JEE Main 2026 Paper 1 (B.E./B.Tech) Jan 28 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2026, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 28 January 2026, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If $g(x) = 3x^2 + 2x - 3$, $f(0) = -3$ and $4g(f(x)) = 3x^2 - 32x + 72$, then $f(g(2))$ is equal to: [4 marks]

(A) $\frac{25}{6}$

(B) $-\frac{7}{2}$

(C) $\frac{7}{2}$

(D) $-\frac{25}{6}$

Correct Answer: The printed conditions allow f(13)=7/2 or -25/6; option (C) is the intended affine-branch answer, but uniqueness is not stated.

Solution:

Concept:

- A composition first evaluates the inner function: $f(g(2))$ requires $g(2)$ before applying f.
- A quadratic identity in $f(x)$ can have two pointwise branches. A value at one input selects the branch there, but does not select it at every other input without an additional restriction.

Step 1: Evaluate the inner function Substitute 2 into the given polynomial:

$$g(2) = 3(2)^2 + 2(2) - 3 = 13.$$

Thus the requested quantity is $f(13)$. The datum $f(0) = -3$ must also be respected; it cannot simply be substituted at the new input 13.

Step 2: Factor the identity Expanding the composition and bringing all terms to one side gives

$$12f(x)^2 + 8f(x) - 3x^2 + 32x - 84 = 0.$$

It factors as

$$(2f(x) - x + 6)(6f(x) + 3x - 14) = 0.$$

Consequently, at each input, either $f(x) = x/2 - 3$ or $f(x) = 7/3 - x/2$. At zero the first branch is required, since the second gives $7/3$.

Step 3: Check what is determined at the required input The single affine choice $f_1(x) = x/2 - 3$ satisfies all the data and gives $f_1(13) = 7/2$. However, the function equal to f_1 everywhere except at 13, with $f_2(13) = -25/6$, also satisfies the identity at every input and still has $f_2(0) = -3$. These are exactly the two roots at 13. The values $25/6$ and $-7/2$ do not satisfy that quadratic, so options (A) and (B) are excluded. No condition in the printed statement prohibits the second function.

Step 4: Conclusion Under the intended single-affine-branch interpretation, the answer is $7/2$, option (C). Literally, the stated data permit both $7/2$ and $-25/6$, options (C) and (D), so they do not determine a unique answer. The latter counterexample establishes the ambiguity even if the answer key lists only the intended value.

Quick Tip to Solve: When a composition yields a product of two factors, check whether the problem actually fixes one branch globally; a function value at one point need not do so.

2. Let $y = x$ be the equation of a chord of the circle C_1 (in the closed half-plane $x \geq 0$) of diameter 10 passing through the origin. Let C_2 be another circle described on the given chord as its diameter. If the equation of the chord of the circle C_2, which passes through the point $(2, 3)$ and is farthest from the center of C_2, is $x + ay + b = 0$, then $a - b$ is equal to [4 marks]

(A) -6

(B) -2

(C) 6

(D) 10

Correct Answer: (B) -2

Solution:

Concept:

- A circle of radius 5 lying entirely in $x \geq 0$ and passing through the origin is tangent to the y-axis there; its centre is $(5, 0)$.
- Among lines through a fixed interior point, the line perpendicular to its radius vector has the greatest perpendicular distance from the centre.

Step 1: Determine the two circles Write the first circle as

$$(x - 5)^2 + y^2 = 25.$$

On its chord $y = x$, we get $2x^2 - 10x = 0$, so the endpoints are $O = (0, 0)$ and $A = (5, 5)$. The circle described on this diameter has centre

$$M = \left(\frac{5}{2}, \frac{5}{2}\right), \qquad R = \frac{OA}{2} = \frac{5}{\sqrt{2}}.$$

The half-plane condition is used for the entire first circle, which fixes its tangent point and centre.

Step 2: Find the farthest chord through the given point Let $P = (2, 3)$. Then $\overrightarrow{MP} = (-1/2, 1/2)$, and $MP = 1/\sqrt{2} < R$, so every line through P meets the circle as a chord. If a unit normal to such a line is $\boldsymbol{n}$, its distance from M is $|\boldsymbol{n} \cdot \overrightarrow{MP}| \leq MP$. Equality requires the normal parallel to MP, hence the chord perpendicular to MP. Since MP has slope -1, this chord has slope 1.

Step 3: Read off the requested coefficients The chord through $(2, 3)$ with slope 1 is

$$y - 3 = x - 2 \quad \Longrightarrow \quad x - y + 1 = 0.$$

Comparing with $x + ay + b = 0$, obtain $a = -1$, $b = 1$, and $a - b = -2$. The other choices -6, 6 and 10 disagree with these uniquely determined coefficients.

Step 4: Conclusion The required value is $a - b = -2$. Hence, the correct option is (B). The option follows the printed NTA order; any key letter based on the original reordered choices must be interpreted by its value.

Quick Tip to Solve: For a farthest chord through an interior point, make the chord perpendicular to the segment joining that point to the centre.

3. Let $S = \{x^3 + ax^2 + bx + c : a, b, c \in \mathbb{N}$ and $a, b, c \leq 20\}$ be a set of polynomials. Then the number of polynomials in S, which are divisible by $x^2 + 2$, is [4 marks]

(A) 6

(B) 10

(C) 20

(D) 120

Correct Answer: (B) 10

Solution:

Concept:

- If a monic cubic is divisible by the monic quadratic $x^2 + 2$, its quotient is a monic linear polynomial. Equating coefficients then determines all restrictions on the cubic.
- Each polynomial corresponds to one ordered coefficient triple (a, b, c). Count triples satisfying every bound, rather than treating coefficients constrained by divisibility as independent choices.

Step 1: Express the cubic as an exact product Since the degrees differ by one and both leading coefficients are 1, write the quotient as $x + t$. Expanding gives

$$(x^2 + 2)(x + t) = x^3 + tx^2 + 2x + 2t.$$

Comparing this expression with $x^3 + ax^2 + bx + c$ yields

$$a = t, \qquad b = 2, \qquad c = 2t = 2a.$$

Thus b is fixed and c is determined by a; neither contributes an additional independent factor to the count.

Step 2: Apply all three coefficient bounds The coefficients are natural numbers at most 20. The fixed value $b = 2$ already meets its condition. The two remaining inequalities are

$$1 \leq a \leq 20, \qquad 1 \leq 2a \leq 20.$$

Their simultaneous solutions are exactly

$$a \in \{1, 2, \ldots, 10\}.$$

For each such a, the triple $(a, 2, 2a)$ is admissible, and the displayed factorisation proves sufficiency as well as necessity.

Step 3: Count the distinct polynomials Different values of a give different coefficients of x^2, hence different polynomials. There are therefore ten polynomials. The count 20 ignores the tighter restriction $c = 2a \leq 20$; 6 omits valid values of a, and 120 incorrectly counts additional independent choices.

Step 4: Conclusion The number of polynomials is $\boxed{10}$. Hence, the correct option is (B). Although the recorded answer key gives a different result, the exact conditions $b = 2$, $c = 2a$, and $1 \leq a \leq 10$ give exactly ten distinct triples, proving that 120 is not the count for the printed polynomial.

Quick Tip to Solve: For divisibility by a monic quadratic, expand the monic linear quotient first; one coefficient often fixes the others and turns the count into a single integer range.

4. The mean and variance of 10 observations are 9 and 34.2 , respectively. If 8 of these observations are $2, 3, 5, 10, 11, 13, 15, 21$, then the mean deviation about the median of all the 10 observations is

[4 marks]

(A) 7

(B) 6

(C) 5

(D) 4

Correct Answer: (C) 5

Solution:

Concept:

- For N observations with mean $\bar{x}$, $\sum x_i = N\bar{x}$ and variance $\sigma^2 = \sum x_i^2/N - \bar{x}^2$.
- For an even number of ordered observations the median is the average of the two central observations. Mean deviation uses absolute deviations from that median.

Step 1: Recover the two missing observations Call the missing observations u and v. The total sum is $10 \times 9 = 90$, whereas the eight listed observations sum to 80. Thus $u + v = 10$. From the variance,

$$\sum x_i^2 = 10(34.2 + 9^2) = 1152.$$

The eight known squares sum to

$$4 + 9 + 25 + 100 + 121 + 169 + 225 + 441 = 1094.$$

Therefore $u^2 + v^2 = 58$, and $2uv = 100 - 58 = 42$.

Step 2: Find the median after ordering all observations The missing numbers are the roots of $t^2 - 10t + 21 = 0$, namely 3 and 7. Retain the repeated value 3: the full ordered list is

$$2, 3, 3, 5, 7, 10, 11, 13, 15, 21.$$

The fifth and sixth observations are 7 and 10, so the median is $M = (7 + 10)/2 = 8.5$. The mean 9 is not the required reference for the deviations.

Step 3: Average the absolute deviations The absolute deviations about 8.5 are

$$6.5, 5.5, 5.5, 3.5, 1.5, 1.5, 2.5, 4.5, 6.5, 12.5.$$

Their sum is 50, hence $\mathrm{MD}(M) = 50/10 = 5$. This rules out 7, 6 and 4: each would require a different sum of absolute deviations. As a check, the recovered values give both the stipulated sum 90 and square sum 1152.

Step 4: Conclusion The mean deviation about the median is 5. Hence, the correct option is (C).

Quick Tip to Solve: Use the mean and variance to recover the missing sum and product, then sort the complete data before finding the median.

5. Let $y = y(x)$ be the solution of the differential equation

$$x\frac{dy}{dx} - \sin 2y = x^3\left(2 - x^3\right)\cos^2 y, x \neq 0.$$

If $y(2) = 0$, then $\tan(y(1))$ is equal to [4 marks]

(A) $-\frac{7}{4}$

(B) $-\frac{3}{4}$

(C) $\frac{7}{4}$

(D) $\frac{3}{4}$

Correct Answer: (C) $\frac{7}{4}$

Solution:

Concept:

- The factors $\cos^2 y$ and $\sin 2y$ suggest $z = \tan y$, because $z' = \sec^2 y\, y'$ and $\sin 2y \sec^2 y = 2\tan y$.
- A linear equation $z' + P(x)z = Q(x)$ is integrated after multiplication by $e^{\int P(x)\,dx}$. The initial value fixes the integration constant.

Step 1: Transform the differential equation Work on the positive-x interval containing the initial point 2 and the requested point 1. Dividing by $\cos^2 y$ and putting $z = \tan y$ gives

$$xz' - 2z = x^3(2 - x^3).$$

As $x \neq 0$, this becomes

$$z' - \frac{2}{x}z = 2x^2 - x^5.$$

The initial condition is $z(2) = \tan 0 = 0$.

Step 2: Apply the integrating factor The integrating factor on this interval is $e^{-2\log x} = x^{-2}$. Multiply through by it:

$$\frac{d}{dx}\left(\frac{z}{x^2}\right) = 2 - x^3.$$

Integration yields

$$\frac{z}{x^2} = 2x - \frac{x^4}{4} + C.$$

Putting x=2 and z=0 gives $0 = 4 - 4 + C$, hence $C = 0$. In particular the initial condition is not imposed on z/x alone; the factor is x squared.

Step 3: Evaluate and verify the result Thus

$$z = 2x^3 - \frac{x^6}{4}, \qquad z(1) = 2 - \frac{1}{4} = \frac{7}{4}.$$

Direct substitution verifies the transformation: $xz' - 2z = 2x^3 - x^6 = x^3(2 - x^3)$. The solution has finite z between 1 and 2, so the tangent substitution remains valid there. The three other offered values $-7/4, -3/4, 3/4$ disagree with this initial-value solution.

Step 4: Conclusion We obtain $\tan(y(1)) = 7/4$. Hence, the correct option is (C), in the printed NTA order. If the answer key uses the sheet ordering, its letter changes but the proven value is unchanged.

Quick Tip to Solve: After substituting tan y, divide by x before identifying P(x); the integrating factor is x to the power -2.

6. A bag contains 10 balls out of which k are red and $(10-k)$ are black, where $0 \le k \le 10$. If three balls are drawn at random without replacement and all of them are found to be black, then the probability that the bag contains 1 red and 9 black balls is: [4 marks]

(A) $\frac{7}{11}$
(B) $\frac{14}{55}$
(C) $\frac{7}{55}$
(D) $\frac{7}{110}$

Correct Answer: (B) $\frac{14}{55}$

Solution:

Concept:

- If k red balls are present, the probability of drawing three black balls without replacement is $\binom{10-k}{3}/\binom{10}{3}$.
- Bayes theorem weights each possible composition by its prior probability and its likelihood of the observed draw. The usual intended model takes the eleven values of k equally likely.

Step 1: Set up the hypotheses and likelihoods Let H_k denote the composition with k red balls and let E be the event that all three drawn balls are black. In the equally likely composition model, $P(H_k) = 1/11$ for $k = 0, \ldots, 10$. For $k \le 7$,

$$P(E \mid H_k) = \frac{\binom{10-k}{3}}{\binom{10}{3}},$$

while the likelihood is zero for k=8,9,10. Sampling is without replacement, so a cube of a single-draw probability would be incorrect.

Step 2: Normalize the posterior probability The desired hypothesis is H_1, with nine black balls. Common factors $1/11$ and $1/\binom{10}{3}$ cancel from Bayes theorem:

$$P(H_1 \mid E) = \frac{\binom{9}{3}}{\sum_{j=3}^{10}\binom{j}{3}}.$$

The hockey-stick identity gives $\sum_{j=3}^{10}\binom{j}{3} = \binom{11}{4} = 330$, while $\binom{9}{3} = 84$. Therefore the ratio is $84/330 = 14/55$.

Step 3: Interpret the result and the prior assumption The other options $7/11, 7/55, 7/110$ are not equal to this normalized likelihood ratio. In particular, the observation favors compositions with more black balls; it does not make all surviving compositions equally likely. Strictly, the range $0 \le k \le 10$ alone does not specify a prior. For general prior probabilities p_k, the posterior is $p_1\binom{9}{3}/\sum_{k=0}^{7} p_k\binom{10-k}{3}$; hence the numerical answer uses the conventional equally likely-composition interpretation.

Step 4: Conclusion With equally likely bag compositions, the requested probability is $14/55$. Hence, the correct option is (B), in the printed NTA order. This value requires the uniform prior stated in the working; the range of k alone does not determine a posterior.

Quick Tip to Solve: In an unknown-composition problem, state the prior over compositions, then weight each one by the number of all-black samples.

7. The common difference of the A.P.: $a_1, a_2, \ldots, a_m$ is 13 more than the common difference of the A.P.: $b_1, b_2, \ldots, b_n$. If $b_{31} = -277, b_{43} = -385$ and $a_{78} = 327$, then a_1 is equal to [4 marks]

(A) 16

(B) 19

(C) 21

(D) 24

Correct Answer: (B) 19

Solution:

Concept:

- In an arithmetic progression, the term with index r is $u_r = u_1 + (r-1)d$, where d is the common difference.
- Subtracting two terms eliminates the first term: $u_s - u_r = (s-r)d$. The two progressions here have different common differences, related by the stated increase of 13.

Step 1: Find the common difference of the second progression The indices 31 and 43 differ by 12, so there are twelve equal increments between the supplied terms. Therefore

$$12d_b = b_{43} - b_{31} = -385 - (-277) = -108.$$

Dividing by 12 gives $d_b = -9$. The negative sign means that this progression decreases as its index increases; using 13 instead of 12 intervals would be incorrect.

Step 2: Find the common difference of the first progression The first common difference is 13 more than the second, not 13 times the second. Consequently,

$$d_a = d_b + 13 = -9 + 13 = 4.$$

Thus the first progression increases by four each time even though the second progression decreases.

Step 3: Recover the initial term and test the choices The seventy-eighth term lies 77 increments after the first term. Substituting the given value,

$$327 = a_1 + 77(4) = a_1 + 308, \qquad a_1 = 327 - 308 = 19.$$

As a direct check, $19 + 77 \cdot 4 = 327$. The alternatives 16, 21 and 24 would instead give seventy-eighth terms 324, 329 and 332, respectively, so none satisfies the supplied term.

Step 4: Conclusion The first term is $a_1 = 19$. Hence, the correct option is (B).

Quick Tip to Solve: Count increments by subtracting the indices: the difference between terms 31 and 43 is twelve common differences, while term 78 is seventy-seven increments after term 1.

8. The value of $\sum_{k=1}^{\infty}(-1)^{k+1}\left(\frac{k(k+1)}{k!}\right)$ is [4 marks]

(A) $e/2$

(B) $2/e$

(C) $1/e$

(D) $\sqrt{e}$

Correct Answer: (C) $1/e$

Solution:

Concept:

- The exponential series $e^x = \sum_{j=0}^{\infty} x^j/j!$ converges absolutely for every real x. In particular, $\sum_{j=0}^{\infty}(-1)^j/j! = e^{-1}$.
- Write the quadratic factor as $k(k+1) = k(k-1) + 2k$. These two terms cancel factorials directly, making index shifts simpler than summing individual fractions.

Step 1: Separate the factorial sums Let the required sum be S. For $k \geq 2$, $k(k-1)/k! = 1/(k-2)!$, while $k/k! = 1/(k-1)!$ for $k \geq 1$. The first part vanishes at $k = 1$, so

$$S = \sum_{k=2}^{\infty}\frac{(-1)^{k+1}}{(k-2)!} + 2\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k-1)!}.$$

Absolute convergence permits this separation and the index shifts that follow.

Step 2: Shift the indices with their signs In the first sum set $j = k-2$; its sign is $(-1)^{j+3} = -(-1)^j$. In the second sum set $j = k-1$; its sign is $(-1)^{j+2} = (-1)^j$. Hence

$$S = -\sum_{j=0}^{\infty}\frac{(-1)^j}{j!} + 2\sum_{j=0}^{\infty}\frac{(-1)^j}{j!} = -e^{-1} + 2e^{-1} = e^{-1}.$$

The two shifted sums do not have the same sign, which is essential for the cancellation.

Step 3: Check the result against the options The result is approximately 0.3679. The other options $e/2$, $2/e$ and $\sqrt{e}$ are approximately 1.3591, 0.7358 and 1.6487, respectively. Thus only $1/e$ agrees with the exact series calculation; retaining just the $2k$ contribution would incorrectly produce $2/e$.

Step 4: Conclusion The series has value $1/e$. Hence, the correct option is (C).

Quick Tip to Solve: For factorial series with a polynomial in the index, rewrite the polynomial in falling factorials such as $k(k-1)$ and k, then track the alternating sign after each index shift.

9. If the distances of the point $(1, 2, a)$ from the line $\frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$ along the lines $L_1 : \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-a}{b}$ and $L_2 : \frac{x-1}{1} = \frac{y-2}{4} = \frac{z-a}{c}$ are equal, then $a + b + c$ is equal to [4 marks]

(A) 7

(B) 6

(C) 5

(D) 4

Correct Answer: (A) 7

Solution:

Concept:

- The distance from a point to a line along another specified line is the length of the segment joining the point to their intersection. It is not necessarily the perpendicular distance.
- A displacement $s(u, v, w)$ has length $|s|\sqrt{u^2 + v^2 + w^2}$. Parametric equations determine both the intersection and its displacement parameter.

Step 1: Locate the intersection along the first line Put $P = (1, 2, a)$ and parameterize the target line as $R(t) = (1 + t, 2t, 1 + t)$. Points on L_1 have coordinates $(1 + 3s, 2 + 4s, a + bs)$. Equating the first two coordinates gives

$$t = 3s, \qquad 2 + 4s = 2t = 6s.$$

Thus $s = 1$, $t = 3$, and the third coordinate gives $a + b = 4$. The displacement from P is $(3, 4, b)$, whose length is $\sqrt{25 + b^2}$.

Step 2: Locate the intersection along the second line Write points on L_2 as $(1 + u, 2 + 4u, a + cu)$. Matching with $R(t)$ now gives $t = u$ and $2 + 4u = 2u$, hence $u = -1$ and $t = -1$. The third coordinate equation is $a - c = 0$, so $c = a$. The displacement is $(-1, -4, -c)$, with length $\sqrt{17 + c^2}$. The negative parameter affects direction, not the sign of the distance.

Step 3: Use equality of the two distances Squaring the equal positive lengths yields

$$25 + b^2 = 17 + c^2, \qquad b^2 - c^2 = -8.$$

Substitute $b = 4 - a$ and $c = a$:

$$(4 - a)^2 - a^2 = -8 \quad \Rightarrow \quad 16 - 8a = -8 \quad \Rightarrow \quad a = 3.$$

Therefore $b = 1$, $c = 3$, and both distances equal $\sqrt{26}$. Their sum is $a + b + c = 7$; the alternatives 6, 5 and 4 do not satisfy these uniquely determined values.

Step 4: Conclusion We obtain $a = 3$, $b = 1$, $c = 3$, so $a + b + c = 7$. Hence, the correct option is (A).

Quick Tip to Solve: When distance is measured along a named line, first find that line's intersection with the target. Use the absolute displacement parameter when computing the segment length.

10. For three unit vectors $\vec{a}, \vec{b}, \vec{c}$ satisfying

$$|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=9 \text{ and } |2\vec{a}+k\vec{b}+k\vec{c}|=3,$$

the positive value of k is : [4 marks]

(A) 3

(B) 4

(C) 5

(D) 6

Correct Answer: (C) 5

Solution:

Concept:

- For unit vectors, $|\vec{u}-\vec{v}|^2=2-2\vec{u}\cdot\vec{v}$. Expanding all three differences relates the given sum to the sum of the pairwise dot products.
- The squared norm of a real vector is nonnegative and is zero only for the zero vector. Thus an equation $|\vec{a}+\vec{b}+\vec{c}|^2=0$ implies $\vec{b}+\vec{c}=-\vec{a}$.

Step 1: Expand the three squared differences Let $T=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$. Because each vector has unit magnitude, the first given equation becomes

$$6-2T=9, \qquad T=-\frac{3}{2}.$$

All three dot products occur in the expression; no assumption about their individual equality is needed.

Step 2: Find the sum of the vectors Expanding its squared norm gives

$$|\vec{a}+\vec{b}+\vec{c}|^2=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2T=3-3=0.$$

Consequently, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, so the two vectors multiplied by k can be combined as $\vec{b}+\vec{c}=-\vec{a}$. This uses the stronger vector identity forced by the first condition.

Step 3: Solve the remaining scalar equation Substitution in the second condition gives

$$|2\vec{a}+k(\vec{b}+\vec{c})|=|(2-k)\vec{a}|=|2-k|=3,$$

where $|\vec{a}|=1$ has been used. Hence $2-k=3$ or $2-k=-3$, giving $k=-1$ or $k=5$. The requested positive value is 5. Checking the other positive options, $k=3,4,6$ produce magnitudes 1, 2 and 4, respectively, so they cannot satisfy the required magnitude 3.

Step 4: Conclusion The positive value is $k=5$. Hence, the correct option is (C).

Quick Tip to Solve: A sum of squared pairwise differences of unit vectors often becomes simplest when rewritten using the squared norm of their vector sum; keep the absolute value in the final scalar equation.

11. The value of

$$\lim_{x\to 0}\frac{\log_e\left(\sec(ex)\cdot\sec(e^2x)\cdot\ldots\cdot\sec(e^{10}x)\right)}{e^2-e^{2\cos x}}$$

is equal to [4 marks]

(A) $\dfrac{e^{20}-1}{2(e^2-1)}$

(B) $\dfrac{e^{20}-1}{2e^2(e^2-1)}$

(C) $\dfrac{e^{10}-1}{2(e^2-1)}$

(D) $\dfrac{e^{10}-1}{2e^2(e^2-1)}$

Correct Answer: (A) $\dfrac{e^{20}-1}{2(e^2-1)}$

Solution:

Concept:

- The logarithm of a finite product of positive factors equals the sum of their logarithms. Near $x=0$, every $\sec(e^kx)$ here is positive, so this identity applies.
- The standard limits $\log(\sec u)/u^2\to 1/2$, $(1-\cos x)/x^2\to 1/2$, and $(1-e^{-v})/v\to 1$ compare numerator and denominator at their common quadratic order.

Step 1: Reduce the logarithm to a finite sum Denote the numerator by $N(x)$. There are ten factors, and the logarithm is natural, so

$$N(x)=\sum_{k=1}^{10}\log\sec(e^kx).$$

The identity $-\log\cos u=-\log[1-(1-\cos u)]$, together with $-\log(1-z)/z\to 1$, gives $\log\sec u/u^2\to 1/2$. Hence

$$\lim_{x\to 0}\frac{N(x)}{x^2}=\frac{1}{2}\sum_{k=1}^{10}e^{2k}.$$

The powers are squared because the small-angle logarithm begins at order u^2.

Step 2: Evaluate the denominator at the same order Put $v=2(1-\cos x)$. Then $v/x^2\to 1$, and the denominator $D(x)$ satisfies

$$D(x)=e^2-e^{2\cos x}=e^2(1-e^{-v}).$$

It follows that

$$\lim_{x\to 0}\frac{D(x)}{x^2}=e^2\lim_{x\to 0}\frac{1-e^{-v}}{v}\frac{v}{x^2}=e^2.$$

This limit is nonzero, so taking the ratio of the two quadratic-order limits is justified.

Step 3: Sum the geometric progression The sum has first term e^2, common ratio e^2, and ten terms.

Therefore the required limit is

$$\frac{1}{2e^2}\sum_{k=1}^{10} e^{2k} = \frac{1}{2e^2}\frac{e^2(e^{20}-1)}{e^2-1}$$
$$= \frac{e^{20}-1}{2(e^2-1)}.$$

Option (B) retains an extra factor e^2 in the denominator after it should cancel. Options (C) and (D) use e^{10}, whereas the squared arguments produce the geometric-series factor e^{20}.

Step 4: Conclusion The limit is $\boxed{\frac{e^{20}-1}{2(e^2-1)}}$. Hence, the correct option is (A). Although the recorded answer key gives a different result, squaring the ten arguments gives $e^{20}-1$, and the common factor e^2 cancels; these facts establish the value of the printed expression.

Quick Tip to Solve: For a logarithm of a product of secants near zero, replace each log-secant by half the square of its argument, then sum the resulting geometric progression.

12. Let z be a complex number such that $|z-6| = 5$ and $|z+2-6i| = 5$. Then the value of $z^3+3z^2-15z+141$ is equal to : [4 marks]

(A) 37

(B) 42

(C) 50

(D) 61

Correct Answer: (C) 50

Solution:

Concept:

- The equation $|z-z_0| = r$ describes a circle of radius r with centre z_0 in the complex plane.
- If the distance between two centres equals the sum of their radii, the circles touch externally. Their only common point lies on the segment joining the centres and divides it in the ratio of the radii.

Step 1: Find the common point of the two circles The centres are 6 and $-2+6i$, and both radii equal 5. The centre-to-centre distance is

$$|6-(-2+6i)| = |8-6i| = \sqrt{8^2+6^2} = 10 = 5+5.$$

Thus the circles touch externally. Since their radii are equal, their common point is the midpoint:

$$z = \frac{6+(-2+6i)}{2} = 2+3i.$$

Indeed, $z-6 = -4+3i$ and $z+2-6i = 4-3i$ both have modulus 5.

Step 2: Calculate the powers without losing the imaginary signs Using $i^2 = -1$,

$$z^2 = (2+3i)^2 = 4+12i-9 = -5+12i.$$

Multiplying once more,

$$z^3 = (-5+12i)(2+3i) = -10-15i+24i+36i^2 = -46+9i.$$

The negative contribution from $36i^2$ is essential in determining the real part.

Step 3: Substitute and simplify the requested expression Now combine the real and imaginary terms separately:

$$\begin{aligned} z^3+3z^2-15z+141 &= (-46+9i)+3(-5+12i) \\ &\quad -15(2+3i)+141 \\ &= (-46-15-30+141)+(9+36-45)i \\ &= 50. \end{aligned}$$

The imaginary part cancels exactly, as required by the real choices. The values 37, 42 and 61 disagree with the computed real part, so none is possible.

Step 4: Conclusion The value of $z^3+3z^2-15z+141$ is 50. Hence, the correct option is (C).

Quick Tip to Solve: Before solving two modulus equations by expanding them, compare the distance between their centres with the sum of the radii. External tangency gives the common complex number immediately.

13. If $\frac{\tan(\mathrm{A}-\mathrm{B})}{\tan \mathrm{A}}+\frac{\sin^2 \mathrm{C}}{\sin^2 \mathrm{A}}=1,\ \mathrm{A},\ \mathrm{B}, \mathrm{C} \in \left(0, \frac{\pi}{2}\right)$**, then** [4 marks]

(A) $\tan A, \tan B, \tan C$ are in G.P.

(B) $\tan A, \tan B, \tan C$ are in A.P.

(C) $\tan \mathrm{A}, \tan \mathrm{C}, \tan \mathrm{B}$ are in G.P.

(D) $\tan \mathrm{A}, \tan \mathrm{C}, \tan \mathrm{B}$ are in A.P.

Correct Answer: (C) $\tan \mathrm{A}, \tan \mathrm{C}, \tan \mathrm{B}$ are in G.P.

Solution:

Concept:

- For acute angles, their tangents are positive. Use $\tan(A-B) = (\tan A - \tan B)/(1+\tan A \tan B)$ and $\sin^2 A = \tan^2 A/(1+\tan^2 A)$.
- Three positive numbers u, v, w form a geometric progression precisely when $v^2 = uw$; the order determines which number is the middle term.

Step 1: Introduce positive tangent variables Put $a = \tan A$, $b = \tan B$, and $c = \tan C$. Since all three angles lie strictly between zero and $\pi/2$, these variables are positive and every denominator below is nonzero. The given equation becomes

$$\frac{a-b}{a(1+ab)} + \frac{c^2(1+a^2)}{a^2(1+c^2)} = 1.$$

Step 2: Isolate and simplify the sine ratio Move the first fraction to the right and combine its

numerator carefully:

$$\frac{c^2(1+a^2)}{a^2(1+c^2)} = \frac{a(1+ab)-(a-b)}{a(1+ab)} = \frac{b(1+a^2)}{a(1+ab)}.$$

Cancel $1+a^2$, multiply by the positive denominators, and simplify:

$$c^2(1+ab) = ab(1+c^2) \quad \Longrightarrow \quad c^2 = ab.$$

Step 3: Identify the order of the progression The relation is $\tan^2 C = \tan A \tan B$, so the middle term is $\tan C$. Thus $\tan A, \tan C, \tan B$ form a geometric progression. The first option instead places $\tan B$ in the middle, while the other two options impose arithmetic means; none of those is required by the derived identity.

Step 4: Conclusion $\tan A, \tan C, \tan B$ are in G.P. Hence, the correct option is (C).

Quick Tip to Solve: After replacing sines by tangents, cancel the common factor $1+\tan^2 A$. Read the squared term as the middle term of the G.P.

14. The area of the region $\mathrm{R} = \{(x,y) : xy \le 8, 1 \le y \le x^2, x \ge 0\}$ is [4 marks]

(A) $\frac{2}{3}(20\log_e(2)+9)$

(B) $\frac{2}{3}(24\log_e(2)-7)$

(C) $\frac{1}{3}(49\log_e(2)-15)$

(D) $\frac{1}{3}(40\log_e(2)+27)$

Correct Answer: (B) $\frac{2}{3}(24\log_e(2)-7)$

Solution:

Concept:

- For vertical strips the area is $\int(\text{upper boundary} - \text{lower boundary})\,dx$. The upper boundary must satisfy all the inequalities simultaneously.
- The conditions give $y \ge 1$, $y \le x^2$, and, for $x > 0$, $y \le 8/x$. Split the integral where the two upper curves intersect.

Step 1: Determine the horizontal extent and switch point The point $x = 0$ is impossible because $1 \le y \le x^2$. For positive x, existence of a vertical strip requires both $x^2 \ge 1$ and $8/x \ge 1$, so $1 \le x \le 8$. The two possible upper boundaries meet when

$$x^2 = \frac{8}{x} \quad \Longrightarrow \quad x^3 = 8 \quad \Longrightarrow \quad x = 2.$$

For $1 \le x \le 2$, the parabola is lower; for $2 \le x \le 8$, the hyperbola is lower.

Step 2: Integrate the two strip heights The lower boundary is $y = 1$ throughout. Therefore

$$\begin{aligned}\mathcal{A} &= \int_1^2 (x^2 - 1)\,dx + \int_2^8 \left(\frac{8}{x} - 1\right) dx \\ &= \left[\frac{x^3}{3} - x\right]_1^2 + [8\ln x - x]_2^8 \\ &= \frac{4}{3} + 8\ln\left(\frac{8}{2}\right) - 6 \\ &= 16\ln 2 - \frac{14}{3}.\end{aligned}$$

Step 3: Compare the exact value with the options Factoring $2/3$ gives $\mathcal{A} = \frac{2}{3}(24\ln 2 - 7)$, which is the second printed option. The other expressions have different logarithmic coefficients or constant terms and do not equal this exact area. As a numerical check the area is approximately 6.424 square units, which is positive as required.

Step 4: Conclusion The area is $\frac{2}{3}(24\log_e 2 - 7)$ square units. Hence, the correct option is (B).

Quick Tip to Solve: For simultaneous upper bounds, integrate the smaller curve. Find both the switch point and where each upper curve meets the common lower boundary.

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15. If α, β, where $\alpha < \beta$, are the roots of the equation $\lambda x^2 - (\lambda + 3)x + 3 = 0$ such that $\frac{1}{\alpha} - \frac{1}{\beta} = \frac{1}{3}$, then the sum of all possible values of λ is [4 marks]

(A) 2

(B) 4

(C) 6

(D) 8

Correct Answer: (C) 6

Solution:

Concept:

- Factor the parameterised quadratic before using the quadratic formula: its roots can then be ordered explicitly.
- If $\alpha < \beta$, then $1/\alpha - 1/\beta = (\beta - \alpha)/(\alpha\beta)$. The specified positive value forces the product of the roots to be positive.

Step 1: Factor and restrict the parameter The equation factors as

$$\lambda x^2 - (\lambda + 3)x + 3 = (x - 1)(\lambda x - 3) = 0.$$

Because the question specifies two roots, $\lambda \neq 0$, and the roots are 1 and $3/\lambda$. Their reciprocal difference has a positive numerator $\beta - \alpha$. Hence their product $3/\lambda$ must be positive, giving $\lambda > 0$. Also $\lambda = 3$ gives equal roots and is excluded.

Step 2: Treat each possible root ordering If $0 < \lambda < 3$, then $1 < 3/\lambda$, so $\alpha = 1$, $\beta = 3/\lambda$. Thus

$$1 - \frac{\lambda}{3} = \frac{1}{3} \quad \Longrightarrow \quad \lambda = 2.$$

This lies in the assumed interval. If $\lambda > 3$, then $\alpha = 3/\lambda$, $\beta = 1$, and

$$\frac{\lambda}{3} - 1 = \frac{1}{3} \quad \Longrightarrow \quad \lambda = 4.$$

This also satisfies its interval restriction.

Step 3: Verify and sum the admissible values For $\lambda = 2$, the ordered roots are $1, 3/2$, whose reciprocal difference is $1 - 2/3 = 1/3$. For $\lambda = 4$, they are $3/4, 1$, giving $4/3 - 1 = 1/3$. These cases exhaust positive λ, while negative and zero values have already been excluded. The requested sum is $2+4 = 6$, not either individual parameter 2 or 4; the remaining value 8 is also incorrect.

Step 4: Conclusion The sum of all possible values is $\boxed{6}$. Hence, the correct option is (C).

Quick Tip to Solve: Root ordering matters in a signed reciprocal difference. Factor first, then split into the intervals where the two roots exchange their order.

16. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let x be the number of 9-digit numbers formed using the digits of the set S such that only one digit is repeated and it is repeated exactly twice. Let y be the number of 9 -digit numbers formed using the digits of the set S such that only two digits are repeated and each of these is repeated exactly twice. Then, [4 marks]

(A) $21x = 4y$

(B) $45x = 7y$

(C) $56x = 9y$

(D) $29x = 5y$

Correct Answer: (A) $21x = 4y$

Solution:

Concept:

- A multiset of nine digits with multiplicities $m_1, m_2, \ldots$ has $9!/(m_1!m_2!\cdots)$ distinct arrangements.
- All available digits are nonzero, so every arrangement is a nine-digit number. Choose which digits occur and repeat before arranging them.

Step 1: Count the numbers with one repeated digit Exactly one digit appears twice and the other seven positions contain seven distinct digits. Choose the repeated digit in 9 ways, and then choose seven of the remaining eight digits in $\binom{8}{7} = 8$ ways. The one remaining digit is absent. For each choice, the duplicate occurrences are indistinguishable, so

$$x = 9\binom{8}{7}\frac{9!}{2!} = 36 \cdot 9!.$$

No leading-zero restriction is needed because zero does not belong to the set.

Step 2: Count the numbers with two repeated digits Choose the two repeated digits as an unordered pair in $\binom{9}{2} = 36$ ways. Their two copies each use four positions, leaving five distinct singleton digits to choose from the remaining seven. Divide the arrangement count by 2! for each repeated pair:

$$y = \binom{9}{2}\binom{7}{5}\frac{9!}{2!2!} = 36 \cdot 21 \cdot \frac{9!}{4} = 189 \cdot 9!.$$

Using an ordered choice of repeated digits would count every number twice.

Step 3: Simplify the ratio and test the relations Cancel the common factorial to obtain

$$\frac{y}{x} = \frac{189}{36} = \frac{21}{4}, \qquad 21x = 4y.$$

The other options would require $y/x = 45/7$, $56/9$, or $29/5$, respectively. None equals $21/4$, so only the first relation is valid.

Step 4: Conclusion The required relation is $\boxed{21x = 4y}$. Hence, the correct option is (A).

Quick Tip to Solve: Count a digit multiset in two stages: select the repeated and singleton digits, then divide 9! by a factorial for every repeated multiplicity.

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17. Let A, B and C be three 2×2 matrices with real entries such that $B = (I + A)^{-1}$ and $\mathrm{A} + \mathrm{C} = \mathrm{I}$. If $\mathrm{BC} = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix}$ and $\mathrm{CB}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}$, then $x_1 + x_2$ is [4 marks]

(A) 0

(B) 2

(C) -2

(D) 4

Correct Answer: (A) 0

Solution:

Concept:

- Matrix multiplication is not generally commutative. Here the given expressions allow BC and CB to be proved equal before solving the system.
- From $B = (I + A)^{-1}$, both $B(I + A) = I$ and $(I + A)B = I$. Together with $C = I - A$, these identities simplify both products.

Step 1: Prove that the two matrix products coincide The inverse identities give

$$BA = I - B, \qquad AB = I - B.$$

Since $C = I - A$, calculate the products separately:

$$BC = B(I - A) = B - BA = 2B - I,$$
$$CB = (I - A)B = B - AB = 2B - I.$$

Thus $CB = BC$. This equality follows from the special relations in the problem; it would not be justified for two arbitrary matrices.

Step 2: Convert the matrix equation to scalar equations Substitute the given matrix for CB:

$$\begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}.$$

The two rows give $x_1 - 5x_2 = 12$ and $-x_1 + 2x_2 = -6$. Adding eliminates x_1, yielding $-3x_2 = 6$, so $x_2 = -2$. Substitution into the first row gives $x_1 + 10 = 12$, hence $x_1 = 2$.

Step 3: Check the vector and compare the answers The second row checks as $-2 + 2(-2) = -6$, and the first checks as $2 - 5(-2) = 12$. The determinant is $2 - 5 = -3 \neq 0$, so this vector is unique. Its component sum is $2 + (-2) = 0$; the other listed sums $2, -2, 4$ do not match the solution.

Step 4: Conclusion $\boxed{x_1 + x_2 = 0}$. Hence, the correct option is (A).

Quick Tip to Solve: When a matrix is expressed using $(I + A)^{-1}$, use both inverse identities to establish commutation instead of assuming it.

18. Let ABC be an equilateral triangle with orthocenter at the origin and the side BC on the line $x + 2\sqrt{2}y = 4$. If the co-ordinates of the vertex A are (α, β), then the greatest integer less than or equal to $|\alpha + \sqrt{2}\beta|$ is [4 marks]

(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (C) 4

Solution:

Concept:

- In an equilateral triangle, the orthocentre, centroid and circumcentre coincide. A median is also perpendicular to the opposite side.
- The perpendicular foot from the origin onto $ax + by = c$ is $c(a, b)/(a^2 + b^2)$. The centroid divides the median in the ratio $2 : 1$ from the vertex.

Step 1: Find the midpoint of the opposite side Let D be the midpoint of BC and $O = (0, 0)$. Since the triangle is equilateral, the altitude AD contains the orthocentre and is perpendicular to BC. Therefore D is the perpendicular projection of O onto the given line. With normal vector $(1, 2\sqrt{2})$ and squared norm $1 + 8 = 9$,

$$D = \frac{4}{9}(1, 2\sqrt{2}) = \left(\frac{4}{9}, \frac{8\sqrt{2}}{9}\right).$$

Substitution gives $4/9 + 2\sqrt{2}(8\sqrt{2}/9) = 4$, confirming that this point lies on the side.

Step 2: Use the centroid to locate the vertex Because O is also the centroid, its position vector satisfies

$3O = A + B + C = A + 2D$. Since $O = 0$, this yields

$$A = -2D = \left(-\frac{8}{9}, -\frac{16\sqrt{2}}{9}\right).$$

The negative sign is essential: the vertex lies on the opposite side of the origin from the midpoint D, with $OA = 2OD$. Thus $\alpha = -8/9$ and $\beta = -16\sqrt{2}/9$.

Step 3: Evaluate the absolute value and its floor Compute

$$\left|\alpha + \sqrt{2}\beta\right| = \left|-\frac{8}{9} - \frac{32}{9}\right| = \frac{40}{9}.$$

Since $4 \le 40/9 < 5$, its greatest integer less than or equal to it is 4. The value 5 rounds upward and is too large, while 2 and 3 are not the greatest integers satisfying the bound.

Step 4: Conclusion The required greatest integer is $\boxed{4}$. Hence, the correct option is (C).

Quick Tip to Solve: For an equilateral triangle centred at the origin, project the origin onto the opposite side and multiply that midpoint vector by -2 to obtain the vertex.

19. If $\int \left(\frac{1-5\cos^2 x}{\sin^5 x \cos^2 x}\right) dx = f(x) + C$, where C is the constant of integration, then $f\left(\frac{\pi}{6}\right) - f\left(\frac{\pi}{4}\right)$ is equal to [4 marks]

(A) $\frac{1}{\sqrt{3}}(26 + \sqrt{3})$

(B) $\frac{4}{\sqrt{3}}(8 - \sqrt{6})$

(C) $\frac{2}{\sqrt{3}}(4 + \sqrt{6})$

(D) $\frac{1}{\sqrt{3}}(26 - \sqrt{3})$

Correct Answer: (B) $\frac{4}{\sqrt{3}}(8 - \sqrt{6})$

Solution:

Concept:

- The indefinite integral determines an antiderivative up to an additive constant. That constant cancels in the difference of two values on the same interval.
- The denominator suggests differentiating a product of negative powers, using the product rule and $\sin^2 x + \cos^2 x = 1$.

Step 1: Identify an antiderivative Set $F(x) = \sin^{-4} x \cos^{-1} x$, where the negative powers denote reciprocals, not inverse trigonometric functions. The product rule gives

$$\begin{aligned} F'(x) &= -4\sin^{-5} x + \sin^{-3} x \cos^{-2} x \\ &= \frac{-4\cos^2 x + \sin^2 x}{\sin^5 x \cos^2 x} \\ &= \frac{1 - 5\cos^2 x}{\sin^5 x \cos^2 x}. \end{aligned}$$

Thus F is exactly an antiderivative of the printed integrand, so it can be used in place of f when computing the difference. Both evaluation points lie in $(0, \pi/2)$, where this expression is defined.

Step 2: Evaluate the two values and simplify Use $\sin(\pi/6) = 1/2$, $\cos(\pi/6) = \sqrt{3}/2$, and $\sin(\pi/4) = \cos(\pi/4) = 1/\sqrt{2}$:

$$F\left(\frac{\pi}{6}\right) = \frac{1}{(1/2)^4(\sqrt{3}/2)} = \frac{32}{\sqrt{3}}, \qquad F\left(\frac{\pi}{4}\right) = 4\sqrt{2}.$$

Consequently,

$$f\left(\frac{\pi}{6}\right) - f\left(\frac{\pi}{4}\right) = \frac{32}{\sqrt{3}} - 4\sqrt{2} = \frac{4}{\sqrt{3}}(8 - \sqrt{6}).$$

Only option (B) has this exact value. The other expressions, $26/\sqrt{3} + 1$, $8/\sqrt{3} + 2\sqrt{2}$, and $26/\sqrt{3} - 1$, have different rational and radical coefficients and do not equal the evaluated difference.

Step 3: Conclusion The required value is $\frac{4}{\sqrt{3}}(8 - \sqrt{6})$. Hence, the correct option is (B).

Quick Tip to Solve: For a numerator containing $1 - m\cos^2 x$, try differentiating a reciprocal product of powers of sine and cosine before expanding the integrand.

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20. Let f be a polynomial function such that $f\left(x^2 + 1\right) = x^4 + 5x^2 + 2$, for all $x \in \mathbb{R}$.

Then $\int_0^3 f(x)dx$ is equal to [4 marks]

(A) $\frac{27}{2}$

(B) $\frac{33}{2}$

(C) $\frac{41}{3}$

(D) $\frac{5}{3}$

Correct Answer: (B) $\frac{33}{2}$

Solution:

Concept:

- Two polynomials that agree at infinitely many real arguments are identical. This allows an identity first obtained for arguments at least one to determine the polynomial everywhere.
- Integrate the identified polynomial term by term, using $\int x^n\,dx = x^{n+1}/(n+1)$ for nonnegative integers n.

Step 1: Determine the polynomial Put $t = x^2 + 1$, so that $x^2 = t - 1$. The given identity becomes

$$f(t) = (t-1)^2 + 5(t-1) + 2 = t^2 + 3t - 2, \qquad t \geq 1.$$

Although this substitution initially gives only $t \geq 1$, the difference $f(t) - (t^2 + 3t - 2)$ is a polynomial with infinitely many zeros. It must therefore be the zero polynomial. Hence

$$f(t) = t^2 + 3t - 2 \quad \text{for every real } t.$$

In particular this formula is valid on the entire integration interval, including $0 \leq t < 1$. A check by

substitution gives $(x^2+1)^2+3(x^2+1)-2 = x^4+5x^2+2$, as required.

Step 2: Integrate over the stated interval Now apply the usual polynomial integration rule:

$$\begin{aligned}\int_0^3 f(x)\,dx &= \int_0^3 (x^2+3x-2)\,dx \\ &= \left[\frac{x^3}{3}+\frac{3x^2}{2}-2x\right]_0^3 \\ &= 9+\frac{27}{2}-6 = \frac{33}{2}.\end{aligned}$$

The lower endpoint contributes zero. Option (A), $27/2$, keeps only the integral of the linear term and misses the net contribution $9-6=3$ of the other terms. Options (C), $41/3$, and (D), $5/3$, also disagree with the exact endpoint evaluation.

Step 3: Conclusion The identity gives $f(x) = x^2+3x-2$, so the integral is $\frac{33}{2}$, printed as option (B). The key's earlier option position differs from the printed order; the computed value is unchanged. Hence, the correct option is (B).

Quick Tip to Solve: After substituting $t = x^2+1$, use the polynomial identity principle before integrating below $t = 1$; the restricted substitution does not restrict the polynomial formula.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. In a G.P., if the product of the first three terms is 27 and the set of all possible values for the sum of its first three terms is $\mathbb{R}-(a,b)$, then a^2+b^2 is equal to

____ .

[4 marks]

Solution:

Concept:

- Three nonzero real terms in a geometric progression can be written as $u/r, u, ur$, where $r \neq 0$ is the common ratio. Their product is u^3.
- For a real nonzero number r, the range of $r+1/r$ is $(-\infty,-2]\cup[2,\infty)$. Both signs of the ratio must be considered.

Step 1: Use the product to determine the middle term Write the first three terms as $u/r, u, ur$. Since their product is 27,

$$\frac{u}{r}\cdot u\cdot ur = u^3 = 27, \qquad u = 3.$$

The sum is consequently

$$S = \frac{3}{r}+3+3r = 3\left(1+r+\frac{1}{r}\right), \qquad r\in\mathbb{R}\setminus\{0\}.$$

The product is nonzero, so none of the terms or the common ratio can be zero. A negative ratio is permitted: it makes the first and third terms negative while the middle term remains positive.

Step 2: Find the complete range of the sum Set $v = r + 1/r$. The equation for r is $r^2 - vr + 1 = 0$, which has real roots precisely when

$$v^2 - 4 \geq 0, \qquad v \leq -2 \text{ or } v \geq 2.$$

These roots are nonzero because their product is 1, so every value in this range is attained. Therefore

$$S \in (-\infty, -3] \cup [9, \infty) = \mathbb{R} - (-3, 9).$$

The endpoints are included: $r = -1$ gives terms $-3, 3, -3$ and sum -3, while $r = 1$ gives $3, 3, 3$ and sum 9. Matching the excluded open interval with (a, b), we obtain $a = -3$, $b = 9$, and

$$a^2 + b^2 = (-3)^2 + 9^2 = 9 + 81 = 90.$$

Step 3: Conclusion The required dimensionless value is $\boxed{90}$.

Quick Tip to Solve: For a fixed product of three real G.P. terms, fix the middle term first. Include negative common ratios when finding the full range of their sum.

22. If $k = \tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{2}{3}\right)\right) + \tan\left(\frac{1}{2}\sin^{-1}\left(\frac{2}{3}\right)\right)$, then the number of solutions of the equation $\sin^{-1}(kx - 1) = \sin^{-1} x - \cos^{-1} x$ is ____. [4 marks]

Solution:

Concept:

- The principal ranges are $\sin^{-1} t \in [-\pi/2, \pi/2]$ and $\cos^{-1} t \in [0, \pi]$; also $\cos^{-1} t = \pi/2 - \sin^{-1} t$.
- Taking sine of an inverse-trigonometric equation gives a necessary algebraic equation. Its roots must still satisfy all argument and principal-range restrictions.

Step 1: Simplify the constant k Let $\alpha = \sin^{-1}(2/3)$, so $0 < \alpha < \pi/2$ and $\cos^{-1}(2/3) = \pi/2 - \alpha$. Then

$$\begin{aligned} k &= \tan\left(\frac{\pi}{2} - \frac{\alpha}{2}\right) + \tan\frac{\alpha}{2} \\ &= \cot\frac{\alpha}{2} + \tan\frac{\alpha}{2} = \frac{1}{\sin(\alpha/2)\cos(\alpha/2)} \\ &= \frac{2}{\sin\alpha} = 3. \end{aligned}$$

All denominators are nonzero because α is strictly between zero and $\pi/2$.

Step 2: Apply the domains and principal ranges The equation becomes

$$\sin^{-1}(3x - 1) = 2\sin^{-1} x - \frac{\pi}{2}.$$

The inverse-sine arguments require $-1 \leq x \leq 1$ and $-1 \leq 3x - 1 \leq 1$; hence $0 \leq x \leq 2/3$. On this interval the right-hand side also lies within $[-\pi/2, \pi/2]$, so the principal-range condition causes no further exclusion.

Step 3: Solve and check the candidates Taking sine and using $\cos(2\sin^{-1}x) = 1 - 2x^2$ gives

$$3x - 1 = \sin\left(2\sin^{-1}x - \frac{\pi}{2}\right) = 2x^2 - 1.$$

Thus $x(2x-3) = 0$, with candidates $x = 0$ and $x = 3/2$. The second is outside the allowed interval and even outside the domain of $\sin^{-1}x$. At $x = 0$, both sides of the original equation equal $-\pi/2$, so this candidate is valid.

Step 4: Conclusion Exactly one real solution exists, namely $x = 0$. The required number is $\boxed{1}$.

Quick Tip to Solve: Simplify complementary inverse-trigonometric angles first, then write the domain before taking sine; the resulting quadratic can contain inadmissible roots.

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23. For some $\theta \in \left(0, \frac{\pi}{2}\right)$, let the eccentricity and the length of the latus rectum of the hyperbola $x^2 - y^2\sec^2\theta = 8$ be e_1 and l_1, respectively, and let the eccentricity and the length of the latus rectum of the ellipse $x^2\sec^2\theta + y^2 = 6$ be e_2 and l_2, respectively. If $e_1^2 = e_2^2\left(\sec^2\theta + 1\right)$, then $\left(\frac{l_1 l_2}{e_1 e_2}\right)\tan^2\theta$ is equal to [4 marks]

Solution:

Concept:

- For a hyperbola $x^2/a^2 - y^2/b^2 = 1$, the eccentricity satisfies $e^2 = 1 + b^2/a^2$ and the full latus-rectum length is $2b^2/a$.
- For an ellipse, a denotes the major semiaxis even when it is along the y-axis. Its eccentricity satisfies $e^2 = 1 - b^2/a^2$ and its latus-rectum length is $2b^2/a$.

Step 1: Express the conic parameters using one variable Put $c = \cos^2\theta$, so $0 < c < 1$. The equations become

$$\frac{x^2}{8} - \frac{y^2}{8c} = 1, \qquad \frac{x^2}{6c} + \frac{y^2}{6} = 1.$$

For the hyperbola, $a_1 = \sqrt{8}$, $b_1^2 = 8c$; therefore

$$e_1^2 = 1 + c, \qquad l_1 = \frac{16c}{\sqrt{8}} = 4\sqrt{2}\,c.$$

For the ellipse, the larger denominator is 6, so its major axis is along the y-axis: $a_2 = \sqrt{6}$, $b_2^2 = 6c$. Hence

$$e_2^2 = 1 - c, \qquad l_2 = \frac{12c}{\sqrt{6}} = 2\sqrt{6}\,c.$$

Step 2: Use the eccentricity relation to find c Substitute $\sec^2\theta = 1/c$ into the given relation:

$$1 + c = (1 - c)\left(\frac{1}{c} + 1\right).$$

Multiplication by the positive number c gives $c + c^2 = 1 - c^2$, or

$$2c^2 + c - 1 = (2c - 1)(c + 1) = 0.$$

Only $c = 1/2$ belongs to $(0, 1)$; $c = -1$ cannot be a squared cosine. Consequently $\tan^2\theta = (1-c)/c = 1$.

Step 3: Evaluate the requested expression At $c = 1/2$,

$$l_1 = 2\sqrt{2}, \qquad l_2 = \sqrt{6}, \qquad e_1e_2 = \sqrt{(1+c)(1-c)} = \frac{\sqrt{3}}{2}.$$

Taking the positive square root is appropriate because eccentricities are positive. Therefore

$$\frac{l_1l_2}{e_1e_2}\tan^2\theta = \frac{(2\sqrt{2})(\sqrt{6})}{\sqrt{3}/2}\cdot 1 = 8.$$

Step 4: Conclusion The requested value is $\boxed{8}$ in the coordinate system of the question.

Quick Tip to Solve: In an ellipse, identify the major semiaxis from the larger denominator before calculating eccentricity; here the major axis lies along y, not x.

24.

The value of $\sum_{r=1}^{20}\left(\left|\sqrt{\pi\left(\int_0^r x|\sin\pi x|dx\right)}\right|\right)$ **is**

[4 marks]

Solution:

Concept:

- The function $|\sin\pi x|$ has period 1 and is symmetric about the midpoint of every interval $[j, j+1]$, where j is an integer.
- For a symmetric weight on an interval, its weighted first moment equals the interval midpoint times its total integral. The square root is taken to be nonnegative.

Step 1: Evaluate the integral on a single unit interval For a nonnegative integer j, put $x = j + t$, with $0 \le t \le 1$. Since $|\sin\pi(j+t)| = \sin\pi t$,

$$\int_j^{j+1} x|\sin\pi x|\,dx = \int_0^1 (j+t)\sin\pi t\,dt.$$

The unweighted integral is

$$\int_0^1 \sin\pi t\,dt = \left[-\frac{\cos\pi t}{\pi}\right]_0^1 = \frac{2}{\pi}.$$

Let $J = \int_0^1 t\sin\pi t\,dt$. The substitution $t \mapsto 1-t$ gives $J = \int_0^1 (1-t)\sin\pi t\,dt$. Adding these identities yields $2J = 2/\pi$, so $J = 1/\pi$. Thus the required unit-interval contribution is $(2j+1)/\pi$.

Step 2: Add the intervals and simplify each summand For each positive integer r, divide $[0, r]$ into its

r unit intervals:

$$\int_0^r x|\sin \pi x|\, dx = \frac{1}{\pi}\sum_{j=0}^{r-1}(2j+1)$$
$$= \frac{1}{\pi}\left(r(r-1)+r\right) = \frac{r^2}{\pi}.$$

The absolute value around the square root does not change its value. Because $r > 0$, each summand is

$$\left|\sqrt{\pi \int_0^r x|\sin \pi x|\, dx}\right| = \sqrt{r^2} = r.$$

Finally the sum of the first 20 positive integers is

$$\sum_{r=1}^{20} r = \frac{20 \cdot 21}{2} = 210.$$

Step 3: Conclusion The required dimensionless sum is $\boxed{210}$.

Quick Tip to Solve: For an integer endpoint and $|\sin \pi x|$, split into unit intervals. Midpoint symmetry converts the factor x into a simple arithmetic progression.

---

25. Let PQR be a triangle such that $\overrightarrow{PQ} = -2\hat{i} - \hat{j} + 2\hat{k}$ and $\overrightarrow{PR} = a\hat{i} + b\hat{j} - 4\hat{k}, a, b \in Z$. Let S be the point on QR , which is equidistant from the lines PQ and PR . If $|\overrightarrow{PR}| = 9$ and $\overrightarrow{PS} = \hat{i} - 7\hat{j} + 2\hat{k}$, then the value of $3a - 4b$ is ____ [4 marks]

Solution:

Concept:

- With P as the origin, write the position vectors of Q, R and S as $\mathbf{q}, \mathbf{r}, \mathbf{s}$. A point on the segment QR has $\mathbf{s} = (1-t)\mathbf{q} + t\mathbf{r}$, where $0 \le t \le 1$.
- Collinearity requires the same parameter t in all three coordinates. A contradiction in even one coordinate prevents the stated triangle and point from existing.

Step 1: Express the point on QR The supplied vectors give $\mathbf{q} = (-2, -1, 2)$, $\mathbf{r} = (a, b, -4)$ and $\mathbf{s} = (1, -7, 2)$. Therefore a necessary condition for S to lie on QR is

$$(1, -7, 2) = (-2, -1, 2) + t(a+2, b+1, -6).$$

This identity must hold component by component, independently of the distance and angle-bisector conditions.

Step 2: Test the third coordinate and the first coordinate The third coordinate gives $2 = 2 - 6t$, hence $t = 0$. But substituting this into the first coordinate gives $1 = -2$, which is impossible. Equivalently, t=0 would make S equal to Q, whereas their given position vectors are different. This contradiction holds for every real a and b, so restricting them to integers cannot remove it.

Step 3: Check why the remaining conditions cannot repair the contradiction The length condition

would give $a^2 + b^2 + 16 = 81$, or $a^2 + b^2 = 65$. However, selecting any pair on that circle still leaves the third-coordinate argument unchanged. Similarly, equal perpendicular distances to PQ and PR can only further restrict a point already on QR; they cannot turn the inconsistent coordinate identity into a valid one.

Step 4: Conclusion There is no triangle and point S satisfying all the printed conditions. Thus $3a - 4b$ has no defined value for this question. The official final key dropped this question and awarded bonus marks to all candidates.

Quick Tip to Solve: Before applying the angle-bisector theorem, test whether the given point can lie on the stated side using one common affine parameter.

Physics – Section A: Multiple Choice Questions (80 marks)

26. The electric current in the circuit is given as $i = i_o(t/T)$. The r.m.s current for the period $t = 0$ to $t = T$ is ____ . [4 marks]

(A) i_o

(B) $\frac{i_o}{\sqrt{2}}$

(C) $\frac{i_o}{\sqrt{3}}$

(D) $\frac{i_o}{\sqrt{6}}$

Correct Answer: (C) $\frac{i_o}{\sqrt{3}}$

Solution:

Concept:

- The root mean square current on an interval T is $I_{\text{rms}} = \sqrt{T^{-1}\int_0^T i(t)^2\,dt}$. It is the constant current producing the same Joule heating in a fixed resistance.
- The waveform here is a linear ramp $i(t) = i_0 t/T$; a sinusoidal-wave result must not be used for a ramp.

Step 1: Square the current before averaging Substitute the given time dependence directly into the mean-square definition:

$$I_{\text{rms}}^2 = \frac{1}{T}\int_0^T \left(\frac{i_0 t}{T}\right)^2 dt = \frac{i_0^2}{T^3}\int_0^T t^2 dt.$$

The factor outside the integral contains T^3, since the averaging factor contributes one power of T in addition to the two powers from squaring the current.

Step 2: Evaluate the integral and take its square root Using $\int_0^T t^2 dt = T^3/3$, we obtain

$$I_{\text{rms}}^2 = \frac{i_0^2}{T^3}\frac{T^3}{3} = \frac{i_0^2}{3}, \qquad I_{\text{rms}} = \frac{i_0}{\sqrt{3}}.$$

Here i_0 denotes the nonnegative peak magnitude. The cancellation of T is expected because stretching the ramp in time does not change the relative time spent at each current.

Step 3: Eliminate the other options Option A, i_0, would require the current to remain at its maximum

throughout the interval. Option B, $i_0/\sqrt{2}$, is the familiar sinusoidal result and does not describe the linear ramp. Option D gives mean square $i_0^2/6$, only half the integral actually obtained. The arithmetic mean $i_0/2$ is also different from the root mean square.

Step 4: Conclusion The r.m.s. current is $\boxed{i_0/\sqrt{3}}$, in the same current unit as i_0. Hence, the correct option is (C). The differing key letter B comes from a different option order; the derived value occurs at (C) in the printed order.

Quick Tip to Solve: For a ramp, square the time dependence first and then integrate; do not substitute the sinusoidal factor $1/\sqrt{2}$.

---

27. The magnitudes of power of a biconvex lens (refractive index 1.5) and that of a plano-concave lens (refractive index $= 1.7$) are same. If the curvature of planoconcave lens exactly matches with the curvature of back surface of the biconvex lens, then ratio of radius of curvature of front and back surface of the biconvex lens is ____ [4 marks]

(A) $5 : 2$

(B) $5 : 12$

(C) $2 : 5$

(D) $12 : 5$

Correct Answer: (A) $5 : 2$

Solution:

Concept:

- For a thin lens in air, $P = (\mu - 1)(1/R_1 - 1/R_2)$, with signed radii. A biconvex lens has positive power; a plano-concave lens has negative power.
- Let r and R be the positive magnitudes of the front and back radii of the biconvex lens. Its power is $(1.5 - 1)(1/r + 1/R)$.

Step 1: Write both power magnitudes For the biconvex lens the two curvatures add, because its signed radii have opposite signs:

$$P_{\text{convex}} = 0.5\left(\frac{1}{r} + \frac{1}{R}\right).$$

The plane surface of the plano-concave lens has zero curvature. Its curved surface has radius magnitude R, since it matches the back surface of the biconvex lens. Hence

$$|P_{\text{concave}}| = \frac{1.7 - 1}{R} = \frac{0.7}{R}.$$

Step 2: Equate the magnitudes and solve The question equates magnitudes, so we use the positive expressions:

$$0.5\left(\frac{1}{r} + \frac{1}{R}\right) = \frac{0.7}{R}.$$

Subtracting $0.5/R$ from both sides gives $0.5/r = 0.2/R$. Cross-multiplication then gives $0.5R = 0.2r$,

so

$$\frac{r}{R} = \frac{0.5}{0.2} = \frac{5}{2}.$$

Step 3: Check the order of the requested ratio The requested order is front radius to back radius, namely r:R, not its reciprocal. For $r = 2.5R$, the convex power is $0.5(0.4 + 1)/R = 0.7/R$, verifying equality. Option C reverses the ratio. Options B and D also fail substitution into the power equation; their ratios are $5/12$ and $12/5$, respectively, rather than $5/2$.

Step 4: Conclusion The front-to-back radius ratio is $\boxed{5 : 2}$. Hence, the correct option is (A). The differing key letter B comes from a different option order; the derived value occurs at (A) in the printed order.

Quick Tip to Solve: Use signed radii in the lens maker formula, but positive magnitudes when comparing powers of a converging and a diverging lens.

---

28. An atom $^{8}_{3}X$ is bombarded by shower of fundamental particles and in 10 s this atom absorbed 10 electrons, 10 protons and 9 neutrons. The percentage growth in the surface area of the nucleons is recorded by : [4 marks]

(A) 150%

(B) 900%

(C) 225%

(D) 250%

Correct Answer: The calculated growth is 125 percent, which is absent from the printed options.

Solution:

Concept:

- The nuclear radius follows $R = R_0 A^{1/3}$, where A counts protons plus neutrons. Thus nuclear surface area is proportional to $A^{2/3}$.
- Electrons are not nucleons. Adding electrons changes the electronic configuration or charge balance, but not the mass number used in the nuclear-radius law.

Step 1: Count the final nucleons The initial mass number is 8. Absorbing ten protons and nine neutrons increases the mass number by 19, while the ten electrons contribute no nucleons. Hence

$$A_i = 8, \qquad A_f = 8 + 10 + 9 = 27.$$

The atomic number would rise from 3 to 13, but it is the total nucleon number A that determines the nuclear radius in this model. The stated time interval is unnecessary because the question asks for a percentage change, not a rate.

Step 2: Calculate the surface-area ratio Using the spherical nuclear area $S = 4\pi R^2$,

$$\frac{S_f}{S_i} = \left(\frac{A_f}{A_i}\right)^{2/3} = \left(\frac{27}{8}\right)^{2/3} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}.$$

Thus the final surface area is 225 percent of the initial surface area. This is the final amount expressed as a percentage, not the percentage growth.

Step 3: Subtract the initial area and compare the options The percentage growth is

$$\frac{S_f - S_i}{S_i} \times 100 = \left(\frac{9}{4} - 1\right) 100 = 125\%.$$

The printed options are 150 percent, 900 percent, 225 percent and 250 percent. None equals 125 percent. In particular, 225 percent omits subtraction of the original 100 percent, while 150 percent is the final radius as a percentage of the initial radius, not the surface-area growth.

Step 4: Conclusion The nuclear surface-area growth is $\boxed{125\%}$. None of the printed options is correct: the last printed option is 250 percent. A key selecting the last option cannot agree with the literal printed choices; the calculation above establishes the discrepancy.

Quick Tip to Solve: Distinguish final area as a percentage of initial area from percentage growth: subtract 100 percent from the former.

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29. Given below are two statements :

Statement I : A plane wave after passing through prism remains as plane wave but passing through small pin hole may become spherical wave.

Statement II : The curvature of a spherical wave emerging from a slit will increase for increasing slit width.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- A wavefront is a surface of constant phase. A plane wave incident on a prism is refracted into a new direction while retaining a plane wavefront in the geometrical-optics description.
- A sufficiently small aperture diffracts an incident wave strongly and acts approximately as a secondary source. Diffraction spreading decreases as the aperture width increases relative to wavelength.

Step 1: Evaluate Statement I A prism has plane refracting faces. An incident collimated plane wave is deviated by refraction, but the emergent rays remain parallel and the emergent wavefront remains plane. A very small pinhole, on the other hand, restricts the transmitted wave to a small region. Its diffracted radiation spreads into many directions and may be treated as a spherical wave from a

localized source. Therefore Statement I is true.

Step 2: Evaluate Statement II For a slit of width a and wavelength λ, the characteristic diffraction angle scales as $\theta \sim \lambda/a$ when the angle is small. Increasing a therefore reduces the spreading, tending toward the plane-wave limit rather than producing greater curvature. Moreover, the local curvature of an ideal spherical wavefront is $1/r$, where r is its distance from its centre; it is not an increasing function of slit width. The claimed increase in Statement II is false.

Step 3: Match the truth values to the options The combination is true for Statement I and false for Statement II. This is exactly option C. Option A wrongly accepts the second statement, option B wrongly rejects the first statement, and option D reverses both truth values. The key distinction is refraction through plane faces versus diffraction through an aperture, and then the decrease of diffraction with increasing aperture size.

Step 4: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: To judge an aperture claim, check the limiting case: a much wider slit gives less diffraction and approaches plane-wave propagation.

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30. When both jaws of vernier callipers touch each other, zero mark of the vernier scale is right to zero mark of main scale, 4^{th} mark on vernier scale coincides with certain mark on the main scale. While measuring the length of a cylinder, observer observes 15 divisions on main scale and 5^{th} division of vernier scale coincides with a main scale division. Measured length of cylinder is ____ mm.

(Least count of Vernier calliper $= 0.1$ mm) [4 marks]

(A) 15.9

(B) 15.5

(C) 15.1

(D) 15.4

Correct Answer: (C) 15.1

Solution:

Concept:

- A vernier reading is the main-scale reading plus the coincident vernier division multiplied by the least count. A positive zero error is subtracted from the observed reading.
- When the vernier zero lies to the right of the main-scale zero with closed jaws, the instrument shows a positive reading despite the true separation being zero.

Step 1: Find the zero error and its correction The fourth vernier division coincides with a main-scale mark when the jaws are closed. With least count $0.1\,\text{mm}$, the positive zero error is

$$e_0 = 4(0.1) = +0.4\,\text{mm}.$$

Hence the zero correction is $-0.4\,\text{mm}$. Its sign matters: this instrument overreads lengths and the measured value must be reduced.

Step 2: Calculate the observed reading The main-scale observation is 15 divisions, corresponding to

$15\,\text{mm}$ on the stated millimetre scale. The fifth vernier division coincides, so the vernier contribution is $5(0.1) = 0.5\,\text{mm}$. Therefore

$$L_{\text{observed}} = 15 + 0.5 = 15.5\,\text{mm}.$$

This is the raw instrument reading, before the correction for its nonzero closed-jaw reading.

Step 3: Apply the zero correction and eliminate alternatives The corrected length is

$$L = L_{\text{observed}} - e_0 = 15.5 - 0.4 = 15.1\,\text{mm}.$$

Option A, 15.9 mm, results from adding the positive error instead of subtracting it. Option B, 15.5 mm, ignores the zero error. Option D, 15.4 mm, does not combine the observed fifth-division contribution with the fourth-division zero offset. Only option C includes both contributions with the correct signs.

Step 4: Conclusion The corrected cylinder length is $\boxed{15.1\,\text{mm}}$. Hence, the correct option is (C). The differing key letter D comes from a different option order; the derived value occurs at (C) in the printed order.

Quick Tip to Solve: Write zero error with its sign first, then always use corrected reading = observed reading minus zero error.

31. In the potentiometer, when the cell in the secondary circuit is shunted with 4Ω resistance, the balance is obtained at the length 120 cm of wire. Now when the same cell is shunted with 12Ω resistance, the balance is shifted to a length of 180 cm . The internal resistance of cell is ____ Ω

[4 marks]

(A) 3

(B) 4

(C) 6

(D) 12

Correct Answer: (B) 4

Solution:

Concept:

- A potentiometer balance length is proportional to the terminal potential difference: $V = k\ell$, where k is the potential gradient of the unchanged primary circuit.
- A cell of emf E and internal resistance r, connected across an external resistance R, supplies current $I = E/(R + r)$ and terminal voltage $V = ER/(R + r)$.

Step 1: Write the two terminal voltages The shunt resistance loads the cell; it is not the resistance of the potentiometer wire. For the two loads, the same emf and internal resistance apply. Thus

$$k(120) = \frac{4E}{4 + r}, \qquad k(180) = \frac{12E}{12 + r}.$$

The lengths may both be retained in centimetres because only their ratio will be used.

Step 2: Eliminate the common quantities Divide the first balance equation by the second. The emf

and potential gradient cancel, leaving an equation involving only the unknown internal resistance:

$$\frac{120}{180} = \frac{4(12+r)}{12(4+r)}.$$

Therefore

$$\frac{2}{3} = \frac{12+r}{3(4+r)} \quad \Longrightarrow \quad 2(4+r) = 12+r.$$

It follows that $8+2r = 12+r$, so $r = 4\,\Omega$.

Step 3: Check the result against the balance ratio For $r = 4\,\Omega$, the terminal voltages are $E/2$ and $3E/4$; their ratio is exactly $2/3$, as required. The other proposed resistances $3, 6, 12\,\Omega$ would give voltage ratios $5/7, 3/5, 1/2$, respectively, and therefore cannot reproduce the observed lengths.

Step 4: Conclusion The internal resistance is $\boxed{4\,\Omega}$, which gives the required voltage ratio $(E/2)/(3E/4) = 2/3$. This value occupies the second position in the printed options, resolving the differently ordered key letter. Hence, the correct option is (B).

Quick Tip to Solve: For two loaded balance lengths, use the ratio of $ER/(R+r)$; the longer balance length belongs to the larger load resistance because the cell loses less voltage internally.

32. Water drops fall from a tap on the floor, 5 m below, at regular intervals of time, the first drop strikes the floor when the sixth drop begins to fall. The height at which the fourth drop will be from ground, at the instant when the first drop strikes the ground is ____ m.

$$\left(\mathrm{g} = 10\ \mathrm{m/s^2}\right)$$

[4 marks]

(A) 4.0

(B) 2.5

(C) 4.2

(D) 3.8

Correct Answer: (C) 4.2

Solution:

Concept:

- A drop released from rest falls a distance $s = \frac{1}{2}gt^2$ after time t, neglecting air resistance.
- Six successive releases contain five equal time intervals. A drop released later has been falling for fewer of these intervals at the same observation instant.

Step 1: Find the flight time of the first drop The first drop travels the full $5\,\mathrm{m}$ from tap to floor. Let its flight time be T. Using the given acceleration,

$$5 = \frac{1}{2}(10)T^2 \quad \Longrightarrow \quad T = 1\,\mathrm{s}.$$

The first drop is released at time zero, whereas the sixth is released at $5\Delta t$, where Δt is the interval between drops. Hence $5\Delta t = T$, giving $\Delta t = 0.2\,\text{s}$.

Step 2: Determine how long the fourth drop has fallen The fourth drop is released after three intervals, at $3\Delta t = 0.6\,\text{s}$. When the first reaches the ground at $1\,\text{s}$, the fourth has therefore fallen for

$$t_4 = T - 3\Delta t = 0.4\,\text{s}.$$

Its downward displacement from the tap is

$$s_4 = \frac{1}{2}(10)(0.4)^2 = 0.8\,\text{m}.$$

Step 3: Convert displacement into height above the ground The question asks for height above the floor, not distance below the tap. Subtract the displacement from the tap height:

$$h_4 = 5 - 0.8 = 4.2\,\text{m}.$$

The release count must use five intervals, not six, and the remaining flight time is two intervals. The alternatives 4.0, 2.5 and $3.8\,\text{m}$ do not satisfy this timing and the quadratic free-fall law.

Step 4: Conclusion The fourth drop is $\boxed{4.2\,\text{m}}$ above the ground. Hence, the correct option is (C).

Quick Tip to Solve: Number release times from zero: drop j starts at $(j-1)\Delta t$. This avoids the common error of assigning six intervals to six drops.

---

33. The electric field of an electromagnetic wave travelling through a medium is given by $\vec{E}(x,t) = 25\sin\left(2.0\times10^{15}t - 10^{7}x\right)\hat{n}$ then the refractive index of the medium is ____ .

(All given measurement are in SI units) [4 marks]

(A) 1.5

(B) 1.2

(C) 2

(D) 1.7

Correct Answer: (A) 1.5

Solution:

Concept:

- A sinusoidal travelling wave has phase $\omega t - kx$, where ω is angular frequency in $\text{rad}\,\text{s}^{-1}$ and k is angular wave number in $\text{rad}\,\text{m}^{-1}$. Its phase speed is $v = \omega/k$.
- The absolute refractive index is $n = c/v$, with vacuum light speed $c = 3.0\times10^{8}\,\text{m}\,\text{s}^{-1}$. The electric-field amplitude does not determine the propagation speed.

Step 1: Read the coefficients of time and position Compare the given phase $2.0\times10^{15}t - 10^{7}x$ directly

with $\omega t - kx$. This identifies

$$\omega = 2.0 \times 10^{15}\,\mathrm{rad\,s^{-1}}, \qquad k = 10^{7}\,\mathrm{rad\,m^{-1}}.$$

The coefficient 25 is the electric-field amplitude, and $\hat{n}$ specifies the electric-field direction. Neither enters the ratio of the phase coefficients.

Step 2: Calculate the wave speed A point of fixed phase satisfies $\omega t - kx =$ constant. Differentiating gives $\omega - k\,dx/dt = 0$, which explains the speed formula. Substituting the coefficients,

$$v = \frac{2.0 \times 10^{15}}{10^{7}} = 2.0 \times 10^{8}\,\mathrm{m\,s^{-1}}.$$

No additional factor of 2π is needed: both coefficients already use angular quantities.

Step 3: Find and check the refractive index The wave travels more slowly than light in vacuum, so the index exceeds one:

$$n = \frac{3.0 \times 10^{8}}{2.0 \times 10^{8}} = 1.5.$$

For comparison, indices 1.2, 2 and 1.7 predict speeds 2.5×10^{8}, 1.5×10^{8} and approximately $1.76 \times 10^{8}\,\mathrm{m\,s^{-1}}$, respectively. None equals the speed read from the given phase.

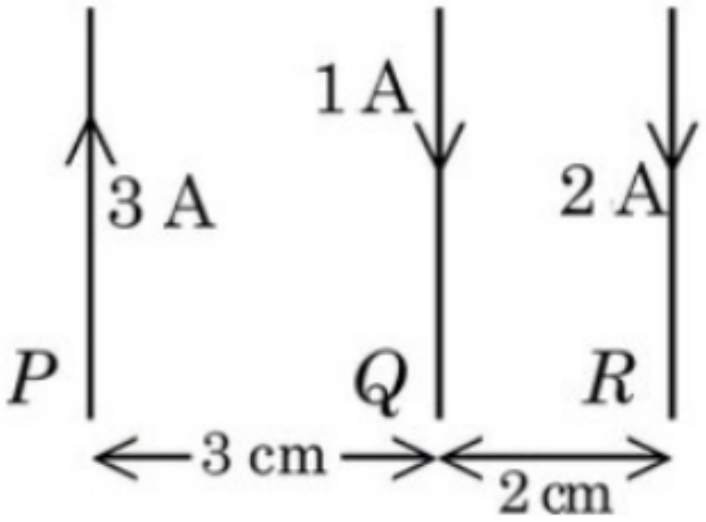
Step 4: Conclusion Since $n = ck/\omega = 1.5$, the medium has dimensionless refractive index $\boxed{1.5}$. This value is first in the printed options, resolving the differently ordered key letter. Hence, the correct option is (A).

Quick Tip to Solve: Read ω from the time coefficient and k from the position coefficient, then use $n = ck/\omega$; the amplitude is irrelevant.

34. Three long straight wires carrying current are arranged mutually parallel as shown in the figure. The force experienced by 15 cm length of wire Q is ____ .

$$\left(\mu_o = 4\pi \times 10^{-7}\ \text{T.m/A}\right)$$

[4 marks]



(A) 6×10^{-7} N towards R

(B) 6×10^{-7} N towards P

(C) 6×10^{-6} N towards P

(D) 6×10^{-6} N towards R

Correct Answer: (D) 6×10^{-6} N towards R

Solution:

Concept:

- Two long parallel wires a distance d apart exert force $F = \mu_0 I_1 I_2 L/(2\pi d)$ on a length L. Parallel currents attract; antiparallel currents repel.
- The total force on wire Q is the vector sum of the forces due to P and R. Convert the wire length and separations to metres before substitution.

Step 1: Determine both force directions In the figure, P carries 3 A upward, whereas Q carries 1 A downward. Their antiparallel currents repel, so P pushes Q rightward, towards R. Wire R carries 2 A downward, parallel to Q, so it attracts Q rightward as well. The two forces therefore add rather than cancel.

Step 2: Calculate the force due to each neighbour Use $L = 0.15\,\text{m}$, $d_{PQ} = 0.03\,\text{m}$, $d_{QR} = 0.02\,\text{m}$, and $\mu_0/(2\pi) = 2 \times 10^{-7}$. Then

$$F_{PQ} = 2 \times 10^{-7}\frac{(3)(1)(0.15)}{0.03} = 3 \times 10^{-6}\,\text{N}.$$

Similarly,

$$F_{RQ} = 2 \times 10^{-7} \frac{(2)(1)(0.15)}{0.02} = 3 \times 10^{-6}\,\text{N}.$$

The magnitudes coincide because $3/0.03 = 2/0.02$, even though the currents differ.

Step 3: Add the directed forces and reject the alternatives Both contributions point towards R, giving

$$F_Q = F_{PQ} + F_{RQ} = 6 \times 10^{-6}\,\text{N}.$$

The alternatives towards P contradict both interaction directions. The remaining alternative towards R has magnitude $6 \times 10^{-7}\,\text{N}$, smaller by a factor of ten; it does not follow from the metre-based substitution.

Step 4: Conclusion Both forces point towards R, and $3 \times 10^{-6} + 3 \times 10^{-6} = 6 \times 10^{-6}\,\text{N}$. Thus the answer is $\boxed{6 \times 10^{-6}\,\text{N towards } R}$, fourth in the printed options, resolving the differently ordered key letter. Hence, the correct option is (D).

Quick Tip to Solve: Fix attraction or repulsion for each pair before calculating magnitudes. Equal magnetic-force magnitudes cancel only if their directions are opposite.

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35. Two wires A and B made of different materials of lengths 6.0 cm and 5.4 cm , respectively and area of cross sections $3.0 \times 10^{-5}\ \text{m}^2$ and $4.5 \times 10^{-5}\ \text{m}^2$, respectively are stretched by the same magnitude under a given load. The ratio of the Young's modulus of A to that of B is $x : 3$. The value of x is ____ [4 marks]

(A) 1

(B) 2

(C) 4

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- Young's modulus is stress divided by longitudinal strain: $Y = (F/A)/(\Delta L/L) = FL/(A\Delta L)$. Here A is cross-sectional area.
- When two wires carry the same tensile load and undergo the same absolute extension, their Young's moduli are proportional to L/A, not to length alone.

Step 1: Form a ratio before inserting numbers For the two wires, write

$$Y_A = \frac{FL_A}{A_A \Delta L}, \qquad Y_B = \frac{FL_B}{A_B \Delta L}.$$

The common load F and equal extension ΔL cancel when these equations are divided. Consequently,

$$\frac{Y_A}{Y_B} = \frac{L_A}{L_B}\frac{A_B}{A_A}.$$

The area ratio is reversed because a greater area lowers the stress produced by the same load.

Step 2: Substitute the measured length and area ratios Both lengths are given in centimetres, so their common conversion to metres cancels. The common factor $10^{-5}\,\mathrm{m}^2$ also cancels from the areas. Thus

$$\frac{Y_A}{Y_B} = \frac{6.0}{5.4}\frac{4.5\times10^{-5}}{3.0\times10^{-5}} = \frac{10}{9}\frac{3}{2} = \frac{5}{3}.$$

Although the extensions are equal, the strains are unequal because the initial lengths differ. Setting the strains equal would therefore be incorrect.

Step 3: Match the requested ratio The problem writes the modulus ratio as $x : 3$, so $x/3 = 5/3$ and $x = 5$. Values 1, 2 and 4 would give ratios $1/3$, $2/3$ and $4/3$, none consistent with the measured L/A factors. The result also has the expected direction: the longer, thinner wire must have a larger modulus to extend by the same amount.

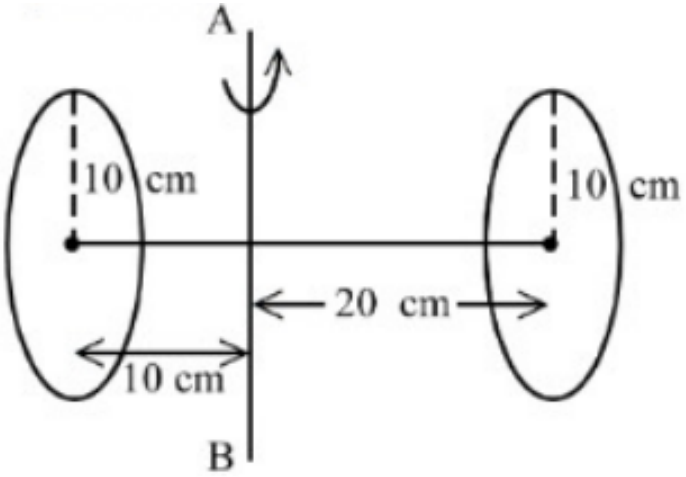
Step 4: Conclusion $Y_A : Y_B = 5 : 3$, so the dimensionless value is $\boxed{x = 5}$. Hence, the correct option is (D).

Quick Tip to Solve: For equal load and equal extension, remember $Y \propto L/A$. Equal extension does not mean equal strain when the original lengths differ.

---

36. Two circular discs of radius each 10 cm are joined at their centres by a rod of length 30 cm and mass 600 gm as shown in figure.

If the mass of each disc is 600 gm and applied torque between two discs is 43×10^5 dyne.cm, the angular acceleration of the discs about the given axis AB is ____ $\mathrm{rad/s^2}$. [4 marks]



(A) 27

(B) 22

(C) 11

(D) 100

Correct Answer: (C) 11

Solution:

Concept:

- For a rigid assembly, $\tau = I\alpha$. Add the moments of inertia of all its components about the stated rotation axis.
- A thin disc has moment of inertia $mR^2/4$ about a diameter. For a parallel displaced axis use $I = I_{\text{cm}} + md^2$; a uniform rod has $I_{\text{cm}} = ML^2/12$ about an axis perpendicular to its length.

Step 1: Read the axis geometry and convert units Axis AB is parallel to a diameter of each disc. The figure places the disc centres $0.10\,\text{m}$ and $0.20\,\text{m}$ from it. Each disc and the rod have mass $0.60\,\text{kg}$; each disc radius is $0.10\,\text{m}$. The $0.30\,\text{m}$ rod has its centre $0.15\,\text{m}$ from its left end, hence $0.05\,\text{m}$ from AB.

Step 2: Add the disc and rod contributions For the two discs the parallel-axis theorem gives

$$I_{\text{discs}} = 2\left(\frac{0.60(0.10)^2}{4}\right) + 0.60\left[(0.10)^2 + (0.20)^2\right] = 0.033\,\text{kg m}^2.$$

For the rod,

$$I_{\text{rod}} = \frac{0.60(0.30)^2}{12} + 0.60(0.05)^2 = 0.006\,\text{kg m}^2.$$

Thus $I = 0.039\,\text{kg m}^2$. The $mR^2/2$ disc formula would apply to the central axis normal to its face, not to the diameter-parallel axis shown here.

Step 3: Convert torque and calculate angular acceleration Since $1\,\text{dyne} = 10^{-5}\,\text{N}$ and $1\,\text{cm} = 10^{-2}\,\text{m}$,

$$\tau = 43 \times 10^5 \times 10^{-7} = 0.43\,\text{N m}.$$

Therefore

$$\alpha = \frac{0.43}{0.039} = \frac{430}{39}\,\text{rad s}^{-2} \simeq 11.03\,\text{rad s}^{-2}.$$

Among the stated integer options this is 11. The alternatives 27, 22 and 100 would require different moments of inertia and do not match the specified geometry.

Step 4: Conclusion The printed data give $\alpha = 0.43/0.039 = 430/39\,\text{rad s}^{-2}$, approximately $\boxed{11\,\text{rad s}^{-2}}$ at the precision of the options. This value is third in the printed options, resolving the differently ordered key letter. Hence, the correct option is (C).

Quick Tip to Solve: Identify whether the rotation axis is parallel to a disc diameter or normal to its face before choosing its central moment of inertia; then include every displaced component.

37. For the two cells having same EMF E and internal resistance r, the current passing through the external resistor 6Ω is same when both the cells are connected either in parallel or in series. The value of internal resistance r is ____ Ω. [4 marks]

(A) 3

(B) 4

(C) 6

(D) 9

Correct Answer: (C) 6

Solution:

Concept:

- For identical cells connected in series with aiding polarities, their EMFs and internal resistances add. The current is the total EMF divided by the total circuit resistance.
- For two identical cells connected in parallel with like terminals together, the equivalent EMF remains E, while the internal resistance becomes $r/2$.

Step 1: Write the current for the series combination Let the external resistance be $R = 6\,\Omega$. The series combination supplies EMF $2E$ and contributes internal resistance $2r$. Hence Ohm's law for the complete circuit gives

$$I_s = \frac{2E}{R + 2r} = \frac{2E}{6 + 2r}.$$

The internal resistances must be included because both cells carry the same series current.

Step 2: Write and equate the parallel current The two equal internal resistances are in parallel, so their equivalent is $r/2$. The terminal arrangement therefore gives

$$I_p = \frac{E}{R + r/2} = \frac{E}{6 + r/2}.$$

The stated equality of the external currents, for a nonzero EMF, yields

$$\frac{2}{6 + 2r} = \frac{1}{6 + r/2} \quad \Rightarrow \quad 12 + r = 6 + 2r.$$

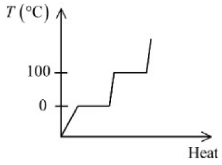
Thus $r = 6\,\Omega$.

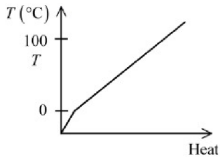
Step 3: Check the result against the options At $r = 6\,\Omega$, the series current is $2E/18 = E/9$, while the parallel current is $E/(6 + 3) = E/9$. Both agree. More generally the equality simplifies to $r = R$, so the values 3, 4 and $9\,\Omega$ cannot satisfy the required equality for $R = 6\,\Omega$.

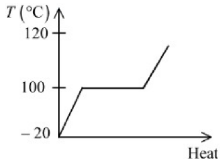
Step 4: Conclusion The internal resistance of each cell is $\boxed{6\,\Omega}$. Hence, the correct option is (C). The option order follows the printed paper; the value follows directly from equality of the two currents.

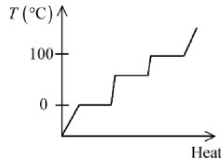
Quick Tip to Solve: For two identical cells, equating the series and parallel external currents immediately gives $r = R$; keep the unchanged parallel EMF separate from the halved internal resistance.

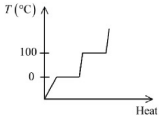
38. Which of the following best represents the temperature versus heat supplied graph for water, in the range of -20°C to 120°C? [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- Within a single phase, supplied heat raises temperature according to $Q = mc\Delta T$. Thus a temperature-versus-heat graph has a positive slope $dT/dQ = 1/(mc)$.
- During a phase change at fixed pressure, heat is absorbed as latent heat, $Q = mL$, while the temperature remains constant. These intervals are horizontal parts of the heating curve.

Step 1: Follow the heating of ice to liquid water In the ordinary heating curve for water, the sample initially at -20°C is ice. Its temperature first rises to 0°C, giving an upward segment. At the melting point, further heat converts ice into liquid without raising the temperature, so a horizontal segment must appear at 0°C. Once melting is complete, the liquid temperature rises from 0°C to 100°C.

Step 2: Include boiling and the heating of steam At 100°C, the supplied heat vaporises the water. A second horizontal segment is required while liquid and vapour coexist. After vaporisation, the steam warms to 120°C, producing another upward segment. Thus the required sequence is rise, plateau at zero, rise, plateau at 100, and rise.

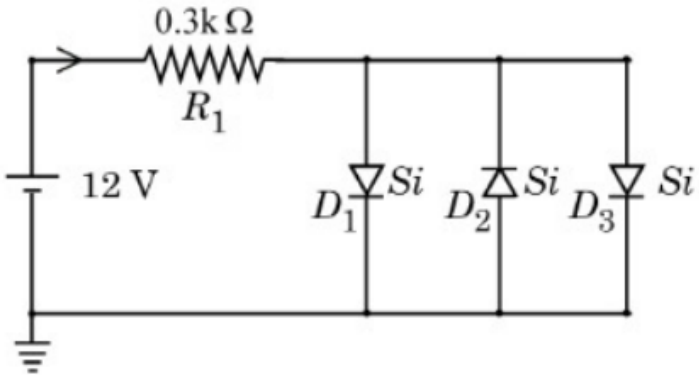
Step 3: Compare all four graphical options Graph (A) contains both required constant-temperature intervals and all three rising portions, so it best represents the heating stages. Graph (B) has no latent-heat plateau. Graph (C) includes boiling but omits the melting plateau at zero. Graph (D) inserts an additional plateau at an intermediate temperature, where pure liquid water undergoes no phase change. The sketch is schematic: the decisive feature is the location and number of phase-change plateaus.

Step 4: Conclusion The graph with exactly two plateaus, at 0°C and 100°C, is the required heating curve. Hence, the correct option is (A).

Quick Tip to Solve: Count the phase changes before comparing the sketches: melting and boiling each require one horizontal temperature interval, with rising segments between and outside them.

39. Assuming in forward bias condition there is a voltage drop of 0.7 V across a silicon diode, the current through diode D_1 in the circuit is ____ mA .

(Assume all diodes in the given circuit are identical) [4 marks]



(A) 17.6

(B) 20.15

(C) 11.7

(D) 18.8

Correct Answer: (D) 18.8

Solution:

Concept:

- A forward-biased silicon diode has the stated voltage drop $0.7\,\text{V}$. A reverse-biased diode carries negligible current in this circuit model.
- Identical forward-biased diodes connected in parallel have equal voltage and share the total current equally. The series resistor carries the sum of their currents.

Step 1: Identify the conducting branches The upper supply terminal is positive relative to the grounded lower rail. From the diode orientations, D_1 and D_3 are forward biased from the upper node to the lower rail, whereas D_2 has the opposite orientation and is reverse biased. Therefore D_2 does not carry the supply current. The conducting parallel pair clamps the upper node to $0.7\,\text{V}$ above ground.

Step 2: Find the current in the series resistor The resistor is $R_1 = 0.3\,\text{k}\Omega = 300\,\Omega$. Its voltage is the supply voltage minus the forward-diode drop, so

$$V_R = 12 - 0.7 = 11.3\,\text{V}.$$

Ohm's law gives the total current reaching the parallel branches:

$$I_R = \frac{11.3}{300}\,\text{A} = 37.67\,\text{mA}.$$

The two diode drops must not be added: the conducting diodes are parallel, not series.

Step 3: Share the current between the identical diodes Since D_1 and D_3 are identical and have the

same forward voltage, $I_1 = I_3$. Kirchhoff's current law gives

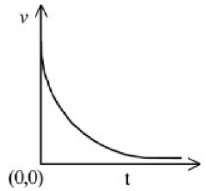
$$I_R = I_1 + I_3 = 2I_1, \qquad I_1 = \frac{37.67}{2} = 18.83\,\text{mA}.$$

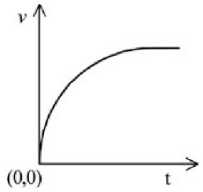
This rounds to $18.8\,\text{mA}$. The alternatives 17.6, 20.15 and $11.7\,\text{mA}$ do not satisfy both the resistor voltage and equal sharing between the two conducting branches.

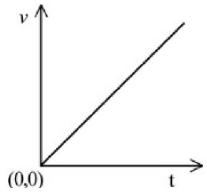
Step 4: Conclusion The current through D_1 is $\boxed{18.8\,\text{mA}}$. Hence, the correct option is (D). The printed option order places the value obtained from Ohm's law and equal current sharing at (D).

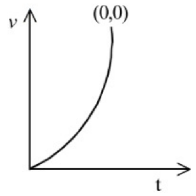
Quick Tip to Solve: First remove the reverse-biased branch, then calculate the resistor current using one forward drop and divide it equally between the identical parallel conducting diodes.

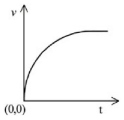
40. A particle of mass m falls from rest through a resistive medium having resistive force, $F = -kv$, where v is the velocity of the particle and k is a constant. Which of the following graphs represents velocity (v) versus time (t) ? [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- Taking downward as positive, gravity contributes mg and the resistive force contributes $-kv$. Newton's second law is $m\,dv/dt = mg - kv$.
- Terminal velocity occurs when the net force vanishes, giving $v_T = mg/k$. The particle begins from rest, so $v(0) = 0$.

Step 1: Determine the initial and limiting slopes At release the resistive force is zero because $v = 0$, and the acceleration is g. As speed increases, the upward resistive force grows and the downward acceleration decreases. Thus the graph must begin at the origin with positive slope and gradually flatten. It cannot begin at a nonzero velocity or have an increasing slope.

Step 2: Solve the equation of motion Rearrange the differential equation as

$$\frac{dv}{dt} + \frac{k}{m}v = g.$$

With $v(0) = 0$, its solution is

$$v(t) = \frac{mg}{k}\left(1 - e^{-kt/m}\right).$$

The exponential decreases from one to zero, so velocity increases continuously from zero toward mg/k, without overshooting that limiting value.

Step 3: Use the curvature to select the graph Differentiating gives

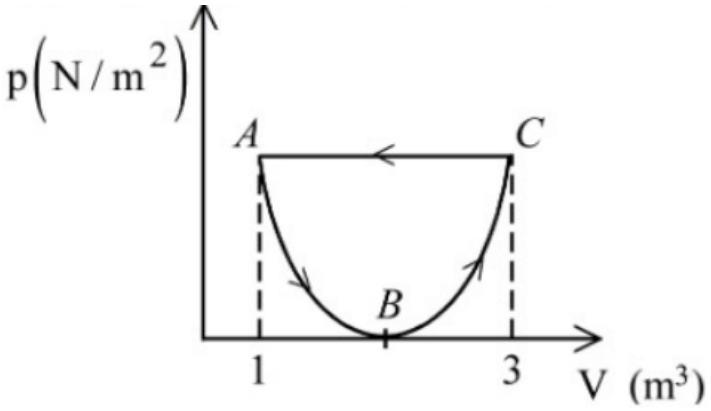
$$\frac{dv}{dt} = ge^{-kt/m} > 0, \qquad \frac{d^2v}{dt^2} = -\frac{kg}{m}e^{-kt/m} < 0.$$

Hence the velocity curve rises but is concave downward, approaching a horizontal asymptote. This is graph (B). Graph (A) decreases from a nonzero initial speed. Graph (C) has constant slope, as if resistance were absent. Graph (D) curves upward and implies increasing acceleration, contrary to the growing resistive force.

Step 4: Conclusion The velocity increases from rest and approaches $v_T = mg/k$ asymptotically, with decreasing slope. Hence, the correct option is (B).

Quick Tip to Solve: For resistance proportional to speed, check three features together: zero initial velocity, initial slope g, and a horizontal terminal-speed asymptote.

41. In the following $p-V$ diagram the equation of state along the curved path is given by $(V-2)^2 = 4ap$ where a is a constant. The total work done in the closed path is [4 marks]



(A) $-\frac{1}{a}$
(B) $-\frac{1}{3a}$
(C) $+\frac{1}{3a}$
(D) $\frac{1}{2a}$

Correct Answer: (B) $-\frac{1}{3a}$

Solution:

Concept:

- The work done by a gas is $W = \int p\,dV$. Expansion gives positive work and compression gives negative work.
- For a closed cycle, the net work equals the signed area enclosed on the p-V diagram. An anticlockwise cycle has negative net work done by the gas.

Step 1: Read the two paths and their pressures The arrows show expansion along the curved path $A \to B \to C$, from $V = 1$ to $V = 3$, followed by compression along the horizontal path $C \to A$. The curve satisfies

$$p = \frac{(V-2)^2}{4a}.$$

At both endpoints, $(V-2)^2 = 1$, so the constant pressure on CA is $p_0 = 1/(4a)$. The curve reaches zero pressure at $V = 2$.

Step 2: Integrate the work along the curved expansion The work on the lower path is

$$W_{ABC} = \int_1^3 \frac{(V-2)^2}{4a}\,dV = \frac{1}{12a}\left[(V-2)^3\right]_1^3 = \frac{1}{6a}.$$

The cubic changes from -1 to 1, so this expansion work is positive. This calculation uses the varying pressure rather than treating the curve as a straight line.

Step 3: Add the negative work of horizontal compression Along the return path the pressure is constant while the volume decreases:

$$W_{CA} = \frac{1}{4a}(1-3) = -\frac{1}{2a}.$$

Therefore

$$W_{\text{cycle}} = \frac{1}{6a} - \frac{1}{2a} = -\frac{1}{3a}.$$

The negative sign agrees with the anticlockwise arrows. The positive alternatives have the wrong sign. The value $-1/a$ has the right sign but the wrong enclosed area; the magnitude is $1/(3a)$, not $1/a$.

Step 4: Conclusion The total work done by the gas in the indicated closed path is $\boxed{-1/(3a)}$, in joules with the stated SI axes. Hence, the correct option is (B). The printed options place this negative enclosed-area result at (B).

Quick Tip to Solve: Check the arrow direction before integrating: expansion on the lower-pressure path and compression on the upper path must give negative net work.

42. The magnetic field at the centre of a current carrying circular loop of radius R is 16μ T. The magnetic field at a distance $x = \sqrt{3}R$ on its axis from the centre is

____ μT. [4 marks]

(A) 2

(B) $2\sqrt{2}$

(C) 4

(D) 8

Correct Answer: (A) 2

Solution:

Concept:

- For a circular loop of radius R carrying current I, the magnetic field at axial distance x is $B(x) = \mu_0 I R^2/[2(R^2 + x^2)^{3/2}]$.
- At the centre, $B_0 = \mu_0 I/(2R)$. Taking the ratio $B(x)/B_0$ removes the unknown current and depends only on x/R.

Step 1: Form the ratio to the field at the centre Divide the axial-field expression by the centre-field expression:

$$\frac{B(x)}{B_0} = \frac{R^3}{(R^2 + x^2)^{3/2}} = \left(1 + \frac{x^2}{R^2}\right)^{-3/2}.$$

No separate value of the loop current or radius is required. The same current and the same loop appear in both measurements, so both cancel from the ratio.

Step 2: Substitute the specified axial distance Here $x = \sqrt{3}R$, so $x^2/R^2 = 3$. Consequently

$$\frac{B(x)}{B_0} = (1 + 3)^{-3/2} = 4^{-3/2} = \frac{1}{8}.$$

The power $3/2$ is essential: $(4R^2)^{3/2} = (2R)^3 = 8R^3$, rather than $4R^3$.

Step 3: Calculate the field and reject the other values Using $B_0 = 16\,\mu$T,

$$B(x) = \frac{16}{8}\,\mu\text{T} = 2\,\mu\text{T}.$$

The answers 4 and $8\,\mu$T would require reduction factors $1/4$ and $1/2$, inconsistent with the axial-field denominator. The value $2\sqrt{2}\,\mu$T also does not equal $16/8$. The obtained field is smaller than the centre field, as expected on moving away along the axis.

Step 4: Conclusion The magnetic field is $\boxed{2\,\mu\text{T}}$. Hence, the correct option is (A). The printed option order places this value at (A); the ratio calculation fixes the result independently of the answer key.

Quick Tip to Solve: Use $B/B_0 = [1 + (x/R)^2]^{-3/2}$ whenever the centre field is known; evaluate the square root before cubing the denominator.

43. A block of mass 5 kg is moving on an inclined plane which makes an angle of 30° with the horizontal. Friction coefficient between the block and inclined plane surface is $\frac{\sqrt{3}}{2}$. The force to be applied on the block so that the block will move down without acceleration is ____ N.

$$\left(g = 10 \text{ m/s}^2\right).$$

[4 marks]

(A) 12.5

(B) 7.5

(C) 25

(D) 15

Correct Answer: (A) 12.5

Solution:

Concept:

- For a block moving down an incline, kinetic friction acts up the plane. Its magnitude is $f_k = \mu N$, where N is the normal reaction.
- Uniform motion requires zero resultant force along the plane. An applied force parallel to the plane leaves $N = mg\cos\theta$ unchanged.

Step 1: Resolve the weight perpendicular and parallel to the plane The weight is $mg = 5 \times 10 = 50\,\text{N}$. The component perpendicular to the incline determines the normal force, while the component along it tends to move the block down. Thus

$$N = 50\cos 30^\circ = 25\sqrt{3}\,\text{N}, \qquad W_{\parallel} = 50\sin 30^\circ = 25\,\text{N}.$$

The normal force balances only the perpendicular component; it is not equal to the full weight.

Step 2: Balance the forces for constant downward speed The block is sliding down, so the opposing friction force is

$$f_k = \frac{\sqrt{3}}{2}(25\sqrt{3}) = 37.5\,\text{N}.$$

Friction exceeds the downward weight component by $12.5\,\text{N}$. Therefore an additional force must act down the plane, rather than up it. Taking downhill as positive,

$$F + 25 - 37.5 = 0 \quad \Rightarrow \quad F = 12.5\,\text{N}.$$

The net force is then zero, so Newton's second law gives zero acceleration without requiring the already moving block to stop.

Step 3: Compare the alternatives Only $12.5\,\text{N}$ supplies exactly the deficit between friction and the downhill weight component. An applied force of $7.5\,\text{N}$ would leave a resultant up the plane; forces of $25\,\text{N}$ or $15\,\text{N}$ would leave a resultant down the plane. These choices therefore do not give uniform motion.

Step 4: Conclusion The required force is $\boxed{12.5\,\text{N}}$, directed down the incline. In the printed option order, this is (A). Hence, the correct option is (A).

Quick Tip to Solve: For uniform sliding, use the direction of motion to set friction, then calculate the difference between friction and the component of weight along the plane.

44. 10 kg of ice at $-10^{\circ}\mathrm{C}$ is added to 100 kg of water to lower its temperature from 25 $^{\circ}\mathrm{C}$. Consider no heat exchange to surroundings. The decrement to the temperature of water is ____ $^{\circ}\mathrm{C}$.

(specific heat of ice $= 2100\ \mathrm{J/Kg.^{\circ}C}$, specific heat of water $= 4200\ \mathrm{J/Kg.^{\circ}C}$, latent heat of fusion of ice $= 3.36 \times 10^5\ \mathrm{J/Kg}$) [4 marks]

(A) 11.6

(B) 15

(C) 6.67

(D) 10

Correct Answer: (D) 10

Solution:

Concept:

- In an insulated mixture, heat lost by the warm water equals heat gained by the ice and by the water produced on melting.
- Heating within a phase uses $Q = mc\Delta T$; melting uses $Q = mL_f$. The melted ice also warms from $0^{\circ}\mathrm{C}$ to the final temperature.

Step 1: Check whether all the ice melts The heat required to bring the ice to its melting point and melt it is

$$Q_1 = 10(2100)(10) = 2.10 \times 10^5\ \mathrm{J},$$

$$Q_2 = 10(3.36 \times 10^5) = 3.36 \times 10^6\ \mathrm{J}.$$

Their sum is 3.57×10^6 J. Cooling the original water to $0^{\circ}\mathrm{C}$ would release $100(4200)(25) = 1.05 \times 10^7$ J, which is sufficient. Thus all the ice melts and the final temperature is above zero.

Step 2: Apply heat balance to the final mixture Let the common final temperature be T in degrees Celsius. The original water loses heat, while the melted ice gains an additional $10(4200)T$. Therefore

$$100(4200)(25 - T) = 3.57 \times 10^6 + 10(4200)T.$$

Collecting the temperature terms gives

$$1.05 \times 10^7 - 3.57 \times 10^6 = 462000T,$$

$$T = \frac{6.93 \times 10^6}{462000} = 15^{\circ}\mathrm{C}.$$

Both portions of water reach this same equilibrium temperature.

Step 3: Calculate the requested decrease The question asks for the decrement, not the final temperature. It is

$$\Delta T = 25 - 15 = 10^{\circ}\mathrm{C}.$$

The choice 15 is the final temperature and hence answers a different quantity. Neither 11.6 nor 6.67 satisfies the complete heat balance including warming the ice, melting it, and warming the melt.

Step 4: Conclusion The temperature of the water decreases by $\boxed{10^{\circ}\mathrm{C}}$. Hence, the correct option is (D).

Quick Tip to Solve: After solving a mixing problem, check whether the question requests the final temperature or the change; include warming of the melted ice in the heat balance.

45. Two point charges of 1 nC and 2 nC are placed at the two corners of equilateral triangle of side 3 cm . The work done in bringing a charge of 3 nC from infinity to the third corner of the triangle is ____ μJ.

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N} \cdot \text{ m}^2/\text{C}^2$$

[4 marks]

(A) 3.3

(B) 5.4

(C) 27

(D) 2.7

Correct Answer: (D) 2.7

Solution:

Concept:

- Electric potential is a scalar. At a point equidistant from charges q_1 and q_2, the potential is $V = k(q_1 + q_2)/r$.
- The external work in bringing a test charge q_3 slowly from infinity is the increase in potential energy, $W = q_3V$, taking potential at infinity as zero.

Step 1: Convert the data and find the potential Each occupied corner is $r = 3\,\text{cm} = 3 \times 10^{-2}\,\text{m}$ from the empty corner. The fixed charges are $q_1 = 10^{-9}\,\text{C}$ and $q_2 = 2 \times 10^{-9}\,\text{C}$. Their potentials add directly because potential has no direction. Hence

$$V = \frac{9 \times 10^9(1 + 2) \times 10^{-9}}{3 \times 10^{-2}} = 900\,\text{V}.$$

There is no factor involving the angle of the equilateral triangle, as would be needed for adding electric-field vectors.

Step 2: Calculate the work for the incoming charge The charge brought in is $q_3 = 3 \times 10^{-9}\,\text{C}$. Its potential energy changes from zero at infinity to q_3V at the third corner:

$$W = (3 \times 10^{-9})(900) = 2.7 \times 10^{-6}\,\text{J}.$$

Using $1\,\mu\text{J} = 10^{-6}\,\text{J}$, this is $2.7\,\mu\text{J}$. The positive sign is consistent with doing external work against repulsion between positive charges.

Step 3: Check what energy has changed The interaction energy of the two charges already in position is unchanged and must not be added to the required work. Only the two new pair interactions contribute. The alternatives 3.3, 5.4, and 27 μJ do not equal $kq_3(q_1 + q_2)/r$; doubling the result or missing a centimetre conversion gives an incorrect value.

Step 4: Conclusion The work required is $\boxed{2.7\,\mu\text{J}}$. Direct evaluation of the printed charges gives this value, which occupies position (D) in the official printed order. Hence, the correct option is (D).

Quick Tip to Solve: When adding one charge to an existing arrangement, calculate its charge times the potential of the fixed charges; do not count their unchanged mutual energy.

Physics – Section B: Numerical Value Questions (20 marks)

46. A convex lens of refractive index 1.5 and focal length $f = 18\ \text{cm}$ is immersed in water. The difference in focal lengths of the given lens when it is in water and in air is $\alpha \times \text{f}$. The value of α is ____ .

(refractive index of water $= 4/3$) [4 marks]

Solution:

Concept:

- For a thin lens in a surrounding medium, $1/f_m = (n_l/n_m - 1)K$, where $K = 1/R_1 - 1/R_2$ depends only on the unchanged lens surfaces.
- Changing the medium changes the relative refractive index and lens power. A smaller positive relative-index contrast produces a longer positive focal length.

Step 1: Compare the refractive-index factors For air, take $n_a = 1$. Since the lens index is $n_l = 1.5 = 3/2$, the factor multiplying its curvature is

$$\frac{n_l}{n_a} - 1 = \frac{3}{2} - 1 = \frac{1}{2}.$$

For water, $n_w = 4/3$, giving

$$\frac{n_l}{n_w} - 1 = \frac{3/2}{4/3} - 1 = \frac{9}{8} - 1 = \frac{1}{8}.$$

The lens remains converging because its refractive index still exceeds that of the water.

Step 2: Eliminate the unchanged curvature The lens has the same radii of curvature before and after immersion, so dividing the two lens-maker equations eliminates K:

$$\frac{1/f_w}{1/f_a} = \frac{1/8}{1/2} = \frac{1}{4}.$$

Thus $f_w = 4f_a$. With $f_a = f = 18\,\text{cm}$,

$$f_w = 72\,\text{cm}.$$

It is the power that falls by a factor of four; the focal length increases by that factor. This direction agrees with the weaker refraction at a glass-water interface.

Step 3: Express the difference in the requested form The difference in focal lengths is positive:

$$f_w - f_a = 72 - 18 = 54\,\text{cm} = 3(18\,\text{cm}) = 3f.$$

Comparing this with αf gives $\alpha = 3$. The ratio $f_w/f_a = 4$ must not be confused with the difference expressed in units of f; subtracting the original focal length is essential.

Step 4: Conclusion $\boxed{\alpha = 3}$ (dimensionless); the focal length increases by $54\,\text{cm}$.

Quick Tip to Solve: Use relative refractive index in the lens-maker formula, take a ratio to cancel the radii, and distinguish the focal-length ratio from the requested difference.

47. A solid sphere of radius 10 cm is rotating about an axis which is at a distance 15 cm from its centre. The radius of gyration about this axis is $\sqrt{n}$ cm. The value of n is [4 marks]

Solution:

Concept:

- A uniform solid sphere has moment of inertia $I_C = \frac{2}{5}MR^2$ about any diameter through its centre.
- For a parallel axis a perpendicular distance d away, $I = I_C + Md^2$. The radius of gyration k is defined by $I = Mk^2$.

Step 1: Identify the central reference axis Take the diameter through the centre parallel to the specified rotation axis. Spherical symmetry makes its central moment of inertia $2MR^2/5$, irrespective of that diameter's orientation. The given radius is $R = 10\,\text{cm}$, and the distance between the parallel axes is $d = 15\,\text{cm}$. This is the distance from the centre, so the radius must not be added to it. No numerical mass is needed because it will cancel.

Step 2: Apply the parallel-axis theorem Adding the translation contribution to the central inertia gives

$$I = \frac{2}{5}M(10\,\text{cm})^2 + M(15\,\text{cm})^2.$$

Factoring out the mass,

$$\frac{I}{M} = \frac{2}{5}(100) + 225 = 40 + 225 = 265\,\text{cm}^2.$$

The added term is positive, as expected for an axis farther from the centre. The solid-sphere coefficient $2/5$, rather than the hollow-sphere coefficient $2/3$, is required by the question.

Step 3: Read the radius of gyration and the coefficient By definition, $k^2 = I/M$. Therefore

$$k = \sqrt{265}\,\text{cm}.$$

Comparing with the stated $\sqrt{n}\,\text{cm}$ gives $n = 265$. It is valid to keep all lengths in centimetres because the requested radius is in centimetres. As a check, $k \approx 16.28\,\text{cm} > d$, consistent with the additional nonzero spread of mass around the centre.

Step 4: Conclusion The radius of gyration is $\sqrt{265}\,\text{cm}$, so the requested dimensionless coefficient is $\boxed{n = 265}$.

Quick Tip to Solve: For a shifted axis, use $k^2 = k_C^2 + d^2$; retain the length unit requested and square both distances before adding.

48. The displacement of a particle, executing simple harmonic motion with time period T, is expressed as $x(t) = A\sin\omega t$, where A is the amplitude. The maximum value of potential energy of this oscillator is found at $t = T/2\beta$. The value of β is ____ . [4 marks]

Solution:

Concept:

- The potential energy of an ideal simple harmonic oscillator, measured from equilibrium, is $U = \frac{1}{2}kx^2$, where $k = m\omega^2$ is the restoring-force constant.
- For $x = A\sin\omega t$, the potential energy is greatest at either extreme position, $|x| = A$. The angular frequency is $\omega = 2\pi/T$.

Step 1: Write the potential energy as a function of time Substitute the given displacement into the energy expression:

$$U(t) = \frac{1}{2}kA^2\sin^2(\omega t).$$

The coefficient is constant, so the maximum value is $U_{\max} = kA^2/2$, attained when $\sin^2(\omega t) = 1$. At these instants the particle is at an extreme and its kinetic energy is zero. At the equilibrium position the reverse is true: potential energy is minimum and kinetic energy is maximum.

Step 2: Find the first positive time of maximum energy At $t = 0$, the sine displacement is zero, so the particle begins at equilibrium. It reaches its first extreme when the phase increases to $\pi/2$:

$$\omega t = \frac{\pi}{2}, \qquad t = \frac{\pi}{2\omega}.$$

Using the time-period relation,

$$t = \frac{\pi}{2(2\pi/T)} = \frac{T}{4}.$$

More generally the energy maxima occur at $t = (2j+1)T/4$, for nonnegative integers j, since both extremes have the same potential energy.

Step 3: Compare with the stated time For the first maximum, comparing $t = T/4$ with the given $T/(2\beta)$ yields

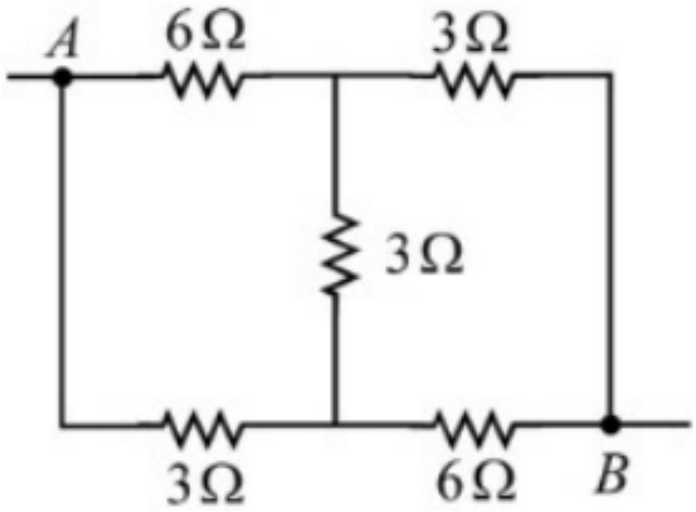
$$\frac{T}{2\beta} = \frac{T}{4} \quad \Rightarrow \quad 2\beta = 4 \quad \Rightarrow \quad \beta = 2.$$

A time of $T/2$ would instead return the particle to equilibrium, where its potential energy is zero. The quarter-period result follows from the sine initial phase, not just from the period alone.

Step 4: Conclusion The first potential-energy maximum occurs at $t = T/4$; hence $\boxed{\beta = 2}$ (dimensionless).

Quick Tip to Solve: For a sine displacement starting at equilibrium, the first extreme is one quarter-period later; energy depends on displacement squared and has maxima at both extremes.

49. The equivalent resistance between the points A and B in the following circuit is $\frac{x}{5}\Omega$. The value of x is ____ . [4 marks]



Solution:

Concept:

- For a resistor connecting potentials V_1 and V_2, the current from the first node is $(V_1 - V_2)/R$. Kirchhoff's junction rule states that the algebraic sum of currents leaving an internal node is zero.
- The equivalent resistance is $R_{AB} = V/I$, where V is the terminal voltage and I is the total current drawn. A bridge resistor can be ignored only if its endpoints are at equal potentials.

Step 1: Assign the junction potentials Apply voltage V, taking $V_A = V$ and $V_B = 0$. Let the upper internal junction C have potential u, and the lower junction D have potential v. The upper branch contains $6\,\Omega$ followed by $3\,\Omega$; the lower branch has $3\,\Omega$ followed by $6\,\Omega$. The arm ratios differ, so this bridge is not balanced.

Step 2: Apply the junction equations At C, the three currents give

$$\frac{u - V}{6} + \frac{u}{3} + \frac{u - v}{3} = 0 \quad \Rightarrow \quad 5u - 2v = V.$$

At D, similarly,

$$\frac{v - V}{3} + \frac{v}{6} + \frac{v - u}{3} = 0 \quad \Rightarrow \quad 5v - 2u = 2V.$$

Solving the simultaneous equations gives $u = 3V/7$, $v = 4V/7$. The unequal potentials confirm that the central resistor conducts.

Step 3: Calculate the terminal current Add the currents through the two resistors attached to A:

$$I = \frac{V - u}{6} + \frac{V - v}{3} = \frac{2V}{21} + \frac{V}{7} = \frac{5V}{21}.$$

The voltage-to-current ratio is therefore $R_{AB} = 21/5\,\Omega$. This is smaller than either outer path's resistance of $9\,\Omega$, consistent with the network providing several conducting paths.

Step 4: Conclusion Comparing $R_{AB} = \frac{21}{5}\,\Omega$ with $\frac{x}{5}\,\Omega$, the required dimensionless value is $\boxed{x = 21}$.

Quick Tip to Solve: Check the arm ratios before ignoring a bridge resistor. When they differ, two junction-potential equations give the answer directly.

50. The ratio of de Broglie wavelength of a deutron with kinetic energy E to that of an alpha particle with kinetic energy $2E$, is $n:1$. The value of n is ____ .

(Assume mass of proton = mass of neutron) : [4 marks]

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and p is momentum. For a nonrelativistic particle, kinetic energy $K = p^2/(2m)$, so $p = \sqrt{2mK}$.
- Consequently $\lambda = h/\sqrt{2mK}$. Both particle mass and kinetic energy affect the wavelength, which varies inversely as the square root of their product.

Step 1: Identify the two masses Let the common proton and neutron mass be m_0. A deuteron contains one proton and one neutron, so $m_d = 2m_0$. An alpha particle contains two protons and two neutrons, so $m_\alpha = 4m_0$. Particle charges do not enter this calculation because the kinetic energies are already specified, rather than being inferred from an accelerating potential.

Step 2: Substitute the kinetic energies For the deuteron the kinetic energy is E, while for the alpha particle it is $2E$. The wavelengths are

$$\lambda_d = \frac{h}{\sqrt{2(2m_0)E}} = \frac{h}{\sqrt{4m_0E}},$$

$$\lambda_\alpha = \frac{h}{\sqrt{2(4m_0)(2E)}} = \frac{h}{\sqrt{16m_0E}}.$$

Thus the alpha particle has four times the product mK, and twice the momentum, of the deuteron.

Step 3: Take the ratio in the requested order The requested ratio has the deuteron wavelength in the numerator. Therefore

$$\frac{\lambda_d}{\lambda_\alpha} = \sqrt{\frac{16m_0E}{4m_0E}} = \sqrt{4} = 2.$$

Both h and the common factor m_0E cancel. The result does not depend on the numerical value of E, since the given energies are specified relative to the same energy scale.

Step 4: Conclusion The wavelength ratio is $\lambda_d : \lambda_\alpha = 2:1$, so $\boxed{n = 2}$. Both the ratio and n are dimensionless.

Quick Tip to Solve: For de Broglie wavelength ratios at specified energies, compare the products mK, take the square roots, and invert the ratio.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Given below are two statements:

Statement I : Griss-Ilosvay test is used for the detection of nitrite ion, which involves the use of sulphanilic acid and α-naphthylamine reagent.

Statement II : In the above test, sulphanilic acid is diazotized by the acidified nitrite ion, which on further coupling with α-naphthylamine forms an azo-dye.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- Acidified nitrite produces nitrous acid, HNO_2, which diazotizes a primary aromatic amine. The resulting diazonium species couples with an activated aromatic ring.
- The Griess–Ilosvay test detects nitrite through formation of a coloured azo dye. Sulphanilic acid supplies the amino group for diazotization, and α-naphthylamine is the coupling component.

Step 1: Check Statement I Nitrite ion, NO_2^-, is the species being detected. Acidification generates the reagent needed to diazotize sulphanilic acid. Sulphanilic acid and α-naphthylamine are indeed the two organic reagents used in this test. The eventual azo-dye colour provides the visible analytical observation. Statement I therefore correctly identifies both the ion and the test reagents.

Step 2: Trace the reactions in Statement II The aromatic primary amino group of sulphanilic acid becomes a diazonium group through reaction with nitrous acid. The general diazotization equation is

$$\mathrm{Ar{-}NH_2 + HNO_2 + H^+ \longrightarrow Ar{-}N_2^+ + 2\,H_2O.}$$

Here Ar represents the sulphonated aromatic group. The diazonium compound then couples with the activated naphthalene ring of α-naphthylamine. This forms the azo linkage, $-N{=}N-$, whose extended conjugation produces the characteristic colour. Statement II correctly describes this sequence.

Step 3: Compare the truth combinations Both statements are true, as stated in option (A). Option (B) rejects both correct facts. Option (C) wrongly rejects diazotization followed by coupling, while option (D) wrongly rejects the identity of the nitrite test and its reagents. There is no contradiction between the reagent list and the proposed reaction mechanism.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: For an azo-dye test, identify the primary aromatic amine that is diazotized first and the activated aromatic compound that couples with it.

52. CORRECT order of stability for the following is

$$CH_2 = CH^-, CH_3 - CH_2^-, CH \equiv C^-$$

[4 marks]

(A) $CH_2 = CH^- > CH \equiv C^- > CH_3 - CH_2^-$
(B) $CH_3 - CH_2^- > CH_2 = CH^- > CH \equiv C^-$
(C) $CH \equiv C^- > CH_3 - CH_2^- > CH_2 = CH^-$
(D) $CH \equiv C^- > CH_2 = CH^- > CH_3 - CH_2^-$

Correct Answer: (D) $CH \equiv C^- > CH_2 = CH^- > CH_3 - CH_2^-$

Solution:

Concept:

- A carbon lone pair is stabilized by increasing the s-character of its orbital, because an s orbital holds excess electron density closer to the nucleus than a p orbital.
- The charged carbon is sp, sp^2, or sp^3 hybridized in an acetylide, vinyl, or alkyl carbanion. The respective s-characters are 50%, $33\frac{1}{3}\%$, and 25%.

Step 1: Identify the charged-carbon orbitals In $HC{\equiv}C^-$, the negative charge is on the terminal carbon of a triple bond, so the lone pair occupies an sp orbital. In $CH_2{=}CH^-$, the charge lies directly on a double-bond carbon and the lone-pair orbital is sp^2. In $CH_3CH_2{}^-$, the saturated carbon has an sp^3 lone-pair orbital. The vinyl ion must not be confused with an allylic ion: its charge is on the double bond, rather than adjacent to it.

Step 2: Rank the negative-charge stabilization Greater s-character increases the attraction of the orbital for its electrons. Thus the acetylide ion is most stable, the vinyl ion is intermediate, and the ethyl ion is least stable:

$$HC{\equiv}C^- > CH_2{=}CH^- > CH_3CH_2{}^-.$$

This order also agrees with the relative acidity of the conjugate hydrocarbons: a terminal alkyne loses its terminal proton more readily than an alkene or an alkane loses the corresponding proton.

Step 3: Eliminate the other options Option (A) incorrectly places vinyl above acetylide. Option (B) places the alkyl carbanion first and reverses the relevant trend. Option (C) starts correctly with acetylide but incorrectly ranks ethyl above vinyl. Only option (D) follows decreasing s-character for all three charged-carbon orbitals.

Step 4: Conclusion The stability order is $HC{\equiv}C^- > CH_2{=}CH^- > CH_3CH_2{}^-$. Hence, the correct option is (D).

Quick Tip to Solve: For these carbanions, compare the orbital containing the lone pair: stability follows $sp > sp^2 > sp^3$, corresponding to decreasing s-character.

53. Given below are two statements :

Statement I : The number of pairs, from the following, in which both the ions are coloured in aqueous solution is 3.

$$\left[Sc^{3+}, Ti^{3+}\right], \left[Mn^{2+}, Cr^{2+}\right], \left[Cu^{2+}, Zn^{2+}\right] \text{ and } \left[Ni^{2+}, Ti^{4+}\right]$$

Statement II : Th^{4+} is the strongest reducing agent among $Th^{4+}, Ce^{4+}, Gd^{3+}$ and Eu^{2+}.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (B) Both Statement I and Statement II are false

Solution:

Concept:

- Partially filled d orbitals in ordinary aqueous transition-metal ions permit visible-light d–d transitions. The d^0 and d^{10} ions in this comparison are colourless.
- A reducing agent donates an electron and is itself oxidized. Europium(II) can donate an electron to form europium(III), whereas cerium(IV) typically accepts an electron.

Step 1: Count pairs in which both ions are coloured In the first pair, Sc^{3+} is d^0 and colourless, while Ti^{3+} is d^1 and coloured. The second pair qualifies: Mn^{2+} is d^5, pale pink, and Cr^{2+} is d^4, coloured. In the third pair, Cu^{2+} is coloured with d^9, but Zn^{2+} is colourless with d^{10}. Finally, Ni^{2+} is coloured with d^8, whereas Ti^{4+} is colourless with d^0. Exactly one pair contains two coloured ions, not three. Statement I is false.

Step 2: Identify the reducing agent Europium(II) readily supplies an electron to form the trivalent ion:

$$Eu^{2+} \longrightarrow Eu^{3+} + e^{-}.$$

Thus Eu^{2+} is the strongest reducing agent listed. Thorium(IV) is already in its stable high oxidation state and is not the strongest reducing agent. Cerium(IV) is characteristically oxidizing, because it is reduced to cerium(III); gadolinium(III) is stabilized by its half-filled $4f^7$ shell. Statement II is also false.

Step 3: Choose the correct combination Option (B) gives the false–false combination. Option (A) incorrectly accepts both assertions. Option (C) wrongly accepts the stated count of coloured pairs, and option (D) wrongly accepts the thorium assertion. Each statement has been checked separately before combining the results.

Step 4: Conclusion Only one pair has both ions coloured, and Eu^{2+}, rather than Th^{4+}, is the strongest reducing agent listed. Both statements are false. Hence, the correct option is (B).

Quick Tip to Solve: Count a pair only when both members are coloured. In redox comparisons, identify which ion donates an electron, rather than which accepts one.

54.

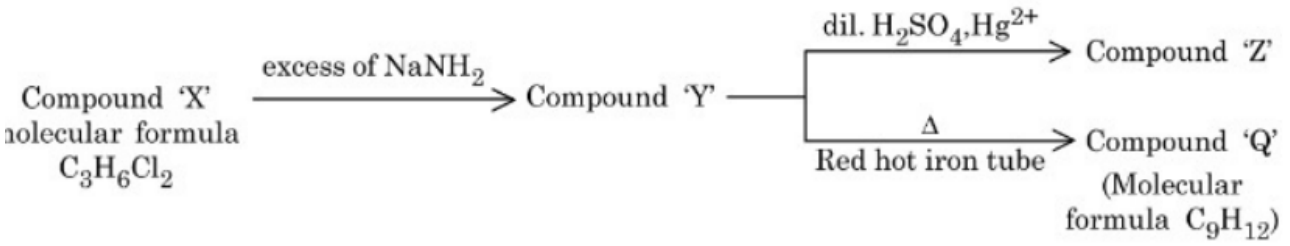
Given below are two statements for the following reaction sequence.

Statement I : Compound ' Z ' will give yellow precipitate with NaOI .

Statement II : Compound ' Q ' has two different types of ' H ' atoms (aromatic : aliphatic) in the ratio $1:3$.

In the light of the above statements, choose the *correct* answer from the options given below :

[4 marks]



(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- Excess $NaNH_2$ promotes dehydrohalogenation of a suitable dihalide to an alkyne. Acidic mercuric-ion catalysed hydration of propyne gives an enol that tautomerizes to propanone.
- A methyl ketone gives the iodoform reaction, forming yellow CHI_3. Alkynes can also undergo cyclic trimerization when passed through a red-hot iron tube.

Step 1: Identify the hydration product The sequence converts the three-carbon dihalide X, $C_3H_6Cl_2$, to the three-carbon alkyne Y, propyne, $CH_3C{\equiv}CH$. Hydration using dilute H_2SO_4 and Hg^{2+} produces the Markovnikov enol, which tautomerizes to acetone:

$$CH_3C{\equiv}CH + H_2O \longrightarrow CH_3COCH_3.$$

Hence Z is propanone, containing a methyl ketone group.

Step 2: Check Statement I Acetone undergoes the haloform reaction under alkaline iodine or hypoiodite conditions. Its methyl ketone group yields yellow iodoform:

$$CH_3COCH_3 + 3\,I_2 + 4\,OH^- \longrightarrow CHI_3 + CH_3COO^- + 3\,I^- + 3\,H_2O.$$

Thus the stated $NaOI$ reagent gives the predicted yellow precipitate, and Statement I is true.

Step 3: Identify the cyclic trimer and count hydrogens Three molecules of propyne form mesitylene, 1,3,5-trimethylbenzene, in the red-hot iron tube. Its formula is C_9H_{12}, matching Q. There are three equivalent aromatic hydrogen atoms on the ring and nine equivalent aliphatic hydrogen atoms in the

three methyl groups. Consequently the two hydrogen environments have the ratio

$$H_{\text{aromatic}} : H_{\text{aliphatic}} = 3 : 9 = 1 : 3.$$

Statement II is therefore true. Options (B), (C), and (D) reject one or both of the independently verified statements; only option (A) accepts both.

Step 4: Conclusion Z is acetone and gives yellow CHI_3; Q is mesitylene with aromatic-to-aliphatic hydrogens in the ratio $1 : 3$. Both statements are true. Hence, the correct option is (A).

Quick Tip to Solve: Recognize propyne as the branching point: hydration yields a methyl ketone, while cyclic trimerization yields mesitylene.

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55. Given below are two statements :

Statement I : The number of species among BF_4^-, SiF_4, XeF_4 and SF_4, that have unequal $E - F$ bond lengths is two. Here, E is the central atom.

Statement II : Among O_2^-, O_2^{2-}, F_2 and O_2^+, O_2^- has the highest bond order.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (B) Both Statement I and Statement II are false

Solution:

Concept:

- Equivalent bonds related by molecular symmetry have equal lengths. Distinguish the electron-pair geometry from the molecular shape when a central atom has lone pairs.
- Molecular orbital bond order is $\text{BO} = (N_b - N_a)/2$, where N_b and N_a count bonding and antibonding electrons. Adding an electron to an antibonding orbital decreases bond order by $1/2$.

Step 1: Count the species with unequal bond lengths Both $BF_4{}^-$ and SiF_4 are tetrahedral with four equivalent fluorine positions. All four E–F bonds are therefore equal within each species. In XeF_4, two lone pairs occupy opposite positions in the octahedral electron-pair arrangement; the four fluorine atoms form a square plane and have equal Xe–F lengths. In see-saw-shaped SF_4, the axial S–F bonds differ in length from the equatorial ones because their lone-pair and bond-pair repulsions differ. Thus only one species, not two, has unequal E–F bonds. Statement I is false.

Step 2: Compare the four bond orders Neutral O_2 has bond order 2. Addition of one or two electrons fills antibonding orbitals, whereas removal of one electron removes an antibonding electron. Consequently,

$$\text{BO}(O_2{}^-) = 1.5, \quad \text{BO}(O_2{}^{2-}) = 1,$$
$$\text{BO}(F_2) = 1, \qquad \text{BO}(O_2{}^+) = 2.5.$$

The largest value belongs to O_2^+, not O_2^-. Statement II is also false. Options (A), (C) and (D) each require at least one of these false statements to be true and are excluded.

Step 3: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (B).

Quick Tip to Solve: For oxygen ions, start with bond order 2 for O_2: each added antibonding electron subtracts one-half, and each removed one adds one-half.

56. 20.0dm^3 **of an ideal gas ' X ' at 600 K and 0.5 MPa undergoes isothermal reversible expansion until pressure of the gas is 0.2 MPa . Which of the following option is correct?**
(Given: $\log 2 = 0.3010$ **and** $\log 5 = 0.6989$ **)** [4 marks]

(A) $\text{w} = -3.9\ \text{kJ}, \Delta\text{U} = 0, \Delta\text{H} = 0; \text{q} = 3.9\ \text{kJ}$
(B) $\text{w} = 9.1\ \text{J}, \Delta\text{U} = 9.1\ \text{J}, \Delta\text{H} = 0; \text{q} = 0$
(C) $\text{w} = -9.1\ \text{kJ}, \Delta\text{U} = 0, \Delta\text{H} = 0, \text{q} = 9.1\ \text{kJ}$
(D) $\text{w} = +4.1\ \text{kJ}, \Delta\text{U} = 0, \Delta\text{H} = 0; \text{q} = -4.1\ \text{kJ}$
Correct Answer: (C) $\text{w} = -9.1\ \text{kJ}, \Delta\text{U} = 0, \Delta\text{H} = 0, \text{q} = 9.1\ \text{kJ}$

Solution:

Concept:

- For an ideal gas, internal energy and enthalpy depend only on temperature. Isothermal expansion therefore gives $\Delta U = \Delta H = 0$.
- Using the chemistry sign convention, reversible expansion work is $w = -nRT\ln(V_2/V_1)$, and the first law is $\Delta U = q + w$. Work is negative when the gas expands against its surroundings.

Step 1: Find the volume ratio and energy scale At fixed temperature, $P_1V_1 = P_2V_2 = nRT$. Therefore,

$$\frac{V_2}{V_1} = \frac{P_1}{P_2} = \frac{0.5}{0.2} = 2.5.$$

Convert the initial pressure and volume before multiplying:

$$nRT = P_1V_1 = (0.5 \times 10^6\,\text{Pa})(20.0 \times 10^{-3}\,\text{m}^3) = 10^4\,\text{J}.$$

The final volume is $50.0\,\text{dm}^3$, confirming that this process is an expansion.

Step 2: Calculate the reversible work Use the logarithm of the volume ratio, not the final pressure times the total volume change:

$$\begin{aligned} w &= -2.303(10^4)\log(2.5) \\ &= -2.303(10^4)(\log 5 - \log 2) \\ &= -2.303(10^4)(0.6989 - 0.3010) \\ &\simeq -9.16 \times 10^3\,\text{J}. \end{aligned}$$

This is approximately $-9.1\,\text{kJ}$ at the precision represented by the options.

Step 3: Apply the first law and check the options Because $\Delta U = 0$, the absorbed heat exactly balances

the work done by the gas: $q = -w \simeq +9.1\,\text{kJ}$. Also $\Delta H = 0$, since the temperature remains constant. Option (A) gives the wrong work magnitude; option (B) has the wrong work sign and a nonzero internal-energy change; option (D) has both the wrong sign and magnitude. The computed set appears in the third printed option.

Step 4: Conclusion $w \simeq -9.1\,\text{kJ}$, $\Delta U = \Delta H = 0$, and $q \simeq +9.1\,\text{kJ}$. The calculated values identify the third option in the printed order. Hence, the correct option is (C).

Quick Tip to Solve: For reversible isothermal expansion, first calculate P_1V_1 in joules and P_1/P_2. A positive expansion work signals the wrong sign convention.

57. Consider a weak base ' B ' of $\text{pK}_\text{b} = 5.699$. ' x ' mL of 0.02 M HCl and ' y ' mL of 0.02 M weak base ' B ' are mixed to make 100 mL of a buffer of pH 9 at 25°C. The values of ' x ' and ' y ' respectively are : [Given : $\log 2 = 0.3010, \log 3 = 0.4771, \log 5 = 0.699$] [4 marks]

(A)

x	y
42.7	57.3

(B)

x	y
11.1	88.9

(C)

x	y
85.7	14.3

(D)

x	y
14.3	85.7

Correct Answer: (D)

x	y
14.3	85.7

Solution:

Concept:

- A weak base and its conjugate acid form a basic buffer. Added strong acid reacts first: $\text{B} + \text{H}^+ \longrightarrow \text{BH}^+$; use the amounts remaining after this reaction.
- At 25°C, $\text{pOH} = 14 - \text{pH}$, and $\text{pOH} = \text{p}K_b + \log([\text{BH}^+]/[\text{B}])$.

Step 1: Set up the amounts after mixing Both starting solutions have concentration $0.02\,\text{mol}\,\text{L}^{-1}$. Hence $x\,\text{mL}$ of acid supplies $0.02x$ millimoles of H^+, while $y\,\text{mL}$ of base supplies $0.02y$ millimoles of B. After neutralisation, the conjugate acid amount is $0.02x$ millimoles and the unreacted base amount is $0.02(y - x)$ millimoles. The buffer requires $y > x$. Since the final volume is common to both species,

$$\frac{[\text{BH}^+]}{[\text{B}]} = \frac{x}{y - x}, \qquad x + y = 100.$$

Step 2: Use the required pH The required pH 9 corresponds to pOH 5. Substitution in the buffer

equation gives

$$5 = 5.699 + \log \frac{x}{y-x},$$
$$\log \frac{x}{y-x} = -0.699 = -\log 5,$$
$$\frac{x}{y-x} = \frac{1}{5}.$$

Thus $5x = y - x$, or $y = 6x$. Together with $x + y = 100$, this gives

$$x = \frac{100}{7} = 14.2857\ldots, \qquad y = \frac{600}{7} = 85.7143\ldots.$$

Step 3: Check neutralisation and the alternatives These volumes leave five times as much free base as conjugate acid, exactly the ratio required by pOH 5. Option (A) gives too much acid relative to the remaining base; option (B) gives a base-to-conjugate-acid ratio of about seven instead of five. In option (C), acid exceeds base, so the weak-base buffer would not remain. The fourth printed option gives the calculated volumes.

Step 4: Conclusion $x = 14.3\,\text{mL}$ and $y = 85.7\,\text{mL}$. Partial neutralisation leaves the required ratio $[\text{B}]/[\text{BH}^+] = 5$, identifying the fourth printed option. Hence, the correct option is (D).

Quick Tip to Solve: In an acid-plus-weak-base buffer, the free base is proportional to $y - x$, not y. Perform neutralisation before applying the buffer equation.

58. Regarding the hydrides of group 15 elements $EH_3(E = N, P, As, Sb)$, select the correct statement from the following :

A. The stability of hydrides decreases down the group.

B. The basicity of hydrides decreases down the group.

C. The reducing character increases down the group.

D. The boiling point increases down the group.

Choose the correct answer from the options given below : [4 marks]

(A) A, B, C& D

(B) B & C only

(C) A & D only

(D) A, B & C only

Correct Answer: (D) A, B & C only

Solution:

Concept:

- Down group 15 the central atom becomes larger and the E–H bond weakens. This affects thermal stability and the readiness of a hydride to act as a reducing agent.
- Basicity depends on the availability of the central-atom lone pair for protonation. Boiling point depends on intermolecular forces, so hydrogen bonding can interrupt the mass trend.

Step 1: Check stability and basicity The E–H bond becomes longer and weaker on moving from nitrogen to phosphorus, arsenic and antimony. Consequently, hydride stability decreases in the order $NH_3 > PH_3 > AsH_3 > SbH_3$. Statement A is true. Basicity also decreases down the group: the lone pair on the larger central atoms is less effectively donated to a proton. Ammonia is the strongest base of these hydrides, while stibine is the weakest. Thus statement B is true.

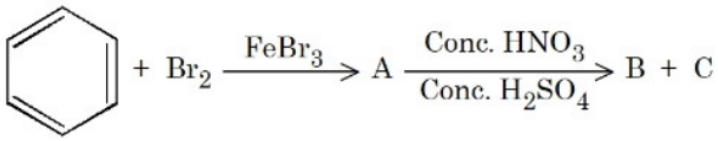
Step 2: Check reducing character The less stable hydrides lose hydrogen more readily, so reducing character increases down the group. Thus SbH_3 is more strongly reducing than NH_3. This is the opposite trend to thermal stability, making statement C true. It is the weakening E–H bonding, rather than increasing basicity, that accounts for this trend.

Step 3: Identify the boiling-point exception For PH_3, AsH_3 and SbH_3, increasing molecular size and polarizability increase dispersion forces and boiling point. However, NH_3 molecules form hydrogen bonds and have an anomalously high boiling point relative to PH_3. The boiling point therefore falls from ammonia to phosphine before rising farther down the group. Statement D, which asserts an uninterrupted increase for all four hydrides, is false. Options (A) and (C) incorrectly include D, while option (B) omits the true statement A.

Step 4: Conclusion Statements A, B and C are true, whereas D is false because of hydrogen bonding in ammonia. Hence, the correct option is (D).

Quick Tip to Solve: For group 15 hydrides, remember three smooth trends: stability down, basicity down, reducing strength up. Test boiling-point claims separately for ammonia hydrogen bonding.

59. Method used for separation of mixture of products (B and C) obtained in the following reaction is [4 marks]



(A) sublimation

(B) steam distillation

(C) fractional distillation

(D) simple distillation

Correct Answer: (C) fractional distillation

Solution:

Concept:

- Bromination of benzene with $Br_2/FeBr_3$ is electrophilic aromatic substitution. A bromine substituent deactivates the ring but directs further substitution mainly to the ortho and para positions.
- Fractional distillation separates volatile components with close boiling points by repeated vaporisation and condensation in a fractionating column. Simple distillation works best when the boiling-point separation is large.

Step 1: Identify intermediate A and the final products Benzene first undergoes substitution with bromine in the presence of the Lewis acid $FeBr_3$, giving bromobenzene as A. The concentrated nitric acid–sulfuric acid mixture then supplies the nitrating electrophile. Although bromine withdraws electron density inductively and slows substitution, its lone-pair resonance donation makes the ortho and para positions the preferred sites. The mixture B and C therefore consists mainly of ortho-bromonitrobenzene and para-bromonitrobenzene.

Step 2: Select the separation principle The two products are positional isomers with the same molecular formula and closely spaced boiling points. A fractionating column gives repeated liquid–vapour equilibrations: each condensation and revaporisation enriches the rising vapour in the more volatile component. This repeated process is required to resolve such a close-boiling mixture more effectively than a single distillation stage.

Step 3: Exclude the other methods Simple distillation provides too little repeated equilibration for close-boiling components. Steam distillation is useful for isolating a steam-volatile, water-immiscible substance from a nonvolatile matrix; it does not provide the needed fractional resolution merely because these products are organic. Sublimation is not the selected boiling-point-based separation for this pair. Among the listed methods, fractional distillation directly addresses their close boiling points and is option (C).

Step 4: Conclusion B and C are the ortho and para bromonitrobenzene products; their close boiling points call for fractional distillation among the listed choices. Hence, the correct option is (C).

Quick Tip to Solve: Identify the products before selecting a separation method. For a close-boiling mixture, the repeated equilibrations in a fractionating column are the decisive feature.

60. In period 4 of the periodic table, the elements with highest and lowest atomic radii are respectively.

[4 marks]

(A) Na & Cl

(B) K&Br

(C) Rb & Br

(D) K&Se

Correct Answer: (B) K&Br

Solution:

Concept:

- For the usual bonded-atom radii, atomic size generally decreases across a period as effective nuclear charge increases while electrons enter the same principal shell.
- First identify which period each candidate belongs to. Noble-gas van der Waals radii are measured on a different basis and should not be substituted into a covalent-radius comparison.

Step 1: Locate the beginning of the fourth period Period 4 starts with potassium, whose outer configuration is $[Ar]4s^1$. Its valence electron occupies a new principal shell beyond the argon core and experiences relatively weak effective nuclear attraction. Potassium therefore has the largest atomic

radius among the period-4 elements in this comparison. Sodium belongs to period 3, and rubidium belongs to period 5; neither can be the required first element.

Step 2: Follow the trend toward the halogen As atomic number increases across period 4, the nucleus attracts the electrons more strongly. Shielding does not fully cancel the increasing nuclear charge, so the bonded-atom radius contracts overall. Among the listed period-4 choices near the right-hand end, bromine follows selenium and is smaller. Thus bromine, rather than selenium, supplies the smallest radius in the conventional periodic-trend comparison used by these choices.

Step 3: Check every pair against both conditions Option (A), Na and Cl, consists of period-3 elements, so it is excluded even though it resembles the analogous trend in that period. Option (C) incorrectly starts with Rb, a period-5 element. Option (D) starts correctly with K but ends with Se rather than the smaller Br. Option (B) alone gives the correct period and both required extremes. A noble-gas size tabulated as a van der Waals radius is not directly comparable to the bonded radii used for this trend.

Step 4: Conclusion The elements are potassium and bromine, respectively: K has the highest radius and Br the lowest in the indicated period-4 comparison. Hence, the correct option is (B).

Quick Tip to Solve: Eliminate wrong-period elements first: Na is in period 3 and Rb is in period 5. Then use the left-to-right contraction to distinguish Br from Se.

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61. At $\mathrm{T(K)}, 2$ moles of liquid A and 3 moles of liquid B are mixed. The vapour pressure of ideal solution formed is 320 mm Hg . At this stage, one mole of A and one mole of B are added to the solution. The vapour pressure is now measured as 328.6 mm Hg . The vapour pressure (in mm Hg) of A and B are respectively: [4 marks]

(A) 400, 300

(B) 500, 200

(C) 600, 400

(D) 300, 200

Correct Answer: (B) 500, 200

Solution:

Concept:

- For an ideal binary liquid solution, Raoult's law gives $P = x_A P_A^\circ + x_B P_B^\circ$, where the mole fractions refer to the liquid and the pure-component pressures are at the same temperature.
- Adding liquid changes both the component mole numbers and their common total. The pure-liquid vapour pressures remain fixed because the temperature remains fixed.

Step 1: Write the equation for the first mixture Initially the total amount is $2+3=5$ mol, so $x_A = 2/5$ and $x_B = 3/5$. The measured total pressure therefore gives

$$320 = \frac{2}{5}P_A^\circ + \frac{3}{5}P_B^\circ \quad \Longrightarrow \quad 2P_A^\circ + 3P_B^\circ = 1600.$$

Step 2: Use the new composition After adding one mole of each, the solution contains 3 mol A and 4

mol B, not equal mole fractions. Hence

$$328.6 = \frac{3}{7}P_A^\circ + \frac{4}{7}P_B^\circ, \qquad 3P_A^\circ + 4P_B^\circ = 2300.2.$$

Eliminating P_A° gives $P_B^\circ = 199.6$ mm Hg and then $P_A^\circ = 500.6$ mm Hg. The small departures from round values reflect the second pressure being stated to one decimal place.

Step 3: Check the offered pressure pairs For $(P_A^\circ, P_B^\circ) = (500, 200)$ mm Hg, the first pressure is exactly

$$\frac{2(500) + 3(200)}{5} = 320 \text{ mm Hg},$$

and the second is $2300/7 = 328.571\ldots$ mm Hg, which rounds to 328.6. The other listed pairs predict initial pressures 340, 480 and 240 mm Hg respectively, so none satisfies even the first measurement.

Step 4: Conclusion The vapour pressures of A and B are respectively $\boxed{500 \text{ mm Hg},\ 200 \text{ mm Hg}}$, consistent with the stated measurement precision. Hence, the correct option is (B).

Quick Tip to Solve: Write a fresh total mole number after each addition, and check rounded experimental pressures by substituting the offered pure-component values.

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62. $\text{Ph} - \text{CH} = \text{CH}_2 \xrightarrow[\text{HBr}]{(\text{PhCOO})_2}$ **Product**

Consider the above reaction

A. The reaction proceeds through a more stable radical intermediate.

B. The role of peroxide is to generate $H^\bullet$ (Hydrogen radical).

C. During this reaction, benzene is formed as a byproduct.

D. 1-Bromo-2-phenylethane is formed as the minor product.

E. The same reaction in absence of peroxide proceeds via carbocation intermediate.

Identify the correct statements. Choose the correct answer from the options given below : [4 marks]

(A) A & E Only

(B) A, B & D Only

(C) A, C & E Only

(D) C, D & E Only

Correct Answer: (C) A, C & E Only

Solution:

Concept:

- In the presence of peroxide, addition of HBr follows a radical chain mechanism. Regioselectivity favours the addition step that produces the more stable carbon radical.
- Benzoyl peroxide initiates radical formation; benzoyloxy radicals can lose CO_2 to give phenyl radicals. In the absence of peroxide, the usual electrophilic addition involves a carbocation.

Step 1: Evaluate the radical intermediate and major product A bromine radical adds to the terminal

carbon of styrene so that the unpaired electron lies at the benzylic carbon:

$$\mathrm{PhCH{=}CH_2} + \mathrm{Br^\bullet} \longrightarrow \mathrm{PhCH^\bullet CH_2Br}.$$

The benzylic radical is resonance stabilised and abstracts hydrogen from HBr, forming $\mathrm{PhCH_2CH_2Br}$ and regenerating $\mathrm{Br^\bullet}$. Thus A is true. This product is 1-bromo-2-phenylethane and is the major product, so D is false.

Step 2: Identify what the peroxide generates Homolysis of the peroxide bond generates benzoyloxy radicals, rather than hydrogen radicals. These initiating radicals ultimately produce bromine radicals from HBr; the hydrogen atom is transferred by abstraction and is not released as the initiating free radical. Hence B is false. Decarboxylation and hydrogen abstraction also give

$$\mathrm{PhCOO^\bullet} \longrightarrow \mathrm{Ph^\bullet} + \mathrm{CO_2}, \qquad \mathrm{Ph^\bullet} + \mathrm{HBr} \longrightarrow \mathrm{PhH} + \mathrm{Br^\bullet}.$$

Consequently benzene can be formed as an initiation byproduct and C is true.

Step 3: Check the reaction without peroxide Without peroxide, protonation of the alkene gives the resonance-stabilised benzylic carbocation, followed by attack of bromide. Thus E is true. The correct statement set is A, C and E: A and E alone omit the true byproduct statement, while both remaining sets include the false statement D; the set A, B and D also includes false B.

Step 4: Conclusion Only statements A, C and E are correct. Hence, the correct option is (C).

Quick Tip to Solve: For peroxide addition, distinguish the initiating radicals from the chain-carrying bromine radical, and place the carbon radical at the resonance-stabilised benzylic position.

63. The wave numbers of three spectral lines of H atom are considered. Identify the set of spectral lines belonging to Balmer series.

(R = Rydberg constant) [4 marks]

(A) $\frac{5R}{36}, \frac{8R}{9}, \frac{15R}{16}$

(B) $\frac{3R}{4}, \frac{3R}{16}, \frac{7R}{144}$

(C) $\frac{5R}{36}, \frac{3R}{16}, \frac{21R}{100}$

(D) $\frac{7R}{144}, \frac{3R}{16}, \frac{16R}{255}$

Correct Answer: (C) $\frac{5R}{36}, \frac{3R}{16}, \frac{21R}{100}$

Solution:

Concept:

- The Rydberg expression for hydrogen emission is $\widetilde{\nu} = R(1/n_f^2 - 1/n_i^2)$, where $n_i > n_f$ are positive integers.
- Every Balmer line terminates at $n_f = 2$; thus $\widetilde{\nu} = R(1/4 - 1/n_i^2)$ with $n_i = 3, 4, 5, \ldots$. The final level, rather than one coincidentally matching fraction, identifies the series.

Step 1: Calculate the first three Balmer wavenumbers Use the common final level 2 and the successive

initial levels 3, 4 and 5:

$$\widetilde{\nu}_{3\to2} = R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36},$$
$$\widetilde{\nu}_{4\to2} = R\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R}{16},$$
$$\widetilde{\nu}_{5\to2} = R\left(\frac{1}{4} - \frac{1}{25}\right) = \frac{21R}{100}.$$

These are exactly the three entries in the third printed option.

Step 2: Exclude each competing set The first set contains $8R/9$ and $15R/16$, which are transitions $3 \to 1$ and $4 \to 1$ of the Lyman series. The second set includes $3R/4$, the Lyman $2 \to 1$ line, and $7R/144 = R(1/9 - 1/16)$, the Paschen $4 \to 3$ line. The fourth set also contains that Paschen line; its printed $16R/255$ does not rescue the set. A set must have all three lines in Balmer, so one correct entry such as $3R/16$ is insufficient. As a further check, Balmer wavenumbers lie between the first line $5R/36$ and the limiting value $R/4$.

Step 3: Conclusion The Balmer set is $\boxed{5R/36,\ 3R/16,\ 21R/100}$, which occupies the third position in the printed paper. The explicit transitions above establish the third printed option despite the differently ordered key. Hence, the correct option is (C).

Quick Tip to Solve: Fix the final level at 2 before calculating Balmer lines; reject any proposed wavenumber at or above the series limit $R/4$.

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64. An organic compound undergoes first order decomposition. The time taken for decomposition to $\left(\frac{1}{8}\right)^{\text{th}}$ and $\left(\frac{1}{10}\right)^{\text{th}}$ of its initial concentration are $t_{1/8}$ and $t_{1/10}$ respectively.

What is the value of $\frac{t_{1/8}}{t_{1/10}} \times 10$?

$$(\log 2 = 0.3)$$

[4 marks]

(A) 3

(B) 9

(C) 30

(D) 0.9

Correct Answer: (B) 9

Solution:

Concept:

- For a first-order reaction, $t = (2.303/k)\log([A]_0/[A]_t)$, where k is the rate constant and the logarithm is base ten.
- If a fraction f of the initial concentration remains, the logarithmic factor is $\log(1/f)$. In a ratio of times for the same reaction at the same conditions, the factor $2.303/k$ cancels.

Step 1: Translate each remaining concentration The wording states that the concentration decreases to

one eighth and one tenth of its initial value. Therefore the ratios of initial to remaining concentration are 8 and 10:

$$t_{1/8} = \frac{2.303}{k}\log 8, \qquad t_{1/10} = \frac{2.303}{k}\log 10.$$

These are not the times for merely one eighth or one tenth of the reactant to have decomposed. That alternative reading would use remaining fractions $7/8$ and $9/10$, which is not what is printed.

Step 2: Evaluate the requested scaled ratio Divide the two expressions and then include the factor 10 explicitly:

$$10\frac{t_{1/8}}{t_{1/10}} = 10\frac{\log 8}{\log 10} = 10\frac{3\log 2}{1} = 10(3)(0.3) = 9.$$

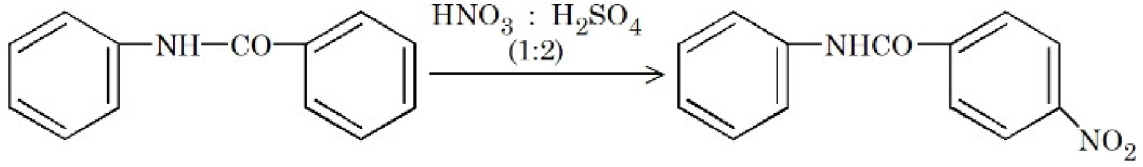
The ratio is dimensionless because both times have the same unit. No numerical value of the rate constant or initial concentration is required.

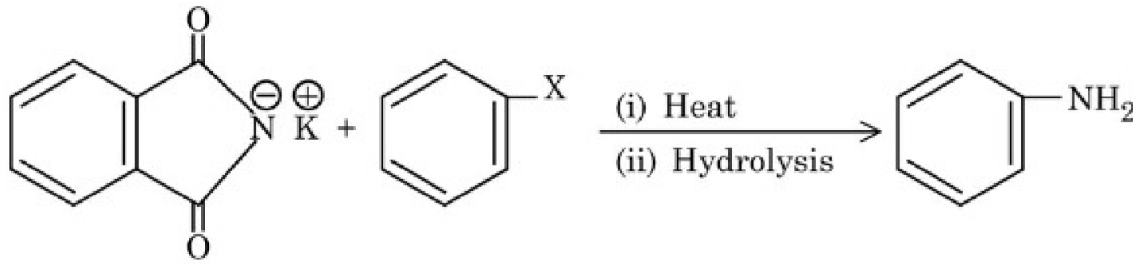
Step 3: Check the alternatives and physical ordering Reaching one eighth remaining requires less time than reaching one tenth remaining, so the unscaled ratio is below one. It equals 0.9 using the supplied logarithm; 0.9 is therefore the unscaled ratio, not the requested answer. The value 3 omits the logarithm and scaling correctly, while 30 ignores the value $\log 2 = 0.3$. Only 9 includes all factors and agrees with the expected ordering.

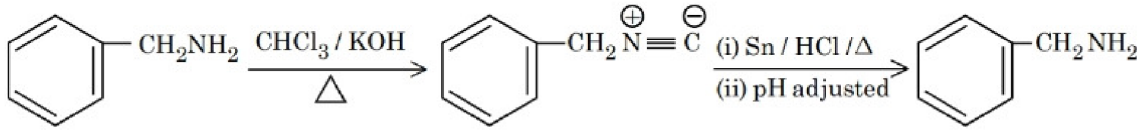
Step 4: Conclusion The requested value is $\boxed{9}$, a dimensionless number. Hence, the correct option is (B).

Quick Tip to Solve: For first-order fraction questions, write the fraction remaining first, cancel the common rate-constant factor, and apply any final multiplier only after forming the ratio.

65. Consider the following reactions giving major product. Identify the correct reaction. [4 marks]

(A) 

(B) 

(C) 

(D) $CH_3CH_2CONH_2 + Br_2 + 4\,KOH \xrightarrow{\Delta} CH_3CH_2NH_2 + 2\,KBr + K_2CO_3 + 2\,H_2O$ (alcoholic KOH)

Correct Answer: (D) $CH_3CH_2CONH_2 + Br_2 + 4\,KOH \xrightarrow{\Delta} CH_3CH_2NH_2 + 2\,KBr + K_2CO_3 + 2\,H_2O$ (alcoholic KOH)

Solution:

Concept:

- Hofmann bromamide degradation converts a primary amide $\mathrm{RCONH_2}$ into $\mathrm{RNH_2}$, losing the carbonyl carbon from the organic product.
- Assess named reactions by their substrate restrictions and product connectivity: Gabriel synthesis requires suitable alkyl substitution, while isocyanide reduction retains its carbon as an N-methyl group.

Step 1: Check nitration and Gabriel synthesis In A, the ring attached directly to nitrogen bears an acylamino group, $-\mathrm{NHCOPh}$, which directs electrophilic substitution mainly to its para position. The ring attached to the carbonyl is deactivated. Nitration on the latter ring as drawn is not the major reaction, so A is incorrect. In B, potassium phthalimide is shown reacting with a haloarene to produce aniline after hydrolysis. The Gabriel substitution does not proceed by ordinary $\mathrm{S_N2}$ displacement at the aromatic sp^2 carbon of an unactivated aryl halide; consequently this is not a valid preparation of aniline.

Step 2: Track the carbon in the isocyanide sequence In C, benzylamine undergoes the carbylamine reaction with $\mathrm{CHCl_3/KOH}$ to form benzyl isocyanide. Reduction of an isocyanide converts its terminal carbon into a methyl group:

$$\mathrm{RNC} \xrightarrow{\text{reduction}} \mathrm{RNHCH_3}.$$

Here $\mathrm{R{=}PhCH_2}$, so the reduced product is N-methylbenzylamine, not the original benzylamine drawn. Thus C is incorrect.

Step 3: Verify the Hofmann reaction Propanamide has $\mathrm{R{=}CH_3CH_2}$. Its reaction with bromine and alkali gives ethylamine, with one fewer carbon:

$$\mathrm{CH_3CH_2CONH_2 + Br_2 + 4\,KOH \longrightarrow CH_3CH_2NH_2 + 2\,KBr + K_2CO_3 + 2\,H_2O}.$$

The carbonyl carbon appears in carbonate, while the ethyl group remains attached to nitrogen. Carbon, hydrogen, nitrogen, bromine, potassium and oxygen all balance in this equation. Therefore the product and stoichiometry shown in D are correct.

Step 4: Conclusion The propanamide-to-ethylamine Hofmann bromamide reaction is correct. Hence, the correct option is (D).

Quick Tip to Solve: Track the carbonyl carbon in Hofmann degradation and the terminal carbon in isocyanide reduction: the former leaves the amine, whereas the latter becomes an N-methyl group.

66. Consider the following reaction sequence:

$$\textbf{Compound } (x) \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } CO_2, NaOH,\ 120^\circ C,\ \textbf{high pressure}} \textbf{Compound } (y) \textbf{ (Major Product)}.$$

Compound (x): 76.6% C, 6.38% H, vapour density 47.

Compound (y) develops characteristic colour with neutral $FeCl_3$ solution.

Identify the INCORRECT statement from the following for the above sequence. [4 marks]

(A) Both compounds x and y will dissolve in NaOH .

(B) Compound y will dissolve in $NaHCO_3$ and evolve a gas.

(C) Both compounds x and y will burn with sooty flame.

(D) Compound x is more acidic than compound y .

Correct Answer: (D) Compound x is more acidic than compound y .

Solution:

Concept:

- Vapour density relative to hydrogen gives molar mass $M = 2 \times \text{VD}$. Elemental percentages then determine the molecular formula.
- Sodium phenoxide reacts with CO_2 under pressure and on acidification gives mainly salicylic acid. A phenolic group gives a characteristic colour with neutral $FeCl_3$, while a carboxylic acid reacts with bicarbonate.

Step 1: Identify compound x from the composition The molar mass is $2(47) = 94\ \text{g mol}^{-1}$. In 94 g, the masses of carbon and hydrogen are approximately $0.766(94) = 72$ g and $0.0638(94) = 6$ g. The remaining 16 g is oxygen. Thus one mole contains 6 mol C atoms, 6 mol H atoms and 1 mol O atoms, giving C_6H_6O. The stated carboxylation conditions identify this compound as phenol, C_6H_5OH, rather than merely identifying a molecular formula.

Step 2: Identify y and test the positive statements Phenol forms phenoxide in $NaOH$, then undergoes Kolbe carboxylation and acidification to give mainly 2-hydroxybenzoic acid, or salicylic acid. Its retained phenolic group explains the neutral $FeCl_3$ colour. Both phenol and salicylic acid form salts with $NaOH$, so A is correct. Salicylic acid has a carboxyl group and reacts as

$$HOC_6H_4COOH + NaHCO_3 \longrightarrow HOC_6H_4COONa + CO_2 + H_2O.$$

Thus B is correct. Both compounds contain an aromatic ring and burn with a sooty flame under the ordinary flame test, making C correct.

Step 3: Compare the acidities The carboxyl group of salicylic acid is substantially more acidic than the phenolic group of phenol. Its conjugate carboxylate distributes charge over two oxygen atoms; phenol is too weak an acid to liberate carbon dioxide from bicarbonate. Hence compound y is more acidic than x. The printed assertion that x is more acidic than y reverses this ordering and is the required incorrect statement.

Step 4: Conclusion Compound x is phenol and y is salicylic acid; y is more acidic than x. Hence, the correct option is (D).

Quick Tip to Solve: Use bicarbonate as the quick distinction: a carboxylic acid releases CO_2, whereas an ordinary phenol does not, even though both dissolve in $NaOH$.

67. Figure 1. electron probability density for 2 s orbital

Figure 2. wave function for 2s orbital

Which of the following point in Figure 2 most accurately represents the nodal surface as shown in Figure 1? [4 marks]

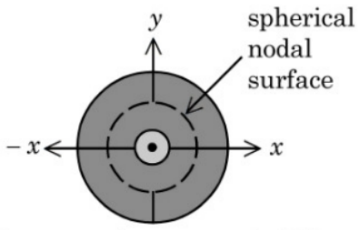


Figure 1. electron probability density for 2s orbital

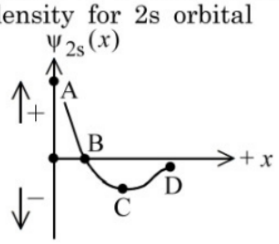


Figure 2. wave function for 2s orbital

(A) A

(B) B

(C) C

(D) D

Correct Answer: (B) B

Solution:

Concept:

- The probability density at a point is proportional to $|\psi|^2$, where ψ is the orbital wave function. A node therefore requires $\psi = 0$, not merely a negative value of ψ.
- For an orbital with principal quantum number n and angular momentum quantum number l, the number of radial nodes is $n - l - 1$. An s orbital has spherical symmetry.

Step 1: Identify the type of node For the 2s orbital, $n = 2$ and $l = 0$, so the number of radial nodes is $2 - 0 - 1 = 1$. All points at the corresponding nonzero radial distance have zero probability density. In a cross-section this spherical nodal surface appears as the dashed circle in Figure 1; it is not the outer boundary of the entire orbital.

Step 2: Locate the zero of the wave function Figure 2 plots the wave function itself, not its square. The curve crosses the horizontal axis at point B. Thus $\psi_{2s} = 0$ and consequently $|\psi_{2s}|^2 = 0$ there. The wave function changes sign as this radial node is crossed, while probability density remains nonnegative on either side.

Step 3: Exclude the other labelled points At A the wave function is positive and nonzero. At C it has a negative minimum, so its square is positive rather than zero. At D the curve is still below the axis and its wave function is nonzero. Neither a negative sign nor a turning point signifies a node; only

the zero at B meets the definition.

Step 4: Conclusion Point B represents the spherical nodal surface of the 2s orbital. Hence, the correct option is (B).

Quick Tip to Solve: On a wave-function graph, find the nonzero-radius axis crossing. On a probability-density graph, find where the density falls to zero; a negative wave function still gives positive density.

68.

Given below are the four isomeric compounds $(\mathrm{P}, \mathrm{Q}, \mathrm{R}, \mathrm{S})$

Identify correct statements from below.

A. Q, R **and S will give precipitate with 2, 4-DNP.**

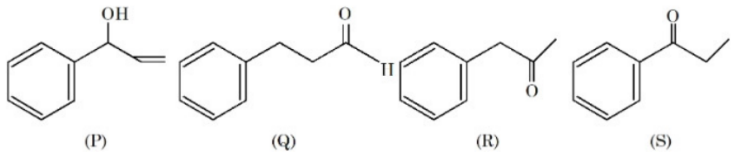
B. P **and** Q **will give positive Bayer's test.**

C. Q and R will give sooty flame.

D. R and S will give yellow precipitate with $\mathrm{I_2/NaOH}$**.**

E. Q alone will deposit silver with Tollen's reagent

Choose the correct option. [4 marks]



(A) C and E only

(B) A, C and E only

(C) A, B, D and E only

(D) A and E only

Correct Answer: No option includes all chemically correct statements A, B, C and E.

Solution:

Concept:

- 2,4-Dinitrophenylhydrazine gives a precipitate with aldehydes and ketones. Tollens reagent is reduced by aldehydes, whereas ordinary ketones do not give the silver-mirror test.
- Baeyer's reagent is cold dilute alkaline $\mathrm{KMnO_4}$. It oxidises a nonaromatic carbon–carbon double bond, but aldehydes also reduce and decolorise it; decolorisation is not exclusive to alkenes.
- The iodoform test requires a methyl ketone $\mathrm{CH_3COR}$, or a suitable compound oxidisable to one. Aromatic compounds commonly burn with a sooty flame.

Step 1: Identify the functional groups The structures are P: $\mathrm{C_6H_5CH(OH)CH{=}CH_2}$, Q: $\mathrm{C_6H_5CH_2CH_2CHO}$, R: $\mathrm{C_6H_5CH_2COCH_3}$, and S: $\mathrm{C_6H_5COCH_2CH_3}$. Thus P is an unsaturated alcohol, Q an aldehyde, and R and S ketones. Only R contains the methyl-ketone unit.

Step 2: Check statements A, C, D and E A is true because Q, R and S each contain an aldehyde

or ketone carbonyl group and give 2,4-DNP derivatives. C is true because both Q and R contain a benzene ring and give a sooty flame. D is false: R gives yellow CHI_3, but S has an ethyl group beside its carbonyl, not a methyl group directly attached to it. E is true because Q is the only aldehyde; neither ordinary ketone nor P gives the stated silver deposition.

Step 3: Test statement B against the actual reagent P decolorises alkaline permanganate by oxidation of its alkene to a vicinal diol. Q also decolorises it because its aldehyde group is oxidised to carboxylate. Writing $RCHO$ for Q, the redox equation is

$$3\,RCHO + 2\,MnO_4^- + OH^- \longrightarrow 3\,RCOO^- + 2\,MnO_2 + 2\,H_2O.$$

Consequently the literal positive-reagent observation makes B true as well. Calling Baeyer's test an unsaturation test does not prevent an aldehyde from reducing the reagent.

Step 4: Compare the complete truth set with the options The chemically correct set is A, B, C and E. Option A omits A and B; option B omits B; option C includes the false D and omits C; option D omits B and C. No printed option includes the full correct set. The keyed combination A, C and E only reflects an interpretation that excludes aldehyde oxidation, rather than the actual behaviour of alkaline permanganate.

Step 5: Conclusion As printed, A, B, C and E are true and D is false, so no listed option is fully correct. The question-sheet key selects option (B), A, C and E only, but Q also reduces alkaline $KMnO_4$ to MnO_2 as the balanced equation shows. Therefore the literal question has no correct listed combination.

Quick Tip to Solve: For organic spot tests, inspect actual reactive groups: aldehydes can also decolorise permanganate, while a ketone gives iodoform only if a methyl group is directly attached to its carbonyl.

69. The correct statement among the following is: [4 marks]

(A) $Ni(CO)_4$ and $[Ni(CN)_4]^{2-}$ are diamagnetic and $[NiCl_4]^{2-}$ is paramagnetic.
(B) $Ni(CO)_4$ and $[NiCl_4]^{2-}$ are diamagnetic and $[Ni(CN)_4]^{2-}$ is paramagnetic.
(C) $[Ni(CN)_4]^{2-}$ and $[NiCl_4]^{2-}$ are diamagnetic and $Ni(CO)_4$ is paramagnetic.
(D) $Ni(CO)_4$ is diamagnetic and $[NiCl_4]^{2-}$ and $[Ni(CN)_4]^{2-}$ are paramagnetic.

Correct Answer: (A) $Ni(CO)_4$ and $[Ni(CN)_4]^{2-}$ are diamagnetic and $[NiCl_4]^{2-}$ is paramagnetic.

Solution:

Concept:

- A complex is diamagnetic when all electrons are paired and paramagnetic when one or more electrons are unpaired. Determine the metal oxidation state before counting its d electrons.
- Nickel has atomic number 28. Ni^{2+} is $3d^8$; the neutral-metal carbonyl $Ni(CO)_4$ has the paired-electron arrangement associated with a d^{10} metal centre.

Step 1: Analyse nickel tetracarbonyl Carbon monoxide is a neutral ligand, so nickel is in oxidation state zero in $Ni(CO)_4$. In the usual valence-bond description its electrons pair in the five 3d orbitals, and four empty sp^3 hybrid orbitals accept ligand electron pairs. The complex is tetrahedral and has

no unpaired electrons, so it is diamagnetic.

Step 2: Compare the two nickel(II) complexes Both anionic complexes contain Ni^{2+}, since four singly charged ligands and overall charge -2 require metal charge $+2$. Cyanide is a strong-field ligand: $[Ni(CN)_4]^{2-}$ is square planar, with a paired d^8 configuration and dsp^2 hybridisation. Chloride is weak field: tetrahedral $[NiCl_4]^{2-}$ retains two unpaired electrons and is paramagnetic.

Step 3: Check the printed alternatives The required pattern is diamagnetic carbonyl, diamagnetic cyanide complex and paramagnetic chloride complex. Option A states exactly this. Option B incorrectly calls the chloride complex diamagnetic and the cyanide complex paramagnetic; option C incorrectly makes the carbonyl paramagnetic; option D incorrectly makes the cyanide complex paramagnetic.

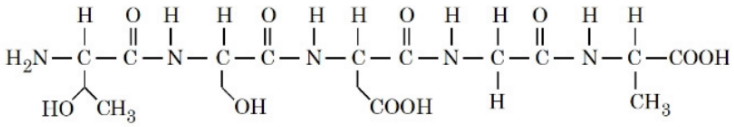
Step 4: Conclusion $Ni(CO)_4$ and $[Ni(CN)_4]^{2-}$ are diamagnetic, whereas $[NiCl_4]^{2-}$ is paramagnetic. Hence, the correct option is (A). The printed order places this statement first, although the question-sheet key associates the same statement with a different option position.

Quick Tip to Solve: For four-coordinate nickel complexes, distinguish strong-field square-planar d^8 cyanide from weak-field tetrahedral d^8 chloride; the latter has two unpaired electrons.

70. In the given pentapeptide, find out an essential amino acid (Y) and the sequence present in the pentapeptide :

Choose the correct answer from the options given below :

[4 marks]



(A)

(Y)	(Sequence)
Threonine	Thr-Ser-Asp-Gly-Ala

(B)

(Y)	(Sequence)
Serine	Thr-Ser-Asp-Ala-Gly

(C)

(Y)	(Sequence)
Serine	Ser-Asp-Thr-Ala-Gly

(D)

(Y)	(Sequence)
Threonine	Ser-Thr-Asp-Gly-Ala

Correct Answer: (A)

(Y)	(Sequence)
Threonine	Thr-Ser-Asp-Gly-Ala

Solution:

Concept:

- Peptide sequences are read from the free amino terminus (N-terminus) to the free carboxyl terminus (C-terminus). Each alpha carbon and its side chain identify one amino-acid residue.
- Threonine is an essential amino acid and has side chain $-CH(OH)CH_3$. Serine has $-CH_2OH$ and is not an essential amino acid in the usual dietary classification.

Step 1: Fix the direction of reading The displayed chain has a free H_2N group at the left and a free $-COOH$ group at the right. Therefore read the five residues from left to right. The intervening $-CO-NH-$ groups are peptide bonds; their carbonyl groups must not be confused with the side chains attached to the alpha carbons.

Step 2: Identify each successive side chain The first alpha carbon carries $-CH(OH)CH_3$, identifying threonine (Thr). The second carries $-CH_2OH$, identifying serine (Ser). The third carries $-CH_2COOH$, identifying aspartic acid (Asp). The fourth carries H, identifying glycine (Gly), and the fifth carries $-CH_3$, identifying alanine (Ala). Thus the sequence is Thr–Ser–Asp–Gly–Ala.

Step 3: Choose the essential residue and compare options Among these five residues, threonine is essential. Option A gives both threonine and the correct sequence. Options B and C incorrectly identify serine as essential and also change the sequence. Option D identifies the essential residue correctly but reverses the first two residues, whereas the first printed side chain unambiguously belongs to threonine.

Step 4: Conclusion Y is threonine and the sequence is Thr–Ser–Asp–Gly–Ala. Hence, the correct option is (A). The correct combination is first in the printed paper; the question-sheet key uses its former option position.

Quick Tip to Solve: First mark the N-terminus, then read side chains only: $CH(OH)CH_3$, CH_2OH, CH_2COOH, H, CH_3 give Thr, Ser, Asp, Gly, Ala.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. 500 mL of 1.2 M KI solution is mixed with 500 mL of $0.2MKMnO_4$ solution in basic medium. The liberated iodine was titrated with standard $0.1MNa_2S_2O_3$ solution in the presence of starch indicator till the blue color disappeared. The volume (in L) of $Na_2S_2O_3$ consumed is ____ . (Nearest integer) [4 marks]

Solution:

Concept:

- In the stated basic-medium iodine-liberation reaction, permanganate is reduced from Mn(VII) to MnO_2, gaining three electrons per MnO_4^-. Iodide is oxidised to iodine.
- Iodometric titration follows $I_2 + 2\,S_2O_3^{2-} \longrightarrow 2\,I^- + S_4O_6^{2-}$. Thus one mole of liberated iodine consumes two moles of thiosulfate.

Step 1: Calculate the available reactant amounts Convert the volumes to litres before using $n = MV$:

$$n(\mathrm{I^-}) = 1.2 \times 0.500 = 0.600 \text{ mol}, \qquad n(\mathrm{MnO_4^-}) = 0.2 \times 0.500 = 0.100 \text{ mol}.$$

KI and permanganate each furnish one mole of the relevant ion per mole of solute. Mixing changes concentrations but does not change these amounts.

Step 2: Balance the iodine-liberation reaction The balanced reaction consistent with the liberated iodine is

$$2\,\mathrm{MnO_4^-} + 6\,\mathrm{I^-} + 4\,\mathrm{H_2O} \longrightarrow 2\,\mathrm{MnO_2} + 3\,\mathrm{I_2} + 8\,\mathrm{OH^-}.$$

Therefore 0.100 mol permanganate consumes 0.300 mol iodide, less than the 0.600 mol available, and permanganate is the limiting reagent. The iodine formed is

$$n(\mathrm{I_2}) = \frac{3}{2}(0.100) = 0.150 \text{ mol}.$$

Step 3: Find the required thiosulfate volume At the starch endpoint all iodine has been reduced to iodide, so the blue iodine–starch colour disappears. The required thiosulfate amount is $2 \times 0.150 = 0.300$ mol. Hence

$$V = \frac{n}{M} = \frac{0.300 \text{ mol}}{0.100 \text{ mol L}^{-1}} = 3.00 \text{ L}.$$

This is the volume delivered from the titrant, not the combined initial volume of the KI and permanganate solutions.

Step 4: Conclusion The volume of $\mathrm{Na_2S_2O_3}$ consumed is 3 L. The required nearest-integer entry is $\boxed{3}$.

Quick Tip to Solve: Combine the two stoichiometric ratios: one mole of permanganate forming $\mathrm{MnO_2}$ liberates enough iodine to consume three moles of thiosulfate. Check iodide is in excess first.

72. Consider the dissociation equilibrium of the following weak acid

$$\mathrm{HA} \rightleftharpoons \mathrm{H^+(aq)} + \mathrm{A^-(aq)}$$

If the pKa of the acid is 4 , then the pH of 10 mM HA solution is ____ .(Nearest integer)
[Given: The degree of dissociation can be neglected with respect to unity] [4 marks]

Solution:

Concept:

- For a monoprotic weak acid of initial concentration C and degree of dissociation α, $K_a = C\alpha^2/(1-\alpha)$. The stated approximation gives $K_a \simeq C\alpha^2$.
- $\mathrm{p}K_a = -\log_{10} K_a$ and $\mathrm{pH} = -\log_{10}[\mathrm{H^+}]$, with concentration expressed in $\mathrm{mol\,L^{-1}}$. The weak-acid approximation is $[\mathrm{H^+}] \simeq \sqrt{K_a C}$.

Step 1: Convert the given quantities From $\mathrm{p}K_a = 4$,

$$K_a = 10^{-4}.$$

The concentration is $10\ \mathrm{mM} = 10 \times 10^{-3}\ \mathrm{mol\,L^{-1}} = 10^{-2}\ \mathrm{mol\,L^{-1}}$. Using the number 10 directly would mix millimolar and molar units and give an incorrect hydrogen-ion concentration.

Step 2: Apply the specified approximation Let the acid dissociate by an amount $x\ \mathrm{mol\,L^{-1}}$. Then $[\mathrm{H^+}] = [\mathrm{A^-}] = x$ and $[\mathrm{HA}] = C - x$. Neglecting the degree of dissociation relative to unity, as instructed, gives

$$K_a = \frac{x^2}{C - x} \simeq \frac{x^2}{C}, \qquad x = \sqrt{10^{-4} \times 10^{-2}} = 10^{-3}\ \mathrm{mol\,L^{-1}}.$$

The acid contribution is much larger than that from water, so water autoionisation does not affect the requested integer.

Step 3: Calculate and round the pH Taking the negative common logarithm,

$$\mathrm{pH} = -\log_{10}(10^{-3}) = 3.$$

As a check, the exact quadratic would give $x \approx 9.51 \times 10^{-4}\ \mathrm{mol\,L^{-1}}$ and pH approximately 3.02; this also rounds to 3. The question explicitly authorises the approximation, so the direct square-root calculation is sufficient.

Step 4: Conclusion The pH is approximately 3, a dimensionless quantity. The required nearest-integer entry is $\boxed{3}$.

Quick Tip to Solve: For a weak monoprotic acid, use $\mathrm{pH} \simeq \frac{1}{2}(\mathrm{p}K_a - \log_{10} C)$ with C in molarity; here $\frac{1}{2}[4 - (-2)] = 3$.

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73. X is the number of geometrical isomers exhibited by $[\mathrm{Pt\,(NH_3)\,(H_2O)\,BrCl}]$.
Y is the number of optically inactive isomer(s) exhibited by $[\mathrm{CrCl_2(ox)_2}]^{3-}$
Z is the number of geometrical isomers exhibited by $[\mathrm{Co\,(NH_3)_3\,(NO_2)_3}]$.
The value of $\mathrm{X + Y + Z}$ is ____ . [4 marks]

Solution:

Concept:

- Four-coordinate $\mathrm{Pt(II)}$ complexes are square planar. In a square-planar complex with four different monodentate ligands, geometrical isomers are distinguished by which pairs occupy opposite, or trans, positions.
- In octahedral complexes, geometrical and optical isomerism must be counted separately. A pair of non-superimposable mirror images consists of two optically active isomers; neither member is an optically inactive isomer.

Step 1: Count the square-planar arrangements In $[\mathrm{Pt(NH_3)(H_2O)BrCl}]$, all four ligands are different.

Fix NH_3 at one position. Its trans partner can be H_2O, Br^-, or Cl^-. Once this partner is chosen, the remaining two ligands occupy the other opposite pair; interchanging them does not create a new geometrical arrangement. Thus the three distinct trans-pair choices give

$$X = 3.$$

Step 2: Identify the optically inactive chromium isomer Each oxalate ion, ox, is bidentate, so the two oxalate ligands and two chloride ligands give an octahedral coordination sphere in $[CrCl_2(ox)_2]^{3-}$. The two chloride ligands can be cis or trans. The cis arrangement forms a pair of optically active mirror-image isomers. The trans arrangement is achiral and is the only optically inactive isomer. The question asks for inactive isomers, not the total number of stereoisomers. Therefore

$$Y = 1.$$

Step 3: Count the cobalt geometrical isomers and add The cobalt complex has the octahedral MA_3B_3 form. In its facial isomer, the three identical ligands occupy the vertices of one triangular face and are mutually cis. In its meridional isomer, two of the three identical ligands are trans and all three lie in a plane through the metal. These are the two geometrical arrangements. Linkage alternatives of nitrite are not additional geometrical isomers for this count. Hence

$$Z = 2, \qquad X + Y + Z = 3 + 1 + 2 = 6.$$

Step 4: Conclusion The required dimensionless integer is $\boxed{6}$, with $X = 3$, $Y = 1$, and $Z = 2$.

Quick Tip to Solve: For square-planar four-different-ligand complexes, choose the trans partner of one fixed ligand. For octahedral chelates, distinguish counting geometrical forms from counting optically inactive forms.

74. 0.53 g of an organic compound (x) when heated with excess of nitric acid (concentrated) and then with silver nitrate gave 0.75 g of silver bromide precipitate. 1.0 g of (x) gave 1.32 g of CO_2 gas on combustion. The percentage of hydrogen in the compound (x) is ____ %. [Nearest Integer]

[Given: Molar mass in $gmol^{-1}$ $H : 1, C : 12, Br : 80, Ag : 108, O : 16$**; Compound (x) :** $C_xH_yBr_z$**]** [4 marks]

Solution:

Concept:

- In halogen estimation, each mole of $AgBr$ precipitate contains one mole of bromine atoms. Its bromine mass fraction is $M(Br)/M(AgBr)$.
- Complete combustion converts every carbon atom in the sample into CO_2. Since the compound contains only carbon, hydrogen and bromine, their mass percentages add to 100%; hydrogen is obtained by difference.

Step 1: Calculate the bromine percentage Using the supplied atomic masses, the molar mass of silver bromide is

$$M(AgBr) = 108 + 80 = 188\,g\,mol^{-1}.$$

Therefore the bromine present in the precipitate, and hence in the original $0.53\,\mathrm{g}$ sample, has mass

$$m(\mathrm{Br}) = 0.75 \times \frac{80}{188} = 0.3191489\,\mathrm{g}.$$

Divide by the mass of the sample used for this particular experiment:

$$\%\mathrm{Br} = \frac{0.3191489}{0.53} \times 100 = 60.2168\%.$$

Step 2: Calculate the carbon percentage A different $1.0\,\mathrm{g}$ sample yields $1.32\,\mathrm{g}$ of carbon dioxide. Since $M(\mathrm{CO_2}) = 12 + 2(16) = 44\,\mathrm{g\,mol^{-1}}$, its carbon mass is

$$m(\mathrm{C}) = 1.32 \times \frac{12}{44} = 0.36\,\mathrm{g}.$$

Thus the carbon percentage in the compound is

$$\%\mathrm{C} = \frac{0.36}{1.0} \times 100 = 36.0\%.$$

Use the appropriate sample mass in each denominator.

Step 3: Obtain hydrogen by mass balance The stated formula $\mathrm{C}_x\mathrm{H}_y\mathrm{Br}_z$ contains no other elements; the oxygen in carbon dioxide comes from combustion. Hence,

$$\%\mathrm{H} = 100 - 36.0 - 60.2168 = 3.7832\%.$$

Round only after subtracting the unrounded percentages.

Step 4: Conclusion The hydrogen content is approximately 3.7832%, so the required nearest-integer percentage is $\boxed{4\%}$.

Quick Tip to Solve: Normalize each analysis to its own sample mass before combining percentages. Find hydrogen by difference only because the formula specifies carbon, hydrogen and bromine as the sole elements.

75. Consider the following redox reaction taking place in acidic medium

$$BH_4^-(aq) + ClO_3^-(aq) \longrightarrow H_2BO_3^-(aq) + Cl^-(aq)$$

If the Nernst equation for the above balanced reaction is

$$E_{cell} = E^\circ_{cell} - \frac{RT}{nF} \ln Q,$$

then the value of n is ____ .(Nearest integer) [4 marks]

Solution:

Concept:

- The quantity n in the Nernst equation is the number of electrons transferred in the balanced overall cell reaction, written with the smallest whole-number coefficients.
- Balance oxygen using H_2O, hydrogen using H^+, and charge using electrons in acidic medium. Multiply the two half-reactions so that their electron counts are equal before adding them.

Step 1: Balance the oxidation half-reaction Hydrogen has oxidation number -1 in borohydride and $+1$ in the oxygen-containing products. Boron remains at $+3$, so looking only at boron's oxidation number would miss the oxidation. The balanced half-reaction is

$$BH_4^- + 3\,H_2O \longrightarrow H_2BO_3^- + 8\,H^+ + 8\,e^-.$$

Both sides have ten H atoms, three O atoms and charge -1; eight electrons are released.

Step 2: Balance the reduction half-reaction Chlorine changes from $+5$ in chlorate to -1 in chloride, gaining six electrons per chlorine atom. The acidic-medium reduction half-reaction is

$$ClO_3^- + 6\,H^+ + 6\,e^- \longrightarrow Cl^- + 3\,H_2O.$$

Atoms and charge are balanced: the left-hand charge is $-1 + 6 - 6 = -1$.

Step 3: Equalize electrons and form the overall reaction The least common multiple of eight and six is twenty-four. Multiply the oxidation half-reaction by three and the reduction half-reaction by four. This produces and consumes

$$3 \times 8 = 4 \times 6 = 24 \text{ electrons.}$$

On addition, the 24 electrons and 24 hydrogen ions cancel. Nine water molecules on the left cancel against nine of the twelve on the right, leaving

$$3\,BH_4^- + 4\,ClO_3^- \longrightarrow 3\,H_2BO_3^- + 4\,Cl^- + 3\,H_2O.$$

These are the smallest whole-number coefficients. The acidic-medium hydrogen ions cancel, but the transferred electron count remains $n = 24$.

Step 4: Conclusion In the stated Nernst equation, the required dimensionless nearest integer is $\boxed{n = 24}$.

Quick Tip to Solve: In borohydride oxidation, track hydrogen as well as boron. Obtain the Nernst electron count from the least common multiple of the balanced half-reaction electron numbers, here eight and six.