JEE Main 2026 Paper 1 (B.E./B.Tech) Jan 24 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2026, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 24 January 2026, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $729, 81, 9, 1, \ldots$ be a sequence and P_n denote the product of the first n terms of this sequence. If $2\sum_{n=1}^{40}(P_n)^{\frac{1}{n}} = \frac{3^{\alpha}-1}{3^{\beta}}$ and $\gcd(\alpha, \beta) = 1$, then $\alpha + \beta$ is equal to [4 marks]

(A) 73

(B) 74

(C) 75

(D) 76

Correct Answer: (A) 73

Solution:

Concept:

- The sequence is geometric with first term $729 = 3^6$ and common ratio $1/9 = 3^{-2}$. Its kth term is $a_k = 3^{8-2k}$.
- To multiply powers of the same base, add their exponents. After taking the positive nth root, use the finite geometric-sum formula $S_N = a(1 - r^N)/(1 - r)$.

Step 1: Find the product exponent All terms are positive, so the real positive root is unambiguous. Summing the exponents of the first n terms gives

$$P_n = 3^{\sum_{k=1}^{n}(8-2k)} = 3^{8n-n(n+1)} = 3^{7n-n^2}.$$

Hence

$$(P_n)^{1/n} = 3^{7-n}.$$

For example, the first value is $3^6 = 729$, while the second is $3^5 = 243$, consistent with the geometric mean of 729 and 81.

Step 2: Sum the resulting geometric progression The forty roots form a geometric progression of first term 3^6 and ratio $1/3$, rather than the original ratio $1/9$. Therefore

$$2\sum_{n=1}^{40}(P_n)^{1/n} = 2 \cdot 3^6 \frac{1 - 3^{-40}}{1 - 1/3} = 3^7(1 - 3^{-40}) = \frac{3^{40} - 1}{3^{33}}.$$

Thus $\alpha = 40$ and $\beta = 33$. The Euclidean algorithm gives $40 = 33 + 7$, $33 = 4 \cdot 7 + 5$, $7 = 5 + 2$, and $5 = 2 \cdot 2 + 1$, so their gcd is indeed one. The required sum is $40 + 33 = 73$; the other listed values 74, 75 and 76 do not equal this sum.

Step 3: Conclusion $\alpha + \beta = \boxed{73}$. Hence, the correct option is (A).

Quick Tip to Solve: Write every term as a power of 3 first: taking the geometric mean changes the ratio from $1/9$ to $1/3$.

2. The value of $\frac{\sqrt{3}\,\text{cosec}\,20^\circ - \sec 20^\circ}{\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ}$ is equal to [4 marks]

(A) 16

(B) 12

(C) 32

(D) 64

Correct Answer: (D) 64

Solution:

Concept:

- Use $\text{cosec}\,\theta = 1/\sin\theta$, $\sec\theta = 1/\cos\theta$, and the angle-addition identities to simplify the numerator.
- Repeated application of $\sin 2\theta = 2\sin\theta\cos\theta$ evaluates the product of cosines. All angles in this question are in degrees.

Step 1: Simplify the numerator Combining the two terms over a common denominator gives

$$\sqrt{3}\,\text{cosec}\,20^\circ - \sec 20^\circ = \frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cos 20^\circ}.$$

Since $\sqrt{3}\cos 20^\circ - \sin 20^\circ = 2\cos 50^\circ = 2\sin 40^\circ$, the double-angle identity yields

$$\frac{2\sin 40^\circ}{\sin 20^\circ \cos 20^\circ} = 4.$$

The numerator is positive; there is no sign change from taking reciprocals.

Step 2: Evaluate the cosine product Multiply $\cos 20^\circ \cos 40^\circ \cos 80^\circ$ by $8\sin 20^\circ$. Applying the double-angle formula three times gives

$$8\sin 20^\circ \cos 20^\circ \cos 40^\circ \cos 80^\circ = \sin 160^\circ = \sin 20^\circ.$$

Since $\sin 20^\circ \neq 0$, the three-cosine product equals $1/8$. The extra factor $\cos 60^\circ = 1/2$ makes the denominator $1/16$. Therefore the requested quotient is $4/(1/16) = 64$. Values 16 and 32 omit a factor in the numerator or denominator, while 12 is inconsistent with the exact identities.

Step 3: Conclusion The expression equals $\boxed{64}$. Hence, the correct option is (D).

Quick Tip to Solve: For a product with successive doubled angles, multiply by the sine of the smallest angle and telescope with the double-angle formula.

3. Let the lines $L_1 : \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k}), \lambda \in \mathbb{R}$ and $L_2 : \vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k}), \mu \in \mathbb{R}$, intersect at the point R . Let P and Q be the points lying on lines L_1 and L_2, respectively, such that $|\overrightarrow{PR}| = \sqrt{29}$ and $|\overrightarrow{PQ}| = \sqrt{\frac{47}{3}}$. If the point P lies in the first octant, then $27(QR)^2$ is equal to [4 marks]

(A) 320

(B) 340

(C) 348

(D) 360

Correct Answer: (D) 360

Solution:

Concept:

- A line in vector form gives its coordinates by equating the coefficients of $\hat{i}, \hat{j}, \hat{k}$. Two intersecting lines have equal coordinates at their common point.
- A displacement $t\mathbf{v}$ has squared length $t^2|\mathbf{v}|^2$. For two displacements use $|t\mathbf{v} - \mathbf{u}|^2 = t^2|\mathbf{v}|^2 - 2t\mathbf{u} \cdot \mathbf{v} + |\mathbf{u}|^2$.

Step 1: Find the intersection and choose P Equating coordinates gives $1+2\lambda = 4+5\mu$, $2+3\lambda = 1+2\mu$, and $3 + 4\lambda = \mu$. Substituting the third into the second gives $\lambda = -1$, then $\mu = -1$; the first equation also holds. Thus $R = (-1, -1, -1)$. Put $\mathbf{u} = (2, 3, 4)$, whose length is $\sqrt{29}$. The points of L_1 at the required distance from R are $R \pm \mathbf{u}$. Only $P = R + \mathbf{u} = (1, 2, 3)$ lies in the first octant; $(-3, -4, -5)$ does not.

Step 2: Use the second distance to locate Q Write $Q = R + t\mathbf{v}$, where $\mathbf{v} = (5, 2, 1)$. Here $|\mathbf{v}|^2 = 30$ and $\mathbf{u} \cdot \mathbf{v} = 20$. Therefore

$$|PQ|^2 = |t\mathbf{v} - \mathbf{u}|^2 = 30t^2 - 40t + 29 = \frac{47}{3}.$$

Multiplying and simplifying gives

$$90t^2 - 120t + 40 = 10(3t - 2)^2 = 0, \qquad t = \frac{2}{3}.$$

Consequently $QR^2 = t^2|\mathbf{v}|^2 = (4/9)30 = 40/3$, and $27QR^2 = 360$. The alternative values 320, 340 and 348 fail this squared-distance equation.

Step 3: Conclusion $27(QR)^2 = \boxed{360}$. Hence, the correct option is (D).

Quick Tip to Solve: After finding R, parametrize both points relative to R. The first-octant condition selects the sign of the displacement to P.

4. The number of the real solutions of the equation: $x|x+3|+|x-1|-2=0$ is [4 marks]

(A) 2

(B) 3

(C) 5

(D) 4

Correct Answer: (B) 3

Solution:

Concept:

- The signs inside the absolute values change at $x=-3$ and $x=1$. Solve separately on intervals cut by these two points.
- A quadratic root is valid only if it belongs to the interval used to remove the absolute values. Count distinct valid real roots, rather than all algebraic candidates.

Step 1: Check the interval below minus three For $x<-3$, $|x+3|=-x-3$ and $|x-1|=1-x$. Hence

$$-x^2-4x-1=0, \qquad x=-2\pm\sqrt{3}.$$

Only $-2-\sqrt{3}<-3$ satisfies this case. The root $-2+\sqrt{3}$ lies outside the assumed interval and must be rejected.

Step 2: Check the middle interval For $-3\le x<1$, $|x+3|=x+3$ and $|x-1|=1-x$, so

$$x^2+2x-1=0, \qquad x=-1\pm\sqrt{2}.$$

Both roots lie in $[-3,1)$: they are approximately -2.414 and 0.414. Thus this interval contributes two further real solutions.

Step 3: Check the interval from one onward For $x\ge 1$, the equation becomes

$$x^2+4x-3=0, \qquad x=-2\pm\sqrt{7}.$$

Even the larger root is less than 1 because $\sqrt{7}<3$; neither candidate is valid. The endpoints give values 2 at $x=-3$ and 2 at $x=1$, so neither is a root. In total there are $1+2+0=3$ distinct real solutions. Two misses a middle-interval root, while four and five incorrectly retain roots outside their case intervals.

Step 4: Conclusion There are $\boxed{3}$ real solutions. Hence, the correct option is (B).

Quick Tip to Solve: Always write the interval beside each quadratic: rejecting roots outside that interval is essential when counting absolute-value solutions.

5. Let A_1 be the bounded area enclosed by the curves $y = x^2 + 2, x + y = 8$ and y-axis that lies in the first quadrant. Let A_2 be the bounded area enclosed by the curves $y = x^2 + 2, y^2 = x, x = 2$, and y-axis that lies in the first quadrant. Then $A_1 - A_2$ is equal to [4 marks]

(A) $\frac{2}{3}(\sqrt{2}+1)$
(B) $\frac{2}{3}(2\sqrt{2}+1)$
(C) $\frac{2}{3}(3\sqrt{2}+1)$
(D) $\frac{2}{3}(4\sqrt{2}+1)$

Correct Answer: (B) $\frac{2}{3}(2\sqrt{2}+1)$

Solution:

Concept:

- For curves expressed as functions of x, the enclosed area over an interval is the integral of the upper curve minus the lower curve.
- In the first quadrant $y^2 = x$ is $y = \sqrt{x}$. The y-axis is $x = 0$; retain only intersections belonging to the stated quadrant.

Step 1: Find the bounds and compute the first area The parabola and line intersect when $x^2+2 = 8-x$, giving $(x+3)(x-2) = 0$. The first-quadrant intersection is at $x = 2$, so the relevant interval is $[0, 2]$. On it the line lies above the parabola because $6 - x - x^2 \geq 0$. Thus

$$A_1 = \int_0^2 (6 - x - x^2)\, dx = \left[6x - \frac{x^2}{2} - \frac{x^3}{3}\right]_0^2 = \frac{22}{3}.$$

The negative intersection $x = -3$ is excluded by the first-quadrant condition.

Step 2: Compute the second area and subtract The second region is bounded vertically by $x = 0$ and $x = 2$. Throughout this interval the parabola lies above $\sqrt{x}$: indeed $x^2 + 2 \geq 2 > \sqrt{2} \geq \sqrt{x}$. Therefore

$$A_2 = \int_0^2 (x^2 + 2 - \sqrt{x})\, dx = \left[\frac{x^3}{3} + 2x - \frac{2}{3}x^{3/2}\right]_0^2 = \frac{20}{3} - \frac{4\sqrt{2}}{3}.$$

Subtracting gives

$$A_1 - A_2 = \frac{22}{3} - \frac{20}{3} + \frac{4\sqrt{2}}{3} = \frac{2}{3}(1 + 2\sqrt{2}).$$

The coefficient of $\sqrt{2}$ inside the parentheses is exactly 2, so the options with coefficients 1, 3 and 4 are excluded.

Step 3: Conclusion $A_1 - A_2 = \boxed{\frac{2}{3}(2\sqrt{2}+1)}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: Both regions use the same x-interval. Check the upper and lower curves first, then subtract the two definite integrals.

6. Let R be a relation defined on the set $\{1,2,3,4\} \times \{1,2,3,4\}$ by

$$\mathrm{R} = \{((a,b),(c,d)) : 2a + 3b = 3c + 4d\}.$$

Then the number of elements in R is [4 marks]

(A) 6

(B) 12

(C) 15

(D) 18

Correct Answer: (B) 12

Solution:

Concept:

- An element of this relation is an ordered pair of ordered pairs, $((a,b),(c,d))$, where all four entries are integers from 1 to 4.
- Group the possible pairs by their common weighted sum. If a sum occurs f times as $2a + 3b$ and g times as $3c + 4d$, it contributes fg elements to the relation.

Step 1: List the sums on both sides For $b = 1, 2, 3, 4$, respectively, the four values of $2a + 3b$ as a runs from 1 to 4 are

$$b = 1 : 5, 7, 9, 11,$$
$$b = 2 : 8, 10, 12, 14,$$
$$b = 3 : 11, 13, 15, 17,$$
$$b = 4 : 14, 16, 18, 20.$$

For $d = 1, 2, 3, 4$, respectively, the values of $3c + 4d$ are

$$d = 1 : 7, 10, 13, 16,$$
$$d = 2 : 11, 14, 17, 20,$$
$$d = 3 : 15, 18, 21, 24,$$
$$d = 4 : 19, 22, 25, 28.$$

Each list entry corresponds to a specific ordered pair, so repeated left-hand sums must be counted with multiplicity.

Step 2: Match equal sums and count multiplicities The common sums are

$$7, 10, 11, 13, 14, 15, 16, 17, 18, 20.$$

Every one occurs once on the right. On the left, 11 comes from $(a,b) = (4,1), (1,3)$, and 14 comes from $(4,2), (1,4)$; every other common sum occurs once. Thus the count is

$$8 \cdot 1 + 2 \cdot 2 = 12.$$

This counts each admissible ordered quadruple once. It is not a count of just the distinct common

sums, nor may the two ordered pairs be freely interchanged. The values 6, 15 and 18 disagree with this exhaustive count.

Step 3: Conclusion The relation contains $\boxed{12}$ elements. Hence, the correct option is (B).

Quick Tip to Solve: Tabulate the weighted sums by one variable, then multiply their frequencies. Equal sums can represent more than one ordered pair.

7. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \vec{a} \times \vec{b}$. Let $\vec{d}$ be a vector such that $|\vec{d} - \vec{a}| = \sqrt{11}, |\vec{c} \times \vec{d}| = 3$ and the angle between $\vec{c}$ and $\vec{d}$ is $\frac{\pi}{4}$. Then $\vec{a} \cdot \vec{d}$ is equal to [4 marks]

(A) 0

(B) 1

(C) 3

(D) 11

Correct Answer: (A) 0

Solution:

Concept:

- The magnitude of a cross product is $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin\theta$, where θ is the angle between the vectors.
- The squared distance identity $|\vec{d} - \vec{a}|^2 = |\vec{d}|^2 + |\vec{a}|^2 - 2\vec{a} \cdot \vec{d}$ determines the required dot product without finding all components of $\vec{d}$.

Step 1: Calculate the known cross product Using the components of the two given vectors, evaluate the determinant with its alternating signs:

$$\vec{c} = \vec{a} \times \vec{b} = (2, -2, 1), \qquad |\vec{c}| = \sqrt{4 + 4 + 1} = 3.$$

Also, the squared length of the first vector is $|\vec{a}|^2 = 2^2 + 1^2 + (-2)^2 = 9$. These are magnitudes of the original vectors, rather than of their difference.

Step 2: Determine the length of the unknown vector Apply the stated angle and cross-product magnitude together:

$$3 = 3|\vec{d}| \sin\frac{\pi}{4} = \frac{3|\vec{d}|}{\sqrt{2}}.$$

Consequently $|\vec{d}| = \sqrt{2}$, so $|\vec{d}|^2 = 2$. The angle condition supplies exactly the information needed about the length; there is no need to solve three simultaneous component equations.

Step 3: Use the distance condition and check the choices Squaring the given distance gives $|\vec{d} - \vec{a}|^2 = 11$. Substitute the two squared lengths:

$$11 = 2 + 9 - 2\vec{a} \cdot \vec{d}.$$

Thus the dot product must be zero. In contrast, proposed dot products $1, 3, 11$ would make this squared distance $9, 5, -11$, respectively, and none agrees with the prescribed value 11.

Step 4: Conclusion $\boxed{\vec{a}\cdot\vec{d}=0}$. Hence, the correct option is (A).

Quick Tip to Solve: When both a distance between vectors and a cross-product magnitude are given, find the unknown length first and then expand the squared distance.

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8. Let each of the two ellipses $\mathrm{E}_1 : \frac{x^2}{a^2}+\frac{y^2}{b^2}=1,(a>b)$ and $\mathrm{E}_2 : \frac{x^2}{\mathrm{A}^2}+\frac{y^2}{\mathrm{B}^2}=1,(\ \mathrm{A}<\mathrm{B})$ have eccentricity $\frac{4}{5}$. Let the lengths of the latus recta of E_1 and E_2 be l_1 and l_2, respectively, such that $2l_1^2=9l_2$. If the distance between the foci of E_1 is 8 , then the distance between the foci of E_2 is [4 marks]

(A) $\frac{8}{5}$
(B) $\frac{16}{5}$
(C) $\frac{32}{5}$
(D) $\frac{96}{5}$

Correct Answer: (C) $\frac{32}{5}$

Solution:

Concept:

- For an ellipse with semi-major axis p, semi-minor axis q and eccentricity e, the distance between its foci is $2pe$.
- The latus rectum has length $2q^2/p = 2p(1-e^2)$. Identify the semi-major axis from the larger denominator, irrespective of whether it lies along the horizontal or vertical axis.

Step 1: Find the axes and latus rectum of the first ellipse Here $e=4/5$, and the focal separation of the first ellipse is 8. Therefore

$$2a\frac{4}{5}=8 \quad\Longrightarrow\quad a=5.$$

Next, $b^2=a^2(1-e^2)=25(1-16/25)=9$. Its latus rectum is consequently

$$l_1=\frac{2b^2}{a}=\frac{18}{5}.$$

Step 2: Calculate the second latus rectum The given relation contains the square of the first latus rectum, not the second. Substituting carefully gives

$$l_2=\frac{2l_1^2}{9}=\frac{2}{9}\left(\frac{18}{5}\right)^2=\frac{72}{25}.$$

Both lengths are positive, as required for lengths of chords.

Step 3: Find the second focal separation and compare the options For E_2, the condition $A<B$ makes B the semi-major axis. Thus

$$l_2=2B(1-e^2)=\frac{18B}{25}=\frac{72}{25}, \qquad B=4.$$

The focal separation is $2Be=2(4)(4/5)=32/5$. Values $8/5, 16/5, 96/5$ would require semi-major

axes $1, 2, 12$, respectively, none of which yields the calculated latus rectum.

Step 4: Conclusion The distance between the foci of E_2 is $\boxed{32/5}$ units. Hence, the correct option is (C).

Quick Tip to Solve: For ellipses of equal eccentricity, the latus rectum and focal separation are both proportional to the semi-major axis; remember to switch axes for a vertical ellipse.

9. Let $S = \left\{ z \in \mathbb{C} : \left|\frac{z-6i}{z-2i}\right| = 1 \text{ and } \left|\frac{z-8+2i}{z+2i}\right| = \frac{3}{5} \right\}$.

Then $\sum_{z \in S} |z|^2$ **is equal to :** [4 marks]

(A) 385

(B) 398

(C) 413

(D) 423

Correct Answer: (A) 385

Solution:

Concept:

- Writing $z = x + iy$ converts a modulus into a distance: $|z - (u + iv)|^2 = (x - u)^2 + (y - v)^2$.
- Equal distances from two points define their perpendicular bisector. After imposing this first locus, the second modulus equation reduces to a quadratic in one real coordinate.

Step 1: Simplify the first locus The first ratio has modulus one, so $|z - 6i| = |z - 2i|$, with $z \neq 2i$. Squaring gives

$$x^2 + (y - 6)^2 = x^2 + (y - 2)^2.$$

Cancelling $x^2 + y^2$ leaves $-12y + 36 = -4y + 4$, hence $y = 4$. This is the horizontal line midway between the two imaginary-axis points.

Step 2: Apply the second modulus condition For $z = x + 4i$, the numerator $z - 8 + 2i$ becomes $(x - 8) + 6i$, while the denominator $z + 2i$ becomes $x + 6i$. Therefore

$$\frac{(x - 8)^2 + 36}{x^2 + 36} = \frac{9}{25}.$$

Clearing the positive denominator gives

$$25(x^2 - 16x + 100) = 9x^2 + 324,$$

$$16x^2 - 400x + 2176 = 0, \quad x^2 - 25x + 136 = 0.$$

Step 3: List the points and sum their squared moduli Factor the quadratic as $(x - 8)(x - 17) = 0$. Thus the set contains exactly $8 + 4i$ and $17 + 4i$; neither is an excluded denominator zero. Their

squared moduli are

$$|8+4i|^2 = 64+16 = 80, \qquad |17+4i|^2 = 289+16 = 305.$$

The sum is 385. The other options $398, 413, 423$ do not equal this sum and fail the two-locus calculation.

Step 4: Conclusion $\boxed{\sum_{z\in S} |z|^2 = 385}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the equal-modulus condition first: its perpendicular bisector often fixes one coordinate and makes an Apollonius-circle calculation a simple quadratic.

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10. Let $f(t) = \int \left(\frac{1-\sin(\log_e t)}{1-\cos(\log_e t)}\right) dt,\ t > 1$**. If** $f(e^{\pi/2}) = -e^{\pi/2}$ **and** $f(e^{\pi/4}) = \alpha e^{\pi/4}$**, then** α **equals:** [4 marks]

(A) $-1-2\sqrt{2}$

(B) $-1-\sqrt{2}$

(C) $1+\sqrt{2}$

(D) $-1+\sqrt{2}$

Correct Answer: (B) $-1-\sqrt{2}$

Solution:

Concept:

- Put $u = \log t$. The half-angle identities $1-\cos u = 2\sin^2(u/2)$ and $\sin u = 2\sin(u/2)\cos(u/2)$ simplify the given integrand.
- The product rule and $d(\log t)/dt = 1/t$ show how a factor t cancels the chain-rule denominator. The two specified points lie in $1 < t < e^{2\pi}$, where the integrand is continuous.

Step 1: Rewrite the integrand in half-angle form Separate the numerator before applying the identities:

$$\frac{1-\sin u}{1-\cos u} = \frac{1}{1-\cos u} - \frac{\sin u}{1-\cos u} = \frac{1}{2}\csc^2\frac{u}{2} - \cot\frac{u}{2}.$$

This combination suggests differentiating a product involving $t\cot(\log t/2)$, rather than treating u as the integration variable without changing dt.

Step 2: Verify the antiderivative and determine the constant Direct differentiation gives

$$\frac{d}{dt}\left[-t\cot\left(\frac{\log t}{2}\right)\right] = -\cot\left(\frac{\log t}{2}\right) + \frac{1}{2}\csc^2\left(\frac{\log t}{2}\right).$$

Therefore $f(t) = -t\cot(\log t/2) + C$ on the interval containing both specified points. At $t = e^{\pi/2}$, the cotangent is $\cot(\pi/4) = 1$. The condition $f(e^{\pi/2}) = -e^{\pi/2}$ consequently gives $C = 0$.

Step 3: Evaluate at the second point Substituting $t = e^{\pi/4}$ gives

$$f(e^{\pi/4}) = -e^{\pi/4}\cot\frac{\pi}{8}.$$

The half-angle value is

$$\cot\frac{\pi}{8}=\frac{1+\cos(\pi/4)}{\sin(\pi/4)}=1+\sqrt{2}.$$

Thus $\alpha=-1-\sqrt{2}$. The positive values in (C) and (D) have the wrong sign; (A) has an extra factor of two multiplying $\sqrt{2}$.

Step 4: Conclusion Differentiating the antiderivative verifies the printed integrand, and the given initial value yields $\boxed{\alpha=-1-\sqrt{2}}$. Hence, the correct option is (B).

Quick Tip to Solve: After half-angle simplification, look for a product derivative: the factor t cancels the $1/t$ from differentiating $\log t$.

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11. Let $\mathrm{S}=\frac{1}{25!}+\frac{1}{3!23!}+\frac{1}{5!21!}+\ldots$ **up to 13 terms. If** $13\,\mathrm{S}=\frac{2^k}{n!}, k\in\mathrm{N}$**, then** $n+k$ **is equal to** [4 marks]

(A) 49

(B) 50

(C) 51

(D) 52

Correct Answer: (A) 49

Solution:

Concept:

- The binomial coefficient $\binom{n}{r}=n!/[r!(n-r)!]$ converts reciprocal products of factorials into a common factorial denominator.
- For any positive integer n, the odd-index binomial coefficients sum to 2^{n-1}. This follows by subtracting the expansions of $(1+1)^n$ and $(1-1)^n$.

Step 1: Express every summand with the same factorial The first term can be written as $1/(1!25!)$. In every denominator the two factorial arguments add to 26, and the first argument takes the odd values $1,3,5,\ldots,25$. There are exactly thirteen such values. Hence

$$S=\frac{1}{26!}\left[\binom{26}{1}+\binom{26}{3}+\cdots+\binom{26}{25}\right].$$

This also confirms that both the first and final terms are included.

Step 2: Sum the odd binomial coefficients The two binomial expansions give

$$2^{26}=\sum_{r=0}^{26}\binom{26}{r},\qquad 0=\sum_{r=0}^{26}(-1)^r\binom{26}{r}.$$

Subtracting the second equation from the first doubles exactly the odd-index terms. Their sum is therefore 2^{25}, and

$$S=\frac{2^{25}}{26!}.$$

Step 3: Absorb the multiplier into the factorial Use $26! = 26 \cdot 25! = 2 \cdot 13 \cdot 25!$, so that

$$13S = \frac{13 \cdot 2^{25}}{26!} = \frac{2^{24}}{25!}.$$

Comparing with the requested form gives $k = 24$ and $n = 25$, hence $n + k = 49$. The alternatives $50, 51, 52$ arise from failing to cancel the factor 2 along with 13; none matches the simplified expression.

Step 4: Conclusion $\boxed{n + k = 49}$. Hence, the correct option is (A).

Quick Tip to Solve: When the factorial arguments in every term have a constant sum, multiply by that sum-factorial and look for an even or odd binomial-coefficient sum.

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12. Let $\alpha, \beta \in \mathbb{R}$ be such that the function $f(x) = \begin{cases} 2\alpha\left(x^2 - 2\right) + 2\beta x & , x < 1 \\ (\alpha + 3)x + (\alpha - \beta) & , x \geq 1 \end{cases}$ be differentiable at all $x \in \mathbb{R}$. Then $34(\alpha + \beta)$ is equal to [4 marks]

(A) 24

(B) 36

(C) 48

(D) 84

Correct Answer: (C) 48

Solution:

Concept:

- A function differentiable at a joining point must first be continuous there. Its left derivative and right derivative must also agree.
- Each branch here is a polynomial, so every point except the join $x = 1$ is automatically differentiable. Two matching conditions determine the two parameters.

Step 1: Impose continuity at the join The left limit comes from the quadratic branch, while the actual value at 1 comes from the linear branch:

$$f(1^-) = 2\alpha(1 - 2) + 2\beta = -2\alpha + 2\beta,$$

$$f(1) = (\alpha + 3) + (\alpha - \beta) = 2\alpha + 3 - \beta.$$

Equating them gives

$$-4\alpha + 3\beta = 3.$$

Matching slopes alone would not exclude a jump, so this equation is essential.

Step 2: Match the one-sided derivatives Differentiate each polynomial on its own interval. The left derivative at the join is $4\alpha + 2\beta$, and the right derivative is the slope $\alpha + 3$. Thus

$$4\alpha + 2\beta = \alpha + 3 \quad \Longrightarrow \quad 3\alpha + 2\beta = 3.$$

Together with continuity, this is sufficient for differentiability at the join.

Step 3: Solve the simultaneous equations and check Multiply the continuity equation by 2 and the derivative equation by 3, then subtract:

$$-8\alpha + 6\beta = 6, \qquad 9\alpha + 6\beta = 9.$$

Hence $17\alpha = 3$, giving $\alpha = 3/17$ and $\beta = 21/17$. Substitution verifies both equations. Finally

$$34(\alpha + \beta) = 34\left(\frac{24}{17}\right) = 48.$$

Values $24, 36, 84$ do not follow from parameters satisfying both matching conditions.

Step 4: Conclusion The required value is $\boxed{48}$. Hence, the correct option is (C).

Quick Tip to Solve: At a piecewise-polynomial join, write the value-matching equation before the slope-matching equation; differentiability requires both, not just equal derivatives.

13. The mean and variance of a data of 10 observations are 10 and 2 , respectively. If an observations α in this data is replaced by β, then the mean and variance become 10.1 and 1.99 , respectively. Then $\alpha + \beta$ equals [4 marks]

(A) 5

(B) 10

(C) 15

(D) 20

Correct Answer: (D) 20

Solution:

Concept:

- For n observations, the mean is $\bar{x} = \frac{1}{n}\sum x_i$, and the variance is $\sigma^2 = \frac{1}{n}\sum x_i^2 - \bar{x}^2$. Here the number of observations remains ten.
- Replacing only α by β changes the sum by $\beta - \alpha$ and the sum of squares by $\beta^2 - \alpha^2$. Factoring the latter connects the desired sum to these two changes.

Step 1: Find the change in the sum The original sum is ten times the original mean, while the new sum is ten times the new mean. Therefore

$$\sum x_i = 10(10) = 100, \qquad \sum x_i' = 10(10.1) = 101.$$

All unchanged observations cancel when these sums are subtracted, giving $\beta - \alpha = 1$. The positive sign is important: the replacement observation is larger.

Step 2: Find the change in the sum of squares Rearrange the variance formula before substituting the

data. For the original observations,

$$\sum x_i^2 = 10(2 + 10^2) = 1020.$$

For the revised observations,

$$\sum (x_i')^2 = 10(1.99 + 10.1^2) = 10(104) = 1040.$$

Thus the sum of squares increases by $1040 - 1020 = 20$, even though the variance decreases slightly.

Step 3: Factor and compare the options Only one square has changed, so

$$\beta^2 - \alpha^2 = (\beta - \alpha)(\beta + \alpha) = 20.$$

Using $\beta - \alpha = 1$ gives $\alpha + \beta = 20$. As a check, the two observations are $\alpha = 9.5$ and $\beta = 10.5$, whose squared difference is indeed twenty. The values 5, 10 and 15 cannot satisfy both changes simultaneously.

Step 4: Conclusion The required sum is $\boxed{\alpha + \beta = 20}$. Hence, the correct option is (D).

Quick Tip to Solve: For a one-observation replacement, compute changes in the sum and sum of squares first; their ratio gives the sum of the old and new observations.

---

14. If the function $f(x) = \frac{e^x\left(e^{\tan x - x} - 1\right) + \log_e(\sec x + \tan x) - x}{\tan x - x}$ is continuous at $x = 0$, then the value of $f(0)$ is equal to [4 marks]

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) $\frac{3}{2}$

(D) 2

Correct Answer: (C) $\frac{3}{2}$

Solution:

Concept:

- Continuity at zero requires $f(0) = \lim_{x \to 0} f(x)$. Split the numerator so that the exponential difference and the logarithmic difference can be treated separately.
- The standard limit $\lim_{u \to 0}(e^u - 1)/u = 1$ handles the first part. For the second part, use $\frac{d}{dx}\log(\sec x + \tan x) = \sec x$, valid in a neighbourhood of zero.

Step 1: Separate the two limits For nonzero x sufficiently close to zero, write

$$f(x) = e^x \frac{e^{\tan x - x} - 1}{\tan x - x} + \frac{\log(\sec x + \tan x) - x}{\tan x - x}.$$

Put $u = \tan x - x$. Then $u \to 0$, and the first term tends to $1 \cdot 1 = 1$. The second numerator and denominator both tend to zero, so their first derivatives can be compared.

Step 2: Evaluate the logarithmic contribution The derivative of the logarithm is obtained by dividing its inner derivative by its argument:

$$\frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \sec x.$$

Consequently, applying the zero-over-zero limit rule and then factoring yields

$$\lim_{x\to 0} \frac{\log(\sec x + \tan x) - x}{\tan x - x} = \lim_{x\to 0} \frac{\sec x - 1}{\sec^2 x - 1}$$
$$= \lim_{x\to 0} \frac{1}{\sec x + 1} = \frac{1}{2}.$$

The cancellation is legitimate on a punctured neighbourhood of zero; it does not substitute zero into a cancelled zero factor.

Step 3: Use continuity and eliminate alternatives Adding the two independent contributions gives $1 + \frac{1}{2} = \frac{3}{2}$. The option $\frac{1}{2}$ omits the exponential contribution. Neither $\frac{2}{3}$ nor 2 equals the limit of the displayed expression. This common two-sided limit is the unique value that fills the removable discontinuity.

Step 4: Conclusion The exponential term tends to 1 and the logarithmic term to $\frac{1}{2}$, so continuity requires $f(0) = 1 + \frac{1}{2} = \boxed{\frac{3}{2}}$. This value is printed as (C); the value 2 is excluded. Hence, the correct option is (C).

Quick Tip to Solve: In a quotient with a sum in its numerator, split off a standard exponential limit before differentiating the remaining zero-over-zero expression.

15. From a lot containing 10 defective and 90 non-defective bulbs, 8 bulbs are selected one by one with replacement. Then the probability of getting at least 7 defective bulbs is [4 marks]

(A) $\frac{73}{10^8}$
(B) $\frac{7}{10^7}$
(C) $\frac{81}{10^8}$
(D) $\frac{67}{10^8}$

Correct Answer: (A) $\frac{73}{10^8}$

Solution:

Concept:

- With replacement, every draw has the same defective-bulb probability $p = 10/100 = 1/10$, and the draws are independent. Hence the defective count X in eight draws has a binomial distribution.
- The binomial formula is $P(X = k) = \binom{8}{k} p^k (1-p)^{8-k}$. At least seven defective bulbs means the two disjoint possibilities $X = 7$ and $X = 8$, both of which must be included.

Step 1: Calculate the probability of exactly seven defects There are $\binom{8}{7} = 8$ choices for the position of the only non-defective bulb. Each such ordered outcome has seven defective draws and one

non-defective draw, so

$$P(X=7)=\binom{8}{7}\left(\frac{1}{10}\right)^{7}\left(\frac{9}{10}\right)=\frac{72}{10^{8}}.$$

The factor eight counts positions rather than different bulbs; replacement allows even the same bulb to be drawn again.

Step 2: Include the all-defective outcome All eight draws can also be defective. This event has only one pattern of defective/non-defective labels, with probability

$$P(X=8)=\binom{8}{8}\left(\frac{1}{10}\right)^{8}=\frac{1}{10^{8}}.$$

These two events cannot occur together, so the probability of their union is their sum, not their product.

Step 3: Add and compare the alternatives Therefore

$$P(X\geq 7)=\frac{72}{10^{8}}+\frac{1}{10^{8}}=\frac{73}{10^{8}}.$$

The other printed values have numerators 70, 81 and 67 over the same denominator 10^{8}, and none equals the sum of the two required events. In particular, a without-replacement calculation would be inappropriate because the question explicitly restores the bulb after every draw.

Step 4: Conclusion The required probability is $\boxed{\frac{73}{10^{8}}}$. Hence, the correct option is (A).

Quick Tip to Solve: For an event near the top of a binomial range, list the few allowed counts directly; here at least seven out of eight means exactly seven or exactly eight.

---

16. Consider an A.P.: $a_1, a_2, \ldots, a_n; a_1 > 0$. **If** $a_2 - a_1 = \frac{-3}{4}$, $a_n = \frac{1}{4}a_1$, **and** $\sum_{i=1}^{n} a_i = \frac{525}{2}$, **then** $\sum_{i=1}^{17} a_i$ **is equal to** [4 marks]

(A) 136

(B) 238

(C) 476

(D) 952

Correct Answer: (B) 238

Solution:

Concept:

- For an arithmetic progression with first term a_1 and common difference d, the last term is $a_n = a_1 + (n-1)d$, and the sum is $S_n = \frac{n}{2}(a_1 + a_n)$.
- The stated last-term relation eliminates a_1 in favour of the integer n. The positive first-term condition and the fact that n counts terms select the admissible quadratic root.

Step 1: Relate the first term to the number of terms Insert $d = -\frac{3}{4}$ and $a_n = \frac{1}{4}a_1$ into the last-term formula:

$$\frac{1}{4}a_1 = a_1 - \frac{3}{4}(n-1).$$

Multiplying by four and rearranging gives $3a_1 = 3(n-1)$, hence $a_1 = n-1$. This uses the number of intervals, $n-1$, rather than the number of terms.

Step 2: Use the given total sum Now

$$\frac{525}{2} = S_n = \frac{n}{2}\left(a_1 + \frac{1}{4}a_1\right) = \frac{5na_1}{8},$$
$$n(n-1) = 420,$$
$$n^2 - n - 420 = (n-21)(n+20) = 0.$$

Since a progression cannot have a negative number of terms, $n = 21$, and therefore $a_1 = 20$. The final term is five, and $\frac{21}{2}(20+5) = \frac{525}{2}$, checking the data.

Step 3: Sum the first seventeen terms Use the original common difference with the first term just found:

$$S_{17} = \frac{17}{2}\left(2(20) + 16\left(-\frac{3}{4}\right)\right)$$
$$= \frac{17}{2}(40-12) = 17(14) = 238.$$

Equivalently, $a_{17} = 8$, so the average of the first and seventeenth terms is fourteen. This check excludes 136, 476 and 952; the last two are multiples produced by losing the half in the sum formula.

Step 4: Conclusion The required sum is $\boxed{238}$. Hence, the correct option is (B).

Quick Tip to Solve: When the last term is a fixed fraction of the first, combine the endpoint formula with the endpoint-average sum before calculating individual terms.

---

17. Let a circle of radius 4 pass through the origin O , the points $\mathrm{A}(-\sqrt{3}a, 0)$ and $\mathrm{B}(0, -\sqrt{2}b)$, where a and b are real parameters and $ab \neq 0$. Then the locus of the centroid of $\triangle\mathrm{OAB}$ is a circle of radius [4 marks]

(A) $\frac{5}{3}$
(B) $\frac{7}{3}$
(C) $\frac{8}{3}$
(D) $\frac{11}{3}$

Correct Answer: (C) $\frac{8}{3}$

Solution:

Concept:

- A circle through the origin has equation $x^2 + y^2 + 2gx + 2fy = 0$, centre $(-g, -f)$, and radius $\sqrt{g^2 + f^2}$. Its intercepts determine g and f.
- The centroid of three vertices is obtained by averaging their coordinates. Eliminating the parameters

from those averages gives the required locus; the restriction $ab \neq 0$ excludes degenerate triangles.

Step 1: Determine the circle coefficients Substitute $A(-\sqrt{3}a, 0)$ into the circle equation. Since $a \neq 0$,

$$3a^2 - 2\sqrt{3}ag = 0 \quad \Longrightarrow \quad g = \frac{\sqrt{3}a}{2}.$$

Similarly, substituting $B(0, -\sqrt{2}b)$, with $b \neq 0$, gives

$$2b^2 - 2\sqrt{2}bf = 0 \quad \Longrightarrow \quad f = \frac{\sqrt{2}b}{2}.$$

Thus the fixed radius four imposes $g^2 + f^2 = 16$, or $3a^2 + 2b^2 = 64$.

Step 2: Express the centroid coordinates Let the centroid be $G(h, k)$. Averaging the coordinates of O, A, B, we get

$$h = -\frac{\sqrt{3}a}{3}, \qquad k = -\frac{\sqrt{2}b}{3}.$$

Therefore $9h^2 = 3a^2$ and $9k^2 = 2b^2$. Substitution into the radius constraint eliminates both parameters:

$$9(h^2 + k^2) = 64.$$

Step 3: Identify the radius and check the options The locus equation is $h^2 + k^2 = \frac{64}{9}$, a circle centred at the origin with radius $\sqrt{64/9} = \frac{8}{3}$. The condition $ab \neq 0$ removes the points on the coordinate axes, since those would collapse one vertex onto the origin; it does not alter this circle's radius. The other proposed radii have squares $25/9$, $49/9$, and $121/9$, none of which satisfies the eliminated equation.

Step 4: Conclusion Eliminating the parameters gives $9(h^2+k^2) = 64$; hence the radius is $\sqrt{64/9} = \boxed{\frac{8}{3}}$. This is printed as (C); the value $\frac{11}{3}$ does not satisfy the radius constraint. Hence, the correct option is (C).

Quick Tip to Solve: For a right triangle with its right-angle vertex at the origin, the centroid is two-thirds of the circumcentre vector; the circumradius condition can then give its distance from the origin directly.

18. Let $A(1,0), B(2,-1)$ and $C\left(\frac{7}{3}, \frac{4}{3}\right)$ be three points. If the equation of the bisector of the angle ABC is $\alpha x + \beta y = 5$, then the value of $\alpha^2 + \beta^2$ is [4 marks]

(A) 5

(B) 8

(C) 10

(D) 13

Correct Answer: (C) 10

Solution:

Concept:

- The internal bisector of an angle has direction equal to the sum of the unit vectors along its two bounding rays. Normalising is essential when the two side lengths differ.
- The angle ABC has vertex B, so its rays are BA and BC. After finding their bisector direction, use the point B to write its line equation and match the specified right-hand side.

Step 1: Find the unit vectors along the two rays The vectors from $B(2,-1)$ to the other vertices are

$$\overrightarrow{BA} = (-1,1), \qquad \overrightarrow{BC} = \left(\frac{1}{3}, \frac{7}{3}\right).$$

Their lengths are $\sqrt{2}$ and $\frac{5\sqrt{2}}{3}$, respectively. Hence the corresponding unit vectors are

$$u = \frac{(-1,1)}{\sqrt{2}}, \qquad v = \frac{(1,7)}{5\sqrt{2}}.$$

Adding the unnormalised vectors would give unequal weight to the two rays and would not generally bisect the angle.

Step 2: Construct the internal bisector The sum of the unit vectors points inside the angle:

$$u + v = \frac{(-5,5) + (1,7)}{5\sqrt{2}} = \frac{(-4,12)}{5\sqrt{2}}.$$

Thus a direction vector is $(-1,3)$, with slope -3. Passing this line through B gives

$$y + 1 = -3(x-2) \quad \Longrightarrow \quad 3x + y = 5.$$

The difference of the unit vectors would describe the external bisector, which is not the ray bisecting the interior angle ABC.

Step 3: Read off the coefficients and compare The equation already has the required constant five, so no rescaling is necessary: $\alpha = 3$ and $\beta = 1$. Consequently

$$\alpha^2 + \beta^2 = 3^2 + 1^2 = 10.$$

The bisector also passes the direct point check $3(2) + (-1) = 5$. Values 5, 8 and 13 do not equal this coefficient sum; using a slope alone without imposing the fixed right-hand side would leave the coefficients incorrectly normalised.

Step 4: Conclusion The internal bisector is $3x + y = 5$, so $\alpha = 3$, $\beta = 1$, and $\alpha^2 + \beta^2 = 9 + 1 = \boxed{10}$. This is printed as (C); the value 8 is excluded. Hence, the correct option is (C).

Quick Tip to Solve: Add unit vectors along the two rays for the internal angle bisector, then normalise the line equation to its prescribed constant before comparing coefficients.

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19. If the domain of the function

$$f(x) = \log_{(10x^2-17x+7)}(18x^2 - 11x + 1)$$

is $(-\infty, a) \cup (b, c) \cup (d, \infty) - \{e\}$**, then** $90(a + b + c + d + e)$ **equals:** [4 marks]

(A) 316

(B) 307

(C) 170

(D) 177

Correct Answer: (A) 316

Solution:

Concept:

- A real logarithm $\log_{b(x)} a(x)$ requires three simultaneous conditions: $a(x) > 0$, $b(x) > 0$, and $b(x) \neq 1$.
- Factor each quadratic and use its sign outside or between its roots. Excluded base-one points must be checked against the intervals that survive the two positivity conditions.

Step 1: Find where the argument is positive Factor the argument and locate its roots:

$$18x^2 - 11x + 1 = (9x - 1)(2x - 1).$$

Since its leading coefficient is positive, the required strict inequality holds for

$$x < \frac{1}{9} \quad \text{or} \quad x > \frac{1}{2}.$$

The two roots themselves cannot belong to the domain because the logarithm of zero is undefined.

Step 2: Require a positive base and intersect the intervals The base factors as

$$10x^2 - 17x + 7 = (10x - 7)(x - 1).$$

It is positive for $x < 7/10$ or $x > 1$. Intersecting with the argument condition gives

$$(-\infty, 1/9) \cup (1/2, 7/10) \cup (1, \infty).$$

Endpoints are open because either the base or the argument vanishes there.

Step 3: Exclude a base of one and evaluate the expression Solve the remaining forbidden-base

equation:

$$10x^2 - 17x + 7 = 1 \quad \Longleftrightarrow \quad (2x-1)(5x-6) = 0.$$

The point $1/2$ is already excluded; only $6/5$ must be removed from the surviving intervals. Thus

$$a = \frac{1}{9}, \quad b = \frac{1}{2}, \quad c = \frac{7}{10}, \quad d = 1, \quad e = \frac{6}{5}.$$

$$90(a+b+c+d+e) = 10 + 45 + 63 + 90 + 108 = 316.$$

Options (B), (C) and (D), giving $307, 170, 177$, respectively, are inconsistent with these five boundary values.

Step 4: Conclusion The three logarithmic-domain conditions prove that the required value is $\boxed{316}$, rather than any of the other printed values. Hence, the correct option is (A).

Quick Tip to Solve: For a variable-base logarithm, intersect argument-positive and base-positive intervals first, then remove only those base-one roots still present.

20. If $\cot x = \frac{5}{12}$ for some $x \in \left(\pi, \frac{3\pi}{2}\right)$, then $\sin 7x \left(\cos \frac{13x}{2} + \sin \frac{13x}{2}\right) + \cos 7x \left(\cos \frac{13x}{2} - \sin \frac{13x}{2}\right)$ is equal to [4 marks]

(A) $\frac{6}{\sqrt{26}}$
(B) $\frac{4}{\sqrt{26}}$
(C) $\frac{5}{\sqrt{13}}$
(D) $\frac{1}{\sqrt{13}}$

Correct Answer: (D) $\frac{1}{\sqrt{13}}$

Solution:

Concept:

- Combine products using $\sin A \cos B - \cos A \sin B = \sin(A-B)$ and $\cos A \cos B + \sin A \sin B = \cos(A-B)$.
- The sign of a half-angle function follows from the quadrant of the half-angle, not the quadrant of the original angle.

Step 1: Reduce the given expression Put $A = 7x$ and $B = 13x/2$. Group the four terms into a sine difference and a cosine difference. The expression E becomes

$$\begin{aligned} E &= (\sin A \cos B - \cos A \sin B) \\ &\quad + (\cos A \cos B + \sin A \sin B) \\ &= \sin(A-B) + \cos(A-B) \\ &= \sin(x/2) + \cos(x/2). \end{aligned}$$

This eliminates the large multiples without expanding seventh-angle formulae.

Step 2: Determine the correct half-angle signs Since x is in the third quadrant and $\cot x = 5/12$, both

sine and cosine are negative. The associated hypotenuse is $\sqrt{5^2+12^2}=13$, so $\cos x=-5/13$. Also $x/2\in(\pi/2,3\pi/4)$, which is in the second quadrant. Hence

$$\sin(x/2)=\sqrt{\frac{1-\cos x}{2}}=\frac{3}{\sqrt{13}},\qquad \cos(x/2)=-\sqrt{\frac{1+\cos x}{2}}=-\frac{2}{\sqrt{13}}.$$

Step 3: Substitute and compare the choices Therefore $E=(3-2)/\sqrt{13}=1/\sqrt{13}$. Choice (C), $5/\sqrt{13}$, would result from incorrectly making the cosine of the half-angle positive. Choices (A) and (B) also disagree with the signed half-angle values; their squares are respectively $18/13$ and $8/13$, while $E^2=1/13$.

Step 4: Conclusion The expression equals $\boxed{1/\sqrt{13}}$. Hence, the correct option is (D).

Quick Tip to Solve: Group the four products into $\sin(A-B)+\cos(A-B)$, then determine the half-angle quadrant before choosing square-root signs.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. The number of 3×2 matrices A , which can be formed using the elements of the set $\{-2,-1,0,1,2\}$ such that the sum of all the diagonal elements of $\mathrm{A^TA}$ is 5 , is

____

[4 marks]

Solution:

Concept:

- For a real matrix $A=(a_{ij})$, $\mathrm{tr}(A^TA)=\sum_{i,j}a_{ij}^2$, the sum of the squares of all its entries.
- Each entry here contributes 0, 1, or 4; a nonzero square has two possible signs for the corresponding entry.

Step 1: Translate the diagonal-sum condition The matrix has six independently labelled positions. The first diagonal entry of A^TA is the sum of squares in the first column, and the second is the corresponding sum for the second column. Thus the required condition is

$$\sum_{i=1}^{3}\sum_{j=1}^{2}a_{ij}^2=5.$$

We must count assignments to all six positions, so entries in different positions are not interchangeable.

Step 2: Count matrices containing an entry of magnitude two Two entries of magnitude two would already contribute $8>5$, so at most one such entry is possible. If there is one, it contributes four and exactly one other entry must have magnitude one. All remaining entries are zero. Choose the position of the magnitude-two entry in six ways, its sign in two ways, the magnitude-one position in five ways,

and its sign in two ways. This gives

$$6 \cdot 2 \cdot 5 \cdot 2 = 120.$$

Step 3: Count matrices without an entry of magnitude two All nonzero entries now have square one. Exactly five of the six positions must therefore contain either $+1$ or -1, with the remaining position zero. There are six choices for the zero position and 2^5 independent sign choices. Hence this case gives

$$6 \cdot 2^5 = 192.$$

The two cases are disjoint and exhaustive because they separate the presence or absence of a squared contribution of four.

Step 4: Conclusion The required number of matrices is $120 + 192 = \boxed{312}$.

Quick Tip to Solve: First partition the required trace into allowed squared values, then count the positions and the independent signs for each partition.

---

22. Let a line L passing through the point $\mathrm{P}(1,1,1)$ be perpendicular to the lines $\frac{x-4}{4} = \frac{y-1}{1} = \frac{z-1}{1}$ and $\frac{x-17}{1} = \frac{y-71}{1} = \frac{z}{0}$. Let the line L intersect the $yz-$ plane at the point Q . Another line parallel to L and passing through the point $\mathrm{S}(1,0,-1)$ intersects the yz-plane at the point R . Then the square of the area of the parallelogram PQRS is equal to ____ . [4 marks]

Solution:

Concept:

- A line perpendicular to two nonparallel directions has direction equal to their vector cross product. The constants locating the given lines do not affect those directions.
- For adjacent side vectors $\mathbf{u}$ and $\mathbf{v}$, the squared parallelogram area is $\|\mathbf{u} \times \mathbf{v}\|^2$.

Step 1: Find the common perpendicular direction The given lines have direction vectors $\mathbf{a} = (4,1,1)$ and $\mathbf{b} = (1,1,0)$. The zero denominator means a zero component in the second direction. Their cross product is

$$\mathbf{a} \times \mathbf{b} = (-1,1,3).$$

Its dot products with the two given directions are $-4 + 1 + 3 = 0$ and $-1 + 1 = 0$, confirming perpendicularity.

Step 2: Locate the intersections with the plane The line through $P = (1,1,1)$ is $(x,y,z) = (1,1,1) + t(-1,1,3)$. The yz-plane has equation $x = 0$, giving $t = 1$ and $Q = (0,2,4)$. Similarly the parallel line through $S = (1,0,-1)$ is $(x,y,z) = (1,0,-1) + u(-1,1,3)$. Setting $x = 0$ gives $u = 1$ and $R = (0,1,2)$.

Step 3: Calculate the area from adjacent sides Take the adjacent sides from P:

$$\mathbf{u} = \overrightarrow{PQ} = (-1,1,3), \qquad \mathbf{v} = \overrightarrow{PS} = (0,-1,-2).$$

These are the appropriate adjacent vectors for the order P, Q, R, S; indeed $\overrightarrow{QR} = \overrightarrow{PS}$. The cross

product is

$$\mathbf{u} \times \mathbf{v} = (1, -2, 1).$$

Thus the squared area is $1^2 + (-2)^2 + 1^2 = 6$. As a check, $\|\mathbf{u}\|^2\|\mathbf{v}\|^2 - (\mathbf{u} \cdot \mathbf{v})^2 = 11 \cdot 5 - (-7)^2 = 6$.

Step 4: Conclusion The square of the parallelogram area is $\boxed{6}$, in fourth powers of the coordinate length unit.

Quick Tip to Solve: Find the cross-product direction once; setting the first coordinate to zero then locates both intersections with the yz-plane.

---

23. Let $(2\alpha, \alpha)$ be the largest interval in which the function $f(t) = \frac{|t+1|}{t^2}, t < 0$, is strictly decreasing. Then the local maximum value of the function $g(x) = 2\log_e(x-2) + \alpha x^2 + 4x - \alpha, x > 2$, is ____ [4 marks]

Solution:

Concept:

- Remove the absolute value separately on each side of $t = -1$, then use the derivative sign to determine intervals of strict decrease.
- A stationary point of g is a strict local maximum when $g' = 0$ and $g'' < 0$ there, provided the point lies in the specified domain.

Step 1: Determine the decreasing interval of the first function For $t < -1$, $f(t) = -(t+1)/t^2$, whereas for $-1 < t < 0$, $f(t) = (t+1)/t^2$. Differentiation gives

$$f'(t) = \begin{cases} (t+2)/t^3, & t < -1, \\ -(t+2)/t^3, & -1 < t < 0. \end{cases}$$

On the negative domain $t^3 < 0$. Consequently the first derivative is positive on $(-\infty, -2)$ and negative on $(-2, -1)$. The second expression is positive on $(-1, 0)$. Thus the largest open interval of decrease is $(-2, -1)$, matching $(2\alpha, \alpha)$, so $\alpha = -1$.

Step 2: Find the admissible stationary point Substitute $\alpha = -1$ into the second function:

$$g(x) = 2\log_e(x-2) - x^2 + 4x + 1, \qquad x > 2.$$

Its derivative is

$$g'(x) = \frac{2}{x-2} - 2x + 4 = \frac{2}{x-2} - 2(x-2).$$

Setting it to zero gives $(x-2)^2 = 1$. Since $x - 2 > 0$, only $x = 3$ belongs to the domain; $x = 1$ is rejected.

Step 3: Classify and evaluate the stationary value The second derivative satisfies

$$g''(x) = -\frac{2}{(x-2)^2} - 2 < 0 \quad (x > 2).$$

Therefore the stationary point is a strict local maximum. Substituting $x = 3$, and using $\log_e 1 = 0$,

gives

$$g(3) = 2\log_e 1 - 9 + 12 + 1 = 4.$$

The question asks for the maximum value of the function, rather than the coordinate at which it occurs.

Step 4: Conclusion The local maximum value is $\boxed{4}$, attained at $x = 3$.

Quick Tip to Solve: With an absolute value, split at its zero before differentiating. For the logarithmic function, put $u = x - 2 > 0$ to reject the extraneous stationary root immediately.

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24. Let a differentiable function f satisfy the equation $\int_0^{36} f\left(\frac{tx}{36}\right) dt = 4\alpha f(x)$. If $y = f(x)$ is a standard parabola passing through the points $(2, 1)$ and $(-4, \beta)$, then β^α is equal to ____ . [4 marks]

Solution:

Concept:

- A standard parabola expressible as a differentiable function of x, with vertex at the origin and axis along the y-axis, has the form $f(x) = kx^2$.
- For a quadratic function, scaling the argument by $t/36$ multiplies its value by $t^2/36^2$; integrate this factor over the given interval.

Step 1: Determine the equation of the parabola Write $f(x) = kx^2$. Since the standard parabola passes through $(2, 1)$,

$$1 = k(2)^2 = 4k, \qquad k = \frac{1}{4}.$$

Hence $f(x) = x^2/4$. This standard form has no horizontal or vertical translation; adding arbitrary linear or constant terms would not represent the specified standard parabola. At $x = -4$, the ordinate is

$$\beta = f(-4) = \frac{(-4)^2}{4} = 4.$$

Step 2: Apply the integral equation Insert the determined function into the left-hand side of the identity:

$$\begin{aligned}\int_0^{36} f\left(\frac{tx}{36}\right) dt &= \int_0^{36} \frac{t^2x^2}{4 \cdot 36^2}\, dt \\ &= \frac{x^2}{4 \cdot 36^2}\left[\frac{t^3}{3}\right]_0^{36} \\ &= \frac{x^2}{4} \cdot \frac{36}{3} \\ &= 12f(x).\end{aligned}$$

The prescribed right-hand side is $4\alpha f(x)$. Comparing at any nonzero x, where $f(x) \neq 0$, gives $4\alpha = 12$, so $\alpha = 3$. The identity also holds at $x = 0$, with both sides zero.

Step 3: Combine the two independently determined values The point condition supplies $\beta = 4$, while the integral identity supplies $\alpha = 3$. They play different roles and should not be interchanged. Raising

the ordinate to the required exponent yields

$$\beta^{\alpha} = 4^3 = 4 \cdot 4 \cdot 4 = 64.$$

No rounding is required because the result is an exact integer.

Step 4: Conclusion The requested value is $\boxed{64}$.

Quick Tip to Solve: For a standard quadratic parabola, factor $f(x)$ out of the scaled-argument integral and integrate the remaining squared scale factor.

---

25. The number of numbers greater than 5000 , less than 9000 and divisible by 3 , that can be formed using the digits $0, 1, 2, 5, 9$, if the repetition of the digits is allowed, is ____ [4 marks]

Solution:

Concept:

- A number is divisible by 3 if and only if its digit sum is divisible by 3. Classifying digits by their remainders avoids listing every number.
- When repetition is allowed, each position can independently use any permitted digit. Arrangements of a residue pattern must still be counted because the positions are distinct.

Step 1: Fix the thousands digit The number lies strictly between 5000 and 9000. Of the allowed digits $0, 1, 2, 5, 9$, only 5 can occupy the thousands place. Thus write the number as $5abc$. The remaining digits must satisfy

$$a + b + c \equiv 1 \pmod 3,$$

since 5 has remainder 2. The candidate 5000 is not divisible by 3 anyway, so the strict lower bound needs no subtraction.

Step 2: Classify and count the possible residues The allowed digits split into residue classes $R_0 = \{0, 9\}$, $R_1 = \{1\}$, and $R_2 = \{2, 5\}$, with sizes 2, 1, and 2. For three positions, the unordered residue patterns whose sum is 1 modulo 3 are exactly $(0, 0, 1)$, $(0, 2, 2)$, and $(1, 1, 2)$. Their sums are 1, 4, and 4 respectively. Each has three distinct position arrangements. Hence their counts are

$$3(2^2)(1) = 12, \qquad 3(2)(2^2) = 24, \qquad 3(1^2)(2) = 6.$$

Repeated digits are included because each factor counts an independent choice, even when the chosen digits coincide.

Step 3: Add the disjoint cases Every permissible suffix belongs to exactly one of these residue patterns, so the cases do not overlap. Therefore

$$N = 12 + 24 + 6 = 42.$$

A leading zero in the suffix is allowed because the full number already begins with 5. A leading 9 for the full number would exceed the upper bound and has correctly been excluded.

Step 4: Conclusion The required number of numbers is $\boxed{42}$. This is a dimensionless count.

Quick Tip to Solve: Fix any digit forced by the interval first, then count the remaining digit sums modulo 3 using residue-class sizes.

26. A boy throws a ball into air at 45° from the horizontal to land it on a roof of a building of height H. If the ball attains maximum height in 2 s and lands on the building in 3 s after launch, then value of H is ____ m.

$$(g = 10\ \text{m/s}^2)$$

[4 marks]

(A) 15

(B) 25

(C) 20

(D) 10

Correct Answer: (A) 15

Solution:

Concept:

- The vertical velocity of a projectile is $v_y = u_y - gt$, where u_y is its initial upward velocity and g is the downward acceleration.
- At the highest point, $v_y = 0$. Its vertical displacement from launch after time t is $y = u_y t - \frac{1}{2}gt^2$; horizontal motion does not change this equation.

Step 1: Use the time to reach the highest point Take upward displacement as positive and set the launch level to $y = 0$. At $t = 2\,\text{s}$, the vertical component of velocity becomes zero. Thus

$$0 = u_y - 10(2), \qquad u_y = 20\,\text{m s}^{-1}.$$

The launch angle is 45°, so $u_x = u_y = 20\,\text{m s}^{-1}$. The total launch speed would be $20\sqrt{2}\,\text{m s}^{-1}$, but only its vertical component is needed for the roof height.

Step 2: Find the vertical position at landing The ball lands at $t = 3\,\text{s}$, measured from launch, not from its highest point. Substituting this total time gives

$$H = 20(3) - \frac{1}{2}(10)(3^2) = 60 - 45 = 15\,\text{m}.$$

Its vertical velocity then is $20 - 10(3) = -10\,\text{m s}^{-1}$, confirming that it reaches the roof while descending.

Step 3: Check the height against the trajectory The maximum height is $u_y^2/(2g) = 400/20 = 20\,\text{m}$. During the one second between the top and landing, the ball falls $\frac{1}{2}g(1)^2 = 5\,\text{m}$, giving $20 - 5 = 15\,\text{m}$ again. The option 20 m is the apex height, 25 m exceeds that maximum, and 10 m does not satisfy the displacement equation at three seconds.

Step 4: Conclusion The roof height is $\boxed{H = 15\,\text{m}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the time to the apex to determine the vertical launch velocity; then substitute the full time from launch into the displacement equation.

27. There are three co-centric conducting spherical shells A, B and C of radii a, b and c respectively ($c > b > a$) and they are charged with charge q_1, q_2 and q_3 respectively. The potentials of the spheres A, B and C respectively, are : [4 marks]

(A) $\frac{1}{4\pi\epsilon_o}\left(\frac{q_1}{a}+\frac{q_2}{b}+\frac{q_3}{c}\right), \frac{1}{4\pi\epsilon_o}\left(\frac{q_1+q_2+q_3}{b}\right), \frac{1}{4\pi\epsilon_o}\left(\frac{q_1+q_2+q_3}{c}\right)$

(B) $\frac{1}{4\pi\epsilon_o}\left(\frac{q_1}{a}+\frac{q_2}{b}+\frac{q_3}{c}\right), \frac{1}{4\pi\epsilon_o}\left(\frac{q_1+q_2}{b}+\frac{q_3}{c}\right), \frac{1}{4\pi\epsilon_o}\left(\frac{q_1+q_2+q_3}{c}\right)$

(C) $\frac{1}{4\pi\epsilon_o}\left(\frac{q_1+q_2+q_3}{a}\right), \frac{1}{4\pi\epsilon_o}\left(\frac{q_1+q_2+q_3}{b}\right), \frac{1}{4\pi\epsilon_o}\left(\frac{q_1+q_2+q_3}{c}\right)$

(D) $\frac{1}{4\pi\epsilon_o}\left(\frac{q_1+q_2+q_3}{a}\right), \frac{1}{4\pi\epsilon_o}\left(\frac{q_1+q_2}{b}+\frac{q_3}{c}\right), \frac{1}{4\pi\epsilon_o}\left(\frac{q_1}{a}+\frac{q_2}{b}+\frac{q_3}{c}\right)$

Correct Answer: (B) $\frac{1}{4\pi\epsilon_o}\left(\frac{q_1}{a}+\frac{q_2}{b}+\frac{q_3}{c}\right), \frac{1}{4\pi\epsilon_o}\left(\frac{q_1+q_2}{b}+\frac{q_3}{c}\right), \frac{1}{4\pi\epsilon_o}\left(\frac{q_1+q_2+q_3}{c}\right)$

Solution:

Concept:

- Taking zero potential at infinity, a spherical shell of charge Q and radius R contributes kQ/R everywhere inside and on it, and kQ/r outside it, where $k = 1/(4\pi\varepsilon_0)$.
- Electric potential is a scalar, so the contributions of the three concentric shells add algebraically. The sign of each charge is retained in its contribution.

Step 1: Evaluate the potential on the innermost shell At $r = a$, shell A contributes its surface potential kq_1/a. The point is inside both B and C, whose respective contributions are the constants kq_2/b and kq_3/c. Hence

$$V_A = k\left(\frac{q_1}{a}+\frac{q_2}{b}+\frac{q_3}{c}\right).$$

The outer-shell contributions must not be evaluated using distance a; a shell has constant potential throughout its interior.

Step 2: Evaluate the potentials on the other shells At $r = b$, A acts as a point charge at the centre, B contributes its own surface potential, and C still gives its interior constant. At $r = c$, all charges are at or inside the observation sphere. Therefore

$$V_B = k\left(\frac{q_1+q_2}{b}+\frac{q_3}{c}\right), \qquad V_C = k\frac{q_1+q_2+q_3}{c}.$$

Induced surface charges do not alter these net-shell results because concentric spherical symmetry is maintained.

Step 3: Compare the three expressions with the choices The required ordered triple must reproduce all three potentials. Option A incorrectly uses q_3/b in the middle potential. Option C places all charge over a and also mishandles the outer shell at B. Option D has an incorrect first potential and assigns the innermost expression to C. Only option B preserves the constant interior potential of each outer shell.

Step 4: Conclusion The potentials are

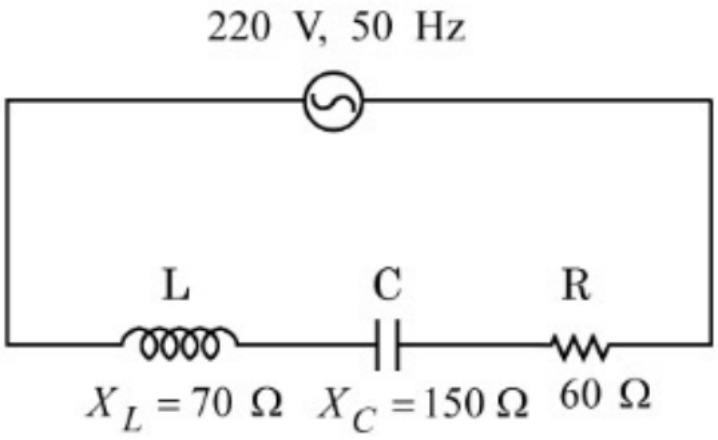
$$\boxed{V_A = k\left(\frac{q_1}{a} + \frac{q_2}{b} + \frac{q_3}{c}\right)},$$

$$\boxed{V_B = k\left(\frac{q_1 + q_2}{b} + \frac{q_3}{c}\right)}, \qquad \boxed{V_C = k\frac{q_1 + q_2 + q_3}{c}}.$$

Hence, the correct option is (B).

Quick Tip to Solve: For each shell separately, use its radius in the denominator when the observation point is inside it, and the observation radius when outside.

28. For the series LCR circuit connected with $220\ \text{V}, 50\ \text{Hz}$ a.c source as shown in the figure, the power factor is $\frac{\alpha}{10}$. The value of α is ____ . [4 marks]



(A) 4

(B) 6

(C) 8

(D) 10

Correct Answer: (B) 6

Solution:

Concept:

- For a series LCR circuit, the net reactance is $X = X_L - X_C$, and the impedance magnitude is $Z = \sqrt{R^2 + X^2}$. All reactances must correspond to the operating frequency.
- The power factor is $\cos\phi = R/Z$, where ϕ is the phase difference between voltage and current. An ideal inductor and capacitor exchange energy but dissipate no average power.

Step 1: Read the circuit values and combine reactances The figure gives $R = 60\,\Omega$, $X_L = 70\,\Omega$, and $X_C = 150\,\Omega$ at the stated supply frequency. Their effects oppose each other, so

$$X = 70 - 150 = -80\,\Omega.$$

The negative sign shows that the circuit is net capacitive and current leads voltage. Adding the two reactances as positive numbers would incorrectly ignore their opposite phases.

Step 2: Calculate the impedance and power factor Resistance and net reactance are perpendicular components of impedance. Consequently,

$$Z = \sqrt{60^2 + (-80)^2} = \sqrt{10000} = 100\,\Omega,$$

$$\cos\phi = \frac{R}{Z} = \frac{60}{100} = 0.6.$$

The source voltage 220 V can determine the current, $I_{\text{rms}} = 220/100 = 2.2\,\text{A}$, but cancels from the ratio defining power factor.

Step 3: Extract the requested parameter The problem writes the power factor as $\alpha/10$. Therefore

$$\frac{\alpha}{10} = 0.6, \qquad \alpha = 6.$$

The value 8 corresponds to $|X|/Z$ multiplied by ten, which uses the reactive component instead of the resistive one. The value 10 would require resonance, contradicted by $X_L \neq X_C$, and 4 does not match the calculated impedance ratio.

Step 4: Conclusion The power factor is 0.6, and $\boxed{\alpha = 6}$, a dimensionless value. Hence, the correct option is (B).

Quick Tip to Solve: Find the difference of the reactances before using the impedance triangle; power factor is its resistance-to-impedance ratio.

---

29. An unpolarised light is incident at an interface of two dielectric media having refractive indices of 2 (incident medium) and $2\sqrt{3}$ (medium) respectively. To satisfy the condition that reflected and refracted rays are perpendicular to each other, the angle of incidence is ____ [4 marks]

(A) 60°

(B) 45°

(C) 30°

(D) 10°

Correct Answer: (A) 60°

Solution:

Concept:

- When the reflected and refracted rays are perpendicular, the incidence and refraction angles obey $i + r = 90^\circ$. Angles are measured from the normal to the interface.
- Snell's law is $n_1 \sin i = n_2 \sin r$. Combining it with the perpendicular-ray condition gives Brewster's law, $\tan i_B = n_2/n_1$.

Step 1: Apply the perpendicular-ray condition The incident-medium refractive index is $n_1 = 2$, while the transmitted-medium index is $n_2 = 2\sqrt{3}$. By the law of reflection, the reflected angle equals the

incident angle. The specified right angle between the outgoing reflected and refracted rays therefore requires

$$r = 90^\circ - i.$$

This relates two angles in different media; it does not say that either angle individually equals 90°.

Step 2: Combine the condition with refraction Substitute $\sin r = \cos i$ in Snell's law:

$$2\sin i = 2\sqrt{3}\cos i.$$

For an incidence angle between zero and 90°, divide by $2\cos i$ to obtain

$$\tan i = \sqrt{3}, \qquad i = 60^\circ.$$

The common factor 2 cancels, so the relevant quantity is the refractive-index ratio, not the transmitted index alone.

Step 3: Check the refraction angle and alternatives The refracted angle is 30°, and indeed $2\sin 60^\circ = 2\sqrt{3}\sin 30^\circ = \sqrt{3}$. Thus the rays are perpendicular. An incidence angle of 45° would require equal refractive indices. The angle 30° reverses the required index ratio, while 10° has a tangent far smaller than $\sqrt{3}$. Both fail the simultaneous conditions.

Step 4: Conclusion The required incidence angle is $\boxed{60^\circ}$. Hence, the correct option is (A).

Quick Tip to Solve: For perpendicular reflected and refracted rays, use transmitted index divided by incident index in Brewster's law, even when neither medium is air.

---

30. The exit surface of a prism with refractive index n is coated with a material having refractive index $\frac{n}{2}$. When this prism is set for minimum angle of deviation, it exactly meets the condition of critical angle. The prism angle is ____ . [4 marks]

(A) 15°

(B) 30°

(C) 60°

(D) 45°

Correct Answer: (C) 60°

Solution:

Concept:

- For light travelling from a denser medium of index n_1 into a rarer medium of index n_2, the critical angle C obeys $\sin C = n_2/n_1$. At this angle the refracted ray grazes the interface.
- For a prism at its minimum-deviation setting, the internal refraction angles are equal: $r_1 = r_2 = A/2$, where A is the prism angle and $r_1 + r_2 = A$.

Step 1: Determine the critical angle at the coated face The incident ray at the exit boundary is inside the prism of refractive index n. The coating has the lower index $n/2$. Applying Snell's law at grazing

refraction gives

$$n \sin C = \frac{n}{2} \sin 90^\circ,$$

$$\sin C = \frac{1}{2}, \qquad C = 30^\circ.$$

The value of n cancels, so no numerical refractive index is needed. Reversing the ratio would give an impossible sine greater than one.

Step 2: Use the stated minimum-deviation setting At minimum deviation, the standard symmetric path through the prism has equal internal angles. The angle incident on the exit surface is thus $r_2 = A/2$. Since this setting exactly meets the critical-angle condition,

$$\frac{A}{2} = C = 30^\circ, \qquad A = 60^\circ.$$

The critical angle is an angle to the exit-face normal inside the prism, whereas the prism angle is the angle between its faces; they must not be equated directly.

Step 3: Check the listed prism angles For $A = 60^\circ$, the internal exit angle is 30°, and $n \sin 30^\circ = n/2$, as required. The other listed prism angles 15, 30, and 45 degrees give internal angles 7.5, 15, and 22.5 degrees in the minimum-deviation setting. All are smaller than the critical angle and therefore fail the condition of grazing emergence.

Step 4: Conclusion The prism angle is $\boxed{A = 60^\circ}$. Hence, the correct option is (C).

Quick Tip to Solve: Calculate the critical angle using the two indices at the exit interface, then double it when the stated minimum-deviation setting makes the internal angles equal.

31. A spring of force constant $15\ \mathrm{N/m}$ is cut into two pieces. If the ratio of their length is $1 : 3$, then the force constant of smaller piece is ____ $\mathrm{N/m}$. [4 marks]

(A) 20

(B) 45

(C) 60

(D) 15

Correct Answer: (C) 60

Solution:

Concept:

- For a uniform spring, the extension under a given force is proportional to its active length. Hence its spring constant is inversely proportional to length: $k\ell =$ constant.
- Cutting a spring changes its stiffness, although the material and coil geometry remain the same. A shorter piece stretches less under the same force and therefore has a larger force constant.

Step 1: Find the shorter length Let the original spring length be ℓ. Write the two piece lengths as x

and $3x$, so that $4x = \ell$. The smaller piece consequently contains one quarter of the original spring:

$$\ell_s = \frac{\ell}{4}.$$

The ratio $1 : 3$ compares the pieces with each other; it does not mean that the smaller piece is one third of the original.

Step 2: Apply the inverse-length relation Using the same force to compare extensions gives $k_s\ell_s = k\ell$. Substitute the original force constant and the smaller length:

$$k_s = 15\frac{\ell}{\ell/4} = 4(15) = 60\,\mathrm{N\,m^{-1}}.$$

As a check, the longer piece has stiffness $15/(3/4) = 20\,\mathrm{N\,m^{-1}}$, and the two pieces joined end-to-end recover $k = 60(20)/(60 + 20) = 15\,\mathrm{N\,m^{-1}}$.

Step 3: Compare the alternatives Option (A), $20\,\mathrm{N\,m^{-1}}$, belongs to the longer piece. Option (B), $45\,\mathrm{N\,m^{-1}}$, incorrectly uses the piece-length ratio as the shortening factor. Option (D) would leave the stiffness unchanged despite removing three quarters of the spring.

Step 4: Conclusion The force constant of the smaller piece is $60\,\mathrm{N\,m^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: For lengths in the ratio $a : b$, the shorter piece has stiffness $k(a + b)/a$, provided $a < b$.

32. Two electrons are moving in orbits of two hydrogen like atoms with speeds 3×10^5 m/s and 2.5×10^5 m/s respectively. If the radii of these orbits are nearly same then the possible order of energy states are ____ respectively. [4 marks]

(A) 9 and 8

(B) 10 and 12

(C) 6 and 5

(D) 8 and 10

Correct Answer: (C) 6 and 5

Solution:

Concept:

- In a Bohr orbit the electron angular momentum is quantized: $m_e v r = n\hbar$, where n is the principal quantum number.
- For the same electron mass and nearly equal orbital radii, this relation makes n proportional to speed. Nuclear charge need not be the same in the two hydrogen-like atoms.

Step 1: Relate the two quantum numbers Apply angular-momentum quantization separately to the two electrons:

$$m_e v_1 r_1 = n_1\hbar, \qquad m_e v_2 r_2 = n_2\hbar.$$

Divide these equations. Since the radii are nearly equal, their ratio is approximately unity, giving

$$\frac{n_1}{n_2} = \frac{v_1 r_1}{v_2 r_2} \simeq \frac{v_1}{v_2}.$$

This avoids assuming equal nuclear charge, an assumption not stated in the question.

Step 2: Substitute the speeds Both velocities have the same units and power of ten, which cancel in the ratio:

$$\frac{n_1}{n_2} \simeq \frac{3 \times 10^5}{2.5 \times 10^5} = \frac{3}{2.5} = \frac{6}{5}.$$

Thus the faster first electron has the larger quantum number when the orbital radii are held nearly equal. Among the listed pairs, $(6, 5)$ gives precisely this ratio.

Step 3: Check the remaining pairs The pair $(9, 8)$ gives $9/8 = 1.125$, rather than 1.2. The pairs $(10, 12)$ and $(8, 10)$ both put the first electron in a lower quantum-number orbit, contradicting $n_1/n_2 > 1$. The familiar rule $v \propto 1/n$ alone applies only when the nuclear charge is fixed; using it across these two atoms would select the wrong order.

Step 4: Conclusion The possible listed energy-state orders are $n_1 = 6$ and $n_2 = 5$, respectively. Hence, the correct option is (C).

Quick Tip to Solve: When two Bohr orbits have equal radii, compare $vr = n\hbar/m_e$, rather than assuming that their nuclear charges are equal.

33. A cylindrical block of mass M and area of cross section A is floating in a liquid of density ρ and with its axis vertical. When depressed a little and released the block starts oscillating. The period of oscillation is ____ [4 marks]

(A) $2\pi\sqrt{\frac{M}{\rho A g}}$

(B) $\pi\sqrt{\frac{2M}{\rho A g}}$

(C) $2\pi\sqrt{\frac{\rho A}{M g}}$

(D) $\pi\sqrt{\frac{\rho A}{M g}}$

Correct Answer: (A) $2\pi\sqrt{\frac{M}{\rho A g}}$

Solution:

Concept:

- Archimedes' principle gives the upward buoyant force as $F_b = \rho g V$, where V is the submerged volume.
- A small vertical displacement of a cylinder changes its submerged volume by Ax. The resulting restoring force has magnitude $\rho g A x$, so the effective spring constant is $k_{\text{eff}} = \rho g A$.

Step 1: Use the floating equilibrium Let the equilibrium submerged depth be h_0. The buoyant force balances the weight, giving

$$\rho g A h_0 = M g.$$

Now displace the cylinder downward by a small distance x, keeping it partly immersed with its axis vertical. Its submerged depth becomes $h_0 + x$, so the upward buoyancy increases by $\rho g A x$.

Step 2: Write the restoring-force equation Take downward displacement as positive. The weight is unchanged, while the extra buoyancy acts upward. Thus

$$M\ddot{x} = Mg - \rho g A(h_0 + x) = -\rho g A x.$$

The equilibrium terms cancel. Comparing with $\ddot{x} + \omega^2 x = 0$, we obtain

$$\omega^2 = \frac{\rho A g}{M}, \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M}{\rho A g}}.$$

Step 3: Check dimensions and options The quantity $\rho A g$ has units $\mathrm{kg\,s^{-2}}$, so $M/(\rho A g)$ has units of time squared, as required. The second option has an incorrect numerical factor. The third and fourth options invert the mass and displaced-volume dependence and are dimensionally unsuitable. The restoring-force derivation therefore selects the first printed expression.

Step 4: Conclusion The period is $T = 2\pi\sqrt{M/(\rho A g)}$. The equation $M\ddot{x} = -\rho A g x$ proves this first printed expression, even if a differently ordered answer list assigns it another letter. Hence, the correct option is (A).

Quick Tip to Solve: For a floating cylinder, cancel the equilibrium weight and buoyancy first; only the change $\rho g A x$ provides the restoring force.

34. Match the LIST-I with LIST-II.

List-I		List-II	
A.	**Magnetic induction**	**I.**	$\mathrm{MLT^{-2}A^{-2}}$
B.	**Magnetic flux**	**II.**	$\mathrm{ML^2T^{-2}A^{-2}}$
C.	**Magnetic permeability**	**III.**	$\mathrm{ML^0T^{-2}A^{-1}}$
D.	**Self inductance**	**IV.**	$\mathrm{ML^2T^{-2}A^{-1}}$

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-IV, C-II, D-I

(B) A-IV, B-III, C-I, D-II

(C) A-I, B-III, C-IV, D-II

(D) A-III, B-IV, C-I, D-II

Correct Answer: (D) A-III, B-IV, C-I, D-II

Solution:

Concept:

- Use defining physical laws to obtain dimensions. Here $\mathrm{M, L, T, A}$ denote mass, length, time and electric current respectively; A in a dimensional formula is not an area.
- Magnetic induction follows from $F = BI\ell$; magnetic flux is $\Phi = BS$; permeability follows from

$B \propto \mu I/r$; self-inductance follows from $\Phi \propto \mathcal{L}I$.

Step 1: Find induction and flux dimensions For a wire perpendicular to a magnetic field, $B = F/(I\ell)$. Hence

$$[B] = \frac{\mathrm{MLT^{-2}}}{\mathrm{AL}} = \mathrm{ML^0T^{-2}A^{-1}},$$

which is entry III. Multiplying by the area of a loop gives

$$[\Phi] = [B]\mathrm{L}^2 = \mathrm{ML^2T^{-2}A^{-1}},$$

which is entry IV. Thus the first two matches are A-III and B-IV.

Step 2: Find permeability and self-inductance dimensions For a long straight wire, $B = \mu I/(2\pi r)$. Numerical constants have no dimensions, so

$$[\mu] = \frac{[B]\mathrm{L}}{\mathrm{A}} = \mathrm{MLT^{-2}A^{-2}},$$

giving C-I. Flux linkage equals self-inductance times current, with the number of turns dimensionless. Therefore

$$[\mathcal{L}] = \frac{[\Phi]}{\mathrm{A}} = \mathrm{ML^2T^{-2}A^{-2}},$$

giving D-II.

Step 3: Combine and check the matching codes The complete matching is A-III, B-IV, C-I, D-II. Option (A) interchanges the permeability and inductance dimensions, which differ by one power of length. Option (B) interchanges induction and flux, which differ by two powers of length. Option (C) already gives induction the wrong current exponent, so it cannot be correct.

Step 4: Conclusion The required code is A-III, B-IV, C-I, D-II. Hence, the correct option is (D).

Quick Tip to Solve: Build the four dimensions in sequence: induction from force, flux by multiplying area, permeability by multiplying length/current, and inductance from flux/current.

---

35. A brass wire of length 2 m and radius 1 mm at $27^\circ\mathrm{C}$ is held taut between two rigid supports. Initially it was cooled to a temperature of $-43^\circ\mathrm{C}$ creating a tension T in the wire. The temperature to which the wire has to be cooled in order to increase the tension in it to $1.4T$, is ____ $^\circ\mathrm{C}$. [4 marks]

(A) -86

(B) -71

(C) -80

(D) -65

Correct Answer: (B) -71

Solution:

Concept:

- A wire held between rigid supports cannot undergo its free thermal contraction. For a temperature decrease $\Delta\theta$, its tensile stress is $Y\alpha\Delta\theta$, within the linear elastic approximation.

- The tensile force is $F = YA\alpha\Delta\theta$, where Y is Young's modulus, A the wire area and α its linear expansion coefficient. For the same wire these factors cancel in a force ratio.

Step 1: Calculate the first temperature drop The wire is initially held taut at 27°C, which is the reference temperature for the thermally generated tension. Cooling to -43°C produces a decrease

$$\Delta\theta_1 = 27 - (-43) = 70^\circ\text{C}.$$

A temperature difference has the same numerical value in kelvin, so $T = YA\alpha(70)$.

Step 2: Relate the increased tension to cooling Let the required final temperature be θ_2. The new cooling from the original reference is $27 - \theta_2$. Since the material, area and constraints remain unchanged,

$$\frac{1.4T}{T} = \frac{27 - \theta_2}{70}.$$

Therefore

$$27 - \theta_2 = 98, \qquad \theta_2 = 27 - 98 = -71^\circ\text{C}.$$

The second cooling is an additional 28°C below the first cooled state.

Step 3: Check the alternatives Cooling to -86°C, -80°C and -65°C gives total drops of 113, 107 and 92°C, respectively. None is $1.4 \times 70 = 98^\circ\text{C}$. The supplied wire length and radius would be needed for an absolute tension, but cancel when comparing tensions in the same wire.

Step 4: Conclusion The wire must be cooled to -71°C to make its tension $1.4T$. Hence, the correct option is (B).

Quick Tip to Solve: Thermal tension is proportional to the drop from the original stress-free temperature, not to the Celsius temperature itself.

36. Two resistors of 100Ω each are connected in series with a 9 V battery. A voltmeter of 400Ω resistance is connected to measure the voltage drop across one of the resistors. The voltmeter reading is ____ V. [4 marks]

(A) 2

(B) 3

(C) 4

(D) 4.5

Correct Answer: (C) 4

Solution:

Concept:

- A real voltmeter has finite resistance and is connected in parallel with the component whose potential difference is measured. Its presence changes that branch's effective resistance.
- Parallel resistances satisfy $R_p = R_1R_2/(R_1 + R_2)$. The measured branch voltage can then be found from the series voltage divider or from $V_p = IR_p$.

Step 1: Combine the measured resistor and voltmeter The $400\,\Omega$ voltmeter is connected across one $100\,\Omega$ resistor. These two share the same terminal voltage and form a parallel combination:

$$R_p = \frac{100 \times 400}{100 + 400} = 80\,\Omega.$$

The other $100\,\Omega$ resistor stays in series with this combination. Thus the battery sees

$$R_{\text{total}} = 100 + 80 = 180\,\Omega.$$

Step 2: Calculate the branch voltage The current supplied by the ideal $9\,\text{V}$ battery is

$$I = \frac{9}{180} = 0.05\,\text{A}.$$

This is the current through the unshunted resistor and into the parallel branch. The voltage across that branch, and hence the voltmeter reading, is

$$V_p = IR_p = 0.05 \times 80 = 4\,\text{V}.$$

The remaining resistor has a $5\,\text{V}$ drop, and the two drops add to $9\,\text{V}$.

Step 3: Check current splitting and alternatives At $4\,\text{V}$, the measured resistor carries $4/100 = 0.04\,\text{A}$ and the meter carries $4/400 = 0.01\,\text{A}$. Their sum agrees with the battery current. The $4.5\,\text{V}$ option neglects meter loading and would require infinite voltmeter resistance; $2\,\text{V}$ and $3\,\text{V}$ do not satisfy the calculated series voltage division.

Step 4: Conclusion The voltmeter reads $4\,\text{V}$. Hence, the correct option is (C).

Quick Tip to Solve: Always replace the measured resistor and finite-resistance voltmeter by their parallel equivalent before applying the voltage-divider formula.

---

37. Two masses 400 g and 350 g are suspended from the ends of a light string passing over a heavy pulley of radius 2 cm . When released from rest the heavier mass is observed to fall 81 cm in 9 s . The rotational inertia of the pulley is ____ $\text{kg} \cdot \text{m}^2$. $(\text{g} = 9.8\ \text{m/s}^2)$ [4 marks]

(A) 8.3×10^{-3}

(B) 9.5×10^{-3}

(C) 1.86×10^{-2}

(D) 4.75×10^{-3}

Correct Answer: (B) 9.5×10^{-3}

Solution:

Concept:

- For uniformly accelerated motion from rest, the displacement is $s = \frac{1}{2}at^2$. Here a is the common magnitude of the linear acceleration of both masses.
- A massive pulley needs a net torque. Thus its two string tensions differ, and $(T_1 - T_2)R = I\alpha$, with

$\alpha = a/R$ for a string that does not slip.

Step 1: Determine the measured acceleration Convert all quantities to SI units: $m_1 = 0.400\,\text{kg}$, $m_2 = 0.350\,\text{kg}$, $R = 0.020\,\text{m}$, and $s = 0.81\,\text{m}$. Since release is from rest,

$$a = \frac{2s}{t^2} = \frac{2(0.81)}{9^2} = 0.020\,\text{m s}^{-2}.$$

The heavier mass moves downward and the lighter mass moves upward with this magnitude.

Step 2: Apply Newton law to each mass Taking the direction of motion as positive separately for each mass gives $m_1 g - T_1 = m_1 a$ and $T_2 - m_2 g = m_2 a$. Subtracting the tensions,

$$T_1 - T_2 = (m_1 - m_2)g - (m_1 + m_2)a = 0.050(9.8) - 0.750(0.020) = 0.475\,\text{N}.$$

The unequal tensions supply the torque that accelerates the pulley.

Step 3: Use the rotational equation With $\alpha = a/R$, the rotational equation gives

$$I = \frac{(T_1 - T_2)R^2}{a} = \frac{0.475(0.020)^2}{0.020} = 9.5 \times 10^{-3}\,\text{kg m}^2.$$

Options (A), (C) and (D) give different inertias and therefore cannot reproduce the measured acceleration with the stated masses and pulley radius.

Step 4: Conclusion The rotational inertia is $9.5 \times 10^{-3}\,\text{kg m}^2$. Hence, the correct option is (B).

Quick Tip to Solve: Use the measured displacement to find acceleration first; never set the two tensions equal when the pulley has rotational inertia.

---

38. Given below are two statements :

Statement I : For all elements, greater the mass of the nucleus, greater is the binding energy per nucleon.

Statement II : For all elements, nuclei with less binding energy per nucleon transforms to nuclei with greater binding energy per nucleon.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Concept:

- The binding energy B is the energy needed to separate a nucleus into free nucleons. The quantity B/A, where A is its mass number, measures average binding per nucleon.
- An increase in total binding energy in a nuclear reaction corresponds to a decrease in rest energy

and releases energy. The binding-energy curve peaks near the iron–nickel region.

Step 1: Test the asserted dependence on nuclear mass Statement I claims that binding energy per nucleon increases with nuclear mass for all elements. The actual curve rises rapidly for light nuclei, reaches roughly $8.8\,\mathrm{MeV}$ per nucleon around the iron–nickel region, and then decreases gradually for heavier nuclei. Very heavy nuclei have a smaller value than these medium-mass nuclei. Consequently, greater nuclear mass does not always mean greater binding energy per nucleon, so Statement I is false.

Step 2: Identify the energetically favoured direction Statement II expresses the tendency of nuclear transformations towards more tightly bound products. Light nuclei can combine by fusion and heavy nuclei can split by fission so that the products have greater average binding energy per nucleon. For a nucleon-conserving reaction, the increase in total binding energy is the energy released:

$$Q = B_{\text{products}} - B_{\text{reactants}} > 0.$$

This is an energetic tendency, not a claim that every stable nucleus transforms spontaneously without overcoming a reaction barrier. In the binding-energy comparison used here, Statement II is true.

Step 3: Match the truth values to the options The required pair is false for Statement I and true for Statement II. Option (A) incorrectly accepts Statement I; option (B) rejects the stabilising tendency in Statement II; option (C) reverses both truth values.

Step 4: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish total binding energy from binding energy per nucleon: the latter reaches a maximum near iron and then falls for heavier nuclei.

---

39. In a microscope of tube length 10 cm two convex lenses are arranged with focal length of 2 cm and 5 cm . Total magnification obtained with this system for normal adjustment is $(5)^k$. The value of k is ____ . [4 marks]

(A) 4

(B) 2

(C) 5

(D) 3.5

Correct Answer: (B) 2

Solution:

Concept:

- A compound microscope uses a short-focal-length objective to form an enlarged real intermediate image and an eyepiece to magnify that image angularly.
- For normal adjustment, the final image is at infinity. The standard magnifying-power formula is $|M| = (L/f_o)(D/f_e)$, where L is tube length and $D = 25\,\mathrm{cm}$ is the least distance of distinct vision.

Step 1: Identify the objective and eyepiece The shorter focal length belongs to the objective, so take $f_o = 2\,\mathrm{cm}$ and $f_e = 5\,\mathrm{cm}$, with $L = 10\,\mathrm{cm}$. Use the normal-adjustment formula because the observer

sees the final image at infinity; the near-point eyepiece factor $1 + D/f_e$ is not applicable here.

Step 2: Calculate the two magnification factors In the standard microscope tube-length approximation, the magnitude of the objective magnification is

$$|m_o| = \frac{L}{f_o} = \frac{10}{2} = 5.$$

The eyepiece angular magnification for a relaxed eye is

$$M_e = \frac{D}{f_e} = \frac{25}{5} = 5.$$

All lengths have been expressed in centimetres, so each ratio is dimensionless.

Step 3: Express the product as the required power The total magnifying power is the product of the objective and eyepiece factors:

$$|M| = |m_o| M_e = 5 \times 5 = 25 = 5^2.$$

The question asks for the magnitude, so the inversion of the microscope image does not change the exponent. Comparing with 5^k gives $k = 2$. The values 4, 5 and 3.5 produce powers of 5 other than 25, so they are excluded.

Step 4: Conclusion The magnification is $25 = 5^2$, hence $k = 2$. In the printed option order, 2 is option (B). Hence, the correct option is (B).

Quick Tip to Solve: For normal adjustment use D/f_e, whereas a final image at the near point uses $1 + D/f_e$; multiply by the objective factor only afterward.

40. The electrostatic potential in a charged spherical region of radius r varies as $V = ar^3 + b$, where a and b are constants. The total charge in the sphere of unit radius is $\alpha \times \pi a \epsilon_0$. The value of α is ____ . (permittivity of vacuum is ϵ_0) [4 marks]

(A) -12

(B) -6

(C) -8

(D) -9

Correct Answer: (A) -12

Solution:

Concept:

- For a spherically symmetric potential, the radial electric field is $E_r = -dV/dr$. The minus sign gives the direction in which potential decreases.
- Gauss law on a concentric sphere of radius r gives $Q_{\text{enc}} = 4\pi\epsilon_0 r^2 E_r$. The radial component is signed, so negative flux corresponds to negative enclosed charge.

Step 1: Differentiate the given potential The potential is $V(r) = ar^3 + b$, where both a and b are

constants. Therefore,

$$E_r = -\frac{d}{dr}(ar^3 + b) = -3ar^2.$$

The additive constant b shifts the reference potential but has no effect on either the field or the charge. If a is positive, the field points inward.

Step 2: Find the electric flux through a sphere Spherical symmetry makes the radial field constant in magnitude on a concentric Gaussian sphere. Its outward flux is

$$\Phi_E = E_r(4\pi r^2) = (-3ar^2)(4\pi r^2) = -12\pi a r^4.$$

The area factor must contain r^2; using the volume of the sphere instead would not be Gauss law.

Step 3: Evaluate the enclosed charge at unit radius Using $Q_{\text{enc}} = \epsilon_0 \Phi_E$ gives

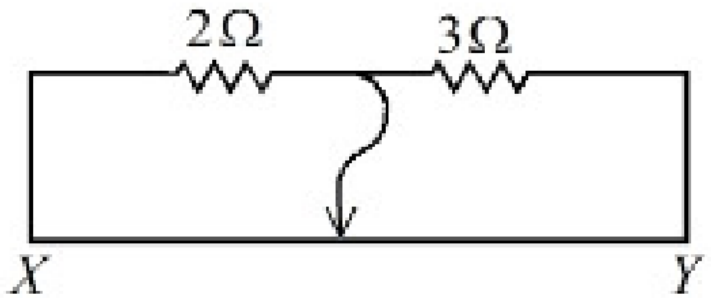
$$Q(r) = -12\pi a \epsilon_0 r^4.$$

At the stated unit radius, $Q = -12\pi a\epsilon_0$. Comparison with $\alpha\pi a\epsilon_0$ gives $\alpha = -12$. The coefficients -6, -8 and -9 do not contain the correct product of the derivative factor 3 and the spherical-area factor 4.

Step 4: Conclusion The required coefficient is $\alpha = -12$. Hence, the correct option is (A).

Quick Tip to Solve: Differentiate potential before using Gauss law; retain the minus sign and multiply the radial field by the spherical surface area.

41. Two resistors 2Ω and 3Ω are connected in the gaps of bridge as shown in figure. The null point is obtained with the contact of jockey at some point on wire XY. When an unknown resistor is connected in parallel with 3Ω resistor, the null point is shifted by 22.5 cm toward Y. The resistance of unknown resistor is ____ Ω. [4 marks]



(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

Solution:

Concept:

- At balance in a metre bridge with uniform wire, the ratio of gap resistances equals the ratio of the wire lengths: $R_L/R_R = l/(100-l)$, with lengths in centimetres.
- Connecting an unknown resistance R in parallel with $3\,\Omega$ gives $R_{\text{eq}} = 3R/(3+R)$. This decreases the right-hand gap resistance.

Step 1: Locate the initial balance point The diagram places $2\,\Omega$ on the left and $3\,\Omega$ on the right. Let l be the distance of the balance point from X. For the metre bridge,

$$\frac{2}{3} = \frac{l}{100-l} \quad \Rightarrow \quad 200 - 2l = 3l \quad \Rightarrow \quad l = 40\,\text{cm}.$$

The remaining length toward Y is initially $60\,\text{cm}$.

Step 2: Apply the stated shift toward the right The new balance point moves toward Y, so its distance from X increases:

$$l_{\text{new}} = 40 + 22.5 = 62.5\,\text{cm}, \qquad 100 - l_{\text{new}} = 37.5\,\text{cm}.$$

The new right-hand equivalent resistance follows from

$$\frac{2}{R_{\text{eq}}} = \frac{62.5}{37.5} = \frac{5}{3}, \qquad R_{\text{eq}} = \frac{6}{5}\,\Omega.$$

The direction is consistent: reducing the right gap resistance increases the left-to-right length ratio.

Step 3: Recover the unknown parallel resistance Use the parallel-resistance formula:

$$\frac{3R}{3+R} = \frac{6}{5} \quad \Rightarrow \quad 15R = 18 + 6R \quad \Rightarrow \quad R = 2\,\Omega.$$

Checking the other options, $R = 1, 3, 4\,\Omega$ gives right-gap equivalents $0.75, 1.5, 12/7\,\Omega$, respectively. None equals the required $1.2\,\Omega$.

Step 4: Conclusion The unknown resistor is $2\,\Omega$, which is option (B) in the printed order. Hence, the correct option is (B).

Quick Tip to Solve: Measure both balance lengths from the same end. A shift toward Y adds to the distance from X, rather than subtracting from it.

42. Three masses $200\ \text{kg}, 300\ \text{kg}$ and 400 kg are placed at the vertices of an equilateral triangle with sides 20 m . They are rearranged on the vertices of a bigger triangle of side 25 m and with the same centre. The work done in this process ____ J. (Gravitational constant $\text{G} = 6.7 \times 10^{-11}\ \text{N m}^2/\text{kg}^2$) [4 marks]

(A) 4.77×10^{-7}

(B) 9.86×10^{-6}

(C) 2.85×10^{-7}

(D) 1.74×10^{-7}

Correct Answer: (D) 1.74×10^{-7}

Solution:

Concept:

- The gravitational potential energy of two point masses separated by r is $-Gm_im_j/r$, taking zero energy at infinite separation.
- For three masses, add the contributions of the three distinct pairs once each. In a quasistatic rearrangement, external work equals the change in total gravitational potential energy.

Step 1: Sum the products for all distinct pairs The three pair products are 200×300, 300×400 and 400×200, all in kg^2. Their sum is

$$S = 60000 + 120000 + 80000 = 260000\,\text{kg}^2.$$

Each pair separation equals the side length because both arrangements are equilateral triangles. Keeping the same centre does not add another energy term; only mutual separations matter.

Step 2: Calculate the change in potential energy Initially the side is $20\,\text{m}$, giving $U_i = -GS/20$. Finally the side is $25\,\text{m}$, giving $U_f = -GS/25$. Thus,

$$W_{\text{ext}} = U_f - U_i = GS\left(\frac{1}{20} - \frac{1}{25}\right).$$

The difference in reciprocal lengths is $(25 - 20)/500 = 0.01\,\text{m}^{-1}$, which is positive.

Step 3: Substitute the gravitational constant Using the specified constant,

$$W_{\text{ext}} = (6.7 \times 10^{-11})(2.6 \times 10^{5})(10^{-2}) = 1.742 \times 10^{-7}\,\text{J}.$$

This is $1.74 \times 10^{-7}\,\text{J}$ to the precision of the options. The sign is positive because separating mutually attracting masses requires external work. The gravitational force itself does negative work. The other three options do not equal the sum of all three pair-energy changes.

Step 4: Conclusion The required external work is $1.74 \times 10^{-7}\,\text{J}$. Hence, the correct option is (D).

Quick Tip to Solve: Count the three distinct mass pairs once each and use the difference of reciprocal separations; the change in side length alone is insufficient.

43. Match the LIST-I with LIST-II.

List-I		List-II	
A.	Radio-wave	I.	is produced by Magnetron valve
B.	Micro-wave	II.	due to change in the vibrational modes of atoms
C.	Infrared-wave	III.	due to inner shell electrons moving from higher energy level to lower energy level
D.	X-ray	IV.	due to rapid acceleration of electrons

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-III, C-I, D-II

(B) A-IV, B-I, C-II, D-III

(C) A-II, B-IV, C-III, D-I

(D) A-IV, B-II, C-I, D-III

Correct Answer: (B) A-IV, B-I, C-II, D-III

Solution:

Concept:

- Accelerating electric charges radiate electromagnetic waves. Radio waves can be generated by rapidly oscillating, and therefore accelerating, electrons in an antenna.
- Different wavelength ranges have characteristic sources: a magnetron generates microwaves, atomic or molecular vibrations generate infrared radiation, and inner-shell electronic transitions generate characteristic X-rays.

Step 1: Match the radio and microwave sources Radio-wave emission corresponds to the rapid acceleration of electrons, so A matches IV. A magnetron is a microwave oscillator: electrons interacting with electric and magnetic fields excite microwave-frequency oscillations in its resonant cavities. Hence B matches I. Neither an inner-shell electronic transition nor a vibrational transition is the characteristic microwave source listed here.

Step 2: Match the infrared and X-ray sources Changes in vibrational modes correspond to infrared radiation, so C matches II. When an inner-shell vacancy is filled by an electron from a higher energy level, the energy difference can emerge as a characteristic X-ray photon. Thus D matches III. Collecting the four assignments gives A-IV, B-I, C-II and D-III.

Step 3: Eliminate the other combinations Option A incorrectly associates microwaves with inner-shell transitions and infrared with a magnetron. Option C assigns radio waves to vibrations and X-rays to a magnetron. Option D assigns microwaves to vibrations and infrared to a magnetron. Each fails at least one characteristic source, whereas option B satisfies all four matches.

Step 4: Conclusion The correct matching is A-IV, B-I, C-II, D-III. Hence, the correct option is (B).

Quick Tip to Solve: Remember the source ladder: antenna acceleration, magnetron, vibrations, inner-shell transitions correspond to radio, microwave, infrared and X-ray radiation.

44. Three charges $+2q, +3q$ and $-4q$ are situated at $(0, -3a), (2a, 0)$ and $(-2a, 0)$ respectively in the xy plane. The resultant dipole moment about origin is ____ . [4 marks]

(A) $2qa(7\hat{i} - 3\hat{j})$

(B) $2qa(3\hat{j} - \hat{i})$

(C) $2qa(3\hat{i} - 7\hat{j})$

(D) $2qa(3\hat{j} - 7\hat{i})$

Correct Answer: (A) $2qa(7\hat{i} - 3\hat{j})$

Solution:

Concept:

- The electric dipole moment about a specified origin is $\vec{p} = \sum_i q_i \vec{r}_i$, where each charge retains its algebraic sign and each position vector is measured from that origin.
- Vector components add separately. For a distribution with nonzero net charge the dipole moment depends on the origin, so the origin specified in this question must be retained.

Step 1: Write each charge-position contribution The position vectors of the three charges are $-3a\hat{j}$, $2a\hat{i}$ and $-2a\hat{i}$, respectively. Therefore their contributions are

$$\vec{p}_1 = (2q)(-3a\hat{j}) = -6qa\hat{j},$$
$$\vec{p}_2 = (3q)(2a\hat{i}) = 6qa\hat{i},$$
$$\vec{p}_3 = (-4q)(-2a\hat{i}) = 8qa\hat{i}.$$

The third contribution points along positive x because both its charge and its x-coordinate are negative.

Step 2: Add the components and factor the result The two charges on the x-axis contribute $(6+8)qa = 14qa$ along positive x. Only the first charge contributes along y, giving $-6qa$. Thus

$$\vec{p} = 14qa\hat{i} - 6qa\hat{j} = 2qa(7\hat{i} - 3\hat{j}).$$

Its dimensions are charge times length. The net charge is q, which is why calculating about the stated origin, rather than silently shifting it, is important.

Step 3: Compare with the alternatives The required vector has a positive x-component with coefficient 7 and a negative y-component with coefficient 3 after extracting $2qa$. Option B has negative x and positive y components. Option C interchanges the required magnitudes. Option D reverses the complete vector. Only option A has both the correct signs and magnitudes.

Step 4: Conclusion $\vec{p} = 2qa(7\hat{i} - 3\hat{j})$, in coulomb metres when q and a are in SI units. Hence, the correct option is (A).

Quick Tip to Solve: For each charge multiply its signed value by its signed coordinates before adding; a negative charge at negative x contributes positively to the x-component.

45. Density of water at $4^\circ\mathrm{C}$ and $20^\circ\mathrm{C}$ are $1000\ \mathrm{kg/m^3}$ and $998\ \mathrm{kg/m^3}$ respectively. The increase in internal energy of 4 kg of water when it is heated from $4^\circ\mathrm{C}$ to $20^\circ\mathrm{C}$ is ____ J.
(specific heat capacity of water $= 4.2\ \mathrm{J/kg}$. and 1 atmospheric pressure $= 10^5\ \mathrm{Pa}$) [4 marks]

(A) 258700.8

(B) 315826.2

(C) 268799.2

(D) 234699.2

Correct Answer: The printed specific heat has inconsistent units and magnitude; its numerical value yields about 268 J, whereas option C requires $4200\,\mathrm{J\,kg^{-1}K^{-1}}$.

Solution:

Concept:

- At constant external pressure, the heat supplied satisfies $Q = \Delta U + P\Delta V$, with $P\Delta V$ the work done by the expanding water. The mass stays fixed, so $V = m/\rho$.
- For a temperature-independent specific heat, $Q = mc\Delta T$. A specific heat requires units of energy per mass per temperature interval; the printed value $4.2\,\mathrm{J/kg}$ omits that interval and is also a thousand times smaller than the usual SI value for water.

Step 1: Calculate the small expansion work The temperature rise is $20 - 4 = 16\,\mathrm{K}$. Using the two supplied densities gives

$$\begin{aligned}\Delta V &= 4\left(\frac{1}{998} - \frac{1}{1000}\right)\mathrm{m^3}\\ &= 8.01603 \times 10^{-6}\,\mathrm{m^3},\\ W &= 10^5\Delta V = 0.801603\,\mathrm{J}.\end{aligned}$$

The water expands, so this positive work must be subtracted from the heat to obtain its internal-energy increase.

Step 2: Check the numerical value as printed Even if the missing temperature unit is supplied and the printed number is used as $c = 4.2\,\mathrm{J\,kg^{-1}K^{-1}}$, the heat is only

$$Q = 4 \times 4.2 \times 16 = 268.8\,\mathrm{J}.$$

Consequently,

$$\Delta U = 268.8 - 0.801603 \simeq 267.9984\,\mathrm{J}.$$

None of the four listed options equals this value. Hence the numerical data as printed do not support any listed answer.

Step 3: Identify the conventional interpretation behind the options The standard specific heat of water is $4.2 \times 10^3\,\mathrm{J\,kg^{-1}K^{-1}}$, equivalently $4.2\,\mathrm{J\,g^{-1}K^{-1}}$. With that value the calculation becomes

$$\Delta U = 4 \times 4200 \times 16 - 0.801603 = 268799.1984\,\mathrm{J}.$$

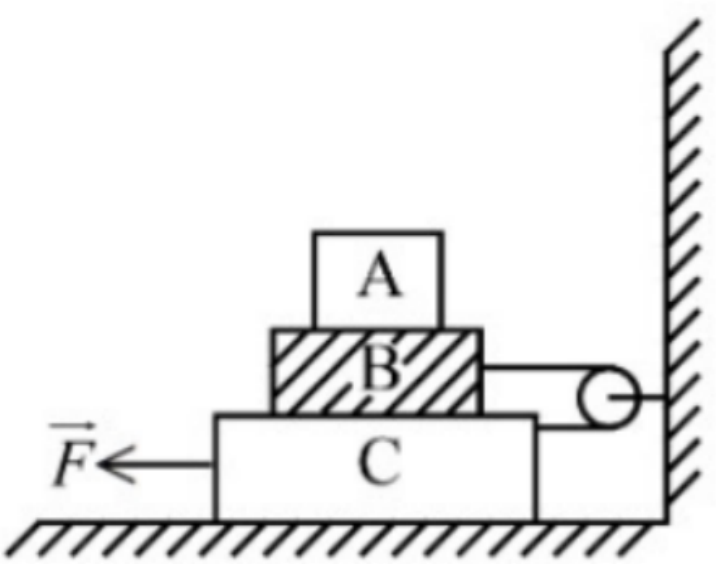
This rounds to $268799.2\,\mathrm{J}$, the number in option C. Options A, B and D match neither the literal printed value nor this conventional specific heat.

Step 4: Conclusion As printed, no listed option follows from the supplied specific heat: its numerical value gives approximately $268\,\text{J}$. The intended answer is option (C), $268799.2\,\text{J}$, only when the specific heat is read as $4.2 \times 10^3\,\text{J}\,\text{kg}^{-1}\text{K}^{-1}$. This explains the key selecting C while retaining the printed-data inconsistency.

Quick Tip to Solve: Check whether the specific heat is per gram or per kilogram before using SI mass, then subtract the small positive expansion work from the heat.

Physics – Section B: Numerical Value Questions (20 marks)

46. In the given figure the blocks A, B and C weigh $4\text{ kg}, 6\text{ kg}$ and 8 kg respectively. The co-efficient of sliding friction between any two surfaces is 0.5 . The force $\vec{F}$ required to slide the block C with constant speed is ____ N . (Use $g = 10\text{ m/s}^2$) [4 marks]



Solution:

Concept:

- For sliding contact the friction magnitude is $f = \mu N$, with N the normal reaction. The bottom contact supports the weights of all blocks above it, not merely the weight of the block touching the floor.
- The string over the fixed pulley makes B move right when C moves left, at equal speeds. During the sliding regime B moves beneath A; friction on B from both its upper and lower contacts acts left. Constant speed requires zero resultant force on B and C.

Step 1: Find the normal reactions and friction forces No block has vertical acceleration. The A-B contact supports A alone, the B-C contact supports A and B, and the ground supports all three blocks.

Therefore

$$N_{AB} = 4(10) = 40\,\text{N}, \qquad f_{AB} = 0.5(40) = 20\,\text{N},$$
$$N_{BC} = (4+6)(10) = 100\,\text{N}, \qquad f_{BC} = 0.5(100) = 50\,\text{N},$$
$$N_g = (4+6+8)(10) = 180\,\text{N}, \qquad f_g = 0.5(180) = 90\,\text{N}.$$

The friction at B-C depends on the supported mass 10 kg, even though C itself has mass 8 kg.

Step 2: Balance the middle block As C is pulled left, the pulley constraint makes B move right. Both the contact with C below and the sliding contact with A above oppose B's rightward motion. The string tension pulls B right. For constant speed of B,

$$T - f_{BC} - f_{AB} = 0, \qquad T = 50 + 20 = 70\,\text{N}.$$

A can accelerate while sliding; the stated constant-speed condition is imposed on C and, through the string, B.

Step 3: Balance the lower block Three forces oppose the applied leftward force on C: ground friction, friction exerted by B, and the rightward pull of the string. The B-C friction on C is rightward by Newton's third law. Consequently,

$$F = f_g + f_{BC} + T = 90 + 50 + 70 = 210\,\text{N}.$$

Counting only the floor friction would miss both the inter-block resistance and the string force. The result pertains to the sliding regime at the contacts specified by the question.

Step 4: Conclusion The required force during sliding is $\boxed{210\,\text{N}}$, directed left as shown. The numerical answer is 210.

Quick Tip to Solve: Draw separate horizontal balances for B and C: first obtain the tension from B, then include that same tension among the resisting forces on C.

47. A voltage regulating circuit consisting of Zener diode, having break-down voltage of 10 V and maximum power dissipation of 0.4 W , is operated at 15 V . The approximate value of protective resistance in this circuit is ____ Ω. [4 marks]

Solution:

Concept:

- In breakdown a Zener diode maintains approximately its rated voltage V_Z. Its dissipated power is $P_Z = V_Z I_Z$, so the maximum safe Zener current is $I_{Z,\max} = P_{Z,\max}/V_Z$.
- A series protective resistor drops the excess of the supply voltage over the Zener voltage. With no load specified, the limiting design uses the maximum no-load current through the Zener: $R = (V_s - V_Z)/I_{Z,\max}$.

Step 1: Convert the power rating to a safe current The Zener voltage is 10 V and the maximum

allowed dissipation is 0.4 W. Therefore

$$I_{Z,\max} = \frac{0.4}{10} = 0.04\,\text{A} = 40\,\text{mA}.$$

It is the voltage across the diode, not the total supply voltage, that appears in this power calculation. Using 15 V here would underestimate the allowed current because part of the supply voltage is dropped across the resistor.

Step 2: Find the voltage across the protective resistor Kirchhoff's voltage law for the supply, resistor and Zener gives

$$V_R = V_s - V_Z = 15 - 10 = 5\,\text{V}.$$

At the limiting safe current, Ohm's law then yields

$$R = \frac{V_R}{I_{Z,\max}} = \frac{5}{0.04} = 125\,\Omega.$$

The resistor limits the current while allowing the diode to remain in its breakdown region.

Step 3: Check the rating at this resistance The series current is $5/125 = 0.04\,\text{A}$, and the resulting Zener power is $10(0.04) = 0.4\,\text{W}$, exactly its stated maximum. A smaller resistance would exceed that dissipation in the no-load case. If a load is connected in parallel with the Zener it takes part of the resistor current, so the no-load case is the relevant worst case for protecting the diode.

Step 4: Conclusion The limiting protective resistance is $\boxed{125\,\Omega}$. Thus the requested numerical value is 125.

Quick Tip to Solve: Use the Zener voltage to turn its power rating into current, but use supply minus Zener voltage to calculate the series resistance.

48. A short bar magnet placed with its axis at 30° with an external field of 800 Gauss, experiences a torque of 0.016 N.m. The work done in moving it from most stable to most unstable position is $\alpha \times 10^{-3}$ J. The value of α is ____ . [4 marks]

Solution:

Concept:

- A magnetic dipole of moment m in a uniform magnetic field B experiences torque $\tau = mB\sin\theta$, where θ is the angle between its axis and the field.
- Its potential energy is $U = -mB\cos\theta$. The most stable orientation is parallel to the field and the most unstable orientation is antiparallel, so the external work for a slow rotation between them is the increase in potential energy.

Step 1: Obtain the product of magnetic moment and field At the stated angle 30°, $\sin 30^\circ = 1/2$. The measured torque gives

$$mB = \frac{\tau}{\sin 30^\circ} = \frac{0.016}{0.5} = 0.032\,\text{J}.$$

Torque and energy share the dimensions of newton metre. If the magnetic moment is wanted

separately, convert $800\,\mathrm{G} = 0.0800\,\mathrm{T}$, giving $m = 0.032/0.0800 = 0.4\,\mathrm{A\,m^2}$. The work calculation itself only needs the product mB.

Step 2: Evaluate the two endpoint energies At the most stable orientation, $\theta = 0^\circ$, and at the most unstable orientation, $\theta = 180^\circ$. Thus

$$U_{\text{stable}} = -mB = -0.032\,\mathrm{J},$$
$$U_{\text{unstable}} = +mB = +0.032\,\mathrm{J}.$$

The work supplied externally is positive because the dipole is being moved from the energy minimum to the energy maximum.

Step 3: Calculate the work and extract the coefficient The required work is the difference between those energies, not the torque multiplied by 180°, since torque changes continuously with angle:

$$W = U_{\text{unstable}} - U_{\text{stable}} = 2mB = 0.064\,\mathrm{J}.$$

Writing this in the requested form gives

$$0.064\,\mathrm{J} = 64 \times 10^{-3}\,\mathrm{J}.$$

The field does the opposite work, $-0.064\,\mathrm{J}$, during this controlled rotation.

Step 4: Conclusion The required external work is $64 \times 10^{-3}\,\mathrm{J}$, so $\boxed{\alpha = 64}$. The coefficient alpha is dimensionless.

Quick Tip to Solve: First calculate mB directly from torque divided by sine of the angle; a stable-to-unstable rotation always costs 2mB in external work.

---

49. A gas of certain mass filled in a closed cylinder at a pressure of 3.23 kPa has temperature $50^\circ\mathrm{C}$. The gas is now heated to double its temperature. The modified pressure is ____ Pa . [4 marks]

Solution:

Concept:

- For a fixed amount of gas in a rigid closed cylinder, the ideal-gas law gives $PV = nRT$. Because n and V remain constant, P/T is constant.
- Temperature ratios in gas laws must use absolute temperature in kelvin: $T = t + 273$, where t is the Celsius reading. Pressure may be in any consistent unit.

Step 1: Identify the final Celsius reading The temperature is initially quoted as $50^\circ\mathrm{C}$. Taking the stated doubling to mean doubling this Celsius reading gives

$$t_2 = 2(50) = 100^\circ\mathrm{C}.$$

Thus the absolute temperatures used in the pressure law are

$$T_1 = 50 + 273 = 323\,\mathrm{K}, \qquad T_2 = 100 + 273 = 373\,\mathrm{K}.$$

The numerical answer depends on the temperature scale meant by the phrase "double its temperature"; one must specify that interpretation before applying the gas law.

Step 2: Apply the constant-volume pressure law Convert the initial pressure and use the ratio of kelvin temperatures:

$$P_1 = 3.23\,\text{kPa} = 3230\,\text{Pa},$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}, \qquad P_2 = 3230\frac{373}{323} = 3730\,\text{Pa}.$$

The pressure increases because heating increases molecular kinetic energy and hence momentum transfer to the cylinder walls. It does not double under the Celsius-reading interpretation because the absolute temperature has not doubled.

Step 3: Distinguish absolute-temperature doubling If the phrase instead meant doubling the absolute temperature, then $T_2 = 646\,\text{K}$ and $P_2 = 6460\,\text{Pa}$. That is a different interpretation. The value $3730\,\text{Pa}$ corresponds specifically to heating from 50°C to 100°C.

Step 4: Conclusion The requested numerical value is $\boxed{3730}$, with pressure $3730\,\text{Pa}$, interpreting doubling as 50°C to 100°C.

Quick Tip to Solve: Convert both stated Celsius readings to kelvin before taking their ratio; doubling a Celsius reading does not double absolute temperature.

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50. Sixty four rain drops of radius 1 mm each falling down with a terminal velocity of $10\ \text{cm/s}$ coalesce to form a bigger drop. The terminal velocity of bigger drop is

____ cm/s. [4 marks]

Solution:

Concept:

- Conservation of liquid volume during coalescence gives $N(4\pi r^3/3) = 4\pi R^3/3$, where r and R are the small- and large-drop radii.
- In the Stokes-law regime, terminal speed is $v_t = 2r^2(\rho - \rho_f)g/(9\eta)$. For the same liquid and surrounding fluid, terminal speed is proportional to the square of the radius.

Step 1: Find the radius of the combined drop All 64 drops have the same radius, and coalescence preserves their total volume. Therefore

$$64\frac{4\pi r^3}{3} = \frac{4\pi R^3}{3}, \qquad R^3 = 64r^3.$$

Taking cube roots gives

$$R = 4r = 4\,\text{mm}.$$

The radius grows by a factor of four, not by a factor of 64: it is volume, which depends on the cube of radius, that is additive.

Step 2: Use the terminal-speed scaling At terminal speed the effective downward weight, after allowing for buoyancy, is balanced by the viscous drag $6\pi\eta r v_t$. The effective weight varies as r^3, while

the drag coefficient varies as r, producing $v_t \propto r^2$. The density difference, viscosity and gravitational acceleration are unchanged, so

$$\frac{V_t}{v_t} = \left(\frac{R}{r}\right)^2 = 4^2 = 16.$$

Substituting the original terminal speed,

$$V_t = 16(10) = 160\,\text{cm}\,\text{s}^{-1}.$$

This is the final terminal speed of the larger drop under the Stokes-law model; it need not be its speed immediately after collision and coalescence.

Step 3: Conclusion The terminal velocity of the bigger drop is $\boxed{160\,\text{cm}\,\text{s}^{-1}}$. The numerical entry is $\boxed{160}$.

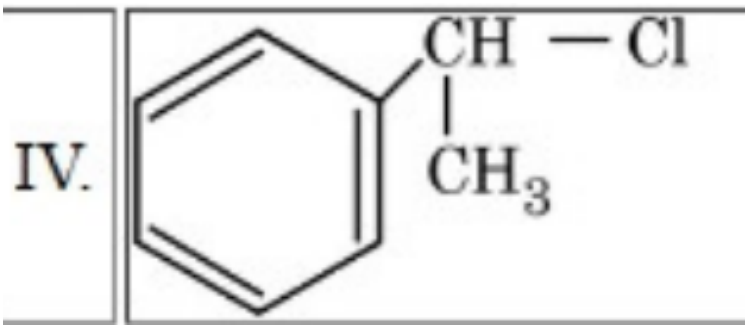
Quick Tip to Solve: For N identical drops merging in the Stokes regime, first use $R/r = N^{1/3}$, then $V_t/v_t = N^{2/3}$.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Match the List-I with List-II.

List-I: Chloro derivative	List-II: Example
A. Vinyl Chloride	I. $CH_2=CH-CH_2Cl$
B. Benzyl Chloride	II. $CH_3-CH(Cl)CH_3$
C. Alkyl Chloride	III. $CH_2=CHCl$
D. Allyl Chloride	IV.

Choose the correct answer from the options given below: [4 marks]

IV. 

(A) A-I, B-II, C-IV, D-III

(B) A-III, B-IV, C-II, D-I

(C) A-III, B-IV, C-I, D-II

(D) A-IV, B-I, C-III, D-II

Correct Answer: (B) A-III, B-IV, C-II, D-I

Solution:

Concept:

- Classify a chloro derivative by the carbon directly bonded to chlorine and its relation to a double bond or aromatic ring. Vinylic chlorine is attached directly to an alkene carbon.
- Benzylic and allylic chlorides have chlorine on a saturated carbon adjacent to an aromatic ring and an alkene, respectively. An ordinary alkyl chloride lacks either adjacent unsaturated system.

Step 1: Identify vinyl and allyl examples Example III is $CH_2=CHCl$. Its chlorine is attached directly to a carbon of the carbon–carbon double bond, so it is the vinyl chloride example: A–III.

Example I is $CH_2=CH-CH_2Cl$. Here the chlorine-bearing CH_2 group lies beside the double bond rather than being part of it. This is allyl chloride: D–I. The extra saturated carbon between chlorine and the double bond is the decisive difference between examples I and III.

Step 2: Identify the benzylic and alkyl examples In the drawn example IV, the chlorine-bearing CH carbon is directly attached to a benzene ring and also bears a CH_3 group. It is a secondary benzylic chloride, $C_6H_5CH(Cl)CH_3$, and therefore matches the benzylic category represented by B: B–IV.

Example II, $CH_3-CH(Cl)CH_3$, is an ordinary secondary alkyl chloride, 2-chloropropane. There is no adjacent aromatic ring or double bond, so C–II. The printed category "Benzyl Chloride" is thus being used for the benzylic chloride class; IV is not unsubstituted $C_6H_5CH_2Cl$.

Step 3: Compare the four combinations Combining the identifications yields A–III, B–IV, C–II, D–I. Option A incorrectly identifies the vinyl example as I; option C interchanges the alkyl and allyl examples; option D identifies the benzylic structure IV as vinyl chloride. Only option B preserves all four classifications.

Step 4: Conclusion The matching is **A–III, B–IV, C–II, D–I**. Hence, the correct option is (B).

Quick Tip to Solve: Locate the carbon bonded to Cl first: on $C=C$ means vinylic; next to $C=C$ means allylic; next to a benzene ring means benzylic.

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52. At $27^\circ C$ in presence of a catalyst, activation energy of a reaction is lowered by $10\ \text{kJ mol}^{-1}$. The logarithm of ratio of $\frac{\text{k (catalysed)}}{\text{k (uncatalysed)}}$ is....
(Consider that the frequency factor for both the reactions is same) [4 marks]

(A) 0.1741

(B) 1.741

(C) 17.41

(D) 3.482

Correct Answer: (B) 1.741

Solution:

Concept:

- The Arrhenius equation is $k = Ae^{-E_a/(RT)}$, where A is the frequency factor and E_a is the activation energy.
- At the same temperature and the same frequency factor, lowering the activation energy increases the rate constant. The common logarithm is related to the natural logarithm by $\ln x = 2.303 \log_{10} x$.

Step 1: Form the ratio of rate constants Write the uncatalysed and catalysed activation energies as E_u and E_c. The given decrease is

$$E_u - E_c = 10\,\text{kJ mol}^{-1} = 10000\,\text{J mol}^{-1}.$$

Because both frequency factors are the same, they cancel:

$$\frac{k_c}{k_u} = \frac{Ae^{-E_c/(RT)}}{Ae^{-E_u/(RT)}} = e^{(E_u - E_c)/(RT)}.$$

The exponent is positive, as expected for an increase in rate constant on adding the catalyst.

Step 2: Convert to the common logarithm Use the absolute temperature $T = 27 + 273 = 300\,\text{K}$ and $R = 8.314\,\text{J mol}^{-1}\text{K}^{-1}$. Taking the common logarithm gives

$$\log_{10} \frac{k_c}{k_u} = \frac{E_u - E_c}{2.303RT} = \frac{10000}{2.303(8.314)(300)} \simeq 1.741.$$

The ratio itself is about $10^{1.741}$, whereas the question asks for its logarithm. The natural logarithm would be about 4.01, which is not the listed common-log value.

Step 3: Check the alternatives The value 0.1741 is ten times too small and 17.41 is ten times too large; both fail the stated energy conversion and temperature substitution. The value 3.482 is twice the correct logarithm and would correspond to twice the stated activation-energy decrease at this temperature. Thus only 1.741 matches the data.

Step 4: Conclusion $\log_{10}\left(\frac{k_{\text{catalysed}}}{k_{\text{uncatalysed}}}\right) = \boxed{1.741}$. Hence, the correct option is (B).

Quick Tip to Solve: When frequency factors are equal, subtract activation energies first. Use joules with $R = 8.314$ and include 2.303 for a common logarithm.

53. A hydroxy compound (X) with molar mass $122\ \text{g mol}^{-1}$ is acetylated with acetic anhydride, using a large excess of the reagent ensuring complete acetylation of all hydroxyl groups. The product obtained has a molar mass of $290\ \text{g mol}^{-1}$. The number of hydroxyl groups present in compound (X) is: [4 marks]

(A) 3

(B) 5

(C) 2

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- Acetylation replaces the hydrogen of each hydroxyl group by an acetyl group, converting ROH into $ROCOCH_3$. The oxygen originally present in the hydroxyl group remains in the product.
- An attached acetyl group CH_3CO has molar mass $43\,\text{g mol}^{-1}$, but it replaces a hydrogen of molar mass $1\,\text{g mol}^{-1}$. The net increase per hydroxyl group is therefore $42\,\text{g mol}^{-1}$.

Step 1: Calculate the increase per hydroxyl group The acetylation reaction can be represented by

$$\text{ROH} + (\text{CH}_3\text{CO})_2\text{O} \longrightarrow \text{ROCOCH}_3 + \text{CH}_3\text{COOH}.$$

Only the mass incorporated into the acetylated molecule is used. The molecule loses one hydroxyl hydrogen and gains $COCH_3$, so

$$\Delta M_{\text{per OH}} = (2 \times 12 + 3 \times 1 + 16) - 1 = 42\,\text{g mol}^{-1}.$$

It would be incorrect to add the mass of an entire acetic anhydride molecule or to use 43 without subtracting the displaced hydrogen.

Step 2: Find the total number of hydroxyl groups The observed increase in molar mass is

$$\Delta M = 290 - 122 = 168\,\text{g mol}^{-1}.$$

Let n be the number of hydroxyl groups. The large excess of reagent ensures that every such group is

acetylated, hence

$$42n = 168, \qquad n = \frac{168}{42} = 4.$$

As a check, four substitutions give $122 + 4(42) = 290\,\mathrm{g\,mol^{-1}}$, exactly the stated product molar mass.

Step 3: Eliminate the other counts Three hydroxyl groups would give a product molar mass of $248\,\mathrm{g\,mol^{-1}}$, five would give $332\,\mathrm{g\,mol^{-1}}$, and two would give $206\,\mathrm{g\,mol^{-1}}$. None equals the observed 290. Thus the alternatives 3, 5 and 2 are excluded by the same mass balance.

Step 4: Conclusion Compound X contains $\boxed{4}$ hydroxyl groups. Hence, the correct option is (D).

Quick Tip to Solve: For complete acetylation, divide the molar-mass increase by 42, not 43: the acetyl group replaces one hydrogen.

54. Consider three metal chlorides $\mathrm{x, y}$ and z , where x is water soluble at room temperature, y is sparingly soluble in water at room temperature and z is soluble in hot water. $\mathrm{x, y}$ and z are respectively [4 marks]

(A) $\mathrm{AlCl_3, PbCl_2}$ and $\mathrm{BaCl_2}$

(B) $\mathrm{CuCl_2, AgCl}$ and $\mathrm{PbCl_2}$

(C) $\mathrm{AgCl, Hg_2Cl_2}$ and $\mathrm{PbCl_2}$

(D) $\mathrm{MgCl_2, AgCl}$ and $\mathrm{AlCl_3}$

Correct Answer: (B) $\mathrm{CuCl_2, AgCl}$ and $\mathrm{PbCl_2}$

Solution:

Concept:

- Most common metal chlorides dissolve readily in water, whereas AgCl, $\mathrm{Hg_2Cl_2}$ and $\mathrm{PbCl_2}$ are important exceptions used in qualitative analysis.
- Lead(II) chloride is sparingly soluble in cold water but dissolves appreciably in hot water. This temperature dependence distinguishes it from silver chloride in the usual group-I chloride separation.

Step 1: Identify the room-temperature soluble chloride The first substance x must dissolve in water at room temperature. $\mathrm{CuCl_2}$ satisfies this requirement, forming an aqueous copper(II) chloride solution. AgCl does not satisfy it because silver chloride is very poorly soluble in water. Therefore the option beginning with AgCl is immediately excluded.

The remaining soluble first members must still be checked against both conditions on y and z; choosing x alone cannot determine the answer.

Step 2: Identify the sparingly soluble and hot-water soluble chlorides AgCl is described as sparingly soluble because its dissolution equilibrium strongly favours the solid:

$$\mathrm{AgCl(s) \rightleftharpoons Ag^+(aq) + Cl^-(aq).}$$

It is thus suitable for y. $\mathrm{PbCl_2}$ has the characteristic increase in solubility on heating and is the

hot-water soluble chloride required for z. This supplies the ordered sequence

$$x = \mathrm{CuCl_2}, \qquad y = \mathrm{AgCl}, \qquad z = \mathrm{PbCl_2}.$$

The order matters: the question assigns a different solubility description to each position.

Step 3: Check the rejected sequences Option A ends with $BaCl_2$, which is already readily water soluble and is not the characteristic chloride separated by hot-water dissolution. Option C puts poorly soluble AgCl in the room-temperature soluble position x. Option D ends with $AlCl_3$, which dissolves and hydrolyses in water rather than representing the characteristic cold-water sparingly soluble, hot-water soluble chloride. Option B alone gives the standard qualitative-analysis sequence.

Step 4: Conclusion $\boxed{x = \mathrm{CuCl_2},\ y = \mathrm{AgCl},\ z = \mathrm{PbCl_2}}$. Hence, the correct option is (B).

Quick Tip to Solve: For chloride-separation questions, remember the hot-water test for $PbCl_2$; then distinguish readily soluble chlorides from AgCl.

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55. Given below are statements about some molecules/ions.

Identify the CORRECT statements.

A. The dipole moment value of NF_3 is higher than that of NH_3.

B. The dipole moment value of BeH_2 is zero.

C. The bond order of O_2^{2-} and F_2 is same.

D. The formal charge on the central oxygen atom of ozone is -1 .

E. In NO_2, all the three atoms satisfy the octet rule, hence it is very stable.

Choose the correct answer from the options given below: [4 marks]

(A) A, B, C, D & E

(B) B & C Only

(C) B, C & D Only

(D) A, C & D Only

Correct Answer: (B) B & C Only

Solution:

Concept:

- Molecular dipole moment is the vector sum of bond moments; a symmetric linear molecule can have polar bonds but zero net moment.
- Bond order is $(N_b - N_a)/2$. Formal charge is valence electrons minus nonbonding electrons minus half the bonding electrons.

Step 1: Check the dipole-moment statements Both NH_3 and NF_3 are pyramidal, but their bond-moment contributions act differently relative to the lone-pair contribution. They reinforce in NH_3 and largely oppose in NF_3. Thus NF_3 has the smaller dipole moment, so A is false. In linear BeH_2, the two identical bond moments point in opposite directions and cancel. Its dipole moment is zero, so B is true.

Step 2: Check bond order and formal charge Adding two electrons to O_2 fills antibonding orbitals and reduces its bond order from 2 to 1. The bond order of F_2 is also 1. Therefore C is true. In an ozone resonance structure, the central oxygen has one lone pair and three shared electron pairs. Its formal charge is

$$6 - 2 - \frac{6}{2} = +1.$$

The singly bonded terminal oxygen carries -1; the central atom does not. Hence D is false.

Step 3: Test the octet claim and compare options NO_2 has $5 + 2(6) = 17$ valence electrons, an odd total. It is an odd-electron species and cannot give complete octets to all three atoms. Hence E is false. Only B and C survive: option (A) includes three false statements, while options (C) and (D) include the incorrect central-oxygen charge statement.

Step 4: Conclusion Only statements B and C are correct. Hence, the correct option is (B).

Quick Tip to Solve: For several bonding statements, test symmetry, antibonding occupancy and formal charge separately; an odd electron total immediately flags an octet exception.

56. $A \rightarrow D$ **is an endothermic reaction occurring in three steps (elementary).**

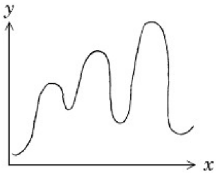
(i) $A \rightarrow B \Delta H_i = +ve$

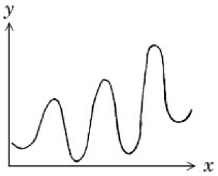
(ii) $B \rightarrow C \Delta H_{ii} = -ve$

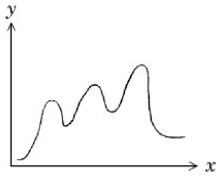
(iii) $C \rightarrow D \Delta H_{iii} = -ve$

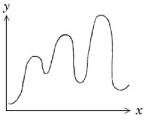
Which of the following graphs between potential energy (y-axis) vs reaction coordinate (x-axis) correctly represents the reaction profile of $A \rightarrow D$? [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- For an elementary step, the enthalpy change is the energy of the product minimum minus that of the reactant minimum. Peak heights describe activation barriers, not reaction enthalpies.
- Three elementary steps require three transition-state peaks, with the stable states read in order as A, B, C and D.

Step 1: Translate each enthalpy sign into an energy inequality Let E_A, E_B, E_C, E_D denote the energies of the starting state, the two intermediate minima and the final state. The first step is endothermic, so $E_B > E_A$. The second and third are exothermic, so $E_C < E_B$ and $E_D < E_C$. The overall reaction is endothermic, giving $E_D > E_A$. Combining these conditions gives

$$E_B > E_C > E_D > E_A.$$

The relative heights of the three peaks need not follow this order.

Step 2: Read the minima in each proposed graph In option (B), the first intermediate minimum is highest, the second is lower, and the product endpoint is lower still but remains above the reactant starting level. It therefore satisfies all four inequalities. In option (A), the final state is above the second intermediate, making the third step endothermic. In option (C), the first intermediate is below A, contrary to the first step. In option (D), the second intermediate lies above the first, making the second step endothermic. Counting peaks alone would not distinguish these graphs; the intermediate minima are decisive.

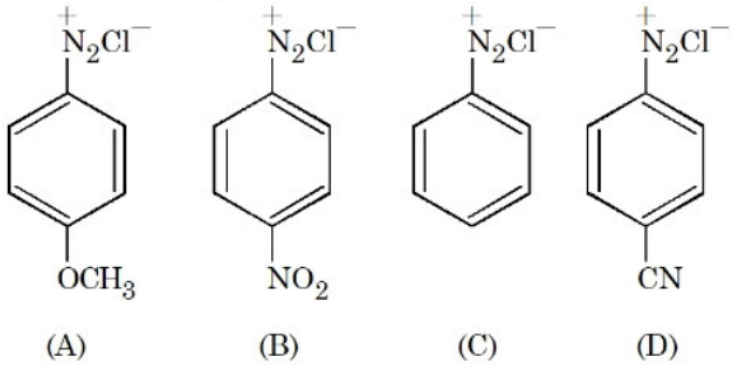
Step 3: Conclusion Option (B) has $E_B > E_C > E_D > E_A$, so it represents all three elementary steps and the endothermic overall reaction. Hence, the correct option is (B).

Quick Tip to Solve: Ignore the tops of the barriers when checking enthalpy signs: compare consecutive minima and then compare the final endpoint with the starting point.

57.

The correct stability order of the following diazonium salts is

[4 marks]



(A) (B) (C) (D)

(A) A > B > C > D

(B) $A > C > D > B$

(C) C > A > D > B

(D) $C > D > B > A$

Correct Answer: (B) $A > C > D > B$

Solution:

Concept:

- Arenediazonium ions have a positively charged diazonium group conjugated with an aromatic ring. Ring substituents change its electronic stabilization through resonance and induction.
- An electron-donating para substituent supports the electron-deficient diazonium system; electron-withdrawing substituents reduce that stabilization.

Step 1: Identify the substituent in each structure A is the para-methoxy-substituted benzenediazonium salt, B is the para-nitro derivative, C is the unsubstituted salt and D is the para-cyano derivative. The chloride counterion is common to all four and does not determine their relative order here. Compare the electronic effects of the para substituents on the same arenediazonium framework.

Step 2: Rank the electronic effects The methoxy group donates a lone pair into the aromatic conjugated system by its positive resonance effect. At the para position this donation stabilizes the electron-deficient diazonium system, so A is more stable than unsubstituted C. Both cyano and nitro withdraw electron density by negative resonance and inductive effects, so both lie below C. The nitro group is the stronger electron-withdrawing substituent in this comparison; therefore the nitro derivative B is less stable than the cyano derivative D.

Step 3: Compare the complete order with the choices Combining the comparisons gives

$$\mathrm{A} > \mathrm{C} > \mathrm{D} > \mathrm{B}.$$

Option (A) incorrectly puts the strongly withdrawing nitro derivative above the unsubstituted salt. Option (C) reverses the methoxy-versus-unsubstituted comparison. Option (D) places the electron-donating methoxy derivative last, the opposite of its stabilizing resonance effect.

Step 4: Conclusion The correct stability order is $\mathrm{A} > \mathrm{C} > \mathrm{D} > \mathrm{B}$. Hence, the correct option is (B).

Quick Tip to Solve: First identify whether the conjugated centre needs or has excess electron density. For this diazonium cation, para resonance donation stabilizes, while strong withdrawal destabilizes.

58.

Arrange the following carbanions in the decreasing order of stability.

Choose the correct answer from the options given below : [4 marks]

I. $p - Br - C_6H_4 - \overset{\ominus}{\underset{\cdot\cdot}{C}}H_2$

II. $C_6H_5 - \overset{\ominus}{\underset{\cdot\cdot}{C}}H_2$

III. $p - CH_3O - C_6H_4 - \overset{\ominus}{\underset{\cdot\cdot}{C}}H_2$

IV. $p - CHO - C_6H_4 - \overset{\ominus}{\underset{\cdot\cdot}{C}}H_2$

V. $p - CH_3 - C_6H_4 - \overset{\ominus}{\underset{\cdot\cdot}{C}}H_2$

(A) IV > I > II > V > III

(B) IV > II > I > III > V

(C) I > IV > II > V > III

(D) I > II > IV > V > III

Correct Answer: (A) IV > I > II > V > III

Solution:

Concept:

- The benzyl carbanion delocalizes its negative charge into the ring, including the para position. Electron withdrawal stabilizes it, whereas electron donation destabilizes it.
- Compare substituent resonance and inductive effects on the same benzyl-anion skeleton; the unsubstituted ion provides a useful reference.

Step 1: Identify the stabilizing substituents IV has a para-formyl group, $-CHO$. This group withdraws by resonance and induction; the delocalized negative charge can be accepted by the carbonyl system, with charge ultimately accommodated on oxygen. It therefore provides the greatest stabilization. I has para-bromine: its electron-withdrawing inductive effect gives net stabilization relative to unsubstituted benzyl carbanion II in this comparison. Thus the upper part of the order is

$$IV > I > II.$$

Step 2: Compare the electron-donating substituents V carries a para-methyl group, which donates electron density by its positive inductive effect and hyperconjugative influence. This destabilizes an already negatively charged system relative to II. III carries para-methoxy, whose strong positive resonance donation at the para position further opposes delocalization of negative charge onto the

ring. Accordingly III is less stable than V, giving

$$\text{II} > \text{V} > \text{III}.$$

Step 3: Eliminate the inconsistent orders Combining both comparisons gives IV, I, II, V, III in decreasing stability. Option (B) reverses both the bromine-versus-hydrogen and methyl-versus-methoxy comparisons. Options (C) and (D) place I above the strongly resonance-stabilized formyl derivative IV, so neither can be correct.

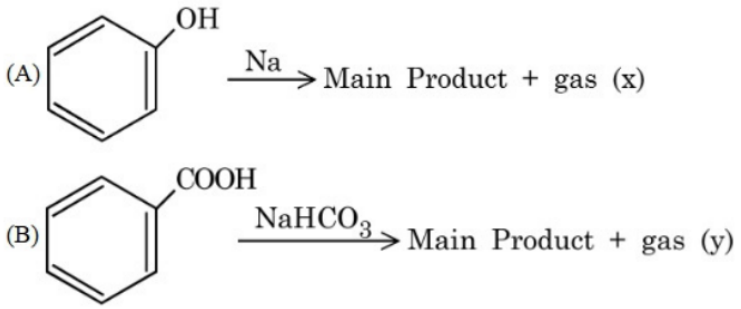
Step 4: Conclusion The decreasing stability order is $\text{IV} > \text{I} > \text{II} > \text{V} > \text{III}$. The verified printed option order places this sequence first. Hence, the correct option is (A).

Quick Tip to Solve: For a benzyl carbanion, put a para resonance-withdrawing carbonyl group near the stable end and a strong para resonance donor near the unstable end.

59.

Consider the following two reactions A and B.

Numerical value of [molar mass of x+ molar mass of y] is ____ [4 marks]



(A) 46

(B) 160

(C) 88

(D) 4

Correct Answer: (A) 46

Solution:

Concept:

- Sodium metal replaces the acidic hydrogen of a phenolic hydroxyl group, producing a phenoxide salt and hydrogen gas.
- A carboxylic acid reacts with sodium hydrogencarbonate to form a carboxylate salt, water and carbon dioxide. Molar mass refers to one mole of the identified molecular species.

Step 1: Identify gas x in reaction A The first reactant is phenol and the reagent is sodium metal. The balanced reaction is

$$2\,\text{C}_6\text{H}_5\text{OH} + 2\,\text{Na} \longrightarrow 2\,\text{C}_6\text{H}_5\text{ONa} + \text{H}_2.$$

Thus gas x is molecular hydrogen, H_2, whose molar mass is $2\,g\,mol^{-1}$. The stoichiometric coefficient of phenol is 2, but it does not change the molar mass of the hydrogen molecule.

Step 2: Identify gas y in reaction B The second reactant is benzoic acid. Being sufficiently acidic, it liberates carbon dioxide from hydrogencarbonate:

$$C_6H_5COOH + NaHCO_3 \longrightarrow C_6H_5COONa + H_2O + CO_2.$$

Hence y is CO_2. Using the usual atomic masses C = 12 and O = 16 gives

$$M_y = 12 + 2(16) = 44\,g\,mol^{-1}.$$

Step 3: Add the molecular molar masses The question asks for the sum of the two gas molar masses, not the molar masses of the organic salts. Therefore

$$M_x + M_y = 2 + 44 = 46\,g\,mol^{-1}.$$

The value 88 would count two carbon dioxide molar masses and ignore hydrogen. The value 4 would count two hydrogen molar masses and ignore carbon dioxide. The value 160 does not equal the sum for the two gases identified by these balanced reactions.

Step 4: Conclusion The requested numerical value is 46, corresponding to $46\,g\,mol^{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: Identify each evolved gas from the reagent before using molar masses: sodium metal gives hydrogen, whereas acid plus hydrogencarbonate gives carbon dioxide.

---

60. ' W ' g of a non-volatile electrolyte solid solute of molar mass ' M ' $gmol^{-1}$ when dissolved in 100 mL water, decreases vapour pressure of water from 640 mm Hg to 600 mm Hg . If aqueous solution of the electrolyte boils at 375 K and K_b for water is $0.52\ K\ kg\ mol^{-1}$, then the mole fraction of the electrolyte solute (x_2) in the solution can be expressed as

(Given : density of water $= 1\ g/mL$ and boiling point of water $= 373\ K$) [4 marks]

(A) $\frac{1.3}{8} \times \frac{M}{W}$
(B) $\frac{2.6}{16} \times \frac{M}{W}$
(C) $\frac{1.3}{8} \times \frac{W}{M}$
(D) $\frac{16}{2.6} \times \frac{W}{M}$

Correct Answer: (C) $\frac{1.3}{8} \times \frac{W}{M}$

Solution:

Concept:

- In the standard dilute-solution treatment of an electrolyte, relative lowering of vapour pressure is ix_2, where i is the van't Hoff factor and x_2 is the solute mole fraction before dissociation.
- Boiling-point elevation is $\Delta T_b = iK_bm$, where molality m is moles of solute per kilogram of solvent.

Step 1: Calculate the relative vapour-pressure lowering The decrease is $640 - 600 = 40\,\text{mm Hg}$. Dividing by the pure-water vapour pressure gives

$$\frac{p^0 - p}{p^0} = \frac{40}{640} = \frac{1}{16} = ix_2.$$

The denominator is 640, not the solution pressure 600. Thus $x_2 = 1/(16i)$, and the boiling-point data determine the factor i.

Step 2: Obtain the van't Hoff factor from the boiling point The temperature rise is $375 - 373 = 2\,\text{K}$. The mass of water is $100 \times 1 = 100\,\text{g} = 0.100\,\text{kg}$, while the solute amount is W/M mol. Consequently

$$m = \frac{W/M}{0.100} = \frac{10W}{M}.$$

Substitution into the boiling-point equation gives

$$2 = i(0.52)\frac{10W}{M}, \qquad i = \frac{M}{2.6W}.$$

Step 3: Eliminate the van't Hoff factor and compare choices Insert this value into the vapour-pressure relation:

$$x_2 = \frac{1}{16}\frac{2.6W}{M} = \frac{2.6}{16}\frac{W}{M} = \frac{1.3}{8}\frac{W}{M}.$$

Options (A) and (B) use the reciprocal ratio M/W, whereas elimination requires W/M. Option (D) inverts the numerical factor $2.6/16$. Thus only the third printed choice follows from both colligative-property relations.

Step 4: Conclusion $x_2 = \dfrac{1.3}{8}\dfrac{W}{M}$, a dimensionless mole fraction in the dilute-solution treatment. Hence, the correct option is (C).

Quick Tip to Solve: Use boiling-point elevation to find the same dissociation factor that appears in vapour-pressure lowering, then eliminate it algebraically before comparing options.

61. Match the LIST-I with LIST-II.

	List-I Isothermal process for ideal gas system		List-II Work done $(V_f > V_i)$
A.	**Reversible expansion**	**I.**	$w = 0$
B.	**Free expansion**	**II.**	$w = -nRT \ln \frac{V_f}{V_i}$
C.	**Irreversible expansion**	**III.**	$w = -p_{ex}(V_f - V_i)$
D.	**Irreversible compression**	**IV.**	$w = -p_{ex}(V_i - V_f)$

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-II, C-III, D-I

(B) A-II, B-I, C-III, D-IV

(C) A-IV, B-I, C-III, D-II

(D) A-I, B-III, C-II, D-IV

Correct Answer: (B) A-II, B-I, C-III, D-IV

Solution:

Concept:

- In the chemistry sign convention, pressure-volume work on a gas is $w = -\int p_{ex}\, dV$. Expansion gives negative work, whereas compression gives positive work.
- For an ideal gas undergoing a reversible isothermal change, $p = nRT/V$, where n, R, and T denote amount, gas constant, and constant absolute temperature.

Step 1: Match reversible and free expansion For reversible expansion, the external pressure differs infinitesimally from the gas pressure, so integration gives

$$w = -\int_{V_i}^{V_f} \frac{nRT}{V}\, dV = -nRT \ln \frac{V_f}{V_i}.$$

Thus A matches II. In free expansion the opposing external pressure is zero throughout. The volume changes, but no pressure-volume work is performed: $w = 0$. Thus B matches I. Isothermal does not itself imply zero work; it means constant temperature.

Step 2: Match the irreversible processes by sign Against constant external pressure, expansion from the smaller volume V_i to the larger volume V_f gives $w = -p_{ex}(V_f - V_i) < 0$. Hence C matches III. The table fixes $V_f > V_i$; compression is the reverse volume change, from the larger to the smaller volume, so its change is $V_i - V_f < 0$. Consequently $w = -p_{ex}(V_i - V_f) > 0$, matching D with IV. The reversed subtraction is essential to the sign of compression work.

Step 3: Check the four combinations Only the second combination has A-II and B-I together with C-III and D-IV. The first and third wrongly assign the positive compression expression to reversible expansion; the fourth wrongly assigns zero work to reversible expansion and nonzero work to free expansion.

Step 4: Conclusion The matching is A-II, B-I, C-III, D-IV. Hence, the correct option is (B).

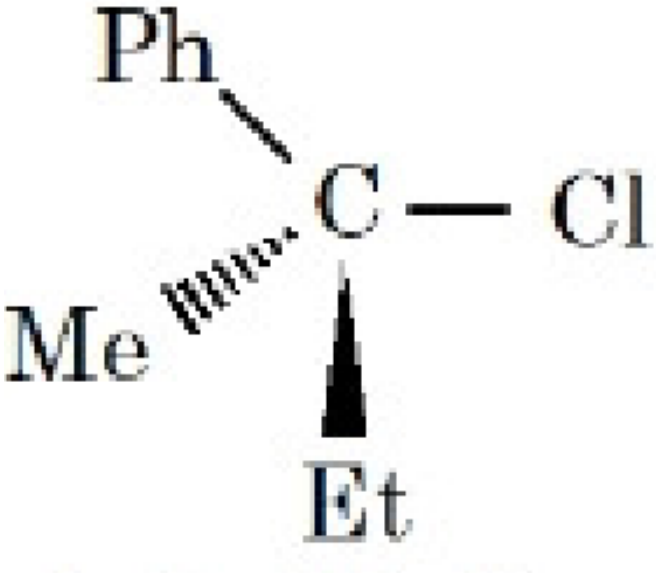
Quick Tip to Solve: First match free expansion to zero work. Then use negative expansion work and positive compression work to distinguish the remaining expressions.

62. Given below are two statements :

Statement I : ' $C-Cl$ **' bond is stronger in** $CH_2=CH-Cl$ **than** CH_3-CH_2-Cl

Statement II : The given optically active molecule, on hydrolysis gives a solution that can rotate the plane polarized light.

In the light of the above statements, choose the correct answer from the options given below [4 marks]



(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- In vinyl chloride, a chlorine lone pair conjugates with the adjacent C=C bond, giving the C−Cl bond partial double-bond character.
- Hydrolysis of a tertiary benzylic chloride proceeds mainly through an S_N1 carbocation intermediate. A planar intermediate permits attack on either face and gives racemisation in the usual stereochemical treatment.

Step 1: Compare the two carbon-chlorine bonds The chlorine-bearing carbon of $CH_2{=}CHCl$ is sp^2-hybridised, while that of CH_3CH_2Cl is sp^3-hybridised. Greater s-character and resonance donation of a chlorine lone pair make the vinylic bond shorter and stronger. Ethyl chloride has an ordinary carbon-chlorine single bond without this conjugation. Statement I is therefore true; it is incorrect to apply the behaviour of an alkyl halide directly to a vinyl halide.

Step 2: Follow hydrolysis at the chiral carbon The displayed carbon is bonded to phenyl, methyl, ethyl and chlorine. Loss of chloride gives a tertiary benzylic carbocation, stabilised by the phenyl ring and alkyl groups. This carbon becomes planar, losing the original tetrahedral stereochemical arrangement. Water can attack either face; after loss of a proton, the two enantiomers of the corresponding alcohol

are formed. In the racemisation model relevant here, their equal and opposite rotations cancel, so the solution is optically inactive.

Step 3: Judge the statements separately An alcohol molecule in the product still has a stereogenic carbon; this does not mean the mixture rotates polarised light. Optical activity of a solution depends on its enantiomeric composition. Statement II asserts a rotating solution and is false in this racemic hydrolysis outcome. Therefore the alternatives saying both statements are true, both are false, or only Statement II is true are excluded.

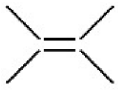
Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (C).

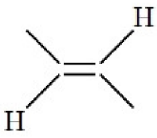
Quick Tip to Solve: Distinguish a chiral product molecule from an optically active mixture: equal amounts of two enantiomers have zero net rotation.

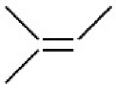
63.

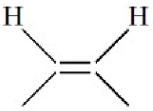
Arrange the following alkenes in decreasing order of stability.

Choose the correct answer from the options given below: [4 marks]

I. 

II. 

III. 

IV. 

(A) III > II > I > IV

(B) III > I > II > IV

(C) I > III > II > IV

(D) I > III > IV > II

Correct Answer: (C) I > III > II > IV

Solution:

Concept:

- Alkyl substitution stabilises an alkene by hyperconjugation and the electron-releasing inductive effect. For the given simple alkenes, greater substitution of the double bond gives greater stability.
- For disubstituted geometrical isomers with the same substituents, the trans arrangement is more

stable than the cis arrangement because its alkyl groups are farther apart.

Step 1: Count the alkyl groups on each double bond Structure I has four methyl groups attached to the two doubly bonded carbons: it is tetrasubstituted. Structure III has three methyl substituents and one implicit hydrogen: it is trisubstituted. Structures II and IV each have two methyl groups and two hydrogens and are disubstituted. Thus the first comparison places I above III, with both above II and IV. The hydrogens on an unlabelled skeletal carbon are inferred by completing carbon valency, not by adding another carbon atom.

Step 2: Compare the disubstituted pair In II the methyl groups are on opposite sides of the double bond, so it is trans-but-2-ene. In IV the two explicitly shown hydrogens are on the same side and both methyl groups are on the other side: it is cis-but-2-ene. The cis arrangement has greater steric repulsion between the methyl groups. Therefore II is more stable than IV, while their alkyl substitution count is the same.

Step 3: Assemble and test the order Combining substitution with the cis-trans comparison gives

$$\mathrm{I} > \mathrm{III} > \mathrm{II} > \mathrm{IV}.$$

The first two alternatives incorrectly put the trisubstituted alkene III above tetrasubstituted I. The last alternative correctly ranks I and III but reverses the trans/cis pair. Only the third option satisfies both independent comparisons.

Step 4: Conclusion The decreasing stability order is $\mathrm{I} > \mathrm{III} > \mathrm{II} > \mathrm{IV}$. Hence, the correct option is (C).

Quick Tip to Solve: Rank the degree of alkyl substitution first; use trans greater than cis only to break a tie between otherwise comparable disubstituted alkenes.

64. Given below are two statements :

Statement I : Hybridisation, shape and spin only magnetic moment of $\mathrm{K_3\,[Co\,(CO_3)_3]}$ is $\mathrm{sp^3\ d^2}$, octahedral and 4.9 BM respectively.

Statement II : Geometry, hybridisation and spin only magnetic moment values (BM) of the ions $[\mathrm{Ni(CN)_4}]^{2-}, [\mathrm{MnBr_4}]^{2-}$ and $[\mathrm{CoF_6}]^{3-}$ respectively are square planar, tetrahedral, octahedral; $\mathrm{dsp^2, sp^3, sp^3\ d^2}$ and $0, 5.9, 4.9$.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Concept:

- Find the metal oxidation state from the complex charge and ligand charges, then determine the d-electron count. Carbonate is bidentate, so three carbonate ligands give coordination number six.
- The spin-only moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons. Pairing depends on the metal ion and the ligand field; ligand identity alone must not be used to assign the spin state.

Step 1: Evaluate the carbonate complex In $K_3[Co(CO_3)_3]$, the complex ion has charge -3, and three carbonate ligands contribute -6. Cobalt is therefore $+3$, with configuration $3d^6$. Three bidentate carbonate ligands provide six donor atoms, giving an octahedral complex. This cobalt(III) carbonate complex is low-spin: the six d electrons pair in the lower t_{2g} set, giving $t_{2g}^6 e_g^0$. It has inner-orbital d^2sp^3 hybridisation and no unpaired electrons, so $\mu = 0$. Statement I is false: only its octahedral shape is correct, while its outer-orbital hybridisation and 4.9 BM moment are incorrect.

Step 2: Check nickel and manganese For $[Ni(CN)_4]^{2-}$, nickel is $+2$, or d^8. Strong-field cyanide gives the square-planar dsp^2 arrangement with no unpaired electrons, so $\mu = 0$. In $[MnBr_4]^{2-}$, manganese is $+2$, or d^5. Bromide is weak-field and the complex is tetrahedral with sp^3 hybridisation and five unpaired electrons. Its moment is $\sqrt{35} \simeq 5.9\,\text{BM}$.

Step 3: Check fluoride and select the truth pair For $[CoF_6]^{3-}$, cobalt is also $+3$, or d^6, but the weak fluoride field gives a high-spin octahedral outer-orbital sp^3d^2 complex. The configuration $t_{2g}^4 e_g^2$ has four unpaired electrons and $\mu = \sqrt{24} \simeq 4.9\,\text{BM}$. Thus all the properties in Statement II agree. The fluoride result must not be transferred to the carbonate complex merely because the oxidation state and coordination number coincide. The alternatives accepting Statement I or rejecting Statement II are excluded.

Step 4: Conclusion The carbonate complex is low-spin t_{2g}^6 and diamagnetic, so Statement I is false; Statement II is true. This differs from the supplied key that marks both statements true: treating the carbonate complex as high-spin is incorrect. Hence, the correct option is (D).

Quick Tip to Solve: Calculate the metal d-count first, then decide ligand field strength and geometry; only afterward count unpaired electrons for the magnetic moment.

65. Given below are two statements :

Statement I : $\mathrm{K} > \mathrm{Mg} > \mathrm{Al} > \mathrm{B}$ **is the correct order in terms of metallic character.**

Statement II : Atomic radius is always greater than the ionic radius for any element.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- Metallic character measures the tendency to lose electrons. It generally increases down a group and decreases from left to right across a period.
- A cation is smaller than its parent atom, while an anion is generally larger. The radius change depends on electron loss or gain, so no one inequality applies to every ion.

Step 1: Test the metallic-character order Potassium is a group-1 metal in period 4 and loses its outer electron readily; it is more metallic than magnesium. Magnesium and aluminium are in period 3, with magnesium to the left of aluminium, so magnesium is more metallic. Boron lies above aluminium in group 13 and is a metalloid, while aluminium is metallic. Combining these comparisons gives

$$\mathrm{K} > \mathrm{Mg} > \mathrm{Al} > \mathrm{B}.$$

This is exactly the order asserted in Statement I.

Step 2: Test the universal radius claim When an atom forms a positive ion, it loses electrons, reducing electron-electron repulsion and sometimes losing its outer shell. Thus, for example, $r(\mathrm{Na}^{+}) < r(\mathrm{Na})$. When an atom gains an electron to form a negative ion, the nuclear charge stays fixed while repulsion and shielding increase. Hence $r(\mathrm{Cl}^{-}) > r(\mathrm{Cl})$. This anion provides a counterexample to the claim that an atomic radius is always greater than an ionic radius.

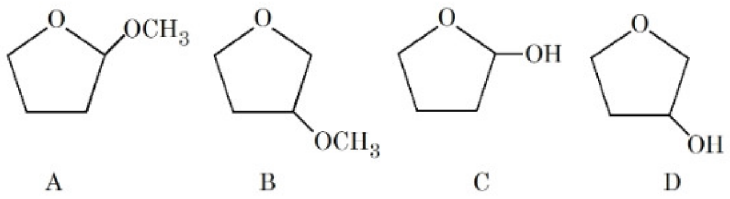
Step 3: Choose the appropriate statement pair Statement I follows the applicable group and period trends and is true. Statement II uses the word "always" and incorrectly extends the cation rule to anions, so it is false. The first option fails because it accepts the universal radius claim, the second rejects the correct metallic order, and the fourth reverses both truth values.

Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: For any claim about ionic size, ask whether the ion is a cation or an anion before applying an atomic-versus-ionic radius comparison.

66. A student is given one compound among the following compounds that gives positive test with Tollen's reagent.

The compound is : [4 marks]



A B C D

(A) A

(B) B

(C) C

(D) D

Correct Answer: (C) C

Solution:

Concept:

- Tollens reagent contains the diamminesilver(I) ion, $[Ag(NH_3)_2]^+$, which is reduced to silver by an aldehyde. The aldehyde is oxidised to a carboxylate in alkaline solution.
- A cyclic hemiacetal has an OH group and an ether oxygen bonded to the same carbon. It can equilibrate with an open-chain aldehyde; a full acetal has two ether groups at that carbon and no free hemiacetal OH.

Step 1: Identify the reactive functional arrangement In compound C, the OH-bearing carbon is directly next to the ring oxygen and bonded to that oxygen. That carbon therefore carries both an OH group and an OR bond, making C a cyclic hemiacetal. Opening the hemiacetal ring gives the hydroxy aldehyde $HO(CH_2)_3CHO$. This aldehyde can react with Tollens reagent even though the drawn cyclic structure does not display a carbonyl group explicitly.

Step 2: Explain the positive silver test The ring-opened aldehyde is oxidised while silver(I) is reduced. For a general aldehyde, the balanced alkaline reaction is

$$RCHO + 2\,[Ag(NH_3)_2]^+ + 3\,OH^- \longrightarrow RCOO^- + 2\,Ag + 4\,NH_3 + 2\,H_2O.$$

Removal of the open-chain aldehyde by oxidation shifts the ring-chain equilibrium toward further opening. Hence a cyclic hemiacetal gives a positive test, just as a reducing sugar can respond despite being drawn predominantly in cyclic form.

Step 3: Exclude the other three structures A has methoxy and ring-oxygen bonds at the same carbon and is a full acetal; it lacks the free hemiacetal OH needed for easy ring opening under the alkaline test conditions. B contains a ring ether and a methoxy substituent at a different carbon, without a hemiacetal centre. D is an ordinary alcohol on a carbon not directly bonded to the ring oxygen; it likewise cannot open to an aldehyde by hemiacetal equilibration. Thus none of A, B or D gives the specified positive test.

Step 4: Conclusion Compound C is the cyclic hemiacetal and gives the positive Tollens test through

its open-chain aldehyde. Compound D is an ordinary alcohol, so it is excluded. Hence, the correct option is (C).

Quick Tip to Solve: Look for an OH and an OR bond on the same carbon: a hemiacetal can reveal a hidden aldehyde, whereas an ordinary alcohol or full acetal cannot do so in Tollens reagent.

67. Consider a mixture ' X ' which is made by dissolving 0.4 mol of $[Co(NH_3)_5SO_4]Br$ and 0.4 mol of $[Co(NH_3)_5Br]SO_4$ in water to make 4 L of solution. When 2 L of mixture ' X ' is allowed to react with excess of $AgNO_3$, it forms precipitate ' Y '. The rest 2 L of mixture ' X ' reacts with excess $BaCl_2$ to form precipitate ' Z '. Which of the following statements is CORRECT? [4 marks]

(A) 0.4 mol of ' Z ' is formed.

(B) 0.1 mol of ' Y ' is formed.

(C) 0.2 mol of ' Z ' is formed.

(D) ' Y ' is $BaSO_4$ and ' Z ' is AgBr .

Correct Answer: (C) 0.2 mol of ' Z ' is formed.

Solution:

Concept:

- Only ions outside the coordination sphere are liberated as counterions when these coordination salts dissolve in water. A coordinated ligand does not undergo the ordinary counterion precipitation test.
- A homogeneous aliquot contains the same fraction of each solute as its volume fraction of the complete solution. Silver ions precipitate free bromide as $AgBr$; barium ions precipitate free sulfate as $BaSO_4$.

Step 1: Identify the available counterions The ionisation of the two salts is

$$[Co(NH_3)_5SO_4]Br \longrightarrow [Co(NH_3)_5SO_4]^+ + Br^-$$

$$[Co(NH_3)_5Br]SO_4 \longrightarrow [Co(NH_3)_5Br]^{2+} + {SO_4}^{2-}.$$

Thus the complete 4 L mixture contains 0.4 mol of free bromide and 0.4 mol of free sulfate. The sulfate ligand of the first complex and the bromide ligand of the second remain inside their respective coordination spheres.

Step 2: Calculate the amount in each aliquot Each 2 L portion is half of the original solution. Therefore, each portion contains

$$n(Br^-) = n({SO_4}^{2-}) = 0.4 \times \frac{2}{4} = 0.2 \text{ mol}.$$

For the first portion, $Ag^+ + Br^- \longrightarrow AgBr\downarrow$, so Y is 0.2 mol of silver bromide. For the second portion, $Ba^{2+} + {SO_4}^{2-} \longrightarrow BaSO_4\downarrow$, so Z is 0.2 mol of barium sulfate.

Step 3: Compare the statements Option (A) incorrectly assigns the full-solution amount of sulfate to a half-volume aliquot. Option (B) halves the bromide amount a second time, giving 0.1 mol instead

of 0.2 mol. Option (D) interchanges the identities of the precipitates: silver nitrate produces AgBr, whereas barium chloride produces barium sulfate.

Step 4: Conclusion Precipitate Z is 0.2 mol of $BaSO_4$. Hence, the correct option is (C).

Quick Tip to Solve: Write the counterion outside each square bracket first, then apply the aliquot fraction before calculating the precipitate.

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68. A solution is prepared by dissolving 0.3 g of a non-volatile non-electrolyte solute 'A' of molar mass $60\ \text{g mol}^{-1}$ and 0.9 g of a non-volatile non-electrolyte solute ' B ' of molar mass $180\ \text{g mol}^{-1}$ in $100\ \text{mL}H_2O$ at 27°C. Osmotic pressure of the solution will be

[Given: $R = 0.082\ \text{L atm K}^{-1}\ \text{mol}^{-1}$ **]** [4 marks]

(A) 0.82 atm

(B) 1.23 atm

(C) 1.47 atm

(D) 2.46 atm

Correct Answer: (D) 2.46 atm

Solution:

Concept:

- For an ideal dilute solution, osmotic pressure is $\pi = CRT = nRT/V$, where n is the total number of moles of dissolved solute particles and V is the solution volume.
- Independent non-electrolytes contribute additively to the number of solute particles. Each has van't Hoff factor 1, so masses must first be divided by their own molar masses.

Step 1: Find the moles of both solutes For solute A,

$$n_A = \frac{0.3}{60} = 0.005\ \text{mol}.$$

For solute B,

$$n_B = \frac{0.9}{180} = 0.005\ \text{mol}.$$

Although B has three times the mass, its molar mass is also three times as large, so it contributes the same number of particles. The combined amount is $n_A + n_B = 0.010$ mol.

Step 2: Convert the temperature and volume The absolute temperature is $T = 27 + 273 = 300$ K. For this dilute-solution calculation, the volume is taken as the given $100\ \text{mL} = 0.100$ L. Thus

$$C = \frac{0.010}{0.100} = 0.100\ \text{mol L}^{-1}.$$

This litre-based concentration is consistent with the given gas constant in litre-atmosphere units.

Step 3: Evaluate the total pressure Substitution gives

$$\pi = (0.100)(0.082)(300) = 2.46\ \text{atm}.$$

Equivalently, each solute contributes 1.23 atm and the two contributions add. Option (B) counts only one solute. Options (A) and (C) do not equal the pressure obtained from the total particle concentration and the specified absolute temperature; they cannot represent the osmotic pressure of this mixture.

Step 4: Conclusion The osmotic pressure is $\boxed{2.46\ \text{atm}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a mixture of non-electrolytes, calculate each mass divided by its molar mass and add those mole amounts before using the osmotic-pressure formula.

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69. Given below are two statements:

Statement I : The number of paramagnetic species among $[CoF_6]^{3-}$, $[TiF_6]^{3-}$, V_2O_5 and $[Fe(CN)_6]^{3-}$ is 3 .

Statement II :

$K_4[Fe(CN)_6] < K_3[Fe(CN)_6] < [Fe(H_2O)_6]SO_4 \cdot H_2O < [Fe(H_2O)_6]Cl_3$ is the correct order in terms of number of unpaired electron(s) present in the complexes.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- A species is paramagnetic if it contains one or more unpaired electrons. Determine the metal oxidation state first, then obtain its d-electron count.
- In these octahedral complexes, fluoride and water give high-spin configurations, whereas cyanide gives low-spin configurations. Pairing therefore depends on both the metal ion and the ligand.

Step 1: Check Statement I In $[CoF_6]^{3-}$, cobalt is +3 and hence d^6. Its high-spin configuration $t_{2g}^4 e_g^2$ has four unpaired electrons. In $[TiF_6]^{3-}$, titanium is +3, so it is d^1 with one unpaired electron. In V_2O_5, vanadium is +5 and d^0, with no unpaired electrons. Finally, $[Fe(CN)_6]^{3-}$ contains low-spin Fe(III), d^5, with configuration t_{2g}^5 and one unpaired electron. Exactly three species are paramagnetic, so Statement I is true.

Step 2: Check Statement II In $K_4[Fe(CN)_6]$, Fe(II) is low-spin d^6, giving t_{2g}^6 and zero unpaired electrons. In $K_3[Fe(CN)_6]$, low-spin Fe(III) has one. The hydrated sulfate contains $[Fe(H_2O)_6]^{2+}$: high-spin d^6 gives four. The chloride contains $[Fe(H_2O)_6]^{3+}$: high-spin d^5 gives five. Water of crystallisation does not change the iron oxidation state. Thus the proposed order is

$$0 < 1 < 4 < 5,$$

which is correct.

Step 3: Select the truth combination Both statements have now been checked independently by electron counting. Option (B) wrongly rejects both, option (C) wrongly rejects the increasing unpaired-electron order, and option (D) wrongly rejects the count of three paramagnetic species.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: Do not count unpaired electrons from oxidation state alone: after finding the d count, distinguish low-spin cyanide from high-spin aqua or fluoride complexes.

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70. Among the following, the CORRECT combinations are

A. $IF_3 \rightarrow$ **T-shaped (** $sp^3\ d$ **)**

B. $IF_5 \rightarrow$ **Square pyramidal** $(sp^3\ d^2)$

C. $IF_7 \rightarrow$ **Pentagonal bipyramidal** $(sp^3\ d^3)$

D. $ClO_4^- \rightarrow$ **Square planar (** $sp^2\ d$ **)**

Choose the correct answer from the options given below: [4 marks]

(A) A, B, C and D

(B) B, C and D Only

(C) A and B Only

(D) A, B and C Only

Correct Answer: (D) A, B and C Only

Solution:

Concept:

- VSEPR distinguishes electron-domain geometry from molecular shape: lone pairs occupy domains but are omitted when naming the positions of the bonded atoms.
- For the stated conventional hybridisation assignments, five, six and seven electron domains correspond to sp^3d, sp^3d^2 and sp^3d^3, respectively. A multiple bond occupies one electron domain.

Step 1: Examine the iodine fluorides Iodine has seven valence electrons. In IF_3, three I–F bonds and two lone pairs give five domains, AX_3E_2. The two lone pairs occupy equatorial sites of a trigonal bipyramid, leaving a T-shaped molecule, so A is correct. In IF_5, five bonds and one lone pair give six domains, AX_5E, an octahedral domain arrangement and a square-pyramidal molecular shape. The stated sp^3d^2 assignment is correct, so B is correct. In IF_7, seven bonds and no lone pair give AX_7, a pentagonal bipyramid, with the stated sp^3d^3 assignment. Thus C is also correct.

Step 2: Check perchlorate separately In ClO_4^-, chlorine has four bonded oxygen atoms and no lone pair in the VSEPR description. The four bonding domains point toward the vertices of a tetrahedron. Resonance and multiple-bond notation do not create additional bond directions. Its molecular shape is tetrahedral, with the conventional sp^3 assignment, rather than square planar sp^2d. Therefore D is incorrect.

Step 3: Match the valid combinations The valid set is A, B and C only. Option (A) and option (B) both include the false perchlorate statement D. Option (C) leaves out the correct pentagonal-bipyramidal assignment for iodine heptafluoride. Only the fourth option includes every correct statement and

excludes D.

Step 4: Conclusion Combinations A, B and C only are correct. Hence, the correct option is (D).

Quick Tip to Solve: Count bonding domains and lone pairs first; do not confuse a square-pyramidal molecule with a square-planar one or count a double bond as two directions.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The hydrogen spectrum consists of several spectral lines in Lyman series ($L_1, L_2, L_3 \ldots; L_1$ has lowest energy among Lyman series). Similarly it consists of several spectral lines in Balmer series ($B_1, B_2, B_3 \ldots; B_1$ has lowest energy among Balmer lines). The energy of L_1 is x times the energy of B_1. The value of x is ____ $\times 10^{-1}$. (Nearest integer) [4 marks]

Solution:

Concept:

- For hydrogen, the photon energy for a downward transition is $\Delta E = 13.6(1/n_f^2 - 1/n_i^2)$ eV, where $n_i > n_f$.
- The Lyman series ends at $n_f = 1$, and the Balmer series ends at $n_f = 2$. Within each series the smallest photon energy comes from the nearest higher initial level.

Step 1: Identify the first lines For the Lyman series, the lowest-energy allowed transition is $n_i = 2$ to $n_f = 1$, not a transition from infinity. Hence

$$E(L_1) = 13.6\left(1 - \frac{1}{4}\right) = 13.6\frac{3}{4}\ \text{eV}.$$

For the Balmer series, the lowest-energy transition is from $n_i = 3$ to $n_f = 2$. Thus

$$E(B_1) = 13.6\left(\frac{1}{4} - \frac{1}{9}\right) = 13.6\frac{5}{36}\ \text{eV}.$$

Higher initial levels increase the energy difference for either fixed final level.

Step 2: Form the dimensionless energy ratio The question defines $E(L_1) = xE(B_1)$, so

$$x = \frac{E(L_1)}{E(B_1)} = \frac{3/4}{5/36} = \frac{3}{4} \times \frac{36}{5} = \frac{27}{5} = 5.4.$$

The common factor 13.6 eV cancels, so neither conversion to joules nor an additional Rydberg constant is needed. The ratio is greater than one, consistent with the larger separation of the lower hydrogen levels.

Step 3: Apply the requested scaling The blank is the coefficient multiplying 10^{-1}; it is not the ratio itself rounded to an integer. If the required entry is N, then

$$5.4 = N \times 10^{-1}, \qquad N = 54.$$

This coefficient is already an integer, so the nearest-integer instruction does not change it.

Step 4: Conclusion The ratio is $x = 5.4 = 54 \times 10^{-1}$. The required dimensionless numerical entry is $\boxed{54}$.

Quick Tip to Solve: Find the nearest higher initial level for the first line of each series, then distinguish the actual ratio from the coefficient requested beside a power of ten.

72. X and Y are the number of electrons involved, respectively during the oxidation of I^- to I_2 and S^{2-} to S by acidified $K_2Cr_2O_7$. The value of $X + Y$ is ____ . [4 marks]

Solution:

Concept:

- In acidic medium, one dichromate ion is reduced to two chromium(III) ions and accepts six electrons. The oxidation half-reaction must supply the same number of electrons.
- Oxidation raises oxidation number and releases electrons. Iodide changes from -1 to 0; sulfide changes from -2 to 0. Balance each complete reaction with the dichromate reduction.

Step 1: Write the reduction half-reaction Chromium has oxidation number +6 in $Cr_2O_7{}^{2-}$ and +3 in Cr^{3+}. There are two chromium atoms, each gaining three electrons. The acidic reduction half-reaction is

$$Cr_2O_7{}^{2-} + 14\,H^+ + 6\,e^- \longrightarrow 2\,Cr^{3+} + 7\,H_2O.$$

Both charge and atoms balance: the charge on the left is $-2 + 14 - 6 = +6$, matching the two trivalent chromium ions.

Step 2: Balance oxidation of iodide The basic oxidation half-reaction is $2\,I^- \longrightarrow I_2 + 2\,e^-$. Multiply it by three to supply the six electrons needed by one dichromate ion:

$$6\,I^- \longrightarrow 3\,I_2 + 6\,e^-.$$

Adding the reduction half-reaction gives

$$Cr_2O_7{}^{2-} + 14\,H^+ + 6\,I^- \longrightarrow 2\,Cr^{3+} + 7\,H_2O + 3\,I_2.$$

The number of electrons transferred in this balanced reaction is $X = 6$.

Step 3: Balance oxidation of sulfide For sulfide, $S^{2-} \longrightarrow S + 2\,e^-$. Multiplying by three gives $3\,S^{2-} \longrightarrow 3\,S + 6\,e^-$, and the complete reaction is

$$Cr_2O_7{}^{2-} + 14\,H^+ + 3\,S^{2-} \longrightarrow 2\,Cr^{3+} + 7\,H_2O + 3\,S.$$

Thus $Y = 6$. The different numbers of iodide and sulfide ions account for their different electron loss per ion; both complete reactions must match the six-electron dichromate reduction.

Step 4: Add the electron-transfer counts The two balanced oxidations therefore give $X + Y = 6 + 6 = 12$. Counting one iodide ion and one sulfide ion would give their individual changes in oxidation number, rather than the electron counts of the balanced reactions with acidified dichromate.

Step 5: Conclusion The electron-transfer counts are $X = 6$ and $Y = 6$, so the required dimensionless value is $\boxed{12}$.

Quick Tip to Solve: Use the six-electron dichromate reduction as the common balancing target; multiply each oxidation half-reaction until its released electrons match.

73. Consider two Group IV metal ions X^{2+} and Y^{2+}.

A solution containing $0.01MX^{2+}$ and $0.01MY^{2+}$ is saturated with H_2S. The pH at which the metal sulphide YS will form as a precipitate is ____ . (Nearest integer)

(Given: $K_{sp}(XS) = 1 \times 10^{-22}$ at $25°C, K_{sp}(YS) = 4 \times 10^{-16}$ at $25°C$, $[H_2S] = 0.1M$ in solution, $K_{a1} \times K_{a2}\,(H_2S) = 1.0 \times 10^{-21}, \log 2 = 0.30$, $\log 3 = 0.48, \log 5 = 0.70$) [4 marks]

Solution:

Concept:

- A metal sulphide starts precipitating when its ionic product reaches its solubility product: for YS, $[Y^{2+}][S^{2-}] = K_{sp}$.
- Multiplying the two acid-dissociation equilibria of H_2S gives $K_{a1}K_{a2} = [H^+]^2[S^{2-}]/[H_2S]$. Increasing pH therefore increases the sulphide-ion concentration.

Step 1: Find the sulphide concentration needed for YS At the onset of precipitation, the dissolved metal concentration is still $0.01\,\text{mol}\,L^{-1}$. Hence

$$[S^{2-}]_{\text{critical}} = \frac{4 \times 10^{-16}}{0.01} = 4 \times 10^{-14}\,\text{mol}\,L^{-1}.$$

Below this sulphide concentration the solution is unsaturated with respect to YS; above it, precipitation is favoured.

Step 2: Relate sulphide concentration to hydrogen-ion concentration Using the stated dissolved H_2S concentration and the product of its two dissociation constants,

$$[H^+]^2 = \frac{K_{a1}K_{a2}[H_2S]}{[S^{2-}]} = \frac{(10^{-21})(0.1)}{4 \times 10^{-14}} = 2.5 \times 10^{-9}.$$

Thus $[H^+] = 5 \times 10^{-5}\,\text{mol}\,L^{-1}$. The less soluble XS begins precipitating earlier, since its initial sulphide threshold is only $10^{-22}/10^{-2} = 10^{-20}\,\text{mol}\,L^{-1}$. Its smaller solubility product is not the threshold requested for YS.

Step 3: Calculate and round the pH With the supplied logarithm,

$$\text{pH} = -\log(5 \times 10^{-5}) = 5 - \log 5 = 5 - 0.70 = 4.30.$$

This is the onset value before rounding; round only the final pH as the question requests.

Step 4: Conclusion YS begins precipitating at $\text{pH} = 4.30$. The required nearest-integer answer is $\boxed{4}$; pH is dimensionless.

Quick Tip to Solve: First calculate the sulphide threshold for the metal actually asked about, then combine the two acid equilibria; the hydrogen-ion concentration appears squared.

74. In Dumas method for estimation of nitrogen, 0.50 g of an organic compound gave 70 mL of nitrogen collected at 300 K and 715 mm pressure. The percentage of nitrogen in the organic compound is ____ %.

(Aqueous tension at 300 K is 15 mm). [4 marks]

Solution:

Concept:

- In Dumas estimation, the nitrogen in the sample is obtained as N_2. Its measured volume determines the mass of nitrogen, using the gas law and the molar mass $28\,\mathrm{g\,mol^{-1}}$.
- For gas collected over water, Dalton's law gives the dry gas pressure as total pressure minus aqueous tension. The percentage by mass is $100 m_{\text{nitrogen}}/m_{\text{sample}}$.

Step 1: Correct the pressure for water vapour The measured pressure includes both nitrogen and water vapour. Subtract the aqueous tension at the collection temperature:

$$p_{N_2} = 715 - 15 = 700\,\mathrm{mm\,Hg} = \frac{700}{760}\,\mathrm{atm}.$$

Use $V = 70\,\mathrm{mL} = 0.070\,\mathrm{L}$ and $T = 300\,\mathrm{K}$. Using 715 mm directly would overestimate the amount of nitrogen.

Step 2: Calculate the mass of nitrogen Apply $pV = nRT$ with $R = 0.0821\,\mathrm{L\,atm\,mol^{-1}\,K^{-1}}$:

$$n(N_2) = \frac{(700/760)(0.070)}{0.0821(300)} \simeq 2.62 \times 10^{-3}\,\mathrm{mol}.$$

Each mole of N_2 has mass 28 g; the nitrogen atoms already occur in pairs in this molar mass. Therefore no additional factor of two is needed:

$$m_{\text{nitrogen}} = (2.62 \times 10^{-3})(28) \simeq 0.0733\,\mathrm{g}.$$

Step 3: Express the result as a mass percentage Divide by the mass of the original organic compound, not by the mass of nitrogen collected:

$$\%\mathrm{N} = \frac{0.0733}{0.50} \times 100 \simeq 14.7\%.$$

The numerical result rounds to 15 when expressed as an integer percentage. Keeping the gas-law calculation unrounded until this final step avoids accumulating rounding error.

Step 4: Conclusion The nitrogen content is approximately 14.7% by mass, giving the integer numerical answer $\boxed{15}$ (i.e. approximately 15%).

Quick Tip to Solve: Subtract aqueous tension before applying the gas law, and multiply moles of nitrogen gas by 28 g/mol rather than 14 g/mol.

75. Electricity is passed through an acidic solution of Cu^{2+} till all the Cu^{2+} was exhausted, leading to the deposition of 300 mg of Cu metal. However, a current of 600 mA was continued to pass through the same solution for another 28 minutes by keeping the total volume of the solution fixed at 200 mL . The total volume of oxygen evolved at STP during the entire process is ____ mL . (Nearest integer) [Given:

$$Cu^{2+}(aq) + 2e^- \rightarrow Cu(\ s) E^{o}_{red} = +0.34\ V$$
$$O_2(\ g) + 4H^+ + 4e^- \rightarrow 2H_2O E^{o}_{red} = +1.23\ V$$

Molar mass of $Cu = 63.54\ g\ mol^{-1}$
Molar mass of $O_2 = 32\ g\ mol^{-1}$
Faraday Constant $= 96500 C mol^{-1}$
Molar volume at $STP = 22.4\ L$ **]** [4 marks]

Solution:

Concept:

- The same charge passes through both electrodes. Copper deposition, $Cu^{2+} + 2\,e^- \longrightarrow Cu$, requires two moles of electrons per mole of copper deposited.
- At the anode, water oxidation is $2\,H_2O \longrightarrow O_2 + 4\,H^+ + 4\,e^-$. Thus one mole of oxygen corresponds to four moles of electrons throughout both stages of the electrolysis.

Step 1: Calculate the electron amount during copper deposition Convert the copper mass to grams and use its stated molar mass:

$$n(Cu) = \frac{0.300}{63.54} = 0.00472144\,mol.$$

The electron amount passed while this copper was deposited is therefore

$$n_{e,1} = 2n(Cu) = 0.00944287\,mol.$$

Oxygen is already evolving at the anode during this stage, so it must be included in the total.

Step 2: Find the extra charge after copper ions are exhausted After the copper ions have been consumed, hydrogen is produced at the cathode while oxygen evolution continues at the anode. Convert the current and time before using $Q = It$:

$$Q_2 = (0.600)(28 \times 60) = 1008\,C, \qquad n_{e,2} = \frac{1008}{96500} = 0.01044560\,mol.$$

The specified solution volume is not required: the deposited copper mass fixes the first-stage charge, and current multiplied by time fixes the second-stage charge.

Step 3: Convert the total electron amount into oxygen volume Add the electron amounts first, then divide by the four-electron oxygen stoichiometry:

$$n(O_2) = \frac{0.00944287 + 0.01044560}{4} = 0.00497212\,mol.$$

At the stated STP molar volume,

$$V(\mathrm{O_2}) = (0.00497212)(22400) = 111.375\,\mathrm{mL}.$$

Equivalently, the two stages contribute approximately 52.88 mL and 58.50 mL of oxygen. Their sum confirms that neither stage has been omitted.

Step 4: Conclusion The total oxygen volume at STP is approximately $111.375\,\mathrm{mL}$. To the nearest integer, the answer is $\boxed{111\,\mathrm{mL}}$.

Quick Tip to Solve: For successive electrolysis stages, add their electron amounts and apply the anode stoichiometry once; copper deposition does not stop oxygen forming at the other electrode.