JEE Main 2026 Paper 1 (B.E./B.Tech) Apr 5 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2026, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 5 April 2026, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let α, β be the roots of the equation $x^2 - x + p = 0$ and γ, δ be the roots the equation $x^2 - 4x + q = 0$; $p, q \in$ Z. If $\alpha, \beta, \gamma, \delta$ are in G.P., then $|p + q|$ equals : [4 marks]

(A) 16

(B) 32

(C) 34

(D) 38

Correct Answer: (C) 34

Solution:

Concept:

- For consecutive terms a, ar, ar^2, ar^3 of a geometric progression, successive pair sums differ by the factor r^2.
- Vieta's formulae give the sum and product of the roots of a monic quadratic: $\alpha + \beta = 1$, $\alpha\beta = p$, $\gamma + \delta = 4$, and $\gamma\delta = q$.

Step 1: Represent the roots in their given order Write $\alpha = a, \beta = ar, \gamma = ar^2, \delta = ar^3$. The two sum equations are

$$a(1 + r) = 1, \qquad ar^2(1 + r) = 4.$$

The first equation is nonzero, so neither a nor $1 + r$ is zero. Dividing the second equation by the first therefore gives $r^2 = 4$, hence $r = 2$ or $r = -2$. Retain both signs initially because a geometric progression may have a negative common ratio.

Step 2: Apply the integer restriction to the products For $r = 2$, the first sum gives $a = 1/3$. Then $p = a^2 r = 2/9$, contradicting $p \in$ **Z**. For $r = -2$, we get $a = -1$, so the roots in order are $-1, 2, -4, 8$. Their sums are indeed $-1 + 2 = 1$ and $-4 + 8 = 4$. Their products give

$$p = (-1)(2) = -2, \qquad q = (-4)(8) = -32.$$

Both are integers, so this is the admissible progression. The other listed values 16, 32 and 38 are excluded by the uniquely determined products; 32 alone would ignore the first quadratic's product.

Step 3: Conclusion Thus $|p + q| = |-2 - 32| = 34$. Hence, the correct option is (C).

Quick Tip to Solve: Divide the sums of consecutive pairs in a four-term GP to find the square of the common ratio; then use the integer restriction to choose its sign.

2. Let $z_1, z_2 \in \mathbb{C}$ be the distinct solutions of the equation $z^2 + 4z - (1 + 12i) = 0$.
Then $|z_1|^2 + |z_2|^2$ is equal to : [4 marks]

(A) 18

(B) 22

(C) 29

(D) 34

Correct Answer: (D) 34

Solution:

Concept:

- Completing the square reduces a quadratic in z to an equation of the form $(z + c)^2 = w$.
- For $z = x + iy$, its squared modulus is the real number $|z|^2 = x^2 + y^2$, not the complex square z^2.

Step 1: Complete the square and identify its roots The given equation is $z^2 + 4z - (1 + 12i) = 0$. Add 4 after moving the constant term to obtain

$$(z + 2)^2 = 5 + 12i.$$

To take this square root, observe that $(3 + 2i)^2 = 9 - 4 + 12i = 5 + 12i$. The square roots are therefore $3 + 2i$ and $-3 - 2i$. Consequently the two distinct solutions are

$$z_1 = -2 + (3 + 2i) = 1 + 2i, \qquad z_2 = -2 - (3 + 2i) = -5 - 2i.$$

The order of the two labels has no effect on the requested sum.

Step 2: Compute the squared moduli separately Use the real and imaginary parts of each root rather than squaring the complex number directly. We obtain

$$|1 + 2i|^2 = 1^2 + 2^2 = 5, \qquad |-5 - 2i|^2 = (-5)^2 + (-2)^2 = 29.$$

Adding gives $5 + 29 = 34$. As a check, the roots sum to -4, while their product is $(1 + 2i)(-5 - 2i) = -1 - 12i$, agreeing with Vieta's formulae. The option 29 is just one root's squared modulus and omits the other contribution. Neither 18 nor 22 equals the sum of the two nonnegative squared distances from the origin computed above.

Step 3: Conclusion The required value is $|z_1|^2 + |z_2|^2 = 34$. Hence, the correct option is (D).

Quick Tip to Solve: After completing the square, recognize a simple complex square and compute each modulus using real-part squared plus imaginary-part squared.

3. If $f : \mathbf{N} \to \mathbf{Z}$ is defined by

$$f(n) = \begin{vmatrix} n & -1 & -5 \\ -2n^2 & 3(2k+1) & 2k+1 \\ -3n^3 & 3k(2k+1) & 3k(k+2)+1 \end{vmatrix}, k \in N,$$

and $\sum_{n=1}^{k} f(n) = 98$, then k is equal to : [4 marks]

(A) 3

(B) 4

(C) 5

(D) 6

Correct Answer: (A) 3

Solution:

Concept:

- A determinant is linear in each column separately. Here only the first column varies with n, so sum that column while keeping columns 2 and 3 fixed.
- The standard sums are $\sum n = k(k+1)/2$, $\sum n^2 = k(k+1)(2k+1)/6$, and $\sum n^3 = k^2(k+1)^2/4$.

Step 1: Sum the varying first column The second and third columns are independent of n. Thus $S = \sum_{n=1}^{k} f(n)$ is the determinant with first column

$$\begin{pmatrix} \frac{k(k+1)}{2} \\ -\frac{k(k+1)(2k+1)}{3} \\ -\frac{3k^2(k+1)^2}{4} \end{pmatrix},$$

and with columns 2 and 3 unchanged. These entries are respectively $\sum n$, $-2\sum n^2$, and $-3\sum n^3$. Equivalently, expanding each determinant along its first row gives

$$f(n) = 3(2k+1)(k^2+5k+1)n + (54k^2+18k-2)n^2 - 42(2k+1)n^3.$$

Here k stays fixed while the summation index n runs from 1 to k.

Step 2: Substitute the sums and test the choices Substituting the three standard sums into the preceding polynomial and collecting terms yields

$$S = \frac{7k(k+1)(2k+1)}{6}.$$

Thus the specified condition becomes

$$k(k+1)(2k+1) = 84.$$

For $k = 3$, the left side is $3 \cdot 4 \cdot 7 = 84$, giving $S = 98$. All three factors are positive and strictly increase with the positive integer k, so no larger value can also work. More explicitly, the other listed choices

$k = 4, 5, 6$ give sums $210, 385, 637$, respectively, and none equals 98.

Step 3: Conclusion The positive integer satisfying the determinant sum is $k = 3$. Hence, the correct option is (A).

Quick Tip to Solve: When only one column of a determinant varies with the summation index, sum that column and leave the other two columns unchanged.

4. Let M be a 3×3 matrix such that $\mathrm{M}\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix}, \mathrm{M}\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\1\\2\end{pmatrix}$ and $\mathrm{M}\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}-1\\1\\1\end{pmatrix}$. If $\mathrm{M}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\7\\11\end{pmatrix}$, then $x + y + z$ equals : [4 marks]

(A) 4

(B) 5

(C) 7

(D) 11

Correct Answer: (B) 5

Solution:

Concept:

- Multiplying a matrix by a standard basis vector extracts the corresponding column of that matrix.
- For any matrix M, linearity gives $M(xe_1 + ye_2 + ze_3) = xMe_1 + yMe_2 + zMe_3$.

Step 1: Recover the columns and form the equations The three given products specify the first, second and third columns of M, in that order. Therefore

$$M = \begin{pmatrix}1 & 0 & -1\\2 & 1 & 1\\3 & 2 & 1\end{pmatrix}.$$

Multiplying this matrix by the unknown column gives

$$x - z = 1, \qquad 2x + y + z = 7, \qquad 3x + 2y + z = 11.$$

It is essential to place the given vectors as columns; placing them as rows would represent a different linear transformation.

Step 2: Eliminate the unknowns and verify the result From the first equation, $z = x - 1$. Substituting into the second equation gives $y = 8 - 3x$. Insert both expressions into the third equation:

$$3x + 2(8 - 3x) + (x - 1) = 11.$$

Thus $15 - 2x = 11$, whence $x = 2$. It follows that $y = 2$ and $z = 1$. Direct substitution checks every

component: $2-1=1$, $4+2+1=7$, and $6+4+1=11$. Hence the requested sum is $2+2+1=5$. The choices 4, 7 and 11 are inconsistent with this unique solution; 7 and 11 are components of the output vector, not sums of the input coordinates.

Step 3: Conclusion We obtain $x+y+z=5$. Hence, the correct option is (B).

Quick Tip to Solve: Treat the images of the standard basis vectors as matrix columns, then solve the resulting three scalar equations by elimination.

5. If the sum of the first 10 terms of the series $\frac{1}{1+1^4\times4}+\frac{2}{1+2^4\times4}+\frac{3}{1+3^4\times4}+\frac{4}{1+4^4\times4}+\ldots\ldots$ is $\frac{m}{n}, \gcd(m,n)=1$, then $m+n$ is equal to : [4 marks]

(A) 256

(B) 264

(C) 276

(D) 284

Correct Answer: (C) 276

Solution:

Concept:

- The factorization $4r^4+1=(2r^2-2r+1)(2r^2+2r+1)$ is the Sophie Germain identity.
- If consecutive terms are written as a difference whose second denominator becomes the next first denominator, the finite sum telescopes.

Step 1: Factor a general denominator Let the summation index be r. Its term is $r/(4r^4+1)$. Since the two quadratic factors differ by $4r$, partial fractions give

$$\frac{r}{4r^4+1}=\frac{1}{4}\left(\frac{1}{2r^2-2r+1}-\frac{1}{2r^2+2r+1}\right).$$

This follows directly by taking the difference over a common denominator: its numerator is $4r$, cancelled by the prefactor $1/4$. Also, $2r^2+2r+1=2(r+1)^2-2(r+1)+1$, which is the needed consecutive-index match.

Step 2: Cancel the intermediate fractions The first ten terms therefore have sum

$$\begin{aligned}S_{10}&=\frac{1}{4}\left(1-\frac{1}{5}+\frac{1}{5}-\frac{1}{13}+\cdots+\frac{1}{181}-\frac{1}{221}\right)\\&=\frac{1}{4}\left(1-\frac{1}{221}\right)=\frac{55}{221}.\end{aligned}$$

Here the final negative denominator is $2(10)^2+2(10)+1=221$. The fraction is already in lowest terms: $55=5\cdot11$, whereas $221=13\cdot17$. Thus $m=55$ and $n=221$. Their sum is 276. The remaining choices 256, 264 and 284 do not equal the numerator-plus-denominator total of this reduced fraction.

Step 3: Conclusion The reduced sum is $55/221$, so $m+n=276$. Hence, the correct option is (C).

Quick Tip to Solve: For a denominator of the form $4r^4+1$, factor into neighboring quadratics and look for cancellation before adding any terms.

6. Let $A_1, A_2, A_3, \ldots\ldots\ldots, A_{39}$ be 39 arithmetic means between the numbers 59 and 159. Then the mean of A_{25}, A_{28}, A_{31} and A_{36} is equal to : [4 marks]

(A) 129

(B) 136

(C) 131.50

(D) 134

Correct Answer: (D) 134

Solution:

Concept:

- Inserting 39 arithmetic means between two endpoints creates an arithmetic progression containing 41 terms and 40 equal intervals.
- The arithmetic mean of selected terms in an arithmetic progression equals the term formula evaluated at the mean of their indices.

Step 1: Find the common difference and indexing formula The progression has endpoints 59 and 159, with the inserted means between them. There are 40 steps from 59 to 159, so the common difference is

$$d = \frac{159 - 59}{40} = \frac{5}{2}.$$

Because A_1 is the first inserted term after 59, the correct formula is

$$A_j = 59 + jd = 59 + \frac{5}{2}j.$$

The endpoint 159 corresponds to $j = 40$, confirming the indexing. Using 39 as the number of intervals would incorrectly omit one gap between consecutive terms.

Step 2: Average the requested four terms The mean of the selected indices is

$$(25 + 28 + 31 + 36)/4 = 120/4 = 30.$$

By the linear term formula, the requested mean equals $59 + (5/2)30 = 134$. One can also calculate the four terms explicitly:

$$A_{25} = 121.5, \quad A_{28} = 129, \quad A_{31} = 136.5, \quad A_{36} = 149.$$

Their sum is 536, and dividing by four again gives 134. Among the distractors, 129 is merely A_{28}, not the mean of all four terms. The values 136 and 131.50 also fail the direct sum check, which fixes the answer uniquely.

Step 3: Conclusion The mean of $A_{25}, A_{28}, A_{31}, A_{36}$ is 134. Hence, the correct option is (D).

Quick Tip to Solve: Count intervals rather than inserted terms: 39 inserted arithmetic means create 40 equal gaps between the endpoints.

7. The coefficient of x^2 in the expansion of $\left(2x^2+\frac{1}{x}\right)^{10}, x \neq 0$, is : [4 marks]

(A) 3240

(B) 3360

(C) 3480

(D) 3600

Correct Answer: (B) 3360

Solution:

Concept:

- The binomial theorem expands $(u+v)^n$ into terms $\binom{n}{k}u^k v^{n-k}$, where k is an integer between zero and n.
- When multiplying powers of the same nonzero variable, add the exponents. First identify the required power; only then evaluate its numerical coefficient.

Step 1: Write a general term Choose k factors of $2x^2$ from the ten factors, and choose $1/x$ from the remaining $10-k$. The resulting term is

$$T_k = \binom{10}{k}(2x^2)^k(x^{-1})^{10-k} = \binom{10}{k}2^k x^{3k-10}.$$

The binomial coefficient counts all ways of making that same selection; the factor 2^k must also be included.

Step 2: Match the exponent and evaluate The required exponent is two, so

$$3k-10=2 \quad \Longrightarrow \quad k=4.$$

This value is an allowed integer index. Consequently the coefficient is

$$\binom{10}{4}2^4 = \frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2\cdot 1}\cdot 16 = 210\cdot 16 = 3360.$$

As a check, four selections of x^2 give x^8, and six selections of x^{-1} give x^{-6}, leaving x^2. No other index produces this exponent because $3k-10$ is strictly increasing in k. The other listed numbers, 3240, 3480 and 3600, therefore cannot be coefficients of the requested term.

Step 3: Conclusion The coefficient of x^2 is $\boxed{3360}$. Hence, the correct option is (B).

Quick Tip to Solve: Keep the numerical factor 2^k separate from the power of x; forgetting it is the main trap after finding the correct binomial index.

8. The probabilities that players A and B of a team are selected for the captaincy for a tournament are 0.6 and 0.4 , respectively. If A is selected the captain, the probability that the team wins the tournament is 0.8 and if B is selected the captain, the probability that the team wins the tournament is 0.7 . Then the probability, that the team wins the tournament, is : [4 marks]

(A) 0.74

(B) 0.76

(C) 0.72

(D) 0.78

Correct Answer: (B) 0.76

Solution:

Concept:

- The law of total probability gives $P(W) = P(A)P(W \mid A) + P(B)P(W \mid B)$ when A and B are disjoint and exhaustive captaincy choices.
- A conditional winning probability applies only after its captain has been selected, so it must be weighted by that captain's selection probability.

Step 1: Separate the possible captains Let W denote winning the tournament. The events A and B denote selection of the corresponding player as captain, not whether that player wins a match. Since exactly one captain is selected and $0.6 + 0.4 = 1$, these events partition the possibilities. Thus W is the disjoint union of $A \cap W$ and $B \cap W$.

Step 2: Weight and add the winning probabilities Use the multiplication rule on each disjoint route to a win:

$$P(A \cap W) = 0.6(0.8) = 0.48, \qquad P(B \cap W) = 0.4(0.7) = 0.28.$$

Therefore

$$P(W) = 0.48 + 0.28 = 0.76.$$

An independent check uses losses: their probability is $0.6(0.2) + 0.4(0.3) = 0.12 + 0.12 = 0.24$, whose complement is again 0.76. The unweighted average 0.75 would incorrectly make the two captains equally likely. Because A has the higher winning probability and is more likely to be captain, the weighted value lies above 0.75 and below 0.8. Options 0.74, 0.72 and 0.78 do not equal the required weighted sum.

Step 3: Conclusion The probability that the team wins is $\boxed{0.76}$. Hence, the correct option is (B).

Quick Tip to Solve: Construct one probability path per captain: selection probability times conditional winning probability. Add those paths because they are mutually exclusive.

9. A box contains 5 blue, 6 yellow and 4 red balls. The number of ways, of drawing 8 balls containing at least two balls of each colour, is : [4 marks]

(A) 4100

(B) 4140

(C) 4230

(D) 4290

Correct Answer: (A) 4100

Solution:

Concept:

- An unordered selection of r distinct balls from m balls can be made in $\binom{m}{r}$ ways. Different colours are independent selections, so multiply their counts.
- After allocating two balls to each colour, two more balls remain to be distributed. Their colour patterns are either $(2,0,0)$ or $(1,1,0)$.

Step 1: List all allowed colour totals Write (b, y, r) for the numbers of blue, yellow and red balls. We need $b+y+r=8$, with each entry at least two. Allocating the extra two balls gives exactly six triples:

$$(4,2,2),\ (2,4,2),\ (2,2,4),\ (3,3,2),\ (3,2,3),\ (2,3,3).$$

Every triple respects the available stocks of five blue, six yellow and four red balls. They are disjoint cases and exhaust all possibilities.

Step 2: Count each case and add For a fixed triple the count is $\binom{5}{b}\binom{6}{y}\binom{4}{r}$. The six counts, in the order above, are

$$\begin{aligned}(4,2,2):\ 5\cdot 15\cdot 6 &= 450,\\ (2,4,2):\ 10\cdot 15\cdot 6 &= 900,\\ (2,2,4):\ 10\cdot 15\cdot 1 &= 150,\\ (3,3,2):\ 10\cdot 20\cdot 6 &= 1200,\\ (3,2,3):\ 10\cdot 15\cdot 4 &= 600,\\ (2,3,3):\ 10\cdot 20\cdot 4 &= 800.\end{aligned}$$

Their sum is $450+900+150+1200+600+800=4100$. There is no additional ordering factor: drawing eight balls here means choosing the set, not arranging it. The remaining options 4140, 4230 and 4290 disagree with the exhaustive case sum.

Step 3: Conclusion The number of selections is $\boxed{4100}$. Hence, the correct option is (A).

Quick Tip to Solve: For an at-least-two condition on three colours, subtract the compulsory two from each count first. Distribute only the remaining balls to list cases without omissions.

10. A variable X takes values

$$0, 0, 2, 6, 12, 20, \ldots, n(n-1)$$

with frequencies

$$\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \binom{n}{3}, \binom{n}{4}, \binom{n}{5}, \ldots, \binom{n}{n},$$

respectively. If the mean of this data is 60, then its median is : [4 marks]

(A) 56

(B) 42

(C) 72

(D) 90

Correct Answer: (A) 56

Solution:

Concept:

- The values are $r(r-1)$ with frequencies $\binom{n}{r}$, for $r = 0, 1, \ldots, n$. Their total frequency is 2^n.
- The identity $r(r-1)\binom{n}{r} = n(n-1)\binom{n-2}{r-2}$ evaluates the weighted sum. Binomial frequencies are symmetric about $r = n/2$.

Step 1: Use the mean to find n The weighted mean is

$$\overline{X} = \frac{\sum_{r=0}^{n} r(r-1)\binom{n}{r}}{2^n}$$
$$= \frac{n(n-1)2^{n-2}}{2^n} = \frac{n(n-1)}{4}.$$

The first two summands vanish, so the shifted binomial identity applies directly to the rest. Given the mean 60,

$$n(n-1) = 240 \quad \Longrightarrow \quad (n-16)(n+15) = 0.$$

Since n is a nonnegative integer counting the binomial index, $n = 16$.

Step 2: Locate the two central observations The total frequency is $2^{16} = 65536$, so the median is the average of observations in positions 32768 and 32769. Symmetry gives equal frequencies below and above $r = 8$. In fact $\binom{16}{8} = 12870$, so the number of observations with $r < 8$ is

$$\frac{65536 - 12870}{2} = 26333.$$

The block at $r = 8$ therefore occupies positions 26334 through 39203, containing both central observations. Its value is $8(8-1) = 56$. The duplicate zero at $r = 0, 1$ does not disturb this ordering. Values 42, 72 and 90 arise from other indices and do not contain the central ranks.

Step 3: Conclusion The median is $\boxed{56}$. Hence, the correct option is (A).

Quick Tip to Solve: First recover the binomial parameter from the mean. Then locate the median using cumulative frequencies; the median need not equal the given mean.

11. Let the point P be the vertex of the parabola $y = x^2 - 6x + 12$. If a line passing through the point P intersects the circle $x^2 + y^2 - 2x - 4y + 3 = 0$ at the points R and S , then the maximum value of $(\mathrm{PR} + \mathrm{PS})^2$ is : [4 marks]

(A) 10

(B) 20

(C) 25

(D) 5

Correct Answer: (B) 20

Solution:

Concept:

- Completing squares identifies a parabola's vertex and a circle's centre and radius. The power of an exterior point is $PR \cdot PS = PO^2 - r^2$.
- For a secant through an exterior point, both intersection points lie along the same ray. Projection on the secant controls the sum of their distances.

Step 1: Find the fixed geometry Write the parabola as $y = (x-3)^2 + 3$, giving $P = (3, 3)$. The circle is

$$(x-1)^2 + (y-2)^2 = 2,$$

so its centre is $O = (1, 2)$ and its radius is $\sqrt{2}$. Therefore $PO = \sqrt{(3-1)^2 + (3-2)^2} = \sqrt{5}$, which exceeds the radius. The point is outside the circle, and every intersecting line has both intersections on the ray directed toward the circle.

Step 2: Maximize the sum of secant distances Let θ be the angle between that ray and PO. Parameterize the ray by distance t from P. The circle condition is

$$t^2 - 2\sqrt{5}\cos\theta\, t + (5-2) = 0.$$

Its two positive roots are PR and PS, so their sum is $2\sqrt{5}\cos\theta$. Hence

$$(PR + PS)^2 = 20\cos^2\theta \le 20.$$

Equality is attained by the line PO, whose two distances are $\sqrt{5} - \sqrt{2}$ and $\sqrt{5} + \sqrt{2}$. Their sum is $2\sqrt{5}$, confirming the bound is achievable. Thus 10 and 5 are smaller than an attainable value, while 25 exceeds the proven bound.

Step 3: Conclusion The maximum of $(PR + PS)^2$ is $\boxed{20}$. Hence, the correct option is (B).

Quick Tip to Solve: For an exterior point and a variable secant, the distance sum is twice the projection of the centre vector on the secant. It is largest on the line through the centre.

12. Let the directrix of the parabola $\mathrm{P}: y^2 = 8x$, cut x-axis at the point A . Let $\mathrm{B}(\alpha, \beta), \alpha > 1$, be a point on P such that the slope of AB is $3/5$. If BC is a focal chord of P, then six times the area of $\triangle ABC$ is : [4 marks]

(A) 80

(B) 160

(C) 174

(D) 192

Correct Answer: (B) 160

Solution:

Concept:

- For $y^2 = 4ax$, the focus is $(a, 0)$, the directrix is $x = -a$, and a parametric point is $(at^2, 2at)$.
- The parameters of the endpoints of a focal chord satisfy $t_1 t_2 = -1$. A triangle's area is half the absolute determinant of two vectors from one vertex.

Step 1: Find A and B from the slope condition Here $4a = 8$, so $a = 2$, the focus is $F = (2, 0)$, and $A = (-2, 0)$. The given slope gives $\beta = 3(\alpha + 2)/5$. Substituting in $\beta^2 = 8\alpha$ yields

$$9(\alpha + 2)^2 = 200\alpha, \qquad 9\alpha^2 - 164\alpha + 36 = 0.$$

The roots are 18 and $2/9$. Since $\alpha > 1$, take $\alpha = 18$, giving $\beta = 12$. Thus $B = (18, 12)$ has parameter $t_B = 3$.

Step 2: Find the other focal endpoint The focal-chord condition gives $t_C = -1/3$, hence

$$C = \left(2\left(-\frac{1}{3}\right)^2, 4\left(-\frac{1}{3}\right)\right) = \left(\frac{2}{9}, -\frac{4}{3}\right).$$

As a direct check, both BF and CF have slope $3/4$, so the chord indeed passes through the focus.

Step 3: Evaluate the triangle area The two vectors from A are $AB = (20, 12)$ and $AC = (20/9, -4/3)$. Therefore

$$[ABC] = \frac{1}{2}\left|20\left(-\frac{4}{3}\right) - 12\left(\frac{20}{9}\right)\right| = \frac{80}{3}.$$

The question asks for six times this area, which is $6(80/3) = 160$. Options 80, 174 and 192 do not match the determinant calculation; retaining the smaller root for α would violate the stated condition.

Step 4: Conclusion Six times the area of $\triangle ABC$ is $\boxed{160}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: Use the slope condition to choose the correct point B before using the focal-chord parameter product. The condition on the x-coordinate removes the unwanted root.

13. Let the eccentricity e of a hyperbola satisfy the equation $6e^2 - 11e + 3 = 0$. If the foci of the hyperbola are $(3, 5)$ and $(3, -4)$, then the length of its latus rectum is : [4 marks]

(A) $11/3$

(B) $17/3$

(C) $15/2$

(D) $17/2$

Correct Answer: (C) $15/2$

Solution:

Concept:

- For a hyperbola, the eccentricity is $e = c/a > 1$, where c is the centre-to-focus distance and a is the transverse semi-axis.
- The conjugate semi-axis satisfies $b^2 = c^2 - a^2$, and the full length of a latus rectum is $2b^2/a$.

Step 1: Select the admissible eccentricity Factor the given quadratic before using the geometry.

$$6e^2 - 11e + 3 = (3e - 1)(2e - 3) = 0.$$

The algebraic possibilities are $e = 1/3$ and $e = 3/2$. Only $e = 3/2$ can describe a hyperbola, since its eccentricity exceeds one. The smaller root must therefore be discarded.

Step 2: Find the semi-axes from the foci The foci have the same first coordinate, so the transverse axis is vertical. Their separation is $5 - (-4) = 9$, which equals $2c$. Hence

$$c = \frac{9}{2}, \qquad a = \frac{c}{e} = \frac{9/2}{3/2} = 3.$$

The centre is $(3, 1/2)$; translating the centre does not change the latus rectum. Now

$$b^2 = c^2 - a^2 = \frac{81}{4} - 9 = \frac{45}{4}.$$

Step 3: Calculate the full latus rectum Use the full chord length through a focus, rather than its half-length:

$$L = \frac{2b^2}{a} = \frac{2(45/4)}{3} = \frac{15}{2}.$$

Thus options (A), (B) and (D) give values inconsistent with the semi-axes determined by the foci and eccentricity; none equals this chord length.

Step 4: Conclusion The length of the latus rectum is $\boxed{15/2}$ units. Hence, the correct option is (C).

Quick Tip to Solve: First reject any eccentricity root at most one. Then use half the focal separation for c, not the full distance between the foci.

14. Let a triangle PQR be such that P and Q lie on the line $\frac{x+3}{8} = \frac{y-4}{2} = \frac{z+1}{2}$ and are at a distance of 6 units from $R(1, 2, 3)$. If (α, β, γ) is the centroid of ΔPQR, then $\alpha + \beta + \gamma$ is equal to : [4 marks]

(A) 4

(B) 5

(C) 6

(D) 8

Correct Answer: (C) 6

Solution:

Concept:

- A point on a symmetric-form line is obtained by equating its three ratios to a parameter t.
- The centroid of a triangle is the coordinate-wise average of its three vertices; the two intersections with a sphere supply the required vertices on the line.

Step 1: Parametrise the line and impose the distance Write a general point on the given line as

$$X(t) = (-3 + 8t,\ 4 + 2t,\ -1 + 2t).$$

Its displacement from $R = (1, 2, 3)$ is $(-4 + 8t, 2 + 2t, -4 + 2t)$. Both P and Q must satisfy the squared-distance condition

$$(-4 + 8t)^2 + (2 + 2t)^2 + (-4 + 2t)^2 = 36.$$

Squaring avoids taking square roots and includes both points at distance six.

Step 2: Find both vertices on the line Expanding gives

$$72t^2 - 72t + 36 = 36, \qquad 72t(t - 1) = 0.$$

The two distinct parameter values are therefore $t = 0$ and $t = 1$. In either order the vertices are

$$P = (-3, 4, -1), \qquad Q = (5, 6, 1).$$

As a check, their displacements from R are $(-4, 2, -4)$ and $(4, 4, -2)$, and each has squared length 36.

Step 3: Average the coordinates The centroid is

$$(\alpha, \beta, \gamma) = \left(\frac{-3 + 5 + 1}{3}, \frac{4 + 6 + 2}{3}, \frac{-1 + 1 + 3}{3}\right) = (1, 4, 1).$$

Consequently $\alpha + \beta + \gamma = 1 + 4 + 1 = 6$. Options (A), (B) and (D) cannot be the coordinate sum of this centroid. Interchanging the labels of P and Q leaves the centroid unchanged.

Step 4: Conclusion $\boxed{\alpha + \beta + \gamma = 6}$. Hence, the correct option is (C).

Quick Tip to Solve: For two vertices on a line, form the distance quadratic once. Their parameter sum can also be used directly to obtain the sum of their position vectors.

15. If the distance of the point $(a, 2, 5)$ from the image of the point $(1, 2, 7)$ in the line $\frac{x}{1} = \frac{y-1}{1} = \frac{z-2}{2}$ is 4 , then the sum of all possible values of a is equal to : [4 marks]

(A) 11

(B) 9

(C) 6

(D) 4

Correct Answer: (C) 6

Solution:

Concept:

- The image S of a point B in a line satisfies $S = 2H - B$, where H is the perpendicular foot of B on the line.
- For a line $A + t\mathbf{d}$, the perpendicular-foot parameter is $t = ((B - A) \cdot \mathbf{d})/(\mathbf{d} \cdot \mathbf{d})$.

Step 1: Locate the perpendicular foot Here choose $A = (0, 1, 2)$, direction $\mathbf{d} = (1, 1, 2)$, and the point to be reflected $B = (1, 2, 7)$. Then $B - A = (1, 1, 5)$. Orthogonality of $B - H$ and the line gives

$$t = \frac{(1, 1, 5) \cdot (1, 1, 2)}{1^2 + 1^2 + 2^2} = \frac{12}{6} = 2.$$

Hence $H = A + 2\mathbf{d} = (2, 3, 6)$. The foot lies on the line and $(B - H) \cdot \mathbf{d} = (-1, -1, 1) \cdot (1, 1, 2) = 0$, as required.

Step 2: Reflect and use the specified distance The foot is the midpoint of the point and its image, so

$$S = 2H - B = (4, 6, 12) - (1, 2, 7) = (3, 4, 5).$$

The point $(a, 2, 5)$ has distance four from this image. The squared-distance equation is therefore

$$(a - 3)^2 + (2 - 4)^2 + (5 - 5)^2 = 16,$$

or $(a - 3)^2 = 12$. This is distance from the reflected point, not distance from the line itself.

Step 3: Add both real possibilities The two real values are $a = 3 + 2\sqrt{3}$ and $a = 3 - 2\sqrt{3}$. Their radicals cancel in the sum, leaving

$$a_1 + a_2 = 6.$$

Thus the alternatives 11, 9 and 4 are ruled out. In particular, using the foot itself in place of the image would be an incorrect reflection construction.

Step 4: Conclusion The sum of all possible values of a is $\boxed{6}$. Hence, the correct option is (C).

Quick Tip to Solve: After finding the perpendicular foot, double it and subtract the original point. The foot is only the midpoint; it is not the reflected image.

16. Let O be the origin, $\overrightarrow{OP} = \vec{a}$ and $\overrightarrow{OQ} = \vec{b}$. If R is the point on $\overrightarrow{OP}$ such that $\overrightarrow{OP} = 5\overrightarrow{OR}$, and M is the point such that $\overrightarrow{OQ} = 5\overrightarrow{RM}$, then $\overrightarrow{PM}$ is equal to : [4 marks]

(A) $\frac{1}{5}(\vec{a} - 4\vec{b})$

(B) $\frac{1}{5}(\vec{b} - 4\vec{a})$

(C) $\frac{1}{5}(-\vec{a} + 4\vec{b})$

(D) $\frac{1}{5}(-\vec{b} + 4\vec{a})$

Correct Answer: (B) $\frac{1}{5}(\vec{b} - 4\vec{a})$

Solution:

Concept:

- Position vectors are measured from the same origin: $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$.
- The triangle rule gives $\overrightarrow{OM} = \overrightarrow{OR} + \overrightarrow{RM}$. Scalar multiples of directed vectors retain their specified directions when the multiplier is positive.

Step 1: Express the two given displacements The statement $\overrightarrow{OP} = 5\overrightarrow{OR}$, together with $\overrightarrow{OP} = \vec{a}$, immediately gives

$$\overrightarrow{OR} = \frac{1}{5}\vec{a}.$$

Thus R lies one fifth of the way from O to P, as the direction and the positive coefficient require. Similarly the second condition is about the displacement from R to M, not the position of M itself:

$$\overrightarrow{RM} = \frac{1}{5}\overrightarrow{OQ} = \frac{1}{5}\vec{b}.$$

Step 2: Find the position vector of M Follow the directed path from O to R and then from R to M. Vector addition yields

$$\overrightarrow{OM} = \overrightarrow{OR} + \overrightarrow{RM} = \frac{1}{5}\vec{a} + \frac{1}{5}\vec{b}.$$

Keeping this position vector separate from $\overrightarrow{RM}$ is essential because the two vectors have different initial points.

Step 3: Subtract the position vector of P The required vector starts at P, so subtract its position vector from that of the terminal point M:

$$\overrightarrow{PM} = \overrightarrow{OM} - \overrightarrow{OP} = \frac{\vec{a} + \vec{b}}{5} - \vec{a} = \frac{\vec{b} - 4\vec{a}}{5}.$$

Option (D) is the opposite vector $\overrightarrow{MP}$, while (A) and (C) interchange the roles of the two original position vectors and have incorrect coefficients.

Step 4: Conclusion $\boxed{\overrightarrow{PM} = \frac{1}{5}(\vec{b} - 4\vec{a})}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert every given displacement to position vectors first. For a requested vector from P to M, always take the position of M minus the position of P.

17. Let $f(x) = \lim\limits_{y\to 0} \frac{(1-\cos(xy))\tan(xy)}{y^3}$. Then the number of solutions of the equation $f(x) = \sin x$, $x \in \mathbf{R}$ is : [4 marks]

(A) 0

(B) 2

(C) 3

(D) 1

Correct Answer: (C) 3

Solution:

Concept:

- The standard limits are $\lim_{u\to 0}(1-\cos u)/u^2 = 1/2$ and $\lim_{u\to 0} \tan u/u = 1$.
- Oddness pairs positive and negative roots. On $(0, \pi)$, the function $\sin x/x$ is strictly decreasing, while $x^2/2$ is strictly increasing.

Step 1: Evaluate the limit for each fixed x For $x \neq 0$, put $u = xy$ and separate the standard limits:

$$f(x) = x^3 \lim_{u\to 0} \frac{1-\cos u}{u^2}\frac{\tan u}{u} = \frac{x^3}{2}.$$

At $x = 0$, the numerator is identically zero for nonzero y, so $f(0) = 0$, also agreeing with this formula. We must therefore count the solutions of $x^3/2 = \sin x$.

Step 2: Prove there is exactly one positive root A solution satisfies $|x|^3/2 \leq 1$, hence $|x| \leq \sqrt[3]{2} < \pi$. For positive x, division by x gives $x^2/2 = \sin x/x$. To justify strict decrease, set $k(x) = \sin x - x\cos x$; then $k(0) = 0$ and $k\prime(x) = x\sin x > 0$ on $(0, \pi)$. Therefore

$$(\sin x/x)\prime = \frac{x\cos x - \sin x}{x^2} < 0.$$

The left side increases strictly. Near zero it is below the right side, whereas at $x = \sqrt[3]{2}$ the original left side is 1 and the sine is strictly below 1. Continuity gives one crossing, and the opposite monotonicities make it unique.

Step 3: Include zero and the negative root Both $x^3/2$ and $\sin x$ are odd, so the unique positive root produces exactly one negative root. Zero is itself a solution. The bound above excludes all larger magnitudes, giving three roots in total. Thus zero, two and one, the other proposed counts, all omit at least one of these established roots.

Step 4: Conclusion The equation has $\boxed{3}$ real solutions. Hence, the correct option is (C).

Quick Tip to Solve: Use the limit first, then bound all roots with the range of sine. Divide by x only after retaining the root x=0 separately.

18. Let $\left(2^{1-a}+2^{1+a}\right), f(a), \left(3^{a}+3^{-a}\right)$ be in A.P. and α be the minimum value of $f(a)$. Then the value of the integral $\int_{\log_e(\alpha-1)}^{\log_e(\alpha)} \frac{dx}{(e^{2x}-e^{-2x})}$ is : [4 marks]

(A) $\frac{1}{2}\log_e\left(\frac{4}{3}\right)$
(B) $\frac{1}{4}\log_e\left(\frac{4}{3}\right)$
(C) $\frac{1}{2}\log_e\left(\frac{8}{5}\right)$
(D) $\frac{1}{4}\log_e\left(\frac{8}{5}\right)$

Correct Answer: (B) $\frac{1}{4}\log_e\left(\frac{4}{3}\right)$

Solution:

Concept:

- The middle term of three terms in arithmetic progression is their arithmetic mean.
- For any positive number u, $u+u^{-1}\geq 2$, with equality exactly when $u=1$.
- The substitution $t=e^{2x}$ turns the integrand into a rational function, integrated by partial fractions.

Step 1: Find the minimum middle term The arithmetic progression condition gives

$$2f(a)=2(2^{-a}+2^{a})+(3^{a}+3^{-a}).$$

Each reciprocal pair is at least two. Hence $2f(a)\geq 2(2)+2=6$. Both equalities hold simultaneously at $a=0$, so the lower bound is attained and $\alpha=3$. The integral limits are consequently $\log 2$ and $\log 3$.

Step 2: Transform the integral and its limits Multiply numerator and denominator by e^{2x}, and let $t=e^{2x}$, so $dt=2e^{2x}dx$. The endpoints become $t=4$ and $t=9$. Thus

$$I=\frac{1}{2}\int_{4}^{9}\frac{dt}{t^2-1}=\frac{1}{4}\int_{4}^{9}\left(\frac{1}{t-1}-\frac{1}{t+1}\right)dt.$$

All values of t here exceed one, so the logarithms have positive arguments and the integral has no singularity in the interval.

Step 3: Evaluate the logarithmic expression The antiderivative and endpoint subtraction give

$$I=\frac{1}{4}\left[\log\frac{t-1}{t+1}\right]_{4}^{9}=\frac{1}{4}\log\left(\frac{8/10}{3/5}\right)=\frac{1}{4}\log\frac{4}{3}.$$

Option (A) doubles the correct coefficient. Options (C) and (D) have an incorrect logarithm argument; the lower endpoint contributes the divisor $3/5$, which must not be omitted.

Step 4: Conclusion The integral is $\boxed{\frac{1}{4}\log_e(4/3)}$. Hence, the correct option is (B).

Quick Tip to Solve: Find the attainable minimum before substituting the limits. When using t=e to the power 2x, change both endpoints and keep the factor one half in dx.

19. Let $f : [1, \infty) \to \mathbf{R}$ be a differentiable function defined as $f(x) = \int_1^x f(\mathrm{t})\mathrm{dt} + (1 - x)\left(\log_{\mathrm{e}} x - 1\right) + \mathrm{e}$. Then the value of $f(f(1))$ is : [4 marks]

(A) $(1 + \mathrm{e}^{\mathrm{e}})$

(B) $(1 + \mathrm{e})$

(C) $(1 + \mathrm{e} + \mathrm{e}^{\mathrm{e}})$

(D) $1 + 2e$

Correct Answer: (A) $(1 + \mathrm{e}^{\mathrm{e}})$

Solution:

Concept:

- The fundamental theorem of calculus gives $\frac{d}{dx}\int_1^x f(t)\,dt = f(x)$, because the differentiable function is continuous.
- After differentiation, a first-order linear differential equation is obtained. Its complementary solution is Ce^x, and the value at the lower integration limit determines C.

Step 1: Find the initial condition At $x = 1$, both the integral and the factor $1 - x$ vanish. Hence $f(1) = e$. In particular, the required composition is $f(f(1)) = f(e)$; the inner function must be evaluated before substituting into the outer function.

Step 2: Differentiate and solve Differentiating the defining equation carefully gives

$$f'(x) = f(x) - (\ln x - 1) + \frac{1 - x}{x} = f(x) - \ln x + \frac{1}{x}.$$

Thus $f' - f = -\ln x + 1/x$. The function $\ln x$ is a particular solution, since its derivative minus itself is precisely this right-hand side. Therefore

$$f(x) = \ln x + Ce^x.$$

Using $f(1) = e$ gives $C = 1$.

Step 3: Evaluate the composition and check the choices Consequently

$$f(f(1)) = f(e) = \ln e + e^e = 1 + e^e.$$

As a check, integrating $f(t) = \ln t + e^t$ from 1 to x and adding the other terms in the question recovers $\ln x + e^x$. Option (B) replaces the exponential evaluation by e; (C) adds an extra e, and (D) also uses the wrong exponential value. None equals the derived expression.

Step 4: Conclusion The value is $\boxed{1 + e^e}$. Hence, the correct option is (A).

Quick Tip to Solve: For an integral equation with upper limit x, first substitute the lower limit to obtain the initial value, then differentiate.

20. Let $f(x)$ and $g(x)$ be twice differentiable functions satisfying $f''(x) = g''(x)$ for all $x \in \mathbf{R}$, $f'(1) = 2g'(1) = 4$ and $g(2) = 3f(2) = 9$. Then $f(25) - g(25)$ is equal to : [4 marks]

(A) 20

(B) 40

(C) -20

(D) -40

Correct Answer: (B) 40

Solution:

Concept:

- If two twice differentiable functions have equal second derivatives on $\mathbb{R}$, their difference is an affine function: $h(x) = f(x) - g(x) = Ax + B$.
- The given first derivatives fix the slope A, while a pair of function values fixes one point of the line. There is no need to determine either original function separately.

Step 1: Determine the slope of the difference Set $h = f - g$. Since $h'' = 0$ everywhere, h' is constant. Read the chained equality as $f'(1) = 4$ and $g'(1) = 2$, not as two equal derivatives. Thus

$$h'(x) = h'(1) = 4 - 2 = 2, \qquad h(x) = 2x + B.$$

Step 2: Use the values at the same argument The second chained equality gives $g(2) = 9$ and $f(2) = 3$. Therefore

$$h(2) = f(2) - g(2) = 3 - 9 = -6.$$

Substituting $x = 2$ into the affine expression yields $-6 = 4 + B$, so $B = -10$ and $h(x) = 2x - 10$.

Step 3: Evaluate and distinguish the options It follows that

$$f(25) - g(25) = h(25) = 2(25) - 10 = 40.$$

Equivalently, moving from 2 to 25 increases the difference by $2(25 - 2) = 46$, turning -6 into 40. The values -40 and -20 have the wrong sign; 20 does not satisfy the line determined by the slope and the point $(2, -6)$. This also checks that the requested order is $f - g$, rather than $g - f$.

Step 4: Conclusion $\boxed{f(25) - g(25) = 40}$. Hence, the correct option is (B).

Quick Tip to Solve: When second derivatives agree, work with the difference directly; it is a straight line fixed by one slope and one value.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let $\mathrm{A} = \{1, 4, 7\}$ **and** $\mathrm{B} = \{2, 3, 8\}$. **Then the number of elements, in the relation** $R = \{((a_1, b_1), (a_2, b_2)) \in ((A \times B) \times (A \times B)) : a_1 + b_2$ **divides** $a_2 + b_1\}$ **is ____ .** [4 marks]

Solution:

Concept:

- A member of the relation is an ordered pair of ordered pairs. The condition depends on $u = a_1 + b_2$ and $v = a_2 + b_1$, and asks whether $u \mid v$.
- The choices determining u are independent of those determining v. If a sum s occurs $m(s)$ times in $A + B$, the ordered sum pair (u, v) therefore occurs $m(u)m(v)$ times.

Step 1: List the possible sums with multiplicities For each element of A, add the three elements of B:

a	$a+2$	$a+3$	$a+8$
1	3	4	9
4	6	7	12
7	9	10	15

Hence the possible sums are $3, 4, 6, 7, 9, 10, 12, 15$. Only 9 has multiplicity 2; every other sum has multiplicity 1. This multiplicity matters because distinct choices of the original coordinates remain distinct relation elements.

Step 2: Count equal sums Every positive integer divides itself. Thus all pairs with $u = v$ contribute

$$\sum_s m(s)^2 = 7(1)^2 + 2^2 = 11.$$

The square for 9 counts its four independent coordinate combinations, not just one sum value.

Step 3: Count the strict divisibilities For $u < v$, the possible divisible pairs are

$$(3, 6), (3, 9), (3, 12), (3, 15), (4, 12), (6, 12).$$

Their multiplicity contributions are respectively $1, 2, 1, 1, 1, 1$, summing to 7. There are no cases with $u > v$, because both sums are positive; and the remaining candidate divisors have no larger multiple in the list. The total is therefore $11 + 7 = 18$.

Step 4: Conclusion The relation contains $\boxed{18}$ elements, a dimensionless count.

Quick Tip to Solve: Group repeated sums before testing divisibility; each valid sum pair contributes the product of its two multiplicities.

22. From the point $(-1,-1)$, two rays are sent making angles of 45° with the line $x+y=0$. These rays get reflected from the mirror $x+2y=1$. If the equations of the reflected rays are $ax+by=9$ and $cx+dy=7, a,b,c,d \in$ Z, then the value of $ad+bc$ is ____ . [4 marks]

Solution:

Concept:

- The line $x+y=0$ has inclination 135°. Lines making 45° with it are horizontal and vertical, so the incident rays lie on $y=-1$ and $x=-1$.
- For a mirror with normal vector n, an incident direction v reflects to $v_r = v - 2\frac{v\cdot n}{n\cdot n}n$. This reverses the normal component and preserves the tangential component.

Step 1: Locate both points of incidence The horizontal ray from $(-1,-1)$ meets $x+2y=1$ at $P=(3,-1)$. The vertical ray meets it at $Q=(-1,1)$. The rays directed towards these points have incident direction vectors $(1,0)$ and $(0,1)$, respectively. These intersections determine where the reflected lines must pass.

Step 2: Reflect the horizontal direction Use the mirror normal $n=(1,2)$, with $n\cdot n=5$. For $v=(1,0)$,

$$v_r = (1,0) - \frac{2}{5}(1,2) = \frac{1}{5}(3,-4).$$

A normal to the reflected line is therefore $(4,3)$. Through P, its equation is

$$4(x-3)+3(y+1)=0, \qquad 4x+3y=9.$$

Hence $a=4, b=3$.

Step 3: Reflect the vertical direction For $v=(0,1)$, the reflected vector is

$$v_r = (0,1) - \frac{4}{5}(1,2) = \frac{1}{5}(-4,-3).$$

A normal is $(3,-4)$. The line through Q is

$$3(x+1)-4(y-1)=0, \qquad -3x+4y=7.$$

The last form matches the prescribed positive right-hand side, giving $c=-3, d=4$. Thus $ad+bc = 4(4)+3(-3)=7$.

Step 4: Conclusion The required dimensionless value is $\boxed{7}$.

Quick Tip to Solve: Write each reflected line through its incidence point, then normalize its constant to the exact right-hand side specified in the question.

23. **If $S = \left\{\theta \in [-\pi, \pi] : \cos\theta \cos\frac{5\theta}{2} = \cos 7\theta \cos\frac{7\theta}{2}\right\}$, then $n(S)$ is equal to ____ .** [4 marks]

Solution:

Concept:

- The product-to-sum identity $2\cos A\cos B = \cos(A+B) + \cos(A-B)$ converts the equation into a difference of two cosines.
- The identity $\cos u - \cos v = -2\sin((u+v)/2)\sin((u-v)/2)$ gives two root families. Their common roots must be subtracted once when counting the union.

Step 1: Reduce the products Multiplying the given equation by 2 gives

$$\cos\frac{7\theta}{2} + \cos\frac{3\theta}{2} = \cos\frac{21\theta}{2} + \cos\frac{7\theta}{2}.$$

Cancel the identical cosine terms from the two sides, obtaining $\cos(3\theta/2) = \cos(21\theta/2)$. Using the difference identity, this is equivalent to

$$2\sin(6\theta)\sin\frac{9\theta}{2} = 0.$$

No division by a sine is used, so no zero is lost.

Step 2: Count the two families in the closed interval The first family is $\theta = k\pi/6$. The condition $-\pi \le \theta \le \pi$ permits all integers $-6 \le k \le 6$, giving 13 roots, including both endpoints. The second family is $\theta = 2m\pi/9$; here $-9/2 \le m \le 9/2$, so $m = -4, -3, \ldots, 4$, giving 9 roots.

Step 3: Remove the overlap A root common to the two families satisfies

$$\frac{k}{6} = \frac{2m}{9}, \qquad 3k = 4m.$$

Hence $k = 4r, m = 3r$ for an integer r. Within the allowed ranges, $r = -1, 0, 1$, corresponding to $-2\pi/3, 0, 2\pi/3$. There are exactly three duplicated roots. Inclusion-exclusion therefore gives $n(S) = 13 + 9 - 3 = 19$.

Step 4: Conclusion The set has $\boxed{19}$ elements, a dimensionless count.

Quick Tip to Solve: After factoring a trigonometric equation, count roots in each family and solve the equality of the two parameterizations to find overlaps.

---

24. **Let $f : \mathbf{R} \to \mathbf{R}$ be a function such that $f(x) + 3f\left(\frac{\pi}{2} - x\right) = \sin x, x \in \mathbf{R}$. Let the maximum value of f on $\mathbf{R}$ be α. If the area of the region bounded by the curves $g(x) = x^2$ and $h(x) = \beta x^3, \beta > 0$, is α^2, then $30\beta^3$ is equal to ____ .** [4 marks]

Solution:

Concept:

- The substitution $x \mapsto \pi/2 - x$ swaps the two arguments and turns the functional equation into two simultaneous linear equations for the two function values.

- The maximum of $A\cos x + B\sin x$ is $\sqrt{A^2+B^2}$. The bounded area between curves is the integral of the upper curve minus the lower one between their intersections.

Step 1: Find the function and its maximum Put $u = f(x)$ and $v = f(\pi/2 - x)$. The original and transformed equations are

$$u + 3v = \sin x, \qquad 3u + v = \cos x.$$

Subtracting the first equation from three times the second gives $8u = 3\cos x - \sin x$. Thus

$$f(x) = \frac{3\cos x - \sin x}{8}, \qquad \alpha = \frac{\sqrt{10}}{8}, \qquad \alpha^2 = \frac{5}{32}.$$

The amplitude is attained for a suitable real phase, so this is the actual maximum, not merely a bound.

Step 2: Identify the bounded region and integrate The intersections satisfy $x^2 = \beta x^3$, hence $x = 0$ and $x = 1/\beta$. Since $\beta > 0$, the quadratic is above the cubic throughout $0 < x < 1/\beta$. Consequently the enclosed area is

$$\begin{aligned} A &= \int_0^{1/\beta} (x^2 - \beta x^3)\, dx \\ &= \left[\frac{x^3}{3} - \frac{\beta x^4}{4}\right]_0^{1/\beta} \\ &= \frac{1}{3\beta^3} - \frac{1}{4\beta^3} = \frac{1}{12\beta^3}. \end{aligned}$$

Step 3: Use the prescribed area Equating the area to α^2 yields

$$\frac{1}{12\beta^3} = \frac{5}{32}, \qquad \beta^3 = \frac{32}{60} = \frac{8}{15}.$$

Therefore $30\beta^3 = 30(8/15) = 16$. Positivity of β is consistent with the positive bounded area and selects the positive cube root if β itself is required.

Step 4: Conclusion The required dimensionless value is $\boxed{30\beta^3 = 16}$.

Quick Tip to Solve: Use the argument swap to solve for f; then integrate between 0 and $1/\beta$ before substituting the maximum amplitude.

---

25. Let $y = y(x)$ be the solution of the differential equation $(\tan x)^{1/2}\, dy = \left(\sec^3 x - (\tan x)^{3/2} y\right) dx, 0 < x < \frac{\pi}{2}, y\left(\frac{\pi}{4}\right) = \frac{6\sqrt{2}}{5}$. If $y\left(\frac{\pi}{3}\right) = \frac{4}{5}\alpha$, then α^4 equals

____ .

[4 marks]

Solution:

Concept:

- A linear equation $y' + P(x)y = Q(x)$ has integrating factor $e^{\int P(x)\,dx}$. Multiplication by this factor makes the left side an exact product derivative.
- Here $0 < x < \pi/2$, so $\tan x > 0$ and $\sec x > 0$. All square roots and fractional powers used below therefore mean their positive real values.

Step 1: Put the equation in linear form Divide by $\sqrt{\tan x}$, which is nonzero on the given interval. The equation becomes

$$y' + (\tan x)y = \frac{\sec^3 x}{\sqrt{\tan x}}.$$

Since $\int \tan x\, dx = \log(\sec x)$, the integrating factor is $\sec x$. Thus

$$\frac{d}{dx}(y \sec x) = \frac{\sec^4 x}{\sqrt{\tan x}}.$$

Step 2: Integrate using the tangent substitution Let $u = \tan x$. Then $du = \sec^2 x\, dx$, and the remaining factor is $\sec^2 x = 1 + u^2$. Hence

$$y \sec x = \int (u^{-1/2} + u^{3/2})\, du = 2\sqrt{u} + \frac{2}{5}u^{5/2} + C.$$

Restoring x, we obtain

$$y \sec x = 2\sqrt{\tan x} + \frac{2}{5}(\tan x)^{5/2} + C.$$

Step 3: Apply the initial condition and evaluate At $x = \pi/4$, the left side is $(6\sqrt{2}/5)\sqrt{2} = 12/5$, while the two explicit terms on the right total $2 + 2/5 = 12/5$. Therefore $C = 0$. At $x = \pi/3$, use $\tan x = \sqrt{3}$ and $\sec x = 2$:

$$2y = 2\,3^{1/4} + \frac{2}{5}\,3^{5/4} = \frac{16}{5}3^{1/4}.$$

Thus $y(\pi/3) = \frac{8}{5}3^{1/4}$. Comparing with $4\alpha/5$ gives $\alpha = 2\,3^{1/4}$.

Step 4: Conclusion The requested dimensionless value is

$$\boxed{\alpha^4 = 2^4 \cdot 3 = 48}.$$

Quick Tip to Solve: After multiplying by the integrating factor, reserve one factor $\sec^2 x\, dx$ for $d(\tan x)$; convert the other to $1 + \tan^2 x$.

26.

Match List - I with List - II.

List - I		List - II	
A.	**Meter (L)**	**I.**	$\sqrt{\frac{hc}{G}}$
B.	**Second (S)**	**II.**	$\sqrt{\frac{Gh}{c^5}}$
C.	**Kilogram (M)**	**III.**	$\sqrt{\frac{K^2L^2c^3}{Gh}}$
D.	**Kelvin (K)**	**IV.**	$\sqrt{\frac{Gh}{c^3}}$

where h (Planck's constant), G (gravitational constant) and c (speed of light in vacuum) as fundamental units.

Choose the correct answer from the options given below : [4 marks]

(A) A-II, B-IV, C-I, D-III

(B) A-IV, B-II, C-I, D-III

(C) A-IV, B-I, C-II, D-III

(D) A-III, B-I, C-II, D-IV

Correct Answer: (B) A-IV, B-II, C-I, D-III

Solution:

Concept:

- The dimensions of Planck's constant, gravitational constant and speed are $[h] = ML^2T^{-1}$, $[G] = M^{-1}L^3T^{-2}$ and $[c] = LT^{-1}$, respectively.
- Dimensions multiply and divide like algebraic powers. Taking a square root halves every exponent; a matching expression must have precisely the dimension of the named unit.

Step 1: Identify the mass expression For expression I,

$$\left[\frac{hc}{G}\right] = \frac{(ML^2T^{-1})(LT^{-1})}{M^{-1}L^3T^{-2}} = M^2.$$

Its square root therefore has dimension M, so kilogram, item C, matches I. This immediately rules out options (C) and (D), both of which assign C to II.

Step 2: Find the time and length expressions For expression II,

$$\left[\frac{Gh}{c^5}\right] = \frac{L^5T^{-3}}{L^5T^{-5}} = T^2.$$

Thus second, item B, matches II. For expression IV,

$$\left[\frac{Gh}{c^3}\right] = \frac{L^5T^{-3}}{L^3T^{-3}} = L^2.$$

Thus meter, item A, matches IV. In particular option (A) interchanges the length and time assignments,

so it cannot be correct.

Step 3: Check the remaining temperature entry Use Θ for the temperature dimension represented by K in the table. The factor $\sqrt{c^3/(Gh)}$ has dimension L^{-1}, the reciprocal of expression IV. Consequently

$$\left[\sqrt{\frac{K^2L^2c^3}{Gh}}\right] = \Theta L \cdot L^{-1} = \Theta.$$

Expression III therefore matches kelvin, item D. The explicit temperature factor in this printed expression supplies the independent temperature dimension; it is not generated by h, G, c alone.

Step 4: Conclusion The matching is A-IV, B-II, C-I, D-III. Hence, the correct option is (B).

Quick Tip to Solve: Calculate $[Gh] = L^5T^{-3}$ once. Dividing it by c^3 gives a length squared, while dividing by c^5 gives a time squared.

---

27. In an experiment to determine the resistance of a given wire using Ohm's law, the voltmeter and ammeter readings are noted as 10 V and 5 A , respectively. The least counts of voltmeter and ammeter are 500 mV and 200 mA , respectively. The estimated error in the resistance measurement is ____ Ω [4 marks]

(A) 0.25

(B) 2

(C) 2.5

(D) 0.18

Correct Answer: (D) 0.18

Solution:

Concept:

- Ohm's law gives resistance as $R = V/I$. For a quotient, the maximum estimated fractional error is the sum of the fractional errors of numerator and denominator.
- Use the specified least counts as the measurement uncertainties: $\Delta V = 500\,\text{mV} = 0.5\,\text{V}$ and $\Delta I = 200\,\text{mA} = 0.2\,\text{A}$.

Step 1: Calculate the measured resistance The central readings give

$$R = \frac{10\,\text{V}}{5\,\text{A}} = 2\,\Omega.$$

This is the measured resistance itself, not the error. Therefore the option $2\,\Omega$ cannot be selected merely by applying Ohm's law; the uncertainty calculation is a separate step.

Step 2: Add the fractional uncertainties For small errors, differentiating $R = V/I$ gives $dR/R = dV/V - dI/I$. The largest magnitude occurs when the voltage and current errors act in opposite directions, so their absolute contributions add:

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{0.5}{10} + \frac{0.2}{5} = 0.05 + 0.04 = 0.09.$$

Thus the estimated percentage error is 9%. Subtracting the two contributions would underestimate the uncertainty.

Step 3: Convert back to absolute error Multiply the fractional uncertainty by the central resistance:

$$\Delta R = (2\,\Omega)(0.09) = 0.18\,\Omega.$$

The measurement can therefore be expressed to this approximation as $R = (2.00 \pm 0.18)\,\Omega$. Neither $0.25\,\Omega$ nor $2.5\,\Omega$ follows from the stated instrument uncertainties; the latter would even exceed the measured resistance. No factor of one-half is applied because the quoted least counts are the uncertainties used in this estimate.

Step 4: Conclusion The estimated absolute error is $\boxed{0.18\,\Omega}$. Hence, the correct option is (D).

Quick Tip to Solve: Keep fractional error and absolute error distinct: add $\Delta V/V$ and $\Delta I/I$ first, then multiply their sum by V/I.

---

28. A mass of 1 kg is kept on a inclined plane with 30° inclination with respect to horizontal plane and it is at rest initially. Then the whole assembly is moved up with constant velocity of $4\ \mathrm{m/s}$. The work done by the frictional force in time 2 s is ____ J. (Take $g = 10\ \mathrm{m/s^2}$) [4 marks]

(A) 20

(B) 25

(C) 30

(D) 10

Correct Answer: (A) 20

Solution:

Concept:

- Constant translational velocity means zero acceleration. A block remaining at rest relative to the incline therefore has balanced forces parallel to the plane.
- Work is $W = \mathbf{F} \cdot \mathbf{s}$, evaluated using displacement in the chosen reference frame. Static friction can do nonzero work on a block carried by a moving surface.

Step 1: Find the frictional force along the incline The component of the weight down the plane is $mg\sin 30^\circ$. Static friction acts up the plane to balance it. Since the assembly has constant velocity, there is no acceleration and no additional inertial force:

$$f = mg\sin 30^\circ = (1)(10)\left(\frac{1}{2}\right) = 5\,\mathrm{N}.$$

This uses equilibrium along the incline; the normal force separately balances the perpendicular weight component.

Step 2: Find the upward displacement In the ground frame the block moves with the assembly through a vertical distance

$$s = vt = (4\,\mathrm{m\,s^{-1}})(2\,\mathrm{s}) = 8\,\mathrm{m}.$$

The frictional force is directed along a plane inclined at 30° above the horizontal. Its angle with this vertical displacement is therefore 60°. Only its vertical component contributes to the dot product.

Step 3: Evaluate the work of friction The work is

$$W_f = fs\cos 60^\circ = (5)(8)\left(\frac{1}{2}\right) = 20\,\text{J}.$$

Equivalently, the vertical component of friction is $f\sin 30^\circ = 2.5\,\text{N}$, giving the same result. Friction does positive work because its upward component acts along the displacement. Zero relative sliding does not imply zero work in the ground frame. The values 25, 30 and 10 J do not follow from the force projection and the stated 8 m displacement; using the full 5 N along the vertical would also give the incorrect value 40 J.

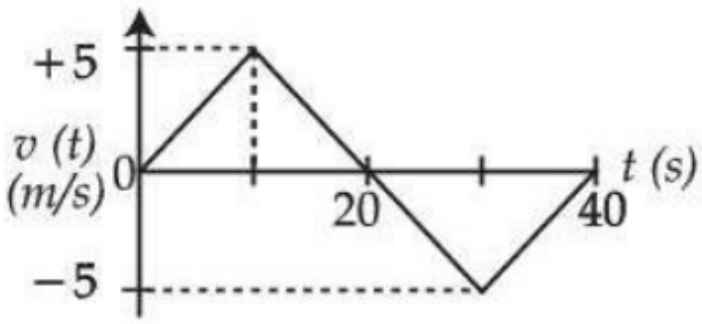
Step 4: Conclusion The work done by friction is $\boxed{20\,\text{J}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a translating incline, distinguish the direction of friction from the direction of the block's ground-frame displacement before using $W = Fs\cos\phi$.

29. The velocity (v) versus time (t) plot of a particle is shown in the figure, for a time interval of 40 s . The total distance travelled by the particle and the average velocity during this period are, respectively

____.

[4 marks]



(A) 25 m and zero

(B) 50 m and zero

(C) 100 m and zero

(D) 100 m and $2.5\ \text{m/s}$

Correct Answer: (C) 100 m and zero

Solution:

Concept:

- The signed area under a velocity-time graph gives displacement. Total distance is the sum of the magnitudes of the areas, because reversing direction cannot cancel distance.

- Average velocity equals net displacement divided by the full elapsed time. It differs from average speed whenever the particle travels in both directions.

Step 1: Calculate the positive area From 0 to 20 s the graph lies above the time axis. The velocity starts at zero, reaches $+5\,\mathrm{m\,s^{-1}}$, and returns to zero, forming one triangle. Its base is 20 s and height is $5\,\mathrm{m\,s^{-1}}$, so

$$A_+ = \frac{1}{2}(20)(5) = 50\,\mathrm{m}.$$

The sloping sides need no separate integration because the area formula already accounts for the linear variation of velocity.

Step 2: Calculate the negative area and total distance From 20 to 40 s the corresponding triangle is below the axis. Its base is again 20 s and its maximum speed is $5\,\mathrm{m\,s^{-1}}$. Its signed area is therefore

$$A_- = -\frac{1}{2}(20)(5) = -50\,\mathrm{m}.$$

The particle travels 50 m in the negative direction during this part. Add magnitudes for distance:

$$d = |A_+| + |A_-| = 50 + 50 = 100\,\mathrm{m}.$$

The answers 25 m and 50 m omit some of the triangular areas.

Step 3: Find the average velocity The net displacement over the complete 40 s is

$$\Delta x = A_+ + A_- = 50 - 50 = 0.$$

Hence

$$\overline{v} = \frac{\Delta x}{40\,\mathrm{s}} = 0\,\mathrm{m\,s^{-1}}.$$

The value $100/40 = 2.5\,\mathrm{m\,s^{-1}}$ is the average speed, not the average velocity, so option (D) combines the correct distance with the wrong second quantity. The equal positive and negative triangular areas show that the particle returns to its starting position.

Step 4: Conclusion The total distance is $\boxed{100\,\mathrm{m}}$, and the average velocity is zero. Hence, the correct option is (C).

Quick Tip to Solve: For a velocity graph, add absolute areas for distance but signed areas for displacement; divide the latter by total time to obtain average velocity.

30. A wheel initially at rest is subjected to a uniform angular acceleration about its axis. In the first 2 s it rotates through an angle θ_1 and in the next 2 s it rotates through an angle θ_2. The ratio $\frac{\theta_2}{\theta_1}$ is ____ . [4 marks]

(A) 6

(B) 3

(C) 4

(D) $\frac{1}{3}$

Correct Answer: (B) 3

Solution:

Concept:

- For constant angular acceleration a, angular displacement after time t is $\theta(t) = \omega_0 t + \frac{1}{2}at^2$. Here $\omega_0 = 0$ because the wheel starts from rest.
- The angle covered in a later time interval is the difference of the cumulative angles at its endpoints. The wheel does not restart from rest at the end of the first interval.

Step 1: Calculate the angle in the first interval Write the angular acceleration as a to avoid confusing it with either angle. For the first 2 s,

$$\theta_1 = \theta(2) - \theta(0) = \frac{1}{2}a(2)^2 = 2a.$$

If a is in $\mathrm{rad\,s^{-2}}$, the resulting angle is in radians. Its numerical value is not needed because the same acceleration appears in both intervals.

Step 2: Subtract endpoint angles for the next interval The cumulative angular displacement at 4 s is

$$\theta(4) = \frac{1}{2}a(4)^2 = 8a.$$

The next 2 s means the interval from 2 s to 4 s. Therefore

$$\theta_2 = \theta(4) - \theta(2) = 8a - 2a = 6a.$$

Using $\theta(4)$ directly for θ_2 would include the first interval again. Alternatively, the angular velocity at 2 s is $2a$, so the next interval gives $(2a)(2) + \frac{1}{2}a(2)^2 = 6a$.

Step 3: Form and check the ratio Dividing the two interval displacements cancels the acceleration:

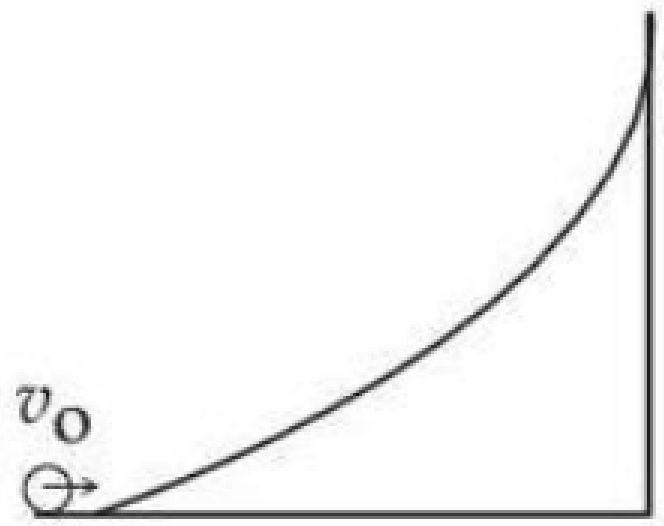
$$\frac{\theta_2}{\theta_1} = \frac{6a}{2a} = 3.$$

The ratio is dimensionless. Option (A), 6, confuses the coefficient in the second displacement with the ratio itself. Option (C), 4, is the cumulative ratio $\theta(4)/\theta(2)$, while option (D), $1/3$, reverses the requested ratio. This agrees with the familiar odd-number sequence 1, 3, 5 for successive equal time intervals under uniform acceleration from rest.

Step 4: Conclusion The required ratio is $\boxed{3}$. Hence, the correct option is (B).

Quick Tip to Solve: For equal successive intervals from rest, subtract squared endpoint times: the first two angular displacements are proportional to 1^2 and $2^2 - 1^2 = 3$.

31. An object of uniform density rolls up the curved path with the initial velocity v_o as shown in the figure. If the maximum height attained by an object is $\frac{7v_0^2}{10\ g}$ (g = acceleration due to gravity), the object is a ____ . [4 marks]



(A) solid cylinder

(B) ring

(C) disc

(D) solid sphere

Correct Answer: (D) solid sphere

Solution:

Concept:

- For pure rolling, the centre-of-mass speed and angular speed satisfy $v = R\omega$, where R is the rolling radius.
- The total kinetic energy is $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. Static friction on a stationary rough track does no work, so mechanical energy is conserved.

Step 1: Write the energy balance Initially the object has both translational and rotational kinetic energy. At its maximum height its speed is zero; pure rolling then gives zero angular speed as well. Taking the initial level as the zero of gravitational potential energy,

$$mgh = \frac{1}{2}mv_0^2 + \frac{1}{2}I\frac{v_0^2}{R^2}.$$

Here the moment of inertia is about the central rolling axis. The mass cancels; density being uniform allows use of the standard moments of inertia.

Step 2: Extract the inertia factor and compare the shapes Put $I = kmR^2$. The energy equation becomes

$$h = \frac{v_0^2}{2g}(1 + k).$$

The given height therefore requires

$$\frac{1+k}{2} = \frac{7}{10}, \qquad k = \frac{2}{5}.$$

A solid sphere has $I = \frac{2}{5}mR^2$, whereas both a solid cylinder and a disc have $k = \frac{1}{2}$, giving $h = 3v_0^2/(4g)$. A ring has $k = 1$, giving $h = v_0^2/g$. Thus none of those three alternatives gives the specified height.

Step 3: Conclusion The object is a solid sphere, whose rotational inertia factor is $2/5$. Hence, the correct option is (D).

Quick Tip to Solve: For rolling uphill, compare $2gh/v_0^2 - 1$ with the standard factor $I/(mR^2)$; include rotational energy before choosing the shape.

---

32. A body of mass m is taken from the surface of earth to a height equal to twice the radius of earth (R_e). The increase in potential energy will be ____ .
(g is acceleration due to gravity at the surface of earth) [4 marks]

(A) $\frac{1}{2}mgR_e$
(B) $\frac{3}{4}mgR_e$
(C) $\frac{1}{4}mgR_e$
(D) $\frac{2}{3}mgR_e$

Correct Answer: (D) $\frac{2}{3}mgR_e$

Solution:

Concept:

- Outside a spherical Earth, $U(r) = -GM_em/r$, with potential energy zero at infinity.
- The surface gravitational acceleration satisfies $g = GM_e/R_e^2$. A height above the surface must be added to R_e to obtain the distance from the centre.

Step 1: Identify the initial and final radii The initial position is on the surface, so $r_i = R_e$. The stated height is $h = 2R_e$, not the final distance from the centre. Hence

$$r_f = R_e + h = 3R_e.$$

Because this height is comparable with Earth́s radius, the acceleration is not approximately constant during the displacement. The near-surface expression mgh, using the surface value of g throughout, is therefore inappropriate.

Step 2: Evaluate the difference in potential energies Use the inverse-distance potential at the two endpoints:

$$\begin{aligned}\Delta U &= U_f - U_i \\ &= -\frac{GM_em}{3R_e} + \frac{GM_em}{R_e} \\ &= \frac{2GM_em}{3R_e} = \frac{2}{3}mgR_e.\end{aligned}$$

The change is positive, as expected when a body is lifted against attractive gravity. The coefficient $1/2$ would correspond to a height R_e, and $3/4$ to a height $3R_e$; $1/4$ also does not follow from the actual endpoint radii. Thus the other coefficients fail the endpoint calculation.

Step 3: Conclusion The increase in gravitational potential energy is $\boxed{\Delta U = \frac{2}{3}mgR_e}$, in joules. Hence, the correct option is (D).

Quick Tip to Solve: For large heights use $\Delta U = mgR_eh/(R_e + h)$, with g measured at the surface, rather than treating gravity as constant.

33. Eight mercury drops, each of radius r, coalesce to form a bigger drop. The surface energy released in this process is ____ - (S is the surface tension of mercury). [4 marks]

(A) $8\pi r^2$ S

(B) $16\pi r^2 S$

(C) $64\pi r^2 S$

(D) $4\pi r^2$ S

Correct Answer: (B) $16\pi r^2 S$

Solution:

Concept:

- A liquid drop has one liquid-air interface, whose surface energy is $E = SA$ for constant surface tension S.
- Coalescence conserves liquid volume. The surface energy released is S times the decrease in total surface area.

Step 1: Find the radius of the combined drop Let the final radius be R. There are eight identical spherical drops initially. Conservation of the volume of mercury gives

$$8\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3, \qquad R^3 = 8r^3, \qquad R = 2r.$$

The radius is doubled, not multiplied by eight: volumes scale as the cube of the radius, whereas surface areas scale as its square.

Step 2: Subtract the final surface energy from the initial energy The initial area includes all eight drops, while the final area is that of one sphere:

$$A_i = 8(4\pi r^2) = 32\pi r^2, \qquad A_f = 4\pi(2r)^2 = 16\pi r^2.$$

Consequently,

$$E_{\text{released}} = S(A_i - A_f) = 16\pi r^2 S.$$

The positive result represents energy liberated as surface area decreases. The alternatives $8\pi r^2 S$ and $4\pi r^2 S$ omit part of the area change. The value $64\pi r^2 S$ exceeds even the initial surface energy $32\pi r^2 S$, so it cannot be the released energy. No factor of two for two surfaces is used, because mercury drops are not soap bubbles.

Step 3: Conclusion The surface energy released is $\boxed{16\pi r^2 S}$, in joules. Hence, the correct option is (B).

Quick Tip to Solve: First conserve volume to obtain the larger radius; only then compare areas. A liquid drop has one interface, unlike a soap bubble.

34. An ideal gas at pressure P and temperature T is expanding such that $PT^3 =$ constant. The coefficient of volume expansion of the gas is ____ . [4 marks]

(A) $\frac{2}{T}$
(B) $\frac{1}{T}$
(C) $\frac{4}{T}$
(D) $\frac{3}{T}$

Correct Answer: (C) $\frac{4}{T}$

Solution:

Concept:

- For a fixed amount of ideal gas, $PV = nRT$, where n is the number of moles and R is the gas constant.
- The fractional volume change per unit temperature along the specified process is $\beta = (1/V)\, dV/dT$. The constraint determines how pressure changes as temperature changes.

Step 1: Express the volume in terms of temperature Write the given process constraint as $PT^3 = C$, with constant C. Therefore

$$P = \frac{C}{T^3}.$$

Substitute this pressure into the ideal-gas equation:

$$\frac{C}{T^3}V = nRT, \qquad V = \frac{nR}{C}T^4.$$

Both the explicit temperature factor of the gas law and the fall in pressure contribute to expansion; the path is not an isobaric one.

Step 2: Differentiate and compare the options Keeping n, R, and C fixed along this process gives

$$\frac{dV}{dT} = 4\frac{nR}{C}T^3.$$

Dividing by $V = (nR/C)T^4$,

$$\beta = \frac{1}{V}\frac{dV}{dT} = \frac{4}{T}.$$

The unit is inverse kelvin. The value $1/T$ applies at constant pressure and disregards the prescribed pressure variation. The value $3/T$ counts only that variation, omitting the direct temperature factor; $2/T$ does not match the power $V \propto T^4$. Equivalently, logarithmic differentiation gives $d\ln V = 4\, d\ln T$.

Step 3: Conclusion Along the stated expansion, the coefficient is $\boxed{\beta = 4/T}$, with units K^{-1}. Hence,

the correct option is (C).

Quick Tip to Solve: Eliminate the pressure using the process equation before differentiating. If $V \propto T^a$, its fractional expansion rate is a/T.

35. Match List-I with List-II.

	List-I		List-II
A.	$\sin^2 \omega t$	**I.**	**Periodic with time period $T = \pi/\omega$ but not simple harmonic motion (SHM)**
B.	$\sin^3(2\omega t)$	**II.**	**Periodic with time period $T = 2\pi/\omega$ but not SHM**
C.	$\sin(\omega t) + \cos(\pi\omega t)$	**III.**	**Periodic with time period $T = \pi/\omega$ and SHM**
D.	$\cos \omega t + \cos 2\omega t$	**IV.**	**Non-periodic**

Choose the correct answer from the options given below: [4 marks]

(A) A-III, B-I, C-IV, D-II

(B) A-II, B-I, C-III, D-IV

(C) A-III, B-II, C-IV, D-I

(D) A-II, B-I, C-IV, D-III

Correct Answer: (A) A-III, B-I, C-IV, D-II

Solution:

Concept:

- SHM is a single-frequency sinusoidal oscillation about an equilibrium position: $x - x_{\text{eq}} = A\cos(\Omega t + \phi)$. A constant offset does not change its harmonic character.
- A sum of sinusoidal terms is periodic only when its frequencies are commensurate. Multiple distinct frequencies generally make the motion non-SHM.

Step 1: Classify expressions A and B For A, use the double-angle identity:

$$\sin^2 \omega t = \frac{1}{2} - \frac{1}{2}\cos 2\omega t.$$

It is SHM about the mean position $1/2$, with angular frequency 2ω and period π/ω; hence A matches III. For B,

$$\sin^3(2\omega t) = \frac{3}{4}\sin 2\omega t - \frac{1}{4}\sin 6\omega t.$$

Its fundamental angular frequency is 2ω, so its period is π/ω. The third harmonic prevents a single SHM equation, so B matches I.

Step 2: Classify expressions C and D and select the mapping For C, the frequencies are ω and $\pi\omega$. Their ratio is irrational; no nonzero common period can make both phases repeat. Thus C is non-periodic and matches IV. For D, the frequencies ω and 2ω have common fundamental frequency ω. Therefore its period is $2\pi/\omega$, but the two-frequency superposition is not SHM, giving D-II. The full matching is

A-III, B-I, C-IV, D-II.

Only option (A) has all four assignments. Options (B) and (D) misclassify A and C or D, while option (C) interchanges the periods assigned to B and D.

Step 3: Conclusion The correct matching is A-III, B-I, C-IV, D-II. Hence, the correct option is (A).

Quick Tip to Solve: Remove constant offsets before testing SHM. For a sum of frequencies, first check whether their ratio is rational, then find the fundamental period.

36. A metal rod of length L rotates about one end at origin with a uniform angular velocity ω. The magnetic field radially falls off as $B(\mathrm{r}) = B_{\mathrm{o}}\mathrm{e}^{-\lambda r}; \lambda$ being a positive constant. The emf induced (neglecting the centripetal force on electrons in the rod) is : [4 marks]

(A) $B_o\omega\left[\frac{1}{\lambda^2} - e^{-\lambda L}\left(\frac{1}{\lambda^2} + \frac{L}{\lambda}\right)\right]$
(B) $B_o\omega\left[\frac{1}{\lambda^2} + e^{-\lambda L}\left(\frac{1}{\lambda^2} + \frac{L}{\lambda}\right)\right]$
(C) $B_o\omega\left[\frac{4}{\lambda^2} - e^{-2\lambda L}\left(\frac{1}{\lambda^2} + \frac{2L}{\lambda}\right)\right]$
(D) $B_0\omega\left[\frac{3}{\lambda^2} - e^{-3\lambda L}\left(\frac{3}{\lambda^2} + \frac{L}{\lambda}\right)\right]$
Correct Answer: (A) $B_o\omega\left[\frac{1}{\lambda^2} - e^{-\lambda L}\left(\frac{1}{\lambda^2} + \frac{L}{\lambda}\right)\right]$

Solution:

Concept:

- An element dr at distance r from the pivot has tangential speed $v = \omega r$. Its motional emf contribution is $d\mathcal{E} = B(r)\omega r\, dr$.
- The total end-to-end emf is found by integration along the rod because both speed and magnetic-field magnitude vary with radius.

Step 1: Set up the radial integral With the magnetic field perpendicular to the plane of rotation, the magnetic force drives charges along the rod. The radial dependence in the question is a variation in its magnitude. Ignoring the stated centripetal contribution, integrate the magnetic motional term:

$$\mathcal{E} = B_0\omega\int_0^L re^{-\lambda r}\, dr.$$

The factor r is essential because the end at the pivot is stationary while the outer end has speed ωL. A single endpoint value of the field cannot represent the whole rod.

Step 2: Integrate by parts and check the endpoints Take $u = r$ and $dv = e^{-\lambda r}dr$, giving

$$\int re^{-\lambda r}dr = -e^{-\lambda r}\left(\frac{r}{\lambda} + \frac{1}{\lambda^2}\right).$$

Evaluation from zero to L yields

$$\mathcal{E} = B_0\omega\left[\frac{1}{\lambda^2} - e^{-\lambda L}\left(\frac{1}{\lambda^2} + \frac{L}{\lambda}\right)\right].$$

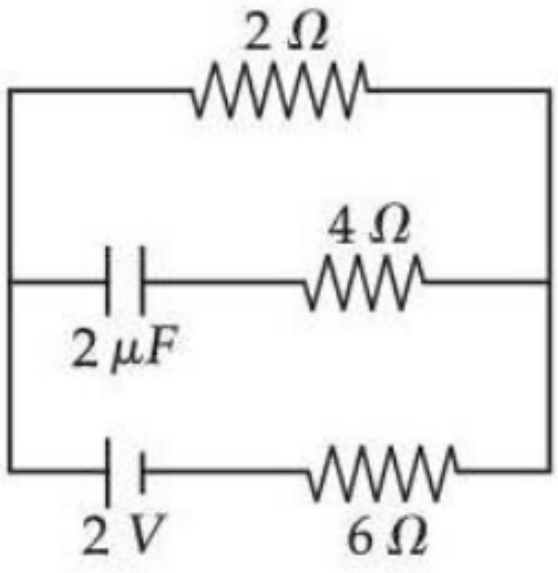
This expression vanishes at $L = 0$, unlike the plus sign in option (B). Option (C) also fails the zero-length check. Option (D) has an incorrect exponential factor and a term linear in small L, whereas the

integral must begin quadratically in L. Expanding the answer for $\lambda L \ll 1$ gives $B_0\omega L^2/2$, recovering the familiar uniform-field limit.

Step 3: Conclusion The induced emf magnitude is $\boxed{B_0\omega[\lambda^{-2} - e^{-\lambda L}(\lambda^{-2} + L/\lambda)]}$, in volts. Hence, the correct option is (A).

Quick Tip to Solve: For a rotating rod in a varying field, integrate $B(r)\omega r\, dr$. Test zero rod length and the uniform-field limit to catch endpoint sign errors.

37. Under steady state condition the potential difference across the capacitor in the circuit is ____ V. [4 marks]



(A) 0.5

(B) 1.5

(C) 0

(D) 2

Correct Answer: (A) 0.5

Solution:

Concept:

- An ideal capacitor in a DC circuit carries no current after the steady state has been reached. Its voltage can still be nonzero because charge remains on its plates.
- For a resistor, the potential drop is IR. Kirchhoff's loop rule determines the current in the conducting outer loop; the capacitor branch is open in steady state.

Step 1: Identify the conducting path The middle branch contains the $2\,\mu$F capacitor and the $4\,\Omega$ resistor in series. At steady state its current is zero. Consequently the drop across this resistor is zero, and the capacitor's right plate is at the potential of the right vertical rail. The outer loop, containing the battery and the other two resistors, continues to carry current.

Step 2: Calculate the outer-loop current The $2\,\Omega$ and $6\,\Omega$ resistors lie in the same conducting loop with the $2\,\mathrm{V}$ battery. Their total resistance is $8\,\Omega$, so

$$I = \frac{2}{2+6} = 0.25\,\mathrm{A}.$$

The battery's positive terminal is on the left, so the top resistor carries current from the left rail to the right rail.

Step 3: Find the rail and capacitor voltage The potential difference between the rails is the drop across the top resistor:

$$V_{\text{left}} - V_{\text{right}} = I(2) = 0.50\,\mathrm{V}.$$

Since the middle resistor has zero voltage drop, this is also the capacitor voltage. Option B confuses the drop across the bottom resistor with the rail voltage; option C confuses zero capacitor current with zero voltage, while option D ignores the series resistance.

Step 4: Conclusion The potential difference across the capacitor is $0.5\,\mathrm{V}$. Hence, the correct option is (A).

Quick Tip to Solve: Open the capacitor branch first for a steady DC circuit, then find its plate potentials from the remaining resistor network.

---

38. A particle of charge q and mass m is projected from origin with an initial velocity $\vec{v} = \left(\frac{v_0}{\sqrt{2}}\hat{x} + \frac{v_0}{\sqrt{2}}\hat{y}\right)$. There exists a uniform magnetic field $\vec{B} = B_0\hat{z}$ and a space varying electric field $\vec{E} = E_0 e^{-\lambda x}\hat{x}$ within the region $0 \leqslant x \leqslant L$. After travelling a distance such that x-coordinate has changed from $x = 0$ to $x = L$, the change in the kinetic energy is ____ . [4 marks]

(A) $\frac{qE_0}{\lambda}\left[1 - e^{-\lambda L}\right]$

(B) $\left(\frac{v_0 qB_0}{2\lambda}\right)\left[2 - e^{-2\lambda L}\right]$

(C) $\frac{qE_0}{\lambda}\left[1 + e^{-\lambda L}\right]$

(D) $q\left(\frac{E_0 + v_0 B_0}{\lambda}\right)\left[1 - e^{-\lambda L/2}\right]$

Correct Answer: (A) $\frac{qE_0}{\lambda}\left[1 - e^{-\lambda L}\right]$

Solution:

Concept:

- The work-energy theorem gives $\Delta K = \int \vec{F} \cdot d\vec{r}$. A magnetic force $q\vec{v} \times \vec{B}$ is perpendicular to the instantaneous velocity and does no work.
- The electric field is along the x-axis and depends only on x, so its work is $q\int_0^L E_0 e^{-\lambda x}\,dx$, irrespective of the curved trajectory.

Step 1: Remove the magnetic contribution to work The total force is $q\vec{E} + q\vec{v} \times \vec{B}$. Taking its scalar product with velocity gives

$$\frac{dK}{dt} = q\vec{E} \cdot \vec{v} + q(\vec{v} \times \vec{B}) \cdot \vec{v} = qE_0 e^{-\lambda x}\frac{dx}{dt}.$$

Thus the magnetic field changes the direction of motion, but it cannot directly change the kinetic energy. The initial components of velocity are not needed for this energy difference.

Step 2: Integrate between the stated positions Integrating the energy equation from the origin to the point with coordinate $x = L$ yields

$$\Delta K = qE_0 \int_0^L e^{-\lambda x}\, dx = \frac{qE_0}{\lambda}\left(1 - e^{-\lambda L}\right).$$

This result depends only on the two x-coordinates. Even if the path bends in the magnetic field, using the electric line integral gives the same endpoint result.

Step 3: Check the expression against the alternatives As $L \to 0$, the change in energy must tend to zero; the expression with $1 + e^{-\lambda L}$ fails this test. Options B and D incorrectly introduce B_0 into magnetic work. For small λL, the answer tends to $qE_0 L$, exactly the work of an approximately uniform electric field, which also checks the sign and dimensions.

Step 4: Conclusion

$$\Delta K = \frac{qE_0}{\lambda}\left(1 - e^{-\lambda L}\right).$$

Hence, the correct option is (A).

Quick Tip to Solve: For kinetic-energy changes in combined fields, discard magnetic work and integrate only the electric-field component along displacement.

39. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : The electromagnetic wave exerts pressure on the surface on which they are allowed to fall.

Reason (R) : There is no mass associated with the electromagnetic waves.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true

Correct Answer: (B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

Solution:

Concept:

- An electromagnetic wave transports both energy and momentum. If energy U is carried along one direction in vacuum, the associated momentum is $p = U/c$.
- Pressure is normal force per unit area. Absorption or reflection of incident radiation transfers momentum to a surface, producing radiation pressure.

Step 1: Evaluate the assertion using momentum transfer When a wave is absorbed by a surface, its incident momentum is delivered to that surface. A continuous incident wave therefore produces a continuous force. For normal incidence on a perfectly absorbing surface with intensity I, the momentum arriving per second per unit area is I/c, giving pressure $P = I/c$. A perfectly reflecting surface reverses the momentum and experiences $P = 2I/c$. Thus Assertion (A) is true.

Step 2: Interpret the reason in its standard physical sense Electromagnetic radiation has no rest mass; equivalently its photons have zero invariant mass. This is the conventional meaning of the reason's statement that no mass is associated with electromagnetic waves. It does not mean zero energy or zero momentum. Indeed the relativistic relation $E^2 = p^2c^2 + m_0^2c^4$, with $m_0 = 0$, gives $p = E/c$. Reason (R), understood as absence of rest mass, is true.

Step 3: Test whether the reason explains the assertion Radiation pressure follows from momentum transfer, rather than merely from having zero rest mass. The given reason does not mention momentum, absorption, reflection or the rate of momentum transfer, so it does not explain the pressure. Option A supplies an explanatory connection that is absent. Options C and D incorrectly deny the reason or the assertion respectively.

Step 4: Conclusion Both statements are true, but the reason does not explain radiation pressure. Hence, the correct option is (B).

Quick Tip to Solve: For radiation-pressure assertion-reason questions, distinguish zero rest mass from zero momentum: pressure is explained by momentum transfer.

40. A thin convex lens and a thin concave lens are kept in contact and are co-axial. Which of the following statements is correct for this combination of two lenses ? [4 marks]

(A) behaves as concave lens if $|f_{\text{convex}}| > |f_{\text{concave}}|$

(B) behaves as concave lens if $|f_{\text{convex}}| < |f_{\text{concave}}|$

(C) behaves as convex lens if $|f_{\text{convex}}| > |f_{\text{concave}}|$

(D) Focal length of the lens system will change if the positions of two lenses are interchanged

Correct Answer: (A) behaves as concave lens if $|f_{\text{convex}}| > |f_{\text{concave}}|$

Solution:

Concept:

- For thin coaxial lenses in contact, powers add: $P = P_1 + P_2$, or $1/F = 1/f_1 + 1/f_2$. The sign of the equivalent focal length determines converging or diverging behaviour.
- A convex lens has positive focal length and a concave lens has negative focal length. The lens with the smaller absolute focal length has the greater absolute optical power.

Step 1: Express both focal lengths with their signs Write $a = |f_{\text{convex}}| > 0$ and $b = |f_{\text{concave}}| > 0$. The signed focal lengths are $+a$ and $-b$. Consequently the equivalent power is

$$\frac{1}{F} = \frac{1}{a} - \frac{1}{b} = \frac{b-a}{ab}.$$

Since $ab > 0$, only the numerator decides the sign of the combination.

Step 2: Apply the condition for a diverging combination A concave equivalent lens requires $F < 0$, hence $b - a < 0$, or $a > b$. In the question's notation this is

$$|f_{\text{convex}}| > |f_{\text{concave}}|.$$

The concave lens then has the larger magnitude of power and dominates. For example, focal lengths $+20\,\text{cm}$ and $-10\,\text{cm}$ give an equivalent focal length of $-20\,\text{cm}$.

Step 3: Reject the remaining statements If $a < b$, the power is positive and the combination is convex, so option B is reversed. Option C assigns converging behaviour to a negative equivalent power. Interchanging the two thin lenses in contact does not change the sum of powers, ruling out option D. When $a = b$, the net power is zero, which is the boundary between the two behaviours rather than a concave combination.

Step 4: Conclusion The combination behaves as a concave lens when $|f_{\text{convex}}| > |f_{\text{concave}}|$. Hence, the correct option is (A).

Quick Tip to Solve: Compare the magnitudes of the lens powers, not their focal lengths: the smaller absolute focal length dominates a contact combination.

---

41. An object AB is placed 15 cm on the left of a convex lens P of focal length 10 cm . Another convex lens Q is now placed 15 cm right of lens P. If the focal length of lens Q is 15 cm , the final image is ____ . [4 marks]

(A) virtual, formed at 7.5 cm right of lens Q, with a size bigger than that of AB

(B) real, formed at 7.5 cm right of lens Q, with a size same as that of AB

(C) formed at infinity.

(D) real, formed at 7 cm right of lens Q, with a size smaller than that of AB

Correct Answer: (B) real, formed at 7.5 cm right of lens Q, with a size same as that of AB

Solution:

Concept:

- Use the Cartesian thin-lens formula $1/v - 1/u = 1/f$, taking distances to the right of each lens as positive for light travelling left to right.
- The image produced by the first lens acts as the object for the second. If the rays arriving at the second lens are converging towards a point on its right, that point is a virtual object with positive u.

Step 1: Locate the image due to lens P For lens P, $u_P = -15\,\text{cm}$ and $f_P = +10\,\text{cm}$. Hence

$$\frac{1}{v_P} = \frac{1}{10} - \frac{1}{15} = \frac{1}{30}, \qquad v_P = 30\,\text{cm}.$$

Without the second lens, the rays would meet 30 cm to the right of P. The first lens's magnification is $m_P = v_P/u_P = -2$.

Step 2: Use the converging rays as a virtual object for Q Lens Q lies only 15 cm to the right of P, so the first image is 15 cm to the right of Q. Therefore $u_Q = +15\,\text{cm}$, not negative. With $f_Q = +15\,\text{cm}$,

$$\frac{1}{v_Q} = \frac{1}{f_Q} + \frac{1}{u_Q} = \frac{1}{15} + \frac{1}{15} = \frac{2}{15}, \qquad v_Q = 7.5\,\text{cm}.$$

A positive image distance means the final image is real and lies to the right of Q.

Step 3: Calculate the final size and compare options The second magnification is $m_Q = v_Q/u_Q = 7.5/15 = 1/2$. Thus

$$m = m_P m_Q = (-2)(1/2) = -1.$$

The final image is inverted with the same size as AB. Option A incorrectly calls this right-side image virtual and enlarged; option C follows from using the wrong sign for the virtual object. Option D has both the wrong distance and the wrong size.

Step 4: Conclusion The final image is real, 7.5 cm to the right of Q, and the same size as AB. Hence, the correct option is (B).

Quick Tip to Solve: When the second lens intercepts converging rays, locate their intended meeting point first: a meeting point to its right is a positive-distance virtual object.

---

42. The maximum intensity in a Young's double slit experiment is I_0. Distance between the slits (d) is 5λ, where λ is the wavelength of light used. The intensity of the fringe, exactly opposite to one of the slits on the screen, placed at $D = 10d$ is ____ . [4 marks]

(A) $\frac{I_0}{4}$

(B) $\frac{I_o}{2}$

(C) I_o

(D) $\frac{3I_0}{4}$

Correct Answer: (B) $\frac{I_o}{2}$

Solution:

Concept:

- For coherent slits of equal intensity, the resultant intensity is $I = I_0 \cos^2(\phi/2)$, where I_0 is the maximum intensity and $\phi = 2\pi\Delta/\lambda$ is the phase difference.
- In the usual paraxial treatment of Young's experiment, the path difference at transverse coordinate y from the central axis is $\Delta \simeq dy/D$.

Step 1: Locate the point opposite one slit The central axis passes through the midpoint between the slits. A point directly opposite either slit is therefore at transverse distance $|y| = d/2$ from the central axis, rather than at zero displacement. With $D = 10d$ and $d = 5\lambda$, the path difference is

$$|\Delta| \simeq \frac{d(d/2)}{D} = \frac{d}{20} = \frac{\lambda}{4}.$$

The sign is immaterial when calculating intensity.

Step 2: Convert the path difference to phase The phase difference between the two waves arriving at the point is

$$|\phi| = \frac{2\pi}{\lambda}|\Delta| = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}.$$

This is neither a completely constructive phase difference nor a completely destructive one. The intensity must therefore lie between zero and the maximum.

Step 3: Evaluate the interference intensity Using the equal-slit interference relation gives

$$I = I_0 \cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}.$$

Option C would require zero phase difference modulo 2π, which occurs on the central axis but not at this point. Options A and D correspond to different phase differences. The standard far-screen approximation is consistent with $d/D = 0.1$; it is the Young-experiment treatment underlying the listed options.

Step 4: Conclusion The intensity at the stated point is $I_0/2$ in the standard Young double-slit approximation. Hence, the correct option is (B).

Quick Tip to Solve: A point opposite a slit is displaced by half the slit separation from the central axis; insert y=d/2 before calculating the phase.

43. An electron is travelling with a velocity v in free space and when it enters a medium, its velocity is reduced by 20%. The de Broglie wavelength of electron in the medium is $\alpha\lambda_0$, where λ_0 is its de Broglie wavelength in free space. The value of α is ____ . [4 marks]

(A) 1.20

(B) 1.0

(C) 1.25

(D) 0.75

Correct Answer: (C) 1.25

Solution:

Concept:

- The de Broglie wavelength of an electron is $\lambda = h/p$, where h is Planck's constant and p is its momentum.
- For the nonrelativistic electron considered here, $p = m_e v$. Its mass remains the same on entering the medium, so wavelength varies inversely with speed.

Step 1: Calculate the speed in the medium A reduction by 20% means that the electron retains 80% of its original speed. Thus the new speed is

$$v_m = v - 0.20v = 0.80v.$$

The speed decreases, so the momentum decreases and the matter wavelength must increase. The reduction is a fraction of the original speed, not a statement about the wavelength.

Step 2: Take the ratio of the wavelengths Write the two wavelengths before cancelling the common constants:

$$\lambda_0 = \frac{h}{m_e v}, \qquad \lambda_m = \frac{h}{m_e(0.80v)}.$$

Therefore

$$\alpha = \frac{\lambda_m}{\lambda_0} = \frac{1}{0.80} = 1.25.$$

Option (A) incorrectly adds the percentage decrease directly to the wavelength; the inverse relation gives a 25% increase instead. Option (B) would require unchanged speed, and option (D) predicts a smaller wavelength despite the reduced momentum.

Step 3: Conclusion The dimensionless multiplier is $\boxed{\alpha = 1.25}$. Hence, the correct option is (C).

Quick Tip to Solve: For a fractional speed decrease f, use $\lambda_{\text{new}}/\lambda_{\text{old}} = 1/(1-f)$; do not add f to 1.

---

44. Assuming the experimental mass of $^{12}_{6}C$ as $12u$, the mass defect of $^{12}_{6}C$ atom is ____ MeV/c^2. (Mass of proton $= 1.00727\text{u}$. mass of neutron $= 1.00866\text{u}, 1\text{u} = 931.5\text{MeV/c}^2$ and c is the speed of the light in vacuum). [4 marks]

(A) 127.5

(B) 89.03

(C) 272.0

(D) 92.0

Correct Answer: (B) 89.03

Solution:

Concept:

- For carbon-12, the atomic number is $Z = 6$ and the mass number is $A = 12$, so it contains six protons and six neutrons.
- Using the masses specified in the question, the mass defect is $\Delta m = Zm_p + (A-Z)m_n - M$. The conversion $1\,\text{u} = 931.5\,\text{MeV}/c^2$ converts a mass, not a dimensionless number.

Step 1: Add the separate nucleon masses The six protons and six neutrons, before binding, have total mass

$$M_{\text{free}} = 6(1.00727) + 6(1.00866)\,\text{u}.$$

Calculate the two contributions independently:

$$6(1.00727) = 6.04362, \qquad 6(1.00866) = 6.05196.$$

Hence

$$M_{\text{free}} = 12.09558\,\text{u}.$$

There are six neutrons rather than twelve because the mass number counts both kinds of nucleons.

Step 2: Subtract the stated mass and convert units Using the prescribed experimental mass $12\,\text{u}$

directly gives

$$\Delta m = 12.09558 - 12 = 0.09558\,\text{u}.$$

Thus

$$\Delta m = 0.09558(931.5) = 89.03277\,\text{MeV}/c^2.$$

The calculation uses exactly the given masses and the question's stated mass convention; no extra electron-mass correction is specified. Rounding to two decimal places gives 89.03. The other three proposed values, 127.5, 272.0, and 92.0, do not follow from the supplied subtraction and conversion.

Step 3: Conclusion With the numerical data given, the mass defect is $\boxed{89.03\,\text{MeV}/c^2}$. Hence, the correct option is (B).

Quick Tip to Solve: First subtract masses in atomic mass units, then multiply only the resulting small mass defect by 931.5.

---

45. In a semiconductor p-n diode, the doping concentrations on p-side and n-side are 10^{15} atoms $/\text{cm}^3$ and 10^{18} atoms $/\text{cm}^3$, respectively. Which one of the following statements is true? [4 marks]

(A) Widths of depletion region on either side of the interface are equal

(B) The depletion region width is more on p -side compared to that in n -side

(C) The depletion region width is more on n-side compared to that in p-side

(D) No depletion region forms because of unequal doping concentrations on p and n -sides

Correct Answer: (B) The depletion region width is more on p -side compared to that in n -side

Solution:

Concept:

- The depletion region consists of uncovered fixed acceptor and donor ions left near a p-n junction after mobile carriers diffuse and recombine.
- Charge neutrality requires equal magnitudes of fixed charge on the two sides: $eN_A A x_p = eN_D A x_n$, where A is junction area and x_p, x_n are depletion widths.

Step 1: Relate widths to doping concentrations Here $N_A = 10^{15}\,\text{cm}^{-3}$ on the p-side and $N_D = 10^{18}\,\text{cm}^{-3}$ on the n-side. After cancelling the common electronic charge and junction area, neutrality becomes

$$N_A x_p = N_D x_n.$$

A lightly doped side has fewer fixed ions per unit volume, so it must contribute a larger volume, and hence a greater width, to balance the charge on the more heavily doped side.

Step 2: Calculate the width ratio and select the statement The ratio is

$$\frac{x_p}{x_n} = \frac{N_D}{N_A} = \frac{10^{18}}{10^{15}} = 10^3.$$

Thus the depletion region extends a thousand times farther into the p-side than into the n-side, within the depletion approximation. Option (A) would apply to equal doping and therefore fails here. Option

(C) reverses the inverse dependence of width on concentration. Option (D) is also false: unequal doping changes the distribution of depletion width, but does not prevent diffusion, recombination or formation of the junction barrier.

Step 3: Conclusion The depletion width is greater on the lightly doped p-side: $x_p = 1000x_n$. Hence, the correct option is (B).

Quick Tip to Solve: The depletion region spreads mainly into the lightly doped side; use equal uncovered charge, rather than equal widths.

Physics – Section B: Numerical Value Questions (20 marks)

46. A copper wire of length 3 m is stretched by 3 mm by applying an external force. The volume of the wire is 600×10^{-6} m^3. The elastic potential energy stored in the wire in stretched condition would be ____ J.

(Given Young modulus of copper $= 1.1 \times 10^{11}$ N/m^2) [4 marks]

Solution:

Concept:

- Young modulus is $Y = (F/A)/(\Delta L/L)$, where L is initial length, A is cross-sectional area and ΔL is extension.
- In the elastic range the tension increases linearly from zero to F. Therefore the work stored is $U = \frac{1}{2}F\Delta L = \frac{1}{2}YV(\Delta L/L)^2$, with $V = AL$.

Step 1: Calculate the strain and cross-sectional area Convert the extension to metres before using the SI value of Young modulus:

$$\Delta L = 3\,\text{mm} = 3 \times 10^{-3}\,\text{m}.$$

The strain is

$$\epsilon = \frac{\Delta L}{L} = \frac{3 \times 10^{-3}}{3} = 10^{-3}.$$

The supplied volume is $V = 600 \times 10^{-6}\,\text{m}^3$, so

$$A = \frac{V}{L} = \frac{600 \times 10^{-6}}{3} = 2 \times 10^{-4}\,\text{m}^2.$$

Strain has no units, whereas area is in square metres.

Step 2: Find the final force and the stored work Hooke's law gives the final stress

$$\sigma = Y\epsilon = (1.1 \times 10^{11})(10^{-3}) = 1.1 \times 10^{8}\,\text{N/m}^2.$$

Thus

$$F = \sigma A = (1.1 \times 10^{8})(2 \times 10^{-4}) = 2.2 \times 10^{4}\,\text{N}.$$

The elastic potential energy is the triangular area under the force-extension graph:

$$U = \frac{1}{2}(2.2 \times 10^4)(3 \times 10^{-3}) = 33\,\text{J}.$$

Using the final force throughout would incorrectly double the energy; its average during stretching is $F/2$.

Step 3: Conclusion The wire stores $\boxed{33\,\text{J}}$ of elastic potential energy. The required numerical entry is 33.

Quick Tip to Solve: When volume and strain are supplied, evaluate $U = YV\epsilon^2/2$ directly and retain the factor of one-half.

47. The heat extracted out of x gram of water initially at 50°C to cool it down to 0°C is sufficient to evaporate $(1000 - x)$ gram of water also initially at 50°C. The value of x (closest integer) is ____ . (Take latent heat of water 2256 kJ/kg.K, specific heat capacity of water 4200 J/kg.K) [4 marks]

Solution:

Concept:

- The sensible heat released by cooling water is $Q = mc\Delta T$. Vaporising liquid requires latent heat mL in addition to the heat needed to bring it to its boiling temperature.
- For the other sample to become steam, it is first heated from 50°C to 100°C, and then vaporised. Equate this total requirement to the heat extracted from the cooling sample.

Step 1: Express heat released and heat required Use grams consistently: $c = 4.2\,\text{J}\,\text{g}^{-1}\text{K}^{-1}$ and the stated latent-heat magnitude is $L = 2256\,\text{J}\,\text{g}^{-1}$. Cooling x grams through $50\,\text{K}$ releases

$$Q_{\text{released}} = x(4.2)(50) = 210x\,\text{J}.$$

Each gram of the second sample needs

$$q = 4.2(100 - 50) + 2256 = 210 + 2256 = 2466\,\text{J/g}.$$

The latent heat is an energy per mass; the temperature change belongs only in the sensible-heat term.

Step 2: Apply the energy balance and round at the end The second sample has mass $(1000 - x)$ grams. Therefore

$$210x = 2466(1000 - x).$$

Rearranging without rounding intermediate values gives

$$2676x = 2466000, \qquad x = \frac{2466000}{2676} = 921.5247\ldots\,\text{g}.$$

This leaves about $78.4753\,\text{g}$ to evaporate. That small mass requires much more energy per gram than the first sample releases per gram, consistent with the result. Omitting the initial heating of the second sample would underestimate the necessary cooled-water mass.

Step 3: Conclusion The closest integer is $\boxed{x = 922}$, corresponding to approximately $922\,\mathrm{g}$ of water cooled. The required numerical entry is 922.

Quick Tip to Solve: For water initially below its boiling point, include both $mc\Delta T$ and mL before rounding the resulting mass.

48. A series LCR circuit with $R = 20\Omega, L = 1.6\mathrm{H}$ and $C = 40\mu\ \mathrm{F}$ is connected to a variable frequency a.c. source. The inductive reactance at resonant frequency is ____ Ω. [4 marks]

Solution:

Concept:

- In a series LCR circuit, resonance occurs when the inductive and capacitive reactances cancel: $X_L = X_C$, with $X_L = \omega L$ and $X_C = 1/(\omega C)$.
- The resonant angular frequency is $\omega_0 = 1/\sqrt{LC}$. Consequently each nonzero reactance at resonance has magnitude $X_L = \sqrt{L/C}$; the net reactance is zero.

Step 1: Convert capacitance and determine angular frequency The given capacitance must be written in farads:

$$C = 40\,\mu\mathrm{F} = 40 \times 10^{-6}\,\mathrm{F}.$$

Equating reactances gives

$$\omega_0 L = \frac{1}{\omega_0 C} \quad \Longrightarrow \quad \omega_0^2 = \frac{1}{LC}.$$

Here

$$LC = (1.6)(40 \times 10^{-6}) = 64 \times 10^{-6},$$

so

$$\omega_0 = \frac{1}{8 \times 10^{-3}} = 125\,\mathrm{rad/s}.$$

This is angular frequency, so no extra factor of 2π is needed when calculating reactance.

Step 2: Evaluate the inductive reactance At this angular frequency the inductive reactance is

$$X_L = \omega_0 L = (125)(1.6) = 200\,\Omega.$$

As a check, the capacitive reactance is

$$X_C = \frac{1}{(125)(40 \times 10^{-6})} = 200\,\Omega.$$

They cancel in the series impedance, leaving $Z = R = 20\,\Omega$. The resistor therefore determines the resonant impedance but does not set the ideal resonant frequency. It would be incorrect to report zero for the individual inductive reactance simply because the net reactive contribution vanishes.

Step 3: Conclusion The inductive reactance at resonance is $\boxed{200\,\Omega}$. The required numerical entry is 200.

Quick Tip to Solve: At series resonance use $X_L = \sqrt{L/C}$; distinguish the individual reactance from the zero net reactance.

49. When an external resistance of 5Ω is connected across terminals of a cell, a current of 0.25 A flows through it. When the 5Ω resistor is replaced by a 2Ω resistor, a current of 0.5 A flows through it. The internal resistance of the cell is ____ Ω. [4 marks]

Solution:

Concept:

- A real cell has an emf E and an internal resistance r. The same current passes through the external resistor and the internal resistance, so $E = I(R + r)$.
- The emf and internal resistance remain the same in the two measurements. The terminal voltage IR, however, changes because the internal voltage drop Ir changes with current.

Step 1: Write the equation for each load For the first measurement, the external resistance is $R_1 = 5\,\Omega$ and the current is $I_1 = 0.25\,\text{A}$. Thus

$$E = 0.25(5 + r) = 1.25 + 0.25r.$$

For the second measurement, use $R_2 = 2\,\Omega$ and $I_2 = 0.50\,\text{A}$:

$$E = 0.50(2 + r) = 1.00 + 0.50r.$$

Here the numerical coefficients give voltages when resistance is measured in ohms.

Step 2: Eliminate the common emf Equate the two expressions because both measurements use the same cell:

$$1.25 + 0.25r = 1.00 + 0.50r.$$

Collect the resistance terms on one side:

$$0.25 = 0.25r \quad \Longrightarrow \quad r = 1\,\Omega.$$

A positive result is physically consistent: the larger current causes the larger internal voltage loss.

Step 3: Check both measurements The first load gives $E = 0.25(5 + 1) = 1.50\,\text{V}$. The second independently gives $E = 0.50(2 + 1) = 1.50\,\text{V}$. Both currents therefore correspond to exactly the same emf. Using only the external terminal voltage would incorrectly identify different emfs for the same cell.

Step 4: Conclusion The internal resistance is $\boxed{1\,\Omega}$. The numerical entry in the blank is $\boxed{1}$.

Quick Tip to Solve: For two loads on one cell, equate $I_1(R_1 + r) = I_2(R_2 + r)$; do not equate the two terminal voltages.

50. A circular loop of radius 20 cm and resistance 2Ω is placed in a time varying magnetic field $\vec{B} = \left(2t^2 + 2t + 3\right) T$. At $t = 0$, for the plane of the loop being perpendicular to the magnetic field and, the induced current in the loop at $t = 3$ s is $\frac{\alpha}{50}$ A. The value of α is ____ . (Take $\pi = 22/7$) [4 marks]

Solution:

Concept:

- Faraday's law gives the magnitude of induced emf as $|\mathcal{E}| = |d\Phi_B/dt|$. For a uniform field perpendicular to a stationary loop, $\Phi_B = BA$, where A is its area.
- The induced current magnitude is $I = |\mathcal{E}|/R$. A constant term in the magnetic field contributes flux but contributes nothing to its time derivative.

Step 1: Calculate the loop area Convert the radius to SI units before squaring it:

$$a = 20\,\text{cm} = 0.20\,\text{m}, \qquad A = \pi a^2 = 0.04\pi\,\text{m}^2.$$

The plane is perpendicular to the magnetic field, so the area vector is parallel to the field and the flux factor is $\cos 0 = 1$. The loop is stationary; only the field changes.

Step 2: Differentiate the field and find the current The given field magnitude is $B(t) = 2t^2 + 2t + 3$. Therefore

$$\frac{dB}{dt} = 4t + 2.$$

At $t = 3\,\text{s}$, its rate of change is $14\,\text{T}\,\text{s}^{-1}$. Hence

$$I = \frac{A}{R}\left|\frac{dB}{dt}\right| = \frac{0.04\pi \times 14}{2} = 0.28\pi\,\text{A}.$$

The value of B itself is not used in place of its derivative.

Step 3: Determine the requested coefficient Using the supplied value $\pi = 22/7$,

$$I = 0.28 \times \frac{22}{7} = 0.88\,\text{A} = \frac{44}{50}\,\text{A}.$$

Comparison with $I = \alpha/50\,\text{A}$ gives $\alpha = 44$. Lenz's law fixes the circulation direction to oppose the increasing flux; the question asks only for the current magnitude.

Step 4: Conclusion The induced current is $0.88\,\text{A}$, so the dimensionless coefficient is $\boxed{\alpha = 44}$.

Quick Tip to Solve: Differentiate $B(t)$ first, then multiply by $\pi a^2/R$. Convert centimetres to metres before calculating area.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. What volume of hydrogen gas at STP would be liberated by action of 50 mL of H_2SO_4 of 50% purity (density $= 1.3\ \text{g mL}^{-1}$) on 20 g of zinc?

Given : Molar mass of H, O, S, Zn are 1, 16, 32, $65\ \text{g mol}^{-1}$ respectively. [4 marks]

(A) 5.824 L

(B) 7.428 L

(C) 6.892 L

(D) 8.375 L

Correct Answer: (C) 6.892 L

Solution:

Concept:

- The reaction is $Zn + H_2SO_4 \longrightarrow ZnSO_4 + H_2$. One mole of zinc reacts with one mole of sulfuric acid and produces one mole of hydrogen.
- The volume of hydrogen is set by the limiting reagent. At the STP convention used by the options, one mole of an ideal gas occupies $22.4\,\text{L}$.

Step 1: Find the amount of pure acid The mass of the supplied acid solution is obtained from density and volume:

$$m_{\text{solution}} = 50 \times 1.3 = 65\,\text{g}.$$

Only half of this is pure H_2SO_4, so

$$m_{\text{acid}} = 0.50 \times 65 = 32.5\,\text{g}.$$

The molar mass is $2(1) + 32 + 4(16) = 98\,\text{g mol}^{-1}$. Thus

$$n_{\text{acid}} = \frac{32.5}{98} = 0.3316\,\text{mol}.$$

Step 2: Identify the limiting reactant The available zinc is

$$n_{\text{Zn}} = \frac{20}{65} = 0.30769\,\text{mol}.$$

Because the stoichiometric ratio of zinc to acid is $1 : 1$, compare the mole amounts directly. Zinc has the smaller amount and is fully consumed; acid remains in excess. The hydrogen formed is consequently $0.30769\,\text{mol}$, not $0.3316\,\text{mol}$.

Step 3: Convert hydrogen amount to volume and compare At STP,

$$V_{H_2} = \frac{20}{65} \times 22.4 = 6.8923\,\text{L} \approx 6.892\,\text{L}.$$

Option (B), about $7.428\,\text{L}$, results from treating the acid as limiting. Option (A) is smaller and option (D) is larger than the volume permitted by the available zinc; neither satisfies the reaction's mole balance.

Step 4: Conclusion The liberated hydrogen volume is $\boxed{6.892\,\mathrm{L}}$ at STP. Hence, the correct option is (C).

Quick Tip to Solve: Calculate the pure solute mass from density and purity, then compare stoichiometric mole amounts before using molar volume.

---

52. Which of the following statement(s) is/are true ?

A. If two orbitals have the same value of ($\mathrm{n}+l$), the orbital with lower value of n will have lower energy.

B. Energies of the orbitals in the same subshell increase with increase in atomic number.

C. The size of $2\mathrm{p}_x$ orbital is less than the size of $3\mathrm{p}_x$ orbital.

D. Among 5f, 6s, 4d, 5p and 5d orbitals, none of the orbitals have 2 radial nodes.

Choose the correct answer from the options given below : [4 marks]

(A) A, B and C only

(B) A and C only

(C) C and D only

(D) A only

Correct Answer: (B) A and C only

Solution:

Concept:

- The $(n+l)$ rule orders orbitals in many-electron atoms: smaller $n+l$ means lower energy, and for equal $n+l$, smaller principal quantum number n means lower energy.
- The number of radial nodes is $n-l-1$. The angular quantum number is $l=0,1,2,3$ for s, p, d and f orbitals, respectively. Orbital energies are measured relative to a free electron.

Step 1: Check statements A and B A is true: the tie in $n+l$ is resolved in favour of the lower n, as stated. B is false. Increasing atomic number generally increases the effective nuclear attraction for a given subshell, making its bound-state energy more negative. Its energy is lowered rather than increased. Confusing greater binding-energy magnitude with greater orbital energy reverses this conclusion.

Step 2: Check the orbital sizes in C C is true. For the same type of orbital, increasing the principal quantum number increases its characteristic radial extent. Thus a $3p_x$ orbital is larger than a $2p_x$ orbital. The subscript x specifies orientation and does not reverse the size trend. Both orbitals have the same angular shape, but their radial distributions differ.

Step 3: Calculate all the radial-node counts in D Apply $n-l-1$ to each listed orbital:

$$\begin{aligned} 5f &: 5-3-1=1, \qquad & 6s &: 6-0-1=5, \\ 4d &: 4-2-1=1, \qquad & 5p &: 5-1-1=3, \\ 5d &: 5-2-1=2. \end{aligned}$$

The $5d$ orbital does have two radial nodes, so D is false. A and C are the only true statements. Option (A) wrongly includes B, option (C) wrongly includes D and excludes A, and option (D) wrongly omits C.

Step 4: Conclusion Only statements A and C are true. Hence, the correct option is (B).

Quick Tip to Solve: For a node statement listing several orbitals, evaluate $n - l - 1$ for every entry; one counterexample disproves a claim of "none".

53. The covalent radii of atoms A and B are r_A and r_B, respectively. The covalent bond length and total length of AB molecule are respectively [4 marks]

(A) $(\mathrm{r_A} + \mathrm{r_B})\,, 2\,(\mathrm{r_A} + \mathrm{r_B})$

(B) $\frac{1}{2}\,(\mathrm{r_A} + \mathrm{r_B})\,, (\mathrm{r_A} + \mathrm{r_B})$

(C) $(\mathrm{r_A} + \mathrm{r_B})\,, (\mathrm{r_A} + \mathrm{r_B})$

(D) $2\,(\mathrm{r_A} + \mathrm{r_B})\,, \frac{1}{2}\,(\mathrm{r_A} + \mathrm{r_B})$

Correct Answer: (A) $(\mathrm{r_A} + \mathrm{r_B})\,, 2\,(\mathrm{r_A} + \mathrm{r_B})$

Solution:

Concept:

- In the covalent-radius model, the internuclear separation in a bond is the sum of the two bonded atoms' covalent radii. Bond length measures centre-to-centre distance, not the full extent of the molecule.
- The total end-to-end length of a diatomic molecule also includes the outer radius on each side of the internuclear segment. All the lengths must be expressed in the same unit.

Step 1: Calculate the internuclear bond length Let the nuclei of A and B be the reference centres. Their covalent radii along the bond are r_A and r_B. In the additive-radius model used in the question, the bonding contact lies between the nuclei, giving

$$d_{AB} = r_A + r_B.$$

For identical atoms this reduces to $d_{AA} = 2r_A$, the definition used to obtain a covalent radius from a homonuclear bond.

Step 2: Include the two outer portions The full molecular length runs from the far outer edge of A to the far outer edge of B. It consists of the outward radius of A, the internuclear bond length, and the outward radius of B:

$$L = r_A + d_{AB} + r_B.$$

Substitute the bond length without replacing it by a diameter:

$$L = r_A + (r_A + r_B) + r_B = 2(r_A + r_B).$$

This distinguishes the molecule's full length from the distance between its nuclei.

Step 3: Reject the other ordered pairs Option (B) makes the bond length half the sum of radii,

contradicting radius additivity. Option (C) uses the correct bond length but equates it with the total end-to-end length, omitting both outer portions. Option (D) doubles the bond length and then makes the entire molecule even shorter than the bond itself. Only option (A) gives the correct quantities in the order requested.

Step 4: Conclusion The bond length is $\boxed{r_A + r_B}$, and the total molecular length is $\boxed{2(r_A + r_B)}$. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish centre-to-centre bond length from outer-edge-to-outer-edge molecular length; add one outer radius at each end.

54. Consider the following data for the reaction

$$X_2(g) + Y_2(g) \rightleftharpoons 2XY(g)$$

at $600\ \mathrm{K}^\circ$**. The** $\Delta_r G^\ominus$ **(in** kJmol^{-1} **) for the reaction is :**

Compound	$\Delta_f H^\ominus_{600\ \mathrm{K}}\ (\mathrm{kJ\ mol}^{-1})$	$S^\ominus_{600\ \mathrm{K}}\ (\ \mathrm{J\ mol}^{-1}\ \mathrm{K}^{-1})$
$\mathrm{XY}(\ \mathrm{g})$	42	200
$\mathrm{X_2}(\ \mathrm{g})$	8	140
$\mathrm{Y_2}(\ \mathrm{g})$	80	250

[4 marks]

(A) -21000

(B) -10

(C) -1000

(D) -9.012

Correct Answer: (B) -10

Solution:

Concept:

- The standard reaction Gibbs energy is $\Delta_r G^\circ = \Delta_r H^\circ - T\Delta_r S^\circ$, with all quantities evaluated at the same temperature.
- Reaction enthalpy and entropy are calculated as stoichiometric sums for products minus those for reactants. Entropy in joules must be converted to kilojoules when enthalpy is in kilojoules.

Step 1: Calculate the reaction enthalpy For $\mathrm{X_2(g) + Y_2(g) \longrightarrow 2\,XY\,(g)}$, use every tabulated value as supplied at $600\,\mathrm{K}$:

$$\Delta_r H^\circ = 2(42) - [8 + 80] = 84 - 88 = -4\,\mathrm{kJ\,mol}^{-1}.$$

The coefficient 2 multiplies the product formation enthalpy. Do not replace the explicitly supplied reactant values by zero; the requested calculation is based on the given data.

Step 2: Calculate the reaction entropy Apply the same products-minus-reactants rule to the entropy

column:

$$\Delta_r S^\circ = 2(200) - [140 + 250] = 400 - 390 = 10\,\text{J mol}^{-1}\,\text{K}^{-1}.$$

Convert this to $0.010\,\text{kJ mol}^{-1}\,\text{K}^{-1}$ before using it with the enthalpy. The small positive entropy change makes the Gibbs energy more negative at this temperature.

Step 3: Combine the quantities in consistent units At $T = 600\,\text{K}$,

$$T\Delta_r S^\circ = 600(0.010) = 6\,\text{kJ mol}^{-1}.$$

Therefore

$$\Delta_r G^\circ = -4 - 6 = -10\,\text{kJ mol}^{-1}.$$

Option (A) and option (C) have incorrect numerical magnitudes, while option (D) does not follow from the tabulated enthalpy and entropy. A mixture of joules and kilojoules would also produce an invalid result.

Step 4: Conclusion The standard reaction Gibbs energy is $\boxed{-10\,\text{kJ mol}^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the reaction coefficients in both thermodynamic sums and convert entropy to kilojoules before calculating $T\Delta S$.

55. The correct order of molar heat capacities measured at 298 K and 1 bar is : [4 marks]

(A) Copper(s) > Bromine(l) > Helium(g)

(B) Bromine(l) > Copper(s) > Helium(g)

(C) Helium(g) > Bromine(l) > Copper(s)

(D) Helium(g) > Bromine(l) = Copper(s)

Correct Answer: (B) Bromine(l) > Copper(s) > Helium(g)

Solution:

Concept:

- Molar heat capacity at constant pressure is $C_{p,m} = (\partial H_m/\partial T)_p$: the heat needed to raise the temperature of one mole by one kelvin at fixed pressure.
- Heat capacity depends on accessible molecular motions and intermolecular interactions, so a molar comparison must not be confused with a comparison per gram.

Step 1: Estimate the heat capacity of helium Helium is a monatomic gas. At 298 K its translational degrees of freedom give $C_{V,m} = 3R/2$, and the ideal-gas relation gives

$$C_{p,m} = C_{V,m} + R = \frac{5}{2}R \simeq 20.8\,\text{J mol}^{-1}\text{K}^{-1}.$$

There are no rotational or vibrational contributions for an isolated helium atom.

Step 2: Compare the condensed substances For solid copper near room temperature, the molar heat capacity is approximately $24.5\,\text{J mol}^{-1}\text{K}^{-1}$, close to the Dulong–Petit value $3R \simeq 24.9\,\text{J mol}^{-1}\text{K}^{-1}$.

Liquid bromine has a substantially larger value, approximately $75.7\,\mathrm{J\,mol^{-1}K^{-1}}$, because it consists of diatomic molecules in a liquid with additional energy-storage contributions. Thus the relevant ordering of the measured molar values is

$$75.7 > 24.5 > 20.8.$$

Step 3: Eliminate the remaining orders Option (A) incorrectly puts copper above liquid bromine. Options (C) and (D) both put helium first, contrary to its smaller monatomic-gas heat capacity; (D) also incorrectly equates copper and bromine. Merely using the rule that gases expand more than liquids does not rank their molar heat capacities.

Step 4: Conclusion The order is Bromine(l) > Copper(s) > Helium(g). Hence, the correct option is (B).

Quick Tip to Solve: Use $5R/2$ for monatomic helium and approximately $3R$ for copper, then place liquid bromine above both; compare per mole, not per gram.

56. The reaction $\mathrm{A(g)} \rightleftharpoons \mathrm{B(g)} + \mathrm{C(g)}$ was initiated with the amount ' a ' of $\mathrm{A(g)}$. At equilibrium it is found that the amount of $\mathrm{A(g)}$ remaining is ($\mathrm{a} - x$) at a total pressure of p .

The equilibrium constant Kp of the reaction can be calculated from the expression : [4 marks]

(A) $\frac{x^2}{\mathrm{a}^2+x^2} \times \mathrm{p}$

(B) $\frac{x^2}{a^2-x^2} \times p$

(C) $\frac{\mathrm{a}+x^2}{x^2} \times \mathrm{p}$

(D) $\frac{\mathrm{a}^2-x^2}{x^2} \times \mathrm{p}$

Correct Answer: (B) $\frac{x^2}{a^2-x^2} \times p$

Solution:

Concept:

- For $\mathrm{A(g)} \rightleftharpoons \mathrm{B(g)} + \mathrm{C(g)}$, the pressure-form equilibrium expression is $K_p = p_{\mathrm{B}} p_{\mathrm{C}} / p_{\mathrm{A}}$, in the pressure convention used in the options.
- For an ideal gas mixture, each partial pressure is its mole fraction multiplied by the total equilibrium pressure: $p_i = y_i p$.

Step 1: Write the equilibrium amounts Initially only A is present, with amount a. Since its remaining amount is $a - x$, the amount reacted is x. The stoichiometric coefficients are all unity, so the amounts of B and C formed are each x. The equilibrium amounts are therefore

$$n_{\mathrm{A}} = a - x, \qquad n_{\mathrm{B}} = x, \qquad n_{\mathrm{C}} = x.$$

The total is $a - x + x + x = a + x$, not a, because the reaction increases the number of gas molecules.

Step 2: Convert amounts to partial pressures Divide each equilibrium amount by the total $a + x$ and

multiply by the given total pressure:

$$p_{\mathrm{A}} = \frac{a-x}{a+x}p, \qquad p_{\mathrm{B}} = p_{\mathrm{C}} = \frac{x}{a+x}p.$$

These expressions also satisfy $p_{\mathrm{A}} + p_{\mathrm{B}} + p_{\mathrm{C}} = p$, providing a useful check on the total amount.

Step 3: Substitute and simplify Using the stoichiometric equilibrium expression gives

$$K_p = \frac{\left(\frac{x}{a+x}p\right)^2}{\frac{a-x}{a+x}p} = \frac{x^2 p}{(a+x)(a-x)} = \frac{x^2}{a^2-x^2}p.$$

The denominator is a difference of squares, which excludes (A). Option (D) reverses the amount ratio, and (C) does not follow from the equilibrium mole fractions. As x tends to zero, the expression tends to zero as expected for negligible dissociation.

Step 4: Conclusion $K_p = \dfrac{x^2}{a^2-x^2}\,p$. Hence, the correct option is (B).

Quick Tip to Solve: Always calculate the new total amount first: one mole dissociating into two makes the total $a + x$, producing $(a + x)(a - x)$ in the denominator.

57. One half cell in a voltaic cell is constructed by dipping silver rod in $\mathrm{AgNO_3}$ solution of unknown concentration, other half cell is Zn rod dipped in 1 molar solution of $\mathrm{ZnSO_4}$.

A voltage of 1.60 V is measured at 298 K for this cell. What is the concentration of $\mathrm{Ag^+}$ ions used in terms of $\log x$ $\left(x = \left[\mathrm{Ag^+}\right]\right)$?

$$\mathrm{E^{\ominus}_{Zn^{2+}/Zn}} = -0.76\ \mathrm{V}, \quad \mathrm{E^{\ominus}_{Ag^{+}/Ag}} = +0.80\ \mathrm{V}, \frac{2.303\mathrm{RT}}{\mathrm{F}} = 0.059\ \mathrm{V}$$

[4 marks]

(A) $\frac{2}{3.9}$
(B) $\frac{4}{5.9}$
(C) $\frac{2.9}{2}$
(D) $\frac{5.9}{4}$

Correct Answer: (B) $\frac{4}{5.9}$

Solution:

Concept:

- The cell potential is the cathode reduction potential minus the anode reduction potential. Silver is reduced and zinc is oxidised because silver has the more positive standard reduction potential.
- At 298 K, $E = E^\circ - (0.059/n)\log Q$, where n is the number of electrons transferred and the logarithm is base ten. Pure solid activities are unity.

Step 1: Write the overall cell reaction The half reactions are $\mathrm{Zn} \longrightarrow \mathrm{Zn^{2+}} + 2\,\mathrm{e^-}$ and $2\,\mathrm{Ag^+} + 2\,\mathrm{e^-} \longrightarrow$

2 Ag. Thus

$$\mathrm{Zn} + 2\,\mathrm{Ag}^{+} \longrightarrow \mathrm{Zn}^{2+} + 2\,\mathrm{Ag}, \qquad n = 2.$$

The standard cell potential is

$$E^{\circ} = 0.80 - (-0.76) = 1.56\,\mathrm{V}.$$

The silver potential is not multiplied by two when balancing electrons because electrode potential is an intensive quantity.

Step 2: Express the reaction quotient With $[\mathrm{Zn}^{2+}] = 1$ and $[\mathrm{Ag}^{+}] = x$, the concentration quotient is

$$Q = \frac{[\mathrm{Zn}^{2+}]}{[\mathrm{Ag}^{+}]^2} = \frac{1}{x^2}.$$

Concentrations are understood relative to the standard concentration in the logarithm. Therefore

$$E = 1.56 - \frac{0.059}{2}\log(x^{-2}) = 1.56 + 0.059\log x.$$

The two from the silver exponent cancels the electron-transfer number.

Step 3: Use the measured voltage Substituting $E = 1.60\,\mathrm{V}$ gives

$$0.04 = 0.059\log x, \qquad \log x = \frac{0.04}{0.059} = \frac{4}{5.9} \simeq 0.678.$$

The positive logarithm is consistent with an emf above the standard emf: increasing the silver-ion concentration favours silver reduction. Options (A), (C) and (D) do not equal this voltage-derived ratio; in particular (D) takes its reciprocal.

Step 4: Conclusion $\log x = \frac{4}{5.9}$, corresponding to $[\mathrm{Ag}^{+}] \simeq 4.76\,\mathrm{mol\,L^{-1}}$ in the concentration approximation. Hence, the correct option is (B).

Quick Tip to Solve: Write the balanced reaction before forming Q: the square on silver-ion concentration cancels the $n = 2$ factor in the Nernst equation.

58. Given below are two statements :

Statement I : The number of pairs among $[Al_2O_3, Cr_2O_3]$, $[Cl_2O_7, Mn_2O_7]$, $[Na_2O, V_2O_3]$ **and** $[CO, N_2O]$ **that contain oxides of same nature (acidic, basic, neutral or amphoteric) is 4 .**

Statement II : Among Na_2O, Al_2O_3, CO **and** Cl_2O_7**, the most basic and acidic oxides are** Na_2O **and** Cl_2O_7**, respectively.**

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- Acidic oxides react with bases, basic oxides react with acids, amphoteric oxides react with both, and neutral oxides do not exhibit the usual acid-base oxide behaviour.
- For transition elements, lower oxidation states generally favour basic oxides, whereas very high oxidation states favour acidic oxides. Amphoterism must be checked for the particular oxide.

Step 1: Classify all four pairs Al_2O_3 and Cr_2O_3 are both amphoteric. Cl_2O_7 and Mn_2O_7 are both acidic oxides, corresponding to chlorine and manganese in their high $+7$ oxidation states. Na_2O and V_2O_3 are both basic: sodium oxide is strongly basic and the lower vanadium oxide has basic character. Finally, CO and N_2O are both neutral oxides. Every listed pair therefore contains oxides of the same nature.

Step 2: Decide Statement I The four classifications can be collected as

$$\begin{aligned}(Al_2O_3, Cr_2O_3) &: \text{ amphoteric},\\ (Cl_2O_7, Mn_2O_7) &: \text{ acidic},\\ (Na_2O, V_2O_3) &: \text{ basic},\\ (CO, N_2O) &: \text{ neutral}.\end{aligned}$$

Hence all four pairs qualify, exactly as Statement I claims, so Statement I is true.

Step 3: Decide Statement II and select the combination Among the four individual oxides named in Statement II, Na_2O is basic, Al_2O_3 is amphoteric, CO is neutral and Cl_2O_7 is acidic. Sodium oxide forms sodium hydroxide with water, while chlorine heptoxide is the acid anhydride of perchloric acid. Thus the most basic and most acidic members are precisely Na_2O and Cl_2O_7. Statement II is also true. Option (B) rejects both true claims, and options (C) and (D) incorrectly reject one of them.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: Classify each oxide separately before counting pairs; remember that V_2O_3 is basic but Cr_2O_3 is amphoteric.

59. Given below are two statements :

Statement I: Aluminium upon reaction with NaOH forms $[Al(OH)_6]^{3-}$ ion.

Statement II : The geometry of ICl_4^-, ClO_3^- and IBr_2^- is square planar, pyramidal and linear respectively.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Concept:

- Aluminium is amphoteric and reacts with aqueous sodium hydroxide to form tetrahydroxoaluminate, $[Al(OH)_4]^-$, with evolution of hydrogen gas.
- VSEPR theory distinguishes electron-pair arrangement from molecular shape: lone pairs influence the arrangement, but the stated molecular geometry describes only the bonded atoms.

Step 1: Test the aluminium statement The reaction with aqueous alkali is represented by

$$2\,Al + 2\,NaOH + 6\,H_2O \longrightarrow 2\,Na[Al(OH)_4] + 3\,H_2.$$

Its complex ion is tetrahydroxoaluminate, $[Al(OH)_4]^-$, rather than the six-hydroxide ion written in Statement I. Charge balance alone cannot establish which species is formed: aluminium has oxidation state $+3$ in both proposed formulas. The standard aqueous reaction and coordination species must be used. Statement I is therefore false.

Step 2: Find each molecular shape in Statement II In ICl_4^-, iodine has four bond pairs and two lone pairs, giving six electron domains. The octahedral electron-pair arrangement places the two lone pairs opposite one another, leaving a square planar molecular shape. In ClO_3^-, chlorine has three sigma-bond domains and one lone pair, so the electron-domain arrangement is tetrahedral and the molecular shape is trigonal pyramidal. Multiple bonding in resonance descriptions does not create extra VSEPR domains.

Step 3: Check the final ion and compare the options In IBr_2^-, iodine has two bond pairs and three lone pairs, or five electron domains. The three lone pairs occupy the equatorial positions of a trigonal bipyramid; the two bromine atoms occupy opposite axial positions, giving a linear ion. Thus all three shapes stated in Statement II are correct. Option (A) wrongly accepts Statement I, (B) wrongly rejects Statement II, and (C) reverses both truth values.

Step 4: Conclusion Statement I is false, but Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: For VSEPR count sigma-bond domains and lone pairs, then name the shape of the bonded atoms; do not call an AX_4E_2 ion octahedral in molecular geometry.

60. Given below are two statements :

Statement I : Presence of large number of unpaired electrons in transition metal atoms results in higher enthalpies of their atomisation.

Statement II : $d_{xy} = d_{xz} = d_{yz} < d_{x^2-y^2} = d_{z^2}$ **and** $d_{x^2-y^2} = d_{z^2} < d_{xy} = d_{xz} = d_{yz}$ **are the d-orbital splittings in** $[Fe(H_2O)_6]^{3+}$ **and** $[Ni(Cl)_4]^{2-}$ **complex ions respectively.**

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are correct

(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (A) Both Statement I and Statement II are correct

Solution:

Concept:

- A larger number of unpaired valence electrons generally allows stronger metallic bonding in transition metals, leading to greater enthalpy required to separate the solid into gaseous atoms.
- Crystal-field splitting depends on ligand geometry. Octahedral and tetrahedral fields reverse the ordering of the two sets of d orbitals; the ordering is distinct from the high-spin or low-spin occupancy.

Step 1: Assess the atomisation statement Atomisation breaks the cohesive metallic interactions and forms isolated gaseous atoms. Transition metals use their valence d and s electrons in metallic bonding. A greater number of unpaired electrons generally strengthens this bonding and is associated with higher enthalpies of atomisation. This is the standard qualitative trend expressed by Statement I, rather than a claim of an exact monotonic numerical law for every neighbouring element. Statement I is correct.

Step 2: Check the octahedral iron complex $[Fe(H_2O)_6]^{3+}$ contains six water ligands in an octahedral arrangement. Ligands approach along the coordinate axes, so $d_{x^2-y^2}$ and d_{z^2}, which point along those axes, experience greater repulsion. The ordering is

$$d_{xy} = d_{xz} = d_{yz} < d_{x^2-y^2} = d_{z^2}.$$

The lower set is called t_{2g}, and the higher set is called e_g. The first ordering in Statement II is therefore correct.

Step 3: Check the tetrahedral nickel complex $[NiCl_4]^{2-}$ is tetrahedral because chloride is a weak-field ligand. In this geometry the e set lies below the t_2 set:

$$d_{x^2-y^2} = d_{z^2} < d_{xy} = d_{xz} = d_{yz}.$$

This matches the second ordering in Statement II. Both geometry-dependent comparisons are correct. Option (B) rejects both correct statements, while options (C) and (D) incorrectly reject Statement II and Statement I respectively.

Step 4: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (A).

Quick Tip to Solve: Remember octahedral $t_{2g} < e_g$ and tetrahedral $e < t_2$; first identify the geometry, then apply the orbital ordering.

61. Identify the correct statements from the following

A. $[Fe\,(C_2O_4)_3]^{3-}$ **is the most stable complex among** $[Fe(OH)_6]^{3-}$, $[Fe\,(C_2O_4)_3]^{3-}$ **and** $[Fe(SCN)_6]^{3-}$

B. The stability of $[Cu\,(NH_3)_4]^{2+}$ **is greater than that of** $[Cu(en)_2]^{2+}$

C. The hybridization of Fe in $K_4\,[Fe(CN)_6]$ **is** d^2sp^3

D. $[Fe\,(NO_2)_3\,Cl_3]^{3-}$ **exhibits linkage isomerism**

E. NO_2^- **and** SCN^- **ligands are NOT ambidentate ligands**

Choose the correct answer from the options given below : [4 marks]

(A) A, B, C, D and E

(B) B, C and D only

(C) A, C and D only

(D) A, C and E only

Correct Answer: (C) A, C and D only

Solution:

Concept:

- Chelating ligands form rings with the central ion and generally stabilize a complex relative to comparable monodentate ligands.
- In valence-bond theory, a strong-field ligand can pair the metal d electrons; ambidentate ligands can coordinate through alternative donor atoms.

Step 1: Evaluate the stability statements Oxalate is bidentate, so three oxalate ions form chelate rings with iron(III). The chelate effect makes the oxalate complex the most stable of the three complexes listed; statement A is true. Ethane-1,2-diamine (en) is also bidentate, whereas ammonia is monodentate. Therefore the bis(en) complex of copper(II) is more stable than its tetraammine complex, contrary to statement B. Statement B is false.

Step 2: Determine the iron configuration In $K_4[Fe(CN)_6]$, the complex ion has charge -4. Six cyanide ions contribute -6, so iron is in oxidation state $+2$, with configuration $3d^6$. Cyanide is a strong-field ligand: the six d electrons pair in three orbitals, leaving two inner d orbitals available. The octahedral complex therefore uses d^2sp^3 hybridisation. Statement C is true.

Step 3: Check linkage isomerism and select the set NO_2^- can coordinate through nitrogen (nitro) or oxygen (nitrito), so the given mixed-ligand iron complex can exhibit linkage isomerism. Statement D is true. Both NO_2^- and SCN^- are ambidentate; thiocyanate binds through sulfur or nitrogen. Statement E is false. Options A and D contain false E, and option B contains false B; only the set A, C and D survives.

Step 4: Conclusion The correct statements are A, C and D only. Hence, the correct option is (C).

Quick Tip to Solve: Check chelation first, then the metal oxidation state; do not confuse an ambidentate ligand with a bidentate one.

62. Match List - I with List - II.

List - I Purification technique	List - II Used to separate
A. Simple distillation	I. Steam volatile compound
B. Fractional distillation	II. Two liquids with large difference in boiling points
C. Steam distillation	III. Liquid decomposing at its boiling point
D. Distillation under reduced pressure	IV. Two liquids with close boiling points

Choose the correct answer from the options given below: [4 marks]

(A) A-II, B-III, C-I, D-IV

(B) A-II, B-IV, C-I, D-III

(C) A-II, B-IV, C-III, D-I

(D) A-IV, B-III, C-II, D-I

Correct Answer: (B) A-II, B-IV, C-I, D-III

Solution:

Concept:

- Distillation separates substances by their differing volatilities; the technique must also avoid thermal decomposition.
- Fractionating columns provide repeated vaporisation and condensation, while reduced pressure lowers a liquid's boiling temperature.

Step 1: Match the ordinary distillation methods Simple distillation is suitable when the boiling points of two liquids differ sufficiently, so the vapour is strongly enriched in the more volatile component. Thus A matches II. When the boiling points are close, a single vaporisation does not separate them effectively. A fractionating column supplies repeated equilibrium stages and improves separation; fractional distillation therefore matches IV, giving B-IV.

Step 2: Match steam and reduced-pressure distillation A steam-volatile compound can be carried over with steam. The combined vapour pressures permit distillation below the normal boiling temperature of the organic compound, so steam distillation corresponds to I: C-I. A liquid that decomposes at its normal boiling point should instead be distilled at reduced pressure. It then boils at a lower temperature, avoiding decomposition: D-III.

Step 3: Compare the complete mapping with the options The combined mapping is A-II, B-IV, C-I, D-III. Option A wrongly assigns a decomposing liquid to fractional distillation and close-boiling liquids to reduced pressure. Option C interchanges the uses of steam and reduced-pressure distillation. Option D also misassigns simple and fractional distillation. Only option B matches all four physical requirements.

Step 4: Conclusion The correct matching is A-II, B-IV, C-I, D-III. Hence, the correct option is (B).

Quick Tip to Solve: Use fractional distillation for close boiling points, steam for steam volatility, and vacuum when heating to the normal boiling point causes decomposition.

63. IUPAC name of the some alkenes are given below.

Find out the correct stability order.

A. 2-Methylbut-2-ene

B. cis-But-2-ene

C. 2,3-Dimethylbut-2-ene

D. Prop-1-ene

Choose the correct answer from the options given below : [4 marks]

(A) $C > A > B > D$

(B) $C > A > D > B$

(C) $B > D > A > C$

(D) $A > B > C > D$

Correct Answer: (A) $C > A > B > D$

Solution:

Concept:

- Alkyl groups stabilize a carbon-carbon double bond by hyperconjugation and their electron-releasing inductive effect.
- For the acyclic alkenes here, greater alkyl substitution of the double bond gives greater stability; compare substitution before considering cis/trans effects.

Step 1: Identify the substitution of each double bond In 2,3-dimethylbut-2-ene (C), each double-bond carbon carries two methyl groups, so it is tetrasubstituted. In 2-methylbut-2-ene (A), the double bond has three alkyl substituents, making it trisubstituted. In cis-but-2-ene (B), each double-bond carbon has one methyl group and one hydrogen; it is disubstituted. Prop-1-ene (D) has just one methyl group attached to the double bond and is monosubstituted.

Step 2: Apply hyperconjugative stabilization The alkyl groups donate electron density and allow stabilizing interaction of adjacent carbon-hydrogen sigma bonds with the alkene pi system. In this series, the numbers of such adjacent hydrogens are 12 for C, 9 for A, 6 for B and 3 for D. The increasing substitution and hyperconjugation both give the order

$$C > A > B > D.$$

The cis arrangement makes B less stable than its trans isomer, but that comparison does not reverse its advantage over monosubstituted propene.

Step 3: Eliminate the inconsistent orders Option B places monosubstituted D above disubstituted B, contrary to the substitution trend. Option C makes disubstituted B the most stable and tetrasubstituted C the least stable. Option D puts trisubstituted A above tetrasubstituted C. None is consistent with the structures; the first order correctly includes all four substitution levels.

Step 4: Conclusion The stability order is $C > A > B > D$. Hence, the correct option is (A).

Quick Tip to Solve: Write the double bond and count the attached carbon groups: tetra > tri > di > mono is the decisive trend for this set.

64. Identify the correct IUPAC name of hydrocarbon (x) containing three primary carbon atoms and with molar mass $72\ \text{g mol}^{-1}$. [4 marks]

(A) 1, 1 - Dimethylcyclopropane

(B) 2, 2 - Dimethylpropane

(C) 2 - Methylbutane

(D) n-pentane

Correct Answer: (C) 2 - Methylbutane

Solution:

Concept:

- A primary carbon atom is directly bonded to exactly one other carbon atom; count carbon neighbours rather than attached hydrogens.
- An acyclic saturated hydrocarbon has formula $\text{C}_n\text{H}_{2n+2}$ and molar mass $14n + 2$ using C = 12 and H = 1.

Step 1: Use the molar mass to identify possible formulae For an alkane,

$$14n + 2 = 72 \quad \Rightarrow \quad n = 5.$$

The molecular formula is therefore C_5H_{12}. The three acyclic pentane isomers in the options have this molar mass. By contrast, 1,1-dimethylcyclopropane is a cycloalkane, with formula C_5H_{10}, and molar mass $70\,\text{g mol}^{-1}$; it fails the mass condition.

Step 2: Count the primary carbon atoms 2-Methylbutane has condensed structure

$$\text{CH}_3{-}\text{CH}(\text{CH}_3){-}\text{CH}_2{-}\text{CH}_3.$$

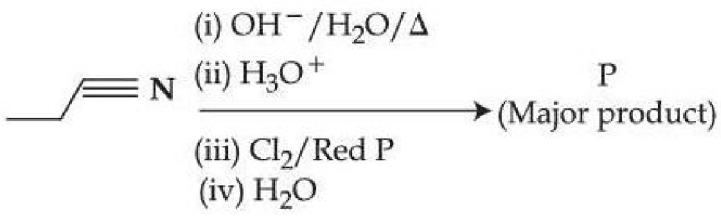
Its two terminal methyl groups and the methyl branch are each bonded to only one carbon, giving three primary carbon atoms. The branching carbon is tertiary and the methylene carbon is secondary. In 2,2-dimethylpropane, all four methyl carbons attach to the central carbon and are primary, so its count is four. In straight-chain pentane, only its two terminal methyl carbons are primary, so its count is two.

Step 3: Check both conditions together The required structure must meet the mass and carbon-count conditions simultaneously. Option A fails the mass and has only two primary methyl carbons; option B has the correct mass but four primary carbons; option D has the correct mass but only two primary carbons. Only 2-methylbutane combines $72\,\text{g mol}^{-1}$ with exactly three primary carbon atoms.

Step 4: Conclusion The hydrocarbon is 2-methylbutane. Hence, the correct option is (C).

Quick Tip to Solve: For primary-carbon counting, mark every carbon with exactly one carbon neighbour; a branch methyl counts just like a terminal methyl.

65. Complete the following reaction sequence and give the name of major product P. [4 marks]



(A) 2-Chloropropanoic acid

(B) 3-Chloropropanoic acid

(C) 1-Chloropropane

(D) 2-Chloropropane

Correct Answer: (A) 2-Chloropropanoic acid

Solution:

Concept:

- Alkaline hydrolysis of a nitrile converts its nitrile carbon into a carboxylate carbon; acidification gives the corresponding carboxylic acid.
- Chlorine and red phosphorus followed by water replace an alpha hydrogen of a carboxylic acid by chlorine: the Hell-Volhard-Zelinsky reaction.

Step 1: Hydrolyse the starting nitrile The starting compound in the sequence is propanenitrile, CH_3CH_2CN. Hydroxide and water on heating hydrolyse the nitrile group to the propanoate ion, and subsequent treatment with H_3O^+ produces propanoic acid. The carbon of the nitrile is retained as the carbon of the carboxyl group, so the chain still contains three carbons:

$$CH_3CH_2CN \xrightarrow{OH^-/H_2O,\ \text{heat}} CH_3CH_2COO^- \xrightarrow{H_3O^+} CH_3CH_2COOH.$$

Step 2: Chlorinate the alpha position Treatment with Cl_2 and red phosphorus enables alpha halogenation of the acid through the corresponding acyl halide and its enol. The alpha carbon is the carbon immediately next to the carboxyl carbon, namely carbon 2 of propanoic acid. One of its hydrogens is replaced by chlorine. Water in the final step hydrolyses the acyl halide group back to the carboxylic acid:

$$CH_3CH_2COOH \xrightarrow[H_2O]{Cl_2/\text{red P}} CH_3CHClCOOH.$$

Step 3: Name the product and exclude other positions The carboxyl carbon has priority and is numbered 1, making the chloro substituent at carbon 2. Thus the product is 2-chloropropanoic acid. The 3-chloro acid would require chlorination at the terminal beta carbon, which is not the HVZ position. Neither chloropropane option retains the carboxyl group, whereas no decarboxylation or reduction occurs in this sequence.

Step 4: Conclusion The major product P is 2-chloropropanoic acid, $CH_3CHClCOOH$. Hence, the correct option is (A).

Quick Tip to Solve: Keep the nitrile carbon in the chain during hydrolysis, then number from COOH; HVZ chlorination occurs at the adjacent carbon.

66. Given below are two statements: [4 marks]

Statement I : The condensation reaction between $CH_3-CH=O$ and $H_2N-\underset{\underset{H}{|}}{N}-\underset{\underset{O}{||}}{C}-NH_2$ under optimum pH will produce $CH_3-CH=N-\underset{\underset{O}{||}}{C}-\overset{\overset{H}{|}}{N}-NH_2$

Statement II : The molecule, $Ph-CH\begin{matrix}O-H\\O-CH_3\end{matrix}$ will generate $Ph-CH=O$ in the presence of dilute acid.

In the light of the above statements, choose the **correct** answer from the options given below :

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Correct Answer: (D) **Statement I** is false but **Statement II** is true

Solution:

Concept:

- An aldehyde reacts with the terminal hydrazino amino group of semicarbazide to form a semicarbazone, eliminating water at an optimum mildly acidic pH.
- A hemiacetal contains OH and OR groups on the same carbon; dilute aqueous acid reverses its formation to regenerate the carbonyl compound and an alcohol.

Step 1: Locate the reactive nitrogen of semicarbazide The reagent shown is semicarbazide, $H_2N-NH-C(=O)-NH_2$. The terminal hydrazino $-NH_2$ group attacks the carbonyl carbon of ethanal. The amino group bonded directly to the carbonyl carbon is amide-like; resonance delocalises its lone pair and makes it much less nucleophilic. The carbonyl derivative must therefore form at the terminal hydrazino end, retaining the original N-N-C connectivity.

Step 2: Compare the actual product with the printed product Addition to ethanal followed by proton transfers and loss of water gives

$$CH_3CHO + H_2NNHCONH_2 \longrightarrow CH_3CH=NNHCONH_2 + H_2O.$$

The correct semicarbazone has the sequence $CH_3CH=N-NH-C(=O)-NH_2$. In contrast, the product drawn in statement I has the sequence $CH_3CH=N-C(=O)-NH-NH_2$. It attaches the imine nitrogen directly to the carbonyl carbon, rather than preserving the hydrazino nitrogen pair next to the imine. The drawn structure is therefore not the condensation product; statement I is false.

Step 3: Test the hemiacetal in statement II The depicted molecule is $Ph-CH(OH)-OCH_3$, the methyl hemiacetal of benzaldehyde. Dilute aqueous acid reverses hemiacetal formation: the alkoxy group is

released as methanol and the carbon-oxygen double bond reforms. Thus

$$\mathrm{PhCH(OH)OCH_3} \overset{\text{dilute acid}}{\rightleftharpoons} \mathrm{PhCHO + CH_3OH.}$$

Benzaldehyde is regenerated, so statement II is true. Option A and option C wrongly declare I true, while option B wrongly declares II false. Only option D gives the correct pair of truth values.

Step 4: Conclusion Statement I is false and statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: Check the exact N-N-C connectivity of a proposed semicarbazone; a plausible-looking C=N bond alone does not make the structure correct.

---

67. Given below are two statements :

Statement I : Heating benzamide with bromine in an ethanolic solution of sodium hydroxide will give benzylamine.

Statement II : Nitration of aniline with $\mathrm{HNO_3/H_2SO_4}$ at 288 K produces m-nitroaniline in higher amount than o-nitroaniline (pH adjusted).

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Concept:

- The Hofmann bromamide reaction converts a primary amide into a primary amine containing one carbon atom fewer. The carbonyl carbon of the amide is lost.
- In a strongly acidic nitrating mixture, aniline is protonated to the anilinium ion, which directs electrophilic substitution predominantly towards the meta position.

Step 1: Check the product of benzamide The group attached to the carbonyl carbon in benzamide is phenyl, not benzyl. Hofmann rearrangement therefore gives aniline, with the nitrogen attached directly to the benzene ring. The overall alkaline reaction is

$$\mathrm{C_6H_5CONH_2 + Br_2 + 4\,NaOH \longrightarrow C_6H_5NH_2 + Na_2CO_3 + 2\,NaBr + 2\,H_2O.}$$

Benzylamine would be $\mathrm{C_6H_5CH_2NH_2}$, which retains the extra carbon and is not this rearrangement product. Thus Statement I is false.

Step 2: Check the directing effect during nitration The nitrating mixture contains strong acids. Protonation gives $\mathrm{C_6H_5NH_3^+}$, so the lone pair is no longer available for donation to the ring. The positively charged ammonium substituent is deactivating and meta-directing. Consequently direct

nitration gives a substantial meta product, exceeding the ortho product under the stated conditions. Adjusting the pH during isolation restores the amino group without changing the position of nitration. Statement II is true. Options (A) and (C) incorrectly accept Statement I, while (B) incorrectly rejects Statement II.

Step 3: Conclusion Statement I is false and Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: For Hofmann rearrangement, delete the carbonyl carbon; for aniline nitration, first check whether the acidic medium protonates nitrogen.

68. Identify the incorrect statement about tertiary structure of proteins. [4 marks]

(A) They can be fibrous or globular in structure.

(B) The main forces that stabilize the structure are hydrogen bonding, disulphide links, van der Waals and electrostatic forces of attraction.

(C) The structure remains intact when exposed to pH changes.

(D) A linear polypeptide chain will convert to a secondary structure and then further folding of the secondary structure will convert to tertiary structure.

Correct Answer: (C) The structure remains intact when exposed to pH changes.

Solution:

Concept:

- Tertiary structure is the overall three-dimensional folding of a polypeptide chain beyond its local secondary structures.
- Hydrogen bonds, disulphide bridges, van der Waals interactions and electrostatic attractions stabilize protein folding. Changes in pH can alter ionisation and disrupt these interactions.

Step 1: Identify the pH-sensitive interactions A change in pH changes the protonation states of acidic and basic amino-acid side chains. Charged groups may become neutral or change their interactions with neighbouring groups. Salt bridges and hydrogen-bond networks can consequently be weakened, and the native three-dimensional conformation can unfold. This is one route to denaturation. It is therefore incorrect to claim, without qualification, that the tertiary structure remains intact on exposure to pH changes. The primary peptide-bond sequence can remain intact even while this higher-order folding is lost.

Step 2: Check the remaining statements Option (A) describes the familiar fibrous and globular forms of proteins and is consistent with their overall structural classification. Option (B) correctly lists interactions responsible for stabilising the folded structure, including covalent disulphide links. Option (D) correctly distinguishes local secondary folding from further overall folding to tertiary structure. Since the question asks for the incorrect statement, these three acceptable descriptions must not be selected merely because they mention different aspects of protein organisation.

Step 3: Conclusion The claim that tertiary structure remains intact when exposed to pH changes is incorrect. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish loss of folding from cleavage of peptide bonds: pH-induced denaturation principally affects secondary and tertiary organisation.

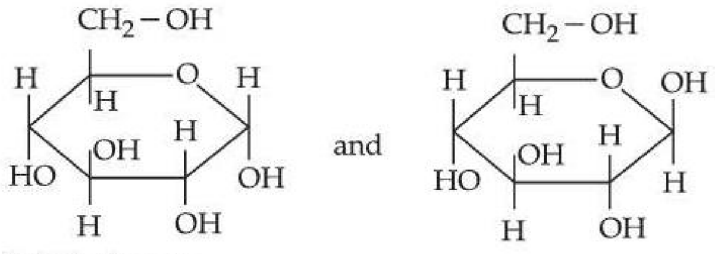
69. Given below are two statements:

Statement I: The two structures shown below are two anomers of D-(+)-glucose.

Statement II: The open chain forms of D-glucose and D-fructose contain three similar chiral carbons at C_3, C_4 and C_5.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]



(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- Anomers are cyclic sugar stereoisomers differing only in configuration at the anomeric carbon, formed from the carbonyl carbon during ring closure.
- Open-chain D-glucose is an aldohexose, whereas D-fructose is a ketohexose. Both have corresponding stereocentres at carbons 3, 4 and 5.

Step 1: Compare the two cyclic structures In the two drawings, the ring size and the configurations at the other stereocentres are unchanged. At carbon 1, one structure has the hydroxyl group below the ring and the other has it above the ring. For D-glucose, the CH_2OH group is above the ring in the conventional Haworth representation. Thus the first drawing is the alpha anomer and the second is the beta anomer. They differ only at the newly formed hemiacetal centre, not at another ring carbon. Statement I is true.

Step 2: Compare the open-chain stereocentres In D-glucose the aldehyde carbon is carbon 1; carbons 2, 3, 4 and 5 are stereogenic. In D-fructose the ketone carbon is carbon 2, which is planar and not stereogenic; carbons 3, 4 and 5 are stereogenic. In their conventional Fischer projections the hydroxyl orientations at these three shared centres are left, right and right respectively. Thus the three corresponding chiral carbons named in Statement II have matching configurations. Statement II is also true. Options (B), (C) and (D) each incorrectly reject at least one of these verified statements.

Step 3: Conclusion Both statements are true: the drawings are glucose anomers and the open-chain sugars share the specified three stereocentres. Hence, the correct option is (A).

Quick Tip to Solve: Locate the carbonyl carbon before counting stereocentres: glucose has anomeric C1, while fructose has carbonyl C2 in its open chain.

70. A paper dipped in a dil. H_2SO_4 solution of ' X ' upon treatment with SO_2 gas turns into green. The compound ' X ' is : [4 marks]

(A) $KI - strach$

(B) $KMnO_4$

(C) $Pb\,(CH_3COO)_2$

(D) $K_2Cr_2O_7$

Correct Answer: (D) $K_2Cr_2O_7$

Solution:

Concept:

- Acidified dichromate contains chromium in oxidation state +6 and is orange. Reduction produces green chromium(III) ions.
- Sulphur dioxide is a reducing agent: its sulphur is oxidised from +4 to +6 while reducing the oxidising agent.

Step 1: Identify the colour-changing ion The diagnostic observation is the formation of a green product in an acidic medium. Dichromate accepts six electrons per ion to produce two chromium(III) ions:

$$Cr_2O_7{}^{2-} + 14\,H^+ + 6\,e^- \longrightarrow 2\,Cr^{3+} + 7\,H_2O.$$

The original chromium(VI) species is orange and the chromium(III) product is green. Thus potassium dichromate is consistent with both the acidified reagent and the observed final colour.

Step 2: Balance the reaction and reject the alternatives Each sulphur dioxide molecule loses two electrons on conversion to sulphate, so three molecules supply the six electrons needed. Combining the half-reactions gives

$$Cr_2O_7{}^{2-} + 3\,SO_2 + 2\,H^+ \longrightarrow 2\,Cr^{3+} + 3\,SO_4{}^{2-} + H_2O.$$

Acidified permanganate in option (B) is reduced to nearly colourless manganese(II), rather than a green product. Iodide-starch in option (A) is associated with iodine and its starch colour, not chromium(III) green. Lead acetate in option (C) does not supply the chromium species responsible for the stated colour change. Therefore none of those reagents fits the observation.

Step 3: Conclusion The compound is potassium dichromate, $K_2Cr_2O_7$. Hence, the correct option is (D).

Quick Tip to Solve: Use the final colour to identify the reduction product: orange dichromate becomes green chromium(III), whereas acidified permanganate loses its purple colour.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The total number of unpaired electrons present in the d^3, d^4 (low spin) d^5 (high spin), d^6 (high spin) and d^7 (low spin) octahedral complex systems is ____ . [4 marks]

Solution:

Concept:

- In an octahedral field, the three lower-energy t_{2g} orbitals precede the two higher-energy e_g orbitals.
- For low spin, pairing in t_{2g} is favoured before occupation of e_g; for high spin, maximum single occupation is favoured before pairing.

Step 1: Fill the first three configurations For d^3, the three electrons occupy separate lower-energy orbitals, giving $t_{2g}^3 e_g^0$ and three unpaired electrons. No high-spin versus low-spin distinction changes this count. For low-spin d^4, the fourth electron pairs in t_{2g}, giving occupancies $(2, 1, 1)$ there and hence two unpaired electrons. For high-spin d^5, all five d orbitals are occupied singly before any pairing occurs. The configuration is $t_{2g}^3 e_g^2$, with five unpaired electrons.

Step 2: Fill the remaining configurations and add For high-spin d^6, the sixth electron pairs in one of the lower orbitals after the five singly occupied orbitals have been established. Thus $t_{2g}^4 e_g^2$ has two unpaired electrons in t_{2g} and two in e_g, giving four altogether. For low-spin d^7, all three lower orbitals are paired before the seventh electron occupies an upper orbital: $t_{2g}^6 e_g^1$, with one unpaired electron. Each configuration represents a separate complex system, so their counts must be added, rather than adding their d-electron populations.

$$N_{\text{unpaired}} = 3 + 2 + 5 + 4 + 1 = 15.$$

Step 3: Conclusion The total number of unpaired electrons is $\boxed{15}$, a dimensionless electron count.

Quick Tip to Solve: Write the orbital occupancies before counting: low-spin d4 has two unpaired electrons, but high-spin d6 has four.

---

72. RMgI when treated with ice cold water liberated a gas which occupied $1.4\mathrm{dm}^3/\mathrm{g}$ at STP. The gas produced is further reacted with iodine in presence of HIO_3 to give compound (X). Compound (X) in presence of Na and dry ether produced compound (Y). Molar mass of compound (Y) is ____ gmol^{-1}. (Nearest integer) [4 marks]

Solution:

Concept:

- A Grignard reagent reacts with water by protonation of its carbon group: $\mathrm{RMgI} + \mathrm{H_2O} \longrightarrow \mathrm{RH} + \mathrm{Mg(OH)I}$.
- At STP, one mole of an ideal gas occupies $22.4\,\mathrm{dm}^3$. Wurtz coupling joins two alkyl groups using sodium in dry ether.

Step 1: Identify the hydrocarbon gas The gas occupies $1.4\,\mathrm{dm}^3$ per gram. Its molar mass is the molar

volume divided by its specific volume:

$$M(\mathrm{RH}) = \frac{22.4\,\mathrm{dm^3\,mol^{-1}}}{1.4\,\mathrm{dm^3\,g^{-1}}} = 16\,\mathrm{g\,mol^{-1}}.$$

This is methane, $\mathrm{CH_4}$, so the original Grignard reagent is methylmagnesium iodide. The reaction with ice-cold water supplies a proton to the methyl group; it does not lengthen the carbon chain.

Step 2: Follow iodination and sodium coupling Methane reacts with iodine in the presence of iodic acid to give methyl iodide, compound X. Iodic acid removes the hydrogen iodide formed, promoting iodination. The relevant organic transformation is

$$\mathrm{CH_4 + I_2 \longrightarrow CH_3I + HI}.$$

Two methyl iodide molecules then undergo the Wurtz reaction with sodium in dry ether:

$$\mathrm{2\,CH_3I + 2\,Na \longrightarrow C_2H_6 + 2\,NaI}.$$

Thus Y is ethane, not methane or an iodine-containing compound. Its molecular formula contains two carbons and six hydrogens, giving

$$M(\mathrm{C_2H_6}) = 2(12) + 6(1) = 30\,\mathrm{g\,mol^{-1}}.$$

The value is already integral, so rounding to the nearest integer does not change it.

Step 3: Conclusion Compound Y is ethane and its molar mass is $\boxed{30\,\mathrm{g\,mol^{-1}}}$. The requested numerical entry is 30.

Quick Tip to Solve: First use molar volume divided by volume per gram to identify RH; then double the alkyl carbon count in the Wurtz product.

73. 20 g hemoglobin in a 1 L aqueous solution (A) at 300 K is separated from pure water by semi permeable membrane. At equilibrium the height of solution in a tube dipped in a solution (A) is found to be 80.0 mm higher than the tube dipped in water.

The molar mass of hemoglobin is ____ $\mathrm{kgmol^{-1}}$. (Nearest integer)

(Given : $\mathrm{g = 10\ m\ s^{-2}, R = 8.3 kPa dm^3 K^{-1}\ mol^{-1}}$**, density of solution** $= 1000\ \mathrm{kg\ m^{-3}}$ **)** [4 marks]

Solution:

Concept:

- A semipermeable membrane passes water but retains hemoglobin. At equilibrium, the hydrostatic pressure of the elevated solution balances its osmotic pressure: $\pi = \rho g h$.
- For a dilute molecular solution, $\pi V = nRT = (m/M)RT$. Here m is the solute mass and M is its molar mass; hemoglobin is treated as a nondissociating solute.

Step 1: Convert the height and calculate the balancing pressure The level difference is the height of the extra solution column, not the entire height of either tube. Convert the measured difference to SI

units:

$$h = 80.0\,\text{mm} = 0.0800\,\text{m}.$$

With the given density and gravitational acceleration, the pressure of this column is

$$\pi = (1000\,\text{kg m}^{-3})(10\,\text{m s}^{-2})(0.0800\,\text{m}) = 800\,\text{Pa} = 0.800\,\text{kPa}.$$

Osmotic flow stops when this pressure opposes the tendency of pure water to enter the solution.

Step 2: Apply the osmotic-pressure equation with consistent units Use $V = 1.00\,\text{dm}^3$, $m = 20.0\,\text{g}$, $T = 300\,\text{K}$ and the stated gas constant in $\text{kPa dm}^3\,\text{K}^{-1}\,\text{mol}^{-1}$. Rearranging before substitution gives

$$M = \frac{mRT}{\pi V} = \frac{(20.0)(8.3)(300)}{(0.800)(1.00)}\,\text{g mol}^{-1} = 62250\,\text{g mol}^{-1}.$$

The units of pressure and volume cancel those in the gas constant. Dividing by 1000 converts grams per mole to kilograms per mole:

$$M = 62.25\,\text{kg mol}^{-1}.$$

Step 3: Conclusion The molar mass is $62.25\,\text{kg mol}^{-1}$, so the required nearest integer is $\boxed{62}\,\text{kg mol}^{-1}$.

Quick Tip to Solve: First turn the level difference into pressure using ρgh. Pair kPa with dm^3 when using the given value of R, and convert the final molar mass to kilograms only at the end.

---

74. At 298 K , the molar conductivity of $x\%$(w/w)MX solution (aqueous) is $123.5\ \text{S cm}^2\ \text{mol}^{-1}$. The conductance of same solution is 1.9×10^{-3} S. The value of x is ____ $\times 10^{-2}$.
(Given: cell constant $= 1.3\ \text{cm}^{-1}$; molar mass of MX is $75\ \text{g mol}^{-1}$, density of aqueous solution of MX at 298 K is $1.0\ \text{g mL}^{-1}$) [4 marks]

Solution:

Concept:

- Conductivity is conductance multiplied by the cell constant: $\kappa = G(l/A)$. Conductance depends on the cell geometry, whereas conductivity describes the solution.
- When κ is in S cm^{-1} and concentration c is in mol L^{-1}, molar conductivity is $\Lambda_m = 1000\kappa/c$.
- Mass percentage is the solute mass divided by the solution mass, multiplied by 100. The density converts a chosen solution volume to its mass.

Step 1: Obtain conductivity from the measured conductance Multiply the conductance by the stated cell constant; do not substitute the conductance directly into the molar-conductivity relation:

$$\kappa = (1.9 \times 10^{-3}\,\text{S})(1.3\,\text{cm}^{-1}) = 2.47 \times 10^{-3}\,\text{S cm}^{-1}.$$

Step 2: Find the molar concentration Rearrange the molar-conductivity equation to isolate c:

$$c = \frac{1000\kappa}{\Lambda_m} = \frac{1000(2.47 \times 10^{-3})}{123.5} = 0.0200\,\text{mol L}^{-1}.$$

The factor 1000 converts a litre to cubic centimetres, matching the centimetre units of the conductivity and molar conductivity.

Step 3: Convert the concentration into mass percentage Choose one litre of solution. It contains 0.0200 mole of MX, whose mass is

$$m_{\text{solute}} = (0.0200)(75) = 1.50\,\text{g}.$$

At density $1.0\,\text{g}\,\text{mL}^{-1}$, the mass of this litre of solution is $1000\,\text{g}$. Hence

$$x = \frac{1.50}{1000} \times 100 = 0.150.$$

This is the percentage number itself: the solution is 0.150% by mass. Express it in the form requested by the blank:

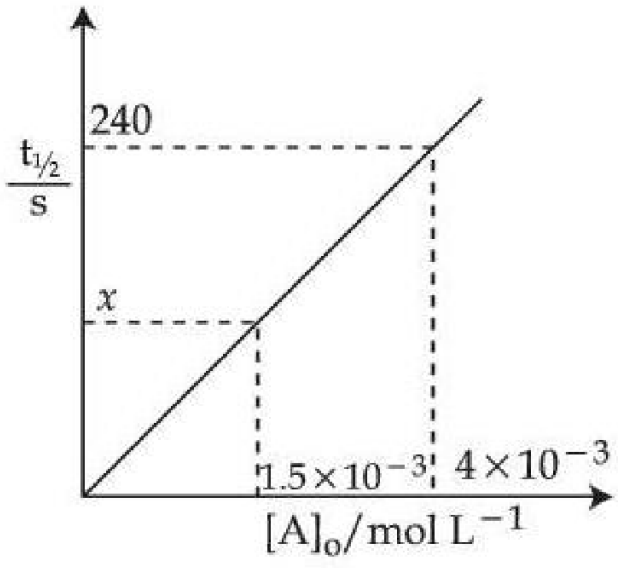
$$x = 0.150 = 15 \times 10^{-2}.$$

Step 4: Conclusion The solution is 0.150% (w/w), and $x = 15 \times 10^{-2}$. Therefore the numerical entry in the blank is $\boxed{15}$ (dimensionless).

Quick Tip to Solve: Keep conductance, conductivity and molar conductivity distinct. After finding the percentage, check the power of ten printed beside the answer blank before entering the coefficient.

75. For a reaction $\text{A} \rightarrow \text{P}$ at T K , the half life $(t_{1/2})$ is plotted as a function of initial concentration $[\text{A}]_0$ of A as given below.

The value of x in the given figure is ____ s (Nearest integer) [4 marks]



Solution:

Concept:

- For a zero-order reaction, $[\mathrm{A}]_t = [\mathrm{A}]_0 - kt$. Setting the remaining concentration to half its initial value gives $t_{1/2} = [\mathrm{A}]_0/(2k)$.
- At a fixed temperature the rate constant is fixed. Thus a straight-line plot of half-life against initial concentration passing through the origin shows direct proportionality, $t_{1/2} \propto [\mathrm{A}]_0$.

Step 1: Read the two concentration values from the graph The vertical axis gives half-life in seconds and the horizontal axis gives initial concentration in $\mathrm{mol\,L^{-1}}$. The upper marked point corresponds to

$$[\mathrm{A}]_0 = 4.0 \times 10^{-3}\,\mathrm{mol\,L^{-1}}, \qquad t_{1/2} = 240\,\mathrm{s}.$$

The lower marked point has initial concentration $1.5 \times 10^{-3}\,\mathrm{mol\,L^{-1}}$, and its half-life is the unknown x. Because the line passes through the origin, no intercept must be subtracted from either reading.

Step 2: Use the common slope to calculate the unknown half-life For the same straight line, the ratio of the vertical coordinates equals the ratio of the horizontal coordinates. Equivalently, the factor $1/(2k)$ in the zero-order half-life expression cancels between the two points:

$$\frac{x}{240\,\mathrm{s}} = \frac{1.5 \times 10^{-3}}{4.0 \times 10^{-3}}.$$

Cancel the equal powers of ten and simplify:

$$x = 240 \times \frac{1.5}{4.0}\,\mathrm{s} = 90\,\mathrm{s}.$$

As a check, the concentration at the lower point is $3/8$ of that at the upper point, so its half-life must also be $3/8$ of $240\,\mathrm{s}$. A first-order reaction would instead give a horizontal half-life plot and cannot describe this graph.

Step 3: Conclusion The indicated half-life is $\boxed{x = 90\,\mathrm{s}}$. Its nearest-integer numerical value is $\boxed{90}$.

Quick Tip to Solve: For a half-life graph that is a straight line through the origin, compare the two concentrations directly; the rate constant cancels and no separate calculation of k is needed.