JEE Main 2026 Paper 1 (B.E./B.Tech) Jan 21 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2026, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 21 January 2026, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $A = \{2, 3, 5, 7, 9\}$. Let R be the relation on A defined by xRy if and only if $2x \leq 3y$. Let l be the number of elements in R, and m be the minimum number of elements required to be added in R to make it a symmetric relation. Then $l + m$ is equal to: [4 marks]

(A) 21

(B) 23

(C) 25

(D) 27

Correct Answer: (C) 25

Solution:

Concept:

- A relation on A is a subset of $A \times A$; each element of the relation is an ordered pair (x, y).
- A symmetric relation contains (y, x) whenever it contains (x, y). Its smallest symmetric extension is $R \cup R^{-1}$.

Step 1: Count the ordered pairs already present For each fixed first coordinate x, the condition becomes $y \geq 2x/3$. Count only values of y belonging to the given set, including equality:

x	Permitted values of y	Count
2	$2, 3, 5, 7, 9$	5
3	$2, 3, 5, 7, 9$	5
5	$5, 7, 9$	3
7	$5, 7, 9$	3
9	$7, 9$	2

Thus $l = 5 + 5 + 3 + 3 + 2 = 18$. The pairs on the diagonal are already included; they must not be counted again.

Step 2: Determine the missing reverse pairs For any two positive members x, y, at least one of $2x \leq 3y$ and $2y \leq 3x$ is true. Indeed, if $x \leq y$, the first inequality certainly holds. Therefore every ordered pair of $A \times A$ belongs either to R or to its inverse. The symmetric closure is all of $A \times A$, with $5^2 = 25$ ordered pairs. Consequently the seven missing ordered pairs must all be added: $(5, 2), (5, 3), (7, 2), (7, 3), (9, 2), (9, 3), (9, 5)$. Each has its reverse in R, so none of these additions is optional. This proves minimality, giving $m = 25 - 18 = 7$. Options 21 and 23 leave required pairs missing; 27 exceeds the number of pairs on this set.

Step 3: Conclusion $l + m = 18 + 7 = 25$. Hence, the correct option is (C).

Quick Tip to Solve: For this positive set, first check whether $R \cup R^{-1} = A \times A$; then $l + m$ is immediately $|A|^2$.

2. Let z be the complex number satisfying $|z-5| \leq 3$ and having maximum positive principal argument. Then $34\left|\frac{5z-12}{5iz+16}\right|^2$ is equal to: [4 marks]

(A) 12

(B) 16

(C) 20

(D) 26

Correct Answer: (C) 20

Solution:

Concept:

- The locus $|z-5| \leq 3$ is the closed disc of centre $C = (5, 0)$ and radius 3 in the complex plane.
- The greatest positive argument occurs where a ray from the origin is tangent to the upper half of the disc. The tangent is perpendicular to the radius at contact.

Step 1: Locate the point of maximum argument Write the contact point as $P = (x, y)$, with $y > 0$. Every point in the disc has $x \geq 2 > 0$, so its principal argument is measured continuously from the positive real axis. The last ray rotating upwards which intersects the disc touches its boundary, hence it is a tangent. The perpendicularity condition $OP \perp CP$ gives

$$x(x-5) + y^2 = 0, \qquad x^2 + y^2 = 5x.$$

The circle equation is $x^2 + y^2 - 10x + 25 = 9$. Combining the two equations gives $x = 16/5$, and then $y^2 = 144/25$. Choose the positive value $y = 12/5$ because the maximum positive argument is required. Thus $z = (16 + 12i)/5$. The lower tangent gives a negative argument and cannot be selected.

Step 2: Evaluate the squared modulus Substitute this exact complex number before taking moduli:

$$5z - 12 = 4 + 12i, \qquad 5iz + 16 = 4 + 16i.$$

For a quotient, the squared modulus is the quotient of the sums of squares of real and imaginary parts. The denominator is nonzero, and therefore

$$34\left|\frac{5z-12}{5iz+16}\right|^2 = 34\frac{4^2+12^2}{4^2+16^2} = 34\frac{160}{272}.$$

Since $272 = 34 \times 8$, the simplification is exact. The alternatives 12, 16 and 26 do not equal this ratio; replacing a squared modulus by a modulus would also give a different expression.

Step 3: Conclusion The required value is 20. Hence, the correct option is (C).

Quick Tip to Solve: Use the tangent condition $x^2 + y^2 = 5x$ with the circle equation to find the maximizing complex number without trigonometric differentiation.

3. Let α and β be the roots of the equation $x^2 + 2ax + (3a + 10) = 0$ such that $\alpha < 1 < \beta$. Then the set of all possible values of a is : [4 marks]

(A) $(-\infty, -2) \cup (5, \infty)$

(B) $(-\infty, -3)$

(C) $(-\infty, -11/5)$

(D) $(-\infty, -11/5) \cup (5, \infty)$

Correct Answer: (C) $(-\infty, -11/5)$

Solution:

Concept:

- For a monic quadratic $f(x) = (x - \alpha)(x - \beta)$ with distinct real roots $\alpha < \beta$, one has $f(t) < 0$ exactly when $\alpha < t < \beta$.
- A negative value of an upward-opening quadratic guarantees two distinct real roots, one on either side of that point.

Step 1: Translate the position of the roots into a sign Let $f(x) = x^2 + 2ax + 3a + 10$. If $\alpha < 1 < \beta$, the factors $1 - \alpha$ and $1 - \beta$ have opposite signs, so

$$f(1) = (1 - \alpha)(1 - \beta) < 0.$$

Direct substitution gives $f(1) = 1 + 2a + 3a + 10 = 5a + 11$. Hence the necessary condition is

$$5a + 11 < 0 \quad \Longrightarrow \quad a < -\frac{11}{5}.$$

The inequality is strict: at the endpoint $a = -11/5$, the number 1 itself is a root, which is excluded by the question.

Step 2: Check sufficiency and the alternatives Conversely, if $a < -11/5$, then $f(1) < 0$, while $f(x) \to +\infty$ at both ends of the real line. Continuity therefore gives one root below 1 and one above 1. A quadratic has only two roots, so this verifies the exact condition without imposing an additional restriction. As an algebraic check, the discriminant is $4(a - 5)(a + 2)$, which is positive throughout $a < -11/5$. Real roots alone would merely give $a < -2$ or $a > 5$; those regions need not place 1 between the roots. Option (A) thus includes invalid values such as $a = 6$. Option (B) misses valid values such as $a = -5/2$, and option (D) again wrongly includes $a > 5$. At all such positive values $f(1) > 0$.

Step 3: Conclusion Since $f(1) = 5a + 11 < 0$ is necessary and sufficient for $\alpha < 1 < \beta$, the set is $\left(-\infty, -\frac{11}{5}\right)$. Hence, the correct option is (C).

Quick Tip to Solve: When a specified point lies strictly between the roots of a monic quadratic, evaluate the polynomial at that point and require a negative value.

4. If the line $\alpha x + 4y = \sqrt{7}$, where $\alpha \in \mathbf{R}$, touches the ellipse $3x^2 + 4y^2 = 1$ at the point P in the first quadrant, then one of the focal distances of P is : [4 marks]

(A) $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{5}}$
(B) $\frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{5}}$
(C) $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{7}}$
(D) $\frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{11}}$

Correct Answer: (C) $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{7}}$

Solution:

Concept:

- For $x^2/a^2 + y^2/b^2 = 1$, with $a > b$, the eccentricity satisfies $e^2 = 1 - b^2/a^2$, and the focal distances at (x, y) are $a \pm ex$.
- The tangent to $3x^2 + 4y^2 = 1$ at (x_0, y_0) is $3x_0x + 4y_0y = 1$.

Step 1: Find the tangent point in the first quadrant Divide the given line by $\sqrt{7}$ and compare its coefficients with the tangent equation:

$$3x_0 = \frac{\alpha}{\sqrt{7}}, \qquad 4y_0 = \frac{4}{\sqrt{7}}.$$

Thus $x_0 = \alpha/(3\sqrt{7})$ and $y_0 = 1/\sqrt{7}$. Because the contact point belongs to the ellipse,

$$\frac{\alpha^2}{21} + \frac{4}{7} = 1, \qquad \alpha^2 = 9.$$

The first-quadrant requirement gives $x_0 > 0$, hence $\alpha = 3$ and $P = (1/\sqrt{7}, 1/\sqrt{7})$. The negative value of α would put the contact point in the second quadrant.

Step 2: Compute the two distances from the foci The standard form is $x^2/(1/3) + y^2/(1/4) = 1$. Therefore

$$a = \frac{1}{\sqrt{3}}, \qquad b = \frac{1}{2}, \qquad e = \sqrt{1 - \frac{1/4}{1/3}} = \frac{1}{2}.$$

The foci lie on the x-axis. Applying the focal-distance formula at the computed point gives the two values

$$\frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{7}}, \qquad \frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{7}}.$$

Their sum is $2a$, as required for an ellipse, and both are positive. Options (A) and (B) use $\sqrt{5}$ instead of the tangent point's $\sqrt{7}$, while option (D) uses $\sqrt{11}$. None equals either actual focal distance.

Step 3: Conclusion The tangent point is $P = (1/\sqrt{7}, 1/\sqrt{7})$ and $e = 1/2$, so $a + ex_P = \frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{7}}$ is one focal distance. Hence, the correct option is (C).

Quick Tip to Solve: Normalize the tangent to have right-hand side 1, compare coefficients to find its contact point, and then use $a \pm ex$.

5. Let $A = \{x : |x^2 - 10| \le 6\}$ and $B = \{x : |x - 2| > 1\}$. Then [4 marks]

(A) $A - B = [2, 3)$

(B) $A \cap B = [-4, -2] \cup [3, 4]$

(C) $A \cup B = (-\infty, 1] \cup (2, \infty)$

(D) $B - A = (-\infty, -4) \cup (-2, 1) \cup (4, \infty)$

Correct Answer: (D) $B - A = (-\infty, -4) \cup (-2, 1) \cup (4, \infty)$

Solution:

Concept:

- An inequality $|u| \le a$ means $-a \le u \le a$, whereas $|u| > a$ means $u < -a$ or $u > a$, for $a > 0$.
- The difference $B - A$ contains elements of B that are outside A; intersection requires membership in both sets and union requires membership in at least one. Strict inequalities exclude their boundary points.

Step 1: Convert both sets to intervals For A, the absolute-value inequality gives

$$-6 \le x^2 - 10 \le 6 \quad \Longrightarrow \quad 4 \le x^2 \le 16.$$

Thus $2 \le |x| \le 4$, and

$$A = [-4, -2] \cup [2, 4].$$

For B, we have $x - 2 < -1$ or $x - 2 > 1$. Hence

$$B = (-\infty, 1) \cup (3, \infty).$$

In particular, neither 1 nor 3 belongs to B.

Step 2: Remove the points of A from B Removing the closed interval $[-4, -2]$ from the part $(-\infty, 1)$ leaves $(-\infty, -4) \cup (-2, 1)$. Removing $[2, 4]$ from $(3, \infty)$ leaves $(4, \infty)$. Therefore

$$B - A = (-\infty, -4) \cup (-2, 1) \cup (4, \infty).$$

The endpoints $-4, -2, 4$ are removed because they belong to A, while 1 was never in B. This verifies every open endpoint in the proposed difference.

Step 3: Check the other three statements The actual difference $A - B = [2, 3]$ includes 3, so (A) incorrectly excludes its right endpoint. Also,

$$A \cap B = [-4, -2] \cup (3, 4],$$

so (B) incorrectly includes 3, which is absent from B. Finally,

$$A \cup B = (-\infty, 1) \cup [2, \infty).$$

Thus (C) incorrectly includes 1 and excludes 2. All three distractors fail at a boundary point, while the fourth statement agrees with the complete interval calculation.

Step 4: Conclusion $\boxed{B - A = (-\infty, -4) \cup (-2, 1) \cup (4, \infty)}$. Hence, the correct option is (D).

Quick Tip to Solve: After solving the inequalities, test the boundary points $1, 2, 3$: their membership immediately distinguishes the three incorrect statements.

6. Let $f(x) = x^3 + x^2 f'(1) + 2x f''(2) + f'''(3)$, $x \in \mathbb{R}$. Then the value of $f'(5)$ is : [4 marks]

(A) $\frac{2}{5}$

(B) $\frac{62}{5}$

(C) $\frac{657}{5}$

(D) $\frac{117}{5}$

Correct Answer: (D) $\frac{117}{5}$

Solution:

Concept:

- Values such as $f\prime(1)$, $f\prime\prime(2)$ and $f\prime\prime\prime(3)$ are constants when differentiating an identity with respect to x.
- Differentiate the polynomial identity first, then evaluate at the specified arguments to form simultaneous equations for its unknown coefficients.

Step 1: Differentiate while keeping evaluated derivatives constant Set $A = f'(1)$, $B = f''(2)$ and $C = f^{(3)}(3)$. The identity becomes

$$f(x) = x^3 + Ax^2 + 2Bx + C.$$

Differentiating successively gives

$$\begin{aligned} f'(x) &= 3x^2 + 2Ax + 2B, \\ f''(x) &= 6x + 2A, \\ f^{(3)}(x) &= 6. \end{aligned}$$

Thus $C = 6$, but it does not enter the first derivative. In particular, A and B must not themselves be differentiated as functions of x: their arguments 1 and 2 are fixed numbers.

Step 2: Find the constants from their defining values Evaluating the first derivative at 1 and the second derivative at 2 gives

$$A = 3 + 2A + 2B, \qquad B = 12 + 2A.$$

The first equation says $A + 2B = -3$. Substitute the second equation into it:

$$A + 2(12 + 2A) = -3, \qquad 5A = -27.$$

Therefore $A = -27/5$ and $B = 12 - 54/5 = 6/5$. These satisfy the original definitions: $3 - 54/5 + 12/5 = -27/5$ and $12 - 54/5 = 6/5$.

Step 3: Evaluate the requested first derivative Substitute $x = 5$ into the derivative expression, retaining the factor $2x$ multiplying A:

$$f'(5) = 75 + 10\left(-\frac{27}{5}\right) + 2\left(\frac{6}{5}\right) = 75 - 54 + \frac{12}{5} = \frac{117}{5}.$$

The values $2/5$, $62/5$ and $657/5$ do not satisfy this direct evaluation. The constant third derivative contributes to f, but disappears in f', so adding it here would be incorrect.

Step 4: Conclusion The derivative identities give $f\prime(1) = -27/5$ and $f\prime\prime(2) = 6/5$, hence $f\prime(5) = 75 - 54 + 12/5 = \dfrac{117}{5}$. Hence, the correct option is (D).

Quick Tip to Solve: Rename evaluated derivatives as constants before differentiating, then use their definitions to solve a small linear system.

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7. Let the line L_1 be parallel to the vector $-3\hat{i} + 2\hat{j} + 4\hat{k}$ and pass through the point $(2, 6, 7)$, and the line L_2 be parallel to the vector $2\hat{i} + \hat{j} + 3\hat{k}$ and pass through the point $(4, 3, 5)$. If the line L_3 is parallel to the vector $-3\hat{i} + 5\hat{j} + 16\hat{k}$ and intersects the lines L_1 and L_2 at the points C and D, respectively, then $\left|\overrightarrow{CD}\right|^2$ is equal to: [4 marks]

(A) 89

(B) 171

(C) 290

(D) 312

Correct Answer: (C) 290

Solution:

Concept:

- A line through a point with direction vector $\mathbf{v}$ has points $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$, where t is a real parameter.
- Parallel vectors are scalar multiples. For $\overrightarrow{CD} = u(-3, 5, 16)$, the squared length is $u^2(9 + 25 + 256)$.

Step 1: Parameterise the two intersection points As C lies on L_1 and D lies on L_2, introduce independent parameters s, t:

$$C = (2 - 3s, 6 + 2s, 7 + 4s), \qquad D = (4 + 2t, 3 + t, 5 + 3t).$$

Subtract in the order $D - C$. The direction condition for their joining line gives

$$(2 + 2t + 3s, -3 + t - 2s, -2 + 3t - 4s) = u(-3, 5, 16).$$

The scalar u must be found; the direction vector alone does not determine the distance.

Step 2: Solve the component equations The three components give

$$\begin{aligned} 2 + 2t + 3s &= -3u, \\ -3 + t - 2s &= 5u, \\ -2 + 3t - 4s &= 16u. \end{aligned}$$

The middle equation yields $t = 5u + 3 + 2s$. Substitution in the first and third gives

$$7s + 13u = -8, \qquad 2s - u = -7.$$

Thus $u = 2s + 7$, so $33s = -99$. Hence $s = -3$, $u = 1$ and $t = 2$.

Step 3: Calculate and check the separation These parameters give $C = (11, 0, -5)$ and $D = (8, 5, 11)$, so the displacement is exactly $(-3, 5, 16)$, as required. Therefore

$$|\overrightarrow{CD}|^2 = (-3)^2 + 5^2 + 16^2 = 9 + 25 + 256 = 290.$$

The other numerical options, 89, 171 and 312, disagree with this sum of squared coordinate differences.

Step 4: Conclusion $|\overrightarrow{CD}|^2 = 290$. Hence, the correct option is (C).

Quick Tip to Solve: Use separate parameters for the two given lines, then equate their displacement to a scalar multiple of the third direction vector.

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8. Let $y = y(x)$ be the solution of the differential equation $\sec x \dfrac{dy}{dx} - 2y = 2 + 3\sin x$, $x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$, $y(0) = -\dfrac{7}{4}$. Then $y\left(\dfrac{\pi}{6}\right)$ is equal to : [4 marks]

(A) $-3\sqrt{3} - 7$

(B) $-3\sqrt{2} - 7$

(C) $-\dfrac{5}{4}$

(D) $-\dfrac{5}{2}$

Correct Answer: (D) $-\dfrac{5}{2}$

Solution:

Concept:

- A first-order linear equation $dy/dt + Py = Q$ has integrating factor $e^{\int P\,dt}$.
- Here $t = \sin x$ gives $dy/dt = \sec x\, dy/dx$, since $\cos x > 0$ on the specified interval.

Step 1: Change the independent variable Put $t = \sin x$. This substitution is valid throughout the given interval and transforms the equation into

$$\frac{dy}{dt} - 2y = 2 + 3t.$$

The initial condition becomes $y = -7/4$ at $t = 0$. The coefficient of y is constant, and the integrating factor is e^{-2t}. Multiplying the equation by it gives

$$\frac{d}{dt}(ye^{-2t}) = (2 + 3t)e^{-2t}.$$

Step 2: Integrate and use the initial value Integration by parts, or direct differentiation of a linear polynomial times e^{-2t}, yields

$$\int (2 + 3t)e^{-2t}\,dt = e^{-2t}\left(-\frac{3}{2}t - \frac{7}{4}\right) + C.$$

Consequently

$$y = -\frac{3}{2}t - \frac{7}{4} + Ce^{2t}.$$

At $t = 0$, the prescribed value gives $-7/4 = -7/4 + C$, hence $C = 0$. In terms of the original variable,

$$y(x) = -\frac{7}{4} - \frac{3}{2}\sin x.$$

For a direct check, $y' = -\frac{3}{2}\cos x$, so $\sec x\, y' - 2y = -\frac{3}{2} + \frac{7}{2} + 3\sin x = 2 + 3\sin x$.

Step 3: Evaluate at the requested argument Since $\sin(\pi/6) = 1/2$,

$$y\left(\frac{\pi}{6}\right) = -\frac{7}{4} - \frac{3}{4} = -\frac{5}{2}.$$

Option (C), $-5/4$, has the wrong value. Options (A) and (B), $-3\sqrt{3} - 7$ and $-3\sqrt{2} - 7$, also disagree with the solution fixed by the initial condition.

Step 4: Conclusion The expression $y(x) = -7/4 - (3/2)\sin x$ satisfies both $\sec x\, y' - 2y = 2 + 3\sin x$ and $y(0) = -7/4$, proving $y(\pi/6) = -5/2$ from the printed question. Hence, the correct option is (D).

Quick Tip to Solve: When the derivative is multiplied by $\sec x$ and the other terms use $\sin x$, set $t = \sin x$ to obtain a constant-coefficient linear equation.

9. If the area of the region $\{(x, y) : 1 - 2x \le y \le 4 - x^2,\ x \ge 0,\ y \ge 0\}$ is $\frac{\alpha}{\beta}$, $\alpha, \beta \in \mathbb{N}, \gcd(\alpha, \beta) = 1$, then the value of $(\alpha + \beta)$ is: [4 marks]

(A) 67

(B) 73

(C) 85

(D) 91

Correct Answer: (B) 73

Solution:

Concept:

- For a region between two graphs, area is the integral of upper ordinate minus lower ordinate over its allowed horizontal interval.
- Two simultaneous lower bounds $y \ge 1 - 2x$ and $y \ge 0$ give the effective lower boundary $\max(1 - 2x, 0)$.

Step 1: Find the interval and the boundary change The upper boundary is $y = 4 - x^2$. Since $x \ge 0$ and some allowed y must be nonnegative, $4 - x^2 \ge 0$, giving $0 \le x \le 2$. The line $y = 1 - 2x$ crosses the horizontal axis at $x = 1/2$. Thus it is the lower boundary from 0 to $1/2$; thereafter the lower boundary is $y = 0$. On the first interval the vertical height is $3 + 2x - x^2 > 0$, so there is no further restriction.

Step 2: Integrate the two pieces The required area is

$$\begin{aligned}\mathcal{A} &= \int_0^{1/2} \left[(4-x^2)-(1-2x)\right] dx + \int_{1/2}^{2} (4-x^2)\, dx \\ &= \int_0^{2} (4-x^2)\, dx - \int_0^{1/2} (1-2x)\, dx \\ &= \left[4x - \frac{x^3}{3}\right]_0^{2} - \left[x - x^2\right]_0^{1/2} \\ &= \frac{16}{3} - \frac{1}{4} = \frac{61}{12}.\end{aligned}$$

This also corresponds to subtracting the small triangle below the line from the entire first-quadrant region under the parabola.

Step 3: Reduce the fraction and identify the option The integers 61 and 12 are coprime, so the reduced fraction has $\alpha = 61$, $\beta = 12$. Their sum is $61 + 12 = 73$. Of the printed choices $67, 73, 85, 91$, only 73 agrees; the other three cannot be obtained from the reduced area.

Step 4: Conclusion $\mathcal{A} = 61/12$ and $\alpha + \beta = 73$. Hence, the correct option is (B).

Quick Tip to Solve: Combine the two lower bounds first: integrate above the line only until it meets the axis, then use the axis as the lower boundary.

10. Let $a_1, \frac{a_2}{2}, \frac{a_3}{2^2}, \ldots, \frac{a_{10}}{2^9}$ be a G.P. of common ratio $\frac{1}{\sqrt{2}}$. If $a_1 + a_2 + \ldots + a_{10} = 62$, then a_1 is equal to: [4 marks]

(A) $2(\sqrt{2}-1)$
(B) $\sqrt{2}-1$
(C) $2(2-\sqrt{2})$
(D) $2-\sqrt{2}$

Correct Answer: (A) $2(\sqrt{2}-1)$

Solution:

Concept:

- If $b_n = a_n/2^{n-1}$ is geometric with ratio r, then a_n is geometric with ratio $2r$.
- For a geometric progression with first term a, ratio $R \neq 1$, and n terms, the sum is $S_n = a(R^n - 1)/(R-1)$.

Step 1: Recover the progression of the original terms Define $b_n = a_n/2^{n-1}$. The given progression has $b_1 = a_1$ and common ratio $1/\sqrt{2}$, so

$$\frac{a_n}{2^{n-1}} = a_1 \left(\frac{1}{\sqrt{2}}\right)^{n-1}.$$

Multiplication by 2^{n-1} gives

$$a_n = a_1(\sqrt{2})^{n-1}.$$

Thus the original terms $a_1, a_2, \ldots, a_{10}$, whose sum is specified, have common ratio $\sqrt{2}$, not $1/\sqrt{2}$. For example, $a_2 = \sqrt{2}a_1$ and $a_3 = 2a_1$.

Step 2: Apply the finite geometric sum Use the ten original terms in the given sum:

$$62 = a_1\left(1 + \sqrt{2} + (\sqrt{2})^2 + \cdots + (\sqrt{2})^9\right)$$
$$= a_1\frac{(\sqrt{2})^{10} - 1}{\sqrt{2} - 1} = a_1\frac{31}{\sqrt{2} - 1}.$$

Here $(\sqrt{2})^{10} = 2^5 = 32$. Solving the equation gives $a_1 = 2(\sqrt{2} - 1)$. Substituting back gives the required sum $2(\sqrt{2} - 1) \cdot 31/(\sqrt{2} - 1) = 62$.

Step 3: Check the remaining choices If $a_1 = \sqrt{2} - 1$, the sum would be 31, excluding (B). Since $2 - \sqrt{2} = \sqrt{2}(\sqrt{2} - 1)$, choices (C) and (D) give sums $62\sqrt{2}$ and $31\sqrt{2}$, respectively. Neither equals 62.

Step 4: Conclusion $a_1 = 2(\sqrt{2} - 1)$. Hence, the correct option is (A).

Quick Tip to Solve: Undo the factors 2^{n-1} before summing: the original common ratio is twice the ratio of the displayed scaled terms.

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11. Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function such that $f''(x) > 0$ for all $x \in \mathbb{R}$ and $f'(a - 1) = 0$, where a is a real number.

Let $g(x) = f(\tan^2 x - 2\tan x + a),\ 0 < x < \frac{\pi}{2}$.

Consider the following two statements:

(I) g is increasing in $\left(0, \frac{\pi}{4}\right)$

(II) g is decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Then, [4 marks]

(A) Only (I) is True

(B) Only (II) is True

(C) Both (I) and (II) are True

(D) Neither (I) nor (II) is True

Correct Answer: (D) Neither (I) nor (II) is True

Solution:

Concept:

- Since $f''(u) > 0$ everywhere, $f'(u)$ is strictly increasing; combined with $f'(a - 1) = 0$, this gives $f'(u) > 0$ for $u > a - 1$.
- For a composite function $g(x) = f(h(x))$, the chain rule gives $g'(x) = f'(h(x))h'(x)$. The sign of this product determines monotonicity.

Step 1: Locate the argument of the outer function Complete the square in the argument:

$$h(x) = \tan^2 x - 2\tan x + a = (\tan x - 1)^2 + a - 1.$$

Therefore $h(x) \geq a-1$, with equality only at $x = \pi/4$ in the given domain. On either side of that point, $h(x) > a-1$. Because f' is strictly increasing and vanishes at $a-1$, the factor $f'(h(x))$ is strictly positive on both open intervals under consideration.

Step 2: Determine the derivative on the first interval Differentiate the composite function:

$$g'(x) = 2(\tan x - 1)\sec^2 x\, f'(h(x)).$$

For $0 < x < \pi/4$, we have $0 < \tan x < 1$. Thus $\tan x - 1 < 0$, while $\sec^2 x > 0$ and $f'(h(x)) > 0$. Consequently $g'(x) < 0$, so g is strictly decreasing on this interval. Statement (I), which asserts increase, is false.

Step 3: Determine the derivative on the second interval For $\pi/4 < x < \pi/2$, $\tan x > 1$. All three factors in the displayed derivative are positive, hence $g'(x) > 0$ and g is strictly increasing. Statement (II), which asserts decrease, is also false. Options (A), (B) and (C) each assert that at least one of these false statements is true, so all three are excluded.

Step 4: Conclusion Neither (I) nor (II) is true. Hence, the correct option is (D).

Quick Tip to Solve: Complete the square inside f first. Its argument stays to the right of $a-1$, so only the factor $\tan x - 1$ changes the derivative sign.

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12. For the matrices $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -29 & 49 \\ -13 & 18 \end{bmatrix}$, if $(A^{15} + B)\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, then among the following which one is true? [4 marks]

(A) $x = 5, y = 7$

(B) $x = 16, y = 3$

(C) $x = 11, y = 2$

(D) $x = 18, y = 11$

Correct Answer: (C) $x = 11, y = 2$

Solution:

Concept:

- If $A = I + N$ with $N^2 = 0$, the binomial expansion terminates: $A^m = I + mN$ for every positive integer m.
- A homogeneous matrix equation is checked by multiplying each matrix row by the column vector. Proportional rows impose a single independent linear condition.

Step 1: Find a nilpotent part of the matrix Subtract the identity matrix from A:

$$N = A - I = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}.$$

Multiply to verify the crucial cancellation:

$$N^2 = \begin{bmatrix} 4-4 & -8+8 \\ 2-2 & -4+4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Since all powers N^k with $k \geq 2$ vanish, the binomial expansion of $(I+N)^{15}$ contains only its first two terms. This avoids multiplying A repeatedly.

Step 2: Evaluate the matrix power and add the given matrix The power is

$$A^{15} = I + 15N = \begin{bmatrix} 31 & -60 \\ 15 & -29 \end{bmatrix}.$$

Adding B entry by entry gives

$$A^{15} + B = \begin{bmatrix} 31-29 & -60+49 \\ 15-13 & -29+18 \end{bmatrix} = \begin{bmatrix} 2 & -11 \\ 2 & -11 \end{bmatrix}.$$

Thus both rows of the given homogeneous equation reduce to the same condition,

$$2x - 11y = 0.$$

There are infinitely many solutions of this condition, and the task is to identify which listed pair belongs to this set.

Step 3: Test all listed pairs Evaluate the expression $2x - 11y$ for each choice:

$$\begin{aligned} \text{(A)}: &\quad 2(5) - 11(7) = -67, \\ \text{(B)}: &\quad 2(16) - 11(3) = -1, \\ \text{(C)}: &\quad 2(11) - 11(2) = 0, \\ \text{(D)}: &\quad 2(18) - 11(11) = -85. \end{aligned}$$

Only the third pair satisfies both identical row equations.

Step 4: Conclusion The pair $x = 11,\ y = 2$ satisfies the matrix equation. Hence, the correct option is (C).

Quick Tip to Solve: For a large power of a small matrix, test $(A-I)^2$. If it vanishes, replace A^{15} directly by $I + 15(A-I)$.

13. Let one end of a focal chord of the parabola $y^2 = 16x$ be $(16, 16)$. If $P(\alpha,\ \beta)$ divides this focal chord internally in the ratio $5 : 2$, then the minimum value of $\alpha + \beta$ is equal to: [4 marks]

(A) 5

(B) 7

(C) 16

(D) 22

Correct Answer: (B) 7

Solution:

Concept:

- The parabola $y^2 = 4ax$ has focus $(a, 0)$. A focal chord is a chord whose line passes through this focus.
- If $AP : PB = m : n$, the internal section formula gives $P = (nA+mB)/(m+n)$, applied separately to each coordinate.

Step 1: Find the other endpoint of the focal chord Here $4a = 16$, so the focus is $S = (4, 0)$. Write the given endpoint as $A = (16, 16)$. The slope of the line through S and A is

$$m = \frac{16 - 0}{16 - 4} = \frac{4}{3}.$$

Thus its equation is $y = \frac{4}{3}(x - 4)$. Substitution in the parabola gives

$$\frac{16}{9}(x - 4)^2 = 16x \quad \Longrightarrow \quad x^2 - 17x + 16 = 0.$$

The roots are 16 and 1. The first is the given endpoint; for the second, the line gives $y = -4$. Therefore the other endpoint is $B = (1, -4)$, which also satisfies $y^2 = 16x$.

Step 2: Compare the two internal division points The endpoints are not ordered in the statement, so a division in the ratio $5 : 2$ permits two locations. For $AP : PB = 5 : 2$,

$$P = \left(\frac{2(16) + 5(1)}{7}, \frac{2(16) + 5(-4)}{7}\right) = \left(\frac{37}{7}, \frac{12}{7}\right).$$

The sum of its coordinates is $49/7 = 7$. Reversing the endpoint order gives

$$P = \left(\frac{5(16) + 2(1)}{7}, \frac{5(16) + 2(-4)}{7}\right) = \left(\frac{82}{7}, \frac{72}{7}\right),$$

whose coordinate sum is $154/7 = 22$. Both points lie internally on the same chord. Thus 22 is the larger possible sum, while 5 and 16 are neither of these sums.

Step 3: Conclusion The minimum value of $\alpha + \beta$ is $\boxed{7}$. Hence, the correct option is (B).

Quick Tip to Solve: Find the second endpoint using the focus, then apply the section formula directly to the endpoint sums 32 and -3 in both orders.

14. Let $y^2 = 12x$ be the parabola with its vertex at O. Let P be a point on the parabola and A be a point on the x-axis such that $\angle OPA = 90^\circ$. Then the locus of the centroid of such triangles OPA is: [4 marks]

(A) $y^2 - 4x + 8 = 0$

(B) $y^2 - 6x + 4 = 0$

(C) $y^2 - 9x + 6 = 0$

(D) $y^2 - 2x + 8 = 0$

Correct Answer: (D) $y^2 - 2x + 8 = 0$

Solution:

Concept:

- A general point of $y^2 = 4ax$ is $(at^2, 2at)$, where t is a real parameter.
- Perpendicular vectors have zero dot product. The centroid coordinates are the arithmetic means of the three vertex coordinates.

Step 1: Express the vertices using one parameter Since $4a = 12$, take $O = (0, 0)$, $P = (3t^2, 6t)$ and $A = (u, 0)$. A genuine triangle requires $t \neq 0$, because $t = 0$ makes P coincide with O. The right angle is at P, so use the two vectors that start there:

$$\overrightarrow{PO} = (-3t^2, -6t), \qquad \overrightarrow{PA} = (u - 3t^2, -6t).$$

Their dot product vanishes, giving

$$-3t^2(u - 3t^2) + 36t^2 = 0.$$

Dividing by $3t^2$ and rearranging gives $u = 3t^2 + 12$. This locates A uniquely for each permitted P.

Step 2: Eliminate the parameter from the centroid Let the centroid be $G = (x, y)$. Averaging the three coordinates gives

$$x = \frac{0 + 3t^2 + (3t^2 + 12)}{3} = 2t^2 + 4, \qquad y = \frac{0 + 6t + 0}{3} = 2t.$$

Therefore $t = y/2$, and substitution in the first equation yields

$$x = \frac{y^2}{2} + 4 \quad \Longleftrightarrow \quad y^2 - 2x + 8 = 0.$$

Every nonzero y supplies a nonzero parameter and hence a triangle. Strictly, the vertex $(4, 0)$ is excluded because the triangle would degenerate. The corresponding parabola is still identified uniquely among the options. For example, $t = 1$ gives $G = (6, 2)$; substitution into (A), (B) and (C) gives respectively $-12, -28, -44$, not zero.

Step 3: Conclusion The centroid lies on $\boxed{y^2 - 2x + 8 = 0}$, excluding $(4, 0)$ for nondegenerate triangles. Hence, the correct option is (D).

Quick Tip to Solve: Use $P = (3t^2, 6t)$ and the dot product at P; the centroid then gives $y = 2t$, making parameter elimination immediate.

15. The positive integer n, for which the solutions of the equation

$x(x+2)+(x+2)(x+4)+\cdots+(x+2n-2)(x+2n)=\frac{8n}{3}$ **are two consecutive even integers, is :** [4 marks]

(A) 3

(B) 6

(C) 9

(D) 12

Correct Answer: (A) 3

Solution:

Concept:

- For $k=0,\ldots,n-1$, use $\sum k=n(n-1)/2$ and $\sum k^2=n(n-1)(2n-1)/6$.
- Two consecutive even integers differ by 2. For a monic quadratic, the discriminant equals the square of the difference between its roots.

Step 1: Sum the products and form a quadratic The general term in the given sum is

$$(x+2k)(x+2k+2)=x^2+(4k+2)x+4k^2+4k,$$

where k runs from 0 to $n-1$. Therefore the coefficient of x in the sum is $4\sum k+2n=2n^2$, while its constant term is

$$\begin{aligned}4\sum k^2+4\sum k&=\frac{2n(n-1)(2n-1)}{3}+2n(n-1)\\&=\frac{4n(n^2-1)}{3}.\end{aligned}$$

Equating the sum to $8n/3$ and dividing by the positive integer n yields

$$x^2+2nx+\frac{4(n^2-3)}{3}=0.$$

Step 2: Impose the root difference and verify parity For this monic quadratic, the discriminant is

$$D=(2n)^2-4\frac{4(n^2-3)}{3}=16-\frac{4n^2}{3}.$$

Since consecutive even integers differ by 2, their squared difference is 4, so

$$16-\frac{4n^2}{3}=4\quad\Longrightarrow\quad n^2=9.$$

Positivity permits only $n=3$. The quadratic then becomes $x^2+6x+8=0$, whose roots are -4 and -2. They are indeed consecutive even integers, so the difference condition has not introduced roots of the wrong parity. Directly, each root makes the three products sum to 8, equal to $8n/3$. The other listed values $6, 9, 12$ give a negative discriminant and hence cannot even produce real roots.

Step 3: Conclusion The required positive integer is $\boxed{n=3}$. Hence, the correct option is (A).

Quick Tip to Solve: After summing to a monic quadratic, set its discriminant to 2^2, then check that both resulting roots are even integers.

16. A random variable X takes values 0, 1, 2, 3 with probabilities $\frac{2a+1}{30}$, $\frac{8a-1}{30}$, $\frac{4a+1}{30}$, b respectively, where $a, b \in \mathbb{R}$.

Let μ and σ respectively be the mean and standard deviation of X such that $\sigma^2 + \mu^2 = 2$.

Then $\frac{a}{b}$ is equal to: [4 marks]

(A) 60

(B) 30

(C) 12

(D) 3

Correct Answer: (A) 60

Solution:

Concept:

- The probabilities of all possible values of a discrete random variable sum to 1, and each probability must be nonnegative.
- Variance satisfies $\sigma^2 = E(X^2) - [E(X)]^2$. Thus $\sigma^2 + \mu^2 = E(X^2)$, where $\mu = E(X)$.

Step 1: Form the normalization and moment equations Adding the four probabilities and equating their sum to unity gives

$$\frac{2a+1+8a-1+4a+1}{30} + b = 1 \quad \Longrightarrow \quad 14a + 30b = 29.$$

The given condition is exactly $E(X^2) = 2$. Multiply each probability by the square of its corresponding value. In particular, the probability b has weight $3^2 = 9$, while the probability at zero contributes nothing:

$$\frac{8a-1}{30} + 4\frac{4a+1}{30} + 9b = 2 \quad \Longrightarrow \quad 24a + 270b = 57.$$

These are two linear equations for the two parameters; calculating the mean separately is unnecessary.

Step 2: Solve the equations and check the distribution Multiplying the first equation by 9 gives $126a + 270b = 261$. Subtracting the second equation eliminates b, leaving

$$102a = 204, \qquad a = 2, \qquad b = \frac{29 - 14(2)}{30} = \frac{1}{30}.$$

The probabilities at $0, 1, 2, 3$ are therefore $5/30, 15/30, 9/30, 1/30$. They are all positive and sum to 1. Their second moment is $(15 + 36 + 9)/30 = 2$, verifying the supplied condition. Consequently the required ratio is $2/(1/30)$. The alternatives $30, 12, 3$ are all inconsistent with the unique values of a and b; 30 is only $1/b$, omitting the numerator $a = 2$.

Step 3: Conclusion The printed probabilities give $14a + 30b = 29$ and $24a + 270b = 57$, hence $a = 2$, $b = 1/30$, and $a/b = \boxed{60}$. The value 60 follows directly from the printed question; its option letter changes when the options are reordered. Hence, the correct option is (A).

Quick Tip to Solve: Replace $\sigma^2 + \mu^2$ by $E(X^2)$ immediately, avoiding a quadratic calculation of the mean and variance separately.

17. Let the line L pass through the point $(-3, 5, 2)$ and make equal angles with the positive coordinate axes. If the distance of L from the point $(-2, r, 1)$ is $\sqrt{\frac{14}{3}}$, then the sum of all possible values of r is: [4 marks]

(A) 6

(B) 10

(C) 12

(D) 16

Correct Answer: (B) 10

Solution:

Concept:

- Equal angles with the positive coordinate axes give equal direction cosines. A direction vector for the line is therefore $\mathbf{d} = (1, 1, 1)$.
- For a line through P with direction vector $\mathbf{d}$, the squared perpendicular distance of Q is $h^2 = |\overrightarrow{PQ}|^2 - (\overrightarrow{PQ} \cdot \mathbf{d})^2/|\mathbf{d}|^2$. The second term removes the squared projection along the line.

Step 1: Express the displacement from the given point Let $P = (-3, 5, 2)$ and $Q = (-2, r, 1)$. The line has equal direction cosines, whose squares sum to one, so a unit vector along it is $(1, 1, 1)/\sqrt{3}$. The displacement from P to Q is

$$\mathbf{v} = \overrightarrow{PQ} = (1, r - 5, -1).$$

Hence $|\mathbf{v}|^2 = 2 + (r - 5)^2$, while $\mathbf{v} \cdot \mathbf{d} = r - 5$ and $|\mathbf{d}|^2 = 3$. The opposite orientation of the same line gives the same distance because the projection is squared.

Step 2: Apply the prescribed perpendicular distance Squaring the given positive distance and substituting gives

$$\begin{aligned} 2 + (r - 5)^2 - \frac{(r - 5)^2}{3} &= \frac{14}{3}, \\ 6 + 2(r - 5)^2 &= 14, \\ (r - 5)^2 &= 4. \end{aligned}$$

Therefore $r = 5 - 2 = 3$ or $r = 5 + 2 = 7$. Both values give squared distance $2 + 8/3 = 14/3$, so both are admissible; squaring has introduced no extraneous value.

Step 3: Compare the requested sum with the options The two possible values add to $3 + 7 = 10$. The question asks for their sum, rather than either value separately. The alternatives 6, 12 and 16 do not equal this sum. Equivalently, expanding the equation gives $r^2 - 10r + 21 = 0$, whose root sum is 10 by Vieta's formula.

Step 4: Conclusion The sum of all possible values of r is $\boxed{10}$. Hence, the correct option is (B).

Quick Tip to Solve: With direction vector $(1, 1, 1)$, subtract one third of the squared sum of the displacement coordinates from its squared length.

18. The largest $n \in \mathbb{N}$, for which 7^n divides $101!$, is : [4 marks]

(A) 19

(B) 18

(C) 16

(D) 15

Correct Answer: (C) 16

Solution:

Concept:

- The largest exponent n for which p^n divides an integer is the number of factors p in its prime factorization.
- For a prime p, Legendre's formula gives $v_p(N!) = \lfloor N/p \rfloor + \lfloor N/p^2 \rfloor + \cdots$, stopping when the powers exceed N.

Step 1: Count one factor from each multiple of seven The factorial $101!$ is the product of all positive integers from 1 through 101. A factor contributes at least one 7 precisely when it is divisible by 7. The relevant multiples are

$$7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98.$$

There are $\lfloor 101/7 \rfloor = 14$ of them, since $14 \cdot 7 = 98 \leq 101$ and $15 \cdot 7 = 105 > 101$. This first count supplies one factor of 7 from each multiple, including 49 and 98, but has not yet counted all their factors of 7.

Step 2: Add the extra factors from higher powers Every multiple of $7^2 = 49$ contributes another factor of 7 in addition to the one already counted. Up to 101, these multiples are 49 and 98, so there are $\lfloor 101/49 \rfloor = 2$ additional factors. Neither contributes a third factor, and no other factor can do so, because $7^3 = 343 > 101$. Hence

$$v_7(101!) = \left\lfloor \frac{101}{7} \right\rfloor + \left\lfloor \frac{101}{49} \right\rfloor + \left\lfloor \frac{101}{343} \right\rfloor = 14 + 2 + 0.$$

After extracting these factors, the remaining product is not divisible by 7. Thus this is the exact exponent, not just a lower bound. The proposed values 18 and 19 require factors that are absent. Although 7^{15} does divide the factorial, 15 is not the largest possible exponent.

Step 3: Conclusion The largest required exponent is $\boxed{n = 16}$. Hence, the correct option is (C).

Quick Tip to Solve: Divide 101 successively by 7 and 49, add the integer quotients, and stop because the next power 343 exceeds 101.

19. For a triangle ABC, let $\vec{p} = \overrightarrow{BC}, \vec{q} = \overrightarrow{CA}$ and $\vec{r} = \overrightarrow{BA}$. If $|\vec{p}| = 2\sqrt{3}, |\vec{q}| = 2$ and $\cos\theta = \frac{1}{\sqrt{3}}$, where θ is the angle between $\vec{p}$ and $\vec{q}$, then $|\vec{p} \times (\vec{q} - 3\vec{r})|^2 + 3|\vec{r}|^2$ is equal to : [4 marks]

(A) 200

(B) 220

(C) 340

(D) 410

Correct Answer: (A) 200

Solution:

Concept:

- The triangle law gives $\overrightarrow{BA} = \overrightarrow{BC} + \overrightarrow{CA}$, so the orientation of each vector determines its sign.
- For vectors $\vec{u}, \vec{v}$, $|\vec{u} + \vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 + 2\vec{u} \cdot \vec{v}$, and $|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2|\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$.

Step 1: Express the third side and find the scalar product Following the path from B through C to A gives $\vec{r} = \vec{p} + \vec{q}$. The given angle is between the directed vectors $\vec{p}$ and $\vec{q}$; hence

$$\vec{p} \cdot \vec{q} = (2\sqrt{3})(2)\frac{1}{\sqrt{3}} = 4.$$

It follows that

$$|\vec{r}|^2 = 12 + 4 + 2(4) = 24.$$

Using a minus sign in this formula would correspond to a different directed side and would not match $\overrightarrow{BA}$.

Step 2: Reduce and evaluate the vector product Substitute the triangle relation before expanding the vector product:

$$\vec{q} - 3\vec{r} = -3\vec{p} - 2\vec{q},$$
$$\vec{p} \times (\vec{q} - 3\vec{r}) = -2(\vec{p} \times \vec{q}).$$

Here $\vec{p} \times \vec{p} = \vec{0}$. Also,

$$|\vec{p} \times \vec{q}|^2 = (12)(4) - 4^2 = 32.$$

Squaring the magnitude multiplies this by $(-2)^2 = 4$, so the first term is 128, while $3|\vec{r}|^2 = 72$. Thus the required sum is $128 + 72$. The alternatives 220, 340 and 410 disagree with these separately determined terms.

Step 3: Conclusion The required value is $128 + 72 = \boxed{200}$. Hence, the correct option is (A).

Quick Tip to Solve: First use $\vec{r} = \vec{p} + \vec{q}$; the cross product then collapses to $-2\vec{p} \times \vec{q}$, whose squared magnitude carries a factor of four.

20. If the system of equations

$$3x + y + 4z = 3$$

$$2x + \alpha y - z = -3$$

$$x + 2y + z = 4$$

has no solution, then the value of α is equal to: [4 marks]

(A) 4

(B) 13

(C) 19

(D) 23

Correct Answer: (C) 19

Solution:

Concept:

- A system of three linear equations is inconsistent when elimination produces a false statement such as $0 = c$, where $c \neq 0$.
- A nonzero coefficient determinant guarantees a unique solution. A zero determinant needs a further consistency check: it can give either no solution or infinitely many solutions.

Step 1: Eliminate two variables using the fixed equations Label the equations in their printed order. Subtract three times the third equation from the first:

$$(3x + y + 4z) - 3(x + 2y + z) = 3 - 12.$$

This simplifies to $-5y + z = -9$, so $z = 5y - 9$. Substituting in the third equation gives

$$x + 2y + 5y - 9 = 4, \qquad x = 13 - 7y.$$

These expressions satisfy the first and third equations for every real y. Therefore the second equation alone now decides whether a common solution exists.

Step 2: Test the remaining equation for a contradiction Substitute the expressions for x and z into $2x + \alpha y - z = -3$:

$$\begin{aligned} 2(13 - 7y) + \alpha y - (5y - 9) &= -3, \\ 35 + (\alpha - 19)y &= -3, \\ (\alpha - 19)y &= -38. \end{aligned}$$

At $\alpha = 19$, this is $0 = -38$, which is impossible. For every other value, $y = -38/(\alpha - 19)$ exists and determines unique values of x and z. In particular, the alternatives 4, 13 and 23 give nonzero coefficients of y, so none makes the system inconsistent. This also excludes the possibility of infinitely many solutions at the singular value.

Step 3: Conclusion The system has no solution exactly when $\boxed{\alpha = 19}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the equations without the parameter to express x, z in terms of y; the remaining equation immediately shows whether its zero coefficient creates a contradiction.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let the maximum value of $\left(\sin^{-1} x\right)^2 + \left(\cos^{-1} x\right)^2$ for $x \in \left[-\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}\right]$ be $\frac{\mathrm{m}}{\mathrm{n}}\pi^2$, where $\gcd(\mathrm{m}, \mathrm{n}) = 1$. Then $\mathrm{m} + \mathrm{n}$ is equal to ____. [4 marks]

Solution:

Concept:

- On the principal branches, $\sin^{-1} x + \cos^{-1} x = \pi/2$ for $-1 \le x \le 1$, and $\sin^{-1} x$ is increasing.
- Completing the square locates the vertex of a quadratic. A positive squared term increases with the distance from that vertex, so the maximum on a closed interval occurs at the farther endpoint.

Step 1: Convert the expression to a quadratic in one angle Set $t = \sin^{-1} x$. The given endpoints are within the principal sine-inverse range, so monotonicity yields

$$-\frac{\pi}{3} \le t \le \frac{\pi}{4}.$$

The other inverse function is $\cos^{-1} x = \pi/2 - t$. Thus the expression becomes

$$\begin{aligned} F(t) &= t^2 + \left(\frac{\pi}{2} - t\right)^2 \\ &= 2t^2 - \pi t + \frac{\pi^2}{4} \\ &= 2\left(t - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{8}. \end{aligned}$$

This rewriting uses principal values; choosing a negative value of $\cos^{-1} x$ would violate its range $[0, \pi]$.

Step 2: Locate the maximum and reduce its coefficient The vertex $t = \pi/4$ is the right endpoint of the interval and gives the minimum. The greatest distance from this vertex occurs at $t = -\pi/3$, corresponding to $x = -\sqrt{3}/2$. At this endpoint the inverse cosine equals $5\pi/6$, and therefore

$$F\left(-\frac{\pi}{3}\right) = \frac{\pi^2}{9} + \frac{25\pi^2}{36} = \frac{29}{36}\pi^2.$$

For comparison, the right endpoint gives only $\pi^2/8$. Since 29 is prime and does not divide 36, the fraction $29/36$ is already in lowest terms. The integers in the question are consequently $m = 29$ and $n = 36$.

Step 3: Conclusion The maximum is $\frac{29}{36}\pi^2$, giving $\boxed{m + n = 65}$.

Quick Tip to Solve: Put $t = \sin^{-1} x$ and complete the square about $\pi/4$; compare distances from the vertex instead of differentiating inverse trigonometric functions.

22. If $\left(\frac{1}{{}^{15}C_0}+\frac{1}{{}^{15}C_1}\right)\left(\frac{1}{{}^{15}C_1}+\frac{1}{{}^{15}C_2}\right)\cdots\left(\frac{1}{{}^{15}C_{12}}+\frac{1}{{}^{15}C_{13}}\right)=\frac{\alpha^{13}}{{}^{14}C_0\,{}^{14}C_1\cdots{}^{14}C_{12}}$**, then** 30α **is equal to ____ .**

[4 marks]

Solution:

Concept:

- The factorial definition is $\binom{n}{r}=n!/[r!(n-r)!]$, for integers $0\le r\le n$. It allows adjacent reciprocal binomial coefficients to be combined.
- In a product of thirteen equal constant factors, that constant appears to the thirteenth power. The indices $r=0,1,\ldots,12$ contain thirteen factors.

Step 1: Simplify a typical adjacent pair Consider the factor indexed by r, where $0\le r\le 12$. Expand each binomial coefficient by factorials and take the common part outside:

$$\begin{aligned}\frac{1}{\binom{15}{r}}+\frac{1}{\binom{15}{r+1}}&=\frac{r!(15-r)!+(r+1)!(14-r)!}{15!}\\&=\frac{r!(14-r)!}{15!}\big((15-r)+(r+1)\big)\\&=\frac{16r!(14-r)!}{15!}.\end{aligned}$$

Since $15!=15\cdot 14!$, this becomes

$$\frac{1}{\binom{15}{r}}+\frac{1}{\binom{15}{r+1}}=\frac{16}{15}\frac{1}{\binom{14}{r}}.$$

The cancellation of r in the bracket is what makes the same constant appear in every factor.

Step 2: Multiply the thirteen identities Apply the identity for every r from 0 through 12. The product is

$$\frac{(16/15)^{13}}{\binom{14}{0}\binom{14}{1}\cdots\binom{14}{12}}.$$

This has exactly the denominator printed on the right-hand side. All its binomial coefficients are positive, so cancelling the common denominator gives $\alpha^{13}=(16/15)^{13}$. Since the exponent is odd, there is a unique real value, $\alpha=16/15$. As a check on the constant, the first factor is $1+1/15=16/15$, in agreement with the formula at $r=0$.

Step 3: Conclusion Thus $30\alpha=30\cdot\dfrac{16}{15}=\boxed{32}$.

Quick Tip to Solve: For adjacent reciprocal binomial coefficients, factor the common factorials first; their leftover numerators add to $n+1$, independently of the index.

23. Let $[\cdot]$ denote the greatest integer function and $f(x) = \lim\limits_{n\to\infty} \frac{1}{n^3} \sum\limits_{k=1}^{n} \left[\frac{k^2}{3^x}\right]$. Then $12 \sum\limits_{j=1}^{\infty} f(j)$ is equal to _____.

[4 marks]

Solution:

Concept:

- For every real u, $u - 1 < [u] \le u$. Thus replacing a greatest integer by its argument introduces an error smaller than one in each term.
- The sum of squares is $\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6$. A geometric series with first term a and ratio r, where $|r| < 1$, has sum $a/(1-r)$.

Step 1: Bound the total greatest-integer error Fix a real number x. Since $3^x > 0$, the greatest-integer inequality gives

$$\frac{k^2}{3^x} - 1 < \left[\frac{k^2}{3^x}\right] \le \frac{k^2}{3^x}.$$

Sum over the n terms and divide by n^3:

$$\frac{1}{3^x n^3} \sum_{k=1}^{n} k^2 - \frac{1}{n^2} < \frac{1}{n^3} \sum_{k=1}^{n} \left[\frac{k^2}{3^x}\right] \le \frac{1}{3^x n^3} \sum_{k=1}^{n} k^2.$$

The difference between the bounding expressions is $1/n^2$, which tends to zero. The floors therefore do not change the limit, even though they cannot be discarded term by term at finite n.

Step 2: Evaluate the limit and the geometric series By the sum-of-squares formula and the squeeze theorem,

$$f(x) = \frac{1}{3^x} \lim_{n\to\infty} \frac{n(n+1)(2n+1)}{6n^3} = \frac{1}{3^{x+1}}.$$

In particular, $f(1) = 1/9$, and successive terms $f(j)$ have ratio $1/3$. Consequently the series converges, with

$$\sum_{j=1}^{\infty} f(j) = \frac{1/9}{1 - 1/3} = \frac{1}{6}.$$

We first evaluated $f(x)$ at fixed x, and then summed its values; no exchange of an infinite series with the defining limit is needed. Starting at $j = 0$ would change the first term and give the wrong sum.

Step 3: Conclusion The requested number is $12 \sum_{j=1}^{\infty} f(j) = 12 \cdot \frac{1}{6} = \boxed{2}$.

Quick Tip to Solve: With n floor terms divided by n^3, their total rounding error is at most $1/n^2$; use this bound before applying the sum-of-squares formula.

24. If P is a point on the circle $x^2 + y^2 = 4, Q$ is a point on the straight line $5x + y + 2 = 0$ and $x - y + 1 = 0$ is the perpendicular bisector of PQ , then 13 times the sum of abscissa of all such points P is ____ . [4 marks]

Solution:

Concept:

- If a line is the perpendicular bisector of PQ, then Q is the reflection of P in that line. Reflection in $Ax + By + C = 0$ maps (a, b) to $(a - 2Ad, b - 2Bd)$, where $d = (Aa + Bb + C)/(A^2 + B^2)$.
- For a quadratic $Au^2 + Bu + C = 0$, the sum of its roots is $-B/A$. Only roots satisfying the original geometric conditions are counted.

Step 1: Reflect the point and impose the line condition Let $P = (a, b)$, so $a^2 + b^2 = 4$. For the reflecting line $x - y + 1 = 0$, the coefficient pair is $(1, -1)$, giving $d = (a - b + 1)/2$. Its reflection is therefore

$$Q = \big(a - (a - b + 1),\ b + (a - b + 1)\big) = (b - 1, a + 1).$$

Since Q lies on $5x + y + 2 = 0$, we obtain

$$5(b - 1) + (a + 1) + 2 = 0, \qquad a + 5b - 2 = 0.$$

Thus the required points P are intersections of the original circle with the reflected line, and $b = (2 - a)/5$.

Step 2: Find and check all admissible abscissae Substitute the expression for b into the circle:

$$a^2 + \frac{(2 - a)^2}{25} = 4,$$
$$26a^2 - 4a - 96 = 0,$$
$$13a^2 - 2a - 48 = 0.$$

This factors as $(a - 2)(13a + 24) = 0$, so the two points are $(2, 0)$ and $(-24/13, 10/13)$. Their values of $a - b + 1$ are 3 and $-21/13$, both nonzero; hence neither point coincides with its reflection. Both give genuine segments with the stated perpendicular bisector. The sum of their abscissae, also given directly by the quadratic root-sum formula, is

$$2 - \frac{24}{13} = \frac{2}{13}.$$

Step 3: Conclusion Thirteen times the sum of all admissible abscissae is $13 \cdot \dfrac{2}{13} = \boxed{2}$.

Quick Tip to Solve: Reflect the line condition onto P, then intersect with the circle; use the root sum of the resulting quadratic instead of finding both points when only their abscissae sum is needed.

25. If $\int_0^1 4\cot^{-1}\left(1-2x+4x^2\right)\mathrm{d}x = \mathrm{a}\tan^{-1}(2) - \mathrm{b}\log_{\mathrm{e}}(5)$**, where** $\mathrm{a},\mathrm{b}\in\mathrm{N}$**, then** $(2\mathrm{a}+\mathrm{b})$ **is equal to ____**
.

[4 marks]

Solution:

Concept:

- For positive z, the principal inverse cotangent is the angle in $(0,\pi/2)$ whose tangent is $1/z$. The tangent subtraction identity can split a quadratic denominator into two inverse tangents.
- An odd integrand integrates to zero over symmetric limits. Also, integration by parts gives $\int \tan^{-1} t\,dt = t\tan^{-1} t - \frac{1}{2}\ln(1+t^2)$.

Step 1: Split the inverse cotangent Let $\alpha = \tan^{-1}(2x)$ and $\beta = \tan^{-1}(2x-1)$. Then

$$\tan(\alpha-\beta) = \frac{2x-(2x-1)}{1+2x(2x-1)} = \frac{1}{1-2x+4x^2}.$$

The denominator is $4(x-1/4)^2+3/4>0$. Since $\alpha>\beta$, their difference lies in $(0,\pi)$; its positive tangent places it in $(0,\pi/2)$. Thus the principal branches give

$$\cot^{-1}(1-2x+4x^2) = \tan^{-1}(2x) - \tan^{-1}(2x-1).$$

Step 2: Exploit symmetry and integrate Write the required integral as

$$I = 4\int_0^1 \tan^{-1}(2x)\,dx - 4\int_0^1 \tan^{-1}(2x-1)\,dx.$$

For the second integral, $t = 2x-1$ changes the limits to $-1, 1$, so it equals $\frac{1}{2}\int_{-1}^{1}\tan^{-1} t\,dt = 0$, because inverse tangent is odd. Integration by parts evaluates the first integral:

$$\begin{aligned} I &= 4\left[x\tan^{-1}(2x) - \frac{1}{4}\ln(1+4x^2)\right]_0^1 \\ &= 4\tan^{-1}(2) - \ln 5. \end{aligned}$$

The lower endpoint contributes zero to both terms.

Step 3: Identify the requested coefficients Comparing the evaluated expression with $a\tan^{-1}(2) - b\log_e 5$ gives the natural-number coefficients $a=4$ and $b=1$. Substitute them into the requested combination $2a+b$; the logarithm already uses the natural base, so no conversion factor is needed.

Step 4: Conclusion $a=4$, $b=1$, and hence $\boxed{2a+b=9}$.

Quick Tip to Solve: Look for consecutive linear arguments: the product $(2x)(2x-1)$ supplies the quadratic in the inverse-tangent subtraction formula.

26. Consider two identical metallic spheres of radius R each having charge Q and mass m. Their centers have an initial separation of $4R$. Both the spheres are given an initial speed of u towards each other. The minimum value of u, so that they can just touch each other is:

(Take $k = \frac{1}{4\pi\epsilon_0}$ and assume $kQ^2 > Gm^2$ where G is the Gravitational constant) [4 marks]

(A) $\sqrt{\frac{kQ^2}{4mR}\left(1+\frac{Gm^2}{kQ^2}\right)}$

(B) $\sqrt{\frac{kQ^2}{4mR}\left(1-\frac{Gm^2}{kQ^2}\right)}$

(C) $\sqrt{\frac{kQ^2}{2mR}\left(1-\frac{Gm^2}{kQ^2}\right)}$

(D) $\sqrt{\frac{kQ^2}{2mR}\left(1-\frac{Gm^2}{2kQ^2}\right)}$

Correct Answer: (B) $\sqrt{\frac{kQ^2}{4mR}\left(1-\frac{Gm^2}{kQ^2}\right)}$

Solution:

Concept:

- For the inverse-square interaction used here, the combined potential energy at centre separation r is $U(r) = (kQ^2 - Gm^2)/r$; electric repulsion is positive and gravitational attraction is negative.
- At the minimum approach speed, initial kinetic energy exactly equals the increase in potential energy up to contact. Both spheres contribute to the initial kinetic energy.

Step 1: Identify the endpoint separations and energies The centres are initially separated by $r_i = 4R$. The spheres touch when their surfaces meet, which means $r_f = 2R$, not R. Their equal and opposite initial momenta make the centre of mass stationary. The total initial kinetic energy is therefore

$$K_i = \frac{1}{2}mu^2 + \frac{1}{2}mu^2 = mu^2.$$

Since $kQ^2 > Gm^2$, the combined interaction is repulsive and its potential energy increases as the centres approach. At the threshold for touching, both spheres come instantaneously to rest at contact.

Step 2: Apply energy conservation With $K_f = 0$, conservation gives

$$\begin{aligned} mu^2 + \frac{kQ^2 - Gm^2}{4R} &= \frac{kQ^2 - Gm^2}{2R}, \\ mu^2 &= \frac{kQ^2 - Gm^2}{4R}, \\ u^2 &= \frac{kQ^2}{4mR}\left(1 - \frac{Gm^2}{kQ^2}\right). \end{aligned}$$

The stated inequality makes the expression positive. A smaller speed leads to a turning point before contact, while a larger speed leaves kinetic energy at contact.

Step 3: Check the alternatives Option (A) adds the gravitational contribution, although gravity assists

the approach and must reduce the required speed. Option (C) doubles the required value of u^2, effectively losing the kinetic-energy contribution of one sphere. Option (D) uses both an incorrect overall factor and an incorrect relative gravitational term. Option (B) has the energy balance and also reduces correctly to $\sqrt{kQ^2/(4mR)}$ when gravity is neglected.

Step 4: Conclusion The printed data give $mu^2 = (kQ^2 - Gm^2)(1/(2R) - 1/(4R))$, so

$$\boxed{u_{\min} = \sqrt{\frac{kQ^2}{4mR}\left(1 - \frac{Gm^2}{kQ^2}\right)}}.$$

This expression is option (B) in the printed order. Hence, the correct option is (B).

Quick Tip to Solve: Use centre separations $4R$ and $2R$, and add the kinetic energies of both moving spheres before solving for their common speed.

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27. Surface tension of two liquids (having same densities), T_1 and T_2, are measured using capillary rise method utilizing two tubes with inner radii of r_1 and r_2 where $r_1 > r_2$. The measured liquid heights in these tubes are h_1 and h_2 respectively. [Ignore the weight of the liquid above the lowest point of miniscus]. The heights h_1 and h_2 and surface tensions T_1 and T_2 satisfy the relation : [4 marks]

(A) $h_1 = h_2$ and $T_1 = T_2$

(B) $h_1 > h_2$ and $T_1 < T_2$

(C) $h_1 > h_2$ and $T_1 = T_2$

(D) $h_1 < h_2$ and $T_1 = T_2$

Correct Answer: (D) $h_1 < h_2$ and $T_1 = T_2$

Solution:

Concept:

- Balancing the upward surface-tension force with the liquid-column weight gives $2\pi r T\cos\theta = \pi r^2 h\rho g$, hence $h = 2T\cos\theta/(\rho g r)$. Here θ is the contact angle.
- For the usual complete-wetting capillary-rise comparison, $\theta = 0$. With equal liquid densities, $h \propto T/r$; at equal surface tension, the narrower tube has the greater rise.

Step 1: Form the ratio of the two rises Ignoring the liquid above the lowest point of the meniscus, as requested, the supported liquid column has volume $\pi r^2 h$. Applying the capillary-rise formula to each tube in the usual wetting case gives

$$h_1 = \frac{2T_1}{\rho g r_1}, \qquad h_2 = \frac{2T_2}{\rho g r_2}.$$

Dividing eliminates the common density and gravitational acceleration:

$$\frac{h_1}{h_2} = \frac{T_1}{T_2}\frac{r_2}{r_1}.$$

Since $r_1 > r_2$, the radius ratio is strictly less than one.

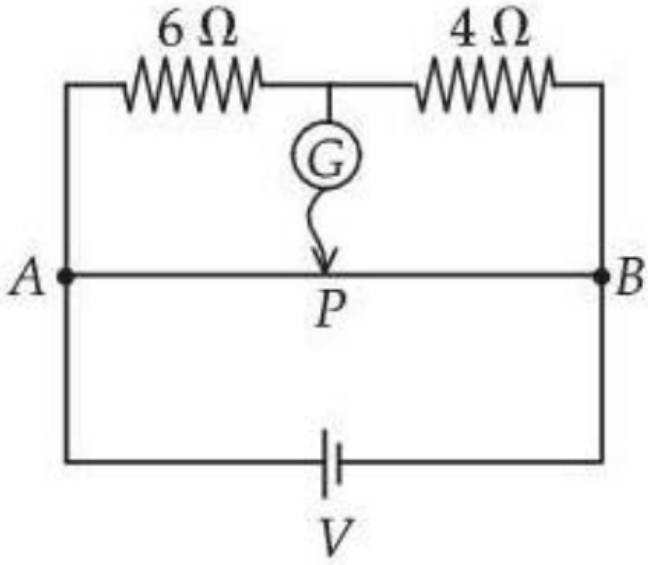
Step 2: Test every proposed pair of relations For $T_1 = T_2$, the ratio becomes $h_1/h_2 = r_2/r_1 < 1$. Thus the equal-surface-tension pairs in (A), with equal heights, and (C), with $h_1 > h_2$, fail. For option (B), $T_1 < T_2$ makes both factors in the ratio less than one, so it also requires $h_1 < h_2$, contrary to its stated height relation. Option (D) is the consistent pair: equal surface tensions and the larger rise in the smaller-radius tube.

Step 3: Distinguish compatibility from an extra deduction Equal density alone does not imply equal surface tension. The question is answered by testing the joint relations supplied in its choices: under the standard capillary-rise comparison, only one of those pairs obeys the force balance. The relation $T_1 = T_2$ is part of that compatible option, rather than a consequence of equal density by itself.

Step 4: Conclusion For equal surface tensions, $h_1/h_2 = r_2/r_1 < 1$; thus the compatible pair is $\boxed{h_1 < h_2 \text{ and } T_1 = T_2}$. This is option (D) in the printed order. Hence, the correct option is (D).

Quick Tip to Solve: Check each entire option using $h_1/h_2 = (T_1/T_2)(r_2/r_1)$; do not infer equal surface tension merely from equal density.

28. The total length of potentiometer wire AB is 50 cm in the arrangement as shown in the figure. If *P* is the point where the galvanometer shows zero reading then the length AP is _____ cm. [4 marks]



(A) 30

(B) 20

(C) 15

(D) 25

Correct Answer: (A) 30

Solution:

Concept:

- At galvanometer null, no current flows through its branch; the resistor junction and the contact point P are at equal potentials.
- A uniform potentiometer wire has resistance proportional to length and a constant potential gradient. Series resistors divide voltage in the ratio of their resistances.

Step 1: Find the junction potential relative to A Let C denote the junction of the $6\,\Omega$ and $4\,\Omega$ resistors. Both the top series branch and the wire AB are connected across the same cell. If the terminal potential difference is V, the current in the top branch at null is

$$I = \frac{V}{6+4} = \frac{V}{10}.$$

No galvanometer current is drawn, so the two resistors carry this same current. The potential drop from A to C, across the $6\,\Omega$ resistor, is

$$V_{AC} = 6I = \frac{3V}{5}.$$

Step 2: Translate the null condition into length At balance, $V_C = V_P$, hence $V_{AP} = V_{AC}$. Over the uniform 50 cm wire,

$$\frac{V_{AP}}{V_{AB}} = \frac{AP}{AB} = \frac{AP}{50\,\text{cm}}.$$

Therefore

$$\frac{AP}{50\,\text{cm}} = \frac{3}{5}, \qquad AP = 30\,\text{cm}.$$

Equivalently, the balanced-bridge condition is $AP/PB = 6/4 = 3/2$, together with $AP + PB = 50$ cm. The supply voltage cancels, which is why its numerical value is unnecessary.

Step 3: Check the possible lengths For $AP = 30$ cm, the remaining segment is $PB = 20$ cm, and their ratio is the required $3:2$. The 20 cm choice confuses AP with PB; 25 cm would require equal resistors; and 15 cm gives $AP/PB = 3/7$, which cannot balance the $6\,\Omega : 4\,\Omega$ ratio.

Step 4: Conclusion $\boxed{AP = 30\,\text{cm}}$. Hence, the correct option is (A).

Quick Tip to Solve: At null, match lengths to the resistors beside the same ends: $AP : PB = 6 : 4$, then divide the total 50 cm in that ratio.

29. Keeping the significant figures in view, the sum of the physical quantities 52.01 m, 153.2 m and 0.123 m is : [4 marks]

(A) 205 m

(B) 205.3 m

(C) 205.33 m

(D) 205.333 m

Correct Answer: (B) 205.3 m

Solution:

Concept:

- For addition or subtraction of measured quantities, retain the decimal place of the least precise measurement, rather than the smallest number of significant figures.
- Carry extra digits through the arithmetic and round only the final result. All three quantities must first be expressed in the same unit.

Step 1: Identify the limiting decimal place The measurement $52.01\,\text{m}$ is reported to the hundredths place, $153.2\,\text{m}$ to the tenths place, and $0.123\,\text{m}$ to the thousandths place. The least precise measurement is $153.2\,\text{m}$, whose final reported digit lies at $0.1\,\text{m}$. Thus the sum must be rounded to one digit after the decimal point. The fact that 0.123 has fewer significant figures does not set the precision of an addition; that counting rule belongs to multiplication and division.

Step 2: Add first and round once All values are already in metres, so no unit conversion is needed. Keeping their digits during addition,

$$\begin{aligned} S &= (52.010 + 153.200 + 0.123)\,\text{m} \\ &= 205.333\,\text{m}. \end{aligned}$$

The zeros added for alignment do not change the stated precision of the original measurements. To report to tenths, inspect the hundredths digit, which is $3 < 5$. Consequently, the tenths digit remains 3, giving $205.3\,\text{m}$.

Step 3: Compare the reported precisions Option (A), $205\,\text{m}$, discards a justified tenths-place digit and is too coarsely rounded. Option (C), $205.33\,\text{m}$, keeps the hundredths place despite the limiting tenths-place input. Option (D), $205.333\,\text{m}$, is the unrounded arithmetic sum but retains still more unsupported precision. Option (B) alone preserves exactly the decimal place justified by all the measurements.

Step 4: Conclusion The properly rounded sum is $\boxed{205.3\,\text{m}}$. Hence, the correct option is (B).

Quick Tip to Solve: Circle the tenths place in $153.2\,\text{m}$; add all three values with guard digits, then round the total to that same place.

30. Two cars A and B each of mass 10^3 kg are moving on parallel tracks separated by a distance of 10 m, in same direction with speeds 72 km/h and 36 km/h. The magnitude of angular momentum of car A with respect to car B is _____ J·s. [4 marks]

(A) 10^5

(B) 3×10^5

(C) 3.6×10^5

(D) 2×10^5

Correct Answer: (A) 10^5

Solution:

Concept:

- With respect to car B, use the relative velocity $\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$. Since B moves uniformly, its translating frame is inertial.
- The angular momentum of A in that frame is $\vec{L} = \vec{r}_{A/B} \times m_A \vec{v}_{A/B}$. Its magnitude is momentum multiplied by the perpendicular distance from the reference point to the line of relative motion.

Step 1: Convert the speeds and find relative motion Both cars move in the same direction along parallel tracks, so their speeds must be subtracted. Converting first to SI units,

$$v_A = 72 \times \frac{5}{18} = 20\,\text{m s}^{-1}, \qquad v_B = 36 \times \frac{5}{18} = 10\,\text{m s}^{-1}.$$

Therefore A moves at $v_{A/B} = 10\,\text{m s}^{-1}$ relative to B. Its relative momentum has magnitude

$$p_{A/B} = m_A v_{A/B} = 10^3 \times 10 = 10^4\,\text{kg m s}^{-1}.$$

Step 2: Use the perpendicular separation The track separation is the perpendicular distance $d = 10\,\text{m}$ between B and A's line of relative motion. Any instantaneous separation along the tracks is parallel to the momentum and contributes nothing to the cross product. Hence

$$L = dp_{A/B} = 10 \times 10^4 = 10^5\,\text{kg m}^2\text{s}^{-1}.$$

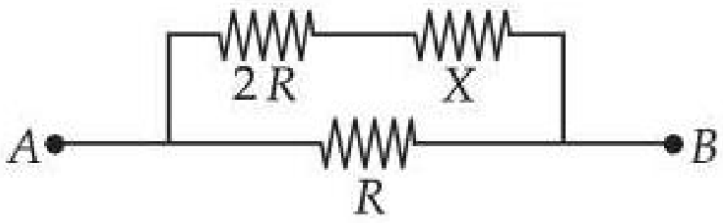
Since $1\,\text{J} = 1\,\text{kg m}^2\text{s}^{-2}$, the angular-momentum unit equals J·s. The direction would be perpendicular to the plane of the tracks, but only its magnitude is requested.

Step 3: Reject the alternative values Option (B), 3×10^5, would follow from adding the speeds as though the cars moved oppositely. Option (D), 2×10^5, incorrectly uses A's ground-frame speed $20\,\text{m s}^{-1}$. Option (C), 3.6×10^5, uses the relative numerical speed 36 in kilometres per hour without converting it to metres per second. Only option (A) uses both the correct relative speed and SI units.

Step 4: Conclusion $L = m(v_A - v_B)d = 10^3(20 - 10)10 = \boxed{10^5\,\text{J}\cdot\text{s}}$. This is option (A) in the printed order. Hence, the correct option is (A).

Quick Tip to Solve: For parallel tracks, compute $L = md|v_A - v_B|$; convert speeds to metres per second before inserting the $10\,\text{m}$ separation.

31. Two known resistances of $R\ \Omega$ and $2R\ \Omega$ and one unknown resistance $X\ \Omega$ are connected in a circuit as shown in the figure. If the equivalent resistance between points A and B in the circuit is $X\ \Omega$, then the value of X is _____ Ω. [4 marks]



(A) R

(B) $(\sqrt{3}+1)R$

(C) $(\sqrt{3}-1)R$

(D) $2(\sqrt{3}-1)R$

Correct Answer: (C) $(\sqrt{3}-1)R$

Solution:

Concept:

- Resistors connected end to end with no junction between them are in series: their resistances add.
- Parallel branches have the same potential difference. For branch resistances R_1, R_2, $R_{\text{eq}} = R_1R_2/(R_1+R_2)$. All resistances here are positive.

Step 1: Identify the two branches The upper path from A to B contains $2R$ and X in series, so its resistance is $2R+X$. The lower path contains only R. These two paths connect the same pair of junctions and are therefore parallel, rather than all three resistors being in series.

Step 2: Apply the stated equivalent resistance The equivalent resistance is required to equal the unknown resistor itself. Thus

$$X = \frac{R(2R+X)}{3R+X}.$$

Multiplying by the positive denominator and collecting terms gives

$$3RX + X^2 = 2R^2 + RX,$$
$$X^2 + 2RX - 2R^2 = 0.$$

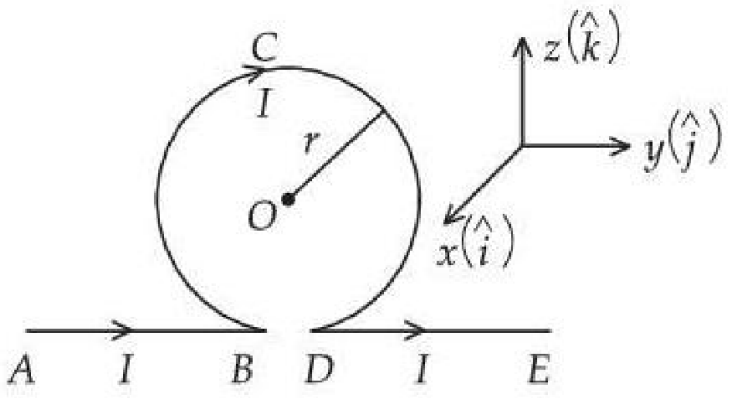
Dividing by R^2 and putting $x = X/R$ reduces the equation to $x^2 + 2x - 2 = 0$.

Step 3: Select the physically admissible root The quadratic formula gives $x = -1 \pm \sqrt{3}$. The root $-1-\sqrt{3}$ is negative and cannot represent a passive resistance. Also a parallel combination with a branch R must be less than R. The positive root $\sqrt{3}-1 \simeq 0.732$ meets this check. Option (A) equals R, while (B) and (D) exceed R; each fails the parallel-resistance bound.

Step 4: Conclusion $X = (\sqrt{3}-1)R\,\Omega$. Hence, the correct option is (C).

Quick Tip to Solve: Before solving the quadratic, note that the equivalent resistance must be below R; this rejects every option except the positive value $(\sqrt{3}-1)R$.

32. An infinitely long straight wire carrying current I is bent in a planar shape as shown in the diagram. The radius of the circular part is r. The magnetic field at the centre O of the circular loop is : [4 marks]



(A) $-\frac{\mu_0}{2\pi}\frac{I}{r}(\pi+1)\hat{i}$
(B) $-\frac{\mu_0}{2\pi}\frac{I}{r}(\pi-1)\hat{i}$
(C) $\frac{\mu_0}{2\pi}\frac{I}{r}(\pi-1)\hat{i}$
(D) $\frac{\mu_0}{2\pi}\frac{I}{r}(\pi+1)\hat{i}$

Correct Answer: (B) $-\frac{\mu_0}{2\pi}\frac{I}{r}(\pi-1)\hat{i}$

Solution:

Concept:

- The field at the centre of a circular loop of radius r carrying current I has magnitude $\mu_0 I/(2r)$; its direction follows the right-hand rule.
- A semi-infinite straight wire at perpendicular distance r produces magnitude $\mu_0 I/(4\pi r)$. Magnetic fields add as vectors.

Step 1: Find the circular contribution The diagram lies in the yz-plane, with $+\hat{i}$ directed out of it. The arrow at the top of the loop points to the right, so the loop current is clockwise when viewed in the diagram. Its field is into the plane, along $-\hat{i}$. Therefore

$$\vec{B}_{\text{loop}} = -\frac{\mu_0 I}{2r}\hat{i} = -\frac{\mu_0 I}{2\pi r}\pi\hat{i}.$$

Step 2: Find the straight-wire contribution The two straight portions each extend from the tangent point to infinity. They carry current to the right, along $+\hat{j}$, and O is above them, in the $+\hat{k}$ direction. Since $\hat{j}\times\hat{k}=\hat{i}$, both produce an outward field. Their contributions add to

$$\vec{B}_{\text{straight}} = 2\frac{\mu_0 I}{4\pi r}\hat{i} = \frac{\mu_0 I}{2\pi r}\hat{i}.$$

Step 3: Superpose and compare the options The loop contribution is π times the straight contribution and opposes it, so the resultant points along $-\hat{i}$. Explicitly,

$$\vec{B} = \frac{\mu_0 I}{2\pi r}(-\pi+1)\hat{i}.$$

Option (A) adds magnitudes even though the fields oppose. Options (C) and (D) have an outward resultant, contradicting the stronger clockwise-loop contribution. Only the negative difference has both the correct magnitude and direction.

Step 4: Conclusion $\vec{B} = -\frac{\mu_0}{2\pi}\frac{I}{r}(\pi - 1)\hat{i}$. Hence, the correct option is (B).

Quick Tip to Solve: Mark the loop and tangent-wire field directions separately: the clockwise loop dominates, and the two straight halves together subtract one unit from its π units.

33. The energy of an electron in an orbit of the Bohr's atom is $-0.04E_0$ eV where E_0 is the ground state energy. If L is the angular momentum of the electron in this orbit and h is the Planck's constant, then

$\frac{2\pi L}{h}$ **is _____ :** [4 marks]

(A) 5

(B) 4

(C) 2

(D) 6

Correct Answer: (A) 5

Solution:

Concept:

- For a Bohr orbit of principal quantum number n, the bound-state energy scales as $E_n = -E_0/n^2$, where E_0 denotes the positive magnitude of the ground-state energy in the given expression.
- Bohr's angular-momentum quantisation is $L = nh/(2\pi)$, with h Planck's constant. Thus the dimensionless ratio $2\pi L/h$ is exactly n.

Step 1: Determine the energy ratio The stated electron energy is negative, as required for a bound orbit. Comparing the given value $-0.04E_0$ with the Bohr energy expression gives

$$-\frac{E_0}{n^2} = -0.04E_0 \quad \Longrightarrow \quad \frac{1}{n^2} = 0.04 = \frac{1}{25}.$$

The energy unit and the common ground-state energy scale cancel in this ratio. Hence $n^2 = 25$. Since principal quantum numbers are positive integers, the allowed root is $n = 5$, rather than the algebraic root -5.

Step 2: Translate to angular momentum Substitute the orbit number into the quantisation condition:

$$L = \frac{5h}{2\pi}, \qquad \frac{2\pi L}{h} = 5.$$

No numerical value of Planck's constant or of the ground-state binding energy is necessary. The requested ratio counts the number of reduced-Planck-constant units in the orbital angular momentum.

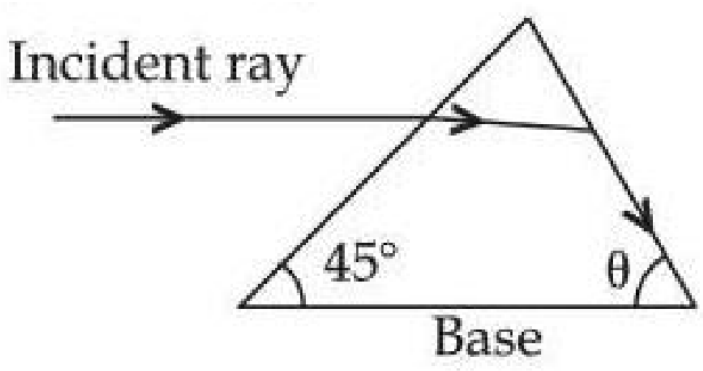
Step 3: Check the remaining possibilities Option (B), corresponding to $n = 4$, would give energy $-E_0/16 = -0.0625E_0$. Option (C), $n = 2$, gives $-0.25E_0$, and option (D), $n = 6$, gives $-E_0/36$. None

equals the specified $-0.04E_0$. The energy fraction therefore uniquely selects the fifth orbit.

Step 4: Conclusion $\frac{2\pi L}{h} = 5$. The printed energy fraction $0.04 = 1/25$ proves $n = 5$; the alternative value 2 would instead give $-0.25E_0$. Hence, the correct option is (A).

Quick Tip to Solve: Recognise $0.04 = 1/25$: Bohr energy varies as $1/n^2$, whereas angular momentum varies as n, so take the square root of 25.

34. As shown in the diagram, when the incident ray is parallel to base of the prism, the emergent ray grazes along the second surface.
If refractive index of the material of prism is $\sqrt{2}$, the angle θ of prism is. [4 marks]



(A) 45°

(B) 60°

(C) 75°

(D) 90°

Correct Answer: (B) 60°

Solution:

Concept:

- Snell's law at an air-prism boundary is $\sin i = \mu \sin r_1$, where i, r_1 are angles to the surface normal and μ is the prism's refractive index relative to air.
- For a prism of apex angle A, the internal refraction angles obey $r_1 + r_2 = A$. Grazing emergence means the internal incidence at the second face is the critical angle, with $\sin r_2 = 1/\mu$.

Step 1: Find refraction at the first face The left face is inclined at 45° to the horizontal base. Since the incident ray is parallel to the base, its acute angle with that face is 45°; the angle to the normal is therefore also $i = 45^\circ$. Snell's law gives

$$\sin r_1 = \frac{\sin 45^\circ}{\sqrt{2}} = \frac{1}{2}, \qquad r_1 = 30^\circ.$$

The ray bends towards the normal as it enters the denser prism.

Step 2: Use the condition for grazing emergence The emergent ray runs along the second face, so its

refracted angle to the normal is 90°. Applying Snell's law at that face,

$$\sqrt{2}\sin r_2 = \sin 90^\circ = 1, \qquad r_2 = 45^\circ.$$

Consequently the apex angle is $A = r_1 + r_2 = 75^\circ$. This angle is at the top vertex and must not be confused with the angle θ marked at the lower right.

Step 3: Use the triangle angle sum The three interior prism angles are 45°, A, and θ. Hence

$$\theta = 180^\circ - 45^\circ - 75^\circ = 60^\circ.$$

Option (A) confuses the requested angle with the critical angle or the known left base angle. Option (C) is the apex angle, not θ. Option (D) is the grazing refraction angle measured from the normal, not an angle of this prism.

Step 4: Conclusion The marked prism angle is $\theta = 60^\circ$. Hence, the correct option is (B).

Quick Tip to Solve: Grazing emergence fixes the second internal angle at 45°; add it to the first refraction angle 30°, then use the triangle sum for the marked base angle.

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35. Given below are two statements :

Statement I : In a Young's double slit experiment, the angular separation of fringes will increase as the screen is moved away from the plane of the slits

Statement II : In a Young's double slit experiment, the angular separation of fringes will increase when monochromatic source is replaced by another monochromatic source of higher wavelength

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Correct Answer: (D) **Statement I** is false but **Statement II** is true

Solution:

Concept:

- For slit separation d, wavelength λ, and a distant screen at distance D, the linear fringe width is $\beta = \lambda D/d$.
- The angular fringe separation in the usual small-angle region is $\Delta\theta = \beta/D = \lambda/d$. It depends on wavelength and slit separation, but not on screen distance.

Step 1: Evaluate Statement I Moving the screen away increases D, so the spacing β measured in metres on the screen increases. However, the question asks about angular separation, measured at

the slits. Dividing the linear separation by the screen distance gives

$$\Delta\theta = \frac{\lambda D/d}{D} = \frac{\lambda}{d}.$$

The factor D cancels, so Statement I is false. More generally, the bright directions are fixed by $d\sin\theta_m = m\lambda$; changing where the screen intercepts those directions does not change their angles.

Step 2: Evaluate Statement II Replace the source by one of wavelength $\lambda' > \lambda$, keeping the slit separation fixed. Then

$$\frac{\Delta\theta'}{\Delta\theta} = \frac{\lambda'/d}{\lambda/d} = \frac{\lambda'}{\lambda} > 1.$$

The angular separation increases, so Statement II is true. Monochromatic light still supplies a single wavelength for the fringe pattern; it is the increase of that wavelength that widens its angular scale.

Step 3: Match the two truth values The established pair is false for Statement I and true for Statement II. Option (A) incorrectly makes the first statement true. Option (B) incorrectly makes the wavelength statement false. Option (C) reverses both truth values. Option (D) alone matches the independent tests of screen distance and wavelength.

Step 4: Conclusion Statement I is false and Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: Keep linear and angular fringe spacings distinct: divide $\beta = \lambda D/d$ by D before deciding whether moving the screen changes the answer.

36. The kinetic energy of a simple harmonic oscillator is oscillating with angular frequency of 176 rad/s. The frequency of this simple harmonic oscillator is _____ Hz. [take $\pi = \frac{22}{7}$] [4 marks]

(A) 176

(B) 28

(C) 14

(D) 88

Correct Answer: (C) 14

Solution:

Concept:

- For displacement $x = A\cos(\omega t + \phi)$, velocity is $v = -A\omega\sin(\omega t + \phi)$, where ω is the oscillator's angular frequency.
- Kinetic energy is $K = mv^2/2$. Squaring the sinusoidal velocity makes the kinetic-energy oscillation have angular frequency 2ω, while the displacement frequency in hertz is $f = \omega/(2\pi)$.

Step 1: Relate energy frequency to motion frequency Substitute the SHM velocity into the kinetic-

energy formula and use the double-angle identity:

$$K = \frac{1}{2}mA^2\omega^2 \sin^2(\omega t + \phi)$$
$$= \frac{1}{4}mA^2\omega^2[1 - \cos(2\omega t + 2\phi)].$$

The constant term does not oscillate. The cosine term has angular frequency 2ω. Physically the particle passes the mean position twice per displacement cycle, giving two kinetic-energy maxima in that cycle.

Step 2: Find the oscillator angular frequency The stated value $176\,\text{rad s}^{-1}$ belongs to kinetic energy, so

$$2\omega = 176\,\text{rad s}^{-1}, \qquad \omega = 88\,\text{rad s}^{-1}.$$

This halving is essential before converting from radians per second to cycles per second.

Step 3: Convert to hertz and compare options Using the stipulated $\pi = 22/7$,

$$f = \frac{88}{2\pi} = \frac{88 \times 7}{44} = 14\,\text{Hz}.$$

Option (A) merely repeats the supplied energy angular frequency. Option (B) converts that energy angular frequency to hertz without halving it. Option (D) halves the supplied value but leaves it in radians per second rather than converting it to hertz. Only option (C) accounts for both distinctions.

Step 4: Conclusion The simple harmonic oscillator has frequency $f = 14\,\text{Hz}$. Hence, the correct option is (C).

Quick Tip to Solve: For a kinetic-energy angular frequency Ω_K, combine both conversions at once: the displacement frequency is $f = \Omega_K/(4\pi)$.

37. A body of mass 2 kg is moving along *x*-direction such that its displacement as function of time is given by $x(t) = \alpha t^2 + \beta t + \gamma$ m, where $\alpha = 1\ m/s^2$, $\beta = 1\ m/s$ and $\gamma = 1\ m$. The work done on the body during the time interval $t = 2\ s$ to $t = 3\ s$, is _____ J. [4 marks]

(A) 24

(B) 49

(C) 12

(D) 42

Correct Answer: (A) 24

Solution:

Concept:

- The net work done on a body equals its change in kinetic energy: $W = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$.
- For one-dimensional motion, velocity and acceleration follow from displacement: $v = dx/dt$ and $a = dv/dt$.

Step 1: Obtain the endpoint velocities Differentiate the given position function before substituting the times. Since $x(t) = \alpha t^2 + \beta t + \gamma$,

$$v(t) = 2\alpha t + \beta = 2t + 1\,\text{m s}^{-1}.$$

Thus $v_i = v(2) = 5\,\text{m s}^{-1}$ and $v_f = v(3) = 7\,\text{m s}^{-1}$. The constant γ fixes the position origin and has no effect on either velocity or kinetic energy.

Step 2: Apply the work-energy theorem With $m = 2\,\text{kg}$, the initial and final kinetic energies are $K_i = 25\,\text{J}$ and $K_f = 49\,\text{J}$. Therefore

$$W = \frac{1}{2}(2)(7^2 - 5^2) = 49 - 25 = 24\,\text{J}.$$

This is positive because the body speeds up during the interval. The quantity required is the change in energy, not the final kinetic energy alone.

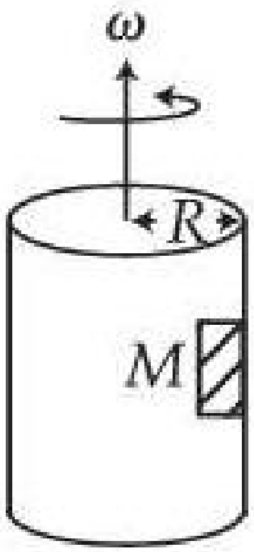
Step 3: Check using force and displacement The acceleration is constant, $a = 2\alpha = 2\,\text{m s}^{-2}$, so the net force is $F = ma = 4\,\text{N}$. Also $x(2) = 7\,\text{m}$ and $x(3) = 13\,\text{m}$, giving $W = F\Delta x = 4(6) = 24\,\text{J}$. Option (B), 49, is just the final kinetic energy; option (C), 12, misses the factor of two arising from differentiating t^2; option (D), 42, equals neither the force-displacement product nor the kinetic-energy change.

Step 4: Conclusion The work done is $\boxed{24\,\text{J}}$. Hence, the correct option is (A).

Quick Tip to Solve: Differentiate the position first, then subtract the two endpoint kinetic energies; the constant position term cancels automatically.

38. A large drum having radius R is spinning around its axis with angular velocity ω, as shown in figure.

The minimum value of ω so that a body of mass M remains stuck to the inner wall of the drum, taking the coefficient of friction between the drum surface and mass M as μ, is : [4 marks]



(A) $\sqrt{\frac{g}{2\mu R}}$

(B) $\sqrt{\frac{g}{\mu R}}$

(C) $\sqrt{\frac{2g}{\mu R}}$

(D) $\sqrt{\frac{\mu g}{R}}$

Correct Answer: (B) $\sqrt{\frac{g}{\mu R}}$

Solution:

Concept:

- A body moving in a horizontal circle of radius R with angular speed ω requires inward force $M\omega^2 R$.
- Static friction adjusts to prevent sliding, but cannot exceed $f_{\max} = \mu N$, where N is the normal reaction from the wall.

Step 1: Resolve the radial and vertical forces The drum rotates about a vertical axis. Its wall exerts a horizontal normal reaction towards the axis, while gravity acts vertically downwards. The body tends to slip down the wall, so static friction acts upwards. The normal reaction supplies the centripetal force:

$$N = M\omega^2 R.$$

Gravity does not supply the centripetal force because it is perpendicular to the horizontal radius.

Step 2: Impose the condition for no vertical slipping Since the body remains at a fixed height, vertical

equilibrium requires $f = Mg$. This is possible only if the required friction is at most its limiting value:

$$Mg \le \mu N = \mu M\omega^2 R.$$

At the minimum angular speed the inequality becomes equality. Cancelling the mass gives $\omega_{\min}^2 = g/(\mu R)$. A larger speed increases the normal reaction and hence the available friction; the actual friction still equals the weight.

Step 3: Compare the alternatives For option (A), $\omega^2 = g/(2\mu R)$, the maximum friction is only $Mg/2$, insufficient to support the body. Option (C) gives $2Mg$ as the maximum friction, so it can hold the body but is not the minimum. Option (D) places μ in the numerator and incorrectly predicts that reducing friction reduces the required speed. The correct dependence is $\omega_{\min} \propto \mu^{-1/2}$; the mass cancels because both the weight and the normal force are proportional to it.

Step 4: Conclusion Since $\mu M\omega_{\min}^2 R = Mg$, $\boxed{\omega_{\min} = \sqrt{\dfrac{g}{\mu R}}}$. This force balance establishes the correct option as (B) in the official printed order.

Quick Tip to Solve: Use the wall's normal force for centripetal motion, then set limiting upward friction equal to weight at the threshold speed.

39. A capacitor C is first charged fully with potential difference of V_0 and disconnected from the battery. The charged capacitor is connected across an inductor having inductance L. In t s, 25% of the initial energy in the capacitor is transferred to the inductor. The value of t is _____ s. [4 marks]

(A) $\pi\sqrt{\dfrac{LC}{2}}$

(B) $\dfrac{\pi\sqrt{LC}}{2}$

(C) $\dfrac{\pi\sqrt{LC}}{3}$

(D) $\dfrac{\pi\sqrt{LC}}{6}$

Correct Answer: (D) $\dfrac{\pi\sqrt{LC}}{6}$

Solution:

Concept:

- In an ideal LC circuit, the angular frequency is $\omega_0 = 1/\sqrt{LC}$; energy oscillates between the capacitor's electric field and the inductor's magnetic field.
- If the capacitor is initially fully charged and the current is zero, $q(t) = CV_0\cos(\omega_0 t)$, so the magnetic energy fraction is $U_L/U_0 = \sin^2(\omega_0 t)$.

Step 1: Express the transferred energy Initially the total stored energy is $U_0 = \frac{1}{2}CV_0^2$, entirely in the capacitor. At time t,

$$U_C = \frac{q^2}{2C} = U_0\cos^2(\omega_0 t).$$

Conservation of energy gives $U_L = U_0 - U_C = U_0 \sin^2(\omega_0 t)$. The fraction transferred is an energy fraction, so it depends on the square of the sine, not the sine itself.

Step 2: Find the first time for one-quarter energy transfer The stated transfer is $25\% = 1/4$. During the first transfer from the capacitor to the inductor, $0 \le \omega_0 t \le \pi/2$, and therefore

$$\sin^2(\omega_0 t) = \frac{1}{4}, \qquad \sin(\omega_0 t) = \frac{1}{2}, \qquad \omega_0 t = \frac{\pi}{6}.$$

Substituting $\omega_0 = 1/\sqrt{LC}$ determines the first such instant. The energy fraction recurs in later oscillations, but the options identify the first transfer time.

Step 3: Check the printed times Option (A) has phase $\pi/\sqrt{2}$, whose sine squared is approximately 0.63, not 0.25. Option (B) has phase $\pi/2$ and transfers all the energy; option (C) has phase $\pi/3$ and transfers $3/4$. Option (D) has phase $\pi/6$, producing precisely $1/4$. Equivalently, the oscillation period is $T = 2\pi\sqrt{LC}$, and the required instant is $T/12$.

Step 4: Conclusion $\boxed{t = \frac{\pi\sqrt{LC}}{6}\ \text{s}}$, when L and C are in SI units. Hence, the correct option is (D).

Quick Tip to Solve: For an initially charged capacitor, set the transferred energy fraction equal to $\sin^2(t/\sqrt{LC})$, not to $\sin(t/\sqrt{LC})$.

40. A spherical body of radius r and density σ falls freely through a viscous liquid having density ρ and viscosity η and attains a terminal velocity v_0. Estimated maximum error in the quantity η is : (Ignore errors associated with σ, ρ and g, gravitational acceleration) [4 marks]

(A) $2\left[\frac{\Delta r}{r} + \frac{\Delta v_0}{v_0}\right]$

(B) $2\left[\frac{\Delta r}{r} - \frac{\Delta v_0}{v_0}\right]$

(C) $\frac{2\Delta r}{r} - \frac{\Delta v_0}{v_0}$

(D) $\frac{2\Delta r}{r} + \frac{\Delta v_0}{v_0}$

Correct Answer: (D) $\frac{2\Delta r}{r} + \frac{\Delta v_0}{v_0}$

Solution:

Concept:

- For slow viscous flow around a sphere, Stokes' drag is $F_d = 6\pi\eta r v_0$. At terminal speed the drag equals the weight minus the buoyant force.
- For a product of measured powers, the estimated maximum fractional error is the sum of absolute powers times their fractional errors. A denominator contributes positively to the maximum error.

Step 1: Obtain the dependence of viscosity on measured quantities At terminal speed, the viscous

drag balances the weight minus the buoyant force:

$$6\pi\eta r v_0 = \frac{4}{3}\pi r^3(\sigma-\rho)g, \qquad \eta = \frac{2(\sigma-\rho)g}{9}\frac{r^2}{v_0}.$$

Ignore uncertainties in σ, ρ and g, so their combined factor is exact.

Step 2: Propagate the uncertainties Logarithmic differentiation gives the signed relation $d\eta/\eta = 2\,dr/r - dv_0/v_0$. For the estimated maximum error, the errors can reinforce each other: overestimating r and underestimating v_0 both overestimate η. Hence their magnitudes must be added, giving

$$\left(\frac{\Delta\eta}{\eta}\right)_{\max} = 2\frac{\Delta r}{r} + \frac{\Delta v_0}{v_0}.$$

The options are dimensionless, so they represent fractional error; the absolute error would be this expression multiplied by η.

Step 3: Eliminate the other expressions Option (A) incorrectly doubles the velocity contribution even though v_0 has exponent -1. Option (B) both doubles that contribution and subtracts its magnitude. Option (C) is the signed differential pattern, not the maximum uncertainty. Option (D) alone uses the absolute exponents 2 and 1 with positive contributions.

Step 4: Conclusion Since $\eta \propto r^2 v_0^{-1}$, adding the absolute fractional-error contributions gives $\boxed{2\frac{\Delta r}{r} + \frac{\Delta v_0}{v_0}}$. This propagation of errors establishes the correct option as (D) in the official printed order.

Quick Tip to Solve: First isolate $\eta \propto r^2/v_0$; maximum fractional errors add with coefficients equal to the absolute exponents.

41. A river of width 200 m is flowing from west to east with a speed of 18 km/h. A boat, moving with speed of 36 km/h in still water, is made to travel one-round trip (bank to bank of the river). Minimum time taken by the boat for this journey and also the displacement along the river bank are _____ and _____ respectively. [4 marks]

(A) 20 s and 100 m

(B) 40 s and 200 m

(C) 40 s and 0 m

(D) 40 s and 100 m

Correct Answer: (B) 40 s and 200 m

Solution:

Concept:

- The boat's velocity relative to the ground is its velocity relative to the water plus the river velocity. The river flows parallel to the banks and supplies no cross-river component.
- The minimum time to reach the opposite bank is obtained by aiming perpendicular to the banks relative to the water, thereby maximizing the cross-river speed.

Step 1: Convert speeds and choose the fastest heading The river speed is $u = 18 \times 5/18 = 5\,\text{m s}^{-1}$, and the boat's still-water speed is $v = 36 \times 5/18 = 10\,\text{m s}^{-1}$. If its heading makes angle θ with the normal to a bank, its crossing component is $v\cos\theta \le v$. The shortest crossing therefore uses $\theta = 0$, with the boat aimed directly across the river in the water frame.

Step 2: Calculate the outward and return crossing times The width of each crossing is $d = 200\,\text{m}$. With the maximum transverse speed $10\,\text{m s}^{-1}$,

$$t_{\text{out}} = \frac{200}{10} = 20\,\text{s}, \qquad t_{\text{back}} = 20\,\text{s}, \qquad t_{\min} = 40\,\text{s}.$$

On the return trip the boat reverses its cross-river heading, while the river continues to carry it eastwards. Reaching the original bank does not require reaching the original point on that bank.

Step 3: Accumulate downstream drift on both crossings At the minimum-time heading the boat has no along-bank velocity relative to the water. Its eastward velocity relative to the ground is therefore u throughout both legs:

$$\Delta x = u(t_{\text{out}} + t_{\text{back}}) = 5(40) = 200\,\text{m}.$$

Option (A) gives only one crossing's time and drift. Option (C) incorrectly cancels the river's drift on the return leg; achieving zero drift would require an upstream heading and a longer crossing time. Option (D) counts the drift of just one leg with the time for two. The transverse displacement cancels, leaving only the eastward displacement.

Step 4: Conclusion The minimum round-trip time is $\boxed{40\,\text{s}}$ and the displacement along the bank is $\boxed{200\,\text{m}}$ eastwards. Hence, the correct option is (B).

Quick Tip to Solve: For the fastest bank-to-bank round trip, use $2d/v$ for time and multiply that entire time by the river speed for the drift.

42. The r.m.s. speed of oxygen molecules at 47 °C is equal to that of the hydrogen molecules kept at _____ °C. (Mass of oxygen molecule/mass of hydrogen molecule = 32/2) [4 marks]

(A) -20

(B) -253

(C) -100

(D) -235

Correct Answer: (B) -253

Solution:

Concept:

- The root-mean-square speed of gas molecules is $v_{\text{rms}} = \sqrt{3k_BT/m}$, where T is absolute temperature, m is molecular mass and k_B is Boltzmann's constant.
- Equal rms speeds imply equal ratios T/m. Temperatures must be expressed in kelvin before taking such ratios.

Step 1: Express equality of the molecular speeds Let T_{O} and T_{H} be the absolute temperatures of

oxygen and hydrogen, with molecular masses m_O and m_H. Equating their squared rms speeds cancels the common factor $3k_B$:

$$\frac{T_O}{m_O} = \frac{T_H}{m_H}, \qquad T_H = T_O \frac{m_H}{m_O}.$$

The supplied mass ratio is $m_O/m_H = 32/2 = 16$, so the lighter hydrogen must be colder by a factor of sixteen in absolute temperature.

Step 2: Substitute the oxygen temperature Convert the oxygen temperature to kelvin using the conventional integer conversion appropriate to the options:

$$T_O = 47 + 273 = 320\,\text{K}, \qquad T_H = 320\left(\frac{2}{32}\right) = 20\,\text{K}.$$

Converting this back to the requested Celsius scale gives $\theta_H = 20 - 273 = -253\,^\circ\text{C}$. Using 273.15 instead changes the result by only about $0.14\,^\circ\text{C}$, leaving the same listed answer.

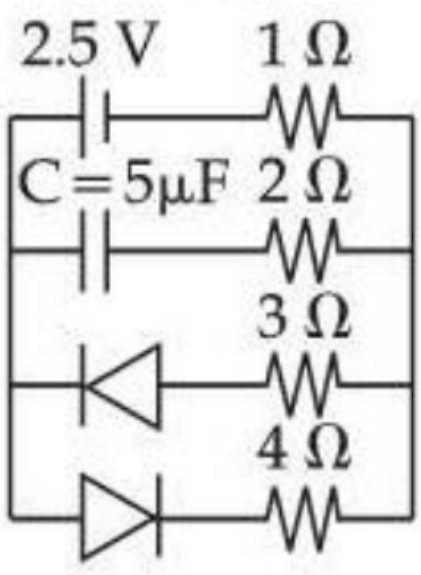
Step 3: Check all candidate temperatures Equal speeds require $T_H/m_H = 320/32 = 10$ in a common mass unit. Option (A), $-20\,^\circ\text{C}$, gives $253/2 = 126.5$; option (C), $-100\,^\circ\text{C}$, gives $173/2 = 86.5$; option (D), $-235\,^\circ\text{C}$, gives $38/2 = 19$. None equals 10. Option (B) gives $20/2 = 10$, exactly as required with the stated conversion.

Step 4: Conclusion The required hydrogen temperature is $\boxed{-253\,^\circ\text{C}}$. Hence, the correct option is (B).

Quick Tip to Solve: For equal rms speeds, scale the kelvin temperature by the molecular-mass ratio first; convert to Celsius only at the end.

43. The charge stored by the capacitor C in the given circuit in the steady state is _____ μC.

[4 marks]



(A) 5

(B) 10

(C) 7.5

(D) 12.5

Correct Answer: (B) 10

Solution:

Concept:

- In a steady DC circuit, a capacitor carries no current and its charge magnitude is $Q = CV_C$, where V_C is its terminal voltage.
- An ideal forward-biased diode acts as a conducting wire; a reverse-biased diode blocks current. A resistor in a branch carrying zero current has zero potential drop.

Step 1: Identify the conducting branch The longer plate of the $2.5\,\text{V}$ cell is on the left, so the left rail is at higher potential. The diode in the $4\,\Omega$ branch conducts from left to right; the oppositely oriented diode in the $3\,\Omega$ branch is reverse biased. The capacitor branch is open at steady state. Thus only the $1\,\Omega$ and $4\,\Omega$ resistors carry the loop current.

Step 2: Calculate the voltage across the rails The two conducting resistors are in series with the cell. Kirchhoff's loop law gives

$$I = \frac{2.5}{1+4} = 0.5\,\text{A}, \qquad V_{LR} = I(4) = 2\,\text{V}.$$

The $1\,\Omega$ resistor drops $0.5\,\text{V}$, leaving $2\,\text{V}$ between the left and right rails. Since the capacitor branch has no current, the $2\,\Omega$ resistor has no voltage drop, and the capacitor is also across this rail voltage.

Step 3: Use the capacitance and check the alternatives The capacitance is $5\,\mu\text{F}$, so

$$Q = (5\,\mu\text{F})(2\,\text{V}) = 10\,\mu\text{C}.$$

The values 5 and $7.5\,\mu\text{C}$ would require capacitor voltages of 1 and $1.5\,\text{V}$, inconsistent with the conducting loop. The value $12.5\,\mu\text{C}$ incorrectly puts the full cell emf across the capacitor and ignores the drop in the $1\,\Omega$ resistor.

Step 4: Conclusion The conducting branch gives $V_C = 2.5(4)/(1+4) = 2\,\text{V}$, hence $Q = 5(2) = 10\,\mu\text{C}$. This is option (B) in the printed order. Hence, the correct option is (B).

Quick Tip to Solve: First open the capacitor branch at steady state, choose the forward-biased diode, and find the rail voltage before using $Q = CV$.

44. A battery with EMF E and internal resistance r is connected across a resistance R. The power consumption in R will be maximum when: [4 marks]

(A) $R = \frac{r}{2}$

(B) $R = r$

(C) $R = 2r$

(D) $R = \sqrt{2}\,r$

Correct Answer: (B) $R = r$

Solution:

Concept:

- For emf E, internal resistance r, and external resistance R, the current is $I = E/(R + r)$.
- The power dissipated in the external resistor alone is $P = I^2R = E^2R/(R + r)^2$. The emf and internal resistance remain fixed while R is varied.

Step 1: Express the load power The same current passes through the external and internal resistances, but the required power is that in R only. Substituting the circuit current gives

$$P(R) = \left(\frac{E}{R + r}\right)^2 R = \frac{E^2R}{(R + r)^2}.$$

Increasing R raises the resistance factor in I^2R, but simultaneously reduces the current, so the competition must be included.

Step 2: Find and classify the stationary point Differentiation with respect to the variable load gives

$$\frac{dP}{dR} = \frac{E^2(r - R)}{(R + r)^3}.$$

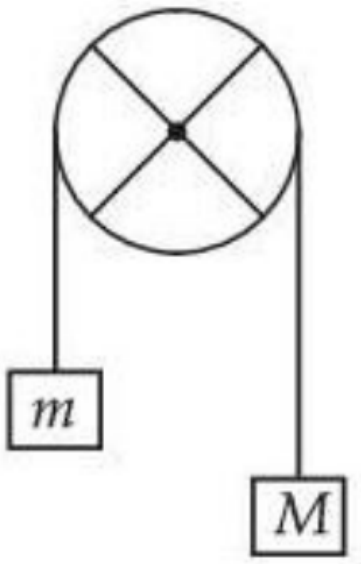
For positive resistances, the derivative is positive when $R < r$, zero at $R = r$, and negative when $R > r$. Thus this stationary point is a maximum. Also P tends to zero as R tends to either zero or infinity, confirming the maximum is global.

Step 3: Compare the proposed load values Writing $x = R/r$ gives $P = (E^2/r)x/(1 + x)^2$. At $x = 1$, the factor is $1/4$. At $x = 1/2$ and $x = 2$, it is $2/9 < 1/4$. At $x = \sqrt{2}$, it is also smaller because $(1 + x)^2 - 4x = (x - 1)^2 > 0$. Hence each of the other three options dissipates less power. The maximum power itself is $E^2/(4r)$.

Step 4: Conclusion Since $dP/dR = E^2(r - R)/(R + r)^3$, the load power is maximum at $R = r$, which is option (B) in the printed order. Hence, the correct option is (B).

Quick Tip to Solve: For a fixed real battery, maximize the load power $E^2R/(R + r)^2$; equal load and internal resistances give its maximum.

45. The pulley shown in figure is made using a thin rim and two rods of length equal to diameter of the rim. The rim and each rod have a mass of M. Two blocks of mass of M and m are attached to two ends of a light string passing over the pulley, which is hinged to rotate freely in vertical plane about its center. The magnitudes of the acceleration experienced by the blocks is _____ (assume no slipping of string on pulley). [4 marks]



(A) $\frac{(M-m)g}{\left[\left(\frac{13}{6}\right)M+m\right]}$

(B) $\frac{(M-m)g}{\left[\left(\frac{8}{3}\right)M+m\right]}$

(C) $\frac{(M-m)g}{M+m}$

(D) $\frac{(M-m)g}{2M+m}$

Correct Answer: (B) $\frac{(M-m)g}{\left[\left(\frac{8}{3}\right)M+m\right]}$

Solution:

Concept:

- For a thin rim of radius R, $I_{\text{rim}} = MR^2$. A thin rod of length $2R$, about its centre perpendicular to its length, has $I_{\text{rod}} = M(2R)^2/12$.
- No slipping gives $\alpha = a/R$. The net torque of the two string tensions is $(T_M - T_m)R = I\alpha$.

Step 1: Add the moments of inertia The rim and each of the two rods have mass M; hence there are three separate contributions. Both rods rotate about their own midpoint, which is the pulley centre. Therefore

$$I = MR^2 + 2\left(\frac{M(2R)^2}{12}\right) = \frac{5}{3}MR^2.$$

The total pulley mass is $3M$, but its moment of inertia must be obtained from its actual mass distribution, not from a solid-disc formula.

Step 2: Write the equations for translation and rotation Take the block of mass M to move down and that of mass m to move up, as implied by the option numerators. If their common acceleration is a, Newton's second law gives

$$Mg - T_M = Ma, \qquad T_m - mg = ma.$$

The tensions are unequal because their difference accelerates the pulley. With no slipping,

$$T_M - T_m = \frac{I}{R^2}a = \frac{5}{3}Ma.$$

Step 3: Eliminate the tensions and check the options Adding the block equations and substituting the tension difference gives

$$(M-m)g = \left(M + m + \frac{5}{3}M\right)a, \qquad a = \frac{(M-m)g}{\frac{8}{3}M + m}.$$

The coefficient $13/6$ uses an incorrect rotational inertia, 2 neglects the rods' contribution, and the denominator $M + m$ ignores pulley inertia altogether. The radius cancels as expected. If the relative masses are reversed, the motion reverses and the magnitude uses $|M - m|$ in the numerator.

Step 4: Conclusion Using $I/R^2 = 5M/3$, the acceleration magnitude for $M > m$ is $a = (M - m)g/[(8/3)M + m]$. This is option (B) in the printed order. Hence, the correct option is (B).

Quick Tip to Solve: For an Atwood machine with a massive pulley, add I/R^2 to the two hanging masses in the acceleration denominator.

Physics – Section B: Numerical Value Questions (20 marks)

46. An electromagnetic wave of frequency 100 MHz propagates through a medium of conductivity, $\sigma = 10\,\text{mho/m}$. The ratio of maximum conduction current density to maximum displacement current density is ____.

$$\left[\textbf{Take } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9\,\text{Nm}^2/\text{C}^2\right]$$

[4 marks]

Solution:

Concept:

- In a conducting medium, the conduction-current density is $J_c = \sigma E$, where σ is conductivity.
- For a sinusoidal field with angular frequency ω, the displacement-current density is $J_d = \epsilon_0\,\partial E/\partial t$, using the permittivity specified by the supplied constant.

Step 1: Write the peak values of both currents Let the electric field at a fixed point be $E = E_0 \cos \omega t$. Its derivative has amplitude ωE_0, so

$$J_{c,\max} = \sigma E_0, \qquad J_{d,\max} = \epsilon_0 \omega E_0.$$

The peaks occur at different instants, but the question asks for the ratio of their maximum magnitudes. The common field amplitude cancels; no knowledge of E_0 is needed.

Step 2: Convert frequency and evaluate the permittivity factor The frequency is $f = 100\,\text{MHz} = 10^8\,\text{Hz}$, and angular frequency is $\omega = 2\pi f$. The supplied electrostatic constant gives

$$\epsilon_0 = \frac{1}{36\pi \times 10^9}\,\text{F m}^{-1}, \qquad \epsilon_0\omega = \frac{2\pi \times 10^8}{36\pi \times 10^9} = \frac{1}{180}\,\text{S m}^{-1}.$$

One mho is one siemens, so the conductivity is $10\,\text{S m}^{-1}$.

Step 3: Take the ratio and check its dimensions Using the peak amplitudes,

$$\frac{J_{c,\max}}{J_{d,\max}} = \frac{\sigma}{\epsilon_0\omega} = \frac{10}{1/180} = 1800.$$

Both the numerator and denominator have conductivity units, so their ratio is dimensionless. The conduction-current amplitude is much larger than the displacement-current amplitude. Keeping 2π in the conversion from frequency to angular frequency is essential; it cancels the π in the supplied constant.

Step 4: Conclusion The required ratio of maximum current densities is $\boxed{1800}$, a dimensionless number.

Quick Tip to Solve: Use $J_{c,\max}/J_{d,\max} = \sigma/(2\pi f\epsilon_0)$; substitute the supplied $1/(4\pi\epsilon_0)$ so the factors of π cancel.

---

47. A diatomic gas $(\gamma = 1.4)$ does 100 J of work when it is expanded isobarically. Then the heat given to the gas ____ J. [4 marks]

Solution:

Concept:

- For an ideal gas expanding at constant pressure, the work done by the gas is $W = P\Delta V = nR\Delta T$.
- The heat supplied at constant pressure is $Q = nC_P\Delta T$. The heat-capacity identities are $C_P - C_V = R$ and $\gamma = C_P/C_V$.

Step 1: Relate heat capacities to the given ratio Using $C_P = \gamma C_V$ in Mayer's relation gives

$$(\gamma - 1)C_V = R, \qquad C_V = \frac{R}{\gamma - 1}, \qquad C_P = \frac{\gamma R}{\gamma - 1}.$$

For $\gamma = 1.4$, these give $C_V = 2.5R$ and $C_P = 3.5R$. These are the heat capacities consistent with the supplied diatomic-gas value.

Step 2: Express the heat in terms of the work Because the process is isobaric, the ideal-gas equation implies $P\Delta V = nR\Delta T$. Eliminating the unknown number of moles and temperature change between this expression and the heat equation yields

$$\frac{Q}{W} = \frac{C_P}{R} = \frac{\gamma}{\gamma - 1}.$$

Consequently,

$$Q = \frac{1.4}{1.4-1}(100\,\text{J}) = 3.5(100\,\text{J}) = 350\,\text{J}.$$

Step 3: Check the energy balance and sign The gas expands and does positive work, so its temperature increases during the constant-pressure process. Its internal-energy increase is

$$\Delta U = nC_V\Delta T = \frac{W}{\gamma-1} = 250\,\text{J}.$$

With work done by the gas taken as positive, the first law is $Q = \Delta U + W$. Thus $250 + 100 = 350\,\text{J}$, confirming both the magnitude and the positive sign of the supplied heat. Heat cannot equal the work here because some energy raises the gas temperature.

Step 4: Conclusion The heat given to the gas is $\boxed{350\,\text{J}}$; the numerical entry is 350.

Quick Tip to Solve: For an isobaric ideal-gas process, multiply the work by $\gamma/(\gamma-1)$; here the multiplier is 3.5.

---

48. In a Young's double slit experiment set up, the two slits are kept 0.4 mm apart and screen is placed at 1 m from slits. If a thin transparent sheet of thickness 20μ m is introduced in front of one of the slits then center bright fringe shifts by 20 mm on the screen.

The refractive index of transparent sheet is given by $\frac{\alpha}{10}$, where α is ____. [4 marks]

Solution:

Concept:

- A transparent sheet of thickness t and refractive index μ adds optical path $(\mu-1)t$ relative to the same thickness of air.
- In Young's experiment, geometric path difference at displacement x is approximately dx/D, where d is slit separation and D is screen distance. The central bright fringe shifts by $x = D(\mu-1)t/d$.

Step 1: Apply the central-fringe condition Before insertion, the central bright fringe has zero net path difference. The sheet delays light through one slit, and the central maximum moves until the geometric path difference compensates this delay. Equating magnitudes gives

$$\frac{dx}{D} = (\mu-1)t, \qquad \mu - 1 = \frac{xd}{Dt}.$$

The wavelength cancels because the displaced central maximum still corresponds to zero net path difference. The shift is toward the slit in front of which the sheet is placed.

Step 2: Convert all lengths and substitute Use metres consistently:

$$d = 0.4\times10^{-3}\,\text{m}, \quad D = 1\,\text{m}, \quad t = 20\times10^{-6}\,\text{m}, \quad x = 20\times10^{-3}\,\text{m}.$$

Hence

$$\mu - 1 = \frac{(20\times10^{-3})(0.4\times10^{-3})}{(1)(20\times10^{-6})} = 0.4, \qquad \mu = 1.4.$$

The ratio is dimensionless, as required for a refractive index. Including the initial 1 is necessary

because the added optical path depends on $\mu - 1$, not on μ.

Step 3: Extract the requested parameter and check the shift The question writes the refractive index as $\mu = \alpha/10$. Therefore $\alpha = 10(1.4) = 14$. As a check, the added path is $(1.4 - 1)(20\,\mu\text{m}) = 8\,\mu\text{m}$; multiplying by $D/d = 2500$ gives $20\,\text{mm}$, exactly the stated displacement. The small-angle approximation is consistent with $x/D = 0.02$.

Step 4: Conclusion The refractive index is 1.4, so the required dimensionless value is $\boxed{\alpha = 14}$.

Quick Tip to Solve: Find $\mu - 1 = xd/(Dt)$, add one to obtain μ, and only then multiply by ten to obtain α.

---

49. A particle having electric charge 3×10^{-19}C and mass 6×10^{-27} kg is accelerated by applying an electric potential of 1.21 V .
Wavelength of the matter wave associated with the particle is $\alpha \times 10^{-12}$ m. The value of α is ____ .
(Take Planck's constant $= 6.6 \times 10^{-34}$ J.s) [4 marks]

Solution:

Concept:

- A particle of charge magnitude q, accelerated from rest through a potential difference V, acquires kinetic energy $K = qV$.
- The de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and $p = \sqrt{2mK}$ is the nonrelativistic momentum of a particle of mass m.

Step 1: Calculate the kinetic energy gained Use the given charge of the particle, rather than substituting the elementary charge. The electrical work becomes its kinetic energy:

$$K = qV = (3 \times 10^{-19})(1.21) = 3.63 \times 10^{-19}\,\text{J}.$$

All the supplied quantities are already in SI units. Since the voltage is an accelerating potential, the energy gain is positive.

Step 2: Find the particle's momentum From $K = p^2/(2m)$,

$$\begin{aligned} p &= \sqrt{2(6 \times 10^{-27})(3.63 \times 10^{-19})} \\ &= \sqrt{4.356 \times 10^{-45}} \\ &= \sqrt{43.56 \times 10^{-46}} = 6.6 \times 10^{-23}\,\text{kg m s}^{-1}. \end{aligned}$$

Writing the exponent as an even number before taking the square root avoids a power-of-ten error. The corresponding speed is $p/m = 1.1 \times 10^4\,\text{m s}^{-1}$, so the nonrelativistic relation is appropriate.

Step 3: Express the wavelength in the requested form Substitute the stated value of Planck's constant:

$$\lambda = \frac{6.6 \times 10^{-34}}{6.6 \times 10^{-23}} = 10^{-11}\,\text{m} = 10 \times 10^{-12}\,\text{m}.$$

The question asks for the coefficient of 10^{-12} m, not the wavelength written in metres or its power of ten.

Step 4: Conclusion The matter wavelength is 10×10^{-12} m. Therefore, $\boxed{\alpha = 10}$.

Quick Tip to Solve: Use $\lambda = h/\sqrt{2mqV}$ with the particle's actual charge, then rewrite the result using exactly the power of ten requested.

50. The terminal velocity of a metallic ball of radius 6 mm in a viscous fluid is 20 cm/s. The terminal velocity of another ball of same material and having radius 3 mm in the same fluid will be ____ cm/s. [4 marks]

Solution:

Concept:

- Stokes' drag on a sphere of radius r moving slowly through a fluid of viscosity η is $F_d = 6\pi\eta r v$.
- At terminal speed the weight balances buoyancy plus viscous drag. For sphere density ρ_s and fluid density ρ_f, $v_t = 2r^2(\rho_s - \rho_f)g/(9\eta)$.

Step 1: Relate terminal speed to the sphere's radius The weight and the buoyant force both depend on the sphere's volume. Their difference is the effective downward force. At terminal velocity the acceleration vanishes, giving

$$\frac{4}{3}\pi r^3(\rho_s - \rho_f)g = 6\pi\eta r v_t.$$

Solving for v_t gives

$$v_t = \frac{2(\rho_s - \rho_f)g}{9\eta}\, r^2.$$

Thus terminal velocity is proportional to the square of the radius, not to the radius or to the volume alone.

Step 2: Cancel the common material and fluid factors Both balls are made of the same metal and move in the same fluid. Their density difference, fluid viscosity and gravitational acceleration are therefore identical. Taking the ratio removes all these unknown quantities:

$$\frac{v_{t2}}{v_{t1}} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{3\,\text{mm}}{6\,\text{mm}}\right)^2 = \frac{1}{4}.$$

The radii can remain in millimetres because their units cancel in the ratio; no conversion of either radius is required.

Step 3: Apply the speed ratio The first ball's terminal speed is $20\,\text{cm s}^{-1}$, so

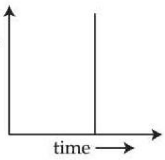
$$v_{t2} = 20 \times \frac{1}{4} = 5\,\text{cm s}^{-1}.$$

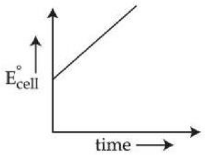
Halving the radius reduces terminal speed by a factor of four. This direction of change is consistent with a smaller sphere having less effective weight relative to its viscous resistance.

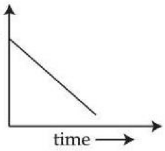
Step 4: Conclusion The second ball has terminal velocity $\boxed{5\,\text{cm s}^{-1}}$. The required numerical entry is 5.

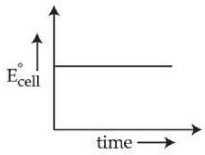
Quick Tip to Solve: For spheres of the same material in the same viscous fluid, square the radius ratio: half the radius gives one-quarter of the terminal speed.

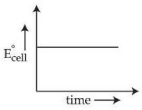
51. For a closed circuit Daniell cell, which of the following plots is the accurate one at a given temperature? [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- The standard cell potential E°_{cell} is fixed for a specified cell reaction at a specified temperature; it is the difference of the standard reduction potentials of the cathode and anode.
- The actual cell emf obeys $E_{\text{cell}} = E^\circ_{\text{cell}} - (RT/nF)\ln Q$, where Q is the reaction quotient and n is the number of electrons transferred.

Step 1: Identify the plotted potential Each vertical axis is labelled E°_{cell}, with a superscript circle. Thus the graph concerns the standard cell potential, rather than the instantaneous potential under changing concentrations. For the Daniell cell the overall reaction is

$$\text{Zn}\,(\text{s}) + \text{Cu}^{2+}(\text{aq}) \longrightarrow \text{Zn}^{2+}(\text{aq}) + \text{Cu}\,(\text{s}).$$

At the fixed temperature in the question, the standard electrode potentials of this cell do not change with elapsed time.

Step 2: Distinguish cell discharge from a change in standard potential Closing the circuit allows zinc to dissolve and copper ions to be reduced. This changes the ionic activities, so the actual emf can decrease according to

$$E_{\text{cell}} = E^\circ_{\text{cell}} - \frac{RT}{2F}\ln\left(\frac{a_{\text{Zn}^{2+}}}{a_{\text{Cu}^{2+}}}\right).$$

Here a denotes ionic activity; pure solid activities are unity. The changing activity ratio affects the second term, not the standard potential itself. Therefore the closed-circuit wording does not imply that the plotted standard potential falls with time.

Step 3: Compare the four graphs A time-independent E°_{cell} requires a horizontal line, as shown in (D). The vertical line in (A) is not a single-valued constant-potential history. The rising line in (B) and falling line in (C) both incorrectly make the standard potential vary with time at fixed temperature. In particular, (C) confuses the actual emf during discharge with the standard emf identified on the axis.

Step 4: Conclusion E°_{cell} remains constant at the given temperature, so the horizontal plot is correct. Hence, the correct option is (D).

Quick Tip to Solve: Check for the superscript circle on a potential axis: changing concentrations alter E, while E° stays fixed at a specified temperature.

52. Given below are four compounds :

(a) n-propyl chloride (b) iso-propyl chloride

(c) sec-butyl chloride (d) neo-pentyl chloride

Percentage of carbon in the one which exhibits optical isomerism is : [4 marks]

(A) 52

(B) 56

(C) 46

(D) 40

Correct Answer: (A) 52

Solution:

Concept:

- For these acyclic alkyl chlorides, a tetrahedral carbon bonded to four different groups is a stereogenic centre and permits a pair of optical isomers.
- Carbon mass percentage is $100 \times (\text{mass of carbon per mole})/(\text{molar mass of compound})$. Use atomic masses C = 12, H = 1 and Cl = 35.5.

Step 1: Examine the carbon environments n-Propyl chloride, $CH_3CH_2CH_2Cl$, has no carbon bearing four different groups: the carbon bonded to chlorine has two hydrogen atoms. In iso-propyl chloride, $CH_3CHClCH_3$, that carbon bears two identical methyl groups, so it is also not a stereogenic centre. In sec-butyl chloride, $CH_3CHClCH_2CH_3$, the carbon bonded to chlorine bears H, Cl, a methyl group and an ethyl group. All four groups differ, so this compound exhibits optical isomerism. Neo-pentyl chloride, $(CH_3)_3CCH_2Cl$, has two hydrogen atoms on the carbon bonded to chlorine and three identical methyl groups on the adjacent carbon; neither is stereogenic.

Step 2: Determine the molecular formula and molar mass The selected compound is sec-butyl chloride, or 2-chlorobutane. It contains four carbons, nine hydrogens and one chlorine atom, giving molecular formula C_4H_9Cl.

$$M = 4(12) + 9(1) + 35.5 = 92.5\,\text{g mol}^{-1}.$$

The mass of carbon in one mole is $4(12) = 48\,\text{g}$. Chlorine must be included in the denominator even though only carbon contributes to the numerator.

Step 3: Calculate and compare the percentages

$$\%\ \text{carbon} = \frac{48}{92.5} \times 100 = 51.89\% \simeq 52\%.$$

This agrees with (A). The values 56, 46 and 40 in (B), (C) and (D) do not equal the carbon percentage of the optically active compound. Selecting a formula before identifying the stereogenic centre would mix up the different carbon counts in the list.

Step 4: Conclusion Sec-butyl chloride exhibits optical isomerism and contains approximately $\boxed{52\%}$ carbon. Hence, the correct option is (A).

Quick Tip to Solve: First find the carbon with H, Cl, methyl and ethyl as four different groups; only then calculate the carbon percentage of C_4H_9Cl.

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53. Given below are two statements :

Statement I : Crystal Field Stabilization Energy (CFSE) of $[Cr(H_2O)_6]^{2+}$ is greater than that of $[Mn(H_2O)_6]^{2+}$.

Statement II : Potassium ferricyanide has a greater spin-only magnetic moment than sodium ferrocyanide.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Correct Answer: (A) Both **Statement I** and **Statement II** are true

Solution:

Concept:

- In an octahedral field, each t_{2g} electron contributes $-0.4\Delta_o$ and each e_g electron contributes $+0.6\Delta_o$. The stabilisation magnitude is the energy lowering relative to the unsplit barycentre.
- Water is a weak-field ligand and cyanide a strong-field ligand. The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_B$, where n is the number of unpaired electrons.

Step 1: Evaluate Statement I using the aqua complexes Water is neutral, so the metal oxidation state in each complex is $+2$. Thus Cr^{2+} is d^4 and Mn^{2+} is d^5. Both aqua complexes are high spin:

$$d^4 : t_{2g}^3 e_g^1, \quad E_{CF} = 3(-0.4\Delta_o) + 0.6\Delta_o = -0.6\Delta_o,$$
$$d^5 : t_{2g}^3 e_g^2, \quad E_{CF} = 3(-0.4\Delta_o) + 2(0.6\Delta_o) = 0.$$

The chromium complex has a nonzero stabilisation of magnitude $0.6\Delta_o$, whereas the manganese complex has zero crystal-field stabilisation. In the usual stabilisation-magnitude sense of CFSE, Statement I is true.

Step 2: Evaluate Statement II using the cyanide complexes Potassium ferricyanide is $K_3[Fe(CN)_6]$, containing Fe^{3+} with d^5 configuration. Strong-field cyanide gives low-spin t_{2g}^5, with one unpaired electron. Sodium ferrocyanide is $Na_4[Fe(CN)_6]$, containing Fe^{2+} with d^6 configuration; its low-spin

t_{2g}^6 configuration has no unpaired electrons.

$$\mu_{\text{ferricyanide}} = \sqrt{1(1+2)}\,\mu_B = \sqrt{3}\,\mu_B, \qquad \mu_{\text{ferrocyanide}} = 0.$$

The potassium and sodium ions do not supply unpaired electrons. Therefore ferricyanide has the greater spin-only magnetic moment, and Statement II is true.

Step 3: Match the truth values with the options Both statements are true, which is the combination in (A). Option (B) incorrectly rejects both statements, (C) incorrectly rejects the magnetic-moment comparison, and (D) incorrectly rejects the crystal-field stabilisation comparison.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: Aqua complexes here are high spin; cyanide complexes are low spin. Determine oxidation state and d-electron count before calculating CFSE or magnetic moment.

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54. The correct statements are :

A. Activation energy for enzyme catalysed hydrolysis of sucrose is lower than that of acid catalysed hydrolysis.

B. During denaturation, secondary and tertiary structures of a protein are destroyed but primary structure remains intact.

C. Nucleotides are joined together by glycosidic linkage between C_1 and C_4 carbons of the pentose sugar.

D. Quaternary structure of proteins represents overall folding of the polypeptide chain.

Choose the correct answer from the options given below : [4 marks]

(A) A and B Only

(B) A, B and D Only

(C) B and C Only

(D) A, C and D Only

Correct Answer: (A) A and B Only

Solution:

Concept:

- Enzymes accelerate specific reactions by providing a pathway with a lower activation-energy barrier.
- Primary protein structure is the amino-acid sequence; secondary and tertiary structures describe its local and overall folding. Quaternary structure describes the arrangement of multiple polypeptide subunits.
- Adjacent nucleotides in nucleic acids are connected through $3'$-$5'$ phosphodiester bonds in the sugar-phosphate backbone.

Step 1: Check the enzyme and denaturation statements Statement A is true. Enzyme-catalysed sucrose hydrolysis has a lower activation energy than the acid-catalysed reaction, explaining the

greater catalytic efficiency of the enzyme. Statement B is true. Denaturation disrupts secondary and tertiary structure and causes loss of biological activity. It does not ordinarily break peptide bonds, so the primary amino-acid sequence remains intact.

Step 2: Check the nucleotide-linkage statement Statement C is false. Successive nucleotides are joined by phosphodiester linkages involving the $3'$-hydroxyl of one pentose and the $5'$-phosphate of the next nucleotide. They are not joined by a glycosidic bond between the C_1 and C_4 atoms of pentose sugars. A glycosidic linkage instead joins the base to its sugar within a nucleotide.

Step 3: Check the protein-level statement and options Statement D is false. The overall three-dimensional folding of a single polypeptide chain is its tertiary structure. Quaternary structure concerns the relative arrangement and association of two or more polypeptide subunits. Thus A and B alone are correct. Option (B) additionally includes the false statement D; (C) includes the false statement C and omits A; and (D) includes both false statements C and D while omitting B.

Step 4: Conclusion The correct statements are A and B only. Hence, the correct option is (A).

Quick Tip to Solve: Keep two distinctions separate: nucleotide chains use phosphodiester bonds, and one chain's overall fold is tertiary while multiple subunits define quaternary structure.

55. By usual analysis, 1.00 g of compound (X) gave 1.79 g of magnesium pyrophosphate. The percentage of phosphorus in compound (X) is : (nearest integer)
(Given, molar mass in g mol^{-1}: O = 16, Mg = 24, P = 31) [4 marks]

(A) 20

(B) 30

(C) 40

(D) 50

Correct Answer: (D) 50

Solution:

Concept:

- In gravimetric analysis, phosphorus is determined from the mass of magnesium pyrophosphate, $Mg_2P_2O_7$, obtained from the sample.
- The mass fraction of an element in a pure compound equals the total mass of its atoms in one mole divided by the compound's molar mass.

Step 1: Calculate the phosphorus fraction of the precipitate Magnesium pyrophosphate contains two Mg atoms, two P atoms and seven O atoms per formula unit. Using the supplied atomic masses,

$$M(Mg_2P_2O_7) = 2(24) + 2(31) + 7(16) = 222\,\text{g mol}^{-1}.$$

Thus, each 222 g of the precipitate contains 62 g of phosphorus. The two phosphorus atoms must both be counted; using only 31 g would halve the phosphorus content.

Step 2: Convert the precipitate mass to a sample percentage All the phosphorus in the analysed

sample is represented by the phosphorus in the recovered pyrophosphate. Hence,

$$m(\mathrm{P}) = 1.79 \times \frac{62}{222} = 0.49991\,\mathrm{g}.$$

The percentage is calculated relative to the original compound, not relative to the precipitate:

$$\%\mathrm{P} = \frac{0.49991}{1.00} \times 100 = 49.991\%.$$

The precipitate can weigh more than the sample because it also contains magnesium and oxygen introduced during analysis. Rounding the calculated percentage to the nearest integer gives 50. Values 20, 30 and 40 do not match the phosphorus mass fixed by the measured precipitate.

Step 3: Conclusion The phosphorus content of compound X is $\boxed{50\%}$, to the nearest integer. Hence, the correct option is (D).

Quick Tip to Solve: For magnesium pyrophosphate, multiply the precipitate mass by $62/222$, then divide by the original sample mass and multiply by 100.

56. Aqueous HCl reacts with MnO_2(s) to form $MnCl_2$(aq), Cl_2(g), and H_2O(l). What is the weight (in g) of Cl_2 liberated when 8.7 g of MnO_2(s) is reacted with excess aqueous HCl solution? (Given Molar mass in g mol^{-1} Mn = 55, Cl = 35.5, O = 16, H = 1) [4 marks]

(A) 14.2

(B) 21.3

(C) 7.1

(D) 71

Correct Answer: (C) 7.1

Solution:

Concept:

- A balanced chemical equation gives the mole ratio of reactants consumed and products formed; coefficients are mole ratios, not mass ratios.
- For a substance of mass m and molar mass M, the amount is $n = m/M$. With hydrochloric acid in excess, the available $\mathrm{MnO_2}$ determines the chlorine yield.

Step 1: Balance the oxidation-reduction reaction Manganese changes from oxidation state +4 to +2, gaining two electrons. Two chloride ions lose two electrons together to produce one chlorine molecule. The balanced reaction is

$$\mathrm{MnO_2} + 4\,\mathrm{HCl} \longrightarrow \mathrm{MnCl_2} + \mathrm{Cl_2} + 2\,\mathrm{H_2O}.$$

There is one mole of $\mathrm{Cl_2}$ formed per mole of $\mathrm{MnO_2}$. Of the four chloride ions supplied by hydrochloric acid, two remain in manganese chloride and two form chlorine gas.

Step 2: Calculate the amount and mass of chlorine From the atomic masses supplied in the question,

$$M(\mathrm{MnO_2}) = 55 + 2(16) = 87\,\mathrm{g\ mol^{-1}},$$

and therefore

$$n(\mathrm{MnO_2}) = \frac{8.7}{87} = 0.100\,\mathrm{mol}.$$

Hydrochloric acid is in excess, so all of this amount can react. The equal mole ratio gives $n(\mathrm{Cl_2}) = 0.100\,\mathrm{mol}$. Since chlorine is diatomic,

$$m(\mathrm{Cl_2}) = 0.100 \times [2(35.5)] = 7.1\,\mathrm{g}.$$

The proposed masses 14.2 and 21.3 g would require twice and three times the available manganese dioxide, respectively. A mass of 71 g corresponds to a full mole of chlorine, whereas only 0.100 mol can be formed here.

Step 3: Conclusion The mass of chlorine liberated is $\boxed{7.1\,\mathrm{g}}$. Hence, the correct option is (C).

Quick Tip to Solve: Balance first: $87\,\mathrm{g}$ of $\mathrm{MnO_2}$ yields $71\,\mathrm{g}$ of $\mathrm{Cl_2}$, so 8.7 g gives one tenth of that chlorine mass.

57. Match List - I with List - II.

List – I: Reagents	List – II: Reaction Name (Involving aldehydes)
A. H_2, Pd–$BaSO_4$	I. Etard Reaction
B. $SnCl_2$, HCl	II. Rosenmund Reduction
C. CrO_2Cl_2, CS_2	III. Gattermann–Koch Reaction
D. CO, HCl, Anhyd. $AlCl_3$	IV. Stephen Reaction

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-I, C-II, D-III

(B) A-IV, B-III, C-I, D-II

(C) A-II, B-IV, C-I, D-III

(D) A-II, B-III, C-IV, D-I

Correct Answer: (C) A-II, B-IV, C-I, D-III

Solution:

Concept:

- Aldehydes can be prepared by controlled reduction of acid chlorides or nitriles, oxidation of an aromatic side chain, or formylation of an aromatic ring.
- Identify a named reaction from its characteristic reagent set and the functional-group transformation; the common aldehyde product alone does not distinguish these reactions.

Step 1: Match the two reducing reagent systems In A, hydrogen with palladium supported on barium

sulphate is the reagent for Rosenmund reduction. An acid chloride is reduced to an aldehyde:

$$\mathrm{RCOCl} + \mathrm{H_2} \longrightarrow \mathrm{RCHO} + \mathrm{HCl}.$$

The controlled catalyst prevents the reduction from continuing readily to the alcohol. Thus A matches II. In B, $\mathrm{SnCl_2}$ and HCl reduce a nitrile to an iminium salt, which yields an aldehyde on hydrolysis. This is the Stephen reaction, so B matches IV.

Step 2: Match oxidation and aromatic formylation In C, chromyl chloride, $\mathrm{CrO_2Cl_2}$, in carbon disulphide is the characteristic Etard reagent. It oxidises the methyl group of toluene through a chromium complex, which gives benzaldehyde after hydrolysis. Thus C matches I. In D, carbon monoxide and hydrogen chloride in the presence of anhydrous aluminium chloride are associated with Gattermann–Koch formylation. An aromatic ring receives a formyl group and gives an aromatic aldehyde. Thus D matches III.

Step 3: Check the complete matching code The four independent identifications give A-II, B-IV, C-I and D-III. In the printed option order, the first and second alternatives both incorrectly assign A to the Stephen reaction. The fourth starts with A-II but incorrectly assigns B to Gattermann–Koch, C to Stephen and D to Etard. Only the third alternative has all four matches correct.

Step 4: Conclusion The correct matching is $\boxed{\text{A-II, B-IV, C-I, D-III}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the distinctive pairs: Pd–$\mathrm{BaSO_4}$ means Rosenmund, $\mathrm{SnCl_2/HCl}$ means Stephen, and chromyl chloride means Etard.

58. Given below are two statements :

Statement I : The correct order in terms of bond dissociation enthalpy is $Cl_2 > Br_2 > F_2 > I_2$.

Statement II : The correct trend in the covalent character of the metal halides is [$SnCl_4 > SnCl_2$], [$PbCl_4 > PbCl_2$] and [$UF_4 > UF_6$].

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Correct Answer: (C) **Statement I** is true but **Statement II** is false

Solution:

Concept:

- Bond dissociation enthalpy measures the energy needed to break one mole of gaseous diatomic molecules into gaseous atoms. Both orbital overlap and nonbonding-electron repulsions affect its magnitude.
- By Fajans' rules, greater charge and smaller size increase a cation's polarising power. Greater distortion of an anion's electron cloud increases the covalent character of the bond.

Step 1: Evaluate the halogen bond-enthalpy order Down the halogen group from chlorine to iodine, increasing atomic size lengthens the bond and weakens effective overlap, so the bond enthalpy decreases. Fluorine is exceptional: its very small atoms place the lone pairs close together, causing appreciable interelectronic repulsion and weakening the F–F bond. Representative bond enthalpies in kJ mol^{-1} are approximately 243 for Cl_2, 193 for Br_2, 159 for F_2, and 151 for I_2. Therefore,

$$D(Cl_2) > D(Br_2) > D(F_2) > D(I_2).$$

Statement I is true; a simple size-only argument would miss fluorine's anomalously weak bond.

Step 2: Evaluate each covalent-character comparison For chlorides of the same metal, the higher oxidation state increases polarising power. Hence the comparisons $SnCl_4 > SnCl_2$ and $PbCl_4 > PbCl_2$ are correct in covalent character. For uranium fluorides, uranium is in oxidation state +6 in UF_6 and +4 in UF_4. The higher oxidation state gives the stronger polarisation, so the correct comparison is

$$\text{covalent character of } UF_6 > UF_4.$$

The last comparison in Statement II is reversed, making that entire statement false. Both-true and both-false options therefore fail; the option declaring only Statement II true also reverses the established truth values.

Step 3: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: Remember fluorine's lone-pair repulsion exception, then check every bracket in Statement II: one reversed uranium-fluoride comparison makes the whole statement false.

59. Consider the following spectral lines for atomic hydrogen :

A. First line of Paschen series

B. Second line of Balmer series

C. Third line of Paschen series

D. Fourth line of Bracket series

The correct arrangement of the above lines in ascending order of energy is : [4 marks]

(A) A < B < C < D

(B) D < A < C < B

(C) D < C < A < B

(D) C < D < B < A

Correct Answer: (B) D < A < C < B

Solution:

Concept:

- The energy of a photon emitted in a hydrogen transition is $E = 13.6(1/n_f^2 - 1/n_i^2)$ eV, where $n_i > n_f$ are the initial and final principal quantum numbers.
- Balmer, Paschen and Brackett lines terminate at $n_f = 2, 3, 4$, respectively. The rth line of a series

starts from $n_i = n_f + r$.

Step 1: Identify the four transitions A, the first Paschen line, is the transition $4 \to 3$. B, the second Balmer line, is $4 \to 2$. C, the third Paschen line, is $6 \to 3$. D, the fourth Brackett line, is $8 \to 4$. The line number counts upward from the lowest possible upper level; it is not itself the initial quantum number.

Step 2: Calculate and compare the energy factors All four energies contain the common positive factor 13.6 eV, so it is enough to compare the reciprocal-square differences:

$$E_A/13.6 = \frac{1}{9} - \frac{1}{16} = \frac{7}{144},$$
$$E_B/13.6 = \frac{1}{4} - \frac{1}{16} = \frac{3}{16},$$
$$E_C/13.6 = \frac{1}{9} - \frac{1}{36} = \frac{1}{12},$$
$$E_D/13.6 = \frac{1}{16} - \frac{1}{64} = \frac{3}{64}.$$

Here the energies are expressed in electron volts. On taking the common denominator 576, the four factors for A, B, C and D become $28/576$, $108/576$, $48/576$ and $27/576$, respectively. In particular, D is slightly below A; comparing only rounded decimal values can obscure this close pair. Thus the numerators increase in the order $27 < 28 < 48 < 108$. Option (A) incorrectly places B before C and D; option (C) reverses A and C; option (D) starts with C although D and A both have lower energies.

Step 3: Conclusion The ascending order of photon energy is $\boxed{D < A < C < B}$. Hence, the correct option is (B).

Quick Tip to Solve: Translate each line into $n_i \to n_f$ first, then compare reciprocal-square differences; a denominator of 576 distinguishes the close D and A energies exactly.

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60. Decomposition of A is a first order reaction at T(K) and is given by A(g) $\to$ B(g) + C(g).

In a closed 1 L vessel, 1 bar A(g) is allowed to decompose at T(K). After 100 minutes, the total pressure was 1.5 bar. What is the rate constant (in min^{-1}) of the reaction? (log 2 = 0.3) [4 marks]

(A) 6.9×10^{-2}

(B) 6.9×10^{-1}

(C) 6.9×10^{-3}

(D) 6.9×10^{-4}

Correct Answer: (C) 6.9×10^{-3}

Solution:

Concept:

- At fixed temperature and volume, a gas's partial pressure is proportional to its amount. Stoichiometry therefore relates the total pressure to the partial pressure of the reacting gas.
- For a first-order reaction, $k = (2.303/t)\log_{10}(p_{A,0}/p_{A,t})$, where $p_{A,0}$ and $p_{A,t}$ are the initial and remaining partial pressures of A.

Step 1: Use the pressure increase to find the remaining reactant Initially only A is present, at pressure $p_0 = 1$ bar. Let x bar be the decrease in its partial pressure after 100 minutes. For $\mathrm{A(g)} \longrightarrow \mathrm{B(g)} + \mathrm{C(g)}$, the partial pressures at that instant are

$$p_A = p_0 - x, \qquad p_B = x, \qquad p_C = x.$$

Consequently, $p_{\text{total}} = p_0 + x$. The measured total pressure gives

$$x = 1.5 - 1.0 = 0.5\,\text{bar}, \qquad p_A = 1.0 - 0.5 = 0.5\,\text{bar}.$$

Thus half of A remains. The total pressure includes both products and must not be substituted directly for the concentration of A.

Step 2: Apply the integrated first-order rate law The vessel volume and temperature are fixed, so the ratio of concentrations equals the ratio of partial pressures. Substituting $t = 100$ min and the supplied $\log 2 = 0.3$,

$$\begin{aligned} k &= \frac{2.303}{100} \log\left(\frac{1.0}{0.5}\right) \\ &= \frac{2.303(0.3)}{100} \\ &= 6.909 \times 10^{-3}\,\text{min}^{-1}. \end{aligned}$$

The time is already in minutes, so no conversion to seconds is required. The other printed rate constants differ by factors of ten or one hundred: they would give half-lives of about 10, 1 or 1000 minutes instead of the observed 100 minutes.

Step 3: Conclusion The printed total pressure gives $p_A = 2(1) - 1.5 = 0.5$ bar, so 100 minutes is one half-life and $k = 2.303(0.3)/100$. Therefore the rate constant is $\boxed{6.9 \times 10^{-3}\,\text{min}^{-1}}$. This proves the third printed option despite the discrepant key position. Hence, the correct option is (C).

Quick Tip to Solve: For $\mathrm{A} \longrightarrow \mathrm{B} + \mathrm{C}$ starting from pure A, use $p_A = 2p_0 - p_{\text{total}}$; here a total pressure of $1.5p_0$ means one half-life has elapsed.

61. The correct order of the rate of the reaction for the following reaction with respect to nucleophiles is :

$$CH_3Br + Nu^- \longrightarrow CH_3Nu + Br^-$$

[4 marks]

(A) $OH^- > PhO^- > CH_3COO^- > ClO_4^-$
(B) $CH_3COO^- > PhO^- > OH^- > ClO_4^-$
(C) $PhO^- > OH^- > CH_3COO^- > ClO_4^-$
(D) $ClO_4^- > CH_3COO^- > OH^- > PhO^-$

Correct Answer: (A) $OH^- > PhO^- > CH_3COO^- > ClO_4^-$

Solution:

Concept:

- Methyl bromide undergoes bimolecular nucleophilic substitution: $v = k[\mathrm{CH_3Br}][\mathrm{Nu^-}]$. At equal concentrations, a stronger nucleophile gives a larger rate constant.
- For these oxygen-centred anions, greater availability of the oxygen lone pair gives greater nucleophilicity. Resonance stabilisation reduces that availability.

Step 1: Identify the common attacking atom All four nucleophiles attack through oxygen, so the comparison is within the same period and is governed mainly by charge localisation and basicity. The methyl carbon is unhindered, allowing direct backside attack with simultaneous loss of Br^-. A methyl carbocation need not form. Thus the comparison concerns the attacking anions, not the leaving group, which is identical in all cases.

Step 2: Compare delocalisation of the negative charge In OH^-, the negative charge remains on its single oxygen atom. In phenoxide, PhO^-, the charge is delocalised partly into the aromatic ring, reducing oxygen lone-pair availability. Acetate, CH_3COO^-, has equivalent resonance forms that share the charge between two electronegative oxygen atoms and stabilise it more strongly. Perchlorate, ClO_4^-, is exceptionally stable and a very poor nucleophile. The corresponding conjugate-acid strengths support the same basicity trend: water is weaker as an acid than phenol, which is weaker than acetic acid, while perchloric acid is very strong.

Step 3: Check the proposed orders The order therefore follows decreasing basicity for this set. Option (B) incorrectly places acetate above hydroxide and phenoxide; option (C) incorrectly places phenoxide above hydroxide; option (D) reverses the essential trend and makes perchlorate strongest.

Step 4: Conclusion Increasing resonance stabilisation from hydroxide through phenoxide and acetate to perchlorate reduces oxygen nucleophilicity, proving the order

$$\mathrm{OH^- > PhO^- > CH_3COO^- > ClO_4^-}.$$

Hence, the correct option is (A).

Quick Tip to Solve: For oxygen-centred nucleophiles, first rank charge localisation: hydroxide is localised, while phenoxide, acetate and perchlorate are increasingly stabilised.

62. Consider the following data :

$\Delta_f H^\circ$ (methane, g) = $-X$ kJ mol^{-1}

Enthalpy of sublimation of graphite = Y kJ mol^{-1}

Dissociation enthalpy of $H_2 = Z$ kJ mol^{-1}

The bond enthalpy of C–H bond is given by : [4 marks]

(A) $X + Y + Z$

(B) $\dfrac{X + Y + 2Z}{4}$

(C) $\dfrac{X + Y + 4Z}{2}$

(D) $\dfrac{-X + Y + Z}{4}$

Correct Answer: (B) $\dfrac{X + Y + 2Z}{4}$

Solution:

Concept:

- Hess law makes the reaction enthalpy independent of the path. Atomising methane breaks four C–H bonds, so its atomisation enthalpy is $4D_{\mathrm{C-H}}$, where $D_{\mathrm{C-H}}$ is the mean bond enthalpy.
- Standard formation of one mole of methane uses graphite and two moles of hydrogen gas: $\mathrm{C(graphite)} + 2\,\mathrm{H_2(g)} \longrightarrow \mathrm{CH_4(g)}$, with enthalpy $-X$.

Step 1: Reverse methane formation To reach free gaseous atoms from methane through the elements, first reverse its formation reaction. Reversing a reaction reverses the sign of its enthalpy:

$$\mathrm{CH_4(g)} \longrightarrow \mathrm{C(graphite)} + 2\,\mathrm{H_2(g)}, \qquad \Delta H = X.$$

The minus sign in the given formation enthalpy must therefore become a plus sign in the atomisation cycle.

Step 2: Atomise the elemental products Subliming one mole of graphite produces one mole of gaseous carbon atoms and costs Y. Dissociating each mole of $\mathrm{H_2}$ costs Z, so the two moles formed in the first step cost $2Z$:

$$\mathrm{C(graphite)} \longrightarrow \mathrm{C\,(g)}, \quad \Delta H = Y,$$
$$2\,\mathrm{H_2(g)} \longrightarrow 4\,\mathrm{H\,(g)}, \quad \Delta H = 2Z.$$

Adding these steps gives $\mathrm{CH_4(g)} \longrightarrow \mathrm{C\,(g)} + 4\,\mathrm{H\,(g)}$, with total enthalpy $X + Y + 2Z$.

Step 3: Relate atomisation to the four bonds One methane molecule contains four C–H bonds, hence

$$4D_{\mathrm{C-H}} = X + Y + 2Z.$$

Dividing by four gives the mean enthalpy per mole of C–H bonds. Option (A) neither counts the two hydrogen molecules nor averages over four bonds; (C) uses four hydrogen dissociations and only two bonds; (D) retains the wrong formation sign and uses only one hydrogen dissociation.

Step 4: Conclusion Reversing methane formation and atomising one carbon plus two hydrogen

molecules costs $X + Y + 2Z$; this breaks four C–H bonds. Therefore

$$D_{\mathrm{C-H}} = \frac{X + Y + 2Z}{4}\ \mathrm{kJ\ mol^{-1}}.$$

Hence, the correct option is (B).

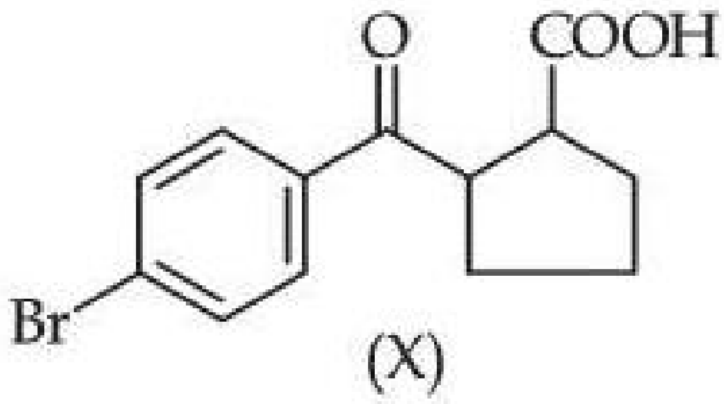
Quick Tip to Solve: For methane, reverse formation, add one carbon sublimation and two H–H dissociations, then divide the total atomisation enthalpy by four.

63. Statement I : Compound (X), shown below, dissolves in $NaHCO_3$ solution and has two chiral carbon atoms

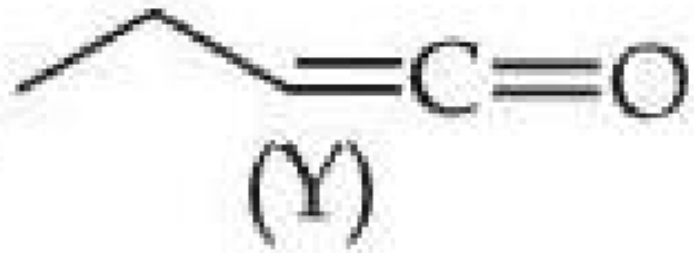
Statement II : Compound (Y), shown below, has two carbons with sp^3 hybridization, one carbon with sp^2 and one carbon with sp hybridization

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]



(X)



(Y)

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Correct Answer: (A) Both **Statement I** and **Statement II** are true

Solution:

Concept:

- A carboxylic acid reacts with aqueous sodium hydrogen carbonate to give a soluble carboxylate salt, carbon dioxide and water.
- A tetrahedral carbon is chiral when its four attached groups are different. Hybridisation depends on the number of sigma-bond directions: four, three and two correspond to sp^3, sp^2 and sp, respectively.

Step 1: Test the acidic group in compound X Compound (X) has a $-$COOH group attached to the cyclopentane ring. It reacts with $NaHCO_3$, forming the ionic sodium carboxylate:

$$RCOOH + NaHCO_3 \longrightarrow RCOONa + CO_2 + H_2O.$$

The carboxyl group is sufficiently acidic for this reaction; neither the aromatic bromine nor the ketone removes that behaviour. Salt formation accounts for its dissolution in the bicarbonate solution.

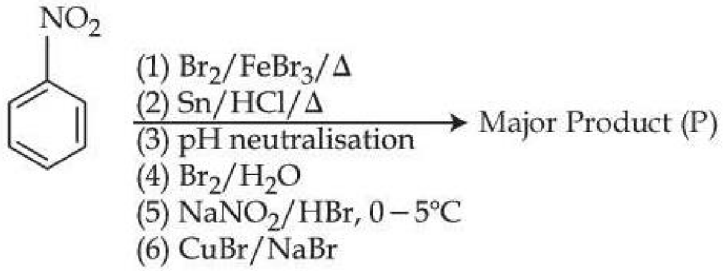
Step 2: Count the chiral carbons in X The two neighbouring substituted carbons of the cyclopentane ring are tetrahedral. At the carbon carrying $-$COOH, the two paths around the ring are different because one immediately encounters the carbon carrying the aroyl group. At the carbon carrying the aroyl group, the paths are likewise distinguished by the neighbouring carboxyl substituent. Each of these carbons also has one hydrogen and its external substituent, giving four different groups. The other ring carbons are CH_2 groups and cannot be chiral, while the carbonyl and aromatic carbons are planar. Statement I is therefore true.

Step 3: Count each hybridisation in Y The displayed cumulene is $CH_3CH_2CH{=}C{=}O$. Its CH_3 and CH_2 carbons have four sigma-bond directions and are sp^3. The CH carbon in CH$=$C has three directions and is sp^2. The central carbon of C$=$C$=$O has two directions, one toward carbon and one toward oxygen, and is sp. Thus Statement II is also true. Options (B), (C) and (D) each incorrectly declare at least one verified statement false.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: For a substituted ring, compare the two ring paths at each tetrahedral carbon; for C$=$C$=$O, count two sigma directions at its central carbon, giving sp.

64. Consider the above sequence of reactions. The number of bromine atom(s) in the final product (P) will be : [4 marks]



(A) 1

(B) 5

(C) 3

(D) 6

Correct Answer: (B) 5

Solution:

Concept:

- The nitro group is deactivating and meta-directing in electrophilic aromatic substitution; the amino group is strongly activating and directs substitution to the ortho and para positions.
- Diazotisation converts an aromatic primary amino group into a diazonium salt. Sandmeyer bromination replaces that diazonium group by one bromine atom.

Step 1: Carry out the first bromination and reduction Number the carbon bearing NO_2 as position 1. Bromination with $Br_2/FeBr_3$ on heating gives meta-bromonitrobenzene, which can be drawn with bromine at position 3. The nitro group directs the incoming bromine meta. Next, Sn/HCl reduces NO_2 to the protonated amino group without replacing the aryl bromine. Neutralisation then liberates 3-bromoaniline. The ring at this stage contains one bromine atom.

Step 2: Apply bromine water to the free amine The free NH_2 group strongly activates the ring. Bromine water introduces bromine at all three available positions ortho and para to NH_2, namely positions 2, 4 and 6. The pre-existing bromine at position 3 is retained. Its ortho/para directing effect also points toward positions 2, 4 and 6. The intermediate is consequently 2,3,4,6-tetrabromoaniline, containing four bromine atoms; position 5 is not an ortho or para site relative to the amino group.

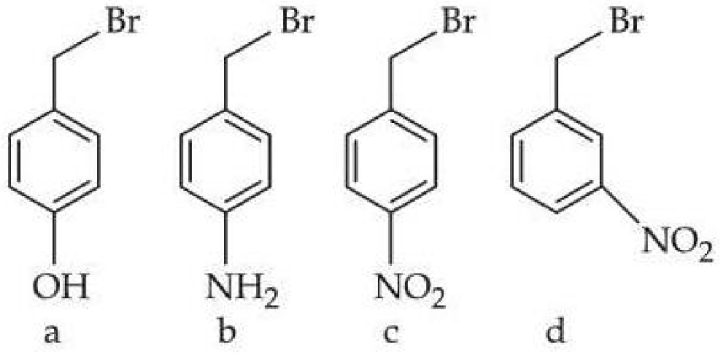
Step 3: Replace the amino-derived diazonium group $NaNO_2$/HBr at 0–$5\,^\circ C$ converts the amino group at position 1 into a diazonium group. Treatment with CuBr/NaBr then replaces this group by bromine with evolution of nitrogen. It adds one bromine to the four already on the ring. Options (A) and (C), giving one and three bromines respectively, omit later substitutions; option (D), giving six, would incorrectly brominate position 5 as well.

Step 4: Conclusion The final product is pentabromobenzene, with $1 + 3 + 1 = 5$ bromine atoms. Hence, the correct option is (B).

Quick Tip to Solve: Keep a running bromine count through the sequence: initial meta bromination adds one, bromine water adds three, and Sandmeyer replacement adds one.

65. The correct order of reactivity of the following benzyl halides towards reaction with KCN is :

[4 marks]



(A) b > a > d > c

(B) a > b > d > c

(C) b > a > c > d

(D) a > b > c > d

Correct Answer: (A) b > a > d > c

Solution:

Concept:

- Benzylic substitution is influenced by conjugation between the aromatic ring and the benzylic reaction centre. Electron donation from an ortho or para substituent can stabilise the charge development associated with C–Br bond cleavage.
- An amino group donates by resonance more strongly than a hydroxyl group. A nitro group withdraws by induction and, at the para position, also by resonance.

Step 1: Identify the four substituents and the common reaction All four substrates are benzyl bromides, so their leaving group and the immediate benzylic structure are the same. Compound (a) has para OH, (b) has para NH_2, (c) has para NO_2, and (d) has meta NO_2. Cyanide supplied by KCN replaces benzylic bromide to produce the corresponding benzyl cyanide. Comparing these substrates therefore requires the electronic effects and positions of their ring substituents, rather than a leaving-group comparison.

Step 2: Compare the two electron-donating substituents The lone pairs of both para NH_2 and para OH can interact with the aromatic system and help stabilise the benzylic substitution pathway. Nitrogen is less electronegative than oxygen, so its lone pair is more readily donated into the ring. The resonance-donating effect of NH_2 is stronger than that of OH; consequently (b) reacts faster than (a). This rules out both options that place (a) before (b).

Step 3: Compare the nitro-substituted substrates The nitro group withdraws electron density. At the para position it withdraws by both its inductive and resonance effects, opposing the stabilisation supplied by the donor groups. At the meta position its direct resonance withdrawal does not act at the benzylic attachment site in the same way, so its deactivating effect is smaller. Therefore (d) reacts faster than (c), while both are below the para donor-substituted compounds. Option (C) incorrectly reverses this last pair; options (B) and (D) reverse the first pair.

Step 4: Conclusion The correct order is $b > a > d > c$. Hence, the correct option is (A).

Quick Tip to Solve: Split the comparison into two pairs: para amino donates more strongly than para hydroxyl, while para nitro withdraws more strongly than meta nitro.

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66. Given below are some of the statements about Mn and Mn_2O_7. Identify the correct statements.

A. Mn forms the oxide Mn_2O_7, in which Mn is in its highest oxidation state.

B. Oxygen stabilizes the Mn in higher oxidation states by forming multiple bonds with Mn.

C. Mn_2O_7 is an ionic oxide.

D. The structure of Mn_2O_7 consists of one bridged oxygen.

Choose the correct answer from the options given below : [4 marks]

(A) A, B, C and D

(B) A, B and D Only

(C) A, C and D Only

(D) A, B and C Only

Correct Answer: (B) A, B and D Only

Solution:

Concept:

- The oxidation number of oxygen in an ordinary oxide is -2; the sum of oxidation numbers in a neutral compound is zero.
- Transition metals can attain high oxidation states in covalent oxides because strong metal–oxygen bonds, including multiple bonding, stabilise these states.

Step 1: Evaluate the oxidation state and oxygen stabilisation Let the oxidation state of manganese in Mn_2O_7 be x. Charge balance gives

$$2x + 7(-2) = 0 \quad \Rightarrow \quad x = +7.$$

Manganese is a group 7 element and $+7$ is its highest oxidation state. Statement A is true. The terminal manganese–oxygen bonds have multiple-bond character, and the strong bonding to oxygen stabilises this high oxidation state. Statement B is therefore true as well. Formal oxidation number is an electron-counting convention; it does not mean that a free Mn^{7+} ion exists in the compound.

Step 2: Distinguish molecular and ionic bonding Mn_2O_7 is a covalent, molecular oxide, rather than an ionic lattice composed of manganese cations and oxide anions. The very high formal oxidation state promotes covalent metal–oxygen bonding. It is the acidic oxide associated with permanganic acid. Thus Statement C, which calls it an ionic oxide, is false. Every answer combination containing C must be rejected.

Step 3: Inspect the oxygen bridge and select the combination The molecular connectivity can be written as $O_3Mn - O - MnO_3$. Each manganese is linked to three terminal oxygen atoms and to the same bridging oxygen atom; the two tetrahedral coordination units share this corner. There are six terminal oxygens and one bridge, giving seven oxygen atoms altogether. Statement D is true. Option

(A) includes false statement C; option (C) includes C and omits true B; option (D) includes C and omits true D.

Step 4: Conclusion The correct statements are A, B and D only. Hence, the correct option is (B).

Quick Tip to Solve: For Mn_2O_7, remember the connectivity $O_3Mn-O-MnO_3$: six terminal oxygens plus one shared bridge, with manganese formally at $+7$.

67. On heating a mixture of common salt and $K_2Cr_2O_7$ in equal amount along with concentrated H_2SO_4 in a test tube, a gas is evolved. Formula of the gas evolved and oxidation state of the central metal atom in the gas respectively are: [4 marks]

(A) CrO_2Cl_2 and $+5$

(B) CrO_2Cl_2 and $+6$

(C) $Cr_2O_2Cl_2$ and $+3$

(D) $Cr_2O_2Cl_2$ and $+6$

Correct Answer: (B) CrO_2Cl_2 and $+6$

Solution:

Concept:

- Heating a dry chloride with potassium dichromate and concentrated sulphuric acid is the chromyl chloride test. The volatile product is red-brown chromyl chloride, CrO_2Cl_2.
- In a neutral compound the sum of oxidation numbers is zero. Oxygen has oxidation number -2, and chlorine bonded to chromium has oxidation number -1.

Step 1: Identify the volatile chromium compound Common salt supplies chloride ions, while $K_2Cr_2O_7$ supplies chromium(VI). On heating in concentrated H_2SO_4, chromyl chloride vapours escape from the mixture. The net formation equation is

$$Cr_2O_7^{2-} + 4\,Cl^- + 6\,H^+ \longrightarrow 2\,CrO_2Cl_2 + 3\,H_2O.$$

This equation balances chromium, oxygen, hydrogen and chlorine as well as net charge. Each product molecule contains one chromium atom, two oxygen atoms and two chlorine atoms. Thus the formula is not the two-chromium formula appearing in the last two options.

Step 2: Calculate chromium's oxidation state and compare options Let chromium's oxidation number in the neutral molecule be x. There is no peroxide linkage, so both oxygen atoms have their usual value -2:

$$x + 2(-2) + 2(-1) = 0, \qquad x = +6.$$

Chromium therefore retains its dichromate oxidation state; formation of the volatile oxychloride does not reduce it to $+5$ or $+3$. Option (A) gives the right molecular formula but the wrong oxidation state. Options (C) and (D) give the wrong molecular formula. Only (B) satisfies both independent checks.

Step 3: Conclusion The evolved vapour is chromyl chloride, CrO_2Cl_2, and chromium has oxidation state $+6$. Hence, the correct option is (B).

Quick Tip to Solve: Recall the chromyl chloride test first, then use $x-4-2=0$ to check the metal's oxidation state independently.

68. Given below are two statements :

Statement I : The correct order in terms of atomic/ionic radii is Al $>$ Mg $>$ Mg^{2+} $>$ Al^{3+}.

Statement II : The correct order in terms of the magnitude of electron gain enthalpy is Cl $>$ Br $>$ S $>$ O.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Correct Answer: (D) **Statement I** is false but **Statement II** is true

Solution:

Concept:

- Atomic radii generally decrease across a period as effective nuclear charge increases. Removing valence electrons makes a cation smaller than its parent atom.
- For isoelectronic ions, a greater nuclear charge means a smaller radius. Electron gain enthalpy measures the enthalpy change on adding an electron to a gaseous atom; its magnitude is the absolute size of that change.

Step 1: Test the atomic and ionic radius order Magnesium and aluminium lie consecutively in period 3. Aluminium has the higher effective nuclear charge, so its atom is smaller than magnesium. The start of Statement I, $\mathrm{Al} > \mathrm{Mg}$, is therefore incorrect. Both Mg^{2+} and Al^{3+} have ten electrons and the neon configuration. Aluminium's nucleus has 13 protons rather than 12, and attracts the same electron population more strongly. Also, these cations have lost their outer third-shell electrons. The correct complete order is

$$\mathrm{Mg} > \mathrm{Al} > \mathrm{Mg}^{2+} > \mathrm{Al}^{3+}.$$

Statement I is false even though its comparison of the two cations is correct.

Step 2: Compare electron gain enthalpy magnitudes Representative first electron gain enthalpies for chlorine, bromine, sulphur and oxygen are approximately $-349, -325, -200$ and $-141\,\mathrm{kJ\ mol^{-1}}$, respectively. Hence their magnitudes obey

$$|\Delta_{\mathrm{eg}}H(\mathrm{Cl})| > |\Delta_{\mathrm{eg}}H(\mathrm{Br})| > |\Delta_{\mathrm{eg}}H(\mathrm{S})| > |\Delta_{\mathrm{eg}}H(\mathrm{O})|.$$

Oxygen's compact $2p$ shell causes greater repulsion for the incoming electron than sulphur's larger $3p$ shell. Statement II is true. Options (A) and (C) incorrectly accept Statement I, while (B) incorrectly rejects Statement II.

Step 3: Conclusion Statement I is false and Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: Check the first pair Al, Mg immediately; for electron gain enthalpy, compare absolute magnitudes rather than signed negative values.

69. **Match List - I with List - II.**

List - IPair of Compounds	List - IIType of Isomers
A. 2-Methylpropene and but-1-ene	I. Stereoisomers
B. Cis-but-2-ene and trans-but-2-ene	II. Position isomers
C. 2-Butanol and diethyl ether	III. Chain isomers
D. But-1-ene and but-2-ene	IV. Functional group isomers

Choose the correct answer from the options given below : [4 marks]

(A) A-III, B-I, C-II, D-IV

(B) A-I, B-IV, C-III, D-II

(C) A-II, B-I, C-IV, D-III

(D) A-III, B-I, C-IV, D-II

Correct Answer: (D) A-III, B-I, C-IV, D-II

Solution:

Concept:

- Structural isomers have the same molecular formula but different connectivity. Chain, position and functional group isomerism distinguish changes in carbon skeleton, group position and functional group, respectively.
- Stereoisomers have the same connectivity but different spatial arrangements. Restricted rotation about a carbon-carbon double bond permits cis-trans isomerism when each double-bonded carbon has two different substituents.

Step 1: Identify the changes in pairs A and B In pair A, 2-methylpropene, $CH_2{=}C(CH_3)_2$, has a branched carbon skeleton, whereas but-1-ene, $CH_2{=}CHCH_2CH_3$, has an unbranched skeleton. Both have molecular formula C_4H_8 and an alkene group. The defining change is therefore chain isomerism: A-III.

In pair B, cis-but-2-ene and trans-but-2-ene have identical atom connectivity and the double bond in the same position. Their methyl groups lie on the same side or opposite sides of the double bond. They are geometrical stereoisomers: B-I.

Step 2: Identify the changes in pairs C and D In pair C, butan-2-ol, $CH_3CH(OH)CH_2CH_3$, is an alcohol, whereas diethyl ether, $CH_3CH_2OCH_2CH_3$, is an ether. Both have formula $C_4H_{10}O$, but their functional groups differ. Thus C-IV, not position or chain isomerism.

In pair D, but-1-ene and but-2-ene have the same straight four-carbon skeleton and the same alkene functional group. Moving the double bond from carbon 1 to carbon 2 makes them position isomers: D-II. Option (A) interchanges the classifications of C and D; option (B) misclassifies A, B and C; option (C) interchanges chain and position isomerism for A and D.

Step 3: Conclusion The matching is A-III, B-I, C-IV, D-II. Pair C has different functional groups (alcohol and ether), while pair D differs in double-bond position. Hence, the correct option is (D).

Quick Tip to Solve: Compare skeleton, functional group and group position before checking spatial arrangement; cis-trans partners retain identical connectivity.

70. The correct increasing order of C–H(A), C–O(B), C=O(C) and C≡N(D) bonds in terms of covalent bond length is : [4 marks]

(A) A < B < C < D

(B) A < D < C < B

(C) D < C < B < A

(D) D < C < A < B

Correct Answer: (B) A < D < C < B

Solution:

Concept:

- A covalent bond length is the equilibrium internuclear distance. It depends on the sizes of the bonded atoms as well as bond order and hybridisation.
- For the same pair of atoms, a multiple bond is shorter than a single bond. When comparing different atom pairs, bond order alone is insufficient because atomic size also changes.

Step 1: Associate the bonds with their typical lengths The usual approximate bond lengths relevant to this comparison are

$$\ell(\mathrm{C-H}) \simeq 109\,\mathrm{pm} \quad \text{(A)},$$
$$\ell(\mathrm{C-O}) \simeq 143\,\mathrm{pm} \quad \text{(B)},$$
$$\ell(\mathrm{C=O}) \simeq 121\,\mathrm{pm} \quad \text{(C)},$$
$$\ell(\mathrm{C\equiv N}) \simeq 116\,\mathrm{pm} \quad \text{(D)}.$$

These are standard characteristic values, rather than an assertion that every chemical environment gives an identical distance. Hydrogen's small covalent radius makes the carbon-hydrogen bond especially short despite its single-bond character.

Step 2: Sort the distances and reject inconsistent orders Comparing the approximate distances directly gives $109 < 116 < 121 < 143$. The carbon-oxygen double bond is shorter than the carbon-oxygen single bond because the greater bond order draws the nuclei closer. The carbon-nitrogen triple bond is shorter still, but remains longer than the carbon-hydrogen bond.

Consequently A must be first and B last. Option (A) incorrectly puts the carbon-oxygen single bond before both multiple bonds. Option (C) incorrectly makes the carbon-hydrogen bond the longest, while option (D) incorrectly places it after the carbon-oxygen double bond. Only option (B) respects all four characteristic distances.

Step 3: Conclusion The increasing covalent bond-length order is $\mathrm{A} < \mathrm{D} < \mathrm{C} < \mathrm{B}$. Hence, the correct option is (B).

Quick Tip to Solve: Remember the approximate sequence $109, 116, 121, 143$ pm; do not rank unlike atom pairs using bond order alone.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The first and second ionization constants of H_2X are 2.5×10^{-8} and 1.0×10^{-13} respectively. The concentration of X^{2-} in 0.1 M H_2X solution is _____ $\times 10^{-15}$ M. (Nearest Integer) [4 marks]

Solution:

Concept:

- A diprotic acid dissociates in two stages: $H_2X \rightleftharpoons H^+ + HX^-$ and $HX^- \rightleftharpoons H^+ + X^{2-}$. Their constants are K_{a1} and K_{a2}, respectively.
- When $K_{a2} \ll K_{a1}$, the first stage supplies nearly all the hydrogen ions. For a weak acid of initial concentration C, $[H^+] \simeq [HX^-] \simeq \sqrt{K_{a1}C}$.

Step 1: Determine the dominant first-stage concentrations Here $K_{a1} = 2.5 \times 10^{-8}$, $K_{a2} = 1.0 \times 10^{-13}$, and $C = 0.1\,\text{mol L}^{-1}$. If x is the concentration ionised in the first stage, then

$$K_{a1} = \frac{x^2}{C - x}, \qquad x \simeq \sqrt{(2.5 \times 10^{-8})(0.1)} = 5.0 \times 10^{-5}\,\text{M}.$$

The fractional ionisation is only $x/C = 5.0 \times 10^{-4}$, so neglecting x beside C is justified. The second stage is much weaker and contributes negligibly to the hydrogen-ion concentration. Thus both $[H^+]$ and $[HX^-]$ are approximately $5.0 \times 10^{-5}\,\text{M}$.

Step 2: Apply the second equilibrium and express the requested coefficient The second dissociation constant gives

$$K_{a2} = \frac{[H^+][X^{2-}]}{[HX^-]}, \qquad [X^{2-}] = K_{a2}\frac{[HX^-]}{[H^+]}.$$

Since the concentration ratio is essentially unity,

$$[X^{2-}] \simeq 1.0 \times 10^{-13}\,\text{M} = 100 \times 10^{-15}\,\text{M}.$$

This is self-consistent: the doubly charged ion concentration is vastly smaller than the first-stage concentrations. Small corrections for further ionisation or water do not change the requested nearest integer. The blank asks for the coefficient of 10^{-15}, not for the exponent or the concentration in unscaled units.

Step 3: Conclusion The concentration is approximately $100 \times 10^{-15}\,\text{M}$; the nearest-integer entry is $\boxed{100}$.

Quick Tip to Solve: For a weak diprotic acid with well-separated dissociation constants, use $[X^{2-}] \simeq K_{a2}$, then convert to the power of ten printed beside the blank.

72. The osmotic pressure of a living cell is 12 atm at 300 K. The strength of sodium chloride solution that is isotonic with the living cell at this temperature is _____ g L^{-1}. (Nearest integer)

Given : R = 0.08 L atm K^{-1} mol^{-1}

Assume complete dissociation of NaCl

(Given: Molar mass of Na and Cl are 23 and 35.5 g mol^{-1} respectively.) [4 marks]

Solution:

Concept:

- Isotonic solutions have equal osmotic pressures at the same temperature. For a dilute electrolyte solution, $\pi = iCRT$, where C is the formula-unit molarity and i is the van't Hoff factor.
- Complete dissociation of $\mathrm{NaCl} \longrightarrow \mathrm{Na^+} + \mathrm{Cl^-}$ produces two ions per formula unit, so $i = 2$. Mass concentration is molarity multiplied by the solute molar mass.

Step 1: Relate isotonicity to sodium chloride molarity The sodium chloride solution must exert the same pressure as the cell, namely $\pi = 12\,\text{atm}$ at $T = 300\,\text{K}$. Each mole of sodium chloride supplies one mole of sodium ions and one mole of chloride ions. Consequently a solution of formula-unit concentration C has total solute-particle concentration $2C$. Substituting the supplied gas constant directly,

$$C = \frac{\pi}{iRT} = \frac{12}{2 \times 0.08 \times 300} = 0.25\,\text{mol L}^{-1}.$$

The pressure unit atm cancels that in R, and the kelvin units cancel too, leaving molarity. Using $i = 1$ would neglect dissociation and incorrectly double the required amount of salt.

Step 2: Convert molarity into strength and round once The given atomic molar masses give

$$M_{\mathrm{NaCl}} = 23 + 35.5 = 58.5\,\text{g mol}^{-1}.$$

Strength means grams of sodium chloride in one litre of solution, so

$$s = CM_{\mathrm{NaCl}} = 0.25 \times 58.5 = 14.625\,\text{g L}^{-1}.$$

As a check, this mass corresponds to 0.25 mole of salt and hence 0.50 mole of dissolved ions per litre. Their osmotic pressure is $0.50 \times 0.08 \times 300 = 12\,\text{atm}$, as required. Keep the unrounded mass concentration until the final nearest-integer instruction is applied.

Step 3: Conclusion The required strength is $14.625\,\text{g L}^{-1}$, or $\boxed{15\,\text{g L}^{-1}}$ to the nearest integer. The numerical entry is 15.

Quick Tip to Solve: Use $s = \pi M/(iRT)$ directly for strength; for fully dissociated sodium chloride, remember the factor $i = 2$.

73. Identify the metal ions among Co^{2+}, Ni^{2+}, Fe^{2+}, V^{3+} and Ti^{2+} having a spin-only magnetic moment value more than 3.0 BM. The sum of unpaired electrons present in the high spin octahedral complexes formed by those metal ions is _____ . [4 marks]

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons.
- In a high-spin octahedral complex, electrons occupy the three t_{2g} and two e_g orbitals with maximum unpaired spins before pairing as required. For transition-metal cations, remove the outer $4s$ electrons before $3d$ electrons.

Step 1: Determine the d-electron counts Cobalt, nickel, iron, vanadium and titanium have atomic numbers 27, 28, 26, 23 and 22 respectively. Removing the electrons indicated by each positive charge gives

$$Co^{2+} : 3d^7, \quad Ni^{2+} : 3d^8, \quad Fe^{2+} : 3d^6,$$
$$V^{3+} : 3d^2, \quad Ti^{2+} : 3d^2.$$

The high-spin assumption matters particularly for the iron and cobalt ions: weak octahedral splitting permits occupation of the upper orbitals before additional pairing.

Step 2: Count unpaired electrons and apply the threshold The high-spin octahedral configurations and moments are

Ion	Configuration	n	μ (BM)
Co^{2+}	$t_{2g}^5 e_g^2$	3	$\sqrt{15} = 3.87$
Ni^{2+}	$t_{2g}^6 e_g^2$	2	$\sqrt{8} = 2.83$
Fe^{2+}	$t_{2g}^4 e_g^2$	4	$\sqrt{24} = 4.90$
V^{3+}	$t_{2g}^2 e_g^0$	2	$\sqrt{8} = 2.83$
Ti^{2+}	$t_{2g}^2 e_g^0$	2	$\sqrt{8} = 2.83$

Thus only the cobalt and iron ions exceed 3.0 BM. Each of the other three has two unpaired electrons and falls below the stated cutoff. Add the unpaired-electron counts of the selected ions, not their magnetic moments.

Step 3: Conclusion The selected ions are Co^{2+} and Fe^{2+}; the required sum is $3 + 4 = \boxed{7}$.

Quick Tip to Solve: For integer unpaired-electron counts, a spin-only moment above 3 BM requires at least three unpaired electrons; screen the d configurations before adding their counts.

74. A substance 'X' (1.5 g) dissolved in 150 g of a solvent 'Y' (molar mass = 300 g mol^{-1}) led to an elevation of the boiling point by 0.5 K. The relative lowering in the vapour pressure of the solvent 'Y' is _____ $\times 10^{-2}$. (nearest integer)

[Given: K_b of the solvent = 5.0 K kg mol^{-1}]

Assume the solution to be dilute and no association or dissociation of X takes place in solution.

[4 marks]

Solution:

Concept:

- Boiling-point elevation obeys $\Delta T_b = iK_b m$, where m is the molality in moles of solute per kilogram of solvent. With no association or dissociation, the van't Hoff factor is $i = 1$.
- For a dilute solution of a nonvolatile solute, Raoult's law gives the relative lowering of solvent vapour pressure as $(p_Y^0 - p_Y)/p_Y^0 = x_X = n_X/(n_X + n_Y)$, where n_X, n_Y are solute and solvent amounts.

Step 1: Find the amount of solute from boiling-point elevation Use the given elevation and ebullioscopic constant to obtain

$$m = \frac{\Delta T_b}{K_b} = \frac{0.5}{5.0} = 0.10\,\text{mol kg}^{-1}.$$

The solvent mass is $150\,\text{g} = 0.150\,\text{kg}$, so

$$n_X = m \times 0.150 = 0.015\,\text{mol}.$$

The stated solute mass would therefore correspond to a molar mass $1.5/0.015 = 100\,\text{g mol}^{-1}$. Determining this molar mass separately is unnecessary because the colligative measurement already supplies the amount of solute.

Step 2: Calculate the solvent amount and mole fraction The molar mass $300\,\text{g mol}^{-1}$ belongs to the solvent. Hence

$$n_Y = \frac{150}{300} = 0.500\,\text{mol}.$$

Consequently the relative lowering is

$$\frac{p_Y^0 - p_Y}{p_Y^0} = \frac{0.015}{0.500 + 0.015} = 0.029126\ldots = 2.9126\ldots \times 10^{-2}.$$

In the dilute approximation, replacing the denominator by n_Y gives $n_X/n_Y = 0.030 = 3.0 \times 10^{-2}$. Both treatments give the same nearest-integer coefficient. Relative lowering is dimensionless; it is not the absolute decrease in vapour pressure.

Step 3: Conclusion The relative lowering is approximately $\boxed{3 \times 10^{-2}}$, so the required nearest-integer entry is $\boxed{3}$.

Quick Tip to Solve: Combine $m = \Delta T_b/K_b$ with the solvent molar mass in kg per mole: the dilute relative lowering is $mM_Y = 0.10 \times 0.300 = 0.030$.

75. MX is a sparingly soluble salt that follows the given solubility equilibrium at 298 K.

$$\mathrm{MX(s)} \rightleftharpoons \mathrm{M^+(aq) + X^-(aq)}; \quad \mathrm{K_{sp}} = 10^{-10}$$

If the standard reduction potential for M^+ (aq) + e^- → M(s) is $\left(\mathrm{E^{\ominus}_{M^+/M}}\right) = 0.79$ V, then the value of the standard reduction potential for the metal/metal insoluble salt electrode $\mathrm{E^{\ominus}_{X^-/MX(s)/M}}$ is _____ mV. (nearest integer)

[Given: $\frac{2.303RT}{F} = 0.059$ V] [4 marks]

Solution:

Concept:

- For $\mathrm{M^+ + e^- \longrightarrow M(s)}$, the Nernst equation is $E = E^\circ_{\mathrm{M^+/M}} + (RT/F)\ln a_{\mathrm{M^+}}$, since the pure solid has unit activity and one electron is transferred.
- In equilibrium with the sparingly soluble solid, $K_{\mathrm{sp}} = a_{\mathrm{M^+}} a_{\mathrm{X^-}}$. Eliminating the metal-ion activity gives the potential of the metal/insoluble-salt electrode.

Step 1: Write the electrode reaction and eliminate the metal-ion activity Add the dissolution equilibrium to the metal-ion reduction:

$$\mathrm{MX(s) \rightleftharpoons M^+(aq) + X^-(aq)},$$
$$\mathrm{M^+(aq) + e^- \longrightarrow M(s)}.$$

The net reduction is $\mathrm{MX(s) + e^- \longrightarrow M(s) + X^-(aq)}$. Since $a_{\mathrm{M^+}} = K_{\mathrm{sp}}/a_{\mathrm{X^-}}$, substitution in the Nernst equation yields

$$E = E^\circ_{\mathrm{M^+/M}} + \frac{RT}{F}\ln K_{\mathrm{sp}} - \frac{RT}{F}\ln a_{\mathrm{X^-}}.$$

The standard potential of the new electrode is the part independent of the anion activity, corresponding to $a_{\mathrm{X^-}} = 1$. This does not mean setting the equilibrium metal-ion activity to unity as well; it is then fixed by the solubility product.

Step 2: Evaluate the standard potential Comparing with the Nernst form for the net reduction gives

$$\begin{aligned} E^\circ_{\text{salt electrode}} &= E^\circ_{\mathrm{M^+/M}} + \frac{2.303RT}{F}\log_{10} K_{\mathrm{sp}} \\ &= 0.79 + 0.059\log_{10}(10^{-10}) \\ &= 0.79 - 0.590 = 0.200\,\mathrm{V}. \end{aligned}$$

The correction is negative because the very small solubility product lowers the available metal-ion activity. The electron number is one, so there is no extra factor of two in the denominator. Finally, multiply volts by 1000 to express the potential in millivolts.

Step 3: Conclusion The standard reduction potential of the metal/metal insoluble salt electrode is $\boxed{200\,\mathrm{mV}}$; the nearest-integer entry is $\boxed{200}$.

Quick Tip to Solve: For a 1:1 salt with a monovalent metal, use $E^\circ_{\mathrm{salt}} = E^\circ_{\mathrm{M^+/M}} + 0.059\log_{10} K_{\mathrm{sp}}$; a small solubility product lowers the reduction potential.