JEE Main 2026 Paper 1 (B.E./B.Tech) Apr 2 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2026, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 2 April 2026, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let α, β be the roots of the equation $x^2 - 3x + r = 0$, and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 + 3x + r = 0$.

If the roots of the equation $x^2 + 6x = m$ are $2\alpha + \beta + 2r$ and $\alpha - 2\beta - \frac{r}{2}$, then m is equal to : [4 marks]

(A) -135

(B) -567

(C) 135

(D) 567

Correct Answer: (D) 567

Solution:

Concept:

- For a monic quadratic $x^2 + px + q = 0$, the sum of its roots is $-p$ and their product is q.
- Use the two given root sums to determine the ordered pair (α, β). In the final equation $x^2+6x-m = 0$, the root product is $-m$, not m.

Step 1: Determine the original roots The first equation gives $\alpha + \beta = 3$. The second gives $\alpha/2 + 2\beta = -3$, or $\alpha + 4\beta = -6$. Subtract the first relation from the latter:

$$3\beta = -9, \qquad \beta = -3, \qquad \alpha = 6.$$

The product relation in either equation now yields

$$r = \alpha\beta = 6(-3) = -18.$$

This also checks the second equation: its proposed roots are 3 and -6, whose sum is -3 and product is -18.

Step 2: Substitute into the final roots Evaluate each expression with its own signs intact:

$$2\alpha + \beta + 2r = 12 - 3 - 36 = -27,$$

$$\alpha - 2\beta - \frac{r}{2} = 6 + 6 + 9 = 21.$$

Their sum is -6, as required by the coefficient of x. Their product determines the constant term:

$$-m = (-27)(21) = -567, \qquad m = 567.$$

Option (B), -567, is the root product rather than the required value of m. Options (A) and (C) have the wrong magnitude and cannot reproduce the product of the two specified roots.

Step 3: Conclusion The equation is $x^2 + 6x - 567 = 0$, so $\boxed{m = 567}$. Hence, the correct option is (D).

Quick Tip to Solve: Move the right-hand side to the left before applying the product-of-roots formula; here the constant term is $-m$.

2. Let the circles $C_1 : |z| = r$ and $C_2 : |z - 3 - 4i| = 5$, $z \in \mathbb{C}$, be such that C_2 lies within C_1. If z_1 moves on C_1, z_2 moves on C_2 and $\min |z_1 - z_2| = 2$, then $\max |z_1 - z_2|$ is equal to : [4 marks]

(A) 12

(B) 17

(C) 22

(D) 24

Correct Answer: (C) 22

Solution:

Concept:

- In the Argand plane, $|z - a| = R$ is a circle with centre a and radius R. A complex modulus $|z_1 - z_2|$ represents the Euclidean distance between the two points.
- For one circle lying inside another, the minimum boundary separation is the outer radius minus the centre distance and the inner radius. The maximum separation is the sum of these three lengths.

Step 1: Find the outer radius from the minimum separation The centre of C_1 is the origin, while that of C_2 is $(3, 4)$. Thus the centre separation is

$$d = \sqrt{3^2 + 4^2} = 5.$$

The radius of C_2 is also 5. Its farthest point from the origin is at distance $d + 5 = 10$. Since C_2 lies inside C_1, the shortest distance to the outer boundary occurs along this outward line of centres:

$$\min |z_1 - z_2| = r - (d + 5) = r - 10 = 2.$$

Therefore $r = 12$, consistent with strict containment.

Step 2: Maximise the distance between the moving points Choose z_2 at the farthest point of the inner circle from the origin and choose z_1 on the opposite side of the outer circle. These collinear positions attain the triangle-inequality upper bound:

$$|z_1 - z_2| \le |z_1| + |z_2| \le 12 + 10 = 22.$$

Equality is attainable, so this bound is the maximum. Option (A), 12, is merely the outer radius. Option (B), 17, omits the inner radius. Option (D), 24, is the outer diameter, but the second point cannot reach the opposite outer boundary.

Step 3: Conclusion The required maximum is $\boxed{22}$. Hence, the correct option is (C).

Quick Tip to Solve: For nested circles, first use $R - d - r$ for the minimum gap; the maximum distance is then $R + d + r$.

3. If the system of equations

$$x + 5y + 6z = 4$$

$$2x + 3y + 4z = 7$$

$$x + 6y + az = b$$

has infinitely many solutions, then the point (a, b) lies on the line [4 marks]

(A) $y - x = 3$

(B) $x - y = 3$

(C) $x + y = 11$

(D) $x + y = 12$

Correct Answer: (B) $x - y = 3$

Solution:

Concept:

- Three linear equations in three unknowns have infinitely many solutions when their coefficient and augmented matrices have the same rank smaller than three.
- If the first two equations are independent, the third must be their linear combination, including its right-hand side. Checking only a zero determinant does not ensure consistency.

Step 1: Express the third equation as a linear combination The first two coefficient rows are independent because their x, y minor is $1 \cdot 3 - 2 \cdot 5 = -7 \neq 0$. Let the third equation be u times the first plus v times the second. Comparing the coefficients of x and y gives

$$u + 2v = 1, \qquad 5u + 3v = 6.$$

Substituting $u = 1 - 2v$ into the second relation yields

$$5 - 7v = 6, \qquad v = -\frac{1}{7}, \qquad u = \frac{9}{7}.$$

These multipliers are fixed by the two independent rows, so they must also reproduce the z-coefficient and constant term.

Step 2: Determine the point and test the listed lines Matching the remaining entries gives

$$a = 6u + 4v = \frac{54 - 4}{7} = \frac{50}{7}, \qquad b = 4u + 7v = \frac{36 - 7}{7} = \frac{29}{7}.$$

The third equation is now redundant, while the first two impose two independent conditions on three variables; hence there is one free variable and infinitely many solutions. For the point $(x, y) = (a, b)$,

$$x - y = a - b = \frac{21}{7} = 3.$$

In contrast, $y - x = -3$, so option (A) fails. Also $x + y = 79/7$, which is neither 11 nor 12, excluding options (C) and (D).

Step 3: Conclusion The point is $(50/7, 29/7)$ and lies on $\boxed{x - y = 3}$. Hence, the correct option is (B).

Quick Tip to Solve: For infinitely many solutions, apply the same row relation to the constants as to the coefficients; this rules out inconsistent systems.

4. Let $a_1, a_2, a_3, \ldots$ be an A.P. and $g_1 = a_1, g_2, g_3, \ldots$ be an increasing G.P. If $a_1 = a_2 + g_2 = 1$ and $a_3 + g_3 = 4$, then $a_{10} + g_5$ is equal to: [4 marks]

(A) 81

(B) 76

(C) 62

(D) 55

Correct Answer: (D) 55

Solution:

Concept:

- An A.P. with first term 1 and common difference d has $a_n = 1 + (n-1)d$. A G.P. with first term 1 and common ratio t has $g_n = t^{n-1}$.
- The condition that the G.P. is increasing selects the admissible common ratio after solving the algebraic equation. A negative ratio produces alternating signs and cannot give this increasing sequence.

Step 1: Translate the given conditions into equations The chained equality states both $a_1 = 1$ and $a_2 + g_2 = 1$. Since $g_1 = a_1 = 1$, write

$$a_2 = 1 + d, \quad a_3 = 1 + 2d, \quad g_2 = t, \quad g_3 = t^2.$$

Thus the first sum gives $1 + d + t = 1$, so $d = -t$. The second sum becomes

$$1 + 2d + t^2 = 4 \quad \Longrightarrow \quad t^2 - 2t - 3 = 0.$$

Factoring gives $(t - 3)(t + 1) = 0$, with candidates $t = 3$ and $t = -1$.

Step 2: Use the increasing condition and evaluate the terms With $t = -1$, the G.P. is $1, -1, 1, -1, \ldots$, which is not increasing. Hence $t = 3$ and $d = -3$. These values satisfy the original sums: $a_2 + g_2 = -2 + 3 = 1$ and $a_3 + g_3 = -5 + 9 = 4$. Therefore

$$a_{10} = 1 + 9(-3) = -26, \qquad g_5 = 3^4 = 81,$$

$$a_{10} + g_5 = -26 + 81 = 55.$$

Option (A) is g_5 alone and omits the A.P. contribution. Option (B) uses $a_3 = -5$ instead of a_{10}. Option (C) also fails the required tenth-term calculation.

Step 3: Conclusion The required sum is $\boxed{55}$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the term indices separate: the tenth A.P. term uses nine differences, while the fifth G.P. term uses the fourth power of its ratio.

5. The sum $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots$ up to 8 terms, is : [4 marks]

(A) 70

(B) 71

(C) 72

(D) 73

Correct Answer: (B) 71

Solution:

Concept:

- The sum of the first k positive odd integers is k^2, while the sum of the first k cubes is $[k(k+1)/2]^2$.
- Simplify the general term before summing. The resulting shifted square series can be evaluated with $\sum_{j=1}^{n} j^2 = n(n+1)(2n+1)/6$.

Step 1: Simplify the general term In the k-th fraction, the numerator includes cubes from 1^3 through k^3, and the denominator includes the first k odd numbers. Therefore

$$T_k = \frac{1^3 + 2^3 + \cdots + k^3}{1 + 3 + \cdots + (2k-1)} = \frac{k^2(k+1)^2/4}{k^2} = \frac{(k+1)^2}{4}.$$

The cancellation is valid because $k \geq 1$. As checks, $T_1 = 1$ and $T_2 = 9/4$, exactly matching the first two displayed fractions.

Step 2: Add the eight simplified terms The index k runs from 1 to 8, so $k+1$ runs from 2 to 9. Consequently

$$S_8 = \frac{1}{4}\sum_{j=2}^{9} j^2 = \frac{1}{4}\left(\frac{9 \cdot 10 \cdot 19}{6} - 1\right) = \frac{285-1}{4} = 71.$$

The subtraction removes 1^2, which is not among the shifted squares. Equivalently, the eight squared values sum to

$$4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 = 284.$$

Dividing this exact sum by four rules out 70, 72, and 73, so options (A), (C), and (D) do not satisfy the stated eight-term sum.

Step 3: Conclusion The sum of the first eight fractions is $\boxed{71}$. Hence, the correct option is (B).

Quick Tip to Solve: After simplifying to $(k+1)^2/4$, shift both limits carefully: eight terms mean the squares from 2^2 to 9^2.

6. If for $3 \le r \le 30$, $\binom{30}{30-r} + 3\binom{30}{31-r} + 3\binom{30}{32-r} + \binom{30}{33-r} = {}^{m}C_{r}$, then m equals : [4 marks]

(A) 31

(B) 32

(C) 33

(D) 34

Correct Answer: (C) 33

Solution:

Concept:

- Binomial symmetry gives $\binom{n}{k} = \binom{n}{n-k}$. It converts the lower indices $30-r, 31-r, 32-r, 33-r$ into $r, r-1, r-2, r-3$.
- The weights $1, 3, 3, 1$ are the coefficients of $(1+x)^3$. Multiplying binomial expansions therefore combines the given four terms into one coefficient.

Step 1: Rewrite the sum using binomial symmetry For $3 \le r \le 30$, all the required indices are valid. Denote the left-hand side by L. Applying symmetry separately to every term gives

$$L = \binom{30}{r} + 3\binom{30}{r-1} + 3\binom{30}{r-2} + \binom{30}{r-3}.$$

This is precisely the coefficient of x^r in

$$(1+x)^{30}(1+3x+3x^2+x^3).$$

The four contributions correspond respectively to taking powers $0, 1, 2, 3$ from the second factor and the complementary powers from the first.

Step 2: Combine the factors and identify the coefficient Since $1+3x+3x^2+x^3 = (1+x)^3$, the product is

$$(1+x)^{30}(1+x)^3 = (1+x)^{33}.$$

Its coefficient of x^r is $\binom{33}{r}$. Hence the given identity holds with $m = 33$. This also excludes the other listed values: at $r = 3$, the coefficients $\binom{m}{3}$ strictly increase as the integer $m \ge 3$ increases. Thus $m = 31, 32, 34$ cannot have the same coefficient as $m = 33$, and options (A), (B), and (D) fail.

Step 3: Conclusion The required upper index is $\boxed{m = 33}$. Hence, the correct option is (C).

Quick Tip to Solve: Recognise weights $1, 3, 3, 1$ as a binomial row: their convolution with row 30 produces row $30+3$.

7. Let p_n denote the total number of triangles formed by joining the vertices of an n-side regular polygon.

If $p_{n+1} - p_n = 66$, then the sum of all distinct prime divisors of n is : [4 marks]

(A) 7

(B) 8

(C) 5

(D) 6

Correct Answer: (C) 5

Solution:

Concept:

- Any three distinct vertices of a regular polygon form a triangle because no three of its vertices are collinear. Thus the number of triangles is a combination, not a permutation.
- Pascal's identity gives $\binom{n+1}{3} - \binom{n}{3} = \binom{n}{2}$. Distinct prime divisors are counted once each, regardless of their powers in a factorisation.

Step 1: Express the increase in the number of triangles There are $\binom{n}{3}$ ways to choose three vertices from the original polygon. For the polygon with one additional vertex, there are $\binom{n+1}{3}$ choices. Subtracting and simplifying gives

$$p_{n+1} - p_n = \binom{n+1}{3} - \binom{n}{3} = \binom{n}{2} = \frac{n(n-1)}{2}.$$

The same count follows by fixing the newly added vertex and choosing the other two vertices from the original n vertices.

Step 2: Determine the admissible number of sides Substitute the stated difference of 66:

$$n(n-1) = 132, \qquad n^2 - n - 132 = (n-12)(n+11) = 0.$$

Hence $n = 12$ or $n = -11$. The negative root cannot represent the number of sides of a polygon, so $n = 12$. As a check, $\binom{13}{3} - \binom{12}{3} = 286 - 220 = 66$.

Step 3: Add only the distinct prime divisors The prime factorisation is $12 = 2^2 \cdot 3$. Its distinct prime divisors are therefore 2 and 3, giving $2 + 3 = 5$. Counting the repeated factor 2 would incorrectly give 7; 8 and 6 are also different from the required distinct-prime sum.

Step 4: Conclusion The sum of all distinct prime divisors of n is $\boxed{5}$. Hence, the correct option is (C).

Quick Tip to Solve: For a change from an n-vertex polygon to an $(n+1)$-vertex polygon, count the new triangles by fixing the new vertex: the increase is immediately $\binom{n}{2}$.

8. A man throws a fair coin repeatedly. He gets 10 points for each head he throws and 5 points for each tail he throws. If the probability that he gets exactly 30 points is $\frac{m}{n}$, gcd $(m, n) = 1$, then m + n is equal to : [4 marks]

(A) 53

(B) 55

(C) 107

(D) 105

Correct Answer: (C) 107

Solution:

Concept:

- With independent fair-coin tosses, each specified sequence of r outcomes has probability 2^{-r}. Sequences of different lengths need not have equal probabilities.
- All score increments are positive. Thus an exact total of 30 is reached at most once, and the events of reaching it after different numbers of tosses are mutually exclusive.

Step 1: List the possible head and tail counts Let h and t be the numbers of heads and tails when the cumulative score reaches 30. Their scores satisfy

$$10h + 5t = 30, \qquad 2h + t = 6.$$

The nonnegative integer solutions (h, t) are $(3, 0), (2, 2), (1, 4), (0, 6)$, requiring respectively 3, 4, 5 and 6 tosses. No other number of tosses can first produce that total. Since every increment is positive, a sequence whose final score is 30 has not previously reached or exceeded 30.

Step 2: Calculate the probability for each length For fixed counts, choose the positions occupied by heads. This gives

$$P_3 = \binom{3}{3}\left(\frac{1}{2}\right)^3 = \frac{1}{8}, \quad P_4 = \binom{4}{2}\left(\frac{1}{2}\right)^4 = \frac{6}{16},$$
$$P_5 = \binom{5}{1}\left(\frac{1}{2}\right)^5 = \frac{5}{32}, \quad P_6 = \binom{6}{0}\left(\frac{1}{2}\right)^6 = \frac{1}{64}.$$

Later tosses are unrestricted once 30 has been reached, so no extra probability factor is needed after these sequences.

Step 3: Add the disjoint probabilities and reduce The four hitting events are disjoint, so their probabilities add:

$$P = P_3 + P_4 + P_5 + P_6 = \frac{8 + 24 + 10 + 1}{64} = \frac{43}{64}.$$

The numerator 43 is prime and the denominator is a power of 2, so the fraction is already in lowest terms. Therefore $m + n = 43 + 64 = 107$, which excludes 53, 55 and 105.

Step 4: Conclusion The required probability is $43/64$, and $\boxed{m + n = 107}$. Hence, the correct option is (C).

Quick Tip to Solve: When positive random increments must hit a target exactly, enumerate the possible counts and add their sequence probabilities; do not assign equal weights to sequences of unequal lengths.

9. The mean and variance of n observations are 8 and 16, respectively. If the sum of the first (n − 1) observations is 48 and the sum of squares of the first (n − 1) observations is 496, then the value of n is : [4 marks]

(A) 21

(B) 16

(C) 13

(D) 7

Correct Answer: (D) 7

Solution:

Concept:

- For n observations with mean $\bar{x}$, their sum is $n\bar{x}$. The last observation is obtained by subtracting the sum of the first $n-1$ observations.
- The variance of the observations is $\sigma^2 = \frac{1}{n}\sum x_i^2 - \bar{x}^2$. This uses the denominator n for the given data, rather than a sample-variance correction.

Step 1: Express the last observation in terms of the count Since the mean of all the observations is 8, their total is $8n$. The first $n-1$ observations add to 48. Therefore the last observation is

$$x_n = 8n - 48.$$

Its square must be added to 496 to obtain the sum of squares of all n observations. The given sum 48 alone does not determine n, because the last observation is not yet known.

Step 2: Apply the variance identity The mean and variance give

$$\frac{\sum x_i^2}{n} = \sigma^2 + \bar{x}^2 = 16 + 8^2 = 80.$$

Consequently,

$$496 + (8n - 48)^2 = 80n.$$

Expanding and moving every term to the left gives

$$64n^2 - 848n + 2800 = 0, \qquad 4n^2 - 53n + 175 = 0.$$

Step 3: Select the integer root and check the data Factorising the quadratic,

$$(n-7)(4n-25) = 0,$$

so $n = 7$ or $n = 25/4$. An observation count must be an integer, which rules out $25/4$. For $n = 7$, the final observation is $56 - 48 = 8$; the total of squares becomes $496 + 64 = 560$. This gives $560/7 - 8^2 = 80 - 64 = 16$, exactly the stated variance. The other offered counts 21, 16 and 13 do not satisfy the derived equation.

Step 4: Conclusion The number of observations is $\boxed{n = 7}$. Hence, the correct option is (D).

Quick Tip to Solve: Convert the mean and variance immediately into a total and a sum of squares. When one observation is omitted, express it from the total before using the sum-of-squares equation.

10. Let a circle pass through the origin and its centre be the point of intersection of two mutually perpendicular lines $x + (k-1)y + 3 = 0$ and $2x + k^2y - 4 = 0$. If the line $x - y + 2 = 0$ intersects the circle at the points A and B, then $(AB)^2$ is equal to : [4 marks]

(A) 10

(B) 27

(C) 18

(D) 34

Correct Answer: (C) 18

Solution:

Concept:

- Lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular when $a_1a_2 + b_1b_2 = 0$. This condition also handles vertical lines without dividing by a slope denominator.
- For a circle of radius r, a chord whose line is at distance d from the centre has squared length $4(r^2 - d^2)$, since the perpendicular from the centre bisects the chord.

Step 1: Determine the real parameter from perpendicularity The two line normals are $(1, k-1)$ and $(2, k^2)$. Their scalar product must be zero:

$$2 + k^2(k-1) = 0, \qquad (k+1)(k^2 - 2k + 2) = 0.$$

Since $k^2 - 2k + 2 = (k-1)^2 + 1$ is positive for every real k, the only real value is $k = -1$.

Step 2: Find the circle centre and radius Substituting $k = -1$, the centre is the intersection of

$$x - 2y + 3 = 0, \qquad 2x + y - 4 = 0.$$

From $x = 2y - 3$, the second equation gives $5y - 10 = 0$. Thus the centre is $C = (1, 2)$. The circle passes through the origin, so its radius satisfies

$$r^2 = CO^2 = 1^2 + 2^2 = 5.$$

Step 3: Use the perpendicular distance to the chord For the chord line $x - y + 2 = 0$, the distance from $C = (1, 2)$ is

$$d = \frac{|1 - 2 + 2|}{\sqrt{1^2 + (-1)^2}} = \frac{1}{\sqrt{2}}.$$

Therefore

$$(AB)^2 = 4\left(5 - \frac{1}{2}\right) = 18.$$

The inequality $d^2 < r^2$ confirms that the line indeed cuts a real chord. Values 10 and 27 do not satisfy this chord relation; 34 is even larger than the squared diameter $4r^2 = 20$, so it cannot be a chord's squared length.

Step 4: Conclusion The required value is $\boxed{(AB)^2 = 18}$. Hence, the correct option is (C).

Quick Tip to Solve: To find a chord length, calculate the centre-to-line distance and use $4(r^2 - d^2)$; solving explicitly for both intersection points is usually unnecessary.

11. Let O be the origin, and P and Q be two points on the rectangular hyperbola $xy = 12$ such that the midpoint of the line segment PQ is $\left(\frac{1}{2}, -\frac{1}{2}\right)$. Then the area of the triangle OPQ equals : [4 marks]

(A) $\frac{3}{2}$

(B) $\frac{5}{2}$

(C) $\frac{7}{2}$

(D) $\frac{9}{2}$

Correct Answer: (C) $\frac{7}{2}$

Solution:

Concept:

- The midpoint of $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ is $((x_1 + x_2)/2, (y_1 + y_2)/2)$. Each point on $xy = 12$ has a nonzero x-coordinate and satisfies $y = 12/x$.
- For a triangle with one vertex at the origin, its area is $\frac{1}{2}|x_1y_2 - x_2y_1|$. The absolute value ensures a positive area regardless of the order of the other vertices.

Step 1: Use both midpoint coordinates The stated midpoint gives

$$x_1 + x_2 = 1, \qquad y_1 + y_2 = -1.$$

Since both endpoints lie on the hyperbola, substitute $y_1 = 12/x_1$ and $y_2 = 12/x_2$ into the second relation:

$$12\left(\frac{1}{x_1} + \frac{1}{x_2}\right) = -1.$$

Using the known sum $x_1 + x_2 = 1$, this becomes $12/(x_1x_2) = -1$, so $x_1x_2 = -12$.

Step 2: Recover the two endpoints Two numbers with sum 1 and product -12 are the roots of

$$u^2 - u - 12 = (u - 4)(u + 3) = 0.$$

Thus we may take $x_1 = 4$ and $x_2 = -3$. Their corresponding ordinates are $y_1 = 12/4 = 3$ and $y_2 = 12/(-3) = -4$, giving $P = (4, 3)$ and $Q = (-3, -4)$. Both products equal 12, and their midpoint is $(1/2, -1/2)$, verifying the required conditions.

Step 3: Evaluate the determinant area With $O = (0, 0)$, use the coordinate area formula:

$$[OPQ] = \frac{1}{2}|4(-4) - (-3)(3)| = \frac{1}{2}|-16 + 9| = \frac{7}{2}.$$

Swapping the names of P and Q only changes the determinant's sign, not the area. The other values $3/2$, $5/2$ and $9/2$ would require determinant magnitudes 3, 5 and 9; the actual magnitude is 7.

Step 4: Conclusion The area of triangle OPQ is $\boxed{\frac{7}{2}}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: On $xy = c$, a chord midpoint supplies the sum of the endpoint abscissae, while the sum of their reciprocals gives the product. Recover the endpoints from a quadratic.

12. Let the parabola $y = x^2 + px + q$ passing through the point $(1, -1)$ be such that the distance between its vertex and the x-axis is minimum. Then the value of $p^2 + q^2$ is : [4 marks]

(A) 2

(B) 4

(C) 5

(D) 8

Correct Answer: (B) 4

Solution:

Concept:

- Completing the square writes $y = x^2 + px + q$ as $y = (x + p/2)^2 + q - p^2/4$. Its vertex is therefore $(-p/2, q - p^2/4)$.
- The distance of a point from the x-axis is the absolute value of its ordinate. A quadratic expressed as a nonnegative square plus a constant can be minimised directly.

Step 1: Apply the condition that the parabola passes through the point Substitute $x = 1$ and $y = -1$ into the parabola equation:

$$-1 = 1 + p + q, \qquad q = -p - 2.$$

Thus only one parameter is free. The vertex ordinate becomes

$$y_V = q - \frac{p^2}{4} = -p - 2 - \frac{p^2}{4}.$$

The vertex abscissa does not affect the perpendicular distance to the horizontal x-axis.

Step 2: Express the distance as a nonnegative square Complete the square in the vertex ordinate:

$$y_V = -\frac{(p+2)^2}{4} - 1.$$

This is always at most -1, so the vertex always lies below the x-axis. Hence its distance from that axis is

$$d = |y_V| = 1 + \frac{(p+2)^2}{4}.$$

The square is nonnegative, with equality only when $p = -2$. Therefore the least distance is 1, attained uniquely at this parameter value. Minimising the signed ordinate instead would be incorrect because the ordinate is negative.

Step 3: Calculate the requested expression and check With $p = -2$, the point condition gives $q = -(-2) - 2 = 0$. Thus the parabola is $y = x^2 - 2x = (x - 1)^2 - 1$, whose vertex is exactly $(1, -1)$. It passes through the required point and has the least possible distance. Finally,

$$p^2 + q^2 = (-2)^2 + 0^2 = 4.$$

The other proposed values 2, 5 and 8 cannot arise at the unique minimiser $(p, q) = (-2, 0)$.

Step 4: Conclusion At the minimum vertex-to-axis distance, $p = -2$ and $q = 0$, giving $\boxed{p^2 + q^2 = 4}$. Hence, the correct option is (B).

Quick Tip to Solve: Before minimising a distance to an axis, determine the sign of the relevant coordinate. Here completing the square shows that the vertex ordinate is always negative.

13. Let $P = \{\theta \in [0, 4\pi] : \tan^2\theta \neq 1\}$ and $S = \{a \in \mathbb{Z} : 2(\cos^8\theta - \sin^8\theta)\sec 2\theta = a^2, \theta \in P\}$. Then $n(S)$ is: [4 marks]

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (A) 0

Solution:

Concept:

- Factor the difference of even powers before using $\sec 2\theta = 1/\cos 2\theta$. The cancellation is valid only where $\cos 2\theta \neq 0$.
- The equation asks for integer values of a, so its right side must be a perfect square. Determining the range of the left side is enough.

Step 1: Factor the trigonometric expression Using $\cos^2\theta + \sin^2\theta = 1$, factor in two stages:

$$\begin{aligned}\cos^8\theta - \sin^8\theta &= (\cos^4\theta - \sin^4\theta)(\cos^4\theta + \sin^4\theta)\\ &= \cos 2\theta\,(\cos^4\theta + \sin^4\theta).\end{aligned}$$

The excluded values $\tan^2\theta = 1$ are precisely the points where this factor $\cos 2\theta$ vanishes. At points where the tangent is undefined the original definition of P does not apply; these do not affect the following range argument.

Step 2: Find the allowed range For every admissible angle, cancellation gives

$$\begin{aligned}2(\cos^8\theta - \sin^8\theta)\sec 2\theta &= 2(\cos^4\theta + \sin^4\theta)\\ &= 2 - 4\sin^2\theta\cos^2\theta\\ &= 2 - \sin^2 2\theta.\end{aligned}$$

Since $\sin^2 2\theta = 1$ occurs only at the excluded angles, the value is strictly greater than 1 and at most 2. The upper endpoint is attained, for example, at $\theta = 0$.

Step 3: Count the integer squares We therefore require $1 < a^2 \leq 2$. No integer square lies in this interval: $a = 0$ gives 0, $a = \pm 1$ gives the excluded value 1, and $|a| \geq 2$ gives at least 4. Thus the proposed counts 1, 2 and 3 all include values that fail the equation or its domain restriction.

Step 4: Conclusion The set S is empty, so $n(S) = 0$. Hence, the correct option is (A).

Quick Tip to Solve: Keep the excluded angles after cancelling $\cos 2\theta$; including the lower endpoint would incorrectly admit $a = \pm 1$.

14. Let the vectors $\vec{a} = -\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$. For some $\lambda, \mu \in \mathbb{R}$, let $\vec{c} = \lambda\vec{a} + \mu\vec{b}$. If $\vec{c} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 10$ and $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = -2$, then $|\vec{c}|^2$ is equal to : [4 marks]

(A) 8

(B) 12

(C) 14

(D) 15

Correct Answer: (B) 12

Solution:

Concept:

- The scalar product is linear: $(\lambda\vec{a} + \mu\vec{b}) \cdot \vec{d} = \lambda(\vec{a} \cdot \vec{d}) + \mu(\vec{b} \cdot \vec{d})$.
- Two independent scalar-product conditions determine λ and μ. The squared magnitude is the sum of the squares of the three Cartesian components.

Step 1: Convert both conditions into linear equations For $\vec{d} = (3, -6, 2)$, compute

$$\vec{a} \cdot \vec{d} = -3 - 6 + 6 = -3, \qquad \vec{b} \cdot \vec{d} = 3 - 18 + 2 = -13.$$

The first condition therefore becomes $-3\lambda - 13\mu = 10$. For $\vec{e} = (1, 1, 1)$, the products are

$$\vec{a} \cdot \vec{e} = -1 + 1 + 3 = 3, \qquad \vec{b} \cdot \vec{e} = 1 + 3 + 1 = 5,$$

so the second condition is $3\lambda + 5\mu = -2$. Keeping the negative components is essential here.

Step 2: Solve for the coefficients Add the two equations to eliminate λ:

$$-8\mu = 8 \quad \Longrightarrow \quad \mu = -1.$$

Substitute this into the second equation:

$$3\lambda - 5 = -2 \quad \Longrightarrow \quad \lambda = 1.$$

The elimination gives a unique pair, so there is no remaining family of possible vectors.

Step 3: Compute and check the required magnitude Hence

$$\vec{c} = \vec{a} - \vec{b} = -2\hat{i} - 2\hat{j} + 2\hat{k}, \qquad |\vec{c}|^2 = 4 + 4 + 4 = 12.$$

As a check, $(-2, -2, 2) \cdot (3, -6, 2) = -6 + 12 + 4 = 10$, while its dot product with $(1, 1, 1)$ is $-2 - 2 + 2 = -2$. Both original constraints hold. The values 8, 14 and 15 are not the squared magnitude of this uniquely determined vector.

Step 4: Conclusion The required squared magnitude is $|\vec{c}|^2 = 12$. Hence, the correct option is (B).

Quick Tip to Solve: Use the two given dot products to solve for the coefficients first; expand $\vec{c}$ into components only after finding them.

15. **Let the point A be the foot of perpendicular drawn from the point P$(a, b, 0)$ on the line**

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-\alpha}{3}.$$

If the midpoint of the line segment PA is $\left(0, \frac{3}{4}, -\frac{1}{4}\right)$, then the value of $a^2+b^2+\alpha^2$ is equal to : [4 marks]

(A) 1

(B) 2

(C) 6

(D) 9

Correct Answer: (A) 1

Solution:

Concept:

- A point on a symmetric-form line can be written using one parameter. Here its direction vector is $(2, 1, 3)$.
- The midpoint condition determines the endpoints componentwise. A perpendicular displacement has zero scalar product with the line direction.

Step 1: Use the midpoint to express the foot Let $M = (0, 3/4, -1/4)$. Since $M = (P + A)/2$ and $P = (a, b, 0)$,

$$A = 2M - P = \left(-a, \frac{3}{2} - b, -\frac{1}{2}\right).$$

The foot also lies on the given line. Introduce its common parameter t:

$$A = (1 + 2t,\ 2 + t,\ \alpha + 3t).$$

Equating coordinates gives

$$a = -1 - 2t, \qquad b = -\frac{1}{2} - t, \qquad \alpha = -\frac{1}{2} - 3t.$$

Step 2: Impose the perpendicular condition The displacement from P to A is

$$A - P = \left(-2a, \frac{3}{2} - 2b, -\frac{1}{2}\right) = \left(2 + 4t, \frac{5}{2} + 2t, -\frac{1}{2}\right).$$

Because PA is perpendicular to the line, its dot product with $(2, 1, 3)$ vanishes:

$$2(2 + 4t) + \left(\frac{5}{2} + 2t\right) - \frac{3}{2} = 0.$$

Thus $5 + 10t = 0$, giving $t = -1/2$. This is the geometric condition that distinguishes the foot from an arbitrary point on the line.

Step 3: Recover the parameters and verify Substitution yields $a = 0$, $b = 0$ and $\alpha = 1$. Thus

$P = (0, 0, 0)$ and $A = (0, 3/2, -1/2)$. Their midpoint is the specified point, and

$$(0, 3/2, -1/2) \cdot (2, 1, 3) = \frac{3}{2} - \frac{3}{2} = 0.$$

Consequently $a^2 + b^2 + \alpha^2 = 0 + 0 + 1 = 1$. The alternatives 2, 6 and 9 do not follow from the uniquely determined parameters.

Step 4: Conclusion The value of $a^2 + b^2 + \alpha^2$ is 1. Hence, the correct option is (A).

Quick Tip to Solve: For a known midpoint, write the unknown foot as $2M - P$, then combine the line parameter with one perpendicular dot product.

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16. Two adjacent sides of a parallelogram PQRS are given by $\overrightarrow{PQ} = \hat{j} + \hat{k}$ and $\overrightarrow{PS} = \hat{i} - \hat{j}$. If the side PS is rotated about the point P by an acute angle α in the plane of the parallelogram so that it becomes perpendicular to the side PQ, then $\sin^2\left(\frac{5\alpha}{2}\right) - \sin^2\left(\frac{\alpha}{2}\right)$ is equal to: [4 marks]

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{\sqrt{3}}{4}$

(D) $\frac{2\sqrt{3}}{5}$

Correct Answer: (B) $\frac{\sqrt{3}}{2}$

Solution:

Concept:

- The angle β between two vectors satisfies $\cos\beta = (\vec{u} \cdot \vec{v})/(|\vec{u}||\vec{v}|)$. Use the directed sides that start at the same vertex.
- The identity $\sin^2 A - \sin^2 B = \sin(A + B)\sin(A - B)$ simplifies the difference of squared sines without evaluating half angles separately.

Step 1: Find the initial angle between the sides Here $\vec{u} = \overrightarrow{PQ} = (0, 1, 1)$ and $\vec{v} = \overrightarrow{PS} = (1, -1, 0)$. Their scalar product and magnitudes are

$$\vec{u} \cdot \vec{v} = -1, \qquad |\vec{u}| = |\vec{v}| = \sqrt{2}.$$

Therefore

$$\cos\beta = \frac{-1}{\sqrt{2}\sqrt{2}} = -\frac{1}{2}, \qquad \beta = \frac{2\pi}{3}.$$

The angle between the directed adjacent sides is obtuse, even though the requested rotation is acute.

Step 2: Determine the acute rotation The nearest direction in the plane perpendicular to PQ makes an angle $\pi/2$ with PQ. Hence the required acute rotation of PS is

$$\alpha = \frac{2\pi}{3} - \frac{\pi}{2} = \frac{\pi}{6}.$$

The other perpendicular direction would require a rotation of $5\pi/6$, which is not acute. Thus the

condition in the question fixes the appropriate rotation.

Step 3: Evaluate the trigonometric difference Apply the product identity with $A = 5\alpha/2$ and $B = \alpha/2$:

$$\sin^2\frac{5\alpha}{2} - \sin^2\frac{\alpha}{2} = \sin 3\alpha \, \sin 2\alpha$$
$$= \sin\frac{\pi}{2}\,\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}.$$

This is positive, as expected from 75° and 15°. The values $1/2$, $\sqrt{3}/4$ and $2\sqrt{3}/5$ do not equal this product; in particular no extra factor of one-half belongs in the identity.

Step 4: Conclusion The required value is $\sqrt{3}/2$. Hence, the correct option is (B).

Quick Tip to Solve: Find the angle between the directed sides before taking its difference from 90°; then use $\sin^2 A - \sin^2 B$ as a product.

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17. The value of $\int_0^{20\pi} (\sin^4 x + \cos^4 x)dx$ is equal to: [4 marks]

(A) $\frac{15\pi}{2}$

(B) 25π

(C) 15π

(D) $\frac{25\pi}{2}$

Correct Answer: (C) 15π

Solution:

Concept:

- Use $\sin^2 x + \cos^2 x = 1$ to reduce fourth powers to a double-angle expression.
- Over an integral number of complete periods, a cosine term has zero integral. The constant term gives the mean value of the integrand.

Step 1: Reduce the fourth powers Square the basic identity and rearrange:

$$\sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$$
$$= 1 - \frac{1}{2}\sin^2 2x$$
$$= 1 - \frac{1}{4}(1 - \cos 4x)$$
$$= \frac{3}{4} + \frac{1}{4}\cos 4x.$$

This separates the constant mean value from an oscillating term. Both the factor 2 in the squared identity and the factor $1/2$ in the power-reduction formula are necessary.

Step 2: Integrate over the stated limits Let the required integral be I. An antiderivative of the reduced

expression is $3x/4 + \sin 4x/16$, because differentiating the sine term multiplies it by 4. Therefore

$$\begin{aligned} I &= \left[\frac{3x}{4} + \frac{\sin 4x}{16}\right]_0^{20\pi} \\ &= \frac{3}{4}(20\pi) + \frac{\sin 80\pi - \sin 0}{16} \\ &= 15\pi. \end{aligned}$$

The endpoint sine terms vanish exactly; no approximation is involved.

Step 3: Check the size and exclude the other options The integrand lies between $1/2$ and 1, so the integral must lie between 10π and 20π. This immediately rules out $15\pi/2$ and 25π. Although $25\pi/2$ lies within that bound, it would give the wrong average value. The interval contains 40 complete periods of $\cos 4x$, so the exact average is $3/4$, not $5/8$.

Step 4: Conclusion The integral equals 15π. Hence, the correct option is (C).

Quick Tip to Solve: Over complete periods, replace $\sin^4 x + \cos^4 x$ by its mean value $3/4$, then multiply by the interval length.

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18. Let $f(x)$ be a polynomial of degree 5, and have extrema at $x = 1$ and $x = -1$. If $\lim\limits_{x\to 0}\left(\frac{f(x)}{x^3}\right) = -5$, then $f(2) - f(-2)$ is equal to: [4 marks]

(A) 0

(B) 50

(C) 92

(D) 112

Correct Answer: (D) 112

Solution:

Concept:

- A finite limit of $f(x)/x^3$ at zero forces the constant, linear and quadratic coefficients of the polynomial to vanish.
- A polynomial is differentiable everywhere, so each local extremum satisfies $f'(x) = 0$. Apply this separately at both specified points.

Step 1: Use the limit to restrict the polynomial Write the degree-five polynomial with undetermined coefficients. Since dividing by x^3 has a finite limit, the coefficients of $1, x, x^2$ must all be zero. The limit also fixes the coefficient of x^3 as -5. Thus

$$f(x) = Ax^5 + Bx^4 - 5x^3, \qquad A \neq 0.$$

Indeed, after division by x^3, the expression becomes $Ax^2 + Bx - 5$, whose limit is exactly -5.

Step 2: Apply both extremum conditions Differentiate:

$$f'(x) = 5Ax^4 + 4Bx^3 - 15x^2.$$

The conditions at 1 and -1 give

$$5A + 4B - 15 = 0, \qquad 5A - 4B - 15 = 0.$$

Subtracting gives $8B = 0$, hence $B = 0$. Adding gives $10A = 30$, hence $A = 3$. Therefore $f(x) = 3x^5 - 5x^3$. To confirm that the stationary points really are extrema, observe that $f'(x) = 15x^2(x^2 - 1)$ changes from positive to negative at -1, and from negative to positive at 1.

Step 3: Evaluate the requested difference The resulting polynomial is odd, so $f(-2) = -f(2)$. Direct substitution gives

$$f(2) = 3(32) - 5(8) = 96 - 40 = 56, \qquad f(-2) = -56.$$

Consequently

$$f(2) - f(-2) = 56 - (-56) = 112.$$

The value 0 would require equality instead of opposite signs, and neither 50 nor 92 matches the coefficient equations and direct substitution. The requested quantity is the difference, not just $f(2)$.

Step 4: Conclusion The value of $f(2) - f(-2)$ is 112. Hence, the correct option is (D).

Quick Tip to Solve: Use the limit to remove low-degree terms first. The symmetric stationary points then eliminate the even-power term, making the final difference easy.

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19. Let $f(x) = \int \left(\frac{16x+24}{x^2+2x-15}\right) dx$. If $f(4) = 14\log_e(3)$ and $f(7) = \log_e(2^\alpha \cdot 3^\beta)$, $\alpha, \beta \in \mathbb{N}$, then $\alpha + \beta$ is equal to : [4 marks]

(A) 31

(B) 37

(C) 39

(D) 41

Correct Answer: (C) 39

Solution:

Concept:

- A rational function with distinct linear factors in the denominator can be resolved into partial fractions. Integrating each reciprocal linear factor produces a natural logarithm.
- The given function value fixes the constant of integration. Since both evaluation points exceed 3, the factors $x - 3$ and $x + 5$ are positive there.

Step 1: Resolve the integrand Factor the quadratic and write

$$\frac{16x + 24}{(x - 3)(x + 5)} = \frac{a}{x - 3} + \frac{b}{x + 5}.$$

Equating the numerator coefficients gives $a+b=16$ and $5a-3b=24$. Thus $a=9$ and $b=7$. The numerator check is $9(x+5)+7(x-3)=16x+24$.

Step 2: Integrate and determine the constant Therefore, on the interval containing 4 and 7,

$$f(x)=9\ln(x-3)+7\ln(x+5)+C.$$

At $x=4$, this becomes $f(4)=9\ln 1+7\ln 9+C=14\ln 3+C$. The given value is $14\ln 3$, so $C=0$. This constant must be fixed before evaluating the second function value.

Step 3: Collect the prime powers Substitute $x=7$ and use the logarithm laws:

$$\begin{aligned} f(7) &= 9\ln 4+7\ln 12 \\ &= 18\ln 2+7(2\ln 2+\ln 3) \\ &= 32\ln 2+7\ln 3=\ln(2^{32}3^7). \end{aligned}$$

Unique prime factorisation gives $\alpha=32$ and $\beta=7$. Their sum is 39; the other proposed sums 31, 37 and 41 are inconsistent with these exponents.

Step 4: Conclusion $\alpha+\beta=32+7=\boxed{39}$. Hence, the correct option is (C).

Quick Tip to Solve: After fixing the integration constant, expand every logarithm into prime factors; the coefficients of $\ln 2$ and $\ln 3$ are the required exponents.

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20. Let $x=x(y)$ be the solution of the differential equation $2y^2\frac{dx}{dy}-2xy+x^2=0$, $y>1$, $x(e)=e$. Then $x(e^2)$ is equal to: [4 marks]

(A) $\frac{3}{2}e^2$

(B) $\frac{2}{3}e^2$

(C) e^2

(D) $2e^2$

Correct Answer: (B) $\frac{2}{3}e^2$

Solution:

Concept:

- When a first-order equation depends on the ratio x/y, the substitution $x=vy$, with v a function of y, can make it separable.
- The product rule gives $dx/dy=v+y\,dv/dy$. The initial condition determines the integration constant and selects the relevant nonzero solution.

Step 1: Substitute the ratio Put $x=vy$ in the given equation. Then

$$2y^2\left(v+y\frac{dv}{dy}\right)-2vy^2+v^2y^2=0.$$

The two terms linear in v cancel. Since $y > 1$, division by y^2 is valid and yields

$$2y\frac{dv}{dy} + v^2 = 0.$$

Also, $x(e) = e$ implies $v(e) = 1$, so the zero solution is not the required one.

Step 2: Integrate the separated equation Separate the variables and integrate:

$$\frac{dv}{v^2} = -\frac{dy}{2y}, \qquad -\frac{1}{v} = -\frac{1}{2}\ln y + C.$$

Equivalently, $1/v = \frac{1}{2}\ln y + C_1$. Using $v(e) = 1$ gives $1 = \frac{1}{2} + C_1$, hence $C_1 = \frac{1}{2}$. Consequently,

$$v = \frac{2}{1 + \ln y}, \qquad x(y) = \frac{2y}{1 + \ln y}.$$

The denominator is positive for $y > 1$, so this expression is valid throughout the stated domain.

Step 3: Evaluate the requested value At $y = e^2$, the logarithm equals 2, and therefore

$$x(e^2) = \frac{2e^2}{1+2} = \frac{2}{3}e^2.$$

The coefficient must be $2/3$, ruling out $3/2$, 1 and 2 in the other options. Substitution at $y = e$ also returns e, confirming the initial condition.

Step 4: Conclusion $x(e^2) = \boxed{\frac{2}{3}e^2}$. Hence, the correct option is (B).

Quick Tip to Solve: For an equation homogeneous in x and y, try $x = vy$; retain both terms in $dx/dy = v + yv'$ to obtain the cancellation.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let $A = \{2, 3, 4, 5, 6\}$. Let R be a relation on the set $A \times A$ given by $(x, y)R(z, w)$ if and only if x divides z and $y \le w$. Then the number of elements in R is _____. [4 marks]

Solution:

Concept:

- An element of this relation is an ordered pair $((x, y), (z, w))$, where all four coordinates belong to A. It must satisfy both specified conditions.
- The restrictions $x \mid z$ and $y \le w$ involve independent pairs of coordinates. Their numbers of choices therefore multiply.

Step 1: Count the divisibility pairs For each possible first coordinate x, list the elements z of A

divisible by it:

x	permitted z	number
2	2, 4, 6	3
3	3, 6	2
4	4	1
5	5	1
6	6	1

Every number divides itself, so the equal-coordinate cases must be included. The total number of ordered pairs (x, z) is $3 + 2 + 1 + 1 + 1 = 8$. Reversing such a pair is not automatically allowed; for example, 2 divides 4 but 4 does not divide 2.

Step 2: Count the ordered inequality pairs For $y = 2, 3, 4, 5, 6$, respectively, the number of choices of $w \geq y$ in A is $5, 4, 3, 2, 1$. Hence

$$\#\{(y, w) : y \leq w\} = 5 + 4 + 3 + 2 + 1 = 15.$$

Equivalently, the 10 pairs with distinct elements have exactly one increasing order, and the five equal pairs are also permitted. The weak inequality is why these diagonal pairs are retained.

Step 3: Combine the independent choices Each of the 8 admissible pairs (x, z) can be combined with any of the 15 admissible pairs (y, w). Such a choice uniquely produces the relation element $((x, y), (z, w))$; conversely, every relation element gives exactly these two coordinate pairs. Thus there is neither an omission nor double counting, and

$$|R| = 8 \times 15 = 120.$$

Step 4: Conclusion The number of elements in R is $\boxed{120}$. This is a dimensionless count.

Quick Tip to Solve: When a relation on a Cartesian product imposes separate coordinate conditions, count each ordered coordinate pair separately and then multiply.

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22. Consider the matrices $A = \begin{bmatrix} 2 & -2 \\ 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix}$. If matrices P and Q are such that $PA = B$ and $AQ = B$, then the absolute value of the sum of the diagonal elements of $2(P + Q)$ is _____.

[4 marks]

Solution:

Concept:

- For an invertible matrix A, the equations $PA = B$ and $AQ = B$ give $P = BA^{-1}$ and $Q = A^{-1}B$, with multiplication on the appropriate side.
- The sum of diagonal entries is the trace. It is linear, so $\operatorname{tr}[2(P + Q)] = 2 \operatorname{tr} P + 2 \operatorname{tr} Q$.

Step 1: Find the inverse of the coefficient matrix Compute the determinant first:

$$\det A = 2(-2) - (-2)(4) = -4 + 8 = 4 \neq 0.$$

Thus the inverse exists, and the two matrices are uniquely determined. For a two-by-two matrix, interchange the diagonal entries, negate the other entries, and divide by the determinant:

$$A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 2 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ -1 & \frac{1}{2} \end{bmatrix}.$$

Step 2: Multiply in the correct order Right-multiply $PA = B$ by A^{-1}, and left-multiply $AQ = B$ by A^{-1}. This gives

$$P = BA^{-1} = \begin{bmatrix} -\frac{21}{2} & 6 \\ -\frac{7}{2} & 2 \end{bmatrix}, \qquad Q = A^{-1}B = \begin{bmatrix} -1 & -3 \\ -\frac{5}{2} & -\frac{15}{2} \end{bmatrix}.$$

For example, the first diagonal entry of P is $3(-1/2) + 9(-1) = -21/2$, whereas the second diagonal entry of Q is $(-1)9 + (1/2)3 = -15/2$. Matrix multiplication cannot generally be commuted.

Step 3: Add the diagonal entries and take the magnitude The diagonal sums are

$$\operatorname{tr} P = -\frac{21}{2} + 2 = -\frac{17}{2}, \qquad \operatorname{tr} Q = -1 - \frac{15}{2} = -\frac{17}{2}.$$

Therefore

$$\operatorname{tr}[2(P+Q)] = 2\left(-\frac{17}{2} - \frac{17}{2}\right) = -34.$$

The question requests its absolute value, so the negative sign is removed only after the complete diagonal sum is formed.

Step 4: Conclusion The required absolute value is $\boxed{34}$.

Quick Tip to Solve: For this diagonal-sum question, $\operatorname{tr}(BA^{-1}) = \operatorname{tr}(A^{-1}B)$ also gives the shortcut $\left|4\operatorname{tr}(BA^{-1})\right|$.

23. Let A be the point (3, 0) and circles with variable diameter AB touch the circle $x^2 + y^2 = 36$ internally. Let the curve C be the locus of the point B. If the eccentricity of C is e, then $72e^2$ is equal to _____.

[4 marks]

Solution:

Concept:

- For a circle of radius r internally tangent to a circle of radius 6, the distance d between the centres satisfies $d + r = 6$.
- An ellipse consists of points whose distances from two fixed foci have constant sum $2a$. Its eccentricity is $e = c/a$, where the focal separation is $2c$.

Step 1: Express the moving centre and radius Let $B = (u, v)$. Since AB is a diameter and $A = (3, 0)$, the moving circle has centre and radius

$$M = \left(\frac{u+3}{2}, \frac{v}{2}\right), \qquad r = \frac{1}{2}\sqrt{(u-3)^2 + v^2}.$$

The fixed circle has centre $O = (0, 0)$ and radius 6. Its distance from the moving centre is

$$OM = \frac{1}{2}\sqrt{(u+3)^2 + v^2}.$$

These expressions use the midpoint and half-length of the diameter, rather than treating B itself as the centre.

Step 2: Apply internal tangency The moving circle must be inside the fixed circle: if it enclosed the fixed disk, every point on its circumference, including A, would have distance at least 6 from O, contrary to $OA = 3$. Therefore internal tangency gives $OM + r = 6$. Multiplying by 2 yields

$$\sqrt{(u+3)^2 + v^2} + \sqrt{(u-3)^2 + v^2} = 12.$$

This is the sum of distances of B from the fixed points $(-3, 0)$ and $(3, 0)$. It exceeds their separation 6, so the locus is a nondegenerate ellipse.

Step 3: Read the ellipse parameters The focal separation is 6, giving $c = 3$, and the constant distance sum is 12, giving $a = 6$. Hence

$$e = \frac{c}{a} = \frac{3}{6} = \frac{1}{2}, \qquad 72e^2 = 72\left(\frac{1}{2}\right)^2 = 18.$$

As a check, its semi-minor axis satisfies $b^2 = a^2 - c^2 = 36 - 9 = 27 > 0$, consistent with an ellipse.

Step 4: Conclusion The required value is $\boxed{18}$. Eccentricity and the requested expression are dimensionless.

Quick Tip to Solve: For a moving diameter circle, multiply the internal-tangency equation by 2; the midpoint distance then becomes a distance from the reflected fixed endpoint.

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24. If the area of the region bounded by $16x^2 - 9y^2 = 144$ and $8x - 3y = 24$ is A, then $3(A + 6\log_e(3))$ is equal to _____. [4 marks]

Solution:

Concept:

- The area between two curves is the integral of the upper ordinate minus the lower ordinate between their intersection abscissae.
- For $x \geq a > 0$, use $\int \sqrt{x^2 - a^2}\, dx = \frac{1}{2}[x\sqrt{x^2 - a^2} - a^2 \ln(x + \sqrt{x^2 - a^2})] + C$.

Step 1: Locate the intersections and the enclosed arc The line is $y = (8x - 24)/3$. Substitute it in the hyperbola:

$$16x^2 - (8x - 24)^2 = 144 \quad \Longrightarrow \quad x^2 - 8x + 15 = 0.$$

Thus $x = 3, 5$, with points $(3, 0)$ and $(5, 16/3)$. Between these points the relevant hyperbola arc is its upper right branch,

$$y = \frac{4}{3}\sqrt{x^2 - 9}.$$

It lies above the line: both ordinates are nonnegative, and the difference of their squares is $-\frac{16}{3}(x-3)(x-5) > 0$ for $3 < x < 5$.

Step 2: Integrate the upper-minus-lower difference Consequently the bounded area is

$$A = \int_3^5 \left(\frac{4}{3}\sqrt{x^2-9} - \frac{8}{3}x + 8\right) dx.$$

An antiderivative of the integrand is

$$\frac{2}{3}\left[x\sqrt{x^2-9} - 9\ln\left(x + \sqrt{x^2-9}\right)\right] - \frac{4}{3}x^2 + 8x.$$

At the bounds, $\sqrt{5^2-9} = 4$ and $\sqrt{3^2-9} = 0$. The logarithmic difference is $\ln 9 - \ln 3 = \ln 3$, so

$$A = \frac{2}{3}(20 - 9\ln 3) - \frac{4}{3}(25-9) + 8(5-3) = 8 - 6\ln 3.$$

Step 3: Evaluate the requested combination Insert this exact area before making any numerical approximation:

$$3(A + 6\ln 3) = 3(8 - 6\ln 3 + 6\ln 3) = 24.$$

The area itself is positive, since $8 - 6\ln 3 \approx 1.408$, which checks the choice of upper and lower curves. The logarithms cancel exactly in the expression asked for.

Step 4: Conclusion The area is $A = 8 - 6\ln 3$ square units, and the required value is $\boxed{24}$.

Quick Tip to Solve: Keep logarithmic terms exact in area integrals. Here their endpoint ratio is $9/3 = 3$, and the logarithm cancels in the final requested combination.

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25. The number of points in the interval $[2, 4]$, at which the function $f(x) = \left[x^2 - x - \frac{1}{2}\right]$, where $[\cdot]$ denotes the greatest integer function, is discontinuous, is _____. [4 marks]

Solution:

Concept:

- The greatest integer function $\lfloor u \rfloor$ is constant between consecutive integers and jumps when a continuous, strictly increasing argument crosses an integer.
- For $g(x) = x^2 - x - \frac{1}{2}$, monotonicity determines whether each integer value is reached exactly once in the given interval.

Step 1: Find the range of the inner function On $[2, 4]$, the derivative is $g'(x) = 2x - 1 \geq 3 > 0$. Thus g is continuous and strictly increasing throughout the interval. Its endpoint values are

$$g(2) = 4 - 2 - \frac{1}{2} = \frac{3}{2}, \qquad g(4) = 16 - 4 - \frac{1}{2} = \frac{23}{2}.$$

Consequently its range is exactly $[1.5, 11.5]$. Neither endpoint value is an integer, so neither endpoint creates a jump.

Step 2: Identify every jump For each integer n between 2 and 11 inclusive, continuity and strict monotonicity give exactly one point $x_n \in (2,4)$ with $g(x_n) = n$. Just to the left, the floor equals $n-1$; at the point and just to its right, it equals n. The unequal one-sided limits prove a discontinuity. Explicitly, the relevant root is

$$x_n = \frac{1+\sqrt{4n+3}}{2}.$$

The negative root lies outside the interval.

Step 3: Count the points No other points can be discontinuities: near a noninteger value of g, continuity keeps $g(x)$ between the same consecutive integers, so its floor is locally constant. The admissible integer levels are $2, 3, \ldots, 11$, giving

$$N = 11 - 2 + 1 = 10.$$

Step 4: Conclusion The function is discontinuous at exactly $\boxed{10}$ points in $[2,4]$. This is a dimensionless count.

Quick Tip to Solve: For the floor of a continuous strictly monotone function, count the integer levels crossed by its inner function; check endpoint levels separately.

Physics – Section A: Multiple Choice Questions (80 marks)

26. Dimensions of universal gravitational constant (G) in terms of Planck's constant (h), distance (L), mass (M) and time (T) are _____. [4 marks]

(A) $[hTLM^{-2}]$

(B) $[hT^{-1}LM^{-2}]$

(C) $[hTL^2M^{-2}]$

(D) $[h^{-1}T^{-1}LM^{-2}]$

Correct Answer: (B) $[hT^{-1}LM^{-2}]$

Solution:

Concept:

- Newton's gravitational law is $F = Gm_1m_2/r^2$; therefore $[G] = M^{-1}L^3T^{-2}$.
- Planck's constant has dimensions of action: $[h] = [E][T] = ML^2T^{-1}$. Replace h by these dimensions when testing an option.

Step 1: Derive the required dimensions From the force law, isolate the constant before combining dimensions:

$$G = \frac{Fr^2}{m_1m_2}, \qquad [G] = \frac{(MLT^{-2})L^2}{M^2} = M^{-1}L^3T^{-2}.$$

Here M, L, T denote the dimensions of mass, length and time. This step fixes the powers independently of the proposed expressions.

Step 2: Test the expression containing Planck's constant The dimensional formula of h follows from

energy times time. For option (B),

$$[hT^{-1}LM^{-2}] = (ML^2T^{-1})T^{-1}LM^{-2} = M^{-1}L^3T^{-2}.$$

All three exponents agree with those of G. This is a dimensional relation, not an assertion of a numerical identity between the quantities.

Step 3: Rule out the other expressions Option (A) gives $M^{-1}L^3$, with no inverse-square time factor. Option (C) gives $M^{-1}L^4$, with both the length and time exponents wrong. Option (D), using $[h^{-1}] = M^{-1}L^{-2}T$, gives $M^{-3}L^{-1}$. It has the wrong mass and length powers and also lacks T^{-2}. Therefore none of these can represent the dimensions of the universal gravitational constant.

Step 4: Conclusion The required expression is $\boxed{[hT^{-1}LM^{-2}]}$. Hence, the correct option is (B).

Quick Tip to Solve: Use $[h] = ML^2T^{-1}$, then add exponents separately for mass, length and time; do not treat h as dimensionless.

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27. A 0.5 kg mass is in contact against the inner wall of a cylindrical drum of radius 4 m rotating about its vertical axis. The minimum rotational speed of the drum to enable the mass to remain stuck to the wall (without falling) is 5 rad/s. The coefficient of friction between the drum's inner wall surface and mass is _____. (Take $g = 10\ \text{m/s}^2$) [4 marks]

(A) 0.1

(B) 0.5

(C) 0.7

(D) 0.3

Correct Answer: (A) 0.1

Solution:

Concept:

- The normal reaction from the vertical drum wall supplies the radial centripetal force: $N = m\omega^2 r$.
- Static friction acts upward to balance weight. At the minimum angular speed for no slipping, $f_{\max} = \mu N = mg$.

Step 1: Calculate the wall reaction The mass moves in a horizontal circle while remaining at a fixed height. The radial acceleration is $\omega^2 r$, directed toward the axis. The wall reaction is horizontal and radial; it does not directly balance the weight. Using the given mass, radius and angular speed,

$$N = m\omega^2 r = 0.5 \times 5^2 \times 4 = 50\,\text{N}.$$

Step 2: Apply the vertical equilibrium condition Gravity would make the mass slide downward, so static friction acts upward along the wall. The weight is

$$mg = 0.5 \times 10 = 5\,\text{N}.$$

At the specified minimum speed the friction is at its limiting value. Therefore

$$\mu N = mg \quad \Rightarrow \quad \mu = \frac{5}{50} = 0.1.$$

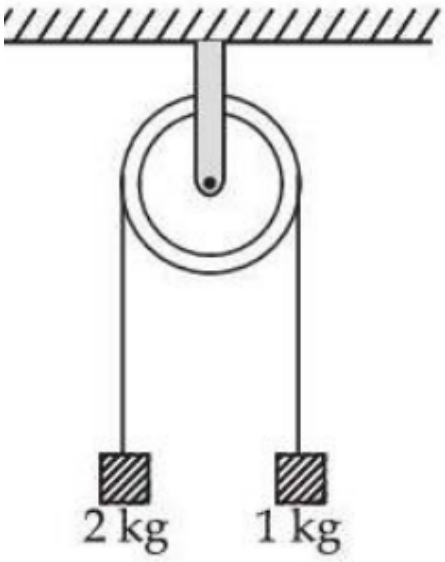
Equivalently, the mass cancels and $\mu = g/(\omega^2 r)$. The coefficient is dimensionless.

Step 3: Check the minimum-speed requirement The condition for sticking is $\mu\omega^2 r \geq g$; equality gives the threshold speed. A larger coefficient would allow the mass to remain stuck even at an angular speed below $5\,\mathrm{rad\,s^{-1}}$. Specifically the other choices, 0.5, 0.7 and 0.3, give threshold speeds $\sqrt{5}$, $\sqrt{25/7}$ and $\sqrt{25/3}\,\mathrm{rad\,s^{-1}}$, respectively. Each is less than 5, contradicting the stated minimum.

Step 4: Conclusion The coefficient of friction is $\boxed{\mu = 0.1}$. Hence, the correct option is (A).

Quick Tip to Solve: In a rotating drum, use the horizontal normal force for centripetal acceleration and the vertical friction force to support the weight.

28. Two blocks of masses 2 kg and 1 kg respectively, are tied to the ends of a string which passes over a light frictionless pulley as shown in the figure below. The masses are held at rest at the same horizontal level and then released. The distance traversed by the centre of mass in 2 s is _____ m. (Take $g = 10\ m/s^2$) [4 marks]



(A) 3.33

(B) 3.12

(C) 2.22

(D) 1.42

Correct Answer: (C) 2.22

Solution:

Concept:

- For an ideal Atwood machine with masses $m_1 > m_2$, the two blocks have equal and opposite accelerations of magnitude $a = (m_1 - m_2)g/(m_1 + m_2)$.
- The centre-of-mass displacement is the mass-weighted average of the signed displacements: $\Delta y_{\text{CM}} = (m_1\Delta y_1 + m_2\Delta y_2)/(m_1 + m_2)$.

Step 1: Find the acceleration of each block Take the downward motion of the heavier block as positive for that block. If T is the common string tension,

$$2g - T = 2a, \qquad T - g = a.$$

Adding these equations eliminates the tension and gives $g = 3a$, so

$$a = \frac{10}{3}\,\text{m}\,\text{s}^{-2}.$$

The 2 kg mass accelerates downward and the 1 kg mass upward. Both start from rest.

Step 2: Calculate the signed displacements In $t = 2\,\text{s}$, each block travels a distance

$$s = \frac{1}{2}at^2 = \frac{1}{2}\frac{10}{3}(2)^2 = \frac{20}{3}\,\text{m}.$$

Use one common coordinate system, with downward positive and the initial common level as zero. Then the displacements are $\Delta y_1 = 20/3\,\text{m}$ and $\Delta y_2 = -20/3\,\text{m}$. The opposite signs are essential.

Step 3: Take the mass-weighted average The horizontal positions of the blocks are fixed, so the centre of mass moves only vertically. Its displacement is

$$\Delta y_{\text{CM}} = \frac{2(20/3) + 1(-20/3)}{2+1} = \frac{20}{9}\,\text{m}.$$

Its direction remains downward, hence its distance travelled equals this displacement magnitude, approximately $2.22\,\text{m}$. The values 3.33, 3.12 and 1.42 m do not satisfy the signed mass-weighted displacement formula; 3.33 is instead the numerical magnitude of the individual acceleration in SI units.

Step 4: Conclusion The centre of mass travels $\boxed{20/9\,\text{m} \simeq 2.22\,\text{m}}$ downward. Hence, the correct option is (C).

Quick Tip to Solve: Use opposite signs for the two block displacements before averaging; the centre of mass does not travel the full distance of either block.

29. A particle having charge 10^{-9} C moving in x-y plane in fields of $0.4\hat{j}$ N/C and $4\times10^{-3}\hat{k}$ T experiences a force of $(4\hat{i}+2\hat{j})\times10^{-10}$ N. The velocity of the particle at that instant is _____ m/s. [4 marks]

(A) $50\hat{i}+100\hat{j}$

(B) $100\hat{i}+50\hat{j}$

(C) $-50\hat{i}+100\hat{j}$

(D) $50\hat{i}-100\hat{j}$

Correct Answer: (A) $50\hat{i}+100\hat{j}$

Solution:

Concept:

- The total electromagnetic force is $\vec{F}=q(\vec{E}+\vec{v}\times\vec{B})$. Subtract the electric contribution before inferring the velocity.
- For $\vec{v}=v_x\hat{i}+v_y\hat{j}$ and $\vec{B}=B\hat{k}$, the cross product is $\vec{v}\times\vec{B}=Bv_y\hat{i}-Bv_x\hat{j}$.

Step 1: Separate the magnetic contribution The charge is positive and equals 10^{-9} C. Divide the given total force by the charge:

$$\frac{\vec{F}}{q}=0.4\hat{i}+0.2\hat{j}\quad(\text{N/C}).$$

The electric field is $0.4\hat{j}$ N/C, so

$$\vec{v}\times\vec{B}=\frac{\vec{F}}{q}-\vec{E}=0.4\hat{i}-0.2\hat{j}\quad(\text{N/C}).$$

The magnetic contribution along y is negative even though the total force along y is positive.

Step 2: Compare the two vector components Using $B=4\times10^{-3}$ T and the right-handed cross-product identities $\hat{i}\times\hat{k}=-\hat{j}$, $\hat{j}\times\hat{k}=\hat{i}$,

$$4\times10^{-3}v_y=0.4,\qquad -4\times10^{-3}v_x=-0.2.$$

Hence

$$v_y=100\,\text{m/s},\qquad v_x=50\,\text{m/s}.$$

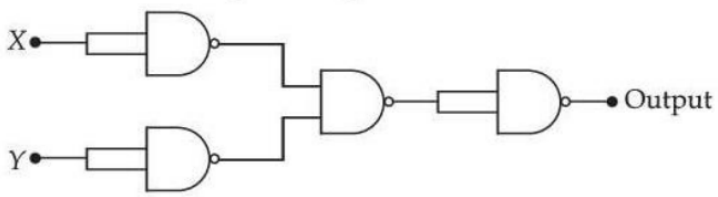
There is no z-component because the particle moves in the x-y plane.

Step 3: Verify and exclude the alternatives Substitution gives $q\vec{v}\times\vec{B}=(4\hat{i}-2\hat{j})\times10^{-10}$ N; adding $q\vec{E}=4\hat{j}\times10^{-10}$ N reproduces the stated force. Option (B) interchanges the component magnitudes and gives $(2\hat{i}+0\hat{j})\times10^{-10}$ N. Option (C) gives $(4\hat{i}+6\hat{j})\times10^{-10}$ N, while option (D) reverses the required x-force. All three fail the given vector equation.

Step 4: Conclusion The velocity is $\boxed{\vec{v}=(50\hat{i}+100\hat{j})\,\text{m/s}}$. Hence, the correct option is (A).

Quick Tip to Solve: Write the cross product componentwise: with the field along $+z$, $F_{B,x}=qBv_y$ and $F_{B,y}=-qBv_x$.

30. If X and Y are the inputs, the given circuit works as _____. [4 marks]



(A) OR gate

(B) AND gate

(C) NAND gate

(D) NOR gate

Correct Answer: (D) NOR gate

Solution:

Concept:

- A NAND gate gives $\overline{AB}$. When its two inputs are joined to the same signal, its output is $\overline{AA} = \overline{A}$, so it acts as an inverter.
- De Morgan's law gives $\overline{\overline{X}\,\overline{Y}} = X + Y$, where multiplication denotes AND and addition denotes Boolean OR.

Step 1: Read the input gates Each gate in the diagram has an AND-shaped body with an output bubble, identifying it as a NAND gate. Both inputs of the upper first-stage gate are connected to X, and both inputs of the lower gate are connected to Y. Thus their outputs are

$$A = \overline{XX} = \overline{X}, \qquad B = \overline{YY} = \overline{Y}.$$

The joined inputs are one repeated logical signal, not two independent inputs.

Step 2: Simplify the middle and final stages The middle NAND gate receives A and B, giving

$$C = \overline{AB} = \overline{\overline{X}\,\overline{Y}} = X + Y.$$

Both inputs of the last NAND gate receive this same output C, so the final gate inverts it:

$$Z = \overline{CC} = \overline{C} = \overline{X + Y}.$$

This last inversion must be included; stopping at the middle gate would incorrectly identify the circuit as OR.

Step 3: Check the truth table and alternatives The output is 1 only when both inputs are 0. The complete truth table is

X	Y	Z
0	0	1
0	1	0
1	0	0
1	1	0

An OR gate instead outputs 1 for mixed inputs. An AND gate outputs 1 for inputs $(1,1)$. A NAND gate outputs 1 for each mixed pair as well as $(0,0)$. These disagree with the circuit, so options (A), (B) and (C) are excluded.

Step 4: Conclusion The circuit performs $\boxed{Z = \overline{X+Y}}$, the NOR operation. Hence, the correct option is (D).

Quick Tip to Solve: A NAND gate with tied inputs is a NOT gate. Simplify each stage in order, including the final tied-input NAND.

---

31. If a body of mass 1 kg falls on the earth from infinity, it attains velocity (v) and kinetic energy (k) on reaching the surface of earth. The values of v and k respectively are _____.
(Take radius of earth to be 6400 km and g = 9.8 m/s^2) [4 marks]

(A) 11.2 km/s; 6.27×10^7 J

(B) 11.2 km/s; 12.54×10^7 J

(C) 8.8 km/s; 6.27×10^7 J

(D) 8.8 km/s; 12.54×10^7 J

Correct Answer: (A) 11.2 km/s; 6.27×10^7 J

Solution:

Concept:

- For a body released from rest at infinity, gravitational potential energy and kinetic energy are both zero initially. Mechanical energy is conserved during the fall when air resistance is neglected.
- At Earth's surface the potential energy is $-GMm/R$, where M and R are Earth's mass and radius. Using $g = GM/R^2$, the kinetic energy gained is $K = mgR$.

Step 1: Apply conservation of energy The body's initial total energy is zero, so at the surface

$$0 = \frac{1}{2}mv^2 - \frac{GMm}{R}.$$

Consequently,

$$v = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}.$$

This equals Earth's escape speed: a body falling from rest at infinity gains exactly the energy needed to escape back to infinity with zero final speed. The mass cancels from the speed expression.

Step 2: Substitute the radius in SI units Convert $6400\,\text{km} = 6.4 \times 10^6\,\text{m}$. Thus

$$v = \sqrt{2(9.8)(6.4 \times 10^6)} = \sqrt{1.2544 \times 10^8} = 1.12 \times 10^4\,\text{m s}^{-1}.$$

Therefore the speed is $11.2\,\text{km s}^{-1}$, not $8.8\,\text{km s}^{-1}$; this eliminates options (C) and (D).

Step 3: Calculate the kinetic energy With $m = 1\,\text{kg}$,

$$K = mgR = (1)(9.8)(6.4 \times 10^6) = 6.272 \times 10^7\,\text{J}.$$

To the precision of the options this is 6.27×10^7 J. Option (B) is twice this value because it omits the factor $1/2$ in $K = mv^2/2$.

Step 4: Conclusion The speed and kinetic energy are $11.2\,\mathrm{km\,s^{-1}}$ and 6.27×10^7 J, respectively. Hence, the correct option is (A).

Quick Tip to Solve: For a fall from rest at infinity, use $v = \sqrt{2gR}$ and $K = mgR$; a constant-g formula over an infinite height is invalid.

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32. In a screw gauge the zero of main scale reference line coincides with the fifth division of the circular scale when two studs are in contact. There are 100 divisions in circular scale and pitch of screw gauge is 0.1 mm. When diameter of a sphere is measured, the reading of main scale is 5 mm and 50th division of circular scale coincides with the reference line of main scale. The diameter of sphere is _____ mm. [4 marks]

(A) 5.045

(B) 5.055

(C) 5.450

(D) 5.550

Correct Answer: (A) 5.045

Solution:

Concept:

- The least count of a screw gauge is its pitch divided by the number of divisions on the circular scale.
- The observed reading is the main-scale reading plus the circular-scale contribution. The corrected reading equals the observed reading minus the zero error.

Step 1: Determine the least count One complete turn advances the screw by the pitch, $0.1\,\mathrm{mm}$. Since the circular scale has 100 equal divisions,

$$\mathrm{LC} = \frac{0.1}{100} = 0.001\,\mathrm{mm}.$$

Each circular division represents one thousandth of a millimetre.

Step 2: Find the positive zero error When the studs touch, the actual separation is zero, but the instrument shows five circular divisions. Its zero error is therefore positive:

$$e = 5(0.001) = +0.005\,\mathrm{mm}.$$

The instrument overestimates a length by this amount, so the zero correction is $-0.005\,\mathrm{mm}$.

Step 3: Correct the measured diameter The observed reading of the sphere is

$$d_{\mathrm{obs}} = 5 + 50(0.001) = 5.050\,\mathrm{mm}.$$

Subtracting the positive zero error gives

$$d = d_{\text{obs}} - e = 5.050 - 0.005 = 5.045\,\text{mm}.$$

Option (B), $5.055\,\text{mm}$, results from applying the correction with the wrong sign. Options (C) and (D) use a circular-scale contribution about ten times too large and therefore disagree with the stated pitch and least count.

Step 4: Conclusion The corrected diameter of the sphere is $5.045\,\text{mm}$. Hence, the correct option is (A).

Quick Tip to Solve: First calculate the least count from the stated pitch; then subtract the reading obtained with the studs in contact from the observed reading.

33. The surface tension of a soap bubble is 0.03 N/m. The work done in increasing the diameter of bubble from 2 cm to 6 cm is $\alpha\pi \times 10^{-4}$ J. The value of α is _____. (Take $\pi = 3.14$) [4 marks]

(A) 0.86

(B) 0.64

(C) 1.92

(D) 7.68

Correct Answer: (C) 1.92

Solution:

Concept:

- Surface tension T is the surface energy per unit area. The reversible work needed to increase the area is $W = T\Delta A$, with T treated as constant.
- A soap bubble has two liquid-air surfaces, inner and outer. Neglecting film thickness, their combined area is $A = 2(4\pi r^2) = 8\pi r^2$.

Step 1: Convert diameters into radii The initial and final diameters are $2\,\text{cm}$ and $6\,\text{cm}$, so the corresponding radii are

$$r_1 = 1\,\text{cm} = 0.01\,\text{m}, \qquad r_2 = 3\,\text{cm} = 0.03\,\text{m}.$$

Using the stated diameters directly as radii would multiply the area increase, and hence the calculated work, by four.

Step 2: Find the increase in total surface area Both surfaces enlarge when the bubble expands. Thus

$$\begin{aligned}\Delta A &= 8\pi(r_2^2 - r_1^2)\\ &= 8\pi\left[(0.03)^2 - (0.01)^2\right]\\ &= 8\pi(8 \times 10^{-4})\\ &= 64\pi \times 10^{-4}\,\text{m}^2.\end{aligned}$$

The difference of the squared radii is required; squaring the radius difference would describe a

different area.

Step 3: Calculate the work and compare coefficients Multiplying the area change by $T = 0.03\,\mathrm{N\,m^{-1}}$ gives

$$W = 0.03(64\pi \times 10^{-4}) = 1.92\pi \times 10^{-4}\,\mathrm{J}.$$

Therefore $\alpha = 1.92$. If $\pi = 3.14$ is substituted, the work is $6.0288 \times 10^{-4}\,\mathrm{J}$, but π is already factored out in the requested form. Options (A) and (B) do not equal the required coefficient; option (D), 7.68, is four times too large and follows from confusing diameters with radii.

Step 4: Conclusion The work is $1.92\pi \times 10^{-4}\,\mathrm{J}$, so $\alpha = 1.92$. Hence, the correct option is (C).

Quick Tip to Solve: A soap bubble has two surfaces: use $8\pi T(r_2^2 - r_1^2)$, or equivalently $2\pi T(d_2^2 - d_1^2)$, with all lengths in metres.

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34. A mixture of carbon dioxide and oxygen has volume 8310 cm^3, temperature 300 K, pressure 100 kPa and mass 13.2 g. The number of moles of carbon dioxide and oxygen gases in the mixture respectively are _____.

(Assume both carbon dioxide and oxygen gases behave like ideal gases) [R = 8.31 J/mol K] [4 marks]

(A) 0.15 and 0.18

(B) 0.25 and 0.08

(C) 0.21 and 0.12

(D) 0.13 and 0.20

Correct Answer: (C) 0.21 and 0.12

Solution:

Concept:

- For an ideal-gas mixture, the total number of moles satisfies $PV = nRT$. At a common pressure, volume and temperature, this equation gives the sum of the component mole amounts.
- Mass is additive: if x and y are the moles of CO_2 and O_2, respectively, then the mixture mass in grams is $44x + 32y$, using their molar masses in $\mathrm{g\,mol^{-1}}$.

Step 1: Calculate the total number of moles Use SI units with the supplied gas constant:

$$P = 10^5\,\mathrm{Pa}, \qquad V = 8310 \times 10^{-6}\,\mathrm{m^3}.$$

At $T = 300\,\mathrm{K}$,

$$x + y = \frac{PV}{RT} = \frac{10^5(8310 \times 10^{-6})}{8.31(300)} = \frac{831}{2493} = \frac{1}{3}.$$

Thus the total is approximately $0.3333\,\mathrm{mol}$. This constraint alone is insufficient to find the composition, because the gases have different molar masses.

Step 2: Use the measured mass to separate the gases The second independent relation is

$$44x + 32y = 13.2.$$

Substitute $y = 1/3 - x$:

$$12x + \frac{32}{3} = 13.2, \qquad x = \frac{19}{90} = 0.21111\ldots \text{ mol}.$$

It follows that

$$y = \frac{1}{3} - \frac{19}{90} = \frac{11}{90} = 0.12222\ldots \text{ mol}.$$

Step 3: Match the precision of the options To two decimal places these amounts are 0.21 and $0.12\,\text{mol}$. As a check, the exact values give $44(19/90) + 32(11/90) = 13.2\,\text{g}$. The pairs in options (A), (B) and (D) give masses 12.36, 13.56 and $12.12\,\text{g}$, respectively, so they do not represent the required composition. The displayed two-decimal values are rounded values, not the exact mole amounts.

Step 4: Conclusion The mixture contains approximately $0.21\,\text{mol}$ of CO_2 and $0.12\,\text{mol}$ of O_2, respectively. Hence, the correct option is (C).

Quick Tip to Solve: Combine the total-moles equation from $PV = nRT$ with the mass equation; retain the exact total until the final rounding.

35. If an air bubble of diameter 2 mm rises steadily through a liquid of density 2000 kg/m^3 at a rate of 0.5 cm/s, then the coefficient of viscosity of liquid is _____ Poise. (Take g = 10 m/s^2) [4 marks]

(A) 0.88

(B) 8.8

(C) 88.8

(D) 0.088

Correct Answer: (B) 8.8

Solution:

Concept:

- A bubble rising steadily has zero net force. Its upward buoyant force balances the downward viscous resistance, with the mass and density of air neglected relative to the displaced liquid.
- In the Stokes-law model used here, the viscous force is $6\pi\eta r v$, where η is viscosity, r is bubble radius and v is terminal speed. The buoyant force is $(4/3)\pi r^3 \rho g$.

Step 1: Convert radius and speed into SI units The diameter is $2\,\text{mm}$, so

$$r = 1\,\text{mm} = 10^{-3}\,\text{m}.$$

The given steady speed is

$$v = 0.5\,\text{cm}\,\text{s}^{-1} = 5 \times 10^{-3}\,\text{m}\,\text{s}^{-1}.$$

The liquid density is $\rho = 2000\,\text{kg}\,\text{m}^{-3}$, and $g = 10\,\text{m}\,\text{s}^{-2}$. The radius, rather than diameter, must enter the drag and volume expressions.

Step 2: Balance buoyancy and viscous force At the terminal speed,

$$\frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v.$$

Cancelling common factors gives

$$\eta = \frac{2r^2\rho g}{9v} = \frac{2(10^{-3})^2(2000)(10)}{9(5\times10^{-3})} = \frac{0.04}{0.045} = 0.888\ldots\ \mathrm{Pa\,s}.$$

The force balance includes no acceleration term, because the question states that the bubble rises steadily.

Step 3: Convert viscosity to poise Since $1\,\mathrm{Pa\,s} = 10\,\mathrm{P}$,

$$\eta = 8.888\ldots\ \mathrm{P}.$$

This is about $8.9\,\mathrm{P}$; the matching approximate choice supplied is $8.8\,\mathrm{P}$, option (B). Option (A) incorrectly keeps the SI numerical value while calling it poise. Options (C) and (D) introduce further factors of ten and are inconsistent with the unit conversion.

Step 4: Conclusion The calculation gives $\eta = 8.888\ldots\ \mathrm{P}$, corresponding to the supplied approximate value $8.8\,\mathrm{P}$. Hence, the correct option is (B).

Quick Tip to Solve: After applying Stokes' law with radius in metres, multiply the viscosity in $\mathrm{Pa\,s}$ by 10 to express it in poise.

36. A spherical ball of mass 2 kg falls from a height of 10 m and is brought to rest after penetrating 10 cm into sand.

The average force exerted by sand on the ball is _____ N.

(Take $g = 10\ \mathrm{m/s^2}$) [4 marks]

(A) 1980

(B) 2020

(C) 2000

(D) 1000

Correct Answer: (B) 2020

Solution:

Concept:

- The work-energy theorem states that the net work on a body equals its change in kinetic energy. The ball starts and ends at rest, so the total work over its complete downward journey is zero.
- Gravity acts during both the fall through air and the penetration into sand. The sand's upward resisting force does negative work only over the penetration distance.

Step 1: Find the kinetic energy on reaching the sand Neglecting air resistance, the loss of gravitational

potential energy over the initial height $h = 10\,\text{m}$ becomes kinetic energy:

$$K_{\text{entry}} = mgh = (2)(10)(10) = 200\,\text{J}.$$

The ball then travels a further $d = 10\,\text{cm} = 0.10\,\text{m}$ downward before coming to rest. This second part of the motion is where the sand supplies the stopping force.

Step 2: Apply work-energy during penetration Take downward displacement as positive, and let F be the average magnitude of the upward force exerted by the sand. Gravity does positive work mgd; the sand does negative work $-Fd$. Hence

$$mgd - Fd = K_{\text{final}} - K_{\text{entry}} = -200\,\text{J}.$$

Equivalently, $(F - mg)d = 200\,\text{J}$. The required quantity is the sand force itself, rather than the net upward stopping force.

Step 3: Solve for the average sand force Rearranging and substituting the penetration distance,

$$F = mg + \frac{200}{0.10} = 20 + 2000 = 2020\,\text{N}.$$

A check over the whole path gives $Fd = mg(h + d) = (20)(10.1) = 202\,\text{J}$, yielding the same result. Option (C), $2000\,\text{N}$, omits gravity's work during penetration. Option (A) subtracts the weight instead of adding it. Option (D) does not supply enough opposing work to stop the ball over $0.10\,\text{m}$.

Step 4: Conclusion The average force exerted by the sand on the ball is $2020\,\text{N}$, directed upward. Hence, the correct option is (B).

Quick Tip to Solve: For a fall through height h followed by penetration d, the average resistance is $mg(1+h/d)$; include gravity during penetration.

---

37. An electromagnetic wave travels in free space along the *x*-direction. At a particular point in space and time, $\vec{B} = 2 \times 10^{-7}\hat{j}$ T is associated with this wave. The value of corresponding electric field $\vec{E}$ at this point is _____ V/m. [4 marks]

(A) $60\ \hat{k}$

(B) $-60\ \hat{k}$

(C) $30\ \hat{k}$

(D) $-600\ \hat{k}$

Correct Answer: (B) $-60\ \hat{k}$

Solution:

Concept:

- In free space, the electric and magnetic fields of a plane electromagnetic wave satisfy $E = cB$, where $c = 3 \times 10^8\,\text{m s}^{-1}$.
- The direction of propagation is the direction of $\vec{E} \times \vec{B}$; the two fields and the propagation direction are mutually perpendicular.

Step 1: Calculate the electric-field magnitude The given magnetic-field magnitude is $2 \times 10^{-7}\,\mathrm{T}$. Therefore, at the same point and instant,

$$E = cB = (3 \times 10^{8})(2 \times 10^{-7}) = 60\,\mathrm{V\,m^{-1}}.$$

This is an instantaneous relation, so no root-mean-square conversion is needed.

Step 2: Determine the vector direction The wave travels along the positive x-direction and $\vec{B}$ points along $+\hat{j}$. Since $\hat{k} \times \hat{j} = -\hat{i}$, the electric field must point along $-\hat{k}$, giving $(-\hat{k}) \times \hat{j} = +\hat{i}$. Option (A) has the opposite propagation direction. Options (C) and (D) have magnitudes different from $cB = 60\,\mathrm{V\,m^{-1}}$.

Step 3: Conclusion $\boxed{\vec{E} = -60\hat{k}\,\mathrm{V\,m^{-1}}}$. Hence, the correct option is (B).

Quick Tip to Solve: Find the magnitude with $E = cB$, then check the sign using $\vec{E} \times \vec{B}$, in that order.

---

38. Two resistors of 200 Ω and 400 Ω are connected in series with a battery of 100 V. A bulb rated at 200 V, 100 W is connected across the 400 Ω resistance. The potential drop across the bulb is _____ V. [4 marks]

(A) 25

(B) 50

(C) 66.6

(D) 100

Correct Answer: (B) 50

Solution:

Concept:

- A bulb rated at voltage V_r and power P_r has resistance $R_b = V_r^2/P_r$, treated as constant in the ideal circuit model.
- Components connected across the same two points are in parallel and have equal potential differences.

Step 1: Replace the bulb by its resistance Use the rated values only to determine the bulb resistance:

$$R_b = \frac{200^2}{100} = 400\,\Omega.$$

The bulb is connected across the existing $400\,\Omega$ resistor, so their parallel equivalent is

$$R_p = \frac{400 \times 400}{400 + 400} = 200\,\Omega.$$

Step 2: Find the voltage across the parallel branch The $200\,\Omega$ series resistor and the $200\,\Omega$ parallel

equivalent divide the $100\,\mathrm{V}$ supply equally. Thus

$$I = \frac{100}{200 + 200} = 0.25\,\mathrm{A}, \qquad V_b = IR_p = 0.25(200) = 50\,\mathrm{V}.$$

The $66.6\,\mathrm{V}$ value neglects the loading caused by the bulb; $100\,\mathrm{V}$ neglects the drop across the series resistor. The value $25\,\mathrm{V}$ incorrectly divides the branch voltage between its parallel components.

Step 3: Conclusion The potential drop across the bulb is $\boxed{50\,\mathrm{V}}$. Hence, the correct option is (B).

Quick Tip to Solve: A bulb rating determines its resistance; it does not impose that rated voltage when the bulb is placed in another circuit.

---

39. Two metal plates (A, B) are kept horizontally with separation of $\left(\frac{12}{\pi}\right)$ cm, with plate A on the top. An atomizer jet sprays oil (density 1.5 g/cm^3) droplets of radius 1 mm horizontally. All oil droplets carry a charge 5 nC. The potentials V_A and V_B are required on plates A and B respectively in order to ensure the droplets do not descend. The values of V_A and V_B are _____.
(Neglect the air resistance to the droplets and take g = 10 m/s^2) [4 marks]

(A) 100 V and 580 V

(B) 580 V and 100 V

(C) 60 V and 400 V

(D) 0 V and -200 V

Correct Answer: (A) 100 V and 580 V

Solution:

Concept:

- A positively charged droplet remains at constant height when the upward electric force balances its weight: $qE = mg$.
- For parallel plates separated by d, the field magnitude is $|V_B - V_A|/d$, directed from higher to lower potential.

Step 1: Calculate the required field Convert all data to SI units: $\rho = 1500\,\mathrm{kg\,m^{-3}}$, $r = 10^{-3}\,\mathrm{m}$, and $q = 5 \times 10^{-9}\,\mathrm{C}$. The droplet mass is

$$m = \frac{4}{3}\pi r^3 \rho = 2\pi \times 10^{-6}\,\mathrm{kg}.$$

Hence

$$E = \frac{mg}{q} = \frac{2\pi \times 10^{-6} \times 10}{5 \times 10^{-9}} = 4000\pi\,\mathrm{V\,m^{-1}}.$$

Step 2: Fix the potential difference and polarity The plate spacing is $d = 0.12/\pi\,\mathrm{m}$, so

$$V_B - V_A = Ed = 4000\pi \frac{0.12}{\pi} = 480\,\mathrm{V}.$$

The positive droplet needs an upward force, requiring the lower plate B to be at higher potential than

A. Only $V_A = 100\,\text{V}$, $V_B = 580\,\text{V}$ has both the correct difference and polarity. Reversing these values accelerates the drop downward. The remaining pairs give differences $340\,\text{V}$ and $-200\,\text{V}$, which do not satisfy equilibrium.

Step 3: Conclusion $\boxed{V_A = 100\,\text{V}, \quad V_B = 580\,\text{V}}$. Hence, the correct option is (A).

Quick Tip to Solve: Balance the vertical forces first, then determine which plate must be at higher potential from the sign of the droplet charge.

---

40. Two point charges $8\ \mu C$ and $-2\ \mu C$ are located at $x = 2$ cm and $x = 4$ cm, respectively on the x-axis. The ratio of electric flux due to these charges through two spheres of radii 3 cm and 5 cm with their centers at the origin is _____. [4 marks]

(A) 4 : 1

(B) 3 : 4

(C) 4 : 3

(D) 4 : 5

Correct Answer: (C) 4 : 3

Solution:

Concept:

- The net outward electric flux through a closed surface is $\Phi = Q_{\text{enclosed}}/\varepsilon_0$.
- An external point charge contributes zero net flux through a closed surface, although its field need not vanish on the surface.

Step 1: Identify the enclosed charges The smaller sphere has radius $3\,\text{cm}$ and is centred at the origin. It encloses the $+8\,\mu\text{C}$ charge at $x = 2\,\text{cm}$, while the $-2\,\mu\text{C}$ charge at $x = 4\,\text{cm}$ is outside. Thus $Q_3 = 8\,\mu\text{C}$. The larger sphere has radius $5\,\text{cm}$ and encloses both charges, so $Q_5 = 8 - 2 = 6\,\mu\text{C}$.

Step 2: Form the flux ratio Apply Gauss's law separately:

$$\Phi_3 = \frac{8 \times 10^{-6}}{\varepsilon_0}, \qquad \Phi_5 = \frac{6 \times 10^{-6}}{\varepsilon_0}.$$

Therefore

$$\Phi_3 : \Phi_5 = 8 : 6 = 4 : 3.$$

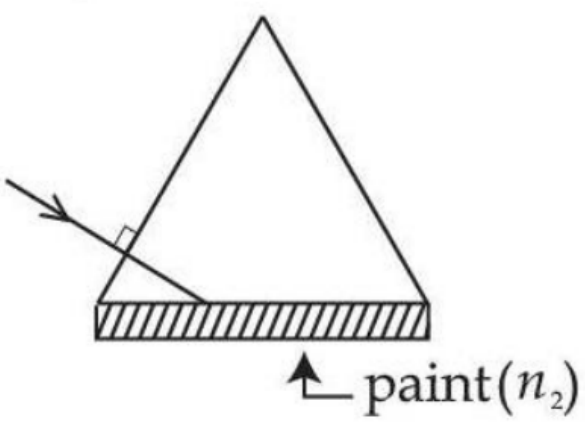
The sphere radii determine which charges are enclosed, not a direct area factor in the net-flux ratio. Option (B) reverses the requested order. Options (A) and (D) do not equal the ratio of the signed enclosed charges.

Step 3: Conclusion The flux ratio for the $3\,\text{cm}$ and $5\,\text{cm}$ spheres is $\boxed{4 : 3}$. Hence, the correct option is (C).

Quick Tip to Solve: For closed-surface flux, list enclosed charges with their signs before considering any field or surface-area calculation.

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41. One side of an equilateral prism is painted by a transparent material of refractive index n_2. The refractive index of prism is 1.6. The minimum value of n_2 required for total internal reflection from painted face is _____. [4 marks]



(A) $\frac{3\sqrt{3}}{1.6}$

(B) $\sqrt{3}$

(C) $\frac{3.2}{\sqrt{3}}$

(D) $\frac{4\sqrt{3}}{5}$

Correct Answer: No option is a minimum for total internal reflection; option (D) is the critical upper threshold, and strict total internal reflection requires a smaller coating index.

Solution:

Concept:

- At an interface from a prism of index n_1 into a coating of index n_2, total internal reflection requires $n_1 \sin i > n_2$. Equality gives the critical limiting case.
- A ray incident normally on the first prism face enters undeviated. The angle between the normals to two faces of an equilateral prism is $60°$.

Step 1: Determine incidence at the painted face The right-angle mark shows normal entry through the left sloping face. The ray therefore travels inside along that face's inward normal and meets the horizontal painted face at incidence $i = 60°$, measured from its vertical normal.

Step 2: Apply the strict condition and assess the wording With $n_1 = 1.6$, Snell's law gives the limiting coating index

$$n_{2,\text{crit}} = 1.6 \sin 60° = \frac{4\sqrt{3}}{5} \approx 1.386.$$

Total internal reflection actually requires

$$n_2 < \frac{4\sqrt{3}}{5}.$$

Thus reducing the coating index favours total internal reflection; the displayed value is an upper threshold, not a minimum. At equality the transmitted ray is grazing. Options (A), (B) and (C) all exceed even the prism index and cannot permit total internal reflection from prism to coating. Option

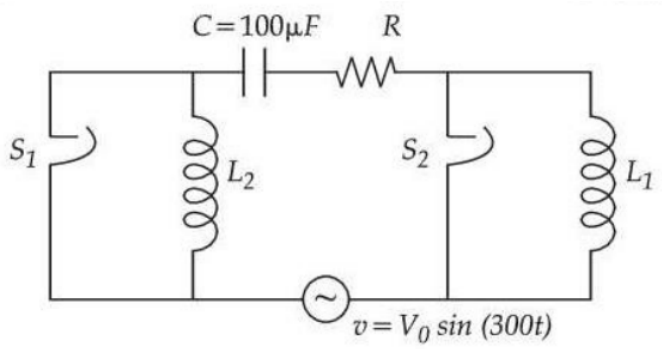
(D) is the critical boundary intended by the listed answers, but it is not a minimum that ensures total internal reflection.

Step 3: Conclusion As literally worded, no listed option gives a minimum coating index ensuring total internal reflection. The condition is $\boxed{n_2 < 4\sqrt{3}/5}$. The key's option (D) is the limiting upper value, $4\sqrt{3}/5$, with equality corresponding to critical refraction.

Quick Tip to Solve: Write the inequality for total internal reflection before interpreting a question that asks for a minimum or maximum refractive index.

42. The figure given below shows an *LCR* series circuit with two switches S_1 and S_2. When switch S_1 is closed keeping S_2 open, the phase difference (ϕ) between the current and source voltage is 30° and phase difference is 60° when S_2 is closed keeping S_1 open. The value of ($3L_1 - L_2$) is _____ H.

[4 marks]



(A) $\frac{9}{2}$

(B) $\frac{2}{9}$

(C) $\frac{1}{3}$

(D) 3

Correct Answer: (B) $\frac{2}{9}$

Solution:

Concept:

- For a series LCR circuit, the signed impedance phase satisfies $\tan\phi = (\omega L - 1/(\omega C))/R$.
- A closed ideal switch shorts the inductor connected across it. In $v = V_0\sin(300t)$, the angular frequency is $\omega = 300\,\text{rad}\,\text{s}^{-1}$.

Step 1: Write the two phase equations Closing S_1 bypasses L_2, leaving L_1 in the series circuit. Closing S_2 bypasses L_1, leaving L_2. With $X_C = 1/(\omega C)$, the stated positive phase angles give

$$\omega L_1 - X_C = R\tan 30^\circ = \frac{R}{\sqrt{3}},$$

$$\omega L_2 - X_C = R\tan 60^\circ = \sqrt{3}R.$$

Step 2: Eliminate resistance and substitute Multiply the first equation by 3 and subtract the second. The resistance terms cancel:

$$\omega(3L_1 - L_2) - 2X_C = 0.$$

Hence, with $C = 100\,\mu\text{F} = 10^{-4}\,\text{F}$,

$$3L_1 - L_2 = \frac{2}{\omega^2 C} = \frac{2}{(300)^2(10^{-4})} = \frac{2}{9}\,\text{H}.$$

None of $9/2$, $1/3$, or $3\,\text{H}$ satisfies the resulting equation $9(3L_1 - L_2) = 2$. In particular, $9/2$ is the reciprocal of the required numerical value.

Step 3: Conclusion $\boxed{3L_1 - L_2 = \frac{2}{9}\,\text{H}}$. Hence, the correct option is (B).

Quick Tip to Solve: Identify which inductor each switch bypasses, then eliminate the unknown resistance using the ratio of the two tangents.

43. A circular current loop of radius R is placed inside square loop of side length L ($L \gg R$) such that they are co-planar and their centers coincide. The permeability of free space is μ_0. The mutual inductance between circular loop and square loop is _____. [4 marks]

(A) $2\sqrt{2}\frac{\mu_0 L^2}{R}$

(B) $\sqrt{2}\frac{\mu_0 L^2}{R}$

(C) $\sqrt{2}\frac{\mu_0 R^2}{L}$

(D) $2\sqrt{2}\frac{\mu_0 R^2}{L}$

Correct Answer: (D) $2\sqrt{2}\frac{\mu_0 R^2}{L}$

Solution:

Concept:

- Mutual inductance is the flux linked with one loop per unit current in the other: $M = \Phi/I$. Reciprocity gives the same M whichever loop carries the current.
- The magnetic field of a finite straight wire at perpendicular distance d is $B = \mu_0 I(\sin\theta_1 + \sin\theta_2)/(4\pi d)$, where the angles are measured from the perpendicular.

Step 1: Choose the square loop as the source Pass a current I through the square. Since $R \ll L$, its field over the small circular loop is approximately uniform and equal to the field at the common centre. The perpendicular distance of each square side from the centre is $L/2$, and the two endpoint angles for each side are $45°$.

Step 2: Add the fields from all four sides All four contributions point perpendicular to the plane in

the same direction. For one side and then the whole square,

$$B_1 = \frac{\mu_0 I}{4\pi(L/2)}(2\sin 45^\circ) = \frac{\sqrt{2}\mu_0 I}{2\pi L}, \qquad B = 4B_1 = \frac{2\sqrt{2}\mu_0 I}{\pi L}.$$

Step 3: Calculate the flux through the circular loop The area vector of the circle is parallel to the square's magnetic field, so the linked flux is field times area:

$$\Phi = B\pi R^2 = \frac{2\sqrt{2}\mu_0 I R^2}{L}, \qquad M = \frac{\Phi}{I} = \frac{2\sqrt{2}\mu_0 R^2}{L}.$$

Options (A) and (B) have the wrong dependence on the small-loop area and would grow as its radius decreases. Option (C) has the correct scaling but is smaller by a factor of two.

Step 4: Conclusion The mutual inductance in the stated small-loop approximation is $\boxed{2\sqrt{2}\mu_0 R^2/L}$. Hence, the correct option is (D).

Quick Tip to Solve: Use reciprocity to put the current in the large loop; its field is nearly constant over the smaller loop, making the flux calculation immediate.

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44. The binding energy per nucleon of $^{209}_{83}Bi$ is _____ MeV.

[Take $m(^{209}_{83}Bi) = 208.980388$ u, $m_p = 1.007825$ u, $m_n = 1.008665$ u, 1 u $= 931$ MeV/c^2] [4 marks]

(A) 7.48

(B) 7.84

(C) 8.79

(D) 6.94

Correct Answer: (B) 7.84

Solution:

Concept:

- The mass defect of a nucleus is the difference between the sum of the masses of its separated constituents and its bound mass. Its binding energy is $E_b = \Delta m c^2$.
- For mass number A and atomic number Z, there are Z protons and $A - Z$ neutrons. Binding energy per nucleon is the total binding energy divided by A.

Step 1: Count the constituent particles For bismuth-209, $Z = 83$ and $A = 209$. Hence the neutron number is $N = 209 - 83 = 126$. Use all the masses supplied in the question consistently. The supplied value 1.007825 u is the hydrogen-atom mass convention; when used with the atomic mass of bismuth, the electron masses cancel in the mass difference to the required accuracy.

Step 2: Evaluate the mass defect Add the constituent masses first and subtract the mass of the bound atom:

$$\begin{aligned}\Delta m &= 83(1.007825) + 126(1.008665) - 208.980388 \\ &= 83.649475 + 127.091790 - 208.980388 \\ &= 1.760877\,\text{u}.\end{aligned}$$

The positive difference means energy must be supplied to separate the nucleus into its constituents.

Step 3: Convert to energy and divide by the nucleon number The conversion given in the question yields

$$E_b = 1.760877 \times 931 = 1639.376487\,\text{MeV}, \qquad \frac{E_b}{A} = \frac{1639.376487}{209} = 7.8439066\,\text{MeV}.$$

Rounding to two decimal places gives 7.84. Option (A), 7.48, interchanges digits; options (C), 8.79, and (D), 6.94, do not equal the mass-defect result. Dividing by 83 or 126 would give energy per proton or per neutron, not per nucleon.

Step 4: Conclusion The binding energy per nucleon is $\boxed{7.84\,\text{MeV}}$. Hence, the correct option is (B).

Quick Tip to Solve: Keep all supplied mass digits until the last step: a small mass difference is multiplied by 931 before division by the total nucleon number.

45. The equation of motion of a particle is given by $x = a\sin(50t + \pi/3)$ cm. The particle will come to rest at time t_1 and it will have zero acceleration at time t_2. The t_1 and t_2 respectively are _____.

[4 marks]

(A) $\frac{\pi}{300}$ s, $\frac{\pi}{75}$ s
(B) $\frac{\pi}{75}$ s, $\frac{\pi}{300}$ s
(C) $\frac{\pi}{300}$ s, $\frac{\pi}{25}$ s
(D) $\frac{\pi}{50}$ s, $\frac{\pi}{100}$ s

Correct Answer: (A) $\frac{\pi}{300}$ s, $\frac{\pi}{75}$ s

Solution:

Concept:

- For $x = a\sin(\omega t + \phi)$, velocity is $v = a\omega\cos(\omega t + \phi)$, and acceleration is $\ddot{x} = -\omega^2 x$.
- The particle is instantaneously at rest at an extreme position, whereas its acceleration vanishes at the mean position. The first future occurrence is found by choosing the next appropriate phase after the initial phase.

Step 1: Differentiate the displacement Here the angular frequency is $50\,\text{rad}\,\text{s}^{-1}$ and the initial phase is $\pi/3$. Differentiation with respect to time gives

$$v = 50a\cos(50t + \pi/3), \qquad \ddot{x} = -2500a\sin(50t + \pi/3).$$

The centimetre unit of the amplitude does not affect the times obtained from the phase conditions.

Step 2: Find the first instant of rest Rest requires $v = 0$, so the cosine must vanish. Starting from phase $\pi/3$, the next such phase is $\pi/2$:

$$50t_1 + \frac{\pi}{3} = \frac{\pi}{2}, \qquad t_1 = \frac{\pi/6}{50} = \frac{\pi}{300}\,\text{s}.$$

Later instants of rest differ from this by $\pi/50\,\text{s}$.

Step 3: Find the first zero of acceleration Acceleration vanishes when the sine vanishes. The first accessible phase greater than $\pi/3$ is π, rather than zero:

$$50t_2 + \frac{\pi}{3} = \pi, \qquad t_2 = \frac{2\pi/3}{50} = \frac{\pi}{75}\,\text{s}.$$

Option (B) reverses the rest and zero-acceleration conditions. Option (C) has the correct first time but its second time gives phase $7\pi/3$, whose sine is nonzero. Option (D) satisfies neither required first-time condition.

Step 4: Conclusion The first positive times are $\boxed{t_1 = \pi/300\,\text{s}, \quad t_2 = \pi/75\,\text{s}}$, respectively. Hence, the correct option is (A).

Quick Tip to Solve: Advance the phase from its initial value: the next odd multiple of $\pi/2$ gives rest, and the next multiple of π gives zero acceleration.

Physics – Section B: Numerical Value Questions (20 marks)

46. In a Young's double slit experiment, the intensity at some point on the screen is found to be $\frac{3}{4}$ times of the maximum of the interference pattern. The path difference between the interfering waves at this point is $\frac{\lambda}{x}$ where λ is wavelength of the incident light. The value of x is _____. [4 marks]

Solution:

Concept:

- For two coherent waves of equal individual intensity I_0, the resultant intensity is $I = 4I_0\cos^2(\delta/2)$, where δ is their phase difference. The maximum intensity is $I_{\max} = 4I_0$.
- A path difference Δ produces phase difference $\delta = 2\pi\Delta/\lambda$. Consequently, $I/I_{\max} = \cos^2(\pi\Delta/\lambda)$.

Step 1: Form the normalized intensity equation The stated intensity is three-fourths of the maximum. Dividing by $I_{\max}$ removes the individual slit intensity:

$$\frac{I}{I_{\max}} = \cos^2\left(\frac{\pi\Delta}{\lambda}\right) = \frac{3}{4}.$$

On the first interval outward from the central bright fringe, the cosine is positive, so its value is $\sqrt{3}/2$. Taking the intensity ratio itself as the amplitude ratio would give the wrong phase.

Step 2: Determine the principal path difference The smallest positive angle with cosine $\sqrt{3}/2$ is $\pi/6$. Thus the principal positive path difference is

$$\frac{\pi\Delta}{\lambda} = \frac{\pi}{6}, \qquad \Delta = \frac{\lambda}{6}.$$

This corresponds to phase difference $\delta = \pi/3$. Substitution checks the intensity: $\cos^2(\pi/6) = (\sqrt{3}/2)^2 = 3/4$.

Step 3: Compare with the requested form Comparing $\Delta = \lambda/6$ with $\Delta = \lambda/x$ gives $x = 6$. The intensity repeats across the screen: the general path differences are $\Delta/\lambda = n \pm 1/6$, for integral n.

The requested numerical value uses the principal positive path difference, which is the point nearest the central maximum.

Step 4: Conclusion For the principal positive path difference, $\boxed{x = 6}$. The quantity x is dimensionless, while the associated path difference is $\lambda/6$.

Quick Tip to Solve: Normalize by the maximum intensity first, then take the square root before identifying the half-phase angle. Then use $\Delta = \lambda\delta/(2\pi)$ to convert the full phase difference to path difference.

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47. Using Bohr's model, calculate the ratio of the magnetic fields generated due to the motion of the electrons in the 2nd and 4th orbits of hydrogen atom _____. [4 marks]

Solution:

Concept:

- In the Bohr model of hydrogen, the radius and speed in the nth orbit scale as $r_n \propto n^2$ and $v_n \propto 1/n$, respectively.
- A revolving electron constitutes a current of magnitude $I = e/T = ev/(2\pi r)$. The magnetic field at the centre of a circular current loop is $B = \mu_0 I/(2r)$.

Step 1: Express the orbital current in terms of speed and radius During one orbital period, one electronic charge passes any fixed point of the orbit. The period equals the circumference divided by the speed:

$$T_n = \frac{2\pi r_n}{v_n}, \qquad I_n = \frac{e}{T_n} = \frac{ev_n}{2\pi r_n}.$$

Conventional current has the opposite direction to electron motion, but only field magnitudes enter the requested ratio.

Step 2: Obtain the dependence of the field on the orbit number Substitute the orbital current into the circular-loop field formula:

$$B_n = \frac{\mu_0 I_n}{2r_n} = \frac{\mu_0 e v_n}{4\pi r_n^2}.$$

Since $v_n \propto n^{-1}$ and $r_n^2 \propto n^4$, the field at the nucleus obeys $B_n \propto n^{-5}$. The current alone scales as n^{-3}; the extra inverse radius in the field formula supplies two more powers of n.

Step 3: Compare the second and fourth orbits All constants cancel in the ratio, leaving

$$\frac{B_2}{B_4} = \frac{2^{-5}}{4^{-5}} = \left(\frac{4}{2}\right)^5 = 32.$$

Equivalently, the second-orbit speed is twice the fourth-orbit speed and the fourth-orbit radius is four times the second-orbit radius, giving $2 \times 4^2 = 32$. The smaller orbit therefore produces the stronger field, as expected.

Step 4: Conclusion The ratio of the field magnitudes is $\boxed{B_2 : B_4 = 32 : 1}$, so the requested numerical value is $\boxed{32}$, a dimensionless ratio.

Quick Tip to Solve: Remember the chain $I_n \propto n^{-3}$ and $B_n \propto I_n/r_n \propto n^{-5}$; using the current ratio alone misses the radius factor.

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48. 5 moles of unknown gas is heated at constant volume from 10°C to 20°C. The molar specific heat of this gas at constant pressure $c_p = 8$ cal/mol.°C and $R = 8.36$ J/mol.°C. The change in the internal energy of the gas is _____ calorie. [4 marks]

Solution:

Concept:

- For an ideal gas, internal energy depends on temperature, and its change is $\Delta U = nc_v\Delta T$, where c_v is the molar heat capacity at constant volume.
- Mayer's relation is $c_p - c_v = R$. All three quantities must be expressed in the same energy units before this subtraction is performed.

Step 1: Convert the gas constant to calorie units The supplied c_p is in calories per mole per degree, but R is in joules per mole per degree. Using $1\,\text{cal} = 4.18\,\text{J}$, convert the gas constant:

$$R = \frac{8.36}{4.18} = 2\,\text{cal}\,\text{mol}^{-1}\,\text{K}^{-1}.$$

A temperature interval of one degree Celsius equals an interval of one kelvin, so no additional factor is needed in the denominator. Subtracting 8.36 directly from 8 would incorrectly mix joules and calories.

Step 2: Find the heat capacity at constant volume Apply Mayer's relation with consistent units:

$$c_v = c_p - R = 8 - 2 = 6\,\text{cal}\,\text{mol}^{-1}\,\text{K}^{-1}.$$

Although the question gives c_p, the required internal energy change uses c_v. At constant volume the gas does no expansion work, since $W = \int p\,dV = 0$; thus the heat supplied equals the increase in internal energy.

Step 3: Calculate the increase in internal energy The amount of gas is $n = 5\,\text{mol}$, and the temperature rise is $\Delta T = 20 - 10 = 10\,\text{K}$. Therefore,

$$\Delta U = nc_v\Delta T = 5 \times 6 \times 10 = 300\,\text{cal}.$$

The sign is positive because the temperature increases. As a unit check, moles and kelvins cancel against the units of the molar heat capacity, leaving calories. Using $nc_p\Delta T$ instead would give 400 calories, which is not the internal energy change.

Step 4: Conclusion The gas gains internal energy $\boxed{\Delta U = +300\,\text{cal}}$. The numerical value to enter is $\boxed{300}$.

Quick Tip to Solve: Convert R to the units of the stated heat capacity before using $c_v = c_p - R$; a Celsius temperature rise already has the correct kelvin magnitude.

49. If sunlight is focused on a paper using convex lens, it starts burning the paper in shortest time when the lens is kept at 30 cm above the paper. If the radius of curvature of the lens is 60 cm then the refractive index of the lens material is $\frac{\alpha}{10}$. The value of α is _____. [4 marks]

Solution:

Concept:

- Sunlight arrives as an approximately parallel beam. A converging lens gives its smallest and most intense spot at its principal focus, so the paper burns fastest when it lies in the focal plane.
- For a thin lens in air, the lens maker relation is $1/f = (\mu - 1)(1/R_1 - 1/R_2)$, where μ is the refractive index relative to air and the radii carry Cartesian signs.

Step 1: Identify the focal length The shortest burning time corresponds to maximum concentration of the sunlight, rather than simply the smallest lens-to-paper distance. Thus the given height of the lens above the paper is its focal length:

$$f = 30\,\text{cm}.$$

Rays from the distant Sun converge approximately in this plane; away from it, their energy spreads over a larger spot.

Step 2: Apply the signed radii to the lens maker formula The single curvature magnitude given for the convex lens is used for its two equally curved surfaces. With light travelling from left to right, the first surface has $R_1 = +60\,\text{cm}$, while the second has $R_2 = -60\,\text{cm}$. Therefore

$$\frac{1}{30} = (\mu - 1)\left(\frac{1}{60} - \frac{1}{-60}\right) = \frac{\mu - 1}{30}.$$

All lengths have been kept in centimetres, so no unit conversion is needed in this ratio. Multiplication by 30 gives $\mu - 1 = 1$, hence $\mu = 2$.

Step 3: Extract the requested numerical parameter The refractive index is specified as $\mu = \alpha/10$. Equating this expression with the computed index gives

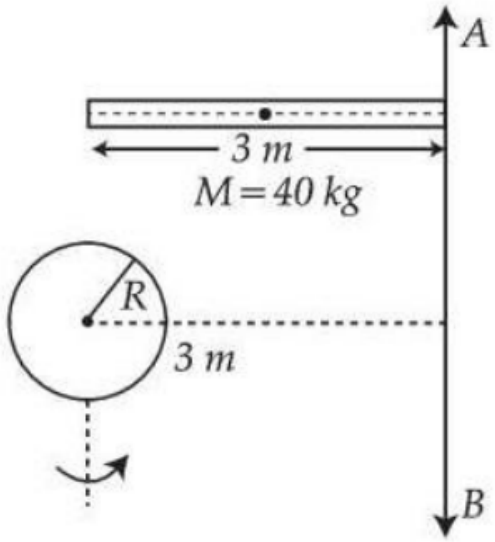
$$\frac{\alpha}{10} = 2 \quad \Rightarrow \quad \alpha = 20.$$

Both refractive index and α are dimensionless.

Step 4: Conclusion The refractive index is 2, and the required numerical value is $\boxed{\alpha = 20}$.

Quick Tip to Solve: For an equally curved biconvex lens in air, use $f = R/[2(\mu - 1)]$; opposite signs of the two radii produce the factor of two.

50. Moment of inertia about an axis *AB* for a rod of mass 40 kg and length 3 m is same as that of a solid sphere of mass of 10 kg and radius R about an axis parallel to *AB* axis with separation of 3 m as shown in figure below. The value of R is given as $\sqrt{\frac{\alpha}{2}}$. The value of α is _____. [4 marks]



Solution:

Concept:

- A uniform rod of mass M and length L has moment of inertia $ML^2/3$ about an axis perpendicular to it through one end.
- A uniform solid sphere of mass m and radius R has moment of inertia $2mR^2/5$ about a diameter. An offset contributes md^2 only when the rotation axis is displaced from its own centre.

Step 1: Identify the two rotation axes In the diagram, AB passes through the right-hand end of the rod and is perpendicular to its length. The sphere rotates about the dashed vertical diameter shown through its centre. This diameter is parallel to AB, and the separation of the two axes is $3\,\text{m}$. The separation locates the two different axes; it does not make the sphere rotate about AB.

Step 2: Calculate the rod inertia For the rod, $M = 40\,\text{kg}$ and $L = 3\,\text{m}$, so

$$I_{\text{rod}} = \frac{ML^2}{3} = \frac{40(3)^2}{3} = 120\,\text{kg m}^2.$$

This is the end-axis value, not the central-axis value $ML^2/12$. Equivalently, adding $M(L/2)^2$ to the central value gives the same result.

Step 3: Equate it to the sphere inertia The solid sphere has $m = 10\,\text{kg}$, hence its inertia about the indicated central axis is

$$I_{\text{sphere}} = \frac{2}{5}(10)R^2 = 4R^2.$$

Equality of the specified inertias gives

$$4R^2 = 120 \quad \Rightarrow \quad R^2 = 30\,\text{m}^2.$$

Comparing $R = \sqrt{30}\,\mathrm{m}$ with the stated form $R = \sqrt{\alpha/2}$ in metres, we find $\alpha/2 = 30$.

Step 4: Conclusion The radius is $R = \sqrt{30}\,\mathrm{m}$, so the required numerical value is $\boxed{\alpha = 60}$.

Quick Tip to Solve: Assign each object its own indicated rotation axis before using the parallel-axis theorem; the distance between two different objects' axes is not automatically an offset term.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. The ratio of mass percentage (w/w) of C : H in a hydrocarbon is 12 : 1. It has two carbon atoms. The weight (in g) of CO_2(g) formed when 3.38 g of this hydrocarbon is completely burnt in oxygen is : (Given : Molar mass in g mol^{-1} C : 12, H : 1, O : 16) [4 marks]

(A) 5.68

(B) 11.44

(C) 22.74

(D) 17.05

Correct Answer: (B) 11.44

Solution:

Concept:

- Mass ratios must be divided by atomic molar masses to obtain atom ratios. A hydrocarbon contains only carbon and hydrogen.
- In complete combustion, every carbon atom in the fuel produces one molecule of $\mathrm{CO_2}$. Thus a fuel molecule with two carbon atoms yields two molecules of $\mathrm{CO_2}$.

Step 1: Determine the molecular formula The mass ratio is $m_{\mathrm{C}} : m_{\mathrm{H}} = 12 : 1$. Dividing by the respective atomic molar masses gives

$$n_{\mathrm{C}} : n_{\mathrm{H}} = \frac{12}{12} : \frac{1}{1} = 1 : 1.$$

The empirical formula is therefore CH. Since each hydrocarbon molecule contains two carbon atoms, its molecular formula is $\mathrm{C_2H_2}$, with molar mass $2(12) + 2(1) = 26\,\mathrm{g\,mol^{-1}}$.

Step 2: Convert the fuel mass into carbon dioxide mass The balanced complete-combustion equation is

$$2\,\mathrm{C_2H_2} + 5\,\mathrm{O_2} \longrightarrow 4\,\mathrm{CO_2} + 2\,\mathrm{H_2O}.$$

The amount of fuel burned is

$$n(\mathrm{C_2H_2}) = \frac{3.38}{26} = 0.13\,\mathrm{mol}.$$

Consequently, $n(\mathrm{CO_2}) = 2(0.13) = 0.26\,\mathrm{mol}$. Using $M(\mathrm{CO_2}) = 12 + 2(16) = 44\,\mathrm{g\,mol^{-1}}$, its mass is

$$m(\mathrm{CO_2}) = 0.26(44) = 11.44\,\mathrm{g}.$$

Step 3: Check the options against carbon balance Option (B) equals the calculated mass. Option

(A), 5.68 g, is too small; option (C), 22.74 g, and option (D), 17.05 g, are too large. None of those values corresponds to the required 0.26 mol of carbon dioxide. The product can be heavier than the fuel because its oxygen comes from the oxygen supplied for combustion.

Step 4: Conclusion The mass of CO_2 formed is $\boxed{11.44\,\text{g}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a combustion mass calculation, conserve carbon: multiply the mass of carbon in the fuel by $44/12$ to obtain the carbon dioxide mass.

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52. The first and second ionization constants of a weak dibasic acid H_2A are 8.1×10^{-8} and 1.0×10^{-13} respectively. 0.1 mol of H_2A was dissolved in 1 L of 0.1 M HCl solution. The concentration of HA^- in the resultant solution is: [4 marks]

(A) 0.1 M

(B) 9.53×10^{-6} M

(C) 8.1×10^{-8} M

(D) 1.0×10^{-13} M

Correct Answer: (C) 8.1×10^{-8} M

Solution:

Concept:

- For the first dissociation $H_2A \rightleftharpoons H^+ + HA^-$, $K_{a1} = [H^+][HA^-]/[H_2A]$. Added strong acid supplies a common ion and suppresses this dissociation.
- The second dissociation obeys $K_{a2} = [H^+][A^{2-}]/[HA^-]$. Its much smaller constant makes loss of the second proton negligible here.

Step 1: Identify the dominant hydrogen-ion source The 0.1 M hydrochloric acid is a strong acid and dissociates essentially completely. Therefore $[H^+] \simeq 0.1$ M. Dissolving 0.1 mol of H_2A in the stated litre gives an acid concentration of approximately 0.1 M. Because the weak acid is strongly suppressed by the common hydrogen ion, $[H_2A] \simeq 0.1$ M.

Step 2: Calculate the singly ionized acid concentration Rearranging the first-ionization equilibrium gives

$$\begin{aligned}[HA^-] &= K_{a1}\frac{[H_2A]}{[H^+]} \\ &\simeq (8.1 \times 10^{-8})\frac{0.1}{0.1} \\ &= 8.1 \times 10^{-8}\,\text{M}.\end{aligned}$$

This is much smaller than 0.1 M, confirming the concentration approximations. Also, $[A^{2-}]/[HA^-] = 10^{-13}/0.1 = 10^{-12}$, so the second dissociation cannot materially deplete HA^-.

Step 3: Eliminate the alternative concentrations Option (A) would imply almost complete first ionization despite the common ion. Option (B) does not satisfy the first-ionization expression with the specified concentrations. Option (D) merely copies the second ionization constant and is not the concentration of HA^-. Only option (C) satisfies the applicable equilibrium.

Step 4: Conclusion The resultant concentration is $\boxed{[\mathrm{HA}^-] \simeq 8.1 \times 10^{-8}\,\mathrm{M}}$. Hence, the correct option is (C).

Quick Tip to Solve: When strong acid supplies the hydrogen ions, use $[\mathrm{HA}^-] \simeq K_{a1}C/[\mathrm{H}^+]$; do not use the isolated weak-acid formula $\sqrt{K_aC}$.

53. SF_4 is isostructural with :

A. $BrF_4^{\ominus}$

B. CH_4

C. $IF_4^{\oplus}$

D. XeF_4

E. XeO_2F_2

Choose the correct answer from the options given below : [4 marks]

(A) C Only

(B) C and E Only

(C) A and D Only

(D) B and E Only

Correct Answer: (B) C and E Only

Solution:

Concept:

- Isostructural species have the same molecular geometry. In VSEPR theory, count bonding domains and lone pairs on the central atom; a multiple bond counts as one directional domain.
- Four bonded atoms and one central lone pair give an AX_4E arrangement: a trigonal-bipyramidal electron-domain geometry and a seesaw molecular shape.

Step 1: Establish the shape of sulfur tetrafluoride In SF_4, sulfur has six valence electrons. Four are used in four single bonds to fluorine, leaving two electrons as one lone pair. Thus there are five domains: four bonds plus one lone pair. The lone pair occupies an equatorial site to reduce repulsions, and the four fluorine atoms form a seesaw shape.

Step 2: Classify each proposed species BrF_4^- has $7 + 1 = 8$ central-atom electrons available in the usual counting: four bonds leave two lone pairs. It is AX_4E_2, square planar. CH_4 has four bonds and no lone pair, so it is tetrahedral. IF_4^+ has $7 - 1 = 6$ such electrons: four bonds leave one lone pair, giving AX_4E, seesaw. XeF_4 has four bonds and two lone pairs and is square planar. In XeO_2F_2, xenon forms two Xe=O double bonds and two Xe−F single bonds, leaving one central lone pair. Each double bond counts as one domain, so this species is also AX_4E, seesaw.

Step 3: Select the complete matching set The matching species are C, IF_4^+, and E, XeO_2F_2. Option (A) leaves out E. Option (C) contains two square-planar species rather than seesaw species. Option (D) incorrectly includes tetrahedral methane. Option (B) lists both and only the matching species.

Step 4: Conclusion SF_4 is isostructural with C and E only. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish molecular shape from electron-domain geometry, and count a double bond as one domain when comparing VSEPR shapes.

54. Gas 'A' undergoes change from state 'X' to state 'Y'. In this process, the heat absorbed and work done by the gas is 10 J and 18 J respectively. Now gas is brought back to state 'X' by another process during which 6 J of heat is evolved. In the reverse process of 'Y' to 'X', [4 marks]

(A) 18 J of the work is done by the gas 'A'.

(B) 2 J of the work is done by the gas 'A'.

(C) 12 J of the work is done on the gas 'A' by the surrounding.

(D) 14 J of the work is done on the gas 'A' by the surrounding.

Correct Answer: (D) 14 J of the work is done on the gas 'A' by the surrounding.

Solution:

Concept:

- Using the chemistry convention, $\Delta U = q + w$, where q is positive for heat absorbed and w is positive for work done on the gas.
- Internal energy is a state function. Reversing the initial and final states reverses ΔU, even if the return path has different heat and work exchanges.

Step 1: Find the forward internal-energy change For the change $X \to Y$, the gas absorbs $10\,\text{J}$, so $q_{XY} = +10\,\text{J}$. The gas does $18\,\text{J}$ of work on its surroundings, hence the work done on it is $w_{XY} = -18\,\text{J}$. The first law gives

$$\Delta U_{XY} = q_{XY} + w_{XY} = 10 - 18 = -8\,\text{J}.$$

Thus state Y has internal energy 8 J less than state X.

Step 2: Use the return heat to determine the work For the return $Y \to X$, the internal-energy change must be $\Delta U_{YX} = +8\,\text{J}$. The gas evolves 6 J of heat, so $q_{YX} = -6\,\text{J}$. Substituting into the first law,

$$8 = -6 + w_{YX} \quad \Rightarrow \quad w_{YX} = 14\,\text{J}.$$

The positive value means the surroundings perform 14 J of work on the gas. Of this input, 6 J leaves as heat and 8 J increases its internal energy.

Step 3: Check the direction and magnitude in each option Option (D) states exactly this result. Options (A) and (B) describe positive work done by the gas, which would further lower its internal energy while it is already releasing heat. Option (C) gives only 12 J of work input; subtracting the 6 J heat loss would raise the internal energy by 6 J, not the required 8 J. The work itself need not reverse sign with unchanged magnitude because the return follows another process.

Step 4: Conclusion In the reverse process, $\boxed{14\,\text{J}}$ of work is done on the gas by the surroundings. Hence, the correct option is (D).

Quick Tip to Solve: Reverse the sign of the state-function change ΔU first; then use the return-path heat to calculate its work independently.

55. Solution A is prepared by dissolving 1 g of a protein (molar mass = 50,000 g mol^{-1}) in 0.5 L of water at 300 K. Its osmotic pressure is x bar. Solution B is made by dissolving 2 g of the same protein in 1 L of water at 300 K. Osmotic pressure of solution B is y bar. Entire solution of A is mixed with entire solution of B at the same temperature. The osmotic pressure of resultant solution is z bar. x, y and z respectively are:

(R = 0.083 L bar mol^{-1} K^{-1}) [4 marks]

(A) 9.96×10^{-4}; 9.96×10^{-4}; 9.96×10^{-4}

(B) 9.96×10^{-4}; 9.96×10^{-4}; 19.92×10^{-4}

(C) 4.98×10^{-4}; 4.98×10^{-4}; 9.96×10^{-4}

(D) 4.98×10^{-4}; 4.98×10^{-4}; 4.98×10^{-4}

Correct Answer: (A) 9.96×10^{-4}; 9.96×10^{-4}; 9.96×10^{-4}

Solution:

Concept:

- For a dilute solution of a non-electrolyte, osmotic pressure is $\pi = cRT = nRT/V$, where c is molarity, n is solute amount and V is solution volume.
- The protein is the same in both solutions and does not dissociate. When solutions are mixed, add their solute amounts and volumes before finding the new concentration.

Step 1: Calculate the pressure of solution A The amount of protein in A is its mass divided by its molar mass. With the dilute-solution volume given in the question,

$$n_A = \frac{1}{50000} = 2.0 \times 10^{-5} \text{ mol}, \qquad c_A = \frac{2.0 \times 10^{-5}}{0.5} = 4.0 \times 10^{-5} \text{ mol L}^{-1}.$$

Using the supplied gas constant in litre-bar units gives

$$x = c_A RT = (4.0 \times 10^{-5})(0.083)(300) = 9.96 \times 10^{-4} \text{ bar}.$$

Step 2: Calculate the pressure of solution B Both mass and volume are doubled, so concentration remains unchanged:

$$n_B = \frac{2}{50000} = 4.0 \times 10^{-5} \text{ mol}, \qquad c_B = \frac{4.0 \times 10^{-5}}{1} = 4.0 \times 10^{-5} \text{ mol L}^{-1}.$$

Therefore $y = c_B RT = 9.96 \times 10^{-4}$ bar. Doubling the amount alone would double the pressure, but here the volume doubles too.

Step 3: Combine the solutions and test the options The mixture contains 3 g of protein in 1.5 L:

$$c_{\text{mix}} = \frac{(1+2)/50000}{0.5+1} = 4.0 \times 10^{-5} \text{ mol L}^{-1}, \qquad z = 9.96 \times 10^{-4} \text{ bar}.$$

Option B incorrectly adds the two pressures. Options C and D give initial pressures half the calculated values. Osmotic pressure is an intensive concentration-dependent property; it is not additive on mixing equal-concentration solutions.

Step 4: Conclusion $x = y = z = 9.96 \times 10^{-4}$ bar. Hence, the correct option is (A).

Quick Tip to Solve: Compare mass divided by volume first. For the same solute at the same temperature, equal mass concentrations give equal osmotic pressures, including after mixing.

56. At 25°C, 20.0 mL of 0.2 M weak monoprotic acid HX is titrated against 0.2 M NaOH. The pH of the solution (a) at the start of the titration (when NaOH has not been added) and (b) when 10 mL of NaOH is added respectively, are :

Given :

$$K_a = 5 \times 10^{-4}$$

$$\mathrm{p}K_a = 3.3$$

$$\alpha \ll 1$$

[4 marks]

(A) (a) 0.7; (b) 2.0

(B) (a) 2.0; (b) 3.3

(C) (a) 1.1; (b) 2.2

(D) (a) 3.0; (b) 2.2

Correct Answer: (B) (a) 2.0; (b) 3.3

Solution:

Concept:

- A weak monoprotic acid satisfies $K_a = [\mathrm{H}^+][\mathrm{X}^-]/[\mathrm{HX}]$. Under the given small-ionization approximation, $[\mathrm{H}^+] \simeq \sqrt{K_a C}$.
- Before equivalence, partial neutralization produces an acid/conjugate-base buffer: $\mathrm{pH} = \mathrm{p}K_a + \log([\mathrm{X}^-]/[\mathrm{HX}])$. Equal acid and conjugate-base amounts give $\mathrm{pH} = \mathrm{p}K_a$.

Step 1: Find the initial pH Initially the analytical acid concentration is $C = 0.2\ \mathrm{mol\,L^{-1}}$. The question supplies $\alpha \ll 1$, so use the weak-acid approximation rather than treating HX as completely ionized:

$$[\mathrm{H}^+] \simeq \sqrt{(5 \times 10^{-4})(0.2)} = \sqrt{10^{-4}} = 10^{-2}\ \mathrm{mol\,L^{-1}}.$$

$$\mathrm{pH} = -\log_{10}(10^{-2}) = 2.0.$$

The corresponding approximate ionization fraction is $0.01/0.2 = 0.05$, consistent with the intended approximation.

Step 2: Account for the added hydroxide Neutralization occurs according to $\mathrm{HX} + \mathrm{OH}^- \longrightarrow \mathrm{X}^- + \mathrm{H_2O}$. Initial acid and added hydroxide amounts are

$$n_{\mathrm{HX},0} = 0.2(0.020) = 0.004\ \mathrm{mol}, \qquad n_{\mathrm{OH}^-} = 0.2(0.010) = 0.002\ \mathrm{mol}.$$

Thus 0.002 mol HX remains and 0.002 mol conjugate base forms. This is half-equivalence, not equivalence; no excess strong base is present.

Step 3: Calculate the buffer pH and choose the pair Both species occupy the same total volume, so

their concentration ratio equals their mole ratio:

$$\text{pH} = 3.3 + \log_{10}\left(\frac{0.002}{0.002}\right) = 3.3.$$

Only option B gives the ordered pair $(2.0, 3.3)$. Options A, C and D fail the initial weak-acid calculation and also fail the half-equivalence condition. The first and second pH values use different equilibrium situations.

Step 4: Conclusion The initial pH is 2.0, and after adding $10\ \text{mL}$ NaOH it is 3.3, using the stated approximation. Hence, the correct option is (B).

Quick Tip to Solve: Locate half-equivalence by comparing moles of base with half the initial acid moles; there the buffer pH is the supplied pKa, independent of dilution.

57. Consider the reaction $aX \rightarrow bY$, for which the rate constant at 30°C is $1 \times 10^{-3}\ \text{mol}^{-1}\,\text{L}\,\text{s}^{-1}$. Which of the following statements are true?

A. When concentration of 'X' is increased to four times, the rate of reaction becomes 16 times.

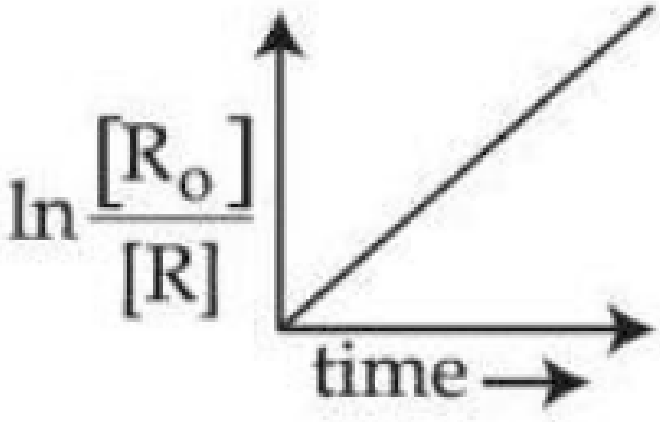
B. The reaction is a second order reaction.

C. The half-life period is independent of the concentration of X.

D. Decomposition of N_2O_5 is an example of the above reaction.

E. The graph of $\ln([R_0]/[R])$ vs time shown below is valid for the above reaction.

Choose the correct answer from the options given below: [4 marks]



(A) A and B Only

(B) A, B and C Only

(C) A, B, D and E Only

(D) C and D Only

Correct Answer: (A) A and B Only

Solution:

Concept:

- For $r = k[X]^n$, the units of k are $(\text{mol}\,\text{L}^{-1})^{1-n}\text{s}^{-1}$. Thus $\text{L}\,\text{mol}^{-1}\,\text{s}^{-1}$ identifies second-order kinetics.
- A second-order single-reactant rate law gives $t_{1/2} \propto 1/[X]_0$, while a straight-line plot of $\ln([X]_0/[X])$ against time identifies first-order kinetics.

Step 1: Infer the order and test A and B The stated rate constant has units $\text{mol}^{-1}\,\text{L}\,\text{s}^{-1}$, so the exponent in the rate law is two:

$$r = k[X]^2.$$

At the same temperature, changing only the reactant concentration to four times gives

$$\frac{r_{\text{new}}}{r_{\text{old}}} = \frac{k(4[X])^2}{k[X]^2} = 16.$$

Statement A is therefore true, and statement B correctly names the reaction order. The stoichiometric coefficient a alone is not used to infer the order.

Step 2: Check the half-life and the example For a second-order disappearance law, integration gives $1/[X] - 1/[X]_0 = k't$, with a constant k' incorporating any stoichiometric rate convention. Substituting $[X] = [X]_0/2$ yields

$$t_{1/2} = \frac{1}{k'[X]_0}.$$

The half-life depends inversely on the initial concentration, so C is false. The usual decomposition of N_2O_5 follows first-order kinetics, not the second-order law identified here; D is false.

Step 3: Interpret the graph and eliminate the distractors The graph in E is a straight line of $\ln([R_0]/[R])$ against time. The first-order integrated law is

$$\ln\left(\frac{[R_0]}{[R]}\right) = kt.$$

For a second-order reaction, the linear concentration plot is instead $1/[R]$ against time. E is false. Option B incorrectly includes C; option C incorrectly includes D and E; option D includes two false statements. Only the set consisting of A and B survives.

Step 4: Conclusion Only statements A and B are true. Hence, the correct option is (A).

Quick Tip to Solve: Read the units of the rate constant before examining the statements: inverse concentration per second immediately indicates a second-order rate law.

58. The correct set that contains all kinds (basic, acidic, amphoteric and neutral) of oxides is :

[4 marks]

(A) Na_2O, K_2O, Al_2O_3 and As_2O_3

(B) Al_2O_3, As_2O_3, CO and NO

(C) K_2O, Cl_2O_7, As_2O_3 and NO

(D) Na_2O, N_2O, Al_2O_3 and CO

Correct Answer: (C) K_2O, Cl_2O_7, As_2O_3 and NO

Solution:

Concept:

- Basic oxides react with acids; acidic oxides react with bases. Amphoteric oxides react with both, whereas neutral oxides show neither ordinary acidic nor basic oxide behaviour.
- Alkali-metal oxides such as K_2O are basic; the high-oxidation-state non-metal oxide Cl_2O_7 is acidic. As_2O_3 is amphoteric and NO is neutral.

Step 1: Classify the four members of option C The required set must contain one representative of each category. In option C the assignments are:

$$\mathrm{K_2O}:\ \text{basic}, \qquad \mathrm{Cl_2O_7}:\ \text{acidic},$$
$$\mathrm{As_2O_3}:\ \text{amphoteric}, \qquad \mathrm{NO}:\ \text{neutral}.$$

Potassium oxide gives an alkaline solution in water, whereas dichlorine heptoxide is the acid anhydride of perchloric acid:

$$\mathrm{K_2O + H_2O \longrightarrow 2\,KOH}, \qquad \mathrm{Cl_2O_7 + H_2O \longrightarrow 2\,HClO_4}.$$

Arsenic(III) oxide can react with acids and with strong bases; nitric oxide is a neutral oxide.

Step 2: Check every competing set In option A, Na_2O and K_2O are both basic, while Al_2O_3 and As_2O_3 are amphoteric; acidic and neutral representatives are absent. In option B, the two amphoteric oxides are accompanied by neutral CO and NO, so no basic or acidic oxide occurs. In option D, Na_2O is basic, Al_2O_3 is amphoteric, and both N_2O and CO are neutral; there is no acidic oxide. The presence of non-metals alone does not make all their oxides acidic.

Step 3: Conclusion K_2O, Cl_2O_7, As_2O_3 and NO represent basic, acidic, amphoteric and neutral oxides, respectively. Hence, the correct option is (C).

Quick Tip to Solve: Treat CO, NO and N2O as the familiar neutral-oxide exceptions; then look for a clearly acidic oxide to eliminate incomplete sets quickly.

59. Given below are two statements :

Statement I: The second ionization enthalpy of B, Al and Ga is in the order of B $>$ Al $>$ Ga.

Statement II: The correct order in terms of first ionization enthalpy is Si $<$ Ge $<$ Pb $<$ Sn.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Correct Answer: (B) Both **Statement I** and **Statement II** are false

Solution:

Concept:

- The second ionization enthalpy removes an electron from a gaseous singly charged ion, so compare the electronic structures of the ions rather than applying a neutral-atom trend mechanically.
- Ionization enthalpy generally falls with increasing size down a group, but poor shielding by filled d and f subshells causes important exceptions in the heavier p-block elements.

Step 1: Evaluate the second-ionization order After the first electron is removed, the relevant configurations are $B^+ : [He]2s^2$, $Al^+ : [Ne]3s^2$ and $Ga^+ : [Ar]3d^{10}4s^2$. The compact boron ion binds its remaining valence electrons strongly. Gallium's intervening filled d subshell shields imperfectly, increasing the effective nuclear attraction compared with the simple down-group prediction. The approximate second ionization enthalpies, in $\mathrm{kJ\,mol^{-1}}$, are

$$\mathrm{B : 2427}, \qquad \mathrm{Al : 1817}, \qquad \mathrm{Ga : 1979}.$$

Hence the order is $\mathrm{B > Ga > Al}$, not $\mathrm{B > Al > Ga}$. Statement I is false.

Step 2: Evaluate the first-ionization order For the group-14 elements, the approximate first ionization enthalpies in $\mathrm{kJ\,mol^{-1}}$ are

$$\mathrm{Si : 786}, \qquad \mathrm{Ge : 762}, \qquad \mathrm{Sn : 709}, \qquad \mathrm{Pb : 716}.$$

The slight increase from tin to lead reflects the stronger effective attraction associated with poor shielding by intervening d and f electrons. Thus the ascending order is

$$\mathrm{Sn < Pb < Ge < Si}.$$

The printed ascending order starts with silicon, although silicon has the largest value of these four, and ends with tin, which has the smallest. Statement II is false.

Step 3: Combine the truth values Since both statements are false, option A cannot hold; option C incorrectly declares I true, and option D incorrectly declares II true. The classification required is the option in which both statements are false.

Step 4: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (B).

Quick Tip to Solve: For these p-block comparisons, check the Al/Ga and Sn/Pb exceptions explicitly; a uniform decrease down the group gives the wrong answer.

60. Given below are two statements :

Statement I : Among Zn, Mn, Sc and Cu, the energy required to remove the third valence electron is highest for Zn and lowest for Sc.

Statement II : The correct order of the following complexes in terms of CFSE is $[Co(H_2O)_6]^{2+} < [Co(H_2O)_6]^{3+} < [Co(en)_3]^{3+}$.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Correct Answer: (A) Both **Statement I** and **Statement II** are true

Solution:

Concept:

- The third ionization removes an electron from M^{2+}. Filled d^{10} and half-filled d^5 configurations resist electron removal, while removal of the lone d electron from Sc^{2+} forms a noble-gas configuration.
- For octahedral complexes, increased metal oxidation state strengthens the crystal field, and ethane-1,2-diamine (en) is a stronger-field ligand than water. CFSE comparisons here refer to stabilization magnitudes.

Step 1: Compare the third ionizations The divalent ions have configurations

$$Zn^{2+} : 3d^{10}, \quad Mn^{2+} : 3d^5, \quad Sc^{2+} : 3d^1, \quad Cu^{2+} : 3d^9.$$

Removing an electron from Zn^{2+} breaks its filled d subshell and requires the greatest energy of this set. In contrast, Sc^{2+} loses its sole d electron to form Sc^{3+} with the stable argon configuration, giving the lowest third ionization enthalpy. The approximate values in $kJ\,mol^{-1}$ are Zn 3833, Cu 3555, Mn 3248 and Sc 2389. These establish both extremes claimed in Statement I, which is true.

Step 2: Compare the cobalt complexes The ligands water and en are neutral, so cobalt has oxidation states +2, +3 and +3 in the three complexes, respectively. Raising the charge from +2 to +3 strengthens metal-ligand attraction and increases octahedral splitting. The aqua Co^{2+} complex is high-spin d^7, with $t_{2g}^5 e_g^2$ and stabilization magnitude $0.8\Delta_o$. The Co^{3+} complexes are d^6; their larger splitting favours the strongly stabilized low-spin t_{2g}^6 arrangement, with orbital stabilization magnitude $2.4\Delta_o$. At the same +3 oxidation state, en produces greater splitting than water. Therefore the stated order of increasing stabilization is correct:

$$[Co(H_2O)_6]^{2+} < [Co(H_2O)_6]^{3+} < [Co(en)_3]^{3+}.$$

Statement II is true.

Step 3: Select the combined verdict Both statements are supported. Option B wrongly rejects both statements; option C wrongly rejects the cobalt-complex comparison; option D wrongly rejects the third-ionization comparison. When CFSE is written as a negative energy, greater stabilization

corresponds to a more negative value; the comparison in the question uses its magnitude.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: For a third ionization, write the divalent-ion configuration first. For complexes, compare metal charge before using the spectrochemical order of the ligands.

61. Which of the following complexes will show coordination isomerism?

A. $[Ag(NH_3)_2][Ag(CN)_2]$

B. $[Co(NH_3)_6][Cr(CN)_6]$

C. $[Co(NH_3)_6][Co(CN)_6]$

D. $[Fe(NH_3)_6][Co(CN)_6]$

E. $[Co(NH_3)_6][Fe(CN)_6]$

Choose the correct answer from the options given below: [4 marks]

(A) B, C and D Only

(B) B, D and E Only

(C) A, C and D Only

(D) C, D and E Only

Correct Answer: (B) B, D and E Only

Solution:

Concept:

- Coordination isomerism concerns salts in which both the cation and the anion are complex ions. Interchanging the ligand sets between their metal centres gives a different coordination compound with the same overall composition.
- Test whether the interchange produces a genuinely different compound. Simply reversing the written order of the same two ions does not produce an isomer.

Step 1: Examine the two salts with identical metal centres In A, the two metal centres are both silver. Exchanging the complete ammonia and cyanide ligand sets simply gives the same pair, $[Ag(NH_3)_2]^+$ and $[Ag(CN)_2]^-$. Likewise, C contains cobalt at both centres: exchanging the complete ligand sets just reproduces the original hexaamminecobalt and hexacyanidocobalt ions. Thus A and C do not give distinct coordination isomers by this interchange.

Step 2: Exchange the ligand sets in the other salts B has cobalt and chromium centres, so its interchange gives $[Cr(NH_3)_6][Co(CN)_6]$, distinct from $[Co(NH_3)_6][Cr(CN)_6]$. D and E similarly contain iron and cobalt: $[Fe(NH_3)_6][Co(CN)_6]$ and $[Co(NH_3)_6][Fe(CN)_6]$ have the same overall composition but different allocations of ligands to the two metals. They form the required coordination-isomer pair.

Step 3: Compare the complete selection with the options The qualifying entries are B, D and E. Option (A) incorrectly includes C and omits E; option (C) includes both A and C; option (D) includes C and omits B. Only option (B) lists precisely the three qualifying complexes.

Step 4: Conclusion Complexes B, D and E show coordination isomerism. Hence, the correct option is

(B).

Quick Tip to Solve: Exchange the full ligand sets and rewrite the resulting ions: a mere reversal of the same cation and anion is not a new isomer.

62. Complete combustion of X g of an organic compound gave 0.25 g of CO_2 and 0.12 g of H_2O. If the % of carbon is 25% and of hydrogen is 4.89%, then $X =$ ___ $\times 10^{-3}$ g. (Nearest integer)
(Molar mass of C, H and O are 12, 1 and 16 g mol^{-1} respectively.) [4 marks]

(A) 273

(B) 27

(C) 2730

(D) 227

Correct Answer: (A) 273

Solution:

Concept:

- Complete combustion converts all carbon in the sample to CO_2, and all hydrogen to H_2O. Use the mass fraction of the element in its combustion product.
- For sample mass X, an elemental mass percentage p means $m_{\text{element}} = (p/100)X$. Convert the final sample mass into the requested multiple of 10^{-3} g.

Step 1: Calculate the carbon mass The molar mass of CO_2 is $12 + 2(16) = 44\,\text{g mol}^{-1}$. Therefore the carbon fraction in carbon dioxide is $12/44$, not $1/44$. The carbon originally present in the sample has mass

$$m_C = 0.25 \times \frac{12}{44} = \frac{3}{44} = 0.0681818\,\text{g}.$$

Step 2: Use the carbon percentage to find the sample mass Carbon constitutes 25 percent of the original sample, so

$$0.25X = 0.0681818, \qquad X = 0.272727\,\text{g}.$$

Expressing this in the requested form gives

$$X = 272.727 \times 10^{-3}\,\text{g} \simeq 273 \times 10^{-3}\,\text{g}.$$

The nearest integer is therefore 273.

Step 3: Check the hydrogen data and reject the alternatives The hydrogen mass is $0.12(2/18) = 0.0133333\,\text{g}$. With the calculated sample mass, its percentage is $100(0.0133333/0.272727) = 4.8889\%$, matching the printed 4.89% after rounding. Options (B) and (C) differ by factors of ten from the required coefficient. Option (D), 227, would not reproduce the given carbon percentage.

Step 4: Conclusion The sample mass is approximately $273 \times 10^{-3}\,\text{g}$; the requested integer is 273. Hence, the correct option is (A).

Quick Tip to Solve: First recover the element mass from its combustion product, then divide by its fractional percentage; do not treat the product mass as the sample mass.

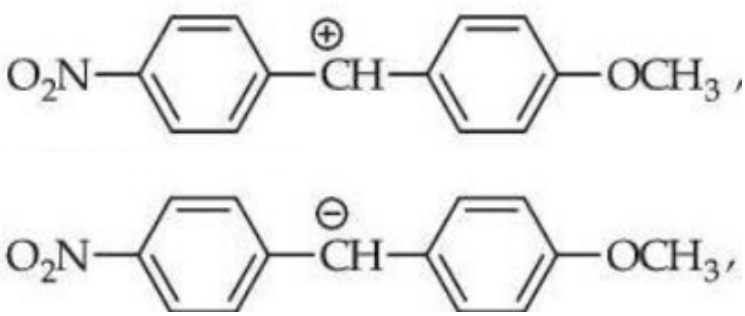
63. Given below are two statements :

Statement I : In , the carbocation is stabilised by +R effect of –OCH_3 group.

Statement II : In , the carbanion is stabilised by –R effect of –NO_2 group.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]



(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Correct Answer: (A) Both **Statement I** and **Statement II** are true

Solution:

Concept:

- A para substituent communicates with a benzylic charged centre through the conjugated aromatic ring. Resonance can spread the charge over a larger part of the molecule.
- The methoxy group, $-\mathrm{OCH_3}$, donates an oxygen lone pair by the $+R$ effect. The nitro group, $-\mathrm{NO_2}$, withdraws electron density by the $-R$ effect.

Step 1: Test Statement I for the carbocation The positively charged central carbon is bonded to two aromatic rings. On the methoxy-substituted ring, the methoxy group is para to the benzylic centre, so its lone-pair donation is conjugated with the electron-deficient carbon. Resonance donation from oxygen through the ring supplies electron density toward this centre and allows contributors in which the original positive charge is dispersed. Thus the stated stabilising $+R$ effect of $-\mathrm{OCH_3}$ is present, and Statement I is true.

Step 2: Test Statement II for the carbanion The negatively charged central carbon has a lone pair that can enter conjugation with the aromatic rings. On the nitro-substituted ring, delocalisation carries electron density toward the para nitro group. Its $-R$ effect allows the excess electron density to be accommodated in resonance contributors involving the electronegative oxygen atoms. This stabilises the carbanion, so Statement II is also true. The presence of the other substituent does not remove either of the stated resonance interactions.

Step 3: Select the truth-value combination Both statements identify the appropriate direction of electron movement: donation stabilises an electron-deficient carbocation, while withdrawal stabilises an electron-rich carbanion. Option (B) wrongly rejects both interactions; (C) wrongly rejects nitro stabilisation; (D) wrongly rejects methoxy stabilisation.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: For a benzylic charge, check conjugation and substituent position first; para donors favour cations, while para resonance-withdrawing groups favour anions.

64. The compound (X) on

(i) on heating in the presence of anhydrous $AlCl_3$ and HCl gas gives 2,4-dimethyl pentane

(ii) aromatization gives toluene and

(iii) cyclisation gives methyl cyclohexane

The correct name of compound (X) is: [4 marks]

(A) Hept-2-ene

(B) Hept-1,3,5-triene

(C) Heptane

(D) Hept-2,4,6-triene

Correct Answer: (C) Heptane

Solution:

Concept:

- Skeletal isomerisation of an alkane changes its carbon skeleton while preserving its molecular formula. Anhydrous $AlCl_3$ with HCl promotes the conversion of straight-chain alkanes into branched isomers.
- A seven-carbon straight-chain alkane can undergo cyclisation to a methyl-substituted six-membered ring and further dehydrogenation to the corresponding aromatic hydrocarbon.

Step 1: Use the isomerisation product to identify the formula The product 2,4-dimethylpentane contains seven carbon atoms and is an acyclic saturated hydrocarbon, C_7H_{16}. The first reaction is skeletal isomerisation, so X must also have formula C_7H_{16}. Among the options, only heptane has this formula. Hept-2-ene has C_7H_{14}, while the two listed trienes have C_7H_{10}; their conversion to this alkane would require hydrogenation as well.

Step 2: Verify the cyclisation and aromatisation clues Joining the appropriate chain positions of normal heptane yields a six-membered ring with one methyl substituent. The net cyclisation is

$$C_7H_{16} \longrightarrow C_7H_{14} + H_2,$$

where C_7H_{14} is methylcyclohexane. Further loss of three hydrogen molecules produces toluene. The overall aromatisation is

$$C_7H_{16} \longrightarrow C_7H_8 + 4\,H_2.$$

Both transformations retain the seven carbon atoms and agree with the stated products.

Step 3: Reject the unsaturated choices Option (A) is an alkene rather than the alkane required by the first clue. Options (B) and (D) describe trienes and likewise fail the unchanged molecular-formula test for isomerisation. Satisfying only an aromatisation possibility is insufficient: X must meet all three

clues simultaneously.

Step 4: Conclusion Compound X is heptane, $CH_3(CH_2)_5CH_3$. Hence, the correct option is (C).

Quick Tip to Solve: When several transformations identify one hydrocarbon, start with the reaction that preserves its molecular formula; skeletal isomerisation gives the quickest filter.

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65. Correct statements regarding alkyl halides (R–X) among the following are :

A. Alcohol being less polar solvent as compared to water, alcoholic KOH favours elimination reaction with R–X.

B. Order of reactivity towards S_N1 mechanism is C_6H_5–CH_2–Cl > C_6H_5–CHCl–C_6H_5.

C. Non substituted aryl halides exhibit properties similar to alkyl halides.

D. Vinyl chloride is an example of haloalkene and allyl chloride is an example of haloalkyne.

E. R–Cl can be prepared by reacting R–OH with $SOCl_2$ but Ar–Cl cannot be prepared by reacting Ar–OH with $SOCl_2$.

Choose the correct answer from the options given below : [4 marks]

(A) A, B and C Only

(B) B and D Only

(C) A and E Only

(D) D and E Only

Correct Answer: (C) A and E Only

Solution:

Concept:

- Alcoholic KOH favours removal of a beta hydrogen and elimination of halide, whereas aqueous hydroxide commonly favours substitution in suitable alkyl halides.
- An S_N1 reaction is controlled by carbocation formation. Resonance stabilisation increases carbocation stability; an aryl carbon–halogen bond behaves differently from an ordinary alkyl carbon–halogen bond.

Step 1: Assess statements A and B A is correct: the less polar alcoholic medium favours the basic behaviour associated with beta elimination, giving an alkene from a suitable alkyl halide. B is false. Benzhydryl chloride, $C_6H_5CHClC_6H_5$, gives a carbocation stabilised by two phenyl rings, whereas benzyl chloride gives one stabilised by one phenyl ring. The benzhydryl compound therefore reacts faster by S_N1, opposite to the printed inequality.

Step 2: Assess statements C and D C is false. In an aryl halide the halogen is attached directly to an sp^2 aromatic carbon; resonance gives the carbon–halogen bond partial double-bond character. Such unsubstituted aryl halides resist the ordinary substitution reactions of alkyl halides. D is false: vinyl chloride is $CH_2{=}CHCl$, while allyl chloride is $CH_2{=}CHCH_2Cl$. Both contain a carbon–carbon double bond; allyl chloride contains no triple bond and is not a haloalkyne.

Step 3: Assess E and select the set E is correct. Thionyl chloride converts an alcohol into an alkyl

chloride:

$$\mathrm{ROH} + \mathrm{SOCl_2} \longrightarrow \mathrm{RCl} + \mathrm{SO_2} + \mathrm{HCl}.$$

Phenol does not undergo the analogous ordinary replacement of its aryl hydroxyl group by $\mathrm{SOCl_2}$. Thus only A and E are true. Options (A), (B) and (D) each include at least one of the false statements B, C or D.

Step 4: Conclusion The correct statements are A and E only. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish the carbon bearing halogen from nearby unsaturation: allylic halides have an alkyl C–X bond next to a double bond, while aryl halides have X directly on the ring.

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66. An organic compound "x" where molar ratio of C, O and H are equal, on treatment with 50% KOH under reflux followed by acidification produced "y". The most likely structure of "y" is:
[Molar mass of 'x' is 58 g mol^{-1}] [4 marks]

(A) $\mathrm{CH_2{=}CHCOOH}$

(B) $\mathrm{CH_3CH{=}CHCHO}$

(C) $\mathrm{HOCH_2COOH}$

(D) $\mathrm{CH_3COOH}$

Correct Answer: (C) $\mathrm{HOCH_2COOH}$

Solution:

Concept:

- Equal molar amounts of carbon, hydrogen and oxygen give the empirical formula CHO, whose formula mass is $12 + 1 + 16 = 29\,\mathrm{g\,mol^{-1}}$.
- Aldehydes without an alpha hydrogen undergo the Cannizzaro reaction in concentrated alkali: one aldehyde group is oxidised to carboxylate while another is reduced to a primary alcohol. Acidification converts carboxylate to the carboxylic acid.

Step 1: Determine the molecular formula and reactive structure The molecular mass is $58\,\mathrm{g\,mol^{-1}}$, twice the empirical-formula mass. Hence

$$n = \frac{58}{29} = 2, \qquad x:\ \mathrm{C_2H_2O_2}.$$

The dialdehyde glyoxal, $\mathrm{OHC{-}CHO}$, has this formula and contains two adjacent aldehyde groups. There is no enolisable alpha carbon bearing hydrogen; the aldehydic hydrogens are attached to carbonyl carbons and are not alpha hydrogens.

Step 2: Apply disproportionation in concentrated alkali In 50 percent KOH, glyoxal undergoes an intramolecular Cannizzaro reaction. Hydroxide addition at one aldehyde group is followed by hydride transfer to the other, so one carbon becomes carboxylate and the other becomes $\mathrm{CH_2OH}$:

$$\mathrm{OHC{-}CHO} + \mathrm{KOH} \longrightarrow \mathrm{HOCH_2COOK}.$$

Acidification then gives

$$HOCH_2COO^- + H^+ \longrightarrow HOCH_2COOH.$$

Thus y is glycolic acid, also called 2-hydroxyethanoic acid.

Step 3: Identify the displayed product and reject the others Option (C) displays $HOCH_2COOH$, with the alcohol and acid groups required by the disproportionation. Option (A), acrylic acid, has three carbon atoms; option (B), but-2-enal, has four. Neither fits this two-carbon intramolecular transformation. Option (D), acetic acid, lacks the hydroxyl group produced by reduction of the second aldehyde group.

Step 4: Conclusion The product y is glycolic acid, $HOCH_2COOH$, the structure shown in option (C). Hence, the correct option is (C).

Quick Tip to Solve: Use the empirical-formula multiplier before choosing a carbonyl compound; for glyoxal in concentrated alkali, track one aldehyde becoming acid and the other becoming alcohol.

67. A molecule (X) with following structure under mild acidic condition is hydrolysed to produce (Y) and (Z). Identify the correct statements about (Y) and (Z).

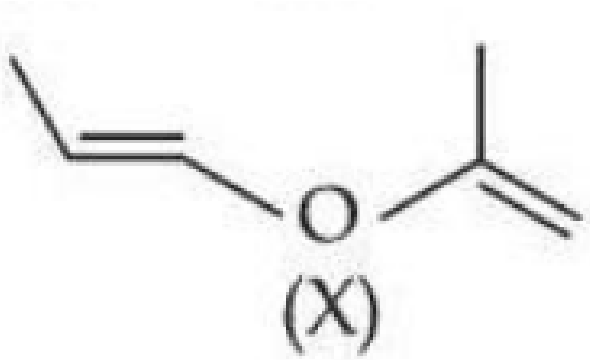
A. Both (Y) and (Z) have same molar mass.

B. (Y) and (Z) can be distinguished from each other by $NaHCO_3$.

C. (Y) and (Z) react with HCN with same rates.

D. (Y) and (Z) undergo addition reaction with 2,4-DNP.

Choose the correct answer from the options given below : [4 marks]



(X)

(A) A, B and C Only

(B) B and C Only

(C) C and D Only

(D) A and D Only

Correct Answer: (D) A and D Only

Solution:

Concept:

- An enol ether undergoes acid-catalysed hydrolysis. The resulting enols tautomerise to the more stable aldehyde or ketone.
- Aldehydes and ketones undergo nucleophilic addition at the carbonyl carbon. Aldehydes generally react faster than ketones because they are less hindered and less electron-rich.

Step 1: Identify the two carbonyl products The left-hand fragment of X is $CH_3CH{=}CH{-}O^-$, while the right-hand fragment is $-O-C(CH_3){=}CH_2$. Hydrolysis gives the corresponding enolic fragments, which furnish propanal and propanone (acetone):

$$CH_3CH{=}CHOH \longrightarrow CH_3CH_2CHO, \qquad CH_2{=}C(OH)CH_3 \longrightarrow CH_3COCH_3.$$

Thus Y and Z are an aldehyde and a ketone with the same molecular formula C_3H_6O; interchanging their labels does not change any statement.

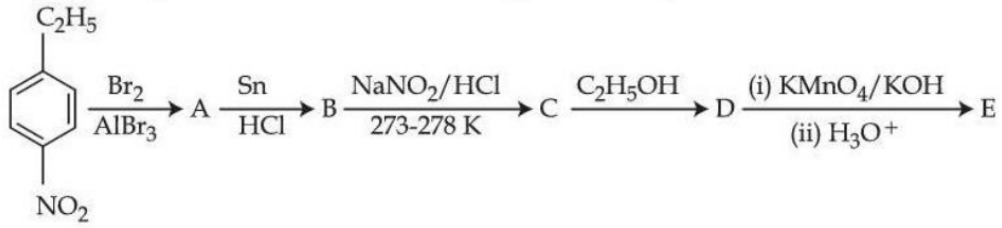
Step 2: Test statements A and B Each product has molar mass $3(12) + 6(1) + 16 = 58\,\mathrm{g\,mol^{-1}}$, so A is true. Neither product contains a carboxylic acid group. Neither liberates carbon dioxide from aqueous $NaHCO_3$; consequently this reagent cannot distinguish them, and B is false.

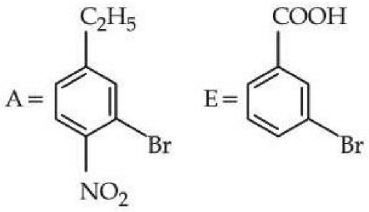
Step 3: Test statements C and D Propanal reacts with HCN faster than propanone. Its carbonyl carbon bears only one alkyl group, giving less steric obstruction and a smaller electron-donating effect than the two alkyl groups of propanone. C is therefore false. Both carbonyl compounds react with 2,4-dinitrophenylhydrazine: nucleophilic addition is followed by loss of water to give the corresponding hydrazone. Hence D is true in the carbonyl-reaction sense used here. Options (A) and (B) include the false statements B and C; option (C) also incorrectly includes C.

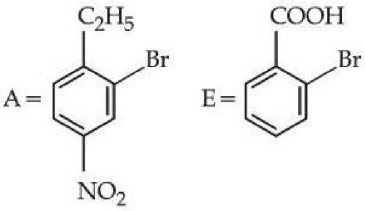
Step 4: Conclusion Only statements A and D are correct. Hence, the correct option is (D).

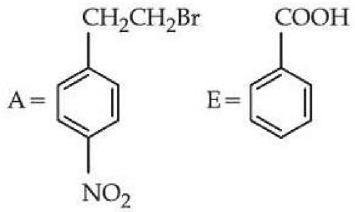
Quick Tip to Solve: Convert each enol fragment into its carbonyl tautomer first; then compare molecular formula, acidity and carbonyl reactivity separately.

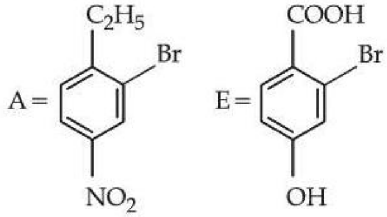
68. Identify compounds A and E in the following reaction sequence. [4 marks]



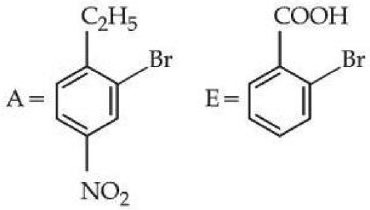
(A) 

(B) 

(C) 

(D) 

Correct Answer: (B)



Solution:

Concept:

- An ethyl group is ortho/para directing, whereas a nitro group is meta directing. Positions favoured by both groups determine ring bromination with $Br_2/AlBr_3$.

- Sn/HCl reduces an aromatic nitro group to an amino group. Diazotisation followed by ethanol replaces that amino group by hydrogen; hot alkaline permanganate oxidises an alkyl side chain having a benzylic hydrogen to a carboxyl group.

Step 1: Locate the bromine in A Number the carbon bearing the ethyl group as 1 and the carbon bearing the nitro group as 4. The ethyl group favours positions 2 and 6, and these same positions are meta to the nitro group. They are equivalent by symmetry. Bromination therefore gives 2-bromo-1-ethyl-4-nitrobenzene. This is the A structure in options (B) and (D). The bromine in option (A) is wrongly placed meta to ethyl and ortho to nitro; option (C) wrongly shows side-chain bromination.

Step 2: Follow the amino and diazonium intermediates Reduction by Sn/HCl changes $-NO_2$ at position 4 into $-NH_2$, giving B. Treatment with $NaNO_2$/HCl at 273–278 K forms the corresponding arenediazonium salt C. Ethanol reduces the diazonium group to hydrogen, with nitrogen evolved and ethanol oxidised to ethanal. The aryl bromine and ethyl group remain, so D is ortho-bromoethylbenzene. Ethanol in this sequence does not introduce a phenolic hydroxyl group.

Step 3: Oxidise the remaining side chain $KMnO_4$/KOH oxidises the ethyl substituent, which has benzylic hydrogen, to a carboxylate group. Acidification with H_3O^+ yields 2-bromobenzoic acid. Ring bromine remains ortho to the new $-COOH$ group. Option (D) incorrectly retains an additional $-OH$ group, and option (C) lacks the ring bromine. Only option (B) has both the correct brominated precursor and the correct final acid.

Step 4: Conclusion A is 2-bromo-1-ethyl-4-nitrobenzene and E is 2-bromobenzoic acid. Hence, the correct option is (B).

Quick Tip to Solve: Track ring positions throughout the sequence: diazonium plus ethanol removes the nitrogen substituent, while permanganate changes the ethyl side chain into carboxyl.

69. Identify the correct pair having amino acid (A) and the hormone (B) that is iodinated derivative of the amino acid (A).

(T and Y represent one letter code for amino acids) [4 marks]

(A) Amino acid (A) - T Hormone (B) - Insulin

(B) Amino acid (A) - T Hormone (B) - Thyroxine

(C) Amino acid (A) - Y Hormone (B) - Thyroxine

(D) Amino acid (A) - Y Hormone (B) - Insulin

Correct Answer: (C) Amino acid (A) - Y Hormone (B) - Thyroxine

Solution:

Concept:

- The one-letter amino-acid code Y denotes tyrosine, whereas T denotes threonine. A one-letter code is a conventional identifier and is not necessarily the first letter of the amino-acid name.
- Thyroxine is an iodine-containing thyroid hormone derived from tyrosine residues. Insulin is a peptide hormone consisting of amino-acid chains, not an iodinated derivative of a single amino acid.

Step 1: Identify the relevant amino acid Tyrosine is the aromatic amino acid containing a para-hydroxyphenyl side chain. Its condensed structure is shown below, with hydroxyl para to the side chain:

$$HO-C_6H_4-CH_2-CH(NH_2)-COOH.$$

This phenolic aromatic group supplies the framework involved in thyroid-hormone formation. The code for tyrosine is Y. The code T instead means threonine, whose side chain is $-CH(OH)CH_3$; it is not the precursor specified here.

Step 2: Identify the iodinated hormone Iodination of tyrosine residues and coupling of iodinated residues give the thyroid hormones. Thyroxine, also called T_4, contains four iodine atoms. The notation T_4 names the hormone and must not be confused with the single-letter amino-acid code T. Insulin is structurally a peptide hormone, with two polypeptide chains connected by disulfide bonds, so it does not satisfy the description of an iodinated tyrosine derivative.

Step 3: Check each proposed pair Option (A) combines threonine with insulin, so both identifications fail. Option (B) names the correct hormone but gives the wrong amino-acid code T. Option (D) correctly chooses tyrosine by its code Y but pairs it with insulin, which is not iodinated tyrosine. Option (C) alone gives Y for tyrosine and thyroxine for the hormone, satisfying both requirements simultaneously.

Step 4: Conclusion The required pair is Y (tyrosine) and thyroxine. Hence, the correct option is (C).

Quick Tip to Solve: Remember Y for tyrosine and T for threonine; do not read the hormone abbreviation T_4 as an amino-acid code.

70. Among Fe^{2+}, Fe^{3+}, Cr^{2+} and Zn^{2+}, the ion that shows positive borax bead test and with highest ionisation enthalpy is : [4 marks]

(A) Fe^{2+}

(B) Zn^{2+}

(C) Cr^{2+}

(D) Fe^{3+}

Correct Answer: (D) Fe^{3+}

Solution:

Concept:

- A positive borax bead test is associated with formation of a coloured borate bead by suitable metal ions. Iron and chromium give characteristic coloured beads; zinc does not give the characteristic coloured test.
- For an ion, ionisation enthalpy refers to removing another electron from that ion in the gaseous state. A larger positive charge and a particularly stable electronic configuration make further electron removal harder.

Step 1: Write the ionic configurations Remove the $4s$ electrons before $3d$ electrons in forming these

ions. Their configurations are

$$\mathrm{Fe}^{2+} : [\mathrm{Ar}]\, 3d^6, \quad \mathrm{Fe}^{3+} : [\mathrm{Ar}]\, 3d^5,$$
$$\mathrm{Cr}^{2+} : [\mathrm{Ar}]\, 3d^4, \quad \mathrm{Zn}^{2+} : [\mathrm{Ar}]\, 3d^{10}.$$

In particular, ferric ion has a half-filled $3d$ subshell, whereas zinc ion has a fully filled one.

Step 2: Apply the bead-test requirement Iron and chromium compounds can produce coloured borax beads. Zinc ion has a filled $3d^{10}$ configuration and does not produce the characteristic coloured bead required here, so option (B) is eliminated. The test alone does not distinguish the three remaining candidates; the ionisation-enthalpy condition must also be used.

Step 3: Compare further electron removal Removing an electron from Fe^{3+} changes its stable half-filled $3d^5$ configuration to $3d^4$, while acting against the attraction of a triply charged ion. It requires more energy than removal from either divalent iron or divalent chromium. Removal from Fe^{2+} actually produces the stable $3d^5$ ferric ion, so it is easier than removing the next electron from ferric ion. Cr^{2+} has neither that tripositive charge nor a half-filled configuration to resist removal. Thus options (A) and (C) fail the highest-enthalpy condition, leaving ferric ion.

Step 4: Conclusion Fe^{3+} satisfies both requirements. Hence, the correct option is (D).

Quick Tip to Solve: Compare ionisation of the ions actually listed, not first ionisation of neutral atoms; the stable $3d^5$ configuration of Fe^{3+} is decisive.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The surface of sodium metal is irradiated with radiation of wavelength x nm. The kinetic energy of ejected electrons is 2.8×10^{-20} J. The work function of sodium is 2.3 eV. The value of x is _____ $\times 10^2$ nm. (Nearest integer)

(Given: $h = 6.6 \times 10^{-34}$ J s ; 1 eV $= 1.6 \times 10^{-19}$ J ; $c = 3.0 \times 10^8$ m s^{-1}) [4 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $hc/\lambda = \phi + K_{\max}$, where ϕ is the work function and $K_{\max}$ is the kinetic energy of an emitted electron.
- All energies must use the same unit before addition. With h in joule seconds and c in metres per second, the calculated wavelength is in metres; $1\,\mathrm{nm} = 10^{-9}\,\mathrm{m}$.

Step 1: Convert the work function into joules The work function is the minimum energy spent releasing an electron from the metal. Using the given conversion,

$$\phi = 2.3 \times 1.6 \times 10^{-19} = 3.68 \times 10^{-19}\,\mathrm{J}.$$

The stated kinetic energy is additional energy retained by the liberated electron, so it must be added to the work function rather than subtracted.

Step 2: Calculate the incident photon energy Express the kinetic energy with the same power of ten:

$$K_{\max} = 2.8 \times 10^{-20} = 0.28 \times 10^{-19}\,\text{J}.$$

Hence the incident photon energy is

$$E = \phi + K_{\max} = (3.68 + 0.28) \times 10^{-19} = 3.96 \times 10^{-19}\,\text{J}.$$

This exceeds the threshold energy, consistently with emission of electrons having positive kinetic energy.

Step 3: Find the wavelength and requested coefficient Substitute the supplied constants without changing their specified values:

$$\lambda = \frac{hc}{E} = \frac{(6.6 \times 10^{-34})(3.0 \times 10^{8})}{3.96 \times 10^{-19}} = 5.0 \times 10^{-7}\,\text{m}.$$

Converting to nanometres gives $\lambda = 500\,\text{nm} = 5 \times 10^{2}\,\text{nm}$. The response blank asks for the coefficient multiplying $10^{2}\,\text{nm}$, not the wavelength in unscaled nanometres; that coefficient is already an integer.

Step 4: Conclusion The incident wavelength is $500\,\text{nm} = 5 \times 10^{2}\,\text{nm}$. The required nearest-integer entry is $\boxed{5}$.

Quick Tip to Solve: Add work function and kinetic energy in joules, then convert the wavelength to the exact power-of-ten unit requested in the answer blank.

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72. Consider the following gas phase reaction being carried out in a closed vessel at 25°C.

$$2\,\text{A}\,(\text{g}) \longrightarrow 4\,\text{B}\,(\text{g}) + \text{C}\,(\text{g})$$

Time (min)	Total pressure of the system (mm Hg)
30	300
∞	600

The pressure of C (g) at 30 minutes time interval would be _____ mm Hg. (nearest integer)

[4 marks]

Solution:

Concept:

- At fixed temperature and vessel volume, the partial pressure of each ideal gas is proportional to its amount. Reaction stoichiometry may therefore be applied directly to pressure changes.
- For $2\,\text{A}\,(\text{g}) \longrightarrow 4\,\text{B}\,(\text{g}) + \text{C}\,(\text{g})$, formation of C with partial pressure p consumes A by pressure $2p$ and forms B with pressure $4p$. Dalton's law gives total pressure as the sum of partial pressures.

Step 1: Express all partial pressures using one variable Let the initial pressure of reactant A be P_0, with products initially absent. If the partial pressure of C at some time is p, the stoichiometric pressure

relations are

$$P_A = P_0 - 2p, \qquad P_B = 4p, \qquad P_C = p.$$

Consequently,

$$P_{\text{total}} = (P_0 - 2p) + 4p + p = P_0 + 3p.$$

The total pressure increases because two reactant molecules generate five product molecules; the net increase is three per molecule of C formed.

Step 2: Use the completion pressure to find the initial pressure At infinite time the reaction is complete, so A is exhausted and $P_0 - 2p_\infty = 0$. Thus $p_\infty = P_0/2$, giving

$$600 = P_\infty = P_0 + 3\left(\frac{P_0}{2}\right) = \frac{5}{2}P_0.$$

It follows that $P_0 = 240\,\text{mm Hg}$. This determination uses the final gas-mole ratio, rather than a kinetic rate law.

Step 3: Calculate and check the pressure of C at 30 minutes The total pressure at 30 minutes is $300\,\text{mm Hg}$. Therefore

$$300 = 240 + 3p_{30}, \qquad p_{30} = \frac{60}{3} = 20\,\text{mm Hg}.$$

As a check, $P_A = 240 - 40 = 200$, $P_B = 80$, and $P_C = 20$, all in mm Hg; their sum is the given pressure 300. No reaction order or rate constant is needed.

Step 4: Conclusion The partial pressure of C at 30 minutes is $\boxed{20\,\text{mm Hg}}$, so the nearest-integer entry is 20.

Quick Tip to Solve: First recover the initial pressure from the completion mole ratio; the increase in total pressure is three times the partial pressure of C.

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73. Consider the following two half-cell reactions along with the standard reduction potential given :

$$CO_2 + 6H^+ + 6e^- \longrightarrow CH_3OH + H_2O \qquad E^\circ_{\text{red}} = 0.02\ \text{V}$$

$$\frac{1}{2}O_2 + 2H^+ + 2e^- \longrightarrow H_2O \qquad E^\circ_{\text{red}} = 1.23\ \text{V}$$

A fuel cell was set up using the above two reactions such that the cell operates under the standard condition of 1 bar pressure and 298 K temperature. The fuel cell works with 80% efficiency. If the work derived from the cell using 1 mol of CH_3OH is used to compress an ideal gas isothermally against a constant pressure of 1 kPa, then the change in the volume of the gas,

$\Delta V =$ _____ **m^3. (nearest integer)**

Given : $F = 96500\ \text{C mol}^{-1}$ [4 marks]

Solution:

Concept:

- A fuel cell oxidises its fuel at the anode and reduces oxygen at the cathode. Its standard emf is the cathode reduction potential minus the anode reduction potential.
- The maximum electrical work available from one mole of reaction is nFE°_{cell}, where n is the number of electrons transferred and F is the Faraday constant. The usable work is this value multiplied by the efficiency.
- For compression against constant external pressure, the positive work done on the gas is $w_{\text{on}} = P_{\text{ext}}(V_i - V_f)$. Thus the volume decrease is work divided by pressure; the signed change $V_f - V_i$ is negative.

Step 1: Identify the cell reaction and electron transfer Reverse the methanol reduction half-reaction to oxidise methanol, and multiply the oxygen reduction half-reaction by three to consume the six electrons:

$$\mathrm{CH_3OH + H_2O \longrightarrow CO_2 + 6\,H^+ + 6\,e^-}$$

$$\frac{3}{2}\,\mathrm{O_2 + 6\,H^+ + 6\,e^- \longrightarrow 3\,H_2O}.$$

Their sum is $\mathrm{CH_3OH} + \frac{3}{2}\,\mathrm{O_2} \longrightarrow \mathrm{CO_2} + 2\,\mathrm{H_2O}$. Therefore one mole of methanol transfers $n = 6$ moles of electrons. Electrode potentials are intensive quantities, so multiplying a half-reaction does not multiply its potential.

Step 2: Calculate the usable electrical work Using the two potentials exactly as given,

$$E^\circ_{\text{cell}} = 1.23 - 0.02 = 1.21\ \text{V}.$$

The ideal work available and the actual usable work are

$$W_{\max} = 6(96500)(1.21) = 700590\ \text{J},$$
$$W_{\text{use}} = 0.80(700590) = 560472\ \text{J}.$$

This positive available energy supplies the work done on the gas. The fuel cell's own Gibbs-energy change has the opposite sign.

Step 3: Convert the work into a volume decrease Since $1\ \text{kPa} = 1000\ \text{Pa}$ and $1\ \text{Pa}\,\text{m}^3 = 1\ \text{J}$,

$$V_i - V_f = \frac{560472}{1000} = 560.472\ \text{m}^3.$$

Constant external pressure gives $P_{\text{ext}}\Delta V$ work even though the gas remains isothermal; a reversible logarithmic-work formula is not applicable to this specified constant-pressure compression.

Step 4: Conclusion The gas volume decreases by $560.472\ \text{m}^3$, whose nearest-integer magnitude is $\boxed{560\ \text{m}^3}$. The numerical entry is $\boxed{560}$ when the blank denotes the amount of compression. With the signed convention $\Delta V = V_f - V_i$, the change is $-560.472\ \text{m}^3$, or $-560\ \text{m}^3$ to the nearest integer.

Quick Tip to Solve: Count six electrons per methanol, apply the efficiency once, and convert kPa to Pa before dividing energy by pressure. Distinguish a positive volume decrease from a negative signed volume change.

74. Number of paramagnetic ions among the following d- and f-block metal ions is _____.

Mn^{2+}, Cu^{2+}, Zn^{2+}, Yb^{2+}, Sc^{3+}, La^{3+}, Gd^{3+}, Lu^{3+}, Ti^{4+}, Ce^{4+}

(Atomic number of Mn = 25, Cu = 29, Zn = 30, Yb = 70, Sc = 21, La = 57, Gd = 64, Lu = 71, Ti = 22, Ce = 58) [4 marks]

Solution:

Concept:

- An ion is paramagnetic if its electronic configuration contains one or more unpaired electrons. An ion with only closed shells or filled subshells is diamagnetic.
- In forming transition-metal cations, remove the outer ns electrons before the $(n-1)d$ electrons. For lanthanoid ions, track the $6s$, $5d$ and $4f$ occupations rather than subtracting electrons blindly from $4f$.
- Hund's rule gives five unpaired electrons in an isolated d^5 ion, one in d^9, and seven in f^7. Empty d^0 or f^0, and filled d^{10} or f^{14}, subshells contribute no unpaired electrons.

Step 1: Classify the d-block ions Manganese loses its two $4s$ electrons to form Mn^{2+}, with configuration $[Ar]3d^5$. Its five singly occupied d orbitals make it paramagnetic. Copper loses its $4s$ electron and one $3d$ electron to give Cu^{2+}, $[Ar]3d^9$, with one unpaired electron. Zinc forms Zn^{2+}, $[Ar]3d^{10}$, whose d shell is completely filled and hence diamagnetic.

Scandium loses $4s^23d^1$ to form Sc^{3+}, $[Ar]$. Titanium loses $4s^23d^2$ to form Ti^{4+}, also $[Ar]$. Both are d^0 and diamagnetic. The d-block portion therefore supplies exactly two paramagnetic ions.

Step 2: Classify the lanthanoid ions Yb^{2+} retains the filled $[Xe]4f^{14}$ configuration after losing its two $6s$ electrons, so it is diamagnetic. La^{3+} loses $6s^25d^1$, giving the closed-shell configuration $[Xe]$, and is also diamagnetic.

Neutral gadolinium has $[Xe]4f^75d^16s^2$. Removing three electrons produces Gd^{3+} with $[Xe]4f^7$. Its seven unpaired f electrons make it paramagnetic. In contrast, Lu^{3+} has $[Xe]4f^{14}$, with all f electrons paired, while Ce^{4+} has the closed-shell configuration $[Xe]$. These last two ions are diamagnetic.

Step 3: Count only ions with unpaired electrons The complete paramagnetic list is Mn^{2+}, Cu^{2+} and Gd^{3+}. The other seven ions have empty or completely filled relevant subshells. The number of unpaired electrons is different for the three magnetic ions, but the question asks for the number of ions, so each is counted once.

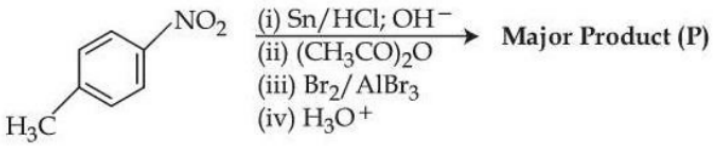
Step 4: Conclusion There are $\boxed{3}$ paramagnetic ions: Mn^{2+}, Cu^{2+} and Gd^{3+}. The answer is a dimensionless count.

Quick Tip to Solve: Write the ionic configuration before deciding magnetism. In particular, distinguish diamagnetic Yb^{2+} with $4f^{14}$ from paramagnetic Gd^{3+} with $4f^7$.

75. Consider the following reactions sequence

When the product (P) is subjected to Carius analysis using $AgNO_3$, 1.0 g of the product (P) will produce _____ g of the precipitate of AgBr. (Nearest Integer)

(Given : molar mass in g mol^{-1} C : 12, H : 1, O : 16, N : 14, Br : 80, Ag : 108) [4 marks]



Solution:

Concept:

- Sn/HCl reduces an aromatic nitro group to an amino group; alkaline work-up gives the free amine. Acetic anhydride then protects the amino group as an acetamide.
- An acetamido substituent directs electrophilic substitution to its ortho and para positions. Acidic hydrolysis subsequently removes the acetyl protecting group without removing the aromatic bromine substituent.
- In Carius halogen analysis, each mole of bromine atoms in the organic sample gives one mole of $AgBr$. Thus the precipitate mass follows from the molecular formula and the number of bromine atoms per molecule.

Step 1: Follow reduction and amino-group protection The starting compound is para-nitrotoluene. The first operation changes $-NO_2$ to $-NH_2$, leaving the methyl group unchanged, and the basic work-up gives para-toluidine. Treatment with $(CH_3CO)_2O$ converts the amino group into $-NHCOCH_3$, producing para-methylacetanilide. This protection controls the strong activation caused by a free amino group during bromination.

Step 2: Locate bromination and remove the protecting group The position para to $-NHCOCH_3$ is already occupied by the methyl group. Therefore bromination occurs at an ortho position relative to the acetamido group. The two such positions are equivalent by symmetry in this para-disubstituted starting ring, so they lead to the same monobrominated major product.

The final H_3O^+ treatment hydrolyses the acetamide back to the amino group. Product P is consequently 2-bromo-4-methylaniline, containing one bromine atom per molecule. Its formula is C_7H_8BrN: the ring contributes C_6H_3, the methyl group contributes CH_3, and the amino group contributes NH_2.

Step 3: Calculate the mass of silver bromide Using the supplied atomic masses, the molar masses are

$$M(P) = 7(12) + 8(1) + 80 + 14 = 186\ \text{g mol}^{-1},$$
$$M(\text{AgBr}) = 108 + 80 = 188\ \text{g mol}^{-1}.$$

A 1.0 g sample contains $1/186$ mole of P. Each molecule contains one bromine atom, so the same number of moles of $AgBr$ precipitates:

$$m(\text{AgBr}) = \frac{1.0}{186} \times 188 = 1.01075\ldots\ \text{g}.$$

The precipitate can weigh slightly more than the original sample because silver from the reagent is

incorporated into it. The acetyl group must not be included in the final product's molar mass because it was removed in the last step.

Step 4: Conclusion The mass of AgBr obtained is approximately $1.011\ \mathrm{g}$. Rounded to the nearest integer, the requested result is $\boxed{1\ \mathrm{g}}$, so the numerical entry is $\boxed{1}$.

Quick Tip to Solve: Determine the product after the final hydrolysis before calculating its molar mass. Then multiply moles of product by its bromine-atom count and by the molar mass of AgBr.