JEE Main 2025 Paper 1 (B.E./B.Tech) Jan 24 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2025, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 24 January 2025, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. In an arithmetic progression, if $S_{40} = 1030$ and $S_{12} = 57$, then $S_{30} - S_{10}$ is equal to : [4 marks]

(A) 525

(B) 505

(C) 510

(D) 515

Correct Answer: (D) 515

Solution:

Concept:

- For an arithmetic progression with first term a and common difference d, the sum of its first n terms is $S_n = \frac{n}{2}[2a + (n-1)d]$.
- The difference $S_{30} - S_{10}$ is the sum of terms numbered 11 through 30. First determine the same progression from both given sums; do not treat the two sums as individual terms.

Step 1: Form equations from the given sums Substitute the two values of n in the sum formula, retaining the factors outside the brackets. We obtain

$$20(2a + 39d) = 1030, \qquad 6(2a + 11d) = 57.$$

Dividing each equation by its prefactor gives

$$2a + 39d = \frac{103}{2}, \qquad 2a + 11d = \frac{19}{2}.$$

These are two linear equations for the first term and common difference.

Step 2: Find the first term and common difference Subtract the second equation from the first to eliminate a:

$$28d = 42 \quad \Rightarrow \quad d = \frac{3}{2}.$$

Returning to the second equation,

$$2a = \frac{19}{2} - 11 \cdot \frac{3}{2} = -7, \qquad a = -\frac{7}{2}.$$

A negative first term is permissible; the positive common difference makes later terms larger.

Step 3: Evaluate the required difference Use the values just found in both sums:

$$S_{30} = 15\left(-7 + 29 \cdot \frac{3}{2}\right) = \frac{1095}{2},$$

$$S_{10} = 5\left(-7 + 9 \cdot \frac{3}{2}\right) = \frac{65}{2}.$$

Therefore

$$S_{30} - S_{10} = \frac{1095 - 65}{2} = 515.$$

As a direct check, terms 11 and 30 are $23/2$ and 40, so the sum of these twenty consecutive terms is $10(23/2 + 40) = 515$. The other listed values fail this endpoint-average check.

Step 4: Conclusion $S_{30} - S_{10} = 515$. Hence, the correct option is (D).

Quick Tip to Solve: Divide S_n by $n/2$ before subtracting equations; this eliminates the first term immediately.

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2. Let $\mathrm{A} = [\mathrm{a_{ij}}]$ be a square matrix of order 2 with entries either 0 or 1 . Let E be the event that A is an invertible matrix. Then the probability $\mathrm{P(E)}$ is : [4 marks]

(A) $\frac{3}{8}$
(B) $\frac{1}{8}$
(C) $\frac{3}{16}$
(D) $\frac{5}{8}$

Correct Answer: (A) $\frac{3}{8}$

Solution:

Concept:

- A square matrix is invertible precisely when its determinant is nonzero. For $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, this determinant is $ad - bc$.
- Each of the four entries independently has two possible values, so the equally likely sample space contains $2^4 = 16$ matrices. The probability is the number of invertible matrices divided by 16.

Step 1: Identify all possible nonzero determinants The products ad and bc can each be only 0 or 1, because every entry is binary. Consequently, their difference is nonzero in exactly two disjoint situations:

$$ad = 1,\ bc = 0, \qquad \text{or} \qquad ad = 0,\ bc = 1.$$

Equal products give determinant zero and therefore must be excluded. This criterion counts the matrices directly without any assumption about typical real-valued matrices.

Step 2: Count the two disjoint cases If $ad = 1$, then $a = d = 1$ is forced. To make $bc = 0$, the pair (b, c) can be $(0, 0), (0, 1), (1, 0)$: three choices. Similarly, if $bc = 1$, then $b = c = 1$ is forced, while (a, d) has the same three choices making $ad = 0$. Thus

$$N(E) = 3 + 3 = 6.$$

These cases do not overlap, since a determinant cannot simultaneously be positive and negative.

Step 3: Divide by the sample-space size The favorable six matrices are among the sixteen possible matrices, hence

$$P(E) = \frac{6}{16} = \frac{3}{8}.$$

Equivalently, ten matrices are singular and have probability $10/16 = 5/8$; that is the complement rather than the requested event. The values $1/8$ and $3/16$ omit some of the six invertible matrices and therefore cannot be the required probability.

Step 4: Conclusion The probability of invertibility is $3/8$. Hence, the correct option is (A).

Quick Tip to Solve: For binary entries, count the cases $ad = 1, bc = 0$ and $ad = 0, bc = 1$ separately; each contributes three matrices.

3. Let $(2, 3)$ be the largest open interval in which the function $f(x) = 2\log_e(x-2) - x^2 + ax + 1$ is strictly increasing and (b, c) be the largest open interval, in which the function $g(x) = (x-1)^3(x+2-a)^2$ is strictly decreasing. Then $100(a+b-c)$ is equal to : [4 marks]

(A) 360

(B) 420

(C) 160

(D) 280

Correct Answer: (A) 360

Solution:

Concept:

- On an interval where a differentiable function has positive derivative it is strictly increasing; a negative derivative gives strict decrease. The real domain of the logarithmic function here is $x > 2$.
- At an interior finite boundary between increase and decrease, continuity of the derivative forces it to vanish. A repeated even-multiplicity factor of a derivative does not change its sign.

Step 1: Determine the parameter from the increasing interval Differentiate on $x > 2$:

$$f\prime(x) = \frac{2}{x-2} - 2x + a.$$

Since the largest interval of strict increase ends at 3, the continuous derivative must be zero there; otherwise its sign would persist locally on the same side of that endpoint. Thus

$$f\prime(3) = 2 - 6 + a = 0 \quad \Rightarrow \quad a = 4.$$

Verify the full interval rather than using only the endpoint:

$$f\prime(x) = \frac{2[1-(x-2)^2]}{x-2} = \frac{-2(x-1)(x-3)}{x-2}.$$

This is positive for $2 < x < 3$ and negative for $x > 3$, as required.

Step 2: Find the decreasing interval of the second function With $a = 4$, write $g(x) = (x-1)^3(x-2)^2$. The product rule gives

$$\begin{aligned} g\prime(x) &= 3(x-1)^2(x-2)^2 + 2(x-1)^3(x-2) \\ &= (x-1)^2(x-2)(5x-8). \end{aligned}$$

The square factor is nonnegative, so the derivative is negative exactly when the two remaining factors have opposite signs:

$$\frac{8}{5} < x < 2.$$

Outside this interval their product is positive. The isolated zero at $x = 1$ creates no decreasing interval, because its square factor does not reverse the sign.

Step 3: Combine the interval endpoints The largest decreasing interval is therefore $(b, c) = (8/5, 2)$. Substitute these endpoints together with $a = 4$:

$$100(a + b - c) = 100\left(4 + \frac{8}{5} - 2\right) = 100 \cdot \frac{18}{5} = 360.$$

The other options would require different endpoints or a different parameter, each contradicting the derivative sign tests above.

Step 4: Conclusion $100(a + b - c) = 360$. Hence, the correct option is (A).

Quick Tip to Solve: Factor derivatives fully and separate even-power factors before drawing a sign chart; isolated zeros need not change monotonicity.

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4. Let $\mathrm{A} = \left\{x \in (0, \pi) - \left\{\frac{\pi}{2}\right\} : \log_{(2/\pi)} |\sin x| + \log_{(2/\pi)} |\cos x| = 2\right\}$ **and** $\mathrm{B} = \{x \geqslant 0 : \sqrt{x}(\sqrt{x} - 4) - 3|\sqrt{x} - 2| + 6 = 0\}$**. Then** $\mathrm{n}(\mathrm{A} \cup \mathrm{B})$ **is equal to :** [4 marks]

(A) 4

(B) 8

(C) 6

(D) 2

Correct Answer: (B) 8

Solution:

Concept:

- For a positive base different from 1, $\log_k u + \log_k v = \log_k(uv)$ when both arguments are positive. The excluded points ensure this condition for the trigonometric logarithms.
- Count a union with $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. In the second equation the substitution $t = \sqrt{x} \geq 0$ makes the absolute-value expression a quadratic in $|t - 2|$.

Step 1: Count the trigonometric solutions Combining logarithms and exponentiating to the same base gives

$$|\sin x \cos x| = \left(\frac{2}{\pi}\right)^2, \qquad |\sin 2x| = \frac{8}{\pi^2}.$$

The right-hand side lies strictly between 0 and 1. As x runs through $(0, \pi)$, the angle $2x$ runs through $(0, 2\pi)$; the absolute sine assumes each value strictly between 0 and 1 four times. Thus $n(A) = 4$. Writing $\alpha = \frac{1}{2}\arcsin(8/\pi^2)$, these four values are $\alpha, \pi/2 - \alpha, \pi/2 + \alpha, \pi - \alpha$. None is an excluded endpoint.

Step 2: Solve the equation defining the second set Put $t = \sqrt{x} \geq 0$ and $u = t - 2$. The equation becomes

$$t^2 - 4t - 3|t - 2| + 6 = 0 \quad \Longleftrightarrow \quad u^2 - 3|u| + 2 = 0.$$

Since $u^2 = |u|^2$, factor it as

$$(|u| - 1)(|u| - 2) = 0.$$

Hence $u = \pm 1, \pm 2$, giving $t = 1, 3, 0, 4$, all nonnegative. Squaring yields

$$B = \{0, 1, 9, 16\}, \qquad n(B) = 4.$$

No extraneous root is introduced because each retained value satisfies $t \geq 0$.

Step 3: Check the intersection before adding The set A lies in $(0, \pi)$, so only 1 from B could lie in A. At $x = 1$, however, $|\sin 2x| = \sin 2 \approx 0.90930$, whereas $8/\pi^2 \approx 0.81057$. Thus 1 fails its equation and $A \cap B$ is empty. Therefore

$$n(A \cup B) = 4 + 4 - 0 = 8.$$

Counting only one set gives 4; 6 or 2 cannot result from the verified empty intersection.

Step 4: Conclusion $n(A \cup B) = 8$. Hence, the correct option is (B).

Quick Tip to Solve: For a square-root equation, substitute $t = \sqrt{x}$ first and retain $t \geq 0$; check overlap before adding set sizes.

5. The area of the region enclosed by the curves $y = \mathrm{e}^x, y = |\mathrm{e}^x - 1|$ and y-axis is : [4 marks]

(A) $1 + \log_{\mathrm{e}} 2$

(B) $\log_{\mathrm{e}} 2$

(C) $1 - \log_{\mathrm{e}} 2$

(D) $2 \log_{\mathrm{e}} 2 - 1$

Correct Answer: (C) $1 - \log_{\mathrm{e}} 2$

Solution:

Concept:

- The area between an upper curve $y = u(x)$ and a lower curve $y = l(x)$ from a to b is $\int_a^b [u(x) - l(x)]\, dx$, provided their order is identified correctly.
- The absolute value changes form at $x = 0$: $|e^x - 1| = 1 - e^x$ on the negative half-line and $|e^x - 1| = e^x - 1$ on the positive half-line. The stated vertical boundary is the y-axis, or $x = 0$.

Step 1: Find where the two curves meet For $x \geq 0$, the equation $e^x = |e^x - 1|$ would give $e^x = e^x - 1$, which is impossible. For $x < 0$, it instead becomes

$$e^x = 1 - e^x \quad \Rightarrow \quad e^x = \frac{1}{2} \quad \Rightarrow \quad x = -\ln 2.$$

The intersection is $(-\ln 2, 1/2)$. Together with the y-axis, this determines the bounded region on the left of that axis. The positive half-line provides no second closed region.

Step 2: Identify the upper and lower boundaries On the interval $[-\ln 2, 0]$, the exponential e^x ranges from $1/2$ to 1. Thus

$$e^x - (1 - e^x) = 2e^x - 1 \geq 0.$$

Therefore the upper curve is $y = e^x$, and the lower curve is $y = 1 - e^x$. This sign check prevents reversing the integral or mistakenly removing the absolute value as $e^x - 1$ in the negative interval.

Step 3: Integrate the vertical separation The required area is

$$\begin{aligned}\mathcal{A} &= \int_{-\ln 2}^{0} (2e^x - 1)\, dx \\ &= [2e^x - x]_{-\ln 2}^{0} \\ &= 2 - (1 + \ln 2) \\ &= 1 - \ln 2.\end{aligned}$$

It is positive, approximately 0.30685 square units. The values $1 + \ln 2$ and $\ln 2$ do not subtract the lower boundary correctly. The expression $2\ln 2 - 1$ also differs from the evaluated integral and fails the direct antiderivative calculation.

Step 4: Conclusion The enclosed area is $1 - \ln 2$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Resolve an absolute value on the actual bounded interval before subtracting curves; here the entire closed region has $x \le 0$.

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6. For some a, b, let $f(x) = \begin{vmatrix} a + \frac{\sin x}{x} & 1 & b \\ a & 1 + \frac{\sin x}{x} & b \\ a & 1 & b + \frac{\sin x}{x} \end{vmatrix}, x \neq 0, \lim_{x \to 0} f(x) = \lambda + \mu a + \nu b$. Then $(\lambda + \mu + v)^2$ is equal to : [4 marks]

(A) 25

(B) 16

(C) 9

(D) 36

Correct Answer: (B) 16

Solution:

Concept:

- The standard limit $\lim_{x \to 0} (\sin x)/x = 1$ allows substitution in every entry here because a determinant is a polynomial in its entries and hence continuous.
- Elementary row replacement does not change a determinant. Subtracting the first row from the other two isolates the repeated entries and makes expansion particularly short.

Step 1: Take the limit entry by entry Set $t = \sin x/x$. As $x \to 0$, $t \to 1$, so the limiting determinant is

$$L = \begin{vmatrix} a+1 & 1 & b \\ a & 2 & b \\ a & 1 & b+1 \end{vmatrix}.$$

The original function need not be defined at $x = 0$ for this limit to exist. All nine entries have finite

limits, and polynomial continuity of the determinant justifies this operation without dividing by an expression involving the unknown parameters.

Step 2: Evaluate the simplified determinant Apply $R_2 \leftarrow R_2 - R_1$ and $R_3 \leftarrow R_3 - R_1$, keeping the first row unchanged:

$$L = \begin{vmatrix} a+1 & 1 & b \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}.$$

Expanding along the first row, with the alternating cofactor signs, gives

$$L = (a+1)(1) - 1(-1) + b(1) = a + b + 2.$$

As a check before taking the limit, the same row operations with t in place of 1 give $f(x) = t^2(a+b+1+t)$, whose limit is again $a+b+2$.

Step 3: Compare the coefficients and calculate The stated limiting expression is $\lambda+\mu a+\nu b$. Comparing its constant and linear coefficients with the evaluated determinant yields

$$\lambda = 2, \qquad \mu = 1, \qquad \nu = 1.$$

Consequently

$$(\lambda + \mu + \nu)^2 = (2+1+1)^2 = 16.$$

Squaring only the constant would give 4, while using a wrong cofactor sign could lead to other totals. None of 25, 9 or 36 agrees with the verified coefficient sum of 4.

Step 4: Conclusion $(\lambda + \mu + \nu)^2 = 16$. Hence, the correct option is (B).

Quick Tip to Solve: When each diagonal receives the same term, subtract the first row from the others before expanding the determinant.

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7. If $\alpha > \beta > \gamma > 0$, then the expression $\cot^{-1}\left\{\beta + \frac{(1+\beta^2)}{(\alpha-\beta)}\right\} + \cot^{-1}\left\{\gamma + \frac{(1+\gamma^2)}{(\beta-\gamma)}\right\} + \cot^{-1}\left\{\alpha + \frac{(1+\alpha^2)}{(\gamma-\alpha)}\right\}$ is equal to : [4 marks]

(A) 3π

(B) $\frac{\pi}{2} - (\alpha + \beta + \gamma)$

(C) π

(D) 0

Correct Answer: (C) π

Solution:

Concept:

- The principal value of $\cot^{-1} t$ lies in $(0, \pi)$. Thus an angle outside that interval must be adjusted by a multiple of π, even if its cotangent is correct.
- The subtraction formula is $\cot(u-v) = (\cot u \cot v + 1)/(\cot v - \cot u)$. Introduce inverse-cotangent

angles to recognise the three rational expressions.

Step 1: Introduce ordered angles Put $A = \cot^{-1}\alpha$, $B = \cot^{-1}\beta$, and $C = \cot^{-1}\gamma$. Since cotangent is decreasing and all three numbers are positive, $0 < A < B < C < \pi/2$. Simplify the first argument:

$$\beta + \frac{1+\beta^2}{\alpha-\beta} = \frac{1+\alpha\beta}{\alpha-\beta} = \cot(B-A).$$

Here $B - A$ lies in the principal interval, so the first inverse cotangent equals $B - A$.

Step 2: Resolve the other principal values Similarly, the second argument is $(1+\beta\gamma)/(\beta-\gamma) = \cot(C-B)$, giving $C - B$. The third argument is

$$\alpha + \frac{1+\alpha^2}{\gamma-\alpha} = \frac{1+\alpha\gamma}{\gamma-\alpha} = \cot(A-C).$$

But $-\pi/2 < A - C < 0$, so this negative angle is not a principal inverse-cotangent value. Add π: its principal value is $\pi + A - C$, between $\pi/2$ and π.

Step 3: Add and compare the options The angle differences cancel exactly:

$$(B-A) + (C-B) + (\pi + A - C) = \pi.$$

A sum of three principal values cannot be zero, ruling out (D), and the exact cancellation excludes 3π. The result is constant, whereas (B) depends on the three original numbers.

Step 4: Conclusion The expression equals $\boxed{\pi}$. Hence, the correct option is (C).

Quick Tip to Solve: For inverse-cotangent sums, write each argument as a cotangent difference and check the principal interval before cancelling the angles.

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8. If the system of equations

$$\begin{aligned} x + 2y - 3z &= 2 \\ 2x + \lambda y + 5z &= 5 \\ 14x + 3y + \mu z &= 33 \end{aligned}$$

has infinitely many solutions, then $\lambda + \mu$ is equal to : [4 marks]

(A) 13

(B) 10

(C) 12

(D) 11

Correct Answer: (C) 12

Solution:

Concept:

- Infinitely many solutions require the coefficient matrix and augmented matrix to have the same rank, smaller than the number of unknowns. Thus the third equation must be a consistent linear combination of two independent equations.
- Using the constants as well as the coefficients prevents the mistake of requiring only a zero determinant, which may also describe an inconsistent system.

Step 1: Find the required linear combination Let the third equation equal a times the first plus b times the second. Matching the coefficient of x and the right-hand side gives

$$a + 2b = 14, \qquad 2a + 5b = 33.$$

Subtracting twice the first relation from the second yields $b = 5$, and then $a = 4$. Thus the entire third augmented row must be four times the first plus five times the second.

Step 2: Match the remaining coefficients The coefficient of y in this combination is $4(2) + 5\lambda$, and that of z is $4(-3) + 5(5)$. Therefore

$$8 + 5\lambda = 3 \quad \Longrightarrow \quad \lambda = -1,$$

$$\mu = -12 + 25 = 13.$$

The constants also match: $4(2) + 5(5) = 33$, so no contradiction is introduced.

Step 3: Verify the rank and select the value At $\lambda = -1$, the first two equations have the coefficient minor

$$\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -5 \neq 0.$$

They are independent. The third is their linear combination, so both ranks are exactly two; with three unknowns there is one free parameter and infinitely many solutions. Finally $\lambda + \mu = -1 + 13 = 12$, excluding 13, 10 and 11.

Step 4: Conclusion $\boxed{\lambda + \mu = 12}$. Hence, the correct option is (C).

Quick Tip to Solve: For an infinite-solution parameter problem, match a full augmented row as a linear combination, then verify that two remaining rows are independent.

9. Suppose A and B are the coefficients of 30^{th} and 12^{th} terms respectively in the binomial expansion of $(1+x)^{2n-1}$. If $2\,\mathrm{A} = 5\,\mathrm{B}$, then n is equal to: [4 marks]

(A) 20

(B) 19

(C) 22

(D) 21

Correct Answer: (D) 21

Solution:

Concept:

- In $(1+x)^m$, the coefficient of the $(r+1)$th term is $\binom{m}{r}$. The term number is one greater than the lower index.
- Use symmetry $\binom{m}{r} = \binom{m}{m-r}$ and the adjacent-coefficient ratio $\binom{m}{r+1}/\binom{m}{r} = (m-r)/(r+1)$ to avoid large factorial calculations.

Step 1: Translate the given coefficients Write $m = 2n-1$. The 30th term has power x^{29}, while the 12th has power x^{11}. Hence

$$A = \binom{m}{29}, \qquad B = \binom{m}{11}, \qquad \frac{A}{B} = \frac{5}{2}.$$

In particular $m \geq 29$, since the expansion must contain its 30th term. Using 30 and 12 directly as lower indices would give the wrong equation.

Step 2: Test the symmetric adjacent pair For the listed value $n = 21$, we have $m = 41$. Symmetry makes the required coefficients adjacent:

$$\binom{41}{29} = \binom{41}{12}.$$

Therefore

$$\frac{A}{B} = \frac{\binom{41}{12}}{\binom{41}{11}} = \frac{41-11}{12} = \frac{30}{12} = \frac{5}{2}.$$

This proves that $n = 21$ satisfies the exact relation $2A = 5B$, without approximating either large coefficient.

Step 3: Establish uniqueness among the options For integers $m \geq 29$, cancellation of factorials gives

$$\frac{\binom{m}{29}}{\binom{m}{11}} = \frac{11!}{29!}(m-28)(m-27)\cdots(m-11).$$

Every factor on the right is positive and strictly increases with m. The ratio is therefore strictly increasing. Values $n = 19$ and 20 yield ratios smaller than $5/2$, while $n = 22$ yields a larger one. Thus none of the other listed choices satisfies the equation.

Step 4: Conclusion $\boxed{n = 21}$. Hence, the correct option is (D).

Quick Tip to Solve: First reduce the term numbers by one. Then exploit coefficient symmetry to turn a distant pair into adjacent binomial coefficients.

10. Let the position vectors of three vertices of a triangle be $4\vec{p}+\vec{q}-3\vec{r}, -5\vec{p}+\vec{q}+2\vec{r}$ and $2\vec{p}-\vec{q}+2\vec{r}$. If the position vectors of the orthocenter and the circumcenter of the triangle are $\frac{\vec{p}+\vec{q}+\vec{r}}{4}$ and $\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}$ respectively, then $\alpha+2\beta+5\gamma$ is equal to : [4 marks]

(A) 4

(B) 3

(C) 1

(D) 6

Correct Answer: (B) 3

Solution:

Concept:

- For triangle vertex vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$, circumcentre $\mathbf{o}$, and orthocentre $\mathbf{h}$, the Euler relation is $\mathbf{h} = \mathbf{a}+\mathbf{b}+\mathbf{c}-2\mathbf{o}$.
- The relation holds for position vectors measured from any common origin. It allows the circumcentre to be obtained without solving perpendicular-bisector equations.

Step 1: Justify the centre relation Take the circumcentre temporarily as origin and let the vertex vectors be $\mathbf{u}, \mathbf{v}, \mathbf{w}$. They have equal lengths. The vector $\mathbf{u}+\mathbf{v}+\mathbf{w}$ is the orthocentre because the line from the first vertex has direction $\mathbf{v}+\mathbf{w}$, perpendicular to the opposite side: $(\mathbf{v}+\mathbf{w})\cdot(\mathbf{v}-\mathbf{w})=0$. The same check holds for the other altitudes. Translating back gives $\mathbf{h}=\mathbf{a}+\mathbf{b}+\mathbf{c}-2\mathbf{o}$.

Step 2: Add the vertex vectors Collect coefficients in the three given positions:

$$\mathbf{a}+\mathbf{b}+\mathbf{c} = (4-5+2)\vec{p}+(1+1-1)\vec{q}+(-3+2+2)\vec{r}.$$

Thus the sum is $\vec{p}+\vec{q}+\vec{r}$. The given orthocentre is one quarter of this sum. Consequently

$$2\mathbf{o} = \left(1-\frac{1}{4}\right)(\vec{p}+\vec{q}+\vec{r}), \qquad \mathbf{o} = \frac{3}{8}(\vec{p}+\vec{q}+\vec{r}).$$

Step 3: Evaluate the requested combination Reading the coefficients gives $\alpha=\beta=\gamma=3/8$. Hence

$$\alpha+2\beta+5\gamma = \frac{3}{8}(1+2+5) = 3.$$

The weights 1, 2 and 5 must be applied after finding the circumcentre coefficients; summing them without these weights would not answer the question. The other numerical options differ from this exact value.

Step 4: Conclusion $\boxed{\alpha+2\beta+5\gamma=3}$. Hence, the correct option is (B).

Quick Tip to Solve: When vertex vectors and one triangle centre are supplied, try the Euler relation before calculating individual perpendicular bisectors.

11. If $7 = 5 + \frac{1}{7}(5+\alpha) + \frac{1}{7^2}(5+2\alpha) + \frac{1}{7^3}(5+3\alpha) + \ldots\ldots\ldots\ldots\infty$, then the value of α is : [4 marks]

(A) $\frac{1}{7}$

(B) 1

(C) $\frac{6}{7}$

(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- For $|r| < 1$, $\sum_{k=0}^{\infty} r^k = 1/(1-r)$ and $\sum_{k=0}^{\infty} kr^k = r/(1-r)^2$. The second identity follows by differentiating the geometric sum and multiplying by r.
- A series whose numerator is linear in its index can be split into an ordinary geometric sum and an index-weighted geometric sum. Here the common ratio is $r = 1/7$, so both sums converge.

Step 1: Express the general term correctly The initial term 5 corresponds to $k = 0$; it must be included rather than added a second time. The complete right-hand side is

$$\sum_{k=0}^{\infty}(5+k\alpha)r^k = 5\sum_{k=0}^{\infty} r^k + \alpha\sum_{k=0}^{\infty} kr^k, \qquad r = \frac{1}{7}.$$

Since $|r| < 1$, the two convergent sums can be evaluated separately.

Step 2: Evaluate the two sums Substitute $r = 1/7$ into the standard expressions:

$$5\sum_{k=0}^{\infty} r^k = \frac{5}{1-1/7} = \frac{35}{6},$$

$$\alpha\sum_{k=0}^{\infty} kr^k = \alpha\frac{1/7}{(1-1/7)^2} = \frac{7\alpha}{36}.$$

Thus the printed equality reduces to $7 = 35/6 + 7\alpha/36$. The denominator in the second sum is squared; overlooking this square changes the answer.

Step 3: Solve and check the parameter Isolate the parameter term:

$$\frac{7\alpha}{36} = 7 - \frac{35}{6} = \frac{7}{6}, \qquad \alpha = \frac{7}{6}\frac{36}{7} = 6.$$

Putting it back gives $35/6 + 42/36 = 35/6 + 7/6 = 7$, as required. The coefficient of α is nonzero, so the equation has only one solution; the choices $1/7$, 1 and $6/7$ therefore cannot satisfy it.

Step 4: Conclusion $\boxed{\alpha = 6}$. Hence, the correct option is (D).

Quick Tip to Solve: For numerators such as $a + kd$, split the infinite sum into $a/(1-r) + dr/(1-r)^2$, taking care to start the index at zero.

12. If the equation of the parabola with vertex $V\left(\frac{3}{2}, 3\right)$ and the directrix $x + 2y = 0$ is $\alpha x^2 + \beta y^2 - \gamma xy - 30x - 60y + 225 = 0$, then $\alpha + \beta + \gamma$ is equal to : [4 marks]

(A) 6

(B) 8

(C) 7

(D) 9

Correct Answer: (D) 9

Solution:

Concept:

- A parabola consists of points whose distance from the focus equals their perpendicular distance from the directrix. Its vertex lies halfway between the focus and the foot of the perpendicular from the focus to the directrix.
- The distance from (x, y) to $ax + by + c = 0$ is $|ax + by + c|/\sqrt{a^2 + b^2}$. Squaring the equality removes the absolute value without losing any distance condition.

Step 1: Locate the foot and focus The directrix $x + 2y = 0$ has normal vector $(1, 2)$. The vertex $V = (3/2, 3) = (3/2)(1, 2)$ lies on the normal through the origin, and the origin itself lies on the directrix. Therefore the perpendicular foot is $D = (0, 0)$. Since the vertex is the midpoint of D and the focus,

$$F = 2V - D = (3, 6).$$

The focus is on the same side of the directrix as the vertex.

Step 2: Apply the distance definition For an arbitrary point (x, y) on the parabola, the focus distance and directrix distance satisfy

$$\sqrt{(x-3)^2 + (y-6)^2} = \frac{|x + 2y|}{\sqrt{5}}.$$

Squaring and multiplying by 5 gives

$$5\left((x-3)^2 + (y-6)^2\right) = (x + 2y)^2.$$

Both original distances are nonnegative, so this squared equation is equivalent to their equality.

Step 3: Expand and compare coefficients Expanding the left side gives $5x^2 + 5y^2 - 30x - 60y + 225$; the right side is $x^2 + 4xy + 4y^2$. Move all terms to the left:

$$4x^2 + y^2 - 4xy - 30x - 60y + 225 = 0.$$

The linear and constant terms agree with the form in the question, so no rescaling is needed. Comparing coefficients yields $\alpha = 4$, $\beta = 1$, and $\gamma = 4$, since the mixed term is written with a minus sign. Their sum is 9, excluding 6, 8 and 7.

Step 4: Conclusion $\boxed{\alpha + \beta + \gamma = 9}$. Hence, the correct option is (D).

Quick Tip to Solve: Project the vertex onto the directrix first; reflecting that foot in the vertex gives the focus, after which the distance definition supplies the equation.

13. Let $\overrightarrow{a} = 3\hat{i} - \hat{j} + 2\hat{k}$, $\overrightarrow{b} = \overrightarrow{a} \times (\hat{i} - 2\hat{k})$ and $\overrightarrow{c} = \overrightarrow{b} \times \hat{k}$. Then the projection of $\overrightarrow{c} - 2\hat{j}$ on $\vec{a}$ is : [4 marks]

(A) $2\sqrt{7}$

(B) $3\sqrt{7}$

(C) $\sqrt{14}$

(D) $2\sqrt{14}$

Correct Answer: (D) $2\sqrt{14}$

Solution:

Concept:

- For vectors $\vec{u}$ and nonzero $\vec{a}$, the scalar projection of $\vec{u}$ on $\vec{a}$ is $\vec{u} \cdot \vec{a}/|\vec{a}|$. It is a scalar, unlike the vector projection.
- Evaluate each cross product in its specified order using the determinant rule. Reversing that order changes its sign, so both intermediate vectors must be checked before taking the dot product.

Step 1: Calculate the first cross product Write the vectors as ordered Cartesian components: $\vec{a} = (3, -1, 2)$ and $\hat{i} - 2\hat{k} = (1, 0, -2)$. Hence

$$\begin{aligned}\vec{b} &= \vec{a} \times (\hat{i} - 2\hat{k}) \\ &= \big((-1)(-2) - 2(0),\ 2(1) - 3(-2),\ 3(0) - (-1)(1)\big) \\ &= (2, 8, 1).\end{aligned}$$

As a sign check, $\vec{a} \cdot \vec{b} = 6 - 8 + 2 = 0$, as required because a cross product is perpendicular to each factor.

Step 2: Obtain the vector to be projected The next cross product is with $\hat{k} = (0, 0, 1)$, on the right. Thus

$$\vec{c} = (2, 8, 1) \times (0, 0, 1) = (8, -2, 0).$$

Subtracting $2\hat{j}$ changes only the second component:

$$\vec{u} = \vec{c} - 2\hat{j} = (8, -4, 0).$$

In particular, the second component is negative; replacing it by zero would incorrectly cancel a term that has the same sign.

Step 3: Divide the dot product by the magnitude Now compute the dot product and the length of the direction vector separately:

$$\vec{u} \cdot \vec{a} = 8(3) + (-4)(-1) + 0(2) = 28, \qquad |\vec{a}| = \sqrt{9 + 1 + 4} = \sqrt{14}.$$

Therefore the scalar projection is

$$\frac{28}{\sqrt{14}} = 2\sqrt{14}.$$

Options (A) and (B) have the wrong radical, while option (C) is half the required projection. Only (D) agrees with both the dot product and the normalization.

Step 4: Conclusion The projection of $\vec{c} - 2\hat{j}$ on $\vec{a}$ is $\boxed{2\sqrt{14}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use $(u, v, w) \times \hat{k} = (v, -u, 0)$ for the second cross product, then divide the dot product by $|\vec{a}|$, not by $|\vec{a}|^2$.

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14. The equation of the chord, of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, whose mid-point is $(3, 1)$ is : [4 marks]

(A) $5x + 16y = 31$

(B) $48x + 25y = 169$

(C) $4x + 122y = 134$

(D) $25x + 101y = 176$

Correct Answer: (B) $48x + 25y = 169$

Solution:

Concept:

- For the ellipse $x^2/a^2 + y^2/b^2 = 1$, a chord whose midpoint is (h, k) has equation $hx/a^2 + ky/b^2 = h^2/a^2 + k^2/b^2$, usually abbreviated as $T = S_1$.
- The midpoint equation differs from the tangent equation, whose right-hand side is 1. The midpoint must also lie inside the ellipse for a nondegenerate chord to exist.

Step 1: Establish the midpoint chord equation Let the endpoints of a chord be $(h + u, k + v)$ and $(h - u, k - v)$. Substituting these two points in the ellipse and subtracting their equations gives

$$\frac{4hu}{a^2} + \frac{4kv}{b^2} = 0.$$

Thus every direction (u, v) along this chord satisfies $hu/a^2 + kv/b^2 = 0$. A general point on the chord therefore satisfies

$$\frac{h(x - h)}{a^2} + \frac{k(y - k)}{b^2} = 0.$$

Expanding this equation yields the stated midpoint formula. This derivation also shows why using the tangent formula at an interior midpoint would be inappropriate.

Step 2: Substitute the ellipse and midpoint data Here $a^2 = 25$, $b^2 = 16$, $h = 3$ and $k = 1$. First verify that the midpoint is interior:

$$\frac{3^2}{25} + \frac{1^2}{16} = \frac{144 + 25}{400} = \frac{169}{400} < 1.$$

Therefore a genuine chord is possible. Its equation is

$$\frac{3x}{25} + \frac{y}{16} = \frac{169}{400}.$$

Multiplying throughout by 400 clears the denominators without changing the line:

$$48x + 25y = 169.$$

Step 3: Check the line and distinguish the options Substitution of $(3,1)$ gives $48(3)+25=169$, so the line passes through the required midpoint. Its normal is proportional to $(3/25, 1/16)$, which is the direction required by the midpoint condition. Passing through the midpoint alone is insufficient: the other listed lines also pass through $(3,1)$, but their normal ratios differ. Only option (B) has the correct coefficients.

Step 4: Conclusion The required chord is $\boxed{48x+25y=169}$. Hence, the correct option is (B).

Quick Tip to Solve: For a chord with a known midpoint, use $T = S_1$; using $T = 1$ instead gives a tangent and misses the required constant.

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15. Let the points $\left(\frac{11}{2}, \alpha\right)$ lie on or inside the triangle with sides $x+y=11, x+2y=16$ and $2x+3y=29$. Then the product of the smallest and the largest values of α is equal to : [4 marks]

(A) 22

(B) 33

(C) 55

(D) 44

Correct Answer: (B) 33

Solution:

Concept:

- Three intersecting nonparallel lines determine a triangle through their pairwise intersection points. For a fixed abscissa, the allowed ordinates form the vertical segment inside that triangle.
- A boundary line's intersection with the vertical line must lie on the actual side segment, not merely on its extension. Since the triangle includes its boundary, the extreme ordinates are included.

Step 1: Find the three vertices Denote the given lines by $L_1 : x+y=11$, $L_2 : x+2y=16$ and $L_3 : 2x+3y=29$. Solving them in pairs gives

$$\begin{aligned} L_1 \cap L_2 &: y=5, \quad x=6, \\ L_1 \cap L_3 &: y=7, \quad x=4, \\ L_2 \cap L_3 &: y=3, \quad x=10. \end{aligned}$$

Thus the triangle has vertices $(6,5)$, $(4,7)$ and $(10,3)$. The required abscissa $11/2 = 5.5$ is between 4 and 6, so its vertical section meets the two sides issuing from $(4,7)$.

Step 2: Identify the lower and upper boundary ordinates The side joining $(4,7)$ to $(6,5)$ lies on L_1. At $x = 11/2$, it gives

$$\alpha = 11 - \frac{11}{2} = \frac{11}{2}.$$

The side joining $(4,7)$ to $(10,3)$ lies on L_3. At the same abscissa,

$$11 + 3\alpha = 29 \quad \Rightarrow \quad \alpha = 6.$$

Both intersections lie within their respective side segments. Although L_2 gives $\alpha = 21/4$, that side has abscissae between 6 and 10. Its intersection at 5.5 is on an extension and is not part of the triangle.

Step 3: Multiply the included endpoint values Convexity of the triangle means that every point on the vertical segment between these two boundary points is on or inside it. Consequently,

$$\frac{11}{2} \le \alpha \le 6.$$

The requested product is therefore

$$\alpha_{\min}\alpha_{\max} = \frac{11}{2} \times 6 = 33.$$

This agrees with option (B); the other products 22, 55 and 44 do not use the two actual boundary ordinates of the section.

Step 4: Conclusion The smallest and largest values are $11/2$ and 6, respectively, and their product is $\boxed{33}$. Hence, the correct option is (B).

Quick Tip to Solve: After intersecting a vertical line with the three side equations, reject any intersection on a side's extension by checking the endpoint abscissae.

16. Group A consists of 7 boys and 3 girls, while group B consists of 6 boys and 5 girls. The number of ways, 4 boys and 4 girls can be invited for a picnic if 5 of them must be from group A and the remaining 3 from group B, is equal to : [4 marks]

(A) 8925

(B) 9100

(C) 8575

(D) 8750

Correct Answer: (A) 8925

Solution:

Concept:

- Choosing r distinct people from n without regard to order gives $\binom{n}{r}$ possibilities. Multiply independent choices from separate categories and add counts from mutually exclusive cases.
- There must be exactly 5 people from group A and 3 from group B, while the complete party contains exactly 4 boys and 4 girls. Tracking one group's girls determines all four category counts.

Step 1: Parametrize the permissible selections Let g girls be invited from group A. Since A contributes 5 people, it contributes $5 - g$ boys. The remaining 4 girls require $4 - g$ girls from B, leaving $g - 1$ boys from B. Thus the count for a fixed g is

$$\binom{3}{g}\binom{7}{5-g}\binom{5}{4-g}\binom{6}{g-1}.$$

Nonnegative category counts and A's supply of only 3 girls force $1 \le g \le 3$. There are precisely three

cases. In particular, $g = 0$ would require a negative number of B boys, and $g = 4$ exceeds A's available girls.

Step 2: Evaluate the three disjoint cases For $g = 1$, select 4 boys and 1 girl from A, and 0 boys and 3 girls from B:

$$N_1 = \binom{7}{4}\binom{3}{1}\binom{6}{0}\binom{5}{3} = 35 \cdot 3 \cdot 1 \cdot 10 = 1050.$$

For $g = 2$, select 3 boys and 2 girls from A, and 1 boy and 2 girls from B:

$$N_2 = \binom{7}{3}\binom{3}{2}\binom{6}{1}\binom{5}{2} = 35 \cdot 3 \cdot 6 \cdot 10 = 6300.$$

For $g = 3$, select 2 boys and 3 girls from A, and 2 boys and 1 girl from B:

$$N_3 = \binom{7}{2}\binom{3}{3}\binom{6}{2}\binom{5}{1} = 21 \cdot 1 \cdot 15 \cdot 5 = 1575.$$

Each case contains exactly 5 A members, 3 B members, 4 boys and 4 girls; no selection can belong to two different values of g.

Step 3: Add the case counts The cases are exhaustive and disjoint, so the addition rule gives

$$N = N_1 + N_2 + N_3 = 1050 + 6300 + 1575 = 8925.$$

No factorial for arranging these people is used because an invitation list is an unordered selection. The remaining offered totals 9100, 8575 and 8750 disagree with this complete enumeration.

Step 4: Conclusion The number of valid invitation lists is $\boxed{8925}$. Hence, the correct option is (A).

Quick Tip to Solve: Fix the number of girls from A first; the group totals and sex totals then determine every remaining category, preventing missing or overlapping cases.

17. Let $[x]$ denote the greatest integer function, and let m and n respectively be the numbers of the points, where the function $f(x) = [x] + |x - 2|, -2 < x < 3$, is not continuous and not differentiable. Then $\mathrm{m} + \mathrm{n}$ is equal to : [4 marks]

(A) 6

(B) 9

(C) 8

(D) 7

Correct Answer: (C) 8

Solution:

Concept:

- The greatest integer function $[x]$ is locally constant between consecutive integers and has an upward jump of 1 at each integer. Adding a continuous function cannot remove such a jump.
- Differentiability implies continuity. The absolute-value term $|x - 2|$ has its only corner at 2, so

every possible exceptional point is either an integer or 2, within the given open interval.

Step 1: Locate and count the discontinuities The integers strictly inside $(-2, 3)$ are $-1, 0, 1, 2$. Let r be any one of these integers. Since $|x-2|$ is continuous everywhere, its limits from both sides at r equal $|r-2|$. Therefore

$$\lim_{x \to r^-} f(x) = r - 1 + |r-2|,$$
$$\lim_{x \to r^+} f(x) = r + |r-2|.$$

The right-hand limit exceeds the left-hand limit by 1. Thus the function is discontinuous at all four listed integers, and $m = 4$. The interval endpoints -2 and 3 are excluded and must not be counted.

Step 2: Examine differentiability at every other point At each of the four discontinuities the function cannot be differentiable. Away from integers, $[x]$ is constant in a sufficiently small neighbourhood. Also, for $x < 2$, $|x-2| = 2 - x$, while for $x > 2$, $|x-2| = x - 2$. Hence at every noninteger point in the domain,

$$f'(x) = \begin{cases} -1, & x < 2, \\ 1, & x > 2. \end{cases}$$

So no additional nondifferentiable point occurs. The corner of the absolute value is at $x = 2$, but that point is already among the floor function's jumps and is counted only once in n. Consequently $n = 4$.

Step 3: Add the two counts The quantities m and n count two specified properties separately; the same point may contribute once to each count. Thus

$$m + n = 4 + 4 = 8.$$

Option (B), 9, would incorrectly count the corner at 2 as an additional nondifferentiability beyond the existing four points. Options (A) and (D), 6 and 7, omit one or more jumps in the open interval.

Step 4: Conclusion There are four discontinuity points and four nondifferentiability points, giving $\boxed{m + n = 8}$. Hence, the correct option is (C).

Quick Tip to Solve: Take the union of the floor function's integer jumps and the absolute value's corners before counting nondifferentiable points; a shared point is counted once.

18. The number of real solution(s) of the equation $x^2 + 3x + 2 = \min\{|x-3|, |x+2|\}$ is : [4 marks]

(A) 2

(B) 3

(C) 1

(D) 0

Correct Answer: (A) 2

Solution:

Concept:

- For two nonnegative distances, comparing their squares preserves their order. Thus $|x+2| \le |x-3|$ exactly when $x \le 1/2$.
- Resolve the minimum first, then split at the zero of the selected absolute value. Each quadratic root is valid only when it lies in the interval used to derive that quadratic.

Step 1: Resolve the smaller distance The equality of distances occurs midway between -2 and 3. Algebraically,

$$(x+2)^2 \le (x-3)^2 \quad \Longleftrightarrow \quad 10x \le 5.$$

Therefore the right-hand side is $|x+2|$ for $x \le 1/2$, and $|x-3|$ for $x \ge 1/2$. The two formulas agree at the boundary, so assigning equality to either branch does not lose a possible solution.

Step 2: Solve on the left-hand branches For $x < -2$, the equation becomes

$$x^2 + 3x + 2 = -x - 2 \quad \Rightarrow \quad (x+2)^2 = 0.$$

Its only root is -2, outside this strict interval. For $-2 \le x \le 1/2$, the equation instead becomes

$$x^2 + 3x + 2 = x + 2 \quad \Rightarrow \quad x(x+2) = 0.$$

Both roots $x = -2$ and $x = 0$ belong to this interval, giving two distinct candidates.

Step 3: Exclude additional roots on the right-hand branches For $1/2 \le x \le 3$, the selected absolute value is $3 - x$, so

$$x^2 + 4x - 1 = 0 \quad \Rightarrow \quad x = -2 \pm \sqrt{5}.$$

Neither root belongs to this interval: the positive root is less than $1/2$, since $\sqrt{5} < 5/2$, and the other root is negative. For $x > 3$, the selected absolute value is $x - 3$, giving

$$x^2 + 2x + 5 = (x+1)^2 + 4 = 0,$$

which has no real roots. Thus all real intervals have been checked.

Step 4: Verify and count the distinct roots At $x = -2$, both sides of the original equation are zero. At $x = 0$, the left-hand side is 2 and the minimum of the distances 3 and 2 is also 2. Hence both candidates work. The repeated appearance of -2 at a branch boundary never creates a second distinct solution. The count is 2, excluding the offered counts 3, 1 and 0.

Step 5: Conclusion The real solutions are $x = -2$ and $x = 0$, so their number is $\boxed{2}$. Hence, the correct option is (A).

Quick Tip to Solve: Split at -2, $1/2$ and 3, and test every quadratic root against its branch interval before counting distinct solutions.

19. Let $f : (0, \infty) \to \mathbf{R}$ be a function which is differentiable at all points of its domain and satisfies the condition $x^2 f'(x) = 2xf(x) + 3$, with $f(1) = 4$. Then $2f(2)$ is equal to : [4 marks]

(A) 19

(B) 23

(C) 29

(D) 39

Correct Answer: (D) 39

Solution:

Concept:

- A first-order linear differential equation is written as $f' + P(x)f = Q(x)$. Multiplication by the integrating factor $e^{\int P(x)\,dx}$ makes its left side an exact product derivative.
- Here the domain is $x > 0$, so division by x^2 and the integrating factor x^{-2} are valid throughout the given domain. The initial value determines the arbitrary constant uniquely.

Step 1: Put the equation in linear form Divide the given equation by x^2 and bring the term involving the function to the left:

$$f'(x) - \frac{2}{x} f(x) = \frac{3}{x^2}.$$

Thus $P(x) = -2/x$ and the integrating factor is $e^{-2\log x} = x^{-2}$. Multiplying by this factor gives

$$\frac{d}{dx}\left(\frac{f(x)}{x^2}\right) = \frac{3}{x^4}.$$

Notice that the right side becomes $3/x^4$, because it must also be multiplied by the integrating factor.

Step 2: Integrate and use the initial value Integrate the product derivative:

$$\frac{f(x)}{x^2} = \int 3x^{-4}\,dx = -x^{-3} + C, \qquad f(x) = Cx^2 - \frac{1}{x}.$$

At $x = 1$, the condition $f(1) = 4$ yields $C - 1 = 4$, so $C = 5$. Therefore

$$f(2) = 5(2)^2 - \frac{1}{2} = \frac{39}{2}, \qquad 2f(2) = 39.$$

As a check, $f' = 10x + x^{-2}$, so $x^2 f' = 10x^3 + 1 = 2xf + 3$. The values 19, 23 and 29 do not equal the required doubled value; in particular, the question asks for $2f(2)$, not $f(2)$.

Step 3: Conclusion $\boxed{2f(2) = 39}$. Hence, the correct option is (D).

Quick Tip to Solve: For an equation containing $x^2 f' - 2xf$, recognise $\left(f/x^2\right)'$ after dividing by x^4.

20. The function $f : (-\infty, \infty) \to (-\infty, 1)$, defined by $f(x) = \frac{2^x - 2^{-x}}{2^x + 2^{-x}}$ is : [4 marks]

(A) One-one but not onto

(B) Onto but not one-one

(C) Both one-one and onto

(D) Neither one-one nor onto

Correct Answer: (A) One-one but not onto

Solution:

Concept:

- A function is one-one when two different inputs cannot produce the same output. Strict monotonicity on the entire domain is sufficient to establish this property.
- A function is onto only if its actual range equals its declared codomain. A horizontal limiting value is not included in the range unless some finite input attains it.

Step 1: Find the range using a positive substitution Multiply the numerator and denominator by 2^x, which is positive for all real x, and put $t = 2^{2x} = 4^x$. Then t ranges over $(0, \infty)$, and

$$f(x) = \frac{t-1}{t+1} = 1 - \frac{2}{t+1}.$$

Since $t > 0$, this expression is strictly between -1 and 1. Conversely, for any $u \in (-1, 1)$, solving $u = (t-1)/(t+1)$ gives $t = (1+u)/(1-u) > 0$, which equals 4^x for a real input. Hence the range is exactly $(-1, 1)$.

Step 2: Check injectivity and compare with the codomain The function $t = 4^x$ is strictly increasing, and $h(t) = (t-1)/(t+1)$ has derivative

$$h'(t) = \frac{2}{(t+1)^2} > 0 \quad (t > 0).$$

Their composition is strictly increasing and therefore one-one. However, the declared codomain is $(-\infty, 1)$, larger than the actual range $(-1, 1)$. For example, -2 belongs to the codomain but has no preimage. Thus the function is not onto. Options (B) and (C) incorrectly assert surjectivity; option (D) incorrectly denies injectivity. Neither endpoint -1 nor 1 is attained at a finite input.

Step 3: Conclusion The function is one-one but not onto the stated codomain $(-\infty, 1)$. Hence, the correct option is (A).

Quick Tip to Solve: Always compare the computed range with the codomain actually printed: this formula has range $(-1, 1)$, although its declared codomain extends below -1.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let P be the image of the point $\mathrm{Q}(7,-2,5)$ in the line $\mathrm{L}: \frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{4}$ and $\mathrm{R}(5,\mathrm{p},\mathrm{q})$ be a point on L. Then the square of the area of $\triangle PQR$ is _____. [4 marks]

Solution:

Concept:

- The image of a point in a line is obtained by a half-turn about that line. If H is the perpendicular projection of Q onto the line, then H is the midpoint of PQ, so $P = 2H - Q$.
- The projection parameter onto a line $A + t\mathbf{d}$ is $t = (Q - A) \cdot \mathbf{d}/|\mathbf{d}|^2$. When the triangle base is PQ, its altitude from a point on the reflecting line is its distance from H.

Step 1: Find the foot of the perpendicular and the reflected point Write the line parametrically as

$$A + t\mathbf{d} = (1 + 2t, -1 + 3t, 4t), \quad A = (1, -1, 0), \quad \mathbf{d} = (2, 3, 4).$$

We have $Q - A = (6, -1, 5)$, and therefore

$$t_H = \frac{(6,-1,5)\cdot(2,3,4)}{2^2+3^2+4^2} = \frac{12-3+20}{29} = 1.$$

Consequently $H = (3, 2, 4)$, $Q - H = (4, -4, 1)$, and $P = 2H - Q = (-1, 6, 3)$. The dot product $(4, -4, 1) \cdot (2, 3, 4) = 8 - 12 + 4 = 0$ verifies perpendicularity.

Step 2: Determine the altitude and compute the squared area Because $R = (5, p, q)$ lies on the line, $1 + 2t_R = 5$, giving $t_R = 2$. Thus $R = (5, 5, 8)$ and $R - H = (2, 3, 4)$. The base PQ lies along QH, and RH, being along the reflecting line, is perpendicular to it. Hence

$$|QH|^2 = 4^2 + (-4)^2 + 1^2 = 33, \qquad |RH|^2 = 2^2 + 3^2 + 4^2 = 29.$$

Since $|PQ| = 2|QH|$,

$$\mathrm{Area}(PQR) = \frac{1}{2}(2|QH|)|RH|, \qquad \mathrm{Area}(PQR)^2 = 33 \cdot 29 = 957.$$

The factor of two from reflection cancels the one-half in the triangle formula; the requested quantity is already the square, so no square root is needed.

Step 3: Conclusion The square of the area of $\triangle PQR$ is $\boxed{957}$.

Quick Tip to Solve: With a reflected pair P, Q and a vertex R on the mirror line, use $\mathrm{Area}^2 = QH^2\, RH^2$, where H is the perpendicular foot.

22. If $\int \frac{2x^2+5x+9}{\sqrt{x^2+x+1}}\,dx = x\sqrt{x^2+x+1} + \alpha\sqrt{x^2+x+1} + \beta \log_e \left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right| + C$**, where** C **is the constant of integration, then** $\alpha + 2\beta$ **is equal to _____ .** [4 marks]

Solution:

Concept:

- If an antiderivative is already supplied with unknown constants, differentiate its right side and equate it to the integrand. Equality of polynomial coefficients then determines the constants.
- Writing $s = x^2+x+1 = (x+\frac{1}{2})^2 + \frac{3}{4}$ gives $s > 0$ for every real x, and $\frac{d}{dx}\log|x+\frac{1}{2}+\sqrt{s}| = 1/\sqrt{s}$.

Step 1: Differentiate the proposed antiderivative Combine its first two terms as $(x+\alpha)\sqrt{s}$, where $s = x^2+x+1$. By the product and chain rules,

$$\frac{d}{dx}\big((x+\alpha)\sqrt{s}\big) = \sqrt{s} + \frac{(x+\alpha)(2x+1)}{2\sqrt{s}}.$$

For the logarithmic term, put $u = x+1/2$, so $s = u^2+3/4$. Differentiating directly gives

$$\frac{d}{dx}\log|u+\sqrt{s}| = \frac{1+u/\sqrt{s}}{u+\sqrt{s}} = \frac{1}{\sqrt{s}}.$$

Here $\sqrt{s} > |u|$, so the logarithm has a nonzero argument everywhere; the differentiation is valid on the entire real line.

Step 2: Equate coefficients and evaluate the required combination Combining the derivatives over the common denominator gives

$$\frac{2x^2 + (\frac{3}{2}+\alpha)x + 1 + \frac{\alpha}{2} + \beta}{\sqrt{s}} = \frac{2x^2+5x+9}{\sqrt{s}}.$$

Equate the coefficient of x and the constant term:

$$\alpha + \frac{3}{2} = 5 \quad\Longrightarrow\quad \alpha = \frac{7}{2},$$

$$1 + \frac{\alpha}{2} + \beta = 9 \quad\Longrightarrow\quad \beta = 9 - 1 - \frac{7}{4} = \frac{25}{4}.$$

The coefficient of x^2 is already 2, agreeing with the numerator. Thus

$$\alpha + 2\beta = \frac{7}{2} + \frac{25}{2} = 16.$$

The arbitrary integration constant disappears under differentiation and does not affect this coefficient comparison.

Step 3: Conclusion The required numerical value is $\boxed{\alpha + 2\beta = 16}$.

Quick Tip to Solve: For an integral supplied in a proposed closed form, differentiate that form first; matching the numerator often avoids a long integration.

23. Let $y = y(x)$ be the solution of the differential equation

$2\cos x\frac{dy}{dx} = \sin 2x - 4y\sin x, x \in \left(0, \frac{\pi}{2}\right)$**. If** $y\left(\frac{\pi}{3}\right) = 0$**, then** $y'\left(\frac{\pi}{4}\right) + y\left(\frac{\pi}{4}\right)$ **is equal to _____.** [4 marks]

Solution:

Concept:

- A linear equation $y' + P(x)y = Q(x)$ is solved by multiplying by $e^{\int P(x)\,dx}$. On $(0, \pi/2)$, $\cos x > 0$, so division by $\cos x$ is permitted.
- Use $\sin 2x = 2\sin x\cos x$, $\int \tan x\,dx = -\log(\cos x)$, and $(\sec x)' = \sec x\tan x$ to integrate the resulting product derivative.

Step 1: Reduce the equation and integrate Divide by $2\cos x$ and collect the term involving y:

$$y' + 2y\tan x = \sin x.$$

The integrating factor is

$$e^{\int 2\tan x\,dx} = e^{-2\log(\cos x)} = \sec^2 x.$$

After multiplication,

$$\frac{d}{dx}(y\sec^2 x) = \sin x\sec^2 x = \sec x\tan x.$$

Integrating gives $y\sec^2 x = \sec x + C$, so

$$y = \cos x + C\cos^2 x.$$

All these operations are valid throughout the stated interval because neither $\cos x$ nor the integrating factor vanishes.

Step 2: Apply the initial condition and evaluate both terms The initial value at $x = \pi/3$ gives

$$0 = \frac{1}{2} + C\left(\frac{1}{2}\right)^2, \qquad C = -2.$$

Hence $y = \cos x - 2\cos^2 x$, and differentiating yields

$$y' = -\sin x + 4\sin x\cos x.$$

At $x = \pi/4$, $\sin x = \cos x = 1/\sqrt{2}$, so

$$y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} - 1, \qquad y'\left(\frac{\pi}{4}\right) = 2 - \frac{1}{\sqrt{2}}.$$

Their sum is 1. The radical terms cancel exactly; rounding the two terms separately is unnecessary. Substitution into the reduced differential equation also recovers $\sin x$, verifying the signs in the solution.

Step 3: Conclusion $\boxed{y'(\pi/4) + y(\pi/4) = 1}$.

Quick Tip to Solve: After finding the constant, keep the values of y and y' exact: their $1/\sqrt{2}$ terms cancel in this sum.

24. Number of functions $f : \{1, 2, \ldots, 100\} \to \{0, 1\}$, that assign 1 to exactly one of the positive integers less than or equal to 98 , is equal to _____. [4 marks]

Solution:

Concept:

- A function from a finite set is specified by choosing exactly one output for every domain element. Outputs assigned to different domain elements are independent unless a stated condition links them.
- The multiplication principle counts a sequence of independent choices. Choosing the unique element mapped to 1 among n elements has $\binom{n}{1} = n$ possibilities; each unrestricted binary output has two choices.

Step 1: Count the assignments on the first 98 integers The condition says that exactly one of the integers $1, 2, \ldots, 98$ must have image 1. Choose that unique integer in

$$\binom{98}{1} = 98$$

ways. Once it is chosen, all the other 97 integers in this group must have image 0; they provide no additional freedom. There is no ordering factor because specifying which input receives 1 already identifies the assignment on the entire first group. In particular, assigning 1 to two of these inputs would violate the condition.

Step 2: Include the two unrestricted inputs The words less than or equal to 98 restrict only the first group. The remaining domain elements 99 and 100 can each independently be mapped to either 0 or 1. Their ordered output pairs are

$$(f(99), f(100)) = (0, 0), (0, 1), (1, 0), (1, 1),$$

giving $2^2 = 4$ choices for each assignment already counted. Thus the multiplication principle gives

$$N = \binom{98}{1} \cdot 2^2 = 98 \cdot 4 = 392.$$

Every valid function is counted once, since it has one uniquely determined input among the first 98 mapped to 1 and one uniquely determined ordered pair of outputs for the final two inputs. Requiring a unique 1 over all 100 inputs would impose an additional restriction absent from the question.

Step 3: Conclusion The number of functions is $\boxed{392}$.

Quick Tip to Solve: Separate constrained inputs from unrestricted inputs: choose the unique 1 among the first 98, then multiply by 2^2 for the last two.

25. Let $H_1 : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $H_2 : -\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$ be two hyperbolas having length of latus rectums $15\sqrt{2}$ and $12\sqrt{5}$ respectively. Let their ecentricities be $e_1 = \sqrt{\frac{5}{2}}$ and e_2 respectively. If the product of the lengths of their transverse axes is $100\sqrt{10}$, then $25e_2^2$ is equal to _____ . [4 marks]

Solution:

Concept:

- For a horizontal hyperbola $x^2/a^2 - y^2/b^2 = 1$, the eccentricity satisfies $e^2 = 1 + b^2/a^2$, the transverse axis has length $2a$, and its latus rectum has length $2b^2/a$.
- For the vertical hyperbola $y^2/B^2 - x^2/A^2 = 1$, the transverse semiaxis is B, so its eccentricity is $e_2^2 = 1 + A^2/B^2$ and its latus rectum has length $2A^2/B$.

Step 1: Determine the first transverse semiaxis The given eccentricity fixes the ratio of the squared semiaxes of the first hyperbola. Thus

$$\frac{b^2}{a^2} = e_1^2 - 1 = \frac{5}{2} - 1 = \frac{3}{2}.$$

Substitute this into the first latus-rectum length:

$$\frac{2b^2}{a} = 3a = 15\sqrt{2}, \qquad a = 5\sqrt{2}.$$

The positive value is taken because a semiaxis is a length.

Step 2: Use the product of the transverse axes The two transverse axes have lengths $2a$ and $2B$, not $2a$ and $2A$. Hence

$$4aB = 100\sqrt{10}.$$

Using $a = 5\sqrt{2}$, obtain

$$B = \frac{100\sqrt{10}}{20\sqrt{2}} = 5\sqrt{5}, \qquad B^2 = 125.$$

This identifies the correct denominator for the vertical hyperbola.

Step 3: Calculate the second eccentricity The second latus rectum supplies the other squared semiaxis:

$$\frac{2A^2}{B} = 12\sqrt{5} \quad \Rightarrow \quad A^2 = 6\sqrt{5}\,(5\sqrt{5}) = 150.$$

Therefore

$$e_2^2 = 1 + \frac{150}{125} = 1 + \frac{6}{5} = \frac{11}{5}.$$

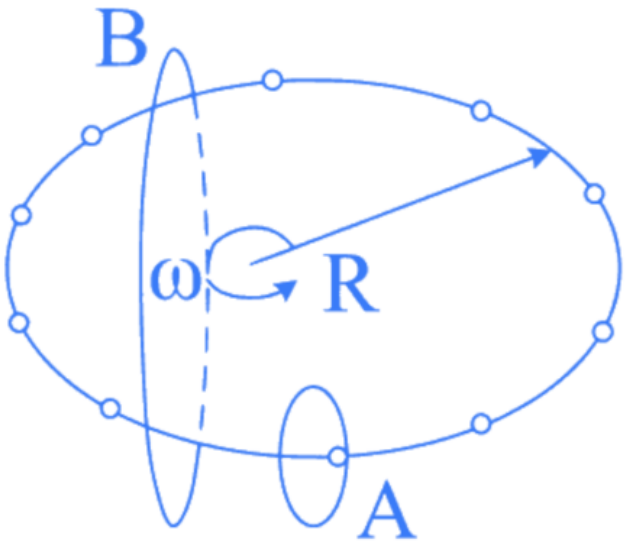
As a check, $e_2^2 > 1$, as required for a hyperbola; substituting $A^2 = 150$ and $B = 5\sqrt{5}$ also reproduces its stated latus rectum.

Step 4: Conclusion The required numerical value is $25e_2^2 = 25 \times \frac{11}{5} = \boxed{55}$.

Quick Tip to Solve: Identify the positive squared term before choosing the transverse semiaxis: for the second hyperbola it is B, even though its other denominator is written first.

Physics – Section A: Multiple Choice Questions (80 marks)

26. N equally spaced charges each of value q , are placed on a circle of radius R . The circle rotates about its axis with an angular velocity ω as shown in the figure. A bigger Amperian loop B encloses the whole circle where as a smaller Amperian loop A encloses a small segment. The difference between enclosed currents, $I_A - I_B$, for the given Amperian loops is [4 marks]



(A) $\frac{N^2}{2\pi}q\omega$
(B) $\frac{N}{2\pi}q\omega$
(C) $\frac{2\pi}{N}q\omega$
(D) $\frac{N}{\pi}q\omega$

Correct Answer: (B) $\frac{N}{2\pi}q\omega$

Solution:

Concept:

- A uniformly rotating total charge Q constitutes a current $I = Q/T$, where its rotation period is $T = 2\pi/\omega$.
- The current enclosed by an Amperian loop is the algebraic current through a surface bounded by it. Opposite crossings of that surface cancel; it is not the total charge geometrically inside the loop.

Step 1: Calculate the circulating current All N charges pass a fixed cross-section of the circular orbit during one complete revolution. Thus the charge transported in a period is Nq, and

$$I = \frac{Nq}{T} = \frac{Nq\omega}{2\pi}.$$

Equivalently, adjacent charges pass the cross-section at intervals T/N, so their current is $q/(T/N)$. There is only one factor of N; multiplying by N once again would double-count the passing charges.

Step 2: Count the crossings of the two surfaces The small loop A links one segment of the orbit, so its

spanning surface is crossed once by the circulating current. With the indicated current orientation,

$$I_A = I.$$

The surface bounded by the larger loop B is crossed twice by the circular current, once in each sense. Consequently

$$I_B = I - I = 0.$$

Even though B surrounds the whole orbit in the drawing, this does not imply that it encloses a nonzero algebraic current. Current leaving the surface balances current entering it.

Step 3: Compare the proposed expressions The required difference is therefore the single circulating current:

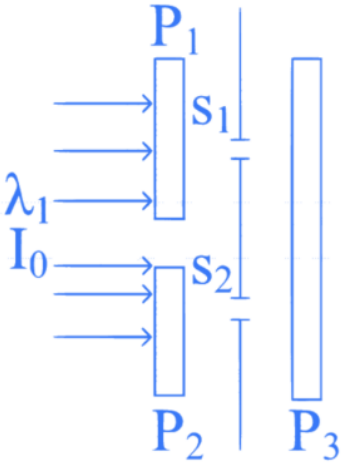
$$I_A - I_B = \frac{Nq\omega}{2\pi} - 0.$$

Option A has an extra factor of N; option C incorrectly makes the current decrease as more charges are added. Option D is twice the current because it counts both crossings without their opposite signs. The radius cancels because a revolution transports the same charge regardless of its circumference.

Step 4: Conclusion $\boxed{I_A - I_B = \frac{Nq\omega}{2\pi}}$. Hence, the correct option is (B).

Quick Tip to Solve: For an Amperian surface, count signed crossings of the current path. A closed current orbit crossing the same surface twice can contribute zero net enclosed current.

27. In a Young's double slit experiment, three polarizers are kept as shown in the figure. The transmission axes of P_1 and P_2 are orthogonal to each other. The polarizer P_3 covers both the slits with its transmission axis at 45° to those of P_1 and P_2. An unpolarized light of wavelength λ and intensity I_0 is incident on P_1 and P_2. The intensity at a point after P_3 where the path difference between the light waves from s_1 and s_2 is $\frac{\lambda}{3}$, is [4 marks]



(A) I_0
(B) $\frac{I_0}{3}$
(C) $\frac{I_0}{4}$
(D) $\frac{I_0}{2}$

Correct Answer: (D) $\frac{I_0}{2}$

Solution:

Concept:

- An ideal polarizer transmits half the intensity of unpolarized incident light. Malus' law gives the further factor $\cos^2\theta$ through an analyzer at angle θ.
- Interference requires mutual coherence, not merely parallel output polarizations. For unpolarized light the two orthogonal field components have zero average correlation: $\langle E_x E_y^* \rangle = 0$.

Step 1: Find each transmitted intensity Choose the axes of P_1 and P_2 as the x and y axes. Each first polarizer transmits $I_0/2$, since the incident light is unpolarized. The common analyzer P_3 makes 45° with each axis. Therefore the separate slit contributions after it are

$$I_1 = I_2 = \frac{I_0}{2}\cos^2 45^\circ = \frac{I_0}{4}.$$

The total intensity must include any averaged interference cross term, so equal polarization after P_3 alone does not yet justify using a fully coherent two-beam formula.

Step 2: Evaluate the mutual correlation Apart from propagation phases, the two fields emerging from P_3 are proportional to $E_x/\sqrt{2}$ and $E_y/\sqrt{2}$, along its common transmission axis. Their average product is proportional to $\langle E_x E_y^* \rangle/2$. For the stated unpolarized input this is zero. A passive analyzer projects the fields but cannot create correlation between statistically uncorrelated orthogonal components. Thus

$$I = I_1 + I_2 + 2\,\mathrm{Re}\langle E_1 E_2^* \rangle = \frac{I_0}{4} + \frac{I_0}{4} + 0 = \frac{I_0}{2}.$$

The path difference contributes a phase $2\pi/3$, but multiplying a zero correlation by this phase still gives zero.

Step 3: Distinguish the coherent alternative If the two projected amplitudes were mutually coherent with fixed relative phase, the familiar Young formula would instead give

$$I_{\text{coherent}} = \frac{I_0}{2} + \frac{I_0}{2}\cos\frac{2\pi}{3} = \frac{I_0}{4}.$$

That calculation introduces correlation absent for unpolarized incident light selected into orthogonal components by P_1, P_2. It therefore explains the tempting option C but does not follow from the printed premise. Option A would require a coherent bright fringe, and option B follows from neither Malus' law nor the zero-correlation sum.

Step 4: Conclusion For the stated unpolarized incident light, $\boxed{I = I_0/2}$. Hence, the correct option is (D). The supplied key gives $I_0/4$; that value presumes mutually coherent projected fields, whereas the printed unpolarized premise gives $\langle E_x E_y^* \rangle = 0$ and eliminates the interference term.

Quick Tip to Solve: After applying Malus' law, check mutual coherence before adding an interference term. A common analyzer aligns polarization but cannot correlate initially uncorrelated field components.

28. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : A electron in a certain region of uniform magnetic field is moving with constant velocity in a straight line path.

Reason (R) : The magnetic field in that region is along the direction of velocity of the electron. In the light of the above statements, choose the correct answer from the options given below : [4 marks]

(A) (A) is true but (R) is false

(B) Both (A) and (R) are true and (R) is the correct explanation of (A)

(C) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(D) (A) is false but (R) is true

Correct Answer: (B) Both (A) and (R) are true and (R) is the correct explanation of (A)

Solution:

Concept:

- The magnetic force on a charge is $\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$; its magnitude is $|q|vB\sin\theta$, with θ the angle between velocity and magnetic field.
- A magnetic field causes no acceleration when velocity is parallel or antiparallel to it. Constant velocity means that both the speed and direction remain unchanged.

Step 1: Evaluate the assertion The assertion describes an electron following a straight path with constant velocity in a magnetic field. Such motion is physically possible: it occurs when the velocity has no component perpendicular to the field. With zero magnetic force the electron has no magnetic acceleration and retains its initial velocity. The assertion is therefore true for the stated situation. A magnetic field does not necessarily bend every charged-particle trajectory.

Step 2: Evaluate the reason The reason specifies that the magnetic field lies along the electron velocity. This gives $\theta = 0$, so

$$\mathbf{v} \times \mathbf{B} = \mathbf{0}, \qquad \mathbf{F} = -e\,(\mathbf{v} \times \mathbf{B}) = \mathbf{0}.$$

The electron has negative charge, but changing the charge sign cannot turn a zero cross product into a nonzero force. The reason is therefore true. An antiparallel arrangement would also give zero force, although the stated parallel arrangement is sufficient here.

Step 3: Check the explanation link With only the specified magnetic interaction, Newton's second law yields

$$m\mathbf{a} = \mathbf{0} \quad \Rightarrow \quad \mathbf{a} = \mathbf{0}.$$

Hence the velocity remains constant and the path stays straight. The reason directly supplies the zero-force condition needed by the assertion, so the two true statements are causally connected. Option A wrongly rejects the reason, option D wrongly rejects the assertion, and option C misses this direct explanation. Constant speed alone would be insufficient, since perpendicular motion in a magnetic field has constant speed but changing direction.

Step 4: Conclusion Both the assertion and the reason are true, and the reason correctly explains the assertion. Hence, the correct option is (B).

Quick Tip to Solve: Check the cross product before deciding the trajectory: parallel velocity gives zero force, while perpendicular velocity gives a curved path at constant speed.

29. The temperature of a body in air falls from 40°C to 24°C in 4 minutes. The temperature of the air is 16°C. The temperature of the body in the next 4 minutes will be : [4 marks]

(A) $\frac{28}{3}^\circ\text{C}$
(B) $\frac{56}{3}^\circ\text{C}$
(C) $\frac{42}{3}^\circ\text{C}$
(D) $\frac{14}{3}^\circ\text{C}$

Correct Answer: (B) $\frac{56}{3}^\circ\text{C}$

Solution:

Concept:

- Newton's law of cooling gives $d(T - T_a)/dt = -k(T - T_a)$ for a body cooling in fixed surroundings, where T_a is the ambient temperature and $k > 0$ is the cooling constant.
- The excess temperature $\theta = T - T_a$ decays exponentially: $\theta(t) = \theta(0)e^{-kt}$. Equal time intervals multiply the excess temperature by the same factor.

Step 1: Measure temperature above the surroundings The surrounding air remains at 16°C. The initial and four-minute excess temperatures are therefore

$$\theta_0 = 40 - 16 = 24^\circ\text{C}, \qquad \theta_4 = 24 - 16 = 8^\circ\text{C}.$$

It is these excesses that obey a constant decay ratio. The actual Celsius temperatures cannot be multiplied directly, since the body approaches the air temperature rather than zero Celsius.

Step 2: Find the equal-interval cooling factor During the first interval of four minutes,

$$e^{-4k} = \frac{\theta_4}{\theta_0} = \frac{8}{24} = \frac{1}{3}.$$

Over the next interval of the same duration, the excess above ambient is again divided by three. Thus at eight minutes from the start,

$$\theta_8 = \theta_4 e^{-4k} = 8 \times \frac{1}{3} = \frac{8}{3}^\circ\text{C}.$$

No explicit evaluation of the logarithm or of the cooling constant is needed.

Step 3: Restore the ambient temperature Add the ambient temperature back to convert excess temperature into the body temperature:

$$T_8 = 16 + \frac{8}{3} = \frac{48 + 8}{3} = \frac{56}{3}^\circ\text{C}.$$

This is about 18.67°C, between the current body temperature 24°C and ambient temperature 16°C, as required. Every other offered temperature is below the air temperature, which passive cooling from above cannot attain under this model. Subtracting the previous fall of 16°C again would also be wrong because cooling slows as the temperature difference decreases.

Step 4: Conclusion The temperature after the next four minutes is $\boxed{\frac{56}{3}^\circ\text{C}}$. Hence, the correct option is (B).

Quick Tip to Solve: For equal cooling intervals, form a ratio of temperatures above ambient. Multiply the excess by that ratio, then add the ambient temperature back.

30. The output of the circuit is low (zero) for :

(A) $X = 0, Y = 0$

(B) $X = 0, Y = 1$

(C) $X = 1, Y = 0$

(D) $X = 1, Y = 1$

Choose the correct answer from the options given below : [4 marks]



(A) (A), (B) and (C) only

(B) (A), (C) and (D) only

(C) (B), (C) and (D) only

(D) (A), (B) and (D) only

Correct Answer: (C) (B), (C) and (D) only

Solution:

Concept:

- An OR gate produces $Z = X + Y$, where Boolean addition means that the output is one if at least one input is one.
- A NOR gate is an OR gate followed by inversion. With both its inputs tied together at Z, its output is $\overline{Z + Z} = \overline{Z}$, using the Boolean identity $Z + Z = Z$.

Step 1: Identify the two gates The first symbol in the diagram is an OR gate: it has the curved input side and no output inversion bubble. Its two inputs are X and Y, giving the intermediate signal $Z = X + Y$. That signal is connected to both inputs of the second gate. The output bubble on the second OR-shaped symbol makes it a NOR gate, so it acts as an inverter when its inputs are joined.

Step 2: Write the overall output If F denotes the final output, the two-stage circuit gives

$$F = \overline{Z + Z} = \overline{Z} = \overline{X + Y}.$$

The circuit as a whole is therefore a NOR operation on the original inputs. It produces zero whenever at least one of X, Y is one, and produces one only when both original inputs are zero. The tied input branches do not constitute an additional independent input.

Step 3: Check every listed input pair Evaluating all four cases makes the required combination explicit:

Case	X	Y	$Z = X + Y$	$F = \overline{Z}$
A	0	0	0	1
B	0	1	1	0
C	1	0	1	0
D	1	1	1	0

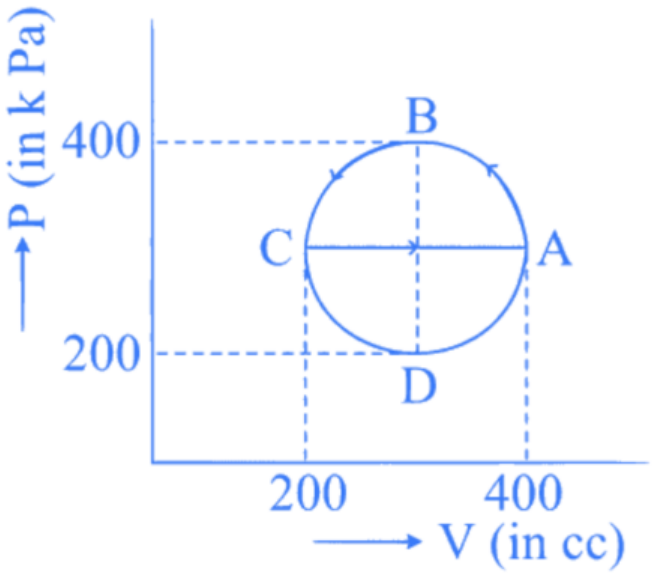
Thus case A must be excluded, whereas B, C and D must all be included. The first, second and fourth

answer combinations each incorrectly include A and omit at least one zero-output case. The third combination includes exactly the three cases that make the final output low.

Step 4: Conclusion The output is low for the last three input cases, namely 01, 10 and 11. Hence, the correct option is (C).

Quick Tip to Solve: A NOR gate with its inputs tied is a NOT gate. Simplify that stage first, then test whether the first OR gate produces zero or one.

31. The magnitude of heat exchanged by a system for the given cyclic process ABCA (as shown in figure) is (in SI unit) : [4 marks]



(A) zero

(B) 40π

(C) 5π

(D) 10π

Correct Answer: (C) 5π

Solution:

Concept:

- The first law is $Q = \Delta U + W$, where $W = \int P\,dV$ is work done by the system. Internal energy is a state function, so its change over a complete cycle is zero.
- The magnitude of net work in a closed pressure-volume cycle equals the area enclosed. A circle drawn on differently scaled axes represents an ellipse in physical P, V coordinates; use its pressure and volume semiaxes with their units.

Step 1: Identify the actual closed path The stated cycle is ABCA: it goes from A to B to C along the upper semicircular arc and returns from C to A along the horizontal diameter. The lower arc through D is not part of this cycle. Thus the enclosed area is half the area of the ellipse, not the full area shown

by the complete outline.

Step 2: Convert the two semiaxes to SI units The pressure extremes are 200 and 400 kPa, giving a pressure semiaxis $b = (400 - 200)/2 = 100\,\text{kPa} = 10^5\,\text{Pa}$. The volume extremes are 200 and 400 cc, giving $a = 100\,\text{cc} = 10^{-4}\,\text{m}^3$. Here $1\,\text{cc} = 10^{-6}\,\text{m}^3$, so the numerical scale on the volume axis must not be used directly in joules.

Step 3: Apply the first law around the cycle Because the final state is A again, $\Delta U_{\text{cycle}} = 0$. Hence

$$|Q| = |W| = \frac{\pi ab}{2} = \frac{\pi}{2}(10^{-4})(10^5) = 5\pi\,\text{J}.$$

The traversal A to B to C is a compression at the higher pressures, so net work done by the gas and net heat absorbed are negative. Their magnitudes are positive. Zero ignores the enclosed area, while 10π counts the full ellipse; 40π incorrectly uses diameters as semiaxes.

Step 4: Conclusion The magnitude of heat exchanged is $\boxed{5\pi\,\text{J}}$; heat is released by the system over the cycle. Hence, the correct option is (C).

Quick Tip to Solve: Trace only the named cycle before finding area: ABCA uses the upper half, whereas ABCDA would enclose the full ellipse.

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32. In photoelectric effect, the stopping potential $(\text{V}_0)\,\text{v/s}$ frequency (v) curve is plotted.
(h is the Planck's constant and ϕ_0 is work function of metal)
(A) $\text{V}_0\text{v/s}v$ is linear.
(B) The slope of $\text{V}_0\text{v/s}v$ curve $= \frac{\phi_0}{\text{h}}$
(C) h constant is related to the slope of $\text{V}_0\text{v/s}v$ line.
(D) The value of electric charge of electron is not required to determine h using the $\text{V}_0\text{v/s}v$ curve.
(E) The work function can be estimated without knowing the value of h.
Choose the correct answer from the options given below : [4 marks]

(A) (A), (C) and (E) only
(B) (C) and (D) only
(C) (A), (B) and (C) only
(D) (D) and (E) only
Correct Answer: (A) (A), (C) and (E) only

Solution:

Concept:

- Einstein's photoelectric equation is $eV_0 = h\nu - \phi_0$, where e is the magnitude of electron charge, h is Planck's constant and ϕ_0 is the work function.
- Writing this as $V_0 = (h/e)\nu - \phi_0/e$ identifies the slope and vertical intercept of the stopping-potential graph. The straight line is physically measured for frequencies at or above the threshold.

Step 1: Read the slope and test statements A to C The equation is of the form $y = mx + c$, with

$y = V_0$ and $x = \nu$. Therefore statement (A), that the graph is linear, is true. Its slope is

$$m = \frac{\Delta V_0}{\Delta \nu} = \frac{h}{e},$$

not ϕ_0/h, so statement (B) is false. The latter ratio is the threshold frequency, with units of frequency rather than slope. Since $h = em$, statement (C) is true.

Step 2: Test the charge and work-function claims To obtain h numerically in SI units from the measured slope, one must multiply the slope by the electron charge e. Thus the claim in statement (D) that the charge value is unnecessary is false. The extrapolated vertical intercept is $c = -\phi_0/e$, so

$$\phi_0 = -ec.$$

Consequently the work function can be found from the intercept using the known electron charge without separately knowing h. Statement (E) is true. This is different from using only the threshold frequency, which would require h in $\phi_0 = h\nu_0$.

Step 3: Select the complete set of true statements The truth pattern is A true, B false, C true, D false and E true. Only the first option contains exactly (A), (C) and (E). The second and fourth options incorrectly accept (D), and the third option incorrectly accepts (B).

Step 4: Conclusion The correct statements are A, C and E only. Hence, the correct option is (A).

Quick Tip to Solve: For a stopping-potential versus frequency graph, remember slope h/e and vertical intercept $-\phi_0/e$; do not confuse the slope with the threshold frequency.

33. A small uncharged conducting sphere is placed in contact with an identical sphere but having 4×10^{-8}C charge and then removed to a distance such that the force of repulsion between them is 9×10^{-3} N. The distance between them is (Take $\frac{1}{4\pi\epsilon_0}$ as 9×10^9 in SI units) [4 marks]

(A) 1 cm

(B) 2 cm

(C) 4 cm

(D) 3 cm

Correct Answer: (B) 2 cm

Solution:

Concept:

- When identical conducting spheres touch, charge redistributes until their potentials are equal. Their equal radii then imply equal final charges; total electric charge is conserved.
- For small separated charged spheres, Coulomb's law gives $F = kq_1q_2/r^2$, where r is the separation of their centres and $k = 9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$.

Step 1: Find the charge on each sphere after contact Initially one sphere has zero charge and the other has 4×10^{-8} C. Their combined charge remains 4×10^{-8} C. As the two conductors are identical,

each receives half of this total:

$$q_1 = q_2 = \frac{4 \times 10^{-8}}{2} = 2 \times 10^{-8}\,\text{C}.$$

Both charges have the same sign, consistent with the stated repulsive force. The initially uncharged sphere does not remain uncharged after contact.

Step 2: Solve Coulomb's law for the separation Use the final charges, since the force is measured after contact and separation. Rearranging gives

$$r^2 = \frac{kq^2}{F} = \frac{(9 \times 10^9)(2 \times 10^{-8})^2}{9 \times 10^{-3}}.$$

The squared charge is $4 \times 10^{-16}\,\text{C}^2$, so

$$r^2 = \frac{36 \times 10^{-7}}{9 \times 10^{-3}} = 4 \times 10^{-4}\,\text{m}^2, \qquad r = 2 \times 10^{-2}\,\text{m}.$$

The positive square root is required because distance is positive.

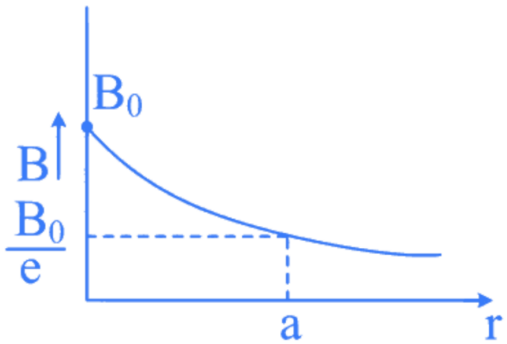
Step 3: Convert and check the alternatives The separation is $2\,\text{cm}$. Substitution gives $F = (3.6 \times 10^{-6})/(4 \times 10^{-4}) = 9 \times 10^{-3}\,\text{N}$, matching the question. Distances 1, 3 and 4 cm would give 0.036, 0.004 and 0.00225 N respectively, so none of those options reproduces the stated force.

Step 4: Conclusion The distance between the sphere centres is $\boxed{2\,\text{cm}}$. Hence, the correct option is (B).

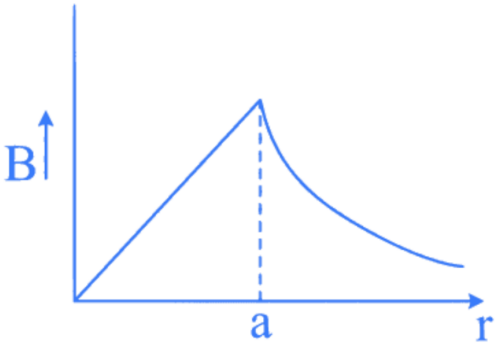
Quick Tip to Solve: For identical spheres, first halve the conserved total charge; insert the redistributed charges into Coulomb's law, not the original charge.

34. A long straight wire of a circular cross-section with radius ' a ' carries a steady current I . The current I is uniformly distributed across this cross-section. The plot of magnitude of magnetic field B with distance r from the centre of the wire is given by [4 marks]

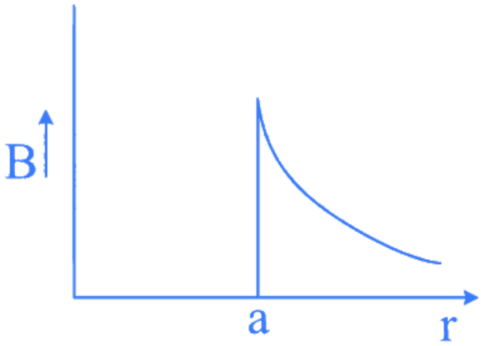
(A)



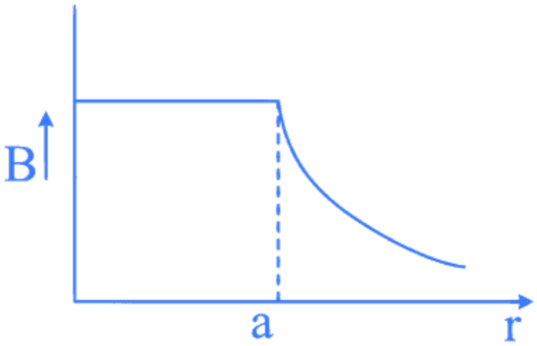
(B)



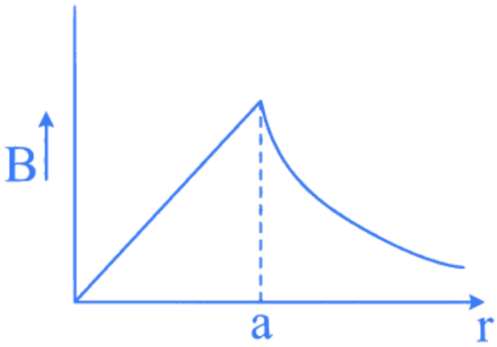
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- Ampere's circuital law is $\oint \mathbf{B} \cdot d\ell = \mu_0 I_{\text{enc}}$. Cylindrical symmetry makes B constant on a circle of radius r concentric with a long straight wire.

- Uniform current density is $J = I/(\pi a^2)$. Inside the wire only the current through the enclosed area contributes to I_{enc}; outside, the entire current is enclosed.

Step 1: Derive the field inside the conductor For $0 \le r \le a$, the enclosed cross-sectional area is πr^2. Uniform distribution gives

$$I_{\text{enc}} = J\pi r^2 = I\frac{r^2}{a^2}.$$

Ampere's law then becomes

$$B(2\pi r) = \mu_0 I\frac{r^2}{a^2}, \qquad B = \frac{\mu_0 I}{2\pi a^2}r.$$

Thus the field is zero at the axis and increases linearly up to the surface. A finite nonzero field at the centre is incompatible with this uniform solid-wire model.

Step 2: Derive the field outside and join the branches For $r \ge a$, the circular path encloses all the current, giving

$$B = \frac{\mu_0 I}{2\pi r}.$$

At $r = a$, both expressions give $B(a) = \mu_0 I/(2\pi a)$, so the graph is continuous and has its maximum there. Beyond the surface the field decreases as $1/r$, approaching zero at large distance. The exterior curve is not an exponential decay.

Step 3: Match the four plotted possibilities The second graph starts at the origin, rises in a straight line to $r = a$, and then falls continuously along an inverse-distance curve. It therefore represents both required regions. The first starts with a nonzero central field, the third incorrectly has zero field throughout the interior followed by a jump, and the fourth incorrectly makes the interior field constant and nonzero. None of those can describe a uniformly distributed current in a solid wire.

Step 4: Conclusion The field rises linearly inside and falls as $1/r$ outside, with a continuous maximum at $r = a$. Hence, the correct option is (B).

Quick Tip to Solve: For a uniformly conducting solid wire, use enclosed current proportional to r^2; division by the Amperian circumference leaves an interior field proportional to r.

---

35. The energy E and momentum p of a moving body of mass m are related by some equation. Given that c represents the speed of light, identify the correct equation [4 marks]

(A) $\mathrm{E}^2 = \mathrm{pc}^2 + \mathrm{m}^2\mathrm{c}^4$

(B) $\mathrm{E}^2 = \mathrm{pc}^2 + \mathrm{m}^2\mathrm{c}^2$

(C) $\mathrm{E}^2 = \mathrm{p}^2\mathrm{c}^2 + \mathrm{m}^2\mathrm{c}^4$

(D) $\mathrm{E}^2 = \mathrm{p}^2\mathrm{c}^2 + \mathrm{m}^2\mathrm{c}^2$

Correct Answer: (C) $\mathrm{E}^2 = \mathrm{p}^2\mathrm{c}^2 + \mathrm{m}^2\mathrm{c}^4$

Solution:

Concept:

- A body of rest mass m has total relativistic energy $E = \gamma mc^2$ and momentum $p = \gamma mv$, where $\gamma = (1 - v^2/c^2)^{-1/2}$.

- Total energy includes rest energy. Eliminating velocity between these expressions gives the energy-momentum relation; all terms must have the dimensions of energy squared.

Step 1: Eliminate the velocity Square the energy and momentum equations, then form their difference:

$$E^2 - p^2c^2 = \gamma^2 m^2 c^4 - \gamma^2 m^2 v^2 c^2.$$

Factor out $\gamma^2 m^2 c^4$:

$$E^2 - p^2c^2 = \gamma^2 m^2 c^4 \left(1 - \frac{v^2}{c^2}\right).$$

By the definition of γ, the product $\gamma^2(1 - v^2/c^2)$ is unity. Hence the remaining term is m^2c^4, independent of the speed.

Step 2: Write the invariant relation and check dimensions Rearranging gives

$$E^2 = p^2c^2 + m^2c^4.$$

The combination pc has the dimensions of energy, and mc^2 is also an energy; therefore their squares can be added. In options (A) and (B), pc^2 is not energy squared. In option (D), m^2c^2 has the dimensions of momentum squared rather than energy squared. Only option (C) has both the correct powers and the correct dimensions.

Step 3: Check the rest and massless limits If the body is at rest, $p = 0$, and the relation gives $E = mc^2$ for positive total energy, recovering its rest energy. For a massless particle it reduces to $E = pc$. These limits provide useful independent checks of the squared factors of c and p; replacing total energy by kinetic energy would not satisfy the same equation.

Step 4: Conclusion The correct equation is $\boxed{E^2 = p^2c^2 + m^2c^4}$. Hence, the correct option is (C).

Quick Tip to Solve: Memorise the relation as $E^2 = (pc)^2 + (mc^2)^2$; the grouped factors prevent missing the square on momentum or a power of c.

36. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : In an insulated container, a gas is adiabatically shrunk to half of its initial volume. The temperature of the gas decreases.

Reason (R) : Free expansion of an ideal gas is an irreversible and an adiabatic process.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(C) (A) is false but (R) is true

(D) (A) is true but (R) is false

Correct Answer: (C) (A) is false but (R) is true

Solution:

Concept:

- The first law can be written $\Delta U = Q + W_{\text{on}}$. For an ideal gas, internal energy depends only on temperature, with $\Delta U = nC_V\Delta T$.
- An adiabatic process has no heat transfer, $Q = 0$. This does not imply constant temperature: work can change internal energy. Free expansion into a vacuum differs from compression because no external resisting pressure is present.

Step 1: Evaluate the assertion about compression The gas is shrunk to half its initial volume in an insulated container, so it is compressed without heat exchange. Compression requires positive work on the gas. Therefore

$$\Delta U = W_{\text{on}} > 0, \qquad \Delta T > 0.$$

The temperature rises rather than falls, so Assertion (A) is false. For reversible adiabatic compression this is also seen from $TV^{\gamma-1} =$ constant, giving $T_f = T_i 2^{\gamma-1} > T_i$ because $\gamma > 1$. The first-law argument explains the temperature change without confusing it with heat supplied.

Step 2: Evaluate the reason about free expansion In free expansion, gas expands into an evacuated space. The external pressure is zero, so $W = \int P_{\text{ext}}\, dV = 0$. In an insulated free expansion, $Q = 0$ as well; hence an ideal gas has $\Delta U = 0$ and no temperature change. The process is irreversible: simply restoring the surroundings does not spontaneously reconfine the gas, and its entropy increases. Thus the usual free expansion of an isolated ideal gas is both irreversible and adiabatic, making Reason (R) true.

Step 3: Choose the correct truth combination The assertion describes compression and predicts the wrong temperature change, while the reason correctly describes a different process, free expansion. Therefore the required combination is a false assertion with a true reason. The two options claiming both statements true fail because of the assertion, and the option claiming a true assertion with a false reason reverses both judgements.

Step 4: Conclusion The assertion is false, whereas the reason is true. Hence, the correct option is (C).

Quick Tip to Solve: Insulation means $Q = 0$, not $\Delta T = 0$: adiabatic compression heats the gas, while insulated ideal-gas free expansion has no work and no temperature change.

37. A solid sphere and a hollow sphere of the same mass and of same radius are rolled on an inclined plane. Let the time taken to reach the bottom by the solid sphere and the hollow sphere be t_1 and t_2, respectively, then [4 marks]

(A) $t_1 < t_2$

(B) $t_1 = 2t_2$

(C) $t_1 > t_2$

(D) $t_1 = t_2$

Correct Answer: (A) $t_1 < t_2$

Solution:

Concept:

- For rolling without slipping, the centre-of-mass acceleration down a slope of angle θ is $a = g\sin\theta/[1 + I/(mR^2)]$, where I is the moment of inertia about the centre.
- A solid sphere has $I_s = 2mR^2/5$, whereas a thin hollow sphere has $I_h = 2mR^2/3$. Greater rotational inertia reduces the translational acceleration.

Step 1: Find the acceleration of each sphere For the same inclined plane, the component of gravity along the slope is identical for the two bodies. Substituting their moments of inertia gives

$$a_s = \frac{g\sin\theta}{1 + 2/5} = \frac{5}{7}g\sin\theta, \qquad a_h = \frac{g\sin\theta}{1 + 2/3} = \frac{3}{5}g\sin\theta.$$

Since $5/7 > 3/5$, the solid sphere accelerates faster. Static friction supplies the required torque; it does not imply that both bodies have the same acceleration.

Step 2: Compare the times for the same distance Both spheres start from rest and cover the same slope length L, so $L = at^2/2$. Therefore

$$\frac{t_1}{t_2} = \sqrt{\frac{a_h}{a_s}} = \sqrt{\frac{3/5}{5/7}} = \frac{\sqrt{21}}{5} < 1.$$

Thus the solid sphere reaches the bottom first. Equal mass and radius do not give equal times because the mass distributions differ. Options (B) and (C) incorrectly make the solid sphere slower; option (D) ignores rotational inertia.

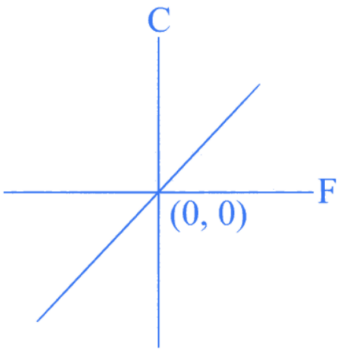
Step 3: Conclusion $t_1 < t_2$: the solid sphere takes less time to reach the bottom. Hence, the correct option is (A).

Quick Tip to Solve: For bodies rolling down the same slope, compare $I/(mR^2)$: the smaller value means a larger acceleration and a shorter arrival time.

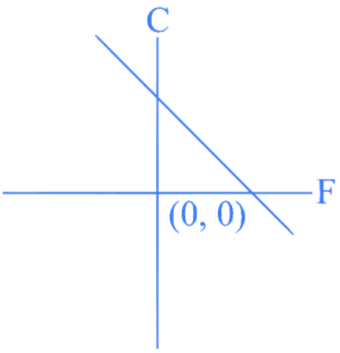
38. Which of the following figure represents the relation between Celsius and Fahrenheit temperatures?

[4 marks]

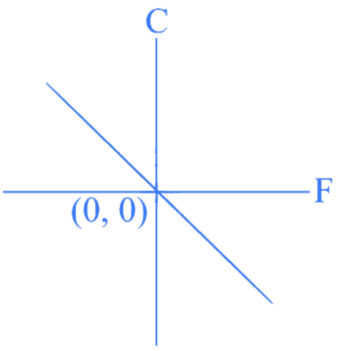
(A)



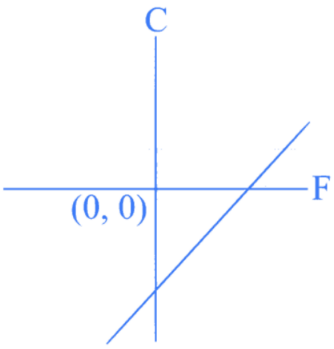
(B)



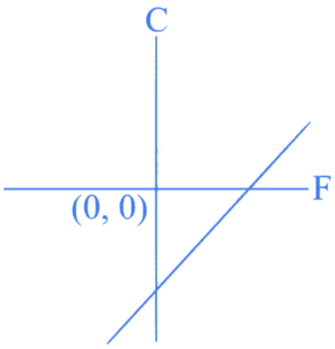
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- The Celsius and Fahrenheit scales are linearly related by $C/5 = (F-32)/9$, with C plotted vertically and F horizontally in the given figures.
- A straight line is determined by its slope and intercept. Here $C = (5/9)F - 160/9$, so the slope is positive and the vertical intercept is negative.

Step 1: Calculate the intercepts of the temperature line At the ice point, $C = 0$ and $F = 32$. Thus the graph must meet the positive Fahrenheit axis to the right of the origin. At $F = 0$, the Celsius reading is

$$C = \frac{5}{9}(0 - 32) = -\frac{160}{9} \simeq -17.8.$$

The line therefore cuts the vertical Celsius axis below the origin. It cannot pass through $(0, 0)$, since zero readings on the two scales represent different temperatures.

Step 2: Compare the slope and intercepts with all four graphs Increasing Fahrenheit temperature increases Celsius temperature, so the required graph rises from left to right. Graph (A) has the right sign of slope but incorrectly passes through the origin. Graphs (B) and (C) have negative slopes, which would make one temperature decrease as the other increases. Graph (D) alone rises to the right, crosses the vertical axis below zero and crosses the horizontal axis at a positive value. As an additional check, the boiling point $(F, C) = (212, 100)$ also lies on this positive-slope line.

Step 3: Conclusion The correct graph has positive slope, a negative Celsius intercept and a positive Fahrenheit intercept. Hence, the correct option is (D).

Quick Tip to Solve: Check the ice point first: $0°\text{C} = 32°\text{F}$, so a Celsius-versus-Fahrenheit line cannot pass through the origin.

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39. Arrange the following in the ascending order of wavelength (λ) :

(A) Microwaves (λ_1)

(B) Ultraviolet rays (λ_2)

(C) Infrared rays (λ_3)

(D) X-rays (λ_4)

Choose the most appropriate answer from the options given below : [4 marks]

(A) $\lambda_4 < \lambda_3 < \lambda_2 < \lambda_1$

(B) $\lambda_4 < \lambda_2 < \lambda_3 < \lambda_1$

(C) $\lambda_3 < \lambda_4 < \lambda_2 < \lambda_1$

(D) $\lambda_4 < \lambda_3 < \lambda_1 < \lambda_2$

Correct Answer: (B) $\lambda_4 < \lambda_2 < \lambda_3 < \lambda_1$

Solution:

Concept:

- All electromagnetic waves travel at speed c in vacuum and obey $c = \nu\lambda$, where ν is frequency and λ is wavelength. Higher frequency means shorter wavelength.
- In increasing wavelength order, the relevant regions are X-rays, ultraviolet, visible light, infrared and microwaves. Visible light helps locate ultraviolet and infrared on opposite sides of the optical band.

Step 1: Order the named regions by wavelength X-rays have the shortest wavelengths among the four listed regions. Ultraviolet radiation has wavelengths longer than X-rays but shorter than visible light. Infrared lies beyond the red end of visible light and therefore has a longer wavelength than ultraviolet. Microwaves have still longer wavelengths than infrared. The physical sequence required by the word ascending is consequently

$$\text{X-rays} < \text{ultraviolet} < \text{infrared} < \text{microwaves}.$$

Do not reverse this list using the frequency order, which runs in the opposite direction.

Step 2: Replace the radiation names by their assigned symbols The question assigns λ_4 to X-rays, λ_2 to ultraviolet rays, λ_3 to infrared rays and λ_1 to microwaves. Substitution gives

$$\lambda_4 < \lambda_2 < \lambda_3 < \lambda_1.$$

Option (A) wrongly places infrared below ultraviolet. Option (C) wrongly places infrared below X-rays. Option (D) also puts infrared below ultraviolet and makes ultraviolet longer than microwaves, so it fails both comparisons.

Step 3: Conclusion The ascending wavelength order is $\lambda_4 < \lambda_2 < \lambda_3 < \lambda_1$. Hence, the correct option is (B).

Quick Tip to Solve: Insert visible light mentally: X-rays, ultraviolet, visible, infrared, microwaves is the increasing-wavelength sequence; then substitute the question's labels.

40. Young's double slit inteference apparatus is immersed in a liquid of refractive index 1.44. It has slit separation of 1.5 mm . The slits are illuminated by a parallel beam of light whose wavelength in air is 690 nm . The fringe-width on a screen placed behind the plane of slits at a distance of 0.72 m , will be: [4 marks]

(A) 0.63 mm

(B) 0.33 mm

(C) 0.46 mm

(D) 0.23 mm

Correct Answer: (D) 0.23 mm

Solution:

Concept:

- In a medium of refractive index n, the frequency of light is unchanged but its wavelength becomes $\lambda = \lambda_{\text{air}}/n$, taking the refractive index of air as approximately one.
- The fringe width in Young's experiment is $\beta = \lambda D/d$, where D is the slit-to-screen distance and d is the slit separation. Immersion decreases fringe width by the factor n.

Step 1: Convert the dimensions and use the wavelength in the liquid The supplied values in SI units are

$$\lambda_{\text{air}} = 690 \times 10^{-9}\,\text{m}, \quad d = 1.5 \times 10^{-3}\,\text{m}, \quad D = 0.72\,\text{m}.$$

The apparatus is immersed, so the wavelength that controls the optical path difference is

$$\lambda = \frac{690 \times 10^{-9}}{1.44}\,\text{m} \simeq 479.17 \times 10^{-9}\,\text{m}.$$

Using the air wavelength unchanged would overestimate the fringe width.

Step 2: Calculate the fringe width and select its rounded value Substitute directly into the interference formula:

$$\beta = \frac{(690 \times 10^{-9})(0.72)}{(1.44)(1.5 \times 10^{-3})} = 2.30 \times 10^{-4}\,\text{m} = 0.23\,\text{mm}.$$

For comparison, the air fringe width would be $0.3312\,\text{mm}$, close to option (B); that option neglects the reduction on immersion. Options (A) and (C) are larger than the air value and cannot describe immersion in a medium with refractive index greater than one. The computed liquid value matches option (D).

Step 3: Conclusion The fringe width in the liquid is $0.23\,\text{mm}$. Hence, the correct option is (D).

Quick Tip to Solve: First calculate or recognise the air fringe width, then divide it by the liquid's refractive index; frequency does not change on immersion.

41. A solid sphere is rolling without slipping on a horizontal plane. The ratio of the linear kinetic energy of the centre of mass of the sphere and rotational kinetic energy is : [4 marks]

(A) $\frac{3}{4}$
(B) $\frac{4}{3}$
(C) $\frac{5}{2}$
(D) $\frac{2}{5}$

Correct Answer: (C) $\frac{5}{2}$

Solution:

Concept:

- The kinetic energy of a rolling rigid body is the sum of translational energy $K_{\text{lin}} = mv^2/2$ and rotational energy $K_{\text{rot}} = I\omega^2/2$, with the rotation taken about its centre of mass.
- For rolling without slipping, $v = \omega R$. A uniform solid sphere has central moment of inertia $I = 2mR^2/5$.

Step 1: Express both energies using the centre-of-mass speed Let the sphere have mass m, radius R and centre-of-mass speed v. Its translational energy is directly $mv^2/2$. Use the rolling constraint in the rotational energy:

$$K_{\text{rot}} = \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{5}mv^2.$$

The radius cancels because increasing radius increases the moment of inertia but reduces the angular speed corresponding to the same linear speed. The central moment of inertia is the appropriate one when translation and rotation are counted separately.

Step 2: Form the ratio in the order requested The question asks for linear energy divided by rotational energy, so

$$\frac{K_{\text{lin}}}{K_{\text{rot}}} = \frac{\frac{1}{2}mv^2}{\frac{1}{5}mv^2} = \frac{5}{2}.$$

It is dimensionless and independent of the sphere's mass, radius and nonzero rolling speed. Option (D), $2/5$, is the inverse ratio and would answer a different question. Options (A) and (B) do not follow from the solid-sphere moment of inertia. The translational energy is larger than the rotational energy, consistent with the ratio exceeding one.

Step 3: Conclusion The ratio of linear to rotational kinetic energy is $5/2$, or $5 : 2$. Hence, the correct option is (C).

Quick Tip to Solve: For any rolling body, $K_{\text{lin}}/K_{\text{rot}} = mR^2/I$. Check the order before choosing between a ratio and its reciprocal.

42. The position vector of a moving body at any instant of time is given as $\vec{r} = \left(5t^2\hat{i} - 5t\hat{j}\right)$ m. The magnitude and direction of velocity at $t = 2s$ is, [4 marks]

(A) $5\sqrt{17}$ m/s, making an angle of $\tan^{-1} 4$ with - ve Y axis

(B) $5\sqrt{15}$ m/s, making an angle of $\tan^{-1} 4$ with + ve X axis

(C) $5\sqrt{17}$ m/s, making an angle of $\tan^{-1} 4$ with + ve X axis

(D) $5\sqrt{15}$ m/s, making an angle of $\tan^{-1} 4$ with $-$ ve Y axis

Correct Answer: (A) $5\sqrt{17}$ m/s, making an angle of $\tan^{-1} 4$ with - ve Y axis

Solution:

Concept:

- Instantaneous velocity is the time derivative of position: $\vec{v} = d\vec{r}/dt$. Differentiate each Cartesian component separately while keeping the fixed unit vectors unchanged.
- The speed is $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$. The signs of the components determine the quadrant, and an angle must be measured from the reference axis specified in the option.

Step 1: Differentiate the position vector and evaluate at two seconds The position is $\vec{r} = (5t^2\hat{i} - 5t\hat{j})$ m, with time in seconds. Hence

$$\vec{v} = (10t\hat{i} - 5\hat{j})\,\mathrm{m\,s^{-1}}, \qquad \vec{v}(2) = (20\hat{i} - 5\hat{j})\,\mathrm{m\,s^{-1}}.$$

Its magnitude is

$$|\vec{v}| = \sqrt{20^2 + (-5)^2} = \sqrt{425} = 5\sqrt{17}\,\mathrm{m\,s^{-1}}.$$

Thus any option containing $5\sqrt{15}$ is eliminated; the squared components add even though the vertical component is negative.

Step 2: Identify the direction relative to the negative vertical axis The vector points rightward and downward, in the fourth quadrant. If α is the acute angle from the negative Y-axis towards positive X, the transverse component is 20 and the component along the reference axis is 5. Therefore

$$\tan\alpha = \frac{20}{5} = 4, \qquad \alpha = \tan^{-1} 4.$$

Relative to the positive X-axis its downward angle is instead $\tan^{-1}(1/4)$, so option (C) has the correct speed but the wrong direction. Option (A) gives both quantities correctly.

Step 3: Conclusion The velocity has magnitude $5\sqrt{17}\,\mathrm{m\,s^{-1}}$ and makes an angle $\tan^{-1} 4$ with the negative Y-axis towards positive X. Hence, the correct option is (A).

Quick Tip to Solve: Write the signed velocity components before finding an angle. An angle from the vertical axis uses horizontal component divided by vertical-component magnitude.

43. A photograph of a landscape is captured by a drone camera at a height of 18 km . The size of the camera film is $2\ \text{cm} \times 2\ \text{cm}$ and the area of the landscape photographed is $400\ \text{km}^2$. The focal length of the lens in the drone camera is : [4 marks]

(A) 2.8 cm

(B) 0.9 cm

(C) 2.5 cm

(D) 1.8 cm

Correct Answer: (D) 1.8 cm

Solution:

Concept:

- A camera lens forms a real, diminished image of the landscape on its film. The magnitude of linear magnification is $m = v/u = h_i/h_o$, where u and v are object and image distances.
- For a distant landscape, $u \gg f$, so the film lies approximately in the focal plane: $v \simeq f$. Area magnification is the square of linear magnification, not the linear magnification itself.

Step 1: Convert area to a linear scale The square film has side $2\,\text{cm}$. The corresponding square landscape has side $\sqrt{400} = 20\,\text{km}$. Thus the ratio of corresponding lengths is

$$m = \frac{2\,\text{cm}}{20\,\text{km}} = 10^{-6}.$$

The ratio of the areas would be 10^{-12}; using that directly as the linear magnification would give an incorrect focal length.

Step 2: Find the image distance and focal length The object distance is the drone height, $u = 18\,\text{km}$. Therefore

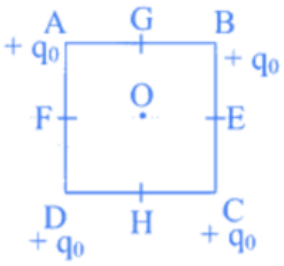
$$v = mu = 10^{-6}(18 \times 10^3)\,\text{m} = 0.018\,\text{m} = 1.8\,\text{cm}.$$

Using positive distance magnitudes, the thin-lens equation gives $f = uv/(u+v)$. Since $v/u = 10^{-6}$, this differs from v by only about one part in a million. Hence $f \simeq 1.8\,\text{cm}$. The values 2.8, 0.9 and 2.5 cm do not reproduce the specified image scale at the stated height.

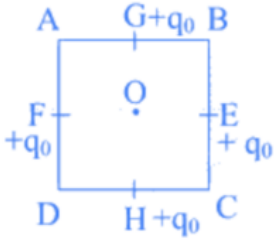
Step 3: Conclusion The focal length is approximately $\boxed{1.8\,\text{cm}}$. Hence, the correct option is (D).

Quick Tip to Solve: Take the square root of the area ratio before using camera magnification; for a distant scene, multiply that linear ratio by the camera height.

44. In the first configuration (1) as shown in the figure, four identical charges (q_0) are kept at the corners A, B, C and D of square of side length ' a '. In the second configuration (2), the same charges are shifted to mid points G, E, H and F, of the square. If $K = \frac{1}{4\pi\epsilon_0}$, the difference between the potential energies of configuration (2) and (1) is given by : [4 marks]



Configuration (1)



Configuration (2)

(A) $\frac{Kq_0^2}{a}(4-2\sqrt{2})$

(B) $\frac{Kq_0^2}{a}(4\sqrt{2}-2)$

(C) $\frac{Kq_0^2}{a}(3-\sqrt{2})$

(D) $\frac{Kq_0^2}{a}(3\sqrt{2}-2)$

Correct Answer: (D) $\frac{Kq_0^2}{a}(3\sqrt{2}-2)$

Solution:

Concept:

- The energy of a system of point charges is the sum over distinct pairs: $U = \sum_{i<j} Kq_iq_j/r_{ij}$. Four charges form $\binom{4}{2} = 6$ pairs.
- The midpoint charges form a smaller square whose side is $a/\sqrt{2}$ and whose diagonal is a. All charges have the same sign, so every pair energy is positive.

Step 1: Sum the six corner-pair energies In configuration (1), four adjacent pairs are separated by the original side a. The other two pairs lie along diagonals of length $a\sqrt{2}$. Counting each pair just once gives

$$U_1 = Kq_0^2\left(\frac{4}{a} + \frac{2}{a\sqrt{2}}\right) = \frac{Kq_0^2}{a}(4+\sqrt{2}).$$

There is no additional factor of one-half here because the sum already includes each unordered pair only once.

Step 2: Sum the midpoint energies and subtract In configuration (2), adjacent midpoints are separated by $\sqrt{(a/2)^2+(a/2)^2} = a/\sqrt{2}$. Opposite midpoints are separated by a. Thus

$$U_2 = Kq_0^2\left(\frac{4}{a/\sqrt{2}} + \frac{2}{a}\right) = \frac{Kq_0^2}{a}(4\sqrt{2}+2).$$

The requested difference has the order configuration (2) minus configuration (1):

$$U_2 - U_1 = \frac{Kq_0^2}{a}\left[(4\sqrt{2}+2) - (4+\sqrt{2})\right] = \frac{Kq_0^2}{a}(3\sqrt{2}-2).$$

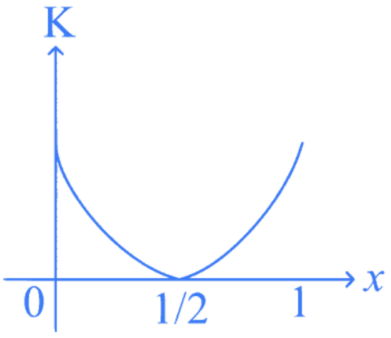
This is positive, as bringing like charges closer requires work. The other coefficients result from incorrect pair counts or separations.

Step 3: Conclusion The energy difference is $\boxed{\frac{Kq_0^2}{a}(3\sqrt{2}-2)}$. Hence, the correct option is (D).

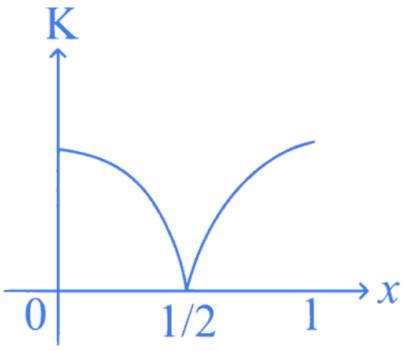
Quick Tip to Solve: For four charges on a square, group the six pairs into four sides and two diagonals before calculating either energy.

45. A particle oscillates along the x-axis according to the law, $x(t) = x_0 \sin^2\left(\frac{t}{2}\right)$ where $x_0 = 1$ m. The kinetic energy (K) of the particle as a function of x is correctly represented by the graph [4 marks]

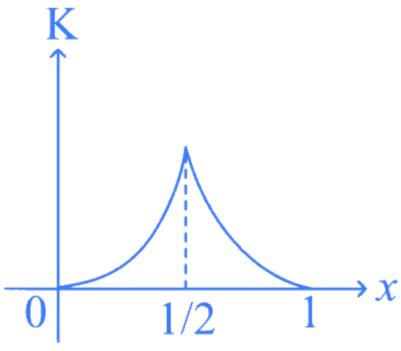
(A)



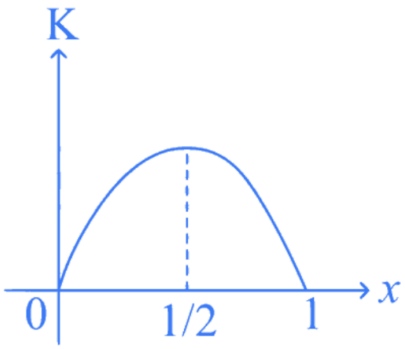
(B)



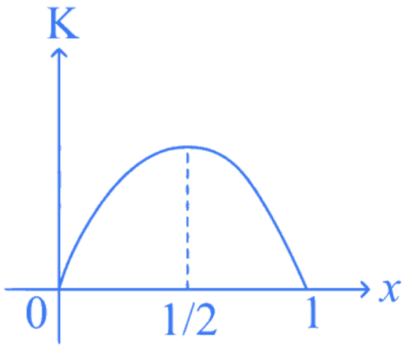
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- The velocity is the time derivative of displacement, $v = dx/dt$, and the kinetic energy is $K = \frac{1}{2}mv^2$. Eliminate time to find the shape of K versus x.
- The identity $\sin^2(t/2) = (1 - \cos t)/2$ shows that the mean position is $x_0/2$, not zero. The motion is restricted to $0 \le x \le x_0$.

Step 1: Differentiate the displacement Writing the displacement in terms of $\cos t$,

$$x = \frac{x_0}{2}(1 - \cos t), \qquad v = \frac{x_0}{2}\sin t.$$

Thus $\cos t = 1 - 2x/x_0$. Squaring the velocity removes its sign, as required for kinetic energy, and gives

$$v^2 = \frac{x_0^2}{4}\left[1 - \left(1 - \frac{2x}{x_0}\right)^2\right] = x_0 x - x^2.$$

The time argument has the angular-frequency factor implicit in the stated law.

Step 2: Identify the graph from its zeros and curvature Therefore

$$K = \frac{m}{2}(x_0 x - x^2) = \frac{m}{2}\left[\frac{x_0^2}{4} - \left(x - \frac{x_0}{2}\right)^2\right].$$

This is a smooth, downward-opening parabola. With $x_0 = 1\,\text{m}$, it vanishes at $x = 0$ and $x = 1\,\text{m}$, and has its maximum at $x = 0.5\,\text{m}$. Option (D) has exactly these features. Options (A) and (B) put a minimum at the midpoint, where the speed is actually largest; option (C) has a pointed maximum

and the wrong curvature. A quadratic kinetic-energy expression cannot produce that cusp.

Step 3: Conclusion The kinetic-energy curve is the downward-opening parabola with zeros at 0 and $1\,\mathrm{m}$ and a smooth maximum at $0.5\,\mathrm{m}$. Hence, the correct option is (D).

Quick Tip to Solve: For shifted SHM, locate the two turning points first: kinetic energy is zero there and is greatest at their midpoint.

Physics – Section B: Numerical Value Questions (20 marks)

46. The ratio of the power of a light source S_1 to that the light source S_2 is $2.S_1$ is emitting 2×10^{15} photons per second at 600 nm . If the wavelength of the source S_2 is 300 nm , then the number of photons per second emitted by S_2 is _____ $\times 10^{14}$. [4 marks]

Solution:

Concept:

- A photon of wavelength λ has energy $E = hc/\lambda$. If $\dot{N}$ photons are emitted each second, the optical power is $P = \dot{N}hc/\lambda$.
- The emission rate counts photons per unit time, whereas power counts energy per unit time. Equal photon rates do not imply equal powers when wavelengths differ.

Step 1: Relate power to the photon emission rates Let the photon rates for the two sources be $\dot{N}_1$ and $\dot{N}_2$. Since h and c are common,

$$\frac{P_1}{P_2} = \frac{\dot{N}_1}{\dot{N}_2}\frac{\lambda_2}{\lambda_1}.$$

The stated power ratio is $P_1/P_2 = 2$, not the photon-rate ratio. The second wavelength is half the first, so each photon from S_2 carries twice as much energy as a photon from S_1.

Step 2: Substitute the wavelengths and isolate the rate Insert $\lambda_1 = 600\,\mathrm{nm}$, $\lambda_2 = 300\,\mathrm{nm}$, and $\dot{N}_1 = 2 \times 10^{15}\,\mathrm{s}^{-1}$:

$$2 = \frac{2 \times 10^{15}}{\dot{N}_2}\frac{300}{600}.$$

Hence

$$\dot{N}_2 = \frac{2 \times 10^{15}}{4} = 5 \times 10^{14}\,\mathrm{s}^{-1}.$$

This also checks physically: source S_2 has half the power but twice the energy per photon, so its photon rate must be one-quarter of that of S_1. The blank asks for the coefficient of 10^{14}, rather than the full emission rate.

Step 3: Conclusion Source S_2 emits 5×10^{14} photons per second. The required numerical value is $\boxed{5}$.

Quick Tip to Solve: Use photon rate proportional to power times wavelength: halving both power and wavelength reduces the photon rate to one-quarter.

47. The increase in pressure required to decrease the volume of a water sample by 0.2% is $\mathrm{P}\times10^5\mathrm{Nm}^{-2}$. Bulk modulus of water is $2.15\times10^9\mathrm{Nm}^{-2}$. The value of P is _____ . [4 marks]

Solution:

Concept:

- The bulk modulus is $B=-\Delta p/(\Delta V/V)$. The negative sign makes B positive because increasing pressure decreases volume.
- For a small compression, the required pressure rise is $\Delta p=B|\Delta V|/V$. A percentage strain must be divided by 100 before substitution.

Step 1: Convert the stated compression into fractional strain The sample loses 0.2% of its original volume, so

$$\frac{\Delta V}{V}=-\frac{0.2}{100}=-0.002=-2\times10^{-3}.$$

The relevant strain is dimensionless. Using 0.2 itself would represent a 20 percent compression and would overestimate the pressure by a factor of 100. The minus sign denotes the decrease of volume and does not mean that the applied pressure decreases.

Step 2: Calculate the pressure rise and the requested coefficient Use the given bulk modulus of water, $B=2.15\times10^9\,\mathrm{N\,m^{-2}}$:

$$\Delta p=-B\frac{\Delta V}{V}=(2.15\times10^9)(2\times10^{-3})=4.30\times10^6\,\mathrm{N\,m^{-2}}.$$

The positive result is an increase in pressure, consistent with compression. The question expresses this as $P\times10^5\,\mathrm{N\,m^{-2}}$, so

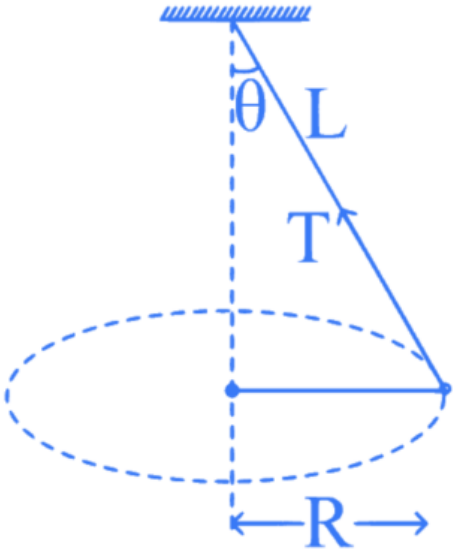
$$P=\frac{4.30\times10^6}{10^5}=43.$$

The coefficient is dimensionless; the pressure unit belongs to the full pressure expression. The fractional compression is small, making the linear bulk-modulus relation appropriate.

Step 3: Conclusion The required pressure increase is $4.30\times10^6\,\mathrm{N\,m^{-2}}=43\times10^5\,\mathrm{N\,m^{-2}}$. Thus $\boxed{P=43}$.

Quick Tip to Solve: Convert percent to a fraction first, calculate the pressure in pascals, then divide by the power of ten specified beside the blank.

48. A string of length L is fixed at one end and carries a mass of M at the other end. The mass makes $\left(\frac{3}{\pi}\right)$ rotations per second about the vertical axis passing through end of the string as shown. The tension in the string is _____ ML. [4 marks]



Solution:

Concept:

- For a conical pendulum, the mass moves in a horizontal circle while the string makes a constant angle θ with the vertical. Its radius is $R = L\sin\theta$.
- Tension supplies the horizontal centripetal force: $T\sin\theta = M\omega^2 R$. The angular speed is $\omega = 2\pi f$, where f is rotations per second.

Step 1: Convert rotation frequency into angular speed The given frequency is $f = 3/\pi\,\mathrm{s}^{-1}$. Each complete rotation corresponds to 2π radians, so

$$\omega = 2\pi\left(\frac{3}{\pi}\right) = 6\,\mathrm{rad\,s}^{-1}.$$

Using the numerical frequency directly in the centripetal-force formula would omit the factor 2π. The vertical component of tension balances the weight, $T\cos\theta = Mg$; the mass has no vertical acceleration.

Step 2: Use the horizontal component of tension The horizontal force equation and the geometry give

$$T\sin\theta = M\omega^2 R = M\omega^2 L\sin\theta.$$

For the nonzero circular radius shown, $\sin\theta \neq 0$, and cancellation yields

$$T = M\omega^2 L = ML(6)^2 = 36ML.$$

The angle is unnecessary because both the horizontal component of tension and the orbital radius contain the same factor $\sin\theta$. Gravity determines the permissible cone angle through $\cos\theta = g/(\omega^2 L)$, while the given angular speed and length fix the tension. Since $ML\omega^2$ has units $\mathrm{kg\,m\,s}^{-2}$, it is a force

as required.

Step 3: Conclusion The string tension is $\boxed{T = 36ML}$ in SI units. The numerical coefficient required in the blank is $\boxed{36}$.

Quick Tip to Solve: For a conical pendulum with known frequency and string length, use $T = M(2\pi f)^2 L$; the cone angle cancels.

49. Acceleration due to gravity on the surface of earth is ' g '. If the diameter of earth is reduced to one third of its original value and mass remains unchanged, then the acceleration due to gravity on the surface of the earth is _____ g. [4 marks]

Solution:

Concept:

- For a spherical Earth, the gravitational acceleration at the surface is $g = GM/R^2$, where G is the gravitational constant, M is the total mass and R is the radius.
- Diameter and radius change by the same factor because $D = 2R$. At fixed mass, the surface acceleration varies inversely as the square of the radius.

Step 1: Express the new radius Let the original diameter be $D = 2R$. Reducing the diameter to one third gives $D' = D/3$, so the new surface is at a radius

$$R' = \frac{D'}{2} = \frac{D}{6} = \frac{R}{3}.$$

The mass is unchanged, so the numerator GM in the gravitational acceleration does not change. It would be incorrect to hold the density fixed and reduce the mass with the volume.

Step 2: Compare the two surface accelerations Use the same gravitational law at the original and new surfaces:

$$g = \frac{GM}{R^2}, \qquad g' = \frac{GM}{(R/3)^2} = \frac{9GM}{R^2}.$$

Consequently,

$$\frac{g'}{g} = \left(\frac{R}{R'}\right)^2 = 3^2 = 9.$$

The increase is quadratic rather than linear: a factor of three reduction in distance gives a factor of nine increase in acceleration. Both accelerations have the same units, so their ratio is dimensionless.

Step 3: Conclusion The new surface acceleration is $g' = 9g$. Therefore, the number to enter in the blank is $\boxed{9}$.

Quick Tip to Solve: When the mass is fixed, square the inverse radius ratio. A diameter ratio can be used directly because it equals the radius ratio.

50. A tightly wound long solenoid carries a current of 1.5 A . An electron is executing uniform circular motion inside the solenoid with a time period of 75 ns . The number of turns per metre in the solenoid is _____.

[Take mass of electron $m_e = 9 \times 10^{-31}\,\text{kg}$**, charge of electron** $|q_e| = 1.6 \times 10^{-19}\,\text{C}$**,** $\mu_0 = 4\pi \times 10^{-7}\,\text{N A}^{-2}$**,** $1\,\text{ns} = 10^{-9}\,\text{s}$**.]** [4 marks]

Solution:

Concept:

- A tightly wound long solenoid produces a uniform interior magnetic field $B = \mu_0 n I$, where n is the number of turns per metre and I is its current.
- For an electron moving in a circle perpendicular to a uniform magnetic field, $T = 2\pi m_e/(|q_e|B)$. The magnitude of the charge determines the period; its sign determines the sense of rotation.

Step 1: Relate the period to the magnetic field The magnetic force supplies the centripetal force:

$$|q_e| v B = \frac{m_e v^2}{r}.$$

Since $T = 2\pi r/v$, eliminating r/v gives

$$T = \frac{2\pi m_e}{|q_e| B}.$$

The speed and orbit radius cancel, so neither is required. Convert the period before substitution: $75\,\text{ns} = 75 \times 10^{-9}\,\text{s}$.

Step 2: Calculate the turn density Substitute $B = \mu_0 n I$ and solve for n:

$$n = \frac{2\pi m_e}{|q_e| \mu_0 I T}.$$

Using precisely the constants given in the question,

$$n = \frac{2\pi(9 \times 10^{-31})}{(1.6 \times 10^{-19})(4\pi \times 10^{-7})(1.5)(75 \times 10^{-9})}.$$

The denominator is $720\pi \times 10^{-35}$, while the numerator is $18\pi \times 10^{-31}$. Thus

$$n = \frac{18}{720} \times 10^{4} = 250\,\text{m}^{-1}.$$

The positive turn density is physically required; inserting the negative electron charge instead of its magnitude would give a spurious negative result.

Step 3: Conclusion The solenoid has $\boxed{250 \text{ turns per metre}}$. The numerical entry is 250.

Quick Tip to Solve: Combine the cyclotron period and solenoid formulas before inserting numbers; the factors of pi cancel and no electron speed is needed.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. The successive 5 ionisation energies of an element are $800, 2427, 3658, 25024$ and 32824 kJ/mol, respectively. By using the above values predict the group in which the above element is present :

[4 marks]

(A) Group 4

(B) Group 2

(C) Group 13

(D) Group 14

Correct Answer: (C) Group 13

Solution:

Concept:

- Successive ionisation energies remove electrons one at a time from a gaseous atom and then its increasingly positive ions. They generally increase because the remaining electrons are more strongly attracted.
- A particularly large jump occurs when removal of the outer valence electrons has been completed and the next electron must be removed from an inner, closed shell.

Step 1: Locate the exceptional increase The first five ionisation energies are $800, 2427, 3658, 25024$ and $32824\,\text{kJ mol}^{-1}$. The successive increases are

$$1627, \quad 1231, \quad 21366, \quad 7800\,\text{kJ mol}^{-1}.$$

The striking discontinuity is between the third and fourth removals: the fourth value is about 6.84 times the third. This is much more significant than the ordinary increase associated with the earlier removals.

Step 2: Identify the valence electron count and group Three electrons can therefore be removed from the outer shell before an inner-shell electron must be taken. The stable core is reached after forming the tripositive ion, so the element has three valence electrons. A main-group element with valence configuration ns^2np^1 belongs to Group 13. Group 2 would show its major discontinuity after the second ionisation, whereas Group 14 would show it after the fourth. Group 4 is a transition-metal group and is not the three-valence-electron main-group assignment indicated by these values.

Step 3: Conclusion The large jump after the third ionisation identifies $\boxed{\text{Group 13}}$. Hence, the correct option is (C).

Quick Tip to Solve: Locate the largest jump first. The number of electrons removed before that jump gives the number of outer electrons.

52. Match List - I with List - II.

	List - I(Transition metal ion)		List - II(Spin only magnetic moment (B.M.))
(A)	Ti^{3+}	**(I)**	**3.87**
(B)	V^{2+}	**(II)**	**0.00**
(C)	Ni^{2+}	**(III)**	**1.73**
(D)	Sc^{3+}	**(IV)**	**2.84**

Choose the correct answer from the options given below : [4 marks]

(A) $(A)-(III),(B)-(I),(C)-(IV),(D)-(II)$

(B) $(A)-(II),(B)-(IV),(C)-(I),(D)-(III)$

(C) $(A)-(III),(B)-(I),(C)-(II),(D)-(IV)$

(D) $(A)-(IV),(B)-(II),(C)-(III),(D)-(I)$

Correct Answer: (A) $(A)-(III),(B)-(I),(C)-(IV),(D)-(II)$

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{B.M.}$, where n is the number of unpaired electrons and B.M. denotes the Bohr magneton.
- When forming these transition-metal ions, remove the $4s$ electrons before the $3d$ electrons. Distribute the remaining d electrons according to Hund's rule.

Step 1: Find the electronic configurations Titanium has neutral configuration $[Ar]3d^2 4s^2$, so Ti^{3+} is $3d^1$, with one unpaired electron. Vanadium is $[Ar]3d^3 4s^2$, giving V^{2+} as $3d^3$, with three unpaired electrons. Nickel is $[Ar]3d^8 4s^2$; its dipositive ion is $3d^8$, containing two unpaired electrons. Scandium loses its two $4s$ electrons and its one $3d$ electron to form Sc^{3+}, which is $3d^0$.

Step 2: Calculate and match the moments For the four ions, the spin-only values are

$$\begin{aligned} Ti^{3+}: &\quad \sqrt{1(1+2)} = \sqrt{3} \simeq 1.73, \\ V^{2+}: &\quad \sqrt{3(3+2)} = \sqrt{15} \simeq 3.87, \\ Ni^{2+}: &\quad \sqrt{2(2+2)} = \sqrt{8} \simeq 2.83, \\ Sc^{3+}: &\quad 0. \end{aligned}$$

The listed value 2.84 corresponds to the approximate $\sqrt{8}$ value. Hence the matching is A–III, B–I, C–IV and D–II. Options (B) and (D) already misidentify titanium, while (C) incorrectly assigns zero moment to nickel and a nonzero moment to scandium.

Step 3: Conclusion The correct matching is $\boxed{A \to III,\ B \to I,\ C \to IV,\ D \to II}$. Hence, the correct option is (A).

Quick Tip to Solve: Start with the unmistakable d0 ion: scandium(III) has zero moment. Then check the d1 and d3 values to identify the complete match.

53.

Find the compound ' A ' from the following reaction sequences.

$$\mathrm{A} \xrightarrow{\textbf{aqua-regia}} \mathrm{B} \xrightarrow[\textbf{(2)}\ \mathrm{AcOH}]{\textbf{(1)}\ \mathrm{KNO_2|NH_4OH}} \textbf{yellow ppt}$$

[4 marks]

(A) NiS

(B) MnS

(C) ZnS

(D) CoS

Correct Answer: (D) CoS

Solution:

Concept:

- In qualitative salt analysis, the identity of a metal ion is established by its characteristic reaction with a confirmatory reagent after the original salt has been dissolved.
- Cobalt forms a yellow potassium hexanitritocobaltate(III) precipitate with potassium nitrite in an acetate medium. Its formula is $\mathrm{K_3[Co(NO_2)_6]}$.

Step 1: Identify the diagnostic final precipitate The decisive observation is the yellow precipitate obtained after treatment with $\mathrm{KNO_2}$ and then acetic acid, $\mathrm{CH_3COOH}$. This is the cobalt confirmatory test: nitrite coordinates to cobalt, with the cobalt in the final complex present in the $+3$ oxidation state. The resulting hexanitritocobaltate(III) anion is $\mathrm{[Co(NO_2)_6]^{3-}}$, and three potassium ions give the yellow salt $\mathrm{K_3[Co(NO_2)_6]}$. Its charge follows directly from $+3 + 6(-1) = -3$.

Step 2: Trace the metal back to the starting sulphide Aqua regia dissolves the starting sulphide and brings its metal into solution. Subsequent reagent treatment changes the cobalt species and produces the characteristic complex, but it does not replace cobalt by another metal. Thus the starting compound must contain cobalt, which identifies CoS. Nickel sulphide would give nickel ions, commonly confirmed by a red dimethylglyoxime precipitate. Manganese and zinc salts do not give the yellow potassium cobaltinitrite test. Therefore NiS, MnS and ZnS are excluded by the identity of the final precipitate.

Step 3: Conclusion The compound A is $\boxed{\mathrm{CoS}}$, cobalt sulphide. Hence, the correct option is (D).

Quick Tip to Solve: In a reaction sequence for salt analysis, identify the final confirmatory precipitate first and then trace the unchanged metal back to the starting salt.

54.

$$\mathrm{S}(\mathrm{g}) + \frac{3}{2}\mathrm{O}_2(\mathrm{g}) \rightarrow \mathrm{SO}_3(\mathrm{g}) + 2x\,\mathrm{kcal}$$

$$\mathrm{SO}_2(\mathrm{g}) + \frac{1}{2}\mathrm{O}_2(\mathrm{g}) \rightarrow \mathrm{SO}_3(\mathrm{g}) + y\,\mathrm{kcal}$$

The heat of formation of $\mathrm{SO}_2(\mathrm{g})$ is given by : [4 marks]

(A) $2x + y$ kcal

(B) $\frac{2x}{y}$ kcal

(C) $y - 2x$ kcal

(D) $x + y$ kcal

Correct Answer: (C) $y - 2x$ kcal

Solution:

Concept:

- Hess's law states that reaction enthalpy depends only on the initial and final states. Reversing a reaction reverses the sign of its enthalpy change.
- Heat written on the product side is released, so the enthalpy changes of the two displayed reactions are $-2x$ and $-y$ kcal, respectively.

Step 1: Reverse the second reaction The given reactions can be written in enthalpy notation as

$$\mathrm{S}(\mathrm{g}) + \frac{3}{2}\mathrm{O}_2(\mathrm{g}) \longrightarrow \mathrm{SO}_3(\mathrm{g}), \qquad \Delta H_1 = -2x\,\mathrm{kcal},$$

$$\mathrm{SO}_2(\mathrm{g}) + \frac{1}{2}\mathrm{O}_2(\mathrm{g}) \longrightarrow \mathrm{SO}_3(\mathrm{g}), \qquad \Delta H_2 = -y\,\mathrm{kcal}.$$

To obtain sulphur dioxide as a product, reverse the second reaction:

$$\mathrm{SO}_3(\mathrm{g}) \longrightarrow \mathrm{SO}_2(\mathrm{g}) + \frac{1}{2}\mathrm{O}_2(\mathrm{g}), \qquad \Delta H_{\mathrm{rev}} = +y\,\mathrm{kcal}.$$

Keep the states exactly as printed; no change of state is included in this reversal.

Step 2: Add the reactions and cancel common species Add the first reaction to the reversed second reaction. Sulphur trioxide cancels, and half a mole of oxygen cancels from the oxygen terms. The net reaction is

$$\mathrm{S}(\mathrm{g}) + \mathrm{O}_2(\mathrm{g}) \longrightarrow \mathrm{SO}_2(\mathrm{g}).$$

Its enthalpy change is

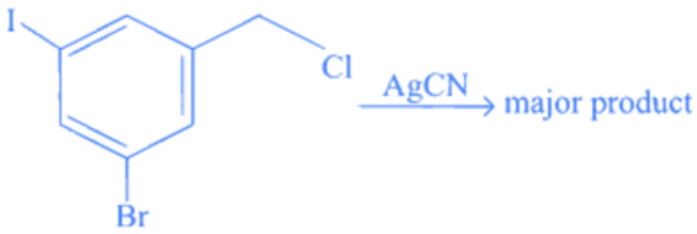
$$\Delta H = \Delta H_1 - \Delta H_2 = -2x - (-y) = y - 2x\,\mathrm{kcal}.$$

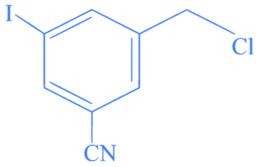
The sum $2x + y$ ignores the reversal, the quotient is not a Hess-law combination, and $x + y$ loses the coefficient 2. Since the starting sulphur is printed as gaseous sulphur, this calculation refers to that stated reactant, rather than formation from standard-state solid sulphur.

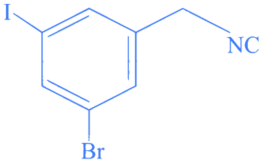
Step 3: Conclusion For the formation reaction using the sulphur state specified in the question, the enthalpy change is $\boxed{y - 2x\,\mathrm{kcal}}$. Hence, the correct option is (C).

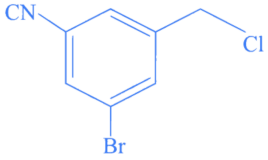
Quick Tip to Solve: Translate heat on the product side into a negative enthalpy first. Then reverse the unwanted product reaction before adding.

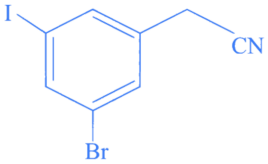
55. The structure of the major product formed in the following reaction is : [4 marks]



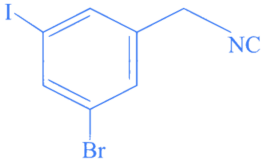
(A) 1-I−3-(CH_2Cl)−5-CN−C_6H_3 

(B) 1-I−3-(CH_2NC)−5-Br−C_6H_3 

(C) 1-CN−3-(CH_2Cl)−5-Br−C_6H_3 

(D) 1-I−3-(CH_2CN)−5-Br−C_6H_3 

Correct Answer: (B)

1-I−3-(CH_2NC)−5-Br−C_6H_3 

Solution:

Concept:

- Cyanide is an ambident nucleophile: bonding through carbon gives a nitrile, $R-CN$, whereas bonding through nitrogen gives an isocyanide, $R-NC$.
- With predominantly covalent $AgCN$, the nitrogen end is available for attack. A benzylic carbon-halogen bond undergoes substitution much more readily than an unactivated aryl carbon-halogen bond.

Step 1: Locate the reactive carbon-halogen bond The molecule contains a benzylic $-CH_2Cl$ side chain, together with iodine and bromine bonded directly to the aromatic ring. The side-chain carbon is saturated and benzylic, so substitution occurs at this carbon. In contrast, the aryl $C-I$ and $C-Br$ bonds are not replaced under these conditions. Their partial double-bond character and the unfavourable

ordinary substitution pathways must be distinguished from the benzylic chloride.

Step 2: Apply the reagent selectivity For $AgCN$, carbon is strongly associated with silver and attack occurs predominantly through nitrogen. Writing Ar for the unchanged iodo-bromophenyl group, the transformation is

$$Ar{-}CH_2Cl + AgCN \longrightarrow Ar{-}CH_2{-}NC + AgCl.$$

The product therefore retains both ring halogens and changes only the terminal chlorine into an isocyano group. Option (B) shows precisely this connectivity.

Step 3: Eliminate the other structures Options (A) and (C) replace a ring halogen by a cyano group while retaining the benzylic chloride, so they identify the wrong reaction site. Option (D) has the correct site but gives a nitrile, $-CH_2-CN$, which is characteristic of carbon attack with an ionic cyanide reagent such as KCN. The printed reagent is instead $AgCN$.

Step 4: Conclusion The major product is the benzylic isocyanide, $Ar{-}CH_2{-}NC$, with both aryl halogens unchanged. Hence, the correct option is (B).

Quick Tip to Solve: First distinguish aryl halides from the benzylic halide; then remember that $AgCN$ favours isocyanide, while KCN favours nitrile.

---

56. The elemental composition of a compound is $54.2\%C, 9.2\%H$ and $36.6\%O$. If the molar mass of the compound is $132\ g\ mol^{-1}$, the molecular formula of the compound is : [Given : The relative atomic mass of $C : H : O = 12 : 1 : 16$] [4 marks]

(A) $C_4H_8O_2$

(B) $C_6H_{12}O_6$

(C) $C_4H_9O_3$

(D) $C_6H_{12}O_3$

Correct Answer: (D) $C_6H_{12}O_3$

Solution:

Concept:

- Taking a $100\,g$ sample converts percentage composition directly into masses. Divide each mass by its atomic mass to obtain relative mole amounts.
- The molecular formula is an integral multiple of the empirical formula: $n = M/M_{emp}$, where M is the given molar mass.

Step 1: Convert masses to mole ratios For a hypothetical $100\,g$ sample, the carbon, hydrogen and oxygen masses are 54.2, 9.2 and $36.6\,g$, respectively. Hence

$$n_C = \frac{54.2}{12} = 4.5167, \quad n_H = 9.2, \quad n_O = \frac{36.6}{16} = 2.2875.$$

Dividing by the smallest amount gives approximately $1.9745 : 4.0219 : 1$, or $2 : 4 : 1$. The small deviations arise from the percentages being rounded.

Step 2: Find the integral molecular multiplier The empirical formula is therefore C_2H_4O, whose

formula mass is

$$M_{\text{emp}} = 2(12) + 4(1) + 16 = 44.$$

The molecular mass is 132, so

$$n = \frac{132}{44} = 3, \qquad (C_2H_4O)_3 = C_6H_{12}O_3.$$

Multiply every empirical subscript by the same integer; do not multiply only the oxygen count.

Step 3: Check the proposed options For $C_6H_{12}O_3$, the mass contributions are 72, 12, and 48, totalling 132. The corresponding percentages are approximately 54.55%, 9.09%, and 36.36%, consistent with the slightly rounded printed composition. The other options have formula masses 88, 180, and 105, respectively, so none can satisfy the stated molar mass. This mass check fixes the unique answer despite the imprecision of the percentages.

Step 4: Conclusion The molecular formula is $C_6H_{12}O_3$. Hence, the correct option is (D).

Quick Tip to Solve: After finding the empirical ratio, use the given molar mass as a decisive check; every subscript must be multiplied by the same integer.

---

57. Based on the data given below :

$$E^{\circ}_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33\ V \quad E^{\circ}_{Cl_2/Cl^{(-)}} = 1.36\ V$$
$$E^{0}_{MnO_4^{-}/Mn^{2+}} = 1.51\ V \quad E^{\circ}_{Cr^{3+}/Cr} = -0.74\ V$$

the strongest reducing agent is : [4 marks]

(A) Cl^-

(B) MnO_4^-

(C) Cr

(D) Mn^{2+}

Correct Answer: (C) Cr

Solution:

Concept:

- A reducing agent donates electrons and is itself oxidised. A species more readily oxidised is a stronger reducing agent.
- For a conjugate reduction couple, reversing the half-reaction changes the sign of E°: $E^{\circ}_{\text{ox}} = -E^{\circ}_{\text{red}}$. A more negative reduction potential makes the reduced member a stronger reductant.

Step 1: Identify the reduced members The listed potentials describe reduction of dichromate, chlorine, permanganate and chromium(III), respectively. The relevant candidate reducing species are Cl^-, Mn^{2+}, and metallic Cr. Permanganate, $MnO_4{}^-$, has manganese in its highest common oxidation state and is a strong oxidising agent in acidic solution, rather than the strongest reducing agent.

Step 2: Reverse the appropriate half-reactions For chromium,

$$Cr^{3+} + 3\,e^- \longrightarrow Cr, \qquad E^\circ_{red} = -0.74\,V.$$

Thus oxidation of the metal has $E^\circ_{ox} = +0.74\,V$. By comparison, oxidation of chloride to chlorine has $E^\circ_{ox} = -1.36\,V$, and oxidation of Mn^{2+} to permanganate in the stated acidic couple has $E^\circ_{ox} = -1.51\,V$. Chromium therefore has the greatest tendency to lose electrons among these candidates.

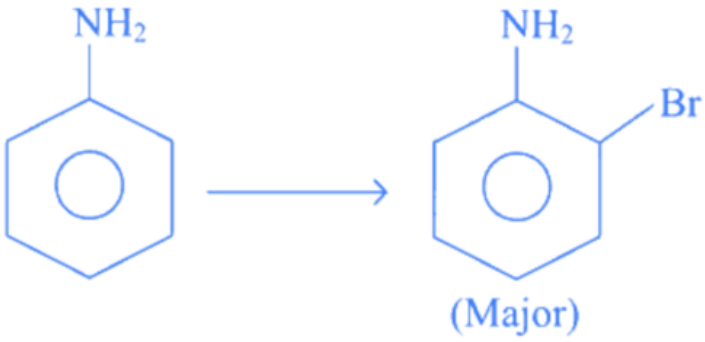
Step 3: Use the sign convention consistently The large positive reduction potential $1.51\,V$ signals a strong oxidant, namely permanganate; it does not make the reduced species Mn^{2+} a strong reductant. Likewise, chlorine has a strong tendency to accept electrons, so chloride has only a weak tendency to reverse that process. The negative chromium reduction potential instead favours electron donation by chromium metal relative to the other listed reduced species.

Step 4: Conclusion Metallic chromium, Cr, is the strongest reducing agent. Hence, the correct option is (C).

Quick Tip to Solve: For reducing strength, look at the reduced member of each couple and reverse the reduction potential; do not select the largest positive reduction potential.

58. For reaction

The correct order of set of reagents for the above conversion is : [4 marks]



(A) $Ac_2O, Br_2, H_2O(\Delta), NaOH$

(B) $H_2SO_4, Ac_2O, Br_2, H_2O(\Delta), NaOH$

(C) $Br_2 \mid FeBr_3, H_2O(\Delta), NaOH$

(D) $Ac_2O, H_2SO_4, Br_2, NaOH$

Correct Answer: (B) $H_2SO_4, Ac_2O, Br_2, H_2O(\Delta), NaOH$

Solution:

Concept:

- The amino group strongly activates a benzene ring towards electrophilic substitution. Direct bromination of aniline can give polysubstitution, so controlled monobromination requires protection.
- Acetylation converts $-NH_2$ into the less strongly activating $-NHCOCH_3$ group. A temporary para-sulphonic acid group blocks para substitution and can later be removed by hot aqueous

treatment.

Step 1: Block the para position first The required product is ortho-bromoaniline, not the normally favoured para-brominated product of protected aniline. Sulphonation of aniline with H_2SO_4 provides a para $-SO_3H$ blocking group. This reserves the para position while leaving the ortho sites available for subsequent bromination. The amino group must also be protected to control its otherwise excessive activating effect.

Step 2: Protect and brominate the amino compound Treatment with acetic anhydride, $(CH_3CO)_2O$, acetylates the amino group. Bromination then occurs ortho to the acetamido group because its para position is occupied by $-SO_3H$. The two ortho positions in this para-substituted intermediate are equivalent, so this directs formation of the single required ring-substitution pattern rather than an unrelated positional isomer.

Step 3: Remove temporary groups and compare sequences Hot aqueous treatment removes the protecting acetyl group and the temporary sulphonyl blocking group under the sequence indicated; final $NaOH$ liberates the free amino group from its salt. The full listed order is therefore H_2SO_4, Ac_2O, Br_2, hot water, then $NaOH$. Option (A) lacks para blocking and mainly leads to the para product. Option (C) lacks amino protection and para blocking. Option (D) omits the stated hot aqueous deprotection step and does not give the required complete sequence.

Step 4: Conclusion The required reagent order is H_2SO_4, Ac_2O, Br_2, H_2O with heat, and $NaOH$. Hence, the correct option is (B).

Quick Tip to Solve: To obtain ortho bromination of aniline selectively, block para with $-SO_3H$, protect the amino group, brominate, and then remove the temporary groups.

59. Given below are two statements :

Statement (I) : The first ionization energy of Pb is greater than that of Sn .

Statement (II) : The first ionization energy of Ge is greater than that of Si .

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are false

(C) Both Statement I and Statement II are true

(D) Statement I is true but Statement II is false

Correct Answer: (D) Statement I is true but Statement II is false

Solution:

Concept:

- First ionization energy is the energy required to remove the outermost electron from an isolated gaseous atom: $M(g) \longrightarrow M^+(g) + e^-$.
- Atomic size generally increases down a group, lowering the first ionization energy. Poor shielding by intervening d and f electrons can modify this trend, so actual group-14 comparisons must be

checked individually.

Step 1: Evaluate the lead-tin comparison The first ionization energies of tin and lead are approximately 709 and 716 $\mathrm{kJ\,mol^{-1}}$, respectively. Thus the first ionization energy of Pb is slightly greater than that of Sn. Poor shielding by intervening electrons and the increased effective nuclear attraction in the heavy atom counter the simple size trend. Therefore Statement (I) is true; applying a strictly decreasing trend all the way down this group would give the wrong result.

Step 2: Evaluate the germanium-silicon comparison The first ionization energies of silicon and germanium are approximately 786 and 762 $\mathrm{kJ\,mol^{-1}}$, respectively. Although the intervening d electrons shield imperfectly, germanium still has a lower first ionization energy than silicon. Its outer electron occupies the next principal shell, and the net result is easier removal than in silicon. Hence the claim $IE_1(\mathrm{Ge}) > IE_1(\mathrm{Si})$ is false.

Step 3: Select the matching truth combination The verified comparisons are

$$IE_1(\mathrm{Pb}) > IE_1(\mathrm{Sn}), \qquad IE_1(\mathrm{Ge}) < IE_1(\mathrm{Si}).$$

The question asks for two independent truth values, not an assertion-reason explanation. Options (A), (B), and (C) each assign an incorrect truth value to at least one statement. Only option (D) identifies the first statement as true and the second as false.

Step 4: Conclusion Statement (I) is true but Statement (II) is false. Hence, the correct option is (D).

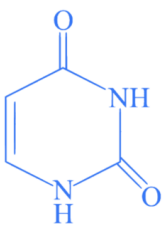
Quick Tip to Solve: Remember that first ionization energy decreases from carbon to silicon to germanium to tin, followed by a small rise to lead; do not assume every step decreases.

60. Match List - I with List - II.

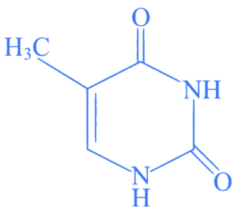
	List - I
(A)	Adenine
(B)	Cytosine
(C)	Thymine
(D)	Uracil

List - II consists of the following structures (I), (II), (III) and (IV), respectively. Choose the correct answer from the options given below : [4 marks]

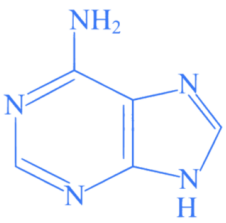
(I)



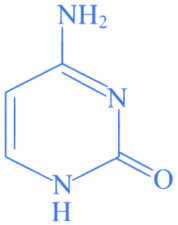
(II)



(III)



(IV)



(A) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(B) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(C) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(D) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Correct Answer: (B) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Solution:

Concept:

- Adenine is a purine base with fused six- and five-membered rings. Cytosine, thymine and uracil are pyrimidine bases with a single six-membered ring.
- Thymine is 5-methyluracil: both have two carbonyl groups, but only thymine has a methyl substituent. Cytosine has an amino group and one carbonyl group; adenine has an amino group and no carbonyl group.

Step 1: Identify the fused-ring base Structure (III) contains the fused purine ring system and an exocyclic $-NH_2$ group, with no oxygen atom. It is adenine, so List-I entry (A) matches (III). This immediately excludes option (A), which incorrectly pairs adenine with the single-ring structure (IV). The ring count is a faster and more reliable first test than following every double bond.

Step 2: Distinguish the three pyrimidines Structure (IV) is a single ring bearing one carbonyl and one exocyclic amino group, so it is cytosine and (B) matches (IV). Structures (I) and (II) each have two carbonyl groups. Structure (II) additionally has a $-CH_3$ substituent on the ring, identifying thymine, whereas structure (I) lacks that methyl group and is uracil. Therefore (C) matches (II), and (D) matches (I).

Step 3: Check the complete mapping The four independently established pairs are

$$\text{Adenine} \to (\text{III}), \quad \text{Cytosine} \to (\text{IV}),$$

$$\text{Thymine} \to (\text{II}), \quad \text{Uracil} \to (\text{I}).$$

Option (B) contains all four pairs. Option (C) misidentifies cytosine and interchanges other pyrimidine assignments. Option (D) reverses thymine and uracil; that would ignore the methyl group that distinguishes their otherwise similar rings. Each printed structure thus has one specific match, with no duplicate assignment.

Step 4: Conclusion Adenine matches III, cytosine IV, thymine II and uracil I. Hence, the correct option is (B).

Quick Tip to Solve: Identify the fused purine ring first, then count carbonyl groups and look for the methyl group: thymine is 5-methyluracil.

61. Which of the following mixing of 1 M base and 1 M acid leads to the largest increase in temperature?

[4 marks]

(A) 50 mL HCl and 20 mL NaOH

(B) 30 mL HCl and 30 mL NaOH

(C) 45 mL CH_3COOH and 25 mL NaOH

(D) 30 mL CH_3COOH and 30 mL NaOH

Correct Answer: (B) 30 mL HCl and 30 mL NaOH

Solution:

Concept:

- Neutralisation releases heat when hydrogen ions and hydroxide ions form water: $H^+ + OH^- \longrightarrow H_2O$. The amount of water formed is set by the limiting reactant.
- The temperature rise is $\Delta T = q/(mc)$. For dilute aqueous solutions of equal concentration, compare heat released per total volume, taking their densities and specific heats as approximately equal.
- A strong acid is already ionised, whereas a weak acid such as ethanoic acid uses some energy for ionisation. Its neutralisation therefore releases less heat per mole of water.

Step 1: Compare the two strong-acid mixtures Let h be the magnitude of the molar enthalpy of strong acid–strong base neutralisation. In (A), 0.050 mol of HCl reacts with only 0.020 mol of $NaOH$, forming 0.020 mol of water in 70 mL. In (B), both reactants supply 0.030 mol, forming 0.030 mol of water in 60 mL. Thus the factors proportional to temperature rise are

$$\frac{0.020h}{70} \quad \text{and} \quad \frac{0.030h}{60}.$$

The second is larger; excess acid in (A) contributes thermal mass without contributing further neutralisation heat.

Step 2: Check both weak-acid alternatives In (C), only 0.025 mol of water forms in a total volume of 70 mL. Even if its heat per mole equalled h, its factor $0.025h/70$ would be smaller than $0.030h/60$. Its actual heat is smaller still because CH_3COOH is weak. In (D), the water formed and total volume equal those in (B), but weak-acid ionisation reduces the net heat released. Consequently neither weak-acid mixture can surpass (B).

Step 3: Conclusion The largest temperature increase occurs on mixing $30\,\text{mL}$ of $1\,\text{M}$ HCl with $30\,\text{mL}$ of $1\,\text{M}$ $NaOH$. Hence, the correct option is (B).

Quick Tip to Solve: Compare limiting moles divided by total volume first; then account for the smaller heat of neutralisation of a weak acid.

62. In the given structure, number of sp and sp^2 hybridized carbon atoms present respectively are :

[4 marks]



(A) 3 and 5

(B) 3 and 6

(C) 4 and 5

(D) 4 and 6

Correct Answer: (A) 3 and 5

Solution:

Concept:

- An sp-hybridised carbon has two electron domains, as in a carbon–carbon triple bond or the carbon of a nitrile group $-C{\equiv}N$. A triple bond is one domain for hybridisation counting.
- An sp^2-hybridised carbon has three electron domains. Every carbon of an ordinary carbon–carbon double bond and the carbonyl carbon of $C{=}O$ belongs to this category.
- Count carbon atoms rather than bonds, and exclude oxygen and nitrogen atoms even when they occur in multiple bonds.

Step 1: Count the linear carbon centres The chain contains one internal $C{\equiv}C$ unit. Both ends of this triple bond are carbon atoms, so it contributes two sp centres. The terminal $-C{\equiv}N$ group contributes one more sp carbon: its nitrogen is not a carbon and must not be added. Therefore

$$N(sp) = 2 + 1 = 3.$$

The single bonds attached to these centres do not change their hybridisation, because each centre still has only two directions of sigma bonding.

Step 2: Count the trigonal carbon centres There are two $C{=}C$ double bonds elsewhere in the chain, and each contributes two sp^2 carbon atoms. The left-hand carbonyl contributes one additional sp^2 carbon, while the oxygen itself is excluded. Thus

$$N(sp^2) = 2 + 2 + 1 = 5.$$

Saturated end or linking carbons carry only single bonds and are sp^3, so they do not enter either requested count. Options giving four sp centres overcount a heteroatom or a saturated centre, and those giving six sp^2 centres overcount the carbonyl oxygen or another non-alkenic centre.

Step 3: Conclusion The numbers of sp- and sp^2-hybridised carbon atoms are respectively 3 and 5. Hence, the correct option is (A).

Quick Tip to Solve: Mark both carbons of each carbon double or triple bond, but count only the carbon in carbonyl and nitrile groups.

63. The conditions and consequence that favours the $t_{2\,g}{}^{3}e_{g}{}^{1}$ configuration in a metal complex are :

[4 marks]

(A) strong field ligand, low spin complex

(B) strong field ligand, high spin complex

(C) weak field ligand, high spin complex

(D) weak field ligand, low spin complex

Correct Answer: (C) weak field ligand, high spin complex

Solution:

Concept:

- In an octahedral crystal field, the five metal d orbitals split into the lower-energy t_{2g} set and the higher-energy e_g set, separated by Δ_o.
- The choice between occupying an upper orbital and pairing in a lower orbital depends on the competition between the splitting energy Δ_o and the pairing energy P. Weak-field ligands give $\Delta_o < P$.
- A high-spin configuration maximises unpaired electrons when excitation to the upper set costs less than pairing in the lower set.

Step 1: Identify the electron count and the competing configurations The stated occupation $t_{2g}^3 e_g^1$ contains four d electrons. The first three occupy the three lower t_{2g} orbitals singly. For the fourth electron, there are two possibilities: pair it in a lower orbital to give $t_{2g}^4 e_g^0$, or promote it into an empty upper orbital to give $t_{2g}^3 e_g^1$. The stated configuration therefore describes the case in which promotion is energetically preferred to pairing.

Step 2: Determine the ligand field and spin With a weak-field ligand, $\Delta_o < P$, so the fourth electron occupies e_g without pairing. All four electrons then remain unpaired; this is the high-spin d^4 arrangement. Its crystal-field energy is

$$3(-0.4\Delta_o) + 1(0.6\Delta_o) = -0.6\Delta_o,$$

with no additional paired electrons. A strong field instead makes pairing favourable and gives the low-spin t_{2g}^4 configuration. Thus both strong-field alternatives fail, and the weak-field/low-spin alternative incorrectly assigns the spin consequence of this occupation.

Step 3: Conclusion The configuration $t_{2g}^3 e_g^1$ is favoured by a weak-field ligand and corresponds to a high-spin complex. Hence, the correct option is (C).

Quick Tip to Solve: For octahedral d4, an electron in eg before any pairing in t2g immediately identifies the weak-field, high-spin case.

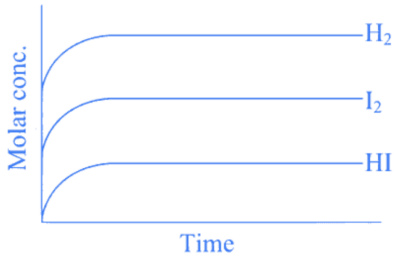
64. For the reaction,

$$H_2(\,g) + I_2(\,g) \rightleftharpoons 2HI(\,g)$$

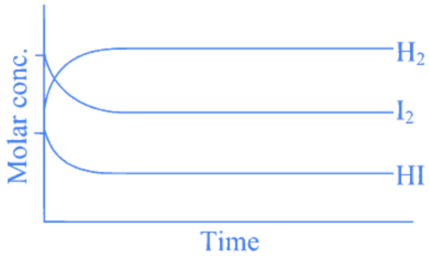
Attainment of equilibrium is predicted correctly by :

[4 marks]

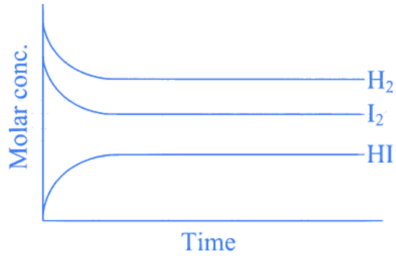
(A)



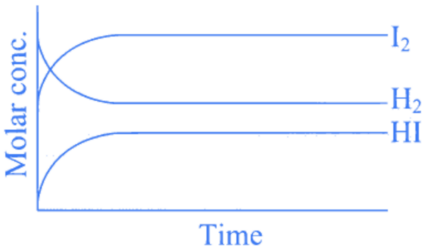
(B)



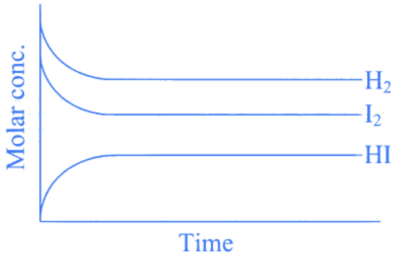
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- For $H_2(g)+I_2(g) \rightleftharpoons 2\,HI\,(g)$, hydrogen and iodine are consumed together in the forward direction, while hydrogen iodide is produced. The reverse direction reverses all three changes.
- At fixed volume the stoichiometric concentration changes obey $\Delta[H_2] = \Delta[I_2] = -\frac{1}{2}\Delta[HI]$. The reactants therefore change in the same direction, opposite to the product.
- At dynamic equilibrium the forward and reverse rates are equal. Each concentration becomes

constant with time, but the three equilibrium concentrations need not be equal.

Step 1: Identify the physically consistent approach When the reaction starts with hydrogen and iodine, both reactant concentrations decrease as hydrogen iodide is formed; the concentration of HI increases. All curves must eventually become horizontal as the composition ceases to change. Graph (C) has exactly these directions: the curves labelled H_2 and I_2 fall, whereas that labelled HI rises. Their different final levels are permissible because equilibrium requires constant concentrations, not equal concentrations.

Step 2: Eliminate the other graphs by stoichiometry Graph (A) shows all three concentrations increasing together. That cannot follow from this single reaction in a closed, fixed-volume mixture. In graph (B), H_2 increases while I_2 decreases; the two reactants must instead be produced or consumed together. Graph (D) also makes H_2 and I_2 change in opposite directions, even though it shows HI increasing. A plateau alone does not make a graph correct: its changing portions must also respect the reaction stoichiometry. Only (C) satisfies both requirements.

Step 3: Conclusion Graph (C) correctly shows both reactants decreasing and the product increasing until all concentrations become constant. Hence, the correct option is (C).

Quick Tip to Solve: Check the directions of all curves before looking at their plateaus: reactants on the same side of one reaction must change together.

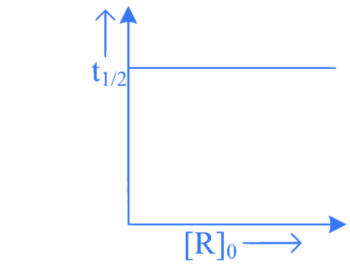
65. Given below are two statements :

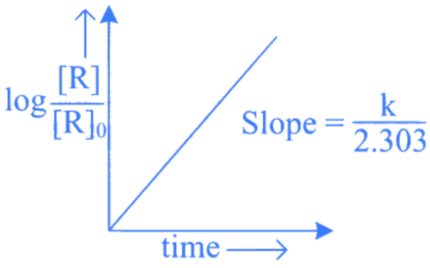
Statement (I) : is valid for first order reaction.

Statement (II) : is valid for first order reaction.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]





(A) Both Statement I and Statement II are false

(B) Statement I is true but Statement II is false

(C) Statement I is false but Statement II is true

(D) Both Statement I and Statement II are true

Correct Answer: (B) Statement I is true but Statement II is false

Solution:

Concept:

- For a first-order reaction with reactant concentration $[R]$, the rate law is $-d[R]/dt = k[R]$, where $k > 0$ is the rate constant.
- Its integrated form is $\ln([R]/[R]_0) = -kt$, or $\log_{10}([R]/[R]_0) = -kt/2.303$. The sign depends on which concentration is in the numerator.
- The first-order half-life is $t_{1/2} = \ln 2/k$, independent of the initial reactant concentration $[R]_0$.

Step 1: Test the half-life graph in Statement I At the half-life, substitute $[R] = [R]_0/2$ into the integrated rate law:

$$\ln \frac{[R]_0}{[R]_0/2} = kt_{1/2} \quad \Rightarrow \quad t_{1/2} = \frac{0.693}{k}.$$

No initial concentration remains in this expression. Consequently a plot of $t_{1/2}$ against $[R]_0$ is horizontal at a fixed temperature, where k is constant. The first graph shows exactly this independence. Statement I is therefore true.

Step 2: Check the slope and sign in Statement II The second graph labels the vertical axis as $\log([R]/[R]_0)$, yet shows a positive slope $k/2.303$. During reactant consumption, $[R] < [R]_0$, so this logarithm is negative for positive time. In fact

$$\log \frac{[R]}{[R]_0} = -\frac{k}{2.303}t.$$

The required line passes through the origin with a negative slope. A positive slope would be correct for the reciprocal ratio $\log([R]_0/[R])$, which is not the ratio printed here. Statement II is false. This rules out the alternatives that accept both statements or reject Statement I.

Step 3: Conclusion Statement I is true, but Statement II is false because the printed logarithmic concentration ratio requires a negative slope. Hence, the correct option is (B).

Quick Tip to Solve: Before memorising a kinetic graph slope, check the ratio: remaining over initial gives a negative logarithm; initial over remaining gives a positive one.

66. Given below are two statements :

Statement (I) : Experimentally determined oxygen-oxygen bond lengths in the O_3 are found to be same and the bond length is greater than that of a $O = O$ (double bond) but less than that of a single $(O - O)$ bond.

Statement (II) : The strong lone pair-lone pair repulsion between oxygen atoms is solely responsible for the fact that the bond length in ozone is smaller than that of a double bond $(O = O)$ but more than that of a single bond $(O - O)$.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are true

(C) Statement I is true but Statement II is false

(D) Both Statement I and Statement II are false

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- Ozone is represented by two equivalent resonance contributors, $O{=}O^{+}{-}O^{-}$ and $O^{-}{-}O^{+}{=}O$. Neither contributor alone describes the actual molecule.
- The resonance hybrid has two equivalent oxygen–oxygen bonds with bond order intermediate between a single and a double bond. Increasing bond order generally decreases bond length.
- Lone-pair repulsions influence molecular geometry, but equivalence of the ozone bonds follows from delocalised bonding and cannot be explained solely by repulsion.

Step 1: Evaluate the measured bond-length comparison In the resonance hybrid, the double-bond character is distributed over both oxygen–oxygen links. Thus neither link remains a pure single bond or a pure double bond, and their measured lengths are equal. The correct ordering is

$$r(\mathrm{O{=}O}) < r(\mathrm{O{-}O\ in\ ozone}) < r(\mathrm{O{-}O\ single}).$$

This agrees with Statement I: ozone bonds are longer than an oxygen double bond and shorter than an oxygen single bond. Statement I is true.

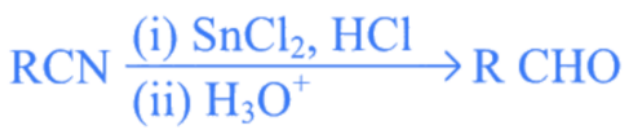
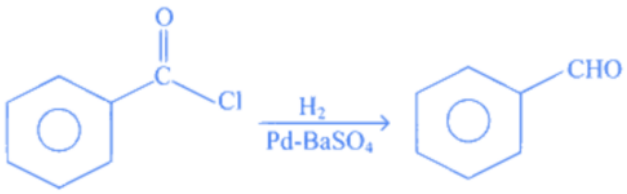
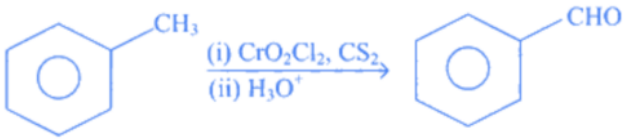
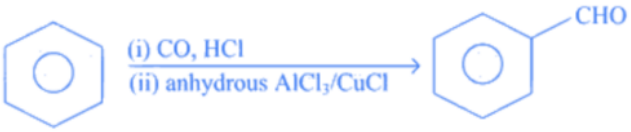
Step 2: Test both claims in Statement II Statement II reverses that physical ordering, saying the ozone bond is smaller than a double bond but greater than a single bond. A single oxygen–oxygen bond is itself longer than a double bond, so the stated comparison is inconsistent. Moreover, attributing the intermediate and equal lengths solely to lone-pair repulsion misses the central role of resonance and delocalisation. Either defect makes Statement II false; together they rule it out decisively. The options accepting Statement II therefore fail, while the option rejecting both statements incorrectly rejects the valid experimental description in Statement I.

Step 3: Conclusion Statement I is true and Statement II is false. The equal ozone bonds have lengths intermediate between ordinary oxygen single and double bonds because of resonance. Hence, the correct option is (C).

Quick Tip to Solve: For equivalent resonance contributors, assign the same intermediate bond order to corresponding bonds before comparing their lengths.

67. Match List - I with List - II.

Choose the correct answer from the options given below : [4 marks]

	List - I		List - II
(A)		(I)	Etard reaction
(B)		(II)	Gatterman-Koch reaction
(C)		(III)	Rosenmund reduction
(D)		(IV)	Stephen reaction

(A) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(B) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(C) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

(D) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Correct Answer: (B) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Solution:

Concept:

- A named reaction is identified by its starting functional group and its characteristic reagent, not merely by the aldehyde product. Several different routes give benzaldehyde.
- Stephen reduction starts from a nitrile; Rosenmund reduction starts from an acyl chloride. Etard oxidation changes an aromatic methyl group to an aldehyde, whereas Gattermann-Koch formylation adds a formyl group to an aromatic ring.

Step 1: Identify the two reduction routes In reaction (A), the nitrile RCN is treated with $SnCl_2$ and HCl, followed by hydrolysis. Partial reduction gives an iminium salt, which hydrolyses to RCHO: this is the Stephen reaction, so (A) matches (IV). In (B), benzoyl chloride is hydrogenated over palladium supported on barium sulphate. This poisoned catalyst stops the reduction at benzaldehyde instead of allowing further reduction to an alcohol. It is Rosenmund reduction, so (B) matches (III).

Step 2: Identify oxidation and formylation In (C), toluene reacts with chromyl chloride, CrO_2Cl_2, in CS_2, followed by acidic hydrolysis. Oxidation of the benzylic methyl group to CHO is the Etard reaction, giving (C)-(I). In (D), benzene is treated with carbon monoxide and hydrogen chloride in the presence of anhydrous $AlCl_3/CuCl$. Introduction of CHO on the ring is Gattermann-Koch formylation, giving (D)-(II).

Step 3: Compare the complete matching The four matches in order are (A)-(IV), (B)-(III), (C)-(I), (D)-(II). The first and fourth choices interchange reduction names and also misassign the aromatic reactions; the third choice assigns Etard oxidation to a nitrile. Only the second choice preserves all four substrate-reagent combinations.

Step 4: Conclusion The reactions are Stephen, Rosenmund, Etard, and Gattermann-Koch in the order A, B, C, D. Hence, the correct option is (B).

Quick Tip to Solve: First separate nitrile and acyl-chloride reductions: $SnCl_2/HCl$ signals Stephen, while H_2/poisoned Pd signals Rosenmund.

68. When Ethane-1,2-diamine is added progressively to an aqueous solution of Nickel (II) chloride, the sequence of colour change observed will be: [4 marks]

(A) Pale Blue → Blue → Green → Violet

(B) Pale Blue → Blue → Violet → Green

(C) Green → Pale Blue → Blue → Violet

(D) Violet → Blue → Pale Blue → Green

Correct Answer: (C) Green → Pale Blue → Blue → Violet

Solution:

Concept:

- Ethane-1,2-diamine, abbreviated en, is a neutral bidentate ligand: both nitrogen lone pairs coordinate to a metal ion. One en ligand replaces two coordinated water molecules.
- The colour of an octahedral nickel(II) complex depends on the ligand field splitting. Progressive substitution changes the ligand environment even though nickel remains in oxidation state $+2$.

Step 1: Start with the aqueous nickel complex Aqueous nickel(II) chloride contains the green hexaaquanickel(II) ion, $[Ni(H_2O)_6]^{2+}$. Chloride supplies the counterions in this description. Thus, the sequence must begin with green, not pale blue or violet. Addition of the first equivalent of en forms the mono-chelate complex $[Ni(en)(H_2O)_4]^{2+}$, whose colour is pale blue.

Step 2: Follow the successive ligand substitutions As more en is added, a second ligand replaces two further waters, giving blue $[Ni(en)_2(H_2O)_2]^{2+}$. A third en then replaces the remaining two waters to give violet $[Ni(en)_3]^{2+}$. Each substitution preserves six donor atoms around nickel: the coordinated nitrogen count increases from zero to two, four and six, respectively. The final species therefore has three bidentate ligands, not six en molecules.

Step 3: Select the observed sequence The complete progression is green for the aqua ion, pale blue for the mono-en complex, blue for the bis-en complex, and violet for the tris-en complex. The first

two choices start with pale blue and place green after ligand addition, so they miss the initial aqua ion. The fourth choice reverses the substitution trend. Only the third sequence has both the correct starting colour and the correct final tris-chelate colour.

Step 4: Conclusion The colour sequence is Green → Pale Blue → Blue → Violet. Hence, the correct option is (C).

Quick Tip to Solve: Count en ligands as 0, 1, 2, 3; associate these with green, pale blue, blue, violet while keeping nickel in the same oxidation state.

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69. For hydrogen atom, the orbital/s with lowest energy is/are :

(A) 4s

(B) $3p_x$

(C) 3 $d_{x^2-y^2}$

(D) 3 d_{z^2}

(E) $4p_z$

Choose the correct answer from the options given below : [4 marks]

(A) (A) only

(B) (B) only

(C) (B), (C) and (D) only

(D) (A) and (E) only

Correct Answer: (C) (B), (C) and (D) only

Solution:

Concept:

- For an isolated hydrogen atom, a one-electron system, orbital energy depends only on the principal quantum number n: $E_n = -13.6/n^2\,\text{eV}$. It does not depend on the azimuthal quantum number or orbital orientation.
- The usual subshell energy ordering used for many-electron atoms cannot be applied to split hydrogen orbitals having the same principal quantum number.

Step 1: Group the orbitals by principal quantum number The orbital in statement (A), $4s$, and that in (E), $4p_z$, both have $n = 4$. The orbitals in (B), $3p_x$, (C), $3d_{x^2-y^2}$, and (D), $3d_{z^2}$, all have $n = 3$. Their different shapes and orientations do not change their energy for hydrogen, since there are no other electrons producing shielding and penetration-dependent subshell splitting.

Step 2: Compare the two energy levels Calculate the two possible energies:

$$E_3 = -\frac{13.6}{9}\,\text{eV} \simeq -1.51\,\text{eV}, \qquad E_4 = -\frac{13.6}{16}\,\text{eV} = -0.85\,\text{eV}.$$

The more negative value is the lower energy. Thus, every listed $n = 3$ orbital has the same energy, below that of either $n = 4$ orbital. The question asks for the lowest among the listed orbitals, rather than the absolute ground-state orbital $1s$.

Step 3: Check the combinations Statements (B), (C), and (D) must all be included. Choosing (B) alone would incorrectly lift the degeneracy between the $3p$ and $3d$ subshells. Choosing (A) alone, or (A) and (E), instead selects the higher $n = 4$ energy level. Hence, the third combination alone lists all the orbitals at the lowest available energy.

Step 4: Conclusion The orbitals $3p_x$, $3d_{x^2-y^2}$, and $3d_{z^2}$ are degenerate and lowest among those listed. Hence, the correct option is (C).

Quick Tip to Solve: For hydrogen, read only the leading integer n to compare orbital energies; do not use the many-electron $n + l$ ordering.

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70. Identify correct statement/s :

(A) $-OCH_3$ **and** $-NHCOCH_3$ **are activating group.**

(B) $-CN$ **and -OH are meta directing group.**

(C) -CN and $-SO_3H$ **are meta directing group.**

(D) Activating groups act as ortho - and para directing groups.

(E) Halides are activating groups.

Choose the correct answer from the options given below : [4 marks]

(A) (A), (C) and (D) only

(B) (A), (B) and (E) only

(C) (A) and (C) only

(D) (A) only

Correct Answer: (A) (A), (C) and (D) only

Solution:

Concept:

- Electron-donating substituents generally activate an aromatic ring towards electrophilic substitution and direct incoming groups to ortho and para positions. Electron-withdrawing groups generally deactivate it and direct meta.
- Halogens are the standard exception: their inductive withdrawal deactivates the ring, but lone-pair resonance donation produces ortho/para orientation.

Step 1: Evaluate the electron-donating groups In statement (A), $-OCH_3$ donates an oxygen lone pair into the aromatic ring by resonance. The group $-NHCOCH_3$, when attached through nitrogen as written, also donates a nitrogen lone pair into the ring; acylation reduces this donation relative to $-NH_2$, but it remains activating. Thus, (A) is true. Statement (D) states the corresponding orientation rule: activating groups favour ortho and para substitution. It is also true.

Step 2: Evaluate the proposed meta directors The cyano group $-CN$ is electron withdrawing and meta directing. The sulfonic acid group $-SO_3H$ is likewise deactivating and meta directing, so (C) is true. However, the hydroxyl group $-OH$ strongly donates a lone pair by resonance and is activating and ortho/para directing. Therefore, statement (B), which calls both cyano and hydroxyl meta directing, is false.

Step 3: Check halogens and the answer sets Statement (E) is false: halogen substituents deactivate benzene through their strong negative inductive effect, despite being ortho/para directing through resonance. The true set is therefore (A), (C), and (D). The second option includes false statements (B) and (E); the third leaves out true statement (D); the fourth leaves out both (C) and (D). Only the first option contains the complete correct set.

Step 4: Conclusion The true statements concern the activating methoxy and acetamido groups, the meta-directing cyano and sulfonic acid groups, and the ortho/para orientation of activating groups. Hence, the correct option is (A).

Quick Tip to Solve: Treat activation and orientation separately for halogens: they are deactivating yet ortho/para directing, whereas $-\mathrm{OH}$ is activating and ortho/para directing.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. Consider a complex reaction taking place in three steps with rate constants $\mathrm{k}_1, \mathrm{k}_2$ and k_3 respectively. The overall rate constant k is given by the expression $k = \sqrt{\frac{k_1 k_3}{k_2}}$. If the activation energies of the three steps are 60, 30 and $10\ \mathrm{kJ\ mol^{-1}}$ respectively, then the overall energy of activation in $\mathrm{kJmol^{-1}}$ is _____ . (Nearest integer) [4 marks]

Solution:

Concept:

- For each elementary step, the Arrhenius relation is $k_i = A_i \exp[-E_i/(RT)]$, where E_i is the activation energy and A_i the pre-exponential factor.
- Taking a logarithm converts products, quotients, and powers of rate constants into a linear combination. The effective activation energy follows from the coefficient of $-1/(RT)$, assuming the usual temperature-independent pre-exponential factors.

Step 1: Take the logarithm of the given rate constant The square root applies to the entire ratio $k_1 k_3/k_2$, so

$$\ln k = \frac{1}{2}(\ln k_1 + \ln k_3 - \ln k_2).$$

Substitute $\ln k_i = \ln A_i - E_i/(RT)$. The denominator rate constant enters with a minus sign, while the square root multiplies every contribution by one-half. This sign and factor must be applied to the activation energies as well as to the logarithms.

Step 2: Collect the temperature-dependent term After substitution,

$$\ln k = \frac{1}{2}\ln\left(\frac{A_1 A_3}{A_2}\right) - \frac{E_1 + E_3 - E_2}{2RT}.$$

Comparison with $\ln k = \ln A - E_{\text{overall}}/(RT)$ gives

$$E_{\text{overall}} = \frac{E_1 + E_3 - E_2}{2}.$$

Thus, the overall barrier is not the arithmetic mean of all three positive barriers. The denominator

contributes negatively to the temperature dependence of the combined expression.

Step 3: Insert the three given energies Use $E_1 = 60$, $E_2 = 30$, and $E_3 = 10$, all in $\mathrm{kJ\,mol^{-1}}$:

$$E_{\text{overall}} = \frac{60 + 10 - 30}{2} = \frac{40}{2} = 20\,\mathrm{kJ\,mol^{-1}}.$$

All energies already share the same units, so no conversion is required. The result is already an integer and nearest-integer rounding does not change it.

Step 4: Conclusion The overall activation energy is $20\,\mathrm{kJ\,mol^{-1}}$. The required numerical entry is $\boxed{20}$.

Quick Tip to Solve: For $k = k_1^a k_2^b k_3^c$, combine activation energies with the same signed powers: $E = aE_1 + bE_2 + cE_3$.

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72. The hydrocarbon (X) with molar mass $80\ \mathrm{g\ mol^{-1}}$ and 90% carbon has _____ degree of unsaturation.

[4 marks]

Solution:

Concept:

- A hydrocarbon contains only carbon and hydrogen. Mass percentages combined with molar mass determine the numbers of carbon and hydrogen atoms in one molecule.
- For a neutral hydrocarbon $\mathrm{C}_c\mathrm{H}_h$, degree of unsaturation is $\mathrm{DBE} = (2c + 2 - h)/2$. A ring or double bond contributes one, and a triple bond contributes two.

Step 1: Determine the molecular formula Consider one mole of the hydrocarbon, whose mass is $80\,\mathrm{g}$. Carbon accounts for 90% of this mass:

$$m_{\mathrm{C}} = 0.90(80) = 72\,\mathrm{g}.$$

Using the standard atomic molar mass $12\,\mathrm{g\,mol^{-1}}$, the carbon atoms per molecule number $72/12 = 6$. The remaining mass is hydrogen:

$$m_{\mathrm{H}} = 80 - 72 = 8\,\mathrm{g}, \qquad h = 8/1 = 8.$$

Hence, the molecular formula is $\mathrm{C_6H_8}$. It has molar mass $6(12) + 8 = 80\,\mathrm{g\,mol^{-1}}$, consistent with the question.

Step 2: Compare with the saturated acyclic formula A saturated acyclic hydrocarbon containing six carbon atoms would be $\mathrm{C_6H_{14}}$, by the general formula $\mathrm{C}_n\mathrm{H}_{2n+2}$. The actual formula has six fewer hydrogen atoms. Each unit of unsaturation removes a pair of hydrogens relative to the saturated acyclic reference, so the deficit corresponds to $6/2 = 3$ units.

Step 3: Apply the general formula and interpret it Direct substitution confirms the count:

$$\mathrm{DBE} = \frac{2(6) + 2 - 8}{2} = \frac{6}{2} = 3.$$

The molecular formula alone does not determine whether these units are rings, double bonds, or a combination involving a triple bond. No particular structure needs to be assumed, because the question asks only for their total degree of unsaturation. The value is dimensionless, since it counts structural units rather than mass or moles.

Step 4: Conclusion The hydrocarbon has molecular formula C_6H_8, and its degree of unsaturation is $\boxed{3}$.

Quick Tip to Solve: Use molar mass to obtain C_6H_8 first, then compare its hydrogen count with C_6H_{14}; divide the hydrogen deficit by two.

73. In Carius method of estimation of halogen, 0.25 g of an organic compound gave 0.15 g of silver bromide (AgBr). The percentage of Bromine in the organic compound is _____ $\times 10^{-1}\%$ (Nearest integer).

(Given : Molar mass of Ag is 108 and Br is $80\ \text{g mol}^{-1}$) [4 marks]

Solution:

Concept:

- In the Carius method, the halogen in the organic sample is converted quantitatively into a silver halide. Each mole of $AgBr$ contains exactly one mole of bromine atoms.
- The mass fraction of bromine in $AgBr$ is $M(Br)/M(AgBr)$. The percentage in the original compound is the recovered bromine mass divided by the original sample mass, multiplied by 100.

Step 1: Calculate the molar mass of silver bromide Use the atomic molar masses supplied in the question, rather than a rounded value taken from memory. For one silver atom and one bromine atom,

$$M(AgBr) = 108 + 80 = 188\,\text{g mol}^{-1}.$$

Thus $188\,\text{g}$ of precipitated silver bromide contains $80\,\text{g}$ of bromine; the remaining mass belongs to silver supplied by the reagent.

Step 2: Find the mass and percentage of bromine The $0.15\,\text{g}$ precipitate therefore represents a bromine mass of

$$m(Br) = 0.15 \times \frac{80}{188} = 0.0638298\,\text{g}.$$

The original organic sample weighed $0.25\,\text{g}$. Hence its bromine percentage is

$$\%Br = \frac{0.0638298}{0.25} \times 100 = 25.5319\%.$$

Using the entire precipitate mass as bromine would incorrectly include the silver mass in the analysis.

Step 3: Express the result in the requested scale Let the requested blank be x. The question expresses the percentage as $x \times 10^{-1}\%$, so

$$x \times 10^{-1} = 25.5319, \qquad x = 255.319.$$

Round this coefficient only at the end. Its nearest integer is 255, rather than rounding the percentage

first.

Step 4: Conclusion The bromine content is approximately 25.5319%, or $255.319 \times 10^{-1}\%$. The required nearest-integer entry is $\boxed{255}$.

Quick Tip to Solve: First multiply the silver-halide mass by the halogen's molar-mass fraction. Convert the resulting percentage to the scale of the blank before rounding.

74. The observed and normal molar masses of compound MX_2 are 65.6 and 164 respectively. The percent degree of ionisation of MX_2 is _____%. (Nearest integer) [4 marks]

Solution:

Concept:

- Ionisation increases the number of dissolved particles and hence increases a colligative property relative to its value for an undissociated solute. The van't Hoff factor is $i = M_{\text{normal}}/M_{\text{observed}}$.
- If one formula unit produces ν ions and the fraction ionised is α, particle counting gives $i = 1 + (\nu - 1)\alpha$. The percent degree of ionisation is 100α.

Step 1: Determine the van't Hoff factor The normal molar mass describes intact formula units. The smaller observed molar mass results from the additional particles produced by ionisation. Therefore take the normal mass in the numerator:

$$i = \frac{164}{65.6} = 2.5.$$

The reciprocal ratio would be less than one and would describe a reduction in particle number, contrary to the ionisation described here.

Step 2: Count the particles formed by the electrolyte The dissociation of the compound is represented by

$$\mathrm{MX_2} \longrightarrow \mathrm{M^{2+}} + 2\,\mathrm{X^-}.$$

Each ionised formula unit gives three ions. Starting with one mole of formula units, the undissociated amount is $1 - \alpha$ mole, while the cations and anions contribute α and 2α mole respectively. The total particle amount is consequently

$$(1 - \alpha) + \alpha + 2\alpha = 1 + 2\alpha.$$

This counts the surviving molecules as well as all the ions, avoiding the incorrect use of $i = 3\alpha$.

Step 3: Solve for the fraction ionised Equate the particle-count expression to the measured factor:

$$1 + 2\alpha = 2.5, \qquad \alpha = \frac{2.5 - 1}{2} = 0.75.$$

Thus the percentage ionisation is

$$100\alpha = 75\%.$$

This lies between zero and complete ionisation, and its factor 2.5 lies between the limits 1 and 3, as required.

Step 4: Conclusion The percent degree of ionisation of MX_2 is $\boxed{75\%}$. The required nearest-integer entry is $\boxed{75}$.

Quick Tip to Solve: For an electrolyte giving three ions, use $\alpha = (i-1)/2$, with i equal to normal molar mass divided by observed molar mass.

75. The possible number of stereoisomers for 5-phenylpent-4-en-2-ol is _____. [4 marks]

Solution:

Concept:

- A tetrahedral carbon bearing four different substituents is a stereogenic centre and ordinarily permits two configurations, R and S.
- An alkene permits E/Z geometrical isomerism when each double-bonded carbon has two different substituents. Independent stereogenic elements multiply the number of configurations unless symmetry makes some identical.

Step 1: Identify the connectivity from the name Number the five-carbon chain so that the hydroxyl group has locant 2. The double bond joins carbons 4 and 5, with a phenyl group attached to carbon 5. The condensed connectivity is

$$CH_3{-}CH(OH){-}CH_2{-}CH{=}CH{-}C_6H_5.$$

This identifies both kinds of stereogenic element without changing the positions in the given name. The phenyl ring itself contributes no additional stereogenic centre.

Step 2: Count the configurations at carbon 2 Carbon 2 is attached to four different groups: hydrogen, hydroxyl, methyl, and the chain $-CH_2{-}CH{=}CH{-}C_6H_5$. It is therefore a stereogenic carbon, with two possible configurations, R and S. Carbon 3 is a methylene carbon with two identical hydrogens, while carbon 1 is a methyl carbon, so neither supplies another stereogenic centre.

Step 3: Count the alkene configurations and combine them At carbon 4 the two substituents are hydrogen and the chain towards carbon 3. At carbon 5 they are hydrogen and phenyl. Each alkene carbon thus has different substituents, giving two distinct geometrical configurations, E and Z. Either alkene configuration can coexist with either configuration at carbon 2:

$$(R,E),\quad (S,E),\quad (R,Z),\quad (S,Z).$$

The molecule has only one stereogenic carbon and no internal symmetry that could produce a meso form. The R/S pair for a fixed alkene geometry consists of enantiomers, while changing E to Z gives distinct diastereomers. Hence all four combinations are different.

Step 4: Conclusion There are two optical configurations and two geometrical configurations. The possible number of stereoisomers is $2 \times 2 = \boxed{4}$.

Quick Tip to Solve: After counting stereogenic carbons, inspect every double bond separately: both alkene carbons must have different substituents before an E/Z factor is included.