JEE Main 2026 Paper 1 (B.E./B.Tech) Jan 23 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2026, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 23 January 2026, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The system of linear equations

$$x + y + z = 6$$
$$2x + 5y + az = 36$$
$$x + 2y + 3z = b$$

has : [4 marks]

(A) unique solution for $a = 8$ and $b = 14$

(B) unique solution for $a = 8$ and $b = 16$

(C) infinitely many solutions for $a = 8$ and $b = 14$

(D) infinitely many solutions for $a = 8$ and $b = 16$

Correct Answer: (C) infinitely many solutions for $a = 8$ and $b = 14$

Solution:

Concept:

- A linear system has a unique solution when its coefficient determinant is nonzero. If the determinant vanishes, consistency must be checked separately.
- Elementary elimination preserves the solution set. A redundant equation leaves a free variable, whereas contradictory equations give no solution.

Step 1: Eliminate the first variable All four options specify $a = 8$. Subtract twice the first equation from the second, and subtract the first equation from the third. This gives

$$3y + 6z = 24, \qquad y + 2z = b - 6.$$

The first reduced equation is equivalent to $y + 2z = 8$, so the two reduced left sides are dependent.

Step 2: Test consistency for each value of b For $b = 14$, the third equation also gives $y + 2z = 8$; hence it imposes no further restriction. For $b = 16$, it gives $y + 2z = 10$, contradicting the second equation. Thus $b = 16$ gives no solution, ruling out both (B) and (D).

Step 3: Exhibit the family of solutions For $b = 14$, set $z = t$, where t is any real number. Then

$$y = 8 - 2t, \qquad x = 6 - y - z = t - 2.$$

These values satisfy all three original equations. There is one free parameter, so there are infinitely many solutions, not the unique solution asserted in (A).

Step 4: Conclusion For $a = 8$ and $b = 14$, the solutions are $(x, y, z) = (t - 2, 8 - 2t, t)$, $t \in \mathbb{R}$. Hence, the correct option is (C).

Quick Tip to Solve: When the determinant vanishes, compare the reduced right sides before deciding between no solution and infinitely many solutions.

2. If the mean and the variance of the data

Class	$4-8$	$8-12$	$12-16$	$16-20$
Frequency	3	λ	4	7

are μ and 19 respectively, then the value of $\lambda+\mu$ is : [4 marks]

(A) 18

(B) 19

(C) 20

(D) 21

Correct Answer: (B) 19

Solution:

Concept:

- Use the class midpoints as the representative observations of grouped data: $6, 10, 14, 18$.
- For total frequency N, the mean is $\mu = \sum fx/N$, and the variance is $\sigma^2 = \sum fx^2/N - \mu^2$. Frequencies cannot be negative.

Step 1: Calculate the weighted sums The total frequency and the two weighted sums are

$$N = 14+\lambda, \qquad \sum fx = 200+10\lambda,$$

$$\sum fx^2 = 3(36)+100\lambda+4(196)+7(324) = 3160+100\lambda.$$

Thus $\mu = (200+10\lambda)/(14+\lambda)$.

Step 2: Apply the stated variance Substitute the sums into the variance formula:

$$\frac{3160+100\lambda}{14+\lambda} - \left(\frac{200+10\lambda}{14+\lambda}\right)^2 = 19.$$

Multiplying by $(14+\lambda)^2$ and collecting terms gives

$$19\lambda^2 - 28\lambda - 516 = 0,$$

$$(\lambda-6)(19\lambda+86) = 0.$$

The root $-86/19$ is inadmissible because it is a negative frequency, so $\lambda = 6$.

Step 3: Evaluate and check the mean With $\lambda = 6$, we obtain

$$N = 20, \qquad \mu = \frac{260}{20} = 13.$$

Also $\sum fx^2/N = 3760/20 = 188$, so the variance is $188 - 13^2 = 19$, as required. Therefore the requested sum is $6+13 = 19$; the alternatives 18, 20 and 21 do not equal this sum.

Step 4: Conclusion $\lambda = 6$, $\mu = 13$, and $\lambda+\mu = 19$. Hence, the correct option is (B).

Quick Tip to Solve: For grouped data, compute both weighted sums from class midpoints; reject any algebraic root that makes a frequency negative.

3. Let $\mathrm{I}(x)=\int \frac{3dx}{(4x+6)\left(\sqrt{4x^2+8x+3}\right)}$ **and** $\mathrm{I}(0)=\frac{\sqrt{3}}{4}+20$**. If**

$\mathrm{I}\left(\frac{1}{2}\right)=\frac{a\sqrt{2}}{b}+\mathrm{c}$**, where** $a,b,\mathrm{c}\in\mathrm{N},\gcd(a,b)=1$**, then** $a+b+c$ **is equal to :** [4 marks]

(A) 28

(B) 29

(C) 30

(D) 31

Correct Answer: (D) 31

Solution:

Concept:

- Factor the quadratic under the radical: $4x^2+8x+3=(2x+1)(2x+3)$. The interval from 0 to $1/2$ lies wholly in the real domain.
- When the denominator contains one linear factor times the square root of two linear factors, try the square root of their ratio as a substitution.

Step 1: Choose a substitution adapted to the denominator Put

$$t=\sqrt{\frac{2x+1}{2x+3}}.$$

Logarithmic differentiation gives

$$\frac{t'}{t}=\frac{1}{2x+1}-\frac{1}{2x+3},\qquad t'=\frac{2}{(2x+3)\sqrt{(2x+1)(2x+3)}}.$$

Since $4x+6=2(2x+3)$, the printed integrand is exactly $3t'/4$.

Step 2: Integrate and determine the constant Consequently,

$$I(x)=\frac{3}{4}\sqrt{\frac{2x+1}{2x+3}}+C.$$

At $x=0$, its radical term is $3/(4\sqrt{3})=\sqrt{3}/4$. Comparing with the given $I(0)=\sqrt{3}/4+20$ yields $C=20$. Differentiating the expression above recovers the original integrand, verifying the coefficient.

Step 3: Substitute the requested argument At $x=1/2$,

$$I\left(\frac{1}{2}\right)=\frac{3}{4}\sqrt{\frac{2}{4}}+20=\frac{3\sqrt{2}}{8}+20.$$

Thus $a=3$, $b=8$, and $c=20$, with $\gcd(3,8)=1$. Their sum is 31, so 28, 29 and 30 are excluded.

Step 4: Conclusion $a+b+c=3+8+20=31$. Hence, the correct option is (D).

Quick Tip to Solve: After a radical-ratio substitution, verify its derivative explicitly; a missing factor of two changes the requested numerator and denominator.

4. An equilateral triangle OAB is inscribed in the parabola $y^2 = 4x$ with the vertex O at the vertex of the parabola. Then the minimum distance of the circle having AB as a diameter from the origin is [4 marks]

(A) $2(8 - 3\sqrt{3})$

(B) $4(6 + \sqrt{3})$

(C) $2(3 + \sqrt{3})$

(D) $4(3 - \sqrt{3})$

Correct Answer: (D) $4(3 - \sqrt{3})$

Solution:

Concept:

- An equilateral triangle has equal distances $OA = OB = AB$. On $y^2 = 4x$, a point has squared distance $x^2 + 4x$ from the origin.
- For a circle of centre C and radius r, an external point O has minimum distance $OC - r$ from the circumference.

Step 1: Establish the symmetry of the two vertices Write $A = (u, v)$ and $B = (s, t)$, with $u, s \geq 0$. As $OA = OB$,

$$u^2 + 4u = s^2 + 4s, \qquad (u - s)(u + s + 4) = 0.$$

The second factor is positive, hence $u = s$. Since the vertices are distinct and lie on the parabola, they have opposite ordinates. We may therefore write $A = (u, v)$, $B = (u, -v)$, with $v > 0$.

Step 2: Use equality of the side lengths Now $AB = 2v$, whereas $OA^2 = u^2 + v^2$. The equilateral condition gives

$$u^2 + v^2 = 4v^2, \qquad u^2 = 3v^2.$$

Also $v^2 = 4u$, so $u^2 = 12u$. The root $u = 0$ would collapse the triangle; thus $u = 12$ and $v = 4\sqrt{3}$.

Step 3: Find the nearest point of the circle The circle with diameter AB has centre $(12, 0)$ and radius $4\sqrt{3}$. Because $12 > 4\sqrt{3}$, the origin is outside it. Its nearest point lies along the line from the centre toward the origin, giving distance

$$12 - 4\sqrt{3} = 4(3 - \sqrt{3}).$$

This is about 5.07; none of the other three expressions has this value.

Step 4: Conclusion The minimum distance is $4(3 - \sqrt{3})$. Hence, the correct option is (D).

Quick Tip to Solve: Prove the symmetry from equal distances to the parabola vertex; then the diameter circle has its centre directly on the axis.

5. The sum of all the real solutions of the equation

$$\log_{(x+3)}\left(6x^2+28x+30\right)=5-2\log_{(6x+10)}\left(x^2+6x+9\right)$$

is equal to : [4 marks]

(A) 0

(B) 1

(C) 2

(D) 4

Correct Answer: (A) 0

Solution:

Concept:

- Logarithm bases must be positive and different from 1, and arguments must be positive. Change of base gives $\log_a b \, \log_b a = 1$.
- Factor the argument as $6x^2+28x+30=(x+3)(6x+10)$, and note that $x^2+6x+9=(x+3)^2$.

Step 1: State the domain and simplify the logarithms Put $t=x+3$ and $s=6x+10=6t-8$. The domain requires $t>0$, $t\neq 1$, $s>0$, and $s\neq 1$. In particular, $x>-5/3$ and $x\neq -3/2$; these also ensure the two arguments are positive. Let $u=\log_t s$. Since $s\neq 1$, we have $u\neq 0$. Then

$$\log_t(ts)=1+u, \qquad \log_s(t^2)=\frac{2}{u}.$$

Step 2: Solve the transformed equation The original equation becomes

$$1+u=5-\frac{4}{u}.$$

Multiplying by the nonzero u,

$$u^2-4u+4=0, \qquad (u-2)^2=0.$$

Thus $\log_t s=2$, which is equivalent to $s=t^2$. Substituting $s=6t-8$ gives

$$t^2-6t+8=(t-2)(t-4)=0.$$

Step 3: Check both roots in the original domain For $t=2,4$, respectively, $x=-1,1$ and $s=4,16$. Both bases are admissible, and in each case $s=t^2$, making both sides of the original equation equal to 3. The complete set of roots is therefore $\{-1,1\}$, whose sum is zero. The other listed sums 1, 2 and 4 are not the sum of all roots.

Step 4: Conclusion The sum of all real solutions is $-1+1=0$. Hence, the correct option is (A).

Quick Tip to Solve: Look for one logarithm argument that is a product of the two bases; a reciprocal-log substitution can reduce the equation to a quadratic.

6. The least value of $\left(\cos^2\theta - 6\sin\theta\cos\theta + 3\sin^2\theta + 2\right)$ is [4 marks]

(A) $4+\sqrt{10}$

(B) $4-\sqrt{10}$

(C) -1

(D) 1

Correct Answer: (B) $4-\sqrt{10}$

Solution:

Concept:

- The identities $\cos^2\theta = (1+\cos 2\theta)/2$, $\sin^2\theta = (1-\cos 2\theta)/2$ and $2\sin\theta\cos\theta = \sin 2\theta$ convert this quadratic expression to a single-angle combination.
- For real A, B, the range of $A\cos t + B\sin t$ is $[-\sqrt{A^2+B^2}, \sqrt{A^2+B^2}]$. Its endpoints are attained.

Step 1: Express the function in terms of twice the angle Let the given expression be $F(\theta)$. Combining the squared terms gives

$$\cos^2\theta + 3\sin^2\theta = 2 - \cos 2\theta.$$

The cross term is $-6\sin\theta\cos\theta = -3\sin 2\theta$. Therefore

$$F(\theta) = 4 - \cos 2\theta - 3\sin 2\theta.$$

The added constant 2 in the question is included in the constant 4 here.

Step 2: Maximise the subtracted combination By the Cauchy–Schwarz inequality,

$$\cos 2\theta + 3\sin 2\theta \le \sqrt{1^2+3^2}\sqrt{\cos^2 2\theta + \sin^2 2\theta} = \sqrt{10}.$$

Equality is possible when $\cos 2\theta = 1/\sqrt{10}$ and $\sin 2\theta = 3/\sqrt{10}$, since these squares sum to 1. Hence the lower bound $4-\sqrt{10}$ is achieved.

Step 3: Compare the alternatives The full range is $[4-\sqrt{10}, 4+\sqrt{10}]$. Thus $4+\sqrt{10}$ is the maximum, not the minimum. The value -1 lies below the range, while 1 is greater than the attained minimum $4-\sqrt{10} \approx 0.838$. Neither can be the least value.

Step 4: Conclusion The least value is $4-\sqrt{10}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert quadratic sine-cosine expressions to double angles, then use the amplitude; remember to prove the bound can be attained.

7. Let $\mathrm{A} = \{0, 1, 2, \ldots, 9\}$. Let R be a relation on A defined by $(x, y) \in \mathrm{R}$ if and only if $|x - y|$ is a multiple of 3.

Given below are two statements :

Statement I : $n(\mathrm{R}) = 36$.

Statement II : R is an equivalence relation.

In the light of the above statements, choose the *correct* answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are correct

(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (D) Statement I is incorrect but Statement II is correct

Solution:

Concept:

- Two integers differ by a multiple of 3 exactly when they have the same remainder on division by 3. Thus the relation partitions the set into residue classes.
- A relation is an equivalence relation when it is reflexive, symmetric and transitive. A class containing k elements contributes k^2 ordered pairs.

Step 1: Count the ordered pairs The three classes are $C_0 = \{0, 3, 6, 9\}$, $C_1 = \{1, 4, 7\}$ and $C_2 = \{2, 5, 8\}$. Every element is related to every element in its own class, including itself, and to no element outside that class. Therefore

$$n(R) = 4^2 + 3^2 + 3^2 = 16 + 9 + 9 = 34.$$

The pairs are ordered, so both $(0, 3)$ and $(3, 0)$ count. Statement I, which gives 36, is false.

Step 2: Check the three defining properties For every $x \in A$, $|x - x| = 0 = 3 \cdot 0$, so R is reflexive. Since $|x - y| = |y - x|$, it is symmetric. If xRy and yRz, then $x - y$ and $y - z$ are integer multiples of 3. Their sum $x - z$ is also a multiple of 3, hence so is $|x - z|$. Thus R is transitive. Statement II is true.

Step 3: Select the matching truth combination Options (A) and (C) require Statement I to be true, contradicting the count 34. Option (B) requires Statement II to be false, contradicting all three verified properties. Only (D) has false I and true II.

Step 4: Conclusion $n(R) = 34$, and R is an equivalence relation. Hence, the correct option is (D).

Quick Tip to Solve: For a congruence relation, list the residue-class sizes and add their squares; do not count only unordered pairs.

8. The area of the region enclosed between the circles $x^2 + y^2 = 4$ and $x^2 + (y-2)^2 = 4$ is: [4 marks]

(A) $\frac{4}{3}(2\pi - 3\sqrt{3})$

(B) $\frac{2}{3}(4\pi - 3\sqrt{3})$

(C) $\frac{2}{3}(2\pi - 3\sqrt{3})$

(D) $\frac{4}{3}(2\pi - \sqrt{3})$

Correct Answer: (B) $\frac{2}{3}(4\pi - 3\sqrt{3})$

Solution:

Concept:

- The circles have radius 2, centres $O_1 = (0,0)$ and $O_2 = (0,2)$, and centre separation 2. Their enclosed common region is a symmetric lens.
- A circular segment equals its sector area minus its triangle area. For radius r and central angle θ in radians, these areas are $r^2\theta/2$ and $r^2 \sin\theta/2$.

Step 1: Find the common chord Subtracting the circle equations gives $-4y + 4 = 0$, hence $y = 1$. Substitution into $x^2 + y^2 = 4$ gives the intersection points $(\pm\sqrt{3}, 1)$. The common chord is therefore one unit from each centre. Each half of the sector angle satisfies $\cos\alpha = 1/2$, so $\alpha = \pi/3$. The full angle is $\theta = 2\pi/3$.

Step 2: Calculate both congruent segments The segment belonging to the first circle has area

$$\frac{1}{2}(2)^2\frac{2\pi}{3} - \frac{1}{2}(2)^2 \sin\frac{2\pi}{3} = \frac{4\pi}{3} - \sqrt{3}.$$

The other segment has the same area by reflection in $y = 1$. Consequently the total lens area is

$$A = 2\left(\frac{4\pi}{3} - \sqrt{3}\right) = \frac{2}{3}(4\pi - 3\sqrt{3}).$$

Step 3: Compare the listed expressions Option (A) subtracts $4\sqrt{3}$ instead of $2\sqrt{3}$; option (C) halves the required sector contribution; option (D) subtracts only $4\sqrt{3}/3$. None equals the two-segment result. Option (B) matches both contributions.

Step 4: Conclusion The area is $\frac{2}{3}(4\pi - 3\sqrt{3})$ square units. Hence, the correct option is (B).

Quick Tip to Solve: For equal intersecting circles, find one segment from the common chord and double it; use radians in the sector formula.

9. Bag A contains 9 white and 8 black balls, while bag B contains 6 white and 4 black balls. One ball is randomly picked up from the bag B and mixed up with the balls in the bag A . Then a ball is randomly drawn from the bag A . If the probability, that the ball drawn is white, is $\frac{p}{q}, \gcd(p, q) = 1$, then $p + q$ is equal to [4 marks]

(A) 23

(B) 22

(C) 21

(D) 24

Correct Answer: (A) 23

Solution:

Concept:

- Condition on the colour of the transferred ball. These two cases are mutually exclusive and exhaustive, so their contributions add by the law of total probability.
- Bag A has 18 balls after the transfer, regardless of the transferred colour. Its white-ball count is 10 after a white transfer and 9 after a black transfer.

Step 1: Write the probabilities of the transfer Let T_W and T_B denote a white or black transfer from bag B, and let W denote a white final draw. Since bag B has 6 white and 4 black balls,

$$P(T_W) = \frac{6}{10} = \frac{3}{5}, \qquad P(T_B) = \frac{4}{10} = \frac{2}{5}.$$

The transfer is made before the draw from A, so the original denominator 17 for A must be replaced by 18.

Step 2: Combine the conditional probabilities After T_W, bag A contains 10 white and 8 black balls, giving $P(W \mid T_W) = 10/18$. After T_B, it contains 9 white and 9 black balls, giving $P(W \mid T_B) = 9/18$. Hence

$$P(W) = \frac{3}{5}\frac{10}{18} + \frac{2}{5}\frac{9}{18} = \frac{30 + 18}{90} = \frac{8}{15}.$$

Step 3: Reduce the fraction and choose the sum The integers 8 and 15 share no prime factor, so this is already in lowest terms: $p = 8$ and $q = 15$. Thus $p + q = 23$. Option (B), 22, option (C), 21, and option (D), 24, each differs from this reduced-fraction sum; no further cancellation is possible.

Step 4: Conclusion $P(W) = 8/15$ and $p + q = 23$. Hence, the correct option is (A).

Quick Tip to Solve: After a one-ball transfer, use the new total in the receiving bag for both conditional probabilities, then reduce the final fraction.

10. If the points of intersection of the ellipses $x^2 + 2y^2 - 6x - 12y + 23 = 0$ and $4x^2 + 2y^2 - 20x - 12y + 35 = 0$ lie on a circle of radius r and centre (a, b), then the value of $ab + 18r^2$ is : [4 marks]

(A) 51

(B) 55

(C) 53

(D) 52

Correct Answer: (B) 55

Solution:

Concept:

- If $F_1 = 0$ and $F_2 = 0$ are two curves, every common point satisfies $F_2 + \lambda F_1 = 0$. Choose λ so that the coefficients of x^2 and y^2 agree.
- For $x^2 + y^2 + 2gx + 2fy + c = 0$, the centre is $(-g, -f)$ and the squared radius is $g^2 + f^2 - c$.

Step 1: Form a circle through the common points Write the first ellipse equation as $F_1 = 0$ and the second as $F_2 = 0$. In $F_2 + \lambda F_1$, the squared-term coefficients are $4 + \lambda$ and $2 + 2\lambda$. Equating them gives $\lambda = 2$. Thus every intersection lies on

$$6x^2 + 6y^2 - 32x - 36y + 81 = 0.$$

There is no mixed xy term, so equal nonzero squared coefficients give a circle.

Step 2: Read the centre and radius Dividing by 6 gives

$$x^2 + y^2 - \frac{16}{3}x - 6y + \frac{27}{2} = 0.$$

Completing squares,

$$\left(x - \frac{8}{3}\right)^2 + (y - 3)^2 = \frac{64}{9} + 9 - \frac{27}{2} = \frac{47}{18}.$$

Therefore $a = 8/3$, $b = 3$, and $r^2 = 47/18$.

Step 3: Evaluate the requested expression Substitution now yields

$$ab + 18r^2 = \frac{8}{3} \cdot 3 + 18 \cdot \frac{47}{18} = 8 + 47 = 55.$$

Option (A), 51, option (C), 53, and option (D), 52, do not equal this result. In particular, the expression requires the squared radius, so taking a square root is unnecessary.

Step 4: Conclusion $ab + 18r^2 = 55$. Hence, the correct option is (B).

Quick Tip to Solve: To find a circle through conic intersections, combine the equations to equalise the squared-term coefficients before solving for any intersection coordinates.

11. If $f(x) = \begin{cases} \frac{a|x|+x^2-2(\sin|x|)(\cos|x|)}{x} & , x \neq 0 \\ b & , x = 0 \end{cases}$ **is continuous at** $x = 0$**, then** $a + b$ **is equal to :** [4 marks]

(A) 0

(B) 1

(C) 2

(D) 4

Correct Answer: (C) 2

Solution:

Concept:

- Continuity at zero requires both one-sided limits to exist, agree with each other, and equal the assigned value $f(0) = b$.
- Use $2\sin|x|\cos|x| = \sin(2|x|)$, $\sin t/t \to 1$, and the different signs of $|x|/x$ on the two sides of zero.

Step 1: Calculate the right-hand limit For $x > 0$, $|x| = x$, and hence

$$f(x) = a + x - \frac{\sin(2x)}{x}.$$

Since $\sin(2x)/x = 2\sin(2x)/(2x) \to 2$,

$$\lim_{x\to 0^+} f(x) = a - 2.$$

The x^2/x term tends to zero, so it contributes nothing to this limit.

Step 2: Calculate the left-hand limit For $x < 0$, $|x| = -x$. Then $\sin(2|x|) = \sin(-2x) = -\sin(2x)$, giving

$$f(x) = -a + x + \frac{\sin(2x)}{x}.$$

Therefore

$$\lim_{x\to 0^-} f(x) = 2 - a.$$

The change of sign affects both the absolute-value term and the sine term; overlooking either gives an incorrect condition.

Step 3: Impose continuity and check the choices Equating the limits gives $a - 2 = 2 - a$, so $a = 2$. Their common value is zero, which must equal b. Thus $b = 0$ and $a + b = 2$. The values 0 in (A), 1 in (B), and 4 in (D) cannot be the sum of these uniquely determined parameters. The matching choice is (C).

Step 4: Conclusion $a = 2$, $b = 0$, and $a + b = 2$. Hence, the correct option is (C).

Quick Tip to Solve: In a continuity problem containing $|x|/x$, compute the two one-sided expressions before using the standard sine limit.

12. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $\vec{a} \times \vec{b} = 2(\vec{a} \times \vec{c})$. If $|\vec{a}| = 1, |\vec{b}| = 4, |\vec{c}| = 2$, and the angle between $\vec{b}$ and $\vec{c}$ is $60°$, then $|\vec{a} \cdot \vec{c}|$ is equal to [4 marks]

(A) 0

(B) 1

(C) 2

(D) 4

Correct Answer: (B) 1

Solution:

Concept:

- The equation $\vec{a} \times (\vec{b} - 2\vec{c}) = \vec{0}$ implies that the nonzero vector $\vec{b} - 2\vec{c}$ is parallel to $\vec{a}$.
- The scalar product is $\vec{b} \cdot \vec{c} = |\vec{b}||\vec{c}| \cos 60°$. A unit vector parallel to a nonzero vector $\vec{d}$ is $\pm\vec{d}/|\vec{d}|$.

Step 1: Compute the scalar product and the parallel vector From the given lengths and angle,

$$\vec{b} \cdot \vec{c} = 4 \cdot 2 \cdot \frac{1}{2} = 4.$$

Set $\vec{d} = \vec{b} - 2\vec{c}$. Bilinearity of the cross product converts the stated equality to $\vec{a} \times \vec{d} = \vec{0}$. Its squared length is

$$|\vec{d}|^2 = |\vec{b}|^2 + 4|\vec{c}|^2 - 4\vec{b} \cdot \vec{c} = 16 + 16 - 16 = 16.$$

Thus $\vec{d} \neq \vec{0}$ and $|\vec{d}| = 4$.

Step 2: Use the unit length of the first vector Since $|\vec{a}| = 1$, parallelism gives $\vec{a} = \pm\vec{d}/4$. Taking the scalar product with $\vec{c}$,

$$\vec{a} \cdot \vec{c} = \pm\frac{\vec{b} \cdot \vec{c} - 2|\vec{c}|^2}{4} = \pm\frac{4 - 8}{4} = \mp 1.$$

The two directions of $\vec{a}$ change the sign but leave the requested absolute value unchanged.

Step 3: Check the answer against the options The magnitude is exactly 1. Option (A), 0, would require $\vec{d} \cdot \vec{c} = 0$, but that dot product is -4. Option (C), 2, would require complete alignment of $\vec{a}$ with $\vec{c}$, contrary to the computed projection. Option (D), 4, even exceeds $|\vec{a}||\vec{c}| = 2$.

Step 4: Conclusion $|\vec{a} \cdot \vec{c}| = 1$. Hence, the correct option is (B).

Quick Tip to Solve: Move cross-product terms to one side to reveal a parallel vector, then normalise it before calculating the requested projection.

13. Let $\vec{a} = \hat{\mathrm{i}} - 2\hat{\mathrm{j}} + 3\hat{\mathrm{k}}, \vec{b} = 2\hat{\mathrm{i}} + \hat{\mathrm{j}} - \hat{\mathrm{k}}, \vec{c} = \lambda\hat{\mathrm{i}} + \hat{\mathrm{j}} + \hat{\mathrm{k}}$ and $\vec{v} = \vec{a} \times \vec{b}$. If $\vec{v} \cdot \vec{c} = 11$ and the length of the projection of $\vec{b}$ on $\vec{c}$ is p, then $9p^2$ is equal to [4 marks]

(A) 4

(B) 6

(C) 9

(D) 12

Correct Answer: (D) 12

Solution:

Concept:

- The cross product gives a vector perpendicular to both given vectors. Its components are obtained by expanding the determinant in the order $\vec{a} \times \vec{b}$.
- The length of the projection of $\vec{b}$ on a nonzero vector $\vec{c}$ is $p = |\vec{b} \cdot \vec{c}|/|\vec{c}|$. The absolute value ensures a nonnegative length.

Step 1: Calculate the cross product Using the components in their stated order,

$$\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 1 & -1 \end{vmatrix} = -\hat{i} + 7\hat{j} + 5\hat{k}.$$

For example, its second component is $3(2) - 1(-1) = 7$; this is where a sign error in determinant expansion is easy to make.

Step 2: Determine the parameter Substitute $\vec{c} = (\lambda, 1, 1)$ in the given scalar-product condition:

$$\vec{v} \cdot \vec{c} = -\lambda + 7 + 5 = 11.$$

Thus $\lambda = 1$, so $\vec{c} = \hat{i} + \hat{j} + \hat{k}$ and $|\vec{c}| = \sqrt{3}$. The vector onto which we project is therefore nonzero.

Step 3: Find the projection and compare the choices The numerator is $\vec{b} \cdot \vec{c} = 2 + 1 - 1 = 2$. Consequently,

$$p = \frac{2}{\sqrt{3}}, \qquad p^2 = \frac{4}{3}, \qquad 9p^2 = 12.$$

The other proposed values, $4, 6, 9$, do not equal the scaled square of this projection. In particular, the scalar product itself is not the projection length: division by $|\vec{c}|$ is essential.

Step 4: Conclusion The required value is $9p^2 = 12$. Hence, the correct option is (D).

Quick Tip to Solve: Find the parameter from the scalar triple product first; then square the projection formula to avoid unnecessary radicals.

14. Let PQ be a chord of the hyperbola $\frac{x^2}{4} - \frac{y^2}{b^2} = 1$, perpendicular to the x -axis such that OPQ is an equilateral triangle, O being the centre of the hyperbola. If the eccentricity of the hyperbola is $\sqrt{3}$, then the area of the triangle OPQ is [4 marks]

(A) $\frac{8\sqrt{3}}{5}$

(B) $2\sqrt{3}$

(C) $\frac{9}{5}$

(D) $\frac{11}{5}$

Correct Answer: (A) $\frac{8\sqrt{3}}{5}$

Solution:

Concept:

- For $x^2/a^2 - y^2/b^2 = 1$, the eccentricity satisfies $e^2 = 1 + b^2/a^2$. Here the transverse axis is the x-axis and $a^2 = 4$.
- A vertical chord has endpoints (h, k) and $(h, -k)$. An equilateral triangle with these endpoints and the origin has equal lengths $2|k|$ and $\sqrt{h^2 + k^2}$.

Step 1: Use the eccentricity to determine the hyperbola Since $e = \sqrt{3}$,

$$3 = 1 + \frac{b^2}{4} \quad \Longrightarrow \quad b^2 = 8.$$

The equation is therefore $x^2/4 - y^2/8 = 1$. A nondegenerate vertical chord has two endpoints symmetric about the x-axis, so write them as $P(h, k)$ and $Q(h, -k)$, with $k \neq 0$.

Step 2: Impose the equilateral condition The side PQ has length $2|k|$, while OP has length $\sqrt{h^2 + k^2}$. Equating their squares gives

$$h^2 + k^2 = 4k^2 \quad \Longrightarrow \quad h^2 = 3k^2.$$

Because P is on the hyperbola,

$$\frac{3k^2}{4} - \frac{k^2}{8} = 1 \quad \Longrightarrow \quad \frac{5k^2}{8} = 1, \qquad k^2 = \frac{8}{5}.$$

Step 3: Calculate the area Take PQ as the base. Its length is $2|k|$, and the perpendicular distance from the origin to its supporting line $x = h$ is $|h|$. Hence

$$\text{Area} = \frac{1}{2}(2|k|)|h| = |hk| = \sqrt{3}\,k^2 = \frac{8\sqrt{3}}{5}.$$

The choices $2\sqrt{3}$, $9/5$, and $11/5$ differ from this value. The reflected chord on the other branch gives the same positive area.

Step 4: Conclusion The area of $\triangle OPQ$ is $8\sqrt{3}/5$ square units. Hence, the correct option is (A).

Quick Tip to Solve: For an equilateral triangle formed by the origin and a vertical symmetric chord, use $h^2 = 3k^2$ before substituting in the conic.

15. Let $\frac{\pi}{2} < \theta < \pi$ and $\cot\theta = -\frac{1}{2\sqrt{2}}$. Then the value of

$$\sin\left(\frac{15\theta}{2}\right)(\cos 8\theta + \sin 8\theta) + \cos\left(\frac{15\theta}{2}\right)(\cos 8\theta - \sin 8\theta)$$

is equal to : [4 marks]

(A) $\frac{\sqrt{2}-1}{\sqrt{3}}$

(B) $\frac{1-\sqrt{2}}{\sqrt{3}}$

(C) $\frac{\sqrt{2}}{\sqrt{3}}$

(D) $-\frac{\sqrt{2}}{\sqrt{3}}$

Correct Answer: (B) $\frac{1-\sqrt{2}}{\sqrt{3}}$

Solution:

Concept:

- Use the sine-difference and cosine-difference identities to combine the four products into functions of $u - v$.
- The quadrant fixes the signs of sine and cosine. Half-angle square roots must likewise use the quadrant of $\theta/2$, not arbitrary signs.

Step 1: Combine the expression before evaluating angles Put $u = 15\theta/2$ and $v = 8\theta$. Group the four terms as

$$\begin{aligned} E &= (\sin u\cos v - \cos u\sin v) \\ &\quad + (\sin u\sin v + \cos u\cos v) \\ &= \sin(u-v) + \cos(u-v). \end{aligned}$$

Since $u - v = -\theta/2$, the expression simplifies to

$$E = \cos(\theta/2) - \sin(\theta/2).$$

This avoids expanding either of the large multiple angles.

Step 2: Find the signed trigonometric ratios The angle lies in the second quadrant, where sine is positive and cosine is negative. From $\cot\theta = -1/(2\sqrt{2})$, a reference triangle has hypotenuse $\sqrt{1+8} = 3$. Thus

$$\cos\theta = -\frac{1}{3}, \qquad \sin\theta = \frac{2\sqrt{2}}{3}.$$

Also $\pi/4 < \theta/2 < \pi/2$, so both half-angle functions are positive.

Step 3: Apply the half-angle formulae Using the positive square roots,

$$\cos(\theta/2) = \sqrt{\frac{1+\cos\theta}{2}} = \frac{1}{\sqrt{3}},$$

$$\sin(\theta/2) = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{2}{3}}.$$

Therefore $E = (1-\sqrt{2})/\sqrt{3}$. The opposite-sign choice reverses the subtraction, while $\pm\sqrt{2/3}$ omit

the cosine half-angle term. Moreover, $E < 0$, consistent with $\theta/2 > \pi/4$.

Step 4: Conclusion The value is $(1 - \sqrt{2})/\sqrt{3}$. Hence, the correct option is (B).

Quick Tip to Solve: When two large angles differ by a simple fraction of the original angle, group products into sum-and-difference identities first.

16. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}, i = \sqrt{-1}$, then $\left(z^{201} - i\right)^8$ is equal to [4 marks]

(A) 0

(B) -1

(C) 1

(D) 256

Correct Answer: (D) 256

Solution:

Concept:

- A complex number of unit modulus can be written as $\cos\phi + i\sin\phi$. De Moivre's theorem multiplies its argument by an integer exponent.
- Powers on the unit circle repeat after an argument change of 2π; reduce large exponents using this period before subtracting other complex numbers.

Step 1: Express the number in trigonometric form The real and imaginary parts satisfy $3/4 + 1/4 = 1$, so $|z| = 1$. Both parts are positive, and

$$z = \frac{\sqrt{3}}{2} + \frac{i}{2} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}.$$

The argument is $\pi/6$, not $\pi/3$: the imaginary component is $\sin(\pi/6) = 1/2$. In particular, $z^{12} = 1$, which supplies a convenient exponent cycle.

Step 2: Reduce the exponent using the cycle Since $201 = 12 \cdot 16 + 9$,

$$z^{201} = (z^{12})^{16} z^9 = z^9.$$

By De Moivre's theorem,

$$z^9 = \cos\frac{9\pi}{6} + i\sin\frac{9\pi}{6} = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = -i.$$

It is important to retain the negative sign: the resulting point lies on the negative imaginary axis, not the positive one.

Step 3: Subtract before taking the final power The bracket becomes $z^{201} - i = -i - i = -2i$, so

$$(z^{201} - i)^8 = (-2i)^8 = 2^8 i^8 = 256.$$

Here $i^8 = (i^4)^2 = 1$. The zero choice would require $z^{201} = i$, which the argument calculation disproves.

The choices -1 and 1 ignore the magnitude factor 2^8: the bracket has modulus 2, so its eighth power has modulus 256.

Step 4: Conclusion The required complex-number expression equals 256. Hence, the correct option is (D).

Quick Tip to Solve: Write a unit complex number in polar form, reduce the exponent modulo its cycle, and only then perform subtraction and the outer power.

17. Consider two sets $\mathrm{A} = \{x \in \mathrm{Z} : |(|x-3|-3)| \le 1\}$ **and** $\mathrm{B} = \left\{x \in \mathbb{R} - \{1,2\} : \frac{(x-2)(x-4)}{x-1} \log_e(|x-2|) = 0\right\}$**. Then the number of onto functions** $f : \mathrm{A} \to \mathrm{B}$ **is equal to :** [4 marks]

(A) 32

(B) 62

(C) 79

(D) 81

Correct Answer: (B) 62

Solution:

Concept:

- An inequality $||x-3|-3| \le 1$ bounds the nonnegative quantity $|x-3|$ between 2 and 4; the restriction $x \in \mathbb{Z}$ then selects integers.
- An onto function must hit every codomain element. From a set with m elements to a two-element set there are $2^m - 2$ onto functions, since only the two constant functions fail.

Step 1: Enumerate the domain Unpack the outer absolute value:

$$-1 \le |x-3| - 3 \le 1 \quad \Longrightarrow \quad 2 \le |x-3| \le 4.$$

Thus $x - 3 \in [-4,-2] \cup [2,4]$, or

$$x \in [-1,1] \cup [5,7].$$

Keeping only integers gives $A = \{-1, 0, 1, 5, 6, 7\}$, so $|A| = 6$. Endpoints are included because the inequality is non-strict.

Step 2: Find the codomain with all exclusions The stated domain excludes $x = 1, 2$, making the denominator and logarithm meaningful. The product is zero if its algebraic numerator vanishes or its logarithm vanishes. The numerator gives $x = 2, 4$, of which only 4 is allowed. The logarithm gives

$$\log_e |x-2| = 0 \quad \Longrightarrow \quad |x-2| = 1,$$

so $x = 1, 3$, of which only 3 is allowed. Both surviving values satisfy the original equation, and therefore $B = \{3, 4\}$.

Step 3: Count functions that use both values Each of the six domain elements has two possible

images, giving $2^6 = 64$ functions in all. Exactly two are not onto: the function constantly equal to 3, and the function constantly equal to 4. Thus

$$N = 2^6 - 2 = 62.$$

The value 32 is too small; 79 and 81 even exceed the total of 64 functions. No ordering or injectivity requirement is imposed on the domain images.

Step 4: Conclusion The number of onto functions is 62. Hence, the correct option is (B).

Quick Tip to Solve: Apply domain exclusions before counting the codomain; for a two-element codomain, subtract just the two constant functions from the total.

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18. Let $\mathrm{A}(1,2)$ and $\mathrm{C}(-3,-6)$ be two diagonally opposite vertices of a rhombus, whose sides AD and BC are parallel to the line $7x - y = 14$. If $\mathrm{B}(\alpha,\beta)$ and $\mathrm{D}(\gamma,\delta)$ are the other two vertices, then $|\alpha+\beta+\gamma+\delta|$ is equal to : [4 marks]

(A) 1

(B) 3

(C) 6

(D) 9

Correct Answer: (C) 6

Solution:

Concept:

- A rhombus is a parallelogram, so its diagonals bisect one another. The midpoint of one diagonal therefore equals the midpoint of the other.
- To determine a sum of coordinates, use midpoint relations directly. Finding the individual remaining vertices is unnecessary when their coordinate sums alone are asked.

Step 1: Identify the common diagonal midpoint The given opposite vertices are $A(1,2)$ and $C(-3,-6)$. Their diagonal has midpoint

$$M = \left(\frac{1+(-3)}{2}, \frac{2+(-6)}{2}\right) = (-1,-2).$$

Because all rhombi are parallelograms, BD must pass through this same midpoint. This property holds irrespective of which pair of parallel sides has the given slope.

Step 2: Relate the unknown coordinate sums Writing $B = (\alpha,\beta)$ and $D = (\gamma,\delta)$, the midpoint of BD is

$$\left(\frac{\alpha+\gamma}{2}, \frac{\beta+\delta}{2}\right).$$

Equating coordinates with $M = (-1,-2)$ yields

$$\alpha+\gamma = -2, \qquad \beta+\delta = -4.$$

These are exactly the two sums needed in the requested expression. The line $7x - y = 14$ specifies a side direction, but it cannot change the common-midpoint equations.

Step 3: Evaluate the absolute value and test the options Add the two coordinate identities before taking the absolute value:

$$\alpha + \beta + \gamma + \delta = (-2) + (-4) = -6,$$

$$|\alpha + \beta + \gamma + \delta| = |-6| = 6.$$

The other choices $1, 3, 9$ violate the coordinate-sum constraint. Taking separate absolute values of individual coordinates would answer a different question; the absolute value here surrounds the whole sum. The result also remains unchanged if the names B and D are interchanged.

Step 4: Conclusion The required absolute value is 6. Hence, the correct option is (C).

Quick Tip to Solve: For opposite vertices of any parallelogram, use $B + D = A + C$ coordinatewise; this often avoids solving for either unknown vertex.

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19. Let $\sum_{k=1}^{n} a_k = \alpha n^2 + \beta n$. If $a_{10} = 59$ and $a_6 = 7a_1$, then $\alpha + \beta$ is equal to : [4 marks]

(A) 3

(B) 5

(C) 7

(D) 12

Correct Answer: (B) 5

Solution:

Concept:

- If S_n is the sum of the first n terms, then $a_n = S_n - S_{n-1}$, with $S_0 = 0$.
- A quadratic partial-sum formula gives an arithmetic progression, because its successive differences are linear in n.

Step 1: Obtain the general term Use the given sum, keeping the same constants in both consecutive partial sums:

$$\begin{aligned} a_n &= \alpha n^2 + \beta n - \{\alpha(n-1)^2 + \beta(n-1)\} \\ &= \alpha(2n-1) + \beta. \end{aligned}$$

This expression also gives $a_1 = \alpha + \beta$, the quantity asked for.

Step 2: Use the relation between the first and sixth terms Substitute $n = 6$ and $n = 1$ in the general term:

$$11\alpha + \beta = 7(\alpha + \beta) \quad \Longrightarrow \quad 4\alpha = 6\beta \quad \Longrightarrow \quad \beta = \frac{2\alpha}{3}.$$

The relation concerns individual terms, so the partial sums must not be substituted directly.

Step 3: Use the tenth term and check the result The other condition gives

$$19\alpha + \beta = 59 \quad \Longrightarrow \quad \frac{59\alpha}{3} = 59.$$

Therefore $\alpha = 3$ and $\beta = 2$. These values give $a_1 = 5$, $a_6 = 35 = 7a_1$, and $a_{10} = 59$, checking both conditions. Among the four offered values, only 5 satisfies them; 3, 7 and 12 are excluded.

Step 4: Conclusion $\alpha + \beta = 3 + 2 = \boxed{5}$. Hence, the correct option is (B).

Quick Tip to Solve: When a sum S_n is given, first take $S_n - S_{n-1}$; substituting n into S_n does not give the individual term.

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20. The number of ways, in which 16 oranges can be distributed to four children such that each child gets at least one orange, is [4 marks]

(A) 384

(B) 403

(C) 455

(D) 429

Correct Answer: (C) 455

Solution:

Concept:

- A distribution of identical oranges among four distinct children is specified by the ordered numbers (x_1, x_2, x_3, x_4) that they receive.
- The number of positive integer solutions of $x_1 + \cdots + x_r = N$ is $\binom{N-1}{r-1}$; requiring each recipient to receive at least one excludes empty groups.

Step 1: Translate the condition into positive integers Let x_i be the number of oranges received by child i. Then

$$x_1 + x_2 + x_3 + x_4 = 16, \qquad x_i \geq 1.$$

Each ordered solution corresponds to exactly one allocation of quantities to the four named children. Oranges are counted by quantity; individual oranges are not labelled.

Step 2: Place three separators Arrange the 16 oranges in a row. There are 15 internal gaps. Choose three distinct gaps for separators, forming four nonempty consecutive groups assigned to the four children in their fixed order. Thus

$$N = \binom{15}{3} = \frac{15 \cdot 14 \cdot 13}{3 \cdot 2 \cdot 1} = 455.$$

There is no extra factor of $4!$: the order of the four groups already specifies the recipients.

Step 3: Check the count by removing the compulsory oranges Give one orange to each child first and put $y_i = x_i - 1$. The remaining equation is $y_1 + y_2 + y_3 + y_4 = 12$, with all $y_i \geq 0$. Stars and bars gives $\binom{12+4-1}{4-1} = \binom{15}{3} = 455$ again. Therefore 384, 403 and 429 do not count all the permitted

positive allocations.

Step 4: Conclusion The number of distributions is $\boxed{455}$. Hence, the correct option is (C).

Quick Tip to Solve: For at least one identical object per recipient, choose the separators only from the internal gaps: $\binom{N-1}{r-1}$.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let S denote the set of 4-digit numbers $abcd$ such that $a > b > c > d$ and P denote the set of 5 -digit numbers having product of its digits equal to 20 . Then $n(\mathrm{S}) + n(\mathrm{P})$ is equal to ____ [4 marks]

Solution:

Concept:

- A strictly decreasing digit sequence uses distinct digits; once its digit set is selected, its descending order is unique.
- For a multiset of five digits with multiplicities $r_1, r_2, \ldots$, the number of arrangements is $5!/(r_1!r_2!\cdots)$.

Step 1: Count the decreasing four-digit numbers Choose any four different digits from $\{0, 1, \ldots, 9\}$ and arrange them in descending order. The largest chosen digit is positive, so even a selection containing zero gives a valid four-digit number. Hence

$$n(S) = \binom{10}{4} = 210.$$

No factor of $4!$ is needed because the required inequality fixes the order.

Step 2: List every possible digit multiset with product 20 Since $20 = 2^2 \cdot 5$, zero cannot occur and exactly one digit must be 5. The remaining four digits have product 4. They are either $4, 1, 1, 1$ or $2, 2, 1, 1$. Thus the complete five-digit multisets are

$$\{5, 4, 1, 1, 1\}, \qquad \{5, 2, 2, 1, 1\}.$$

There are no other possibilities: a digit containing a prime factor other than 2 or 5 is excluded, and digits 8 or larger contribute too many factors or a forbidden prime.

Step 3: Arrange the multisets and add the counts All the listed digits are nonzero, so every arrangement is a five-digit number. Taking account of repeated digits gives

$$n(P) = \frac{5!}{3!} + \frac{5!}{2!2!} = 20 + 30 = 50.$$

The two multisets are distinct, so these counts do not overlap. The question asks for the sum of the two cardinalities, which is

$$n(S) + n(P) = 210 + 50 = 260.$$

Step 4: Conclusion The required numerical answer is $\boxed{260}$.

Quick Tip to Solve: For a digit-product condition, factor the target first and list digit multisets before permuting them; this prevents missing repeated-digit cases.

22. If the image of the point $\mathrm{P}(a, 2, a)$ in the line $\frac{x}{2} = \frac{y+a}{1} = \frac{z}{1}$ is Q and the image of Q in the line $\frac{x-2b}{2} = \frac{y-a}{1} = \frac{z+2b}{-5}$ is P , then $a + b$ is equal to ____ . [4 marks]

Solution:

Concept:

- When a point is reflected in a line, the midpoint of the point and its image lies on the line, and their joining segment is perpendicular to that line.
- Both given reflections interchange P and Q, so their common midpoint M lies on both lines and PM is perpendicular to both direction vectors.

Step 1: Parameterise the common midpoint The first line has direction $\mathbf{d}_1 = (2, 1, 1)$, so write

$$M = (2t, t - a, t), \qquad P - M = (a - 2t, a + 2 - t, a - t).$$

Perpendicularity to $\mathbf{d}_1$ gives

$$2(a - 2t) + (a + 2 - t) + (a - t) = 0, \qquad 6t = 4a + 2.$$

Step 2: Use perpendicularity to the second line The second line has direction $\mathbf{d}_2 = (2, 1, -5)$. Therefore

$$\begin{aligned}(P - M) \cdot \mathbf{d}_2 &= 2(a - 2t) + (a + 2 - t) - 5(a - t) \\ &= 2 - 2a = 0.\end{aligned}$$

Thus $a = 1$, and the first perpendicularity equation gives $t = 1$. Consequently $M = (2, 0, 1)$.

Step 3: Locate this midpoint on the second line A point on the second line is

$$(2b + 2s, a + s, -2b - 5s).$$

Its y-coordinate at M yields $0 = 1 + s$, so $s = -1$. The x-coordinate then gives $2 = 2b - 2$, hence $b = 2$. The z-coordinate checks: $-2b - 5s = -4 + 5 = 1$. Finally, $P = (1, 2, 1)$ and $Q = 2M - P = (3, -2, 1)$; their joining direction is perpendicular to both axes, so both reflection conditions are satisfied.

Step 4: Conclusion $a = 1$, $b = 2$, and therefore $a + b = \boxed{3}$.

Quick Tip to Solve: For two axes producing the same reflection pair, use the common midpoint and two perpendicularity conditions instead of expanding two full reflection formulas.

23. Let $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$ and B be a matrix such that $B(I-A) = I+A$. Then the sum of the diagonal elements of $\mathrm{B^TB}$ is equal to ____ [4 marks]

Solution:

Concept:

- The displayed matrix is skew-symmetric: $A^T = -A$. Consequently $(I-A)^T = I+A$ and $(I+A)^T = I-A$.
- The sum of diagonal entries is the trace. If $B^TB = I_3$, this trace is 3, independently of the individual entries of B.

Step 1: Check that the required inverse exists Put $C = I-A$ and $D = I+A$. Direct expansion gives

$$\det C = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ -3 & 1 & 1 \end{vmatrix} = 2 - 2 + 15 = 15 \neq 0.$$

Thus C is invertible, so cancellation by C and C^T is legitimate. This avoids assuming an inverse without checking it.

Step 2: Compare the two matrix products The given equation is $BC = D$. Transpose it and multiply the transposed equation by the original equation:

$$C^TB^TBC = D^TD.$$

Since $A^T = -A$, we have

$$D^TD = (I-A)(I+A) = I - A^2,$$
$$C^TC = (I+A)(I-A) = I - A^2.$$

Both products are equal because each expands to $I - A^2$; this does not require a general assumption that arbitrary matrices commute.

Step 3: Cancel the invertible factors Substitution gives $C^TB^TBC = C^TC$. Multiply on the left by $(C^T)^{-1}$ and on the right by C^{-1} to obtain

$$B^TB = I_3.$$

Hence the three diagonal entries are each 1. This proves orthogonality directly, without the longer calculation of all nine entries of B.

Step 4: Conclusion The sum of the diagonal elements of B^TB is $1+1+1 = \boxed{3}$.

Quick Tip to Solve: In an expression involving $I+A$ and $I-A$, check first whether A is skew-symmetric; the corresponding matrix ratio can be orthogonal.

24. The number of elements in the set $S = \{x : x \in [0, 100]$ **and** $\int_0^x t^2 \sin(x-t)dt = x^2\}$ **is ____** [4 marks]

Solution:

Concept:

- Integration by parts evaluates $\int_0^x t^2 \sin(x-t)\,dt$, treating x as a constant during integration with respect to t.
- The real solutions of $\cos x = 1$ are $x = 2k\pi$ for integers k; endpoints must be included when counting on a closed interval.

Step 1: Integrate by parts once Denote the given integral by $F(x)$. Since $\frac{d}{dt}\cos(x-t) = \sin(x-t)$, integration by parts gives

$$F(x) = [t^2 \cos(x-t)]_0^x - 2\int_0^x t\cos(x-t)\,dt = x^2 - 2J,$$

where $J = \int_0^x t\cos(x-t)\,dt$. Keeping the sign in this antiderivative correct is essential.

Step 2: Evaluate the remaining integral For J, use the antiderivative $-\sin(x-t)$ of $\cos(x-t)$:

$$\begin{aligned} J &= [-t\sin(x-t)]_0^x + \int_0^x \sin(x-t)\,dt \\ &= [\cos(x-t)]_0^x = 1 - \cos x. \end{aligned}$$

Both boundary terms in the first bracket vanish. Therefore

$$F(x) = x^2 - 2 + 2\cos x.$$

Step 3: Solve the equation and count the permitted values The required equality $F(x) = x^2$ becomes $\cos x = 1$. Hence

$$x = 2k\pi, \qquad 0 \le 2k\pi \le 100.$$

Since $30\pi < 100 < 32\pi$, the allowed integers are $k = 0, 1, \ldots, 15$. There are 16 such integers. In particular, $x = 0$ must be counted: its integral and its square are both zero.

Step 4: Conclusion The set contains $\boxed{16}$ elements.

Quick Tip to Solve: After reducing an integral equation to a periodic trigonometric equation, count the integer parameter carefully and test both endpoints explicitly.

25. If the solution curve $y = f(x)$ of the differential equation

$$\left(x^2 - 4\right) y' - 2xy + 2x\left(4 - x^2\right)^2 = 0, x > 2,$$

passes through the point $(3, 15)$, then the local maximum value of f is ____ [4 marks]

Solution:

Concept:

- A first-order linear differential equation can be solved with an integrating factor. Here $x > 2$, so $x^2 - 4 > 0$ and division by this factor is valid.
- A stationary point is a local maximum when the derivative changes from positive to negative; completing the square can also establish its value exactly.

Step 1: Convert the equation to an exact derivative Since $(4 - x^2)^2 = (x^2 - 4)^2$, divide the equation by $(x^2 - 4)^2$. The first two terms are a quotient derivative:

$$\frac{(x^2 - 4)y' - 2xy}{(x^2 - 4)^2} = -2x, \qquad \frac{d}{dx}\left(\frac{y}{x^2 - 4}\right) = -2x.$$

Equivalently, the integrating factor of the linear equation is $1/(x^2 - 4)$. Integrating gives

$$\frac{y}{x^2 - 4} = C - x^2.$$

Step 2: Use the point on the solution curve The initial condition is $f(3) = 15$, and hence

$$\frac{15}{9 - 4} = C - 9 \quad \Rightarrow \quad C = 12.$$

Therefore the particular solution, throughout the given domain, is

$$f(x) = (x^2 - 4)(12 - x^2) = 16 - (x^2 - 8)^2.$$

Step 3: Locate and classify the stationary point Differentiating the particular solution yields

$$f'(x) = 4x(8 - x^2).$$

For $x > 2$, the only zero is $x = \sqrt{8} = 2\sqrt{2}$. The derivative is positive for $2 < x < 2\sqrt{2}$ and negative for $x > 2\sqrt{2}$, proving a local maximum. Moreover, the squared expression shows directly that $f(x) \leq 16$, with equality precisely at this point.

Step 4: Conclusion The local maximum is $f(2\sqrt{2}) = 16$. Thus the required numerical answer is $\boxed{16}$.

Quick Tip to Solve: Look for the quotient derivative when the coefficients are $(x^2 - a)y' - 2xy$; it immediately gives $d[y/(x^2 - a)]/dx$.

26. A small metallic sphere of diameter 2 mm and density $10.5\ \mathrm{g/cm^3}$ is dropped in glycerine having viscosity 10 Poise and density $1.5\ \mathrm{g/cm^3}$ respectively. The terminal velocity attained by the sphere is ____ $\mathrm{cm/s}$.

$\left(\pi = \frac{22}{7}\right.$ **and** $\left. g = 10\ \mathrm{m/s^2}\right)$ [4 marks]

(A) 1.0

(B) 1.5

(C) 2.0

(D) 3.0

Correct Answer: (C) 2.0

Solution:

Concept:

- At terminal speed, the downward weight is balanced by the upward buoyant force and viscous drag, so the net force and acceleration are zero.
- For a small sphere in viscous flow, Stokes' law gives drag $6\pi\eta r v$, where η is viscosity, r is radius and v is speed.

Step 1: Balance the forces on the sphere Let ρ_s be the density of the sphere and ρ_f that of glycerine. Their difference enters because the displaced liquid provides buoyancy:

$$\frac{4}{3}\pi r^3 \rho_s g = \frac{4}{3}\pi r^3 \rho_f g + 6\pi\eta r v_t.$$

Cancelling common factors gives

$$v_t = \frac{2r^2(\rho_s - \rho_f)g}{9\eta}.$$

The factor π cancels, so its supplied value is not needed.

Step 2: Convert the data and calculate The given diameter is $2\,\mathrm{mm}$, so $r = 10^{-3}\,\mathrm{m}$. In SI units,

$$\rho_s - \rho_f = (10.5 - 1.5) \times 10^3 = 9000\,\mathrm{kg\,m^{-3}}, \qquad \eta = 10\,\text{poise} = 1\,\mathrm{Pa\,s}.$$

Substitute these values:

$$v_t = \frac{2(10^{-3})^2(9000)(10)}{9(1)} = 0.02\,\mathrm{m\,s^{-1}} = 2.0\,\mathrm{cm\,s^{-1}}.$$

Step 3: Check the alternatives The calculated downward terminal speed is $2.0\,\mathrm{cm\,s^{-1}}$. Thus 1.0, 1.5 and $3.0\,\mathrm{cm\,s^{-1}}$ do not satisfy the terminal force balance with the stated radius, density difference and viscosity. Using the diameter in place of the radius would incorrectly multiply the answer by four.

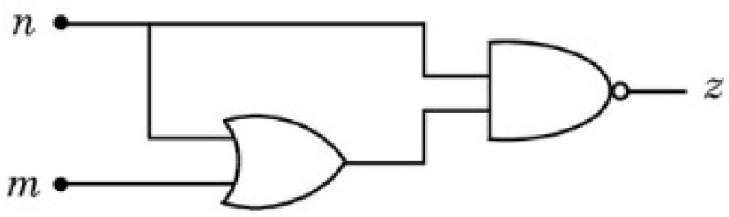
Step 4: Conclusion The terminal velocity is $2.0\,\mathrm{cm\,s^{-1}}$ downward, which is option (C) in the printed official order. Hence, the correct option is (C).

Quick Tip to Solve: Use radius squared, subtract the fluid density, and convert poise to SI before substituting in Stokes' formula.

27.

For the given logic gate circuit, which of the following is the correct truth table?

[4 marks]



(A)

n	m	z
0	0	1
0	1	0
1	1	0
1	0	0

(B)

n	m	z
0	0	1
0	1	0
1	1	1
1	0	0

(C)

n	m	z
0	0	1
0	1	1
1	1	0
1	0	0

(D)

n	m	z
0	0	0
0	1	1
1	1	0
1	0	1

Correct Answer: (C)

n	m	z
0	0	1
0	1	1
1	1	0
1	0	0

Solution:

Concept:

- An OR gate gives $n + m$, where $+$ denotes logical OR. A NAND gate complements the logical product of its two inputs.
- The absorption identity $n(n + m) = n$ follows because $n = 0$ makes the product zero, while $n = 1$ makes the OR output one.

Step 1: Trace the two inputs to the final gate The lower gate has the OR symbol and receives inputs n and m, giving $n + m$. The upper wire brings n directly to the final gate. That gate has an AND outline with a small output circle, so it is a NAND gate, not an AND gate. Its output is therefore

$$z = \overline{n(n+m)}.$$

The output circle complements the entire product, rather than just one input.

Step 2: Simplify and form the truth table Apply the absorption identity to obtain

$$z = \overline{n}.$$

Consequently the output does not depend on m. In the exact input-row order used in the question, the complete working is

n	m	$n+m$	$n(n+m)$	z
0	0	0	0	1
0	1	1	0	1
1	1	1	1	0
1	0	1	1	0

Step 3: Compare all four printed tables The required output sequence is $1, 1, 0, 0$, matching the third printed table. The first table fails for $(n, m) = (0, 1)$. The second fails for both $(0, 1)$ and $(1, 1)$. The fourth starts with output zero for $(0, 0)$, whereas the NAND gate must output one when its direct input is zero. These checks also rule out confusing this circuit with NOR, XNOR or XOR.

Step 4: Conclusion The circuit implements $z = \overline{n}$ and has output sequence $1, 1, 0, 0$. This is option (C) in the printed official order. Hence, the correct option is (C).

Quick Tip to Solve: Before constructing all rows, simplify repeated-input combinations using absorption: $A(A + B) = A$. Keep the final inversion bubble.

28. An air bubble of volume $2.9\ \text{cm}^3$ rises from the bottom of a swimming pool of 5 m deep. At the bottom of the pool water temperature is 17°C. The volume of the bubble when it reaches the surface, where the water temperature is 27°C, is ____ cm^3.

($g = 10\ \text{m/s}^2$, density of water $= 10^3\ \text{kg/m}^3$, and 1 atm pressure is $10^5\ \text{Pa}$) [4 marks]

(A) 4.2

(B) 2.0

(C) 3.0

(D) 4.5

Correct Answer: (D) 4.5

Solution:

Concept:

- For a fixed amount of ideal gas, PV/T remains constant. Temperatures must be absolute temperatures in kelvin.
- At depth h in water, the bubble pressure is $P_{\text{atm}} + \rho gh$; at the surface it is atmospheric pressure in the model used here.

Step 1: Find the initial and final pressures The hydrostatic contribution at the bottom is

$$\rho gh = (10^3)(10)(5) = 5 \times 10^4\,\text{Pa}.$$

This is a gauge pressure and must be added to atmospheric pressure for the gas-law calculation:

$$P_1 = 10^5 + 5 \times 10^4 = 1.5 \times 10^5\,\text{Pa}, \qquad P_2 = 10^5\,\text{Pa}.$$

Thus pressure decreases as the bubble rises, tending to increase its volume.

Step 2: Use the combined gas law The temperatures are $T_1 = 17 + 273 = 290\,\text{K}$ and $T_2 = 27 + 273 = 300\,\text{K}$. Since the amount of gas remains the same,

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}, \qquad V_2 = V_1 \frac{P_1}{P_2}\frac{T_2}{T_1}.$$

The volume can remain in cubic centimetres because it appears in a ratio. Substituting,

$$V_2 = 2.9 \times 1.5 \times \frac{300}{290} = 4.5\,\text{cm}^3.$$

Step 3: Check the direction and options Both the pressure drop and the temperature increase expand the bubble. The value $2.0\,\text{cm}^3$ is therefore impossible here. The alternatives 3.0 and $4.2\,\text{cm}^3$ also fail the pressure-temperature ratio $V_2/V_1 = 45/29$; only $4.5\,\text{cm}^3$ satisfies it. Using Celsius directly would give an incorrect temperature factor.

Step 4: Conclusion The surface volume is $\boxed{4.5\,\text{cm}^3}$. Hence, the correct option is (D).

Quick Tip to Solve: For rising bubbles, use absolute pressure at the bottom and kelvin temperatures; both factors belong in $V_2/V_1 = (P_1/P_2)(T_2/T_1)$.

29. A circular loop of radius 7 cm is placed in uniform magnetic field of 0.2 T directed perpendicular to plane of loop. The loop is converted into a square loop in 0.5 s . The EMF induced in the loop is ____ mV. [4 marks]

(A) 1.32

(B) 13.2

(C) 6.6

(D) 8.25

Correct Answer: (A) 1.32

Solution:

Concept:

- The average induced emf has magnitude $|\mathcal{E}| = |\Delta\Phi|/\Delta t$. For a loop perpendicular to a uniform magnetic field, $\Phi = BA$.
- Reshaping the same wire preserves its perimeter, not its enclosed area. The decrease in area changes the flux even though the field remains constant.

Step 1: Calculate the square side from the wire length The initial circle has radius $r = 7\,\text{cm}$. Using $\pi = 22/7$, its circumference, and hence the perimeter of the final square, is

$$L = 2\pi r = 44\,\text{cm}.$$

The square side is therefore

$$a = \frac{L}{4} = 11\,\text{cm}.$$

It would be incorrect to set this side equal to the radius or the diameter: the conserved quantity during reshaping is total wire length.

Step 2: Find the change in magnetic flux The two enclosed areas are

$$A_i = \pi r^2 = 154\,\text{cm}^2, \qquad A_f = a^2 = 121\,\text{cm}^2.$$

Hence the magnitude of the area reduction is

$$|\Delta A| = 33\,\text{cm}^2 = 33 \times 10^{-4}\,\text{m}^2.$$

With unchanged perpendicular field, the flux change has magnitude $B|\Delta A|$. The induced current, if the loop is conducting and closed, opposes this decrease in flux.

Step 3: Calculate and compare the emf values Divide the flux change by the stated reshaping time:

$$|\mathcal{E}| = \frac{0.2(33 \times 10^{-4})}{0.5} = 1.32 \times 10^{-3}\,\text{V} = 1.32\,\text{mV}.$$

Thus 13.2, 6.6 and $8.25\,\text{mV}$ are inconsistent with the conserved perimeter and the stated time. The quantity determined from total change and total time is the average induced emf.

Step 4: Conclusion The magnitude of the average induced emf is $\boxed{1.32\,\text{mV}}$, option (A) in the printed official order. Hence, the correct option is (A).

Quick Tip to Solve: In reshaping-loop questions, first conserve wire length to obtain the new dimensions; then use the area difference in Faraday' law.

30. A body of mass 14 kg initially at rest explodes and breaks into three fragments of masses in the ratio $2:2:3$. The two pieces of equal masses fly off perpendicular to each other with a speed of $18\ \mathrm{m/s}$ each. The velocity of the heavier fragment is

____ $\mathrm{m/s}$. [4 marks]

(A) $12\sqrt{2}$

(B) $10\sqrt{2}$

(C) 12

(D) $24\sqrt{2}$

Correct Answer: (A) $12\sqrt{2}$

Solution:

Concept:

- An explosion involves internal forces, so total momentum is conserved when external impulse is negligible. The initial momentum here is zero.
- Momenta must be added as vectors. Two equal perpendicular momenta of magnitude p have resultant magnitude $p\sqrt{2}$.

Step 1: Calculate the fragment masses The mass ratio $2:2:3$ contains seven equal shares. Since the total mass is $14\,\mathrm{kg}$, one share is $2\,\mathrm{kg}$. Thus

$$m_1 = m_2 = 4\,\mathrm{kg}, \qquad m_3 = 6\,\mathrm{kg}.$$

Choose the positive x- and y-axes along the velocities of the two lighter fragments. Their perpendicular momenta are then

$$\vec{p}_1 = 72\hat{i}\,\mathrm{kg\,m\,s^{-1}}, \qquad \vec{p}_2 = 72\hat{j}\,\mathrm{kg\,m\,s^{-1}}.$$

Step 2: Apply vector momentum conservation The system was initially at rest, so the third fragment must cancel the sum of these two momenta:

$$\vec{p}_3 = -(\vec{p}_1 + \vec{p}_2) = -72\hat{i} - 72\hat{j}.$$

Dividing by its mass gives the velocity components

$$\vec{v}_3 = -12\hat{i} - 12\hat{j} \quad \mathrm{m\,s^{-1}}.$$

It travels opposite the resultant of the two lighter fragments, along the bisector of the opposite quadrant.

Step 3: Find the speed and eliminate alternatives The options specify the magnitude of the velocity. Therefore

$$|\vec{v}_3| = \sqrt{(-12)^2 + (-12)^2} = 12\sqrt{2}\,\mathrm{m\,s^{-1}}.$$

The value 12 is only one component, not the speed. The value $24\sqrt{2}$ doubles the required magnitude, while $10\sqrt{2}$ fails to balance the $72\,\mathrm{kg\,m\,s^{-1}}$ momentum in each direction. Treating perpendicular momenta as collinear would also give an incorrect result.

Step 4: Conclusion The heavier fragment has speed $\boxed{12\sqrt{2}\,\mathrm{m\,s^{-1}}}$, directed opposite the resultant momentum of the lighter fragments. Hence, the correct option is (A).

Quick Tip to Solve: For explosions from rest, resolve momentum along the known fragment directions before taking the magnitude of the unknown velocity.

31. Which of the following pair of nuclei are isobars of the element? [4 marks]

(A) ${}^{2}_{1}\mathrm{H}$ and ${}^{3}_{1}\mathrm{H}$

(B) ${}^{236}_{92}\mathrm{U}$ and ${}^{238}_{92}\mathrm{U}$

(C) ${}^{3}_{1}\mathrm{H}$ and ${}^{3}_{2}\mathrm{He}$

(D) ${}^{198}_{80}\mathrm{Hg}$ and ${}^{197}_{79}\mathrm{Au}$

Correct Answer: (C) ${}^{3}_{1}\mathrm{H}$ and ${}^{3}_{2}\mathrm{He}$

Solution:

Concept:

- A nucleus is written ${}^{A}_{Z}X$, where Z is its proton number and A is its mass number, the total number of protons and neutrons.
- Isobars have the same mass number A but different atomic numbers Z. Isotopes have the same Z but different A.

Step 1: Compare mass and atomic numbers For ${}^{3}_{1}\mathrm{H}$ and ${}^{3}_{2}\mathrm{He}$, both nuclei have mass number 3, but their atomic numbers are 1 and 2. Thus one nucleus has one proton and two neutrons, while the other has two protons and one neutron. Their common nucleon total and different proton numbers satisfy both conditions for isobars.

Step 2: Eliminate the other pairs Option (A) contains hydrogen nuclei of mass numbers 2 and 3 with the same atomic number 1; they are isotopes. Option (B) similarly contains uranium isotopes with mass numbers 236 and 238 and atomic number 92. Option (D) has different mass numbers, 198 and 197, so it cannot be an isobaric pair even though the atomic numbers also differ. Only option (C) has equal mass numbers and unequal atomic numbers.

Step 3: Conclusion The isobars are ${}^{3}_{1}\mathrm{H}$ and ${}^{3}_{2}\mathrm{He}$. Hence, the correct option is (C).

Quick Tip to Solve: For isobars compare the upper numbers first: they must match; then check that the lower numbers differ.

32. The ratio of speeds of electromagnetic waves in vacuum and a medium, having dielectric constant $k = 3$ and permeability of $\mu = 2\mu_0$, is (μ_0 = permeability of vacuum) [4 marks]

(A) $3 : 2$

(B) $6 : 1$

(C) $36 : 1$

(D) $\sqrt{6} : 1$

Correct Answer: (D) $\sqrt{6} : 1$

Solution:

Concept:

- In a medium, electromagnetic waves travel at $u = 1/\sqrt{\mu\varepsilon}$; in vacuum, $c = 1/\sqrt{\mu_0\varepsilon_0}$.
- The dielectric constant is $k = \varepsilon/\varepsilon_0$, and relative permeability is $\mu_r = \mu/\mu_0$. The speed ratio is $c/u = \sqrt{\mu_r k}$.

Step 1: Identify the relative material constants The given dielectric constant is $k = 3$, so $\varepsilon = 3\varepsilon_0$. The permeability is $\mu = 2\mu_0$, hence $\mu_r = 2$. Both changes must be included: it would be incorrect to use only the dielectric constant as for a nonmagnetic material.

Step 2: Form the speed ratio Substitute the medium constants into the wave-speed relation:

$$u = \frac{1}{\sqrt{(2\mu_0)(3\varepsilon_0)}} = \frac{c}{\sqrt{6}}.$$

Consequently,

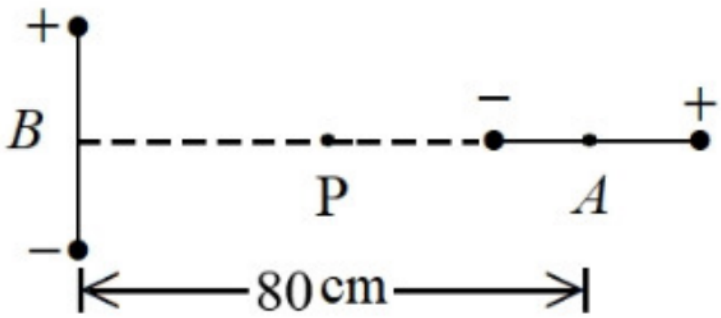
$$\frac{c}{u} = \sqrt{6}.$$

The factor is a square root of the product, not the product itself, so (B) is excluded. Dividing the two relative constants gives 3:2, which is not the speed law and excludes (A). Squaring their product produces 36:1, also incorrect, excluding (C).

Step 3: Conclusion The vacuum-to-medium speed ratio is $\sqrt{6} : 1$. Hence, the correct option is (D).

Quick Tip to Solve: Use $c/u = \sqrt{\varepsilon_r\mu_r}$; omit μ_r only when the medium is explicitly nonmagnetic.

33. Two shorts dipoles (A, B), A having charges $\pm 2\mu C$ and length 1 cm and B having charges $\pm 4\mu C$ and length 1 cm are placed with their centres 80 cm apart as shown in the figure. The electric field at a point P, equi-distant from the centres of both dipoles is ____ N/C. [4 marks]



(A) $9\sqrt{2} \times 10^4$

(B) $\frac{9}{16}\sqrt{2} \times 10^4$

(C) $\frac{9}{16}\sqrt{2} \times 10^5$

(D) $4.5\sqrt{2} \times 10^4$

Correct Answer: (B) $\frac{9}{16}\sqrt{2} \times 10^4$

Solution:

Concept:

- A short dipole has moment $p = q\ell$, where ℓ is the full separation of its charges. Its axial and equatorial field magnitudes are $2kp/r^3$ and kp/r^3, respectively.
- The axial field is along the dipole moment; the equatorial field is opposite to it. Perpendicular field components add by the Pythagorean rule.

Step 1: Find the moments and observation distances Point P is midway between the centres, so its distance from either centre is $r = 0.40\,\text{m}$. Using the given full dipole length $\ell = 0.01\,\text{m}$,

$$p_A = 2 \times 10^{-6} \times 0.01 = 2 \times 10^{-8}\,\text{C m}, \qquad p_B = 4 \times 10^{-8}\,\text{C m}.$$

The figure places P on the axial line of A and on the equatorial line of B.

Step 2: Calculate and combine the fields With $k = 9 \times 10^9\,\text{N m}^2\,\text{C}^{-2}$,

$$E_A = \frac{2kp_A}{r^3} = \frac{360}{0.064} = 5625\,\text{N/C}, \qquad E_B = \frac{kp_B}{r^3} = 5625\,\text{N/C}.$$

The moment of A points right, so its field at P points right. The moment of B points up, making its equatorial field point down. Therefore

$$E = \sqrt{E_A^2 + E_B^2} = 5625\sqrt{2} = \frac{9}{16}\sqrt{2} \times 10^4\,\text{N/C}.$$

Option (A) is sixteen times this result, (C) is ten times it, and (D) is eight times it; none has the required magnitude.

Step 3: Conclusion The field magnitude is $\frac{9}{16}\sqrt{2} \times 10^4\,\text{N/C}$, directed rightward and downward at

$45°$. Hence, the correct option is (B).

Quick Tip to Solve: Locate the observation point relative to each dipole separately; an axial factor of 2 can exactly compensate a smaller dipole moment.

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34. The internal energy of a monoatomic gas is 3nRT. One mole of helium is kept in a cylinder having internal cross section area of $17\ \text{cm}^2$ and fitted with a light movable frictionless piston. The gas is heated slowly by suppling 126 J heat. If the temperature rises by $4°\text{C}$, then the piston will move ____ cm.

(atmospheric pressure $= 10^5\ \text{Pa}$) [4 marks]

(A) 15.5

(B) 14.5

(C) 1.55

(D) 1.45

Correct Answer: (A) 15.5

Solution:

Concept:

- The first law is $Q = \Delta U + W$, where W is work done by the gas. Use the internal-energy expression explicitly supplied in the question.
- A light frictionless piston expands against atmospheric pressure, so $W = P_{\text{atm}}\Delta V = P_{\text{atm}}Ax$, where x is its displacement.

Step 1: Calculate the stipulated internal-energy increase The problem gives $U = 3nRT$. Although the usual ideal monoatomic formula is $3nRT/2$, the calculation here must use the supplied expression. A temperature rise of $4°\text{C}$ equals $4\,\text{K}$. Thus, using $n = 1$ and $R = 8.31\,\text{J mol}^{-1}\,\text{K}^{-1}$,

$$\Delta U = 3nR\Delta T = 3(1)(8.31)(4) = 99.72\,\text{J}.$$

The energy available for expansion work is

$$W = 126 - 99.72 = 26.28\,\text{J}.$$

Step 2: Convert the expansion work into piston travel Convert the cross-sectional area before substituting:

$$A = 17\,\text{cm}^2 = 17 \times 10^{-4}\,\text{m}^2.$$

Then

$$x = \frac{W}{P_{\text{atm}}A} = \frac{26.28}{10^5(17 \times 10^{-4})} = 0.1546\,\text{m} \simeq 15.5\,\text{cm}.$$

This matches (A). The 14.5 cm value in (B) does not follow from the heat balance; the values 1.55 cm and 1.45 cm in (C) and (D) also have the wrong scale. The result follows the internal-energy relation stipulated in the printed problem.

Step 3: Conclusion Using the given $U = 3nRT$, the piston moves approximately $15.5\,\text{cm}$. Hence, the correct option is (A).

Quick Tip to Solve: Subtract the internal-energy increase from the supplied heat before using $W = PAx$; convert square centimetres with a factor 10^{-4}.

35. To compare EMF of two cells using potentiometer the balancing lengths obtained are 200 cm and 150 cm . The least count of scale is 1 cm . The percentage error in the ratio of EMFs is ____ [4 marks]

(A) 1.45

(B) 1.55

(C) 1.65

(D) 1.75

Correct Answer: The percentage error is 1.17%. None of the printed options (A)–(D) equals this result; there is no correct printed option. The numerical result follows directly from $100(1/200 + 1/150)$.

Solution:

Concept:

- For the same potentiometer potential gradient, the EMF ratio is $r = E_1/E_2 = l_1/l_2$. The gradient cancels because both balances use the same wire conditions.
- For a quotient, the maximum fractional error is the sum of the fractional errors: $\Delta r/r = \Delta l_1/l_1 + \Delta l_2/l_2$.

Step 1: Write the measured ratio and length uncertainties The balance lengths are $l_1 = 200\,\text{cm}$ and $l_2 = 150\,\text{cm}$, giving $r = 200/150 = 4/3$. Taking the stated least count as the absolute uncertainty of either length gives $\Delta l_1 = \Delta l_2 = 1\,\text{cm}$. The uncertainties are divided by their own measured lengths, not by the difference of the lengths.

Step 2: Propagate and compare with the printed options The maximum fractional error is

$$\frac{\Delta r}{r} = \frac{1}{200} + \frac{1}{150} = \frac{3}{600} + \frac{4}{600} = \frac{7}{600}.$$

Therefore the percentage error is

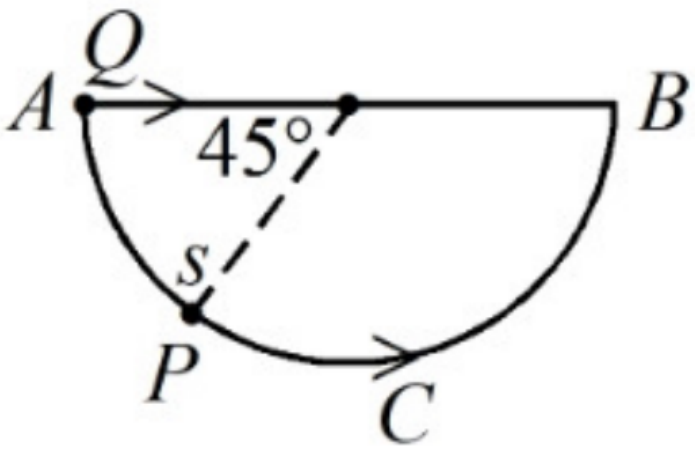
$$100\frac{\Delta r}{r} = \frac{7}{6}\% = 1.1666\ldots\% \simeq 1.17\%.$$

The printed alternatives are 1.45, 1.55, 1.65 and 1.75. Each differs from 1.17, so none is supported by the given measurements. Taking a ratio does not mean subtracting the uncertainty contributions: a numerator increase and denominator decrease reinforce each other in the maximum-error calculation.

Step 3: Conclusion The percentage error is 1.17%. None of the printed options (A)–(D) equals this result; there is no correct printed option. The numerical result follows directly from $100(1/200 + 1/150)$.

Quick Tip to Solve: For a product or quotient, add fractional uncertainties and multiply by 100 only at the final step.

36. A bead P sliding on a frictionless semi-circular string (ACB) and it is at point S at $t = 0$ and at this instant the horizontal component of its velocity is v. Another bead Q of the same mass as P is ejected from point A at $t = 0$ along the horizontal string AB, with the speed v, friction between the beads and the respective strings may be neglected in both cases. Let t_P and t_Q be the respective times taken by beads P and Q to reach the point B, then the relation between t_P and t_Q is [4 marks]



(A) $t_P < t_Q$

(B) $t_P > t_Q$

(C) $t_P = t_Q$

(D) $t_P > 1.25t_Q$

Correct Answer: (A) $t_P < t_Q$

Solution:

Concept:

- For the horizontal-plane interpretation of the shown guides, gravity has no tangential component. A smooth guide provides only a normal constraint force, so the speed along each path is constant. Smoothness alone would not ensure constant speed in a vertical plane; the time comparison here uses horizontal-plane motion.
- On a circular arc, distance equals $R\theta$ with θ in radians. Compare travel times as path length divided by total speed, not horizontal speed.

Step 1: Recover the speed of the bead on the arc Let the semicircle have radius R and centre O. The figure gives $\angle AOS = 45°$. The tangent at S therefore makes $45°$ with the horizontal. If u is the speed of bead P, its stated horizontal component is

$$v = u\cos 45°, \qquad u = \sqrt{2}\,v.$$

Bead Q travels along the diameter with speed v, so

$$t_Q = \frac{2R}{v}.$$

Step 2: Use the remaining arc and compare the times The arc from S through C to B subtends $180^\circ - 45^\circ = 135^\circ = 3\pi/4$. Its length and travel time are

$$s_P = \frac{3\pi R}{4}, \qquad t_P = \frac{3\pi R}{4\sqrt{2}\,v}.$$

Thus

$$\frac{t_P}{t_Q} = \frac{3\pi}{8\sqrt{2}} \simeq 0.833 < 1.$$

Although the arc from S to B is longer than the diameter, bead P has the larger total speed. The ratio excludes equality in (C) and both greater-than statements (B) and (D).

Step 3: Conclusion $t_P = \frac{3\pi}{8\sqrt{2}} t_Q \simeq 0.833 t_Q$, so $t_P < t_Q$. Hence, the correct option is (A).

Quick Tip to Solve: When only a velocity component is given, recover the full tangent speed before comparing arc and straight-line travel times.

37. A parallel plate capacitor with plate separation 5 mm is charged by a battery. On introducing a mica sheet of 2 mm and maintaining the connections of the plates with the terminals of the battery, it is found that it draws 25% more charge from the battery. The dielectric constant of mica is ____

[4 marks]

(A) 2.5

(B) 1.0

(C) 1.5

(D) 2.0

Correct Answer: (D) 2.0

Solution:

Concept:

- For a battery-connected capacitor, potential difference V remains fixed, so $Q = CV$ makes the charge ratio equal the capacitance ratio.
- A dielectric slab of thickness t and relative permittivity K changes the effective plate spacing from d to $d - t + t/K$. The air gap and dielectric behave as two capacitors in series.

Step 1: Write the capacitance ratio Initially $C_0 = \varepsilon_0 A/d$. After insertion, $C = \varepsilon_0 A/(d - t + t/K)$, with unchanged area A. Drawing 25 percent more charge means the final charge is $1.25Q_0$, not $0.25Q_0$. Thus

$$\frac{C}{C_0} = \frac{5}{3 + 2/K} = 1.25.$$

All lengths in this ratio are in millimetres, so their common unit cancels.

Step 2: Solve and test the options Rearranging gives

$$3 + \frac{2}{K} = 4, \qquad K = 2.$$

The effective separation is then 4 mm, giving the required ratio $5/4$. The alternatives $K = 2.5, 1.0, 1.5$ give capacitance ratios $25/19, 1, 15/13$, respectively; none gives 1.25.

Step 3: Conclusion The dielectric constant of mica is $\boxed{K = 2.0}$, a dimensionless quantity. Hence, the correct option is (D).

Quick Tip to Solve: With the battery attached, use the charge increase as a capacitance increase; replace the slab thickness by t/K in the effective gap.

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38. One mole of an ideal diatomic gas expands from volume V to $2V$ isothermally at a temperature $27^\circ\mathrm{C}$ and does W joule of work. If the gas undergoes same magnitude of expansion adiabatically from $27^\circ\mathrm{C}$ doing the same amount of work W, then its final temperature will be (close to) ____ $^\circ\mathrm{C}$.

$$(\log_e 2 = 0.693)$$

[4 marks]

(A) -117

(B) -56

(C) -30

(D) -189

Correct Answer: Formal equal-work answer: (B), $-56^\circ\mathrm{C}$; the additional volume-doubling condition is incompatible with a closed adiabatic process.

Solution:

Concept:

- For one mole, reversible isothermal work is $W = RT\ln(V_f/V_i)$. For a diatomic gas near room temperature, $C_V = 5R/2$.
- Adiabatic energy conservation gives $W = C_V(T_i - T_f)$. A closed adiabatic process must also satisfy $\Delta S \geq 0$; energy conservation alone cannot guarantee physical consistency.

Step 1: Use the stipulated equal work The initial temperature is $300\,\mathrm{K}$. The isothermal work for doubling the volume is

$$W = 300R\ln 2 = 207.9R.$$

Equating this to the stipulated adiabatic work yields

$$\frac{5}{2}R(300 - T_f) = 207.9R, \qquad T_f = 216.84\,\mathrm{K}.$$

Thus the formal equal-work result is $-56.16^\circ\mathrm{C}$, close to option (B). The other listed temperatures do not satisfy this equal-work equation.

Step 2: Check the additional volume condition The printed statement also requires the adiabatic

volume to double. With the temperature just found, the entropy change would be

$$\frac{\Delta S}{R} = \frac{5}{2}\ln\left(\frac{216.84}{300}\right) + \ln 2 \simeq -0.1184.$$

This is negative and therefore impossible for a closed adiabatic expansion. Even the reversible adiabatic expansion to $2V$ ends at $300/2^{2/5} \simeq 227.36\,\mathrm{K}$, with less work than the isothermal process. An irreversible adiabatic expansion cannot extract more work between the same volumes from this initial state.

Step 3: Conclusion The equal-work calculation gives $\boxed{-56^\circ\mathrm{C}}$, option (B), agreeing with the listed key. However, no physical adiabatic process satisfies both the stated equal work and the stated volume doubling; the printed conditions are mutually inconsistent.

Quick Tip to Solve: Check both the first and second laws when a question specifies work, final volume and adiabatic behaviour simultaneously.

---

39. A block is sliding down on an inclined plane of slope θ and at an instant $t = 0$ this block is given an upward momentum so that it starts moving up on the inclined surface with velocity u. The distance (S) travelled by the block before its velocity become zero, is ____ .
(g = gravitational acceleration) [4 marks]

(A) $\dfrac{u^2}{4g\sin\theta}$

(B) $\dfrac{u^2}{2g\cos\theta}$

(C) $\dfrac{2u^2}{g\cos\theta}$

(D) $\dfrac{u^2}{\sqrt{2}g\cos\theta}$

Correct Answer: No unique answer: friction or uniform downward motion is not specified. Option (A) follows only if the initial downward speed was constant.

Solution:

Concept:

- The downslope component of gravity is $mg\sin\theta$. If kinetic friction is present, its magnitude is $\mu_k mg\cos\theta$, and it opposes the instantaneous motion.
- For upward motion, gravity and friction both act down the plane. The stopping distance follows from $v^2 = u^2 + 2aS$, with $v = 0$.

Step 1: Keep the unspecified friction coefficient explicit Taking uphill as positive,

$$a = -g(\sin\theta + \mu_k\cos\theta).$$

Consequently,

$$S = \frac{u^2}{2g(\sin\theta + \mu_k \cos\theta)}.$$

The statement that the block was sliding down does not specify μ_k, its downward acceleration, or that its downward speed was constant. It therefore does not determine a numerical expression involving only u, g, θ.

Step 2: Identify the extra condition behind the first option If the preceding downward motion had been uniform, force balance would give $\mu_k = \tan\theta$. Only with that additional condition would

$$S = \frac{u^2}{4g\sin\theta},$$

the printed option (A), follow. If the plane were smooth instead, the result would be $u^2/(2g\sin\theta)$, which is absent from the printed options. The three expressions using only $g\cos\theta$ do not represent the general along-plane retardation either. None can be selected uniquely from the supplied information.

Step 3: Conclusion The stopping distance depends on the unspecified friction coefficient as derived above. Option (A) requires uniform initial sliding, which is not stated. The given information therefore does not determine a unique stopping distance or justify selecting any single printed option.

Quick Tip to Solve: Do not infer force balance merely from the word sliding. Constant speed must be stated before using $\mu_k = \tan\theta$.

40. A paratrooper jumps from an aeroplane and opens a parachute after 2 s of free fall and starts deaccelerating with $3\ \mathrm{m/s^2}$. At 10 m height from ground, while descending with the help of parachute, the speed of paratrooper is $5\ \mathrm{m/s}$. The initial height of the airplane is ____ m.

$$\left(\mathrm{g} = 10\ \mathrm{m/s^2}\right)$$

[4 marks]

(A) 82.5

(B) 62.5

(C) 92.5

(D) 20

Correct Answer: (C) 92.5

Solution:

Concept:

- For constant acceleration, $v = u + at$, $s = ut + at^2/2$, and $v^2 = u^2 + 2as$. Apply them separately before and after the parachute opens.
- Take downward displacement as positive. The initial free fall has acceleration $+g$; the stated deceleration after opening means net acceleration $-3\,\mathrm{m\,s^{-2}}$.

Step 1: Calculate the free-fall stage The initial vertical velocity is zero. In the first $2\,\mathrm{s}$,

$$v_1 = 10 \times 2 = 20\,\mathrm{m\,s^{-1}}, \qquad s_1 = \frac{1}{2}(10)(2)^2 = 20\,\mathrm{m}.$$

This speed becomes the initial downward speed for the parachute-controlled stage.

Step 2: Find the descent with the parachute open Let s_2 be the distance travelled while the speed decreases from 20 to $5\,\mathrm{m\,s^{-1}}$. Then

$$5^2 = 20^2 + 2(-3)s_2, \qquad s_2 = \frac{400 - 25}{6} = 62.5\,\mathrm{m}.$$

At that instant the paratrooper is still 10 m above the ground, so

$$H = s_1 + s_2 + 10 = 20 + 62.5 + 10 = 92.5\,\mathrm{m}.$$

Option (A) omits the final height above ground; option (B) counts only the decelerated descent; option (D) counts only the free fall.

Step 3: Conclusion The initial height is $\boxed{92.5\,\mathrm{m}}$. Hence, the correct option is (C).

Quick Tip to Solve: At a stated height above the ground, add the remaining height to all descent distances already travelled.

---

41. Two charges $7\mu\mathrm{C}$ and $-2\mu\mathrm{C}$ are placed at $(-9,0,0)\mathrm{cm}$ and $(9,0,0)\mathrm{cm}$ respectively in an external field $E = \frac{\mathrm{A}}{r^2}\hat{r}$, where $A = 9 \times 10^5\ \mathrm{N/C.m^2}$. Considering the potential at infinity is 0 , the electrostatic energy of the configuration is ____ J. [4 marks]

(A) -90.7

(B) 1.4

(C) 49.3

(D) 24.3

Correct Answer: (C) 49.3

Solution:

Concept:

- For the outward radial field $E_r = A/r^2$, $E_r = -dV/dr$ and $V(\infty) = 0$ give $V(r) = A/r$.
- The configuration energy is the sum of each charge in the prescribed external potential and their mutual interaction: $U = q_1V_1 + q_2V_2 + kq_1q_2/r_{12}$.

Step 1: Evaluate the external-field contribution Both charges are 9 cm from the origin, while their separation is 18 cm. Hence

$$V_1 = V_2 = \frac{9 \times 10^5}{0.09} = 10^7\,\mathrm{V}.$$

The total external energy is

$$U_{\mathrm{ext}} = (7 - 2) \times 10^{-6} \times 10^7 = 50\,\mathrm{J}.$$

The negative charge contributes negative energy, so it must be added algebraically.

Step 2: Add the mutual interaction once Using $k = 9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$,

$$U_{12} = \frac{(9 \times 10^9)(7 \times 10^{-6})(-2 \times 10^{-6})}{0.18} = -0.7\,\mathrm{J}.$$

Therefore $U = 50 - 0.7 = 49.3\,\mathrm{J}$. The mutual term occurs once, not twice, and the external contribution has no factor of one-half because the field is prescribed independently. The other choices do not equal this sum; a negative total would also conflict with the dominant positive 50 J contribution.

Step 3: Conclusion The electrostatic energy of the configuration is $\boxed{49.3\,\mathrm{J}}$. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish the distance from the external-field centre from the intercharge separation; here they are 0.09 m and 0.18 m.

---

42. Suppose a long solenoid of 100 cm length, radius 2 cm having 500 turns per unit length, carries a current $I = 10\sin(\omega t)\mathrm{A}$, where $\omega = 1000\mathrm{rad./s}$. A circular conducting loop (B) of radius 1 cm coaxially slided through the solenoid at a speed $v = 1\ \mathrm{cm/s}$. The r.m.s. current through the loop when the coil B is inserted 10 cm inside the solenoid is $\frac{\alpha}{\sqrt{2}}\mu A$. The value of α is ____ .

[Resistance of the loop $= 10\Omega$] [4 marks]

(A) 100

(B) 280

(C) 197

(D) 80

Correct Answer: (C) 197

Solution:

Concept:

- Well inside a long solenoid, $B(t) = \mu_0 n I(t)$, where n is the number of turns per metre.
- The small coaxial loop has area $A_l = \pi a^2$. Faraday's law gives $\varepsilon = -A_l\, dB/dt$, and a sinusoidal current has rms value equal to its amplitude divided by $\sqrt{2}$.

Step 1: Find the changing flux through the loop The loop radius is $a = 0.01\,\mathrm{m}$, smaller than the solenoid radius. The relevant area is therefore $\pi(0.01)^2$, not the area of the solenoid. Ten centimetres inside the long solenoid, the ideal interior field is spatially uniform, so axial translation does not change the intercepted area or produce an extra flux change. Thus

$$\Phi(t) = \mu_0 n \pi a^2 I_0 \sin\omega t.$$

Step 2: Calculate the current amplitude and rms value With $I_0 = 10\,\mathrm{A}$, $n = 500\,\mathrm{m^{-1}}$, $\omega = 1000\,\mathrm{s^{-1}}$

and resistance $R_l = 10\,\Omega$,

$$\begin{aligned} i_0 &= \frac{\mu_0 n\pi a^2 I_0 \omega}{R_l} \\ &= \frac{(4\pi \times 10^{-7})(500)\pi(0.01)^2(10)(1000)}{10} \\ &= 2\pi^2 \times 10^{-5}\,\text{A} \simeq 197.4\,\mu\text{A}. \end{aligned}$$

Hence $i_{\text{rms}} = 197.4/\sqrt{2}\,\mu$A. The coefficient requested is the amplitude in microamperes, not the rms value itself. The alternatives 100, 280 and 80 do not match this coefficient.

Step 3: Conclusion Comparing with $i_{\text{rms}} = \alpha/\sqrt{2}\,\mu$A gives $\boxed{\alpha \simeq 197}$. Hence, the correct option is (C).

Quick Tip to Solve: For a loop smaller than the solenoid, use the loop area and differentiate the solenoid current; axial speed is irrelevant in a uniform interior field.

43. The current passing through a conducting loop in the form of equilateral triangle of side $4\sqrt{3}$ cm is 2 A . The magnetic field at its centroid is $\alpha \times 10^{-5}$ T. The value of α is ____ .

(Given : $\mu_o = 4\pi \times 10^{-7}$ SI units) [4 marks]

(A) $\sqrt{3}$

(B) $2\sqrt{3}$

(C) $3\sqrt{3}$

(D) $\frac{\sqrt{3}}{2}$

Correct Answer: (C) $3\sqrt{3}$

Solution:

Concept:

- For a finite straight wire, the magnetic field at perpendicular distance d is $B = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$, where the endpoint angles are measured from the perpendicular.
- The centroid of an equilateral triangle is also its incentre. All three sides produce equal magnetic fields normal to the loop, in the same direction by the right-hand rule.

Step 1: Find the perpendicular distance to one side Let the side length be $s = 4\sqrt{3}\,\text{cm}$. The altitude is $h = \sqrt{3}s/2 = 6\,\text{cm}$. The centroid lies one-third of the altitude above the opposite side, so

$$d = \frac{h}{3} = 2\,\text{cm} = 2 \times 10^{-2}\,\text{m}.$$

The distance along a side from its midpoint to an endpoint is $s/2 = 2\sqrt{3}\,\text{cm}$. Hence $\tan\theta = (s/2)/d = \sqrt{3}$, giving $\theta_1 = \theta_2 = 60^\circ$.

Step 2: Add the fields of the three sides Using $I = 2\,\text{A}$, the contribution of one side is

$$B_1 = \frac{4\pi \times 10^{-7} \times 2}{4\pi(2 \times 10^{-2})}(2\sin 60^\circ) = \sqrt{3} \times 10^{-5}\,\text{T}.$$

The three contributions add rather than cancel because their normal directions agree:

$$B = 3B_1 = 3\sqrt{3} \times 10^{-5}\,\text{T}.$$

Thus the single-side result in option (A) omits two sides; (B) and (D) likewise do not include the correct three-side sum.

Step 3: Conclusion Comparing with $B = \alpha \times 10^{-5}\,\text{T}$, we obtain $\boxed{\alpha = 3\sqrt{3}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a regular polygon, find the field of one side at the centre using the apothem, then multiply by the number of sides.

---

44. When an unpolarized light falls at a particular angle on a glass plate (placed in air), it is observed that the reflected beam is linearly polarized. The angle of refracted beam with respect to the normal is ____ .

$(\tan^{-1}(1.52) = 57.7^\circ$, refractive indices of air and glass are 1.00 and 1.52, respectively.) [4 marks]

(A) 32.3°

(B) 36.3°

(C) 39.6°

(D) 42.6°

Correct Answer: (A) 32.3°

Solution:

Concept:

- When the reflected light is completely linearly polarized, the incidence angle is the Brewster angle i_B, with $\tan i_B = n_2/n_1$.
- At Brewster incidence the reflected and refracted rays are perpendicular, giving $i_B + r = 90^\circ$, where r is measured from the normal in the glass.

Step 1: Determine the incident angle The light travels from air, $n_1 = 1.00$, into glass, $n_2 = 1.52$. Therefore

$$\tan i_B = \frac{1.52}{1.00} = 1.52.$$

Using the inverse tangent supplied in the question,

$$i_B = \tan^{-1}(1.52) = 57.7^\circ.$$

This is the angle of the incident ray with the normal; it is not the angle of refraction requested.

Step 2: Use the perpendicular-ray condition The refracted ray is closer to the normal because glass has the larger refractive index. At Brewster incidence,

$$r = 90^\circ - i_B = 90^\circ - 57.7^\circ = 32.3^\circ.$$

The same relation follows from Snell's law: $n_1 \sin i_B = n_2 \sin r$. Substituting $n_2/n_1 = \tan i_B$ gives $\sin r = \cos i_B$, so the acute refracted angle equals $90^\circ - i_B$. The values 36.3°, 39.6° and 42.6° do not satisfy this condition with 57.7°.

Step 3: Conclusion The refracted beam makes $\boxed{32.3^\circ}$ with the normal. Hence, the correct option is (A).

Quick Tip to Solve: At Brewster incidence, first calculate the incident angle with the index ratio, then subtract it from 90° to obtain the refracted angle.

---

45. A prism of angle 75° and refractive index $\sqrt{3}$ is coated with thin film of refractive index 1.5 only at the back exit surface. To have total internal reflection at the back exit surface the incident angle must be ____

($\sin 15^\circ = 0.25$ **and** $\sin 25^\circ = 0.43$) [4 marks]

(A) 15°

(B) $> 25^\circ$

(C) between 15° and 20°

(D) $< 15^\circ$

Correct Answer: (A) 15°

Solution:

Concept:

- Total internal reflection at a boundary from index n_p to smaller index n_f requires internal incidence greater than $C = \sin^{-1}(n_f/n_p)$.
- For a prism of apex angle A, the internal refraction angles satisfy $r_1 + r_2 = A$. At the air-prism entrance, Snell's law is $\sin i = \sqrt{3} \sin r_1$.

Step 1: Find the critical angle at the coating The light meets the film directly at the back face. Use the film index 1.5, rather than the surrounding air index, at this boundary:

$$\sin C = \frac{1.5}{\sqrt{3}} = \frac{\sqrt{3}}{2}, \qquad C = 60^\circ.$$

For total internal reflection, $r_2 > 60^\circ$. As $r_1 + r_2 = 75^\circ$, this gives $r_1 < 15^\circ$.

Step 2: Translate the condition to external incidence At the entrance,

$$\sin i = \sqrt{3} \sin r_1 < \sqrt{3} \sin 15^\circ.$$

Using the supplied rounded value $\sin 15^\circ = 0.25$, the upper limiting sine is about 0.433; thus an incident angle up to roughly 25.7° meets the condition, with equality at the critical limit excluded. The exact trigonometric value gives a slightly higher upper limit, without changing the accepted choices. An incidence of 15°, every incidence between 15° and 20°, and incidences below 15° all satisfy the requirement. The unrestricted statement $i > 25^\circ$ does not guarantee total internal reflection: for example, a sufficiently large incidence violates the inequality.

Step 3: Conclusion An incidence of 15° satisfies the condition. Hence, the correct option is (A). This is a multiple-accepted-answer question: in the printed option order, the final key accepts (A), (C) and (D).

Quick Tip to Solve: Apply the critical-angle condition at the prism-film interface first, then use the prism-angle relation and entrance refraction to test the external angles.

Physics – Section B: Numerical Value Questions (20 marks)

46. The average energy released per fission for the nucleus of $^{235}_{92}\mathrm{U}$ is 190 MeV . When all the atoms of 47 g pure $^{235}_{92}\mathrm{U}$ undergo fission process, the energy released is $\alpha \times 10^{23}\mathrm{MeV}$. The value of α is ____ . (Avogadro Number $= 6 \times 10^{23}$ per mole) [4 marks]

Solution:

Concept:

- The number of nuclei in a pure sample is $N = (m/\mathcal{M})N_A$, where m is the sample mass and $\mathcal{M}$ its molar mass in matching units.
- If every nucleus undergoes one fission, the total released energy equals the number of fissions multiplied by the average energy per fission. Energies may remain in MeV throughout this calculation.

Step 1: Count the uranium nuclei For the isotope $^{235}_{92}\mathrm{U}$, take the molar mass as $235\,\mathrm{g\,mol^{-1}}$. The pure sample contains

$$n = \frac{47\,\mathrm{g}}{235\,\mathrm{g\,mol^{-1}}} = 0.2\,\mathrm{mol}.$$

With the supplied Avogadro constant,

$$N = nN_A = 0.2 \times 6 \times 10^{23} = 1.2 \times 10^{23}.$$

This is the number of uranium nuclei, and therefore the number of fissions under the stated complete-fission condition.

Step 2: Multiply by the energy per fission Each fission releases an average of $190\,\mathrm{MeV}$, so

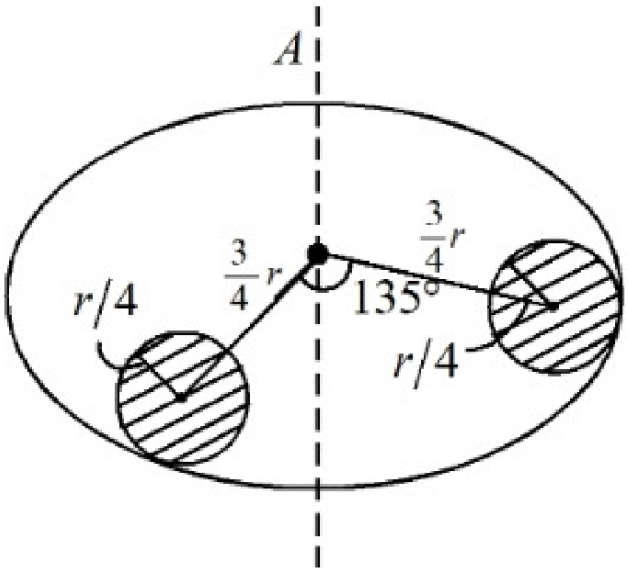
$$E = (1.2 \times 10^{23})(190\,\mathrm{MeV}) = 228 \times 10^{23}\,\mathrm{MeV}.$$

Equivalently, $E = 2.28 \times 10^{25}\,\mathrm{MeV}$. The coefficient requested is tied specifically to $10^{23}\,\mathrm{MeV}$, so it is 228, not the coefficient 2.28 of the normalized scientific-notation form. No MeV-to-joule conversion is needed because the answer is requested in MeV.

Step 3: Conclusion The released energy is $\boxed{228 \times 10^{23}\,\mathrm{MeV}}$. Therefore, $\boxed{\alpha = 228}$.

Quick Tip to Solve: Keep the energy unit supplied and match the final power of ten to the question before extracting its numerical coefficient.

47. Suppose there is a uniform circular disc of mass M kg and radius r m shown in figure. The shaded regions are cut out from the disc. The moment of inertia of the remainder about the axis A of the disc is given by $\frac{x}{256}Mr^2$. The value of x is ____ . [4 marks]



Solution:

Concept:

- For a uniform disc about its central perpendicular axis, $I_0 = Mr^2/2$. Removed portions are subtracted about that same axis.
- Uniform surface density makes mass proportional to area. For each circular cut-out, the parallel-axis theorem gives $I_{\text{cut}} = \frac{1}{2}ma^2 + md^2$, with radius a and centre offset d.

Step 1: Determine the mass and position of each cut-out The diagram shows two circular holes, each of radius $a = r/4$, whose centres are each a distance $d = 3r/4$ from the original disc centre. Since the original mass is M, each removed mass is

$$m = M\frac{\pi(r/4)^2}{\pi r^2} = \frac{M}{16}.$$

Axis A is the central axis perpendicular to the disc, as drawn in perspective. The angle $135°$ between the two centre positions does not change either perpendicular distance to this axis.

Step 2: Calculate the inertia removed Use the parallel-axis theorem for one hole:

$$\begin{aligned} I_{\text{cut}} &= \frac{1}{2}\frac{M}{16}\left(\frac{r}{4}\right)^2 + \frac{M}{16}\left(\frac{3r}{4}\right)^2 \\ &= \frac{Mr^2}{512} + \frac{9Mr^2}{256} = \frac{19Mr^2}{512}. \end{aligned}$$

The two identical holes therefore remove $19Mr^2/256$. It would be incorrect to use only their central moments, because both hole centres are displaced from axis A.

Step 3: Subtract from the original disc Keep M as the mass of the original complete disc:

$$I_A = \frac{1}{2}Mr^2 - \frac{19}{256}Mr^2 = \frac{128-19}{256}Mr^2 = \frac{109}{256}Mr^2.$$

The remaining mass is smaller than M, but the expression in the question explicitly uses the original M, so no rescaling of the final coefficient is needed.

Step 4: Conclusion The remainder has $\boxed{I_A = \frac{109}{256}Mr^2}$, in $\mathrm{kg\,m^2}$ when M and r use SI units. Hence, $\boxed{x = 109}$.

Quick Tip to Solve: For holes in a uniform lamina, obtain their masses from area ratios and subtract their parallel-axis moments about the requested fixed axis.

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48. A ball of radius r and density ρ dropped through a viscous liquid of density σ and viscosity η attains its terminal velocity at time t, given by $t = A\rho^a r^b \eta^c \sigma^d$, where A is a constant and a, b, c and d are integers. The value of $\frac{b+c}{a+d}$ is ____ . [4 marks]

Solution:

Concept:

- Dimensional homogeneity requires the dimensions of both sides of $t = A\rho^a r^b \eta^c \sigma^d$ to match, with A dimensionless.
- Both densities have dimensions $\mathsf{ML^{-3}}$, while dynamic viscosity has dimensions $\mathsf{ML^{-1}T^{-1}}$. Consequently dimensional analysis determines $a + d$, rather than a and d separately.

Step 1: Write and collect the dimensions Use $[t] = \mathsf{T}$, $[r] = \mathsf{L}$, $[\rho] = [\sigma] = \mathsf{ML^{-3}}$ and $[\eta] = \mathsf{ML^{-1}T^{-1}}$. Then

$$\mathsf{T} = (\mathsf{ML^{-3}})^a \mathsf{L}^b (\mathsf{ML^{-1}T^{-1}})^c (\mathsf{ML^{-3}})^d.$$

Collecting powers gives

$$\mathsf{T} = \mathsf{M}^{a+c+d}\mathsf{L}^{-3a+b-c-3d}\mathsf{T}^{-c}.$$

The mass and length exponents must be zero, and the time exponent must be one.

Step 2: Solve only the combinations needed Equating powers of time gives $-c = 1$, so $c = -1$. The mass equation then yields

$$a + c + d = 0 \quad \Rightarrow \quad a + d = 1.$$

Using this in the length equation,

$$-3(a+d) + b - c = 0 \quad \Rightarrow \quad -3 + b + 1 = 0,$$

so $b = 2$. There is no need to assign separate values to the density exponents: their sum is enough for the requested expression.

Step 3: Evaluate the dimensionless ratio Substitute the determined combinations:

$$\frac{b+c}{a+d} = \frac{2+(-1)}{1} = 1.$$

The denominator is nonzero. Dimensional analysis fixes this ratio even though it cannot distinguish the contributions of two quantities with identical dimensions.

Step 4: Conclusion The required dimensionless numerical value is $\boxed{1}$.

Quick Tip to Solve: When two variables have identical dimensions, group their exponents immediately; the requested result may depend only on their sum.

---

49. The velocity of sound in air is doubled when the temperature is raised from $0°\mathrm{C}$ to $\alpha°\mathrm{C}$. The value of α is ____ . [4 marks]

Solution:

Concept:

- For an ideal gas of fixed composition, the speed of sound is $v = \sqrt{\gamma RT/M}$, where T is the absolute temperature, M the molar mass and $\gamma = C_p/C_v$.
- With the gas properties unchanged, $v \propto \sqrt{T}$. A ratio of sound speeds must therefore use kelvin temperatures, not Celsius temperatures.

Step 1: Write the ratio in kelvin The initial temperature is $0\,°\mathrm{C}$, corresponding to $T_1 = 273\,\mathrm{K}$. The final temperature is $\alpha\,°\mathrm{C}$, so $T_2 = (273+\alpha)\,\mathrm{K}$. Dividing the two speed expressions cancels γ, R and M:

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{273+\alpha}{273}}.$$

The statement that the velocity is doubled means $v_2/v_1 = 2$, not that the temperature itself is doubled.

Step 2: Square the ratio and solve Squaring both sides makes the final absolute temperature four times the initial absolute temperature:

$$4 = \frac{273+\alpha}{273}, \qquad 273+\alpha = 1092.$$

Therefore,

$$\alpha = 1092 - 273 = 819.$$

As a check, $\sqrt{1092/273} = \sqrt{4} = 2$, which reproduces the given speed change. The increase of $819\,\mathrm{K}$ has the same numerical value as the Celsius increase, but the final absolute temperature is $1092\,\mathrm{K}$. A linear small-temperature approximation for sound speed is inappropriate for such a large increase.

Step 3: Conclusion The required temperature is $\boxed{\alpha = 819\,°\mathrm{C}}$. The numerical entry is $\boxed{819}$.

Quick Tip to Solve: For a speed increase by a factor n, multiply the absolute temperature by n^2, then convert back to Celsius.

50. The size of the images of an object, formed by a thin lens are equal when the object is placed at two different positions 8 cm and 24 cm from the lens. The focal length of the lens is ____ cm. [4 marks]

Solution:

Concept:

- The Cartesian thin-lens formula is $1/f = 1/v - 1/u$, and the signed linear magnification is $m = v/u$. Equal image sizes mean equal magnitudes $|m|$, allowing opposite signs.
- For a real object at distance $d > 0$, $u = -d$. Combining the two formulae gives $v = fd/(d - f)$ and $m = f/(f - d)$.

Step 1: Express the two magnifications The two object distances are $d_1 = 8\,\text{cm}$ and $d_2 = 24\,\text{cm}$. The images have equal size for the same object, so

$$\left|\frac{f}{f-8}\right| = \left|\frac{f}{f-24}\right|.$$

A thin lens has a finite nonzero focal length. Canceling $|f|$ gives $|f - 8| = |f - 24|$. The distances from the focal point to the two object positions must therefore be equal. Equating signed magnifications directly would incorrectly discard the real-image/virtual-image possibility.

Step 2: Solve and verify both image distances Square the equation and expand:

$$(f-8)^2 = (f-24)^2, \qquad -16f + 64 = -48f + 576.$$

Thus $32f = 512$, giving $f = 16\,\text{cm}$. At $d = 8\,\text{cm}$,

$$v = \frac{16 \times 8}{8 - 16} = -16\,\text{cm}, \qquad m = +2.$$

At $d = 24\,\text{cm}$,

$$v = \frac{16 \times 24}{24 - 16} = 48\,\text{cm}, \qquad m = -2.$$

One image is virtual and erect, while the other is real and inverted. Both are twice the object's size, exactly as required. The positive focal length identifies a convex lens; a concave lens cannot provide these equal magnifications at distinct real-object distances.

Step 3: Conclusion The focal length is $\boxed{16\,\text{cm}}$; hence the numerical entry is $\boxed{16}$.

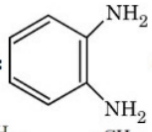
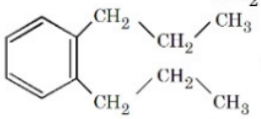
Quick Tip to Solve: For equal image sizes at two distinct real-object distances, the convex lens has $f = (d_1 + d_2)/2$; the images have opposite orientations.

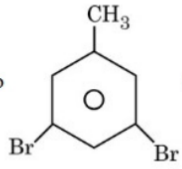
Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Given below are two statements:

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

Statement I:  can be synthesized from  using simpler reagents in the order i) Acidic $KMnO_4$, ii) Ammonia, iii) Bromine and alkali

Statement II:  can be converted into  using reagents in the order i) Bromine-H_2O ii) $NaNO_2$/HCl (0 - 5 °C) (iii) Aq. H_3PO_2 .

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- An alkyl side chain with benzylic hydrogen is oxidised by hot acidic $KMnO_4$ to a carboxyl group. Primary amides give amines with one fewer carbon atom in the Hofmann bromamide reaction.
- The $-NH_2$ group strongly activates its ortho and para ring positions. Diazotisation followed by aqueous H_3PO_2 replaces an aromatic amino group by hydrogen.

Step 1: Follow the conversions in Statement I Both propyl substituents contain benzylic hydrogen, so their oxidation gives adjacent carboxyl groups on the ring, forming benzene-1,2-dicarboxylic acid. Conversion of the acid groups to primary amide groups using ammonia gives the corresponding diamide. Each $-CONH_2$ group then undergoes Hofmann rearrangement with bromine and alkali:

$$ArCONH_2 + Br_2 + 4\,NaOH \longrightarrow ArNH_2 + 2\,NaBr + Na_2CO_3 + 2\,H_2O.$$

The carbonyl carbon is removed in each conversion, while the two points of attachment to the benzene ring stay adjacent. Thus the required product is benzene-1,2-diamine, and Statement I is true.

Step 2: Track substitution in Statement II In p-toluidine, the para position relative to $-NH_2$ is already occupied by methyl. Bromine water therefore substitutes both free ortho positions relative to $-NH_2$. Relative to the methyl carbon, these are positions 3 and 5. Treatment with $NaNO_2$/HCl at 0–$5\,^{\circ}C$

converts $-NH_2$ into a diazonium group. Aqueous H_3PO_2 then replaces that group by hydrogen without removing the two bromines. The product is 3,5-dibromotoluene, so Statement II is also true. Option B incorrectly rejects both routes; C rejects the valid bromination/deamination sequence; D rejects the valid oxidation and Hofmann sequence.

Step 3: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: In bromination followed by deamination, first mark the positions directed by $-NH_2$; remove the amino group only after placing the bromines.

52. In Carius method 0.2425 g of an organic compound gave 0.5253 g silver chloride. The percentage of chlorine in the organic compound is [4 marks]

(A) 37.57%

(B) 34.79%

(C) 53.58%

(D) 87.65%

Correct Answer: (C) 53.58%

Solution:

Concept:

- In the Carius method, chlorine in an organic compound is converted quantitatively into AgCl. Each mole of AgCl contains one mole of chlorine atoms.
- The chlorine mass fraction in AgCl is $M_{Cl}/M_{AgCl} = 35.5/143.5$, using $M_{Ag} = 108\,g\,mol^{-1}$.

Step 1: Find the chlorine in the precipitate The precipitate mass is $0.5253\,g$. Only the chlorine component of this mass came from the chlorine in the organic sample; the silver was supplied by the reagent. Therefore,

$$m_{Cl} = 0.5253 \times \frac{35.5}{108 + 35.5} = 0.12995\,g \quad \text{(approximately)}.$$

This is also obtained by first finding the moles of AgCl, $0.5253/143.5$, and multiplying by the chlorine molar mass. A precipitate heavier than the original sample is entirely possible because it contains reagent-derived silver.

Step 2: Calculate the percentage and compare choices Divide the chlorine mass by the organic sample mass, $0.2425\,g$, and multiply by 100:

$$\%Cl = \frac{0.5253 \times 35.5}{143.5 \times 0.2425} \times 100 = 53.58\%.$$

The result lies below 100% and corresponds to approximately $0.1300\,g$ of chlorine in the sample. Option A, 37.57%, and option B, 34.79%, would each imply substantially less chlorine than the measured precipitate contains. Option D, 87.65%, would imply about $0.2126\,g$ of chlorine and therefore too much AgCl. Only option C obeys the one-to-one chloride stoichiometry and the measured masses.

Step 3: Conclusion The chlorine content is $\boxed{53.58\%}$. Hence, the correct option is (C).

Quick Tip to Solve: For a silver halide precipitate, multiply its mass by the halogen-to-silver-halide molar-mass ratio before dividing by the sample mass.

53. It is noticed that Pb^{2+} is more stable than Pb^{4+} but Sn^{2+} is less stable than Sn^{4+}. Observe the following reactions.

$$PbO_2 + Pb \rightarrow 2PbO; \Delta_r G^\circ(1)$$
$$SnO_2 + Sn \rightarrow 2SnO; \Delta_r G^\circ(2)$$

Identify the correct set from the following [4 marks]

(A) $\Delta_r G^\circ(1) < 0; \quad \Delta_r G^\circ(2) > 0$

(B) $\Delta_r G^\circ(1) < 0; \quad \Delta_r G^\circ(2) < 0$

(C) $\Delta_r G^\circ(1) > 0; \quad \Delta_r G^\circ(2) > 0$

(D) $\Delta_r G^\circ(1) > 0; \quad \Delta_r G^\circ(2) < 0$

Correct Answer: (A) $\Delta_r G^\circ(1) < 0; \quad \Delta_r G^\circ(2) > 0$

Solution:

Concept:

- The inert-pair effect stabilises the lower oxidation state down group 14. Thus lead favours $+2$, whereas tin favours $+4$ over $+2$.
- A reaction favoured in the stated direction has $\Delta_r G^\circ < 0$; the reverse of a favoured reaction has $\Delta_r G^\circ > 0$.

Step 1: Assess the lead reaction In PbO_2, lead is in oxidation state $+4$, while elemental lead has oxidation state zero. In PbO, lead has oxidation state $+2$. The reaction

$$PbO_2 + Pb \longrightarrow 2\,PbO$$

combines the higher and lower states to form the more stable intermediate state, a comproportionation. Because Pb^{2+} is favoured relative to Pb^{4+}, the printed direction is favourable:

$$\Delta_r G^\circ(1) < 0.$$

The positive sign in options C and D is therefore inconsistent with the lead trend.

Step 2: Assess the tin reaction and select the sign pair Tin(II) is less stable relative to tin(IV). Tin(II) oxide tends towards disproportionation into tin metal and tin(IV) oxide:

$$2\,SnO \longrightarrow SnO_2 + Sn.$$

The printed reaction SnO_2 + Sn $\longrightarrow$ 2 SnO is the reverse of this favoured direction, so

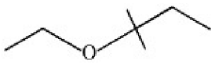
$$\Delta_r G^\circ(2) > 0.$$

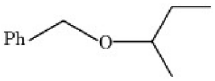
The desired pair is therefore negative for lead and positive for tin. Option B incorrectly makes tin comproportionation favourable; C has both signs positive; D reverses both required signs. These signs follow the relative oxidation-state stability given in the question, without assuming the same behaviour for every element in the group.

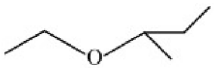
Step 3: Conclusion The required signs are $\Delta_r G^\circ(1) < 0$ and $\Delta_r G^\circ(2) > 0$. Hence, the correct option is (A).

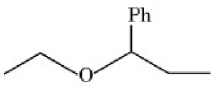
Quick Tip to Solve: Check whether the printed equation forms or consumes the stable oxidation state; reversing an equation reverses the sign of its Gibbs-energy change.

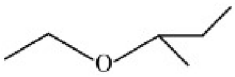
54. A mixed ether (P), when heated with excess of hot concentrated hydrogen iodide produces two different alkyl iodides which when treated with aq. NaOH give compounds (Q) and (R). Both (Q) and (R) give yellow precipitate with NaOI . Identify the mixed ether (P) : [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- Excess hot concentrated HI cleaves an ether into the two corresponding alkyl iodides. Aqueous NaOH replaces iodide by hydroxyl to give the corresponding alcohols.
- Ethanol and secondary alcohols containing $CH_3-CH(OH)-R$ give the iodoform test. Oxidation produces ethanal or a methyl ketone, which yields yellow CHI_3 with hypoiodite.

Step 1: Identify the two alcohols required Both fragments of the ether must yield alcohols that pass the iodoform test. In option C, the two alkyl groups are ethyl and sec-butyl. Cleavage and hydrolysis give ethanol and butan-2-ol:

$$CH_3CH_2-O-CH(CH_3)CH_2CH_3 + 2\,HI$$

$$\longrightarrow CH_3CH_2I + CH_3CH(I)CH_2CH_3 + H_2O.$$

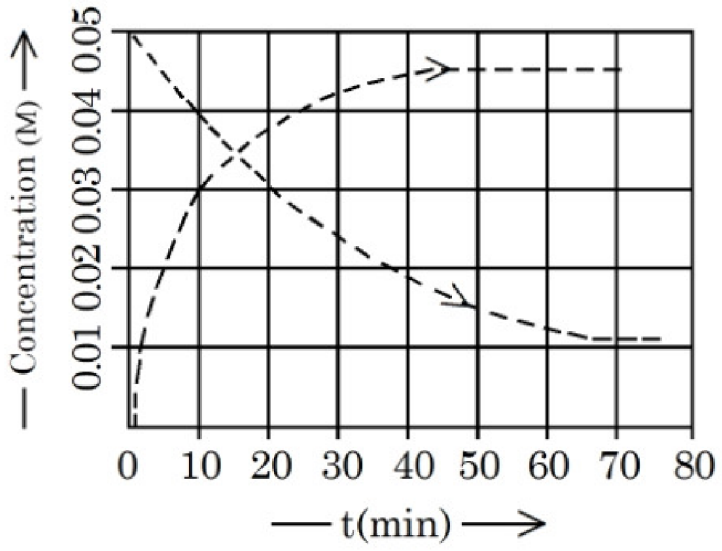
Aqueous alkali gives CH_3CH_2OH and $CH_3CH(OH)CH_2CH_3$. Ethanol oxidises to ethanal, while butan-2-ol oxidises to butan-2-one. Both consequently form yellow iodoform with NaOI.

Step 2: Reject the remaining ether structures Option A gives ethanol and 2-methylbutan-2-ol. The latter is a tertiary alcohol and lacks the required $CH_3-CH(OH)^-$ unit, so both products do not pass. Option B gives benzyl alcohol and butan-2-ol: benzyl alcohol oxidises to benzaldehyde, which has no methyl-carbonyl group, so it fails. Option D gives ethanol and 1-phenylpropan-1-ol, $C_6H_5CH(OH)CH_2CH_3$. The secondary alcohol gives propiophenone, $C_6H_5COCH_2CH_3$, rather than a methyl ketone, so it also fails. Being a secondary alcohol alone is insufficient; a methyl group must be bonded directly to its hydroxyl-bearing carbon. Only option C satisfies the requirement for both fragments.

Step 3: Conclusion The mixed ether is ethyl sec-butyl ether, $CH_3CH_2-O-CH(CH_3)-CH_2CH_3$. Hence, the correct option is (C).

Quick Tip to Solve: Cut the ether at oxygen and convert both carbon fragments into alcohols; each must be ethanol or contain $CH_3-CH(OH)-R$.

55. Given above is the concentration vs time plot for a dissociation reaction : $\mathrm{A} \rightarrow \mathrm{nB}$.
Based on the data of the initial phase of the reaction (initial 10 min), the value of n is ____ . [4 marks]



(A) 4
(B) 5
(C) 3
(D) 2

Correct Answer: (C) 3

Solution:

Concept:

- For $\mathrm{A} \longrightarrow \mathrm{nB}$ at constant volume, the increase in the concentration of B is n times the decrease in that of A: $\Delta[B] = -n\Delta[A]$.
- A reactant concentration decreases with time, whereas a product concentration increases. Stoichiometric coefficients are obtained from concentration changes over the same time interval, not from the ratio of the concentrations themselves.

Step 1: Read the initial ten-minute changes The descending curve belongs to A. At $t = 0$, $[A] = 0.05\,\mathrm{mol\,L^{-1}}$, and at $t = 10\,\mathrm{min}$, $[A] = 0.04\,\mathrm{mol\,L^{-1}}$. The ascending curve belongs to B, whose concentration increases from zero to $0.03\,\mathrm{mol\,L^{-1}}$ during the same interval. Thus,

$$-\Delta[A] = 0.05 - 0.04 = 0.01\,\mathrm{mol\,L^{-1}}, \qquad \Delta[B] = 0.03 - 0 = 0.03\,\mathrm{mol\,L^{-1}}.$$

Step 2: Apply the stoichiometric ratio Every mole of A consumed produces n moles of B. Therefore,

$$n = \frac{\Delta[B]}{-\Delta[A]} = \frac{0.03}{0.01} = 3.$$

The average disappearance rate of A is $0.001\,\mathrm{mol\,L^{-1}\,min^{-1}}$, while the average formation rate of B is $0.003\,\mathrm{mol\,L^{-1}\,min^{-1}}$, confirming the same ratio. Values 4, 5 and 2 would instead predict B

concentrations of 0.04, 0.05 and $0.02\,\text{mol}\,\text{L}^{-1}$, respectively, after this consumption of A; none agrees with the plotted initial interval.

Step 3: Conclusion The dissociation produces three moles of B per mole of A consumed, so $n = 3$. Hence, the correct option is (C).

Quick Tip to Solve: Read both curves at the same two times and divide product formed by reactant consumed; use only the interval requested.

56. The work functions of two metals $(\text{M}_\text{A}$ and $\text{M}_\text{B})$ are in the $1 : 2$ ratio. When these metals are exposed to photons of energy 6 eV , the kinetic energy of liberated electrons of $M_A : M_B$ is in the ratio of $2.642 : 1$. The work functions (in eV) of M_A and M_B are respectively. [4 marks]

(A) $2.3, 4.6$

(B) $3.1, 6.2$

(C) $1.5, 3.0$

(D) $1.4, 2.8$

Correct Answer: (A) $2.3, 4.6$

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = E_{\text{photon}} - \phi$, where ϕ is the metal's work function, the minimum energy needed to remove an electron.
- The two metals receive photons of the same energy. Their different kinetic energies therefore arise from their different work functions; emission requires $E_{\text{photon}} > \phi$.

Step 1: Express both work functions using one variable Since $\phi_A : \phi_B = 1 : 2$, let $\phi_A = x\,\text{eV}$ and $\phi_B = 2x\,\text{eV}$. Subtracting each work function from the incident energy gives

$$K_A = (6 - x)\,\text{eV}, \qquad K_B = (6 - 2x)\,\text{eV}.$$

Metal B must have a positive kinetic energy, so $x < 3$. Using the stated ratio,

$$\frac{6 - x}{6 - 2x} = 2.642.$$

Step 2: Solve and check the listed pairs Cross-multiplication yields

$$6 - x = 15.852 - 5.284x, \qquad 4.284x = 9.852, \qquad x \approx 2.300\,\text{eV}.$$

Thus the pair is $2.3, 4.6\,\text{eV}$. It gives kinetic energies 3.7 and $1.4\,\text{eV}$, whose ratio is $2.642857\ldots$, consistent with the quoted approximate value. Pair (B) would require $6.2\,\text{eV}$ to remove an electron from metal B, exceeding the incident energy. Pair (C) gives $4.5/3 = 1.5$, and pair (D) gives $4.6/3.2 = 1.4375$; neither matches the stated ratio.

Step 3: Conclusion The work functions are $\phi_A = 2.3\,\text{eV}$ and $\phi_B = 4.6\,\text{eV}$, respectively. Hence, the

correct option is (A).

Quick Tip to Solve: Subtract the work function from photon energy before taking a kinetic-energy ratio; also reject any work function above the incident energy.

57. Observe the following reactions at $\mathrm{T(K)}$.

I. $\mathrm{A} \rightarrow$ **products.**

II. $5\mathrm{Br}^-(\mathrm{aq}) + \mathrm{BrO_3}^-(\mathrm{aq}) + 6\mathrm{H}^+(\mathrm{aq}) \rightarrow 3\mathrm{Br_2}(\mathrm{aq}) + 3\mathrm{H_2O}(\mathrm{l})$

Both the reactions are started at 10.00 am . The rates of these reactions at 10.10 am are same. The value of $-\frac{\Delta[\mathrm{Br}^-]}{\Delta \mathrm{t}}$ at 10.10 am is $2 \times 10^{-4}\ \mathrm{mol\ L^{-1}\ min^{-1}}$. The concentration of A at 10.10 am is $10^{-2}\ \mathrm{mol\ L^{-1}}$. What is the first order rate constant (in $\mathrm{min^{-1}}$) of reaction I ? [4 marks]

(A) 4×10^{-3}

(B) 2×10^{-3}

(C) 10^{-2}

(D) 10^{-3}

Correct Answer: (A) 4×10^{-3}

Solution:

Concept:

- The rate of a reaction is obtained by dividing a species' disappearance or formation rate by its stoichiometric coefficient. For bromide in reaction II, $r_{\mathrm{II}} = -\frac{1}{5}\Delta[\mathrm{Br}^-]/\Delta t$.
- For a first-order reaction $\mathrm{A} \longrightarrow$ products, its rate is $r_{\mathrm{I}} = k[A]$. Therefore the rate constant is the rate divided by the concentration at the same instant.

Step 1: Convert bromide disappearance into reaction rate The balanced equation consumes five moles of bromide for each stoichiometric occurrence of reaction II. Thus the numerical bromide disappearance rate is five times the reaction rate. At 10.10 am,

$$r_{\mathrm{II}} = \frac{1}{5}\left(2 \times 10^{-4}\right) = 4 \times 10^{-5}\,\mathrm{mol\,L^{-1}\,min^{-1}}.$$

The two reaction rates are stated to be equal at this time, so r_{I} also has this value. Equating r_{I} directly to bromide disappearance would omit the coefficient 5.

Step 2: Use the first-order rate law The concentration of A at the same time is $10^{-2}\,\mathrm{mol\,L^{-1}}$. Hence,

$$k = \frac{r_{\mathrm{I}}}{[A]} = \frac{4 \times 10^{-5}}{10^{-2}} = 4 \times 10^{-3}\,\mathrm{min^{-1}}.$$

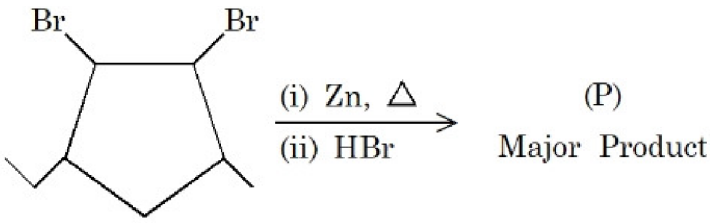
Concentration units cancel, leaving inverse minutes as required for a first-order constant. The other proposed constants, 2×10^{-3}, 10^{-2}, and $10^{-3}\,\mathrm{min^{-1}}$, would predict reaction rates of 2×10^{-5}, 10^{-4}, and $10^{-5}\,\mathrm{mol\,L^{-1}\,min^{-1}}$, respectively, rather than the required 4×10^{-5}. The ten-minute elapsed time need not be inserted separately because a rate and concentration at that instant are already supplied.

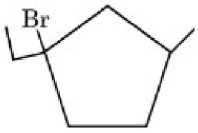
Step 3: Conclusion The first-order rate constant of reaction I is $4 \times 10^{-3}\,\mathrm{min^{-1}}$. Hence, the correct

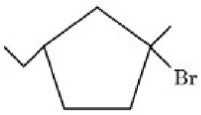
option is (A).

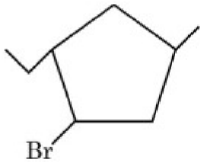
Quick Tip to Solve: Normalize a species rate by its stoichiometric coefficient before comparing reaction rates, then use $k = r/[A]$ for first order.

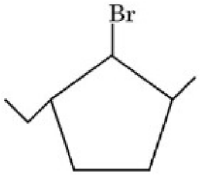
58. Identify (P). [4 marks]

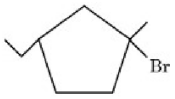


(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- Zinc removes the two bromine atoms of a vicinal dibromide to form a double bond between the carbons that carried them. The carbon skeleton remains unchanged in this dehalogenation.
- Addition of HBr without peroxide proceeds through a carbocation. A neighbouring hydride can migrate when this converts a secondary carbocation into a more stable tertiary carbocation.
- Among comparable alkyl carbocations, hyperconjugation by adjacent carbon–hydrogen bonds helps identify the more stable intermediate and hence the major product.

Step 1: Form the cyclopentene intermediate The bromine atoms occupy two adjacent ring carbons at the top of the starting structure. Treatment with Zn and heat removes them as $ZnBr_2$, creating the double bond along that top edge. The ethyl and methyl groups remain on the neighbouring ring carbons. Thus the first step is dehalogenation, not removal or migration of an alkyl group.

Step 2: Locate the rearranged carbocations Protonation of the double bond by HBr initially gives a secondary carbocation. A 1,2-hydride shift from an adjacent alkyl-bearing ring carbon can move the positive charge to that carbon, giving a tertiary carbocation. The two possibilities place the charge at the methyl-bearing carbon or the ethyl-bearing carbon. Both retain the original five-membered ring and both are more stable than an unrearranged secondary ion.

Step 3: Compare the tertiary ions and trap the major one At the methyl-bearing carbon, the neighbouring methyl group supplies three alpha hydrogens and the two adjacent ring methylene groups supply two each, giving seven alpha hydrogens. At the ethyl-bearing carbon, the ethyl methylene supplies two and the two ring methylene groups supply two each, giving six. Greater hyperconjugative stabilisation favours the former tertiary ion. Attack by Br^- therefore places bromine on the same ring carbon as the methyl group, as in option (B). Option (A) corresponds to the competing, less stabilised tertiary ion; options (C) and (D) retain bromine at secondary ring carbons and do not represent the major rearranged product.

Step 4: Conclusion The major product has bromine and methyl attached to the same ring carbon, with the ethyl group retained at its original ring position. Hence, the correct option is (B).

Quick Tip to Solve: After zinc dehalogenation, check for a hydride shift during ionic HBr addition before choosing the bromide; compare the tertiary carbocations by their adjacent C–H bonds.

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59. Given below are two statements :

Statement I : $(CH_3)_3\overset{\oplus}{C}$ **is more stable than** $\overset{\oplus}{C}H_3$ **as nine hyperconjugation interactions are possible in** $(CH_3)_3\overset{\oplus}{C}$**.**

Statement II : $\overset{\oplus}{C}H_3$ **is less stable than** $(CH_3)_3\overset{\oplus}{C}$ **as only three hyperconjugation interactions are possible in** $\overset{\oplus}{C}H_3$**.**

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- Hyperconjugation stabilizes a carbocation through overlap of an adjacent C–H sigma bond with the empty p orbital on the positively charged carbon. The relevant hydrogens are attached to neighbouring carbon atoms.
- Alkyl groups also donate electron density through the positive inductive effect. Among simple alkyl carbocations, the usual stability order is tertiary, secondary, primary, then methyl.

Step 1: Evaluate Statement I In the tert-butyl carbocation, $(CH_3)_3C^+$, the charged carbon is bonded to three methyl groups. Each neighbouring methyl carbon carries three hydrogens, giving

$$N_{\alpha\text{-H}} = 3 \times 3 = 9.$$

These nine adjacent C–H bonds can participate in hyperconjugative interactions with the empty p orbital. Together with the electron-donating effect of the alkyl groups, this stabilizes the tert-butyl

carbocation relative to the methyl carbocation. Statement I is therefore true.

Step 2: Evaluate Statement II and compare the choices The methyl carbocation, $CH_3{}^+$, has three hydrogens bonded directly to the positively charged carbon. It has no neighbouring carbon and hence no adjacent C–H bonds available for the usual hyperconjugation discussed here:

$$N_{\alpha\text{-H}}(CH_3{}^+) = 0.$$

Its three directly attached hydrogens must not be counted as three hyperconjugative interactions. Although Statement II correctly says methyl is less stable, its proposed reason is false; thus the statement as a whole is false. Option (A) incorrectly calls II true, (B) incorrectly calls I false, and (D) reverses the two truth values.

Step 3: Conclusion Statement I is true but Statement II is false: tert-butyl has nine hyperconjugative interactions, whereas methyl has none of this kind. Hence, the correct option is (C).

Quick Tip to Solve: For carbocation hyperconjugation, count hydrogens on adjacent carbons, never hydrogens attached directly to the charged carbon.

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60. Which statements are NOT TRUE about XeO_2F_2?

A. It has a see-saw shape.

B. Xe has 5 electron pairs in its valence shell in XeO_2F_2.

C. The $O - Xe - O$ bond angle is close to $180°$.

D. The $F - Xe - F$ bond angle is close to $180°$.

E. Xe has 16 valence electrons in XeO_2F_2.

Choose the *correct* answer from the options given below : [4 marks]

(A) B, C and E Only

(B) B and D Only

(C) A and D Only

(D) B, D and E Only

Correct Answer: (A) B, C and E Only

Solution:

Concept:

- In VSEPR theory, a multiple bond counts as one electron domain for predicting geometry, even though it contains more than one shared electron pair. Electron-domain count and total electron-pair count are different.
- The Lewis structure of XeO_2F_2 has two Xe=O double bonds, two Xe–F single bonds and one lone pair on Xe. Its five electron domains have a trigonal-bipyramidal arrangement.

Step 1: Determine the molecular shape and angles The lone pair occupies an equatorial position to reduce repulsion. The two double-bonded oxygen atoms occupy the other equatorial positions, while the fluorine atoms occupy axial positions. Omitting the lone pair from the shape description gives a

see-saw molecule, so A is true. The O–Xe–O angle is associated with two equatorial positions and is near 120°, with distortion from the lone pair; it is not near 180°, so C is false. The axial F–Xe–F angle is close to 180°, so D is true.

Step 2: Count actual electron pairs around xenon The two double bonds contribute four shared pairs, the two single bonds contribute two shared pairs, and the lone pair contributes one further pair. Thus,

$$\text{electron pairs around Xe} = 4 + 2 + 1 = 7, \qquad \text{electrons around Xe} = 2(7) = 14.$$

Statement B confuses five VSEPR domains with five actual electron pairs; it is false. Statement E gives 16 rather than 14 electrons, so it is also false. This count refers to electrons surrounding Xe in the Lewis structure; it does not imply that an isolated Xe atom has 14 valence electrons. Options (B), (C) and (D) all include statement D among the false statements, even though the axial fluorine angle is close to 180°.

Step 3: Conclusion The statements that are not true are B, C and E only. Hence, the correct option is (A).

Quick Tip to Solve: Count each multiple bond once for VSEPR geometry, but count both shared pairs of a double bond when counting electrons around the central atom.

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61. Identify the INCORRECT statements from the following :

A. Notation $^{24}_{12}\mathrm{Mg}$ represents 24 protons and 12 neutrons.

B. Wavelength of a radiation of frequency $4.5 \times 10^{15}\ \mathrm{s}^{-1}$ is $6.7 \times 10^{-8}\ \mathrm{m}$.

C. One radiation has wavelength $= \lambda_1(900\ \mathrm{nm})$ and energy $= \mathrm{E}_1$. Other radiation has wavelength $= \lambda_2(300\ \mathrm{nm})$ and energy $= \mathrm{E}_2 \cdot \mathrm{E}_1 : \mathrm{E}_2 = 3 : 1$.

D. Number of photons of light of wavelength 2000 pm that provides 1 J of energy is 1.006×10^{16}.

Choose the correct answer from the options given below : [4 marks]

(A) A and B Only

(B) A and C Only

(C) A and D Only

(D) B and C Only

Correct Answer: (B) A and C Only

Solution:

Concept:

- In the notation $^{A}_{Z}\mathrm{X}$, the atomic number Z is the number of protons and the mass number A is the total number of protons and neutrons. Hence the neutron count is $A - Z$.
- Electromagnetic radiation obeys $c = \nu\lambda$ and each photon has energy $E = h\nu = hc/\lambda$, where $c = 3.0 \times 10^{8}\,\mathrm{m\,s^{-1}}$ and $h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$.

Step 1: Test the nuclear notation in statement A For $^{24}_{12}\mathrm{Mg}$, the lower number gives 12 protons, while the neutron count is $24 - 12 = 12$. The upper number 24 is the mass number, not the proton count. Therefore statement A is incorrect even though its stated neutron count is right.

Step 2: Calculate the wavelength in statement B Divide the speed of light by the stated frequency:

$$\lambda = \frac{c}{\nu} = \frac{3.0 \times 10^{8}}{4.5 \times 10^{15}} = 6.67 \times 10^{-8}\,\text{m}.$$

This rounds to $6.7 \times 10^{-8}\,\text{m}$, so statement B is correct.

Step 3: Compare photon energies in statement C Photon energy is inversely proportional to wavelength. Thus

$$\frac{E_1}{E_2} = \frac{\lambda_2}{\lambda_1} = \frac{300}{900} = \frac{1}{3}.$$

The actual ratio is $E_1 : E_2 = 1 : 3$, not $3 : 1$. Radiation of wavelength 900 nm has the lower photon energy. Statement C is therefore incorrect.

Step 4: Count the photons in statement D Convert the wavelength before calculating the photon energy:

$$2000\,\text{pm} = 2.000 \times 10^{-9}\,\text{m}, \qquad E_{\text{ph}} = \frac{hc}{\lambda} = 9.939 \times 10^{-17}\,\text{J}.$$

The number of such photons required for 1 J is

$$N = \frac{1}{9.939 \times 10^{-17}} \simeq 1.006 \times 10^{16}.$$

Statement D is correct. Options (A) and (D) wrongly include B among the incorrect statements, and option (C) wrongly includes D while omitting the incorrect statement C.

Step 5: Conclusion Only statements A and C are incorrect: magnesium has 12 protons, and the energy ratio is $1 : 3$. Hence, the correct option is (B).

Quick Tip to Solve: Read the lower nuclide index as the proton count, and reverse the wavelength ratio when comparing photon energies; convert picometres to metres before counting photons.

62. Both human DNA and RNA are chiral molecules. The chirality in DNA and RNA arises due to the presence of [4 marks]

(A) D-sugar component

(B) L-sugar component

(C) Base unit

(D) Chiral phosphate unit

Correct Answer: (A) D-sugar component

Solution:

Concept:

- A chiral molecule is not superimposable on its mirror image. The pentose sugars in nucleotides contain stereogenic carbon atoms bonded to four different groups.
- RNA contains D-ribose, whereas DNA contains 2-deoxy-D-ribose. The D designation describes relative configuration; it does not by itself specify the sign of optical rotation.

Step 1: Identify the sugars in the two nucleic acids A nucleotide contains a nitrogenous base, a pentose sugar and a phosphate group. In RNA the sugar is D-ribose, usually present in its furanose ring form. In DNA the corresponding sugar is 2-deoxy-D-ribose. Removal of the oxygen at the second carbon does not remove every stereogenic centre, so the deoxyribose component remains chiral. Both polymers therefore contain sugars with the D configuration.

Step 2: Locate the origin of molecular chirality The sugar carbon skeleton has an asymmetric arrangement of its attached groups. When sugars join bases and phosphate groups to form nucleotides, their stereogenic centres retain this configurational information. Repetition of these chiral sugar units gives the nucleic-acid backbone a handed molecular environment. Thus the structural component common to both DNA and RNA that accounts for the stated chirality is the D-sugar component.

Step 3: Reject the other proposed components Option (B) specifies L-sugar, whereas the natural sugars in human DNA and RNA are D sugars. The usual nitrogenous bases are planar heterocyclic units and are not the source of the sugar stereocentres, rejecting (C). The ordinary phosphate unit is not a chiral phosphate group of the kind proposed in (D). Helical organisation is consistent with the handedness of the polymer, but the question asks which chemical component introduces the chirality.

Step 4: Conclusion Both nucleic acids contain chiral D-pentose sugar units. Hence, the correct option is (A).

Quick Tip to Solve: Identify the pentose first: RNA has D-ribose and DNA has 2-deoxy-D-ribose; deoxygenation at C2 does not make the entire sugar achiral.

63. The oxidation state of chromium in the final product formed in the reaction between KI and acidified $K_2Cr_2O_7$ solution is : [4 marks]

(A) $+4$

(B) $+6$

(C) $+2$

(D) $+3$

Correct Answer: (D) $+3$

Solution:

Concept:

- Acidified dichromate is an oxidising agent: it accepts electrons while iodide ions lose electrons to form iodine.
- In $Cr_2O_7^{2-}$, oxygen has oxidation number -2. If chromium has oxidation number x, then $2x + 7(-2) = -2$, giving $x = +6$.

Step 1: Write the reduction half-reaction The stable chromium product of dichromate reduction in acidic solution is the chromium(III) ion. Balance oxygen using water and hydrogen using hydrogen ions, then balance charge using electrons:

$$Cr_2O_7^{2-} + 14\,H^+ + 6\,e^- \longrightarrow 2\,Cr^{3+} + 7\,H_2O.$$

The two chromium atoms each accept three electrons, so six electrons are consumed in total. Chromium is consequently reduced from $+6$ to $+3$.

Step 2: Balance the iodide oxidation and combine Potassium iodide supplies I^-; potassium is a spectator ion. Iodide undergoes the oxidation

$$2\,I^- \longrightarrow I_2 + 2\,e^-.$$

Multiply this by three to cancel the six electrons in the reduction half-reaction. The net ionic equation is

$$Cr_2O_7{}^{2-} + 14\,H^+ + 6\,I^- \longrightarrow 2\,Cr^{3+} + 3\,I_2 + 7\,H_2O.$$

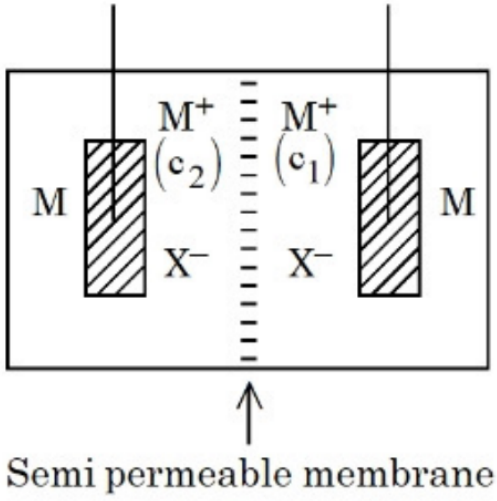
The charge is $+6$ on each side, and all atoms balance. This confirms the identity and oxidation state of the final chromium product.

Step 3: Compare the proposed oxidation states Option (B), $+6$, describes chromium in the initial dichromate ion rather than the reduced product. Neither $+4$ in option (A) nor $+2$ in option (C) is the final chromium oxidation state in the stated acidified-dichromate oxidation of iodide. The product is Cr^{3+}, whose charge directly gives oxidation number $+3$.

Step 4: Conclusion The oxidation state of chromium in the final product is $+3$. Hence, the correct option is (D).

Quick Tip to Solve: For acidified dichromate reactions, write the $Cr_2O_7{}^{2-}/Cr^{3+}$ half-reaction first; distinguish the starting $+6$ state from the final $+3$ state.

64. Consider the above electrochemical cell where a metal electrode (M) is undergoing redox reaction by forming M^+ $(M \rightarrow M^+ + e^-)$. The cation M^+ is present in two different concentrations c_1 and c_2 as shown above. Which of the following statement is correct for generating a positive cell potential? [4 marks]



(A) If c_1 is present at cathode, then $c_1 < c_2$.

(B) If c_1 is present at anode, then $c_1 > c_2$.

(C) If c_1 is present at cathode, then $c_1 > c_2$.

(D) If c_1 is present at anode, then $c_1 = c_2$.

Correct Answer: (C) If c_1 is present at cathode, then $c_1 > c_2$.

Solution:

Concept:

- The two electrodes contain the same metal, so their standard reduction potentials cancel. The cell voltage depends on the ratio of metal-ion activities, approximated here by concentrations.
- For $M^+ + e^- \longrightarrow M$, the Nernst equation is $E = E^\circ + (RT/F)\ln[M^+]$, since the activity of the solid metal is unity.

Step 1: Compare the reduction potentials Because R, T and F are positive, increasing the metal-ion concentration increases the reduction potential of a metal electrode. In a spontaneously operating cell, the electrode with higher reduction potential acts as the cathode, where M^+ is reduced to metal. The less concentrated half-cell acts as the anode, where dissolution of metal increases the ion concentration.

Step 2: Derive the condition for a positive voltage The diagram labels the right half-cell c_1 and the left half-cell c_2; neither location alone determines electrode polarity. Let the cathode and anode concentrations be c_{cath} and c_{an}. Subtract their reduction potentials:

$$\begin{aligned} E_{\text{cell}} &= E_{\text{cath}} - E_{\text{an}} \\ &= \frac{RT}{F}\ln\left(\frac{c_{\text{cath}}}{c_{\text{an}}}\right). \end{aligned}$$

A positive cell potential therefore requires $c_{\text{cath}} > c_{\text{an}}$. When c_1 is at the cathode and c_2 at the anode, this becomes $c_1 > c_2$. Discharge consumes ions on the concentrated side and generates ions on the dilute side, tending to equalise the concentrations.

Step 3: Check all four alternatives Option (A) places the smaller concentration at the cathode, making the displayed potential difference negative. Option (B) places the larger concentration at the anode, again reversing the spontaneous assignment. Option (C) places the larger concentration at the cathode and gives a positive logarithm. In (D), equal concentrations make the logarithm of the ratio zero, so no concentration-cell voltage is generated.

Step 4: Conclusion For c_1 at the cathode, a positive cell potential requires $c_1 > c_2$. Hence, the correct option is (C).

Quick Tip to Solve: In a metal/metal-ion concentration cell, the concentrated solution is the cathode; metal dissolution at the dilute anode helps equalise concentrations.

65. A student has been given a compound " x " of molecular formula- C_6H_7N. ' x ' is sparingly soluble in water. However, on addition of dilute mineral acid, ' x ' becomes soluble in water. ' x ' when treated with $CHCl_3$ and $KOH(alc)$, ' y ' is produced. ' y ' has a specific unpleasant smell. On treatment with benzenesulphonyl chloride, ' x ' gives a compound ' z ' which is soluble in alkali. The number of different " H " atoms present in ' z ' is:- [4 marks]

(A) 5

(B) 4

(C) 7

(D) 8

Correct Answer: (C) 7

Solution:

Concept:

- The carbylamine reaction is given by primary amines: heating with $CHCl_3$ and alcoholic KOH gives an unpleasant-smelling isocyanide.
- A primary amine reacts with benzenesulphonyl chloride to form a sulphonamide retaining one N–H bond. This hydrogen is acidic enough for the product to dissolve in alkali.
- Chemically equivalent hydrogen atoms form one environment. Each monosubstituted phenyl ring has an ortho pair, a meta pair and one para hydrogen.

Step 1: Identify the starting amine The formula C_6H_7N, limited water solubility and dissolution in dilute mineral acid are consistent with aniline, $C_6H_5NH_2$. Protonation gives a water-soluble anilinium salt. The positive carbylamine test confirms a primary amino group:

$$C_6H_5NH_2 + CHCl_3 + 3\,KOH \longrightarrow C_6H_5NC + 3\,KCl + 3\,H_2O.$$

The isocyanide is compound y; its unpleasant odour is the diagnostic observation.

Step 2: Form the sulphonamide product Aniline reacts with benzenesulphonyl chloride to form N-phenylbenzenesulphonamide:

$$C_6H_5SO_2Cl + H_2NC_6H_5 \longrightarrow C_6H_5SO_2NHC_6H_5 + HCl.$$

This is compound z. Its N–H proton can be removed by alkali because the sulphonyl group stabilises the resulting anion. The two phenyl rings are not chemically identical: one is bonded directly to sulfur, while the other is bonded directly to nitrogen.

Step 3: Count the distinct hydrogen environments On the sulfur-bonded phenyl ring, its two ortho hydrogens are equivalent, its two meta hydrogens are equivalent, and its para hydrogen is distinct: three environments. The nitrogen-bonded phenyl ring likewise contributes three environments, but these differ from those of the first ring because the attached groups differ. The N–H hydrogen contributes one more environment. Thus

$$N_{\text{environments}} = 3 + 3 + 1 = 7.$$

The count is not five or four because the two rings cannot be treated as one equivalent ring. Eight

would incorrectly split an equivalent aromatic pair. The question asks for different kinds of hydrogen atoms, not the total eleven hydrogens.

Step 4: Conclusion Compound z, $C_6H_5SO_2NHC_6H_5$, has seven different hydrogen environments. Hence, the correct option is (C).

Quick Tip to Solve: For a sulphonamide bearing two phenyl rings, check which atom each ring attaches to before combining proton environments; also count the N–H proton.

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66. Which of the following statements are TRUE about Haloform reaction?

A. Sodium hypochlorite reacts with KI to give KOI.

B. KOI is a reducing agent.

C. α,β-unsaturated methylketone ($CH_3-CH=CH-C(=O)-CH_3$) will give iodoform reaction.

D. Isopropyl alcohol will not give iodoform test.

E. Methanoic acid will give positive iodoform test.

Choose the correct answer from the options given below: [4 marks]

(A) A, B & C Only

(B) A, C & E Only

(C) B, D & E Only

(D) A & C Only

Correct Answer: (D) A & C Only

Solution:

Concept:

- A methyl ketone, $RCOCH_3$, gives the iodoform test with iodine in alkali, producing yellow CHI_3. The reaction requires an intact methyl group attached to the carbonyl carbon.
- Alcohols oxidisable to methyl ketones also respond. Hypoiodite supplies oxidising and iodinating conditions; it is not a reducing agent.

Step 1: Evaluate statements A and B Sodium hypochlorite oxidises iodide to hypoiodite in alkaline medium. The formal salt equation is

$$NaOCl + KI \longrightarrow NaCl + KOI.$$

Iodine changes from -1 in iodide to $+1$ in hypoiodite. Statement A is therefore true. Hypoiodite can oxidise suitable alcohols to carbonyl compounds, so calling KOI a reducing agent is incorrect. Statement B is false.

Step 2: Examine the unsaturated methyl ketone in C The displayed compound is

$$CH_3-CH=CH-C(=O)-CH_3.$$

Although it contains an adjacent carbon–carbon double bond, it retains the $-COCH_3$ group needed for the haloform reaction. Iodination of this methyl group followed by cleavage gives iodoform and

the corresponding carboxylate. Its relevant net reaction can be expressed as

$$\mathrm{RCOCH_3 + 3\,I_2 + 4\,OH^- \longrightarrow RCOO^- + CHI_3 + 3\,I^- + 3\,H_2O},$$

with R representing the $CH_3CH{=}CH$ group. Thus statement C is true; the unsaturation does not remove the reactive methyl-ketone group.

Step 3: Test the alcohol and acid claims Isopropyl alcohol, $CH_3CH(OH)CH_3$, is oxidised to acetone, CH_3COCH_3, under the test conditions. Acetone gives iodoform, so the claim in D that isopropyl alcohol will not give the test is false. Methanoic acid, HCOOH, has neither a methyl-ketone group nor an alcohol group that can be oxidised into one. It does not give the iodoform test, making E false.

Step 4: Select the true set Only statements A and C survive all the checks. Option (A) includes the false statement B. Option (B) includes the false statement E, while option (C) consists of the false statements B, D and E. Option (D) alone lists exactly A and C.

Step 5: Conclusion Statements A and C only are true. Hence, the correct option is (D).

Quick Tip to Solve: For iodoform, search for $-COCH_3$ or an alcohol oxidisable to it; a positive result is not a general test for all alcohols, carbonyl compounds or acids.

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67. Elements X and Y belong to Group 15. The difference between the electronegativity values of ' X ' and phosphorus is higher than that of the difference between phosphorus and ' Y '. ' X ' & ' Y ' are respectively [4 marks]

(A) Bi & N

(B) As & Bi

(C) N & As

(D) As & Sb

Correct Answer: (C) N & As

Solution:

Concept:

- Electronegativity measures an atom's attraction for the shared electron pair. In Group 15, nitrogen is appreciably more electronegative than phosphorus, whereas the changes among the heavier members are comparatively small.
- The stated comparison is between the magnitudes of two differences: $|\chi_X - \chi_P| > |\chi_P - \chi_Y|$. Here χ denotes electronegativity on a common scale.

Step 1: Compare nitrogen and arsenic with phosphorus The familiar rounded electronegativity values are approximately $\chi_N = 3.0$, $\chi_P = 2.1$ and $\chi_{As} = 2.0$. Thus, nitrogen differs markedly from phosphorus, while arsenic is close to phosphorus:

$$|\chi_N - \chi_P| \approx 0.9, \qquad |\chi_P - \chi_{As}| \approx 0.1.$$

Hence the inequality holds when X is nitrogen and Y is arsenic. The large change from nitrogen to

phosphorus reflects the increase in atomic size and shielding on entering the third period.

Step 2: Check the other proposed pairs Antimony and bismuth have rounded values near 1.9, so their differences from phosphorus are only about 0.2. For option (A), bismuth's difference from phosphorus is smaller than nitrogen's, reversing the required inequality. In option (B), arsenic's difference is smaller than bismuth's. In option (D), arsenic's difference is also smaller than antimony's. Thus none of these pairs satisfies the printed ordering. Small variations between tabulated electronegativity scales do not change the intended large nitrogen–phosphorus contrast.

Step 3: Conclusion $X = \mathrm{N}$ and $Y = \mathrm{As}$, since nitrogen's electronegativity differs from that of phosphorus much more than arsenic's does. Hence, the correct option is (C).

Quick Tip to Solve: In Group 15, first notice the unusually large nitrogen-to-phosphorus drop; the heavier members have much closer electronegativities.

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68. Iodoform test can differentiate between

A. Methanol and Ethanol

B. CH_3COOH **and** CH_3CH_2COOH

C. Cyclohexene and cyclohexanone

D. Diethyl ether and Pentan-3-one

E. Anisole and acetone

Choose the correct answer from the options given below : [4 marks]

(A) A, B & E Only

(B) B, C & E Only

(C) A & E Only

(D) A & D Only

Correct Answer: (C) A & E Only

Solution:

Concept:

- Iodine in alkali gives a yellow precipitate of iodoform, CHI_3, with methyl ketones containing $CH_3-C(=O)-R$. Ethanal and compounds oxidised to suitable carbonyl compounds, especially ethanol and $CH_3CH(OH)R$ alcohols, also respond.
- A test differentiates a pair only when one member gives a positive result and the other does not. Two negative results do not distinguish the compounds.

Step 1: Identify the pairs giving different results In pair A, ethanol is oxidised to ethanal, CH_3CHO, and therefore gives the iodoform test. Methanol forms methanal, which lacks the required methyl group, and gives no iodoform precipitate. Thus A can be differentiated. In pair E, acetone, CH_3COCH_3, is a methyl ketone and gives a positive result. Anisole, $C_6H_5OCH_3$, is an aromatic ether and lacks the required carbonyl or alcohol grouping, so it is negative. Thus E can also be differentiated.

Step 2: Eliminate the three pairs with identical negative results In pair B, both acetic acid and propanoic acid are carboxylic acids. The CH_3COOH group does not behave as a methyl ketone, so

neither produces iodoform. In pair C, cyclohexene is an alkene and cyclohexanone has no methyl group directly bonded to its carbonyl carbon; both are negative. In pair D, diethyl ether lacks a carbonyl group, while pentan-3-one is $CH_3CH_2COCH_2CH_3$, an ethyl ketone on both sides rather than a methyl ketone. Both are negative. Therefore the combinations in options (A) and (B) wrongly include B, and option (D) wrongly includes D. Only the pair of statements A and E survives.

Step 3: Conclusion Iodoform testing distinguishes methanol from ethanol and anisole from acetone: A and E only. Hence, the correct option is (C).

Quick Tip to Solve: Search for $CH_3-C(=O)-R$ or an alcohol that oxidises to it, and remember ethanol as the important positive primary-alcohol case.

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69. Identify the CORRECT set of details from the following :

A. $[Co\,(NH_3)_6]^{3+}$ **: Inner orbital complex;** d^2sp^3 **hybridized**

B. $[MnCl_6]^{3-}$ **: Outer orbital complex;** $sp^3\ d^2$ **hybridized**

C. $[CoF_6]^{3-}$ **: Outer orbital complex;** d^2sp^3 **hybridized**

D. $[FeF_6]^{3-}$ **: Outer orbital complex;** $sp^3\ d^2$ **hybridized**

E. $[Ni(CN)_4]^{2-}$ **: Inner orbital complex;** sp^3 **hybridized**

Choose the correct answer from the options given below : [4 marks]

(A) A, B, C, D & E

(B) A, B & D Only

(C) A, C & E Only

(D) C & D Only

Correct Answer: (B) A, B & D Only

Solution:

Concept:

- An inner-orbital octahedral complex uses two inner $3d$ orbitals and has d^2sp^3 hybridisation. An outer-orbital octahedral complex uses outer $4d$ orbitals and has sp^3d^2 hybridisation.
- Oxidation state fixes the metal's d-electron count. Ligand field strength determines whether pairing provides vacant inner orbitals; four-coordinate d^8 nickel with strong-field cyanide is square planar and dsp^2 hybridised.

Step 1: Verify the three correct descriptions In A, Co^{3+} is $3d^6$. Ammonia with cobalt(III) produces the low-spin arrangement, leaving two inner $3d$ orbitals available. Thus $[Co(NH_3)_6]^{3+}$ is an inner-orbital d^2sp^3 complex. In B, Mn^{3+} is $3d^4$. Chloride is weak field, so the high-spin configuration does not supply the two vacant inner orbitals required for d^2sp^3 bonding. The octahedral complex is described as outer-orbital sp^3d^2. In D, Fe^{3+} is $3d^5$. Weak-field fluoride maintains five unpaired electrons in the inner d orbitals, and the octahedral complex likewise uses outer-orbital sp^3d^2 hybridisation.

Step 2: Correct the inconsistent descriptions In C, fluoride gives high-spin Co^{3+} in $[CoF_6]^{3-}$. Its outer-orbital description is correct, but its hybridisation must be sp^3d^2, not the printed d^2sp^3. Hence the complete statement C is false. In E, Ni^{2+} is $3d^8$; strong-field cyanide yields a square-planar

inner-orbital dsp^2 complex, not a tetrahedral sp^3 complex. Thus E is false. Options (A) and (C) include these incorrect statements, while option (D) includes incorrect statement C and omits correct A and B.

Step 3: Conclusion The fully correct descriptions are A, B and D only. Hence, the correct option is (B).

Quick Tip to Solve: Check the geometry and hybridisation together: inner octahedral means d^2sp^3, outer octahedral means sp^3d^2, and square planar means dsp^2.

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70. Given below are two statements :

Statement I : The second ionisation enthalpy of Na is larger than the corresponding ionisation enthalpy of Mg .

Statement II : The ionic radius of O^{2-} is larger than that of F^-.

In the light of the above statements, choose the *correct* answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- The second ionisation enthalpy is the energy required to remove an electron from an isolated gaseous singly charged cation: $M^+(g) \longrightarrow M^{2+}(g) + e^-$. The configuration after the first ionisation must therefore be examined.
- For an isoelectronic series, electron number is fixed. Increasing nuclear charge attracts the same electron cloud more strongly and reduces the ionic radius.

Step 1: Compare the second ionisations of sodium and magnesium Sodium has configuration $[Ne]3s^1$, so after its first electron is removed, Na^+ has the closed-shell configuration $[Ne]$. Its second ionisation removes an electron from the compact $n = 2$ core and breaks this noble-gas configuration. Magnesium has configuration $[Ne]3s^2$, so Mg^+ is $[Ne]3s^1$. Its second ionisation removes the remaining outer $3s$ electron and produces the stable $[Ne]$ configuration. This is much easier than removing a core electron from Na^+. Therefore,

$$IE_2(Na) > IE_2(Mg),$$

and Statement I is true. The usual first-ionisation comparison cannot be applied to this second-ionisation question.

Step 2: Compare the two ten-electron anions Both O^{2-} and F^- contain ten electrons. Their nuclear charges are $Z = 8$ and $Z = 9$, respectively. Fluoride's larger nuclear charge holds the identical number

of electrons more tightly, making it smaller:

$$r(O^{2-}) > r(F^{-}).$$

Statement II is therefore also true. Option (B) rejects two correct statements; option (C) incorrectly rejects the radius comparison; option (D) incorrectly rejects the second-ionisation comparison.

Step 3: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: For successive ionisations, write the ion's configuration first. For isoelectronic ions, the smaller nuclear charge gives the larger radius.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. Total number of unpaired electrons present in the central metal atoms/ions of $[Ni(CO)_4]$, $[NiCl_4]^{2-}$, $[PtCl_2(NH_3)_2]$, $[Ni(CN)_4]^{2-}$ and $[Pt(CN)_4]^{2-}$ is ____. [4 marks]

Solution:

Concept:

- Find the oxidation state and metal d-electron count before assigning geometry and spin. Nickel(II) and platinum(II) are d^8, but their complexes need not have the same number of unpaired electrons.
- Tetrahedral d^8 complexes have two unpaired electrons. Square-planar d^8 complexes have their eight electrons paired in the four lower-energy d orbitals; d^{10} configurations are also fully paired.

Step 1: Count the unpaired electrons in the nickel complexes In $[Ni(CO)_4]$, carbon monoxide is neutral, so nickel is in oxidation state zero. In the complex it adopts a filled $3d^{10}$ arrangement, with tetrahedral sp^3 bonding and no unpaired electrons. In $[NiCl_4]^{2-}$, four chloride ions give nickel oxidation state $+2$, hence $3d^8$. Chloride is weak field and the complex is tetrahedral. Its occupation is $e^4t_2^4$: the three higher orbitals hold one pair and two single electrons. This contributes two unpaired electrons. In $[Ni(CN)_4]^{2-}$, nickel is again $+2$, but strong-field cyanide gives square-planar dsp^2 bonding. The electrons are paired, contributing zero.

Step 2: Count the platinum contributions and add Both $[PtCl_2(NH_3)_2]$ and $[Pt(CN)_4]^{2-}$ contain platinum in oxidation state $+2$, with a $5d^8$ configuration. Platinum(II) characteristically forms square-planar complexes with strong enough splitting to pair the eight electrons. Consequently both complexes are diamagnetic, each contributing zero, even though chloride is a weak-field ligand in the spectrochemical series. Cis or trans arrangement of the first platinum complex does not change this electron count. In the order listed in the question, the contributions are therefore

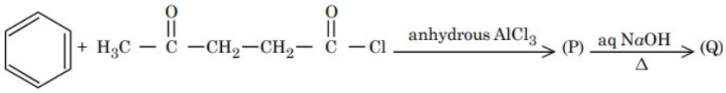
$$0 + 2 + 0 + 0 + 0 = 2.$$

Step 3: Conclusion The total number of unpaired electrons is $\boxed{2}$, a dimensionless count. Only $[NiCl_4]^{2-}$ contributes unpaired electrons.

Quick Tip to Solve: Do not assign spin from the ligand alone: tetrahedral nickel(II) and square-planar platinum(II) can behave differently even with chloride.

72. Consider the following reaction of benzene.

In compound (Q), the percentage of oxygen is ____ %. (Nearest integer) [4 marks]



Solution:

Concept:

- An acid chloride acylates benzene in the presence of anhydrous $AlCl_3$, replacing a ring hydrogen by the acyl group. The pre-existing ketone group in the side chain remains present.
- A diketone with suitable alpha hydrogens can undergo intramolecular aldol condensation. Base generates an enolate, ring closure forms a beta-hydroxy ketone, and heating removes water to form a conjugated enone.

Step 1: Determine the acylation product The reagent is $CH_3COCH_2CH_2COCl$. Acylation occurs at its acid-chloride carbonyl, giving

$$C_6H_6 + CH_3COCH_2CH_2COCl \longrightarrow C_6H_5COCH_2CH_2COCH_3 + HCl.$$

Thus P is the diketone $C_6H_5COCH_2CH_2COCH_3$, with molecular formula $C_{11}H_{12}O_2$. Its two carbonyl groups and terminal methyl group permit an intramolecular aldol reaction.

Step 2: Follow ring formation and dehydration Aqueous NaOH removes an alpha hydrogen from the terminal methyl group. The resulting enolate carbon attacks the benzoyl carbonyl carbon within the same molecule. The new bond closes a five-membered ring: the attacking methyl carbon, its neighbouring carbonyl carbon, the two intervening methylene carbons and the attacked carbonyl carbon form the ring. Protonation gives the beta-hydroxy ketone. On heating, loss of water gives 3-phenylcyclopent-2-en-1-one, a conjugated cyclic enone. No carbon atoms are lost. The overall formula change is

$$C_{11}H_{12}O_2 \longrightarrow C_{11}H_{10}O + H_2O.$$

Therefore Q contains one oxygen atom rather than the two present in P.

Step 3: Calculate the oxygen mass percentage Using atomic masses $C = 12$, $H = 1$ and $O = 16$, the molar mass of Q is

$$M = 11(12) + 10(1) + 16 = 158\ \text{g mol}^{-1}.$$

Its oxygen contribution is $16\ \text{g mol}^{-1}$, so

$$\%O = \frac{16}{158} \times 100 = 10.1266\ldots\%.$$

Step 4: Conclusion The percentage of oxygen in Q, to the nearest integer, is $\boxed{10\%}$. The numerical entry is 10.

Quick Tip to Solve: In a heated intramolecular aldol reaction, subtract one water molecule before calculating composition; the carbon count stays unchanged.

73. 200 cc of $x \times 10^{-3}$M potassium dichromate is required to oxidise 750 cc of 0.6 M Mohr's salt solution in acidic medium.

Here $x =$ ____ . [4 marks]

Solution:

Concept:

- In acidic medium, each dichromate ion accepts six electrons when its two chromium atoms are reduced from oxidation state +6 to +3.
- Each formula unit of Mohr's salt supplies one Fe^{2+} ion. Oxidation to Fe^{3+} releases one electron, so one mole of dichromate oxidises six moles of Mohr's salt.

Step 1: Establish the redox mole ratio Write the reduction half-reaction and the oxidation half-reaction:

$$Cr_2O_7^{2-} + 14\,H^+ + 6\,e^- \longrightarrow 2\,Cr^{3+} + 7\,H_2O,$$

$$Fe^{2+} \longrightarrow Fe^{3+} + e^-.$$

Multiplying the second half-reaction by six balances the electrons. Therefore, the number of moles of dichromate required is one-sixth the number of moles of ferrous ions. The ammonium and sulfate ions of Mohr's salt do not alter this redox ratio.

Step 2: Calculate the amount of ferrous ions Convert the volume from cubic centimetres to litres before multiplying by molarity:

$$V_{\text{Mohr}} = 750\,\text{cm}^3 = 0.750\,\text{L}.$$

$$n(Fe^{2+}) = 0.6 \times 0.750 = 0.450\,\text{mol}.$$

Hence the required amount of potassium dichromate is

$$n(K_2Cr_2O_7) = \frac{0.450}{6} = 0.0750\,\text{mol}.$$

Step 3: Find the dichromate molarity and the coefficient The dichromate volume is $200\,\text{cm}^3 = 0.200\,\text{L}$, giving

$$M = \frac{0.0750}{0.200} = 0.375\,\text{mol L}^{-1}.$$

The question expresses this concentration as $x \times 10^{-3}$ M. Thus

$$x \times 10^{-3} = 0.375, \qquad x = 375.$$

Step 4: Conclusion The required dichromate concentration is 0.375 M, so the numerical value to enter is $\boxed{375}$.

Quick Tip to Solve: In acidic dichromate titrations, equate $6MV$ for dichromate to MV for ferrous ions; Mohr's salt contributes only one ferrous ion per formula unit.

74. Two liquids A and B form an ideal solution. At 320 K , the vapour pressure of the solution, containing 3 mol of A and 1 mol of B is 500 mm Hg . At the same temperature, if 1 mol of A is further added to this solution, vapour pressure of the solution increases by 20 mm Hg . Vapour pressure (in mm Hg) of B in pure state is ____ . (Nearest integer) [4 marks]

Solution:

Concept:

- For an ideal binary liquid solution, Raoult's law gives total vapour pressure $P = x_A p_A^\circ + x_B p_B^\circ$, where x_A, x_B are liquid-phase mole fractions and p_A°, p_B° are pure-liquid vapour pressures.
- At a fixed temperature, the pure-liquid vapour pressures remain constant. Adding liquid A changes both mole fractions because it changes the total number of moles.

Step 1: Write the equation for the original solution Initially the solution contains 3 mol of A and 1 mol of B, so the total is 4 mol. Its mole fractions are

$$x_A = \frac{3}{4}, \qquad x_B = \frac{1}{4}.$$

At $320\,\mathrm{K}$, Raoult's law therefore gives

$$500 = \frac{3}{4}p_A^\circ + \frac{1}{4}p_B^\circ, \qquad 3p_A^\circ + p_B^\circ = 2000.$$

All pressures in these equations are measured in $\mathrm{mm\,Hg}$.

Step 2: Account for the added mole of A After adding 1 mol of A, the amounts become 4 mol of A and 1 mol of B. The new total is 5 mol, giving mole fractions $4/5$ and $1/5$. The pressure increases by 20, so its new value is $500 + 20 = 520\,\mathrm{mm\,Hg}$. Thus

$$520 = \frac{4}{5}p_A^\circ + \frac{1}{5}p_B^\circ, \qquad 4p_A^\circ + p_B^\circ = 2600.$$

Keeping the old denominator of 4 here would incorrectly ignore dilution of B.

Step 3: Solve the two pressure equations Subtract the original equation from the new equation:

$$(4p_A^\circ + p_B^\circ) - (3p_A^\circ + p_B^\circ) = 2600 - 2000.$$

Hence $p_A^\circ = 600\,\mathrm{mm\,Hg}$, and substitution gives

$$p_B^\circ = 2000 - 3(600) = 200\,\mathrm{mm\,Hg}.$$

The initial pressure checks as $(3\times600+200)/4 = 500$, while the final pressure is $(4\times600+200)/5 = 520$, as required.

Step 4: Conclusion The vapour pressure of pure B at $320\,\mathrm{K}$ is $\boxed{200\,\mathrm{mm\,Hg}}$. The nearest-integer entry is $\boxed{200}$.

Quick Tip to Solve: Recalculate both liquid mole fractions after adding a component. At constant temperature, use the same two pure-liquid vapour pressures in both Raoult's-law equations.

75.

$$X_2(g) + Y_2(g) \rightleftharpoons 2Z(g)$$

$X_2(g)$ and $Y_2(g)$ are added to a 1 L flask and it is found that the system attains the above equilibrium at $T(K)$ with the number of moles of $X_2(g), Y_2(g)$ and $Z(g)$ being 3,3 and 9 mol respectively (equilibrium moles). Under this condition of equilibrium, 10 mol of $Z(g)$ is added to the flask and the temperature is maintained at $T(K)$. Then the number of moles of $Z(g)$ in the flask when the new equilibrium is established is ____ . (Nearest integer) [4 marks]

Solution:

Concept:

- For $X_2(g) + Y_2(g) \rightleftharpoons 2\,Z(g)$, the equilibrium constant is $K_c = [Z]^2/([X_2][Y_2])$. Its value remains unchanged when composition changes at the same temperature.
- Adding product initially increases the reaction quotient above K_c, so the reaction shifts in the reverse direction. Each mole of X_2 and Y_2 formed consumes two moles of Z.

Step 1: Calculate the equilibrium constant The flask volume is $1\,L$, so the initial equilibrium concentrations numerically equal the given mole amounts. Therefore

$$K_c = \frac{9^2}{3 \times 3} = 9.$$

Because the temperature remains T, this same equilibrium constant must hold after the added product has redistributed.

Step 2: Set up the new equilibrium amounts Immediately after adding 10 mol of Z, its amount becomes $9 + 10 = 19\,mol$; each reactant still has $3\,mol$. The reaction quotient is then $19^2/(3 \times 3) > 9$, confirming a reverse shift. Let a mol of each reactant form. At the new equilibrium,

$$n(X_2) = 3 + a, \quad n(Y_2) = 3 + a, \quad n(Z) = 19 - 2a.$$

The coefficient 2 in the product amount is essential: forming one mole of each reactant consumes two moles of product.

Step 3: Apply the unchanged constant and solve Since the flask remains at $1\,L$, substitute these numerical concentrations:

$$\frac{(19 - 2a)^2}{(3 + a)^2} = 9.$$

All physical concentrations are positive. Taking the positive square root gives

$$19 - 2a = 3(3 + a) = 9 + 3a, \qquad a = 2.$$

The product amount is consequently

$$n(Z) = 19 - 2(2) = 15\,mol.$$

As a check, the final reactant amounts are 5 mol each and $15^2/(5 \times 5) = 9$. The total remains 25 mol

because the forward and reverse reactions have no net change in gaseous mole number.

Step 4: Conclusion At the new equilibrium, the flask contains $\boxed{15\,\text{mol}}$ of $\text{Z}\,(\text{g})$. The nearest-integer numerical entry is $\boxed{15}$.

Quick Tip to Solve: Update the amount immediately after addition before introducing the reaction shift. For equal reactant amounts, taking the positive square root of the equilibrium expression avoids a quadratic equation.