JEE Main 2026 Paper 1 (B.E./B.Tech) Jan 22 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2026, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 22 January 2026, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Two distinct numbers a and b are selected at random from $1, 2, 3, \ldots, 50$. The probability, that their product ab is divisible by 3 , is [4 marks]

(A) $\frac{664}{1225}$
(B) $\frac{272}{1225}$
(C) $\frac{561}{1225}$
(D) $\frac{8}{25}$

Correct Answer: (A) $\frac{664}{1225}$

Solution:

Concept:

- For equally likely selections, probability is the number of favourable pairs divided by the total number of pairs. Distinct numbers are selected without replacement.
- Since 3 is prime, a product is divisible by 3 exactly when at least one factor is divisible by 3. The complementary event is that neither factor is divisible by 3.

Step 1: Count the available numbers The multiples of 3 in the set are $3, 6, \ldots, 48$, so their number is $48/3 = 16$. The remaining $50 - 16 = 34$ numbers are not divisible by 3. Unordered pairs can be used because each ordered selection corresponds to exactly two orders of the same distinct pair.

Step 2: Count the complementary pairs There are $\binom{50}{2} = 50 \cdot 49/2 = 1225$ possible pairs. A pair whose product is not divisible by 3 must select both numbers from the group of 34 nonmultiples. Thus its count is $\binom{34}{2} = 34 \cdot 33/2 = 561$, and the desired probability is

$$1 - \frac{561}{1225} = \frac{664}{1225}.$$

Step 3: Check the favourable count directly Exactly one multiple gives $16 \cdot 34 = 544$ pairs, and two multiples give $\binom{16}{2} = 120$ pairs. Their sum is 664, confirming the complement calculation. The fraction $561/1225$ is the opposite event, while $8/25 = 16/50$ concerns only one selected number. The remaining fraction $272/1225$ also fails the favourable-pair count.

Step 4: Conclusion The required probability is $\boxed{\frac{664}{1225}}$. The direct count is 664 favourable pairs out of 1225; $561/1225$ instead counts products not divisible by 3. Hence, the correct option is (A).

Quick Tip to Solve: For divisibility of a product by a prime, count pairs containing no multiple and subtract their probability from 1; do not treat the two draws as independent.

2. If a random variable x has the probability distribution

x	0	1	2	3	4	5	6	7
$P(x)$	0	$2k$	k	$3k$	$2k^2$	$2k$	k^2+k	$7k^2$

then $P(3 < x \leq 6)$ is equal to

[4 marks]

(A) 0.34

(B) 0.64

(C) 0.22

(D) 0.33

Correct Answer: (D) 0.33

Solution:

Concept:

- The probabilities in a discrete distribution are nonnegative and sum to 1. These two requirements determine an admissible value of the constant k.
- To find the probability of an interval, add only the probabilities of supported values lying in that interval; the strict lower endpoint is excluded.

Step 1: Normalize the distribution Adding all eight entries gives

$$0 + 2k + k + 3k + 2k^2 + 2k + (k^2 + k) + 7k^2 = 1.$$

The coefficients of k total 9 and those of k^2 total 10. Consequently,

$$10k^2 + 9k - 1 = (10k - 1)(k + 1) = 0.$$

The two algebraic possibilities are $k = 1/10$ and $k = -1$.

Step 2: Reject the negative-probability root If $k = -1$, then, for instance, $P(X = 1) = 2k = -2$, which cannot be a probability. Hence $k = 0.1$. With this root, every table entry is nonnegative and their sum is 1, so the normalization is valid.

Step 3: Sum the required entries The event $3 < X \leq 6$ includes precisely $X = 4, 5, 6$, not $X = 3$ or $X = 7$. Therefore

$$\begin{aligned} P(3 < X \leq 6) &= 2k^2 + 2k + (k^2 + k) \\ &= 3k^2 + 3k = 3(0.01) + 3(0.1) = 0.33. \end{aligned}$$

In particular, 0.22 omits the contribution $P(X = 6) = 0.11$. Neither 0.34 nor 0.64 equals the sum of the three specified entries.

Step 4: Conclusion The required probability is $\boxed{0.33}$. Hence, the correct option is (D).

Quick Tip to Solve: Normalize first and reject roots that make any probability negative. Then list the allowed integer values before adding entries to avoid endpoint errors.

3. Let $f : [1, \infty) \to \mathbb{R}$ be a differentiable function. If $6\int_1^x f(t)dt = 3xf(x) + x^3 - 4$ for all $x \geq 1$, then the value of $f(2) - f(3)$ is : [4 marks]

(A) 4

(B) 3

(C) -3

(D) -4

Correct Answer: (B) 3

Solution:

Concept:

- The fundamental theorem of calculus gives $\frac{d}{dx}\int_1^x f(t)\,dt = f(x)$. Differentiating the given identity converts it into a first-order linear differential equation.
- An initial value must be taken from the original integral identity, because differentiation loses its constant information. Here the integral vanishes at $x = 1$.

Step 1: Find the initial value Put $x = 1$ in the given identity. Since the lower and upper limits coincide,

$$0 = 3f(1) + 1 - 4, \qquad f(1) = 1.$$

This value will determine the integration constant uniquely on the stated domain.

Step 2: Differentiate and solve Using the product rule on $3xf(x)$, we obtain

$$6f(x) = 3f(x) + 3xf'(x) + 3x^2.$$

Thus $xf'(x) - f(x) = -x^2$. Since $x \geq 1$, division by x^2 is allowed, and

$$\left(\frac{f(x)}{x}\right)' = -1.$$

Integration gives $f(x)/x = -x + C$, or $f(x) = -x^2 + Cx$. From $f(1) = 1$, we get $C = 2$.

Step 3: Evaluate and verify Accordingly, $f(x) = 2x - x^2$, so $f(2) = 0$ and $f(3) = -3$. Their difference is $0 - (-3) = 3$. As a check,

$$6\int_1^x (2t - t^2)\,dt = 6x^2 - 2x^3 - 4,$$

which also equals $3x(2x - x^2) + x^3 - 4$. Thus the original identity is satisfied, not merely its derivative. The value -3 reverses the requested subtraction, while 4 and -4 are inconsistent with these function values.

Step 4: Conclusion $\boxed{f(2) - f(3) = 3}$. Hence, the correct option is (B).

Quick Tip to Solve: For an integral identity, substitute its lower limit before differentiating. Recognize $(xf' - f)/x^2$ as the derivative of f/x.

4. If the image of the point $\mathrm{P}(1,2,a)$ in the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{7-z}{2}$ is $\mathrm{Q}(5,b,\mathrm{c})$, then $a^2+b^2+c^2$ is equal to [4 marks]

(A) 264

(B) 283

(C) 293

(D) 298

Correct Answer: (D) 298

Solution:

Concept:

- Reflection of a point in a line makes the line the perpendicular bisector of the segment joining the point and its image. Thus the midpoint lies on the line, and the joining vector is perpendicular to its direction.
- The given symmetric form corresponds to $(x,y,z)=(6+3t,7+2t,7-2t)$. Its direction vector is $(3,2,-2)$; the negative sign in its third component is essential.

Step 1: Place the midpoint on the line Let M be the midpoint of $P(1,2,a)$ and $Q(5,b,c)$. Its coordinates are

$$M=\left(3,\frac{2+b}{2},\frac{a+c}{2}\right).$$

Since M lies on the line, $3=6+3t$, so $t=-1$. At this parameter, the line point is $(3,5,9)$. Equating the other coordinates yields

$$b=8, \qquad a+c=18.$$

Step 2: Impose perpendicularity The joining vector is $\overrightarrow{PQ}=(4,b-2,c-a)=(4,6,c-a)$. Its dot product with the line direction must vanish:

$$4(3)+6(2)+(c-a)(-2)=0.$$

Hence $c-a=12$. Combining this with $a+c=18$ gives $c=15$ and $a=3$. The midpoint-to-point vector $(-2,-3,-6)$ also has zero dot product with $(3,2,-2)$, confirming that the midpoint is the perpendicular foot.

Step 3: Compute the requested sum Substitution gives

$$a^2+b^2+c^2=3^2+8^2+15^2=9+64+225=298.$$

Both the midpoint and perpendicularity conditions fixed the values uniquely. Therefore 264, 283 and 293 cannot satisfy the requested sum; changing the sign of the direction in the third coordinate would solve a different line.

Step 4: Conclusion The required value is $\boxed{298}$. Hence, the correct option is (D).

Quick Tip to Solve: For reflection in a line, use the midpoint-on-line condition first, then one dot product. Rewrite every symmetric denominator with a consistent coordinate sign.

5. If the chord joining the points $\mathrm{P}_1(x_1, y_1)$ and $\mathrm{P}_2(x_2, y_2)$ on the parabola $y^2 = 12x$ subtends a right angle at the vertex of the parabola, then $x_1x_2 - y_1y_2$ is equal to [4 marks]

(A) 280

(B) 284

(C) 288

(D) 292

Correct Answer: (C) 288

Solution:

Concept:

- The vertex of $y^2 = 12x$ is the origin. A right angle subtended there means the nonzero position vectors of the two chord endpoints have zero dot product.
- Each endpoint satisfies the parabola equation. Multiplying these two equations relates the products x_1x_2 and y_1y_2, without needing the individual coordinates.

Step 1: Translate the right angle into an equation Since OP_1 and OP_2 are perpendicular,

$$x_1x_2 + y_1y_2 = 0.$$

Write $u = x_1x_2$ and $v = y_1y_2$. Then $v = -u$. Neither endpoint can be the vertex itself, because the angle there would then not be defined. Every nonvertex point on this parabola has $x > 0$, and therefore $u > 0$.

Step 2: Use the parabola equations The two endpoint equations are $y_1^2 = 12x_1$ and $y_2^2 = 12x_2$. On multiplying,

$$(y_1y_2)^2 = 144x_1x_2, \qquad v^2 = 144u.$$

Substitute $v = -u$ to obtain $u^2 = 144u$, or $u(u - 144) = 0$. The root $u = 0$ would put an endpoint at the vertex and is excluded by the right-angle condition. Thus $u = 144$ and $v = -144$.

Step 3: Form the difference and compare options We now get

$$x_1x_2 - y_1y_2 = u - v = 144 - (-144) = 288.$$

For example, the endpoints $(12, 12)$ and $(12, -12)$ lie on the parabola and have perpendicular position vectors, confirming this invariant value. The alternatives 280, 284 and 292 differ from the fixed value enforced by the two equations and so cannot be correct.

Step 4: Conclusion The required value is $\boxed{288}$. Hence, the correct option is (C).

Quick Tip to Solve: When a chord subtends a right angle at the origin, combine the zero dot product with the product of the curve equations. Exclude a zero endpoint before cancelling.

6. If the domain of the function $f(x) = \sin^{-1}\left(\frac{5-x}{3+2x}\right) + \frac{1}{\log_e(10-x)}$ is $(-\infty, \alpha] \cup [\beta, \gamma) - \{\delta\}$, then $6(\alpha+\beta+\gamma+\delta)$ is equal to [4 marks]

(A) 66

(B) 67

(C) 68

(D) 70

Correct Answer: (D) 70

Solution:

Concept:

- A real inverse sine requires its argument to lie in $[-1, 1]$, with any denominator in that argument nonzero.
- The logarithm $\log_e(10-x)$ requires $10 - x > 0$; because it is itself a denominator, it must also be nonzero, excluding $10 - x = 1$. The domain is the intersection of all these conditions.

Step 1: Solve the inverse-sine restriction For $x \neq -3/2$, the restriction is

$$\left|\frac{5-x}{3+2x}\right| \leq 1.$$

Squaring and multiplying by the positive square $(3+2x)^2$ gives

$$(5-x)^2 \leq (3+2x)^2, \qquad (3x-2)(x+8) \geq 0.$$

A sign chart therefore gives $x \leq -8$ or $x \geq 2/3$. The excluded denominator zero $-3/2$ lies in neither interval. The boundary values are allowed: the inverse-sine arguments there are -1 and 1.

Step 2: Apply the logarithm conditions The positive-argument condition gives $x < 10$, while the nonzero-logarithm condition gives $x \neq 9$. Intersecting these with the previous intervals yields

$$(-\infty, -8] \cup [2/3, 10) \ - \{9\}.$$

The endpoint 10 is open because its logarithm is undefined, whereas 9 is an isolated exclusion because its logarithm equals zero.

Step 3: Identify the four parameters Comparing with the stated domain,

$$\alpha = -8, \qquad \beta = \frac{2}{3}, \qquad \gamma = 10, \qquad \delta = 9.$$

Hence

$$6(\alpha+\beta+\gamma+\delta) = 6\left(-8 + \frac{2}{3} + 10 + 9\right) = 6 \cdot \frac{35}{3} = 70.$$

The other proposed values 66, 67 and 68 do not equal the sum from these endpoints and the excluded point.

Step 4: Conclusion The required value is $\boxed{70}$. Hence, the correct option is (D).

Quick Tip to Solve: Solve an inverse-sine rational bound by comparing squares, retaining denominator exclusions. For a reciprocal logarithm, exclude both nonpositive arguments and argument 1.

7. Let $\mathrm{P}(\alpha,\beta,\gamma)$ be the point on the line $\frac{x-1}{2}=\frac{y+1}{-3}=z$ at a distance $4\sqrt{14}$ from the point $(1,-1,0)$ and nearer to the origin. Then the shortest distance, between the lines $\frac{x-\alpha}{1}=\frac{y-\beta}{2}=\frac{z-\gamma}{3}$ and $\frac{x+5}{2}=\frac{y-10}{1}=\frac{z-3}{1}$, is equal to [4 marks]

(A) $4\sqrt{\frac{5}{7}}$

(B) $2\sqrt{\frac{7}{4}}$

(C) $7\sqrt{\frac{5}{4}}$

(D) $4\sqrt{\frac{7}{5}}$

Correct Answer: (D) $4\sqrt{\frac{7}{5}}$

Solution:

Concept:

- A point on a line through $\mathbf{a}$, with direction $\mathbf{d}$, has position $\mathbf{a}+t\mathbf{d}$. Its distance from $\mathbf{a}$ is $|t|\|\mathbf{d}\|$.
- For nonparallel lines through $\mathbf{a}$ and $\mathbf{b}$, with directions $\mathbf{u}$ and $\mathbf{v}$, their shortest distance is $d=|(\mathbf{b}-\mathbf{a})\cdot(\mathbf{u}\times\mathbf{v})|/\|\mathbf{u}\times\mathbf{v}\|$.

Step 1: Determine the two possible points Put the common line parameter equal to t. Then $P=(1+2t,-1-3t,t)$. Its displacement from $(1,-1,0)$ is $t(2,-3,1)$, so the distance condition gives

$$|t|\sqrt{2^2+(-3)^2+1^2}=4\sqrt{14},\qquad t=\pm 4.$$

The two candidates are $(9,-13,4)$ and $(-7,11,-4)$. Their squared distances from the origin are respectively 266 and 186, so the nearer point is $P=(-7,11,-4)$.

Step 2: Find a common perpendicular direction The required lines pass through $P=(-7,11,-4)$ and $Q=(-5,10,3)$, with directions $\mathbf{u}=(1,2,3)$ and $\mathbf{v}=(2,1,1)$. Hence

$$Q-P=(2,-1,7),\qquad \mathbf{u}\times\mathbf{v}=(-1,5,-3).$$

The nonzero cross product confirms that the lines are not parallel.

Step 3: Project the displacement onto the normal The absolute scalar triple product is $|-2-5-21|=28$, and the normal has length $\sqrt{1+25+9}=\sqrt{35}$. Therefore

$$d=\frac{28}{\sqrt{35}}=4\sqrt{\frac{7}{5}}.$$

Its square is $112/5$; the squares of options (A), (B) and (C) are $80/7$, 7 and $245/4$, respectively, so none equals the calculated distance.

Step 4: Conclusion The printed lines give the shortest distance $28/\sqrt{35}=4\sqrt{7/5}$, which is option (D). The differing keyed choice refers to $4\sqrt{5/7}$ in the printed order, whose square is $80/7$, not the required $112/5$. Hence, the correct option is (D).

Quick Tip to Solve: Choose the required point by comparing squared distances first; then use the scalar triple product, avoiding unnecessary square roots.

8. If $\mathrm{A} = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$**, then the determinant of the matrix** $\left(\mathrm{A}^{2025} - 3\,\mathrm{A}^{2024} + \mathrm{A}^{2023}\right)$ **is** [4 marks]

(A) 12

(B) 16

(C) 24

(D) 28

Correct Answer: (B) 16

Solution:

Concept:

- Factor a common matrix power exactly as for a polynomial: $A^{2025} - 3A^{2024} + A^{2023} = A^{2023}(A^2 - 3A + I)$.
- For square matrices, $\det(BC) = \det B \det C$ and $\det(A^n) = (\det A)^n$. For a two-by-two matrix, $\det(cA) = c^2 \det A$.

Step 1: Calculate the basic matrix invariants The given matrix is $A = \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}$. Its determinant is $2 \cdot 5 - 3 \cdot 3 = 1$, and its trace is 7. Thus its characteristic polynomial is $\lambda^2 - 7\lambda + 1$. Cayley–Hamilton gives $A^2 - 7A + I = 0$. This identity can also be verified by multiplying the small matrix directly.

Step 2: Reduce the bracket to a multiple of the matrix From the identity, $A^2 = 7A - I$. Substituting into the polynomial factor gives

$$A^2 - 3A + I = (7A - I) - 3A + I = 4A.$$

Consequently the entire matrix, not merely its determinant, simplifies to

$$A^{2023}(4A) = 4A^{2024}.$$

All powers concern the same matrix, so the factorization is valid without any issue of noncommuting matrices.

Step 3: Take the determinant with the correct scaling power The determinant is now

$$\det(4A^{2024}) = 4^2 \det(A^{2024}) = 16(\det A)^{2024} = 16.$$

The scalar factor is squared because the matrix has two rows. In particular, adding or subtracting the determinants of the three original terms would be invalid: determinants are multiplicative, but not additive. The values 12, 24, and 28 do not satisfy this exact reduction.

Step 4: Conclusion The required determinant is 16. Hence, the correct option is (B).

Quick Tip to Solve: For a very high power of a two-by-two matrix, factor the smallest power and reduce the remaining quadratic using the trace and determinant.

9. Let the relation R on the set $\mathrm{M} = \{1, 2, 3, \ldots, 16\}$ be given by $\mathrm{R} = \{(x, y) : 4y = 5x - 3, x, y \in \mathrm{M}\}$. Then the minimum number of elements required to be added in R , in order to make the relation symmetric, is equal to [4 marks]

(A) 2

(B) 3

(C) 4

(D) 1

Correct Answer: (A) 2

Solution:

Concept:

- A relation is symmetric if $(x, y) \in R$ implies $(y, x) \in R$. A diagonal pair (x, x) already satisfies this condition.
- The smallest symmetric relation containing R is $R \cup R^{-1}$; therefore count only reverse pairs that are actually missing. Both coordinates must belong to the stated set.

Step 1: Enumerate all admissible original pairs The defining equation gives $y = (5x - 3)/4$. Since y is an integer, $5x - 3 \equiv 0 \pmod 4$, or $x \equiv 3 \pmod 4$. The candidates in M are therefore $3, 7, 11, 15$. Substitution gives $y = 3, 8, 13, 18$, respectively. The last candidate is excluded because $18 \notin M$. Thus

$$R = \{(3, 3), (7, 8), (11, 13)\}.$$

Step 2: Identify the reverse pairs that are absent The reverse of $(3, 3)$ is itself, so this pair requires no addition. The other reverse pairs are $(8, 7)$ and $(13, 11)$, neither of which appears in the displayed relation. They are distinct, and all four of their coordinates are in M, so adding them is permitted.

Step 3: Prove that the additions are both necessary and sufficient Symmetry forces $(8, 7)$ because $(7, 8)$ is already present, and independently forces $(13, 11)$ because $(11, 13)$ is present. Hence fewer than two additions cannot work. After adding these two pairs, each off-diagonal pair has its reverse, while $(3, 3)$ remains self-reversing. Thus two additions suffice. Option (D), one addition, leaves a missing reverse; options (B) and (C), three and four, exceed the minimum. Symmetry does not require adding every diagonal pair.

Step 4: Conclusion The relation is $\{(3, 3), (7, 8), (11, 13)\}$, so exactly the two missing reverses $(8, 7)$ and $(13, 11)$ must be added. Thus the minimum is 2, printed as (A), whereas the differing keyed choice denotes 1 in the printed order and cannot supply both reverses. Hence, the correct option is (A).

Quick Tip to Solve: List the relation with both domain restrictions enforced, cross out self-pairs, and count missing reverses; do not confuse symmetry with reflexivity.

10. Let the set of all values of r, for which the circles $(x+1)^2+(y+4)^2=r^2$ and $x^2+y^2-4x-2y-4=0$ intersect at two distinct points be the interval (α,β). Then $\alpha\beta$ is equal to [4 marks]

(A) 25

(B) 24

(C) 21

(D) 20

Correct Answer: (A) 25

Solution:

Concept:

- Two circles with positive radii r and R, whose centres are distance d apart, intersect at two distinct points precisely when $|r-R|<d<r+R$.
- Equality in either bound gives tangency, so the endpoints must be excluded. Completing squares identifies the centre and radius of a general circle.

Step 1: Identify the centres and the fixed radius The first circle has centre $C_1=(-1,-4)$ and radius r. Completing the squares in the second equation gives

$$(x-2)^2+(y-1)^2=9.$$

Thus its centre is $C_2=(2,1)$ and its radius is 3. The centre separation is

$$d=\sqrt{(2+1)^2+(1+4)^2}=\sqrt{34}>3.$$

Step 2: Apply the strict intersection inequalities The external-separation condition $d<r+3$ gives $r>\sqrt{34}-3$. The internal-separation condition $|r-3|<\sqrt{34}$ gives $3-\sqrt{34}<r<3+\sqrt{34}$. Combining them, the stronger lower bound is the positive one, and the complete radius interval is

$$\sqrt{34}-3<r<\sqrt{34}+3.$$

At either endpoint there is only one common point, so neither endpoint is included.

Step 3: Multiply the interval endpoints We therefore have $\alpha=\sqrt{34}-3$ and $\beta=\sqrt{34}+3$. Their product simplifies by the difference-of-squares identity:

$$\alpha\beta=(\sqrt{34}-3)(\sqrt{34}+3)=34-9=25.$$

The radius is the nonnegative geometric parameter of the circle; the strict lower bound also makes it positive. The other proposed products 24, 21, and 20 disagree with the exact value d^2-3^2.

Step 4: Conclusion The endpoint product is $\alpha\beta=25$. Hence, the correct option is (A).

Quick Tip to Solve: When the centre separation exceeds the fixed radius, the variable-radius interval is $(d-R,d+R)$, so its endpoint product is immediately d^2-R^2.

11. Let the solution curve of the differential equation $xdy - ydx = \sqrt{x^2+y^2}dx, x > 0$, $y(1) = 0$, be $y = y(x)$. Then $y(3)$ is equal to [4 marks]

(A) 1

(B) 2

(C) 4

(D) 6

Correct Answer: (C) 4

Solution:

Concept:

- A differential equation involving y/x can be reduced by $y = vx$, so that $dy/dx = v + x\,dv/dx$.
- Here $x > 0$, hence $\sqrt{x^2+y^2} = x\sqrt{1+v^2}$. Separating variables then uses $\int dv/\sqrt{1+v^2} = \ln(v + \sqrt{1+v^2})$.

Step 1: Make the homogeneous substitution Divide by dx and put $y = vx$. The terms containing v cancel:

$$x(v + xv') - vx = x\sqrt{1+v^2}, \qquad \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}.$$

The initial value $y(1) = 0$ corresponds to $v(1) = 0$. The positive value of x is essential when removing its square from the radical.

Step 2: Integrate and determine the constant Integration yields

$$\ln\left(v + \sqrt{1+v^2}\right) = \ln x + C.$$

At $x = 1, v = 0$, both logarithms are zero, so $C = 0$. Exponentiating gives $v+\sqrt{1+v^2} = x$. The left side is positive for every real v, consistent with the stated domain. Since $(\sqrt{1+v^2}+v)(\sqrt{1+v^2}-v) = 1$, the conjugate equation is $\sqrt{1+v^2} - v = 1/x$.

Step 3: Recover the solution and check the required value Subtracting the conjugate equation gives $2v = x - 1/x$, hence

$$y = xv = \frac{x^2-1}{2}, \qquad y(3) = \frac{9-1}{2} = 4.$$

For verification, $y' = x$, so $xy' - y = (x^2+1)/2$. Also $\sqrt{x^2+y^2} = (x^2+1)/2$, and $y(1) = 0$. Thus the equation and initial value are satisfied exactly. The alternatives 1, 2, and 6 are incompatible with this uniquely determined solution.

Step 4: Conclusion The initial condition gives $y(x) = (x^2-1)/2$, hence $y(3) = (9-1)/2 = 4$, printed as (C). The differing keyed choice denotes 2 in the printed order, which disagrees with the solution verified in the differential equation. Hence, the correct option is (C).

Quick Tip to Solve: After exponentiating an integral of $1/\sqrt{1+v^2}$, use the conjugate product equal to one to isolate v without introducing extraneous signs.

12. Let the line $x = -1$ divide the area of the region $\{(x, y) : 1 + x^2 \le y \le 3 - x\}$ in the ratio $m : n, \gcd(m, n) = 1$. Then $m + n$ is equal to [4 marks]

(A) 25

(B) 26

(C) 27

(D) 28

Correct Answer: (C) 27

Solution:

Concept:

- For a region between an upper curve $y = u(x)$ and lower curve $y = l(x)$, its area on $[a, b]$ is $\int_a^b [u(x) - l(x)]\, dx$.
- A vertical dividing line splits the integral at its abscissa. Reduce the ratio of the two positive areas to coprime integers before adding its terms.

Step 1: Find the horizontal limits of the bounded region The curves intersect where $1 + x^2 = 3 - x$, or

$$x^2 + x - 2 = (x + 2)(x - 1) = 0.$$

Their intersection abscissas are -2 and 1. Throughout this interval the line lies above the parabola, and the vertical height of the region is $2 - x - x^2$. The dividing line $x = -1$ is strictly between the two endpoints.

Step 2: Integrate the area to the left of the dividing line An antiderivative of the height is

$$F(x) = 2x - \frac{x^2}{2} - \frac{x^3}{3}.$$

Its values at the relevant points are $F(-2) = -10/3$, $F(-1) = -13/6$, and $F(1) = 7/6$. The left area is therefore

$$A_L = F(-1) - F(-2) = -\frac{13}{6} + \frac{20}{6} = \frac{7}{6}.$$

Step 3: Calculate the other area and reduce the ratio The area to the right is

$$A_R = F(1) - F(-1) = \frac{7}{6} + \frac{13}{6} = \frac{10}{3}.$$

Thus $A_L : A_R = (7/6) : (20/6) = 7 : 20$. The total $7/6 + 20/6 = 9/2$ also equals the integral from -2 to 1, providing an area check. Since $\gcd(7, 20) = 1$, the required integers sum to 27. Reversing the order of the two regions reverses the ratio but leaves this sum unchanged. The alternatives 25, 26, and 28 are not the sum of the reduced terms.

Step 4: Conclusion The areas $7/6$ and $10/3$ give the coprime ratio $7 : 20$, hence $m + n = 27$, printed as (C). The differing keyed choice denotes 25 in the printed order and disagrees with these exact integrals. Hence, the correct option is (C).

Quick Tip to Solve: Integrate the height between the curves, not either ordinate alone; use one antiderivative at all three endpoints to keep the ratio calculation consistent.

13. The number of solutions of $\tan^{-1} 4x + \tan^{-1} 6x = \frac{\pi}{6}$, where $-\frac{1}{2\sqrt{6}} < x < \frac{1}{2\sqrt{6}}$, is equal to : [4 marks]

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (B) 1

Solution:

Concept:

- The principal function $\tan^{-1} u$ is continuous and strictly increasing for every real u. A sum of increasing functions can cross a fixed level at most once.
- For positive a, b with $ab = 1$, $\tan^{-1} a + \tan^{-1} b = \pi/2$. We use continuity to check that the required level is actually reached inside the stated interval.

Step 1: Show that the left side is strictly increasing Put $F(x) = \tan^{-1}(4x) + \tan^{-1}(6x)$. Its derivative is

$$F'(x) = \frac{4}{1 + 16x^2} + \frac{6}{1 + 36x^2} > 0.$$

Thus F is strictly increasing throughout the interval. In particular, two different values of x cannot give the same value $\pi/6$. This already excludes the options suggesting two or three solutions.

Step 2: Prove that a solution lies inside the interval Let $b = 1/(2\sqrt{6})$. At zero, $F(0) = 0$. At the positive endpoint, the two positive tangent arguments have product

$$(4b)(6b) = 24b^2 = 1.$$

Consequently $F(b) = \pi/2$. Since $0 < \pi/6 < \pi/2$, continuity gives a root strictly between 0 and b. The endpoint itself is not used, so the open interval causes no difficulty. Negative x gives two negative inverse tangents and cannot satisfy the equation. Hence the zero-solution option is also excluded.

Step 3: Check the root algebraically On $(0, b)$, the tangent addition formula is valid with positive denominator. Therefore

$$\frac{10x}{1 - 24x^2} = \frac{1}{\sqrt{3}}, \qquad 24x^2 + 10\sqrt{3}\,x - 1 = 0.$$

Its positive root is $x = (3\sqrt{11} - 5\sqrt{3})/24$, about 0.054, and lies inside the allowed interval. The monotonicity proof prevents any extraneous second root from being counted.

Step 4: Conclusion There is exactly 1 solution in the given open interval. In the printed option order, 1 is option (B). Hence, the correct option is (B).

Quick Tip to Solve: For a question asking only the number of inverse-tangent solutions, combine strict monotonicity with endpoint values before solving the resulting quadratic.

14. Let $\overrightarrow{AB} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\overrightarrow{AD} = \hat{i} + 2\hat{j} + \lambda\hat{k}, \lambda \in \mathbb{R}$. Let the projection of the vector $\vec{v} = \hat{i} + \hat{j} + \hat{k}$ on the diagonal $\overrightarrow{AC}$ of the parallelogram ABCD be of length one unit. If α, β, where $\alpha > \beta$, be the roots of the equation $\lambda^2 x^2 - 6\lambda x + 5 = 0$, then $2\alpha - \beta$ is equal to [4 marks]

(A) 1

(B) 3

(C) 4

(D) 6

Correct Answer: (B) 3

Solution:

Concept:

- The diagonal from a vertex of a parallelogram is the sum of its two adjacent side vectors: $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AD}$.
- The length of the projection of $\vec{v}$ on a nonzero vector $\vec{d}$ is $|\vec{v} \cdot \vec{d}|/|\vec{d}|$. The absolute value is necessary because a length cannot be negative.

Step 1: Form the diagonal and impose the projection condition Adding the given adjacent sides gives

$$\vec{d} = \overrightarrow{AC} = 3\hat{i} + 6\hat{j} + (\lambda - 5)\hat{k}.$$

This vector is nonzero for every real λ. With $\vec{v} = \hat{i} + \hat{j} + \hat{k}$, we get

$$\vec{v} \cdot \vec{d} = \lambda + 4, \qquad |\vec{d}| = \sqrt{45 + (\lambda - 5)^2}.$$

The stated unit projection length therefore requires

$$\frac{|\lambda + 4|}{\sqrt{45 + (\lambda - 5)^2}} = 1.$$

Step 2: Determine the parameter and check it Squaring the length equation and expanding both sides yields

$$\lambda^2 + 8\lambda + 16 = 45 + \lambda^2 - 10\lambda + 25.$$

Thus $18\lambda = 54$, so $\lambda = 3$. At this value, the numerator is 7 and the denominator is $\sqrt{45 + 4} = 7$; hence the original unsquared projection condition is satisfied. There is no extraneous parameter from squaring.

Step 3: Order the roots and evaluate the expression The quadratic now becomes

$$9x^2 - 18x + 5 = (3x - 1)(3x - 5) = 0.$$

The roots are $1/3$ and $5/3$. Because the problem specifies $\alpha > \beta$, take $\alpha = 5/3$ and $\beta = 1/3$. Therefore

$$2\alpha - \beta = \frac{10}{3} - \frac{1}{3} = 3.$$

The values 1, 4 and 6 in the other options do not equal this expression; in particular, 6 is the coefficient

in the original equation, not the required root combination.

Step 4: Conclusion The required value is $2\alpha - \beta = 3$, which is option (B) in the printed order. Hence, the correct option is (B).

Quick Tip to Solve: Use the diagonal sum first, apply the absolute scalar-projection formula, and only then solve the quadratic with the larger root assigned to α.

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15. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{[x]+4}\right) dx$, where $[\cdot]$ denotes the greatest integer function, is [4 marks]

(A) $\frac{7}{60}(3\pi - 1)$
(B) $\frac{7}{60}(\pi - 3)$
(C) $\frac{1}{60}(\pi - 7)$
(D) $\frac{1}{60}(21\pi - 1)$

Correct Answer: (A) $\frac{7}{60}(3\pi - 1)$

Solution:

Concept:

- The greatest integer function $[x]$ is constant on each interval $[k, k+1)$, where k is an integer. An integral involving it is evaluated by splitting at the integer boundaries.
- Changing the integrand at finitely many endpoints does not change a definite integral. Since $1 < \pi/2 < 2$, the relevant breaks are $-1, 0, 1$.

Step 1: Identify the four constant values On $[-\pi/2, -1)$, $[x] = -2$, so the integrand is $1/2$. On $[-1, 0)$, it is $1/3$; on $[0, 1)$, it is $1/4$; and on $[1, \pi/2]$, it is $1/5$. All four denominators are positive, and no singularity occurs in the integration interval. In particular, the floor of a negative noninteger must be rounded downward: for example, $[-1.2] = -2$, not -1.

Step 2: Integrate over each subinterval Each contribution is its constant height multiplied by its interval length. Writing the requested integral as I,

$$\begin{aligned} I &= \frac{1}{2}\left(\frac{\pi}{2} - 1\right) + \frac{1}{3}(1) + \frac{1}{4}(1) + \frac{1}{5}\left(\frac{\pi}{2} - 1\right) \\ &= \frac{7}{10}\left(\frac{\pi}{2} - 1\right) + \frac{7}{12}. \end{aligned}$$

The two outer intervals have the same length, which is why their contributions can be combined before simplifying.

Step 3: Simplify and compare the options Putting the constant terms over denominator 60 gives

$$I = \frac{7\pi}{20} - \frac{7}{10} + \frac{7}{12} = \frac{21\pi - 7}{60} = \frac{7}{60}(3\pi - 1).$$

This is positive, as the original integrand is positive. The expressions involving $\pi - 3$ and $\pi - 7$ have the wrong magnitude or sign, and $(21\pi - 1)/60$ has an incorrect constant term. Only the first printed expression matches all four interval contributions.

Step 4: Conclusion $I = \frac{7}{60}(3\pi - 1)$. Hence, the correct option is (A).

Quick Tip to Solve: Split a greatest-integer integral at every integer in its range; on negative intervals, take the floor toward minus infinity, not toward zero.

16. Let $f(x) = x^{2025} - x^{2000}, x \in [0, 1]$ and the minimum value of the function $f(x)$ in the interval $[0, 1]$ be $(80)^{80}(n)^{-81}$. Then n is equal to [4 marks]

(A) -80

(B) -81

(C) -40

(D) -41

Correct Answer: (B) -81

Solution:

Concept:

- A continuous function on a closed interval attains its minimum. Compare all interior critical points with the endpoint values.
- The common exponent factor suggests the substitution $t = x^{25}$. As x runs over $[0, 1]$, t runs over the same interval, so this substitution preserves the minimum problem.

Step 1: Reduce the large exponents Since $2025 = 25 \times 81$ and $2000 = 25 \times 80$, write

$$f(x) = t^{81} - t^{80} = g(t), \qquad 0 \le t \le 1.$$

The endpoints give $g(0) = g(1) = 0$. Every interior value satisfies $t^{80} > 0$ and $t - 1 < 0$, so $g(t) < 0$; consequently the minimum is strictly negative and occurs inside the interval.

Step 2: Find and classify the interior critical point Differentiation gives

$$g'(t) = 81t^{80} - 80t^{79} = t^{79}(81t - 80).$$

For $0 < t < 80/81$, the derivative is negative, whereas for $80/81 < t < 1$, it is positive. Thus $t = 80/81$ is the unique interior minimum. This derivative sign change, together with the zero endpoint values, establishes the global minimum on the full closed interval.

Step 3: Match the minimum with the stated form At the minimizing value,

$$g\left(\frac{80}{81}\right) = \left(\frac{80}{81}\right)^{80}\left(\frac{80}{81} - 1\right) = -\frac{80^{80}}{81^{81}}.$$

Equating this with $80^{80}n^{-81}$ and cancelling the nonzero factor 80^{80} gives $n^{-81} = -81^{-81}$. Since 81 is odd, the real solution is $n = -81$. The other listed negative numbers have a different magnitude and therefore cannot yield the required denominator 81^{81}.

Step 4: Conclusion The minimum is $-80^{80}/81^{81}$, so $n = -81$. Hence, the correct option is (B).

Quick Tip to Solve: When two large powers share a common factor in their exponents, substitute that power first; retain the negative sign when matching an odd reciprocal power.

17. If the sum of the first four terms of an A.P. is 6 and the sum of its first six terms is 4 , then the sum of its first twelve terms is [4 marks]

(A) -20

(B) -22

(C) -24

(D) -26

Correct Answer: (B) -22

Solution:

Concept:

- For an arithmetic progression with first term a and common difference d, $S_n = \frac{n}{2}[2a + (n-1)d]$. This formula applies equally to increasing and decreasing progressions.
- Two specified partial sums give two linear equations in a and d. Divide out the factors $n/2$ before subtracting the equations.

Step 1: Translate the two supplied sums The condition $S_4 = 6$ gives

$$2(2a + 3d) = 6, \qquad 2a + 3d = 3.$$

Similarly, $S_6 = 4$ gives

$$3(2a + 5d) = 4, \qquad 2a + 5d = \frac{4}{3}.$$

The second sum being smaller does not contradict an arithmetic progression: later terms can become negative. Therefore the signs of a and d must be obtained from these equations rather than assumed.

Step 2: Solve for the common difference and first term Subtracting the first reduced equation from the second eliminates $2a$:

$$2d = \frac{4}{3} - 3 = -\frac{5}{3}, \qquad d = -\frac{5}{6}.$$

Substitute this into $2a + 3d = 3$:

$$2a = 3 - 3\left(-\frac{5}{6}\right) = \frac{11}{2}, \qquad a = \frac{11}{4}.$$

As a check, $2a+5d = 11/2-25/6 = 4/3$, which reproduces the stated six-term sum after multiplication by 3.

Step 3: Evaluate the twelve-term sum Using the same sum formula with $n = 12$,

$$\begin{aligned} S_{12} &= 6(2a + 11d) \\ &= 6\left(\frac{11}{2} - \frac{55}{6}\right) \\ &= 6\left(-\frac{22}{6}\right) = -22. \end{aligned}$$

The other choices $-20, -24, -26$ do not follow from the uniquely determined values of a and d. The negative total is consistent with the decreasing progression and its increasingly negative later terms.

Step 4: Conclusion The sum of the first twelve terms is -22. Hence, the correct option is (B).

Quick Tip to Solve: For two given A.P. sums, first work with $2S_n/n = 2a + (n-1)d$; subtracting these reduced equations gives the common difference immediately.

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18. The coefficient of x^{48} in $(1+x) + 2(1+x)^2 + 3(1+x)^3 + \ldots + 100(1+x)^{100}$ is equal to [4 marks]

(A) $100\binom{100}{49} - \binom{100}{50}$
(B) $\binom{100}{50} + \binom{101}{49}$
(C) $100\binom{101}{49} - \binom{101}{50}$
(D) $100\binom{100}{49} - \binom{100}{48}$

Correct Answer: (C) $100\binom{101}{49} - \binom{101}{50}$

Solution:

Concept:

- The binomial theorem gives the coefficient of x^r in $(1+x)^k$ as $\binom{k}{r}$, with zero contribution when $k < r$.
- Pascal's identity gives $\binom{k}{48} = \binom{k+1}{49} - \binom{k}{49}$. This turns the required weighted binomial sum into a telescoping expression.

Step 1: Express the coefficient as a finite sum Each summand $k(1+x)^k$ contributes $k\binom{k}{48}$ to the coefficient of x^{48}. The first 47 summands have degree less than 48 and contribute nothing. Thus the desired coefficient is

$$C = \sum_{k=48}^{100} k\binom{k}{48}.$$

The factor k must be retained; using only an unweighted hockey-stick identity would omit this coefficient multiplying each power.

Step 2: Apply the binomial recurrence and telescope Substitute the difference form and shift the

index in the first sum:

$$C = \sum_{k=48}^{100} k\left[\binom{k+1}{49} - \binom{k}{49}\right]$$
$$= \sum_{j=49}^{101} (j-1)\binom{j}{49} - \sum_{j=49}^{100} j\binom{j}{49}$$
$$= 100\binom{101}{49} - \sum_{j=49}^{100} \binom{j}{49}.$$

Here the term with $k = 48$ in the second original sum vanishes because $\binom{48}{49} = 0$. All overlapping terms have coefficient $(j-1) - j = -1$.

Step 3: Apply the hockey-stick identity and check the choices The remaining sum satisfies

$$\sum_{j=49}^{100} \binom{j}{49} = \binom{101}{50}.$$

Therefore $C = 100\binom{101}{49} - \binom{101}{50}$. For an exact comparison, put $U = \binom{100}{49}$. Then $\binom{101}{49} = 101U/52$, $\binom{101}{50} = 101U/50$, and $\binom{100}{50} = 51U/50$. The required value is $124937U/650$; the first, second and fourth printed choices equal $4949U/50$, $3851U/1300$ and $5151U/52$, respectively. These distinct rational multiples of the same positive number can be compared exactly. Hence none of those alternatives is equal to the derived expression.

Step 4: Conclusion The coefficient is $100\binom{101}{49} - \binom{101}{50}$, which is the third printed option. Hence, the correct option is (C).

Quick Tip to Solve: For a weighted binomial sum, replace each coefficient by a Pascal difference and shift indices; the end term survives and the remaining sum is a hockey-stick sum.

19. The number of distinct real solutions of the equation $x|x+4| + 3|x+2| + 10 = 0$ is [4 marks]

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (B) 1

Solution:

Concept:

- An absolute value changes its algebraic form only when its argument changes sign. Here the dividing points are $x = -4$ and $x = -2$.
- Solve separately on each interval and retain only roots belonging to that interval. A quadratic with negative discriminant has no real roots.

Step 1: Examine the interval to the right of both break points For $x \geq -2$, both $x + 4$ and $x + 2$ are

nonnegative. The equation becomes

$$x(x+4)+3(x+2)+10=x^2+7x+16=0.$$

Its discriminant is $49-64=-15<0$, so this interval contributes no root, including its endpoint.

Step 2: Check the middle interval For $-4\le x<-2$, we have $|x+4|=x+4$ and $|x+2|=-x-2$. Therefore

$$x(x+4)-3(x+2)+10=x^2+x+4=0.$$

Again the discriminant is negative: $1-16=-15$. Thus there is no root in the middle interval; the point $x=-4$ is covered by this check.

Step 3: Solve on the remaining interval and count For $x<-4$, both absolute-value arguments are negative. Thus

$$-x(x+4)-3(x+2)+10=0 \quad\Longrightarrow\quad x^2+7x-4=0.$$

The candidate roots are

$$x=\frac{-7\pm\sqrt{65}}{2}.$$

Since $8<\sqrt{65}<9$, the negative-sign root lies between -8 and -7.5, and is valid. The positive-sign root lies between 0.5 and 1, so it does not belong to this interval. Only one distinct real root survives; counts zero, two and three all fail the interval test.

Step 4: Conclusion There is exactly 1 distinct real solution, namely $x=(-7-\sqrt{65})/2$. Hence, the correct option is (B).

Quick Tip to Solve: Write the permitted interval beside each quadratic before solving; a real quadratic root may still be invalid for the original absolute-value equation.

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20. If the line $\alpha x+2y=1$, where $\alpha\in\mathbb{R}$, does not meet the hyperbola $x^2-9y^2=9$, then a possible value of α is : [4 marks]

(A) 0.5

(B) 0.6

(C) 0.7

(D) 0.8

Correct Answer: (D) 0.8

Solution:

Concept:

- A line meets a hyperbola at a real point precisely when substitution of the line equation gives a real coordinate satisfying the conic.
- For a genuine quadratic $Ax^2+Bx+C=0$, no real root exists when $B^2-4AC<0$. Check separately when the leading coefficient vanishes.

Step 1: Eliminate the second coordinate From the line, $y=(1-\alpha x)/2$. Substitute this expression

into $x^2 - 9y^2 = 9$, multiplying by 4 to clear the denominator:

$$4x^2 - 9(1 - \alpha x)^2 = 36.$$

On expanding and collecting terms, the intersection equation is

$$(4 - 9\alpha^2)x^2 + 18\alpha x - 45 = 0.$$

Every real root of this equation gives a real point on both curves, since the expression for y involves no additional restriction.

Step 2: Find the condition for no real intersection Provided $4 - 9\alpha^2 \neq 0$, the discriminant is

$$\begin{aligned}\Delta &= (18\alpha)^2 - 4(4 - 9\alpha^2)(-45) \\ &= 324\alpha^2 + 180(4 - 9\alpha^2) \\ &= 144(5 - 9\alpha^2).\end{aligned}$$

Hence no intersection requires $\alpha^2 > 5/9$, or $|\alpha| > \sqrt{5}/3$. At $\alpha = \pm 2/3$ the equation is linear with a nonzero coefficient of x, so that exceptional case still gives an intersection.

Step 3: Test the four proposed values The threshold is $\sqrt{5}/3 \approx 0.745$. The values $0.5, 0.6, 0.7$ have squares $0.25, 0.36, 0.49$, all below $5/9$, so they produce real intersections. For $\alpha = 0.8$, $\alpha^2 = 0.64 > 5/9$; equivalently $5 - 9(0.64) = -0.76 < 0$. This value gives no real point of intersection.

Step 4: Conclusion A possible value is $\alpha = 0.8$. Hence, the correct option is (D).

Quick Tip to Solve: Use the discriminant after substituting the line, but always inspect any parameter value that makes the resulting quadratic linear.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let A be a 3×3 matrix such that $\mathrm{A} + \mathrm{A}^{\mathrm{T}} = \mathrm{O}$. If $\mathrm{A}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}, \mathrm{A}^2\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 19 \\ -24 \end{bmatrix}$ and $\det(\mathrm{adj}(2\,\mathrm{adj}(\,\mathrm{A} + \mathrm{I}))) = (2)^{\alpha} \cdot (3)^{\beta} \cdot (11)^{\gamma}, \alpha, \beta, \gamma$ are non-negative integers, then $\alpha + \beta + \gamma$ is equal to ____

[4 marks]

Solution:

Concept:

- The condition $A + A^T = O$ makes A skew-symmetric: diagonal entries are zero and opposite off-diagonal entries have opposite signs.
- For a 3×3 matrix B, $\det(\mathrm{adj}\, B) = (\det B)^2$ and $\det(cB) = c^3 \det B$. These identities avoid computing the adjugate entries.

Step 1: Determine the skew-symmetric entries Write

$$A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}, \qquad v = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}.$$

Then $Av = (-a, -a, -b + c)^T = (3, 3, 2)^T$. Thus $a = -3$ and $c - b = 2$. Also $A^2 v = A(3, 3, 2)^T$, giving

$$3a + 2b = -3, \quad -3a + 2c = 19, \quad -3b - 3c = -24.$$

Substituting $a = -3$ yields $b = 3$ and $c = 5$. These satisfy both $c - b = 2$ and the third equation, checking consistency.

Step 2: Evaluate the basic determinant Put $B = A + I$. Its determinant is

$$\det B = \begin{vmatrix} 1 & -3 & 3 \\ 3 & 1 & 5 \\ -3 & -5 & 1 \end{vmatrix} = 26 + 54 - 36 = 44.$$

Equivalently, the determinant of identity plus this skew-symmetric matrix is $1 + a^2 + b^2 + c^2 = 1 + 9 + 9 + 25 = 44$. Both evaluations agree.

Step 3: Apply the determinant identities in their nested order First, $\det(\operatorname{adj} B) = 44^2$. Multiplying that adjugate matrix by 2 multiplies its determinant by 2^3, and the outer adjugate squares the determinant once more:

$$\begin{aligned} \det\big(\operatorname{adj}(2 \operatorname{adj} B)\big) &= (2^3 \, 44^2)^2 \\ &= 2^6 \, 44^4 \\ &= 2^{14} \, 3^0 \, 11^4. \end{aligned}$$

Therefore $\alpha = 14$, $\beta = 0$, and $\gamma = 4$; zero is permitted because the exponents are nonnegative integers.

Step 4: Conclusion The required numerical value is $\alpha + \beta + \gamma = 14 + 0 + 4 = \boxed{18}$.

Quick Tip to Solve: For nested adjugates, replace each adjugate determinant by a power of the preceding determinant, keeping scalar-matrix factors separate.

22. Let $\alpha = \frac{-1+i\sqrt{3}}{2}$ and $\beta = \frac{-1-i\sqrt{3}}{2}, i = \sqrt{-1}$. If

$$(7 - 7\alpha + 9\beta)^{20} + (9 + 7\alpha - 7\beta)^{20} + (-7 + 9\alpha + 7\beta)^{20} + (14 + 7\alpha + 7\beta)^{20} = m^{10},$$

then m is ____ [4 marks]

Solution:

Concept:

- The numbers α, β are the non-real cube roots of unity: $\alpha^3 = \beta^3 = 1$, $\alpha\beta = 1$, and $1 + \alpha + \beta = 0$. In particular $\alpha^2 = \beta$ and $\beta^2 = \alpha$.
- If three bases are $z, \alpha z, \beta z$, their nth powers sum to $z^n(1 + \alpha^n + \beta^n)$. This vanishes when n is not divisible by 3.

Step 1: Recognise a cyclic triple among the first three bases Let $z = 7 - 7\alpha + 9\beta$. Multiplication by α gives

$$\alpha z = 7\alpha - 7\alpha^2 + 9\alpha\beta = 9 + 7\alpha - 7\beta.$$

Similarly, multiplying by β gives

$$\beta z = 7\beta - 7\alpha\beta + 9\beta^2 = -7 + 9\alpha + 7\beta.$$

These are exactly the second and third bases in the question, including every coefficient and sign.

Step 2: Cancel their twentieth powers Since $20 \equiv 2 \pmod 3$, we have $\alpha^{20} = \alpha^2 = \beta$ and $\beta^{20} = \beta^2 = \alpha$. Hence the sum of the first three terms is

$$z^{20} + (\alpha z)^{20} + (\beta z)^{20} = z^{20}(1 + \beta + \alpha) = 0.$$

No expansion of any twentieth power is needed; the cancellation follows exactly from the cube-root identity.

Step 3: Evaluate the remaining power The last base simplifies using $\alpha + \beta = -1$:

$$14 + 7\alpha + 7\beta = 14 - 7 = 7.$$

Thus the equation reduces to

$$m^{10} = 7^{20} = (7^2)^{10} = 49^{10}.$$

Its nonnegative value of m is 49, agreeing with the numerical answer key. Because the exponent is even, the equation alone also permits $m = -49$ if arbitrary real values of m are allowed.

Step 4: Conclusion The nonnegative numerical answer is $\boxed{49}$. Strictly, without a sign restriction the displayed equation determines $|m| = 49$, so its real roots are $m = \pm 49$.

Quick Tip to Solve: Before expanding high powers of expressions in cube roots of unity, multiply one base by each root and look for a cyclic triple.

23. If $\int (\sin x)^{\frac{-11}{2}} (\cos x)^{\frac{-5}{2}}\, dx = -\frac{p_1}{q_1}(\cot x)^{\frac{9}{2}} - \frac{p_2}{q_2}(\cot x)^{\frac{5}{2}} - \frac{p_3}{q_3}(\cot x)^{\frac{1}{2}} + \frac{p_4}{q_4}(\cot x)^{\frac{-3}{2}} + C$, where p_i and q_i are positive integers with $\gcd(p_i, q_i) = 1$ for $i = 1, 2, 3, 4$ and C is the constant of integration, then $\frac{15p_1p_2p_3p_4}{q_1q_2q_3q_4}$ is equal to ____ [4 marks]

Solution:

Concept:

- Use $t = \cot x$, so $dt = -\csc^2 x\, dx$ and $1 + t^2 = \csc^2 x$. This substitution is suited to the half-integer powers in the proposed answer.
- The power rule is $\int t^r\, dt = t^{r+1}/(r+1)$ for $r \neq -1$. Work on an interval on which the displayed real fractional powers are defined; in particular $\sin x, \cos x > 0$ gives $t > 0$.

Step 1: Convert the integrand to powers of cotangent With $t = \cot x = \cos x / \sin x$, write $\cos x = t \sin x$. Also $dx = -\sin^2 x\, dt$. Therefore

$$\begin{aligned}(\sin x)^{-11/2}(\cos x)^{-5/2}\, dx &= -t^{-5/2}(\sin x)^{-6}\, dt \\ &= -t^{-5/2}(1+t^2)^3\, dt.\end{aligned}$$

The extra factor $\sin^2 x$ from the differential is essential in getting the exponent -6.

Step 2: Expand and integrate each power Expand the cubic before integrating:

$$-t^{-5/2}(1+t^2)^3 = -\left(t^{7/2} + 3t^{3/2} + 3t^{-1/2} + t^{-5/2}\right).$$

Applying the power rule term by term gives

$$I = -\frac{2}{9}t^{9/2} - \frac{6}{5}t^{5/2} - 6t^{1/2} + \frac{2}{3}t^{-3/2} + C.$$

The last sign is positive because the exponent after integration is $-3/2$, which divides the negative coefficient.

Step 3: Read the reduced fractions and compute the product Replace t by $\cot x$ and compare with the given form. The four positive reduced coefficients are

$$\frac{p_1}{q_1} = \frac{2}{9}, \quad \frac{p_2}{q_2} = \frac{6}{5}, \quad \frac{p_3}{q_3} = \frac{6}{1}, \quad \frac{p_4}{q_4} = \frac{2}{3}.$$

Each numerator is coprime to its denominator. Consequently

$$\frac{15p_1p_2p_3p_4}{q_1q_2q_3q_4} = \frac{15 \cdot 2 \cdot 6 \cdot 6 \cdot 2}{9 \cdot 5 \cdot 1 \cdot 3} = 16.$$

Differentiating the four powers with respect to t restores the expanded integrand, checking all coefficients and signs.

Step 4: Conclusion The required numerical value is $\boxed{16}$.

Quick Tip to Solve: For products of powers of sine and cosine, try to reserve the differential of cotangent first; the remaining even sine power becomes a polynomial in $1 + \cot^2 x$.

24. If $\dfrac{\cos^2 48^\circ - \sin^2 12^\circ}{\sin^2 24^\circ - \sin^2 6^\circ} = \dfrac{\alpha + \beta\sqrt{5}}{2}$, where $\alpha, \beta \in \mathbb{N}$, then $\alpha + \beta$ is equal to _____. [4 marks]

Solution:

Concept:

- The difference-of-squares identities $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$ and $\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$ convert the numerator and denominator to products.
- The exact values $\cos 36^\circ = (1+\sqrt{5})/4$ and $\sin 18^\circ = (\sqrt{5}-1)/4$ are linked by $\sin 18^\circ = \cos 72^\circ$.

Step 1: Simplify each difference of squares Apply the cosine-sine identity with $A = 48^\circ$, $B = 12^\circ$:

$$\cos^2 48^\circ - \sin^2 12^\circ = \cos 60^\circ \cos 36^\circ = \tfrac{1}{2}\cos 36^\circ.$$

For the denominator, use $A = 24^\circ$, $B = 6^\circ$:

$$\sin^2 24^\circ - \sin^2 6^\circ = \sin 30^\circ \sin 18^\circ = \tfrac{1}{2}\sin 18^\circ.$$

The common factor $1/2$ cancels, and the denominator is nonzero because $\sin 18^\circ > 0$.

Step 2: Confirm the exact angle values For completeness, put $c = \cos 36^\circ$. Since $\cos 108^\circ = -\cos 72^\circ$, the triple-angle and double-angle formulas give

$$4c^3 - 3c = -(2c^2 - 1).$$

Thus

$$(c+1)(4c^2 - 2c - 1) = 0.$$

As $0 < c < 1$, this yields $c = (1+\sqrt{5})/4$. Substitution into $\sin 18^\circ = \cos 72^\circ = 2c^2 - 1$ gives $(\sqrt{5}-1)/4$.

Step 3: Rationalise and identify the two integers The requested ratio is therefore

$$\begin{aligned}\frac{\cos 36^\circ}{\sin 18^\circ} &= \frac{1+\sqrt{5}}{\sqrt{5}-1} \\ &= \frac{(1+\sqrt{5})^2}{5-1} \\ &= \frac{6+2\sqrt{5}}{4} = \frac{3+\sqrt{5}}{2}.\end{aligned}$$

Comparing the rational and irrational parts with $(\alpha + \beta\sqrt{5})/2$ gives $\alpha = 3$, $\beta = 1$, both natural numbers.

Step 4: Conclusion The required sum is $\alpha + \beta = 3 + 1 = \boxed{4}$.

Quick Tip to Solve: When squared trigonometric terms involve angle pairs whose sums are 30° or 60°, product identities often cancel the simple factor immediately.

25. Let ABC be a triangle. Consider four points $\mathrm{p}_1, \mathrm{p}_2, \mathrm{p}_3, \mathrm{p}_4$ on the side AB , five points p_5, p_6, p_7, p_8, p_9 on the side BC, and four points $p_{10}, p_{11}, p_{12}, p_{13}$ on the side AC . None of these points is a vertex of the triangle ABC . Then the total number of pentagons, that can be formed by taking all the vertices from the points $p_1, p_2, \ldots, p_{13}$, is ____ [4 marks]

Solution:

Concept:

- A pentagon requires five distinct vertices with no redundant vertex on a straight side. Hence at most two selected points can lie on any one side of the triangle.
- The number of unordered selections of r points from n distinct points is $\binom{n}{r}$. Independent selections on different sides multiply; mutually exclusive cases add.

Step 1: Identify the possible distribution The five vertices must be selected from the three sides containing 4, 5 and 4 points. Selecting three points on one side would put the middle one on a straight segment, so they would not form five genuine corners. With at most two vertices per side, the only possible distribution is $(2, 2, 1)$. Every such selection gives a convex pentagon in its cyclic boundary order. There is no additional factorial factor because changing the starting vertex does not create a new pentagon.

Step 2: Count according to the side contributing one point If the single point is on AC, choose two from AB and two from BC. If it is on BC, choose two from each of AB and AC. If it is on AB, choose two from BC and two from AC. Thus

$$\begin{aligned} N &= \binom{4}{2}\binom{5}{2}\binom{4}{1} + \binom{4}{2}\binom{5}{1}\binom{4}{2} + \binom{4}{1}\binom{5}{2}\binom{4}{2} \\ &= 6 \cdot 10 \cdot 4 + 6 \cdot 5 \cdot 6 + 4 \cdot 10 \cdot 6 \\ &= 240 + 180 + 240 = 660. \end{aligned}$$

These cases are disjoint because each pentagon has exactly one side of the original triangle contributing only one vertex.

Step 3: Conclusion The total number of pentagons is $\boxed{660}$. This is a dimensionless count.

Quick Tip to Solve: For vertices on the three sides of a triangle, first restrict each side to at most two selected points; then count the allowed distributions instead of subtracting many overlapping invalid cases.

26. A projectile is thrown upward at an angle 60° with the horizontal. The speed of the projectile is $20\ \mathrm{m/s}$ when its direction of motion is 45° with the horizontal. The initial speed of the projectile is ____ $\mathrm{m/s}$. [4 marks]

(A) 40

(B) $40\sqrt{2}$

(C) $20\sqrt{2}$

(D) $20\sqrt{3}$

Correct Answer: (C) $20\sqrt{2}$

Solution:

Concept:

- In ideal projectile motion gravity acts vertically, so the horizontal velocity remains constant: $v_x = u\cos\theta$, where u is the launch speed.
- At a later instant with speed v and direction ϕ to the horizontal, the horizontal component is $v\cos\phi$. The speed magnitude itself need not remain constant.

Step 1: Resolve the velocity at the specified instant The projectile has speed $20\,\mathrm{m\,s^{-1}}$ when its velocity makes 45° with the horizontal. Its horizontal component is therefore

$$v_x = 20\cos 45^\circ = \frac{20}{\sqrt{2}} = 10\sqrt{2}\,\mathrm{m\,s^{-1}}.$$

This component is unchanged whether the specified instant is on the upward or downward part of the trajectory, because gravity does not alter horizontal motion.

Step 2: Equate the initial and later horizontal components The initial direction is 60°, so the initial horizontal component is $u\cos 60^\circ = u/2$. Equating the two horizontal components gives

$$\frac{u}{2} = 10\sqrt{2} \quad \Rightarrow \quad u = 20\sqrt{2}\,\mathrm{m\,s^{-1}}.$$

As a check, the initial vertical component is $u\sin 60^\circ = 10\sqrt{6}$, greater than the later upward vertical component $10\sqrt{2}$, as expected under gravity. The proposed speeds in (A), (B) and (D) would give horizontal components 20, $20\sqrt{2}$ and $10\sqrt{3}$, respectively, rather than the required $10\sqrt{2}$.

Step 3: Conclusion The initial speed is $\boxed{20\sqrt{2}\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (C).

Quick Tip to Solve: When a projectile speed and its instantaneous direction are given, conserve the horizontal component directly; there is usually no need to calculate flight time or height.

27. Three identical coils C_1, C_2 and C_3 are closely placed such that they share a common axis. C_2 is exactly midway. C_1 carries current I in anti-clockwise direction while C_3 carries current I in clockwise direction. An induced current flows through C_2 will be in clockwise direction when [4 marks]

(A) C_1 and C_3 move with equal speeds towards C_2

(B) C_1 and C_3 move with equal speeds away from C_2

(C) C_1 moves towards C_2 and C_3 moves away from C_2

(D) C_1 moves away from C_2 and C_3 moves towards C_2

Correct Answer: (C) C_1 moves towards C_2 and C_3 moves away from C_2

Solution:

Concept:

- Faraday's law gives $\mathcal{E} = -d\Phi/dt$. Lenz's law means the induced magnetic field opposes the change in external flux, rather than necessarily opposing the external field itself.
- Using the same viewing direction for all three coils, an anticlockwise current produces positive axial flux and a clockwise current produces negative axial flux. The magnitude increases when a source coil approaches.

Step 1: Determine the flux change required Take the flux through C_2 due to the anticlockwise current in C_1 as positive. The clockwise current in C_3 gives negative flux. A clockwise induced current in C_2 produces negative induced flux; it must therefore oppose an increasing positive external flux. Thus the required condition is $d\Phi_{\text{ext}}/dt > 0$. Initially the equal, oppositely directed contributions cancel because C_2 is midway between identical coils.

Step 2: Test the motions in the printed options When C_1 approaches, its positive flux grows. When C_3 recedes, its negative flux decreases in magnitude, which also increases the net positive flux. These motions reinforce the required positive rate of change and induce a clockwise current. Equal-speed approach of both coils, or equal-speed recession of both, preserves equal opposite contributions and gives zero net flux change; these eliminate (A) and (B). In (D), the positive contribution weakens while the negative one strengthens, so the net flux decreases and the induced current is anticlockwise.

Step 3: Conclusion C_1 must move towards C_2 while C_3 moves away from it. This is option (C) in the printed official order. Hence, the correct option is (C).

Quick Tip to Solve: Assign opposite signs to the two source fluxes before applying Lenz's law. A negative flux becoming weaker is a positive change of net flux.

28. $7.9\text{MeV}\,\alpha$**-particle scatters from a target material of atomic number 79 . From the given data the estimated diameter of nuclei of the target material is (approximately) ____ m.**

$$\left[\frac{1}{4\pi\epsilon_o} = 9 \times 10^9 \text{Nm}^2/\text{C}^2 \text{ and electron charge } = 1.6 \times 10^{-19}\text{C}\right]$$

[4 marks]

(A) 1.69×10^{-12}

(B) 2.88×10^{-14}

(C) 1.44×10^{-13}

(D) 5.76×10^{-14}

Correct Answer: (D) 5.76×10^{-14}

Solution:

Concept:

- For a head-on approach, the initial kinetic energy of an alpha particle is converted to electrostatic potential energy at its distance of closest approach: $K = k(2e)(Ze)/r_0$.
- The alpha particle has charge $+2e$, the target nucleus has charge $+Ze$, and $1\,\text{MeV} = 10^6(1.6 \times 10^{-19})\,\text{J}$. A diameter estimate is twice the inferred radial distance.

Step 1: Convert the kinetic energy and set up the turning point For the target $Z = 79$, while the alpha-particle energy is $7.9\,\text{MeV}$. At the closest point in a head-on encounter the particle momentarily stops, so

$$K = 7.9 \times 10^6 \times 1.6 \times 10^{-19} = 1.264 \times 10^{-12}\,\text{J}.$$

Using a head-on trajectory gives the closest approach, since angular momentum would prevent an oblique trajectory from getting as close at the same energy.

Step 2: Calculate the radial estimate and double it Insert the supplied Coulomb constant and elementary charge:

$$r_0 = \frac{(9 \times 10^9)(2)(79)(1.6 \times 10^{-19})^2}{1.264 \times 10^{-12}}$$
$$= 2.88 \times 10^{-14}\,\text{m}.$$

The diameter estimate requested is

$$D \simeq 2r_0 = 5.76 \times 10^{-14}\,\text{m}.$$

The value in (B) is the radial distance, not the requested diameter; (A) and (C) do not follow from the supplied charge-energy calculation. This closest-approach argument gives an estimated upper scale for nuclear size, rather than a precise measurement of the nuclear material boundary.

Step 3: Conclusion The estimated nuclear diameter is $\boxed{5.76 \times 10^{-14}\,\text{m}}$. Hence, the correct option is (D).

Quick Tip to Solve: Convert MeV to joules before using SI Coulomb constants, include the alpha charge $2e$, and check whether the question requests a radius or a diameter.

29. Consider an equilateral prism (refractive index $\sqrt{2}$). A ray of light is incident on its one surface at a certain angle i. If the emergent ray is found to graze along the other surface then the angle of refraction at the incident surface is close to ____ [4 marks]

(A) 15°

(B) 30°

(C) 20°

(D) 40°

Correct Answer: (A) 15°

Solution:

Concept:

- For a prism, the internal angles measured from the normals obey $r_1 + r_2 = A$, where A is the prism angle. An equilateral prism has $A = 60^\circ$.
- At the second surface, Snell's law for emergence into air is $\mu \sin r_2 = \sin e$. A grazing emergent ray has $e = 90^\circ$, so the internal angle is the critical angle.

Step 1: Find the angle at the second surface The refractive index of the prism is $\mu = \sqrt{2}$. The emergent ray grazes the surface, which means its angle with the normal is 90°, not zero. Snell's law therefore gives

$$\sqrt{2} \sin r_2 = \sin 90^\circ = 1.$$

Hence

$$\sin r_2 = \frac{1}{\sqrt{2}}, \qquad r_2 = 45^\circ.$$

The physical internal angle lies between zero and 90°, so the relevant inverse-sine value is unambiguous.

Step 2: Use the prism-angle relation Inside the prism, the two normal-based angles sum to the apex angle. Thus

$$r_1 = 60^\circ - 45^\circ = 15^\circ.$$

This is the angle of refraction at the first surface, which is what is asked. The external incidence angle i is a different quantity and is not needed. The alternatives 30°, 20° and 40° would require second-surface angles 30°, 40° and 20°, respectively. None equals the critical angle 45°, so none would give grazing emergence.

Step 3: Conclusion The first-surface angle of refraction is $\boxed{15^\circ}$, option (A) in the printed official order. Hence, the correct option is (A).

Quick Tip to Solve: For grazing emergence, start at the exit face: set its internal angle equal to the critical angle, then subtract it from the prism angle to obtain the first refraction angle.

30. Given below are two statements :

Statement I : Pressure of a fluid is exerted only on a solid surface in contact as the fluid-pressure does not exist everywhere in a still fluid.

Statement II : Excess potential energy of the molecules on the surface of a liquid, when compared to interior, results in surface tension.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Concept:

- A static fluid has pressure at every interior point. Pressure is normal force per unit area across any imagined surface within the fluid as well as across a solid boundary.
- Molecules at a liquid surface have higher potential energy than molecules in the interior because their cohesive environment is incomplete. Surface tension is associated with the energy required to increase surface area.

Step 1: Evaluate Statement I Fluid pressure is not confined to locations where the fluid contacts a solid. Consider an imaginary small plane element inside a still liquid: the fluid on either side exerts a normal force on the other side. The local pressure describes this force per unit area. In hydrostatic equilibrium the pressure generally changes with depth according to $p = p_0 + \rho g h$, and at a given point it acts equally in all directions. The absence of a solid wall at that point does not make the pressure vanish. Therefore Statement I is false.

Step 2: Evaluate Statement II and select the combination A molecule deep inside a liquid is surrounded by neighbouring molecules, whereas one at the surface has fewer neighbours above it. Moving molecules from the interior to the surface therefore requires work and raises the potential energy. Increasing the area by dA at fixed conditions requires reversible work $dW = \gamma\, dA$, where γ is surface tension. This energetic cost makes the liquid resist an increase of surface area. Statement II correctly connects the excess energy of surface molecules with surface tension, so it is true. Options (A) and (C) incorrectly accept Statement I; option (B) incorrectly rejects Statement II.

Step 3: Conclusion Statement I is false and Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish pressure, which exists throughout a fluid, from surface tension, which concerns the energetic cost of a liquid interface. Test the two statements independently.

31. The volume of an ideal gas increases 8 times and temperature becomes $(1/4)^{th}$ of initial temperature during a reversible change. If there is no exchange of heat in this process $(\Delta Q = 0)$ then identify the gas from the following options (Assuming the gases given in the options are ideal gases) :

[4 marks]

(A) NH_3

(B) O_2

(C) He

(D) CO_2

Correct Answer: (C) He

Solution:

Concept:

- A reversible change with no heat transfer is adiabatic. For an ideal gas with constant heat-capacity ratio $\gamma = C_P/C_V$, its temperature and volume satisfy $TV^{\gamma-1} =$ constant.
- The number of active degrees of freedom f determines $C_V = fR/2$ and $\gamma = 1 + 2/f$. A monatomic gas has three translational degrees of freedom.

Step 1: Apply the adiabatic relation Write the relation for the initial and final states, using absolute temperature. The volume increases by a factor of eight while the final temperature is one quarter of the initial temperature:

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}, \qquad 1 = \frac{1}{4}\, 8^{\gamma-1}.$$

Thus $8^{\gamma-1} = 4$. This is an expansion, so the fall in temperature is consistent with work being done by the gas at the expense of its internal energy.

Step 2: Find the heat-capacity ratio Express both sides as powers of two and equate the exponents:

$$2^{3(\gamma-1)} = 2^2 \quad \Longrightarrow \quad 3(\gamma - 1) = 2 \quad \Longrightarrow \quad \gamma = \frac{5}{3}.$$

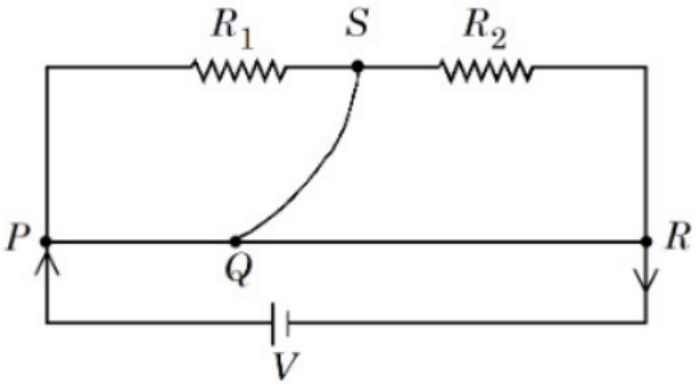
Equivalently, $1 + 2/f = 5/3$ gives $f = 3$. This is the monatomic ideal-gas value, not the ordinary diatomic value $7/5$.

Step 3: Identify the gas and eliminate the alternatives Helium is monatomic and therefore has the required ratio $5/3$. Oxygen is diatomic, while ammonia and carbon dioxide are polyatomic; their rotational degrees of freedom raise C_V and lower γ below the monatomic value under the usual ideal-gas heat-capacity model. Substituting helium back gives $T_f/T_i = 8^{-2/3} = 1/4$, exactly the stated change.

Step 4: Conclusion The gas is helium, He, with $\gamma = 5/3$. Hence, the correct option is (C).

Quick Tip to Solve: For a reversible adiabatic change, determine $\gamma - 1 = \log(T_i/T_f)/\log(V_f/V_i)$ before identifying the gas.

32. A meter bridge with two resistances R_1 and R_2 as shown in figure was balanced (null point) at 40 cm from the point P. The null point changed to 50 cm from the point P, when 16Ω resistance is connected in parallel to R_2. The values of resistances R_1 and R_2 are ____ . [4 marks]



(A) $R_2 = 4\,\Omega,\ R_1 = \frac{4}{3}\,\Omega$

(B) $R_2 = 8\,\Omega,\ R_1 = \frac{16}{3}\,\Omega$

(C) $R_2 = 12\,\Omega,\ R_1 = \frac{12}{3}\,\Omega$

(D) $R_2 = 16\,\Omega,\ R_1 = \frac{16}{3}\,\Omega$

Correct Answer: (B) $R_2 = 8\,\Omega,\ R_1 = \frac{16}{3}\,\Omega$

Solution:

Concept:

- A balanced metre bridge is a Wheatstone bridge. With R_1 in the left gap and R_2 in the right gap, $R_1/R_2 = l/(100 - l)$, where l is measured from the left end in centimetres.
- When a resistance r is added in parallel with R_2, the effective right-gap resistance becomes $R_2 r/(R_2 + r)$. The left-gap resistance remains unchanged.

Step 1: Use the first balance point The null point is initially 40 cm from P, so the right-hand length is $100 - 40 = 60$ cm. Uniformity of the bridge wire makes its resistance proportional to length. Hence

$$\frac{R_1}{R_2} = \frac{40}{60} = \frac{2}{3}, \qquad R_1 = \frac{2}{3}R_2.$$

Do not interchange the ratio: the figure places R_1 on the same side as the length measured from P.

Step 2: Use the new balance condition Connecting $16\,\Omega$ in parallel changes only the right gap to

$$R_2' = \frac{16R_2}{16 + R_2}.$$

The new balance at 50 cm divides the wire into equal resistances, so $R_1 = R_2'$. Combining both equations and cancelling the nonzero R_2,

$$\frac{2}{3} = \frac{16}{16 + R_2} \quad \Longrightarrow \quad 32 + 2R_2 = 48.$$

Therefore $R_2 = 8\,\Omega$ and $R_1 = 16/3\,\Omega$.

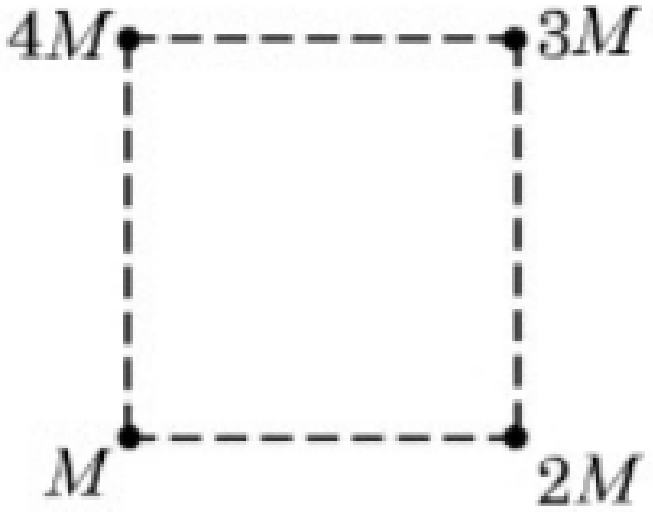
Step 3: Check both balances and the options The original ratio is $(16/3)/8 = 2/3$, giving the required 40 cm balance. The new right-gap resistance is $(8 \times 16)/(8 + 16) = 16/3\,\Omega$, equal to the left gap and giving 50 cm. Each other printed pair has $R_1/R_2 = 1/3$, so it already fails the first balance condition; no further adjustment can make it the stated original pair.

Step 4: Conclusion $R_1 = \dfrac{16}{3}\,\Omega$ and $R_2 = 8\,\Omega$. Hence, the correct option is (B).

Quick Tip to Solve: A 50 cm null point means equal gap resistances. Combine that equality with the original wire-length ratio.

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33. Net gravitational force at the center of a square is found to be F_1 when four particles having mass $M, 2M, 3M$ and $4M$ are placed at the four corners of the square as shown in figure and it is F_2 when the positions of $3M$ and $4M$ are interchanged. The ratio $\frac{F_1}{F_2}$ is $\frac{\alpha}{\sqrt{5}}$. The value of α is ____ . [4 marks]



(A) 1

(B) 2

(C) $2\sqrt{5}$

(D) 3

Correct Answer: (B) 2

Solution:

Concept:

- Gravitational forces add as vectors. All four corners of a square are equally far from its centre, so each force magnitude is proportional to the mass at that corner.
- Choose horizontal and vertical axes through the centre. Each corner force makes 45° with either axis; its components have equal magnitudes, with signs set by the corner.

Step 1: Set a common component scale Let the test mass at the centre be m_0, and let each corner be at distance r. Define $K = GMm_0/(\sqrt{2}r^2)$, the magnitude of either component of the attraction

due to mass M. Thus mass nM contributes components $\pm nK$. Take right and up as positive. The common factors, including the unspecified test mass, will cancel from the requested ratio.

Step 2: Calculate the original resultant Initially the bottom-left, bottom-right, top-right and top-left masses are $M, 2M, 3M, 4M$, respectively. Therefore

$$F_{1x} = K(-1+2+3-4) = 0, \qquad F_{1y} = K(-1-2+3+4) = 4K.$$

The horizontal forces cancel, while the vertical contributions give $F_1 = 4K$. Adding magnitudes without directions would miss this cancellation.

Step 3: Interchange the top masses and form the ratio After interchanging $3M$ and $4M$, the right-top mass is $4M$ and the left-top mass is $3M$. Hence

$$F_{2x} = K(-1+2+4-3) = 2K, \qquad F_{2y} = K(-1-2+4+3) = 4K.$$

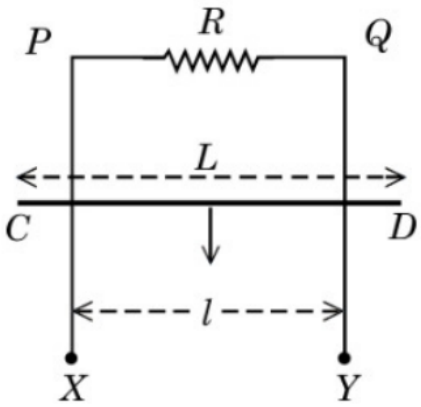
$$F_2 = \sqrt{(2K)^2 + (4K)^2} = 2\sqrt{5}K, \qquad \frac{F_1}{F_2} = \frac{2}{\sqrt{5}}.$$

Comparing with $\alpha/\sqrt{5}$ gives $\alpha = 2$. Values 1 and 3 give incorrect component ratios, while $2\sqrt{5}$ confuses the coefficient with a different ratio.

Step 4: Conclusion $\alpha = 2$. Hence, the correct option is (B).

Quick Tip to Solve: For equal centre-to-corner distances, add signed mass coefficients in each direction; the gravitational constant and distance cancel.

34. **$XPQY$ is a vertical smooth long loop having a total resistance R where PX is parallel to QY and separation between them is l. A constant magnetic field B perpendicular to the plane of the loop exists in the entire space. A rod CD of length $L(L > l)$ and mass m is made to slide down from rest under the gravity as shown in figure. The terminal speed acquired by the rod is ____ $\mathrm{m/s}$.(g = acceleration due to gravity)** [4 marks]



(A) $\frac{mgR}{B^2l^2}$
(B) $\frac{2mgR}{B^2l^2}$
(C) $\frac{2mgR}{B^2L^2}$
(D) $\frac{8mgR}{B^2l^2}$

Correct Answer: (A) $\frac{mgR}{B^2l^2}$

Solution:

Concept:

- A conductor moving perpendicular to a magnetic field develops a motional emf $\mathcal{E} = Blv$ across the length l belonging to the closed circuit.
- The induced current produces a force opposing the motion by Lenz's law. At terminal speed the upward magnetic force equals the downward weight, so the acceleration is zero.

Step 1: Identify the active length of the rod The rod has total length L, but its contacts with the two rails are separated by l. Only the segment between those contacts completes the conducting loop. The overhanging ends carry no steady circuit current. As the rod falls a distance $v\,dt$, the loop area changes by $lv\,dt$; Faraday's law therefore gives

$$|\mathcal{E}| = B\frac{dA}{dt} = Blv.$$

The length relevant to the circuit is l, not the full rod length L.

Step 2: Calculate the magnetic braking force Using the loop resistance R, the current magnitude is

$$I = \frac{Blv}{R}.$$

The current-carrying rod segment is perpendicular to the magnetic field, so its magnetic force is

$$F_B = BIl = \frac{B^2l^2v}{R}.$$

This force acts upward, opposing the downward movement that causes the induced current. The smooth rails exert no frictional braking force.

Step 3: Apply the terminal-speed condition The equation of motion, taking downward as positive, is

$$m\frac{dv}{dt} = mg - \frac{B^2l^2v}{R}.$$

At terminal speed $dv/dt = 0$, and hence

$$v_t = \frac{mgR}{B^2l^2}.$$

A larger resistance weakens the induced current and allows a larger terminal speed, while a stronger magnetic field reduces it. Options containing factors 2 or 8 do not satisfy force balance; the expression containing L additionally uses the wrong current-carrying length.

Step 4: Conclusion The terminal speed is $v_t = \frac{mgR}{B^2l^2}\,\mathrm{m\,s^{-1}}$ when the quantities are in SI units. Hence, the correct option is (A).

Quick Tip to Solve: Use the rail separation for both the induced emf and magnetic force; overhanging rod ends do not carry the loop current.

35. The escape velocity from a spherical planet A is $10\ \mathrm{km/s}$. The escape velocity from another planet B whose density and radius are 10% of those of planet A, is ____ $\mathrm{m/s}$. [4 marks]

(A) 1000

(B) $1000\sqrt{2}$

(C) $100\sqrt{10}$

(D) $200\sqrt{5}$

Correct Answer: (C) $100\sqrt{10}$

Solution:

Concept:

- Ignoring atmosphere and rotation, escape speed from the surface of a spherical planet is $v_e = \sqrt{2GM/R}$, found by setting the total mechanical energy to zero at infinity.
- For a spherical planet of average density ρ, $M = (4\pi/3)\rho R^3$. Substitution shows that escape speed scales as $R\sqrt{\rho}$, not simply as the square root of density.

Step 1: Express escape speed in terms of density Insert the mass of the spherical planet into the escape-speed formula:

$$v_e = \sqrt{\frac{2G}{R}\left(\frac{4\pi}{3}\rho R^3\right)} = R\sqrt{\frac{8\pi G\rho}{3}}.$$

This form is convenient because both radius and density ratios are specified. The proportionality constants cancel between the two planets, so neither absolute radius nor absolute mass is required.

Step 2: Substitute both ratios Planet B has $R_B/R_A = 0.1$ and $\rho_B/\rho_A = 0.1$. Consequently,

$$\frac{v_B}{v_A} = \frac{R_B}{R_A}\sqrt{\frac{\rho_B}{\rho_A}} = 0.1\sqrt{0.1} = \frac{1}{10\sqrt{10}}.$$

As a check, the mass ratio is $0.1(0.1)^3 = 10^{-4}$. Therefore $v_B/v_A = \sqrt{10^{-4}/10^{-1}} = \sqrt{10^{-3}}$, giving the same result.

Step 3: Convert the given speed and compare the options The initial escape speed is $10\,\mathrm{km\,s^{-1}} = 10^4\,\mathrm{m\,s^{-1}}$. Thus

$$v_B = \frac{10^4}{10\sqrt{10}} = \frac{1000}{\sqrt{10}} = 100\sqrt{10}\,\mathrm{m\,s^{-1}}.$$

Numerically this is about $316\,\mathrm{m\,s^{-1}}$. The choices 1000, $1000\sqrt{2}$, and $200\sqrt{5}$ do not equal this value. In particular, 1000 ignores the additional reduction caused by the lower density.

Step 4: Conclusion The escape speed is $100\sqrt{10}\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: Write $v_e \propto R\sqrt{\rho}$ when density and radius ratios are given, then convert kilometres per second to metres per second.

36. A thin convex lens of focal length 5 cm and a thin concave lens of focal length 4 cm are combined together (without any gap) and this combination has magnification m_1 when an object is placed 10 cm before the convex lens. Keeping the positions of convex lens and object undisturbed a gap of 1 cm is introduced between the lenses by moving the concave lens away, which lead to a change in magnification of total lens system to m_2. The value of $\left|\frac{m_1}{m_2}\right|$ is ____ . [4 marks]

(A) $\frac{5}{9}$

(B) $\frac{25}{27}$

(C) $\frac{5}{27}$

(D) $\frac{3}{2}$

Correct Answer: The stated lens data give 5/6, which is absent from the printed options.

Solution:

Concept:

- Use the Cartesian thin-lens formula $1/v - 1/u = 1/f$ and transverse magnification $m = v/u$. Convex and concave focal lengths are positive and negative, respectively.
- For lenses in contact their powers add. For separated lenses, the image formed by the first lens acts as the object for the second, and total magnification is the product of their individual magnifications.

Step 1: Calculate the contact-system magnification The focal lengths are $f_c = +5\,\text{cm}$ and $f_d = -4\,\text{cm}$. In contact,

$$\frac{1}{F} = \frac{1}{5} - \frac{1}{4} = -\frac{1}{20}, \qquad F = -20\,\text{cm}.$$

With $u = -10\,\text{cm}$,

$$\frac{1}{v} = \frac{1}{F} + \frac{1}{u} = -\frac{1}{20} - \frac{1}{10} = -\frac{3}{20}.$$

Thus $v = -20/3\,\text{cm}$ and $m_1 = v/u = 2/3$.

Step 2: Track the image through the separated lenses The convex lens and object remain fixed. For the convex lens alone,

$$\frac{1}{v_c} = \frac{1}{5} - \frac{1}{10} = \frac{1}{10}, \qquad v_c = 10\,\text{cm}, \qquad m_c = -1.$$

The concave lens is now 1 cm to its right, so the converging rays are directed toward a point 9 cm to the right of that lens. Its object is virtual: $u_d = +9\,\text{cm}$, not $-9\,\text{cm}$. Therefore

$$\frac{1}{v_d} = \frac{1}{u_d} + \frac{1}{f_d} = \frac{1}{9} - \frac{1}{4} = -\frac{5}{36}, \qquad v_d = -\frac{36}{5}\,\text{cm}.$$

Its magnification is $m_d = v_d/u_d = -4/5$, giving $m_2 = m_c m_d = 4/5$.

Step 3: Compare the derived ratio with the printed choices The required absolute ratio is

$$\left|\frac{m_1}{m_2}\right| = \frac{2/3}{4/5} = \frac{5}{6}.$$

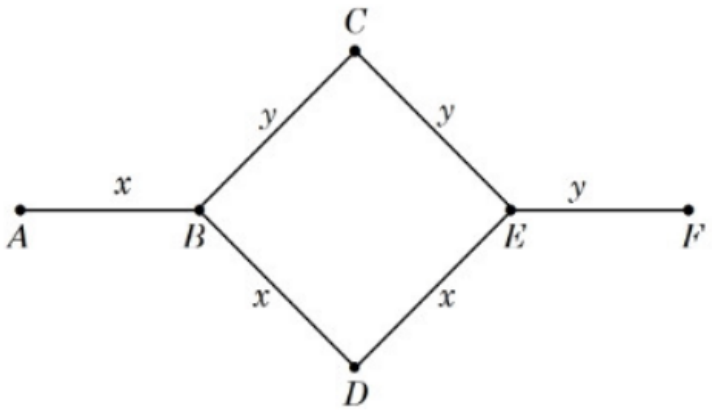
The printed choices are $5/9$, $25/27$, $5/27$, and $3/2$. None equals $5/6$; the first choice must not be

changed to a different denominator to force an answer. Both calculations use the stated focal lengths, unchanged object distance, and the specified 1 cm separation.

Step 4: Conclusion The stated data give $|m_1/m_2| = 5/6$. No printed option is correct.

Quick Tip to Solve: For the second lens, locate the first lens's image relative to the second lens. Converging incident rays correspond to a positive virtual-object distance.

37. Rods x and y of equal dimensions but of different materials are joined as shown in figure. Temperatures of end points A and F are maintained at 100°C and 40°C respectively. Given the thermal conductivity of rod x is three times of that of rod y, the temperature at junction points B and E are (close to): [4 marks]



(A) 80°C and 60°C respectively

(B) 60°C and 45°C respectively

(C) 89°C and 73°C respectively

(D) 80°C and 70°C respectively

Correct Answer: (C) 89°C and 73°C respectively

Solution:

Concept:

- The thermal resistance of a rod is $R_{\text{th}} = L/(kS)$, where L is its length, S its cross-sectional area and k its thermal conductivity.
- At steady state, rods in series carry the same heat current, while parallel branches have the same temperature difference. Heat current is $H = \Delta T/R_{\text{th}}$.

Step 1: Assign the rod resistances Let the resistance of each rod of material x be r. All rods have equal dimensions and $k_x = 3k_y$, so every y-rod has resistance $3r$. Thus AB has resistance r, the upper path BCE has $6r$, the lower path BDE has $2r$, and EF has $3r$. The two middle paths are parallel, not series with one another.

Step 2: Reduce the middle branches The equivalent resistance between B and E is

$$R_{BE} = \frac{(6r)(2r)}{6r + 2r} = \frac{3r}{2}.$$

The complete network therefore has

$$R_{AF} = r + \frac{3r}{2} + 3r = \frac{11r}{2}.$$

With the end temperatures held at 100°C and 40°C, the steady heat current is

$$H = \frac{100 - 40}{11r/2} = \frac{120}{11r}.$$

Step 3: Find the junction temperatures The temperature drop across AB is $Hr = 120/11$ degrees, giving

$$T_B = 100 - \frac{120}{11} = \frac{980}{11} \simeq 89.1^\circ\text{C}.$$

Across EF, the drop is $H(3r) = 360/11$ degrees, so

$$T_E = 40 + \frac{360}{11} = \frac{800}{11} \simeq 72.7^\circ\text{C}.$$

The pairs in (A), (B) and (D) fail these steady-state resistance drops; the higher-conductivity rod next to A produces the smaller end drop.

Step 4: Conclusion The junction temperatures are approximately $T_B = 89^\circ\text{C}$ and $T_E = 73^\circ\text{C}$. Hence, the correct option is (C).

Quick Tip to Solve: Replace each rod by $L/(kS)$, combine the two paths between the same junctions in parallel, then divide the total temperature drop in proportion to the series resistances.

38. Match the LIST-I with LIST-II.

List-I		List-II	
A.	**Spring constant**	**I.**	$ML^2T^{-2}K^{-1}$
B.	**Thermal conductivity**	**II.**	ML^0T^{-2}
C.	**Boltzmann constant**	**III.**	$ML^2T^{-3}A^{-2}$
D.	**Inductive reactance**	**IV.**	$MLT^{-3}K^{-1}$

Choose the correct answer from the options given below: [4 marks]

(A) A-I, B-IV, C-II, D-III

(B) A-III, B-II, C-IV, D-I

(C) A-II, B-IV, C-I, D-III

(D) A-II, B-I, C-IV, D-III

Correct Answer: (C) A-II, B-IV, C-I, D-III

Solution:

Concept:

- Dimensional formulae follow from defining physical equations. Here M, L, T, K, A denote the dimensions of mass, length, time, temperature and electric current.
- Products add powers and quotients subtract them. Dimensionless numerical constants, including 2π, do not affect a dimensional formula.

Step 1: Derive the spring and conductivity dimensions For a spring, Hooke's law gives $F = k_s x$, where x is extension. Therefore

$$[k_s] = \frac{[F]}{[x]} = \frac{MLT^{-2}}{L} = ML^0T^{-2}.$$

Hence A matches II. For heat conduction, $P = k_{\text{th}} S\Delta\theta/\ell$. Using $[P] = ML^2T^{-3}$, $[S] = L^2$ and $[\Delta\theta] = K$,

$$[k_{\text{th}}] = \frac{[P][\ell]}{[S][\Delta\theta]} = MLT^{-3}K^{-1}.$$

Hence B matches IV.

Step 2: Derive the Boltzmann and reactance dimensions Boltzmann's constant relates energy to absolute temperature, as in $E = k_B\theta$. Its dimensions are therefore

$$[k_B] = \frac{ML^2T^{-2}}{K} = ML^2T^{-2}K^{-1},$$

so C matches I. Inductive reactance has the same dimensions as resistance, since $X_L = V/I$. From $P = VI$, voltage has dimensions $ML^2T^{-3}A^{-1}$; dividing by current again gives

$$[X_L] = ML^2T^{-3}A^{-2}.$$

Hence D matches III.

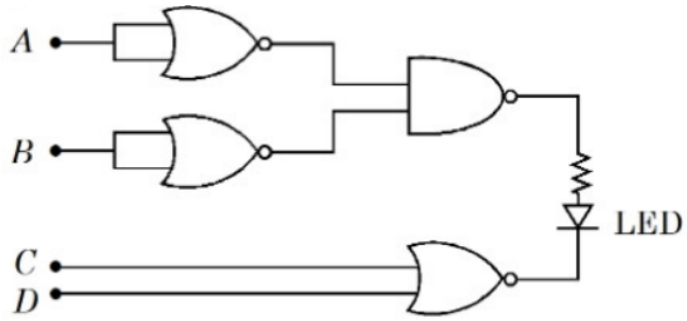
Step 3: Check the proposed matchings The complete matching is A-II, B-IV, C-I, D-III. Option (A) wrongly gives spring constant the dimensions of energy per temperature. Option (B) wrongly assigns

electrical resistance dimensions to spring constant. Option (D) interchanges thermal conductivity and Boltzmann constant: these differ in both the power of length and the power of time. Only (C) agrees with all four defining equations.

Step 4: Conclusion The matching is A-II, B-IV, C-I, D-III. Hence, the correct option is (C).

Quick Tip to Solve: For unfamiliar dimensions, start from the defining equation: $F = k_s x$, $P = k_{\text{th}} S\Delta\theta/\ell$, $E = k_B\theta$, and $V = IX_L$.

39. Find the correct combination of $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D inputs which can cause the LED to glow. [4 marks]



(A) 0011

(B) 1101

(C) 0100

(D) 1000

Correct Answer: (B) 1101

Solution:

Concept:

- A NOR gate produces the complement of the OR of its inputs. Joining both inputs of a NOR gate makes an inverter: $\overline{A + A} = \overline{A}$.
- A NAND output is the complement of an AND product. The LED conducts when its upper anode is at logic 1 and its lower cathode is at logic 0, giving forward bias through the resistor.

Step 1: Simplify the upper output The first two gates have their input terminals joined. Their outputs are therefore $\overline{A}$ and $\overline{B}$. These feed the upper NAND gate, whose output U is

$$U = \overline{\overline{A}\,\overline{B}} = A + B,$$

by De Morgan's law. Thus the top of the resistor-LED branch is high if at least one of A and B is 1. The small bubbles on the gate outputs are essential: omitting them would give a different circuit.

Step 2: Determine the lower output and bias condition The lower gate is a NOR gate with inputs C and D, so its output is

$$L = \overline{C + D}.$$

To light the LED, we require $U = 1$ and $L = 0$. Therefore at least one input in the pair (A, B) and at

least one input in the pair (C, D) must be 1. Equal output levels provide no forward voltage across the LED.

Step 3: Test all four input strings Read each string in the order $ABCD$. For 0011, $U = 0, L = 0$, so the LED is off. For 1101, $U = 1, L = 0$, so it is forward biased and glows. For 0100, $U = 1, L = 1$, so it is off. For 1000, $U = 1, L = 1$, so it is again off. Thus only the second string supplies a high potential above and a low potential below the LED.

Step 4: Conclusion The input combination is $ABCD = 1101$. Hence, the correct option is (B).

Quick Tip to Solve: Simplify tied-input gates first, then compare the potentials at both LED terminals; a high output alone does not guarantee that the LED glows.

---

40. Electric field in a region is given by $\vec{E} = Ax\hat{i} + By\hat{j}$, where $A = 10\ \mathrm{V/m^2}$ and $B = 5\ \mathrm{V/m^2}$. If the electric potential at a point $(10, 20)$ is 500 V , then the electric potential at origin is ____ V. [4 marks]

(A) 0

(B) 500

(C) 1000

(D) 2000

Correct Answer: (D) 2000

Solution:

Concept:

- Electrostatic field and potential satisfy $\vec{E} = -\nabla V$. Consequently, $V(P) - V(O) = -\int_O^P \vec{E} \cdot d\vec{r}$.
- Here $E_x = Ax$ and $E_y = By$ depend only on their own coordinates, so the potential separates into an x-term and a y-term plus a constant.

Step 1: Integrate the field with the correct sign Take the origin as $O = (0, 0)$, and let its unknown potential be V_0. Moving first along the x-axis and then parallel to the y-axis gives

$$V(x, y) - V_0 = -\int_0^x Ax\,dx - \int_0^y By\,dy.$$

The integrals yield

$$V(x, y) = V_0 - \frac{Ax^2}{2} - \frac{By^2}{2}.$$

The negative signs express that potential falls in the direction of the electric field. A direct derivative verifies both components of the given field.

Step 2: Use the specified reference point At $(10, 20)$, use $A = 10\,\mathrm{V/m^2}$, $B = 5\,\mathrm{V/m^2}$, and $V = 500\,\mathrm{V}$. The contributions are

$$\frac{Ax^2}{2} = \frac{10(10)^2}{2} = 500\,\mathrm{V},$$

$$\frac{By^2}{2} = \frac{5(20)^2}{2} = 1000\,\mathrm{V}.$$

Thus

$$500 = V_0 - 500 - 1000, \qquad V_0 = 2000\,\text{V}.$$

The units are consistent because each coefficient in V/m^2 multiplies a squared distance.

Step 3: Check the options and reference The origin lies at a potential $1500\,\text{V}$ above the specified point, so 0, 500 and 1000 V cannot satisfy the given condition. Substituting $V_0 = 2000\,\text{V}$ back gives exactly $500\,\text{V}$ at $(10, 20)$. Zero potential cannot be imposed at the origin independently, since the problem already specifies a potential reference.

Step 4: Conclusion The electric potential at the origin is $\boxed{2000\,\text{V}}$. In the printed option order, 2000 V is (D). Hence, the correct option is (D).

Quick Tip to Solve: Use $V(x, y) = V_0 - Ax^2/2 - By^2/2$; keep the integration constant until the supplied reference potential determines it.

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41. A simple pendulum has a bob with mass m and charge q. The pendulum string has negligible mass. When a uniform and horizontal electric field $\vec{E}$ is applied, the tension in the string changes. The final tension in the string, when pendulum attains an equilibrium position is ____.

(g: acceleration due to gravity) [4 marks]

(A) $mg + qE$

(B) $\sqrt{m^2g^2 + q^2E^2}$

(C) $mg - qE$

(D) $\sqrt{m^2g^2 - q^2E^2}$

Correct Answer: (B) $\sqrt{m^2g^2 + q^2E^2}$

Solution:

Concept:

- At equilibrium, the vector sum of the forces on the bob is zero. The gravitational force mg is vertical and the electric force $q\vec{E}$ is horizontal.
- Tension acts along the string toward the support. Its magnitude balances the resultant of the two mutually perpendicular external forces, not their scalar sum or difference.

Step 1: Resolve the equilibrium forces Let the string make an angle θ with the vertical in its final equilibrium position. The vertical component of tension supports the weight, while the horizontal component opposes the electric force. Therefore

$$T\cos\theta = mg, \qquad T\sin\theta = |q|E.$$

The absolute value in the second equation specifies the magnitude; reversing the sign of the charge reverses the side to which the pendulum deflects, but does not change the tension magnitude.

Step 2: Eliminate the equilibrium angle Square and add the two component equations:

$$T^2\cos^2\theta + T^2\sin^2\theta = m^2g^2 + q^2E^2.$$

Since $\sin^2\theta + \cos^2\theta = 1$,

$$T^2 = m^2g^2 + q^2E^2, \qquad T = \sqrt{m^2g^2 + q^2E^2}.$$

We choose the positive square root because tension is a force magnitude. Both terms under the root have dimensions of force squared. The equilibrium angle could also be found from $\tan\theta = |q|E/(mg)$, but it is unnecessary for the requested magnitude.

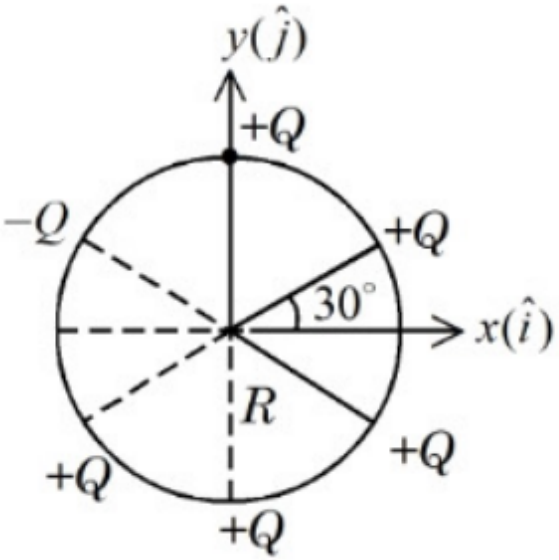
Step 3: Reject scalar addition and subtraction Options $mg + qE$ and $mg - qE$ would treat the two applied forces as collinear; here they are perpendicular. The expression with a negative sign under the square root would make tension smaller than mg, contradicting $T\cos\theta = mg$ for a tilted string. The derived result also correctly reduces to $T = mg$ when $E = 0$.

Step 4: Conclusion The final equilibrium tension is $\boxed{T = \sqrt{m^2g^2 + q^2E^2}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a horizontal electric force on a suspended charged bob, use perpendicular force components; tension is the hypotenuse of the force triangle.

42. Six point charges are kept 60° apart from each other on the circumference of a circle of radius R as shown in figure. The net electric field at the center of the circle is ____ .

(ϵ_0 is permittivity of free space) [4 marks]



(A) $-\frac{5Q}{8\pi\epsilon_0 R^2}(\hat{i} + \sqrt{3}\hat{j})$

(B) $-\frac{5Q}{8\pi\epsilon_0 R^2}(\hat{i} - 3\hat{j})$

(C) $-\frac{Q}{4\pi\epsilon_0 R^2}(\sqrt{3}\hat{i} - \hat{j})$

(D) $\frac{Q}{4\pi\epsilon_0 R^2}(\sqrt{3}\hat{i} - \hat{j})$

Correct Answer: (C) $-\frac{Q}{4\pi\epsilon_0 R^2}(\sqrt{3}\hat{i} - \hat{j})$

Solution:

Concept:

- The electric field at the centre due to a positive charge on the circle points away from that charge, while the field due to a negative charge points toward it.
- For six identical charges placed at equal angular intervals, fields at the centre cancel pairwise. A single changed charge can therefore be treated as a difference from this symmetric arrangement.

Step 1: Choose a symmetric reference arrangement In the figure, five positions carry $+Q$, while the upper-left position at 150° from the positive x-axis carries $-Q$. Imagine first replacing that charge by $+Q$. All six charges would then be equal, and each charge would have an equal diametrically opposite partner, making the net field at the centre zero. To restore the actual arrangement, add a charge $-2Q$ at the upper-left position.

Step 2: Resolve the remaining field Let $k = 1/(4\pi\epsilon_0)$. The field due to the added negative charge points from the centre toward that position, with magnitude $2kQ/R^2$. Its direction has components

$$\cos 150^\circ = -\frac{\sqrt{3}}{2}, \qquad \sin 150^\circ = \frac{1}{2}.$$

Consequently,

$$\vec{E} = \frac{2kQ}{R^2}\left(-\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right)$$

$$= -\frac{Q}{4\pi\epsilon_0 R^2}(\sqrt{3}\hat{i} - \hat{j}).$$

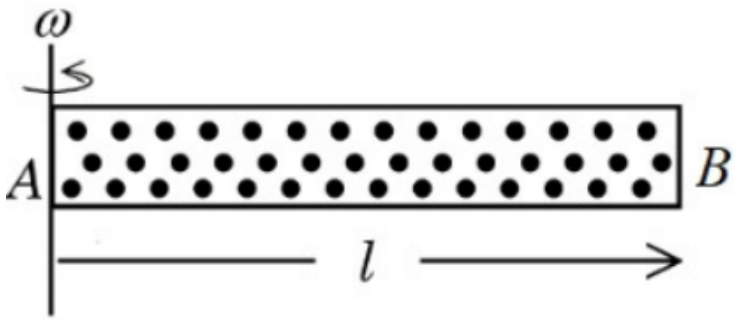
The negative horizontal component and positive vertical component agree with a field directed toward the upper-left negative charge.

Step 3: Check magnitude and alternatives The magnitude of the final vector is $2kQ/R^2$, since $\sqrt{3+1} = 2$. The option with the opposite sign points down and to the right, so it is wrong. The two expressions containing $5Q/(8\pi\epsilon_0 R^2)$ have incorrect component ratios or magnitudes. The symmetric-reference method accounts for all six charges without neglecting the five positive ones.

Step 4: Conclusion The net field is $\boxed{-\frac{Q}{4\pi\epsilon_0 R^2}(\sqrt{3}\hat{i} - \hat{j})}$. Hence, the correct option is (C).

Quick Tip to Solve: When one charge in an otherwise symmetric ring changes from $+Q$ to $-Q$, retain the zero-field reference and add only the field of $-2Q$ at that position.

43. A cylindrical tube AB of length l, closed at both ends contains an ideal gas of 1 mol having molecular weight M. The tube is rotated in a horizontal plane with constant angular velocity ω about an axis perpendicular to AB and passing through the edge at end A, as shown in the figure. If P_A and P_B are the pressures at A and B respectively, then (Consider the temperature is same at all points in the tube) [4 marks]



(A) $P_B = P_A$

(B) $P_B = P_A \exp\left(\frac{M\omega^2 l^2}{2RT}\right)$

(C) $P_B = P_A \exp\left(\frac{M\omega^2 l^2}{3RT}\right)$

(D) $P_B = P_A \exp\left(\frac{M\omega^2 l^2}{RT}\right)$

Correct Answer: (B) $P_B = P_A \exp\left(\frac{M\omega^2 l^2}{2RT}\right)$

Solution:

Concept:

- An element of gas rotating at radius r requires centripetal acceleration $\omega^2 r$. A pressure gradient supplies the inward resultant force.
- For an ideal gas at uniform temperature T, mass density is $\rho = MP/(RT)$, where M is the molar mass and R is the gas constant.

Step 1: Balance forces on a thin gas element Let the tube cross-sectional area be S. A slice of thickness dr at distance r from end A has mass $dm = \rho S\, dr$. Pressure must increase towards B, so the greater pressure on the outer face provides an inward force $S\, dP$. Thus

$$S\, dP = \rho S\, dr\, \omega^2 r, \qquad \frac{dP}{dr} = \rho\omega^2 r.$$

The horizontal rotation requires no gravitational pressure term along the tube.

Step 2: Integrate the isothermal pressure relation Substitute the ideal-gas density and separate variables:

$$\frac{dP}{P} = \frac{M\omega^2}{RT} r\, dr.$$

Since A lies on the rotation axis, its coordinate is zero; B has coordinate l. Therefore

$$\int_{P_A}^{P_B} \frac{dP}{P} = \frac{M\omega^2}{RT} \int_0^l r\, dr = \frac{M\omega^2 l^2}{2RT}.$$

Exponentiating gives the required ratio. The amount of gas fixes the absolute pressures but cancels from this ratio.

Step 3: Check the alternatives For nonzero angular speed and length, the exponent is positive, so $P_B > P_A$; option A cannot apply. The factor $1/2$ arises from the integral of r. Options C and D replace that factor by $1/3$ and 1, respectively, and therefore give the wrong pressure gradient.

Step 4: Conclusion

$$\boxed{P_B = P_A \exp\left(\frac{M\omega^2 l^2}{2RT}\right)}.$$

Hence, the correct option is (B).

Quick Tip to Solve: Use a differential pressure balance for a rotating gas: its density varies with pressure, so a constant-density pressure difference cannot be used.

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44. A solid sphere of mass 5 kg and radius 10 cm is kept in contact with another solid sphere of mass 10 kg and radius 20 cm . The moment of inertia of this pair of spheres about the tangent passing through the point of contact is ____ $\mathrm{kg \cdot m^2}$. [4 marks]

(A) 0.63

(B) 0.18

(C) 0.36

(D) 0.72

Correct Answer: (A) 0.63

Solution:

Concept:

- A uniform solid sphere has moment of inertia $I_{\mathrm{cm}} = 2mR^2/5$ about any diameter.
- The parallel-axis theorem gives $I = I_{\mathrm{cm}} + md^2$. A tangent at the contact point is a distance R from the centre of each sphere.

Step 1: Identify the distance from each centre to the axis The two spheres touch externally. Their centres and the contact point are collinear, and the common tangent axis through that point is perpendicular to this line. Its perpendicular distances from the two centres are therefore their individual radii, not the sum of their radii. Convert the dimensions before squaring them:

$$R_1 = 10\,\mathrm{cm} = 0.10\,\mathrm{m}, \qquad R_2 = 20\,\mathrm{cm} = 0.20\,\mathrm{m}.$$

Step 2: Apply the parallel-axis theorem to each sphere For either sphere the tangent-axis inertia is

$$I_{\tan} = \frac{2}{5}mR^2 + mR^2 = \frac{7}{5}mR^2.$$

Consequently,

$$I_1 = \frac{7}{5}(5)(0.10)^2 = 0.07\,\text{kg m}^2, \qquad I_2 = \frac{7}{5}(10)(0.20)^2 = 0.56\,\text{kg m}^2.$$

Both contributions are positive and refer to the same physical axis, so they must be added.

Step 3: Add the contributions and compare options

$$I_{\text{pair}} = I_1 + I_2 = 0.07 + 0.56 = 0.63\,\text{kg m}^2.$$

The value $0.18\,\text{kg m}^2$ in option B is merely the sum of the two centre-axis inertias; it omits both axis shifts. Options C and D also differ from the tangent-axis sum and do not satisfy the parallel-axis theorem.

Step 4: Conclusion The moment of inertia is $\boxed{0.63\,\text{kg m}^2}$. Hence, the correct option is (A).

Quick Tip to Solve: For a solid sphere about a tangent, combine the diameter inertia and the axis shift first: $I_{\tan} = 7mR^2/5$.

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45. The minimum frequency of photon required to break a particle of mass 15.348 amu into 4α particles is ____ kHz .

[mass of He nucleus = $4.002\text{amu}, 1\text{amu} = 1.66 \times 10^{-27}$ kg, $\text{h} = 6.6 \times 10^{-34}$ J.s **and** $\text{c} = 3 \times 10^8$ m/s **]** [4 marks]

(A) 9×10^{20}

(B) 14.94×10^{19}

(C) 14.94×10^{20}

(D) 9×10^{19}

Correct Answer: (B) 14.94×10^{19}

Solution:

Concept:

- The minimum energy needed for breakup is the increase in total rest energy, $E_{\min} = (m_{\text{products}} - m_{\text{initial}})c^2$.
- A photon has energy $E = h\nu$. Use the supplied atomic-mass conversion and report frequency in kHz, with $1\,\text{kHz} = 10^3\,\text{Hz}$.

Step 1: Calculate the required increase in mass Each alpha particle is a helium nucleus of mass $4.002\,\text{amu}$, so the four products have total mass

$$m_{\text{products}} = 4(4.002) = 16.008\,\text{amu}.$$

The products are heavier than the original particle. Energy must therefore be supplied, with

$$\Delta m = 16.008 - 15.348 = 0.660\,\text{amu} = 0.660(1.66 \times 10^{-27})\,\text{kg} = 1.0956 \times 10^{-27}\,\text{kg}.$$

Step 2: Convert the mass increase to photon energy At the breakup threshold in the mass-defect treatment, the supplied energy accounts for this rest-mass increase:

$$E_{\min} = \Delta m c^2 = (1.0956 \times 10^{-27})(3 \times 10^8)^2 = 9.8604 \times 10^{-11}\ \text{J}.$$

Use the given value of Planck's constant rather than substituting a more precise one, since the options reflect the stated constants.

Step 3: Calculate frequency and convert its unit

$$\nu_{\min} = \frac{9.8604 \times 10^{-11}}{6.6 \times 10^{-34}} = 1.494 \times 10^{23}\ \text{Hz}.$$

Dividing by 10^3 converts Hz to kHz:

$$\nu_{\min} = 1.494 \times 10^{20}\ \text{kHz} = 14.94 \times 10^{19}\ \text{kHz}.$$

Option C is ten times this result. Options A and D have a different coefficient and do not equal the calculated energy divided by h.

Step 4: Conclusion $\boxed{\nu_{\min} = 14.94 \times 10^{19}\ \text{kHz}}$. Hence, the correct option is (B).

Quick Tip to Solve: First subtract the initial mass from the total product mass; only after evaluating $\Delta m c^2/h$ convert Hz to the requested kHz.

Physics – Section B: Numerical Value Questions (20 marks)

46. A circular disc has radius R_1 and thickness T_1. Another circular disc made of the same material has radius R_2 and thickness T_2. If the moment of inertia of both discs are same and $\frac{R_1}{R_2} = 2$ then $\frac{T_1}{T_2} = \frac{1}{\alpha}$. The value of α is ____ . [4 marks]

Solution:

Concept:

- For a uniform circular disc about its central symmetry axis, $I = \frac{1}{2}mR^2$.
- Discs made from the same material have the same mass density ρ, and their masses are $m = \rho\pi R^2 T$, where T is thickness.

Step 1: Express inertia in terms of radius and thickness The disc volume is its circular face area multiplied by its thickness:

$$V = \pi R^2 T, \qquad m = \rho\pi R^2 T.$$

Substitute this mass into the central-axis formula:

$$I = \frac{1}{2}(\rho\pi R^2 T)R^2 = \frac{\rho\pi}{2}R^4 T.$$

The fourth power occurs because radius contributes both to mass through the area and to the squared distance from the rotation axis.

Step 2: Use equality of the two moments of inertia For the two discs,

$$\frac{\rho\pi}{2}R_1^4T_1 = \frac{\rho\pi}{2}R_2^4T_2.$$

Cancel the common density and numerical factors. Rearrangement then gives

$$\frac{T_1}{T_2} = \left(\frac{R_2}{R_1}\right)^4.$$

One must not cancel the masses: equal material does not mean equal mass when radii and thicknesses differ.

Step 3: Substitute the radius ratio and identify alpha The given ratio $R_1/R_2 = 2$ yields

$$\frac{T_1}{T_2} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}.$$

Compare this directly with the specified form $T_1/T_2 = 1/\alpha$. Thus the denominator is 16, not $1/16$. Both ratios and the requested parameter are dimensionless. The larger-radius disc must be thinner, consistent with the calculated ratio.

Step 4: Conclusion

$$\boxed{\alpha = 16}.$$

The parameter is dimensionless.

Quick Tip to Solve: For equal-density discs about their symmetry axes, remember $I \propto R^4T$; doubling the radius requires dividing thickness by 16.

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47. Inductance of a coil with 10^4 turns is 10 mH and it is connected to a dc source of 10 V with internal resistance of 10Ω. The energy density in the inductor when the current reaches $\left(\frac{1}{e}\right)$ of its maximum value is $\alpha\pi \times \frac{1}{e^2}$ J/m^3. The value of α is

____ .

$$\left(\mu_0 = 4\pi \times 10^{-7}\text{Tm/A}\right).$$

[4 marks]

Solution:

Concept:

- In a dc RL circuit the steady current is $I_{\max} = V/r$ when the stated internal resistance is the circuit resistance; stored magnetic energy is $U = LI^2/2$.
- For a uniform air-core solenoid, energy density is $u = B^2/(2\mu_0) = \mu_0(N/\ell)^2I^2/2$, where N is total turns and ℓ is coil length. Total turns and turns per metre are different quantities.

Step 1: Calculate the stated current and total stored energy The maximum current and the requested

fraction are

$$I_{\max} = \frac{10\,\text{V}}{10\,\Omega} = 1\,\text{A}, \qquad I = \frac{1}{e}\,\text{A}.$$

Since $L = 10\,\text{mH} = 10^{-2}\,\text{H}$, the total magnetic energy is determined:

$$U = \frac{1}{2}LI^2 = \frac{5 \times 10^{-3}}{e^2}\,\text{J}.$$

Energy density, however, requires this energy divided by a magnetic-field volume, which is not given.

Step 2: Test whether the number of turns determines the density Even with the standard uniform-solenoid model, substituting $N = 10^4$ gives

$$u = \frac{\mu_0 N^2 I^2}{2\ell^2} = \frac{20\pi}{e^2}\left(\frac{1\,\text{m}}{\ell}\right)^2 \text{J m}^{-3}.$$

Thus comparison with the requested expression gives $\alpha = 20(1\,\text{m}/\ell)^2$. The supplied inductance does not fix ℓ: $L = \mu_0 N^2 A/\ell$ fixes only A/ℓ, leaving different lengths and cross-sectional areas possible.

Step 3: Identify the condition behind the numerical value 20 If the intended winding density were $n = 10^4\,\text{m}^{-1}$, then

$$u = \frac{1}{2}(4\pi \times 10^{-7})(10^4)^2\frac{1}{e^2} = \frac{20\pi}{e^2}\,\text{J m}^{-3},$$

and $\alpha = 20$. This also follows from $N = 10^4$ with the additional condition $\ell = 1\,\text{m}$. Neither a winding density nor that length is specified in the printed question.

Step 4: Conclusion The printed total turn count does not determine a unique energy density or α. The numerical value 20 requires 10^4 turns per metre (or a 1 m coil length), an additional condition absent from the question. For a uniform solenoid, $\alpha = 20(1\,\text{m}/\ell)^2$.

Quick Tip to Solve: Use $LI^2/2$ for total magnetic energy. For energy per unit volume, check that a field volume or the turn density N/ℓ is actually specified.

48. A parallel beam of light travelling in air (refractive index 1.0) is incident on a convex spherical glass surface of radius of curvature 50 cm . Refractive index of glass is 1.5 . The rays converge to a point at a distance x cm from the centre of the curvature of the spherical surface. The value of x is ____ cm . [4 marks]

Solution:

Concept:

- For refraction from refractive index n_1 to n_2 at a spherical surface, the Cartesian-sign formula is $n_2/v - n_1/u = (n_2 - n_1)/R$.
- Distances u, v and R are measured from the surface pole. The final distance from the centre of curvature must be found separately.

Step 1: Assign the signs and use the parallel incident beam Take the direction of incident light into

the glass as positive. The convex surface has its centre of curvature inside the glass, so

$$R = +50\,\text{cm}, \qquad n_1 = 1.0, \qquad n_2 = 1.5.$$

A parallel incident beam corresponds to an object at infinity on the air side: $u \to -\infty$, hence $n_1/u = 0$. The rays refract towards the normals and converge within the glass.

Step 2: Find the convergence distance from the surface Substitute the values into the spherical-refraction formula:

$$\frac{1.5}{v} - 0 = \frac{1.5 - 1.0}{50\,\text{cm}}.$$

Therefore

$$v = \frac{1.5(50\,\text{cm})}{0.5} = 150\,\text{cm}.$$

The positive image distance means the focus lies in the glass, on the same side of the pole as the centre of curvature. No second refracting surface is specified, so a thin-lens formula is inappropriate.

Step 3: Convert to distance from the centre of curvature The centre is $50\,\text{cm}$ from the pole and the convergence point is $150\,\text{cm}$ from the pole. They lie on the same directed axis, with the focus beyond the centre. Their separation is therefore

$$x = v - R = 150 - 50 = 100\,\text{cm}.$$

Adding these distances would place the centre on the wrong side of the surface; reporting 150 would answer the distance from the pole rather than the one requested.

Step 4: Conclusion The convergence point is $\boxed{x = 100\,\text{cm}}$ from the centre of curvature. The requested numerical entry is 100.

Quick Tip to Solve: In a spherical-surface problem, first find the image distance from the pole; then subtract the radius if the question asks for distance from the centre.

49. The electric field of a plane electromagnetic wave, travelling in an unknown nonmagnetic medium is given by,

$$E_{\text{y}} = 20\sin\left(3 \times 10^6 x - 4.5 \times 10^{14}\text{t}\right)\,\text{V/m}$$

(where x, t and other values have S.I. units). The dielectric constant of the medium is ____
(speed of light in free space is 3×10^8 m/s) [4 marks]

Solution:

Concept:

- A travelling sinusoidal wave has phase $kx - \omega t$, where k is the angular wave number and ω is the angular frequency. Its phase speed is $v = \omega/k$.
- For an electromagnetic wave, $v = c/\sqrt{\mu_r \varepsilon_r}$. A nonmagnetic medium has relative permeability $\mu_r = 1$, so its dielectric constant is $K = \varepsilon_r = (c/v)^2$.

Step 1: Read the wave parameters Compare the given field with $E_y = E_0 \sin(kx - \omega t)$. The coefficient of position is the angular wave number and the coefficient of time is the angular frequency:

$$k = 3 \times 10^6 \text{ rad m}^{-1}, \qquad \omega = 4.5 \times 10^{14} \text{ rad s}^{-1}.$$

The field amplitude is 20 V m^{-1}; it determines the field strength but does not enter the speed calculation. Both phase coefficients already contain the angular factor, so no extra 2π is required.

Step 2: Calculate the phase speed At a fixed phase, $kx - \omega t$ is constant. Differentiating gives $k\,dx/dt = \omega$, and therefore

$$v = \frac{4.5 \times 10^{14}}{3 \times 10^6} = 1.5 \times 10^8 \text{ m s}^{-1}.$$

This is one-half of the speed of light in vacuum. The minus sign before the time term indicates propagation along positive x, without changing this speed.

Step 3: Determine the relative permittivity Use the nonmagnetic condition to separate the dielectric contribution from permeability:

$$K = \left(\frac{3 \times 10^8}{1.5 \times 10^8}\right)^2 = 2^2 = 4.$$

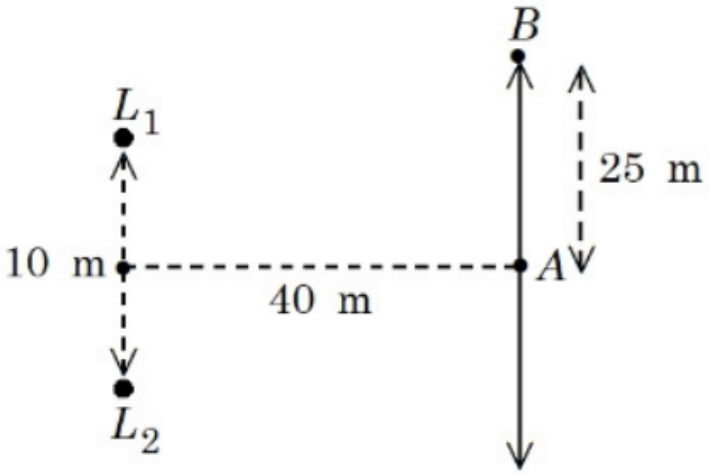
The refractive index is $n = c/v = 2$; the required dielectric constant is its square, not the refractive index itself.

Step 4: Conclusion The dielectric constant of the medium is $\boxed{4}$. It is dimensionless.

Quick Tip to Solve: First find ω/k, then square c/v for a nonmagnetic medium; do not confuse refractive index with dielectric constant.

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50. Two loudspeakers (L_1 and L_2) are placed with a separation of 10 m , as shown in figure. Both speakers are fed with an audio input signal of same frequency with constant volume. A voice recorder, initially at point A, at equidistance to both loud speakers, is moved by 25 m along the line AB while monitoring the audio signal. The measured signal was found to undergo 10 cycles of minima and maxima during the movement. The frequency of the input signal is ____ Hz (Speed of sound in air is 324 m/s and $\sqrt{5} = 2.23$) [4 marks]



Solution:

Concept:

- For coherent speakers, a full intensity cycle occurs whenever their path difference changes by one wavelength: $\Delta(\delta) = N\lambda$.
- The wavelength and frequency satisfy $v = f\lambda$. Path lengths must be calculated from the actual source positions, rather than identified with the recorder displacement.

Step 1: Calculate the distances at the endpoints Let the midpoint of the two speakers be the origin, with speakers at $(0, 5)$ and $(0, -5)$ metres. The diagram puts $A = (40, 0)$ and $B = (40, 25)$. Initially both distances equal $\sqrt{40^2 + 5^2}$, so the initial path difference is zero. At the final position, the vertical separations from the upper and lower speakers are 20 m and 30 m respectively. Thus

$$L_1B = \sqrt{40^2 + 20^2} = 20\sqrt{5}, \qquad L_2B = \sqrt{40^2 + 30^2} = 50 \text{ m}.$$

Step 2: Find the path-difference change Use the supplied approximation $\sqrt{5} = 2.23$, as required by the numerical data:

$$L_1B = 20(2.23) = 44.6 \text{ m}, \qquad \delta_B = 50 - 44.6 = 5.4 \text{ m}.$$

The path difference increases continuously from zero during the movement. The 25 m displacement of the recorder is not the path difference, because both source-to-recorder distances increase.

Step 3: Convert the fringe count to frequency Ten complete cycles of maxima and minima correspond to ten wavelengths of path-difference change. Therefore

$$10\lambda = 5.4 \text{ m} \quad \Rightarrow \quad \lambda = 0.54 \text{ m}.$$

With the given speed of sound,

$$f = \frac{v}{\lambda} = \frac{324}{0.54} = 600 \text{ Hz}.$$

A minimum and the next maximum are half a cycle apart; the stated ten cycles already count complete repetitions, so no extra factor of two is needed.

Step 4: Conclusion The frequency of the input signal is $\boxed{600 \text{ Hz}}$.

Quick Tip to Solve: Count full interference cycles using the change in path difference, and use the supplied approximation for $\sqrt{5}$ before evaluating the frequency.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. The correct order of reactivity of CH_3Br in methanol with the following nucleophiles is $F^-, I^-, C_2H_5O^-$ and $C_6H_5O^-$ [4 marks]

(A) $I^- > F^- > C_6H_5O^- > C_2H_5O^-$

(B) $I^- > C_6H_5O^- > F^- > C_2H_5O^-$

(C) $I^- > C_2H_5O^- > C_6H_5O^- > F^-$

(D) $I^- > C_2H_5O^- > F^- > C_6H_5O^-$

Correct Answer: (C) $I^- > C_2H_5O^- > C_6H_5O^- > F^-$

Solution:

Concept:

- Methyl bromide reacts with anionic nucleophiles mainly through an S_N2 displacement. For a fixed substrate and solvent, the relative rate reflects nucleophilicity.
- Methanol is a polar protic solvent. Hydrogen bonding strongly solvates small anions, while resonance delocalisation reduces the availability of a nucleophilic lone pair.

Step 1: Account for the protic solvent Methanol forms hydrogen bonds to anions. The small fluoride ion has a high charge density and becomes strongly surrounded by solvent molecules; this solvation shell hinders its attack on the carbon atom. Iodide is large and polarizable, and its charge is distributed over a larger volume. It is less tightly solvated and attacks readily. Hence I^- is much more nucleophilic than F^- in this solvent; gas-phase basicity alone would give a misleading comparison.

Step 2: Compare the oxygen nucleophiles In ethoxide, $C_2H_5O^-$, the negative charge is localised on oxygen and is available for bond formation. In phenoxide, $C_6H_5O^-$, the oxygen lone pair is delocalised into the benzene ring by resonance. This stabilises phenoxide but makes its electrons less available for nucleophilic attack. Ethoxide is therefore more reactive than phenoxide. Combining these effects gives

$$I^- > C_2H_5O^- > C_6H_5O^- > F^-.$$

Step 3: Check the alternatives Option (C) gives precisely this sequence. Option (A) wrongly places strongly solvated fluoride above both oxygen nucleophiles and puts phenoxide above ethoxide. Option (B) incorrectly places ethoxide last. Option (D) incorrectly puts fluoride above phenoxide. The observed order is a solvent-dependent nucleophilicity order, not a simple order of anion basic strengths.

Step 4: Conclusion The required order is $I^- > C_2H_5O^- > C_6H_5O^- > F^-$. Hence, the correct option is (C), in the printed option order.

Quick Tip to Solve: In a protic solvent, check hydrogen-bond solvation first, then resonance: fluoride is strongly solvated and phenoxide is resonance-stabilised.

52. Match the LIST-I with LIST-II.

List-I: Reagents	List-II: Name of Reaction involving carbonyl compounds
A. NH_2-NH_2, KOH	I. Tollen's Test
B. $Ag(NH_3)_2OH$	II. Clemmensen Reduction
C. Aq. $CuSO_4$, Sodium Potassium tartarate, KOH	III. Wolff–Kishner Reduction
D. Zn – Hg, HCl	IV. Fehling's Test

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-III, C-II, D-I

(B) A-III, B-I, C-IV, D-II

(C) A-II, B-I, C-IV, D-III

(D) A-III, B-IV, C-I, D-II

Correct Answer: (B) A-III, B-I, C-IV, D-II

Solution:

Concept:

- Carbonyl compounds can be reduced to methylene compounds by Wolff–Kishner conditions (basic) or Clemmensen conditions (acidic).
- Tollens and Fehling reagents are oxidising test reagents: aldehydes reduce silver(I) to silver metal or copper(II) to copper(I) oxide, respectively.

Step 1: Identify the reduction reagents Reagent A contains hydrazine, NH_2-NH_2, and KOH. Hydrazine forms a hydrazone with the carbonyl compound, which undergoes reduction on heating under basic conditions with evolution of nitrogen. This is Wolff–Kishner reduction, so A matches III. Reagent D contains zinc amalgam and hydrochloric acid, the acidic conditions used in Clemmensen reduction. Thus D matches II. Both methods replace the carbonyl oxygen by two hydrogens, but their reagent conditions are distinct.

Step 2: Identify the qualitative test reagents Reagent B is ammoniacal silver hydroxide, written as $Ag(NH_3)_2OH$ in the question. It is Tollens reagent, whose silver(I) complex is reduced to metallic silver in the silver-mirror test. Therefore B matches I. Reagent C contains aqueous $CuSO_4$, sodium potassium tartrate and KOH. The tartrate keeps copper(II) complexed in alkaline solution; this is Fehling reagent. An appropriate aldehyde produces a brick-red precipitate of Cu_2O, so C matches IV.

Step 3: Assemble and verify the matching The complete mapping is

$$\text{A-III, B-I, C-IV, D-II.}$$

Option (A) mismatches both reduction reagents and both tests. Option (C) interchanges the two reduction methods. Option (D) interchanges Tollens and Fehling tests. Only option (B) preserves all four reagent identities.

Step 4: Conclusion The matching is A-III, B-I, C-IV and D-II. Hence, the correct option is (B).

Quick Tip to Solve: Pair hydrazine/base with Wolff–Kishner and zinc amalgam/acid with Clemmensen; silver identifies Tollens, while alkaline copper/tartrate identifies Fehling.

53. As compared with chlorocyclohexane, which of the following statements correctly apply to chlorobenzene?

A. The magnitude of negative charge is more on chlorine atom.

B. The $C-Cl$ bond has partial double bond character.

C. $C-Cl$ bond is less polar.

D. $C-Cl$ bond is longer due to repulsion between delocalised electrons of the aromatic ring and lone pairs of electrons of chlorine.

E. The $C-Cl$ bond is formed using sp^2 hybridised orbital of carbon.

Choose the correct answer from the options given below : [4 marks]

(A) B, C and E Only

(B) A, D and E Only

(C) B, C and D Only

(D) A, C and E Only

Correct Answer: (A) B, C and E Only

Solution:

Concept:

- In chlorobenzene, a chlorine lone pair overlaps with the aromatic π system, giving the carbon–chlorine bond partial double bond character.
- The carbon bonded to chlorine in chlorobenzene is sp^2-hybridised, whereas that in chlorocyclohexane is sp^3-hybridised. Greater s character increases the electronegativity of the bonded carbon.

Step 1: Check the resonance and hybridisation statements Delocalisation of a chlorine lone pair into the benzene ring allows resonance contributors containing a carbon–chlorine double bond. The actual bond therefore has partial double bond character, confirming B. The ring carbon uses an sp^2 hybrid orbital for the sigma bond to chlorine, confirming E. Chlorocyclohexane instead has a saturated, tetrahedral carbon at this position and lacks the corresponding aromatic conjugation.

Step 2: Check the charge and polarity statements The sp^2 carbon of chlorobenzene is more electronegative than the sp^3 carbon of chlorocyclohexane. The carbon–chlorine electronegativity difference, and hence bond polarity, is smaller in chlorobenzene, so C is correct. Donation of the chlorine lone pair into the ring also works against accumulation of negative charge on chlorine. Statement A, which claims a greater magnitude of negative charge on chlorine, is therefore incorrect.

Step 3: Check the bond-length claim and select the set Partial double bond character and the higher s character of the carbon orbital make the carbon–chlorine bond shorter and stronger, rather than longer. The proposed repulsion explanation in D does not describe the actual bond-length trend, so D is incorrect. The true statements are B, C and E. Option (B) includes false A and D, option (C) includes false D, and option (D) includes false A.

Step 4: Conclusion Only statements B, C and E apply. Hence, the correct option is (A).

Quick Tip to Solve: For an aryl carbon–halogen bond, combine resonance with sp^2 carbon: expect partial double character, a shorter bond and reduced polarity.

54. The energy required by electrons, present in the first Bohr orbit of hydrogen atom to be excited to second Bohr orbit is ____ Jmol^{-1}.

Given: $R_H = 2.18 \times 10^{-11}\text{ergs}$**.** [4 marks]

(A) 1.635×10^{-11}

(B) 9.835×10^{5}

(C) 9.835×10^{12}

(D) 1.635×10^{-18}

Correct Answer: (B) 9.835×10^{5}

Solution:

Concept:

- For hydrogen, the energy of the nth orbit is $E_n = -R_H/n^2$, where the given R_H is the ground-state binding energy per atom.
- Excitation requires positive energy $\Delta E = E_2 - E_1$. Use $1\text{ erg} = 10^{-7}$ J, then multiply the single-atom energy by Avogadro's constant to obtain energy per mole.

Step 1: Find the energy difference per atom The first orbit has $E_1 = -R_H$, while the second has $E_2 = -R_H/4$. Consequently,

$$\Delta E = \left(-\frac{R_H}{4}\right) - (-R_H) = \frac{3}{4}R_H.$$

Inserting the given value,

$$\Delta E = \frac{3}{4}(2.18 \times 10^{-11}) = 1.635 \times 10^{-11}\text{ erg atom}^{-1}.$$

The positive sign is consistent with absorption: the excited electron has a less negative energy.

Step 2: Convert ergs into joules The supplied constant is in ergs, so its numerical value cannot be treated as joules. Using the conversion factor,

$$\Delta E = 1.635 \times 10^{-11} \times 10^{-7} = 1.635 \times 10^{-18}\text{ J atom}^{-1}.$$

This is the energy for one electron in one hydrogen atom, not for a mole of such electrons.

Step 3: Calculate the molar energy and compare options Multiply by $N_A \simeq 6.02 \times 10^{23}\text{ mol}^{-1}$:

$$\Delta E_{\text{mol}} = (1.635 \times 10^{-18})(6.02 \times 10^{23}) \simeq 9.84 \times 10^{5}\text{ J mol}^{-1}.$$

This corresponds to the listed 9.835×10^{5}. Option (A) retains the single-atom value in ergs. Option (D) gives the single-atom value in joules. Option (C) misses the erg-to-joule factor when converting to a molar quantity.

Step 4: Conclusion The required molar excitation energy is approximately $\boxed{9.835 \times 10^{5}\text{ J mol}^{-1}}$, using the precision of the listed choices. Hence, the correct option is (B), in the printed option order.

Quick Tip to Solve: Keep the two conversions separate: first ergs to joules, then per atom to per mole. Excitation from $n = 1$ to $n = 2$ uses three-quarters of the binding energy.

55. Consider the transition metal ions $Mn^{3+}, Cr^{3+}, Fe^{3+}$ and Co^{3+} and all form low spin octahedral complexes. The correct decreasing order of unpaired electrons in their respective d-orbitals of the complexes is [4 marks]

(A) $Cr^{3+} > Fe^{3+} > Co^{3+} > Mn^{3+}$

(B) $Mn^{3+} > Fe^{3+} > Co^{3+} > Cr^{3+}$

(C) $Fe^{3+} > Co^{3+} > Mn^{3+} > Cr^{3+}$

(D) $Cr^{3+} > Mn^{3+} > Fe^{3+} > Co^{3+}$

Correct Answer: (D) $Cr^{3+} > Mn^{3+} > Fe^{3+} > Co^{3+}$

Solution:

Concept:

- In an octahedral field the five d-orbitals split into three lower-energy t_{2g} orbitals and two higher-energy e_g orbitals. In a low-spin complex, pairing in t_{2g} is preferred to occupation of e_g.
- Find the d-electron count of each tripositive ion by removing the 4s electrons before the 3d electrons. Hund's rule still distributes electrons singly among degenerate orbitals before pairing them.

Step 1: Find the d-electron counts The neutral configurations are $Cr : 3d^5 4s^1$, $Mn : 3d^5 4s^2$, $Fe : 3d^6 4s^2$ and $Co : 3d^7 4s^2$. Removal of three electrons gives

$$Cr^{3+} : d^3, \quad Mn^{3+} : d^4, \quad Fe^{3+} : d^5, \quad Co^{3+} : d^6.$$

Step 2: Fill the low-spin orbitals The corresponding configurations are t_{2g}^3, t_{2g}^4, t_{2g}^5 and t_{2g}^6, with no e_g occupation. The three t_{2g} orbitals contain respectively $(1, 1, 1)$, $(2, 1, 1)$, $(2, 2, 1)$ and $(2, 2, 2)$ electrons. Therefore their numbers of unpaired electrons are 3, 2, 1 and 0.

Step 3: Compare the proposed orders The required order is consequently $Cr^{3+} > Mn^{3+} > Fe^{3+} > Co^{3+}$. Option (A) incorrectly places Mn^{3+} last. Options (B) and (C) put Cr^{3+}, which has three unpaired electrons, last. Only option (D) follows the counts.

Step 4: Conclusion The decreasing order is $\boxed{Cr^{3+} > Mn^{3+} > Fe^{3+} > Co^{3+}}$, corresponding to $3 > 2 > 1 > 0$ unpaired electrons. Hence, the correct option is (D).

Quick Tip to Solve: For low-spin octahedral d^3 to d^6, fill only the three t_{2g} orbitals: the unpaired counts decrease as 3, 2, 1, 0.

56. A first row transition metal (M) does not liberate H_2 gas from dilute HCl .1 mol of aqueous solution of MSO_4 is treated with excess of aqueous KCN and then $H_2S(g)$ is passed through the solution. The amount of MS (metal sulphide) formed from the above reaction is ____ mol. [4 marks]

(A) 3

(B) 2

(C) 0

(D) 1

Correct Answer: (C) 0

Solution:

Concept:

- A metal below hydrogen in the electrochemical series does not normally displace hydrogen from dilute non-oxidising HCl. Copper is the relevant first-row transition metal.
- Complex formation lowers the concentration of free metal ions. Copper forms a particularly stable soluble cyanide complex in excess cyanide, preventing its usual sulphide precipitation.

Step 1: Identify the metal Copper does not liberate H_2 from dilute HCl, whereas reactive first-row metals such as iron and zinc do. Thus $M = Cu$, and the initial sulphate is $CuSO_4$, containing copper(II) ions. The amount initially present does not itself establish how much sulphide will precipitate.

Step 2: Follow the effect of excess cyanide Cyanide reduces copper(II) to copper(I), with cyanogen formation, and then coordinates to copper(I). A representation of the reduction is

$$2\,Cu^{2+} + 4\,CN^- \longrightarrow 2\,CuCN + (CN)_2.$$

Excess cyanide dissolves the copper(I) cyanide through complex formation, commonly represented as

$$CuCN + 3\,CN^- \longrightarrow [Cu(CN)_4]^{3-}.$$

The relevant point is that copper remains bound in a stable cyanide complex.

Step 3: Decide whether sulphide precipitates Passing H_2S through this cyanide-containing solution does not precipitate copper sulphide under the stated qualitative-analysis conditions: cyanide masks copper by making its free-ion concentration too small. Hence none of the initial 1 mol of metal is obtained as the requested sulphide. Values 1, 2 and 3 mol all require precipitation that the complex prevents; 2 and 3 mol also exceed the initial amount of copper.

Step 4: Conclusion The amount of metal sulphide formed is $\boxed{0\ \text{mol}}$. Hence, the correct option is (C).

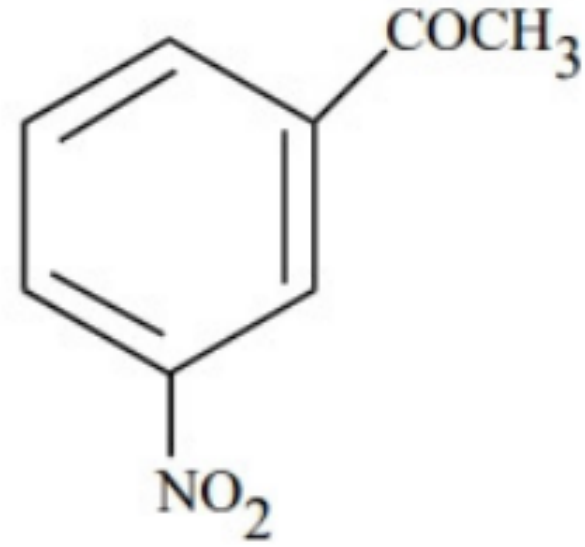
Quick Tip to Solve: Check complex formation before applying a sulphide precipitation rule: excess cyanide masks copper and prevents its usual sulphide test.

57. Given below are two statements:

Statement I: Benzene is nitrated to give nitrobenzene, which on further treatment with $CH_3COCl/AlCl_3$ will give

Statement II: $-NO_2$ group is a m-directing, and deactivating group.

In the light of the above statements, choose the most appropriate answer from the options given below [4 marks]



(A) Both Statement I and Statement II are correct

(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (D) Statement I is incorrect but Statement II is correct

Solution:

Concept:

- Nitration of benzene is an electrophilic aromatic substitution that yields nitrobenzene. A substituent can control both the reactivity of the ring and the preferred position of a later substitution.
- The $-NO_2$ group strongly withdraws electrons by inductive and resonance effects. Nitrobenzene is too strongly deactivated to undergo the usual Friedel–Crafts alkylation or acylation.

Step 1: Test the proposed Friedel–Crafts reaction The first part of Statement I is valid: nitration converts benzene into nitrobenzene. The next reagents, $CH_3COCl/AlCl_3$, are Friedel–Crafts acylation reagents. However, nitrobenzene does not undergo this reaction under the usual conditions because the nitro group greatly reduces ring electron density. Therefore the depicted meta-nitroacetophenone is not obtained by this sequence, and Statement I is incorrect.

Step 2: Check the directing effect of the nitro group The $-NO_2$ group withdraws electron density from the aromatic ring through both $-I$ and $-M$ effects, making electrophilic substitution slower than in benzene. For substitutions that do occur under suitable conditions, ortho and para attack have especially destabilised cationic resonance contributors; meta attack avoids those contributors. Thus

the group is both deactivating and meta-directing, so Statement II is correct.

Step 3: Select the matching truth combination Meta direction does not imply that every electrophilic substitution is feasible. This distinction eliminates option (A), which accepts the impossible acylation. Option (B) wrongly rejects the nitro group's directing effect, and option (C) reverses both truth values. Option (D) alone matches the two assessments.

Step 4: Conclusion Statement I is incorrect, while Statement II is correct. Hence, the correct option is (D).

Quick Tip to Solve: Before using an ortho/meta/para rule, ask whether the proposed reaction occurs at all: nitrobenzene fails the usual Friedel–Crafts reaction.

58. Given below are two statements :

Statement I : The Henry's law constant $\mathrm{K_H}$ is constant with respect to variations in solution's concentration over the range for which the solution is ideally dilute.

Statement II : $\mathrm{K_H}$ does not differ for the same solute in different solvents.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- At a fixed temperature, Henry's law in mole-fraction form is $p = K_H x$, where p is the gas partial pressure and x is its mole fraction in solution.
- The proportionality constant K_H characterises a particular solute–solvent pair at that temperature in the ideally dilute range. It is not a universal constant for the solute alone.

Step 1: Evaluate the concentration statement For an ideally dilute solution at fixed temperature, changing the dissolved gas concentration changes x, and the equilibrium partial pressure changes proportionally. Thus

$$\frac{p_1}{x_1} = \frac{p_2}{x_2} = K_H.$$

Within this range the same constant applies rather than varying with concentration. The restriction to ideally dilute solutions matters because appreciable non-ideality can cause deviations at higher concentrations. Statement I is therefore true.

Step 2: Evaluate the solvent statement Dissolution depends on the interactions between gas molecules and solvent molecules. Changing the solvent changes those interactions and generally changes the equilibrium solubility at the same temperature and gas pressure. Since $K_H = p/x$, a different equilibrium x at the same p implies a different K_H. In this convention, a gas that is more soluble has a smaller K_H. Thus Statement II is false.

Step 3: Compare the choices Option (A) incorrectly treats the constant as solvent-independent. Option (B) incorrectly rejects its concentration independence in the specified dilute range. Option (D) makes both incorrect judgements. The true/false combination required here is given only by option (C).

Step 4: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: For $p = K_H x$, hold temperature and the solute–solvent pair fixed before treating K_H as constant; changing the solvent changes solubility.

59. Two p-block elements X and Y form fluorides of the type EF_3. The fluoride compound XF_3 is a Lewis acid and YF_3 is a Lewis base. The hybridizations of the central atoms of XF_3 and YF_3 respectively are [4 marks]

(A) Both sp^2

(B) Both sp^3

(C) sp^3 and sp^2

(D) sp^2 and sp^3

Correct Answer: (D) sp^2 and sp^3

Solution:

Concept:

- A Lewis acid accepts an electron pair, whereas a Lewis base donates an available lone pair. Three bonds alone do not determine hybridisation; the central atom's lone pairs must also be counted.
- In the usual valence-bond description, three electron domains give sp^2 hybridisation, while four electron domains give sp^3 hybridisation.

Step 1: Describe the Lewis-acid fluoride The electron-deficient trihalide represented by BF_3 has three central-atom bond pairs and no central-atom lone pair. Boron has only six electrons around it and can accept a donor electron pair into its vacant orbital. Its three sigma bonds have trigonal-planar arrangement, giving

$$XF_3: \quad 3 \text{ bond pairs} + 0 \text{ lone pairs} \Rightarrow sp^2.$$

Step 2: Describe the Lewis-base fluoride A group-15 trifluoride, represented by NF_3, has three central-atom bond pairs and one lone pair. The lone pair supplies the electron pair required for Lewis-basic behaviour. Four electron domains give tetrahedral electron-pair geometry and a trigonal-pyramidal molecular shape, with

$$YF_3: \quad 3 \text{ bond pairs} + 1 \text{ lone pair} \Rightarrow sp^3.$$

Step 3: Preserve the order asked in the question The question asks for the Lewis acid first and the Lewis base second. Their hybridisations are therefore sp^2 followed by sp^3. Both-sp^2 omits the lone pair on Y, both-sp^3 incorrectly adds a fourth domain to X, and sp^3, sp^2 reverses the required assignments.

Step 4: Conclusion The central atoms of XF_3 and YF_3 are $\boxed{sp^2 \text{ and } sp^3}$, respectively. The acid has three electron domains and the base has four, so both cannot be sp^2 as the first listed choice states.

Hence, the correct option is (D).

Quick Tip to Solve: Count sigma bonds plus central-atom lone pairs: an electron-deficient trifluoride has three domains, whereas a lone-pair-donating trifluoride has four.

60. A 'p'-block element (E) and hydrogen form a binary cation $(EH_x)^+$, while EH_3 on treatment with K_2HgI_4 in alkaline medium gives a precipitate of basic mercury(II)amido-iodine. Given below are first ionisation enthalpy values $(kJmol^{-1})$ for first element each from group 13, 14, 15 and 16. Identify the correct first ionisation enthalpy value for element E. [4 marks]

(A) 801

(B) 1086

(C) 1402

(D) 1312

Correct Answer: (C) 1402

Solution:

Concept:

- Alkaline potassium tetraiodomercurate(II), $K_2[HgI_4]$, is Nessler's reagent. It detects ammonia by forming a coloured precipitate containing basic mercury(II) amido-iodide.
- The first ionisation enthalpy is the energy required for $E(g) \longrightarrow E^+(g) + e^-$. The first element of group 15 is nitrogen, whose half-filled $2p^3$ subshell is relatively stable.

Step 1: Identify the hydride from the diagnostic test The stated reagent and precipitate identify EH_3 as ammonia, NH_3. Thus the central element E is nitrogen. This chemical test is more specific than merely observing that an element forms a hydride with three hydrogen atoms; other p-block elements can also form trihydrides.

Step 2: Check the cation clue Ammonia accepts a proton through its nitrogen lone pair:

$$NH_3 + H^+ \longrightarrow NH_4{}^+.$$

Consequently the binary cation is $NH_4{}^+$, so $x = 4$, consistent with the first clue. Both independent descriptions therefore point to the same element.

Step 3: Select its first ionisation enthalpy For nitrogen, $1s^2 2s^2 2p^3$, the first ionisation enthalpy is approximately $1402\ kJ\,mol^{-1}$. The values 801 and $1086\ kJ\,mol^{-1}$ correspond to boron and carbon, respectively, so neither fits the ammonia test. The remaining value 1312 is not nitrogen's first ionisation enthalpy. The especially stable half-filled p-subshell also explains why nitrogen has a comparatively high value rather than simply following a smooth trend.

Step 4: Conclusion Element E is nitrogen, and its first ionisation enthalpy is $\boxed{1402\ kJ\,mol^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use Nessler's test to identify ammonia first, then read the ionisation enthalpy for nitrogen; the ammonium-ion clue provides an independent cross-check.

61. In the reaction,

$$2\mathrm{Al}(\mathrm{s}) + 6\mathrm{HCl}(\mathrm{aq}) \rightarrow 2\mathrm{Al}^{3+}(\mathrm{aq}) + 6\mathrm{Cl}^{-}(\mathrm{aq}) + 3\mathrm{H}_2(\mathrm{g})$$

[4 marks]

(A) $12\ \mathrm{L}\ \mathrm{HCl}(\mathrm{aq})$ is consumed for every $6\ \mathrm{L}\ \mathrm{H}_2(\mathrm{g})$ produced.

(B) $33.6\ \mathrm{L}\ \mathrm{H}_2(\mathrm{g})$ is produced regardless of temperature and pressure for every mole of Al that reacts.

(C) $67.2\ \mathrm{L}\ \mathrm{H}_2(\mathrm{g})$ at STP is produced for every mole of Al that reacts.

(D) $11.2\ \mathrm{L}\ \mathrm{H}_2(\mathrm{g})$ at STP is produced for every mole of HCl consumed.

Correct Answer: (D) $11.2\ \mathrm{L}\ \mathrm{H}_2(\mathrm{g})$ at STP is produced for every mole of HCl consumed.

Solution:

Concept:

- The coefficients of a balanced chemical equation give mole ratios, not volume ratios between an aqueous solution and a gas.
- At STP, the conventional molar volume used here is $22.4\ \mathrm{L\ mol^{-1}}$. At other conditions, gas volume follows $V = nRT/P$.

Step 1: Read the mole ratio The reaction consumes six moles of HCl and two moles of aluminium to produce three moles of $\mathrm{H_2}$. Hence

$$\frac{n(\mathrm{H_2})}{n(\mathrm{HCl})} = \frac{3}{6} = \frac{1}{2}, \qquad \frac{n(\mathrm{H_2})}{n(\mathrm{Al})} = \frac{3}{2}.$$

The aqueous state of hydrochloric acid matters: its solution volume also depends on its concentration, which is not specified.

Step 2: Convert the hydrogen amount at STP For each mole of acid consumed, the hydrogen amount is 0.5 mol. Its STP volume is

$$V = \frac{1}{2} \times 22.4 = 11.2\ \mathrm{L}.$$

For one mole of aluminium, the amount is 1.5 mol, giving

$$V = \frac{3}{2} \times 22.4 = 33.6\ \mathrm{L} \quad \text{at STP}.$$

Step 3: Reject the other statements Option (A) incorrectly treats the coefficients as volumes of aqueous acid and hydrogen. Option (B) gives the STP volume per mole of aluminium but incorrectly makes it independent of temperature and pressure. Option (C) assigns $67.2\ \mathrm{L}$ to one mole of aluminium, whereas that is the volume produced by two moles at STP.

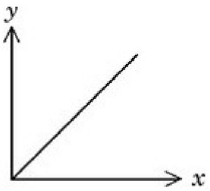
Step 4: Conclusion Each mole of HCl consumed produces $11.2\ \mathrm{L}$ of $\mathrm{H_2}$ at STP. Hence, the correct option is (D).

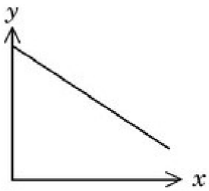
Quick Tip to Solve: First convert the balanced coefficients into moles of gas, then use the molar volume only at the stated temperature and pressure.

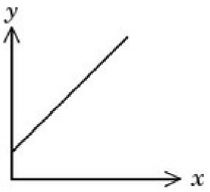
62. Consider a solution of $CO_2(g)$ dissolved in water in a closed container.

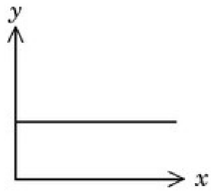
Which one of the following plots correctly represents variation of log (partial pressure of CO_2 in vapour phase above water) [y-axis] with log (mole fraction of CO_2 in water) [x-axis] at $25^\circ C$?

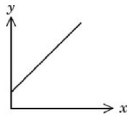
[4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- At a fixed temperature, Henry's law gives $p = K_H x_{CO_2}$, where p is the gas partial pressure, x_{CO_2} is its dissolved mole fraction, and K_H is constant.
- A log–log plot of $p = Kx^n$ is a straight line with slope n and intercept $\log K$, using consistent pressure units.

Step 1: Take logarithms of Henry's law Write the dissolved mole fraction as x_{CO_2}, to distinguish it from the plotted horizontal coordinate. Taking logarithms on both sides gives

$$\log p = \log K_H + \log x_{CO_2}.$$

Thus, if $Y = \log p$ and $X = \log x_{CO_2}$,

$$Y = X + \log K_H.$$

Keeping the temperature at $25^\circ C$ fixes the Henry constant and therefore the intercept.

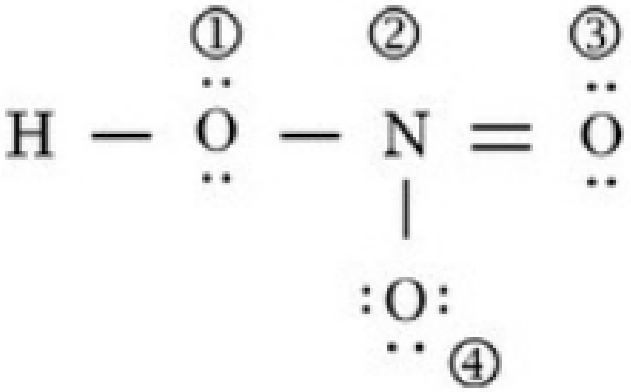
Step 2: Identify the slope and intercept The coefficient of X is $+1$, so the line must rise with increasing horizontal coordinate. Its intercept is $\log K_H$; it is not forced to pass through the origin. With the conventional pressure units for the Henry constant of carbon dioxide in water, the schematic graph has a positive intercept. Option (C) shows the required rising straight line with a nonzero intercept.

Step 3: Compare the remaining graphs Option (A) passes through the origin and would require the numerical value $K_H = 1$, which is not the Henry constant here. Option (B) has negative slope, contrary to $p \propto x_{CO_2}$. Option (D) is horizontal and would incorrectly make the partial pressure independent of the dissolved mole fraction.

Step 4: Conclusion The relation is $\log p = \log x_{CO_2} + \log K_H$, a line of slope $+1$ and nonzero intercept. Hence, the correct option is (C).

Quick Tip to Solve: Do not confuse a direct proportionality plot with its log–log plot: the multiplicative constant becomes an additive intercept.

63. The formal charges on the atoms marked as (1) to (4) in the Lewis representation of HNO_3 molecule respectively are [4 marks]



(A) $0, +1, 0, -1$

(B) $0, -1, 0, +1$

(C) $0, 0, -1, +1$

(D) $+1, 0, 0, -1$

Correct Answer: (A) $0, +1, 0, -1$

Solution:

Concept:

- Formal charge is $\mathrm{FC} = V - N_{\mathrm{nb}} - \frac{1}{2}N_{\mathrm{b}}$, where V is the valence-electron count, N_{nb} counts nonbonding electrons, and N_{b} counts bonding electrons.
- Equivalently, subtract the number of nonbonding electrons and the sum of bond orders from the valence-electron count. The total formal charge must equal the molecular charge.

Step 1: Evaluate the hydroxyl oxygen and nitrogen Atom (1) is the oxygen bonded to hydrogen and nitrogen. It has two lone pairs, hence four nonbonding electrons, and two single bonds:

$$\mathrm{FC}(1) = 6 - 4 - 2 = 0.$$

Atom (2) is nitrogen. It has no lone pair and has two single bonds plus one double bond, giving a bond-order sum of four:

$$\mathrm{FC}(2) = 5 - 0 - 4 = +1.$$

Step 2: Evaluate the remaining oxygen atoms Atom (3) is double-bonded oxygen. It has two lone pairs and a bond-order sum of two:

$$\mathrm{FC}(3) = 6 - 4 - 2 = 0.$$

Atom (4) is singly bonded oxygen with three lone pairs:

$$\mathrm{FC}(4) = 6 - 6 - 1 = -1.$$

The sum is $0 + 1 + 0 - 1 = 0$, as required for the neutral nitric acid molecule.

Step 3: Match the printed options The ordered sequence is $(0, +1, 0, -1)$. Option (B) reverses the signs on nitrogen and the singly bonded terminal oxygen. Option (C) wrongly charges the double-bonded oxygen and leaves nitrogen neutral. Option (D) wrongly gives a positive charge to the hydroxyl oxygen. Only the first printed sequence reproduces all four electron counts.

Step 4: Conclusion The formal charges on atoms (1), (2), (3), and (4) are $0, +1, 0, -1$, respectively. Hence, the correct option is (A).

Quick Tip to Solve: For oxygen, two bonds usually mean zero formal charge; one bond with three lone pairs means minus one. Always verify the total molecular charge.

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64. Given below are two statements :

Statement I : The halogen that makes longest bond with hydrogen in HX, has the smallest covalent radius in its group.

Statement II : A group 15 element's hydride EH_3 has the lowest boiling point among corresponding hydrides of other group 15 elements. The maximum covalency of that element E is 4 .

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (B) Both Statement I and Statement II are false

Solution:

Concept:

- A hydrogen–halogen bond generally becomes longer as the halogen covalent radius increases down the group.
- Hydride boiling points depend on intermolecular forces as well as mass. Hydrogen bonding makes NH_3 anomalous, so the lightest group-15 hydride need not have the lowest boiling point.

Step 1: Test the bond-length statement Among the usual hydrogen halides, the bond-length trend is

$$H{-}F < H{-}Cl < H{-}Br < H{-}I.$$

The longest bond is associated with the larger halogen atom, not the smallest one. Fluorine has the smallest covalent radius and forms the shortest hydrogen–halogen bond. Statement I therefore reverses the size trend and is false.

Step 2: Identify the lowest-boiling group-15 hydride Ammonia forms intermolecular hydrogen bonds, raising its boiling point above that of phosphine. Phosphine, PH_3, has the lowest boiling point among the corresponding group-15 hydrides. Downward from phosphine, increasing size and polarizability

strengthen dispersion forces; the approximate order is

$$PH_3 < AsH_3 < NH_3 < SbH_3 < BiH_3.$$

Consequently, the element described as E is phosphorus, not nitrogen.

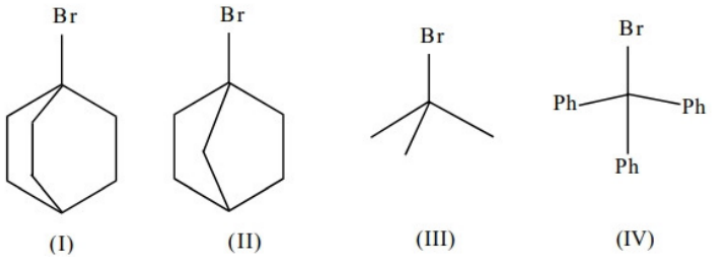
Step 3: Check phosphorus covalency Phosphorus can form five covalent bonds, as illustrated by PCl_5. It can even attain covalency six in PF_6^-, so the claimed maximum of four is incorrect. The value four characterises nitrogen's maximum covalency, for example in NH_4^+, but nitrogen does not give the lowest-boiling hydride. Statement II is also false. Options (A), (C), and (D) each call at least one false statement true.

Step 4: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (B).

Quick Tip to Solve: Identify E from the actual boiling-point exception before using a covalency fact; the nitrogen fact cannot be transferred to phosphorus.

65. The correct order of the rate of reaction of the following reactants with nucleophile by S_N1 mechanism is :

(Given : Structures I and II are rigid) [4 marks]



(A) III < I < II < IV

(B) I < II < III < IV

(C) IV < III < II < I

(D) II < I < III < IV

Correct Answer: (D) II < I < III < IV

Solution:

Concept:

- In an S_N1 reaction, the slow step is carbon–bromine bond ionisation. A substrate reacts faster when formation of its carbocation is more favourable.
- A classical carbocation favours a trigonal-planar carbon with an empty p orbital. Rigid bridgehead frameworks resist this geometry; resonance and hyperconjugation stabilise accessible carbocations.

Step 1: Compare the rigid bridgehead substrates Compounds I and II would place positive charge on their bromine-bearing bridgehead carbons. Their rigid bridges prevent the carbon from freely becoming planar, so both ionise less readily than an ordinary tertiary alkyl bromide. Framework II has the shorter bridge and greater geometric constraint than I. Formation of the planar cationic centre is

therefore least favourable in II:

$$\text{II} < \text{I}.$$

Merely counting the carbon neighbours as tertiary would miss the restriction explicitly given in the question.

Step 2: Compare the unconstrained substrates Compound III is tert-butyl bromide. Its carbon can approach trigonal-planar geometry upon ionisation, and the resulting tert-butyl cation is stabilised by the electron-releasing methyl groups and hyperconjugation. It therefore reacts faster than either rigid bridgehead substrate. Compound IV is triphenylmethyl bromide. Its cation delocalises positive charge into three phenyl rings, providing strong resonance stabilisation and making ionisation most favourable:

$$\text{I} < \text{III} < \text{IV}.$$

Step 3: Combine the comparisons Together, the constraints and stabilisation give

$$\text{II} < \text{I} < \text{III} < \text{IV}.$$

Option (A) incorrectly places III below the bridgehead substrates. Option (B) reverses I and II. Option (C) puts the resonance-stabilised triphenylmethyl substrate last and reverses the overall tendency. The fourth printed order satisfies both comparisons.

Step 4: Conclusion The increasing rate order is $\text{II} < \text{I} < \text{III} < \text{IV}$. Hence, the correct option is (D).

Quick Tip to Solve: Before applying the usual tertiary-carbocation rule, check whether a rigid bridgehead can attain the geometry needed for ionisation.

66. Given below are two statements :

Statement I : Phenol on treatment with $CHCl_3$ /aq. KOH under refluxing condition, followed by acidification produces p-hydroxy benzaldehyde as the major product and o-hydroxy benzaldehyde as the minor product.

Statement II : The mixture of p-hydroxybenzaldehyde and o hydroxybenzaldehyde can be easily separated through steam distillation.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Concept:

- Phenol reacts with chloroform and aqueous alkali in the Reimer–Tiemann reaction; formylation gives chiefly the ortho hydroxybenzaldehyde after acidification.

- Intramolecular hydrogen bonding reduces intermolecular association and helps steam volatility, whereas intermolecular hydrogen bonding makes molecules more strongly associated.

Step 1: Identify the major formylation product In aqueous alkali phenol forms phenoxide, and chloroform generates the reactive dichlorocarbene intermediate. Reaction at the activated ring followed by hydrolysis and acidification introduces the formyl group. The major product is o-hydroxybenzaldehyde (salicylaldehyde), while p-hydroxybenzaldehyde is the minor product. Statement I interchanges these roles and is false. The reagent combination identifies this as Reimer–Tiemann formylation, rather than a reaction that makes the para isomer predominant.

Step 2: Compare hydrogen bonding in the two isomers In the ortho isomer, the phenolic $\mathrm{O-H}$ group lies next to the aldehyde group. It can hydrogen-bond to the carbonyl oxygen within the same molecule, forming a favourable six-membered arrangement. This reduces intermolecular association and makes the ortho isomer appreciably steam volatile. The para groups are too far apart for such an intramolecular bond, so the para isomer associates through intermolecular hydrogen bonding.

Step 3: Use the volatility difference On steam distillation, the ortho isomer passes into the distillate more readily, while the strongly associated para isomer largely remains behind. The mixture can therefore be separated by this method, making Statement II true. Option (A) wrongly accepts Statement I, option (B) wrongly rejects Statement II, and option (C) reverses both truth values.

Step 4: Conclusion Statement I is false, while Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: Link the ortho arrangement to intramolecular hydrogen bonding: it explains why salicylaldehyde is steam volatile despite its polar groups.

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67. Given below are two statements :

Statement I : Sucrose is dextrorotatory. However, sucrose upon hydrolysis gives a solution having mixture of products. This solution shows laevorotation.

Statement II : Hydrolysis of sucrose gives glucose and fructose. Since the laevorotation of glucose is more than the dextrorotation of fructose, the resulting solution becomes laevorotatory.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- Optical rotation is the rotation of plane-polarised light. Dextrorotatory substances rotate it to the right, whereas laevorotatory substances rotate it to the left.
- Hydrolysis of one mole of sucrose gives one mole each of D-glucose and D-fructose. The D/L

configurational designation does not by itself specify the sign of optical rotation.

Step 1: Identify the hydrolysis products Sucrose contains glucose and fructose units joined by a glycosidic bond. On hydrolysis this linkage breaks:

$$C_{12}H_{22}O_{11} + H_2O \longrightarrow C_6H_{12}O_6\,(\text{glucose}) + C_6H_{12}O_6\,(\text{fructose}).$$

The products occur in equal molar amounts and, because they have the same molar mass, equal masses.

Step 2: Check Statement I Sucrose is dextrorotatory. Glucose is also dextrorotatory, but fructose is laevorotatory and its negative specific rotation has the greater magnitude. Their mixture therefore has a net negative rotation. This change from positive to negative rotation is called inversion of sugar, and Statement I is true.

Step 3: Check Statement II and eliminate the alternatives Statement II correctly names the products but reverses their directions of rotation: glucose does not provide the larger laevorotation, and fructose is not dextrorotatory. It is the laevorotation of fructose that outweighs the dextrorotation of glucose. Statement II is consequently false. Option (A) incorrectly accepts Statement II; options (B) and (D) incorrectly reject Statement I.

Step 4: Conclusion Statement I is true, but Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: Remember inversion as sucrose (+) giving glucose (+) and a more strongly rotating fructose (-); never infer the rotation sign from D or L.

68. Match the LIST-I with LIST-II.

	List-I: Thermodynamic Process	**List-II Magnitude in kJ**	
A.	**Work done in reversible, isothermal expansion of 2 mol of ideal gas from $2\,dm^3$ to $20\,dm^3$ at 300 K.**	**I.**	**4**
B.	**Work done in irreversible isothermal expansion of 1 mol ideal gas from $1\,m^3$ to $3\,m^3$ at 300 K against a constant pressure of 3 kPa.**	**II.**	**11.5**
C.	**Change in internal energy for adiabatic expansion of a 1 mol ideal gas with change of temperature $= 320\,K$ and $\overline{C}_V = \frac{3}{2}R$.**	**III.**	**6**
D.	**Change in enthalpy at constant pressure of 1 mol ideal gas with change of temperature $= 337\,K$ and $\overline{C}_p = \frac{5}{2}R$.**	**IV.**	**7**

Choose the correct answer from the options given below: [4 marks]

(A) A-II, B-III, C-I, D-IV

(B) A-II, B-I, C-III, D-IV

(C) A-I, B-II, C-III, D-IV

(D) A-III, B-II, C-IV, D-I

Correct Answer: (A) A-II, B-III, C-I, D-IV

Solution:

Concept:

- For reversible isothermal expansion of an ideal gas, the magnitude of work is $nRT\ln(V_f/V_i)$. Against constant external pressure, it is $P_{\text{ext}}(V_f - V_i)$.
- For an ideal gas, $|\Delta U| = n\overline{C}_V|\Delta T|$ and $|\Delta H| = n\overline{C}_p|\Delta T|$. The matching list asks for magnitudes, not signed changes.

Step 1: Calculate the reversible work in A Use $R = 8.314\,\text{J}\,\text{mol}^{-1}\text{K}^{-1}$. Only the volume ratio is required, so both volumes may remain in cubic decimetres:

$$|w_A| = 2(8.314)(300)\ln(20/2) \simeq 11486\,\text{J} \simeq 11.5\,\text{kJ}.$$

Thus A matches II.

Step 2: Calculate the constant-pressure work in B The volume increase is $3 - 1 = 2\,\text{m}^3$. Since $1\,\text{kPa}\,\text{m}^3 = 1\,\text{kJ}$,

$$|w_B| = 3(3 - 1) = 6\,\text{kJ}.$$

Thus B matches III. The reversible logarithmic formula cannot replace the constant-external-pressure formula here.

Step 3: Calculate the internal-energy and enthalpy magnitudes For process C, use the given molar heat capacity:

$$|\Delta U_C| = 1 \times \frac{3}{2}(8.314) \times 320 = 3990.72\,\text{J} \simeq 4\,\text{kJ}.$$

Hence C matches I. For process D,

$$|\Delta H_D| = 1 \times \frac{5}{2}(8.314) \times 337 = 7004.545\,\text{J} \simeq 7\,\text{kJ}.$$

Hence D matches IV. Option (B) swaps the values for B and C, while (C) and (D) assign an incorrect value to A.

Step 4: Conclusion The matching is A-II, B-III, C-I, D-IV. Hence, the correct option is (A).

Quick Tip to Solve: Choose the work formula from the process before substituting: reversible isothermal uses a logarithm, while constant external pressure uses the volume difference.

69. $A \rightarrow$ **product (First order reaction).**

Three sets of experiment were performed for a reaction under similar experimental conditions:

Run $1 \Rightarrow 100\ \text{mL}$ **of 10 M solution of reactant A**

Run $2 \Rightarrow 200\ \text{mL}$ **of 10 M solution of reactant A**

Run $3 \Rightarrow 100\ \text{mL}$ **of 10 M solution of reactant** $A + 100\ \text{mL}$ **of** H_2O **added.**

The correct variation of rate of reaction is [4 marks]

(A) Run 1 = Run 2 = Run 3

(B) Run 3 < Run 1 = Run 2

(C) Run 3 < Run 1 < Run 2

(D) Run 1 < Run 2 < Run 3

Correct Answer: (B) Run 3 < Run 1 = Run 2

Solution:

Concept:

- The rate of a first-order reaction is $r = k[A]$, where $[A]$ is concentration and k is the rate constant. Under identical experimental conditions, the same k applies to all runs.
- Dilution changes concentration according to $C_f = C_i V_i / V_f$, with volumes in consistent units. Reaction rate per unit volume is controlled by concentration, not by the total moles initially present.

Step 1: Compare Runs 1 and 2 Both solutions have concentration $10\,\text{mol}\,\text{L}^{-1}$. Doubling the solution volume from 100 mL to 200 mL doubles the total moles but leaves the concentration unchanged. Consequently,

$$r_1 = k(10), \qquad r_2 = k(10), \qquad r_1 = r_2.$$

The first-order rate is not doubled merely because a larger sample is used.

Step 2: Account for dilution in Run 3 Adding 100 mL water to 100 mL of the original solution gives a total volume of 200 mL while preserving the moles of A. Therefore,

$$[A]_3 = 10 \times \frac{100}{100 + 100} = 5\,\text{mol}\,\text{L}^{-1}, \qquad r_3 = k(5).$$

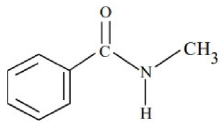
Thus the initial rate in Run 3 is half that in either of the other runs.

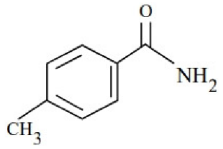
Step 3: Select the comparison and reject the others The rate ratio is $r_1 : r_2 : r_3 = 2 : 2 : 1$. Option (A) ignores dilution. Option (C) incorrectly makes Run 2 faster than Run 1 merely because of its larger volume. Option (D) incorrectly makes the diluted solution the fastest. Only the second printed option preserves equality of Runs 1 and 2 and the lower rate of Run 3.

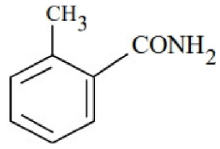
Step 4: Conclusion The printed concentrations give $r_1 = r_2 = 10k$ and $r_3 = 5k$, so the correct order is Run 3 < Run 1 = Run 2. Hence, the correct option is (B).

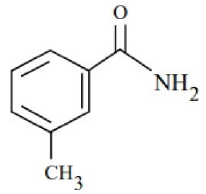
Quick Tip to Solve: For a concentration-based rate law, compare molarity after mixing; never replace concentration by the total moles in differently sized samples.

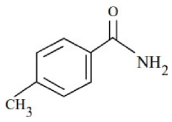
70. 'A' is a neutral organic compound (M. F.: C_8H_9ON). On treatment with aqueous Br_2/HO^-, 'A' forms a compound 'B' which is soluble in dilute acid. 'B' on treatment with aqueous $NaNO_2/HCl$ (0–5°C) produces a compound 'C' which on treatment with $CuCN/NaCN$ produces 'D'. Hydrolysis of 'D' produces 'E' which is also obtainable from the hydrolysis of 'A'. 'E' on treatment with acidified $KMnO_4$ produces 'F'. 'F' contains two different types of hydrogen atoms. The structure of 'A' is [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- Hofmann bromamide degradation converts a primary amide $RCONH_2$ into RNH_2, with loss of the carbonyl carbon. The amine dissolves in dilute acid by forming an ammonium salt.
- An aromatic primary amine undergoes diazotisation at 0–5°C. Replacing its diazonium group by cyanide and hydrolysing the nitrile restores a carboxyl group at that same ring position.
- Acidified permanganate oxidises a benzylic methyl group to COOH. The number of distinct hydrogen environments in the final acid identifies the substitution pattern.

Step 1: Identify the suitable amide and reaction sequence The second structure is 4-methylbenzamide, p-$CH_3C_6H_4CONH_2$. It gives p-toluidine (B) by Hofmann degradation. Diazotisation produces p-methylbenzenediazonium chloride (C), and cyanide substitution gives p-methylbenzonitrile (D). Hydrolysis of D gives p-methylbenzoic acid (E). Direct hydrolysis of A gives this same acid, as required.

Step 2: Use the final oxidation and symmetry test Oxidation of the methyl group of E produces terephthalic acid, p-$HOOCC_6H_4COOH$. Its identical para substituents make all four aromatic hydrogens chemically equivalent. The two carboxylic-acid hydrogens form a second equivalent set. Thus F has exactly two types of hydrogen atoms: one aromatic type and one carboxylic-acid type.

Step 3: Eliminate the remaining structures Option (A), N-methylbenzamide, is a secondary amide rather than the primary amide needed for this Hofmann sequence. Option (C), the ortho-methylbenzamide, would produce phthalic acid, with two aromatic hydrogen environments plus the acid hydrogens. Option (D), the meta isomer, would produce isophthalic acid, with three aromatic hydrogen environments plus the acid hydrogens. Neither has only two total hydrogen types.

Step 4: Conclusion Compound A is 4-methylbenzamide (p-toluamide), which gives terephthalic acid as F. Hence, the correct option is (B).

Quick Tip to Solve: For this sequence, use Hofmann degradation to locate the primary amide, then count symmetry-related hydrogens in the final dicarboxylic acid, including its acidic hydrogens.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The temperature at which the rate constants of the given below two gaseous reactions become equal is ____ K. (Nearest integer)

$$\mathrm{X} \longrightarrow \mathrm{Y}, \quad \mathrm{k}_1 = 10^6 e^{\frac{-30000}{\mathrm{T}}}$$
$$\mathrm{P} \longrightarrow \mathrm{Q}, \quad \mathrm{k}_2 = 10^4 e^{\frac{-24000}{\mathrm{T}}}$$

Given : $\ln 10 = 2.303$ [4 marks]

Solution:

Concept:

- The Arrhenius form is $k = Ae^{-E_a/(RT)}$. In the given expressions, the activation-energy-to-gas-constant ratio is already contained in the numerical coefficient of $1/T$; it must not be divided by R again.
- Equality of two positive exponential expressions can be solved by dividing one by the other and taking natural logarithms. The temperature in an Arrhenius exponent is absolute temperature in kelvin.

Step 1: Equate and divide the rate constants At the required temperature,

$$10^6 e^{-30000/T} = 10^4 e^{-24000/T}.$$

Divide by the positive quantity $10^4 e^{-24000/T}$:

$$10^2 e^{(-30000+24000)/T} = 1, \qquad 100 e^{-6000/T} = 1.$$

The negative sign in the exponent is essential; the higher pre-exponential factor must be balanced by the greater activation barrier.

Step 2: Take logarithms and isolate temperature Taking the natural logarithm gives

$$\ln 100 - \frac{6000}{T} = 0.$$

Using the supplied value $\ln 10 = 2.303$,

$$\ln 100 = 2 \ln 10 = 4.606, \qquad T = \frac{6000}{4.606} = 1302.6487\,\mathrm{K}.$$

This is a positive physical temperature; retaining the stated logarithm avoids a rounding discrepancy.

Step 3: Check the equality and round once At this temperature, $6000/T = \ln 100$, so $e^{-6000/T} = 1/100$. Therefore the ratio $k_1/k_2 = 100 e^{-6000/T}$ is exactly unity before rounding. Rounding the calculated temperature only at the final step gives the nearest integer requested.

Step 4: Conclusion The temperature is $\boxed{1303\,\mathrm{K}}$, to the nearest integer.

Quick Tip to Solve: When two Arrhenius constants are equated, subtract their exponent coefficients and take the logarithm of the prefactor ratio; do not introduce an extra gas constant.

72. Consider the following electrochemical cell at 298 K

$\mathrm{Pt}\left|\mathrm{HSnO_2^-(aq)}\right|\mathrm{Sn(OH)_6^{2-}(aq)}\left|\mathrm{OH^-(aq)}\right|\mathrm{Bi_2O_3(s)}\mid\mathrm{Bi(s)}$.

If the reaction quotient at a given time is 10^6, then the cell EMF $(\mathrm{E_{cell}})$ is ____ $\times 10^{-1}$ V (Nearest integer).

Given the standard half-cell reduction potential as

$$\mathrm{E^\circ_{Bi_2O_3/Bi,OH^-}} = -0.44\ \mathrm{V}\ \textbf{and}\ \mathrm{E^\circ_{Sn(OH)_6^{2-}/HSnO_2^-,OH^-}} = -0.90\ \mathrm{V}$$

[4 marks]

Solution:

Concept:

- The standard cell potential is the cathode reduction potential minus the anode reduction potential. Multiplying a half-reaction does not multiply its electrode potential.
- At 298 K, the Nernst equation is $E_{\text{cell}} = E^\circ_{\text{cell}} - (0.0591/n)\log_{10} Q$, where n is the number of electrons in the balanced overall cell reaction.

Step 1: Determine the standard potential The bismuth couple has the more positive reduction potential, so bismuth(III) is reduced at the cathode and tin(II) is oxidised at the anode. Thus

$$E^\circ_{\text{cell}} = (-0.44) - (-0.90) = 0.46\,\mathrm{V}.$$

Using the given reduction potentials with their signs avoids incorrectly adding the two negative values.

Step 2: Balance the electron transfer Tin changes from oxidation state +2 in $\mathrm{HSnO_2^-}$ to +4 in $\mathrm{Sn(OH)_6^{2-}}$, losing two electrons per tin. The alkaline half-reactions are

$$\mathrm{HSnO_2^- + 3\,OH^- + H_2O \longrightarrow Sn(OH)_6^{2-} + 2\,e^-},$$

$$\mathrm{Bi_2O_3 + 3\,H_2O + 6\,e^- \longrightarrow 2\,Bi + 6\,OH^-}.$$

Multiply the tin half-reaction by three to cancel six electrons. Therefore $n = 6$, not 2 or 3.

Step 3: Apply the reaction quotient and express the requested coefficient With $Q = 10^6$,

$$E_{\text{cell}} = 0.46 - \frac{0.0591}{6}\log_{10}(10^6) = 0.46 - 0.0591 = 0.4009\,\mathrm{V}.$$

The value is below the standard potential, as expected for $Q > 1$. In the requested form, $0.4009\,\mathrm{V} = 4.009 \times 10^{-1}\,\mathrm{V}$. Round the coefficient 4.009, rather than rounding the voltage directly to a whole volt.

Step 4: Conclusion The cell EMF is approximately $\boxed{4 \times 10^{-1}\,\mathrm{V}}$; the required nearest-integer entry is $\boxed{4}$.

Quick Tip to Solve: Balance the full electron exchange before using Nernst: this cell needs six electrons. Then convert the voltage to the specified power-of-ten form before rounding.

73. The cycloalkene (X) on bromination consumes one mole of bromine per mole of (X) and gives the product (Y) in which $C : Br$ ratio is 3:1. The percentage of bromine in the product (Y) is ____ %. (Nearest integer)

(Given : molar mass in $gmol^{-1} H : 1, C : 12, O : 16, Br : 80$) [4 marks]

Solution:

Concept:

- Bromine adds across an alkene double bond. One mole of Br_2 supplies two bromine atoms to each molecule without removing hydrogen atoms.
- A monocyclic alkene with one double bond has formula C_nH_{2n-2}. Its dibromo addition product has formula $C_nH_{2n-2}Br_2$.
- An elemental mass percentage is the mass of that element in one mole of the compound divided by the compound's molar mass, multiplied by 100.

Step 1: Determine the carbon count from the product ratio Consumption of one mole of bromine per mole of X means that one Br_2 molecule adds to each molecule of the cycloalkene. Thus Y contains two bromine atoms. The stated $C : Br = 3 : 1$ atomic ratio gives

$$\frac{n}{2} = \frac{3}{1} \quad \Rightarrow \quad n = 6.$$

The starting cycloalkene therefore has six carbon atoms. Its ring and its double bond each account for one degree of unsaturation, giving ten hydrogen atoms.

Step 2: Write the addition product and its molar mass The molecular change is

$$C_6H_{10} + Br_2 \longrightarrow C_6H_{10}Br_2.$$

Bromination removes the double bond but leaves the ring intact. No hydrogen is lost in this addition, so the product is $C_6H_{10}Br_2$, not $C_6H_{12}Br_2$. Using the given atomic masses,

$$M_Y = 6(12) + 10(1) + 2(80) = 72 + 10 + 160 = 242\,\text{g mol}^{-1}.$$

Step 3: Calculate and round the bromine percentage One mole of product contains $160\,g$ of bromine in a total mass of $242\,g$. Hence

$$\%Br = \frac{160}{242} \times 100 = 66.1157\ldots\%.$$

Keep the unrounded molar mass throughout and round only the final percentage to the nearest integer.

Step 4: Conclusion The bromine content of Y is $66.1157\ldots\%$, so the required nearest-integer answer is $\boxed{66}$ (that is, 66%).

Quick Tip to Solve: Convert one mole of added Br_2 into two bromine atoms before using the C:Br ratio; retain the original hydrogen count in an addition reaction.

74. Dissociation of a gas A_2 takes place according to the following chemical reaction. At equilibrium, the total pressure is 1 bar at 300 K .

$$A_2(g) \rightleftharpoons 2\,A(g)$$

The standard Gibbs energy of formation of the involved substances has been provided below:

Substance	ΔG_f°/kJ mol^{-1}
A_2	-100.00
A	-50.832

The degree of dissociation of $A_2(g)$ is given by $\left(x \times 10^{-2}\right)^{1/2}$ where $x =$ ____ . (Nearest integer).

[Given : $R = 8$ J mol^{-1} K^{-1}, $\log 2 = 0.3010, \log 3 = 0.48$ **]**

Assume degree of dissociation is not negligible. [4 marks]

Solution:

Concept:

- The standard reaction Gibbs energy is the sum of stoichiometric formation Gibbs energies of products minus that of reactants: $\Delta_r G^\circ = \sum \nu \Delta_f G^\circ$.
- The dimensionless equilibrium constant satisfies $\Delta_r G^\circ = -RT \ln K_p$. Express energies and R in compatible units before substitution.
- For dissociation of an initially pure diatomic gas, equilibrium partial pressures must include the increase in total moles. The degree of dissociation α is not small here.

Step 1: Calculate the standard reaction Gibbs energy For $A_2(g) \rightleftharpoons 2\,A(g)$, the product formation energy is counted twice:

$$\begin{aligned}\Delta_r G^\circ &= 2(-50.832) - (-100.00) \\ &= -1.664\,\text{kJ mol}^{-1} = -1664\,\text{J mol}^{-1}.\end{aligned}$$

The negative value implies $K_p > 1$, which provides a useful sign check.

Step 2: Obtain the equilibrium constant at 300 K Using the specified $R = 8\,\text{J mol}^{-1}\,\text{K}^{-1}$,

$$\ln K_p = \frac{1664}{8(300)} = 0.69333\ldots.$$

Equivalently, $\log K_p = 1664/[2.303(8)(300)] \simeq 0.3010$. The supplied $\log 2 = 0.3010$ therefore gives $K_p \simeq 2$.

Step 3: Express the partial pressures without a small-dissociation approximation Start with one mole of A_2. At equilibrium, the amounts are $1-\alpha$ mole of A_2, 2α moles of A, and $1+\alpha$ moles in total. Thus

$$p_{A_2} = \frac{1-\alpha}{1+\alpha}P, \qquad p_A = \frac{2\alpha}{1+\alpha}P.$$

With standard pressure $p^\circ = 1\,\text{bar}$,

$$K_p = \frac{(p_A/p^\circ)^2}{p_{A_2}/p^\circ} = \frac{4\alpha^2}{1-\alpha^2}\frac{P}{p^\circ}.$$

Step 4: Solve for the requested parameter Since $P = 1\,\text{bar}$, the pressure ratio is unity. Therefore

$$2 = \frac{4\alpha^2}{1-\alpha^2} \quad \Rightarrow \quad \alpha^2 = \frac{1}{3}.$$

The question writes $\alpha = (x \times 10^{-2})^{1/2}$, so

$$x = 100\alpha^2 = \frac{100}{3} = 33.333\ldots.$$

Keeping the unrounded exponential for K_p changes only the small decimal correction and leaves the nearest integer unchanged.

Step 5: Conclusion The degree of dissociation is approximately 0.577, and the requested dimensionless parameter is $\boxed{x = 33}$ to the nearest integer.

Quick Tip to Solve: For $A_2 \rightleftharpoons 2\,A$ at one bar, use $\alpha^2 = K_p/(K_p + 4)$; do not replace $1 - \alpha^2$ by unity when dissociation is appreciable.

75. Sodium fusion extract of an organic compound (Y) with $CHCl_3$ and chlorine water gives violet color to the $CHCl_3$ layer. 0.15 g of (Y) gave 0.12 g of the silver halide precipitate in Carius method. Percentage of halogen in the compound (Y) is

____ . (Nearest integer)

(Given : molar mass $gmol^{-1}$ $C : 12, H : 1, Cl : 35.5, Br : 80, I : 127$ **)** [4 marks]

Solution:

Concept:

- In the sodium fusion test, an organic halogen is converted into an inorganic halide. Chlorine water oxidises iodide to iodine, which gives a violet solution in chloroform.
- The Carius method converts the halogen in a weighed organic sample into a silver halide precipitate. One mole of silver iodide contains one mole of iodine atoms.
- The halogen mass is the precipitate mass multiplied by the halogen's mass fraction in that precipitate; divide by the original sample mass to obtain the percentage.

Step 1: Identify the halogen from the violet layer The violet colour of the $CHCl_3$ layer identifies molecular iodine. Chlorine water liberates it from iodide ions in the sodium fusion extract:

$$Cl_2 + 2\,I^- \longrightarrow 2\,Cl^- + I_2.$$

Iodine dissolves in the chloroform layer and imparts the characteristic violet colour. Bromine would instead give an orange-brown organic layer. Therefore the halogen in Y is iodine and the silver halide

weighed in the Carius method is AgI.

Step 2: Convert the silver iodide mass into iodine mass Using the standard atomic molar mass $M_{\mathrm{Ag}} = 108\,\mathrm{g\,mol^{-1}}$, together with the given iodine value,

$$M_{\mathrm{AgI}} = 108 + 127 = 235\,\mathrm{g\,mol^{-1}}.$$

Thus $235\,\mathrm{g}$ of AgI contains $127\,\mathrm{g}$ of iodine. The $0.12\,\mathrm{g}$ precipitate therefore contains

$$m_{\mathrm{I}} = 0.12 \times \frac{127}{235} = 0.064851\ldots\ \mathrm{g}.$$

The rest of the precipitate mass is silver introduced during the test; it must not be counted as halogen originally present in the sample.

Step 3: Calculate the iodine percentage in the original sample Divide the recovered iodine mass by the $0.15\,\mathrm{g}$ sample mass, not by the precipitate mass:

$$\%\mathrm{I} = \frac{0.064851\ldots}{0.15} \times 100 = \frac{0.12 \times 127 \times 100}{235 \times 0.15} = 43.234\ldots\,\%.$$

The measured masses refer to the sample and its precipitate, respectively; the silver iodide stoichiometry links the two. Round only after this final calculation.

Step 4: Conclusion The compound contains iodine, and its halogen percentage is $43.234\ldots\,\%$. The required nearest-integer answer is $\boxed{43}$ (that is, 43%).

Quick Tip to Solve: Identify the halogen from the colour first, then multiply the precipitate mass by $M_{\text{halogen}}/M_{\text{silver halide}}$ before dividing by the sample mass.