JEE Main 2026 Paper 1 (B.E./B.Tech) Apr 4 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2026, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 4 April 2026, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. For the function $f : [1, \infty) \to [1, \infty)$ defined by $f(x) = (x-1)^4 + 1$, among the two statements:
(I) The set $S = \{x \in [1, \infty) : f(x) = f^{-1}(x)\}$ contains exactly two elements, and
(II) The set $S = \{x \in [1, \infty) : f(x) = f^{-1}(x+1)\}$ is an empty set, [4 marks]

(A) only (I) is TRUE

(B) only (II) is TRUE

(C) both (I) and (II) are TRUE

(D) neither (I) nor (II) is TRUE

Correct Answer: (A) only (I) is TRUE

Solution:

Concept:

- The function is strictly increasing on its domain, so it is one-one and has an inverse. Solving $y = (x-1)^4 + 1$ with $x \geq 1$ gives $f^{-1}(y) = 1 + (y-1)^{1/4}$.
- Use $t = x - 1 \geq 0$ to remove the shift. A continuous function with opposite signs at two endpoints has a zero between them.

Step 1: Check statement (I) Equating the function and its inverse gives $1 + t^4 = 1 + t^{1/4}$, or $t^4 = t^{1/4}$. The value $t = 0$ is a solution. If $t > 0$, dividing by $t^{1/4}$ gives $t^{15/4} = 1$, hence $t = 1$. Thus the only corresponding inputs are $x = 1$ and $x = 2$. Both lie in the domain, and direct substitution verifies both equalities. Therefore statement (I) is true.

Step 2: Test whether the second set is empty Now $f^{-1}(x+1) = 1 + x^{1/4} = 1 + (t+1)^{1/4}$. The required equality is $t^4 = (t+1)^{1/4}$. Both sides are nonnegative, so taking fourth powers is reversible and gives $t^{16} - t - 1 = 0$. Define $g(t) = t^{16} - t - 1$. It is continuous and satisfies

$$g(1) = -1 < 0, \qquad g(2) = 65533 > 0.$$

Therefore some $t \in (1, 2)$ solves the equation, yielding an admissible $x = t + 1 \in (2, 3)$. The second set is consequently nonempty and statement (II) is false.

Step 3: Select the truth combination Only statement (I) holds. Option (B) reverses both truth values, option (C) incorrectly calls the second set empty, and option (D) incorrectly rejects the two solutions of the first equation.

Step 4: Conclusion The first set is $\{1, 2\}$, while the second set is nonempty. Hence, the correct option is (A).

Quick Tip to Solve: For a shifted power function, substitute the shifted variable first; use a sign change to prove existence without solving a high-degree equation.

2. Let $S = \{z \in C : z^2 + 4z + 16 = 0\}$**. Then** $\sum_{z \in S} |z + \sqrt{3}i|^2$ **is equal to :** [4 marks]

(A) 42

(B) 23

(C) 27

(D) 38

Correct Answer: (D) 38

Solution:

Concept:

- For $z = a + bi$, the squared modulus is $|z|^2 = a^2 + b^2$. It is real and nonnegative; it is not the ordinary square z^2.
- The quadratic formula gives both roots of a quadratic. With real coefficients, nonreal roots occur as a conjugate pair, and each distinct root contributes once to the sum.

Step 1: Find both elements of the set The discriminant of $z^2 + 4z + 16 = 0$ is $4^2 - 4(1)(16) = -48$. Hence

$$z = \frac{-4 \pm \sqrt{-48}}{2} = -2 \pm 2\sqrt{3}\, i.$$

The discriminant is nonzero, so these are two distinct elements of S. Both have modulus 4, which also checks their product 16.

Step 2: Add the imaginary shift to each root For $z_1 = -2 + 2\sqrt{3}\, i$, the shifted number is $z_1 + \sqrt{3}\, i = -2 + 3\sqrt{3}\, i$. Therefore

$$|z_1 + \sqrt{3}\, i|^2 = (-2)^2 + (3\sqrt{3})^2 = 4 + 27 = 31.$$

For $z_2 = -2 - 2\sqrt{3}\, i$, the shifted number is $-2 - \sqrt{3}\, i$, giving

$$|z_2 + \sqrt{3}\, i|^2 = (-2)^2 + (-\sqrt{3})^2 = 4 + 3 = 7.$$

The shift is the same for both roots, so the shifted numbers are not a conjugate pair.

Step 3: Form the required sum and compare Adding the two contributions gives $31 + 7 = 38$. Equivalently, the original squared moduli sum to 32, the two shifts add $2(3) = 6$, and the cross terms cancel because the original imaginary parts are opposite. Thus neither 42, 23, nor 27 equals the required sum; in particular, 27 counts only one imaginary-part square.

Step 4: Conclusion $\sum_{z \in S} |z + \sqrt{3}\, i|^2 = 38$. Hence, the correct option is (D).

Quick Tip to Solve: For conjugate roots shifted by the same imaginary number, sum the squared moduli together: the cross terms cancel.

3. If the system of equations :

$$x + y + z = 5$$
$$x + 2y + 3z = 9$$
$$x + 3y + \lambda z = \mu$$

has infinitely many solutions, then the value of $\lambda + \mu$ is : [4 marks]

(A) 16

(B) 18

(C) 19

(D) 21

Correct Answer: (B) 18

Solution:

Concept:

- A system of three linear equations has infinitely many solutions when its equations are consistent but do not determine all three variables independently. The coefficient and augmented matrices must have the same rank, smaller than three.
- Subtracting one equation from another preserves the solution set. Once two independent equations remain, the third must impose no additional restriction for a free variable to exist.

Step 1: Eliminate the common variable Subtract the first equation from the second and from the third. This eliminates x and gives

$$y + 2z = 4, \qquad 2y + (\lambda - 1)z = \mu - 5.$$

The first two original equations are independent because their coefficients are not proportional. Thus they already give two independent constraints; infinitely many solutions require the remaining constraint to be redundant.

Step 2: Impose the dependence condition Subtract twice the reduced equation $y + 2z = 4$ from the other reduced equation:

$$(\lambda - 5)z = \mu - 13.$$

If $\lambda \neq 5$, this equation fixes z, after which y and x are also fixed. If $\lambda = 5$ but $\mu \neq 13$, it gives an inconsistency. Therefore the equation must be the identity $0 = 0$, requiring

$$\lambda = 5, \qquad \mu = 13.$$

Step 3: Verify the free parameter and compare the options With these values, let $z = t$, where t is any real number. Then $y = 4 - 2t$, and the first equation gives $x = 1 + t$. Substitution into the third equation yields

$$(1 + t) + 3(4 - 2t) + 5t = 13,$$

independently of t. Hence there really are infinitely many solutions, and

$$\lambda + \mu = 5 + 13 = 18.$$

The other offered values, 16, 19, and 21, do not equal the sum forced by consistency and dependence.

Step 4: Conclusion The required value is $\boxed{18}$. Hence, the correct option is (B).

Quick Tip to Solve: After elimination, an equation of the form $az = b$ leaves z free only when both a and b vanish; a zero determinant alone is not enough.

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4. If $\alpha = 1$ and $\beta = 1 + i\sqrt{2}$, where $i = \sqrt{-1}$, are two roots of the equation $x^3 + ax^2 + bx + c = 0$, $a, b, c \in \mathbb{R}$, then $\int_{-1}^{1} (x^3 + ax^2 + bx + c)\, dx$ is equal to: [4 marks]

(A) -2

(B) -4

(C) -8

(D) -10

Correct Answer: (C) -8

Solution:

Concept:

- A polynomial with real coefficients has conjugate nonreal roots: if $1 + i\sqrt{2}$ is a root, then $1 - i\sqrt{2}$ is also a root.
- On the symmetric interval $[-1, 1]$, the integral of an odd function is zero. Thus only the even-power and constant terms of the polynomial contribute.

Step 1: Construct the cubic from its roots The three roots are 1, $1 + i\sqrt{2}$ and $1 - i\sqrt{2}$. Since the leading coefficient is one, the cubic must be

$$\begin{aligned} P(x) &= (x - 1)(x - 1 - i\sqrt{2})(x - 1 + i\sqrt{2}) \\ &= (x - 1)\big((x - 1)^2 + 2\big) \\ &= (x - 1)(x^2 - 2x + 3) \\ &= x^3 - 3x^2 + 5x - 3. \end{aligned}$$

Consequently $a = -3$, $b = 5$ and $c = -3$. The signs can be checked from the sum of roots 3 and their product 3.

Step 2: Use symmetry before integrating The terms x^3 and $5x$ are odd, so their integrals cancel between the negative and positive halves of the interval. The remaining expression is

$$\int_{-1}^{1} P(x)\, dx = -3 \int_{-1}^{1} x^2\, dx - 3 \int_{-1}^{1} 1\, dx.$$

Since $\int_{-1}^{1} x^2\,dx = 2/3$ and the interval length is 2, this is

$$(-3)\frac{2}{3} + (-3)(2) = -2 - 6 = -8.$$

Step 3: Check the proposed values The value -2 retains only the quadratic contribution and loses the constant contribution. The values -4 and -10 also disagree with the exact two contributions -2 and -6. Their sum uniquely selects -8.

Step 4: Conclusion $\int_{-1}^{1} (x^3 + ax^2 + bx + c)\,dx = -8$. Hence, the correct option is (C).

Quick Tip to Solve: For a real cubic with one nonreal root, insert its conjugate first; on symmetric limits discard odd terms before integrating.

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5. If the quadratic equation $(\lambda+2)x^2 - 3\lambda x + 4\lambda = 0, \lambda \neq -2$, has two positive roots, then the number of possible integral values of λ is : [4 marks]

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

Solution:

Concept:

- For a real quadratic to have two positive real roots, its discriminant must be nonnegative, and both the sum and product of the roots must be strictly positive.
- Vieta's formulae give the sum as minus the linear coefficient divided by the quadratic coefficient, and the product as the constant coefficient divided by the quadratic coefficient.

Step 1: Impose reality of the roots The equation is quadratic because $\lambda \neq -2$. Its discriminant is

$$D = (-3\lambda)^2 - 4(\lambda+2)(4\lambda) = -7\lambda^2 - 32\lambda = -\lambda(7\lambda + 32).$$

Therefore real roots require $\lambda(7\lambda + 32) \leq 0$, which gives

$$-\frac{32}{7} \leq \lambda \leq 0.$$

A repeated root at an endpoint does not affect the eventual integer count because the nonzero endpoint is not an integer.

Step 2: Require both roots to be positive If the roots are r_1, r_2, then

$$r_1 + r_2 = \frac{3\lambda}{\lambda+2}, \qquad r_1 r_2 = \frac{4\lambda}{\lambda+2}.$$

Both are positive precisely when $\lambda/(\lambda+2) > 0$, meaning $\lambda < -2$ or $\lambda > 0$. A positive product makes real roots have the same sign, while a positive sum selects the positive sign. Intersecting with the discriminant interval leaves

$$-\frac{32}{7} \le \lambda < -2.$$

The value $\lambda = 0$ gives zero roots, and $\lambda = -2$ is explicitly excluded.

Step 3: Count and verify the integers The integers in this interval are only -4 and -3. At $\lambda = -4$, the equation reduces to $x^2 - 6x + 8 = 0$, whose roots are $2, 4$. At $\lambda = -3$, it reduces to $x^2 - 9x + 12 = 0$, whose roots $(9 \pm \sqrt{33})/2$ are both positive. Thus one value would omit an admissible integer, while three or four would include values outside the required interval.

Step 4: Conclusion There are exactly two integral values, $\lambda = -4, -3$. Hence, the correct option is (B).

Quick Tip to Solve: Combine all three conditions: real roots, positive sum, and positive product. The discriminant alone cannot ensure positive roots.

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6. Let $A = \begin{bmatrix} 1 & 2 & 7 \\ 4 & -2 & 8 \\ 3 & 8 & -7 \end{bmatrix}$ **and** $\det(A - \alpha I) = 0$**, where** α **is a real number. If the largest possible value of** α **is** p**, then the circle** $(x-p)^2 + (y-2p)^2 = 320$**, intersects the co-ordinate axes at** [4 marks]

(A) 1 point

(B) 2 points

(C) 3 points

(D) 4 points

Correct Answer: (C) 3 points

Solution:

Concept:

- The possible values of α are the real roots of the characteristic equation $\det(A - \alpha I) = 0$. Factor the determinant and compare all roots.
- For a circle, substitute $y = 0$ to find intersections with the horizontal axis and $x = 0$ for the vertical axis. If the origin appears in both lists, count it only once.

Step 1: Expand and factor the determinant Expanding along the first row gives

$$\begin{aligned} \det(A - \alpha I) &= (1-\alpha)\big((\alpha+2)(\alpha+7) - 64\big) \\ &\quad - 2\big(4(-7-\alpha) - 24\big) \\ &\quad + 7\big(32 + 3(2+\alpha)\big) \\ &= -\alpha^3 - 8\alpha^2 + 88\alpha + 320 \\ &= -(\alpha - 8)(\alpha^2 + 16\alpha + 40). \end{aligned}$$

Thus $\alpha = 8, -8 + 2\sqrt{6}, -8 - 2\sqrt{6}$. The latter two are negative, so the largest root is $p = 8$.

Step 2: Find intersections with each axis The circle becomes $(x-8)^2+(y-16)^2=320$. Setting $y=0$ yields

$$(x-8)^2=320-256=64,$$

so $x=0$ or 16. The horizontal-axis intersections are $(0,0)$ and $(16,0)$. Setting $x=0$ instead gives

$$(y-16)^2=320-64=256,$$

so $y=0$ or 32. The vertical-axis intersections are $(0,0)$ and $(0,32)$.

Step 3: Count distinct intersection points The union of the two lists is $\{(0,0),(16,0),(0,32)\}$. There are three distinct points because the origin lies on both axes. Counting four incorrectly counts the origin twice; counting one or two misses at least one of the two nonzero intercepts. As a check, the origin satisfies $8^2+16^2=320$.

Step 4: Conclusion The circle intersects the coordinate axes at three distinct points. Hence, the correct option is (C).

Quick Tip to Solve: When a circle passes through the origin, the two axis-intersection lists share that point; subtract the duplicate when counting distinct points.

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7. Let $\alpha=\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\infty$ and $\beta=\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots\infty$. Then the value of $(0.2)^{\log_{\sqrt{5}}(\alpha)}+(0.04)^{\log_5(\beta)}$ is equal to : [4 marks]

(A) 4

(B) 5

(C) 8

(D) 25

Correct Answer: (C) 8

Solution:

Concept:

- An infinite geometric progression with first term a and common ratio r, where $|r|<1$, has sum $a/(1-r)$.
- The change-of-base identity gives $\log_{\sqrt{5}}t=2\log_5 t$. Also, $5^{\log_5 t}=t$ for positive t, so logarithmic powers can be evaluated exactly.

Step 1: Sum the two geometric progressions For α, the first term is $1/4$ and the common ratio is $1/2$. For β, both the first term and the common ratio are $1/3$. Each ratio has magnitude less than one, so both infinite sums converge:

$$\alpha=\frac{1/4}{1-1/2}=\frac{1}{2},\qquad \beta=\frac{1/3}{1-1/3}=\frac{1}{2}.$$

Notice that equal sums do not mean the two progressions have equal first terms or ratios.

Step 2: Rewrite the decimal bases as powers of five Since $0.2 = 5^{-1}$, combine this exponent with the change of logarithmic base:

$$(0.2)^{\log_{\sqrt{5}} \alpha} = 5^{-2\log_5 \alpha} = \alpha^{-2} = 4.$$

Similarly, $0.04 = 5^{-2}$, and therefore

$$(0.04)^{\log_5 \beta} = 5^{-2\log_5 \beta} = \beta^{-2} = 4.$$

Each logarithm is negative because its argument is between zero and one; a base below one raised to that negative exponent indeed gives a value above one.

Step 3: Add the contributions and compare the options The required expression is $4 + 4 = 8$. Option (A) counts only one contribution. Options (B) and (D) do not equal the sum obtained after correctly converting both logarithmic bases.

Step 4: Conclusion The required value is $\boxed{8}$. Hence, the correct option is (C).

Quick Tip to Solve: Convert decimal bases into powers of the logarithm's base before evaluating; this turns both terms directly into reciprocal squares.

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8. For 10 observations $x_1, x_2, \ldots, x_{10}$, if $\sum_{i=1}^{10} (x_i + 2)^2 = 180$ and $\sum_{i=1}^{10} (x_i - 1)^2 = 90$, then their standard deviation is : [4 marks]

(A) 2

(B) $\sqrt{3}$

(C) $2\sqrt{2}$

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- For n observations, the mean is $\bar{x} = \sum x_i/n$, and the variance is $\sigma^2 = \sum x_i^2/n - \bar{x}^2$.
- Standard deviation is the nonnegative square root of the variance. Expanding the two given shifted sums produces linear equations in $\sum x_i$ and $\sum x_i^2$.

Step 1: Expand the given sums Put $S = \sum_{i=1}^{10} x_i$ and $T = \sum_{i=1}^{10} x_i^2$. The constants in each expansion must be counted ten times, once for each observation. Thus

$$T + 4S + 40 = 180 \quad \Rightarrow \quad T + 4S = 140,$$

and

$$T - 2S + 10 = 90 \quad \Rightarrow \quad T - 2S = 80.$$

These relations determine the two moments even though the individual observations are not specified.

Step 2: Obtain the mean and the mean square Subtract the second relation from the first to eliminate

T:

$$6S = 60 \quad \Rightarrow \quad S = 10.$$

Substitution into $T - 2S = 80$ gives $T = 100$. Consequently,

$$\bar{x} = \frac{10}{10} = 1, \qquad \frac{1}{10}\sum_{i=1}^{10} x_i^2 = \frac{100}{10} = 10.$$

As a check, the original sums are $100 + 40 + 40 = 180$ and $100 - 20 + 10 = 90$.

Step 3: Calculate the standard deviation Use the variance of the ten observations, with denominator ten:

$$\sigma^2 = 10 - 1^2 = 9, \qquad \sigma = \sqrt{9} = 3.$$

The squared values of options (A), (B) and (C) are respectively 4, 3 and 8, none of which equals the required variance 9. There is no sample-estimation correction in this question.

Step 4: Conclusion The standard deviation is $\boxed{3}$. Hence, the correct option is (D).

Quick Tip to Solve: Subtract shifted square-sum equations first to obtain the mean; then recover the sum of squares and use mean square minus square of mean.

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9. In the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}, x > 0$, if the term independent of x is (221)k, then k is equal to: [4 marks]

(A) 84

(B) 78

(C) 168

(D) 198

Correct Answer: (A) 84

Solution:

Concept:

- The general term of $(a+b)^n$ is $T_{r+1} = \binom{n}{r} a^{n-r} b^r$, where $r = 0, 1, \ldots, n$.
- A term independent of x has total exponent zero. The numerical coefficient and the sign must be evaluated after finding the required value of r.

Step 1: Write the general term with its power of x Choose r factors of $-1/(3\sqrt{x})$ and $18 - r$ factors of $9x$. Because $x > 0$, the fractional powers are well-defined. The resulting term is

$$\begin{aligned} T_{r+1} &= \binom{18}{r} (9x)^{18-r} \left(-\frac{1}{3\sqrt{x}}\right)^r \\ &= \binom{18}{r} (-1)^r 3^{36-3r} x^{18-3r/2}. \end{aligned}$$

Both the x in the first factor and the square root in the denominator contribute to this exponent.

Step 2: Set the exponent to zero The constant term requires

$$18 - \frac{3r}{2} = 0 \quad \Rightarrow \quad r = 12.$$

This integer is in the allowed range from zero to eighteen. It identifies the thirteenth term, since the term index is $r + 1$. Its sign is positive, and its power of three is zero:

$$T_{13} = \binom{18}{12}(-1)^{12}3^{36-36} = \binom{18}{6}.$$

Step 3: Compare the coefficient with the given multiple Evaluate the binomial coefficient and divide by 221:

$$\binom{18}{6} = \frac{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13}{720} = 18564, \qquad k = \frac{18564}{221} = 84.$$

Indeed, $221 \times 84 = 18564$. The proposed values 78, 168 and 198 give different multiples of 221, so options (B), (C) and (D) cannot represent this constant term.

Step 4: Conclusion The term independent of x is $18564 = 221 \times 84$, so $\boxed{k = 84}$. Hence, the correct option is (A).

Quick Tip to Solve: Find the zero exponent before evaluating coefficients; here the same value of r also cancels every power of three.

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10. Let $\mathrm{P}(3\cos\alpha, 2\sin\alpha), \alpha \neq 0$, be a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1, \mathrm{Q}$ be a point on the circle $x^2 + y^2 - 14x - 14y + 82 = 0$ and R be a point on the line $x + y = 5$ such that the centroid of the triangle PQR is $\left(2 + \cos\alpha, 3 + \frac{2}{3}\sin\alpha\right)$. Then the sum of the ordinates of all possible points R is: [4 marks]

(A) 6

(B) 2

(C) 4

(D) 8

Correct Answer: (D) 8

Solution:

Concept:

- A triangle's centroid is the coordinate-wise average of its three vertices: $3G = P + Q + R$.
- A point on $x + y = 5$ can be written as $(t, 5 - t)$. Substituting the corresponding coordinates of Q into its circle gives all possible values of t.

Step 1: Eliminate the ellipse parameter using the centroid Three times the given centroid is $(6 + 3\cos\alpha, 9 + 2\sin\alpha)$. Subtract the coordinates of $P = (3\cos\alpha, 2\sin\alpha)$:

$$Q + R = 3G - P = (6, 9).$$

Thus the restrictions on Q and R are independent of α; no trigonometric equation needs to be solved.

The ellipse coordinates cancel exactly, rather than being approximated.

Step 2: Express Q in terms of the abscissa of R Write $R = (t, 5 - t)$, which automatically satisfies its line equation. The coordinate sum just found gives

$$Q = (6 - t, 9 - (5 - t)) = (6 - t, 4 + t).$$

Complete squares in the equation of the circle:

$$(x - 7)^2 + (y - 7)^2 = 16.$$

Consequently, the coordinates of Q must satisfy

$$(t + 1)^2 + (t - 3)^2 = 16.$$

Step 3: Find the two possible ordinates Expand and factor the last equation:

$$2t^2 - 4t - 6 = 0 \quad \Rightarrow \quad (t - 3)(t + 1) = 0.$$

For $t = 3$, $R = (3, 2)$ and $Q = (3, 7)$. For $t = -1$, $R = (-1, 6)$ and $Q = (7, 3)$. Both corresponding points Q lie on the circle, each four units from its centre. Both choices can form a nondegenerate triangle with an allowed point P; for example, take $\alpha = \pi/2$. The required sum of ordinates is therefore $2 + 6 = 8$. Options (A) and (B) each give only one ordinate; option (C) is neither their sum nor another possible total.

Step 4: Conclusion The sum of the ordinates of all possible points R is $\boxed{8}$. Hence, the correct option is (D).

Quick Tip to Solve: Compute three times the centroid minus the known vertex first; matching trigonometric terms can cancel before any conic substitution.

11. Let $\mathrm{H} : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be a hyperbola such that the distance between its foci is 6 and the distance between its directrices is $\frac{8}{3}$. If the line $x = \alpha$ intersects the hyperbola H at the points A and B such that the area of the triangle AOB is $4\sqrt{15}$, where O is the origin, then α^2 equals [4 marks]

(A) 12

(B) 16

(C) 24

(D) 25

Correct Answer: (B) 16

Solution:

Concept:

- For $x^2/a^2 - y^2/b^2 = 1$, the foci are $(\pm c, 0)$, where $c^2 = a^2 + b^2$. Its eccentricity is $e = c/a$, and its directrices are $x = \pm a/e = \pm a^2/c$.

- A vertical chord at $x = \alpha$ has endpoints $(\alpha, \pm y_0)$. With the origin as the third vertex, its triangle has area $\frac{1}{2}(2y_0)|\alpha|$.

Step 1: Determine the hyperbola from the two distances The distance between the foci is $2c = 6$, so $c = 3$. The distance between the directrices gives

$$\frac{2a^2}{c} = \frac{8}{3} \quad \Rightarrow \quad \frac{2a^2}{3} = \frac{8}{3} \quad \Rightarrow \quad a^2 = 4.$$

Therefore $b^2 = c^2 - a^2 = 9 - 4 = 5$, and the hyperbola is

$$\frac{x^2}{4} - \frac{y^2}{5} = 1.$$

The directrix distance involves a^2/c, not the focal distance c.

Step 2: Express the chord length and the area At $x = \alpha$, the positive ordinate satisfies

$$y_0^2 = \frac{5}{4}(\alpha^2 - 4).$$

Two distinct intersection points require $\alpha^2 > 4$. The base AB has length $2y_0$, while the perpendicular distance of the origin from its vertical line is $|\alpha|$. Hence

$$|\alpha|\sqrt{\frac{5}{4}(\alpha^2 - 4)} = 4\sqrt{15}.$$

The absolute value permits a chord on either branch of the hyperbola.

Step 3: Solve for the square of alpha Both sides of the area equation are nonnegative. Squaring and setting $t = \alpha^2$ gives

$$\frac{5}{4}t(t-4) = 240 \quad \Rightarrow \quad t^2 - 4t - 192 = 0 \quad \Rightarrow \quad (t-16)(t+12) = 0.$$

Since $t > 4$, retain $t = 16$ and reject $t = -12$. Indeed, $|\alpha| = 4$ gives $y_0 = \sqrt{15}$ and the stated area $4\sqrt{15}$. Options (A), (C) and (D), namely 12, 24 and 25, do not satisfy $t(t-4) = 192$.

Step 4: Conclusion The required value is $\boxed{\alpha^2 = 16}$. Hence, the correct option is (B).

Quick Tip to Solve: For a vertical hyperbola chord, area with the origin is absolute abscissa times positive ordinate; solve directly for the squared abscissa.

12. $\max\limits_{0\le x\le\pi}\left(16\sin\left(\frac{x}{2}\right)\cos^3\left(\frac{x}{2}\right)\right)$ **is equal to:** [4 marks]

(A) $\dfrac{3\sqrt{3}}{2}$

(B) $3\sqrt{3}$

(C) $4\sqrt{3}$

(D) $6\sqrt{3}$

Correct Answer: (B) $3\sqrt{3}$

Solution:

Concept:

- A continuous function on a closed interval attains a maximum. To locate it, compare the values at interior stationary points and at the endpoints.
- The substitution $\theta = x/2$ changes the interval $0 \le x \le \pi$ into $0 \le \theta \le \pi/2$. It is one-to-one, so maximizing in θ gives the same maximum value.

Step 1: Rewrite the function and differentiate Put $\theta = x/2$. The expression becomes

$$F(\theta) = 16\sin\theta\cos^3\theta.$$

Applying the product rule with respect to θ,

$$\begin{aligned} F'(\theta) &= 16\big(\cos^4\theta - 3\sin^2\theta\cos^2\theta\big) \\ &= 16\cos^2\theta\big(1 - 4\sin^2\theta\big). \end{aligned}$$

The domain restriction is essential: both sine and cosine are nonnegative throughout this interval.

Step 2: Find the interior stationary point For $0 < \theta < \pi/2$, the factor $\cos^2\theta$ is positive. Therefore an interior stationary point must satisfy

$$1 - 4\sin^2\theta = 0 \quad\Longrightarrow\quad \sin\theta = \frac{1}{2} \quad\Longrightarrow\quad \theta = \frac{\pi}{6}.$$

The derivative is positive before this point and negative after it, since $\sin\theta$ increases on the interval. Thus this is the unique interior maximum, corresponding to $x = \pi/3$.

Step 3: Evaluate the maximum and check the endpoints The endpoint values are $F(0) = F(\pi/2) = 0$. At the interior point,

$$F\left(\frac{\pi}{6}\right) = 16\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)^3 = 16\cdot\frac{1}{2}\cdot\frac{3\sqrt{3}}{8} = 3\sqrt{3}.$$

This positive value exceeds both endpoint values and hence is the global maximum. Option (A) is smaller than this attained value, while the values in (C) and (D) exceed the established maximum and cannot be attained.

Step 4: Conclusion The maximum is $\boxed{3\sqrt{3}}$, attained at $x = \pi/3$. Hence, the correct option is (B).

Quick Tip to Solve: For a half-angle expression, first substitute $\theta = x/2$ and transform the interval. Always compare the stationary value with both endpoint values.

13. The shortest distance between the lines

$$\vec{r} = \left(\frac{1}{3}\hat{i} + 2\hat{j} + \frac{8}{3}\hat{k}\right) + \lambda(2\hat{i} - 5\hat{j} + 6\hat{k})$$

and $\vec{r} = \left(-\frac{2}{3}\hat{i} - \frac{1}{3}\hat{k}\right) + \mu(\hat{j} - \hat{k}), \lambda, \mu \in \mathbb{R}$**, is:** [4 marks]

(A) $\sqrt{5}$

(B) 3

(C) $2\sqrt{3}$

(D) $\sqrt{15}$

Correct Answer: (B) 3

Solution:

Concept:

- For lines $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{c} + \mu\vec{d}$, a common perpendicular has direction $\vec{b} \times \vec{d}$.
- The shortest distance is the magnitude of the projection of $\vec{c} - \vec{a}$ on that direction: $D = |(\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d})|/|\vec{b} \times \vec{d}|$, when the direction vectors are not parallel.

Step 1: Identify the points and directions Read the constant vectors as points on the two lines and the parameter coefficients as their directions:

$$\vec{a} = (1/3, 2, 8/3), \qquad \vec{b} = (2, -5, 6),$$
$$\vec{c} = (-2/3, 0, -1/3), \quad \vec{d} = (0, 1, -1).$$

The vector joining the selected points is therefore

$$\vec{c} - \vec{a} = (-1, -2, -3).$$

These points need not be the closest points; projection onto the common normal removes their components along the lines.

Step 2: Compute the common normal and projection Calculate the cross product carefully, retaining its signs:

$$\vec{b} \times \vec{d} = (5 - 6,\ 0 - (-2),\ 2 - 0) = (-1, 2, 2).$$

Its magnitude is $\sqrt{1 + 4 + 4} = 3$, so the lines are not parallel. The scalar triple product is

$$(-1, -2, -3) \cdot (-1, 2, 2) = 1 - 4 - 6 = -9.$$

Taking the absolute value gives the non-negative distance:

$$D = \frac{|-9|}{3} = 3.$$

The other listed values $\sqrt{5}$, $2\sqrt{3}$ and $\sqrt{15}$ differ from this exact projection and are excluded.

Step 3: Conclusion The shortest distance is 3 units. Hence, the correct option is (B).

Quick Tip to Solve: Use any point on each line, but project their joining vector onto the unit common normal. Dividing by the cross-product magnitude is essential.

14. If $\left(2\alpha+1, \alpha^2-3\alpha, \frac{\alpha-1}{2}\right)$ is the image of $(\alpha, 2\alpha, 1)$ in the line $\frac{x-2}{3}=\frac{y-1}{2}=\frac{z}{1}$, then the possible value(s) of α is (are) [4 marks]

(A) Only 3

(B) Only 3 and - 1

(C) Only $3, \frac{1}{4}$ and -1

(D) Only 3 and $\frac{1}{4}$

Correct Answer: (A) Only 3

Solution:

Concept:

- If Q is the image of P in a line, the midpoint of PQ lies on that line and PQ is perpendicular to its direction.
- Both conditions must hold. A root obtained from perpendicularity alone is only a candidate for the reflection.

Step 1: Impose the midpoint condition Let $P=(\alpha, 2\alpha, 1)$ and $Q=(2\alpha+1, \alpha^2-3\alpha, (\alpha-1)/2)$. Their midpoint is

$$M=\left(\frac{3\alpha+1}{2}, \frac{\alpha^2-\alpha}{2}, \frac{\alpha+1}{4}\right).$$

A point on the given line has coordinates $(2+3t, 1+2t, t)$. From the third coordinate of M, $t=(\alpha+1)/4$. Substitution in its first coordinate gives

$$\frac{3\alpha+1}{2}=2+\frac{3(\alpha+1)}{4}.$$

Thus $6\alpha+2=11+3\alpha$, which forces $\alpha=3$. No other value can have its midpoint on this line.

Step 2: Verify the reflection and reject extraneous values For $\alpha=3$, the points are $P=(3,6,1)$, $Q=(7,0,1)$, and their midpoint is $M=(5,3,1)$. It lies on the line at $t=1$. Also,

$$\overrightarrow{PQ}=(4,-6,0), \qquad (4,-6,0)\cdot(3,2,1)=12-12=0.$$

Hence the midpoint is the perpendicular foot, proving that the two points are indeed images. For comparison, perpendicularity alone yields

$$3(\alpha+1)+2(\alpha^2-5\alpha)+\frac{\alpha-3}{2}=0,$$

$$(4\alpha-1)(\alpha-3)=0.$$

The additional root $1/4$ fails the midpoint condition; -1 fails it too. Consequently every choice including either of these extra values is incorrect.

Step 3: Conclusion The only possible value is $\alpha=3$. Hence, the correct option is (A).

Quick Tip to Solve: For reflection in a line, check both the midpoint and perpendicularity. Perpendicularity by itself can introduce a spurious root.

15. Let $\hat{u}$ and $\hat{v}$ be unit vectors inclined at an acute angle such that $|\hat{u} \times \hat{v}| = \frac{\sqrt{3}}{2}$. If $\overrightarrow{A} = \lambda\hat{u} + \hat{v} + (\hat{u} \times \hat{v})$, then λ is equal to: [4 marks]

(A) $\frac{4}{3}(\overrightarrow{A} \cdot \hat{u}) - \frac{2}{3}(\overrightarrow{A} \cdot \hat{v})$

(B) $\frac{2}{3}(\overrightarrow{A} \cdot \hat{u}) - \frac{1}{3}(\overrightarrow{A} \cdot \hat{v})$

(C) $\frac{4}{3}(\overrightarrow{A} \cdot \hat{u}) + \frac{2}{3}(\overrightarrow{A} \cdot \hat{v})$

(D) $(\overrightarrow{A} \cdot \hat{u}) - \frac{1}{2}(\overrightarrow{A} \cdot \hat{v})$

Correct Answer: (A) $\frac{4}{3}(\overrightarrow{A} \cdot \hat{u}) - \frac{2}{3}(\overrightarrow{A} \cdot \hat{v})$

Solution:

Concept:

- For unit vectors making an angle θ, $|\hat{u} \times \hat{v}| = \sin\theta$ and $\hat{u} \cdot \hat{v} = \cos\theta$. The acute-angle restriction fixes the positive sign of the cosine.
- The cross product $\hat{u} \times \hat{v}$ is perpendicular to both original vectors, so dotting with either vector removes that component.

Step 1: Find the mutual scalar product The given magnitude implies $\sin\theta = \sqrt{3}/2$. Since θ is acute,

$$\cos\theta = +\sqrt{1 - \sin^2\theta} = \frac{1}{2}.$$

Thus $\hat{u} \cdot \hat{v} = 1/2$, while $\hat{u} \cdot \hat{u} = \hat{v} \cdot \hat{v} = 1$. Set $p = \vec{A} \cdot \hat{u}$ and $q = \vec{A} \cdot \hat{v}$. Taking the two scalar products in the defining expression for $\vec{A}$ gives

$$p = \lambda + \frac{1}{2}, \qquad q = \frac{\lambda}{2} + 1.$$

There is no contribution from the cross-product term in either equation.

Step 2: Eliminate the unwanted component Subtract half of the second projection from the first:

$$p - \frac{q}{2} = \lambda + \frac{1}{2} - \frac{\lambda}{4} - \frac{1}{2} = \frac{3\lambda}{4}.$$

Hence

$$\lambda = \frac{4}{3}p - \frac{2}{3}q = \frac{4}{3}(\vec{A} \cdot \hat{u}) - \frac{2}{3}(\vec{A} \cdot \hat{v}).$$

This identity holds for every allowed λ. The expression in option (B) equals $\lambda/2$, and that in (D) equals $3\lambda/4$, so neither is the required general expression. The plus sign in option (C) gives $(5\lambda+4)/3$; it fails to cancel the contribution of $\hat{v}$. Agreement at an isolated value would not make any of these a valid identity.

Step 3: Conclusion We obtain $\lambda = \frac{4}{3}(\vec{A} \cdot \hat{u}) - \frac{2}{3}(\vec{A} \cdot \hat{v})$. Hence, the correct option is (A).

Quick Tip to Solve: Dot with the original unit vectors to remove their cross product, then eliminate the unwanted scalar projection. Use the angle restriction to choose the cosine sign.

16. Let for some $\alpha \in \mathbb{R}, f : \mathbb{R} \to \mathbb{R}$ be a function satisfying $f(x+y) = f(x) + 2y^2 + y + \alpha xy$ for all $x, y \in \mathbb{R}$. If $f(0) = -1$ and $f(1) = 2$, then the value of $\sum_{n=1}^{5} (\alpha + f(n))$ is : [4 marks]

(A) 110

(B) 140

(C) 150

(D) 170

Correct Answer: (B) 140

Solution:

Concept:

- An identity valid for all real x, y can be evaluated at $x = 0$ to determine f, then compared with the original identity to find its parameter.
- The finite sums needed are $\sum_{n=1}^{m} n = m(m+1)/2$ and $\sum_{n=1}^{m} n^2 = m(m+1)(2m+1)/6$. A constant inside a sum contributes once for each term.

Step 1: Determine the function and its parameter Put $x = 0$ into the given relation. Since $f(0) = -1$,

$$f(y) = -1 + 2y^2 + y.$$

Therefore $f(t) = 2t^2 + t - 1$ for every real t. This also gives $f(1) = 2$, as required, so it is consistent with the second stated value. Now expand the difference using this formula:

$$\begin{aligned} f(x+y) - f(x) &= 2(x+y)^2 + (x+y) - 1 - (2x^2 + x - 1) \\ &= 2y^2 + y + 4xy. \end{aligned}$$

The supplied identity gives the same difference with αxy. Comparing these for all x, y, or simply choosing $x = y = 1$, gives $\alpha = 4$.

Step 2: Evaluate all five terms of the sum For an integer n, the summand is

$$\alpha + f(n) = 4 + (2n^2 + n - 1) = 2n^2 + n + 3.$$

Consequently

$$\begin{aligned} \sum_{n=1}^{5} (\alpha + f(n)) &= 2\sum_{n=1}^{5} n^2 + \sum_{n=1}^{5} n + 3 \cdot 5 \\ &= 2(55) + 15 + 15 = 140. \end{aligned}$$

As a direct check, the five summands are $6, 13, 24, 39, 58$, which also add to 140. The values 110, 150, and 170 fail this direct substitution. In particular, 110 retains only the quadratic contribution and misses the linear and constant terms.

Step 3: Conclusion The required sum is 140. Hence, the correct option is (B).

Quick Tip to Solve: Set one variable to zero first. When summing $\alpha + f(n)$, remember that α is included in every summand, not just once.

17. Let

$\mathrm{A} = \{(a, b, c) : a, b, c$ **are non-negative integers and** $a + b + 2c = 22\}$**.**

Then $n(\mathrm{A})$ **is equal to :** [4 marks]

(A) 121

(B) 124

(C) 144

(D) 169

Correct Answer: (C) 144

Solution:

Concept:

- Fixing one variable partitions the set into disjoint cases. For a non-negative integer m, the equation $a + b = m$ has $m + 1$ ordered non-negative integer solutions.
- The sum of the first k positive odd integers is k^2. Equivalently, an arithmetic progression sums to half the number of terms times the sum of its end terms.

Step 1: Count the pairs for each possible third coordinate Since all three entries are non-negative and $a + b + 2c = 22$, we must have

$$0 \le 2c \le 22, \qquad c = 0, 1, \ldots, 11.$$

For a fixed c, let $m = 22 - 2c$. Then $a + b = m$, and choosing a from 0 through m determines $b = m - a$ uniquely. Thus the number of pairs is

$$m + 1 = 23 - 2c.$$

The entries of a triple are ordered: for instance, $(0, 22, 0)$ and $(22, 0, 0)$ are distinct. Both zero endpoints are allowed, which is why the count is $m + 1$, not m.

Step 2: Sum the disjoint cases Different values of c give different triples, so there is no overlap when we add the counts:

$$\begin{aligned} n(A) &= \sum_{c=0}^{11} (23 - 2c) \\ &= 23 + 21 + 19 + \cdots + 3 + 1 \\ &= \frac{12(23 + 1)}{2} = 144. \end{aligned}$$

One can also check the arithmetic as $12 \cdot 23 - 2(11 \cdot 12/2) = 276 - 132 = 144$. There are twelve cases, including $c = 11$, for which the single pair is $(a, b) = (0, 0)$. The options $121 = 11^2$ and $169 = 13^2$ correspond to the wrong number of odd terms. The remaining value 124 also fails the explicit case sum.

Step 3: Conclusion The set contains 144 ordered triples. Hence, the correct option is (C).

Quick Tip to Solve: For a weighted equation, fix the variable with the non-unit coefficient first; count the remaining two-variable solutions including both zero endpoints.

18. The area of the region bounded by the curves $x + 3y^2 = 0$ and $x + 4y^2 = 1$ is equal to : [4 marks]

(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) $\frac{4}{3}$
(D) $\frac{5}{3}$

Correct Answer: (C) $\frac{4}{3}$

Solution:

Concept:

- When curves are written as x in terms of y, horizontal strips give area $\int (x_{\text{right}} - x_{\text{left}})\, dy$. The bounds are the intersection ordinates.
- An area is non-negative. Determine which curve is on the right throughout the interval before subtracting; symmetry may halve the integration work.

Step 1: Find the intersections and the strip width Rewrite the curves without changing their signs:

$$x = -3y^2, \qquad x = 1 - 4y^2.$$

At an intersection the two expressions for x agree:

$$-3y^2 = 1 - 4y^2 \quad \Longrightarrow \quad y^2 = 1.$$

Thus the intersections are $(-3, 1)$ and $(-3, -1)$, and the bounded region lies between $y = -1$ and $y = 1$. Within this interval,

$$(1 - 4y^2) - (-3y^2) = 1 - y^2 \geq 0.$$

Therefore $x = 1 - 4y^2$ is the right-hand boundary and $x = -3y^2$ is the left-hand boundary. This inequality confirms the boundary order across the entire region, including where both x-coordinates are negative.

Step 2: Integrate the non-negative horizontal width Use the width $1 - y^2$ found above:

$$\begin{aligned} \mathcal{A} &= \int_{-1}^{1} (1 - y^2)\, dy \\ &= \left[y - \frac{y^3}{3} \right]_{-1}^{1} \\ &= \left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right) = \frac{4}{3}. \end{aligned}$$

The region is symmetric about the x-axis. Its upper half has area $\int_0^1 (1 - y^2)\, dy = 2/3$, which explains why option (B) counts only half the requested region. Doubling gives $4/3$; the values $1/3$ and $5/3$ do not agree with the integral over the full interval.

Step 3: Conclusion The bounded area is $\frac{4}{3}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Use horizontal strips when both curves give x directly in terms of y. Integrate right minus left between the two intersection ordinates.

19. Let $y = y(x)$ be the solution of the differential equation :

$$\frac{dy}{dx} + \left(\frac{6x^2 + \left(3x^2 + 2x^3 + 4\right) e^{-2x}}{\left(x^3 + 2\right)\left(2 + e^{-2x}\right)} \right) y = 2 + e^{-2x}$$

$x \in (-1, 2)$, satisfying $y(0) = \frac{3}{2}$. If $y(1) = \alpha\left(2 + e^{-2}\right)$, then α is equal to : [4 marks]

(A) $\frac{13}{8}$

(B) $\frac{6}{13}$

(C) $\frac{12}{13}$

(D) $\frac{13}{12}$

Correct Answer: (D) $\frac{13}{12}$

Solution:

Concept:

- For a linear differential equation $y' + P(x)y = Q(x)$, the integrating factor is $I(x) = e^{\int P(x)\,dx}$, and $(Iy)' = IQ$.
- A coefficient written as a sum of logarithmic derivatives makes the integrating factor easy to identify. Here both $x^3 + 2$ and $2 + e^{-2x}$ are positive on the given interval.

Step 1: Split the coefficient into logarithmic derivatives Combining the following fractions over their common denominator reproduces the given numerator exactly:

$$P(x) = \frac{3x^2}{x^3 + 2} + \frac{2e^{-2x}}{2 + e^{-2x}} = \frac{d}{dx}\left[\log(x^3 + 2) - \log(2 + e^{-2x})\right].$$

The minus sign before the second logarithm is necessary because the derivative of e^{-2x} is negative. Therefore choose

$$I(x) = \frac{x^3 + 2}{2 + e^{-2x}}.$$

Step 2: Integrate and apply the initial value Multiply the differential equation by this integrating factor. The right side simplifies to a polynomial:

$$\frac{d}{dx}\left(\frac{(x^3 + 2)y}{2 + e^{-2x}}\right) = x^3 + 2.$$

Integrating gives

$$\frac{(x^3 + 2)y}{2 + e^{-2x}} = \frac{x^4}{4} + 2x + C.$$

At $x = 0$, the left side equals $(2/3)(3/2) = 1$, so $C = 1$.

Step 3: Evaluate the required coefficient At $x = 1$, the polynomial on the right is $1/4 + 2 + 1 = 13/4$. Thus

$$y(1) = \frac{2 + e^{-2}}{3} \cdot \frac{13}{4} = \frac{13}{12}(2 + e^{-2}).$$

Comparing coefficients gives $\alpha = 13/12$. None of $13/8$, $6/13$, or $12/13$ equals this value; the reciprocal $12/13$ would result from reversing the final ratio.

Step 4: Conclusion $\alpha = \boxed{13/12}$. Hence, the correct option is (D).

Quick Tip to Solve: Before integrating a complicated linear coefficient, look for the logarithmic derivatives of its denominator factors.

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20. The integral $\int_0^1 \cot^{-1}\left(1+x+x^2\right) dx$ is equal to : [4 marks]

(A) $2\tan^{-1}2 + \frac{1}{2}\log_e\left(\frac{5}{4}\right) + \frac{\pi}{2}$

(B) $2\tan^{-1}2 + \frac{1}{2}\log_e\left(\frac{5}{4}\right) - \frac{\pi}{2}$

(C) $2\tan^{-1}2 - \frac{1}{2}\log_e\left(\frac{5}{4}\right) + \frac{\pi}{2}$

(D) $2\tan^{-1}2 - \frac{1}{2}\log_e\left(\frac{5}{4}\right) - \frac{\pi}{2}$

Correct Answer: (D) $2\tan^{-1}2 - \frac{1}{2}\log_e\left(\frac{5}{4}\right) - \frac{\pi}{2}$

Solution:

Concept:

- For positive arguments, $\cot^{-1}u = \tan^{-1}(1/u)$. The tangent subtraction identity can turn a quadratic reciprocal into a difference of inverse tangents.
- Integration by parts gives $F(t) = t\tan^{-1}t - \frac{1}{2}\log(1+t^2)$ as an antiderivative of $\tan^{-1}t$.

Step 1: Rewrite the inverse cotangent For $0 \le x \le 1$, let $\theta = \tan^{-1}(x+1) - \tan^{-1}x$. Then

$$\tan\theta = \frac{(x+1)-x}{1+x(x+1)} = \frac{1}{1+x+x^2}.$$

Since $0 < \theta < \pi/2$, there is no branch correction. Consequently

$$\cot^{-1}(1+x+x^2) = \tan^{-1}(x+1) - \tan^{-1}x.$$

This identity also agrees at $x = 0$, where both sides are $\pi/4$.

Step 2: Integrate the two terms Using $t = x+1$ in the first integral,

$$I = \int_1^2 \tan^{-1}t\,dt - \int_0^1 \tan^{-1}t\,dt = F(2) - 2F(1) + F(0).$$

Integration by parts justifies the stated primitive: take $u = \tan^{-1}t$, $dv = dt$, so the remaining integral is $\int t/(1+t^2)\,dt = \frac{1}{2}\log(1+t^2)$.

Step 3: Substitute the endpoints and compare options The required endpoint values are

$$F(2) = 2\tan^{-1}2 - \tfrac{1}{2}\log 5, \qquad F(1) = \tfrac{\pi}{4} - \tfrac{1}{2}\log 2, \qquad F(0) = 0.$$

Therefore

$$I = 2\tan^{-1}2 - \frac{\pi}{2} - \frac{1}{2}\log\left(\frac{5}{4}\right).$$

Options (A) and (C) have the wrong sign for the $\pi/2$ term and exceed the bound $I \le \pi/4$. Option (B)

has the wrong logarithm sign; the endpoint subtraction yields $\log 2 - \frac{1}{2}\log 5$, which is negative.

Step 4: Conclusion $I = \boxed{2\tan^{-1}2 - \frac{1}{2}\log_e\left(\frac{5}{4}\right) - \frac{\pi}{2}}$. Hence, the correct option is (D).

Quick Tip to Solve: When an inverse cotangent contains x^2+x+1, test the difference $\tan^{-1}(x+1) - \tan^{-1}x$ before using integration by parts.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. From a month of 31 days, 3 different dates are selected at random. If the probability that these dates are in an increasing A.P. is equal to $\frac{a}{b}$, where $a, b \in$ N and $\gcd(a,b) = 1$, then $a+b$ is equal to ____

[4 marks]

Solution:

Concept:

- Choosing three different dates uniformly gives $\binom{31}{3}$ equally likely three-element subsets. Sort each subset into increasing order to test the arithmetic-progression condition.
- Every increasing three-term arithmetic progression has the unique form $r, r+d, r+2d$, with integer common difference $d \geq 1$. Its last term must not exceed 31.

Step 1: Count all possible selections The dates are the integers from 1 through 31. Order of selection is irrelevant because the selected dates can be listed increasingly. Thus the sample-space size is

$$N = \binom{31}{3} = \frac{31 \cdot 30 \cdot 29}{6} = 4495.$$

We must use the same unordered-selection convention for favourable cases; multiplying either count alone by 3! would give an incorrect probability.

Step 2: Count progressions by their common difference For a fixed positive difference d, the first term satisfies

$$1 \leq r \leq 31 - 2d.$$

There are therefore $31 - 2d$ choices. Such a choice exists precisely for $1 \leq d \leq 15$. Each progression has a unique first term and difference, so none is counted twice. Hence

$$M = \sum_{d=1}^{15}(31 - 2d) = 31 \cdot 15 - 2\frac{15 \cdot 16}{2} = 225.$$

Equivalently, the counts form $29 + 27 + \cdots + 1$, the sum of the first 15 odd numbers.

Step 3: Reduce the probability to lowest terms The required probability is

$$\frac{M}{N} = \frac{225}{4495} = \frac{45}{899}.$$

Here $45 = 3^2 \cdot 5$, whereas $899 = 29 \cdot 31$, so the numerator and denominator are coprime. Thus $a = 45$,

$b = 899$, and

$$a + b = 45 + 899 = 944.$$

The answer is a pure integer, with no physical unit.

Step 4: Conclusion The required value is $\boxed{944}$.

Quick Tip to Solve: Count three-term progressions by their positive common difference; for dates 1 through n, each difference contributes $n - 2d$ starting dates.

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22. Let $f(x) = \begin{cases} e^{x-1} & , x < 0 \\ x^2 - 5x + 6 & , x \geq 0 \end{cases}$ **and** $g(x) = f(|x|) + |f(x)|$**. If the number of points where** g **is not continuous and is not differentiable are** α **and** β **respectively, then** $\alpha + \beta$ **is equal to ____** [4 marks]

Solution:

Concept:

- The possible exceptional points of $g(x) = f(|x|) + |f(x)|$ occur at the branch junction $x = 0$ and at zeros where the expression inside an absolute value changes sign.
- A discontinuity always prevents differentiability. At a continuous junction, compare the left and right derivatives to detect a corner.

Step 1: Write the function on its smooth intervals For $x < 0$, $|x| = -x$, so $f(|x|) = x^2 + 5x + 6$. Also $f(x) = e^{x-1} > 0$, giving

$$g(x) = x^2 + 5x + 6 + e^{x-1} \quad (x < 0).$$

For $x \geq 0$, put $p(x) = x^2 - 5x + 6 = (x - 2)(x - 3)$. Then $g(x) = p(x) + |p(x)|$, and hence

$$g(x) = \begin{cases} 2(x^2 - 5x + 6), & 0 \leq x \leq 2, \\ 0, & 2 < x < 3, \\ 2(x^2 - 5x + 6), & x \geq 3. \end{cases}$$

Every open interval in these formulas is smooth; only 0, 2 and 3 need checking.

Step 2: Check continuity at the three candidates At the origin,

$$\lim_{x \to 0^-} g(x) = 6 + e^{-1}, \qquad \lim_{x \to 0^+} g(x) = g(0) = 12.$$

The one-sided limits differ, so 0 is a discontinuity. At 2 and 3, the adjoining formulas both give 0 and the corresponding limits are also 0. Thus those two points are continuous, and the number of discontinuities is $\alpha = 1$.

Step 3: Check differentiability and count the points At 0 the discontinuity already proves non-differentiability. The derivative of $2p(x)$ is $4x - 10$, whereas the derivative on $(2, 3)$ is zero. Therefore

$$g'_-(2) = -2, \quad g'_+(2) = 0; \qquad g'_-(3) = 0, \quad g'_+(3) = 2.$$

Both junctions have corners. There are no other candidates: the negative branch is a polynomial plus

an exponential and is smooth throughout $x < 0$. Hence $\beta = 3$, counting the points 0, 2 and 3.

Step 4: Conclusion $\alpha = 1$ and $\beta = 3$, so $\boxed{\alpha + \beta = 4}$.

Quick Tip to Solve: For $p + |p|$, use $2p$ where $p \geq 0$ and zero where $p < 0$; then inspect the sign-changing roots and any original branch junction.

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23. Let A, B be points on the two half-lines $x - \sqrt{3}|y| = \alpha, \alpha > 0$ at a distance of α from their point of intersection P. The line segment AB meets the angle bisector of the given half-lines at the point Q. If $PQ = \frac{9}{2}$ and R is the radius of the circumcircle of $\triangle\mathrm{PAB}$, then $\frac{\alpha^2}{R}$ is equal to ____ [4 marks]

Solution:

Concept:

- The equation $x - \sqrt{3}|y| = \alpha$ represents two rays with common vertex $P = (\alpha, 0)$, symmetric about the positive horizontal direction.
- An isosceles triangle with equal sides $PA = PB = \alpha$ and included angle 60° is equilateral. Its altitude is $\sqrt{3}\alpha/2$ and its circumradius is $\alpha/\sqrt{3}$.

Step 1: Identify the rays and their angle For the upper ray, $y = (x - \alpha)/\sqrt{3}$ with $x \geq \alpha$; the lower ray has $y = -(x - \alpha)/\sqrt{3}$. They make angles 30° and -30° with the positive horizontal axis. Thus

$$\angle APB = 60^\circ.$$

Because A and B lie a distance α from P on their respective rays, their coordinates are

$$A = \left(\alpha + \frac{\sqrt{3}\alpha}{2}, \frac{\alpha}{2}\right), \quad B = \left(\alpha + \frac{\sqrt{3}\alpha}{2}, -\frac{\alpha}{2}\right).$$

In particular, $AB = \alpha$, confirming that the triangle is equilateral.

Step 2: Use the given angle-bisector distance The internal angle bisector is $y = 0$, and it meets AB at its midpoint

$$Q = \left(\alpha + \frac{\sqrt{3}\alpha}{2}, 0\right).$$

Therefore the given length yields

$$PQ = \frac{\sqrt{3}\alpha}{2} = \frac{9}{2} \quad \Longrightarrow \quad \alpha = \frac{9}{\sqrt{3}} = 3\sqrt{3}.$$

The positive root agrees with the stated condition $\alpha > 0$.

Step 3: Find the circumradius and the requested ratio Using the extended sine rule opposite angle P,

$$2R = \frac{AB}{\sin 60^\circ} = \frac{\alpha}{\sqrt{3}/2}, \qquad R = \frac{\alpha}{\sqrt{3}} = 3.$$

Finally,

$$\frac{\alpha^2}{R} = \frac{(3\sqrt{3})^2}{3} = \frac{27}{3} = 9.$$

The equal-side and angle information are essential: the radius is the circumradius, not the smaller inradius of the equilateral triangle.

Step 4: Conclusion The required value is $\boxed{9}$ in the coordinate length units.

Quick Tip to Solve: Resolve an absolute-value line into its two rays first; their symmetry often makes the angle-bisector segment an altitude of an isosceles triangle.

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24. Let A, B and C be the vertices of a variable right angled triangle inscribed in the parabola $y^2 = 16x$. Let the vertex B containing the right angle be $(4, 8)$ and the locus of the centroid of $\triangle ABC$ be a conic C_0. Then three times the length of latus rectum of C_0 is ____ [4 marks]

Solution:

Concept:

- A point on $y^2 = 4ax$ has parameter form $(at^2, 2at)$. Here $a = 4$, so the points are $(4t^2, 8t)$, and B has parameter 1.
- The centroid is the coordinate-wise average of the three vertices. Perpendicular chord vectors have zero dot product, and a parabola $Y^2 = 4A(X - h)$ has latus rectum length $4A$.

Step 1: Express the right-angle condition in parameters Let

$$A = (4t^2, 8t), \quad B = (4, 8), \quad C = (4s^2, 8s).$$

For a nondegenerate triangle, neither t nor s equals 1. The chord vectors based at B are

$$\overrightarrow{BA} = 4(t-1)(t+1, 2), \qquad \overrightarrow{BC} = 4(s-1)(s+1, 2).$$

Their dot product must vanish. Cancelling the nonzero factors gives

$$(t+1)(s+1) + 4 = 0, \qquad ts + t + s = -5.$$

Using vectors also handles a possible vertical chord without dividing by its horizontal component.

Step 2: Write the centroid using symmetric sums Let the centroid be (X, Y), and put $u = t + s$. The perpendicularity condition gives $ts = -5 - u$. Hence

$$X = \frac{4}{3}(t^2 + s^2 + 1) = \frac{4}{3}\left(u^2 - 2ts + 1\right) = \frac{4}{3}(u^2 + 2u + 11),$$

while

$$Y = \frac{8}{3}(t + s + 1) = \frac{8}{3}(u + 1).$$

Thus both coordinates now depend on the single symmetric parameter u; the individual endpoint parameters no longer need to be solved.

Step 3: Eliminate the parameter and read the latus rectum Complete the square in X and substitute $u+1=3Y/8$:

$$X=\frac{4}{3}\left((u+1)^2+10\right)=\frac{3Y^2}{16}+\frac{40}{3}.$$

Therefore the conic containing the centroid locus is

$$Y^2=\frac{16}{3}\left(X-\frac{40}{3}\right).$$

Its horizontal translation changes the vertex but not its latus rectum. Comparing with $Y^2=4A(X-h)$, the latus rectum length is $16/3$. Consequently three times this length is $3(16/3)=16$.

Step 4: Conclusion Three times the latus rectum length of C_0 is $\boxed{16}$ coordinate length units.

Quick Tip to Solve: For a centroid locus on a parabola, introduce the sum and product of the endpoint parameters; the right-angle condition usually eliminates the product.

25. Let f be a twice differentiable function such that

$$f(x)=\int_0^x \tan(t-x)dt-\int_0^x f(t)\tan t dt, x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$

Then $f''\left(\frac{\pi}{6}\right)+12f'\left(-\frac{\pi}{6}\right)+f\left(\frac{\pi}{6}\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- The fundamental theorem of calculus converts an integral equation into a differential equation; the original equation also supplies the initial value.
- A linear equation $f'+P(x)f=Q(x)$ has integrating factor $e^{\int P(x)\,dx}$. Here $\cos x>0$ throughout the given interval, so logarithms and $\sec x$ are well defined.

Step 1: Evaluate the first integral and find the initial value Put $u=t-x$, treating x as fixed in this integration. The limits become $-x$ and 0, so

$$\int_0^x \tan(t-x)\,dt=[-\ln(\cos u)]_{-x}^{0}=\ln(\cos x).$$

Consequently, $f(x)=\ln(\cos x)-\int_0^x f(t)\tan t\,dt$. At $x=0$, both integrals vanish, giving $f(0)=0$.

Step 2: Solve the resulting linear differential equation Differentiate with respect to x:

$$f'(x)+f(x)\tan x=-\tan x.$$

The integrating factor is $e^{\int \tan x\,dx}=\sec x$. Multiplying and integrating gives

$$(f\sec x)'=-\sec x\tan x,\qquad f\sec x=-\sec x+C.$$

Thus $f(x)=-1+C\cos x$. The initial value gives $C=1$, hence $f(x)=\cos x-1$.

Step 3: Substitute the requested arguments Since $f'(x) = -\sin x$ and $f''(x) = -\cos x$, the three contributions are

$$f''(\pi/6) = -\frac{\sqrt{3}}{2}, \qquad 12f'(-\pi/6) = 6, \qquad f(\pi/6) = \frac{\sqrt{3}}{2} - 1.$$

The cosine terms cancel, leaving $6 - 1 = 5$. The negative argument in the first derivative is essential for its positive contribution.

Step 4: Conclusion The required value is $\boxed{5}$, a dimensionless number.

Quick Tip to Solve: Take the initial value directly from the integral equation before differentiating; it fixes the integration constant without additional work.

Physics – Section A: Multiple Choice Questions (80 marks)

26. Match the LIST-I with LIST-II.

List-I		List-II	
A.	**Planck's constant**	**I.**	$\mathrm{ML^2T^{-2}}$
B.	**Stopping potential**	**II.**	$\mathrm{T^{-1}}$
C.	**Work function**	**III.**	$\mathrm{ML^2T^{-1}}$
D.	**Threshold frequency**	**IV.**	$\mathrm{ML^2T^{-3}A^{-1}}$

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-IV, C-I, D-II

(B) A-I, B-II, C-III, D-IV

(C) A-IV, B-III, C-I, D-II

(D) A-I, B-IV, C-III, D-II

Correct Answer: (A) A-III, B-IV, C-I, D-II

Solution:

Concept:

- Dimensions follow from defining equations: energy has dimensions $[E] = \mathrm{ML^2T^{-2}}$, and frequency has dimensions $\mathrm{T^{-1}}$.
- Electric potential is energy per unit charge. Since charge is current multiplied by time, $[q] = \mathrm{AT}$; the symbol A in dimensional expressions denotes electric current.

Step 1: Match Planck's constant and threshold frequency The photon-energy relation is $E = h\nu$. Dividing the dimensions of energy by those of frequency gives

$$[h] = \frac{\mathrm{ML^2T^{-2}}}{\mathrm{T^{-1}}} = \mathrm{ML^2T^{-1}}.$$

Hence Planck's constant, entry A, matches III. Threshold frequency is a frequency irrespective of the material, so its dimensions are $\mathrm{T^{-1}}$; entry D therefore matches II.

Step 2: Match stopping potential and work function The maximum photoelectron kinetic energy equals eV_s. Therefore the stopping potential has dimensions

$$[V_s] = \frac{[E]}{[e]} = \frac{\mathrm{ML^2T^{-2}}}{\mathrm{AT}} = \mathrm{ML^2T^{-3}A^{-1}}.$$

Thus B matches IV. The work function is the minimum energy required to release an electron; it has energy dimensions $\mathrm{ML^2T^{-2}}$, giving C-I.

Step 3: Compare the complete matching with the options Together the matches are A-III, B-IV, C-I and D-II. Option (B) incorrectly assigns energy dimensions to Planck's constant and inverse time to potential. Option (C) interchanges the dimensions of Planck's constant and stopping potential. Option (D) interchanges Planck's constant and work function. Only option (A) satisfies all four defining relations.

Step 4: Conclusion The matching is $\boxed{\text{A-III, B-IV, C-I, D-II}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use $E = h\nu$ and $E = qV$ as anchors: they distinguish energy, action and potential without memorising separate dimensional formulae.

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27. Two cars A and B are moving in the same direction along a straight line with speeds $100\ \mathrm{km/h}$ and $80\ \mathrm{km/h}$, respectively such that car A is moving ahead of car B. A person in car B throws a stone with a speed v so that it hits the car A with a speed of $5\ \mathrm{m/s}$. The value of v is ____ $\mathrm{km/h}$. [4 marks]

(A) 18

(B) 28

(C) 38

(D) 48

Correct Answer: (C) 38

Solution:

Concept:

- In one-dimensional motion, relative velocity is the algebraic difference of velocities measured in the same frame: $u_{S/A} = u_S - u_A$.
- A speed imparted by a person in car B is measured relative to B. For a forward throw, the stone's ground velocity is $u_S = u_B + v$.

Step 1: Express the impact speed in consistent units Take the direction of both cars as positive. The stone approaches car A from behind, so its velocity relative to A must be positive. The stated impact speed relative to that car is

$$u_{S/A} = 5 \times \frac{18}{5} = 18\,\mathrm{km\,h^{-1}}.$$

Car A is faster than car B by $100 - 80 = 20\,\mathrm{km\,h^{-1}}$. The throw must first overcome this difference before providing a positive approach speed.

Step 2: Add the velocities in the correct frames In the ground frame the stone must move at

$$u_S = u_A + u_{S/A} = 100 + 18 = 118\,\text{km h}^{-1}.$$

Since the throw takes place from car B moving at $80\,\text{km h}^{-1}$, the required speed relative to the thrower is

$$v = u_S - u_B = 118 - 80 = 38\,\text{km h}^{-1}.$$

Equivalently, in B's frame car A moves forward at 20, and the stone must move forward at $20 + 18$.

Step 3: Reject the other listed speeds A throw at 18 would give the stone a ground speed of 98, so it could not catch car A ahead. A throw at 28 would yield relative impact speed $108 - 100 = 8\,\text{km h}^{-1}$. A throw at 48 would yield $128 - 100 = 28\,\text{km h}^{-1}$. Neither is the required $18\,\text{km h}^{-1}$, while 38 gives exactly that value.

Step 4: Conclusion The required throwing speed is $\boxed{38\,\text{km h}^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: Work in the thrower's frame: add the target's recession speed to the required approach speed, after converting both to the same unit.

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28. At $t = 0$, a body of mass 100 g starts moving under the influence of a force $(5\hat{\text{i}} + 10\hat{\text{j}})\text{N}\cdot$ After 2 s its position is $(2x\hat{\text{i}} + 5y\hat{\text{j}})\text{m}$. The ratio $x : y$ is ____ . [4 marks]

(A) $1 : 2$

(B) $2 : 5$

(C) $5 : 2$

(D) $5 : 4$

Correct Answer: (D) $5 : 4$

Solution:

Concept:

- Newton's second law is $\vec{F} = m\vec{a}$. A constant force on a fixed mass produces constant acceleration in the direction of the force.
- For a body starting from rest at the origin, $\vec{r} = \frac{1}{2}\vec{a}t^2$. The actual position components here are $2x$ and $5y$, not x and y.

Step 1: Convert the mass and resolve the acceleration The phrase starts moving specifies zero initial velocity; the starting point is the position origin. Convert the mass to SI units:

$$m = 100\,\text{g} = 0.100\,\text{kg}.$$

Dividing each component of the force by the mass gives

$$\vec{a} = \frac{5\hat{\text{i}} + 10\hat{\text{j}}}{0.100} = (50\hat{\text{i}} + 100\hat{\text{j}})\,\text{m s}^{-2}.$$

Step 2: Calculate the position after two seconds Both components have the same elapsed time and zero initial velocity. Thus

$$\vec{r} = \frac{1}{2}(50\hat{\mathrm{i}} + 100\hat{\mathrm{j}})(2)^2 = (100\hat{\mathrm{i}} + 200\hat{\mathrm{j}})\,\mathrm{m}.$$

Comparing this expression with the given position $(2x\hat{\mathrm{i}} + 5y\hat{\mathrm{j}})\,\mathrm{m}$ requires matching the coefficients of each unit vector separately:

$$2x = 100, \qquad 5y = 200, \qquad x = 50, \quad y = 40.$$

Step 3: Form the requested ratio The resulting ratio is $x : y = 50 : 40 = 5 : 4$. Option (A), $1 : 2$, is the ratio of the actual displacements and force components, but ignores the factors 2 and 5 multiplying the variables. Options (B) and (C) also fail the component equations: they would give $(2x)/(5y) = 4/25$ and 1 respectively, rather than the required $1/2$.

Step 4: Conclusion The ratio is $\boxed{x : y = 5 : 4}$. Hence, the correct option is (D).

Quick Tip to Solve: For motion from rest under constant force, compare $2x : 5y = 5 : 10$ directly; distinguish the named variables from the actual displacement components.

29. If x and y coordinates of a projectile as a function of time (t) are given as $24t$ and $43.6t - 4.9t^2$, respectively, then the angle (in degrees) made by the projectile with horizontal when $t = 2$ s is ____ . [4 marks]

(A) 60

(B) 45

(C) 30

(D) 75

Correct Answer: (B) 45

Solution:

Concept:

- The instantaneous velocity is the time derivative of position: $v_x = dx/dt$ and $v_y = dy/dt$. Its direction is tangent to the projectile's path.
- The angle θ of motion with the positive horizontal satisfies $\tan\theta = v_y/v_x$. The ratio y/x instead gives the direction of the position vector from the origin.

Step 1: Differentiate the given coordinates The horizontal coordinate is $x(t) = 24t$, so the horizontal velocity is constant:

$$v_x = \frac{d}{dt}(24t) = 24\,\mathrm{m\,s^{-1}}.$$

From $y(t) = 43.6t - 4.9t^2$, differentiation gives

$$v_y = \frac{d}{dt}(43.6t - 4.9t^2) = 43.6 - 9.8t\,\mathrm{m\,s^{-1}}.$$

The negative term reflects the reduction of upward velocity due to gravitational acceleration; it does not change the horizontal component.

Step 2: Evaluate the velocity direction at two seconds Substitute $t = 2\,\text{s}$ into the vertical component:

$$v_y(2) = 43.6 - 9.8(2) = 43.6 - 19.6 = 24\,\text{m s}^{-1}.$$

Both components are positive and equal. Therefore the projectile is moving upward and to the right, with

$$\tan\theta = \frac{v_y(2)}{v_x} = \frac{24}{24} = 1.$$

The first-quadrant angle satisfying this relation is 45°.

Step 3: Check the competing angles An angle of 60° would require $v_y/v_x = \sqrt{3}$; 30° would require $1/\sqrt{3}$; and 75° would require $2+\sqrt{3}$. None equals the calculated ratio 1. Using the initial vertical velocity 43.6 would give the launch direction, not the direction at the requested time.

Step 4: Conclusion At $t = 2\,\text{s}$, the projectile moves at $\boxed{45^\circ}$ above the horizontal. Hence, the correct option is (B).

Quick Tip to Solve: Differentiate coordinate equations before finding the direction of motion. Equal positive velocity components immediately imply an upward angle of 45°.

30. The height in terms of radius of the earth (R), at which the acceleration due to gravity becomes $\frac{g}{9}$, where g is acceleration due to gravity on earth's surface, is

____ . [4 marks]

(A) $\sqrt{3}R$

(B) $2\sqrt{2}R$

(C) $2R$

(D) $\frac{4}{9}R$

Correct Answer: (C) $2R$

Solution:

Concept:

- Outside a spherical Earth, the gravitational acceleration at distance r from its centre is GM/r^2, where M is Earth's mass and G is the gravitational constant.
- An altitude h above the surface corresponds to centre-distance $r = R + h$. Consequently $g_h/g = R^2/(R+h)^2$, with $g = GM/R^2$ at the surface.

Step 1: Form the ratio of accelerations Use the inverse-square law at the surface and at the required height:

$$g = \frac{GM}{R^2}, \qquad g_h = \frac{GM}{(R+h)^2}.$$

Dividing cancels the common factors G and M. Substituting the given value $g_h = g/9$ gives

$$\frac{g_h}{g} = \left(\frac{R}{R+h}\right)^2 = \frac{1}{9}.$$

This exact relation is needed because the height need not be small compared with Earth's radius.

Step 2: Find the distance from the centre and then the height Both radii are positive, so take the positive square root:

$$\frac{R}{R+h} = \frac{1}{3}, \qquad R + h = 3R.$$

The centre-distance is three Earth radii. Subtracting the radius of the surface gives

$$h = 3R - R = 2R.$$

Keeping this distinction prevents reporting the centre-distance as the height above the surface.

Step 3: Verify the answer and exclude the alternatives At $h = 2R$, the denominator is $(3R)^2$, so $g_h = GM/(9R^2) = g/9$, as required. Options (A) and (B) give centre-distances $(1 + \sqrt{3})R$ and $(1 + 2\sqrt{2})R$, neither equal to $3R$. Option (D) gives $r = 13R/9$, for which $g_h/g = 81/169$, also not $1/9$.

Step 4: Conclusion The required height above Earth's surface is $\boxed{h = 2R}$. Hence, the correct option is (C).

Quick Tip to Solve: Reducing gravity by a factor of 9 triples the distance from Earth's centre; subtract one Earth radius to convert that distance into altitude.

---

31. A metal string A is suspended from a rigid support and its free end is attached to a block of mass M. Second block having mass 2 M is suspended at the bottom of the first block using a string B. The area of cross sections of strings A and B are same. The ratio of lengths of strings of A to B is 2 and the ratio of their Young's moduli (Y_A/Y_B) is 0.5 . The ratio of elongations in A to B is ____ . [4 marks]

(A) 1

(B) 4

(C) 8

(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- Young's modulus relates longitudinal stress to strain: $Y = (F/S)/(\Delta L/L)$, so a string's extension is $\Delta L = FL/(SY)$. Here S is its cross-sectional area.
- Each string supports the total load below it. Static equilibrium determines its tension before the extension formula is applied.

Step 1: Determine the two tensions The lower string B supports only the block of mass $2M$, whereas A supports both blocks. Neglecting the strings' own weight, as in the given string model, their tensions

are therefore

$$F_B = 2Mg, \qquad F_A = Mg + 2Mg = 3Mg.$$

The upper block is pulled downward by its own weight and by the tension in B; these two forces explain why the tension in A is not merely Mg.

Step 2: Form the ratio of extensions Let the unstretched lengths be L_A, L_B and the common cross-sectional area be S. Division cancels the common area:

$$\frac{\Delta L_A}{\Delta L_B} = \frac{F_A}{F_B}\frac{L_A}{L_B}\frac{Y_B}{Y_A} = \frac{3}{2} \times 2 \times \frac{1}{0.5} = 6.$$

A is twice as long and has half the Young's modulus of B, making its extension four times as large for the same tension. Its actual tension is another factor of $3/2$ larger. Thus 4 would omit the unequal loads, while 1 and 8 also fail the combined load, length and modulus ratio.

Step 3: Conclusion The elongation ratio is $\Delta L_A : \Delta L_B = 6 : 1$. Hence, the correct option is (D).

Quick Tip to Solve: For vertically connected strings, first count all masses below each string; then use $\Delta L \propto FL/Y$ when their areas are equal.

---

32. A water spray gun is attached to a hose of cross sectional area $30\ \text{cm}^2$. The gun comprises of 10 perforations each of cross sectional area of $15\ \text{mm}^2$. If the water flows in the hose with the speed of $50\ \text{cm/s}$, calculate the speed at which the water flows out from each perforation. (Neglect any edge effects) [4 marks]

(A) $100\ \text{m/s}$

(B) $10\ \text{m/s}$

(C) $1000\ \text{m/s}$

(D) $15 \times 10^2\ \text{m/s}$

Correct Answer: (B) $10\ \text{m/s}$

Solution:

Concept:

- For steady incompressible flow, volume entering per second equals volume leaving per second: $Q = Av$. Here A is the flow area and v is the speed.
- For n identical outlets of area a, the combined discharge is nav. The outlet area in continuity is the sum of all perforation areas, not just one hole.

Step 1: Convert all quantities to SI units The hose area and water speed are

$$A = 30 \times 10^{-4} = 3.0 \times 10^{-3}\,\text{m}^2, \qquad u = 50 \times 10^{-2} = 0.50\,\text{m s}^{-1}.$$

Each perforation has area $a = 15 \times 10^{-6}\,\text{m}^2$. For ten identical perforations, their total area is

$$na = 10 \times 15 \times 10^{-6} = 1.5 \times 10^{-4}\,\text{m}^2.$$

Remember that converting an area requires squaring the length conversion factor.

Step 2: Apply conservation of volume flow The incoming discharge is

$$Au = (3.0 \times 10^{-3})(0.50) = 1.5 \times 10^{-3}\,\mathrm{m^3\,s^{-1}}.$$

Equating this with the total discharge through the holes gives

$$Au = nav, \qquad v = \frac{1.5 \times 10^{-3}}{1.5 \times 10^{-4}} = 10\,\mathrm{m\,s^{-1}}.$$

The total outlet area is twenty times smaller than the hose area, so its speed must be twenty times greater than $0.50\,\mathrm{m\,s^{-1}}$. The value $100\,\mathrm{m\,s^{-1}}$ wrongly sends all water through only one perforation. The values 1000 and 1500 likewise violate the given volume-flow balance.

Step 3: Conclusion Water leaves each perforation at $10\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: Add the areas of identical outlets before applying continuity, and convert square centimetres and square millimetres as areas.

33. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : If the average kinetic energy of H_2 and O_2 molecules, kept in two different sized containers are same, then their temperatures will be same.

Reason R : The r.m.s speed of H_2 and O_2 molecules are same at same temperature.

Choose the correct answer from the options given below [4 marks]

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(C) A is true but **R** is false

(D) A is false but **R** is true

Correct Answer: (C) A is true but **R** is false

Solution:

Concept:

- The mean translational kinetic energy per molecule is $\langle K \rangle = \frac{3}{2}k_BT$, where k_B is Boltzmann's constant and T is absolute temperature.
- The rms molecular speed is $v_{\mathrm{rms}} = \sqrt{3k_BT/m} = \sqrt{3RT/\mathcal{M}}$, where m is molecular mass and $\mathcal{M}$ is molar mass. Equal energy does not mean equal speed for different masses.

Step 1: Evaluate Assertion A For hydrogen and oxygen, equating the stated average kinetic energies gives

$$\frac{3}{2}k_BT_{H_2} = \frac{3}{2}k_BT_{O_2} \quad \Longrightarrow \quad T_{H_2} = T_{O_2}.$$

The molecular masses and container volumes do not enter this energy-temperature relation. Different container sizes can affect density or pressure, but do not change this conclusion about equal average

molecular energies. Assertion A is true.

Step 2: Evaluate Reason R At the same temperature, the speed ratio depends on molar masses, approximately 2 and $32\,\mathrm{g\,mol^{-1}}$:

$$\frac{v_{\mathrm{rms,H_2}}}{v_{\mathrm{rms,O_2}}} = \sqrt{\frac{32}{2}} = 4.$$

Hydrogen molecules therefore have four times the rms speed of oxygen molecules. The speeds are not equal, so Reason R is false.

Step 3: Check the explanatory link and choices A false statement cannot explain the true assertion. The equality of average kinetic energies implies equal temperatures through $\langle K\rangle = 3k_BT/2$, independently of the false speed claim. Options (A) and (B) both require R to be true, and option (D) reverses the truth values; all three are excluded.

Step 4: Conclusion Assertion A is true and Reason R is false. Hence, the correct option is (C).

Quick Tip to Solve: At equal temperature, compare molecular energies directly but compare rms speeds using the inverse square root of molecular mass.

34. The temperature of a metal strip having coefficient of linear expansion α is increased from T_1 to T_2 resulting in increase of its length by ΔL_1. The temperature is further increased from T_2 to T_3 such that the increase in its length is ΔL_2.

Given $T_3 + T_1 = 2T_2$ and $T_2 - T_1 = \Delta T$, the value of ΔL_2 is ____ . [4 marks]

(A) $\Delta L_1\left[1 + 2\alpha^2(\Delta T)^2\right]$

(B) $\Delta L_1\left[1 + \alpha^2(\Delta T)^2\right]$

(C) $\Delta L_1[1 + 2\alpha\Delta T]$

(D) $\Delta L_1[1 + \alpha\Delta T]$

Correct Answer: (D) $\Delta L_1[1 + \alpha\Delta T]$

Solution:

Concept:

- For a temperature increment ΔT, the linear-expansion relation is $\Delta L = \alpha L\Delta T$, with L the length at the beginning of that heating interval.
- For successive heating intervals, use the length reached after the first interval as the starting length for the second. The relation $T_3 + T_1 = 2T_2$ makes the two temperature increments equal.

Step 1: Identify equal temperature increments Rearranging the supplied temperature condition gives

$$T_3 - T_2 = T_2 - T_1 = \Delta T.$$

Let L_1 denote the length at T_1. Applying the given linear-expansion model to the first interval,

$$\Delta L_1 = \alpha L_1\Delta T, \qquad L_2 = L_1 + \Delta L_1 = L_1(1 + \alpha\Delta T).$$

The quantity ΔL_1 is the increase in length, whereas L_2 is the entire length at the intermediate

temperature.

Step 2: Apply the expansion relation to the second interval The second interval starts with length L_2, while its temperature rise is again ΔT. Therefore,

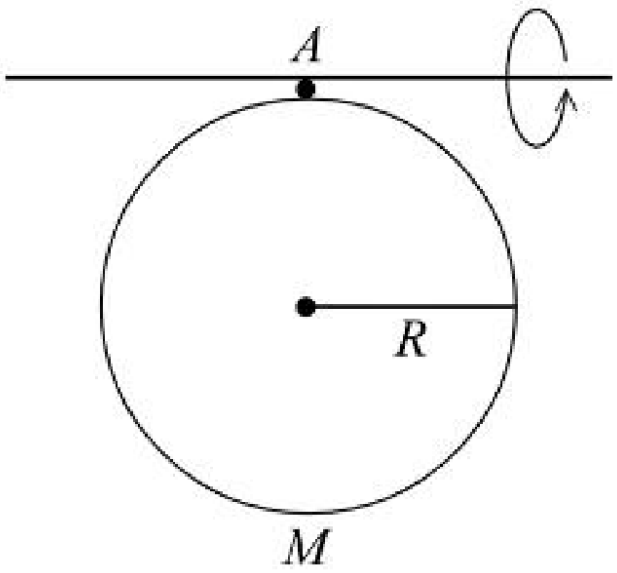
$$\begin{aligned}\Delta L_2 &= \alpha L_2 \Delta T \\ &= \alpha L_1(1+\alpha\Delta T)\Delta T \\ &= \Delta L_1(1+\alpha\Delta T).\end{aligned}$$

The bracket contains a single factor $\alpha\Delta T$, which is dimensionless. The derived ratio is $1+\alpha\Delta T$, not $1+2\alpha\Delta T$; option (C) incorrectly doubles the correction term. Options (A) and (B) omit the linear correction to the starting length and instead insert quadratic corrections, so neither follows from the successive-interval expansion relation.

Step 3: Conclusion The second increase is $\Delta L_2 = \Delta L_1[1+\alpha\Delta T]$. Hence, the correct option is (D).

Quick Tip to Solve: For successive expansion steps, keep the total current length separate from its increment; the second step begins with the already expanded strip.

35. A uniform disc of radius R and mass M is free to oscillate about the axis A as shown in the figure. For small oscillations the time period is ____ .
(g is acceleration due to gravity) [4 marks]



(A) $2\pi\sqrt{\frac{5R}{4g}}$
(B) $2\pi\sqrt{\frac{2R}{3g}}$
(C) $2\pi\sqrt{\frac{3R}{2g}}$
(D) $2\pi\sqrt{\frac{3R}{g}}$

Correct Answer: (A) $2\pi\sqrt{\frac{5R}{4g}}$

Solution:

Concept:

- A rigid body oscillating under gravity about a horizontal fixed axis is a physical pendulum. For small angles, $T = 2\pi\sqrt{I_A/(Mgd)}$, where d is the distance from the axis to the centre of mass.
- For a uniform thin disc, the moment of inertia about an in-plane diameter is $MR^2/4$. The parallel-axis theorem gives $I_A = I_{\text{cm}} + Md^2$ for a parallel displaced axis.

Step 1: Read the axis and calculate its moment of inertia The figure places axis A tangent to the upper edge of the disc and in the plane of the disc. The centre of mass is at the disc centre, a distance $d = R$ below this axis. Thus the parallel central axis is a diameter, not the axis normal to the disc. Hence

$$I_A = \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2.$$

Using $MR^2/2$ instead would describe rotation about a perpendicular central axis and would not match the illustrated support.

Step 2: Write the small-angle restoring equation After a small angular displacement θ, gravity exerts restoring torque $-MgR\sin\theta$. With $\sin\theta \simeq \theta$, rotational dynamics gives

$$I_A\ddot{\theta} = -MgR\theta, \qquad \omega^2 = \frac{MgR}{I_A} = \frac{4g}{5R}.$$

Consequently,

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{5R}{4g}}.$$

The mass cancels, as it should for a gravitational pendulum. Option (C) would result from using the wrong central moment of inertia $MR^2/2$. Options (B) and (D) also fail the required ratio $I_A/(MR) = 5R/4$; they cannot describe the shown axis.

Step 3: Conclusion The small-oscillation period is $2\pi\sqrt{5R/(4g)}$. Hence, the correct option is (A).

Quick Tip to Solve: Before using the physical-pendulum formula, decide whether the disc rotates about a diameter-parallel or a normal-parallel axis; their moments of inertia differ.

36. A rigid dipole undergoes a simple harmonic motion about its centre in the presence of an electric field $\vec{E}_1 = E_0\hat{x}$. If another electric field $\vec{E}_2 = 2E_0(\hat{y}+\hat{z})$ is introduced to the system, what will be the percentage change in the frequency of the oscillation (approximate)? [4 marks]

(A) 73%

(B) 63%

(C) 83%

(D) 53%

Correct Answer: (A) 73%

Solution:

Concept:

- A dipole of moment magnitude p in a uniform field E experiences restoring torque $-pE\sin\theta$ about its aligned stable equilibrium.
- For small angular oscillations of a rigid dipole of moment of inertia I, $I\ddot{\theta} = -pE\theta$, so $f = (2\pi)^{-1}\sqrt{pE/I}$. Thus frequency is proportional to $\sqrt{E}$ when the dipole is unchanged.

Step 1: Find the magnitude of the resultant field The two fields add as vectors, not as their magnitudes. Their combined Cartesian components are

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = E_0\hat{x} + 2E_0\hat{y} + 2E_0\hat{z}.$$

Since the unit vectors are mutually perpendicular,

$$E = \sqrt{E_0^2 + (2E_0)^2 + (2E_0)^2} = 3E_0.$$

The stable equilibrium direction changes to the resultant-field direction. The relevant new frequency is that of small oscillations about this new aligned equilibrium.

Step 2: Compare the frequencies and calculate the increase The dipole moment and moment of inertia remain unchanged. Dividing the frequency expressions eliminates both:

$$\frac{f_{\text{new}}}{f_{\text{old}}} = \sqrt{\frac{E}{E_0}} = \sqrt{3}.$$

The percentage change is measured relative to the original frequency:

$$\frac{f_{\text{new}} - f_{\text{old}}}{f_{\text{old}}} \times 100 = (\sqrt{3} - 1) \times 100 \simeq 73.2\%.$$

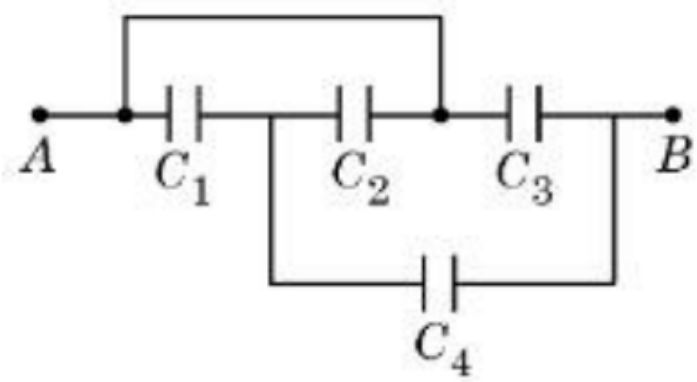
The field is stronger, so the change is an increase. Values of 63, 83 or 53 percent would imply frequency ratios 1.63, 1.83 or 1.53; none equals the required $\sqrt{3} \simeq 1.732$. The ratio of field strengths itself is 3, but this is not the frequency ratio.

Step 3: Conclusion The oscillation frequency increases by approximately 73%. Hence, the correct option is (A).

Quick Tip to Solve: Add electric-field vectors first, take the square root of the field-magnitude ratio second, and subtract 1 only when converting that ratio into a percentage change.

37. From the circuit given below, the capacitance between terminals A and B shown in the circuit is ____ μF.

(take $C_1 = C_2 = C_3 = 1\mu$ F and $C_4 = 2\mu$ F.) [4 marks]



(A) 2

(B) 7/2

(C) 7/3

(D) 5/2

Correct Answer: (A) 2

Solution:

Concept:

- An ideal connecting wire joins points at the same potential. Identify the circuit nodes before applying a series or parallel formula.
- Capacitors connected across the same two nodes are in parallel: $C_p = C_1 + C_2$. For two capacitors in series, $C_s = C_a C_b/(C_a + C_b)$.

Step 1: Identify the common nodes Call the junction between C_1 and C_2 node X. The upper wire connects terminal A directly to the junction between C_2 and C_3. Thus C_1 and C_2 both join nodes A and X, despite their apparent left-to-right arrangement. They have the same potential difference and are in parallel.

Step 2: Reduce the branch through the lower capacitor The parallel combination has capacitance

$$C_{12} = C_1 + C_2 = 1 + 1 = 2\,\mu\text{F}.$$

The lower wire connects C_4 between X and B. Therefore C_{12} and C_4 form a series branch from A to B:

$$C_s = \frac{C_{12}C_4}{C_{12} + C_4} = \frac{2 \times 2}{2 + 2} = 1\,\mu\text{F}.$$

Step 3: Include the direct branch Capacitor C_3 is connected directly between A and B, in parallel with the reduced series branch. Hence

$$C_{AB} = C_3 + C_s = 1 + 1 = 2\,\mu\text{F}.$$

The values $7/2$, $7/3$ and $5/2\,\mu\text{F}$ do not follow from these node connections; treating the three horizontal capacitors as a simple series chain ignores the upper wire.

Step 4: Conclusion The equivalent capacitance is $\boxed{2\,\mu\text{F}}$. Hence, the correct option is (A).

Quick Tip to Solve: Mark every point joined by an uninterrupted wire with the same node letter; parallel capacitors need not look parallel in the drawing.

38. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : In electrostatics, a conductor does not store any net charge inside.

Reason R : Inside the capacitor (with no dielectric medium), the free charge carriers, if placed between the plates of capacitor, experience force and drift.

Choose the correct answer from the options given below [4 marks]

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(C) A is true but **R** is false

(D) A is false but **R** is true

Correct Answer: (B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

Solution:

Concept:

- In electrostatic equilibrium the electric field within the conducting material is zero. Otherwise its mobile charges would continue to move.
- A free charge q in an electric field experiences $\vec{F} = q\vec{E}$. A vacuum gap between charged capacitor plates can sustain a nonzero field.

Step 1: Evaluate the assertion An isolated conductor in electrostatic equilibrium has no excess volume charge in its conducting bulk. If excess charge were initially placed there, its mobile carriers would redistribute until the internal field vanished. Gauss's law for a closed surface wholly within the conducting material then gives

$$Q_{\text{enc}} = \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = 0.$$

The excess charge resides on the surface, so Assertion A is true in its usual electrostatic sense.

Step 2: Evaluate the reason The space between the plates of a charged capacitor is not conducting material. Even without a dielectric it contains an electric field, approximately $E = V/d$ for ideal parallel plates separated by d. A free carrier introduced into this region experiences $\vec{F} = q\vec{E}$ and moves under this force; negative and positive charges move in opposite directions. Thus Reason R is also true.

Step 3: Test the explanatory connection Reason R describes the response of a test charge in the field of a capacitor gap. It does not state the redistribution of free charges within a conductor or the condition of zero internal field at equilibrium, which explains Assertion A. Both statements are true,

but R is not the explanation of A. Option (A) incorrectly asserts an explanatory link, while (C) and (D) incorrectly declare one statement false.

Step 4: Conclusion Both A and R are true, but R is not the correct explanation of A. Hence, the correct option is (B).

Quick Tip to Solve: For assertion-reason questions, judge each statement first, then check whether the reason actually establishes the assertion in the same physical system.

39. A solenoid has a core made of material with relative permeability 400 . The magnetic field produced in the interior of solenoid is 1.0 T . The magnetic intensity in SI units is $\alpha \times 10^5$. The value of α is ____ .

(Free space permeability $\mu_0 = 4\pi \times 10^{-7}$ SI units.) [4 marks]

(A) $\frac{25}{\pi}$

(B) $\frac{1}{16\pi}$

(C) $\frac{1}{\pi}$

(D) $\frac{1}{4\pi}$

Correct Answer: (B) $\frac{1}{16\pi}$

Solution:

Concept:

- For the core material, magnetic flux density B and magnetic field intensity H are related by $B = \mu H = \mu_0 \mu_r H$.
- B is measured in tesla, H in $\mathrm{A\,m^{-1}}$, and the relative permeability μ_r is dimensionless. The requested coefficient multiplies 10^5.

Step 1: Calculate the core permeability Use the given relative permeability rather than the vacuum value alone. The absolute permeability of the core is

$$\mu = \mu_0 \mu_r = (4\pi \times 10^{-7})(400) = 1600\pi \times 10^{-7} = 1.6\pi \times 10^{-4}\,\mathrm{H\,m^{-1}}.$$

The large relative permeability means that a given flux density requires a smaller applied magnetic field intensity than in vacuum.

Step 2: Find the magnetic intensity Rearranging the constitutive relation and substituting $B = 1.0\,\mathrm{T}$,

$$\begin{aligned} H &= \frac{1.0}{1.6\pi \times 10^{-4}} \\ &= \frac{6250}{\pi}\,\mathrm{A\,m^{-1}} \\ &= \frac{1}{16\pi} \times 10^5\,\mathrm{A\,m^{-1}}. \end{aligned}$$

The numerical magnitude is about $1.99 \times 10^3\,\mathrm{A\,m^{-1}}$; this is consistent with the reduced field intensity in a high-permeability core.

Step 3: Compare the coefficient with the options Comparing with $H = \alpha \times 10^5$ gives $\alpha = 1/(16\pi)$. The coefficients $25/\pi$, $1/\pi$ and $1/(4\pi)$ would make μH equal to 400, 16 and 4 T, respectively, instead of the stated 1.0 T. They are therefore excluded.

Step 4: Conclusion $\boxed{\alpha = \dfrac{1}{16\pi}}$, with $H = \dfrac{10^5}{16\pi}\ \text{A m}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: Compute $H = B/(\mu_0\mu_r)$ first; only then rewrite its power of ten to match the requested coefficient.

---

40. A magnetic field vector in an electromagnetic wave is represented by $\vec{B} = B_0 \sin\left(2\pi vt - \frac{2\pi x}{\lambda}\right)\hat{j}$. Its associated electric field vector is ____ . [4 marks]

(A) $\vec{E} = -v\lambda B_0 \sin\left(2\pi vt - \frac{2\pi x}{\lambda}\right)\hat{k}$

(B) $\vec{E} = -v\lambda B_0 \sin\left(2\pi vt - \frac{2\pi x}{\lambda}\right)\hat{i}$

(C) $\vec{E} = v\lambda B_0 \sin\left(2\pi vt - \frac{2\pi x}{\lambda}\right)\hat{k}$

(D) $\vec{E} = v\lambda B_0 \sin\left(2\pi vt - \frac{2\pi x}{\lambda}\right)\hat{i}$

Correct Answer: (A) $\vec{E} = -v\lambda B_0 \sin\left(2\pi vt - \frac{2\pi x}{\lambda}\right)\hat{k}$

Solution:

Concept:

- For a plane electromagnetic wave the electric field, magnetic field and propagation direction are mutually perpendicular. Energy travels along $\vec{E} \times \vec{B}$.
- The field amplitudes satisfy $E_0 = uB_0$, where the wave speed is $u = v\lambda$ and v denotes the frequency in the given expression.

Step 1: Find the direction of propagation Write the common phase as $\phi = 2\pi vt - 2\pi x/\lambda$. A point of constant phase satisfies

$$2\pi vt - \frac{2\pi x}{\lambda} = \text{constant}, \qquad \frac{dx}{dt} = v\lambda > 0.$$

Thus the wave travels along $+\hat{i}$. Since $\vec{B}$ is along $\hat{j}$, its associated electric field must be along the z-axis rather than the x-axis.

Step 2: Fix the sign of the electric field When $\sin\phi > 0$, the magnetic field points along $+\hat{j}$. To make the energy flow along $+\hat{i}$, the electric field must point along $-\hat{k}$, because

$$(-\hat{k}) \times \hat{j} = +\hat{i}.$$

With both fields carrying the same sinusoidal factor, their cross product is proportional to $\sin^2\phi$, so this propagation direction remains unchanged when the fields reverse.

Step 3: Use the amplitude relation The electric amplitude is $E_0 = v\lambda B_0$. Combining its magnitude, phase and direction yields

$$\vec{E} = -v\lambda B_0 \sin\left(2\pi vt - \frac{2\pi x}{\lambda}\right)\hat{k}.$$

Options (B) and (D) give a longitudinal electric field and violate transversality. Option (C) has the

opposite sign, making $\vec{E} \times \vec{B}$ point along $-\hat{i}$.

Step 4: Conclusion $\boxed{\vec{E} = -v\lambda B_0 \sin(2\pi vt - 2\pi x/\lambda)\hat{k}}$. Hence, the correct option is (A).

Quick Tip to Solve: Determine propagation from the phase first, then choose the sign of the other field using $\vec{E} \times \vec{B}$.

41. A convex lens is made from glass material having refractive index of 1.4 with same radius of curvature on both sides. The ratio of its focal length and radius of curvature is ____ . [4 marks]

(A) 0.5

(B) 2.5

(C) 0.8

(D) 1.25

Correct Answer: (D) 1.25

Solution:

Concept:

- For a thin lens in air, the lens maker formula is $1/f = (\mu - 1)(1/R_1 - 1/R_2)$, where μ is the refractive index of the lens material.
- For a symmetric biconvex lens, the two radii have equal magnitudes but opposite signs: $R_1 = +R$ and $R_2 = -R$ for light travelling left to right.

Step 1: Assign signed radii to the two surfaces The centre of curvature of the first convex surface lies to its right, whereas that of the second lies to its left. Therefore use $R_1 = R$ and $R_2 = -R$, with $R > 0$ the common radius magnitude. Equal curvature magnitudes do not mean equal signed radii. Giving both surfaces the same sign would incorrectly predict zero lens power.

Step 2: Substitute the refractive index Using $\mu = 1.4$,

$$\begin{aligned}\frac{1}{f} &= (1.4 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) \\ &= 0.4\left(\frac{2}{R}\right) \\ &= \frac{0.8}{R}.\end{aligned}$$

The result is positive, as expected for a convex lens made of glass in air, so the lens is converging.

Step 3: Form the requested ratio Taking the reciprocal and dividing by R gives

$$f = \frac{R}{0.8}, \qquad \frac{f}{R} = \frac{1}{0.8} = 1.25.$$

The value 0.8 in option (C) is R/f, the inverse of the requested ratio. The value 2.5 in option (B) neglects the contribution of one curved surface. The value 0.5 in option (A) would require a different refractive index, namely $\mu = 2$, for this symmetric lens.

Step 4: Conclusion The dimensionless ratio is $\boxed{f/R = 1.25}$. Hence, the correct option is (D).

Quick Tip to Solve: For a symmetric biconvex lens in air, remember $f/R = 1/[2(\mu - 1)]$; equal radii mean equal magnitudes, not equal signs.

42. An unpolarized light of certain intensity passes through a combination of two polarizers whose transmission axes are at 30° and 90°, respectively, with respect to the horizontal axis. A third polarizer with its transmission axis at 60° with the horizontal axis is placed between the two existing polarizers. The ratio of the output intensities with and without the third polarizer is ____ . [4 marks]

(A) 3/4

(B) 4/3

(C) 9/4

(D) 4/9

Correct Answer: (C) 9/4

Solution:

Concept:

- An ideal polarizer transmits half the intensity of unpolarized incident light. The transmitted beam is linearly polarized along its transmission axis.
- Malus law gives $I_{\text{out}} = I_{\text{in}} \cos^2\theta$ for polarized input, where θ is the angle between successive transmission axes.

Step 1: Calculate the output without the extra polarizer Let the original unpolarized intensity be I_0. After the first polarizer at 30°, the intensity is $I_0/2$. The angle to the second polarizer at 90° is 60°, so

$$I_{\text{without}} = \frac{I_0}{2}\cos^2 60^\circ = \frac{I_0}{8}.$$

The angles must be measured between the axes, rather than individually from the horizontal.

Step 2: Calculate the output with the middle polarizer Insert the polarizer at 60° between the existing pair. The successive angular differences are now $60^\circ - 30^\circ = 30^\circ$ and $90^\circ - 60^\circ = 30^\circ$. Applying Malus law at both stages gives

$$I_{\text{with}} = \frac{I_0}{2}\cos^2 30^\circ \cos^2 30^\circ = \frac{I_0}{2}\left(\frac{3}{4}\right)^2 = \frac{9I_0}{32}.$$

The middle polarizer changes the polarization direction of the surviving light before it reaches the last polarizer.

Step 3: Take the ratio in the stated order The required ratio is

$$\frac{I_{\text{with}}}{I_{\text{without}}} = \frac{9I_0/32}{I_0/8} = \frac{9}{4}.$$

The value $4/9$ is the reverse ratio. The values $3/4$ and $4/3$ represent one angular transmission factor

and its reciprocal; they omit the complete comparison of the two arrangements. The ratio exceeds unity without violating energy conservation, since both outputs remain below I_0.

Step 4: Conclusion The ratio of output intensities with and without the inserted polarizer is $\boxed{9/4}$. Hence, the correct option is (C).

Quick Tip to Solve: For a chain of polarizers, multiply one $\cos^2$ factor for each adjacent pair; use the initial factor $1/2$ only once for unpolarized input.

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43. In Rutherford's alpha-particle scattering experiment, only a few alpha particles rebound back because

A. The size of gold nucleus is very small as compared to the size of gold atom.

B. Alpha particle and gold nucleus have equal charge.

C. The impact parameter is minimum for a few alpha particles.

D. A few alpha particles have very high kinetic energy.

E. Only a few alpha particles undergo head-on collision with the nuclei.

Choose the correct answer from the options given below : [4 marks]

(A) A, B Only

(B) B, E Only

(C) C, D Only

(D) A, C, E Only

Correct Answer: (D) A, C, E Only

Solution:

Concept:

- An alpha particle has positive charge $+2e$; a gold nucleus has charge $+79e$. Their interaction is a repulsive electrostatic force concentrated near the tiny nucleus.
- The impact parameter is the perpendicular separation between the incident trajectory and the nuclear centre. A small impact parameter produces a large scattering angle; a head-on approach has zero impact parameter.

Step 1: Connect rare backward scattering with nuclear size The nucleus occupies a very small region compared with the atom. Most incident alpha particles therefore pass far from a nucleus and suffer little deflection. Only a small fraction approach sufficiently close to be turned through a large angle or sent back. Thus statement A is correct.

Step 2: Examine the impact parameter and head-on approach Only a few trajectories have an impact parameter close to zero, so C is correct. For an exactly head-on approach, repulsion slows the alpha particle to rest at its distance of closest approach and then accelerates it backwards along the incident line. Such encounters are rare because the nucleus is so small. Hence E is also correct.

Step 3: Reject the charge and energy claims Statement B is false: the charges are $+2e$ and $+79e$, not equal. Statement D does not explain backward scattering. At a given nonzero impact parameter, higher kinetic energy makes the trajectory harder to deflect, rather than making reversal more likely.

The decisive factor is a close, nearly head-on encounter. Options (A) and (B) include the false statement B, while option (C) includes D.

Step 4: Conclusion The correct statements are A, C and E only. Hence, the correct option is (D).

Quick Tip to Solve: Associate rare large-angle scattering with the small nucleus and a nearly zero impact parameter; check the actual nuclear charges separately.

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44. The de Broglie wavelength associated with an electron accelerated through a potential difference V is λ_e and the de Broglie wavelength associated with a proton accelerated through the same potential difference is λ_p. If their corresponding masses are m_e and m_p, respectively, then the ratio of their de Broglie wavelengths $\left(\frac{\lambda_e}{\lambda_p}\right)$ is ____ . [4 marks]

(A) $\sqrt{\frac{m_p}{m_e}}$

(B) $\sqrt{\frac{m_e}{m_p}}$

(C) $\frac{m_p}{m_e}$

(D) $\left(\frac{m_p}{m_e}\right)^2$

Correct Answer: (A) $\sqrt{\frac{m_p}{m_e}}$

Solution:

Concept:

- A particle accelerated from rest through a potential difference of magnitude V gains kinetic energy $K = |q|V$. The electron and proton have equal charge magnitudes e, although their charge signs differ.
- In the nonrelativistic relation used here, $K = p^2/(2m)$ and the de Broglie wavelength is $\lambda = h/p$, where p is momentum and h is Planck's constant.

Step 1: Express the wavelength using the accelerating voltage Equating the electrical energy gained to the kinetic energy gives

$$\frac{p^2}{2m} = eV, \qquad p = \sqrt{2meV}.$$

Therefore,

$$\lambda = \frac{h}{\sqrt{2meV}}.$$

The equal voltages give equal kinetic energies, not equal speeds or momenta. This distinction fixes the power of the mass in the wavelength.

Step 2: Divide the electron wavelength by the proton wavelength Substituting the respective masses and cancelling the common factors yields

$$\frac{\lambda_e}{\lambda_p} = \frac{h/\sqrt{2m_e eV}}{h/\sqrt{2m_p eV}} = \sqrt{\frac{m_p}{m_e}}.$$

Since the electron is lighter, its momentum is smaller at the same kinetic energy, so its wavelength

must be larger. Option (B) incorrectly predicts the reverse. Options (C) and (D) use incorrect powers of the mass ratio: momentum varies as the square root of mass at fixed kinetic energy.

Step 3: Conclusion The dimensionless wavelength ratio is $\lambda_e/\lambda_p = \sqrt{m_p/m_e}$. Hence, the correct option is (A).

Quick Tip to Solve: For equal charge magnitudes and equal accelerating voltages, use $\lambda \propto m^{-1/2}$; the lighter particle has the longer wavelength.

45. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : A diode under reverse-biased condition provides very small current which is nearly independent of voltage until a critical limit at which the current increases drastically.

Reason R : Below the critical voltage limit, only majority charge carriers flow which increases drastically above critical voltage.

choose the correct answer from the options given below [4 marks]

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(C) A is true but **R** is false

(D) A is false but **R** is true

Correct Answer: (C) A is true but **R** is false

Solution:

Concept:

- Reverse bias increases the potential barrier and widens the depletion region of a p-n junction. It suppresses the majority-carrier diffusion current across the junction.
- A small reverse saturation current is carried by thermally generated minority carriers. At a fixed temperature it is nearly independent of reverse voltage until breakdown begins.

Step 1: Evaluate Assertion A Below breakdown, the reverse current is very small because the supply of thermally generated minority carriers is limited. Increasing the reverse voltage sweeps these carriers across the junction but changes the current only slightly. At the critical reverse voltage, Zener or avalanche breakdown produces a sharp rise in current. This is precisely the behaviour described by Assertion A, so A is true.

Step 2: Evaluate Reason R The reason assigns the small reverse current to majority charge carriers. That is incorrect: majority electrons on the n side and majority holes on the p side are pulled away from the junction by reverse bias. The reverse saturation current instead involves minority electrons on the p side and minority holes on the n side. Breakdown increases carrier production or permits tunnelling; it does not validate the claim about majority-carrier flow below the critical voltage. Thus R is false.

Step 3: Check the explanation and select the combination Since R is false, it cannot explain the true assertion. Options (A) and (B) both require R to be true and are therefore excluded. Option (D)

incorrectly declares the assertion false. Only option (C) gives the correct truth values. In practical reverse-breakdown operation a series resistance limits the current and prevents excessive heating.

Step 4: Conclusion Assertion A is true, whereas Reason R is false. Hence, the correct option is (C).

Quick Tip to Solve: Remember: forward conduction is dominated by majority carriers, while the small pre-breakdown reverse current is due to minority carriers.

Physics – Section B: Numerical Value Questions (20 marks)

46. A diode has Zener voltage of 10 V and maximum power dissipation of 0.5 W , then the minimum resistance to be used in series with this diode for safety when it is connected to a 25 V power supply is ____ Ω. [4 marks]

Solution:

Concept:

- In reverse breakdown a Zener diode maintains an approximately constant voltage V_Z. Its power dissipation is $P_Z = V_Z I_Z$, where I_Z is the current through it.
- The series resistance drops the excess supply voltage and limits the current. With only the diode and resistor in series, $I_Z = (V_s - V_Z)/R$.

Step 1: Calculate the maximum permissible Zener current The given maximum diode power is $P_{\max} = 0.5\,\text{W}$, while its breakdown voltage is $V_Z = 10\,\text{V}$. The largest safe current is therefore

$$I_{Z,\max} = \frac{P_{\max}}{V_Z} = \frac{0.5}{10} = 0.05\,\text{A}.$$

A greater current would dissipate more than the stated power rating in the diode. The supply voltage must not be used in this power formula because the diode itself has only the Zener voltage across it.

Step 2: Find the voltage that the resistor must absorb The supply and Zener voltage are not both across the resistor. Kirchhoff's voltage law gives

$$V_R = V_s - V_Z = 25 - 10 = 15\,\text{V}.$$

This is the voltage responsible for driving the series current through the resistor.

Step 3: Impose the current limit Safety requires $V_R/R \le I_{Z,\max}$. Consequently,

$$R \ge \frac{15}{0.05} = 300\,\Omega.$$

At this limiting resistance, the diode dissipates $10 \times 0.05 = 0.5\,\text{W}$, exactly its rated maximum. A smaller resistance would increase its current and power. The resistor itself dissipates $15 \times 0.05 = 0.75\,\text{W}$, a separate component-rating consideration.

Step 4: Conclusion The minimum series resistance is $\boxed{300\,\Omega}$. The numerical value to enter is 300.

Quick Tip to Solve: First use $I_{\max} = P_{\max}/V_Z$, then divide the excess supply voltage $V_s - V_Z$ by that current.

47. A gun mounted on the ground fires bullets in all directions with same speed. The farthest distance the bullets could reach is 6.4 m . The speed of the bullets from the gun is ____ $\mathrm{m/s}$.

(take $\mathrm{g} = 10\ \mathrm{m/s^2}$ **)**

[4 marks]

Solution:

Concept:

- For projectile motion under uniform gravity with launch and landing at ground level, the horizontal range is $R = u^2 \sin(2\theta)/g$. Here u is launch speed and θ is the elevation angle.
- At fixed speed, changing the firing direction changes $\sin(2\theta)$. Its greatest possible value is one, attained at $\theta = 45°$, so $R_{\max} = u^2/g$.

Step 1: Relate the farthest landing point to maximum range All bullets leave the gun with the same speed, while their launch directions vary. The farthest ground point is reached by the direction that maximises the horizontal range. The horizontal and vertical components are $u\cos\theta$ and $u\sin\theta$. The flight time back to ground level is

$$T = \frac{2u\sin\theta}{g}.$$

Multiplying by the constant horizontal velocity gives

$$R = (u\cos\theta)T = \frac{u^2\sin(2\theta)}{g}.$$

Step 2: Maximise the range and substitute the data Since $\sin(2\theta) \le 1$, the greatest range is obtained at $45°$. Thus the given farthest distance $6.4\,\mathrm{m}$ obeys

$$6.4 = \frac{u^2}{10}, \qquad u^2 = 64\,\mathrm{m^2\,s^{-2}}.$$

Taking the positive square root is necessary because speed is nonnegative:

$$u = \sqrt{64} = 8\,\mathrm{m\,s^{-1}}.$$

As a check, substituting $u = 8\,\mathrm{m\,s^{-1}}$ into u^2/g returns $6.4\,\mathrm{m}$. The given distance is a horizontal range, not the vertical height, so the maximum-height formula would not apply.

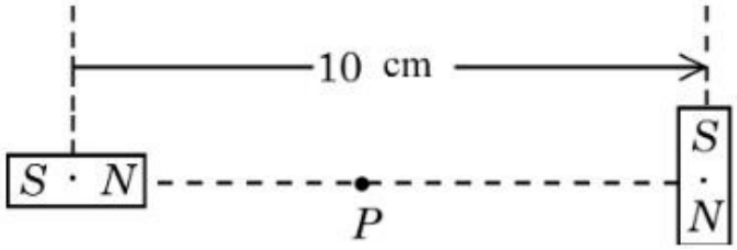
Step 3: Conclusion The launch speed of the bullets is $\boxed{8\,\mathrm{m\,s^{-1}}}$. The numerical value to enter is 8.

Quick Tip to Solve: When all launch angles are available and the farthest ground distance is given, use $u = \sqrt{gR_{\max}}$ directly.

48. Two identical small bar magnets each of dipole moment $3\sqrt{5}$ J/T are placed at a center to center separation of 10 cm , with their axes perpendicular to each other as shown in figure. The value of magnetic field at the point P midway between the magnets is $\alpha \times 10^{-3}$ T. The value of α is ____

$$\left(\mu_0 = 4\pi \times 10^{-7}\text{Tm/A}\right)$$

[4 marks]



Solution:

Concept:

- For a small bar magnet of dipole moment m, the axial field magnitude at distance r is $B_{\text{a}} = (\mu_0/4\pi)(2m/r^3)$, whereas the equatorial magnitude is $B_{\text{e}} = (\mu_0/4\pi)(m/r^3)$.
- Magnetic fields add as vectors. At an axial point the field is along the magnetic moment, while at an equatorial point it is opposite to the magnetic moment.

Step 1: Identify the two field directions at P The midpoint P lies on the axis of the horizontal magnet and on the equatorial line of the vertical magnet. The horizontal magnet has its north pole towards P, so its field at P points to the right. The vertical magnet has its north pole below its south pole, so its magnetic moment points downwards and its equatorial field at P points upwards. The two contributions are therefore perpendicular.

Step 2: Calculate the common field factor Each magnet is half the centre-to-centre separation from P:

$$r = \frac{10}{2}\,\text{cm} = 0.05\,\text{m}, \qquad r^3 = 1.25 \times 10^{-4}\,\text{m}^3.$$

With $m = 3\sqrt{5}\,\text{J}\,\text{T}^{-1}$ and $\mu_0/(4\pi) = 10^{-7}$, define

$$C = \frac{\mu_0 m}{4\pi r^3} = \frac{10^{-7}(3\sqrt{5})}{1.25 \times 10^{-4}} = 2.4\sqrt{5} \times 10^{-3}\,\text{T}.$$

Thus the rightward axial field is $2C$, and the upward equatorial field is C.

Step 3: Add the perpendicular fields Use the Pythagorean sum, rather than adding the magnitudes arithmetically:

$$B = \sqrt{(2C)^2 + C^2} = \sqrt{5}\,C = \sqrt{5}(2.4\sqrt{5} \times 10^{-3}) = 12 \times 10^{-3}\,\text{T}.$$

Comparing this expression with the given form $\alpha \times 10^{-3}$ T determines the numerical coefficient. Using $10\,\text{cm}$ for r would incorrectly use the whole separation instead of the distance from either magnet to P.

Step 4: Conclusion The resultant magnetic field is $12 \times 10^{-3}\,\mathrm{T}$, so the required dimensionless coefficient is $\boxed{\alpha = 12}$.

Quick Tip to Solve: At an axial-equatorial midpoint of equal perpendicular dipoles, the fields have magnitudes $2C$ and C; their resultant is $\sqrt{5}\,C$.

49. A circular coil of radius 2 cm and 125 turns carries a current of 1 A . The coil is placed in a uniform magnetic field of magnitude 0.4 T . The axis of the coil makes an angle of 30° with the direction of the magnetic field. The torque acting on the coil is $\alpha \times 10^{-4}$ N.m. The value of α is ____ .

$$(\pi = 3.14)$$

[4 marks]

Solution:

Concept:

- A current loop acts as a magnetic dipole with magnetic moment $\mu = NIA$, where N is its number of turns, I its current and A the area of one turn.
- The torque magnitude is $\tau = \mu B \sin\theta$. The angle θ is measured between the coil axis (the normal to its plane) and the magnetic field.

Step 1: Convert the radius and find the area The radius, rather than the diameter, is given. Convert centimetres into metres before calculating the area, because the magnetic field and required torque are in SI units.

$$r = 2 \times 10^{-2}\,\mathrm{m}, \qquad A = \pi r^2 = 3.14(2 \times 10^{-2})^2 = 1.256 \times 10^{-3}\,\mathrm{m}^2.$$

The area in this expression is for one circular turn; the factor of 125 is included separately.

Step 2: Calculate the magnetic moment and torque The stated angle of 30° is already measured from the coil axis, so use its sine without taking the complementary angle.

$$\mu = 125 \times 1 \times 1.256 \times 10^{-3} = 0.157\,\mathrm{A\,m}^2.$$

$$\tau = 0.157 \times 0.4 \times \sin 30^\circ = 0.0314\,\mathrm{N\,m}.$$

The torque would vanish for an axis parallel to the field and be greatest for a perpendicular axis, consistent with the sine factor.

Step 3: Extract the requested coefficient Write the calculated torque using the power of ten specified in the question:

$$0.0314\,\mathrm{N\,m} = 314 \times 10^{-4}\,\mathrm{N\,m}.$$

The requested number is the coefficient of this power of ten, rather than the SI numerical value 0.0314 itself.

Step 4: Conclusion The torque is $3.14 \times 10^{-2}\,\mathrm{N\,m}$, so $\boxed{\alpha = 314}$ (dimensionless).

Quick Tip to Solve: For coil-torque questions, identify whether the given angle is with the axis or the plane, and square the radius only after converting it to metres.

50. In a double slit experiment, when one of the slits is covered by a transparent mica sheet of refractive index 1.56 , the central fringe shifts to the position of 7^{th} bright fringe, obtained with both slits uncovered. If the light source wavelength is 450 nm , the thickness of mica sheet is $\alpha \times 10^{-9}$ m. The value of α is

____ .

[4 marks]

Solution:

Concept:

- A transparent sheet of thickness t and refractive index μ replaces the same thickness of air, adding optical path difference $(\mu - 1)t$.
- One fringe-width displacement corresponds to one wavelength of additional optical path difference. Thus a displacement through m bright-fringe positions gives $(\mu - 1)t = m\lambda$.

Step 1: Relate the fringe shift to path difference The central bright fringe corresponds to zero net phase difference. Inserting mica delays the wave through one slit. The new central fringe occurs where the geometrical path difference compensates this delay. Its displacement to the old seventh bright fringe therefore means a shift by seven fringe widths. In magnitude,

$$(\mu - 1)t = 7\lambda.$$

There is no extra half-wavelength term, because both the initial and final positions under comparison are bright fringes.

Step 2: Substitute the optical data Use $\mu = 1.56$, so the excess refractive index relative to air is $\mu - 1 = 0.56$. With $\lambda = 450 \times 10^{-9}\,\text{m}$,

$$t = \frac{7 \times 450 \times 10^{-9}}{0.56}\,\text{m} = 5625 \times 10^{-9}\,\text{m}.$$

Dividing by 1.56 instead would incorrectly count the optical path in the displaced air as an additional delay.

Step 3: Check the thickness and identify the coefficient The thickness is $5.625 \times 10^{-6}\,\text{m}$. As a check, the added path is

$$0.56 \times 5625\,\text{nm} = 3150\,\text{nm} = 7 \times 450\,\text{nm}.$$

The slit separation and screen distance cancel when displacement is expressed as a number of fringes, so neither is required.

Step 4: Conclusion The mica sheet thickness is $5.625\,\mu\text{m} = 5625 \times 10^{-9}\,\text{m}$. Hence $\boxed{\alpha = 5625}$ (dimensionless).

Quick Tip to Solve: For a transparent-sheet fringe shift, use the added optical path $(\mu - 1)t$; the number of shifted fringes is this path divided by the wavelength.

51. The correct order of total number of atoms present in

(A) 2 moles of cyclohexane

(B) 684 g of sucrose

(C) 90.8 L of dihydrogen at STP

is : [4 marks]

(A) $\mathrm{C} > \mathrm{A} > \mathrm{B}$

(B) $\mathrm{C} > \mathrm{B} > \mathrm{A}$

(C) $\mathrm{B} > \mathrm{C} > \mathrm{A}$

(D) $\mathrm{B} > \mathrm{A} > \mathrm{C}$

Correct Answer: (D) $\mathrm{B} > \mathrm{A} > \mathrm{C}$

Solution:

Concept:

- Total atoms equal the number of molecules multiplied by the atoms in each molecule. One mole contains N_{A} entities.
- For a molecular sample, moles of atoms equal moles of molecules times the sum of all subscripts in its molecular formula. Mass is converted using molar mass; gas volume is converted using the molar volume at STP.

Step 1: Count atoms in cyclohexane and sucrose Cyclohexane is $\mathrm{C_6H_{12}}$, with $6 + 12 = 18$ atoms per molecule. Therefore sample A contains

$$N_A = 2 \times 18 N_{\mathrm{A}} = 36 N_{\mathrm{A}} \text{ atoms.}$$

Sucrose is $\mathrm{C_{12}H_{22}O_{11}}$, with molar mass $12(12) + 22(1) + 11(16) = 342\,\mathrm{g\,mol^{-1}}$ and $12 + 22 + 11 = 45$ atoms per molecule. Thus

$$n_B = \frac{684}{342} = 2\,\mathrm{mol}, \qquad N_B = 2 \times 45 N_{\mathrm{A}} = 90 N_{\mathrm{A}} \text{ atoms.}$$

Although A and B each contain two moles of molecules, their atom counts differ.

Step 2: Count atoms in dihydrogen and compare Each $\mathrm{H_2}$ molecule contains two atoms. Using the standard STP molar volume of approximately $22.7\,\mathrm{L\,mol^{-1}}$,

$$n_C = \frac{90.8}{22.7} = 4\,\mathrm{mol}, \qquad N_C = 8 N_{\mathrm{A}} \text{ atoms.}$$

The older $22.4\,\mathrm{L\,mol^{-1}}$ convention gives approximately $8.11 N_{\mathrm{A}}$ atoms and leaves the ranking unchanged. Hence

$$90 N_{\mathrm{A}} > 36 N_{\mathrm{A}} > 8 N_{\mathrm{A}}.$$

Options (A) and (B) incorrectly place C first; option (C) incorrectly places C above A.

Step 3: Conclusion The total number of atoms follows $\boxed{\mathrm{B} > \mathrm{A} > \mathrm{C}}$. Hence, the correct option is (D).

Quick Tip to Solve: Compare moles of atoms, not just moles of molecules: multiply each molecular mole count by the total number of atoms in its formula.

52. The species having identical radii according to the Bohr's theory are :

A. H (first orbit)

B. He^+ **(first orbit)**

C. He^+ **(Second orbit)**

D. Li^{2+} **(first orbit)**

E. Be^{3+} **(Second orbit)**

Choose the correct answer from the options given below : [4 marks]

(A) A and C Only

(B) A and E Only

(C) B and E Only

(D) C and D Only

Correct Answer: (B) A and E Only

Solution:

Concept:

- For a one-electron atom or ion, the Bohr radius of orbit n is $r_n = a_0 n^2/Z$, where Z is nuclear charge and a_0 is the first-orbit radius of hydrogen.
- All listed species have one electron, so compare their dimensionless factors n^2/Z. Equal factors imply equal radii even when the nuclear charges differ.

Step 1: Evaluate the radius factor for each species For H, $Z = 1$; for He^+, $Z = 2$; for Li^{2+}, $Z = 3$; and for Be^{3+}, $Z = 4$. The ionic charge is not substituted for Z. The given orbits yield

$$r_A = a_0\frac{1^2}{1} = a_0, \quad r_B = a_0\frac{1^2}{2} = \frac{a_0}{2},$$
$$r_C = a_0\frac{2^2}{2} = 2a_0, \quad r_D = a_0\frac{1^2}{3} = \frac{a_0}{3},$$
$$r_E = a_0\frac{2^2}{4} = a_0.$$

The second orbit carries the factor $n^2 = 4$, rather than 2, which offsets the higher nuclear charge of beryllium.

Step 2: Identify the equal pair and reject the alternatives Only A and E both have radius a_0. A and C are unequal because C has twice the radius of A. B and E are unequal because B has half the radius of E. C and D differ by a factor of six, so they cannot be the required pair. These comparisons directly exclude options (A), (C) and (D). No numerical value of a_0 is needed, since it is a common factor for every listed species.

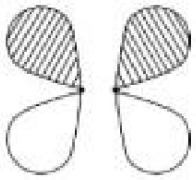
Step 3: Conclusion H in its first orbit and Be^{3+} in its second orbit have identical radii: A and E only. Hence, the correct option is (B).

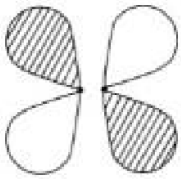
Quick Tip to Solve: For hydrogen-like radii, compare n^2/Z; use atomic number for Z, not the positive charge on the ion.

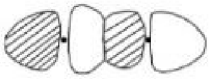
53. Which of the following pictorial diagram most correctly represents the π^* (π antibonding) molecular orbital between two atoms if the internuclear axis is taken to be in the z-direction ($\xrightarrow{z\text{-axis}}$) ?

[4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- A π molecular orbital arises from sideways overlap of two parallel p orbitals perpendicular to the internuclear axis. Here that axis is the horizontal z-direction.
- An antibonding π^* orbital has opposite phases in facing lobes. Destructive interference produces an additional node between the nuclei. Shading denotes wavefunction phase, not positive or negative electric charge.

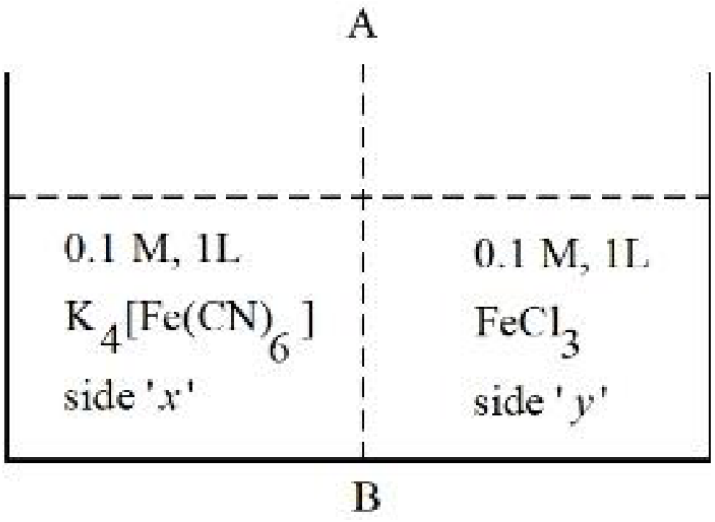
Step 1: Locate the phase change required for antibonding overlap Each atomic p orbital has two lobes of opposite phase. For a π^* combination, the upper lobe on one atom must have the opposite phase from the upper lobe on its neighbour, and the lower pair must also have opposite phases. Consequently, the four-lobe picture has matching phases diagonally and unlike phases horizontally across the internuclear gap. This creates a nodal plane between the nuclei, in addition to the nodal plane containing the internuclear axis characteristic of a π orbital.

Step 2: Compare the four pictorial options Option (C) shows two perpendicular-to-axis p orbitals with diagonal lobes shaded alike. The upper facing pair and the lower facing pair have opposite shading, giving the required destructive sideways overlap. Option (A) shows continuous density above and below the internuclear region, characteristic of bonding π overlap. Option (B) shows the same phase on the two upper lobes and likewise on the lower lobes, so their sideways combination is bonding rather than antibonding. Option (D) places its lobes along the internuclear direction, corresponding to axial overlap of sigma symmetry instead of pi symmetry.

Step 3: Conclusion The transverse four-lobe diagram with opposite phases on facing lobes is the π^* orbital. Hence, the correct option is (C).

Quick Tip to Solve: For π^*, look for sideways p-orbital alignment and opposite phases across the internuclear gap; equal diagonal shading is a useful visual check.

54. At $27^{\circ}C, 0.1M, 1$ L $K_4[Fe(CN)_6]$ aqueous solution and $0.1M, 1$ L$FeCl_3$ aqueous solution are placed in a container separated by a semi permeable membrane AB . Assume complete dissociation of both the solutes. Which of the following statement is correct? [4 marks]



(A) Blue color is formed on both sides.

(B) Ionic solutes in aqueous solution can pass through semi-permeable membrane.

(C) Solution on side ' y ' is hypotonic.

(D) To cause the reverse flow of solvent during osmosis, external pressure (any value) should be applied to side ' x '.

Correct Answer: (C) Solution on side ' y ' is hypotonic.

Solution:

Concept:

- For an ideal dilute electrolyte solution, osmotic pressure is $\Pi = iCRT$, where i is the van't Hoff factor, C the solute molarity and T the absolute temperature.
- A semipermeable membrane allows solvent through but excludes dissolved solute particles. At equal temperature, solvent flows towards the solution with larger effective particle concentration iC.

Step 1: Calculate the particle concentrations on both sides On side x, potassium ferrocyanide dissociates as

$$K_4[Fe(CN)_6] \longrightarrow 4\,K^+ + [Fe(CN)_6]^{4-},$$

giving five ions per formula unit, so $i_x = 5$. The complex ion remains one particle. On side y,

$$FeCl_3 \longrightarrow Fe^{3+} + 3\,Cl^-,$$

giving $i_y = 4$. Therefore

$$i_x C_x = 0.50\,\text{mol}\,L^{-1}, \qquad i_y C_y = 0.40\,\text{mol}\,L^{-1}.$$

At the common temperature of 300 K, $\Pi_x > \Pi_y$, so y is hypotonic relative to x.

Step 2: Check each proposed statement Water tends to move from y to x. Option (C) correctly identifies y as the side of lower osmotic pressure. Option (B) is false because the membrane excludes the ionic solutes. Consequently the iron(III) and ferrocyanide ions do not mix across it to form a blue product, so option (A) is false. For reverse osmosis, pressure applied to x must exceed the osmotic-pressure difference, rather than being of any arbitrary value:

$$\Delta P > \Pi_x - \Pi_y = (0.50 - 0.40)RT.$$

A pressure below this threshold cannot reverse the solvent flow, which excludes option (D).

Step 3: Conclusion Side y, containing $FeCl_3$, is hypotonic relative to side x. Hence, the correct option is (C).

Quick Tip to Solve: At equal molarity, compare the number of independently dissolved particles. Keep a complex ion intact when counting the dissociation products.

55. 20 mL of a solution of acetic acid required 28.4 mL of 0.1 M NaOH for its neutralization. A solution (X) was prepared by mixing 20 mL of the above acetic acid and 14.2 mL of 0.1 M NaOH solution. What is the pH of the solution (X) ? (pK_a value of acetic acid is 4.75). [4 marks]

(A) 7.0

(B) 4.75

(C) 3.5

(D) 4.82

Correct Answer: (B) 4.75

Solution:

Concept:

- A weak acid and its conjugate base form a buffer. For acetic acid,

$$\text{pH} = \text{p}K_a + \log_{10}\frac{[CH_3COO^-]}{[CH_3COOH]}.$$

- Neutralization is a one-to-one reaction: $CH_3COOH + OH^- \longrightarrow CH_3COO^- + H_2O$. At half-neutralization, equal amounts of acid and acetate are present.

Step 1: Find the original acid amount The complete titration tells us the amount of acetic acid in the 20 mL sample. Since each mole of acid requires one mole of hydroxide,

$$n_0(CH_3COOH) = 0.100 \times 28.4 \times 10^{-3} = 2.84 \times 10^{-3}\ \text{mol}.$$

The second 20 mL portion contains this same amount of acid because it is taken from the same solution.

Step 2: Account for partial neutralization The added base provides

$$n(\mathrm{OH^-}) = 0.100 \times 14.2 \times 10^{-3} = 1.42 \times 10^{-3}\ \mathrm{mol}.$$

It converts exactly half of the acid into acetate. After reaction, the remaining acid and the acetate formed each amount to 1.42×10^{-3} mol. There is no excess strong base. Both species occupy the same total volume, so their concentration ratio equals their mole ratio, namely one.

Step 3: Calculate the buffer pH and compare the options Substitution into the buffer equation gives

$$\mathrm{pH} = 4.75 + \log_{10}\left(\frac{1.42 \times 10^{-3}}{1.42 \times 10^{-3}}\right) = 4.75.$$

The value 7.0 incorrectly treats half-neutralization as a neutral solution. Values 3.5 and 4.82 require unequal acid and acetate concentrations, contrary to the stated volumes. The result is a buffer pH, not the equivalence-point pH.

Step 4: Conclusion The solution is at half-neutralization, so $\boxed{\mathrm{pH} = 4.75}$. Hence, the correct option is (B).

Quick Tip to Solve: Compare the added base volume with the full titration volume first: exactly half gives $\mathrm{pH} = \mathrm{p}K_a$, independent of the final buffer volume.

56. Match LIST-I with LIST-II.

List-I: Reaction		List-II: Mechanism	
A.	Williamson Synthesis	I.	Electrophilic addition
B.	Friedel Craft Reaction	II.	Free radical substitution
C.	Bromination of vinyl benzene	III.	Nucleophilic substitution
D.	Chlorination of toluene in light	IV.	Electrophilic substitution

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-I, C-II, D-IV

(B) A-III, B-IV, C-II, D-I

(C) A-III, B-IV, C-I, D-II

(D) A-I, B-III, C-IV, D-II

Correct Answer: (C) A-III, B-IV, C-I, D-II

Solution:

Concept:

- Reaction mechanisms are classified by the attacking species and by whether a group is replaced or added. Nucleophiles donate an electron pair; electrophiles accept one.
- An alkene generally undergoes addition at its carbon-carbon double bond, an aromatic ring preserves aromaticity by substitution, and light can initiate radical chain reactions.

Step 1: Identify the ether synthesis and aromatic reaction In Williamson synthesis, an alkoxide ion

attacks the carbon bearing the leaving group in an alkyl halide. A halide ion departs and an ether forms, generally by the S_N2 pathway:

$$C_2H_5O^- + CH_3Br \longrightarrow C_2H_5OCH_3 + Br^-.$$

Thus A matches III, nucleophilic substitution. In a Friedel-Crafts reaction, an electrophile attacks the benzene ring and replaces a ring hydrogen after aromaticity is restored. Hence B matches IV, electrophilic substitution.

Step 2: Use the double bond and light conditions Vinyl benzene is styrene, $C_6H_5CH{=}CH_2$. Bromination at the side-chain double bond adds bromine across that bond through electrophilic addition; it does not require replacing a ring hydrogen. Therefore C matches I. In chlorination of toluene in light, photochemical homolysis of Cl_2 generates chlorine radicals. Benzylic hydrogen replacement proceeds through radical propagation steps, so D matches II, free radical substitution.

Step 3: Check the four proposed matchings The complete correspondence is

$$A-III, \quad B-IV, \quad C-I, \quad D-II.$$

Option (A) incorrectly calls Friedel-Crafts an addition and assigns radical substitution to bromination of the alkene. Option (B) also interchanges the last two mechanisms. Option (D) incorrectly classifies Williamson synthesis as electrophilic addition. Only option (C) preserves all four mechanisms.

Step 4: Conclusion The matching is A-III, B-IV, C-I, D-II. Hence, the correct option is (C).

Quick Tip to Solve: For side chains on benzene, check both the functional group and the condition: an alkene adds bromine, whereas light drives benzylic radical chlorination.

57. The 1^{st} ionization enthalpy for Mg is $+737$ kJ/mol. The most probable estimated value of the 2^{nd} ionization enthalpy of Mg is ____ . [4 marks]

(A) -906 kJ/mol

(B) -856 kJ/mol

(C) $+1450$ kJ/mol

(D) $+590$ kJ/mol

Correct Answer: (C) $+1450$ kJ/mol

Solution:

Concept:

- Ionization enthalpy is the energy needed to remove an electron from an isolated gaseous atom or ion. Electron removal is endothermic, so the ionization enthalpy is positive.
- Successive ionizations remove electrons from increasingly positive ions. With the same nuclear charge and fewer screening electrons, the remaining electron is more strongly bound.

Step 1: Write the relevant configurations Magnesium has atomic number 12, so its ground-state

configuration is $[\mathrm{Ne}]3s^2$. The first ionization removes one of its two valence electrons:

$$\mathrm{Mg(g)} \longrightarrow \mathrm{Mg^+(g)} + \mathrm{e^-}, \qquad \Delta H_1 = +737\ \mathrm{kJ\,mol^{-1}}.$$

The resulting ion has configuration $[\mathrm{Ne}]3s^1$. The second ionization removes that remaining $3s$ electron, rather than an electron from a neutral atom.

Step 2: Apply the sign and magnitude tests The second process is

$$\mathrm{Mg^+(g)} \longrightarrow \mathrm{Mg^{2+}(g)} + \mathrm{e^-}.$$

The electron is attracted to a net-positive ion, and the loss of the first electron reduces electron-electron repulsion. Thus more energy is required for the second removal than for the first:

$$\Delta H_2 > \Delta H_1 = 737\ \mathrm{kJ\,mol^{-1}}.$$

Reaching a stable noble-gas configuration does not make electron removal exothermic; energy must still be supplied.

Step 3: Eliminate the incompatible estimates The proposed values -906 and $-856\ \mathrm{kJ\,mol^{-1}}$ are negative, contradicting endothermic ionization. Although $+590\ \mathrm{kJ\,mol^{-1}}$ is positive, it is smaller than the first ionization enthalpy and therefore fails the successive-ionization trend. The only positive value exceeding 737 is $+1450\ \mathrm{kJ\,mol^{-1}}$. No empirical interpolation is necessary to select it.

Step 4: Conclusion The most probable second ionization enthalpy is $\boxed{+1450\ \mathrm{kJ\,mol^{-1}}}$. Hence, the correct option is (C).

Quick Tip to Solve: For successive ionization estimates, reject negative values first, then require the next ionization enthalpy to exceed the preceding one.

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58. The electronegativity of a group 13 element ' E ' is same as that of Ge (on Pauling scale and upto one decimal point). The CORRECT statements about $\mathrm{E^{3+}}$ are

A. It can act as a reducing agent.

B. It can act as an oxidizing agent.

C. $\mathrm{E^{3+}}$ is more stable than $\mathrm{E^+}$.

D. The standard electrode potential value for $\mathrm{E^{3+}/E}$ is positive.

Choose the correct answer from the options given below : [4 marks]

(A) A and C Only

(B) B and C Only

(C) B and D Only

(D) A and D Only

Correct Answer: (C) B and D Only

Solution:

Concept:

- The rounded Pauling electronegativity used for Ge is 1.8. Among the group 13 elements, Tl has the same value and is therefore the element E.
- The inert pair effect stabilizes the lower oxidation state in heavy p-block elements. In thallium, the $6s^2$ pair tends to remain unshared, making Tl(I) more stable than Tl(III).
- An oxidizing agent gains electrons and is reduced. A positive standard reduction potential denotes a reduction favourable relative to the standard hydrogen electrode under standard conditions.

Step 1: Identify the ion and its preferred oxidation state The electronegativity clue identifies Tl, so E^{3+} means Tl^{3+}. The valence configuration of neutral thallium is $6s^2 6p^1$. Loss of the $6p$ electron gives the comparatively stable $+1$ state, retaining the inert $6s^2$ pair. Consequently the $+3$ state tends to accept two electrons:

$$Tl^{3+} + 2\,e^- \longrightarrow Tl^+.$$

This establishes that statement B is correct and statement C is false.

Step 2: Judge the reducing-agent claim A reducing agent donates electrons and is itself oxidized. The characteristic redox behaviour of Tl^{3+} is reduction to the more stable Tl^+, not oxidation to a still higher ordinary oxidation state. It therefore acts as an oxidizing agent in this comparison. Statement A is false; it reverses the electron-transfer role of the trivalent ion.

Step 3: Check the electrode potential and choose the pair The standard reduction potential for Tl^{3+}/Tl, referring to $Tl^{3+} + 3\,e^- \longrightarrow Tl$, is positive (about $+0.72$ V). Thus statement D is true. The true statements are B and D only. Options (A) and (D) contain the false reducing-agent claim A, while option (B) contains the false stability claim C.

Step 4: Conclusion Here E is Tl, and statements B and D are correct. Hence, the correct option is (C).

Quick Tip to Solve: For Tl, remember that the inert pair effect favours $+1$; the less stable $+3$ ion tends to gain electrons and therefore behaves as an oxidizing agent.

59. Pairs of elements with the same number of electrons in their respective 4f orbital are

[Atomic number. Eu-63, Gd-64, Dy-66, Ho-67, Tm-69, Yb-70, Lu-71, Hf-72]

A. (Eu and Gd)

B. (Dy and Ho)

C. (Yb and Hf)

D. (Lu and Tm)

Choose the correct answer from the options given below : [4 marks]

(A) B and C Only

(B) A and B Only

(C) A and D Only

(D) A and C Only

Correct Answer: (D) A and C Only

Solution:

Concept:

- The question compares the number of electrons in the $4f$ subshell of neutral atoms. Use ground-state configurations, including the exceptional half-filled $4f^7$ configuration of Gd.
- The xenon core contains 54 electrons. In these atoms the remaining electrons occupy $4f$, $5d$, and $6s$; a filled $4f$ subshell contains 14 electrons.

Step 1: Compare europium with gadolinium The ground-state configurations are

$$\text{Eu } (63) : [\text{Xe}]4f^7 6s^2,$$
$$\text{Gd } (64) : [\text{Xe}]4f^7 5d^1 6s^2.$$

Gadolinium places its extra electron in $5d$, retaining the stable half-filled $4f^7$ arrangement. Both therefore contain seven $4f$ electrons, and pair A is correct. Simply increasing the $4f$ exponent by one between these neighbours would miss this exception.

Step 2: Compare dysprosium and holmium For the next listed pair,

$$\text{Dy} : [\text{Xe}]4f^{10}6s^2, \qquad \text{Ho} : [\text{Xe}]4f^{11}6s^2.$$

Their $4f$ populations are 10 and 11, respectively. Since the populations differ, pair B is incorrect. The question concerns total $4f$ electrons, not just the unpaired electrons.

Step 3: Check the pairs near the filled subshell Ytterbium is $[\text{Xe}]4f^{14}6s^2$, whereas hafnium is $[\text{Xe}]4f^{14}5d^2 6s^2$. Both have 14 electrons in $4f$, making pair C correct. Lutetium is $[\text{Xe}]4f^{14}5d^1 6s^2$, but thulium is $[\text{Xe}]4f^{13}6s^2$; therefore pair D is incorrect. Thus only A and C survive. Options (A) and (B) include false pair B; option (C) includes false pair D.

Step 4: Conclusion The equal-population pairs are Eu/Gd and Yb/Hf, namely A and C only. Hence, the correct option is (D).

Quick Tip to Solve: Check Gd separately: its extra electron enters $5d$, so Eu and Gd both retain $4f^7$; after Yb, the $4f^{14}$ count stays filled.

60. Consider the metal complexes $[Ni(en)_3]^{2+}$ (A), $[NiCl_4]^{2-}$ (B) and $[Ni(NH_3)_6]^{2+}$ (C). Choose the CORRECT option by considering the number of unpaired electrons present in (A), (B) and (C) respectively and the order of frequency of absorption. [4 marks]

(A) 2, 2, 2 and (A) > (C) > (B)

(B) 0, 2, 0 and (A) > (C) > (B)

(C) 2, 2, 0 and (B) > (C) > (A)

(D) 2, 2, 2 and (C) > (A) > (B)

Correct Answer: (A) 2, 2, 2 and (A) > (C) > (B)

Solution:

Concept:

- In all three complexes nickel is in the +2 oxidation state, with a $3d^8$ configuration. Count the electrons after identifying both oxidation state and coordination geometry.
- An octahedral d^8 ion has occupation $t_{2g}^6 e_g^2$; a tetrahedral d^8 ion has $e^4 t_2^4$. Both arrangements contain two unpaired electrons.
- For a crystal-field transition, $h\nu = \Delta$. A larger crystal-field splitting corresponds to a larger absorbed frequency; ligand strength follows $Cl^- < NH_3 < en$.

Step 1: Determine geometry and unpaired electrons Each ethylenediamine ligand (en) is neutral and bidentate, so complex A has six donor atoms and an octahedral geometry. The six neutral ammonia ligands make C octahedral as well. In each case, the two electrons in the degenerate e_g orbitals occupy them singly, giving two unpaired electrons. For B, four chloride ligands around Ni^{2+} give a tetrahedral complex. Its t_2^4 population has one pair and two singly occupied orbitals, again giving two unpaired electrons.

Step 2: Compare the absorption frequencies For the two octahedral complexes, en produces a stronger field than ammonia, hence

$$\Delta_A > \Delta_C.$$

The chloride complex is tetrahedral and has the weak-field ligand Cl^-; its splitting is smaller than those of the octahedral complexes. Therefore

$$\Delta_A > \Delta_C > \Delta_B \quad \Rightarrow \quad \nu_A > \nu_C > \nu_B.$$

The order is for frequency; wavelength would have the reverse order.

Step 3: Reject the alternative combinations The electron counts are $(2, 2, 2)$. Options (B) and (C) incorrectly assign zero unpaired electrons to an octahedral d^8 complex. Stronger octahedral ligands do not change the two singly occupied e_g orbitals into the square-planar arrangement. Option (D) has the correct counts but incorrectly places the ammonia complex above the en complex in absorption frequency.

Step 4: Conclusion The numbers of unpaired electrons are $2, 2, 2$, and $\nu_A > \nu_C > \nu_B$. Hence, the correct option is (A).

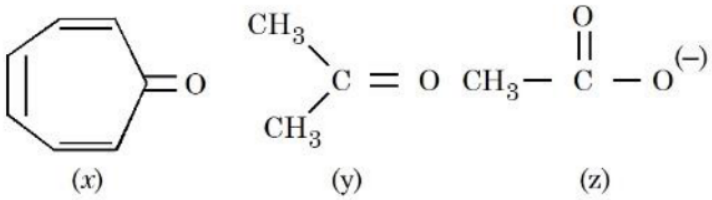
Quick Tip to Solve: For nickel(II), distinguish octahedral or tetrahedral d^8 from square-planar d^8; ligand strength alone does not imply a diamagnetic complex.

61.

Consider the following molecules/species :

The correct order of carbon-oxygen double bond length is :

[4 marks]



(A) $x > y > z$

(B) $\mathrm{y} > \mathrm{z} > x$

(C) $\mathrm{z} > x > \mathrm{y}$

(D) $x > \mathrm{z} > \mathrm{y}$

Correct Answer: (D) $x > \mathrm{z} > \mathrm{y}$

Solution:

Concept:

- Bond length decreases as bond order increases. Delocalisation that gives a carbonyl bond more single-bond character therefore lengthens that bond.
- Resonance contributors are especially important when they produce equivalent bonding arrangements or an aromatic ring containing six delocalised π electrons.

Step 1: Identify the three carbonyl environments The species labelled x is tropone, the seven-membered conjugated cyclic ketone; y is acetone; and z is the acetate ion. These cannot be compared merely by counting the double bonds drawn in one Lewis structure, because the drawing of a delocalised species represents only one contributor.

Step 2: Compare their resonance effects Acetone has the most localised carbonyl double bond of the three, so y has the shortest carbon–oxygen bond. In acetate, the negative charge is shared by two oxygen atoms through two equivalent resonance contributors. Both carbon–oxygen bonds are consequently equivalent, with bond order intermediate between one and two; the bond labelled z is longer than the carbonyl bond in acetone.

Step 3: Use aromatic stabilisation for tropone In tropone, displacement of the carbonyl π-electron pair towards oxygen gives a charge-separated contributor with an O^- group and a positively charged seven-membered ring. This ring has six π electrons and gains aromatic stabilisation. The resulting substantial single-bond character of its carbon–oxygen linkage makes x longest in the comparison. Hence

$$x > z > y.$$

Option A incorrectly puts acetate below acetone, option B makes acetone longest, and option C puts acetate above tropone.

Step 4: Conclusion The carbon–oxygen bond-length order is $x > z > y$. Hence, the correct option is (D).

Quick Tip to Solve: Look for aromatic stabilisation of a charge-separated contributor before comparing an unusual cyclic carbonyl with a carboxylate and an ordinary ketone.

62. Consider $|x|$ is the difference in oxidation states of Mn in highest manganese fluoride and highest manganese oxide. The ions with $|x|$ number of unpaired electrons from the following are :

A. Sc^{3+}

B. Zn^{2+}

C. V^{2+}

D. Fe^{2+}

E. Co^{2+}

Choose the correct answer from the options given below : [4 marks]

(A) A and B Only

(B) C, D and E Only

(C) C and E Only

(D) B and E Only

Correct Answer: (C) C and E Only

Solution:

Concept:

- The highest stable fluoride of manganese is MnF_4, whereas its highest oxide is Mn_2O_7. Fluorine has oxidation state -1 and oxygen has oxidation state -2.
- For transition-metal cations remove the $4s$ electrons before the $3d$ electrons. Apply Hund's rule to count unpaired electrons in the resulting free-ion d configuration.

Step 1: Calculate the required electron count Let the oxidation state of manganese be m. In MnF_4, $m - 4 = 0$, giving $+4$. In Mn_2O_7, $2m - 14 = 0$, giving $+7$. The difference required by the question is therefore

$$|x| = |4 - 7| = 3.$$

We must select ions with three unpaired electrons, not ions whose oxidation state happens to be three.

Step 2: Write and count the ionic configurations For A, Sc^{3+} has configuration $[Ar]3d^0$, so its count is zero. For B, Zn^{2+} is $[Ar]3d^{10}$, again with zero unpaired electrons. For C, V^{2+} is $[Ar]3d^3$; its three electrons occupy separate d orbitals, giving three unpaired electrons. For D, Fe^{2+} is $[Ar]3d^6$, giving four unpaired electrons. For E, Co^{2+} is $[Ar]3d^7$, giving three unpaired electrons.

Step 3: Select only the matching ions The required pair is therefore C and E. No ligand or coordination complex is specified, so a low-spin ligand-field arrangement must not be imposed on these free ions. Options A and D include closed-shell ions, and option B includes Fe^{2+}, whose unpaired-electron count is four rather than three.

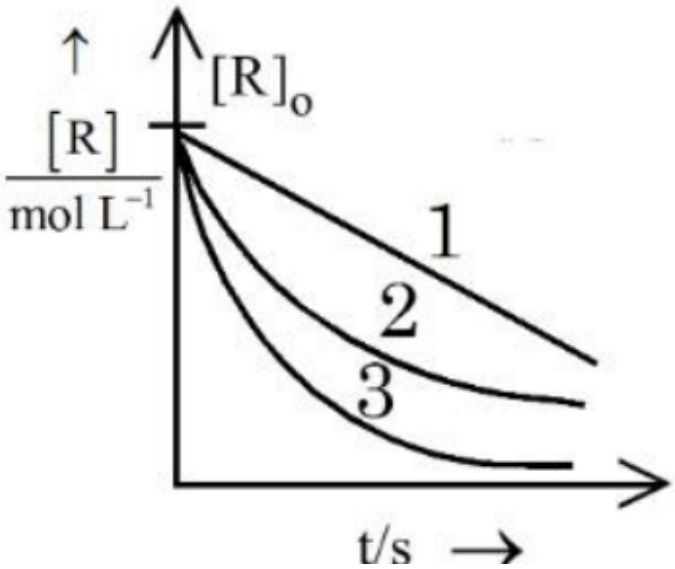
Step 4: Conclusion Only V^{2+} and Co^{2+}, namely C and E, have $|x| = 3$ unpaired electrons. Hence,

the correct option is (C).

Quick Tip to Solve: For a free d^n ion the unpaired count is n up to d^5, then $10 - n$; remove $4s$ electrons first.

63. Consider the given graph showing variation of reactant concentration with time.

Three different reactions were started with identical initial concentration of reactants. Which of the following statement is correct? [4 marks]



(A) The order of all the three reactions is same.

(B) The rate constant of reaction 3 is larger than the rate constant of reaction 2 if the order of reaction is same for both.

(C) The SI unit of rate constant of reaction 1 is s^{-1}.

(D) Thermal decomposition of HI on gold surface is an example of reaction 2 .

Correct Answer: (B) The rate constant of reaction 3 is larger than the rate constant of reaction 2 if the order of reaction is same for both.

Solution:

Concept:

- A zero-order reaction obeys $[R] = [R]_0 - kt$, so its concentration–time graph is a straight line of negative slope.
- For a common rate law $-d[R]/dt = k[R]^n$, reactions with the same order and initial concentration consume reactant more rapidly when k is larger.

Step 1: Identify reaction 1 The curve labelled 1 is a straight declining line on a plot of concentration against time. Its slope is constant, so its rate is independent of concentration and it represents zero-order kinetics. Curves 2 and 3 are curved, with their slope magnitudes decreasing as the concentration falls. Thus the graph does not support the claim that all three orders are the same, eliminating option A.

Step 2: Compare reactions 2 and 3 under the stated condition The two reactions start at the same concentration. Curve 3 reaches any chosen lower concentration sooner than curve 2. For a fixed order n, integration gives

$$kt = \int_{[R]}^{[R]_0} \frac{dC}{C^n}.$$

At the same initial and final concentrations the right side is identical. The shorter time for reaction 3 therefore requires $k_3 > k_2$. This establishes option B without assuming that both curved traces must have a particular order.

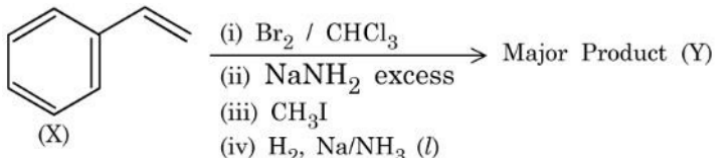
Step 3: Check the remaining statements The units of a zero-order rate constant are concentration divided by time: $\mathrm{mol\,m^{-3}\,s^{-1}}$ in SI, or $\mathrm{mol\,L^{-1}\,s^{-1}}$ when concentration is quoted in molarity. They are not simply $\mathrm{s^{-1}}$, so C is false. Thermal decomposition of HI on a gold surface is the standard zero-order surface reaction under surface-saturation conditions; it corresponds to the straight trace 1 rather than curved trace 2. Hence D is also false.

Step 4: Conclusion If reactions 2 and 3 have the same order, then $k_3 > k_2$. Hence, the correct option is (B).

Quick Tip to Solve: Identify which quantity is plotted: a straight $[R]$-versus-time line signals zero order; a straight $\ln[R]$-versus-time line signals first order.

64. Compound (X) is subjected to the sequence of reactions as shown above. Molar mass of the major product (Y) formed is ____ $\mathrm{gmol^{-1}}$.

(Given molar mass in $\mathrm{gmol^{-1}}$ C : 12, H : 1, O : 16) [4 marks]



(A) 90

(B) 118

(C) 160

(D) 125

Correct Answer: (B) 118

Solution:

Concept:

- Addition of Br_2 to an alkene gives a vicinal dibromide. Strong base can then eliminate two molecules of HBr to form an alkyne.
- Excess $NaNH_2$ deprotonates a terminal alkyne, allowing alkylation with CH_3I. Dissolving-metal reduction of an alkyne gives the corresponding trans alkene.

Step 1: Convert the side-chain double bond into an acetylide The starting structure is styrene, $C_6H_5CH{=}CH_2$. Bromine in chloroform adds across its side-chain double bond to give

$C_6H_5CHBrCH_2Br$. Treatment with excess $NaNH_2$ removes two equivalents of HBr, producing phenylacetylene, $C_6H_5C{\equiv}CH$. Further base removes the terminal proton to produce the phenylacetylide ion, $C_6H_5C{\equiv}C^-$. The excess-base condition is therefore essential for the next step.

Step 2: Add the methyl group and reduce the triple bond The acetylide attacks methyl iodide by an S_N2 reaction, adding one carbon:

$$C_6H_5C{\equiv}C^- + CH_3I \longrightarrow C_6H_5C{\equiv}CCH_3 + I^-.$$

Reduction under the stated sodium/liquid-ammonia conditions converts this side-chain triple bond into a double bond, yielding predominantly trans-$C_6H_5CH{=}CHCH_3$. The benzene ring remains the phenyl group in this transformation. The product formula is C_9H_{10}.

Step 3: Calculate and compare the molar mass The product contains nine carbon atoms and ten hydrogen atoms, with no bromine, nitrogen, iodine or oxygen retained. Using the given atomic masses,

$$M(Y) = 9(12) + 10(1) = 108 + 10 = 118\ \text{g mol}^{-1}.$$

Thus 90, 160 and 125 do not match the product formula. In particular, the supplied oxygen atomic mass does not mean oxygen must occur in the product.

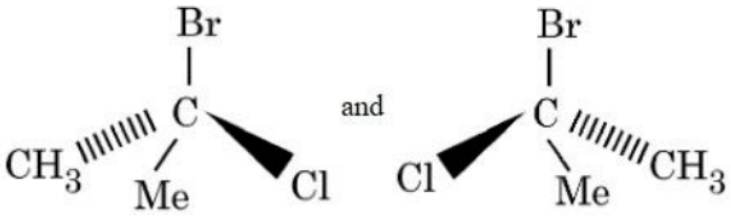
Step 4: Conclusion The major product is trans-$C_6H_5CH{=}CHCH_3$, with molar mass $118\ \text{g mol}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: After double dehydrohalogenation, check whether excess base also deprotonates a terminal alkyne; subsequent methylation increases the carbon count by one.

65.

The following structures are

[4 marks]



(A) enantiomers.

(B) identical molecules.

(C) diastereomers.

(D) meso compounds.

Correct Answer: (B) identical molecules.

Solution:

Concept:

- A tetrahedral carbon is a stereogenic centre only when its four attached substituents are different. Different positions on a drawing do not make chemically identical groups different.
- The abbreviation Me means methyl, CH_3. Enantiomers are non-superimposable mirror images; an achiral molecule and its mirror image represent the same compound.

Step 1: Read all four substituents at the central carbon In each structure the central carbon is bonded to bromine, chlorine, a group written CH_3, and a group written Me. Since Me is another notation for CH_3, the actual substituent set is Br, Cl, CH_3, CH_3. There are only three different substituent types, not four. The differently written methyl groups must be recognised as equivalent before using the wedge-and-dash bonds.

Step 2: Test whether the apparent mirror images are distinct The drawings exchange the apparent left and right positions of chlorine and one methyl group. They look like mirror images, but the central carbon is not a stereogenic centre because two substituents are identical. Interchanging those two indistinguishable methyl groups allows the mirror depiction to be superimposed on the original by spatial rotation. Both drawings therefore specify the same achiral molecule; no R or S configuration can be assigned to that carbon.

Step 3: Exclude the other stereochemical descriptions Option A is false because enantiomers must be non-superimposable. Option C is false because diastereomers must be distinct stereoisomers, whereas these two structures are identical. Option D is false because a meso compound has stereogenic centres but is achiral owing to internal compensation; the displayed structure has no stereogenic centre at all. Merely seeing wedge and dashed bonds does not establish either chirality or meso character.

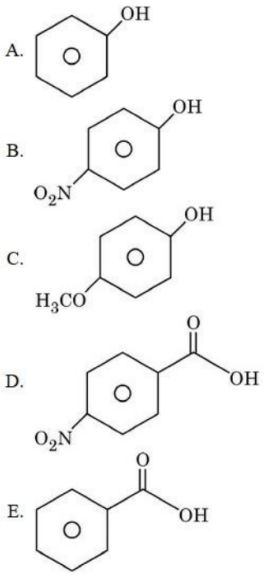
Step 4: Conclusion The structures are identical molecules because Me and CH_3 are the same substituent. Hence, the correct option is (B).

Quick Tip to Solve: Expand abbreviations such as Me, Et and Ph before counting distinct groups at a possible stereogenic carbon.

66.

The descending order of acidity among the following compounds is :

Choose the correct answer from the options given below : [4 marks]



(A) $B > D > E > A > C$

(B) D > B > E > A > C

(C) $C > A > B > D > E$

(D) $D > E > B > A > C$

Correct Answer: (D) $D > E > B > A > C$

Solution:

Concept:

- Acid strength increases when the conjugate base is more stable. Carboxylate ions distribute negative charge over two oxygen atoms and are generally more stabilised than phenoxide ions.
- A para nitro group withdraws electrons by inductive and resonance effects, stabilising the conjugate base. A para methoxy group donates by resonance and decreases phenol acidity.

Step 1: Identify the five acids and separate functional groups A is phenol, B is para-nitrophenol, C is para-methoxyphenol, D is para-nitrobenzoic acid, and E is benzoic acid. D and E contain a carboxylic acid group. A, B and C contain a phenolic hydroxyl group. Loss of a proton from a carboxylic acid forms a carboxylate with two equivalent resonance contributors, each placing the negative charge on oxygen. This stabilisation makes D and E stronger acids than the three phenols in this comparison.

Step 2: Compare the two carboxylic acids The para NO_2 substituent in D withdraws electron density and stabilises its carboxylate conjugate base relative to unsubstituted benzoate. Therefore $D > E$. Although B also contains a nitro group, it is still a nitrophenol; that substituent does not make its phenoxide more stable than the benzoate ion in this set. Hence $E > B$, which immediately rules out options A and B.

Step 3: Order the phenols and assemble the result For the phenols, the para nitro group stabilises the

phenoxide ion by withdrawing electron density and allowing further delocalisation, so $B > A$. The para methoxy group has a resonance-donating effect that dominates its inductive withdrawal here. It increases electron density and destabilises the phenoxide relative to phenol, so $A > C$. Combining the comparisons gives

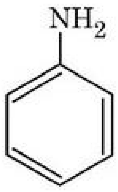
$$D > E > B > A > C.$$

Option C reverses the key functional-group and substituent trends and is therefore also incorrect.

Step 4: Conclusion The descending acidity order is $D > E > B > A > C$. Hence, the correct option is (D).

Quick Tip to Solve: First rank carboxylic acids against phenols, then apply electron-withdrawing or electron-donating substituent effects within each functional-group family.

67. The strongest conjugate acid will result from: [4 marks]

(A) Aniline 

(B) p-Methoxyaniline 

(C) p-Nitroaniline 

(D) p-Methylaniline 

Correct Answer: (C) p-Nitroaniline 

Solution:

Concept:

- For a base B and its conjugate acid BH^+, $K_a(BH^+)K_b(B) = K_w$. Thus the weakest base produces the strongest conjugate acid.
- The basicity of an aromatic amine depends on the availability of its nitrogen lone pair. Electron-withdrawing groups decrease this availability, whereas electron-donating groups generally increase it.

Step 1: Reverse the comparison for conjugate acids Each option contains an amino group that accepts a proton to give an anilinium ion. The question asks about the acidity of this protonated species, not the basicity of the starting amine. A base that binds a proton least strongly has a conjugate acid that releases the proton most readily. We must therefore identify the weakest of the four anilines.

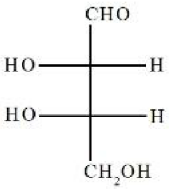
Step 2: Compare the para substituents Aniline in (A) is the reference compound. In (B), the para methoxy group donates electron density through resonance. In (D), the para methyl group donates through its positive inductive effect and hyperconjugation. Both enhance basicity relative to aniline.

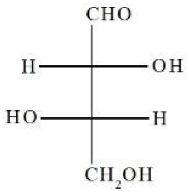
In (C), the nitro group withdraws strongly through both its negative inductive and resonance effects; delocalisation towards this group makes the amino lone pair least available for protonation. Thus p-nitroaniline is the weakest base. Options (A), (B) and (D) yield weaker conjugate acids because their parent bases are more basic.

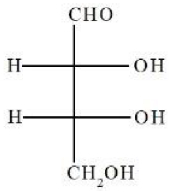
Step 3: Conclusion The strongest conjugate acid is the p-nitroanilinium ion, formed from p-nitroaniline. Hence, the correct option is (C).

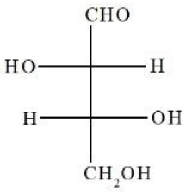
Quick Tip to Solve: For a conjugate-acid question, first rank the starting bases and then reverse that order; a strong electron-withdrawing group on aniline strengthens its conjugate acid.

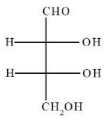
68. A D-aldotetrose on oxidation with concentrated HNO_3 resulted in optically inactive dicarboxylic acid. The structure of the D-aldotetrose is : [4 marks]

(A) L-Erythrose 

(B) L-Threose 

(C) D-Erythrose 

(D) D-Threose 

Correct Answer: (C) D-Erythrose 

Solution:

Concept:

- Concentrated HNO_3 oxidises both the aldehyde end and the terminal primary alcohol of an aldose into carboxyl groups without changing the configurations of its internal stereocentres.
- An aldotetrose is D when the hydroxyl group on its lowest stereocentre in a Fischer projection lies on the right. A meso dicarboxylic acid is optically inactive by internal compensation.

Step 1: Apply the D-configuration requirement In the displayed Fischer projections, (A) has both hydroxyl groups on the left and (B) has the lower hydroxyl group on the left. These are L sugars, so neither satisfies the stated D-aldotetrose condition. Both (C) and (D) have the lower hydroxyl on the right and are D sugars. In (C) both hydroxyl groups are on the right, whereas in (D) they are on opposite sides.

Step 2: Compare the oxidation products Oxidation converts the top CHO and bottom CH_2OH groups into identical COOH ends, giving $HOOC-CH(OH)-CH(OH)-COOH$. The same-side hydroxyl arrangement of (C), D-erythrose, then gives meso-tartaric acid: its two stereocentres have opposite absolute configurations and the molecule has internal symmetry. It is therefore optically inactive. The opposite-side arrangement in (D), D-threose, gives an optically active tartaric-acid stereoisomer.

Although (A) also yields the meso acid, it fails the independent D requirement.

Step 3: Conclusion The required D-aldotetrose is D-erythrose, whose two hydroxyl groups lie on the right in the displayed Fischer projection. Hence, the correct option is (C).

Quick Tip to Solve: Use both clues: the lowest hydroxyl fixes D/L, while the relative positions of the two hydroxyls decide whether oxidation gives meso-tartaric acid.

69. Among $Fe^{3+}, Pb^{2+}, Cu^{2+}$ and Mn^{2+}, identify the one that gets precipitated out while passing H_2S in presence of NH_4OH as group reagent. The highest possible oxidation state of the corresponding metal is [4 marks]

(A) +3

(B) +4

(C) +2

(D) +7

Correct Answer: (D) +7

Solution:

Concept:

- In systematic cation analysis, the group reagent distinguishes ions by their precipitation under specified conditions, after earlier groups have been removed.
- An alkaline medium increases the concentration of S^{2-} supplied by H_2S, allowing precipitation of comparatively more soluble sulphides such as MnS.

Step 1: Identify the analytical group Passing H_2S in the presence of NH_4OH is the group reagent for group-IV cations, including Mn^{2+}. Its precipitate is manganese(II) sulphide:

$$Mn^{2+} + S^{2-} \longrightarrow MnS\,(s).$$

In the systematic separation scheme, Pb^{2+} is removed in the earlier chloride/sulphide groups and Cu^{2+} is precipitated as sulphide in acidic medium. Fe^{3+} belongs to the hydroxide group and is removed as $Fe(OH)_3$. Thus the specified group reagent identifies manganese.

Step 2: Find the maximum oxidation state of that metal Manganese has valence configuration $3d^5 4s^2$, and its highest oxidation state is $+7$, realised in permanganate. Assigning oxygen its usual oxidation state verifies this directly:

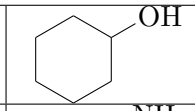
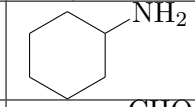
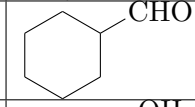
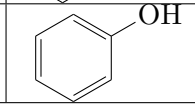
$$x + 4(-2) = -1 \quad \Rightarrow \quad x = +7 \quad \text{in } MnO_4^-.$$

The value $+2$ describes the ion in the precipitate, not the maximum state asked for. Values $+3$ and $+4$ are also below manganese's known maximum, so (A), (B) and (C) are excluded.

Step 3: Conclusion The cation is Mn^{2+}, and manganese has the highest oxidation state $+7$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the two tasks separate: identify the metal using its group reagent, then report its highest oxidation state rather than the charge of the precipitated cation.

70. Match the LIST-I with LIST-II.

List-I: Compound		List-II: Test	
A.		**I.**	**Hinsberg's reagent test**
B.		**II.**	**Phthalein dye test**
C.		**III.**	**Lucas test**
D.		**IV.**	**Tollen's test**

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-I, C-IV, D-II

(B) A-III, B-IV, C-I, D-II

(C) A-I, B-III, C-II, D-IV

(D) A-I, B-II, C-III, D-IV

Correct Answer: (A) A-III, B-I, C-IV, D-II

Solution:

Concept:

- Functional-group tests depend on the reacting group: Lucas reagent tests alcohols, Hinsberg reagent classifies amines, Tollens reagent detects aldehydes, and the phthalein dye test identifies phenols.
- The ring alone is not decisive. Distinguish an alcohol with $-\mathrm{OH}$ on a saturated carbon from a phenol with $-\mathrm{OH}$ directly attached to an aromatic carbon.

Step 1: Match the saturated-ring compounds A is cyclohexanol, a secondary alcohol. Concentrated HCl with anhydrous $\mathrm{ZnCl_2}$, the Lucas reagent, replaces its hydroxyl group with chlorine and gives turbidity from the insoluble chloride; therefore A matches III. B is cyclohexylamine, a primary amine. It reacts with benzenesulphonyl chloride, Hinsberg reagent, to form a sulphonamide containing an acidic N–H bond; therefore B matches I.

Step 2: Match the aldehyde and phenol C has a $-\mathrm{CHO}$ group attached to the cyclohexyl ring. It reduces Tollens reagent to metallic silver while the aldehyde is oxidised, so C matches IV. D is phenol. Heating it with phthalic anhydride in concentrated sulphuric acid forms phenolphthalein, which gives a pink colour in alkali; therefore D matches II. Combining the four matches gives $\mathrm{A}-\mathrm{III},\ \mathrm{B}-\mathrm{I},\ \mathrm{C}-\mathrm{IV},\ \mathrm{D}-\mathrm{II}$. Option (B) interchanges the amine and aldehyde tests, while (C) and (D) incorrectly assign the amine-specific Hinsberg test to the alcohol A.

Step 3: Conclusion The matching is A-III, B-I, C-IV, D-II. Hence, the correct option is (A).

Quick Tip to Solve: Identify the group attached to each ring first: alcohol, amine, aldehyde and phenol immediately suggest Lucas, Hinsberg, Tollens and phthalein tests, respectively.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. If 3.365 g of ethanol (l) is burnt completely in a bomb calorimeter at 298.15 K , the heat produced is 99.472 kJ . The $|\Delta H_f^\circ|$ of ethanol at 298.15 K is

____ $\times 10^2$ kJ mol^{-1}. (Nearest integer)

Given: Standard enthalpy for combustion of graphite $= -393.5$ kJ mol^{-1}

Standard enthalpy of formation of water (l) $= -285.8$ kJ mol^{-1}

Molar mass in gmol^{-1} **of** C, H, O **are 12,1 and 16 respectively** [4 marks]

Solution:

Concept:

- A bomb calorimeter operates at constant volume, so the reaction heat is $q_V = \Delta U$. For gaseous species, $\Delta H = \Delta U + \Delta n_g RT$.
- Hess law gives $\Delta H_c^\circ = \sum \nu \Delta H_f^\circ(\text{products}) - \sum \nu \Delta H_f^\circ(\text{reactants})$; the standard formation enthalpy of elemental oxygen is zero.

Step 1: Convert the measured heat to molar internal energy The molar mass of ethanol is $2(12) + 6(1) + 16 = 46\,\text{g mol}^{-1}$. The amount burnt is

$$n = \frac{3.365}{46} = 0.0731522\,\text{mol}.$$

The calorimeter receives heat, so the reacting ethanol loses it. Thus

$$\Delta U_c^\circ = \frac{-99.472}{0.0731522} = -1359.80\,\text{kJ mol}^{-1}.$$

Step 2: Obtain the molar enthalpy of combustion The balanced combustion is

$$\text{C}_2\text{H}_5\text{OH}\,(\text{l}) + 3\,\text{O}_2(\text{g}) \longrightarrow 2\,\text{CO}_2(\text{g}) + 3\,\text{H}_2\text{O}\,(\text{l}).$$

Only gases enter Δn_g, so $\Delta n_g = 2 - 3 = -1$. Using $R = 8.314\,\text{J mol}^{-1}\,\text{K}^{-1}$,

$$\Delta H_c^\circ = -1359.80 - \frac{8.314(298.15)}{1000} = -1362.27\,\text{kJ mol}^{-1}.$$

Step 3: Use the formation-enthalpy cycle and round Combustion of graphite gives the formation enthalpy of $\text{CO}_2(\text{g})$. Therefore

$$-1362.27 = 2(-393.5) + 3(-285.8) - \Delta H_f^\circ(\text{ethanol}),$$
$$\Delta H_f^\circ(\text{ethanol}) = -282.13\,\text{kJ mol}^{-1}.$$

Its magnitude is $2.8213 \times 10^2\,\text{kJ mol}^{-1}$. The blank asks for this coefficient rounded to the nearest integer, rather than the unscaled enthalpy.

Step 4: Conclusion $\left|\Delta H_f^\circ\right| = 2.8213 \times 10^2\,\text{kJ mol}^{-1}$, so the required nearest-integer coefficient is $\boxed{3}$.

Quick Tip to Solve: Bomb heat gives internal energy. Count only gaseous moles in the conversion to enthalpy, then divide the requested magnitude by its printed power of ten before rounding.

72. For the following reaction at $50^{\circ}C$ and at 2 atm pressure,

$$2\ N_2O_5(\ g) \rightleftharpoons 2\ N_2O_4(\ g) + O_2(\ g)$$

N_2O_5 is 50% dissociated.

The magnitude of standard free energy change at this temperature is x.

$x =$ ____ $Jmol^{-1}$ [Nearest integer].

Given : $R = 8.314\ J\ mol^{-1}\ K^{-1}, \log 2 = 0.30, \log 3 = 0.48, \ln 10 = 2.303, {}^{\circ}C + 273 = K$ [4 marks]

Solution:

Concept:

- For a gas mixture, the partial pressure is its mole fraction times the total pressure. The pressure equilibrium constant must use the powers in the balanced reaction.
- The standard reaction free energy satisfies $\Delta_r G^{\circ} = -RT \ln K_p$, with pressures referred to the conventional standard pressure when evaluating the dimensionless equilibrium constant.

Step 1: Write the equilibrium amounts Start with $2\,mol$ of N_2O_5. At 50% dissociation, $1\,mol$ remains and the reaction produces $1\,mol$ of N_2O_4 and $0.5\,mol$ of O_2. The total is $2.5\,mol$. Hence, at a total pressure of $2\,atm$,

$$p_{N_2O_5} = p_{N_2O_4} = \frac{1}{2.5}(2) = 0.8\,atm, \qquad p_{O_2} = \frac{0.5}{2.5}(2) = 0.4\,atm.$$

Step 2: Evaluate the equilibrium constant For the reaction exactly as written,

$$K_p = \frac{(p_{N_2O_4})^2 p_{O_2}}{(p_{N_2O_5})^2} = \frac{(0.8)^2(0.4)}{(0.8)^2} = 0.4,$$

using the conventional 1-atm pressure reference for the stated data. The equal partial pressures cancel. It is essential not to halve the reaction, because doing so would also change its equilibrium constant and its standard free-energy change.

Step 3: Use the supplied logarithms The temperature is $T = 50 + 273 = 323\,K$. Using only the supplied rounded logarithms,

$$\log(0.4) = \log 4 - 1 = 2(0.30) - 1 = -0.40, \qquad \ln(0.4) = -0.40(2.303) = -0.9212.$$

Therefore

$$\Delta_r G^{\circ} = -8.314(323)(-0.9212) = 2473.8107\,J\,mol^{-1}.$$

This is positive because $K_p < 1$; its magnitude has the same numerical value.

Step 4: Conclusion The magnitude is $x = 2473.8107\,J\,mol^{-1}$, giving the nearest integer $\boxed{2474\,J\,mol^{-1}}$.

Quick Tip to Solve: At 50% dissociation the two nitrogen-oxide partial pressures are equal, so their squared ratio cancels; use the supplied logarithm values to reproduce the required integer.

73. An electrochemical cell, consist of the following two redox couples, $M^{x+}(aq)/M(s)$ $\left[E^{\ominus}_{red} = +0.15\ V\right]$ and $Fe^{3+}(aq)/Fe(s)$ $\left[E^{\ominus}_{red} = -0.036\ V\right]$. The cell EMF (E_{cell}) is recorded to be 0.2057 V . If the reaction quotient of the electrochemical reaction is found to be 10^{-2}, then the value of x is ____ .(Nearest integer)

[Given : M is a p-block metal and $\frac{2.303RT}{F} = 0.059\ V$] [4 marks]

Solution:

Concept:

- The couple with the higher standard reduction potential is reduced at the cathode. The standard cell potential is the cathode reduction potential minus the anode reduction potential.
- For a balanced cell reaction, the Nernst equation is $E_{cell} = E^{\circ}_{cell} - (0.059/n)\log_{10} Q$, where n is the number of electrons transferred and Q is its reaction quotient.

Step 1: Identify the electrodes and standard potential Since $+0.15\,V$ is greater than $-0.036\,V$, the metal ion is reduced and iron is oxidised. Subtract the two reduction potentials without multiplying either potential by a stoichiometric coefficient:

$$E^{\circ}_{cell} = 0.15 - (-0.036) = 0.186\,V.$$

The positive standard potential confirms that this is the spontaneous direction under standard conditions.

Step 2: Determine the electron transfer from the measured potential The reaction quotient is 10^{-2}, so its logarithm is -2. Consequently the measured potential is larger than the standard potential:

$$0.2057 = 0.186 - \frac{0.059}{n}(-2) = 0.186 + \frac{0.118}{n}.$$

Therefore

$$n = \frac{0.118}{0.2057 - 0.186} = \frac{0.118}{0.0197} \simeq 5.99 \simeq 6.$$

The small departure from the integer is caused by the given decimal precision, since electron transfer must be integral.

Step 3: Relate electron transfer to the ionic charge Iron oxidation releases three electrons per atom, while reduction of one metal ion consumes x electrons. The smallest balanced cell reaction therefore transfers

$$n = \mathrm{lcm}(3, x) = 6.$$

For positive integral x, this permits $x = 2$ or $x = 6$. The stated metal is a p-block metal with a simple aqueous cation M^{x+}. A bare aqueous M^{6+} ion is not a viable species: high positive oxidation states of p-block elements occur in covalent compounds or oxo species, not as such a simple hexavalent aqueous metal cation. Thus $x = 2$. The resulting smallest electron balance is

$$2\,Fe \longrightarrow 2\,Fe^{3+} + 6\,e^{-},$$
$$3\,M^{2+} + 6\,e^{-} \longrightarrow 3\,M.$$

This gives $n = 6$ as required. As a check, $0.186 + 0.118/6 = 0.205666\ldots\ V$, which rounds to the

recorded potential.

Step 4: Conclusion The charge number is $\boxed{x = 2}$. The requested nearest integer is $\boxed{2}$, a dimensionless value.

Quick Tip to Solve: First subtract the reduction potentials, then use the sign of $\log Q$. Here $Q < 1$, so the Nernst correction must increase the cell potential.

74.

For a first order reaction $\mathrm{A} \rightarrow \mathrm{B}$

t/ min	[A]/M
0	0.6500
x	0.0650
20	0.00065

$x =$ ____ **min. (Nearest integer)** [4 marks]

Solution:

Concept:

- For a first-order reaction, the integrated rate law is $kt = \ln([A]_0/[A]_t)$, where k is the rate constant and $[A]_0$ and $[A]_t$ are the initial and remaining concentrations.
- Equal fractional decreases in concentration require equal time intervals in a first-order reaction. Ratios of elapsed times equal ratios of the logarithms of the corresponding concentration ratios.

Step 1: Calculate the rate constant using the known time The concentration falls from $0.6500\,\mathrm{M}$ initially to $0.00065\,\mathrm{M}$ after $20\,\mathrm{min}$. Use the remaining concentration in the rate law, rather than the amount that has reacted:

$$k(20) = \ln\left(\frac{0.6500}{0.00065}\right) = \ln 1000 = 3\ln 10.$$

Thus the rate constant, when time is measured in minutes, is

$$k = \frac{3\ln 10}{20}\,\mathrm{min}^{-1} \simeq 0.34539\,\mathrm{min}^{-1}.$$

The concentration ratio is dimensionless, as required inside a logarithm.

Step 2: Find the time for the first tenfold decrease At time x, the remaining concentration is $0.0650\,\mathrm{M}$, which is one tenth of its initial value. Applying the same rate constant gives

$$kx = \ln\left(\frac{0.6500}{0.0650}\right) = \ln 10.$$

Divide this equation by the equation for the twenty-minute reading. This eliminates k and avoids

unnecessary rounding:

$$\frac{x}{20} = \frac{\ln 10}{\ln 1000} = \frac{1}{3}, \qquad x = \frac{20}{3} = 6.666\ldots \text{ min}.$$

A consistency check is that three successive tenfold decreases take twenty minutes: 0.6500 to 0.0650, then to 0.00650, and finally to $0.000650\,\text{M}$. Each decrease therefore takes one third of the total elapsed time.

Step 3: Conclusion The elapsed time is $6.666\ldots$ min. Rounded to the nearest integer as requested, $\boxed{x = 7\,\text{min}}$.

Quick Tip to Solve: Recognise powers of ten in the concentration ratios: a thousandfold decrease takes three times as long as a tenfold decrease for a first-order reaction.

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75. In sulphur estimation, 2.0×10^{-3} mol of an organic compound (X) (molar mass $76\ \text{g mol}^{-1}$) gave 0.4813 g of barium sulphate (molar mass $233\ \text{g mol}^{-1}$). The percentage of sulphur in the compound (X) is ____ $\times 10^{-1}\%$ (Nearest integer) [4 marks]

Solution:

Concept:

- In gravimetric sulphur estimation, sulphur in the organic sample is converted to sulphate and precipitated as BaSO_4. Each mole of BaSO_4 contains one mole of sulphur atoms.
- The sulphur mass fraction in BaSO_4 is $32/233$. The percentage of sulphur in the organic sample is its sulphur mass divided by the original sample mass, multiplied by 100.

Step 1: Determine the mass of the organic sample Convert the given amount of the compound into mass using its molar mass. This sample mass, rather than the mass of the precipitate, is the denominator for the final composition percentage:

$$m_X = n_X M_X = (2.0 \times 10^{-3})(76) = 0.152\,\text{g}.$$

No molecular formula of the compound is needed, because the experiment directly gives the mass of sulphur contained in this amount of sample.

Step 2: Calculate the sulphur mass from the precipitate One formula unit of BaSO_4 contains one sulphur atom, so the mole ratio between sulphur and the measured precipitate is $1 : 1$. Using the atomic molar mass $32\,\text{g mol}^{-1}$ for sulphur,

$$n_{\text{S}} = n_{\text{BaSO}_4} = \frac{0.4813}{233}\,\text{mol},$$

$$m_{\text{S}} = \frac{0.4813 \times 32}{233} = 0.0661013\ldots\ \text{g}.$$

The barium and oxygen masses in the precipitate are excluded: they are not sulphur originally present in the organic sample.

Step 3: Express the percentage in the requested form The mass percentage of sulphur is

$$\%\mathrm{S} = \frac{0.0661013\ldots}{0.152} \times 100 = 43.487689\ldots\%.$$

If the blank is y, the question expresses this result as $y \times 10^{-1}\%$. Hence

$$y \times 10^{-1} = 43.487689\ldots, \qquad y = 434.87689\ldots.$$

Round this coefficient only at the end. Rounding the percentage itself first would discard the precision needed for the blank.

Step 4: Conclusion The sulphur content is approximately 43.49%, or $434.87689\ldots \times 10^{-1}\%$. The requested nearest-integer coefficient is $\boxed{435}$.

Quick Tip to Solve: Use $m(\mathrm{S}) = (32/233)m(\mathrm{BaSO_4})$, then divide by the organic sample mass. Check the power of ten printed after the blank before rounding.