JEE Main 2026 Paper 1 (B.E./B.Tech) Jan 24 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2026, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 24 January 2026, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The largest value of n, for which 40^n divides $60!$, is [4 marks]

(A) 12

(B) 13

(C) 14

(D) 11

Correct Answer: (C) 14

Solution:

Concept:

- For a prime p, the exponent of p in $N!$ is $v_p(N!) = \lfloor N/p \rfloor + \lfloor N/p^2 \rfloor + \cdots$. Each term counts an additional factor of that prime.
- Since $40 = 2^3 \cdot 5$, divisibility by 40^n requires at least $3n$ factors of 2 and n factors of 5 simultaneously.

Step 1: Count the factors of 2 Apply the factorial formula, stopping once the prime power exceeds 60:

$$v_2(60!) = 30 + 15 + 7 + 3 + 1 = 56.$$

Thus the condition from the prime 2 is $3n \leq 56$, or $n \leq 18$ for integral n. Counting only multiples of 2 would miss the extra factors contributed by multiples of 4, 8 and higher powers.

Step 2: Count the factors of 5 The multiples of 5 supply twelve factors, and the multiples of 25 supply two additional factors:

$$v_5(60!) = \left\lfloor \frac{60}{5} \right\rfloor + \left\lfloor \frac{60}{25} \right\rfloor = 12 + 2 = 14.$$

Hence $n \leq 14$. There is no contribution from 125 because it exceeds 60.

Step 3: Apply both restrictions The smaller bound controls the answer:

$$n_{\max} = \min(18, 14) = 14.$$

This value is attainable because $40^{14} = 2^{42}5^{14}$, with $42 \leq 56$. The next power requires fifteen factors of 5, which are unavailable. The other listed values 12, 13 and 11 also give divisors, but none is the largest.

Step 4: Conclusion The bound $v_5(60!) = 14$, together with $v_2(60!) = 56$, proves that the largest value is $\boxed{14}$, which is (C) in the printed option order. Hence, the correct option is (C).

Quick Tip to Solve: For a composite base, factor it first and divide each factorial prime exponent by the corresponding exponent in the base; take the smallest integer bound.

2. Consider the following three statements for the function $f : (0, \infty) \to \mathbb{R}$ defined by $f(x) = |\log_e x| - |x - 1|$:

(I) f is differentiable at all $x > 0$.

(II) f is increasing in $(0, 1)$.

(III) f is decreasing in $(1, \infty)$.

Then. [4 marks]

(A) Only (I) is TRUE.

(B) Only (II) and (III) are TRUE.

(C) Only (I) and (III) are TRUE.

(D) All (I), (II) and (III) are TRUE.

Correct Answer: (C) Only (I) and (III) are TRUE.

Solution:

Concept:

- Remove absolute values separately on intervals where their arguments have fixed signs. Both $\ln x$ and $x - 1$ change sign at $x = 1$.
- Differentiability at the joining point requires equal one-sided derivatives, while the sign of the derivative determines monotonicity on each open interval.

Step 1: Write the function on each interval For $0 < x < 1$, both arguments are negative; for $x > 1$, both are positive. Therefore

$$f(x) = \begin{cases} -\ln x + x - 1, & 0 < x < 1, \\ \ln x - x + 1, & x > 1. \end{cases}$$

Also $f(1) = 0$, and both expressions approach zero there, so the function is continuous at 1.

Step 2: Check differentiability at the joining point Away from 1, the formulas are differentiable and give

$$f\prime(x) = \begin{cases} 1 - 1/x, & 0 < x < 1, \\ 1/x - 1, & x > 1. \end{cases}$$

At 1, the left-hand derivative is $-1 + 1 = 0$, and the right-hand derivative is $1 - 1 = 0$. Thus $f\prime(1) = 0$, so (I) is true. The two individual absolute-value cusps cancel in their difference.

Step 3: Determine the monotonicity If $0 < x < 1$, then $1/x > 1$, so $f\prime(x) < 0$: the function is decreasing, and (II) is false. If $x > 1$, then $1/x < 1$, again giving $f\prime(x) < 0$, so (III) is true. Option (A) omits the true statement (III); options (B) and (D) include the false statement (II).

Step 4: Conclusion Only statements (I) and (III) are true. Hence, the correct option is (C).

Quick Tip to Solve: An absolute value need not make the whole expression nondifferentiable: check whether the one-sided slope jumps cancel before rejecting differentiability.

3. Let $P = [p_{ij}]$ and $Q = [q_{ij}]$ be two square matrices of order 3 such that $q_{ij} = 2^{(i+j-1)}p_{ij}$ and $\det(Q) = 2^{10}$. Then the value of $\det(\text{adj}(\text{adj}\, P))$ is: [4 marks]

(A) 16

(B) 81

(C) 32

(D) 124

Correct Answer: (A) 16

Solution:

Concept:

- Multiplying a row or column by a scalar multiplies the determinant by that scalar. A factor depending on both indices can be separated into a row factor and a column factor.
- For an $m \times m$ matrix A, $\det(\text{adj}\, A) = (\det A)^{m-1}$. Here $m = 3$, so each adjugate operation squares the determinant.

Step 1: Separate the entry multipliers Write $2^{i+j-1} = 2^i 2^{j-1}$. Thus

$$Q = \text{diag}(2, 4, 8)\, P\, \text{diag}(1, 2, 4).$$

The left diagonal matrix multiplies rows 1, 2 and 3 by 2, 4 and 8; the right one multiplies columns by 1, 2 and 4. This reproduces every prescribed entry of Q.

Step 2: Find the determinant of P Take determinants and multiply all row and column factors:

$$\det Q = (2 \cdot 4 \cdot 8)(1 \cdot 2 \cdot 4) \det P = 2^9 \det P.$$

Since $\det Q = 2^{10}$, we obtain

$$\det P = \frac{2^{10}}{2^9} = 2.$$

In particular P is nonsingular, so the adjugate determinant identity applies directly through the inverse identity as well.

Step 3: Apply the adjugate identity twice The adjugate remains a matrix of order 3. Hence

$$\det(\text{adj}\, P) = 2^2 = 4, \qquad \det(\text{adj}(\text{adj}\, P)) = 4^2 = 16.$$

The total exponent is $(3-1)^2 = 4$, rather than 3 or 5. Thus 81, 32 and 124 do not satisfy the determinant identities; no entrywise calculation of the adjugate is necessary.

Step 4: Conclusion The required determinant is $\boxed{16}$. Hence, the correct option is (A).

Quick Tip to Solve: For a double adjugate of an order-m matrix, its determinant is $(\det A)^{(m-1)^2}$; keep this distinct from the formula for the double adjugate matrix itself.

4. Let $\mathrm{X} = \{x \in \mathrm{N} : 1 \le x \le 19\}$ and for some $a, b \in \mathbb{R}, \mathrm{Y} = \{ax + b : x \in \mathrm{X}\}$. If the mean and variance of the elements of Y are 30 and 750 , respectively, then the sum of all possible values of b is [4 marks]

(A) 20

(B) 60

(C) 80

(D) 100

Correct Answer: (B) 60

Solution:

Concept:

- For equally weighted observations transformed by $y = ax + b$, the mean becomes $a\bar{x} + b$, while the variance becomes $a^2\sigma_x^2$. Adding a constant changes the mean but not the variance.
- The integers $1, 2, \ldots, n$ have mean $(n+1)/2$ and variance $(n^2-1)/12$. Both positive and negative values of the scale factor must be retained.

Step 1: Find the original mean and variance For the nineteen integers,

$$\bar{x} = \frac{1+19}{2} = 10.$$

Also, using the sum of squares,

$$\frac{1}{19}\sum_{x=1}^{19} x^2 = \frac{20 \cdot 39}{6} = 130, \qquad \sigma_x^2 = 130 - 10^2 = 30.$$

The variance here is the average squared deviation of the complete set, with denominator 19.

Step 2: Determine both scale factors The given variance of Y yields

$$750 = a^2 \cdot 30 \quad \Longrightarrow \quad a^2 = 25 \quad \Longrightarrow \quad a = 5 \text{ or } a = -5.$$

Neither value is zero, so the affine transformation is one-to-one and Y has nineteen distinct elements in either case. A negative scale reverses their order but leaves the variance positive.

Step 3: Use the mean to obtain the translations The mean equation is $10a + b = 30$. Thus

$$a = 5 \Rightarrow b = -20, \qquad a = -5 \Rightarrow b = 80.$$

Both pairs satisfy the stated mean and variance, and there are no further possibilities because $a^2 = 25$ has only these two real roots. Their sum is

$$(-20) + 80 = 60.$$

The option 80 retains only the negative-scale branch; 20 and 100 likewise are not the sum of the two valid translations.

Step 4: Conclusion The sum of all possible values of b is $\boxed{60}$. Hence, the correct option is (B).

Quick Tip to Solve: Variance determines $|a|$, not the sign of a. In an affine-transformation problem, use both signs before applying the mean condition.

5. Let the angles made with the positive x-axis by two straight lines drawn from the point $\mathrm{P}(2,3)$ and meeting the line $x+y=6$ at a distance $\sqrt{\frac{2}{3}}$ from the point P be θ_1 and θ_2. Then the value of $(\theta_1+\theta_2)$ is: [4 marks]

(A) $\frac{\pi}{3}$
(B) $\frac{\pi}{2}$
(C) $\frac{\pi}{6}$
(D) $\frac{\pi}{12}$

Correct Answer: (B) $\frac{\pi}{2}$

Solution:

Concept:

- Write a point on a line through $P(2,3)$ as $(2+u,3+v)$. The displacements u,v determine both its distance from P and the slope v/u.
- For positive slopes that are reciprocals, the inclinations lie in $(0,\pi/2)$ and add to $\pi/2$. Checking the quadrant removes the ambiguity in tangent identities.

Step 1: Express the intersection conditions If $Q=(2+u,3+v)$ lies on $x+y=6$, then $u+v=1$. Its prescribed distance from P gives

$$u^2+v^2=\frac{2}{3}.$$

Squaring the first relation and subtracting the second gives

$$2uv=1-\frac{2}{3}=\frac{1}{3},\qquad uv=\frac{1}{6}.$$

Hence u,v have positive product and positive sum, so both are positive.

Step 2: Find the two slopes The two displacements are the roots of $t^2-t+1/6=0$, namely

$$u,v=\frac{1\pm 1/\sqrt{3}}{2}.$$

Interchanging these coordinates gives the two intersection points, and therefore two slopes $m_1=v/u$ and $m_2=u/v$. Their product is $m_1m_2=1$. Both are finite and positive, because neither displacement is zero.

Step 3: Resolve the inclination sum Since both inclinations are acute, $\tan\theta_2=1/\tan\theta_1=\cot\theta_1$ implies

$$\theta_2=\frac{\pi}{2}-\theta_1.$$

Thus their sum is exactly $\pi/2$, not $3\pi/2$ or another tangent-equivalent angle. The alternatives $\pi/3$, $\pi/6$ and $\pi/12$ are incompatible with the reciprocal positive slopes.

Step 4: Conclusion The positive reciprocal slopes prove $\boxed{\theta_1+\theta_2=\pi/2}$, which is (B) in the printed option order. Hence, the correct option is (B).

Quick Tip to Solve: For intersections with $x+y=c$, displacement coordinates often reveal a symmetry under $u\leftrightarrow v$; this converts the two slopes into reciprocals.

6. Let a_1, a_2, a_3, a_4 be an A.P. of four terms such that each term of the A.P. and its common difference l are integers. If $a_1 + a_2 + a_3 + a_4 = 48$ and $a_1a_2a_3a_4 + l^4 = 361$, then the largest term of the A.P. is equal to [4 marks]

(A) 21

(B) 23

(C) 24

(D) 27

Correct Answer: (D) 27

Solution:

Concept:

- Four terms of an arithmetic progression can be placed symmetrically around their mean. Here the mean is $48/4 = 12$.
- The integer condition on the terms restricts the common difference. Pairing equally spaced terms about the mean turns the product into a quadratic in a square.

Step 1: Use the sum and integer conditions Let the common difference be l. From $4a_1 + 6l = 48$,

$$a_1 = 12 - \frac{3l}{2}.$$

Since a_1 and l are integers, l must be even. Put $l = 2k$, where k is an integer. The four terms are then

$$12 - 3k, \quad 12 - k, \quad 12 + k, \quad 12 + 3k.$$

This representation covers increasing and decreasing progressions.

Step 2: Translate the product condition Pair the outside terms and the inside terms:

$$(144 - 9k^2)(144 - k^2) + 16k^4 = 361.$$

Expanding and moving 361 to the left gives

$$25k^4 - 1440k^2 + 20375 = 0.$$

With $z = k^2$, divide by 5 and factor:

$$5z^2 - 288z + 4075 = (z - 25)(5z - 163) = 0.$$

Hence $k^2 = 25$ or $k^2 = 163/5$.

Step 3: Apply integrality and identify the largest term The value $163/5$ is not the square of an integer, so it is inadmissible. Thus $k = \pm 5$, and the terms, in one order or its reverse, are

$$-3, \quad 7, \quad 17, \quad 27.$$

As a check, their product is -9639, and $l^4 = (\pm 10)^4 = 10000$, giving $-9639 + 10000 = 361$. Their sum is 48. The largest term is 27; 21, 23 and 24 cannot be the largest term of either admissible

progression.

Step 4: Conclusion The largest term is $\boxed{27}$. Hence, the correct option is (D).

Quick Tip to Solve: For four-term progressions with a known sum, center the terms at the mean and pair opposite terms. Enforce integer conditions before accepting roots of the resulting polynomial.

7. The letters of the word "UDAYPUR" are written in all possible ways with or without meaning and these words are arranged as in a dictionary. The rank of the word "UDAYPUR" is [4 marks]

(A) 1578

(B) 1579

(C) 1580

(D) 1581

Correct Answer: (C) 1580

Solution:

Concept:

- Dictionary order compares words at their first differing position. Count the distinct words before the given word, then add one for its rank.
- The letters of UDAYPUR in alphabetical order are A, D, P, R, U, U, Y. If both identical U's remain in a suffix of length n, its distinct arrangements number $n!/2!$.

Step 1: Count words differing at the first two positions A first letter smaller than U can be A, D, P or R. For each choice both U's remain among the six letters, giving

$$4\frac{6!}{2!} = 4(360) = 1440.$$

Now fix the first U. To precede D in the second position, only A is available. The remaining five letters are distinct, so this contributes $5! = 120$ words. Words starting UD have not yet been counted; this separation prevents double counting.

Step 2: Continue through the fixed prefix With UD fixed, A is already the smallest remaining letter, so there is no contribution at position three. With UDA fixed, the available letters are P, R, U, Y. Before Y we may place P, R or U, and arrange the three remaining letters in $3!$ ways. Thus this position contributes $3 \times 6 = 18$. With UDAY fixed, P is the smallest available letter and adds zero. With UDAYP fixed, the suffix is UR; the smaller suffix RU gives exactly one preceding word. The final position adds nothing.

Step 3: Convert the preceding count to rank The number of words strictly preceding UDAYPUR is

$$1440 + 120 + 18 + 1 = 1579.$$

Dictionary ranks start at one, so the required rank is $1579 + 1 = 1580$. The value 1579 counts earlier words but omits the word itself; 1578 and 1581 likewise disagree with the complete prefix count.

Step 4: Conclusion The rank of UDAYPUR is $\boxed{1580}$. Hence, the correct option is (C).

Quick Tip to Solve: Update the repeated-letter factorial after fixing each prefix; only then count smaller available letters. Add one after counting all earlier words.

8. The sum of all values of α, for which the shortest distance between the lines $\frac{x+1}{\alpha} = \frac{y-2}{-1} = \frac{z-4}{-\alpha}$ and $\frac{x}{\alpha} = \frac{y-1}{2} = \frac{z-1}{2\alpha}$ is $\sqrt{2}$, is [4 marks]

(A) -6

(B) 6

(C) -8

(D) 8

Correct Answer: (A) -6

Solution:

Concept:

- For nonparallel lines through points P, Q, with direction vectors $\vec{u}, \vec{w}$, the shortest distance is $d = |(Q - P) \cdot (\vec{u} \times \vec{w})|/|\vec{u} \times \vec{w}|$.
- The absolute value makes distance nonnegative. Squaring an equation between nonnegative quantities is valid, but any parameter excluded during cancellation must be checked separately.

Step 1: Identify the points and directions From the symmetric equations take

$$P = (-1, 2, 4), \quad Q = (0, 1, 1), \quad Q - P = (1, -1, -3).$$

The direction vectors are $\vec{u} = (\alpha, -1, -\alpha)$ and $\vec{w} = (\alpha, 2, 2\alpha)$. Their cross product is

$$\vec{u} \times \vec{w} = (0, -3\alpha^2, 3\alpha).$$

Hence its magnitude is $3|\alpha|\sqrt{\alpha^2 + 1}$, and its scalar product with $Q - P$ is $3\alpha^2 - 9\alpha$.

Step 2: Solve the distance condition For $\alpha \neq 0$, cancellation gives

$$d = \frac{|\alpha - 3|}{\sqrt{\alpha^2 + 1}} = \sqrt{2}.$$

Squaring and collecting terms yields

$$(\alpha - 3)^2 = 2(\alpha^2 + 1), \qquad \alpha^2 + 6\alpha - 7 = 0.$$

Thus $(\alpha - 1)(\alpha + 7) = 0$, giving $\alpha = 1, -7$. Substitution gives distances $2/\sqrt{2}$ and $10/\sqrt{50}$, respectively; both equal $\sqrt{2}$.

Step 3: Check the exceptional case and add the roots If the lines are interpreted parametrically at $\alpha = 0$, they are parallel to the y-axis. Their fixed (x, z) coordinates differ by $(1, -3)$, so their separation is $\sqrt{10}$, not $\sqrt{2}$. Therefore this exceptional value adds no solution. The required sum is $1 - 7 = -6$. The other offered sums 6, -8 and 8 do not equal the sum of the two verified roots.

Step 4: Conclusion The sum of all admissible values is $\boxed{-6}$. Hence, the correct option is (A).

Quick Tip to Solve: Factor the parameter out of the cross product before solving, but check its zero value separately because the lines can then become parallel.

9. If the domain of the function $f(x) = \sin^{-1}\left(\frac{1}{x^2-2x-2}\right)$, is $(-\infty, \alpha] \cup [\beta, \gamma] \cup [\delta, \infty)$, then $\alpha + \beta + \gamma + \delta$ is equal to [4 marks]

(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (C) 4

Solution:

Concept:

- The real function $\sin^{-1} u$ is defined exactly when $-1 \le u \le 1$. Its endpoint values are allowed.
- For a real nonzero denominator D, the condition $|1/D| \le 1$ is equivalent to $|D| \ge 1$, or $D \le -1$ or $D \ge 1$. This avoids multiplying inequalities by a denominator of unknown sign.

Step 1: Express the domain condition without a fraction Write the quadratic denominator in completed-square form:

$$D = x^2 - 2x - 2 = (x-1)^2 - 3.$$

The required condition is $|D| \ge 1$. It automatically excludes $D = 0$, where the original expression would be undefined. We therefore solve the two disjoint inequalities $D \ge 1$ and $D \le -1$, retaining their equality endpoints.

Step 2: Solve both quadratic inequalities The first inequality gives

$$(x-1)^2 - 3 \ge 1 \quad \Longrightarrow \quad |x-1| \ge 2,$$

so $x \le -1$ or $x \ge 3$. The second gives

$$(x-1)^2 - 3 \le -1 \quad \Longrightarrow \quad (x-1)^2 \le 2,$$

so $1 - \sqrt{2} \le x \le 1 + \sqrt{2}$. At the outer endpoints the denominator is 1, and at the inner endpoints it is -1, so all four endpoints indeed belong to the domain.

Step 3: Match and add the four endpoints The domain is

$$(-\infty, -1] \cup [1 - \sqrt{2}, 1 + \sqrt{2}] \cup [3, \infty).$$

Thus $\alpha = -1$, $\beta = 1 - \sqrt{2}$, $\gamma = 1 + \sqrt{2}$, and $\delta = 3$. Their sum is

$$-1 + (1 - \sqrt{2}) + (1 + \sqrt{2}) + 3 = 4.$$

The radical terms cancel in the middle interval. Therefore the offered values 2, 3 and 5 cannot be the sum of these endpoints.

Step 4: Conclusion The requested sum is $\boxed{4}$. Hence, the correct option is (C).

Quick Tip to Solve: For an inverse sine of a reciprocal, impose an absolute-value condition on the denominator first; completing the square then reveals the endpoint symmetry.

10. Let the length of the latus rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,(a>b)$, be 30 . If its eccentricity is the maximum value of the function $f(t)=-\frac{3}{4}+2t-t^2$, then $\left(a^2+b^2\right)$ is equal to [4 marks]

(A) 256

(B) 496

(C) 516

(D) 276

Correct Answer: (B) 496

Solution:

Concept:

- For an ellipse with semi-major axis a, semi-minor axis b, and eccentricity e, the relation is $b^2=a^2(1-e^2)$.
- The full length of a latus rectum is $2b^2/a$. A quadratic with a negative squared term reaches its maximum when that square is zero.

Step 1: Determine the eccentricity Complete the square in the given function:

$$f(t)=-\frac{3}{4}+2t-t^2=\frac{1}{4}-(t-1)^2.$$

Since $(t-1)^2\geq 0$ for real t, the maximum is $1/4$, attained at $t=1$. Thus $e=1/4$, not $e^2=1/4$. This is a valid ellipse eccentricity because it lies strictly between zero and one.

Step 2: Combine the two ellipse relations Using the square of the eccentricity gives

$$b^2=a^2\left(1-\frac{1}{16}\right)=\frac{15}{16}a^2.$$

The specified latus rectum is the full chord, so

$$\frac{2b^2}{a}=30 \quad\Longrightarrow\quad b^2=15a.$$

Equating the expressions for b^2, we obtain $15a^2/16=15a$. Since a semi-axis is positive, division by $15a$ gives $a=16$. Therefore $a^2=256$ and $b^2=240$, with $a>b$ as required.

Step 3: Evaluate and verify the requested sum Add the squared semi-axes:

$$a^2+b^2=256+240=496.$$

As a check, $2(240)/16=30$, and $\sqrt{1-240/256}=1/4$, reproducing both original conditions. Option 256 gives only a^2; the values 516 and 276 do not equal the sum obtained from the two conditions.

Step 4: Conclusion The value of a^2+b^2 is $\boxed{496}$. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish the eccentricity from its square, and remember that the full latus rectum is twice b^2/a.

11. Let $\vec{a} = 2\hat{i} - \hat{j} - \hat{k}, \vec{b} = \hat{i} + 3\hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} + 3\hat{k}$. Let $\vec{v}$ be the vector in the plane of the vectors $\vec{a}$ and $\vec{b}$, such that the length of its projection on the vector $\vec{c}$ is $\frac{1}{\sqrt{14}}$. Then $|\vec{v}|$ is equal to [4 marks]

(A) 7

(B) 13

(C) $\frac{\sqrt{21}}{2}$

(D) $\frac{\sqrt{35}}{2}$

Correct Answer: The stated projection does not determine a unique magnitude; all four listed lengths are possible.

Solution:

Concept:

- A vector in the plane spanned by two nonparallel vectors has the form $\vec{v} = \lambda\vec{a} + \mu\vec{b}$, with real coefficients λ, μ.
- The length of the projection of $\vec{v}$ on $\vec{c}$ is $|\vec{v} \cdot \vec{c}|/|\vec{c}|$. A projection fixes only one scalar component and need not determine the entire vector or its length.

Step 1: Evaluate the relevant scalar products Using the printed coordinates, we find

$$\vec{a} \cdot \vec{c} = 4 - 1 - 3 = 0, \qquad \vec{b} \cdot \vec{c} = 2 + 3 - 3 = 2.$$

Also, $|\vec{c}| = \sqrt{4 + 1 + 9} = \sqrt{14}$, while

$$|\vec{a}|^2 = 6, \qquad |\vec{b}|^2 = 11, \qquad \vec{a} \cdot \vec{b} = 2 - 3 + 1 = 0.$$

In particular, changing the component along $\vec{a}$ cannot change the projection on $\vec{c}$.

Step 2: Impose the projection condition Set $\vec{v} = \lambda\vec{a} + \mu\vec{b}$. The given length requires

$$\frac{|2\mu|}{\sqrt{14}} = \frac{1}{\sqrt{14}}, \qquad \mu = \pm\frac{1}{2}.$$

The coefficient λ remains arbitrary. Since $\vec{a}$ and $\vec{b}$ are perpendicular, the squared magnitude is

$$|\vec{v}|^2 = 6\lambda^2 + 11\mu^2 = 6\lambda^2 + \frac{11}{4}.$$

Consequently every length at least $\sqrt{11}/2$ is possible; there is no single forced magnitude.

Step 3: Test all four offered lengths For any proposed length L, choose $\mu = 1/2$ and $\lambda^2 = (L^2 - 11/4)/6$. In the printed option order $L = 7, 13, \sqrt{21}/2, \sqrt{35}/2$, this gives respectively

$$\lambda^2 = \frac{185}{24}, \quad \frac{665}{24}, \quad \frac{5}{12}, \quad 1.$$

Every value is nonnegative, so each option can occur. In particular $\lambda = 1$ produces $\sqrt{35}/2$, but nothing in the question requires that coefficient. Selecting it uniquely would add a condition absent from the statement.

Step 4: Conclusion As printed, the question has no unique answer: $\boxed{|\vec{v}| \geq \sqrt{11}/2}$, and each of (A),

(B), (C) and (D) is attainable. The explicit family $\vec{v} = \lambda\vec{a} + \vec{b}/2$ proves why no single listed option can be imposed by the given projection.

Quick Tip to Solve: Before solving for a vector's length from one projection, count the free coefficients. A direction perpendicular to the projection vector can change the length without changing the projection.

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12. Let f be a function such that $3f(x) + 2f\left(\frac{m}{19x}\right) = 5x, x \neq 0$, where $m = \sum_{i=1}^{9}(i)^2$. Then $f(5) - f(2)$ is equal to [4 marks]

(A) -9

(B) 9

(C) 18

(D) 36

Correct Answer: (C) 18

Solution:

Concept:

- The sum of the first n positive squares is $n(n+1)(2n+1)/6$. Evaluating this sum first identifies the reciprocal substitution in the functional equation.
- The transformation $x \mapsto k/x$ is an involution for nonzero x, k: applying it twice returns x. This produces two linear equations for $f(x)$ and $f(k/x)$.

Step 1: Simplify the reciprocal argument The constant in the question is

$$m = \sum_{i=1}^{9} i^2 = \frac{9 \cdot 10 \cdot 19}{6} = 285, \qquad \frac{m}{19} = 15.$$

Therefore the equation becomes $3f(x) + 2f(15/x) = 5x$, valid for every nonzero x. In particular, the values 5 and 2 requested in the question lie in its domain.

Step 2: Form the companion equation and eliminate Replace x by $15/x$, which is also nonzero. Since $15/(15/x) = x$, this gives

$$2f(x) + 3f(15/x) = \frac{75}{x}.$$

Multiply the original equation by 3 and this companion equation by 2, then subtract. The coefficients of $f(15/x)$ cancel:

$$5f(x) = 15x - \frac{150}{x}, \qquad f(x) = 3x - \frac{30}{x}.$$

Substitution verifies the formula: $f(15/x) = 45/x - 2x$, so $3f(x) + 2f(15/x) = 9x - 90/x + 90/x - 4x = 5x$.

Step 3: Evaluate the required difference in order The two values are

$$f(5) = 15 - 6 = 9, \qquad f(2) = 6 - 15 = -9.$$

Consequently $f(5) - f(2) = 9 - (-9) = 18$. The values -9 and 9 are the separate function values, not their difference; 36 would double the requested difference. The minus sign before the negative value $f(2)$ is essential.

Step 4: Conclusion The functional equation gives $\boxed{f(5) - f(2) = 18}$, which is (C) in the printed option order. Hence, the correct option is (C).

Quick Tip to Solve: For equations coupling $f(x)$ and $f(k/x)$, replace x by k/x and solve the resulting two-by-two linear system.

13. Let $f(\alpha)$ denote the area of the region in the first quadrant bounded by $x = 0, x = 1, y^2 = x$ and $y = |\alpha x - 5| - |1 - \alpha x| + \alpha x^2$. Then $(f(0) + f(1))$ is equal to [4 marks]

(A) 7

(B) 9

(C) 12

(D) 14

Correct Answer: (A) 7

Solution:

Concept:

- In the first quadrant, $y^2 = x$ is the upper branch $y = \sqrt{x}$. The area between two curves on $[0, 1]$ is the integral of the upper ordinate minus the lower ordinate.
- Evaluate $f(0)$ and $f(1)$ separately. The signs inside each absolute value must be determined over the whole interval before integrating.

Step 1: Simplify the upper curve for each parameter For $\alpha = 0$, the second curve is $y = |-5| - |1| = 4$. For $\alpha = 1$, throughout $0 \le x \le 1$, we have $x - 5 < 0$ and $1 - x \ge 0$. Therefore

$$y = (5 - x) - (1 - x) + x^2 = 4 + x^2.$$

Both ordinates are at least 4, whereas $0 \le \sqrt{x} \le 1$. Thus the same curve is uppermost throughout the interval; there is no intersection requiring the area integral to be split.

Step 2: Integrate the two vertical differences Using the vertical boundaries $x = 0$ and $x = 1$,

$$f(0) = \int_0^1 (4 - \sqrt{x})\, dx = \left[4x - \frac{2}{3}x^{3/2}\right]_0^1 = \frac{10}{3}.$$

Similarly,

$$f(1) = \int_0^1 (4 + x^2 - \sqrt{x})\, dx = 4 + \frac{1}{3} - \frac{2}{3} = \frac{11}{3}.$$

Adding gives $f(0) + f(1) = 21/3 = 7$. The other proposed values 9, 12 and 14 do not equal this area sum. Including the negative branch of the parabola would violate the first-quadrant restriction.

Step 3: Conclusion The required sum is 7 square units. Hence, the correct option is (A).

Quick Tip to Solve: First remove the absolute values using the fixed interval; for both parameter values here the parabola is the lower boundary.

14. The smallest positive integral value of a, for which all the roots of $x^4 - ax^2 + 9 = 0$ are real and distinct, is equal to [4 marks]

(A) 3

(B) 7

(C) 4

(D) 9

Correct Answer: (B) 7

Solution:

Concept:

- Set $t = x^2$. Four distinct real roots in x require two distinct, strictly positive roots in t; a zero or repeated root does not give four distinct values.
- For $t^2 - at + 9 = 0$, the discriminant is $a^2 - 36$, the sum of the roots is a, and their product is 9.

Step 1: Impose distinctness and positivity Two distinct real values of t require

$$a^2 - 36 > 0.$$

Because a is positive, this is $a > 6$. The product $t_1 t_2 = 9 > 0$ makes the two real roots have the same sign. Their sum $t_1 + t_2 = a > 0$ then forces both to be positive. These conditions are sufficient as well as necessary: each positive t_i gives the two different roots $\pm\sqrt{t_i}$, and distinct t_i give different magnitudes.

Step 2: Choose and check the least integer The smallest positive integer exceeding 6 is 7. At $a = 7$,

$$t_1 = \frac{7 + \sqrt{13}}{2}, \qquad t_2 = \frac{7 - \sqrt{13}}{2}.$$

They are positive and unequal, so their positive and negative square roots provide four distinct real roots of the original quartic. At the boundary $a = 6$, the quartic is $(x^2 - 3)^2$, which has repeated roots and must be excluded. The choices 3 and 4 have negative discriminant in t, while 9 satisfies the condition but is not the smallest allowed integer.

Step 3: Conclusion The smallest possible positive integral value is $a = 7$. Hence, the correct option is (B).

Quick Tip to Solve: For a biquadratic, check that both roots in x^2 are positive and unequal; checking only a nonnegative discriminant is insufficient.

15. Let $\vec{a} = 2\hat{i} - 5\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + 3\hat{k}$. If $\vec{c}$ is a vector such that $2(\vec{a} \times \vec{c}) + 3(\vec{b} \times \vec{c}) = \vec{0}$ and $(\vec{a} - \vec{b}) \cdot \vec{c} = -97$, then $|\vec{c} \times \hat{k}|^2$ is equal to [4 marks]

(A) 193

(B) 205

(C) 218

(D) 233

Correct Answer: (C) 218

Solution:

Concept:

- The cross product is distributive: $2(\vec{a} \times \vec{c}) + 3(\vec{b} \times \vec{c}) = (2\vec{a} + 3\vec{b}) \times \vec{c}$. A zero cross product with a nonzero vector makes the vectors parallel.
- If $\vec{c} = (c_x, c_y, c_z)$, then $\vec{c} \times \hat{k} = (c_y, -c_x, 0)$, so its squared magnitude is $c_x^2 + c_y^2$.

Step 1: Write the unknown vector as a scalar multiple Calculate the nonzero vector

$$2\vec{a} + 3\vec{b} = (4, -10, 10) + (3, -3, 9) = (7, -13, 19).$$

The cross-product condition therefore gives

$$\vec{c} = \lambda(7, -13, 19).$$

The second condition determines both its length and direction, so parallelism alone does not yet imply that the scalar is positive.

Step 2: Use the dot product to determine the scalar We have $\vec{a} - \vec{b} = (1, -4, 2)$. Hence

$$(\vec{a} - \vec{b}) \cdot \vec{c} = \lambda\{7 + 52 + 38\} = 97\lambda = -97.$$

Thus $\lambda = -1$ and $\vec{c} = (-7, 13, -19)$. Substitution checks the given negative dot product, while a scalar multiple of $2\vec{a} + 3\vec{b}$ automatically satisfies the original cross-product equation.

Step 3: Calculate the requested squared magnitude Crossing with the unit vector along the third axis removes the third component:

$$\vec{c} \times \hat{k} = (13, 7, 0), \qquad |\vec{c} \times \hat{k}|^2 = 13^2 + 7^2 = 218.$$

The alternatives 193, 205 and 233 fail this direct sum-of-squares check. Adding the square of 19 would calculate $|\vec{c}|^2$, a different quantity.

Step 4: Conclusion $|\vec{c} \times \hat{k}|^2 = 218$. Hence, the correct option is (C).

Quick Tip to Solve: Combine the cross products before introducing components; then use the dot product to find the single unknown scalar.

16. Let $[t]$ denote the greatest integer less than or equal to t. If the function

$$f(x) = \begin{cases} b^2 \sin\left(\frac{\pi}{2}\left[\frac{\pi}{2}(\cos x + \sin x)\cos x\right]\right), & x < 0, \\ \frac{\sin x - \frac{1}{2}\sin 2x}{x^3}, & x > 0, \\ a, & x = 0 \end{cases}$$

is continuous at $x = 0$, then $a^2 + b^2$ is equal to [4 marks]

(A) $\frac{3}{4}$

(B) $\frac{9}{16}$

(C) $\frac{1}{2}$

(D) $\frac{5}{8}$

Correct Answer: (A) $\frac{3}{4}$

Solution:

Concept:

- Continuity at zero requires the left-hand limit, the right-hand limit and the assigned value $f(0) = a$ to be equal.
- The greatest-integer function is locally constant when its argument approaches a noninteger. Also, $\sin x/x \to 1$ and $(1 - \cos x)/x^2 \to 1/2$ as $x \to 0$.

Step 1: Evaluate the left-hand limit The argument inside the greatest-integer brackets approaches

$$\frac{\pi}{2}(\cos 0 + \sin 0)\cos 0 = \frac{\pi}{2}.$$

Since $1 < \pi/2 < 2$ and this argument is continuous, it stays strictly between 1 and 2 throughout a sufficiently small neighbourhood of zero. Its greatest integer is therefore 1 on the left of zero. Consequently,

$$\lim_{x \to 0^-} f(x) = b^2 \sin\frac{\pi}{2} = b^2.$$

There is no jump of the greatest-integer term at this limiting argument.

Step 2: Evaluate the right-hand limit Use $\sin 2x = 2\sin x \cos x$ to factor the numerator:

$$\frac{\sin x - \frac{1}{2}\sin 2x}{x^3} = \frac{\sin x(1 - \cos x)}{x^3} = \frac{\sin x}{x}\frac{1 - \cos x}{x^2}.$$

Thus $\lim_{x \to 0^+} f(x) = 1 \cdot \frac{1}{2} = \frac{1}{2}$. Equality of both one-sided limits with $f(0)$ gives $a = b^2 = \frac{1}{2}$.

Step 3: Form the requested expression The requested expression contains a^2, but only b^2, so

$$a^2 + b^2 = \left(\frac{1}{2}\right)^2 + \frac{1}{2} = \frac{3}{4}.$$

The sign of b is irrelevant. The alternatives $9/16$, $1/2$ and $5/8$ do not equal this sum; in particular, squaring b^2 again would use the wrong expression.

Step 4: Conclusion $a^2 + b^2 = \frac{3}{4}$. Hence, the correct option is (A).

Quick Tip to Solve: Check the limiting argument of a floor function before expanding: a noninteger limit makes the floor constant locally.

17. Let $f(x)=\int \frac{7x^{10}+9x^8}{(1+x^2+2x^9)^2}dx, x>0, \lim\limits_{x\to 0} f(x)=0$ **and** $f(1)=\frac{1}{4}$**.**

If $\mathrm{A}=\begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{4} & f'(1) & 1 \\ \alpha^2 & 4 & 1 \end{bmatrix}$ **and** $\mathrm{B}=\mathrm{adj}(\mathrm{adj}\,\mathrm{A})$ **be such that** $|\mathrm{B}|=81$**, then** α^2 **is equal to** [4 marks]

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- If f is an antiderivative of the given integrand, $f'(1)$ follows by substituting 1 directly into the integrand. The additive integration constant does not affect the derivative.
- For a square matrix of order n, $\det(\mathrm{adj}\,A)=(\det A)^{n-1}$. Applying this twice for a matrix of order 3 gives $\det(\mathrm{adj}(\mathrm{adj}\,A))=(\det A)^4$.

Step 1: Obtain the derivative entry Differentiating the given indefinite integral and setting $x=1$ yields

$$f'(1)=\frac{7+9}{(1+1+2)^2}=\frac{16}{16}=1.$$

The values of f fix its constant consistently but are unnecessary for this entry. Thus the matrix becomes

$$A=\begin{pmatrix} 0 & 0 & 1 \\ 1/4 & 1 & 1 \\ \alpha^2 & 4 & 1 \end{pmatrix}.$$

Step 2: Expand the determinant and use the adjugate identity Expansion along the first row has only one nonzero term, whose cofactor sign is positive:

$$\det A=\begin{vmatrix} 1/4 & 1 \\ \alpha^2 & 4 \end{vmatrix}=1-\alpha^2.$$

The condition on B therefore reads

$$(1-\alpha^2)^4=81=3^4.$$

For real α, this gives $1-\alpha^2=3$ or -3, hence $\alpha^2=-2$ or 4. A real square cannot be negative, leaving 4.

Step 3: Check the offered values For $\alpha^2=4$, $\det A=-3$, giving $\det B=(-3)^4=81$ as required. The other choices $\alpha^2=1,2,3$ instead give $\det B=0,1,16$, respectively. None satisfies the stated determinant condition.

Step 4: Conclusion The required value is $\alpha^2 = 4$. Hence, the correct option is (D).

Quick Tip to Solve: Use the determinant identity twice instead of computing either adjugate; the derivative entry needs no actual integration.

18. $\left(\frac{1}{3}+\frac{4}{7}\right)+\left(\frac{1}{3^2}+\frac{1}{3}\times\frac{4}{7}+\frac{4^2}{7^2}\right)+\left(\frac{1}{3^3}+\frac{1}{3^2}\times\frac{4}{7}+\frac{1}{3}\times\frac{4^2}{7^2}+\frac{4^3}{7^3}\right)+\ldots$ **upto infinite terms, is equal to**

[4 marks]

(A) $\frac{6}{5}$
(B) $\frac{4}{3}$
(C) $\frac{5}{2}$
(D) $\frac{7}{4}$

Correct Answer: (C) $\frac{5}{2}$

Solution:

Concept:

- Write $r = 1/3$ and $s = 4/7$. The parenthesis of total degree n is $r^n + r^{n-1}s + \cdots + s^n$, for $n \geq 1$.
- For $|r|, |s| < 1$, geometric sums converge. The product $(1 + r + r^2 + \cdots)(1 + s + s^2 + \cdots)$ groups by total degree into exactly these parentheses, together with the constant term 1.

Step 1: Identify the product behind the series In the product of the two geometric series, each monomial $r^i s^j$, with nonnegative integers i, j, appears once. Grouping by $i + j$ gives

$$1 + (r + s) + (r^2 + rs + s^2) + (r^3 + r^2 s + rs^2 + s^3) + \cdots .$$

The printed series starts at total degree 1, so its sum S is this product minus 1. Both ratios are positive and less than 1, and the product series is absolutely convergent, which justifies this grouping.

Step 2: Evaluate the geometric sums The two sums, including their constant terms, are

$$\sum_{i=0}^{\infty} r^i = \frac{1}{1 - 1/3} = \frac{3}{2}, \qquad \sum_{j=0}^{\infty} s^j = \frac{1}{1 - 4/7} = \frac{7}{3}.$$

Their product is $7/2$. Removing the unprinted constant term gives

$$S = \frac{3}{2} \cdot \frac{7}{3} - 1 = \frac{7}{2} - 1 = \frac{5}{2}.$$

The choices $6/5$, $4/3$ and $7/4$ do not match this exact product calculation. Summing just the pure powers would miss all mixed terms, while retaining the constant term would produce $7/2$.

Step 3: Conclusion The infinite series has sum $\frac{5}{2}$. Hence, the correct option is (C).

Quick Tip to Solve: A bracket containing all mixed powers of the same total degree suggests the product of two geometric series; check whether its constant term is present.

19. **Let $y = y(x)$ be a differentiable function in the interval $(0, \infty)$ such that $y(1) = 2$, and $\lim\limits_{t \to x} \left(\frac{t^2 y(x) - x^2 y(t)}{x - t} \right) = 3$ for each $x > 0$. Then $2y(2)$ is equal to :** [4 marks]

(A) 23

(B) 12

(C) 18

(D) 27

Correct Answer: (A) 23

Solution:

Concept:

- A difference quotient at a differentiability point determines a derivative. The denominator here is $x - t = -(t - x)$, so its sign must be retained.
- The linear equation $y' - 2y/x = 3/x^2$ has integrating factor x^{-2} on $x > 0$. Equivalently, its left side becomes the derivative of y/x^2.

Step 1: Convert the limit into a differential equation Hold x fixed and rewrite the numerator by adding and subtracting $x^2 y(x)$:

$$\frac{t^2 y(x) - x^2 y(t)}{x - t} = -(t + x)y(x) + x^2 \frac{y(t) - y(x)}{t - x}.$$

Differentiability makes the second quotient tend to $y'(x)$. Thus the given limit yields

$$x^2 y' - 2xy = 3, \qquad y' - \frac{2}{x} y = \frac{3}{x^2}.$$

Step 2: Integrate and impose the initial value Multiply the equation by x^{-2}, which is defined throughout the specified interval:

$$\frac{d}{dx} \left(\frac{y}{x^2} \right) = \frac{3}{x^4}.$$

Integration gives $y/x^2 = -x^{-3} + C$, or $y = Cx^2 - 1/x$. Since $y(1) = 2$, we have $C - 1 = 2$, hence $C = 3$.

Step 3: Evaluate the requested multiple Substitute $x = 2$ into the obtained function:

$$y(2) = 3(2)^2 - \frac{1}{2} = \frac{23}{2}, \qquad 2y(2) = 23.$$

The other listed numbers $12, 18, 27$ do not equal this value. As a check, $y' = 6x + x^{-2}$ gives $x^2 y' - 2xy = 3$, so the original limit condition is satisfied.

Step 4: Conclusion The initial condition fixes $y = 3x^2 - 1/x$, proving $2y(2) = \boxed{23}$, rather than any of $12, 18, 27$. Hence, the correct option is (A).

Quick Tip to Solve: When a difference quotient has $x - t$ below, first change it to $t - x$ with a minus sign; this prevents reversing the differential equation.

20. Let the image of parabola $x^2 = 4y$, in the line $x - y = 1$ be $(y + a)^2 = b(x - c)$, $a, b, c \in$ N. Then $a + b + c$ is equal to [4 marks]

(A) 4

(B) 6

(C) 8

(D) 12

Correct Answer: (B) 6

Solution:

Concept:

- Reflection in $x - y = 1$, equivalently $y = x - 1$, interchanges the coordinate directions while also translating them. The reflected point of (u, v) is $(v + 1, u - 1)$.
- To find the image of a curve, express the original coordinates in terms of its reflected coordinates and substitute them in the original equation. Reflection is its own inverse.

Step 1: Establish the reflection map Take an arbitrary point (u, v) on $u^2 = 4v$. Its reflection (x, y) in the line $u - v - 1 = 0$ is obtained using the point-reflection formula:

$$x = u - \frac{2(u - v - 1)}{2} = v + 1, \qquad y = v + \frac{2(u - v - 1)}{2} = u - 1.$$

These formulas also show that the midpoint lies on the mirror line and the joining segment is perpendicular to it. Inverting them gives $u = y + 1$, $v = x - 1$.

Step 2: Transform the parabola equation Substitute the inverse coordinates in $u^2 = 4v$:

$$(y + 1)^2 = 4(x - 1).$$

Thus the image opens to the right and has vertex $(1, -1)$. This agrees with reflecting the original vertex $(0, 0)$, providing a direct check on the translation signs.

Step 3: Identify the three positive parameters Compare the equation with $(y + a)^2 = b(x - c)$:

$$a = 1, \qquad b = 4, \qquad c = 1, \qquad a + b + c = 1 + 4 + 1 = 6.$$

The values $4, 8, 12$ in the remaining options are excluded by this exact comparison. Merely swapping x and y would reflect in $y = x$, which is a different line and would miss the translations.

Step 4: Conclusion Reflection gives $(y + 1)^2 = 4(x - 1)$, so $a + b + c = \boxed{6}$. Hence, the correct option is (B).

Quick Tip to Solve: For reflection in $y = x + d$, use $(u, v) \mapsto (v - d, u + d)$; check the reflected vertex before transforming the whole curve.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. The number of elements in the set $\{x \in [0, 180^\circ] : \tan(x + 100^\circ) = \tan(x + 50^\circ)\tan x \tan(x - 50^\circ)\}$ is ____ . [4 marks]

Solution:

Concept:

- An equation containing tangents is defined only when each relevant cosine is nonzero. Clearing denominators is reversible only on that restricted domain.
- The identities $2\cos A\cos B = \cos(A+B)+\cos(A-B)$ and $2\sin A\sin B = \cos(A-B)-\cos(A+B)$ reduce symmetric products. All angles below are in degrees.

Step 1: Record the excluded angles and clear denominators On $[0^\circ, 180^\circ]$, the four tangents exclude $x = 40^\circ, 90^\circ, 140^\circ, 170^\circ$. At all other points the equation is equivalent to

$$\sin(x + 100^\circ)\cos(x + 50^\circ)\cos x\cos(x - 50^\circ) - \cos(x + 100^\circ)\sin(x + 50^\circ)\sin x\sin(x - 50^\circ) = 0.$$

Step 2: Reduce the symmetric products Twice the cosine product with shifts $\pm 50^\circ$ is $\cos 2x + \cos 100^\circ$, while twice the corresponding sine product is $\cos 100^\circ - \cos 2x$. Substitution and the sine addition formulas therefore give

$$\cos 2x \sin(2x + 100^\circ) + \cos 100^\circ \sin 100^\circ = 0.$$

Multiplying by two and using product-to-sum again yields

$$\sin(4x + 100^\circ) + \sin 100^\circ + \sin 200^\circ = 0.$$

Since $\sin 100^\circ + \sin 200^\circ = \sin 100^\circ - \sin 20^\circ = \sin 40^\circ$, the reduced equation is $\sin(4x + 100^\circ) = -\sin 40^\circ$.

Step 3: List and verify the roots in the interval The general solutions are

$$4x + 100^\circ = 220^\circ + 360^\circ k \quad \text{or} \quad 4x + 100^\circ = 320^\circ + 360^\circ k,$$

where k is an integer. Hence $x = 30^\circ + 90^\circ k$ or $x = 55^\circ + 90^\circ k$. The interval contains exactly $30^\circ, 55^\circ, 120^\circ, 145^\circ$. None is excluded, and neither endpoint is a root. Because the denominator-clearing step was reversible at these points, all four satisfy the original equation.

Step 4: Conclusion The set is $\{30^\circ, 55^\circ, 120^\circ, 145^\circ\}$, so its number of elements is $\boxed{4}$.

Quick Tip to Solve: For shifted tangent products, clear denominators after noting their zeros, then pair the symmetric shifts before using product-to-sum.

22. Let $z = (1+i)(1+2i)(1+3i)\ldots(1+ni)$, where $i = \sqrt{-1}$. If $|z|^2 = 44200$, then n is equal to ____ [4 marks]

Solution:

Concept:

- The squared modulus of a complex number is its product with its conjugate: $|w|^2 = w\overline{w}$. Thus $|1+ki|^2 = 1+k^2$ for real k.
- Modulus is multiplicative, so the squared modulus of a product is the product of the squared moduli. The argument of each factor does not affect this calculation.

Step 1: Replace each complex factor by its squared modulus Since $i^2 = -1$, conjugate multiplication gives

$$(1+ki)(1-ki) = 1 - k^2 i^2 = 1 + k^2.$$

Consequently,

$$|z|^2 = \prod_{k=1}^{n} |1+ki|^2 = \prod_{k=1}^{n} (1+k^2).$$

There is no need to expand z itself; doing so would introduce many cancelling imaginary terms. Also, the stated value is already $|z|^2$, so no additional squaring of 44200 is required.

Step 2: Evaluate the successive positive products For $k = 1, 2, 3, 4, 5$, the factors $1 + k^2$ are $2, 5, 10, 17, 26$, respectively. Their running products are

$$P_1 = 2, \qquad P_2 = 2 \cdot 5 = 10,$$
$$P_3 = 10 \cdot 10 = 100, \qquad P_4 = 100 \cdot 17 = 1700,$$
$$P_5 = 1700 \cdot 26 = 44200.$$

Thus stopping after the fifth factor gives exactly the required squared modulus.

Step 3: Check that no other positive integer works Every new factor $1 + k^2$ is strictly greater than one. Hence P_n is strictly increasing for positive integral n. Values with $n < 5$ are smaller than 44200, while $P_6 = 44200 \cdot 37$ and every later product are larger. This proves uniqueness rather than merely exhibiting one possible value.

Step 4: Conclusion The unique positive integer satisfying $|z|^2 = 44200$ is $\boxed{n = 5}$.

Quick Tip to Solve: For a product of factors $1 + ki$, compute $\prod(1 + k^2)$ directly when the squared modulus is given; avoid expanding the complex product.

23. Let (h, k) lie on the circle $C : x^2 + y^2 = 4$ and the point $(2h + 1, 3k + 2)$ lie on an ellipse with eccentricity e.

Then the value of $\frac{5}{e^2}$ is equal to ____ . [4 marks]

Solution:

Concept:

- A coordinate translation changes the centre of an ellipse but not its semiaxes or eccentricity. Unequal scaling along two perpendicular coordinate directions transforms a circle into an ellipse.
- For an ellipse with major semiaxis a and minor semiaxis b, where $a > b > 0$, the eccentricity satisfies $e^2 = 1 - b^2/a^2$. The larger denominator in standard form is always a^2.

Step 1: Eliminate the original circle coordinates Denote the point on the ellipse by (X, Y). The given transformation is

$$X = 2h + 1, \qquad Y = 3k + 2.$$

Its inverse is $h = (X - 1)/2$, $k = (Y - 2)/3$. Substitute these expressions into $h^2 + k^2 = 4$:

$$\frac{(X-1)^2}{4} + \frac{(Y-2)^2}{9} = 4.$$

Dividing every term by four yields the standard ellipse equation

$$\frac{(X-1)^2}{16} + \frac{(Y-2)^2}{36} = 1.$$

Step 2: Read the semiaxes in their correct order The centre is $(1, 2)$. The horizontal semiaxis is 4 and the vertical semiaxis is 6. Thus the major axis is vertical and $a = 6$, $b = 4$; these names are determined by size, not by which variable appears first. This also matches the circle radius 2 scaled horizontally by 2 and vertically by 3.

Step 3: Calculate the eccentricity and required ratio Substitute the squared semiaxes into the eccentricity formula:

$$e^2 = 1 - \frac{16}{36} = \frac{20}{36} = \frac{5}{9}.$$

This lies between zero and one, as it must for an ellipse. Therefore

$$\frac{5}{e^2} = \frac{5}{5/9} = 9.$$

The constants 1 and 2 in the coordinate transformation affect only the centre and therefore do not enter this final ratio.

Step 4: Conclusion The ellipse has $e^2 = 5/9$, and the required dimensionless value is $\boxed{5/e^2 = 9}$.

Quick Tip to Solve: After eliminating the circle coordinates, divide by the circle's radius squared before reading the ellipse semiaxes; label the larger one a.

24. If $f(x)$ satisfies the relation $f(x) = e^x + \int_0^1 (y + xe^x)\, f(y)dy$, then $e + f(0)$ is equal to ____ . [4 marks]

Solution:

Concept:

- In an integral with fixed limits and dummy variable y, expressions such as $\int_0^1 f(y)\, dy$ are constants with respect to x. Separating these constants reduces the integral equation to an algebraic relation.
- Integrate the resulting expression for $f(x)$ over the same interval to obtain consistency conditions. Only the constants needed for $f(0)$ have to be determined.

Step 1: Name the two constant integrals Set

$$A = \int_0^1 yf(y)\, dy, \qquad B = \int_0^1 f(y)\, dy.$$

The factor xe^x does not depend on y, so it can be taken outside the integral. The given equation becomes

$$f(x) = e^x + A + Bxe^x.$$

In particular, $f(0) = 1 + A$; the term containing B vanishes at $x = 0$.

Step 2: Integrate the reduced equation from zero to one The left side integrates to B. On the right,

$$\int_0^1 e^x\, dx = e - 1, \qquad \int_0^1 xe^x\, dx = [(x-1)e^x]_0^1 = 1.$$

Since the interval has length one, the constant A integrates to A. Hence

$$B = (e - 1) + A + B.$$

The two B terms cancel, giving $A = 1 - e$. No division by a possibly zero unknown constant is involved.

Step 3: Evaluate the requested expression Substitution into the value at zero gives

$$f(0) = 1 + (1 - e) = 2 - e, \qquad e + f(0) = 2.$$

For consistency, the other moment equation would give $A = 1+A/2+B(e-2)$, using $\int_0^1 x^2e^x\, dx = e-2$. It determines $B = -(e+1)/(2(e-2))$, a finite value, but that constant is unnecessary for the requested expression.

Step 4: Conclusion The integral identity fixes $f(0) = 2 - e$, so the required value is $\boxed{e + f(0) = 2}$.

Quick Tip to Solve: Replace fixed-limit integrals of the unknown function by constants, then integrate once; here the coefficient integral is one, making the unwanted constant cancel.

25. Let S be a set of 5 elements and $\mathrm{P(S)}$ denote the power set of S . Let E be an event of choosing an ordered pair (A, B) from the set $\mathrm{P(S)} \times \mathrm{P(S)}$ such that $\mathrm{A} \cap \mathrm{B} = \emptyset$. If the probability of the event E is $\frac{3^p}{2^q}$, where $p, q \in N$, then $p + q$ is equal to [4 marks]

Solution:

Concept:

- For a set with n elements, the power set has 2^n subsets. An ordered pair distinguishes the first subset from the second.
- Count disjoint pairs by assigning each element independently to A only, B only, or neither. Membership in both is forbidden.

Step 1: Count all equally likely ordered pairs Here $|S| = 5$, so $|\mathrm{P}(S)| = 2^5 = 32$. Each first subset can be paired with any of the 32 second subsets, including itself. Thus the sample space contains

$$N = 32 \times 32 = 2^{10}.$$

The pair is ordered, so we neither divide by two nor exclude equal subsets. In particular, the pair of two empty sets is permitted.

Step 2: Count the pairs with empty intersection For each element of S, record its membership using two indicators. The four possibilities are $(0,0), (1,0), (0,1), (1,1)$, where the positions refer to A and B. Disjointness removes just $(1,1)$. Therefore there are three independent choices for each of the five distinct elements, giving

$$N(E) = 3^5 = 243.$$

This count includes an empty A or an empty B and counts each ordered pair exactly once. It does not require that the union be all of S.

Step 3: Form the probability and identify the exponents The probability is the favourable count divided by the entire ordered sample space:

$$\mathrm{P}(E) = \frac{3^5}{2^{10}}.$$

The numerator and denominator are coprime because their only prime factors are 3 and 2. Hence comparison with $3^p/2^q$ gives $p = 5$ and $q = 10$.

Step 4: Conclusion Therefore $p + q = 5 + 10 = \boxed{15}$. This is a dimensionless integer.

Quick Tip to Solve: For two disjoint subsets of an n-element set, each element has three allowed membership states out of four, giving probability $(3/4)^n$.

26. The binding energy for the following nuclear reactions are expressed in MeV .

$$_2\mathrm{He}^3 + {}_0\mathrm{n}^1 \rightarrow {}_2\mathrm{He}^4 + 20\mathrm{MeV}$$
$$_2\mathrm{He}^4 + {}_0\mathrm{n}^1 \rightarrow {}_2\mathrm{He}^5 - 0.9\mathrm{MeV}$$

If $\mathrm{X}_3, \mathrm{X}_4, \mathrm{X}_5$ denote the stability of $_2\mathrm{He}^3, {}_2\mathrm{He}^4$ and $_2\mathrm{He}^5$, respectively, then the correct order is :

[4 marks]

(A) $X_4 < X_5 < X_3$

(B) $X_4 > X_5 > X_3$

(C) $X_4 = X_5 = X_3$

(D) $X_4 > X_5 < X_3$

Correct Answer: (B) $X_4 > X_5 > X_3$

Solution:

Concept:

- The energy released when a free neutron is added equals the increase in total nuclear binding energy. A free neutron has no nuclear binding energy of its own.
- A positive reaction Q-value means energy is released; a negative Q-value means energy must be supplied. Compare the stated binding-energy differences with their signs intact.

Step 1: Translate the first reaction into a binding-energy difference Write the total binding energies of the three nuclei as B_3, B_4, B_5, respectively. The reaction producing helium-4 releases 20 MeV, so the final nucleus is more tightly bound by this amount:

$$B_4 - B_3 = 20 \text{ MeV}.$$

Therefore $B_4 = B_3 + 20$ MeV. The incident neutron contributes mass energy but is not a pre-bound nucleus, so no neutron binding term is subtracted.

Step 2: Use the negative energy in the second reaction The second reaction has $Q = -0.9$ MeV. Forming helium-5 from helium-4 and a neutron requires this energy, giving

$$B_5 - B_4 = -0.9 \text{ MeV}.$$

Combining the two reactions yields

$$B_5 = B_3 + 19.1 \text{ MeV}.$$

Thus helium-4 has the largest total binding, and helium-5 still has greater total binding than helium-3 in the comparison supplied by these reactions.

Step 3: Compare the listed orders The differences are $B_4 - B_5 = 0.9 \text{ MeV} > 0$ and $B_5 - B_3 = 19.1 \text{ MeV} > 0$. Accordingly the binding-based stability order intended here is $X_4 > X_5 > X_3$. Option (A) incorrectly puts helium-4 lowest, option (C) makes unequal bindings equal, and option (D) places

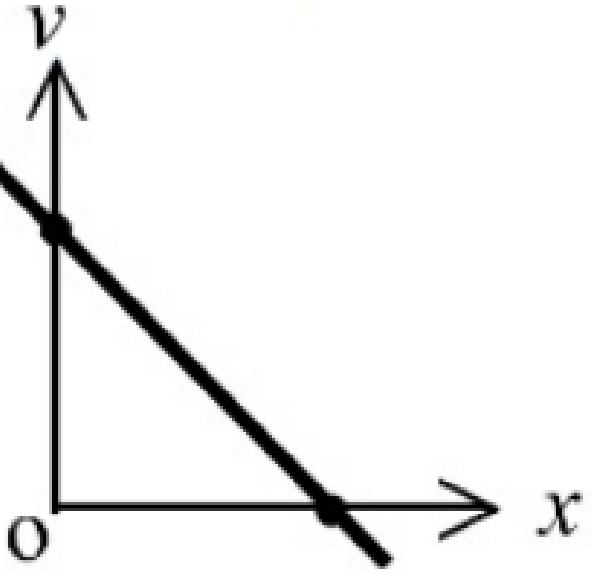
helium-5 below helium-3. These contradict the reaction-energy differences. This comparison concerns the binding energies stated in the problem; it is not a claim that helium-5 is stable against neutron emission.

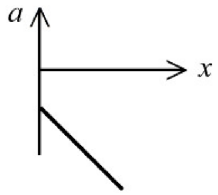
Step 4: Conclusion The stated binding-energy comparison gives $\boxed{X_4 > X_5 > X_3}$. Hence, the correct option is (B).

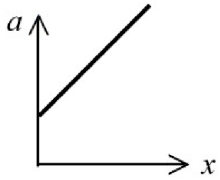
Quick Tip to Solve: Add the signed Q-values: the net change from helium-3 to helium-5 is $20 - 0.9 = 19.1$ MeV, not $20 + 0.9$.

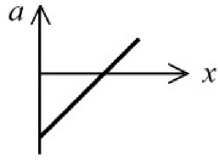
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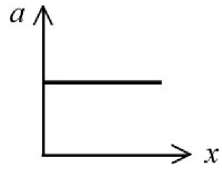
27. The velocity (v) - Distance (x) graph is shown in figure. Which graph represents acceleration(a) versus distance (x) variation of this system? [4 marks]

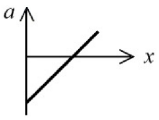


(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- Acceleration is $a = dv/dt$. When velocity is given as a function of position, the chain rule gives $a = v\,dv/dx$.
- A straight descending velocity-position graph can be written $v = v_0 - kx$, where the positive constants v_0 and k are the velocity intercept and magnitude of its slope.

Step 1: Write the equation of the given straight line The graph has a positive vertical intercept and slopes down to a positive x-intercept. Thus

$$v = v_0 - kx, \qquad v_0 > 0, \quad k > 0.$$

Its slope is $dv/dx = -k$. Because the horizontal axis is distance x, this slope is not acceleration; a velocity-time graph would be needed for that direct interpretation.

Step 2: Apply the chain rule Multiply the position derivative by the velocity:

$$a = v\frac{dv}{dx} = (v_0 - kx)(-k) = k^2x - kv_0.$$

Hence acceleration varies linearly with x, has positive slope k^2, and has negative vertical intercept $-kv_0$. At the point $x = v_0/k$, the velocity vanishes and this acceleration expression also becomes zero. These three features fix the required line.

Step 3: Compare every graphical option Option (C) starts below the x-axis and rises linearly through it, exactly as the derived equation requires. Option (A) has a negative slope instead of $k^2 > 0$. Option (B) has the right slope sign but a positive initial acceleration, whereas $a(0) = -kv_0 < 0$. Option (D) represents a constant positive acceleration and cannot equal the nonconstant function obtained above. The falling v-x graph therefore does not imply a falling a-x graph.

Step 4: Conclusion The acceleration graph is the rising straight line with negative vertical intercept: $\boxed{a = k^2x - kv_0}$. Hence, the correct option is (C).

Quick Tip to Solve: On a v-x graph use $a = v\,dv/dx$, then check both the slope and the intercept of the resulting a-x graph.

28. A regular hexagon is formed by six wires each of resistance $r\Omega$ and the corners are joined to the centre by wires of same resistance. If the current enters at one corner and leaves at the opposite corner, the equivalent resistance of the hexagon between the two opposite corners will be [4 marks]

(A) $\frac{3}{4}r$
(B) $\frac{3}{5}r$
(C) $\frac{4}{5}r$
(D) $\frac{5}{8}r$

Correct Answer: (C) $\frac{4}{5}r$

Solution:

Concept:

- Equal-potential nodes can be identified by reflection symmetry. The centre of this symmetric network is at half the applied potential.
- Apply Kirchhoff's current law at a node: the sum of currents $(V_i - V_j)/r$ through its connected resistors is zero.

Step 1: Assign potentials using the hexagon symmetry Label the vertices consecutively A, B, C, D, E, F, with current entering A and leaving the opposite vertex D. Apply $V_A = V$ and $V_D = 0$. Reflection about AD gives $V_B = V_F = b$ and $V_C = V_E = c$. Interchanging the opposite terminals gives $V_O = V/2$ at the centre O and $c = V - b$. Each outer edge and each spoke has resistance r.

Step 2: Find the two intermediate vertex potentials At B the three connected nodes are A, C and O. Kirchhoff's law gives

$$\frac{b - V}{r} + \frac{b - c}{r} + \frac{b - V/2}{r} = 0.$$

Therefore $3b = 3V/2 + c$. Substituting $c = V - b$, we obtain

$$4b = \frac{5V}{2}, \qquad b = \frac{5V}{8}, \qquad c = \frac{3V}{8}.$$

These potentials decrease from the source to the sink, as expected.

Step 3: Calculate the current drawn at the source Three branches leave A: the two equal outer-edge branches AB and AF, and the spoke AO. Their combined current is

$$I = 2\frac{V - 5V/8}{r} + \frac{V - V/2}{r} = \frac{3V}{4r} + \frac{V}{2r} = \frac{5V}{4r}.$$

Hence

$$R_{\text{eq}} = \frac{V}{I} = \frac{4}{5}r.$$

The values $3r/4$, $3r/5$, and $5r/8$ would give currents different from this Kirchhoff-law result, so the other three options fail.

Step 4: Conclusion The equivalent resistance is $\boxed{R_{\text{eq}} = \frac{4}{5}r\ \Omega}$, when r is the numerical resistance in ohms. Hence, the correct option is (C).

Quick Tip to Solve: Use reflection to reduce a hexagon with equal spokes to two unknown vertex potentials; do not treat all spokes as independent parallel branches.

29. Distance between an object and three times magnified real image is 40 cm . The focal length of the mirror used is ____ cm . [4 marks]

(A) -10

(B) $-15/2$

(C) -15

(D) -20

Correct Answer: (C) -15

Solution:

Concept:

- For a spherical mirror, linear magnification is $m = -v/u$, and the mirror formula is $1/f = 1/u + 1/v$.
- A real image of a real object in a concave mirror is inverted. With the Cartesian sign convention both object distance u and real image distance v are negative.

Step 1: Use the signed magnification Threefold magnification of a real image means $m = -3$, not $+3$, since the image is inverted. Thus

$$-\frac{v}{u} = -3 \quad \Longrightarrow \quad v = 3u.$$

Since both distances are negative, the image is three times as far from the mirror as the object, on the same side of the reflecting surface.

Step 2: Convert the object-image separation into distances The separation is the magnitude of the difference of their signed positions:

$$|v - u| = 40 \text{ cm}.$$

Using $v = 3u$, this becomes $2|u| = 40$ cm. Hence

$$u = -20 \text{ cm}, \qquad v = -60 \text{ cm}.$$

The separation is $60 - 20 = 40$ cm; adding the distances would incorrectly place the real image on the opposite side of the mirror.

Step 3: Apply the mirror formula and check the options Substitute the two signed distances:

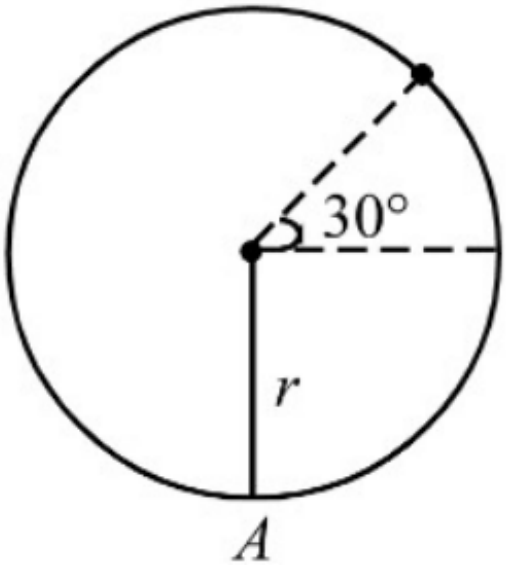
$$\frac{1}{f} = -\frac{1}{20} - \frac{1}{60} = -\frac{4}{60} = -\frac{1}{15} \text{ cm}^{-1}.$$

Therefore $f = -15$ cm. This also places the object between F and C because $15 < 20 < 30$, consistent with an enlarged real image. The other choices, -10, $-15/2$, and -20 cm, do not satisfy the mirror formula for these distances.

Step 4: Conclusion The required focal length is $\boxed{-15 \text{ cm}}$. Hence, the correct option is (C).

Quick Tip to Solve: For an enlarged real mirror image, use negative magnification and take the difference of object and image distances, since both lie in front of the mirror.

30. In case of vertical circular motion of a particle by a thread of length r if the tension in the thread is zero at an angle 30° shown in figure, the velocity at the bottom point (A) of the circular path is (g = gravitational acceleration) [4 marks]



(A) $\sqrt{\frac{5}{2}gr}$

(B) $\sqrt{5gr}$

(C) $\sqrt{\frac{7}{2}gr}$

(D) $\sqrt{4gr}$

Correct Answer: (C) $\sqrt{\frac{7}{2}gr}$

Solution:

Concept:

- The inward radial force supplies mv^2/r. When the thread tension is zero, only the inward component of gravity supplies this force.
- Tension does no work while the thread is taut, so mechanical energy is conserved. The angle in this diagram is measured above the horizontal radius.

Step 1: Resolve gravity at the zero-tension point Let the speed at the marked point P be v_P. The radius OP is 30° above the horizontal. Gravity has inward radial component $mg\sin 30^\circ = mg/2$. Thus

$$T + mg\sin 30^\circ = \frac{mv_P^2}{r}.$$

With $T = 0$, we find

$$v_P^2 = gr\sin 30^\circ = \frac{gr}{2}.$$

Zero tension does not mean zero velocity: gravity still provides a centripetal force at that instant.

Step 2: Find the height gained from the bottom The bottom point A is a distance r below the centre.

Point P is $r\sin 30^\circ = r/2$ above it. Consequently the total rise is

$$h = r + r\sin 30^\circ = \frac{3r}{2}.$$

Using $r\cos 30^\circ$ here would treat the marked angle as measured from the vertical, contrary to the diagram.

Step 3: Conserve energy and compare the choices From A to P, energy conservation gives

$$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_P^2 + mgh.$$

Substitution yields

$$v_A^2 = \frac{gr}{2} + 2g\frac{3r}{2} = \frac{7}{2}gr.$$

Therefore the speed is the positive square root. The alternatives $5gr/2$, $5gr$, and $4gr$ for the squared speed fail the combined radial-force and height equations. In particular, $5gr$ is the familiar minimum bottom speed for a complete taut loop, a different condition from zero tension at this marked point.

Step 4: Conclusion The velocity magnitude at the bottom is $\boxed{v_A = \sqrt{\frac{7}{2}gr}}$. For SI values of g and r it is in $\mathrm{m\,s^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: For zero tension at angle α above the horizontal, combine $v_P^2 = gr\sin\alpha$ with height $r(1+\sin\alpha)$.

31. The fifth harmonic of a closed organ pipe is found to be in unison with the first harmonic of an open pipe. The ratio of lengths of closed pipe to that of the open pipe is $5/x$. The value of x is ____

[4 marks]

(A) 2

(B) 3

(C) 4

(D) 1

Correct Answer: (A) 2

Solution:

Concept:

- A closed organ pipe supports odd harmonics of its fundamental frequency: $f_n = nv/(4L_c)$, where $n = 1, 3, 5, \ldots$, v is sound speed and L_c is its length.
- An open organ pipe has fundamental frequency $f_1 = v/(2L_o)$. Two notes in unison have equal frequencies; the speed of sound is the same in the two pipes.

Step 1: Identify the two frequencies The fifth harmonic means five times the closed pipe's fundamental, not its fifth allowed mode. Thus the closed pipe has frequency $5v/(4L_c)$. The first harmonic of the open pipe is simply its fundamental, $v/(2L_o)$. The distinction matters because the fifth allowed

closed-pipe mode would be the ninth harmonic, which is not what is stated.

Step 2: Equate the frequencies and find the length ratio Use the unison condition and cancel the common nonzero sound speed:

$$\frac{5v}{4L_c} = \frac{v}{2L_o} \quad \Longrightarrow \quad 10L_o = 4L_c.$$

$$\frac{L_c}{L_o} = \frac{10}{4} = \frac{5}{2}.$$

The required ratio is given as $5/x$, so matching the denominator gives $x = 2$.

Step 3: Check the other choices The alternatives $x = 3, 4, 1$ give length ratios $5/3, 5/4, 5$. Substitution into $f_{5,c}/f_{1,o} = 5L_o/(2L_c)$ gives $3/2, 2, 1/2$, respectively, instead of unity. Only $x = 2$ makes the two frequencies equal.

Step 4: Conclusion The required dimensionless value is $\boxed{x = 2}$. Hence, the correct option is (A).

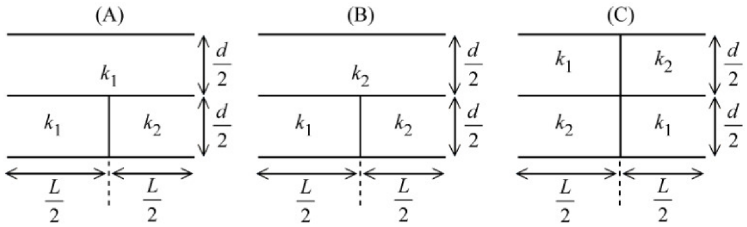
Quick Tip to Solve: For unison, write the actual harmonic frequency for each pipe before taking a length ratio; a closed pipe has denominator $4L$, while an open pipe has $2L$.

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32. Three parallel plate capacitors each with area A and separation d are filled with two dielectric (k_1 and k_2) in the following fashion. Which of the following is true?

$$(k_1 > k_2)$$

[4 marks]



(A) $C_A > C_C > C_B$

(B) $C_C > C_B > C_A$

(C) $C_B > C_C > C_A$

(D) $C_C > C_A > C_B$

Correct Answer: (A) $C_A > C_C > C_B$

Solution:

Concept:

- A dielectric slab gives capacitance $C = \varepsilon_0 kA/d$. Dielectric layers along the field act in series, while side-by-side portions spanning the plates contribute in parallel.
- Compare the arrangements by following each vertical half-area column between the plates. Set $C_0 = \varepsilon_0 A/d$, where A and d are the common plate area and separation.

Step 1: Evaluate a mixed column Each half-area column contains two layers of thickness $d/2$. Their individual capacitances are $\varepsilon_0 k_1 (A/2)/(d/2) = k_1 C_0$ and $k_2 C_0$. Their series combination is therefore

$$C_m = C_0 \frac{k_1 k_2}{k_1 + k_2}.$$

Reversing the order of the two layers leaves this result unchanged.

Step 2: Combine the two columns in each arrangement In A, the left half contains only k_1 across the full separation; in B, the right half contains only k_2. Each remaining half is a mixed column. In C both halves are mixed columns. Hence

$$C_A = C_0 \left(\frac{k_1}{2} + \frac{k_1 k_2}{k_1 + k_2} \right),$$

$$C_B = C_0 \left(\frac{k_2}{2} + \frac{k_1 k_2}{k_1 + k_2} \right), \qquad C_C = C_0 \frac{2k_1 k_2}{k_1 + k_2}.$$

Step 3: Compare positive differences Subtract the expressions rather than guessing from the picture:

$$C_A - C_C = C_0 \frac{k_1 (k_1 - k_2)}{2(k_1 + k_2)} > 0,$$

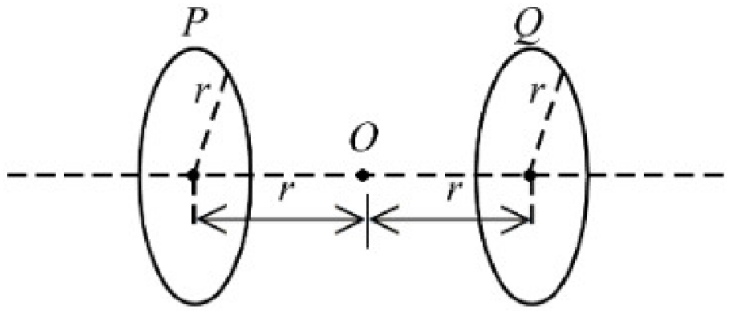
$$C_C - C_B = C_0 \frac{k_2 (k_1 - k_2)}{2(k_1 + k_2)} > 0.$$

Both inequalities follow from $k_1 > k_2 > 0$. Thus A exceeds C, and C exceeds B. Every other listed ordering reverses at least one of these strict inequalities.

Step 4: Conclusion The capacitances satisfy $\boxed{C_A > C_C > C_B}$. Hence, the correct option is (A).

Quick Tip to Solve: For mixed dielectric pictures, identify complete paths between the plates: combine layers along each path in series, then add side-by-side paths in parallel.

33. Two identical circular loops P and Q each of radius r are lying in parallel planes such that they have common axis. The current through P and Q are I and $4I$ respectively in clockwise direction as seen from O. The net magnetic field at O is : [4 marks]



(A) $\frac{3\mu_0 I}{4\sqrt{2}r}$ towards P

(B) $\frac{3\mu_0 I}{4\sqrt{2}r}$ towards Q

(C) $\frac{\mu_0 I}{4\sqrt{2}r}$ towards P

(D) $\frac{\mu_0 I}{4\sqrt{2}r}$ towards Q

Correct Answer: (B) $\frac{3\mu_0 I}{4\sqrt{2}r}$ towards Q

Solution:

Concept:

- The magnetic field at axial distance x from a circular loop of radius r carrying current I has magnitude $B = \mu_0 I r^2/[2(r^2+x^2)^{3/2}]$.
- The right-hand rule determines the direction. A loop appearing clockwise to an observer produces an axial magnetic field directed away from that observer and towards the loop. Magnetic fields add as vectors.

Step 1: Calculate the two field magnitudes The diagram places O a distance r from the centre of each loop. For P, substitute $x = r$ and current I:

$$B_P = \frac{\mu_0 I r^2}{2(r^2+r^2)^{3/2}} = \frac{\mu_0 I}{4\sqrt{2}\,r}.$$

Q has identical geometry and four times the current, so

$$B_Q = 4B_P = \frac{4\mu_0 I}{4\sqrt{2}\,r}.$$

Step 2: Determine the directions from O When O looks towards P, its current appears clockwise, so P produces a field at O directed towards P. When O looks the other way towards Q, the current there also appears clockwise, so its field points towards Q. Thus the two field vectors oppose one another. Clockwise as seen from O does not mean that both loop magnetic moments point in the same spatial direction.

Step 3: Subtract and select the stronger direction Q produces the larger field. Therefore

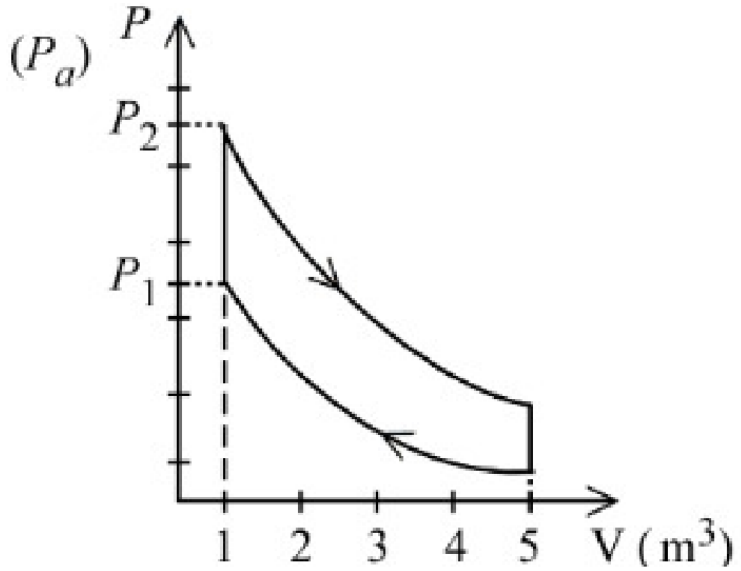
$$B_{\text{net}} = B_Q - B_P = \frac{(4-1)\mu_0 I}{4\sqrt{2}\,r} = \frac{3\mu_0 I}{4\sqrt{2}\,r},$$

directed towards Q. The choice with the same magnitude towards P has the wrong direction; both choices with coefficient one fail to subtract the fields of currents $4I$ and I correctly.

Step 4: Conclusion The net magnetic field is $\boxed{\frac{3\mu_0 I}{4\sqrt{2}\,r}}$, directed towards Q. Hence, the correct option is (B).

Quick Tip to Solve: For loops on opposite sides of an observer, apply the right-hand rule separately to each view before adding the axial fields; identical apparent rotation can give opposite fields.

34. 10 mole of an ideal gas is undergoing the process shown in the figure. The heat involved in the process from P_1 to P_2 is α Joule ($P_1 = 21.7$ Pa and $P_2 = 30$ Pa, $C_v = 21$ J/K.mol, $R = 8.3$ J/mol.K). The value of α is ____ . [4 marks]



(A) 15

(B) 21

(C) 24

(D) 28

Correct Answer: (B) 21

Solution:

Concept:

- The ideal-gas equation is $PV = nRT$. At fixed volume, $\Delta T = V\Delta P/(nR)$.
- The first law in the work-done-by-gas convention is $Q = \Delta U + W$. For an ideal gas, $\Delta U = nC_v\Delta T$, and a constant-volume path has $W = \int P\,dV = 0$.

Step 1: Read the requested part of the graph The transition from P_1 to P_2 is the vertical segment at $V = 1\,\mathrm{m}^3$. The curved branches and the segment at $V = 5\,\mathrm{m}^3$ belong to other parts of the cycle. The requested heat is for the vertical pressure rise only, so the volume does not change and no boundary work is done.

Step 2: Find the temperature increase The pressure increases by

$$\Delta P = 30 - 21.7 = 8.3\,\mathrm{Pa}.$$

With $n = 10\,\mathrm{mol}$ and $R = 8.3\,\mathrm{J\,mol^{-1}\,K^{-1}}$,

$$\Delta T = \frac{V\Delta P}{nR} = \frac{1 \times 8.3}{10 \times 8.3} = 0.1\,\mathrm{K}.$$

The units are consistent because $1\,\mathrm{Pa\,m^3} = 1\,\mathrm{J}$.

Step 3: Apply the first law and compare the choices Since $W = 0$, all the supplied heat increases internal energy:

$$Q = nC_v\Delta T = 10 \times 21 \times 0.1 = 21\,\mathrm{J}.$$

Thus $\alpha = 21$. The values 15, 24 and 28 do not satisfy $Q = (C_v/R)V\Delta P$ for the stated data. Using the area enclosed by the full cycle would answer a different question.

Step 4: Conclusion The gas absorbs $\boxed{Q = 21\,\mathrm{J}}$, so $\boxed{\alpha = 21}$. Hence, the correct option is (B).

Quick Tip to Solve: A vertical segment on a P-V plot has zero work: directly use $Q = (C_v/R)V(P_2 - P_1)$ for an ideal gas.

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35. In a vernier callipers, 50 vernier scale divisions are equal to 48 main scale divisions. If one main scale division $= 0.05$ mm, then the least count of the vernier callipers is ____ mm. [4 marks]

(A) 0.02

(B) 0.002

(C) 0.05

(D) 0.005

Correct Answer: (B) 0.002

Solution:

Concept:

- For a direct vernier, the least count is the difference between one main-scale division (MSD) and one vernier-scale division (VSD): $\mathrm{LC} = 1\,\mathrm{MSD} - 1\,\mathrm{VSD}$.
- The given equality is a length comparison of complete sets of divisions. First divide by the number of vernier divisions to find the length of one VSD, retaining millimetres throughout.

Step 1: Convert the division equality to lengths The length of one main-scale division is $0.05\,\mathrm{mm}$.

Hence the span of 48 such divisions is

$$48\,\text{MSD} = 48 \times 0.05 = 2.4\,\text{mm}.$$

This is the same span as 50 vernier divisions, so

$$1\,\text{VSD} = \frac{2.4}{50} = 0.048\,\text{mm}.$$

Step 2: Subtract the two single-division lengths The vernier divisions are slightly shorter than the main-scale divisions. Their difference is

$$\text{LC} = 0.05 - 0.048 = 0.002\,\text{mm}.$$

Equivalently, the fractional difference is $1 - 48/50 = 2/50$, giving $\text{LC} = (2/50)(0.05\,\text{mm})$. It is not appropriate to use $\text{MSD}/50$, since that shortcut would require 50 VSD to equal 49 MSD.

Step 3: Check the scale and exclude the alternatives The least count must be much smaller than the $0.05\,\text{mm}$ main-scale division. The option $0.05\,\text{mm}$ is the main-scale interval itself; $0.02\,\text{mm}$ is ten times the calculated difference, and $0.005\,\text{mm}$ also disagrees with the exact division equality. Only $0.002\,\text{mm}$ reproduces the measured difference.

Step 4: Conclusion The least count is $\boxed{0.002\,\text{mm}}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the actual division relation to calculate one VSD; the familiar MSD/N shortcut applies only when N VSD equal $N - 1$ MSD.

36. A flexible chain of mass m hangs between two fixed points at the same level. The inclination of the chain with the horizontal at the two points of support is $30°$. Considering the equilibrium of each half of the chain, the tension of the chain at the lowest point is ____ . [4 marks]

(A) $\frac{\sqrt{3}}{2}mg$
(B) $\sqrt{3}mg$
(C) $\frac{1}{2}mg$
(D) mg

Correct Answer: (A) $\frac{\sqrt{3}}{2}mg$

Solution:

Concept:

- For a flexible chain in equilibrium, tension acts along its local tangent. At the lowest point, the tangent and hence the tension are horizontal.
- Symmetry of the chain hanging between equal-level supports gives half of its weight, $mg/2$, to each half. Resolve the forces on one half horizontally and vertically.

Step 1: Choose one half as the system Consider the half-chain between the lowest point and the

right support. Its forces are its downward weight $mg/2$, the support-end tension T_s inclined at 30° to the horizontal, and the horizontal tension T_0 at the lowest point, acting towards the left. No vertical component of T_0 exists because the tangent is horizontal there.

Step 2: Balance the vertical forces Only the support tension provides an upward component. Vertical equilibrium requires

$$T_s \sin 30^\circ = \frac{mg}{2}.$$

Since $\sin 30^\circ = 1/2$,

$$T_s = mg.$$

This is the tension at a support, not the tension at the lowest point.

Step 3: Balance the horizontal forces and test the options Horizontal equilibrium gives

$$T_0 = T_s \cos 30^\circ = mg\frac{\sqrt{3}}{2}.$$

Alternatively, divide the horizontal and vertical balances to obtain $T_0 = (mg/2)\cot 30^\circ$. The value mg confuses the support tension with its horizontal component; $mg/2$ is the half-chain weight, and $\sqrt{3}mg$ incorrectly uses the full-chain weight in the half-chain balance.

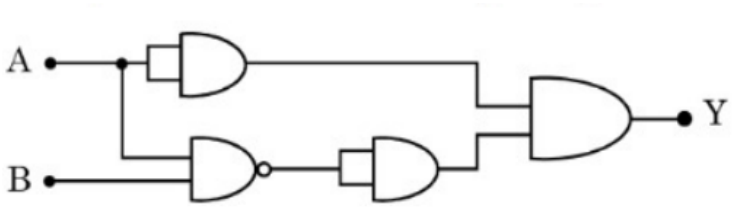
Step 4: Conclusion The tension at the lowest point is $\boxed{T_0 = \frac{\sqrt{3}}{2}mg}$. Hence, the correct option is (A).

Quick Tip to Solve: At the bottom of a symmetric hanging chain, tension is horizontal. For support angle θ, equilibrium of one half gives $T_0 = (mg/2)\cot\theta$.

37.

Identify the correct truth table of the given logic circuit.

[4 marks]



(A)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(B)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(C)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	0

(D)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Correct Answer: (C)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	0

Solution:

Concept:

- An AND gate outputs 1 only when both its inputs are 1; a NAND gate gives the complement of their product.
- Connecting both inputs of an AND gate to the same signal leaves that signal unchanged: $X \cdot X = X$.

Boolean multiplication represents AND.

Step 1: Trace the two branches The top gate has both inputs connected to A, so its output is $A \cdot A = A$. The lower left gate takes A and B and has an output bubble, so it is a NAND gate with output $X = \overline{AB}$. Both inputs of the next AND gate receive X, giving $X \cdot X = X$. Thus that gate does not invert the NAND output.

Step 2: Simplify the final output The final gate is an AND gate, receiving A from above and $\overline{AB}$ from below. Therefore, using De Morgan's law,

$$Y = A\overline{AB} = A(\overline{A} + \overline{B}) = A\overline{A} + A\overline{B} = A\overline{B}.$$

The output can be 1 only when A is 1 and B is 0.

Step 3: Evaluate all input combinations For $(A, B) = (0,0), (0,1), (1,0), (1,1)$, the respective values of $A\overline{B}$ are $0, 0, 1, 0$. In the printed order, option (A) has output 1 for input 01 and so is incorrect. Options (B) and (D) both have output 1 for input 00, contradicting the final AND gate's zero A input. Only option (C) gives all four required outputs.

Step 4: Conclusion The required output column is $0, 0, 1, 0$, since $Y = A\overline{B}$. Hence, the correct option is (C). The selected table follows the printed option order; its output is determined directly from the circuit.

Quick Tip to Solve: Collapse an AND gate with tied inputs to a wire, then write a Boolean expression before making the truth table.

---

38. A moving coil galvanometer of resistance 100Ω shows a full scale deflection for a current of 1 mA . The value of resistance required to convert this galvanometer into an ammeter, showing full scale deflection for a current of 5 mA , is ____ Ω [4 marks]

(A) 2.5

(B) 0.5

(C) 10

(D) 25

Correct Answer: (D) 25

Solution:

Concept:

- An ammeter is made by connecting a low shunt resistance S in parallel with a galvanometer of resistance G; the two branches have equal potential differences.
- At full scale, the galvanometer carries I_g while the shunt carries $I - I_g$, where I is the desired total range. Hence $I_g G = (I - I_g)S$.

Step 1: Find the shunt current The specified 5 mA is the total current through the converted ammeter, not the current in its shunt. The galvanometer must still carry only its full-scale current of 1 mA. Consequently,

$$I_s = I - I_g = 5 - 1 = 4\,\text{mA}.$$

A series resistor would reduce current in both components and would be used for a voltmeter, so a parallel connection is essential.

Step 2: Equate the branch voltages The galvanometer voltage at full scale is

$$V_g = I_g G = (1 \times 10^{-3})(100) = 0.100\,\text{V}.$$

The shunt must carry 4 mA at this same voltage:

$$S = \frac{0.100}{4 \times 10^{-3}} = 25\,\Omega.$$

Equivalently, $S = G/(I/I_g - 1) = 100/(5 - 1)$.

Step 3: Check the proposed values The desired range is five times the galvanometer range, so the shunt must carry four times its current; this requires its resistance to be one-quarter of 100 ohms. Shunts of 2.5, 0.5 and 10 ohms would instead carry 40, 200 and 10 mA respectively at 0.100 V. None would give the required total current of 5 mA.

Step 4: Conclusion The required parallel shunt is $25\,\Omega$, which is fourth in the printed option order. Hence, the correct option is (D).

Quick Tip to Solve: For a range multiplier $n = I/I_g$, use $S = G/(n - 1)$; subtract the galvanometer current before finding the shunt current.

39. A point source is kept at the center of a spherically enclosed detector. If the volume of the detector increased by 8 times, the intensity will [4 marks]

(A) increase by 8 times

(B) increase by 64 times

(C) decrease by 8 times

(D) decrease by 4 times

Correct Answer: (D) decrease by 4 times

Solution:

Concept:

- For an isotropic point source of fixed power P, intensity at radius r is power per unit spherical area: $I = P/(4\pi r^2)$.
- The volume enclosed by a sphere is $V = 4\pi r^3/3$; intensity therefore varies as $V^{-2/3}$, not as inverse volume.

Step 1: Translate the volume change into a radius change Let the original and enlarged detector radii be r_1 and r_2. The stated eightfold volume means $V_2 = 8V_1$. Cancelling the common factor $4\pi/3$,

$$\left(\frac{r_2}{r_1}\right)^3 = 8, \qquad \frac{r_2}{r_1} = 2.$$

The detector surface is twice as far from the central point source after the enlargement.

Step 2: Compare the power per unit area The emitted power is unchanged, but it spreads over a greater area. The ratio of detector surface areas is

$$\frac{A_2}{A_1} = \left(\frac{r_2}{r_1}\right)^2 = 4.$$

It follows that

$$\frac{I_2}{I_1} = \frac{P/A_2}{P/A_1} = \frac{1}{4}.$$

This is a decrease by a factor of four. The total power intercepted by the enclosing detector need not decrease; the question concerns its intensity per unit area.

Step 3: Eliminate the other options Options (A) and (B) predict increased intensity despite the increased distance from a source of fixed power. Option (C) incorrectly uses inverse volume scaling and predicts an eightfold decrease. Radiation spreads through the enclosing spherical surface, so its area, rather than its enclosed volume, determines intensity.

Step 4: Conclusion The new intensity is $I_2 = I_1/4$, a decrease by four times. Hence, the correct option is (D).

Quick Tip to Solve: For spherical spreading, first cube-root the volume factor to get the radius factor, then square and invert it for intensity.

40. Five persons P_1, P_2, P_3, P_4 and P_5 recorded object distance (u) and image distance (v) using same convex lens having power +5 D as $(25, 96), (30, 62), (35, 37), (45, 35)$ and $(50, 32)$ respectively. Identify correct statement [4 marks]

(A) Readings recorded by all persons are correct

(B) Readings recorded by P_3 person are incorrect

(C) Readings recorded by P_3 and P_2 persons are incorrect

(D) Readings recorded by P_4 and P_5 persons are incorrect

Correct Answer: (B) Readings recorded by P_3 person are incorrect

Solution:

Concept:

- The power P of a lens in dioptres equals the reciprocal of its focal length in metres: $f = 1/P$.
- For a real object and a real image, using the positive measured distances U and V gives $1/f = 1/U + 1/V$, or $f = UV/(U + V)$.

Step 1: Find the expected focal length The same lens has power $+5\,\mathrm{D}$, so

$$f = \frac{1}{5}\,\mathrm{m} = 20\,\mathrm{cm}.$$

The tabulated positive distances are read as distance magnitudes. In the Cartesian convention the object coordinate is negative, which is why the reciprocal magnitudes add in this formula.

Step 2: Check each recorded pair The focal lengths inferred from the five measured pairs, in centimetres, are

$$f_1 = \frac{25 \times 96}{25 + 96} = 19.83, \quad f_2 = \frac{30 \times 62}{30 + 62} = 20.22,$$
$$f_3 = \frac{35 \times 37}{35 + 37} = 17.99, \quad f_4 = \frac{45 \times 35}{45 + 35} = 19.69,$$
$$f_5 = \frac{50 \times 32}{50 + 32} = 19.51.$$

The first, second, fourth and fifth readings cluster close to 20 cm, whereas the third gives approximately 18 cm.

Step 3: Identify the inconsistent observation The printed data are experimental readings, so small measurement deviations are present rather than exact ideal pairs. The conspicuous outlier is P_3: at $U = 35\,\text{cm}$, the lens predicts

$$V = \frac{fU}{U - f} = \frac{20 \times 35}{35 - 20} = 46.67\,\text{cm},$$

far from the recorded 37 cm. Thus all readings cannot be accepted, but there is no comparable discrepancy requiring rejection of P_2, or of P_4 and P_5. Those statements eliminate options (A), (C) and (D) in the printed order.

Step 4: Conclusion On comparing the experimental readings with the 20 cm focal length, P_3 is the inconsistent observation. Hence, the correct option is (B).

Quick Tip to Solve: Calculate $UV/(U + V)$ for each observation; comparing inferred focal lengths makes a bad experimental reading easy to spot.

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41. In the Young's double slit experiment the intensity produced by each one of the individual slits is I_o. The distance between two slits is 2 mm . The distance of screen from slits is 10 m . The wavelength of light is 6000 A°. The intensity of light on the screen in front of one of the slits is ____ [4 marks]

(A) $\frac{I_o}{2}$

(B) I_o

(C) $2I_o$

(D) $4I_o$

Correct Answer: (B) I_o

Solution:

Concept:

- For two coherent slits each producing intensity I_0, their resultant intensity at phase difference ϕ is $I = 2I_0(1 + \cos\phi) = 4I_0\cos^2(\phi/2)$.
- At a screen point at transverse distance y from the central axis, the small-angle path difference is $\Delta = dy/D$, and $\phi = 2\pi\Delta/\lambda$.

Step 1: Locate the observation point correctly The central axis passes halfway between the slits. A

point directly in front of either slit is therefore displaced by half their separation, rather than by the whole separation:

$$y = \frac{d}{2} = 1 \times 10^{-3}\,\text{m}.$$

Here $d = 2 \times 10^{-3}\,\text{m}$, $D = 10\,\text{m}$, and $\lambda = 6000 \times 10^{-10} = 6 \times 10^{-7}\,\text{m}$.

Step 2: Calculate the phase difference Using the given geometry,

$$\Delta = \frac{dy}{D} = \frac{(2 \times 10^{-3})(1 \times 10^{-3})}{10} = 2 \times 10^{-7}\,\text{m} = \frac{\lambda}{3}.$$

Thus

$$\phi = \frac{2\pi\Delta}{\lambda} = \frac{2\pi}{3}.$$

The screen is very far away compared with the slit separation, so the small-angle interference expression is applicable.

Step 3: Evaluate the intensity and reject the alternatives Substituting the phase difference gives

$$I = 2I_0\left(1 + \cos\frac{2\pi}{3}\right) = 2I_0\left(1 - \frac{1}{2}\right) = I_0.$$

The value $4I_0$ requires zero phase difference modulo 2π, while $2I_0$ requires a zero cosine. Neither condition occurs here. The value $I_0/2$ is also inconsistent with the calculated cosine of $-1/2$.

Step 4: Conclusion The intensity directly in front of either slit is I_0. Hence, the correct option is (B).

Quick Tip to Solve: In front of one slit means $y = d/2$ from the central axis; use this displacement before finding the phase difference.

---

42. A cubical block of density $\rho_b = 600\ \text{kg/m}^3$ floats in a liquid of density $\rho_e = 900\text{kg/m}^3$. If the height of block is $H = 8.0\ \text{cm}$ then height of the submerged part is

____ cm . [4 marks]

(A) 4.3

(B) 5.3

(C) 6.3

(D) 7.3

Correct Answer: (B) 5.3

Solution:

Concept:

- A freely floating body is in vertical equilibrium: its weight equals the buoyant force, which is the weight of the displaced liquid.
- For a uniform block with constant horizontal cross-sectional area A, total height H and submerged height h, the balance is $\rho_b AHg = \rho_e Ahg$.

Step 1: Express the weight and buoyant force The whole block contributes to its mass, so its weight is

$W = \rho_b AHg$. Only the submerged part displaces liquid, so the upward buoyant force is $F_B = \rho_e Ahg$. It would be incorrect to use the full block volume in the buoyancy expression, because the block floats with part of its height above the liquid surface.

Step 2: Use floating equilibrium Setting the two forces equal and cancelling their common factors A and g gives

$$\rho_b H = \rho_e h, \qquad \frac{h}{H} = \frac{\rho_b}{\rho_e}.$$

The density ratio is $600/900 = 2/3$, so two-thirds of the block volume, and therefore two-thirds of its height, must be immersed.

Step 3: Substitute the dimensions and compare the options The required height is

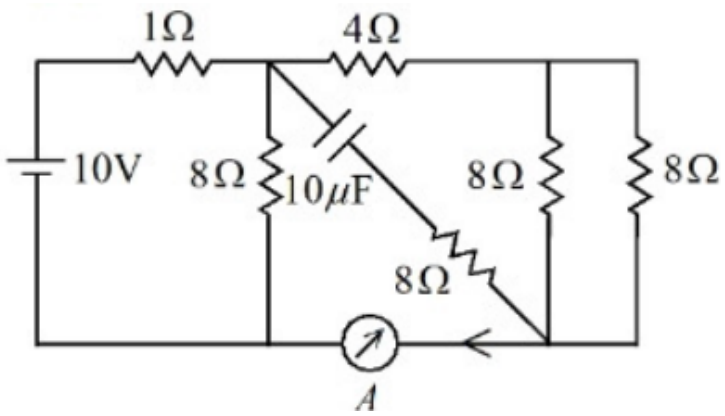
$$h = \frac{600}{900} \times 8.0 = \frac{16}{3}\,\text{cm} = 5.333\ldots\,\text{cm} \approx 5.3\,\text{cm}.$$

This is below the full height of 8.0 cm, consistent with the block being less dense than the liquid. Options 4.3, 6.3 and 7.3 cm would make the displaced-liquid weight respectively too small, too large and too large to balance the specified block weight. The exposed height would be about 2.7 cm, which is not the quantity asked for.

Step 4: Conclusion The height of the submerged part is $5.3\,\text{cm}$. Hence, the correct option is (B).

Quick Tip to Solve: For a uniform prism floating upright, the submerged height fraction equals the density ratio; distinguish immersed height from freeboard.

43. The reading of the ammeter (A) in steady state in the following circuit (assuming negligible internal resistance of the ammeter) is ____ A. [4 marks]



(A) 1

(B) 1/2

(C) 2

(D) 0

Correct Answer: (A) 1

Solution:

Concept:

- In a direct-current steady state, a charged ideal capacitor carries no current. Its branch is an open circuit when calculating the ammeter reading.
- Series resistances add, while two equal resistances in parallel have half the individual resistance. The ammeter measures only the current in its own wire.

Step 1: Remove the capacitive branch The diagonal branch contains the $10\,\mu\text{F}$ capacitor in series with an $8\,\Omega$ resistor. Its steady current is zero, so that resistor provides no additional conducting path. The two right-hand $8\,\Omega$ resistors are in parallel:

$$R_{\text{right}} = \frac{8 \times 8}{8 + 8} = 4\,\Omega.$$

Step 2: Reduce the conducting network This combination is in series with the upper $4\,\Omega$ resistor, giving $8\,\Omega$. That entire path is in parallel with the left vertical $8\,\Omega$ resistor. Hence the network after the $1\,\Omega$ resistor has resistance $4\,\Omega$. The battery sees

$$R_{\text{total}} = 1 + 4 = 5\,\Omega, \qquad I_{\text{battery}} = \frac{10}{5} = 2\,\text{A}.$$

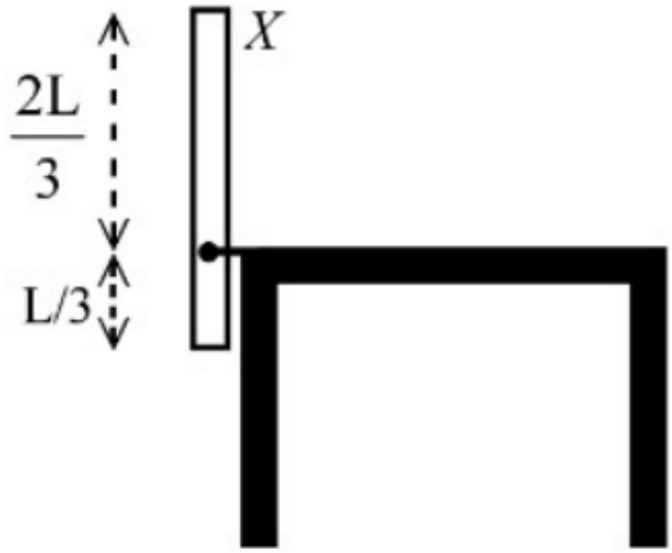
Step 3: Find the current through the meter The current divides equally between the two $8\,\Omega$ paths, so each receives $1\,\text{A}$. The ammeter lies in the return wire of the right-hand path and reads $1\,\text{A}$. Each individual right-hand resistor carries $1/2\,\text{A}$, but these currents combine before passing through the meter. Thus $1/2\,\text{A}$ is an individual branch current, $2\,\text{A}$ is the battery current, and zero applies only to the capacitive branch.

Step 4: Conclusion The ammeter reading is $\boxed{1\,\text{A}}$. Hence, the correct option is (A).

Quick Tip to Solve: Open the capacitor branch first, then trace the meter's own wire; it need not carry the total battery current.

44. A thin uniform rod (X) of mass M and length L is pivoted at a height $\left(\frac{L}{3}\right)$ as shown in the figure. The rod is allowed to fall from a vertical position and lie horizontally on the table. The angular velocity of this rod when it hits the table top, is ____ .

($g =$ gravitational acceleration) [4 marks]



(A) $\sqrt{\frac{3g}{2L}}$

(B) $\frac{3}{\sqrt{2}}\sqrt{\frac{g}{L}}$

(C) $\sqrt{\frac{3g}{L}}$

(D) $\frac{1}{\sqrt{2}}\sqrt{\frac{g}{L}}$

Correct Answer: (C) $\sqrt{\frac{3g}{L}}$

Solution:

Concept:

- A uniform rod has its centre of mass at its midpoint and central transverse moment of inertia $I_{\text{cm}} = ML^2/12$.
- About a fixed pivot at distance d from the centre, $I_p = I_{\text{cm}} + Md^2$. The loss of gravitational potential energy becomes rotational kinetic energy before impact.

Step 1: Locate the centre of mass relative to the pivot The diagram places the pivot $L/3$ above the lower end, leaving $2L/3$ of the rod above it. The centre of mass is $L/2$ from the lower end, so initially it lies above the pivot by

$$d = \frac{L}{2} - \frac{L}{3} = \frac{L}{6}.$$

When the rod is horizontal, its centre is at the pivot's level. Its vertical fall is therefore $L/6$, not $L/2$ or $L/3$.

Step 2: Calculate the moment of inertia about the pivot The parallel-axis theorem gives

$$I_p = \frac{ML^2}{12} + M\left(\frac{L}{6}\right)^2 = ML^2\left(\frac{3+1}{36}\right) = \frac{ML^2}{9}.$$

The fixed pivot supplies a constraint force but does no work because its point of application does not move.

Step 3: Apply conservation of energy before contact Starting from rest and taking the angular speed immediately before the rod strikes the table, we obtain

$$Mg\frac{L}{6} = \frac{1}{2}\frac{ML^2}{9}\omega^2, \qquad \omega^2 = \frac{3g}{L}.$$

The mass cancels as expected. Squaring the other options gives $3g/(2L)$, $9g/(2L)$, and $g/(2L)$; none satisfies this energy equation. Impact losses occur after this instant and do not enter the calculation.

Step 4: Conclusion The angular velocity just before impact is $\boxed{\omega = \sqrt{3g/L}}$, in $\mathrm{rad\,s^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: Measure the centre-of-mass height from the actual pivot; the same offset enters both the energy loss and the parallel-axis theorem.

45. When a light of a given wavelength falls on a metallic surface the stopping potential for photoelectrons is 3.2 V . If a second light having wavelength twice of first light is used, the stopping potential drops to 0.7 V . The wavelength of first light is ____ m .

$$\left(\mathrm{h} = 6.63 \times 10^{-34}\ \mathrm{J.s}, \mathrm{e} = 1.6 \times 10^{-19}\mathrm{C}, \mathrm{c} = 3 \times 10^{8}\ \mathrm{m/s}\right)$$

[4 marks]

(A) 3.1×10^{-7}

(B) 2.5×10^{-7}

(C) 2.9×10^{-8}

(D) 2.2×10^{-8}

Correct Answer: (B) 2.5×10^{-7}

Solution:

Concept:

- Einstein's photoelectric equation is $eV_s = hc/\lambda - \phi$, where V_s is the stopping potential and ϕ is the metal's work function.
- Doubling the incident wavelength halves the photon energy. Subtracting equations for the same metal eliminates its unchanged work function.

Step 1: Write the two photoelectric equations Let the first wavelength be λ. For the given stopping

potentials, the maximum photoelectron kinetic energies give

$$e(3.2\,\mathrm{V}) = \frac{hc}{\lambda} - \phi, \qquad e(0.7\,\mathrm{V}) = \frac{hc}{2\lambda} - \phi.$$

The second wavelength is 2λ, so its photon energy is smaller; this agrees with its smaller stopping potential.

Step 2: Eliminate the work function and calculate the wavelength Subtraction yields

$$e(2.5\,\mathrm{V}) = \frac{hc}{2\lambda}, \qquad \lambda = \frac{hc}{2e(2.5\,\mathrm{V})}.$$

Substituting the stated constants,

$$\lambda = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{2(1.6 \times 10^{-19})(2.5)} = 2.48625 \times 10^{-7}\,\mathrm{m}.$$

Step 3: Check the energy scale and select the option The first photon has energy $5.0\,\mathrm{eV}$, so the work function is $5.0 - 3.2 = 1.8\,\mathrm{eV}$. The second photon then has $2.5\,\mathrm{eV}$, leaving $0.7\,\mathrm{eV}$, as required. The value $3.1 \times 10^{-7}\,\mathrm{m}$ would give an energy difference of about $2.0\,\mathrm{eV}$; the two options of order $10^{-8}\,\mathrm{m}$ give much larger differences. Only the wavelength near $2.5 \times 10^{-7}\,\mathrm{m}$ fits both observations.

Step 4: Conclusion The first wavelength is $\boxed{2.5 \times 10^{-7}\,\mathrm{m}}$ to the precision of the options. Hence, the correct option is (B).

Quick Tip to Solve: For two wavelengths on the same metal, subtract the stopping-potential equations before substituting constants; the work function cancels.

Physics – Section B: Numerical Value Questions (20 marks)

46. A soap bubble of surface tension $0.04\ \mathrm{N/m}$ is blown to a diameter of 7 cm . If $(15000 - x)\mu\mathrm{J}$ of work is done in blowing it further to make its diameter 14 cm , then the value of x is ____ .

$$(\pi = 22/7)$$

[4 marks]

Solution:

Concept:

- Surface tension T is also surface energy per unit area. A soap bubble has two liquid-air surfaces, so its total area is $2(4\pi r^2) = 8\pi r^2$.
- The work required to increase the bubble's size is the increase in its surface energy: $W = 8\pi T(r_2^2 - r_1^2)$, with both radii measured in metres.

Step 1: Convert the diameters to radii The initial diameter is $7\,\mathrm{cm}$, and the final diameter is $14\,\mathrm{cm}$.

Their radii are therefore

$$r_1 = 3.5\,\text{cm} = 0.035\,\text{m}, \qquad r_2 = 7\,\text{cm} = 0.070\,\text{m}.$$

Only the additional expansion is relevant: the energy used to form the initial bubble is not part of the requested work.

Step 2: Calculate the increase in surface energy With $T = 0.04\,\text{N m}^{-1}$ and the specified $\pi = 22/7$,

$$\begin{aligned} W &= 8\left(\frac{22}{7}\right)(0.04)\left[(0.070)^2 - (0.035)^2\right] \\ &= 8\left(\frac{22}{7}\right)(0.04)(0.003675) \\ &= 0.003696\,\text{J} = 3696\,\mu\text{J}. \end{aligned}$$

The units are consistent: surface tension times area is $\text{N m} = \text{J}$. Doubling the radius makes the final area four times the initial area, so the additional area is three times the initial area of both surfaces.

Step 3: Equate the work to the given expression Since the given expression already uses microjoules, compare its numerical coefficient with 3696:

$$15000 - x = 3696, \qquad x = 15000 - 3696 = 11304.$$

No extra factor of two is needed here; the two surfaces were already included in the energy formula.

Step 4: Conclusion The required numerical value is $\boxed{x = 11304}$, dimensionless in the coefficient $(15000 - x)\,\mu\text{J}$. The additional work is $3696\,\mu\text{J}$.

Quick Tip to Solve: For a soap bubble, count both surfaces and use radii, not diameters; compute the change of squared radius before converting the final energy to microjoules.

47. A uniform solid cylinder of length L and radius R has moment of inertia about its axis equal to I_1. A small co-centric cylinder of length $L/2$ and radius $R/3$ carved from this cylinder has moment of inertia about its axis equals to I_2. The ratio I_1/I_2 is ____ . [4 marks]

Solution:

Concept:

- A uniform solid cylinder of mass M and radius R has moment of inertia $I = MR^2/2$ about its own longitudinal symmetry axis.
- For cylinders made of the same uniform material, mass is proportional to volume, $M = \rho\pi R^2 L$. Changing the radius affects both mass and the squared lever arm.

Step 1: Find the mass of the carved cylinder Let the original cylinder have mass M, and let the smaller cylinder have mass m. Both have the same density because the smaller one is carved from the

original material. Using its length $L/2$ and radius $R/3$,

$$\frac{m}{M} = \frac{\pi(R/3)^2(L/2)}{\pi R^2 L} = \frac{1}{9} \times \frac{1}{2} = \frac{1}{18}.$$

Thus the small cylinder has only $M/18$ of the original mass; its mass cannot be treated as unchanged.

Step 2: Write the two moments of inertia about their axes The moment of inertia of the original cylinder is

$$I_1 = \frac{1}{2}MR^2.$$

The carved solid cylinder itself, rather than the material left behind, is the body whose moment is I_2. Therefore

$$I_2 = \frac{1}{2}\left(\frac{M}{18}\right)\left(\frac{R}{3}\right)^2 = \frac{MR^2}{324}.$$

The axes are concentric, so there is no parallel-axis correction. Shortening the cylinder affects this axial moment only through its mass.

Step 3: Form and verify the ratio Dividing the two expressions gives

$$\frac{I_1}{I_2} = \frac{MR^2/2}{MR^2/324} = 162.$$

Equivalently, the axial moment scales as LR^4 for fixed density. Halving L and reducing R by a factor of three gives $I_2/I_1 = (1/2)(1/3)^4 = 1/162$, independently checking the result.

Step 4: Conclusion The required ratio is $\boxed{I_1/I_2 = 162}$. It is dimensionless because both moments of inertia have units $\mathrm{kg\,m^2}$.

Quick Tip to Solve: For a solid cylinder of fixed density, remember the axial scaling $I \propto LR^4$; radius changes both its mass and its rotational lever arm.

---

48. In a meter bridge experiment to determine the value of unknown resistance, first the resistances 2Ω and 3Ω are connected in the left and right gaps of the bridge and the null point is obtained at a distance l cm from the left. Now when an unknown resistance $x\Omega$ is connected in parallel to 3Ω resistance, the null point is shifted by 10 cm to the right of wire. The value of unknown resistance x is ____ Ω. [4 marks]

Solution:

Concept:

- For a uniform metre-bridge wire at balance, the ratio of the gap resistances equals the ratio of the corresponding wire lengths: $R_{\mathrm{L}}/R_{\mathrm{R}} = l/(100-l)$, with l measured from the left end in centimetres.
- A resistance x in parallel with $3\,\Omega$ gives $R_{\mathrm{R}} = 3x/(3+x)$. Adding a parallel path lowers the right-gap resistance.

Step 1: Find the initial balance length Initially the left gap has $2\,\Omega$ and the right gap has $3\,\Omega$. Apply

the balance condition using the length from the left:

$$\frac{2}{3} = \frac{l}{100 - l}, \qquad 200 - 2l = 3l, \qquad l = 40\,\text{cm}.$$

The remaining right-hand length is $60\,\text{cm}$, consistent with the resistance ratio $2 : 3$.

Step 2: Use the shifted null point The null point shifts $10\,\text{cm}$ to the right, so the new left-hand length is $40 + 10 = 50\,\text{cm}$. Both wire segments now have equal lengths. Consequently the new right-gap equivalent resistance must equal the unchanged left-gap resistance:

$$\frac{2}{R_{\text{R}}} = \frac{50}{100 - 50} = 1, \qquad R_{\text{R}} = 2\,\Omega.$$

This is smaller than its original value of $3\,\Omega$, as a parallel connection requires.

Step 3: Solve for the added resistance Taking x as the numerical value in ohms, the parallel-resistance relation becomes

$$\frac{3x}{3 + x} = 2, \qquad 3x = 6 + 2x, \qquad x = 6.$$

As a check, $3\,\Omega$ in parallel with $6\,\Omega$ gives $18/9 = 2\,\Omega$, placing the null point exactly at the midpoint. A shift to the left would give the wrong sign of change in the right-gap resistance.

Step 4: Conclusion The unknown resistance is $\boxed{6\,\Omega}$, so the required numerical entry is $x = 6$.

Quick Tip to Solve: A midpoint balance means equal gap resistances. Track the direction of the null-point shift before applying the parallel-resistance formula.

---

49. When 300 J of heat given to an ideal gas with $C_p = \frac{7}{2}R$ its temperature raises from $20°\text{C}$ to $50°\text{C}$ keeping its volume constant. The mass of the gas is (approximately) ____ g. $(\text{R} = 8.314\ \text{J/mol.K})$

[4 marks]

Solution:

Concept:

- At constant volume, no expansion work is done, so the supplied heat is $Q = nC_V\Delta T$, where n is the amount in moles.
- For an ideal gas, the molar heat capacities satisfy Mayer's relation $C_P - C_V = R$. Mass requires the separate molar mass M: $m = nM$.

Step 1: Find the constant-volume heat capacity The stated $C_P = 7R/2$ is a molar heat capacity because it is expressed as a multiple of the universal gas constant. Thus

$$C_V = C_P - R = \frac{5R}{2}.$$

The temperature change is $50 - 20 = 30\,\text{K}$. A Celsius temperature difference has the same numerical value as a kelvin difference; using either absolute temperature in place of this difference would be incorrect.

Step 2: Calculate the amount of gas Apply the constant-volume heat equation rather than $Q = nC_P\Delta T$:

$$n = \frac{300}{(5/2)(8.314)(30)} = 0.4811\,\text{mol}.$$

This uniquely determines the number of moles. The heat-capacity ratio identifies the active degrees of freedom, but does not determine the chemical identity or the mass of each molecule.

Step 3: Check whether a unique mass follows If M is expressed in g mol^{-1}, the mass is

$$m = 0.4811M\,\text{g}.$$

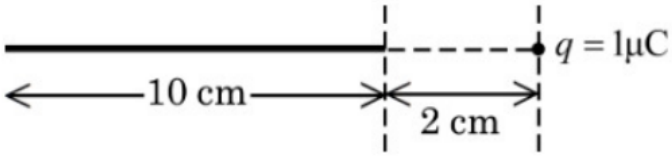
No molar mass or gas identity is supplied. For example, a gas with $M = 2\,\text{g mol}^{-1}$ would give approximately $0.962\,\text{g}$, whereas $M = 28\,\text{g mol}^{-1}$ would give approximately $13.47\,\text{g}$; both can have the stated ideal-gas heat capacity. Hence a numerical mass cannot be inferred from these data.

Step 4: Conclusion The data determine $n \approx 0.4811\,\text{mol}$, but no unique mass in grams. The printed question is underdetermined because the molar mass is missing; the official answer key dropped this question and awarded bonus credit to all candidates.

Quick Tip to Solve: A molar heat capacity converts heat into moles. Check for a molar mass before converting that result into grams.

50. A point charge $q = 1\mu\text{C}$ is located at a distance 2 cm from one end of a thin insulating wire of length 10 cm having a charge $Q = 24\mu\text{C}$, distributed uniformly along its length, as shown in figure. Force between q and wire is ____ N.

(Use : $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\ \text{N}\cdot\ \text{m}^2/\text{C}^2$) [4 marks]



Solution:

Concept:

- For a uniformly charged rod of length L, the linear charge density is $\lambda = Q/L$. A length element dx carries $dq = \lambda\,dx$.
- All elements lie on the same line as the external point charge. Their Coulomb forces therefore add algebraically in one direction: $dF = kq\,dq/x^2$.

Step 1: Express the geometry in SI units The charge lies on the extension of the rod, $a = 2\,\text{cm} = 0.02\,\text{m}$ from its near end. The rod length is $L = 0.10\,\text{m}$, so the distances of its elements from the point charge range from a to $a + L = 0.12\,\text{m}$. Also, $q = 10^{-6}\,\text{C}$ and $Q = 24 \times 10^{-6}\,\text{C}$. Thus

$$\lambda = \frac{24 \times 10^{-6}}{0.10} = 2.4 \times 10^{-4}\,\text{C m}^{-1}.$$

Step 2: Integrate the contributions along the wire Choose x as distance measured from the point charge towards the rod. Both charges are positive, so all elementary forces on q point away from the wire. Their magnitudes sum to

$$\begin{aligned} F &= kq\lambda \int_a^{a+L} \frac{dx}{x^2} \\ &= kq\lambda \left(\frac{1}{a} - \frac{1}{a+L} \right) \\ &= \frac{kqQ}{a(a+L)}. \end{aligned}$$

Treating the extended wire as a point charge at its midpoint would not be exact, since the distance varies substantially over its length.

Step 3: Substitute and evaluate the force Using the given Coulomb constant,

$$F = \frac{(9 \times 10^9)(10^{-6})(24 \times 10^{-6})}{(0.02)(0.12)} = \frac{0.216}{0.0024} = 90\,\text{N}.$$

The force on the wire is equal in magnitude and opposite in direction to the force on the point charge, by Newton's third law.

Step 4: Conclusion The force between the point charge and the wire is $\boxed{90\,\text{N}}$, repulsive. The required numerical entry is 90.

Quick Tip to Solve: For an external charge on a uniform rod's axis, use $F = kqQ/[a(a+L)]$, with both endpoint distances measured from the charge.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. In the Group analysis of cations, Ba^{2+} & Ca^{2+} are precipitated respectively as [4 marks]

(A) sulphide & sulphide

(B) hydroxide & carbonate

(C) carbonate & carbonate

(D) chromate & sulphide

Correct Answer: (C) carbonate & carbonate

Solution:

Concept:

- Qualitative group analysis separates cations by selective precipitation under specified reagent conditions, rather than by any salt that can be made in a later confirmatory test.
- Group V cations Ba^{2+}, Sr^{2+} and Ca^{2+} precipitate as carbonates with ammonium carbonate in the presence of ammonium chloride and ammonium hydroxide.

Step 1: Identify the common group reagent Barium and calcium belong to the same analytical group. The reagent $(NH_4)_2CO_3$ supplies carbonate ions; the accompanying ammonium salts establish the conditions for the group separation. Both metal ions form sparingly soluble carbonates, so their group

precipitates have the same anion.

Step 2: Write the two precipitation reactions The net ionic equations are

$$Ba^{2+}(aq) + CO_3{}^{2-}(aq) \longrightarrow BaCO_3(s)$$

and

$$Ca^{2+}(aq) + CO_3{}^{2-}(aq) \longrightarrow CaCO_3(s).$$

Each equation combines one divalent metal ion with one divalent carbonate ion to form a neutral solid. The required pair is therefore carbonate and carbonate, in the order barium then calcium.

Step 3: Eliminate the other proposed pairs Option (A) incorrectly assigns both cations to sulphide precipitation. Option (B) incorrectly makes barium hydroxide the group precipitate. Option (D) mixes a possible barium confirmatory precipitate, chromate, with an incorrect calcium sulphide group assignment. Although $BaCrO_4$ can help identify barium after separation, that does not replace $BaCO_3$ as its Group V precipitate.

Step 4: Conclusion Barium and calcium are precipitated respectively as $BaCO_3$ and $CaCO_3$, i.e. carbonate and carbonate. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish the common group reagent from the individual confirmatory test: Group V collects Ba, Sr and Ca as carbonates.

52. Given below are two statements:

Statement I: The dipole moment of R–CN is greater than R–NC and R–NC can undergo hydrolysis under acidic medium to produce R–C(=O)–OH.

Statement II: R–CN hydrolyses under acidic medium to produce a compound which on treatment with $SOCl_2$, followed by the addition of NH_3 gives another compound (x). This compound (x) on treatment with NaOCl/NaOH gives a product, that on treatment with $CHCl_3$/KOH/Δ produces R–NC.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Concept:

- Nitriles R–CN and isocyanides R–NC differ in which atom of the cyanide unit is attached to the organic group; this changes their hydrolysis products.
- Hofmann bromamide-type rearrangement with alkaline hypochlorite converts $RCONH_2$ into RNH_2, losing the carbonyl carbon. A primary amine gives an isocyanide in the carbylamine reaction.

Step 1: Evaluate Statement I as a complete claim The quoted dipole-moment comparison does not make the entire statement true, because its hydrolysis assertion is incorrect. Acidic hydrolysis of an isocyanide gives a primary amine (protonated in acid) and formic acid, rather than the stated RCOOH:

$$\mathrm{RNC} + 2\,\mathrm{H_2O} + \mathrm{H^+} \longrightarrow \mathrm{RNH_3{}^+} + \mathrm{HCOOH}.$$

The organic group remains bonded to nitrogen. Formation of RCOOH is characteristic of nitrile hydrolysis. Thus Statement I is false.

Step 2: Follow the nitrile through the acid and amide Acidic hydrolysis of the nitrile produces the carboxylic acid RCOOH. Thionyl chloride converts that acid to RCOCl; ammonia then replaces chlorine to form the primary amide:

$$\mathrm{RCN} \xrightarrow{\mathrm{H_2O/H^+}} \mathrm{RCOOH} \xrightarrow{\mathrm{SOCl_2}} \mathrm{RCOCl} \xrightarrow{\mathrm{NH_3}} \mathrm{RCONH_2}.$$

Consequently, the intermediate (x) is $\mathrm{RCONH_2}$, and its carbon skeleton is precisely the one needed for the next rearrangement.

Step 3: Complete the rearrangement and carbylamine reaction Alkaline hypochlorite converts the primary amide into $\mathrm{RNH_2}$, removing the amide carbonyl carbon. The primary amine then reacts with chloroform and alcoholic potassium hydroxide on heating:

$$\mathrm{RNH_2} + \mathrm{CHCl_3} + 3\,\mathrm{KOH} \longrightarrow \mathrm{RNC} + 3\,\mathrm{KCl} + 3\,\mathrm{H_2O}.$$

This is the stated final isocyanide, so Statement II is true. Options (A) and (C) wrongly call Statement I true; option (B) wrongly calls Statement II false.

Step 4: Conclusion Statement I is false and Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: Track the bond to R: nitrile hydrolysis keeps R bonded to carbon, whereas isocyanide hydrolysis keeps R bonded to nitrogen.

53. " X " is an oxoanion of the lightest element of group 7 (in the periodic table). The metal is in +6 oxidation state in " X ". The color of the potassium salt of X is [4 marks]

(A) green

(B) purple

(C) yellow

(D) orange

Correct Answer: (A) green

Solution:

Concept:

- The lightest element of modern group 7 is manganese, Mn; this group is distinct from the halogens of group 17.
- Manganese forms the manganate oxoanion $\mathrm{MnO_4{}^{2-}}$ in oxidation state +6 and the permanganate

oxoanion MnO_4^- in oxidation state +7. Their colours differ.

Step 1: Identify the metal and its oxoanion The group number and the word lightest identify manganese, rather than chromium. Let the metal oxidation number in MnO_4^{2-} be x. Taking each oxygen as -2,

$$x + 4(-2) = -2 \quad \Rightarrow \quad x = +6.$$

Thus the stated oxoanion X is manganate, MnO_4^{2-}, which satisfies the specified oxidation state.

Step 2: Form the potassium salt and recall its colour Two potassium ions are required to balance the dianion, giving potassium manganate, K_2MnO_4. This salt is green. The prefix manganate must be distinguished from permanganate: the latter has only one potassium ion in $KMnO_4$ because its anion carries a single negative charge.

Step 3: Check the competing colour choices Purple belongs to potassium permanganate, whose manganese is +7, so option (B) conflicts with the oxidation state in the question. Yellow and orange are associated with chromate and dichromate, respectively; those involve chromium, a group 6 element. Therefore options (C) and (D) do not meet the group-7 condition. Green alone matches both the correct element and its +6 oxoanion.

Step 4: Conclusion The potassium salt is K_2MnO_4, and its colour is green. Hence, the correct option is (A).

Quick Tip to Solve: Use oxidation state before colour: Mn(+6) gives green manganate, while Mn(+7) gives purple permanganate.

54. Choose the INCORRECT statement [4 marks]

(A) Carbon cannot exceed its covalency more than four.

(B) Carbon exhibits negative oxidation states along with +4 and +2 .

(C) Among the isotopes of carbon, ^{13}C is a radioactive isotope.

(D) CO_2 is the most acidic oxide among the dioxides of group of 14 elements.

Correct Answer: (C) Among the isotopes of carbon, ^{13}C is a radioactive isotope.

Solution:

Concept:

- An isotope has the same proton number as the element but a different neutron number. The common carbon isotopes ^{12}C and ^{13}C are stable, while ^{14}C is radioactive.
- Carbon is a second-period element with normal maximum covalency four. In group 14, increasing metallic character down the group decreases the acidic character of the dioxides.

Step 1: Check the covalency and oxidation-state statements Option (A) states the usual maximum covalency of carbon correctly: it forms at most four ordinary covalent bonds in the standard valence description. Option (B) is also correct. Carbon is +4 in CO_2, +2 in CO, and has negative oxidation states, for example -4 in CH_4. Its oxidation state therefore is not restricted to positive values.

Step 2: Identify the incorrect isotope claim Carbon has atomic number 6. The isotope ^{13}C contains

six protons and seven neutrons and is stable; it is widely encountered as a naturally occurring carbon isotope. The radioactive isotope used in radiocarbon dating is ^{14}C, with eight neutrons. Hence the assertion in option (C) that ^{13}C is radioactive is false.

Step 3: Verify the oxide trend before selecting Option (D) agrees with the group trend. Carbon dioxide is an acidic oxide and reacts with bases to form carbonates or bicarbonates. On descending group 14, metallic character increases, and the dioxides become less acidic and eventually amphoteric. Thus CO_2 is the most acidic member of this series. Since the question asks for the incorrect statement, the three true statements must be rejected as answers.

Step 4: Conclusion The incorrect statement is that ^{13}C is radioactive; it is stable, whereas ^{14}C is radioactive. Hence, the correct option is (C).

Quick Tip to Solve: For carbon isotope questions, remember stable 12 and 13 versus radioactive 14, and check whether the question asks for the incorrect claim.

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55. Two liquids A and B form an ideal solution at temperature T K . At T K , the vapour pressures of pure A and B are 55 and $15\,\mathrm{kNm^{-2}}$ respectively. What is the mole fraction of A in solution of A and B in equilibrium with a vapour in which the mole fraction of A is 0.8? [4 marks]

(A) 0.480

(B) 0.663

(C) 0.5217

(D) 0.340

Correct Answer: (C) 0.5217

Solution:

Concept:

- For an ideal binary liquid solution, Raoult's law gives partial pressures $p_A = x_A p_A^\circ$ and $p_B = (1 - x_A)p_B^\circ$, where x_A is the liquid mole fraction.
- Dalton's law gives the vapour mole fraction $y_A = p_A/(p_A + p_B)$. Liquid and vapour compositions are generally different because the two components have different volatilities.

Step 1: Express the equilibrium vapour composition Let x be the mole fraction of A in the liquid. The complementary liquid fraction of B is $1 - x$. At the stated temperature, the partial pressures and total pressure, all in $\mathrm{kN\,m^{-2}}$, are

$$p_A = 55x, \qquad p_B = 15(1 - x), \qquad P = 15 + 40x.$$

Using the given vapour fraction rather than incorrectly setting the liquid fraction to 0.8 gives

$$0.8 = \frac{55x}{15 + 40x}.$$

Step 2: Solve for the liquid fraction and check the result Cross-multiply and collect the terms

containing x:

$$12 + 32x = 55x, \qquad 23x = 12, \qquad x = \frac{12}{23} = 0.521739\ldots.$$

As a check, $p_A = 660/23$ and $p_B = 165/23$, giving $p_A/P = 660/825 = 0.8$. A is more volatile, so the vapour must be richer in A than the liquid; indeed $0.8 > 0.5217$. The alternatives 0.480, 0.663 and 0.340 do not satisfy the vapour-composition equation.

Step 3: Conclusion The mole fraction of A in the liquid solution is $x_A = 0.5217$, a dimensionless quantity. Hence, the correct option is (C).

Quick Tip to Solve: Use $x_A/(1 - x_A) = [y_A/(1 - y_A)](p_B^\circ/p_A^\circ)$ to convert vapour composition to liquid composition directly.

56. The number of possible tripeptides formed involving alanine (ala), glycine (gly) and valine (val), where no amino acid has been used more than once is: [4 marks]

(A) 3

(B) 4

(C) 6

(D) 8

Correct Answer: (C) 6

Solution:

Concept:

- A tripeptide contains three amino-acid residues joined by two peptide bonds. Its sequence is read from the free amino end (N-terminus) to the free carboxyl end (C-terminus).
- When three different amino acids are each used once, different orders give different tripeptides. The number is a permutation, $3! = 3 \times 2 \times 1$, rather than a selection of a set.

Step 1: Count the choices at each position The available residues are alanine (ala), glycine (gly) and valine (val). A tripeptide has exactly three positions, and no residue can be repeated. Therefore all three must occur once. There are three choices for the N-terminal position. Once that residue is chosen, only two choices remain for the middle position, and the final remaining residue must occupy the C-terminal position.

$$N = 3 \times 2 \times 1 = 6.$$

Step 2: List the distinct sequences to verify the count The six sequences, always written from N-terminus to C-terminus, are

ala–gly–val, ala–val–gly,
gly–ala–val, gly–val–ala,
val–ala–gly, val–gly–ala.

Reversing a sequence does not make the same peptide: its chemically distinct amino and carboxyl termini exchange the residues attached to them. Thus the three reversal pairs must not be counted as only three products. The choices 3 and 4 omit valid sequences, whereas 8 includes possibilities

beyond the six allowed permutations. No extra stereoisomers or repeated-residue chains are specified in the question.

Step 3: Conclusion Exactly six tripeptide sequences can be formed without repeating an amino acid. Hence, the correct option is (C).

Quick Tip to Solve: For peptides made by using each of n distinct residues exactly once, count ordered N-to-C sequences with $n!$; do not divide by two for reversed chains.

57. One mole of $Cl_2(g)$ was passed into 2 L of cold 2 M KOH solution. After the reaction, the concentrations of Cl^-, ClO^- and OH^- are respectively (assume volume remains constant) [4 marks]

(A) $1M, 1M, 1M$

(B) $0.5M, 0.5M, 1M$

(C) $0.5M, 0.5M, 0.5M$

(D) $0.75M, 0.75M, 1M$

Correct Answer: (B) $0.5M, 0.5M, 1M$

Solution:

Concept:

- Chlorine disproportionates in cold aqueous alkali to chloride and hypochlorite: $Cl_2 + 2\,OH^- \longrightarrow Cl^- + ClO^- + H_2O$. Chlorine changes from oxidation state 0 to both -1 and $+1$.
- Molar concentration is $c = n/V$, with n in moles and V in litres. First determine the limiting reactant and remaining moles, then divide every ionic amount by the common final volume.

Step 1: Calculate the initial and consumed hydroxide The initial hydroxide concentration equals the KOH concentration because KOH dissociates completely in water. Hence

$$n_0(OH^-) = 2\,\text{mol}\,L^{-1} \times 2\,L = 4\,\text{mol}.$$

From the balanced equation, one mole of Cl_2 consumes two moles of OH^-. There is sufficient hydroxide to react with all the chlorine, so chlorine is limiting. The reaction produces one mole each of Cl^- and ClO^-, leaving $4 - 2 = 2$ moles of hydroxide.

Step 2: Convert the final ionic amounts to concentrations The final volume remains $2\,L$, as specified. Therefore

$$[Cl^-] = \frac{1}{2} = 0.5\,M,$$
$$[ClO^-] = \frac{1}{2} = 0.5\,M,$$
$$[OH^-] = \frac{2}{2} = 1\,M.$$

The result also respects charge balance: the total anion concentration is $2\,M$, equal to the unchanged potassium-ion concentration. Option (A) overcounts the products, (C) removes too much hydroxide, and (D) overcounts both chlorine-containing ions. Chlorate formation belongs to hot concentrated alkali, not these cold conditions.

Step 3: Conclusion The concentrations of Cl^-, ClO^- and OH^- are respectively $0.5\,M$, $0.5\,M$ and $1\,M$. Hence, the correct option is (B).

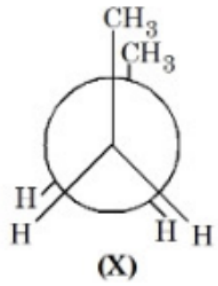
Quick Tip to Solve: For chlorine in cold alkali, use the 1:2:1:1 mole ratio for chlorine, hydroxide, chloride and hypochlorite before dividing by the final volume.

58. Given below are two statements :

Statement I : There are several conformers for n -butane. Out of those conformers,

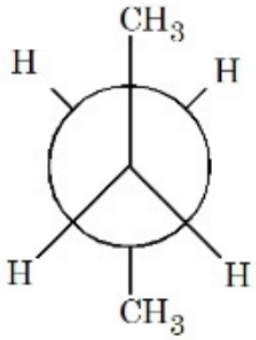
Statement II : As the dihedral angle increases, torsional strain decreases from (X) to (Y).

In the light of the above statements, choose the correct answer from the options given below [4 marks]



(X)

is the least stable and most stable conformer is



(Y)

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- Rotation about the central carbon–carbon single bond of n-butane changes the dihedral angle between its methyl groups, producing eclipsed and staggered conformations without changing connectivity.
- Eclipsing increases torsional strain; close methyl groups also increase steric repulsion. The fully eclipsed methyl–methyl arrangement is highest in energy, while the anti staggered arrangement is lowest.

Step 1: Identify the conformers in Statement I In X, the front and rear methyl groups eclipse each other: their dihedral angle is 0°. This arrangement combines eclipsing interactions with the closest approach of the two bulky methyl groups, making it the least stable conformer of n-butane. In Y, the methyl groups lie opposite one another in a staggered arrangement, with dihedral angle 180°. Their separation is greatest and eclipsing interactions are absent. Y is therefore the anti conformer and is the most stable. Thus Statement I is true.

Step 2: Compare the strain at the stated endpoints On comparing X with Y, the methyl-group dihedral

angle increases from 0° to 180°, and the eclipsed torsional strain of X is relieved in the staggered conformer Y. Statement II is therefore true as the comparison of the two named conformers. This endpoint comparison should not be mistaken for a claim of a smooth monotonic fall throughout rotation: intermediate gauche minima and eclipsed maxima occur. Both named endpoints nevertheless have exactly the strain and stability relationship stated. Option (B) rejects both valid statements, while (C) and (D) each incorrectly reject one of them.

Step 3: Conclusion X is the least stable fully eclipsed conformer, and Y is the most stable anti staggered conformer. Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: In a Newman projection of n-butane, first locate the two methyl groups: eclipsed methyls identify the highest-energy form, and opposite staggered methyls identify anti.

59. At 298 K , the mole percentage of $N_2(g)$ in air is 80%. Water is in equilibrium with air at a pressure of 10 atm . What is the mole fraction of $N_2(g)$ in water at 298 K ? (K_H for N_2 is 6.5×10^7 mmHg)

[4 marks]

(A) 9.35×10^{-5}

(B) 1.23×10^{-7}

(C) 9.35×10^{5}

(D) 1.17×10^{-4}

Correct Answer: (A) 9.35×10^{-5}

Solution:

Concept:

- Henry's law in the convention given is $p = K_H x$, where p is the gas partial pressure above the solution and x its mole fraction dissolved in the liquid.
- Dalton's law gives $p_{N_2} = y_{N_2} P$. The gas-phase mole fraction y differs from the dissolved mole fraction x; pressure units must match those of K_H.

Step 1: Find the partial pressure of nitrogen Nitrogen constitutes 80 mole percent of air, so its gas-phase mole fraction is 0.80. The total air pressure is $10\,\text{atm}$, giving

$$p_{N_2} = 0.80 \times 10 = 8\,\text{atm}.$$

The Henry constant is supplied in millimetres of mercury, so convert the partial pressure using $1\,\text{atm} = 760\,\text{mm Hg}$:

$$p_{N_2} = 8 \times 760 = 6080\,\text{mm Hg}.$$

Step 2: Apply Henry law with consistent units At the stated temperature,

$$x_{N_2} = \frac{p_{N_2}}{K_H} = \frac{6080}{6.5 \times 10^7} = 9.3538 \times 10^{-5}.$$

The pressure units cancel, as required for a mole fraction. The small positive result is consistent with the low solubility of nitrogen. The value 1.23×10^{-7} comes from dividing 8 atm directly by a constant

in mm Hg; 9.35×10^5 cannot be a mole fraction because it exceeds one; and 1.17×10^{-4} incorrectly uses the total air pressure instead of the nitrogen partial pressure. These checks eliminate options (B), (C) and (D), respectively.

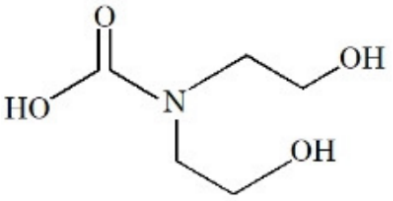
Step 3: Conclusion The mole fraction of nitrogen dissolved in water at $298\,\mathrm{K}$ is 9.35×10^{-5}, a dimensionless quantity. Hence, the correct option is (A).

Quick Tip to Solve: Before using $x = p/K_H$, multiply total pressure by the gas mole fraction and convert that partial pressure into the units used for K_H.

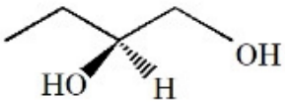
60.

From the following, how many compounds contain at least one secondary alcohol?

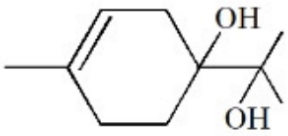
Choose the correct answer from the options given below : [4 marks]



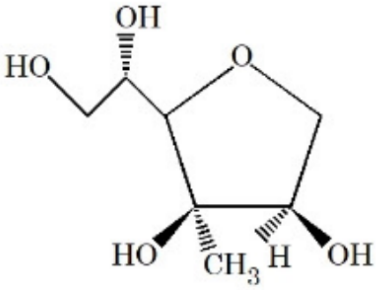
(I)



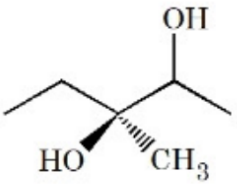
(II)



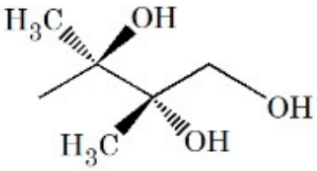
(III)



(IV)



(V)



(VI)

(A) Three

(B) Five

(C) Two

(D) Four

Correct Answer: (A) Three

Solution:

Concept:

- An alcohol is secondary when its hydroxyl-bearing saturated carbon is bonded directly to two other carbon atoms and one hydrogen: $\mathrm{R{-}CH(OH){-}R'}$. Count carbon neighbours of that particular carbon.
- A molecule qualifies if it contains at least one secondary alcohol group, even when other hydroxyl

groups are primary or tertiary. A carboxylic acid hydroxyl is not an alcohol group.

Step 1: Inspect compounds I, II and III In I, both alcoholic hydroxyl groups occur on terminal CH_2OH carbons, each attached to only one carbon, so they are primary. The $C(=O)OH$ group is not an alcohol. I does not qualify. In II, the central carbon bearing OH is attached to an ethyl group, a hydroxymethyl group and a hydrogen: it has two carbon neighbours, so II contains a secondary alcohol. In III, each hydroxyl-bearing carbon has three carbon neighbours. Both alcohol groups are tertiary, and III does not qualify.

Step 2: Inspect compounds IV, V and VI and count molecules In IV, the side-chain $CH(OH)$ carbon is bonded to the terminal carbon and a ring carbon. It is secondary; the lower-right hydroxyl-bearing ring carbon is also secondary. Thus IV qualifies but counts only once. In V, the right-hand $CH(OH)$ carbon is attached to a methyl group and the adjacent central carbon, making it secondary; the other alcohol is tertiary. V qualifies. In VI, the two internal hydroxyl-bearing carbons each have three carbon neighbours and are tertiary, while the terminal CH_2OH group is primary. VI does not qualify. The qualifying compounds are therefore II, IV and V: three molecules. Two omits a qualifying molecule; four and five include molecules having no secondary alcohol.

Step 3: Conclusion Three compounds, II, IV and V, contain at least one secondary alcohol group. Hence, the correct option is (A).

Quick Tip to Solve: Circle each OH-bearing carbon and count its directly attached carbon neighbours. Then count qualifying molecules once each, not the number of secondary OH groups.

61. The wavelength of light absorbed for the following complexes are in the order

$[Co(NH_3)_6]^{3+}$ **(I)** $[Co(H_2O)_6]^{3+}$ **(II)** $[Co(CN)_6]^{3-}$ **(III)**

$[Co(NH_3)_5(H_2O)]^{3+}$ **(IV)** $[CoF_6]^{3-}$ **(V)**

[4 marks]

(A) III $<$ IV $<$ I $<$ II $<$ V

(B) III $<$ I $<$ II $<$ IV $<$ V

(C) III $<$ I $<$ IV $<$ II $<$ V

(D) III $<$ I $<$ IV $<$ V $<$ II

Correct Answer: (C) III $<$ I $<$ IV $<$ II $<$ V

Solution:

Concept:

- For an octahedral complex, the energy absorbed in a d–d transition is set by the crystal-field splitting Δ_o, with $\Delta_o = hc/\lambda$. A stronger ligand field therefore means a shorter absorbed wavelength.
- For the ligands here, the spectrochemical order is $F^- < H_2O < NH_3 < CN^-$. All five complexes contain cobalt in oxidation state $+3$, so ligand strength is the relevant comparison.

Step 1: Identify the common metal oxidation state The ammine and aqua ligands are neutral, whereas each cyanide or fluoride ligand has charge -1. Thus complexes I, II and IV have cobalt at +3 directly, and in III and V the equation $x - 6 = -3$ again gives $x = +3$. The oxidation-state contribution does not reverse the ligand-field comparison.

Step 2: Order the splitting energies Cyanide produces the strongest field, so III has the largest splitting. I contains six ammonia ligands. Replacing one of them by the weaker water ligand gives IV a slightly smaller splitting. II has only water ligands, and V has the weakest-field fluoride ligands. Hence

$$\Delta_o(\mathrm{III}) > \Delta_o(\mathrm{I}) > \Delta_o(\mathrm{IV}) > \Delta_o(\mathrm{II}) > \Delta_o(\mathrm{V}).$$

Step 3: Reverse the order for wavelength Since $\lambda = hc/\Delta_o$, the wavelength order is the reverse of the splitting order. Option A incorrectly places IV before I, even though replacing ammonia by water weakens the field. Option B puts II before IV, and option D puts V before II; both also contradict the ligand-strength order.

Step 4: Conclusion The order is $\mathrm{III} < \mathrm{I} < \mathrm{IV} < \mathrm{II} < \mathrm{V}$. Hence, the correct option is (C).

Quick Tip to Solve: First rank the ligand-field splitting, then reverse that ranking for wavelength; do not confuse absorbed energy with absorbed wavelength.

62. Consider the following gaseous equilibrium in a closed container of volume ' V ' at $\mathrm{T(K)}$.

$$\mathrm{P_2(\ g) + Q_2(\ g) \rightleftharpoons 2PQ(\ g)}$$

2 moles each of $\mathrm{P_2(\ g), Q_2(\ g)}$ and $\mathrm{PQ(g)}$ are present at equilibrium. Now one mole each of ' $\mathrm{P_2}$ ' and ' $\mathrm{Q_2}$ ' are added to the equilibrium keeping the temperature at $\mathrm{T(K)}$. The number of moles of $\mathrm{P_2, Q_2}$ and PQ at the new equilibrium, respectively, are [4 marks]

(A) $1.66, 1.66, 1.66$

(B) $2.67, 2.67, 2.67$

(C) 1.21, 2.24, 1.56

(D) $2.56, 1.62, 2.24$

Correct Answer: (B) $2.67, 2.67, 2.67$

Solution:

Concept:

- At a fixed temperature the equilibrium constant is unchanged by adding reactants. For $\mathrm{P_2(g) + Q_2(g) \rightleftharpoons 2\,PQ(g)}$, $K_c = [\mathrm{PQ}]^2/([\mathrm{P_2}][\mathrm{Q_2}])$.
- The total stoichiometric change in gaseous moles is zero, so the volume factors cancel in the equilibrium expression. Equal initial amounts of the two reactants remain equal because they react in a 1:1 ratio.

Step 1: Determine the original equilibrium constant Initially all three species have 2 mol in volume

V. Therefore

$$K_c = \frac{(2/V)^2}{(2/V)(2/V)} = 1.$$

Immediately after the additions, their amounts are 3, 3 and 2 mol. The reaction quotient is $Q_c = 4/9 < 1$, so the reaction proceeds forward to increase the product amount.

Step 2: Set up the new equilibrium amounts Let ξ mol each of P_2 and Q_2 react. At the new equilibrium the amounts are $3-\xi$, $3-\xi$ and $2+2\xi$ mol. Thus

$$\frac{(2+2\xi)^2}{(3-\xi)^2} = 1.$$

All equilibrium amounts are positive, so taking the positive square root gives $2+2\xi = 3-\xi$. Hence $3\xi = 1$, or $\xi = 1/3$.

Step 3: Evaluate and check the three amounts Substitution gives

$$n(P_2) = n(Q_2) = n(PQ) = \frac{8}{3} = 2.666\ldots \text{ mol}.$$

Their sum is 8 mol, equal to the amount just after addition, as required by zero change in total gaseous moles. Option A has the wrong total. Options C and D make the two reactant amounts unequal, violating their equal starting quantities and equal consumption.

Step 4: Conclusion The new amounts are $2.67,\ 2.67,\ 2.67$ mol, respectively. Hence, the correct option is (B).

Quick Tip to Solve: With equal reactant amounts and $K_c = 1$, the positive equilibrium root makes the product amount equal to each reactant amount; use total moles as a quick check.

63. Given below are two statements :

Statement I : Cross aldol condensation between two different aldehydes will always produce four different products.

Statement II : When semicarbazide reacts with a mixture of benzaldehyde and acetophenone under optimum pH , it forms a condensation product with acetophenone only.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (B) Both Statement I and Statement II are false

Solution:

Concept:

- Aldol reactions require an enolizable carbonyl compound with an alpha hydrogen. Four possible self- and cross-condensation products are not a universal result for every pair of different aldehydes.

- Semicarbazide reacts through its terminal amino group with aldehydes and ketones to form semicarbazones by addition followed by loss of water. Aldehydes are generally more reactive than comparable ketones toward nucleophilic addition.

Step 1: Test the universal claim in Statement I The word "always" is decisive. When both aldehydes possess suitable alpha hydrogens, each may act as the enolate donor and each as the carbonyl acceptor; self and crossed reactions can then produce a mixture. However, benzaldehyde has no alpha hydrogen and cannot furnish the usual enolate donor. A mixture containing benzaldehyde therefore does not automatically produce all four products. If neither aldehyde has an alpha hydrogen, the ordinary aldol reaction is unavailable. Consequently Statement I is false.

Step 2: Examine the selectivity claimed in Statement II Benzaldehyde contains an aldehyde carbonyl that can react with semicarbazide. Acetophenone is a ketone; its carbonyl is more sterically hindered, and its methyl group also reduces carbonyl electrophilicity by electron donation. Under suitable mildly acidic conditions, semicarbazone formation is not restricted to acetophenone. In particular, the more reactive benzaldehyde cannot be excluded as the statement claims. Thus the assertion of an acetophenone-only condensation is false.

Step 3: Match the truth values to the options Both statements have failed independently: I incorrectly turns a conditional four-product possibility into an absolute rule, and II excludes the reactive aldehyde without justification. Option A requires both to be true, C requires I to be true, and D requires II to be true; each conflicts with these chemical tests.

Step 4: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (B).

Quick Tip to Solve: For aldol claims, check alpha hydrogens before counting products. For carbonyl condensation, do not mistake a ketone for the inherently more reactive aldehyde.

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64. The wavelength of spectral line obtained in the spectrum of Li^{2+} ion, when the transition takes place between two levels whose sum is 4 and difference is 2 , is [4 marks]

(A) 2.28×10^{-6} cm

(B) 1.14×10^{-6} cm

(C) 1.14×10^{-7} cm

(D) 2.28×10^{-7} cm

Correct Answer: (B) 1.14×10^{-6} cm

Solution:

Concept:

- For a hydrogen-like ion, the Rydberg relation is $1/\lambda = RZ^2(1/n_1^2 - 1/n_2^2)$, where $n_2 > n_1$, Z is the nuclear charge and $R \simeq 1.097 \times 10^7$ m^{-1}.
- Li^{2+} has one electron and nuclear charge $Z = 3$, so its spectrum follows the hydrogen-like formula. Convert metres to centimetres only after evaluating the wavelength.

Step 1: Identify the two principal quantum numbers The stated sum and difference give

$$n_2 + n_1 = 4, \qquad n_2 - n_1 = 2.$$

Adding yields $2n_2 = 6$, hence $n_2 = 3$ and $n_1 = 1$. The emitted spectral line corresponds to the downward transition from the third level to the first. Its photon energy equals the magnitude of the energy difference between those levels.

Step 2: Substitute into the Rydberg expression Using $Z^2 = 9$ gives

$$\frac{1}{\lambda} = R(3)^2 \left(1 - \frac{1}{9}\right) = 8R.$$

Therefore

$$\lambda = \frac{1}{8(1.097 \times 10^7)} \simeq 1.1395 \times 10^{-8}\ \text{m}.$$

The factor of nine is essential: using the hydrogen formula without the nuclear-charge factor would give a wavelength nine times too large.

Step 3: Convert units and eliminate alternatives Since $1\ \text{m} = 100\ \text{cm}$,

$$\lambda \simeq 1.1395 \times 10^{-6}\ \text{cm} \simeq 1.14 \times 10^{-6}\ \text{cm}.$$

Option A is twice this wavelength and would imply half the required photon energy. Option C is smaller by a factor of ten, and option D also has the wrong numerical factor and exponent. Neither conversion nor the allowed transition supports those alternatives.

Step 4: Conclusion The wavelength is $1.14 \times 10^{-6}\ \text{cm}$. Hence, the correct option is (B).

Quick Tip to Solve: Solve the sum-and-difference equations for the levels first, retain the Z^2 factor, and make the metre-to-centimetre conversion last.

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65. The heat of atomisation of methane and ethane are ' x ' kJmol^{-1} and ' y ' kJmol^{-1} respectively. The longest wavelength (λ) of light capable of breaking the $\text{C} - \text{C}$ bond can be expressed in SI unit as : [4 marks]

(A) $\frac{\text{N}_\text{A}\text{hc}}{250(4y-6x)}$

(B) $\frac{\text{hc}}{1000}\left(\frac{y-6x}{4}\right)^{-1}$

(C) $\frac{\text{N}_\text{A}\text{hc}}{250(y-6x)}$

(D) $\text{N}_\text{A}\text{hc}\left(y - \frac{6x}{4}\right)^{-1}$

Correct Answer: (A) $\frac{\text{N}_\text{A}\text{hc}}{250(4y-6x)}$

Solution:

Concept:

- Atomisation breaks all bonds to form separated gaseous atoms. In the bond-enthalpy model used here, methane has four C–H bonds and ethane has six C–H bonds plus one C–C bond.

- The longest dissociating wavelength is the threshold wavelength: hc/λ equals the bond energy per molecule. Molar energies in $\mathrm{kJ\,mol^{-1}}$ must be multiplied by 1000 and divided by N_A.

Step 1: Extract the C–H bond enthalpy Let $D_{\mathrm{C-H}}$ denote the C–H bond enthalpy in the stated bond-energy model. Methane contains four such bonds, so

$$4D_{\mathrm{C-H}} = x, \qquad D_{\mathrm{C-H}} = \frac{x}{4} \quad (\mathrm{kJ\,mol^{-1}}).$$

Ethane has six C–H bonds, not four, and its atomisation energy also includes cleavage of the single bond between the carbon atoms.

Step 2: Isolate the C–C bond enthalpy Writing the ethane atomisation balance,

$$y = 6D_{\mathrm{C-H}} + D_{\mathrm{C-C}}, \qquad D_{\mathrm{C-C}} = y - \frac{6x}{4}.$$

Convert this molar energy to the energy of one bond in joules:

$$\varepsilon = \frac{1000}{N_A}\left(y - \frac{6x}{4}\right) = \frac{250(4y - 6x)}{N_A}.$$

Step 3: Use the photon threshold and check the options At threshold $hc/\lambda_{\max} = \varepsilon$, hence

$$\lambda_{\max} = \frac{N_A hc}{250(4y - 6x)}.$$

With h and c in SI units the result is in metres. Option B omits the Avogadro conversion and uses the wrong bond-energy combination. Option C omits the factor four multiplying y after simplifying the denominator. Option D lacks the conversion from kilojoules to joules and so is larger by a factor of 1000.

Step 4: Conclusion $\lambda_{\max} = \dfrac{N_A hc}{250(4y - 6x)}$ m, with the given numerical x, y in $\mathrm{kJ\,mol^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: Count bonds before subtracting atomisation energies. In a photon calculation, convert molar kilojoules to joules per bond before applying $E = hc/\lambda$.

66. Pair of species among the following having same bond order as well as paramagnetic character will be- [4 marks]

(A) O_2^-, N_2^+

(B) O_2^-, N_2^-

(C) $O_2{}^+, N_2{}^{2-}$

(D) O_2^+, N_2^-

Correct Answer: (D) O_2^+, N_2^-

Solution:

Concept:

- Molecular-orbital bond order is $(N_b - N_a)/2$, where N_b and N_a are bonding and antibonding electron counts. An unpaired electron makes a species paramagnetic.
- Removing one antibonding electron increases bond order by $1/2$; adding one antibonding electron decreases it by $1/2$. Degenerate antibonding orbitals fill singly before pairing, according to Hund's rule.

Step 1: Evaluate the oxygen ions Neutral O_2 has bond order 2 and two unpaired electrons in its degenerate π^* orbitals. Removing one of these antibonding electrons gives $O_2{}^+$, with bond order $2 + 1/2 = 2.5$ and one unpaired electron. Adding an electron instead gives $O_2{}^-$, whose bond order is $2 - 1/2 = 1.5$, again with one unpaired electron. Thus both oxygen ions are paramagnetic, but their bond orders differ.

Step 2: Evaluate the nitrogen ions Neutral N_2 has bond order 3 and all electrons paired. Removing its highest occupied bonding electron gives $N_2{}^+$ with bond order $3 - 1/2 = 2.5$ and one unpaired electron. Adding one electron occupies a π^* antibonding orbital, giving $N_2{}^-$ with bond order 2.5 and one unpaired electron. Adding two electrons gives $N_2{}^{2-}$ with bond order $3 - 2/2 = 2$; the two electrons occupy the degenerate π^* orbitals singly, so this ion is also paramagnetic.

Step 3: Compare both requirements for each pair Option A compares bond orders 1.5 and 2.5, while option B again compares 1.5 and 2.5. Option C compares 2.5 and 2. These fail the equal-bond-order requirement even though the species are paramagnetic. Only option D pairs $O_2{}^+$ and $N_2{}^-$, both with bond order 2.5 and one unpaired electron; it satisfies both requirements simultaneously.

Step 4: Conclusion $O_2{}^+$ and $N_2{}^-$ each have bond order 2.5 and are paramagnetic. Hence, the correct option is (D).

Quick Tip to Solve: Check bond order and unpaired electrons separately. Equal electron counts can be useful, but whether the added or removed electron is bonding or antibonding decides the bond order.

67. The unsaturated ether on acidic hydrolysis produces carbonyl compounds as shown below :-

Based on this, predict the solution/reagent that will help to distinguish "P" and "Q" obtained in the following reaction :- [4 marks]

$$CH_3-CH=CH-O-CH=CH_2 \xrightarrow{H_3O^{(+)}} CH_3-CH_2-CH=O \text{ and } O=CH-CH_3$$

$$CH_3-CH=CH-O-\underset{\displaystyle CH_3}{\overset{|}{C}}=CH_2 \xrightarrow[\Delta]{H_3O^{(+)}} P + Q$$

(A) Saturated $NaHSO_3$ solution

(B) Fehling solution

(C) 2, 4 - DNP reagent

(D) Lucas reagent

Correct Answer: (B) Fehling solution

Solution:

Concept:

- An enol ether undergoes acid-catalysed hydrolysis to a carbonyl compound. The initially formed enol changes into its more stable carbonyl tautomer.
- Fehling's solution oxidises an aliphatic aldehyde to a carboxylate while its blue copper(II) complex is reduced to brick-red Cu_2O. A simple ketone does not give this test.

Step 1: Identify the two carbonyl products The left-hand unsaturated fragment is $CH_3-CH=CH-O^-$. Its enol product is $CH_3-CH=CH-OH$, which tautomerises to propanal, CH_3CH_2CHO. The right-hand fragment is $-O-C(CH_3)=CH_2$; its enol is $CH_2=C(OH)CH_3$, giving propanone, CH_3COCH_3. Thus the pair consists of an aliphatic aldehyde and a methyl ketone.

Step 2: Apply the selective oxidation test Warm separate samples with Fehling's solution. Propanal gives a brick-red precipitate, whereas propanone does not. The aldehyde reaction in alkaline medium is

$$CH_3CH_2CHO + 2\,Cu^{2+} + 5\,OH^- \longrightarrow CH_3CH_2COO^- + Cu_2O\downarrow + 3\,H_2O.$$

This different observation distinguishes the products without requiring their boiling points.

Step 3: Reject the other reagents Both propanal and propanone give carbonyl derivatives with 2,4-DNP, so that reagent confirms a carbonyl group but does not selectively distinguish this pair. Both also form bisulfite addition compounds with saturated $NaHSO_3$. Lucas reagent is used to classify alcohols, whereas neither final product is an alcohol. Thus options (A), (C) and (D) lack the required selective response.

Step 4: Conclusion The products are propanal and propanone; Fehling's solution gives a positive test only with propanal. Hence, the correct option is (B).

Quick Tip to Solve: First convert each enol fragment into its carbonyl tautomer; then choose a test that reacts with only one of the two products.

68. Find out the statements which are not true.

A. Resonating structures with more number of covalent bonds and lesser charge separation are more stable.

B. In electromeric effect, an unsaturated system shows +E effect with nucleophile and -E effect with electrophile.

C. Inductive effect is responsible for high melting point, boiling point and dipole moment of polar compounds.

D. The greater the number of alkyl groups attached to the doubly bonded carbon atoms, higher is the heat of hydrogenation.

E. Stability of carbanion increases with the increase in $s-$ character of the carbon carrying the negative charge.

Choose the correct answer from the options given below : [4 marks]

(A) B & D only

(B) A, C & D only

(C) B, D & E only

(D) A, D & E only

Correct Answer: (A) B & D only

Solution:

Concept:

- Resonance contributors with more covalent bonds and less charge separation are generally more stable when other factors are comparable.
- The electromeric effect is a temporary complete shift of a π-electron pair during reagent attack; hyperconjugation stabilises alkyl-substituted alkenes.
- A carbon orbital with greater s-character holds negative charge closer to the nucleus and stabilises a carbanion.

Step 1: Check statements A and B A is true: greater bonding and reduced charge separation favour a resonance contributor. B is false because it reverses the usual effects. An electrophile induces a +E effect when the electron pair moves toward the atom to which it attaches. A nucleophile attacking a carbonyl carbon induces a -E effect as the π pair shifts toward oxygen, away from the incoming nucleophile.

Step 2: Check statements C and D C is accepted as true in this electronic-effect classification: permanent bond polarisation underlies dipole moments and the dipole interactions that influence melting and boiling behaviour. This does not mean every polar compound has a larger melting point than every nonpolar one. D is false: more alkyl substitution generally stabilises an alkene through hyperconjugation and electron donation. Greater initial stability means less heat is liberated on hydrogenation, not more.

Step 3: Check statement E and compare the sets E is true. Increasing s-character increases the effective electronegativity of the carbon bearing the lone pair. The usual hybridisation comparison is $sp > sp^2 > sp^3$ for stabilising that negative charge. Only B and D are false. Option (B) wrongly includes true A and C, while options (C) and (D) wrongly include true E; option (D) also includes

true A.

Step 4: Conclusion The statements that are not true are B and D only. Hence, the correct option is (A).

Quick Tip to Solve: For alkene stability, remember that more stabilisation means a smaller magnitude of heat released on hydrogenation.

69. The correct order of C, N, O and F in terms of second ionisation potential is [4 marks]

(A) $F < N < C < O$

(B) $C < O < N < F$

(C) $C < F < N < O$

(D) $C < N < F < O$

Correct Answer: (D) $C < N < F < O$

Solution:

Concept:

- The second ionisation process is $X^+(g) \longrightarrow X^{2+}(g) + e^-$. Therefore compare the configurations of singly charged cations, rather than neutral atoms.
- Effective nuclear charge generally increases across a period, but removing an electron from a half-filled subshell is particularly difficult.

Step 1: Write the singly charged configurations After the first electron has been removed, the relevant valence configurations are

$$C^+ : 2s^2 2p^1, \quad N^+ : 2s^2 2p^2,$$
$$O^+ : 2s^2 2p^3, \quad F^+ : 2s^2 2p^4.$$

In every case the second electron is removed from a 2p subshell. Oxygen is exceptional here because its cation, not its neutral atom, has the half-filled configuration.

Step 2: Compare successive removals From C^+ to N^+, increasing effective nuclear charge makes removal harder, so carbon has the lower second ionisation potential. Removing from O^+ breaks a stable half-filled $2p^3$ arrangement. In F^+, removal of a paired electron changes $2p^4$ to the stable half-filled $2p^3$ arrangement and relieves pairing repulsion. Consequently oxygen exceeds fluorine, while fluorine still exceeds nitrogen because of its stronger nuclear attraction.

Step 3: Select the matching sequence The required sequence is

$$C < N < F < O.$$

Option (A) incorrectly places fluorine below carbon and nitrogen. Option (B) puts oxygen below nitrogen and fluorine, overlooking the half-filled oxygen cation. Option (C) incorrectly places fluorine below nitrogen. Only (D) respects both the general increase and the oxygen exception.

Step 4: Conclusion The increasing order of second ionisation potential is $C < N < F < O$. Hence, the correct option is (D).

Quick Tip to Solve: For the nth ionisation energy, write the configuration after n minus one electrons have already been removed.

70. A student has planned to prepare acetanilide from aniline using acetic anhydride.
The student has started from 9.3 g of aniline. However, the student has managed to obtain 11 g of dry acetanilide.
The % yield of this reaction is :- [4 marks]

(A) 72.5%

(B) 81.5%

(C) 97.5%

(D) 59.5%

Correct Answer: (B) 81.5%

Solution:

Concept:

- Acetylation converts one mole of aniline into one mole of acetanilide. The theoretical yield follows from this mole ratio, using the amount of aniline supplied.
- Percentage yield is $100 \times m_{\text{actual}}/m_{\text{theoretical}}$, where both masses refer to the same product.

Step 1: Calculate moles of aniline Aniline is $C_6H_5NH_2$, or C_6H_7N. Taking the usual atomic masses C = 12, H = 1 and N = 14 gives

$$M_{\text{aniline}} = 6(12) + 7(1) + 14 = 93\,\text{g mol}^{-1}.$$

Therefore the supplied amount is

$$n_{\text{aniline}} = \frac{9.3}{93} = 0.100\,\text{mol}.$$

Step 2: Find the theoretical product mass The acetylation stoichiometry is

$$C_6H_5NH_2 + (CH_3CO)_2O \longrightarrow C_6H_5NHCOCH_3 + CH_3COOH.$$

Acetanilide has formula C_8H_9NO, so its molar mass is

$$8(12) + 9(1) + 14 + 16 = 135\,\text{g mol}^{-1}.$$

One mole of aniline produces one mole of acetanilide; thus the maximum product mass from 0.100 mol is $0.100 \times 135 = 13.5\,\text{g}$. The product is heavier than the starting aniline because an acetyl group is incorporated.

Step 3: Compare actual and theoretical yield The isolated dry product weighs 11 g. Hence

$$\text{yield} = \frac{11}{13.5} \times 100 = 81.481\ldots\% = 81.5\% \text{ (approximately)}.$$

Using the starting 9.3 g in the denominator would compare different substances and is incorrect. Options (A), (C) and (D) do not equal the actual-to-theoretical product mass ratio.

Step 4: Conclusion The percentage yield of acetanilide is approximately 81.5%. Hence, the correct option is (B).

Quick Tip to Solve: Convert the starting material to moles first, use the balanced mole ratio, then compare the isolated product with its theoretical mass.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The half-life of ^{65}Zn is 245 days. After x days, 75% of original activity remained. The value of x in days is ____ . (Nearest integer)

(Given: $\log 3 = 0.4771$ **and** $\log 2 = 0.3010$ **)** [4 marks]

Solution:

Concept:

- Radioactive decay obeys the first-order law $N = N_0 e^{-kt}$, where k is the decay constant and N is the number of undecayed nuclei.
- Activity is proportional to N for the same isotope, so $A/A_0 = N/N_0$. The half-life satisfies $k = \ln 2/t_{1/2}$.

Step 1: Use the remaining activity fraction The statement says that 75 percent of the original activity remains, not that 75 percent has decayed. Thus

$$\frac{A}{A_0} = 0.75 = \frac{3}{4}, \qquad \frac{A_0}{A} = \frac{4}{3}.$$

The time must be smaller than one half-life, because the remaining activity is greater than one-half of its initial value.

Step 2: Eliminate the decay constant Combining the first-order equation and the half-life relation gives

$$x = \frac{\ln(A_0/A)}{k}$$
$$= 245\,\frac{\ln(4/3)}{\ln 2} = 245\,\frac{\log(4/3)}{\log 2}.$$

The logarithm base cancels in this ratio, allowing direct use of the supplied common logarithms. The numerical factor 245 carries the unit days.

Step 3: Substitute and round only at the end Using the supplied logarithms,

$$\log(4/3) = 2\log 2 - \log 3 = 2(0.3010) - 0.4771 = 0.1249.$$

Therefore

$$x = 245 \times \frac{0.1249}{0.3010} = 101.662\ldots \text{ days}.$$

This is about 0.415 of one half-life, consistent with only one-quarter of the nuclei having decayed. Rounding the final value, rather than the intermediate logarithmic ratio, gives the requested integer.

Step 4: Conclusion The elapsed time is $x = 102$ days to the nearest integer. The numerical entry is $\boxed{102}$.

Quick Tip to Solve: In decay questions distinguish the fraction remaining from the fraction decayed before placing a ratio inside the logarithm.

72. Molar conductivity of a weak acid HQ of concentration 0.18 M was found to be $1/30$ of the molar conductivity of another weak acid HZ with concentration of 0.02 M . If $\lambda^\circ Q^-$happened to be equal with $\lambda^\circ Z^-$, then the difference of the pK_a values of the two weak acids $(pK_a(HQ) - pK_a(HZ))$ is ____ (Nearest integer).

[Given: degree of dissociation $(\alpha) \ll 1$ for both weak acids, λ° : limiting molar conductivity of ions] [4 marks]

Solution:

Concept:

- For a weak monobasic acid, the dissociation fraction is $\alpha = \Lambda_m/\Lambda_m^\circ$, where Λ_m is molar conductivity at the stated concentration.
- For $\alpha \ll 1$, Ostwald's dilution relation gives $K_a = c\alpha^2/(1-\alpha) \simeq c\alpha^2$, and $pK_a = -\log K_a$.

Step 1: Compare limiting conductivities Both acids furnish the same cation, H^+, and the question states that their anion limiting conductivities are equal. By independent migration of ions,

$$\Lambda_m^\circ(HQ) = \lambda_{H^+}^\circ + \lambda_{Q^-}^\circ,$$
$$\Lambda_m^\circ(HZ) = \lambda_{H^+}^\circ + \lambda_{Z^-}^\circ.$$

These sums are equal. Therefore the ratio of dissociation fractions equals the stated molar conductivity ratio: $\alpha_{HQ}/\alpha_{HZ} = 1/30$.

Step 2: Form the ratio of acid dissociation constants The concentrations are 0.18 M and 0.02 M, giving a concentration ratio of 9. Since both degrees of dissociation are small,

$$\frac{K_a(HQ)}{K_a(HZ)} \simeq \frac{0.18}{0.02}\left(\frac{1}{30}\right)^2 = \frac{9}{900} = \frac{1}{100}.$$

The degree-of-dissociation ratio must be squared; the conductivity ratio alone is not the ratio of the acid constants.

Step 3: Take the logarithm with the correct sign The required difference is

$$pK_a(HQ) - pK_a(HZ) = -\log\left[\frac{K_a(HQ)}{K_a(HZ)}\right]$$
$$= -\log(10^{-2}) = 2.$$

The result is positive because HQ has the smaller acid constant and is the weaker acid. A pKa difference is dimensionless.

Step 4: Conclusion The required pKa difference, to the nearest integer, is $\boxed{2}$ (dimensionless).

Quick Tip to Solve: Equal limiting molar conductivities make the conductivity ratio an alpha ratio; square it and include concentration before taking the pKa difference.

73. A chromium complex with a formula $CrCl_3 \cdot 6H_2O$ has a spin only magnetic moment value of 3.87 BM and its solution conductivity corresponds to $1:2$ electrolyte. 2.75 g of the complex solution was initially passed through a cation exchanger. The solution obtained after the process was reacted with excess of $AgNO_3$. The amount of AgCl formed in the above process is ____ g. (Nearest integer)

[Given: Molar mass in $gmol^{-1}$ $Cr: 52; Cl: 35.5, Ag: 108, O: 16, H: 1$] [4 marks]

Solution:

Concept:

- The spin-only moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons. Chromium(III) is a d^3 ion with three unpaired electrons.
- A $1:2$ electrolyte gives one complex cation and two counter-anions. A cation exchanger retains the complex cation; chloride ions outside its coordination sphere remain available for precipitation with $AgNO_3$.

Step 1: Identify the coordination formula Since $\sqrt{3(3+2)} = \sqrt{15} \simeq 3.87$, the stated moment agrees with chromium(III). The two ionisable chlorides required by the conductivity give the hydrate formula

$$[CrCl(H_2O)_5]Cl_2 \cdot H_2O.$$

There is one coordinated chloride, two outer-sphere chlorides, five coordinated waters and one water of crystallisation. Its dissolution therefore yields $[CrCl(H_2O)_5]^{2+}$ and $2Cl^-$.

Step 2: Count the chloride ions after ion exchange The exchanger removes the positively charged complex, including its coordinated chloride. The two free chloride ions per formula unit pass into the solution. Excess silver nitrate then precipitates them according to

$$Ag^+ + Cl^- \longrightarrow AgCl\,(s).$$

Thus each mole of the complex gives two moles of silver chloride, not three. The chloride inside the retained complex is not included in the precipitate.

Step 3: Calculate the precipitate mass Using the given atomic masses, the molar masses are

$$M_{\text{complex}} = 52 + 3(35.5) + 6(18) = 266.5\,\text{g mol}^{-1},$$

$$M_{AgCl} = 108 + 35.5 = 143.5\,\text{g mol}^{-1}.$$

For the stated $2.75\,\text{g}$ of complex used in solution,

$$m_{AgCl} = \frac{2.75}{266.5} \times 2 \times 143.5 = 2.9615\,\text{g}.$$

Step 4: Conclusion The silver chloride precipitate has mass $2.9615\,\text{g}$, which is $\boxed{3\,\text{g}}$ to the nearest integer. The required numerical entry is 3.

Quick Tip to Solve: After cation exchange, count only the outer-sphere chlorides: the coordinated chloride leaves with the complex cation.

74. 0.25 g of an organic compound "A" containing carbon, hydrogen and oxygen was analysed using the combustion method. There was an increase in mass of $CaCl_2$ tube and potash tube at the end of the experiment. The amount was found to be 0.15 g and 0.1837 g , respectively. The percentage of oxygen in compound A is

____ %. (Nearest integer)

(Given: molar mass in $gmol^{-1}$ H : 1, C : 12, O : 16) [4 marks]

Solution:

Concept:

- Complete combustion converts the carbon in an organic compound into CO_2 and its hydrogen into H_2O. The masses of these products determine the original masses of carbon and hydrogen.
- The $CaCl_2$ tube absorbs water, whereas the potash tube absorbs carbon dioxide. For a compound containing only carbon, hydrogen and oxygen, the oxygen mass is obtained by subtracting the carbon and hydrogen masses from the sample mass.

Step 1: Assign the absorption-tube gains The gains in the two tubes represent different combustion products. Hence,

$$m_{H_2O} = 0.15\,\text{g}, \qquad m_{CO_2} = 0.1837\,\text{g}.$$

The whole water gain cannot be counted as hydrogen, and the whole carbon dioxide gain cannot be counted as carbon, because both products also contain oxygen supplied during combustion.

Step 2: Calculate the carbon and hydrogen masses From the given atomic masses, $18\,\text{g}$ of water contains $2\,\text{g}$ of hydrogen and $44\,\text{g}$ of carbon dioxide contains $12\,\text{g}$ of carbon. Thus

$$m_{H} = \frac{2}{18}(0.15) = 0.0166667\,\text{g},$$

$$m_{C} = \frac{12}{44}(0.1837) = 0.0501\,\text{g}.$$

These are the masses of the corresponding elements in the original $0.25\,\text{g}$ sample.

Step 3: Obtain the oxygen percentage by difference Since the compound contains no elements other than carbon, hydrogen and oxygen, the remaining sample mass is oxygen:

$$m_{O} = 0.25 - 0.0501 - 0.0166667 = 0.1832333\,\text{g}.$$

Therefore,

$$\%O = \frac{0.1832333}{0.25} \times 100 = 73.2933\%.$$

Retain the fractional hydrogen mass until this final calculation, then round the percentage as requested.

Step 4: Conclusion The oxygen content of compound A is 73.2933%, or $\boxed{73\%}$ to the nearest integer. The required numerical entry is 73.

Quick Tip to Solve: Associate calcium chloride with water and potash with carbon dioxide; use the elemental mass fractions before subtracting from the sample mass.

75. Grignard reagent $\mathrm{RMgBr(P)}$ reacts with water and forms a gas (Q). One gram of Q occupies $1.4\mathrm{dm}^3$ at STP. (P) on reaction with dry ice in dry ether followed by $\mathrm{H_3O^+}$ forms a compound (Z). 0.1 mole of (Z) will weigh ____ g. (Nearest integer) [4 marks]

Solution:

Concept:

- Water protonates a Grignard reagent to give the hydrocarbon RH: RMgBr + $H_2O \longrightarrow$ RH + Mg(OH)Br. The carbon skeleton of R is retained.
- At STP, one mole of an ideal gas occupies approximately $22.4\,\mathrm{dm}^3$. Reaction of a Grignard reagent with dry ice, followed by acidic work-up, produces RCOOH, adding one carbon atom to the organic group.

Step 1: Find the molar mass of the gas One gram of gas Q occupies $1.4\,\mathrm{dm}^3$. Its amount and molar mass are therefore

$$n_Q = \frac{1.4}{22.4} = 0.0625\,\mathrm{mol},$$

$$M_Q = \frac{1}{0.0625} = 16\,\mathrm{g\,mol^{-1}}.$$

The hydrocarbon of molar mass $16\,\mathrm{g\,mol^{-1}}$ is methane, CH_4. Thus RH is methane and the group R is methyl, CH_3.

Step 2: Identify the Grignard reagent and carboxylation product Consequently, P is methylmagnesium bromide, CH_3MgBr. Its two relevant reactions are

$$\mathrm{CH_3MgBr + H_2O \longrightarrow CH_4 + Mg(OH)Br},$$

$$\mathrm{CH_3MgBr + CO_2 \longrightarrow CH_3COOMgBr}.$$

The second reaction takes place in dry ether. On acidic work-up, the magnesium carboxylate is protonated:

$$\mathrm{CH_3COO^- + H_3O^+ \longrightarrow CH_3COOH + H_2O}.$$

Hence Z is ethanoic acid, CH_3COOH. Dry ice supplies the carboxyl carbon, so Z has two carbon atoms although the initial gas contains only one.

Step 3: Convert the amount of acid into mass Ethanoic acid has molecular formula $C_2H_4O_2$, giving

$$M_Z = 2(12) + 4(1) + 2(16) = 60\,\mathrm{g\,mol^{-1}}.$$

Apply $m = nM$ to the stated amount of Z:

$$m_Z = 0.1\,\mathrm{mol} \times 60\,\mathrm{g\,mol^{-1}} = 6\,\mathrm{g}.$$

Step 4: Conclusion Z is ethanoic acid, and 0.1 mole weighs $\boxed{6\,\mathrm{g}}$. The nearest-integer numerical entry is 6.

Quick Tip to Solve: Hydrolysis identifies the group R through RH; carboxylation then adds exactly one carbon to give RCOOH before you calculate its molar mass.