

JEE Main 2026 Paper 1 (B.E./B.Tech) Jan 23 Shift 1

Question Paper with Solutions PDF



Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours

Maximum Marks: 300

General Instructions

- (i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2026, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 23 January 2026, Shift 1. The total duration is 3 hours.
- (ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.
- (iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.
- (iv) Marking scheme (multiple choice questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (v) Marking scheme (numerical value questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let the line $y - x = 1$ intersect the ellipse $\frac{x^2}{2} + \frac{y^2}{1} = 1$ at the points A and B. Then the angle made by the line segment AB at the center of the ellipse is : [4 marks]

- (A) $\pi - \tan^{-1}(1/4)$
(B) $\pi/2 + \tan^{-1}(1/4)$
(C) $\pi/2 - \tan^{-1}(1/4)$
(D) $\pi/2 + 2 \tan^{-1}(1/4)$

Correct Answer: (B) $\pi/2 + \tan^{-1}(1/4)$

Solution:

Concept:

- The centre of the ellipse is the origin. The angle subtended by the chord is the angle between the position vectors of its two endpoints.
- For vectors $\mathbf{u}, \mathbf{v}$, $\cos \phi = (\mathbf{u} \cdot \mathbf{v}) / (|\mathbf{u}| |\mathbf{v}|)$. The sign of the dot product determines whether the angle is acute or obtuse.

Step 1: Find the chord endpoints Substitute $y = x + 1$ in the ellipse equation. Then

$$\frac{x^2}{2} + (x + 1)^2 = 1 \implies 3x^2 + 4x = 0.$$

Thus $x = 0$ or $x = -4/3$, giving $A = (0, 1)$ and $B = (-4/3, -1/3)$. Both points satisfy the line and the ellipse, so these are the required endpoints.

Step 2: Compute the angle at the origin The position vectors give

$$\vec{OA} \cdot \vec{OB} = -\frac{1}{3}, \quad |OA| = 1, \quad |OB| = \frac{\sqrt{17}}{3}.$$

Hence $\cos \phi = -1/\sqrt{17}$, so ϕ is obtuse. If $t = \tan^{-1}(1/4)$, then $\sin t = 1/\sqrt{17}$. Therefore $\cos(\pi/2 + t) = -1/\sqrt{17}$, which identifies $\phi = \pi/2 + t$.

Step 3: Compare the alternatives Option (C) is acute and has a positive cosine. Option (A) has cosine $-4/\sqrt{17}$, while option (D) has cosine $-\sin(2t) = -8/17$. Neither equals the calculated value. Only option (B) gives the correct central angle.

Step 4: Conclusion The angle is $\boxed{\pi/2 + \tan^{-1}(1/4)}$. Hence, the correct option is (B).

Quick Tip to Solve: For an angle subtended at the origin, use endpoint vectors; a negative dot product prevents choosing the acute supplementary angle.

2. Number of solutions of $\sqrt{3} \cos 2\theta + 8 \cos \theta + 3\sqrt{3} = 0, \theta \in [-3\pi, 2\pi]$ is :

[4 marks]

- (A) 5
- (B) 4
- (C) 3
- (D) 0

Correct Answer: (A) 5

Solution:

Concept:

- Replace $\cos 2\theta$ by $2\cos^2 \theta - 1$ to turn the equation into a quadratic in $\cos \theta$.
- A real cosine lies in $[-1, 1]$. For $\cos \theta = c$, the full solution is $\theta = 2k\pi \pm \arccos c$, with integer k ; then enforce both interval endpoints.

Step 1: Solve the quadratic in cosine Put $c = \cos \theta$. The equation becomes

$$2\sqrt{3}c^2 + 8c + 2\sqrt{3} = 0 \implies \sqrt{3}c^2 + 4c + \sqrt{3} = 0.$$

Thus

$$c = \frac{-4 \pm \sqrt{16 - 12}}{2\sqrt{3}} = -\frac{1}{\sqrt{3}} \text{ or } -\sqrt{3}.$$

The second value is outside the cosine range and gives no real angle.

Step 2: Count every permitted angle Let $a = \arccos(-1/\sqrt{3})$; then $\pi/2 < a < \pi$. For $2k\pi + a$ in $[-3\pi, 2\pi]$, only $k = -1, 0$ work, giving $-2\pi + a$ and a . For $2k\pi - a$, only $k = -1, 0, 1$ work, giving $-2\pi - a, -a, 2\pi - a$. These five angles are distinct. Neither endpoint is a solution: their cosines are -1 and 1 , respectively.

Step 3: Check the count against the options The first sign contributes two roots and the second contributes three. Counting only complete periods would miss the extra root in the remaining interval. Thus the values 4 and 3 undercount, and 0 wrongly discards the admissible cosine root.

Step 4: Conclusion There are $\boxed{5}$ solutions. Hence, the correct option is (A).

Quick Tip to Solve: After rejecting inadmissible cosine roots, count the two families separately using bounds on the principal angle; do not round the number of periods.

3. Let the direction cosines of two lines satisfy the equations : $4l + m - n = 0$ and $2mn + 10nl + 3lm = 0$. Then the cosine of the acute angle between these lines is : [4 marks]

(A) $20/(3\sqrt{38})$

(B) $10/(7\sqrt{38})$

(C) $10/\sqrt{38}$

(D) $10/(3\sqrt{38})$

Correct Answer: (D) $10/(3\sqrt{38})$

Solution:

Concept:

- Direction cosines also serve as direction ratios. Homogeneous equations therefore determine the ratio $l : m : n$ without first imposing $l^2 + m^2 + n^2 = 1$.
- The cosine of the acute angle between direction vectors $\mathbf{u}$ and $\mathbf{v}$ is $|\mathbf{u} \cdot \mathbf{v}|/(|\mathbf{u}||\mathbf{v}|)$. The absolute value removes orientation dependence.

Step 1: Eliminate one direction cosine The linear equation gives $n = 4l + m$. Substitution into the quadratic equation yields

$$2m(4l + m) + 10l(4l + m) + 3lm = 0,$$

or

$$2m^2 + 21lm + 40l^2 = (m + 8l)(2m + 5l) = 0.$$

If $l = 0$, these equations also force $m = n = 0$, which cannot represent a direction. Thus the two factors provide the two valid lines.

Step 2: Choose direction vectors From $m = -8l$, we get $n = -4l$, so take $\mathbf{u} = (1, -8, -4)$. From $m = -5l/2$, take $l = 2$, giving $\mathbf{v} = (2, -5, 3)$. Their scalar product and lengths are

$$\mathbf{u} \cdot \mathbf{v} = 2 + 40 - 12 = 30, \quad |\mathbf{u}| = 9, \quad |\mathbf{v}| = \sqrt{38}.$$

Step 3: Calculate and compare the cosine Therefore

$$\cos \phi = \frac{|\mathbf{u} \cdot \mathbf{v}|}{|\mathbf{u}||\mathbf{v}|} = \frac{30}{9\sqrt{38}} = \frac{10}{3\sqrt{38}}.$$

Options (A) and (C) even exceed 1 and cannot be cosines. Option (B) uses the wrong normalization factor; the first vector has length 9, not 21. The calculated ratio agrees only with option (D).

Step 4: Conclusion The direction vectors give $\cos \phi = 30/(9\sqrt{38}) = \boxed{10/(3\sqrt{38})}$, which is the fourth printed option. Hence, the correct option is (D).

Quick Tip to Solve: Factor the homogeneous equation after eliminating one variable, then normalize the resulting direction ratios only when taking the angle.

4. Let α and β respectively be the maximum and the minimum values of the function $f(\theta) = 4 \left(\sin^4 \left(\frac{7\pi}{2} - \theta \right) + \sin^4(11\pi + \theta) \right) - 2 \left(\sin^6 \left(\frac{3\pi}{2} - \theta \right) + \sin^6(9\pi - \theta) \right)$, $\theta \in \mathbf{R}$.

Then $\alpha + 2\beta$ is equal to :

[4 marks]

(A) 3

(B) 6

(C) 4

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- Even powers remove signs introduced by shifts through integer multiples of π . Thus all shifted sine terms reduce to powers of $\sin \theta$ or $\cos \theta$.
- If $u = \sin^2 \theta$ and $v = \cos^2 \theta$, then $u + v = 1$, $u^2 + v^2 = 1 - 2uv$ and $u^3 + v^3 = 1 - 3uv$. Also $0 \leq uv \leq 1/4$.

Step 1: Remove the phase shifts The four sines reduce as follows:

$$\begin{aligned}\sin(7\pi/2 - \theta) &= -\cos \theta, & \sin(11\pi + \theta) &= -\sin \theta, \\ \sin(3\pi/2 - \theta) &= -\cos \theta, & \sin(9\pi - \theta) &= \sin \theta.\end{aligned}$$

Since the powers are even, their signs do not affect the expression. Consequently

$$f = 4(\cos^4 \theta + \sin^4 \theta) - 2(\cos^6 \theta + \sin^6 \theta).$$

Step 2: Express the function through a bounded product Write $p = \sin^2 \theta \cos^2 \theta = \frac{1}{4} \sin^2 2\theta$, so $0 \leq p \leq 1/4$. The identities in the concept block give

$$f = 4(1 - 2p) - 2(1 - 3p) = 2 - 2p.$$

This decreases as p increases. Its maximum is $\alpha = 2$ at $p = 0$, attained for example at $\theta = 0$. Its minimum is $\beta = 3/2$ at $p = 1/4$, attained at $\theta = \pi/4$.

Step 3: Evaluate the requested combination Use the maximum and minimum in the specified order:

$$\alpha + 2\beta = 2 + 2 \left(\frac{3}{2} \right) = 5.$$

The alternatives 3, 6 and 4 do not equal this combination; in particular, the minimum is not 1 and the maximum is not 3.

Step 4: Conclusion The attained extrema $\alpha = 2$ and $\beta = 3/2$ give $\boxed{\alpha + 2\beta = 5}$, the fourth printed option. Hence, the correct option is (D).

Quick Tip to Solve: For symmetric fourth and sixth powers of sine and cosine, use their squared product; it turns an extrema calculation into a linear expression.

5. Let $f(x) = \begin{cases} \frac{ax^2+2ax+3}{4x^2+4x-3} & , x \neq -\frac{3}{2}, \frac{1}{2} \\ b & , x = -\frac{3}{2}, \frac{1}{2} \end{cases}$ be continuous at $x = -\frac{3}{2}$. If $f \circ f(x) = \frac{7}{5}$, then x is equal to: [4 marks]

- (A) 0
(B) 1
(C) 1.4
(D) 2

Correct Answer: (B) 1

Solution:

Concept:

- A rational function can have a finite limit at a zero of its denominator only if the numerator also vanishes there. Continuity then fixes the assigned value.
- Composition requires applying the function twice, including its separately defined values at exceptional inputs. A cancelled rational expression must not silently replace those definitions.

Step 1: Use continuity to determine the constants At $x = -3/2$, the numerator is $3 - 3a/4$, which must vanish. Thus $a = 4$. For $x \neq -3/2, 1/2$, factor and cancel to obtain

$$f(x) = \frac{4x^2 + 8x + 3}{4x^2 + 4x - 3} = \frac{(2x+1)(2x+3)}{(2x-1)(2x+3)} = \frac{2x+1}{2x-1}.$$

The limit at $-3/2$ equals $(-2)/(-4) = 1/2$, so continuity gives $b = 1/2$.

Step 2: Solve the composition with its exceptional cases If $x = -3/2$ or $x = 1/2$, then $f(x) = 1/2$ and $f(f(x)) = 1/2$, not $7/5$. If any other input makes the inner value an exceptional argument, the outer value again equals $b = 1/2$. All solutions of the requested equation therefore use the rational rule twice, giving

$$f(f(x)) = \frac{2\frac{2x+1}{2x-1} + 1}{2\frac{2x+1}{2x-1} - 1} = \frac{6x+1}{2x+3}.$$

Solving $5(6x+1) = 7(2x+3)$ gives $16x = 16$, hence $x = 1$.

Step 3: Verify and reject the other choices At $x = 1$, $f(1) = 3$ and $f(3) = 7/5$, so no exceptional value intervenes. For the other listed inputs 0, 1.4, 2, the composition is respectively $1/3, 47/29, 13/7$; none is $7/5$.

Step 4: Conclusion The required value is $x = 1$: $f(1) = 3$ and $f(3) = 7/5$, verifying the second printed option directly. Hence, the correct option is (B).

Quick Tip to Solve: After cancelling a factor, retain the original exceptional inputs separately; check both the inner and outer arguments before using a composition formula.

6. If α and β ($\alpha < \beta$) are the roots of the equation $(-2 + \sqrt{3})(|\sqrt{x} - 3|) + (x - 6\sqrt{x}) + (9 - 2\sqrt{3}) = 0, x \geq 0$, then $\sqrt{\frac{\beta}{\alpha}} + \sqrt{\alpha\beta}$ is equal to : [4 marks]

- (A) 8
(B) 9
(C) 10
(D) 11

Correct Answer: (C) 10

Solution:

Concept:

- For $x \geq 0$, set $u = \sqrt{x}$, so $u \geq 0$. Completing the square gives $x - 6\sqrt{x} + 9 = (u - 3)^2$.
- With $t = |u - 3|$, we have $(u - 3)^2 = t^2$ and $t \geq 0$. A negative root in t must be rejected before recovering the possible values of u .

Step 1: Reduce the equation to a quadratic The given equation can be written as

$$(\sqrt{3} - 2)|u - 3| + (u - 3)^2 - 2\sqrt{3} = 0.$$

Put $t = |u - 3|$. It becomes

$$t^2 + (\sqrt{3} - 2)t - 2\sqrt{3} = 0 \implies (t - 2)(t + \sqrt{3}) = 0.$$

The roots are $t = 2$ and $t = -\sqrt{3}$, but the latter is impossible because t is an absolute value.

Step 2: Recover the two roots in the original variable The admissible condition is $|u - 3| = 2$, so $u = 1$ or $u = 5$. Both satisfy $u \geq 0$. Squaring these nonnegative values gives $x = 1$ and $x = 25$; therefore $\alpha = 1, \beta = 25$, consistent with $\alpha < \beta$. Substitution also checks them: both have $|\sqrt{x} - 3| = 2$ and $x - 6\sqrt{x} = -5$, making the original left side zero.

Step 3: Evaluate the expression using the ordered roots Now

$$\sqrt{\frac{\beta}{\alpha}} + \sqrt{\alpha\beta} = \sqrt{25} + \sqrt{25} = 5 + 5 = 10.$$

The other alternatives 8, 9 and 11 do not match this sum. Using the roots in u rather than the roots in x would incorrectly change both radicals.

Step 4: Conclusion The roots $\alpha = 1$ and $\beta = 25$ give $\sqrt{\beta/\alpha} + \sqrt{\alpha\beta} = 5 + 5 = \boxed{10}$, the third printed option. Hence, the correct option is (C).

Quick Tip to Solve: Complete the square around the absolute-value centre, then reject negative absolute-value roots before squaring back to the original variable.

7. The vertices B and C of a triangle ABC lie on the line $\frac{x}{1} = \frac{1-y}{-2} = \frac{z-2}{3}$. The coordinates of A and B are (1, 6, 3) and (4, 9, α) respectively and C is at a distance of 10 units from B. The area (in sq. units) of $\triangle ABC$ is : [4 marks]

- (A) $10\sqrt{13}$
 (B) $5\sqrt{13}$
 (C) $20\sqrt{13}$
 (D) $15\sqrt{13}$

Correct Answer: (B) $5\sqrt{13}$

Solution:

Concept:

- The area of a triangle is half its base times the perpendicular distance of the opposite vertex from the base line.
- For a point displacement $\mathbf{v}$ from a point on a line of direction $\mathbf{d}$, the perpendicular distance is $h = |\mathbf{v} \times \mathbf{d}|/|\mathbf{d}|$.

Step 1: Read the direction of the line Put the three ratios equal to t . Then $x = t$, $1 - y = -2t$, and $z = 2 + 3t$, so the line has direction $\mathbf{d} = (1, 2, 3)$. In particular, the second direction component is positive: the numerator is $1 - y$, not $y - 1$. At $B = (4, 9, \alpha)$, $t = 4$, giving $\alpha = 14$.

Step 2: Find the perpendicular height With $A = (1, 6, 3)$ and $B = (4, 9, 14)$, take $\mathbf{v} = \overrightarrow{AB} = (3, 3, 11)$. The cross product is

$$\mathbf{v} \times \mathbf{d} = (9 - 22, 11 - 9, 6 - 3) = (-13, 2, 3).$$

Consequently,

$$h = \frac{\sqrt{169 + 4 + 9}}{\sqrt{1 + 4 + 9}} = \frac{\sqrt{182}}{\sqrt{14}} = \sqrt{13}.$$

Reversing the displacement vector changes only the sign of the cross product, not this distance.

Step 3: Use the given base length The distance $BC = 10$ is the base, and both possible positions of C on the line have the same perpendicular height from A. Therefore

$$[ABC] = \frac{1}{2}(10)(\sqrt{13}) = 5\sqrt{13}.$$

The alternatives $10\sqrt{13}$, $20\sqrt{13}$, and $15\sqrt{13}$ do not equal half the specified base times this height; the first of them omits the factor one-half.

Step 4: Conclusion The area is $5\sqrt{13}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: When two vertices lie on a known line, use their separation as the base and a point-to-line cross product for the height; finding both possible third coordinates is unnecessary.

8. Among the statements :

I: If $\begin{vmatrix} 1 & \cos \alpha & \cos \beta \\ \cos \alpha & 1 & \cos \gamma \\ \cos \beta & \cos \gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos \alpha & \cos \beta \\ \cos \alpha & 0 & \cos \gamma \\ \cos \beta & \cos \gamma & 0 \end{vmatrix}$, **then** $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \frac{3}{2}$, **and**

II: If $\begin{vmatrix} x^2 + x & x + 1 & x - 2 \\ 2x^2 + 3x - 1 & 3x & 3x - 3 \\ x^2 + 2x + 3 & 2x - 1 & 2x - 1 \end{vmatrix} = px + q$, **then** $p^2 = 196q^2$,

[4 marks]

- (A) both are true
(B) both are false
(C) only I is true
(D) only II is true

Correct Answer: (B) both are false

Solution:

Concept:

- Expand a determinant along a row, or use operations $R_i \rightarrow R_i + kR_j$, which preserve its value.
- For a statement asserting a determinant identity, compare the resulting coefficients exactly; the two statements must be tested separately.

Step 1: Test statement I Write $a = \cos \alpha$, $b = \cos \beta$, and $c = \cos \gamma$. Direct expansion gives

$$\begin{vmatrix} 1 & a & b \\ a & 1 & c \\ b & c & 1 \end{vmatrix} = 1 - a^2 - b^2 - c^2 + 2abc, \quad \begin{vmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{vmatrix} = 2abc.$$

Equality therefore implies $a^2 + b^2 + c^2 = 1$, not $3/2$. Thus statement I is false. The condition is possible, for instance with $a = 1$, $b = c = 0$, so this is not an empty condition.

Step 2: Reduce the determinant in statement II Apply $R_2 \rightarrow R_2 - 2R_1$ and $R_3 \rightarrow R_3 - R_1$. The last two rows become $(x - 1, x - 2, x + 1)$ and $(x + 3, x - 2, x + 1)$. Next replace R_3 by $R_3 - R_2$, giving $(4, 0, 0)$. Expansion along this last row has a positive cofactor sign:

$$\begin{aligned} D &= 4 \begin{vmatrix} x + 1 & x - 2 \\ x - 2 & x + 1 \end{vmatrix} \\ &= 4((x + 1)^2 - (x - 2)^2) \\ &= 4(6x - 3) = 24x - 12. \end{aligned}$$

Step 3: Compare the coefficient relation Here $p = 24$ and $q = -12$, so $p^2 = 576$ and $q^2 = 144$. Thus $p^2 = 4q^2$, whereas $196q^2 = 28224$. Statement II is also false. Options saying both true, only I true, or only II true each contradict at least one of these direct calculations.

Step 4: Conclusion Both statements are false. Hence, the correct option is (B).

Quick Tip to Solve: For the second determinant, row subtraction exposes two equal pairs of entries; their difference leaves a row with two zeros and avoids a cubic expansion.

9. The value of the integral $\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1 + \sqrt[3]{\tan 2x}}$ is :

[4 marks]

- (A) $\frac{\pi}{3}$
- (B) $\frac{\pi}{6}$
- (C) $\frac{\pi}{12}$
- (D) $\frac{\pi}{18}$

Correct Answer: (C) $\frac{\pi}{12}$

Solution:

Concept:

- For an integrable function, reflection of an interval gives $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$.
- The complementary-angle identity $\tan(\pi/2 - \theta) = 1/\tan \theta$ turns the positive cube root into its reciprocal.

Step 1: Check the interval and its reflection Let $a = \pi/24$, $b = 5\pi/24$. Their sum is $a + b = \pi/4$. Throughout this interval, $2x$ lies between $\pi/12$ and $5\pi/12$, where tangent is finite and positive. The cube root and the denominator are consequently well-defined, with no improper endpoint.

Step 2: Write a second expression for the integral Under $x \mapsto \pi/4 - x$, the argument $2x$ becomes $\pi/2 - 2x$. If $t = \sqrt[3]{\tan 2x} > 0$, the reflected integrand is

$$\frac{1}{1 + \sqrt[3]{\tan(\pi/2 - 2x)}} = \frac{1}{1 + 1/t} = \frac{t}{1 + t}.$$

Hence the same integral I can be written as the integral of $t/(1+t)$ over the original limits.

Step 3: Add the two integrals Adding pointwise complementary integrands eliminates the cube root:

$$\begin{aligned} 2I &= \int_{\pi/24}^{5\pi/24} \left(\frac{1}{1+t} + \frac{t}{1+t} \right) dx \\ &= \int_{\pi/24}^{5\pi/24} 1 dx = \frac{4\pi}{24} = \frac{\pi}{6}. \end{aligned}$$

Therefore $I = \pi/12$. The option $\pi/6$ is the interval length, twice the answer; $\pi/3$ exceeds that length despite the integrand being below one. The remaining value $\pi/18$ contradicts the exact reflection identity.

Step 4: Conclusion The reflection identity gives $2I = \pi/6$, so the integral equals $\boxed{\pi/12}$. Hence, the correct option is (C).

Quick Tip to Solve: When the limits add to $\pi/4$ and the integrand involves $\tan 2x$, reflect about the midpoint before trying an antiderivative.

10. Let the domain of the function $f(x) = \log_3 \log_5 \log_7 (9x - x^2 - 13)$ be the interval (m, n) . Let the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ have eccentricity $\frac{n}{3}$ and the length of the latus rectum $\frac{8m}{3}$. Then $b^2 - a^2$ is equal to : [4 marks]

- (A) 5
(B) 7
(C) 9
(D) 11

Correct Answer: (B) 7

Solution:

Concept:

- Each real logarithm requires a strictly positive argument. For bases greater than one, logarithmic inequalities preserve their direction.
- For $x^2/a^2 - y^2/b^2 = 1$, eccentricity satisfies $e^2 = 1 + b^2/a^2$, and the full latus rectum has length $2b^2/a$, with $a, b > 0$.

Step 1: Determine the nested logarithm domain Put $U = 9x - x^2 - 13$. The outer logarithm requires $\log_5(\log_7 U) > 0$. Since the base 5 exceeds one, this is equivalent to $\log_7 U > 1$, and hence to $U > 7$. This strongest condition also ensures the inner logarithms exist. Therefore

$$9x - x^2 - 13 > 7 \iff x^2 - 9x + 20 < 0 \iff (x - 4)(x - 5) < 0.$$

The domain is $(4, 5)$, so $m = 4, n = 5$. Equality is excluded because it would make the outer logarithm argument zero.

Step 2: Use the eccentricity Here $e = n/3 = 5/3$. Substituting into the hyperbola identity gives

$$\frac{25}{9} = 1 + \frac{b^2}{a^2}, \quad b^2 = \frac{16a^2}{9}.$$

This determines the ratio of the squared semiaxes, not their individual lengths.

Step 3: Use the latus rectum to fix the scale The full latus rectum is $8m/3 = 32/3$, so

$$\frac{2b^2}{a} = \frac{32}{3} \implies \frac{32a}{9} = \frac{32}{3} \implies a = 3.$$

It follows that $a^2 = 9$, $b^2 = 16$, and $b^2 - a^2 = 7$. The alternatives 5, 9, and 11 fail this difference; in particular, 9 is a^2 , not the expression requested.

Step 4: Conclusion The printed data give $a^2 = 9$ and $b^2 = 16$, hence $b^2 - a^2 = 7$. Hence, the correct option is (B).

Quick Tip to Solve: Work from the outermost logarithm inward. For the hyperbola, eccentricity fixes a ratio and latus rectum then fixes the scale.

11. Let $f(x) = \int \frac{(2-x^2)e^x}{\sqrt{1+x}(1-x)^{3/2}} dx$. If $f(0) = 0$, then $f\left(\frac{1}{2}\right)$ is equal to:

[4 marks]

- (A) $\sqrt{3e} + 1$
- (B) $\sqrt{3e} - 1$
- (C) $\sqrt{2e} + 1$
- (D) $\sqrt{2e} - 1$

Correct Answer: (B) $\sqrt{3e} - 1$

Solution:

Concept:

- To integrate an exponential multiplied by an algebraic expression, look for a product $e^x g(x)$. Its derivative is $e^x [g(x) + g'(x)]$.
- The real integrand is defined on $-1 < x < 1$. The radical $g(x) = \sqrt{(1+x)/(1-x)}$ has a derivative containing the same denominator as the integrand.

Step 1: Differentiate the radical factor Put $g(x) = (1+x)^{1/2}(1-x)^{-1/2}$. Applying the product and chain rules gives

$$\begin{aligned} g'(x) &= \frac{1}{2\sqrt{1+x}\sqrt{1-x}} + \frac{\sqrt{1+x}}{2(1-x)^{3/2}} \\ &= \frac{(1-x) + (1+x)}{2\sqrt{1+x}(1-x)^{3/2}} \\ &= \frac{1}{\sqrt{1+x}(1-x)^{3/2}}. \end{aligned}$$

The positive sign in the second term follows from differentiating both the negative exponent and the inner factor $1-x$.

Step 2: Recognise the complete derivative Expressing $g(x)$ over the same denominator, its numerator becomes $(1+x)(1-x) = 1-x^2$. Therefore

$$\begin{aligned} \frac{d}{dx} [e^x g(x)] &= e^x [g(x) + g'(x)] \\ &= \frac{e^x [(1-x^2) + 1]}{\sqrt{1+x}(1-x)^{3/2}}. \end{aligned}$$

This is exactly the given integrand, so

$$f(x) = e^x \sqrt{\frac{1+x}{1-x}} + C.$$

Step 3: Apply the initial value and evaluate At $x = 0$, the exponential and the radical both equal 1. Hence $0 = f(0) = 1 + C$, so $C = -1$. At $x = 1/2$, the ratio inside the radical equals $(3/2)/(1/2) = 3$, giving

$$f\left(\frac{1}{2}\right) = e^{1/2} \sqrt{3} - 1 = \sqrt{3e} - 1.$$

Option A has the wrong integration constant. Options C and D have $\sqrt{2e}$ instead of $\sqrt{3e}$, which cannot arise from the radical evaluated at $1/2$.

Step 4: Conclusion The exact derivative check and the condition $f(0) = 0$ give $f(1/2) = \sqrt{3e} - 1$. Hence, the correct option is (B).

Quick Tip to Solve: When an integrand contains e^x and neighbouring half-powers, test the derivative of e^x times the corresponding radical ratio before trying integration by parts.

12. A building construction work can be completed by two masons A and B together in 22.5 days. Mason A alone can complete the construction work in 24 days less than mason B alone. Then mason A alone will complete the construction work in : [4 marks]

- (A) 24 days
- (B) 30 days
- (C) 36 days
- (D) 42 days

Correct Answer: (C) 36 days

Solution:

Concept:

- If a worker takes t days to complete one job, the daily work rate is $1/t$ jobs per day. Rates add when two workers work together.
- Time must be positive. A quadratic equation obtained from work rates may have an algebraic root that cannot represent an elapsed duration.

Step 1: Represent the individual completion times Let mason A alone take t days. Since A takes 24 days less than B, mason B takes $t + 24$ days. Their combined time is $22.5 = 45/2$ days, giving combined rate $2/45$ jobs per day. Thus

$$\frac{1}{t} + \frac{1}{t+24} = \frac{2}{45}, \quad t > 0.$$

Adding times instead of their reciprocals would not describe simultaneous work.

Step 2: Form and solve the quadratic Combine the fractions and cross-multiply, which is valid for positive t :

$$\begin{aligned} 45(2t + 24) &= 2t(t + 24), \\ 90t + 1080 &= 2t^2 + 48t, \\ t^2 - 21t - 540 &= 0, \\ (t - 36)(t + 15) &= 0. \end{aligned}$$

The algebraic roots are 36 and -15 . A negative duration is inadmissible, leaving $t = 36$ days.

Step 3: Check the result against the original data B would then take $36 + 24 = 60$ days, and

$$\frac{1}{36} + \frac{1}{60} = \frac{5+3}{180} = \frac{2}{45}.$$

The reciprocal of this combined rate is exactly 22.5 days. A alone must take longer than the combined

time, and the selected value satisfies that requirement. Substituting the other proposed times 24, 30, or 42 into $t^2 - 21t - 540$ gives respectively -468 , -270 , and 342 , so none satisfies the work equation.

Step 4: Conclusion Mason A alone completes the construction in 36 days. Hence, the correct option is (C).

Quick Tip to Solve: Assign the faster worker the time t and the slower worker $t + 24$, add their daily rates, and reject negative roots after solving.

13. Let $y = y(x)$ be the solution of the differential equation $x^4 dy + (4x^3 y + 2 \sin x) dx = 0, x > 0, y(\frac{\pi}{2}) = 0$.

Then $\pi^4 y(\frac{\pi}{3})$ is equal to :

[4 marks]

(A) 81

(B) 72

(C) 64

(D) 92

Correct Answer: (A) 81

Solution:

Concept:

- The product rule gives $\frac{d}{dx}(x^4 y) = x^4 y' + 4x^3 y$. Recognising this derivative avoids a lengthy integration-factor calculation.
- An initial condition determines the integration constant uniquely on the interval $x > 0$; division by x^4 is valid throughout that interval.

Step 1: Identify the exact derivative Divide the stated equation by dx , retaining the sign of the sine term. This gives

$$x^4 y' + 4x^3 y = -2 \sin x.$$

The two terms on the left are precisely the derivative of $x^4 y$, because differentiating the product contributes both $x^4 y'$ and $4x^3 y$. Hence

$$\frac{d}{dx}(x^4 y) = -2 \sin x.$$

Step 2: Integrate and apply the condition Integrating with respect to x yields

$$x^4 y = 2 \cos x + C.$$

The sign is positive because the derivative of $2 \cos x$ is $-2 \sin x$. At $x = \pi/2$, the given value of y is zero and $\cos(\pi/2) = 0$. Therefore $C = 0$, so

$$y(x) = \frac{2 \cos x}{x^4}.$$

Step 3: Evaluate the required expression Now substitute $x = \pi/3$, for which $2 \cos(\pi/3) = 1$:

$$y\left(\frac{\pi}{3}\right) = \frac{1}{(\pi/3)^4} = \frac{81}{\pi^4}.$$

Multiplication by the stated factor π^4 gives 81. The alternatives 72, 64, 92 do not satisfy this exact evaluation; the fourth power of 3 is essential, rather than a cubic or square factor.

Step 4: Conclusion $\pi^4 y(\pi/3) = \boxed{81}$. Hence, the correct option is (A).

Quick Tip to Solve: When the coefficients are x^n and nx^{n-1} , test the product derivative $(x^n y)'$ before finding an integrating factor.

14. A rectangle is formed by the lines $x = 0, y = 0, x = 3$ and $y = 4$. Let the line L be perpendicular to $3x + y + 6 = 0$ and divide the area of the rectangle into two equal parts. Then the distance of the point $(\frac{1}{2}, -5)$ from the line L is equal to : [4 marks]

- (A) $2\sqrt{5}$
- (B) $3\sqrt{10}$
- (C) $\sqrt{10}$
- (D) $2\sqrt{10}$

Correct Answer: (D) $2\sqrt{10}$

Solution:

Concept:

- A line through the centre of a rectangle bisects its area: a half-turn about the centre interchanges the two portions.
- For a point (x_0, y_0) , the perpendicular distance from $ax + by + c = 0$ is $|ax_0 + by_0 + c|/\sqrt{a^2 + b^2}$.

Step 1: Determine the direction and centre The rectangle has vertices $(0, 0), (3, 0), (3, 4), (0, 4)$, so its centre is $(3/2, 2)$. The line $3x + y + 6 = 0$ has slope -3 ; a perpendicular line therefore has slope $1/3$. For this fixed direction, the bisecting line passes through the centre. Translating a parallel line away from the centre strictly increases one portion of the rectangle and decreases the other, so it cannot remain an equal-area cut.

Step 2: Write and check the bisector Using point-slope form,

$$y - 2 = \frac{1}{3} \left(x - \frac{3}{2} \right), \quad L : x - 3y + \frac{9}{2} = 0.$$

This line meets the vertical sides at heights $3/2$ and $5/2$, both within the rectangle. The area below it is the trapezium area

$$\frac{1}{2} \left(\frac{3}{2} + \frac{5}{2} \right) 3 = 6,$$

which is half the total area $3 \times 4 = 12$.

Step 3: Compute the perpendicular distance Substitute $(x_0, y_0) = (1/2, -5)$ into the distance formula:

$$d = \frac{|\frac{1}{2} - 3(-5) + \frac{9}{2}|}{\sqrt{1 + 9}} = \frac{20}{\sqrt{10}} = 2\sqrt{10}.$$

The alternatives $2\sqrt{5}$, $3\sqrt{10}$, $\sqrt{10}$ disagree with the numerator 20 and the normal-vector length $\sqrt{10}$.

Step 4: Conclusion The bisector is $x - 3y + 9/2 = 0$, giving distance $20/\sqrt{10} = \boxed{2\sqrt{10}}$ units from the stated point. Hence, the correct option is (D).

Quick Tip to Solve: For a rectangle, first draw the required direction through its centre; then verify the cut using a trapezium area if needed.

15. Let the mean and variance of 8 numbers $-10, -7, -1, x, y, 9, 2, 16$ be $\frac{7}{2}$ and $\frac{293}{4}$, respectively. Then the mean of 4 numbers $x, y, x + y + 1, |x - y|$ is: [4 marks]

- (A) 9
- (B) 10
- (C) 11
- (D) 12

Correct Answer: (C) 11

Solution:

Concept:

- For n observations with mean μ and variance σ^2 , the sum is $n\mu$, and the sum of squares is $n(\sigma^2 + \mu^2)$.
- Once $x + y$ and $x^2 + y^2$ are known, the identity $(x - y)^2 = 2(x^2 + y^2) - (x + y)^2$ gives the absolute difference without assigning separate values to x and y .

Step 1: Determine the sum of the unknown observations The mean of the eight original observations is $7/2$. Their total is therefore $8 \times 7/2 = 28$. The six known observations have sum

$$-10 - 7 - 1 + 9 + 2 + 16 = 9.$$

Thus $x + y = 28 - 9 = 19$. The order in which the observations are listed does not affect their mean.

Step 2: Use the variance to find the sum of squares The variance equals the mean square minus the square of the mean. Hence the sum of the eight squared observations is

$$8 \left(\frac{293}{4} + \left(\frac{7}{2} \right)^2 \right) = 8 \left(\frac{342}{4} \right) = 684.$$

The known squares contribute

$$100 + 49 + 1 + 81 + 4 + 256 = 491.$$

Subtracting gives $x^2 + y^2 = 684 - 491 = 193$. Using only the variance without adding μ^2 would omit

part of the total mean square.

Step 3: Obtain the absolute difference and the new average Using the two results,

$$(x - y)^2 = 2(193) - 19^2 = 386 - 361 = 25, \quad |x - y| = 5.$$

The third new observation is $x + y + 1 = 20$. Thus the sum of the four new observations is $19 + 20 + 5 = 44$, and their mean is

$$\frac{x + y + (x + y + 1) + |x - y|}{4} = \frac{44}{4} = 11.$$

As a check, x, y are 12 and 7 in either order; the new list is therefore 12, 7, 20, 5. Options A, B and D give 9, 10 and 12, none of which equals this four-number average.

Step 4: Conclusion The sum of the new observations is 44, so the required mean is 11. Hence, the correct option is (C).

Quick Tip to Solve: Find $x + y$ from the mean and $x^2 + y^2$ from the variance, then use the difference-square identity. There is no need to decide which unknown is the larger one.

16. Let $A = \{-2, -1, 0, 1, 2, 3, 4\}$. Let R be a relation on A defined by xRy if and only if $2x + y \leq 2$. Let l be the number of elements in R . Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations respectively. Then $l + m + n$ is equal to : [4 marks]

- (A) 35
- (B) 32
- (C) 33
- (D) 34

Correct Answer: (C) 33

Solution:

Concept:

- A relation on A consists of ordered pairs. For each fixed first coordinate x , count the permitted second coordinates from $y \leq 2 - 2x$.
- Reflexivity requires every diagonal pair (x, x) . Symmetry requires the reverse (y, x) of every existing pair (x, y) ; only missing reverse pairs need to be added.

Step 1: Count the pairs already present The elements of A are $-2, -1, 0, 1, 2, 3, 4$. Applying the bound $y \leq 2 - 2x$ for each row gives

x	-2	-1	0	1	2	3	4
$2 - 2x$	6	4	2	0	-2	-4	-6
permitted y count	7	7	5	3	1	0	0

Thus $l = 7 + 7 + 5 + 3 + 1 = 23$. A row with a bound below -2 has no allowed second coordinate.

Step 2: Complete the diagonal for reflexivity A diagonal pair is already present exactly when

$2x + x \leq 2$, or $x \leq 2/3$. Thus the pairs for $x = -2, -1, 0$ are present. The missing pairs are $(1, 1), (2, 2), (3, 3), (4, 4)$, and each must be added. Consequently $m = 4$.

Step 3: Count the missing reverse pairs For symmetry, inspect reverses independently of the reflexive completion. The pairs that must be added are

$$(2, -1), (2, 0), (3, -2), \\ (3, -1), (4, -2), (4, -1).$$

Each is absent although its reverse belongs to R ; all other existing pairs already have their reverse. Hence $n = 6$, and

$$l + m + n = 23 + 4 + 6 = 33.$$

The other totals 35, 32, 34 do not follow from these counts. Missing diagonal pairs are not counted again in the symmetry calculation, since diagonal pairs are their own reverses.

Step 4: Conclusion $l + m + n = 33$. Hence, the correct option is (C).

Quick Tip to Solve: Count missing reverse pairs as $R^{-1} \setminus R$. Do not double-count pairs whose reverse is already in the relation.

17. Let $\vec{a} = -\hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} - \hat{j} - 3\hat{k}$, $\vec{c} = \vec{a} \times \vec{b}$ **and** $\vec{d} = \vec{c} \times \vec{a}$. **Then** $(\vec{a} - \vec{b}) \cdot \vec{d}$ **is equal to :** [4 marks]

- (A) 4
(B) -4
(C) 2
(D) -2

Correct Answer: (D) -2

Solution:

Concept:

- The cross product is perpendicular to each factor, so $\vec{d} = (\vec{a} \times \vec{b}) \times \vec{a}$ satisfies $\vec{a} \cdot \vec{d} = 0$.
- The vector triple-product identity is $(\vec{a} \times \vec{b}) \times \vec{a} = \vec{b}(\vec{a} \cdot \vec{a}) - \vec{a}(\vec{a} \cdot \vec{b})$. The order of the factors controls the sign.

Step 1: Find the scalar products Using $\vec{a} = (-1, 1, 2)$ and $\vec{b} = (1, -1, -3)$, compute

$$\vec{a} \cdot \vec{a} = 1 + 1 + 4 = 6, \quad \vec{a} \cdot \vec{b} = -1 - 1 - 6 = -8.$$

These values use the negative third component of $\vec{b}$, which is particularly important for the sign in the next step.

Step 2: Evaluate the second cross product Apply the stated identity in the actual order $(\vec{a} \times \vec{b}) \times \vec{a}$:

$$\vec{d} = 6\vec{b} + 8\vec{a} = (6, -6, -18) + (-8, 8, 16) = (-2, 2, -2).$$

As a direct check, $\vec{c} = \vec{a} \times \vec{b} = (-1, -1, 0)$, and $\vec{c} \times \vec{a} = (-2, 2, -2)$ gives the same vector. Also $\vec{a} \cdot \vec{d} = 2 + 2 - 4 = 0$, as perpendicularity requires.

Step 3: Take the final dot product The difference of the given vectors is

$$\vec{a} - \vec{b} = (-2, 2, 5).$$

Therefore

$$(\vec{a} - \vec{b}) \cdot \vec{d} = (-2)(-2) + (2)(2) + (5)(-2) = 4 + 4 - 10 = -2.$$

Equivalently, $\vec{b} \cdot \vec{d} = -2 - 2 + 6 = 2$ and $\vec{a} \cdot \vec{d} = 0$, so the result is $0 - 2$. The value $+2$ would result from reversing one cross product; ± 4 do not equal the computed dot product.

Step 4: Conclusion From the printed vectors, $\vec{d} = (-2, 2, -2)$, so $(\vec{a} - \vec{b}) \cdot \vec{d} = 4 + 4 - 10 = \boxed{-2}$. Hence, the correct option is (D).

Quick Tip to Solve: Check a double cross product by its perpendicularity: here $\vec{a} \cdot \vec{d} = 0$, reducing the final expression to $-\vec{b} \cdot \vec{d}$.

18. Let $S = \{z : 3 \leq |2z - 3(1 + i)| \leq 7\}$ be a set of complex numbers.

Then $\min_{z \in S} \left| z + \frac{1}{2}(5 + 3i) \right|$ is equal to :

[4 marks]

- (A) $\frac{1}{2}$
- (B) $\frac{3}{2}$
- (C) 2
- (D) $\frac{5}{2}$

Correct Answer: (B) $\frac{3}{2}$

Solution:

Concept:

- The modulus $|z - c|$ is the Euclidean distance from the point representing z to the point representing c .
- For a point outside a circle of radius R with centre C , the least distance to the disk is $PC - R$, attained on the radius towards the point.

Step 1: Describe the allowed annulus Factor 2 out of the given modulus:

$$|2z - 3(1 + i)| = 2 \left| z - \left(\frac{3}{2} + \frac{3}{2}i \right) \right|.$$

Thus S is the closed annulus centred at $C = 3/2 + (3/2)i$, with inner radius $3/2$ and outer radius $7/2$:

$$\frac{3}{2} \leq |z - C| \leq \frac{7}{2}.$$

Both boundary circles are included.

Step 2: Locate the point whose distance is minimized The required expression can be written as

$$\left| z + \frac{5+3i}{2} \right| = |z - P|, \quad P = -\frac{5}{2} - \frac{3}{2}i.$$

The vector from P to C is $4 + 3i$, so

$$|C - P| = \sqrt{4^2 + 3^2} = 5.$$

Since $5 > 7/2$, the point P lies outside the outer circle.

Step 3: Establish and attain the lower bound The reverse triangle inequality gives, for every allowed z ,

$$|z - P| \geq |C - P| - |z - C| \geq 5 - \frac{7}{2} = \frac{3}{2}.$$

Equality is attained at the point on the outer circle facing P :

$$z = C + \frac{7}{10}(P - C) = -\frac{13}{10} - \frac{3}{5}i.$$

It satisfies $|z - C| = 7/2$, so it is allowed despite the inner hole. Therefore the minimum is $3/2$. The value $1/2$ is below the proven bound, while 2 and $5/2$ exceed this attained distance.

Step 4: Conclusion The target point is 5 units from the annulus centre, so the attained minimum is $5 - 7/2 = \boxed{\frac{3}{2}}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert a modulus inequality into a centre and radii first. If the target point is outside the annulus, only its outer radius determines the minimum distance.

19. The sum of all possible values of $n \in \mathbb{N}$, so that the coefficients of x, x^2 and x^3 in the expansion of $(1+x^2)^2(1+x)^n$, are in arithmetic progression is : [4 marks]

- (A) 3
- (B) 7
- (C) 9
- (D) 12

Correct Answer: (C) 9

Solution:

Concept:

- The coefficient of a power in a product is found by adding every product of terms whose exponents have that total. Here $(1+x^2)^2 = 1 + 2x^2 + x^4$.
- Three numbers a, b, c form an arithmetic progression precisely when $2b = a+c$. Binomial coefficients with lower index greater than n are zero.

Step 1: Extract the three coefficients Only the terms 1 and $2x^2$ of the first factor can contribute up to

degree three. Thus, writing the coefficients in increasing degree as a, b, c ,

$$a = n, \quad b = \binom{n}{2} + 2, \quad c = \binom{n}{3} + 2n.$$

The term x^4 cannot contribute to these coefficients, so no term from it should enter the progression condition.

Step 2: Impose and factor the progression condition Substitute these expressions into $2b = a + c$, and multiply by six to clear the binomial denominators:

$$2 \left(\frac{n(n-1)}{2} + 2 \right) = 3n + \frac{n(n-1)(n-2)}{6}.$$

$$\begin{aligned} 0 &= n(n-1)(n-2) - 6n(n-1) + 18n - 24 \\ &= n^3 - 9n^2 + 26n - 24 \\ &= (n-2)(n-3)(n-4). \end{aligned}$$

Consequently the only possible natural numbers are $n = 2, 3, 4$.

Step 3: Check the roots and add them For $n = 2$, the coefficients are 2, 3, 4; for $n = 3$, they are 3, 5, 7; and for $n = 4$, they are 4, 8, 12. Each triple has equal consecutive differences. Their required sum is $2 + 3 + 4 = 9$. The other listed values 3, 7, 12 are not this sum; in particular, an individual coefficient or one root is not the requested quantity.

Step 4: Conclusion The sum of all admissible values of n is $\boxed{9}$. Hence, the correct option is (C). The progression condition gives $n = 2, 3, 4$, whose sum is 9; the printed option order places 9 at (C).

Quick Tip to Solve: For low-degree coefficients of a product, discard higher-degree terms first; then apply the progression condition before expanding the resulting polynomial.

20. The value of $\frac{{}^{100}C_{50}}{51} + \frac{{}^{100}C_{51}}{52} + \dots + \frac{{}^{100}C_{100}}{101}$ is:

[4 marks]

- (A) $\frac{2^{100}}{100}$
- (B) $\frac{2^{100}}{101}$
- (C) $\frac{2^{101}}{100}$
- (D) $\frac{2^{101}}{101}$

Correct Answer: (B) $\frac{2^{100}}{101}$

Solution:

Concept:

- The factorial identity $\binom{n}{r}/(r+1) = \binom{n+1}{r+1}/(n+1)$ converts a varying denominator into a constant one.
- Binomial coefficients are symmetric: $\binom{N}{j} = \binom{N}{N-j}$. Their complete sum is 2^N ; when N is odd, the coefficients divide into two equal halves with no unpaired central term.

Step 1: Raise the upper index by one Let the given sum be S . For each integer r from 50 to 100, use

factorials to obtain

$$\frac{\binom{100}{r}}{r+1} = \frac{100!}{(r+1)!(100-r)!} = \frac{1}{101} \binom{101}{r+1}.$$

Therefore, changing the index to $j = r + 1$,

$$S = \frac{1}{101} \sum_{j=51}^{101} \binom{101}{j}.$$

The first lower index is 51, not 50, because the denominator has been absorbed by increasing the lower index.

Step 2: Pair the two halves of the binomial expansion For every j between 51 and 101, the index $101 - j$ lies between 0 and 50. The corresponding coefficients are equal, so

$$\sum_{j=51}^{101} \binom{101}{j} = \sum_{j=0}^{50} \binom{101}{j} = \frac{1}{2} \sum_{j=0}^{101} \binom{101}{j} = \frac{2^{101}}{2} = 2^{100}.$$

There is no middle coefficient left over: a self-paired index would have to be $101/2$, which is not an integer.

Step 3: Substitute and compare the options It follows that $S = 2^{100}/101$. Option (A) incorrectly keeps denominator 100, while option (C) both keeps that denominator and uses the full coefficient sum. Option (D) uses the full binomial sum rather than its upper half. Only option (B) has both the correct denominator and the factor of one-half.

Step 4: Conclusion The required value is $\boxed{\frac{2^{100}}{101}}$. Hence, the correct option is (B).

Quick Tip to Solve: When a binomial coefficient is divided by its lower index plus one, raise both indices by one and then look for symmetry in the new row.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let the area of the region bounded by the curve $y = \max\{\sin x, \cos x\}$, lines $x = 0, x = \frac{3\pi}{2}$, and the x -axis be A . Then, $A + A^2$ is equal to _____. [4 marks]

Solution:

Concept:

- The area between a curve $y = g(x)$ and the x -axis is $\int |g(x)| dx$. Negative function values must contribute positive area.
- To integrate $\max\{\sin x, \cos x\}$, first locate the crossings $\sin x = \cos x$, then determine which function is larger on each interval.

Step 1: Find the changes in the upper curve The equality $\sin x = \cos x$ gives $x = \pi/4 + k\pi$. On the given interval the crossings are $\pi/4$ and $5\pi/4$. Thus the maximum is $\cos x$ on $[0, \pi/4]$, $\sin x$ on $[\pi/4, 5\pi/4]$, and $\cos x$ on $[5\pi/4, 3\pi/2]$. It is nonnegative up to $x = \pi$, but nonpositive from π to $3\pi/2$.

The sign change at π therefore requires an additional split for geometrical area.

Step 2: Integrate the positive areas separately Using the signs just established,

$$A = \int_0^{\pi/4} \cos x \, dx + \int_{\pi/4}^{\pi} \sin x \, dx - \int_{\pi}^{5\pi/4} \sin x \, dx - \int_{5\pi/4}^{3\pi/2} \cos x \, dx.$$

The four contributions are

$$\frac{\sqrt{2}}{2}, \quad 1 + \frac{\sqrt{2}}{2}, \quad 1 - \frac{\sqrt{2}}{2}, \quad 1 - \frac{\sqrt{2}}{2},$$

respectively. Each is positive, as required for a geometrical area. Adding them cancels every radical term and gives $A = 3$ square units.

Step 3: Evaluate the requested expression The question asks for a function of the area rather than the area alone. Substitute $A = 3$:

$$A + A^2 = 3 + 3^2 = 3 + 9 = 12.$$

Using the signed integral instead would incorrectly subtract the portions below the axis. The absolute-value treatment is necessary even though the curve is defined by a maximum.

Step 4: Conclusion The area is $A = 3$ square units, and the requested numerical value is 12.

Quick Tip to Solve: For an area involving a maximum, split at both curve crossings and axis crossings: the larger of two negative values still lies below the axis.

22. The number of 4-letter words, with or without meaning, which can be formed using the letters PQRQRSTUVP, is ____ . [4 marks]

Solution:

Concept:

- Identical copies of a letter do not produce different words. If a word has multiplicities $a_1, a_2, \dots$, its distinct arrangements number $4!/(a_1!a_2!\dots)$.
- The multiset PQRQRSTUVP contains seven distinct letters: P occurs three times, Q and R twice each, and S, T, U, V once each. Count separately by the repetition pattern of the four chosen letters.

Step 1: Count words with four distinct letters Choose any four of the seven distinct letter types, and arrange them in all possible orders. This contributes

$$\binom{7}{4} 4! = 35 \times 24 = 840.$$

Availability of extra copies does not affect these words, since each selected letter is used only once. Choosing particular physical copies would overcount identical words.

Step 2: Count the patterns containing pairs For pattern $2 + 1 + 1$, the doubled letter must be P, Q or R, giving three choices. Choose two different singleton letters from the six remaining types, and

arrange the resulting multiset:

$$3 \binom{6}{2} \frac{4!}{2!} = 3 \times 15 \times 12 = 540.$$

For pattern $2 + 2$, choose the two doubled letter types from P, Q and R. Its contribution is

$$\binom{3}{2} \frac{4!}{2!2!} = 3 \times 6 = 18.$$

These cases are disjoint: the first has three distinct letter types and the second has two.

Step 3: Count triples and combine the cases For pattern $3 + 1$, only P can be tripled. The singleton may be any of the other six letters. Hence this case contributes

$$6 \frac{4!}{3!} = 6 \times 4 = 24.$$

Pattern 4 is impossible, because no letter occurs four times. The patterns $1 + 1 + 1 + 1$, $2 + 1 + 1$, $2 + 2$ and $3 + 1$ exhaust all possible multiplicity partitions of a four-letter word under the available counts. Therefore

$$840 + 540 + 18 + 24 = 1422.$$

Step 4: Conclusion The number of distinct four-letter words is 1422.

Quick Tip to Solve: Partition word-counting problems by multiplicity pattern, then separately count the choices of letter types and the arrangements of each multiset.

23. Let f be a twice differentiable non-negative function such that

$$(f(x))^2 = 25 + \int_0^x ((f(t))^2 + (f'(t))^2) dt.$$

Then the mean of $f(\log_e(1)), f(\log_e(2)), \dots, f(\log_e(625))$ is equal to ____.

[4 marks]

Solution:

Concept:

- The fundamental theorem of calculus gives the derivative of an integral with variable upper limit: $\frac{d}{dx} \int_0^x h(t) dt = h(x)$ for continuous h .
- The identity $u^2 + v^2 - 2uv = (u - v)^2$ can turn the differentiated equation into a simple differential equation. The original relation at $x = 0$ supplies the initial value.

Step 1: Differentiate and complete a square Differentiating the given relation, using the assumed differentiability, yields

$$2f(x)f'(x) = f(x)^2 + f'(x)^2.$$

Moving all terms to one side and factoring gives

$$(f'(x) - f(x))^2 = 0.$$

Since these are real values, a square is zero only when its base is zero. Hence $f'(x) = f(x)$ throughout the interval of interest; there is no independent sign choice at this step.

Step 2: Determine the function from its initial value At $x = 0$, the integral vanishes, so $f(0)^2 = 25$. Non-negativity selects $f(0) = 5$. Solving $f' = f$ gives $f(x) = Ce^x$, and therefore

$$f(x) = 5e^x.$$

This function satisfies the original equation as well: its squared value and squared derivative both equal $25e^{2t}$, giving

$$25 + \int_0^x 50e^{2t} dt = 25 + 25(e^{2x} - 1) = 25e^{2x} = f(x)^2.$$

Thus differentiation has not introduced an extraneous solution.

Step 3: Average the logarithmic arguments For every integer k from 1 through 625,

$$f(\log_e k) = 5e^{\log_e k} = 5k.$$

The requested 625 values therefore form the arithmetic progression 5, 10, ..., 3125. Its mean is

$$\frac{1}{625} \sum_{k=1}^{625} 5k = \frac{5}{625} \frac{625 \times 626}{2} = 5 \times 313 = 1565.$$

Equivalently, the average is half the sum of the first and last terms; both methods give the same result.

Step 4: Conclusion The mean of the specified function values is 1565.

Quick Tip to Solve: After differentiating an integral equation, check for a perfect square before separating variables; use the original equation to determine the initial value.

24. From the first 100 natural numbers, two numbers first a and then b are selected randomly without replacement. If the probability that $a - b \geq 10$ is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to

[4 marks]

Solution:

Concept:

- Selecting first a and then b makes the outcome an ordered pair. Without replacement, there are 100×99 equally likely pairs with distinct entries.
- The event $a - b \geq 10$ has a direction: it requires the first number to exceed the second by at least ten. It is not the absolute-difference event $|a - b| \geq 10$.

Step 1: Set up the ordered sample space There are 100 possibilities for the first number and 99 for the second once the first has been removed. Thus

$$N = 100 \times 99 = 9900.$$

The inequality automatically guarantees $a \neq b$, so favorable outcomes need no additional correction

for drawing the same number twice. Unordered pairs would not directly represent which selected number is subtracted from which.

Step 2: Count every favorable pair Fix the second number b . The admissible first numbers are $b + 10, b + 11, \dots, 100$, so their count is $91 - b$. This is positive precisely for $b = 1, 2, \dots, 90$. Therefore the total number of favorable outcomes is

$$F = \sum_{b=1}^{90} (91 - b) = 90 + 89 + \dots + 1 = \frac{90 \times 91}{2} = 4095.$$

As an endpoint check, $b = 1$ permits $a = 11$ through 100, giving 90 pairs, while $b = 90$ permits only $a = 100$. No value $b > 90$ can satisfy the inequality.

Step 3: Reduce the probability and add the integers Consequently,

$$P(a - b \geq 10) = \frac{4095}{9900} = \frac{91}{220},$$

since the numerator and denominator share the factor 45. The remaining factors are $91 = 7 \times 13$ and $220 = 2^2 \times 5 \times 11$; hence their greatest common divisor is one. Thus the requested coprime integers are $m = 91$ and $n = 220$, and

$$m + n = 91 + 220 = 311.$$

Doubling the favorable count would instead count both directions of the difference, which is not the given event.

Step 4: Conclusion The probability is $91/220$, so $\boxed{m + n = 311}$.

Quick Tip to Solve: For a directional difference in sequential sampling, keep the pairs ordered and count allowed first values for each fixed second value.

25. Let $|A| = 6$, where A is a 3×3 matrix. If $|\text{adj}(3 \text{adj}(A^2 \cdot \text{adj}(2A)))| = 2^m \cdot 3^n$, $m, n \in \mathbf{N}$, then $m + n$ is equal to

_____.

[4 marks]

Solution:

Concept:

- For a square matrix M of order n , $\det(\text{adj } M) = (\det M)^{n-1}$. All matrices here have order three, so taking an adjugate squares the determinant.
- For an order-three matrix, $\det(cM) = c^3 \det M$, whereas $\det(MN) = \det M \det N$. Scalar multiplication and the adjugate therefore affect prime exponents differently.

Step 1: Evaluate the determinant of the innermost adjugate Start from the innermost bracket and keep every determinant as a product of powers of 2 and 3. Since $\det A = 6$, we have

$$\det(2A) = 2^3 \det A = 48 = 2^4 \cdot 3.$$

Thus the determinant of its adjugate is

$$\det(\operatorname{adj}(2A)) = 48^2 = 2^8 \cdot 3^2.$$

The exponent 3 in the scalar rule comes from the number of rows, while the exponent 2 in the adjugate rule is one less than the order.

Step 2: Form the determinant of the matrix product Put $B = A^2 \operatorname{adj}(2A)$. Multiplicativity applies even without any separate commutativity assumption. Hence

$$\begin{aligned}\det B &= (\det A)^2 \det(\operatorname{adj}(2A)) \\ &= 6^2 (2^8 \cdot 3^2) = 2^{10} \cdot 3^4.\end{aligned}$$

Taking the next adjugate gives $\det(\operatorname{adj} B) = 2^{20} \cdot 3^8$.

Step 3: Apply the outer scalar and adjugate The factor 3 multiplies the matrix $\operatorname{adj} B$, so it contributes 3^3 to its determinant:

$$\det(3 \operatorname{adj} B) = 3^3 2^{20} \cdot 3^8 = 2^{20} \cdot 3^{11}.$$

The final adjugate squares this entire determinant:

$$\det(\operatorname{adj}(3 \operatorname{adj} B)) = (2^{20} \cdot 3^{11})^2 = 2^{40} \cdot 3^{22}.$$

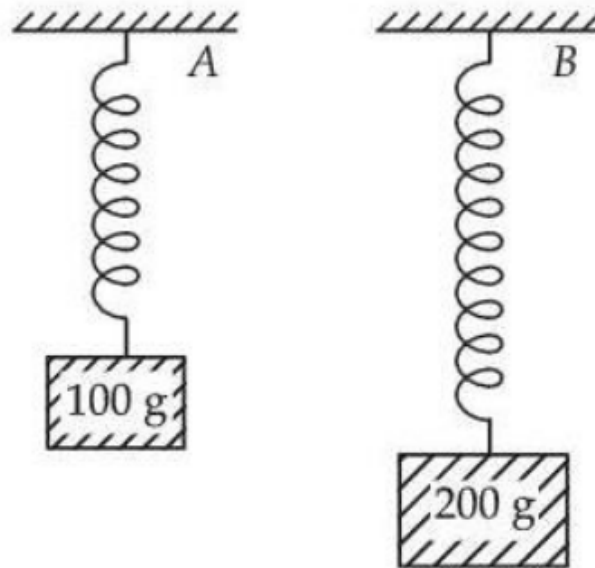
Comparing the two prime powers separately gives $m = 40$ and $n = 22$.

Step 4: Conclusion The required numerical value is $m + n = 40 + 22 = \boxed{62}$, a dimensionless integer.

Quick Tip to Solve: Work from the innermost matrix outward: a scalar contributes its cube to a determinant of order three, and each adjugate doubles every prime exponent.

Physics – Section A: Multiple Choice Questions (80 marks)

26. Two blocks with masses 100 g and 200 g are attached to the ends of springs A and B as shown in figure. The energy stored in A is E . The energy stored in B , when spring constants k_A, k_B of A and B , respectively satisfy the relation $4k_A = 3k_B$, is : [4 marks]



- (A) $4E$
- (B) $3E$
- (C) $2E$
- (D) $\frac{4}{3}E$

Correct Answer: (B) $3E$

Solution:

Concept:

- A vertically suspended mass at rest stretches its spring until the upward elastic force balances its weight: $kx = mg$, where x is the extension from natural length.
- The energy stored in an ideal spring is $U = \frac{1}{2}kx^2$. Combining this with equilibrium gives $U = \frac{m^2g^2}{(2k)}$, so both the mass and the spring constant affect the energy.

Step 1: Find the extensions of the two springs The two springs carry separate masses, so the force on each is the weight of its own block. For spring A , $m_A = 100$ g, while for spring B , $m_B = 200$ g. Thus

$$x_A = \frac{m_A g}{k_A}, \quad x_B = \frac{m_B g}{k_B}.$$

The supplied relation $4k_A = 3k_B$ means $k_B = 4k_A/3$, not $3k_A/4$. Consequently, B is the stiffer spring even though its extension is larger because it carries twice the mass.

Step 2: Compare the elastic energies Substitute the equilibrium extensions into the energy formula

and divide the results. The common factors $g^2/2$ cancel:

$$\frac{U_B}{U_A} = \left(\frac{m_B}{m_A}\right)^2 \frac{k_A}{k_B} = \left(\frac{200}{100}\right)^2 \frac{3}{4} = 3.$$

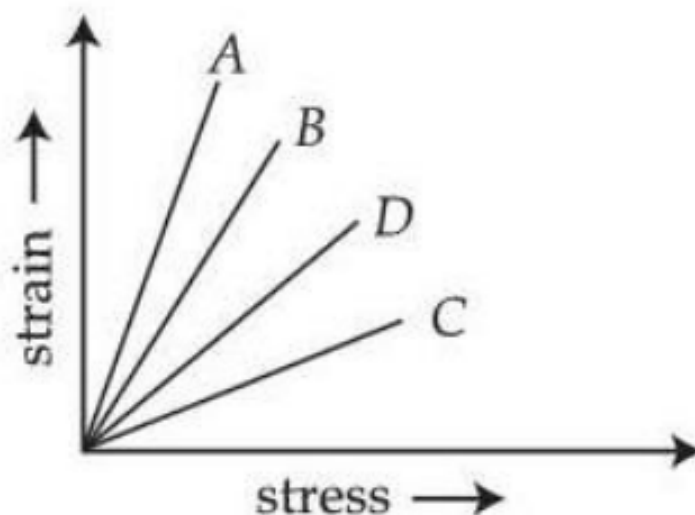
Since $U_A = E$, the second spring stores $U_B = 3E$. As a check, $x_B/x_A = 2(3/4) = 3/2$, giving $(k_B/k_A)(x_B/x_A)^2 = (4/3)(9/4) = 3$ again.

Step 3: Check the alternatives The value $4E$ would result from using the squared mass ratio while ignoring the different spring constants. The value $2E$ incorrectly makes the elastic energy directly proportional to the load. The value $4E/3$ is merely the stiffness ratio times E and omits the change in extension. Only $3E$ satisfies both force balance and the spring-energy expression.

Step 4: Conclusion The energy stored in spring B is $3E$. The equilibrium calculation gives $U_B/U_A = 4(3/4) = 3$, placing this value second in the printed option order. Hence, the correct option is (B).

Quick Tip to Solve: For separate hanging masses in equilibrium, compare m^2/k . Convert a given stiffness relation into k_A/k_B before multiplying by the squared mass ratio.

27. The strain-stress plot for materials A, B, C and D is shown in the figure. Which material has the largest Young's modulus ? [4 marks]



- (A) A
- (B) B
- (C) C
- (D) D

Correct Answer: (C) C

Solution:

Concept:

- Young's modulus is the ratio of longitudinal stress σ to longitudinal strain ε in the linear elastic region: $Y = \sigma/\varepsilon$. Stress has pressure units and strain is dimensionless.
- The plotted vertical coordinate is strain and the horizontal coordinate is stress. Therefore the slope is $\varepsilon/\sigma = 1/Y$, the reciprocal of Young's modulus.

Step 1: Read the axes before comparing the lines The figure explicitly puts strain on the vertical axis and stress on the horizontal axis. Let a line make angle θ with the positive stress axis. Its positive slope is

$$\tan \theta = \frac{\text{strain}}{\text{stress}} = \frac{1}{Y}.$$

A steeper line therefore represents a material that strains more under the same applied stress. Such a material has a smaller, rather than larger, Young's modulus.

Step 2: Order the slopes shown in the graph All four lines start at the origin. Reading them from steepest to shallowest gives A, B, D and C. Hence

$$s_A > s_B > s_D > s_C > 0,$$

where s denotes the strain-versus-stress slope. Taking reciprocals reverses the inequalities:

$$Y_A < Y_B < Y_D < Y_C.$$

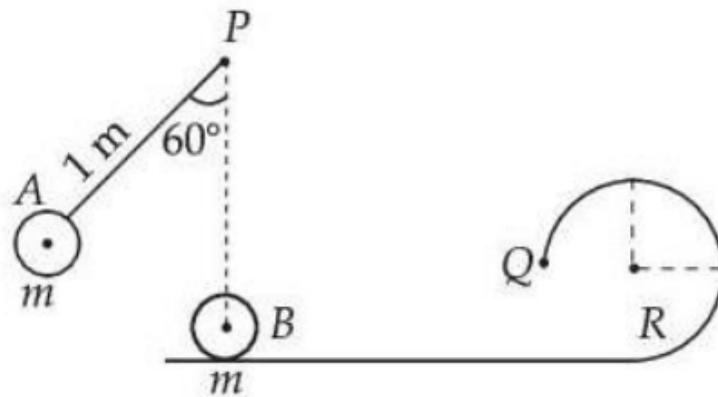
Equivalently, choose any common stress on the horizontal axis. The C line then has the smallest vertical coordinate, so C undergoes the least strain and resists extension most strongly.

Step 3: Select the material with greatest modulus Material C has the smallest positive slope and therefore the largest reciprocal slope. A is the steepest line and actually has the smallest modulus, so selecting A confuses this graph with a stress-versus-strain plot. B and D have intermediate slopes; both have larger strain than C at a given stress and cannot have the maximum modulus.

Step 4: Conclusion Material $\boxed{C}$ has the largest Young's modulus because $Y = 1/\text{slope}$ for the printed strain-stress axes. Material C is the third printed choice. Hence, the correct option is (C).

Quick Tip to Solve: Check which quantity is vertical: the slope of stress against strain is Y , but the slope of strain against stress is $1/Y$.

28. A small bob A of mass m is attached to a massless rigid rod of length 1 m pivoted at point P and kept at an angle of 60° with vertical as shown in figure. At distance of 1 m below point P, an identical bob B is kept at rest on a smooth horizontal surface that extends to a circular track of radius R as shown in figure. If bob B just manages to complete the circular path of radius R upto a point Q after being hit elastically by bob A, then radius R is ____ m. [4 marks]



- (A) $\frac{3}{5}$
 (B) $\frac{1}{5}$
 (C) $\frac{2-\sqrt{3}}{5}$
 (D) $\frac{2+\sqrt{3}}{5}$

Correct Answer: (B) $\frac{1}{5}$

Solution:

Concept:

- During the frictionless swing, loss of gravitational potential energy becomes kinetic energy. A rod of length l , initially at angle θ to the downward vertical, lets the bob fall through $l(1 - \cos \theta)$.
- In a head-on elastic collision between identical masses, with one initially at rest, the velocities exchange. On the inside of a circular track the limiting top condition is $v_t^2 = gR$, obtained when the normal reaction just vanishes.

Step 1: Find the speed just before the collision The bob A starts at 60° to the downward vertical. Its fall to the level of B is

$$h = l(1 - \cos 60^\circ) = 1 \left(1 - \frac{1}{2} \right) = \frac{1}{2} \text{ m.}$$

The rod's constraint force does no work along the motion. Conservation of mechanical energy gives

$$mgh = \frac{1}{2}mu^2 \Rightarrow u^2 = 2gh = g(1 \text{ m}).$$

The collision occurs at the lowest point, where A's velocity is horizontal.

Step 2: Transfer the speed to bob B For equal masses in a one-dimensional elastic collision, momentum and kinetic-energy conservation give $v_A = 0$ and $v_B = u$ after impact. Thus B enters the circular track with bottom speed satisfying $v_b^2 = u^2$. No energy is lost on the smooth horizontal portion.

Step 3: Apply the limiting condition at the top The path to Q passes through the top of the circular

arc. At the threshold for retaining contact there, $N = 0$, so $mg = mv_t^2/R$ and $v_t^2 = gR$. The rise from bottom to top is $2R$, giving

$$\frac{1}{2}mv_b^2 = \frac{1}{2}mgR + mg(2R), \quad v_b^2 = 5gR.$$

Consequently $g(1 \text{ m}) = 5gR$, or $R = 1/5 \text{ m}$. The descending part from the top to Q does not impose a stricter contact condition. The other three radii fail the equality $5R = 2l(1 - \cos 60^\circ) = 1 \text{ m}$; the square-root choices would arise from a different initial angle.

Step 4: Conclusion The limiting radius is $R = \frac{1}{5} \text{ m}$. Energy transfer and the top contact condition give $5gR = g(1 \text{ m})$, selecting the second printed choice. Hence, the correct option is (B).

Quick Tip to Solve: First calculate the actual vertical fall, then transfer the speed for an equal-mass elastic collision. Use $v_b^2 = 5gR$ when an unconstrained particle must retain contact through the top.

29. A thin prism with angle 5° of refractive index 1.72 is combined with another prism of refractive index 1.9 to produce dispersion without deviation. The angle of second prism is ____ . [4 marks]

- (A) 4°
- (B) 5°
- (C) 6°
- (D) 4.5°

Correct Answer: (A) 4°

Solution:

Concept:

- For a thin prism in air, the deviation of a representative wavelength is $\delta = (\mu - 1)A$, where μ is its refractive index and A its small prism angle.
- Dispersion without net deviation requires oppositely oriented prisms whose mean deviations have equal magnitudes. Thus $(\mu_1 - 1)A_1 = (\mu_2 - 1)A_2$; their wavelength-dependent spreads need not cancel.

Step 1: Write the zero-deviation condition Take the first prism's deviation as positive. The second prism must be reversed so that its deviation is negative. The total deviation is then

$$\delta_{\text{net}} = (\mu_1 - 1)A_1 - (\mu_2 - 1)A_2.$$

Setting this to zero cancels the mean bending while allowing angular separation of colours. It is the factor $\mu - 1$, rather than μ itself, which governs thin-prism deviation.

Step 2: Calculate the angle of the second prism Insert $\mu_1 = 1.72$, $A_1 = 5^\circ$ and $\mu_2 = 1.90$:

$$(1.72 - 1) 5^\circ = (1.90 - 1)A_2.$$

Therefore

$$A_2 = \frac{0.72}{0.90} 5^\circ = 0.8 \times 5^\circ = 4^\circ.$$

Using degrees consistently is valid in this proportional thin-prism approximation. Both deviations have magnitude 3.6° , so the opposite orientations make their algebraic sum zero.

Step 3: Check the other proposed angles With the first prism fixed, the second must supply exactly 3.6° of opposite deviation. Angles 5° , 6° and 4.5° would instead give 4.5° , 5.4° and 4.05° , respectively. None cancels the first prism. The higher refractive index of the second prism also provides a useful check: its angle should be smaller than 5° .

Step 4: Conclusion The second prism must have angle 4° , oriented oppositely to the first. The calculation $0.72 \times 5/0.90 = 4$ selects the first printed choice. Hence, the correct option is (A).

Quick Tip to Solve: For zero mean deviation, equate $(\mu - 1)A$ for the two oppositely oriented thin prisms; do not equate μA .

30. Four persons measure the length of a rod as 20.00 cm, 19.75 cm, 17.01 cm and 18.25 cm . The relative error in the measurement of average length of the rod is : [4 marks]

- (A) 0.08
- (B) 0.18
- (C) 0.06
- (D) 0.24

Correct Answer: (C) 0.06

Solution:

Concept:

- For repeated measurements L_i , the best estimate is their arithmetic mean $\bar{L}$. The absolute error of each reading is its positive deviation $|L_i - \bar{L}|$.
- The mean absolute error is $\Delta L = (1/n) \sum |L_i - \bar{L}|$. The relative error of the mean measurement is $\Delta L/\bar{L}$, a dimensionless ratio, rather than the percentage error or the standard deviation.

Step 1: Calculate the average rod length All readings are already in centimetres, so they can be averaged directly:

$$\bar{L} = \frac{20.00 + 19.75 + 17.01 + 18.25}{4} = \frac{75.01}{4} = 18.7525 \text{ cm.}$$

Keep the extra digits during intermediate calculations, since rounding the average too early changes all four deviations. The final choice requires only two decimal places.

Step 2: Find the mean absolute error Subtract the mean from each reading and take the magnitude.

The four absolute deviations, in centimetres, are

$$|20.00 - 18.7525| = 1.2475,$$

$$|19.75 - 18.7525| = 0.9975,$$

$$|17.01 - 18.7525| = 1.7425,$$

$$|18.25 - 18.7525| = 0.5025.$$

Their sum is 4.4900 cm, hence

$$\Delta L = \frac{4.4900}{4} = 1.1225 \text{ cm.}$$

Using signed deviations instead would give zero by construction and would not measure the spread of the readings.

Step 3: Convert the absolute error into relative error Divide by the average length so that the centimetre units cancel:

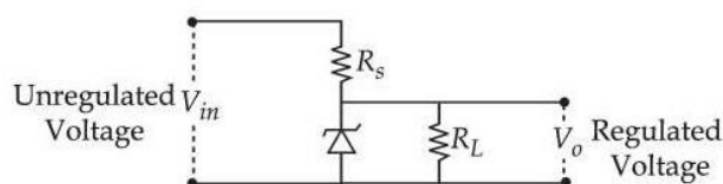
$$\frac{\Delta L}{\bar{L}} = \frac{1.1225}{18.7525} \simeq 0.05986 \simeq 0.06.$$

The corresponding percentage is about 5.986%, but the question asks for the ratio itself. The alternatives 0.08, 0.18 and 0.24 do not equal the mean absolute deviation divided by the mean length; using the full range or the largest deviation would be a different error prescription.

Step 4: Conclusion The relative error is 0.06, dimensionless, to the precision of the options. Hence, the correct option is (C).

Quick Tip to Solve: For repeated readings, average the absolute deviations first, then divide by the mean. Multiply by 100 only when the question requests a percentage.

31. The following diagram shows a Zener diode as a voltage regulator. The Zener diode is rated at $V_z = 5 \text{ V}$ and the desired current in load is 5 mA . The unregulated voltage source can supply upto 25 V . Considering the Zener diode can withstand four times of the load current, the value of resistor R_s (shown in circuit) should be $\underline{\hspace{1cm}} \Omega$. [4 marks]



- (A) 1000
- (B) 100
- (C) 10
- (D) 4000

Correct Answer: (A) 1000

Solution:

Concept:

- A reverse-biased Zener diode in breakdown keeps the load voltage at $V_Z = 5\text{ V}$. The series resistor therefore drops $V_{\text{in}} - V_Z$.
- Kirchhoff's current law gives $I_s = I_L + I_Z$. A safe resistor must keep the Zener current below its maximum while still supplying the specified load current.

Step 1: Find the permitted Zener current The desired load current is $I_L = 5\text{ mA}$. Four times this current gives the maximum permitted Zener current:

$$I_{Z,\text{max}} = 4(5) = 20\text{ mA}.$$

The resistor carries the sum of the two branch currents, not just the Zener current.

Step 2: Apply the maximum input voltage At the largest input, the resistor voltage is $25 - 5 = 20\text{ V}$. With the stated load connected, the limiting total current is $20 + 5 = 25\text{ mA}$, so

$$R_s \geq \frac{20}{25 \times 10^{-3}} = 800\ \Omega.$$

Thus the exact limiting resistance is not one of the choices; the offered resistance must be tested for safe operation.

Step 3: Test the printed resistor values For $R_s = 1000\ \Omega$, $I_s = 20\text{ mA}$, leaving $I_Z = 15\text{ mA}$, safely below the limit. It also limits the no-load Zener current to 20 mA . Values $100\ \Omega$ and $10\ \Omega$ give total currents 200 mA and 2 A , which overload the diode. At $4000\ \Omega$, the total current is only 5 mA even at maximum input, leaving no Zener current and hence no regulation margin.

Step 4: Conclusion The suitable listed resistor is $R_s = 1000\ \Omega$: it supplies the load and keeps the Zener conducting safely. Hence, the correct option is (A).

Quick Tip to Solve: Compute the total series current before selecting a Zener resistor; then check that some current remains in the diode at the required load.

32. The moment of inertia of a square loop made of four uniform solid cylinders, each having radius R and length L ($R < L$) about an axis passing through the mid points of opposite sides, is (Take the mass of the entire loop as M) :

[4 marks]

- (A) $\frac{3}{8}MR^2 + \frac{1}{6}ML^2$
 (B) $\frac{3}{4}MR^2 + \frac{7}{12}ML^2$
 (C) $\frac{3}{8}MR^2 + \frac{7}{12}ML^2$
 (D) $\frac{3}{4}MR^2 + \frac{1}{6}ML^2$

Correct Answer: (A) $\frac{3}{8}MR^2 + \frac{1}{6}ML^2$

Solution:

Concept:

- For a uniform solid cylinder of mass m_c , radius R and length L , the axial moment is $m_c R^2/2$, while the transverse central moment is $m_c(3R^2 + L^2)/12$.
- The four cylinders have equal mass $m_c = M/4$. Moments of inertia about the same axis add, and a parallel displacement a adds $m_c a^2$.

Step 1: Identify the two types of cylinders The specified axis lies in the plane of the square and joins the midpoints of two opposite sides. It crosses those two cylinders transversely through their centres. The other two cylinders lie parallel to this axis, each with its central axis displaced by $L/2$. Thus the two pairs must be treated with different cylinder formulae.

Step 2: Calculate the transverse pair For each of the crossed cylinders the rotation axis is a transverse central axis. Their combined contribution is

$$I_1 = 2 \left[\frac{M/4}{12} (3R^2 + L^2) \right] = \frac{MR^2}{8} + \frac{ML^2}{24}.$$

Keeping the radius term is essential: these are solid cylinders, not massless-thickness rods.

Step 3: Calculate the parallel pair and add For each parallel cylinder, use its axial moment and then the parallel-axis theorem:

$$I_2 = 2 \left[\frac{1}{2} \frac{M}{4} R^2 + \frac{M}{4} \left(\frac{L}{2} \right)^2 \right] = \frac{MR^2}{4} + \frac{ML^2}{8}.$$

Therefore

$$I = I_1 + I_2 = \frac{3}{8}MR^2 + \frac{1}{6}ML^2.$$

Options B and C have the incorrect length coefficient $7/12$; option D doubles the radius contribution. As a check, $R \rightarrow 0$ gives the familiar thin square-frame result $ML^2/6$.

Step 4: Conclusion Adding the two distinct cylinder pairs gives $I = \frac{3}{8}MR^2 + \frac{1}{6}ML^2$, which matches the first printed expression. Hence, the correct option is (A).

Quick Tip to Solve: For a frame made of thick rods, first identify which members are parallel to the rotation axis and which are crossed by it; retain each member's own central inertia.

33. In a perfectly inelastic collision, two spheres made of the same material with masses 15 kg and 25 kg, moving in opposite directions with speeds of 10 m/s and 30 m/s, respectively, strike each other and stick together. The rise in temperature (in $^{\circ}\text{C}$), if all the heat produced during the collision is retained by these spheres, is :

(specific heat of sphere material $31\text{ cal/kg}^{\circ}\text{C}$ and $1\text{ cal} = 4.2\text{ J}$)

[4 marks]

- (A) 1.15
- (B) 1.44
- (C) 1.75
- (D) 1.95

Correct Answer: (B) 1.44

Solution:

Concept:

- Linear momentum is conserved during the collision, but kinetic energy is not conserved when the spheres stick together.
- The lost kinetic energy becomes heat. Since both spheres have the same specific heat c , the temperature rise satisfies $Q = (m_1 + m_2)c\Delta T$.

Step 1: Find the common final velocity Take the initial direction of the 15 kg sphere as positive. The velocities are then $+10$ and -30 m s^{-1} . Momentum conservation gives

$$15(10) + 25(-30) = 40v, \quad v = -15\text{ m s}^{-1}.$$

The negative sign means that the combined spheres move in the original direction of the heavier sphere.

Step 2: Find the kinetic energy converted to heat The initial and final kinetic energies are

$$K_i = \frac{1}{2}(15)(10)^2 + \frac{1}{2}(25)(30)^2 = 12000\text{ J},$$

$$K_f = \frac{1}{2}(40)(15)^2 = 4500\text{ J}.$$

Hence $Q = 12000 - 4500 = 7500\text{ J}$. The final kinetic energy must be subtracted: sticking together does not mean that the combined body comes to rest.

Step 3: Convert the heat capacity and calculate The given specific heat is per kilogram, so no gram conversion is required. In SI units,

$$c = 31(4.2) = 130.2\text{ J kg}^{-1}\text{ }^{\circ}\text{C}^{-1}.$$

Therefore

$$\Delta T = \frac{7500}{40(130.2)} = 1.440\dots\text{ }^{\circ}\text{C}.$$

The values 1.15, 1.75 and 1.95 do not satisfy the heat balance with this conserved momentum and the stated specific heat.

Step 4: Conclusion The temperature rise is approximately 1.44°C . Hence, the correct option is (B).

Quick Tip to Solve: Assign opposite signs to opposite velocities before finding the final speed; convert only the kinetic energy lost, not the entire initial kinetic energy, into heat.

34. Two small balls with masses m and $2m$ are attached to both ends of a rigid rod of length d and negligible mass. If angular momentum of this system is L about an axis (A) passing through its centre of mass and perpendicular to the rod then angular velocity of the system about A is : [4 marks]

- (A) $\frac{2L}{5md^2}$
 (B) $\frac{2L}{md^2}$
 (C) $\frac{3L}{2md^2}$
 (D) $\frac{4L}{3md^2}$

Correct Answer: (C) $\frac{3L}{2md^2}$

Solution:

Concept:

- For rotation about a fixed axis, angular momentum and angular speed obey $L = I\omega$. The rod has negligible mass, so only the two balls contribute to I .
- The centre of mass lies closer to the heavier ball. For two masses separated by d , their distances from it satisfy both $r_1 + r_2 = d$ and $m_1r_1 = m_2r_2$.

Step 1: Locate the centre of mass Let r_1 be the distance from the ball of mass m to the centre of mass and r_2 the distance from the ball of mass $2m$. Then $mr_1 = 2mr_2$, so $r_1 = 2r_2$. Combining this with $r_1 + r_2 = d$ gives

$$r_1 = \frac{2d}{3}, \quad r_2 = \frac{d}{3}.$$

These distances are unequal; using the midpoint would select the wrong rotation axis.

Step 2: Calculate the moment of inertia The balls are small, so their own radii can be neglected and each contributes its mass times the square of its distance from the axis. Thus

$$I = m \left(\frac{2d}{3} \right)^2 + 2m \left(\frac{d}{3} \right)^2 = \frac{4md^2}{9} + \frac{2md^2}{9} = \frac{2}{3}md^2.$$

This also agrees with the two-particle formula $I = \mu d^2$, where the reduced mass is $\mu = m(2m)/(3m) = 2m/3$.

Step 3: Use the given angular momentum Now divide the angular momentum by the moment of inertia:

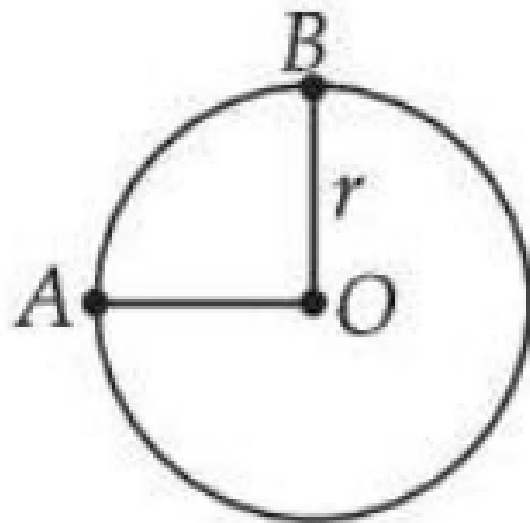
$$\omega = \frac{L}{I} = \frac{L}{(2/3)md^2} = \frac{3L}{2md^2}.$$

The other printed coefficients, $2/5$, 2 and $4/3$, imply incorrect moments of inertia and fail the direct sum above. The dimensions are $(\text{kg m}^2 \text{ s}^{-1})/(\text{kg m}^2) = \text{s}^{-1}$, as required for angular speed.

Step 4: Conclusion The angular velocity about the stated axis is $\omega = \frac{3L}{2md^2}$, the third printed expression. Hence, the correct option is (C).

Quick Tip to Solve: For two masses on a light rod, use the reduced-mass shortcut $I = \mu d^2$ only after confirming that the axis passes through their centre of mass.

35. A wire of uniform resistance $\lambda \Omega/\text{m}$ is bent into a circle of radius r and another piece of wire with length $2r$ is connected between points A and B (AOB) as shown in figure. The equivalent resistance between points A and B is ____ Ω . [4 marks]



- (A) $\frac{3\pi\lambda r}{8}$
 (B) $\frac{6\pi\lambda r}{3\pi+16}$
 (C) $2\pi\lambda r$
 (D) $(\pi + 1)2r\lambda$

Correct Answer: (B) $\frac{6\pi\lambda r}{3\pi+16}$

Solution:

Concept:

- A uniform wire with resistance per unit length λ has resistance $\lambda\ell$ for a piece of length ℓ .
- Paths connected between the same two terminals are in parallel. The drawing gives three independent paths from A to B : the short arc, the long arc and the bent wire through O .

Step 1: Find the resistances of the two arcs The radii OA and OB are perpendicular, so the shorter arc subtends $\pi/2$ radians and has length $\pi r/2$. The remaining arc has length $3\pi r/2$. Therefore

$$R_1 = \frac{\pi r \lambda}{2}, \quad R_2 = \frac{3\pi r \lambda}{2}.$$

They are not semicircles: terminals A and B are separated by a right angle at the centre.

Step 2: Include the wire through the centre The added wire follows AOB , with total length $2r$, so $R_3 = 2r\lambda$. Its two radial portions are in series with each other. This entire path is in parallel with both circular arcs because all three branches have the same terminal potential difference.

Step 3: Add the three conductances Using the parallel formula,

$$\frac{1}{R_{AB}} = \frac{2}{\pi r \lambda} + \frac{2}{3\pi r \lambda} + \frac{1}{2r \lambda} = \frac{16 + 3\pi}{6\pi r \lambda}.$$

Hence

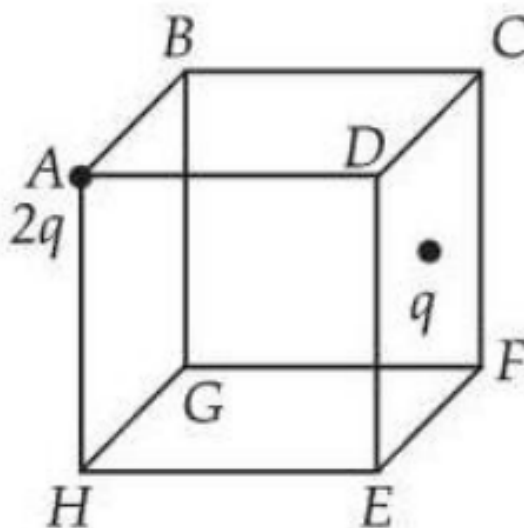
$$R_{AB} = \frac{6\pi r\lambda}{3\pi + 16}.$$

Option A, $3\pi r\lambda/8$, accounts for only the two arcs and omits the added wire. Options C and D treat all or part of the wire lengths as though they were in series; they also exceed the smallest branch resistance, impossible for a parallel combination.

Step 4: Conclusion The equivalent resistance is $\frac{6\pi r\lambda}{3\pi + 16} \Omega$ when r and λ are in the stated units. Hence, the correct option is (B).

Quick Tip to Solve: Count distinct terminal-to-terminal paths before combining a wire network; geometry determines each path length, while the connections determine series or parallel.

36. Two point charges $2q$ and q are placed at vertex A and centre of face $CDEF$ of the cube as shown in figure. The electric flux passing through the cube is: [4 marks]



- (A) $\frac{q}{\epsilon_0}$
- (B) $\frac{3q}{\epsilon_0}$
- (C) $\frac{3q}{2\epsilon_0}$
- (D) $\frac{3q}{4\epsilon_0}$

Correct Answer: (D) $\frac{3q}{4\epsilon_0}$

Solution:

Concept:

- Gauss's law relates total outward electric flux to enclosed charge: $\Phi = Q_{\text{enc}}/\epsilon_0$. Boundary charges are treated by symmetry, using the fraction of space occupied by the cube around the charge.
- Electric fields and their fluxes obey superposition. Find the contribution of each charge separately and then add them.

Step 1: Evaluate the charge at a vertex Eight identical cubes can be arranged around the vertex A , putting the charge $2q$ at the centre of the assembled larger cube. By symmetry, one original cube accounts for one eighth of the charge's total outward flux. Thus

$$\Phi_A = \frac{1}{8} \frac{2q}{\epsilon_0} = \frac{q}{4\epsilon_0}.$$

The three faces meeting the charge are boundary planes, so this is the usual symmetric boundary-charge interpretation.

Step 2: Evaluate the charge at the face centre Place an identical cube on the other side of face $CDEF$. The charge q then lies inside the combined symmetric region, halfway between its two halves. Each cube accounts for half of the total outward flux due to that charge:

$$\Phi_{\text{face}} = \frac{1}{2} \frac{q}{\epsilon_0} = \frac{q}{2\epsilon_0}.$$

A face-centre charge therefore contributes a different fraction from a vertex charge.

Step 3: Superpose the two flux contributions Adding the two contributions gives

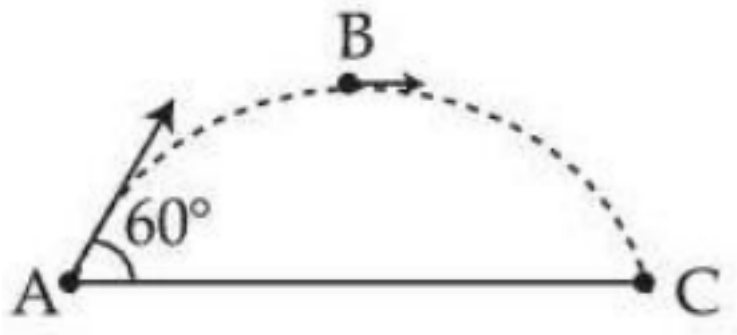
$$\Phi = \frac{q}{4\epsilon_0} + \frac{q}{2\epsilon_0} = \frac{3q}{4\epsilon_0}.$$

Treating both charges as fully inside would give option B, $3q/\epsilon_0$, which ignores their boundary positions. Treating both as face-centre charges gives option C, also incorrectly. Option A likewise fails the separate one-eighth and one-half sharing factors. Flux depends on the locations relative to the closed surface, not on the cube's side length.

Step 4: Conclusion The net outward electric flux is $\Phi = \frac{3q}{4\epsilon_0}$, the fourth printed expression. Hence, the correct option is (D).

Quick Tip to Solve: Use sharing factors separately: a charge at a cube vertex contributes one eighth, while a charge at a face centre contributes one half of its total flux.

37. An object is projected with kinetic energy K from a point A at an angle 60° with the horizontal. The ratio of the difference in kinetic energies at points B and C to that at point A (see figure), in the absence of air friction is : [4 marks]



- (A) 1 : 2
- (B) 2 : 3
- (C) 3 : 4
- (D) 1 : 4

Correct Answer: (C) 3 : 4

Solution:

Concept:

- With no air resistance, horizontal velocity stays constant, while gravity changes the vertical component. Kinetic energy is $K = \frac{1}{2}mu^2$, where u is the launch speed.
- The highest point has zero vertical velocity. At equal heights, conservation of mechanical energy gives equal kinetic energies.

Step 1: Read the heights and velocity components The figure places B at the top of the trajectory and C at the same horizontal level as A. Resolve the launch velocity at A:

$$u_x = u \cos 60^\circ = \frac{u}{2}, \quad u_y = u \sin 60^\circ.$$

The horizontal component remains unchanged throughout the flight. At B, the vertical component is zero, but the projectile is still moving horizontally; its kinetic energy is therefore not zero.

Step 2: Calculate the kinetic energy at B and C At B the speed equals $u/2$, hence

$$K_B = \frac{1}{2}m \left(\frac{u}{2}\right)^2 = \frac{K}{4}.$$

A and C have equal gravitational potential energies, so conservation of total mechanical energy gives

$$K_C = K_A = K.$$

The downward vertical velocity at C has the opposite sign to its initial value, but its square, and hence its contribution to kinetic energy, is the same.

Step 3: Form the requested difference ratio The positive difference between the kinetic energies at C

and B is

$$K_C - K_B = K - \frac{K}{4} = \frac{3K}{4}.$$

Dividing by the kinetic energy at A gives

$$\frac{|K_B - K_C|}{K_A} = \frac{3}{4}.$$

Option (D), 1 : 4, is the ratio of the energy at B itself to the initial energy, not the difference requested. Options (A) and (B) also fail the component calculation.

Step 4: Conclusion The ratio is 3 : 4. Hence, the correct option is (C).

Quick Tip to Solve: At a projectile apex, retain the horizontal kinetic energy: the fraction remaining is $\cos^2 \theta$, and the fraction lost is $\sin^2 \theta$.

38. A 20 m long uniform copper wire held horizontally is allowed to fall under the gravity ($g = 10 \text{ m/s}^2$) through a uniform horizontal magnetic field of 0.5 Gauss perpendicular to the length of the wire. The induced EMF across the wire when it travels a vertical distance of 200 m is ____ mV . [4 marks]

(A) $200\sqrt{10}$

(B) $20\sqrt{10}$

(C) $2\sqrt{10}$

(D) $0.2\sqrt{10}$

Correct Answer: (B) $20\sqrt{10}$

Solution:

Concept:

- A straight conductor of length l moving at speed v , with its length, velocity and magnetic field mutually perpendicular, has motional emf $\varepsilon = Blv$.
- A body released from rest through height h in uniform gravity has $v^2 = 2gh$. Use tesla for the magnetic field: $1 \text{ G} = 10^{-4} \text{ T}$.

Step 1: Calculate the speed after falling The wire is released rather than projected, so its initial speed is zero. Its speed after the stated vertical displacement is

$$v = \sqrt{2gh} = \sqrt{2(10)(200)} = \sqrt{4000} = 20\sqrt{10} \text{ m s}^{-1}.$$

The emf at this instant depends on this instantaneous speed, not on the average speed over the fall. The question supplies free fall under gravity; no closed circuit carrying a retarding current is specified.

Step 2: Convert the field and apply the motional emf formula The field is horizontal and perpendicular to the horizontal wire, while velocity is vertical. Thus the component of $\mathbf{v} \times \mathbf{B}$ along the wire has magnitude vB . Convert the field before substitution:

$$B = 0.5 \times 10^{-4} = 5 \times 10^{-5} \text{ T}.$$

Hence

$$\varepsilon = (5 \times 10^{-5})(20)(20\sqrt{10}) = 0.020\sqrt{10} \text{ V}.$$

The length is already in metres.

Step 3: Express the result in millivolts and compare Since $1 \text{ V} = 1000 \text{ mV}$,

$$\varepsilon = 20\sqrt{10} \text{ mV}.$$

This is option (B) in the printed order. Option (A) is ten times too large, (C) is ten times too small, and (D) is one hundred times too small; none follows the gauss-to-tesla and volt-to-millivolt conversions.

Step 4: Conclusion The induced emf is $20\sqrt{10} \text{ mV}$. Hence, the correct option is (B). The value follows directly from $Bl\sqrt{2gh}$; option letters follow the printed order.

Quick Tip to Solve: Convert gauss to tesla first, calculate Blv in volts, and convert to millivolts only at the end to avoid powers-of-ten errors.

39. In hydrogen atom spectrum, ($R \rightarrow$ Rydberg's constant)

A. the maximum wavelength of the radiation of Lyman series is $\frac{4}{3R}$

B. the Balmer series lies in the visible region of the spectrum

C. the minimum wavelength of the radiation of Paschen series is $\frac{9}{R}$

D. the minimum wavelength of Lyman series is $\frac{5}{4R}$

Choose the correct answer from the options given below :

[4 marks]

(A) A, B and D Only

(B) A, B Only

(C) A, B and C Only

(D) B, D Only

Correct Answer: (C) A, B and C Only

Solution:

Concept:

- The hydrogen wavelengths satisfy $1/\lambda = R(1/n_f^2 - 1/n_i^2)$, with upper level $n_i > n_f$. Lyman, Balmer and Paschen have $n_f = 1, 2, 3$, respectively.
- For a fixed final level, the longest wavelength comes from the smallest energy gap, $n_i = n_f + 1$; the shortest is the series limit $n_i \rightarrow \infty$.

Step 1: Check the Lyman wavelengths in A and D For the first Lyman transition, $n_i = 2$ and $n_f = 1$, so

$$\frac{1}{\lambda_{\max}} = R \left(1 - \frac{1}{4} \right) = \frac{3R}{4}, \quad \lambda_{\max} = \frac{4}{3R}.$$

Statement A is correct. At the Lyman limit, the initial electron energy approaches the ionisation limit:

$$\frac{1}{\lambda_{\min}} = R(1 - 0) = R, \quad \lambda_{\min} = \frac{1}{R}.$$

Thus D is false: $5/(4R)$ is not the series-limit wavelength.

Step 2: Check the Balmer and Paschen statements Balmer transitions end at $n_f = 2$ and give the familiar visible hydrogen lines, for example the $3 \rightarrow 2$ red line near 656 nm. Thus B uses the usual spectral-series classification as visible. The high-order Balmer lines converge into the near ultraviolet, but this does not change the conventional classification tested here. For the Paschen limit,

$$\frac{1}{\lambda_{\min}} = R \left(\frac{1}{3^2} - 0 \right) = \frac{R}{9}, \quad \lambda_{\min} = \frac{9}{R}.$$

Statement C is correct.

Step 3: Choose the complete correct set The accepted statements are A, B and C, with D excluded. Option (A) and option (D) both include the incorrect Lyman-limit statement D. Option (B) omits the correct Paschen-limit statement C. Only option (C) includes precisely A, B and C.

Step 4: Conclusion Statements A, B and C are correct under the usual hydrogen-series classification. Hence, the correct option is (C).

Quick Tip to Solve: For a series ending at n_f , memorise the limit $\lambda_{\min} = n_f^2/R$; calculate the longest wavelength using $n_i = n_f + 1$.

40. The de Broglie wavelength of an oxygen molecule at 27°C is $x \times 10^{-12}$ m. The value of x is (take Planck's constant = 6.63×10^{-34} J.s, Boltzmann constant = 1.38×10^{-23} J/K, mass of oxygen molecule = 5.31×10^{-26} kg) [4 marks]

- (A) 20
- (B) 24
- (C) 26
- (D) 30

Correct Answer: (C) 26

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, where p is the molecular momentum. For the thermal rms speed, $\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}k_B T$.
- Therefore the rms momentum is $p_{\text{rms}} = \sqrt{3mk_B T}$ and the corresponding wavelength is $h/\sqrt{3mk_B T}$. Temperature must be absolute.

Step 1: Express the thermal momentum at the given temperature Convert 27°C to $T = 300$ K. The standard thermal speed used in this question is the rms speed associated with translational kinetic energy. Rotational degrees of freedom of oxygen do not replace the translational value $3k_B T/2$ in the momentum relation. Thus

$$p_{\text{rms}} = \sqrt{3(5.31 \times 10^{-26})(1.38 \times 10^{-23})(300)}.$$

The given mass is the mass of one molecule, so no molar-mass conversion is needed.

Step 2: Evaluate the square root and wavelength The quantity under the square root is

$$3mk_BT = 6.59502 \times 10^{-46} \text{ kg}^2 \text{ m}^2 \text{ s}^{-2}.$$

Hence

$$p_{\text{rms}} \simeq 2.568 \times 10^{-23} \text{ kg m s}^{-1}.$$

Substitution into the de Broglie relation gives

$$\lambda = \frac{6.63 \times 10^{-34}}{2.568 \times 10^{-23}} \simeq 2.582 \times 10^{-11} \text{ m} = 25.82 \times 10^{-12} \text{ m}.$$

The dimensions are metres, as required.

Step 3: Identify the closest listed value Comparing with $x \times 10^{-12} \text{ m}$, the coefficient is $x \simeq 25.82$, which corresponds to 26 among the choices. Values 20, 24 and 30 do not reproduce the calculated rms thermal momentum using the supplied constants. In particular, using a different characteristic molecular speed instead of the rms speed changes the numerical coefficient.

Step 4: Conclusion The coefficient is $x \simeq 26$, so $\lambda \simeq 26 \times 10^{-12} \text{ m}$. Hence, the correct option is (C).

Quick Tip to Solve: Combine $p = mv_{\text{rms}}$ with $v_{\text{rms}} = \sqrt{3k_BT/m}$ before substituting; this avoids an extra speed calculation.

41. A simple pendulum of string length 30 cm performs 20 oscillations in 10 s . The length of the string required for the pendulum to perform 40 oscillations in the same time duration is ____ cm . [Assume that the mass of the pendulum remains same.] [4 marks]

- (A) 0.75
- (B) 7.5
- (C) 15
- (D) 120

Correct Answer: (B) 7.5

Solution:

Concept:

- For a simple pendulum executing small oscillations at fixed gravity, $T = 2\pi\sqrt{L/g}$. Thus the period is proportional to $\sqrt{L}$ and is independent of the bob mass.
- The number of complete oscillations N in a fixed interval t obeys $T = t/N$. Consequently, $L \propto 1/N^2$ when the interval is unchanged.

Step 1: Calculate the old and required periods The original pendulum completes 20 oscillations in 10 s, so

$$T_1 = \frac{10}{20} = 0.50 \text{ s}.$$

The required number is 40 in the same interval:

$$T_2 = \frac{10}{40} = 0.25 \text{ s.}$$

Therefore the period must be halved. The frequency doubles, but length is not directly proportional to either the period or the frequency.

Step 2: Use the squared period ratio Taking the ratio of the period formula for the two lengths cancels both 2π and the unchanged local gravitational acceleration:

$$\frac{T_2}{T_1} = \sqrt{\frac{L_2}{L_1}}.$$

Square and substitute the ratio before multiplying by the initial length:

$$L_2 = L_1 \left(\frac{T_2}{T_1} \right)^2 = 30 \left(\frac{0.25}{0.50} \right)^2 = \frac{30}{4} = 7.5 \text{ cm.}$$

The given data are sufficient as a ratio; a separate numerical value of gravity is unnecessary.

Step 3: Check the direction and other options A shorter pendulum has a shorter period and hence completes more oscillations in the same time, consistent with the result. Option (C), 15 cm, incorrectly halves length instead of quartering it. Option (D), 120 cm, quadruples length and would halve the frequency. Option (A), 0.75 cm, is a factor of ten smaller than the calculated length and fails the required period ratio.

Step 4: Conclusion The required string length is 7.5 cm. Hence, the correct option is (B).

Quick Tip to Solve: For a fixed observation time, write $L_2/L_1 = (N_1/N_2)^2$. Doubling the oscillation count requires one quarter of the length.

42. Consider light travelling from a medium A to medium B separated by a plane interface. If the light undergoes total internal reflection during its travel from medium A to B and the speed of light in media A and B are $2.4 \times 10^8 \text{ m/s}$ and $2.7 \times 10^8 \text{ m/s}$, respectively, then the value of critical angle is : [4 marks]

- (A) $\sin^{-1}(9/8)$
- (B) $\tan^{-1}(8/\sqrt{17})$
- (C) $\cos^{-1}(8/9)$
- (D) $\cot^{-1}(3/\sqrt{13})$

Correct Answer: (B) $\tan^{-1}(8/\sqrt{17})$

Solution:

Concept:

- The refractive index of a medium is $n = c/v$. A smaller light speed means a larger refractive index, so total internal reflection is possible from the slower medium to the faster medium.

- At the critical angle i_c , the refracted ray grazes the interface: $n_A \sin i_c = n_B \sin 90^\circ$. Thus $\sin i_c = n_B/n_A = v_A/v_B$.

Step 1: Determine the correct refractive-index ratio The supplied speeds obey $v_A = 2.4 \times 10^8 < v_B = 2.7 \times 10^8 \text{ m s}^{-1}$. Therefore A is optically denser, as required for total internal reflection from A to B. The ratio must be less than one:

$$\frac{n_B}{n_A} = \frac{c/v_B}{c/v_A} = \frac{v_A}{v_B} = \frac{2.4}{2.7} = \frac{8}{9}.$$

Inverting this ratio would give an impossible sine greater than unity.

Step 2: Convert the sine expression to the printed tangent form Snell's law at critical incidence gives

$$\sin i_c = \frac{8}{9}.$$

Since $0 < i_c < 90^\circ$, the cosine is positive:

$$\cos i_c = \sqrt{1 - \frac{64}{81}} = \frac{\sqrt{17}}{9}.$$

Hence

$$\tan i_c = \frac{\sin i_c}{\cos i_c} = \frac{8}{\sqrt{17}}, \quad i_c = \tan^{-1} \left(\frac{8}{\sqrt{17}} \right).$$

The inverse tangent is taken in its acute principal range.

Step 3: Eliminate the alternative expressions Option (A), $\sin^{-1}(9/8)$, is not a real angle. Option (C), $\cos^{-1}(8/9)$, is the complementary angle, not the incidence angle measured from the normal. For option (D), $\cot i_c = 3/\sqrt{13}$ would imply $\sin i_c = \sqrt{13/22}$, different from $8/9$. Only option (B) has the required sine and satisfies Snell's law.

Step 4: Conclusion The critical angle is $\tan^{-1}(8/\sqrt{17})$, equivalently $\sin^{-1}(8/9)$. Hence, the correct option is (B). This follows from the printed speeds and the printed option order.

Quick Tip to Solve: Use $\sin i_c = v_{\text{slower}}/v_{\text{faster}}$, then draw the corresponding right-triangle ratios mentally if the options use another inverse function.

43. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Consider a ferromagnetic material :

Assertion (A) : The individual atoms in a ferromagnetic material possess a magnetic dipole moment and interact with one another in such a way that they spontaneously align themselves forming domains.

Reason (R) : At high enough temperature, the domain structure of ferromagnetic material disintegrates. Thus, magnetization will disappear at high enough temperature known as Curie temperature.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Both (A) and (R) are true and (R) is the correct explanation of (A)
- (B) Both (A) and (R) are true but (R) is not the correct explanation of (A)
- (C) (A) is true but (R) is false
- (D) (A) is false but (R) is true

Correct Answer: (B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

Solution:

Concept:

- Ferromagnetic atoms have permanent magnetic dipole moments. Exchange interactions favour parallel alignment of neighbouring moments within a magnetic domain.
- Thermal agitation opposes magnetic ordering. Above the Curie temperature, spontaneous ferromagnetic magnetization vanishes and the material becomes paramagnetic.

Step 1: Evaluate the assertion The assertion describes the origin of magnetic domains. A large number of neighbouring atomic moments align spontaneously because the interaction between them favours an ordered state. Thus each domain has a large magnetic moment even without an externally applied field. Different domains can point in different directions, so an unmagnetized specimen need not have a nonzero net magnetic moment. Assertion (A) is therefore true.

Step 2: Evaluate the reason As temperature increases, thermal disorder increasingly competes with the interaction responsible for alignment. At the Curie temperature the long-range ferromagnetic order is lost, so the spontaneous domain magnetization disappears. This statement refers to spontaneous ferromagnetism, not to the complete disappearance of each atom's magnetic moment. Reason (R) is therefore true.

Step 3: Test the explanatory connection The reason states when the domain structure is destroyed; it does not explain why atomic moments interact so as to form domains in the first place. The interaction described in the assertion provides that cause. Thus option (A) incorrectly claims an explanatory connection, while options (C) and (D) incorrectly declare one of the two true statements false.

Step 4: Conclusion The assertion and the reason are both true, but the reason does not explain the assertion. Hence, the correct option is (B).

Quick Tip to Solve: For assertion–reason questions, test the truth of each statement first, then separately test whether the reason gives the cause of the assertion.

44. Match List - I with List - II.

	List - I: Relation		List - II: Law
A.	$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \oint \vec{B} \cdot d\vec{a}$	I.	Ampere's circuital law
B.	$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I + \epsilon_0 \frac{d\phi_E}{dt} \right)$	II.	Faraday's laws of electromagnetic induction
C.	$\oint \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0} \int_V \rho \, dv$	III.	Ampere-Maxwell law
D.	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$	IV.	Gauss's law of electrostatics

Choose the correct answer from the options given below:

[4 marks]

(A) A-II, B-III, C-I, D-IV

(B) A-II, B-III, C-IV, D-I

(C) A-I, B-IV, C-III, D-II

(D) A-IV, B-I, C-II, D-III

Correct Answer: (B) A-II, B-III, C-IV, D-I

Solution:

Concept:

- An electric-field circulation related to changing magnetic flux expresses Faraday's law; magnetic-field circulation due to current expresses Ampere's law.
- Maxwell adds displacement current $I_d = \epsilon_0 d\phi_E/dt$. Electric flux through a closed surface equals enclosed charge divided by ϵ_0 .

Step 1: Identify relations A and B Relation A equates induced electric circulation to minus the rate of magnetic-flux change. The minus sign represents Lenz's law, so it is Faraday's law: A-II. Relation B includes both conduction current I and displacement current $\epsilon_0 d\phi_E/dt$. That additional term distinguishes the Ampere-Maxwell law from the steady-current form, so B-III.

Step 2: Identify relations C and D In relation C, the volume integral $\int_V \rho \, dv$ is the charge enclosed by the surface. Therefore the electric flux is $Q_{\text{enc}}/\epsilon_0$, which is Gauss's law of electrostatics: C-IV. Relation D contains only $\mu_0 I$ on the right of the magnetic circulation integral. It is Ampere's circuital law: D-I.

Step 3: Compare all four combinations The required correspondence is A-II, B-III, C-IV, D-I. Option (A) swaps the laws for C and D and therefore fails even though its first two matches agree. Option (C) assigns A to Ampere's law and B to Gauss's law, both incorrectly. Option (D) also assigns incorrect laws to A and B. Only option (B) preserves all four matches.

Step 4: Conclusion The matching is A-II, B-III, C-IV, D-I. Hence, the correct option is (B).

Quick Tip to Solve: Look for the displacement-current term to identify Ampere-Maxwell immediately; electric flux through a closed surface identifies Gauss.

45. In a screw gauge, the zero of the circular scale lies 3 divisions above the horizontal pitch line when their metallic studs are brought in contact. Using this instrument thickness of a sheet is measured. If pitch scale reading is 1 mm and the circular scale reading is 51 then the correct thickness of the sheet is ____ mm.

[Assume least count is 0.01 mm]

[4 marks]

- (A) 1.48
- (B) 1.54
- (C) 1.51
- (D) 1.50

Correct Answer: (B) 1.54

Solution:

Concept:

- A screw-gauge reading is $x_{\text{obs}} = \text{PSR} + \text{CSR} \times \text{LC}$, where PSR is the pitch-scale reading, CSR the circular-scale reading and LC the least count.
- The corrected value is observed reading minus signed zero error. A circular-scale zero above the reference line gives a negative zero error.

Step 1: Determine the zero error With the metallic studs touching, the true separation is zero. The circular-scale zero lies three divisions above the reference line, corresponding to a negative offset of three least counts. Therefore

$$e_0 = -3(0.01) = -0.03 \text{ mm.}$$

The zero correction is the opposite of this error: +0.03 mm. Keeping the error signed prevents confusing it with the correction.

Step 2: Calculate the observed reading The pitch scale contributes 1 mm, while 51 circular divisions contribute $51 \times 0.01 = 0.51$ mm. Thus

$$x_{\text{obs}} = 1 + 0.51 = 1.51 \text{ mm.}$$

This is the uncorrected instrument reading, not yet the thickness of the sheet.

Step 3: Apply the correction and reject alternatives Subtract the signed zero error:

$$x = x_{\text{obs}} - e_0 = 1.51 - (-0.03) = 1.54 \text{ mm.}$$

Option (A), 1.48 mm, subtracts the magnitude of a negative error and uses the wrong sign. Option (C), 1.51 mm, ignores the zero error entirely. Option (D), 1.50 mm, does not follow from a three-division correction. Only option (B) applies the positive correction correctly.

Step 4: Conclusion The correct thickness of the sheet is 1.54 mm. Hence, the correct option is (B).

Quick Tip to Solve: Write zero error with its sign before using true reading = observed reading minus zero error; an above-line zero requires addition.

Physics – Section B: Numerical Value Questions (20 marks)

46. A simple pendulum made of mass 10 g and a metallic wire of length 10 cm is suspended vertically in a uniform magnetic field of 2 T. The magnetic field direction is perpendicular to the plane of oscillations of the pendulum. If the pendulum is released from an angle of 60° with vertical, then maximum induced EMF between the point of suspension and point of oscillation is

_____ mV. (Take $g = 10 \text{ m/s}^2$)

[4 marks]

Solution:

Concept:

- For an ideal pendulum, mechanical energy gives $v^2 = 2gL(\cos \theta - \cos \theta_0)$, where L is its length and θ_0 its release angle.
- A conducting wire rotating in a magnetic field perpendicular to its plane has end-to-end emf $\mathcal{E} = \frac{1}{2}B\omega L^2 = \frac{1}{2}BLv$, because speed increases linearly with distance from the pivot.

Step 1: Find the greatest bob speed The bob descends by $h = L(1 - \cos 60^\circ)$ to the lowest point. Convert $L = 10 \text{ cm} = 0.10 \text{ m}$. Conservation of energy gives

$$\frac{1}{2}mv_{\max}^2 = mgL(1 - \cos 60^\circ).$$

The mass cancels, and hence

$$v_{\max} = \sqrt{2(10)(0.10)(1 - 1/2)} = 1 \text{ m s}^{-1}.$$

At this point the gravitational potential energy is least and the angular speed is greatest.

Step 2: Account for the speed variation along the wire A small wire element at distance r from the suspension has speed ωr , with $\omega = v/L$. Since the magnetic field is perpendicular to the motion plane, the motional force is directed along the wire. The magnitude of its emf contribution is $d\mathcal{E} = B\omega r dr$. Integrating from pivot to bob gives

$$\mathcal{E} = \int_0^L B\omega r dr = \frac{1}{2}B\omega L^2 = \frac{1}{2}BLv.$$

Using BLv without the factor one-half would incorrectly give every element the bob speed.

Step 3: Substitute and convert to millivolts The field and wire length are constant, so maximum emf occurs at maximum speed, at the bottom of the swing:

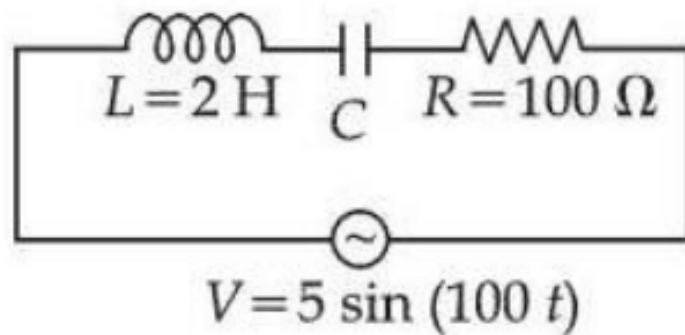
$$\mathcal{E}_{\max} = \frac{1}{2}(2)(0.10)(1) = 0.10 \text{ V}.$$

As $1 \text{ V} = 1000 \text{ mV}$, this equals 100 mV . The given bob mass does not affect this result for the ideal pendulum.

Step 4: Conclusion The maximum induced emf is 100 mV, so the required numerical value is 100.

Quick Tip to Solve: A pivoted conductor has speeds from zero to the tip speed; its emf uses the average speed, giving the factor one-half.

47. Using a variable frequency a.c. voltage source the maximum current measured in the given LCR circuit is 50 mA for $V = 5 \sin(100t)$. The values of L and R are shown in the figure. The capacitance of the capacitor (C) used is ____ μF . [4 marks]



Solution:

Concept:

- For a series LCR circuit, $Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}$ and the current amplitude is $I_0 = V_0/Z$.
- At resonance, inductive and capacitive reactances cancel: $\omega_0 L = 1/(\omega_0 C)$. Thus $\omega_0^2 LC = 1$, the impedance is R , and current is maximum.

Step 1: Read the source and circuit values Comparing $V = 5 \sin(100t)$ with $V_0 \sin(\omega t)$ gives $V_0 = 5 \text{ V}$ and $\omega = 100 \text{ rad s}^{-1}$. The figure supplies $L = 2 \text{ H}$ and $R = 100 \Omega$. The number 100 in the sine argument is angular frequency, so no additional factor of 2π is needed.

Step 2: Use the maximum-current condition The frequency has been adjusted to the maximum current, so the impedance is at its minimum. Since the squared reactance term cannot be negative, this minimum is achieved when $\omega L - 1/(\omega C) = 0$. The resulting current amplitude is

$$I_{0,\text{max}} = \frac{5}{100} = 0.050 \text{ A} = 50 \text{ mA},$$

consistent with the stated value. Thus the operating angular frequency is the resonant value.

Step 3: Calculate the capacitance Rearranging the resonance relation gives

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{100^2 \times 2} = 5.0 \times 10^{-5} \text{ F}.$$

Convert farads to microfarads using $1 \text{ F} = 10^6 \mu\text{F}$:

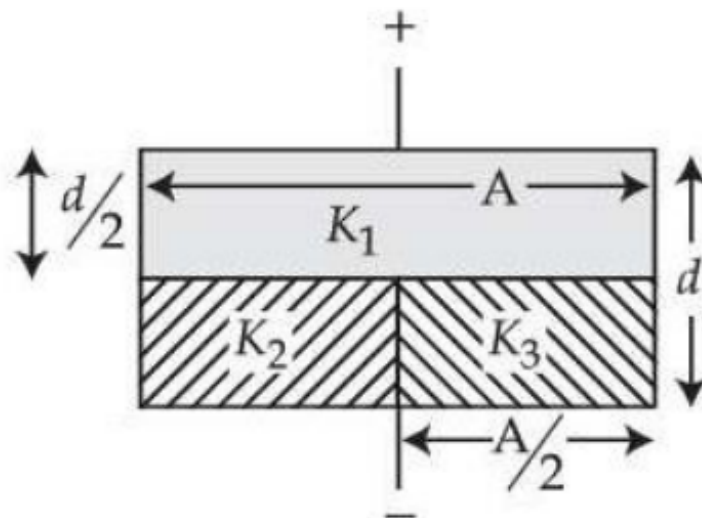
$$C = (5.0 \times 10^{-5})(10^6) = 50 \mu\text{F}.$$

As a check, $X_L = 100 \times 2 = 200 \Omega$ and $X_C = 1/(100 \times 5 \times 10^{-5}) = 200 \Omega$, so the reactances cancel exactly.

Step 4: Conclusion The required capacitance is $50 \mu\text{F}$, so the numerical entry is 50.

Quick Tip to Solve: In a variable-frequency series LCR circuit, maximum current identifies resonance. Read angular frequency directly from the coefficient of time in the sine.

48. The space between the plates of a parallel plate capacitor of capacitance C (without any dielectric) is now filled with three dielectric slabs of dielectric constants $K_1 = 2$, $K_2 = 3$ and $K_3 = 5$ (as shown in figure). If new capacitance is $\frac{n}{3}C$ then the value of n is ____ . [4 marks]



Solution:

Concept:

- For a dielectric-filled parallel-plate region, $C_i = K_i \epsilon_0 A_i / d_i$. The original capacitor has $C = \epsilon_0 A / d$.
- Capacitive regions side by side between the same two potential levels combine in parallel; successive layers along the field combine in series in the usual ideal slab model.

Step 1: Read the slab geometry The upper slab with $K_1 = 2$ covers the full plate area A and has thickness $d/2$. The lower half contains two side-by-side slabs with $K_2 = 3$ and $K_3 = 5$; each has area $A/2$ and thickness $d/2$. The lower slabs therefore form the parallel portion, and that portion is in series with the full upper layer. Use the thickness of each layer, rather than the full plate separation, in each capacitance.

Step 2: Express each capacitance in terms of the original value For the upper slab,

$$C_1 = \frac{2\epsilon_0 A}{d/2} = 4 \frac{\epsilon_0 A}{d} = 4C.$$

For the two lower slabs,

$$C_2 = \frac{3\epsilon_0 (A/2)}{d/2} = 3C, \quad C_3 = \frac{5\epsilon_0 (A/2)}{d/2} = 5C.$$

Their parallel equivalent is $C_{23} = C_2 + C_3 = 8C$. Their areas add to A , consistent with the geometry.

Step 3: Combine the layers and determine n The upper layer and the lower equivalent have equal series charge, and their voltage drops add. Therefore

$$\frac{1}{C_{\text{new}}} = \frac{1}{4C} + \frac{1}{8C} = \frac{3}{8C}, \quad C_{\text{new}} = \frac{8}{3}C.$$

The equivalent is smaller than either series member, as it should be. Comparing this with the stated form $C_{\text{new}} = (n/3)C$ gives $n = 8$. Since n is a ratio coefficient, it has no unit.

Step 4: Conclusion The new capacitance is $\frac{8}{3}C$; hence the required dimensionless value is $n = 8$.

Quick Tip to Solve: First label every slab with its own area and thickness. Combine the two lower side-by-side slabs before combining the upper layer in series.

49. The equation of the electric field of an electromagnetic wave propagating through free space is given by : $E = \sqrt{377} \sin(6.27 \times 10^3 t - 2.09 \times 10^{-5} x) \text{ N/C}$

The average power of the electromagnetic wave is $(\frac{1}{\alpha}) \text{ W/m}^2$. The value of α is

$$\left(\text{Take } \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \text{ in SI units} \right)$$

[4 marks]

Solution:

Concept:

- The intensity of an electromagnetic wave is the time-averaged energy crossing unit area per unit time. In free space the instantaneous Poynting flux is $S = E^2/Z_0$, where $Z_0 = \sqrt{\mu_0/\epsilon_0}$ is the wave impedance.
- For a sinusoidal electric field with peak amplitude E_0 , $\langle \sin^2 \phi \rangle = 1/2$, so $I = \langle S \rangle = E_0^2/(2Z_0)$. The amplitude and the root-mean-square field must not be confused.

Step 1: Identify the peak field and wave impedance Compare the given expression with $E = E_0 \sin(\omega t - kx)$. The coefficient of the sine is the maximum electric field, so $E_0 = \sqrt{377} \text{ N C}^{-1}$, and hence $E_0^2 = 377$ in SI units. The question directly supplies $Z_0 = 377 \Omega$. The values of ω and k describe propagation and do not enter the average intensity once the electric-field amplitude and impedance are known.

Step 2: Average the energy flux over a cycle At any fixed position, the electric field varies sinusoidally and its square has mean $E_0^2/2$. Therefore

$$I = \frac{E_0^2}{2Z_0} = \frac{377}{2 \times 377} = \frac{1}{2} \text{ W m}^{-2}.$$

Using E_0^2/Z_0 without the factor of two would give the peak flux rather than the average. Equivalently, one can use $E_{\text{rms}} = E_0/\sqrt{2}$ in $I = E_{\text{rms}}^2/Z_0$.

Step 3: Compare with the stated reciprocal form The given average power per unit area is $(1/\alpha) \text{ W m}^{-2}$. Equating the numerical coefficients gives

$$\frac{1}{\alpha} = \frac{1}{2} \Rightarrow \alpha = 2.$$

The intensity carries the units; α is a dimensionless numerical parameter.

Step 4: Conclusion The average intensity is 0.5 W m^{-2} , and the required dimensionless value is

$$\alpha = 2.$$

Quick Tip to Solve: For a sinusoidal wave, square the peak electric field and divide by twice the free-space impedance; the cycle average supplies the factor of two.

50. In two separate Young's double-slit experimental set-ups and two monochromatic light sources of different wavelengths are used to get fringes of equal width. The ratios of the slits separations and that of the wavelengths of light used are $2 : 1$ and $1 : 2$ respectively. The corresponding ratio of the distances between the slits and the respective screens (D_1/D_2) is _____. [4 marks]

Solution:

Concept:

- For a Young double-slit arrangement in the small-angle regime, the fringe width is $\beta = \lambda D/d$, where λ is the wavelength, D the slit-to-screen distance and d the slit separation.
- When two arrangements have equal fringe widths, their ratios satisfy $1 = (\lambda_1/\lambda_2)(D_1/D_2)(d_2/d_1)$. Keep the same ordering of arrangements throughout the calculation.

Step 1: Write the condition for equal fringe widths Denote the two experiments by subscripts 1 and 2. Since the observed fringe spacings are equal,

$$\beta_1 = \beta_2 \Rightarrow \frac{\lambda_1 D_1}{d_1} = \frac{\lambda_2 D_2}{d_2}.$$

Both formulae use the same physical definition of fringe width, the separation between successive bright fringes or successive dark fringes. No absolute wavelength or distance is needed because this is a ratio problem.

Step 2: Isolate the ratio of screen distances Dividing the first expression by the second and rearranging gives

$$\frac{D_1}{D_2} = \frac{d_1 \lambda_2}{d_2 \lambda_1}.$$

The wavelength ratio must be inverted here: the question supplies $\lambda_1/\lambda_2 = 1/2$, whereas the required expression contains λ_2/λ_1 . The slit-separation ratio remains $d_1/d_2 = 2$.

Step 3: Substitute and check the physical trend Insert the stated ratios:

$$\frac{D_1}{D_2} = 2 \times \frac{2}{1} = 4.$$

For the first experiment, halving the wavelength and doubling the slit separation would together reduce the fringe width to one quarter if the screen distance stayed fixed. Increasing its screen distance by a factor of four restores the same fringe width, confirming the result. Since both numerator and denominator are distances, their ratio has no unit.

Step 4: Conclusion The required dimensionless ratio is $D_1/D_2 = 4$, or $D_1 : D_2 = 4 : 1$.

Quick Tip to Solve: For fixed fringe width, remember $D \propto d/\lambda$: a larger slit separation or a shorter wavelength requires a more distant screen.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. The correct statements from the following are :

- A. Ionic radii of trivalent cations of group 13 elements decreases down the group.
- B. Electronegativity of group 13 elements decreases down the group.
- C. Among the group 13 elements, Boron has highest first ionisation enthalpy.
- D. The trichloride and triiodide of group 13 elements are covalent in nature.

Choose the correct answer from the options given below :

[4 marks]

- (A) A and C Only
- (B) B and D Only
- (C) C and D Only
- (D) A and D Only

Correct Answer: (C) C and D Only

Solution:

Concept:

- Periodic trends down group 13 are affected by the addition of electron shells and by the incomplete shielding of intervening *d* and *f* electrons. A claim of a uniform decrease must therefore be checked rather than assumed.
- The small size of boron gives a high first ionisation enthalpy. Trivalent group 13 centres strongly polarise the large, polarisable chloride and iodide ions, favouring covalent trihalides.

Step 1: Check the two proposed downward trends Statement A is false. Trivalent-cation radii do not decrease continuously down the group: addition of shells generally enlarges the ions, although contraction effects produce irregularities. In particular, the heavier trivalent ions are not all successively smaller than the lighter ones. Statement B is also false. Electronegativity decreases from boron to aluminium but then shows irregular behaviour, with the heavier elements influenced by poor shielding of the intervening electrons. Thus it is incorrect to describe the entire group by a continuous downward decrease.

Step 2: Check the ionisation and bonding statements Statement C is true. Boron has the highest first ionisation enthalpy among these group 13 elements because its outer electron is in a compact second-shell orbital and is strongly attracted to the nucleus. The later elements show anomalies, but none exceeds boron. Statement D is true in the stated group-13 trihalide classification: the trichlorides and triiodides are covalent. A trivalent centre has substantial polarising power, and chloride and iodide are sufficiently polarisable to favour covalent bonding.

Step 3: Select the combination of true statements The truth pattern is A false, B false, C true and D true. Therefore only C and D should be selected. Option (A) includes false statement A, option (B) includes false statement B, and option (D) also includes false statement A. Option (C) alone contains both true statements without either false trend.

Step 4: Conclusion Only statements C and D are correct. Hence, the correct option is (C).

Quick Tip to Solve: For group 13, do not extend the boron-to-aluminium trend uniformly down the group; check the effects of poor *d*- and *f*-electron shielding.

52. Given below are two statements :

Statement I : Sublimation is used for the separation and purification of compounds with low melting point.

Statement II : The boiling point of a liquid increases as the external pressure is reduced. In the light of the above statements,

choose the correct answer from the options given below :

[4 marks]

- (A) Both Statement I and Statement II are true
- (B) Both Statement I and Statement II are false
- (C) Statement I is true but Statement II is false
- (D) Statement I is false but Statement II is true

Correct Answer: (B) Both Statement I and Statement II are false

Solution:

Concept:

- Sublimation separates a solid that passes directly into vapour on heating and deposits again as a solid on cooling from non-sublimable impurities. A low melting point alone does not establish this behaviour.
- A liquid boils when its saturated vapour pressure equals the external pressure. As vapour pressure rises with temperature, lowering the external pressure lowers the boiling temperature.

Step 1: Assess the criterion for sublimation Statement I attributes the use of sublimation to a low melting point. This is not the defining requirement and the statement is false. The useful distinction is whether the compound sublimates under the chosen conditions while its impurities do not. A low-melting solid may simply melt into a liquid when heated and therefore cannot be selected for sublimation merely on the basis of its melting point. In a sublimation apparatus, vapour from the sublimable solid condenses as solid on a cooler surface, leaving non-volatile impurities behind.

Step 2: Apply the condition for boiling At the boiling temperature T_b ,

$$P_{\text{vap}}(T_b) = P_{\text{ext}}.$$

If P_{ext} is reduced, the liquid needs to reach a smaller vapour pressure. That pressure is attained at a lower temperature, so the boiling point decreases rather than increases. This is why reduced-pressure distillation can distil a high-boiling liquid at a lower temperature and helps avoid thermal decomposition. Statement II reverses this relationship and is false.

Step 3: Match the two truth values to the options Both statements are false. Option (A) wrongly accepts both, option (C) wrongly accepts the low-melting-point criterion in Statement I, and option (D) wrongly accepts the pressure trend in Statement II. Only option (B) records the two false truth values correctly.

Step 4: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (B).

Quick Tip to Solve: Choose sublimation by direct solid-to-vapour behaviour, not melting point; for boiling, remember that a vacuum lowers the temperature needed to boil.

53. The correct trend in the first ionization enthalpies of the elements in the 3rd period of periodic table is :

[4 marks]

(A) $\text{Si} < \text{S} < \text{Al} < \text{P} < \text{Cl}$

(B) $\text{Al} < \text{Si} < \text{S} < \text{P} < \text{Cl}$

(C) $\text{S} < \text{Si} < \text{Al} < \text{P} < \text{Cl}$

(D) $\text{Al} < \text{S} < \text{P} < \text{Si} < \text{Cl}$

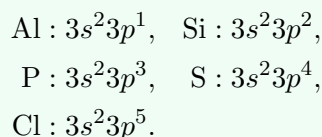
Correct Answer: (B) $\text{Al} < \text{Si} < \text{S} < \text{P} < \text{Cl}$

Solution:

Concept:

- First ionisation enthalpy is the energy needed to remove the most loosely bound electron from an isolated gaseous atom. Across period 3 it generally increases as effective nuclear charge rises.
- Phosphorus has the relatively stable half-filled $3p^3$ configuration. Sulfur has $3p^4$, containing one paired set of electrons, whose repulsion makes the first removal easier than from phosphorus.

Step 1: Apply the general trend to the selected atoms The relevant valence configurations are



The increase in nuclear attraction across the period makes aluminium the easiest of these atoms to ionise. Silicon requires more energy than aluminium. Chlorine, near the right end of the period, has the highest first ionisation enthalpy among the listed elements.

Step 2: Include the phosphorus-sulfur exception In phosphorus the three $3p$ electrons occupy separate orbitals, giving a half-filled subshell. In sulfur the fourth $3p$ electron must pair in one orbital. Removing an electron from this paired orbital relieves repulsion and leaves the stable $3p^3$ configuration. Thus sulfur has a lower first ionisation enthalpy than phosphorus despite being farther right. Sulfur still lies above silicon; the local pairing effect does not reverse the whole across-period trend.

Step 3: Construct the order and reject the alternatives The increasing order is



Option (A) incorrectly places aluminium above both silicon and sulfur. Option (C) incorrectly puts sulfur below silicon and aluminium. Option (D) incorrectly puts silicon above phosphorus. Only option (B) preserves the overall trend and the specific sulfur-phosphorus inversion.

Step 4: Conclusion The correct order is $\text{Al} < \text{Si} < \text{S} < \text{P} < \text{Cl}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the p-electron counts first: the first paired p electron in sulfur explains why its ionisation enthalpy is lower than that of phosphorus.

54. In the given electrochemical cell, $\text{Ag(s)}|\text{AgCl(s)}|\text{FeCl}_2(\text{aq}), \text{FeCl}_3(\text{aq}) | \text{Pt(s)}$ at 298 K, the cell potential (E_{cell}) will increase when :

- A. Concentration of Fe^{2+} is increased.
- B. Concentration of Fe^{3+} is decreased.
- C. Concentration of Fe^{2+} is decreased.
- D. Concentration of Fe^{3+} is increased.
- E. Concentration of Cl^- is increased.

Choose the correct answer from the options given below :

[4 marks]

- (A) A and B Only
- (B) A and E Only
- (C) B Only
- (D) C, D and E Only

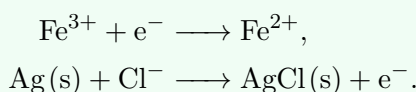
Correct Answer: (D) C, D and E Only

Solution:

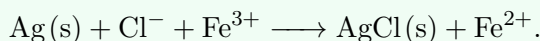
Concept:

- The cell potential for the written cell is the right-hand reduction potential minus the left-hand reduction potential. Pure solids have unit activity and do not appear in the reaction quotient.
- At 298 K, the Nernst equation is $E_{\text{cell}} = E_{\text{cell}}^{\circ} - (0.0591/n) \log Q$. Here one electron is transferred, and dilute-solution concentrations represent the ionic activities.

Step 1: Write the half-reactions and reaction quotient The right electrode reduces iron(III), while the left electrode oxidises silver:



The net reaction is



Consequently, with the activities of the pure solids omitted,

$$Q = \frac{[\text{Fe}^{2+}]}{[\text{Fe}^{3+}][\text{Cl}^-]}.$$

Chloride is a reactant at the silver-silver chloride electrode; it must not be left out of the quotient.

Step 2: Determine which changes increase the potential For this one-electron reaction,

$$E_{\text{cell}} = E_{\text{cell}}^{\circ} - 0.0591 \log \left(\frac{[\text{Fe}^{2+}]}{[\text{Fe}^{3+}][\text{Cl}^-]} \right).$$

A decrease in Q raises the potential. Decreasing $[\text{Fe}^{2+}]$ lowers the numerator, so C is correct. Increasing either $[\text{Fe}^{3+}]$ or $[\text{Cl}^-]$ increases the denominator, so D and E are correct. These comparisons concern each listed concentration change with the other quantities fixed.

Step 3: Rule out the changes that lower the potential A increases the numerator of Q , and B decreases its denominator. Both therefore increase Q and lower the cell potential, so neither is correct. Options

(A), (B) and (C) all contain at least one of these incorrect statements. The combination C, D and E in option (D) contains exactly the changes that raise the potential.

Step 4: Conclusion The cell potential increases for changes C, D and E. Hence, the correct option is (D).

Quick Tip to Solve: Write the complete cell reaction before using Nernst: the silver-silver chloride electrode makes chloride concentration part of the reaction quotient.

55. A cup of water at 5°C (system) is placed in a microwave oven and the oven is turned on for one minute during which the water begins to boil. Which of the following option is true? [4 marks]

(A) $q = +ve, w = -ve, \Delta U = +ve$

(B) $q = +ve, w = 0, \Delta U = -ve$

(C) $q = +ve, w = -ve, \Delta U = -ve$

(D) $q = -ve, w = -ve, \Delta U = -ve$

Correct Answer: (A) $q = +ve, w = -ve, \Delta U = +ve$

Solution:

Concept:

- Use the chemistry sign convention: heat absorbed by the system has $q > 0$, while work done by the system on its surroundings has $w < 0$.
- The first law is $\Delta U = q + w$. Heating liquid water to its boiling point raises its internal energy; expansion against external pressure gives $w = -P_{\text{ext}}\Delta V$.

Step 1: Identify the direction of energy transfer The water is the system and receives energy while the microwave oven operates. In the heat-and-expansion bookkeeping used by these choices, energy supplied to heat the water is positive: $q > 0$. The water is warmed from 5°C to the onset of boiling; it is not cooling or losing energy overall.

Step 2: Determine the work and internal-energy signs As heating and the onset of vaporisation cause expansion, the system displaces its surroundings. Thus $\Delta V > 0$ and the pressure-volume work is negative. The absorbed energy exceeds the magnitude of the expansion work, so the internal energy rises:

$$w = -P_{\text{ext}}\Delta V < 0, \quad \Delta U = q - |w| > 0.$$

Negative work by itself does not imply a negative change in internal energy.

Step 3: Eliminate the incompatible alternatives Option (B) assigns zero work and a decrease in internal energy, contrary to expansion and heating. Option (C) correctly gives positive heat and negative work but incorrectly makes ΔU negative. Option (D) additionally assigns negative heat, which would describe energy leaving the water. Only (A) has all three signs consistent with the process.

Step 4: Conclusion $q = +ve, w = -ve, \Delta U = +ve$. Hence, the correct option is (A).

Quick Tip to Solve: Fix the system first, then use heat in as positive and expansion work as negative; determine ΔU from the actual heating, not from the sign of work alone.

56. Identify the molecule (X) with maximum number of lone pairs of electrons (obtained using Lewis dot structure) among HNO_3 , H_2SO_4 , NF_3 and O_3 . Choose the correct bond angle made by the central atom of the molecule (X). [4 marks]

- (A) 116°
(B) 102°
(C) 107°
(D) 120°

Correct Answer: (B) 102°

Solution:

Concept:

- Count lone pairs on every atom in the Lewis structure when comparing the total number in a molecule. Count only the central atom's electron domains when determining its shape.
- An AX_3E molecule has three bond pairs and one central lone pair. Its molecular shape is trigonal pyramidal, with an angle smaller than the tetrahedral value.

Step 1: Count the total lone pairs For HNO_3 , the usual resonance structure has two lone pairs on the hydroxyl oxygen, two on the double-bonded oxygen and three on the singly bonded terminal oxygen: seven in total. Nitrogen has no lone pair. In H_2SO_4 , the conventional expanded-octet structure has two lone pairs on each of its four oxygens, giving eight; sulfur has none. Ozone has six lone pairs in a resonance contributor: one on central oxygen, two on the double-bonded terminal oxygen and three on the other terminal oxygen.

Step 2: Select the molecule with the greatest count In NF_3 , each fluorine has three lone pairs and nitrogen has one. Therefore

$$N_{\text{LP}}(\text{NF}_3) = 3(3) + 1 = 10.$$

This exceeds the counts for nitric acid, sulfuric acid and ozone, so $X = \text{NF}_3$. The fluorine lone pairs contribute to this total even though they do not determine the central-atom domain count.

Step 3: Identify its bond angle Nitrogen in NF_3 has three N–F bonds and one lone pair, giving a trigonal-pyramidal molecule. The F–N–F angle is approximately 102° . Fluorine draws bonding electron density away from nitrogen, reducing bond-pair repulsion near nitrogen and permitting a smaller angle than in ammonia. The 107° choice describes ammonia approximately; 116° is associated with ozone, and 120° is the ideal trigonal-planar value.

Step 4: Conclusion $X = \text{NF}_3$, with ten lone pairs in total and a bond angle of approximately 102° . Hence, the correct option is (B).

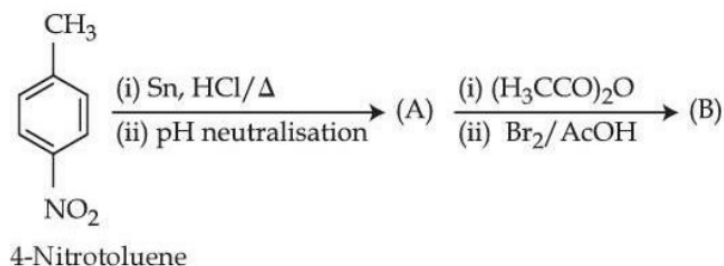
Quick Tip to Solve: Distinguish total molecular lone pairs from the lone pairs on the central atom: the former selects X , while the latter helps determine its shape.

57. Consider the following sequence of reactions.

Assuming that the reaction proceeds to completion, then 137 mg of 4-nitrotoluene will produce ____ mg of B.

(Given molar mass in g mol^{-1} H : 1, C : 12, N : 14, O : 16, Br : 80)

[4 marks]



(A) 208

(B) 228

(C) 146

(D) 301

Correct Answer: (B) 228

Solution:

Concept:

- Sn/HCl reduces an aromatic nitro group to an amine; neutralisation releases the free amine. Acetic anhydride converts $-\text{NH}_2$ into the less activating $-\text{NHCOCH}_3$ group.
- Acylated amino groups remain ortho/para directing. Under the stated bromination conditions, the protected amine undergoes monobromination; the mass is obtained from the one-to-one mole ratio.

Step 1: Identify the intermediate amine and its protected derivative Reduction of 4-nitrotoluene followed by neutralisation gives 4-methylaniline, A, also called para-toluidine. Acetic anhydride acetylates its amino nitrogen to give 4-methylacetanilide. The ring methyl group remains unchanged throughout these transformations. Protection moderates the strong activating effect of the free amino group and controls the subsequent bromination.

Step 2: Determine the bromination product and molecular formula The $-\text{NHCOCH}_3$ group directs bromination ortho and para to itself. Its para position is already occupied by methyl, so bromine enters an ortho position; the two ortho positions are equivalent. Product B is 2-bromo-4-methylacetanilide. The protected precursor has formula $\text{C}_9\text{H}_{11}\text{NO}$; replacing one ring hydrogen by bromine gives $\text{C}_9\text{H}_{10}\text{BrNO}$. Hence

$$M_B = 9(12) + 10(1) + 80 + 14 + 16 = 228 \text{ g mol}^{-1}.$$

Step 3: Convert the starting mass into product mass 4-Nitrotoluene has formula $\text{C}_7\text{H}_7\text{NO}_2$, so its molar mass is

$$M = 7(12) + 7 + 14 + 2(16) = 137 \text{ g mol}^{-1}.$$

Thus 137 mg = 0.137 g represents 0.001 mol. One mole of starting material gives one mole of B, so

complete conversion gives

$$m_B = 0.001(228) \text{ g} = 228 \text{ mg}.$$

The other masses, 208, 146 and 301 mg, do not equal the mass of one millimole of this singly brominated, acetylated product.

Step 4: Conclusion The mass of B is 228 mg. Hence, the correct option is (B).

Quick Tip to Solve: After identifying each functional-group change, count atoms in the final protected product before applying the one-to-one mole ratio.

58. Given below are two statements :

Statement I : $[\text{CoBr}_4]^{2-}$ ion will absorb light of lower energy than $[\text{CoCl}_4]^{2-}$ ion.

Statement II : In $[\text{CoI}_4]^{2-}$ ion, the energy separation between the two set of d-orbitals is more than $[\text{CoCl}_4]^{2-}$ ion.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Both Statement I and Statement II are true
- (B) Both Statement I and Statement II are false
- (C) Statement I is true but Statement II is false
- (D) Statement I is false but Statement II is true

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- For complexes of the same metal ion and geometry, a stronger-field ligand produces a larger splitting of the metal d orbitals. Halide field strength follows $\text{I}^- < \text{Br}^- < \text{Cl}^-$.
- For a crystal-field transition, the absorbed photon energy is set by the orbital energy gap: $E = h\nu = \Delta$. A smaller splitting therefore requires lower-energy light.

Step 1: Establish the common metal and geometry Each complex contains four halide ions and has charge -2 , so cobalt has oxidation state $+2$: $z + 4(-1) = -2$ gives $z = +2$. These tetrahalocobaltate(II) complexes are tetrahedral. Comparing like metal ions in the same geometry allows their splittings to be ranked directly using the spectrochemical series.

Step 2: Assess Statement I Bromide is a weaker-field ligand than chloride. Consequently the tetrahedral splitting satisfies

$$\Delta_t([\text{CoBr}_4]^{2-}) < \Delta_t([\text{CoCl}_4]^{2-}).$$

Because the required transition energy follows this gap, the bromide complex absorbs lower-energy light. Statement I is therefore true; lower energy corresponds to lower frequency and longer wavelength.

Step 3: Assess Statement II and select the combination Iodide is weaker-field than chloride, so

$$\Delta_t([\text{CoI}_4]^{2-}) < \Delta_t([\text{CoCl}_4]^{2-}).$$

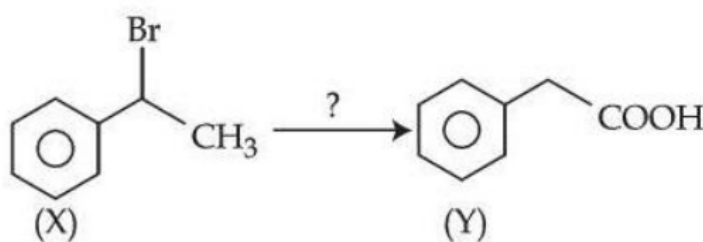
The statement claims the reverse inequality and is false. Hence the combination is true for I and false for II. Option (A) incorrectly accepts II, (B) incorrectly rejects I, and (D) reverses both truth values; only (C) preserves the ligand-strength order.

Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: For comparable halide complexes, write $\text{I}^- < \text{Br}^- < \text{Cl}^-$ first; the splitting and absorbed-energy orders follow the same direction.

59. The correct sequence of reagents for the above conversion of X to Y is :

[4 marks]



- (A) (i) $\text{B}_2\text{H}_6/\text{H}_2\text{O}_2$
(ii) NaOEt
(iii) Jones reagent
(B) (i) Jones reagent
(ii) NaOEt
(iii) Hot KMnO_4/KOH
(C) (i) NaOEt
(ii) $\text{B}_2\text{H}_6/\text{H}_2\text{O}_2$
(iii) Jones reagent
(D) (i) NaOH(aq)
(ii) Jones reagent
(iii) H_3O^+

Correct Answer: (C) (i) NaOEt

- (ii) $\text{B}_2\text{H}_6/\text{H}_2\text{O}_2$
(iii) Jones reagent

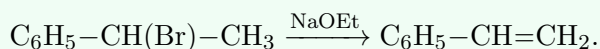
Solution:

Concept:

- A strong base can eliminate hydrogen bromide from an alkyl bromide to form an alkene. Hydroboration followed by oxidation installs the hydroxyl group at the less substituted alkene carbon.

- Jones reagent oxidises a primary alcohol to a carboxylic acid. Tracking which carbon bears the functional group is essential because oxidation of a secondary alcohol gives a ketone instead.

Step 1: Form the alkene from X The figure shows $X = \text{C}_6\text{H}_5-\text{CH}(\text{Br})-\text{CH}_3$ and $Y = \text{C}_6\text{H}_5-\text{CH}_2-\text{COOH}$. Sodium ethoxide removes a beta hydrogen while bromide leaves, forming styrene:



This creates the carbon-carbon double bond required for the subsequent hydroboration step without changing the two-carbon side-chain skeleton.

Step 2: Place hydroxyl on the terminal carbon Hydroboration with diborane, followed by peroxide oxidation, gives anti-Markovnikov hydration of styrene. Boron bonds preferentially to the terminal carbon and is replaced by hydroxyl during oxidation, producing the primary alcohol $\text{C}_6\text{H}_5-\text{CH}_2-\text{CH}_2\text{OH}$. The hydroxyl group must be on this terminal carbon to obtain the carboxyl group at the position shown in Y.

Step 3: Oxidise and reject the other sequences Jones oxidation converts the primary alcohol into phenylacetic acid:

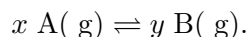


Thus elimination must precede hydroboration, which must precede oxidation. Option (A) applies hydroboration before an alkene exists, and (B) begins with an oxidant that does not perform the required elimination. In (D), aqueous hydroxide substitutes bromide to give the secondary alcohol $\text{C}_6\text{H}_5-\text{CH}(\text{OH})-\text{CH}_3$; its oxidation gives acetophenone, not the required acid.

Step 4: Conclusion The sequence is (i) NaOEt, (ii) $\text{B}_2\text{H}_6/\text{H}_2\text{O}_2$, and (iii) Jones reagent. Hence, the correct option is (C).

Quick Tip to Solve: Work backward from the acid: its precursor must be a primary alcohol, which here requires anti-Markovnikov hydration after elimination.

60. Consider the general reaction given below at 400 K



The values of K_p and K_c are studied under the same condition of temperature but variation in x and y .

(i) $K_p = 85.87$ and $K_c = 2.586$ appropriate units

(ii) $K_p = 0.862$ and $K_c = 28.62$ appropriate units

The values of x and y in (i) and (ii) respectively are :

[4 marks]

(A) (i) (ii)
1,2 2,1

(B) (i) (ii)
1,3 2,1

(C) (i) (ii)
3,1 3,1

(D) (i) (ii)
4,1 4,1

Correct Answer: (A) (i) (ii)
1,2 2,1

Solution:

Concept:

- For a gaseous reaction, $K_p = K_c(RT)^{\Delta n_g}$, where Δn_g is the sum of gaseous product coefficients minus the sum of gaseous reactant coefficients.
- For $x\text{A(g)} \rightleftharpoons y\text{B(g)}$, the exponent is $\Delta n_g = y - x$. Compare K_p/K_c with RT and its reciprocal to find this difference.

Step 1: Determine the common RT factor At $T = 400 \text{ K}$, using $R \simeq 0.083 \text{ L bar mol}^{-1} \text{ K}^{-1}$ gives $RT \simeq 33.2$ in matching units. The quoted constants carry the appropriate pressure and concentration units, so their numerical ratio is compared consistently with this factor. We need the change in gaseous coefficients, not the separate coefficients immediately.

Step 2: Use the first pair of constants For case (i),

$$\frac{K_p}{K_c} = \frac{85.87}{2.586} \simeq 33.21 \simeq RT.$$

Thus $(RT)^{y-x} = RT$, so $y - x = +1$. The proposed pair $(x, y) = (1, 2)$ satisfies this. The other offered first-case pairs give differences $+2$, -2 and -3 , and therefore cannot reproduce this ratio.

Step 3: Check the second pair and select the offered coefficients For case (ii),

$$\frac{K_p}{K_c} = \frac{0.862}{28.62} \simeq 0.03012 \simeq \frac{1}{33.2}.$$

This requires $y - x = -1$, which is satisfied by $(x, y) = (2, 1)$. Option (A) therefore matches both cases.

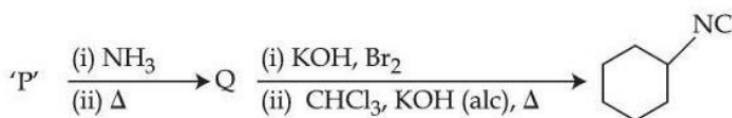
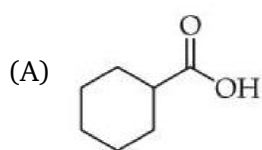
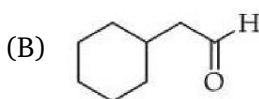
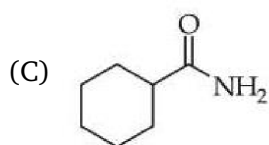
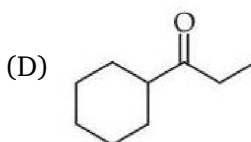
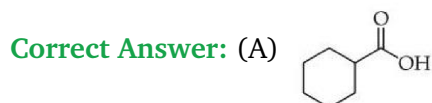
Option (B) matches case (ii) but not case (i); (C) and (D) give negative differences of magnitude two and three in both cases. The ratios determine coefficient differences, and the listed alternatives identify the requested pairs.

Step 4: Conclusion Case (i): $(x, y) = (1, 2)$; case (ii): $(x, y) = (2, 1)$. Hence, the correct option is (A).

Quick Tip to Solve: When $RT > 1$, a ratio $K_p/K_c > 1$ signals more gaseous product moles; a reciprocal ratio signals fewer. Then test the exact coefficient differences.

61. Compound *P* undergoes the following sequence of reactions. Identify *P*.

[4 marks]



Solution:

Concept:

- Heating an ammonium carboxylate gives the corresponding primary amide: $\text{RCOOH} \xrightarrow[\text{heat}]{\text{NH}_3} \text{RCONH}_2$.
- Hofmann bromamide degradation converts RCONH_2 into RNH_2 , removing the carbonyl carbon. A primary amine then gives RNC in the carbylamine reaction.

Step 1: Work backwards from the isocyanide The final structure is cyclohexyl isocyanide, $\text{C}_6\text{H}_{11}\text{NC}$: the ring is bonded to nitrogen, not to the terminal carbon. The reaction with chloroform and hot alcoholic potassium hydroxide therefore requires cyclohexylamine, $\text{C}_6\text{H}_{11}\text{NH}_2$, immediately before this step.

Step 2: Identify the amide and its precursor The preceding Br_2/KOH treatment converts cyclohexanecarboxamide into cyclohexylamine. Hence *Q* must be $\text{C}_6\text{H}_{11}\text{CONH}_2$. The carboxylic acid in option (A) reacts with ammonia to form its ammonium salt, and heating eliminates water to form precisely this amide. The carbon skeleton follows the sequence



The final carbylamine reaction is

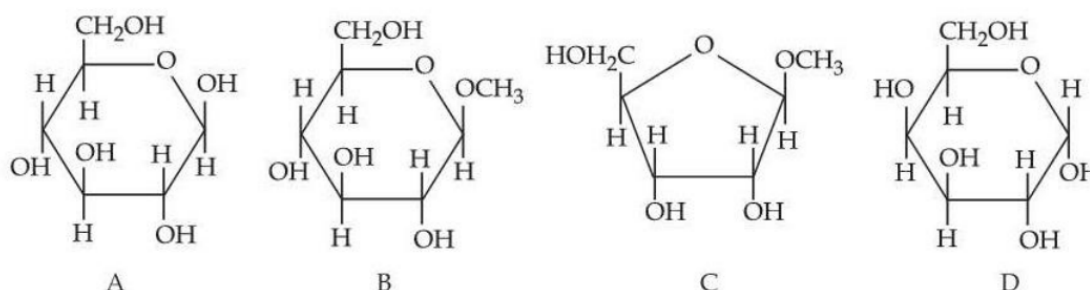


Step 3: Exclude the other starting compounds Option (B) is an aldehyde and option (D) is a ketone; their reactions with ammonia do not generate the required primary carboxamide by ammonium-salt dehydration. Option (C) is already the amide *Q*, whereas the stated first conversion starts from its carboxylic acid. Thus only (A) accounts for the whole prescribed sequence.

Step 4: Conclusion *P* is cyclohexanecarboxylic acid, $\text{C}_6\text{H}_{11}\text{COOH}$. Hence, the correct option is (A).

Quick Tip to Solve: For an isocyanide formed using chloroform and alcoholic KOH, work back to the primary amine, then restore one carbonyl carbon to identify the Hofmann amide.

62. From the given following (A to D) cyclic structures, those which will not react with Tollen's reagent are: [4 marks]



(A) A and B

(B) A and D

(C) B and D

(D) B and C

Correct Answer: (D) B and C

Solution:

Concept:

- Tollens' reagent oxidises an accessible aldehyde group. A cyclic sugar with a free anomeric hydroxyl group can open to its carbonyl form and is reducing.
- Replacing the anomeric hydroxyl group by OCH_3 creates a glycosidic acetal. It cannot undergo the ordinary ring-chain equilibrium in the alkaline Tollens test.

Step 1: Locate each anomeric centre The anomeric carbon is the ring carbon bonded to the ring oxygen and also to an external oxygen substituent. In structures A and D, this carbon bears a free OH group. In B and C, the corresponding substituent is OCH_3 . This local bonding feature, rather than the size of the ring or the orientation of the other hydroxyl groups, determines the reducing behaviour.

Step 2: Classify the four structures A and D are cyclic hemiacetals. Their rings can open to give aldehydic forms; removal of the open-chain form by oxidation allows more of it to form. They

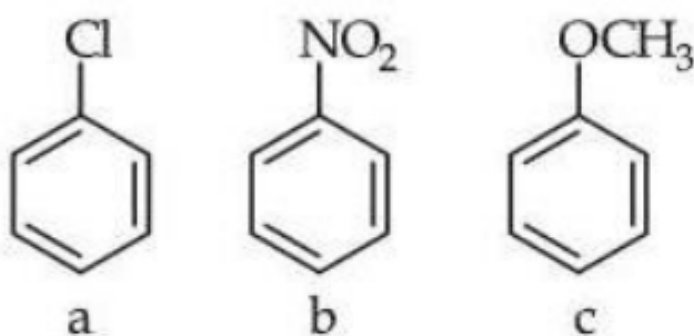
therefore reduce Tollens reagent to metallic silver. B and C are methyl glycosides: the anomeric centres are acetals rather than free hemiacetals. Their acetal bonds are stable under the alkaline test conditions, so neither produces the required free carbonyl group.

Step 3: Choose the nonreacting pair The required pair is B and C. The choice A and B includes reducing sugar A; A and D consists of two reducing sugars; B and D includes reducing sugar D. Each of those choices therefore contains a structure that does react with Tollens reagent. Only the pair B and C has both anomeric centres blocked.

Step 4: Conclusion B and C do not react with Tollens reagent. Hence, the correct option is (D).

Quick Tip to Solve: Find the carbon attached to two oxygens: a free anomeric OH allows ring opening, but an anomeric OMe group blocks the ordinary reducing-sugar test.

63. Consider the following compounds. Arrange these compounds in the increasing order of reactivity with nitrating mixture. [4 marks]



- (A) $c < b < a$
(B) $b < c < a$
(C) $c < a < b$
(D) $b < a < c$

Correct Answer: (D) $b < a < c$

Solution:

Concept:

- Nitration is electrophilic aromatic substitution. The nitrating mixture generates NO_2^+ , and an electron-rich ring attacks this electrophile more readily.
- Reaction rate depends on ring activation or deactivation; the ortho/para-directing character of a substituent does not by itself imply activation.

Step 1: Compare the three substituents Compound a is chlorobenzene, b is nitrobenzene and c is anisole. The nitro group in b withdraws electron density strongly by both its inductive and resonance effects. It destabilises the positively charged intermediate of electrophilic substitution, making b strongly deactivated. Chlorine in a also withdraws inductively; its lone-pair resonance donation determines orientation but does not overcome its net deactivating effect.

Step 2: Identify the most activated ring The methoxy oxygen in c donates a lone pair into the aromatic ring by resonance. This increases electron density and stabilises the substitution intermediate, particularly for ortho and para attack. Although oxygen has an electron-withdrawing inductive effect, resonance donation makes methoxy a strong activating group overall. Consequently anisole reacts faster than chlorobenzene, and chlorobenzene reacts faster than nitrobenzene.

Step 3: Write and test the increasing order The resulting order is



Options (A) and (C) put anisole first, incorrectly treating the methoxy group as the strongest deactivator. Option (B) places chlorobenzene above anisole, reversing their activation. Only option (D) has both comparisons correct.

Step 4: Conclusion The increasing order of reactivity is $b < a < c$. Hence, the correct option is (D).

Quick Tip to Solve: For nitration rates, rank net activation first: methoxy activates, chlorine weakly deactivates, and nitro strongly deactivates.

64. Given,

(A) $n = 5, m_l = -1$

(B) $n = 3, l = 2, m_l = -1, m_s = +\frac{1}{2}$

The maximum number of electron(s) in an atom that can have the quantum numbers as given in (A) and (B) respectively are: [4 marks]

(A) 26 and 1

(B) 2 and 4

(C) 8 and 1

(D) 4 and 1

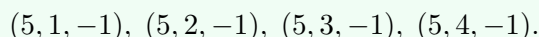
Correct Answer: (C) 8 and 1

Solution:

Concept:

- For a principal quantum number n , the allowed azimuthal values are $l = 0, 1, \dots, n - 1$; for each l , $m_l = -l, \dots, +l$.
- Each orbital, specified by (n, l, m_l) , holds at most two electrons with opposite spins. A complete set of four quantum numbers specifies only one electron state.

Step 1: Count the orbitals allowed in case A For $n = 5$, the possible values of l are 0, 1, 2, 3, 4. The condition $m_l = -1$ excludes $l = 0$, since an s orbital has only $m_l = 0$. It is permitted once in each of $l = 1, 2, 3, 4$. Thus the four allowed orbitals are specified by



The magnetic quantum number is fixed, so we do not count the other magnetic orientations in those

subshells.

Step 2: Include spin and evaluate case B In case A, spin is unrestricted. Each of those four orbitals accommodates $m_s = +1/2$ and $m_s = -1/2$, giving

$$N_A = 4 \times 2 = 8.$$

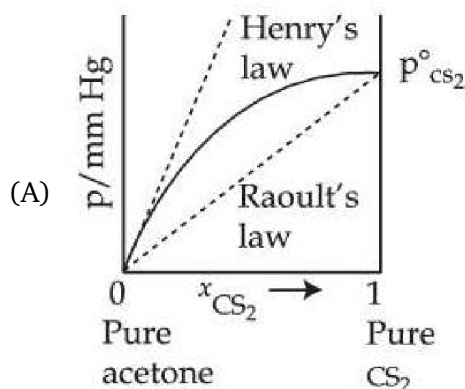
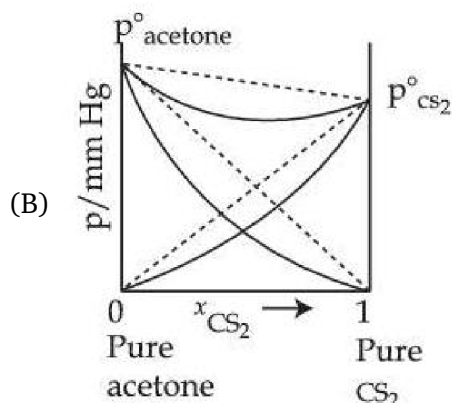
For case B, $n = 3, l = 2, m_l = -1$ specify one 3d orbital, and $m_s = +1/2$ further specifies a single spin state. Pauli exclusion forbids two electrons with this same complete set, so $N_B = 1$.

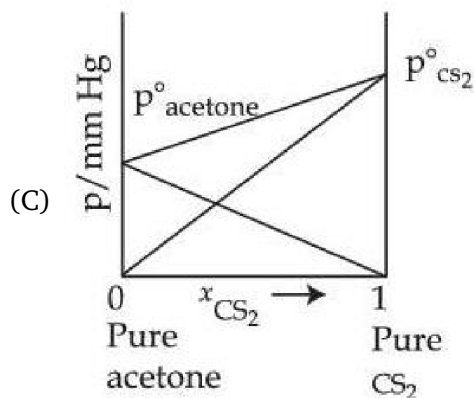
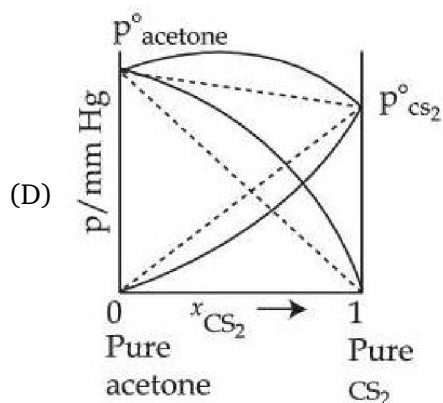
Step 3: Reject the alternative counts The pair 26 and 1 overcounts case A by including states with other magnetic quantum numbers. The pair 2 and 4 fails both the four-orbital count in A and the unique-state restriction in B. The pair 4 and 1 counts orbitals in A but omits their two spin possibilities. Therefore the only consistent pair is 8 and 1.

Step 4: Conclusion The maximum numbers are 8 and 1, respectively. Hence, the correct option is (C).

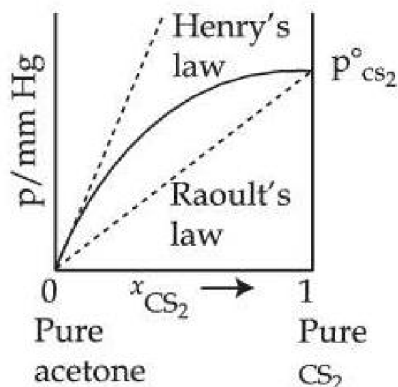
Quick Tip to Solve: If n and a nonzero magnetic quantum number are fixed but spin is free, count allowed l values first and multiply their number by two.

65. Which one of the following graphs accurately represents the plot of partial pressure of CS_2 vs its mole fraction in a mixture of acetone and CS_2 at constant temperature? [4 marks]





Correct Answer: (A)



Solution:

Concept:

- An ideal solution obeys $p_{\text{CS}_2} = x_{\text{CS}_2} p_{\text{CS}_2}^\circ$, a straight line joining the zero-pressure and pure-component endpoints.
- Acetone and carbon disulfide form a solution showing positive deviation from Raoult's law: the unlike interactions are weaker than the interactions disrupted on mixing.

Step 1: Determine the sign of deviation Pure acetone has appreciable dipole–dipole attractions. Introducing comparatively nonpolar CS_2 disrupts these attractions, and the unlike interactions do not compensate fully. The molecules escape more readily into the vapour. Therefore, at an intermediate

composition, the carbon disulfide partial pressure lies above its ideal Raoult-law line:

$$p_{\text{CS}_2} > x_{\text{CS}_2} p_{\text{CS}_2}^\circ$$

Step 2: Check both composition limits At $x_{\text{CS}_2} = 0$, no carbon disulfide is present and its partial pressure is zero. At $x_{\text{CS}_2} = 1$, it is pure and the partial pressure equals $p_{\text{CS}_2}^\circ$. Close to infinite dilution the partial pressure follows Henry's law, $p_{\text{CS}_2} \simeq K_H x_{\text{CS}_2}$. The initial slope for positive deviation exceeds the Raoult-line slope. The first graph has precisely these endpoints and a curve above the ideal line.

Step 3: Compare the other graphs The second graph shows the rising carbon disulfide partial-pressure curve below its ideal line, corresponding to negative deviation. The third graph uses straight ideal partial-pressure lines. In the fourth graph the total-pressure curve is above its reference, but the rising carbon disulfide partial-pressure curve is below its own dashed Raoult line. The question asks for that component's partial pressure, so a total-pressure maximum cannot make the fourth graph correct.

Step 4: Conclusion The first graph correctly represents the positive deviation of CS_2 partial pressure. Hence, the correct option is (A).

Quick Tip to Solve: Trace the curve starting at zero when the component mole fraction is zero; compare that partial-pressure curve, rather than the total-pressure curve, with Raoult's line.

66. But-2-yne and hydrogen (one mole each) are separately treated with (i) Pd/C and (ii) Na/ liq. NH_3 to give the products X and Y respectively.

Identify the incorrect statements.

A. X and Y are stereoisomers.

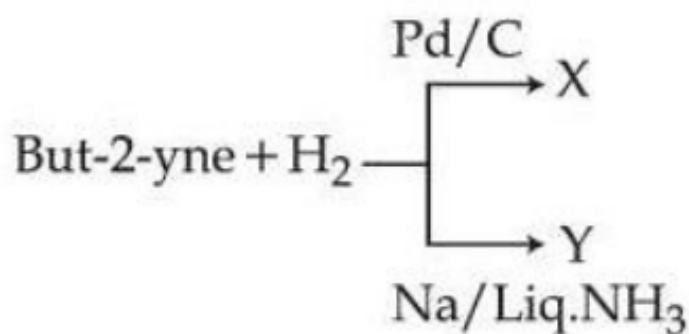
B. Dipole moment of X is zero.

C. Boiling point of X is higher than Y .

D. X and Y react with $\text{O}_3/\text{Zn} + \text{H}_2\text{O}$ to give different products.

Choose the correct answer from the options given below :

[4 marks]



(A) A and C Only

(B) B and D Only

(C) B and C Only

(D) A and B Only

Correct Answer: (B) B and D Only

Solution:

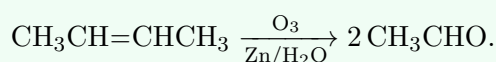
Concept:

- Addition of one equivalent of hydrogen to an alkyne supplies the two hydrogen atoms needed for an alkene. Catalytic surface addition is syn, whereas dissolving-metal reduction gives anti addition.
- Cis and trans alkenes have the same connectivity but differ in geometry. Reductive ozonolysis cleaves the double bond, so its carbonyl products depend on substituents on the two alkene carbons.

Step 1: Identify X and Y With the stated one-mole hydrogen amount, catalytic syn addition to but-2-yne gives cis-but-2-ene as X. The Na/liquid NH_3 reduction gives trans-but-2-ene as Y. Both have the connectivity $\text{CH}_3\text{CH}=\text{CHCH}_3$ and differ in the relative sides occupied by the methyl groups. Thus statement A, that they are stereoisomers, is correct.

Step 2: Compare polarity and boiling points In cis-but-2-ene the bond-dipole contributions do not cancel by symmetry, so X has a nonzero dipole moment. Statement B is incorrect. Trans-but-2-ene is more symmetric and has zero resultant dipole moment. The additional polar attractions in cis-but-2-ene make its boiling point higher than that of trans-but-2-ene, so statement C is correct. This comparison concerns boiling point, not melting-point trends arising from crystal packing.

Step 3: Check reductive ozonolysis Each carbon of the double bond in either isomer bears one hydrogen and one methyl group. Cleavage followed by the stated reductive work-up therefore gives two molecules of ethanal from either X or Y:



The cis/trans distinction disappears on cleavage. Statement D, claiming different products, is incorrect. Hence the incorrect statements are B and D; every other offered pair includes at least one of the true statements A or C.

Step 4: Conclusion Statements B and D are incorrect. Hence, the correct option is (B).

Quick Tip to Solve: For ozonolysis, split the double bond and keep each carbon's substituents; cis/trans geometry changes polarity but not the resulting carbonyl identities.

67. The statements that are incorrect about the nickel(II) complex of dimethylglyoxime are :

A. It is red in colour.

B. It has a high solubility in water at $\text{pH} = 9$.

C. The Ni ion has two unpaired d-electrons.

D. The $\text{N} - \text{Ni} - \text{N}$ bond angle is almost close to 90° .

E. The complex contains four five-membered metallacycles (metal containing rings).

Choose the correct answer from the options given below :

[4 marks]

(A) B, C and E Only

(B) A, D and B Only

(C) C and D Only

(D) C and E Only

Correct Answer: (A) B, C and E Only

Solution:

Concept:

- Dimethylglyoximate is a bidentate ligand: each ligand binds Ni^{2+} through two nitrogen donor atoms, producing a square-planar complex.
- For square-planar d^8 nickel(II), the occupied lower-energy orbitals contain paired electrons; chelate rings must be counted by tracing closed paths through the metal.

Step 1: Check colour and solubility The nickel(II) dimethylglyoxime complex is a characteristic red precipitate. Thus statement A is correct. It is precipitated from ammoniacal solution, including conditions near pH 9, because it is sparingly soluble in water. Statement B, which claims high water solubility at this pH, is incorrect. The appearance of the red precipitate is the basis of the analytical test for nickel(II).

Step 2: Check magnetism and geometry Nickel(II) has the $3d^8$ configuration. In this square-planar complex the eight d-electrons occupy four orbitals as pairs, leaving the highest-energy orbital empty. There are no unpaired electrons, so the complex is diamagnetic: C is incorrect. Adjacent nitrogen donors lie approximately at right angles in the square plane, so the stated nearly 90° angle in D is correct.

Step 3: Count the metal-containing rings and select the set Each of the two bidentate ligands forms one five-membered chelate ring, with the sequence $\text{Ni}-\text{N}-\text{C}-\text{C}-\text{N}$. There are therefore two such metallacycles, not four; E is incorrect. Hydrogen bonding between oxime groups does not turn these two chelate rings into four five-membered metal-containing rings. The incorrect set is B, C and E. Option B wrongly includes the true statements A and D; option C wrongly includes D; option D omits the false statement B.

Step 4: Conclusion B, C and E are incorrect, while A and D are correct. Hence, the correct option is (A).

Quick Tip to Solve: For the red nickel–dimethylglyoxime precipitate, remember square-planar d^8 , no unpaired electrons, and one five-membered chelate ring per bidentate ligand.

68. Which of the following statements regarding the energy of the stationary state is true in the following one - electron systems ? [4 marks]

(A) -1.09×10^{-18} J for second orbit of H atom.

(B) $+2.18 \times 10^{-18}$ J for second orbit of He^+ ion

(C) $+8.72 \times 10^{-18}$ J for first orbit of He^+ ion

(D) -2.18×10^{-18} J for third orbit of Li^{2+} ion

Correct Answer: (D) -2.18×10^{-18} J for third orbit of Li^{2+} ion

Solution:

Concept:

- For a hydrogen-like ion with nuclear charge Z and principal quantum number n , $E_n = -2.18 \times 10^{-18} Z^2/n^2$ J.
- The zero of energy is a separated stationary electron and nucleus. Every bound stationary state has negative energy; a positive energy cannot describe one of these bound orbits.

Step 1: Evaluate the hydrogen statement For hydrogen, $Z = 1$, and its second orbit has $n = 2$. Therefore

$$E_2 = -2.18 \times 10^{-18} \frac{1}{4} = -5.45 \times 10^{-19} \text{ J.}$$

Option A quotes -1.09×10^{-18} J, which is twice the magnitude of the correct energy. It incorrectly corresponds to dividing by 2 rather than by 2^2 .

Step 2: Evaluate both helium-ion statements The one-electron helium ion is He^+ , with $Z = 2$. In its second orbit,

$$E_2 = -2.18 \times 10^{-18} \frac{2^2}{2^2} = -2.18 \times 10^{-18} \text{ J.}$$

In its first orbit,

$$E_1 = -2.18 \times 10^{-18} \frac{2^2}{1^2} = -8.72 \times 10^{-18} \text{ J.}$$

Options B and C quote these magnitudes with positive signs, so both are false.

Step 3: Evaluate the lithium-ion statement For Li^{2+} , $Z = 3$, and the third orbit has $n = 3$. Thus

$$E_3 = -2.18 \times 10^{-18} \frac{3^2}{3^2} = -2.18 \times 10^{-18} \text{ J.}$$

The ratio Z^2/n^2 is exactly unity. Both the negative sign and the numerical magnitude agree with option D, making it the only true statement.

Step 4: Conclusion The third orbit of Li^{2+} has energy -2.18×10^{-18} J. Hence, the correct option is (D).

Quick Tip to Solve: Reject positive energies for bound Bohr orbits first; then compare Z/n , whose square sets the energy magnitude.

69. Match List - I with List - II.

List - I Functional group (detection)	List - II Change observed during detection
A. Unsaturation (Baeyer's test)	I. Red colour appears
B. Alcoholic group (Ceric ammonium nitrate test)	II. Silver mirror appears
C. Aldehyde group (Tollen's reagent)	III. Violet colour appears
D. Phenolic group (FeCl_3 test)	IV. Discharge of pink colour

Choose the correct answer from the options given below:

[4 marks]

- (A) A-IV, B-III, C-II, D-I
 (B) A-III, B-IV, C-II, D-I
 (C) A-IV, B-I, C-II, D-III
 (D) A-III, B-IV, C-I, D-II

Correct Answer: (C) A-IV, B-I, C-II, D-III

Solution:

Concept:

- A functional-group test is identified by the observable change caused by its reagent, rather than by the colour of an unrelated test.
- Baeyer's test uses dilute alkaline KMnO_4 ; Tollens' reagent contains an ammoniacal silver(I) complex that aldehydes reduce to metallic silver.

Step 1: Match the unsaturation and alcohol tests In Baeyer's test, an unsaturated compound reduces permanganate while its multiple bond is oxidised. The pink or purple permanganate colour is discharged, often with brown MnO_2 formation. Thus A matches IV. In the ceric ammonium nitrate test, an alcoholic group gives a characteristic red-coloured complex; hence B matches I. The red observation must not be exchanged with the violet colour associated with a phenolic test.

Step 2: Match the aldehyde and phenol tests An aldehyde reduces Tollens' reagent to metallic silver, which deposits on the clean inner wall of the test tube as a silver mirror. Hence C matches II. Phenols form coloured complexes with neutral ferric chloride; the violet observation listed here is the characteristic positive result. Thus D matches III. The observations are therefore assigned one-to-one, using each Roman label once.

Step 3: Compare the completed mapping with every option The full matching is

A:IV, B:I, C:II, D:III.

Option A has A and C right but exchanges the alcohol and phenol observations. Option B gives the wrong observations for A, B and D. Option D also exchanges the aldehyde silver mirror and gives an incorrect unsaturation observation. Only option C preserves all four experimental matches.

Step 4: Conclusion The matching is A-IV, B-I, C-II, D-III. In the printed option order this is option C. Hence, the correct option is (C).

Quick Tip to Solve: Use the distinctive pair first: aldehyde–silver mirror. Then separate alcohol–red CAN test from phenol–violet ferric chloride test.

70. 'x' is the product which is obtained from propanenitrile and stannous chloride in the presence of hydrochloric acid followed by hydrolysis. 'y' is the product which is obtained from the but-2-ene by the ozonolysis followed by hydrolysis. From the following, which product is not obtained when one mole of 'x' and one mole of 'y' react with each other in the presence of alkali followed by heating?

[4 marks]

- (A) 3-Methylbut-2-enal
- (B) Pent-2-enal
- (C) 2-Methylpent-2-enal
- (D) 2-Methylbut-2-enal

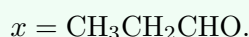
Correct Answer: (A) 3-Methylbut-2-enal

Solution:

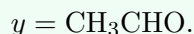
Concept:

- Stephen reduction converts a nitrile $R-CN$ to $R-CHO$ after hydrolysis. The nitrile carbon remains in the aldehyde.
- An aldehyde with an alpha hydrogen forms an enolate in alkali. Aldol addition followed by heating gives an alpha,beta-unsaturated aldehyde, with the donor retaining its aldehyde carbon.

Step 1: Identify the two aldehydes Propanenitrile, CH_3CH_2CN , undergoes reduction with $SnCl_2/HCl$, followed by hydrolysis, to give propanal:



Ozonolysis cleaves the central double bond of but-2-ene symmetrically; the stated hydrolytic work-up gives ethanal:



Both aldehydes have alpha hydrogens, so both can act as enolate donor or carbonyl acceptor.

Step 2: List the crossed-condensation products When ethanal supplies the enolate and propanal supplies the carbonyl acceptor, dehydration gives $OHC-CH=CH-CH_2CH_3$, pent-2-enal. Reversing donor and acceptor gives $OHC-C(CH_3)=CH-CH_3$, 2-methylbut-2-enal. These are options B and D. The methyl substituent from the propanal donor remains at the alpha carbon, C2; it does not migrate to C3.

Step 3: Include the self-condensations and eliminate the impossible product Propanal self-condensation gives $OHC-C(CH_3)=CH-CH_2CH_3$, 2-methylpent-2-enal, so option C can also occur. Ethanal self-condensation gives but-2-enal, which is not among the choices. In contrast, 3-methylbut-2-enal has the skeleton $OHC-CH=C(CH_3)_2$: its beta carbon has two methyl groups and no hydrogen. The aldehyde acceptors available here each retain a hydrogen on that beta carbon. This skeleton would require a ketone acceptor such as acetone, which is absent.

Step 4: Conclusion 3-Methylbut-2-enal is not obtained in this mixture; the other listed aldehydes are possible aldol-condensation products. Hence, the correct option is (A).

Quick Tip to Solve: In a crossed aldol product, label the donor alpha carbon and the acceptor carbonyl carbon before naming the product; this prevents shifting a methyl group from C2 to C3.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The crystal field splitting energy of $[\text{Co}(\text{oxalate})_3]^{3-}$ complex is ' n ' times that of the $[\text{Cr}(\text{oxalate})_3]^{3-}$ complex. Here ' n ' is _____. (Assume $\Delta_o \gg P$) [4 marks]

Solution:

Concept:

- Each oxalate ligand has charge -2 . The complex charge -3 therefore makes the metal oxidation state $+3$ in both complexes.
- In an octahedral field, each t_{2g} electron contributes $-0.4\Delta_o$ and each e_g electron contributes $+0.6\Delta_o$ to the orbital crystal-field stabilisation energy (CFSE).
- The gap Δ_o between the two orbital sets is the splitting energy. It is distinct from the sum of the occupied-electron contributions, the CFSE.

Step 1: Find the configurations in the stated strong-field limit Cobalt has atomic number 27, so Co^{3+} is d^6 . Chromium has atomic number 24, so Cr^{3+} is d^3 . With $\Delta_o \gg P$, cobalt(III) is low-spin and has $t_{2g}^6 e_g^0$. Chromium(III) has $t_{2g}^3 e_g^0$; its three electrons occupy the three lower orbitals singly.

Step 2: Compute the stabilisation coefficients The orbital CFSE contributions are

$$\text{CFSE}_{\text{Co}} = 6(-0.4\Delta_{o,\text{Co}}) = -2.4\Delta_{o,\text{Co}},$$

$$\text{CFSE}_{\text{Cr}} = 3(-0.4\Delta_{o,\text{Cr}}) = -1.2\Delta_{o,\text{Cr}}.$$

On the conventional comparison in units of the respective splitting, the stabilisation coefficients have ratio $2.4/1.2 = 2$. Pairing energy is not part of these orbital CFSE coefficients.

Step 3: Distinguish the intended numerical comparison from the literal wording The numerical answer 2 is obtained by comparing these CFSE coefficients, or by comparing CFSE magnitudes if a common Δ_o is assumed. The actual splitting energies of two different metal ions are not fixed by counting their d-electrons. In particular, $\Delta_o \gg P$ establishes low spin, but does not specify $\Delta_{o,\text{Co}}/\Delta_{o,\text{Cr}}$. Consequently the literal splitting-energy ratio in the wording cannot be determined from the supplied information.

Step 4: Conclusion The intended CFSE-coefficient comparison gives $n = 2$, a dimensionless number. Strictly, the ratio of the actual splitting energies $\Delta_{o,\text{Co}}/\Delta_{o,\text{Cr}}$ is not determined by the stated data.

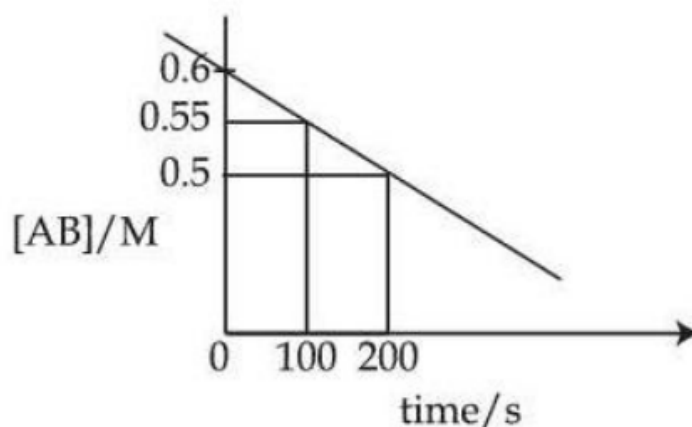
Quick Tip to Solve: Distinguish a splitting gap from a stabilisation sum: electron counts determine the CFSE coefficient, but do not determine the actual splitting gap for a different metal.

72. For the thermal decomposition of reactant $\text{AB}(\text{g})$, the following plot is constructed.

The half life of the reaction is 'x' min.

$x = \underline{\hspace{1cm}}$ min. (Nearest integer)

[4 marks]



Solution:

Concept:

- A straight-line concentration-versus-time plot with negative slope follows the zero-order integrated rate law $[\text{AB}]_t = [\text{AB}]_0 - kt$. The slope is $-k$.
- For a zero-order reaction, $t_{1/2} = [\text{AB}]_0/(2k)$. Unlike a first-order half-life, it depends on the initial concentration.

Step 1: Read the concentration change from the graph The initial concentration at $t = 0$ is 0.60 mol L^{-1} . At 100 s it is 0.55 mol L^{-1} , and at 200 s it is 0.50 mol L^{-1} . The equal concentration decrease of 0.05 mol L^{-1} in each 100-second interval confirms the constant slope. The horizontal axis is in seconds, although the required answer is in minutes.

Step 2: Calculate the zero-order rate constant Using the initial point and the 200-second point,

$$k = -\frac{\Delta[\text{AB}]}{\Delta t} = \frac{0.60 - 0.50}{200} = 5.0 \times 10^{-4} \text{ mol L}^{-1} \text{ s}^{-1}.$$

The positive value of k measures the disappearance rate. Its concentration-per-time unit is consistent with a zero-order reaction.

Step 3: Find the time to reach half the initial concentration At the half-life, the concentration is $0.60/2 = 0.30 \text{ mol L}^{-1}$. The integrated law therefore gives

$$t_{1/2} = \frac{0.60 - 0.30}{5.0 \times 10^{-4}} = 600 \text{ s}.$$

Convert to minutes only after using the graph's time unit:

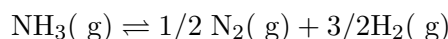
$$t_{1/2} = \frac{600}{60} = 10 \text{ min}.$$

The first-order expression $0.693/k$ is inapplicable because the plotted ordinate is concentration itself, not its logarithm.

Step 4: Conclusion The half-life is 10 min, so the nearest-integer value is $x = 10$.

Quick Tip to Solve: For a linear concentration–time graph, use the intercept and slope directly: half-life is half the intercept divided by the magnitude of the slope.

73. For the following gas phase equilibrium reaction at constant temperature,



if the total pressure is $\sqrt{3}$ atm and the pressure equilibrium constant (K_p) is 9 atm, then the degree of dissociation is given as $(x \times 10^{-2})^{-1/2}$. The value of x is ____ . (nearest integer) [4 marks]

Solution:

Concept:

- The degree of dissociation, α , is the fraction of the initial ammonia that decomposes. Starting with one mole makes every equilibrium amount directly expressible in terms of α .
- Each partial pressure equals its mole fraction times the total pressure P . The equilibrium expression uses the stoichiometric coefficients as powers: $K_p = p_{\text{N}_2}^{1/2} p_{\text{H}_2}^{3/2} / p_{\text{NH}_3}$.

Step 1: Write the equilibrium mole amounts For $\text{NH}_3(\text{g}) \rightleftharpoons \frac{1}{2} \text{N}_2(\text{g}) + \frac{3}{2} \text{H}_2(\text{g})$, start with one mole of ammonia. The equilibrium amounts are

$$n_{\text{NH}_3} = 1 - \alpha, \quad n_{\text{N}_2} = \frac{\alpha}{2}, \quad n_{\text{H}_2} = \frac{3\alpha}{2}.$$

Hence the total is $1 + \alpha$ moles, not one mole: dissociation increases the number of gas molecules.

Step 2: Express the equilibrium constant using total pressure The partial pressures are

$$p_{\text{NH}_3} = \frac{(1 - \alpha)P}{1 + \alpha}, \quad p_{\text{N}_2} = \frac{\alpha P}{2(1 + \alpha)}, \quad p_{\text{H}_2} = \frac{3\alpha P}{2(1 + \alpha)}.$$

Substitution and simplification give

$$\begin{aligned} K_p &= \frac{\sqrt{\alpha P / [2(1 + \alpha)]} [3\alpha P / (2(1 + \alpha))]^{3/2}}{(1 - \alpha)P / (1 + \alpha)} \\ &= \frac{3\sqrt{3} \alpha^2 P}{4(1 - \alpha^2)}. \end{aligned}$$

The remaining first power of pressure agrees with the stated pressure unit of K_p .

Step 3: Solve for dissociation and convert to the requested parameter Using $P = \sqrt{3}$ atm and $K_p = 9$ atm,

$$9 = \frac{9\alpha^2}{4(1 - \alpha^2)} \Rightarrow 4(1 - \alpha^2) = \alpha^2 \Rightarrow \alpha^2 = \frac{4}{5}.$$

The positive root is physical, since a degree of dissociation lies between zero and one. The question

writes $\alpha = (x \times 10^{-2})^{-1/2}$, so

$$\alpha^2 = \frac{100}{x} = \frac{4}{5} \Rightarrow x = 125.$$

Step 4: Conclusion The degree of dissociation is $\alpha = 2/\sqrt{5} \approx 0.8944$. Thus the requested nearest integer is 125; x is dimensionless.

Quick Tip to Solve: For ammonia dissociation, calculate the changing total moles before using partial pressures. Square the given expression for α only after finding α^2 .

74. x mg of pure HCl was used to make an aqueous solution. 25.0 mL of 0.1M Ba(OH)₂ solution is used when the HCl solution was titrated against it. The numerical value of x is ____ $\times 10^{-1}$. (Nearest integer)

Given : Molar mass of HCl and Ba(OH)₂ are 36.5 and 171.0 g mol⁻¹ respectively.

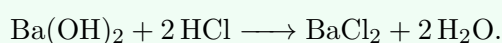
[4 marks]

Solution:

Concept:

- Neutralisation requires equal amounts of hydrogen ions and hydroxide ions. One mole of Ba(OH)₂ supplies two moles of OH⁻, whereas one mole of HCl supplies one mole of H⁺.
- Molarity is amount per litre, so $n = MV$ when V is in litres. Convert the required acid amount to mass using its molar mass, then express that mass in the numerical format requested.

Step 1: Balance the neutralisation reaction The reaction is



Therefore the acid-to-base mole ratio is 2 : 1. A one-to-one ratio would leave one hydroxide ion unneutralised per formula unit of barium hydroxide. The solution is aqueous, so the calculation concerns neutralisation of both hydroxide ions.

Step 2: Calculate the amount of acid consumed Convert the measured volume before multiplying by the molarity:

$$V_{\text{base}} = 25.0 \times 10^{-3} \text{ L} = 0.0250 \text{ L}.$$

Thus

$$n_{\text{Ba(OH)}_2} = 0.1 \times 0.0250 = 0.00250 \text{ mol},$$

and the balanced reaction gives

$$n_{\text{HCl}} = 2(0.00250) = 0.00500 \text{ mol}.$$

The given molar mass of the base is unnecessary because its molarity already specifies moles per litre; it is not a mass concentration.

Step 3: Convert the mass to the requested numerical entry With the given acid molar mass,

$$m_{\text{HCl}} = 0.00500 \times 36.5 = 0.1825 \text{ g} = 182.5 \text{ mg}.$$

Since the original mass is x mg, its numerical value is $x = 182.5$. The blank multiplies 10^{-1} , so if the blank is N ,

$$x = N \times 10^{-1}, \quad N = 10x = 1825.$$

This scaling is essential: entering 183 would round the mass itself instead of the number requested in the blank.

Step 4: Conclusion The acid mass is $182.5 \text{ mg} = 1825 \times 10^{-1} \text{ mg}$. Hence the nearest integer to enter in the blank is 1825.

Quick Tip to Solve: Count the two hydroxide groups in barium hydroxide before using the titration ratio, and apply the printed power-of-ten factor only after finding the mass in milligrams.

75. Consider all the structural isomers with molecular formula $\text{C}_5\text{H}_{11}\text{Br}$ are separately treated with KOH(aq) to give respective substitution products, without any rearrangement. The number of products which can exhibit optical isomerism from these is ____. [4 marks]

Solution:

Concept:

- Aqueous KOH substitutes the bromine atom of an alkyl bromide by an OH group. With rearrangement excluded, the carbon skeleton and the position of substitution remain unchanged.
- For these saturated open-chain alcohols, optical isomerism requires a tetrahedral carbon bonded to four different groups. Count the distinct constitutional products capable of enantiomerism, not the two enantiomers of each such product separately.

Step 1: Enumerate the products on every carbon skeleton The five-carbon skeletons are pentane, 2-methylbutane and 2,2-dimethylpropane. Replacing bromine with hydroxyl at inequivalent positions gives eight distinct alcohols. Equivalent positions related by symmetry must not be counted again.

Carbon skeleton	Distinct alcohol products
Pentane	Pentan-1-ol, pentan-2-ol, pentan-3-ol
2-Methylbutane	2-Methylbutan-1-ol, 3-methylbutan-1-ol 3-Methylbutan-2-ol, 2-methylbutan-2-ol
2,2-Dimethylpropane	2,2-Dimethylpropan-1-ol

Step 2: Identify the products with four different substituents Pentan-2-ol has a stereogenic C_2 bonded to OH , H , methyl and propyl groups. In 2-methylbutan-1-ol, C_2 is bonded to H , methyl, ethyl and CH_2OH ; the stereogenic carbon need not be the carbon bearing hydroxyl. In 3-methylbutan-2-ol, C_2 is bonded to OH , H , methyl and isopropyl groups. Each of these three products can therefore exist as a pair of enantiomers.

Step 3: Exclude the remaining products Pentan-1-ol has no carbon bearing four different groups. Pentan-3-ol has two identical ethyl groups on its hydroxyl-bearing carbon. In 3-methylbutan-1-ol, the branched carbon has two methyl groups, so it is not stereogenic. The hydroxyl-bearing carbon of 2-methylbutan-2-ol also has two methyl groups. Finally, 2,2-dimethylpropan-1-ol has three identical methyl groups on its central carbon and two hydrogens on the hydroxyl-bearing carbon. None of

these five products exhibits optical isomerism.

Step 4: Conclusion Pentan-2-ol, 2-methylbutan-1-ol and 3-methylbutan-2-ol are the products capable of optical isomerism. The requested number of products is $\boxed{3}$, a dimensionless count.

Quick Tip to Solve: Check every tetrahedral carbon, including carbons away from the hydroxyl group. Count chiral constitutional products once each, rather than doubling the count for their enantiomers.