

JEE Main 2026 Paper 1 (B.E./B.Tech) Jan 22 Shift 2

Question Paper with Solutions PDF



Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours

Maximum Marks: 300

General Instructions

- (i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2026, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 22 January 2026, Shift 2. The total duration is 3 hours.
- (ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.
- (iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.
- (iv) Marking scheme (multiple choice questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (v) Marking scheme (numerical value questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Among the statements

(S1) : If $A(5, -1)$ and $B(-2, 3)$ are two vertices of a triangle, whose orthocentre is $(0, 0)$, then its third vertex is $(-4, -7)$

and

(S2) : If positive numbers $2a, b, c$ are three consecutive terms of an A.P., then the lines $ax + by + c = 0$ are concurrent at $(2, -2)$, [4 marks]

(A) both are correct

(B) both are incorrect

(C) only (S1) is correct

(D) only (S2) is correct

Correct Answer: (A) both are correct

Solution:

Concept:

- An altitude of a triangle passes through a vertex and is perpendicular to the opposite side. The orthocentre is the common point of its altitudes.
- For three consecutive terms of an arithmetic progression, twice the middle term equals the sum of the other two. A fixed point satisfying every member of a family of lines is their point of concurrence.

Step 1: Find the third vertex using the altitudes Write the third vertex as $C(u, v)$ and the orthocentre as $H(0, 0)$. Since $AH \perp BC$ and $BH \perp AC$, the corresponding scalar products vanish:

$$(5, -1) \cdot (u + 2, v - 3) = 0,$$

$$(-2, 3) \cdot (u - 5, v + 1) = 0.$$

Thus $5u - v + 13 = 0$ and $-2u + 3v + 13 = 0$. Substitute $v = 5u + 13$ into the second equation:

$$13u + 52 = 0 \Rightarrow u = -4, \quad v = -7.$$

The two altitude conditions therefore give exactly the vertex stated in (S1), so (S1) is true.

Step 2: Test the proposed common point Because $2a, b, c$ are consecutive terms of an A.P.,

$$2b = 2a + c \Rightarrow 2a - 2b + c = 0.$$

At $(x, y) = (2, -2)$, the left side of every line $ax + by + c = 0$ is $2a - 2b + c$, which is zero by this identity. Hence all these lines pass through $(2, -2)$, so (S2) is true as well. Options (C) and (D) each incorrectly reject one true statement, while option (B) incorrectly rejects both.

Step 3: Conclusion Both (S1) and (S2) are correct. Hence, the correct option is (A).

Quick Tip to Solve: Use dot products for altitude conditions; for concurrence, substitute the suggested point and compare the result with the relation between the coefficients.

2. Let n be the number obtained on rolling a fair die. If the probability that the system

$$x - ny + z = 6$$

$$x + (n - 2)y + (n + 1)z = 8$$

$$(n - 1)y + z = 1$$

has a unique solution is $\frac{k}{6}$, then the sum of k and all possible values of n is :

[4 marks]

(A) 20

(B) 21

(C) 22

(D) 24

Correct Answer: (C) 22

Solution:

Concept:

- A square linear system has a unique solution precisely when the determinant of its coefficient matrix is nonzero. This criterion does not depend on the constant terms.
- A fair die has six equally likely outcomes. If exactly k outcomes satisfy a condition, its probability is $k/6$.

Step 1: Compute and factor the determinant The coefficient determinant is

$$D = \begin{vmatrix} 1 & -n & 1 \\ 1 & n-2 & n+1 \\ 0 & n-1 & 1 \end{vmatrix}.$$

Subtract the first row from the second, which leaves the determinant unchanged. Expanding along the first column then gives

$$\begin{aligned} D &= \begin{vmatrix} 2n-2 & n \\ n-1 & 1 \end{vmatrix} \\ &= 2n-2 - n(n-1) \\ &= (n-1)(2-n). \end{aligned}$$

Thus the determinant vanishes only at $n = 1$ or $n = 2$. Neither outcome can yield a unique solution, irrespective of whether the corresponding singular system is consistent.

Step 2: Count the admissible die outcomes and form the sum The die outcomes are 1, 2, 3, 4, 5, 6. Removing the two singular cases leaves

$$n \in \{3, 4, 5, 6\}.$$

There are four favourable outcomes, so the probability of a unique solution is $4/6$, giving $k = 4$. The possible values of n here are the values for which the system is uniquely solvable; their sum is

$$3 + 4 + 5 + 6 = 18.$$

Therefore the requested sum is $k + 18 = 4 + 18 = 22$. The other offered totals, 20, 21 and 24, do not equal the sum of the four admissible outcomes plus their count.

Step 3: Conclusion The required sum is $\boxed{22}$. Hence, the correct option is (C).

Quick Tip to Solve: Factor the determinant before checking individual die outcomes. Count only the nonsingular outcomes, then add that count to their sum.

3. Let the domain of the function $f(x) = \log_3 \log_5 (7 - \log_2 (x^2 - 10x + 85)) + \sin^{-1} \left(\left| \frac{3x-7}{17-x} \right| \right)$ be $(\alpha, \beta]$. Then $\alpha + \beta$ is equal to : [4 marks]

- (A) 8
- (B) 9
- (C) 10
- (D) 12

Correct Answer: (B) 9

Solution:

Concept:

- A real logarithm requires a strictly positive argument. When its base exceeds 1, its value is positive exactly when its argument exceeds 1.
- The real inverse sine requires its argument to lie in $[-1, 1]$. For an absolute-value argument, this reduces to an upper bound of 1; intersect all domain conditions.

Step 1: Impose both logarithm conditions Let $Q = x^2 - 10x + 85 = (x - 5)^2 + 60$, which is positive for every real x . The argument of the outer logarithm is $\log_5(7 - \log_2 Q)$, so it must be positive. As $5 > 1$, this is equivalent to

$$7 - \log_2 Q > 1 \Rightarrow \log_2 Q < 6.$$

Together with $Q > 0$, this gives $Q < 64$. Therefore

$$(x - 5)^2 + 60 < 64 \Rightarrow |x - 5| < 2,$$

so the logarithmic term is defined precisely for $3 < x < 7$. The endpoints are excluded because the outer logarithm would have argument zero.

Step 2: Impose the inverse-sine condition and intersect We require $x \neq 17$ and

$$\left| \frac{3x - 7}{17 - x} \right| \leq 1.$$

Squaring the nonnegative sides after multiplying by $|17 - x|$ gives

$$\begin{aligned} (3x - 7)^2 &\leq (17 - x)^2, \\ (4x - 24)(2x + 10) &\leq 0, \\ (x - 6)(x + 5) &\leq 0. \end{aligned}$$

Hence $-5 \leq x \leq 6$; the forbidden denominator value 17 is outside this interval. Intersecting with $(3, 7)$ yields $(3, 6]$. Thus $\alpha = 3$, $\beta = 6$, and their sum is 9. The values 8, 10 and 12 fail this endpoint calculation.

Step 3: Conclusion The domain is $(3, 6]$, so $\alpha + \beta = \boxed{9}$. Hence, the correct option is (B), using the printed official option order.

Quick Tip to Solve: For a logarithm of a logarithm, impose positivity on the inner logarithm itself, not merely on its argument; then intersect with the trigonometric domain.

4. Let $[\cdot]$ denote the greatest integer function, and let $f(x) = \min \{\sqrt{2}x, x^2\}$.

Let $S = \{x \in (-2, 2) : \text{the function } g(x) = |x| [x^2] \text{ is discontinuous at } x\}$.

Then $\sum_{x \in S} f(x)$ equals

[4 marks]

(A) $2 - \sqrt{2}$

(B) $1 - \sqrt{2}$

(C) $\sqrt{6} - 2\sqrt{2}$

(D) $2\sqrt{6} - 3\sqrt{2}$

Correct Answer: (B) $1 - \sqrt{2}$

Solution:

Concept:

- The greatest-integer function jumps when its argument crosses an integer. Multiplication by a continuous, nonzero factor preserves a nonzero jump.
- To evaluate a minimum of two expressions, compare their values at each required input. For negative x , $\sqrt{2}x < 0 \leq x^2$, so the linear expression is always the minimum.

Step 1: Locate every discontinuity in the open interval For $-2 < x < 2$, we have $0 \leq x^2 < 4$. The positive integer levels crossed by x^2 are therefore 1, 2 and 3. They occur at

$$S = \{-\sqrt{3}, -\sqrt{2}, -1, 1, \sqrt{2}, \sqrt{3}\}.$$

At each point, $|x|$ is positive and continuous, so it cannot remove the jump in $[x^2]$. Although $x^2 = 0$ at the origin, $[x^2] = 0$ throughout a neighbourhood of zero; hence g is continuous there. The values ± 2 lie outside the specified open interval and are not included.

Step 2: Evaluate the minimum on the negative and positive points At the three negative points, $f(x) = \sqrt{2}x$, giving

$$f(-1) = -\sqrt{2}, \quad f(-\sqrt{2}) = -2, \quad f(-\sqrt{3}) = -\sqrt{6}.$$

For positive x , the inequality $x^2 \leq \sqrt{2}x$ holds exactly when $x \leq \sqrt{2}$. Consequently

$$f(1) = 1, \quad f(\sqrt{2}) = 2, \quad f(\sqrt{3}) = \sqrt{6}.$$

Adding all six values gives

$$-\sqrt{2} - 2 - \sqrt{6} + 1 + 2 + \sqrt{6} = 1 - \sqrt{2}.$$

The $\sqrt{6}$ and constant 2 contributions cancel in pairs. Thus options retaining a $\sqrt{6}$ term, or replacing the surviving constant 1 by 2, are incorrect.

Step 3: Conclusion The required sum is $1 - \sqrt{2}$. Hence, the correct option is (B).

Quick Tip to Solve: List integer levels of the inner function first. For symmetric discontinuities, pair $f(t)$ with $f(-t)$ to expose cancellations.

5. If the mean deviation about the median of the numbers $k, 2k, 3k, \dots, 1000k$ is 500, then k^2 is equal to : [4 marks]

- (A) 9
- (B) 16
- (C) 4
- (D) 1

Correct Answer: (C) 4

Solution:

Concept:

- For an even number of ordered observations, the median is the average of the two central observations. Reversing the order does not change this average.
- Mean deviation about a median M is the arithmetic mean of $|x_i - M|$. Scaling every observation by k scales absolute deviations by $|k|$, not by k^2 .

Step 1: Express the median and absolute deviations There are 1000 observations $k, 2k, \dots, 1000k$. For positive k , the middle observations are $500k$ and $501k$. For negative k , the ordering reverses but the same two observations are central. In either case a median is

$$M = \frac{500k + 501k}{2} = 500.5k.$$

Thus the absolute deviation of the observation jk is

$$|jk - M| = |k| |j - 500.5|.$$

The case $k = 0$ would give zero mean deviation and cannot satisfy the given value 500.

Step 2: Sum the symmetric deviations and solve for the scale The 1000 numbers $|j - 500.5|$ occur in equal pairs. Their distinct values are

$$0.5, 1.5, 2.5, \dots, 499.5.$$

This arithmetic progression has 500 terms and sum

$$\frac{500}{2}(0.5 + 499.5) = 125000.$$

Therefore the sum over all 1000 observations is 250000, and the mean deviation is

$$\frac{|k|}{1000}(250000) = 250|k|.$$

Equating this to 500 yields $|k| = 2$, hence $k^2 = 4$. The other squared values 9, 16 and 1 would give $|k| = 3, 4, 1$, and mean deviations 750, 1000 and 250 respectively, none equal to 500.

Step 3: Conclusion The required value is $k^2 = 4$. Hence, the correct option is (C), using the printed official option order.

Quick Tip to Solve: In a scaled data set, factor $|k|$ out of the absolute deviations. Pair observations equally far from the median before summing.

6. Let $P(10, 2\sqrt{15})$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, whose foci are S and S' . If the length of its latus rectum is 8, then the square of the area of $\triangle PSS'$ is equal to : [4 marks]

- (A) 900
- (B) 2700
- (C) 1462
- (D) 4200

Correct Answer: (B) 2700

Solution:

Concept:

- For $x^2/a^2 - y^2/b^2 = 1$, with $a, b > 0$, the foci are $(\pm c, 0)$, where $c^2 = a^2 + b^2$. The full length of a latus rectum is $2b^2/a$.
- The segment joining the foci lies on the x -axis. A triangle having this segment as base and a vertex (x, y) therefore has area $\frac{1}{2}(2c)|y| = c|y|$.

Step 1: Use the latus rectum and the point on the hyperbola The given latus-rectum length gives

$$\frac{2b^2}{a} = 8 \Rightarrow b^2 = 4a.$$

Since $P(10, 2\sqrt{15})$ lies on the hyperbola, its coordinates satisfy the equation:

$$\frac{100}{a^2} - \frac{60}{b^2} = 1.$$

Substituting $b^2 = 4a$ and multiplying by a^2 yields

$$100 - 15a = a^2 \Rightarrow (a + 20)(a - 5) = 0.$$

The semitransverse-axis length must be positive, so $a = 5$, not -20 . Thus $a^2 = 25$ and $b^2 = 20$. Substitution checks the point: $100/25 - 60/20 = 4 - 3 = 1$.

Step 2: Find the focal base and square the triangle area The focal parameter obeys

$$c^2 = a^2 + b^2 = 25 + 20 = 45.$$

The base SS' is $2c$, and the perpendicular distance of P from the focal axis is $2\sqrt{15}$. If the area is Δ , then

$$\Delta = \frac{1}{2}(2c)(2\sqrt{15}) = 2c\sqrt{15}, \quad \Delta^2 = 60c^2 = 60(45) = 2700.$$

The horizontal coordinate 10 is needed to determine the hyperbola but is not the triangle's height. The other offered values 900, 1462 and 4200 do not satisfy this base-height calculation.

Step 3: Conclusion The square of the area of $\triangle PSS'$ is $\boxed{2700}$. Hence, the correct option is (B).

Quick Tip to Solve: For a focal triangle of a horizontal hyperbola, use $\text{area}^2 = c^2 y_P^2$. This avoids taking an unnecessary square root of c^2 .

7. The area of the region $A = \{(x, y) : 4x^2 + y^2 \leq 8 \text{ and } y^2 \leq 4x\}$ is:

[4 marks]

- (A) $\frac{\pi}{2} + \frac{1}{3}$
- (B) $\pi + \frac{2}{3}$
- (C) $\frac{\pi}{2} + 2$
- (D) $\pi + 4$

Correct Answer: (B) $\pi + \frac{2}{3}$

Solution:

Concept:

- The required region is the intersection of the ellipse $4x^2 + y^2 \leq 8$ and the region $y^2 \leq 4x$. For a fixed y , the latter condition gives the left boundary $x = y^2/4$.
- The right boundary of the ellipse is $x = \frac{1}{2}\sqrt{8 - y^2}$. Since both inequalities involve y^2 , the enclosed region is symmetric about the x -axis; integrate its horizontal width and double the upper half.

Step 1: Find the boundary intersections At an intersection, substitute $x = y^2/4$ into the ellipse. Writing $u = y^2 \geq 0$, we obtain

$$\frac{u^2}{4} + u = 8, \quad u^2 + 4u - 32 = (u - 4)(u + 8) = 0.$$

Only $u = 4$ is admissible, so the intersection points are $(1, 2)$ and $(1, -2)$. The horizontal interval between the two curves exists precisely for $-2 \leq y \leq 2$.

Step 2: Integrate the horizontal widths The area is the integral of right boundary minus left boundary,

not their sum:

$$\begin{aligned} A &= 2 \int_0^2 \left(\frac{\sqrt{8-y^2}}{2} - \frac{y^2}{4} \right) dy \\ &= \int_0^2 \sqrt{8-y^2} dy - \frac{1}{2} \int_0^2 y^2 dy. \end{aligned}$$

Using the circular-segment antiderivative gives

$$\left[\frac{y}{2} \sqrt{8-y^2} + 4 \sin^{-1} \left(\frac{y}{2\sqrt{2}} \right) \right]_0^2 = 2 + \pi.$$

The second integral contributes $\frac{1}{2} \cdot \frac{8}{3} = \frac{4}{3}$. Hence $A = 2 + \pi - \frac{4}{3} = \pi + \frac{2}{3}$. Option (A) is exactly half this area and misses the lower half; (C) and (D) also disagree with the evaluated integral.

Step 3: Conclusion The required area is $\pi + \frac{2}{3}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: Use horizontal strips here: both curves give a single positive x -boundary, and symmetry reduces the integral to $0 \leq y \leq 2$.

8. Let the locus of the mid-point of the chord through the origin O of the parabola $y^2 = 4x$ be the curve S . Let P be any point on S . Then the locus of the point, which internally divides OP in the ratio $3 : 1$, is : [4 marks]

- (A) $2y^2 = 3x$
- (B) $3y^2 = 2x$
- (C) $2x^2 = 3y$
- (D) $3x^2 = 2y$

Correct Answer: (A) $2y^2 = 3x$

Solution:

Concept:

- A chord through the vertex $O = (0, 0)$ of $y^2 = 4x$ joins O to another point Q on the parabola. Parametrize Q as $(t^2, 2t)$, where t is real.
- The midpoint is obtained by averaging endpoint coordinates. A point dividing OP internally in the ratio $3 : 1$ lies three quarters of the way from O to P ; both its coordinates must be scaled by the same factor.

Step 1: Determine the midpoint curve Let $P = (h, k)$ be the midpoint of the chord OQ . Then

$$h = \frac{t^2}{2}, \quad k = t.$$

Eliminating t gives $k^2 = 2h$, the equation of the midpoint curve S . Equivalently, the other chord endpoint is $(2h, 2k)$, and substituting it in the original parabola gives $4k^2 = 8h$. This check avoids confusing the original parabola with the midpoint locus.

Step 2: Apply the internal division and eliminate the parameter Write the required dividing point as

$R = (x, y)$. Since $OR : RP = 3 : 1$, the section formula gives

$$x = \frac{3h}{4} = \frac{3t^2}{8}, \quad y = \frac{3k}{4} = \frac{3t}{4}.$$

Thus $t = 4y/3$, and substitution into the expression for x yields

$$x = \frac{3}{8} \left(\frac{4y}{3} \right)^2 = \frac{2}{3} y^2, \quad 2y^2 = 3x.$$

This is a right-opening parabola. Option (B) has the wrong scaling coefficient, while (C) and (D) interchange the squared and linear coordinates and therefore describe parabolas with the wrong axis. For a nondegenerate chord $t \neq 0$; the origin is the limiting point of the stated locus.

Step 3: Conclusion The required locus has equation $2y^2 = 3x$. Hence, the correct option is (A).

Quick Tip to Solve: When a locus is scaled from the origin, scale both coordinates first; a squared coordinate acquires the square of that scale factor.

9. If $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is a solution of the system of equations $AX = B$, where $\text{adj } A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$, then $|x + y + z|$ is equal to :

[4 marks]

- (A) 1
- (B) $\frac{3}{2}$
- (C) 2
- (D) 3

Correct Answer: (C) 2

Solution:

Concept:

- For a square matrix A , $(\text{adj } A)A = (\det A)I$. Multiplying $AX = B$ by the adjugate gives $(\det A)X = (\text{adj } A)B$.
- For a 3×3 matrix, $\det(\text{adj } A) = (\det A)^2$. The given adjugate therefore determines the magnitude, though not necessarily the sign, of $\det A$. The absolute value requested removes that sign ambiguity.

Step 1: Evaluate the determinant of the given adjugate Let $M = \text{adj } A$. Expanding its determinant along the first row gives

$$\begin{aligned} \det M &= 4(0 \cdot 3 - 5(-2)) \\ &\quad - 2((-5) \cdot 3 - 5 \cdot 1) \\ &\quad + 2((-5)(-2) - 0 \cdot 1) \\ &= 40 + 40 + 20 = 100. \end{aligned}$$

Hence $(\det A)^2 = 100$, so $\det A = \pm 10$. In particular, A is nonsingular and the system has a unique solution for each admissible A . It is unnecessary to reconstruct A itself.

Step 2: Multiply by the adjugate and sum the coordinates The needed matrix-vector product is

$$MB = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 20 \\ -10 \\ 10 \end{pmatrix}.$$

Consequently

$$x + y + z = \frac{20 - 10 + 10}{\det A} = \frac{20}{\det A}, \quad |x + y + z| = \frac{20}{10} = 2.$$

The two possible determinant signs reverse all coordinates together, but leave the required magnitude unchanged. Values 1, $3/2$, and 3 in the other options do not equal this exact ratio; using 100 as $\det A$ would also misuse the adjugate identity.

Step 3: Conclusion The required value is $|x + y + z| = 2$. Hence, the correct option is (C).

Quick Tip to Solve: Compute the determinant of the adjugate first, then multiply it by B . For an absolute-value answer, retain $|\det A|$ and avoid choosing an unsupported sign.

10. Let α, β be the roots of the quadratic equation $12x^2 - 20x + 3\lambda = 0, \lambda \in \mathbf{Z}$. If $\frac{1}{2} \leq |\beta - \alpha| \leq \frac{3}{2}$, then the sum of all possible values of λ is : [4 marks]

- (A) 1
- (B) 3
- (C) 4
- (D) 6

Correct Answer: The intended real-root interpretation gives 3, option (B); allowing complex roots as the printed wording does gives 12, which is unlisted.

Solution:

Concept:

- By Vieta's relations, $\alpha + \beta = 5/3$ and $\alpha\beta = \lambda/4$. Thus $(\beta - \alpha)^2 = (\alpha + \beta)^2 - 4\alpha\beta = 25/9 - \lambda$.
- For real roots, the square of their distance is $25/9 - \lambda$. For possibly complex roots, the modulus instead satisfies $|\beta - \alpha|^2 = |25/9 - \lambda|$. The printed question does not explicitly restrict the roots to be real, so these interpretations must be distinguished.

Step 1: Obtain the listed answer under the real-root interpretation If both roots are real, squaring the nonnegative bounds gives

$$\frac{1}{4} \leq \frac{25}{9} - \lambda \leq \frac{9}{4}.$$

Rearranging each side yields

$$\frac{19}{36} \leq \lambda \leq \frac{91}{36}.$$

The integer possibilities are 1 and 2; each also gives positive discriminant $400 - 144\lambda$. Their sum is 3,

the value in option (B). The other listed values 1, 4, 6 are not this sum. This is the intended real-root answer.

Step 2: Check the literal modulus condition without a real-root restriction Taking absolute values of the squared root difference gives

$$\frac{1}{4} \leq \left| \frac{25}{9} - \lambda \right| \leq \frac{9}{4}.$$

The positive branch gives $\lambda = 1, 2$ as above. The negative branch gives

$$\frac{1}{4} \leq \lambda - \frac{25}{9} \leq \frac{9}{4}, \quad \frac{109}{36} \leq \lambda \leq \frac{181}{36},$$

so $\lambda = 4, 5$ also qualify. For these two values the roots are conjugate complex numbers, and their difference has modulus $\sqrt{11}/3$ or $\sqrt{20}/3$, respectively. Both lie between $1/2$ and $3/2$. Thus the full literal set is $\{1, 2, 4, 5\}$, whose sum is 12, absent from all four choices. These are ordinary complex moduli, so the inequality alone cannot exclude them.

Step 3: Conclusion With the intended assumption that the roots are real, the sum is $\boxed{3}$, corresponding to option (B). As literally printed without that restriction, complex roots also allow $\lambda = 4, 5$, and the sum is $\boxed{12}$; hence no listed option covers the unrestricted question. The difference from the keyed real-root answer is established by the additional admissible complex-root cases.

Quick Tip to Solve: Before replacing $|\beta - \alpha|^2$ by $(\beta - \alpha)^2$, check that the roots are real. For complex roots keep the modulus around the squared difference.

11. Let C_r denote the coefficient of x^r in the binomial expansion of $(1+x)^n$, $n \in \mathbb{N}$, $0 \leq r \leq n$. If

$P_n = C_0 - C_1 + \frac{2^2}{3}C_2 - \frac{2^3}{4}C_3 + \dots + \frac{(-2)^n}{n+1}C_n$, then the value of $\sum_{n=1}^{25} \frac{1}{P_{2n}}$ equals. [4 marks]

- (A) 675
(B) 525
(C) 650
(D) 580

Correct Answer: (A) 675

Solution:

Concept:

- The coefficient C_r is $\binom{n}{r}$. Since $\int_0^1 t^r dt = 1/(r+1)$, denominators $r+1$ in a binomial sum suggest integrating its generating polynomial.
- Here the numerator $(-2)^r$ shows that the relevant polynomial is $(1-2t)^n$. Its $r=1$ integrated term is $-2C_1/2 = -C_1$, agreeing with the separately written second term.

Step 1: Express the finite sum as an integral The binomial theorem gives

$$(1-2t)^n = \sum_{r=0}^n C_r (-2)^r t^r.$$

Termwise integration is valid because this is a finite polynomial. Therefore

$$P_n = \sum_{r=0}^n \frac{(-2)^r}{r+1} C_r = \int_0^1 (1-2t)^n dt.$$

Substitute $u = 1 - 2t$, so $dt = -du/2$ and the endpoints become 1 and -1 . Reversing the limits yields

$$P_n = \frac{1}{2} \int_{-1}^1 u^n du = \frac{1 - (-1)^{n+1}}{2(n+1)}.$$

Step 2: Use even indices and add the resulting progression For the requested even index $2n$, the power $2n + 1$ is odd. Thus

$$P_{2n} = \frac{1 - (-1)^{2n+1}}{2(2n+1)} = \frac{1}{2n+1}, \quad \frac{1}{P_{2n}} = 2n+1.$$

The reciprocal sum is consequently

$$\sum_{n=1}^{25} \frac{1}{P_{2n}} = 2 \sum_{n=1}^{25} n + 25 = 2 \cdot \frac{25 \cdot 26}{2} + 25 = 675.$$

It is the sum of the 25 odd numbers from 3 to 51, also equal to $25(3 + 51)/2$. Option (C), 650, omits the added 1 in each of the 25 terms; (B) and (D) likewise fail the exact progression sum.

Step 3: Conclusion The value of the sum is $\boxed{675}$. Hence, the correct option is (A).

Quick Tip to Solve: A binomial coefficient divided by $r + 1$ is an integration cue. Convert the sum to a polynomial integral before substituting an even or odd index.

12. The number of elements in the relation $R = \{(x, y) : 4x^2 + y^2 < 52, x, y \in \mathbf{Z}\}$ is

[4 marks]

- (A) 67
- (B) 77
- (C) 86
- (D) 89

Correct Answer: (B) 77

Solution:

Concept:

- An element of the relation is an ordered pair of integers (x, y) . Count all permitted y -values for each allowed x , keeping positive, negative and zero values distinct.
- The strict inequality $4x^2 + y^2 < 52$ excludes boundary points. For a fixed x , first find the largest nonnegative integer m with $m^2 < 52 - 4x^2$; there are then $2m + 1$ choices for y .

Step 1: Find the allowable horizontal coordinates Since $y^2 \geq 0$, a necessary condition is $4x^2 < 52$, or

$x^2 < 13$. Therefore

$$x \in \{-3, -2, -1, 0, 1, 2, 3\}.$$

The bound depends only on x^2 , so opposite values of x contribute equally. We can count $x = 0, 1, 2, 3$ and double the last three contributions. Values $|x| \geq 4$ already contribute at least 64 to the left side and are impossible.

Step 2: Count each vertical slice and add For $x = 0$, the condition is $y^2 < 52$, giving $y = -7, -6, \dots, 7$: 15 pairs. For $x = \pm 1$, it becomes $y^2 < 48$, giving $-6 \leq y \leq 6$: 13 pairs for each sign. For $x = \pm 2$, it becomes $y^2 < 36$, so $-5 \leq y \leq 5$: 11 pairs for each sign. The values $y = \pm 6$ are excluded because equality is forbidden. For $x = \pm 3$, it becomes $y^2 < 16$, so $-3 \leq y \leq 3$: 7 pairs for each sign, excluding $y = \pm 4$. Thus the total is

$$|R| = 15 + 2(13 + 11 + 7) = 15 + 62 = 77.$$

Every allowed pair belongs to exactly one of these slices, so there is no double counting. The counts 67, 86, and 89 in the other options disagree with this complete enumeration; moreover the total must be odd because the origin is unpaired under $(x, y) \mapsto (-x, -y)$.

Step 3: Conclusion The relation has $\boxed{77}$ elements. Hence, the correct option is (B).

Quick Tip to Solve: In strict lattice inequalities, inspect perfect-square bounds separately: $y^2 < 36$ permits only $|y| \leq 5$, not $|y| \leq 6$.

13. Let $f(x) = [x]^2 - [x + 3] - 3$, $x \in \mathbf{R}$, where $[.]$ is the greatest integer function. Then

[4 marks]

- (A) $f(x) = 0$ for finitely many values of x
- (B) $f(x) < 0$ only for $x \in [-1, 3)$
- (C) $f(x) > 0$ only for $x \in [4, \infty)$
- (D) $\int_0^2 f(x) dx = -6$

Correct Answer: (B) $f(x) < 0$ only for $x \in [-1, 3)$

Solution:

Concept:

- For an integer shift, $[x + 3] = [x] + 3$. Write $n = [x]$, so that $n \leq x < n + 1$.
- The sign of a factored quadratic is determined by its two roots; the integer restriction on n then determines the intervals in x .

Step 1: Reduce the function to an integer quadratic Substitute the shift identity before studying the sign. This gives

$$f(x) = n^2 - (n + 3) - 3 = n^2 - n - 6 = (n - 3)(n + 2).$$

The roots of this quadratic in n are -2 and 3 , and it is negative strictly between these roots.

Step 2: Translate the negative integer values into intervals The integers satisfying $-2 < n < 3$ are

$-1, 0, 1, 2$. Their corresponding intervals are $[-1, 0), [0, 1), [1, 2), [2, 3)$. Taking the union gives

$$f(x) < 0 \iff x \in [-1, 3).$$

At $x = -1$ the value is negative, whereas at $x = 3$ it is zero, explaining the different endpoint brackets.

Step 3: Check the remaining statements The zero set is $[-2, -1) \cup [3, 4)$, which contains infinitely many real numbers; hence (A) is false. The positive set is $(-\infty, -2) \cup [4, \infty)$, so (C) misses the negative half-line. Finally, on each of $[0, 1)$ and $[1, 2)$, the value is -6 . Consequently

$$\int_0^2 f(x) dx = -6(1) - 6(1) = -12,$$

not -6 ; thus (D) is false.

Step 4: Conclusion The negative set is exactly $[-1, 3)$. Hence, the correct option is (B).

Quick Tip to Solve: For a floor-function polynomial, first solve in the integer $n = [x]$, then replace each allowed integer by its interval $[n, n + 1)$.

14. Let $S = \{z \in \mathbb{C} : 4z^2 + \bar{z} = 0\}$. Then $\sum_{z \in S} |z|^2$ is equal to:

[4 marks]

- (A) $\frac{5}{64}$
- (B) $\frac{3}{16}$
- (C) $\frac{1}{16}$
- (D) $\frac{7}{64}$

Correct Answer: (B) $\frac{3}{16}$

Solution:

Concept:

- The modulus of a product is the product of the moduli, and $|\bar{z}| = |z|$.
- For $z = re^{i\theta}$, the conjugate is $re^{-i\theta}$. This separates the magnitude condition from the possible arguments.

Step 1: Separate the zero and nonzero solutions The equation $4z^2 + \bar{z} = 0$ clearly has $z = 0$. For a nonzero solution, taking moduli in $4z^2 = -\bar{z}$ yields

$$4|z|^2 = |z|, \quad |z| = \frac{1}{4}.$$

Division by $|z|$ is valid only after separating the zero solution. Thus every nonzero solution contributes $1/16$ to the required sum.

Step 2: Count the possible arguments Put $z = \frac{1}{4}e^{i\theta}$ in the original equation. Multiplying by $4e^{i\theta}$ gives

$$e^{3i\theta} + 1 = 0.$$

Therefore $3\theta = (2k + 1)\pi$. Modulo 2π , the three distinct arguments are $\pi/3, \pi, 5\pi/3$. They give three distinct nonzero roots, with no others possible. Explicitly these roots are $-1/4$ and $(1 \pm i\sqrt{3})/8$.

Step 3: Add all squared moduli Including the zero root, the sum is

$$\sum_{z \in S} |z|^2 = 0 + 3 \left(\frac{1}{4}\right)^2 = \frac{3}{16}.$$

The value $1/16$ counts only one nonzero root. The values $5/64$ and $7/64$ are also excluded because the sum is exactly $12/64$. Counting arguments is essential: the modulus equation alone does not say there is only one root.

Step 4: Conclusion The required sum is $3/16$, which is the second printed option. Hence, the correct option is (B).

Quick Tip to Solve: After using moduli, return to the argument equation to count distinct roots; equal moduli do not mean equal complex numbers.

15. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \lambda\hat{j} + 2\hat{k}$, $\lambda \in \mathbb{Z}$ be two vectors. Let $\vec{c} = \vec{a} \times \vec{b}$ and $\vec{d}$ be a vector of magnitude 2 in yz -plane. If $|\vec{c}| = \sqrt{53}$, then the maximum possible value of $(\vec{c} \cdot \vec{d})^2$ is equal to : [4 marks]

- (A) 26
(B) 52
(C) 104
(D) 208

Correct Answer: (D) 208

Solution:

Concept:

- A cross product can be evaluated by its determinant, and its squared magnitude is the sum of the squared components.
- If $\vec{d}$ is restricted to a plane, only the projection of $\vec{c}$ onto that plane contributes to $\vec{c} \cdot \vec{d}$. The largest absolute dot product is the product of their magnitudes.

Step 1: Determine the admissible integer parameter Expanding the cross product gives

$$\vec{c} = (-2 - \lambda)\hat{i} - 4\hat{j} + 2\lambda\hat{k}.$$

Using $|\vec{c}|^2 = 53$,

$$(\lambda + 2)^2 + 16 + 4\lambda^2 = 53, \quad 5\lambda^2 + 4\lambda - 33 = 0.$$

The factorisation $(\lambda + 3)(5\lambda - 11) = 0$ gives $\lambda = -3$ or $11/5$. Since the problem requires an integer, only $\lambda = -3$ is allowed, and $\vec{c} = \hat{i} - 4\hat{j} - 6\hat{k}$.

Step 2: Maximise the dot product within the prescribed plane Write $\vec{d} = u\hat{j} + v\hat{k}$, where $u^2 + v^2 = 4$. Then

$$\vec{c} \cdot \vec{d} = -4u - 6v.$$

Cauchy-Schwarz gives

$$(-4u - 6v)^2 \leq (16 + 36)(u^2 + v^2) = 52 \cdot 4 = 208.$$

Equality is achievable: choose $(u, v) = 2(-4, -6)/\sqrt{52}$, which has the required magnitude and lies in the yz -plane.

Step 3: Check that the bound is a maximum The equality vector is admissible, so the bound is attained rather than merely approached. Values 26, 52 and 104 are smaller than this attained squared dot product, so none is the requested maximum. Using the full magnitude $\sqrt{53}$ would incorrectly retain the x -component, which cannot contribute to a vector in the yz -plane.

Step 4: Conclusion The maximum possible value of $(\vec{c} \cdot \vec{d})^2$ is 208. Hence, the correct option is (D).

Quick Tip to Solve: For a dot-product maximum with a plane restriction, project the fixed vector onto that plane before applying Cauchy-Schwarz.

16. If $y = y(x)$ satisfies the differential equation $16(\sqrt{x+9\sqrt{x}})(4 + \sqrt{9+\sqrt{x}}) \cos y \, dy = (1 + 2 \sin y) dx, x > 0$ and $y(256) = \frac{\pi}{2}, y(49) = \alpha$, then $2 \sin \alpha$ is equal to : [4 marks]

- (A) $2\sqrt{2} - 1$
(B) $\sqrt{2} - 1$
(C) $2(\sqrt{2} - 1)$
(D) $3(\sqrt{2} - 1)$

Correct Answer: No printed option satisfies the literal differential equation; its integrated relation gives approximately 1.465675805.

Solution:

Concept:

- Separate the variables using $u = 1 + 2 \sin y$, so $du = 2 \cos y \, dy$. The initial value gives $u = 3$.
- Radical bounds must be preserved: with $t = \sqrt{9 + \sqrt{x}}$, one has $x = (t^2 - 9)^2$ and $\sqrt{x + 9\sqrt{x}} = t\sqrt{t^2 - 9}$, not $t(t^2 - 9)$.

Step 1: Separate the literal printed equation Dividing the equation and using $u = 1 + 2 \sin y$ gives

$$\frac{du}{u} = \frac{dx}{8\sqrt{x+9\sqrt{x}}(4 + \sqrt{9+\sqrt{x}})}.$$

Now $dx = 4t(t^2 - 9) \, dt$, so

$$\frac{du}{u} = \frac{\sqrt{t^2 - 9}}{2(t + 4)} \, dt.$$

The endpoints $x = 256$ and $x = 49$ correspond to $t = 5$ and $t = 4$, respectively. All radicals are positive on this interval.

Step 2: Evaluate the elementary integral Put $r = \sqrt{t^2 - 9}$ and $v = r/(t + 3)$, and define

$$F(t) = r - 4 \log(t + r) + \sqrt{7} \log\left(\frac{\sqrt{7} + v}{\sqrt{7} - v}\right).$$

This elementary primitive can be obtained with $w = t + r$: then $t = (w + 9/w)/2$, $r = (w - 9/w)/2$, and the integrand becomes the rational function

$$\frac{r}{t + 4} dt = \frac{(w^2 - 9)^2}{2w^2(w^2 + 8w + 9)} dw,$$

which integrates by partial fractions. To verify the primitive directly, use $r' = t/r$, $v' = 3/[r(t + 3)]$, and $7 - v^2 = 6(t + 4)/(t + 3)$. These give

$$F'(t) = \frac{t - 4}{r} + \frac{7}{r(t + 4)} = \frac{r}{t + 4}.$$

Thus no conjectured change to the radical is needed.

Step 3: Apply the initial value and test the options Integration from 5 to 4 gives

$$1 + 2 \sin \alpha = 3 \exp\left(\frac{F(4) - F(5)}{2}\right).$$

Here $F(4) \approx -4.169024468$ and $F(5) \approx -3.776731751$, hence the integrated relation gives

$$2 \sin \alpha \approx 1.465675805.$$

This is none of the four printed values. The exclusion is also exact: $h(t) = \sqrt{t^2 - 9}/(t + 4)$ is increasing on $[4, 5]$, and $1/4 < h(t) < 1/2$. Therefore the exponent is between $-1/4$ and $-1/8$. Using $e^{-1/4} > 3/4$ and $e^{-1/8} < 8/9$ gives

$$\frac{5}{4} < 2 \sin \alpha < \frac{5}{3}.$$

Option (A) exceeds $5/3$; options (B), (C) and (D) are below $5/4$, so all are excluded.

Step 4: Check the endpoint interpretation There is an additional endpoint issue if the equation is required to hold for a differentiable $y(x)$ at $x = 256$. Writing it in derivative form and substituting $y = \pi/2$ makes the left side zero for a finite derivative, while the right side is $1 + 2 \sin(\pi/2) = 3$. Thus no classical differentiable solution passes through that point. The integrated value above uses the usual one-sided continuous endpoint interpretation; even with that interpretation none of the offered answers holds.

Step 5: Conclusion As printed, no option is correct. The literal integrated relation gives $2 \sin \alpha = 3 \exp((F(4) - F(5))/2) - 1 \approx 1.465675805$; this differs from the key value $2\sqrt{2} - 1$ because the printed first radical produces the factor $\sqrt{t^2 - 9}$. A classical solution differentiable at the stated initial point does not exist.

Quick Tip to Solve: When substituting into nested radicals, square the proposed simplification to check it. An omitted square root changes the separated integral and can falsely reproduce an answer choice.

17. If $\lim_{x \rightarrow 0} \frac{e^{(a-1)x} + 2 \cos bx + (c-2)e^{-x}}{x \cos x - \log_e(1+x)} = 2$, then $a^2 + b^2 + c^2$ is equal to :

[4 marks]

(A) 3

(B) 5

(C) 7

(D) 9

Correct Answer: (C) 7

Solution:

Concept:

- A finite limit requires the numerator to vanish to at least the same order as the denominator.
- Use $e^{kx} = 1 + kx + \frac{k^2 x^2}{2} + O(x^3)$, $\cos(bx) = 1 - \frac{b^2 x^2}{2} + O(x^4)$, and $\log(1+x) = x - \frac{x^2}{2} + O(x^3)$.

Step 1: Find the leading denominator term Near zero, $x \cos x = x + O(x^3)$. Subtracting the logarithm gives

$$x \cos x - \log(1+x) = \frac{x^2}{2} + O(x^3).$$

The denominator therefore has a nonzero quadratic leading coefficient. A nonzero constant or linear numerator term would make the quotient fail to have a finite limit.

Step 2: Cancel the lower-order numerator terms The numerator expands as

$$N(x) = (c+1) + (a-c+1)x + \left[\frac{(a-1)^2}{2} - b^2 + \frac{c-2}{2} \right] x^2 + O(x^3).$$

Thus $c+1=0$, giving $c=-1$, and $a-c+1=0$, giving $a=-2$. These cancellations follow from finiteness, before using the specified value of the limit.

Step 3: Match the prescribed limit With these values the quadratic coefficient is $9/2 - b^2 - 3/2 = 3 - b^2$. Hence

$$\lim_{x \rightarrow 0} \frac{N(x)}{x \cos x - \log(1+x)} = \frac{3-b^2}{1/2} = 2.$$

It follows that $b^2 = 2$. Therefore

$$a^2 + b^2 + c^2 = 4 + 2 + 1 = 7.$$

Both real signs of b give the same sum; the other offered totals 3, 5 and 9 contradict the coefficient equations.

Step 4: Conclusion The required value is 7, the third option in the printed official order. Hence, the correct option is (C).

Quick Tip to Solve: When a denominator starts at order x^2 , cancel the constant and linear numerator terms before matching the quadratic coefficient.

18. Let f and g be functions satisfying $f(x+y) = f(x)f(y)$, $f(1) = 7$ and $g(x+y) = g(xy)$, $g(1) = 1$, for all $x, y \in \mathbb{N}$. If $\sum_{x=1}^n \left(\frac{f(x)}{g(x)} \right) = 19607$, then n is equal to : [4 marks]

- (A) 4
(B) 5
(C) 6
(D) 7

Correct Answer: (B) 5

Solution:

Concept:

- Setting one argument equal to 1 often converts a functional equation on the positive integers into a recurrence.
- The finite geometric sum is $a(r^n - 1)/(r - 1)$ for first term a and ratio $r \neq 1$.

Step 1: Determine both functions on the positive integers Put $y = 1$ into $f(x+y) = f(x)f(y)$. Since $f(1) = 7$,

$$f(x+1) = 7f(x), \quad f(x) = 7^x \quad (x \in \mathbb{N}).$$

The last equality follows by induction from the value at 1. For the other equation, putting $y = 1$ gives

$$g(x+1) = g(x \cdot 1) = g(x).$$

Starting with $g(1) = 1$, induction therefore yields $g(x) = 1$ for every positive integer. This constant function also satisfies the original two-variable equation.

Step 2: Evaluate the finite sum The ratio in the summand is $f(x)/g(x) = 7^x$, so

$$\sum_{x=1}^n \frac{f(x)}{g(x)} = 7 + 7^2 + \dots + 7^n = \frac{7(7^n - 1)}{6}.$$

Equating this to the given number yields

$$7^n - 1 = \frac{6 \cdot 19607}{7} = 16806, \quad 7^n = 16807 = 7^5.$$

Therefore $n = 5$.

Step 3: Verify the value and exclude the other choices Direct addition confirms

$$7 + 49 + 343 + 2401 + 16807 = 19607.$$

The sum through 4 is only 2800. Every subsequent term is strictly positive, so the sums through 6 or 7 are larger than the required value. Thus the natural-number solution is unique and none of the other three options can satisfy the equation.

Step 4: Conclusion The required value is $n = 5$, the second option in the printed official order. Hence, the correct option is (B).

Quick Tip to Solve: For functional equations on $\mathbb{N}$, try $y = 1$ first: it can reveal an exponential sequence or a constant sequence immediately.

19. Let L be the line $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z+3}{6}$ and let S be the set of all points (a, b, c) on L , whose distance from the line $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z-9}{0}$ along the line L is 7. Then $\sum_{(a,b,c) \in S} (a + b + c)$ is equal to : [4 marks]

- (A) 6
(B) 28
(C) 34
(D) 40

Correct Answer: (C) 34

Solution:

Concept:

- A point on a line with direction vector $(2, 3, 6)$ can be written using one parameter. A parameter change Δt corresponds to distance $\sqrt{2^2 + 3^2 + 6^2}|\Delta t| = 7|\Delta t|$.
- The stated distance is measured along L , so its starting point is the intersection of L with the second line. There are two possible directions of travel along L .

Step 1: Parametrize both lines Write the points of L as

$$P(t) = (-1 + 2t, -1 + 3t, -3 + 6t).$$

The zero denominator in the second line means its z -coordinate is fixed at 9, not that we divide by zero. Its points are

$$Q(u) = (-1 + 2u, -1 + 3u, 9).$$

At an intersection the first two coordinates give $t = u$, and the last coordinate gives $-3 + 6t = 9$. Thus $t = u = 2$, and the intersection is $I = (3, 5, 9)$.

Step 2: Locate the two required points The displacement from $I = P(2)$ to $P(t)$ equals $(t - 2)(2, 3, 6)$. Its length along the straight line is therefore

$$IP(t) = 7|t - 2|.$$

Imposing the stated distance 7 gives $|t - 2| = 1$, hence $t = 1$ or $t = 3$. These give the two distinct points $P(1) = (1, 2, 3)$ and $P(3) = (5, 8, 15)$. Both lie on L , and each is exactly 7 units from the intersection.

Step 3: Sum over the complete set The requested sum includes both points, so

$$\sum_{(a,b,c) \in S} (a + b + c) = (1 + 2 + 3) + (5 + 8 + 15) = 6 + 28 = 34.$$

The values 6 and 28 count only one of the points each. The value 40 is neither their sum nor consistent with the parameter calculation. Equivalently, the two points are symmetric about I , so the sum is twice $3 + 5 + 9$.

Step 4: Conclusion The two points give $6 + 28 = \boxed{34}$, which is the printed option (C). Hence, the correct option is (C).

Quick Tip to Solve: For distance measured along an intersecting line, find the intersection first; use the norm of its direction vector to convert distance into a parameter change.

20. Let S and S' be the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and $P(\alpha, \beta)$ be a point on the ellipse in the first quadrant. If $(SP)^2 + (S'P)^2 - SP \cdot S'P = 37$, then $\alpha^2 + \beta^2$ is equal to : [4 marks]

- (A) 11
(B) 13
(C) 15
(D) 17

Correct Answer: (B) 13

Solution:

Concept:

- For an ellipse with semi-major axis a , the sum of the distances from any point on it to the two foci is $2a$. Here $a = 5$, the semi-minor axis is 3, and the focal distance is $c = \sqrt{25 - 9} = 4$.
- Expanding squared distances to the symmetric foci $(4, 0)$ and $(-4, 0)$ cancels the terms linear in the horizontal coordinate. This relates their sum directly to $\alpha^2 + \beta^2$.

Step 1: Find the product of focal distances Put $u = SP$ and $v = S'P$. Since P lies on the ellipse, $u + v = 10$. The additional condition becomes

$$u^2 + v^2 - uv = (u + v)^2 - 3uv = 37.$$

Therefore

$$100 - 3uv = 37 \implies uv = 21.$$

It follows that $u^2 + v^2 = 37 + 21 = 58$. In particular the positive distances are 3 and 7, which also satisfy their required sum.

Step 2: Express the squared distances in coordinates Using the two foci,

$$\begin{aligned} u^2 + v^2 &= (\alpha - 4)^2 + \beta^2 + (\alpha + 4)^2 + \beta^2 \\ &= 2(\alpha^2 + \beta^2) + 32. \end{aligned}$$

Equating this to 58 gives

$$2(\alpha^2 + \beta^2) = 26, \quad \alpha^2 + \beta^2 = 13.$$

No separate calculation of the first-quadrant coordinates is needed for this symmetric expression.

Step 3: Check the option and admissibility For a proposed value $R = \alpha^2 + \beta^2$, the squared-distance sum would be $2R + 32$. Thus 11, 15 and 17 give 54, 62 and 66, respectively, whereas the given condition requires 58. Only 13 works. It is also compatible with a first-quadrant point: solving with the ellipse gives $\alpha^2 = 25/4$ and $\beta^2 = 27/4$, both positive.

Step 4: Conclusion $\alpha^2 + \beta^2 = 13$. Hence, the correct option is (B).

Quick Tip to Solve: Combine the fixed focal-distance sum with the given expression before using coordinates; symmetry then cancels the linear terms.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Suppose a, b, c are in A.P. and $a^2, 2b^2, c^2$ are in G.P. If $a < b < c$ and $a+b+c = 1$, then $9(a^2 + b^2 + c^2)$ is equal to ____ . [4 marks]

Solution:

Concept:

- For three terms in arithmetic progression, write them as $b-d, b, b+d$. The strict ordering requires $d > 0$. Their sum determines the middle term immediately.
- For three terms $a^2, 2b^2, c^2$ in geometric progression, the square of the middle term equals the product of the outer terms: $(2b^2)^2 = a^2 c^2$. Taking the square root gives two possible signs for ac , and both must be checked.

Step 1: Use the arithmetic progression Since $a+c = 2b$ and $a+b+c = 1$, we have $3b = 1$, so $b = 1/3$. Write

$$a = \frac{1}{3} - d, \quad c = \frac{1}{3} + d, \quad d > 0.$$

Consequently

$$ac = \left(\frac{1}{3} - d\right) \left(\frac{1}{3} + d\right) = \frac{1}{9} - d^2.$$

We do not assume that all three terms are positive: the question states only their order and their sum.

Step 2: Apply the geometric progression condition The given GP implies

$$a^2 c^2 = 4b^4 \implies ac = \pm 2b^2 = \pm \frac{2}{9}.$$

If $ac = 2/9$, then $1/9 - d^2 = 2/9$, giving $d^2 = -1/9$, impossible for real terms. Therefore $ac = -2/9$, and

$$\frac{1}{9} - d^2 = -\frac{2}{9} \implies d^2 = \frac{1}{3}.$$

The positive choice $d = 1/\sqrt{3}$ satisfies the strict ordering; in particular $a < 0 < c$, which explains the negative product.

Step 3: Evaluate the requested symmetric sum The cross terms cancel in the sum of squares:

$$\begin{aligned} a^2 + b^2 + c^2 &= (b-d)^2 + b^2 + (b+d)^2 \\ &= 3b^2 + 2d^2 = \frac{1}{3} + \frac{2}{3} = 1. \end{aligned}$$

Multiplying by 9 gives the requested numerical value. As a check, the square terms are positive and their GP condition is unchanged by the negative sign of a .

Step 4: Conclusion The required dimensionless value is $\boxed{9}$.

Quick Tip to Solve: When a GP involves squares, retain both signs after taking a square root. The AP parametrization quickly eliminates the impossible sign.

22. Let S be the set of the first 11 natural numbers. Then the number of elements in $A = \{B \subseteq S : n(B) \geq 2 \text{ and the product of all elements of } B \text{ is even}\}$ is ____ . [4 marks]

Solution:

Concept:

- A product of natural numbers is even exactly when at least one factor is even. Thus an even-product subset must contain a nonempty selection from the even elements.
- A set with m elements has 2^m subsets, including the empty set. Count the parity condition first, then exclude any counted subsets that violate the minimum size.

Step 1: Separate the odd and even elements Here $S = \{1, 2, 3, \dots, 11\}$. Its five even elements are 2, 4, 6, 8, 10, while its six odd elements are 1, 3, 5, 7, 9, 11. A subset containing no even number has an odd product whenever it is nonempty. Conversely, one even member suffices to make the entire product even, regardless of the other members.

Step 2: Count all subsets containing an even element Choose a nonempty subset of the five evens in $2^5 - 1 = 31$ ways. Independently choose any subset of the six odds in $2^6 = 64$ ways. Different choices produce different subsets, and every subset with an even product appears once. Hence the preliminary count is

$$(2^5 - 1)2^6 = 31 \cdot 64 = 1984.$$

This count automatically excludes the empty subset, because at least one even member has been selected.

Step 3: Enforce the size restriction The condition $n(B) \geq 2$ excludes singletons as well. Among the 1984 subsets counted above, precisely the five singletons $\{2\}, \{4\}, \{6\}, \{8\}, \{10\}$ have only one element. Thus

$$n(A) = 1984 - 5 = 1979.$$

For an independent complement check, all subsets of size at least two number $2^{11} - 1 - 11 = 2036$. The all-odd subsets of that size number $2^6 - 1 - 6 = 57$, so their difference is again $2036 - 57 = 1979$.

Step 4: Conclusion The number of elements in A is 1979.

Quick Tip to Solve: Count nonempty even selections times arbitrary odd selections, then remove only the even singletons; the empty subset has already been excluded.

23. Let $\cos(\alpha + \beta) = -\frac{1}{10}$ and $\sin(\alpha - \beta) = \frac{3}{8}$, where $0 < \alpha < \frac{\pi}{3}$ and $0 < \beta < \frac{\pi}{4}$. If $\tan 2\alpha = \frac{3(1-r\sqrt{5})}{\sqrt{11}(s+\sqrt{5})}$, $r, s \in \mathbb{N}$, then $r + s$ is equal to ____ . [4 marks]

Solution:

Concept:

- Introduce $A = \alpha + \beta$ and $D = \alpha - \beta$, so $2\alpha = A + D$. The addition formula for tangent can be evaluated through the sine and cosine of this sum.
- The angle bounds determine the signs when recovering a sine from a cosine, or a cosine from a sine. Those signs are essential before simplifying the radicals.

Step 1: Determine the missing trigonometric values The given bounds imply $0 < A < 7\pi/12$. Since $\cos A = -1/10$, A lies in the second quadrant within this interval and has positive sine. Thus

$$\sin A = \sqrt{1 - \frac{1}{100}} = \frac{3\sqrt{11}}{10}.$$

Also $-\pi/4 < D < \pi/3$, where cosine is positive. Hence

$$\cos D = \sqrt{1 - \frac{9}{64}} = \frac{\sqrt{55}}{8}, \quad \sin D = \frac{3}{8}.$$

In fact the positive sine ensures $D > 0$ in this interval.

Step 2: Expand the sine and cosine of twice alpha Using $2\alpha = A + D$, we obtain

$$\begin{aligned} \sin 2\alpha &= \sin A \cos D + \cos A \sin D \\ &= \frac{3\sqrt{11}\sqrt{55} - 3}{80} = \frac{33\sqrt{5} - 3}{80}, \\ \cos 2\alpha &= \cos A \cos D - \sin A \sin D \\ &= -\frac{\sqrt{55} + 9\sqrt{11}}{80}. \end{aligned}$$

The sine is positive and the cosine negative, so the resulting tangent must be negative, consistent with these values.

Step 3: Match the required form Dividing and factoring gives

$$\tan 2\alpha = \frac{3(11\sqrt{5} - 1)}{-\sqrt{11}(\sqrt{5} + 9)} = \frac{3(1 - 11\sqrt{5})}{\sqrt{11}(9 + \sqrt{5})}.$$

Comparing this with the stated representation gives $r = 11$ and $s = 9$, both natural numbers. Their sum is therefore $11 + 9 = 20$. The negative sign has been absorbed in the numerator, so it must not be discarded while identifying r .

Step 4: Conclusion The required value is $\boxed{r + s = 20}$.

Quick Tip to Solve: Use sum and difference angles as new variables, and settle their quadrants before taking square roots or comparing radical coefficients.

24. Let $[.]$ be the greatest integer function. If $\alpha = \int_0^{64} (x^{1/3} - [x^{1/3}]) dx$, then $\frac{1}{\pi} \int_0^{\alpha\pi} \left(\frac{\sin^2 \theta}{\sin^6 \theta + \cos^6 \theta} \right) d\theta$ is equal to ____ . [4 marks]

Solution:

Concept:

- The floor of $x^{1/3}$ is constant on each interval $[k^3, (k+1)^3)$. Splitting at cube values therefore removes the greatest-integer function from the first integral.

- The function $f(\theta) = \sin^2 \theta / (\sin^6 \theta + \cos^6 \theta)$ has period π . On the interval $0 \leq \theta \leq \pi/2$, the substitution $t = \tan \theta$ converts its denominator into a rational expression.

Step 1: Compute alpha by splitting at cubes The relevant intervals have endpoints 0, 1, 8, 27, 64, with floor values 0, 1, 2 and 3. Endpoint values do not affect an integral. Therefore

$$\begin{aligned}\alpha &= \int_0^{64} x^{1/3} dx - \sum_{k=0}^3 k((k+1)^3 - k^3) \\ &= \frac{3}{4} 64^{4/3} - (0 + 7 + 38 + 111) \\ &= 192 - 156 = 36.\end{aligned}$$

Step 2: Reduce one period using symmetry Let $I = \int_0^\pi f(\theta) d\theta$. Reflection about $\pi/2$ gives $I = 2 \int_0^{\pi/2} f(\theta) d\theta$. On $[0, \pi/2]$, replacing θ by $\pi/2 - \theta$ interchanges the sine and cosine numerators while keeping the denominator fixed. Adding the two equal integrals yields

$$I = \int_0^{\pi/2} \frac{d\theta}{\sin^6 \theta + \cos^6 \theta}.$$

Step 3: Evaluate the period integral exactly Set $t = \tan \theta$. Since $1 + t^6 = (1 + t^2)(t^4 - t^2 + 1)$,

$$I = \int_0^\infty \frac{1 + t^2}{t^4 - t^2 + 1} dt.$$

Now put $u = t - 1/t$. Then $du = (1 + t^{-2})dt$ and $u^2 + 1 = t^2 - 1 + t^{-2}$, so the integrand becomes $du/(1 + u^2)$. As t increases from 0 to infinity, u increases from minus infinity to infinity. Thus

$$I = \int_{-\infty}^\infty \frac{du}{1 + u^2} = \pi.$$

Step 4: Combine the complete periods The upper limit is $\alpha\pi = 36\pi$, exactly 36 periods. Consequently

$$\frac{1}{\pi} \int_0^{36\pi} f(\theta) d\theta = \frac{36I}{\pi} = 36.$$

The denominator is strictly positive throughout, so all symmetry and periodicity steps are valid without singularity exclusions.

Step 5: Conclusion The required dimensionless value is $\boxed{36}$.

Quick Tip to Solve: First split a floor of a cube root at perfect cubes. For the resulting trigonometric integral, use symmetry before the tangent substitution.

25. Let a vector $\vec{a} = \sqrt{2}\hat{i} - \hat{j} + \lambda\hat{k}$, $\lambda > 0$, make an obtuse angle with the vector $\vec{b} = -\lambda^2\hat{i} + 4\sqrt{2}\hat{j} + 4\sqrt{2}\hat{k}$ and an angle θ , $\frac{\pi}{6} < \theta < \frac{\pi}{2}$, with the positive z -axis. If the set of all possible values of λ is $(\alpha, \beta) - \{\gamma\}$, then $\alpha + \beta + \gamma$ is equal to ____ . [4 marks]

Solution:

Concept:

- For nonzero vectors, an obtuse angle means a strictly negative scalar product: $\vec{a} \cdot \vec{b} < 0$. A zero scalar product gives a right angle and must be excluded.
- The cosine of the angle with the positive z -axis is the positive z -component divided by the magnitude: $\cos \theta = \lambda / \sqrt{3 + \lambda^2}$.

Step 1: Apply the obtuse-angle condition Using the corresponding Cartesian components,

$$\begin{aligned}\vec{a} \cdot \vec{b} &= -\sqrt{2}\lambda^2 - 4\sqrt{2} + 4\sqrt{2}\lambda \\ &= -\sqrt{2}(\lambda - 2)^2.\end{aligned}$$

This is negative for every real $\lambda \neq 2$, and zero at $\lambda = 2$. Thus the first angle condition removes only the value 2 from the already stated domain $\lambda > 0$.

Step 2: Convert the direction-angle interval Cosine decreases on $(0, \pi)$. Therefore $\pi/6 < \theta < \pi/2$ gives

$$0 < \frac{\lambda}{\sqrt{3 + \lambda^2}} < \frac{\sqrt{3}}{2}.$$

Both sides of the upper inequality are positive, so squaring preserves its direction:

$$\frac{\lambda^2}{3 + \lambda^2} < \frac{3}{4} \implies 4\lambda^2 < 9 + 3\lambda^2 \implies \lambda^2 < 9.$$

Combining this with positivity yields $0 < \lambda < 3$. The endpoints are excluded because the given angular inequalities are strict.

Step 3: Identify the interval and excluded point Intersect the two conditions to obtain $(0, 3) \setminus \{2\}$. Thus $\alpha = 0$, $\beta = 3$ and $\gamma = 2$. In particular, 2 lies inside the interval but is removed because the two vectors would be perpendicular there.

Step 4: Conclusion The required dimensionless value is $\alpha + \beta + \gamma = 0 + 3 + 2 = \boxed{5}$.

Quick Tip to Solve: Factor the dot product before solving inequalities; a negative perfect square excludes its zero without splitting the remaining domain.

Physics – Section A: Multiple Choice Questions (80 marks)

26. Which of the following are true for a single slit diffraction?

- A. Width of central maxima increases with increase in wavelength keeping slit width constant.
- B. Width of central maxima increases with decrease in wavelength keeping slit width constant.
- C. Width of central maxima increases with decrease in slit width at constant wavelength.
- D. Width of central maxima increases with increase in slit width at constant wavelength.
- E. Brightness of central maxima increases for decrease in wavelength at constant slit width. [4 marks]

(A) A, D, E only

(B) A, D only

(C) B, D only

(D) B, C only

Correct Answer: No printed option is correct; the correct statement set is A, C, E.

Solution:

Concept:

- The first minima of a slit of width a satisfy $a \sin \theta = \pm \lambda$. In the small-angle screen geometry, the central width is $w = 2D\lambda/a$, where D is the screen distance.
- For fixed illumination and geometry, the usual Fraunhofer central irradiance varies as $I(0) \propto a^2/\lambda^2$. Width and peak brightness are distinct quantities.

Step 1: Test the wavelength statements With a and D fixed, $w \propto \lambda$. Increasing wavelength spreads the first minima farther apart, so A is true. Decreasing wavelength narrows the central maximum, so B is false. The same proportionality can also be obtained from the angular width $2\lambda/a$.

Step 2: Test the slit-width statements At fixed wavelength and screen distance, $w \propto 1/a$. A narrower slit diffracts the beam more widely, making C true. A wider slit reduces the central width, so D is false. These conclusions follow directly from the first-minimum condition and do not depend on the brightness convention.

Step 3: Check brightness and the printed choices Under otherwise unchanged Fraunhofer illumination, reducing λ increases $I(0)$, so E is true in the usual comparison. The resulting set is A, C and E. However, printed option (A) contains A, D and E; it therefore includes the false statement D and omits C. Option (B) also includes D, option (C) includes both false statements B and D, and option (D) includes false statement B. Thus none of the printed combinations is correct.

Step 4: Conclusion The true set is A, C and E under the usual fixed-illumination comparison. No printed option gives this set. The answer key selects (A), but the printed (A) says A, D, E, which contradicts $w \propto 1/a$; hence no unique valid printed option can be selected.

Quick Tip to Solve: Use $w \propto \lambda/a$ to test width statements first, then check every letter in the offered combination rather than choosing by a partial match.

27. In an open organ pipe ν_3 and ν_6 are 3rd and 6th harmonic frequencies, respectively. If $\nu_6 - \nu_3 = 2200$ Hz then length of the pipe is ____ mm .

(Take velocity of sound in air is 330 m/s.)

[4 marks]

(A) 200

(B) 225

(C) 250

(D) 275

Correct Answer: (B) 225

Solution:

Concept:

- An organ pipe open at both ends supports all positive integer harmonics: $\nu_n = nv/(2L)$, with sound speed v and pipe length L .
- The difference between the sixth and third harmonics is three times the fundamental frequency. Convert the final length from metres to millimetres.

Step 1: Express the given frequency difference For the same pipe and the same sound speed,

$$\nu_6 = \frac{6v}{2L}, \quad \nu_3 = \frac{3v}{2L}.$$

Subtracting these expressions gives

$$\nu_6 - \nu_3 = \frac{(6-3)v}{2L} = \frac{3v}{2L} = 2200 \text{ Hz}.$$

We use harmonic numbers directly; third harmonic does not mean third overtone. Nor should the odd-harmonic restriction of a pipe closed at one end be used.

Step 2: Calculate the length and convert units Rearranging and inserting $v = 330 \text{ m s}^{-1}$,

$$L = \frac{3v}{2(2200)} = \frac{3 \times 330}{4400} \text{ m} = 0.225 \text{ m}.$$

Since $1 \text{ m} = 1000 \text{ mm}$, the length is 225 mm. The units are consistent because velocity divided by frequency has dimensions of length.

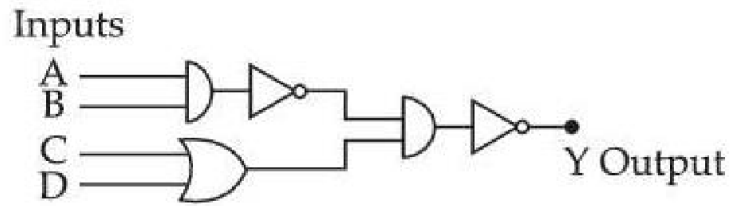
Step 3: Check the candidate lengths Substituting $L = 0.225 \text{ m}$ gives $3v/(2L) = 990/0.450 = 2200 \text{ Hz}$, exactly as required. The lengths 200, 250 and 275 mm instead give 2475, 1980 and 1800 Hz, respectively, so they do not satisfy the given difference.

Step 4: Conclusion The pipe length is 225 mm. Hence, the correct option is (B).

Quick Tip to Solve: For two harmonics of the same open pipe, subtract their harmonic numbers first: $\Delta\nu = (n-m)v/(2L)$.

28. The correct truth table for the given input data of the following logic gate is :

[4 marks]



(A)

Inputs				Output
A	B	C	D	Y
1	1	0	1	1
0	0	1	1	0
1	0	1	0	1
1	1	1	1	0

(B)

Inputs				Output
A	B	C	D	Y
1	1	0	1	0
0	0	1	1	0
1	0	1	0	1
1	1	1	1	1

(C)

Inputs				Output
A	B	C	D	Y
1	1	0	1	1
0	0	1	1	0
1	0	1	0	0
1	1	1	1	1

(D)

Inputs				Output
A	B	C	D	Y
1	1	0	1	0
0	0	1	1	1
1	0	1	0	1
1	1	1	1	1

Correct Answer: (C)

Inputs				Output
A	B	C	D	Y
1	1	0	1	1
0	0	1	1	0
1	0	1	0	0
1	1	1	1	1

Solution:

Concept:

- AND gives the product AB , OR gives the Boolean sum $C + D$, and NOT complements its input. The small output circle represents inversion.

- The output must be evaluated after both NOT gates. De Morgan's law gives $\overline{PQ} = \overline{P} + \overline{Q}$.

Step 1: Write the output of each branch The upper branch first forms AB , then inverts it, giving $P = \overline{AB}$. The lower branch is an OR gate, giving $Q = C + D$. These two signals enter an AND gate followed by a NOT gate. Therefore

$$Y = \overline{\overline{AB}(C + D)} = AB + \overline{C + D} = AB + \overline{C} \overline{D}.$$

The final inversion acts on the product of the entire two branch outputs, not on either original input alone.

Step 2: Evaluate the four printed input rows For each row, evaluate the intermediate signals before taking the final complement:

A	B	C	D	AB	$\overline{AB}$	$C + D$	Y
1	1	0	1	1	0	1	1
0	0	1	1	0	1	1	0
1	0	1	0	0	1	1	0
1	1	1	1	1	0	1	1

Every listed row has $C + D = 1$, so for these particular inputs the network reduces to $Y = AB$. This shortcut verifies the complete calculation without assuming it for all possible inputs.

Step 3: Match the printed table order The required output column is 1, 0, 0, 1. It appears in the third printed table, option (C). The first table gives 1, 0, 1, 0, wrong in the last two rows. The second gives 0, 0, 1, 1, wrong in rows one and three. The fourth gives 0, 1, 1, 1, wrong in the first three rows.

Step 4: Conclusion The correct output sequence is $\boxed{1, 0, 0, 1}$. Hence, the correct option is (C). The sheet key selects the first table, but the printed table order places this sequence in (C), as verified by $Y = AB + \overline{C} \overline{D}$.

Quick Tip to Solve: Label intermediate gate outputs before making the truth table; here $C + D = 1$ in every given row, so each output is simply AB .

29. Given below are two statements :

Statement I : An object moves from position r_1 to position r_2 under a conservative force field $\vec{F}$. The work done by the force is $W = - \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$.

Statement II : Any object moving from one location to another location can follow infinite number of paths. Therefore, the amount of work done by the object changes with the path it follows for a conservative force.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Both Statement I and Statement II are true
- (B) Both Statement I and Statement II are false
- (C) Statement I is true but Statement II is false
- (D) Statement I is false but Statement II is true

Correct Answer: (B) Both Statement I and Statement II are false

Solution:

Concept:

- Work done by a force during a displacement is $dW = \vec{F} \cdot d\vec{r}$, so $W = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$, without an extra minus sign.
- For a conservative force, $\vec{F} = -\nabla U$, so the change of potential energy is $\Delta U = -W$, and the force's work depends only on the endpoints.

Step 1: Check the sign in Statement I The stated expression places a minus sign in front of the force integral. That expression is the change in potential energy, not the work done by the force:

$$W_{\text{force}} = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} = U(r_1) - U(r_2).$$

For example, if an object moves along a constant force through a positive distance, the dot product and the work are positive; the printed negative integral would incorrectly make that work negative. Statement I is therefore false.

Step 2: Check path dependence in Statement II There may indeed be many possible paths between the same endpoints, but conservative work is path independent. For any two paths joining r_1 and r_2 , each work integral equals $U(r_1) - U(r_2)$. Equivalently, the integral around the closed curve formed by going along one path and returning along the other is zero. The existence of different paths therefore does not imply different work. Statement II is false.

Step 3: Select the combination Both complete statements are false. Option (A) requires both to be true, option (C) requires I to be true, and option (D) requires II to be true. Each contradicts at least one of the checks above; only option (B) matches both results.

Step 4: Conclusion Statement I and Statement II are both false. Hence, the correct option is (B).

Quick Tip to Solve: Keep $W_{\text{force}} = -\Delta U$ separate from the definition of work; the minus sign belongs to the potential-energy relation.

30. Light is incident on a metallic plate having work function 110×10^{-20} J. If the produced photoelectrons have zero kinetic energy then the angular frequency of the incident light is ____ rad/s. ($h = 6.63 \times 10^{-34}$ J.s). [4 marks]

(A) 1.04×10^{13}

(B) 1.66×10^{15}

(C) 1.66×10^{16}

(D) 1.04×10^{16}

Correct Answer: (D) 1.04×10^{16}

Solution:

Concept:

- Einstein's photoelectric equation is $h\nu = \phi + K_{\max}$, where ϕ is the work function and ν is the ordinary frequency.
- Angular frequency is $\omega = 2\pi\nu$, so photon energy can also be written $\hbar\omega$, where $\hbar = h/(2\pi)$.

Step 1: Use the zero-kinetic-energy condition At the threshold, the photon supplies just enough energy to liberate an electron and leaves no kinetic energy. Setting $K_{\max} = 0$ gives

$$h\nu_0 = \phi, \quad \nu_0 = \frac{\phi}{h}.$$

The work function is $110 \times 10^{-20} = 1.10 \times 10^{-18}$ J. Keeping the given value in joules avoids an unnecessary conversion to electron-volts.

Step 2: Calculate ordinary and angular frequency Substitute the supplied Planck constant:

$$\nu_0 = \frac{1.10 \times 10^{-18}}{6.63 \times 10^{-34}} \text{ s}^{-1} = 1.659 \times 10^{15} \text{ Hz}.$$

The requested quantity is angular frequency, so multiply by 2π :

$$\omega_0 = 2\pi(1.659 \times 10^{15}) \simeq 1.04 \times 10^{16} \text{ rad s}^{-1}.$$

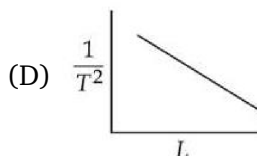
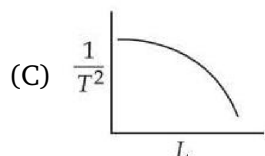
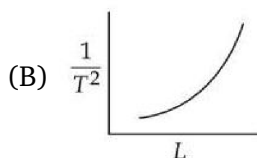
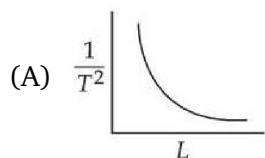
Using $h\omega$ as the photon energy would omit the factor 2π and produce the ordinary frequency instead.

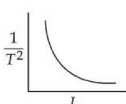
Step 3: Compare the printed options Option (B), 1.66×10^{15} , is the ordinary frequency and has not been multiplied by 2π . Option (C), 1.66×10^{16} , introduces a factor of ten instead of 2π . Option (A), 1.04×10^{13} , is three orders of magnitude too small. Only option (D) has the required numerical value.

Step 4: Conclusion The threshold angular frequency is $1.04 \times 10^{16} \text{ rad s}^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: When a photoelectric question asks for rad/s, calculate ϕ/h first and then multiply by 2π , or use $\phi/\hbar$ directly.

31. Using a simple pendulum experiment g is determined by measuring its time period T . Which of the following plots represent the correct relation between the pendulum length L and time period T ? [4 marks]



Correct Answer: (A) 

Solution:

Concept:

- For small oscillations of a simple pendulum, $T = 2\pi\sqrt{L/g}$, where L is its effective length and g is the local gravitational acceleration.
- At fixed g , squaring the period formula gives $T^2 \propto L$, so its reciprocal satisfies $1/T^2 \propto 1/L$. Read the axes before deciding the shape of a graph.

Step 1: Express the plotted ordinate in terms of length All four graphs plot $1/T^2$ vertically against L horizontally. Squaring and inverting the pendulum relation gives

$$T^2 = \frac{4\pi^2 L}{g}, \quad \frac{1}{T^2} = \frac{g}{4\pi^2 L}.$$

Thus the product of the two plotted quantities is constant. This is an inverse relation, not a straight-line relation between the displayed coordinates.

Step 2: Check the direction and curvature Write $y = C/L$, where $C = g/(4\pi^2) > 0$. Increasing the length decreases the ordinate; doubling the length halves it. Also,

$$\frac{dy}{dL} = -\frac{C}{L^2} < 0, \quad \frac{d^2y}{dL^2} = \frac{2C}{L^3} > 0.$$

The curve falls steeply at small lengths and becomes flatter at larger lengths, approaching the horizontal axis without crossing it. Graph (A) has this shape. Graph (B) increases, graph (C) has the opposite curvature, and graph (D) incorrectly assumes a constant negative slope. The positive reciprocal-square period never becomes zero at a finite positive length.

Step 3: Conclusion The $1/T^2$ versus L plot is the decreasing rectangular hyperbola shown in (A). Hence, the correct option is (A).

Quick Tip to Solve: Rewrite the period formula using the quantities on the axes: T^2 versus L is linear, but $1/T^2$ versus L is an inverse curve.

32. The smallest wavelength of Lyman series is 91 nm . The difference between the largest wavelengths of Paschen and Balmer series is nearly ____ nm. [4 marks]

(A) 1550

(B) 1875

(C) 1784

(D) 1217

Correct Answer: (D) 1217

Solution:

Concept:

- The Rydberg relation is $1/\lambda = R(1/n_f^2 - 1/n_i^2)$, where the electron falls from n_i to n_f . The final level fixes the spectral series.
- The shortest wavelength in a series occurs for $n_i \rightarrow \infty$; its longest wavelength occurs for the adjacent-level transition $n_i = n_f + 1$.

Step 1: Find the Rydberg reciprocal from the Lyman limit Lyman transitions end at $n_f = 1$. At the series limit the initial level tends to infinity, so

$$\frac{1}{\lambda_{L,\min}} = R, \quad \frac{1}{R} = 91 \text{ nm.}$$

This given value can be substituted directly into all subsequent wavelength expressions; an external numerical value of the Rydberg constant is unnecessary.

Step 2: Calculate the two longest wavelengths The longest Paschen wavelength is from $4 \rightarrow 3$, whereas the longest Balmer wavelength is from $3 \rightarrow 2$. Therefore,

$$\lambda_{P,\max} = \frac{1}{R(1/9 - 1/16)} = \frac{144}{7R} = 1872 \text{ nm,}$$

$$\lambda_{B,\max} = \frac{1}{R(1/4 - 1/9)} = \frac{36}{5R} = 655.2 \text{ nm.}$$

Subtracting gives $1872 - 655.2 = 1216.8 \text{ nm}$, nearly 1217 nm. Option (B), near 1875 nm, approximates the Paschen wavelength alone. Option (C) resembles subtracting the Lyman limit instead of the Balmer wavelength. Option (A) does not equal the required difference.

Step 3: Conclusion The difference is approximately 1217 nm. Hence, the correct option is (D).

Quick Tip to Solve: For a series ending at level n , use $(n + 1) \rightarrow n$ for the longest wavelength and $\infty \rightarrow n$ for the shortest.

33. An electric power line having total resistance of 2Ω , delivers 1 kW of power at 250 V. The percentage efficiency of transmission line is ____ . [4 marks]

- (A) 100
- (B) 86.5
- (C) 92.5
- (D) 96.9

Correct Answer: (D) 96.9

Solution:

Concept:

- The power delivered to a load is $P_{\text{out}} = VI$, using the voltage across that load and the current through the line.
- A line with total resistance R dissipates I^2R . The input power is the useful output plus this loss, and $\eta = 100P_{\text{out}}/P_{\text{in}}$.

Step 1: Calculate the current and the line loss The stated 1 kW is the power delivered at the receiving end, where the voltage is 250 V. Convert the power to watts and calculate

$$I = \frac{P_{\text{out}}}{V} = \frac{1000}{250} = 4 \text{ A.}$$

The resistance given is the total line resistance, so it must not be doubled for the return conductor. Its heating loss is

$$P_{\text{loss}} = I^2R = 4^2 \times 2 = 32 \text{ W.}$$

Step 2: Compare useful output with input power The source must supply both the load power and the heat generated along the line:

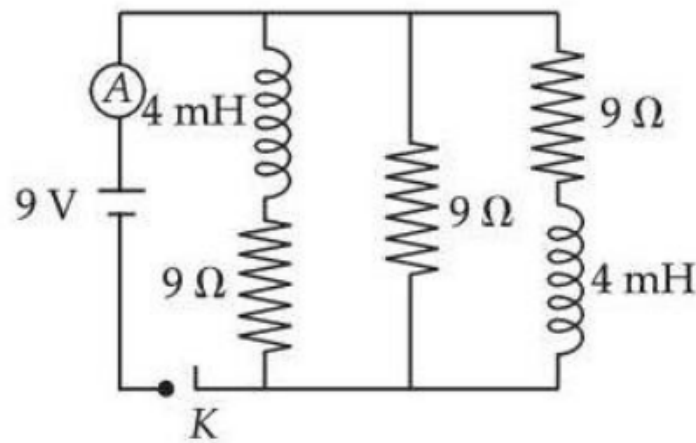
$$P_{\text{in}} = 1000 + 32 = 1032 \text{ W,} \quad \eta = \frac{1000}{1032} \times 100 = 96.899 \dots \%$$

As a check, the line drop is $IR = 8 \text{ V}$, so the sending-end voltage is 258 V; multiplying by 4 A again gives 1032 W. Efficiency is below 100 percent because the resistance is nonzero, rejecting (A). Options (B) and (C) imply losses much greater than the calculated 32 W. Dividing loss by output instead of input would not be the definition of transmission efficiency.

Step 3: Conclusion The transmission efficiency is approximately 96.9%. Hence, the correct option is (D).

Quick Tip to Solve: When delivered power and voltage are specified, calculate line current from them first, then add I^2R to the delivered power to obtain input power.

34. Figure shows the circuit that contains three resistances (9Ω each) and two inductors (4 mH each). The reading of ammeter at the moment switch K is turned ON , is ____ A. [4 marks]



- (A) 1
- (B) 2
- (C) 3
- (D) zero

Correct Answer: (A) 1

Solution:

Concept:

- The current through an ideal inductor cannot change instantaneously: $V_L = L \frac{di}{dt}$. A sudden jump of current would require an unbounded voltage.
- Before this initially unenergised circuit is switched on, each inductor current is zero. At $t = 0^+$, each branch containing an inductor therefore still carries zero current.

Step 1: Identify the conducting branches immediately after switching The 9 V battery feeds three parallel branches. The left and right branches each contain a 4 mH inductor in series with a 9 ohm resistor. Since their inductor currents were zero immediately before closing K , both currents remain zero at the instant after closing it. For finding this instantaneous current, those two branches behave as open circuits. The middle branch contains only a 9 ohm resistor, so it has no inductive restriction on its initial current.

Step 2: Apply Ohm's law to the remaining branch The middle resistor is directly across the battery. Its current, which is also the initial total current measured by the ammeter, is

$$I(0^+) = \frac{V}{R} = \frac{9\text{ V}}{9\Omega} = 1\text{ A}.$$

The result is not zero because the middle resistive path remains closed. It is not 2 A because neither inductive branch contributes an instantaneous current. The 3 A answer would apply only after a long time with ideal inductors, when all three 9 ohm branches conduct in parallel. The 4 mH inductances determine how rapidly the branch currents rise, but do not alter their zero initial values.

Step 3: Conclusion The initial ammeter reading is 1 A , corresponding to (A) in the printed option

order. Hence, the correct option is (A).

Quick Tip to Solve: At switch-on, preserve every inductor's pre-switch current. Replace an initially unenergised inductive branch by an open circuit only for the instantaneous-current calculation.

35. The wavelength of light, while it is passing through water is 540 nm . The refractive index of water is $4/3$. The wavelength of the same light when it is passing through a transparent medium having refractive index of $3/2$ is ____ nm. [4 marks]

- (A) 480
- (B) 840
- (C) 540
- (D) 380

Correct Answer: (A) 480

Solution:

Concept:

- For a medium of refractive index n , the speed of light is $v = c/n$. The frequency f remains unchanged when light passes between stationary transparent media.
- Since $v = f\lambda$, the wavelength is $\lambda = c/(nf)$. Thus $n\lambda$ is constant for the same light in different media.

Step 1: Express the invariant vacuum wavelength Let subscripts w and m denote water and the second medium. The stated 540 nm is the wavelength in water, not in vacuum. The corresponding vacuum wavelength is

$$\lambda_0 = n_w \lambda_w = \frac{4}{3} \times 540 = 720 \text{ nm.}$$

This multiplication follows from $\lambda_w = \lambda_0/n_w$. The light source fixes the frequency, so this same vacuum wavelength must be used for the second medium.

Step 2: Calculate the wavelength in the second medium Using its refractive index $n_m = 3/2$,

$$\lambda_m = \frac{\lambda_0}{n_m} = \frac{720}{3/2} = 480 \text{ nm.}$$

Equivalently, $\lambda_m/\lambda_w = n_w/n_m = 8/9$, giving $540 \times 8/9 = 480$ nm. The second medium has a larger index than water, so its light speed and wavelength must both be smaller. This rules out the larger value 840 nm and the unchanged value 540 nm. The value 380 nm also fails the required index ratio: multiplying it by $3/2$ would not recover the common vacuum wavelength 720 nm.

Step 3: Conclusion The wavelength in the transparent medium is 480 nm. Hence, the correct option is (A).

Quick Tip to Solve: Across media, frequency stays fixed and $n\lambda$ stays constant. A larger refractive index must give a shorter wavelength.

36. Given below are two statements :

Statement I : For a mechanical system of many particles total kinetic energy is the sum of kinetic energies of all the particles.

Statement II : The total kinetic energy can be the sum of kinetic energy of the center of mass w.r.t to the origin and the kinetic energy of all the particles w.r.t. the center of mass as the reference.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Both Statement I and Statement II are true
- (B) Both Statement I and Statement II are false
- (C) Statement I is true but Statement II is false
- (D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- The total kinetic energy of a nonrelativistic system is $K = \sum_i \frac{1}{2} m_i v_i^2$, with all velocities measured in the same reference frame.
- Writing $\mathbf{v}_i = \mathbf{V} + \mathbf{u}_i$, where $\mathbf{V}$ is the centre-of-mass velocity and $\mathbf{u}_i$ is the velocity relative to it, gives $\sum_i m_i \mathbf{u}_i = 0$.

Step 1: Evaluate the first statement Kinetic energy is an additive scalar quantity for the particles of the system. Adding the individual values measured relative to the origin gives

$$K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \dots$$

Interactions may introduce potential energy, but do not invalidate this expression for total kinetic energy. Hence Statement I is true.

Step 2: Expand the energy using centre-of-mass velocities Let $M = \sum_i m_i$. Substitute $\mathbf{v}_i = \mathbf{V} + \mathbf{u}_i$ in the sum:

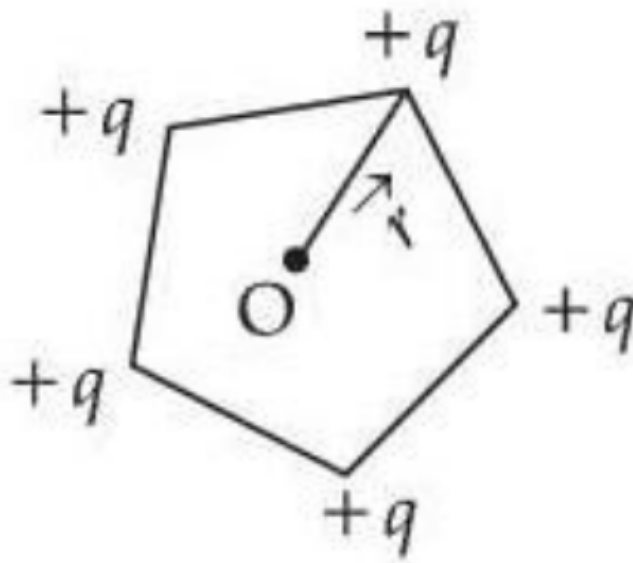
$$\begin{aligned} K &= \frac{1}{2} \sum_i m_i (\mathbf{V} + \mathbf{u}_i)^2 \\ &= \frac{1}{2} M V^2 + \mathbf{V} \cdot \sum_i m_i \mathbf{u}_i + \frac{1}{2} \sum_i m_i u_i^2 \\ &= \frac{1}{2} M V^2 + \frac{1}{2} \sum_i m_i u_i^2. \end{aligned}$$

The cross term vanishes by the definition of the centre of mass. The first term represents the translational kinetic energy of the total mass moving with the centre of mass, while the second is the kinetic energy of the particles relative to it. Statement II is therefore also true; rigid-body motion is not required for this decomposition. Options (B), (C), and (D) each incorrectly declare at least one of these identities false.

Step 3: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: To split kinetic energy into centre-of-mass and relative parts, expand $\mathbf{v}_i = \mathbf{V} + \mathbf{u}_i$; the mixed term vanishes because relative total momentum is zero.

37. Five positive charges each having charge q are placed at the vertices of a pentagon as shown in the figure. The electric potential (V) and the electric field ($\vec{E}$) at the center O of the pentagon due to these five positive charges are : [4 marks]



- (A) $V = 0$ and $\vec{E} = 0$
 (B) $V = \frac{5q}{4\pi\epsilon_0 r}$ and $\vec{E} = \frac{5q}{4\pi\epsilon_0 r^2} \hat{r}$
 (C) $V = \frac{5q}{4\pi\epsilon_0 r}$ and $\vec{E} = 0$
 (D) $V = \frac{5q}{4\pi\epsilon_0 r}$ and $\vec{E} = \frac{5\sqrt{3}q}{8\pi\epsilon_0 r^2} \hat{r}$

Correct Answer: (C) $V = \frac{5q}{4\pi\epsilon_0 r}$ and $\vec{E} = 0$

Solution:

Concept:

- Electric potential is a scalar: the potential of a point charge is $V = kq/r$, where $k = 1/(4\pi\epsilon_0)$ and zero potential is chosen at infinity.
- Electric field is a vector, so the five contributions must be added with their directions. Equal charges at the vertices of a regular pentagon have fivefold rotational symmetry about its centre.

Step 1: Add the five potentials Each vertex is at the same distance r from O . All five charges are positive, so each contributes the same positive potential; their potentials cannot cancel. Thus

$$V_O = \sum_{j=1}^5 \frac{kq}{r} = \frac{5kq}{r} = \frac{5q}{4\pi\epsilon_0 r}.$$

The radius shown in the figure is the centre-to-vertex distance, not the side length.

Step 2: Sum the field vectors using symmetry Each field at O has magnitude kq/r^2 and points away from its positive source charge, along the corresponding vertex-to-centre line. The directions are separated by 72° . A rotation by 72° leaves the entire charge arrangement unchanged, so it must leave the resultant field at the centre unchanged. No nonzero in-plane vector has that property; therefore $\vec{E}_O = 0$. Equivalently, the sine and cosine components of five equally spaced unit vectors each sum to

zero.

Step 3: Check the options Option (A) incorrectly sets the positive scalar potential to zero. Options (B) and (D) give nonzero fields and violate the rotational symmetry. Option (C) alone combines the positive summed potential with the zero vector field.

Step 4: Conclusion

$$V_O = \frac{5q}{4\pi\epsilon_0 r}, \quad \vec{E}_O = 0.$$

Hence, the correct option is (C). The option position follows the printed order.

Quick Tip to Solve: At a symmetry centre, sum potentials as scalars and fields as vectors; zero field does not imply zero potential.

38. Consider two boxes containing ideal gases A and B such that their temperatures, pressures and number densities are same. The molecular size of A is half of that of B and mass of molecule A is four times that of B. If the collision frequency in gas B is $32 \times 10^{18}/\text{s}$ then collision frequency in gas A is ____ /s. [4 marks]

- (A) 2×10^8
- (B) 4×10^8
- (C) 8×10^8
- (D) 32×10^8

Correct Answer: The printed data give $4 \times 10^{18} \text{ s}^{-1}$; none of the printed options with exponent 8 is correct.

Solution:

Concept:

- For identical hard-sphere molecules within one gas, the collision frequency per molecule is $z = \sqrt{2} n \pi d^2 \bar{c}$, where n is number density, d molecular diameter and $\bar{c}$ mean speed.
- At fixed temperature, $\bar{c} = \sqrt{8k_B T / (\pi m)}$, hence $z \propto n d^2 / \sqrt{m}$. Molecular size enters through a cross-sectional area, not linearly.

Step 1: Compare collision cross-sections and speeds The equal number densities cancel in the ratio. Because $d_A = d_B/2$, the cross-section ratio is $d_A^2/d_B^2 = 1/4$. Also, $m_A = 4m_B$ at equal temperature gives

$$\frac{\bar{c}_A}{\bar{c}_B} = \sqrt{\frac{m_B}{m_A}} = \frac{1}{2}.$$

Both the smaller collision area and the slower speed reduce the collision frequency in gas A.

Step 2: Calculate the collision frequency Combining the independent factors,

$$\frac{z_A}{z_B} = \frac{n_A}{n_B} \left(\frac{d_A}{d_B} \right)^2 \sqrt{\frac{m_B}{m_A}} = 1 \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}.$$

Using the stated value,

$$z_A = \frac{32 \times 10^{18}}{8} = 4 \times 10^{18} \text{ s}^{-1}.$$

The same ratio applies to collisions per unit volume since the number densities are equal.

Step 3: Compare the literal result with the printed options All four printed options have exponent 10^8 : their coefficients are 2, 4, 8 and 32. None equals 4×10^{18} . In particular, option (B), 4×10^8 , is smaller than the calculated result by a factor of 10^{10} . Thus selecting its coefficient alone would disregard the printed exponent.

Step 4: Conclusion The literal printed data give $z_A = 4 \times 10^{18} \text{ s}^{-1}$. No printed option is correct. This differs from the keyed choice because the stem states $32 \times 10^{18} \text{ s}^{-1}$, whereas every printed option uses 10^8 .

Quick Tip to Solve: For collision-frequency ratios at fixed temperature and density, multiply the diameter ratio squared by the inverse square root of the mass ratio; then check the exponent in the options.

39. Given below are two statements :

Statement I : A satellite is moving around earth in the orbit very close to the earth surface. The time period of revolution of satellite depends upon the density of earth.

Statement II : The time period of revolution of the satellite is $T = 2\pi \sqrt{\frac{R_e}{g}}$ (for satellite very close to the earth surface), where R_e radius of earth and g acceleration due to gravity. In the light of the above statements, choose the correct answer from the options given below : [4 marks]

- (A) Both Statement I and Statement II are true
- (B) Both Statement I and Statement II are false
- (C) Statement I is true but Statement II is false
- (D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- For a circular orbit, gravity supplies centripetal force: $GM_em/r^2 = mv^2/r$. The satellite mass cancels.
- For an orbit very close to the surface, $r \simeq R_e$ and $g = GM_e/R_e^2$. Earth's mean density is $\rho = 3M_e/(4\pi R_e^3)$.

Step 1: Derive the near-surface orbital period At the surface-radius approximation, $v^2/R_e = g$, so $v = \sqrt{gR_e}$. The satellite travels one circumference per revolution. Therefore

$$T = \frac{2\pi R_e}{v} = \frac{2\pi R_e}{\sqrt{gR_e}} = 2\pi \sqrt{\frac{R_e}{g}}.$$

This establishes Statement II. The approximation neglects orbital altitude relative to Earth's radius; it does not mean that the satellite touches the ground.

Step 2: Express the period in terms of mean density Substitute $M_e = (4/3)\pi R_e^3 \rho$ into surface gravity:

$$g = \frac{GM_e}{R_e^2} = \frac{4\pi G \rho R_e}{3}.$$

Then

$$T = 2\pi \sqrt{\frac{R_e}{(4\pi G \rho / 3) R_e}} = \sqrt{\frac{3\pi}{G \rho}}.$$

Thus the period varies as $\rho^{-1/2}$, proving Statement I. Mean density is sufficient; a uniform internal density is not required for the external field of a spherically symmetric Earth.

Step 3: Evaluate all statement combinations Both derived expressions describe the same near-surface circular orbit. Hence the option declaring both statements true is correct. Option (B) rejects both valid results, while (C) and (D) each incorrectly reject one of them.

Step 4: Conclusion Both Statement I and Statement II are true: $T = 2\pi \sqrt{R_e/g} = \sqrt{3\pi/(G\rho)}$. Hence, the correct option is (A).

Quick Tip to Solve: For an orbit close to a planet, eliminate mass with $M = (4/3)\pi R^3 \rho$; the radius cancels from the orbital period.

40. In parallax method for the determination of focal length of a concave mirror, the object should always be placed: [4 marks]

- (A) between the pole (P) and the focus (F) of the concave mirror ONLY
- (B) between the focus (F) and the centre of curvature (C) of the mirror ONLY
- (C) beyond the centre of the curvature (C) of the mirror ONLY
- (D) at any point beyond the focus (F) of the mirror

Correct Answer: (D) at any point beyond the focus (F) of the mirror

Solution:

Concept:

- In the object-pin and image-pin parallax method, a real image is located by moving the image pin until it shows no relative displacement against the image when the eye moves sideways.
- A concave mirror forms a finite real image of a real object only when its object distance exceeds the focal length. Using positive distance magnitudes, $1/f = 1/u + 1/v$ for this real-object, real-image arrangement.

Step 1: Identify the condition for a real image Let u and f denote positive magnitudes of the object and focal distances. Rearranging the mirror relation gives

$$v = \frac{fu}{u - f}.$$

For $u > f$, the denominator is positive and the image is real, on the same side of the mirror as the object. An image pin can then be placed at the image position and adjusted until parallax disappears.

Measuring u and v gives $f = uv/(u + v)$.

Step 2: Check positions relative to the centre of curvature If $f < u < 2f$, the real image forms beyond the centre of curvature. If $u > 2f$, it forms between the focus and the centre of curvature. Both placements are usable; at $u = 2f$, object and image distances both equal $2f$. Thus neither of these two intervals alone is compulsory. The practical alignment and choice of pin position do not change the real-image requirement.

Step 3: Exclude the positions inside and at the focus For $u < f$, the image is virtual and lies behind the mirror, so this real-image pin method cannot locate it in front of the mirror. At $u = f$, reflected rays are parallel and the image is at infinity. Option (A) therefore fails, and the word ONLY makes (B) and (C) unnecessarily restrictive. The complete condition is a point beyond the focus.

Step 4: Conclusion The object may be placed at any point beyond the focus F , so that a finite real image is obtained. Hence, the correct option is (D).

Quick Tip to Solve: For the usual parallax experiment, first ask where the mirror produces a finite real image; a concave mirror requires $u > f$.

41. When a part of a straight capillary tube is placed vertically in a liquid, the liquid raises upto certain height h . If the inner radius of the capillary tube, density of the liquid and surface tension of the liquid decrease by 1% each, then the height of the liquid in the tube will change by ____ %. [4 marks]

- (A) +1
- (B) -1
- (C) +3
- (D) -3

Correct Answer: (A) +1

Solution:

Concept:

- Balancing the upward surface-tension force with the liquid column's weight gives $h = 2S \cos \theta / (r\rho g)$, where S is surface tension, r tube radius and ρ liquid density.
- For small changes with contact angle and gravity unchanged, $\Delta h/h \simeq \Delta S/S - \Delta r/r - \Delta \rho/\rho$. Denominator decreases increase the height.

Step 1: Substitute the signed percentage changes The radius, density and surface tension all decrease, so each fractional change equals -0.01 . Apply the signs from the proportionality $h \propto S/(r\rho)$:

$$\frac{\Delta h}{h} \simeq -0.01 - (-0.01) - (-0.01) = +0.01.$$

The decrease in surface tension alone reduces the rise, but the decreases in radius and density each increase it. The net first-order change is positive.

Step 2: Check the result using exact factors Write $S' = 0.99S$, $r' = 0.99r$ and $\rho' = 0.99\rho$. The ratio is

$$\frac{h'}{h} = \frac{0.99}{0.99 \times 0.99} = \frac{1}{0.99}.$$

Hence the exact increase is

$$\left(\frac{1}{0.99} - 1 \right) 100\% = 1.0101 \dots \%$$

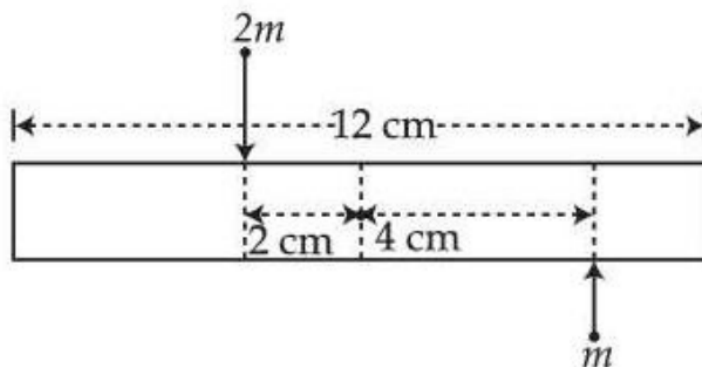
To the accuracy intended by the integer percentage options, this is +1%. The first-order treatment is adequate because each change is only one percent.

Step 3: Reject the incorrect signs and magnitude The options -1 and -3 predict a decrease, contrary to the exact ratio being greater than one. The option $+3$ treats all three decreases as increasing the rise, but surface tension is in the numerator and contributes a negative change. Only the $+1$ option matches the calculation.

Step 4: Conclusion The height increases by approximately 1% (exactly $1.0101 \dots \%$ under the stated unchanged contact angle). Hence, the correct option is (A).

Quick Tip to Solve: For percentage changes in a product or quotient, retain the sign of each change and subtract changes of factors in the denominator.

42. A uniform bar of length 12 cm and mass 20 m lies on a smooth horizontal table. Two point masses m and $2m$ are moving in opposite directions with same speed of v and in the same plane as the bar, as shown in figure. These masses strike the bar simultaneously and get stuck to it. After collision the entire system is rotating with angular frequency ω . The ratio of v and ω is : [4 marks]



- (A) 32
- (B) 33
- (C) $2\sqrt{88}$
- (D) 66

Correct Answer: (B) 33

Solution:

Concept:

- On the smooth table there is no external impulse in the plane, and no external torque about the perpendicular axis through the system's centre of mass. Linear and angular momentum are conserved during the collision.
- After the particles stick, the system undergoes translation of its centre of mass and rotation about it. The rod's central moment of inertia is $ML^2/12$, and each attached particle contributes mr^2 .

Step 1: Locate the final centre of mass Take the bar's centre as $x = 0$, with the right direction positive. From the figure, $2m$ strikes at $x = -2$ cm, and m strikes at $x = +4$ cm. Thus

$$x_{\text{CM}} = \frac{20m(0) + 2m(-2) + m(4)}{23m} = 0.$$

The final centre of mass therefore coincides with the initial bar centre. There is translation because the incoming linear momenta do not cancel, but it contributes no angular momentum about this centre.

Step 2: Calculate the initial angular momentum Let upward velocity be positive. The mass m has momentum $+mv$ at $+4$ cm, while $2m$ has momentum $-2mv$ at -2 cm. Both give the same rotational sense:

$$L_i = mv(4 \text{ cm}) + 2mv(2 \text{ cm}) = 8mv \text{ cm}.$$

Use consistent units for v and the lever arms; taking v in centimetres per second matches the listed numerical choices.

Step 3: Find the combined inertia and conserve angular momentum The final moment of inertia about the centre is

$$I = \frac{20m(12 \text{ cm})^2}{12} + 2m(2 \text{ cm})^2 + m(4 \text{ cm})^2 = 264m \text{ cm}^2.$$

Therefore $I\omega = L_i$, giving

$$264m\omega \text{ cm}^2 = 8mv \text{ cm}, \quad \frac{v}{\omega} = 33 \text{ cm}.$$

The values 32, $2\sqrt{88}$ and 66 do not satisfy this angular-momentum equation with all three inertia contributions included.

Step 4: Conclusion $v/\omega = 33 \text{ cm} = 0.33 \text{ m}$. The listed numerical value uses centimetres. Hence, the correct option is (B).

Quick Tip to Solve: Before applying angular momentum conservation, locate the combined centre of mass; include both stuck masses in the final moment of inertia.

43. A laser beam has intensity of $4.0 \times 10^{14} \text{ W/m}^2$. The amplitude of magnetic field associated with beam is ____ T.

(Take $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$ and $c = 3 \times 10^8 \text{ m/s}$)

[4 marks]

- (A) 5.5
- (B) 18.3
- (C) 1.83
- (D) 2.0

Correct Answer: (C) 1.83

Solution:

Concept:

- The intensity of a sinusoidal electromagnetic wave is its time-averaged energy flux: $I = \frac{1}{2}\epsilon_0 c E_0^2$, where E_0 is the peak electric field.
- The electric and magnetic field amplitudes obey $E_0 = cB_0$. Hence $I = \frac{1}{2}\epsilon_0 c^3 B_0^2$; the factor one-half distinguishes peak amplitude from an rms field.

Step 1: Express the intensity using magnetic amplitude Substitute the relation between the field amplitudes into the intensity formula and solve for the positive magnitude.

$$I = \frac{1}{2}\epsilon_0 c (cB_0)^2 = \frac{1}{2}\epsilon_0 c^3 B_0^2, \quad B_0 = \sqrt{\frac{2I}{\epsilon_0 c^3}}.$$

The specified intensity is an average, whereas the question asks for the amplitude, so the numerator must contain $2I$.

Step 2: Insert the supplied constants Use SI units for every quantity. First evaluate the denominator to avoid losing powers of ten:

$$\epsilon_0 c^3 = (8.85 \times 10^{-12})(3 \times 10^8)^3 = 2.3895 \times 10^{14}.$$

$$B_0^2 = \frac{8.0 \times 10^{14}}{2.3895 \times 10^{14}} = 3.348 \text{ T}^2, \quad B_0 \simeq 1.83 \text{ T}.$$

Step 3: Compare all four values The value 1.83 T agrees with the calculation. The alternatives 5.5 T, 18.3 T and 2.0 T give squared amplitudes 30.25, 334.89 and 4.00 T², respectively; none equals the required 3.348 T².

Step 4: Conclusion The magnetic field amplitude is $B_0 = 1.83 \text{ T}$. Hence, the correct option is (C). The intensity formula gives a squared amplitude of 3.348 square tesla, which selects the third printed value.

Quick Tip to Solve: For a magnetic amplitude obtained from intensity, use $B_0 = \sqrt{2I/(\epsilon_0 c^3)}$; using c instead of c^3 gives the electric-field calculation instead.

44. Three small identical bubbles of water having same charge on each coalesce to form a bigger bubble. Then the ratio of the potentials on one initial bubble and that on the resultant bigger bubble is :

[4 marks]

(A) $1 : 3^{2/3}$

(B) $1 : 3^{1/3}$

(C) $1 : 2^{2/3}$

(D) $3^{2/3} : 1$

Correct Answer: (A) $1 : 3^{2/3}$

Solution:

Concept:

- The potential at the surface of an isolated charged sphere of radius r is $V = kq/r$, where $k = 1/(4\pi\epsilon_0)$ and q is its charge.
- When three identical spherical water bodies coalesce, their total volume and total charge are conserved. The resulting radius scales with the cube root of their number.

Step 1: Determine the radius and charge after coalescence Let each original bubble have radius r and charge q . Let the resultant sphere have radius R and charge Q . Conservation gives

$$\frac{4\pi}{3}R^3 = 3\frac{4\pi}{3}r^3, \quad R = 3^{1/3}r, \quad Q = 3q.$$

The radius is not multiplied by three: it is the volume, proportional to the cube of the radius, that adds.

Step 2: Form the ratio in the requested order The potential of one initial bubble is $V_i = kq/r$. The final potential is

$$V_f = \frac{k(3q)}{3^{1/3}r} = 3^{2/3}\frac{kq}{r} = 3^{2/3}V_i.$$

Therefore

$$\frac{V_i}{V_f} = 3^{-2/3}, \quad V_i : V_f = 1 : 3^{2/3}.$$

The same ratio follows for equal negative charges because their common sign cancels.

Step 3: Eliminate the other ratios The ratio $1 : 3^{1/3}$ uses the radius scaling but misses the factor of three in charge. The factor $2^{2/3}$ would correspond to two identical bodies, whereas three coalesce here. Finally, $3^{2/3} : 1$ reverses the initial-to-final order specifically requested in the question.

Step 4: Conclusion The ratio of the initial potential to the resultant potential is $1 : 3^{2/3}$. Hence, the correct option is (A).

Quick Tip to Solve: For n identical charged spherical drops coalescing, $R = n^{1/3}r$ and $Q = nq$, so the final surface potential is $n^{2/3}$ times the initial value.

45. If ϵ , E and t represent the free space permittivity, electric field and time respectively, then the unit of $\frac{\epsilon E}{t}$ will be : [4 marks]

- (A) A/m^2
- (B) A m^2
- (C) A m
- (D) A/m

Correct Answer: (A) A/m^2

Solution:

Concept:

- Free-space permittivity has SI unit $\text{C}^2/(\text{N m}^2)$, while electric field has SI unit N/C .
- An ampere is one coulomb per second: $\text{A} = \text{C/s}$. Unit algebra must retain all powers of length during cancellation.

Step 1: Multiply the units of permittivity and electric field Write each factor separately rather than guessing from the electric-field unit. Their product has units

$$[\epsilon E] = \frac{\text{C}^2}{\text{N m}^2} \frac{\text{N}}{\text{C}} = \frac{\text{C}}{\text{m}^2}.$$

One power of charge and the full inverse-area factor remain after the newton and one coulomb cancel. This is the unit of electric displacement, or charge per area.

Step 2: Divide by the time unit Time t is measured in seconds. Thus

$$\left[\frac{\epsilon E}{t} \right] = \frac{\text{C}}{\text{m}^2 \text{s}} = \frac{\text{A}}{\text{m}^2}.$$

This is a current-density unit. It is also consistent with the displacement-current density $\epsilon \partial E / \partial t$, which has the same dimensions as the expression in the question.

Step 3: Check the length dependence of each alternative The option A m^2 has positive rather than negative area power. The option A m has a positive single length power. The option A/m loses one required inverse-length factor. Only A/m^2 preserves both the charge-per-time and inverse-area factors found above.

Step 4: Conclusion The SI unit is A/m^2 , or ampere per square metre. Hence, the correct option is (A). The cancellation leaves charge per unit area per unit time, selecting the first printed option.

Quick Tip to Solve: Remember that ϵE has the units of surface charge density. Dividing it by time immediately gives current per unit area.

Physics – Section B: Numerical Value Questions (20 marks)

46. A capacitor P with capacitance 10×10^{-6} F is fully charged with a potential difference of 6.0 V and disconnected from the battery. The charged capacitor P is connected across another capacitor Q with capacitance 20×10^{-6} F. The charge on capacitor Q when equilibrium is established will be $\alpha \times 10^{-5}$ C (assume capacitor Q does not have any charge initially), the value of α is _____. [4 marks]

Solution:

Concept:

- For a capacitor, $q = CV$, with capacitance in farads and potential difference in volts. An initially uncharged capacitor contributes no initial charge.
- When disconnected capacitors are connected across one another, charge is redistributed until their potential differences are equal. Total charge on the connected positive-plate node is conserved.

Step 1: Calculate the available charge Initially only capacitor P is charged. Its capacitance and voltage give

$$q_{\text{initial}} = C_P V_0 = (10 \times 10^{-6})(6.0) = 60 \times 10^{-6} \text{ C}.$$

The battery is disconnected, so it cannot supply additional charge during redistribution. Capacitor Q starts with zero charge.

Step 2: Find the common equilibrium voltage Connecting the capacitors across each other places them in parallel. If their final common voltage is V_f , conservation gives

$$(C_P + C_Q)V_f = q_{\text{initial}}.$$

$$V_f = \frac{60 \times 10^{-6}}{(10 + 20) \times 10^{-6}} = 2.0 \text{ V}.$$

The voltage drops below its initial value because the same total charge is now shared by a larger total capacitance.

Step 3: Determine the charge on the second capacitor The requested quantity belongs to Q , so multiply the common voltage by C_Q , not by C_P :

$$q_Q = C_Q V_f = (20 \times 10^{-6})(2.0) = 40 \times 10^{-6} = 4.0 \times 10^{-5} \text{ C}.$$

As a check, $q_P = (10 \times 10^{-6})(2) = 20 \times 10^{-6}$ C, and the two final charges sum to the initial 60×10^{-6} C.

Step 4: Conclusion The final charge on capacitor Q is 4.0×10^{-5} C. Comparing with $\alpha \times 10^{-5}$ C gives $\alpha = 4$, a dimensionless coefficient.

Quick Tip to Solve: For charge sharing after the battery is removed, first find the common voltage from total charge. Do not retain the original battery voltage or impose conservation of electrostatic energy.

47. A conducting circular loop is rotated about its diameter at a constant angular speed of 100 rad/s in a magnetic field of 0.5 T perpendicular to the axis of rotation. When the loop is rotated by 30° from the horizontal position, the induced EMF is 15.4 mV . The radius of the loop is ____ mm.

$$\left(\text{Take } \pi = \frac{22}{7} \right)$$

[4 marks]

Solution:

Concept:

- The magnetic flux through a single-turn circular loop is $\Phi = BA \cos \theta$, where $A = \pi r^2$ and θ is the angle between its normal and the magnetic field.
- Faraday's law gives the induced-emf magnitude $e = BA\omega |\sin \theta|$ for uniform rotation with angular speed ω . The given emf is instantaneous, not its maximum value.

Step 1: Relate the emf to the angle of rotation Taking the horizontal starting position as the maximum-flux position, a rotation through 30° turns the area normal through the same angle. Differentiating $BA \cos \theta$, with $d\theta/dt = \omega$, gives

$$e = B\pi r^2 \omega \sin 30^\circ.$$

Since $\sin 30^\circ = 1/2$, the observed emf is half the maximum emf for this loop.

Step 2: Substitute in SI units and isolate the radius Convert the emf before substituting: $15.4\text{ mV} = 15.4 \times 10^{-3}\text{ V}$. Thus

$$15.4 \times 10^{-3} = 0.5 \times \frac{22}{7} \times r^2 \times 100 \times \frac{1}{2}.$$

$$r^2 = \frac{15.4 \times 10^{-3} \times 7}{550} = 1.96 \times 10^{-4}\text{ m}^2.$$

The sine factor is essential because the loop has not reached the orientation of maximum emf.

Step 3: Take the square root and convert the length The radius is positive, so

$$r = \sqrt{1.96 \times 10^{-4}} = 1.4 \times 10^{-2}\text{ m} = 14\text{ mm}.$$

Substitution checks the result: $(22/7)(0.014)^2 = 0.000616\text{ m}^2$, and $0.5(0.000616)(100)(1/2) = 0.0154\text{ V}$, the stated emf.

Step 4: Conclusion The radius of the loop is 14 mm. The numerical value to enter is 14.

Quick Tip to Solve: Use the instantaneous value $e = BA\omega \sin \theta$ at the stated angle. Convert millivolts to volts before finding the area, then take a square root to obtain the radius.

48. A cylindrical conductor of length 2 m and area of cross-section 0.2 mm^2 carries an electric current of 1.6 A when its ends are connected to a 2 V battery. Mobility of electrons in the conductor is $\alpha \times 10^{-3} \text{ m}^2/\text{V.s}$. The value of α is :

(electron concentration = $5 \times 10^{28}/\text{m}^3$ and electron charge = $1.6 \times 10^{-19} \text{ C}$)

[4 marks]

Solution:

Concept:

- The magnitude of drift current is $I = neAv_d$, where n is the electron number density, e the charge magnitude, A the cross-sectional area and v_d the drift-speed magnitude.
- Electron mobility is $\mu = v_d/E$. For a uniform conductor of length L with potential difference V , the electric-field magnitude is $E = V/L$.

Step 1: Convert the area and calculate the electric field The square in the area unit must also be converted:

$$A = 0.2 \text{ mm}^2 = 0.2 \times 10^{-6} \text{ m}^2 = 2.0 \times 10^{-7} \text{ m}^2.$$

Across the 2 m conductor,

$$E = \frac{V}{L} = \frac{2}{2} = 1 \text{ V m}^{-1}.$$

This is a field, not the battery voltage itself.

Step 2: Obtain the electron drift speed Rearrange the current relation using the magnitude of the electron charge:

$$v_d = \frac{I}{neA} = \frac{1.6}{(5 \times 10^{28})(1.6 \times 10^{-19})(2 \times 10^{-7})}.$$

The denominator is $16 \times 10^2 = 1600$, giving

$$v_d = \frac{1.6}{1600} = 1.0 \times 10^{-3} \text{ m s}^{-1}.$$

Electron motion is opposite the electric field; mobility uses the positive speed magnitude.

Step 3: Compute the mobility and identify the coefficient Divide this speed by the field strength:

$$\mu = \frac{v_d}{E} = \frac{1.0 \times 10^{-3} \text{ m s}^{-1}}{1 \text{ V m}^{-1}} = 1.0 \times 10^{-3} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}.$$

The units follow directly: dividing metres per second by volts per metre supplies a second power of length. Matching the coefficient of 10^{-3} gives the required number.

Step 4: Conclusion The electron mobility is $1.0 \times 10^{-3} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$, so $\boxed{\alpha = 1}$. Here α is dimensionless.

Quick Tip to Solve: Combine the relations into $\mu = IL/(neAV)$, but convert mm^2 using 10^{-6} , not 10^{-3} . A missed square changes the result by a factor of a thousand.

49. Two masses m and $2m$ are connected by a light string going over a pulley (disc) of mass $30m$ with radius $r = 0.1$ m. The pulley is mounted in a vertical plane and it is free to rotate about its axis. The $2m$ mass is released from rest and its speed when it has descended through a height of 3.6 m is _____ m/s. (Assume string does not slip and $g = 10$ m/s²) [4 marks]

Solution:

Concept:

- The two masses have the same speed v , because the connecting string is light and inextensible. No slipping implies $\omega = v/r$ for the pulley.
- A uniform disc of mass M has moment of inertia $I = Mr^2/2$ about its central axis. The decrease in gravitational potential energy supplies both translational and rotational kinetic energy.

Step 1: Find the net loss of potential energy When the heavier mass $2m$ moves down through $h = 3.6$ m, the lighter mass m rises through the same height. The energy released is therefore the difference of the two changes, not their sum:

$$E_{\text{released}} = 2mgh - mgh = mgh.$$

Both masses and the disc start from rest, so there is no initial kinetic energy.

Step 2: Include the kinetic energy of the disc The pulley mass is $30m$, giving $I = 15mr^2$. The total final kinetic energy is

$$\begin{aligned} K &= \frac{1}{2}mv^2 + \frac{1}{2}(2m)v^2 + \frac{1}{2}I\omega^2 \\ &= \frac{3}{2}mv^2 + \frac{1}{2}(15mr^2)\frac{v^2}{r^2} = 9mv^2. \end{aligned}$$

The radius cancels because the rim speed is fixed by the string speed. Treating the pulley as massless would omit most of the kinetic energy.

Step 3: Apply conservation of mechanical energy Equate the net potential-energy loss to the total kinetic energy:

$$mgh = 9mv^2 \quad \Rightarrow \quad v^2 = \frac{10 \times 3.6}{9} = 4 \text{ m}^2\text{s}^{-2}.$$

The mass m cancels. Since speed is non-negative, take the positive square root.

Step 4: Conclusion The speed of the descending mass is $\boxed{2 \text{ m s}^{-1}}$. The numerical entry is 2.

Quick Tip to Solve: For a massive pulley, add the effective mass I/r^2 to the translating masses before finding the acceleration or kinetic energy.

50. An insulated cylinder of volume 60 cm^3 is filled with a gas at 27°C and 2 atmospheric pressure. Then the gas is compressed making the final volume as 20 cm^3 while allowing the temperature to rise to 77°C . The final pressure is ____ atmospheric pressure. [4 marks]

Solution:

Concept:

- For a fixed amount of an ideal gas, $PV = nRT$. Hence PV/T has the same value in the initial and final states.
- Temperatures in gas-law ratios must be absolute temperatures in kelvin. Pressure and volume may remain in the stated units when only their ratios are used.

Step 1: Write the initial and final state variables The gas remains in the cylinder, so its amount is unchanged. Using the usual Celsius-to-kelvin conversion for these data,

$$T_1 = 27 + 273 = 300 \text{ K}, \quad T_2 = 77 + 273 = 350 \text{ K}.$$

The initial pressure is $P_1 = 2 \text{ atm}$, and the volumes are $V_1 = 60 \text{ cm}^3$ and $V_2 = 20 \text{ cm}^3$. The volume falls to one third while the temperature rises.

Step 2: Eliminate the constant amount of gas Apply the ideal-gas equation at both endpoints and divide:

$$\frac{P_1 V_1}{T_1} = nR = \frac{P_2 V_2}{T_2}.$$

Rearranging isolates the unknown pressure:

$$P_2 = P_1 \frac{V_1}{V_2} \frac{T_2}{T_1}.$$

The cylinder being insulated concerns heat transfer during compression; it does not invalidate the ideal-gas equation at either endpoint. Since the endpoint temperature is given, no heat-capacity ratio is required.

Step 3: Substitute and check the pressure change Using the volume ratio in the same units gives

$$P_2 = 2 \times \frac{60}{20} \times \frac{350}{300} = 2 \times 3 \times \frac{7}{6} = 7 \text{ atm}.$$

At unchanged temperature, reducing the volume by a factor of three would give 6 atm. The stated temperature rise increases it by another factor $7/6$, consistent with the result.

Step 4: Conclusion The final pressure is 7 atm. The numerical entry is 7.

Quick Tip to Solve: Use the combined gas law when both endpoint temperatures are supplied. Convert Celsius to kelvin before taking a temperature ratio.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. $[\text{Ni}(\text{PPh}_3)_2\text{Cl}_2]$ is a paramagnetic complex. Identify the INCORRECT statements about this complex.

- A. The complex exhibits geometrical isomerism.
- B. The complex is white in colour.
- C. The calculated spin-only magnetic moment of the complex is 2.84 BM .
- D. The calculated CFSE (Crystal Field Stabilization Energy) of Ni in this complex is $-0.8\Delta_o$.
- E. The geometrical arrangement of ligands in this complex is similar to that in $\text{Ni}(\text{CO})_4$.

Choose the correct answer from the options given below :

[4 marks]

- (A) A, B and D Only
- (B) C and D Only
- (C) A and B Only
- (D) C, D and E Only

Correct Answer: (A) A, B and D Only

Solution:

Concept:

- In neutral $[\text{Ni}(\text{PPh}_3)_2\text{Cl}_2]$, the two chloride ligands give nickel oxidation state +2, so the metal has a $3d^8$ configuration.
- A paramagnetic four-coordinate d^8 nickel complex is tetrahedral in this context. The square-planar d^8 arrangement would have all electrons paired.

Step 1: Determine the geometry and unpaired electrons Tetrahedral splitting places the two e orbitals below the three t_2 orbitals. Eight electrons occupy them as $e^4t_2^4$, leaving two unpaired electrons. Thus

$$\mu_{\text{so}} = \sqrt{n(n+2)} = \sqrt{2(2+2)} \approx 2.83 \text{ BM.}$$

The value 2.84 BM in statement C is the corresponding customary approximation, so C is treated as correct.

Step 2: Assess isomerism and colour For tetrahedral MA_2B_2 , all ligand positions are equivalent and there is no distinct cis/trans pair. Statement A is therefore incorrect. This partially filled d -shell complex is coloured, rather than white, so statement B is also incorrect. One must use the geometry indicated by paramagnetism rather than imposing square-planar geometrical isomerism.

Step 3: Compute the stabilization and compare the reference complex The tetrahedral orbital energies are $-0.6\Delta_t$ for each e electron and $+0.4\Delta_t$ for each t_2 electron. Therefore

$$\text{CFSE} = 4(-0.6\Delta_t) + 4(+0.4\Delta_t) = -0.8\Delta_t.$$

Statement D incorrectly uses the octahedral splitting Δ_o , so it is false. $\text{Ni}(\text{CO})_4$ is also tetrahedral, making E true. Options (B) and (D) wrongly classify C as incorrect; option (C) omits the incorrect statement D.

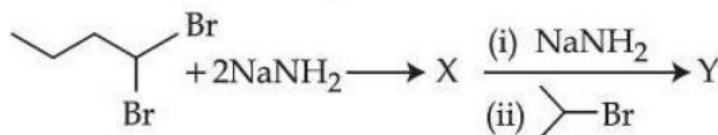
Step 4: Conclusion The incorrect statements are A, B and D only. Hence, the correct option is (A).

Quick Tip to Solve: Keep Δ_t and Δ_o distinct: a correct numerical coefficient with the wrong splitting parameter still gives an incorrect CFSE.

52. Consider the following reaction :

The product Y formed is :

[4 marks]



(A) 5-methylhex-2-yne

(B) 2-methylhex-3-yne

(C) 2-methylhex-2-yne

(D) Isopropylbut-1-yne

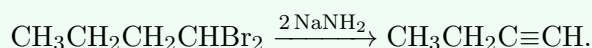
Correct Answer: (B) 2-methylhex-3-yne

Solution:

Concept:

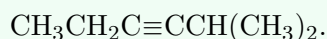
- A geminal dibromide can undergo successive dehydrohalogenations with NaNH_2 to form an alkyne. The carbon skeleton is retained while two molecules of HBr are eliminated.
- A terminal alkyne has an acidic terminal hydrogen. Its acetylide ion provides the carbon nucleophile in the alkylation sequence shown; the final name uses the longest chain containing the triple bond.

Step 1: Identify the terminal alkyne X The starting structure is $\text{CH}_3\text{CH}_2\text{CH}_2\text{CHBr}_2$, a four-carbon geminal dibromide. Two eliminations across the terminal carbon pair form



Thus X is but-1-yne. The two carbon atoms of the triple bond were already present in the starting material; the first stage does not add any carbon.

Step 2: Form the acetylide and attach the alkyl group The additional NaNH_2 removes the terminal alkyne proton, producing $\text{CH}_3\text{CH}_2\text{C}\equiv\text{C}^-$. The alkyl bromide shown is isopropyl bromide, $(\text{CH}_3)_2\text{CHBr}$. The carbon-coupling product represented by the given sequence has connectivity



It contains seven carbon atoms in total, with the new bond joining the terminal acetylide carbon to the carbon bearing bromine.

Step 3: Choose the parent chain and lowest locants The longest chain containing the triple bond has six carbons. Numbering from either end puts the triple bond at position 3; the tie is resolved by giving the methyl group position 2 instead of 5. The name is therefore 2-methylhex-3-yne. Option

(A) puts the triple bond at position 2, changing connectivity. Option (C) also assigns an incompatible triple-bond position. Option (D) does not use the longest six-carbon parent chain.

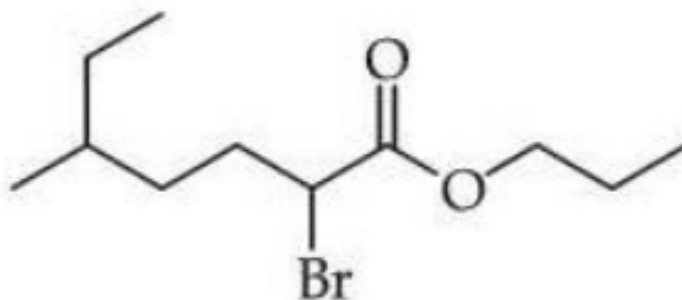
Step 4: Conclusion The product Y is 2-methylhex-3-yne. Hence, the correct option is (B).

Quick Tip to Solve: After joining the two carbon fragments, redraw the full chain before naming it. When triple-bond locants tie, minimize the substituent locant.

53.

The IUPAC name of the following compound is :

[4 marks]



- (A) 2-bromo-5-methylpropanoate
(B) n-propyl-2-bromo-5-methylheptanoate
(C) n-propyl-1-bromo-4-methylhexanoate
(D) 2-bromo-5-methylhexylpropanoate

Correct Answer: (B) n-propyl-2-bromo-5-methylheptanoate

Solution:

Concept:

- An ester is named as alkyl alkanoate: first name the group bonded to oxygen, then the acid-derived part containing the carbonyl carbon.
- For the acid-derived chain, the ester carbonyl carbon is carbon 1. Choose the longest carbon chain containing it, and number all substituents from that end.

Step 1: Identify the oxygen-linked alkyl group The right side of the ester linkage is $\text{-OCH}_2\text{CH}_2\text{CH}_3$. It is an unbranched three-carbon group, namely propyl (or n-propyl). This group is attached through oxygen and must be named separately. Its three carbons cannot be included in the carbon chain on the acyl side of the ester.

Step 2: Count the longest acid-derived chain Begin at the carbonyl carbon as C1 and follow the carbon chain to the left. The next carbon, C2, carries bromine. Continuing gives C3 and C4, then the branching carbon C5. At this branch, follow the two-carbon upward path to C6 and C7, rather than stopping along the shorter one-carbon left branch. The parent acid-derived chain consequently contains seven carbons and is heptanoate. The unused one-carbon branch on C5 is a methyl substituent.

Step 3: Assemble the substituents and eliminate the alternatives The substituents are 2-bromo and 5-methyl; they are cited alphabetically, with bromo before methyl. Combining these with the oxygen-linked group gives n-propyl 2-bromo-5-methylheptanoate. Option (A) omits the propyl group and uses the wrong parent chain. Option (C) starts numbering past the carbonyl carbon, producing a six-carbon parent and wrong locants. Option (D) reverses the roles of the alcohol-derived and acid-derived fragments.

Step 4: Conclusion The compound is n-propyl 2-bromo-5-methylheptanoate (propyl 2-bromo-5-methylheptanoate). Hence, the correct option is (B).

Quick Tip to Solve: In ester naming, separate the fragments at the single C–O bond next to the carbonyl. Count the carbonyl carbon in the alkanoate chain, never in the alkyl group.

54. Given below are two statements :

Statement I : The first ionization enthalpy of Cr is lower than that of Mn .

Statement II : The second and third ionization enthalpies of Cr are higher than those of Mn .

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Both Statement I and Statement II are true
- (B) Both Statement I and Statement II are false
- (C) Statement I is true but Statement II is false
- (D) Statement I is false but Statement II is true

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- Ionization enthalpy refers to removing an electron from the gaseous species at the stated stage. The electron configuration must be updated after every removal.
- Chromium has configuration $[\text{Ar}]3d^54s^1$, whereas manganese has $[\text{Ar}]3d^54s^2$. The stability of a half-filled $3d^5$ subshell is central to their ionization trends.

Step 1: Compare the first ionizations The first electron removed from chromium is its single $4s$ electron, giving Cr^+ with $3d^5$. Manganese loses one of its two $4s$ electrons to give Mn^+ with $3d^54s^1$. Chromium has the lower first ionization enthalpy (approximately 653 versus 717 kJ mol^{-1}). Thus Statement I is true.

Step 2: Track the second ionizations The second ionization of chromium removes a $3d$ electron from the stable $3d^5$ configuration: $\text{Cr}^+ \longrightarrow \text{Cr}^{2+} + e^-$, leaving $3d^4$. The second ionization of manganese removes its remaining $4s$ electron and yields Mn^{2+} with stable $3d^5$. Hence the second ionization enthalpy of chromium is indeed higher than that of manganese.

Step 3: Check the third ionizations separately At the third stage, Cr^{2+} changes from $3d^4$ to $3d^3$, whereas Mn^{2+} must lose an electron from its half-filled $3d^5$ subshell, becoming $3d^4$. The third ionization enthalpy of manganese is therefore higher, not lower, than that of chromium. Statement II

joins two claims with "and"; its false third-ionization comparison makes the whole statement false. Thus options asserting both statements true or Statement I false are eliminated.

Step 4: Conclusion Statement I is true, but Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: Write the configuration of the ion before each electron is removed. The half-filled subshell is protected at different ionization stages for chromium and manganese.

55. At T(K), 100 g of 98% H_2SO_4 (w/w) aqueous solution is mixed with 100 g of 49% H_2SO_4 (w/w) aqueous solution. What is the mole fraction of H_2SO_4 in the resultant solution?

(Given : Atomic mass H = 1u; S = 32u; O = 16u).

(Assume that temperature after mixing remains constant)

[4 marks]

(A) 0.663

(B) 0.1

(C) 0.9

(D) 0.337

Correct Answer: (D) 0.337

Solution:

Concept:

- Mass percentage gives the mass of solute in 100 g of solution. Add solute masses and solvent masses separately before converting either to moles.
- The mole fraction of sulphuric acid is $x_{\text{H}_2\text{SO}_4} = n_{\text{H}_2\text{SO}_4} / (n_{\text{H}_2\text{SO}_4} + n_{\text{H}_2\text{O}})$. It is dimensionless and differs from the mass fraction.

Step 1: Find the masses after mixing The first 100 g solution contains 98 g of H_2SO_4 and $100 - 98 = 2$ g of water. The second contains 49 g of H_2SO_4 and $100 - 49 = 51$ g of water. Therefore,

$$m_{\text{H}_2\text{SO}_4} = 98 + 49 = 147 \text{ g}, \quad m_{\text{H}_2\text{O}} = 2 + 51 = 53 \text{ g}.$$

Their sum is 200 g, equal to the total mass mixed. No solution volume or density is needed for a mole-fraction calculation.

Step 2: Convert the component masses to moles Using the given atomic masses,

$$M_{\text{H}_2\text{SO}_4} = 2(1) + 32 + 4(16) = 98 \text{ g mol}^{-1}, \quad M_{\text{H}_2\text{O}} = 18 \text{ g mol}^{-1}.$$

Thus the amounts of the two components are

$$n_{\text{H}_2\text{SO}_4} = \frac{147}{98} = \frac{3}{2} \text{ mol}, \quad n_{\text{H}_2\text{O}} = \frac{53}{18} \text{ mol}.$$

Step 3: Calculate and compare the mole fractions Substitute the component amounts into the definition:

$$x_{\text{H}_2\text{SO}_4} = \frac{3/2}{3/2 + 53/18} = \frac{27}{80} = 0.3375.$$

The matching listed value is 0.337. The complementary water mole fraction is $53/80 = 0.6625$, explaining the distractor 0.663. Neither 0.1 nor 0.9 follows from these component mole amounts. The acid mass fraction, $147/200 = 0.735$, must not be used in place of the mole fraction.

Step 4: Conclusion The mole fraction of H_2SO_4 is 0.3375, corresponding to the listed value 0.337. Hence, the correct option is (D).

Quick Tip to Solve: For mixtures specified by mass percentage, choose the stated masses as the basis, add acid and water separately, and convert each total to moles only once.

56. The compound A, $\text{C}_8\text{H}_8\text{O}_2$ reacts with acetophenone to form a single product via cross-Aldol condensation. The compound A on reaction with conc. NaOH forms a substituted benzyl alcohol as one of the two products. The compound A is : [4 marks]

- (A) 4-methoxy benzaldehyde
- (B) 4-hydroxy benzylaldehyde
- (C) 2-hydroxy acetophenone
- (D) 4-methyl benzoic acid

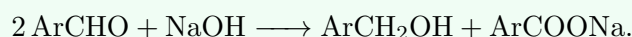
Correct Answer: (A) 4-methoxy benzaldehyde

Solution:

Concept:

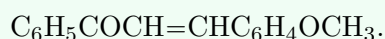
- An aromatic aldehyde without an alpha hydrogen can undergo Cannizzaro disproportionation in concentrated alkali, yielding an aromatic alcohol and a carboxylate salt.
- In a selective crossed aldol condensation, acetophenone supplies the enolate, while a non-enolisable aromatic aldehyde supplies the electrophilic carbonyl carbon.

Step 1: Use the Cannizzaro reaction to identify the functional group Formation of a substituted benzyl alcohol in concentrated NaOH points to an aromatic aldehyde, ArCHO , without an alpha hydrogen. One aldehyde molecule is reduced to ArCH_2OH ; another is oxidised to ArCOO^- . The balanced transformation is



Thus the required compound must have a $-\text{CHO}$ group attached directly to the aromatic ring, rather than being an aromatic ketone or carboxylic acid.

Step 2: Check the formula and crossed aldol behaviour 4-Methoxybenzaldehyde is $p\text{-CH}_3\text{OC}_6\text{H}_4\text{CHO}$. Counting its atoms gives $\text{C}_8\text{H}_8\text{O}_2$, as required. Its methoxy hydrogens are not alpha hydrogens of the carbonyl group, so it does not form a competing carbonyl enolate. Acetophenone, $\text{C}_6\text{H}_5\text{COCH}_3$, has alpha hydrogens and forms the enolate. Addition to the aldehyde and subsequent dehydration give the crossed condensation product



Its Cannizzaro alcohol is 4-methoxybenzyl alcohol, also matching the second observation.

Step 3: Eliminate the remaining choices The hydroxy-substituted benzaldehyde in (B) has only seven carbon atoms and does not have formula $C_8H_8O_2$. The compound in (C), 2-hydroxyacetophenone, is a ketone with alpha hydrogens; it cannot give the stated benzyl alcohol by the aldehyde Cannizzaro reaction. 4-Methylbenzoic acid in (D) is a carboxylic acid: it forms a carboxylate salt with NaOH, rather than undergoing the two stated carbonyl reactions.

Step 4: Conclusion Compound A is 4-methoxybenzaldehyde, $p\text{-CH}_3\text{OC}_6\text{H}_4\text{CHO}$. Hence, the correct option is (A).

Quick Tip to Solve: Use concentrated alkali plus benzyl alcohol as the Cannizzaro clue; then check the molecular formula before choosing among aromatic carbonyl compounds.

57. Correct statements regarding Arrhenius equation among the following are :

A. Factor $e^{-E_a/RT}$ corresponds to fraction of molecules having kinetic energy less than E_a .

B. At a given temperature, lower the E_a , faster is the reaction.

C. Increase in temperature by about 10°C doubles the rate of reaction.

D. Plot of $\log k$ vs $\frac{1}{T}$ gives a straight line with slope $= -\frac{E_a}{R}$.

Choose the correct answer from the options given below :

[4 marks]

(A) A and C Only

(B) B and D Only

(C) B and C Only

(D) A and B Only

Correct Answer: (C) B and C Only

Solution:

Concept:

- The Arrhenius equation is $k = Ae^{-E_a/(RT)}$, where A is the pre-exponential factor, E_a the activation energy, R the gas constant and T the absolute temperature.
- For a common logarithm, $\log_{10} k = \log_{10} A - E_a/(2.303RT)$; for a natural logarithm, $\ln k = \ln A - E_a/(RT)$.

Step 1: Assess the energy factor and activation energy The exponential factor represents, in the elementary Arrhenius interpretation, the fraction able to surmount the activation-energy barrier. It does not represent the fraction with energy less than E_a , so statement A is false. At a fixed temperature, lowering E_a makes the exponent less negative and increases k , for comparable pre-exponential factors. This increases the reaction rate at fixed reactant concentrations, so statement B is correct in the intended comparison.

Step 2: Assess the temperature rule Statement C expresses the usual empirical rule that a temperature rise of about 10°C approximately doubles the rate of many reactions near ordinary temperatures. The Arrhenius expression for a 10 K rise is

$$\frac{k(T+10)}{k(T)} = \exp\left[\frac{E_a}{R}\left(\frac{1}{T} - \frac{1}{T+10}\right)\right].$$

The exact factor depends on E_a and T ; the statement is the standard approximate temperature-coefficient rule, not a universal exact equality. It is accepted as C in this question.

Step 3: Check the logarithmic slope and choose the pair For the plotted common logarithm, the slope is

$$\frac{d(\log_{10} k)}{d(1/T)} = -\frac{E_a}{2.303R}.$$

The slope $-E_a/R$ belongs to a plot of $\ln k$, not $\log_{10} k$. Hence D is false. Options (A) and (D) include false statement A, while option (B) includes false statement D. Only option (C) contains the accepted statements B and C.

Step 4: Conclusion B and C are the correct statements, with C understood as the usual approximate temperature rule. Hence, the correct option is (C).

Quick Tip to Solve: Before reading an Arrhenius slope, identify the logarithm: common log requires the factor 2.303, while natural log does not.

58. Which of the following mixture gives a buffer solution with $\text{pH} = 9.25$?

Given : $\text{p}K_b(\text{NH}_4\text{OH}) = 4.75$

[4 marks]

- (A) 0.4 M NH_4OH (1 L) + 0.1 M HCl (1 L)
- (B) 0.2 M NH_4OH (0.4 L) + 0.1 M HCl (1 L)
- (C) 0.2 M NH_4OH (0.5 L) + 0.1 M HCl (0.5 L)
- (D) 0.5 M NH_4OH (0.2 L) + 0.2 M HCl (0.5 L)

Correct Answer: (C) 0.2 M NH_4OH (0.5 L) + 0.1 M HCl (0.5 L)

Solution:

Concept:

- A basic buffer contains a weak base and its conjugate-acid salt. Partial neutralisation of NH_4OH by HCl produces NH_4Cl while leaving some weak base.
- The buffer equation is $\text{pOH} = \text{p}K_b + \log([\text{salt}]/[\text{base}])$. After mixing, the common volume cancels, so the ratio can be calculated from moles.

Step 1: Find the required base-to-salt ratio Using $\text{pH} + \text{pOH} = 14$, the target pOH is

$$\text{pOH} = 14 - 9.25 = 4.75 = \text{p}K_b.$$

The logarithmic term must therefore be zero: salt and unreacted base must have equal amounts. If the initial weak-base amount is b and the acid amount is h , complete neutralisation of the strong acid gives salt h and residual base $b - h$. Thus

$$h = b - h, \quad b = 2h.$$

Both quantities must be positive for a buffer to remain.

Step 2: Test the printed options using moles For option (A), $b = 0.4 \times 1 = 0.40$ mol and $h = 0.1 \times 1 =$

0.10 mol. The residual base is 0.30 mol and salt is 0.10 mol, which are unequal. For option (B), $b = 0.2 \times 0.4 = 0.08$ mol and $h = 0.1 \times 1 = 0.10$ mol: strong acid remains in excess, so no basic buffer results. For option (C),

$$b = 0.2 \times 0.5 = 0.10 \text{ mol}, \quad h = 0.1 \times 0.5 = 0.05 \text{ mol}.$$

Neutralisation leaves 0.05 mol base and produces 0.05 mol salt. Option (D) has $b = 0.5 \times 0.2 = 0.10$ mol and $h = 0.2 \times 0.5 = 0.10$ mol, so all the weak base is consumed.

Step 3: Verify the selected mixture Only (C) retains equal nonzero amounts of the weak base and its conjugate-acid salt. Consequently,

$$\text{pOH} = 4.75 + \log \frac{0.05}{0.05} = 4.75, \quad \text{pH} = 9.25.$$

Option (A) is a buffer but has a different pH; (B) has excess strong acid, and (D) contains the ammonium salt without the required unreacted weak base.

Step 4: Conclusion The required mixture is 0.2 M NH_4OH (0.5 L) and 0.1 M HCl (0.5 L). Hence, the correct option is (C).

Quick Tip to Solve: When the target pOH equals pK_b, neutralise exactly half the initial weak base: the leftover base and newly formed salt then have equal moles.

59. Among H_2S , H_2O , NF_3 , NH_3 and CHCl_3 , identify the molecule (X) with lowest dipole moment value. The number of lone pairs of electrons present on the central atom of the molecule (X) is :

[4 marks]

- (A) 0
- (B) 1
- (C) 2
- (D) 3

Correct Answer: (B) 1

Solution:

Concept:

- The molecular dipole moment is the vector sum of bond contributions and the effect of the electron distribution. It depends on molecular geometry as well as individual bond polarities.
- In trigonal-pyramidal NF_3 , the resultant of the three N–F bond dipoles opposes the contribution associated with the nitrogen lone pair, making the net moment unusually small.

Step 1: Identify the molecule with the smallest net dipole Both NH_3 and NF_3 have a trigonal-pyramidal geometry, with one lone pair on nitrogen. In NH_3 , the N–H bond-dipole resultant reinforces the lone-pair contribution, giving an appreciable net dipole. In NF_3 , fluorine attracts the bonding electrons away from nitrogen, so the bond resultant acts in the opposite sense and largely cancels the lone-pair contribution. Therefore NF_3 , despite strongly polar N–F bonds, has the lowest molecular

dipole moment among the listed molecules.

Step 2: Compare with the other molecular shapes Water and hydrogen sulphide are bent, so their two bond dipoles do not cancel by symmetry. Chloroform is tetrahedral but its substituents are not all identical: three C–Cl bonds and one C–H bond do not give a zero resultant. These molecules have larger moments than NF_3 . Typical values illustrate the comparison: NF_3 has about 0.24 D, while H_2S , CHCl_3 , NH_3 and H_2O have moments roughly 0.95, 1.0, 1.47 and 1.85 D, respectively. The required identification is thus $X = \text{NF}_3$.

Step 3: Count the central-atom lone pairs Nitrogen has five valence electrons. In neutral NF_3 , it forms three single covalent bonds with three fluorine atoms. Three nitrogen electrons participate in these bonds, leaving two nonbonding electrons on nitrogen:

$$\text{number of lone pairs on N} = \frac{5 - 3}{2} = 1.$$

Zero lone pairs would not give the stated neutral Lewis structure; two or three would exceed nitrogen's available valence electrons with three bonds. The printed choices 0, 1, 2 and 3 therefore select the second option.

Step 4: Conclusion The molecule is NF_3 , and its central nitrogen atom has one lone pair. Hence, the correct option is (B).

Quick Tip to Solve: Do not rank molecular dipoles by bond polarity alone: compare the direction of the bond-dipole resultant with the lone-pair contribution in pyramidal molecules.

60. Given below are two statements :

Statement I : Elements ' X ' and ' Y ' are the most and least electronegative elements, respectively among N, As, Sb and P . The nature of the oxides X_2O_3 and Y_2O_3 is acidic and amphoteric, respectively.

Statement II : BCl_3 is covalent in nature and gets hydrolysed in water. It produces $[\text{B}(\text{OH})_4]^-$ and $[\text{B}(\text{H}_2\text{O})_6]^{3+}$ in aqueous medium.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Both Statement I and Statement II are true
- (B) Both Statement I and Statement II are false
- (C) Statement I is true but Statement II is false
- (D) Statement I is false but Statement II is true

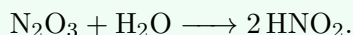
Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

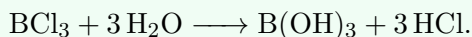
- Electronegativity generally decreases down Group 15, while metallic character increases. The oxides consequently change from acidic towards amphoteric and basic behaviour.
- Boron trichloride is a covalent, electron-deficient compound. Its hydrolysis and the Lewis-acid behaviour of boric acid do not imply formation of a hydrated free boron(III) cation.

Step 1: Identify X and Y and test statement I Among the given Group 15 elements, nitrogen is the most electronegative and antimony the least electronegative. Thus $X = \text{N}$ and $Y = \text{Sb}$. The oxide N_2O_3 is acidic: it is the acid anhydride of nitrous acid,

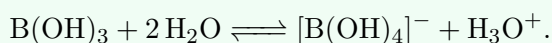


Antimony(III) oxide, Sb_2O_3 , is amphoteric and can react with both acids and strong bases. The oxide characters stated in I therefore agree with the identified elements. Statement I is true.

Step 2: Test each claim in statement II The opening claims are correct: BCl_3 is covalent and undergoes hydrolysis in water. Its overall hydrolysis is



Boric acid accepts a hydroxide ion from water, represented by



Thus tetrahydroxyborate may occur through this aqueous equilibrium. However, the proposed hexaaquaboron(III) ion, $[\text{B}(\text{H}_2\text{O})_6]^{3+}$, is not a product of this hydrolysis. Boron does not form the stated stable six-coordinate hydrated cation in water; the claim incorrectly transfers a hydrated-metal-ion picture to boron.

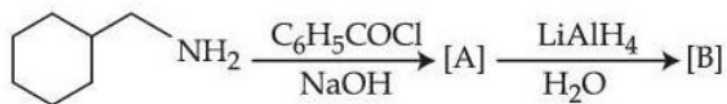
Step 3: Combine the truth values A statement containing a false product assignment is false even if its earlier clauses about covalency and hydrolysis are correct. Therefore statement II is false, while statement I is true. This eliminates (A), which makes II true, (B), which makes I false, and (D), which reverses both truth values. The remaining combination is the one in (C).

Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: For boron halide hydrolysis, write boric acid and hydrogen halide first; do not invent a hexaaqua boron cation by analogy with hydrated metal ions.

61. The final product [B] is (C_6H_{11} denotes cyclohexyl):

[4 marks]



- (A) $C_6H_5CH_2NHCOC_6H_{11}$
(B) $C_6H_{11}CH_2NHCOC_6H_5$
(C) $C_6H_{11}CH_2NHCH(OH)C_6H_5$
(D) $C_6H_{11}CH_2NHCH_2C_6H_5$

Correct Answer: (D) $C_6H_{11}CH_2NHCH_2C_6H_5$

Solution:

Concept:

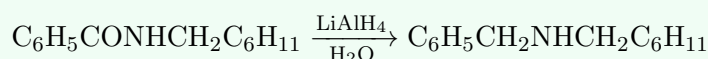
- A primary amine reacts with benzoyl chloride in alkaline medium to form an N-substituted benzamide. The base neutralizes the hydrogen chloride produced.
- Lithium aluminium hydride reduces the carbonyl group of an amide to a methylene group while retaining the carbon-nitrogen bond. Here C_6H_{11} represents cyclohexyl and C_6H_5 represents phenyl.

Step 1: Identify the amine and form intermediate A The starting amine is cyclohexylmethanamine, $C_6H_{11}CH_2NH_2$. Its nitrogen attacks benzoyl chloride, so the benzoyl group is attached to nitrogen; the cyclohexylmethyl group already on nitrogen is unchanged.



Thus A is $C_6H_5CONHCH_2C_6H_{11}$, an amide, rather than the final product.

Step 2: Reduce the amide and compare all structures On reduction with $LiAlH_4$, followed by water, the amide carbonyl carbon becomes CH_2 . Its bond to phenyl and its bond to nitrogen both remain.



Option A retains a carbonyl and puts it next to the cyclohexyl group, contrary to benzoylation. Option B is the unreduced amide A. Option C retains oxygen as an alcohol at the former carbonyl carbon, whereas complete amide reduction gives a methylene group. Only option D has the required secondary-amine connectivity.

Step 3: Conclusion Product B is $C_6H_{11}CH_2NHCH_2C_6H_5$, N-benzylcyclohexylmethanamine. Hence, the correct option is (D).

Quick Tip to Solve: Track the carbonyl carbon: benzoylation inserts $-COC_6H_5$ on nitrogen, and amide reduction converts that same group to $-CH_2C_6H_5$.

62. Match List-I with List-II.

List-I: Reaction of glucose with	List-II: Product formed
A. Hydroxylamine	I. Gluconic acid
B. Br ₂ water	II. Glucose pentacetate
C. Excess acetic anhydride	III. Saccharic acid
D. Concentrated HNO ₃	IV. Glucosime

Choose the correct answer from the options given below:

[4 marks]

(A) A-IV, B-I, C-II, D-III

(B) A-IV, B-III, C-II, D-I

(C) A-III, B-I, C-IV, D-II

(D) A-I, B-III, C-IV, D-II

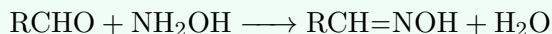
Correct Answer: (A) A-IV, B-I, C-II, D-III

Solution:

Concept:

- Open-chain glucose contains an aldehyde group, five alcoholic hydroxyl groups and a terminal primary-alcohol group. Different reagents distinguish these functional groups.
- Mild oxidation converts only the aldehyde to a carboxylic acid; strong oxidation also converts the terminal primary alcohol to a carboxylic acid. Acetylation modifies the hydroxyl groups.

Step 1: Match the carbonyl and mild-oxidation reactions Hydroxylamine reacts with the aldehyde carbonyl to form an oxime, called glucosime. Therefore A matches IV:



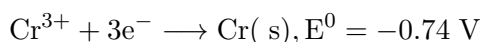
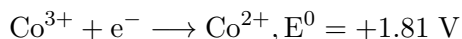
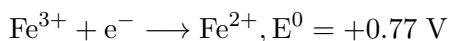
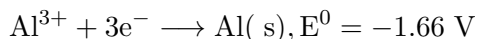
Bromine water is a mild oxidizing agent. It converts the aldehyde end of glucose to a carboxylic acid without oxidizing its terminal CH₂OH group, giving gluconic acid. Thus B matches I.

Step 2: Match acetylation and vigorous oxidation Excess acetic anhydride acetylates all five alcoholic hydroxyl groups of glucose. The product is glucose pentaacetate, called glucose pentacetate in the printed list; therefore C matches II. Concentrated nitric acid oxidizes both the aldehyde end and the terminal primary-alcohol end to carboxylic acid groups. This gives the dicarboxylic acid called saccharic acid, so D matches III. The complete matching is A-IV, B-I, C-II, D-III. Option B interchanges the mild and strong oxidation products. Options C and D incorrectly assign an acid to hydroxylamine and fail the acetylation match. Only option A satisfies all four reactions.

Step 3: Conclusion The matching is A-IV, B-I, C-II, D-III. Hence, the correct option is (A).

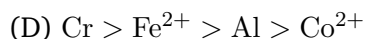
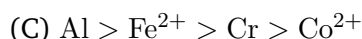
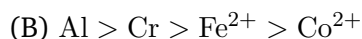
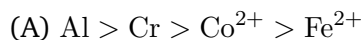
Quick Tip to Solve: Distinguish the two oxidants first: bromine water oxidizes one end of glucose, whereas concentrated nitric acid oxidizes both ends.

63. Consider the following reduction processes :



The tendency to act as reducing agent decreases in the order :

[4 marks]



Correct Answer: (B) $\text{Al} > \text{Cr} > \text{Fe}^{2+} > \text{Co}^{2+}$

Solution:

Concept:

- A reducing agent donates electrons and is itself oxidized. Its reducing strength is compared using the oxidation potential of the appropriate reduced species.
- Reversing a reduction half-reaction changes the sign of its electrode potential: $E_{\text{ox}}^{\circ} = -E_{\text{red}}^{\circ}$. A more positive oxidation potential indicates a stronger tendency to lose electrons.

Step 1: Reverse the given reduction processes The species that act as reducing agents are the products of the listed reduction reactions: Al , Fe^{2+} , Co^{2+} and Cr . Reverse each process to obtain its electron-donating tendency:

$$\begin{aligned} E_{\text{ox}}^{\circ}(\text{Al}) &= +1.66 \text{ V}, & E_{\text{ox}}^{\circ}(\text{Cr}) &= +0.74 \text{ V}, \\ E_{\text{ox}}^{\circ}(\text{Fe}^{2+}) &= -0.77 \text{ V}, & E_{\text{ox}}^{\circ}(\text{Co}^{2+}) &= -1.81 \text{ V}. \end{aligned}$$

The electron coefficient is not a multiplier for electrode potential, since electrode potential is an intensive quantity.

Step 2: Arrange the reducing agents and test the options Sorting these oxidation potentials from highest to lowest gives

$$+1.66 > +0.74 > -0.77 > -1.81.$$

Consequently aluminium donates electrons most readily, followed by chromium, iron(II) and cobalt(II). Option A incorrectly reverses the iron(II)/cobalt(II) comparison. Option C places iron(II) ahead of chromium, contrary to their potentials. Option D fails to put aluminium first and also misorders chromium and iron(II) relative to aluminium. The order in option B agrees with all four numerical comparisons.

Step 3: Conclusion The decreasing reducing tendency is $\text{Al} > \text{Cr} > \text{Fe}^{2+} > \text{Co}^{2+}$. Hence, the correct option is (B).

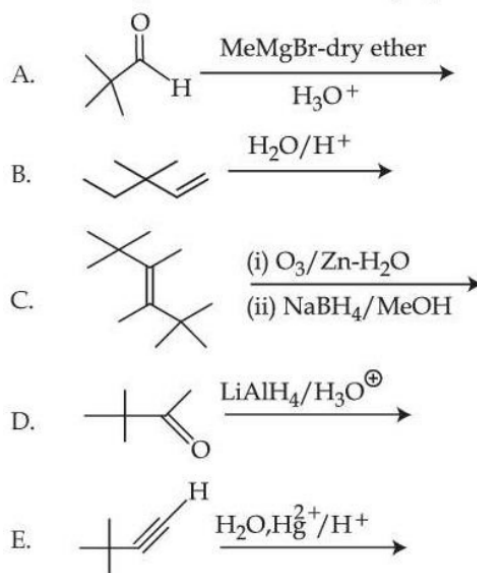
Quick Tip to Solve: For reducing agents on the right of reduction equations, read the reduction potentials from most negative to most positive; do not multiply them by electron counts.

64.

3, 3-Dimethyl-2-butanol cannot be prepared by :

Choose the correct answer from the options given below :

[4 marks]



(A) B and C Only

(B) B Only

(C) B, C and E Only

(D) B and E Only

Correct Answer: (D) B and E Only**Solution:****Concept:**

- The target 3,3-dimethylbutan-2-ol has structure $(\text{CH}_3)_3\text{CCH}(\text{OH})\text{CH}_3$. A successful route must give both its carbon skeleton and its secondary-alcohol group.
- Grignard addition to an aldehyde and hydride reduction of the corresponding ketone can produce a secondary alcohol. Ozonolysis gives carbonyl compounds, while mercury-catalysed alkyne hydration gives a ketone after tautomerization.

Step 1: Check routes A, C and D against the target In A, methylmagnesium bromide adds a methyl group to $(\text{CH}_3)_3\text{CCHO}$. Acidic work-up gives $(\text{CH}_3)_3\text{CCH}(\text{OH})\text{CH}_3$, exactly the target. In C, cleavage of the symmetric alkene $(\text{CH}_3)_3\text{C}-\text{C}(\text{CH}_3)=\text{C}(\text{CH}_3)-\text{C}(\text{CH}_3)_3$ by reductive ozonolysis gives two molecules of $(\text{CH}_3)_3\text{CCOCH}_3$. Sodium borohydride then reduces each ketone to the target secondary alcohol. In D, the starting ketone is already $(\text{CH}_3)_3\text{CCOCH}_3$. Lithium aluminium hydride followed by acidic work-up reduces its carbonyl to $\text{CH}(\text{OH})$, again giving the target.

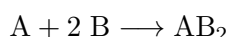
Step 2: Identify the routes that fail Route B starts from $\text{CH}_2=\text{CHC}(\text{CH}_3)_2\text{CH}_2\text{CH}_3$, which contains seven carbon atoms; the target contains six. Acid-catalysed hydration adds water and may rearrange

a carbocation, but it does not remove a carbon atom, so this route cannot give the target. In E, mercury(II)-catalysed hydration of $(\text{CH}_3)_3\text{CC}\equiv\text{CH}$ first gives an enol, which tautomerizes to $(\text{CH}_3)_3\text{CCOCH}_3$. This is the ketone, and no subsequent reduction is supplied. Thus E also fails. The unsuccessful routes are B and E only. Options A and C wrongly exclude successful route C; option B overlooks the failure of E.

Step 3: Conclusion The target alcohol cannot be prepared by routes B and E only. Hence, the correct option is (D).

Quick Tip to Solve: Check carbon count before mechanisms, then check the final functional group: alkyne hydration stops at a ketone unless a reduction step follows.

65.



36.0 g of 'A' (Molar mass : 60 g mol^{-1}) and 56.0 g of 'B' (Molar mass : 80 g mol^{-1}) are allowed to react. Which of the following statements are correct ?

A. 'A' is the limiting reagent.

B. 77.0 g of AB_2 is formed.

C. Molar mass of AB_2 is 140 g mol^{-1} .

D. 15.0 g of A is left unreacted after the completion of reaction.

Choose the correct answer from the options given below :

[4 marks]

(A) A and B Only

(B) B and D Only

(C) C and D Only

(D) A and C Only

Correct Answer: (B) B and D Only

Solution:

Concept:

- For $\text{A} + 2 \text{B} \longrightarrow \text{AB}_2$, one mole of A consumes two moles of B. The limiting reagent is found by comparing the available moles divided by the stoichiometric coefficients.
- The molar mass of AB_2 is $M_A + 2M_B$. Product mass and excess-reactant mass must also satisfy conservation of mass.

Step 1: Convert masses to moles and find the limiting reagent The available amounts are

$$n_A = \frac{36.0}{60} = 0.600 \text{ mol}, \quad n_B = \frac{56.0}{80} = 0.700 \text{ mol}.$$

Compare reaction extents:

$$\frac{n_A}{1} = 0.600, \quad \frac{n_B}{2} = 0.350.$$

B permits the smaller reaction extent and is therefore limiting. A is in excess, so statement A is false.

The amount of product formed is 0.350 mol, and the same number of moles of A is consumed.

Step 2: Calculate product and leftover masses The product contains one A and two B units, so

$$M_{AB_2} = 60 + 2(80) = 220 \text{ g mol}^{-1}.$$

Statement C, which gives 140 g mol^{-1} , is false. The product mass is

$$m_{AB_2} = 0.350(220) = 77.0 \text{ g}.$$

Thus statement B is true. The excess A remaining is

$$m_{A,\text{left}} = (0.600 - 0.350)(60) = 15.0 \text{ g},$$

so statement D is also true. As a check, $77.0 + 15.0 = 92.0 \text{ g}$, the initial total mass $36.0 + 56.0$. The options containing A or C are excluded because those statements are false.

Step 3: Conclusion Only statements B and D are correct: 77.0 g of product forms and 15.0 g of A remains. Hence, the correct option is (B).

Quick Tip to Solve: Compare moles divided by coefficients, not initial masses; use the subscript 2 again when calculating the product molar mass.

66. When 1 g of compound (X) is subjected to Kjeldahl's method for estimation of nitrogen, 15 mL 1 M H_2SO_4 was neutralized by ammonia evolved. The percentage of nitrogen in compound (X) is :

[4 marks]

- (A) 0.21
- (B) 21
- (C) 42
- (D) 0.42

Correct Answer: (C) 42

Solution:

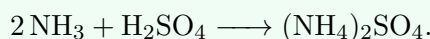
Concept:

- In Kjeldahl estimation, nitrogen in a suitable organic compound is converted to ammonia. Each mole of evolved ammonia represents one mole of nitrogen atoms in the original sample.
- Sulfuric acid is diprotic: one mole of H_2SO_4 neutralizes two moles of NH_3 . Nitrogen percentage is the recovered nitrogen mass divided by the sample mass, multiplied by 100.

Step 1: Use the acid-ammonia stoichiometry The amount of sulfuric acid neutralized is

$$n_{\text{H}_2\text{SO}_4} = MV = 1.00 \times \frac{15}{1000} = 0.0150 \text{ mol}.$$

The neutralization equation is



Therefore the evolved ammonia is

$$n_{\text{NH}_3} = 2(0.0150) = 0.0300 \text{ mol}.$$

The factor of two is essential: the given acid concentration is molarity, not normality, and each sulfuric acid molecule provides two acid equivalents in this reaction.

Step 2: Convert ammonia to nitrogen percentage There is one nitrogen atom per ammonia molecule, so the nitrogen amount is also 0.0300 mol. Using 14 g mol^{-1} for nitrogen,

$$m_N = 0.0300(14) = 0.420 \text{ g}.$$

The original compound sample weighed 1.00 g, hence

$$\%N = \frac{0.420}{1.00} \times 100 = 42.0\%.$$

Option B, 21, misses the diprotic-acid factor. Option D, 0.42, gives the nitrogen mass or mass fraction rather than the percentage; option A, 0.21, also omits the factor of two. Only option C includes both correct neutralization stoichiometry and conversion to percent.

Step 3: Conclusion The nitrogen content of compound X is 42%. Hence, the correct option is (C).

Quick Tip to Solve: For Kjeldahl calculations with sulfuric acid, first convert molarity to acid equivalents by multiplying by two, then convert nitrogen mass fraction to percent.

67. The energy of first (lowest) Balmer line of H atom is x J. The energy (in J) of second Balmer line of H atom is :

[4 marks]

- (A) $2x$
- (B) $1.35x$
- (C) $\frac{x}{1.35}$
- (D) x^2

Correct Answer: (B) $1.35x$

Solution:

Concept:

- The Balmer series consists of transitions from an upper level $n > 2$ to the common lower level $n = 2$. Its first and second lines correspond to $3 \rightarrow 2$ and $4 \rightarrow 2$, respectively.
- The emitted photon energy is $E = K(1/2^2 - 1/n^2)$, where K is the positive ground-state binding-energy constant of hydrogen. The same constant cancels when two line energies are divided.

Step 1: Write the first-line energy For the lowest Balmer line, the upper state is $n = 3$; lowest refers

to the smallest energy gap within this series, not a transition to the ground state. Thus

$$x = K \left(\frac{1}{4} - \frac{1}{9} \right) = K \frac{5}{36}.$$

Both fractions describe energy levels of the same hydrogen atom.

Step 2: Calculate the second-line energy ratio The second Balmer line starts from $n = 4$ and ends at $n = 2$, giving

$$E_2 = K \left(\frac{1}{4} - \frac{1}{16} \right) = K \frac{3}{16}.$$

Dividing by the given first-line energy,

$$\frac{E_2}{x} = \frac{3/16}{5/36} = \frac{108}{80} = \frac{27}{20} = 1.35.$$

The photon energy therefore increases, as expected for the larger initial level.

Step 3: Check the alternatives The levels vary as $-1/n^2$, so consecutive line energies are not proportional to their line numbers: $2x$ is incorrect. The expression $x/1.35$ is smaller than the first-line energy and reverses the ratio. Squaring an energy does not give the required energy dependence, so x^2 is also unsuitable.

Step 4: Conclusion The second Balmer-line photon energy is $1.35x \text{ J}$, with x the numerical energy of the first line in joules. Hence, the correct option is (B).

Quick Tip to Solve: For two lines in one hydrogen series, retain the same lower level and take the ratio of the inverse-square energy gaps; the Rydberg energy cancels.

68. Identify the correct statements :

- A. Hydrated salts can be used as primary standard.
- B. Primary standard should not undergo any reaction with air.
- C. Reactions of primary standard with another substance should be instantaneous and stoichiometric.
- D. Primary standard should not be soluble in water.
- E. Primary standard should have low relative molar mass.

Choose the correct answer from the options given below :

[4 marks]

- (A) A, B, C and E Only
- (B) A, B and E Only
- (C) A, B and C Only
- (D) D and E Only

Correct Answer: (C) A, B and C Only

Solution:

Concept:

- A primary standard is a sufficiently pure, stable substance whose accurately weighed amount fixes the number of moles used to prepare or standardise a solution.
- Its composition must remain known during storage and weighing. In an aqueous titration it should dissolve adequately and react rapidly with a definite stoichiometry; a high molar mass reduces relative weighing error.

Step 1: Check statements A, B and C A is correct: a stable hydrated salt with a definite hydration state can serve as a primary standard. For example, borax, $\text{Na}_2\text{B}_4\text{O}_7 \cdot 10\text{H}_2\text{O}$, is used in acid-base standardisation. This does not mean that every hydrate is suitable; a salt that gains or loses water unpredictably would be unsuitable. B is correct because reaction with oxygen, carbon dioxide or moisture in air would change the weighed composition. C is correct in the titration context: the reaction should be effectively instantaneous on the experimental timescale and follow a known stoichiometric equation, so the endpoint accurately measures the analyte amount.

Step 2: Check statements D and E D is incorrect. In the stated aqueous titration setting the primary standard needs sufficient water solubility to give a homogeneous solution; insolubility is not a desirable requirement. E is also incorrect. For a required amount n , the weighed mass is $m = nM$. A larger molar mass M permits a larger mass to be weighed, reducing the relative uncertainty $\Delta m/m$ for the same balance uncertainty.

Step 3: Select the complete correct set The true statements are A, B and C only. Both the first and second options include the false statement E. The fourth option contains only D and E, both false. The third option alone includes all three true statements without adding a false one.

Step 4: Conclusion A, B and C only are correct. Hence, the correct option is (C).

Quick Tip to Solve: Test a primary standard against four practical needs: known composition, air stability, adequate solubility and a fast quantitative reaction; high molar mass helps accurate weighing.

69. Given below are two statements :

Statement I : $C < O < N < F$ is the correct order in terms of first ionization enthalpy values.

Statement II : $S > Se > Te > Po > O$ is the correct order in terms of the magnitude of electron gain enthalpy values.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Both Statement I and Statement II are true
- (B) Both Statement I and Statement II are false
- (C) Statement I is true but Statement II is false
- (D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- First ionization enthalpy is the energy required to remove an electron from a gaseous atom. It generally rises across a period, with exceptions caused by subshell stability and electron pairing.
- Electron gain enthalpy is the enthalpy change for adding an electron to a gaseous atom. When this change is negative, its magnitude measures the energy released; compare magnitudes separately from signed values.

Step 1: Evaluate Statement I The valence configurations are C : $2s^2 2p^2$, N : $2s^2 2p^3$, O : $2s^2 2p^4$ and F : $2s^2 2p^5$. Nitrogen has a half-filled $2p$ subshell. Oxygen has one paired $2p$ orbital, and removing an electron from that pair relieves repulsion. Consequently oxygen has a lower first ionization enthalpy than nitrogen, despite lying to its right. Carbon remains lower than both, whereas fluorine has the largest value because of its greater effective nuclear attraction. Thus

$$C < O < N < F,$$

so Statement I is true.

Step 2: Evaluate Statement II In group 16, adding an electron to the compact oxygen $2p$ shell creates relatively strong electron-electron repulsion. Sulphur accepts the electron into a larger $3p$ orbital with less repulsion and releases more energy. Below sulphur, increasing size and shielding reduce the attraction for the incoming electron. The magnitudes therefore follow

$$S > Se > Te > Po > O.$$

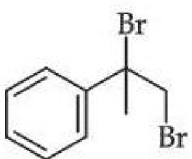
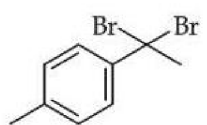
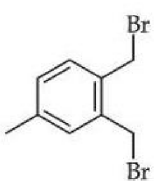
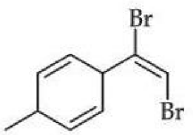
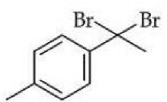
The position of oxygen is the important exception to a simple size-only argument. Statement II is true.

Step 3: Match the truth values Both statements agree with their respective periodic trends and exceptions. The second option rejects both valid orders; the third wrongly rejects the electron-gain order; and the fourth wrongly rejects the ionization order. Only the first option retains both correct statements.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: Remember two separate exceptions: half-filled nitrogen raises its ionization enthalpy above oxygen, while compact oxygen has a smaller electron-gain magnitude than sulphur.

70. The dibromo compound [P] (molecular formula: $C_9H_{10}Br_2$) when heated with excess sodamide followed by treatment with dilute HCl gives [Q]. On warming [Q] with mercuric sulphate and dilute sulphuric acid yield [R] which gives positive Iodoform test but negative Tollen's test. The compound [P] is : [4 marks]

- (A) $C_6H_5C(Br)(CH_3)CH_2Br$ 
- (B) $p\text{-CH}_3C_6H_4CBr_2CH_3$ 
- (C) 1,2-Bis(bromomethyl)-4-methylbenzene 
- (D) $p\text{-CH}_3C_6H_4C(Br)=CHBr$ 
- Correct Answer:** (B) $p\text{-CH}_3C_6H_4CBr_2CH_3$ 

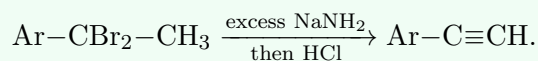
Solution:

Concept:

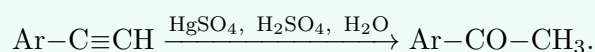
- A suitable geminal or vicinal dihalide undergoes two dehydrohalogenations with excess $NaNH_2$, producing an alkyne. Acid work-up protonates any terminal acetylide formed by the excess base.
- Mercuric-ion-catalysed acidic hydration of a terminal alkyne $Ar-C\equiv CH$ gives an enol that tautomerises to the methyl ketone $ArCOCH_3$. A methyl ketone gives the iodoform test and ordinarily does not reduce Tollens reagent.

Step 1: Infer the required carbonyl product The positive iodoform test requires a group that forms CHI_3 under iodine/alkali conditions. Since the product is obtained by alkyne hydration and gives a negative Tollens test, the suitable product is a methyl ketone rather than ethanal. Work backwards from $ArCOCH_3$ to the terminal alkyne $Ar-C\equiv CH$.

Step 2: Test the second structure Let Ar denote the para-methylphenyl group, $p\text{-CH}_3C_6H_4$. Option (B) is the geminal dibromide $Ar-CBr_2-CH_3$, which has formula $C_9H_{10}Br_2$. Both eliminations use hydrogens on its methyl carbon:



Hydration followed by keto-enol tautomerism gives



This is para-methylacetophenone, a methyl ketone: it gives a yellow iodoform precipitate and a negative Tollens test.

Step 3: Eliminate the other structures In option (A), the brominated carbon attached to phenyl and methyl has no hydrogen. The adjacent dibromide arrangement therefore cannot make the required triple bond between those two brominated carbons by two ordinary eliminations; the substituted carbon would exceed its allowed valence. Option (C) has separate bromomethyl groups on the aromatic ring, not the necessary geminal or adjacent alkyl dibromide chain. Option (D) is already a dibromoalkene and has formula $C_9H_8Br_2$, two hydrogens fewer than specified. These structures do not satisfy the complete sequence and molecular formula.

Step 4: Conclusion [P] is $p\text{-CH}_3C_6H_4CBr_2CH_3$, which yields para-methylacetophenone through the terminal alkyne. Hence, the correct option is (B).

Quick Tip to Solve: For this reaction sequence, work backwards: iodoform-positive ketone means a methyl ketone, hydration then points to a terminal alkyne, and double elimination identifies its dibromide precursor.

Chemistry – Section B: Numerical Value Questions (20 marks)

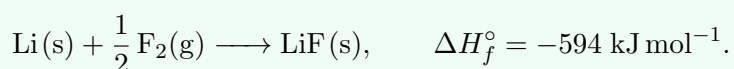
71. If the enthalpy of sublimation of Li is 155 kJ mol^{-1} , enthalpy of dissociation of F_2 is 150 kJ mol^{-1} , ionization enthalpy of Li is 520 kJ mol^{-1} , electron gain enthalpy of F is -313 kJ mol^{-1} , standard enthalpy of formation of LiF is -594 kJ mol^{-1} . The magnitude of lattice enthalpy of LiF is ____ kJ mol^{-1} . (Nearest Integer) [4 marks]

Solution:

Concept:

- Hess's law makes the enthalpy change independent of the path. The formation of one mole of $LiF(s)$ can be expressed through gaseous atoms, gaseous ions and finally the ionic solid.
- Lattice formation enthalpy is negative for combining gaseous ions into a solid. The reverse lattice dissociation enthalpy is positive and has the same magnitude; the question asks for that magnitude.

Step 1: Write the elemental formation reaction The standard formation reaction is



Only one fluorine atom is required per formula unit, so only half a mole of F_2 is dissociated. Using the full dissociation enthalpy would double the fluorine contribution.

Step 2: Assemble the gaseous-ion steps Sublimation $Li(s) \longrightarrow Li(g)$ requires 155 kJ mol^{-1} . Formation of one mole of fluorine atoms requires $150/2 = 75 \text{ kJ mol}^{-1}$. Ionization $Li(g) \longrightarrow Li^+(g) + e^-$ requires 520 kJ mol^{-1} , and electron attachment $F(g) + e^- \longrightarrow F^-(g)$ releases 313 kJ mol^{-1} . Therefore the total to form the separated gaseous ions is

$$155 + 75 + 520 - 313 = 437 \text{ kJ mol}^{-1}.$$

The negative electron-gain sign must be retained.

Step 3: Solve for the lattice contribution Let L_f be the lattice formation enthalpy for $Li^+(g) +$

$\text{F}^-(\text{g}) \longrightarrow \text{LiF}(\text{s})$. Hess's law gives

$$-594 = 437 + L_f, \quad L_f = -1031 \text{ kJ mol}^{-1}.$$

As a sign check, the release on lattice formation exceeds the energy needed to generate the ions, giving the specified negative overall formation enthalpy. Its magnitude is already an integer.

Step 4: Conclusion The required magnitude of the lattice enthalpy is 1031 kJ mol^{-1} . The lattice formation enthalpy itself is $-1031 \text{ kJ mol}^{-1}$.

Quick Tip to Solve: Write the formation equation before the Born-Haber sum: one mole of LiF needs half a mole of fluorine molecules, so the bond dissociation contribution is halved.

72. Consider $\text{A} \xrightarrow{k_1} \text{B}$ and $\text{C} \xrightarrow{k_2} \text{D}$ are two reactions. If the rate constant (k_1) of the $\text{A} \longrightarrow \text{B}$ reaction can be expressed by the following equation $\log_{10} k = 14.34 - \frac{1.5 \times 10^4}{T/\text{K}}$ and activation energy of $\text{C} \longrightarrow \text{D}$ reaction (E_{a2}) is $\frac{1}{5}$ th of the $\text{A} \longrightarrow \text{B}$ reaction (E_{a1}), then the value of (E_{a2}) is _____ kJ mol^{-1} . (Nearest Integer)

[4 marks]

Solution:

Concept:

- The Arrhenius equation is $k = Ae^{-E_a/(RT)}$, where A is the pre-exponential factor, E_a the activation energy, R the gas constant and T the absolute temperature.
- Taking logarithms to base ten gives $\log_{10} k = \log_{10} A - E_a/(2.303RT)$. Thus the coefficient of reciprocal temperature determines activation energy; the intercept determines the pre-exponential factor.

Step 1: Identify the reciprocal-temperature coefficient The given expression is

$$\log_{10} k_1 = 14.34 - \frac{1.5 \times 10^4}{T/\text{K}}.$$

Since T/K is the numerical kelvin temperature, comparison with the common-log Arrhenius form gives

$$\frac{E_{a1}}{2.303R} = 1.5 \times 10^4 \text{ K}.$$

The intercept 14.34 equals $\log_{10} A$ and is not needed to calculate the activation energy.

Step 2: Calculate the first activation energy Using $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$,

$$\begin{aligned} E_{a1} &= 2.303(8.314)(1.5 \times 10^4) \\ &= 2.8721 \times 10^5 \text{ J mol}^{-1} \\ &= 287.21 \text{ kJ mol}^{-1}. \end{aligned}$$

The kelvin factor cancels the inverse-kelvin unit of R . The factor 2.303 is essential because the supplied logarithm is base ten rather than natural.

Step 3: Use the given one-fifth relation and round The second reaction has one-fifth of the first activation energy, not one-fifth of its rate constant. Therefore

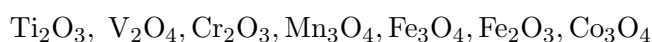
$$E_{a2} = \frac{E_{a1}}{5} = \frac{287.21}{5} = 57.442 \text{ kJ mol}^{-1}.$$

Rounding only this final value to the nearest integer gives 57. No temperature value is required because the complete reciprocal-temperature dependence is already specified.

Step 4: Conclusion The activation energy of $C \longrightarrow D$, to the nearest integer, is $\boxed{57 \text{ kJ mol}^{-1}}$.

Quick Tip to Solve: For a base-ten Arrhenius relation, multiply the magnitude of the reciprocal-temperature coefficient by $2.303R$, then apply any stated energy ratio and round at the end.

73. Among the following oxides of $3d$ elements, the number of mixed oxides are ____ .



[4 marks]

Solution:

Concept:

- A mixed oxide contains the same metal in two oxidation states and can be represented as a combination of its simpler oxides. Oxygen has oxidation number -2 in the oxides listed here.
- For the common mixed oxides M_3O_4 in this list, charge balance gives one M^{2+} and two M^{3+} ions per formula unit: $2 + 2(3) = 8$.

Step 1: Identify the mixed-valence oxides Manganese, iron and cobalt each form the listed oxide containing both the $+2$ and $+3$ oxidation states. Their formal component-oxide representations are



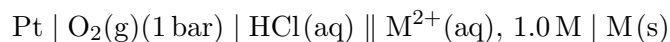
These expressions describe the stoichiometry of the mixed oxides; they do not mean that the solids are merely physical mixtures of separate oxide powders.

Step 2: Exclude the single-valence oxides In Ti_2O_3 , Cr_2O_3 and Fe_2O_3 , balancing the charge of three oxide ions gives the metal oxidation number $+3$. These are the ordinary single-valence oxides, not mixed oxides. In V_2O_4 , vanadium is $+4$; the formula is equivalent to 2VO_2 . A formula with several metal atoms is not automatically a mixed oxide. Thus the titanium, vanadium, chromium and iron(III) oxides contribute nothing to the count.

Step 3: Conclusion The mixed oxides are Mn_3O_4 , Fe_3O_4 and Co_3O_4 . Their number is $\boxed{3}$, a dimensionless count.

Quick Tip to Solve: Recognise Mn_3O_4 , Fe_3O_4 and Co_3O_4 as the common $+2/+3$ mixed oxides; do not classify an oxide solely by the size of its subscripts.

74. Consider the following electrochemical cell :



The pH above which, oxygen gas would start to evolve at anode is _____ (nearest integer).

$$\text{Given: } \left. \begin{array}{l} E_{\text{M}^{2+}/\text{M}}^\circ = 0.994 \text{ V} \\ E_{\text{O}_2/\text{H}_2\text{O}}^\circ = 1.23 \text{ V} \end{array} \right\} \text{standard reduction potential}$$

$$\text{and } \frac{RT}{F}(2.303) = 0.059 \text{ V} \quad \text{at the given condition}$$

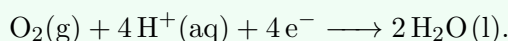
[4 marks]

Solution:

Concept:

- Write both electrode potentials as reduction potentials. For a spontaneous cell, $E_{\text{cell}} = E_{\text{cathode}} - E_{\text{anode}} > 0$; the reaction at the anode itself is oxidation.
- The Nernst equation is $E = E^\circ - (0.059/n) \log_{10} Q$ under the stated conditions, where n is the number of transferred electrons. Pure liquid water has unit activity and $\text{pH} = -\log_{10}[\text{H}^+]$.

Step 1: Express the oxygen electrode potential in terms of pH Use the reduction half-reaction corresponding to the given standard potential:



With oxygen at 1 bar, its dimensionless pressure ratio is unity. Thus

$$\begin{aligned} E_{\text{O}_2/\text{H}_2\text{O}} &= 1.23 - \frac{0.059}{4} \log_{10} \frac{1}{[\text{H}^+]^4} \\ &= 1.23 + 0.059 \log_{10}[\text{H}^+] \\ &= 1.23 - 0.059 \text{ pH} \quad \text{V.} \end{aligned}$$

The oxygen reduction potential therefore decreases as pH increases, making the reverse reaction, oxygen evolution, easier relative to the metal reduction.

Step 2: Apply the condition for spontaneous oxygen evolution The metal-ion concentration is 1.0 M, so the metal cathode has reduction potential $E_{\text{M}^{2+}/\text{M}} = 0.994 \text{ V}$. Consequently,

$$E_{\text{cell}} = 0.994 - (1.23 - 0.059 \text{ pH}) = -0.236 + 0.059 \text{ pH}.$$

At the threshold, $E_{\text{cell}} = 0$, which gives

$$\text{pH}_{\text{threshold}} = \frac{0.236}{0.059} = 4.$$

For $\text{pH} > 4$, the cell potential is positive and water oxidation at the left-hand electrode produces oxygen. At exactly pH 4 the ideal cell is at equilibrium.

Step 3: Conclusion Oxygen evolution is thermodynamically favourable above $\text{pH} = 4$. The requested

threshold, to the nearest integer, is 4; pH is dimensionless.

Quick Tip to Solve: Keep both tabulated potentials in reduction form and subtract the left electrode from the right. At unit oxygen pressure, the oxygen electrode shifts by -0.059 V per pH unit.

75. The mass of benzanilide obtained from the benzoylation reaction of 5.8 g of aniline, if yield of product is 82%, is ____ g (nearest integer).

(Given molar mass in g mol^{-1} H : 1, C : 12, N : 14, O : 16)

[4 marks]

Solution:

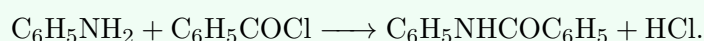
Concept:

- Benzoylation replaces one hydrogen attached to the nitrogen of aniline by a benzoyl group, $\text{C}_6\text{H}_5\text{CO}^-$. One mole of aniline produces one mole of benzanilide.
- The theoretical product mass follows from the reacting mole ratio. Percentage yield relates the isolated mass to this theoretical value: $m_{\text{actual}} = (\text{percentage yield}/100)m_{\text{theoretical}}$.

Step 1: Determine the molecular formulae and molar masses Aniline is $\text{C}_6\text{H}_5\text{NH}_2$, with formula $\text{C}_6\text{H}_7\text{N}$. Using the stated atomic masses,

$$M_{\text{aniline}} = 6(12) + 7(1) + 14 = 93\text{ g mol}^{-1}.$$

The benzoylation reaction can be written as



Benzanilide therefore has formula $\text{C}_{13}\text{H}_{11}\text{NO}$, not the formula obtained by simply adding all atoms of benzoyl chloride without subtracting HCl. Its molar mass is

$$M_{\text{benzanilide}} = 13(12) + 11(1) + 14 + 16 = 197\text{ g mol}^{-1}.$$

Step 2: Convert the aniline mass to the actual product yield The available aniline amount is $5.8/93$ mol. From the one-to-one mole ratio, the theoretical benzanilide mass is

$$m_{\text{theoretical}} = \frac{5.8}{93} \times 197 = 12.2860\text{ g}.$$

Apply the 82% yield only after finding this theoretical amount:

$$m_{\text{actual}} = \frac{82}{100} \times 12.2860 = 10.0745\text{ g}.$$

Retain the decimal values until the final rounding. The product mass exceeds the starting aniline mass because the benzoyl group contributes additional atoms; an 82% yield is not 82% of the aniline mass.

Step 3: Conclusion The benzanilide obtained is 10.0745 g, or 10 g to the nearest integer. The numerical entry is 10.

Quick Tip to Solve: For a one-to-one conversion, multiply the reactant mass by the product/reactant molar-mass ratio and then by the fractional yield. Round only the final mass.