

JEE Main 2026 Paper 1 (B.E./B.Tech) Apr 6 Shift 1

Question Paper with Solutions PDF



Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours

Maximum Marks: 300

General Instructions

- (i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2026, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 6 April 2026, Shift 1. The total duration is 3 hours.
- (ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.
- (iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.
- (iv) Marking scheme (multiple choice questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (v) Marking scheme (numerical value questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $[\bullet]$ denote the greatest integer function. If the domain of the function $f(x) = \sin^{-1} \left(\frac{x + [x]}{3} \right)$ is $[\alpha, \beta)$, then $\alpha^2 + \beta^2$ is equal to: [4 marks]

- (A) 2
- (B) 5
- (C) 10
- (D) 13

Correct Answer: (B) 5

Solution:

Concept:

- The principal inverse sine is real exactly when its argument lies in $[-1, 1]$. Thus the condition is $-3 \leq x + [x] \leq 3$.
- On each integer interval $[n, n + 1)$, the floor is the constant n , while $x + [x] = x + n$ increases with x . At an integer it jumps upward, so endpoint inclusion must be checked directly.

Step 1: Translate the inverse-sine condition The denominator is positive, so multiplying the two inequalities by 3 preserves their directions. For $-1 \leq x < 0$, the floor is -1 and the sum ranges from -2 to -1 . For $0 \leq x < 1$, the sum ranges from 0 to 1 . For $1 \leq x < 2$, it ranges from 2 to 3 , with 3 excluded. Every point of these three intervals is therefore allowed.

Step 2: Check both ends of the domain At $x = -1$, the argument is $-2/3$, which is valid. If $x < -1$, then $[x] \leq -2$, giving $x + [x] < -3$. At $x = 2$, the sum equals 4 and is invalid; for larger x it only increases. Hence the entire domain is exactly $[-1, 2)$. Thus $\alpha = -1$, $\beta = 2$, and $\alpha^2 + \beta^2 = 1 + 4 = 5$. Values 2 , 10 and 13 fail this endpoint calculation.

Step 3: Conclusion The required value is 5 . Hence, the correct option is (B).

Quick Tip to Solve: Check the floor at the integer endpoint itself: its jump can exclude an endpoint even when the left-hand limiting value is allowed.

2. Let one root of the quadratic equation in x :

$$(k^2 - 15k + 27)x^2 + 9(k - 1)x + 18 = 0$$

be twice the other. Then the length of the latus rectum of the parabola $y^2 = 6kx$ is equal to: [4 marks]

- (A) 4
- (B) 6
- (C) 8
- (D) 12

Correct Answer: (D) 12

Solution:

Concept:

- For a quadratic $Ax^2 + Bx + C = 0$, Vieta gives the root sum $-B/A$ and product C/A , provided $A \neq 0$.
- If the roots are t and $2t$, then their squared sum divided by their product is $9/2$. A parabola $y^2 = 4ax$ has latus rectum length $|4a|$.

Step 1: Eliminate the unknown root Put $A = k^2 - 15k + 27$. The root sum is $-9(k - 1)/A$, and the root product is $18/A$. Since the constant term is nonzero, neither root is zero. Therefore

$$\frac{[-9(k - 1)/A]^2}{18/A} = \frac{9}{2}.$$

Multiplying by $2A$ and simplifying gives

$$81(k - 1)^2 = 81A, \quad (k - 1)^2 = k^2 - 15k + 27.$$

Step 2: Find the parameter and the geometric length Expand and cancel the quadratic terms:

$$-2k + 1 = -15k + 27, \quad 13k = 26, \quad k = 2.$$

For this value $A = 1$, so the equation remains genuinely quadratic: $x^2 + 9x + 18 = (x + 3)(x + 6) = 0$. Its roots -3 and -6 do have the required ratio, verifying that the elimination introduced no extraneous value. The parabola becomes $y^2 = 12x$, so $4a = 12$. The lengths 4, 6 and 8 do not equal this coefficient and are rejected.

Step 3: Conclusion The length of the latus rectum is 12. Hence, the correct option is (D).

Quick Tip to Solve: For roots in a given ratio, use sum squared divided by product to eliminate their scale before applying the parabola formula.

3. Let e_1 and e_2 be two distinct roots of the equation $x^2 - ax + 2 = 0$. Let the sets $\{a \in \mathbb{R} : e_1 \text{ and } e_2 \text{ are the eccentricities of hyperbolas}\} = (\alpha, \beta)$, and $\{a \in \mathbb{R} : e_1 \text{ and } e_2 \text{ are the eccentricities of an ellipse and a hyperbola, respectively}\} = (\gamma, \infty)$.

Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to:

[4 marks]

- (A) 18
- (B) 22
- (C) 26
- (D) 34

Correct Answer: (C) 26

Solution:

Concept:

- An ellipse has eccentricity $0 \leq e < 1$, whereas a hyperbola has eccentricity $e > 1$. Here the nonzero root product rules out eccentricity zero.
- The roots have sum a , product 2, and discriminant $a^2 - 8$. Distinct real positive eccentricities require $a > 2\sqrt{2}$.

Step 1: Require both roots to exceed one Let the smaller root be t . Since the product is 2, the larger is $2/t$. Distinct hyperbolic eccentricities require $1 < t < \sqrt{2}$, and their sum is $a = t + 2/t$. On this interval its derivative $1 - 2/t^2$ is negative. The sum decreases from the excluded value 3 to the excluded value $2\sqrt{2}$. Consequently

$$(\alpha, \beta) = (2\sqrt{2}, 3).$$

At the lower endpoint the roots coincide; at the upper endpoint one root is 1, which is not hyperbolic.

Step 2: Put one root inside the ellipse range For an ellipse and a hyperbola, take $0 < t < 1$; then the other root $2/t > 2$ is automatically hyperbolic. The same decreasing sum $t + 2/t$ takes every value in $(3, \infty)$, so $\gamma = 3$. Conversely, for any $a > 3$, the smaller positive root lies below 1 and the larger exceeds 2, so this is both necessary and sufficient. Finally

$$\alpha^2 + \beta^2 + \gamma^2 = (2\sqrt{2})^2 + 3^2 + 3^2 = 8 + 9 + 9 = 26.$$

The other totals 18, 22 and 34 do not satisfy these strict eccentricity bounds.

Step 3: Conclusion The required sum is 26. Hence, the correct option is (C).

Quick Tip to Solve: With fixed positive root product, write the larger eccentricity as product divided by the smaller one; the conic ranges then become simple intervals.

4. Let the set of all values of $k \in \mathbb{R}$ such that the equation $z(\bar{z} + 2 + i) + k(2 + 3i) = 0, z \in \mathbb{C}$, has at least one solution, be the interval $[\alpha, \beta]$. Then $9(\alpha + \beta)$ is equal to: [4 marks]

- (A) -10
(B) -8
(C) $10\sqrt{13}$
(D) $8\sqrt{13}$

Correct Answer: (A) -10

Solution:

Concept:

- Write $z = x + iy$, so $z\bar{z} = x^2 + y^2$. A complex equation holds precisely when both its real and imaginary parts vanish.
- A line intersects a real circle if its perpendicular distance from the centre is at most the radius. Equality includes the tangent endpoints of the allowed parameter interval.

Step 1: Separate the equation into a circle and a line Expanding $(2 + i)(x + iy)$ gives real part $2x - y$ and imaginary part $x + 2y$. Thus

$$x^2 + y^2 + 2x - y + 2k = 0, \quad x + 2y + 3k = 0.$$

Completing squares yields

$$(x + 1)^2 + (y - \frac{1}{2})^2 = \frac{5}{4} - 2k.$$

The circle centre is $(-1, 1/2)$, and its squared radius is $5/4 - 2k$.

Step 2: Apply the line-circle intersection condition The distance from this centre to the line is

$$d = \frac{|-1 + 2(1/2) + 3k|}{\sqrt{1^2 + 2^2}} = \frac{3|k|}{\sqrt{5}}.$$

Hence existence is equivalent to

$$\frac{9k^2}{5} \leq \frac{5}{4} - 2k, \quad 36k^2 + 40k - 25 \leq 0.$$

This inequality also ensures the squared radius is nonnegative, so no extra circle condition was lost. The upward quadratic is nonpositive between its two real roots, including both. These roots are α, β , and Vieta gives $\alpha + \beta = -40/36 = -10/9$. Multiplication by 9 gives -10 ; the other three proposed values fail the root-sum relation.

Step 3: Conclusion The required value $9(\alpha + \beta)$ is -10 . Hence, the correct option is (A).

Quick Tip to Solve: When only the sum of feasible parameter endpoints is needed, derive their boundary quadratic and use Vieta instead of calculating its radicals.

5. Let $0 < \alpha < 1, \beta = \frac{1}{3\alpha}$ and $\tan^{-1}(1 - \alpha) + \tan^{-1}(1 - \beta) = \frac{\pi}{4}$. Then $6(\alpha + \beta)$ is equal to: [4 marks]

- (A) 6
- (B) 7
- (C) 8
- (D) 9

Correct Answer: (B) 7

Solution:

Concept:

- For principal inverse tangents, each angle lies in $(-\pi/2, \pi/2)$; the tangent addition identity must be checked against this branch information.
- Here $\alpha\beta = 1/3$. If $S = \alpha + \beta$, then the sum of the tangent arguments is $2 - S$, and their product is $4/3 - S$.

Step 1: Take the tangent of the given angle sum Set $u = 1 - \alpha$ and $v = 1 - \beta$. Since the given sum is $\pi/4$, its tangent equals 1 and its addition-formula denominator is nonzero. Consequently

$$\frac{u + v}{1 - uv} = 1.$$

Using $\alpha\beta = 1/3$, we obtain

$$\frac{2 - S}{S - 1/3} = 1, \quad 2 - S = S - \frac{1}{3}, \quad S = \frac{7}{6}.$$

The denominator at this value is $5/6$, so the algebra is valid.

Step 2: Verify the allowed parameters and the angle branch The numbers α, β have sum $7/6$ and product $1/3$, hence solve

$$t^2 - \frac{7}{6}t + \frac{1}{3} = 0, \quad (3t - 2)(2t - 1) = 0.$$

They are $1/2$ and $2/3$, in either order, and both satisfy $0 < \alpha < 1$ and $\beta = 1/(3\alpha)$. Both u and v are positive; their inverse tangents sum to an angle in $(0, \pi)$. Its tangent is 1 and the positive addition denominator places it in the first quadrant, confirming $\pi/4$, not a shifted branch. Therefore $6S = 7$, excluding 6, 8 and 9.

Step 3: Conclusion The required value is 7. Hence, the correct option is (B).

Quick Tip to Solve: Use the known product to express the tangent addition formula in the sum alone, then check the principal inverse-tangent branch.

6. The sum of the first ten terms of an A.P. is 160 and the sum of the first two terms of a G.P. is 8 . If the first term of the A.P. is equal to the common ratio of the G.P. and the first term of the G.P. is equal to common difference of the A.P., then the sum of all possible values of the first term of the G.P. is:

[4 marks]

- (A) $\frac{34}{9}$
 (B) $\frac{34}{13}$
 (C) $\frac{32}{9}$
 (D) $\frac{32}{13}$

Correct Answer: (A) $\frac{34}{9}$

Solution:

Concept:

- For an A.P. with first term a and common difference d , the first ten terms sum to $5(2a + 9d)$.
- For a G.P. with first term b and ratio r , its first two terms sum to $b(1 + r)$. The stated cross-identifications are $a = r$ and $b = d$.

Step 1: Translate the two progression conditions Using the first sum,

$$5(2r + 9d) = 160, \quad 2r + 9d = 32.$$

Using the second sum gives

$$d(1 + r) = 8.$$

Thus $r = (32 - 9d)/2$, and substitution into the second equation yields

$$d \left(1 + \frac{32 - 9d}{2} \right) = 8.$$

Multiplying through by 2 gives $d(34 - 9d) = 16$, or

$$9d^2 - 34d + 16 = 0.$$

Step 2: Sum all admissible first terms The required first term of the G.P. is exactly d , so its possible values are the roots of this quadratic. Its discriminant is

$$(-34)^2 - 4(9)(16) = 580 > 0,$$

and the roots are $(17 + \sqrt{145})/9$ and $(17 - \sqrt{145})/9$. Both are real and nonzero. For either one, defining $r = (32 - 9d)/2$ satisfies both original sums, so neither root is extraneous. Vieta therefore gives

$$d_1 + d_2 = \frac{34}{9}.$$

The alternatives with denominator 13 or numerator 32 do not agree with the quadratic coefficients; they arise from losing the factor 2 or the extra d in $d(1 + r)$.

Step 3: Conclusion The sum of all possible first terms of the G.P. is $34/9$. Hence, the correct option is (A).

Quick Tip to Solve: Name the shared A.P. difference and G.P. first term with one variable, eliminate the ratio, and use Vieta when only the sum is asked.

7. The number of 4-letter words, with or without meaning, each consisting of two vowels and two consonants that can be formed from the letters of the word INCONSEQUENTIAL, without repeating any letter, is: [4 marks]

- (A) 2670
- (B) 2840
- (C) 2920
- (D) 3600

Correct Answer: (D) 3600

Solution:

Concept:

- A word is determined by its ordered letters. Choose the required vowel and consonant sets first, then arrange the four chosen distinct letters in $4!$ ways.
- Without repeating any letter means that repeated occurrences of a letter in the given word do not create new letter choices. The selection count is therefore based on distinct letters, using $\binom{n}{r}$.

Step 1: List the available distinct letters The vowels appearing in INCONSEQUENTIAL are I, O, E, U, A , giving five distinct vowel choices. Although some occur more than once in the original word, the condition prevents using a letter twice in the new word. The distinct consonants are N, C, S, Q, T, L , giving six choices. In particular, the repeated occurrences of N are not different consonants for forming a word.

Step 2: Select the two vowels and two consonants Choose an unordered pair of vowels and an unordered pair of consonants. These choices are independent, so the multiplication principle gives

$$\binom{5}{2} \binom{6}{2} = 10 \times 15 = 150.$$

Each selection contains four different letters. No word has been counted yet because the positions of these selected letters have not been fixed.

Step 3: Arrange each selection and compare the options The four selected letters can occupy the four ordered positions in $4! = 24$ ways. Thus

$$150 \times 24 = 3600.$$

Every allowed word determines exactly one pair of vowels, one pair of consonants and one arrangement, so this product neither omits nor duplicates words. The other totals 2670, 2840 and 2920 do not equal the required selection-and-arrangement count.

Step 4: Conclusion The number of required words is 3600. Hence, the correct option is (D).

Quick Tip to Solve: For a no-repetition word problem, list distinct letter types before using combinations; repeated letters in the source word do not multiply the choices.

8. If the coefficients of the middle terms in the binomial expansions of $(1 + \alpha x)^{26}$ and $(1 - \alpha x)^{28}$, $\alpha \neq 0$, are equal, then the value of α is: [4 marks]

- (A) 1
(B) $\frac{14}{13}$
(C) $\frac{27}{7}$
(D) $\frac{7}{27}$

Correct Answer: (D) $\frac{7}{27}$

Solution:

Concept:

- For an even exponent $2m$, the unique middle term of $(1 + z)^{2m}$ is the $(m + 1)$ th term, namely $\binom{2m}{m} z^m$.
- The coefficient of a power of x includes the corresponding power of the constant α . Factorial ratios simplify adjacent binomial coefficients without large-number evaluation.

Step 1: Identify both middle coefficients The expansion with exponent 26 contains 27 terms, so its middle term is the 14th term and its coefficient of x^{13} is $\binom{26}{13} \alpha^{13}$. The expansion with exponent 28 contains 29 terms. Its middle term is the 15th term, with coefficient of x^{14} equal to $\binom{28}{14} (-\alpha)^{14} = \binom{28}{14} \alpha^{14}$. The minus sign disappears because 14 is even.

Step 2: Equate the coefficients and cancel the nonzero power The given equality becomes

$$\binom{26}{13} \alpha^{13} = \binom{28}{14} \alpha^{14}.$$

Since $\alpha \neq 0$, division by α^{13} is valid. Therefore

$$\alpha = \frac{\binom{26}{13}}{\binom{28}{14}}.$$

There is no additional zero solution because it was explicitly excluded in the question.

Step 3: Simplify the factorial ratio Using the factorial definition,

$$\alpha = \frac{26!}{13!13!} \frac{14!14!}{28!} = \frac{14^2}{28 \times 27} = \frac{7}{27}.$$

This is a positive number below one, consistent with the larger central binomial coefficient in the denominator. Options 1, $14/13$ and $27/7$ fail the coefficient equation; the reciprocal $27/7$ arises if the ratio is reversed.

Step 4: Conclusion We obtain $\alpha = \frac{7}{27}$. Hence, the correct option is (D).

Quick Tip to Solve: Locate the middle term by its index, then retain the power of the parameter in its coefficient; cancel that power before simplifying factorials.

9. A data consists of 20 observations $x_1, x_2, \dots, x_{20}$. If $\sum_{i=1}^{20} (x_i + 5)^2 = 2500$ and $\sum_{i=1}^{20} (x_i - 5)^2 = 100$, then the ratio of mean to standard deviation of this data is : [4 marks]

- (A) 2 : 1
(B) 3 : 1
(C) 3 : 2
(D) 4 : 1

Correct Answer: (B) 3 : 1

Solution:

Concept:

- For n observations, the mean is $\bar{x} = \sum x_i / n$ and the population variance is $\sigma^2 = \sum x_i^2 / n - \bar{x}^2$.
- Adding and subtracting the two shifted-square sums isolates the first and second raw moments. The constant term must be counted once for each observation.

Step 1: Subtract the given sums to find the mean For every observation, $(x_i + 5)^2 - (x_i - 5)^2 = 20x_i$. Summing this identity over all 20 observations gives

$$20 \sum_{i=1}^{20} x_i = 2500 - 100 = 2400.$$

Consequently $\sum x_i = 120$, and

$$\bar{x} = \frac{120}{20} = 6.$$

The positive difference also confirms that the mean is positive.

Step 2: Add the sums to find the mean square The sum of the two square expressions for one observation is $2x_i^2 + 50$. There are 20 observations, so the total constant contribution is $50 \times 20 = 1000$. Hence

$$2 \sum x_i^2 + 1000 = 2500 + 100 = 2600,$$

$$\sum x_i^2 = 800, \quad \frac{\sum x_i^2}{20} = 40.$$

Using 50 only once would give an incorrect second moment.

Step 3: Calculate the standard deviation and form the ratio The variance is the mean square minus the square of the mean:

$$\sigma^2 = 40 - 6^2 = 4, \quad \sigma = 2.$$

We take the nonnegative square root because standard deviation cannot be negative. Thus

$$\bar{x} : \sigma = 6 : 2 = 3 : 1.$$

The ratio 3 : 2 would incorrectly compare the mean with the variance, while 2 : 1 and 4 : 1 are inconsistent with the two calculated moments.

Step 4: Conclusion The mean-to-standard-deviation ratio is $\boxed{3 : 1}$. Hence, the correct option is (B).

Quick Tip to Solve: When both positive and negative shifts are given, subtract to get the mean and add to get the mean square; take the square root of the variance before forming the ratio.

10. A bag contains $(N + 1)$ coins - N fair coins, and one coin with 'Head' on both sides. A coin is selected at random and tossed. If the probability of getting 'Head' is $\frac{9}{16}$, then N is equal to: [4 marks]

(A) 5

(B) 7

(C) 8

(D) 9

Correct Answer: (B) 7

Solution:

Concept:

- The law of total probability weights each conditional probability by the probability of selecting that class of coin.
- A fair coin has head probability $1/2$, whereas a coin bearing heads on both faces has head probability 1. Uniform selection is over coins, not over the two coin types.

Step 1: Identify the selection probabilities There are $N + 1$ coins in all. The probability of selecting any fair coin is $N/(N + 1)$, since N of the available coins are fair. The probability of selecting the one double-headed coin is $1/(N + 1)$. These events are mutually exclusive and exhaust all possible selections. They are not equally likely unless there is only one fair coin.

Step 2: Apply total probability to the head event If F denotes a fair-coin selection and D the double-headed selection, then

$$P(H) = P(F)P(H | F) + P(D)P(H | D).$$

Substitution gives

$$P(H) = \frac{N}{N + 1} \frac{1}{2} + \frac{1}{N + 1} = \frac{N + 2}{2(N + 1)}.$$

The double-headed coin contributes certainty of a head, rather than another factor of one-half.

Step 3: Solve and verify the integer count Equating this expression to the stated probability,

$$\frac{N + 2}{2(N + 1)} = \frac{9}{16} \implies 16N + 32 = 18N + 18.$$

Therefore $2N = 14$, so $N = 7$. With seven fair coins and one double-headed coin, the probability is $(7/2 + 1)/8 = 9/16$, as required. The choices 5, 8 and 9 instead give $7/12$, $5/9$ and $11/20$, respectively, none of which equals $9/16$.

Step 4: Conclusion The number of fair coins is $N = 7$. Hence, the correct option is (B).

Quick Tip to Solve: View the double-headed coin as increasing the fair-coin baseline: $P(H) = 1/2 + 1/[2(N + 1)]$, which gives the count immediately.

11. If the eccentricity e of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, passing through $(6, 4\sqrt{3})$, satisfies $15(e^2 + 1) = 34e$, then the length of the latus rectum of the hyperbola $\frac{x^2}{b^2} - \frac{y^2}{2(a^2+1)} = 1$ is: [4 marks]

- (A) 10
(B) 20
(C) 25
(D) 30

Correct Answer: (A) 10

Solution:

Concept:

- For the hyperbola $x^2/a^2 - y^2/b^2 = 1$, $e^2 = 1 + b^2/a^2$ and $e > 1$. Its latus rectum has length $2b^2/a$.
- For the second hyperbola, identify the new transverse semiaxis and the new conjugate semiaxis directly from its denominators; they are not the original a and b .

Step 1: Select the admissible eccentricity The eccentricity equation is

$$15e^2 - 34e + 15 = 0 = (5e - 3)(3e - 5).$$

Its roots are $3/5$ and $5/3$. A hyperbola requires $e > 1$, so only $e = 5/3$ is admissible. It follows that

$$\frac{b^2}{a^2} = e^2 - 1 = \frac{25}{9} - 1 = \frac{16}{9}.$$

The smaller root would instead have the eccentricity range of an ellipse.

Step 2: Use the given point to determine the semiaxes Substitute $(6, 4\sqrt{3})$ into the first hyperbola:

$$\frac{36}{a^2} - \frac{48}{b^2} = 1.$$

With $b^2 = 16a^2/9$, this becomes

$$\frac{36}{a^2} - \frac{27}{a^2} = 1 \implies a^2 = 9.$$

Therefore $b^2 = 16$, and the positive semiaxis length b is 4.

Step 3: Compute the latus rectum of the new hyperbola The new equation is

$$\frac{x^2}{16} - \frac{y^2}{20} = 1.$$

Write it as $x^2/A^2 - y^2/B^2 = 1$, where $A = 4$ and $B^2 = 20$. Its latus rectum length is therefore

$$\frac{2B^2}{A} = \frac{2 \times 20}{4} = 10.$$

The options 20, 25 and 30 do not use the required semiaxis ratio. In particular, 20 is the conjugate semiaxis squared, not the latus rectum length.

Step 4: Conclusion The required length is 10 units. Hence, the correct option is (A).

Quick Tip to Solve: For a transformed hyperbola, rename its semiaxes before applying $2B^2/A$; the denominator is a length, not its square.

12. Let chord PQ of length $3\sqrt{13}$ of the parabola $y^2 = 12x$ be such that the ordinates of points P and Q are in the ratio 1:2. If the chord PQ subtends an angle α at the focus of the parabola, then $\sin \alpha$ is equal to : [4 marks]

- (A) $\frac{3}{5}$
 (B) $\frac{4}{5}$
 (C) $\frac{5}{13}$
 (D) $\frac{12}{13}$

Correct Answer: (A) $\frac{3}{5}$

Solution:

Concept:

- For $y^2 = 4ax$, a parametric point is $(at^2, 2at)$ and the focus is $(a, 0)$. Here $a = 3$.
- The sine of the angle between two plane vectors u and v is $|u_x v_y - u_y v_x| / (|u||v|)$. This avoids finding the angle itself.

Step 1: Parametrize the endpoints using the ordinate ratio Because the ordinates are in the ratio 1 : 2, write

$$P = (3t^2, 6t), \quad Q = (12t^2, 12t).$$

Here $t \neq 0$, since the chord has nonzero length. Either sign of t is allowed; changing it reflects the whole configuration across the x-axis and leaves the required sine unchanged.

Step 2: Use the chord length to determine the parameter The coordinate differences are $9t^2$ and $6t$, so

$$PQ^2 = 81t^4 + 36t^2 = (3\sqrt{13})^2 = 117.$$

Put $u = t^2 \geq 0$. Dividing by 9 gives

$$9u^2 + 4u - 13 = 0 = (u - 1)(9u + 13).$$

The negative root $-13/9$ cannot be a square, hence $u = 1$. Thus one may take $t = 1$, giving $P = (3, 6)$ and $Q = (12, 12)$.

Step 3: Find the angle subtended at the focus The focus is $F = (3, 0)$, so

$$\overrightarrow{FP} = (0, 6), \quad \overrightarrow{FQ} = (9, 12).$$

Their magnitudes are 6 and 15. Therefore

$$\sin \alpha = \frac{|0 \cdot 12 - 6 \cdot 9|}{6 \times 15} = \frac{54}{90} = \frac{3}{5}.$$

The value $4/5$ is the cosine for these vectors, rather than the sine. The ratios $5/13$ and $12/13$ do not result from the focal vectors, even though 13 appears in the chord length.

Step 4: Conclusion The required value is $\sin \alpha = \frac{3}{5}$. Hence, the correct option is (A).

Quick Tip to Solve: When a parabola chord has ordinates in a simple ratio, use parameters in that same ratio, then apply the determinant formula to its focal vectors.

13. The value of $1^3 - 2^3 + 3^3 - \dots + 15^3$ is:

[4 marks]

- (A) 1706
- (B) 1856
- (C) 1982
- (D) 2403

Correct Answer: (B) 1856

Solution:

Concept:

- An alternating sum can be evaluated as the sum of all cubes minus twice the sum of the even cubes. This subtracts precisely the terms whose signs must change.
- The identity $\sum_{k=1}^n k^3 = [n(n+1)/2]^2$ converts each required cubic sum to a square of an arithmetic sum.

Step 1: Separate the positive and negative terms Let the required sum be T . There are fifteen terms, beginning and ending with a positive cube. The negative terms are exactly $2^3, 4^3, \dots, 14^3$; there are seven of them. Therefore, instead of adding seven separate pairs, write

$$T = \sum_{k=1}^{15} k^3 - 2 \sum_{j=1}^7 (2j)^3.$$

The factor two is essential: the full sum initially contains each even cube positively, so twice that cube must be removed to leave its negative.

Step 2: Evaluate the two sums Using the cube-sum identity with the correct upper limits gives

$$\sum_{k=1}^{15} k^3 = \left(\frac{15 \cdot 16}{2} \right)^2 = 120^2 = 14400.$$

Since $(2j)^3 = 8j^3$, the even-cube sum is

$$\sum_{j=1}^7 (2j)^3 = 8 \left(\frac{7 \cdot 8}{2} \right)^2 = 8 \cdot 28^2 = 6272.$$

Consequently,

$$T = 14400 - 2(6272) = 14400 - 12544 = 1856.$$

This exact integer equals option (B). The other three numbers, 1706, 1982 and 2403, do not equal the alternating sum and are ruled out by this calculation.

Step 3: Conclusion The alternating sum is 1856. Hence, the correct option is (B).

Quick Tip to Solve: For an alternating power sum ending in a positive odd term, take the full power sum and subtract twice the even-term sum.

14. Let $S = \{\theta \in (-2\pi, 2\pi) : \cos \theta + 1 = \sqrt{3} \sin \theta\}$.

Then $\sum_{\theta \in S} \theta$ is equal to :

[4 marks]

(A) $-\frac{2\pi}{3}$

(B) $-\frac{4\pi}{3}$

(C) $\frac{2\pi}{3}$

(D) $\frac{4\pi}{3}$

Correct Answer: (B) $-\frac{4\pi}{3}$

Solution:

Concept:

- Use the half-angle identities $1 + \cos \theta = 2 \cos^2(\theta/2)$ and $\sin \theta = 2 \sin(\theta/2) \cos(\theta/2)$.
- When an equation factors, retain every factor that can vanish. Dividing by a trigonometric factor without checking its zeros can discard solutions.

Step 1: Factor the equation without division The defining equation for S becomes

$$2 \cos^2(\theta/2) = 2\sqrt{3} \sin(\theta/2) \cos(\theta/2).$$

Bring both sides together and factor:

$$2 \cos(\theta/2) [\cos(\theta/2) - \sqrt{3} \sin(\theta/2)] = 0.$$

Thus either $\cos(\theta/2) = 0$, giving $\theta = \pi + 2k\pi$, or the second factor is zero. In the second case cosine cannot vanish, so $\tan(\theta/2) = 1/\sqrt{3}$, giving $\theta = \pi/3 + 2k\pi$, where k is an integer.

Step 2: Restrict both families to the stated interval The open interval is $(-2\pi, 2\pi)$. From the first family the permitted values are $-\pi$ and π . From the second family the permitted values are $-5\pi/3$ and $\pi/3$. No endpoints are included, and moving one more period in either direction leaves the interval. The four angles are distinct, so

$$\sum_{\theta \in S} \theta = -\pi + \pi - \frac{5\pi}{3} + \frac{\pi}{3} = -\frac{4\pi}{3}.$$

The opposite-sign alternatives cannot match the negative sum. Option (A) has only half its magnitude; it can arise from omitting a root or an incorrect period.

Step 3: Conclusion The required sum is $-\frac{4\pi}{3}$. Hence, the correct option is (B).

Quick Tip to Solve: Factor the half-angle equation before dividing: the zeros of cosine supply the extra roots that a tangent-only approach loses.

15. Let the image of the point $P(1, 6, a)$ in the line $L : \frac{x}{1} = \frac{y-1}{2} = \frac{z-a+1}{b}, b > 0$, be $(\frac{a}{3}, 0, a+c)$. If $S(\alpha, \beta, \gamma), \alpha > 0$, is the point on L such that the distance of S from the foot of perpendicular from the point P on L is $2\sqrt{14}$, then $\alpha + \beta + \gamma$ is equal to: [4 marks]

- (A) 19
(B) 20
(C) 21
(D) 22

Correct Answer: (C) 21

Solution:

Concept:

- If P reflects to P' in a line, their midpoint H lies on the line and $\overrightarrow{HP}$ is perpendicular to its direction.
- For the line $(x, y, z) = (t, 1+2t, a-1+bt)$, a parameter change Δt gives a distance $|\Delta t|\sqrt{1+4+b^2}$.

Step 1: Locate the midpoint of the reflected pair The midpoint of $P = (1, 6, a)$ and $P' = (a/3, 0, a+c)$ is

$$H = \left(\frac{1+a/3}{2}, 3, a + \frac{c}{2} \right).$$

Since H lies on the line, its second coordinate gives $1+2t=3$, hence $t=1$. Its first coordinate must therefore be 1, so $(1+a/3)/2=1$, giving $a=3$. The third coordinate then gives $3+c/2=2+b$, or $c/2=b-1$.

Step 2: Apply the perpendicularity condition At parameter 1 the foot is $H = (1, 3, 2+b)$. Thus $P-H = (0, 3, 1-b)$. Taking its scalar product with the direction $(1, 2, b)$ gives

$$6 + b(1-b) = 0, \quad b^2 - b - 6 = 0.$$

The roots are 3 and -2; the given restriction $b > 0$ selects $b=3$. Consequently $c=4$, $H = (1, 3, 5)$, and the line direction has length $\sqrt{14}$.

Step 3: Find the admissible point S The required distance $SH = 2\sqrt{14}$ gives $|t-1|\sqrt{14} = 2\sqrt{14}$. Hence $t=3$ or $t=-1$. Since $\alpha = x = t > 0$, only $t=3$ is permitted. This yields

$$S = (3, 1+6, 2+9) = (3, 7, 11).$$

Therefore $\alpha + \beta + \gamma = 3 + 7 + 11 = 21$. The other options 19, 20 and 22 fail this explicit coordinate sum; the negative-parameter point also violates the stated sign restriction.

Step 4: Conclusion The required sum is 21. Hence, the correct option is (C).

Quick Tip to Solve: Use the midpoint of a point and its line reflection as the perpendicular foot; the simplest coordinate often fixes the line parameter immediately.

16. Let a line L be perpendicular to both the lines $L_1 : \frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $L_2 : \frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}$.

If θ is the acute angle between the lines L and $L_3 : \frac{x-8}{2} = \frac{y-4}{1} = \frac{z}{2}$, then $\tan \theta$ is equal to: [4 marks]

- (A) $\frac{3}{2}\sqrt{2}$
(B) $\frac{5}{2}\sqrt{2}$
(C) $\frac{5}{3}\sqrt{2}$
(D) $\frac{4}{3}\sqrt{2}$

Correct Answer: (B) $\frac{5}{2}\sqrt{2}$

Solution:

Concept:

- A line perpendicular to two nonparallel lines has a direction parallel to the cross product of their direction vectors.
- For direction vectors $\mathbf{u}, \mathbf{v}$, the acute angle satisfies $\cos \theta = |\mathbf{u} \cdot \mathbf{v}| / (|\mathbf{u}||\mathbf{v}|)$. Points on the lines do not affect their angle.

Step 1: Find a direction for the common perpendicular The given symmetric equations supply the direction vectors $\mathbf{a} = (3, 5, 7)$ and $\mathbf{b} = (1, 4, 7)$. Their cross product is

$$\mathbf{a} \times \mathbf{b} = (35 - 28, 7 - 21, 12 - 5) = (7, -14, 7).$$

The first component, for example, is the determinant $5 \cdot 7 - 7 \cdot 4$. Dividing by the nonzero factor 7 gives the simpler direction $\mathbf{u} = (1, -2, 1)$ for L . Directly checking $\mathbf{u} \cdot \mathbf{a} = 3 - 10 + 7 = 0$ and $\mathbf{u} \cdot \mathbf{b} = 1 - 8 + 7 = 0$ verifies perpendicularity to both lines.

Step 2: Compute the angle with the third line The third line has direction $\mathbf{v} = (2, 1, 2)$. Its coordinate offsets $8/7$ and $4/7$ locate it but do not enter the angle. Now

$$\mathbf{u} \cdot \mathbf{v} = 2 - 2 + 2 = 2, \quad |\mathbf{u}| = \sqrt{6}, \quad |\mathbf{v}| = 3.$$

Thus $\cos \theta = 2/(3\sqrt{6})$. Since the angle is acute, its sine and tangent are positive. Therefore

$$\tan \theta = \frac{\sqrt{1 - \cos^2 \theta}}{\cos \theta} = \frac{\sqrt{54 - 4}}{2} = \frac{5\sqrt{2}}{2}.$$

Only option (B) has this coefficient. The coefficients in (A), (C) and (D) do not satisfy the dot-product value computed above.

Step 3: Conclusion The tangent of the acute angle is $\boxed{5\sqrt{2}/2}$. Hence, the correct option is (B).

Quick Tip to Solve: For a line normal to two given directions, take their cross product first and simplify it before computing the final angle.

17.

The value of $\lim_{x \rightarrow 0} \left(\frac{x^2 \sin^2 x}{x^2 - \sin^2 x} \right)$ is :

[4 marks]

- (A) 2
(B) 3
(C) 4
(D) 6

Correct Answer: (B) 3

Solution:

Concept:

- Near the origin, $\sin x = x - x^3/6 + O(x^5)$. Retain enough terms to capture the first nonzero term in a difference whose leading terms cancel.
- The standard limit $\sin x/x \rightarrow 1$ controls the numerator, while the fourth-order term of $x^2 - \sin^2 x$ controls the denominator.

Step 1: Find the first nonzero denominator term Squaring the sine expansion gives

$$\sin^2 x = \left(x - \frac{x^3}{6} + O(x^5) \right)^2 = x^2 - \frac{x^4}{3} + O(x^6).$$

The cross term is $2x(-x^3/6) = -x^4/3$; all omitted terms have order at least six. Subtracting this expression from x^2 cancels the quadratic terms, leaving

$$x^2 - \sin^2 x = \frac{x^4}{3} + O(x^6).$$

Using only $\sin x \sim x$ inside this difference would give zero and conceal the actual leading order.

Step 2: Divide by the common fourth power The numerator is

$$x^2 \sin^2 x = x^4 - \frac{x^6}{3} + O(x^8).$$

For nonzero x near zero, divide numerator and denominator by x^4 . The expression then becomes

$$\frac{1 - x^2/3 + O(x^4)}{1/3 + O(x^2)}.$$

Both the left-hand and right-hand limits equal $1/(1/3) = 3$, since the denominator tends to a nonzero constant. In particular the original ratio is positive near zero, consistently with this result. Options (A), (C) and (D), namely 2, 4 and 6, are excluded by the exact leading coefficients; neither the numerator nor the denominator has an extra factor of two.

Step 3: Conclusion The required limit is $\boxed{3}$. Hence, the correct option is (B).

Quick Tip to Solve: When $x^2 - \sin^2 x$ occurs, the quadratic terms cancel: expand through the cubic term of sine to reveal the fourth-order denominator.

18. The value of the integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{32 \cos^4 x}{1 + e^{\sin x}} \right) dx$ is :

[4 marks]

- (A) $4\pi + 2$
- (B) $3\pi + 8$
- (C) $3\pi + 4$
- (D) $4\pi + 3$

Correct Answer: (B) $3\pi + 8$

Solution:

Concept:

- For an integral over $[-a, a]$, substitute $x \mapsto -x$ and add the resulting expressions to exploit symmetry.
- The identity $1/(1 + e^t) + 1/(1 + e^{-t}) = 1$ removes the exponential factor when its exponent is an odd function and the remaining numerator is even.

Step 1: Pair the integrand at opposite arguments Let the given integral be I , and put $a = \pi/4$. Since cosine is even and sine is odd, reversing the integration variable gives

$$I = \int_{-a}^a \frac{32 \cos^4 x}{1 + e^{-\sin x}} dx.$$

Adding this to the original expression and using the reciprocal-exponential identity yields

$$2I = 32 \int_{-a}^a \cos^4 x dx.$$

Thus $I = 16 \int_{-a}^a \cos^4 x dx$. The factor one half comes from adding two copies of the same integral, not from any approximation to the denominator.

Step 2: Integrate the even trigonometric polynomial The power-reduction formula is

$$\cos^4 x = \frac{3 + 4 \cos 2x + \cos 4x}{8}.$$

An antiderivative is $3x/8 + \sin 2x/4 + \sin 4x/32$. At the symmetric endpoints, this gives

$$\int_{-\pi/4}^{\pi/4} \cos^4 x dx = \frac{3\pi}{16} + \frac{1}{2},$$

because $\sin(\pi/2) = 1$, $\sin(-\pi/2) = -1$, and both fourth-angle sine terms vanish. Consequently

$$I = 16 \left(\frac{3\pi}{16} + \frac{1}{2} \right) = 3\pi + 8.$$

Option (B) matches both contributions. Option (C) halves the nonconstant contribution incorrectly; (A) and (D) have the wrong coefficient of π as well.

Step 3: Conclusion The integral equals $\boxed{3\pi + 8}$. Hence, the correct option is (B).

Quick Tip to Solve: For $1 + e^{\text{odd function}}$ over symmetric bounds, add the reflected integral first; then reduce the remaining even trigonometric power.

19. The area of the region $\{(x, y) : 0 \leq y \leq 6 - x, y^2 \geq 4x - 3, x \geq 0\}$ is :

[4 marks]

- (A) 8
- (B) 9
- (C) 12
- (D) 15

Correct Answer: (B) 9

Solution:

Concept:

- For a horizontal strip, area equals its horizontal width times dy . The inequality $y^2 \geq 4x - 3$ gives an upper bound on x , not on y .
- Both upper bounds must hold: $0 \leq x \leq \min\{6 - y, (y^2 + 3)/4\}$. The width changes where the line and parabola intersect.

Step 1: Find the change in the right boundary Since $x \geq 0$ and $0 \leq y \leq 6 - x$, the vertical range is $0 \leq y \leq 6$. Equate the two right boundaries:

$$\frac{y^2 + 3}{4} = 6 - y \implies y^2 + 4y - 21 = (y - 3)(y + 7) = 0.$$

Only $y = 3$ lies in the allowed range. For $0 \leq y \leq 3$, the parabola gives the smaller upper bound; for $3 \leq y \leq 6$, the line does.

Step 2: Integrate the two strip widths The left boundary is the y -axis throughout, so no lower-bound term is subtracted from the widths. Therefore

$$\begin{aligned} S &= \int_0^3 \frac{y^2 + 3}{4} dy + \int_3^6 (6 - y) dy \\ &= \left[\frac{y^3}{12} + \frac{3y}{4} \right]_0^3 + \left[6y - \frac{y^2}{2} \right]_3^6 \\ &= \frac{9}{2} + \frac{9}{2} = 9. \end{aligned}$$

Integrating only the line would include points excluded by the parabolic inequality near the bottom. Integrating only the parabola would include points to the right of the line near the top. The resulting value rules out 8, 12 and 15.

Step 3: Conclusion The required area is 9 square units. Hence, the correct option is (B).

Quick Tip to Solve: Rewrite both inequalities as upper bounds on the same variable, then integrate the smaller bound on each interval.

20. Let e be the base of natural logarithm and let $f : \{1, 2, 3, 4\} \rightarrow \{1, e, e^2, e^3\}$ and $g : \{1, e, e^2, e^3\} \rightarrow \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\}$ be two bijective functions such that f is strictly decreasing and g is strictly increasing. If $\phi(x) = \lceil f^{-1} \{g^{-1}(\frac{1}{2})\} \rceil^x$, then the area of the region $R = \{(x, y) : x^2 \leq y \leq \phi(x), 0 \leq x \leq 1\}$ is :

[4 marks]

- (A) $\frac{3 - \log_e(2)}{3 \log_e(2)}$
 (B) $\frac{1}{3 \log_e(2)}$
 (C) $3 + \log_e(2)$
 (D) $\frac{3 + \log_e(2)}{2 + \log_e(3)}$

Correct Answer: (A) $\frac{3 - \log_e(2)}{3 \log_e(2)}$

Solution:

Concept:

- A strictly monotone bijection between finite ordered sets is determined by the order of their elements. A decreasing map reverses the order, while an increasing map preserves it.
- The area between an upper function and a lower function is the integral of their difference. Here $\int a^x dx = a^x / \ln a$ for $a > 0, a \neq 1$.

Step 1: Determine the inverse images The decreasing bijection must satisfy

$$f(1) = e^3, \quad f(2) = e^2, \quad f(3) = e, \quad f(4) = 1.$$

For the increasing bijection, order the target numerically as $1/4 < 1/3 < 1/2 < 1$. Thus

$$g(1) = \frac{1}{4}, \quad g(e) = \frac{1}{3}, \quad g(e^2) = \frac{1}{2}, \quad g(e^3) = 1.$$

Consequently $g^{-1}(1/2) = e^2$ and $f^{-1}(e^2) = 2$, giving $\phi(x) = 2^x$.

Step 2: Integrate the upper curve minus the lower curve On $[0, 1]$, $x^2 \leq 1$ and $2^x \geq 1$, so the exponential is indeed above the parabola on the whole interval. No interior crossing requires a split. The area is

$$\begin{aligned} S &= \int_0^1 (2^x - x^2) dx \\ &= \left[\frac{2^x}{\ln 2} - \frac{x^3}{3} \right]_0^1 \\ &= \frac{1}{\ln 2} - \frac{1}{3} = \frac{3 - \ln 2}{3 \ln 2}. \end{aligned}$$

Numerically this is about 1.1094. Option (B) gives only about 0.4809, and the expressions in (C) and (D) also differ from the exact integral.

Step 3: Conclusion The area is $\frac{3 - \ln 2}{3 \ln 2}$ square units. Hence, the correct option is (A).

Quick Tip to Solve: For increasing finite-set maps, sort fractions by their actual size before pairing them with the ordered domain.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let $A = \begin{bmatrix} -1 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ satisfy

$$A^2 + \alpha(\text{adj}(\text{adj}(A))) + \beta(\text{adj}(A)(\text{adj}(\text{adj}(A)))) = \begin{bmatrix} 2 & -2 & 2 \\ -2 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix} \text{ for some } \alpha, \beta \in \mathbb{R}.$$

Then $(\alpha - \beta)^2$ is equal to _____

[4 marks]

Solution:

Concept:

- For an invertible matrix, $A \text{adj}(A) = (\det A)I$. For a matrix of order three, $\text{adj}(\text{adj}(A)) = (\det A)A$.
- Matrix equations can be compared entry by entry after simplifying the adjugates. An off-diagonal entry isolates the coefficient of A , while a diagonal entry then gives the coefficient of I .

Step 1: Simplify the adjugate terms Expansion along the last row gives $\det A = (-1)(0) - (1)(1) = -1$, so A is invertible. Therefore

$$\text{adj}(\text{adj}(A)) = -A.$$

Multiplying on the left by $\text{adj}(A)$ gives

$$\text{adj}(A) \text{adj}(\text{adj}(A)) = -\text{adj}(A)A = I.$$

The original left-hand side is consequently $A^2 - \alpha A + \beta I$, with the sign of the identity term positive.

Step 2: Compare entries and determine the parameters Direct multiplication yields

$$A^2 = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

In position (1, 2), the stated matrix equation reads $-1 - \alpha = -2$, so $\alpha = 1$. Position (2, 2) then reads $1 + \beta = 0$, giving $\beta = -1$. As a consistency check,

$$A^2 - A - I = \begin{pmatrix} 2 & -2 & 2 \\ -2 & 0 & -1 \\ 0 & 0 & -1 \end{pmatrix},$$

which reproduces every entry of the right-hand side, not merely the two used to solve for the parameters.

Step 3: Conclusion Thus $(\alpha - \beta)^2 = (1 - (-1))^2 = 4$. The required numerical value is 4, a dimensionless number.

Quick Tip to Solve: Compute the determinant first: for a three-by-three matrix, the double adjugate reduces immediately to a scalar multiple of the original matrix.

22. Let the centre of the circle $x^2 + y^2 + 2gx + 2fy + 25 = 0$ be in the first quadrant and lie on the line $2x - y = 4$. Let the area of an equilateral triangle inscribed in the circle be $27\sqrt{3}$. Then the square of the length of the chord of the circle on the line $x = 1$ is ____ . [4 marks]

Solution:

Concept:

- For a circle with centre (h, k) , the general equation gives $h = -g$, $k = -f$ and $R^2 = h^2 + k^2 - 25$. A point in the first quadrant has both coordinates positive.
- An equilateral triangle inscribed in a circle of radius R has side $\sqrt{3}R$ and area $3\sqrt{3}R^2/4$. A chord at perpendicular distance d from the centre has squared length $4(R^2 - d^2)$.

Step 1: Find the radius and the permitted centre Use the given triangle area to obtain

$$\frac{3\sqrt{3}}{4}R^2 = 27\sqrt{3} \implies R^2 = 36.$$

Hence $h^2 + k^2 = 61$. Since the centre lies on $2x - y = 4$, we have $k = 2h - 4$. Substitution gives

$$h^2 + (2h - 4)^2 = 61 \implies 5h^2 - 16h - 45 = 0.$$

Factoring as $(h - 5)(5h + 9) = 0$ gives $h = 5$ or $h = -9/5$. Only $h = 5$, with $k = 6$, puts the centre in the first quadrant. The negative root is therefore excluded by the stated condition.

Step 2: Use the perpendicular distance to the chord The distance from $(5, 6)$ to the vertical line $x = 1$ is $d = |5 - 1| = 4$. This is less than the radius 6, confirming that the line cuts the circle in a genuine chord. The perpendicular from the centre bisects the chord. If its length is L , the right triangle gives

$$\left(\frac{L}{2}\right)^2 = R^2 - d^2 = 36 - 16 = 20.$$

Therefore $L^2 = 4(20) = 80$. Equivalently, substituting $x = 1$ into $(x - 5)^2 + (y - 6)^2 = 36$ gives endpoints with $y = 6 \pm \sqrt{20}$.

Step 3: Conclusion The square of the chord length is 80 square units. The required numerical entry is 80.

Quick Tip to Solve: For a vertical chord, use the horizontal distance from the centre; calculate the squared length directly to avoid unnecessary radicals.

23. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{j} - \hat{k}$ and $\vec{c}$ be three vectors such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$, then $\vec{c} \cdot (\vec{a} - 2\vec{b})$ is equal to ____ . [4 marks]

Solution:

Concept:

- A cross product $\vec{u} \times \vec{v}$ is perpendicular to each factor. Thus $\vec{v} \cdot (\vec{u} \times \vec{v}) = 0$, a scalar triple product with a repeated vector.

- The dot product is distributive: $\vec{c} \cdot (\vec{a} - 2\vec{b}) = \vec{c} \cdot \vec{a} - 2\vec{c} \cdot \vec{b}$. The supplied scalar product determines the first term directly.

Step 1: Obtain the missing scalar product The given relation $\vec{b} = \vec{a} \times \vec{c}$ makes $\vec{b}$ perpendicular to $\vec{c}$. Accordingly

$$\vec{c} \cdot \vec{b} = \vec{c} \cdot (\vec{a} \times \vec{c}) = 0.$$

This conclusion follows from the cross product itself, so it does not require solving separately for the three components of $\vec{c}$. The other supplied condition is $\vec{a} \cdot \vec{c} = 3$, and symmetry of the dot product gives $\vec{c} \cdot \vec{a} = 3$.

Step 2: Evaluate the requested combination Apply distributivity with the coefficient and sign exactly as printed:

$$\begin{aligned}\vec{c} \cdot (\vec{a} - 2\vec{b}) &= \vec{c} \cdot \vec{a} - 2(\vec{c} \cdot \vec{b}) \\ &= 3 - 2(0) = 3.\end{aligned}$$

A component check is also possible. Writing $\vec{c} = (u, v, w)$ gives $w - v = 0$, $u - w = 1$ and $v - u = -1$. Together with $u + v + w = 3$, these yield $\vec{c} = (5/3, 2/3, 2/3)$. Since $\vec{a} - 2\vec{b} = (1, -1, 3)$, the dot product is $5/3 - 2/3 + 2 = 3$, confirming the shorter orthogonality argument.

Step 3: Conclusion The requested scalar product is 3. The numerical answer is the dimensionless value 3.

Quick Tip to Solve: Before solving for vector components, check whether a cross-product relation already forces a needed dot product to vanish.

24. For the functions $f(\theta) = \alpha \tan^2 \theta + \beta \cot^2 \theta$, and $g(\theta) = \alpha \sin^2 \theta + \beta \cos^2 \theta$, $\alpha > \beta > 0$, let $\min_{0 < \theta < \frac{\pi}{2}} f(\theta) = \max_{0 < \theta < \pi} g(\theta)$. If the first term of a G.P. is $\left(\frac{\alpha}{2\beta}\right)$, its common ratio is $\left(\frac{2\beta}{\alpha}\right)$ and the sum of its first 10 terms is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to ____ . [4 marks]

Solution:

Concept:

- For positive t , $\alpha t + \beta/t \geq 2\sqrt{\alpha\beta}$, with equality when $t = \sqrt{\beta/\alpha}$. Since $\tan^2 \theta$ covers all positive values on $(0, \pi/2)$, equality is attainable.
- A finite geometric progression with first term a and common ratio $r \neq 1$ has sum $S_n = a(1 - r^n)/(1 - r)$. Reduce the resulting fraction before adding its numerator and denominator.

Step 1: Relate the two positive parameters using the extrema Apply the inequality with $t = \tan^2 \theta$ to find $\min f = 2\sqrt{\alpha\beta}$. Rewrite the other function as

$$g(\theta) = \beta + (\alpha - \beta) \sin^2 \theta.$$

Because $\alpha > \beta$, its maximum is α , attained at the allowed point $\theta = \pi/2$ in $(0, \pi)$. The given equality of extrema therefore yields

$$2\sqrt{\alpha\beta} = \alpha \implies 4\alpha\beta = \alpha^2 \implies \alpha = 4\beta,$$

where division by α is valid because it is positive.

Step 2: Compute and reduce the ten-term sum The first term and common ratio now become

$$a = \frac{\alpha}{2\beta} = 2, \quad r = \frac{2\beta}{\alpha} = \frac{1}{2}.$$

Consequently

$$\begin{aligned} S_{10} &= \frac{2(1 - (1/2)^{10})}{1 - 1/2} \\ &= 4 \left(1 - \frac{1}{1024} \right) = \frac{1023}{256}. \end{aligned}$$

The numerator 1023 is odd, whereas $256 = 2^8$, so their greatest common divisor is 1. This is already the required lowest-terms form: $m = 1023$ and $n = 256$.

Step 3: Conclusion Therefore $m + n = 1023 + 256 = 1279$. The required numerical value is 1279, a dimensionless integer.

Quick Tip to Solve: Find extrema before evaluating the progression: the equality fixes the ratio of parameters, which is all the GP needs.

25. Let $y = y(x)$ be the solution of the differential equation

$$\begin{aligned} (x^2 - x\sqrt{x^2 - 1}) dy \\ + (y(x - \sqrt{x^2 - 1}) - x) dx = 0, \quad x \geq 1. \end{aligned}$$

If $y(1) = 1$, then the greatest integer less than $y(\sqrt{5})$ is ____.

[4 marks]

Solution:

Concept:

- A first-order linear equation has the form $y' + P(x)y = Q(x)$. Multiplication by the integrating factor $e^{\int P(x) dx}$ makes the left side an exact derivative.
- For $x \geq 1$, the identity $(x - \sqrt{x^2 - 1})(x + \sqrt{x^2 - 1}) = 1$ rationalises the coefficient without introducing a zero denominator.

Step 1: Reduce the equation to linear form Divide the given equation by $x(x - \sqrt{x^2 - 1})$, which is positive on the stated domain. This gives

$$y' + \frac{y}{x} = \frac{1}{x - \sqrt{x^2 - 1}} = x + \sqrt{x^2 - 1}.$$

The integrating factor is $e^{\int dx/x} = x$, since x is positive. Thus the differential equation becomes

$$\frac{d}{dx}(xy) = x^2 + x\sqrt{x^2 - 1}.$$

Step 2: Integrate and apply the initial condition Integrating the first term directly and using $u = x^2 - 1$

for the second gives

$$xy = \frac{x^3}{3} + \frac{(x^2 - 1)^{3/2}}{3} + C.$$

At $x = 1$, the condition $y(1) = 1$ yields $1 = 1/3 + C$, so $C = 2/3$. Therefore

$$y(x) = \frac{x^3 + (x^2 - 1)^{3/2} + 2}{3x}.$$

This expression satisfies the initial condition and the integrated equation, so it is the required solution.

Step 3: Evaluate and locate between consecutive integers At $x = \sqrt{5}$, we have $(x^2 - 1)^{3/2} = 4^{3/2} = 8$. Hence

$$y(\sqrt{5}) = \frac{5\sqrt{5} + 10}{3\sqrt{5}} = \frac{5 + 2\sqrt{5}}{3} \approx 3.1574.$$

Since this number lies strictly between 3 and 4, its greatest integer below it is 3.

Step 4: Conclusion The required greatest integer is $\boxed{3}$. This is a dimensionless integer.

Quick Tip to Solve: Factor the coefficient of dy as $x(x - \sqrt{x^2 - 1})$; rationalisation immediately reveals the simple integrating factor x .

Physics – Section A: Multiple Choice Questions (80 marks)

26. The density ρ of a uniform cylinder is determined by measuring its mass m , length l and diameter d . The measured values of m, l and d are 97.42 ± 0.02 g, 8.35 ± 0.05 mm and 20.20 ± 0.02 mm, respectively. Calculated percentage fractional error in ρ is ____ . [4 marks]

- (A) 0.63%
- (B) 0.82%
- (C) 0.72%
- (D) 0.25%

Correct Answer: (B) 0.82%

Solution:

Concept:

- The density of a uniform cylindrical body is mass divided by volume: $\rho = m/(\pi d^2 l/4) = 4m/(\pi d^2 l)$. The measured diameter therefore enters with power two.
- The maximum fractional uncertainty in a product or quotient is the sum of the absolute fractional uncertainties multiplied by their powers. The constants 4 and π contribute no measurement uncertainty.

Step 1: Write the fractional error formula Taking logarithms gives $\ln \rho = \ln(4/\pi) + \ln m - 2 \ln d - \ln l$. For the maximum measurement error, the signs of the changes cannot be assumed to cancel. We add their magnitudes:

$$\frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + 2 \frac{\Delta d}{d} + \frac{\Delta l}{l}.$$

Using the diameter rather than the radius changes neither the factor two nor the relative error rule.

Step 2: Substitute the measurements consistently Each uncertainty must be divided by its own measured quantity in matching units. The mass is already paired in grams and both length measurements are paired in millimetres, so no conversion is needed for these dimensionless ratios. Thus

$$\frac{\Delta\rho}{\rho} = \frac{0.02}{97.42} + 2\frac{0.02}{20.20} + \frac{0.05}{8.35}.$$

The three contributions are approximately 0.0002053, 0.0019802, and 0.0059880, respectively.

Step 3: Convert to a percentage and select the option Adding the contributions and multiplying by 100 gives

$$\frac{\Delta\rho}{\rho} \times 100 \approx 0.81735\% \approx 0.82\%.$$

The length measurement contributes most of the uncertainty. The alternatives 0.63%, 0.72%, and 0.25% do not equal the propagated sum; ignoring the diameter power or a contribution cannot give the required maximum error.

Step 4: Conclusion The percentage fractional error is 0.82%. Hence, the correct option is (B).

Quick Tip to Solve: Write density in terms of the actually measured diameter first, then add the relative errors with powers as multipliers.

27. The potential energy of a particle changes with distance x from a fixed origin as $V = \frac{A\sqrt{x}}{x+B}$, where A and B are constant with appropriate dimensions. The dimensions of AB are ____ [4 marks]

- (A) $[M^1 L^{5/2} T^{-2}]$
(B) $[M^{3/2} L^{5/2} T^{-2}]$
(C) $[M^1 L^2 T^{-2}]$
(D) $[M^1 L^{7/2} T^{-2}]$

Correct Answer: (D) $[M^1 L^{7/2} T^{-2}]$

Solution:

Concept:

- Quantities added in a physical expression must have identical dimensions. Because the denominator is $x + B$ and x is a distance, $[B] = L$.
- Potential energy has dimensions $[V] = ML^2T^{-2}$. Numerical coefficients and dimensionless functions do not change the dimensional balance.

Step 1: Find the dimension carried by the denominator The two terms x and B appear in a sum, so they must both represent lengths. Consequently

$$[x + B] = L, \quad [\sqrt{x}] = L^{1/2}.$$

The square root acts on distance alone; it is not the square root of an energy or of the whole fraction. This distinction fixes the half-integer exponent in the answer.

Step 2: Use the energy dimension to determine A Apply dimensional balance to the entire given expression:

$$[V] = \frac{[A][x]^{1/2}}{[x+B]} = [A]L^{-1/2}.$$

Multiplying by $L^{1/2}$ therefore gives

$$[A] = ML^2T^{-2}L^{1/2} = ML^{5/2}T^{-2}.$$

Both mass and time exponents are inherited unchanged from energy because all other factors involve only length.

Step 3: Multiply by the dimension of B The question asks for the product AB , rather than A alone. Thus

$$[AB] = [A][B] = ML^{5/2}T^{-2}L = ML^{7/2}T^{-2}.$$

Option (A) gives the dimension of A alone. Option (B) has an unjustified mass exponent $3/2$, while option (C) simply gives the energy dimension and misses the extra length factors.

Step 4: Conclusion The required dimensions are $[M^1L^{7/2}T^{-2}]$. Hence, the correct option is (D).

Quick Tip to Solve: Use dimensional homogeneity in sums before balancing a formula, and check whether the requested quantity is a constant or a product of constants.

28. The rain drop of mass 1 g , starts with zero velocity from a height of 1 km . It hits the ground with a speed of 5 m/s. The work done by the unknown resistive force is ____ J.

(take $g = 10 \text{ m/s}^2$)

[4 marks]

- (A) -8.75
- (B) -8.35
- (C) -9.55
- (D) -9.98

Correct Answer: (D) -9.98

Solution:

Concept:

- The work-energy theorem states that the total work of all forces on a particle equals its change in kinetic energy: $W_{\text{net}} = K_f - K_i$.
- Gravity does positive work mgh during downward motion. An unknown resistive force need not be constant; its total work can still be obtained from the energy balance without finding its force law.

Step 1: Convert the data and calculate the kinetic energy gain The drop has mass $m = 1 \text{ g} = 10^{-3} \text{ kg}$,

falls through $h = 1 \text{ km} = 10^3 \text{ m}$, and starts from rest. Its final speed is 5 m s^{-1} , so

$$\Delta K = \frac{1}{2}mv^2 - 0 = \frac{1}{2}(10^{-3})(5)^2 = 0.0125 \text{ J}.$$

This small positive gain is the net work, not the work of resistance alone.

Step 2: Calculate the work supplied by gravity Gravity acts in the direction of the downward displacement. With the specified $g = 10 \text{ m s}^{-2}$, its work is

$$W_g = mgh = (10^{-3})(10)(10^3) = 10 \text{ J}.$$

Most of this energy must be removed by resistance because only 0.0125 J remains as kinetic energy at impact. There is no need to infer the distance dependence of the resistive force.

Step 3: Isolate the resistive work and compare options The work-energy equation is $W_g + W_r = \Delta K$. Therefore

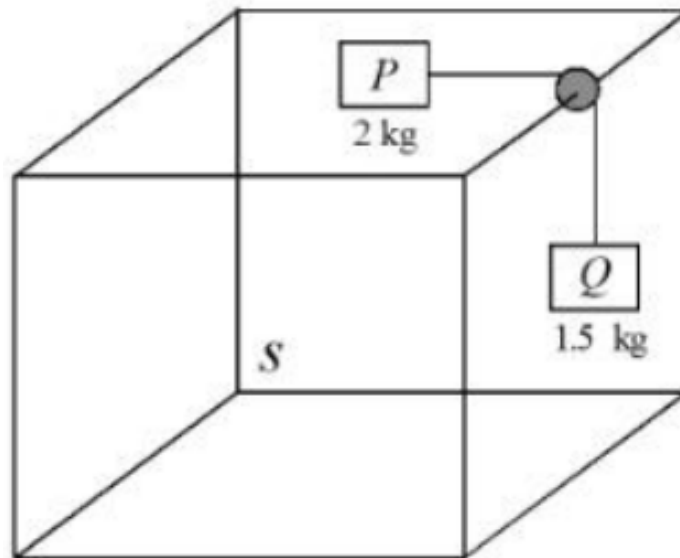
$$W_r = 0.0125 - 10 = -9.9875 \text{ J}.$$

The negative sign means that resistance removes mechanical energy during the descent. Among the printed values, -9.98 J is the intended closest choice; -8.75 , -8.35 , and -9.55 J would leave much larger final kinetic energies than the specified speed allows.

Step 4: Conclusion The calculated work is -9.9875 J , corresponding to the closest printed value -9.98 J . Hence, the correct option is (D).

Quick Tip to Solve: Compute net work from the speed change, then subtract gravitational work; the loss of potential energy alone is not the resistive work.

29. Two blocks (P and Q) with respectively masses 2 kg and 1.5 kg are joined by a massless thread. These blocks are mounted on a frictionless pulley which is fixed on the edge of a cube (S), as shown in the figure below. Block P is positioned on the top surface which has no friction and block Q is in contact with side-surface, having coefficient friction μ . The cube (S) moves towards the right with acceleration of $\frac{g}{2}$, where g is gravitational acceleration. During this movement the block P and Q remain stationary. The value of μ is ____ (take $g = 10 \text{ m/s}^2$) [4 marks]



- (A) 0.33
- (B) 0.67
- (C) 1
- (D) 0.5

Correct Answer: (B) 0.67

Solution:

Concept:

- A block stationary relative to the accelerating cube has the same horizontal acceleration as the cube in the ground frame. In the cube frame one can equivalently balance real forces with a leftward pseudo-force ma .
- The string and pulley are ideal, so both string segments have the same tension. On the side block, friction supplies the vertical force needed in addition to tension, and the horizontal normal reaction supplies its horizontal acceleration.

Step 1: Find the tension from block P Here stationary means at rest relative to the moving cube; both blocks move horizontally with it at $a = g/2 = 5 \text{ m/s}^2$. The top surface is smooth, so the only horizontal force on P is the string tension. Newton's second law gives

$$T = m_P a = 2(5) = 10 \text{ N.}$$

The vertical normal reaction on P balances its weight and has no role in this horizontal equation.

Step 2: Find the normal force and required friction on Q The string segment at Q is vertical. The horizontal normal reaction from the side of the cube must accelerate Q to the right, so

$$N = m_Q a = 1.5(5) = 7.5 \text{ N.}$$

Vertically, Q has no acceleration. Its weight is $m_Q g = 15 \text{ N}$, exceeding the upward tension of 10 N . Friction must therefore act upward with magnitude

$$f = m_Q g - T = 15 - 10 = 5 \text{ N.}$$

Step 3: Apply the friction condition and interpret the coefficient The required ratio is

$$\frac{f}{N} = \frac{5}{7.5} = \frac{2}{3} \approx 0.67.$$

At limiting equilibrium, $f = \mu N$, giving $\mu = 2/3$. More generally static equilibrium requires $\mu_s \geq 2/3$, so this is the minimum coefficient implied by the options. The values 0.33 and 0.5 cannot supply enough friction. A value 1 can support equilibrium but is not the required minimum; the intended option is the limiting value.

Step 4: Conclusion The minimum coefficient is $\mu_{\min} = 2/3 \approx 0.67$, so (B) is the intended limiting value. As printed, the question does not state limiting friction: $\mu_s \geq 2/3$, and (C), $\mu_s = 1$, also permits equilibrium. Thus there is no unique coefficient without that extra condition. Hence, the correct option is (B) under the intended minimum-coefficient interpretation.

Quick Tip to Solve: For the side block, use the imposed horizontal acceleration to obtain its normal force before applying the vertical friction balance.

30. A lift of mass 1600 kg is supported by thick iron wire. If the maximum stress which the wire can withstand is $4 \times 10^8 \text{ N/m}^2$ and its radius is 4 mm , then maximum acceleration the lift can take is _____ m/s^2 .

(take $g = 10 \text{ m/s}^2$ and $\pi = 3.14$)

[4 marks]

- (A) 2.56
- (B) 3.89
- (C) 4.32
- (D) 5.16

Correct Answer: (A) 2.56

Solution:

Concept:

- Tensile stress is force divided by the cross-sectional area: $\sigma = T/A$. A circular wire of radius r has area $A = \pi r^2$.
- For a lift accelerating upward, Newton's second law gives $T - mg = ma$, or $T = m(g + a)$. Upward acceleration increases the tension and is therefore the limiting direction for the supporting wire.

Step 1: Calculate the wire cross-sectional area The stated dimension is the radius, not the diameter. Convert it to SI units before squaring: $r = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$. With the prescribed $\pi = 3.14$,

$$A = \pi r^2 = 3.14(4 \times 10^{-3})^2 = 5.024 \times 10^{-5} \text{ m}^2.$$

Using millimetres directly with a stress in N m^{-2} would produce an incorrect power of ten.

Step 2: Find the largest permissible tension The stress reaches its allowable maximum when the wire carries its maximum safe tension. Thus

$$T_{\max} = \sigma_{\max} A = (4 \times 10^8)(5.024 \times 10^{-5}) = 20096 \text{ N}.$$

For comparison, the stationary weight of the lift is $mg = 1600(10) = 16000 \text{ N}$. The difference between these forces, rather than the whole tension, is available to accelerate the lift upward.

Step 3: Apply Newton law and compare the choices At the limiting upward acceleration,

$$1600a_{\max} = T_{\max} - mg = 20096 - 16000 = 4096.$$

It follows that

$$a_{\max} = \frac{4096}{1600} = 2.56 \text{ m s}^{-2}.$$

The alternatives 3.89 , 4.32 and 5.16 m s^{-2} would require tensions larger than 20096 N , and hence stresses exceeding the permitted value. Subtracting g is essential because tension also supports the weight.

Step 4: Conclusion The maximum safe upward acceleration is $\boxed{2.56 \text{ m s}^{-2}}$. Hence, the correct option is (A).

Quick Tip to Solve: First turn stress into a maximum tension using the area, then use $a = T/m - g$ for the upward acceleration limit.

31. A solid sphere of radius 4 cm and mass 5 kg is rotating (rotation axis is passing through the centre of the sphere) with an angular velocity of 1200 rpm . It is brought to rest in 10 s by applying a constant torque. The torque applied and the number of rotations it made before it comes to rest are ____ and ____ respectively. [4 marks]

- (A) $0.128\pi\text{Nm}$, 100
 (B) $0.0128\pi\text{Nm}$, 50
 (C) $0.128\pi\text{Nm}$, 50
 (D) $0.0128\pi\text{Nm}$, 100

Correct Answer: (D) $0.0128\pi\text{Nm}$, 100

Solution:

Concept:

- A solid sphere rotating about a diameter has moment of inertia $I = \frac{2}{5}MR^2$. The radius must be expressed in metres before evaluating the torque.
- A constant torque gives constant angular acceleration: $\tau = I\alpha$. The angle turned is the average angular velocity multiplied by the stopping time.

Step 1: Convert the speed and calculate the moment of inertia The initial rate is $1200/60 = 20$ revolutions per second. One revolution corresponds to 2π radians, so

$$\omega_0 = 40\pi \text{ rad s}^{-1}, \quad I = \frac{2}{5}(5)(0.04)^2 = 0.0032 \text{ kg m}^2.$$

The rotation axis passes through the centre, so no parallel-axis correction is needed.

Step 2: Find the retarding torque The final angular speed is zero after 10 s. Hence

$$\alpha = \frac{0 - 40\pi}{10} = -4\pi \text{ rad s}^{-2}, \quad \tau = 0.0032(-4\pi) = -0.0128\pi \text{ N m}.$$

The minus sign specifies that the applied torque opposes the initial rotation; the options state its magnitude.

Step 3: Count the rotations before stopping For uniform retardation the mean angular speed is half the initial speed. Thus

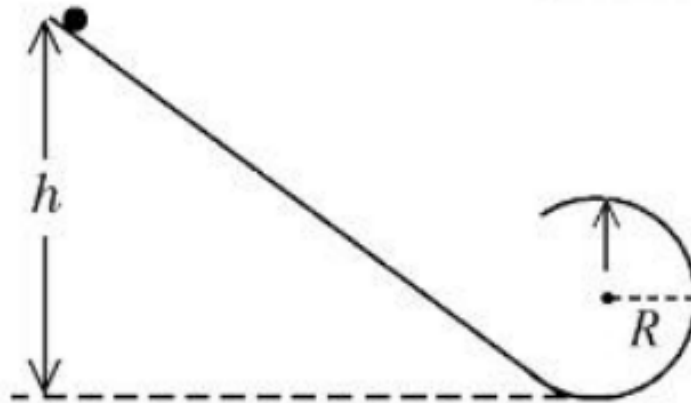
$$\theta = \frac{40\pi + 0}{2}(10) = 200\pi \text{ rad}, \quad N = \frac{200\pi}{2\pi} = 100.$$

Options (A) and (C) overestimate the torque by a factor of ten. Option (B) gives only half the correct rotation count.

Step 4: Conclusion The torque magnitude is $0.0128\pi \text{ N m}$, opposite the rotation, and the sphere completes 100 rotations. Hence, the correct option is (D).

Quick Tip to Solve: Convert rpm to revolutions per second first; with constant braking the average rotation rate is half the initial rate.

32. A smooth inclined plane ends in a vertical circular loop, as shown in the figure. A small body is released from height h as shown. If the body exerts a force of three times its weight on the plane at the highest point of circle then the height $h = \alpha R$. The value of α is ____ [4 marks]



- (A) 2
- (B) 4
- (C) 3
- (D) 6

Correct Answer: (B) 4

Solution:

Concept:

- At the highest point of a vertical loop, both the normal reaction and gravity point towards the centre. The radial force equation is $N + mg = mv^2/R$.
- The track is smooth, so mechanical energy is conserved. The normal reaction does no work because it is perpendicular to the instantaneous displacement.

Step 1: Use the contact force at the top The body exerts a force $3mg$ on the track. By Newton's third law the track exerts an equal normal reaction $N = 3mg$ on the body, directed downwards at the top. Gravity also acts downwards. Consequently,

$$3mg + mg = \frac{mv_{\text{top}}^2}{R} \Rightarrow v_{\text{top}}^2 = 4gR.$$

It would be incorrect to subtract the normal force from the weight here, since both forces are centripetal.

Step 2: Apply energy conservation from release to the top Take the lowest point of the loop as the zero of gravitational potential energy. The body starts from rest at height h ; the top of the loop is at height $2R$. Therefore

$$mgh = mg(2R) + \frac{1}{2}mv_{\text{top}}^2 = 2mgR + \frac{1}{2}m(4gR) = 4mgR.$$

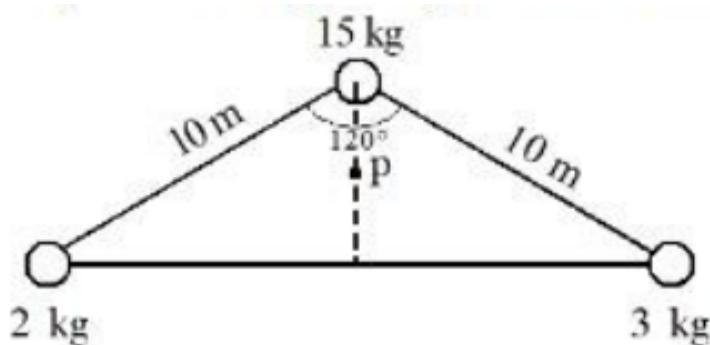
The mass cancels, leaving $h = 4R$. The kinetic energy at the top is positive, so this is not the limiting just-contact condition.

Step 3: Compare the possible heights The requested coefficient is $\alpha = h/R = 4$. A height $2R$ would leave no speed at the top and cannot maintain the specified contact force. A height $3R$ gives $v^2 = 2gR$, hence $N = mg$, while $6R$ gives $N = 7mg$. Only $4R$ gives $N = 3mg$.

Step 4: Conclusion The release height is $h = 4R$, so $\alpha = 4$. Hence, the correct option is (B).

Quick Tip to Solve: At the top of an inside loop add weight and normal reaction; then include the full height gain of two radii in the energy equation.

33. The position of center of mass of three masses 2 kg, 3 kg and 15 kg placed with respect to mid point (p) of normal bisector, as shown in the figure is ____ . [4 marks]



(A) $\left(\frac{\sqrt{3}}{4}, 1.25\right)$

(B) $\left(\frac{\sqrt{3}}{4}, 1.0\right)$

(C) (0, 0)

(D) (1.25, 0)

Correct Answer: (A) $\left(\frac{\sqrt{3}}{4}, 1.25\right)$

Solution:

Concept:

- The centre of mass has coordinates $x_{\text{cm}} = \sum m_i x_i / \sum m_i$ and $y_{\text{cm}} = \sum m_i y_i / \sum m_i$.
- The equal sides form an isosceles triangle with vertex angle 120° . Its altitude bisects that angle and the base.

Step 1: Set coordinates relative to the marked point Take the marked point p , the midpoint of the perpendicular bisector shown, as origin. Let positive x point towards the 3 kg mass and positive y towards the 15 kg mass. Half the vertex angle is 60° , and each sloping side is 10 m. Thus the altitude is $10 \cos 60^\circ = 5$ m and the half-base is $10 \sin 60^\circ = 5\sqrt{3}$ m. Since p is halfway up the altitude, the coordinates are

$$(x_1, y_1) = (-5\sqrt{3}, -2.5), \quad (x_2, y_2) = (5\sqrt{3}, -2.5), \quad (x_3, y_3) = (0, 2.5)$$

in metres.

Step 2: Compute both mass-weighted averages The total mass is $2 + 3 + 15 = 20$ kg. The horizontal

coordinate is

$$x_{\text{cm}} = \frac{2(-5\sqrt{3}) + 3(5\sqrt{3}) + 15(0)}{20} = \frac{\sqrt{3}}{4} \text{ m.}$$

The vertical coordinate is

$$y_{\text{cm}} = \frac{2(-2.5) + 3(-2.5) + 15(2.5)}{20} = \frac{25}{20} = 1.25 \text{ m.}$$

The result lies above p , towards the much heavier upper mass, and slightly to the right because the right base mass is larger. Options with zero horizontal coordinate ignore the unequal base masses; option (B) gives the wrong vertical weighted average.

Step 3: Conclusion The centre of mass relative to p is $\left(\frac{\sqrt{3}}{4}, 1.25\right)$ m. Hence, the correct option is (A).

Quick Tip to Solve: Locate the specified origin before averaging coordinates: the midpoint of the altitude is different from the midpoint of the base.

34. The two wires A and B of equal cross-section but of different materials are joined together. The ratio of Young's modulus of wire A and wire B is $20/11$. When the joined wire is kept under certain tension the elongations in the wires A and B are equal. If the length of wire A is 2.2 m, then the length of wire B is

_____ m.

[4 marks]

- (A) 1.1
- (B) 2.22
- (C) 1.21
- (D) 4.44

Correct Answer: (C) 1.21

Solution:

Concept:

- For a uniform wire in its elastic regime, $\Delta L = FL/(AY)$, where F is tension, A is cross-sectional area and Y is Young's modulus.
- Wires joined in series carry the same tension. Equal areas therefore give equal stress, but equal strain follows only if their Young's moduli are equal.

Step 1: Apply the common tension condition Let L_A, L_B be the original lengths and Y_A, Y_B their Young's moduli. At the joint, force balance makes the tensile force the same in both wires; the statement also specifies equal cross-sectional areas. Their extensions are therefore

$$\Delta L_A = \frac{FL_A}{AY_A}, \quad \Delta L_B = \frac{FL_B}{AY_B}.$$

The problem gives equality of the absolute elongations, rather than equality of the fractional extensions.

Step 2: Use equal elongations to find the unknown length Setting these two expressions equal cancels

the common force and area:

$$\frac{L_A}{Y_A} = \frac{L_B}{Y_B} \Rightarrow L_B = L_A \frac{Y_B}{Y_A}.$$

With $Y_A/Y_B = 20/11$,

$$L_B = 2.2 \left(\frac{11}{20} \right) = \frac{22 \times 11}{100 \times 2} = 1.21 \text{ m}.$$

The stiffer material A must be longer to produce the same extension as B under the same stress, which agrees with $2.2 > 1.21$.

Step 3: Check the proposed numerical values The required ratio is $L_B/L_A = 11/20 = 0.55$. Option (A) would give a ratio of 0.5, while (B) and (D) would make B longer than A despite its smaller Young's modulus. Only 1.21 m satisfies the original equal-extension equation exactly.

Step 4: Conclusion The length of wire B is 1.21 m. Hence, the correct option is (C).

Quick Tip to Solve: For series wires of equal area and equal extension, their lengths are proportional to their Young's moduli, not inversely proportional.

35. Two closed vessels of same volume are joined through a narrow tube and both vessels are filled with air of pressure 90 kPa and temperature 400 K . Keeping the temperature of one vessel constant at 400 K the second vessel temperature is raised to 500 K . The final pressure in the vessels is ____ kPa . [4 marks]

- (A) 100
- (B) 120
- (C) 90
- (D) 105

Correct Answer: (A) 100

Solution:

Concept:

- Connected vessels in equilibrium have a common pressure even when their temperatures are held at different values. For each vessel, $n = PV/(RT)$.
- The combined apparatus is closed, so its total number of moles remains constant; gas can redistribute between the vessels through the narrow tube.

Step 1: Count the initial total amount of gas Let each vessel have volume V , and neglect the volume of the narrow connecting tube. Initially both vessels have pressure $P_0 = 90 \text{ kPa}$ and temperature $T_0 = 400 \text{ K}$. The initial total number of moles is

$$n_{\text{total}} = 2 \frac{P_0 V}{R(400)}.$$

One must conserve this total, rather than separately fixing the amount in each vessel, since the connection allows gas transfer.

Step 2: Express the final mole total at common pressure Let the equilibrium pressure be P . One

vessel remains at 400 K, while the other is maintained at 500 K. Consequently

$$n_{\text{total}} = \frac{PV}{400R} + \frac{PV}{500R}.$$

Equating initial and final totals and cancelling V/R gives

$$\frac{2(90)}{400} = P \left(\frac{1}{400} + \frac{1}{500} \right), \quad P = \frac{0.45}{0.0045} = 100 \text{ kPa}.$$

The hotter vessel contains fewer moles at the final common pressure.

Step 3: Check the pressure against the options The pressure must rise above 90 kPa, since some of the gas is heated with the apparatus volume fixed. However, it cannot reach the pressure obtained by heating all the gas to 500 K, namely 112.5 kPa; this rules out 120 kPa. Substitution into the mole-conservation equation distinguishes 100 from 105 kPa.

Step 4: Conclusion The final common pressure in both vessels is 100 kPa. Hence, the correct option is (A).

Quick Tip to Solve: For connected fixed-volume vessels at different temperatures, conserve the sum of PV/RT ; do not average temperatures arithmetically.

36. In interference experiment the path difference between two interfering waves at a point A on the screen is $\lambda/3$, where λ is the wavelength of these waves, and at another point B the path difference is $\lambda/6$. The ratio of intensities at points A and B is ____ . [4 marks]

- (A) 3
- (B) 4
- (C) 1/3
- (D) 1/4

Correct Answer: (C) 1/3

Solution:

Concept:

- For two coherent interfering waves of equal individual intensity I_0 , the resultant intensity is $I = 4I_0 \cos^2(\phi/2)$.
- A path difference Δ corresponds to phase difference $\phi = 2\pi\Delta/\lambda$. The interference formula therefore contains $\cos^2(\pi\Delta/\lambda)$.

Step 1: Convert the two path differences to phase differences At A the path difference is $\lambda/3$, so

$$\phi_A = \frac{2\pi}{\lambda} \frac{\lambda}{3} = \frac{2\pi}{3}.$$

At B it is $\lambda/6$, giving

$$\phi_B = \frac{2\pi}{\lambda} \frac{\lambda}{6} = \frac{\pi}{3}.$$

The trigonometric factor in the intensity uses half these phase differences. Squaring a full-phase cosine would incorrectly give equal intensities here.

Step 2: Evaluate the intensities and their ratio Using the standard equal-intensity two-wave interference relation,

$$I_A = 4I_0 \cos^2 \left(\frac{\pi}{3} \right) = 4I_0 \left(\frac{1}{2} \right)^2 = I_0,$$
$$I_B = 4I_0 \cos^2 \left(\frac{\pi}{6} \right) = 4I_0 \left(\frac{\sqrt{3}}{2} \right)^2 = 3I_0.$$

Therefore

$$\frac{I_A}{I_B} = \frac{I_0}{3I_0} = \frac{1}{3}.$$

The intensity scale cancels, so neither the wave amplitude nor its numerical wavelength is needed.

Step 3: Reject the other ratios Point B has the smaller positive path difference and is closer to the central constructive condition, so B must be brighter than A. This immediately excludes ratios 3 and 4. The remaining value $1/4$ does not equal the ratio of the squared cosines, which is $(1/4)/(3/4) = 1/3$.

Step 4: Conclusion The intensity ratio is $I_A : I_B = 1 : 3$, or $I_A/I_B = 1/3$. Hence, the correct option is (C).

Quick Tip to Solve: Use half the phase difference in the cosine-squared formula; path differences of one third and one sixth wavelength give intensities in the ratio 1:3.

37. A particle is executing simple harmonic motion. Its amplitude is A and time period is 5 sec . The time required by it to move from $x = A$ to $x = \frac{A}{\sqrt{2}}$ is ____ sec. [4 marks]

- (A) $1/4$
- (B) $5/4$
- (C) $5/8$
- (D) $3/8$

Correct Answer: (C) $5/8$

Solution:

Concept:

- An SHM displacement can be written as $x = A \cos \omega t$ when the time origin is chosen at the positive extreme. The angular frequency is $\omega = 2\pi/T$, where T is the period.
- The time taken between positions is obtained from the change in phase, not from the fraction of the distance travelled. The particle moves fastest near the mean position and has zero speed at an extreme.

Step 1: Choose the phase at the extreme Take $t = 0$ when the particle is at $x = A$, just beginning its inward motion. Then the initial phase is zero and the first arrival at the specified positive displacement occurs during the first quarter-cycle. The amplitude cancels on dividing the displacement equation by

A:

$$\frac{x}{A} = \cos\left(\frac{2\pi t}{5}\right).$$

Step 2: Find the first inward arrival At the required position, $x/A = 1/\sqrt{2}$. In the interval from phase zero to $\pi/2$, the cosine is $1/\sqrt{2}$ at $\pi/4$. Hence

$$\frac{2\pi t}{5} = \frac{\pi}{4}, \quad t = \frac{5}{8} \text{ s.}$$

This is one eighth of a full cycle. The value $5/4$ s would bring the particle to the mean position, while $1/4$ s and $3/8$ s give smaller phase advances and therefore do not reach the stated displacement.

Step 3: Conclusion The required time is $5/8$ s. Hence, the correct option is (C).

Quick Tip to Solve: When motion starts at an extreme, use cosine and convert the target displacement ratio directly into a phase angle.

38. A thin half ring of radius 35 cm is uniformly charged with a total charge of Q coulomb. If the magnitude of the electric field at centre of the half ring is 100 V/m , then the value of Q is ____ nC .

$$(\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2 \text{ and } \pi = 3.14)$$

[4 marks]

- (A) 2.14
- (B) 2.44
- (C) 3.25
- (D) 0.7

Correct Answer: (A) 2.14

Solution:

Concept:

- For a uniformly charged semicircular arc of radius r , its length is πr , so its linear charge density is $\lambda = Q/(\pi r)$.
- At the centre, components perpendicular to the symmetry axis cancel in pairs. Components along the symmetry axis add, giving $E = 2k\lambda/r = 2kQ/(\pi r^2)$, where $k = 1/(4\pi\epsilon_0)$.

Step 1: Add the surviving field components Consider an element subtending angle $d\theta$ at the centre. Its charge is $dq = \lambda r d\theta$, and its field magnitude is $k\lambda d\theta/r$. Taking the symmetry axis through the midpoint of the arc, integrate its axial component from $-\pi/2$ to $\pi/2$:

$$E = \frac{k\lambda}{r} \int_{-\pi/2}^{\pi/2} \cos \theta d\theta = \frac{2k\lambda}{r}.$$

This vector addition is essential; adding all element magnitudes would overestimate the resultant.

Step 2: Calculate the total charge Rearrange the resultant-field expression and use SI units, with

$r = 35 \text{ cm} = 0.35 \text{ m}$:

$$Q = \frac{E\pi r^2}{2k} = 2\pi^2\epsilon_0 E r^2.$$

$$Q = 2(3.14)^2(8.85 \times 10^{-12})(100)(0.35)^2 \text{ C} \simeq 2.14 \times 10^{-9} \text{ C}.$$

Since $1 \text{ nC} = 10^{-9} \text{ C}$, this is 2.14 nC . Each other option gives a different charge and therefore a different field at the same radius; the field is directly proportional to charge.

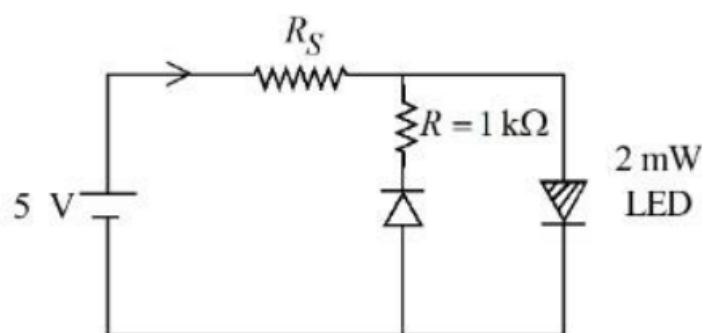
Step 3: Conclusion The charge magnitude is $Q \simeq 2.14 \text{ nC}$. Hence, the correct option is (A).

Quick Tip to Solve: For a semicircular charge distribution, cancel transverse components first; use the arc length πr , not $2\pi r$.

39. The maximum rated power of the LED is 2 mW and it is used in the circuit with input voltage of 5 V as shown in the figure below. The current through resistance R_S is 0.5 mA .

The minimum value of the resistance of R_S , to ensure that the LED is not damaged is ____ $\text{k}\Omega$.

[4 marks]



- (A) 6
- (B) 2
- (C) 4
- (D) 5

Correct Answer: (B) 2

Solution:

Concept:

- The electrical power dissipated in a conducting LED is $P = V_{\text{LED}} I_{\text{LED}}$. Its rated power sets the largest safe voltage at the specified current.
- In the shown circuit the diode in the middle branch is reverse biased by the supply polarity. In the ideal-diode model that branch is open, so all current through R_S passes through the LED. Kirchhoff's voltage law gives $5 = IR_S + V_{\text{LED}}$.

Step 1: Identify the conducting path The supply sends conventional current along the upper wire from left to right. The LED in the right-hand branch is forward biased, whereas the ordinary diode below the $1 \text{ k}\Omega$ resistor in the middle branch opposes the applied polarity. Therefore the middle branch carries no current, and its resistor does not form a parallel load. The stated series-resistor

current is consequently the LED current:

$$I_{\text{LED}} = I = 0.5 \text{ mA} = 5.0 \times 10^{-4} \text{ A}.$$

Step 2: Use the maximum safe LED power At the limiting safe operating point, the LED dissipates its full rated power:

$$V_{\text{LED,max}} = \frac{2.0 \times 10^{-3}}{5.0 \times 10^{-4}} = 4 \text{ V}.$$

The resistor must then drop at least $5 - 4 = 1 \text{ V}$. Thus

$$R_{S,\text{min}} = \frac{1}{5.0 \times 10^{-4}} = 2000 \Omega = 2 \text{ k}\Omega.$$

A larger series resistance provides a larger protective drop at the specified current, so 4, 5, and 6 k Ω are not the minimum. Treating the reverse-biased branch as conducting would incorrectly change the LED current.

Step 3: Conclusion The minimum resistance is $R_S = 2 \text{ k}\Omega$. Hence, the correct option is (B).

Quick Tip to Solve: Check each diode's polarity before using the stated supply current in the LED power formula.

40. A point light source emits E.M. waves in free space. A detector, placed at a distance of L m, measures the intensity as I_0 . The detector is now shifted to another location on the same spherical surface ensuring the angle between original location and new location as 45° . The measured intensity at new location will be ____ . [4 marks]

- (A) $\frac{I_0}{4}$
- (B) I_0
- (C) $\frac{I_0}{\sqrt{2}}$
- (D) $\frac{I_0}{2}$

Correct Answer: (B) I_0

Solution:

Concept:

- An isotropic point light source distributes its emitted power P over a spherical surface of area $4\pi r^2$. The intensity at distance r is therefore $I = P/(4\pi r^2)$.
- Intensity is the magnitude of energy flux per unit area normal to propagation. Changing a detector's position around a sphere does not change its distance from the source; it changes the propagation direction at that position.

Step 1: Write the original intensity The original detector lies at distance L from the point source. For the freely propagating radiation, the power crossing a sphere centred at the source is spread uniformly over its area. Consequently

$$I_0 = \frac{P}{4\pi L^2}.$$

The inverse-square dependence compares different radii, rather than the angular separation between two points on a single sphere. No change in source power or absorbing medium is specified.

Step 2: Compare the radii of the two locations The new position is on the same spherical surface, so its radius is again L . Applying the same expression gives

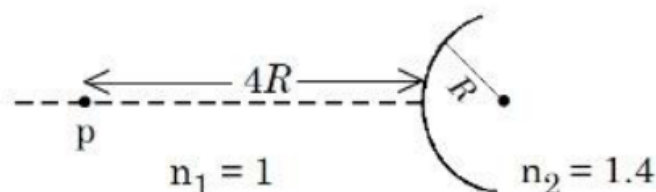
$$I_{\text{new}} = \frac{P}{4\pi L^2} = I_0, \quad \frac{I_{\text{new}}}{I_0} = 1.$$

The 45° angle only specifies the separation of the two radial directions. It is not the angle between a fixed detector area normal and the local ray, and it is not a polarizer angle. Thus neither a cosine projection nor Malus's law applies. The fractions $1/4$, $1/\sqrt{2}$, and $1/2$ would require a change of distance or an additional orientation or polarization condition that is absent here.

Step 3: Conclusion The intensity at the new location remains I_0 . Hence, the correct option is (B).

Quick Tip to Solve: For a point source, first compare distances from the source; angular movement on the same sphere leaves intensity unchanged.

41. A spherical interface lens of radius R separates two media of refractive indices 1 and 1.4 respectively as shown in the figure below. A point source is placed at a distance of $4R$ in front of spherical interface. The magnitude of the magnification of point source image is ____ . [4 marks]



- (A) 1.66
- (B) 2.33
- (C) 2.66
- (D) 1.33

Correct Answer: (A) 1.66

Solution:

Concept:

- For refraction at one spherical surface, the Cartesian formula is $n_2/v - n_1/u = (n_2 - n_1)/R$. Distances are measured from the pole and are positive along the incident-light direction.
- The transverse magnification for a refracting surface is $m = n_1 v / (n_2 u)$. The index factor matters: the thin-lens relation $m = v/u$ does not apply to a single interface between different media.

Step 1: Assign the signed object distance and radius Light travels from the source on the left toward the interface and enters the medium of index 1.4. In the figure, the centre of curvature is on the right

of the pole. Therefore choose rightward as positive and use

$$n_1 = 1, \quad n_2 = 1.4, \quad u = -4R, \quad R_{\text{signed}} = +R.$$

Here R without a sign is the positive radius magnitude. These signs follow the actual surface geometry rather than a memorized lens sign rule.

Step 2: Find the image distance and magnification Substitute the values into the refraction equation:

$$\frac{1.4}{v} - \frac{1}{-4R} = \frac{0.4}{R}, \quad \frac{1.4}{v} = \frac{0.15}{R}.$$

Hence $v = (28/3)R$, a real image in the second medium. The transverse magnification is

$$m = \frac{1 \times (28R/3)}{1.4 \times (-4R)} = -\frac{5}{3}.$$

Its magnitude is $5/3 \simeq 1.67$, represented by the approximate printed value 1.66. The negative sign describes inversion of a small transverse object about the axis; the question asks only for magnitude. The other options do not satisfy the spherical-refraction equation with the specified refractive indices and object distance.

Step 3: Conclusion The magnification magnitude is $5/3 \simeq 1.66$ to the approximation represented in the choices. Hence, the correct option is (A).

Quick Tip to Solve: For a single refracting surface, keep both the signed radius and the index factor in magnification; it is not a thin lens.

42. A small cube of side 1 mm is placed at the centre of a circular loop of radius 10 cm carrying a current of 2 A . The magnetic energy stored inside the cube is $\alpha \times 10^{-14}$ J. The value of α is ____ .

$$(\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}, \pi = 3.14)$$

[4 marks]

- (A) 6.28
- (B) 6.28×10^{-6}
- (C) 628
- (D) 6.28×10^{-4}

Correct Answer: (A) 6.28

Solution:

Concept:

- The magnetic field at the centre of a circular loop carrying current I is $B = \mu_0 I / (2r)$, where r is the loop radius.
- The magnetic energy density in free space is $u_B = B^2 / (2\mu_0)$. The stored energy in a small volume V is $U = u_B V$ when the field varies negligibly across that volume.

Step 1: Calculate the nearly uniform central field Convert the loop radius to $r = 10 \text{ cm} = 0.10 \text{ m}$. The cube side is only 1 mm, much smaller than the loop radius, so the field inside the cube may be taken as its central value. For the single loop,

$$B = \frac{(4\pi \times 10^{-7})(2)}{2(0.10)} = 4\pi \times 10^{-6} \text{ T}.$$

The cube volume requires cubing the converted length, not just converting its numerical value once:

$$V = (10^{-3} \text{ m})^3 = 10^{-9} \text{ m}^3.$$

Step 2: Calculate the stored magnetic energy Insert the field and volume into the energy-density formula:

$$U = \frac{(4\pi \times 10^{-6})^2}{2(4\pi \times 10^{-7})} \times 10^{-9} \text{ J}.$$

Canceling one factor of π and collecting the powers of ten gives

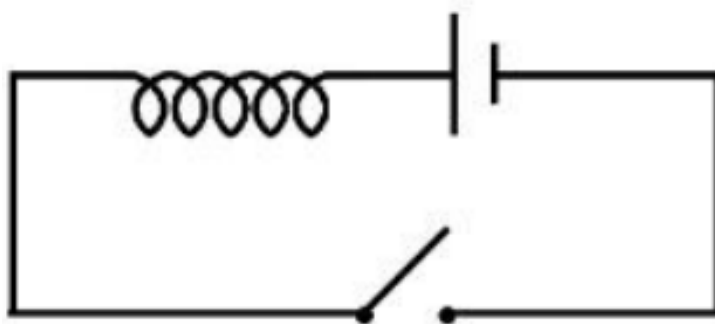
$$U = 2\pi \times 10^{-14} \text{ J} = 6.28 \times 10^{-14} \text{ J}.$$

Comparing with $U = \alpha \times 10^{-14} \text{ J}$, the coefficient is $\alpha = 6.28$. The much smaller coefficients in options (B) and (D), and the larger coefficient in (C), result from incorrect length, volume, or exponent conversions and cannot reproduce this energy.

Step 3: Conclusion The stored energy is $6.28 \times 10^{-14} \text{ J}$, so $\alpha = 6.28$. Hence, the correct option is (A).

Quick Tip to Solve: Convert the cube side to metres before cubing it; the energy uses volume and the square of the loop field.

43. An inductor of inductance 10 mH having resistance of 100Ω is connected to battery of E.M.F. 1.0 V through a switch as shown in the figure below. After switch is closed, the ratio of instantaneous voltages across the inductor when the current passing through it is 2 mA and 4 mA is ____ . [4 marks]



- (A) 4/3
- (B) 3/4
- (C) 5/3
- (D) 3/5

Correct Answer: (A) 4/3

Solution:

Concept:

- For a coil of inductance L and resistance R , Kirchhoff's loop law during current growth is $E = iR + L di/dt$. The induced voltage opposes the increase of current.
- The instantaneous inductive voltage has magnitude $V_L = L di/dt = E - iR$. Distinguish this induced voltage from the total terminal voltage of the resistive coil, which includes its resistive drop.

Step 1: Identify the voltage to compare The resistance is $R = 100\ \Omega$ and the battery emf is $E = 1.0\text{ V}$. As the switch closes, current rises from zero towards $E/R = 10\text{ mA}$. Both specified currents are below this limiting value, so the induced emf opposes the battery at both instants. We compare the inductive voltage, found by subtracting the resistive voltage from the applied emf.

Step 2: Evaluate the two induced voltages Convert the currents into amperes before multiplying by the resistance. At the first instant,

$$V_{L1} = 1.0 - (2 \times 10^{-3})(100) = 0.8\text{ V}.$$

At the second instant,

$$V_{L2} = 1.0 - (4 \times 10^{-3})(100) = 0.6\text{ V}.$$

The larger current gives the larger resistive drop and hence the smaller induced voltage. The inductance controls how rapidly these instants are reached, but cancels from their voltage ratio.

Step 3: Compare with the alternatives Taking the voltages in the order specified,

$$\frac{V_{L1}}{V_{L2}} = \frac{0.8}{0.6} = \frac{4}{3}.$$

The reciprocal $3/4$ reverses the two instants. Neither $5/3$ nor $3/5$ follows from the loop equation and the stated resistance.

Step 4: Conclusion The required ratio of instantaneous inductive voltages is $\boxed{4/3}$. Hence, the correct option is (A).

Quick Tip to Solve: In a growing RL current, calculate the induced voltage as $E - iR$, rather than using the resistive voltage iR .

44. The ratio of momentum of the photons of the 1st and 2nd line of Balmer series of Hydrogen atoms is α/β . The possible values of α and β are:- [4 marks]

- (A) 27 and 20
(B) 3 and 16
(C) 5 and 36
(D) 20 and 27

Correct Answer: (D) 20 and 27

Solution:

Concept:

- A photon has momentum $p = h/\lambda = E_\gamma/c$. Thus photon momentum is proportional to the wavenumber, not to the wavelength.
- The Balmer series consists of hydrogen transitions terminating at $n = 2$. Its first and second lines arise from $n = 3 \rightarrow 2$ and $n = 4 \rightarrow 2$, respectively; the Rydberg relation is $1/\lambda = R_H(1/2^2 - 1/n^2)$.

Step 1: Identify the transitions The first Balmer line is the lowest-energy transition to the second orbit, so its upper orbit is $n = 3$. The second line starts from $n = 4$. These are not the transitions ending at the first orbit, which belong to the Lyman series. Both photons have the same constants h , c and R_H , which cancel when their momenta are divided.

Step 2: Calculate the two wavenumber factors For the first line,

$$\frac{1}{\lambda_1} = R_H \left(\frac{1}{4} - \frac{1}{9} \right) = R_H \frac{5}{36}.$$

For the second line,

$$\frac{1}{\lambda_2} = R_H \left(\frac{1}{4} - \frac{1}{16} \right) = R_H \frac{3}{16}.$$

The second transition releases more energy, so the second photon must have the larger momentum; the required ratio should therefore be less than unity.

Step 3: Form the momentum ratio Use the wavenumbers in the same order as the photons:

$$\frac{p_1}{p_2} = \frac{5/36}{3/16} = \frac{5 \times 16}{36 \times 3} = \frac{20}{27}.$$

The pair (27, 20) gives the reciprocal and fails the energy-order check. The pair (3, 16) describes only the second wavenumber factor, while (5, 36) describes only the first; neither is their ratio.

Step 4: Conclusion One valid pair is $\alpha = 20, \beta = 27$. Hence, the correct option is (D).

Quick Tip to Solve: For spectral photon momentum ratios, divide the Rydberg wavenumber factors directly; invert them only for wavelength ratios.

45. A LCR series circuit driven with $E_{rms} = 90 \text{ V}$ at frequency $f_d = 30 \text{ Hz}$ has resistance $R = 80\Omega$, an inductance with inductive reactance $X_L = 20.0\Omega$ and capacitance with capacitive reactance $X_C = 80.0\Omega$. The power factor of the circuit is ____ . [4 marks]

- (A) 0.8
(B) 0.64
(C) 0.9
(D) 0.5

Correct Answer: (A) 0.8

Solution:

Concept:

- For a series LCR circuit, the net reactance is $X = X_L - X_C$, and the impedance magnitude is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. Resistive and reactive voltage contributions are in quadrature.
- The power factor is $\cos \phi = R/Z$, where ϕ is the phase difference between the source voltage and current. Only resistance consumes average power.

Step 1: Find the net reactance At the stated driving frequency, the component reactances have already been supplied. Therefore there is no need to calculate the inductance or capacitance separately. The net reactance is

$$X = 20.0 - 80.0 = -60.0 \Omega.$$

Its negative sign means that the circuit is predominantly capacitive: current leads the applied voltage. This changes the sign of the phase angle, but does not make the power factor negative.

Step 2: Calculate the impedance and power factor The impedance follows by adding resistance and net reactance as perpendicular phasor components:

$$Z = \sqrt{80^2 + (-60)^2} = \sqrt{10000} = 100 \Omega.$$

Hence

$$\cos \phi = \frac{80}{100} = 0.8.$$

Adding 80 and 60 arithmetically would be incorrect because they are not in phase.

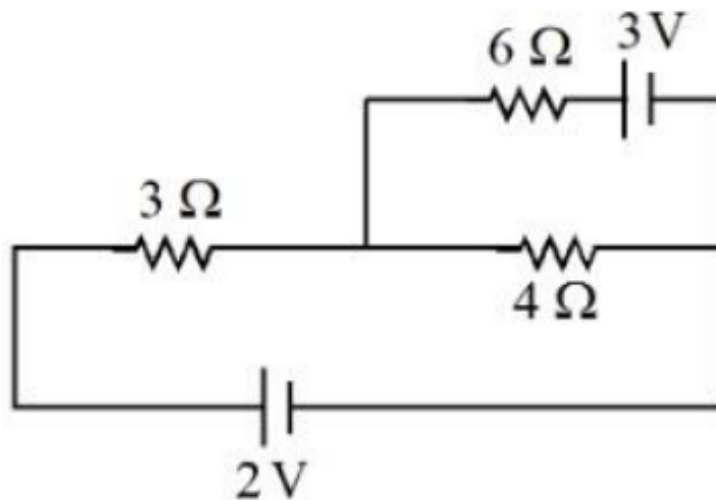
Step 3: Check the result using power The rms current is $I_{rms} = 90/100 = 0.90 \text{ A}$. Thus the real power is $I_{rms}^2 R = 64.8 \text{ W}$, while apparent power is $E_{rms} I_{rms} = 81 \text{ VA}$. Their ratio is again $64.8/81 = 0.8$. The alternative 0.64 is the square of the power factor; 0.9 is the current numerically, and 0.5 is inconsistent with the impedance.

Step 4: Conclusion The circuit power factor is $\boxed{0.8}$, with current leading voltage. Hence, the correct option is (A).

Quick Tip to Solve: Use $R/\sqrt{R^2 + (X_L - X_C)^2}$; the source voltage is unnecessary when the reactances are already given.

Physics – Section B: Numerical Value Questions (20 marks)

46. Refer to the circuit diagram given below. The heat generated across the 6Ω resistance in 100 second is $\frac{\alpha}{100}$ J. The value of α is ____ . (Nearest integer) [4 marks]



Solution:

Concept:

- An ideal battery fixes the potential difference between its terminals, with the longer line representing the positive terminal. At a junction, the algebraic sum of outgoing currents is zero.
- The heat dissipated by a resistor carrying constant current is $H = I^2 R t$. The current through the specified resistor must be found from the whole network, not from the battery voltage divided by that resistance alone.

Step 1: Assign node potentials from the batteries Take the right-hand vertical wire as zero potential. The lower 2 V battery has its positive terminal on the left, so the left-hand wire is at 2 V. The upper 3 V battery is also positive on the left, so the point between that battery and the 6Ω resistor is at 3 V. Let the junction of the 3Ω , 4Ω and 6Ω resistors have potential V .

Step 2: Apply the junction equation Writing the three currents away from the junction gives

$$\frac{V - 2}{3} + \frac{V}{4} + \frac{V - 3}{6} = 0.$$

Multiplication by 12 gives

$$4(V - 2) + 3V + 2(V - 3) = 0, \quad 9V = 14.$$

Therefore $V = 14/9$ V. The current assigned towards the upper battery is

$$I_6 = \frac{14/9 - 3}{6} = -\frac{13}{54} \text{ A}.$$

Its negative sign simply means that the actual current through this resistor flows towards the junction.

Step 3: Calculate the heat and the requested coefficient Heating depends on the square of the current, so its direction has no effect:

$$H = \left(\frac{13}{54}\right)^2 (6)(100) = \frac{8450}{243} \text{ J} \approx 34.7737 \text{ J}.$$

Since the problem defines $H = \alpha/100 \text{ J}$,

$$\alpha = 100H = 3477.366 \dots$$

Round the coefficient only at the last stage, rather than rounding the branch current first.

Step 4: Conclusion The heat is approximately 34.7737 J; the requested nearest-integer coefficient is $\alpha = 3477$.

Quick Tip to Solve: Use the battery polarities to fix two node potentials, solve one junction equation, then square the current only when calculating heat.

47. An unpolarized light of intensity I_0 passes through polarizer and then through a certain optically active solution and finally it goes to analyser. If the angle between analyser and polariser is 0° and intensity of light emerged from analyser is $\frac{3}{8}I_0$, the angle of rotation of the light by the solution with respect to analyser is ____ degrees. [4 marks]

Solution:

Concept:

- An ideal polarizer transmits half the incident intensity of unpolarized light, giving a plane-polarized beam of intensity $I_0/2$.
- Optical activity rotates the plane of polarization. For an ideal analyser, Malus law gives $I = I_{\text{incident}} \cos^2 \theta$, where θ is the angle between its transmission axis and the arriving polarization direction.

Step 1: Track the polarization direction The polarizer and analyser have parallel transmission axes because their stated relative angle is 0° . In the absence of the optically active solution, the analyser would therefore transmit the entire plane-polarized beam incident on it. The solution rotates the polarization plane through an angle θ , so that same angle is now made with the analyser axis. Optical rotation changes the polarization direction; the intensity reduction specified here is obtained at the analyser.

Step 2: Apply Malus law after the first intensity reduction The intensity arriving after the polarizer is $I_p = I_0/2$. Consequently,

$$I = \frac{I_0}{2} \cos^2 \theta.$$

Substitute the given final intensity:

$$\frac{3}{8}I_0 = \frac{1}{2}I_0 \cos^2 \theta \implies \cos^2 \theta = \frac{3}{4}.$$

The initial factor of one-half is essential because the entering light is unpolarized, whereas Malus law applies to the polarized beam arriving at the analyser.

Step 3: Select the principal rotation angle For the acute angle between the polarization direction and the analyser transmission axis,

$$\cos \theta = \frac{\sqrt{3}}{2}, \quad \theta = 30^\circ.$$

This gives $(I_0/2)(3/4) = 3I_0/8$, reproducing the stated transmitted intensity. Intensity alone cannot determine the handedness of rotation, since the square of the cosine is unchanged when the angle changes sign; the requested angle is its principal magnitude.

Step 4: Conclusion The principal angle of rotation relative to the analyser is 30° , so the numerical answer in degrees is 30 .

Quick Tip to Solve: First halve the intensity for an unpolarized incident beam; then apply Malus law using the rotation relative to the analyser.

48. The energy released when $\frac{7}{17.13}$ kg of ${}^7_3\text{Li}$ is converted into ${}^4_2\text{He}$ by proton bombardment is $\alpha \times 10^{32}$ eV. The value of α is ____ . (Nearest integer) (Mass of ${}^7_3\text{Li} = 7.0183\text{u}$, mass of ${}^4_2\text{He} = 4.004\text{u}$, mass of proton = 1.008u and $1\text{u} = 931\text{MeV}/c^2$ and Avogadro number = 6.0×10^{23}) [4 marks]

Solution:

Concept:

- Conservation of nucleon number and charge gives the reaction ${}^7_3\text{Li} + {}^1_1\text{H} \rightarrow 2{}^4_2\text{He}$. One lithium nucleus is consumed per reaction.
- The reaction energy is $Q = \Delta mc^2$, using the supplied conversion $1\text{u}c^2 = 931\text{MeV}$. The total energy is the energy per reaction multiplied by the number of lithium nuclei.

Step 1: Find the mass defect and energy per reaction The initial mass is $7.0183 + 1.008 = 8.0263\text{u}$, while the two helium products have total mass $2(4.004) = 8.0080\text{u}$. Therefore

$$\Delta m = 8.0263 - 8.0080 = 0.0183\text{u}.$$

The positive mass defect means energy is released. Each proton bombardment produces two helium nuclei, so counting only one helium mass would violate nucleon conservation. The released energy per reaction is

$$Q = 0.0183(931) = 17.0373\text{MeV} = 17.0373 \times 10^6\text{eV}.$$

Step 2: Count the lithium nuclei Convert the supplied lithium mass into grams:

$$m = \frac{7000}{17.13}\text{g}.$$

Using the mass-number molar mass 7g mol^{-1} gives

$$n = \frac{1000}{17.13}\text{mol}, \quad N = \frac{1000}{17.13}(6.0 \times 10^{23}).$$

Each of these nuclei undergoes one reaction, so there is no extra factor of two from the helium products. Using $7.0183 \text{ g mol}^{-1}$ instead changes the intermediate total slightly and gives the same requested nearest integer.

Step 3: Compute the total energy and round The total released energy is

$$E = \frac{1000}{17.13} (6.0 \times 10^{23}) (17.0373 \times 10^6) \text{ eV} \approx 5.9675 \times 10^{32} \text{ eV}.$$

Thus the coefficient of 10^{32} eV is approximately 5.97. The nearest-integer instruction applies to this coefficient, after both the number of reactions and the per-reaction energy have been included.

Step 4: Conclusion The total energy is approximately $5.97 \times 10^{32} \text{ eV}$. Hence the requested coefficient is $\boxed{\alpha = 6}$.

Quick Tip to Solve: Balance the nuclear reaction before computing the mass defect, and multiply the per-reaction energy by the number of lithium nuclei, not by the number of product nuclei.

49. A three coulomb charge moves from the point $(0, -2, -5)$ to the point $(5, 1, 2)$ in an electric field expressed as $\vec{E} = 2x\hat{i} + 3y^2\hat{j} + 4z\hat{k} \text{ N/C}$. The work done in moving the charge is ____ J. [4 marks]

Solution:

Concept:

- The work done by an electric field on a charge is $W = q \int_A^B \vec{E} \cdot d\vec{r}$, where $d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$.
- Here each field component depends only on its own coordinate, so the integral separates. Equivalently, $\vec{E} = \nabla(x^2 + y^3 + 4z)$, making the result independent of the path.

Step 1: Set up the coordinate integrals Take the initial point $A = (0, -2, -5)$, final point $B = (5, 1, 2)$, and charge $q = 3 \text{ C}$. The displacement has three components, so using only the straight-line distance would lose the direction dependence of electric work. The required integral is

$$W = 3 \left[\int_0^5 2x \, dx + \int_{-2}^1 3y^2 \, dy + \int_{-5}^2 4z \, dz \right].$$

Step 2: Evaluate all three contributions The antiderivatives are x^2 , y^3 , and $4z$, respectively. Keep the negative lower limits before subtracting:

$$\begin{aligned} W &= 3 [(25 - 0) + (1 - (-8)) + 4(2 - (-5))] \\ &= 3(25 + 9 + 28) = 186 \text{ J}. \end{aligned}$$

Each contribution is positive for this motion, including the y -term, because $3y^2$ is nonnegative and the charge moves toward increasing y .

Step 3: Check the sign convention The calculated value is work done by the field. It equals the decrease of electrostatic potential energy: $\Delta U = -186 \text{ J}$. An external agent moving the charge quasistatically would instead do -186 J ; the positive numerical entry corresponds to the field work asked for here.

Step 4: Conclusion The work done by the electric field is $\boxed{186 \text{ J}}$. The numerical answer is $\boxed{186}$.

Quick Tip to Solve: For a separable field, integrate each component between its own coordinate limits; never multiply the field magnitude by the endpoint distance.

50. A certain gas is isothermally compressed to $\left(\frac{1}{3}\right)^{\text{rd}}$ of its initial volume ($V_o = 3 \text{ litre}$) by applying required pressure. If the bulk modulus of the gas is $3 \times 10^5 \text{ N/m}^2$, the magnitude of work done on the gas is ____ J. [4 marks]

Solution:

Concept:

- The work done on a gas in quasistatic compression is $W_{\text{on}} = -\int_{V_i}^{V_f} P dV$. A constant effective pressure gives $W_{\text{on}} = P(V_i - V_f)$.
- The isothermal bulk modulus is $B_T = -V(dP/dV)_T$. For an ideal gas, $PV = nRT$, so $B_T = P$; it varies during compression.

Step 1: Calculate the volume reduction The initial volume is $V_i = 3 \text{ L} = 3 \times 10^{-3} \text{ m}^3$. Compression to one-third of this volume means

$$V_f = 1 \text{ L} = 1 \times 10^{-3} \text{ m}^3, \quad V_i - V_f = 2 \times 10^{-3} \text{ m}^3.$$

The change in volume is negative, while the magnitude of work done on the gas is positive.

Step 2: Recover the intended numerical entry The stated numerical answer treats the supplied modulus as an effective pressure for the volume reduction. With that convention,

$$W_{\text{on}} = B(V_i - V_f) = (3 \times 10^5)(2 \times 10^{-3}) = 600 \text{ J}.$$

The units check is $\text{Pa m}^3 = \text{J}$. The factor $1/3$ determines the final volume; the volume reduction is $2/3$ of the initial volume, not $1/3$.

Step 3: Distinguish the exact isothermal relation The effective-pressure convention is needed for the quoted entry. Strictly, a reversible isothermal ideal-gas calculation requires

$$W_{\text{on}} = nRT \ln \frac{V_i}{V_f} = nRT \ln 3.$$

If the quoted bulk modulus were the initial value, this would be $900 \ln 3 \text{ J}$; if it were the final value, it would be $300 \ln 3 \text{ J}$. The question does not specify that state or a pressure-volume law for another gas. Thus the exact isothermal work is not uniquely fixed by the wording alone, and 600 J uses the intended constant effective pressure approximation.

Step 4: Conclusion The intended numerical entry is $\boxed{600}$, corresponding to 600 J under the constant effective pressure convention. Exact reversible isothermal work requires the additional state information described above.

Quick Tip to Solve: Convert litres to cubic metres first. In exact isothermal ideal-gas work, use $nRT \ln(V_i/V_f)$, and identify the state at which a quoted bulk modulus applies.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. An oxide of iron contains 69.9% iron, its empirical formula, is :

(Given : Molar mass of Fe and O are 56 and 16 g mol⁻¹ respectively.)

[4 marks]

- (A) FeO
- (B) Fe₂O₃
- (C) Fe₃O₄
- (D) FeO₃

Correct Answer: (B) Fe₂O₃

Solution:

Concept:

- An empirical formula gives the simplest whole-number ratio of atoms. Convert element masses to amounts using $n = m/M$ before comparing their ratio.
- In a binary oxide containing 69.9% iron, oxygen accounts for $100 - 69.9 = 30.1\%$ by mass. A 100 g sample turns these percentages directly into masses.

Step 1: Convert masses to moles Choose 100 g of the oxide. It contains 69.9 g of Fe and 30.1 g of O. Using the given molar masses,

$$n(\text{Fe}) = \frac{69.9}{56} = 1.2482 \text{ mol}, \quad n(\text{O}) = \frac{30.1}{16} = 1.88125 \text{ mol}.$$

Mass percentages cannot be used as the atomic ratio because Fe and O atoms have different masses.

Step 2: Reduce to whole-number subscripts Divide both amounts by the smaller amount:

$$\text{Fe} : \text{O} = 1 : \frac{1.88125}{1.2482} \simeq 1 : 1.507 \simeq 2 : 3.$$

The slight deviation from exactly 1.5 reflects the rounded percentage. Multiplying both relative amounts by two gives the smallest whole-number subscripts, so the empirical formula is Fe₂O₃.

Step 3: Check the alternatives by composition For Fe₂O₃, the predicted iron percentage is

$$\frac{2(56)}{2(56) + 3(16)} \times 100 = 70.0\%,$$

consistent with the given 69.9% to the accuracy of the supplied values. In contrast, FeO, Fe₃O₄, and FeO₃ contain about 77.8%, 72.4%, and 53.8% iron, respectively; none matches the observation.

Step 4: Conclusion The empirical formula is Fe₂O₃. Hence, the correct option is (B).

Quick Tip to Solve: When the mole ratio is near 1 : 1.5, multiply every ratio by two rather than rounding 1.5 to an integer.

52. If shortest wavelength of hydrogen atom in Lyman series is x , then longest wavelength in Balmer series of He^+ is: [4 marks]

- (A) $\frac{9x}{5}$
(B) $\frac{36x}{5}$
(C) $\frac{x}{4}$
(D) $\frac{5x}{9}$

Correct Answer: (A) $\frac{9x}{5}$

Solution:

Concept:

- The Rydberg relation for a hydrogen-like species is $1/\lambda = RZ^2(1/n_f^2 - 1/n_i^2)$, where Z is nuclear charge and $n_i > n_f$.
- The shortest wavelength in a series is its limit $n_i \rightarrow \infty$. The longest wavelength is the first line, with $n_i = n_f + 1$, because it has the smallest transition energy.

Step 1: Express the Lyman limit in terms of x For hydrogen, $Z = 1$, and the Lyman series ends at $n_f = 1$. Its shortest wavelength occurs when the initial level tends to infinity:

$$\frac{1}{x} = R \left(1 - \frac{1}{\infty^2} \right) = R.$$

Thus $x = 1/R$. A finite initial level would produce a smaller wavenumber and hence a longer wavelength, so it cannot define the required limit.

Step 2: Find the first Balmer line of helium ion The ion He^+ has one electron and $Z = 2$, so it is hydrogen-like. The Balmer series ends at $n_f = 2$; its longest-wavelength line is the transition $3 \rightarrow 2$:

$$\frac{1}{\lambda} = 4R \left(\frac{1}{4} - \frac{1}{9} \right) = 4R \frac{5}{36} = \frac{5R}{9}.$$

Consequently $\lambda = 9/(5R) = 9x/5$. The nuclear-charge factor multiplies the wavenumber, not the wavelength.

Step 3: Exclude the other ratios The ratio $36x/5$ omits the factor $Z^2 = 4$ appropriate to helium ion. The value $x/4$ corresponds to a hydrogenic Lyman limit for $Z = 2$, not the first Balmer line. The ratio $5x/9$ reverses the wavelength ratio obtained from the wavenumber calculation. Only $9x/5$ uses both the correct series transition and nuclear charge.

Step 4: Conclusion The longest wavelength in the Balmer series of He^+ is $\boxed{9x/5}$. Hence, the correct option is (A).

Quick Tip to Solve: Mark the final level first: Lyman ends at 1 and Balmer at 2. Then use infinity for the shortest wavelength and the adjacent initial level for the longest.

53.

Match the LIST-I with LIST-II

List-I: Orbital		List-II: Radial nodes and nodal plane	
A.	2s	I.	1 Radial node + two nodal planes
B.	3s	II.	1 Radial node + one nodal plane
C.	3p	III.	2 Radial nodes + No nodal plane
D.	4d	IV.	1 Radial node + No nodal plane

Choose the correct answer from the options given below:

[4 marks]

(A) A-IV, B-I, C-III, D-II

(B) A-IV, B-II, C-III, D-I

(C) A-III, B-I, C-IV, D-II

(D) A-IV, B-III, C-II, D-I

Correct Answer: (D) A-IV, B-III, C-II, D-I**Solution:****Concept:**

- For an orbital with principal quantum number n and azimuthal quantum number l , the number of radial nodes is $n - l - 1$. The number of angular nodes is l .
- The values of l for s, p and d orbitals are 0, 1 and 2. The listed d-orbital entry uses the two nodal-plane description of the usual real d orbitals; for the special d_{z^2} orbital, the two angular nodal surfaces are cones rather than planes.

Step 1: Match the two s orbitals For $2s$, take $n = 2$ and $l = 0$. The radial-node count is $2 - 0 - 1 = 1$, and there is no angular nodal plane. Thus A matches IV. For $3s$, $n = 3$ and $l = 0$, giving $3 - 0 - 1 = 2$ radial nodes and no angular nodal plane. Therefore B matches III. An s orbital is spherically symmetric, so assigning any nodal plane to it would be incorrect.

Step 2: Match the p and d orbitals For $3p$, $n = 3$ and $l = 1$. It has $3 - 1 - 1 = 1$ radial node and one nodal plane, so C matches II. For $4d$, $n = 4$ and $l = 2$, giving $4 - 2 - 1 = 1$ radial node and two angular nodes, represented by the two-plane entry I in the list. Therefore D matches I. In both cases the total node count remains $n - 1$, providing a separate check.

Step 3: Assemble and test the matching code The complete matching is A-IV, B-III, C-II, D-I. Option (A) wrongly assigns B to I, although a $3s$ orbital has no nodal planes. Option (B) wrongly assigns B to II, again giving an s orbital a nodal plane. Option (C) already fails at A, because $2s$ has one radial node rather than the two radial nodes in III. The remaining listed code agrees with all four calculations.

Step 4: Conclusion The matching is A-IV, B-III, C-II, D-I. Hence, the correct option is (D).

Quick Tip to Solve: Calculate $n - l - 1$ and l separately for each orbital; their sum must equal $n - 1$.

54. The pairs among

A = $[\text{SO}_3^{2-}, \text{CO}_3^{2-}]$, B = $[\text{O}_2^{2-}, \text{F}_2]$, C = $[\text{CN}^-, \text{CO}]$, D = $[\text{NH}_3, \text{H}_3\text{O}^+]$ and E = $[\text{MnO}_4^{2-}, \text{CrO}_4^{2-}]$ that do not have similar Lewis dot structure are [4 marks]

(A) A, B and E

(B) A and E

(C) B, C and D

(D) C and D

Correct Answer: (B) A and E

Solution:

Concept:

- Similar Lewis structures require the same valence-electron arrangement and analogous bonding and lone pairs. Count valence electrons, adding electrons for negative charge and subtracting for positive charge.
- Isoelectronic pairs with analogous atom connectivity generally have similar Lewis patterns. Extra electrons can alter lone-pair or unpaired-electron occupancy, even when the skeleton has the same number of atoms.

Step 1: Identify the two unequal electron counts In pair A, sulfite has $6 + 3(6) + 2 = 26$ valence electrons, whereas carbonate has $4 + 3(6) + 2 = 24$. Sulfite has a central lone pair and a pyramidal arrangement; carbonate has no central lone pair and a planar arrangement. Their Lewis structures are different. In pair E, manganate has $7 + 4(6) + 2 = 33$ valence electrons, while chromate has $6 + 4(6) + 2 = 32$. Manganate has an unpaired electron whereas chromate is closed-shell, so their complete electron-dot descriptions differ despite the tetrahedral oxygen skeleton.

Step 2: Verify the three similar pairs For B, peroxide and fluorine each contain 14 valence electrons: $2(6) + 2 = 14$ for O_2^{2-} , and $2(7) = 14$ for F_2 . Both have one single bond and three lone pairs on each atom. For C, CN^- and CO each have 10 valence electrons and a triple bond with one lone pair on each atom. For D, NH_3 and H_3O^+ each have 8 valence electrons, three single bonds and one central lone pair.

Step 3: Select only the dissimilar pairs The pairs that do not have similar Lewis structures are consequently A and E. Option (A) incorrectly includes B: the peroxide and fluorine pair has the same single-bond and lone-pair pattern. Options (C) and (D) include C and D, although both pairs have matching valence-electron counts and bonding patterns. Thus those choices reverse the required distinction. Similar molecular shape alone is insufficient for pair E because the odd electron must also be counted.

Step 4: Conclusion The dissimilar pairs are A and E. Hence, the correct option is (B).

Quick Tip to Solve: Include ionic charge in the valence-electron count. An odd total, such as 33 for manganate, immediately distinguishes it from a closed-shell analogue.

55. Arrange the following isothermal processes in order of the magnitude of the work ($p - V$) involved between states 1 and 2.

A. Expansion in single stage w_A

B. Expansion in multi stages w_B

C. Compression in single stage w_C

D. Compression in multi stages w_D

Choose the correct option.

[4 marks]

(A) $|w_B| > |w_A| > |w_C| > |w_D|$

(B) $|w_C| > |w_D| > |w_A| > |w_B|$

(C) $|w_C| > |w_D| > |w_B| > |w_A|$

(D) $|w_B| > |w_A| > |w_D| > |w_C|$

Correct Answer: (C) $|w_C| > |w_D| > |w_B| > |w_A|$

Solution:

Concept:

- Pressure-volume work in the chemistry sign convention is $w = - \int p_{\text{ext}} dV$. Its magnitude is the area under the external-pressure path, so the number and direction of the pressure steps matter.
- For an ideal gas at fixed temperature the reversible path is $p = nRT/V$. Reversible expansion gives the greatest expansion work; reversible compression requires the least compression work between the same endpoints.

Step 1: Compare the expansion paths Let the endpoint volumes be $V_1 < V_2$, with $p_1 > p_2$. A single expansion against the final pressure has magnitude $p_2(V_2 - V_1)$. Multiple stages use successively decreasing external pressures, each larger than the final pressure during part of the expansion. Their total rectangular area is therefore larger:

$$|w_B| > |w_A|.$$

Increasing the stages approaches $nRT \ln(V_2/V_1)$ from below.

Step 2: Compare compression and join the inequalities A single compression back to the initial volume works against the high final pressure throughout: $|w_C| = p_1(V_2 - V_1)$. Dividing compression into stages lowers the external pressure over part of the path, so $|w_C| > |w_D|$. Compression-stage pressures lie above the isothermal equilibrium curve, whereas expansion-stage pressures lie below it. Thus multistage compression requires more work than multistage expansion delivers. Combining these comparisons gives

$$|w_C| > |w_D| > |w_B| > |w_A|.$$

Options (A) and (D) incorrectly make expansion work exceed compression work; (B) reverses the expansion comparison.

Step 3: Conclusion The decreasing order is $|w_C| > |w_D| > |w_B| > |w_A|$. Hence, the correct option is (C).

Quick Tip to Solve: Think of rectangles above or below the isotherm: compression rectangles are above it, expansion rectangles below it; finer steps approach the curve.

56. When 0.25 moles of a non-volatile, non-ionizable solute was dissolved in 1 mole of a solvent the vapor pressure of solution was $x\%$ of vapor pressure of pure solvent. What is $x\%$? [4 marks]

- (A) 50%
- (B) 60%
- (C) 70%
- (D) 80%

Correct Answer: (D) 80%

Solution:

Concept:

- For an ideal solution containing a non-volatile solute, only the solvent contributes to the vapour pressure. Raoult's law gives $p = X_{\text{solvent}}p^\circ$, where p° is the pure-solvent vapour pressure at the same temperature.
- Mole fraction counts particles: $X_{\text{solvent}} = n_{\text{solvent}}/(n_{\text{solvent}} + n_{\text{solute}})$. Non-ionizability means the stated solute moles are not multiplied by a dissociation factor.

Step 1: Calculate the solvent mole fraction The solvent amount is 1 mol, while the solute amount is 0.25 mol. The total amount in the liquid is therefore 1.25 mol, not one mole. Hence

$$X_{\text{solvent}} = \frac{1}{1 + 0.25} = \frac{1}{1.25} = 0.80.$$

The solute mole fraction is correspondingly 0.20; the two fractions add to unity. The solvent is the component whose mole fraction multiplies the pure-solvent pressure.

Step 2: Express the pressure as a percentage Substitute this mole fraction directly into Raoult's law:

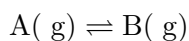
$$\frac{p}{p^\circ} = 0.80, \quad \frac{p}{p^\circ} \times 100\% = 80\%.$$

Thus the vapour pressure remaining is eighty percent of its original value. The relative lowering is twenty percent, but the question asks for the pressure of the solution as a percentage of the pure-solvent pressure. Options (A), (B) and (C) would require solvent mole fractions 0.50, 0.60 and 0.70, respectively, all inconsistent with the given mole amounts. No molecular mass, volume or temperature value is required because the ratio is fixed by composition.

Step 3: Conclusion The solution vapour pressure is 80% of that of the pure solvent, so $x = 80$. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish the pressure remaining, $100X_{\text{solvent}}$, from the percentage lowering, $100X_{\text{solute}}$.

57. One mole each of He and A(g) are taken in a 10 L closed flask and heated to 400 K to establish the following equilibrium.



K_c for this reaction at 400 K is 4.0. The partial pressures (in atm) of He and B(g) are respectively (at equilibrium)

(Assume He, A(g) and B(g) behave as ideal gases)

(Given : $R = 0.082 \text{ L atm K}^{-1} \text{ mol}^{-1}$)

[4 marks]

(A) 3.28, 2.624

(B) 2.624, 3.28

(C) 3.28, 0.656

(D) 0.656, 6.56

Correct Answer: (A) 3.28, 2.624

Solution:

Concept:

- For $A(g) \rightleftharpoons B(g)$, $K_c = [B]/[A]$. Since the gases occupy the same volume, this concentration ratio also equals their mole ratio.
- Each ideal gas has partial pressure $p_i = n_i RT/V$. An inert gas does not enter the equilibrium expression, and at fixed volume its own partial pressure depends only on its unchanged mole amount and temperature.

Step 1: Find the equilibrium mole amounts Initially $n_A = 1 \text{ mol}$ and no B is present. Let ξ moles of A convert to B. The equilibrium amounts are $1 - \xi$ and ξ , respectively. Therefore

$$4.0 = \frac{\xi/10}{(1-\xi)/10} = \frac{\xi}{1-\xi}.$$

Solving gives $5\xi = 4$, or $\xi = 0.8 \text{ mol}$. Thus $n_B = 0.8 \text{ mol}$ and $n_A = 0.2 \text{ mol}$. The helium amount remains exactly one mole.

Step 2: Calculate the two requested partial pressures Using the supplied gas constant and the flask volume,

$$\frac{RT}{V} = \frac{0.082 \times 400}{10} = 3.28 \text{ atm mol}^{-1}.$$

Hence

$$p_{\text{He}} = 1 \times 3.28 = 3.28 \text{ atm}, \quad p_B = 0.8 \times 3.28 = 2.624 \text{ atm}.$$

As a check, $p_A = 0.656 \text{ atm}$, giving $p_B/p_A = 4$. Option (B) interchanges the requested gases. Option (C) uses the pressure of unreacted A instead of B, while (D) neither preserves helium's partial pressure nor the equilibrium mole balance.

Step 3: Conclusion The partial pressures of helium and B are 3.28 atm, 2.624 atm, respectively. Hence, the correct option is (A).

Quick Tip to Solve: For a one-to-one gas equilibrium, obtain the mole ratio first; then multiply each mole amount by the common factor RT/V .

58.

Consider the following data.

Electrolyte	$\Lambda_m^\circ (\text{S cm}^2 \text{ mol}^{-1})$
BaCl ₂	x_1
H ₂ SO ₄	x_2
HCl	x_3

BaSO₄ is sparingly soluble in water. If the conductivity of the saturated BaSO₄ solution is $x \text{ S cm}^{-1}$ then the solubility product of BaSO₄ can be given as (Here $\Lambda_m = \Lambda_m^\circ$) [4 marks]

(A) $\frac{10^6 x^2}{\alpha^2 (x_1 + x_2 - 2x_3)^2}$

(B) $\frac{x^2}{(x_1 + x_2 - 2x_3)^2}$

(C) $\frac{\alpha^2 (x_1 + x_2 - 2x_3)^2}{10^6 x^2}$

(D) $\frac{x^2}{(x_1 + x_2 + 2x_3)^2}$

Correct Answer: (A) $\frac{10^6 x^2}{\alpha^2 (x_1 + x_2 - 2x_3)^2}$

Solution:**Concept:**

- Kohlrausch's law expresses limiting molar conductivity as the stoichiometric sum of ionic conductivities. Combining electrolytes can cancel unwanted ions to obtain that of a sparingly soluble salt.
- With conductivity κ in S cm^{-1} and concentration c in mol L^{-1} , $\Lambda_m = 1000\kappa/c$. For a fully dissociated 1:1 salt with molar solubility s , $K_{sp} = s^2$.

Step 1: Eliminate the hydrogen and chloride contributions Write the limiting ionic sums:

$$x_1 = \lambda_{\text{Ba}^{2+}}^\circ + 2\lambda_{\text{Cl}^-}^\circ,$$

$$x_2 = 2\lambda_{\text{H}^+}^\circ + \lambda_{\text{SO}_4^{2-}}^\circ,$$

$$2x_3 = 2\lambda_{\text{H}^+}^\circ + 2\lambda_{\text{Cl}^-}^\circ.$$

Subtracting the third line from the sum of the first two gives

$$L = x_1 + x_2 - 2x_3 = \Lambda_m^\circ(\text{BaSO}_4).$$

Adding $2x_3$, as in option (D), would retain rather than cancel the unwanted ions.

Step 2: Convert conductivity to dissolved concentration The question explicitly states $\Lambda_m = \Lambda_m^\circ = L$, corresponding to complete dissociation, $\alpha = 1$. With $\kappa = x$,

$$s = \frac{1000x}{L}, \quad K_{sp} = s^2 = \frac{10^6 x^2}{(x_1 + x_2 - 2x_3)^2}.$$

Option (A) contains an additional α^2 in the denominator; under the stated equality $\alpha = \Lambda_m/\Lambda_m^\circ = 1$, it reduces exactly to this result. No unspecified dissociation value is needed. Option (B) omits the litre-

to-cubic-centimetre factor squared, and option (C) inverts the required dependence on conductivity. The factor 10^6 is essential because the ionic product is expressed using molar concentrations.

Step 3: Conclusion With the stipulated $\alpha = 1$, $K_{sp} = 10^6 x^2 / (x_1 + x_2 - 2x_3)^2$, which is the expression in (A). Hence, the correct option is (A).

Quick Tip to Solve: First obtain the salt's limiting conductivity by ionic cancellation; then remember the factor 1000 before squaring the molar solubility.

59. Given below are two statements :

Statement I : Aluminium is more electropositive than thallium as the standard electrode potential value of $E^\circ \text{Al}^{3+}/\text{Al}$ is negative and $E^\circ \text{Tl}^{3+}/\text{Tl}$ is positive.

Statement II : The sum of first three ionization enthalpies of boron is very high when compared to that of aluminium. Due to this reason boron forms covalent compounds only and aluminium forms Al^{3+} ion.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

- (A) Both Statement I and Statement II are true
- (B) Both Statement I and Statement II are false
- (C) Statement I is true but Statement II is false
- (D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- A more negative standard reduction potential for M^{3+}/M means greater tendency for the reverse oxidation under standard conditions. This provides an electrochemical comparison of metallic electropositivity.
- The energy needed to form a tripositive gaseous ion includes the first three ionization enthalpies. Boron's very small size makes this sum especially large; aluminium is larger and its tripositive ion can be stabilized in ionic environments.

Step 1: Assess Statement I using electrode potentials The given comparison is $E^\circ_{\text{Al}^{3+}/\text{Al}} < 0$, whereas $E^\circ_{\text{Tl}^{3+}/\text{Tl}} > 0$. Reversing each reduction to oxidation reverses its potential sign. Aluminium therefore has a stronger tendency to lose electrons than thallium in this comparison. This supports the stated greater electropositivity of aluminium, so Statement I is true. One should compare the specified electrode couples; the separate stability of the +1 oxidation state of thallium does not reverse the given comparison.

Step 2: Assess Statement II and select the combination Boron has configuration $1s^2 2s^2 2p^1$. Removing all three valence electrons from this compact atom requires a very large total ionization enthalpy, so formation of a simple B^{3+} ion is energetically unfavourable. Boron instead forms covalent compounds by sharing electrons. Aluminium's larger size reduces the corresponding ionization cost, and lattice or hydration stabilization permits formation of Al^{3+} . Thus Statement II is also true. This does not say

that every aluminium compound must be purely ionic: some aluminium compounds have substantial covalent character. Options (B), (C) and (D) each reject at least one of the two correct statements and are therefore unsuitable.

Step 3: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: Keep the two comparisons separate: reduction potentials assess metallic oxidation tendency, while the three-ionization-energy sum explains the difficulty of forming B^{3+} .

60. The correct statements among the following are :

A. Basic vanadium oxide is used in the manufacture of H_2SO_4 .

B. The spin-only magnetic moment value of the transition metal halide employed in Ziegler-Natta polymerization is 2.84 BM .

C. The p-block metal compound employed in Ziegler-Natta polymerization has the metal in +3 oxidation state.

D. The number of electrons present in the outer most ' d ' orbital of metal halide employed in Wacker process is 8.

Choose the correct answer from the options given below :

[4 marks]

(A) A and B Only

(B) A, C and D Only

(C) C and D Only

(D) B, C and D Only

Correct Answer: (C) C and D Only

Solution:

Concept:

- Catalyst questions require the actual oxidation state and electron configuration, rather than the configuration of the neutral metal. The spin-only moment is $\mu = \sqrt{n(n+2)}$ BM, where n is the number of unpaired electrons.
- The contact process uses V_2O_5 , Ziegler-Natta catalysts contain a titanium halide and an organoaluminium cocatalyst, and the Wacker process employs palladium(II) chloride with copper(II) chloride.

Step 1: Test statements A and B In the contact process, V_2O_5 catalyses oxidation of SO_2 to SO_3 . It is an amphoteric high-oxidation-state vanadium oxide, not a basic vanadium oxide as asserted, so A is false. The titanium halide commonly specified for Ziegler-Natta polymerization is $TiCl_4$. Titanium(IV) is d^0 , so its spin-only moment is zero. Titanium(III) chloride, used in related catalyst formulations, is d^1 and has moment $\sqrt{3}$ BM, also not 2.84 BM. The quoted 2.84 BM corresponds to two unpaired electrons; B is false.

Step 2: Test statements C and D The p-block cocatalyst is triethylaluminium, $Al(C_2H_5)_3$. Assigning each ethyl group charge -1 gives aluminium oxidation state $+3$, so C is true. In the Wacker catalyst $PdCl_2$, palladium is $+2$. Neutral palladium has a filled $4d$ shell; palladium(II) has configuration $4d^8$.

Its outermost occupied d subshell therefore contains eight electrons, so D is true. Options (A) and (B) include the false statement A, and option (D) includes the false statement B. Only the combination of C and D survives all four checks.

Step 3: Conclusion The correct statements are C and D only. Hence, the correct option is (C).

Quick Tip to Solve: Write the catalyst formula before counting d electrons: TiCl_4 gives d^0 , AlEt_3 gives $\text{Al}(+3)$, and PdCl_2 gives d^8 .

61. Match LIST-I with LIST-II.

LIST-I		LIST-II	
Tetrahedral metal-ion configuration		CFSE (in units of Δ_t)	
A	d^2	I	-0.6
B	d^4	II	-0.8
C	d^6	III	-1.2
D	d^8	IV	-0.4

Choose the correct answer from the options given below:

[4 marks]

- (A) A-III, B-IV, C-II, D-I
 (B) A-III, B-I, C-IV, D-II
 (C) A-III, B-IV, C-I, D-II
 (D) A-II, B-I, C-IV, D-III

Correct Answer: (C) A-III, B-IV, C-I, D-II

Solution:

Concept:

- In a tetrahedral field, the two e orbitals lie at $-0.6\Delta_t$ each and the three t_2 orbitals lie at $+0.4\Delta_t$ each, relative to the barycentre.
- Tetrahedral complexes are normally high spin because their splitting is small. Fill all five d orbitals singly before pairing; then calculate $\text{CFSE} = (-0.6n_e + 0.4n_{t_2})\Delta_t$.

Step 1: Fill the lower configurations For A, the two electrons of d^2 occupy the two lower e orbitals, giving $e^2t_2^0$. Thus its CFSE is $2(-0.6)\Delta_t = -1.2\Delta_t$, so A matches III. For B, the high-spin d^4 arrangement is $e^2t_2^2$, since the third and fourth electrons enter separate upper orbitals instead of pairing below. Its CFSE is $[2(-0.6) + 2(0.4)]\Delta_t = -0.4\Delta_t$, so B matches IV.

Step 2: Fill the higher configurations For C, distribute five electrons singly first; the sixth pairs in the lower set. The d^6 arrangement is therefore $e^3t_2^3$, giving $[3(-0.6) + 3(0.4)]\Delta_t = -0.6\Delta_t$, hence C matches I. For D, the d^8 arrangement is $e^4t_2^4$, giving $[4(-0.6) + 4(0.4)]\Delta_t = -0.8\Delta_t$, hence D matches II.

Step 3: Compare all four combinations The required sequence is A-III, B-IV, C-I, D-II. Option A incorrectly interchanges the energies of C and D. Option B assigns $-0.6\Delta_t$ to B instead of $-0.4\Delta_t$. Option D already gives the wrong result for d^2 , which must have two electrons in the lower set.

Step 4: Conclusion The matching is A-III, B-IV, C-I, D-II. Hence, the correct option is (C).

Quick Tip to Solve: Remember the tetrahedral order: e below t_2 , with coefficients -0.6 and +0.4. Determine occupancies before multiplying.

62. Which of the following are true about the energy of the given d-orbitals of a tetrahedral complex?

- A. $d_{xy} = d_{xz} > d_{x^2-y^2}$
- B. $d_{xy} = d_{yz} > d_{z^2}$
- C. $d_{x^2-y^2} > d_{z^2} > d_{xz}$
- D. $d_{x^2-y^2} = d_{z^2} < d_{xz}$

Choose the correct answer from the given below :

[4 marks]

- (A) A, B and D only
- (B) A and B only
- (C) B and D only
- (D) B, C and D only

Correct Answer: (A) A, B and D only

Solution:

Concept:

- Tetrahedral ligands approach between the Cartesian axes. The d_{xy}, d_{xz}, d_{yz} orbitals form the higher-energy triply degenerate t_2 set.
- The $d_{x^2-y^2}$ and d_{z^2} orbitals form the lower-energy doubly degenerate e set. Degenerate orbitals have equal energies in the ideal tetrahedral field.

Step 1: Establish the two energy levels The complete ordering needed here is

$$E(d_{xy}) = E(d_{xz}) = E(d_{yz}) > E(d_{x^2-y^2}) = E(d_{z^2}).$$

The upper orbitals point more nearly towards the directions of ligand approach and experience greater repulsion. The two lower orbitals remain equal in energy; there is no intermediate third level in this ideal splitting pattern.

Step 2: Test statements A and B Statement A places d_{xy} and d_{xz} at the same energy above $d_{x^2-y^2}$. Both belong to the upper set, whereas the last belongs to the lower set, so A is true. Statement B likewise equates the upper-set orbitals d_{xy} and d_{yz} and puts them above lower-set d_{z^2} ; B is true.

Step 3: Test statements C and D and select Statement C is false: it splits the two e orbitals and reverses their position relative to d_{xz} . Statement D correctly equates the two lower e orbitals and places them below d_{xz} , so D is true. Thus B-only combinations missing A are incomplete, A-and-B-only omits true D, and a combination containing C includes a false statement.

Step 4: Conclusion A, B and D are true, while C is false. Hence, the correct option is (A).

Quick Tip to Solve: Group the orbitals into e and t_2 before testing inequalities; within a group write equality, and place e lower for a tetrahedron.

63. R_f value for 2-methylpropene in a solvent system (Ethyl acetate + ether) is 0.42. 2-methylpropene is treated with dilute H_2SO_4 to give major organic product (X). R_f value for (X) in the same solvent system under identical condition will be:

[4 marks]

- (A) 0.42
- (B) 0.82
- (C) 0.62
- (D) 0.12

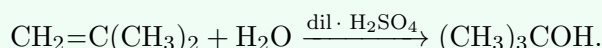
Correct Answer: (D) 0.12

Solution:

Concept:

- Acid-catalysed hydration of an alkene adds water in Markovnikov orientation. 2-Methylpropene therefore gives the tertiary alcohol 2-methylpropan-2-ol.
- In normal adsorption chromatography, the polar stationary phase retains a more polar compound more strongly. R_f is the distance travelled by solute divided by the distance travelled by the solvent front.

Step 1: Identify the hydration product Protonation of $CH_2=C(CH_3)_2$ produces the stable tertiary carbocation $(CH_3)_3C^+$. Water attacks it, and loss of a proton regenerates the acid to give $(CH_3)_3COH$. The overall change is



The product contains a hydroxyl group, whereas the starting alkene is a nonpolar hydrocarbon.

Step 2: Compare adsorption under identical conditions The alcohol can form strong polar and hydrogen-bonding interactions with the usual polar adsorbent. It therefore spends a larger fraction of its time held on the stationary phase and travels a shorter distance up the plate than the alkene in the same solvent system. Keeping the solvent composition and other conditions identical makes this polarity comparison meaningful; a change in eluent could otherwise change both values.

Step 3: Select the smaller retention factor The alkene has $R_f = 0.42$, so the more strongly retained alcohol must have $R_f < 0.42$. The value 0.42 would imply no change in migration, while 0.82 and 0.62 would indicate weaker retention and greater migration. Among the listed values, only 0.12 satisfies the required decrease. Polarity establishes the direction of change; the options identify the suitable value.

Step 4: Conclusion The suitable value for X is $R_f = 0.12$, a dimensionless ratio. Hence, the correct option is (D).

Quick Tip to Solve: On a polar adsorbent with unchanged eluent, converting an alkene into an alcohol normally lowers its R_f .

64. Given below are two statements :

Statement I : 2, 6-diethylcyclohexanone and 6-methyl-2-n-propylcyclohexanone are metamers.

Statement II : 2, 2, 6, 6 - tetramethylcyclohexanone exhibits keto-enol tautomerism.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

- (A) Both Statement I and Statement II are true
(B) Both Statement I and Statement II are false
(C) Statement I is true but Statement II is false
(D) Statement I is false but Statement II is true

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- Metamers have the same molecular formula and functional group but differ in the distribution of carbon groups on the two sides of a polyvalent functional group such as the ketonic carbonyl.
- Ordinary keto-enol tautomerism requires at least one alpha hydrogen: a hydrogen on a carbon directly adjacent to the carbonyl carbon. An enol forms by transfer of that hydrogen to oxygen and formation of an adjacent C=C bond.

Step 1: Compare the two substituted ketones Both molecules retain the cyclohexanone carbonyl and have four additional alkyl carbons, so their molecular formula is the same. In 2,6-diethylcyclohexanone the alkyl carbon distribution at the two sides of the carbonyl is ethyl/ethyl, whereas in 6-methyl-2-n-propylcyclohexanone it is methyl/n-propyl. The carbon framework is redistributed about the same ketone functionality without changing its molecular formula. This is the metameric relationship specified in Statement I, so Statement I is true.

Step 2: Count the alpha hydrogens in the second statement In cyclohexanone the alpha carbons are C-2 and C-6. In 2,2,6,6-tetramethylcyclohexanone each of these ring carbons has bonds to two ring neighbours and to two methyl groups. Each therefore already has four carbon bonds and bears no hydrogen. The hydrogens on the methyl substituents are not alpha hydrogens of the carbonyl: their carbons are one further bond away.

Step 3: Apply the tautomerism condition With no hydrogen on either alpha carbon, the normal proton transfer and adjacent double-bond formation needed for keto-enol tautomerism cannot occur. Statement II is false. A choice declaring both statements true overlooks this absence, both false rejects the metameric pair, and I-false/II-true reverses both evaluations.

Step 4: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: For keto-enol tautomerism, inspect the two carbons directly next to C=O; hydrogens elsewhere do not satisfy the alpha-hydrogen requirement.

65. Given below are two statements :

Statement I : Methane can be prepared by decarboxylation of sodium ethanoate, Kolbe's electrolysis of sodium acetate and reaction of CH_3MgBr with water.

Statement II : Methane cannot be prepared from unsaturated hydrocarbons and by Wurtz reaction.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

- (A) Both Statement I and Statement II are true
(B) Both Statement I and Statement II are false
(C) Statement I is true but Statement II is false
(D) Statement I is false but Statement II is true

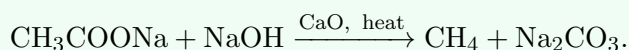
Correct Answer: (D) Statement I is false but Statement II is true

Solution:

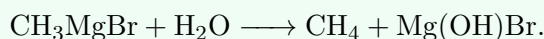
Concept:

- Soda-lime decarboxylation removes the carboxyl carbon of a sodium carboxylate, giving an alkane with one fewer carbon atom. Protonation of an alkyl Grignard reagent gives the corresponding alkane.
- Kolbe electrolysis couples the radicals formed after decarboxylation. Wurtz reaction also couples alkyl groups, and simple hydrogenation preserves the carbon skeleton of an unsaturated hydrocarbon.

Step 1: Check each preparation in Statement I Sodium ethanoate gives methane with soda lime:

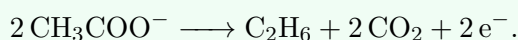


Methylmagnesium bromide also gives methane on hydrolysis:



These two preparations are valid, but every preparation in the compound statement must be valid for Statement I to be true.

Step 2: Identify the Kolbe product At the anode, acetate ions lose carbon dioxide to generate methyl radicals, and two methyl radicals combine:



Thus Kolbe electrolysis of sodium acetate yields ethane as its coupled hydrocarbon, not methane. This single false claim makes Statement I false even though its other two methods work.

Step 3: Evaluate Statement II Ordinary alkenes and alkynes require at least two carbon atoms. Hydrogenating their multiple bonds cannot produce the one-carbon alkane methane because hydrogenation does not remove carbon atoms. Wurtz coupling joins two alkyl groups; even coupling two methyl groups produces ethane, not methane. Therefore methane is not prepared by either standard method stated in II. Statement II is true. Options A and C incorrectly accept I, while B incorrectly rejects II.

Step 4: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish loss of one carbon in soda-lime decarboxylation from radical coupling in Kolbe electrolysis: sodium acetate gives methane in the first and ethane in the second.

66. Given below are two statements :

Statement I : 3-phenylpropene reacts with HBr and gives secondary alkyl bromide having a chiral carbon atom as the major product.

Statement II : Aryl chlorides and aryl cyanides can be prepared by Sandmeyer reaction as well as Gattermann reaction.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

- (A) Both Statement I and Statement II are true
(B) Both Statement I and Statement II are false
(C) Statement I is true but Statement II is false
(D) Statement I is false but Statement II is true

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- Without a peroxide condition, HBr adds to an alkene through electrophilic protonation and carbocation capture. Carbocation rearrangement can favour a resonance-stabilized benzylic cation.
- Sandmeyer replacement of a diazonium group uses cuprous salts such as CuCl or CuCN. The Gattermann method uses copper powder with a hydrogen halide for chloride or bromide replacement; cyanation is not this method.

Step 1: Follow HBr addition to 3-phenylpropene The alkene is $\text{C}_6\text{H}_5\text{CH}_2\text{CH}=\text{CH}_2$. Markovnikov protonation initially gives the secondary carbocation $\text{C}_6\text{H}_5\text{CH}_2\text{CH}^+\text{CH}_3$. A hydride shift from the adjacent benzylic carbon yields the resonance-stabilized benzylic carbocation $\text{C}_6\text{H}_5\text{CH}^+\text{CH}_2\text{CH}_3$, which is then captured by bromide. The favoured product is $\text{C}_6\text{H}_5\text{CH}(\text{Br})\text{CH}_2\text{CH}_3$, a secondary benzylic bromide.

Step 2: Check chirality and the first statement The carbon bearing bromine is bonded to Br, H, a phenyl group and an ethyl group. These are four different substituents, so it is a chiral carbon. Attack on the planar carbocation can occur from either face, giving a racemic mixture; a racemate still consists of molecules possessing a chiral carbon. Statement I is therefore true. Even tracking the unrearranged addition requires checking four different groups rather than equating secondary with achiral.

Step 3: Distinguish the named diazonium reactions An aryl diazonium salt can give aryl chloride with CuCl and aryl cyanide with CuCN by Sandmeyer replacement. Gattermann replacement with copper powder and HCl gives aryl chloride, but does not provide the corresponding general preparation of aryl cyanide. Thus the claim that both compounds are accessible by both named reactions is false. Options A and D wrongly accept II, while B wrongly rejects I.

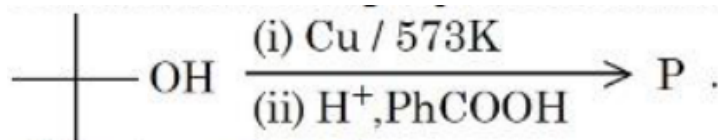
Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: Sandmeyer includes CuCN cyanation; keep this distinct from the copper-powder/HCl or HBr conditions of Gattermann replacement.

67. Consider the following sequence of reactions

The major product P is :

[4 marks]



- (A) $p\text{-(CH}_3)_3\text{C-C}_6\text{H}_4\text{-COOH}$
 (B) $m\text{-(CH}_3)_3\text{C-C}_6\text{H}_4\text{-COOH}$
 (C) $\text{C}_6\text{H}_5\text{COOC(CH}_3)_3$
 (D) $\text{C}_6\text{H}_5\text{COOCH}_2\text{C(CH}_3)=\text{CH}_2$

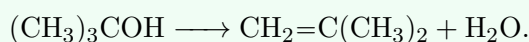
Correct Answer: (C) $\text{C}_6\text{H}_5\text{COOC(CH}_3)_3$

Solution:

Concept:

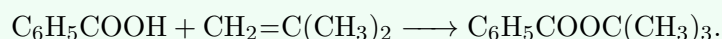
- A tertiary alcohol has no hydrogen on the carbon bearing OH; it cannot undergo the ordinary copper-catalysed dehydrogenation of primary or secondary alcohols.
- On hot copper at 573 K, a tertiary alcohol undergoes dehydration. An alkene then adds a carboxylic acid under acid catalysis with Markovnikov orientation.

Step 1: Identify the first product The starting compound is tert-butyl alcohol, $(\text{CH}_3)_3\text{COH}$. Its hydroxyl-bearing carbon is bonded to three methyl groups. Loss of water removes the hydroxyl group and a hydrogen from a neighbouring methyl group, producing 2-methylpropene:



A ketone is not formed because the required hydrogen on the alcohol carbon is absent.

Step 2: Add benzoic acid to the alkene In the presence of H^+ , the terminal CH_2 carbon of 2-methylpropene accepts a proton. This puts the positive charge on the more substituted carbon and gives the stable tert-butyl carbocation. Benzoic acid supplies the benzoate oxygen that traps this carbocation; loss of a proton regenerates the catalyst and yields the ester



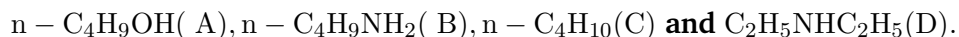
The linkage is between the carboxyl oxygen and the tert-butyl group.

Step 3: Compare the four structures The product is tert-butyl benzoate, the structure in option (C). Options (A) and (B) place the tert-butyl group directly on the aromatic ring while retaining a free carboxylic acid group; neither represents addition across the alkene through oxygen. Option (D) contains an unsaturated methallyl group and an oxygen bonded to a primary carbon, whereas Markovnikov addition consumes the double bond and attaches oxygen at the tertiary carbon.

Step 4: Conclusion The major product is tert-butyl benzoate, $\text{C}_6\text{H}_5\text{COOC(CH}_3)_3$. Hence, the correct option is (C).

Quick Tip to Solve: Check the alcohol class before applying hot-copper reactions: a tertiary alcohol dehydrates, and the resulting alkene undergoes Markovnikov addition of the acid.

68. Arrange the following compounds according to increasing order of boiling points.



[4 marks]

(A) $C < B < A < D$

(B) $D < C < B < A$

(C) $C < D < B < A$

(D) $D < B < A < C$

Correct Answer: (C) $C < D < B < A$

Solution:

Concept:

- For compounds of comparable molar mass, intermolecular hydrogen bonding raises the boiling point relative to dispersion forces alone.
- Alcohols form stronger hydrogen bonds than amines because oxygen is more electronegative than nitrogen; a primary amine can form a more extensive network than a comparable secondary amine.

Step 1: Identify the intermolecular attractions Compound C is $n - C_4H_{10}$, a nonpolar alkane with no hydrogen-bond donor or acceptor. Its molecules interact mainly through London dispersion forces, so it has the lowest boiling point. Compounds B and D are amines and can hydrogen-bond through their N–H bonds and nitrogen lone pairs. Compound A is an alcohol whose O–H group supports stronger hydrogen bonding than the amine groups.

Step 2: Compare the two isomeric amines B is butan-1-amine, $n - C_4H_9NH_2$, a primary amine with two N–H hydrogens. D is diethylamine, $C_2H_5NHC_2H_5$, a secondary amine with one N–H hydrogen. Both have the same molecular formula, so their molar masses are equal. The primary amine forms a more extensive intermolecular hydrogen-bond network and has a higher boiling point than diethylamine. Thus the order within the pair is D before B.

Step 3: Assemble the increasing order Combine the weakly interacting alkane, the two amines, and the strongly hydrogen-bonded alcohol:



This is option (C). Option (A) incorrectly puts diethylamine above butanol. Option (B) incorrectly puts diethylamine below the alkane. Option (D) incorrectly makes the alkane the highest-boiling compound. The functional groups and hydrogen-bond capacities therefore distinguish the correct sequence.

Step 4: Conclusion The increasing boiling-point order is $C < D < B < A$. Hence, the correct option is (C).

Quick Tip to Solve: For these similar-sized molecules, rank alkane below secondary amine below primary amine below alcohol; count the available N–H bonds before comparing the two amines.

69. Match List-I with List-II.

List-I: Deficiency disease	List-II: Vitamin
A. Scurvy	I. Pyridoxine
B. Convulsions	II. Vitamin A
C. Cheilosis	III. Ascorbic acid
D. Xerophthalmia	IV. Riboflavin

Choose the correct answer from the options given below:

[4 marks]

(A) A-I, B-III, C-II, D-IV

(B) A-I, B-III, C-IV, D-II

(C) A-III, B-I, C-IV, D-II

(D) A-III, B-I, C-II, D-IV

Correct Answer: (C) A-III, B-I, C-IV, D-II

Solution:

Concept:

- Water-soluble vitamins include vitamin C (ascorbic acid), vitamin B₆ (pyridoxine), and vitamin B₂ (riboflavin). Their deficiencies cause distinct characteristic disorders.
- Vitamin A is a fat-soluble vitamin required for normal vision and maintenance of epithelial tissue; deficiency can cause xerophthalmia.

Step 1: Match the first two deficiency diseases Scurvy is caused by deficiency of vitamin C, also called ascorbic acid. Inadequate vitamin C impairs normal collagen formation and can lead to bleeding gums and poor wound healing. Therefore A matches III. Convulsions are associated with deficiency of pyridoxine, vitamin B₆, which is important in amino-acid metabolism and normal nervous-system function. Therefore B matches I.

Step 2: Match the remaining two diseases Cheilosis, involving cracking or inflammation at the corners of the mouth, is a characteristic deficiency symptom of riboflavin, vitamin B₂. Thus C matches IV. Xerophthalmia is associated with vitamin A deficiency; this vitamin supports the eye and epithelial tissues, so D matches II. These associations refer to the vitamins in the right-hand list rather than to the order in which the diseases are written.

Step 3: Test the option combinations The complete mapping is

$$A - III, B - I, C - IV, D - II.$$

Option (C) gives all four matches. Options (A) and (B) both interchange the causes of scurvy and convulsions by assigning scurvy to pyridoxine and convulsions to ascorbic acid. Option (D) correctly matches those first two diseases but interchanges vitamin A and riboflavin for cheilosis and xerophthalmia. Hence none of these alternatives satisfies the full matching table.

Step 4: Conclusion The correct matching is A-III, B-I, C-IV, D-II. Hence, the correct option is (C).

Quick Tip to Solve: Anchor the table with scurvy–vitamin C and xerophthalmia–vitamin A; then use cheilosis–riboflavin to finish the remaining matches.

70. Match List-I with List-II.

List-I: Amino acid	List-II: Positive reaction/test for side-chain functional group
A. Glutamine	I. Hinsberg's test
B. Lysine	II. Neutral FeCl_3 test
C. Tyrosine	III. Ceric ammonium nitrate test
D. Serine	IV. Hoffman bromamide degradation

Choose the correct answer from the options given below:

[4 marks]

(A) A-IV, B-II, C-I, D-III

(B) A-IV, B-I, C-II, D-III

(C) A-III, B-II, C-I, D-IV

(D) A-IV, B-I, C-III, D-II

Correct Answer: (B) A-IV, B-I, C-II, D-III

Solution:

Concept:

- The question asks about the side chain, so distinguish its functional group from the amino and carboxyl groups common to all amino acids.
- Primary amines respond to Hinsberg reagent, phenols give a neutral FeCl_3 colour test, alcohols respond to ceric ammonium nitrate, and primary amides undergo Hofmann bromamide degradation.

Step 1: Classify glutamine and lysine side chains Glutamine contains the side chain $-\text{CH}_2\text{CH}_2\text{CONH}_2$, whose terminal group is a primary amide. It can undergo Hofmann bromamide degradation, converting the amide group into an amine with one fewer carbon. Hence A matches IV. Lysine contains a terminal primary amino group in its side chain, $-(\text{CH}_2)_4\text{NH}_2$. This group responds to the Hinsberg test through reaction with benzenesulfonyl chloride, so B matches I.

Step 2: Classify tyrosine and serine side chains Tyrosine has a phenolic OH group bonded directly to its aromatic ring. Phenols form coloured complexes with neutral ferric chloride, so C matches II. Serine has the side chain $-\text{CH}_2\text{OH}$, which is an alcohol group. The ceric ammonium nitrate test detects this alcoholic hydroxyl group, so D matches III. The different bonding environments of the two hydroxyl groups decide their distinct tests.

Step 3: Check the listed mappings The required combination is

$$A - IV, B - I, C - II, D - III.$$

It appears in option (B). Option (A) exchanges lysine and tyrosine tests, assigning the phenol test to the amino side chain. Option (C) incorrectly assigns the alcohol test to glutamine and amide degradation to serine. Option (D) interchanges the phenol and alcohol tests for tyrosine and serine. Only the second combination matches every side-chain functional group.

Step 4: Conclusion The correct matching is A-IV, B-I, C-II, D-III. Hence, the correct option is (B).

Quick Tip to Solve: Ignore the common backbone when a test is specified for the side chain: glutamine is amide, lysine amine, tyrosine phenol, and serine alcohol.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. First and second ionization enthalpies of lithium are 520 kJ mol^{-1} and 7297 kJ mol^{-1} respectively. Energy required to convert 3.5 mg lithium (g) into $\text{Li}^{2+}(\text{g})$ [$\text{Li}(\text{g}) \rightarrow \text{Li}^{2+}(\text{g})$] is ____ kJ mol^{-1} . (nearest integer)

[Molar mass of Li = 7 g mol^{-1}]

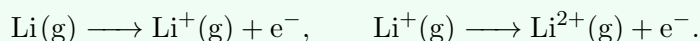
[4 marks]

Solution:

Concept:

- The first ionization removes an electron from neutral gaseous lithium; the second removes an electron from gaseous Li^+ . Both energies must be supplied to obtain Li^{2+} .
- For a sample containing n moles, the required energy is $E = n(\Delta_i H_1 + \Delta_i H_2)$. Molar enthalpies are multiplied by the actual number of moles.

Step 1: Add the two ionization enthalpies The conversion occurs in two successive stages:



The enthalpies refer to consecutive steps, so they add rather than subtract. For one mole the total is

$$\Delta H = 520 + 7297 = 7817 \text{ kJ mol}^{-1}.$$

The much larger second value reflects removal from the stable closed-shell lithium ion.

Step 2: Convert the given mass to moles The sample mass is in milligrams and must first be expressed in grams:

$$m = 3.5 \text{ mg} = 3.5 \times 10^{-3} \text{ g}.$$

With the stated molar mass,

$$n = \frac{3.5 \times 10^{-3} \text{ g}}{7 \text{ g mol}^{-1}} = 5.0 \times 10^{-4} \text{ mol}.$$

Using 3.5 directly as grams would overestimate the sample and its energy by a factor of one thousand.

Step 3: Calculate and round the sample energy Multiply the molar enthalpy by the sample amount:

$$E = (5.0 \times 10^{-4})(7817) = 3.9085 \text{ kJ}.$$

Rounding to the nearest integer gives 4. The sample energy has unit kJ: the mole in the sample amount cancels the inverse mole in the ionization enthalpy. The printed blank carries a molar-unit label, but a quantity depending on 3.5 mg is a total energy; the corresponding molar enthalpy itself remains 7817 kJ mol^{-1} .

Step 4: Conclusion The energy required for 3.5 mg of lithium is 3.9085 kJ, so the nearest-integer numerical entry is 4. The printed kJ mol^{-1} beside the blank is dimensionally inconsistent with a sample energy.

Quick Tip to Solve: Add successive ionization enthalpies first, then multiply by moles; do not confuse the molar value 7817 with the energy of the small lithium sample.

72.

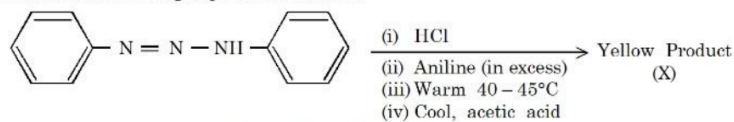
Consider the following sequence of reactions.

The percentage of nitrogen in the yellow product (X) formed is ____ %.

(Nearest Integer)

(Given Molar mass in g mol^{-1} H : 1, C : 12, N : 14)

[4 marks]

**Solution:****Concept:**

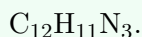
- Diazoaminobenzene, $\text{C}_6\text{H}_5-\text{N}=\text{N}-\text{NH}-\text{C}_6\text{H}_5$, rearranges in acid with excess aniline on warming to give predominantly para-aminoazobenzene.
- The nitrogen mass percentage is $100 \times \text{mass of nitrogen in one mole/molar mass}$. Count every nitrogen atom, including the amino nitrogen.

Step 1: Identify the yellow rearrangement product The starting structure contains a triazene linkage, $-\text{N}=\text{N}-\text{NH}-$, connecting two phenyl groups. Treatment with hydrochloric acid in excess aniline and warming at $40-45^\circ\text{C}$ brings about the diazoamino-to-aminoazo rearrangement. Coupling places the azo substituent predominantly para to the amino group of aniline. Cooling and treatment with acetic acid gives the yellow aminoazo product rather than changing the number of nitrogen atoms.

Step 2: Determine the molecular formula The product is para-aminoazobenzene, with condensed structure



The unsubstituted phenyl group contributes C_6H_5 , the disubstituted ring contributes C_6H_4 , and the amino group contributes NH_2 . The azo linkage contains two further nitrogen atoms. Adding these contributions gives



Thus one mole of product contains three moles of nitrogen atoms; counting only the two azo nitrogens would omit the amino group.

Step 3: Calculate the nitrogen percentage Using the given atomic molar masses, the product molar mass is

$$M = 12(12) + 11(1) + 3(14) = 144 + 11 + 42 = 197 \text{ g mol}^{-1}.$$

Nitrogen accounts for 42 g in each mole. Therefore

$$\% \text{N} = \frac{42}{197} \times 100 = 21.3198 \dots \%$$

The question requests the nearest integer, so the decimal value rounds to 21 rather than 22.

Step 4: Conclusion The yellow product is para-aminoazobenzene, $\text{C}_{12}\text{H}_{11}\text{N}_3$, with nitrogen content 21.32%. The required nearest-integer answer is 21%.

Quick Tip to Solve: For an aminoazo compound count three nitrogen atoms: two in the azo linkage and one in the amino group; then divide their mass by the full molecular mass.

73. 4.7 g of phenol is heated with Zn to give product X . If this reaction goes to 60% completion then the number of moles of compound X formed will be

_____ $\times 10^{-2}$. (Nearest Integer)

(Given molar mass in g mol^{-1} : H : 1, C : 12, O : 16)

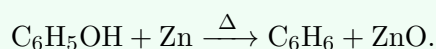
[4 marks]

Solution:

Concept:

- Heating phenol with zinc dust removes its oxygen as zinc oxide and forms benzene. The balanced reaction determines the mole ratio, independently of the masses of the products.
- Percentage completion applies to the initial amount of phenol. For the 1:1 reaction, moles of benzene formed equal moles of phenol that actually react.

Step 1: Identify the product and its stoichiometry The product X is benzene. Zinc removes oxygen from the phenolic group; it does not replace the aromatic ring or change its number of carbon atoms. The balanced equation is



Thus one mole of phenol yields one mole of benzene when it reacts completely. The completion percentage must therefore be applied to this theoretical benzene amount, rather than to its molar mass.

Step 2: Calculate the initial amount of phenol Phenol has formula $\text{C}_6\text{H}_6\text{O}$. With the supplied atomic masses, its molar mass is

$$M = 6(12) + 6(1) + 16 = 94 \text{ g mol}^{-1}.$$

The starting amount is consequently

$$n_0 = \frac{4.7 \text{ g}}{94 \text{ g mol}^{-1}} = 0.050 \text{ mol}.$$

This is also the theoretical benzene amount at complete conversion, provided zinc is available for the stated reaction.

Step 3: Apply the stated completion and express the result Only 60 percent of the initial phenol reacts, so

$$n_X = \frac{60}{100}(0.050) = 0.030 \text{ mol} = 3.0 \times 10^{-2} \text{ mol}.$$

The unreacted phenol is 0.020 mol; it contributes no benzene. Since the blank asks for the coefficient multiplying 10^{-2} , enter 3 rather than 0.03 or 30.

Step 4: Conclusion Benzene formed is 3.0×10^{-2} mol. The required nearest-integer entry is 3.

Quick Tip to Solve: Write the balanced 1:1 reaction first, then multiply the initial phenol moles by 0.60; finally match the power of ten printed outside the blank.

74. Sucrose hydrolyses in acidic medium into glucose and fructose by first order rate law with $t_{1/2} = 3$ hour. The percentage of sucrose remaining after 6 hours is _____. (Nearest integer)

(Given : $\log 2 = 0.3010$ and $\log 3 = 0.4771$)

[4 marks]

Solution:

Concept:

- For a first-order reaction, the fraction of reactant remaining after time t is $N_t/N_0 = e^{-kt}$, where k is the rate constant.
- The first-order half-life is constant: every interval of length $t_{1/2}$ halves the amount remaining, so $N_t/N_0 = (1/2)^{t/t_{1/2}}$.

Step 1: Count the elapsed half-lives The half-life for sucrose hydrolysis is 3 h, and the elapsed time is 6 h. Therefore the number of half-lives is

$$n = \frac{t}{t_{1/2}} = \frac{6}{3} = 2.$$

The law concerns the sucrose that remains, not the combined amount of glucose and fructose formed. Acid catalysis does not alter the stated first-order time dependence used here.

Step 2: Apply successive halving to sucrose Choose an initial sucrose amount of 100 units, since only a percentage is required. After the first three hours, 50 units remain. During the next three hours, half of those 50 units react, leaving 25 units. Algebraically,

$$\frac{N_6}{N_0} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}.$$

Thus the percentage remaining is

$$100 \frac{N_6}{N_0} = 100 \left(\frac{1}{4}\right) = 25\%.$$

The percentage hydrolysed is 75 percent, which is different from the quantity requested.

Step 3: Check with the integrated rate equation The supplied logarithm also gives

$$k = \frac{2.303 \log 2}{t_{1/2}} = \frac{2.303(0.3010)}{3} \simeq 0.2311 \text{ h}^{-1}.$$

Then

$$\log \frac{N_0}{N_6} = \frac{k(6)}{2.303} = 2 \log 2 = \log 4.$$

Hence $N_0/N_6 = 4$, confirming that one quarter remains. No rounding changes the integer result.

Step 4: Conclusion The percentage of sucrose remaining after 6 hours is 25%; the required numerical entry is 25.

Quick Tip to Solve: When time is an exact multiple of a first-order half-life, use successive halving and distinguish percentage remaining from percentage reacted.

75. Consider the reaction $X \rightleftharpoons Y$ at 300 K . If ΔH^θ and K are $28.40 \text{ kJ mol}^{-1}$ and 1.8×10^{-7} at the same temperature, then the magnitude of ΔS^θ for the reaction in $\text{JK}^{-1} \text{ mol}^{-1}$ is ____ . (Nearest integer)
(Given : $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$, $\ln 10 = 2.3$, $\log 3 = 0.48$, $\log 2 = 0.30$) [4 marks]

Solution:

Concept:

- The standard reaction Gibbs energy is related to the dimensionless equilibrium constant by $\Delta G^\circ = -RT \ln K$, at the temperature at which K is specified.
- The thermodynamic relation $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$ gives $\Delta S^\circ = (\Delta H^\circ - \Delta G^\circ)/T$. Energy units must agree before subtraction.

Step 1: Evaluate the logarithm of the equilibrium constant For $K = 1.8 \times 10^{-7}$, convert to a decimal logarithm first. Using only the supplied approximations,

$$\log(1.8) = \log(18) - 1 = 2 \log 3 + \log 2 - 1 = 2(0.48) + 0.30 - 1 = 0.26.$$

It follows that

$$\log K = 0.26 - 7 = -6.74, \quad \ln K = (-6.74)(2.3) = -15.502.$$

The negative logarithm is consistent with the very small equilibrium constant, which favours X under standard-state conditions.

Step 2: Compute the standard Gibbs energy Use $T = 300 \text{ K}$ and $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$:

$$\Delta G^\circ = -(8.3)(300)(-15.502) = 38599.98 \text{ J mol}^{-1}.$$

Thus ΔG° is positive, approximately $38.60 \text{ kJ mol}^{-1}$. Retaining the sign matters: replacing this value by a negative one would contradict both the formula and $K < 1$.

Step 3: Find the entropy change and its magnitude Convert the given enthalpy to joules: $\Delta H^\circ = 28.40 \text{ kJ mol}^{-1} = 28400 \text{ J mol}^{-1}$. Hence

$$\Delta S^\circ = \frac{28400 - 38599.98}{300} = -33.9999 \text{ J K}^{-1} \text{ mol}^{-1}.$$

The entropy change is negative because the Gibbs energy exceeds the enthalpy at this temperature. The question asks for its magnitude, so take the absolute value only after completing this signed calculation.

Step 4: Conclusion The magnitude is $|\Delta S^\circ| \simeq \boxed{34 \text{ J K}^{-1} \text{ mol}^{-1}}$. The nearest-integer entry is $\boxed{34}$.

Quick Tip to Solve: Use the supplied logarithm approximations consistently, convert kJ to J before subtracting, and take the absolute value only after finding the signed entropy change.