JEE Main 2026 Paper 1 (B.E./B.Tech) Apr 5 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2026, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 5 April 2026, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $a, b \in \mathbb{C}$. Let α, β be the roots of the equation $x^2 + ax + b = 0$. If $\beta - \alpha = \sqrt{11}$ and $\beta^2 - \alpha^2 = 3i\sqrt{11}$, then $\left(\beta^3 - \alpha^3\right)^2$ is equal to: [4 marks]

(A) 160

(B) 176

(C) 194

(D) 187

Correct Answer: (B) 176

Solution:

Concept:

- The identities $u^2 - v^2 = (u - v)(u + v)$ and $u^3 - v^3 = (u - v)(u^2 + uv + v^2)$ hold for complex numbers as well as real numbers.
- If $s = \alpha + \beta$ and $p = \alpha\beta$, then $(\beta - \alpha)^2 = s^2 - 4p$ and $\alpha^2 + \alpha\beta + \beta^2 = s^2 - p$. These reduce the calculation without finding the roots separately.

Step 1: Determine the sum of the roots The given difference $\beta - \alpha = \sqrt{11}$ is nonzero, so division by it is permitted. Factor the difference of squares and cancel this factor:

$$\alpha + \beta = \frac{\beta^2 - \alpha^2}{\beta - \alpha} = \frac{3i\sqrt{11}}{\sqrt{11}} = 3i.$$

Consequently $s^2 = (3i)^2 = -9$, since $i^2 = -1$.

Step 2: Obtain the product and the difference of cubes Squaring the given difference gives $11 = s^2 - 4p = -9 - 4p$, hence $p = -5$. Therefore

$$\beta^3 - \alpha^3 = (\beta - \alpha)(s^2 - p) = \sqrt{11}\,[-9 - (-5)] = -4\sqrt{11}.$$

This negative real value is consistent with the complex sum of the roots; the question asks for its square, not its absolute value or the cube of their difference.

Step 3: Square the required expression and compare

$$(\beta^3 - \alpha^3)^2 = (-4\sqrt{11})^2 = 16 \cdot 11 = 176.$$

The exact calculation excludes 160, 194 and 187, so options (A), (C) and (D) do not agree with the two given identities.

Step 4: Conclusion The required value is $\boxed{176}$. Hence, the correct option is (B).

Quick Tip to Solve: Recover the sum from the difference of squares, then use $s^2 - p$ in the difference of cubes; explicit complex roots are unnecessary.

2. Let the sum of the first n terms of an A.P. be $3n^2 + 5n$. Then the sum of squares of the first 10 terms of the A.P. is: [4 marks]

(A) 10220

(B) 12860

(C) 15220

(D) 19780

Correct Answer: (C) 15220

Solution:

Concept:

- If S_n is the sum of the first n terms, the individual term is $a_n = S_n - S_{n-1}$, with $S_0 = 0$.
- The standard sums are $\sum_{n=1}^{N} n = N(N+1)/2$ and $\sum_{n=1}^{N} n^2 = N(N+1)(2N+1)/6$. Expand each squared term before applying these formulae.

Step 1: Recover the individual terms Substitute $n-1$ into the given sum formula and subtract:

$$\begin{aligned} a_n &= (3n^2 + 5n) - [3(n-1)^2 + 5(n-1)] \\ &= (3n^2 + 5n) - (3n^2 - n - 2) = 6n + 2. \end{aligned}$$

Thus the progression starts at $a_1 = 8$ and has common difference 6. Its tenth term is 62. The formula also works at $n = 1$, because the given polynomial gives $S_0 = 0$.

Step 2: Expand and sum the squares The requested quantity is the sum of individual squares, so square $6n + 2$, not the sum S_{10}:

$$\sum_{n=1}^{10} a_n^2 = 36 \sum_{n=1}^{10} n^2 + 24 \sum_{n=1}^{10} n + 4 \cdot 10.$$

Here $\sum n = 10 \cdot 11/2 = 55$ and $\sum n^2 = 10 \cdot 11 \cdot 21/6 = 385$. Consequently

$$\sum_{n=1}^{10} a_n^2 = 36(385) + 24(55) + 40 = 13860 + 1320 + 40 = 15220.$$

Step 3: Check the alternative values The positive cross term contributes 1320 and the constant terms contribute 40; both must be included alongside 13860. Only 15220 gives their full sum. The proposed values 10220, 12860 and 19780 therefore fail the direct expansion and are excluded.

Step 4: Conclusion The sum of the squares of the first ten terms is $\boxed{15220}$. Hence, the correct option is (C).

Quick Tip to Solve: Take the first difference of a supplied partial-sum formula before squaring; the square of the total is a different quantity.

3. Let A be a 3×3 matrix such that

$$\mathrm{A}^{\mathrm{T}}\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}5\\2\\2\end{bmatrix},\mathrm{A}^{\mathrm{T}}\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}3\\1\\1\end{bmatrix},\mathrm{A}\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}3\\4\\4\end{bmatrix}\ \textbf{and}\ \mathrm{A}\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}1\\3\\1\end{bmatrix}.$$

If $\det(A)=1$, then $\det\left(\operatorname{adj}\left(A^2+A\right)\right)$ is equal to: [4 marks]

(A) 16

(B) 25

(C) 49

(D) 64

Correct Answer: (D) 64

Solution:

Concept:

- Multiplication by a coordinate column selects a column of a matrix. Applying the same idea to A^{T} selects a row of A, written as a column.
- For a 3×3 matrix M, $\det(\operatorname{adj} M)=(\det M)^2$. Also $A^2+A=A(A+I)$, so its determinant can be evaluated without squaring the matrix.

Step 1: Recover the rows and columns of the matrix Write the coordinate columns as e_1, e_2, e_3. Subtracting the given $A^{\mathrm{T}}e_3$ from $A^{\mathrm{T}}(e_1+e_3)$ gives the first row $(2,1,1)$; the third row is $(3,1,1)$. Similarly, subtracting Ae_3 from $A(e_1+e_3)$ gives the first column $(2,1,3)^{\mathrm{T}}$, while the third column is $(1,3,1)^{\mathrm{T}}$. Only the central entry remains unknown:

$$A=\begin{pmatrix}2&1&1\\1&k&3\\3&1&1\end{pmatrix}.$$

Step 2: Use the determinant condition Expanding along the first row,

$$\det A=2(k-3)-(1-9)+(1-3k)=3-k.$$

Since $\det A=1$, we obtain $k=2$. Therefore

$$\det(A+I)=\begin{vmatrix}3&1&1\\1&3&3\\3&1&2\end{vmatrix}=3(6-3)-(2-9)+(1-9)=8.$$

Step 3: Apply the product and adjugate identities

$$\det(A^2+A)=\det A\det(A+I)=1\cdot 8=8,\qquad \det(\operatorname{adj}(A^2+A))=8^2=64.$$

The exponent is 2 because the matrices have order 3. Values 16, 25 and 49 would require determinant magnitudes 4, 5 and 7 respectively, contrary to the computed value 8.

Step 4: Conclusion The required determinant is $\boxed{64}$. Hence, the correct option is (D).

Quick Tip to Solve: Read matrix-vector products as selected columns or rows, then factor $A^2 + A$ before taking its determinant.

4. Consider the system of linear equations in x, y, z :

$$\begin{aligned} x + 2y + tz &= 0 \\ 6x + y + 5tz &= 0 \\ 3x + t^2y + f(t)z &= 0 \end{aligned}$$

where $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function. If this system has infinitely many solutions for all $t \in \mathbb{R}$, then f [4 marks]

(A) is a constant function

(B) is strictly increasing on $\mathbb{R}$

(C) is strictly decreasing on R

(D) has two critical points

Correct Answer: (B) is strictly increasing on $\mathbb{R}$

Solution:

Concept:

- A homogeneous system of three linear equations in three unknowns has infinitely many solutions precisely when its coefficient matrix is singular, that is, its determinant vanishes.
- A differentiable function with strictly positive derivative everywhere on $\mathbb{R}$ is strictly increasing there, by the mean value theorem.

Step 1: Impose singularity for every parameter value The zero vector always solves a homogeneous system, so consistency alone adds no condition. Infinitely many solutions require a nonzero nullspace. Thus, for each real t,

$$\begin{vmatrix} 1 & 2 & t \\ 6 & 1 & 5t \\ 3 & t^2 & f(t) \end{vmatrix} = 0.$$

Expanding along the first row keeps the unknown function separate from the polynomial terms:

$$[f(t) - 5t^3] - 2[6f(t) - 15t] + t[6t^2 - 3] = 0.$$

Step 2: Determine and differentiate the function Collecting the terms gives $-11f(t) + t^3 + 27t = 0$. Since this holds for every real parameter and the coefficient of $f(t)$ is nonzero,

$$f(t) = \frac{t^3 + 27t}{11}, \qquad f'(t) = \frac{3t^2 + 27}{11} > 0 \quad (t \in \mathbb{R}).$$

The numerator is at least 27, so the derivative cannot vanish. This formula also satisfies the determi-

nant condition for every t, including $t = 0$; no division by t has discarded a parameter value.

Step 3: Classify the function and exclude the alternatives For any $u < v$, the mean value theorem gives $f(v) - f(u) = f'(c)(v - u) > 0$ for some c between them. Hence the increase is strict on all of $\mathbb{R}$. A positive derivative excludes both a constant function and a strictly decreasing one. Since the derivative exists everywhere and is never zero, the function has no critical points, rather than two.

Step 4: Conclusion $f(t) = (t^3 + 27t)/11$ is strictly increasing on $\mathbb{R}$. Hence, the correct option is (B).

Quick Tip to Solve: For infinitely many solutions of a square homogeneous system, set the coefficient determinant to zero before analysing the parameter function.

---

5. $\sum_{n=1}^{10} \left(\frac{528}{n(n+1)(n+2)}\right)$ **is equal to:** [4 marks]

(A) 65

(B) 130

(C) 220

(D) 440

Correct Answer: (B) 130

Solution:

Concept:

- Consecutive factors in a denominator often produce a telescoping difference. Here $\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)} = \frac{2}{n(n+1)(n+2)}$.
- In a finite telescoping sum, each interior term cancels with the next difference; the first positive term and the final negative term remain. The upper endpoint must be retained.

Step 1: Decompose the summand The difference of the two simpler fractions has numerator $(n + 2) - n = 2$ when put over the common denominator. Consequently

$$\frac{528}{n(n+1)(n+2)} = 264\left[\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)}\right].$$

The coefficient is 264, not 528, because the bracket supplies an additional factor of 2. All denominators are nonzero over the stated range $1 \le n \le 10$.

Step 2: Cancel the interior terms Writing the first two and the last differences displays the cancellation:

$$S = 264\left[\left(\frac{1}{1\cdot 2} - \frac{1}{2\cdot 3}\right) + \left(\frac{1}{2\cdot 3} - \frac{1}{3\cdot 4}\right) + \cdots + \left(\frac{1}{10\cdot 11} - \frac{1}{11\cdot 12}\right)\right].$$

Therefore

$$S = 264\left(\frac{1}{2} - \frac{1}{132}\right) = 132 - 2 = 130.$$

Step 3: Check the size of the result The first term alone is $528/6 = 88$, so a positive sum cannot equal

65. The infinite telescoping sum would be 132, so the finite sum is smaller than 132; this excludes 220 and 440. The value 130 has both the correct endpoints and the expected size.

Step 4: Conclusion The finite sum equals $\boxed{130}$. Hence, the correct option is (B).

Quick Tip to Solve: Split three consecutive factors into the difference of two adjacent reciprocal products, and keep the last negative term when the sum is finite.

6. Let $\tan A, \tan B$, where $A, B \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, be the roots of the quadratic equation $x^2 - 2x - 5 = 0$. Then $20\sin^2\left(\frac{A+B}{2}\right)$ is equal to: [4 marks]

(A) $10 + \sqrt{10}$

(B) $10 - 2\sqrt{10}$

(C) 10-3 $\sqrt{10}$

(D) $10 - \sqrt{10}$

Correct Answer: (C) 10-3 $\sqrt{10}$

Solution:

Concept:

- Vieta's formulae for $x^2 - 2x - 5 = 0$ give $\tan A + \tan B = 2$ and $\tan A \tan B = -5$.
- The identity $2\sin^2(\theta/2) = 1 - \cos\theta$ reduces the requested expression to a cosine. The given intervals imply $\cos A > 0$ and $\cos B > 0$, fixing the sign of the necessary square roots.

Step 1: Evaluate the product of the cosines Set $u = \tan A$ and $v = \tan B$. From $u + v = 2$ and $uv = -5$,

$$u^2 + v^2 = (u+v)^2 - 2uv = 4 + 10 = 14.$$

Since both angles lie strictly between $-\pi/2$ and $\pi/2$, the positive cosine values give

$$\cos A \cos B = \frac{1}{\sqrt{(1+u^2)(1+v^2)}} = \frac{1}{\sqrt{1+14+25}} = \frac{1}{\sqrt{40}}.$$

Step 2: Find the cosine of the sum Use the cosine addition identity in tangent form:

$$\cos(A+B) = \cos A \cos B(1 - \tan A \tan B) = \frac{1-(-5)}{\sqrt{40}} = \frac{3}{\sqrt{10}}.$$

This value is positive. Merely finding $\tan(A+B) = 1/3$ without using the angle intervals would leave an avoidable sign ambiguity for this cosine.

Step 3: Apply the half-angle identity

$$20\sin^2\left(\frac{A+B}{2}\right) = 10[1 - \cos(A+B)] = 10 - \frac{30}{\sqrt{10}} = 10 - 3\sqrt{10}.$$

Option (A) has the opposite sign for the radical correction, while (B) and (D) use incorrect coefficients 2 and 1. The derived coefficient is exactly 3; the result is positive because $3\sqrt{10} < 10$.

Step 4: Conclusion The requested value is $\boxed{10-3\sqrt{10}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the allowed angle intervals to fix cosine signs before applying a half-angle identity; tangent alone does not determine the quadrant.

7. A letter is known to have arrived by post either from KANPUR or from ANANTPUR. On the envelope just two consecutive letters AN are visible. The probability, that the letter came from ANANTPUR, is: [4 marks]

(A) $\frac{7}{10}$
(B) $\frac{10}{17}$
(C) $\frac{12}{19}$
(D) $\frac{7}{19}$

Correct Answer: (B) $\frac{10}{17}$

Solution:

Concept:

- Bayes' theorem updates the probability of an origin after the visible pair of letters is observed. If A denotes ANANTPUR and E denotes seeing AN, then $P(A \mid E) = P(A)P(E \mid A)/P(E)$.
- The two possible origins are taken as equally likely, and the visible consecutive pair is equally likely to occupy any adjacent position within the name. A name with n letters has $n-1$ such positions.

Step 1: Count the adjacent pairs in each name KANPUR contains six letters, so its five consecutive pairs are KA, AN, NP, PU and UR. Exactly one is AN. ANANTPUR contains eight letters, giving the seven pairs AN, NA, AN, NT, TP, PU and UR. Two of these are AN; the repeated appearance must be counted separately. Hence

$$P(E \mid K) = \frac{1}{5}, \qquad P(E \mid A) = \frac{2}{7}.$$

Step 2: Apply Bayes theorem with the equal priors The observed AN is more likely under ANANTPUR, but the different numbers of adjacent positions must also be included. With $P(K) = P(A) = 1/2$,

$$\begin{aligned} P(A \mid E) &= \frac{(1/2)(2/7)}{(1/2)(2/7) + (1/2)(1/5)} \\ &= \frac{2/7}{2/7 + 1/5} = \frac{10/35}{17/35} = \frac{10}{17}. \end{aligned}$$

The complementary posterior is $P(K \mid E) = 7/17$, so the two probabilities sum to one. Of the given fractions, only $10/17$ has the required likelihood ratio $10 : 7$; $7/10$, $12/19$ and $7/19$ do not.

Step 3: Conclusion The probability that the letter came from ANANTPUR is $\boxed{10/17}$. Hence, the correct option is (B).

Quick Tip to Solve: For a visible adjacent pair, count its occurrences and divide by the number of adjacent positions in each word before applying Bayes' theorem.

8. The mean deviation about the mean for the data

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

is equal to:

[4 marks]

(A) $\frac{40}{13}$
(B) $\frac{42}{13}$
(C) $\frac{44}{13}$
(D) $\frac{46}{13}$

Correct Answer: (C) $\frac{44}{13}$

Solution:

Concept:

- The arithmetic mean of frequency data is $\bar{x} = \sum f_i x_i / \sum f_i$. Each listed value contributes as many observations as its frequency.
- The mean deviation about the mean is $\text{MD} = \sum f_i |x_i - \bar{x}| / \sum f_i$. Absolute differences, rather than signed differences or squared differences, are required.

Step 1: Calculate the total frequency and weighted mean There are six different values, but the number of observations is their total frequency:

$$N = 8 + 6 + 2 + 2 + 2 + 6 = 26.$$

The sum of all observations is

$$\begin{aligned} \sum f_i x_i &= 8(5) + 6(7) + 2(9) + 2(10) + 2(12) + 6(15) \\ &= 40 + 42 + 18 + 20 + 24 + 90 = 234. \end{aligned}$$

Therefore $\bar{x} = 234/26 = 9$. Dividing by six would give the mean of the six displayed values without their prescribed frequencies, which is not the required mean.

Step 2: Find and average the absolute deviations The absolute deviations from 9, in the given order, are $4, 2, 0, 1, 3, 6$. Multiplying by the corresponding frequencies gives $32, 12, 0, 2, 6, 36$. Thus

$$\begin{aligned} \sum f_i |x_i - 9| &= 32 + 12 + 0 + 2 + 6 + 36 = 88, \\ \text{MD} &= \frac{88}{26} = \frac{44}{13}. \end{aligned}$$

The middle observation value 9 contributes zero deviation even though its frequency is nonzero. The other options $40/13$, $42/13$ and $46/13$ would require total absolute deviations of 80, 84 and 92 respectively; none equals the directly computed total 88.

Step 3: Conclusion The mean deviation about the mean is $\boxed{44/13}$. Hence, the correct option is (C).

Quick Tip to Solve: After finding the weighted mean, make a short row of absolute deviations and a second row of frequency-weighted deviations; divide their sum by the total frequency.

9. Let a focus of the ellipse $\mathrm{E}: \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be $\mathrm{S}(4,0)$ and its eccentricity be $\frac{4}{5}$. If the point $\mathrm{P}(3,\alpha)$ lies on E and O is the origin, then the area of $\triangle\mathrm{POS}$ is equal to: [4 marks]

(A) 12/5

(B) 14/5

(C) 24/5

(D) 48/5

Correct Answer: (C) 24/5

Solution:

Concept:

- For an ellipse with its major axis along the x-axis, the foci are $(\pm c, 0)$, where $c = ae$, e is the eccentricity, and $b^2 = a^2 - c^2$.
- The area of a triangle is half its base times its perpendicular height. With O and S on the x-axis, the height from $P(3,\alpha)$ is $|\alpha|$, regardless of the sign of α.

Step 1: Determine the ellipse parameters The focus $S(4,0)$ gives $c = 4$. Since $e = 4/5$,

$$a = \frac{c}{e} = \frac{4}{4/5} = 5, \qquad b^2 = a^2 - c^2 = 25 - 16 = 9.$$

The ellipse is therefore

$$\frac{x^2}{25} + \frac{y^2}{9} = 1.$$

The major semiaxis is 5 and the minor semiaxis is 3; using the focal distance 4 as a semiaxis would give an incorrect equation.

Step 2: Find the height and calculate the area Substitute the coordinates of P into the ellipse:

$$\frac{9}{25} + \frac{\alpha^2}{9} = 1 \quad \Longrightarrow \quad \alpha^2 = 9\left(\frac{16}{25}\right) = \frac{144}{25}.$$

Hence $|\alpha| = 12/5$. The base OS has length 4, so

$$\text{Area}(\triangle POS) = \frac{1}{2} \times 4 \times \frac{12}{5} = \frac{24}{5}.$$

Both possible points $(3, 12/5)$ and $(3, -12/5)$ give the same positive area. Option (A) is merely the height, option (B) does not follow from the ellipse equation, and option (D) omits the factor of one-half in the triangle-area formula.

Step 3: Conclusion The area of $\triangle POS$ is $\boxed{24/5}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: When two vertices lie on a coordinate axis, use their separation as the base and the absolute other coordinate of the third vertex as the height.

10. Let P be a moving point on the circle $x^2 + y^2 - 6x - 8y + 21 = 0$. Then, the maximum distance of P from the vertex of the parabola $x^2 + 6x + y + 13 = 0$ is equal to: [4 marks]

(A) 8

(B) 10

(C) 12

(D) 9

Correct Answer: (C) 12

Solution:

Concept:

- Completing squares identifies a circle's centre and radius and a parabola's vertex. These geometric quantities are sufficient; a parametrisation of the moving point is unnecessary.
- For a fixed point V, centre C and a point P on a circle of radius r, the triangle inequality gives $VP \le VC + CP = VC + r$. Equality occurs on the line through V and C, beyond C.

Step 1: Find the circle and parabola parameters Complete squares in the circle equation:

$$x^2 - 6x + y^2 - 8y + 21 = 0,$$
$$(x-3)^2 + (y-4)^2 = 4.$$

Thus its centre is $C(3, 4)$ and its radius is $r = 2$. For the parabola,

$$x^2 + 6x + y + 13 = 0 \quad \Longrightarrow \quad y = -(x+3)^2 - 4.$$

Its vertex is therefore $V(-3, -4)$; both signs matter when locating it.

Step 2: Use the centre-to-vertex distance The distance between the fixed vertex and the circle centre is

$$VC = \sqrt{(3+3)^2 + (4+4)^2} = \sqrt{36 + 64} = 10.$$

Consequently every point on the circle satisfies $VP \le 10 + 2 = 12$. This bound is attained by moving from C a distance 2 in the direction away from V, so it is the actual maximum, not merely an upper estimate. The minimum distance is $10 - 2 = 8$, which explains option (A). Option (B) is the distance only to the centre, while option (D) is an intermediate distance and cannot be the maximum.

Step 3: Conclusion The maximum distance of P from the parabola's vertex is $\boxed{12}$ units. Hence, the correct option is (C).

Quick Tip to Solve: For distance extrema from a fixed point to a circle, first find the centre distance d. The maximum is $d + r$, while the minimum is $|d - r|$.

11. In an equilateral triangle PQR, let the vertex P be at $(3,5)$ and the side QR be along the line $x+y=4$. If the orthocentre of the triangle PQR is (α,β), then $9(\alpha+\beta)$ is equal to: [4 marks]

(A) 16

(B) 27

(C) 36

(D) 48

Correct Answer: (D) 48

Solution:

Concept:

- In an equilateral triangle, the altitude from a vertex is also a median. The orthocentre, centroid, incentre and circumcentre coincide.
- The centroid divides a median in the ratio $2:1$ measured from the vertex. If M is the midpoint of the opposite side, the common centre is $H=(P+2M)/3$.

Step 1: Locate the foot of the altitude The side QR lies on $x+y=4$, a line with slope -1. The altitude from $P(3,5)$ is perpendicular to this line, so its slope is 1 and its equation is

$$y-5=x-3 \quad \Longrightarrow \quad y=x+2.$$

Intersecting the two lines gives $x+(x+2)=4$, hence $x=1$ and $y=3$. Thus the perpendicular foot is $M(1,3)$. Because the triangle is equilateral, this foot is also the midpoint of QR.

Step 2: Use the centroid ratio to find the orthocentre The common centre lies two-thirds of the way from the vertex to the side midpoint:

$$\begin{aligned} H &= P+\frac{2}{3}(M-P) \\ &= (3,5)+\frac{2}{3}(-2,-2) \\ &= \left(\frac{5}{3},\frac{11}{3}\right). \end{aligned}$$

Therefore $\alpha+\beta=16/3$, and

$$9(\alpha+\beta)=9\left(\frac{16}{3}\right)=48.$$

This is consistent with the centre being closer to the side than to the vertex. The other values 16, 27 and 36 do not equal the required scaled sum; in particular 36 would incorrectly use the side's equation $x+y=4$ for an interior centre.

Step 3: Conclusion The required value is $\boxed{48}$. Hence, the correct option is (D).

Quick Tip to Solve: For an equilateral triangle, find the perpendicular foot from the known vertex to the opposite side and then use the centroid's two-to-one ratio.

12. The sum of all the integral values of p such that the equation $3\sin^2 x + 12\cos x - 3 = p, x \in \mathbb{R}$, has at least one solution, is: [4 marks]

(A) -54

(B) -60

(C) -75

(D) -84

Correct Answer: (C) -75

Solution:

Concept:

- Replace $\sin^2 x$ by $1 - \cos^2 x$ to express the left-hand side using a single variable $t = \cos x$. Its allowed domain is the closed interval $[-1, 1]$.
- A continuous function on an interval takes every value between its minimum and maximum. Thus, after finding the range, all integers in that range are admissible values of p.

Step 1: Reduce the trigonometric expression to a quadratic Using the identity $\sin^2 x = 1 - \cos^2 x$,

$$\begin{aligned} 3\sin^2 x + 12\cos x - 3 &= 3(1 - \cos^2 x) + 12\cos x - 3 \\ &= -3\cos^2 x + 12\cos x. \end{aligned}$$

Let $t = \cos x$. We need the range of $f(t) = -3t^2 + 12t$ for $-1 \le t \le 1$, rather than the range of this quadratic over all real t.

Step 2: Find the attainable endpoints The derivative is $f'(t) = 12 - 6t$, which is at least 6 throughout $[-1, 1]$. Hence the function is strictly increasing there. Its endpoint values are

$$f(-1) = -3 - 12 = -15, \qquad f(1) = -3 + 12 = 9.$$

Both occur for real x, and continuity ensures every value between them occurs. Therefore the equation has a real solution exactly when $-15 \le p \le 9$. The unrestricted quadratic vertex at $t = 2$ is outside the cosine domain and cannot supply the maximum.

Step 3: Sum all the admissible integers The integers are $-15, -14, \ldots, 8, 9$, including both endpoints. There are $9 - (-15) + 1 = 25$ terms, giving

$$S = \frac{25}{2}(-15 + 9) = -75.$$

Equivalently, the integers from -9 to 9 cancel in pairs, leaving $-15 - 14 - 13 - 12 - 11 - 10 = -75$. Thus the other proposed sums $-54, -60, -84$ are excluded.

Step 4: Conclusion The sum of all integral values of p is $\boxed{-75}$. Hence, the correct option is (C).

Quick Tip to Solve: Always retain $-1 \le \cos x \le 1$ when replacing a trigonometric expression by a polynomial; an unrestricted quadratic vertex may lie outside the allowed domain.

13. The square of the distance of the point $\mathrm{P}(5,6,7)$ from the line $\frac{x-2}{2}=\frac{y-5}{3}=\frac{z-2}{4}$ is equal to:

[4 marks]

(A) 3

(B) 5

(C) 6

(D) 8

Correct Answer: (C) 6

Solution:

Concept:

- A line in symmetric form supplies a point A on it and a direction vector $\vec{d}$. The displacement $\overrightarrow{AP}$ splits into components parallel and perpendicular to the line.
- The squared perpendicular distance is $h^2 = |\overrightarrow{AP}|^2 - (\overrightarrow{AP}\cdot\vec{d})^2/|\vec{d}|^2$. This subtracts the squared length of the parallel projection, so no square root is needed.

Step 1: Extract the point and direction From the given symmetric equation take $A=(2,5,2)$ and $\vec{d}=(2,3,4)$. For $P=(5,6,7)$, the displacement from the line point is

$$\overrightarrow{AP}=(5-2,6-5,7-2)=(3,1,5).$$

The direction ratios need not form a unit vector: division by $|\vec{d}|^2$ accounts for their length.

Step 2: Compute the projection and perpendicular part The required scalar quantities are

$$|\overrightarrow{AP}|^2=3^2+1^2+5^2=35, \qquad |\vec{d}|^2=2^2+3^2+4^2=29,$$

$$\overrightarrow{AP}\cdot\vec{d}=3(2)+1(3)+5(4)=29.$$

Consequently,

$$h^2=35-\frac{29^2}{29}=35-29=6.$$

As a direct check, the foot occurs at line parameter $t=29/29=1$, hence is $H=(4,8,6)$. The vector $\overrightarrow{HP}=(1,-2,1)$ is perpendicular to $(2,3,4)$, since its dot product is $2-6+4=0$, and its squared length is $1+4+1=6$. Thus options (A), (B) and (D), giving 3, 5 and 8, do not equal the perpendicular squared distance.

Step 3: Conclusion The square of the perpendicular distance is $\boxed{6}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: When the question asks for squared distance, subtract the squared projection directly; calculating the distance and squaring it again adds unnecessary work.

14. Let $\vec{a} = \sqrt{7}\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + 2\hat{k}$. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} + \vec{a} \times \vec{b} = \overrightarrow{0}$ and $\vec{r} \cdot \vec{a} = 0$, then $|3\vec{r}|^2$ is equal to: [4 marks]

(A) 44

(B) 54

(C) 86

(D) 132

Correct Answer: (A) 44

Solution:

Concept:

- The cross product is antisymmetric: $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$. If a vector has zero cross product with a nonzero vector $\vec{a}$, it is parallel to $\vec{a}$.
- The condition $\vec{r} \cdot \vec{a} = 0$ makes $\vec{r}$ the component of $\vec{b}$ perpendicular to $\vec{a}$. Its squared magnitude is $|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2/|\vec{a}|^2$.

Step 1: Rewrite the cross-product equation Using antisymmetry, the stated equation becomes

$$(\vec{r} - \vec{b}) \times \vec{a} = \vec{0}.$$

Since $\vec{a} \neq \vec{0}$, write $\vec{r} = \vec{b} + t\vec{a}$ for a real scalar t. This form satisfies the cross-product equation and leaves the perpendicularity condition to determine t.

Step 2: Apply the dot-product condition For $\vec{a} = (\sqrt{7}, 1, -1)$ and $\vec{b} = (0, 1, 2)$,

$$|\vec{a}|^2 = 7 + 1 + 1 = 9, \quad |\vec{b}|^2 = 1 + 4 = 5, \quad \vec{a} \cdot \vec{b} = 1 - 2 = -1.$$

Taking the dot product of $\vec{r} = \vec{b} + t\vec{a}$ with $\vec{a}$ gives

$$0 = -1 + 9t, \qquad t = \frac{1}{9}.$$

In particular, the positive sign of t follows from the negative dot product; reversing the cross-product order without its sign would give an incorrect relation.

Step 3: Evaluate the required magnitude The perpendicular component therefore has squared magnitude

$$|\vec{r}|^2 = 5 - \frac{(-1)^2}{9} = \frac{44}{9}.$$

Scaling a vector by 3 scales its squared magnitude by $3^2 = 9$. Thus

$$|3\vec{r}|^2 = 9 \cdot \frac{44}{9} = 44.$$

The alternatives 54, 86 and 132 do not satisfy this projection calculation; in particular 132 would introduce an extra factor of 3 after the required square scaling.

Step 4: Conclusion $\boxed{|3\vec{r}|^2 = 44}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert a zero cross product into a parallel-vector relation, then use the dot-product condition to remove the parallel component.

15. The square of the distance of the point of intersection of the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(a\hat{i} - \hat{j}),\ a \neq 0$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + a\hat{k})$ from the origin is: [4 marks]

(A) 5

(B) 10

(C) 17

(D) 26

Correct Answer: (C) 17

Solution:

Concept:

- At the intersection of two parametrised lines, their three Cartesian coordinates are equal. The parameters λ and μ belong to different lines and must be solved independently.
- The square of the distance of (x, y, z) from the origin is $x^2 + y^2 + z^2$. The condition $a \neq 0$ is essential when the coordinate equations are simplified.

Step 1: Write the coordinate equations The first line has a general point

$$(x, y, z) = (1 + a\lambda,\ 1 - \lambda,\ -1),$$

and the second line has a general point

$$(x, y, z) = (4 + 2\mu,\ 0,\ -1 + a\mu).$$

Equating corresponding coordinates gives

$$1 + a\lambda = 4 + 2\mu, \quad 1 - \lambda = 0, \quad -1 = -1 + a\mu.$$

These equations must hold simultaneously; equality of only two coordinates would not establish an intersection in space.

Step 2: Determine the intersection point The second equation gives $\lambda = 1$. The third gives $a\mu = 0$, and the stated condition $a \neq 0$ forces $\mu = 0$. Substituting into the first equation yields

$$1 + a = 4, \qquad a = 3.$$

The intersection is therefore the point corresponding to $\mu = 0$ on the second line:

$$P = (4, 0, -1).$$

Checking the first line with $a = 3$ and $\lambda = 1$ gives the same point, confirming that no coordinate condition has been lost.

Step 3: Compute the squared distance The displacement from the origin to P is $(4, 0, -1)$, so

$$OP^2 = 4^2 + 0^2 + (-1)^2 = 16 + 1 = 17.$$

The distance itself is $\sqrt{17}$, but the question asks for its square. The other choices 5, 10 and 26 are not

the sum of the squared Cartesian coordinates, and 5 would incorrectly use the sum of their absolute values.

Step 4: Conclusion The required square of the distance is $\boxed{17}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Use coordinates with a missing parameter first: here the zero y-coordinate fixes λ, and the common z-coordinate then fixes μ.

---

16. The area of the region $\mathrm{R} = \{(x, y) : xy \le 27, 1 \le y \le x^2\}$ is equal to : [4 marks]

(A) $78 \log_e 3 - \frac{52}{3}$

(B) $54 \log_e 3 - \frac{52}{3}$

(C) $54 \log_e 3 - \frac{26}{3}$

(D) $54 \log_e 3 + \frac{26}{3}$

Correct Answer: The printed region is unbounded with infinite area; option (B) describes only its positive-x component.

Solution:

Concept:

- Area calculations must include every point allowed by the defining inequalities. The bound $1 \le y \le x^2$ allows both $x \ge 1$ and $x \le -1$; it does not by itself restrict the region to the first quadrant.
- For $x < 0$ and $y \ge 1$, the product xy is negative, so $xy \le 27$ holds automatically. A region containing rectangles of arbitrarily large area has infinite area.

Step 1: Check the negative-x component For every $x \le -2$, all points with $1 \le y \le 2$ satisfy $y \le x^2$ and $xy < 0 < 27$. Hence, for every $M > 2$, the rectangle

$$-M \le x \le -2, \qquad 1 \le y \le 2$$

is contained in R. Its area is $M - 2$, which increases without bound as $M \to \infty$. Therefore the full region written in the question has infinite area. None of the four finite expressions can be its area.

Step 2: Identify the finite component behind the listed options If one separately considers only the positive-x component, its bounds are $1 \le x \le 27$ and $1 \le y \le \min(x^2, 27/x)$. The two upper curves meet where $x^3 = 27$, namely $x = 3$. Thus the area of this component alone is

$$\begin{aligned} A_+ &= \int_1^3 (x^2 - 1)\, dx + \int_3^{27} (27/x - 1)\, dx \\ &= \left[\frac{x^3}{3} - x\right]_1^3 + [27 \ln x - x]_3^{27} \\ &= \frac{20}{3} + 54 \ln 3 - 24 = 54 \ln 3 - \frac{52}{3}. \end{aligned}$$

This is option (B), but it excludes the entire negative-x component. Options (A), (C) and (D) do not

even equal the area of the finite component. The printed definition includes no condition allowing the negative component to be discarded.

Step 3: Conclusion As printed, R has $\boxed{\text{infinite area}}$, so no listed option is correct. The answer key's option (B), $54\ln 3 - 52/3$, is the area only after imposing the additional restriction $x > 0$, as the component calculation proves.

Quick Tip to Solve: Before integrating a region bounded using x^2, test both signs of x. A product inequality with a positive right side may leave a whole negative-x branch unbounded.

17. The product of all possible values of α, for which $\lim_{x\to 0}\left(\frac{1-\cos(\alpha x)\cos((\alpha+1)x)\cos((\alpha+2)x)}{\sin^2((\alpha+1)x)}\right) = 2$, is : [4 marks]

(A) -2

(B) 1

(C) -1

(D) $\frac{5}{4}$

Correct Answer: (C) -1

Solution:

Concept:

- Near zero, $\cos(kx) = 1 - k^2x^2/2 + O(x^4)$ and $\sin(kx) = kx + O(x^3)$ for a fixed real constant k. In a product of cosines, the quadratic contributions add.
- The denominator must be nonzero for nearby nonzero x. The parameter $\alpha = -1$ makes it identically zero and must be excluded before dividing by $(\alpha+1)^2$.

Step 1: Extract the quadratic terms For fixed α, multiplying the three cosine expansions gives

$$\cos(\alpha x)\cos((\alpha+1)x)\cos((\alpha+2)x) = 1 - \frac{\alpha^2 + (\alpha+1)^2 + (\alpha+2)^2}{2}x^2 + O(x^4).$$

Products of two quadratic correction terms have order x^4 and do not affect the limit. Also,

$$\sin^2((\alpha+1)x) = (\alpha+1)^2x^2 + O(x^4).$$

For $\alpha \neq -1$, cancellation of x^2 therefore gives the limit as

$$\frac{\alpha^2 + (\alpha+1)^2 + (\alpha+2)^2}{2(\alpha+1)^2}.$$

Step 2: Solve for the allowed parameters Put $t = \alpha+1$, so the consecutive factors become $t-1, t, t+1$. The condition on the limit becomes

$$\frac{(t-1)^2 + t^2 + (t+1)^2}{2t^2} = 2, \qquad \frac{3t^2+2}{2t^2} = 2.$$

Since $t \neq 0$, this is equivalent to $t^2 = 2$. Therefore

$$\alpha = -1 + \sqrt{2} \quad \text{or} \quad \alpha = -1 - \sqrt{2}.$$

Both values are admissible, since neither makes $\alpha + 1 = 0$. Their product is

$$(-1 + \sqrt{2})(-1 - \sqrt{2}) = 1 - 2 = -1.$$

The alternatives -2, 1 and $5/4$ are not the product of these roots. In particular, -2 is their sum, not their product.

Step 3: Conclusion The product of all admissible values of α is $\boxed{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: For three consecutive angle multipliers, centre them using $t = \alpha + 1$; the sum of squares then becomes $3t^2 + 2$.

---

18. The value of the integral $\displaystyle\int_0^{\infty} \frac{\log_e(x)}{x^2+4}\,dx$ is: [4 marks]

(A) $\dfrac{\pi \log_e(2)}{2}$

(B) $\dfrac{\pi \log_e(2)}{4}$

(C) $1 + \pi \log_e(2)$

(D) $2 + \pi \log_e(2)$

Correct Answer: (B) $\dfrac{\pi \log_e(2)}{4}$

Solution:

Concept:

- Rescaling the variable separates a logarithmic integral into a constant multiple of a standard arctangent integral and a reciprocal-symmetric integral.
- For $J = \int_0^{\infty} \ln t/(1+t^2)\,dt$, the substitution $t = 1/u$ changes J into $-J$, so $J = 0$, provided the endpoint integrals converge.

Step 1: Check convergence and rescale the variable Near zero the integrand behaves like $\ln x/4$, whose improper integral is finite. At infinity it behaves like $\ln x/x^2$, which is also integrable. Thus the following substitutions and splitting are justified. Put $x = 2t$, so $dx = 2\,dt$ and $x^2 + 4 = 4(1 + t^2)$. The endpoints remain zero and infinity. Therefore

$$I = \frac{1}{2}\int_0^{\infty} \frac{\ln 2 + \ln t}{1+t^2}\,dt = \frac{\ln 2}{2}\int_0^{\infty} \frac{dt}{1+t^2} + \frac{1}{2}J.$$

Step 2: Cancel the reciprocal logarithmic integral In J, set $t = 1/u$. Then $dt = -du/u^2$, $\ln t = -\ln u$,

and $1+t^2=(1+u^2)/u^2$. Reversing the transformed endpoints gives

$$J=-\int_0^\infty \frac{\ln u}{1+u^2}\,du=-J.$$

Since J is finite, $2J=0$, hence $J=0$. This cancellation does not say the original integral vanishes: rescaling contributed the nonzero constant $\ln 2$.

Step 3: Evaluate the remaining standard integral The antiderivative of $1/(1+t^2)$ is $\tan^{-1}t$, so

$$\int_0^\infty \frac{dt}{1+t^2}=\frac{\pi}{2},\qquad I=\frac{\ln 2}{2}\cdot\frac{\pi}{2}=\frac{\pi\ln 2}{4}.$$

Option (A) is twice the result and misses the scale factor from the denominator. Options (C) and (D) contain additive constants that arise in neither the substitution nor the endpoint evaluation.

Step 4: Conclusion The integral equals $\boxed{\pi\ln 2/4}$. Hence, the correct option is (B). The supplied key selects the third choice, but that expression differs from the exact value of this printed logarithmic integral.

Quick Tip to Solve: For $\int_0^\infty \ln x/(x^2+a^2)\,dx$, first set $x=at$; reciprocal substitution cancels the remaining $\ln t$ term.

19. Let $f:\mathbb{R}\to\mathbb{R}$ be a differentiable function such that $f\left(\frac{x+y}{3}\right)=\frac{f(x)+f(y)}{3}$ for all $x,y\in\mathbb{R}$, and $f'(0)=3$. Then the minimum value of the function $g(x)=3+e^xf(x)$, is: [4 marks]

(A) $3\left(\frac{e+1}{e}\right)$
(B) $3\left(\frac{e-1}{e}\right)$
(C) $\frac{3-e}{e}$
(D) 3e

Correct Answer: (B) $3\left(\frac{e-1}{e}\right)$

Solution:

Concept:

- A differentiable functional equation can be differentiated with respect to one independent variable while the other is held fixed. The chain rule supplies the factor from the inner argument.
- A stationary point of a differentiable real function is a global minimum if the derivative changes from negative to positive there and has no other sign change.

Step 1: Find the value and slope of the function Set $x=y=0$ in the given identity. It gives $f(0)=2f(0)/3$, so $f(0)=0$. Next differentiate the original identity with respect to y, holding x fixed. By the chain rule,

$$\frac{1}{3}f'\left(\frac{x+y}{3}\right)=\frac{1}{3}f'(y).$$

Setting $y=0$ now gives $f'(x/3)=f'(0)=3$ for every real x. Because $x/3$ ranges over all real numbers, the derivative is constantly 3.

Step 2: Obtain the function and differentiate the objective Integrating the constant derivative and using $f(0) = 0$, we obtain $f(x) = 3x$. This also satisfies the original equation directly, so no extra solution has been introduced. Therefore

$$g(x) = 3 + 3xe^x, \qquad g'(x) = 3e^x(x + 1).$$

Since e^x is positive for every real x, the derivative is negative for $x < -1$, zero at $x = -1$, and positive for $x > -1$. Thus the function decreases up to -1 and increases thereafter, establishing its global minimum.

Step 3: Evaluate and compare the options Substitute the unique minimising point:

$$g(-1) = 3 - \frac{3}{e} = 3\left(\frac{e-1}{e}\right).$$

Option (A) has the wrong sign for the contribution $3xe^x$ at $x = -1$. Option (C) is not the value of $3 - 3/e$, while option (D) is much larger than $g(0) = 3$, so it cannot be the minimum.

Step 4: Conclusion The minimum value is $\boxed{3(e-1)/e}$, attained at $x = -1$. Hence, the correct option is (B).

Quick Tip to Solve: Differentiate the functional identity in one variable and then set that variable to zero; this uses the given slope without guessing the function.

---

20. The value of the integral $\displaystyle\int_{\pi/6}^{\pi/3} \frac{4 - \operatorname{cosec}^2 x}{\cos^4 x}\,dx$ is: [4 marks]

(A) $\dfrac{11}{\sqrt{3}}$

(B) $\dfrac{16}{\sqrt{3}}$

(C) $\dfrac{32}{3\sqrt{3}}$

(D) $\dfrac{64}{3\sqrt{3}}$

Correct Answer: (C) $\dfrac{32}{3\sqrt{3}}$

Solution:

Concept:

- The substitution $t = \tan x$ gives $dt = \sec^2 x\,dx$, $\sec^2 x = 1 + t^2$, and $\operatorname{cosec}^2 x = (1 + t^2)/t^2$.
- Transform both limits; tangent is continuous and one-one on the given first-quadrant interval.

Step 1: Transform the bounds and integrand Write the integral as $I = \int_{\pi/6}^{\pi/3} (4 - \operatorname{cosec}^2 x) \sec^4 x\,dx$. With $t = \tan x$, the lower and upper bounds become $1/\sqrt{3}$ and $\sqrt{3}$. One factor $\sec^2 x\,dx$ becomes dt,

while the other becomes $1+t^2$. Consequently

$$I = \int_{1/\sqrt{3}}^{\sqrt{3}} \left(4 - \frac{1+t^2}{t^2}\right)(1+t^2)\,dt.$$

Step 2: Expand and integrate each term The product simplifies to $(3-t^{-2})(1+t^2) = 2+3t^2-t^{-2}$. Integrating gives:

$$I = \left[2t + t^3 + \frac{1}{t}\right]_{1/\sqrt{3}}^{\sqrt{3}}.$$

The antiderivative of $-t^{-2}$ is $+t^{-1}$; differentiation checks the bracket as $2+3t^2-t^{-2}$.

Step 3: Evaluate the endpoints exactly At the upper limit, the bracket equals $2\sqrt{3}+3\sqrt{3}+1/\sqrt{3} = 16/\sqrt{3}$. At the lower limit, it equals $2/\sqrt{3}+1/(3\sqrt{3})+\sqrt{3} = 16/(3\sqrt{3})$. Hence

$$I = \frac{16}{\sqrt{3}} - \frac{16}{3\sqrt{3}} = \frac{32}{3\sqrt{3}}.$$

Option (B) is only the upper endpoint value, not the definite integral. The other two values do not equal this difference.

Step 4: Conclusion The integral is $\boxed{32/(3\sqrt{3})}$. Hence, the correct option is (C). The key instead gives $16/\sqrt{3}$, omitting the lower-limit subtraction.

Quick Tip to Solve: After setting $t = \tan x$, reserve one factor of $\sec^2 x$ for the differential before expanding the remaining factors.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let $A = \{1,2,3,4,5,6\}$. The number of one-one functions $f : A \to A$ such that $f(1) \geq 3, f(3) \leq 4$ and $f(2)+f(3) = 5$, is ____ . [4 marks]

Solution:

Concept:

- A one-one function from a finite set of six elements to itself assigns six distinct images; equivalently it is a permutation of the six possible outputs.
- Use the restrictions on $f(2)$ and $f(3)$ first. Once their images and $f(1)$ are chosen distinctly, the remaining three inputs can receive the unused outputs in 3! ways.

Step 1: List all admissible ordered pairs The outputs are positive integers from 1 to 6. The equality $f(2)+f(3) = 5$, together with $f(3) \leq 4$, permits exactly

$$(f(2), f(3)) = (4,1), (3,2), (2,3), (1,4).$$

Every listed pair has distinct entries, so it is consistent with injectivity. There are no further pairs: an output 5 would force the other output to be 0, outside the set, and any larger output would force a

negative value.

Step 2: Count the possibilities for the first input The inequality $f(1) \geq 3$ initially gives the four candidates $3, 4, 5, 6$. For the pairs $(4, 1)$ and $(1, 4)$, the output 4 is already used, leaving $3, 5, 6$. For the pairs $(3, 2)$ and $(2, 3)$, the output 3 is already used, leaving $4, 5, 6$. Thus each of the four ordered pairs allows exactly three choices for $f(1)$. The ordering of each pair matters because the inputs 2 and 3 are different.

Step 3: Complete the bijection on the remaining inputs For any fixed allowed values of $f(1), f(2), f(3)$, exactly three outputs remain unused. Assign them bijectively to the distinct inputs 4, 5 and 6. This can be done in $3! = 6$ ways, with no further restrictions. The choices are independent after the first three images are fixed, so the multiplication principle gives

$$N = 4 \times 3 \times 3! = 4 \times 3 \times 6 = 72.$$

Different choices produce different functions, and each admissible function occurs exactly once in this construction.

Step 4: Conclusion The required number of one-one functions is $\boxed{72}$.

Quick Tip to Solve: Handle the linked image condition first, exclude its used outputs from the restricted image, and finish the unused inputs with a factorial.

22. Two players A and B play a series of games of badminton. The player, who wins 5 games first, wins the series. Assuming that no game ends in a draw, the number of ways, in which player A wins the series is ____ . [4 marks]

Solution:

Concept:

- A series ends immediately when one player earns a fifth win. Therefore an A-winning series must end with an A win, and A has exactly four wins before that final game.
- If B wins k games in an A-winning series, then $0 \leq k \leq 4$. The preceding $4 + k$ games contain four A wins and k B wins, whose order is counted by a binomial coefficient.

Step 1: Classify by the number of games won by B Let k denote B's number of wins when A wins the series. Since B has not already won the series, k can only be 0, 1, 2, 3 or 4. The series has $5 + k$ games in total, and the last one is necessarily won by A. Otherwise A would either not yet have five wins or would already have ended the series earlier. Thus the final outcome is fixed in each case.

Step 2: Count the possible prefixes Before the final A win, the record consists of four A wins and k B wins. Choose the positions of the four A wins among the first $4 + k$ positions: this gives $\binom{4+k}{4}$ sequences. Every such prefix is legitimate, since it contains at most four wins for either player; no earlier stage could have reached five wins. Conversely every A-winning series with this value of k has exactly one such prefix, so this count neither omits nor duplicates any series.

Step 3: Add the mutually exclusive cases The five possible values of k correspond to different lengths

of the series and are disjoint. Therefore

$$\begin{aligned} N &= \sum_{k=0}^{4} \binom{4+k}{4} \\ &= \binom{4}{4} + \binom{5}{4} + \binom{6}{4} + \binom{7}{4} + \binom{8}{4} \\ &= 1 + 5 + 15 + 35 + 70 = 126. \end{aligned}$$

The same sum equals $\binom{9}{5}$ by the hockey-stick identity. Merely arranging five A wins and four B wins would count only the longest series and would miss the shorter ways A can win.

Step 4: Conclusion The number of possible series won by A is $\boxed{126}$.

Quick Tip to Solve: Fix the last winning game first; then classify by how many wins the losing player has before that game.

---

23. If the sum of the coefficients of x^7 and x^{14} in the expansion of $\left(\frac{1}{x^3} - x^4\right)^n, x \neq 0$, is zero, then the value of n is ____ . [4 marks]

Solution:

Concept:

- The term obtained by selecting $-x^4$ from r factors in $(x^{-3} - x^4)^n$ is $(-1)^r \binom{n}{r} x^{7r-3n}$, where r is an integer between 0 and n.
- Consecutive binomial coefficients satisfy $\binom{n}{r+1}/\binom{n}{r} = (n-r)/(r+1)$. Their alternating signs allow cancellation only when their magnitudes agree.

Step 1: Locate the two powers in the general term In the binomial expansion the general contribution is

$$T_{r+1} = \binom{n}{r} (x^{-3})^{n-r} (-x^4)^r = (-1)^r \binom{n}{r} x^{7r-3n}.$$

Let the index of the x^7 term be r, so $7r - 3n = 7$. The x^{14} term then has index $r+1$, because raising the index by one raises the exponent by seven. Accordingly, $r = 3n/7 + 1$. These indices must correspond to terms in the stated expansion.

Step 2: Apply cancellation of the coefficients The two coefficients are $(-1)^r \binom{n}{r}$ and $(-1)^{r+1} \binom{n}{r+1}$. Since their signs are opposite, their sum is zero precisely when

$$\binom{n}{r} = \binom{n}{r+1}.$$

Using the ratio of consecutive binomial coefficients gives $(n-r)/(r+1) = 1$, hence $n = 2r + 1$. Substitute $r = 3n/7 + 1$:

$$n = \frac{6n}{7} + 3 \quad \Longrightarrow \quad n = 21.$$

Step 3: Check the indices and the signs For $n = 21$, the two relevant indices are $r = 10$ and $r+1 = 11$,

both within the permissible range. Their exponents are $70-63=7$ and $77-63=14$. The coefficients are therefore $+\binom{21}{10}$ and $-\binom{21}{11}$, equal in magnitude by binomial symmetry, so they do cancel exactly. This verifies both the exponent matching and the sign requirement, rather than merely equating two unsigned coefficient values.

Step 4: Conclusion The required value is $\boxed{n=21}$.

Quick Tip to Solve: First express the power of x as $7r-3n$; powers differing by seven correspond to adjacent binomial coefficients.

---

24. If $\frac{\pi}{4}+\sum_{p=1}^{11}\tan^{-1}\left(\frac{2^{p-1}}{1+2^{2p-1}}\right)=\alpha$, then $\tan\alpha$ is equal to ____ . [4 marks]

Solution:

Concept:

- For positive numbers $u>v$, the difference $\tan^{-1}u-\tan^{-1}v$ lies in $(0,\pi/2)$, and its tangent is $(u-v)/(1+uv)$. This range identifies the principal inverse tangent unambiguously.
- A sum of differences of consecutive terms telescopes: every interior term cancels, leaving only the last term minus the first.

Step 1: Express each summand as a difference For each integer p from 1 to 11, take $u=2^p$ and $v=2^{p-1}$. Then

$$\frac{u-v}{1+uv}=\frac{2^{p-1}}{1+2^{2p-1}}.$$

Both inverse tangents lie in $(0,\pi/2)$, with $u>v$; their positive difference is less than $\pi/2$. The tangent subtraction identity thus implies exactly

$$\tan^{-1}\left(\frac{2^{p-1}}{1+2^{2p-1}}\right)=\tan^{-1}(2^p)-\tan^{-1}(2^{p-1}).$$

No additional multiple of π is required because the difference is in the principal range.

Step 2: Cancel the intermediate inverse tangents Summing the identity from $p=1$ through $p=11$ produces

$$\sum_{p=1}^{11}\tan^{-1}\left(\frac{2^{p-1}}{1+2^{2p-1}}\right)=\tan^{-1}(2^{11})-\tan^{-1}(1)$$
$$=\tan^{-1}(2048)-\frac{\pi}{4}.$$

Every term $\tan^{-1}(2^p)$ for $1\le p\le 10$ occurs once with each sign. The initial $\pi/4$ in the definition of α is precisely what cancels the remaining negative endpoint.

Step 3: Evaluate the requested tangent It follows that $\alpha=\tan^{-1}(2048)$, an angle strictly between zero and $\pi/2$. Applying tangent to this equality gives $\tan\alpha=2048$. The endpoint is 2^{11}, not 2^{10}, because the final summand involves the difference between inverse tangents of 2^{11} and 2^{10}. The range check also confirms that tangent is defined at the resulting angle.

Step 4: Conclusion The required numerical value is $\boxed{\tan\alpha=2048}$.

Quick Tip to Solve: Match the numerator to the difference 2^p-2^{p-1} and the denominator to $1+2^p2^{p-1}$; the sum then cancels term by term.

25. Let $y = y(x)$ be the solution of the differential equation $x \sin\left(\frac{y}{x}\right) dy = \left(y \sin\left(\frac{y}{x}\right) - x\right) dx, y(1) = \frac{\pi}{2}$ and let $\alpha = \cos\left(\frac{y(e^{12})}{e^{12}}\right)$. Then the number of integral value of p, for which the equation $x^2 + y^2 - 2px + 2py + \alpha + 2 = 0$ represents a circle of radius $r \leq 6$, is ____ . [4 marks]

Solution:

Concept:

- For a homogeneous differential equation involving y/x, set $v = y/x$, so $y = xv$ and $dy = v\,dx + x\,dv$.
- A real solution must satisfy $-1 \leq \cos v \leq 1$. The domain of an implicit solution must be checked before evaluating it at a specified argument.

Step 1: Reduce the differential equation Substituting $y = xv$ in the printed equation gives

$$x \sin v\,(v\,dx + x\,dv) = (xv \sin v - x)\,dx.$$

The terms $xv \sin v\,dx$ cancel. Since we are considering the interval containing $x = 1 > 0$, divide by x^2:

$$\sin v\,dv = -\frac{dx}{x}.$$

This reduction retains the printed minus sign before x; changing that sign would define a different problem.

Step 2: Apply the initial condition and check the domain Integrating gives $-\cos v = -\ln x + C$, or $\cos v = \ln x + C_1$. The condition $y(1) = \pi/2$ means $v(1) = \pi/2$, so $C_1 = 0$. Therefore every real solution through the given initial point must obey

$$\cos\left(\frac{y}{x}\right) = \ln x.$$

At $x = e^{12}$, the right-hand side is 12, outside the range of a real cosine. Hence $y(e^{12})$ does not exist as a real value of this initial-value solution. In fact the necessary range condition already restricts x to $[e^{-1}, e]$; the requested argument lies far outside it.

Step 3: Assess the proposed circle Completing the squares in the later equation would give

$$(x - p)^2 + (y + p)^2 = 2p^2 - \alpha - 2.$$

For a real circle, α must first be a defined real constant. The preceding domain failure means it is undefined as posed. Substituting 12 for a real cosine would conceal this failure and cannot supply a valid count of integers p.

Step 4: Conclusion The real initial-value solution cannot be evaluated at e^{12}, so α and the requested integer count are undefined. The final answer key dropped this question and awarded bonus credit to all candidates.

Quick Tip to Solve: After solving an implicit trigonometric differential equation, check the range of sine or cosine before substituting a large argument.

26. In a Vernier calipers, when both jaws touch each other, zero of the Vernier scale is shifted to the right of zero of the main scale and 7^{th} Vernier division coincides with a main scale reading. If the value of 1 main scale division is 1 mm and there are 10 Vernier scale divisions, then the Vernier caliper has [4 marks]

(A) 0.07 cm negative zero error

(B) 0.7 cm negative zero error

(C) 0.07 cm positive zero error

(D) 0.7 cm positive zero error

Correct Answer: (C) 0.07 cm positive zero error

Solution:

Concept:

- For a standard direct vernier with ten vernier divisions equal to nine main-scale divisions, least count is $\mathrm{LC} = 1\,\mathrm{MSD} - 1\,\mathrm{VSD} = \mathrm{MSD}/10$.
- Zero error is the reading when the jaws are closed. A vernier zero to the right of the main-scale zero indicates a positive zero error; the correction has the opposite sign.

Step 1: Find the least count Here one main-scale division is $1\,\mathrm{mm}$, and the standard ten-division vernier has

$$1\,\mathrm{VSD} = \frac{9}{10}\,\mathrm{mm}, \qquad \mathrm{LC} = 1 - 0.9 = 0.1\,\mathrm{mm}.$$

The difference between corresponding main-scale and vernier spacings is the fine increment resolved by the instrument. The count of ten divisions does not make the least count $1\,\mathrm{mm}$; that would omit the vernier subdivision.

Step 2: Determine the sign and magnitude The jaws touch, so the true separation is zero. Nevertheless the vernier zero lies to the right of the main zero, making the indicated reading positive. The seventh vernier line coincides, giving

$$e_0 = 7\,\mathrm{LC} = 7(0.1) = 0.7\,\mathrm{mm}.$$

Convert millimetres to centimetres using $10\,\mathrm{mm} = 1\,\mathrm{cm}$:

$$e_0 = +0.07\,\mathrm{cm}.$$

This is the instrument error, whereas the correction to a subsequent observed reading would be $-0.07\,\mathrm{cm}$.

Step 3: Check the alternatives Option (A) has the correct magnitude but the wrong sign: it confuses the error with the correction. Option (B) has both the wrong sign and a tenfold unit-conversion error. Option (D) keeps the positive sign but uses $0.7\,\mathrm{cm}$ instead of $0.7\,\mathrm{mm}$. Only option (C) has both the required positive sign and the converted magnitude.

Step 4: Conclusion The vernier caliper has a positive zero error of $0.07\,\mathrm{cm}$. Hence, the correct option is (C).

Quick Tip to Solve: First determine the sign from the relative zeros, then multiply the coinciding division by the least count and convert units.

27. L, C **and** R **represents physical quantities inductance, capacitance and resistance respectively. The dimensional formula** $\mathrm{ML}^2\ \mathrm{T}^{-4}\ \mathrm{A}^{-2}$ **corresponds to ____ .** [4 marks]

(A) $\frac{R}{\sqrt{LC}}$

(B) $\frac{R}{LC}$

(C) $\frac{C}{\sqrt{LR}}$

(D) $\frac{1}{R}\sqrt{\frac{L}{C}}$

Correct Answer: (A) $\frac{R}{\sqrt{LC}}$

Solution:

Concept:

- Resistance has dimension $[R] = \mathrm{ML}^2\mathrm{T}^{-3}\mathrm{A}^{-2}$, from electric potential divided by current.
- Inductance and capacitance have dimensions $[L] = \mathrm{ML}^2\mathrm{T}^{-2}\mathrm{A}^{-2}$ and $[C] = \mathrm{M}^{-1}\mathrm{L}^{-2}\mathrm{T}^4\mathrm{A}^2$. Here the symbol L for inductance is distinct from the dimension of length.

Step 1: Simplify the product of inductance and capacitance Using the dimensions of each factor, the powers of mass, length and current cancel:

$$[LC] = (\mathrm{ML}^2\mathrm{T}^{-2}\mathrm{A}^{-2})(\mathrm{M}^{-1}\mathrm{L}^{-2}\mathrm{T}^4\mathrm{A}^2) = \mathrm{T}^2.$$

Consequently $[\sqrt{LC}] = \mathrm{T}$. This also agrees with the appearance of $1/\sqrt{LC}$ as the angular-frequency scale of an ideal LC oscillator; its dimension is inverse time.

Step 2: Evaluate the first two alternatives For option (A),

$$\left[\frac{R}{\sqrt{LC}}\right] = \frac{\mathrm{ML}^2\mathrm{T}^{-3}\mathrm{A}^{-2}}{\mathrm{T}} = \mathrm{ML}^2\mathrm{T}^{-4}\mathrm{A}^{-2},$$

exactly the dimension requested. In contrast, dividing by LC rather than its square root gives

$$\left[\frac{R}{LC}\right] = \mathrm{ML}^2\mathrm{T}^{-5}\mathrm{A}^{-2}.$$

Thus option (B) has an extra inverse power of time and cannot be selected.

Step 3: Exclude the remaining expressions The product LR has dimension $\mathrm{M}^2\mathrm{L}^4\mathrm{T}^{-5}\mathrm{A}^{-4}$. Hence

$$\left[\frac{C}{\sqrt{LR}}\right] = \mathrm{M}^{-2}\mathrm{L}^{-4}\mathrm{T}^{13/2}\mathrm{A}^4,$$

which differs in every basic exponent. Finally $\sqrt{L/C}$ has dimension $\mathrm{ML}^2\mathrm{T}^{-3}\mathrm{A}^{-2}$, the same as resistance. Therefore $(1/R)\sqrt{L/C}$ is dimensionless, eliminating option (D).

Step 4: Conclusion The required dimensional formula corresponds to $R/\sqrt{LC}$. Hence, the correct option is (A).

Quick Tip to Solve: Remember that LC has the dimension of time squared; this makes expressions containing $\sqrt{LC}$ quick to check.

28. When one moves from a point 16 km below the earth's surface to a point 16 km above the earth's surface. The change in g is approximately $\alpha\%$. The value of α is ____ .

(Take radius of the earth $= 6400$ km.) [4 marks]

(A) 0.12

(B) 0.25

(C) 0.50

(D) 0.75

Correct Answer: (B) 0.25

Solution:

Concept:

- At a small depth d in a uniformly dense spherical Earth, $g_d = g_0(1 - d/R)$, where g_0 is surface gravity.
- At a small height $h \ll R$, $g_h = g_0R^2/(R + h)^2 \simeq g_0(1 - 2h/R)$. Both locations have gravity below the surface value.

Step 1: Calculate gravity at the initial depth The initial location is $d = 16\,\text{km}$ below the surface, with $R = 6400\,\text{km}$. Thus

$$\frac{d}{R} = \frac{16}{6400} = 0.0025, \qquad g_d = 0.9975g_0.$$

The linear depth formula follows because enclosed mass decreases as the cube of distance from the centre while the inverse-square field divides by its square. This is different from the outside-Earth formula.

Step 2: Calculate gravity at the final height For the final location, $h = 16\,\text{km}$ and $h/R = 0.0025$. Keeping first-order terms,

$$g_h \simeq g_0(1 - 2 \times 0.0025) = 0.9950g_0.$$

The final gravity is therefore slightly smaller than the initial gravity, although both positions are equally far from the surface. The outside variation has twice the first-order fractional slope of the inside variation.

Step 3: Compare the two locations The change in moving from depth to height is

$$\Delta g = g_h - g_d \simeq (0.9950 - 0.9975)g_0 = -0.0025g_0.$$

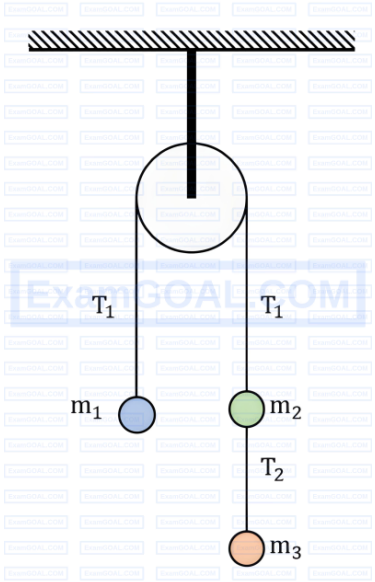
Its magnitude as a percentage is approximately 0.25%. Referencing the initial value rather than surface gravity gives $100(0.0025/0.9975)\% \simeq 0.2506\%$, the same listed approximation. The values 0.50% and 0.75% would respectively use only the height reduction or add the two reductions; the required operation is their difference. The value 0.12% is too small.

Step 4: Conclusion Gravity decreases by approximately 0.25%, so $\alpha = 0.25$. Hence, the correct option is (B).

Quick Tip to Solve: For equal small height and depth, subtract the depth reduction from the height reduction; do not add their magnitudes.

29. Three masses $m_1 = 4\ \text{kg}, m_2 = 4\ \text{kg}$ and $m_3 = 6\ \text{kg}$ are suspended from a fixed smooth frictionless pully as shown in the figure below. The value of T_1/T_2 is

(take $g = 10\ \text{m/s}^2$) [4 marks]



(A) 5/3

(B) 2/3

(C) 3/5

(D) 2/5

Correct Answer: (A) 5/3

Solution:

Concept:

- A light inextensible string over a fixed smooth pulley has the same tension on its two sides. Connected bodies on opposite sides have equal acceleration magnitudes.
- The two masses on the right move together because the short connecting string has fixed length. Apply Newton's second law to the whole right-hand group, then separately to its bottom mass.

Step 1: Find the common acceleration The left side carries $m_1 = 4\,\text{kg}$, whereas the right side carries $m_2 + m_3 = 10\,\text{kg}$. Therefore the right side moves down and the left mass moves up. Taking these motion directions as positive,

$$T_1 - 4g = 4a, \qquad 10g - T_1 = 10a.$$

Adding eliminates the main-string tension:

$$6g = 14a, \qquad a = \frac{3g}{7} = \frac{30}{7}\,\text{m s}^{-2}.$$

The acceleration is smaller than g, as expected because neither side is freely falling.

Step 2: Calculate both tensions For the upward-accelerating left mass,

$$T_1 = 4(g + a) = 4\left(10 + \frac{30}{7}\right) = \frac{400}{7}\,\text{N}.$$

The lower right mass $m_3 = 6\,\text{kg}$ accelerates down. Its forces are weight $6g$ down and T_2 up, so

$$6g - T_2 = 6a, \qquad T_2 = 6(g - a) = \frac{240}{7}\,\text{N}.$$

The two tensions need not be equal: they belong to different strings, and the intermediate mass separates them.

Step 3: Take the ratio and verify the middle mass Therefore

$$\frac{T_1}{T_2} = \frac{400/7}{240/7} = \frac{5}{3}.$$

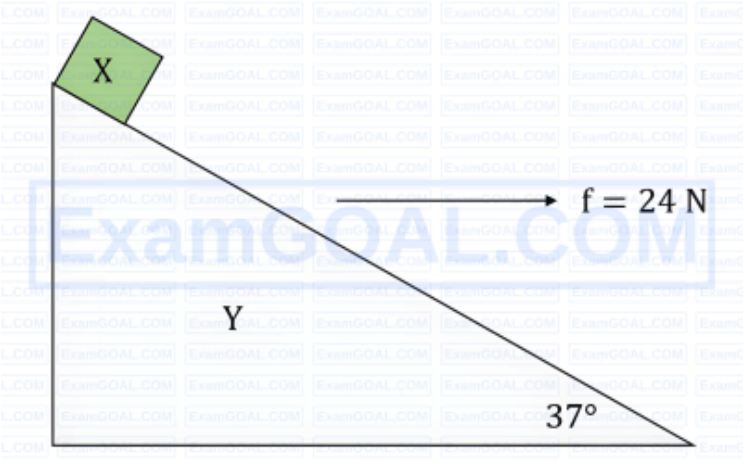
As a consistency check, the middle mass has downward forces $4g + T_2$ and upward force T_1. Their resultant is $40 + 240/7 - 400/7 = 120/7\,\text{N} = 4a$, satisfying its equation. The reciprocal $3/5$ reverses the requested ratio, while $2/3$ and $2/5$ incorrectly compare masses without allowing for the common acceleration.

Step 4: Conclusion The tension ratio is $T_1/T_2 = 5/3$. Hence, the correct option is (A).

Quick Tip to Solve: Treat the two right-hand masses as one body to get acceleration, then isolate the bottom mass to find the lower-string tension.

30. A wedge Y with mass of 10 kg and all frictionless surfaces and the inclined surface making 37° with horizontal. A block X with mass 2 kg is placed at the highest point of the wedge as shown in figure is at rest. At $t = 0$ wedge (Y) is pulled toward right with constant force (f) of 24 N . Taking the block X at rest at $t = 0$, the time taken by it to slide down 8.8 m on the slope, while Y is on the move, is ____ s.

(**take** $\tan(37^\circ) = 3/4$ **and** $g = 10\ \text{m/s}^2$) [4 marks]



(A) 2

(B) 4

(C) $\sqrt{2}$

(D) $2\sqrt{2}$

Correct Answer: The exact time is 1.890 s, absent from the options; the key selects the nearest option, 2 s.

Solution:

Concept:

- Let A be the wedge's rightward acceleration and u the block's displacement down the slope relative to the wedge. The block's inertial acceleration is $(A + \ddot{u}\cos\theta, -\ddot{u}\sin\theta)$.
- With friction absent, the only force along the slope in the wedge frame is $mg\sin\theta - mA\cos\theta$. Horizontal momentum balance for the full block-wedge system uses the external force F.

Step 1: Write the two coupled equations Projection of the block equation down the slope gives

$$\ddot{u} = g\sin\theta - A\cos\theta.$$

The wedge and block do not have equal horizontal acceleration, because the block slides. The system horizontal equation is consequently

$$(M + m)A + m\ddot{u}\cos\theta = F.$$

Here $M = 10\,\text{kg}$, $m = 2\,\text{kg}$, $F = 24\,\text{N}$, $\sin 37^\circ = 3/5$, and $\cos 37^\circ = 4/5$. The slope descends toward the right, as in the printed figure.

Step 2: Solve for the relative acceleration Substituting the along-slope equation into the horizontal

equation gives

$$(M + m\sin^2\theta)A = F - mg\sin\theta\cos\theta,$$

$$A = \frac{24 - 2(10)(3/5)(4/5)}{10 + 2(3/5)^2} = \frac{90}{67}\,\mathrm{m\,s^{-2}}.$$

It follows that

$$\ddot{u} = 6 - \frac{4}{5}\frac{90}{67} = \frac{330}{67}\,\mathrm{m\,s^{-2}}.$$

All quantities are constant, and the initial relative velocity is zero because both bodies start at rest.

Step 3: Calculate the time and assess the listed key For $u = 8.8\,\mathrm{m}$, use $u = \frac{1}{2}\ddot{u}\,t^2$:

$$t = \sqrt{\frac{2(8.8)(67)}{330}} = \sqrt{\frac{2948}{825}}\,\mathrm{s} \simeq 1.890\,\mathrm{s}.$$

None of the four options equals this value. The supplied final key selects (A), $2\,\mathrm{s}$, which is the nearest listed value but is not the exact result. Using $A = 24/(10+2) = 2\,\mathrm{m\,s^{-2}}$ would give $\ddot{u} = 4.4\,\mathrm{m\,s^{-2}}$ and 2 s; that shortcut incorrectly discards the block's horizontal motion relative to the wedge. No rounding instruction in the question makes that substitution exact.

Step 4: Conclusion The printed freely moving wedge system gives $t = \sqrt{2948/825}\,\mathrm{s} \simeq 1.890\,\mathrm{s}$; no option is exactly correct. The final answer key lists (A), $2\,\mathrm{s}$, the nearest offered value, but the literal coupled dynamics give the value above.

Quick Tip to Solve: For a sliding block on a free wedge, include its relative horizontal acceleration in the system equation before applying distance-time kinematics.

---

31. The Young's modulus of steel wire of radius r and length L is Y.
If the radius r and length L of the wire are doubled then the value of Y [4 marks]

(A) increases by two times

(B) reduces by half

(C) remains unchanged

(D) becomes one fourth

Correct Answer: (C) remains unchanged

Solution:

Concept:

- Young's modulus is the ratio of longitudinal stress to longitudinal strain in the elastic range: $Y = (F/A)/(\Delta L/L)$, where A is cross-sectional area and ΔL is extension.
- For a specified material at unchanged temperature, Young's modulus is a material property. The stiffness $k = YA/L$ of a particular wire also depends on its geometry.

Step 1: Identify what the change affects The wire remains steel. Doubling its radius changes its cross-sectional area from $A = \pi r^2$ to $A\prime = \pi(2r)^2 = 4A$, while its length becomes $L\prime = 2L$. These are changes in specimen dimensions, not changes in the elastic nature of steel. Thus the proportionality

between stress and strain stays the same.

Step 2: Check the stress and strain consistently For a useful comparison, apply the same tensile force F to both wires in the elastic regime. The original extension is $\Delta L = FL/(AY)$. The new extension is

$$\Delta L\prime = \frac{F(2L)}{(4A)Y} = \frac{\Delta L}{2}.$$

The new stress is one fourth of the old stress. The new strain is also one fourth:

$$\frac{\Delta L\prime}{L\prime} = \frac{\Delta L/2}{2L} = \frac{1}{4}\frac{\Delta L}{L}.$$

Their ratio therefore remains Y.

Step 3: Distinguish modulus from stiffness The force per unit extension does change: $k\prime = Y(4A)/(2L) = 2k$. Confusing this doubled stiffness with the modulus would incorrectly give option (A). Options (B) and (D) likewise confuse a change of length, area or extension with a change in the material constant.

Step 4: Conclusion The Young's modulus of the steel wire remains unchanged. Hence, the correct option is (C).

Quick Tip to Solve: When wire dimensions change, recompute area and stiffness if needed, but keep the elastic modulus fixed for the same material at the same temperature.

32. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Statement I : Change in internal energy of a system containing n mole of ideal gas can be written as $\Delta U = nC_v\,(T_f - T_i) = \frac{nR}{\gamma - 1}\,(T_f - T_i)$, where $\gamma = \frac{C_p}{C_v}$, T_i = initial temperature, T_f = final temperature.

Statement II : Relation between degree of freedom f and $\gamma\,(= C_p/C_v)$ is $\left(\gamma = 1 + \frac{2}{f}\right)$

Choose the correct answer from the options given below [4 marks]

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(C) A is true but **R** is false

(D) A is false but **R** is true

Correct Answer: (A) Both **A** and **R** are true and **R** is the correct explanation of **A**

Solution:

Concept:

- For n moles of an ideal gas with f active quadratic degrees of freedom per molecule, equipartition gives $U = \frac{f}{2}nRT$, up to a temperature-independent reference.
- The molar heat capacities obey $C_v = fR/2$, $C_p - C_v = R$ and $\gamma = C_p/C_v$. The printed formulas apply over a range in which these heat capacities are constant.

Step 1: Test Assertion A For an ideal gas the internal energy depends only on temperature. Using

constant molar C_v,

$$\Delta U = nC_v(T_f - T_i).$$

Also, Mayer's relation gives $(\gamma - 1)C_v = C_p - C_v = R$, so $C_v = R/(\gamma - 1)$. Therefore

$$\Delta U = \frac{nR}{\gamma - 1}(T_f - T_i).$$

Both equalities in Assertion A are true under the stated ideal-gas heat-capacity model; no restriction to a constant-volume process is needed for the energy difference.

Step 2: Test Reason R Equipartition gives $C_v = fR/2$, while $C_p = C_v + R = (f + 2)R/2$. Thus

$$\gamma = \frac{C_p}{C_v} = \frac{f + 2}{f} = 1 + \frac{2}{f}.$$

The relation stated in Reason R is true. Here f counts active degrees of freedom, so a temperature range that activates new modes needs a revised heat-capacity model.

Step 3: Check whether R explains A Rearranging the reason gives $f/2 = 1/(\gamma - 1)$. Substituting this directly in the equipartition energy change, $\Delta U = (f/2)nR(T_f - T_i)$, produces exactly the asserted gamma form. Thus R supplies the relation that converts the degrees-of-freedom expression into A. Option (B) incorrectly denies this explanatory link; (C) and (D) incorrectly mark one of the true statements false.

Step 4: Conclusion The supplied final key selects the second choice. However, under the ideal-gas equipartition model, R gives $f/2 = 1/(\gamma - 1)$; inserting this in $\Delta U = (f/2)nR(T_f - T_i)$ produces A directly. Thus both are true and R explains A, so the retained answer is (A).

Quick Tip to Solve: For ideal-gas assertion-reason questions, link $U = fnRT/2$, $C_v = fR/2$ and $f/2 = 1/(\gamma - 1)$ before judging the explanation.

33. Consider the following statements:

A. Zeroth law of thermodynamics gives concept of temperature

B. First law of thermodynamics gives concept of internal energy

C. In isothermal expansion of ideal gas, $\Delta Q \neq \Delta W$

D. Product of intensive and extensive variables is extensive

E. The ratio of any extensive variable to mass will be an extensive variable

Choose the correct combination of statements from the options given below: [4 marks]

(A) C, D and E Only

(B) A, B and C Only

(C) A, B and D Only

(D) B, C and D Only

Correct Answer: (C) A, B and D Only

Solution:

Concept:

- The zeroth law establishes thermal equilibrium and the basis of temperature; the first law expresses energy conservation as $\Delta U = Q - W$, with W the work done by the system.
- An extensive quantity scales with system size, whereas an intensive quantity remains unchanged when identical subsystems are combined. Ideal-gas internal energy depends only on temperature.

Step 1: Check the two laws and isothermal expansion Statement A is true: if two systems are separately in thermal equilibrium with a third, they are in equilibrium with each other. This gives a consistent temperature variable. Statement B is true: the first law identifies internal energy as the state function whose change is heat supplied minus work done. For statement C, an isothermal ideal-gas process has $\Delta T = 0$, hence $\Delta U = 0$. The first law then gives

$$0 = Q - W \quad \Longrightarrow \quad Q = W.$$

Thus the printed inequality in C is false.

Step 2: Use scaling to judge D and E Let the amount of an otherwise identical homogeneous system be multiplied by s. An intensive variable x stays x, while an extensive variable X changes to sX. Their product becomes sxX, so it is extensive: D is true. Mass also becomes sm, so

$$\frac{X\prime}{m\prime} = \frac{sX}{sm} = \frac{X}{m}.$$

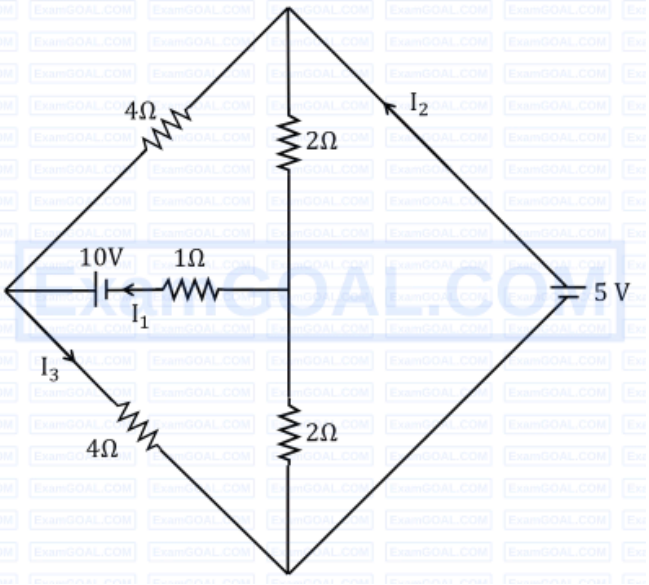
The specific quantity X/m is unchanged and is intensive, not extensive. Hence E is false.

Step 3: Select the valid combination The truth pattern is A true, B true, C false, D true and E false. Therefore only A, B and D form the required combination. Option (A) contains both false statements C and E. Options (B) and (D) each contain the false isothermal claim C, so neither is acceptable.

Step 4: Conclusion The correct statements are A, B and D only. Hence, the correct option is (C).

Quick Tip to Solve: Test intensive versus extensive character by doubling the whole system: extensive quantities double, while a specific quantity such as energy per unit mass stays fixed.

34. **Refer to the figure given below. The values of I_1, I_2 and I_3 are ____ .** [4 marks]



(A) $I_1 = 2.5$ A, $I_2 = 1.875$ A, $I_3 = 1.875$ A

(B) $I_1 = 1.875$ A, $I_2 = 2.5$ A, $I_3 = 1.875$ A

(C) $I_1 = 1.875$ A, $I_2 = 1.875$ A, $I_3 = 2.5$ A

(D) $I_1 = 2.5$ A, $I_2 = 2.5$ A, $I_3 = 1.875$ A

Correct Answer: (A) $I_1 = 2.5$ A, $I_2 = 1.875$ A, $I_3 = 1.875$ A

Solution:

Concept:

- Kirchhoff's junction law states that the algebraic sum of currents leaving a node is zero. For a resistor from potential V_a to V_b, the current is $(V_a - V_b)/R$.
- An ideal cell fixes a potential difference, with its longer plate positive. Label the top, bottom, left and middle junctions T, B, L, M, respectively; the central horizontal branch contains both the 10 V cell and the 1 ohm resistor.

Step 1: Set the reference and cell polarities Take $V_B = 0$. The 5 V cell has its positive plate connected to the top junction, so $V_T = 5\,\text{V}$. The 10 V cell is positive towards the left junction. Consequently the potential immediately to its right is $V_L - 10$, and the current from left to middle through the horizontal branch is $V_L - V_M - 10$ amperes. The marked I_1 points in the opposite direction, from middle to left.

Step 2: Apply the junction law at left and middle The left junction connects two 4 ohm resistors and the horizontal branch. Its equation is

$$\frac{V_L - 5}{4} + \frac{V_L}{4} + V_L - V_M - 10 = 0, \qquad \frac{3}{2}V_L - V_M = \frac{45}{4}.$$

At the middle junction, the two vertical resistors are each 2 ohms. The current towards the left is

$V_M - V_L + 10$, giving

$$\frac{V_M - 5}{2} + \frac{V_M}{2} + V_M - V_L + 10 = 0, \qquad 2V_M - V_L = -\frac{15}{2}.$$

Solving these simultaneous equations gives $V_L = 7.5\,\text{V}$ and $V_M = 0$.

Step 3: Find the three marked currents In their indicated directions,

$$I_1 = V_M - V_L + 10 = 2.5\,\text{A}, \qquad I_3 = \frac{V_L - V_B}{4} = 1.875\,\text{A}.$$

At the top junction the current supplied by the right branch is

$$I_2 = \frac{V_T - V_L}{4} + \frac{V_T - V_M}{2} = -0.625 + 2.5 = 1.875\,\text{A}.$$

The negative first term means 0.625 A actually enters the top junction from the left resistor. Options (B) and (C) wrongly place the 2.5 A value in another branch; (D) overestimates the right-branch current.

Step 4: Conclusion $I_1 = 2.5\,\text{A}$, $I_2 = 1.875\,\text{A}$, and $I_3 = 1.875\,\text{A}$, all in the marked directions. Hence, the correct option is (A).

Quick Tip to Solve: Fix the two battery polarities before writing node equations; the horizontal branch is not simply a 1 ohm resistor between the left and central nodes.

---

35. An electron of mass m is moving in an electric field $\vec{E} = -2E_o\hat{i}$ (E_o = constant > 0), with an initial velocity $\vec{V} = v_o\hat{i}$ (v_o = constant > 0). If $\lambda_o = \frac{h}{4mv_o}$, its de Broglie wavelength at time t is

____ .

(e = charge of electron) [4 marks]

(A) $\frac{4\lambda_o}{\left[1-\frac{E_o e}{2m}\frac{t}{v_o}\right]}$

(B) $\frac{4\lambda_o}{\left[1+\frac{E_o e}{2m}\frac{t}{v_o}\right]}$

(C) $\frac{4\lambda_o}{\left[1+\frac{2E_o e}{m}\frac{t}{v_o}\right]}$

(D) $\frac{4\lambda_o}{\left[1-\frac{2E_o e}{m}\frac{t}{v_o}\right]}$

Correct Answer: (C) $\frac{4\lambda_o}{\left[1+\frac{2E_o e}{m}\frac{t}{v_o}\right]}$

Solution:

Concept:

- The electric force is $\vec{F} = q\vec{E}$. An electron has charge $-e$, where $e > 0$ is the magnitude of its charge, so its acceleration is opposite to the electric field.
- For nonrelativistic motion the de Broglie wavelength is $\lambda = h/p = h/(mv)$. A constant force gives $\vec{v}(t) = \vec{v}(0) + \vec{a}t$; the wavelength depends on the magnitude of this velocity.

Step 1: Determine the force and its direction The field points along the negative x direction, but the electron is negatively charged. Therefore its force and acceleration point along the positive x direction:

$$\vec{F} = (-e)(-2E_o\hat{i}) = 2eE_o\hat{i}, \qquad \vec{a} = \frac{2eE_o}{m}\hat{i}.$$

Since the initial velocity is also along positive x, the electron speeds up continuously for $t \geq 0$. Its wavelength must therefore decrease with time. A denominator with a minus sign would instead predict initial slowing and is inconsistent with the force direction.

Step 2: Calculate the speed and substitute into the wavelength With a constant electric field, the speed is

$$v(t) = v_o + \frac{2eE_o}{m}t = v_o\left(1 + \frac{2eE_ot}{mv_o}\right).$$

Substitution into the de Broglie relation gives

$$\lambda(t) = \frac{h}{mv_o}\frac{1}{1 + \dfrac{2eE_ot}{mv_o}}.$$

The quantity $2eE_ot/(mv_o)$ is dimensionless: the numerator is an impulse and the denominator is the initial momentum.

Step 3: Express the result in terms of the supplied constant The question defines $\lambda_o = h/(4mv_o)$, which is one fourth of the actual initial wavelength. Thus $h/(mv_o) = 4\lambda_o$, and

$$\lambda(t) = \frac{4\lambda_o}{1 + \dfrac{2E_oe}{m}\dfrac{t}{v_o}}.$$

This has the required initial value $\lambda(0) = 4\lambda_o$. Options (A) and (D) have the wrong force sign. Option (B) has a positive sign but an acceleration coefficient four times too small, using $eE_o/(2m)$ in place of $2eE_o/m$.

Step 4: Conclusion The wavelength is $\lambda(t) = \dfrac{4\lambda_o}{1 + \frac{2E_oe}{m}\frac{t}{v_o}}$. Hence, the correct option is (C).

Quick Tip to Solve: First multiply the field by the electron's negative charge; then check the initial wavelength separately, because the supplied λ_o need not equal $h/(mv_o)$.

36. In the hydrogen atom, the electron makes a transition from the higher orbit (i) to a lower orbit (f). The ratio of the radius of the orbits in given by $r_i : r_f = 16 : 4$. The wavelength of photon emitted due to this transition is ____ nm.

(Given Rydberg constant $= 1.0973 \times 10^7/\text{m}$) [4 marks]

(A) 121

(B) 242

(C) 486

(D) 974

Correct Answer: The radius ratio alone does not fix the orbit numbers; both 121 nm and 486 nm are possible.

Solution:

Concept:

- For hydrogen the Bohr radius of orbit n is $r_n = a_0 n^2$. A ratio of radii therefore fixes a ratio of principal quantum numbers, rather than their individual values.
- An emitted photon obeys the Rydberg formula $1/\lambda = R(1/n_f^2 - 1/n_i^2)$, where $n_i > n_f$ are positive integers and $R = 1.0973 \times 10^7\,\text{m}^{-1}$.

Step 1: Extract the actual information supplied by the radius ratio Applying the Bohr radius relation to the printed ratio gives

$$\frac{r_i}{r_f} = \frac{n_i^2}{n_f^2} = \frac{16}{4} = 4, \qquad n_i = 2n_f.$$

This equation permits $(n_i, n_f) = (2,1), (4,2), (6,3)$, and further integer multiples. The ratio 16:4 is numerically the same as 4:1; writing it in unreduced form does not uniquely specify that the orbit numbers themselves are 4 and 2.

Step 2: Calculate the family of allowed wavelengths Put $n_f = k$ and $n_i = 2k$, where k is a positive integer. The Rydberg equation becomes

$$\frac{1}{\lambda} = R\left(\frac{1}{k^2} - \frac{1}{4k^2}\right) = \frac{3R}{4k^2}, \qquad \lambda = \frac{4k^2}{3R}.$$

For $k = 1$, the transition is $2 \to 1$, and

$$\lambda = \frac{4}{3(1.0973 \times 10^7)}\,\text{m} \simeq 121.51\,\text{nm},$$

which corresponds to the approximately stated 121 nm option (A).

Step 3: Check the commonly intended transition without assuming it If the intended transition were $4 \to 2$, then $k = 2$, and

$$\lambda = \frac{16}{3(1.0973 \times 10^7)}\,\text{m} \simeq 486.04\,\text{nm},$$

matching option (C). Both transitions satisfy exactly the supplied radius ratio. Option (B) would require k^2 close to 2, while (D) would require it close to 8; neither is the square of an integer. A stated orbit number or absolute radius would resolve the ambiguity.

Step 4: Conclusion The supplied final key selects (C), corresponding to the intended transition. However, the printed ratio does not determine a unique wavelength: $2 \to 1$ gives approximately 121 nm, while $4 \to 2$ gives approximately 486 nm. Thus both (A) and (C) are compatible; (C) follows only if the intended orbits are explicitly taken as 4 and 2.

Quick Tip to Solve: Reduce a radius ratio before inferring quantum numbers: $r \propto n^2$ fixes their ratio, and an absolute orbit number is needed to choose one spectral line.

---

37. A displacement current of 4.0 A can be set up in the space between two parallel plates of 6μ F capacitor. The rate of change of potential difference across the plates of the capacitor is nearly $\alpha \times 10^6$ V/s. The value of α is ____ . [4 marks]

(A) 0.58

(B) 0.67

(C) 0.82

(D) 0.75

Correct Answer: (B) 0.67

Solution:

Concept:

- Displacement current between capacitor plates is $I_d = \varepsilon_0 \, d\Phi_E/dt$. It accounts for the changing electric field even though charge does not cross the insulating gap.
- For a fixed capacitor, $Q = CV$, so $I_d = dQ/dt = C \, dV/dt$, where C is capacitance and V is the potential difference.

Step 1: Relate the changing field to voltage For parallel plates of area A and separation d, the field is $E = V/d$. Therefore $\Phi_E = AV/d$, and differentiation gives

$$I_d = \frac{\varepsilon_0 A}{d}\frac{dV}{dt} = C\frac{dV}{dt}.$$

The geometrical factor is already contained in the specified capacitance; no separate plate area or separation is needed.

Step 2: Substitute the capacitance in SI units Convert microfarads to farads before dividing:

$$C = 6 \times 10^{-6}\,\text{F}, \qquad I_d = 4.0\,\text{A}.$$

Hence the required rate is

$$\frac{dV}{dt} = \frac{4.0}{6 \times 10^{-6}} = \frac{2}{3} \times 10^6\,\text{V s}^{-1}.$$

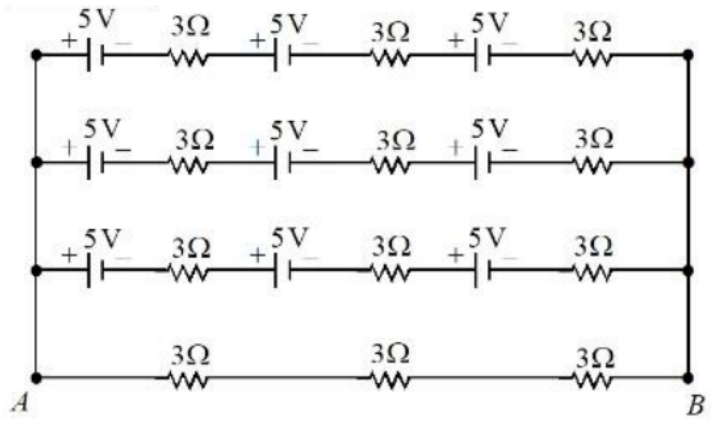
The unit is correct because $\text{A/F} = \text{V/s}$.

Step 3: Identify the coefficient and check the choices Comparing with $\alpha \times 10^6\,\text{V s}^{-1}$ gives $\alpha = 0.666\ldots$, which rounds to 0.67. As a reverse check, this rate produces $6 \times 0.67 = 4.02\,\text{A}$, agreeing with the specified current to the stated precision. The other coefficients give 3.48, 4.92 and $4.50\,\text{A}$, respectively, so they do not satisfy the data.

Step 4: Conclusion The voltage changes at approximately $0.67 \times 10^6\,\mathrm{V\,s^{-1}}$, so $\alpha = 0.67$. Hence, the correct option is (B).

Quick Tip to Solve: For a capacitor with fixed geometry, use $I_d = C\,dV/dt$; converting μF first prevents a factor-of-a-million error.

38. Refer to the figure given below, current between terminals A and B is ____ A. [4 marks]



(A) 12.5

(B) 1.25

(C) 7.5

(D) 5

Correct Answer: (B) 1.25

Solution:

Concept:

- Each of the three upper branches contains three identical cells in series: its emf is $E = 3(5) = 15\,\mathrm{V}$, and its resistance is $r = 3(3) = 9\,\Omega$.
- Identical source branches connected in parallel have the same equivalent emf, while their resistances combine in parallel. The lower, cell-free branch is the external load.

Step 1: Reduce each horizontal branch The three cells in any upper row are oriented alike, with their positive sides toward A, so their emfs add. The resistors in that row carry one branch current and add in series. Each source row is therefore a 15 V source in series with $9\,\Omega$. The lower row has three resistors in series and no cell, so $R_L = 9\,\Omega$. The vertical wires are ideal common nodes A and B; they do not short the horizontal rows.

Step 2: Combine the three source branches Because the source rows have equal emfs with the same polarity, connecting them in parallel does not multiply the emf. Their effective source resistance is

$$r_{\mathrm{eq}} = \left(\frac{1}{9} + \frac{1}{9} + \frac{1}{9}\right)^{-1} = 3\,\Omega.$$

The complete network is consequently a 15 V source with series resistance $3\,\Omega$, driving the $9\,\Omega$ load between A and B.

Step 3: Find the load current and check Applying the loop law gives

$$I = \frac{E}{r_{\text{eq}} + R_L} = \frac{15}{3+9} = 1.25\,\text{A}.$$

Its direction in the lower branch is from A to B, since A is at the higher potential. The terminal voltage is $IR_L = 11.25\,\text{V}$; each upper row supplies $(15 - 11.25)/9 = 5/12\,\text{A}$, and the sum is again 1.25 A. The other choices do not satisfy this current balance or the loop voltage.

Step 4: Conclusion The current through the lower branch between A and B is $1.25\,\text{A}$, directed from A to B. Hence, the correct option is (B).

Quick Tip to Solve: Combine the cells and resistors within each row first; parallel identical source rows reduce resistance, not emf.

39. In Young's double slit experiment, the fringe width of the interference pattern produced on the screen is 2.4μ m. If the experiment is carried out in another medium having refractive index 1.2 , the fringe width will be ____ μm. [4 marks]

(A) 1.2

(B) 2

(C) 2.4

(D) 2.88

Correct Answer: (B) 2

Solution:

Concept:

- For Young's double-slit experiment, the fringe width is $\beta = \lambda D/d$, where D is the screen distance and d is slit separation.
- In a uniform medium of refractive index n, frequency is unchanged but wavelength becomes $\lambda_m = \lambda/n$. Thus the fringe width decreases in the same ratio.

Step 1: Identify what changes in the experiment The source and the positions of the slits and screen remain the same. Consequently D and d are unchanged. Light travels more slowly in the medium, but its frequency is fixed by the source. Using $v = f\lambda$, the reduction of speed from c to c/n therefore reduces the wavelength to λ/n. This is the wavelength that enters the path-difference condition for interference.

Step 2: Take the ratio of fringe widths Substitute the wavelength in the medium into the fringe formula and divide by the original value:

$$\frac{\beta_m}{\beta} = \frac{(\lambda/n)D/d}{\lambda D/d} = \frac{1}{n}.$$

Hence

$$\beta_m = \frac{2.4}{1.2}\,\mu\text{m} = 2.0\,\mu\text{m}.$$

The micrometre unit is retained because this is a dimensionless ratio; converting both widths to metres would give exactly the same result.

Step 3: Check the direction of the change A refractive index greater than unity must make the fringe spacing smaller, which immediately excludes 2.4 and 2.88 micrometres. Multiplying by the index would incorrectly predict the latter value. The value 1.2 micrometres would halve the spacing and corresponds to index 2, rather than the stated 1.2. Only 2.0 micrometres gives the required ratio $\beta/\beta_m = 1.2$.

Step 4: Conclusion The new fringe width is $2.0\,\mu\text{m}$. Hence, the correct option is (B).

Quick Tip to Solve: With the apparatus fixed, divide the air fringe width by the refractive index; frequency does not change on entering the medium.

40. A ray of light passing through an equilateral prism is having velocity 2.12×10^8 m/s in the prism material, then the minimum angle of deviation is ____ degrees. [4 marks]

(A) 45

(B) 30

(C) 28

(D) 58

Correct Answer: (B) 30

Solution:

Concept:

- The refractive index relative to air is $n = c/v$, where v is the speed of light in the prism.
- At minimum deviation the ray passes symmetrically: $r_1 = r_2 = A/2$, and $n = \sin[(A + \delta_m)/2]/\sin(A/2)$.

Step 1: Determine the index and prism angle An equilateral prism has apex angle $A = 60^\circ$. Taking the speed of light in air as $3.00 \times 10^8\,\text{m}\,\text{s}^{-1}$, the specified speed in glass gives

$$n = \frac{3.00 \times 10^8}{2.12 \times 10^8} = 1.4151\ldots.$$

This is very close to $\sqrt{2}$, the small difference being consistent with the rounded speed supplied in the question.

Step 2: Use symmetric refraction at minimum deviation At minimum deviation, both internal refraction angles are 30°, and the angle of incidence equals the angle of emergence. Snell's law then gives

$$\sin i = n \sin 30^\circ = \frac{1.4151\ldots}{2} = 0.70755\ldots.$$

Therefore $i \simeq 45.04^\circ$. Since $\delta_m = i + e - A = 2i - A$,

$$\delta_m \simeq 2(45.04^\circ) - 60^\circ = 30.07^\circ.$$

To the precision of the choices, this is 30 degrees.

Step 3: Compare the possible angles The 45-degree value is the approximate incidence angle, not the deviation: the prism angle must be subtracted from twice the incidence angle. The values 28 and 58 degrees would imply refractive indices $2\sin 44^\circ$ and $2\sin 59^\circ$, respectively, and neither agrees with the measured speed. Thus the rounded data select a unique listed answer.

Step 4: Conclusion The minimum angle of deviation is approximately 30°. Hence, the correct option is (B).

Quick Tip to Solve: For an equilateral prism, first calculate $n = c/v$, then use $\delta_m = 2\sin^{-1}(n/2) - 60^\circ$.

41. Light source having wavelength 331 nm is used to generate photo-electrons whose stopping potential is 0.2 V . The work function of the used metal in the experiment is $\alpha \times 10^{-19}$ J. The value of α is ____ .

$$\left(\mathrm{h} = 6.62 \times 10^{-34}\ \mathrm{J\ s}, \mathrm{e} = 1.6 \times 10^{-19}\mathrm{C}\ \textbf{and}\ \mathrm{c} = 3 \times 10^{8}\ \mathrm{m/s}\right)$$

[4 marks]

(A) 3.68

(B) 4.68

(C) 5.68

(D) 2.68

Correct Answer: (C) 5.68

Solution:

Concept:

- Einstein's photoelectric equation is $hc/\lambda = \phi + K_{\max}$, where ϕ is the work function and $K_{\max}$ is maximum electron kinetic energy.
- The stopping potential V_s determines the maximum kinetic energy by $K_{\max} = eV_s$. It is the excess energy after the electron escapes the metal.

Step 1: Compute the incident photon energy Convert the wavelength to SI units: $331\,\mathrm{nm} = 331 \times 10^{-9}\,\mathrm{m}$. Using only the constants supplied in the question,

$$E_\gamma = \frac{(6.62 \times 10^{-34})(3 \times 10^{8})}{331 \times 10^{-9}} = 6.00 \times 10^{-19}\,\mathrm{J}.$$

This is the energy delivered by one photon; it is shared between overcoming the surface work function and producing electron kinetic energy.

Step 2: Subtract the maximum kinetic energy The stopping potential is 0.2 V, so

$$K_{\max} = eV_s = (1.6 \times 10^{-19})(0.2) = 0.32 \times 10^{-19}\ \text{J}.$$

Rearranging the photoelectric equation gives

$$\phi = E_\gamma - K_{\max} = (6.00 - 0.32) \times 10^{-19} = 5.68 \times 10^{-19}\ \text{J}.$$

The subtraction is essential: the work function is smaller than the photon energy because the emitted electrons still possess kinetic energy.

Step 3: Read the coefficient and reject the distractors Comparison with $\phi = \alpha \times 10^{-19}$ J gives $\alpha = 5.68$. As a reverse check, $(6.00 - 5.68) \times 10^{-19}/(1.6 \times 10^{-19}) = 0.2\,\text{V}$, exactly reproducing the stopping potential. The values 3.68, 4.68 and 2.68 would instead leave too much kinetic energy and imply larger stopping potentials, so they fail the given experimental condition.

Step 4: Conclusion The work function is 5.68×10^{-19} J, and therefore $\alpha = 5.68$. Hence, the correct option is (C).

Quick Tip to Solve: Calculate the photon energy in joules and subtract eV_s; do not subtract a voltage directly from an energy.

42. A compound microscope is designed with two symmetric biconvex lenses. The objective lens is cut vertically, creating two identical plano-convex lenses. One of them is used in place of original objective lens. To retain same magnification keeping the object distance unchanged, the tube length has to be [4 marks]

(A) increased two times

(B) increased $\frac{3}{2}$ times

(C) decreased two times

(D) decreased $\frac{3}{2}$ times

Correct Answer: The intended key is (A), but fixed object distance is incompatible with unchanged microscope magnification under the usual viewing conditions.

Solution:

Concept:

- The lens-maker relation for a thin symmetric biconvex lens is $1/f = (n-1)(2/R)$. Replacing one curved face by a plane halves its power, so a plano-convex half has focal length $f\prime = 2f$.
- The objective obeys $1/f = 1/u + 1/v$ in terms of positive object and real-image distances. Its magnification magnitude is $m_o = v/u$. The familiar microscope approximation $M \simeq LD/(f_o f_e)$ presumes the object can be adjusted near the objective focus.

Step 1: Find the changed objective focal length Cutting the original symmetric lens into two identical plano-convex pieces removes one curved surface from each piece. For the piece used as the new

objective,

$$\frac{1}{f\prime} = (n-1)\left(\frac{1}{R} - 0\right) = \frac{1}{2f}, \qquad f\prime = 2f.$$

The eyepiece is unchanged. If one uses the usual approximate microscope formula alone and keeps its viewing factor unchanged, retaining M would require $L\prime/f\prime = L/f$, or $L\prime = 2L$. This gives the intended listed answer (A).

Step 2: Check the printed fixed-object-distance condition The question also explicitly keeps the object distance unchanged. In the usual fixed viewing arrangement the eyepiece magnification is unchanged, so equal total magnification requires equal objective magnification. Since $u\prime = u$, the relation $m_o = v/u$ then requires $v\prime = v$. But the two lens equations would demand both

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}, \qquad \frac{1}{2f} = \frac{1}{u} + \frac{1}{v},$$

which is impossible for a finite focal length. More directly, $v = fu/(u-f)$, whereas $v\prime = 2fu/(u-2f)$; these are not equal.

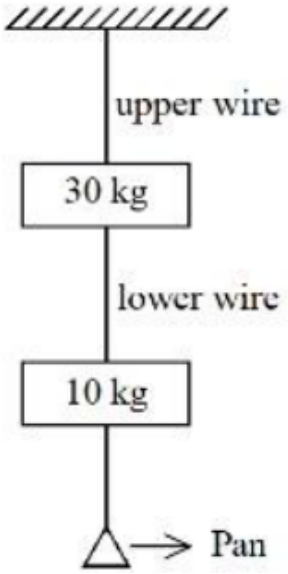
Step 3: Distinguish the intended approximation from a consistent microscope For a normal microscope the object is just beyond the original objective focus, so $f < u < 2f$. After the focal length doubles, that same object lies inside the new focus and the objective produces a virtual image, rather than the real intermediate image of the intended compound microscope. Moving the eyepiece to double the tube length does not repair this. If arbitrary changes to final viewing conditions are allowed instead, the eyepiece factor is unspecified and no unique tube-length ratio follows. Thus the fixed-object clause cannot be satisfied by selecting any of the four ratios; it must be relaxed to justify the conventional result.

Step 4: Conclusion The final key selects (A): double the tube length, using $M \simeq LD/(f_o f_e)$. However, with the printed requirement of unchanged object distance, none of the listed ratios is generally consistent with the lens equation and the same microscope viewing arrangement. Thus (A) is the intended approximate answer, while the literal conditions do not give a physically consistent unique option.

Quick Tip to Solve: After changing an objective focal length, check the object position with the lens equation before applying the approximate tube-length formula.

43. Two wires as shown in the figure below, made of steel and have breaking stress of $12 \times 10^8\ \mathrm{N/m^2}$. Area of cross-section of upper wire is $0.008\ \mathrm{cm^2}$ and of lower wire is $0.004\ \mathrm{cm^2}$. The maximum mass that can be added to pan without breaking any wire is ____ kg.

(Take $g = 10\,\mathrm{m\,s^{-2}}$.) [4 marks]



(A) 56

(B) 38

(C) 96

(D) 5.6

Correct Answer: (B) 38

Solution:

Concept:

- The limiting tension of a wire is its breaking stress multiplied by its cross-sectional area: $T_{\max} = \sigma_b A$. Both wires must remain within their separate limits.
- In static equilibrium, a wire supports the total weight hanging below it. The upper wire supports both blocks and the added mass; the lower wire supports the lower block and the added mass.

Step 1: Convert the areas and find the limiting tensions Because $1\,\mathrm{cm^2} = 10^{-4}\,\mathrm{m^2}$, the upper and lower areas are $8 \times 10^{-7}\,\mathrm{m^2}$ and $4 \times 10^{-7}\,\mathrm{m^2}$, respectively. Hence

$$T_{u,\max} = (12 \times 10^8)(8 \times 10^{-7}) = 960\,\mathrm{N}, \qquad T_{l,\max} = 480\,\mathrm{N}.$$

The thinner lower wire can therefore sustain only half the upper wire's limiting tension.

Step 2: Apply both load limits Let m kg be added to the pan. Using $g = 10\,\mathrm{m\,s^{-2}}$, the tensions are

$$T_u = 10(m + 30 + 10), \qquad T_l = 10(m + 10).$$

The upper-wire condition gives $m + 40 \le 96$, or $m \le 56$. The lower-wire condition gives $m + 10 \le 48$, or $m \le 38$. The smaller bound controls the entire arrangement.

Step 3: Check the allowed load and the other options At $m = 38\,\text{kg}$, the lower tension reaches $480\,\text{N}$, while the upper tension is only $780\,\text{N} < 960\,\text{N}$. The value 56 kg tests only the upper wire and would break the lower one. The value 96 kg neglects the existing loads and the lower-wire limit. A load of 5.6 kg is safe but is not the maximum.

Step 4: Conclusion The maximum additional mass is $\boxed{38\,\text{kg}}$, limited by the lower wire. Hence, the correct option is (B).

Quick Tip to Solve: Write one inequality for each wire using all the masses below that wire, then choose the smaller permitted added mass.

---

44. An a.c. source of angular frequency ω is connected across a resistor R and a capacitor C in series. The current is observed as I. Now the frequency of the source is changed to $\omega/4$, (keeping the voltage unchanged) the current is found to be $I/3$. The ratio of resistance to reactance at frequency ω is [4 marks]

(A) $\sqrt{\frac{6}{7}}$

(B) $\sqrt{\frac{3}{5}}$

(C) $\sqrt{\frac{7}{8}}$

(D) $\sqrt{\frac{3}{4}}$

Correct Answer: (C) $\sqrt{\frac{7}{8}}$

Solution:

Concept:

- For a series resistor-capacitor circuit, the impedance magnitude is $Z = \sqrt{R^2 + X_C^2}$, where $X_C = 1/(\omega C)$ is the capacitive reactance.
- With the source voltage unchanged, the current is inversely proportional to the impedance. Lowering angular frequency increases capacitive reactance; the resistance stays unchanged.

Step 1: Express the original and changed impedances Let $X = 1/(\omega C)$ be the reactance at the original angular frequency. The original current is

$$I = \frac{V}{\sqrt{R^2 + X^2}}.$$

At angular frequency $\omega/4$, the new reactance is $X' = 1/[(\omega/4)C] = 4X$. Thus the changed current is

$$\frac{I}{3} = \frac{V}{\sqrt{R^2 + 16X^2}}.$$

The same voltage convention, rms or peak, is used in both equations.

Step 2: Use the measured current ratio Dividing the current expressions eliminates the voltage. Since

the current falls by a factor of three, the impedance grows by three:

$$\sqrt{R^2 + 16X^2} = 3\sqrt{R^2 + X^2}.$$

Squaring and rearranging gives

$$R^2 + 16X^2 = 9R^2 + 9X^2, \qquad 7X^2 = 8R^2.$$

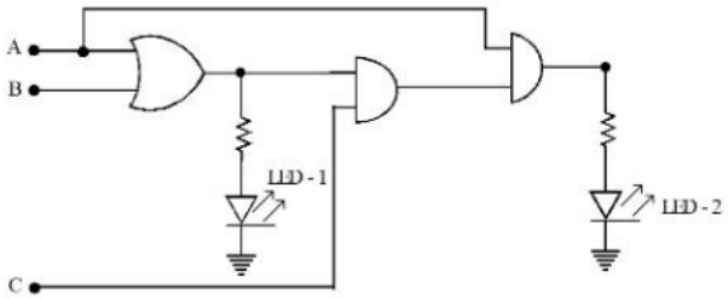
Therefore $(R/X)^2 = 7/8$.

Step 3: Select the positive ratio Resistance and reactance magnitudes are positive, so $R/X = \sqrt{7/8}$. The other proposed squared ratios, $6/7$, $3/5$ and $3/4$, do not satisfy $8R^2 = 7X^2$. As a consistency check, a purely capacitive circuit would reduce its current to one quarter; the resistor makes the observed reduction less severe, to one third.

Step 4: Conclusion At the original angular frequency, $\boxed{R/X_C = \sqrt{7/8}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the current ratio to obtain the inverse impedance ratio, and square only after replacing the capacitive reactance by its new value.

45. For the given logic circuit, which of the following inputs combination will make both LED-1 and LED-2 to glow? [4 marks]



(A) $\mathrm{A} = 0,\ \mathrm{B} = 1, \mathrm{C} = 1$

(B) $\mathrm{A} = 1,\ \mathrm{B} = 0, \mathrm{C} = 0$

(C) $\mathrm{A} = 1,\ \mathrm{B} = 0, \mathrm{C} = 1$

(D) $\mathrm{A} = 1,\ \mathrm{B} = 1, \mathrm{C} = 0$

Correct Answer: (C) $\mathrm{A} = 1,\ \mathrm{B} = 0, \mathrm{C} = 1$

Solution:

Concept:

- An OR gate outputs 1 when either input is 1, whereas an AND gate outputs 1 only when both inputs are 1. A grounded LED in this circuit glows when its driving output is high.
- Trace the intermediate gate outputs separately. In Boolean expressions, $+$ denotes OR and multiplication denotes AND; these are logical operations rather than ordinary arithmetic.

Step 1: Find the signal that drives LED-1 The leftmost gate is an OR gate with inputs A and B. Its output is

$$P = A + B.$$

LED-1 is connected to this node through its resistor, so it glows exactly when $P = 1$. This signal also supplies one input of the middle AND gate. The other middle-gate input is C, giving $Q = PC$.

Step 2: Find the signal that drives LED-2 The final AND gate receives Q and the direct branch from A. Its output is

$$Y = AQ = A(A + B)C = AC.$$

The simplification follows because if $A = 1$, the OR output is automatically 1; if $A = 0$, the final AND output is 0. Both LEDs therefore glow when $A = 1$ and $C = 1$, regardless of B.

Step 3: Test every offered input combination For option (A), $(A, B, C) = (0, 1, 1)$ gives $P = 1$ but $Y = 0$, so only LED-1 glows. Option (B), $(1, 0, 0)$, also gives $P = 1, Y = 0$, because $C = 0$. Option (C), $(1, 0, 1)$, gives $P = 1, Q = 1, Y = 1$, lighting both LEDs. Option (D), $(1, 1, 0)$, gives $P = 1, Y = 0$ because its zero C input blocks the middle AND gate.

Step 4: Conclusion Both LEDs glow for $\boxed{A = 1,\ B = 0,\ C = 1}$. Hence, the correct option is (C).

Quick Tip to Solve: For a cascaded logic circuit, label each gate output and simplify the final expression before testing the listed inputs.

Physics – Section B: Numerical Value Questions (20 marks)

46. A cube has side length 5 cm and modulus of rigidity 10^5 N/m^2. The displacement produced by a force of 10 N in the upper face of cube is ____ mm. [4 marks]

Solution:

Concept:

- The modulus of rigidity, or shear modulus, is $G = (\text{shear stress})/(\text{shear strain})$. Shear stress is F/A, where F acts tangentially on area A.
- For a block of height L, a small relative sideways displacement x of its top face gives shear strain x/L. Hence $G = FL/(Ax)$.

Step 1: Identify the face area and separation The side length is $L = 5\,\text{cm} = 0.05\,\text{m}$. Since the body is a cube, the area of its upper face is

$$A = L^2 = (0.05)^2 = 2.5 \times 10^{-3}\,\text{m}^2.$$

The separation between the upper and opposite face is also $0.05\,\text{m}$. The specified rigidity modulus concerns tangential deformation, so the upper-face displacement is a shear displacement relative to the opposite face.

Step 2: Compute shear stress and shear strain The applied force produces the shear stress

$$\tau = \frac{F}{A} = \frac{10}{2.5 \times 10^{-3}} = 4.0 \times 10^{3}\,\mathrm{N\,m^{-2}}.$$

Dividing this stress by the given shear modulus yields the dimensionless strain

$$\gamma = \frac{\tau}{G} = \frac{4.0 \times 10^{3}}{10^{5}} = 0.04.$$

This strain is the displacement-to-height ratio, not the displacement itself.

Step 3: Convert the displacement to millimetres Using $\gamma = x/L$,

$$x = \gamma L = 0.04(0.05) = 0.002\,\mathrm{m} = 2\,\mathrm{mm}.$$

Equivalently, for a cube the formula reduces to $x = F/(GL)$, which gives $10/(10^5 \times 0.05) = 0.002\,\mathrm{m}$. Both calculations have dimensions of length and agree.

Step 4: Conclusion The displacement of the upper face is $\boxed{2\,\mathrm{mm}}$; the required numerical entry is $\boxed{2}$.

Quick Tip to Solve: For a cube, substitute $A = L^2$ into the shear-modulus equation to get $x = F/(GL)$; convert centimetres before calculating.

---

47. From 18 m height above the ground a ball is dropped from rest . The height above the ground at which the magnitude of velocity equal to the magnitude of acceleration (in the same set of units) due to gravity is ____ m.

(Take $g = 10\ \mathrm{m/s^2}$ and neglect the air resistance) [4 marks]

Solution:

Concept:

- For free fall from rest with negligible air resistance, the speed after descending a distance s satisfies $v^2 = u^2 + 2gs$, with $u = 0$.
- The question compares numerical magnitudes in the specified SI units: the acceleration is $10\,\mathrm{m\,s^{-2}}$, so the required speed is $10\,\mathrm{m\,s^{-1}}$. Speed and acceleration retain their different physical dimensions.

Step 1: Translate the numerical condition into a speed The ball is released at a height of $18\,\mathrm{m}$ above the ground. Taking downward displacement as positive, its initial speed is zero and its constant acceleration is $g = 10\,\mathrm{m\,s^{-2}}$. Numerical equality with this acceleration therefore occurs when its speed reaches $v = 10\,\mathrm{m\,s^{-1}}$. It is not a dimensional equality between two different physical quantities.

Step 2: Find how far the ball has fallen Apply the velocity-displacement equation:

$$v^2 = 2gs \quad \Rightarrow \quad s = \frac{v^2}{2g} = \frac{10^2}{2(10)} = 5\,\mathrm{m}.$$

This is the downward distance travelled from the release point. The value 5 m must not be reported as the height above ground, since those two distances are measured from different reference levels.

Step 3: Calculate the remaining height and check the time The height above ground is

$$h = 18 - s = 18 - 5 = 13\,\text{m}.$$

A separate check uses $v = gt$, which gives $t = 1\,\text{s}$. Then $s = \frac{1}{2}gt^2 = 5\,\text{m}$, confirming the same height. Since the fallen distance is less than 18 m, this condition is reached before the ball hits the ground.

Step 4: Conclusion The ball is $\boxed{13\,\text{m}}$ above the ground; the required numerical entry is $\boxed{13}$.

Quick Tip to Solve: After finding the distance fallen with $v^2 = 2gs$, subtract it from the initial height whenever the question asks for height above ground.

---

48. A transverse wave on a string is described by $y = 3\sin(36t + 0.018x + \pi/4)$. where x, y are in cm and t in seconds. The least distance between the two successive crests in the wave is ____ cm . (Nearest integer)

$$(\pi = 3.14)$$

[4 marks]

Solution:

Concept:

- For a sinusoidal progressive wave $y = A\sin(\omega t + kx + \phi)$, the coefficient k is the angular wave number. Its units are radians per unit length used for x.
- At a fixed time, consecutive crests have phases differing by 2π. Their least positive separation is the wavelength $\lambda = 2\pi/|k|$.

Step 1: Read the wave number in the stated length unit Comparing $y = 3\sin(36t + 0.018x + \pi/4)$ with the standard form gives $k = 0.018\,\text{rad}\,\text{cm}^{-1}$, because x is measured in centimetres. The amplitude is 3 cm and the angular frequency is $36\,\text{rad}\,\text{s}^{-1}$. Neither of those two quantities is the spatial phase coefficient needed for crest separation.

Step 2: Use equal-phase crest positions At fixed time, one crest has phase $\pi/2 + 2n\pi$, and the next has phase $\pi/2 + 2(n+1)\pi$. Subtracting these two phase equations cancels both $36t$ and $\pi/4$:

$$0.018(x_2 - x_1) = 2\pi.$$

Hence the least distance between successive crests is

$$\lambda = x_2 - x_1 = \frac{2\pi}{0.018}.$$

A phase difference of π would instead connect a crest and a trough, only half a wavelength apart.

Step 3: Substitute the given value of pi and round Using the prescribed $\pi = 3.14$,

$$\lambda = \frac{6.28}{0.018} = 348.888\ldots\ \text{cm}.$$

Rounding to the nearest integer gives 349 cm. The plus sign before kx specifies propagation toward decreasing x, but it does not alter the positive distance between neighbouring crests. Keeping the original centimetre unit avoids an unnecessary conversion.

Step 4: Conclusion The least separation is approximately $\boxed{349\,\text{cm}}$; the nearest-integer entry is $\boxed{349}$.

Quick Tip to Solve: Read the unit of x before using $\lambda = 2\pi/k$. A constant phase shift changes crest positions but never their separation.

---

49. The charged particle moving in a uniform magnetic field of $(3\hat{i} + 2\hat{j})\text{T}$ has an acceleration $\left(4\hat{i} - \frac{x}{2}\hat{j}\right)\text{m/s}^2$. The value of x is [4 marks]

Solution:

Concept:

- A particle of charge q, mass m and velocity $\vec{v}$ in a magnetic field experiences $\vec{F} = q(\vec{v} \times \vec{B})$. The cross product is perpendicular to $\vec{B}$, irrespective of the sign of the charge.
- By Newton's second law, $m\vec{a} = \vec{F}$, so the magnetic acceleration satisfies $\vec{a} \cdot \vec{B} = 0$. For Cartesian components, the dot product is the sum of products of corresponding components.

Step 1: Use the perpendicularity condition The stated field has components $B_x = 3$, $B_y = 2$, and $B_z = 0$. The acceleration components are $a_x = 4$, $a_y = -x/2$, and $a_z = 0$. Since the acceleration is caused by the magnetic force, its component along the magnetic field must vanish. Thus we can determine the unknown directly without knowing the particle's charge, mass, speed or velocity direction.

$$\vec{a} \cdot \vec{B} = (4)(3) + \left(-\frac{x}{2}\right)(2) = 0.$$

Step 2: Solve and check the component relation The two nonzero contributions to the dot product must cancel, giving

$$12 - x = 0 \quad \Longrightarrow \quad x = 12.$$

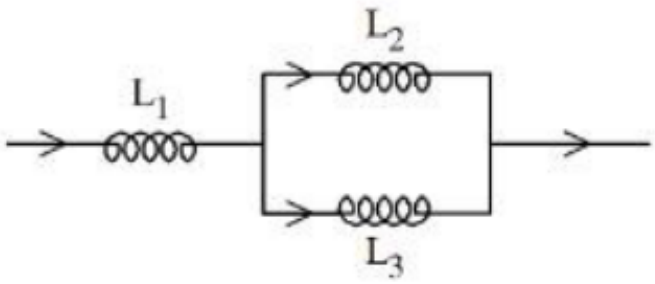
With this value, the acceleration becomes $\vec{a} = (4\hat{i} - 6\hat{j})\,\text{m s}^{-2}$. Its dot product with the given magnetic field is $4 \times 3 - 6 \times 2 = 0$, confirming that the acceleration is perpendicular to the field. The negative sign in the $\hat{j}$ component is essential: using $+x/2$ would give the opposite sign for the requested parameter. The numerical value is consistent with the component units specified in the question.

Step 3: Conclusion The required numerical value is $\boxed{x = 12}$.

Quick Tip to Solve: For acceleration due only to a magnetic field, set $\vec{a} \cdot \vec{B} = 0$; charge, mass and velocity need not be found.

---

50. In the given circuit below inductance values of L_1, L_2 and L_3 are same. The magnetic energy stored in the entire circuit is (U_t) and that stored in the L_2 inductor is (U_l). U_t/U_l is ____ .
(Ignore the mutual inductance if any) [4 marks]



Solution:

Concept:

- The magnetic energy of an ideal inductor of inductance L carrying current I is $U = \frac{1}{2}LI^2$. When mutual inductance is ignored, the total stored energy is the sum of the individual energies.
- The circuit has L_1 in series with the parallel combination of equal inductors L_2 and L_3. Equal parallel branches share the supplied current equally under the usual zero-initial-current condition.

Step 1: Find the currents in the three inductors Let the common inductance be L, and let I be the current through the series inductor L_1. At the junction, this current splits between L_2 and L_3. Since these inductances are equal and have the same voltage across them, their currents are equal. Current conservation then gives

$$I_2 = I_3 = \frac{I}{2}, \qquad I_1 = I.$$

The relevant energy depends on the square of the current, so the series inductor stores more energy than either parallel inductor even though all three inductances are identical.

Step 2: Calculate the total energy and form the ratio The separate stored energies are

$$U_1 = \frac{1}{2}LI^2, \qquad U_2 = U_3 = \frac{1}{2}L\left(\frac{I}{2}\right)^2 = \frac{1}{8}LI^2.$$

Consequently, the energy of the complete circuit is

$$U_t = U_1 + U_2 + U_3 = \left(\frac{1}{2} + \frac{1}{8} + \frac{1}{8}\right)LI^2 = \frac{3}{4}LI^2.$$

The quantity denoted by U_l is the energy in L_2, so $U_l = LI^2/8$. Dividing gives

$$\frac{U_t}{U_l} = \frac{3LI^2/4}{LI^2/8} = 6.$$

The inductance and the overall current cancel, as required for this dimensionless ratio. Equivalently, the three energies are in the ratio $4:1:1$, whose sum is six times either branch energy.

Step 3: Conclusion The required ratio is $\boxed{U_t/U_l = 6}$.

Quick Tip to Solve: Compare energies using squared currents: equal inductors carrying currents $I, I/2, I/2$ store energies in the ratio $4:1:1$.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. How many grams of residue is obtained by heating 2.76 g of silver carbonate?
(Given : Molar mass of C, O and Ag are 12, 16 and $108\ g\ mol^{-1}$ respectively) [4 marks]

(A) $1.08\ g$
(B) $2.16\ g$
(C) $3.24\ g$
(D) $4.32\ g$

Correct Answer: (B) $2.16\ g$

Solution:

Concept:

- Silver carbonate decomposes on strong heating to silver metal, carbon dioxide and oxygen. Silver oxide may appear as an intermediate, but it also decomposes on heating, leaving metallic silver as the final solid residue.
- The balanced overall reaction is $2\,Ag_2CO_3 \longrightarrow 4\,Ag + 2\,CO_2 + O_2$. Thus each mole of Ag_2CO_3 produces two moles of solid Ag.

Step 1: Calculate the molar mass and amount of carbonate Use the atomic masses supplied in the question. Silver carbonate has two silver atoms, one carbon atom and three oxygen atoms per formula unit, so

$$M(Ag_2CO_3) = 2(108) + 12 + 3(16) = 276\,g\,mol^{-1}.$$

The number of moles in the given sample is therefore

$$n(Ag_2CO_3) = \frac{2.76}{276} = 0.0100\,mol.$$

This is an amount of the compound, not of silver atoms; the factor of two in the formula must still be included when finding the metal residue.

Step 2: Convert the amount into the mass of silver residue The decomposition can be viewed in two stages, $Ag_2CO_3 \longrightarrow Ag_2O + CO_2$ followed by $2\,Ag_2O \longrightarrow 4\,Ag + O_2$. Carbon dioxide and oxygen escape as gases, while silver remains as the residue. The overall stoichiometry gives

$$n(Ag) = 2(0.0100) = 0.0200\,mol,$$

$$m(Ag) = 0.0200(108) = 2.16\,g.$$

The residue must weigh less than the initial carbonate because carbon and oxygen are lost as gases. Option (A), $1.08\,g$, misses one of the two silver atoms per formula unit; options (C) and (D) both exceed the initial mass and cannot be the residue from this decomposition.

Step 3: Conclusion The final solid residue is $\boxed{2.16\,g}$ of silver. Hence, the correct option is (B).

Quick Tip to Solve: For complete heating of silver carbonate, use the silver mass fraction $216/276$; do not stop at the intermediate silver oxide.

52. Arrange the following atomic orbitals of multi electron atoms in order of increasing energy.

A. $n = 3, l = 2, m = +1$

B. $n = 4, l = 0, m = 0$

C. $n = 6, l = 1, m = 0$

D. $n = 5, l = 1, m = +1$

E. $n = 2, l = 1, m = +1$

Choose the correct answer from the options given below: [4 marks]

(A) C $<$ D $<$ B $<$ A $<$ E

(B) B $<$ A $<$ E $<$ C $<$ D

(C) E $<$ C $<$ D $<$ B $<$ A

(D) E $<$ B $<$ A $<$ D $<$ C

Correct Answer: (D) E $<$ B $<$ A $<$ D $<$ C

Solution:

Concept:

- For the usual filling order in a multielectron atom, orbital energies are compared using the $(n + l)$ rule: the smaller the sum of principal and azimuthal quantum numbers, the lower the orbital energy.
- If two orbitals have the same $n + l$, the orbital with the smaller principal quantum number n is lower in energy. The magnetic quantum number m specifies orientation and does not affect this ordering in the absence of an external field.

Step 1: Identify the orbitals and their ordering parameters The azimuthal quantum numbers $l = 0, 1, 2$ identify s, p and d orbitals, respectively. The five given entries can therefore be written as

Entry	Orbital	$n + l$	n
A	$3d$	5	3
B	$4s$	4	4
C	$6p$	7	6
D	$5p$	6	5
E	$2p$	3	2

The supplied m values are allowed for their respective l values and only pick a member of each subshell. They do not change the comparison between these subshells.

Step 2: Arrange in ascending energy and select the option The sums are all different: $3 < 4 < 5 < 6 < 7$. There is therefore no need for the equal-sum tie-breaking rule. The increasing order is

$$2p < 4s < 3d < 5p < 6p,$$

$$\text{E} < \text{B} < \text{A} < \text{D} < \text{C}.$$

This order is the sequence asked for, rather than its reverse. Option (A) incorrectly places the highest-sum orbital first; option (B) puts $2p$ after $3d$, although its sum is the smallest. Option (C) puts $6p$ immediately after $2p$, even though $6p$ has the largest sum. Only option (D) preserves the ascending

order of every pair. Ordering by n alone would incorrectly interchange the $4s$ and $3d$ entries.

Step 3: Conclusion The increasing energy order is $\boxed{\text{E} < \text{B} < \text{A} < \text{D} < \text{C}}$. Hence, the correct option is (D).

Quick Tip to Solve: Write $n + l$ beside each entry before comparing; ignore m unless an external field that removes degeneracy is specified.

53. Identify the correct statements from the following :

A. Heisenberg uncertainty principle is applicable to electrons.

B. The size of $2p_x$ orbital is less than the size of $3p_x$ orbital.

C. The energy of 2 s orbital of H atom is equal to the energy of 2 s orbital of Li .

D. The electronic configuration of Cr is $[\text{Ar}]3\ \text{d}^5 4\ \text{s}^1$

Choose the correct answer from the options given below: [4 marks]

(A) A, B and C Only

(B) A, B and D Only

(C) B, C and D Only

(D) A, C and D Only

Correct Answer: (B) A, B and D Only

Solution:

Concept:

- Quantum mechanics applies to electrons, with position and momentum uncertainties satisfying $\Delta x\,\Delta p \geq \hbar/2$. For orbitals of the same type, increasing principal quantum number generally increases radial extent.
- Orbital energies depend on the atomic system as well as the quantum numbers. Chromium has the exceptional ground-state configuration $[\text{Ar}]\,3d^5 4s^1$, associated with the stability of its half-filled d subshell.

Step 1: Evaluate statements A and B Statement A is correct. An electron is a microscopic quantum particle, so it is subject to the Heisenberg uncertainty principle. The principle does not assert that an electron lacks momentum or position; it limits the simultaneous precision with which the two can be specified. Statement B is also correct: both orbitals are p orbitals with the same orientation label, but $3p_x$ has the larger principal quantum number and extends farther from the nucleus than $2p_x$. The subscript x describes orientation and does not reverse this size comparison.

Step 2: Evaluate statements C and D Statement C is false. The $2s$ orbital in hydrogen and the $2s$ orbital in lithium belong to different atoms. Lithium has a different nuclear charge, shielding and electron-electron repulsion, so the orbital energies are not equal. The familiar equality of energies for orbitals of a given n in hydrogen is an intra-atomic degeneracy; it cannot be applied to orbitals in different atoms. Statement D is correct: chromium has atomic number 24, and its ground state is $[\text{Ar}]\,3d^5 4s^1$, rather than $[\text{Ar}]\,3d^4 4s^2$. Thus the correct set is A, B and D only. Options (A), (C) and (D) all include false statement C and each omits one true statement, so none is acceptable.

Step 3: Conclusion The correct statements are A, B and D only. Hence, the correct option is (B).

Quick Tip to Solve: Equal orbital labels do not imply equal energies across different elements; check the atomic system before using degeneracy.

54. What is the mole fraction of water in 10% by weight $(\mathrm{w/w})$ of aqueous urea solution? [Given: Molar mass of $\mathrm{H, O, C}$ and N are $1, 16, 12$ and $14\ \mathrm{g\ mol^{-1}}$ respectively.] [4 marks]

(A) 0.825

(B) 0.032

(C) 0.867

(D) 0.967

Correct Answer: (D) 0.967

Solution:

Concept:

- A 10% by mass aqueous urea solution contains $10\,\mathrm{g}$ of urea per $100\,\mathrm{g}$ of solution. The remaining $90\,\mathrm{g}$ is water; the percentage refers to the total solution mass.
- The mole fraction of water is $x_w = n_w/(n_w + n_u)$, where n_w and n_u are the amounts of water and urea in moles. Masses must be converted to moles before taking this ratio.

Step 1: Choose a mass basis and calculate both amounts Take $100\,\mathrm{g}$ of solution so that the given mass percentage translates directly into $10\,\mathrm{g}$ urea and $90\,\mathrm{g}$ water. From the supplied atomic masses,

$$M(\mathrm{H_2O}) = 2(1) + 16 = 18\,\mathrm{g\,mol^{-1}},$$

$$M(\mathrm{CO(NH_2)_2}) = 12 + 16 + 2(14) + 4(1) = 60\,\mathrm{g\,mol^{-1}}.$$

Therefore, the amounts are

$$n_w = \frac{90}{18} = 5\,\mathrm{mol}, \qquad n_u = \frac{10}{60} = \frac{1}{6}\,\mathrm{mol}.$$

Urea is a molecular solute, so it is counted as urea molecules rather than broken into its constituent atoms.

Step 2: Calculate the mole fraction and compare the options The denominator includes moles of both components, not just the solvent. Thus

$$x_w = \frac{5}{5 + 1/6} = \frac{30}{31} = 0.967741\ldots.$$

The offered value corresponding to this result is 0.967. The complementary urea mole fraction is $1/31$, approximately 0.0323, which explains the tempting option (B); that is the solute fraction rather than the requested water fraction. Options (A) and (C) do not follow from the specified masses and molar masses. As a further check, $x_w + x_u = 30/31 + 1/31 = 1$, and water has the larger fraction because its number of moles is much greater than the urea amount. The result is dimensionless.

Step 3: Conclusion The mole fraction of water is $\boxed{30/31 \approx 0.968}$, corresponding to the listed value 0.967. Hence, the correct option is (D).

Quick Tip to Solve: Use a $100\,\text{g}$ solution basis: $10\,\text{g}$ urea means $90\,\text{g}$ water, then convert both masses to moles.

55. $\text{M}_3\,\text{A}_2$ **is a sparingly soluble salt of molar mass** $y\ \text{g mol}^{-1}$ **and solubility** $x\ \text{g L}^{-1}$**. The ratio of the molar concentration of the anion** (A^{3-}) **to the solubility product of the salt is** [4 marks]

(A) $\frac{1}{54} \cdot \frac{y^4}{x^4}$

(B) $\frac{y^5}{108x^4}$

(C) $108 \cdot \frac{x^5}{y^5}$

(D) $\frac{1}{108}\frac{y^4}{x^4}$

Correct Answer: (A) $\frac{1}{54} \cdot \frac{y^4}{x^4}$

Solution:

Concept:

- The molar solubility s is the mass dissolved per litre divided by the molar mass: $s = x/y$.
- The solubility product uses equilibrium ion concentrations, with each concentration raised to its stoichiometric coefficient.

Step 1: Convert the mass solubility The given x is in grams per litre, whereas ionic concentrations must be in moles per litre. Therefore,

$$s = \frac{x\ \text{g L}^{-1}}{y\ \text{g mol}^{-1}} = \frac{x}{y}\ \text{mol L}^{-1}.$$

One mole of dissolved salt produces three moles of cations and two moles of anions.

Step 2: Write the dissolution equilibrium The charges balance when the metal ion is divalent:

$$\text{M}_3\text{A}_2(\text{s}) \rightleftharpoons 3\,\text{M}^{2+}(\text{aq}) + 2\,\text{A}^{3-}(\text{aq}).$$

Hence $[\text{M}^{2+}] = 3s$ and $[\text{A}^{3-}] = 2s$. The solid has unit activity and does not appear as a concentration factor in the solubility product.

Step 3: Form the requested ratio Substitute both ionic concentrations before simplifying:

$$K_{\text{sp}} = (3s)^3(2s)^2 = 108s^5.$$

Thus

$$\frac{[\text{A}^{3-}]}{K_{\text{sp}}} = \frac{2s}{108s^5} = \frac{1}{54s^4} = \frac{y^4}{54x^4}.$$

Option (C) gives the solubility product itself, not the requested ratio. Option (D) omits the factor of two in the anion concentration. Option (B) contains an extra power of molar mass and does not follow from dividing $2s$ by $108s^5$.

Step 4: Conclusion The required ratio is $\boxed{y^4/(54x^4)}$. Hence, the correct option is (A).

Quick Tip to Solve: For a salt with unequal ion coefficients, write both ion concentrations explicitly before forming the solubility product or any ratio.

56. Arrange the following resultant mixtures in increasing order of their pH values

A. $10\ \text{mL}0.2\text{M}\text{Ca(OH)}_2 + 25\ \text{mL}0.1\text{M}\text{HCl}$

B. $10\ \text{mL}0.01\text{M}\text{H}_2\text{SO}_4 + 10\ \text{mL}0.01\text{M}\text{Ca(OH)}_2$

C. $10\ \text{mL}0.1\text{M}\text{H}_2\text{SO}_4 + 10\ \text{mL}0.1\text{M}\text{KOH}$

Choose the correct answer from the options given below : [4 marks]

(A) $\text{B} < \text{C} < \text{A}$

(B) $\text{C} < \text{A} < \text{B}$

(C) $\text{C} < \text{B} < \text{A}$

(D) $\text{A} < \text{C} < \text{B}$

Correct Answer: (C) $\text{C} < \text{B} < \text{A}$

Solution:

Concept:

- Compare the amounts of acid and base in chemical equivalents before comparing pH. Each mole of Ca(OH)_2 supplies two moles of OH^-.
- Complete neutralisation of one mole of H_2SO_4 requires two moles of OH^-; neutralisation by only one mole leaves acidic $\text{HSO}_4{}^-$.

Step 1: Find the excess base in mixture A In millimoles, the hydroxide amount is $10 \times 0.2 \times 2 = 4$, whereas the hydrochloric acid supplies $25 \times 0.1 = 2.5$ millimoles of H^+. After $\text{H}^+ + \text{OH}^- \longrightarrow \text{H}_2\text{O}$, the excess hydroxide is 1.5 millimoles in 35 mL:

$$[\text{OH}^-] = \frac{1.5}{35} = 0.0429\ \text{mol}\,\text{L}^{-1}.$$

Thus A is strongly alkaline.

Step 2: Identify the result of mixture B The acid contains $10 \times 0.01 = 0.10$ millimoles of H_2SO_4, requiring 0.20 millimoles of hydroxide for full neutralisation. The base supplies exactly $10 \times 0.01 \times 2 = 0.20$ millimoles of hydroxide. There is no excess strong acid or base:

$$\text{H}_2\text{SO}_4 + \text{Ca(OH)}_2 \longrightarrow \text{CaSO}_4 + 2\,\text{H}_2\text{O}.$$

B is approximately neutral in the usual neutralisation treatment; weak sulfate hydrolysis does not alter the ordering.

Step 3: Identify the acid remaining in mixture C The mixture has 1.0 millimole of sulfuric acid but only 1.0 millimole of hydroxide. Only the first acidic proton is neutralised:

$$\text{H}_2\text{SO}_4 + \text{KOH} \longrightarrow \text{KHSO}_4 + \text{H}_2\text{O}.$$

The resulting hydrogen sulfate is acidic because $HSO_4^- \rightleftharpoons H^+ + SO_4^{2-}$. C therefore has the lowest pH, B lies between C and strongly alkaline A. Options (A), (B) and (D) each reverse at least one of these acid-neutral-base comparisons.

Step 4: Conclusion The increasing pH order is $\boxed{C < B < A}$. Hence, the correct option is (C).

Quick Tip to Solve: Count hydroxide ions and acidic protons, not just moles of the compounds: equal moles of sulfuric acid and potassium hydroxide leave an acidic salt.

---

57. First order gas phase reaction

$$A \rightarrow B + C$$

$p_i =$ **initial pressure of gas** A, $p_t =$ **total pressure of the reaction mixture at time** t

Expression of rate constant (k) is [4 marks]

(A) $\frac{1}{t} \ln \frac{p_i}{2p_i - p_t}$

(B) $\frac{1}{t} \ln \frac{2p_i}{p_i - p_t}$

(C) $\frac{1}{t} \ln \frac{p_i}{3p_i - 2p_t}$

(D) $\frac{1}{t} \ln \frac{3p_i}{4p_i - p_t}$

Correct Answer: (A) $\frac{1}{t} \ln \frac{p_i}{2p_i - p_t}$

Solution:

Concept:

- For a first-order reaction, $k = t^{-1} \ln([A]_0/[A]_t)$. At constant volume and temperature, reactant partial pressure is proportional to concentration.
- Total pressure counts all gaseous species. In A $\longrightarrow$ B + C, one mole of reactant produces two moles of products, so the total pressure increases.

Step 1: Relate the pressure increase to reaction progress Let u be the pressure corresponding to the amount of A consumed at time t. Initially only A is present with pressure p_i. The partial pressures at time t are

$$p_A = p_i - u, \qquad p_B = u, \qquad p_C = u.$$

Adding them gives

$$p_t = p_i + u, \qquad u = p_t - p_i.$$

The increase in total pressure equals the pressure-equivalent amount of reactant consumed.

Step 2: Find the remaining reactant pressure Substitute $u = p_t - p_i$ into the expression for A:

$$p_A = p_i - (p_t - p_i) = 2p_i - p_t.$$

This is positive while the reaction is incomplete. It decreases from p_i towards zero as the total pressure increases from p_i towards $2p_i$.

Step 3: Apply the integrated rate law and test the options Use the partial pressure of unreacted A in

the denominator, not the total mixture pressure:

$$k = \frac{1}{t}\ln\left(\frac{p_i}{p_A}\right) = \frac{1}{t}\ln\left(\frac{p_i}{2p_i - p_t}\right).$$

The logarithm is dimensionless and positive for $p_i < p_t < 2p_i$, so k has units of inverse time. Option (B) has a negative denominator once reaction begins. Option (C) uses a different stoichiometric pressure relation and becomes undefined too early. Option (D) does not diverge as the remaining A tends to zero, contradicting the integrated law.

Step 4: Conclusion $k = \frac{1}{t}\ln\frac{p_i}{2p_i - p_t}$. Hence, the correct option is (A).

Quick Tip to Solve: Build a partial-pressure table first. The integrated law needs the pressure of the surviving reactant, which here is twice the initial pressure minus the total pressure.

58. Given below are two statements:

Statement I: The correct order of electronegativity of fluorine, oxygen and nitrogen is $\mathrm{F} > \mathrm{O} > \mathrm{N}$.

Statement II: The oxidation state of oxygen in $\mathrm{OF_2}$ **is +2 and in** $\mathrm{Na_2O}$ **is -2 .**

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- Electronegativity is the tendency of an atom in a chemical bond to attract the shared electron pair. It generally increases across a period as effective nuclear charge increases and atomic size decreases.
- The sum of oxidation states in a neutral compound is zero. Fluorine has oxidation state -1 in its compounds, whereas sodium has oxidation state $+1$.

Step 1: Check the electronegativity order Fluorine, oxygen and nitrogen all belong to the second period. Moving from nitrogen to oxygen to fluorine, the nuclear attraction for bonding electrons increases. Their Pauling electronegativities are approximately 3.0, 3.5 and 4.0, respectively. Thus

$$\mathrm{F} > \mathrm{O} > \mathrm{N}.$$

Statement I is true. This trend concerns attraction for a shared pair, rather than the numerical oxidation state assigned in a compound.

Step 2: Calculate oxygen oxidation states Let the oxidation state of oxygen in $\mathrm{OF_2}$ be x. Each fluorine

atom contributes -1, so charge balance gives

$$x + 2(-1) = 0 \quad \Rightarrow \quad x = +2.$$

For Na_2O, let oxygen have oxidation state y. The two sodium atoms each contribute $+1$:

$$2(+1) + y = 0 \quad \Rightarrow \quad y = -2.$$

Oxygen is positive in its fluoride because fluorine is more electronegative than oxygen. Therefore Statement II is also true.

Step 3: Choose the matching combination Both statements have been checked separately and are true. Option (B) rejects both true statements; option (C) incorrectly rejects the oxidation-state calculation; option (D) incorrectly reverses the verdict on the electronegativity order. Only option (A) agrees with both checks.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: In oxygen fluorides, assign fluorine -1 first and then balance the total charge; do not automatically assign oxygen -2.

---

59. Correct statements from the following are:

A. Nitrogen in oxidation states from +1 to +4 disproportionates in acid medium.

B. Nitrogen has the ability to form $d\pi$ - $p\pi$ multiple bonds with itself and other elements with small size and high electronegativity.

C. N-N single bond is stronger than P-P single bond.

D. Nitrogen has highest density in its group due to small size.

E. The maximum covalency of nitrogen is four since it has only four valence orbitals for bonding.

Choose the correct answer from the options given below: [4 marks]

(A) B, C and D Only

(B) C, D and E Only

(C) A, C and E Only

(D) A and E Only

Correct Answer: (D) A and E Only

Solution:

Concept:

- Nitrogen has valence-shell configuration $2s^2 2p^3$, comprising one $2s$ and three $2p$ orbitals. Its small size favours strong $p\pi$-$p\pi$ multiple bonding, but there are no valence d orbitals.
- Disproportionation converts an element in an intermediate oxidation state into products containing both a higher and a lower oxidation state. Bond strength and density must be assessed separately from atomic size.

Step 1: Identify the valid oxidation-state statement Statement A is the standard group-15 trend:

nitrogen in intermediate positive oxidation states from $+1$ to $+4$ tends to disproportionate in acid medium. Nitrous acid provides a representative example:

$$3\,HNO_2 \longrightarrow HNO_3 + 2\,NO + H_2O.$$

Here nitrogen in $+3$ changes to $+5$ in nitric acid and $+2$ in nitric oxide. Thus A is taken as correct in the stated periodic-property classification.

Step 2: Test the bonding and density claims B is false: nitrogen forms $p\pi$-$p\pi$, rather than $d\pi$-$p\pi$, multiple bonds with suitably small electronegative atoms. C is false: the short $N-N$ single bond brings nonbonding electron pairs close together, causing substantial repulsion; it is weaker than the $P-P$ single bond. The exceptional strength of $N\equiv N$ must not be confused with a nitrogen single bond. D is false: elemental nitrogen is a gas under ordinary conditions and does not have the highest density in its group.

Step 3: Check maximum covalency and select the set E is correct: only four valence orbitals, one $2s$ and three $2p$, are available to nitrogen, and its maximum covalency is four, as in NH_4^+. Nitrogen cannot expand its second-shell octet using valence d orbitals. The correct statements are therefore A and E only. Options (A) and (B) contain the false density claim D, while option (C) contains the false single-bond claim C.

Step 4: Conclusion The correct statements are A and E only. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish the weak $N-N$ single bond from the very strong $N\equiv N$ triple bond; nitrogen multiple bonding uses only p orbitals.

60. Which of the following is NOT a physical or chemical characteristics of interstitial compounds?

[4 marks]

(A) They have high melting points, higher than those of pure metals.

(B) They are very soft and ionic in nature.

(C) They retain metallic conductivity.

(D) They are chemically inert and usually non-stoichiometric.

Correct Answer: (B) They are very soft and ionic in nature.

Solution:

Concept:

- Interstitial compounds form when small atoms such as hydrogen, carbon or nitrogen occupy the holes, or interstices, in a metal crystal lattice without completely replacing the metal framework.
- The resulting solids retain many metallic properties, especially electrical conductivity, while the small inserted atoms strongly hinder movement within the lattice and commonly increase hardness and melting point.

Step 1: Recall the characteristic property pattern The metal lattice persists when small atoms enter its interstitial sites. Mobile electrons remain available for electrical conduction, so these compounds usually retain metallic conductivity. However, the inserted atoms impede the relative motion of

metal layers. Consequently, interstitial compounds are generally very hard, often much harder than the parent metal, rather than very soft. Carbides of transition metals illustrate this combination of hardness and retained metallic character.

Step 2: Check all four alternatives Option (A) describes the usual high melting points of interstitial compounds and is a recognised characteristic. Option (C) correctly states that the compounds retain metallic conductivity. Option (D) is also a standard description: these materials are chemically inert and commonly non-stoichiometric, since interstitial sites can have variable occupancy without requiring a simple fixed atomic ratio. These three options therefore describe accepted physical or chemical properties of the class.

Step 3: Select the statement that is not a characteristic Option (B) says the compounds are very soft and ionic. Both parts conflict with their characteristic behaviour. They are typically hard, and insertion of small atoms does not convert the whole metal framework into an ordinary ionic crystal. The wording asks for the property that is NOT characteristic, so the incorrect description must be selected rather than one of the true properties in the other options.

Step 4: Conclusion Being very soft and ionic in nature is not a characteristic of interstitial compounds. Hence, the correct option is (B).

Quick Tip to Solve: Associate interstitial compounds with hardness, high melting point and retained conductivity; a description calling them soft and ionic is the mismatch.

---

61. The correct statements about metal carbonyls are

A. The metal-carbon bonds in metal carbonyls possess both σ and π character.

B. Due to synergic bonding interactions between metal and CO ligand, the metal-carbon bond becomes weak.

C. The metal-carbon σ bond is formed by the donation of lone pair of electrons on the carbonyl carbon into a vacant orbital of metal.

D. The metal-carbon π bond is formed by the donation of electrons from filled d-orbital of metal into vacant π^* orbital of CO .

Choose the correct answer from the options given below : [4 marks]

(A) A and B Only

(B) A, C and D Only

(C) B and C Only

(D) A and D Only

Correct Answer: (B) A, C and D Only

Solution:

Concept:

- Carbon monoxide is both a sigma donor and a pi acceptor. Its carbon-end lone pair can form a coordinate bond with a vacant metal orbital.
- Back-donation transfers electron density from filled metal d orbitals into the vacant antibonding π^*

orbitals of CO. The two interactions reinforce each other.

Step 1: Identify the sigma component The carbon atom of coordinated CO donates a lone pair into an available orbital on the metal. The donation occurs along the metal–carbon axis and therefore gives a σ bond. Statement C describes this direction of donation correctly. It is not donation from the metal to a filled carbon lone-pair orbital.

Step 2: Identify the pi component A filled metal d orbital can overlap with a suitably oriented vacant π^* orbital of the carbonyl ligand. Metal-to-ligand back-donation produces the π contribution to the metal–carbon bond. Thus statement D is true, and the presence of both donation pathways also makes statement A true.

Step 3: Check the effect of synergy and eliminate the other choices Sigma donation increases the metal electron density available for back-donation; back-donation in turn helps the ligand accept the bonding interaction. Together they strengthen the metal–carbon bond, so statement B is false. Back-donation weakens the internal carbon–oxygen bond because it populates a CO antibonding orbital; this must not be confused with weakening the metal–carbon bond. Options (A) and (C) incorrectly include B, while option (D) omits the true statement C.

Step 4: Conclusion Statements A, C and D are correct; B is incorrect. Hence, the correct option is (B).

Quick Tip to Solve: In metal carbonyls, back-donation strengthens M–C but weakens C–O; identifying which bond is being discussed avoids the trap.

---

62. Given below are two statements:

Statement I: Each electron in e_g orbitals destabilizes the orbitals by $+0.6\Delta_o$ and each electron in the t_{2g} orbitals stabilizes the orbitals by $-0.4\Delta_0$ in an octahedral field on the basis of crystal field theory.

Statement II: All the d - orbitals of the transition metals have the same energy in their free atomic state but when a complex is formed the ligands destroy the degeneracy of these orbitals on the basis of crystal field theory.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are correct

(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (A) Both Statement I and Statement II are correct

Solution:

Concept:

- In an isolated transition-metal atom or ion, the five d orbitals are degenerate. A directional ligand field removes this fivefold degeneracy.
- For an octahedral field, t_{2g} lies at $-0.4\Delta_o$ and e_g lies at $+0.6\Delta_o$, relative to the d-orbital barycentre.

Step 1: Identify the two orbital sets In an octahedral arrangement the six ligands approach along the coordinate axes. The $d_{x^2-y^2}$ and d_{z^2} orbitals point towards these ligands, experience greater repulsion,

and form the higher-energy e_g set. The d_{xy}, d_{yz}, and d_{xz} orbitals lie between the axes and form the lower-energy t_{2g} set.

Step 2: Check the energy shifts in Statement I The energy separation of the two sets is Δ_o. With the barycentre unchanged, three lower orbitals and two upper orbitals give

$$3(-0.4\Delta_o) + 2(+0.6\Delta_o) = 0.$$

Thus each electron in a lower orbital contributes $-0.4\Delta_o$, while each electron in an upper orbital contributes $+0.6\Delta_o$, to the crystal-field energy. Statement I is correct; pairing energy is a separate contribution.

Step 3: Check Statement II and select the pair Without ligands, no particular spatial direction is selected and all five d orbitals have equal energy. Ligands create unequal electrostatic repulsions for different orbital orientations, splitting this degenerate set. Statement II is therefore correct. Option (B) rejects both correct facts, option (C) incorrectly rejects Statement II, and option (D) incorrectly rejects Statement I.

Step 4: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (A).

Quick Tip to Solve: Check octahedral shifts using $3(-0.4) + 2(0.6) = 0$; their difference must also equal one Δ_o.

63. Given below are two statements :

Statement I : On the basis of inductive effect, the order of stability of alkyl carbanions is $CH_3^- > CH_3 - CH_2^- > (CH_3)_2\,CH^- > (CH_3)_3\,C^-$.

Statement II : Allyl and benzyl carbanions are more stabilised by inductive effect and not by resonance effect.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are correct

(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (C) Statement I is correct but Statement II is incorrect

Solution:

Concept:

- A carbanion has an electron-rich carbon centre. Electron-donating alkyl groups destabilize its localized negative charge by the positive inductive effect.
- Resonance can stabilize a carbanion by spreading its negative charge over a conjugated system; this differs from an inductive effect transmitted through sigma bonds.

Step 1: Compare the alkyl carbanions Among the listed localized alkyl carbanions, adding alkyl

substituents increases electron donation towards the already negative carbon. Therefore the stability decreases from methyl through primary and secondary to tertiary:

$$CH_3^- > CH_3CH_2^- > (CH_3)_2CH^- > (CH_3)_3C^-.$$

This is exactly Statement I. It is the reverse of the usual alkyl-carbocation trend, because electron donation helps a positively charged centre but destabilizes a negatively charged one.

Step 2: Identify the stabilization of allyl and benzyl carbanions An allylic carbanion has its lone pair next to a carbon–carbon double bond. The lone pair participates in conjugation, so the negative charge can occur at either terminal carbon in resonance contributors. In a benzyl carbanion the lone pair similarly conjugates with the benzene ring, allowing charge delocalization onto ring positions. The important extra stabilization is therefore resonance, not an assertion that resonance is absent.

Step 3: Evaluate the statement pair Statement II specifically attributes the extra stability to induction and denies resonance; that claim is false. Since I is true and II is false, option (A) fails by accepting II, option (B) fails by rejecting I, and option (D) reverses both truth values. The third listed choice is the only matching combination.

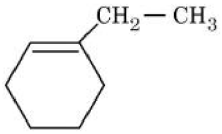
Step 4: Conclusion Statement I is correct but Statement II is incorrect. Hence, the correct option is (C).

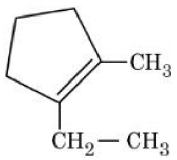
Quick Tip to Solve: For a negative carbon, alkyl donation is destabilizing; before applying that trend, check whether the lone pair is conjugated with a pi system.

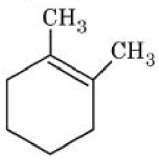
64. "P" is a hydrocarbon of molecular formula C_8H_{14}. On ozonolysis, "P" forms "Q". "Q" on treatment with alkali under reflux condition produces "R", which on treatment with $I_2/NaOH$ gives a yellow precipitate. Acidification of the solution gives "S". The structure of "S" is given below. The correct structure of "P" is: [4 marks]

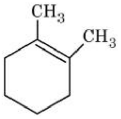


(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- Ozonolysis cleaves an alkene double bond and changes each alkene carbon into a carbonyl carbon. A cyclic alkene therefore gives an open-chain dicarbonyl compound.
- Intramolecular aldol condensation under reflux forms a favourable ring and then eliminates water. A methyl ketone, $RCOCH_3$, undergoes the iodoform reaction to give yellow CHI_3 and a carboxylate, which gives the acid on acidification.

Step 1: Work backwards from the acid The displayed acid is 2-methylcyclopent-1-ene-1-carboxylic acid. The iodoform step replaces an acetyl group, $-COCH_3$, by $-COO^-$, without changing its attachment to the ring. Therefore R must be 1-acetyl-2-methylcyclopent-1-ene. Its acetyl methyl is removed as the yellow precipitate CHI_3, while the carbonyl carbon remains in the acid group.

Step 2: Test the fourth alkene Option (D) is 1,2-dimethylcyclohex-1-ene. Cleaving its double bond opens the six-membered ring and gives

$$CH_3CO(CH_2)_4COCH_3.$$

This diketone is Q. An enolate formed at a chain methylene adjacent to one carbonyl attacks the other carbonyl intramolecularly. Counting that enolate carbon, the three intervening methylenes and the attacked carbonyl carbon gives a five-membered ring. Dehydration produces the conjugated enone R, with an acetyl group at one alkene carbon and a methyl group at the other.

Step 3: Complete the sequence and reject the other structures Treatment of the acetyl group with $I_2/NaOH$ gives the carboxylate and CHI_3; acidification then gives exactly S. Option (A) instead opens to a substituted dialdehyde and does not supply the required methyl-ketone route. Option (B) gives an aldehyde and an ethyl ketone rather than the required symmetrical dimethyl diketone. Option (C) opens the five-membered ring to a diketone with only three intervening methylenes and unequal terminal alkyl groups; it cannot give the stated five-membered enone with the required acetyl and methyl arrangement by this condensation.

Step 4: Conclusion P is 1,2-dimethylcyclohex-1-ene, the structure in (D). Its ozonolysis, intramolecular aldol condensation and iodoform reaction give the displayed acid S. Hence, the correct option is (D).

Quick Tip to Solve: For an iodoform sequence ending in an acid, first replace $-COOH$ in the product by $-COCH_3$, then reverse the aldol ring closure.

65. For the following Friedel Craft's alkylation reaction, which of the statements are correct?

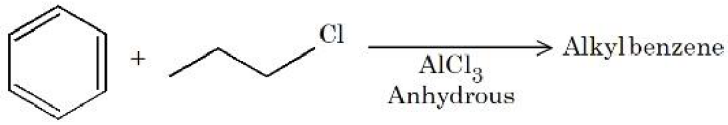
A. Major product is n-propyl benzene.

B. iso-propyl carbocation intermediate is also generated.

C. Multiple substitution is inevitable.

D. Introducing electron-donating substituent on benzene will not produce any alkyl benzene.

Choose the correct answer from the options given below : [4 marks]



(A) A and D only

(B) B and C only

(C) A and C only

(D) B and D only

Correct Answer: (B) B and C only

Solution:

Concept:

- Anhydrous $AlCl_3$ activates an alkyl halide to generate an electrophile for electrophilic aromatic substitution. Carbocation rearrangement can change the carbon skeleton of the attached alkyl group.
- An alkyl group activates a benzene ring by electron donation. Consequently, an alkylbenzene can react further, making polyalkylation a characteristic difficulty of Friedel–Crafts alkylation.

Step 1: Check the electrophile and the major product The figure uses benzene and 1-chloropropane in the presence of anhydrous $AlCl_3$. The initially generated primary propyl electrophile rearranges by a hydride shift to the more stable secondary isopropyl carbocation. Attack of benzene on this secondary electrophile, followed by loss of a proton and restoration of aromaticity, gives isopropylbenzene (cumene) as the major product. Thus A, which assigns n-propylbenzene as the major product, is false; B is true.

Step 2: Check the possibility of repeated substitution The first alkyl group donates electron density to the ring and increases its reactivity towards another electrophile. The alkylated product therefore competes effectively with benzene for further substitution. This is the usual polyalkylation problem of the reaction and is the point of statement C. Hence C is accepted as correct in the stated Friedel–Crafts context. Excess benzene is commonly used in practice to reduce such multiple substitution; it does not change the activating effect of the alkyl group.

Step 3: Evaluate the substituent claim and the choices An electron-donating substituent generally activates an aromatic ring towards electrophilic substitution, so merely introducing such a substituent does not imply that no alkylbenzene can form. Statement D is therefore false. Specific substituents can interact with a Lewis acid, but that cannot justify the blanket claim in D. The true pair is B and C. Option (A) contains two false statements, (C) includes false A, and (D) includes false D.

Step 4: Conclusion Statements B and C are correct: the isopropyl carbocation is generated and

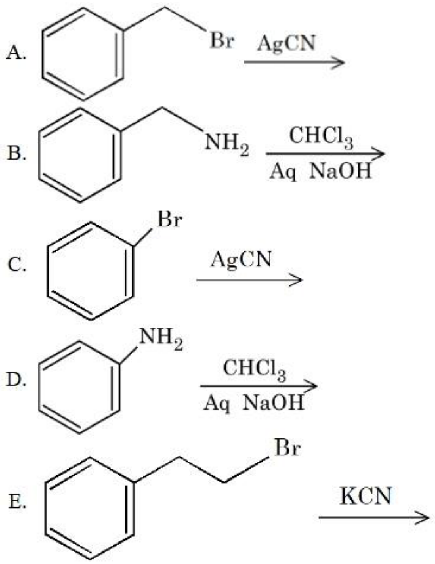
polyalkylation is a characteristic tendency. Hence, the correct option is (B).

Quick Tip to Solve: Check rearrangement before naming the Friedel–Crafts product, then remember that the newly added alkyl group makes further substitution easier.

66.

Benzyl isocyanide can be obtained from

Choose the correct answer from the options given below : [4 marks]



(A) A and B Only

(B) A and C Only

(C) B and D Only

(D) D and E Only

Correct Answer: (A) A and B Only

Solution:

Concept:

- An alkyl halide reacts with predominantly covalent $AgCN$ through nitrogen to form an isocyanide, $R-NC$. Ionic KCN usually gives the carbon-bonded nitrile, $R-CN$.
- A primary amine gives an isocyanide with chloroform and strong alkali in the carbylamine reaction. The group already attached to the amine nitrogen remains attached to nitrogen in the product.

Step 1: Identify the required carbon skeleton Benzyl isocyanide has the connectivity $C_6H_5CH_2-NC$. The methylene group between the benzene ring and nitrogen is essential: phenyl isocyanide, C_6H_5-NC, is a different compound. We must therefore check both the reagent and the carbon group of every starting material, rather than select every route that can produce some isocyanide.

Step 2: Check reactions A and B In A the substrate is benzyl bromide. Reaction with $AgCN$ replaces

bromide by nitrogen-bonded cyanide and produces $C_6H_5CH_2NC$, so A gives the target. In B the substrate is benzylamine, $C_6H_5CH_2NH_2$. Its primary amino group undergoes the carbylamine reaction with the displayed chloroform and alkali, preserving the benzyl group and yielding benzyl isocyanide. Thus B also gives the target.

Step 3: Exclude C, D and E C starts from bromobenzene, an aryl halide that does not undergo the ordinary alkyl-halide substitution with $AgCN$; moreover, it lacks the benzyl methylene carbon. D starts from aniline, so its carbylamine product is phenyl isocyanide, not benzyl isocyanide. E starts from a longer-chain bromoethylbenzene and uses KCN, which gives a nitrile through carbon attack rather than the required isocyanide. Therefore neither the reagent nor the carbon skeleton in E leads to the named target. Only A and B survive all connectivity checks.

Step 4: Conclusion Benzyl isocyanide can be obtained from reactions A and B only. Hence, the correct option is (A).

Quick Tip to Solve: Write the target as $C_6H_5CH_2{-}NC$: check the benzyl methylene first, then distinguish $AgCN$ (isocyanide) from KCN (nitrile).

67. Consider compounds A, B and C with following structural formulae

$$A = CH_3 - CH_2 - CH_2 - CH_2 - CH_2 - OH$$
$$B = CH_2 = CH - CH_2 - CH_2 - CH_3$$
$$C = HO - CH_2 - CH_2 - CH(OH) - CH_3$$

For the conversion of B from A , reagent (D) required is ____ and structural formula of product (E) obtained when C undergoes same reaction using excess reagent (D) is ____ . [4 marks]

(A)

D	E
Conc. H_2SO_4	$CH_2 = CH - CH(OH)CH_3$

(B)

D	E
PCC	$HO - CH_2 - CH_2 - CH = CH_2$

(C)

D	E
PCC	$CH_2 = CH - CH = CH_2$

(D)

D	E
Conc. H_2SO_4 or H_3PO_4	$CH_2 = CH - CH = CH_2$

Correct Answer: (D)

D	E
Conc. H_2SO_4 or H_3PO_4	$CH_2 = CH - CH = CH_2$

Solution:

Concept:

- Dehydration removes the elements of water from an alcohol: the hydroxyl group and a hydrogen on an adjacent carbon are lost, producing a carbon-carbon double bond.
- Concentrated H_2SO_4 or H_3PO_4, with heating, acts as an acid dehydrating reagent; PCC is an oxidising reagent rather than an alkene-forming dehydrating reagent.

Step 1: Identify the transformation of A into B A is pentan-1-ol and B is pent-1-ene. Both have five carbon atoms, so the carbon skeleton is retained while the terminal hydroxyl group is replaced by a double bond. The required change is elimination of water, not oxidation to an aldehyde.

$$CH_3CH_2CH_2CH_2CH_2OH \xrightarrow[\text{heat}]{\text{acid}} CH_2{=}CHCH_2CH_2CH_3 + H_2O$$

Thus reagent D must be a dehydrating acid. This immediately excludes both options containing PCC.

Step 2: Apply dehydration to both hydroxyl groups of C The compound C is butane-1,3-diol, $HOCH_2CH_2CH(OH)CH_3$. With excess dehydrating reagent, both hydroxyl groups can be eliminated with adjacent hydrogens. One elimination creates the terminal double bond at the first two carbon atoms; the other creates the double bond at the third and fourth carbon atoms. The resulting product is the conjugated diene buta-1,3-diene.

$$HOCH_2CH_2CH(OH)CH_3 \xrightarrow[\text{heat}]{\text{excess acid}} CH_2{=}CHCH{=}CH_2 + 2\,H_2O$$

The mono-dehydrated alcohol in option (A) still contains an hydroxyl group and does not represent complete dehydration with excess reagent.

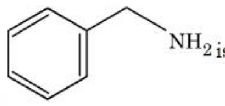
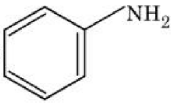
Step 3: Conclusion D is concentrated H_2SO_4 or H_3PO_4, and E is $CH_2{=}CH{-}CH{=}CH_2$, buta-1,3-diene. Hence, the correct option is (D).

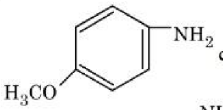
Quick Tip to Solve: First distinguish dehydration from oxidation; then use the word excess to check that neither hydroxyl group remains in the final product.

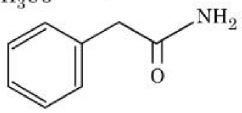
68.

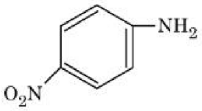
Identify the incorrect statements.

Choose the correct answer from the options given below: [4 marks]

A.  is a stronger base than 

B.  can be synthesized by Gabriel phthalimide synthesis.

C.  $\xrightarrow{Br_2,\ NaOH}$ primary aromatic amine

D.  $\xrightarrow[(ii)\ \Delta]{(i)\ NaNO_2,\ HCl,0°C}$ product will dissolve in NaOH

(A) A and D Only

(B) A and C Only

(C) B and C Only

(D) A and B Only

Correct Answer: (C) B and C Only

Solution:

Concept:

- A nitrogen lone pair is less available for protonation when it is conjugated directly with a benzene ring; an intervening saturated carbon breaks that conjugation.
- Gabriel synthesis gives primary alkyl amines, whereas Hofmann bromamide degradation converts $RCONH_2$ to RNH_2, removing the carbonyl carbon.

Step 1: Check the basicity and Gabriel statements In statement A, the left compound is benzylamine, $C_6H_5CH_2NH_2$, and the right compound is aniline, $C_6H_5NH_2$. In benzylamine the nitrogen is separated from the ring by CH_2, so its lone pair is not delocalised into the aromatic ring. In aniline it is delocalised by resonance. Benzylamine is therefore the stronger base and A is correct. Statement B proposes making an aromatic amine, p-anisidine, by Gabriel phthalimide synthesis. Aryl halides do not undergo the required ordinary nucleophilic substitution with the phthalimide ion. Gabriel synthesis is used for primary alkyl amines, so B is incorrect.

Step 2: Identify the products in the remaining statements The amide in C is phenylacetamide, $C_6H_5CH_2CONH_2$. Hofmann degradation with $Br_2/NaOH$ produces $C_6H_5CH_2NH_2$, not an amine whose nitrogen is directly attached to the aromatic ring. Benzylamine is a primary aralkyl amine, so the claim of a primary aromatic amine is incorrect. In D, p-nitroaniline is diazotised at $0°C$ and the diazonium salt is hydrolysed on heating to p-nitrophenol. This phenol reacts with NaOH to give the soluble sodium phenoxide salt; the nitro group further increases its acidity. Thus D is correct.

Step 3: Conclusion The incorrect statements are B and C only. Hence, the correct option is (C).

Quick Tip to Solve: In Hofmann degradation remove only the carbonyl carbon: a remaining CH_2 between the ring and nitrogen makes the product aralkyl, not aryl, amine.

---

69. Identify the correct statements.

A. Glucose exists in two anomeric forms.

B. Anomers of glucose differ in configuration at $C-1$ in cyclic hemiacetal structure.

C. Melting point of α - anomer of glucose is greater than β - anomer.

D. Specific rotation of α - anomer is $+19°$ while for β - anomer is $+112°$

E. α and β - anomers of glucose are prepared by crystallization of saturated glucose solution at 303 K and 371 K respectively.

Choose the correct answer from the options given below : [4 marks]

(A) A and B Only

(B) B and C Only

(C) A, B and D Only

(D) A, B and E Only

Correct Answer: (D) A, B and E Only

Solution:

Concept:

- The cyclic hemiacetal of glucose contains an anomeric carbon at C-1. Its two configurations are the α and β anomers.
- Anomers have different physical properties. For D-glucose the α form has specific rotation about $+112^\circ$, and the β form about $+19^\circ$.

Step 1: Check the structural statements A and B Open-chain glucose has an aldehyde group at C-1. Intramolecular addition of an hydroxyl group forms a cyclic hemiacetal, creating a new stereogenic centre at C-1. Two arrangements at this centre give α-D-glucose and β-D-glucose, while the other stereocentres retain their configurations. Thus glucose exists in two anomeric forms, so A is correct. Their difference is specifically at C-1 in the cyclic hemiacetal structure, so B is also correct; they do not differ at every chiral carbon.

Step 2: Check the physical data and preparation conditions The melting point of the α form is about 419 K, whereas that of the β form is about 423 K. The alpha form therefore has the lower melting point, making C false. Statement D reverses the rotations: $+112^\circ$ belongs to the alpha form and $+19^\circ$ to the beta form, so D is false. Crystallisation of a concentrated glucose solution at about 303 K gives the alpha form; crystallisation from a hot saturated aqueous solution at about 371 K gives the beta form. Hence E is correct. These solid forms can interconvert after dissolution through the open-chain form, which explains mutarotation without changing the identity of the isolated crystals.

Step 3: Conclusion The correct statements are A, B and E only. Hence, the correct option is (D).

Quick Tip to Solve: Associate alpha glucose with the larger positive specific rotation but the lower melting point; this prevents swapping the two pairs of data.

---

70. Given below are two statements :

Statement I : Sodium dichromate and potassium dichromate are classified as primary standards in titrimetric analysis.

Statement II : Phenolphthalein is a weak base, therefore it dissociates in acidic medium.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (B) Both Statement I and Statement II are false

Solution:

Concept:

- A primary standard must have accurately known composition, high purity and stability, and should not absorb atmospheric moisture during weighing.
- Phenolphthalein behaves as a weak acid indicator. Its colourless acid form and pink basic form are

related by an acid-base equilibrium.

Step 1: Evaluate the primary-standard statement Potassium dichromate, $K_2Cr_2O_7$, is a suitable primary standard in redox titrations because it can be obtained pure and is sufficiently stable and non-hygroscopic for accurate weighing. Sodium dichromate, $Na_2Cr_2O_7$, is hygroscopic and does not satisfy the same requirement for direct preparation of a solution of accurately known concentration by simple weighing. The statement groups both salts as primary standards. Since this classification is not valid for sodium dichromate, Statement I is false. Similar redox chemistry alone does not establish primary-standard suitability; physical stability during storage and weighing also matters.

Step 2: Evaluate the indicator statement Phenolphthalein is a weak acid rather than a weak base. Its simplified indicator equilibrium may be represented by

$$\mathrm{HIn} \rightleftharpoons \mathrm{H^+} + \mathrm{In^-}.$$

Here the acid form is colourless and the deprotonated form is pink in the usual indicator range. In an acidic medium, an increased hydrogen-ion concentration shifts the equilibrium towards the undissociated acid form. In a basic medium, removal of hydrogen ions favours the coloured deprotonated form. Statement II incorrectly calls phenolphthalein a weak base and incorrectly attributes its dissociation to acidic conditions. It is therefore false. Options claiming either statement to be true are excluded.

Step 3: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (B).

Quick Tip to Solve: For primary standards check hygroscopicity as well as purity; for phenolphthalein remember colourless in acid and pink in base.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. Consider the following species:

$$\mathrm{BrF_5}, \mathrm{XeF_5^-}, \mathrm{BF_4^-}, \mathrm{ICl_4^-}, \mathrm{XeF_4}, \mathrm{SF_4}, \mathrm{NH_4^+}, \mathrm{ClF_3}, \mathrm{XeF_2}, \mathrm{ICl_2^-}$$

Number of species having $\mathrm{sp^3\ d}$ hybridized central atom is ____ . [4 marks]

Solution:

Concept:

- In the conventional hybridisation description, five electron domains around a central atom correspond to sp^3d. Count both sigma-bonding pairs and lone pairs.
- For these main-group species with monovalent terminal atoms, the steric number is $\frac{1}{2}(V + M + q_- - q_+)$, where V is the central valence-electron count and M is the number of terminal atoms.

Step 1: Find the species with five electron domains For SF_4, the steric number is $(6+4)/2 = 5$: it has four bonding pairs and one lone pair. For ClF_3, it is $(7+3)/2 = 5$: three bonding pairs and two lone pairs. For XeF_2, it is $(8+2)/2 = 5$: two bonding pairs and three lone pairs. For ICl_2^-, the negative charge adds one electron and gives $(7+2+1)/2 = 5$: two bonding pairs and three lone pairs. All four therefore have the conventional sp^3d assignment, even though their molecular shapes are different.

Step 2: Exclude the remaining species explicitly For BrF_5, $(7+5)/2 = 6$; for ICl_4^-, $(7+4+1)/2 = 6$; and for XeF_4, $(8+4)/2 = 6$. Their conventional assignment is sp^3d^2. The ion XeF_5^- has $(8+5+1)/2 = 7$ domains, hence sp^3d^3. Finally, BF_4^- has $(3+4+1)/2 = 4$ domains and NH_4^+ has $(5+4-1)/2 = 4$; both are sp^3. Thus none of these six adds to the required count. It is electron-domain geometry, rather than just the number of attached atoms, that determines this classification.

Step 3: Conclusion The required species are SF_4, ClF_3, XeF_2 and ICl_2^-. The number of species is $\boxed{4}$.

Quick Tip to Solve: Count lone pairs before assigning hybridisation: two bonds can still give five domains, as in XeF_2 and ICl_2^-.

72. In an estimation of sulphur by Carius method 0.2 g of the substance gave 0.6 g of $BaSO_4$. The percentage of sulphur in the substance is ____%.

(Given molar mass in $gmol^{-1}$ S : 32, $BaSO_4$: 231) [4 marks]

Solution:

Concept:

- In the Carius method, sulphur is oxidised to sulphate and precipitated as $BaSO_4$. Each mole of $BaSO_4$ contains one mole of sulphur atoms.
- The mass percentage is $100 m_S/m_{sample}$. Use the molar masses supplied in the question for the numerical calculation.

Step 1: Calculate the mass of sulphur in the precipitate The mass of precipitated $BaSO_4$ is 0.6 g. Using the printed molar mass $231\,g\,mol^{-1}$, its amount is

$$n(BaSO_4) = \frac{0.6}{231}\,\text{mol}.$$

One formula unit contains one sulphur atom, so the amount of sulphur is the same. The sulphur mass in the original 0.2 g sample is therefore

$$m_S = \frac{0.6}{231} \times 32 = 0.0831169\,g.$$

The full precipitate mass is not sulphur mass: it also contains barium and oxygen. The molar-mass ratio isolates the sulphur contribution.

Step 2: Convert the sulphur mass to a percentage Divide the sulphur mass by the mass of the original organic substance, not by the mass of precipitate, and multiply by 100:

$$\begin{aligned}\%S &= \frac{0.0831169}{0.2} \times 100 \\ &= \frac{0.6 \times 32 \times 100}{231 \times 0.2} \\ &= 41.5584\%.\end{aligned}$$

Thus the unrounded percentage is approximately 41.56%. The numerical entry to the nearest integer is 42. The calculation retains the supplied value 231 for the molar mass of barium sulphate throughout;

substituting a different tabulated molar mass would not follow the data given here. As a check, the sulphur mass is below the sample mass and the resulting percentage lies between 0 and 100.

Step 3: Conclusion The sulphur content calculated from the supplied data is 41.56%, giving the nearest-integer numerical answer $\boxed{42}$.

Quick Tip to Solve: Use precipitate mass times $32/231$ to obtain sulphur mass first, then divide by the original sample mass and multiply by 100.

73. One mole of phenol is treated with dilute HNO_3 at 298 K to give a mixture of products. The mixture is separated by steam distillation. The steam volatile compound (X) is separated. The increase in percentage of oxygen in (X) with respect to phenol is ____ $\times 10^{-1}\%$

(Given molar mass in $gmol^{-1}$ H : 1, C : 12, N : 14, O : 16) [4 marks]

Solution:

Concept:

- Dilute HNO_3 nitrates phenol at room temperature to give ortho- and para-nitrophenol because the hydroxyl group activates the ortho and para positions.
- Ortho-nitrophenol has intramolecular hydrogen bonding and is steam volatile. Para-nitrophenol forms stronger intermolecular associations and is less volatile.
- The oxygen mass percentage is $100 \times$ mass of oxygen per mole/molar mass. The increase in a percentage composition is obtained by subtracting the initial percentage from the final percentage.

Step 1: Identify the steam-volatile product The product separated by steam distillation is $X =$ ortho-nitrophenol. Nitration replaces one ring hydrogen by NO_2, so phenol C_6H_6O becomes $C_6H_5NO_3$. The hydroxyl oxygen remains present: the product contains three oxygen atoms, not two. The ortho and para isomers have identical molecular formulae, but their different hydrogen bonding explains the separation.

Step 2: Calculate the two oxygen percentages With the given atomic masses, the molar masses are

$$M(C_6H_6O) = 6(12) + 6(1) + 16 = 94\,\text{g mol}^{-1},$$

$$M(C_6H_5NO_3) = 6(12) + 5(1) + 14 + 3(16) = 139\,\text{g mol}^{-1}.$$

Hence the oxygen mass percentages are

$$P_{\text{phenol}} = \frac{16}{94} \times 100 = 17.0213\%, \qquad P_X = \frac{48}{139} \times 100 = 34.5324\%.$$

The one-mole quantity does not affect these composition percentages; each is a ratio of masses within its own compound.

Step 3: Express the difference in the requested scale Subtract the percentage compositions before applying the factor printed after the blank:

$$P_X - P_{\text{phenol}} = 34.5324 - 17.0213 = 17.5111\%.$$

If the blank is n, then $0.1n = 17.5111$, giving $n = 175.111$. This is the change in percentage composition, not the relative percentage increase obtained by dividing the change by P_{phenol}.

Step 4: Conclusion X is ortho-nitrophenol. The oxygen percentage increases by approximately $17.51\% = 175.1 \times 10^{-1}\%$, so the integer numerical entry is $\boxed{175}$.

Quick Tip to Solve: Count the retained hydroxyl oxygen as well as the two nitro oxygens. Find the difference of the two mass percentages first, then divide by the printed 10^{-1} factor.

---

74. The values of pressure equilibrium constant recorded at different temperatures for the following equilibrium reaction have been given below $\mathrm{A(g)} \rightleftharpoons \mathrm{B(g)} + \mathrm{C(g)}$

$\frac{1}{\mathrm{T}}\ (\mathrm{K}^{-1})$	$\log_{10}\ \mathrm{K_p}$
0.05	3.5
0.06	2.5
0.07	1.5

The magnitude of $\frac{\Delta \mathrm{H}^\circ}{\mathrm{R}}$ calculated from the above data is ____ . (Nearest integer) [4 marks]

Solution:

Concept:

- The van't Hoff equation relates the temperature dependence of the equilibrium constant to the standard reaction enthalpy: $\mathrm{d}\ln K_p/\mathrm{d}(1/T) = -\Delta H^\circ/R$.
- For a base-ten logarithm, $\ln K_p = (\ln 10)\log_{10} K_p$. Therefore the slope of a plot of $\log_{10} K_p$ against $1/T$ is $-\Delta H^\circ/(R\ln 10)$.

Step 1: Calculate the slope from the tabulated values Take the first two rows with $x = 1/T$ and $y = \log_{10} K_p$. The slope is

$$m = \frac{2.5 - 3.5}{0.06 - 0.05} = \frac{-1.0}{0.01} = -100\,\mathrm{K}.$$

Using the first and third rows independently gives

$$m = \frac{1.5 - 3.5}{0.07 - 0.05} = \frac{-2.0}{0.02} = -100\,\mathrm{K}.$$

Thus all three points have the same slope. The horizontal variable is reciprocal temperature, so its change must not be replaced by a temperature difference.

Step 2: Apply the logarithm conversion The integrated van't Hoff form for this straight-line data is

$$\log_{10} K_p = -\frac{\Delta H^\circ}{R\ln 10}\frac{1}{T} + C,$$

where C is independent of temperature over the interval. Comparing its coefficient of $1/T$ with the measured slope gives

$$\frac{\Delta H^\circ}{R} = -m\ln 10 = 100(2.302585) = 230.2585\,\mathrm{K}.$$

The factor $\ln 10$ is essential because the table uses $\log_{10}$, whereas the differential van't Hoff equation uses a natural logarithm.

Step 3: Check the sign and round the magnitude As $1/T$ decreases, temperature increases and the table shows a larger K_p. Heating therefore favours products, consistent with the positive enthalpy found above. The requested magnitude is already positive. Rounding 230.2585 to the nearest integer yields 230. Since enthalpy per mole divided by the gas constant has units of temperature, the ratio has unit kelvin.

Step 4: Conclusion The magnitude is $|\Delta H^\circ/R| \approx \boxed{230\,\mathrm{K}}$. The numerical entry is $\boxed{230}$.

Quick Tip to Solve: For a $\log_{10} K_p$ versus $1/T$ table, multiply the magnitude of its slope by 2.303. Use the negative slope to confirm a positive reaction enthalpy.

---

75. If the half life of a first order reaction is 6.93 minutes then the time required for completion of 99% of the reaction will be ____ minutes.

(Given : $\log 2 = 0.3010$ **)** [4 marks]

Solution:

Concept:

- A first-order reaction obeys $\ln([A]_0/[A]_t) = kt$, where $[A]_0$ and $[A]_t$ are the initial and remaining reactant concentrations and k is the rate constant.
- Its half-life is $t_{1/2} = \ln 2/k$, independent of initial concentration. Combining this expression with the integrated rate law eliminates k and gives $t/t_{1/2} = \log_{10}([A]_0/[A]_t)/\log_{10} 2$.

Step 1: Find the fraction of reactant remaining Completion of 99% means that 99% of the initial reactant has been consumed. The remaining fraction is therefore $1\% = 0.01$, not 0.99. Thus

$$[A]_t = 0.01[A]_0, \qquad \frac{[A]_0}{[A]_t} = 100, \qquad \log_{10} 100 = 2.$$

The actual initial concentration is unnecessary because only the concentration ratio enters the rate law.

Step 2: Eliminate the rate constant using the half-life Write both time relations in the same logarithm base:

$$t = \frac{2.303}{k}\log_{10}\frac{[A]_0}{[A]_t}, \qquad t_{1/2} = \frac{2.303}{k}\log_{10} 2.$$

Dividing cancels both k and the logarithm-conversion factor. Substituting the given half-life and $\log 2$ gives

$$t = 6.93\frac{2}{0.3010} = \frac{13.86}{0.3010} = 46.0465\,\mathrm{min}.$$

The units remain minutes because the ratio of logarithms is dimensionless and the given half-life is in minutes.

Step 3: Check the result against successive half-lives The required number of half-lives is $2/0.3010 = 6.6445$. After six half-lives, the remaining fraction is $1/64 = 1.5625\%$; after seven it is $1/128 =$

0.78125%. A remaining fraction of 1% must occur between these two times. Indeed, $6(6.93) = 41.58$ minutes and $7(6.93) = 48.51$ minutes bracket the calculated value. First-order decay slows as reactant is consumed, so the time is not proportional to the percentage completed.

Step 4: Conclusion The time required is $\boxed{46.05\,\text{min}}$, approximately $\boxed{46\,\text{min}}$. The integer numerical entry is 46.

Quick Tip to Solve: Convert percentage completion into the fraction remaining first. Then use $t = t_{1/2}\log_{10}(1/f)/\log_{10}2$; for 99% completion, $f = 0.01$.