JEE Main 2026 Paper 1 (B.E./B.Tech) Apr 2 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2026, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 2 April 2026, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $\alpha, \alpha+2, \alpha \in \mathbf{Z}$, be the roots of the quadratic equation $x(x+2)+(x+1)(x+3)+(x+2)(x+4)+\ldots.+(x+\mathrm{n}-1)(x+\mathrm{n}+1)=4\mathrm{n}$ for some $\mathrm{n} \in \mathbf{N}$. Then $\mathrm{n}+\alpha$ is equal to : [4 marks]

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (C) 2

Solution:

Concept:

- The identity $(u-1)(u+1)=u^2-1$ turns each product into a square.
- Completing the square locates the midpoint of the roots. The roots differ by twice their distance from that midpoint.

Step 1: Express the sum using consecutive squares Index the terms by $k=1,\ldots,n$. Since $(x+k-1)(x+k+1)=(x+k)^2-1$, the equation is

$$\sum_{k=1}^{n}(x+k)^2=5n.$$

Use $\sum k=n(n+1)/2$ and $\sum k^2=n(n+1)(2n+1)/6$, then divide by the positive integer n:

$$x^2+(n+1)x+\frac{(n+1)(2n+1)}{6}=5.$$

Step 2: Use the prescribed difference between the roots Completing the square gives

$$\left(x+\frac{n+1}{2}\right)^2=5-\frac{n^2-1}{12}.$$

The roots are symmetric about $-(n+1)/2$. The prescribed roots α and $\alpha+2$ differ by 2, so each is at distance 1 from their midpoint. Consequently the right-hand side must equal 1^2:

$$5-\frac{n^2-1}{12}=1 \quad \Longrightarrow \quad n^2=49.$$

As $n \in \mathbb{N}$, take $n=7$, not -7.

Step 3: Identify the smaller root and compare the options Substitution gives $(x+4)^2=1$, whose roots are $-5,-3$. Since the roots are $\alpha, \alpha+2$, the smaller root is $\alpha=-5$, also satisfying the integer restriction. Therefore $n+\alpha=7-5=2$. The values 0, 1 and 3 in the other options do not equal this uniquely determined sum. As a direct check, the monic quadratic is $x^2+8x+15=0$, which factors as $(x+5)(x+3)=0$.

Step 4: Conclusion $n=7$, $\alpha=-5$, and $n+\alpha=2$. Hence, the correct option is (C).

Quick Tip to Solve: When factors differ by 2, rewrite their product as a square minus 1; the root gap then determines the completed-square constant directly.

2. Let x and y be real numbers such that $50\left(\frac{2x}{1+3i}-\frac{y}{1-2i}\right)=31+17i, i=\sqrt{-1}$. Then the value of $10(x-3y)$ is : [4 marks]

(A) 20

(B) 31

(C) 35

(D) 75

Correct Answer: (D) 75

Solution:

Concept:

- Multiply by the conjugate to remove a complex denominator: $1/(a+bi)=(a-bi)/(a^2+b^2)$.
- Two complex numbers are equal precisely when their real parts and imaginary parts are separately equal.

Step 1: Rationalise both denominators Use the conjugate of each denominator, reversing the sign of its imaginary part:

$$\frac{2x}{1+3i}=\frac{2x(1-3i)}{10}=\frac{x}{5}(1-3i), \qquad \frac{y}{1-2i}=\frac{y(1+2i)}{5}.$$

Thus the expression on the left becomes

$$50\left(\frac{2x}{1+3i}-\frac{y}{1-2i}\right)=10(x-y)-10(3x+2y)i.$$

The minus sign preceding the second fraction affects both its real and imaginary contributions.

Step 2: Equate coefficients and solve the real equations Matching with $31+17i$ gives

$$x-y=\frac{31}{10}, \qquad 3x+2y=-\frac{17}{10}.$$

Substitute $x=y+31/10$ into the second equation:

$$3y+\frac{93}{10}+2y=-\frac{17}{10} \quad \Longrightarrow \quad 5y=-11.$$

Hence $y=-11/5$ and $x=9/10$. These satisfy both equations: $x-y=31/10$ and $3x+2y=-17/10$, so the imaginary sign is consistent.

Step 3: Calculate the requested combination Use the actual requested expression, which involves $3y$, rather than merely the real part $x-y$:

$$10(x-3y)=10\left(\frac{9}{10}+\frac{33}{5}\right)=10\cdot\frac{75}{10}=75.$$

The values 20 and 35 do not match this calculation. The distractor 31 is the real part of the given complex number, equal to $10(x-y)$, not $10(x-3y)$.

Step 4: Conclusion The required value is 75. Hence, the correct option is (D).

Quick Tip to Solve: After rationalising, collect the full real and imaginary coefficients before solving; retain the minus sign on every term of a subtracted fraction.

3. Let $\alpha, \beta \in \mathbb{R}$ be such that the system of linear equations

$$x + 2y + z = 5$$
$$2x + y + \alpha z = 5$$
$$8x + 4y + \beta z = 18$$

has no solution. Then $\frac{\beta}{\alpha}$ is equal to : [4 marks]

(A) -4

(B) 4

(C) 8

(D) -8

Correct Answer: (B) 4

Solution:

Concept:

- A system is inconsistent if elimination produces a contradiction such as $0 = -2$.
- If elimination fixes z and the remaining two equations in x, y have a nonzero determinant, the system has a unique solution.

Step 1: Eliminate the first two unknowns The coefficients of x and y in the third equation are four times those in the second. Subtract four times the second equation from the third:

$$(8x + 4y + \beta z) - 4(2x + y + \alpha z) = 18 - 4(5),$$

so

$$(\beta - 4\alpha)z = -2.$$

This relation isolates the only possible obstruction to consistency.

Step 2: Exclude the case that gives a solution If $\beta - 4\alpha \neq 0$, the last relation fixes

$$z = \frac{-2}{\beta - 4\alpha}.$$

For that value of z, the first two equations reduce to

$$x + 2y = 5 - z, \qquad 2x + y = 5 - \alpha z.$$

Their coefficient determinant is $1 \cdot 1 - 2 \cdot 2 = -3 \neq 0$; thus they always give unique real values of x, y. Consequently this case cannot produce the stated absence of a solution.

Step 3: Apply the inconsistency condition The required case is therefore $\beta - 4\alpha = 0$. Then the eliminated equation reads $0 = -2$, an outright contradiction, so the condition is also sufficient. It follows that $\beta = 4\alpha$. For the quotient asked in the question to be defined, $\alpha \neq 0$, and therefore $\beta/\alpha = 4$. The other offered ratios $-4, 8, -8$ do not satisfy this equality for nonzero α. Strictly, the printed real-parameter condition also permits $\alpha = \beta = 0$, where the system is inconsistent but the quotient is undefined; the requested quotient implicitly excludes that case.

Step 4: Conclusion Whenever the requested quotient is defined, $\beta/\alpha = 4$. Hence, the correct option is (B).

Quick Tip to Solve: When two coefficient rows are proportional, subtract their corresponding equations first; unequal proportional constants reveal inconsistency immediately.

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4. Let $A = \begin{bmatrix} 1 & 2 \\ 1 & \alpha \end{bmatrix}$ **and** $B = \begin{bmatrix} 3 & 3 \\ \beta & 2 \end{bmatrix}$**. If** $A^2 - 4A + I = O$ **and** $B^2 - 5B - 6I = O$**, then among the two statements:**

(S1) : $[(B - A)(B + A)]^T = \begin{bmatrix} 13 & 15 \\ 7 & 10 \end{bmatrix}$

and

(S2) : $\det(\text{adj}(A + B)) = -5$ [4 marks]

(A) only (S1) is correct

(B) only (S2) is correct

(C) both (S1) and (S2) are correct

(D) both (S1) and (S2) are wrong

Correct Answer: (B) only (S2) is correct

Solution:

Concept:

- Matrix polynomial identities can be evaluated entry by entry to determine unknown entries.
- Matrix multiplication is not generally commutative. For a 2×2 matrix M, $\det(\text{adj}\, M) = \det M$.

Step 1: Find the unknown matrix entries The upper-right entry of $A^2 - 4A + I$ is $2 + 2\alpha - 8 = 2\alpha - 6$, giving $\alpha = 3$. The upper-left entry of $B^2 - 5B - 6I$ is $9 + 3\beta - 15 - 6 = 3\beta - 12$, giving $\beta = 4$. Therefore

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}, \qquad B = \begin{bmatrix} 3 & 3 \\ 4 & 2 \end{bmatrix}.$$

These values satisfy the entire matrix identities: $A^2 = 4A - I$ and $B^2 = 5B + 6I$.

Step 2: Check the product and its transpose Keep the factor order exactly as printed:

$$B - A = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}, \qquad B + A = \begin{bmatrix} 4 & 5 \\ 5 & 5 \end{bmatrix}.$$

Their product and its transpose are

$$(B - A)(B + A) = \begin{bmatrix} 13 & 15 \\ 7 & 10 \end{bmatrix}, \qquad [(B - A)(B + A)]^T = \begin{bmatrix} 13 & 7 \\ 15 & 10 \end{bmatrix}.$$

Statement (S1) displays the untransposed product. Its off-diagonal entries are therefore incorrect, and (S1) is false.

Step 3: Check the determinant statement Here $A+B=\begin{bmatrix}4 & 5\\ 5 & 5\end{bmatrix}$, so

$$\det(A+B)=4\cdot 5-5\cdot 5=-5.$$

Since the matrix has order 2, its adjugate has the same determinant. Thus $\det(\text{adj}(A+B))=-5$, and (S2) is true. Options (A) and (C) require the false first statement to be true, while option (D) incorrectly rejects the second statement.

Step 4: Conclusion Only (S2) is correct. Hence, the correct option is (B).

Quick Tip to Solve: Do not apply the scalar difference-of-squares rule to matrices without checking commutation, and transpose the final product before comparing entries.

5. Let A be the set of first 101 terms of an A.P., whose first term is 1 and the common difference is 5 and let B be the set of first 71 terms of an A.P., whose first term is 9 and the common difference is 7. Then the number of elements in $A\cap B$, which are divisible by 3, is : [4 marks]

(A) 4

(B) 5

(C) 6

(D) 7

Correct Answer: (B) 5

Solution:

Concept:

- The rth term of an arithmetic progression is $a+(r-1)d$, so finite-term limits must be included.
- Common terms satisfy both congruences; their spacing is the least common multiple of the two common differences.

Step 1: Write the progressions and their bounds The two finite sets are

$$A=\{1+5r:0\le r\le 100\},\qquad B=\{9+7s:0\le s\le 70\}.$$

Their largest elements are 501 and 499, respectively. A common element is therefore at most 499 and at least 9. Membership requires the element to be congruent to 1 modulo 5 and to 2 modulo 7.

Step 2: Determine the common progression The first common term is 16, since $16=1+5(3)=9+7(1)$. Because 5 and 7 are coprime, all common terms differ from 16 by a multiple of 35. Thus

$$A\cap B=\{16+35t:t=0,1,\ldots,13\},$$

where the upper bound follows from $16+35t\le 499$, giving $t\le 483/35=13.8$. The previous candidate $16-35=-19$ is outside both finite sets, and the next term after $t=13$ is 506, exceeding the upper bound.

Step 3: Impose divisibility by three Reduce the common term modulo 3:

$$16 + 35t \equiv 1 + 2t \equiv 0 \pmod 3 \quad \Longleftrightarrow \quad t \equiv 1 \pmod 3.$$

In the permitted range the indices are $t = 1, 4, 7, 10, 13$, giving the five terms

$$51,\ 156,\ 261,\ 366,\ 471.$$

Each is divisible by 3 and lies in both progressions. There are exactly five, so the counts 4, 6 and 7 in the remaining options respectively omit or add admissible terms.

Step 4: Conclusion The number of required elements is 5. Hence, the correct option is (B).

Quick Tip to Solve: First intersect the two progressions using their finite endpoints, then apply the extra divisibility condition to the common-term index.

6. The number of seven-digit numbers, that can be formed by using the digits 1, 2, 3, 5 and 7 such that each digit is used at least once, is: [4 marks]

(A) 15400

(B) 17800

(C) 16800

(D) 29400

Correct Answer: (C) 16800

Solution:

Concept:

- With repeated counts $r_1, \ldots, r_k$, the number of arrangements of n positions is $n!/(r_1! \cdots r_k!)$.
- Every allowed digit is nonzero, so every arrangement of seven positions is a valid seven-digit number; no leading-zero correction is needed.

Step 1: List the possible multiplicity patterns Each of the five distinct digits must occur at least once. Placing one copy of each uses five positions, leaving two extra occurrences. Either both extra occurrences belong to one digit, giving the pattern $(3, 1, 1, 1, 1)$, or they belong to two different digits, giving $(2, 2, 1, 1, 1)$. These cases are disjoint and exhaustive; no digit can occur four times without leaving fewer than one position for another required digit.

Step 2: Count numbers with a digit occurring three times Choose the tripled digit in 5 ways. For any fixed choice, permuting the seven positions gives $7!/3!$ distinct numbers, because the three equal copies are indistinguishable. The contribution is

$$5 \cdot \frac{7!}{3!} = 5 \cdot 840 = 4200.$$

The other four digits occur once each, so there are no further factorial divisors.

Step 3: Count numbers with two doubled digits Choose the two doubled digits as an unordered pair

in $\binom{5}{2} = 10$ ways. For each pair, divide by 2! for each doubled digit:

$$\binom{5}{2}\frac{7!}{2!\,2!} = 10 \cdot 1260 = 12600.$$

The total is $4200 + 12600 = 16800$. Choosing the doubled digits in an ordered fashion would double-count this case, while ignoring repeated copies would overcount arrangements. Since the multiplicity cases exhaust all possibilities, the alternative totals 15400, 17800 and 29400 cannot be correct.

Step 4: Conclusion There are 16800 such seven-digit numbers. Hence, the correct option is (C).

Quick Tip to Solve: After assigning one copy of every compulsory digit, distribute the extra copies; count each resulting multiplicity pattern with the appropriate repeated-factorial divisor.

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7. The number of elements in the set $S = \left\{(r,k) \ : k \in \mathbb{Z}$ and ${}^{36}C_{r+1} = \frac{6\binom{35}{r}}{(k^2-3)}\right\}$ is : [4 marks]

(A) 2

(B) 4

(C) 8

(D) 16

Correct Answer: (B) 4

Solution:

Concept:

- For ordinary binomial coefficients, the lower index is an integer in its allowed range. Here both coefficients are nonzero when $0 \le r \le 35$.
- The adjacent-row identity $\binom{36}{r+1} = \frac{36}{r+1}\binom{35}{r}$ reduces the equation without expanding factorials.

Step 1: Cancel the common binomial coefficient Since $\binom{35}{r} > 0$ throughout the allowed range, division by this coefficient is valid. The equation becomes

$$\frac{36}{r+1} = \frac{6}{k^2-3} \quad \Longrightarrow \quad r+1 = 6(k^2-3).$$

The denominator cannot vanish for integer k, because 3 is not a perfect square. Moreover, the left side is positive, so $k^2 - 3$ must be positive.

Step 2: Use the index range to enumerate the pairs The condition $0 \le r \le 35$ gives $1 \le r+1 \le 36$. Hence

$$1 \le 6(k^2-3) \le 36, \qquad \frac{19}{6} \le k^2 \le 9.$$

The only perfect squares in this interval are 4 and 9. For $k = \pm 2$, we obtain $r = 5$; for $k = \pm 3$, we obtain $r = 35$. Thus

$$S = \{(5,2), (5,-2), (35,3), (35,-3)\}.$$

Each pair satisfies the original equation and the binomial-index restrictions. Counting only the two values of r would miss the distinct signs of k. Counts 8 and 16 would require extra admissible squares

or repeated counting, neither of which occurs.

Step 3: Conclusion The set contains 4 ordered pairs. Hence, the correct option is (B).

Quick Tip to Solve: Cancel adjacent binomial coefficients first, then impose the lower-index range before counting both signs of an integer square root.

8. If the mean of the data

Class	5-10	10-15	15-20	20-25	25-30	30-35
Frequency	2	k	28	54	k + 1	5

is 21, then k is one of the roots of the equation: [4 marks]

(A) $2x^2 - 23x - 10 = 0$

(B) $4x^2 - 35x + 24 = 0$

(C) $2x^2 - 19x - 10 = 0$

(D) $2x^2 - 35x + 98 = 0$

Correct Answer: (C) $2x^2 - 19x - 10 = 0$

Solution:

Concept:

- For grouped data, use the class midpoint m_i as the representative observation and compute $\bar{x} = \frac{\sum f_i m_i}{\sum f_i}$.
- A value is a root of an equation precisely when substitution makes its left-hand side zero.

Step 1: Form the frequency and weighted totals The midpoints, in class order, are:

$$7.5, \quad 12.5, \quad 17.5, \quad 22.5, \quad 27.5, \quad 32.5.$$

In the printed order, the frequencies are $2, k, 28, 54, k+1, 5$. Therefore

$$\sum f_i = 2 + k + 28 + 54 + (k+1) + 5 = 90 + 2k.$$

Multiplying each frequency by its own midpoint gives

$$\begin{aligned} \sum f_i m_i &= 2(7.5) + 12.5k + 28(17.5) + 54(22.5) \\ &\quad + 27.5(k+1) + 5(32.5) \\ &= 1910 + 40k. \end{aligned}$$

The extra 1 in the fifth frequency contributes 27.5 to this total; it must not be discarded.

Step 2: Solve for the frequency and test the equations Using the stated mean,

$$21 = \frac{1910 + 40k}{90 + 2k} \quad \Longrightarrow \quad 1890 + 42k = 1910 + 40k \quad \Longrightarrow \quad k = 10.$$

This is a nonnegative integer, as required for a frequency. The resulting total frequency is 110

and weighted sum is 2310, whose quotient is indeed 21. At $x = 10$, the four left-hand sides are, respectively,

$$-40, \qquad 74, \qquad 0, \qquad -52.$$

Only the third equation vanishes. Thus the other three options fail the defining test for a root.

Step 3: Conclusion $k = 10$ is a root of $2x^2 - 19x - 10 = 0$. Hence, the correct option is (C).

Quick Tip to Solve: Write both totals before applying the mean; a frequency such as $k + 1$ contributes to the constant as well as the coefficient of k.

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9. Let the mid points of the sides of a triangle ABC be $\left(\frac{5}{2}, 7\right)$, $\left(\frac{5}{2}, 3\right)$ and $(4, 5)$. If its incentre is (h, k), then $3h + k$ is equal to : [4 marks]

(A) 11

(B) 12

(C) 13

(D) 14

Correct Answer: (C) 13

Solution:

Concept:

- If the side midpoints are M, N, P, the vertices are $M + N - P$, $N + P - M$, and $P + M - N$. This follows by adding two midpoint equations and subtracting the third.
- The incentre is the weighted average of the vertices with weights equal to the opposite side lengths: $I = (aA + bB + cC)/(a + b + c)$.

Step 1: Recover the three vertices Put $M = (5/2, 7)$, $N = (5/2, 3)$, and $P = (4, 5)$. A convenient labelling of the original triangle is

$$A = M + N - P = (1, 5),$$
$$B = N + P - M = (4, 1),$$
$$C = P + M - N = (4, 9).$$

The midpoints of AB, BC, CA are then N, P, M, respectively, which checks that the reconstruction uses the given triangle rather than the medial triangle.

Step 2: Weight the vertices by the opposite sides The base BC has length 8, while

$$CA = AB = \sqrt{3^2 + 4^2} = 5.$$

The equal sides also show that the incentre lies on the symmetry line $y = 5$. Applying the weighted-coordinate formula gives

$$h = \frac{8(1) + 5(4) + 5(4)}{8 + 5 + 5} = \frac{8}{3},$$
$$k = \frac{8(5) + 5(1) + 5(9)}{18} = 5.$$

The weight 8 belongs to A, the vertex opposite BC, not to either endpoint of that side. Consequently

$$3h + k = 3\left(\frac{8}{3}\right) + 5 = 13.$$

The values 11, 12 and 14 do not equal this weighted-coordinate result. Using the centroid formula instead would answer a different centre problem.

Step 3: Conclusion The incentre is $(8/3, 5)$, and $3h + k = 13$. Hence, the correct option is (C).

Quick Tip to Solve: Reconstruct the vertices from the midpoint vectors before using an incentre formula; its weights are the opposite side lengths, not equal weights.

10. Let an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a < b$, pass through the point (4, 3) and have eccentricity $\frac{\sqrt{5}}{3}$. Then the length of its latus rectum is : [4 marks]

(A) $\frac{4\sqrt{5}}{3}$

(B) $2\sqrt{5}$

(C) $\frac{7\sqrt{5}}{3}$

(D) $\frac{8\sqrt{5}}{3}$

Correct Answer: (D) $\frac{8\sqrt{5}}{3}$

Solution:

Concept:

- Since $a < b$, the major semiaxis is b along the y-axis. Thus $e^2 = 1 - a^2/b^2$, with a the minor semiaxis.
- The full latus rectum of an ellipse has length twice the square of the minor semiaxis divided by the major semiaxis: $L = 2a^2/b$.

Step 1: Relate the semiaxes and use the given point Squaring the given eccentricity yields

$$\frac{5}{9} = 1 - \frac{a^2}{b^2} \quad \Longrightarrow \quad a^2 = \frac{4}{9}b^2.$$

The point $(4, 3)$ lies on the ellipse, so its coordinates satisfy

$$\frac{16}{a^2} + \frac{9}{b^2} = 1.$$

Substituting the semiaxis relation,

$$\frac{36}{b^2} + \frac{9}{b^2} = 1 \quad \Longrightarrow \quad b^2 = 45, \qquad a^2 = 20.$$

As lengths are positive, $b = 3\sqrt{5}$ and $a = 2\sqrt{5}$. These satisfy $a < b$, consistent with the orientation stated in the question.

Step 2: Calculate the full focal chord length The latus rectum is perpendicular to the major axis and

passes through a focus. Its full length is

$$L = \frac{2a^2}{b} = \frac{40}{3\sqrt{5}} = \frac{8\sqrt{5}}{3}.$$

As a direct check, the focal distance satisfies $c^2 = b^2 - a^2 = 25$. At a focus level $y = c = 5$, the ellipse equation gives $x^2/20 = 1 - 25/45 = 4/9$; doubling the positive x-coordinate gives the same result. Option (A) is only half of this full length. Options (B) and (C) do not equal $2a^2/b$, so neither satisfies the focal-chord calculation.

Step 3: Conclusion The length of the latus rectum is $\frac{8\sqrt{5}}{3}$. Hence, the correct option is (D).

Quick Tip to Solve: Identify the larger semiaxis before using eccentricity or latus-rectum formulas; the letter a need not denote the major semiaxis.

11. If $\sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right) = K$, then the value of $\sin\left(\frac{10K\pi}{3}\right)$ is: [4 marks]

(A) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(B) $\frac{\sqrt{3}-1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{1}{2}$

Correct Answer: (A) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

Solution:

Concept:

- The triple-angle product identity is $4\sin\theta\sin(\pi/3+\theta)\sin(\pi/3-\theta) = \sin 3\theta$.
- After evaluating the product, use $\sin(A+B) = \sin A\cos B + \cos A\sin B$ to find the requested exact value.

Step 1: Reduce the three-factor product Here the angles are $10^\circ, 50^\circ, 70^\circ$, so choose $\theta = \pi/18$. The other angles are $\pi/3 - \theta$ and $\pi/3 + \theta$. For completeness, the product of these last two sines is

$$\sin(\pi/3+\theta)\sin(\pi/3-\theta) = \frac{3}{4} - \sin^2\theta.$$

Multiplication by $4\sin\theta$ yields $3\sin\theta - 4\sin^3\theta = \sin 3\theta$. Thus

$$4K = \sin(\pi/6) = \frac{1}{2}, \qquad K = \frac{1}{8}.$$

All three factors are positive, which also confirms the sign of K.

Step 2: Evaluate the resulting compound angle The angle requested in the question is

$$\frac{10K\pi}{3} = \frac{10\pi}{24} = \frac{5\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}.$$

Therefore

$$\sin\frac{5\pi}{12} = \frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\frac{1}{2}$$
$$= \frac{\sqrt{3}+1}{2\sqrt{2}}.$$

The angle is 75°, so its sine is greater than $\sin 60^\circ = \sqrt{3}/2$; this rules out (C) and (D). Option (B) is approximately 0.518 and also cannot equal the sine of 75°. The exact compound-angle evaluation singles out (A).

Step 3: Conclusion $K = 1/8$ and $\sin(10K\pi/3) = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$. Hence, the correct option is (A).

Quick Tip to Solve: For three sines separated around 60°, try the triple-angle product identity before expanding each trigonometric value separately.

12. Let $S = \{x \in [-\pi, \pi] \ : \sin x(\sin x + \cos x) = a,\ a \in \mathbb{Z}\}$. Then $n(S)$ is equal to: [4 marks]

(A) 3

(B) 6

(C) 7

(D) 9

Correct Answer: (D) 9

Solution:

Concept:

- The expression can be written as $\sin^2 x + \sin x \cos x = \frac{1}{2} + \frac{\sin 2x - \cos 2x}{2}$, whose range determines the possible integer values.
- Count distinct values of x in the specified closed interval. In particular, $-\pi$ and π are different elements even though their trigonometric values coincide.

Step 1: Find the possible integer right-hand sides Since $\sin 2x - \cos 2x$ has amplitude $\sqrt{2}$,

$$\frac{1-\sqrt{2}}{2} \le \sin x(\sin x + \cos x) \le \frac{1+\sqrt{2}}{2}.$$

The lower bound lies strictly between -1 and 0, and the upper bound strictly between 1 and 2. Consequently the only possible integers are $a = 0$ and $a = 1$. No other integer needs to be tested.

Step 2: Solve and count each integer case For $a = 0$, factor directly:

$$\sin x = 0 \quad \text{or} \quad \sin x + \cos x = 0.$$

In $[-\pi, \pi]$, these give

$$\{-\pi, 0, \pi\} \quad \text{and} \quad \{-\pi/4, 3\pi/4\},$$

respectively: five distinct solutions. For $a = 1$, use $1 = \sin^2 x + \cos^2 x$ to obtain

$$\cos x(\sin x - \cos x) = 0.$$

This gives $x = \pm\pi/2$ or $x = \pi/4, -3\pi/4$: four distinct solutions. The two integer cases cannot overlap, since the same expression cannot equal both 0 and 1. Hence the total is $5 + 4 = 9$. Options 3 and 6 omit valid solutions; option 7 would result from incorrectly discarding the two included endpoints.

Step 3: Conclusion There are 9 distinct elements in S. Hence, the correct option is (D).

Quick Tip to Solve: Bound the trigonometric expression first to restrict its integer values, and then count the roots of each case with both closed-interval endpoints included.

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13. If the point of intersection of the lines $\frac{x+1}{3} = \frac{y+a}{5} = \frac{z+b+1}{7}$ and $\frac{x-2}{1} = \frac{y-b}{4} = \frac{z-2a}{7}$ lies on xy-plane, then the value of $a + b$ is: [4 marks]

(A) 2

(B) 5

(C) 7

(D) 9

Correct Answer: (C) 7

Solution:

Concept:

- A point on the xy-plane has $z = 0$. Use a separate parameter for each line and equate all three coordinates at the intersection.
- The parameters in two symmetric equations need not be equal: the direction vectors are different. The common point, rather than the parameters, must agree.

Step 1: Parameterise the two lines Write the common point using parameters t and s:

$$(x, y, z) = (3t - 1, 5t - a, 7t - b - 1)$$

$$(x, y, z) = (s + 2, 4s + b, 7s + 2a).$$

Equality of the first coordinates gives $s = 3t - 3$. Since the point lies in the xy-plane, the two expressions for its third coordinate both vanish. Hence

$$b = 7t - 1, \qquad a = -\frac{7s}{2} = \frac{21 - 21t}{2}.$$

Step 2: Use equality of the remaining coordinate The second coordinates give $5t - a = 4s + b$, and consequently

$$a + b = 5t - 4(3t - 3) = 12 - 7t.$$

The expressions obtained from $z = 0$ also give

$$a + b = \frac{21 - 21t}{2} + 7t - 1 = \frac{19 - 7t}{2}.$$

Equating these results yields $19 - 7t = 24 - 14t$, so $t = 5/7$, $s = -6/7$, $a = 3$ and $b = 4$.

Step 3: Check the common point and options Both lines now give the point $(8/7, 4/7, 0)$, confirming the plane condition as well as all coordinate equations. Thus the sum is exactly 7; the alternatives 2, 5 and 9 contradict the unique solution of these linear equations.

Step 4: Conclusion $a + b = 3 + 4 = 7$. Hence, the correct option is (C).

Quick Tip to Solve: For an intersection on a coordinate plane, set the missing coordinate to zero on both lines before equating the other coordinates.

14. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 2$ and $|\vec{b}| = 3$, then the maximum value of $3|(3\vec{a} + 2\vec{b})| + 4|(3\vec{a} - 2\vec{b})|$ is : [4 marks]

(A) 30

(B) 36

(C) 60

(D) 72

Correct Answer: (C) 60

Solution:

Concept:

- Put $\vec{u} = 3\vec{a}$ and $\vec{v} = 2\vec{b}$. Then $|\vec{u}| = |\vec{v}| = 6$, and the parallelogram identity fixes the sum of the squared lengths of $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$.
- Cauchy–Schwarz gives $(3p + 4q)^2 \le (3^2 + 4^2)(p^2 + q^2)$. Equality requires $p : q = 3 : 4$, and that ratio must be attainable by the vectors.

Step 1: Find the invariant sum of squares Let $p = |3\vec{a} + 2\vec{b}|$ and $q = |3\vec{a} - 2\vec{b}|$. Adding their squared magnitudes cancels the scalar-product terms:

$$p^2 + q^2 = 2|3\vec{a}|^2 + 2|2\vec{b}|^2 = 2(36) + 2(36) = 144.$$

The requested expression is $3p + 4q$, with both lengths nonnegative.

Step 2: Apply and attain the upper bound By Cauchy–Schwarz,

$$3p + 4q \le \sqrt{25}\sqrt{144} = 60.$$

Equality holds when $p = 36/5$ and $q = 48/5$. To check feasibility, let θ be the angle between $\vec{a}$ and $\vec{b}$. Then

$$p^2 = 72(1 + \cos\theta), \qquad q^2 = 72(1 - \cos\theta).$$

The equality values require $\cos\theta = -7/25$, a permissible cosine. Thus the upper bound is actually

reached.

Step 3: Compare the alternatives The options 30 and 36 are smaller than the attainable value 60, so neither can be the maximum. The value 72 exceeds the proven upper bound. Maximising the two magnitudes separately would be invalid because increasing one affects the other.

Step 4: Conclusion The maximum value is 60. Hence, the correct option is (C).

Quick Tip to Solve: For weighted sums of vector lengths, first look for a fixed sum of squares; then use Cauchy–Schwarz and check its equality condition.

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15. Let a line L passing through the point (1, 1, 1) be perpendicular to both the vectors $2\hat{i}+2\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+2\hat{k}$. If $P(a,b,c)$ is the foot of perpendicular from the origin on the line L, then the value of $34(a+b+c)$ is : [4 marks]

(A) 50

(B) 80

(C) 100

(D) 120

Correct Answer: (C) 100

Solution:

Concept:

- A direction vector perpendicular to two nonparallel vectors is their cross product.
- If the foot P lies on $\vec{r}=\vec{r}_0+t\vec{d}$, then $\overrightarrow{OP}\cdot\vec{d}=0$. This orthogonality determines the projection parameter.

Step 1: Find a direction for the line The two given vectors are $(2,2,1)$ and $(1,2,2)$. Their cross product is

$$(2,2,1)\times(1,2,2)=(2,-3,2).$$

Indeed, its scalar product with each original vector is zero. Therefore every point of L, which passes through $(1,1,1)$, has coordinates

$$(x,y,z)=(1,1,1)+t(2,-3,2).$$

Step 2: Impose the perpendicular-foot condition At the foot, $P=(1+2t,1-3t,1+2t)$. The vector from the origin to this point must be perpendicular to $(2,-3,2)$. Thus

$$2(1+2t)-3(1-3t)+2(1+2t)=0,$$

$$1+17t=0,\qquad t=-\frac{1}{17}.$$

It follows that $a=15/17$, $b=20/17$, and $c=15/17$.

Step 3: Evaluate and verify the requested quantity The sum of the foot coordinates is $a+b+c=50/17$,

giving

$$34(a+b+c) = 34 \times \frac{50}{17} = 100.$$

As a check, $2a - 3b + 2c = (30 - 60 + 30)/17 = 0$, so the computed point satisfies the required perpendicularity. The other values 50, 80 and 120 do not equal this uniquely determined projection result.

Step 4: Conclusion $34(a+b+c) = 100$. Hence, the correct option is (C).

Quick Tip to Solve: For the foot from the origin onto a line, use $t = -\vec{r}_0 \cdot \vec{d}/|\vec{d}|^2$ after finding any nonzero direction vector.

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16. If $\lim\limits_{x\to 2} \frac{\sin\left(x^3-5x^2+ax+b\right)}{\left(\sqrt{x-1}-1\right)\log_e(x-1)} = m$**, then** $a+b+m$ **is equal to :** [4 marks]

(A) 5

(B) 6

(C) 8

(D) 10

Correct Answer: The finite limit permits infinitely many values; option (B), 6, is obtained only for the branch k=0.

Solution:

Concept:

- A finite limit with a denominator vanishing quadratically requires the constant and linear terms in the numerator to vanish.
- For real u, $\sin u = 0$ means $u = k\pi$, $k \in \mathbb{Z}$, not necessarily $u = 0$. This distinction matters because no restriction on a, b is printed.

Step 1: Expand the denominator and sine argument Set $h = x - 2$. Then

$$(\sqrt{x-1} - 1)\log_e(x-1) = \frac{h^2}{2} + O(h^3).$$

Writing $P(x) = x^3 - 5x^2 + ax + b$, direct expansion gives

$$P(2+h) = 2a + b - 12 + (a-8)h + h^2 + h^3.$$

For a finite limit we need $\sin P(2) = 0$, so $2a + b - 12 = k\pi$ for an integer k.

Step 2: Cancel the linear term and calculate the limit At $P(2) = k\pi$, the coefficient of h in $\sin P(2+h)$ is $(-1)^k(a-8)$. It must vanish, giving $a = 8$ and $b = k\pi - 4$. Consequently

$$\sin P(2+h) = (-1)^k \sin(h^2 + h^3) = (-1)^k h^2 + O(h^3),$$

$$m = 2(-1)^k, \qquad a + b + m = 4 + k\pi + 2(-1)^k.$$

Step 3: Distinguish the listed answer from uniqueness Choosing $k = 0$ gives $a = 8$, $b = -4$, $m = 2$,

hence $a+b+m=6$, option (B). But $k=1$ gives $a=8$, $b=\pi-4$, $m=-2$, and the different value $2+\pi$, while satisfying the same finite-limit condition. Among the listed integers, only 6 belongs to the derived family: for nonzero integer k, $k\pi$ is irrational. Thus options $5, 8, 10$ cannot occur, but the unrestricted printed premise does not force a unique value.

Step 4: Conclusion The literal conditions give $a+b+m=4+k\pi+2(-1)^k$, $k\in\mathbb{Z}$, so the value is not uniquely determined. The listed answer is (B), 6, corresponding to $k=0$.

Quick Tip to Solve: When a sine tends to zero, retain every integer multiple of π; selecting zero alone requires an additional restriction on the argument.

17. If the curve $y=f(x)$ passes through the point $(1,e)$ and satisfies the differential equation $dy=y(2+\log_e x)\,dx$, $x>0$, then $f(e)$ is equal to: [4 marks]

(A) e^e

(B) e^{e^2}

(C) e^{2e}

(D) e^{2^e}

Correct Answer: (C) e^{2e}

Solution:

Concept:

- The equation $dy/dx=y(2+\ln x)$, $x>0$, is separable. The initial value $y(1)=e>0$ selects the positive solution.
- Use $\int \ln x\,dx=x\ln x-x$, and apply the initial condition after integration to fix the multiplicative constant.

Step 1: Separate variables and integrate For the solution through $(1,e)$, separate the variables:

$$\frac{dy}{y}=(2+\ln x)\,dx.$$

Integrating both sides gives

$$\ln y=2x+x\ln x-x+C=x+x\ln x+C.$$

The zero solution of the differential equation is irrelevant because it cannot pass through the given point. Since the coefficient $2+\ln x$ is continuous for $x>0$, the positive initial value determines a solution throughout that interval.

Step 2: Use the initial condition and evaluate At $x=1$, $y=e$ and $\ln 1=0$. Thus $1=1+C$, so $C=0$. Therefore

$$f(x)=e^{x+x\ln x},\qquad f(e)=e^{e+e\ln e}=e^{2e}.$$

A direct derivative check gives $f'(x)/f(x)=1+(\ln x+1)=2+\ln x$, which verifies the original equation.

Step 3: Check the other exponential forms Option (A) has exponent e, omitting one of the two equal terms at $x = e$. Option (B) has exponent e^2, which is not $2e$. In option (D), the exponent is 2^e, rather than $2e$; these differ as well. For example, $\log(2^e/(2e)) = (e-1)\ln 2 - 1 > 0$. Only option (C) has the exponent obtained by substitution.

Step 4: Conclusion The requested function value is e^{2e}. Hence, the correct option is (C).

Quick Tip to Solve: After solving a separable equation, differentiate its logarithm to check the differential equation before substituting the requested input.

18. The number of critical points of the function

$f(x) = \begin{cases} |\frac{\sin x}{x}|, & x \neq 0 \\ 1, & x = 0 \end{cases}$ **in the interval** $(-2\pi, 2\pi)$ **is equal to :** [4 marks]

(A) 1

(B) 3

(C) 5

(D) 7

Correct Answer: (C) 5

Solution:

Concept:

- A critical point is an interior domain point where the derivative vanishes or does not exist. Absolute values can create critical points at zeros of their inner function.
- The continuous extension of $\sin x/x$ at zero is even and differentiable there. Away from zeros, stationary points satisfy $x\cos x - \sin x = 0$.

Step 1: Check zero and the absolute-value corners Near zero, $\sin x/x$ is positive and

$$f(x) = 1 - \frac{x^2}{6} + O(x^4).$$

Thus $f'(0) = 0$, so zero is one critical point. The zeros $x = \pm\pi$ lie inside the given interval. Since $(\sin x/x)'$ is nonzero at each of them, taking the absolute value creates a corner. Hence both π and $-\pi$ are critical points. The endpoints $\pm 2\pi$ are excluded.

Step 2: Count the remaining stationary points For $x \neq 0, \pm\pi$, the derivative is zero exactly when

$$g(x) = x\cos x - \sin x = 0.$$

For $0 < x < \pi$, $g'(x) = -x\sin x < 0$ and $g(0) = 0$, so there is no positive root there. On $(\pi, 2\pi)$, $g'(x) > 0$; also

$$g(\pi) = -\pi < 0, \qquad g(2\pi) = 2\pi > 0.$$

Continuity and strict increase therefore give exactly one root in that interval (in fact before $3\pi/2$). Evenness gives exactly one corresponding negative stationary point.

Step 3: Combine the distinct critical points There are three stationary points: zero and the symmetric nonzero pair. There are also the two nondifferentiable minima at $\pm\pi$. The total is $3 + 2 = 5$. Option (A) misses the outer lobes and corners; option (B) counts only stationary points; option (D) would overcount, for example by including the excluded endpoints.

Step 4: Conclusion The number of critical points in $(-2\pi, 2\pi)$ is 5. Hence, the correct option is (C).

Quick Tip to Solve: For an absolute-value function, count both stationary points and corners; a derivative-zero calculation alone can miss critical points.

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19. Let [.] denote the greatest integer function. Then the value of $\int_0^3 \left(\frac{e^x+e^{-x}}{[x]!}\right) dx$ is : [4 marks]

(A) $e^2 + e^3 - \frac{1}{e^2} - \frac{1}{e^3}$
(B) $\frac{1}{2}\left(e^2 + e^3 - \frac{1}{e^2} - \frac{1}{e^3}\right)$
(C) $e^2 + e^3 - \frac{1}{2e^2} - \frac{1}{2e^3}$
(D) $\frac{1}{2}(e^2 + e^3) - \frac{1}{e^2} - \frac{1}{e^3}$
Correct Answer: (B) $\frac{1}{2}\left(e^2 + e^3 - \frac{1}{e^2} - \frac{1}{e^3}\right)$

Solution:

Concept:

- The greatest integer $[x]$ is constant on each interval between consecutive integers. Split an integral wherever this value changes.
- Both $0!$ and $1!$ equal 1, whereas $2! = 2$. An antiderivative of $e^x + e^{-x}$ is $F(x) = e^x - e^{-x}$.

Step 1: Split the integral at the integers For $0 \le x < 1$, the denominator is $0! = 1$; for $1 \le x < 2$, it is $1! = 1$; and for $2 \le x < 3$, it is $2! = 2$. Values at the isolated endpoints do not change the definite integral. Consequently,

$$I = \int_0^1 (e^x + e^{-x})\,dx + \int_1^2 (e^x + e^{-x})\,dx + \frac{1}{2}\int_2^3 (e^x + e^{-x})\,dx.$$

The first two pieces can be combined because their integrands have the same coefficient.

Step 2: Evaluate and combine the boundary terms Using $F(0) = 0$, evaluate the two resulting integrals:

$$\begin{aligned} I &= F(2) - F(0) + \frac{1}{2}\{F(3) - F(2)\} \\ &= \frac{1}{2}F(2) + \frac{1}{2}F(3) \\ &= \frac{1}{2}\left(e^2 + e^3 - e^{-2} - e^{-3}\right). \end{aligned}$$

Thus all four exponential terms have coefficient magnitude $1/2$. Option (A) is twice this value, while (C) and (D) weight the positive and negative exponential terms unequally. Only (B) retains the correct coefficients and signs.

Step 3: Conclusion The integral equals $\frac{1}{2}\left(e^2+e^3-\frac{1}{e^2}-\frac{1}{e^3}\right)$. Hence, the correct option is (B).

Quick Tip to Solve: Combine the intervals with denominator $0!$ and $1!$ before integrating; they have identical weights.

20. Let $y=y(x)$ be the solution curve of the differential equation $(1+\sin x)\frac{dy}{dx}+(y+1)\cos x=0,\ y(0)=0$. If the curve $y=y(x)$ passes through the point $\left(\alpha,\frac{-1}{2}\right)$, then a value of α is: [4 marks]

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Correct Answer: (D) $\frac{\pi}{2}$

Solution:

Concept:

- The product rule gives $\frac{d}{dx}[(y+1)(1+\sin x)]=(1+\sin x)y'+(y+1)\cos x$.
- A zero derivative makes the product constant along the solution curve. Use the initial condition to determine that constant before substituting the required ordinate.

Step 1: Integrate the product derivative Recognise the entire left side of the differential equation as one derivative:

$$\frac{d}{dx}\big[(y+1)(1+\sin x)\big]=0.$$

Hence $(y+1)(1+\sin x)=C$. At $x=0$, the initial condition gives $y=0$, so

$$C=(0+1)(1+\sin 0)=1.$$

Therefore, on the solution interval containing zero,

$$y=\frac{1}{1+\sin x}-1=-\frac{\sin x}{1+\sin x}.$$

The denominator is nonzero throughout the interval from 0 to $\pi/2$, so the required candidate point is on this solution branch.

Step 2: Impose the required ordinate Substitute $y=-1/2$ into the constant-product relation:

$$\frac{1}{2}(1+\sin\alpha)=1 \quad\Longrightarrow\quad \sin\alpha=1.$$

Thus a possible value is $\alpha=\pi/2$. The other three listed angles have sine $1/2$, $1/\sqrt{2}$ and $\sqrt{3}/2$,

respectively; each is less than 1 and therefore fails the required equation. Direct substitution at $\pi/2$ gives $y = -1/(1+1) = -1/2$, confirming the point.

Step 3: Conclusion A value of α is $\boxed{\pi/2}$. Hence, the correct option is (D).

Quick Tip to Solve: Before separating variables, check whether the two terms form a product derivative; this avoids unnecessary logarithms.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. If the domain of the function

$f(x) = \sqrt{\log_{(0.6)}\left(\left|\frac{2x-5}{x^2-4}\right|\right)}$ **is** $(-\infty, a] \cup \{b\} \cup [c, d) \cup (e, \infty)$**, then the value of** $a + b + c + d + e$ **is** _____.

[4 marks]

Solution:

Concept:

- A real square root requires its radicand to be nonnegative. Since $0 < 0.6 < 1$, the logarithm $\log_{0.6} t$ is nonnegative precisely when $0 < t \leq 1$.
- The rational expression must also be defined: $x \neq \pm 2$. Its absolute value must be nonzero, excluding $x = 5/2$.

Step 1: Translate the domain conditions Combining the logarithm and square-root restrictions gives

$$0 < \left|\frac{2x-5}{x^2-4}\right| \leq 1, \qquad x \neq -2, 2, \frac{5}{2}.$$

For a nonzero denominator, squaring the upper inequality and multiplying by its positive square is reversible. Thus

$$(2x-5)^2 \leq (x^2-4)^2.$$

Factor the difference of squares rather than expanding to a general quartic:

$$\begin{aligned}(x^2-4)^2 - (2x-5)^2 &= (x^2-2x+1)(x^2+2x-9) \\ &= (x-1)^2\big((x+1)^2 - 10\big).\end{aligned}$$

Step 2: Solve the sign condition and retain the isolated point The squared factor is positive except at $x = 1$, where the product is zero. Therefore the inequality holds when

$$x \leq -1 - \sqrt{10}, \qquad x = 1, \qquad x \geq -1 + \sqrt{10}.$$

At $x = 1$, the absolute ratio is exactly 1, so this isolated point is valid. Neither pole ± 2 lies in these intervals. However, $5/2 > -1 + \sqrt{10}$ must be removed because it gives a zero logarithm argument. The domain is consequently

$$(-\infty, -1 - \sqrt{10}] \cup \{1\} \cup [-1 + \sqrt{10}, 5/2) \cup (5/2, \infty).$$

Reading the five endpoints in the stated order gives

$$a+b+c+d+e=(-1-\sqrt{10})+1+(-1+\sqrt{10})+\frac{5}{2}+\frac{5}{2}=4.$$

Step 3: Conclusion The requested numerical value is $\boxed{4}$.

Quick Tip to Solve: When a sign inequality contains a squared factor, inspect its zero separately: here it creates the isolated domain point $x=1$.

22. If $\sum_{k=1}^{n} a_k = 6n^3$, then $\sum_{k=1}^{6} \left(\frac{a_{k+1}-a_k}{36}\right)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- For partial sums $S_n=\sum_{k=1}^{n} a_k$, individual terms satisfy $a_k=S_k-S_{k-1}$, taking $S_0=0$.
- The difference of successive quadratic terms is linear. After simplifying that difference, use $\sum_{k=1}^{n} k^2=n(n+1)(2n+1)/6$.

Step 1: Recover the general term from the partial sums The statement specifies $S_n=6n^3$, which is a partial sum, not the formula for an individual term. Subtract consecutive partial sums:

$$\begin{aligned} a_k &= 6k^3-6(k-1)^3 \\ &= 6\left(3k^2-3k+1\right) \\ &= 18k^2-18k+6. \end{aligned}$$

This formula also applies at $k=1$, giving $a_1=6=S_1$, because the empty partial sum is zero. Keeping this first term in the same formula allows all six terms in the requested sum to be handled uniformly.

Step 2: Take the next difference and sum its squares Replace k by $k+1$ in the expression for a_k:

$$a_{k+1}=18(k+1)^2-18(k+1)+6=18k^2+18k+6.$$

Consequently,

$$a_{k+1}-a_k=36k, \qquad \left(\frac{a_{k+1}-a_k}{36}\right)^2=k^2.$$

The requested sum is therefore

$$\sum_{k=1}^{6} k^2=\frac{6\cdot 7\cdot 13}{6}=91.$$

As a direct arithmetic check, the six summands are $1,4,9,16,25,36$, whose sum is also 91. The upper limit requires a_7, which is already covered by the general expression; no extra term is added to the six-term sum.

Step 3: Conclusion The value of the given sum is $\boxed{91}$.

Quick Tip to Solve: Use finite differences twice: a cubic partial sum gives quadratic terms, then a linear successive difference.

23. Let a, b, c ∈ {1, 2, 3, 4}. If the probability, that $ax^2+2\sqrt{2}bx+c>0$ for all $x \in \mathbb{R}$, is $\frac{m}{n}$, $\gcd(m,n)=1$, then $m+n$ is equal to _____. [4 marks]

Solution:

Concept:

- A quadratic $ax^2 + Bx + c$ is strictly positive for every real x when $a > 0$ and its discriminant $B^2 - 4ac$ is strictly negative.
- Each of a, b, c is chosen from four values, so there are $4^3 = 64$ equally likely ordered triples. Count ordered pairs (a, c) for each fixed b.

Step 1: Convert positivity to a counting inequality All possible leading coefficients a are positive. Thus the only additional condition is a negative discriminant:

$$(2\sqrt{2}\,b)^2 - 4ac < 0 \quad \Longleftrightarrow \quad ac > 2b^2.$$

Equality is excluded, since a zero discriminant would make the quadratic zero at its vertex, violating strict positivity. We now count products ac among the sixteen ordered pairs from $\{1, 2, 3, 4\}^2$.

Step 2: Count favourable triples for each value of b For $b = 1$, the condition is $ac > 2$. Only $(1, 1), (1, 2), (2, 1)$ fail, so there are $16 - 3 = 13$ favourable pairs.

For $b = 2$, the condition is $ac > 8$. The favourable pairs are exactly

$$(3, 3),\ (3, 4),\ (4, 3),\ (4, 4),$$

giving 4 more triples. Products equal to 8, from $(2, 4)$ and $(4, 2)$, do not qualify.

For $b = 3$ or 4, the required product exceeds 18 or 32 respectively. Since $ac \le 16$, neither value contributes any favourable triple. The total count is therefore $13 + 4 = 17$.

Step 3: Reduce the probability and add its numerator and denominator Hence the probability is $17/64$. Since 17 is prime and does not divide 64, this fraction is already in lowest terms. Therefore $m = 17$, $n = 64$, and

$$m + n = 17 + 64 = 81.$$

The ordering of a and c matters in the sample space, so pairs such as $(3, 4)$ and $(4, 3)$ were correctly counted separately.

Step 4: Conclusion The probability is $17/64$, and the required value is $\boxed{81}$.

Quick Tip to Solve: For strict positivity of an upward quadratic, use $D < 0$, not $D \le 0$; boundary product cases must be excluded.

24. Let a circle C have its centre in the first quadrant, intersect the coordinate axes at exactly three points and cut off equal intercepts from the coordinate axes. If the length of the chord of C on the line $x + y = 1$ is $\sqrt{14}$, then the square of the radius of C is _____. [4 marks]

Solution:

Concept:

- A circle with centre (h, k) and radius r cuts axis chords of lengths $2\sqrt{r^2 - k^2}$ and $2\sqrt{r^2 - h^2}$.
- A chord at perpendicular distance d from the centre has length $L = 2\sqrt{r^2 - d^2}$. A circle passing through the origin satisfies $r^2 = h^2 + k^2$.

Step 1: Use equal intercepts and the number of intersections Equal axis intercept lengths imply $h^2 = k^2$. The centre is in the first quadrant, so $h = k = t > 0$. Equal intercepts also give the same number of intersections with each axis: two with each when the chords are nonzero, or one with each in the tangent case.

The tangent case cannot yield three distinct points. Therefore each axis meets the circle twice. To obtain exactly three distinct points in their union, one intersection must belong to both axes, and their only common point is the origin. Hence the circle passes through $(0, 0)$, giving

$$r^2 = t^2 + t^2 = 2t^2.$$

Its other axis intersections are then $(2t, 0)$ and $(0, 2t)$, consistent with the required count and equal intercepts.

Step 2: Apply the chord length on the given line The perpendicular distance from (t, t) to $x + y = 1$ is

$$d = \frac{|2t - 1|}{\sqrt{2}}.$$

Using the specified chord length $L = \sqrt{14}$,

$$\begin{aligned} 14 &= 4(r^2 - d^2) \\ &= 4\left(2t^2 - \frac{(2t-1)^2}{2}\right) \\ &= 8t - 2. \end{aligned}$$

Thus $t = 2$, which is positive as required, and $r^2 = 2(2)^2 = 8$. As a check, $d^2 = 9/2$, so $4(r^2 - d^2) = 4(8 - 9/2) = 14$, reproducing the stated chord length.

Step 3: Conclusion The square of the radius of the circle is $\boxed{8}$.

Quick Tip to Solve: Three distinct axis intersections with equal nonzero intercepts force the circle through the origin; use this before the chord formula.

25. If $\alpha = \int_0^{2\sqrt{3}} \log_2(x^2+4)\,dx + \int_2^4 \sqrt{2^x-4}\,dx$, then α^2 is equal to _____. [4 marks]

Solution:

Concept:

- For a continuous increasing function $y = f(t)$, its inverse satisfies $\int_a^b f(t)\,dt + \int_{f(a)}^{f(b)} f^{-1}(y)\,dy = bf(b) - af(a)$.
- The identity follows from integration by parts: the two integrals account for the complementary areas bounded by a function and its inverse. The interval and inverse branch must be checked before using it.

Step 1: Identify the inverse function Put $f(t) = \log_2(t^2+4)$ for $t \geq 0$. Solving $y = f(t)$ gives $2^y = t^2+4$, hence $t = \sqrt{2^y-4}$. We take the nonnegative square root because the first integral uses only nonnegative values of its variable. Also, f is strictly increasing on the integration interval, so this is its single-valued inverse.

Step 2: Check the endpoints and combine the integrals At the endpoints, $f(0) = \log_2 4 = 2$ and $f(2\sqrt{3}) = \log_2 16 = 4$. Thus the second integral has exactly the required inverse-function limits. More explicitly, set $y = f(t)$ in it and integrate by parts:

$$\int_2^4 \sqrt{2^y-4}\,dy = \int_0^{2\sqrt{3}} tf\prime(t)\,dt$$
$$= [tf(t)]_0^{2\sqrt{3}} - \int_0^{2\sqrt{3}} f(t)\,dt.$$

Adding the first integral cancels the remaining integral and yields $\alpha = 4(2\sqrt{3}) = 8\sqrt{3}$.

Step 3: Square the resulting value The required quantity is the square of the sum, not the sum itself. Therefore

$$\alpha^2 = (8\sqrt{3})^2 = 64 \times 3 = 192.$$

No rounding is required, and the expression is dimensionless.

Step 4: Conclusion The required numerical value is $\boxed{192}$.

Quick Tip to Solve: When two integrals contain inverse functions, verify that their limits correspond; their sum may reduce directly to an endpoint product.

26. The dimensional formula of $\frac{1}{2}\epsilon_0 E^2$ (ϵ_0 = permittivity of vacuum and E = electric field) is $M^a L^b T^c$. The value of $2a - b + c =$ _____. [4 marks]

(A) 0

(B) 1

(C) -1

(D) 2

Correct Answer: (B) 1

Solution:

Concept:

- The electric-field energy density in vacuum is $u = \frac{1}{2}\epsilon_0 E^2$. Energy density means energy per unit volume.
- The dimensional formula of energy is $[ML^2T^{-2}]$, while volume has dimensions $[L^3]$. Numerical constants such as $1/2$ have no dimensions.

Step 1: Obtain the dimensional formula Dividing energy by volume gives

$$\left[\frac{1}{2}\epsilon_0 E^2\right] = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}.$$

This is also the dimensional formula of pressure, as expected for field energy per unit volume. It is not the formula for total electrostatic energy: the latter would contain L^2, whereas the energy density contains L^{-1}.

Step 2: Read the exponents and evaluate the combination Comparison with the given form $M^a L^b T^c$ gives $a = 1$, $b = -1$ and $c = -2$. Substitution then gives

$$2a - b + c = 2(1) - (-1) + (-2) = 2 + 1 - 2 = 1.$$

In particular, subtracting b adds one because b is negative. The alternatives 0, -1 and 2 do not equal this combination and are therefore ruled out.

Step 3: Conclusion The value of $2a - b + c$ is $\boxed{1}$. Hence, the correct option is (B).

Quick Tip to Solve: Recognise $\epsilon_0 E^2/2$ as energy density before expanding the separate dimensions of permittivity and electric field.

27. The diameter of a wire measured by a screw gauge of least count 0.001 cm is 0.08 cm. The length measured by a scale of least count 0.1 cm is 150 cm. When a weight of 100 N is applied to the wire, the extension in length is 0.5 cm, measured by a micrometer of least count 0.001 cm. The error in the measured Young's modulus is $\alpha \times 10^9$ N/m^2. The value of α is _____.

(Ignore the contribution of the load to Young's modulus error calculation) [4 marks]

(A) 1.3

(B) 1.65

(C) 0.13

(D) 0.25

Correct Answer: (B) 1.65

Solution:

Concept:

- For a wire of diameter d, original length L, extension ℓ and applied force F, Young's modulus is $Y = 4FL/(\pi d^2 \ell)$.
- The maximum fractional error in a product or quotient is the sum of the absolute fractional errors multiplied by their powers. Ignoring load uncertainty gives $\Delta Y/Y = \Delta L/L + 2\Delta d/d + \Delta \ell/\ell$.

Step 1: Calculate the measured modulus Convert the dimensions to SI units: $L = 1.50\,\text{m}$, $d = 8.0 \times 10^{-4}\,\text{m}$ and $\ell = 5.0 \times 10^{-3}\,\text{m}$. Then

$$Y = \frac{4(100)(1.50)}{\pi(8.0 \times 10^{-4})^2(5.0 \times 10^{-3})} = 5.968 \times 10^{10}\,\text{N/m}^2.$$

The diameter must be squared because the cross-sectional area is $\pi d^2/4$.

Step 2: Add the fractional measurement errors Take each specified least count as the absolute uncertainty in that measurement. Ratios can be calculated directly in centimetres, since each numerator and denominator has the same unit:

$$\begin{aligned}\frac{\Delta Y}{Y} &= \frac{0.1}{150} + 2\frac{0.001}{0.08} + \frac{0.001}{0.5} \\ &= 0.0006667 + 0.025 + 0.002 \\ &= 0.0276667.\end{aligned}$$

The diameter contributes the largest uncertainty, and its contribution is doubled by the square in the modulus formula.

Step 3: Convert the fractional error to absolute error Multiply by the measured modulus:

$$\Delta Y = (5.968 \times 10^{10})(0.0276667) \approx 1.65 \times 10^9\,\text{N/m}^2.$$

Thus $\alpha = 1.65$. The other coefficients 1.3, 0.13 and 0.25 do not result from including all three specified uncertainties with their correct powers.

Step 4: Conclusion The error is approximately $1.65 \times 10^9\,\text{N/m}^2$, so $\boxed{\alpha = 1.65}$. Hence, the correct option is (B).

Quick Tip to Solve: For a wire modulus, write the diameter as d^2 before propagating errors: its fractional uncertainty is counted twice.

28. The velocity of a particle is given as $\vec{v} = -x\hat{i} + 2y\hat{j} - z\hat{k}$ m/s. The magnitude of acceleration at point (1, 2, 4) is _____ m/s^2. [4 marks]

(A) $\sqrt{6}$

(B) 9

(C) $\sqrt{33}$

(D) 0

Correct Answer: (B) 9

Solution:

Concept:

- Acceleration is the total time derivative of velocity along the particle's motion: $\vec{a} = d\vec{v}/dt$. A velocity with no explicit time variable can still change as the position changes.
- The Cartesian velocity components satisfy $\dot{x} = v_x$, $\dot{y} = v_y$ and $\dot{z} = v_z$. Apply the chain rule to each component before inserting the coordinates.

Step 1: Differentiate the three velocity components The given field implies $\dot{x} = -x$, $\dot{y} = 2y$ and $\dot{z} = -z$. Therefore

$$a_x = \frac{d(-x)}{dt} = -\dot{x} = x,$$
$$a_y = \frac{d(2y)}{dt} = 2\dot{y} = 4y,$$
$$a_z = \frac{d(-z)}{dt} = -\dot{z} = z.$$

Thus the acceleration vector is $\vec{a} = x\hat{i} + 4y\hat{j} + z\hat{k}$ in SI units. Taking only an explicit partial derivative with respect to time would incorrectly discard the change caused by motion through the field.

Step 2: Evaluate the vector and its magnitude At $(x, y, z) = (1, 2, 4)$, $\vec{a} = \hat{i} + 8\hat{j} + 4\hat{k}\,\text{m/s}^2$. Its magnitude follows from the sum of squares of its perpendicular components:

$$|\vec{a}| = \sqrt{1^2 + 8^2 + 4^2} = \sqrt{81} = 9\,\text{m/s}^2.$$

Zero is excluded because the particle moves through a nonuniform velocity field. The values $\sqrt{6}$ and $\sqrt{33}$ are also excluded by the computed acceleration components.

Step 3: Conclusion The acceleration magnitude is $\boxed{9\,\text{m/s}^2}$. Hence, the correct option is (B).

Quick Tip to Solve: For $v_x = f(x)$, use $a_x = v_x\, dv_x/dx$; differentiate along the motion rather than holding position fixed.

29. The position of an object having mass 0.1 kg as a function of time t is given as $\vec{r} = \left(10t^2\hat{i} + 5t^3\hat{j}\right)$ m. At $t = 1$ s, which of the following statements are correct?

A. The linear momentum $\vec{p} = \left(2\hat{i} + 1.5\hat{j}\right)$ kg·m/s.

B. The force acting on the object $\vec{F} = \left(2\hat{i} + 3\hat{j}\right)$ N.

C. The angular momentum of the object about its origin $\vec{L} = 15\hat{k}$ J·s.

D. The torque acting on the object about its origin $\vec{\tau} = 20\hat{k}$ N·m.

Choose the correct answer from the options given below: [4 marks]

(A) A, B and C only

(B) B, C and D only

(C) A, C and D only

(D) A, B and D only

Correct Answer: (D) A, B and D only

Solution:

Concept:

- For constant mass, $\vec{p} = m\dot{\vec{r}}$ and $\vec{F} = m\ddot{\vec{r}}$. Differentiate the given position before substituting the time.
- Angular momentum and torque about the origin are $\vec{L} = \vec{r} \times \vec{p}$ and $\vec{\tau} = \vec{r} \times \vec{F}$. In the xy-plane, their z-components are $xp_y - yp_x$ and $xF_y - yF_x$.

Step 1: Determine momentum and force Differentiating the position gives

$$\vec{v} = 20t\hat{i} + 15t^2\hat{j}, \qquad \vec{a} = 20\hat{i} + 30t\hat{j}.$$

At $t = 1\,\text{s}$, multiplication by $m = 0.1\,\text{kg}$ gives

$$\vec{p} = (2\hat{i} + 1.5\hat{j})\,\text{kg m/s}, \qquad \vec{F} = (2\hat{i} + 3\hat{j})\,\text{N}.$$

Hence statements A and B are both correct.

Step 2: Calculate the cross products about the origin At the same instant the position is $\vec{r} = (10\hat{i} + 5\hat{j})\,\text{m}$. Thus

$$\vec{L} = [10(1.5) - 5(2)]\hat{k} = 5\hat{k}\,\text{J s},$$
$$\vec{\tau} = [10(3) - 5(2)]\hat{k} = 20\hat{k}\,\text{N m}.$$

Statement C is false: its value 15 omits the second term in the cross product. Statement D is correct, including its positive $\hat{k}$ direction.

Step 3: Select the combination of true statements The true statements are A, B and D. Each of options (A), (B) and (C) includes the false statement C, so all three are eliminated. Option (D) includes exactly the three true statements and excludes C.

Step 4: Conclusion Only statements A, B and D are correct. Hence, the correct option is (D).

Quick Tip to Solve: In planar motion always retain both terms $xp_y - yp_x$; using only xp_y gives the wrong angular momentum.

30. A planet (P_1) is moving around the star of mass $2M$ in the orbit of radius R. Another planet (P_2) is moving around another star of mass $4M$ in a orbit of radius $2R$. Ratio of time periods of revolution of P_2 and P_1 is _____. [4 marks]

(A) $\frac{1}{2}$

(B) 2

(C) 4

(D) $\frac{1}{4}$

Correct Answer: (B) 2

Solution:

Concept:

- For a circular orbit of radius r about a star of mass M_s, gravity supplies centripetal force: $GM_s m/r^2 = mv^2/r$.
- The resulting period is $T = 2\pi\sqrt{r^3/(GM_s)}$. Thus periods around different stars depend on the central masses as well as on orbital radii.

Step 1: Write the period ratio with both central masses Let the stellar masses for the two systems be $M_1 = 2M$ and $M_2 = 4M$, while their orbital radii are $r_1 = R$ and $r_2 = 2R$. Using the period formula cancels the common factors 2π and G:

$$\frac{T_2}{T_1} = \sqrt{\frac{r_2^3/M_2}{r_1^3/M_1}} = \left(\frac{r_2}{r_1}\right)^{3/2}\left(\frac{M_1}{M_2}\right)^{1/2}.$$

The mass of either planet cancels in deriving its period under the usual negligible-planet-mass approximation.

Step 2: Insert the ratios and choose the option The orbital radius doubles and the central stellar mass also doubles. Therefore

$$\frac{T_2}{T_1} = \sqrt{2^3\frac{2M}{4M}} = \sqrt{8 \times \frac{1}{2}} = \sqrt{4} = 2.$$

The value 4 is the ratio of the squared periods, not the periods. The values $1/2$ and $1/4$ reverse the relevant ratios and do not represent the requested T_2/T_1. The larger orbit therefore takes twice as long despite the more massive central star.

Step 3: Conclusion The requested ratio is $\boxed{T_2/T_1 = 2}$. Hence, the correct option is (B).

Quick Tip to Solve: When two planets orbit different stars, use $T^2 \propto r^3/M_s$; the simpler radius-only relation applies to a common central mass.

31. A particle is rotating in a circular path and at any instant its motion can be described as $\theta = \frac{5t^4}{40} - \frac{t^3}{3}$.

The angular acceleration of the particle after 10 seconds is _____ rad/s^2. [4 marks]

(A) 150

(B) 120

(C) 130

(D) 170

Correct Answer: (C) 130

Solution:

Concept:

- Angular velocity is the rate of change of angular displacement: $\omega = d\theta/dt$. Angular acceleration is its second time derivative, $\alpha = d\omega/dt = d^2\theta/dt^2$.
- Apply the power rule to each term before substituting the specified time. The signs of both terms must be retained when differentiating.

Step 1: Simplify and differentiate the angular displacement The given angular displacement can be written as $\theta = t^4/8 - t^3/3$. Therefore,

$$\omega = \frac{d}{dt}\left(\frac{t^4}{8} - \frac{t^3}{3}\right) = \frac{t^3}{2} - t^2.$$

This expression is angular velocity, in radians per second; it is not yet the angular acceleration requested.

Step 2: Differentiate again and substitute the time Differentiating the angular velocity gives

$$\alpha = \frac{d}{dt}\left(\frac{t^3}{2} - t^2\right) = \frac{3t^2}{2} - 2t.$$

At $t = 10\,\text{s}$,

$$\alpha = \frac{3}{2}(10)^2 - 2(10) = 150 - 20 = 130\,\text{rad}\,\text{s}^{-2}.$$

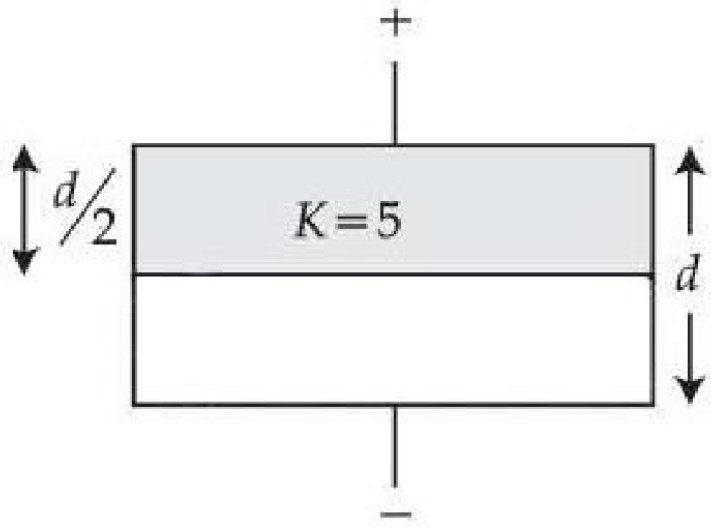
The positive value means that angular velocity is increasing in the chosen positive angular direction at that instant.

Step 3: Check the listed alternatives The value 150 retains only the contribution of the fourth-power term and omits the contribution -20. The value 170 adds that contribution with the wrong sign. The remaining value 120 does not follow from the second derivative. Thus only 130 satisfies the given time-dependent motion.

Step 4: Conclusion The angular acceleration after 10 seconds is $\boxed{130\,\text{rad}\,\text{s}^{-2}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a given angular displacement function, differentiate twice before substituting time; substituting first destroys the time dependence needed for the derivative.

32. A parallel plate air capacitor has a capacitance C. When it is half filled as shown in figure with a dielectric constant $K = 5$, the percentage increase in the capacitance is _____. [4 marks]



(A) 33.34

(B) 66.67

(C) 200

(D) 400

Correct Answer: (B) 66.67

Solution:

Concept:

- For a parallel-plate air capacitor of plate area A and separation d, $C = \varepsilon_0 A/d$. A dielectric multiplies the capacitance of a completely filled region by its relative permittivity K.
- Layers stacked along the electric field have the same free charge and their potential drops add. They therefore behave as capacitors in series, not in parallel.

Step 1: Identify the two layers in the figure The dielectric occupies the full plate area but only half the separation, $d/2$. The remaining air layer also has thickness $d/2$. Their separate capacitances are

$$C_K = \frac{K\varepsilon_0 A}{d/2} = 2KC, \qquad C_{\text{air}} = \frac{\varepsilon_0 A}{d/2} = 2C.$$

The arrangement is in series because a field line crosses both layers between the plates.

Step 2: Combine the capacitances Let C_f denote the final capacitance. Adding the reciprocals gives

$$\frac{1}{C_f} = \frac{1}{2KC} + \frac{1}{2C} = \frac{K+1}{2KC}.$$

Hence, for $K = 5$,

$$C_f = \frac{2K}{K+1}C = \frac{10}{6}C = \frac{5}{3}C.$$

This is smaller than $5C$, as expected because the gap is not filled entirely with dielectric.

Step 3: Calculate the percentage increase and reject the alternatives The requested increase is relative to the original capacitance:

$$\frac{C_f - C}{C} \times 100 = \left(\frac{5}{3} - 1\right) 100 = \frac{200}{3}\% = 66.67\%.$$

The value 200% would apply to half the plate area filled across the full gap, a parallel arrangement. The value 400% requires the entire gap to be filled. The value 33.34% is not the fractional increase obtained here.

Step 4: Conclusion The capacitance increases by $\boxed{66.67\%}$. Hence, the correct option is (B).

Quick Tip to Solve: Decide series versus parallel from the field direction: layers along the plate separation are in series; regions side by side across the plate area are in parallel.

33. Heat is supplied to a diatomic gas at constant pressure. Then the ratio of ΔQ : ΔU : ΔW is _____. [4 marks]

(A) 2 : 3 : 5

(B) 5 : 3 : 2

(C) 2 : 5 : 7

(D) 7 : 5 : 2

Correct Answer: (D) 7 : 5 : 2

Solution:

Concept:

- In the usual ideal-diatomic-gas model, three translational and two rotational degrees of freedom contribute, so $C_V = 5R/2$ and $C_P = C_V + R = 7R/2$.
- With work defined as work done by the gas, the first law is $\Delta Q = \Delta U + \Delta W$. During constant-pressure heating, $\Delta W = P\Delta V = nR\Delta T$.

Step 1: Write the internal-energy change For n moles heated through ΔT, the internal energy of an ideal gas changes only with temperature. Thus,

$$\Delta U = nC_V\Delta T = \frac{5}{2}nR\Delta T.$$

This includes the translational and rotational energy of the diatomic molecules; no vibrational contribution is included in the standard model used for these heat capacities.

Step 2: Write the heat supplied and expansion work At constant pressure, the heat input is

$$\Delta Q = nC_P\Delta T = \frac{7}{2}nR\Delta T.$$

The ideal-gas equation at fixed pressure gives $P\Delta V = nR\Delta T$, and hence

$$\Delta W = nR\Delta T.$$

Their signs are positive during heating and expansion. The heat supplied equals the sum of the internal-energy increase and the work done.

Step 3: Form the ratio in the requested order Cancel the common positive factor $nR\Delta T/2$ to obtain

$$\Delta Q : \Delta U : \Delta W = 7 : 5 : 2.$$

The alternatives $2 : 3 : 5$ and $2 : 5 : 7$ do not satisfy heat input equal to the sum of the other two entries. The ratio $5 : 3 : 2$ satisfies energy conservation but uses the monatomic value $C_V = 3R/2$, so it does not describe this diatomic gas.

Step 4: Conclusion The required ratio is $\boxed{7 : 5 : 2}$. Hence, the correct option is (D).

Quick Tip to Solve: For an ideal gas with f active degrees of freedom heated at constant pressure, use $\Delta Q : \Delta U : \Delta W = (f + 2) : f : 2$.

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34. Two charged conducting spheres S_1 and S_2 of radii 8 cm and 18 cm are connected to each other by a wire. After equilibrium is established, the ratio of electric fields on S_1 and S_2 spheres are E_{S1} and E_{S2} respectively. The value of $\frac{E_{S1}}{E_{S2}}$ is _____. [4 marks]

(A) $\frac{3}{2}$

(B) $\frac{2}{3}$

(C) $\frac{4}{9}$

(D) $\frac{9}{4}$

Correct Answer: (D) $\frac{9}{4}$

Solution:

Concept:

- A conducting wire allows charge to redistribute until the connected conductors are at the same electric potential. Equal potential does not require equal charge or equal surface electric field.
- For the usual spherical-conductor model, the surface potential is $V = kQ/R$ and the electric field immediately outside is $E = kQ/R^2 = V/R$, where $k = 1/(4\pi\varepsilon_0)$.

Step 1: Apply the equilibrium condition Let the equilibrium charges on the spheres be Q_1 and Q_2, and their radii be $R_1 = 8\,\text{cm}$ and $R_2 = 18\,\text{cm}$. Their potentials are equal, so

$$\frac{kQ_1}{R_1} = \frac{kQ_2}{R_2} \quad \Rightarrow \quad \frac{Q_1}{Q_2} = \frac{R_1}{R_2} = \frac{4}{9}.$$

This result is the charge ratio; it must not be confused with the field ratio.

Step 2: Calculate the field ratio The fields just outside the surfaces obey

$$\frac{E_{S1}}{E_{S2}} = \frac{Q_1}{Q_2}\frac{R_2^2}{R_1^2} = \frac{R_1}{R_2}\frac{R_2^2}{R_1^2} = \frac{R_2}{R_1}.$$

Substituting the radii in the same units,

$$\frac{E_{S1}}{E_{S2}} = \frac{18}{8} = \frac{9}{4}.$$

The smaller sphere therefore has the larger surface field, consistent with $E = V/R$ at a common potential.

Step 3: Distinguish the competing options The value $4/9$ is the charge ratio and the inverse of the required field ratio. The values $3/2$ and $2/3$ introduce a square root not present in the relation $E = V/R$. None of those three values is consistent with both equal potentials and the inverse-square expression for the surface field.

Step 4: Conclusion The ratio is $\boxed{E_{S1}/E_{S2} = 9/4}$. Hence, the correct option is (D).

Quick Tip to Solve: When spherical conductors are connected, first set their potentials equal; at a common potential their surface fields vary inversely with their radii.

35. The equation of a plane progressive wave is given by $y = 5\cos\pi\left(200t - \frac{x}{150}\right)$ where x and y are in cm and t is in second. The velocity of the wave is _____ m/s. [4 marks]

(A) 120

(B) 150

(C) 200

(D) 300

Correct Answer: (D) 300

Solution:

Concept:

- A progressive wave written as $y = A\cos(\omega t - kx)$ has speed $v = \omega/k$, angular frequency ω, and wave number k. The relative minus sign indicates travel along increasing x.
- The length unit used for x fixes the length unit of v. If x is in centimetres, ω/k initially gives centimetres per second and must be converted to metres per second.

Step 1: Read the angular frequency and wave number Expand the phase of the given expression:

$$y = 5\cos\left(200\pi t - \frac{\pi}{150}x\right).$$

Comparison with the standard progressive-wave form gives

$$\omega = 200\pi\ \text{rad s}^{-1}, \qquad k = \frac{\pi}{150}\ \text{rad cm}^{-1}.$$

The coefficient 5 is the displacement amplitude and does not determine the propagation speed.

Step 2: Find and convert the wave speed Dividing the phase coefficients yields

$$v = \frac{\omega}{k} = \frac{200\pi}{\pi/150} = 30000\,\text{cm}\,\text{s}^{-1}.$$

Since $100\,\text{cm} = 1\,\text{m}$,

$$v = \frac{30000}{100} = 300\,\text{m}\,\text{s}^{-1}.$$

The velocity is in the positive x-direction: holding the phase constant requires x to increase as t increases.

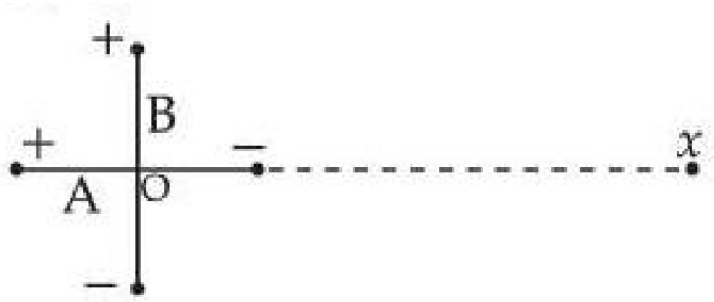
Step 3: Cross-check using frequency and wavelength The frequency is $f = \omega/(2\pi) = 100\,\text{Hz}$, while $\lambda = 2\pi/k = 300\,\text{cm} = 3\,\text{m}$. Hence $f\lambda = 300\,\text{m}\,\text{s}^{-1}$, confirming the result. The alternatives 120, 150 and 200 do not equal the ratio of the two phase coefficients after conversion to the required units.

Step 4: Conclusion The wave velocity is $\boxed{300\,\text{m}\,\text{s}^{-1}}$ along positive x. Hence, the correct option is (D).

Quick Tip to Solve: Take the ratio of the time and position coefficients in the phase; then convert the length unit used for the position coordinate to the unit requested.

36. Two short electric dipoles A and B having dipole moment p_1 and p_2 respectively are placed with their axis mutually perpendicular as shown in the figure. The resultant electric field at a point x is making an angle of 60° with the line joining points O and x. The ratio of the dipole moments p_2/p_1 is _____.

[4 marks]



(A) $\frac{\sqrt{3}}{2}$

(B) $2\sqrt{3}$

(C) $\frac{1}{\sqrt{3}}$

(D) $\sqrt{3}$

Correct Answer: (B) $2\sqrt{3}$

Solution:

Concept:

- At distance r from a short electric dipole of moment magnitude p, the axial field magnitude is $2kp/r^3$, while the equatorial field magnitude is kp/r^3, with $k = 1/(4\pi\varepsilon_0)$.

- For perpendicular field components, the tangent of the acute angle made with the horizontal line is the magnitude of the perpendicular component divided by the magnitude of the horizontal component.

Step 1: Identify the axial and equatorial fields The point x lies on the axis of horizontal dipole A and on the perpendicular bisector of vertical dipole B. Let $r = Ox$. Therefore,

$$E_A = \frac{2kp_1}{r^3}, \qquad E_B = \frac{kp_2}{r^3}.$$

The common centre makes the same distance r appear in both expressions. The factor of two belongs only to the axial field of A.

Step 2: Use the resultant direction In the diagram, A has its positive charge on the left and negative charge on the right, so its axial field at x is leftward. B has its positive charge above O, so its equatorial field is downward. The two components are perpendicular. The acute angle of the resultant with the horizontal line Ox is 60°, giving

$$\tan 60^\circ = \frac{E_B}{E_A} = \frac{kp_2/r^3}{2kp_1/r^3} = \frac{p_2}{2p_1}.$$

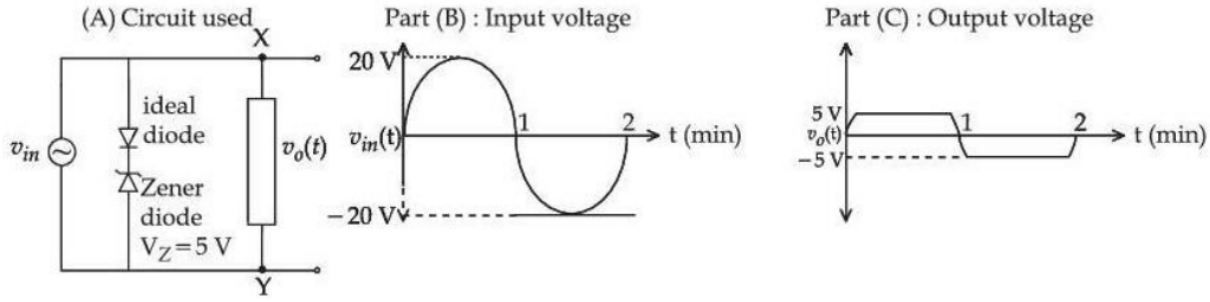
Thus $p_2/p_1 = 2\sqrt{3}$.

Step 3: Check the other proposed ratios If p_2/p_1 were $\sqrt{3}/2$, $1/\sqrt{3}$, or $\sqrt{3}$, the corresponding values of E_B/E_A would be $\sqrt{3}/4$, $1/(2\sqrt{3})$, or $\sqrt{3}/2$. None equals $\tan 60^\circ = \sqrt{3}$. This also shows why treating the axial and equatorial fields as having equal coefficients gives a wrong answer.

Step 4: Conclusion The required ratio is $\boxed{p_2/p_1 = 2\sqrt{3}}$. Hence, the correct option is (B).

Quick Tip to Solve: Mark each observation point as axial or equatorial before resolving dipole fields; the axial field carries an extra factor of two.

37. For the given circuit (shown in part (A)) the time dependent input voltage $v_{in}(t)$ and corresponding output $v_o(t)$ are shown in part (B) and part (C), respectively. Identify the components that are used in the circuit between points X and Y. [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- An ideal diode conducts in its forward direction with zero voltage drop and blocks in reverse bias.
- A Zener diode conducts in reverse breakdown at its Zener voltage. Oppositely oriented diode–Zener branches limit opposite polarities of a voltage.

Step 1: Read the two clipping levels The input reaches $+20\,\text{V}$ and $-20\,\text{V}$, but the output is limited to $+5\,\text{V}$ and $-5\,\text{V}$. Thus a limiter must act during both half-cycles. A resistor alone cannot produce the two horizontal plateaus: it does not define a fixed clipping voltage.

Step 2: Identify the branch already supplied In the existing vertical branch, the ordinary diode conducts from the upper node towards the lower node. For positive input it is forward biased while the Zener is reverse biased. At the breakdown voltage, this combination limits the positive output to $5\,\text{V}$, since the ideal diode contributes no forward drop. In the negative half-cycle the ordinary diode blocks that branch.

Step 3: Supply the complementary negative limiter The missing branch must conduct from Y towards X during the negative half-cycle. It therefore needs an ordinary diode with its cathode towards X and a Zener with its cathode towards Y, connected in opposition. This is precisely the circuit shown in option (B). At negative breakdown, its ordinary diode is forward biased and its Zener sustains the reverse voltage, producing the $-5\,\text{V}$ plateau. Option (A) lacks a Zener threshold; option (C) lacks the blocking ordinary diode required for the opposite polarity; and option (D) has an ordinary diode and a Zener pointing in the same direction from Y towards X. Both are forward biased during the negative half-cycle, so that pair cannot establish the required reverse-breakdown $-5\,\text{V}$ plateau.

Step 4: Conclusion The complementary series combination of an ideal diode and an oppositely directed Zener diode is required. Hence, the correct option is (B).

Quick Tip to Solve: For a symmetric clipper, trace the conducting diode–Zener pair separately for positive and negative input; the two branches must face opposite ways.

38. When a coil is placed in a time dependent magnetic field the power dissipated in it is P. The number of turns, area of the coil and radius of the coil wire are N, A and r respectively. For a second coil number of turns, area of the coil and radius of the coil wire are $2N$, $2A$ and $3r$ respectively. When the first coil is replaced with second coil the power dissipated in it is $\sqrt{2}\,\alpha P$. The value of α is _____. [4 marks]

(A) 36

(B) 128 $\sqrt{2}$

(C) 16

(D) 64

Correct Answer: (A) 36

Solution:

Concept:

- Faraday's law gives the induced emf magnitude $\mathcal{E} = NA|dB/dt|$ for a coil with its normal along the varying field.
- The wire resistance is $R = \rho L/(\pi r^2)$, where L is total wire length and r is the wire radius. The dissipated power is $P = \mathcal{E}^2/R$.

Step 1: Compare the induced emfs The magnetic field changes at the same rate for both coils. Both the number of turns and the area enclosed by one turn double, so their effects multiply:

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{(2N)(2A)}{NA} = 4.$$

The squared-emf contribution to the power ratio is consequently 16.

Step 2: Account for the change in wire resistance For geometrically similar turns, perimeter is proportional to the square root of the enclosed area. Doubling the turn area therefore multiplies the length per turn by $\sqrt{2}$; doubling the number of turns multiplies total wire length by another factor 2. The wire cross-sectional area increases by $3^2 = 9$, not by 3. For the same material,

$$\frac{R_2}{R_1} = \frac{2\sqrt{2}}{9}.$$

Step 3: Calculate the power ratio and identify the coefficient Use the resistance ratio in the denominator:

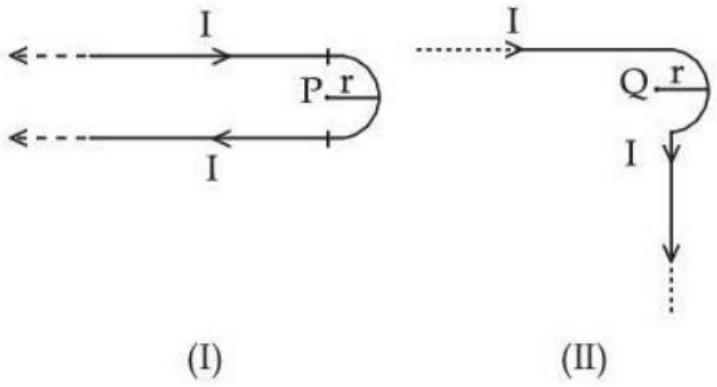
$$\frac{P_2}{P_1} = \left(\frac{\mathcal{E}_2}{\mathcal{E}_1}\right)^2 \frac{R_1}{R_2} = 16\frac{9}{2\sqrt{2}} = 36\sqrt{2}.$$

The question writes this as $\sqrt{2}\,\alpha$, so the remaining numerical coefficient is 36. Option (C), 16, ignores the resistance change. Options (B) and (D) are incompatible with the length and cross-section factors above.

Step 4: Conclusion $P_2 = 36\sqrt{2}\,P$, so $\alpha = 36$. Hence, the correct option is (A).

Quick Tip to Solve: Keep coil area and wire cross-sectional area separate: one controls magnetic flux, while the other controls electrical resistance.

39. Two identical long current carrying wires are bent into the shapes shown in the following figures. If the magnitude of magnetic fields at the centres P and Q of a semicircular arc are B_1 and B_2 respectively, then the ratio $\frac{B_1}{B_2}$ is _____. [4 marks]



(A) $\frac{2+\pi}{1+\pi}$
(B) $\frac{1+\pi}{1-\pi}$
(C) $\frac{2+\pi}{1-\pi}$
(D) $\frac{1+\pi}{2-\pi}$

Correct Answer: (A) $\frac{2+\pi}{1+\pi}$

Solution:

Concept:

- The magnetic field at the centre of a circular arc of angle θ is $B = \mu_0 I\theta/(4\pi r)$. For a semicircle, $\theta = \pi$.
- A semi-infinite straight wire whose end is at the foot of the perpendicular contributes $\mu_0 I/(4\pi r)$. A radial segment contributes zero because $d\ell$ and the displacement to the centre are parallel.

Step 1: Add the contributions at P Both long straight portions in figure (I) are tangent to the semicircle and are a distance r from P. The current goes right along the upper wire, clockwise around the arc, and left along the lower wire. The right-hand rule puts all three fields into the page at P. Therefore,

$$B_1 = \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I\pi}{4\pi r} + \frac{\mu_0 I}{4\pi r} = \frac{\mu_0 I}{4\pi r}(2+\pi).$$

Step 2: Identify the zero contribution at Q In figure (II), the upper straight wire is again tangent and produces a field into the page, as does the clockwise semicircular arc. The downward wire begins at the lower endpoint of the semicircle and lies along the line through Q. It is radial, so every element gives zero cross product in the Biot–Savart law. Thus

$$B_2 = \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I\pi}{4\pi r} = \frac{\mu_0 I}{4\pi r}(1+\pi).$$

Step 3: Cancel common factors and check the alternatives The same current and radius occur in both expressions, so their common prefactor cancels. All nonzero fields in either figure reinforce; subtracting an arc contribution would be incorrect. Also, the ratios in options (B), (C) and (D) are

negative because their denominators contain $1-\pi$ or $2-\pi$. A ratio of the stated magnitudes must be positive.

Step 4: Conclusion

$$\frac{B_1}{B_2}=\frac{2+\pi}{1+\pi}.$$

Hence, the correct option is (A).

Quick Tip to Solve: Before applying straight-wire formulas, check whether a segment is radial from the observation point; such a segment produces no field there.

40. For a thin symmetric prism made of glass (refractive index 1.5), the ratio of incident angle and minimum deviation will be _____. [4 marks]

(A) 3 : 4

(B) 3 : 2

(C) 2 : 1

(D) 1 : 2

Correct Answer: (B) 3 : 2

Solution:

Concept:

- At minimum deviation the ray path is symmetric: the angle of incidence equals the emergence angle, and both internal refraction angles equal half the prism angle.
- For a thin prism in air, the small-angle approximation gives $\delta_{\min}=(\mu-1)A$, where A is the prism angle and μ its refractive index.

Step 1: Express the incidence angle using symmetry Write the prism angle as A, to distinguish it from the angle of incidence i. At minimum deviation, $r_1=r_2=A/2$. Snell's law at the first face is $\sin i=\mu\sin(A/2)$. Since the prism is thin, the relevant angles are small and this reduces to

$$i\simeq\frac{\mu A}{2}.$$

For the given glass, this is $i\simeq 3A/4$.

Step 2: Express the minimum deviation The general deviation formula is $\delta=i+e-A$. At minimum deviation $e=i$, hence

$$\delta_{\min}=2i-A\simeq(\mu-1)A.$$

Substituting $\mu=1.5$ gives $\delta_{\min}\simeq A/2$. The same approximation has been used consistently in both expressions.

Step 3: Take the requested ratio Cancel the prism angle:

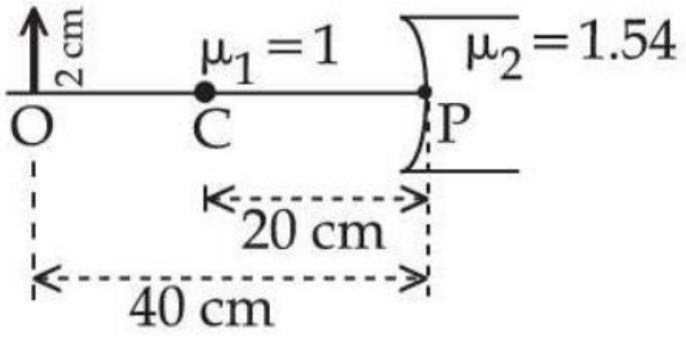
$$\frac{i}{\delta_{\min}}=\frac{\mu}{2(\mu-1)}=\frac{1.5}{2(0.5)}=\frac{3}{2}.$$

Option (A) gives $3/4$, which is the ratio of incidence to prism angle, not to deviation. Option (C) would confuse the prism angle with the incidence angle. Option (D) is also incompatible with the symmetric-path relation $2i = A + \delta_{\min}$.

Step 4: Conclusion The ratio of incidence angle to minimum deviation is $3 : 2$. Hence, the correct option is (B).

Quick Tip to Solve: For a thin prism at minimum deviation, use the pair $i = \mu A/2$ and $\delta_{\min} = (\mu - 1)A$; the unknown prism angle cancels.

41. Refer the figure given below. μ_1 and μ_2 are refractive indices of air and lens material. The height of image will be _____ cm. [4 marks]



(A) 1

(B) 0.5

(C) 1.2

(D) 0.25

Correct Answer: (A) 1

Solution:

Concept:

- For refraction at one spherical interface, $\mu_2/v - \mu_1/u = (\mu_2 - \mu_1)/R$, using Cartesian signs measured from the pole.
- The transverse magnification is $m = \mu_1 v/(\mu_2 u)$. It is not simply v/u when the incident and refracted media have different refractive indices.

Step 1: Assign the distances and refractive indices The object height is $h_o = 2\,\text{cm}$. Light travels from left to right, from air into glass. The object and centre of curvature C are both to the left of pole P, so

$$u = -40\,\text{cm}, \qquad R = -20\,\text{cm}, \qquad \mu_1 = 1, \quad \mu_2 = 1.54.$$

The drawing represents the curved air–glass interface; applying the thin-lens formula would omit the change in medium.

Step 2: Find the image position Insert the signed quantities into the spherical refraction equation:

$$\frac{1.54}{v} - \frac{1}{-40} = \frac{0.54}{-20}.$$

$$\frac{1.54}{v} = -0.027 - 0.025 = -0.052, \qquad v = -\frac{385}{13}\,\text{cm}.$$

The negative image distance means a virtual image on the incident side of the surface.

Step 3: Calculate the height and compare the offered values The magnification is positive, so the image is erect:

$$m = \frac{1(-385/13)}{1.54(-40)} = \frac{25}{52}.$$

$$h_i = mh_o = \frac{25}{52} \times 2 = \frac{25}{26}\,\text{cm} \simeq 0.962\,\text{cm}.$$

With the printed refractive index 1.54, the exact result is not an exact listed number; the corresponding approximate choice is $1\,\text{cm}$. The alternatives 0.5, 1.2 and $0.25\,\text{cm}$ do not agree with this calculated value. In particular, using magnification v/u would incorrectly omit the refractive-index factor.

Step 4: Conclusion The image is virtual and erect, with height $25/26\,\text{cm} \simeq 0.962\,\text{cm}$, approximately $1\,\text{cm}$ among the given choices. Hence, the correct option is (A).

Quick Tip to Solve: For a spherical refracting surface, attach signs to the object and centre first, then include the index ratio in the magnification.

42. For a certain metal, when monochromatic light of wavelength λ is incident, the stopping potential for photoelectrons is $3V_0$. When the same metal is illuminated by light of wavelength 2λ, then the stopping potential becomes V_0. The threshold wavelength for photoelectric emission for the given metal is $\alpha\lambda$. The value of α is _____. [4 marks]

(A) 1

(B) 4

(C) 2

(D) 3

Correct Answer: (B) 4

Solution:

Concept:

- Einstein's equation is $hc/\lambda = \phi + eV_s$, where ϕ is the work function and V_s the stopping potential.
- At threshold the maximum kinetic energy is zero, so $hc/\lambda_{\text{th}} = \phi$. Doubling wavelength halves photon energy.

Step 1: Write the two energy equations The same metal has the same work function in both experiments. For wavelength λ, the stopping potential is $3V_0$, whereas for wavelength 2λ, it is V_0.

Therefore,

$$\frac{hc}{\lambda} = \phi + 3eV_0, \qquad \frac{hc}{2\lambda} = \phi + eV_0.$$

Stopping potential must first be multiplied by the electron charge to represent an energy.

Step 2: Eliminate the work function and then determine it Subtract the second equation from the first:

$$\frac{hc}{2\lambda} = 2eV_0 \quad \Rightarrow \quad \frac{hc}{\lambda} = 4eV_0.$$

Substitution into the first equation now yields

$$\phi = 4eV_0 - 3eV_0 = eV_0.$$

As a consistency check, photons of wavelength 2λ have energy $2eV_0$, leaving kinetic energy eV_0, exactly as given.

Step 3: Find the threshold wavelength At the longest wavelength that just permits emission, all photon energy supplies the work function:

$$\frac{hc}{\lambda_{\text{th}}} = eV_0 = \frac{hc}{4\lambda}.$$

Thus the threshold wavelength is four times the original wavelength. Options (A) and (C) would predict zero kinetic energy at one of the wavelengths for which the question explicitly gives a positive stopping potential. Option (D) gives a work function $hc/(3\lambda)$, inconsistent with the two measured potentials.

Step 4: Conclusion $\lambda_{\text{th}} = 4\lambda$, hence $\alpha = 4$. Hence, the correct option is (B).

Quick Tip to Solve: Subtract photoelectric equations for the same metal to remove its work function before solving for the threshold wavelength.

43. An electromagnetic wave travelling in x-direction is described by field equation $E_y = 300 \sin\omega\left(t - \frac{x}{c}\right)$.

If the electron is restricted to move in y-direction only with speed of 1.5×10^6 m/s then ratio of maximum electric and magnetic forces acting on the electron is _____. [4 marks]

(A) 200

(B) 150

(C) 400

(D) 300

Correct Answer: (A) 200

Solution:

Concept:

- In a plane electromagnetic wave in vacuum, the electric and magnetic fields are mutually perpendicular, transverse to propagation, and satisfy $E_0 = cB_0$.

- The electric-force magnitude is $F_E = eE$; the magnetic-force magnitude is $F_B = evB\sin\theta$, where θ is the angle between velocity and magnetic field.

Step 1: Identify the field directions The wave travels along the positive x-direction and its electric field is along y. Therefore its magnetic field lies along z, since $\vec{E} \times \vec{B}$ points along the propagation direction. The electron moves along y, so its velocity is perpendicular to the magnetic field: $\sin\theta = 1$. The restriction on motion does not remove the magnetic force acting on it; a constraining force balances its transverse effect.

Step 2: Compare the maximum force magnitudes The electric-field amplitude is $E_0 = 300\,\mathrm{V\,m^{-1}}$, and the magnetic amplitude is $B_0 = E_0/c$. Thus

$$\frac{F_{E,\max}}{F_{B,\max}} = \frac{eE_0}{evB_0} = \frac{c}{v} = \frac{3.0 \times 10^8}{1.5 \times 10^6} = 200.$$

The charge magnitude and field amplitude cancel. The negative electron charge reverses force directions but has no effect on this ratio of magnitudes. The other options, 150, 400 and 300, do not equal c/v for the specified speed.

Step 3: Conclusion The ratio of maximum electric to magnetic force is $200 : 1$. Hence, the correct option is (A).

Quick Tip to Solve: For motion perpendicular to the magnetic field of a vacuum electromagnetic wave, use $F_E/F_B = c/v$; check the velocity direction before cancelling the fields.

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44. Angular momentum of an electron in a hydrogen atom is $\frac{3h}{\pi}$, then the energy of the electron is _____ eV. [4 marks]

(A) -1.51

(B) -0.85

(C) -0.38

(D) -0.28

Correct Answer: (C) -0.38

Solution:

Concept:

- Bohr quantisation gives the orbital angular momentum $L = n\hbar = nh/(2\pi)$, where n is a positive integer.
- The total energy of a hydrogen electron in orbit n is $E_n = -13.6/n^2\,\mathrm{eV}$. The negative sign denotes a bound electron, with zero energy at infinite separation.

Step 1: Determine the principal quantum number Compare the given angular momentum with the quantisation condition, keeping the factor 2π in $\hbar$:

$$\frac{nh}{2\pi} = \frac{3h}{\pi} \quad \Rightarrow \quad n = 6.$$

Thus the electron occupies the sixth Bohr orbit. Reading the coefficient 3 directly as the quantum number would be incorrect because the supplied angular momentum is expressed in units of h/π, rather than $\hbar$.

Step 2: Calculate the bound-state energy and check the options Substitute $n = 6$ into the hydrogen energy formula:

$$E_6 = -\frac{13.6}{6^2} = -\frac{13.6}{36} = -0.377\overline{7}\,\text{eV} \simeq -0.38\,\text{eV}.$$

The options $-1.51\,\text{eV}$ and $-0.85\,\text{eV}$ correspond approximately to $n = 3$ and $n = 4$, respectively, and $-0.28\,\text{eV}$ corresponds approximately to $n = 7$. None has the given angular momentum. The energy remains negative because the orbit is bound; its magnitude is much smaller than the ground-state binding energy.

Step 3: Conclusion The electron energy is $-0.38\,\text{eV}$. Hence, the correct option is (C).

Quick Tip to Solve: First express angular momentum as a multiple of $\hbar = h/(2\pi)$; only then use the resulting integer in $E_n = -13.6/n^2$.

45. A liquid drop of diameter 2 mm breaks into 512 droplets. The change in surface energy is $\alpha \times 10^{-6}$ J. The value of α is _____. (Take surface tension of liquid = 0.08 N/m) [4 marks]

(A) 10

(B) 7

(C) 8

(D) 11

Correct Answer: (B) 7

Solution:

Concept:

- For a liquid drop, surface energy is $U = TA$, where T is surface tension and A is the area of its single liquid-air interface.
- On breaking a drop into equal droplets, volume is conserved. For N droplets, $R^3 = Nr^3$, so their total area is $4\pi R^2 N^{1/3}$.

Step 1: Find the original and final radii The given 2 mm is the diameter, so the original radius is $R = 1\,\text{mm} = 10^{-3}\,\text{m}$. For $N = 512 = 8^3$ equal droplets, volume conservation gives

$$\frac{4\pi R^3}{3} = 512\frac{4\pi r^3}{3} \quad \Rightarrow \quad r = \frac{R}{8}.$$

Hence the total final area is $512(4\pi R^2/64) = 8(4\pi R^2)$: the surface area becomes eight times its initial value.

Step 2: Calculate the increase in energy The change in area is the final area minus the initial area, rather than the final area alone:

$$\Delta A = (8-1)4\pi R^2 = 28\pi \times 10^{-6}\,\text{m}^2.$$

Therefore

$$\Delta U = T\Delta A = 0.08 \times 28\pi \times 10^{-6} = 7.04 \times 10^{-6}\,\text{J}.$$

Thus $\alpha \simeq 7$, to the integer precision of the choices. The option 8 confuses the final-area factor with the energy coefficient, while 10 and 11 do not follow from $T\Delta A$. A liquid drop has one surface; a soap-bubble two-surface factor is inapplicable.

Step 3: Conclusion The surface energy increases by approximately 7×10^{-6} J, so $\alpha = 7$. Hence, the correct option is (B).

Quick Tip to Solve: Use the radius, conserve volume first, and subtract the original area: $\Delta U = 4\pi TR^2(N^{1/3} - 1)$ for equal liquid droplets.

Physics – Section B: Numerical Value Questions (20 marks)

46. In single slit diffraction pattern, the wavelength of light used is 628 nm and slit width is 0.2 mm, the angular width of central maximum is $\alpha \times 10^{-2}$ degrees. The value of α is _____. [4 marks]

Solution:

Concept:

- For slit width a, the first diffraction minima satisfy $a \sin\theta_1 = \lambda$. The central maximum extends between the first minima on opposite sides of the axis.
- The full angular width is $2\theta_1 \simeq 2\lambda/a$ in radians when $\lambda/a \ll 1$. Convert radians into degrees by multiplying by $180/\pi$.

Step 1: Locate the first minima Convert the wavelength and slit width into the same units:

$$\lambda = 628 \times 10^{-9}\,\text{m}, \qquad a = 0.2 \times 10^{-3}\,\text{m}.$$

Then

$$\sin\theta_1 = \frac{\lambda}{a} = \frac{628 \times 10^{-9}}{0.2 \times 10^{-3}} = 3.14 \times 10^{-3}.$$

This ratio is very small, so the small-angle approximation is appropriate: $\theta_1 \simeq 3.14 \times 10^{-3}$ rad. This is the angle from the central axis to one first minimum, only half the requested width.

Step 2: Convert the full angular width to degrees The first minima lie at $-\theta_1$ and $+\theta_1$. Their angular separation is therefore

$$\Delta\theta = 2\theta_1 \simeq 6.28 \times 10^{-3}\ \text{rad}.$$

Converting to degrees gives

$$\Delta\theta = 6.28 \times 10^{-3}\frac{180}{\pi} \simeq 0.360^\circ = 36.0 \times 10^{-2}\ \text{degrees}.$$

Using the exact inverse sine changes only insignificant higher decimal places and gives the same integer coefficient. Comparing this expression with $\alpha \times 10^{-2}$ degrees identifies the requested value.

Step 3: Conclusion The full angular width of the central maximum is approximately 0.36°, and the

required dimensionless coefficient is $\boxed{\alpha = 36}$.

Quick Tip to Solve: The central maximum has two sides: use $2\lambda/a$, not λ/a, and convert from radians before matching a coefficient stated in degrees.

47. A vessel contains 0.15 m^3 of a gas at pressure 8 bar and temperature 140 °C with $c_p = 3R$ and $c_v = 2R$. It is expanded adiabatically till pressure falls to 1 bar. The work done during this process is _____ kJ. (R is gas constant) [4 marks]

Solution:

Concept:

- The adiabatic index is $\gamma = c_p/c_v$. For the quasistatic ideal-gas adiabatic process, PV^γ is constant.
- The work done by the expanding gas is $W = (P_1V_1 - P_2V_2)/(\gamma - 1)$. With pressure in pascals and volume in cubic metres, each pressure-volume product has units of joules.

Step 1: Find the adiabatic index and final volume From the supplied heat capacities,

$$\gamma = \frac{3R}{2R} = \frac{3}{2}.$$

The ratio of pressures is $P_1/P_2 = 8$. Applying $P_1V_1^\gamma = P_2V_2^\gamma$,

$$V_2 = V_1\left(\frac{P_1}{P_2}\right)^{1/\gamma} = 0.15(8)^{2/3} = 0.60\,\text{m}^3.$$

The volume increases by a factor of four as the pressure falls, consistent with expansion.

Step 2: Evaluate the work with consistent units Use $1\,\text{bar} = 10^5\,\text{Pa}$. The endpoint pressure-volume products are

$$P_1V_1 = (8 \times 10^5)(0.15) = 1.20 \times 10^5\,\text{J},$$

$$P_2V_2 = (1 \times 10^5)(0.60) = 6.0 \times 10^4\,\text{J}.$$

Thus

$$W = \frac{1.20 \times 10^5 - 6.0 \times 10^4}{1.5 - 1} = 1.20 \times 10^5\,\text{J} = 120\,\text{kJ}.$$

The work is positive for work done by the gas. No heat enters during the adiabatic process, so this energy comes from the decrease in internal energy. The initial temperature is unnecessary because the endpoint pressure-volume relation already determines the work; equivalently, $T_2/T_1 = P_2V_2/(P_1V_1) = 1/2$.

Step 3: Conclusion The work done by the gas during the expansion is $\boxed{120\,\text{kJ}}$. The requested numerical entry is 120.

Quick Tip to Solve: For an adiabatic pressure change, obtain the final volume from $PV^\gamma =$ constant before using the endpoint work formula; convert bar to pascal.

48. 1 μC charge moving with velocity $\vec{v} = (\hat{i} - 2\hat{j} + 3\hat{k})$ m/s in the region of magnetic field $\vec{B} = (2\hat{i} + 3\hat{j} - 5\hat{k})$ T. The magnitude of force acting on it is $\sqrt{\alpha} \times 10^{-6}$ N. The value of α is _____. [4 marks]

Solution:

Concept:

- The magnetic force on a point charge is $\vec{F} = q(\vec{v} \times \vec{B})$. A vector cross product, rather than a dot product, determines the force.
- For Cartesian components F_x, F_y, F_z, the magnitude is $|\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$. The charge conversion is $1\,\mu\text{C} = 10^{-6}\,\text{C}$.

Step 1: Evaluate the cross product component by component For $\vec{v} = (1, -2, 3)\,\text{m s}^{-1}$ and $\vec{B} = (2, 3, -5)\,\text{T}$,

$$(\vec{v} \times \vec{B})_x = (-2)(-5) - (3)(3) = 1,$$
$$(\vec{v} \times \vec{B})_y = (3)(2) - (1)(-5) = 11,$$
$$(\vec{v} \times \vec{B})_z = (1)(3) - (-2)(2) = 7.$$

Therefore $\vec{v} \times \vec{B} = \hat{i} + 11\hat{j} + 7\hat{k}$ in the corresponding SI units. In particular, the y-component is positive; retaining the alternating sign correctly in a determinant gives the same result.

Step 2: Find the magnitude and extract the coefficient Multiplying by the positive charge gives

$$\vec{F} = 10^{-6}(\hat{i} + 11\hat{j} + 7\hat{k})\,\text{N}.$$

Its magnitude is

$$|\vec{F}| = 10^{-6}\sqrt{1^2 + 11^2 + 7^2} = \sqrt{171} \times 10^{-6}\,\text{N}.$$

As a check, $v^2 = 14$, $B^2 = 38$, and $\vec{v} \cdot \vec{B} = 2 - 6 - 15 = -19$, so

$$|\vec{v} \times \vec{B}|^2 = v^2 B^2 - (\vec{v} \cdot \vec{B})^2 = 14(38) - 19^2 = 171.$$

The requested coefficient is the quantity under the square root, rather than the numerical magnitude $\sqrt{171}$.

Step 3: Conclusion The force magnitude is $\sqrt{171} \times 10^{-6}\,\text{N}$. Hence the required dimensionless coefficient is $\boxed{\alpha = 171}$.

Quick Tip to Solve: Keep the cross-product component signs, then square and add. To check a result quickly, use $|\vec{v} \times \vec{B}|^2 = v^2 B^2 - (\vec{v} \cdot \vec{B})^2$.

49. A uniform wire of length l of weight w is suspended from the roof with a weight of W at the other end. The stress in the wire at $\frac{l}{3}$ distance from the top is $\left(\frac{W}{A}+\frac{2}{\gamma}\frac{w}{A}\right)$, where A is the cross sectional area of the wire. The value of γ is _____. [4 marks]

Solution:

Concept:

- Longitudinal stress at a cross-section is $\sigma = T/A$, where T is the tension transmitted across that section and A is its area.
- For a uniform wire, weight is proportional to length. The tension at any section supports the attached load and the part of the wire below that section.

Step 1: Identify the portion supported by the section The specified section lies at a distance $l/3$ from the top. Therefore the length hanging below it is

$$l_{\text{below}} = l - \frac{l}{3} = \frac{2l}{3}.$$

Since the weight per unit length is w/l, the weight of this lower portion is

$$w_{\text{below}} = \frac{w}{l}\frac{2l}{3} = \frac{2w}{3}.$$

The upper third is supported by the roof and does not add to the load carried by this section.

Step 2: Apply equilibrium to the lower portion Isolate the wire below the cut together with the weight W at its lower end. The tension T acts upwards at the cut, while both weights act downwards. Static equilibrium gives

$$T - W - \frac{2w}{3} = 0, \qquad T = W + \frac{2w}{3}.$$

Here W and w already denote weights, so no extra factor of gravitational acceleration is needed.

Step 3: Compare the stress with the given expression Dividing the supporting tension by the cross-sectional area gives

$$\sigma = \frac{T}{A} = \frac{W}{A} + \frac{2}{3}\frac{w}{A}.$$

Comparing its coefficient of w/A with the stated coefficient $2/\gamma$,

$$\frac{2}{\gamma} = \frac{2}{3} \quad \Rightarrow \quad \gamma = 3.$$

As a check, tension decreases from $W + w$ at the top to W at the bottom; the calculated value lies between these limits.

Step 4: Conclusion The required value is $\boxed{\gamma = 3}$, a dimensionless number.

Quick Tip to Solve: At a cut in a hanging wire, count only the wire and external loads below that cut. Measure the remaining length from the cut to the bottom.

50. A tub is filled with water and a wooden cube 10 cm $\times$ 10 cm $\times$ 10 cm is placed in the water. The wooden cube is found to float on the water with a part of it submerged in water. When a metal coin is placed on the wooden cube, the submerged part is increased by 3.87 cm. The mass of the metal coin is _____ gram.

(Take water density as 1 g/cm^3 and density of wood as 0.4 g/cm^3) [4 marks]

Solution:

Concept:

- For a floating body in equilibrium, the weight of displaced water equals the total weight supported: $\rho_{\text{water}} V_{\text{sub}} g = mg$.
- When a load is added without changing the horizontal area, its mass equals the mass of the additional water displaced: $m_{\text{load}} = \rho_{\text{water}} A\Delta h$.

Step 1: Calculate the initial immersion The cube has volume $V = 10^3 = 1000\,\text{cm}^3$, so its mass is

$$m_{\text{wood}} = 0.4 \times 1000 = 400\,\text{g}.$$

Its horizontal area is $A = 10 \times 10 = 100\,\text{cm}^2$. Initially the displaced water has mass $400\,\text{g}$, corresponding to volume $400\,\text{cm}^3$ and submerged depth

$$h_1 = \frac{400}{100} = 4\,\text{cm}.$$

Step 2: Use the increase in displaced water With the coin on top, the submerged depth increases by $\Delta h = 3.87\,\text{cm}$. The additional displaced volume is therefore

$$\Delta V = A\Delta h = 100 \times 3.87 = 387\,\text{cm}^3.$$

Subtracting the original flotation equation from the loaded flotation equation cancels the weight of the wood:

$$m_{\text{coin}} g = \rho_{\text{water}} \Delta V g.$$

Thus

$$m_{\text{coin}} = 1 \times 387 = 387\,\text{g}.$$

Step 3: Check that the loaded cube still floats partly submerged The final immersion is $h_2 = 4 + 3.87 = 7.87\,\text{cm}$, smaller than the cube height of $10\,\text{cm}$. Its displaced water mass is then $787\,\text{g}$, equal to $400 + 387\,\text{g}$, the combined mass of wood and coin. The coin remains above the water, so its own volume does not enter the displaced-water calculation.

Step 4: Conclusion The mass of the metal coin is $\boxed{387\,\text{g}}$. The required numerical entry is 387.

Quick Tip to Solve: For an extra load on a floating block, use only the extra displaced volume, $A\Delta h$. The original block mass cancels when the two equilibrium equations are subtracted.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. The mass of iron converted into Fe_3O_4 by the action of 18 g of steam is:
(Given: Molar mass of H, O and Fe are 1, 16 and 56 g mol^{-1} respectively)
Assume iron is present in excess: [4 marks]

(A) 2.1 g
(B) 4.2 g
(C) 21 g
(D) 42 g

Correct Answer: (D) 42 g

Solution:

Concept:

- A balanced chemical equation gives mole ratios, not mass ratios. Convert the available reactant mass to moles before applying its coefficients.
- Iron reacts with steam to form magnetite, Fe_3O_4, and hydrogen. Since iron is in excess, steam limits the amount of iron that reacts.

Step 1: Balance the reaction The reaction is

$$3\,Fe + 4\,H_2O\,(g) \longrightarrow Fe_3O_4 + 4\,H_2(g).$$

There are three iron atoms, four oxygen atoms and eight hydrogen atoms on each side. Hence four moles of steam consume three moles of iron; the coefficient of iron cannot be replaced by a one-to-one ratio.

Step 2: Convert the steam mass into moles Using the supplied atomic masses,

$$M(H_2O) = 2(1) + 16 = 18\,g\,mol^{-1}.$$

Therefore

$$n(H_2O) = \frac{18\,g}{18\,g\,mol^{-1}} = 1\,mol.$$

All this steam can react because enough iron is available. The amount of iron consumed is

$$n(Fe) = \frac{3}{4}\,n(H_2O) = \frac{3}{4}\,mol.$$

Step 3: Find the mass of iron consumed and compare the options Multiplying by the given iron molar mass,

$$m(Fe) = \frac{3}{4} \times 56 = 42\,g.$$

Equivalently, $72\,g$ of steam reacts with $168\,g$ of iron, so $18\,g$ of steam reacts with one quarter of that iron mass. Options (A) and (B), 2.1 and $4.2\,g$, are too small by factors of twenty and ten; option (C), $21\,g$, is half the required mass. The question asks for iron consumed, not the mass of the oxide formed.

Step 4: Conclusion The mass of iron converted into Fe_3O_4 is $\boxed{42\,g}$. Hence, the correct option is (D).

Quick Tip to Solve: For iron with steam, remember the mole ratio $Fe : H_2O = 3 : 4$. Balance the oxide first, then convert the limiting reactant to moles.

52. What is the energy (in J atom^{-1}) required for the following process?

Li^{2+} (g) $\rightarrow$ Li^{3+} (g) + e^-

(Take the ionization energy for the H atom in the ground state as 2.18×10^{-18} J atom^{-1}) [4 marks]

(A) 8.72×10^{-18}

(B) 1.962×10^{-18}

(C) 1.962×10^{-17}

(D) 6.54×10^{-17}

Correct Answer: (C) 1.962×10^{-17}

Solution:

Concept:

- A one-electron ion with nuclear charge Z has Bohr energy $E_n = -Z^2 E_H/n^2$, where $E_H = 2.18 \times 10^{-18}$ J is the magnitude of the hydrogen ground-state energy.
- Ionization removes the electron to infinity, where its energy is zero. The required energy is positive and equals $0 - E_n$.

Step 1: Identify the one-electron ion and its nuclear charge Lithium has atomic number $Z = 3$. The ion Li^{2+} has lost two of its three electrons, leaving exactly one electron. It therefore obeys the hydrogen-like Bohr expression. The ionic charge $+2$ is not the nuclear charge: the nucleus still contains three protons. With no excited state specified, use the ground state $n = 1$.

Step 2: Calculate the energy needed to remove the electron The initial electron energy is

$$E_1 = -3^2(2.18 \times 10^{-18}) = -1.962 \times 10^{-17}\,J.$$

For $Li^{2+}(g) \longrightarrow Li^{3+}(g) + e^-$, the final free-electron energy is zero, so

$$\Delta E = 0 - E_1 = 9(2.18 \times 10^{-18}) = 1.962 \times 10^{-17}\,J\,atom^{-1}.$$

This is the energy per ion, so multiplication by the Avogadro constant is unnecessary.

Step 3: Check the power of ten and exclude the other choices The product before normalizing scientific notation is 19.62×10^{-18}, which equals 1.962×10^{-17}. Option (A) uses a factor of four, appropriate to $Z = 2$, not lithium. Option (B) is ten times too small because of its exponent. Option (D) also does not equal nine times the hydrogen ionization energy. The positive sign is consistent with energy being supplied to free a bound electron.

Step 4: Conclusion The required energy is $\boxed{1.962 \times 10^{-17}\,J\,atom^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: For any hydrogen-like ion, scale the hydrogen ground-state ionization energy by Z^2. Use atomic number, not the ion charge, for Z.

53. Given below are two statements :

Statement (I): The correct sequence of bond lengths in the following species is :

$$O_2^+ < O_2 < O_2^- < O_2^{2-}$$

Statement (II): The correct sequence of number of unpaired electrons in the following species is :

$$O_2 > O_2^+ > O_2^- > O_2^{2-}$$

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Correct Answer: (C) **Statement I** is true but **Statement II** is false

Solution:

Concept:

- Molecular orbital bond order is $(N_b - N_a)/2$, where N_b and N_a count bonding and antibonding electrons. Greater bond order corresponds to a shorter bond for these related oxygen species.
- In O_2, the two highest occupied electrons are in the degenerate antibonding π^* orbitals. Removing or adding electrons changes their occupancy and therefore both bond order and unpaired-electron count.

Step 1: Compare the bond orders and judge Statement I Neutral O_2 has bond order 2. Removing one antibonding electron produces O_2^+ with bond order 2.5. Adding one or two antibonding electrons gives O_2^- and O_2^{2-}, with bond orders 1.5 and 1, respectively. Thus

$$\mathrm{BO}(O_2^+) > \mathrm{BO}(O_2) > \mathrm{BO}(O_2^-) > \mathrm{BO}(O_2^{2-}).$$

Bond lengths follow the reverse order, exactly as in Statement I, so Statement I is true.

Step 2: Count unpaired electrons and judge Statement II The two π^* orbitals contain one electron each in O_2, giving two unpaired electrons by Hund's rule. In O_2^+, one electron remains unpaired. In O_2^-, three electrons occupy the pair of orbitals: two are paired and one is unpaired. In O_2^{2-}, all four are paired. The counts are therefore

$$O_2 : 2, \quad O_2^+ : 1, \quad O_2^- : 1, \quad O_2^{2-} : 0.$$

Consequently, the correct relation is $2 > 1 = 1 > 0$, not $2 > 1 > 1 > 0$; Statement II is false.

Step 3: Select the combination of truth values Option (A) incorrectly accepts Statement II. Options (B) and (D) incorrectly reject Statement I. Only option (C) combines the verified true bond-length order with the false strict unpaired-electron order. Equal unpaired-electron counts do not imply equal bond orders or equal bond lengths.

Step 4: Conclusion Statement I is true, but Statement II is false because O_2^+ and O_2^- each have one unpaired electron. Hence, the correct option is (C).

Quick Tip to Solve: Keep bond order and unpaired-electron count separate: oxygen cation and superoxide each have one unpaired electron, but their bond orders are 2.5 and 1.5.

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54. Consider the following data.

(i) $2\mathbf{Al(s)} + 6\mathbf{HCl(aq)} \rightarrow \mathbf{Al_2Cl_6(aq)} + 3\mathbf{H_2(g)} + 1200$ **kJ/mol**

(ii) $\mathbf{H_2(g)} + \mathbf{Cl_2(g)} \rightarrow 2\mathbf{HCl(g)} + 164$ **kJ/mol**

(iii) $\mathbf{HCl(g)} + \mathbf{aq} \rightarrow \mathbf{HCl(aq)} + 83$ **kJ/mol**

(iv) $\mathbf{Al_2Cl_6(s)} + \mathbf{aq} \rightarrow \mathbf{Al_2Cl_6(aq)} + 663$ **kJ/mol**

The enthalpy of formation of anhydrous solid Al_2Cl_6 is : [4 marks]

(A) -648 kJ mol^{-1}

(B) -1350 kJ mol^{-1}

(C) -2002 kJ mol^{-1}

(D) -1527 kJ mol^{-1}

Correct Answer: (D) -1527 kJ mol^{-1}

Solution:

Concept:

- By Hess's law, reaction enthalpies add when the corresponding chemical equations are added. Reversing an equation changes the sign of its enthalpy.
- Heat written on the product side represents an exothermic reaction, so its ΔH is negative. Formation of one mole of anhydrous solid requires the target reaction $2\,\mathrm{Al(s)} + 3\,\mathrm{Cl_2(g)} \longrightarrow \mathrm{Al_2Cl_6(s)}$.

Step 1: Convert the given heat terms into signed enthalpies For the four equations as written,

$$\Delta H_1 = -1200, \quad \Delta H_2 = -164, \quad \Delta H_3 = -83, \quad \Delta H_4 = -663 \quad (\mathrm{kJ\,mol^{-1}}).$$

The target contains solid Al_2Cl_6, whereas equation (i) produces its aqueous form. We must therefore reverse the dissolution step (iv).

Step 2: Generate and cancel the hydrochloric acid Multiply equation (ii) by three and equation (iii) by six:

$$3\,\mathrm{H_2(g)} + 3\,\mathrm{Cl_2(g)} \longrightarrow 6\,\mathrm{HCl(g)}, \qquad \Delta H = 3(-164),$$

$$6\,\mathrm{HCl(g)} + \mathrm{aq} \longrightarrow 6\,\mathrm{HCl(aq)}, \qquad \Delta H = 6(-83).$$

Adding these to equation (i) cancels $3\,H_2$, $6\,HCl(g)$ and $6\,HCl(aq)$. The net chemical change is formation of aqueous Al_2Cl_6 from $2\,Al(s)$ and $3\,Cl_2(g)$.

Step 3: Reverse dissolution and calculate the target enthalpy Reverse equation (iv), $Al_2Cl_6(aq) \longrightarrow Al_2Cl_6(s) + aq$, with enthalpy $+663\,\mathrm{kJ\,mol^{-1}}$. The solvent notation cancels on completing the Hess

cycle. Therefore

$$\begin{aligned}\Delta H_f &= \Delta H_1 + 3\Delta H_2 + 6\Delta H_3 - \Delta H_4\\ &= -1200 - 492 - 498 + 663\\ &= -1527\,\text{kJ mol}^{-1}.\end{aligned}$$

The equation forms one mole of $Al_2Cl_6(s)$, so no further division by two is required. Options (A), (B) and (C) disagree with the signed sum for this balanced target reaction; they cannot be obtained by a valid cancellation of the given equations.

Step 4: Conclusion The enthalpy of formation of anhydrous $Al_2Cl_6(s)$ is $\boxed{-1527\,\text{kJ mol}^{-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: Write the target state before adding Hess-law equations. To obtain an anhydrous solid from an aqueous product, reverse its dissolution equation and reverse the heat sign.

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55. 19.5 g of fluoro acetic acid (molar mass = 78 g mol^{-1}) is dissolved in 500 g of water at 298 K. The depression in the freezing point of water was 1°C. What is K_a of fluoro acetic acid? (For water, $K_f = 1.86$ K kg mol^{-1}). Assume molarity and molality to have same values. [4 marks]

(A) 10^{-6}

(B) 4×10^{-4}

(C) 3×10^{-5}

(D) 3×10^{-3}

Correct Answer: (D) 3×10^{-3}

Solution:

Concept:

- Freezing-point depression depends on the number of dissolved particles: $\Delta T_f = iK_f m$, where m is the analytical molality and i is the van't Hoff factor.
- For a monoprotic acid $HA \rightleftharpoons H^+ + A^-$, dissociation of a fraction α gives $i = 1 + \alpha$. Its acid constant is $K_a = C\alpha^2/(1-\alpha)$.

Step 1: Calculate the analytical concentration The acid amount and solvent mass give

$$n = \frac{19.5}{78} = 0.250\ \text{mol}, \qquad m = \frac{0.250}{0.500} = 0.500\ \text{mol kg}^{-1}.$$

The question permits molarity and molality to have the same numerical value, so use $C = 0.500\ \text{mol L}^{-1}$ in the equilibrium expression. The denominator in molality is the mass of water, not the total mass of solution.

Step 2: Find the degree of dissociation Without dissociation, the depression would be $1.86 \times 0.500 = 0.930$ K. The measured value is larger, indicating additional ions:

$$i = \frac{1.000}{1.86 \times 0.500} = \frac{100}{93}, \qquad \alpha = i - 1 = \frac{7}{93} = 0.07527.$$

Thus about 7.53% of the acid dissociates. Each dissociated acid molecule produces two ions in place of one molecule.

Step 3: Evaluate the acid constant and compare the choices At equilibrium, $[\mathrm{H^+}] = [\mathrm{A^-}] = C\alpha$, while $[\mathrm{HA}] = C(1-\alpha)$. Therefore

$$K_a = \frac{0.500(7/93)^2}{1 - 7/93} = 3.06 \times 10^{-3} \approx 3 \times 10^{-3}.$$

Only option (D) has this value. Options (A), (B) and (C) are too small and would predict less dissociation, and hence a depression closer to 0.930 K.

Step 4: Conclusion The acid dissociation constant is approximately $K_a = 3 \times 10^{-3}$. Hence, the correct option is (D).

Quick Tip to Solve: First calculate the depression for an undissociated solute. For a monoprotic acid, the excess particle factor directly gives $\alpha = i - 1$.

56. The solubility product constants of $\mathrm{Ag_2CrO_4}$ and AgBr are $32x$ and $4y$ respectively at 298 K. The value of $\left(\frac{\text{molarity of } \mathrm{Ag_2CrO_4}}{\text{molarity of } \mathrm{AgBr}}\right)$ can be expressed as : [4 marks]

(A) $\frac{2\sqrt[3]{x}}{y}$

(B) $2\sqrt{\frac{x}{y}}$

(C) $\sqrt{\frac{x}{y}}$

(D) $\frac{\sqrt[3]{x}}{\sqrt{y}}$

Correct Answer: (D) $\frac{\sqrt[3]{x}}{\sqrt{y}}$

Solution:

Concept:

- The solubility product is formed from equilibrium ion concentrations, with each concentration raised to its stoichiometric coefficient.
- Molar solubility is the concentration of formula units dissolved in a saturated solution. The ion concentrations need not equal this solubility when a salt yields more than one ion of a given type.

Step 1: Express the solubility of silver chromate Let the molar solubility of $\mathrm{Ag_2CrO_4}$ be s_1. Its dissolution is

$$\mathrm{Ag_2CrO_4(s)} \rightleftharpoons 2\,\mathrm{Ag^+(aq)} + \mathrm{CrO_4{}^{2-}(aq)}.$$

Consequently, $[\mathrm{Ag^+}] = 2s_1$ and $[\mathrm{CrO_4{}^{2-}}] = s_1$. Substituting these concentrations gives

$$K_{sp} = (2s_1)^2 s_1 = 4s_1^3 = 32x, \qquad s_1 = \sqrt[3]{8x} = 2\sqrt[3]{x}.$$

The factor 4 comes from squaring the silver-ion concentration; it must not be omitted.

Step 2: Express the solubility of silver bromide Let the molar solubility of AgBr be s_2. Here the

dissolution is

$$\mathrm{AgBr(s)} \rightleftharpoons \mathrm{Ag^+(aq) + Br^-(aq)}.$$

Both ion concentrations equal s_2, so

$$K_{sp} = s_2^2 = 4y, \qquad s_2 = 2\sqrt{y}.$$

These are the separate saturated solutions referred to by the solubility products; no common-ion concentration is supplied.

Step 3: Take the ratio and reject the alternatives The required molarity ratio is therefore

$$\frac{s_1}{s_2} = \frac{2\sqrt[3]{x}}{2\sqrt{y}} = \frac{\sqrt[3]{x}}{\sqrt{y}}.$$

Option (A) incorrectly uses y rather than its square root and retains an extra factor 2. Options (B) and (C) use a square root of x, although the chromate solubility must depend on the cube root because its solubility-product expression contains s_1^3.

Step 4: Conclusion The required ratio is $\sqrt[3]{x}/\sqrt{y}$. Hence, the correct option is (D).

Quick Tip to Solve: Write the ion concentrations in terms of molar solubility before taking roots: a 2:1 salt gives $4s^3$, whereas a 1:1 salt gives s^2.

57. An electrochemical cell is constructed using half cells in the direction of spontaneous change:
$Fe(OH)_2(s) + 2e^- \rightarrow Fe(s) + 2OH^-(aq)$ $E^\theta = -0.88$ V
and $AgBr(s) + e^- \rightarrow Ag(s) + Br^-(aq)$ $E^\theta = +0.07$ V
Which of the following option is correct? [4 marks]

(A) Overall reaction: $\mathrm{Fe(s) + 2\,OH^-(aq) + 2\,AgBr(s)} \rightleftharpoons \mathrm{Fe(OH)_2(s) + 2\,Ag(s) + 2\,Br^-(aq)}$

(B) $E^\theta_{cell} = -0.95\ V$

(C) Fe is reduced in the electrochemical cell

(D) E^θ_{cell} is an extensive property

Correct Answer: (A) Overall reaction: $\mathrm{Fe(s) + 2\,OH^-(aq) + 2\,AgBr(s)} \rightleftharpoons \mathrm{Fe(OH)_2(s) + 2\,Ag(s) + 2\,Br^-(aq)}$

Solution:

Concept:

- The half-cell having the higher reduction potential acts as the cathode in the spontaneous galvanic cell. The other half-reaction is reversed to give oxidation at the anode.
- For standard reduction potentials, $E^\circ_{\text{cell}} = E^\circ_{\text{cathode}} - E^\circ_{\text{anode}}$. Potential is an intensive quantity and is not multiplied when a half-reaction is scaled.

Step 1: Identify the oxidation and reduction half-reactions The silver bromide reduction potential,

$+0.07$ V, is higher than the iron hydroxide reduction potential, -0.88 V. Silver bromide is therefore reduced, while iron is oxidised. Balancing the electrons gives

$$\mathrm{Fe\,(s) + 2\,OH^-(aq) \longrightarrow Fe(OH)_2(s) + 2\,e^-}$$

$$\mathrm{2\,AgBr\,(s) + 2\,e^- \longrightarrow 2\,Ag\,(s) + 2\,Br^-(aq).}$$

The second equation is doubled only to balance electron transfer, not to double its electrode potential.

Step 2: Combine the reactions and calculate the cell potential Cancel the two electrons to obtain

$$\mathrm{Fe\,(s) + 2\,OH^-(aq) + 2\,AgBr\,(s)} \longrightarrow \mathrm{Fe(OH)_2(s) + 2\,Ag\,(s) + 2\,Br^-(aq).}$$

This is the forward direction of the reaction written in option (A). The standard potential is

$$E^\circ_{\text{cell}} = 0.07 - (-0.88) = +0.95 \text{ V}.$$

The positive sign agrees with a spontaneous standard cell reaction, since $\Delta G^\circ = -nFE^\circ_{\text{cell}} < 0$.

Step 3: Check every remaining assertion Option (B) gives the negative of the actual cell potential and corresponds to the reverse direction. Option (C) is false because iron changes from oxidation state 0 to $+2$ and loses electrons. Option (D) is false because cell potential is intensive: scaling the reaction scales both ΔG° and n, leaving their ratio, and hence E°, unchanged.

Step 4: Conclusion Iron is oxidised and silver bromide is reduced; the overall reaction is the one in (A), with $E^\circ_{\text{cell}} = +0.95$ V. Hence, the correct option is (A).

Quick Tip to Solve: Choose the more positive reduction potential for the cathode. Balance electrons separately; never multiply an electrode potential by a stoichiometric coefficient.

58. $t_{100\%}$ is the time required for the 100% completion of the reaction while $t_{1/2}$ is the time required for 50% of the reaction to be completed. Which of the following option correctly represents the relation between $t_{100\%}$ and $t_{1/2}$ for zero and first order reactions respectively? [4 marks]

(A) $t_{100\%} = (t_{1/2})^2$ and $t_{100\%} = (t_{1/2})^{-\infty}$
(B) $t_{100\%} = 2t_{1/2}$ and $t_{100\%} = (t_{1/2})^{\infty}$
(C) $t_{100\%} = 2t_{1/2}$ and $t_{100\%} = (2t_{1/2})^2$
(D) $t_{100\%} = (t_{1/2})^{\infty}$ and $t_{100\%} = 2t_{1/2}$

Correct Answer: (B) $t_{100\%} = 2t_{1/2}$ and $t_{100\%} = (t_{1/2})^{\infty}$

Solution:

Concept:

- For a zero-order reaction, $[A]_t = [A]_0 - kt$. Its constant rate gives a finite ideal completion time as long as the zero-order law applies.
- For a first-order reaction, $[A]_t = [A]_0 e^{-kt}$, with $t_{1/2} = \ln 2/k$. At any finite time the exponential

remains positive, so exact completion is approached only as $t \to \infty$.

Step 1: Relate the times for the zero-order reaction Setting the concentration to zero in the integrated zero-order law gives

$$0 = [A]_0 - kt_{100\%}, \qquad t_{100\%} = \frac{[A]_0}{k}.$$

At half completion, $[A]_t = [A]_0/2$, so

$$t_{1/2} = \frac{[A]_0}{2k}, \qquad t_{100\%} = 2t_{1/2}.$$

Thus the required first relation must contain a factor 2, not a square of the half-life or an infinite time.

Step 2: Find the first-order completion limit After n half-lives, the remaining fraction is

$$\frac{[A]_t}{[A]_0} = 2^{-n}, \qquad t = nt_{1/2}.$$

No finite n makes this fraction exactly zero. Therefore an infinite number of half-lives, and hence an infinite time, is required for mathematically exact completion. A practical reaction may be regarded as complete at a small residual concentration, but that is not the stated 100% limit.

Step 3: Match the intended relation in the options Option (B) combines $t_{100\%} = 2t_{1/2}$ for zero order with the first-order infinite-completion-time result. Its notation $(t_{1/2})^{\infty}$ should be read as indicating infinite time, not as a dimensionally meaningful power of a time. The precise first-order statement is $t_{100\%} \to \infty$. Option (A) has the wrong zero-order relation, (C) predicts finite first-order completion, and (D) reverses the two behaviours.

Step 4: Conclusion Zero order gives $t_{100\%} = 2t_{1/2}$; first order requires $t_{100\%} \to \infty$. Interpreting the printed infinity notation in this sense, the correct option is (B).

Quick Tip to Solve: For first order, each half-life halves what remains; it never removes the final fraction in a finite number of half-lives.

59. Given below are two statements :

Statement (I) : The first ionisation enthalpy of the elements Na, Mg, Cl and Ar follows the order Na $>$ Mg $>$ Cl $>$ Ar.

Statement (II) : Among Ca, Al, Fe and B, the third ionisation enthalpy is very high for Ca.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Correct Answer: (D) **Statement I** is false but **Statement II** is true

Solution:

Concept:

- First ionisation enthalpy is the energy needed to remove the first electron from an isolated gaseous atom. Across a period it generally rises as effective nuclear attraction increases.
- A large jump in successive ionisation enthalpies occurs when the next electron must be removed from a closed-shell core rather than the valence shell.

Step 1: Test the first statement against the configurations The relevant outer configurations are $\mathrm{Na} : 3s^1$, $\mathrm{Mg} : 3s^2$, $\mathrm{Cl} : 3s^2 3p^5$, and $\mathrm{Ar} : 3s^2 3p^6$. Sodium readily loses its single $3s$ electron, whereas argon has a stable complete valence shell. For these four elements the first ionisation enthalpies follow

$$\mathrm{Na} < \mathrm{Mg} < \mathrm{Cl} < \mathrm{Ar}.$$

This is the reverse of the printed order. Hence Statement (I) is false; the general rise is not reversed by the familiar smaller anomalies elsewhere in the period.

Step 2: Identify the electron removed in the third ionisation Calcium has configuration $[\mathrm{Ar}]4s^2$. Removal of two electrons gives $\mathrm{Ca}^{2+} = [\mathrm{Ar}]$, a noble-gas configuration. The third removal must disrupt that closed shell:

$$\mathrm{Ca}^{2+}(\mathrm{g}) \longrightarrow \mathrm{Ca}^{3+}(\mathrm{g}) + \mathrm{e}^-.$$

It removes a core electron from the $n = 3$ shell, which is strongly bound. In contrast, aluminium and boron still lose a valence-shell electron at their third ionisation, while iron loses a $3d$ electron after its two $4s$ electrons. Calcium therefore has the exceptionally high third ionisation enthalpy among those listed, making Statement (II) true.

Step 3: Select the correct truth-value combination The required pair is false for (I) and true for (II). Option (A) incorrectly accepts the reversed first-ionisation order. Option (B) incorrectly denies the closed-shell jump for calcium. Option (C) gets both truth values wrong. Only option (D) agrees with both electronic-configuration arguments.

Step 4: Conclusion Statement (I) is false and Statement (II) is true. Hence, the correct option is (D).

Quick Tip to Solve: For a successive ionisation question, write the configuration of the ion immediately before the specified electron is removed; look for a closed-shell core.

60. Given below are two statements:

Statement (I): Oxidising power of halogens decreases in the order $F_2 > Cl_2 > Br_2 > I_2$, which is the basis of "Layer test".

Statement (II): "Layer test" to identify Br_2 and I_2 in aqueous solution involves the oxidation of bromide or iodide into Br_2 or I_2 respectively with Cl_2, which is a type of displacement redox reaction.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Correct Answer: (A) Both **Statement I** and **Statement II** are true

Solution:

Concept:

- An oxidising agent accepts electrons and is itself reduced. Halogen oxidising strength decreases in the order $\mathrm{F_2 > Cl_2 > Br_2 > I_2}$.
- A stronger halogen oxidant can oxidise the halide ion of a weaker halogen. The liberated molecular halogen can be extracted into an immiscible organic layer, where its characteristic colour is readily observed.

Step 1: Evaluate the halogen strength statement Chlorine is a stronger oxidising agent than bromine and iodine. It can therefore accept electrons from $\mathrm{Br^-}$ or $\mathrm{I^-}$, liberating $\mathrm{Br_2}$ or $\mathrm{I_2}$. This follows the stated decreasing halogen oxidising-power order, so Statement (I) correctly describes the redox basis of the layer test. The relevant comparison is oxidising power of the neutral halogens, not reducing power of their halide ions.

Step 2: Write the reactions involved in the test For the two halides the displacement reactions are

$$\mathrm{Cl_2(aq) + 2\,Br^-(aq) \longrightarrow 2\,Cl^-(aq) + Br_2(aq),}$$

$$\mathrm{Cl_2(aq) + 2\,I^-(aq) \longrightarrow 2\,Cl^-(aq) + I_2(aq).}$$

Bromide or iodide changes from oxidation state -1 to 0 and is oxidised. Chlorine changes from 0 to -1 and is reduced. These simultaneous oxidation and reduction changes establish that each reaction is a displacement redox reaction, as Statement (II) says.

Step 3: Connect the reactions to the layer and compare options On shaking with a suitable immiscible organic solvent, the liberated halogen partitions into that layer: bromine gives an orange-brown colour and iodine a violet colour. Thus the colour identifies the halogen produced from the corresponding aqueous halide. Both the order of oxidising strengths and the chlorine displacement explanation are correct. Options (B), (C) and (D) each declare at least one of these verified statements false, so none gives the required truth-value pair.

Step 4: Conclusion Both Statement (I) and Statement (II) are true. Hence, the correct option is (A).

Quick Tip to Solve: In a halogen displacement test, write $\mathrm{X_2 + 2\,Y^- \longrightarrow 2\,X^- + Y_2}$ only when $\mathrm{X_2}$ is the stronger oxidising halogen.

61. Which of the following sets includes all the species that will change the orange colour of $K_2Cr_2O_7$ in acidic medium? [4 marks]

(A) Fe^{2+}, Sn^{2+}, I^-, S^{2-}

(B) S^{2-}, Fe^{3+}, I^-, $C_2O_4^{2-}$

(C) Fe^{2+}, NO_2^-, SO_2, Sn^{4+}

(D) Fe^{3+}, SO_4^{2-}, S^{2-}, Sn^{4+}

Correct Answer: (A) Fe^{2+}, Sn^{2+}, I^-, S^{2-}

Solution:

Concept:

- Acidified dichromate contains chromium(VI), which accepts electrons to form chromium(III). The orange dichromate colour changes as the green chromium(III) species is formed.
- A species causes this reduction only if it can itself be oxidised under these conditions; it must therefore act as a reducing agent.

Step 1: Write the dichromate reduction The relevant half-reaction in acidic solution is

$$Cr_2O_7^{2-} + 14\,H^+ + 6\,e^- \longrightarrow 2\,Cr^{3+} + 7\,H_2O.$$

The six electrons must be supplied by the species added. Merely being an ion or containing a variable-valency element does not guarantee that it will reduce dichromate.

Step 2: Test the species in option A Each member of the first set supplies electrons on oxidation:

$$\begin{aligned} Fe^{2+} &\longrightarrow Fe^{3+} + e^-, \\ Sn^{2+} &\longrightarrow Sn^{4+} + 2\,e^-, \\ 2\,I^- &\longrightarrow I_2 + 2\,e^-, \\ S^{2-} &\longrightarrow S + 2\,e^-. \end{aligned}$$

The sulfide oxidation shown is sufficient to demonstrate its reducing character. Thus all four species can consume orange dichromate in acidic medium.

Step 3: Eliminate the other sets Option B contains Fe^{3+}, which is not the reducing form of the iron pair here. Option C contains Sn^{4+}, already the oxidised tin state under these conditions. Option D contains both these non-reducing species and $SO_4{}^{2-}$, with sulfur already at its usual highest oxidation state, +6. The presence of one reducing species in a set is insufficient: every listed member must qualify.

Step 4: Conclusion Only the set Fe^{2+}, Sn^{2+}, I^-, S^{2-} consists entirely of reducing agents for acidified dichromate. Hence, the correct option is (A).

Quick Tip to Solve: For a set of reducing agents, find one species already in a stable high oxidation state; that single member eliminates the entire set.

62. Match List - I with List - II.

	List - IChromium (III) Complexes (en = ethylene diamine)	List - IIΔ_0 (cm^{-1})
A	$[Cr(CN)_6]^{3-}$	**I. 15,060**
B	$[CrF_6]^{3-}$	**II. 17,400**
C	$[Cr(H_2O)_6]^{3+}$	**III. 22,300**
D	$[Cr(en)_3]^{3+}$	**IV. 26,600**

Choose the correct answer from the options given below: [4 marks]

(A) A-I, B-II, C-III, D-IV

(B) A-II, B-III, C-IV, D-I

(C) A-III, B-IV, C-I, D-II

(D) A-IV, B-I, C-II, D-III

Correct Answer: (D) A-IV, B-I, C-II, D-III

Solution:

Concept:

- For complexes with the same metal, oxidation state and octahedral coordination, the ligand-field splitting Δ_o increases with ligand strength in the spectrochemical series.
- The relevant increasing ligand-strength order is $F^- < H_2O < en < CN^-$, where en is the neutral bidentate ligand ethylenediamine.

Step 1: Hold the metal factors constant All four complexes contain chromium(III). Fluoride and cyanide complexes each have six monodentate ligands; the aqua complex also has six donors, and three bidentate en ligands supply six donor atoms. They therefore allow a direct comparison of octahedral splitting by ligand strength. The different overall complex charges do not change the chromium oxidation state from +3.

Step 2: Arrange the listed splitting values The values increase in the order

$$15060 < 17400 < 22300 < 26600 \quad (cm^{-1}).$$

Match the weakest ligand to the smallest value and the strongest to the largest. This gives

$$\begin{aligned} [CrF_6]^{3-} &: 15060 \text{ (I)}, \\ [Cr(H_2O)_6]^{3+} &: 17400 \text{ (II)}, \\ [Cr(en)_3]^{3+} &: 22300 \text{ (III)}, \\ [Cr(CN)_6]^{3-} &: 26600 \text{ (IV)}. \end{aligned}$$

Step 3: Restore the order of List I List I places cyanide first, fluoride second, water third and en fourth. Consequently A matches IV, B matches I, C matches II and D matches III. Option A incorrectly gives cyanide the smallest splitting, option B places water above cyanide, and option C gives fluoride the largest splitting. Each contradicts the ligand-strength order.

Step 4: Conclusion The matching is A-IV, B-I, C-II, D-III. Hence, the correct option is (D).

Quick Tip to Solve: When the metal and oxidation state are fixed, rank ligands first and numerical splittings second; then restore the original letter order.

63. Given below are two statements :

Statement (I) : 1,2,3-Trihydroxypropane can be separated from water by simple distillation.

Statement (II) : An azeotropic mixture cannot be separated by fractional distillation.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Correct Answer: (A) Both **Statement I** and **Statement II** are true

Solution:

Concept:

- Simple distillation can separate a volatile component from a much less volatile liquid by collecting the volatile component as distillate and retaining the other as residue.
- An azeotrope has the same equilibrium vapour and liquid composition, so ordinary fractional distillation at fixed pressure cannot separate its components.

Step 1: Evaluate the first statement 1,2,3-Trihydroxypropane is glycerol. Its structure is

$$HOCH_2-CHOH-CH_2OH.$$

Strong hydrogen bonding makes it much less volatile than water. On distilling an aqueous mixture, water passes preferentially into the vapour and is collected after condensation, while glycerol remains in the flask. Thus the two components can be separated without requiring glycerol itself to be collected as distillate. Statement I, which says only separated from water, is true on this literal reading.

Step 2: Distinguish separation from distilling glycerol Glycerol is normally distilled under reduced pressure when it is the liquid to be vaporised and collected, because its high normal boiling temperature can cause decomposition. That fact does not invalidate separating water from glycerol by vaporising the water. The printed statement specifies no demand to distil glycerol itself, so replacing its wording with that more restrictive demand would change the question.

Step 3: Evaluate the azeotrope statement A fractionating column enriches a component through repeated vaporisation and condensation only when the vapour and liquid compositions differ. At an azeotropic composition they coincide; repeated equilibrium stages therefore reproduce that composition. Statement II is true. Hence B wrongly rejects both statements, C wrongly rejects the azeotrope limitation, and D rejects I only by imposing the unprinted requirement that glycerol must be recovered as distillate.

Step 4: Conclusion For the literal wording, both statements are true. Hence, the correct option is (A). The supplied key treats the first statement as false, corresponding to a claim that glycerol itself should be distilled at atmospheric pressure; that additional requirement is not stated. Removing water as distillate while retaining glycerol as residue establishes the literal result.

Quick Tip to Solve: Distinguish separating a mixture from vaporising a named high-boiling component: the less volatile component can remain as the residue.

64. Given below are two statements :

Statement (I) : Benzyl chloride reacts faster in S_N1 mechanism than ethyl chloride.

Statement (II) : Ethyl carbocation intermediate is less stabilized by hyperconjugation than benzyl carbocation by resonance.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Correct Answer: (A) Both **Statement I** and **Statement II** are true

Solution:

Concept:

- In an S_N1 reaction, carbon-halogen bond ionisation produces a carbocation in the slow step. A more stable carbocation generally lowers the barrier to this ionisation.
- Resonance delocalisation in a benzyl carbocation stabilises positive charge more effectively than the limited hyperconjugation available to a primary ethyl carbocation.

Step 1: Identify the intermediates Benzyl chloride and ethyl chloride would give the following carbocations upon ionisation:

$$C_6H_5CH_2Cl \longrightarrow C_6H_5CH_2{}^+ + Cl^-,$$
$$CH_3CH_2Cl \longrightarrow CH_3CH_2{}^+ + Cl^-.$$

The carbon bearing chlorine in benzyl chloride is adjacent to the aromatic ring. It is not a ring carbon, so this comparison must not be confused with the resistance of chlorobenzene to ordinary substitution.

Step 2: Compare the stabilisation mechanisms The vacant p orbital of the benzyl carbocation overlaps with the aromatic pi system. Resonance distributes the electron deficiency between the benzylic carbon and ring positions, substantially stabilising the intermediate and the developing cation in the ionisation transition state. The ethyl carbocation has no adjacent pi system. Its neighbouring carbon-hydrogen bonds can provide hyperconjugative stabilisation, but this is weaker than the benzyl resonance stabilisation. Statement II is therefore true.

Step 3: Infer the relative reaction rate Because formation of the benzyl carbocation is energetically easier, benzyl chloride undergoes the S_N1 pathway faster than ethyl chloride under comparable conditions. Statement I is also true. Option B denies both correct statements, C wrongly rejects the stability comparison, and D wrongly rejects the resulting rate order. The question asks for their truth values rather than a separately labelled assertion-reason code.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: Identify the carbocation formed after the leaving group departs; a benzylic carbon is adjacent to the ring and benefits from resonance.

65. In IUPAC nomenclature, the correct order of decreasing priority of functional group is : [4 marks]

(A) $-CONH_2,\ >C=O,\ -CHO,\ -NH_2,\ -C\equiv C-$

(B) $-CONH_2,\ -COOCH_3,\ -CHO,\ -NH_2,\ -OH$

(C) $-CONH_2,\ -CHO,\ >C=O,\ -NH_2,\ -C\equiv C-$

(D) $-CONH_2,\ -CHO,\ -CN,\ -NH_2,\ -C\equiv C-$

Correct Answer: (C) $-CONH_2,\ -CHO,\ >C=O,\ -NH_2,\ -C\equiv C-$

Solution:

Concept:

- In naming a molecule with several functional groups, the senior characteristic group supplies the principal suffix. Other groups are described by appropriate prefixes.
- For the groups appearing here, the relevant decreasing seniority is ester, amide, nitrile, aldehyde, ketone, alcohol, amine, followed by carbon-carbon multiple bonds for the present comparison.

Step 1: Compare the carbonyl groups in the candidate sequence Amide $-CONH_2$ has higher priority than aldehyde $-CHO$, and an aldehyde has higher priority than a ketone $>C=O$. The group priority is a nomenclature convention, not an order of reaction rate or carbonyl electrophilicity. Therefore the carbonyl portion of the correct listed sequence must begin amide, aldehyde, ketone.

Step 2: Complete the decreasing order An amine group $-NH_2$ is lower in priority than the ketone group. Carbon-carbon triple bonds are expressed by unsaturation infixes and do not outrank these principal characteristic groups. The sequence among the five groups in option C is therefore

$$-CONH_2 > -CHO > >C=O > -NH_2 > -C\equiv C-.$$

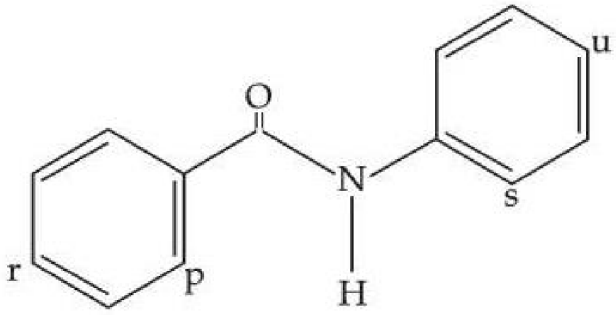
Here the greater-than signs between groups mean decreasing nomenclature priority; the angled bonds at the ketone carbon denote its two carbon substituents.

Step 3: Locate the error in each other option Option A puts ketone ahead of aldehyde, reversing their seniority. Option B puts amide ahead of ester $-COOCH_3$, although ester is senior; it also places amine ahead of alcohol, another reversal. Option D puts aldehyde ahead of nitrile $-CN$, although nitrile is senior to aldehyde. Thus none of these sequences is a consistently decreasing priority order.

Step 4: Conclusion The correct listed sequence is amide, aldehyde, ketone, amine, alkyne. Hence, the correct option is (C).

Quick Tip to Solve: Check adjacent pairs instead of memorising every option: aldehyde outranks ketone, nitrile outranks aldehyde, and ester outranks amide.

66. For the given molecule, "x", the preferred site for the attack of the electrophile is : [4 marks]

$x =$ 

(A) Predominantly at "r"

(B) "r" and "u"

(C) "p" and "s"

(D) Predominantly at "u"

Correct Answer: (D) Predominantly at "u"

Solution:

Concept:

- An aromatic substituent that donates electron density by resonance generally favours ortho and para electrophilic substitution. A carbonyl group bonded to the ring through its carbonyl carbon withdraws electron density and deactivates that ring.
- When both ortho and para positions are activated, steric hindrance commonly makes the para position the preferred site for a bulky substituent already attached to the ring.

Step 1: Distinguish the two aromatic attachments The molecule is $C_6H_5-CO-NH-C_6H_5$. The left ring is attached directly to the carbonyl carbon, so it carries the electron-withdrawing $-CONHC_6H_5$ group. The right ring is attached directly to nitrogen, so its substituent is $-NHCOC_6H_5$. These are electronically different attachments even though they belong to the same amide linkage.

Step 2: Apply the directing effects to the labelled positions The carbonyl group deactivates the left ring and directs substitution meta; the labelled left-ring sites p and r are ortho and para to the carbonyl attachment. On the right ring, the nitrogen lone pair can donate into the aromatic system, stabilising intermediates for attack at the ortho and para positions. Acylation reduces this donation compared with an amino group, but the amide nitrogen remains ortho/para directing. In the drawing, s is ortho and u is para to the nitrogen attachment.

Step 3: Choose the predominant site The nitrogen-attached ring is the preferred ring for electrophilic substitution, and its para site u avoids the steric crowding experienced at the ortho site s next to the bulky amide substituent. Option A selects the deactivated left ring; B incorrectly treats r and u as similarly favoured; C includes the disfavoured left-ring ortho site p and omits the preferred right-ring para site. Hence the predominant attack is at u.

Step 4: Conclusion The electrophile attacks predominantly at the para position labelled u on the nitrogen-attached ring. Hence, the correct option is (D).

Quick Tip to Solve: For an amide joining two rings, check which atom touches each ring: ring-CO withdraws, whereas ring-NHCO directs ortho/para, with para usually less crowded.

67. Match List - I with List - II.

	List - IMixture of Compounds		List - IIReagent used to distinguish
A	Diethyl amine + Ethyl amine	I	Bromine water
B	Acetaldehyde + Acetone	II	$CHCl_3$ + KOH, Δ
C	Ethanol + Phenol	III	Neutral $FeCl_3$
D	Benzoic acid + Cinnamic acid	IV	Ammoniacal silver nitrate

Choose the correct answer from the options given below : [4 marks]

(A) A-IV, B-II, C-I, D-III

(B) A-IV, B-II, C-III, D-I

(C) A-II, B-IV, C-I, D-III

(D) A-II, B-IV, C-III, D-I

Correct Answer: (D) A-II, B-IV, C-III, D-I

Solution:

Concept:

- A distinguishing reagent must produce different observations for the two compounds. Primary and secondary amines differ in the carbylamine test; aldehydes and ordinary ketones differ in Tollens' test.
- Phenols form coloured complexes with neutral ferric chloride. An isolated carbon–carbon double bond consumes bromine by addition.

Step 1: Distinguish the amines and carbonyl compounds Ethylamine is a primary amine, so heating it with chloroform and alcoholic potassium hydroxide produces an offensive-smelling isocyanide. Diethylamine is secondary and does not give this test. Thus A–II.

$$C_2H_5NH_2 + CHCl_3 + 3\,KOH \longrightarrow C_2H_5NC + 3\,KCl + 3\,H_2O$$

Acetaldehyde reduces ammoniacal silver nitrate to metallic silver, whereas acetone does not. Thus B–IV; the aldehyde group, rather than the methyl group common to these compounds, decides this test.

Step 2: Distinguish the alcohol and unsaturated acid pairs Phenol produces a violet colour with neutral $FeCl_3$, whereas ethanol does not form the corresponding coloured phenolic complex. Therefore C–III. Cinnamic acid contains the side-chain double bond $C_6H_5CH{=}CHCOOH$, so it decolourises bromine water by addition. Benzoic acid has no such side-chain double bond and does not give this observation under the test conditions. Hence D–I.

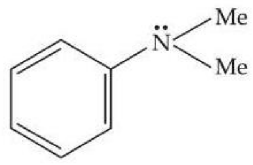
Step 3: Check the complete matching sequence The four matches are A–II, B–IV, C–III and D–I. Options (A) and (B) already assign the wrong reagent to the amine pair. Option (C) interchanges the required matches for C and D: neutral ferric chloride is the characteristic phenol test, while bromine detects the alkene unit in cinnamic acid.

Step 4: Conclusion The correct matching is A–II, B–IV, C–III, D–I. Hence, the correct option is (D).

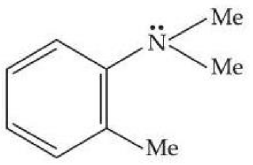
Quick Tip to Solve: Identify one diagnostic functional group in each pair first: primary amine, aldehyde, phenol and alkene give the four matches directly.

68. Consider the three aromatic molecules (P, Q and R) whose structures have been given below : The correct order regarding the reactivity of these compounds with $\mathrm{Ph-\underset{(+)}{N}\equiv NCl^{(-)}}$ under optimum but slightly acidic medium is : [4 marks]

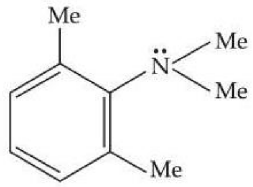
P.



Q.



R.



(A) P > Q > R

(B) R > P > Q

(C) R > Q > P

(D) P > R > Q

Correct Answer: (A) P > Q > R

Solution:

Concept:

- Azo coupling with a benzenediazonium ion is electrophilic aromatic substitution. The aromatic partner reacts faster when its ring is activated by donation of an amino nitrogen lone pair.
- Resonance donation requires overlap of the nitrogen orbital with the aromatic pi system. Bulky ortho substituents can twist the dimethylamino group and inhibit this overlap.

Step 1: Identify the steric environments in P, Q and R P is N,N-dimethylaniline, with no ortho methyl substituent on the ring. Q has one methyl group ortho to $N(CH_3)_2$, while R has methyl groups at both ortho positions. Each compound has an available para position for coupling, so blockage of that reaction site is not the deciding factor.

Step 2: Compare donation into the aromatic ring In P, the nitrogen lone pair can conjugate effectively with the ring and strongly activates the para position. One ortho methyl group in Q crowds the dimethylamino group and reduces its effective coplanarity. The two flanking methyl groups in R impose still greater steric inhibition of resonance. Consequently the activating lone-pair donation decreases in the sequence P, Q, R.

Step 3: Apply the coupling conditions and eliminate alternatives The stated optimum, slightly acidic medium is the usual condition for coupling with an aromatic amine; excessive protonation would remove the donating lone pair. Under the specified conditions, the key comparison is loss of amino-group resonance through ortho crowding, rather than simply counting the electron-donating methyl groups. Both (B) and (C) incorrectly put the most crowded compound R first. Option (D) incorrectly puts R above the less crowded Q.

Step 4: Conclusion The reactivity order is $\boxed{P > Q > R}$. Hence, the correct option is (A).

Quick Tip to Solve: For ortho-substituted dimethylanilines, check whether steric crowding suppresses the powerful nitrogen resonance effect before applying methyl-group activation.

69. Match List - I with List - II.

	List - IVitamin		List - IIName
A	Vitamin B_1	I	Pyridoxine
B	Vitamin B_2	II	Ascorbic acid
C	Vitamin B_6	III	Thiamine
D	Vitamin C	IV	Riboflavin

Choose the correct answer from the options given below: [4 marks]

(A) A-II, B-I, C-III, D-IV

(B) A-IV, B-III, C-II, D-I

(C) A-III, B-IV, C-I, D-II

(D) A-I, B-III, C-II, D-IV

Correct Answer: (C) A-III, B-IV, C-I, D-II

Solution:

Concept:

- Vitamin labels and chemical names are standard identities: vitamin B_1 is thiamine and vitamin B_2 is riboflavin.
- Vitamin B_6 is pyridoxine, while vitamin C is ascorbic acid. Matching requires the Roman numeral attached to each name in List II.

Step 1: Match the first two B vitamins For entry A, vitamin B_1 is thiamine, which is listed as III. Therefore A–III. For entry B, vitamin B_2 is riboflavin, which is listed as IV. Therefore B–IV. Do not assign these numbers by their positions in the vitamin series: the Roman numerals identify the shuffled entries in List II.

Step 2: Match vitamin B6 and vitamin C Entry C is vitamin B_6, whose listed name is pyridoxine. Pyridoxine is entry I, giving C–I. Entry D is vitamin C, also called ascorbic acid. Ascorbic acid is entry II, giving D–II. The completed mapping is

$$A \longrightarrow \text{III}, \quad B \longrightarrow \text{IV},$$
$$C \longrightarrow \text{I}, \quad D \longrightarrow \text{II}.$$

Step 3: Check each offered sequence Option (C) contains all four identities in the required order. Option (A) wrongly calls vitamin B_1 ascorbic acid; option (B) wrongly calls it riboflavin; option (D) wrongly calls it pyridoxine. Hence each alternative already fails the first match, and the remaining three correct matches independently confirm the selected sequence. No chemical reaction or numerical conversion is needed for this nomenclature question.

Step 4: Conclusion The matches are A–III, B–IV, C–I, D–II. Hence, the correct option is (C).

Quick Tip to Solve: Recall the sequence B1–thiamine, B2–riboflavin, B6–pyridoxine and C–ascorbic acid, then translate it into the shuffled Roman labels.

70. A salt with few drops of conc. HCl gives apple green colour in flame test. The group precipitate of the salt is dissolved in acetic acid and treated with K_2CrO_4 to give yellow precipitate. When the sodium carbonate extract of the salt solution is heated with conc. HNO_3 and ammonium molybdate, it resulted a canary yellow precipitate. The cation and anion present in the salt are respectively,

[4 marks]

(A) Ca^{2+} and SO_4^{2-}
(B) Ba^{2+} and PO_4^{3-}
(C) Mn^{2+} and PO_4^{3-}
(D) Ba^{2+} and SO_4^{2-}

Correct Answer: (B) Ba^{2+} and PO_4^{3-}

Solution:

Concept:

- A characteristic flame colour is a preliminary cation test; formation of a characteristic precipitate supplies an independent confirmation.
- Barium gives an apple-green flame and yellow barium chromate. Phosphate gives canary-yellow ammonium phosphomolybdate with ammonium molybdate in nitric acid.

Step 1: Identify and confirm the cation The apple-green flame points to Ba^{2+}. Concentrated hydrochloric acid helps produce the volatile chloride needed for a clear flame test. The subsequent test with potassium chromate confirms the identification: dissolved barium ions form yellow barium chromate.

$$Ba^{2+} + {CrO_4}^{2-} \longrightarrow BaCrO_4 \downarrow$$

This combines a flame observation with a chemically independent precipitation observation, rather than relying on colour alone.

Step 2: Identify the anion from the molybdate test Heating the sodium carbonate extract with nitric acid and ammonium molybdate gives the characteristic canary-yellow ammonium phosphomolybdate precipitate. It is the confirmatory test for phosphate, ${PO_4}^{3-}$, not sulphate. The precipitate can be represented as $(NH_4)_3[PMo_{12}O_{40}]$; its formation incorporates phosphate into the phosphomolybdate species.

Step 3: Reject the other cation–anion combinations Option (A) gives calcium, whose characteristic flame is brick-red, and sulphate, which does not explain the molybdate observation. Option (C) includes phosphate but assigns manganese instead of the barium established by the flame and chromate tests. Option (D) identifies barium correctly but incorrectly assigns sulphate. Only the pair in (B) explains all three observations consistently.

Step 4: Conclusion The cation is Ba^{2+} and the anion is ${PO_4}^{3-}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the flame and chromate tests together for barium; reserve the nitric-acid/ammonium-molybdate canary-yellow test for phosphate.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. 5.33 g of $CrCl_3 \cdot 6H_2O$, which is a 1 : 3 electrolyte, is dissolved in water and is passed through a cation exchanger. The chloride ions in the eluted solution, on treatment with $AgNO_3$ results in 8.61 g of AgCl. The ratio of moles of complex reacted and moles of AgCl formed is _____ $\times 10^{-2}$. (Nearest integer)

[Molar mass in g mol^{-1} Cr : 52, Ag : 108, Cl : 35.5, H : 1, O : 16] [4 marks]

Solution:

Concept:

- A $1:3$ electrolyte has one complex cation and three counter-anions per formula unit. Here the complex is $[Cr(H_2O)_6]Cl_3$, with three ionisable chloride ions.
- Moles equal mass divided by molar mass. Each mole of chloride precipitates one mole of AgCl on treatment with silver nitrate.

Step 1: Calculate the amount of hydrated chromium chloride Use all six waters in the formula mass; their mass is part of the weighed sample.

$$\begin{aligned} M(\mathrm{CrCl_3 \cdot 6\,H_2O}) &= 52 + 3(35.5) + 6[2(1) + 16] \\ &= 52 + 106.5 + 108 = 266.5\ \mathrm{g\,mol^{-1}}. \end{aligned}$$

Therefore

$$n_{\mathrm{complex}} = \frac{5.33}{266.5} = 0.0200\ \mathrm{mol}.$$

The cation exchanger retains the complex cation, while the counter-anion chloride remains in the eluted solution.

Step 2: Calculate the amount of silver chloride The molar mass of silver chloride is

$$M(\mathrm{AgCl}) = 108 + 35.5 = 143.5\ \mathrm{g\,mol^{-1}}.$$

Thus the given precipitate contains

$$n_{\mathrm{AgCl}} = \frac{8.61}{143.5} = 0.0600\ \mathrm{mol}.$$

This is three times the complex amount, exactly as expected for three free chloride ions per formula unit. It also confirms that the ratio should use formula units of complex, not the total number of ions in solution.

Step 3: Express the ratio in the requested form The ratio is dimensionless because it divides moles by moles:

$$\frac{n_{\mathrm{complex}}}{n_{\mathrm{AgCl}}} = \frac{0.0200}{0.0600} = \frac{1}{3} = x \times 10^{-2}.$$

Hence

$$x = \frac{100}{3} = 33.333\ldots.$$

Rounding the requested coefficient, rather than the ratio itself, gives the nearest integer.

Step 4: Conclusion The ratio is $\frac{1}{3} = 33.333\ldots \times 10^{-2}$. The required nearest integer is $\boxed{33}$ (dimensionless).

Quick Tip to Solve: For a 1:3 electrolyte with three chloride counter-ions, the complex-to-silver-chloride mole ratio is 1:3; apply the printed power-of-ten factor last.

72. Consider the isomers of hydrocarbon with molecular formula C_5H_{10}. These isomers do not decolourise $KMnO_4$ solution. These isomers are subjected to chlorination with chlorine in presence of light to give monochloro compounds. The total number of monochloro compounds (structural isomers only) formed is _____. [4 marks]

Solution:

Concept:

- For C_5H_{10}, the degree of unsaturation is one. Failure to decolourise permanganate excludes the alkene isomers, leaving saturated monocyclic hydrocarbons.
- Monochlorination replaces one hydrogen with chlorine. Count constitutionally distinct hydrogen-bearing carbon positions, treating symmetry-related positions as equivalent and ignoring stereoisomers.

Step 1: List the possible cyclic carbon skeletons A five-membered ring gives cyclopentane. A four-membered ring with the remaining carbon gives methylcyclobutane. A three-membered ring can carry either one ethyl group or two methyl groups, giving ethylcyclopropane, 1,1-dimethylcyclopropane and 1,2-dimethylcyclopropane. On a three-membered ring, the apparent 1,3 arrangement is identical to 1,2 by renumbering. Thus there are five constitutional starting skeletons.

Step 2: Count the distinct substitution sites in each skeleton **Cyclopentane:** all five ring carbons are equivalent, giving 1 product.

Methylcyclobutane: the methyl carbon, substituted ring carbon, two equivalent adjacent ring carbons and opposite ring carbon give 4 products.

Ethylcyclopropane: terminal methyl, side-chain methylene, substituted ring carbon and the two equivalent remaining ring carbons give 4 products.

1,1-Dimethylcyclopropane: the two equivalent methyl groups and two equivalent ring methylene groups give 2 products; the substituted ring carbon has no hydrogen.

1,2-Dimethylcyclopropane: the equivalent methyl groups, equivalent substituted ring carbons and remaining ring methylene give 3 products.

Step 3: Add only constitutionally different products Chlorination preserves each carbon skeleton, so products from the five different skeletons are distinct. Cis/trans forms or different spatial arrangements at a substituted carbon are not counted separately, because only structural isomers are requested. The total is

$$N = 1 + 4 + 4 + 2 + 3 = 14.$$

In particular, two different hydrogen orientations on the same carbon do not create additional constitutional products.

Step 4: Conclusion The total number of monochloro compounds, counting structural isomers only, is $\boxed{14}$.

Quick Tip to Solve: List cyclic skeletons first, then mark inequivalent carbon atoms that actually bear hydrogen. Ignore cis/trans doubling and exclude quaternary carbons.

73. One mole of an alkane (x) requires 8 mole oxygen for complete combustion. Sum of number of carbon and hydrogen atoms in the alkane (x) is _____. [4 marks]

Solution:

Concept:

- An acyclic saturated hydrocarbon (alkane) has molecular formula C_nH_{2n+2}, where n is the number of carbon atoms in one molecule.
- Complete combustion converts every carbon atom into CO_2 and every pair of hydrogen atoms into H_2O. Balance these products before calculating the oxygen requirement.

Step 1: Write the balanced combustion equation One mole of C_nH_{2n+2} produces n moles of carbon dioxide and $n+1$ moles of water. Thus the products contain $2n+(n+1)=3n+1$ moles of oxygen atoms. Each mole of oxygen gas provides two moles of oxygen atoms, giving

$$C_nH_{2n+2}+\frac{3n+1}{2}O_2 \longrightarrow nCO_2+(n+1)H_2O.$$

Step 2: Use the stated oxygen consumption The coefficient of oxygen is its molar requirement per mole of alkane. Since the question gives eight moles of oxygen for one mole of fuel,

$$\frac{3n+1}{2}=8 \quad \Rightarrow \quad 3n+1=16 \quad \Rightarrow \quad n=5.$$

The alkane therefore has formula C_5H_{12}. Its particular chain arrangement does not affect the combustion stoichiometry.

Step 3: Count the atoms and verify the balance A molecule has five carbon atoms and $2(5)+2=12$ hydrogen atoms. The check equation is

$$C_5H_{12}+8\,O_2 \longrightarrow 5\,CO_2+6\,H_2O.$$

Both sides contain five carbon, twelve hydrogen and sixteen oxygen atoms. The requested sum refers to atoms in the molecular formula, rather than the macroscopic number of atoms in one mole.

Step 4: Conclusion The alkane is C_5H_{12}, so the required sum is $5+12=\boxed{17}$.

Quick Tip to Solve: For C_nH_{2n+2}, remember the oxygen coefficient $(3n+1)/2$; after finding n, the total number of carbon and hydrogen atoms is $3n+2$.

74. For reaction A $\rightarrow$ P, rate constant $k = 1.5 \times 10^3\ \text{s}^{-1}$ at 27°C

If activation energy for the above reaction is $60\ \text{kJ mol}^{-1}$, then the temperature (in $^\circ\text{C}$) at which rate constant, $k = 4.5 \times 10^3\ \text{s}^{-1}$ is _____. (Nearest integer)

Given : $\log 2 = 0.30$, $\log 3 = 0.48$, $R = 8.3\ \text{J K}^{-1}\ \text{mol}^{-1}$, $\ln 10 = 2.3$ [4 marks]

Solution:

Concept:

- The Arrhenius equation is $k = Ae^{-E_a/(RT)}$, where E_a is activation energy, R is the gas constant and T is absolute temperature in kelvin.
- Eliminating the common pre-exponential factor at two temperatures gives $\ln(k_2/k_1) = (E_a/R)(1/T_1 - 1/T_2)$. The energy units of E_a must match those of R.

Step 1: Convert the data and find the rate-constant ratio Use the given constants and take

$$T_1 = 27 + 273 = 300\,\text{K}, \qquad E_a = 60 \times 10^3\,\text{J mol}^{-1}.$$

The rate constant triples, so its units cancel in the ratio:

$$\frac{k_2}{k_1} = \frac{4.5 \times 10^3}{1.5 \times 10^3} = 3, \qquad \ln 3 = (\ln 10)(\log 3) = 2.3(0.48) = 1.104.$$

Step 2: Substitute in the two-temperature Arrhenius relation Since the rate constant increases for a positive activation energy, the desired temperature must exceed $300\,\text{K}$. Rearranging before numerical substitution keeps the sign clear:

$$\frac{1}{T_2} = \frac{1}{T_1} - \frac{R}{E_a}\ln\frac{k_2}{k_1} = \frac{1}{300} - \frac{8.3(1.104)}{60000}.$$

Hence

$$\frac{1}{T_2} = 0.00333333 - 0.00015272 = 0.00318061\,\text{K}^{-1}, \qquad T_2 \approx 314.40\,\text{K}.$$

Step 3: Convert back to the requested Celsius scale The Arrhenius substitution must use kelvin, but the answer is requested in degrees Celsius. Therefore

$$t_2 = T_2 - 273 \approx 41.40\,^\circ\text{C}.$$

Round only after this conversion. Using the more precise conversion 273.15 also gives the same nearest integer. The positive temperature increase is consistent with the larger rate constant.

Step 4: Conclusion The required temperature, to the nearest integer, is $\boxed{41\,^\circ\text{C}}$. The numerical entry is 41.

Quick Tip to Solve: Use kelvin inside the Arrhenius equation and keep activation energy in joules when R is in joules. Convert to Celsius only after solving for T_2.

75. At the transition temperature T, $A \rightleftharpoons B$ and $\Delta G^0 = 105 - 35 \log T$ where A and B are two states of substance X. The transition temperature in $^\circ C$ when pressure is 1 atm is _____. (Nearest integer) [4 marks]

Solution:

Concept:

- At the transition temperature and the specified pressure, the two states of a pure substance coexist in equilibrium. Their molar Gibbs energies are equal, so the Gibbs energy change for converting one state into the other is zero.
- In the given standard-state relation, impose $\Delta G^0 = 0$ at this equilibrium. The symbol $\log$ denotes the common logarithm (base ten), and the thermodynamic temperature T is in kelvin.

Step 1: Apply the equilibrium condition to the supplied relation For $A \rightleftharpoons B$, neither direction has a Gibbs-energy advantage at the transition temperature. Thus the stated expression becomes

$$0 = 105 - 35 \log T.$$

This uses the supplied temperature dependence directly. There is no need to infer separate enthalpy and entropy changes or to replace the printed logarithmic dependence with a linear expression.

Step 2: Solve for the absolute temperature Move the logarithmic term to the other side and divide by its coefficient:

$$35 \log T = 105 \quad \Rightarrow \quad \log T = 3.$$

Taking the base-ten antilogarithm gives

$$T = 10^3 = 1000\,\text{K}.$$

A substitution verifies the equilibrium condition: $105 - 35 \log(1000) = 105 - 35(3) = 0$. Using e^3 here would incorrectly treat the printed common logarithm as a natural logarithm.

Step 3: Express the result in degrees Celsius The temperature obtained from the thermodynamic formula is an absolute temperature. Convert it to the scale requested in the question:

$$t = T - 273.15 = 1000 - 273.15 = 726.85\,^\circ\text{C}.$$

Rounding this value to the nearest integer gives 727. The usual examination conversion $T - 273$ yields the same integer; 1000 itself is the kelvin value and is not the requested entry.

Step 4: Conclusion The transition temperature is $1000\,\text{K}$, or $\boxed{727\,^\circ\text{C}}$ to the nearest integer. The numerical entry is 727.

Quick Tip to Solve: At phase equilibrium, set the Gibbs energy difference to zero, solve in kelvin, and then convert to Celsius. Distinguish $\log$ from $\ln$ before taking the antilogarithm.