JEE Main 2026 Paper 1 (B.E./B.Tech) Apr 4 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2026, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 4 April 2026, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let [•] denote the greatest integer function. If the domain of the function $f(x) = \cos^{-1}\left(\frac{4x+2[x]}{3}\right)$ is $[\alpha, \beta]$, then $12(\alpha+\beta)$ is equal to : [4 marks]

(A) 6

(B) 8

(C) 9

(D) 4

Correct Answer: (A) 6

Solution:

Concept:

- The real-valued inverse cosine requires its argument to lie in $[-1, 1]$, including both endpoints.
- The greatest integer $[x]$ is constant on each interval $[n, n+1)$. The domain must satisfy the inverse-cosine restriction and the corresponding interval simultaneously.

Step 1: Impose the inverse-cosine restriction Since the denominator is positive, the condition for a real output is

$$-1 \le \frac{4x+2[x]}{3} \le 1 \quad \Longleftrightarrow \quad -3 \le 4x+2[x] \le 3.$$

For $x \ge 1$, the middle expression is at least 6, so these values are excluded. For $x \le -1$, it is at most -6, so those values are excluded as well. We therefore need only inspect $-1 < x < 1$.

Step 2: Solve on the two relevant integer intervals For $-1 < x < 0$, we have $[x] = -1$. Thus

$$-3 \le 4x-2 \le 3 \quad \Longleftrightarrow \quad -\frac{1}{4} \le x \le \frac{5}{4}.$$

Intersecting with $(-1, 0)$ gives $[-1/4, 0)$. For $0 \le x < 1$, we have $[x] = 0$, and the same restriction becomes $-3 \le 4x \le 3$. Its intersection with $[0, 1)$ is $[0, 3/4]$. These intervals join without a gap at zero.

Step 3: Evaluate the requested expression The domain is consequently $[-1/4, 3/4]$. Both outer endpoints are allowed because the inverse-cosine arguments there are -1 and 1, respectively. Hence

$$12(\alpha+\beta) = 12\left(-\frac{1}{4}+\frac{3}{4}\right) = 6.$$

The values $8, 9, 4$ in the other options do not equal this endpoint sum.

Step 4: Conclusion The domain is $[-1/4, 3/4]$, and $12(\alpha+\beta) = 6$. Hence, the correct option is (A).

Quick Tip to Solve: Split a greatest-integer domain at integer boundaries, and intersect each algebraic answer with the interval on which that integer value is valid.

2. If the set of all solutions of $|x^2+x-9| = |x| + |x^2-9|$ is $[\alpha, \beta] \cup [\gamma, \infty)$, then ($\alpha^2+\beta^2+\gamma^2$) is equal to: [4 marks]

(A) 9

(B) 18

(C) 36

(D) 72

Correct Answer: (B) 18

Solution:

Concept:

- For real numbers u, v, the triangle inequality is $|u+v| \le |u| + |v|$. Equality holds exactly when $uv \ge 0$, so the numbers have the same sign or at least one is zero.
- A product inequality can be solved by marking its real zeros and checking the sign on each interval. Equality includes every zero of the product.

Step 1: Recognise the equality condition Write the expression inside the left modulus as $x + (x^2 - 9)$. The printed equation is

$$|x + (x^2 - 9)| = |x| + |x^2 - 9|.$$

Using $u = x$ and $v = x^2 - 9$, its exact condition is

$$x(x^2 - 9) \ge 0 \quad \Longleftrightarrow \quad x(x-3)(x+3) \ge 0.$$

This avoids separately solving several quadratic absolute-value equations. In particular, opposite nonzero signs give strict inequality, so no solutions from those intervals can be included.

Step 2: Determine every allowed interval The critical values are $-3, 0, 3$. On $(-\infty, -3)$, x is negative and $x^2 - 9$ is positive, so their product is negative. On $(-3, 0)$, both factors are negative, and their product is positive. On $(0, 3)$, the signs differ again. On $(3, \infty)$, both are positive. Each critical value gives product zero and satisfies the original equation. Therefore the complete solution set is

$$[-3, 0] \cup [3, \infty).$$

Step 3: Identify the endpoints and compare the options Matching this set to the form in the question gives $\alpha = -3$, $\beta = 0$, and $\gamma = 3$. Consequently,

$$\alpha^2 + \beta^2 + \gamma^2 = (-3)^2 + 0^2 + 3^2 = 18.$$

The other listed values $9, 36, 72$ do not equal the sum of the three endpoint squares.

Step 4: Conclusion The required value is 18. Hence, the correct option is (B).

Quick Tip to Solve: Whenever $|u+v| = |u| + |v|$ appears with real expressions, replace it by $uv \ge 0$ and solve the resulting sign inequality.

3. Let z be a complex number such that $|z+2| = |z-2|$ and $\arg\left(\frac{z+3}{z-i}\right) = \frac{\pi}{4}$. Then $|z|^2$ is equal to: [4 marks]

(A) 9

(B) 4

(C) 5

(D) 1

Correct Answer: (A) 9

Solution:

Concept:

- The modulus $|z-a|$ is the distance from z to the point a in the complex plane. Equal distances from two points define their perpendicular bisector.
- A nonzero complex number with argument $\pi/4$ has equal positive real and imaginary parts. Equality of the parts alone must be supplemented by the quadrant condition.

Step 1: Use the equal-modulus condition Put $z = x + iy$, where x, y are real. Squaring the two equal moduli gives

$$(x+2)^2 + y^2 = (x-2)^2 + y^2.$$

After cancelling like terms, $8x = 0$, so $x = 0$ and $z = iy$. This agrees with the imaginary axis being the perpendicular bisector of the segment joining -2 and 2. The quotient in the second condition requires $y \neq 1$.

Step 2: Write the quotient in rectangular form Substitute $z = iy$ and divide by $i(y-1)$:

$$\frac{z+3}{z-i} = \frac{3+iy}{i(y-1)} = \frac{y}{y-1} - i\frac{3}{y-1}.$$

For argument $\pi/4$, the two displayed components must be equal and positive. Equality yields

$$\frac{y}{y-1} = -\frac{3}{y-1} \quad \Longrightarrow \quad y = -3.$$

The denominator is nonzero for this value, so the algebra has not introduced a forbidden point.

Step 3: Check the argument and evaluate the modulus At $y = -3$, the quotient is $3/4 + 3i/4$. Its equal positive components confirm that its argument is precisely $\pi/4$, rather than a third-quadrant angle with the same tangent. Thus $z = -3i$, and

$$|z|^2 = 0^2 + (-3)^2 = 9.$$

The values $4, 5, 1$ in the remaining options cannot satisfy this derived modulus condition.

Step 4: Conclusion We obtain $z = -3i$ and $|z|^2 = 9$. Hence, the correct option is (A).

Quick Tip to Solve: After replacing an argument condition by an equation between real and imaginary parts, always check the required quadrant and any excluded denominator.

4. The number of functions $f : \{1, 2, 3, 4\} \to \{a, b, c\}$, which are not onto, is : [4 marks]

(A) 48

(B) 45

(C) 51

(D) 35

Correct Answer: (B) 45

Solution:

Concept:

- A function is onto when every element of the codomain occurs as an image. A non-onto function omits at least one codomain element.
- If a domain has m elements and each image may be chosen independently from n elements, there are n^m functions. Inclusion-exclusion corrects repeated counting of functions that omit several elements.

Step 1: Count functions omitting a specified element Let E_a, E_b, E_c be the sets of functions omitting a, b, c, respectively. The required count is $|E_a \cup E_b \cup E_c|$. If a is omitted, each of the four inputs can map independently to either b or c, giving $2^4 = 16$ functions. The same count holds for each of the other omissions. Adding these three counts first gives $3 \cdot 16 = 48$, but includes repeated counts.

Step 2: Apply inclusion-exclusion to the overlaps A function in $E_a \cap E_b$ can only take the value c, so this intersection consists of one constant function. Similarly, each of the other two pairwise intersections contains one constant function. There is no function omitting all three codomain values, because the domain is nonempty and every input must have an image. Hence

$$|E_a \cup E_b \cup E_c| = 3 \cdot 2^4 - 3 \cdot 1^4 + 0 = 48 - 3 = 45.$$

Step 3: Check by the possible image sizes A non-onto function has image size one or two. There are three constant functions. For image size two, first choose the two used values in $\binom{3}{2} = 3$ ways; for each chosen pair, exclude its two constant functions from the 2^4 assignments. This gives

$$3 + 3(16 - 2) = 3 + 42 = 45.$$

Thus 48 counts the omissions without correcting overlaps; 51 and 35 likewise do not match the independently checked total.

Step 4: Conclusion There are 45 functions which are not onto. Hence, the correct option is (B).

Quick Tip to Solve: Count non-onto functions by missing codomain values, but subtract the intersections: a constant function omits more than one value.

5. Let $S = \left\{A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \{0, 1, 2, 3, 4\} \text{ and } A^2 - 4A + 3I = 0\right\}$ **be a set of** 2×2 **matrices. Then the number of matrices in S , for which the sum of the diagonal elements is equal to 4 , is :** [4 marks]

(A) 20

(B) 17

(C) 21

(D) 19

Correct Answer: (D) 19

Solution:

Concept:

- For a 2×2 matrix with trace t and determinant Δ, the Cayley-Hamilton identity is $A^2 - tA + \Delta I = 0$.
- The required diagonal sum fixes $a + d = 4$. The matrix equation then fixes the determinant, reducing the count to ordered pairs of off-diagonal entries with a specified product.

Step 1: Reduce the matrix equation to an entry condition Since $a + d = 4$, the Cayley-Hamilton identity gives $A^2 - 4A + (ad - bc)I = 0$. Comparing with the required $A^2 - 4A + 3I = 0$ yields $ad - bc = 3$. Conversely, trace 4 and determinant 3 imply the required polynomial identity, so these conditions are sufficient as well as necessary. With $d = 4 - a$, we must count

$$bc = a(4 - a) - 3, \qquad a, b, c, d \in \{0, 1, 2, 3, 4\}.$$

Step 2: Check all possible diagonal pairs For $a = 0$ or $a = 4$, the required product is $bc = -3$, impossible because b, c are nonnegative. For $a = 1$ or $a = 3$, it is $bc = 0$. For $a = 2$, it is $bc = 1$. These are all five possibilities for a; in each case $d = 4 - a$ lies in the permitted entry set.

Step 3: Count the ordered off-diagonal pairs When $bc = 0$, there are five choices with $b = 0$ and five with $c = 0$. The pair $(0, 0)$ is in both groups, so the count is $5 + 5 - 1 = 9$. This occurs for both diagonal pairs $(1, 3)$ and $(3, 1)$, contributing 18 matrices. When $bc = 1$, the only allowed integer pair is $(b, c) = (1, 1)$, contributing one further matrix. Therefore

$$N = 9 + 1 + 9 = 19.$$

The total is not 20: counting the zero-product pairs without removing $(0, 0)$ from their overlap overcounts. The other totals 17 and 21 also disagree with the exhaustive cases.

Step 4: Conclusion The required number of matrices is 19. Hence, the correct option is (D).

Quick Tip to Solve: For a quadratic identity in a 2×2 matrix, compare with Cayley-Hamilton first; then count entry pairs, remembering that (b, c) is ordered.

6. Let $A = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 1 \\ 1 & 3 & 5 \end{bmatrix}$. Then the sum of all elements of the matrix $\text{adj}\left(\text{adj}\left(2(\text{adj } A)^{-1}\right)\right)$ is equal to: [4 marks]

(A) 3

(B) 4

(C) -4

(D) -3

Correct Answer: (D) -3

Solution:

Concept:

- For an invertible matrix A, $\text{adj } A = (\det A)A^{-1}$, and therefore $(\text{adj } A)^{-1} = A/\det A$.
- For a 3×3 matrix B, $\text{adj}(\text{adj } B) = (\det B)B$. Also $\det(cA) = c^3 \det A$ in this order.

Step 1: Compute the determinant of the given matrix Expanding along the first row, with alternating cofactor signs, gives

$$\begin{aligned} \det A &= 1(0 \cdot 5 - 1 \cdot 3) - 1((-2) \cdot 5 - 1 \cdot 1) \\ &\quad + 2((-2) \cdot 3 - 0 \cdot 1) \\ &= -3 + 11 - 12 = -4. \end{aligned}$$

The nonzero value confirms that the inverse appearing in the expression exists.

Step 2: Simplify the inner matrix before taking adjugates Denote the inner matrix by $B = 2(\text{adj } A)^{-1}$. The inverse-adjugate identity yields

$$B = \frac{2A}{\det A} = -\frac{A}{2}.$$

Because the matrices have order three, its determinant is

$$\det B = \left(-\frac{1}{2}\right)^3 \det A = -\frac{1}{8}(-4) = \frac{1}{2}.$$

The scalar is cubed in this step; treating it as a single factor would give an incorrect determinant.

Step 3: Apply the double-adjugate identity and sum For order three the double adjugate is the original matrix multiplied by its determinant. Thus the requested matrix is

$$\text{adj}(\text{adj } B) = (\det B)B = \frac{1}{2}\left(-\frac{A}{2}\right) = -\frac{A}{4}.$$

The row sums of A are $4, -1, 9$, respectively, so the sum of all its entries is 12. The requested entry sum is consequently $-12/4 = -3$. This fixes both the sign and magnitude: the positive options $3, 4$ and the value -4 do not match it.

Step 4: Conclusion The sum of all elements of the requested matrix is -3. Hence, the correct option is (D).

Quick Tip to Solve: Collapse nested adjugates using the matrix order and determinant identities before computing individual cofactors; keep the determinant scaling power explicit.

7. The first term of an A.P. of 30 non-negative terms is $\frac{10}{3}$. If the sum of this A.P. is the cube of its last term, then its common difference is: [4 marks]

(A) $\frac{5}{87}$
(B) $\frac{25}{83}$
(C) $\frac{15}{29}$
(D) $\frac{5}{29}$

Correct Answer: (A) $\frac{5}{87}$

Solution:

Concept:

- For an arithmetic progression with first term a, common difference d and N terms, the last term is $l = a + (N-1)d$.
- The sum expressed using the end terms is $S_N = \frac{N}{2}(a+l)$. This form is convenient because the given sum is the cube of the last term.

Step 1: Form an equation in the last term Put $a = \frac{10}{3}$ and $N = 30$. Using the given relation $S_{30} = l^3$, we obtain

$$15\left(\frac{10}{3} + l\right) = l^3 \quad \Longrightarrow \quad l^3 - 15l - 50 = 0.$$

This replaces the common difference temporarily by the last term and reduces the condition to a single cubic equation.

Step 2: Find the admissible last term Substituting $l = 5$ gives $125 - 75 - 50 = 0$, so $l - 5$ is a factor. Thus

$$l^3 - 15l - 50 = (l-5)(l^2 + 5l + 10) = 0.$$

The quadratic factor has discriminant $25 - 40 = -15 < 0$, so it contributes no real root. Therefore the unique real last term is $l = 5$, which satisfies the non-negativity requirement.

Step 3: Recover and check the common difference From the last-term formula,

$$5 = \frac{10}{3} + 29d \quad \Longrightarrow \quad 29d = \frac{5}{3} \quad \Longrightarrow \quad d = \frac{5}{87}.$$

Every term is positive since the first term and common difference are positive. The sum is $15(10/3 + 5) = 125 = 5^3$, confirming the condition. The other three differences produce last terms different from the unique real solution 5, so none can satisfy the required sum.

Step 4: Conclusion The common difference is $\boxed{\frac{5}{87}}$. Hence, the correct option is (A).

Quick Tip to Solve: When the sum is related to the last term, first use $S_N = N(a+l)/2$; recover d only after finding l.

8. The number of ways, of forming a queue of 4 boys and 3 girls such that all the girls are not together, is: [4 marks]

(A) 5040

(B) 3050

(C) 3410

(D) 4320

Correct Answer: (D) 4320

Solution:

Concept:

- A queue is a linear ordering of distinct people. Therefore seven people can be arranged in $7!$ unrestricted ways.
- The excluded event is that all three girls occupy one consecutive block. Count this excluded event using the block method.

Step 1: Count all unrestricted queues There are $4 + 3 = 7$ distinct people. The first position can be filled in seven ways, the next in six ways, and so on. Hence the total number of queues before imposing the restriction is

$$7! = 5040.$$

There is no division by any factorial because exchanging two boys or two girls gives a different queue.

Step 2: Count queues with all girls together Treat the three girls as one block. Together with the four individual boys, there are five objects to arrange, giving $5!$ orders. Within the girls' block, the three distinct girls can appear in $3!$ orders. Thus the number of excluded queues is

$$5! \times 3! = 120 \times 6 = 720.$$

Each excluded queue determines exactly one position and internal order of the girls' block, so this product does not overcount.

Step 3: Subtract the excluded queues The required count is

$$7! - 5!3! = 5040 - 720 = 4320.$$

The condition excludes only queues in which all three girls are consecutive; it does not forbid a pair of girls from standing together. Consequently, a method requiring every girl to be isolated would impose an extra restriction. Option (A) counts all queues without removing the forbidden block. Options (B) and (C) are neither the complement count nor the result of the stated subtraction.

Step 4: Conclusion The required number of queues is $\boxed{4320}$. Hence, the correct option is (D).

Quick Tip to Solve: To exclude all girls being together, subtract the single-block arrangements from all arrangements; this still allows a pair of girls to be adjacent.

9. Let the smallest value of $k \in \mathbb{N}$, for which the coefficient of x^3 in $(1+x)^3+(1+x)^4+(1+x)^5+\ldots+(1+x)^{99}+(1+kx)^{100}, x \neq 0$, is $\left(43n+\frac{101}{4}\right)\left({}^{100}C_3\right)$ for some $n \in \mathbb{N}$, be p. Then the value of $p+n$ is : [4 marks]

(A) 10

(B) 11

(C) 12

(D) 13

Correct Answer: (B) 11

Solution:

Concept:

- By the binomial theorem, the coefficient of x^3 in $(1+x)^r$ is $\binom{r}{3}$; in $(1+kx)^{100}$ it is $k^3\binom{100}{3}$.
- The hockey-stick identity gives $\sum_{r=3}^{99}\binom{r}{3}=\binom{100}{4}$. A coefficient comparison then yields an integer divisibility condition.

Step 1: Combine the coefficients The requested coefficient is

$$\sum_{r=3}^{99}\binom{r}{3}+k^3\binom{100}{3}=\binom{100}{4}+k^3\binom{100}{3}.$$

The summation identity follows by telescoping $\binom{r}{3}=\binom{r+1}{4}-\binom{r}{4}$, so both endpoints of the given sum are included.

Step 2: Reduce the equation to natural numbers Divide the given coefficient equation by the nonzero number $\binom{100}{3}$. Since

$$\frac{\binom{100}{4}}{\binom{100}{3}}=\frac{97}{4},$$

we get

$$\frac{97}{4}+k^3=43n+\frac{101}{4} \quad \Longrightarrow \quad k^3-1=43n.$$

Thus k^3-1 must be a positive multiple of 43. The value $k=1$ gives $n=0$, which is not a positive natural number in this problem.

Step 3: Find the first admissible value For $k=2,3,4,5$, the values of k^3-1 are respectively $7, 26, 63, 124$; none is divisible by 43. At $k=6$,

$$6^3-1=215=43\times 5.$$

Therefore the smallest possible k is $p=6$, with $n=5$. This checks every smaller positive integer, establishing minimality rather than merely finding a solution. The required sum is $p+n=6+5=11$, excluding $10, 12, 13$.

Step 4: Conclusion We have $p=6$, $n=5$, and hence $\boxed{p+n=11}$. Hence, the correct option is (B).

Quick Tip to Solve: For a sum of consecutive binomial coefficients, apply the hockey-stick identity before expanding or testing the parameter.

10. Suppose that the mean and median of the non-negative numbers $21, 8, 17, a, 51, 103, b, 13, 67, (a > b)$, are 40 and 21 , respectively. If the mean deviation about the median is 26 , then $2a$ is equal to :

[4 marks]

(A) 109

(B) 117

(C) 161

(D) 131

Correct Answer: (D) 131

Solution:

Concept:

- The mean of nine observations is their sum divided by nine. The median is the fifth observation after arranging the values in increasing order.
- Mean deviation about the median M is $\frac{1}{9}\sum|x_i - M|$. Determine which side of M the unknown values lie on before removing absolute-value signs.

Step 1: Use the mean to find the sum of the unknowns The seven known observations have sum

$$21 + 8 + 17 + 51 + 103 + 13 + 67 = 280.$$

Since the mean of all nine observations is 40, their total is $9 \times 40 = 360$. Therefore

$$a + b = 360 - 280 = 80.$$

Step 2: Locate the unknowns using the median Among the known values, $8, 13, 17$ are below 21, while $51, 67, 103$ are above it. For the fifth ordered observation to be 21, the two remaining values cannot both lie strictly above 21 or both strictly below it. As $a > b$, the required placement is $b \le 21 \le a$. It follows that

$$|a - 21| + |b - 21| = (a - 21) + (21 - b) = a - b.$$

Step 3: Use the mean deviation and solve The sum of absolute deviations of the known observations from 21 is

$$0 + 13 + 4 + 30 + 82 + 8 + 46 = 183.$$

The stated mean deviation yields

$$\frac{183 + a - b}{9} = 26 \quad \Longrightarrow \quad a - b = 234 - 183 = 51.$$

Adding this to $a + b = 80$ gives $2a = 131$. Thus $a = 65.5$ and $b = 14.5$, both non-negative and with $a > b$. Their ordered list has fifth value 21, and the absolute deviations sum to 234, verifying all the conditions. The other options do not solve these two simultaneous equations.

Step 4: Conclusion The required value is $\boxed{2a = 131}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the median to locate unknown observations relative to it, then simplify the absolute deviations into a linear equation.

11. Let the line $L_1 : x + 3 = 0$ intersect the lines $L_2 : x - y = 0$ and $L_3 : 3x + y = 0$ at the points A and B , respectively. Let the bisector of the obtuse angle between the lines L_2 and L_3 intersect the line L_1 at the point C. Then $BC^2 : AC^2$ is equal to: [4 marks]

(A) 5:1

(B) 1:5

(C) 2:3

(D) 3:2

Correct Answer: (A) 5:1

Solution:

Concept:

- In a triangle, the internal angle bisector divides the opposite side in the ratio of the adjacent sides: $AC/CB = OA/OB$.
- A negative scalar product between two rays means their included angle is obtuse. This identifies the correct angle bisector when two intersecting lines have both acute and obtuse angles.

Step 1: Find the vertices of the relevant triangle The lines $L_2 : x - y = 0$ and $L_3 : 3x + y = 0$ meet at $O = (0, 0)$. Intersecting each with $L_1 : x = -3$ gives

$$A = (-3, -3), \qquad B = (-3, 9).$$

Thus A and B lie on the same vertical line, and OAB is the triangle whose angle at O must be examined.

Step 2: Identify the obtuse angle and its bisector The vectors along the two rays are $\overrightarrow{OA} = (-3, -3)$ and $\overrightarrow{OB} = (-3, 9)$. Their scalar product is

$$\overrightarrow{OA} \cdot \overrightarrow{OB} = 9 - 27 = -18 < 0.$$

Therefore $\angle AOB$ is obtuse, so the specified bisector is the internal angle bisector of this triangle. Its intersection C with the segment AB lies between A and B.

Step 3: Apply the angle bisector theorem and square The side lengths adjacent to the obtuse angle are

$$OA = \sqrt{9 + 9} = 3\sqrt{2}, \qquad OB = \sqrt{9 + 81} = 3\sqrt{10}.$$

Consequently,

$$\frac{AC}{BC} = \frac{OA}{OB} = \frac{1}{\sqrt{5}}, \qquad \frac{BC^2}{AC^2} = 5.$$

The requested order is $BC^2 : AC^2$, so the ratio is $5 : 1$, not its reciprocal $1 : 5$. Ratios $2 : 3$ and $3 : 2$ also fail the angle-bisector relation. Squaring is essential because the question asks for squares of lengths, rather than the lengths themselves.

Step 4: Conclusion The required ratio is $\boxed{BC^2 : AC^2 = 5 : 1}$. Hence, the correct option is (A).

Quick Tip to Solve: Check the sign of the dot product to select the obtuse angle, then use the angle bisector theorem without finding the bisector equation.

12. Let the vertex A of a triangle ABC be $(1, 2)$, and the mid-point of the side AB be $(5, -1)$. If the centroid of this triangle is $(3, 4)$ and its circumcenter is (α, β), then $21(\alpha + \beta)$ is equal to : [4 marks]

(A) 309

(B) 403

(C) 497

(D) 524

Correct Answer: (C) 497

Solution:

Concept:

- The midpoint formula gives $B = 2M - A$. The centroid formula gives $A + B + C = 3G$, interpreted coordinate by coordinate.
- The circumcenter is equidistant from all three vertices. Subtracting squared-distance equations cancels the quadratic terms and produces two linear equations.

Step 1: Recover the other vertices With $A = (1, 2)$ and the midpoint $M = (5, -1)$ of AB,

$$B = (2 \cdot 5 - 1,\ 2(-1) - 2) = (9, -4).$$

The centroid $G = (3, 4)$ then gives

$$C = 3G - A - B = (9 - 1 - 9,\ 12 - 2 + 4) = (-1, 14).$$

These coordinates reproduce both the supplied midpoint and centroid.

Step 2: Equate distances from the circumcenter Write the circumcenter as $O = (\alpha, \beta)$. Equating OA^2 with OB^2, and then with OC^2, gives

$$(\alpha - 1)^2 + (\beta - 2)^2 = (\alpha - 9)^2 + (\beta + 4)^2,$$
$$(\alpha - 1)^2 + (\beta - 2)^2 = (\alpha + 1)^2 + (\beta - 14)^2.$$

After expansion and cancellation, these reduce to

$$16\alpha - 12\beta = 92, \qquad -4\alpha + 24\beta = 192,$$

or $4\alpha - 3\beta = 23$ and $-\alpha + 6\beta = 48$.

Step 3: Solve the linear equations and evaluate The second equation gives $\alpha = 6\beta - 48$. Substitution in the first yields

$$4(6\beta - 48) - 3\beta = 23 \quad \Longrightarrow \quad 21\beta = 215.$$

Hence $\beta = 215/21$ and $\alpha = 282/21$. The two independent perpendicular-bisector equations have a unique solution, since the triangle is non-collinear. Therefore

$$21(\alpha + \beta) = 21\left(\frac{282 + 215}{21}\right) = 497.$$

The values $309, 403, 524$ are inconsistent with this uniquely determined sum, eliminating the remaining

options.

Step 4: Conclusion The required value is $\boxed{21(\alpha+\beta)=497}$. Hence, the correct option is (C).

Quick Tip to Solve: Find missing vertices with midpoint and centroid formulas, then subtract squared distances to obtain linear circumcenter equations.

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13. Suppose that two chords, drawn from the point $(1,2)$ on the circle $x^2+y^2+x-3y=0$ are bisected by the y-axis. If the other ends of these chords are R and S , and the mid point of the line segment RS is (α,β), then $6(\alpha+\beta)$ is equal to : [4 marks]

(A) 1

(B) 3

(C) 4

(D) 6

Correct Answer: (B) 3

Solution:

Concept:

- The midpoint of endpoints (x_1,y_1) and (x_2,y_2) is $((x_1+x_2)/2,(y_1+y_2)/2)$. A midpoint on the y-axis has zero x-coordinate.
- Each endpoint of a chord satisfies the circle equation. Combining the midpoint condition with this equation determines the two possible other endpoints.

Step 1: Use the condition that the chords are bisected Write the common endpoint as $P=(1,2)$, and let an arbitrary other endpoint be $T=(u,v)$. The midpoint of PT lies on the y-axis, so

$$\frac{1+u}{2}=0 \quad\Longrightarrow\quad u=-1.$$

Thus both R and S must lie on the vertical line $x=-1$. This condition concerns the midpoint of each chord separately; it does not put the endpoints themselves on the y-axis.

Step 2: Determine the endpoints and their midpoint Substitute $x=-1$ into the circle equation:

$$1+y^2-1-3y=0 \quad\Longrightarrow\quad y(y-3)=0.$$

Hence the two endpoints are $R=(-1,0)$ and $S=(-1,3)$, in either order. Their midpoint is therefore

$$(\alpha,\beta)=\left(\frac{-1-1}{2},\frac{0+3}{2}\right)=\left(-1,\frac{3}{2}\right).$$

As a check, the midpoints of PR and PS are $(0,1)$ and $(0,5/2)$, both on the required axis.

Step 3: Evaluate the requested expression The sum is $\alpha+\beta=-1+3/2=1/2$, giving $6(\alpha+\beta)=3$. The other proposed values, $1,4,6$, do not follow from the uniquely determined midpoint and are excluded.

Step 4: Conclusion $6(\alpha+\beta)=\boxed{3}$. Hence, the correct option is (B).

Quick Tip to Solve: A chord midpoint on an axis fixes the opposite endpoint's corresponding coordinate immediately; use that before solving the circle equation.

14. A line with direction ratios $1,-1,2$ intersects the lines $\frac{x}{2}=\frac{y}{3}=\frac{z+1}{3}$ and $\frac{x+1}{-1}=\frac{y-2}{1}=\frac{z}{4}$ at the points P and Q , respectively. If the length of the line segment PQ is α, then $225\alpha^2$ is equal to: [4 marks]

(A) 1024

(B) 1014

(C) 1104

(D) 1204

Correct Answer: (B) 1014

Solution:

Concept:

- A point on a line in symmetric form is obtained by setting its three ratios equal to one parameter.
- If a segment is parallel to direction ratios $(1,-1,2)$, its displacement is $k(1,-1,2)$, and its squared length is $6k^2$.

Step 1: Parametrise the two intersection points Let the common ratio for the first given line be t, and for the second be s. Then

$$P=(2t,3t,3t-1), \qquad Q=(-s-1,s+2,4s).$$

Since the joining line has direction ratios $1,-1,2$, write $Q-P=k(1,-1,2)$. Comparing its three components gives

$$\begin{aligned} -s-1-2t&=k,\\ s+2-3t&=-k,\\ 4s-3t+1&=2k. \end{aligned}$$

Step 2: Solve for the displacement parameter Adding the first two equations eliminates both s and k: $1-5t=0$, so $t=1/5$. Thus $k=-s-7/5$, and the third equation becomes

$$4s+\frac{2}{5}=-2s-\frac{14}{5} \quad\Longrightarrow\quad s=-\frac{8}{15}, \qquad k=-\frac{13}{15}.$$

The actual points are $P=(2/5,3/5,-2/5)$ and $Q=(-7/15,22/15,-32/15)$. Their difference is indeed $(-13/15,13/15,-26/15)$, checking all three coordinates.

Step 3: Calculate the squared length The negative sign of k only indicates the orientation from P to Q; length is positive. Therefore

$$\alpha^2=\left(\frac{13}{15}\right)^2(1^2+(-1)^2+2^2)=\frac{1014}{225}.$$

Multiplication by 225 gives 1014. The values $1024, 1104, 1204$ fail the squared-length calculation.

Step 4: Conclusion $225\alpha^2 = \boxed{1014}$. Hence, the correct option is (B).

Quick Tip to Solve: Use a separate parameter for each given line and a third for the joining displacement; add coordinate equations to eliminate parameters before finding the distance.

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15. The square of the distance of the point $(-2, -8, 6)$ from the line $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z}{-1}$ along the line $\frac{x+5}{1} = \frac{y+5}{-1} = \frac{z}{2}$ is equal to: [4 marks]

(A) 3

(B) 6

(C) 8

(D) 12

Correct Answer: (B) 6

Solution:

Concept:

- Distance from a point to a line along a specified line is the length of the segment in that specified direction joining the point to the target line. It need not be the perpendicular distance.
- For a displacement $k(a, b, c)$, the squared Euclidean length is $k^2(a^2 + b^2 + c^2)$.

Step 1: Identify the line along which the distance is measured Denote the given point by $A = (-2, -8, 6)$. The second line has parametric form $(-5 + u, -5 - u, 2u)$. Taking $u = 3$ gives A, so this line really passes through the stated point. Its direction vector is $(1, -1, 2)$. A point reached from A along it can therefore be written as

$$B = (-2 + k, -8 - k, 6 + 2k).$$

Step 2: Locate the intersection with the target line A general point on the first line is $(1+t, 1+2t, -t)$. For this to be B, equate the coordinates:

$$\begin{aligned} -2 + k &= 1 + t, \\ -8 - k &= 1 + 2t, \\ 6 + 2k &= -t. \end{aligned}$$

The first equation gives $k = t + 3$. Substitution into the second yields $-11 - t = 1 + 2t$, hence $t = -4$ and $k = -1$. The last equation checks as $6 - 2 = 4$. Thus the required intersection is $B = (-3, -7, 4)$.

Step 3: Find the squared distance along this segment The displacement is $B - A = (-1, 1, -2)$. Consequently,

$$AB^2 = (-1)^2 + 1^2 + (-2)^2 = 1 + 1 + 4 = 6.$$

The distance itself is $\sqrt{6}$, whereas the question requests its square. The alternatives $3, 8, 12$ do not equal the sum of the three squared coordinate differences.

Step 4: Conclusion The required square of the distance is $\boxed{6}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: When distance is specified along another line, first intersect that line with the target; a perpendicular-distance formula answers a different question.

16. If $y = \tan^{-1}\left(\frac{3\cos x - 4\sin x}{4\cos x + 3\sin x}\right) + 2\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$**, then** $\frac{dy}{dx}$ **at** $x = \frac{\sqrt{3}}{2}$ **is equal to :** [4 marks]

(A) 3

(B) -1

(C) 1

(D) 2

Correct Answer: (C) 1

Solution:

Concept:

- For a differentiable real function $u(x)$, $\frac{d}{dx}\tan^{-1} u = \frac{u'}{1+u^2}$ wherever the expression is defined.
- For $-1 < x < 1$, put $x = \sin\theta$ with $-\pi/2 < \theta < \pi/2$. Then $\sqrt{1-x^2} = \cos\theta > 0$, so the half-angle identity identifies the second inverse-tangent term with $\sin^{-1} x$.

Step 1: Differentiate the first inverse tangent directly Set $N = 3\cos x - 4\sin x$ and $D = 4\cos x + 3\sin x$. These satisfy $N' = -D$ and $D' = N$. Hence, wherever $D \neq 0$,

$$\frac{d}{dx}\tan^{-1}\left(\frac{N}{D}\right) = \frac{N'D - ND'}{D^2 + N^2} = \frac{-D^2 - N^2}{D^2 + N^2} = -1.$$

At the given positive $x = \sqrt{3}/2 < \pi/2$, both sine and cosine are positive, so $D > 0$. Thus this differentiation is valid at the required point without any inverse-tangent branch ambiguity.

Step 2: Simplify the half-angle expression With $x = \sin\theta$ on the principal interval,

$$\frac{x}{1+\sqrt{1-x^2}} = \frac{\sin\theta}{1+\cos\theta} = \tan\frac{\theta}{2}.$$

Since $\theta/2$ is inside the principal range of inverse tangent, twice its inverse tangent is exactly $\theta = \sin^{-1} x$. Its derivative is $1/\sqrt{1-x^2}$.

Step 3: Evaluate the derivative at the stated point Combining the derivatives gives

$$y' = -1 + \frac{1}{\sqrt{1-x^2}}, \qquad y'\left(\frac{\sqrt{3}}{2}\right) = -1 + \frac{1}{1/2} = 1.$$

The option -1 keeps only the first term, and 2 keeps only the second. The value 3 incorrectly changes the first derivative's sign.

Step 4: Conclusion $\left.\frac{dy}{dx}\right|_{x=\sqrt{3}/2} = \boxed{1}$. Hence, the correct option is (C).

Quick Tip to Solve: In a rotated sine-cosine ratio, check whether the numerator derivative is minus the denominator; this often reduces an inverse-tangent derivative to a constant.

17. Let f be a real polynomial of degree n such that $f(x) = f'(x)f''(x)$, for all $x \in \mathbb{R}$. If $f(0) = 0$, then $36\left(f'(2) + f''(2) + \int_0^2 f(x)dx\right)$ is equal to: [4 marks]

(A) 42

(B) 46

(C) 56

(D) 66

Correct Answer: (C) 56

Solution:

Concept:

- Equal nonzero polynomials have equal degrees and matching coefficients. If f has degree $n \geq 2$, then $f'f''$ has degree $2n - 3$.
- Once the polynomial is determined, differentiate and integrate its powers term by term. The zero polynomial has no degree n, so it is not a polynomial of the stated degree.

Step 1: Determine the degree and leading coefficient A nonzero constant or linear polynomial cannot satisfy the identity, because its second derivative is zero. For degree at least two, comparison of degrees gives $n = 2n - 3$, hence $n = 3$. Write

$$f(x) = ax^3 + bx^2 + cx + d, \qquad a \neq 0.$$

The leading coefficient in $f'f''$ is $18a^2$; therefore $a = 18a^2$, giving $a = 1/18$.

Step 2: Use the remaining coefficients and the initial value Expanding the product gives

$$f'f'' = 18a^2x^3 + 18abx^2 + (4b^2 + 6ac)x + 2bc.$$

The coefficient of x yields $c = 4b^2 + c/3$, so $c = 6b^2$. The constant coefficient gives $d = 2bc = 12b^3$. Since $f(0) = d = 0$, real b must be zero; then $c = 0$. Consequently $f(x) = x^3/18$. Directly, $(x^2/6)(x/3) = x^3/18$, confirming the identity.

Step 3: Evaluate all three contributions We obtain

$$f'(2) = \frac{2^2}{6} = \frac{2}{3}, \qquad f''(2) = \frac{2}{3}, \qquad \int_0^2 f(x)\,dx = \left[\frac{x^4}{72}\right]_0^2 = \frac{2}{9}.$$

Hence the requested expression is

$$36\left(\frac{2}{3} + \frac{2}{3} + \frac{2}{9}\right) = 36 \cdot \frac{14}{9} = 56.$$

This checks each derivative and the integral separately; none of $42, 46, 66$ equals their scaled sum.

Step 4: Conclusion The required value is $\boxed{56}$. Hence, the correct option is (C).

Quick Tip to Solve: For a polynomial equalling a product of its derivatives, compare degrees before coefficients; this fixes the polynomial's possible shape immediately.

18. The area of the region $\{(x, y) : y \leq \pi - |x|, y \leq |x \sin x|, y \geq 0\}$ is: [4 marks]

(A) $1 + \frac{\pi^2}{8}$
(B) $2 + \frac{\pi^2}{4}$
(C) $\frac{\pi^2}{8} - 1$
(D) $4 + \frac{\pi^2}{2}$

Correct Answer: (B) $2 + \frac{\pi^2}{4}$

Solution:

Concept:

- When $0 \leq y \leq g(x)$ and $y \leq h(x)$, the vertical height of the region is the smaller nonnegative upper bound, $\min(g(x), h(x))$.
- Both bounds here are even functions. The full area is twice the area for $x \geq 0$, and integration by parts gives $\int x \sin x \, dx = -x \cos x + \sin x$.

Step 1: Restrict the domain and identify the first upper boundary The conditions $y \geq 0$ and $y \leq \pi - |x|$ force $|x| \leq \pi$. For $0 \leq x \leq \pi$, sine is nonnegative and the two upper bounds become $\pi - x$ and $x \sin x$. On $[0, \pi/2]$,

$$x \sin x \leq x \leq \pi - x,$$

so the height is $x \sin x$. The two bounds agree at $x = \pi/2$.

Step 2: Check the boundary on the second half For $\pi/2 \leq x \leq \pi$, put $t = \pi - x$, so $0 \leq t \leq \pi/2$. Since sine is concave on this interval, it lies above its endpoint chord: $\sin t \geq 2t/\pi$. Also $\pi - t \geq \pi/2$. Thus

$$x \sin x = (\pi - t) \sin t \geq (\pi - t)\frac{2t}{\pi} \geq t = \pi - x.$$

The smaller bound on this half is consequently $\pi - x$; no additional crossing changes the area calculation.

Step 3: Integrate the two pieces and restore symmetry The full area is

$$\begin{aligned} A &= 2\left(\int_0^{\pi/2} x \sin x \, dx + \int_{\pi/2}^{\pi} (\pi - x) \, dx\right) \\ &= 2\left(1 + \frac{\pi^2}{8}\right) = 2 + \frac{\pi^2}{4}. \end{aligned}$$

The first integral is $[-x \cos x + \sin x]_0^{\pi/2} = 1$. The second is the area of a triangle with base and height both $\pi/2$. Option (A) counts only the right half; (D) doubles the full answer again, and (C) has the wrong contribution from the sine boundary.

Step 4: Conclusion The area is $\boxed{2 + \pi^2/4}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: For simultaneous upper bounds, integrate the lower curve and split where that curve changes; use even symmetry only after finding the height correctly.

19. Let $\int_{-2}^{2}(|\sin x| + [x \sin x])dx = 2(3 - \cos 2) + \beta$, where $[\cdot]$ is the greatest integer function. Then $\beta \sin\left(\frac{\beta}{2}\right)$ equals: [4 marks]

(A) 1

(B) 2

(C) 4

(D) 8

Correct Answer: (B) 2

Solution:

Concept:

- Both $|\sin x|$ and $x \sin x$ are even functions, so the integral over $[-2, 2]$ is twice the integral over $[0, 2]$.
- The greatest integer function is constant between its jump points. Let α be the point in $(0, \pi/2)$ satisfying $\alpha \sin \alpha = 1$.

Step 1: Locate the single integer crossing On $[0, \pi/2]$, $x \sin x$ increases from zero to $\pi/2 > 1$, so α exists and is unique there. On $[\pi/2, 2]$, both $x \geq \pi/2$ and $\sin x \geq \sin 2$ give $x \sin x > 1$. Also $x \sin x < 2$ throughout $[0, 2]$. Thus the floor is zero below α and one above it; its endpoint value does not affect an integral.

$$\int_0^2 [x \sin x], dx = 2 - \alpha.$$

Step 2: Evaluate the even integral and identify beta Since $0 \leq x \leq 2 < \pi$, the sine is nonnegative on this interval. Hence

$$\begin{aligned} I &= 2\left(\int_0^2 \sin x, dx + 2 - \alpha\right) \\ &= 2(1 - \cos 2 + 2 - \alpha) \\ &= 2(3 - \cos 2) - 2\alpha. \end{aligned}$$

Comparison with the given expression yields $\beta = -2\alpha$. There is no need to approximate the transcendental root α.

Step 3: Use the defining equation of the crossing The two minus signs cancel because sine is odd:

$$\beta \sin(\beta/2) = (-2\alpha)\sin(-\alpha) = 2\alpha \sin \alpha = 2.$$

The other proposed numbers, 1, 4 and 8, cannot satisfy this exact relation; they arise from losing or repeating the factor of two from symmetry.

Step 4: Conclusion The required value is $\boxed{2}$. Hence, the correct option is (B).

Quick Tip to Solve: Introduce the floor-function jump point symbolically; the requested expression may eliminate it using its own defining equation.

20. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = \left(1 + x + x^2\right)\left(1 - y + y^2\right), y(0) = \frac{1}{2}$. Then $(2y(1) - 1)$ is equal to [4 marks]

(A) $\sqrt{3}\tan\left(\frac{11\sqrt{3}}{6}\right)$

(B) $\frac{\sqrt{3}}{2}\tan\left(\frac{11\sqrt{3}}{12}\right)$

(C) $\sqrt{3}\tan\left(\frac{11\sqrt{3}}{12}\right)$

(D) $\frac{\sqrt{3}}{2}\tan\left(\frac{11\sqrt{3}}{6}\right)$

Correct Answer: No finite value exists for the stated initial-value solution at $x = 1$; formal substitution gives option (C).

Solution:

Concept:

- A separable differential equation can be integrated after placing the y-dependent factor with dy. Here $y^2 - y + 1 = (y - 1/2)^2 + 3/4 > 0$.
- An initial-value solution must exist on an interval containing the initial point. A tangent expression cannot be continued through its pole as a finite differentiable solution.

Step 1: Separate variables and use the initial condition Completing the square and integrating gives

$$\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2y-1}{\sqrt{3}}\right) = x + \frac{x^2}{2} + \frac{x^3}{3} + C.$$

At $x = 0$, $y = 1/2$, so $C = 0$. On the interval of existence containing zero, put

$$\phi(x) = \frac{\sqrt{3}}{2}\left(x + \frac{x^2}{2} + \frac{x^3}{3}\right).$$

Then $2y - 1 = \sqrt{3}\tan\phi(x)$, initially with $-\pi/2 < \phi(x) < \pi/2$.

Step 2: Identify the expression suggested by the options Algebraic substitution of $x = 1$ gives

$$\phi(1) = \frac{11\sqrt{3}}{12}, \qquad \sqrt{3}\tan\phi(1) = \sqrt{3}\tan\left(\frac{11\sqrt{3}}{12}\right).$$

This is option (C), the formal answer in the key. Options (A) and (D) double the phase, while (B) and (D) halve the coefficient. However, this calculation must still be checked against the solution's interval of existence.

Step 3: Check whether the solution reaches one The phase is strictly increasing because

$$\phi'(x) = \frac{\sqrt{3}}{2}(1 + x + x^2) > 0.$$

Also $\phi(0) = 0$ and $\phi(1) = 11\sqrt{3}/12 > \pi/2$. Therefore there is a unique $c \in (0, 1)$ with $\phi(c) = \pi/2$. As $x \to c^-$, $y(x) \to +\infty$. Thus no finite differentiable solution through $(0, 1/2)$ exists on an interval reaching 1. Evaluating a separate tangent branch beyond this pole does not extend the initial-value solution.

Step 4: Conclusion Strictly, $y(1)$ is undefined for the stated initial-value problem, so none of the four options is its value. The formal expression in option (C) is the key's answer, but the pole before $x = 1$ prevents it from being the value of the given solution.

Quick Tip to Solve: When separation produces a tangent, check for a pole between the initial point and the requested point before substituting.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. A coin is tossed 8 times. If the probability that exactly 4 heads appear in the first six tosses and exactly 3 heads appear in the last five tosses is p, then $96p$ is equal to ____ . [4 marks]

Solution:

Concept:

- For eight independent tosses of a fair coin, the $2^8 = 256$ ordered head-tail sequences are equally likely.
- The first six and last five tosses overlap in tosses 4, 5 and 6. Their head counts are therefore dependent; count the shared heads before combining the events.

Step 1: Divide the tosses into three disjoint blocks Let u, v and w be the head counts in tosses 1–3, tosses 4–6 and tosses 7–8, respectively. The required conditions are

$$u + v = 4, \qquad v + w = 3.$$

Thus $u = 4 - v$ and $w = 3 - v$. Since $0 \le u, v \le 3$ and $0 \le w \le 2$, only $v = 1, 2, 3$ are possible.

Step 2: Count the sequences for each shared head count Within each disjoint block, choose the positions of its heads. For a fixed v, the number is

$$\binom{3}{4-v}\binom{3}{v}\binom{2}{3-v}.$$

The three cases are disjoint and exhaust the event:

$$\begin{aligned} v = 1: &\quad \binom{3}{3}\binom{3}{1}\binom{2}{2} = 3, \\ v = 2: &\quad \binom{3}{2}\binom{3}{2}\binom{2}{1} = 18, \\ v = 3: &\quad \binom{3}{1}\binom{3}{3}\binom{2}{0} = 3. \end{aligned}$$

Hence there are $3 + 18 + 3 = 24$ favourable sequences. No factor for ordering the blocks is required because the positions of all three blocks are already fixed.

Step 3: Convert the count into the requested quantity Divide by the total number of equally likely sequences:

$$p = \frac{24}{256} = \frac{3}{32}, \qquad 96p = 96 \cdot \frac{3}{32} = 9.$$

Multiplying the two separate binomial probabilities would be invalid, since both events constrain the same middle three tosses.

Step 4: Conclusion The required numerical value is $\boxed{96p = 9}$, a dimensionless number.

Quick Tip to Solve: For head-count conditions on overlapping runs of tosses, condition on the shared block and count each disjoint block separately.

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22. Consider the parabola $\mathrm{P} : y^2 = 4kx$ and the ellipse $\mathrm{E} : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Let the line segment joining the points of intersection of P and E , be their latus rectums. If the eccentricity of E is e, then $e^2 + 2\sqrt{2}$ is equal to ____ . [4 marks]

Solution:

Concept:

- For the parabola $y^2 = 4kx$, the latus rectum lies on $x = k$, with endpoints $(k, \pm 2|k|)$.
- For an ellipse with horizontal major axis, $e^2 = 1 - b^2/a^2$. Its latus recta lie through the foci $x = \pm ae$, and each has semilength b^2/a.

Step 1: Match the locations of the common chord The parabola's latus rectum is vertical. Since the same segment is also an ellipse's latus rectum, the ellipse's major axis is horizontal. Its focal coordinate must coincide with that of the parabola's latus rectum. Writing $h = |k| > 0$ allows either opening direction and gives

$$h = ae.$$

The sign of k only chooses which of the ellipse's two latus recta is shared; it cannot change the eccentricity.

Step 2: Equate the semilengths The parabola's latus rectum has semilength $2h$. For the ellipse, substituting $x = \pm ae$ into its equation gives

$$\frac{y^2}{b^2} = 1 - e^2 = \frac{b^2}{a^2}, \qquad |y| = \frac{b^2}{a}.$$

Therefore the common endpoints require

$$2h = \frac{b^2}{a}, \qquad b^2 = 2ah = 2a^2e.$$

Dividing by a^2 eliminates the overall size of both conics.

Step 3: Solve for the eccentricity and simplify Combining $b^2/a^2 = 2e$ with $b^2/a^2 = 1 - e^2$, we obtain

$$e^2 + 2e - 1 = 0, \qquad e = -1 \pm \sqrt{2}.$$

Eccentricity is nonnegative and less than one, so only $e = \sqrt{2} - 1$ is admissible. Hence

$$e^2 = 3 - 2\sqrt{2}, \qquad e^2 + 2\sqrt{2} = 3.$$

The discarded root is negative and cannot describe an ellipse's eccentricity.

Step 4: Conclusion The requested dimensionless value is $\boxed{3}$.

Quick Tip to Solve: A common latus rectum imposes two equations: match its focal coordinate and its semilength before eliminating the scale parameters.

23. If $\mathrm{A} = \frac{\sin 3^\circ}{\cos 9^\circ} + \frac{\sin 9^\circ}{\cos 27^\circ} + \frac{\sin 27^\circ}{\cos 81^\circ}$ **and** $\mathrm{B} = \tan 81^\circ - \tan 3^\circ$**, then** $\frac{\mathrm{B}}{\mathrm{A}}$ **is equal to ____ .**

[4 marks]

Solution:

Concept:

- The tangent subtraction identity is $\tan u - \tan v = \sin(u-v)/(\cos u \cos v)$, when both denominators are nonzero.
- For angles in geometric progression with ratio three, use $\sin 2\theta = 2\sin\theta\cos\theta$ to turn $\sin\theta/\cos 3\theta$ into a difference of tangents.

Step 1: Derive the useful identity Set $u = 3\theta$ and $v = \theta$ in the subtraction formula. Then

$$\begin{aligned}\tan 3\theta - \tan\theta &= \frac{\sin 2\theta}{\cos 3\theta \cos\theta} \\ &= \frac{2\sin\theta\cos\theta}{\cos 3\theta\cos\theta} \\ &= \frac{2\sin\theta}{\cos 3\theta}.\end{aligned}$$

Thus each summand of A is half a difference. All the cosines involved at $3^\circ, 9^\circ, 27^\circ, 81^\circ$ are positive, so these divisions are valid.

Step 2: Apply the identity to the three terms Substituting $\theta = 3^\circ, 9^\circ, 27^\circ$ gives

$$\begin{aligned}2A = &(\tan 9^\circ - \tan 3^\circ) \\ &+ (\tan 27^\circ - \tan 9^\circ) \\ &+ (\tan 81^\circ - \tan 27^\circ).\end{aligned}$$

The two interior tangent values cancel pairwise. This leaves

$$2A = \tan 81^\circ - \tan 3^\circ = B.$$

Keeping the factor of two outside the entire sum avoids a common reversal of the final ratio.

Step 3: Evaluate the ratio Every summand in A is positive, since both its sine numerator and cosine denominator are positive. In particular $A \neq 0$, so dividing the identity $B = 2A$ by A is legitimate and gives $B/A = 2$, without calculating any individual trigonometric value.

Step 4: Conclusion The required dimensionless ratio is $\boxed{\frac{B}{A} = 2}$.

Quick Tip to Solve: When the angles progress as $\theta, 3\theta, 9\theta$, test a tangent-difference identity to make adjacent terms cancel.

24. Let $\overrightarrow{a_k} = (\tan\theta_k)\,\hat{i} + \hat{j}$ and $\overrightarrow{b_k} = \hat{i} - (\cot\theta_k)\,\hat{j}$, where $\theta_k = \frac{2^{k-1}\pi}{2^n+1}$, for some $n \in \mathbb{N}, n > 5$. Then the value of $\frac{\sum_{k=1}^{n} |\overrightarrow{a_k}|^2}{\sum_{k=1}^{n} |\overrightarrow{b_k}|^2}$ is [4 marks]

Solution:

Concept:

- For a vector with orthogonal unit components, its squared magnitude is the sum of the squares of the components. Thus $|\vec{a}_k|^2 = \sec^2\theta_k$ and $|\vec{b}_k|^2 = \csc^2\theta_k$.
- The double-angle formula gives $\sec^2\theta + \csc^2\theta = 4\csc^2(2\theta)$. The prescribed angles form a cycle under doubling when squared cosecants are used.

Step 1: Express the numerator and denominator as trigonometric sums Define

$$S = \sum_{k=1}^{n} \csc^2\theta_k, \qquad T = \sum_{k=1}^{n} \sec^2\theta_k.$$

Since $0 < \theta_k \le 2^{n-1}\pi/(2^n+1) < \pi/2$, all sine and cosine denominators are nonzero. The squared vector magnitudes are

$$|\vec{a}_k|^2 = 1 + \tan^2\theta_k = \sec^2\theta_k, \quad |\vec{b}_k|^2 = 1 + \cot^2\theta_k = \csc^2\theta_k.$$

The required ratio is therefore T/S, with $S > 0$.

Step 2: Derive and sum the double-angle identity Combine the two reciprocal squares over a common denominator:

$$\sec^2\theta + \csc^2\theta = \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta} = \frac{4}{\sin^2(2\theta)}.$$

Applying this to every θ_k yields

$$T + S = 4\sum_{k=1}^{n} \csc^2(2\theta_k).$$

Step 3: Close the doubling cycle For $k < n$, $2\theta_k = \theta_{k+1}$. At the last angle,

$$2\theta_n = \frac{2^n\pi}{2^n+1} = \pi - \theta_1, \qquad \csc^2(2\theta_n) = \csc^2\theta_1.$$

Consequently the doubled-angle sum contains exactly the same terms as S, merely shifted cyclically. Hence $T + S = 4S$, so $T = 3S$. No individual angle or sum needs to be evaluated, and the result is independent of the allowed choice of n.

Step 4: Conclusion The required dimensionless ratio is $\boxed{\frac{T}{S} = 3}$.

Quick Tip to Solve: In a sum over successive doubled angles, check whether the final angle becomes the supplement of the first; squared sines then close the cycle.

25. The number of points, at which the function $f(x) = \max\left\{6x, 2+3x^2\right\} + |x-1|\cos\left|x^2 - \frac{1}{4}\right|, x \in (-\pi, \pi)$, is not differentiable, is

____ .

[4 marks]

Solution:

Concept:

- The maximum of two smooth functions can have a corner only where they are equal and the active branch changes. Compare their one-sided derivatives at every such point.
- Cosine is even, so $\cos|u| = \cos u$. Thus the inner absolute value does not by itself create a nondifferentiable point.

Step 1: Find the branch-switching points Equating the two entries of the maximum gives

$$3x^2 - 6x + 2 = 0, \qquad x = 1 \pm \frac{1}{\sqrt{3}}.$$

Write these roots as a and b, with $a < b$. Since $3x^2 - 6x + 2$ is positive outside $[a, b]$ and negative inside, the maximum is $2 + 3x^2$ outside and $6x$ inside. Both roots lie in $(-\pi, \pi)$.

Step 2: Check both corners of the maximum The quadratic branch has derivative $6x$, whereas the linear branch has derivative 6. At $a = 1 - 1/\sqrt{3}$, the left and right derivatives are $6a$ and 6; at $b = 1 + 1/\sqrt{3}$, they are 6 and $6b$. Each pair differs. The remaining term is smooth at both roots because neither equals 1, so it cannot remove either derivative jump.

Step 3: Check the remaining absolute value Use evenness to write the second term as $h(x) = |x-1|\cos(x^2 - 1/4)$. It is smooth except possibly at $x = 1$. Its one-sided derivatives there are

$$h'_-(1) = -\cos(3/4), \qquad h'_+(1) = \cos(3/4).$$

Since $\cos(3/4) \neq 0$, these derivatives differ. The maximum equals $6x$ in a neighbourhood of 1, so this corner also remains. At $x = \pm 1/2$, the apparent inner-modulus corners disappear because cosine is even. There are no other possible interior points.

Step 4: Conclusion The three nondifferentiable points are $1 - 1/\sqrt{3}$, 1, and $1 + 1/\sqrt{3}$. The required number is $\boxed{3}$.

Quick Tip to Solve: First simplify an even outer function applied to an absolute value; otherwise $x = \pm 1/2$ would be counted incorrectly.

26. In a screw gauge when the circular scale is given five complete rotations it moves linearly by 2.5 mm . If the circular scale has 100 divisions, the least count of screw gauge is ____ mm. [4 marks]

(A) 1×10^{-2}

(B) 1×10^{-3}

(C) 5×10^{-2}

(D) 5×10^{-3}

Correct Answer: (D) 5×10^{-3}

Solution:

Concept:

- The pitch of a screw gauge is the axial distance moved by its screw in one complete revolution. A total displacement over several turns must first be divided by the number of turns.
- If the circular scale contains N equal divisions, its least count is $\text{LC} = \text{pitch}/N$. This is the displacement corresponding to one circular-scale division.

Step 1: Calculate the pitch per revolution Five complete rotations advance the screw by $2.5\,\text{mm}$. Therefore the advance in one complete rotation is

$$p = \frac{2.5\,\text{mm}}{5} = 0.5\,\text{mm}.$$

The value $2.5\,\text{mm}$ is the travel for all five turns, not the pitch. Dividing by five is essential before using the circular-scale divisions.

Step 2: Convert one turn to one division There are 100 equally spaced divisions in a full turn. Hence moving through one division turns the screw through $1/100$ of a revolution and gives

$$\text{LC} = \frac{0.5\,\text{mm}}{100} = 0.005\,\text{mm} = 5 \times 10^{-3}\,\text{mm}.$$

This is also $5\,\mu\text{m}$. As a check, 100 such advances give $0.5\,\text{mm}$, and five of those advances give the stated $2.5\,\text{mm}$.

Step 3: Compare the listed values Option (A), $0.01\,\text{mm}$, is twice the calculated least count; option (B), $0.001\,\text{mm}$, is one-fifth of it; and option (C), $0.05\,\text{mm}$, is ten times it. None would reproduce the given travel with 100 divisions per turn. Only option (D) gives the required advance of one division.

Step 4: Conclusion The least count is $\boxed{5 \times 10^{-3}\,\text{mm}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a stated travel over several turns, use $\text{LC} = \text{total travel}/(\text{turns} \times \text{divisions per turn})$.

27. The increase in the pressure required to decrease the volume (ΔV) of water is 6.3×10^7 N/m^2. The percentage decrease in the volume is ____ .

(Bulk modulus of water $= 2.1 \times 10^9$ N/m^2.) [4 marks]

(A) 2 %

(B) 3 %

(C) 6 %

(D) 4 %

Correct Answer: (B) 3 %

Solution:

Concept:

- Bulk modulus relates an increase in pressure to the fractional decrease in volume: $B = -\Delta P/(\Delta V/V)$, when ΔV denotes signed change in volume.
- For the positive magnitude of the decrease, $|\Delta V|/V = \Delta P/B$. Multiplying this fraction by 100 expresses the change as a percentage.

Step 1: Use the compressive strain relation The pressure increases by $6.3 \times 10^7\,\mathrm{N\,m^{-2}}$, and water has bulk modulus $2.1 \times 10^9\,\mathrm{N\,m^{-2}}$. The volume decreases, so its signed change is negative. The requested percentage decrease is a positive magnitude:

$$\frac{|\Delta V|}{V} = \frac{\Delta P}{B} = \frac{6.3 \times 10^7}{2.1 \times 10^9}.$$

No initial volume is required because the modulus directly gives the fractional change.

Step 2: Evaluate the fraction and percentage The numerical coefficients give $6.3/2.1 = 3$, while the powers give $10^7/10^9 = 10^{-2}$. Thus

$$\frac{|\Delta V|}{V} = 3 \times 10^{-2} = 0.03, \qquad \text{percentage decrease} = 0.03 \times 100 = 3\%.$$

Pressure and bulk modulus have the same units, so their ratio is dimensionless. This also means the final volume is $0.97V$ within the stated bulk-modulus relation.

Step 3: Eliminate the other percentages A decrease of 2% would require $4.2 \times 10^7\,\mathrm{N\,m^{-2}}$; 6% would require $12.6 \times 10^7\,\mathrm{N\,m^{-2}}$; and 4% would require $8.4 \times 10^7\,\mathrm{N\,m^{-2}}$. These disagree with the stated pressure change. The negative sign in the definition indicates compression and does not make the magnitude of a percentage decrease negative.

Step 4: Conclusion The volume decreases by $\boxed{3\%}$. Hence, the correct option is (B).

Quick Tip to Solve: Divide the pressure change by the bulk modulus before multiplying by 100; 0.03 is a fractional change, whereas 3% is its percentage.

28. The time taken by a block of mass m to slide down from the highest point to the lowest point on a rough inclined plane is 50% more compared to the time taken by the same block on identical inclined smooth plane. Both inclined planes are at 45° with the horizontal. The coefficient of kinetic friction between the rough inclined surface and block is ____ [4 marks]

(A) 3/4

(B) 2/3

(C) 5/9

(D) 4/9

Correct Answer: (C) 5/9

Solution:

Concept:

- For a block released from rest and travelling a fixed distance s with constant acceleration, $s = \frac{1}{2}at^2$; therefore its travel time is proportional to $1/\sqrt{a}$.
- On an incline at angle θ, smooth-plane acceleration is $g\sin\theta$. With kinetic friction coefficient μ, it becomes $g(\sin\theta - \mu\cos\theta)$.

Step 1: Translate the increased time into an acceleration ratio Let the smooth and rough travel times be t_s and t_r. A 50 percent increase means $t_r = 1.5t_s = 3t_s/2$. Both planes have the same length and the block starts sliding from rest, so

$$a_s t_s^2 = a_r t_r^2, \qquad \frac{a_r}{a_s} = \left(\frac{t_s}{t_r}\right)^2 = \frac{4}{9}.$$

The acceleration ratio is the inverse square of the time ratio, not its inverse.

Step 2: Write the force equation on the rough plane The normal reaction is $N = mg\cos\theta$, and kinetic friction μN acts uphill, opposite the motion. Thus

$$ma_r = mg\sin\theta - \mu mg\cos\theta.$$

Dividing by $a_s = g\sin\theta$ gives $a_r/a_s = 1 - \mu\cot\theta$. At $\theta = 45^\circ$, $\cot\theta = 1$, and hence

$$1 - \mu = \frac{4}{9}, \qquad \mu = \frac{5}{9}.$$

Both mass and the common plane length cancel.

Step 3: Check the proposed coefficients For $\mu = 5/9$, the time ratio is $1/\sqrt{1 - 5/9} = 3/2$, as required. The coefficients $3/4$, $2/3$, and $4/9$ instead give time ratios 2, $\sqrt{3}$, and $3/\sqrt{5}$, respectively. They do not give a 50 percent increase. Also $\mu < 1$, so the net force remains downhill at this angle.

Step 4: Conclusion The coefficient of kinetic friction is $\boxed{5/9}$, a dimensionless quantity. Hence, the correct option is (C).

Quick Tip to Solve: For equal distances from rest, square the inverse time ratio first; then use $\mu = \tan\theta\,[1 - (t_s/t_r)^2]$.

29. Two nuclei of mass number 3 combine with another nucleus of mass number 4 to yield a nucleus of mass number 10. If the binding energy per nucleon for the mass numbers 3,4 and 10 are $5.6\text{MeV}, 7.4\text{MeV}$ and 6.1 MeV , respectively, then in the process, ΔMc^2 = ____ MeV . [4 marks]

(A) 6.9

(B) 7.9

(C) 2.2

(D) 4.3

Correct Answer: (C) 2.2

Solution:

Concept:

- Total binding energy equals mass number multiplied by binding energy per nucleon. Multiply separately for each nucleus and include the number of nuclei taking part.
- For a reaction conserving the constituent nucleons, energy release is $Q = B_{\text{final}} - B_{\text{initial}}$. A decrease in total binding energy means energy must be supplied and the final rest mass is larger.

Step 1: Add the binding energies of the initial nuclei There are two nuclei with mass number 3 and one nucleus with mass number 4. Their total nucleon count is $2 \times 3 + 4 = 10$, equal to the product mass number. Each mass-3 nucleus has binding energy $3 \times 5.6 = 16.8\,\text{MeV}$. Thus

$$B_i = 2(3 \times 5.6) + (4 \times 7.4) = 33.6 + 29.6 = 63.2\,\text{MeV}.$$

The factor of two belongs to the entire binding energy of the mass-3 nucleus.

Step 2: Calculate the final binding energy and mass increase The mass-10 nucleus has total binding energy

$$B_f = 10 \times 6.1 = 61.0\,\text{MeV}.$$

Consequently $Q = B_f - B_i = -2.2\,\text{MeV}$. The product is less tightly bound overall, so the process absorbs $2.2\,\text{MeV}$. Equivalently, with $\Delta M = M_f - M_i$,

$$\Delta Mc^2 = B_i - B_f = 63.2 - 61.0 = 2.2\,\text{MeV}.$$

If one uses initial mass minus final mass instead, the reaction energy is negative; the physical conclusion is the same.

Step 3: Compare with the options The listed positive values represent the magnitude of the mass-energy change. Only $2.2\,\text{MeV}$ equals the difference between the correctly weighted total binding energies. Values 6.9, 7.9, and $4.3\,\text{MeV}$ do not satisfy this energy balance. Adding or averaging the per-nucleon energies directly would incorrectly ignore the different mass numbers.

Step 4: Conclusion The rest-mass increase corresponds to $\boxed{\Delta Mc^2 = 2.2\,\text{MeV}}$; this much energy is absorbed, rather than released. Hence, the correct option is (C).

Quick Tip to Solve: Convert every binding energy per nucleon into a total before taking the difference; the larger total binding energy corresponds to the smaller rest mass.

30. A solid sphere of mass M and radius R is divided into two unequal parts. The smaller part having mass $M/8$ is converted into a sphere of radius r and the larger part is converted into a circular disc of thickness t and radius $2R$. If I_1 is moment of inertia of a sphere having radius r about an axis through its centre and I_2 is the moment of inertia of a disc about its diameter, the ratio of their moment of inertia $I_2/I_1 =$ ____ [4 marks]

(A) 35

(B) 70

(C) 140

(D) 210

Correct Answer: (B) 70

Solution:

Concept:

- For material of unchanged uniform density, mass is proportional to volume. A solid sphere of mass m and radius r has moment of inertia $2mr^2/5$ about a diameter.
- A circular disc of mass m_d and radius a has moment of inertia $m_d a^2/4$ about a diameter in the usual thin-disc model. A finite cylinder includes an additional thickness contribution $m_d t^2/12$.

Step 1: Find the radius and inertia of the smaller sphere The smaller sphere contains one-eighth of the original mass, so its volume is also one-eighth:

$$\frac{r^3}{R^3} = \frac{1}{8}, \qquad r = \frac{R}{2}.$$

Its central moment of inertia is therefore

$$I_1 = \frac{2}{5}\left(\frac{M}{8}\right)\left(\frac{R}{2}\right)^2 = \frac{MR^2}{80}.$$

For a sphere, every axis through the centre is equivalent, so no further orientation factor is needed.

Step 2: Use the diameter formula for the disc The remaining mass is $m_d = 7M/8$, and the disc radius is $a = 2R$. In the thin-disc treatment,

$$I_2 = \frac{1}{4}\left(\frac{7M}{8}\right)(2R)^2 = \frac{7}{8}MR^2, \qquad \frac{I_2}{I_1} = \frac{(7/8)MR^2}{MR^2/80} = 70.$$

The value 140 would use the perpendicular central axis, whose disc inertia is twice the diameter inertia. Values 35 and 210 have neither the stated mass allocation nor the required diameter factor.

Step 3: Identify the role of the stated thickness The listed answer uses the standard thin-disc approximation. If the nonzero thickness is retained exactly, volume conservation gives

$$\pi(2R)^2 t = \frac{7}{8}\frac{4\pi R^3}{3}, \qquad t = \frac{7R}{24}.$$

Then $I_2 = (7M/8)(R^2 + t^2/12)$, and the ratio becomes $70(1 + 49/6912) \approx 70.496$, which is not among the options. Thus 70 is the intended disc-model value; it is not the exact finite-thickness cylinder result.

Step 4: Conclusion Using the thin-disc model implicit in the options, $\boxed{I_2/I_1 = 70}$. Hence, the correct option is (B). Retaining finite thickness gives approximately 70.496, so the listed result entails neglecting that correction.

Quick Tip to Solve: Distinguish a disc diameter from its perpendicular central axis, and state the thin-disc approximation when a thickness is also mentioned.

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31. The two projectiles are projected with the same initial velocities at the 15° and 30° with respect to the horizontal. The ratio of their ranges is $1 : x$. The value of x is [4 marks]

(A) $\sqrt{2}$

(B) $\sqrt{3}$

(C) $2\sqrt{3}$

(D) $\frac{1}{\sqrt{2}}$

Correct Answer: (B) $\sqrt{3}$

Solution:

Concept:

- For a projectile launched and landing at the same horizontal level, the horizontal range is $R = u^2 \sin(2\theta)/g$, where u is the launch speed and θ is the angle of projection.
- With the same speed and gravitational acceleration, ranges are proportional to $\sin(2\theta)$. The angle must be doubled before evaluating the sine.

Step 1: Write the two ranges Let R_{15} and R_{30} be the ranges for launch angles 15° and 30°, respectively. Using the same value of u in both cases gives

$$R_{15} = \frac{u^2 \sin 30^\circ}{g} = \frac{u^2}{2g}, \qquad R_{30} = \frac{u^2 \sin 60^\circ}{g} = \frac{\sqrt{3}u^2}{2g}.$$

The larger angle here increases the range because both angles lie below the angle of maximum range, 45°.

Step 2: Normalize the range ratio Cancel the common factor $u^2/(2g)$:

$$R_{15} : R_{30} = 1 : \sqrt{3}.$$

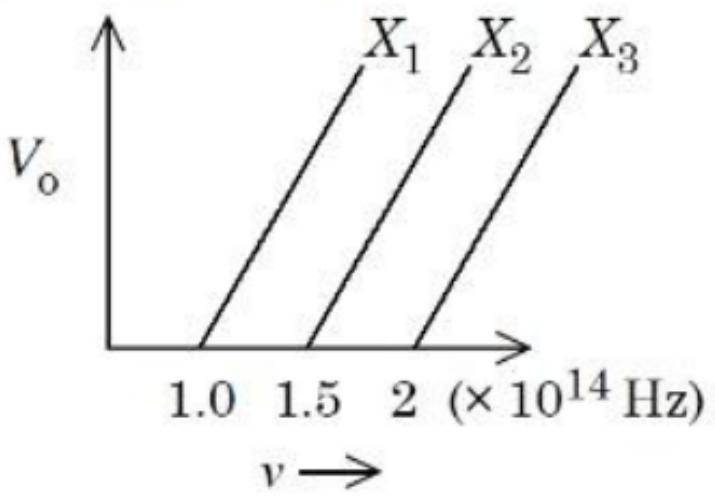
The given ratio is $1 : x$, so $x = R_{30}/R_{15} = \sqrt{3}$, not its reciprocal. Option (A), $\sqrt{2}$, does not equal the ratio of the two sine values; option (C) introduces an extra factor of two; option (D) would predict a smaller range for the second projectile and is inconsistent with the calculation.

Step 3: Conclusion The dimensionless value is $\boxed{x = \sqrt{3}}$. Hence, the correct option is (B).

Quick Tip to Solve: For equal launch speeds, cancel u^2/g first and compare $\sin 2\theta$; keep the order of the ratio exactly as asked.

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32. The graph shows variation of stopping potential V_o with the frequency v of the incident radiation for three photosensitive metals X_1, X_2 and X_3. Which metal will give out electrons with greater kinetic energy, for the same wavelength of incident radiation? [4 marks]



(A) X_1

(B) X_2

(C) X_3

(D) All the metals will give out photo electrons with same kinetic energies.

Correct Answer: (A) X_1

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi = eV_0$, where ϕ is the metal's work function and e is the magnitude of electron charge.
- The threshold frequency $\nu_{\text{th}} = \phi/h$ is the horizontal-axis intercept of a stopping-potential graph. All such lines have the common slope h/e.

Step 1: Compare the work functions from the graph The horizontal intercepts for X_1, X_2, X_3 are 1.0, 1.5 and 2.0, respectively, in units of 10^{14} Hz. Thus

$$\nu_{\text{th},1} < \nu_{\text{th},2} < \nu_{\text{th},3}, \qquad \phi_1 < \phi_2 < \phi_3.$$

Metal X_1 requires the least energy to release an electron. The parallel lines do not mean equal stopping potentials: their different intercepts represent different work functions.

Step 2: Compare the electron energies at a common frequency The same incident wavelength gives the same frequency, $\nu = c/\lambda$, and hence the same photon energy $h\nu$. Subtracting the smallest work function leaves the greatest maximum kinetic energy:

$$K_{\max,1} = h\nu - \phi_1 > K_{\max,2} > K_{\max,3}$$

when all three emit. Equivalently, a vertical line at a common frequency meets the X_1 graph at the greatest stopping potential. Options (B) and (C) have larger energy costs of emission. Option

(D) confuses equal incident photon energies with equal outgoing electron energies. Below a metal's threshold it does not emit at all.

Step 3: Conclusion Metal $\boxed{X_1}$ gives photoelectrons with the greatest maximum kinetic energy. Hence, the correct option is (A).

Quick Tip to Solve: On a stopping-potential versus frequency plot, the leftmost threshold belongs to the smallest work function and gives the largest $K_{\max}$ at a common frequency.

33. A slit of width a is illuminated by light of wavelength λ. The linear separation between 1^{st} and 3^{rd} minima in the diffraction pattern produced on a screen placed at a distance D from the slit system is ____ . [4 marks]

(A) $\frac{D\lambda}{a}$

(B) $1.5\frac{D\lambda}{a}$

(C) $2\frac{D\lambda}{a}$

(D) $3\frac{D\lambda}{a}$

Correct Answer: (C) $2\frac{D\lambda}{a}$

Solution:

Concept:

- Dark fringes for a single slit of width a satisfy $a\sin\theta_m = m\lambda$, where $m = 1, 2, 3, \ldots$ counts minima outward on either side of the central maximum.
- In the usual small-angle diffraction arrangement, $y_m \simeq D\theta_m = mD\lambda/a$. Here y_m is measured from the centre of the pattern on the screen.

Step 1: Locate the first and third minima For the minima on the same side of the central maximum, take the positive orders $m = 1$ and $m = 3$. Their screen positions are

$$y_1 = \frac{D\lambda}{a}, \qquad y_3 = \frac{3D\lambda}{a}.$$

The small-angle relation uses $\sin\theta \simeq \tan\theta$, so these are linear distances on the screen, rather than angular separations. Both positions are measured from the same reference point.

Step 2: Subtract the positions and compare the choices The separation is the difference of the two positions:

$$\Delta y = y_3 - y_1 = \frac{(3-1)D\lambda}{a} = \frac{2D\lambda}{a}.$$

Option (A) is the spacing between consecutive minima, so it counts only one interval instead of two. Option (B) is not the difference of the integer orders. Option (D) gives the distance of the third minimum from the centre, rather than from the first minimum. Opposite-side minima would have their distances added, but the first-to-third order spacing here uses the same side. The result has units of length, since $D\lambda/a$ is a length.

Step 3: Conclusion The required linear separation is $\boxed{2D\lambda/a}$. Hence, the correct option is (C).

Quick Tip to Solve: For two same-side single-slit minima, multiply $D\lambda/a$ by the difference of their orders; do not measure the farther minimum from the centre.

34. A string A of length 0.314 m and Young's modulus $2 \times 10^{10}\ \mathrm{N/m^2}$ is connected to another string B of length and Young's modulus both twice of those of A. This series combination of strings is then suspended from a rigid support and its free end is fixed to a load of mass 0.8 kg . The net change in length of the combination is ____ mm.

(radius of both the strings is 0.2 mm and acceleration due to gravity $= 10\ \mathrm{m/s^2}$) (Mass of both strings is to be neglected as compared to the mass of load) [4 marks]

(A) 3

(B) 2

(C) 1.9

(D) 1

Correct Answer: (B) 2

Solution:

Concept:

- The extension of a uniform elastic string is $\Delta L = FL/(SY)$, where F is tensile force, L is original length, S is cross-sectional area and Y is Young's modulus.
- Massless strings joined in series carry the same tension $F = mg$. The total extension is the sum of their individual extensions.

Step 1: Find the common tension and area The load is supported by both strings in series, so the tension in each is

$$F = 0.8 \times 10 = 8\,\mathrm{N}.$$

The stated radius is $r = 0.2\,\mathrm{mm} = 2 \times 10^{-4}\,\mathrm{m}$. Using $\pi \simeq 3.14$, the area is

$$S = \pi r^2 = 3.14(2 \times 10^{-4})^2 = 1.256 \times 10^{-7}\,\mathrm{m^2}.$$

Step 2: Calculate and add the extensions For string A,

$$\Delta L_A = \frac{8(0.314)}{(1.256 \times 10^{-7})(2 \times 10^{10})} = 10^{-3}\,\mathrm{m} = 1\,\mathrm{mm}.$$

For string B, both length and Young's modulus double while the tension and cross-sectional area remain the same. Therefore

$$\Delta L_B = \frac{F(2L_A)}{S(2Y_A)} = \Delta L_A = 1\,\mathrm{mm}, \qquad \Delta L_{\text{total}} = 2\,\mathrm{mm}.$$

Option (D) counts just one string. Option (A) would result from doubling the second length but forgetting its doubled modulus. Option (C) is inconsistent with the given values; using a more precise value of π still gives approximately $2.00\,\mathrm{mm}$, not $1.9\,\mathrm{mm}$.

Step 3: Conclusion The net increase in length is $\boxed{2\,\mathrm{mm}}$. Hence, the correct option is (B).

Quick Tip to Solve: For equal-area strings in series, compare L/Y. Doubling both length and Young's modulus leaves the extension unchanged.

35. One gas of n_1 mole of molecules at temperature T_1, volume V_1, and pressure P_1, and another gas of n_2 mole of molecules at temperature T_2, volume V_2, and pressure P_2, are mixed resulting in pressure P and volume V of the mixture. The temperature of the mixture is ____ . [4 marks]

(A) $\frac{(T_1+T_2)}{2}$

(B) $\frac{T_1T_2PV}{T_2P_1V_1+T_1P_2V_2}$

(C) $\frac{(T_2P_1V_1+T_1P_2V_2)}{T_1T_2PV}$

(D) $\frac{[T_1-T_2]}{2}$

Correct Answer: (B) $\frac{T_1T_2PV}{T_2P_1V_1+T_1P_2V_2}$

Solution:

Concept:

- Each ideal-gas sample obeys $PV = nRT$, where n is the amount in moles, R is the molar gas constant and T is absolute temperature.
- For a non-reacting mixture the total amount is conserved: $n = n_1 + n_2$. The final pressure and volume are the specified P, V, so its temperature follows from the final ideal-gas equation.

Step 1: Express each amount using its initial state Apply the ideal-gas equation separately before mixing:

$$n_1 = \frac{P_1V_1}{RT_1}, \qquad n_2 = \frac{P_2V_2}{RT_2}.$$

Consequently,

$$n_1 + n_2 = \frac{1}{R}\left(\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2}\right).$$

There is no reason to replace this sum by twice either amount: the initial sample sizes and temperatures need not be equal.

Step 2: Use the final state of the mixture For the mixture, $PV = (n_1 + n_2)RT$. Substitute the preceding expression and cancel R:

$$T = \frac{PV}{P_1V_1/T_1 + P_2V_2/T_2} = \frac{T_1T_2PV}{T_2P_1V_1 + T_1P_2V_2}.$$

Option (A), the arithmetic mean, needs additional special conditions and is not general. Option (C) is the reciprocal of the required expression and has units of inverse temperature. Option (D) can be zero for equal initial temperatures and cannot represent the general final absolute temperature. The selected expression has units of kelvin and remains valid without specifying a heat-transfer path, because the final P, V are already given.

Step 3: Conclusion The final temperature is $\boxed{T = \frac{T_1T_2PV}{T_2P_1V_1 + T_1P_2V_2}}$. Hence, the correct option is (B).

Quick Tip to Solve: When the final pressure and volume of an ideal-gas mixture are given, add the moles from the initial states before applying $PV = nRT$.

36. An ideal gas undergoes a process maintaining relation between pressure (P) and volume(V) as $P = P_o\left(1 + \left(\frac{V_o}{V}\right)^2\right)^{-1}$, where P_o and V_o are constants. If two samples A and B (two moles each) with initial volumes V_o and $3V_o$ respectively undergo above mentioned process and attain same pressure, then the difference at the temperatures of these samples, $T_B - T_A$ is ____ .

(R = gas constant) [4 marks]

(A) $\frac{9P_oV_o}{8R}$

(B) $\frac{11P_oV_o}{10R}$

(C) $\frac{7P_oV_o}{6R}$

(D) $\frac{13P_oV_o}{11R}$

Correct Answer: No listed option is correct: equal attained pressure fixes equal volume and hence equal temperature; the official key dropped the question.

Solution:

Concept:

- The prescribed process can be written as $P = P_0V^2/(V^2 + V_0^2)$. For positive volumes and the same constants, a given pressure fixes a unique volume.
- Each sample contains two moles, so its instantaneous temperature is $T = PV/(2R)$. Compare the states asked for in the question, rather than silently replacing final states by initial ones.

Step 1: Apply the common final-pressure condition Let the common attained pressure be p, and let the final volumes be V_A and V_B. Inverting the process relation gives

$$p(V^2 + V_0^2) = P_0V^2, \qquad V^2 = \frac{pV_0^2}{P_0 - p}.$$

For finite positive physical volumes, $0 < p < P_0$, and the positive square root is unique. Since both samples have the same constants, they must have $V_A = V_B$ when their pressures are equal.

Step 2: Compare their temperatures and identify the distractor Both amounts are two moles, so at that common pressure

$$T_B - T_A = \frac{p(V_B - V_A)}{2R} = 0.$$

None of options (A)–(D) equals zero. To see why option (B) may look tempting, calculate the temperatures at the stated initial volumes instead:

$$P_{A,i} = \frac{P_0}{2}, \quad P_{B,i} = \frac{9P_0}{10}, \quad T_{A,i} = \frac{P_0V_0}{4R}, \quad T_{B,i} = \frac{27P_0V_0}{20R}.$$

Their initial difference is $11P_0V_0/(10R)$, but their initial pressures are unequal. That result does not answer the printed equal-pressure condition. The other three positive expressions also cannot represent the final difference.

Step 3: Conclusion The printed equal-pressure condition gives $\boxed{T_B - T_A = 0}$, so no listed option is correct. The official answer key dropped this question and awarded bonus marks to all candidates.

Quick Tip to Solve: First identify which states are being compared. A common one-to-one $P(V)$ relation gives equal volumes at equal pressures, and equal mole numbers then give equal temperatures.

37. A voltmeter with internal resistance of $x\Omega$ can be used to measure upto 20 V . In order to increase its measuring range to 30 V , the required modification is to ____ . [4 marks]

(A) connect resistor of $\frac{x}{2}\Omega$, in series with voltmeter.

(B) connect resistor of $\frac{x}{2}\Omega$, in parallel to voltmeter.

(C) connect a resistor of $x\Omega$ in series with voltmeter.

(D) connect resistor of $2x\Omega$ in parallel to voltmeter.

Correct Answer: (A) connect resistor of $\frac{x}{2}\Omega$, in series with voltmeter.

Solution:

Concept:

- A voltmeter reaches full-scale deflection at a fixed current I_g. Its voltage range is the product of that current and the total resistance in its measuring branch.
- To measure a larger voltage without increasing the current through the instrument, a multiplier resistance is connected in series. A parallel resistance does not divide the applied voltage.

Step 1: Find the original full-scale current The internal resistance is $x\,\Omega$, and the original maximum reading is $20\,\text{V}$. Thus the largest permitted current through the voltmeter is

$$I_g = \frac{20}{x}\,\text{A}.$$

This current must remain unchanged after the range is extended, because the same instrument and full-scale deflection are being used.

Step 2: Calculate the required series resistance Let the added series resistance be R_s. At the new full-scale voltage of $30\,\text{V}$, the series combination must satisfy

$$30 = I_g(x + R_s) = \frac{20}{x}(x + R_s).$$

Consequently,

$$x + R_s = \frac{3x}{2}, \qquad R_s = \frac{x}{2}\,\Omega.$$

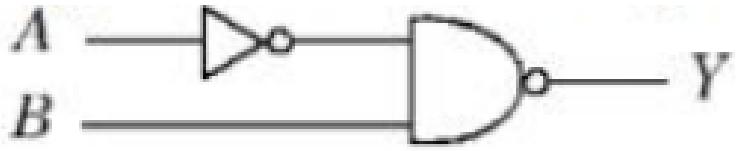
The voltmeter itself still has $20\,\text{V}$ across it, while the added resistor drops the remaining $10\,\text{V}$.

Step 3: Check the alternative connections Adding $x\,\Omega$ in series would double the original range to $40\,\text{V}$, so option (C) is unsuitable. Either parallel connection in (B) or (D) leaves the full applied voltage across the voltmeter; it therefore cannot protect its movement or extend its voltage range.

Step 4: Conclusion Connect a resistance $\boxed{x/2\,\Omega}$ in series with the voltmeter. Hence, the correct option is (A).

Quick Tip to Solve: For a voltmeter range multiplier $m = V_{\text{new}}/V_{\text{old}}$, add $R_s = (m-1)R_v$ in series.

38. Two 4 bits binary numbers, $A = 1101$ and $B = 1010$ are given in the inputs of a logic circuit shown in figure below. The output (Y) will be : [4 marks]



(A) $Y = 1101$

(B) $Y = 0010$

(C) $Y = 0111$

(D) $Y = 1000$

Correct Answer: (A) $Y = 1101$

Solution:

Concept:

- A NOT gate complements its input bit: 0 becomes 1, and 1 becomes 0. A NAND gate gives the complement of the AND of its two input bits.
- The gates operate on corresponding bit positions independently. By De Morgan's law, $\overline{\overline{A} \cdot B} = A + \overline{B}$, where the plus sign denotes logical OR, not binary addition.

Step 1: Read the gates in their printed order The upper input A first passes through an inverter, so the upper input to the final gate is $\overline{A}$. The lower input B reaches that gate directly. Its AND-shaped body and output bubble identify it as a NAND gate. Hence

$$Y = \overline{\overline{A} \cdot B}.$$

The output bubble must be included; omitting it would give the intermediate AND result instead.

Step 2: Evaluate all four bit positions Keep the bit positions aligned from left to right. Complementing $A = 1101$ gives $\overline{A} = 0010$. The bitwise operations are

$$\overline{A} \cdot B = 0010 \text{ AND } 1010 = 0010,$$
$$Y = \overline{0010} = 1101.$$

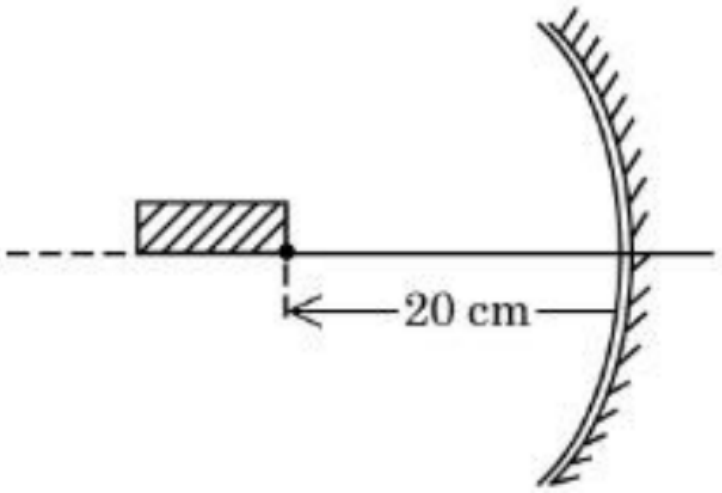
For the four input pairs $(1, 1), (1, 0), (0, 1), (1, 0)$, the outputs are respectively $1, 1, 0, 1$. There is no carry between positions because this is a logic-gate operation, not addition.

Step 3: Rule out the other outputs Option (B), 0010, is the AND result before the final inversion. Option (C), 0111, would be obtained from an exclusive-OR operation on the original inputs, which is not the printed circuit. Option (D), 1000, is their direct AND and ignores both inversion operations.

Step 4: Conclusion The four-bit output is $\boxed{Y = 1101}$. Hence, the correct option is (A).

Quick Tip to Solve: Write the Boolean expression from the gate symbols before manipulating binary strings, and complement every output that carries a bubble.

39. A rod of length 10 cm lies along the principle axis of a concave mirror of focal length 10 cm as shown in figure. The length of the image is ____ cm. [4 marks]



(A) 2.5

(B) 5

(C) 7.5

(D) 7

Correct Answer: (B) 5

Solution:

Concept:

- For a spherical mirror, the Cartesian mirror formula is $1/f = 1/u + 1/v$, where u and v are signed distances from the pole.
- A rod lying along the principal axis has its two ends at different object distances. Find the image of each end separately; the image length is the absolute difference of the two image coordinates.

Step 1: Identify the two endpoint distances The figure places the nearer end of the rod $20\,\text{cm}$ in front of the mirror. Since the rod extends another $10\,\text{cm}$ away from it, the farther end is $30\,\text{cm}$ from the pole. Taking incident light to travel towards the mirror, the Cartesian distances are

$$f = -10\,\text{cm}, \qquad u_1 = -20\,\text{cm}, \qquad u_2 = -30\,\text{cm}.$$

The marked $20\,\text{cm}$ is measured to the nearer end, not to the centre of the rod.

Step 2: Find the image of each end For the nearer end, the mirror formula gives

$$\frac{1}{v_1} = \frac{1}{f} - \frac{1}{u_1} = -\frac{1}{10} + \frac{1}{20} = -\frac{1}{20}, \qquad v_1 = -20\,\text{cm}.$$

For the farther end, it gives

$$\frac{1}{v_2} = -\frac{1}{10} + \frac{1}{30} = -\frac{1}{15}, \qquad v_2 = -15\,\text{cm}.$$

Both images are real and in front of the mirror. The end farther from the mirror produces the image

closer to the focus.

Step 3: Take the image separation The image length along the axis is

$$L_{\text{image}} = |v_2 - v_1| = |-15-(-20)| = 5\,\text{cm}.$$

A single transverse magnification cannot be applied to the entire longitudinal rod, because its object distance changes along its length. The endpoint images fix the separation at $5\,\text{cm}$, excluding 2.5, 7.5 and $7\,\text{cm}$.

Step 4: Conclusion The image of the rod is $\boxed{5\,\text{cm}}$ long. Hence, the correct option is (B).

Quick Tip to Solve: For an object extending along the principal axis, image its two endpoints separately; do not multiply its full length by a transverse magnification.

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40. A parallel plate air capacitor is connected to a battery. The plates are pulled apart at uniform speed v. If x is the separation between the plates at any instant, then the time rate of change of electrostatic energy of the capacitor is proportional to x^{α}, where α is ____ . [4 marks]

(A) -2

(B) 1

(C) -1

(D) 2

Correct Answer: (A) -2

Solution:

Concept:

- A parallel-plate air capacitor of plate area S and separation x has capacitance $C = \varepsilon_0 S/x$, neglecting fringing at the edges.
- The connected battery keeps the potential difference V constant while charge can flow between the battery and the plates. Its stored electrostatic energy is therefore $U = CV^2/2$.

Step 1: Express the energy in terms of separation Substitute the capacitance into the energy formula while holding the battery voltage fixed:

$$U(x) = \frac{1}{2}\frac{\varepsilon_0 S}{x}V^2 = \frac{\varepsilon_0 S V^2}{2}x^{-1}.$$

Thus the energy itself decreases inversely with the separation. The plate area, air permittivity and voltage do not change as the plates are pulled apart. The charge is not constant: $Q = CV = \varepsilon_0 SV/x$ decreases as the separation increases.

Step 2: Differentiate with respect to time The plates separate uniformly, so $dx/dt = v$. Use the chain

rule rather than identifying the requested exponent directly from the energy:

$$\frac{dU}{dt} = \frac{dU}{dx}\frac{dx}{dt} = -\frac{\varepsilon_0 S V^2}{2x^2}\, v.$$

All factors except x^{-2} are constants. The negative sign indicates a decrease in the stored field energy; the magnitude of the rate also varies as x^{-2}.

Step 3: Compare the possible exponents The exponent -1 in option (C) describes the energy, not its time derivative. Positive exponents 1 and 2 in (B) and (D) do not follow from differentiating the constant-voltage energy. Using a fixed-charge energy expression here would be inappropriate because the battery remains connected.

Step 4: Conclusion Since $dU/dt \propto x^{-2}$, $\boxed{\alpha = -2}$. Hence, the correct option is (A).

Quick Tip to Solve: First identify whether voltage or charge stays constant; then write $U(x)$ and use $dU/dt = (dU/dx)(dx/dt)$.

41. An insulated wire is wound so that it forms a flat coil with $N = 200$ turns. The radius of the innermost turn is $r_1 = 3$ cm, and of the outermost turn $r_2 = 6$ cm. If 20 mA current flows in it then the magnetic moment will be $\alpha \times 10^{-2}$ A.m^2. The value of α is ____ . [4 marks]

(A) 4.4

(B) 2.64

(C) 3.25

(D) 1.2

Correct Answer: (B) 2.64

Solution:

Concept:

- The magnetic moment of one circular turn carrying current I is $IA = I\pi r^2$. The moments of the coplanar turns point in the same direction and add.
- For a uniformly wound flat coil, the turn density in the radial direction is $N/(r_2 - r_1)$. Since the radii vary, the areas must be summed; using the outer area for every turn overestimates the moment.

Step 1: Sum the areas of the turns Let dN be the number of turns in a narrow radial interval dr. For the closely spaced winding,

$$dN = \frac{N}{r_2 - r_1}\, dr.$$

The magnetic moment of those turns is $dm = I\pi r^2 dN$. Integrating across the coil gives

$$\begin{aligned} m &= \frac{NI\pi}{r_2 - r_1}\int_{r_1}^{r_2} r^2\,dr \\ &= \frac{NI\pi}{3}(r_1^2 + r_1 r_2 + r_2^2). \end{aligned}$$

The factor in brackets accounts for all the different turn areas rather than treating the flat winding as N identical circles.

Step 2: Convert the data and substitute Use SI units: $I = 20\,\text{mA} = 0.020\,\text{A}$, $r_1 = 0.030\,\text{m}$ and $r_2 = 0.060\,\text{m}$. Then

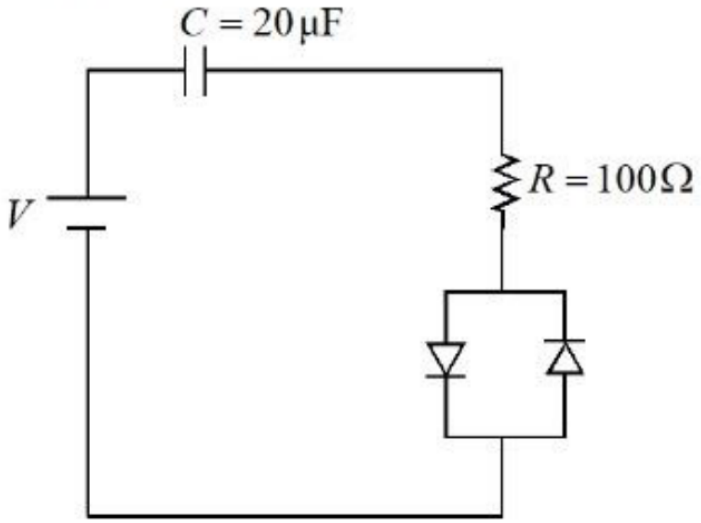
$$\begin{aligned} r_1^2 + r_1 r_2 + r_2^2 &= 0.0009 + 0.0018 + 0.0036 \\ &= 0.0063\,\text{m}^2, \\ m &= \frac{200(0.020)\pi}{3}(0.0063) \\ &= 0.0084\pi\,\text{A m}^2 \\ &\simeq 2.64 \times 10^{-2}\,\text{A m}^2. \end{aligned}$$

Step 3: Identify the coefficient Comparing with $m = \alpha \times 10^{-2}\,\text{A m}^2$ yields $\alpha = 2.64$. Option (A) is close to the overestimate obtained by assigning the outer radius to all turns, and option (D) is near the inner-radius underestimate. Option (C) also differs from the required sum of the turn areas.

Step 4: Conclusion The magnetic moment is approximately $2.64 \times 10^{-2}\,\text{A m}^2$, so $\boxed{\alpha = 2.64}$. Hence, the correct option is (B).

Quick Tip to Solve: For uniformly spaced radii, use the average $r^2 = (r_1^2 + r_1 r_2 + r_2^2)/3$, not the square of the average radius.

42. Consider a circuit consisting of a capacitor $(20\mu\ \mathrm{F})$, resistor (100Ω) and two identical diodes as shown in figure. The resistance of diode under forward biasing condition is 10Ω. The time constant of the circuit is $\alpha \times 10^{-3}$ s. The value of α is ____ [4 marks]



(A) 2.2

(B) 2.0

(C) 2.1

(D) 2.4

Correct Answer: (A) 2.2

Solution:

Concept:

- For a capacitor charging through an effective series resistance R_{eff}, the time constant is $\tau = R_{\text{eff}}C$.
- A forward-biased diode contributes its stated resistance, whereas a reverse-biased diode is treated as an open circuit here. Two oppositely oriented parallel diodes do not both conduct for a single applied polarity.

Step 1: Determine which diode carries current The longer battery line is the positive terminal, at the top of the battery in the figure. During charging, conventional current goes through the capacitor branch and the $100\,\Omega$ resistor, then down through the diode combination. The left diode is oriented to conduct downwards and is forward biased; the right diode points oppositely and is reverse biased. Only the forward diode supplies a conducting path through this part of the circuit.

Step 2: Find the resistance seen by the capacitor The conducting diode has resistance $10\,\Omega$. It is in series with the $100\,\Omega$ resistor in the charging path, while the reverse-biased diode carries no current. Thus

$$R_{\text{eff}} = 100 + 10 = 110\,\Omega.$$

The forward diode resistance is specified as a fixed value in the question, so it is included directly in the RC model. The two diode resistances must not be combined in parallel as if both devices were ordinary conducting resistors.

Step 3: Calculate the time constant and compare Convert the capacitance to farads and multiply:

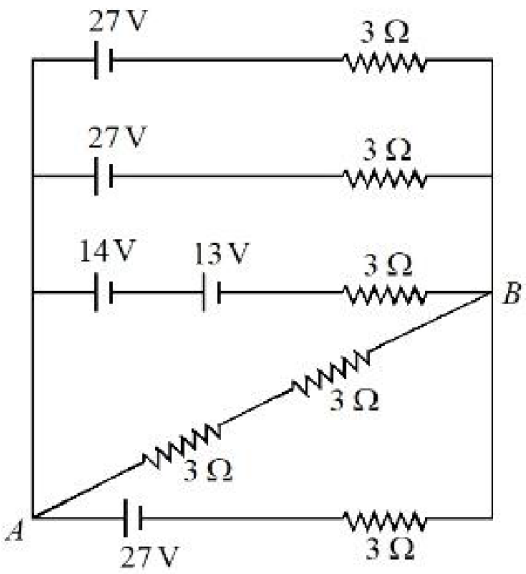
$$\begin{aligned} C &= 20 \times 10^{-6}\,\text{F}, \\ \tau &= (110)(20 \times 10^{-6})\,\text{s} \\ &= 2.2 \times 10^{-3}\,\text{s}. \end{aligned}$$

Ignoring the diode resistance would give $2.0\,\text{ms}$, option (B). Treating both diodes as conducting in parallel gives $2.1\,\text{ms}$, option (C), and incorrectly placing both diode resistances in series gives $2.4\,\text{ms}$, option (D). None matches the actual conducting path.

Step 4: Conclusion The time constant is $\boxed{2.2 \times 10^{-3}\,\text{s}}$, hence $\boxed{\alpha = 2.2}$. Hence, the correct option is (A).

Quick Tip to Solve: Identify the forward-biased path before simplifying a diode network; then include its stated resistance when calculating RC.

43. The voltage and the current between A and B points shown in the circuit are ____ . [4 marks]



(A) $24\ \text{V}, 12\ \text{A}$

(B) $24\ \text{V}, 4\ \text{A}$

(C) $18\ \text{V}, 12\ \text{A}$

(D) $27\ \text{V}, 4\ \text{A}$

Correct Answer: (B) $24\ \text{V}, 4\ \text{A}$

Solution:

Concept:

- Ideal connecting wires make all points on the left rail equipotential with A and all points on the right rail equipotential with B. Thus branches connecting these rails are in parallel.

- Identical cells with series resistance, connected in parallel with the same polarity, retain their emf while their equivalent resistance decreases. The load current is determined by the complete circuit.

Step 1: Identify the four identical source branches The upper two and lowest horizontal branches each contain a $27\,\text{V}$ source and a $3\,\Omega$ resistor. In the remaining horizontal branch, the $14\,\text{V}$ and $13\,\text{V}$ cells aid each other, giving $14 + 13 = 27\,\text{V}$. Its series resistance is also $3\,\Omega$. All four sources have their positive terminals toward A. Their equivalent emf and resistance are therefore

$$E = 27\,\text{V}, \qquad r = \frac{3}{4}\,\Omega.$$

Step 2: Calculate the diagonal current and terminal voltage The two diagonal resistors lie in series along the A-to-B branch, so the load is $R = 3 + 3 = 6\,\Omega$. The current through this branch is

$$I = \frac{E}{R + r} = \frac{27}{6 + 3/4} = 4\,\text{A}.$$

Hence $V_A - V_B = IR = 4 \times 6 = 24\,\text{V}$. Each source branch supplies $(27 - 24)/3 = 1\,\text{A}$, confirming that their total current is $4\,\text{A}$. A current of $12\,\text{A}$ is inconsistent with the load resistance; $18\,\text{V}$ fails the junction equation, and $27\,\text{V}$ is the unloaded emf, not the terminal voltage with current flowing.

Step 3: Conclusion The voltage across A and B is $24\,\text{V}$, and the diagonal current is $4\,\text{A}$, from A to B. Hence, the correct option is (B).

Quick Tip to Solve: First combine the two series cells, then replace the four identical source branches by one source with resistance divided by four.

44. A telescope with objective diameter R is used to observe a distant star emitting light of wavelength 500 nm , at a resolution of 5×10^{-7} radian. The value of R is

____ cm . [4 marks]

(A) 61

(B) 122

(C) 244

(D) 305

Correct Answer: (B) 122

Solution:

Concept:

- Diffraction at a circular objective produces an Airy pattern. By the Rayleigh criterion, the smallest resolvable angular separation is $\theta_{\text{min}} = 1.22\lambda/D$, where D is the objective diameter.
- The wavelength and diameter must be expressed in the same length unit. Increasing the aperture diameter decreases the limiting angular separation and improves resolution.

Step 1: Identify the diameter and convert the wavelength The question uses R for the objective's diameter, despite the common use of that letter for a radius. Therefore set $D = R$, without inserting

an additional factor of two. The given quantities in SI units are

$$\lambda = 500 \times 10^{-9}\,\text{m} = 5 \times 10^{-7}\,\text{m}, \qquad \theta_{\text{min}} = 5 \times 10^{-7}\,\text{rad}.$$

Step 2: Solve the circular-aperture resolution equation Rearranging the Rayleigh formula gives

$$R = \frac{1.22\lambda}{\theta_{\text{min}}} = \frac{1.22 \times 5 \times 10^{-7}}{5 \times 10^{-7}} = 1.22\,\text{m} = 122\,\text{cm}.$$

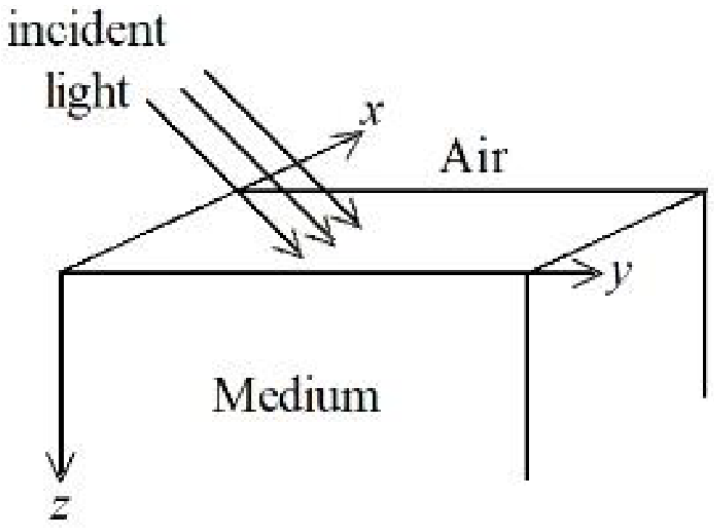
This aperture gives exactly the stated angular resolution. A diameter of $61\,\text{cm}$ gives twice the stated limiting angle, while $244\,\text{cm}$ gives half that angle. A diameter of $305\,\text{cm}$ gives $2 \times 10^{-7}\,\text{rad}$. Thus those alternatives do not yield the specified value of the resolution.

Step 3: Conclusion The required objective diameter is $R = 122\,\text{cm}$. Hence, the correct option is (B).

Quick Tip to Solve: In telescope questions, the factor 1.22 belongs to a circular aperture; read whether the stated symbol denotes radius or diameter before substituting.

45. An unpolarized light is incident on the plane interface of air-dielectric medium shown in figure. If the incident angle is equal to Brewster angle, identify the expression representing reflected wave.

[4 marks]



(A) $\left(E_x\hat{i} + E_y\hat{j}\right)\sin(kx - kz - \omega t)$

(B) $\left(E_x\hat{i} + E_z\hat{k}\right)\sin(kx + ky - \omega t)$

(C) $\left(E_x\hat{j} + E_y\hat{k}\right)\sin(ky + kz - \omega t)$

(D) $\left(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}\right)\sin(kx + ky - kz - \omega t)$

Correct Answer: (A) $\left(E_x\hat{i} + E_y\hat{j}\right)\sin(kx - kz - \omega t)$

Solution:

Concept:

- At Brewster incidence, the reflected light is completely plane-polarized perpendicular to the plane of incidence. The component polarized parallel to that plane has zero reflected amplitude.
- For a plane wave with phase $\mathbf{k} \cdot \mathbf{r} - \omega t$, the spatial coefficients give its propagation direction. Its electric field obeys $\mathbf{k} \cdot \mathbf{E} = 0$.

Step 1: Determine the reflected propagation plane In the figure, the interface is the xy-plane and the positive z-axis points into the medium. The incident ray lies in the xz-plane. Reflection preserves its tangential propagation direction but reverses its normal component, so the reflected ray propagates toward positive x and negative z. The corresponding phase must involve x and $-z$, as in $kx - kz - \omega t$.

Step 2: Apply the Brewster polarization condition The electric field is perpendicular to the xz-plane, hence it is along $\hat{j}$. Of the listed expressions, (A) has the appropriate reflected phase and admits precisely this polarization when $E_x = 0$:

$$\mathbf{E}_r = E_y \hat{j} \sin(kx - kz - \omega t).$$

The zero x-component is essential: a nonzero E_x with that phase would violate transversality. Options (B) and (C) propagate in the wrong coordinate planes, and (D) includes a nonzero y-component of the wave vector, outside the plane of incidence. More generally the phase is $k_x x - k_z z - \omega t$; the option uses the same symbol k for its displayed spatial coefficients.

Step 3: Conclusion The reflected wave is polarized along $\hat{j}$ and propagates outward in the xz-plane. Option (A) represents it with $E_x = 0$. Hence, the correct option is (A).

Quick Tip to Solve: Read the wave's direction from its phase first, then enforce both perpendicular polarization at Brewster incidence and the transverse-wave condition.

Physics – Section B: Numerical Value Questions (20 marks)

46. A 1 kg block subjected to two simultaneous forces $(2\hat{i} + 3\hat{j} + 4\hat{k})$N and $(3\hat{i} - \hat{j} - 2\hat{k})$N is moved a distance of 25 m along $(3\hat{i} - 4\hat{j})$ direction. The work done in this process is ____ J. [4 marks]

Solution:

Concept:

- The total work done by simultaneous constant forces is $W = (\mathbf{F}_1 + \mathbf{F}_2) \cdot \mathbf{s}$, where $\mathbf{s}$ is the displacement vector.
- A direction vector gives orientation, not displacement magnitude. Divide it by its magnitude to obtain the unit direction before multiplying by the stated distance.

Step 1: Add the forces component by component Using the common Cartesian unit vectors, the

resultant of the two specified forces is

$$\begin{aligned}\mathbf{F} &= (2\hat{i} + 3\hat{j} + 4\hat{k}) + (3\hat{i} - \hat{j} - 2\hat{k}) \\ &= (5\hat{i} + 2\hat{j} + 2\hat{k})\,\text{N}.\end{aligned}$$

Work is a scalar, so it is this resultant dotted with displacement, rather than the sum of the magnitudes of the two forces, that is required.

Step 2: Normalize the given direction and find the work The magnitude of $3\hat{i} - 4\hat{j}$ is $\sqrt{3^2 + (-4)^2} = 5$. Therefore a displacement of magnitude $25\,\text{m}$ along it is

$$\mathbf{s} = 25\frac{3\hat{i} - 4\hat{j}}{5} = (15\hat{i} - 20\hat{j})\,\text{m}.$$

The work done is

$$W = 5(15) + 2(-20) + 2(0) = 75 - 40 = 35\,\text{J}.$$

The z-component does no work because there is no z-displacement. As a check, the separate works are $-30\,\text{J}$ and $65\,\text{J}$, whose sum is $35\,\text{J}$. The block's mass is unnecessary for this force-displacement calculation.

Step 3: Conclusion The total work done by the two specified forces is $35\,\text{J}$; the required numerical entry is 35.

Quick Tip to Solve: Normalize a stated direction vector before scaling it to the actual displacement; then use the dot product, keeping negative components negative.

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47. The surface tension of a soap solution is $3.5 \times 10^{-2}\ \text{N/m}$. The work required to increase the radius of a soap bubble from 1 cm to 2 cm is $\alpha \times 10^{-6}$ J. The value of α is ____ .

$$(\pi = 22/7)$$

[4 marks]

Solution:

Concept:

- Surface tension S equals surface energy per unit area. The work required to increase the surface area at constant surface tension is $W = S\Delta A$.
- A thin soap bubble has two liquid-air surfaces, inner and outer. Neglecting film thickness, its total area is $A = 2(4\pi r^2) = 8\pi r^2$.

Step 1: Calculate the increase in total surface area Express both radii in metres before squaring them:

$$r_1 = 1 \times 10^{-2}\,\text{m}, \qquad r_2 = 2 \times 10^{-2}\,\text{m}.$$

The change in squared radius is

$$r_2^2 - r_1^2 = (4-1) \times 10^{-4} = 3 \times 10^{-4}\,\mathrm{m}^2.$$

Consequently the increase in area of the two surfaces is $\Delta A = 8\pi(3 \times 10^{-4})\,\mathrm{m}^2$. Using only $4\pi\Delta(r^2)$ would count a single surface and underestimate the work by a factor of two.

Step 2: Convert the increase in area into work Substitute the stated surface tension and the supplied value of π:

$$\begin{aligned} W &= 8\pi S(r_2^2 - r_1^2) \\ &= 8 \times \frac{22}{7} \times 3.5 \times 10^{-2} \times 3 \times 10^{-4} \\ &= 264 \times 10^{-6}\,\mathrm{J}. \end{aligned}$$

The dimensions also check: $\mathrm{N\,m^{-1}}$ multiplied by m^2 gives $\mathrm{N\,m} = \mathrm{J}$. The result is positive because expansion creates additional surface and increases surface energy.

Step 3: Conclusion The required work is $264 \times 10^{-6}\,\mathrm{J}$, so the dimensionless coefficient is $\alpha = 264$.

Quick Tip to Solve: For a soap bubble use two surfaces, and subtract the squared radii; the square of the change in radius is not the change in squared radius.

48. The velocity of a particle executing simple harmonic motion along x-axis is described as $v^2 = 50 - x^2$, where x represents displacement. If the time period of motion is $\frac{x}{7}$ s, the value of x is ____ . [4 marks]

Solution:

Concept:

- For simple harmonic motion, $v^2 = \omega^2(A^2 - X^2)$, where A is amplitude, X is instantaneous displacement and ω is angular frequency.
- The period is $T = 2\pi/\omega$. It depends on the coefficient of the squared displacement in the velocity relation, not on the instantaneous displacement itself.

Step 1: Compare the coefficients in the velocity relation The printed question uses x both for displacement and for the numerical coefficient in the period. To distinguish them during the calculation, call displacement X and write the requested period as $T = n/7\,\mathrm{s}$. The given relation then reads

$$v^2 = 50 - X^2 = \omega^2 A^2 - \omega^2 X^2.$$

Comparing the coefficients of X^2 gives $\omega^2 = 1\,\mathrm{s}^{-2}$, hence $\omega = 1\,\mathrm{rad\,s}^{-1}$. The constant term fixes $A^2 = 50$ in the corresponding squared length unit; it is not the squared angular frequency.

Step 2: Find the period and the requested coefficient With the angular frequency established,

$$T = \frac{2\pi}{1}\,\mathrm{s} = 2\pi\,\mathrm{s}.$$

Equating this with the period's stated form yields

$$\frac{n}{7} = 2\pi, \qquad n = 14\pi \approx 43.9823.$$

Thus its integer numerical value is 44. The usual approximation $\pi \approx 22/7$ gives the same value directly. The exact period is about $6.2832\,\mathrm{s}$, which is consistent with $44/7\,\mathrm{s}$ to the accuracy of that approximation. The period remains fixed throughout the oscillation.

Step 3: Conclusion The time period is $T = 2\pi\,\mathrm{s}$. The coefficient denoted by x in the period is exactly 14π, giving the nearest-integer numerical entry 44.

Quick Tip to Solve: In $v^2 = C - DX^2$, read $\omega = \sqrt{D}$ from the displacement coefficient; use C only to determine the amplitude.

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49. A body of mass 2 kg begins to move under the influence of time dependent force $\vec{F} = \left(2t\hat{i} + 6t^2\hat{j}\right)$ N, where $\hat{i}$ and $\hat{j}$ are unit vectors along x and y -axis respectively. The power produced by the force at $t = 2$ s is ____ W. [4 marks]

Solution:

Concept:

- Newton's second law gives each acceleration component separately: $a_x = F_x/m$ and $a_y = F_y/m$. Velocity is obtained by integrating acceleration with respect to time.
- The instantaneous power transferred by a force is the scalar product $P = \vec{F} \cdot \vec{v} = F_x v_x + F_y v_y$, evaluated at the same instant. The words begins to move specify zero initial velocity.

Step 1: Integrate the two acceleration components For the mass $m = 2\,\mathrm{kg}$, division of the supplied force by the mass gives $a_x = t$ and $a_y = 3t^2$ in SI units. Since the body starts from rest at $t = 0$, both integration constants vanish. Thus

$$v_x(t) = \int_0^t t\,dt = \frac{t^2}{2}, \qquad v_y(t) = \int_0^t 3t^2\,dt = t^3.$$

The two components must be integrated separately because the direction of the force changes with time.

Step 2: Evaluate the force and velocity at two seconds Substituting $t = 2\,\mathrm{s}$ into the velocity components gives $v_x = 2\,\mathrm{m\,s^{-1}}$ and $v_y = 8\,\mathrm{m\,s^{-1}}$. At that same instant the force components are

$$F_x = 2(2) = 4\,\mathrm{N}, \qquad F_y = 6(2)^2 = 24\,\mathrm{N}.$$

The velocity and force are not parallel, so multiplying their magnitudes would give the wrong power.

Step 3: Take the scalar product Only corresponding components contribute to the scalar product because $\hat{i} \cdot \hat{j} = 0$. Therefore

$$P = (4)(2) + (24)(8) = 8 + 192 = 200\,\mathrm{W}.$$

Both terms are positive, in agreement with the increase in kinetic energy. Equivalently, differentiating $K = \frac{1}{2}m(v_x^2 + v_y^2)$ gives the same result.

Step 4: Conclusion The instantaneous power at $t = 2\,\text{s}$ is $\boxed{200\,\text{W}}$. The required numerical entry is 200.

Quick Tip to Solve: For a force specified as a function of time, integrate $\vec{F}/m$ first; use $\vec{F} \cdot \vec{v}$, not the product of magnitudes.

50. An inductor of 10 mH , capacitor of 0.1μ F and a resistor of 100Ω are connected in series across an $a.c$ power supply 220 V, 70 Hz. The power factor of the given circuit is 0.5 . The difference in the inductive reactance and capacitance reactance is $\sqrt{3}\alpha\Omega$. The value of α is ____ . [4 marks]

Solution:

Concept:

- For a series RLC circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where R is resistance and $X_L - X_C$ is the net reactance.
- The power factor is $\cos\phi = R/Z$. Its value determines the magnitude of the net reactance, but does not by itself specify whether the circuit is inductive or capacitive.

Step 1: Use the specified power factor to find the impedance The supplied power factor is 0.5, and the resistance is $100\,\Omega$. Hence

$$0.5 = \frac{100}{Z}, \qquad Z = 200\,\Omega.$$

The source voltage cancels from this relation: the ratio of real power to apparent power depends on the phase difference, not the amplitude of the supply.

Step 2: Find the magnitude of the reactance difference Rearranging the impedance relation gives

$$\begin{aligned} |X_L - X_C| &= \sqrt{Z^2 - R^2} \\ &= \sqrt{200^2 - 100^2} \\ &= \sqrt{30000} = 100\sqrt{3}\,\Omega. \end{aligned}$$

Comparing this positive difference with $\sqrt{3}\alpha\,\Omega$ gives $\alpha = 100$. Here α is dimensionless because the unit ohm is already written in the expression.

Step 3: Check the consistency of the extra component data The stated $L = 10\,\text{mH}$, $C = 0.1\,\mu\text{F}$ and $f = 70\,\text{Hz}$ independently give $X_L = 2\pi fL \approx 4.40\,\Omega$ and $X_C = 1/(2\pi fC) \approx 22736\,\Omega$. They would give a power factor about 0.0044, not 0.5. Thus these extra data are inconsistent with the explicitly supplied power factor. The value asked for follows from the specified power factor and resistance, as shown above; all the printed data cannot hold simultaneously.

Step 4: Conclusion Using the explicitly stated power factor 0.5, the required value is $\boxed{\alpha = 100}$, matching the numerical key. The listed component values and frequency are inconsistent with that power factor.

Quick Tip to Solve: When R and the power factor are given, use $|X_L - X_C| = R\sqrt{1/\cos^2\phi - 1}$; do not confuse the magnitude with the signed reactance.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Number of moles and number of molecules in 1.4187 L of SO_2 at STP respectively are [4 marks]

(A) $0.1266; 3.812 \times 10^{22}$

(B) $0.0633; 3.812 \times 10^{22}$

(C) $0.1266; 7.6238 \times 10^{22}$

(D) $0.0633; 7.6238 \times 10^{22}$

Correct Answer: (B) $0.0633; 3.812 \times 10^{22}$

Solution:

Concept:

- The amount of an ideal gas is $n = V/V_m$, where V_m is its molar volume at the stated temperature and pressure.
- The number of molecules is $N = nN_A$, with $N_A = 6.022 \times 10^{23}\,\text{mol}^{-1}$. Count molecules of SO_2, not individual oxygen atoms.

Step 1: Convert the gas volume into moles Using the conventional STP molar volume $V_m \approx 22.414\,\text{L mol}^{-1}$ appropriate to the options,

$$n = \frac{1.4187\,\text{L}}{22.414\,\text{L mol}^{-1}} \approx 0.0633\,\text{mol}.$$

The litre units cancel, leaving moles. The simpler rounded value $22.4\,\text{L mol}^{-1}$ gives the same selected answer to the precision needed here. The molecular mass of sulphur dioxide is unnecessary because a gas volume is supplied directly.

Step 2: Convert moles into the number of molecules Multiply the amount by Avogadro's constant:

$$N = (0.0633)(6.022 \times 10^{23}) = 3.812 \times 10^{22}.$$

Each counted entity is one SO_2 molecule. Its two oxygen atoms do not introduce a factor of two in this calculation; that factor would be needed only if the question asked for the number of oxygen atoms.

Step 3: Compare both entries in each option Option (B) has both $0.0633\,\text{mol}$ and 3.812×10^{22} molecules. Option (A) doubles the amount but leaves the molecular count unchanged, so its two entries are inconsistent. Option (C) doubles both required values. Option (D) has the correct amount but doubles the number of molecules. Matching only the first entry is therefore insufficient.

Step 4: Conclusion The sample contains $\boxed{0.0633\,\text{mol}}$ and $\boxed{3.812 \times 10^{22}}$ molecules of SO_2. Hence, the correct option is (B).

Quick Tip to Solve: For paired mole-and-molecule options, first find V/V_m, then check that the second entry equals the first multiplied by N_A.

52. What is the ratio of wave number of first line (lowest energy line) of Balmer series of H atomic spectrum to first line of its Brackett series? [4 marks]

(A) 5 : 1

(B) 5 : 0.81

(C) 5 : 1.75

(D) 5 : 27

Correct Answer: (B) 5 : 0.81

Solution:

Concept:

- The wave number of a hydrogen emission line is $\bar{\nu} = R_H(1/n_f^2 - 1/n_i^2)$, with $n_i > n_f$. Wave number is the reciprocal of wavelength.
- The first line of a series has the smallest energy difference, so its initial level is $n_i = n_f + 1$. Balmer lines terminate at $n_f = 2$, while Brackett lines terminate at $n_f = 4$.

Step 1: Write the first Balmer wave number The lowest-energy Balmer transition is $3 \to 2$; it is not a transition from infinity. Substitution into the Rydberg formula gives

$$\bar{\nu}_B = R_H\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R_H}{36}.$$

The series limit would instead set $1/n_i^2 = 0$, which is inappropriate for the first line requested.

Step 2: Write the first Brackett wave number The corresponding Brackett transition is $5 \to 4$. Therefore

$$\bar{\nu}_{Br} = R_H\left(\frac{1}{16} - \frac{1}{25}\right) = \frac{9R_H}{400}.$$

Both expressions refer to the same hydrogen atom, so the same Rydberg constant appears and will cancel when their ratio is formed.

Step 3: Simplify the ratio and compare the choices Taking the ratio in the order asked,

$$\bar{\nu}_B : \bar{\nu}_{Br} = \frac{5}{36} : \frac{9}{400} = 5 : \frac{324}{400} = 5 : 0.81.$$

Equivalently, the ratio as a single fraction is $500/81 \approx 6.173$. Option (A) gives 5, option (C) gives about 2.857 and option (D) gives about 0.185; none equals this value. Do not invert the result: that would compare wavelengths instead of wave numbers.

Step 4: Conclusion The required ratio is $\boxed{5 : 0.81}$. Hence, the correct option is (B).

Quick Tip to Solve: For the first line use adjacent levels: $3 \to 2$ for Balmer and $5 \to 4$ for Brackett. The common Rydberg constant cancels.

53. Which of the following is correct set of 4 quantum numbers of 19^{th} electron in Chromium (Atomic number $= 24$) in accordance with Aufbau principle? [4 marks]

(A) $n = 3, l = 2, m = +2, s = +\frac{1}{2}$
(B) $n = 3, l = 2, m = -2, s = +\frac{1}{2}$
(C) $n = 4, l = 1, m = 0, s = +\frac{1}{2}$
(D) $n = 4, l = 0, m = 0, s = +\frac{1}{2}$

Correct Answer: (D) $n = 4, l = 0, m = 0, s = +\frac{1}{2}$

Solution:

Concept:

- The Aufbau rule fills orbitals in increasing $n + l$; a tie is resolved in favour of the smaller n. After $3p$, the next subshell in this order is $4s$.
- For an s orbital, $l = 0$ and its only allowed magnetic quantum number is $m_l = 0$. The first electron is conventionally assigned $m_s = +1/2$.

Step 1: Locate the electron after the argon core Filling the first eighteen electrons gives

$$1s^2\,2s^2\,2p^6\,3s^2\,3p^6.$$

The cumulative counts are 2, 4, 10, 12 and 18. The nineteenth electron is therefore the first electron outside the argon core. The question explicitly asks for its position according to the Aufbau principle, so use the filling order at this point.

Step 2: Choose the subshell and its quantum numbers For $4s$, $n + l = 4 + 0 = 4$, while for $3d$, $n + l = 3 + 2 = 5$. Thus the nineteenth electron enters $4s$ before $3d$. Its quantum numbers are

$$n = 4, \qquad l = 0, \qquad m_l = 0, \qquad m_s = +\frac{1}{2}.$$

Chromium's stable ground-state configuration is $[\mathrm{Ar}]3d^5 4s^1$, but this exception does not change the identification of the nineteenth electron in the Aufbau filling sequence requested here.

Step 3: Eliminate the other orbital assignments Options (A) and (B) both have $n = 3, l = 2$, so they describe electrons in a $3d$ orbital. Those are not the nineteenth electron in Aufbau order. Option (C) has $n = 4, l = 1$, which denotes $4p$, not $4s$. Only option (D) simultaneously gives the correct principal, orbital, magnetic and spin quantum numbers.

Step 4: Conclusion The required set is $\boxed{n = 4,\ l = 0,\ m_l = 0,\ m_s = +\frac{1}{2}}$. Hence, the correct option is (D).

Quick Tip to Solve: Count the electrons through the preceding closed shell before assigning quantum numbers; $l = 0$, not n, identifies an s orbital.

54. Given below are two statements :

Statement I : For an ideal gas, heat capacity at constant volume is always greater than the heat capacity at constant pressure.

Statement II : In a constant volume process, no work is produced and all the heat withdrawn goes into the chaotic motion and is reflected by a temperature increase of the ideal gas.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Concept:

- For one mole of an ideal gas, Mayer's relation is $C_{P,m} - C_{V,m} = R > 0$. For any fixed positive amount, heat capacity at constant pressure exceeds that at constant volume.
- For pressure-volume work, $w = -P_{\text{ext}}\Delta V$. At fixed volume $w = 0$, so the first law gives $\Delta U = q_V$; for an ideal gas $\Delta U = nC_{V,m}\Delta T$.

Step 1: Test Statement I with Mayer's relation At constant pressure, heating the gas requires energy both to raise its internal energy and to do expansion work. At constant volume there is no expansion work. Consequently

$$C_P = C_V + nR > C_V.$$

Statement I claims the reverse inequality and is therefore false. This conclusion does not depend on whether the ideal gas is monatomic or polyatomic.

Step 2: Interpret the heat flow described in Statement II Statement II describes heat going into molecular motion and producing a temperature rise. In that description, heat is drawn from the surroundings into the gas. At constant volume,

$$w = 0, \qquad q_V = \Delta U = nC_{V,m}\Delta T.$$

For heat supplied to the gas, $q_V > 0$, so $\Delta T > 0$. The microscopic energy increases without the gas performing expansion work, which is the physical process described.

Step 3: Check the wording and select the truth combination The word withdrawn must here be read as drawn from the surroundings, consistent with the rest of Statement II saying that the heat goes into the gas and increases its temperature. If heat were withdrawn from the gas instead, $q_V < 0$ and its temperature would decrease; that would describe a different process. For the stated heating process, Statement II is true. Options (A) and (C) incorrectly make Statement I true; option (B) also rejects the constant-volume heating result.

Step 4: Conclusion Statement I is false and Statement II describes the correct constant-volume heating process, with heat drawn into the gas. Hence, the correct option is (D).

Quick Tip to Solve: Use $C_P - C_V = nR$ to compare heat capacities, and identify the direction of heat transfer before deciding the sign of ΔT.

55. At $\mathrm{T(K)}$, the equilibrium constant of

$\mathrm{A_2}(g) + \mathrm{B_2}(g) \rightleftharpoons \mathrm{C}(g)$ **is** 2.7×10^{-5}**.**

What is the equilibrium constant for

$\frac{1}{3}\ \mathrm{A_2}(\ \mathrm{g}) + \frac{1}{3}\ \mathrm{B_2}(\ \mathrm{g}) \rightleftharpoons \frac{1}{3}\mathrm{C(g)}$ **at the same temperature?** [4 marks]

(A) $\left(2.7 \times 10^{-5}\right)^3$

(B) 6×10^{-2}

(C) $\sqrt{2.7 \times 10^{-5}}$

(D) 3×10^{-2}

Correct Answer: (D) 3×10^{-2}

Solution:

Concept:

- The equilibrium constant contains the activities of the products and reactants raised to their stoichiometric coefficients. At fixed temperature, changing those coefficients changes the numerical constant.
- If every coefficient of a reaction is multiplied by a positive factor n, its new equilibrium constant is $K_{\text{new}} = K_{\text{old}}^{n}$. Reversing the reaction instead gives the reciprocal.

Step 1: Compare the two equations The original reaction is $\mathrm{A_2(g)} + \mathrm{B_2(g)} \rightleftharpoons \mathrm{C(g)}$. In the second equation, the coefficient of every species is divided by three. Its direction has not been reversed and its temperature is unchanged, so the required transformation is $n = 1/3$, not $n = 3$ or $n = -1/3$.

Step 2: Apply the coefficient rule Writing the original constant as $K = a_{\mathrm{C}}/(a_{\mathrm{A_2}} a_{\mathrm{B_2}})$, the new expression is

$$K_{\text{new}} = \frac{a_{\mathrm{C}}^{1/3}}{a_{\mathrm{A_2}}^{1/3}\, a_{\mathrm{B_2}}^{1/3}} = K^{1/3}.$$

Therefore,

$$K_{\text{new}} = (2.7 \times 10^{-5})^{1/3} = (27 \times 10^{-6})^{1/3} = 3 \times 10^{-2}.$$

Step 3: Check the alternatives Cubing the original constant, as in (A), would correspond to tripling all coefficients. The square root in (C) would correspond to halving them. Option (B) gives twice the calculated cube root and does not satisfy $(K_{\text{new}})^3 = 2.7 \times 10^{-5}$. Option (D) alone satisfies that check.

Step 4: Conclusion The required equilibrium constant is $\boxed{3 \times 10^{-2}}$. Hence, the correct option is (D).

Quick Tip to Solve: Apply the multiplier to every stoichiometric coefficient first; use that same multiplier as the exponent on K.

56. In order to oxidise a mixture of 1 mole each of $FeC_2O_4, Fe_2(C_2O_4)_3, FeSO_4$ and $Fe_2(SO_4)_3$ in acidic medium, the number of moles of $KMnO_4$ required is [4 marks]

(A) 3

(B) 2

(C) 5

(D) 7

Correct Answer: (B) 2

Solution:

Concept:

- In acidic solution, permanganate is reduced from manganese(VII) to manganese(II), accepting five electrons per mole: $MnO_4^- + 8\,H^+ + 5\,e^- \longrightarrow Mn^{2+} + 4\,H_2O$.
- Ferrous ions lose one electron each on oxidation to ferric ions. Each oxalate ion loses two electrons on oxidation to carbon dioxide; ferric ions and sulfate do not supply electrons in this reaction.

Step 1: Count the electrons from each salt One mole of FeC_2O_4 contains one mole of Fe^{2+} and one mole of oxalate. Their half-reactions are

$$Fe^{2+} \longrightarrow Fe^{3+} + e^-, \qquad C_2O_4^{2-} \longrightarrow 2\,CO_2 + 2\,e^-.$$

Thus it supplies $1 + 2 = 3$ moles of electrons. One mole of $Fe_2(C_2O_4)_3$ already contains ferric iron, but its three oxalate ions supply $3 \times 2 = 6$ moles of electrons.

Step 2: Include the sulfate salts One mole of $FeSO_4$ contains one mole of ferrous iron, giving one mole of electrons. In $Fe_2(SO_4)_3$, the iron is already ferric and the sulfur is already in its highest usual oxidation state, so this salt supplies no electrons. The total reducing capacity is therefore

$$n(e^-) = 3 + 6 + 1 + 0 = 10 \text{ mol}.$$

Step 3: Convert electron demand to permanganate Each mole of $KMnO_4$ accepts five moles of electrons in the specified acidic medium. Hence

$$n(KMnO_4) = \frac{10}{5} = 2 \text{ mol}.$$

The alternatives 3, 5 and 7 moles would accept 15, 25 and 35 moles of electrons, respectively, all exceeding the available 10 moles.

Step 4: Conclusion Exactly $\boxed{2 \text{ mol}}$ of $KMnO_4$ is required. Hence, the correct option is (B).

Quick Tip to Solve: Count electrons from both the metal ion and the anion: ferric oxalate still reduces permanganate because its oxalate ions are oxidised.

57. Consider the first order reaction $\mathrm{R} \rightarrow \mathrm{P}$.

The fraction of molecules decomposed in the given first order reaction can be expressed as [4 marks]

(A) $1 - e^{k_1 t}$

(B) $1 + e^{k_1 t}$

(C) $1 + e^{-k_1 t}$

(D) $1 - e^{-k_1 t}$

Correct Answer: (D) $1 - e^{-k_1 t}$

Solution:

Concept:

- For a first-order decomposition, the rate of disappearance is proportional to the amount remaining: $-dN/dt = k_1 N$, where N is the number of reactant molecules and $k_1 > 0$ is the rate constant.
- The fraction decomposed is the number consumed divided by the initial number. It is the complement of the fraction remaining, and must lie between zero and one.

Step 1: Obtain the fraction remaining Let N_0 molecules be present at $t = 0$, and N_t remain after time t. Separating and integrating the rate equation gives

$$\int_{N_0}^{N_t} \frac{dN}{N} = -k_1 \int_0^t dt, \qquad \ln \frac{N_t}{N_0} = -k_1 t.$$

Thus $N_t = N_0 e^{-k_1 t}$, so the fraction remaining is $N_t/N_0 = e^{-k_1 t}$. The same result follows using concentration at constant volume.

Step 2: Subtract the remaining fraction from unity The number decomposed is $N_0 - N_t$, not N_t. Dividing by the initial number yields

$$f = \frac{N_0 - N_t}{N_0} = 1 - \frac{N_t}{N_0} = 1 - e^{-k_1 t}.$$

The exponent is dimensionless because k_1 has units of inverse time. This expression increases smoothly as the reaction proceeds.

Step 3: Test the options at physical limits At $t = 0$, no molecules have decomposed, so $f = 0$; at very long times, $f \rightarrow 1$. Option (D) satisfies both limits. Option (A) is negative for positive time because $e^{k_1 t} > 1$. Options (B) and (C) both equal 2 initially and exceed 1, so neither can represent a decomposed fraction.

Step 4: Conclusion The fraction decomposed is $\boxed{1 - e^{-k_1 t}}$. Hence, the correct option is (D).

Quick Tip to Solve: For first-order kinetics, remember the remaining fraction first: e^{-kt}. Subtract it from one to obtain the decomposed fraction.

58. A monoatomic anion A^- has 45 neutrons and 36 electrons. Atomic mass, group in the periodic table and physical state at room temperature of the element (A) respectively are [4 marks]

(A) 80, 17, liquid

(B) 81, 16, solid

(C) 80,16, gas

(D) 81, 15, gas

Correct Answer: (A) 80, 17, liquid

Solution:

Concept:

- For a singly negative monatomic ion, the electron count is one greater than the proton count: $N_e = Z + 1$. Formation of an ion changes the number of electrons, not the number of protons or neutrons.
- The isotope mass number is the sum of protons and neutrons, $M = Z + N$. The atomic number identifies the element, whose group and ordinary physical state then follow.

Step 1: Determine the atomic number The given ion is A^-, with 36 electrons. Its neutral atom therefore has one fewer electron and contains 35 protons:

$$Z = 36 - 1 = 35.$$

The element with atomic number 35 is bromine, Br. The 36-electron anion is bromide, isoelectronic with krypton; this does not make the parent element krypton.

Step 2: Calculate the mass and identify the group The nucleus has 45 neutrons, so its mass number is

$$M = 35 + 45 = 80.$$

Here the integer atomic mass requested in the options is the isotope mass number, rather than the naturally weighted atomic weight. Neutral bromine has outer configuration $4s^2 4p^5$, giving seven valence electrons. It belongs to group 17, the halogens.

Step 3: Identify the physical state and reject alternatives At room temperature, elemental bromine consists of Br_2 molecules and is a liquid. Thus all three requested entries are 80, 17 and liquid. Option (B) has the wrong mass, group and state. Option (C) has mass 80 but places bromine in group 16 and calls it a gas. Option (D) also gives the wrong mass and group and an incorrect gaseous state.

Step 4: Conclusion The required triplet is $\boxed{80,\ 17,\ \text{liquid}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a negative ion, subtract the magnitude of its charge from its electron count to find Z; then add neutrons to find the mass number.

59. Given below are two statements :

Statement I : The covalency of oxygen is generally two but it can exceed upto four. The oxidation state of oxygen in SO_2 is -2 and in OF_2 it is +2 .

Statement II : The anomalous behaviour of oxygen when compared to the other elements of group 16 is due to its small size and high electronegativity.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- Oxygen has the valence-shell configuration $2s^2 2p^4$. Covalency counts the covalent bonds formed, whereas oxidation state is a formal electron-bookkeeping quantity; the two need not have the same value.
- Oxygen is unusually small and highly electronegative compared with the other group 16 elements. Its second shell has only the $2s$ and $2p$ valence orbitals and cannot expand its octet.

Step 1: Examine the covalency claim in Statement I Oxygen normally forms two covalent bonds, as in H_2O, completing an octet through shared electron pairs and lone pairs. Its covalency can exceed two, with four as the octet-limited upper bound, though it rarely does so in ordinary compounds. Four shared pairs correspond to eight electrons around the atom; this does not require an expanded octet. Thus the stated general covalency and upper limit are consistent.

Step 2: Check both oxidation states In SO_2, oxygen is more electronegative than sulfur and has its usual oxidation state -2; sulfur is consequently $+4$. Fluorine is more electronegative than oxygen and is -1 in OF_2. Charge balance gives

$$x + 2(-1) = 0 \quad \Rightarrow \quad x = +2.$$

Both oxidation-state assignments are correct. Together with the covalency statement, this makes Statement I true.

Step 3: Evaluate Statement II and choose the combination The small size of oxygen and its high electronegativity are principal causes of its anomalous behaviour within group 16, including strong hydrogen bonding in water. The lack of valence-shell d orbitals also restricts its bonding. Statement II gives valid causes and is therefore true. Options (B), (C) and (D) each incorrectly declare at least one of the two statements false.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish covalency from oxidation state: oxygen can have oxidation state $+2$ in OF_2 while still forming two bonds.

60. The correct statements among the following are,

A. $\mathrm{Mo(VI)}$ **and** $\mathrm{W(VI)}$ **are less stable than** $\mathrm{Cr(VI)}$**.**

B. $\mathrm{Ce^{4+}}$ **and** $\mathrm{Tb^{4+}}$ **are oxidant while** $\mathrm{Eu^{2+}}$ **and** $\mathrm{Yb^{2+}}$ **are reductant.**

C. Cm and Am have seven unpaired electrons.

D. Actinoid contraction is greater from element to element than lanthanoid contraction.

Choose the correct answer from the options given below : [4 marks]

(A) A and B Only

(B) C and D Only

(C) B and D Only

(D) A and C Only

Correct Answer: (C) B and D Only

Solution:

Concept:

- High oxidation states of transition elements generally become more stable down a group. For lanthanoids, the $+3$ state is predominant, so less usual $+4$ and $+2$ states can act as oxidants and reductants, respectively.
- Unpaired electrons must be counted from the complete ground-state configuration. Contraction across an f series arises from increasing nuclear charge with imperfect shielding by the added f electrons.

Step 1: Test statements A and B Statement A reverses the stability trend: $\mathrm{Mo(VI)}$ and $\mathrm{W(VI)}$ are more stable than $\mathrm{Cr(VI)}$, so A is false. The ions $\mathrm{Ce^{4+}}$ and $\mathrm{Tb^{4+}}$ can accept an electron to form their trivalent ions, and hence act as oxidants. Conversely, $\mathrm{Eu^{2+}}$ and $\mathrm{Yb^{2+}}$ can lose an electron to form their trivalent ions and act as reductants. Therefore B is true.

Step 2: Count the unpaired electrons for C Americium has ground-state configuration $[\mathrm{Rn}]\,5f^7 7s^2$: its seven singly occupied f orbitals give seven unpaired electrons. Curium has $[\mathrm{Rn}]\,5f^7 6d^1 7s^2$, so it has seven unpaired f electrons plus one unpaired d electron, making eight. Since both atoms do not have seven unpaired electrons, C is false.

Step 3: Evaluate the contraction and select the pair Actinoid contraction is greater from element to element than lanthanoid contraction because 5f electrons shield the increasing nuclear charge poorly. Thus D is true. The true statements are B and D only. Option (A) includes false statement A, option (B) includes false statement C, and option (D) contains both false statements A and C.

Step 4: Conclusion Only statements B and D are correct. Hence, the correct option is (C).

Quick Tip to Solve: When counting unpaired electrons in curium, include its $6d^1$ electron as well as the seven electrons in $5f^7$.

61. Correct statements from the following are

A. Potassium dichromate is an oxidising agent and it oxidises $FeSO_4$ to $Fe_2(SO_4)_3$ in acidic medium.

B. Sodium dichromate can be used as primary standard in volumetric estimation.

C. CrO_4^{2-} and $Cr_2O_7^{2-}$ are interconvertible in aqueous solution by varying the pH of the solution.

D. $Cr-O-Cr$ bond angle in $Cr_2O_7^{2-}$ is $126°$.

Choose the correct answer from the options given below : [4 marks]

(A) A, B and C Only

(B) A, C and D Only

(C) A and C Only

(D) B and D Only

Correct Answer: (B) A, C and D Only

Solution:

Concept:

- Acidified dichromate is an oxidising agent: chromium(VI) is reduced to chromium(III), while the reducing species loses electrons.
- A primary standard must have reliable composition and be stable and non-hygroscopic. Chromate–dichromate equilibrium is controlled by hydrogen-ion concentration.

Step 1: Test the oxidation statement In acidic solution, Fe^{2+} loses one electron to form Fe^{3+}. Dichromate accepts six electrons:

$$Cr_2O_7^{2-} + 14\,H^+ + 6\,Fe^{2+} \longrightarrow 2\,Cr^{3+} + 7\,H_2O + 6\,Fe^{3+}.$$

Thus ferrous sulphate is converted into ferric sulphate, so statement A is true. The sulphate ions supply the counterions and do not change the redox conclusion.

Step 2: Distinguish the two dichromate salts Sodium dichromate is hygroscopic, so a weighed sample does not reliably give the required amount of pure salt. It is not the primary standard used here. Potassium dichromate, which is stable and non-hygroscopic, is suitable as a primary standard. Statement B specifically names sodium dichromate and is false.

Step 3: Check the equilibrium and bridge angle The aqueous equilibrium is

$$2\,CrO_4^{2-} + 2\,H^+ \rightleftharpoons Cr_2O_7^{2-} + H_2O.$$

Acidification favours orange dichromate, whereas making the solution alkaline favours yellow chromate. Hence C is true. The dichromate ion consists of two tetrahedral units sharing an oxygen; its bridging $Cr-O-Cr$ angle is approximately $126°$, so D is also true. This is the bridge angle, not the tetrahedral angle around one chromium atom.

Step 4: Select the statement combination Only A, C and D survive all four checks. Options (A) and (D) include the false statement B; option (C) omits the true statement D.

Step 5: Conclusion The correct statements are A, C and D only. Hence, the correct option is (B).

Quick Tip to Solve: Remember that potassium dichromate is the primary standard; do not transfer this property to hygroscopic sodium dichromate.

62. Match the LIST-I with LIST-II.

List-I Complex ion		List-II Calculated spin only magnetic moment (BM)	
A.	$[Cr(H_2O)_6]^{2+}$	**I.**	**3.87**
B.	$[Co(H_2O)_6]^{2+}$	**II.**	**5.92**
C.	$[Cu(H_2O)_6]^{2+}$	**III.**	**4.90**
D.	$[Mn(H_2O)_6]^{2+}$	**IV.**	**1.73**

Choose the correct answer from the options given below: [4 marks]

(A) A-I, B-III, C-IV, D-II

(B) A-II, B-I, C-III, D-IV

(C) A-IV, B-II, C-I, D-III

(D) A-III, B-I, C-IV, D-II

Correct Answer: (D) A-III, B-I, C-IV, D-II

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons.
- Water is a weak-field ligand in these octahedral complexes, so use the high-spin arrangement. Neutral water leaves the metal oxidation state at $+2$.

Step 1: Find the metal-ion electron counts Remove the outer $4s$ electrons before counting the $3d$ electrons. The relevant ions are Cr^{2+} : d^4, Co^{2+} : d^7, Cu^{2+} : d^9, and Mn^{2+} : d^5. Although neutral chromium and copper have exceptional configurations, their divalent ions still have these stated d-electron counts.

Step 2: Count the unpaired electrons For the high-spin octahedral arrangements, chromium(II) has $t_{2g}^3 e_g^1$, giving four unpaired electrons. Cobalt(II) has $t_{2g}^5 e_g^2$, giving three. Copper(II) has $t_{2g}^6 e_g^3$, giving one, and manganese(II) has $t_{2g}^3 e_g^2$, giving five. The requested values are spin-only values, so orbital contributions to measured moments are not added.

Step 3: Evaluate and match the moments Substitution gives

$$\begin{aligned}
\mu(Cr^{2+}) &= \sqrt{4(6)} = 4.90\,\text{BM} \quad \longrightarrow \text{III},\\
\mu(Co^{2+}) &= \sqrt{3(5)} = 3.87\,\text{BM} \quad \longrightarrow \text{I},\\
\mu(Cu^{2+}) &= \sqrt{1(3)} = 1.73\,\text{BM} \quad \longrightarrow \text{IV},\\
\mu(Mn^{2+}) &= \sqrt{5(7)} = 5.92\,\text{BM} \quad \longrightarrow \text{II}.
\end{aligned}$$

Option (A) interchanges the chromium and cobalt values. Option (B) incorrectly gives chromium the five-unpaired-electron value. Option (C) incorrectly gives chromium the one-unpaired-electron value. Only option (D) contains every correct pairing.

Step 4: Conclusion The matching is A-III, B-I, C-IV, D-II. Hence, the correct option is (D).

Quick Tip to Solve: For high-spin aqua ions, count the unpaired electrons first; the useful sequence for one to five is 1.73, 2.83, 3.87, 4.90 and 5.92 BM.

63. Increasing order of electron withdrawing power of following functional groups is :

a. -CN

b. -COOH

c. $-NO_2$

d. -I [4 marks]

(A) $c < b < d < a$

(B) $c < a < b < d$

(C) $d < b < a < c$

(D) $a < b < c < d$

Correct Answer: (C) $d < b < a < c$

Solution:

Concept:

- The negative inductive effect, denoted $-I$, describes withdrawal of electron density through the sigma-bond framework by a substituent.
- For the groups listed, the standard increasing electron-withdrawing order is $-I < -COOH < -CN < -NO_2$. Here $-I$ denotes the iodine substituent, whereas $-I$ in words denotes the effect.

Step 1: Compare the halogen and carbon-containing groups The iodine substituent withdraws electron density because of its electronegativity, but its electron-withdrawing inductive effect is weaker than that of the carboxyl and cyano groups in the given comparison. Thus group d must be first in an increasing order. The carboxyl group has polar carbon–oxygen bonds and withdraws electron density; the strongly polar cyano group, attached through an sp-hybridised carbon, withdraws more strongly. This fixes the relative sequence d, b, a.

Step 2: Place the nitro group The nitro group is a particularly strong electron-withdrawing substituent. Its charge-separated bonding description and the two oxygen atoms produce a strong electron-attracting influence. It comes after the cyano group in this standard order, giving

$$-I < -COOH < -CN < -NO_2.$$

The question asks for the relative withdrawing power of these groups; it does not specify a substituted aromatic reaction for which competing resonance and positional effects would have to be evaluated.

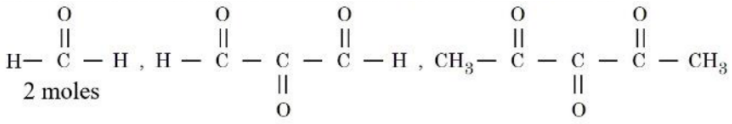
Step 3: Translate back to the given labels Using a = cyano, b = carboxyl, c = nitro and d = iodine gives $d < b < a < c$. Options (A) and (B) put the strongly withdrawing nitro group first, which reverses the required comparison. Option (D) puts iodine last and also reverses the cyano–carboxyl order. Option (C) alone has both the correct weakest and strongest groups and the correct middle pair.

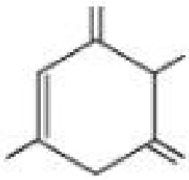
Step 4: Conclusion The increasing order is $d < b < a < c$. Hence, the correct option is (C).

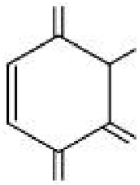
Quick Tip to Solve: Check the weakest and strongest groups first: iodine is first and nitro is last; then place carboxyl before cyano.

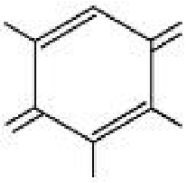
64. An alkene (X) on ozonolysis followed by reduction gives following products.

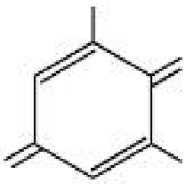
The alkene (X) is: [4 marks]

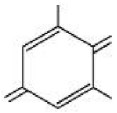


(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- Reductive ozonolysis cleaves each carbon–carbon double bond and converts its two carbons into carbonyl carbons without further oxidising aldehydes.
- An exocyclic $=CH_2$ group gives one molecule of formaldehyde. A double-bond carbon bearing hydrogen gives an aldehyde; one bearing carbon substituents gives a ketone.

Step 1: Read the product information The products are two molecules of $HCHO$, one molecule of $OHC-CO-CHO$, and one molecule of $CH_3CO-CO-COCH_3$. The two formaldehyde molecules require two terminal methylene groups. The remaining products retain two three-carbon portions of the ring skeleton. Their central carbonyl groups come from the two exocyclic double bonds.

Step 2: Test the connectivity in option D Number the six ring vertices of option (D) clockwise from the top as 1 to 6. Vertices 1 and 3 bear methyl groups; vertices 2 and 5 bear exocyclic methylene groups. The ring double bonds are between 1 and 6 and between 3 and 4. Cleaving those ring double bonds separates the ring into the sequences 6–5–4 and 1–2–3. Ozonolysis of the methylene groups simultaneously gives two formaldehyde molecules.

Step 3: Identify the two ring fragments Vertices 6 and 4 each bear hydrogen, so the 6–5–4 fragment becomes $OHC-CO-CHO$. Vertices 1 and 3 each bear methyl groups, so the 1–2–3 fragment becomes $CH_3CO-CO-COCH_3$. All the printed products, including their carbonyl positions, are therefore reproduced. Carbon counting is consistent: the starting compound has ten carbons, while the products contain $2+3+5=10$.

Step 4: Reject the other structures Option (A) has only one ring double bond and cannot split the ring into the two required three-carbon fragments. Option (B) has three exocyclic methylene groups and would give three formaldehyde molecules. Option (C) has an extra methyl substituent and would

not yield the dialdehyde fragment $OHC-CO-CHO$; its carbon count also exceeds that of the stated products.

Step 5: Conclusion Option (D) gives precisely the two formaldehyde molecules and the two stated tricarbonyl fragments. Hence, the correct option is (D).

Quick Tip to Solve: In reverse ozonolysis, count formaldehyde molecules first, then inspect whether each ring-cleavage endpoint bears hydrogen or a methyl group.

65. Match the LIST-I with LIST-II.

List-I Name of reaction		List-II Reagent or catalyst used	
A.	**Finkelstein reaction**	**I.**	SbF_3
B.	**Swarts reaction**	**II.**	**Na, dry ether**
C.	**Sandmeyer's reaction**	**III.**	**NaI**
D.	**Fittig reaction**	**IV.**	Cu_2Cl_2

Choose the correct answer from the options given below: [4 marks]

(A) A-I, B-IV, C-III, D-II

(B) A-III, B-I, C-IV, D-II

(C) A-IV, B-II, C-I, D-III

(D) A-I, B-III, C-II, D-IV

Correct Answer: (B) A-III, B-I, C-IV, D-II

Solution:

Concept:

- Halogen-exchange reactions change the halogen on an alkyl group: Finkelstein introduces iodine, while Swarts introduces fluorine.
- Sandmeyer reactions replace an aromatic diazonium group using a copper(I) reagent. Fittig coupling joins two aryl groups using sodium in dry ether.

Step 1: Match the two halogen exchanges Finkelstein reaction uses sodium iodide, usually in dry acetone, to replace chlorine or bromine in an alkyl halide:

$$R-Cl + NaI \longrightarrow R-I + NaCl.$$

Precipitation of the chloride or bromide salt helps drive the exchange. Thus A matches III. Swarts reaction prepares alkyl fluorides using a metallic fluoride such as SbF_3; consequently B matches I. These two names should not be interchanged merely because both reactions exchange halogens.

Step 2: Match the diazonium substitution In Sandmeyer's reaction, an aromatic diazonium group is replaced by chloride using copper(I) chloride, represented in the list as Cu_2Cl_2. Nitrogen gas is expelled and the aryl chloride forms. Thus C matches IV. Sodium iodide is not the characteristic copper(I) reagent for this named transformation.

Step 3: Match the aromatic coupling Fittig reaction couples aryl halides in the presence of sodium

and dry ether:

$$2\,\mathrm{Ar{-}X} + 2\,\mathrm{Na} \longrightarrow \mathrm{Ar{-}Ar} + 2\,\mathrm{NaX}.$$

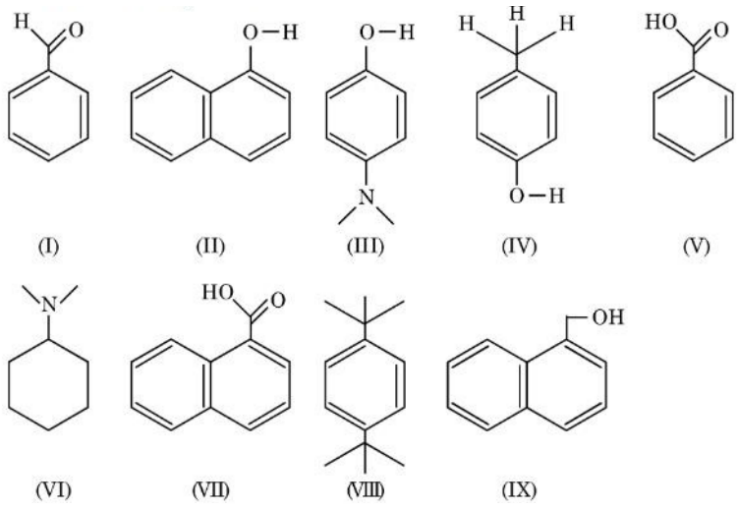
Thus D matches II. Dry conditions matter because water consumes sodium. The aryl–aryl coupling is distinguished from Wurtz coupling, which joins alkyl groups, and Wurtz–Fittig coupling, which joins an alkyl and an aryl group.

Step 4: Compare the complete pairing The required sequence is A-III, B-I, C-IV, D-II. Option (A) starts with the wrong reagent for Finkelstein; option (C) incorrectly assigns copper(I) chloride to that reaction; option (D) again assigns the fluorinating reagent to Finkelstein and gives the wrong remaining matches. Only option (B) reproduces all four named transformations.

Step 5: Conclusion The matching is A-III, B-I, C-IV, D-II. Hence, the correct option is (B).

Quick Tip to Solve: Use the incoming halogen to recall the exchange reagent: Finkelstein uses iodide, Swarts uses fluoride; Fittig requires sodium in dry ether.

66. Amongst the following, the total number of compounds soluble in aqueous NaOH at room temperature is : [4 marks]



(A) 5

(B) 4

(C) 6

(D) 3

Correct Answer: (A) 5

Solution:

Concept:

- Aqueous sodium hydroxide converts carboxylic acids and phenols into ionic sodium salts, increasing their water solubility.
- This acid–base solubility test depends on the functional group. Ordinary alcohols, neutral hydrocarbons and tertiary amines do not form analogous anionic salts with aqueous NaOH.

Step 1: Identify the phenolic compounds Compound II is a naphthol, III is a dimethylamino-substituted phenol, and IV is a methyl-substituted phenol. All three contain an OH group bonded directly to an aromatic ring. Hydroxide removes the phenolic proton to give a resonance-stabilised phenoxide ion:

$$\mathrm{ArOH} + \mathrm{NaOH} \longrightarrow \mathrm{ArONa} + \mathrm{H_2O}.$$

The substituents in III and IV do not change the identification of their phenolic groups. Each is counted as soluble in the stated aqueous-base test.

Step 2: Identify the carboxylic acids Compound V is benzoic acid and VII is a naphthoic acid. Their carboxyl groups react with hydroxide to form sodium carboxylates:

$$\mathrm{ArCOOH} + \mathrm{NaOH} \longrightarrow \mathrm{ArCOONa} + \mathrm{H_2O}.$$

Both therefore join the soluble set. At this stage the counted compounds are II, III, IV, V and VII.

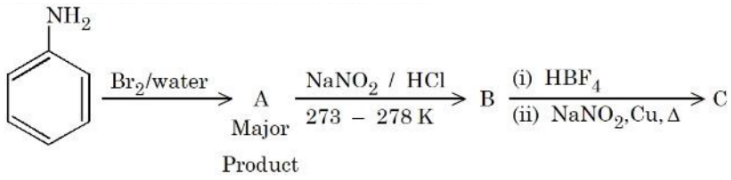
Step 3: Exclude the remaining compounds I is benzaldehyde and lacks a phenolic or carboxylic acidic proton; it is not counted as an acid–base-soluble compound. Cannizzaro reaction under suitable concentrated-base conditions is a different chemical test, not the solubility criterion here. VI is a tertiary amine, which forms salts with acids rather than NaOH. VIII is a hydrocarbon. IX has a hydroxymethyl group: its OH is attached to an aliphatic carbon, so it is an ordinary alcohol rather than naphthol. These four do not form the required sodium salts in aqueous NaOH.

Step 4: Count and choose There are three phenols and two carboxylic acids, giving $3 + 2 = 5$. The alternatives 4 and 3 omit qualifying acidic compounds; 6 incorrectly adds a neutral compound, most temptingly the alcohol IX.

Step 5: Conclusion The soluble compounds are II, III, IV, V and VII: five compounds in total. Hence, the correct option is (A).

Quick Tip to Solve: Inspect where OH is attached: aromatic C–OH is phenolic, whereas aromatic C–CH2OH is an alcohol and is not counted in this NaOH solubility test.

67. Product C of the following reaction sequence will be [4 marks]



(A) 1-Bromo-4-nitrobenzene

(B) 1, 3, 5-Tribromo-2-nitrobenzene

(C) 4-Bromo-1-nitrobenzene

(D) 1, 3, 5-Tribromobenzene

Correct Answer: (B) 1, 3, 5-Tribromo-2-nitrobenzene

Solution:

Concept:

- The amino group in aniline strongly activates its benzene ring and directs electrophilic substitution to the ortho and para positions.
- Diazotisation converts an aromatic primary amino group into a diazonium group. Subsequent replacement reactions change that group while retaining the other ring substituents.

Step 1: Identify the bromination product Bromine water reacts readily with aniline. Because the unprotected $-NH_2$ group strongly activates all three available ortho/para positions, substitution occurs at positions 2, 4 and 6. Thus A is 2,4,6-tribromoaniline, rather than a monobromoaniline.

$$C_6H_5NH_2 + 3\,Br_2 \longrightarrow C_6H_2Br_3NH_2 + 3\,HBr$$

Step 2: Form the diazonium intermediate The combination $NaNO_2/HCl$ generates nitrous acid in situ. At 273–$278\,K$, the amino group is diazotised to give 2,4,6-tribromobenzenediazonium chloride, B. The three bromine atoms already bonded to the ring remain in their original positions.

Step 3: Replace the diazonium group Treatment with HBF_4 gives the diazonium tetrafluoroborate. The following $NaNO_2$, copper and heat replace the diazonium group by $-NO_2$, with loss of nitrogen. The nitro group therefore occupies the carbon that originally bore $-NH_2$; the three bromines are retained.

Step 4: Match the named product Renumbering the resulting ring gives 1,3,5-tribromo-2-nitrobenzene. Options (A) and (C) each contain only one bromine and cannot follow the initial tribromination. Option (D) has no nitro group and would require replacement by hydrogen, which is not the reaction shown.

Step 5: Conclusion C is 1,3,5-tribromo-2-nitrobenzene. Hence, the correct option is (B).

Quick Tip to Solve: First mark all three positions brominated by bromine water; then change only the original amino-group position during diazonium replacement.

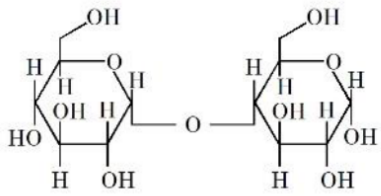
68. Given below are two statements :

Statement I : The structure of Maltose is given below:

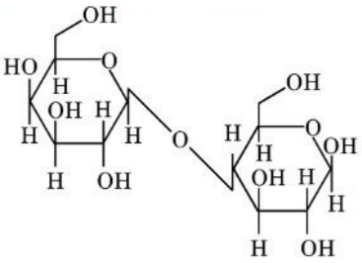
Statement II : The structure of Lactose is given below :

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]



Maltose is a non-reducing sugar.



Lactose is a reducing sugar.

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Concept:

- A disaccharide is reducing when it retains a free anomeric carbon as a hemiacetal; that ring can open to give an aldehydic form.
- Maltose contains two glucose units joined by an $\alpha(1 \rightarrow 4)$ linkage. Lactose contains galactose and glucose joined by a $\beta(1 \rightarrow 4)$ linkage.

Step 1: Examine the maltose statement The displayed maltose structure has a glycosidic bond from the first glucose anomeric carbon to carbon 4 of the second glucose. Its second glucose still carries a free anomeric hydroxyl group. This free hemiacetal can equilibrate with the open-chain form, so maltose is a reducing sugar. The accompanying claim that maltose is non-reducing makes Statement I false.

Step 2: Examine the lactose statement The lactose structure shows a galactose unit linked to a glucose unit. The glycosidic bond uses the galactose anomeric carbon but leaves the glucose anomeric carbon free. The glucose end can consequently open and provide the reducing functionality. Thus the printed assertion that lactose is a reducing sugar is correct, and Statement II is true.

Step 3: Compare all four truth combinations Both structures must be judged together with the words accompanying them. A correctly drawn maltose structure does not make the non-reducing claim true.

Option (A) incorrectly accepts that claim; option (B) incorrectly rejects lactose's reducing nature; option (C) reverses both truth values. Only option (D) gives false for I and true for II.

Step 4: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: Trace the glycosidic linkage and look for an unused anomeric OH: one free reducing end is enough to make the disaccharide reducing.

69. Match the List-I with List-II

List-I Name of amino acid	List-II One letter symbol/type
A. Arginine	I. D/Non-essential
B. Aspartic acid	II. R/Essential
C. Lysine	III. E/Non-essential
D. Glutamic acid	IV. K/Essential

Choose the correct answer from the options given below : [4 marks]

(A) A-II, B-I, C-IV, D-III

(B) A-IV, B-III, C-II, D-I

(C) A-III, B-IV, C-I, D-II

(D) A-II, B-IV, C-I, D-III

Correct Answer: (A) A-II, B-I, C-IV, D-III

Solution:

Concept:

- Each amino acid has a conventional one-letter code; the code uniquely identifies the amino acid and need not be its first letter.
- A matching question must satisfy every entry simultaneously. Here the distinct one-letter symbols determine each pairing directly.

Step 1: Match arginine and aspartic acid Arginine has the one-letter symbol R, so A must match II. Aspartic acid has symbol D, so B must match I. Aspartic acid is one of the acidic, non-essential amino acids, consistent with the printed entry. Do not use A for arginine: A is the symbol for alanine.

Step 2: Match lysine and glutamic acid Lysine is denoted by K, so C matches IV. Lysine is an essential amino acid, agreeing with that entry. Glutamic acid is denoted by E and is non-essential, so D matches III. The symbols D and E distinguish the two acidic amino acids, aspartic acid and glutamic acid.

Step 3: Interpret the nutritional label carefully Arginine is generally described as conditionally essential: its requirement depends on growth and physiological conditions. In this list it is placed under R/Essential. The one-letter identity R is unambiguous and selects II; the other three matches independently confirm the complete mapping. This qualification does not change the pairing asked for by the supplied list.

Step 4: Eliminate the competing mappings The resulting sequence is A-II, B-I, C-IV, D-III. Option (B) assigns K to arginine and E to aspartic acid; option (C) assigns E to arginine and K to aspartic acid.

Option (D) preserves arginine and glutamic acid but interchanges the aspartic-acid and lysine entries. None satisfies all four codes.

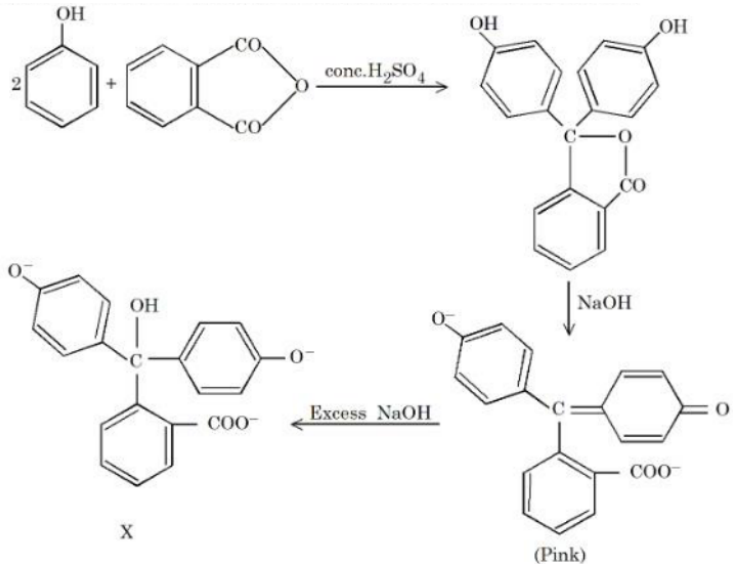
Step 5: Conclusion The matching is A-II, B-I, C-IV, D-III. Hence, the correct option is (A).

Quick Tip to Solve: Remember the acidic pair as Asp = D and Glu = E, then use Arg = R and Lys = K to check the entire matching.

70.

Identify the colour of compound ' X ' in the sequence of the reaction.

[4 marks]



(A) Violet

(B) Green

(C) Red

(D) Colourless

Correct Answer: (D) Colourless

Solution:

Concept:

- Phenol condenses with phthalic anhydride in concentrated sulfuric acid to form phenolphthalein.
- Indicator colour depends on molecular conjugation. The pink quinonoid form of phenolphthalein loses its visible colour when excess hydroxide converts it to the carbinol form.

Step 1: Identify the indicator formed Two molecules of phenol react with phthalic anhydride under the acidic conditions shown. The product is phenolphthalein in its closed lactone form. Its central carbon does not provide a continuous quinonoid conjugation across the rings, so this form is colourless.

Step 2: Follow the first addition of alkali Addition of sodium hydroxide produces the ring-opened, resonance-stabilised quinonoid anion. The reaction scheme itself labels this intermediate pink. Its

extended conjugated system absorbs in the visible region, which explains the familiar pink colour of phenolphthalein in an alkaline solution.

Step 3: Apply the excess-alkali condition The question asks about X after *excess* sodium hydroxide, rather than about the pink intermediate. Hydroxide adds at the central carbon, giving the depicted carbinol species with a central $\mathrm{C{-}OH}$ bond. That carbon is tetrahedral, interrupting the quinonoid conjugation responsible for the pink colour. The strongly alkaline form therefore becomes colourless.

Step 4: Select the final colour Violet, green and red would require a coloured chromophore persisting in X. The displayed central tetrahedral carbon shows why the extended quinonoid chromophore has been destroyed. It is incorrect to stop at the ordinary alkaline pink form when a further reaction with excess alkali is explicitly drawn.

Step 5: Conclusion Compound X is colourless. Hence, the correct option is (D).

Quick Tip to Solve: For phenolphthalein, distinguish ordinary alkali (pink quinonoid form) from excess strong alkali (colourless carbinol form).

Chemistry – Section B: Numerical Value Questions (20 marks)

71. According to Lewis theory, the total number of σ bond-pairs and lone pair of electrons around the central atom of $\mathrm{XeO_6}^{4-}$ ion is ____ . [4 marks]

Solution:

Concept:

- The total valence-electron count includes the valence electrons of all atoms and one additional electron for each unit of negative charge.
- Each Xe–O connection contributes one sigma bond, even when a Lewis representation includes an additional pi bond. Lone pairs on oxygen are not lone pairs on the central xenon.

Step 1: Count the available electrons Xenon contributes 8 valence electrons, the six oxygen atoms contribute $6 \times 6 = 36$, and the $4-$ charge supplies 4 more. Hence the total is

$$8 + 36 + 4 = 48 \text{ valence electrons.}$$

Place Xe at the centre and connect each of the six oxygen atoms to it by one bond.

Step 2: Complete the terminal oxygen octets The six Xe–O single bonds use $6 \times 2 = 12$ electrons. Completing the six terminal oxygen octets requires three lone pairs on each oxygen, or $6 \times 6 = 36$ electrons. Together these account for all $12 + 36 = 48$ available electrons. No pair remains as a lone pair on Xe.

Step 3: Check the equivalent lower-charge representation The all-single-bond bookkeeping gives formal charge +2 on Xe and -1 on each oxygen, summing to -4. Two oxygen lone pairs may instead be represented as Xe=O pi bonds to reduce formal-charge separation: four single-bonded oxygens then carry -1 each. Such resonance representations still have six Xe–O sigma connections and no lone pair on the central atom.

Step 4: Add only the requested central pairs The central atom consequently has six sigma bond-pairs and zero lone pairs. Pi bonds, if shown in a Lewis contributor, are not included in the requested sigma-bond count, and the terminal oxygen lone pairs must not be added to the central-atom total.

$$N = 6 + 0 = 6.$$

Step 5: Conclusion The total number of sigma bond-pairs and lone pairs around xenon is $\boxed{6}$, a dimensionless count.

Quick Tip to Solve: For a central-atom pair count, count one sigma bond per attached atom and add only lone pairs on that central atom.

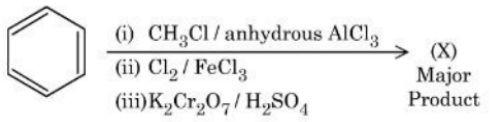
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72. Consider the following sequence of reactions to give the major product (X)**.**

P **g of the major product** (X) **formed is reacted with** $\mathrm{NaHCO_3}$ **solution to liberate a gas which occupied** $11.2\,\mathrm{dm^3}$ **at STP.**

$P =$ _____ g.

(Given molar mass in $\mathrm{g\,mol^{-1}}$**: H: 1, C: 12, O: 16, Cl: 35.5.)** [4 marks]



Solution:

Concept:

- Friedel–Crafts methylation converts benzene into toluene. The methyl group directs subsequent ring chlorination mainly to the ortho and para positions.
- Strong oxidation changes a benzylic methyl group into $-\mathrm{COOH}$. A monocarboxylic acid reacts with bicarbonate to release one mole of $\mathrm{CO_2}$ per mole of acid.

Step 1: Identify the acid product The first reagent pair, $\mathrm{CH_3Cl/AlCl_3}$, gives toluene. Chlorination with $\mathrm{Cl_2/FeCl_3}$ is electrophilic ring chlorination, giving ortho- and para-chlorotoluene. Acidified potassium dichromate then oxidises the methyl side chain to a carboxyl group, producing chlorobenzoic acid with formula $\mathrm{C_7H_5ClO_2}$. Both positional isomers have the same molar mass and each contains one carboxyl group, so their relative yields do not affect the requested mass.

Step 2: Calculate the molar mass Using exactly the supplied atomic masses,

$$M(\mathrm{C_7H_5ClO_2}) = 7(12) + 5(1) + 35.5 + 2(16) = 156.5\,\mathrm{g\,mol^{-1}}.$$

There is one carboxylic acid group per molecule; chlorine substitution does not create another acidic group that liberates carbon dioxide from bicarbonate.

Step 3: Use the evolved carbon dioxide The reaction is

$$\mathrm{ClC_6H_4COOH + NaHCO_3 \longrightarrow ClC_6H_4COONa + H_2O + CO_2}.$$

At STP, using $22.4\,\mathrm{dm^3\,mol^{-1}}$, the gas amount is

$$n(\mathrm{CO_2}) = \frac{11.2}{22.4} = 0.500\,\mathrm{mol}.$$

The 1:1 stoichiometric ratio gives the same amount of X.

Step 4: Determine the required mass Multiply the amount of acid by its molar mass:

$$P = nM = 0.500 \times 156.5 = 78.25\,\mathrm{g}.$$

Thus the mass calculated from the given data is 78.25 g; expressed as the nearest integer for the numerical answer, its value is 78.

Step 5: Conclusion $P = 78.25\,\mathrm{g}$; the nearest-integer numerical answer is $\boxed{78}$ (mass in grams).

Quick Tip to Solve: Each carboxyl group gives one carbon dioxide molecule with bicarbonate: convert gas volume to moles before multiplying by the acid molar mass.

73. 2.0 g of a bromo hydrocarbon (X) was subjected to Carius analysis, gave 3.36 g of AgBr . The percentage of carbon in the compound (X) is 26.7%. Total number of carbon atoms in the empirical formula for compound (X) is ____ .

(Given molar mass in $\mathrm{gmol^{-1}}$ $\mathrm{H:1, C:12, Br:80, Ag:108}$ **)** [4 marks]

Solution:

Concept:

- In Carius analysis, each mole of bromine atoms in the organic compound gives one mole of AgBr. Thus the precipitate determines the mass percentage of bromine.
- A bromo hydrocarbon contains carbon, hydrogen and bromine. Subtract the carbon and bromine percentages from 100 to find hydrogen, then divide each percentage by its atomic mass to obtain the empirical mole ratio.

Step 1: Calculate the bromine content Using the given atomic masses, the molar mass of AgBr is $108 + 80 = 188\,\mathrm{g\,mol^{-1}}$. The mass of bromine in the precipitate, and therefore in the original sample, is

$$m_{\mathrm{Br}} = 3.36 \times \frac{80}{188} = 1.42979\,\mathrm{g}.$$

Since the sample has mass $2.0\,\mathrm{g}$,

$$\%\mathrm{Br} = \frac{1.42979}{2.0} \times 100 = 71.4894\%.$$

Silver is supplied by the reagent; its mass is not part of the original organic compound.

Step 2: Determine hydrogen and the atomic ratio Carbon accounts for 26.7%, so the remaining percentage is hydrogen:

$$\%\mathrm{H} = 100 - 26.7 - 71.4894 = 1.8106\%.$$

On a $100\,\mathrm{g}$ basis, divide the masses by the respective atomic masses:

$$n_{\mathrm{C}} : n_{\mathrm{H}} : n_{\mathrm{Br}} = \frac{26.7}{12} : \frac{1.8106}{1} : \frac{71.4894}{80} = 2.225 : 1.8106 : 0.89362.$$

Dividing by the smallest value gives approximately $2.49 : 2.03 : 1$, which is $2.5 : 2 : 1$ within the precision of the reported experimental data. Multiplication by 2 gives the smallest whole-number ratio $5 : 4 : 2$. Do not round the carbon ratio to 2 before removing the half-integer.

Step 3: Conclusion The empirical formula is $\mathrm{C_5H_4Br_2}$. It contains $\boxed{5}$ carbon atoms, so the required numerical entry is 5.

Quick Tip to Solve: Convert the silver-halide mass to halogen mass first. In an empirical ratio, a value close to 2.5 signals multiplication of every ratio by 2.

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74. The pH of a solution obtained by mixing 5 mL of $0.1\mathrm{M}\,\mathrm{NH_4OH}$ solution with 250 mL of $0.1\mathrm{M}\,\mathrm{NH_4Cl}$ solution is ____ $\times 10^{-2}$.(Nearest integer) Given: $\mathrm{pK_b}\,(\mathrm{NH_4OH}) = 4.74$

$$\log 2 = 0.30$$
$$\log 3 = 0.48$$
$$\log 5 = 0.70$$

[4 marks]

Solution:

Concept:

- A weak base mixed with its conjugate-acid salt forms a basic buffer. For $\mathrm{NH_4OH}$ and $\mathrm{NH_4Cl}$, the buffer relation is $\mathrm{pOH} = \mathrm{p}K_b + \log([\text{salt}]/[\text{base}])$.
- Both components have the same final solution volume, so their concentration ratio equals their mole ratio. At the usual aqueous reference temperature, $\mathrm{pH} + \mathrm{pOH} = 14$; pH is dimensionless.

Step 1: Find the amounts of base and salt Convert the volumes from millilitres to litres before using $n = MV$:

$$n_{\text{base}} = 0.1 \times \frac{5}{1000} = 5.0 \times 10^{-4}\,\mathrm{mol},$$
$$n_{\text{salt}} = 0.1 \times \frac{250}{1000} = 2.50 \times 10^{-2}\,\mathrm{mol}.$$

The salt supplies the common ion $\mathrm{NH_4}^+$, suppressing the weak base's ionisation. In the buffer approximation, the salt and base concentrations are represented by their analytical concentrations. Their ratio is

$$\frac{[\text{salt}]}{[\text{base}]} = \frac{2.50 \times 10^{-2}}{5.0 \times 10^{-4}} = 50.$$

The common final volume cancels, so it need not be inserted separately into the buffer equation.

Step 2: Calculate pOH and convert to the requested entry For a basic buffer the salt concentration

goes in the numerator of the pOH relation. Using the supplied logarithm,

$$\log 50 = \log(5 \times 10) = 0.70 + 1 = 1.70.$$

Therefore,

$$\text{pOH} = 4.74 + 1.70 = 6.44, \qquad \text{pH} = 14 - 6.44 = 7.56.$$

The answer space is the coefficient of 10^{-2}, rather than pH itself. If that coefficient is N, then

$$N \times 10^{-2} = 7.56 \quad \Rightarrow \quad N = 756.$$

This is already an integer, so the requested rounding does not change it.

Step 3: Conclusion The solution has $\text{pH} = 7.56 = 756 \times 10^{-2}$. The required nearest-integer entry is $\boxed{756}$.

Quick Tip to Solve: For a weak-base buffer use salt/base in the pOH equation. Convert pOH to pH before applying any power-of-ten factor printed beside the answer blank.

75. A non-volatile, non-electrolyte solid solute when dissolved in 40 g of a solvent, the vapour pressure of the solvent decreased from 760 mm Hg to 750 mm Hg . If the same solution boils at 320 K , then the number of moles of the solvent present in the solution is ____ . (Nearest integer)

[Given: boiling point of the pure solvent $= 319.5$ K**,**

K_b **of the solvent** $= 0.3$ K kg mol^{-1} **]** [4 marks]

Solution:

Concept:

- For a non-electrolyte, boiling-point elevation is $\Delta T_b = K_b m$, where m is the moles of solute per kilogram of solvent. There is no dissociation factor greater than unity.
- Raoult's law gives $p = p^\circ x_1$ for a non-volatile solute. With solvent and solute amounts n_1 and n_2, $x_1 = n_1/(n_1 + n_2)$, so $(p^\circ - p)/p = n_2/n_1$.

Step 1: Obtain the amount of solute from boiling-point elevation The solution boils at $320\,\text{K}$ while the pure solvent boils at $319.5\,\text{K}$, hence

$$\Delta T_b = 320 - 319.5 = 0.5\,\text{K}.$$

The molality is therefore

$$m = \frac{0.5}{0.3} = \frac{5}{3}\,\text{mol kg}^{-1}.$$

The solvent mass is $40\,\text{g} = 0.040\,\text{kg}$. Multiplying molality by this solvent mass gives the solute amount:

$$n_2 = \frac{5}{3} \times 0.040 = \frac{1}{15}\,\text{mol}.$$

Molality uses the mass of solvent, not the mass of the complete solution; the stated $40\,\text{g}$ is therefore used directly.

Step 2: Use the vapour-pressure ratio to find solvent moles The non-volatile solute does not contribute to the vapour pressure. Thus the measured pressure ratio is the solvent mole fraction:

$$\frac{750}{760} = \frac{n_1}{n_1 + n_2}.$$

Cross-multiplication gives

$$750n_1 + 750n_2 = 760n_1 \quad \Rightarrow \quad n_1 = 75n_2.$$

Substitution of the solute amount found above yields

$$n_1 = 75 \times \frac{1}{15} = 5\,\text{mol}.$$

This uses the full mole-fraction relation. In particular, the denominator in $(p^\circ - p)/p$ is the solution pressure $750\,\text{mm Hg}$, which makes that ratio equal to solute moles divided by solvent moles.

Step 3: Conclusion The solution contains $\boxed{5\,\text{mol}}$ of solvent. The required nearest-integer numerical entry is 5.

Quick Tip to Solve: First obtain solute moles from ΔT_b. Then use $n_1/n_2 = p/(p^\circ - p)$ to get solvent moles without needing the solvent's molar mass.