3$

(B) iso-BuONa + sec-BuBr $\longrightarrow$ sec-Bu–O–iso-Bu

(C) $C_6H_{11}ONa + CH_3Br \longrightarrow C_6H_{11}OCH_3$ (cyclohexyl methyl ether)

(D) tert-BuONa + EtBr $\longrightarrow$ tert-Bu–O–Et

**Correct Answer:** (B) iso-BuONa + sec-BuBr $\longrightarrow$ sec-Bu–O–iso-Bu

**Solution:**

**Concept:**

- Williamson synthesis forms an ether by an alkoxide attacking an alkyl halide through an $S_N2$ displacement. Methyl and primary halides are the most suitable electrophiles.
- Alkoxides are also bases. With a sterically hindered secondary halide, beta-elimination competes strongly and can predominate over the desired ether-forming substitution.

**Step 1:** Examine the electrophile in each reaction In (A), sodium phenoxide attacks n-propyl bromide, a primary halide. The phenyl group remains bonded to oxygen: the substitution occurs at the primary propyl carbon, not at the aromatic carbon. In (C), cyclohexoxide attacks methyl bromide; the methyl carbon is exceptionally accessible and has no beta-carbon for elimination. These are favourable Williamson combinations.

**Step 2:** Identify the unfavourable secondary halide In (B), sodium isobutoxide reacts with sec-butyl bromide. The carbon bearing bromine is secondary and more crowded than a primary carbon. The alkoxide can remove a beta-hydrogen instead of attacking that carbon, so elimination to butenes competes strongly. Consequently, sec-butyl isobutyl ether is not obtained in the desired major proportion. A better synthesis would place the halogen on the primary isobutyl fragment and use sec-butoxide as the nucleophile.

**Step 3:** Check the remaining primary-halide choice In (D), the bulky tert-butyl group is on the alkoxide, while the halide is ethyl bromide, a primary substrate. In the intended Williamson comparison, this permits displacement at the accessible ethyl carbon to give tert-butyl ethyl ether. It is essential not to confuse this with tert-butyl bromide plus ethoxide, where a tertiary electrophile prevents ordinary $S_N2$ ether formation. Thus (A), (C) and (D) are the suitable orientations, whereas (B) places the leaving group on a secondary carbon.

**Step 4:** Conclusion The reaction of sodium isobutoxide with sec-butyl bromide does not give the desired ether in major proportion. Hence, the correct option is (B).

**Quick Tip to Solve:** For an unsymmetrical Williamson ether, put the halide on the less hindered alkyl fragment; the alkoxide can carry the bulkier group.

**69. Match the LIST-I with LIST-II**

| LIST-I | LIST-II |
|---|---|
| A. Carbocation | I. Species that can supply a pair of electrons. |
| B. C-Free radical | II. Species that can receive a pair of electrons. |
| C. Nucleophile | III. $sp^2$ hybridized carbon with empty p-orbital. |
| D. Electrophile | IV. $sp^2/sp^3$ hybridized carbon with one unpaired electron. |

**Choose the correct answer from the options given below:** [4 marks]

(A) A-IV, B-II, C-III, D-I

(B) A-II, B-III, C-I, D-IV

(C) A-III, B-IV, C-I, D-II

(D) A-III, B-IV, C-II, D-I

**Correct Answer:** (C) A-III, B-IV, C-I, D-II

**Solution:**

**Concept:**

- A carbocation is an electron-deficient carbon species, normally with three sigma bonds, trigonal planar $sp^2$ geometry and an empty unhybridized p orbital.
- A carbon free radical contains one unpaired electron. A nucleophile donates an electron pair, whereas an electrophile accepts an electron pair.

**Step 1:** Match the carbon-centred intermediates For A, a carbocation such as $CH_3{}^+$ has only six electrons around carbon. Its three bonds use $sp^2$ orbitals, leaving a vacant p orbital available to accept electron density. Therefore A matches III. For B, a carbon free radical is characterized by an unpaired electron, not by an empty orbital or a donated pair. List-II explicitly allows $sp^2/sp^3$ hybridized radical carbon with one unpaired electron, so B matches IV.

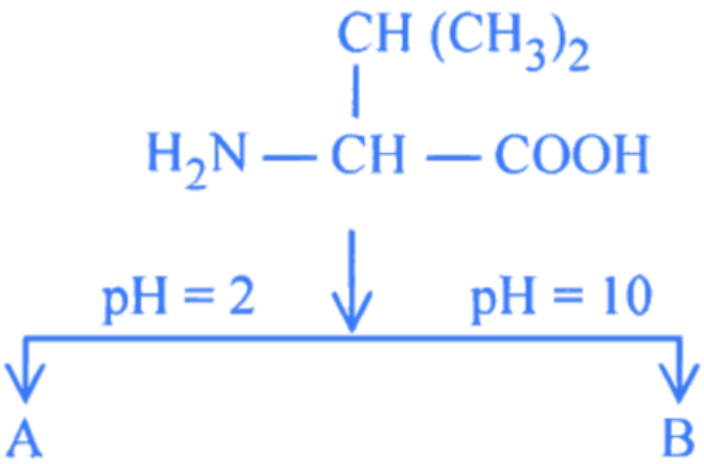
**Step 2:** Use electron-pair transfer to match the reagents A nucleophile has a lone pair or a pi-electron pair that can be supplied to an electron-deficient centre to form a covalent bond. Examples include $OH^-$ and $NH_3$; hence C matches I. An electrophile receives such an electron pair; examples include $H^+$ and electron-deficient Lewis acids. Thus D matches II. These definitions concern the direction of electron-pair transfer rather than simply whether the species is positively or negatively charged.

**Step 3:** Compare the complete combinations The resulting association is A-III, B-IV, C-I, D-II. Option (A) incorrectly identifies the carbocation with the free-radical description and reverses the donor/acceptor roles. Option (B) assigns the carbocation the general electrophile description but misassigns the radical and electrophile entries. Option (D) correctly matches the first two entries but interchanges nucleophile and electrophile. Only option (C) preserves all four definitions simultaneously.

**Step 4:** Conclusion The correct matching is A-III, B-IV, C-I, D-II. Hence, the correct option is (C).

**Quick Tip to Solve:** Remember that nucleophiles donate a pair and electrophiles accept a pair; an empty p orbital and an unpaired electron distinguish cations from radicals.

**70. Choose the correct option for structures of $A$ and $B$, respectively** [4 marks]




(A) $A = H_2N{-}CH(CH(CH_3)_2){-}COO^-$, $B = H_3N^+{-}CH(CH(CH_3)_2){-}COO^-$

(B) $A = H_3N^+{-}CH(CH(CH_3)_2){-}COO^-$, $B = H_3N^+{-}CH(CH(CH_3)_2){-}COOH$

(C) $A = H_3N^+{-}CH(CH(CH_3)_2){-}COOH$, $B = H_2N{-}CH(CH(CH_3)_2){-}COO^-$

(D) $A = H_2N{-}CH(CH(CH_3)_2){-}COO^-$, $B = H_3N^+{-}CH(CH(CH_3)_2){-}COOH$

**Correct Answer:** (C) $A = H_3N^+{-}CH(CH(CH_3)_2){-}COOH$, $B = H_2N{-}CH(CH(CH_3)_2){-}COO^-$

**Solution:**

**Concept:**

- Amino acids contain a basic amino group and an acidic carboxyl group. Their predominant charge state changes with pH through protonation and deprotonation.
- In an acidic medium the amino group is protonated and the carboxyl group is predominantly COOH; in a sufficiently alkaline medium the carboxylate and unprotonated amino group predominate.

**Step 1:** Identify the ionizable groups The given amino acid is valine, with side chain $-CH(CH_3)_2$. This hydrocarbon side chain does not acquire an acid-base charge under the stated conditions. The relevant sites are therefore the alpha-amino group and the carboxyl group. The neutral structural formula shown in the question is a convenient way to identify the connectivity; it does not mean the molecule keeps that charge state at both specified pH values.

**Step 2:** Write the acidic form A At pH 2, the medium is acidic. The amino nitrogen accepts a proton to give $-NH_3^+$, while the carboxyl group is represented in its protonated $-COOH$ form. Thus the acidic structure is

$$A = H_3N^+{-}CH(CH(CH_3)_2){-}COOH.$$

It has overall charge $+1$. A carboxylate with an unprotonated amino group would instead be characteristic of alkaline conditions, so it cannot represent A here.

**Step 3:** Write the alkaline form B and select the pair At pH 10, loss of the carboxylic proton gives

$-\mathrm{COO}-$, and the amino group is represented as $-\mathrm{NH_2}$. Therefore

$$B = \mathrm{H_2N{-}CH(CH(CH_3)_2){-}COO^-}.$$

Its overall charge is -1. Option (C) presents these forms in the required order. Option (A) begins with the alkaline form; option (B) begins with a zwitterion and ends with the acidic form; option (D) reverses the acidic and alkaline assignments. These alternatives contradict the direction of the pH change.

**Step 4:** Conclusion $A = \mathrm{H_3N^+{-}CH(CH(CH_3)_2){-}COOH}$ and $B = \mathrm{H_2N{-}CH(CH(CH_3)_2){-}COO^-}$. Hence, the correct option is (C).

**Quick Tip to Solve:** As pH rises, track proton loss in order: cationic amino acid, then zwitterion, then anionic amino acid. The neutral hydrocarbon side chain keeps the same connectivity throughout.

## Chemistry – Section B: Numerical Value Questions (20 marks)

**71. Consider the following half cell reaction**

$$\mathbf{Cr_2O_7^{2-}\,(aq) + 6e^- + 14H^+\,(aq) \rightarrow 2Cr^{3+}\,(aq) + 7H_2O\,(\ell)}$$

**The reaction was conducted with the ratio of $\frac{[\mathrm{Cr^{3+}}]^2}{[\mathrm{Cr_2O_7^{2-}}]} = 10^{-6}$. The pH value at which the EMF of the half cell will become zero is _____.**
**(nearest integer value)**
**[Given: standard half cell reduction potential $E^\circ_{\mathrm{Cr_2O_7^{2-}, H^+/Cr^{3+}}} = 1.33\,\mathrm{V}$, $\frac{2.303RT}{F} = 0.059\,\mathrm{V}$.]** [4 marks]

**Solution:**

**Concept:**

- For a reduction half-cell, the Nernst equation is $E = E^\circ - (0.059/n)\log_{10} Q$ under the given conditions, with $n$ the number of transferred electrons.
- Pure liquid water has unit activity and is omitted from the reaction quotient. Hydrogen ions appear to the power of their stoichiometric coefficient, and $\mathrm{pH} = -\log_{10}[\mathrm{H^+}]$.

**Step 1:** Form the reaction quotient The balanced half-reaction transfers six electrons, so $n = 6$. Its products contain two chromium(III) ions and its reactants contain one dichromate and fourteen hydrogen ions. Therefore

$$Q = \frac{[\mathrm{Cr^{3+}}]^2}{[\mathrm{Cr_2O_7}^{2-}][\mathrm{H^+}]^{14}}.$$

Using the supplied concentration ratio gives $Q = 10^{-6}/[\mathrm{H^+}]^{14}$. The electrons do not appear as dissolved reactants in this quotient.

**Step 2:** Express the logarithm in terms of pH Take the base-ten logarithm and use $\log_{10}[\mathrm{H^+}] = -\mathrm{pH}$:

$$\log_{10} Q = -6 - 14\log_{10}[\mathrm{H^+}] = -6 + 14\,\mathrm{pH}.$$

This sign is physically consistent: raising the pH lowers the hydrogen-ion concentration, increases the reaction quotient, and lowers the reduction potential for a reaction that consumes hydrogen ions.

**Step 3:** Set the potential to zero and round Insert $E^\circ = 1.33\,\mathrm{V}$ and set $E = 0$:

$$0 = 1.33 - \frac{0.059}{6}(-6 + 14\,\mathrm{pH}).$$

Hence

$$-6 + 14\,\mathrm{pH} = \frac{6(1.33)}{0.059} = 135.2542,$$

$$\mathrm{pH} = \frac{135.2542 + 6}{14} = 10.0896.$$

Rounding only at the end gives the nearest integer 10. The pH itself is dimensionless; the volt units cancel in applying the Nernst equation.

**Step 4:** Conclusion The required pH is approximately $10.09$, so its nearest integer value is $\boxed{10}$ (dimensionless).

**Quick Tip to Solve:** For a proton-consuming reduction, write the hydrogen-ion power explicitly before converting the reaction quotient to pH; this prevents a sign error.

**72. The energy of an electron in the first Bohr orbit of the H-atom is -13.6 eV.**
**The magnitude of energy value of an electron in the first excited state of $Be^{3+}$ is _____ eV (nearest integer value).** [4 marks]

**Solution:**

**Concept:**

- A one-electron ion follows the Bohr energy formula $E_n = -13.6\,Z^2/n^2$ eV, where $Z$ is the atomic number and $n$ is the principal quantum number.
- The ground state is $n = 1$, and the first excited state is $n = 2$. An energy level is negative relative to a free electron at infinity, while its magnitude is positive.

**Step 1:** Identify the hydrogen-like ion Beryllium has atomic number $Z = 4$, meaning that its nucleus contains four protons. The ion $\mathrm{Be}^{3+}$ has lost three of the four electrons of neutral beryllium and therefore has exactly one electron left. The hydrogen-like Bohr expression consequently applies. The charge $+3$ identifies the number of electrons removed; it is not the nuclear charge to insert as $Z$. Using $Z = 3$ would describe the wrong nucleus.

**Step 2:** Select the first excited orbit The question asks for the first excited state, not the first orbit. Starting from the ground state $n = 1$, the next allowed principal quantum number is $n = 2$. Relative to the first hydrogen orbit, the energy is scaled by $Z^2/n^2$:

$$\frac{Z^2}{n^2} = \frac{4^2}{2^2} = \frac{16}{4} = 4.$$

Thus this level is four times as strongly bound as the hydrogen ground level, even though it is an excited state of the beryllium ion.

**Step 3:** Calculate the magnitude and round Substitute into the energy expression:

$$E_2 = -13.6 \times \frac{16}{4}\ \text{eV} = -54.4\ \text{eV}.$$

The negative sign describes a bound state. Since the question explicitly requests the magnitude, use $|E_2| = 54.4\,\text{eV}$, rather than -54.4 eV. Its nearest integer value is 54 eV. This is the magnitude of the level energy, not the energy difference between two beryllium levels; no subtraction of the ground-state energy is required.

**Step 4:** Conclusion The energy magnitude is $54.4\,\text{eV}$, whose nearest integer value is $\boxed{54\,\text{eV}}$.

**Quick Tip to Solve:** For hydrogen-like ions, use the atomic number for Z, set n=2 for the first excited state, and take the absolute value only when a magnitude is requested.

**73. The equilibrium constant for decomposition of $H_2O(g)$**

$$\mathbf{H_2O(g) \rightleftharpoons H_2(g) + \frac{1}{2}O_2(g)} \quad (\Delta G^\circ = 92.34\,\mathbf{kJ\ mol^{-1}})$$

**is $8.0 \times 10^{-3}$ at 2300 K and total pressure at equilibrium is 1 bar. Under this condition, the degree of dissociation ($\alpha$) of water is _____ $\times 10^{-2}$ (nearest integer value).**

**[Assume $\alpha$ is negligible with respect to 1]** [4 marks]

**Solution:**

**Concept:**

- For a gaseous equilibrium, the dimensionless pressure equilibrium constant is the product of partial pressures divided by the standard pressure, raised to their stoichiometric powers. Here the standard pressure is $p^\circ = 1\,\text{bar}$.
- The degree of dissociation $\alpha$ is the fraction of the initial water that decomposes. Start with one mole of $H_2O$; the reaction makes one mole of $H_2$ and half a mole of $O_2$ per mole of water consumed.

**Step 1:** Write the equilibrium amounts The stoichiometric equation is

$$\mathrm{H_2O\,(g)} \rightleftharpoons \mathrm{H_2(g)} + \frac{1}{2}\,\mathrm{O_2(g)}.$$

From an initial one mole of water, the equilibrium amounts of water, hydrogen and oxygen are $1-\alpha$, $\alpha$ and $\alpha/2$ moles, respectively. Thus the total is $1+\alpha/2$ moles, rather than one mole. At total pressure $P$, each partial pressure is its mole fraction multiplied by $P$.

**Step 2:** Express the equilibrium constant Substitute the three partial pressures into the pressure equilibrium expression:

$$K_p = \frac{(p_{\mathrm{H_2}}/p^\circ)(p_{\mathrm{O_2}}/p^\circ)^{1/2}}{p_{\mathrm{H_2O}}/p^\circ} = \frac{\alpha}{1-\alpha}\sqrt{\frac{\alpha}{2(1+\alpha/2)}\frac{P}{p^\circ}}.$$

The question permits neglecting $\alpha$ relative to one. Also $P/p^\circ = 1$, because the total pressure is one

bar. Therefore

$$8.0 \times 10^{-3} = \alpha\sqrt{\frac{\alpha}{2}} = \frac{\alpha^{3/2}}{\sqrt{2}}.$$

**Step 3:** Calculate the requested coefficient Raise both sides after rearrangement to the power $2/3$:

$$\alpha = \left(\sqrt{2} \times 8.0 \times 10^{-3}\right)^{2/3} = 0.0504 = 5.04 \times 10^{-2}.$$

The blank asks for the coefficient of $10^{-2}$, not for $\alpha$ rounded directly to an integer. Rounding $5.04$ gives $5$. The supplied standard free energy is consistent with $K_p = \exp[-\Delta G^\circ/(RT)]$, but the given equilibrium constant already supplies the required information.

**Step 4:** Conclusion The degree of dissociation is $\alpha \simeq 5.04 \times 10^{-2}$ (dimensionless). The nearest-integer entry in the blank is $\boxed{5}$.

**Quick Tip to Solve:** For one mole of water dissociating, retain the half-mole oxygen coefficient: it produces the factor $1/\sqrt{2}$ in $K_p$. Round only the coefficient requested in the blank.

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**74. 20 mL of sodium iodide solution gave 4.74 g silver iodide when treated with excess of silver nitrate solution. The molarity of the sodium iodide solution is _____ M. (Nearest Integer value)**
**(Given : Na = 23, I = 127, Ag = 108, N = 14, O = 16 g mol$^{-1}$)** [4 marks]

**Solution:**

**Concept:**

- Molarity is the amount of solute divided by the volume of its solution in litres: $M = n/V$. The volume used is the original sodium iodide sample, not the volume after addition of precipitating reagent.
- Silver iodide precipitates in a one-to-one molar ratio with iodide: $\mathrm{Ag^+ + I^- \longrightarrow AgI(s)}$. Excess silver nitrate ensures that iodide is the limiting reactant and is precipitated completely.

**Step 1:** Write the precipitation stoichiometry The balanced molecular equation is

$$\mathrm{NaI(aq) + AgNO_3(aq) \longrightarrow AgI(s) + NaNO_3(aq)}.$$

One mole of sodium iodide supplies one mole of iodide ions, and each mole of iodide gives one mole of silver iodide. Thus the moles of the isolated precipitate equal the initial moles of sodium iodide in the aliquot. The masses of sodium, nitrogen and oxygen are not needed because the measured solid is AgI, not sodium nitrate or silver nitrate.

**Step 2:** Convert the precipitate mass to moles Using the given atomic masses, the molar mass of silver iodide is

$$M_{\mathrm{AgI}} = 108 + 127 = 235\,\mathrm{g\,mol^{-1}}.$$

Therefore the amount of silver iodide, and hence of sodium iodide in the sample, is

$$n_{\mathrm{NaI}} = n_{\mathrm{AgI}} = \frac{4.74\,\mathrm{g}}{235\,\mathrm{g\,mol^{-1}}} = 0.0201702\,\mathrm{mol}.$$

No extra factor of two occurs, since both ions are singly charged and the precipitation ratio is one to one.

**Step 3:** Divide by the original solution volume Convert the aliquot to litres before calculating molarity:

$$V = 20\,\mathrm{mL} = 0.020\,\mathrm{L}.$$

Then

$$M = \frac{0.0201702\,\mathrm{mol}}{0.020\,\mathrm{L}} = 1.00851\,\mathrm{mol\,L^{-1}}.$$

The extra decimal places are retained only to make the rounding clear. The prompt asks for the nearest integer rather than the unrounded concentration, so the required numerical entry is one.

**Step 4:** Conclusion The sodium iodide solution has molarity $1.0085\,\mathrm{mol\,L^{-1}}$, which is $\boxed{1\,\mathrm{M}}$ to the nearest integer. The required entry is $\boxed{1}$.

**Quick Tip to Solve:** First convert precipitate mass to moles, then use the balanced mole ratio. In a one-to-one precipitation, the original solute moles equal the precipitate moles, but their masses do not.

**75. Resonance in $X_2Y$ can be represented as**

**The enthalpy of formation of $X_2Y$ $\left(X = X(g) + \frac{1}{2}Y = Y(g) \rightarrow X_2Y(g)\right)$ is 80 kJ mol$^{-1}$. The magnitude of resonance energy of $X_2Y$ is __ kJ mol$^{-1}$ (nearest integer value).**

**Given: Bond energies of $X \equiv X$, $X = X$, $Y = Y$ and $X = Y$ are 940, 410, 500, and 602 kJ mol$^{-1}$ respectively.**

**valence $X$: 3, $Y$: 2** [4 marks]



**Solution:**

**Concept:**

- For a specified bond structure, the bond-energy estimate of reaction enthalpy is $\Delta H_{\mathrm{loc}} = \sum D(\text{bonds broken}) - \sum D(\text{bonds formed})$. Bond breaking requires energy and bond formation releases energy.
- Resonance stabilization is the lowering of the real molecule below a localized contributor. With the same reactant reference, its value is $E_{\mathrm{res}} = \Delta H_{\mathrm{loc}} - \Delta H_{\mathrm{actual}}$. The sign and the reactant bond order must both be checked.

**Step 1:** Identify the localized contributor The first resonance contributor drawn is $\mathrm{X}=\mathrm{X}=\mathrm{Y}$, with one $\mathrm{X}=\mathrm{X}$ bond and one $\mathrm{X}=\mathrm{Y}$ bond. Its bond-energy sum is

$$410+602=1012\,\mathrm{kJ\,mol^{-1}}.$$

The other contributor contains $\mathrm{X}\equiv\mathrm{X}-\mathrm{Y}$, but no single $\mathrm{X}-\mathrm{Y}$ bond energy is supplied. Use the first contributor for the localized-energy estimate; do not add the bond energies of both resonance contributors as though they were separate molecules.

**Step 2:** Check the formation equation exactly as printed The printed formation line writes $\mathrm{X}=\mathrm{X(g)}$, a double bond. Taken literally, its reactant bond-energy sum is

$$410+\frac{1}{2}(500)=660\,\mathrm{kJ\,mol^{-1}}.$$

That gives

$$\Delta H_{\mathrm{loc}}=660-1012=-352\,\mathrm{kJ\,mol^{-1}}.$$

With the stated actual formation enthalpy $+80\,\mathrm{kJ\,mol^{-1}}$, the algebraic difference is $-352-80=-432\,\mathrm{kJ\,mol^{-1}}$. Its magnitude is $432$, but its negative sign would mean destabilization rather than the usual resonance stabilization. Thus the printed double bond creates an internal inconsistency with the accepted answer.

**Step 3:** Explain the accepted numerical answer The accepted entry $98$ follows if the elemental reactant is the triply bonded $\mathrm{X}\equiv\mathrm{X}$, consistent with the supplied $940\,\mathrm{kJ\,mol^{-1}}$ bond energy and valence three for X. Under that interpretation,

$$\begin{aligned}\Delta H_{\mathrm{loc}}&=940+\tfrac{1}{2}(500)-(410+602)\\&=178\,\mathrm{kJ\,mol^{-1}},\\E_{\mathrm{res}}&=178-80=98\,\mathrm{kJ\,mol^{-1}}.\end{aligned}$$

This calculation states the bond-order interpretation explicitly; it does not follow from the displayed double-bond formation equation without that correction.

**Step 4:** Conclusion The accepted numerical entry is $\boxed{98}$, representing $98\,\mathrm{kJ\,mol^{-1}}$ of resonance stabilization when the reactant is $\mathrm{X}\equiv\mathrm{X}$. As literally printed with $\mathrm{X}=\mathrm{X}$, the magnitude of the enthalpy difference is instead $432\,\mathrm{kJ\,mol^{-1}}$; the printed bond order and the accepted key are inconsistent.

**Quick Tip to Solve:** Use the same elemental reference for the localized and actual molecule, then subtract actual formation enthalpy from the localized estimate. Check the reactant bond order before substituting a bond energy.