JEE Main 2026 Paper 1 (B.E./B.Tech) Jan 28 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2026, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 28 January 2026, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $P_1 : y = 4x^2$ and $P_2 : y = x^2 + 27$ be two parabolas. If the area of the bounded region enclosed between P_1 and P_2 is six times the area of the bounded region enclosed between the line $y = \alpha x$, $\alpha > 0$ and P_1, then α is equal to : [4 marks]

(A) 6

(B) 8

(C) 12

(D) 15

Correct Answer: (C) 12

Solution:

Concept:

- The area between two curves is the integral of the upper ordinate minus the lower ordinate over their intersection interval.
- For the line and parabola, find both intersections before integrating; the condition $\alpha > 0$ determines their order.

Step 1: Find the intersections of the parabolas Equating the two ordinates gives $4x^2 = x^2 + 27$, hence $3x^2 = 27$ and $x = \pm 3$. On this interval $x^2 + 27$ lies above $4x^2$, since their difference is $27 - 3x^2 \geq 0$. Thus the area integrand is the upper ordinate of P2 minus the lower ordinate of P1.

Step 2: Calculate the first bounded area Using symmetry about the y-axis, the required area between the parabolas is

$$A_1 = \int_{-3}^{3} (27 - 3x^2)\, dx = 2[27x - x^3]_0^3 = 2(81 - 27) = 108.$$

Both halves are included; integrating only from 0 to 3 would give half the required region.

Step 3: Calculate the area under the line above the parabola The intersections satisfy $4x^2 = \alpha x$, so $x = 0$ and $x = \alpha/4$. Between them the line lies above the parabola. Therefore

$$A_2 = \int_0^{\alpha/4} (\alpha x - 4x^2)\, dx = \left[\frac{\alpha x^2}{2} - \frac{4x^3}{3}\right]_0^{\alpha/4} = \frac{\alpha^3}{32} - \frac{\alpha^3}{48} = \frac{\alpha^3}{96}.$$

Step 4: Apply the area ratio and check the choices The statement says $A_1 = 6A_2$, so $108 = \alpha^3/16$. Hence $\alpha^3 = 1728$, giving the unique positive value $\alpha = 12$. The alternatives 6, 8 and 15 have cubes 216, 512 and 3375 respectively; none satisfies the required area equation.

Step 5: Conclusion The required slope is $\boxed{\alpha = 12}$. Hence, the correct option is (C).

Quick Tip to Solve: For a line $y = mx$ and parabola $y = kx^2$, their enclosed area is $m^3/(6k^2)$ when $m, k > 0$.

2. Let $f(x)=\int \frac{dx}{x^{2/3}+2x^{1/2}}$, be such that $f(0)=-26+24\log_e(2)$. If $f(1)=a+b\log_e(3)$, where $a,b\in\mathbb{Z}$, then $a+b$ is equal to : [4 marks]

(A) -26

(B) -18

(C) -11

(D) -5

Correct Answer: (C) -11

Solution:

Concept:

- Fractional powers $x^{2/3}$ and $x^{1/2}$ become integer powers after the substitution $t=x^{1/6}$, because 6 is the least common multiple of 3 and 2.
- An antiderivative contains an arbitrary additive constant. Its continuous limiting value at zero determines that constant here.

Step 1: Remove the fractional powers For $x>0$, put $x=t^6$, with $t>0$. Then $dx=6t^5\,dt$, $x^{2/3}=t^4$ and $x^{1/2}=t^3$. Consequently,

$$f(x)=\int \frac{6t^5}{t^4+2t^3}\,dt=6\int \frac{t^2}{t+2}\,dt.$$

The factor t^3 is cancelled only on this positive interval; the value at zero will be supplied by the continuous limit.

Step 2: Divide and integrate Polynomial division gives $t^2=(t+2)(t-2)+4$, so

$$\frac{t^2}{t+2}=t-2+\frac{4}{t+2}.$$

Integrating the three terms yields

$$f(x)=3t^2-12t+24\log_e(t+2)+C,\qquad t=x^{1/6}.$$

There is no absolute-value ambiguity for the logarithm because $t+2>0$.

Step 3: Use the specified value at zero As $x\to 0^+$, also $t\to 0^+$, and the formula tends to $24\log_e 2+C$. Equating this with $-26+24\log_e 2$ gives $C=-26$. The original integrand is undefined at zero, but the antiderivative has this finite continuous extension; that is the supplied endpoint value.

Step 4: Evaluate at one and identify both integers At $x=1$, we have $t=1$, hence

$$f(1)=3-12-26+24\log_e 3=-35+24\log_e 3.$$

Thus $a=-35$, $b=24$, and $a+b=-11$. Neither the integration constant -26 nor either of the other listed numbers $-18,-5$ equals this sum; the logarithm coefficient must be included.

Step 5: Conclusion $\boxed{a+b=-11}$. Hence, the correct option is (C).

Quick Tip to Solve: Choose the substitution from the least common multiple of the fractional denominators, and determine the integration constant before evaluating the requested endpoint.

3. Given below are two statements :

Statement I :

$25^{13} + 20^{13} + 8^{13} + 3^{13}$ **is divisible by 7.**

Statement II :

The integral part of $(7 + 4\sqrt{3})^{25}$ **is an odd number.**

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Correct Answer: (A) Both **Statement I** and **Statement II** are true

Solution:

Concept:

- To test divisibility by 7, replace every base by its remainder modulo 7. Opposite residues cancel when raised to the same odd power.
- Adding conjugate expressions $(a + b\sqrt{c})^n$ and $(a - b\sqrt{c})^n$ cancels all odd powers of the radical; the surviving sum is twice an integer when a, b, c are integers.

Step 1: Test Statement I modulo seven The residues of the four bases are $25 \equiv -3$, $20 \equiv -1$, $8 \equiv 1$ and $3 \equiv 3 \pmod 7$. Since 13 is odd,

$$25^{13} + 20^{13} + 8^{13} + 3^{13} \equiv (-3)^{13} + (-1)^{13} + 1^{13} + 3^{13} = 0 \pmod 7.$$

The first and last terms cancel, as do the middle two. Therefore the complete sum is divisible by 7, and Statement I is true.

Step 2: Construct the conjugate sum for Statement II Let $u = 7 + 4\sqrt{3}$ and $v = 7 - 4\sqrt{3}$. By the binomial theorem,

$$N = u^{25} + v^{25} = 2\sum_{j=0}^{12} \binom{25}{2j} 7^{25-2j} 4^{2j} 3^j.$$

Every term inside the summation is an integer, so N is an even integer. This proves its parity exactly, without estimating the very large power.

Step 3: Locate the power between consecutive integers Because $6 < 4\sqrt{3} < 7$, we have $0 < v < 1$, and hence $0 < v^{25} < 1$. Now $u^{25} = N - v^{25}$ lies strictly between $N - 1$ and N. Its integral part is therefore $N - 1$, which is odd because N is even. Statement II is true as well. Choices asserting that either statement is false are ruled out by these two independent calculations.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: For large powers of quadratic surds, add the conjugate power to find an integer, then use whether the small conjugate lies between 0 and 1 to determine the floor.

4. Let the ellipse $E : \frac{x^2}{144} + \frac{y^2}{169} = 1$ and the hyperbola $H : \frac{x^2}{16} - \frac{y^2}{\lambda^2} = -1$ have the same foci. If e and L respectively denote the eccentricity and the length of the latus rectum of H, then the value of $24(e + L)$ is : [4 marks]

(A) 67

(B) 126

(C) 148

(D) 296

Correct Answer: (D) 296

Solution:

Concept:

- For an ellipse with semiaxes $a > b$, the focal distance from the centre satisfies $c^2 = a^2 - b^2$. The larger denominator gives the major-axis direction.
- For a hyperbola in the form $y^2/A^2 - x^2/B^2 = 1$, its focal distance satisfies $c^2 = A^2 + B^2$, its eccentricity is c/A, and its full latus-rectum length is $2B^2/A$.

Step 1: Locate the common foci For the ellipse $x^2/144 + y^2/169 = 1$, the major axis is the y-axis, with $a = 13$ and $b = 12$. Hence

$$c^2 = 169 - 144 = 25, \qquad c = 5.$$

The foci are $(0, 5)$ and $(0, -5)$. Confocality requires the hyperbola to have exactly the same focal distance and axis.

Step 2: Put the hyperbola in standard form Multiply its equation by -1 to obtain

$$\frac{y^2}{\lambda^2} - \frac{x^2}{16} = 1.$$

Thus the squared transverse semiaxis is $A^2 = \lambda^2$, while $B^2 = 16$. Applying the hyperbola focal relation gives

$$25 = \lambda^2 + 16 \quad \Longrightarrow \quad \lambda^2 = 9.$$

The positive semiaxis length is $A = |\lambda| = 3$, regardless of the possible sign of the parameter itself.

Step 3: Evaluate eccentricity and latus rectum Using the semiaxes of the hyperbola, rather than those of the ellipse, gives

$$e = \frac{5}{3}, \qquad L = \frac{2(16)}{3} = \frac{32}{3}.$$

Here L is the full chord through a focus perpendicular to the transverse axis; using a half-length would give the wrong result. Therefore

$$24(e + L) = 24\left(\frac{5 + 32}{3}\right) = 8 \times 37 = 296.$$

The other values 67, 126 and 148 do not equal this expression; in particular 148 is only half the required result.

Step 4: Conclusion $\boxed{24(e+L)=296}$. Hence, the correct option is (D).

Quick Tip to Solve: First rewrite a hyperbola with right-hand side $+1$; this identifies its transverse semiaxis and prevents confusing it with the conjugate semiaxis.

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5. Let the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$ be $\frac{5}{16}$, $a>2$. If α is such that a, 4, α, b are in A.P., then the equation $\alpha x^2-ax+2(\alpha-2b)=0$ has : [4 marks]

(A) complex roots of magnitude less than 2

(B) both roots in the interval $(-2,0)$

(C) one root in $(1,4)$ and another in $(-2,0)$

(D) one root in $(0,2)$ and another in $(-4,-2)$

Correct Answer: (C) one root in $(1,4)$ and another in $(-2,0)$

Solution:

Concept:

- In an arithmetic progression, consecutive differences are equal. Specifying two adjacent terms determines every remaining term in terms of one variable.
- The arithmetic mean of two reciprocals is $(1/a+1/b)/2$; it is not the reciprocal of the arithmetic mean. The restriction $a>2$ selects the admissible solution.

Step 1: Express the progression in terms of a The common difference of $a,4,\alpha,b$ is $d=4-a$. Therefore

$$\alpha=4+d=8-a, \qquad b=4+2d=12-2a.$$

The reciprocal condition also requires $a\neq 0$ and $b\neq 0$, which will be checked for the selected value.

Step 2: Apply the given arithmetic mean The mean equation gives

$$\frac{1}{a}+\frac{1}{b}=\frac{5}{8}.$$

After substituting $b=12-2a$, this becomes

$$\frac{12-a}{a(12-2a)}=\frac{5}{8}, \qquad 8(12-a)=5a(12-2a).$$

Rearranging and factoring,

$$5a^2-34a+48=(a-2)(5a-24)=0.$$

The value $a=2$ is excluded by the strict inequality, leaving $a=24/5$.

Step 3: Find the coefficients of the requested quadratic It follows that $\alpha=16/5$ and $b=12/5$, both consistent with the progression and nonzero denominators. Indeed $(5/24+5/12)/2=5/16$, verifying

the reciprocal mean. Substitution into the quadratic gives

$$\frac{16}{5}x^2 - \frac{24}{5}x + 2\left(\frac{16}{5} - \frac{24}{5}\right) = 0.$$

Multiplying by $5/8$ reduces this to $2x^2 - 3x - 2 = 0$.

Step 4: Factor and locate the two roots Factorisation gives $(2x+1)(x-2) = 0$, so the roots are $-1/2$ and 2. One lies in $(-2, 0)$ and the other in $(1, 4)$. They are real, not both negative, and the positive root 2 is not in the open interval $(0, 2)$. Thus options (A), (B) and (D) are excluded.

Step 5: Conclusion The roots are $\boxed{2, -\frac{1}{2}}$, lying in $(1, 4)$ and $(-2, 0)$, respectively. Hence, the correct option is (C).

Quick Tip to Solve: Write every term of the progression using one common difference, then enforce strict inequalities before choosing the roots of the resulting parameter equation.

6. The sum of the coefficients of x^{499} and x^{500} in $(1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \ldots + x^{1000}$ is : [4 marks]

(A) $\binom{1002}{501}$
(B) $\binom{1002}{500}$
(C) $\binom{1000}{501}$
(D) $\binom{1001}{501}$

Correct Answer: (B) $\binom{1002}{500}$

Solution:

Concept:

- The finite identity $a^n + a^{n-1}b + \cdots + b^n = (a^{n+1} - b^{n+1})/(a-b)$ collapses a sum of mixed powers.
- In $(1+x)^n$, the coefficient of x^r is $\binom{n}{r}$. Pascal's identity combines adjacent coefficients: $\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$.

Step 1: Collapse the finite mixed-power sum Let $S(x) = \sum_{k=0}^{1000} x^k(1+x)^{1000-k}$. Use the finite power-sum identity with $a = 1+x$, $b = x$, and $n = 1000$. Since $a - b = 1$, the denominator disappears:

$$S(x) = (1+x)^{1001} - x^{1001}.$$

This is a polynomial identity, valid for every real x; no restriction from division by a variable is needed. It also includes the final term x^{1000} of the original expression.

Step 2: Extract the two specified coefficients The subtracted monomial x^{1001} affects only the coefficient of degree 1001. It has no effect on either requested degree. Consequently,

$$[x^{499}]S(x) = \binom{1001}{499}, \qquad [x^{500}]S(x) = \binom{1001}{500}.$$

Here the coefficient notation means the multiplier of that power, not the value obtained by substituting

a number for x.

Step 3: Combine adjacent binomial coefficients Apply Pascal's identity to the adjacent lower indices 499 and 500:

$$\binom{1001}{499} + \binom{1001}{500} = \binom{1002}{500}.$$

This raises the upper index by one and retains 500 as the new lower index. The option $\binom{1002}{501}$ is larger, with ratio $502/501$ to our result. Also $\binom{1001}{501} = \binom{1001}{500}$ is only one of the two positive summands, and $\binom{1000}{501}$ is smaller still. None of those alternatives gives the required sum.

Step 4: Conclusion The required sum is $\boxed{\binom{1002}{500}}$. Hence, the correct option is (B).

Quick Tip to Solve: When consecutive terms replace one factor of $(1+x)$ by x, collapse the entire polynomial first; then apply Pascal's identity to adjacent coefficients.

7. Let $y = y(x)$ be the solution of the differential equation $x\frac{dy}{dx} - y = x^2 \cot x$, $x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$, then

$6y\left(\frac{\pi}{6}\right) - 8y\left(\frac{\pi}{4}\right)$ is equal to : [4 marks]

(A) $-\pi$

(B) π

(C) -3π

(D) 3π

Correct Answer: (A) $-\pi$

Solution:

Concept:

- For a differentiable function, $\frac{d}{dx}(y/x) = (xy' - y)/x^2$. This quotient derivative reduces the given equation directly.
- On $(0, \pi)$, $\sin x > 0$, so $\int \cot x\, dx = \ln(\sin x) + C$; the initial condition determines the constant.

Step 1: Recognise the quotient derivative Divide the equation by x^2, which is nonzero throughout the stated interval. This gives

$$\frac{xy' - y}{x^2} = \cot x, \qquad \frac{d}{dx}\left(\frac{y}{x}\right) = \cot x.$$

Integrating on this interval yields

$$\frac{y}{x} = \ln(\sin x) + C.$$

No absolute-value sign is needed here because the sine is positive on the whole interval.

Step 2: Apply the initial condition At $x = \pi/2$, the ratio y/x equals 1 and $\sin(\pi/2) = 1$. Thus

$$1 = \ln 1 + C = C, \qquad y(x) = x\{1 + \ln(\sin x)\}.$$

The same constant applies at both evaluation points because they belong to the same interval of the solution.

Step 3: Evaluate the requested combination Use $\sin(\pi/6) = 1/2$ and $\sin(\pi/4) = 1/\sqrt{2}$. Then

$$6y(\pi/6) = \pi(1 - \ln 2),$$
$$8y(\pi/4) = 2\pi(1 - \tfrac{1}{2}\ln 2).$$

Subtracting, the logarithmic terms cancel:

$$6y(\pi/6) - 8y(\pi/4) = \pi - \pi\ln 2 - 2\pi + \pi\ln 2 = -\pi.$$

A positive sign would reverse the requested subtraction; the values $\pm 3\pi$ do not satisfy this exact simplification.

Step 4: Conclusion The requested value is $\boxed{-\pi}$. Hence, the correct option is (A).

Quick Tip to Solve: When the left side is $xy' - y$, divide by x^2 and integrate $(y/x)'$; evaluate weighted combinations before expanding logarithms.

8. An ellipse has its center at $(1, -2)$, one focus at $(3, -2)$ and one vertex at $(5, -2)$. Then the length of its latus rectum is : [4 marks]

(A) $6\sqrt{3}$
(B) $\dfrac{16}{\sqrt{3}}$
(C) $4\sqrt{3}$
(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- For an ellipse with semi-major axis a, semi-minor axis b, and focal distance c, $b^2 = a^2 - c^2$. Distances are measured from the centre.
- The full length of a latus rectum is $2b^2/a$. It is a chord through a focus perpendicular to the major axis.

Step 1: Find the two centre-to-point distances The centre, focus and given vertex have the same ordinate -2, so the major axis is horizontal. The centre-to-vertex distance is

$$a = |5 - 1| = 4,$$

while the centre-to-focus distance is

$$c = |3 - 1| = 2.$$

These are distances from the centre, not the raw coordinates 5 and 3. The translation of the ellipse changes its position but not its lengths.

Step 2: Determine the semi-minor axis Apply the focal relation to obtain

$$b^2 = a^2 - c^2 = 16 - 4 = 12, \qquad b = 2\sqrt{3}.$$

Thus its equation may be written as

$$\frac{(x-1)^2}{16} + \frac{(y+2)^2}{12} = 1.$$

This also confirms that the major semi-axis exceeds the minor one, as required.

Step 3: Evaluate the focal chord length The latus rectum through the given focus lies on $x = 3$. Substituting this into the ellipse gives

$$\frac{4}{16} + \frac{(y+2)^2}{12} = 1, \qquad (y+2)^2 = 9.$$

Its endpoints therefore have ordinates $-2+3$ and $-2-3$, whose difference is 6. Equivalently,

$$L = \frac{2b^2}{a} = \frac{24}{4} = 6.$$

The value $4\sqrt{3}$ is the full minor-axis length, not this chord. The other two proposed values also differ from the exact endpoint separation.

Step 4: Conclusion The length of the latus rectum is $\boxed{6}$. Hence, the correct option is (D).

Quick Tip to Solve: First subtract the centre coordinates to obtain a and c. Use $2b^2/a$, not $2b$, for the full latus rectum.

9. Given below are two statements :

Statement I : The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \frac{x}{1+|x|}$ is one-one.

Statement II : The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \frac{x^2+4x-30}{x^2-8x+18}$ is many-one.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Correct Answer: (A) Both **Statement I** and **Statement II** are true

Solution:

Concept:

- A function is one-one if different inputs always have different outputs. Strict increase on the entire domain is sufficient for this property.
- To establish that a function is many-one, one pair of distinct domain points with the same output is

enough; no complete range calculation is needed.

Step 1: Check the first function on both sides of zero For $x \geq 0$, write $f(x) = x/(1+x)$, whose derivative is $1/(1+x)^2 > 0$. For $x < 0$, write $f(x) = x/(1-x)$, whose derivative is $1/(1-x)^2 > 0$. Hence it is strictly increasing separately on both parts of its domain. In addition, every negative input has a negative output, whereas every nonnegative input has a nonnegative output. Thus values from the two parts cannot coincide, and the entire function is one-one. Statement I is true.

Step 2: Find repeated outputs for the second function The denominator satisfies

$$x^2 - 8x + 18 = (x-4)^2 + 2 > 0,$$

so both roots of the numerator belong to the domain. Solving

$$x^2 + 4x - 30 = 0$$

gives the distinct real inputs

$$x = -2 + \sqrt{34}, \qquad x = -2 - \sqrt{34}.$$

At either input the numerator is zero and the denominator is nonzero; both outputs are therefore zero. This explicit pair proves that the second function is many-one. Statement II is true.

Step 3: Compare the truth values with the choices Both statements have been proved independently. The second option rejects both established results; the third incorrectly calls Statement II false, and the fourth incorrectly calls Statement I false. There is no implication or explanation relation to test here: the question asks only for the truth of the two statements.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: For a rational function with a denominator that never vanishes, two distinct real numerator roots immediately prove that it is many-one.

10. Let $f(x) = \lim\limits_{\theta \to 0} \left(\frac{\cos \pi x - x^{\left(\frac{2}{\theta}\right)} \sin(x-1)}{1 + x^{\left(\frac{2}{\theta}\right)}(x-1)} \right)$, $x \in \mathbb{R}$. **Consider the following two statements :**

(I) $f(x)$ **is discontinuous at** $x = 1$.

(II) $f(x)$ **is continuous at** $x = -1$.

Then, [4 marks]

(A) Only (I) is True

(B) Only (II) is True

(C) Both (I) and (II) are True

(D) Neither (I) nor (II) is True

Correct Answer: The literal two-sided parameter limit does not define the stated real function, so none of the listed options is justified.

Solution:

Concept:

- A real-valued function must be defined in a neighbourhood of a point before ordinary two-sided continuity there can be tested.
- The printed limit $\theta \to 0$ is two-sided. For a fixed $x > 1$, $x^{2/\theta} \to \infty$ as $\theta \to 0^+$, but tends to zero as $\theta \to 0^-$.

Step 1: Test the literal definition at a positive input Take $x = 2$, for which every power in the expression is real. The expression becomes

$$\frac{1 - 2^{2/\theta} \sin 1}{1 + 2^{2/\theta}}.$$

Its two one-sided limits are

$$\lim_{\theta \to 0^+} = -\sin 1, \qquad \lim_{\theta \to 0^-} = 1.$$

Since $0 < \sin 1 < 1$, these limits are unequal. The printed formula therefore fails to define $f(2)$. This is an exact obstruction, independent of any rounding or option choice.

Step 2: Check the negative-input domain For negative x, the real power $x^{2/\theta}$ is not defined for arbitrary real nonzero θ. For arbitrarily small nonzero θ, the exponent $2/\theta$ can be irrational. Thus the formula does not define a real function on a neighbourhood of -1, and the claimed continuity there cannot be tested as stated.

Step 3: Check the neighbourhood of the other test point At $x = 1$ the formula itself yields -1, but that isolated value does not establish continuity. Put $x = 1 + t$, where $0 < t < 1/2$. The parameter limits from positive and negative θ are respectively

$$-\frac{\sin t}{t} \quad \text{and} \quad -\cos(\pi t).$$

They are unequal: $\tan t > t$ gives $\sin t/t > \cos t$, while cosine is strictly decreasing between zero and $\pi/2$, giving $\cos t > \cos(\pi t)$. Hence the formula is undefined at points arbitrarily close to 1 as well. Neither stated continuity claim can be decided for the alleged real function without changing its definition; treating the two claims as simply false does not fix the undefined function.

Step 4: Conclusion No listed option is justified by the literal definition. The listed key (D) does not follow from it: at $x = 2$, the one-sided parameter limits are exactly $-\sin 1$ and 1, so even $f(2)$ is undefined. The printed formula has been retained without imposing an extra restriction on the parameter.

Quick Tip to Solve: Before testing continuity of a function defined by a parameter limit, verify that the parameter limit exists and that all powers are real on the claimed domain.

11. Let A be the focus of the parabola $y^2 = 8x$. Let the line $y = mx + c$ intersect the parabola at two distinct points B and C. If the centroid of the triangle ABC is $\left(\frac{7}{3}, \frac{4}{3}\right)$, then $(BC)^2$ is equal to : [4 marks]

(A) 32

(B) 41

(C) 80

(D) 89

Correct Answer: (C) 80

Solution:

Concept:

- The parabola $y^2 = 4ax$ has focus $(a, 0)$. The centroid coordinates are the arithmetic means of the three vertex coordinates.
- The squared distance between $B(x_1, y_1)$ and $C(x_2, y_2)$ is $(x_1 - x_2)^2 + (y_1 - y_2)^2$; symmetric sums avoid solving for the points individually.

Step 1: Use the centroid to obtain coordinate sums Here $4a = 8$, so $A = (2, 0)$. Let the other vertices be $B = (x_1, y_1)$ and $C = (x_2, y_2)$. The given centroid implies

$$\frac{2 + x_1 + x_2}{3} = \frac{7}{3}, \qquad \frac{y_1 + y_2}{3} = \frac{4}{3}.$$

Hence

$$x_1 + x_2 = 5, \qquad y_1 + y_2 = 4.$$

These relations hold regardless of the particular slope-intercept description of the chord.

Step 2: Find the squared difference of the ordinates Both endpoints lie on the parabola, so

$$y_1^2 + y_2^2 = 8(x_1 + x_2) = 40.$$

Squaring the known sum yields

$$16 = 40 + 2y_1y_2, \qquad y_1y_2 = -12.$$

Therefore

$$(y_1 - y_2)^2 = y_1^2 + y_2^2 - 2y_1y_2 = 40 + 24 = 64.$$

The positive result also confirms the endpoints are distinct.

Step 3: Compute the abscissa difference and distance Subtract the two parabola equations and factor:

$$8(x_1 - x_2) = y_1^2 - y_2^2 = (y_1 - y_2)(y_1 + y_2) = 4(y_1 - y_2).$$

Thus

$$(x_1 - x_2)^2 = \frac{1}{4}(y_1 - y_2)^2 = 16.$$

Adding the squared differences gives

$$(BC)^2 = 16 + 64 = 80.$$

The other values 32, 41 and 89 cannot equal this sum. In particular, no square root should be taken, because the requested quantity is already the squared length.

Step 4: Conclusion The squared chord length is $\boxed{80}$. Hence, the correct option is (C).

Quick Tip to Solve: For a parabola chord with a known midpoint or centroid, combine the sum and difference of $y_i^2 = 4ax_i$ to obtain the chord length without finding its endpoints.

12. Let [.] denote the greatest integer function. Then

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{12(3 + [x])}{3 + [\sin x] + [\cos x]} \right) dx$$

is equal to : [4 marks]

(A) $15\pi + 4$

(B) $13\pi + 1$

(C) $11\pi + 2$

(D) $12\pi + 5$

Correct Answer: (C) $11\pi + 2$

Solution:

Concept:

- The greatest integer function $[t]$ is the largest integer at most t; for $-1 < t < 0$, it equals -1, not zero.
- Split the integral where any floor function changes. Values at finitely many endpoints do not affect the integral of this bounded piecewise function.

Step 1: Simplify the trigonometric floors For $-\pi/2 < x < 0$, the sine lies strictly between -1 and zero, while the cosine lies strictly between zero and 1. Thus $[\sin x] = -1$ and $[\cos x] = 0$, making the denominator 2. For $0 < x < \pi/2$, both sine and cosine lie strictly between zero and 1, so their floors are zero and the denominator is 3. The exceptional values at zero and the endpoints may be omitted in computing the integral.

Step 2: Split at the integer changes in x Because $1 < \pi/2 < 2$, the floor of x is $-2, -1, 0, 1$ on the successive open intervals separated by $-1, 0, 1$. Accordingly the integrand is

$$\begin{cases} 6, & -\pi/2 < x < -1, \\ 12, & -1 < x < 0, \\ 12, & 0 < x < 1, \\ 16, & 1 < x < \pi/2. \end{cases}$$

For example, the first value is $12(3-2)/2 = 6$; using zero for a negative floor would give a different and incorrect area.

Step 3: Add the four rectangular areas Integrating each constant over its interval gives

$$\begin{aligned} I &= 6(\pi/2 - 1) + 12(1) + 12(1) + 16(\pi/2 - 1) \\ &= 3\pi - 6 + 24 + 8\pi - 16 \\ &= 11\pi + 2. \end{aligned}$$

This calculation accounts for every interval of positive length. Each of the other proposed expressions has a different coefficient of π or constant term, so none equals the exact sum.

Step 4: Conclusion The integral equals $\boxed{11\pi + 2}$. Hence, the correct option is (C).

Quick Tip to Solve: For integrals containing several floor functions, list all their breakpoints first; negative arguments usually create the most easily missed change.

13. Let P be a point in the plane of the vectors $\overrightarrow{AB} = 3\hat{i} + \hat{j} - \hat{k}$ and $\overrightarrow{AC} = \hat{i} - \hat{j} + 3\hat{k}$ such that P is equidistant from the lines AB and AC. If $|\overrightarrow{AP}| = \frac{\sqrt{5}}{2}$, then the area of the triangle ABP is : [4 marks]

(A) $\frac{3}{2}$

(B) $\frac{\sqrt{26}}{4}$

(C) $\frac{\sqrt{30}}{4}$

(D) 2

Correct Answer: (C) $\frac{\sqrt{30}}{4}$

Solution:

Concept:

- For vectors $\mathbf{u}, \mathbf{p}$, the triangle area is $\frac{1}{2}|\mathbf{u} \times \mathbf{p}|$. The distance to the line directed by $\mathbf{u}$ is $|\mathbf{u} \times \mathbf{p}|/|\mathbf{u}|$.
- A point equidistant from two intersecting lines lies on either angle bisector. Both bisectors must be checked when the angle containing the point is not specified.

Step 1: Calculate the angle between the given vectors Put $\mathbf{u} = (3, 1, -1)$ and $\mathbf{v} = (1, -1, 3)$. Then $|\mathbf{u}| = |\mathbf{v}| = \sqrt{11}$, while $\mathbf{u} \cdot \mathbf{v} = 3 - 1 - 3 = -1$. If $\theta = \angle BAC$, then $\cos\theta = -1/11$. Consequently, $\sin^2(\theta/2) = 6/11$ and $\cos^2(\theta/2) = 5/11$.

Step 2: Evaluate the area on each bisector On the internal bisector, the perpendicular distance from

P to AB is $(\sqrt{5}/2)\sin(\theta/2)$. Thus

$$\Delta_{ABP} = \frac{1}{2}\sqrt{11}\,\frac{\sqrt{5}}{2}\sqrt{\frac{6}{11}} = \frac{\sqrt{30}}{4}.$$

The external bisector also satisfies the stated equal-distance condition. It gives

$$\Delta_{ABP} = \frac{1}{2}\sqrt{11}\,\frac{\sqrt{5}}{2}\sqrt{\frac{5}{11}} = \frac{5}{4}.$$

These are both attainable, since either bisector contains points at the prescribed distance from A.

Step 3: Compare the printed alternatives Only $\sqrt{30}/4$ occurs among the four listed options. The squared possible areas are $30/16$ and $25/16$; the alternatives $3/2$, $\sqrt{26}/4$ and 2 have squares $36/16$, $26/16$ and $64/16$, so none is attainable.

Step 4: Conclusion The listed answer is $\boxed{\sqrt{30}/4}$ square units. Hence, the correct option is (C). Strictly, the wording also permits the external-bisector value $5/4$, which is not listed; the area is unique only if the internal angle bisector is intended.

Quick Tip to Solve: For equal distances from two lines, check both internal and external angle bisectors before claiming that the area is unique.

14. Let Q(a, b, c) be the image of the point P(3, 2, 1) in the line $\frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$. Then the distance of Q from the line $\frac{x-9}{3} = \frac{y-9}{2} = \frac{z-5}{-2}$ is [4 marks]

(A) 8

(B) 5

(C) 6

(D) 7

Correct Answer: (D) 7

Solution:

Concept:

- For reflection of P in a line, its perpendicular foot H is the midpoint of PQ, so $Q = 2H - P$.
- The distance from Q to the line through R with direction $\mathbf{d}$ is $|(Q-R)\times\mathbf{d}|/|\mathbf{d}|$.

Step 1: Locate the perpendicular foot The reflecting line has points $H = (1+t, 2t, 1+t)$ and direction $\mathbf{u} = (1,2,1)$. The perpendicularity condition is

$$(P-H)\cdot\mathbf{u} = (2-t) + 2(2-2t) - t = 6 - 6t = 0.$$

Therefore $t = 1$ and $H = (2,2,2)$. Using a point on the line rather than its direction alone is essential for this projection.

Step 2: Find the reflected point Since H bisects the segment joining the original point and its image,

$$Q = 2(2, 2, 2) - (3, 2, 1) = (1, 2, 3).$$

Indeed, $P - H = (1, 0, -1)$ and $Q - H = (-1, 0, 1)$ are opposites and each is perpendicular to $(1, 2, 1)$. This checks the reflection rather than merely locating a nearby point.

Step 3: Calculate the requested distance The second line passes through $R = (9, 9, 5)$ with direction $\mathbf{d} = (3, 2, -2)$. Thus $Q - R = (-8, -7, -2)$, and

$$(Q - R) \times \mathbf{d} = (18, -22, 5).$$

$$d = \frac{\sqrt{18^2 + (-22)^2 + 5^2}}{\sqrt{3^2 + 2^2 + (-2)^2}} = \sqrt{\frac{833}{17}} = 7.$$

The alternatives 8, 5 and 6 have squares 64, 25 and 36, respectively, whereas the exact squared distance is 49.

Step 4: Conclusion The distance of Q from the second line is $\boxed{7}$ units. Hence, the correct option is (D).

Quick Tip to Solve: First reflect using $Q = 2H - P$; only then apply the point-to-line distance formula to the second line.

15. The probability distribution of a random variable X is given below :

X	4k	$\frac{30}{7}k$	$\frac{32}{7}k$	$\frac{34}{7}k$	$\frac{36}{7}k$	$\frac{38}{7}k$	$\frac{40}{7}k$	6k
P(X)	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$

If E(X) = $\frac{263}{15}$, then P(X < 20) is equal to : [4 marks]

(A) $\frac{8}{15}$
(B) $\frac{3}{5}$
(C) $\frac{11}{15}$
(D) $\frac{14}{15}$

Correct Answer: (C) $\frac{11}{15}$

Solution:

Concept:

- For a discrete random variable, $E(X) = \sum x_i p_i$, where $p_i = P(X = x_i)$. The probabilities must sum to one.
- To find $P(X < 20)$, first determine k, then add only the probabilities at values strictly less than 20.

Step 1: Form the expectation equation Write the eight values as $(k/7)(28, 30, 32, 34, 36, 38, 40, 42)$ and the probabilities as $(1/15)(2, 1, 2, 3, 1, 2, 3, 1)$. The probability numerators sum to 15, confirming

a valid distribution. Hence

$$E(X) = \frac{k}{105}(56 + 30 + 64 + 102 + 36 + 76 + 120 + 42) = \frac{526k}{105}.$$

Each value is multiplied by its own probability; equal spacing does not make the distribution uniform.

Step 2: Determine the possible values of the variable Equating the mean to the given value yields

$$\frac{526k}{105} = \frac{263}{15} \quad \Longrightarrow \quad k = \frac{7}{2}.$$

Substitution gives the ordered values

$$X = 14, 15, 16, 17, 18, 19, 20, 21.$$

In particular the seventh value is exactly 20, so it is excluded by the strict inequality.

Step 3: Add the probabilities in the required event The event $X < 20$ consists of the first six values. Therefore

$$P(X < 20) = \frac{2 + 1 + 2 + 3 + 1 + 2}{15} = \frac{11}{15}.$$

The value $14/15$ would wrongly include $X = 20$, producing $P(X \leq 20)$. The alternatives $8/15$ and $3/5 = 9/15$ omit positive probabilities belonging to this event and cannot equal the required sum.

Step 4: Conclusion The required probability is $\boxed{11/15}$. Hence, the correct option is (C).

Quick Tip to Solve: After solving for the scale parameter, list the actual values: the distinction between $<$ and $\leq$ changes the probability at 20.

16. Considering the principal values of inverse trigonometric functions, the value of the expression

$$\tan\left(2\sin^{-1}\left(\frac{2}{\sqrt{13}}\right) - 2\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)\right)$$

is equal to : [4 marks]

(A) $\frac{16}{63}$
(B) $\frac{33}{56}$
(C) $-\frac{16}{63}$
(D) $-\frac{33}{56}$

Correct Answer: (B) $\frac{33}{56}$

Solution:

Concept:

- The principal values $a = \sin^{-1}(2/\sqrt{13})$ and $b = \cos^{-1}(3/\sqrt{10})$ lie in the first quadrant, fixing positive signs for their other trigonometric ratios.
- Use $\tan(a - b) = (\tan a - \tan b)/(1 + \tan a \tan b)$, followed by $\tan 2t = 2\tan t/(1 - \tan^2 t)$.

Step 1: Convert both principal angles to tangents Because $\sin a = 2/\sqrt{13}$ and a is acute,

$$\cos a = \sqrt{1 - 4/13} = \frac{3}{\sqrt{13}}, \qquad \tan a = \frac{2}{3}.$$

Similarly $\cos b = 3/\sqrt{10}$ with acute b gives

$$\sin b = \frac{1}{\sqrt{10}}, \qquad \tan b = \frac{1}{3}.$$

The positive square roots follow from the principal-value ranges; choosing a negative root would change the answer incorrectly.

Step 2: Calculate the tangent of the angle difference The requested expression is $\tan(2a - 2b) = \tan(2(a - b))$. First simplify

$$\tan(a - b) = \frac{2/3 - 1/3}{1 + (2/3)(1/3)} = \frac{1/3}{11/9} = \frac{3}{11}.$$

Since tangent increases on the first quadrant, $a > b$. Both angles are less than $\pi/4$, so $0 < a - b < \pi/4$, and the required tangent is positive.

Step 3: Apply the double-angle identity Now

$$\tan(2(a - b)) = \frac{2(3/11)}{1 - 9/121} = \frac{6/11}{112/121} = \frac{33}{56}.$$

The negative alternatives are excluded by the angle-range check. The positive alternative $16/63$ differs from the exact fraction: $33 \cdot 63 = 2079$, whereas $16 \cdot 56 = 896$. Thus it cannot be an equivalent simplification.

Step 4: Conclusion The expression equals $\boxed{33/56}$. Hence, the correct option is (B).

Quick Tip to Solve: For doubled inverse-trigonometric differences, calculate the tangent of the difference first; it often keeps the fractions smaller.

17. Let the circle $x^2+y^2=4$ intersect x-axis at the points A$(a,0)$, $a>0$ and B$(b,0)$. Let $P(2\cos\alpha, 2\sin\alpha)$, $0<\alpha<\frac{\pi}{2}$ and $Q(2\cos\beta, 2\sin\beta)$ be two points such that $(\alpha-\beta)=\frac{\pi}{2}$. Then the point of intersection of AQ and BP lies on : [4 marks]

(A) $x^2+y^2-4x-4=0$

(B) $x^2+y^2-4y-4=0$

(C) $x^2+y^2-4x-4y-4=0$

(D) $x^2+y^2-4x-4y=0$

Correct Answer: (B) $x^2+y^2-4y-4=0$

Solution:

Concept:

- The intercepts of $x^2+y^2=4$ on the x-axis are $A=(2,0)$ and $B=(-2,0)$. A point on a line satisfies its point-slope equation.
- Half-angle identities give $\sin\alpha/(1+\cos\alpha)=\tan(\alpha/2)$ and $\sin\beta/(\cos\beta-1)=-\cot(\beta/2)$.

Step 1: Write the two slopes using one parameter Put $t=\tan(\alpha/2)$. Since $0<\alpha<\pi/2$, we have $0<t<1$. The slope of BP is

$$m_{BP}=\frac{2\sin\alpha}{2\cos\alpha+2}=t.$$

Also $\beta=\alpha-\pi/2$, so $\tan(\beta/2)=(t-1)/(1+t)$. Therefore

$$m_{AQ}=-\cot(\beta/2)=\frac{1+t}{1-t}.$$

Step 2: Eliminate the parameter at the intersection Let the intersection have coordinates (x,y). The two line equations are

$$y=t(x+2), \qquad y(1-t)=(1+t)(x-2).$$

The intersection cannot have $x=-2$, since the first equation would give $y=0$, which fails the second for $0<t<1$. Hence $t=y/(x+2)$, and substitution yields

$$y(x+2-y)=(x+2+y)(x-2).$$

Expanding and cancelling xy gives

$$x^2+y^2-4y-4=0.$$

Step 3: Check the competing equations For an explicit admissible point, take $\alpha=\pi/4$ and $\beta=-\pi/4$. The intersection is $(2\sqrt{2},2)$. It satisfies the derived equation, whereas substitution into options (A), (C) and (D) gives respectively $8-8\sqrt{2}$, $-8\sqrt{2}$ and $4-8\sqrt{2}$, all nonzero. Thus those equations are not the required locus.

Step 4: Conclusion The intersection lies on $\boxed{x^2+y^2-4y-4=0}$. Hence, the correct option is (B).

Quick Tip to Solve: With one endpoint at each end of a diameter, chord slopes simplify in half-angle parameters; eliminate that parameter instead of solving for the angles.

18. Let

$A = \{z \in \mathbb{C} : |z - 2| \leq 4\}$ **and**

$B = \{z \in \mathbb{C} : |z - 2| + |z + 2| = 5\}$**.**

Then the max $\{|z_1 - z_2| : z_1 \in A$ **and** $z_2 \in B\}$ **is :** [4 marks]

(A) $\frac{15}{2}$

(B) 8

(C) $\frac{17}{2}$

(D) 9

Correct Answer: (C) $\frac{17}{2}$

Solution:

Concept:

- The set A is the closed disc with centre 2 and radius 4. For fixed z_2, its farthest point in this disc is at distance $4 + |z_2 - 2|$.
- The set B is an ellipse with foci $2, -2$ and sum of focal distances 5. Triangle inequalities bound each focal distance without a parametrisation.

Step 1: Bound the distance from the ellipse to the disc centre For $z_2 \in B$, put $r = |z_2 - 2|$ and $s = |z_2 + 2|$. The defining equation gives $r + s = 5$. The reverse triangle inequality gives

$$r - s \leq |2 - (-2)| = 4.$$

Adding these relations yields $2r \leq 9$, so $r \leq 9/2$. This bound uses the distance from the focus at 2, not the distance from the origin.

Step 2: Obtain an upper bound for the required separation For any $z_1 \in A$, we have $|z_1 - 2| \leq 4$. Thus

$$|z_1 - z_2| \leq |z_1 - 2| + |z_2 - 2| \leq 4 + \frac{9}{2} = \frac{17}{2}.$$

An upper bound alone is not enough for a maximum, so the equality case must also be exhibited.

Step 3: Exhibit points attaining the upper bound Take the real point $z_2 = -5/2$. Its focal distances are $9/2$ and $1/2$, whose sum is 5; therefore it lies in B. Take $z_1 = 6$, which lies on the boundary of A because $|6 - 2| = 4$. These points give

$$|z_1 - z_2| = \left|6 + \frac{5}{2}\right| = \frac{17}{2}.$$

The alternatives $15/2$ and 8 are smaller than this attained separation, while 9 exceeds the proved upper bound. All other choices are therefore excluded.

Step 4: Conclusion The maximum separation is $\boxed{17/2}$ units. Hence, the correct option is (C).

Quick Tip to Solve: For a disc versus another locus, maximise distance from the disc centre and add its radius; always exhibit a pair achieving equality.

19. $\frac{6}{3^{26}} + \frac{10\cdot 1}{3^{25}} + \frac{10\cdot 2}{3^{24}} + \frac{10\cdot 2^2}{3^{23}} + \ldots + \frac{10\cdot 2^{24}}{3}$ **is equal to :** [4 marks]

(A) 3^{25}

(B) 3^{26}

(C) 2^{25}

(D) 2^{26}

Correct Answer: (D) 2^{26}

Solution:

Concept:

- A geometric progression has a constant ratio between consecutive terms. For first term a, common ratio $R \neq 1$ and n terms, its sum is $a(R^n - 1)/(R - 1)$.
- The isolated first fraction is kept separate because it does not belong to the geometric progression starting with $10/3^{25}$.

Step 1: Identify the geometric progression Let the complete expression be S. The terms after the first fraction are indexed by $k = 0, 1, \ldots, 24$, so there are 25 of them. Increasing k multiplies the numerator by 2 and divides the denominator by 3; hence the ratio is 6, not $2/3$. Thus

$$S = \frac{6}{3^{26}} + \frac{10}{3^{25}} \sum_{k=0}^{24} 6^k.$$

The last term of this progression is indeed $10 \cdot 2^{24}/3$, which checks the index and the number of terms.

Step 2: Evaluate and cancel the constant terms Use the finite geometric-sum formula before expanding any large powers:

$$\begin{aligned} S &= \frac{6}{3^{26}} + \frac{10}{3^{25}} \frac{6^{25} - 1}{6 - 1} \\ &= \frac{2}{3^{25}} + \frac{2(6^{25} - 1)}{3^{25}} \\ &= \frac{2 \cdot 6^{25}}{3^{25}} = 2 \cdot 2^{25} = 2^{26}. \end{aligned}$$

The first fraction cancels exactly the negative constant from the progression. The options involving powers of 3 retain a denominator contribution that has cancelled, while 2^{25} misses the factor 2 outside the geometric sum.

Step 3: Conclusion The required sum is $\boxed{2^{26}}$. Hence, the correct option is (D).

Quick Tip to Solve: When numerators grow by 2 while denominator powers of 3 decrease, the term ratio is $2 \times 3 = 6$. Separate any exceptional first term before summing.

20. The sum of all the elements in the range of $f(x) = \textbf{Sgn}(\sin x) + \textbf{Sgn}(\cos x) + \textbf{Sgn}(\tan x) + \textbf{Sgn}(\cot x)$, $x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$, where

$$\textbf{Sgn}(t) = \begin{cases} 1, & \textbf{if } t > 0 \\ -1, & \textbf{if } t < 0 \end{cases}$$

is : [4 marks]

(A) 2

(B) 4

(C) -2

(D) 0

Correct Answer: (A) 2

Solution:

Concept:

- For a nonzero real number, the signum function equals $+1$ when the number is positive and -1 when it is negative. Only signs matter here, not trigonometric magnitudes.
- A range is a set of distinct output values. A value attained in two different quadrants is included only once in the sum of its elements.

Step 1: Determine the signs in all four quadrants The excluded points $x = n\pi/2$ are the coordinate-axis angles. Consequently all four trigonometric quantities in the expression are defined and nonzero. Also, $\tan x$ and $\cot x$ always have the same sign. It is enough to consider one complete period, because these signs repeat. The four quadrant sums are

Quadrant	$\text{Sgn}(\sin x)$	$\text{Sgn}(\cos x)$	$\text{Sgn}(\tan x)$	$\text{Sgn}(\cot x)$	$f(x)$
I	1	1	1	1	4
II	1	-1	-1	-1	-2
III	-1	-1	1	1	0
IV	-1	1	-1	-1	-2

Step 2: Form the range and sum its distinct elements Every open quadrant contains allowed angles, so all the displayed output values are attained. There are no additional axis values because those inputs are excluded. Therefore

$$\text{Range}(f) = \{-2, 0, 4\}, \qquad \sum_{y \in \text{Range}(f)} y = -2 + 0 + 4 = 2.$$

The answer 0 would result from adding the four quadrant outputs and counting -2 twice; that is not a sum over a set. The choices 4 and -2 are individual range elements, not the required sum.

Step 3: Conclusion The sum of all distinct elements in the range is $\boxed{2}$. Hence, the correct option is (A).

Quick Tip to Solve: For a signum expression, tabulate signs by quadrant and remove repeated outputs before summing the range.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. If $\sum_{r=1}^{25}\left(\frac{r}{r^4+r^2+1}\right)=\frac{p}{q}$, where p and q are positive integers such that $\gcd(p,q)=1$, then p + q is equal to _____. [4 marks]

Solution:

Concept:

- A useful factorisation is $r^4+r^2+1=(r^2-r+1)(r^2+r+1)$. The two quadratic factors differ by $2r$, giving a telescoping partial-fraction expression.
- If $A_r=1/(r^2-r+1)$, then $A_{r+1}=1/(r^2+r+1)$. Thus a sum of A_r-A_{r+1} retains only its first and last terms.

Step 1: Split the general term Write the denominator as $(r^2+1)^2-r^2$. Subtracting the two reciprocal factors gives

$$\frac{1}{r^2-r+1}-\frac{1}{r^2+r+1}=\frac{2r}{(r^2-r+1)(r^2+r+1)}.$$

Consequently, the numerator r requires the factor $1/2$:

$$\frac{r}{r^4+r^2+1}=\frac{1}{2}\left(\frac{1}{r^2-r+1}-\frac{1}{r^2+r+1}\right).$$

Both factors are positive for positive integer r, so no term is undefined.

Step 2: Cancel the intermediate terms The negative reciprocal from one term is precisely the positive reciprocal from the next. For instance, the denominators begin $1,3,7,13,\ldots$. Hence

$$\begin{aligned}\sum_{r=1}^{25}\frac{r}{r^4+r^2+1}&=\frac{1}{2}\left(\frac{1}{1}-\frac{1}{3}+\frac{1}{3}-\frac{1}{7}+\cdots-\frac{1}{25^2+25+1}\right)\\&=\frac{1}{2}\left(1-\frac{1}{651}\right)=\frac{325}{651}.\end{aligned}$$

Step 3: Check lowest terms Since $651=2\cdot325+1$, the Euclidean algorithm gives $\gcd(325,651)=1$. Thus the reduced numerator and denominator are $p=325$ and $q=651$, and $p+q=325+651=976$.

Step 4: Conclusion The required numerical value is $\boxed{976}$; it is dimensionless.

Quick Tip to Solve: For r^4+r^2+1, try the conjugate quadratic factors r^2-r+1 and r^2+r+1; their reciprocal difference creates the numerator $2r$.

22. Three persons enter in a lift at the ground floor. The lift will go up to 10^{th} floor. The number of ways, in which the three persons can exit the lift at three different floors, if the lift does not stop at first, second and third floors, is equal to _____. [4 marks]

Solution:

Concept:

- The multiplication principle counts successive choices: multiply the number of available choices at each stage when each complete sequence determines exactly one outcome.
- The three persons are distinguishable. Assigning three different allowed floors to them is a permutation without repetition, rather than merely choosing a set of floors.

Step 1: List the permitted destinations The lift starts at the ground floor and travels upward as far as floor 10. The first, second and third floors are not stops, so a person may exit only at one of

$$\{4, 5, 6, 7, 8, 9, 10\}.$$

There are seven permitted floors. The ground floor is the entry point and is not counted as an exit destination during this upward journey. Each person must leave at exactly one of the allowed floors.

Step 2: Assign different floors to the persons Label the persons A, B, C only to keep track of their identities. Person A has 7 possible exit floors. After that choice, person B has 6 remaining choices, because the floors must differ. Person C then has 5 choices. The assignment therefore has

$$7 \times 6 \times 5 = 210$$

possibilities. Counting choices in the order of the persons does not prescribe their chronological exit order; that order is determined automatically by the selected floor numbers.

Step 3: Verify by choosing and arranging Alternatively, first select three floors from the seven, then assign those floors to the three people in all possible ways:

$$\binom{7}{3} 3! = 35 \times 6 = 210.$$

The combination alone, 35, would treat the people as indistinguishable. On the other hand, 7^3 would allow two or more people to leave on the same floor, contrary to the question.

Step 4: Conclusion The number of possible exit assignments is $\boxed{210}$.

Quick Tip to Solve: When different people must use different destinations, count ordered assignments: choose the destinations and multiply by the factorial of the number of people.

23. If the distance of the point $P(43, \alpha, \beta)$, $\beta < 0$, from the line $\vec{r} = 4\hat{i} - \hat{k} + \mu(2\hat{i} + 3\hat{k}), \mu \in \mathbb{R}$ along a line with direction ratios $3, -1, 0$ is $13\sqrt{10}$, then $\alpha^2 + \beta^2$ is equal to _____ [4 marks]

Solution:

Concept:

- A line through $P = (43, \alpha, \beta)$ with direction ratios $(3, -1, 0)$ has points $P + t(3, -1, 0)$. A displacement of parameter t has length $|t|\sqrt{10}$.
- The distance requested is measured along the specified direction, so the relevant endpoint is the intersection of that directed line with the given line. It is not necessarily a perpendicular foot.

Step 1: Parametrise the intersection Let the intersection point be Q. The two line descriptions give

$$Q = (43 + 3t, \alpha - t, \beta) = (4 + 2\mu, 0, -1 + 3\mu).$$

Equating coordinates yields

$$\alpha = t, \qquad 39 + 3t = 2\mu, \qquad \beta = -1 + 3\mu.$$

The zero third direction ratio means the connecting line preserves the z-coordinate; this is why $Q_z = \beta$.

Step 2: Use the prescribed length The displacement from P to Q is $t(3, -1, 0)$. Therefore

$$PQ = \sqrt{(3t)^2 + (-t)^2} = |t|\sqrt{10} = 13\sqrt{10},$$

so $t = 13$ or $t = -13$. Both signs must be examined because distance is nonnegative and alone does not fix the direction along the line.

Step 3: Select the branch with negative beta For $t = 13$, we obtain $\mu = (39 + 39)/2 = 39$ and $\beta = -1 + 117 = 116$, which violates $\beta < 0$. For $t = -13$, we obtain $\mu = 0$, $\alpha = -13$, and $\beta = -1$. In this case $Q = (4, 0, -1)$ lies on the given line, and $Q - P = (-39, 13, 0) = -13(3, -1, 0)$, confirming both the direction and the length.

Step 4: Conclusion Thus $\alpha^2 + \beta^2 = (-13)^2 + (-1)^2 = 169 + 1 = \boxed{170}$, a dimensionless numerical value.

Quick Tip to Solve: For distance measured along given direction ratios, intersect the corresponding line through the point first. Keep both parameter signs until the extra condition selects one.

24. Let f be a differentiable function satisfying $f(x) = 1 - 2x + \int_0^x e^{(x-t)} f(t)\,dt$, $x \in \mathbb{R}$ and let $g(x) = \int_0^x (f(t)+2)^{15}(t-4)^6(t+12)^{17}\,dt$, $x \in \mathbb{R}$.

If p and q are respectively the points of local minima and local maxima of g, then the value of $|p+q|$ is equal to _____. [4 marks]

Solution:

Concept:

- The fundamental theorem of calculus differentiates an integral with a variable upper limit. For the exponential kernel, writing the integral as $e^x \int_0^x e^{-t} f(t)\,dt$ makes product differentiation direct.
- A local extremum is identified by a sign change of the derivative. A stationary point of even multiplicity need not be a maximum or minimum.

Step 1: Derive and solve the differential equation Let $I(x) = \int_0^x e^{x-t} f(t)\,dt$. Then $I'(x) = I(x) + f(x)$, while the given equation gives $I(x) = f(x) - 1 + 2x$. Hence

$$f'(x) = -2 + I'(x) = 2f(x) + 2x - 3, \qquad f(0) = 1.$$

The function $1 - x$ is a particular solution. Thus the general solution is $f(x) = 1 - x + Ce^{2x}$; the initial value forces $C = 0$. Consequently $f(x) + 2 = 3 - x$.

Step 2: Differentiate the function g The integrand is continuous, so

$$g'(x) = (3-x)^{15}(x-4)^6(x+12)^{17}.$$

Its only zeros are $-12, 3, 4$. The factor $(x-4)^6$ is positive away from 4 and has even multiplicity, whereas the other two factors have odd multiplicity. Away from the zeros, the sign therefore equals the sign of $(3-x)(x+12)$.

Step 3: Check the derivative signs For $x < -12$, the two odd factors have opposite signs and $g' < 0$. For $-12 < x < 3$, both are positive and $g' > 0$. For $x > 3$, the derivative is negative except that it vanishes at $x = 4$; it remains negative on both sides of 4. Thus g changes from decreasing to increasing at -12, giving a local minimum $p = -12$, and from increasing to decreasing at 3, giving a local maximum $q = 3$. The stationary point 4 is not an extremum.

Step 4: Conclusion The requested value is $|p+q| = |-12+3| = \boxed{9}$, a dimensionless numerical value.

Quick Tip to Solve: After finding a derivative as a product, use odd versus even multiplicities to track sign changes. Do not automatically count every zero as a local extremum.

25. Let $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and B be two matrices such that $A^{100} = 100B + I$. Then the sum of all the elements of B^{100} is _____ [4 marks]

Solution:

Concept:

- Write a matrix with repeated eigenvalue 1 as $A = I + N$. If $N^2 = 0$, the binomial expansion terminates after its linear term.
- The identity matrix commutes with every matrix, so the usual binomial expansion applies to $(I + N)^r$. A zero matrix has every entry equal to zero.

Step 1: Subtract the identity and square the remainder Here the identity is the 2×2 identity. Subtract it entry by entry:

$$N = A - I = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}.$$

Direct multiplication gives

$$N^2 = \begin{bmatrix} 4-4 & -8+8 \\ 2-2 & -4+4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Thus N is nonzero but nilpotent of order two; it must not be treated as an ordinary nonzero scalar.

Step 2: Identify the matrix B All powers N^j with $j \geq 2$ vanish. Therefore

$$A^{100} = (I + N)^{100} = I + 100N.$$

Comparing this exact matrix equality with the given $A^{100} = 100B + I$, subtract I and divide every entry by 100:

$$B = N = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}.$$

The comparison is valid without assuming that B is invertible.

Step 3: Evaluate the requested power Since $B^2 = N^2 = 0$,

$$B^{100} = (B^2)^{50} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Adding the entries of B itself would answer a different question. The requested sum is of the entries after raising B to the hundredth power, and every one of those entries vanishes.

Step 4: Conclusion The sum of all elements of B^{100} is $\boxed{0}$, a dimensionless number.

Quick Tip to Solve: For a high power of a small matrix, test $(A - I)^2$ before attempting repeated multiplication; a nilpotent remainder collapses the binomial expansion.

26. For a transparent prism, if the angle of minimum deviation is equal to its refracting angle, the refractive index n of the prism satisfies. [4 marks]

(A) $1 < n < 2$

(B) $n \geq 2$

(C) $\sqrt{2} < n < 2\sqrt{2}$

(D) $\sqrt{2} < n < 2$

Correct Answer: (D) $\sqrt{2} < n < 2$

Solution:

Concept:

- At minimum deviation, the ray path through a prism is symmetric: $i = e$, $r_1 = r_2 = A/2$, and $\delta_m = 2i - A$.
- For a prism in air, Snell's law gives $n = \sin i / \sin(A/2)$. A nondegenerate transmitted ray has $0 < i < 90^\circ$.

Step 1: Use the given equality of angles The problem states $\delta_m = A$. Substituting in the symmetric-path relation,

$$A = 2i - A \quad \Longrightarrow \quad i = A.$$

The refracting angle is positive. Since incidence at a face cannot exceed 90°, and the limiting grazing ray is excluded for transmission through the prism, the allowed range is

$$0 < A < 90^\circ.$$

This restriction is stronger than merely requiring a positive prism angle.

Step 2: Express the refractive index in terms of the prism angle Using $i = A$ in Snell's law and the double-angle identity,

$$n = \frac{\sin A}{\sin(A/2)} = \frac{2\sin(A/2)\cos(A/2)}{\sin(A/2)} = 2\cos(A/2).$$

There is no division by zero because $A > 0$. The half-angle lies strictly between 0° and 45°.

Step 3: Apply the angular bounds and compare options Cosine decreases on this interval, so

$$\frac{1}{\sqrt{2}} < \cos(A/2) < 1 \quad \Longrightarrow \quad \sqrt{2} < n < 2.$$

The interval $1 < n < 2$ is only a weaker bound and includes indices below the allowed minimum; it does not give the full constraint. The choice $n \geq 2$ is impossible for a nonzero prism angle. The interval extending to $2\sqrt{2}$ also permits values above the maximum 2. The tight admissible range is therefore the fourth printed option.

Step 4: Conclusion The required range is $\boxed{\sqrt{2} < n < 2}$. Hence, the correct option is (D).

Quick Tip to Solve: When $\delta_m = A$, first obtain $i = A$; the physical bound on incidence supplies the otherwise easily missed lower bound on refractive index.

27. Which one of the following is not a measurable quantity? [4 marks]

(A) Displacement current

(B) Voltage

(C) Voltage difference

(D) Resistance

Correct Answer: (B) Voltage

Solution:

Concept:

- Electric potential at an individual point is defined relative to a chosen zero. Adding the same constant to every potential changes no electric field or measurement.
- A voltmeter measures the potential difference between two terminals. Resistance and displacement current can be determined from observable circuit or field quantities.

Step 1: Distinguish a point potential from a difference In the contrast used by the options, "voltage" means the individual electric potential, while "voltage difference" means the difference between two points. If the reference is changed by a constant C,

$$V_a \longmapsto V_a + C, \qquad V_b \longmapsto V_b + C.$$

Neither individual value has an absolute, reference-independent measurement. A meter necessarily compares its two terminals, including a reference terminal if one is called ground.

Step 2: Show why the potential difference is observable The change of reference cancels in the difference:

$$(V_a + C) - (V_b + C) = V_a - V_b.$$

The work per unit charge between the terminals determines this difference. Consequently option (C) is measurable. In ordinary circuit terminology, voltage itself usually means this measurable difference; the intended distinction in this question is absolute potential versus potential difference.

Step 3: Check the other physical quantities Resistance can be measured by an ohmmeter or determined from the ratio $R = \Delta V/I$ for an ohmic conductor, so (D) is measurable. Displacement current is

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

in vacuum. It can be inferred from the changing electric flux or its magnetic effect; for a charging capacitor it equals the conduction current in the leads. It need not involve charges crossing the dielectric to be physically measurable, so (A) is not the intended answer.

Step 4: Conclusion An absolute point potential, called "voltage in this question in contrast to voltage difference, has no reference-independent measurement. Hence, the correct option is (B).

Quick Tip to Solve: Ask whether adding a constant to every potential changes the proposed measurement. Potential differences survive this test; an absolute potential does not.

28. Identify the correct statements :

A. Electrostatic field lines form closed loops.

B. The electric field lines point radially outward when charge is greater than zero.

C. The Gauss - Law is valid only for inverse-square force.

D. The work done in moving a charged particle in a static electric field around a closed path is zero.

E. The motion of a particle under Coulomb's force must take place in a plane.

Choose the correct answer from the options given below : [4 marks]

(A) A, B, C, D Only

(B) B, C, D, E Only

(C) A, C, E Only

(D) A, B, D, E Only

Correct Answer: (B) B, C, D, E Only

Solution:

Concept:

- An electrostatic field is conservative: $\oint \mathbf{E} \cdot d\mathbf{l} = 0$. Field lines show the force direction on a positive test charge.
- Coulomb's inverse-square field gives radius-independent flux through a sphere. A central force exerts zero torque about its centre and conserves angular momentum.

Step 1: Test the field-line statements Statement A is false. An electrostatic field line cannot form a closed loop: along such a line the field would have a consistently positive tangential component, contradicting zero circulation. Lines start on positive charges and terminate on negative charges or at infinity. For an isolated positive point charge,

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}, \qquad q > 0,$$

so they point radially outward. Statement B is true.

Step 2: Check Gauss law and closed-path work For the inverse-square radial field, the spherical area factor cancels the distance dependence:

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}.$$

This is the electrostatic flux law stated in C; a different radial power would not yield this radius-independent enclosed-charge result. Thus C is true in the Coulomb-law setting. Since the electrostatic field is conservative,

$$W = q \oint \mathbf{E} \cdot d\mathbf{l} = 0,$$

and D is also true.

Step 3: Establish the plane of Coulomb motion Coulomb attraction or repulsion between a particle and a fixed force centre is a central force, parallel to $\mathbf{r}$. Hence

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = 0, \qquad \mathbf{L} = \mathbf{r} \times m\mathbf{v} = \text{constant}.$$

The trajectory remains in the plane perpendicular to the fixed angular-momentum vector. If angular momentum is zero, the trajectory is radial and is still contained in a plane. Thus E is true. Every other listed combination includes the false statement A; the combination B, C, D and E excludes it.

Step 4: Conclusion Only B, C, D and E are correct. Hence, the correct option is (B).

Quick Tip to Solve: For central-force motion, use zero torque to establish planarity; for electrostatic field lines, use zero circulation to exclude closed loops.

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29. The time period of a simple harmonic oscillator is $T = 2\pi\sqrt{\frac{k}{m}}$. Measured value of mass (m) of the object is 10 g with an accuracy of 10 mg and time for 50 oscillations of the spring is found to be 60 s using a watch of 2 s resolution. Percentage error in determination of spring constant (k) is _____%. [4 marks]

(A) 6.76

(B) 3.35

(C) 7.60

(D) 3.43

Correct Answer: (A) 6.76

Solution:

Concept:

- For products and powers, maximum fractional errors add with the absolute values of the exponents: if $k \propto mT^{\pm 2}$, then $\Delta k/k = \Delta m/m + 2\Delta T/T$.
- If an exact count of N oscillations takes time t, then $T = t/N$ and $\Delta T/T = \Delta t/t$. Use the uncertainty of the entire measured interval.

Step 1: Find the fractional mass error Convert the stated mass accuracy to the same unit as the mass:

$$10\,\text{mg} = 0.010\,\text{g}, \qquad \frac{\Delta m}{m} = \frac{0.010}{10} = 0.001.$$

Therefore the percentage contribution from mass is 0.100%. Using 10 mg directly with 10 g would incorrectly enlarge this error by a factor of one thousand.

Step 2: Find the timing error for one period The measured total time for 50 oscillations is $t = 60\,\text{s}$, with stated resolution $\Delta t = 2\,\text{s}$. Thus

$$T = \frac{60}{50} = 1.2\,\text{s}, \qquad \Delta T = \frac{2}{50} = 0.04\,\text{s}.$$

Consequently

$$\frac{\Delta T}{T} = \frac{0.04}{1.2} = \frac{2}{60} = \frac{1}{30}.$$

The two-second uncertainty belongs to the full 60-second interval, not to each oscillation.

Step 3: Propagate the uncertainty and select the listed value The formula as printed gives $k = mT^2/(4\pi^2)$. The physical spring-oscillator relation is instead $T = 2\pi\sqrt{m/k}$, giving $k = 4\pi^2 m/T^2$.

Either expression has the same absolute error exponents, so the printed inversion does not change this requested percentage:

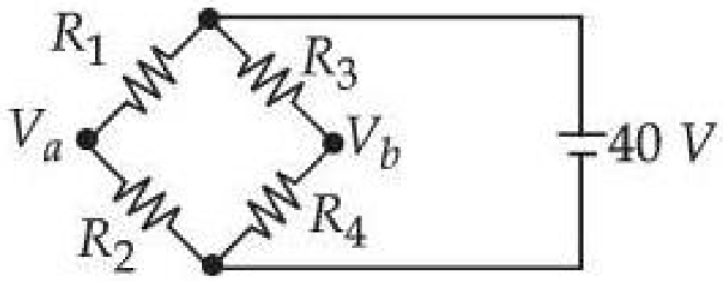
$$\frac{\Delta k}{k} \times 100 = \left(0.001 + 2 \times \frac{2}{60}\right) 100 = 6.7666\ldots\%.$$

The listed value 6.76 is its truncated approximation and is the matching choice. Values near 3.4 omit the factor two on timing error; 7.60 does not follow the stated uncertainties.

Step 4: Conclusion The maximum percentage error is approximately 6.77%, represented by the listed value 6.76%. Hence, the correct option is (A).

Quick Tip to Solve: Divide both total time and its uncertainty by the oscillation count; the count cancels from fractional timing error, which is then doubled for the spring constant.

30. A Wheatstone bridge is initially at room temperature and all arms of the bridge have same value of resistances ($R_1 = R_2 = R_3 = R_4$). When R_3 resistance is heated to some temperature, its resistance value has gone up by 10%. The potential difference ($V_a - V_b$) (after R_3 is heated) is _____ V. [4 marks]



(A) 1.05

(B) 0

(C) 2

(D) 0.95

Correct Answer: (D) 0.95

Solution:

Concept:

- With no conducting link between the two middle points, each side of the bridge is an independent series voltage divider connected across the same supply.
- The potential of a midpoint relative to the lower supply terminal is the supply voltage multiplied by the lower resistance divided by the total branch resistance.

Step 1: Set the reference and update the heated resistor Take the lower terminal of the 40 V supply as zero potential. The upper terminal is positive as drawn, so its potential is $40\,\mathrm{V}$. Initially all four

resistances are R. Heating changes only the upper right resistance:

$$R_1 = R_2 = R_4 = R, \qquad R_3' = 1.10R.$$

There is no connection carrying current from a to b, so the two branch currents can be calculated independently.

Step 2: Calculate the two midpoint potentials The left branch current is $I_L = 40/(2R)$. The voltage across the lower left resistor is therefore

$$V_a = I_L R_2 = \frac{40R}{2R} = 20\,\text{V}.$$

The heated right branch has total resistance $2.10R$, so

$$I_R = \frac{40}{2.10R}, \qquad V_b = I_R R_4 = \frac{40}{2.10} = \frac{400}{21}\,\text{V}.$$

Because the upper resistor increased, the right midpoint falls below its original 20 V value.

Step 3: Subtract in the requested order The required difference is

$$V_a - V_b = 20 - \frac{400}{21} = \frac{20}{21}\,\text{V} = 0.95238\ldots\,\text{V}.$$

The sign is positive because a is now at the higher potential. Zero would require the bridge still to be balanced. The 2 V choice incorrectly treats a ten-percent resistance change as a ten-percent change of the initial midpoint voltage; the series total also changes. The 1.05 V value does not follow the exact divider ratio. The fourth printed value 0.95 V agrees after rounding.

Step 4: Conclusion $\boxed{V_a - V_b \simeq 0.95\,\text{V}}$. Hence, the correct option is (D).

Quick Tip to Solve: For an open Wheatstone bridge, calculate each midpoint with its own voltage-divider ratio; after heating one arm, update both that resistance and its branch total.

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31. The speed of a longitudinal wave in a metallic bar is 400 m/s. If the density and Young's modulus of the bar material are increased by 0.5% and 1%, respectively then the speed of the wave is changed approximately to _____ m/s. [4 marks]

(A) 398

(B) 401

(C) 402

(D) 399

Correct Answer: (B) 401

Solution:

Concept:

- The speed of a longitudinal wave in a thin solid bar is $v = \sqrt{Y/\rho}$, where Y is Young's modulus and ρ is mass density.

- For small changes, $\Delta v/v \simeq \frac{1}{2}(\Delta Y/Y - \Delta\rho/\rho)$. An increased stiffness raises the speed, whereas increased density lowers it.

Step 1: Write the new material parameters The stated increases are relative to the original values, so Y becomes $1.01Y$, while ρ becomes 1.005ρ. They do not contribute with the same sign because density occurs in the denominator of the speed formula. Thus the new speed can also be checked directly from

$$v_{\text{new}} = 400\sqrt{\frac{1.01}{1.005}}\ \text{m s}^{-1}.$$

Step 2: Calculate the first-order speed change Using the small-change formula with percentages gives

$$\frac{\Delta v}{v} \simeq \frac{1}{2}(0.01 - 0.005) = 0.0025 = 0.25\%.$$

Therefore

$$\Delta v = 400\left(\frac{0.25}{100}\right) = 1\ \text{m s}^{-1}, \qquad v_{\text{new}} \simeq 401\ \text{m s}^{-1}.$$

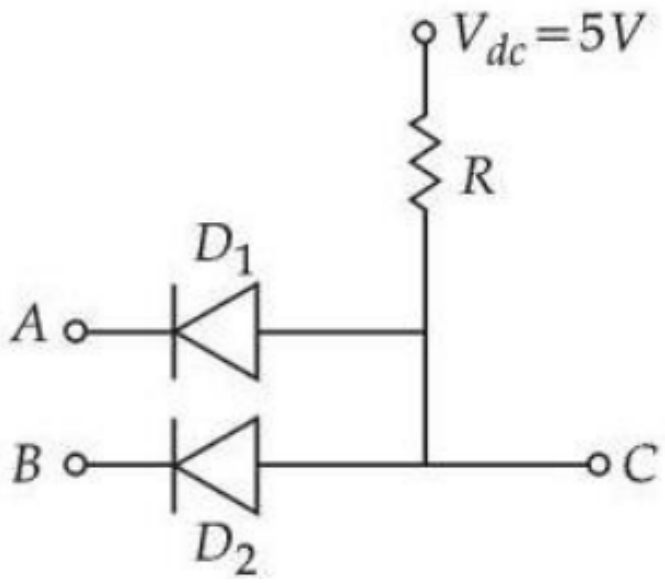
The direct square-root expression gives approximately $400.994\ \text{m s}^{-1}$, confirming the approximation.

Step 3: Compare the proposed speeds The effective fractional increase of Y/ρ is positive, so the lower speeds 398 and 399 cannot be correct. The value 402 corresponds to retaining the modulus increase but overlooking the density increase. Accounting for both changes gives 401.

Step 4: Conclusion The wave speed becomes approximately $\boxed{401\ \text{m s}^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: For $v \propto Y^{1/2}\rho^{-1/2}$, halve the difference of the two percentage changes; do not add them.

32. Two p-n junction diodes D_1 and D_2 are connected as shown in figure. A and B are input signals and C is the output. The given circuit will function as a _____. [4 marks]



(A) OR Gate

(B) NAND Gate

(C) AND Gate

(D) NOR Gate

Correct Answer: (C) AND Gate

Solution:

Concept:

- A forward-biased diode conducts from its anode to its cathode. The diode bars in the figure are at the input terminals, while the anodes meet at output C.
- The resistor connects the output to the positive supply and acts as a pull-up. A conducting diode can hold that output low; high and low correspond to Boolean 1 and 0.

Step 1: Test whether a low input clamps the output If $A = 0$, the pull-up tends to raise C, forward-biasing D_1. Current flows from the supply through R, then through the diode to the low input, holding C low. The same argument applies to D_2 when $B = 0$. Thus one low input is enough to make the output low, irrespective of the other input. In the ideal-diode logic treatment the clamped voltage is zero; a small practical forward drop is still a low logic level.

Step 2: Test both inputs high and form the truth table When both inputs are high, neither diode provides a path to a low terminal. The resistor pulls C high. The complete input-output relation is

A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

Therefore the output is 1 only when both inputs are 1, so $C = A \cdot B$.

Step 3: Eliminate the other gates An OR gate would give a high output for either mixed-input case, but this circuit gives low. A NAND gate would give low for two high inputs, whereas this circuit gives high. A NOR gate would give high for two low inputs, which is also contrary to the truth table.

Step 4: Conclusion The circuit implements the AND relation $\boxed{C = A \cdot B}$. Hence, the correct option is (C).

Quick Tip to Solve: With a pull-up resistor and diode cathodes at the inputs, any low input clamps the output: only all-high inputs produce a high output.

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33. The mean free path of a molecule of diameter 5×10^{-10} m at the temperature $41°$C and pressure 1.38×10^5 Pa, is given as _____ m. (Given $k_B = 1.38 \times 10^{-23}$ J/K). [4 marks]

(A) $10\sqrt{2} \times 10^{-8}$

(B) $2\sqrt{2} \times 10^{-8}$

(C) $2\sqrt{2} \times 10^{-10}$

(D) 2×10^{-8}

Correct Answer: (B) $2\sqrt{2} \times 10^{-8}$

Solution:

Concept:

- For hard-sphere molecules of diameter d and number density n, the mean free path is $\lambda = 1/(\sqrt{2}\,\pi d^2 n)$. The collision cross section uses the diameter squared.
- The ideal-gas relation is $P = nk_BT$. Substituting $n = P/(k_BT)$ gives $\lambda = k_BT/(\sqrt{2}\,\pi d^2 P)$, with T in kelvin.

Step 1: Convert the temperature and square the diameter The absolute temperature is $T = 41 + 273 = 314$ K. The given molecular diameter is already in metres, so

$$d^2 = (5 \times 10^{-10})^2 = 25 \times 10^{-20}\ \text{m}^2.$$

Using 41 directly would incorrectly treat the Celsius zero as the absolute zero. The diameter must not be replaced by its half because the stated collision formula already uses diameter.

Step 2: Substitute and simplify the powers Insert the pressure and Boltzmann constant:

$$\lambda = \frac{1.38 \times 10^{-23} \times 314}{\sqrt{2}\,\pi\,(25 \times 10^{-20})\,(1.38 \times 10^5)} = \frac{314}{25\sqrt{2}\,\pi} \times 10^{-8}\ \text{m}.$$

Taking $\pi \simeq 3.14$ makes the numerical factor $314/(25\pi) \simeq 4$, so

$$\lambda \simeq \frac{4}{\sqrt{2}} \times 10^{-8} = 2\sqrt{2} \times 10^{-8}\ \text{m}.$$

Step 3: Check the scale and options The result is approximately 2.83×10^{-8} m. The value $10\sqrt{2} \times 10^{-8}$ is five times too large; $2\sqrt{2} \times 10^{-10}$ is a hundred times too small. The value 2×10^{-8} loses the required numerical factor $\sqrt{2}$.

Step 4: Conclusion The mean free path is $\boxed{2\sqrt{2} \times 10^{-8}\ \text{m}}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert Celsius to kelvin first, then use d^2 and cancel the common factor 1.38 before handling powers of ten.

---

34. A nucleus has mass number α and radius R_α. Another nucleus has mass number β and radius R_β. If $\beta = 8\alpha$ then R_α/R_β is : [4 marks]

(A) 0.5

(B) 1

(C) 8

(D) 2

Correct Answer: (A) 0.5

Solution:

Concept:

- The empirical nuclear-radius relation is $R = R_0 A^{1/3}$, where A is the mass number and R_0 is approximately the same constant for all nuclei.
- Nuclear volume is proportional to the number of nucleons, so a factor of eight in mass number gives a factor of two in radius, not eight.

Step 1: Write the two radius relations For the nucleus whose mass number is α, $R_\alpha = R_0\alpha^{1/3}$. For the other nucleus, $R_\beta = R_0\beta^{1/3}$. Dividing in the order requested removes the common scale constant:

$$\frac{R_\alpha}{R_\beta} = \frac{R_0\alpha^{1/3}}{R_0\beta^{1/3}} = \left(\frac{\alpha}{\beta}\right)^{1/3}.$$

This ratio is dimensionless because the two radii carry the same length unit.

Step 2: Insert the mass-number relation The given relation is $\beta = 8\alpha$, so

$$\frac{R_\alpha}{R_\beta} = \left(\frac{\alpha}{8\alpha}\right)^{1/3} = \left(\frac{1}{8}\right)^{1/3} = \frac{1}{2} = 0.5.$$

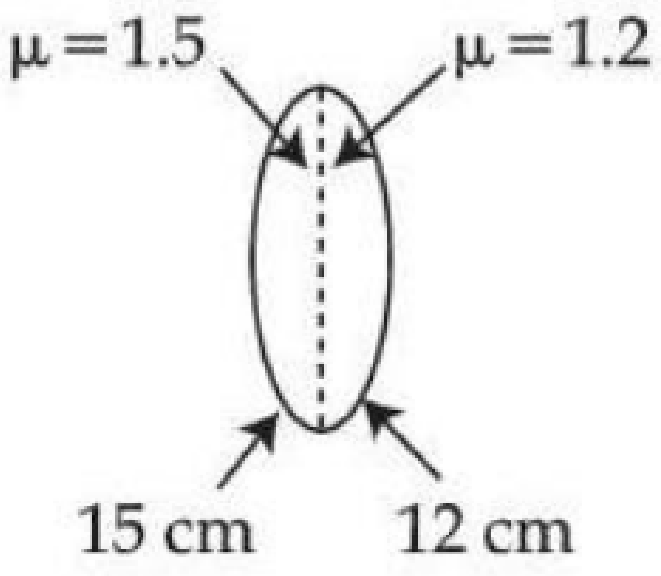
Equivalently, the larger nucleus has eight times the volume and twice the radius, so the radius of the smaller one is half that of the larger one.

Step 3: Reject the reversed or linear ratios The value 1 would require equal nuclear radii and hence equal mass numbers, contrary to the stated factor of eight. The value 2 is the reciprocal ratio R_β/R_α, rather than the one asked for. The value 8 confuses the mass-number ratio with a radius ratio and ignores the cube-root law.

Step 4: Conclusion The required ratio is $\boxed{R_\alpha/R_\beta = 0.5}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the cube root of the mass-number ratio and keep the numerator order unchanged; the nucleus with the smaller mass number must have the smaller radius.

35. A biconvex lens is formed by using two thin planoconvex lenses, as shown in the figure. The refractive index and radius of curved surfaces are also mentioned in figure. When an object is placed on the left side of lens at a distance of 30 cm from the biconvex lens, the magnification of the image will be : [4 marks]



(A) $+2$

(B) -2

(C) -2.5

(D) $+2.5$

Correct Answer: (B) -2

Solution:

Concept:

- For a thin plano-convex lens in air, the positive optical power is $1/f = (\mu - 1)/R$, where R is the magnitude of its curved-surface radius.
- Powers of thin lenses in contact add: $1/f = 1/f_1 + 1/f_2$. With the Cartesian sign convention, $1/f = 1/v - 1/u$ and the transverse magnification is $m = v/u$.

Step 1: Find the combined focal length The left plano-convex element has $\mu_1 = 1.5$ and curved radius 15 cm; the right element has $\mu_2 = 1.2$ and curved radius 12 cm. Both elements converge light. Their flat common interface has zero curvature, so it adds no optical power in the thin-lens limit. Thus

$$\frac{1}{f} = \frac{1.5 - 1}{15} + \frac{1.2 - 1}{12}$$
$$= \frac{0.5}{15} + \frac{0.2}{12} = \frac{1}{30} + \frac{1}{60} = \frac{1}{20}\ \text{cm}^{-1}.$$

Hence the equivalent focal length is $f = +20$ cm.

Step 2: Locate the image using signed distances The object is 30 cm to the left, giving $u = -30$ cm.

The thin-lens equation yields

$$\frac{1}{20} = \frac{1}{v} - \frac{1}{-30} = \frac{1}{v} + \frac{1}{30}, \qquad \frac{1}{v} = \frac{1}{20} - \frac{1}{30} = \frac{1}{60}.$$

Therefore $v = +60$ cm: the image is real and forms on the right.

Step 3: Calculate and interpret the magnification The signed magnification is

$$m = \frac{v}{u} = \frac{60}{-30} = -2.$$

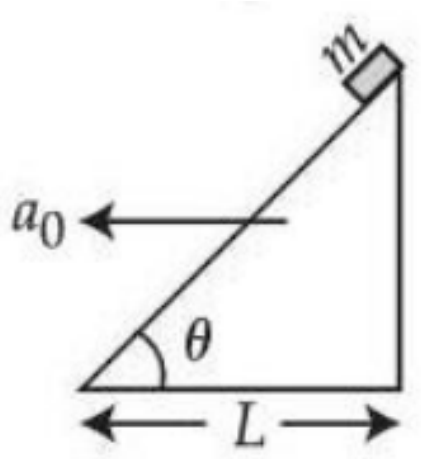
Its negative sign means the image is inverted, and its magnitude means it is twice the object size. The positive options describe an upright image, which is inconsistent with this real image. The magnitude 2.5 does not satisfy the calculated object and image distances.

Step 4: Conclusion The magnification is $\boxed{-2}$, giving a real, inverted image twice the object size. Hence, the correct option is (B).

Quick Tip to Solve: Add the powers of the two plano-convex halves first; then use $u = -30$ cm so the sign of the magnification comes out automatically.

36. A small block of mass m slides down from the top of a frictionless inclined surface, while the inclined plane is moving towards left with constant acceleration a_0. The angle between the inclined plane and ground is θ and its base length is L. Assuming that initially the small block is at the top of the inclined plane, the time it takes to reach the lowest point of the inclined plane is _____.

[4 marks]



(A) $\sqrt{\dfrac{4L}{g\sin 2\theta - a_0(1+\cos 2\theta)}}$

(B) $\sqrt{\dfrac{2L}{g\sin 2\theta - a_0(1+\cos 2\theta)}}$

(C) $\sqrt{\dfrac{4L}{g\cos^2\theta - a_0\sin\theta\cos\theta}}$

(D) $\sqrt{\dfrac{2L}{g\sin\theta - a_0\cos\theta}}$

Correct Answer: (A) $\sqrt{\dfrac{4L}{g\sin 2\theta - a_0(1+\cos 2\theta)}}$

Solution:

Concept:

- In the frame of an incline accelerating left with a_0, a block experiences a pseudo-force ma_0 to the right. Project gravity and this pseudo-force along the incline.
- For a frictionless surface, the normal force has no component along the plane. Starting from rest relative to the plane, the displacement is $s = \frac{1}{2}a_{\text{rel}}t^2$.

Step 1: Resolve the effective force down the incline The figure rises towards the right, so the downhill direction is towards the left. Gravity contributes $mg\sin\theta$ downhill. The rightward pseudo-force has component $ma_0\cos\theta$ uphill, opposing the slide. Hence

$$a_{\text{rel}} = g\sin\theta - a_0\cos\theta.$$

The stated downhill motion from rest requires this quantity to be positive. The mass cancels because both force components are proportional to it.

Step 2: Use the slope length rather than the base length The horizontal base is L, so the distance from the top to the bottom along the incline is $s = L/\cos\theta$. With zero initial relative speed,

$$\frac{L}{\cos\theta} = \frac{1}{2}(g\sin\theta - a_0\cos\theta)t^2.$$

Therefore

$$t^2 = \frac{2L}{g\sin\theta\cos\theta - a_0\cos^2\theta}.$$

Step 3: Express the result using double angles Use $2\sin\theta\cos\theta = \sin 2\theta$ and $2\cos^2\theta = 1 + \cos 2\theta$. Multiplying numerator and denominator by 2 gives

$$t = \sqrt{\frac{4L}{g\sin 2\theta - a_0(1+\cos 2\theta)}}.$$

The option with numerator $2L$ and this same denominator misses a factor of two. The expression with $g\cos^2\theta$ uses the wrong gravitational projection. The expression $\sqrt{2L/(g\sin\theta - a_0\cos\theta)}$ incorrectly treats the base length as the slope length.

Step 4: Conclusion The travel time is $\boxed{t = \sqrt{\frac{4L}{g\sin 2\theta - a_0(1+\cos 2\theta)}}}$, with $g\sin\theta > a_0\cos\theta$. Hence, the correct option is (A).

Quick Tip to Solve: Work in the wedge frame: the pseudo-force opposes the downhill motion here, and the travel distance is $L/\cos\theta$, not L.

37. In an experiment, a set of reading are obtained as follows - 1.24 mm, 1.25 mm, 1.23 mm, 1.21 mm. The expected least count of the instrument used in recording these readings is _____ mm. [4 marks]

(A) 0.1

(B) 0.001

(C) 0.05

(D) 0.01

Correct Answer: (D) 0.01

Solution:

Concept:

- The least count is the smallest change of a measured quantity that an instrument can resolve directly.
- The expected resolution is indicated by the last meaningful decimal place in readings recorded consistently in the same unit.

Step 1: Identify the recorded precision All four observations are reported in millimetres with two digits after the decimal point: $1.24, 1.25, 1.23, 1.21$. Their last reported digit therefore represents hundredths of a millimetre. The precision is associated with this place value, not with the overall size of a reading.

Step 2: Find the smallest indicated increment Order the observations as $1.21, 1.23, 1.24, 1.25$ mm. In particular, successive recorded values include 1.24 and 1.25 mm, whose difference is

$$1.25 - 1.24 = 0.01\,\text{mm}.$$

The expected least count is consequently $10^{-2}\,\text{mm}$. The unequal gaps between some observations do not imply that the least count changes; a reading can skip several divisions.

Step 3: Eliminate the other resolutions A least count of 0.1 mm would resolve only tenths, and 0.05 mm would not permit a directly resolved separation of 0.01 mm. A resolution of 0.001 mm corresponds to thousandths, an extra decimal place not recorded here. The question asks the expected least count from the reporting precision; the spread or average of the observations instead describes their variation or central value.

Step 4: Conclusion The expected least count is $\boxed{0.01\,\text{mm}}$. Hence, the correct option is (D).

Quick Tip to Solve: Read the place value of the last recorded digit in the stated unit; do not confuse least count with the spread between the largest and smallest observations.

38. Number of photons of equal energy emitted per second by a 6 mW laser source operating at 663 nm is _____.

(Given : $h = 6.63 \times 10^{-34}$ J.s and $c = 3 \times 10^{8}$ m/s) [4 marks]

(A) 5×10^{15}

(B) 10×10^{15}

(C) 2×10^{16}

(D) 5×10^{16}

Correct Answer: (C) 2×10^{16}

Solution:

Concept:

- A photon of wavelength λ carries energy $\varepsilon = hc/\lambda$, where h is Planck's constant and c is the speed of light.
- Power is energy delivered per second. For identical photons the emission rate is $\dot{N} = P/\varepsilon = P\lambda/(hc)$.

Step 1: Convert the supplied quantities to SI units The laser power and wavelength are

$$P = 6 \times 10^{-3}\,\mathrm{J\,s^{-1}}, \qquad \lambda = 663 \times 10^{-9}\,\mathrm{m}.$$

The milli- prefix affects the power by 10^{-3}, while nano- affects the wavelength by 10^{-9}. Use the given values $h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$ and $c = 3 \times 10^{8}\,\mathrm{m\,s^{-1}}$.

Step 2: Calculate the energy of one photon Substitution in the photon-energy formula gives

$$\varepsilon = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{663 \times 10^{-9}} = 3.0 \times 10^{-19}\,\mathrm{J}.$$

This is the energy of one photon, whereas the quoted milliwatts describe the combined energy emitted by all the photons each second.

Step 3: Divide the power by the photon energy The number emitted per second is

$$\dot{N} = \frac{6 \times 10^{-3}}{3 \times 10^{-19}} = 2 \times 10^{16}\,\mathrm{s^{-1}}.$$

Multiplying this rate by the individual photon energy recovers 6×10^{-3} W. The other offered rates, 5×10^{15}, 10×10^{15}, and 5×10^{16} per second, would instead yield powers of 1.5, 3, and 15 mW, respectively.

Step 4: Conclusion The emission rate is $\boxed{2 \times 10^{16}\text{ photons per second}}$, the third option in the printed order. Hence, the correct option is (C).

Quick Tip to Solve: Use $\dot{N} = P\lambda/(hc)$ directly, but convert both milliwatts and nanometres before combining powers of ten.

39. A particle starts moving from time $t = 0$ and its coordinate is given as $x(t) = 4t^3 - 3t$

A. The particle returns to its original position (origin) 0.866 units later

B. The particle is 1 unit away from origin at its turning point

C. Acceleration of the particle is non-negative

D. The particle is 0.5 units away from origin at its turning point

E. Particle never turns back as acceleration is non-negative

Choose the correct answer from the options given below : [4 marks]

(A) A, C Only

(B) A, C, D Only

(C) A, B, C Only

(D) C, E Only

Correct Answer: (C) A, B, C Only

Solution:

Concept:

- For one-dimensional motion, velocity is $v = dx/dt$ and acceleration is $a = dv/dt$. A turning point requires velocity to change sign.
- Returning to the origin means $x(t) = 0$ at a positive time. Distance from the origin is $|x|$, irrespective of the direction of motion.

Step 1: Find the return time For $x(t) = 4t^3 - 3t$, the origin condition is

$$t(4t^2 - 3) = 0.$$

Besides the initial instant $t = 0$, the admissible time is $t = \sqrt{3}/2 \simeq 0.866$. The negative root is outside the stated motion beginning at zero. Thus statement A is correct.

Step 2: Locate and check the turning point Differentiating the coordinate gives

$$v(t) = 12t^2 - 3.$$

For $t \geq 0$, this vanishes at $t = 1/2$. Velocity is negative before this instant and positive afterwards, so it is an actual reversal. Its coordinate is

$$x(1/2) = 4(1/2)^3 - 3(1/2) = \tfrac{1}{2} - \tfrac{3}{2} = -1.$$

The distance from the origin is therefore one unit. Statement B is true and statement D is false; 0.5 is the turning time, not its distance.

Step 3: Evaluate acceleration and compare all statements A further derivative gives $a(t) = 24t$, which is non-negative over the stated domain $t \geq 0$. Hence C is true. Nevertheless the initial velocity is -3, so positive acceleration first reduces its negative velocity to zero and then makes it positive. Non-negative acceleration does not prevent reversal: E is false. Only A, B and C hold, ruling out the options that omit B, include D, or include E.

Step 4: Conclusion The correct statements are A, B and C only. Hence, the correct option is (C).

Quick Tip to Solve: A turning point comes from a sign change of velocity, not a sign change of acceleration. Evaluate x at that time to obtain its distance from the origin.

40. Match List - I with List - II.

List – I	List – II
A. Coefficient of viscosity	**I.** $[ML^{-1}T^{-2}]$
B. Surface tension	**II.** $[ML^{2}T^{-2}]$
C. Pressure	**III.** $[ML^{0}T^{-2}]$
D. Surface energy	**IV.** $[ML^{-1}T^{-1}]$

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-III, C-I, D-II

(B) A-I, B-II, C-IV, D-III

(C) A-I, B-III, C-II, D-IV

(D) A-IV, B-I, C-II, D-III

Correct Answer: (A) A-IV, B-III, C-I, D-II

Solution:

Concept:

- Use $[F] = MLT^{-2}$, [area] $= L^2$, and [energy] $= ML^2T^{-2}$ as the basic derived dimensions.
- Dynamic viscosity follows Newton's law $F = \eta A\, dv/dy$; surface tension is force per length, whereas pressure is force per area.

Step 1: Determine the dimensions of viscosity The velocity gradient has dimensions

$$[dv/dy] = (LT^{-1})/L = T^{-1}.$$

Consequently,

$$[\eta] = \frac{[F]}{[A][dv/dy]} = \frac{MLT^{-2}}{L^2T^{-1}} = ML^{-1}T^{-1}.$$

Thus the coefficient of viscosity in A matches IV. The time exponent distinguishes it immediately from pressure.

Step 2: Match surface tension and pressure For surface tension, divide the force by the length along which it acts:

$$[\text{surface tension}] = \frac{MLT^{-2}}{L} = ML^{0}T^{-2}.$$

This gives B–III. Pressure is force per unit area, so

$$[p] = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2},$$

and therefore C–I. Although these quantities involve forces at a surface, their denominators have different powers of length.

Step 3: Match surface energy and test the options Surface energy is the work associated with forming the surface. Its dimensions are those of energy itself:

$$[\text{surface energy}] = [F][L] = ML^{2}T^{-2}.$$

Thus D–II. Do not substitute energy per unit area for the stated surface energy. All four matches are A–IV, B–III, C–I and D–II. Each other offered combination assigns at least one quantity the wrong length or time exponent, so none satisfies the defining relations.

Step 4: Conclusion The matching is $\boxed{\text{A–IV, B–III, C–I, D–II}}$, the first printed option. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish surface energy from surface energy per unit area: the latter has the same dimensions as surface tension, but total surface energy has the dimensions of work.

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41. A plane electromagnetic wave is moving in free space with velocity $c = 3 \times 10^8$ m/s and its electric field is given as $\vec{E} = 54\sin(kz - \omega t)\,\hat{j}$ V/m, where $\hat{j}$ is the unit vector along y-axis. The magnetic field vector $\vec{B}$ of the wave is : [4 marks]

(A) $-1.8 \times 10^{-7}\sin(kz - \omega t)\,\hat{i}$ T

(B) $1.4 \times 10^{-7}\sin(kz - \omega t)\,\hat{k}$ T

(C) $+1.8 \times 10^{-7}\sin(kz - \omega t)\,\hat{i}$ T

(D) $1.4 \times 10^{-7}\sin(kz - \omega t)\,\hat{i}$ T

Correct Answer: (A) $-1.8 \times 10^{-7}\sin(kz - \omega t)\,\hat{i}$ T

Solution:

Concept:

- In a plane electromagnetic wave in free space, $E_0 = cB_0$. The electric and magnetic fields oscillate in phase.
- The propagation direction is that of $\vec{E} \times \vec{B}$. Equivalently, $\vec{B} = (\hat{n} \times \vec{E})/c$, with $\hat{n}$ the propagation unit vector.

Step 1: Read the propagation direction from the phase A point of fixed phase obeys $kz - \omega t =$ constant, giving $z = (\omega/k)t +$ constant. Thus the wave propagates along the positive z-axis and $\hat{n} = \hat{k}$. The electric field is directed along the y-axis, as indicated by $\hat{j}$.

Step 2: Calculate the magnetic-field amplitude The electric-field amplitude is $E_0 = 54\,\text{V m}^{-1}$, so

$$B_0 = \frac{54}{3 \times 10^8} = 1.8 \times 10^{-7}\,\text{T}.$$

Both fields have the same phase factor $\sin(kz - \omega t)$. This fixes the amplitude independently of the vector-direction test.

Step 3: Apply the cross product and reject the alternatives Using $\hat{k} \times \hat{j} = -\hat{i}$, obtain

$$\vec{B} = \frac{\hat{k} \times \vec{E}}{c} = -1.8 \times 10^{-7}\sin(kz - \omega t)\,\hat{i}\,\text{T}.$$

Indeed, $\hat{j} \times (-\hat{i}) = +\hat{k}$, so the energy travels in the required direction. The positive $\hat{i}$ choice with the right amplitude would send the wave in the negative z-direction. A $\hat{k}$-directed magnetic field is not transverse to propagation, and the remaining 1.4×10^{-7} choice has the wrong amplitude.

Step 4: Conclusion $\boxed{\vec{B} = -1.8 \times 10^{-7} \sin(kz - \omega t)\,\hat{i}\ \mathrm{T}}$. Hence, the correct option is (A).

Quick Tip to Solve: Check amplitude with E_0/c, then check the sign independently using $\vec{E} \times \vec{B}$. Perpendicular fields alone do not determine the correct propagation direction.

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42. A long cylindrical conductor with large cross section carries an electric current distributed uniformly over its cross-section. Magnetic field due to this current is:

A. maximum at either ends of the conductor and minimum at the midpoint

B. maximum at the axis of the conductor

C. minimum at the surface of the conductor

D. minimum at the axis of the conductor

E. same at all points in the cross-section of the conductor

Choose the correct answer from the options given below: [4 marks]

(A) A, D Only

(B) D Only

(C) B, C Only

(D) E Only

Correct Answer: (B) D Only

Solution:

Concept:

- Ampere's law gives $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$. Cylindrical symmetry makes B constant on a coaxial circular loop.
- For uniform current density in a cylinder of radius R, the current enclosed at radius $r < R$ is $I_{\text{enc}} = Ir^2/R^2$.

Step 1: Find the field inside the conductor Take a circular Amperian path of radius r centred on the conductor axis. Its circumference is $2\pi r$. Only the current inside this path contributes to the enclosed-current side of Ampere's law:

$$B(2\pi r) = \mu_0 I \frac{r^2}{R^2}.$$

Therefore,

$$B(r) = \frac{\mu_0 I}{2\pi R^2} r \qquad (0 \le r \le R).$$

The magnetic-field magnitude increases linearly with radial distance from the axis.

Step 2: Compare the axis and the surface At the axis $r = 0$, the field is zero. At the surface it is $B(R) = \mu_0 I/(2\pi R)$, its largest value in the cross-section. Outside, the enclosed current is the total current, giving $B(r) = \mu_0 I/(2\pi r)$; the field then decreases. Thus the axis is the minimum within the conductor, not the maximum.

Step 3: Assess each listed statement Statement D is true. B is false because the field vanishes at the

axis, and C is false because the surface field is the maximum rather than the minimum. E is false because the internal field varies with r. A incorrectly attributes the extrema to positions along the length; for a long conductor the cylindrical result describes radial variation and does not give maxima at the ends. Hence the combinations containing A, B, C, or E must be rejected.

Step 4: Conclusion Only D is correct: the field is minimum at the axis of the conductor. Hence, the correct option is (B).

Quick Tip to Solve: For a uniformly filled wire, enclosed current grows as r^2, but the Amperian circumference grows as r; their ratio makes the internal field proportional to r.

43. When the position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ changes sign as $-\vec{r}$, which one of the following vector will not flip under sign change? [4 marks]

(A) Velocity

(B) Acceleration

(C) Angular momentum

(D) Linear momentum

Correct Answer: (C) Angular momentum

Solution:

Concept:

- Under spatial inversion, every position vector changes as $\vec{r} \mapsto -\vec{r}$, while time and mass remain unchanged.
- Polar vectors reverse under inversion. The cross product of two polar vectors is an axial vector and remains unchanged.

Step 1: Transform velocity and acceleration The inversion is applied to the complete trajectory, with the same time coordinate. Differentiating the transformed position gives

$$\vec{v}' = \frac{d(-\vec{r})}{dt} = -\vec{v}.$$

Differentiating once more gives

$$\vec{a}' = \frac{d(-\vec{v})}{dt} = -\vec{a}.$$

Thus both velocity and acceleration reverse their directions. In particular, taking a second time derivative does not introduce a second spatial minus sign: time itself has not been reversed.

Step 2: Transform the two momenta For unchanged mass, linear momentum transforms as

$$\vec{p}' = m\vec{v}' = -m\vec{v} = -\vec{p}.$$

Angular momentum about the same origin is $\vec{L} = \vec{r} \times \vec{p}$. Both factors change sign, so

$$\vec{L}' = (-\vec{r}) \times (-\vec{p}) = \vec{r} \times \vec{p} = \vec{L}.$$

The two negative signs cancel through the bilinearity of the cross product. Consequently the angular momentum is an axial vector. The velocity, acceleration and linear momentum choices all fail because each has exactly one overall minus sign under spatial inversion.

Step 3: Conclusion Angular momentum does not flip under the transformation $\vec{r} \mapsto -\vec{r}$. Hence, the correct option is (C).

Quick Tip to Solve: For inversion questions, transform each factor first: $\vec{r} \times \vec{p}$ has two reversing factors, so its sign is preserved.

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44. Identify the correct statements :

A. Effective capacitance of a series combination of capacitors is always smaller than the smallest capacitance of the capacitor in the combination.

B. When a dielectric medium is placed between the charged plates of a capacitor, displacement of charges cannot occur due to insulation property of dielectric.

C. Increasing of area of capacitor plate or decreasing of thickness of dielectric is an alternate method to increase the capacitance.

D. For a point charge, concentric spherical shells centered at the location of the charge are equipotential surfaces.

Choose the correct answer from the options given below : [4 marks]

(A) A, B and C Only

(B) B and D Only

(C) C and D Only

(D) A, C and D Only

Correct Answer: (D) A, C and D Only

Solution:

Concept:

- For positive capacitances in series, $1/C_{\text{eq}} = \sum_i 1/C_i$. For parallel plates filled with dielectric, $C = \varepsilon A/d$.
- A dielectric permits small displacements of bound charges, producing polarization. The potential of a point charge is $V = kQ/r$.

Step 1: Check statement A using the series formula If C_{min} is the smallest individual capacitance, its reciprocal is one positive term in the reciprocal sum. The other positive terms make

$$\frac{1}{C_{\text{eq}}} > \frac{1}{C_{\text{min}}} \quad \Rightarrow \quad C_{\text{eq}} < C_{\text{min}}.$$

Hence A is true for a series combination of capacitors.

Step 2: Distinguish insulation from polarization An insulating dielectric does not allow charges to travel freely through it as they do through a conductor. Nevertheless, an electric field can slightly displace bound positive and negative charges relative to one another, or orient existing molecular

dipoles. This creates polarization. Thus the claim in B that charge displacement cannot occur is false.

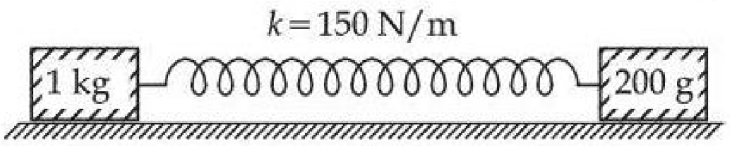
Step 3: Check the geometrical and potential statements For a fixed dielectric, $C = \varepsilon A/d$ increases when plate area A increases or dielectric thickness, equivalently plate separation d, decreases. Statement C is true. For a point charge, $V = kQ/r$ depends only on distance from the charge. All points on a concentric spherical shell have the same r, and hence the same potential. Statement D is also true. Options containing B are excluded; the choice C and D only incorrectly omits the true statement A.

Step 4: Conclusion The correct statements are A, C and D only. Hence, the correct option is (D).

Quick Tip to Solve: Do not confuse absence of free conduction in an insulator with absence of bound-charge displacement: the latter produces dielectric polarization.

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45. As shown in the figure, a spring is kept in a stretched position with some extension by holding the masses 1 kg and 0.2 kg with a separation more than spring natural length and are released. Assuming the horizontal surface to be frictionless, the angular frequency (in SI unit) of the system is : [4 marks]



(A) 5

(B) 30

(C) 27

(D) 20

Correct Answer: (B) 30

Solution:

Concept:

- For two free masses joined by a spring on a frictionless horizontal surface, the centre of mass has no acceleration because the spring forces are internal.
- The extension of the spring oscillates with $\omega^2 = k(1/m_1 + 1/m_2) = k/\mu$, where $\mu = m_1 m_2/(m_1 + m_2)$ is the reduced mass.

Step 1: Write the equation for the spring extension The figure gives $k = 150\,\mathrm{N\,m^{-1}}$, $m_1 = 1\,\mathrm{kg}$ and $m_2 = 0.2\,\mathrm{kg}$. Let $x = x_2 - x_1 - \ell_0$ denote the extension, with ℓ_0 the natural length. Taking rightward as positive, the spring pulls the left mass rightward and the right mass leftward:

$$m_1\ddot{x}_1 = kx, \qquad m_2\ddot{x}_2 = -kx.$$

Subtracting the accelerations gives

$$\ddot{x} = \ddot{x}_2 - \ddot{x}_1 = -k\left(\frac{1}{m_2} + \frac{1}{m_1}\right)x.$$

This is the SHM equation for the relative separation.

Step 2: Calculate the relative angular frequency Compare the equation with $\ddot{x}+\omega^2 x=0$. Substitution gives

$$\omega^2 = 150\left(\frac{1}{1}+\frac{1}{0.2}\right) = 150(1+5) = 900\,\mathrm{s}^{-2},$$

so

$$\omega = 30\,\mathrm{rad\,s}^{-1}.$$

Both masses oscillate at this same angular frequency around the stationary centre of mass, although their amplitudes differ. Treating either end as fixed would be incorrect because both masses are released. The alternatives 5, 27 and 20 do not satisfy $\omega^2 = 900\,\mathrm{s}^{-2}$.

Step 3: Conclusion Because $150(1+5)=900$, the frequency is 30, which is printed option (B), not the alternative value 27. The angular frequency is $\boxed{30\,\mathrm{rad\,s}^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a spring joining two free masses, replace the mass in $\sqrt{k/m}$ by the reduced mass $m_1m_2/(m_1+m_2)$.

Physics – Section B: Numerical Value Questions (20 marks)

46. A fly wheel having mass 3 kg and radius 5 m is free to rotate about a horizontal axis. A string having negligible mass is wound around the wheel and the loose end of the string is connected to 3 kg mass. The mass is kept at rest initially and released. Kinetic energy of the wheel when the mass descends by 3 m is _____ J. ($g = 10\ \mathrm{m/s^2}$) [4 marks]

Solution:

Concept:

- For a uniform disc rotating about its central axis, $I = MR^2/2$. The wheel is treated with this standard disc model.
- When the string unwinds without slipping, its speed equals the rim speed: $v = R\omega$. Gravitational potential energy becomes translational plus rotational kinetic energy.

Step 1: Relate the motion of the mass and the wheel The suspended mass is $m = 3\,\mathrm{kg}$, the wheel mass is $M = 3\,\mathrm{kg}$, and $R = 5\,\mathrm{m}$. Taking the wheel as a uniform disc, its moment of inertia is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(3)(5^2) = 37.5\,\mathrm{kg\,m^2}.$$

Because the string does not slip, $\omega = v/R$. Thus its rotational energy can be expressed using the same speed as the falling mass:

$$K_{\text{wheel}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{I}{R^2}v^2 = \frac{1}{4}Mv^2.$$

The wheel rotates about a fixed axis, so it has no translational kinetic energy.

Step 2: Use conservation of mechanical energy The mass begins at rest and descends through $h = 3\,\mathrm{m}$.

With negligible losses,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}Mv^2.$$

Substituting the given quantities gives

$$3(10)(3) = \left(\frac{3}{2} + \frac{3}{4}\right)v^2 = \frac{9}{4}v^2, \qquad v^2 = 40\,\mathrm{m^2\,s^{-2}}.$$

The rotational part is therefore

$$K_{\text{wheel}} = \frac{1}{4}(3)(40) = 30\,\mathrm{J}.$$

As a check, the falling mass has kinetic energy $\frac{1}{2}(3)(40) = 60\,\mathrm{J}$; the sum is $90\,\mathrm{J}$, precisely the lost gravitational energy.

Step 3: Conclusion The kinetic energy of the wheel is $\boxed{30\,\mathrm{J}}$, so the numerical entry is 30.

Quick Tip to Solve: Use the common speed $v = R\omega$ to divide the lost potential energy between the falling mass and the rotating wheel; do not assign all of mgh to either one.

47. Two tuning forks *A* and *B* are sounded together giving rise to 8 beats in 2 s. When fork *A* is loaded with wax, the beat frequency is reduced to 4 beats in 2 s. If the original frequency of tuning fork *B* is 380 Hz then original frequency of tuning fork *A* is _____ Hz. [4 marks]

Solution:

Concept:

- The beat frequency is $f_{\text{beat}} = |f_A - f_B|$, measured in beats per second. A beat count must first be divided by the observation time.
- Adding wax increases the effective oscillating mass of a tuning fork and lowers its natural frequency.

Step 1: Find the two possible original frequencies The two forks initially produce 8 beats in 2 seconds, so the frequency difference is

$$|f_A - 380| = \frac{8}{2} = 4\,\mathrm{Hz}.$$

This allows two numerical possibilities:

$$f_A = 380 + 4 = 384\,\mathrm{Hz} \quad \text{or} \quad f_A = 380 - 4 = 376\,\mathrm{Hz}.$$

The beat count alone does not indicate which fork has the higher frequency; the loading experiment supplies this missing information.

Step 2: Use the direction of the change on loading After wax is added to A, the observed beat frequency is

$$f'_{\text{beat}} = \frac{4}{2} = 2\,\mathrm{Hz}.$$

The frequency of B remains 380 Hz, while that of A decreases. If A had originally been 376 Hz, lowering it would move it farther below 380 Hz, increasing the difference beyond 4 Hz. That contradicts the observed reduction to 2 Hz. Therefore A must initially have been above B, at 384 Hz. Lowering its

frequency towards 380 Hz then reduces the difference, exactly as observed. The unknown amount of wax need not be used to calculate the original frequency.

Step 3: Conclusion The original frequency of tuning fork A is $\boxed{384\,\text{Hz}}$, so the numerical entry is 384.

Quick Tip to Solve: Wax lowers frequency. If loading a fork reduces the beat rate, the loaded fork was originally the higher-frequency one; first convert beats over a time interval to beats per second.

48. A beam of light consisting of wavelengths 650 nm and 550 nm illuminates the Young's double slits with separation of 2 mm such that the interference fringes are formed on a screen, placed at a distance of 1.2 m from the slits. The least distance of a point from the central maximum, where the bright fringes due to both the wavelengths coincide, is _____ $\times 10^{-5}$ m. [4 marks]

Solution:

Concept:

- In Young's double-slit experiment, the nth bright fringe is at $y = n\lambda D/d$, where D is the screen distance and d the slit separation.
- Two bright fringes coincide when their path differences match: $n_1\lambda_1 = n_2\lambda_2$. The least nonzero coincidence uses the smallest positive integer orders.

Step 1: Find the smallest common bright-fringe orders Let n_1 correspond to $650\,\text{nm}$ and n_2 to $550\,\text{nm}$. Coincidence requires

$$650n_1 = 550n_2 \quad \Rightarrow \quad 13n_1 = 11n_2.$$

Since 11 and 13 are coprime, the smallest positive solution is $n_1 = 11$, $n_2 = 13$. The common path difference is then

$$\Delta = 11(650\,\text{nm}) = 13(550\,\text{nm}) = 7150\,\text{nm}.$$

The common central maximum corresponds to zero orders; the requested distance refers to the first coincidence away from it.

Step 2: Convert the path difference to a screen distance Use $D = 1.2\,\text{m}$, $d = 2 \times 10^{-3}\,\text{m}$, and $\Delta = 7150 \times 10^{-9}\,\text{m}$. The bright-fringe displacement is

$$y = \frac{D\Delta}{d} = \frac{1.2(7150 \times 10^{-9})}{2 \times 10^{-3}} = 4.29 \times 10^{-3}\,\text{m}.$$

To express this in the requested power of ten,

$$4.29 \times 10^{-3}\,\text{m} = 429 \times 10^{-5}\,\text{m}.$$

Both wavelengths give this same position because their orders were chosen to make their optical path differences equal. Any smaller positive pair of integer orders would violate the ratio $11 : 13$.

Step 3: Conclusion The least noncentral coincidence is $\boxed{429 \times 10^{-5}\,\text{m}}$ from the central maximum. The numerical entry is 429.

Quick Tip to Solve: Reduce the wavelength ratio to coprime integers, then reverse that ratio to obtain the two smallest bright-fringe orders. Convert the final distance to the power of ten in the blank.

49. An inductor stores 16 J of magnetic field energy and dissipates 32 W of thermal energy due to its resistance when an a.c. current of 2 A (rms) and frequency 50 Hz flows through it. The ratio of inductive reactance to its resistance is _____. ($\pi = 3.14$) [4 marks]

Solution:

Concept:

- The energy of an inductor at current i is $U = \frac{1}{2}Li^2$, where L is its self-inductance. For an alternating current, the cycle-averaged energy is $\overline{U} = \frac{1}{2}LI_{\text{rms}}^2$.
- Only the resistance dissipates average power: $P = I_{\text{rms}}^2 R$. The inductive reactance is $X_L = \omega L = 2\pi f L$, measured in ohms, like R.

Step 1: Find the resistance from the thermal power The quoted current is already the rms value, so it is the appropriate current for Joule heating. Thus

$$R = \frac{P}{I_{\text{rms}}^2} = \frac{32}{2^2} = 8\,\Omega.$$

The total impedance must not be used in this heating equation: the ideal inductive part stores and returns energy rather than converting it permanently into heat.

Step 2: Find the inductance from the stored energy Using the stated energy with the rms current, equivalently the cycle-averaged magnetic energy, gives

$$16 = \frac{1}{2}L(2)^2, \qquad L = 8\,\text{H}.$$

This follows from averaging $\frac{1}{2}Li^2$, since $\langle i^2 \rangle = I_{\text{rms}}^2$. There is no extra factor of two to insert into this average-energy relation.

Step 3: Compute and divide the reactance At $f = 50\,\text{Hz}$, the angular frequency is $2\pi f = 314\,\text{s}^{-1}$. Therefore

$$X_L = 2(3.14)(50)(8) = 2512\,\Omega,$$

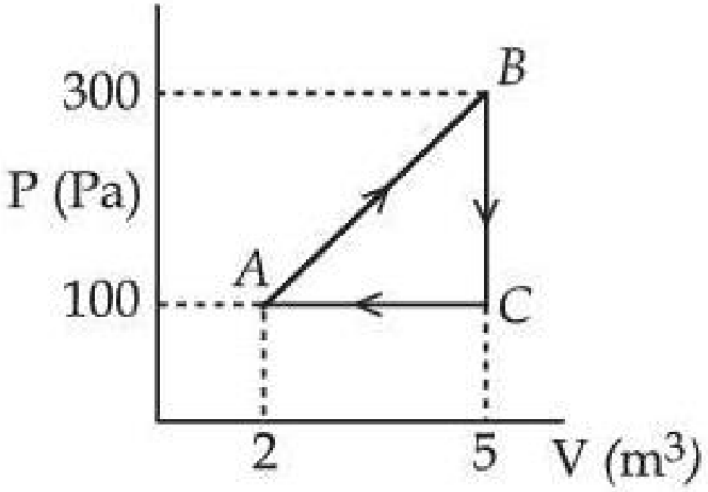
$$\frac{X_L}{R} = \frac{2512}{8} = 314.$$

The units cancel because the numerator and denominator are both resistances in ohms.

Step 4: Conclusion The required ratio of inductive reactance to resistance is $\boxed{314}$, a dimensionless number.

Quick Tip to Solve: Eliminate the rms current directly: $X_L/R = 4\pi f\overline{U}/P$. Keep average energy and rms current consistent.

50. A thermodynamic system is taken through the cyclic process *ABC* as shown in the figure. The total work done by the system during the cycle *ABC* is _____ J. [4 marks]



Solution:

Concept:

- The work done by a system is $W = \int P\,dV$. Expansion contributes positive work and compression contributes negative work.
- For a closed cycle, the net work is the signed area enclosed on the P-V diagram. A clockwise cycle gives positive work done by the system.

Step 1: Read the endpoints and the cycle direction The graph gives $A = (2\,\text{m}^3, 100\,\text{Pa})$, $B = (5\,\text{m}^3, 300\,\text{Pa})$, and $C = (5\,\text{m}^3, 100\,\text{Pa})$. The arrows show $A \to B \to C \to A$. Expansion occurs along the higher-pressure sloping branch and compression along the lower horizontal branch, so the net work must be positive.

Step 2: Evaluate each contribution Along the straight line AB, pressure varies linearly, so its average is the arithmetic mean of its endpoint values:

$$W_{AB} = \frac{100 + 300}{2}(5 - 2) = 600\,\text{J}.$$

The path BC is at constant volume, hence $W_{BC} = 0$. Along CA, the pressure is $100\,\text{Pa}$, while volume decreases:

$$W_{CA} = 100(2 - 5) = -300\,\text{J}.$$

Using the signed volume change is essential; adding the magnitudes would incorrectly count the compression as positive work.

Step 3: Add the work and check the enclosed area The total work done by the system during one complete cycle is

$$W_{ABC} = 600 + 0 - 300 = 300\,\text{J}.$$

Independently, the triangle has base $5 - 2 = 3\,\text{m}^3$ and height $300 - 100 = 200\,\text{Pa}$. Its area is $\frac{1}{2}(3)(200) = 300\,\text{J}$, with a positive sign for the clockwise direction.

Step 4: Conclusion The work done by the system is $\boxed{300\,\text{J}}$; the required numerical entry is 300.

Quick Tip to Solve: For a triangular cycle, use half the pressure difference times the volume difference, then determine the sign from the arrows.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Consider the elements N, P, O, S, Cl and F. The number of valence electrons present in the elements with most and least metallic character from the above list is respectively. [4 marks]

(A) 7 and 5

(B) 6 and 7

(C) 5 and 7

(D) 5 and 6

Correct Answer: (C) 5 and 7

Solution:

Concept:

- Metallic character is the tendency to lose electrons. It generally increases down a group and decreases from left to right across a period.
- For these main-group elements, the valence-electron count follows the outer-shell configuration: group 15 has ns^2np^3, group 16 has ns^2np^4, and group 17 has ns^2np^5.

Step 1: Identify the element with greatest metallic character The period-3 elements in the list are phosphorus, sulphur and chlorine. Among them, phosphorus lies farthest to the left and has the greatest metallic character. Phosphorus is also below nitrogen in group 15, so it is more metallic than nitrogen. The remaining period-2 elements, oxygen and fluorine, lie farther towards the upper right of the periodic table and are less metallic. Thus the most metallic member of the stated list is phosphorus, even though phosphorus itself is classified as a non-metal.

Step 2: Identify the least metallic element and count electrons Fluorine is the topmost listed halogen and lies farthest towards the upper right; it has the least tendency to lose an electron and therefore the least metallic character. The relevant outer configurations are

$$\mathrm{P} : 3s^2 3p^3 \quad \Rightarrow \quad 5 \text{ valence electrons},$$

$$\mathrm{F} : 2s^2 2p^5 \quad \Rightarrow \quad 7 \text{ valence electrons}.$$

The question asks for most metallic first and least metallic second, so the ordered pair is $(5, 7)$.

Step 3: Compare the ordered pair with the alternatives Option (A), $(7, 5)$, reverses the required order. Option (B), $(6, 7)$, incorrectly assigns a group-16 valence count to the most metallic element. Option (D), $(5, 6)$, incorrectly assigns a group-16 count to fluorine. Only option (C) gives both counts in the requested order.

Step 4: Conclusion The elements are phosphorus and fluorine, with 5 and 7 valence electrons respectively. Hence, the correct option is (C).

Quick Tip to Solve: Identify the two elements from their positions first; only then use their group numbers to count the valence electrons.

52. The plot of $\log_{10} K$ vs $\frac{1}{T}$ gives a straight line. The intercept and slope respectively are (where K is equilibrium constant). [4 marks]

(A) $-\frac{\Delta H^\circ}{2.303R}, \frac{\Delta S^\circ}{2.303R}$
(B) $-\frac{\Delta S^\circ R}{2.303}, \frac{\Delta H^\circ R}{2.303}$
(C) $\frac{2.303R}{\Delta H^\circ}, \frac{2.303R}{\Delta S^\circ}$
(D) $\frac{\Delta S^\circ}{2.303R}, -\frac{\Delta H^\circ}{2.303R}$

Correct Answer: (D) $\frac{\Delta S^\circ}{2.303R}, -\frac{\Delta H^\circ}{2.303R}$

Solution:

Concept:

- The standard Gibbs energy change is $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$, and its relation to the equilibrium constant is $\Delta G^\circ = -RT \ln K$.
- The conversion between logarithm bases is $\ln K = 2.303 \log_{10} K$. In a graph $y = mx + c$, the intercept is c and the slope is the coefficient m of the horizontal variable.

Step 1: Express the equilibrium constant in terms of temperature Equate the two expressions for the standard Gibbs energy change:

$$-RT \ln K = \Delta H^\circ - T\Delta S^\circ.$$

Dividing every term by $-RT$ gives

$$\ln K = -\frac{\Delta H^\circ}{RT} + \frac{\Delta S^\circ}{R}.$$

The negative sign multiplies the enthalpy term. The entropy term becomes positive because it already carried a negative sign before division.

Step 2: Convert to the stated axes and identify the line constants Convert the natural logarithm to base ten:

$$\log_{10} K = -\frac{\Delta H^\circ}{2.303R}\left(\frac{1}{T}\right) + \frac{\Delta S^\circ}{2.303R}.$$

For the stated straight-line plot, put $y = \log_{10} K$ and $x = 1/T$. Thus the intercept is $\Delta S^\circ/(2.303R)$, while the slope is $-\Delta H^\circ/(2.303R)$. The straight-line form treats these thermodynamic quantities as constant over the temperature interval concerned.

Step 3: Check the printed alternatives Option (A) exchanges the enthalpy and entropy roles. Option (B) multiplies by R instead of dividing and also gives the wrong signs. Option (C) contains reciprocals of thermodynamic quantities and does not match the derived line. The fourth printed pair, option (D), is precisely the intercept followed by the slope.

Step 4: Conclusion The intercept and slope are $\boxed{\frac{\Delta S^\circ}{2.303R}, -\frac{\Delta H^\circ}{2.303R}}$. Hence, the correct option is (D). This derived pair is the fourth option in the printed order; an answer-key label attached to the earlier option order does not change the thermodynamic result.

Quick Tip to Solve: Write the equation in $y = mx + c$ form before reading off slope and intercept; the horizontal variable here is $1/T$, not T.

53. The reactions which produce alcohol as the product are :

A. $CH_4 + O_2 \xrightarrow{Mo_2O_3,\ \Delta}$

B. $2CH_3CH_3 + 3O_2 \xrightarrow{(CH_3COO)_2Mn,\ \Delta}$

C. $(CH_3)_3CH \xrightarrow{KMnO_4}$

D. $2CH_4 + O_2 \xrightarrow{Cu/523\ K/100\ atm.}$

E. $CH_3 - CH = CH - CH_3 \xrightarrow{KMnO_4/H^+}$

Choose the correct answer from the options given below : [4 marks]

(A) A and D Only

(B) B, D and E Only

(C) C and D Only

(D) A, C and E Only

Correct Answer: (C) C and D Only

Solution:

Concept:

- The product of hydrocarbon oxidation depends on the oxidant and conditions. Controlled oxidation may stop at an alcohol, aldehyde or carboxylic acid rather than complete combustion.
- An alcohol contains a hydroxyl group on a saturated carbon. The hydroxyl in a carboxyl group does not make a carboxylic acid an alcohol.

Step 1: Examine reactions A and B In reaction A, controlled oxidation of methane with the stated molybdenum oxide catalyst produces methanal:

$$CH_4 + O_2 \longrightarrow HCHO + H_2O.$$

Methanal is an aldehyde, not an alcohol. In reaction B, the manganese acetate-catalysed oxidation of ethane produces ethanoic acid:

$$2\,CH_3CH_3 + 3\,O_2 \longrightarrow 2\,CH_3COOH + 2\,H_2O.$$

Although ethanoic acid contains an $O-H$ bond, it belongs to a carboxyl group and is not an alcohol. Therefore neither A nor B qualifies.

Step 2: Examine reactions C and D In reaction C, oxidation of the tertiary hydrogen of 2-methylpropane by permanganate gives the corresponding tertiary alcohol, 2-methylpropan-2-ol:

$$(CH_3)_3CH + [O] \longrightarrow (CH_3)_3COH.$$

In reaction D, methane undergoes controlled oxidation with copper at $523\,K$ and $100\,atm$ to give methanol:

$$2\,CH_4 + O_2 \longrightarrow 2\,CH_3OH.$$

Both C and D therefore give alcohols.

Step 3: Examine E and eliminate the other combinations Acidified permanganate in E oxidatively

cleaves but-2-ene, yielding ethanoic acid from both alkene carbons. These are not the cold, dilute alkaline conditions that produce a vicinal diol. Thus E is excluded. Option (A) incorrectly includes A; option (B) includes B and E; option (D) includes A and E and omits D. Only the combination C and D contains exactly the alcohol-producing reactions.

Step 4: Conclusion Only reactions C and D produce alcohols under the stated conditions. Hence, the correct option is (C).

Quick Tip to Solve: For alkane oxidation, read the catalyst and conditions; for an alkene, distinguish acidic oxidative cleavage from mild alkaline dihydroxylation.

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54. A student has been given 0.314 g of an organic compound and asked to estimate Sulphur. During the experiment, the student has obtained 0.4813 g of barium sulphate. The percentage of sulphur present in the compound is _____. (Given Molar mass in g mol^{-1} S: 32, $BaSO_4$: 233) [4 marks]

(A) 42.10%

(B) 63.15%

(C) 48.24%

(D) 21.05%

Correct Answer: (D) 21.05%

Solution:

Concept:

- In sulphur estimation, the sulphur in the sample is converted into sulphate and weighed as $\mathrm{BaSO_4}$. Each mole of $\mathrm{BaSO_4}$ contains one mole of sulphur atoms.
- The sulphur mass fraction of barium sulphate is $M(\mathrm{S})/M(\mathrm{BaSO_4}) = 32/233$. The percentage in the original compound is $100 m_{\mathrm{S}}/m_{\mathrm{sample}}$.

Step 1: Relate the precipitate mass to the sulphur mass The weighed precipitate contains barium and oxygen in addition to sulphur, so its entire mass cannot be counted as sulphur. One mole of precipitate, of mass $233\,\mathrm{g}$, contains $32\,\mathrm{g}$ of sulphur. Therefore

$$m_{\mathrm{S}} = 0.4813 \times \frac{32}{233}\,\mathrm{g}.$$

This one-to-one molar relation accounts for all the sulphur represented by the precipitate.

Step 2: Calculate the mass percentage Use the mass of the original organic sample, $0.314\,\mathrm{g}$, as the denominator:

$$\%\mathrm{S} = \frac{0.4813 \times 32}{233 \times 0.314} \times 100.$$

The intermediate numerator is $0.4813 \times 32 = 15.4016$, and the denominator is $233 \times 0.314 = 73.162$. Hence

$$\%\mathrm{S} = \frac{15.4016}{73.162} \times 100 \approx 21.0514\%.$$

Rounding to two decimal places gives 21.05%. The sulphur mass itself is about $0.06610\,\mathrm{g}$, which is smaller than the sample mass, as required.

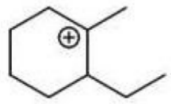
Step 3: Check the other percentages The calculated mass fraction is about 0.2105, not 0.4210, 0.6315 or 0.4824. In particular, the first two incorrect percentages are approximately two and three times the correct value, but each formula unit of $BaSO_4$ contains only one sulphur atom. No multiplier of two or three is justified. Only the fourth option matches the gravimetric calculation.

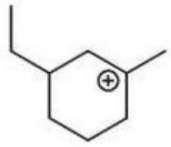
Step 4: Conclusion The percentage of sulphur in the compound is $\boxed{21.05\%}$. Hence, the correct option is (D).

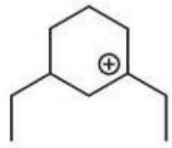
Quick Tip to Solve: Convert precipitate mass to elemental mass with the molar-mass fraction before dividing by the original sample mass.

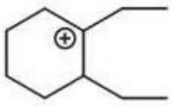
55. The cyclic cations having the same number of hyperconjugation are :

Choose the correct answer from the options given below : [4 marks]

A. 

B. 

C. 

D. 

(A) A and B Only

(B) A, C and D only

(C) B and C Only

(D) A and C Only

Correct Answer: (D) A and C Only

Solution:

Concept:

- For an alkyl carbocation, hyperconjugation involves overlap of an adjacent carbon–hydrogen sigma bond with the vacant p orbital on the positively charged carbon.
- The conventional number of hyperconjugative structures equals the number of hydrogens on carbon atoms directly bonded to the cationic carbon. Hydrogens farther away are not counted.

Step 1: Count the adjacent hydrogens in A and B In A, the positively charged ring carbon is bonded to a methyl group and to two ring carbons. The methyl group contributes three hydrogens; the

unsubstituted neighbouring ring carbon contributes two; the ethyl-substituted neighbouring ring carbon contributes one. Thus

$$n_A = 3 + 2 + 1 = 6.$$

In B, the ethyl substituent is farther from the cationic centre. Both directly adjacent ring carbons are methylene groups, so

$$n_B = 3 + 2 + 2 = 7.$$

Step 2: Count the adjacent hydrogens in C and D In C, the side chain directly attached to the cationic carbon is ethyl, whose first carbon is a methylene group. Both neighbouring ring carbons are also methylene groups. Hence

$$n_C = 2 + 2 + 2 = 6.$$

In D, one neighbouring ring carbon bears an ethyl substituent and has only one hydrogen, while the other neighbouring ring carbon and the attached ethyl carbon each have two:

$$n_D = 2 + 2 + 1 = 5.$$

Step 3: Compare all four counts The counts for A, B, C and D are respectively 6, 7, 6 and 5. Only A and C agree. The pair A and B and the pair B and C each compare 6 with 7; including D would incorrectly equate 5 with 6.

Step 4: Conclusion Only A and C have the same number of hyperconjugative structures, namely six each. Hence, the correct option is (D).

Quick Tip to Solve: Mark the positively charged carbon first, circle only its directly bonded carbon neighbours, and count their hydrogens; a remote alkyl substituent contributes no direct hyperconjugation.

56. The correct order of acidic strength of the major products formed in the given reactions, is :

A. $\mathrm{PhNH_2} \xrightarrow[\text{(3) } \mathrm{H_3O^+}/\Delta]{\text{(1) } \mathrm{NaNO_2{+}HCl}(<5^\circ\mathrm{C})\ \text{(2) } \mathrm{CuCN}} [\mathrm{A}]$

B. $\mathrm{CH_3CH_2CHO} \xrightarrow[\Delta]{[\mathrm{Ag(NH_3)_2}]^+,\mathrm{OH^-}} [\mathrm{B}]$

C. $\mathrm{CH_4 + O_2} \xrightarrow[\text{(ii) } \mathrm{Na_2Cr_2O_7/H^+}]{\text{(i) } \mathrm{Mo_2O_3}} [\mathrm{C}]$

D. $\mathrm{PhCH_2MgBr + CO_2} \xrightarrow[\mathrm{H_3O^+}]{\text{Dry ether}} [\mathrm{D}]$

Choose the correct answer from the options given below : [4 marks]

(A) A > D > B > C

(B) A > D > C > B

(C) C > A > D > B

(D) C > B > A > D

Correct Answer: (C) C > A > D > B

Solution:

Concept:

- Identify the functional-group transformations before comparing acidity. For carboxylic acids, greater stabilisation of the carboxylate conjugate base generally means greater acidic strength.
- An electron-donating alkyl group weakens a carboxylic acid through its positive inductive effect; the effect of a phenyl group depends on its separation from the carboxyl group.

Step 1: Identify products A and B Diazotisation converts aniline into a diazonium salt; replacement by cyanide gives benzonitrile. Acidic hydrolysis of the nitrile gives benzoic acid:

$$\mathrm{C_6H_5NH_2} \longrightarrow \mathrm{C_6H_5N_2^+} \longrightarrow \mathrm{C_6H_5CN} \longrightarrow \mathrm{C_6H_5COOH}.$$

Therefore A is benzoic acid. Tollens reagent oxidises propanal to propanoate in alkaline medium; the corresponding acid used in the acidity comparison is propanoic acid, B, $\mathrm{CH_3CH_2COOH}$.

Step 2: Identify products C and D Controlled oxidation of methane with molybdenum oxide gives methanal, which on oxidation with acidified dichromate gives methanoic acid, C, HCOOH. Carboxylation of benzylmagnesium bromide followed by acidic hydrolysis introduces one additional carbon:

$$\mathrm{C_6H_5CH_2MgBr} + \mathrm{CO_2} \longrightarrow \mathrm{C_6H_5CH_2COOMgBr} \longrightarrow \mathrm{C_6H_5CH_2COOH}.$$

Thus D is phenylacetic acid, not benzoic acid.

Step 3: Rank the acids and eliminate alternatives The approximate aqueous pK_a values provide a direct check: methanoic acid 3.75, benzoic acid 4.20, phenylacetic acid 4.31, and propanoic acid 4.87. Smaller pK_a means stronger acid. The ethyl group weakens propanoic acid, whereas methanoic acid has no electron-donating alkyl group. The intervening methylene group reduces the phenyl-group effect in D relative to A. Therefore

$$C > A > D > B.$$

The first two options incorrectly place A above C, and the fourth incorrectly places B above A and D.

Step 4: Conclusion The correct acidic-strength order is methanoic acid $>$ benzoic acid $>$ phenylacetic acid $>$ propanoic acid, i.e. $C > A > D > B$. Hence, the correct option is (C).

Quick Tip to Solve: In a Grignard carboxylation, retain the entire original carbon skeleton and add the carbon of carbon dioxide; benzylmagnesium bromide therefore gives phenylacetic acid.

57. Total number of alkali insoluble solid sulphonamides obtained by reaction of given amines with Hinsberg's reagent is _____.

Aniline, N-Methylaniline, Methanamine, N,N-Dimethylmethanamine, N-Methyl methanamine, Phenylmethanamine, N-propylaniline, N-phenylaniline, N,N-Dimethylaniline, Allyl amine, Isopropyl amine

[4 marks]

(A) 5

(B) 4

(C) 8

(D) 2

Correct Answer: (B) 4

Solution:

Concept:

- Hinsberg reagent is benzenesulphonyl chloride, $C_6H_5SO_2Cl$. Classify an amine by the number of carbon groups directly attached to nitrogen, not by the size of those groups.
- A primary amine gives a sulphonamide with an acidic N–H bond and hence forms an alkali-soluble salt. A secondary amine gives an N,N-disubstituted sulphonamide with no N–H bond; this product is insoluble in alkali. Tertiary amines do not give sulphonamides in this test.

Step 1: Separate primary and tertiary amines Aniline, methanamine, phenylmethanamine, allyl amine and isopropyl amine all contain an $-NH_2$ group. They are primary amines and their sulphonamide products retain one N–H bond. These five products are not counted as alkali-insoluble sulphonamides. N,N-Dimethylmethanamine is trimethylamine, and N,N-dimethylaniline also has three carbon groups attached to nitrogen. These two tertiary amines do not furnish the required sulphonamide products.

Step 2: Identify every secondary amine The secondary amines are N-methylaniline, N-methylmethanamine, N-propylaniline and N-phenylaniline. Each has two carbon substituents and one hydrogen on nitrogen. Their reaction has the general form

$$R_2NH + C_6H_5SO_2Cl \longrightarrow C_6H_5SO_2NR_2 + HCl.$$

The remaining nitrogen hydrogen is replaced by the sulphonyl group, leaving no acidic N–H bond in the product.

Step 3: Count the qualifying products All four secondary amines produce the alkali-insoluble sulphonamide class. The five primary amines instead explain the tempting count 5, while the two tertiary amines explain the tempting count 2 but give no such sulphonamide. The count 8 does not correspond to the required reaction and solubility criterion. The complete classification is five primary, four secondary and two tertiary amines, accounting for all eleven names.

Step 4: Conclusion Four alkali-insoluble sulphonamides are obtained, one from each of the four listed secondary amines. The value is four; with the options in their printed order, this is (B), irrespective of a differently ordered key label. Hence, the correct option is (B).

Quick Tip to Solve: For the Hinsberg test, first count the N–H bonds of the starting amine: two gives an alkali-soluble product, one gives an insoluble sulphonamide, and zero gives no sulphonamide.

58. Consider the following reactions

$$Na_2B_4O_7 \xrightarrow{\Delta} 2X + Y$$

$$CuSO_4 + Y \xrightarrow{\text{Non-Luminous flame}} Z + SO_3$$

$$2Z + 2X + \textbf{Carbon} \xrightarrow{\text{Luminous flame}} 2Q + Na_2B_4O_7 + CO$$

The oxidation states of Cu in Z and Q, respectively are : [4 marks]

(A) +2 and +2

(B) +2 and +1

(C) +1 and +1

(D) +1 and +2

Correct Answer: (B) +2 and +1

Solution:

Concept:

- On heating, anhydrous borax yields sodium metaborate and boric oxide. Boric oxide combines with metal oxides to produce characteristic metal metaborates in a borax bead.
- The non-luminous flame is oxidising, while the luminous flame provides reducing conditions. Carbon can remove oxygen as carbon monoxide and lower the oxidation state of copper.

Step 1: Identify X, Y and Z Decomposition of borax gives

$$Na_2B_4O_7 \longrightarrow 2\,NaBO_2 + B_2O_3.$$

Hence $X = NaBO_2$ and $Y = B_2O_3$. Copper sulphate reacts with boric oxide in the oxidising flame to form copper(II) metaborate:

$$CuSO_4 + B_2O_3 \longrightarrow Cu(BO_2)_2 + SO_3.$$

Thus $Z = Cu(BO_2)_2$. Each metaborate group has charge -1; two groups require copper in oxidation state $+2$.

Step 2: Use the reducing-flame equation to identify Q Substitute the formulas of X and Z into the final reaction:

$$2\,Cu(BO_2)_2 + 2\,NaBO_2 + C \longrightarrow 2\,CuBO_2 + Na_2B_4O_7 + CO.$$

This equation is balanced for copper, sodium, boron, oxygen and carbon. It identifies Q as copper(I) metaborate, $CuBO_2$. A single metaborate group balances one Cu^+, so copper has oxidation state $+1$ in Q.

Step 3: Check the direction of the redox change The two copper atoms each gain one electron as their oxidation state falls from $+2$ to $+1$. Carbon changes from zero in elemental carbon to $+2$ in carbon monoxide and supplies those two electrons. This confirms reduction in the luminous flame.

The first alternative incorrectly leaves copper unchanged, and the third and fourth incorrectly assign $+1$ to copper in Z. The fourth also implies oxidation in the reducing stage.

Step 4: Conclusion The oxidation states of copper in Z and Q are $+2$ and $+1$, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Use the charge of BO_2^- to assign copper oxidation states, then check that the luminous-flame product has the lower oxidation state.

---

59. The wavelength of photon 'A' is 400 nm. The frequency of photon 'B' is 10^{16} s^{-1}. The wave number of photon 'C' is 10^4 cm^{-1}. The correct order of energy of these photons is : [4 marks]

(A) A > B > C

(B) C > B > A

(C) B > A > C

(D) A > C > B

Correct Answer: (C) B > A > C

Solution:

Concept:

- Photon energy obeys $E = h\nu = hc/\lambda = hc\tilde{\nu}$, where $\tilde{\nu} = 1/\lambda$ is the spectroscopic wave number and h and c are positive constants.
- Convert all three given quantities to the same physical quantity before ordering them. A wave number quoted in cm^{-1} must be converted consistently if the speed of light is used in metres per second.

Step 1: Convert the wavelength of A to frequency The wavelength is $\lambda_A = 400\,\mathrm{nm} = 4.00 \times 10^{-7}\,\mathrm{m}$. Using $c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$,

$$\nu_A = \frac{c}{\lambda_A} = \frac{3.00 \times 10^8}{4.00 \times 10^{-7}} = 7.50 \times 10^{14}\,\mathrm{s^{-1}}.$$

Its energy is proportional to this frequency, not directly to the wavelength.

Step 2: Convert the wave number of C to frequency The supplied wave number is $10^4\,\mathrm{cm^{-1}} = 10^6\,\mathrm{m^{-1}}$, since one reciprocal centimetre equals one hundred reciprocal metres. Thus

$$\nu_C = c\tilde{\nu}_C = (3.00 \times 10^8)(10^6) = 3.00 \times 10^{14}\,\mathrm{s^{-1}}.$$

Equivalently, its wavelength is $10^{-4}\,\mathrm{cm} = 1000\,\mathrm{nm}$, longer than that of A, so C must have lower energy than A.

Step 3: Compare with the supplied frequency of B Photon B has $\nu_B = 10^{16}\,\mathrm{s^{-1}}$. Therefore

$$10^{16} > 7.50 \times 10^{14} > 3.00 \times 10^{14}.$$

Multiplying this order by Planck's positive constant preserves it. The first and fourth options wrongly

put A above B, and the second wrongly makes C the most energetic photon. The wavelength check independently confirms the ordering of A and C.

Step 4: Conclusion The photon energies satisfy $E_B > E_A > E_C$. Hence, the correct option is (C).

Quick Tip to Solve: For mixed wavelength, frequency and wave-number data, convert everything to frequency; remember that $1\,\mathrm{cm}^{-1} = 100\,\mathrm{m}^{-1}$.

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60. A student performed analysis of aliphatic organic compound 'X' which on analysis gave C = 61.01%, H = 15.25%, N = 23.74%.

This compound, on treatment with HNO_2/H_2O produced another compound 'Y' which did not contain any nitrogen atom. However, the compound 'Y' upon controlled oxidation produced another compound 'Z' that responded to iodoform test.

The structure of 'X' is : [4 marks]

(A) $CH_3CH_2CH_2NH_2$

(B) $C_6H_5CH(NH_2)CH_3$

(C) $CH_3CH_2CH(NH_2)CH_3$

(D) $(CH_3)_2CHNH_2$

Correct Answer: (D) $(CH_3)_2CHNH_2$

Solution:

Concept:

- Elemental percentages give mole ratios when divided by atomic masses. Nitrous acid converts an aliphatic primary amine into an alcohol with evolution of nitrogen.
- A positive iodoform test after controlled oxidation indicates a methyl ketone, CH_3COR, or ethanal. A secondary alcohol of the form $CH_3CH(OH)R$ oxidises to the corresponding methyl ketone.

Step 1: Determine the empirical formula Take 100 g of X and divide the masses by the atomic masses C = 12, H = 1 and N = 14:

$$n_C = \frac{61.01}{12} = 5.084, \quad n_H = 15.25, \quad n_N = \frac{23.74}{14} = 1.696.$$

Dividing by the smallest number gives approximately

$$C : H : N = 3 : 9 : 1.$$

Thus the empirical formula is C_3H_9N, with formula mass 59. Its calculated percentages, $100(36/59)\%$, $100(9/59)\%$ and $100(14/59)\%$, agree with the stated data.

Step 2: Use the reactions to select the carbon skeleton Loss of nitrogen with nitrous acid is consistent with a primary aliphatic amine. For propan-2-amine, the sequence is

$$(CH_3)_2CHNH_2 \xrightarrow{HNO_2/H_2O} (CH_3)_2CHOH \xrightarrow{\text{controlled oxidation}} CH_3COCH_3.$$

The alcohol Y is propan-2-ol and the oxidation product Z is propanone. Propanone contains the $CH_3C(=O)-$ group and therefore gives the yellow iodoform precipitate.

Step 3: Eliminate each remaining structure Propan-1-amine has the correct elemental formula, but the corresponding alcohol and controlled oxidation product are propan-1-ol and propanal; propanal lacks a methyl-ketone group and does not give the required iodoform test. The phenyl-substituted amine is not aliphatic and has too many carbon atoms for the analysis. Butan-2-amine can yield a methyl ketone on this reaction sequence, but its formula $C_4H_{11}N$ does not match the percentages. Only propan-2-amine satisfies both the analysis and the functional-group tests.

Step 4: Conclusion X is propan-2-amine (isopropylamine), $(CH_3)_2CHNH_2$. It yields propan-2-ol and then iodoform-positive propanone. Hence, the correct option is (D).

Quick Tip to Solve: Apply independent filters in sequence: elemental analysis fixes the formula, nitrous acid identifies the amine class, and the iodoform test distinguishes the possible carbon skeletons.

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61. Observe the following equilibrium in a 1 L flask.

A(g) $\rightleftharpoons$ B(g)

At T(K), the equilibrium concentrations of A and B are 0.5 M and 0.375 M respectively. 0.1 moles of A is added into the flask and heated to T(K) to establish the equilibrium again. The new equilibrium concentrations (in M) of A and B are respectively [4 marks]

(A) 0.557, 0.418.

(B) 0.742, 0.557.

(C) 0.367, 0.275.

(D) 0.53, 0.4.

Correct Answer: (A) 0.557, 0.418.

Solution:

Concept:

- For the equilibrium $\mathrm{A(g)} \rightleftharpoons \mathrm{B(g)}$, the equilibrium constant is $K_c = [B]/[A]$. It depends only on temperature.
- Adding A disturbs the equilibrium. At the same final temperature, the original value of K_c is restored, while the total concentration $[A] + [B]$ increases by the amount added per litre.

Step 1: Calculate the original equilibrium constant Use the original equilibrium concentrations, not the concentrations immediately after addition:

$$K_c = \frac{0.375}{0.500} = 0.750.$$

The system returns to the same temperature T, so this ratio remains unchanged in the final equilibrium.

Step 2: Write the concentration changes In a 1 L flask, adding 0.1 mol raises the concentration of A by $0.100\,\mathrm{M}$. Immediately after addition, $[A] = 0.600\,\mathrm{M}$ and $[B] = 0.375\,\mathrm{M}$. The reaction quotient is

$0.375/0.600 = 0.625 < K_c$, so the reaction moves towards B. If x M of A is converted, then

$$[A] = 0.600 - x, \qquad [B] = 0.375 + x.$$

Step 3: Solve and check the options Impose the equilibrium ratio:

$$\frac{0.375 + x}{0.600 - x} = 0.750 \quad \Rightarrow \quad 1.750x = 0.075.$$

Thus $x = 0.042857$ M, giving $[A] = 0.557143$ M and $[B] = 0.417857$ M. Their sum is 0.975 M, as required. The other pairs have sums 1.299, 0.642 and 0.930 M, so they violate the conserved total concentration even where their ratios are close to 0.75.

Step 4: Conclusion The new concentrations of A and B are 0.557 M and 0.418 M, respectively. Hence, the correct option is (A).

Quick Tip to Solve: For a one-to-one isomerisation at fixed temperature, combine the unchanged concentration ratio with the new total concentration to avoid a lengthy equilibrium table.

62. Given below are two statements :

Statement I : The increasing order of boiling point of hydrogen halides is HCl < HBr < HI < HF.

Statement II : The increasing order of melting point of hydrogen halides is HCl < HBr < HF < HI.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Correct Answer: (A) Both **Statement I** and **Statement II** are true

Solution:

Concept:

- Boiling points reflect intermolecular attractions in the liquid. For HCl, HBr and HI, dispersion forces strengthen with increasing polarizability.
- HF forms strong intermolecular hydrogen bonds. Melting points also depend on crystal packing, so their order need not duplicate the boiling-point order.

Step 1: Check the boiling-point statement From HCl to HI, the larger, more polarizable electron cloud strengthens dispersion attractions and raises the boiling point. HF is anomalous because its molecules associate through hydrogen bonding. Representative boiling points are approximately

$$\text{HCl}: -85\,^\circ\text{C}, \quad \text{HBr}: -67\,^\circ\text{C},$$
$$\text{HI}: -35\,^\circ\text{C}, \quad \text{HF}: 20\,^\circ\text{C}.$$

Therefore $\mathrm{HCl} < \mathrm{HBr} < \mathrm{HI} < \mathrm{HF}$ is the increasing boiling-point order, and Statement I is true.

Step 2: Check the melting-point statement independently The approximate melting points are $-114\,^\circ\mathrm{C}$ for HCl, $-87\,^\circ\mathrm{C}$ for HBr, $-83\,^\circ\mathrm{C}$ for HF, and $-51\,^\circ\mathrm{C}$ for HI. In increasing numerical order these give

$$\mathrm{HCl} < \mathrm{HBr} < \mathrm{HF} < \mathrm{HI}.$$

Thus Statement II is also true. Hydrogen bonding does not imply that HF must have the highest melting point: melting compares a solid and a liquid and depends on their relative structural stability.

Step 3: Choose the matching truth combination Both statements agree with the corresponding physical-property orders. Option B incorrectly rejects both statements. Option C incorrectly rejects the melting-point order, and option D incorrectly rejects the boiling-point order. Only the option accepting both statements remains.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: Keep separate sequences for hydrogen-halide boiling and melting points: HF comes last in boiling point, whereas HI comes last in melting point.

63. Match List - I with List - II according to shape.

List – I	List – II
A. XeO_3	I. BrF_5
B. XeF_2	II. NH_3
C. XeO_2F_2	III. I_3^-
D. $XeOF_4$	IV. SF_4

Choose the correct answer from the options given below: [4 marks]

(A) A-II, B-I, C-III, D-IV

(B) A-II, B-III, C-I, D-IV

(C) A-III, B-II, C-IV, D-I

(D) A-II, B-III, C-IV, D-I

Correct Answer: (D) A-II, B-III, C-IV, D-I

Solution:

Concept:

- In VSEPR theory, each central-atom bond direction is one electron domain; a multiple bond counts as one domain. Molecular shape describes only the atomic positions, excluding lone pairs.
- Use AX_mE_n, where m is the number of bonded atoms and n the number of central-atom lone pairs, to compare shapes across different elements.

Step 1: Identify the first two xenon shapes In XeO_3, xenon has three Xe–O bonding directions and one lone pair: AX_3E. Its molecular shape is trigonal pyramidal, the same as NH_3. Hence A matches II.

In XeF_2, xenon has two bond pairs and three lone pairs: AX_2E_3. The three lone pairs occupy the

equatorial sites of a trigonal-bipyramidal electron-domain arrangement, leaving the two F atoms axial. Its molecular shape is linear, as in I_3^-. Hence B matches III.

Step 2: Identify the two mixed oxyfluoride shapes In XeO_2F_2, xenon has four bonding directions and one lone pair: AX_4E. Its trigonal-bipyramidal electron-domain arrangement gives a seesaw molecular shape, like SF_4; hence C matches IV.

In $XeOF_4$, there are five bonding directions and one lone pair: AX_5E. An octahedral electron-domain arrangement with one lone pair gives a square-pyramidal shape, like BrF_5; hence D matches I.

Step 3: Compare the complete matching The four matches are

$$\text{A}:\text{II}, \quad \text{B}:\text{III}, \quad \text{C}:\text{IV}, \quad \text{D}:\text{I}.$$

The first option wrongly associates linear XeF_2 with square-pyramidal BrF_5. The second interchanges the shapes assigned to C and D. The third wrongly associates trigonal-pyramidal XeO_3 with linear I_3^-. The fourth is the only option containing all four correct matches.

Step 4: Conclusion The matching is A-II, B-III, C-IV, D-I. In the printed option order, this is the fourth choice. Hence, the correct option is (D).

Quick Tip to Solve: Count bond directions and lone pairs around xenon before comparing shapes; an Xe=O double bond still contributes only one VSEPR direction.

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64. For the given reaction;

$CaCO_3$ + 2HCl $\rightarrow$ $CaCl_2$ + H_2O + CO_2

If 90 g $CaCO_3$ is added to 300 mL of HCl which contains 38.55% HCl by mass and has density 1.13 g mL^{-1}, then which of the following option is correct?

Given molar mass of H, Cl, Ca and O are 1, 35.5, 40 and 16 g mol^{-1} respectively. [4 marks]

(A) 60.32 g of HCl remains unreacted

(B) 64.97 g of HCl remains unreacted

(C) 32.85 g of $CaCO_3$ remains unreacted

(D) 97.30 g of HCl reacted

Correct Answer: (B) 64.97 g of HCl remains unreacted

Solution:

Concept:

- Convert the volume of a solution into its total mass using $m = \rho V$, then use the mass percentage to find the mass of solute.
- The balanced reaction $CaCO_3 + 2\,HCl \longrightarrow CaCl_2 + H_2O + CO_2$ consumes two moles of HCl for every mole of calcium carbonate.

Step 1: Calculate the initial amount of hydrochloric acid The mass of 300 mL solution is

$$m_{\text{solution}} = 300 \times 1.13 = 339\,\text{g}.$$

Its HCl mass is therefore

$$m_{\mathrm{HCl},0} = 339 \times \frac{38.55}{100} = 130.6845\,\mathrm{g}.$$

Using $M(\mathrm{HCl}) = 36.5\,\mathrm{g\,mol^{-1}}$, this is approximately 3.58 mol HCl. The percentage applies to the solution mass, not directly to its volume.

Step 2: Identify the limiting reactant The molar mass of $CaCO_3$ is $100\,\mathrm{g\,mol^{-1}}$, so

$$n_{\mathrm{CaCO_3}} = \frac{90}{100} = 0.900\,\mathrm{mol}.$$

It requires $2(0.900) = 1.800$ mol HCl, less than the available amount. Calcium carbonate is therefore completely consumed and hydrochloric acid is in excess. The mass of HCl consumed is

$$m_{\mathrm{HCl,used}} = 1.800 \times 36.5 = 65.700\,\mathrm{g}.$$

Step 3: Find the excess and compare the options The mass of acid left is

$$m_{\mathrm{HCl,left}} = 130.6845 - 65.700 = 64.9845\,\mathrm{g}.$$

This is approximately $64.98\,\mathrm{g}$. Option B, printed as 64.97 g, is the intended choice, with a small numerical discrepancy of 0.0145 g from the supplied data. Option A gives the wrong residual mass; option C cannot hold because all calcium carbonate reacts; option D gives 97.30 g consumed instead of 65.70 g.

Step 4: Conclusion The given data yield $64.9845\,\mathrm{g}$ of unreacted HCl, approximately $64.98\,\mathrm{g}$. The intended listed value is 64.97 g, a small numerical discrepancy in option B. Hence, the correct option is (B).

Quick Tip to Solve: For mass-percent solutions, calculate solution mass first. Then compare available moles with stoichiometric requirements before subtracting any masses.

65. Consider the following statements about manganate and permanganate ions. Identify the correct statements.

A. The geometry of both manganate and permanganate ions is tetrahedral.

B. The oxidation states of Mn in manganate and permanganate are +7 and +6, respectively.

C. Oxidation of Mn(II) salt by peroxodisulphate gives manganate ion as the final product.

D. Manganate ion is paramagnetic and permanganate ion is diamagnetic.

E. Acidified permanganate ion reduces oxalate, nitrite and iodide ions.

Choose the correct answer from the options given below : [4 marks]

(A) A, B and C Only

(B) A, D and E Only

(C) A, C and D Only

(D) A and D Only

Correct Answer: (D) A and D Only

Solution:

Concept:

- The manganate and permanganate ions are MnO_4^{2-} and MnO_4^{-}. Both have four oxygen atoms arranged tetrahedrally around manganese.
- The oxidation state determines the metal d-electron count: Mn(VI) is d^1, whereas Mn(VII) is d^0. An unpaired electron produces paramagnetism.

Step 1: Check geometry and oxidation states Both oxoanions have tetrahedral geometry, so statement A is correct. Taking the oxidation number of oxygen as -2, the manganese oxidation states follow from

$$x + 4(-2) = -2 \quad \Rightarrow x = +6 \quad (MnO_4^{2-}),$$
$$x + 4(-2) = -1 \quad \Rightarrow x = +7 \quad (MnO_4^{-}).$$

Statement B interchanges these values and is false.

Step 2: Check the oxidation product and magnetic behaviour Peroxodisulphate is a strong oxidising agent. Oxidation of an Mn(II) salt by peroxodisulphate produces permanganate as the final product, not manganate. Statement C is therefore false.

Neutral manganese has a $3d^5 4s^2$ valence configuration. In the +6 state it retains one d electron; manganate is consequently paramagnetic. In the +7 state there are no d electrons; permanganate is diamagnetic. Statement D is correct.

Step 3: Check the direction of redox action Acidified permanganate is an oxidising agent. It oxidises oxalate to CO_2, nitrite to nitrate and iodide to iodine, while manganese is reduced from +7 to +2. Statement E incorrectly says permanganate reduces these ions; it is false.

Thus only A and D are true. The first option includes false B and C; the second includes false E; and the third includes false C. The final option contains exactly the two true statements.

Step 4: Conclusion The correct statements are A and D only. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish what an oxidising agent does from what happens to it: acidified permanganate oxidises the other species while itself being reduced.

66. The correct increasing order of spin-only magnetic moment values of the complex ions $[MnBr_4]^{2-}$ (A), $[Cu(H_2O)_6]^{2+}$ (B), $[Ni(CN)_4]^{2-}$ (C) and $[Ni(H_2O)_6]^{2+}$ (D) is : [4 marks]

(A) A = B < C < D

(B) A = B < D < C

(C) C = D < B < A

(D) C < B < D < A

Correct Answer: (D) C < B < D < A

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_B$, where n is the number of unpaired electrons and μ_B is the Bohr magneton.
- Find the metal oxidation state and d-electron count first. Then account for the coordination geometry and ligand field before counting unpaired electrons.

Step 1: Count unpaired electrons in A and B For A, $[MnBr_4]^{2-}$, four bromide ligands contribute -4, so manganese is $+2$ and has a d^5 configuration. This tetrahedral weak-field complex has five unpaired electrons:

$$\mu_A = \sqrt{5(5+2)} = \sqrt{35} \approx 5.92\,\mu_B.$$

For B, $[Cu(H_2O)_6]^{2+}$, copper is $+2$ and d^9. One electron remains unpaired in the octahedral complex:

$$\mu_B = \sqrt{1(1+2)} = \sqrt{3} \approx 1.73\,\mu_B.$$

Step 2: Distinguish the two nickel complexes In both C and D, nickel is $+2$ and has a d^8 configuration. In C, $[Ni(CN)_4]^{2-}$, the strong-field cyanide ligands produce a square-planar arrangement with all electrons paired. Thus $n_C = 0$ and $\mu_C = 0$.

In D, $[Ni(H_2O)_6]^{2+}$, octahedral Ni(II) has configuration $t_{2g}^6 e_g^2$; the two e_g electrons occupy different orbitals and are unpaired. Hence

$$\mu_D = \sqrt{2(2+2)} = \sqrt{8} \approx 2.83\,\mu_B.$$

Step 3: Arrange the magnetic moments The unpaired-electron counts for C, B, D and A are respectively 0, 1, 2 and 5. Since $\sqrt{n(n+2)}$ increases with n,

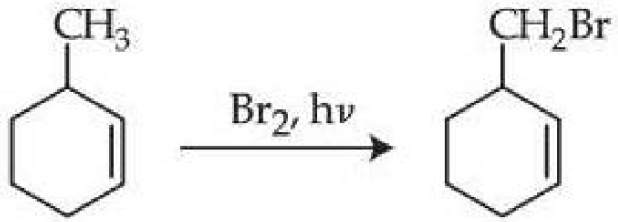
$$\mu_C < \mu_B < \mu_D < \mu_A.$$

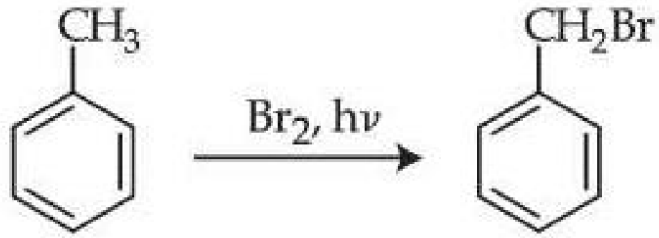
The first two options incorrectly equate A and B despite their five and one unpaired electrons. The third incorrectly equates the diamagnetic cyanide complex C with the paramagnetic aqua complex D. Only the last order is consistent with all four electron counts.

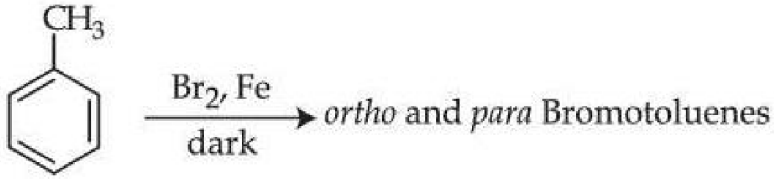
Step 4: Conclusion The increasing order is C < B < D < A, with moments $0, \sqrt{3}, \sqrt{8}, \sqrt{35}$ in Bohr magnetons. Hence, the correct option is (D).

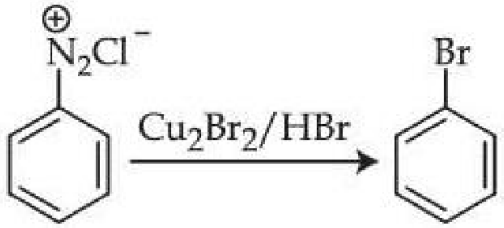
Quick Tip to Solve: The same metal oxidation state does not guarantee the same magnetic moment: square-planar Ni(II) cyanide is diamagnetic, whereas octahedral Ni(II) aqua has two unpaired electrons.

67. Which of the following reaction is NOT correctly represented? [4 marks]

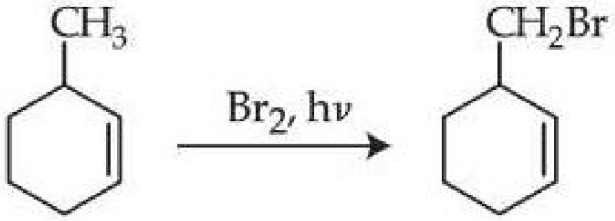
(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- Photochemical bromination proceeds through free radicals. Hydrogen abstraction preferentially generates a resonance-stabilised allylic or benzylic radical.
- Bromine with iron in the dark gives electrophilic aromatic substitution, whereas cuprous bromide replaces an aryl diazonium group by bromine in the Sandmeyer reaction.

Step 1: Inspect the carbon bearing the methyl group in option A The starting ring is a cyclohexene,

not benzene. The ring carbon bearing the methyl group is adjacent to the double bond and has an allylic hydrogen. Removing this hydrogen gives a tertiary allylic radical, stabilised by resonance. In contrast, removing a hydrogen from the attached methyl group gives a primary radical whose radical centre is not directly adjacent to the double bond. The drawn product places bromine on that methyl group, so it does not represent the preferred allylic bromination. This is the incorrect reaction.

Step 2: Check the two reactions of toluene In option B, the methyl group is attached directly to an aromatic ring. Its hydrogen abstraction gives a benzyl radical, which is resonance-stabilised. Thus Br_2 under light converts $C_6H_5CH_3$ to $C_6H_5CH_2Br$, as shown. In option C, iron activates bromine for electrophilic ring substitution in the dark. The methyl group activates the ring and directs substitution to the ortho and para positions. Both reactions are correctly represented.

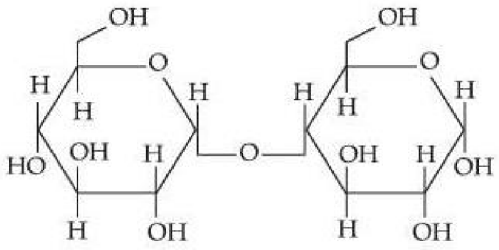
Step 3: Check the diazonium replacement In option D, cuprous bromide in hydrobromic acid replaces the diazonium group by bromine with loss of nitrogen gas. Bromobenzene is therefore the correct product. Since B, C and D each agree with their reagents, they cannot answer the request for the incorrectly represented reaction.

Step 4: Conclusion The methyl-side-chain bromination shown for the cyclohexene in option A is incorrect. Hence, the correct option is (A).

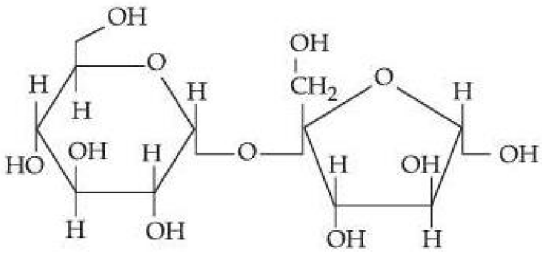
Quick Tip to Solve: Before applying a side-chain bromination rule, distinguish an aromatic ring from an alkene and identify which hydrogen gives an allylic or benzylic radical.

68. Structures of four disaccharides are given below. Among the given disaccharides, the non-reducing sugar is : [4 marks]

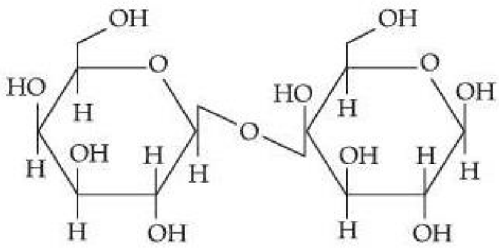
(A)

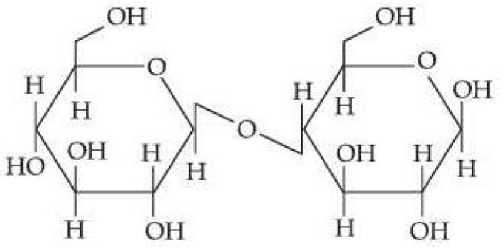


(B)



(C)



(D) 

Correct Answer: No literal option is non-reducing; option B appears intended as sucrose but its printed furanose structure has a free hemiacetal OH.

Solution:

Concept:

- A disaccharide is reducing when at least one anomeric carbon remains a hemiacetal or hemiketal, so its ring can open to a carbonyl form.
- A glycosidic bond involving both anomeric carbons prevents that ring opening. Such a disaccharide is non-reducing even though it contains several other hydroxyl groups.

Step 1: Locate the free hemiacetal centres A carbon bonded to the ring oxygen and also to a hydroxyl group is a hemiacetal centre. Its carbon–oxygen ring bond can cleave as the hydroxyl oxygen forms a carbonyl group; the open-chain form can therefore act as a reducing sugar. A hydroxyl elsewhere on a sugar ring does not by itself give this property. The position and bonding of the hydroxyl group must be checked, rather than simply recognising the overall ring shapes.

Step 2: Inspect the literal drawing in option B The rightmost carbon of the five-membered ring in B is bonded to the ring oxygen, the next ring carbon, H and OH. Thus this drawing contains a free hemiacetal centre. Opening that ring at this centre exposes an aldehyde group. The glycosidic attachment at the other side of the five-membered ring does not remove this free centre. Consequently, the structure exactly as drawn is reducing; it cannot be declared non-reducing merely because its outline resembles sucrose.

Step 3: Check the remaining options In A, C and D, the right-hand six-membered ring also retains a hydroxyl group at its anomeric carbon next to the ring oxygen. Each therefore has a free reducing end. None of these three drawings supplies a non-reducing alternative, and the literal option B has the same decisive free-hemiacetal feature. Hence none of the four drawings, read exactly as printed, satisfies the requested property.

Step 4: Explain the likely intended answer Option B appears intended to depict sucrose. In a correct sucrose structure, the indicated rightmost fructofuranose carbon bears H and CH_2OH, not H and OH. The glycosidic bond joins glucose C1 and fructose C2, an $\alpha(1\rightarrow 2)\beta$ linkage, leaving neither anomeric centre free to open. Correctly drawn sucrose is therefore non-reducing, which explains the intended selection B. This interpretation identifies the diagram error; it does not change the printed option into a different structure.

Step 5: Conclusion As drawn, all four structures have a free reducing centre, so there is no defensible literal non-reducing option. The likely intended answer is (B), sucrose, but its rightmost furanose substituent would need to be CH_2OH instead of the printed OH.

Quick Tip to Solve: Inspect the bonds at each carbon adjacent to the ring oxygen: a free hemiacetal OH gives a reducing end. A familiar-looking sugar outline is not enough to establish that both anomeric centres are locked.

69. Identify the correct statements :

The presence of $-NO_2$ group in benzene ring

A. activates the ring towards electrophilic substitutions.

B. deactivates the ring towards electrophilic substitutions.

C. activates the ring towards nucleophilic substitutions.

D. deactivates the ring towards nucleophilic substitutions.

Choose the correct answer from the options given below : [4 marks]

(A) A and D Only

(B) C and A Only

(C) B and C Only

(D) B and D Only

Correct Answer: (C) B and C Only

Solution:

Concept:

- The nitro group withdraws electron density through both its negative inductive effect and its negative resonance effect.
- Electron withdrawal disfavors attack by electrophiles, but can favour nucleophilic aromatic substitution by stabilising a negatively charged intermediate.

Step 1: Judge statements A and B An electrophile seeks electron density in the aromatic ring. The $-NO_2$ group withdraws electron density and makes formation of the positively charged sigma complex less favourable than in benzene. Hence nitrobenzene reacts more slowly in electrophilic substitution: the nitro group is deactivating. Statement A is false and statement B is true. Its meta-directing character is a separate orientation effect; it does not mean that any position is activated relative to benzene.

Step 2: Judge statements C and D In nucleophilic aromatic substitution, a nucleophile adds electron density to the ring. An electron-withdrawing nitro group stabilises the resulting anionic intermediate, particularly when situated ortho or para to the group being displaced, because the negative charge can delocalise towards the nitro group. This explains the stated activating effect towards nucleophilic substitution. Statement C is true and statement D is false.

Step 3: Compare the statement pairs The truth values are therefore A: false, B: true, C: true and D: false. The pair A and D contains two false statements. The pair C and A includes false A, while B and D includes false D. Only the pair B and C contains both correct statements, and it is the third printed option.

Step 4: Conclusion The correct statements are B and C only. Hence, the correct option is (C).

Quick Tip to Solve: Ask what charge the reacting species brings: electron withdrawal hinders electrophilic attack but helps stabilise the anionic intermediate of nucleophilic aromatic substitution.

70. Consider the following aqueous solutions.

I. 2.2 g Glucose in 125 mL of solution.

II. 1.9 g Calcium chloride in 250 mL of solution.

III. 9.0 g Urea in 500 mL of solution.

IV. 20.5 g Aluminium sulphate in 750 mL of solution.

The correct increasing order of boiling point of these solutions will be :

[Given : Molar mass in g mol^{-1} : H = 1, C = 12, N = 14, O = 16, Cl = 35.5, Ca = 40, Al = 27 and S = 32] [4 marks]

(A) I < II < III < IV

(B) III < I < II < IV

(C) II < III < I < IV

(D) II < III < IV < I

Correct Answer: (A) I < II < III < IV

Solution:

Concept:

- For a dilute solution, boiling-point elevation is $\Delta T_b = iK_b m$, where i counts effective solute particles and m is molality.
- For the usual dilute aqueous comparison, solvent masses are proportional to the stated solution volumes. Thus compare $i(w/M)/V$, taking complete dissociation of the salts and no dissociation of glucose or urea.

Step 1: Find molar masses and particle factors Using the given atomic masses, glucose $C_6H_{12}O_6$ has molar mass 180, $CaCl_2$ has 111, urea $CO(NH_2)_2$ has 60, and $Al_2(SO_4)_3$ has 342, all in $\mathrm{g\,mol^{-1}}$. Glucose and urea have $i = 1$. Calcium chloride gives one calcium and two chloride ions, so $i = 3$. Aluminium sulphate gives two aluminium and three sulphate ions, so $i = 5$.

Step 2: Calculate the relative particle concentrations Convert the four volumes to litres and compute the particle-concentration quantities used for comparison:

$$P_{\mathrm{I}} = \frac{1(2.2/180)}{0.125} = 0.0978,$$
$$P_{\mathrm{II}} = \frac{3(1.9/111)}{0.250} = 0.2054,$$
$$P_{\mathrm{III}} = \frac{1(9.0/60)}{0.500} = 0.3000,$$
$$P_{\mathrm{IV}} = \frac{5(20.5/342)}{0.750} = 0.3996.$$

These are effective moles of particles per litre. Under the dilute aqueous approximation they have the same order as the particle molalities.

Step 3: Order the elevations and boiling points All solutions share water as solvent, so the common ebullioscopic constant and pure-water boiling point do not change the ranking. The calculated values give I below II below III below IV. The second option incorrectly places III below I; the third and fourth incorrectly put II below I. The first printed option alone agrees with all four values.

Step 4: Conclusion The increasing boiling-point order is I $<$ II $<$ III $<$ IV. Hence, the correct option is (A). The printed first option is selected by the particle calculation.

Quick Tip to Solve: Compare particle amount per solvent amount, not solute mass alone; multiply the mole concentration by 3 for calcium chloride and by 5 for aluminium sulphate.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. For strong electrolyte Λ_m increases slowly with dilution and can be represented by the equation

$$\Lambda_m = \Lambda_m^\circ - Ac^{1/2}$$

Molar conductivity values of the solutions of strong electrolyte AB at 18°C are given below :

c [mol L^{-1}]	0.04	0.09	0.16	0.25
Λ_m [S cm^2 mol^{-1}]	96.1	95.7	95.3	94.9

The value of constant A based on the above data [in S cm^2 mol^{-1}/(mol/L)$^{1/2}$] unit is _____.

[4 marks]

Solution:

Concept:

- The equation $\Lambda_m = \Lambda_m^\circ - A\sqrt{c}$ is linear in $\sqrt{c}$, with intercept Λ_m° and slope $-A$.
- Subtracting two conductivity equations eliminates the unknown limiting molar conductivity. Divide the decrease in conductivity by the increase in square-root concentration.

Step 1: Transform the concentration values Take square roots before finding a slope; the independent variable is not concentration itself. The given concentrations $0.04, 0.09, 0.16, 0.25\,\text{mol}\,\text{L}^{-1}$ correspond to $\sqrt{c} = 0.2, 0.3, 0.4, 0.5$ in $(\text{mol}\,\text{L}^{-1})^{1/2}$. Each successive square-root concentration differs by 0.1, while the molar conductivity decreases by $0.4\,\text{S}\,\text{cm}^2\,\text{mol}^{-1}$.

Step 2: Eliminate the intercept and calculate A For the first two entries, the model gives

$$96.1 = \Lambda_m^\circ - 0.2A, \qquad 95.7 = \Lambda_m^\circ - 0.3A.$$

Subtracting the second equation from the first gives

$$0.4 = 0.1A, \qquad A = 4.$$

The slope itself is negative because conductivity falls as concentration rises; the constant A in the printed equation is positive.

Step 3: Check the other two measurements The first reading gives $\Lambda_m^\circ = 96.1 + 4(0.2) = 96.9\,\text{S}\,\text{cm}^2\,\text{mol}^{-1}$. At $\sqrt{c} = 0.4$, the predicted conductivity is $96.9 - 1.6 = 95.3$. At $\sqrt{c} = 0.5$, it is $96.9 - 2.0 = 94.9$. Both match the remaining readings exactly, so all four measurements support the same constant.

Step 4: Conclusion The required constant is $A = 4\,\text{S}\,\text{cm}^2\,\text{mol}^{-1}/(\text{mol}\,\text{L}^{-1})^{1/2}$. The numerical entry is

$\boxed{4}$.

Quick Tip to Solve: For the strong-electrolyte dilution law, calculate the slope against square-root concentration; using concentration directly gives the wrong constant.

72. A $\rightarrow$ B (first reaction)

C $\rightarrow$ D (second reaction)

Consider the above two first-order reactions. The rate constant for first reaction at 500 K is double of the same at 300 K. At 500 K, 50% of the reaction becomes complete in 2 hour. The activation energy of the second reaction is half of that of first reaction. If the rate constant at 500 K of the second reaction becomes double of the rate constant of first reaction at the same temperature; then rate constant for the second reaction at 300 K is _____ $\times$ 10^{-1} $hour^{-1}$ (nearest integer). [4 marks]

Solution:

Concept:

- For a first-order reaction, the half-life is $t_{1/2} = \ln 2/k$. The given 50 percent conversion is therefore a half-life.
- For fixed Arrhenius parameters, $\ln[k(T_h)/k(T_l)] = (E_a/R)(1/T_l - 1/T_h)$. For identical temperatures, halving activation energy halves this logarithmic ratio.

Step 1: Calculate both rate constants at 500 K Write k_1 and k_2 for the first and second reactions. The first reaction has a half-life of 2 hours at 500 K, giving

$$k_1(500) = \frac{\ln 2}{2} = 0.34657\,\mathrm{h}^{-1}.$$

The stated second-reaction rate constant is double this value at the same temperature:

$$k_2(500) = 2k_1(500) = \ln 2 = 0.69315\,\mathrm{h}^{-1}.$$

Step 2: Find the temperature multiplier for the second reaction The first reaction satisfies $k_1(500)/k_1(300) = 2$. Applying the Arrhenius ratio to each reaction and using $E_{a,2} = E_{a,1}/2$,

$$\begin{aligned}\ln\frac{k_2(500)}{k_2(300)} &= \frac{1}{2}\ln\frac{k_1(500)}{k_1(300)} \\ &= \frac{1}{2}\ln 2 = \ln\sqrt{2}.\end{aligned}$$

Thus the second rate constant increases by a factor $\sqrt{2}$, not by a factor of 2, between 300 K and 500 K. The pre-exponential factors need not be equated: each cancels within its own temperature ratio.

Step 3: Convert to the requested coefficient Solve for the lower-temperature rate constant:

$$k_2(300) = \frac{k_2(500)}{\sqrt{2}} = \frac{\ln 2}{\sqrt{2}} = 0.49013\,\mathrm{h}^{-1}.$$

Writing this as $x \times 10^{-1}\,\mathrm{h}^{-1}$ gives $x = 4.9013$. Round this coefficient, rather than rounding the rate constant prematurely: the nearest integer is 5.

Step 4: Conclusion $k_2(300) = 0.49013\,\mathrm{h}^{-1}$, corresponding to $4.9013 \times 10^{-1}\,\mathrm{h}^{-1}$. The required nearest-integer entry is $\boxed{5}$.

Quick Tip to Solve: When activation energy is halved, take the square root of the temperature multiplier; it is the logarithm of the rate ratio that is proportional to activation energy.

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73. The number of isoelectronic species among $Sc^{3+}, Cr^{2+}, Mn^{3+}, Co^{3+}$ and Fe^{3+} is 'n'. If 'n' moles of AgCl is formed during the reaction of complex with formula $CoCl_3(en)_2NH_3$ with excess of $AgNO_3$ solution, then the number of electrons present in the t_{2g} orbital of the complex is _____. [4 marks]

Solution:

Concept:

- Isoelectronic ions have the same total number of electrons. For a cation, subtract its positive charge from the atomic number: $N_e = Z - z$.
- Only chloride ions outside the coordination sphere precipitate immediately as AgCl with excess $AgNO_3$. Ethylenediamine (en) is neutral and bidentate; NH_3 is neutral and monodentate.
- In an octahedral low-spin d^6 complex, pairing fills the lower-energy t_{2g} set before the higher-energy e_g set: $t_{2g}^6 e_g^0$.

Step 1: Count the electrons in the five ions Use the atomic numbers $21, 24, 25, 27, 26$, respectively:

Ion	Sc^{3+}	Cr^{2+}	Mn^{3+}	Co^{3+}	Fe^{3+}
N_e	18	22	22	24	23

Thus Cr^{2+} and Mn^{3+} form the isoelectronic pair. There are two species in this pair, so $n = 2$; the other three ions have distinct electron counts.

Step 2: Locate the chloride ions and identify the metal configuration Formation of two moles of AgCl per mole of the complex means that two chloride ions are counterions. The remaining chloride is coordinated:

$$[\mathrm{CoCl(en)_2(NH_3)}]\mathrm{Cl_2}.$$

Two en ligands supply four donor atoms, ammonia supplies one and the coordinated chloride supplies one, giving coordination number six. The complex cation has charge $+2$. Since only the coordinated chloride contributes -1, the oxidation state of cobalt is $+3$, corresponding to $3d^6$.

Step 3: Fill the octahedral energy levels The high charge of Co^{3+}, together with the en and ammonia ligands, gives the usual low-spin cobalt(III) octahedral complex. The six d electrons pair in the three lower-energy orbitals:

$$(t_{2g})^6 (e_g)^0.$$

The symbol t_{2g} denotes a set of three orbitals, each containing two electrons here; the requested number is their combined occupancy.

Step 4: Conclusion The t_{2g} set contains $\boxed{6}$ electrons. The numerical answer is 6.

Quick Tip to Solve: Count precipitated AgCl to find outer-sphere chloride first; then determine the coordination number and metal oxidation state before filling the d orbitals.

74. A volume of x mL of 5 M $NaHCO_3$ solution was mixed with 10 mL of 2 M H_2CO_3 solution to make an electrolytic buffer. If the same buffer was used in the following electrochemical cell to record a cell potential of 235.3 mV, then the value of $x =$ _____ mL (nearest integer).

$$\mathrm{Sn\,(s)} \mid \mathrm{Sn(OH)_6{}^{2-}\ (0.5\,M)} \mid \mathrm{HSnO_2{}^-\ (0.05\,M)} \mid \mathrm{OH^-} \mid \mathrm{Bi_2O_3(s)} \mid \mathrm{Bi\,(s)}$$

Consider up to one place of decimal for intermediate calculations.

Given:

$$\begin{aligned} E^\circ_{\mathrm{HSnO_2{}^-|Sn(OH)_6{}^{2-}}} &= -0.9\,\mathrm{V} \\ E^\circ_{\mathrm{Bi_2O_3|Bi}} &= -0.44\,\mathrm{V} \\ \mathrm{p}K_a(\mathrm{H_2CO_3}) &= 6.11 \\ \frac{2.303RT}{F} &= 0.059\,\mathrm{V} \\ \mathrm{Antilog}(1.29) &= 19.5 \end{aligned}$$

[4 marks]

Solution:

Concept:

- For a reduction half-cell, $E = E^\circ - (0.059/n)\log Q$, where n is the number of transferred electrons and activities of pure solids and liquid water are unity.
- The cell voltage is the right-hand reduction potential minus the left-hand reduction potential. The hydroxide concentration enters both half-cell quotients and must be retained until they are subtracted.
- For the $H_2CO_3/HCO_3{}^-$ buffer, $\mathrm{pH} = \mathrm{p}K_a + \log(n_{\mathrm{salt}}/n_{\mathrm{acid}})$. Both components have the same final volume, so their concentration ratio equals their mole ratio.

Step 1: Write the reduction half-reactions Let $c = [\mathrm{OH^-}]$, expressed relative to 1 M. The balanced reduction reactions are

$$\mathrm{Sn(OH)_6{}^{2-} + 2\,e^- \longrightarrow HSnO_2{}^- + 3\,OH^- + H_2O},$$

$$\mathrm{Bi_2O_3 + 3\,H_2O + 6\,e^- \longrightarrow 2\,Bi + 6\,OH^-}.$$

Each equation conserves atoms and charge. Using the supplied standard electrode potentials gives

$$E_{\mathrm{Sn}} = -0.90 - \frac{0.059}{2}\log\left(\frac{0.05c^3}{0.5}\right),$$

$$E_{\mathrm{Bi}} = -0.44 - \frac{0.059}{6}\log(c^6) = -0.44 - 0.059\log c.$$

Step 2: Obtain the buffer pH from the cell voltage Subtract the tin potential from the bismuth potential. Since $\log(0.1) = -1$,

$$\begin{aligned} E_{\text{cell}} &= 0.46 - 0.059 \log c + 0.0295 \log(0.1c^3) \\ &= 0.4305 + 0.0295 \log c. \end{aligned}$$

Convert $235.3\,\text{mV}$ to $0.2353\,\text{V}$. Therefore

$$\log c = \frac{0.2353 - 0.4305}{0.0295} \approx -6.6.$$

With the required one-decimal intermediate value, $\text{pOH} = 6.6$ and $\text{pH} = 14.0 - 6.6 = 7.4$. Keep the supplied voltage coefficients intact while calculating this logarithm; rounding them to one decimal beforehand would destroy the precision of the given data.

Step 3: Use the buffer ratio to calculate the volume The acid contains $2 \times 10 = 20$ millimoles and the bicarbonate contains $5x$ millimoles. Apply the given acid dissociation constant and antilogarithm:

$$\begin{aligned} 7.4 &= 6.11 + \log\left(\frac{5x}{20}\right), \\ \log\left(\frac{x}{4}\right) &= 1.29, \\ \frac{x}{4} &= 19.5, \\ x &= 78.0. \end{aligned}$$

The final mixed volume cancels from the salt-to-acid ratio, so it must not be inserted on only one side of that ratio.

Step 4: Conclusion The required volume is $\boxed{x = 78\,\text{mL}}$, to the nearest integer.

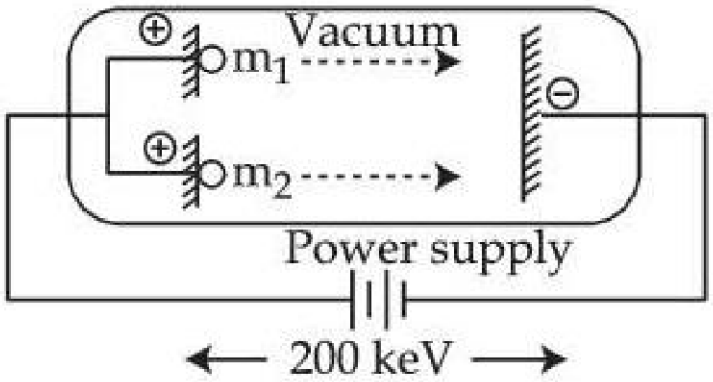
Quick Tip to Solve: Balance each half-reaction before writing its hydroxide exponent. Round the calculated pOH as instructed, then use the supplied antilogarithm to recover the buffer mole ratio.

75. Two positively charged particles m_1 and m_2 have been accelerated across the same potential difference of 200 keV as shown below.

[Given mass of $m_1 = 1$ amu and $m_2 = 4$ amu]

The deBroglie wavelength of m_1 will be x times of m_2. The value of x is _____ (nearest integer)

[4 marks]



Solution:

Concept:

- A particle's de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and p is its momentum.
- For nonrelativistic motion, $K = p^2/(2m)$. A particle of charge q, accelerated from rest through a potential difference V, gains kinetic energy $K = qV$, giving $\lambda = h/\sqrt{2mqV}$.
- For equal charges and a common accelerating potential, $\lambda \propto m^{-1/2}$. Positive particles need not have equal charge magnitudes; a mass of 4 amu alone does not specify a doubly charged alpha particle.

Step 1: Express each wavelength using the accelerating energy Both particles move through the same potential difference. If their charges are q_1 and q_2, the general wavelength ratio is

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2 q_2}{m_1 q_1}} = 2\sqrt{\frac{q_2}{q_1}}.$$

The intended numerical entry uses the equal-charge interpretation, $q_1 = q_2$. On that interpretation both particles acquire the same kinetic energy K, although their speeds differ. Their momenta and wavelengths are

$$p_1 = \sqrt{2m_1K}, \qquad p_2 = \sqrt{2m_2K},$$

$$\lambda_1 = \frac{h}{\sqrt{2m_1K}}, \qquad \lambda_2 = \frac{h}{\sqrt{2m_2K}}.$$

At the indicated energy scale, the kinetic energy is very small compared with the rest energy of an atomic-mass-unit particle, so the nonrelativistic relation is appropriate.

Step 2: Form the wavelength ratio and substitute the masses The factor $h/\sqrt{2K}$ is common. Dividing the two expressions therefore cancels the accelerating energy and avoids any conversion of amu into

kilograms:

$$x = \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{4\,\text{amu}}{1\,\text{amu}}} = \sqrt{4} = 2.$$

The ratio is dimensionless. The lighter particle has the longer wavelength because, at a fixed kinetic energy, it has the smaller momentum. Using the mass ratio itself would give 4, but momentum depends on the square root of mass in this situation, not on mass directly.

Step 3: Conclusion For the intended equal-charge (equal-kinetic-energy) interpretation, $\boxed{x = 2}$, dimensionless. More generally, $x = 2\sqrt{q_2/q_1}$. Positivity and the two masses alone do not specify q_2/q_1; the plus signs label the accelerating plates and do not establish equality of the particle charges.

Quick Tip to Solve: Equal accelerating potentials imply equal kinetic energies only for equal charges. Once that is checked, use $\lambda_1/\lambda_2 = \sqrt{m_2/m_1}$; the lighter particle has the longer wavelength.