

JEE Main 2026 Paper 1 (B.E./B.Tech) Apr 6 Shift 2

Question Paper with Solutions PDF



Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours

Maximum Marks: 300

General Instructions

- (i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2026, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 6 April 2026, Shift 2. The total duration is 3 hours.
- (ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.
- (iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.
- (iv) Marking scheme (multiple choice questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (v) Marking scheme (numerical value questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = \frac{2x^2-3x+2}{3x^2+x+3}$. Then f is :

[4 marks]

- (A) both one-one and onto
- (B) one-one but not onto
- (C) onto but not one-one
- (D) neither one-one nor onto

Correct Answer: (D) neither one-one nor onto

Solution:

Concept:

- A function is one-one if distinct inputs always have distinct outputs. One pair of distinct inputs with equal outputs disproves this property.
- A function from $\mathbb{R}$ to $\mathbb{R}$ is onto only if every real number occurs as an output. A strictly positive function cannot be onto this codomain.

Step 1: Check that the function is defined everywhere Complete the square in the denominator to make its sign explicit:

$$3x^2 + x + 3 = 3\left(x + \frac{1}{6}\right)^2 + \frac{35}{12} > 0.$$

Thus every real input is allowed; a zero denominator does not affect either test.

Step 2: Find two different inputs with the same output The quadratic expressions have matching leading and constant coefficients, suggesting a comparison of reciprocal inputs. Direct substitution gives

$$f(2) = \frac{8 - 6 + 2}{12 + 2 + 3} = \frac{4}{17},$$
$$f\left(\frac{1}{2}\right) = \frac{\frac{1}{2} - \frac{3}{2} + 2}{\frac{3}{4} + \frac{1}{2} + 3} = \frac{4}{17}.$$

The inputs 2 and $1/2$ are distinct, so the function is not one-one.

Step 3: Exclude an output from the stated codomain The numerator is also strictly positive:

$$2x^2 - 3x + 2 = 2\left(x - \frac{3}{4}\right)^2 + \frac{7}{8} > 0.$$

Since both numerator and denominator are positive, $f(x) > 0$ for all real x . In particular, the real number zero has no preimage, so the function is not onto. Options (A) and (B) incorrectly assert injectivity; option (C) incorrectly asserts surjectivity.

Step 4: Conclusion The function is neither one-one nor onto. Hence, the correct option is (D).

Quick Tip to Solve: To disprove these properties, use one repeated output for injectivity and one missing real output for surjectivity; a complete range calculation is unnecessary.

2. Consider the quadratic equation $(n^2 - 2n + 2)x^2 - 3x + (n^2 - 2n + 2) = 0, n \in \mathbf{R}$. Let α be the minimum value of the product of its roots and β be the maximum value of the sum of its roots. Then the sum of the first six terms of the G.P., whose first term is α and the common ratio is $\frac{\alpha}{\beta}$, is : [4 marks]

- (A) $\frac{61}{37}$
 (B) $\frac{121}{81}$
 (C) $\frac{364}{243}$
 (D) $\frac{1093}{729}$

Correct Answer: (C) $\frac{364}{243}$

Solution:

Concept:

- For $ax^2 + bx + c = 0$, the sum of its roots is $-b/a$ and their product is c/a , including when the roots are complex.
- The sum of the first m terms of a geometric progression with first term a and ratio $r \neq 1$ is $S_m = a(1 - r^m)/(1 - r)$.

Step 1: Reduce the parameter to a positive quantity Put $t = n^2 - 2n + 2 = (n - 1)^2 + 1$. As n ranges over the reals, $t \geq 1$, with equality at $n = 1$. The equation becomes

$$tx^2 - 3x + t = 0.$$

The leading coefficient is always nonzero, so the usual sum and product formulas apply for every allowed parameter.

Step 2: Find the required extrema Writing the two roots as u, v , Vieta's formulas yield

$$uv = \frac{t^2}{t} = t, \quad u + v = \frac{3}{t}.$$

The least possible product is therefore $\alpha = 1$, and the greatest possible sum is $\beta = 3$. Both are attained at $t = 1$. We need not optimise individual roots or impose a condition on their being distinct.

Step 3: Calculate the six-term sum The first term is $a = \alpha = 1$, and the common ratio is $r = \alpha/\beta = 1/3$. Hence

$$S_6 = \frac{1 - (1/3)^6}{1 - 1/3} = \frac{1 - 1/729}{2/3} = \frac{364}{243}.$$

The value $121/81$ in (B) is instead the five-term sum. The fractions in (A) and (D) are not equal to the computed six-term sum; all terms must be counted beginning with the initial 1.

Step 4: Conclusion The sum of the first six terms is $\frac{364}{243}$. Hence, the correct option is (C).

Quick Tip to Solve: Complete the square in the parameter first. Vieta's formulas then turn both extrema into simple functions of the same quantity $t \geq 1$.

3. Let $S = \{z \in \mathbb{C} : z^2 + \sqrt{6}iz - 3 = 0\}$. Then $\sum_{z \in S} z^8$ is equal to :

[4 marks]

- (A) 162
- (B) 184
- (C) 262
- (D) 324

Correct Answer: (A) 162

Solution:

Concept:

- The quadratic formula gives both complex roots without changing the sign of the imaginary coefficient.
- If $z = r(\cos \phi + i \sin \phi)$, De Moivre's theorem gives $z^m = r^m(\cos m\phi + i \sin m\phi)$.

Step 1: Find the two distinct roots The discriminant of the equation is

$$(\sqrt{6}i)^2 - 4(1)(-3) = -6 + 12 = 6.$$

Consequently, the roots are

$$z_1 = \frac{\sqrt{6}}{2}(1 - i), \quad z_2 = \frac{\sqrt{6}}{2}(-1 - i).$$

They are distinct since the discriminant is nonzero. Thus the set S contains two elements and the requested sum includes both eighth powers.

Step 2: Express the roots in polar form Each root has modulus $\sqrt{(6/4) + (6/4)} = \sqrt{3}$. The first lies in the fourth quadrant and the second in the third quadrant, so we may write

$$z_1 = \sqrt{3} \left(\cos \frac{-\pi}{4} + i \sin \frac{-\pi}{4} \right),$$

$$z_2 = \sqrt{3} \left(\cos \frac{-3\pi}{4} + i \sin \frac{-3\pi}{4} \right).$$

These arguments preserve the negative imaginary part of both roots.

Step 3: Raise each root to the eighth power and add Multiplying the arguments by eight gives -2π and -6π , respectively. Both angles have cosine 1 and sine 0. Therefore

$$z_1^8 = (\sqrt{3})^8 = 81, \quad z_2^8 = 81,$$

$$\sum_{z \in S} z^8 = 81 + 81 = 162.$$

The answer is a real positive number. Option (D) doubles this two-root sum once again; (B) and (C) do not match the exact powers.

Step 4: Conclusion The requested sum is 162. Hence, the correct option is (A).

Quick Tip to Solve: The roots have arguments that are odd multiples of $\pi/4$. Their eighth powers have zero argument modulo 2π , so only their common modulus remains.

4. The sum of all possible values of $\theta \in [0, 2\pi]$, for which the system of equations :

$$x \cos 3\theta - 8y - 12z = 0$$

$$x \cos 2\theta + 3y + 3z = 0$$

$$x + y + 3z = 0$$

has a non-trivial solution, is equal to :

[4 marks]

(A) π

(B) 2π

(C) 3π

(D) 4π

Correct Answer: (D) 4π

Solution:

Concept:

- A square homogeneous linear system has a non-trivial solution precisely when the determinant of its coefficient matrix is zero.
- Use $\cos 2\theta = 2 \cos^2 \theta - 1$ and $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$ to reduce the determinant condition to a polynomial.

Step 1: Form and expand the determinant The coefficient matrix is

$$M = \begin{pmatrix} \cos 3\theta & -8 & -12 \\ \cos 2\theta & 3 & 3 \\ 1 & 1 & 3 \end{pmatrix}.$$

Expanding along its first row, with the cofactor signs included, gives

$$\begin{aligned} \det M &= 6 \cos 3\theta + 8(3 \cos 2\theta - 3) \\ &\quad - 12(\cos 2\theta - 3) \\ &= 6 \cos 3\theta + 12 \cos 2\theta + 12. \end{aligned}$$

A nonzero solution thus requires $\cos 3\theta + 2 \cos 2\theta + 2 = 0$.

Step 2: Solve the resulting polynomial in cosine Let $c = \cos \theta$. Substitution of the double- and triple-angle identities yields

$$\begin{aligned} 4c^3 - 3c + 2(2c^2 - 1) + 2 &= 0, \\ c(4c^2 + 4c - 3) &= c(2c - 1)(2c + 3) = 0. \end{aligned}$$

Thus $c = 0, 1/2, -3/2$. The last root is impossible because real cosine lies between -1 and 1 .

Step 3: List every angle in the closed interval For $\cos \theta = 0$, the angles are $\pi/2, 3\pi/2$. For $\cos \theta = 1/2$, they are $\pi/3, 5\pi/3$. Neither endpoint 0 nor 2π satisfies the determinant equation. Their sum is

$$\frac{\pi}{2} + \frac{3\pi}{2} + \frac{\pi}{3} + \frac{5\pi}{3} = 4\pi.$$

Options (A), (B) and (C) omit valid contributions or miscount the angles; the two pairs each sum to 2π .

Step 4: Conclusion The sum of all admissible angles is 4π . Hence, the correct option is (D).

Quick Tip to Solve: After factoring in $\cos \theta$, reject roots outside $[-1, 1]$ before listing angles. Pair each angle with $2\pi - \theta$ to add quickly.

5. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $B = [b_{ij}]$, $1 \leq i, j \leq 3$. If $B = A^{99} - I$, then the value of $\frac{b_{31} - b_{21}}{b_{32}}$ is : [4 marks]

(A) 99

(B) 199

(C) 149

(D) 159

Correct Answer: (C) 149

Solution:

Concept:

- Write a triangular matrix with diagonal entries 1 as $A = I + N$. If $N^3 = 0$, the binomial expansion terminates after the N^2 term.
- Since I commutes with N , $(I + N)^m = I + mN + \binom{m}{2}N^2$ when $N^3 = 0$.

Step 1: Find the nilpotent part and its square Subtract the identity matrix from A :

$$N = A - I = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}.$$

Direct multiplication gives

$$N^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{pmatrix}, \quad N^3 = 0.$$

The only contribution to the bottom-left entry of N^2 is the product of its two successive subdiagonal entries, $3 \cdot 3 = 9$.

Step 2: Expand the high power using the finite binomial formula All powers beyond N^2 vanish, so

$$B = A^{99} - I = 99N + \binom{99}{2}N^2.$$

The required entries are therefore

$$b_{21} = 297, \quad b_{32} = 297,$$

$$b_{31} = 99 \cdot 9 + \frac{99 \cdot 98}{2} \cdot 9.$$

The N^2 contribution is essential; treating the power as $I + 99N$ would miss the repeated multiplication through the middle row.

Step 3: Simplify the requested ratio Keep common factors until the last step:

$$\begin{aligned} \frac{b_{31} - b_{21}}{b_{32}} &= \frac{9 \cdot 99 + 9 \cdot 99 \cdot 49 - 3 \cdot 99}{3 \cdot 99} \\ &= 3 + 147 - 1 \\ &= 149. \end{aligned}$$

The denominator is 297, so the ratio is well-defined. None of 99, 199 or 159 agrees with this exact expression. As a general check, replacing 99 by m gives $(3m + 1)/2$, which also gives 149 here.

Step 4: Conclusion The value of $\frac{b_{31} - b_{21}}{b_{32}}$ is 149. Hence, the correct option is (C).

Quick Tip to Solve: For a triangular matrix close to the identity, test whether $(A - I)^3 = 0$. A finite binomial expansion then replaces repeated matrix multiplication.

6. The sum $1 + \frac{1}{2}(1^2 + 2^2) + \frac{1}{3}(1^2 + 2^2 + 3^2) + \dots$ upto 10 terms is equal to :

[4 marks]

- (A) 130
- (B) 155
- (C) $\frac{315}{2}$
- (D) $\frac{325}{2}$

Correct Answer: (C) $\frac{315}{2}$

Solution:

Concept:

- The sum of the first k squares is $1^2 + 2^2 + \dots + k^2 = k(k+1)(2k+1)/6$.
- After simplifying each term, use $\sum_{k=1}^n k = n(n+1)/2$ and $\sum_{k=1}^n k^2 = n(n+1)(2n+1)/6$.

Step 1: Identify the general term including the first one The term in position k is

$$T_k = \frac{1}{k} (1^2 + 2^2 + \dots + k^2).$$

This description includes $T_1 = 1$, so summing ten terms means $k = 1$ through $k = 10$, without adding an extra initial term. Inserting the square-sum formula cancels the factor k :

$$T_k = \frac{(k+1)(2k+1)}{6} = \frac{2k^2 + 3k + 1}{6}.$$

Step 2: Sum the three elementary sequences The required sum is

$$S_{10} = \frac{1}{6} \left(2 \sum_{k=1}^{10} k^2 + 3 \sum_{k=1}^{10} k + \sum_{k=1}^{10} 1 \right).$$

Here the three sums are

$$\sum_{k=1}^{10} k^2 = \frac{10 \cdot 11 \cdot 21}{6} = 385,$$
$$\sum_{k=1}^{10} k = \frac{10 \cdot 11}{2} = 55, \quad \sum_{k=1}^{10} 1 = 10.$$

The constant term contributes ten, rather than one, because it occurs in each of the ten summands.

Step 3: Evaluate and compare the alternatives Substituting these values gives

$$S_{10} = \frac{2(385) + 3(55) + 10}{6} = \frac{945}{6} = \frac{315}{2}.$$

The value is 157.5. Options (A) and (B), 130 and 155, are too small, while option (D), $325/2$, is larger than the exact result. A useful first-term check is $T_1 = (2 + 3 + 1)/6 = 1$, confirming that the polynomial term has the correct starting point.

Step 4: Conclusion The sum up to ten terms is $\frac{315}{2}$. Hence, the correct option is (C).

Quick Tip to Solve: Simplify the general summand before adding. The outside factor $1/k$ cancels the k in the square-sum formula, leaving just a quadratic polynomial.

7. A building has ground floor and 10 more floors. Nine persons enter in a lift at the ground floor. The lift goes up to the 10th floor. The number of ways, in which any 4 persons exit at a floor and the remaining 5 persons exit at a different floor, if the lift does not stop at the first and the second floors, is equal to :

[4 marks]

- (A) 2184
- (B) 3064
- (C) 7056
- (D) 11340

Correct Answer: (C) 7056

Solution:

Concept:

- Distinct people form distinct groups. Choosing four of nine people gives $\binom{9}{4}$ possible four-person groups; the other five people are then determined.
- The multiplication principle combines independent choices of the group, its destination and the other group's destination. The two destinations must be distinct and permitted stops.

Step 1: Identify the permitted destination floors There are ten floors above the ground floor, numbered 1 through 10. The lift does not stop at floors 1 and 2, and all nine people enter at the ground floor. Thus each exit destination belongs to

$$\{3, 4, 5, 6, 7, 8, 9, 10\},$$

giving eight choices. The ground floor is not an exit destination in this upward journey. Reaching floor 10 does not require a passenger to exit there; it is simply one of the available floors.

Step 2: Choose the groups and assign their floors Choose the four people who leave together:

$$\binom{9}{4} = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1} = 126.$$

Their floor can be chosen in eight ways. Once it is selected, the remaining five people must leave at one of the other seven permitted floors. Hence the required count is

$$126 \times 8 \times 7 = 7056.$$

There is no division by two: a destination for the four-person group and a destination for the five-person group are distinguishable assignments. Nor do we multiply by internal arrangements of passengers, since their order inside the lift is irrelevant. Equivalently, choosing two floors first gives $\binom{8}{2}$, then assigning the smaller group to either floor gives a factor of two. The other listed counts do not equal this exhaustive count.

Step 3: Conclusion The number of ways is 7056. Hence, the correct option is (C).

Quick Tip to Solve: When two exit groups have unequal sizes, label the floors by group size: choose the people once, then use ordered distinct destinations.

8. Let the mean and the variance of seven observations $2, 4, \alpha, 8, \beta, 12, 14$, $\alpha < \beta$, be 8 and 16 respectively. Then the quadratic equation whose roots are $3\alpha + 2$ and $2\beta + 1$ is :

[4 marks]

(A) $x^2 - 35x + 306 = 0$

(B) $x^2 - 41x + 420 = 0$

(C) $x^2 - 45x + 506 = 0$

(D) $x^2 - 37x + 342 = 0$

Correct Answer: (B) $x^2 - 41x + 420 = 0$

Solution:

Concept:

- For n observations, the mean is $\bar{x} = \frac{\sum x_i}{n}$, and the variance is $\sigma^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$. Here the denominator is the number of observations, not one less.
- The identity $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$ determines the product once the sum and sum of squares are known. Roots r, s give the monic quadratic $x^2 - (r + s)x + rs = 0$.

Step 1: Use the mean to determine the sum Seven observations have mean 8, so their sum is $7 \times 8 = 56$. The five specified values add to

$$2 + 4 + 8 + 12 + 14 = 40.$$

Consequently,

$$\alpha + \beta = 56 - 40 = 16.$$

This fixes the missing pair's sum but does not by itself fix the individual values; the variance supplies the second equation.

Step 2: Use the variance to find the observations Substitute the mean and variance into the variance formula:

$$\sum x_i^2 = 7(16 + 8^2) = 560.$$

The known squares total $4 + 16 + 64 + 144 + 196 = 424$, hence

$$\alpha^2 + \beta^2 = 136, \quad \alpha\beta = \frac{16^2 - 136}{2} = 60.$$

Thus the missing observations solve

$$t^2 - 16t + 60 = (t - 6)(t - 10) = 0.$$

Since $\alpha < \beta$, take $\alpha = 6$ and $\beta = 10$; reversing them would change the transformed roots.

Step 3: Construct and check the required quadratic The new roots are $3(6) + 2 = 20$ and $2(10) + 1 = 21$. Their sum is 41 and their product is 420, giving

$$(x - 20)(x - 21) = x^2 - 41x + 420.$$

Option (B) has both coefficients. Options (A), (C) and (D) have root sums 35, 45 and 37 respectively, so each fails even the sum check.

Step 4: Conclusion The quadratic equation is $x^2 - 41x + 420 = 0$. Hence, the correct option is (B).

Quick Tip to Solve: Recover the missing pair using sum and sum of squares first; apply the ordering condition before transforming the roots.

9. A bag contains 6 blue and 6 green balls. Pairs of balls are drawn without replacement until the bag is empty. The probability that each drawn pair consists of one blue and one green ball is : [4 marks]

- (A) $\frac{63}{925}$
(B) $\frac{17}{231}$
(C) $\frac{16}{231}$
(D) $\frac{64}{925}$

Correct Answer: (C) $\frac{16}{231}$

Solution:

Concept:

- For successive draws without replacement, multiply the conditional probabilities of success at each stage. A successful pair removes one ball of each colour, preserving equal remaining colour counts.
- From a bag containing k blue and k green balls, there are $\binom{2k}{2}$ equally likely unordered pairs and k^2 mixed-colour pairs. Their ratio is $\frac{k}{2k-1}$.

Step 1: Calculate the probability for one successful pair Initially there are six balls of each colour. The first pair is mixed in

$$\frac{6 \cdot 6}{\binom{12}{2}} = \frac{36}{66} = \frac{6}{11}$$

of all possible selections. More generally, with k balls of each colour, the probability is

$$\frac{k^2}{(2k)(2k-1)/2} = \frac{k}{2k-1}.$$

This counts both possible draw orders automatically, because the pairs are unordered. Treating only blue-then-green would omit half the successful ordered draws.

Step 2: Multiply the conditional success probabilities Conditional on the first pair being mixed, five balls of each colour remain. After another mixed pair, four of each remain, and so on. Thus the probabilities for the six successive pairs are

$$\frac{6}{11}, \frac{5}{9}, \frac{4}{7}, \frac{3}{5}, \frac{2}{3}, 1.$$

The last pair is certainly mixed once all preceding pairs are mixed. Applying the chain rule gives

$$P = \frac{6}{11} \cdot \frac{5}{9} \cdot \frac{4}{7} \cdot \frac{3}{5} \cdot \frac{2}{3} = \frac{48}{693} = \frac{16}{231}.$$

The stages are dependent, which is why the successive fractions change; using the first-pair probability to the sixth power is incorrect. As a separate count, the favourable complete pairings number $6!$, whereas all pairings number $11 \cdot 9 \cdot 7 \cdot 5 \cdot 3 \cdot 1$; their ratio agrees. Only option (C) equals the reduced fraction, so the other three fractions are excluded.

Step 3: Conclusion The required probability is $\frac{16}{231}$. Hence, the correct option is (C).

Quick Tip to Solve: After each successful mixed pair, replace the two equal colour counts by one less and multiply the new conditional probability.

10. Let C be a circle having centre in the first quadrant and touching the x -axis at a distance of 3 units from the origin. If the circle C has an intercept of length $6\sqrt{3}$ on y -axis, then the length of the chord of the circle C on the line $x - y = 3$ is : [4 marks]

- (A) 8
(B) 6
(C) $6\sqrt{2}$
(D) $8\sqrt{2}$

Correct Answer: (C) $6\sqrt{2}$

Solution:

Concept:

- A circle tangent to the x -axis has radius equal to the positive ordinate of its centre. The radius to the point of contact is perpendicular to the axis.
- For a circle of radius r , a line at perpendicular distance d from its centre cuts a chord of length $2\sqrt{r^2 - d^2}$. The perpendicular from the centre bisects that chord.

Step 1: Locate the centre and use the intercept The contact point is three units from the origin on the x -axis. As the centre is in the first quadrant, the contact point must be $(3, 0)$, and the centre is $(3, r)$, where $r > 0$. The circle has equation

$$(x - 3)^2 + (y - r)^2 = r^2.$$

Its perpendicular distance from the y -axis is 3. Half the given intercept is $3\sqrt{3}$, so the right triangle formed by a radius, half-chord and perpendicular gives

$$r^2 = 3^2 + (3\sqrt{3})^2 = 9 + 27 = 36.$$

Therefore $r = 6$, and the centre is $(3, 6)$. Taking the full intercept as a half-chord would give an incorrect radius.

Step 2: Find the distance to the specified line Write the line as $x - y - 3 = 0$. Its distance from $(3, 6)$ is

$$d = \frac{|3 - 6 - 3|}{\sqrt{1^2 + (-1)^2}} = \frac{6}{\sqrt{2}} = 3\sqrt{2}.$$

As $d < 6$, the line indeed intersects the circle in two points. The chord length is therefore

$$L = 2\sqrt{36 - (3\sqrt{2})^2} = 2\sqrt{18} = 6\sqrt{2}.$$

This is a length, so the positive square root is used. Among the options, only (C) has squared length 72. Options (A), (B) and (D) have squared lengths 64, 36 and 128, which fail $L^2 = 4(r^2 - d^2)$. The distance calculation also confirms that the requested chord is not a diameter.

Step 3: Conclusion The chord has length $6\sqrt{2}$ units. Hence, the correct option is (C).

Quick Tip to Solve: Use tangency to write the centre as $(3, r)$, then recover the radius from the known axis intercept before handling the oblique chord.

11. The eccentricity of an ellipse E with centre at the origin O is $\frac{\sqrt{3}}{2}$ and its directrices are $x = \pm \frac{4\sqrt{6}}{3}$. Let $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be a hyperbola whose eccentricity is equal to the length of semi-major axis of E , and whose length of latus rectum is equal to the length of minor axis of E . Then the distance between the foci of H is : [4 marks]

- (A) $\frac{4\sqrt{2}}{\sqrt{7}}$
 (B) $\frac{4\sqrt{2}}{7}$
 (C) $\frac{4}{\sqrt{7}}$
 (D) $\frac{8}{7}$

Correct Answer: (D) $\frac{8}{7}$

Solution:

Concept:

- For an ellipse with major axis along the x -axis, the directrices are $x = \pm A/e_E$, and the semi-minor axis is $B = A\sqrt{1 - e_E^2}$. Here A, B denote the ellipse parameters.
- For $x^2/a^2 - y^2/b^2 = 1$, the hyperbola has eccentricity $e_H = \sqrt{1 + b^2/a^2}$, latus rectum length $2b^2/a$, and distance between foci $2ae_H$.

Step 1: Determine the ellipse parameters The directrices are perpendicular to the x -axis, so its major axis is horizontal. With $e_E = \sqrt{3}/2$, the directrix distance gives

$$\frac{A}{e_E} = \frac{4\sqrt{6}}{3}, \quad A = \frac{\sqrt{3}}{2} \cdot \frac{4\sqrt{6}}{3} = 2\sqrt{2}.$$

The semi-minor axis is then

$$B = A\sqrt{1 - e_E^2} = 2\sqrt{2}\sqrt{1 - \frac{3}{4}} = \sqrt{2}.$$

Therefore the ellipse's minor axis has full length $2B = 2\sqrt{2}$. Distinguishing the full axis length from its semi-axis is essential for the next condition.

Step 2: Transfer the conditions to the hyperbola The hyperbola eccentricity equals the ellipse's semi-major axis length, hence $e_H = 2\sqrt{2}$. Thus

$$1 + \frac{b^2}{a^2} = 8, \quad b^2 = 7a^2.$$

Its latus rectum length equals the ellipse's full minor axis length:

$$\frac{2b^2}{a} = 2\sqrt{2}.$$

Substituting $b^2 = 7a^2$ gives $14a = 2\sqrt{2}$, so

$$a = \frac{\sqrt{2}}{7}.$$

The positive root is required because a is a length, and $e_H > 1$ confirms that this conic is a hyperbola.

Step 3: Calculate the focal separation The distance from the centre to either focus is $c = ae_H$. Consequently,

$$2c = 2 \left(\frac{\sqrt{2}}{7} \right) (2\sqrt{2}) = \frac{8}{7}.$$

Option (D) gives this value. Options (A), (B) and (C) do not equal $8/7$; they retain a different radical factor or denominator and cannot satisfy the derived focal separation.

Step 4: Conclusion The distance between the foci is $\boxed{\frac{8}{7}}$ units. Hence, the correct option is (D).

Quick Tip to Solve: Keep separate symbols for the two conics and translate each stated length carefully: semi-major axis, full minor axis and full latus rectum are different quantities.

12. Let $x = 9$ be a directrix of an ellipse E , whose centre is at the origin and eccentricity is $\frac{1}{3}$. Let $P(\alpha, 0)$, $\alpha > 0$, be a focus of E and AB be a chord passing through P . Then the locus of the mid point of AB is : [4 marks]

- (A) $9y^2 = 8x(1 - x)$
- (B) $3y^2 = 4x(1 - x)$
- (C) $9y^2 = 8x(x - 1)$
- (D) $3y^2 = 4x(x - 1)$

Correct Answer: (A) $9y^2 = 8x(1 - x)$

Solution:

Concept:

- For an ellipse $x^2/a^2 + y^2/b^2 = 1$, the directrix perpendicular to its major axis is $x = a/e$, the focus is $(ae, 0)$, and $b^2 = a^2(1 - e^2)$.
- A chord with midpoint (h, k) has equation $hx/a^2 + ky/b^2 = h^2/a^2 + k^2/b^2$. This follows by writing its endpoints as $(h + u, k + v)$ and $(h - u, k - v)$, then subtracting their ellipse equations.

Step 1: Determine the ellipse and the positive focus The positive directrix is $x = 9$ and $e = 1/3$. Since $a/e = 9$, we obtain $a = 3$, and the focal distance is $ae = 1$. Therefore $P = (1, 0)$. Also,

$$b^2 = 9 \left(1 - \frac{1}{9} \right) = 8,$$

so the ellipse is

$$\frac{x^2}{9} + \frac{y^2}{8} = 1.$$

The focus lies inside the ellipse, so every line through it cuts a genuine chord.

Step 2: Impose the focus condition on the midpoint chord Let the midpoint of the chord be $M = (h, k)$. The midpoint form of its chord is

$$\frac{hx}{9} + \frac{ky}{8} = \frac{h^2}{9} + \frac{k^2}{8}.$$

As the chord passes through $(1, 0)$, substituting this point gives

$$\frac{h}{9} = \frac{h^2}{9} + \frac{k^2}{8}.$$

Multiplication by 72 and rearrangement yield

$$9k^2 = 8h(1 - h).$$

The diameter through the origin and focus has midpoint $(0, 0)$, which is also included in this equation even though its midpoint form reduces to an identity.

Step 3: Identify the locus and exclude the other options Replace the moving midpoint coordinates h, k by x, y :

$$9y^2 = 8x(1 - x).$$

Its points satisfy $0 \leq x \leq 1$, consistent with focal-chord midpoints. Options (C) and (D) reverse the sign and have no nonzero real ordinates for $0 < x < 1$. Option (B) has the wrong coefficient: for instance the derived locus gives $y^2 = 2/9$ at $x = 1/2$, whereas (B) gives $y^2 = 1/3$. Thus only (A) matches the midpoint condition.

Step 4: Conclusion The locus is $\boxed{9y^2 = 8x(1 - x)}$. Hence, the correct option is (A).

Quick Tip to Solve: For a moving chord through a fixed point, use the chord equation in midpoint form and substitute the fixed point directly; this avoids slope elimination.

13. If $\sin(\tan^{-1}(x\sqrt{2})) = \cot(\sin^{-1}\sqrt{1-x^2})$, $x \in (0, 1)$, then the value of x is:

[4 marks]

- (A) $\frac{1}{2}$
- (B) $\frac{1}{3}$
- (C) $\frac{2}{3}$
- (D) $\frac{5}{8}$

Correct Answer: (A) $\frac{1}{2}$

Solution:

Concept:

- For a principal inverse tangent, $\sin(\tan^{-1} u) = u/\sqrt{1+u^2}$. The positive square root follows because the angle lies between $-\pi/2$ and $\pi/2$.
- If $0 < v < 1$, then $\cot(\sin^{-1} v) = \sqrt{1-v^2}/v$. Here $0 < x < 1$, so both radicals and both sides are positive.

Step 1: Rewrite both composite functions Put $u = x\sqrt{2}$ on the left and $v = \sqrt{1-x^2}$ on the right. Since $x > 0$, the identity $\sqrt{x^2} = x$ applies. The given equation therefore becomes

$$\frac{x\sqrt{2}}{\sqrt{1+2x^2}} = \frac{x}{\sqrt{1-x^2}}.$$

Both denominators are nonzero throughout the stated domain; this is why division and cross multiplication are valid.

Step 2: Solve the positive radical equation Divide by the nonzero factor x . Both remaining sides are positive, so squaring is reversible and introduces no sign ambiguity:

$$\frac{2}{1+2x^2} = \frac{1}{1-x^2}.$$

Cross multiplying gives

$$2 - 2x^2 = 1 + 2x^2, \quad 4x^2 = 1.$$

Thus $x = \pm 1/2$, but the domain retains only the positive value.

Step 3: Check the domain and the alternatives At $x = 1/2$, each original side equals $1/\sqrt{3}$, so the solution is verified before selecting an option. The simplified equation requires $x^2 = 1/4$ uniquely in $(0, 1)$; the values $1/3$, $2/3$, and $5/8$ fail that requirement and hence cannot satisfy the original equality.

Step 4: Conclusion The value is $x = 1/2$. Hence, the correct option is (A).

Quick Tip to Solve: Use the domain before cancelling and squaring: here $x > 0$ makes both the cancellation and $\sqrt{x^2} = x$ immediate.

14. The shortest distance between the lines $\frac{x-4}{1} = \frac{y-3}{2} = \frac{z-2}{-3}$ and $\frac{x+2}{2} = \frac{y-6}{4} = \frac{z-5}{-5}$ is: [4 marks]

- (A) $\frac{5\sqrt{6}}{6}$
- (B) $2\sqrt{5}$
- (C) $3\sqrt{5}$
- (D) $4\sqrt{5}$

Correct Answer: (C) $3\sqrt{5}$

Solution:

Concept:

- For lines through $\vec{a}_1, \vec{a}_2$ with nonparallel direction vectors $\vec{d}_1, \vec{d}_2$, their shortest distance is $D = |(\vec{a}_2 - \vec{a}_1) \cdot (\vec{d}_1 \times \vec{d}_2)| / |\vec{d}_1 \times \vec{d}_2|$.
- The cross product is perpendicular to both line directions. Projecting the displacement between any points on the two lines onto its unit vector gives the separation, independently of the chosen points.

Step 1: Read the points and directions The symmetric equations give

$$\vec{a}_1 = (4, 3, 2), \quad \vec{d}_1 = (1, 2, -3),$$

$$\vec{a}_2 = (-2, 6, 5), \quad \vec{d}_2 = (2, 4, -5).$$

In particular, $x + 2 = x - (-2)$, so the second line passes through a point with x-coordinate -2 , not 2 . The displacement is $\vec{a}_2 - \vec{a}_1 = (-6, 3, 3)$.

Step 2: Find the common normal direction Compute the cross product component by component:

$$\vec{d}_1 \times \vec{d}_2 = (2(-5) - (-3)4, (-3)2 - 1(-5), 1(4) - 2(2)) = (2, -1, 0).$$

Its magnitude is $\sqrt{2^2 + (-1)^2} = \sqrt{5}$. Since this vector is nonzero, the lines are not parallel and the skew-line distance formula applies.

Step 3: Project the displacement and compare options The scalar triple product is

$$(-6, 3, 3) \cdot (2, -1, 0) = -12 - 3 = -15.$$

Taking the absolute value ensures a nonnegative distance:

$$D = \frac{15}{\sqrt{5}} = 3\sqrt{5}.$$

Thus the coefficient of $\sqrt{5}$ is exactly 3, excluding the coefficients 2 and 4. The remaining alternative $5\sqrt{6}/6$ also differs from this projection.

Step 4: Conclusion The shortest distance is $\boxed{3\sqrt{5}}$ units. Hence, the correct option is (C).

Quick Tip to Solve: For nearly parallel direction ratios, compute the small cross product first; it often removes most components from the scalar triple product.

15. Let $\vec{a} = 2\hat{i} + 3\hat{j} + 3\hat{k}$ and $\vec{b} = 6\hat{i} + 3\hat{j} + 3\hat{k}$. Then the square of the area of the triangle with adjacent sides determined by the vectors $(2\vec{a} + 3\vec{b})$ and $(\vec{a} - \vec{b})$ is : [4 marks]

- (A) 450
- (B) 900
- (C) 1800
- (D) 2400

Correct Answer: (C) 1800

Solution:

Concept:

- The area of a triangle with adjacent side vectors $\vec{u}$ and $\vec{v}$ is $\frac{1}{2}|\vec{u} \times \vec{v}|$. Consequently its squared area is $\frac{1}{4}|\vec{u} \times \vec{v}|^2$.
- Cross products are bilinear and antisymmetric: $\vec{a} \times \vec{a} = 0$, $\vec{b} \times \vec{b} = 0$, and $\vec{b} \times \vec{a} = -\vec{a} \times \vec{b}$. These facts simplify linear combinations before numerical calculation.

Step 1: Simplify the side-vector cross product Let $\vec{u} = 2\vec{a} + 3\vec{b}$ and $\vec{v} = \vec{a} - \vec{b}$. Expand using bilinearity rather than multiplying large coordinates:

$$\begin{aligned}\vec{u} \times \vec{v} &= 2\vec{a} \times \vec{a} - 2\vec{a} \times \vec{b} + 3\vec{b} \times \vec{a} - 3\vec{b} \times \vec{b} \\ &= -5(\vec{a} \times \vec{b}).\end{aligned}$$

The minus sign affects orientation but disappears on taking the squared magnitude.

Step 2: Calculate the basic cross product Using $\vec{a} = (2, 3, 3)$ and $\vec{b} = (6, 3, 3)$,

$$\vec{a} \times \vec{b} = (3 \cdot 3 - 3 \cdot 3, 3 \cdot 6 - 2 \cdot 3, 2 \cdot 3 - 3 \cdot 6) = (0, 12, -12).$$

Hence

$$|\vec{a} \times \vec{b}|^2 = 0^2 + 12^2 + (-12)^2 = 288.$$

Step 3: Apply the triangle factor before selecting The squared magnitude of the cross product of the adjacent sides is $25 \cdot 288 = 7200$. This is the squared parallelogram area. The required triangle has half that area, so its squared area has a factor of one quarter:

$$\text{Area}^2 = \frac{7200}{4} = 1800.$$

The alternatives 450, 900 and 2400 do not equal this value. In particular, squaring the area factor is essential; using only a factor of one half would be incorrect.

Step 4: Conclusion The square of the triangle area is 1800 square units of area. Hence, the correct option is (C).

Quick Tip to Solve: For sides $p\vec{a} + q\vec{b}$ and $r\vec{a} + s\vec{b}$, replace their cross product by $(ps - qr)(\vec{a} \times \vec{b})$, then remember the triangle factor.

16. Let $\lim_{x \rightarrow 2} \frac{(\tan(x-2))(rx^2 + (p-2)x - 2p)}{(x-2)^2} = 5$ **for some** $r, p \in \mathbb{R}$. **If the set of all possible values of** q **, such that the roots of the equation** $rx^2 - px + q = 0$ **lie in** $(0, 2)$, **be the interval** $(\alpha, \beta]$, **then** $4(\alpha + \beta)$ **equals :** [4 marks]

- (A) 11
(B) 13
(C) 17
(D) 21

Correct Answer: (C) 17

Solution:

Concept:

- The standard limit $\tan h/h \rightarrow 1$ as $h \rightarrow 0$ converts the given expression into a polynomial divided by $x - 2$. A finite limit requires that polynomial to vanish at $x = 2$.
- A real quadratic with roots in an open interval must have real roots and satisfy the strict endpoint restrictions. A repeated root inside the interval is allowed, so the discriminant may be zero.

Step 1: Determine the quadratic coefficient from finiteness Write $P(x) = rx^2 + (p-2)x - 2p$. The given expression is

$$\frac{\tan(x-2)}{x-2} \cdot \frac{P(x)}{x-2}.$$

The first factor tends to 1. For the second to have a finite two-sided limit, $P(2) = 0$. Thus $4r + 2p - 4 - 2p = 4r - 4 = 0$, and $r = 1$.

Step 2: Use the limit value to determine p With $r = 1$, factor the polynomial exactly:

$$P(x) = x^2 + (p - 2)x - 2p = (x - 2)(x + p).$$

After cancellation, the limit becomes $2 + p = 5$, giving $p = 3$. The equation whose roots must lie in the specified interval is therefore $x^2 - 3x + q = 0$.

Step 3: Find the full permitted interval for q Let the smaller root be u . Since the root sum is 3, the larger is $3 - u$, and $u \leq 3/2$. Both roots lie in $(0, 2)$ exactly when $1 < u \leq 3/2$. Their product is

$$q = u(3 - u).$$

This function increases from 2 to $9/4$ on that interval, since its derivative is $3 - 2u \geq 0$. Thus $q \in (2, 9/4]$: 2 is excluded because it gives a root at 2, while $9/4$ gives the permitted repeated root $3/2$.

Step 4: Evaluate the requested endpoint expression The interval endpoints are $\alpha = 2$ and $\beta = 9/4$. Therefore

$$4(\alpha + \beta) = 4\left(2 + \frac{9}{4}\right) = 17.$$

This excludes 11, 13 and 21; those numbers do not follow from the required strict root bounds.

Step 5: Conclusion The requested value is $\boxed{17}$. Hence, the correct option is (C).

Quick Tip to Solve: When a quadratic has fixed root sum, express one root through the other; the interval endpoints and repeated-root endpoint become transparent.

17. Let $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 1 & \alpha \\ 0 & 1 & -1 \end{bmatrix}$ be a singular matrix. Let $f(x) = \int_0^x (t^2 + 2t + 3) dt$, $x \in [1, \alpha]$. If M and m are respectively the maximum and the minimum values of f in $[1, \alpha]$, then $3(M - m)$ is equal to : [4 marks]

- (A) 64
- (B) 68
- (C) 72
- (D) 76

Correct Answer: (B) 68

Solution:

Concept:

- A square matrix is singular precisely when its determinant is zero. Expanding along a row with simple entries determines the parameter α .
- The fundamental theorem of calculus gives $f'(x) = x^2 + 2x + 3$. A strictly positive derivative on an

interval puts the minimum at its left endpoint and the maximum at its right endpoint.

Step 1: Use the singularity condition Expand the determinant along the first row, taking care with the alternating cofactor signs:

$$\begin{aligned}\det A &= 1(-1 - \alpha) - 3(-2) + (-1)(2) \\ &= -1 - \alpha + 6 - 2 = 3 - \alpha.\end{aligned}$$

Since $\det A = 0$, we obtain $\alpha = 3$. The interval for the maximum and minimum is consequently $[1, 3]$.

Step 2: Locate both extrema The derivative satisfies

$$f'(x) = x^2 + 2x + 3 = (x + 1)^2 + 2 > 0.$$

Therefore f is strictly increasing throughout the interval. There is no interior critical point to test: $m = f(1)$ and $M = f(3)$. Their difference can be evaluated directly as one definite integral, with no integration constant.

Step 3: Evaluate the difference and the required multiple Using the endpoint identification,

$$\begin{aligned}M - m &= \int_1^3 (t^2 + 2t + 3) dt \\ &= \left[\frac{t^3}{3} + t^2 + 3t \right]_1^3 \\ &= 27 - \frac{13}{3} = \frac{68}{3}.\end{aligned}$$

Multiplication by 3 yields $3(M - m) = 68$. This eliminates 64, 72 and 76. As an arithmetic check, the separate contributions to $M - m$ are $26/3$, 8 and 6, whose sum is $68/3$.

Step 4: Conclusion The value of $3(M - m)$ is 68. Hence, the correct option is (B).

Quick Tip to Solve: Find the sign of the integrand before integrating; once monotonicity is known, compute the extremal difference as a single endpoint integral.

18. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be such that $f(xy) = f(x)f(y)$, for all $x, y \in \mathbb{R}$ and $f(0) \neq 0$. Let $g : [1, \infty) \rightarrow \mathbb{R}$ be a differentiable function such that

$$x^2 g(x) = \int_1^x (t^2 f(t) - tg(t)) dt$$

Then $g(2)$ is equal to :

[4 marks]

- (A) $\frac{13}{8}$
- (B) $\frac{11}{16}$
- (C) $\frac{15}{32}$
- (D) $\frac{17}{64}$

Correct Answer: (C) $\frac{15}{32}$

Solution:

Concept:

- In a multiplicative equation $f(xy) = f(x)f(y)$, setting one argument equal to zero can determine the whole function when $f(0) \neq 0$.
- The fundamental theorem of calculus differentiates an integral with variable upper limit. A resulting equation $g' + P(x)g = Q(x)$ is solved by multiplying by the integrating factor $e^{\int P(x)dx}$.

Step 1: Determine f and the initial value of g Set $x = 0$ in the functional equation. Then $f(0) = f(0)f(y)$ for every real y . Division by the stated nonzero number $f(0)$ gives $f(y) = 1$ identically. At $x = 1$, the integral has equal limits, so the given relation implies $g(1) = 0$.

Step 2: Differentiate the integral relation The integrand is $t^2 f(t) - tg(t) = t^2 - tg(t)$; the product $tg(t)$ is not a trigonometric function. Differentiation of both sides gives

$$2xg(x) + x^2 g'(x) = x^2 - xg(x).$$

Rearrange and divide by x^2 , which is nonzero on $[1, \infty)$:

$$g'(x) + \frac{3}{x}g(x) = 1.$$

Step 3: Solve with the integrating factor The integrating factor is $e^{\int 3/x dx} = x^3$, since $x \geq 1$. Hence

$$\frac{d}{dx}(x^3 g(x)) = x^3.$$

Integrating from 1 to x and using $g(1) = 0$,

$$x^3 g(x) = \frac{x^4 - 1}{4}, \quad g(x) = \frac{x^4 - 1}{4x^3}.$$

Step 4: Evaluate at the required argument Substitution of $x = 2$ gives

$$g(2) = \frac{16 - 1}{4 \cdot 8} = \frac{15}{32}.$$

The fractions $13/8$, $11/16$ and $17/64$ differ from this value. The solution also satisfies the initial condition and the differentiated equation, so integration has not introduced an arbitrary constant inconsistent with the original relation.

Step 5: Conclusion The required value is $g(2) = 15/32$. Hence, the correct option is (C).

Quick Tip to Solve: Set an argument to zero in the functional equation first; after finding f , differentiate the integral relation and retain the lower-limit initial condition.

19. The area of the region $\{(x, y) : x^2 - 8x \leq y \leq -x\}$ is :

[4 marks]

- (A) $\frac{343}{6}$
- (B) $\frac{637}{6}$
- (C) $\frac{437}{6}$
- (D) $\frac{523}{6}$

Correct Answer: (A) $\frac{343}{6}$

Solution:

Concept:

- The area between two curves on an interval is the integral of the upper ordinate minus the lower ordinate. The inequality in the question fixes which curve is above the other.
- The limits of a bounded region are the intersection abscissae. Here the vertical separation is $(-x) - (x^2 - 8x) = 7x - x^2$, which must be nonnegative.

Step 1: Locate the intersection points Equate the two boundary ordinates to determine where the vertical thickness becomes zero:

$$x^2 - 8x = -x \implies x(x - 7) = 0.$$

Thus the endpoints have abscissae 0 and 7. Also, $7x - x^2 = x(7 - x) \geq 0$ precisely on $[0, 7]$. Outside this interval the lower bound would exceed the upper bound, so no additional portion of the stated region exists there.

Step 2: Integrate the vertical separation Throughout this interval the straight line is the upper boundary and the parabola is the lower boundary. Therefore

$$\begin{aligned} A &= \int_0^7 [-x - (x^2 - 8x)] dx \\ &= \left[\frac{7x^2}{2} - \frac{x^3}{3} \right]_0^7 \\ &= \frac{343}{2} - \frac{343}{3} = \frac{343}{6}. \end{aligned}$$

The integrand stays nonnegative, so this signed integral already equals the geometric area; an extra absolute value or factor of two is unnecessary. The other three fractions have different numerators and do not equal the evaluated integral.

Step 3: Conclusion The area is $\boxed{343/6}$ square units. Hence, the correct option is (A).

Quick Tip to Solve: First factor the vertical gap as $x(7 - x)$. It gives both the valid interval and a direct check that upper minus lower has the correct sign.

20. The value of the integral $\int_{-1}^1 \left(\frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} \right) dx$ is equal to:

[4 marks]

(A) $3 \log_e 2$

(B) $2 \log_e 2$

(C) $5 \log_e 3$

(D) $3 \log_e 3$

Correct Answer: (B) $2 \log_e 2$

Solution:

Concept:

- An odd function has zero integral over a symmetric interval, whereas an even function contributes twice its integral over the positive half of that interval.
- Since $x^2 = |x|^2$, the denominator is $(|x| + 1)^2$, an even and strictly positive function. This allows the odd and even contributions to be separated without introducing a singularity.

Step 1: Separate the odd contribution Rewrite the integrand as

$$\frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} = \frac{x^3}{(|x| + 1)^2} + \frac{1}{|x| + 1}.$$

The first summand is odd: replacing x by $-x$ reverses its numerator and leaves its denominator unchanged. Its integral from -1 to 1 therefore vanishes. This cancellation applies to the cubic term only; discarding the entire numerator would wrongly lose the even contribution.

Step 2: Evaluate the remaining even integral For $x \geq 0$, the absolute value is $|x| = x$. Using evenness and then the logarithmic antiderivative gives

$$\begin{aligned} I &= 2 \int_0^1 \frac{1}{x + 1} dx \\ &= 2 [\ln(x + 1)]_0^1 \\ &= 2(\ln 2 - \ln 1) = 2 \ln 2. \end{aligned}$$

The factor two is the contribution of the negative half of the interval. The upper logarithm is $\ln 2$, not $\ln 3$, because the denominator after cancellation is $x + 1$. Thus the options involving $\ln 3$ fail, and $3 \ln 2$ has an incorrect multiplicative factor.

Step 3: Conclusion The integral is $\boxed{2 \log_e 2}$. Hence, the correct option is (B).

Quick Tip to Solve: When absolute values occur on a symmetric interval, separate odd and even terms before splitting into cases; here the cubic term cancels immediately.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let $R = \{(x, y) \in \mathbb{N} \times \mathbb{N} : \log_e(x + y) \leq 2\}$. Then the minimum number of elements, required to be added in R to make it a transitive relation, is _____. [4 marks]

Solution:

Concept:

- A relation is transitive when (a, b) and (b, c) belonging to it force (a, c) to belong to it. Adding pairs must include every pair forced by such two-step chains.
- Natural numbers here are $1, 2, 3, \dots$. Since $7 < e^2 < 8$, the logarithmic condition is equivalent to the integer condition $x + y \leq 7$.

Step 1: Count the pairs already present The inequality $x + y \leq 7$ with positive integer coordinates permits only $x, y \in \{1, 2, 3, 4, 5, 6\}$. For successive first coordinates $1, 2, \dots, 6$, the allowable numbers of second coordinates are $6, 5, 4, 3, 2, 1$. Hence

$$|R| = 6 + 5 + 4 + 3 + 2 + 1 = 21.$$

No pair in the relation involves a natural number at least seven, since its sum with even the smallest possible positive coordinate would exceed seven.

Step 2: Find all pairs forced through the element one For every $a, c \in \{1, 2, 3, 4, 5, 6\}$, both $(a, 1)$ and $(1, c)$ are already in R : the sums $a + 1$ and $1 + c$ are at most seven. Transitivity therefore forces every ordered pair (a, c) in this six-element set. Its complete relation contains

$$6 \times 6 = 36$$

pairs. Thus any transitive extension must add at least $36 - 21 = 15$ pairs. Conversely, the full relation on these six elements is transitive, and adding exactly those missing pairs attains this bound. Elements outside this set create no chains because no added or original pair uses them.

Step 3: Conclusion The minimum number of ordered pairs to be added is 15. This is a count and has no physical unit.

Quick Tip to Solve: Search for a common intermediate element. Here every active element relates to one in both directions, so transitivity forces the entire six-by-six relation.

22. If $(1 - x^3)^{10} = \sum_{r=0}^{10} a_r x^r (1 - x)^{30-2r}$, then $\frac{9a_9}{a_{10}}$ is equal to _____. [4 marks]

Solution:

Concept:

- The binomial theorem expands $(u + v)^{10}$ as $\sum_{r=0}^{10} \binom{10}{r} u^{10-r} v^r$. Choosing the two summands to reproduce the given powers identifies the requested coefficients directly.
- The factorisation $1 - x^3 = (1 - x)(1 + x + x^2)$, together with $1 + x + x^2 = (1 - x)^2 + 3x$, gives the

required powers of $1 - x$.

Step 1: Rewrite the expression in the required basis Instead of comparing ordinary powers of x , retain the factors of $1 - x$ that occur in the question. The identities above give

$$(1 - x^3)^{10} = (1 - x)^{10}((1 - x)^2 + 3x)^{10}.$$

Apply the binomial theorem to the final factor. A term obtained by choosing $3x$ exactly r times is

$$\binom{10}{r} (3x)^r (1 - x)^{2(10-r)} (1 - x)^{10}.$$

Its exponent of $1 - x$ is $30 - 2r$, exactly the exponent prescribed in the question.

Step 2: Extract the two coefficients and simplify Consequently the expansion has

$$a_r = 3^r \binom{10}{r}, \quad a_9 = 10 \cdot 3^9, \quad a_{10} = 3^{10}.$$

These coefficients are uniquely determined: after factoring out $(1 - x)^{30}$ near zero, the terms are powers of $x/(1 - x)^2$, which varies over an interval. A polynomial identity in that variable has unique coefficients. Hence

$$\frac{9a_9}{a_{10}} = \frac{9 \cdot 10 \cdot 3^9}{3^{10}} = \frac{90}{3} = 30.$$

The extra outside factor $(1 - x)^{10}$ is essential; omitting it would give the wrong target exponent even though the binomial coefficients look similar.

Step 3: Conclusion The required numerical value is $\boxed{30}$, a dimensionless ratio.

Quick Tip to Solve: Match the unusual basis before expanding: $1 + x + x^2 = (1 - x)^2 + 3x$ converts the problem into one direct binomial coefficient comparison.

23. Let the line $x - y = 4$ intersect the circle $C : (x - 4)^2 + (y + 3)^2 = 9$ at the points Q and R . If $P(\alpha, \beta)$ is a point on C such that $PQ = PR$, then $(6\alpha + 8\beta)^2$ is equal to ____ . [4 marks]

Solution:

Concept:

- A point equidistant from two distinct points lies on their perpendicular bisector. The perpendicular bisector of a circle's chord passes through the centre of the circle.
- The given circle has centre $O = (4, -3)$ and radius 3. The chord line $x - y = 4$ has direction $(1, 1)$, so its perpendicular bisector has direction $(1, -1)$.

Step 1: Locate the possible points equidistant from the chord endpoints The distance from the centre to the line is $|4 - (-3) - 4|/\sqrt{2} = 3/\sqrt{2} < 3$, confirming that Q and R are distinct intersection points. Since $PQ = PR$, the point P lies on their perpendicular bisector through O. Write its coordinates as

$$(\alpha, \beta) = (4 + t, -3 - t).$$

This describes the entire perpendicular bisector, rather than the chord itself. The two possible signs of t will correspond to its intersections with the circle.

Step 2: Use the circle and evaluate the expression Substitute the coordinates into the circle equation:

$$(\alpha - 4)^2 + (\beta + 3)^2 = t^2 + (-t)^2 = 9.$$

Therefore $t^2 = 9/2$. The expression asked for simplifies before any square root need be chosen:

$$\begin{aligned} 6\alpha + 8\beta &= 6(4 + t) + 8(-3 - t) = -2t, \\ (6\alpha + 8\beta)^2 &= 4t^2 = 4 \cdot \frac{9}{2} = 18. \end{aligned}$$

Both permissible points give the same squared value. In particular, there is no need to select one intersection or impose an unstated sign condition on P . The cancellation of the centre's contribution, $6(4) + 8(-3) = 0$, explains why the final expression depends only on the radius displacement.

Step 3: Conclusion The required value of $(6\alpha + 8\beta)^2$ is $\boxed{18}$.

Quick Tip to Solve: For equal distances to a chord's endpoints, parameterise its perpendicular bisector through the centre. Evaluate a squared expression using t^2 to avoid unnecessary radicals.

24. Let the image of the point $P(0, -5, 0)$ in the line $\frac{x-1}{2} = \frac{y}{1} = \frac{z+1}{-2}$ be the point R and the image of the point $Q(0, \frac{-1}{2}, 0)$ in the line $\frac{x-1}{-1} = \frac{y+9}{4} = \frac{z+1}{1}$ be the point S . Then the square of the area of the parallelogram $PQRS$ is ____ . [4 marks]

Solution:

Concept:

- For a line $\mathbf{r} = \mathbf{a} + t\mathbf{d}$, the perpendicular foot of a point $\mathbf{p}$ has parameter $t = (\mathbf{p} - \mathbf{a}) \cdot \mathbf{d} / (\mathbf{d} \cdot \mathbf{d})$. Reflection in the line gives $\mathbf{r}_{\text{image}} = 2\mathbf{h} - \mathbf{p}$.
- The area of a parallelogram is the magnitude of the cross product of two adjacent side vectors. Its square is their cross product dotted with itself.

Step 1: Reflect P in the first line For the first line take $\mathbf{a} = (1, 0, -1)$ and $\mathbf{d} = (2, 1, -2)$. With $P = (0, -5, 0)$, its foot parameter and foot H are

$$t = \frac{(-1, -5, 1) \cdot (2, 1, -2)}{4 + 1 + 4} = -1, \quad H = (-1, -1, 1).$$

Therefore $R = 2H - P = (-2, 3, 2)$. The foot is the midpoint of P and its image, not the image itself.

Step 2: Reflect Q in the second line The second line has base point $\mathbf{b} = (1, -9, -1)$ and direction $\mathbf{e} = (-1, 4, 1)$. For $Q = (0, -1/2, 0)$, the parameter is

$$u = \frac{(-1, 17/2, 1) \cdot (-1, 4, 1)}{1 + 16 + 1} = \frac{1 + 34 + 1}{18} = 2.$$

The foot K is $(-1, -1, 1)$, so $S = 2K - Q = (-2, -3/2, 2)$. As a consistency check, P and R have

midpoint H while Q and S have the same midpoint K. Thus the diagonals of the stated quadrilateral bisect one another, confirming its parallelogram structure.

Step 3: Form adjacent sides and find the squared area At vertex P the adjacent sides are

$$\overrightarrow{PQ} = (0, 9/2, 0), \quad \overrightarrow{PS} = (-2, 7/2, 2).$$

Their cross product is $(9, 0, 9)$. Hence the squared area is

$$|\overrightarrow{PQ} \times \overrightarrow{PS}|^2 = 9^2 + 0^2 + 9^2 = 162.$$

Using both diagonals as if they were adjacent sides would incorrectly double the area.

Step 4: Conclusion The square of the area of parallelogram PQRS is $\boxed{162}$ in fourth powers of the coordinate unit.

Quick Tip to Solve: After reflecting points in lines, check diagonal midpoints to confirm the parallelogram, then use adjacent sides in the cross product without a factor of one half.

25. Let $f(x) = \begin{cases} x^3 + 8; & x < 0, \\ x^2 - 4; & x \geq 0, \end{cases}$ and $g(x) = \begin{cases} (x - 8)^{1/3}; & x < 0, \\ (x + 4)^{1/2}; & x \geq 0. \end{cases}$

Then the number of points, where the function $g \circ f$ is discontinuous, is ____ .

[4 marks]

Solution:

Concept:

- The branch used by g depends on the sign of its input $f(x)$, not directly on the sign of x . All cube roots here are real cube roots.
- A piecewise function formed from continuous expressions can be discontinuous only at a branch boundary. Compare both one-sided limits and the actual value at every such boundary.

Step 1: Locate all possible boundaries The formula for f changes at zero. On the negative half-line its sign changes when $x^3 + 8 = 0$, giving $x = -2$. On the nonnegative half-line its sign changes when $x^2 - 4 = 0$, giving $x = 2$. Thus the only candidates are $-2, 0, 2$; every expression used between them is continuous.

Step 2: Write the composition explicitly Substitute each branch of f into the appropriate branch of g :

$$(g \circ f)(x) = \begin{cases} x, & x < -2, \\ \sqrt{x^3 + 12}, & -2 \leq x < 0, \\ \sqrt[3]{x^2 - 12}, & 0 \leq x < 2, \\ x, & x \geq 2. \end{cases}$$

For the last interval, $\sqrt{x^2} = x$ because x is positive. At the two zeros of f , use the nonnegative-input branch of g .

Step 3: Test the one-sided limits At $x = -2$, the left and right limits are -2 and 2 , respectively. At

$x = 0$, they are $\sqrt{12}$ and $-\sqrt[3]{12}$, respectively. At $x = 2$, they are -2 and 2 , respectively. Each pair is unequal, so none of these discontinuities can be removed by the value assigned at the boundary. The square roots have positive radicands on their stated intervals, and real cube roots are continuous everywhere, so no further points occur.

Step 4: Conclusion The discontinuities occur at $x = -2, 0, 2$. Therefore the required number of points is $\boxed{3}$.

Quick Tip to Solve: For $g \circ f$, list the branch boundaries of f and the preimages of the branch boundaries of g , then test only those points.

Physics – Section A: Multiple Choice Questions (80 marks)

26. The percentage error in the calculated volume of a sphere, if there is 2% error in its diameter measurement, is ____ . [4 marks]

- (A) 1
- (B) 2
- (C) 6
- (D) 8

Correct Answer: (C) 6

Solution:

Concept:

- The volume of a sphere in terms of its diameter d is $V = \pi d^3/6$. The factor $\pi/6$ is exact and contributes no measurement error.
- For a power law $y = Cx^n$, the first-order fractional uncertainty is $\Delta y/y = |n|\Delta x/x$. Multiplying by 100 expresses this as a percentage.

Step 1: Express the measured dependence The radius is $r = d/2$. Substitution into the sphere-volume formula gives

$$V = \frac{4}{3}\pi \left(\frac{d}{2}\right)^3 = \frac{\pi d^3}{6}.$$

Thus volume depends on the cube of the one measured length. Converting diameter to radius does not halve the percentage error: the absolute error and the measured length are both halved, leaving their ratio unchanged.

Step 2: Propagate the diameter uncertainty Logarithmic differentiation, or differentiating the power law directly, gives

$$\frac{\Delta V}{V} \simeq 3 \frac{\Delta d}{d}.$$

The given diameter error is 2%, so

$$\text{percentage error in volume} = 3 \times 2\% = 6\%.$$

This is the usual small-error estimate used for propagation of measurement uncertainties. An error signifies an uncertainty in magnitude; its sign is not needed to select the percentage error.

Step 3: Check the listed alternatives The value 1 would incorrectly halve the given percentage when changing diameter to radius. The value 2 would treat volume as proportional to diameter rather than its cube. The value 8 could arise from cubing the percentage number itself, which is not an error-propagation rule. Only 6 follows from the exponent of the measured diameter in the volume formula.

Step 4: Conclusion The percentage error is $\boxed{6\%}$. Hence, the correct option is (C).

Quick Tip to Solve: Write a quantity as a power of the measured variable first; the magnitude of that power multiplies the percentage uncertainty.

27.

Match List - I with List - II.

List - I		List - II	
A.	Boltzmann constant	I.	$[M^{-1} L^3 T^{-2}]$
B.	Stefan's constant	II.	$[M L^2 T^{-1}]$
C.	Planck's constant	III.	$[ML^2 T^{-2} K^{-1}]$
D.	Gravitational constant	IV.	$[M L^0 T^{-3} K^{-4}]$

Choose the correct answer from the options given below :

[4 marks]

- (A) A-I, B-II, C-III, D-IV
- (B) A-IV, B-III, C-II, D-I
- (C) A-III, B-IV, C-II, D-I
- (D) A-II, B-I, C-IV, D-III

Correct Answer: (C) A-III, B-IV, C-II, D-I

Solution:

Concept:

- An equation must have equal dimensions on both sides. Write each constant in terms of familiar physical quantities before assigning dimensions.
- Use $[E] = ML^2T^{-2}$, $[F] = MLT^{-2}$, $[\nu] = T^{-1}$, and temperature dimension K . Radiant power per unit area has dimension MT^{-3} .

Step 1: Find the thermal constants Boltzmann constant relates energy to absolute temperature, for example through $E = k_B T$. Consequently,

$$[k_B] = [E]/[T_{\text{temperature}}] = ML^2T^{-2}K^{-1},$$

which is entry III. Stefan's law gives emitted power per unit area as $j = \sigma T^4$. Since power has

dimension ML^2T^{-3} , dividing it by area removes L^2 . Hence

$$[\sigma] = MT^{-3}K^{-4} = ML^0T^{-3}K^{-4},$$

which is entry IV. Do not confuse total power with power per area in this step.

Step 2: Find the quantum and gravitational constants The photon relation $E = h\nu$ gives

$$[h] = [E]/[\nu] = ML^2T^{-1},$$

so Planck's constant matches II. From Newton's law $F = Gm_1m_2/r^2$,

$$[G] = \frac{[F][r]^2}{[m_1][m_2]} = M^{-1}L^3T^{-2},$$

so gravitational constant matches I. Its inverse mass dimension distinguishes it immediately from all three other constants.

Step 3: Assemble the complete matching Putting these results in List-I order yields A-III, B-IV, C-II, D-I. Option (A) already assigns the gravitational dimensions to Boltzmann constant. Option (B) interchanges the two thermal constants, although their temperature powers differ. Option (D) assigns Planck's dimensions to Boltzmann constant and therefore omits the necessary inverse-temperature factor. Only option (C) satisfies all four physical relations simultaneously.

Step 4: Conclusion The matching is A – III, B – IV, C – II, D – I. Hence, the correct option is (C).

Quick Tip to Solve: Remember k_B as energy per temperature, h as energy times time, and derive the other two from their defining laws.

28. A solid sphere (A) of mass $5m$ and a spherical shell (B) of mass m , both having same radius, are placed on a rough surface. When a force of same magnitude is applied tangentially at the highest points of A and B, they start rolling without slipping with an acceleration of a_A and a_B , respectively. The ratio of a_A and a_B is ____ . [4 marks]

- (A) 5 : 21
- (B) 6 : 10
- (C) 21 : 25
- (D) 1 : 5

Correct Answer: (A) 5 : 21

Solution:

Concept:

- For rolling without slipping, the centre-of-mass acceleration and angular acceleration satisfy $a = \alpha R$. Static friction supplies the torque needed to maintain this constraint.
- The central moment of inertia is $I = (2/5)MR^2$ for a solid sphere and $I = (2/3)MR^2$ for a thin

spherical shell. The applied force at the highest point produces both translation and rotation.

Step 1: Combine translation and rotation Take the applied horizontal force F as positive, and let the signed friction f be positive in that same direction. Translational motion gives

$$F + f = Ma.$$

Taking clockwise torque as positive for rightward rolling, the top force and bottom friction give

$$(F - f)R = I\alpha = \frac{Ia}{R}.$$

Adding the two force equations eliminates the unknown friction:

$$2F = \left(M + \frac{I}{R^2}\right)a.$$

Thus it is not sufficient to compare the masses alone; rotational inertia also contributes to the effective resistance to acceleration.

Step 2: Evaluate each acceleration For solid sphere A, $M_A = 5m$ and $I_A/R^2 = (2/5)(5m) = 2m$. Therefore

$$a_A = \frac{2F}{5m + 2m} = \frac{2F}{7m}.$$

For shell B, $M_B = m$ and $I_B/R^2 = 2m/3$, so

$$a_B = \frac{2F}{m + 2m/3} = \frac{6F}{5m}.$$

The common radius cancels because both inertia formulae contain R^2 . The stated rough surface ensures the required static friction is available.

Step 3: Form the ratio and check the choices The same force magnitude acts on both objects, hence

$$\frac{a_A}{a_B} = \frac{2F}{7m} \frac{5m}{6F} = \frac{5}{21}.$$

A ratio of 1 : 5 would follow from considering only their masses and ignoring their different inertia factors. Neither 6 : 10 nor 21 : 25 agrees with the combined force and torque equations. The shell has the larger acceleration, as its much smaller mass suggests.

Step 4: Conclusion The required ratio is $a_A : a_B = 5 : 21$. Hence, the correct option is (A).

Quick Tip to Solve: For a force applied at the top of a rolling object, eliminate friction to obtain $a = 2F/(M + I/R^2)$ before comparing bodies.

29. A body of mass 1 kg moves along a straight line with a velocity $v = 2x^2$. The work done by the body during displacement from $x = 0$ to 5 m is ____ J. [4 marks]

- (A) 0
(B) 250
(C) 1250
(D) 1000

Correct Answer: (C) 1250

Solution:

Concept:

- The net work done on a body equals the change in its kinetic energy: $W = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$. This avoids assuming constant acceleration.
- If velocity is specified as a function of position, acceleration can also be found from $a = v dv/dx$; integrating ma over displacement gives the same work.

Step 1: Evaluate the endpoint speeds The velocity law is $v = 2x^2$, with x in metres and v in metres per second. At the initial and final positions,

$$v_i = 2(0)^2 = 0, \quad v_f = 2(5)^2 = 50 \text{ m s}^{-1}.$$

The final speed is 50, not 10: square the displacement before multiplying by the coefficient. The acceleration changes with position, so a constant-force calculation would not apply throughout the interval.

Step 2: Apply the work-energy theorem With the mass $m = 1$ kg, the work associated with the motion is

$$W = \frac{1}{2}(1)(50^2 - 0^2) = 1250 \text{ J}.$$

The speed increases, so the kinetic-energy change is positive. In the standard work-energy interpretation of the question, this is the work done by the net driving force on the moving body.

Step 3: Verify with the force integral Differentiating the velocity relation gives $dv/dx = 4x$, and therefore

$$a = v \frac{dv}{dx} = (2x^2)(4x) = 8x^3.$$

Thus

$$W = \int_0^5 ma \, dx = \int_0^5 8x^3 \, dx = [2x^4]_0^5 = 1250 \text{ J}.$$

The two independent methods agree. Zero would require no kinetic-energy change; 250 and 1000 do not equal either the endpoint energy difference or the integrated force. Select the only listed value consistent with the velocity law.

Step 4: Conclusion The work is 1250 J. Hence, the correct option is (C).

Quick Tip to Solve: When speed is given as a function of position, use endpoint kinetic energies for total work; integrate $mv \, dv/dx$ only as a check.

30. A cylinder with adiabatic walls is closed at both ends and is divided into two compartments by a frictionless adiabatic piston. Ideal gas is filled in both (left and right) the compartments at same P, V, T . Heating is started from left side until pressure changes to $27P/8$. If initial volume of each compartment was 9 litres then the final volume in right-hand side compartment is ____ litres. (for this ideal gas $C_P/C_V = 1.5$) [4 marks]

- (A) 3
(B) 4
(C) 14
(D) 9

Correct Answer: (B) 4

Solution:

Concept:

- For a quasistatic adiabatic change of an ideal gas, PV^γ remains constant, where $\gamma = C_P/C_V$. The right-hand gas is insulated by both the piston and the cylinder walls.
- A freely moving frictionless piston establishes equal pressures in the two compartments at equilibrium. Heat supplied to the left gas compresses the right gas by mechanical work; it does not pass through the adiabatic piston.

Step 1: Choose the gas to which the adiabatic law applies The left gas receives heat, so its change cannot be analysed as adiabatic. The right gas receives no heat because all its boundaries are adiabatic. It is compressed as the left side expands. Initially it has pressure $P_i = P$ and volume $V_i = 9$ L; finally its pressure is the common piston-equilibrium pressure

$$P_f = \frac{27P}{8}.$$

The given heat-capacity ratio is $\gamma = 1.5 = 3/2$. The temperature of the right gas rises during compression, so an isothermal pressure-volume relation would be inappropriate.

Step 2: Apply the pressure-volume relation For the right-hand gas,

$$P_i V_i^{3/2} = P_f V_f^{3/2}.$$

Dividing by the final pressure and then raising to the power $2/3$ gives

$$V_f = V_i \left(\frac{P_i}{P_f} \right)^{2/3} = 9 \left(\frac{8}{27} \right)^{2/3} \text{ L}.$$

Since $8 = 2^3$ and $27 = 3^3$, the dimensionless factor is $(2/3)^2 = 4/9$, and the volume is 4 L.

Step 3: Check the physical result and the options The right volume decreases from 9 to 4 litres, consistent with an increased pressure. The total cylinder volume remains 18 litres, so the left final volume is 14 litres; this explains the distractor 14, which is the other compartment. The value 9 would imply no compression. The value 3 does not satisfy $P_f/P_i = (V_i/V_f)^{3/2}$; using 4 gives $(9/4)^{3/2} = 27/8$ exactly.

Step 4: Conclusion The final right-hand volume is $4L$. Hence, the correct option is (B).

Quick Tip to Solve: When only one side of an adiabatic piston is heated, apply $PV^\gamma = \text{constant}$ to the unheated side, using the final common pressure.

31. For an electromagnetic wave propagating through vacuum, $\vec{k}$, $\vec{E}$ and ω represent propagation vector, electric field and angular frequency, respectively. The magnetic field associated with this wave is represented by: [4 marks]

- (A) $\frac{\vec{E} \times \vec{k}}{\omega}$
(B) $\frac{\vec{k} \times \vec{E}}{\omega}$
(C) $\omega(\vec{E} \times \vec{k})$
(D) $\omega(\vec{k} \times \vec{E})$

Correct Answer: (B) $\frac{\vec{k} \times \vec{E}}{\omega}$

Solution:

Concept:

- A plane electromagnetic wave has mutually perpendicular electric field $\vec{E}$, magnetic field $\vec{B}$, and propagation vector $\vec{k}$. Its angular frequency satisfies $\omega = ck$ in vacuum.
- Faraday's law connects the fields: $\nabla \times \vec{E} = -\partial \vec{B} / \partial t$. For a plane wave, spatial differentiation supplies $\vec{k}$, while time differentiation supplies ω .

Step 1: Apply Faraday's law to a plane wave Write the electric field as $\vec{E} = \vec{E}_0 \cos(\vec{k} \cdot \vec{r} - \omega t)$, with the magnetic field having the same phase. Substituting these fields in Faraday's law and cancelling their common sinusoidal factor gives

$$\vec{k} \times \vec{E}_0 = \omega \vec{B}_0.$$

Thus the field relation holds for the instantaneous fields as well as their amplitudes.

Step 2: Check the direction and magnitude Solving the vector relation gives

$$\vec{B} = \frac{\vec{k} \times \vec{E}}{\omega}.$$

For example, propagation along positive x with $\vec{E}$ along positive y gives $\vec{B}$ along positive z . Then $\vec{E} \times \vec{B}$ points along the propagation direction. Since the fields are transverse, its magnitude is $B = kE/\omega = E/c$, the vacuum wave relation.

Step 3: Eliminate the other expressions Option (A) reverses the cross-product order and consequently reverses the magnetic-field direction. Options (C) and (D) multiply by angular frequency rather than divide by it; neither reproduces $B = E/c$, and their dimensions also fail. These checks distinguish both the sign and the frequency dependence of the required expression.

Step 4: Conclusion The magnetic field is $\vec{B} = \frac{\vec{k} \times \vec{E}}{\omega}$. Hence, the correct option is (B).

Quick Tip to Solve: Check both $\vec{E} \times \vec{B} \parallel \vec{k}$ and $B = E/c$; together they fix the cross-product order and the frequency factor.

32. Two identical bodies A and B of equal masses have initial velocities $\vec{v}_1 = 4\hat{i}$ m/s and $\vec{v}_2 = 4\hat{j}$ m/s respectively. The body A has acceleration $\vec{a}_1 = 6\hat{i} + 6\hat{j}$ m/s² while the acceleration of the other body B is zero. The centre of mass of the two bodies moves in ____ path. [4 marks]

- (A) circular
(B) parabolic
(C) straight line
(D) elliptical

Correct Answer: (C) straight line

Solution:

Concept:

- For two equal masses, the velocity and acceleration of their centre of mass are the arithmetic means of the individual velocities and accelerations.
- The path is straight when the constant centre-of-mass acceleration is parallel to its initial velocity. A nonzero acceleration alone does not imply a curved trajectory.

Step 1: Find the initial centre-of-mass velocity Let each body have mass m , and choose the initial centre-of-mass position as the position reference. Mass weighting gives

$$\vec{V}_0 = \frac{m(4\hat{i}) + m(4\hat{j})}{2m} = 2\hat{i} + 2\hat{j} \text{ m s}^{-1}.$$

Thus both Cartesian components of the initial velocity are equal; the initial direction is along $\hat{i} + \hat{j}$.

Step 2: Find the centre-of-mass acceleration Only body A accelerates. Averaging the two acceleration vectors gives

$$\vec{A} = \frac{(6\hat{i} + 6\hat{j}) + \vec{0}}{2} = 3\hat{i} + 3\hat{j} \text{ m s}^{-2}.$$

This vector is parallel to $\vec{V}_0$. Consequently the acceleration changes the magnitude of the velocity without changing its direction.

Step 3: Eliminate time to identify the path With the chosen position reference, the coordinates are

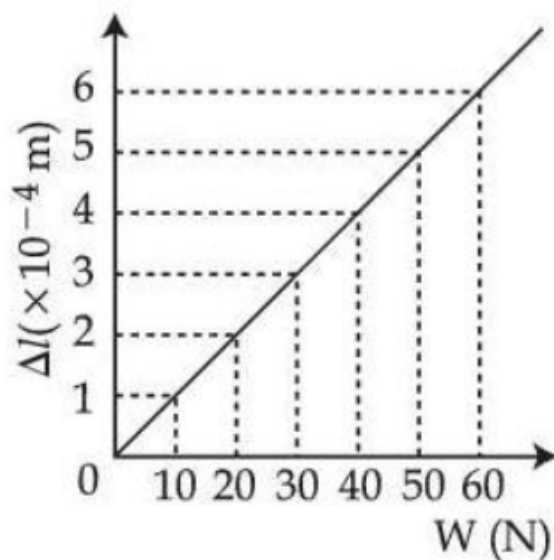
$$X = 2t + \frac{3}{2}t^2, \quad Y = 2t + \frac{3}{2}t^2.$$

Hence $Y = X$, a straight line; an arbitrary initial position merely translates this line. Circular and elliptical paths require a changing velocity direction, while a nondegenerate parabolic projectile path requires independent perpendicular and accelerated components. Neither occurs for the centre of mass here.

Step 4: Conclusion The centre of mass follows a straight-line path parallel to $\hat{i} + \hat{j}$. Hence, the correct option is (C).

Quick Tip to Solve: For constant acceleration, compare its direction with the initial velocity before eliminating time: parallel vectors give a straight path.

33. Figure represents the extension (Δl) of a wire of length 1 meter, suspended from the ceiling of the room at one end with a load W connected to the other end. If the cross-sectional area of the wire is 10^{-5} m^2 then the Young's modulus of the wire is ____ N/m^2 . [4 marks]



- (A) 1.0×10^{11}
- (B) 2.0×10^{10}
- (C) 1.0×10^{10}
- (D) 2.0×10^{11}

Correct Answer: (C) 1.0×10^{10}

Solution:

Concept:

- Young's modulus is longitudinal stress divided by longitudinal strain. Thus $Y = WL/(A\Delta l)$, where W is the force, A the area, and L the initial length.
- The graph plots extension vertically against load horizontally. Its slope is $\Delta l/W = L/(AY)$; the axis multiplier must be included before substitution.

Step 1: Read a point from the straight line The plotted line passes through the origin and the point with load $W = 60 \text{ N}$ and extension $\Delta l = 6 \times 10^{-4} \text{ m}$. The same proportionality can be read from 10 N and $1 \times 10^{-4} \text{ m}$. Thus

$$\frac{W}{\Delta l} = \frac{60}{6 \times 10^{-4}} = 10^5 \text{ N m}^{-1}.$$

The vertical numbers are not extensions in metres by themselves.

Step 2: Calculate the stress and strain The original length is $L = 1 \text{ m}$ and the cross-sectional area is $A = 10^{-5} \text{ m}^2$. At the chosen load,

$$\text{stress} = \frac{60}{10^{-5}} = 6 \times 10^6 \text{ N m}^{-2},$$

$$\text{strain} = \frac{6 \times 10^{-4}}{1} = 6 \times 10^{-4}.$$

The strain is dimensionless because it is a ratio of lengths.

Step 3: Take the ratio and compare the options Therefore

$$Y = \frac{6 \times 10^6}{6 \times 10^{-4}} = 1.0 \times 10^{10} \text{ N m}^{-2}.$$

This agrees with using the reciprocal graph slope multiplied by L/A . Options (A) and (D) are too large by factors of ten and twenty, and option (B) is twice the calculated modulus. No extra factor of two enters the definition of longitudinal strain.

Step 4: Conclusion The Young's modulus is $1.0 \times 10^{10} \text{ N m}^{-2}$. Hence, the correct option is (C).

Quick Tip to Solve: For an extension-versus-load graph, use the reciprocal slope in $Y = L/(A \times \text{slope})$, and retain the extension-axis multiplier.

34. A cylindrical vessel of 40 cm radius is completely filled with water and its capacity is 528 dm^3 (dm : decimeter) The vessel is placed on a solid block of exactly same height as vessel. If a small hole is made at 70 cm below the top of water level, then horizontal range of water falling on the ground in the beginning is ____ cm .

[4 marks]

- (A) $120\sqrt{2}$
- (B) $140\sqrt{2}$
- (C) $140\sqrt{3}$
- (D) $120\sqrt{3}$

Correct Answer: (B) $140\sqrt{2}$

Solution:

Concept:

- Torricelli's law gives the initial horizontal efflux speed from a small hole as $v = \sqrt{2gh}$, where h is its depth below the free water surface.
- After leaving horizontally, water takes $t = \sqrt{2y/g}$ to reach the ground from height y . Its horizontal range is therefore $R = 2\sqrt{hy}$.

Step 1: Determine the vessel height The capacity is $528 \text{ dm}^3 = 528000 \text{ cm}^3$, because one cubic decimetre equals 1000 cm^3 . For radius $r = 40 \text{ cm}$, use the cylindrical volume relation:

$$H = \frac{V}{\pi r^2} = \frac{528000}{(22/7)(40)^2} = 105 \text{ cm}.$$

Here $\pi = 22/7$ gives the intended exact values of the listed choices.

Step 2: Locate the hole relative to the ground The solid block has the same height as the vessel, so the initial water surface is $2H = 210 \text{ cm}$ above ground. The hole is $h = 70 \text{ cm}$ below that surface. Its ground clearance is therefore

$$y = 2H - h = 210 - 70 = 140 \text{ cm}.$$

The depth controlling efflux and the height controlling flight time are different quantities.

Step 3: Combine the efflux speed and flight time For a large vessel with a small hole, the surface speed is negligible initially. Multiplying the horizontal speed by the fall time gives

$$R = \sqrt{2gh} \sqrt{\frac{2y}{g}} = 2\sqrt{hy} = 2\sqrt{70 \times 140} = 140\sqrt{2} \text{ cm.}$$

The gravitational acceleration cancels. Option (A) uses too small a coefficient, while (C) and (D) have the wrong product of the depth and ground clearance. The initial surface height, rather than a later reduced level, is required.

Step 4: Conclusion The initial horizontal range is $140\sqrt{2}$ cm. Hence, the correct option is (B).

Quick Tip to Solve: Label the hole's depth below water and its height above ground separately; their product determines the horizontal range.

35. If 2 mole of an ideal monoatomic gas at temperature T , is mixed with 6 mole of another ideal monoatomic gas at temperature $2T$ then the temperature of mixture is: [4 marks]

- (A) $\frac{5}{2}T$
- (B) $\frac{5}{4}T$
- (C) $\frac{7}{2}T$
- (D) $\frac{7}{4}T$

Correct Answer: (D) $\frac{7}{4}T$

Solution:

Concept:

- For an ideal monoatomic gas, internal energy is $U = (3/2)nRT$, with n the amount in moles and R the gas constant.
- For the usual isolated mixing process with no external work, total internal energy is conserved. Since both gases have the same molar heat capacity, their equilibrium temperature is a mole-weighted average.

Step 1: Write the initial internal energy The first gas contains $n_1 = 2$ moles at temperature T , while the second contains $n_2 = 6$ moles at temperature $2T$. Both are monoatomic, so each has three translational degrees of freedom. Their combined initial energy is

$$U_i = \frac{3}{2}R[2T + 6(2T)] = \frac{3}{2}R(14T).$$

The larger sample carries six times $2T$, not six times T .

Step 2: Apply conservation at the common temperature Let the common final temperature be T_f . The total amount after mixing is $2 + 6 = 8$ moles. Ideal-gas internal energies remain additive, including when the gases are different species:

$$U_f = \frac{3}{2}R(8T_f).$$

Conservation $U_i = U_f$ cancels the common heat-capacity factor and gives $8T_f = 14T$.

Step 3: Evaluate and test the result The resulting temperature is

$$T_f = \frac{14}{8}T = \frac{7}{4}T = 1.75T.$$

It lies between T and $2T$, and it is closer to $2T$ because there are more moles of the hotter gas. Options (A) and (C) exceed both initial temperatures and violate this energy balance. Option (B), although between the endpoints, puts excessive weight on the colder sample and does not conserve the initial internal energy.

Step 4: Conclusion The mixture reaches a temperature $T_f = \frac{7}{4}T$. Hence, the correct option is (D).

Quick Tip to Solve: When ideal gases have equal molar heat capacities, average temperature with mole weights, then check that it lies nearer the larger sample's temperature.

36. A spring stretches by 2 mm when it is loaded with a mass of 200 g . From equilibrium position the mass is further pulled down by 2 mm and released. The frequency associated with the system and maximum energy in the spring are ____ Hz and ____ J, respectively.

(Take $g = 10 \text{ m/s}^2$)

[4 marks]

(A) $\frac{5\sqrt{50}}{\pi}$ and 8×10^{-3}

(B) $\frac{5\sqrt{50}}{\pi}$ and 8

(C) $10\sqrt{50}$ and 2×10^{-3}

(D) $\frac{5\sqrt{50}}{\pi}$ and 16×10^{-3}

Correct Answer: (A) $\frac{5\sqrt{50}}{\pi}$ and 8×10^{-3}

Solution:

Concept:

- The static extension x_0 of a vertical spring satisfies $kx_0 = mg$. Oscillations about this equilibrium have angular frequency $\omega = \sqrt{k/m}$ and frequency $f = \omega/(2\pi)$.
- The spring's elastic potential energy is $U_s = \frac{1}{2}kx^2$, with extension x measured from the natural length. This differs from the oscillation energy $\frac{1}{2}kA^2$, whose displacement amplitude is measured from equilibrium.

Step 1: Find the spring constant and frequency Convert the data to SI units: $m = 0.200 \text{ kg}$, $x_0 = 0.002 \text{ m}$, and $g = 10 \text{ m s}^{-2}$. The static load gives

$$k = \frac{mg}{x_0} = \frac{0.200 \times 10}{0.002} = 1000 \text{ N m}^{-1}.$$

Consequently

$$f = \frac{1}{2\pi} \sqrt{\frac{1000}{0.200}} = \frac{\sqrt{5000}}{2\pi} = \frac{5\sqrt{50}}{\pi} \text{ Hz}.$$

Step 2: Find the maximum extension from natural length The further downward displacement

$A = 2 \text{ mm}$ is the oscillation amplitude. At the lowest turning point the extension from natural length is

$$x_{\max} = x_0 + A = 2 \text{ mm} + 2 \text{ mm} = 4 \times 10^{-3} \text{ m}.$$

The upper turning point has zero extension; the lower one has the greatest spring energy.

Step 3: Compute the largest elastic energy The required elastic energy is

$$U_{s,\max} = \frac{1}{2} k x_{\max}^2 = \frac{1}{2} (1000) (4 \times 10^{-3})^2 = 8 \times 10^{-3} \text{ J}.$$

Using A alone gives $2 \times 10^{-3} \text{ J}$, the oscillation energy relative to equilibrium. Option (C) also misses the angular-frequency conversion; (B) loses the millimetre conversion and (D) doubles the elastic energy.

Step 4: Conclusion The frequency is $\frac{5\sqrt{50}}{\pi} \text{ Hz}$, and the maximum spring energy is $8 \times 10^{-3} \text{ J}$. Hence, the correct option is (A).

Quick Tip to Solve: For a vertical spring, distinguish displacement from equilibrium from extension beyond natural length: use $x_0 + A$ for maximum elastic energy.

37. The electric potential as a function of x, y is given by $V = 5(x^2 - y^2) \text{ V}$. The electric field at a point $(2, 3) \text{ m}$ is ____ V/m . [4 marks]

- (A) $(-20\hat{i} + 30\hat{j})$
- (B) $(20\hat{i} - 30\hat{j})$
- (C) $(20\hat{i} + 45\hat{j})$
- (D) $(-4\hat{i} + 6\hat{j})$

Correct Answer: (A) $(-20\hat{i} + 30\hat{j})$

Solution:

Concept:

- The electric field is the negative spatial gradient of potential: $\mathbf{E} = -\nabla V$. Each component measures the potential decrease per unit distance along its coordinate direction.
- For a potential $V(x, y)$, use $E_x = -\partial V / \partial x$ and $E_y = -\partial V / \partial y$, keeping the other coordinate fixed in each partial derivative. The field has units V m^{-1} .

Step 1: Differentiate with respect to each coordinate The given function is $V = 5x^2 - 5y^2$. Differentiate before inserting the point, since the field depends on how potential changes near the point, not just the potential value there. We obtain

$$\frac{\partial V}{\partial x} = 10x, \quad \frac{\partial V}{\partial y} = -10y.$$

The overall negative sign in the field-potential relation reverses each of these derivatives. Consequently,

$$\mathbf{E} = -10x\hat{i} + 10y\hat{j}.$$

There is no dependence on a third coordinate, so its field component is zero.

Step 2: Evaluate and check the vector signs At $x = 2$ m and $y = 3$ m, substitute into the two components separately:

$$E_x = -10(2) = -20 \text{ V m}^{-1}, \quad E_y = 10(3) = 30 \text{ V m}^{-1}.$$

Thus the field points towards decreasing x and increasing y . Option (B) is the positive gradient rather than the negative gradient. Option (C) has both the wrong x -direction and an incorrect y -component; differentiating y^2 gives $2y$, not a quadratic value. Option (D) omits the coefficient 5 from the potential.

Step 3: Conclusion

$$\mathbf{E} = (-20\hat{i} + 30\hat{j}) \text{ V m}^{-1}.$$

Hence, the correct option is (A).

Quick Tip to Solve: Differentiate the potential first, then apply the minus sign to both components, and only then substitute the coordinates.

38. A current of 30 A each flows in opposite directions in two conducting wires, placed parallel to each other at a distance of 8 cm . The magnetic field at the mid point between the two wires is ____ μT .

$$\left(\frac{\mu_0}{4\pi} = 10^{-7} \text{ N/A}^2\right)$$

[4 marks]

- (A) 30
- (B) 300
- (C) 150
- (D) 0.0

Correct Answer: (B) 300

Solution:

Concept:

- The magnetic field magnitude at perpendicular distance r from a long straight wire is $B = \mu_0 I / (2\pi r)$. Its direction follows the right-hand grip rule.
- The net field is a vector sum. Between parallel wires carrying opposite currents, the two magnetic fields point in the same direction and therefore add.

Step 1: Find the midpoint distance and one-wire field The separation is 8 cm, so the midpoint is half that distance from each wire:

$$r = 4 \text{ cm} = 0.04 \text{ m}.$$

Using $I = 30$ A and the supplied $\mu_0 / (4\pi) = 10^{-7} \text{ N A}^{-2}$, rewrite the field formula to use the given

constant directly:

$$B_1 = \frac{\mu_0}{4\pi} \frac{2I}{r} = 10^{-7} \frac{2(30)}{0.04}.$$

Therefore,

$$B_1 = 1.5 \times 10^{-4} \text{ T} = 150 \mu\text{T}.$$

The second wire gives exactly the same magnitude because its current and distance are equal.

Step 2: Apply the right-hand rule and add the fields To check the directions, imagine the first wire carrying current upward and the second carrying it downward. At a point to the right of the first wire its field points into the page. At the midpoint, which lies to the left of the second wire, that wire also produces a field into the page. Reversing both currents reverses both fields together but does not change the magnitude. Hence,

$$B_{\text{net}} = B_1 + B_2 = 150 + 150 = 300 \mu\text{T}.$$

Option (C) counts only one wire. Option (D) would describe cancellation at the midpoint for equal currents in the same direction, not the stated opposite directions. Option (A) is too small by a factor of ten and does not follow from the given distances.

Step 3: Conclusion The magnetic field magnitude at the midpoint is $300 \mu\text{T}$. Hence, the correct option is (B).

Quick Tip to Solve: At the midpoint, opposite currents give reinforcing fields. Use half the wire separation in each field formula before adding.

39. A square loop of side 2 cm is placed in a time varying magnetic field with magnitude as $B = 0.4 \sin(300t)$ Tesla. The normal to the plane of loop makes an angle of 60° with the field. The maximum induced emf produced in the loop is ____ mV . [4 marks]

(A) 12

(B) 18

(C) 21

(D) 24

Correct Answer: (D) 24

Solution:

Concept:

- For a fixed planar loop in a uniform magnetic field, magnetic flux is $\Phi = BA \cos \theta$, where θ is the angle between the field and the area normal.
- Faraday's law gives $\varepsilon = -d\Phi/dt$ for one turn. If $B = B_0 \sin \omega t$, the maximum emf magnitude is $\varepsilon_{\text{max}} = AB_0\omega |\cos \theta|$.

Step 1: Convert the loop area and write its flux The square has side $a = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$. Its area

must be squared after converting to metres:

$$A = a^2 = (2 \times 10^{-2})^2 = 4 \times 10^{-4} \text{ m}^2.$$

The angle supplied is already the angle with the normal, so $\cos 60^\circ = 1/2$ is the projection factor. With $B(t) = 0.4 \sin(300t)$,

$$\Phi(t) = (4 \times 10^{-4})(0.4)\frac{1}{2} \sin(300t) \text{ Wb}.$$

Neither the area nor the orientation changes with time, so only the sine factor is differentiated.

Step 2: Differentiate the flux and take its amplitude The derivative of $\sin(300t)$ is $300 \cos(300t)$; the factor 300 is the angular frequency, not an extra number of turns. Thus,

$$\varepsilon(t) = -(4 \times 10^{-4})(0.4)\frac{1}{2}(300) \cos(300t).$$

Its amplitude is

$$\varepsilon_{\max} = 2.4 \times 10^{-2} \text{ V} = 24 \text{ mV}.$$

The minus sign expresses Lenz's law and does not make the maximum magnitude negative. Of the proposed values, only option (D) equals this product. Option (A) introduces an additional factor of one-half; options (B) and (C) do not follow from the stated area, angular frequency and normal angle.

Step 3: Conclusion The maximum induced emf is $\boxed{24 \text{ mV}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the angle with the normal in the cosine, and remember that differentiating the sine supplies the angular frequency.

40. A sphere of capacitance 100 pF is charged to a potential of 100 V . Another identical uncharged metal sphere is brought in contact with the charged sphere, then the change in the total energy stored on these spheres, when they touch is $\alpha \times 10^{-7}$ J. The value of α is ____ .

(combined capacitance of spheres is 200 pF)

[4 marks]

- (A) 5
- (B) $\frac{5}{2}$
- (C) $\frac{7}{2}$
- (D) $\frac{9}{2}$

Correct Answer: (B) $\frac{5}{2}$

Solution:

Concept:

- A conductor of capacitance C at potential V stores charge $Q = CV$ and energy $U = \frac{1}{2}CV^2 = Q^2/(2C)$.
- When isolated conducting spheres touch, total charge is conserved and they reach a common potential. The supplied combined capacitance determines the final energy; electrostatic energy itself need not be conserved.

Step 1: Find the initial charge and stored energy Initially only the first sphere is charged. Convert its capacitance as $C = 100 \text{ pF} = 100 \times 10^{-12} \text{ F}$, and use $V_i = 100 \text{ V}$:

$$Q_{\text{total}} = CV_i = (100 \times 10^{-12})(100) = 10^{-8} \text{ C}.$$

The other sphere contributes no initial charge or energy. Thus,

$$U_i = \frac{1}{2}(100 \times 10^{-12})(100)^2 = 5 \times 10^{-7} \text{ J}.$$

The system remains isolated during contact, so this total charge is redistributed rather than replaced by charge from an external voltage source.

Step 2: Use the combined capacitance and compare energies The final capacitance is explicitly $C_f = 200 \text{ pF}$. Therefore,

$$V_f = \frac{Q_{\text{total}}}{C_f} = \frac{10^{-8}}{200 \times 10^{-12}} = 50 \text{ V}.$$

The final total energy is

$$U_f = \frac{1}{2}(200 \times 10^{-12})(50)^2 = 2.5 \times 10^{-7} \text{ J}.$$

Consequently the energy lost is $U_i - U_f = 2.5 \times 10^{-7} \text{ J}$. The signed change $U_f - U_i$ is negative; the positive choices ask for the magnitude of the decrease. Option (A) is the entire initial energy, not the decrease. Options (C) and (D) disagree with charge conservation and the final common potential.

Step 3: Conclusion The total stored energy decreases by $\boxed{\frac{5}{2} \times 10^{-7} \text{ J}}$. Its signed change is $-\frac{5}{2} \times 10^{-7} \text{ J}$; the listed positive coefficient of the loss is $\alpha = \frac{5}{2}$. Hence, the correct option is (B).

Quick Tip to Solve: Conserve charge during contact, not electrostatic energy. With twice the capacitance and the same charge, the stored energy halves.

41. The energy released if hydrogen atoms are combined to form ${}^4_2\text{He}$ is ____ MeV .

(Take binding energies per nucleon of ${}^2_1\text{H}$ and ${}^4_2\text{He}$ as 1.1 MeV and 7.2 MeV , respectively) [4 marks]

- (A) 6.1
- (B) 24.4
- (C) 26.6
- (D) 5

Correct Answer: (B) 24.4

Solution:

Concept:

- Total nuclear binding energy is the binding energy per nucleon multiplied by the mass number: $B = A(B/A)$. The energy released in fusion equals the increase in total binding energy.
- The specified hydrogen isotope is deuterium, ${}^2_1\text{H}$, containing two nucleons. Formation of one ${}^4_2\text{He}$ nucleus therefore combines two deuterium nuclei and conserves both nucleon number and charge.

Step 1: Calculate the binding energy before fusion Each deuterium nucleus has mass number 2 and binding energy per nucleon 1.1 MeV. Its total binding energy is

$$B_D = 2(1.1) = 2.2 \text{ MeV}.$$

Two such nuclei contain four nucleons altogether, matching the helium product. The total initial binding energy is therefore

$$B_i = 2B_D = 2(2.2) = 4.4 \text{ MeV}.$$

The distinction between energy per nucleon and total nuclear binding energy is essential: subtracting the two given numbers directly would describe only a per-nucleon difference, not the energy released by forming one helium nucleus.

Step 2: Find the increase in total binding energy The helium nucleus has mass number 4 and binding energy per nucleon 7.2 MeV. Its binding energy is

$$B_f = 4(7.2) = 28.8 \text{ MeV}.$$

The product is more tightly bound, so its lower rest energy releases the positive difference

$$Q = B_f - B_i = 28.8 - 4.4 = 24.4 \text{ MeV}.$$

Equivalently, all four nucleons gain $7.2 - 1.1 = 6.1$ MeV of binding each, giving $4(6.1) = 24.4$ MeV. Option (A) omits the four-nucleon factor. Option (C) subtracts only one deuterium binding energy, even though two nuclei are required. Option (D) does not equal the binding-energy increase.

Step 3: Conclusion The energy released in forming one helium-4 nucleus is 24.4 MeV. Hence, the correct option is (B).

Quick Tip to Solve: Balance the number of nucleons first, then convert each quoted binding energy per nucleon into total binding energy before subtraction.

42. Angle of minimum deviation is equal to the half of the angle of prism in an equilateral prism. The refractive index of the prism is ____ [4 marks]

- (A) 1.5
- (B) $\sqrt{3}$
- (C) $\sqrt{2}$
- (D) 1.65

Correct Answer: (C) $\sqrt{2}$

Solution:

Concept:

- For a prism in air at minimum deviation, the ray path is symmetric: $i = e$ and the two internal refraction angles satisfy $r_1 = r_2 = A/2$, where A is the prism angle.
- The minimum-deviation relation is $\mu = \sin[(A + \delta_{\min})/2] / \sin(A/2)$, obtained from Snell's law and

$\delta_{\min} = 2i - A$. The refractive index is dimensionless.

Step 1: Determine the prism angle and symmetric ray angles An equilateral prism has an apex angle

$$A = 60^\circ.$$

The minimum deviation is given to be half the prism angle, hence

$$\delta_{\min} = \frac{A}{2} = 30^\circ.$$

At minimum deviation, $r = A/2 = 30^\circ$, and the external incidence angle follows from the total deviation:

$$i = \frac{A + \delta_{\min}}{2} = \frac{60^\circ + 30^\circ}{2} = 45^\circ.$$

The division by two in the incidence relation is necessary because both entry and exit contribute equally to deviation along the symmetric path.

Step 2: Apply Snell law and evaluate the sine ratio At the first surface, with air as the incident medium, Snell's law gives $\sin i = \mu \sin r$. Substitute the two angles:

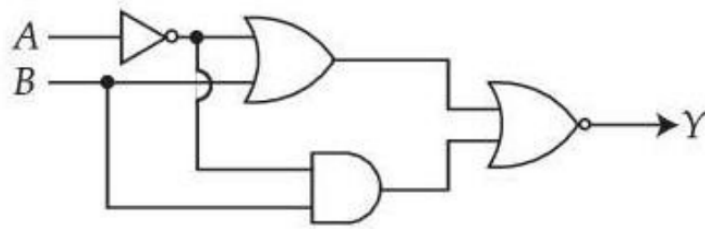
$$\mu = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2}.$$

A direct check is $\sqrt{2} \sin 30^\circ = 1/\sqrt{2} = \sin 45^\circ$, so the computed index produces the required incidence and deviation. Option (B), $\sqrt{3}$, would instead give $i = 60^\circ$ and $\delta_{\min} = 60^\circ$. Options (A) and (D) also yield incidence angles different from 45° , so neither satisfies the specified half-angle deviation.

Step 3: Conclusion The refractive index of the prism is $\boxed{\sqrt{2}}$, a dimensionless quantity. Hence, the correct option is (C).

Quick Tip to Solve: For minimum deviation, first write the symmetric angles: the internal angle is half the prism angle and the incidence angle is half the sum of prism angle and deviation.

43. Refer to the logic circuit given below. For two inputs ($A = 1, B = 1$) and ($A = 0, B = 1$), output (Y) will be ____ . [4 marks]



- (A) 1, 0 respectively
- (B) 0, 1 respectively
- (C) 0, 0 respectively
- (D) 1, 1 respectively

Correct Answer: (C) 0, 0 respectively

Solution:

Concept:

- The NOT gate changes input A to $\bar{A}$. An OR gate outputs 1 if either input is 1, whereas an AND gate outputs 1 only when both inputs are 1.
- The final gate is a NOR gate: its output is the complement of the OR of its two input signals. A crossing with a bridge is not an electrical junction.

Step 1: Identify the two intermediate signals The upper branch receives $\bar{A}$ and B at an OR gate, so its output is $U = \bar{A} + B$. The lower branch receives the same two signals at an AND gate, giving $L = \bar{A}B$. Here addition means Boolean OR, and multiplication means Boolean AND. Thus the final output is

$$Y = \overline{U + L} = \overline{\bar{A} + B + \bar{A}B}.$$

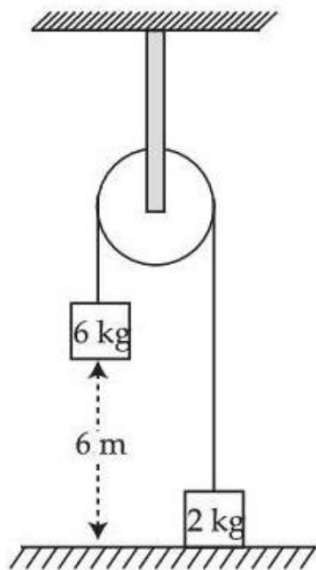
By absorption, $\bar{A} + \bar{A}B = \bar{A}$, so De Morgan's law gives $Y = A\bar{B}$.

Step 2: Evaluate both specified input pairs For $(A, B) = (1, 1)$, the NOT output is 0, the upper OR output is 1 and the lower AND output is 0. The final NOR therefore gives $Y = \overline{1 + 0} = 0$. For $(A, B) = (0, 1)$, the NOT output is 1 and both intermediate outputs are 1, giving $Y = \overline{1 + 1} = 0$. More directly, $B = 1$ makes $\bar{B} = 0$ in both cases, independently of A . Options (A), (B) and (D) each require at least one output to be 1 and therefore contradict the final NOR gate.

Step 3: Conclusion The outputs are 0, 0, respectively. Hence, the correct option is (C).

Quick Tip to Solve: Check the inversion bubble on the final gate before making a truth table: here $B = 1$ forces an OR input to the NOR high and makes its output zero.

44. The velocity at which 6 kg mass (shown in figure) strikes the ground when it is released from a height of 6 m above the ground is ____ m/s. Assume pulley is massless and string is light and inextensible. (Take $g = 10 \text{ m/s}^2$) [4 marks]



- (A) 7.74
- (B) 7.20
- (C) 6.55
- (D) 4.50

Correct Answer: (A) 7.74

Solution:

Concept:

- A light inextensible string over an ideal pulley makes the two connected masses move equal distances with equal speed magnitudes.
- With a massless pulley there is no rotational kinetic energy. The net loss of gravitational potential energy becomes the total translational kinetic energy of both masses.

Step 1: Find the net gravitational energy released Let v be the common speed just before the 6 kg mass reaches the ground. The heavier mass descends $h = 6 \text{ m}$, while the 2 kg mass rises by the same distance. The heavier mass loses potential energy $6gh$, but the lighter mass gains $2gh$. Hence the energy available for kinetic energy is

$$\Delta E = (6 - 2)gh = 4(10)(6) = 240 \text{ J.}$$

The lighter mass lifts off immediately: the string tension in this motion exceeds its weight, so it does not stay on the ground.

Step 2: Equate this energy to the kinetic energy Both masses start from rest. Conservation of

mechanical energy therefore gives

$$240 = \frac{1}{2}(6 + 2)v^2 = 4v^2.$$

Thus

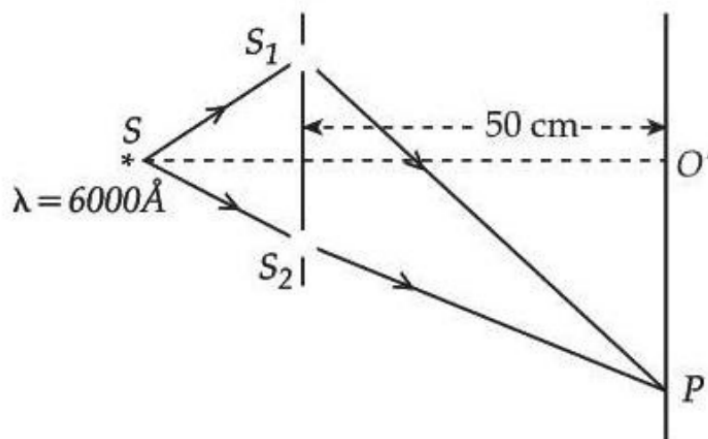
$$v^2 = 60 \text{ m}^2 \text{ s}^{-2}, \quad v = \sqrt{60} = 7.746 \dots \text{ m s}^{-1}.$$

As a check, the acceleration is $a = (6 - 2)g/(6 + 2) = 5 \text{ m s}^{-2}$, and $v^2 = 2ah = 60$ agrees. The quoted value 7.74 is the listed approximation; 7.20, 6.55 and 4.50 do not satisfy the system energy balance.

Step 3: Conclusion The speed is approximately 7.74 m s^{-1} , directed downward for the heavier mass. Hence, the correct option is (A).

Quick Tip to Solve: For two connected hanging masses, subtract their potential-energy changes but add their kinetic energies; using only the falling mass overestimates its speed.

45. In a Young double slit experiment, the wavelength of incident light is $6000 \text{ \AA}$, the separation between slits S_1 and S_2 is 5 cm and the distance between slits plane and screen is 50 cm , as shown in the figure below. If the resultant intensity at P is equal to the intensity due to individual slits, the path difference between interfering waves is ____ $\text{\AA}$. [4 marks]



- (A) 4000
- (B) 3000
- (C) 2000
- (D) 1000

Correct Answer: (C) 2000

Solution:

Concept:

- For equal individual slit intensities I_0 , interference gives $I = I_0 + I_0 + 2I_0 \cos \phi$, where ϕ is the phase difference at the screen.
- Phase difference and total optical path difference Δ are related by $\phi = 2\pi\Delta/\lambda$. The intensity is

periodic in Δ with period λ .

Step 1: Use the specified intensity condition The required resultant intensity is the intensity from either slit alone, so set $I = I_0$. Then

$$I_0 = 2I_0(1 + \cos \phi) \Rightarrow \cos \phi = -\frac{1}{2}.$$

Consequently

$$\phi = 2\pi n \pm \frac{2\pi}{3}, \quad \Delta = n\lambda \pm \frac{\lambda}{3}.$$

The slit spacing and screen distance would locate the screen point for a given order; they are unnecessary for finding the allowed path differences from intensity.

Step 2: Check all four listed path differences With $\lambda = 6000 \text{ \AA}$, the smallest positive solution is $\Delta = \lambda/3 = 2000 \text{ \AA}$. However, $\Delta = 2\lambda/3 = 4000 \text{ \AA}$ also gives $\cos \phi = \cos(4\pi/3) = -1/2$, and hence the same intensity. The value $3000 \text{ \AA} = \lambda/2$ gives zero intensity, while $1000 \text{ \AA} = \lambda/6$ gives $3I_0$. Thus (B) and (D) fail, but (A) is also a valid path difference unless the smallest positive value is intended.

Step 3: Conclusion The keyed answer is $2000 \text{ \AA}$, the smallest positive path difference. Hence, the correct option is (C) under that interpretation. As printed, the condition also admits $4000 \text{ \AA}$, option (A), so it does not select a unique listed answer.

Quick Tip to Solve: Always check the second phase solution when solving an interference intensity condition: $\cos \phi = -1/2$ allows both $2\pi/3$ and $4\pi/3$.

Physics – Section B: Numerical Value Questions (20 marks)

46. A block takes t time to slide down a plane inclined at 45° to the horizontal. If the surface is made smooth (frictionless), the block takes time $\frac{t}{2}$ to slide down the plane. The coefficient of friction between the block and the inclined plane is $\left(\frac{\alpha}{100}\right)$. The value of α is ____ . [4 marks]

Solution:

Concept:

- For a block released from rest and moving a fixed distance s with constant acceleration a , $s = \frac{1}{2}at^2$; therefore acceleration is inversely proportional to the square of the travel time.
- On a plane inclined at θ , the normal reaction is $N = mg \cos \theta$. Sliding friction of magnitude μN opposes the downhill motion.

Step 1: Relate the accelerations using the times Let a_r be the acceleration on the original rough plane and a_s the acceleration after smoothing. The same block traverses the same plane from rest, so

$$s = \frac{1}{2}a_r t^2 = \frac{1}{2}a_s \left(\frac{t}{2}\right)^2.$$

Cancellation of the common factors gives

$$a_r = \frac{a_s}{4}.$$

Halving the time therefore quadruples the acceleration; simply doubling it would be inconsistent with the squared time in the displacement equation.

Step 2: Resolve forces along the plane On the smooth plane, $a_s = g \sin 45^\circ$. On the rough plane the downhill gravitational component is reduced by uphill friction, giving

$$a_r = g(\sin 45^\circ - \mu \cos 45^\circ).$$

Substitute the ratio of accelerations:

$$\sin 45^\circ - \mu \cos 45^\circ = \frac{1}{4} \sin 45^\circ.$$

Hence

$$\mu \cos 45^\circ = \frac{3}{4} \sin 45^\circ, \quad \mu = \frac{3}{4} \tan 45^\circ = 0.75.$$

This is below $\tan 45^\circ = 1$, as required for positive downhill acceleration. Since $\mu = \alpha/100$, multiplying by 100 gives the requested number.

Step 3: Conclusion The coefficient of friction is $\mu = 0.75$, so $\boxed{\alpha = 75}$. Both μ and α are dimensionless.

Quick Tip to Solve: For motion from rest over the same distance, compare at^2 , not at . At 45° , the rough-to-smooth acceleration ratio simplifies directly to $1 - \mu$.

47. The de Broglie wavelength for an electron accelerated through the potential difference of V_1 volt is λ_1 . When the potential difference is changed to V_2 volt, the associated de Broglie wavelength is increased by 50%. If $(V_1/V_2) = (9/\alpha)$, then the value of α is ____. [4 marks]

Solution:

Concept:

- An electron accelerated from rest through potential difference V gains kinetic energy $eV = p^2/(2m_e)$ in the usual nonrelativistic treatment.
- The de Broglie wavelength is $\lambda = h/p = h/\sqrt{2m_e eV}$, so $\lambda \propto V^{-1/2}$ for the same particle.

Step 1: Translate the wavelength increase into a ratio A 50 percent increase means that the new wavelength is the original wavelength plus half of it:

$$\lambda_2 = \lambda_1 + 0.50\lambda_1 = 1.50\lambda_1 = \frac{3}{2}\lambda_1.$$

Thus $\lambda_2/\lambda_1 = 3/2$. This is a ratio of the final value to the initial value, rather than just the fractional increase $1/2$. A longer matter wavelength means a smaller momentum and therefore a lower accelerating potential.

Step 2: Square the ratio and identify the parameter Writing the wavelength formula at both voltages and cancelling h , m_e and e gives

$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{V_1}{V_2}}.$$

Consequently

$$\frac{V_1}{V_2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}.$$

Compare this result with the specified expression $V_1/V_2 = 9/\alpha$:

$$\frac{9}{\alpha} = \frac{9}{4} \Rightarrow \alpha = 4.$$

As a consistency check, $V_2/V_1 = 4/9 < 1$, which agrees with the increase of wavelength. No numerical value of either voltage is needed because the particle constants cancel from the ratio.

Step 3: Conclusion The required dimensionless value is $\boxed{\alpha = 4}$.

Quick Tip to Solve: For a wavelength change in an electron-acceleration problem, square the wavelength ratio and reverse the voltage ratio: increasing wavelength by a factor $3/2$ reduces voltage to $4/9$.

48. A moving coil of galvanometer when shunted with 2Ω resistance gives a full scale deflection for a current of 500 mA . When a resistance of 470Ω is connected in series it gives a full scale deflection for 10 V potential applied on it. The value of resistance of galvanometer coil is _____ Ω . [4 marks]

Solution:

Concept:

- Let G be the galvanometer coil resistance and I_g its full-scale current. The same coil needs the same I_g for full-scale deflection in both arrangements.
- A shunt is connected in parallel with the coil and shares its potential difference. A series resistance carries the coil current and increases the voltage range.

Step 1: Write the shunted-current relation The total full-scale current in the shunted arrangement is $I = 500 \text{ mA} = 0.500 \text{ A}$. The coil carries I_g , leaving $0.500 - I_g$ through the shunt of resistance $S = 2\Omega$. Equal voltages across the parallel branches give

$$I_g G = (0.500 - I_g)2.$$

On rearranging,

$$I_g(G + 2) = 1, \quad I_g = \frac{1}{G + 2}.$$

The numerical relation uses amperes and ohms; the right-hand side before rearrangement is $0.500 \times 2 = 1 \text{ V}$.

Step 2: Use the series-voltmeter relation and eliminate current With 470Ω in series, an applied potential of 10 V produces full-scale current:

$$10 = I_g(G + 470).$$

Eliminate I_g using the previous relation:

$$10 = \frac{G + 470}{G + 2}, \quad 10G + 20 = G + 470.$$

Therefore

$$9G = 450, \quad G = 50 \, \Omega.$$

Checking, the full-scale coil current is $1/52$ A. The shunt current is $25/52$ A, giving total $26/52 = 0.500$ A; the series arrangement gives voltage $(1/52)(520) = 10$ V, as required.

Step 3: Conclusion The resistance of the galvanometer coil is $50 \, \Omega$. The numerical answer to enter is 50.

Quick Tip to Solve: Keep the coil full-scale current identical in the two configurations. Write one parallel-voltage equation and one series-voltage equation, then divide to eliminate that current.

49. Two cells of emfs 1 V and 2 V and internal resistance 2Ω and 1Ω , respectively connected in parallel, gave a current of 1 A through an external resistance. If the polarity of one cell is reversed, then value of current through the external resistance will be $\frac{\alpha}{5}$ A. The value of α is ____. [4 marks]

Solution:

Concept:

- Cells connected in parallel can be replaced by a single equivalent emf and internal resistance. For cells of emfs E_1, E_2 and internal resistances r_1, r_2 , use $E_{eq} = (E_1/r_1 + E_2/r_2)/(1/r_1 + 1/r_2)$.
- The parallel internal resistance is $r_{eq} = r_1 r_2 / (r_1 + r_2)$. Reversing a cell changes its signed emf, while its internal resistance remains unchanged.

Step 1: Find the initial equivalent cell The two sources initially support the same terminal polarity. Thus

$$E_{eq} = \frac{1/2 + 2/1}{1/2 + 1} = \frac{5}{3} \text{ V}, \quad r_{eq} = \frac{2 \times 1}{2 + 1} = \frac{2}{3} \, \Omega.$$

These are the open-circuit voltage and the resistance seen by the external resistor. They include both internal branches, so the emfs must not simply be added as for series cells.

Step 2: Determine the external resistance With the measured external current $I = 1$ A, the complete equivalent circuit gives

$$1 = \frac{5/3}{R + 2/3} \Rightarrow R = 1 \, \Omega.$$

This resistance stays fixed in the second arrangement. The total effective resistance is therefore $R + r_{eq} = 5/3 \, \Omega$.

Step 3: Reverse one emf and recalculate The magnitude of the new emf is independent of which cell is reversed:

$$|E'_{eq}| = \frac{|2/1 - 1/2|}{1 + 1/2} = 1 \text{ V}.$$

Reversing the other cell merely reverses the direction of the equivalent emf and the external current.

Its magnitude is

$$|I'| = \frac{1}{1 + 2/3} = \frac{3}{5} \text{ A.}$$

Comparing this with the specified $\alpha/5 \text{ A}$ determines the required dimensionless number.

Step 4: Conclusion The new current magnitude is $3/5 \text{ A}$, so $\alpha = 3$.

Quick Tip to Solve: For parallel cells, average emfs with conductances as weights; reverse only the sign of the reversed source, never its internal resistance.

50. A concave mirror of focal length 10 cm forms an image which is double the size of object when the object is placed at two different positions. The distance between the two positions of the object is ____ cm. [4 marks]

Solution:

Concept:

- The mirror formula is $1/f = 1/v + 1/u$, and the signed linear magnification is $m = -v/u$. Distances are measured using the Cartesian sign convention.
- For a concave mirror facing the object, $f = -10 \text{ cm}$. Twice the image size allows two cases: an inverted real image with $m = -2$ and an erect virtual image with $m = +2$.

Step 1: Locate the object for the real image An inverted image of twice the size has magnification $m = -2$, so $v = 2u$. Substituting into the mirror formula yields

$$-\frac{1}{10} = \frac{1}{2u} + \frac{1}{u} = \frac{3}{2u}.$$

Hence $u = -15 \text{ cm}$, and $v = -30 \text{ cm}$. The object is between the focus and the centre of curvature, exactly the region in which a concave mirror forms a magnified real image.

Step 2: Locate the object for the virtual image An erect image of twice the size has $m = +2$, so $v = -2u$. The same mirror formula now gives

$$-\frac{1}{10} = -\frac{1}{2u} + \frac{1}{u} = \frac{1}{2u}.$$

Thus $u = -5 \text{ cm}$, with $v = +10 \text{ cm}$. This object lies between the pole and the focus, and the positive image distance correctly indicates a virtual image behind the mirror.

Step 3: Calculate the separation of the positions Both objects are on the same side of the mirror, at distances 15 cm and 5 cm from its pole. Therefore subtract their positions rather than adding their distances:

$$\Delta x = |(-15) - (-5)| = 10 \text{ cm}.$$

Both pairs of distances satisfy the original mirror equation and give magnification magnitude 2, confirming that the two possible signs of magnification have been included.

Step 4: Conclusion The distance between the object positions is 10 cm ; the numerical entry is 10.

Quick Tip to Solve: When only image size is specified for a concave mirror, solve separately for positive and negative magnification before taking the object-position difference.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Which of the following contain the same number of atoms?

(Given : Molar mass in g mol^{-1} of H, He, O and S are 1, 4, 16 and 32 respectively)

- A. 2 g of O_2 gas
- B. 4 g of SO_2 gas
- C. 1400 mL of O_2 at STP
- D. 0.05 L of He at STP
- E. 0.0625 mol of H_2 gas

Choose the correct answer from the options given below :

[4 marks]

- (A) A and B only
- (B) B and C only
- (C) C and D only
- (D) A, C and E only

Correct Answer: (D) A, C and E only

Solution:

Concept:

- The number of atoms is the number of moles of molecules multiplied by the atoms per molecule and Avogadro's constant N_A . Equal masses or equal molecule counts alone need not give equal atom counts.
- Use $n = m/M$ for a mass sample and $n = V/(22.4 \text{ L mol}^{-1})$ for the ideal-gas samples at STP. Here $M(\text{O}_2) = 32$ and $M(\text{SO}_2) = 64 \text{ g mol}^{-1}$.

Step 1: Convert samples A and B into atom amounts Sample A contains $2/32 = 0.0625$ mol of O_2 molecules. Since each molecule has two atoms, its atom amount is

$$n_{\text{atoms}}(A) = 2(0.0625) = 0.125 \text{ mol.}$$

Sample B contains $4/64 = 0.0625$ mol of SO_2 molecules, each containing three atoms. Consequently

$$n_{\text{atoms}}(B) = 3(0.0625) = 0.1875 \text{ mol.}$$

A and B have equal numbers of molecules but unequal numbers of atoms.

Step 2: Evaluate the two volume samples For C, convert 1400 mL into 1.400 L before using the molar volume:

$$n_{\text{atoms}}(C) = 2 \frac{1.400}{22.4} = 0.125 \text{ mol.}$$

Helium in D is monatomic, so no factor of two is used:

$$n_{\text{atoms}}(D) = \frac{0.05}{22.4} = 0.002232 \dots \text{ mol.}$$

Thus C matches A, while D does not.

Step 3: Check E and select the complete set Hydrogen is diatomic. Therefore

$$n_{\text{atoms}}(E) = 2(0.0625) = 0.125 \text{ mol.}$$

A, C and E each contain $0.125N_A$ atoms. Option (A) incorrectly includes B; option (B) pairs unequal amounts B and C; option (C) incorrectly includes D. Only the final option lists all three equal atom counts and excludes both unequal ones.

Step 4: Conclusion A, C and E contain the same number of atoms, namely $0.125N_A$ each. Hence, the correct option is (D).

Quick Tip to Solve: Convert each sample to moles of atoms, not merely moles of molecules; the factor is 2 for oxygen and hydrogen but 3 for sulphur dioxide.

52. The Bohr radius of a hydrogen like species is 70.53 pm . The species and the stationary state (n) are respectively

(Given : Hydrogen atom Bohr radius is 52.9 pm)

[4 marks]

- (A) Li^{2+} , 3
- (B) He^+ , 3
- (C) He^+ , 2
- (D) Li^{2+} , 2

Correct Answer: (D) Li^{2+} , 2

Solution:

Concept:

- A hydrogen-like species contains one electron. In the Bohr model its orbit radius is $r_n = a_0 n^2 / Z$, where Z is the nuclear charge, n is the principal quantum number and $a_0 = 52.9 \text{ pm}$.
- Ion charge is not the value of Z : for He^+ , $Z = 2$, and for Li^{2+} , $Z = 3$. Both ions have one electron, so the same radius formula applies.

Step 1: Compute the required quantum-number ratio Divide the given radius by the hydrogen ground-state Bohr radius:

$$\frac{n^2}{Z} = \frac{70.53}{52.9} = 1.33327 \dots \simeq \frac{4}{3}.$$

The small difference from $4/3$ comes from the radius being printed to two decimal places. We must find the listed pair whose n^2/Z equals this ratio; the radius itself is already in pm, so no unit conversion is necessary.

Step 2: Test the lithium alternatives For Li^{2+} the nuclear charge is $Z = 3$. With $n = 3$, option (A) gives $n^2/Z = 9/3 = 3$ and radius 158.7 pm, too large. With $n = 2$, option (D) gives

$$r_2 = 52.9 \frac{2^2}{3} = 70.5333 \dots \text{ pm.}$$

This rounds to the specified 70.53 pm.

Step 3: Exclude the helium alternatives For He^+ , option (B) has $n = 3$, giving

$$r_3 = 52.9 \frac{9}{2} = 238.05 \text{ pm.}$$

Option (C) has $n = 2$, giving

$$r_2 = 52.9 \frac{4}{2} = 105.8 \text{ pm.}$$

Neither agrees with the measured radius. The increase with n^2 and decrease with Z also provide a useful physical check on these comparisons.

Step 4: Conclusion The species is Li^{2+} , and the stationary state is $n = 2$. Hence, the correct option is (D).

Quick Tip to Solve: First calculate the dimensionless ratio radius divided by 52.9 pm; then match it directly to n squared divided by nuclear charge.

53. Given below are two statements :

Statement I : The number of compounds among $\text{SO}_2, \text{SO}_3, \text{SF}_4, \text{SF}_6$ and H_2S in which sulphur does not obey the Octet rule is 3 .

Statement II : Among $[\text{H}_2\text{O}, \text{ClF}_3, \text{SF}_4], [\text{NH}_3, \text{BrF}_5, \text{SF}_4], [\text{BrF}_5, \text{ClF}_3, \text{XeF}_4]$ and $[\text{XeF}_4, \text{ClF}_3, \text{H}_2\text{O}]$, the number of sets in which all the molecules have one lone pair of electrons on the central atom is 1 .

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Both Statement I and Statement II are true
- (B) Both Statement I and Statement II are false
- (C) Statement I is true but Statement II is false
- (D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Concept:

- Count all bonding electrons and lone-pair electrons around the central atom in the usual Lewis structures. An octet has eight surrounding electrons; expanded-valence structures contain more than eight.
- For the central-atom lone-pair count, use the valence electron count together with the number of bonds. A lone pair is a pair of nonbonding electrons, not an unpaired electron or a bonding pair.

Step 1: Evaluate the sulphur compounds in Statement I In the usual expanded-valence Lewis descriptions, SO_2 has two $\text{S}=\text{O}$ bonds and one lone pair on sulphur, giving ten surrounding electrons. SO_3 has three double bonds, giving twelve. SF_4 has four single bonds and one lone pair, giving ten. SF_6 has six single bonds, giving twelve. In contrast, H_2S has two single bonds and two lone pairs, giving eight electrons. Thus four compounds, not three, exceed the octet, and Statement I is false.

Step 2: Tabulate the central-atom lone-pair counts The counts required by Statement II are

Molecule	H ₂ O	ClF ₃	SF ₄	NH ₃	BrF ₅	XeF ₄
Lone pairs	2	2	1	1	1	2

For example, the six valence electrons of sulphur leave two nonbonding electrons after four single bonds in SF₄; bromine has one lone pair after five single bonds in BrF₅.

Step 3: Inspect each proposed set The first set has counts (2, 2, 1), so it fails the requirement. The second set, [NH₃, BrF₅, SF₄], has counts (1, 1, 1), so it qualifies. The third set has (1, 2, 2), and the fourth has (2, 2, 2); neither qualifies. Exactly one set satisfies the condition, making Statement II true. Options (A) and (C) wrongly call Statement I true, and option (B) wrongly calls Statement II false.

Step 4: Conclusion Statement I is false, while Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: For a set-based lone-pair question, write each molecule's count once and reuse it; one molecule with the wrong count eliminates the entire set.

54. Match List - I with List - II.

Given V_1 and V_2 are initial and final volumes respectively.

List I: Isothermal process		List II: Expression	
A.	Reversible expansion	I.	$q = 0$
B.	Free expansion	II.	$q = nRT \ln \frac{V_2}{V_1}$
C.	Irreversible Compression	III.	$w = -p_{\text{ext}} (V_1 - V_2)$
D.	Cyclic reversible	IV.	$\frac{q_{\text{rev}}}{T} = 0$

Choose the correct answer from the options given below :

[4 marks]

- (A) A-II, B-III, C-I, D-IV
 (B) A-II, B-I, C-IV, D-III
 (C) A-II, B-I, C-III, D-IV
 (D) A-I, B-II, C-III, D-IV

Correct Answer: (C) A-II, B-I, C-III, D-IV

Solution:

Concept:

- For an ideal gas in an isothermal process, internal energy is unchanged. With the chemistry convention $\Delta U = q + w$, the heat is the negative of the work done on the gas.
- Reversible isothermal expansion gives $q = nRT \ln(V_2/V_1)$. Free expansion into a vacuum has zero pressure-volume work; for an ideal gas at unchanged temperature, its heat is also zero.

Step 1: Match the two expansions In a reversible expansion the external pressure follows the gas pressure, and integrating $p = nRT/V$ gives

$$w = -nRT \ln \frac{V_2}{V_1}, \quad q = nRT \ln \frac{V_2}{V_1}.$$

Therefore A matches II. During free expansion $p_{\text{ext}} = 0$, so $w = 0$. Since $\Delta U = 0$, the first law gives $q = 0$, and B matches I. This eliminates option (D), which reverses the two expansion matches.

Step 2: Interpret the compression entry with its printed sign Constant-external-pressure irreversible compression is the process associated with the pressure-volume expression in III. A sign qualification is necessary: as V_1 is initial and V_2 final, compression has $V_1 > V_2$. The chemistry convention gives

$$w_{\text{on}} = -p_{\text{ext}}(V_2 - V_1) > 0.$$

The printed expression $-p_{\text{ext}}(V_1 - V_2)$ is instead the negative quantity, namely work done by the gas. Thus III identifies C when interpreted as work by the gas; it should not be used as positive compression work on the gas.

Step 3: Complete the cyclic match For a reversible cycle the net entropy change is zero, so $\oint \delta q_{\text{rev}}/T = 0$. In an isothermal reversible cycle, the common temperature also permits this to be written as net $q_{\text{rev}}/T = 0$, the meaning of IV. Hence D matches IV. Combining all matches gives A-II, B-I, C-III and D-IV. Option (A) incorrectly assigns zero heat to compression, while option (B) assigns the cyclic entropy condition to compression and the pressure-volume expression to the cycle.

Step 4: Conclusion The process matching, with the work-by-gas sign interpretation of printed III, is A-II, B-I, C-III, D-IV. Hence, the correct option is (C).

Quick Tip to Solve: For compression, first write which convention defines work: work on the gas is positive, while work by the gas is negative. Reversible cycles use a closed-path entropy integral.

55. Given below are two statements :

Statement I : H_2O molecules move from the chamber 1 to chamber 2 .

Statement II : The osmotic pressure of a solution prepared by dissolving 50 mg of potassium sulphate (molar mass = 174 g/mol) in 2 L of water (at 27°C) is 0.0107 bar. (Given: $R = 0.083\text{dm}^3 \text{ bar K}^{-1} \text{ mol}^{-1}$ and assume complete dissociation of electrolyte)

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

Chamber 1		Chamber 2
18 g glucose in 100 mL aqueous solution	Semi-permeable membrane	30 g glucose in 250 mL aqueous solution

- (A) Both Statement I and Statement II are true
- (B) Both Statement I and Statement II are false
- (C) Statement I is true but Statement II is false
- (D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Concept:

- Osmosis carries solvent through a semipermeable membrane from lower solute concentration to higher solute concentration at equal external pressure. The dissolved glucose cannot cross the membrane.
- For a dilute electrolyte solution, $\pi = icRT$, where i is the number of solute particles per formula unit, c is molarity, and T is absolute temperature.

Step 1: Compare the two glucose concentrations The molar mass of glucose is 180 g mol^{-1} . The diagram gives solution volumes, so use them directly:

$$c_1 = \frac{18/180}{0.100} = 1.00 \text{ mol L}^{-1}, \quad c_2 = \frac{30/180}{0.250} = \frac{2}{3} \text{ mol L}^{-1}.$$

Chamber 2 is more dilute. Water therefore moves from chamber 2 to chamber 1, opposite to Statement I. Statement I is false. Comparing masses alone would give the wrong direction because the solution volumes differ.

Step 2: Calculate the electrolyte osmotic pressure Complete dissociation gives $\text{K}_2\text{SO}_4 \longrightarrow 2\text{K}^+ + \text{SO}_4^{2-}$, hence $i = 3$. Convert 50 mg to 0.050 g and 27°C to 300 K. Taking the dilute solution volume as 2 L,

$$\begin{aligned} c &= \frac{0.050}{174 \times 2} \text{ mol L}^{-1}, \\ \pi &= 3 \left(\frac{0.050}{348} \right) (0.083)(300) \\ &= 0.01073 \text{ bar} \simeq 0.0107 \text{ bar}. \end{aligned}$$

Statement II is true. Options (A) and (C) incorrectly accept Statement I; option (B) incorrectly rejects Statement II.

Step 3: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: Compare moles per litre on both sides before deciding the osmotic direction; then include all three ions when calculating the pressure of potassium sulphate.

56. Given is a concentrated solution of a weak electrolyte A_xB_y of concentration ' c ' and dissociation constant ' K '. The degree of dissociation is given by : [4 marks]

(A) $[K \times c^{x+y-1} x^x y^y]^{x+y}$

(B) $\left(\frac{K}{c^{x+y-1} x^x y^y}\right)^{\frac{1}{x+y}}$

(C) $\left(\frac{c^{x+y-1} x^x y^y}{K}\right)^{x+y}$

(D) $\left(\frac{c^{x+y-1} x^x y^y}{K}\right)^{\frac{1}{x+y}}$

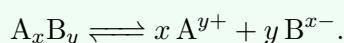
Correct Answer: (B) $\left(\frac{K}{c^{x+y-1} x^x y^y}\right)^{\frac{1}{x+y}}$

Solution:

Concept:

- The degree of dissociation α is the fraction of initial electrolyte formula units that dissociate. Stoichiometric coefficients multiply the ion concentrations in the equilibrium expression.
- For a concentrated solution of a weak electrolyte, $\alpha \ll 1$, so the concentration of undissociated electrolyte is $c(1 - \alpha) \simeq c$. This is the approximation used in Ostwald's dilution law.

Step 1: Write the equilibrium concentrations Let the electrolyte dissociate as



Starting with concentration c , the undissociated concentration is $c(1 - \alpha)$. The two ion concentrations are $xc\alpha$ and $yc\alpha$, respectively. Their powers in the dissociation constant are x and y , not unity. Thus,

$$K = \frac{(xc\alpha)^x (yc\alpha)^y}{c(1 - \alpha)} = \frac{x^x y^y c^{x+y-1} \alpha^{x+y}}{1 - \alpha}.$$

Step 2: Apply weak dissociation and isolate the fraction Use $1 - \alpha \simeq 1$ for the weak electrolyte in a concentrated solution:

$$\alpha^{x+y} = \frac{K}{c^{x+y-1} x^x y^y}.$$

Taking the positive $(x + y)$ -th root gives the physical value of the dissociation fraction. The dependence is sensible: increasing K increases dissociation, whereas increasing c suppresses it. Option (A) multiplies by the concentration factor and has the wrong exponent. Option (C) both inverts the required fraction and uses the wrong exponent. Option (D) takes the root of the reciprocal, predicting the opposite dependence on K . For $x = y = 1$, the result reduces to the familiar $\alpha = \sqrt{K/c}$.

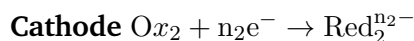
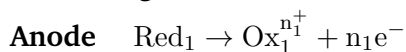
Step 3: Conclusion

$$\alpha = \left(\frac{K}{c^{x+y-1} x^x y^y}\right)^{1/(x+y)}.$$

Hence, the correct option is (B).

Quick Tip to Solve: Write each ionic concentration with its stoichiometric coefficient before forming the equilibrium constant; check the result using the special case of a 1:1 electrolyte.

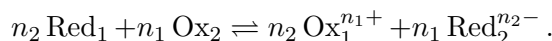
57. For a general redox reaction



Which of the following statement is incorrect?

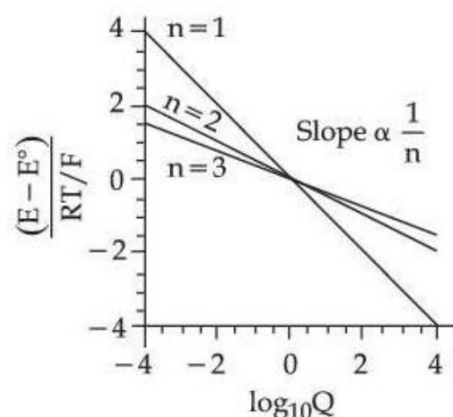
[4 marks]

(A) The overall reaction can be written as



(B) The electrons do not appear in the overall reaction because electrons produced at the anode are consumed at the cathode.

(C) Here n is the number of electrons transferred in redox reaction.



(D) If the reaction is carried out reversibly, the electrical work done is equal to the ratio of charge and potential difference through which charge is moved.

Correct Answer: (D) If the reaction is carried out reversibly, the electrical work done is equal to the ratio of charge and potential difference through which charge is moved.

Solution:

Concept:

- An overall redox equation conserves electrons: multiply the oxidation and reduction half-reactions until they transfer equal numbers of electrons, then add them.
- The Nernst equation is $E = E^\circ - (RT/nF) \ln Q$. Reversible electrical work has magnitude $w = qE$, since potential difference is energy transferred per unit charge.

Step 1: Check electron balance and the graph Multiplying the anodic half-reaction by n_2 produces $n_1 n_2$ electrons; multiplying the cathodic half-reaction by n_1 consumes the same number. On addition, the electrons cancel and the coefficients are exactly those in option (A). Option (B) correctly explains why no free electrons remain in the overall reaction.

For the graph in option (C), divide the Nernst equation by RT/F :

$$\frac{E - E^\circ}{RT/F} = -\frac{\ln Q}{n} = -\frac{2.303}{n} \log_{10} Q.$$

The lines therefore pass through the origin, have negative slopes, and become less steep as the number of transferred electrons n increases. The proportionality of slope magnitude to $1/n$ and the identification of n in option (C) are correct; the sketch conveys these relative slopes.

Step 2: Test the statement about reversible work The definition of potential difference is

$$E = \frac{w}{q} \implies w = qE.$$

For one mole of a cell reaction transferring n moles of electrons, $q = nF$, so the maximum electrical work delivered has magnitude nFE , while the system free-energy change is $\Delta G = -nFE$. A ratio of charge to potential difference is not work: its units are C/V, whereas work requires C V = J. Option (D) incorrectly says ratio instead of product. Because (A), (B) and (C) are valid, they cannot answer the request for the incorrect statement.

Step 3: Conclusion Electrical work is the product of charge and potential difference, not their ratio. Hence, the correct option is (D).

Quick Tip to Solve: Use units to check work statements immediately: coulomb times volt is joule. For Nernst plots, increasing the transferred-electron count reduces the slope magnitude.

58. In a period, the first ionisation enthalpy of the element at extreme left and the negative electron gain enthalpy of the extreme right element, except noble gases, are respectively. [4 marks]

- (A) lowest and lowest
- (B) highest and lowest
- (C) lowest and highest
- (D) highest and highest

Correct Answer: (C) lowest and highest

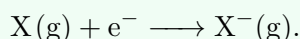
Solution:

Concept:

- First ionisation enthalpy is the energy required to remove the outermost electron from an isolated gaseous atom. A loosely held valence electron gives a small positive ionisation enthalpy.
- Electron gain enthalpy is negative when electron attachment releases energy. Across a period, the halogen near the right-hand end has a strong tendency to complete its valence shell.

Step 1: Identify the left-hand element For the usual alkali-metal-to-halogen span of a period, the extreme-left element has one valence electron outside a closed shell. Losing that electron gives a stable cation. Its comparatively large atomic size and weaker attraction for the outer electron make removal easier than for the elements to its right. Increasing effective nuclear charge across the period generally holds valence electrons more strongly. Thus the first ionisation enthalpy at the left-hand end is the lowest in the period, ruling out options (B) and (D), which call it highest.

Step 2: Interpret the negative electron gain enthalpy at the right With noble gases excluded, the extreme-right element is a halogen, with a valence configuration ns^2np^5 . Gaining one electron completes its valence shell:



This electron attachment is strongly exothermic. The phrase negative electron gain enthalpy refers

here to its large energy-release magnitude: the halogen has the highest tendency to gain an electron, or the most strongly negative electron gain enthalpy. The sign convention must not be confused with a small numerical magnitude. Alkali metals have a much weaker electron-accepting tendency. Option (A) incorrectly assigns the lowest negative electron gain tendency to the right-hand halogen; option (C) combines the correct left-hand minimum with the correct right-hand maximum.

Step 3: Conclusion The required trends are lowest first ionisation enthalpy and highest negative electron gain enthalpy (largest energy-release tendency), respectively. Hence, the correct option is (C).

Quick Tip to Solve: Locate the groups first: the left end loses one electron readily, while the halogen end gains one readily. Distinguish a negative enthalpy value from the magnitude of energy released.

59. Given below are two statements :

Statement I : $F_2O < H_2O < Cl_2O$ is the correct trend in terms of bond angle.

Statement II : SiF_4 , SnF_4 and PbF_4 are ionic in nature.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Both Statement I and Statement II are true
- (B) Both Statement I and Statement II are false
- (C) Statement I is true but Statement II is false
- (D) Statement I is false but Statement II is true

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- In a bent molecule with two oxygen lone pairs, bond angle depends on the distribution of bonding electron density as well as repulsions involving the terminal atoms. Electron-pair geometry alone does not fix one identical angle for every molecule.
- A claim that several compounds are all ionic is false if even one member is molecular and covalent. Silicon tetrafluoride has directional covalent bonds in a tetrahedral molecule.

Step 1: Compare the three oxygen-centred angles In OF_2 , very electronegative fluorine draws bonding electron density away from the central oxygen. This reduces the repulsion exerted by the bonding regions near oxygen and allows a relatively small $F-O-F$ angle, about 103° . In water the $H-O-H$ angle is about 104.5° . In Cl_2O , the larger chlorine substituents and the bonding-electron distribution give a larger $Cl-O-Cl$ angle, about 111° . Consequently,

$$\angle F-O-F < \angle H-O-H < \angle Cl-O-Cl.$$

All three have oxygen as their central atom; identical counts of bonds and lone pairs do not require identical angles. Statement I is true.

Step 2: Test the universal ionic assertion SiF_4 is a covalent molecular compound whose silicon atom

forms four bonds directed tetrahedrally toward fluorine. Polar bonds are not sufficient evidence for an ionic lattice: bond polarity and complete ionic character are different ideas. Hence the assertion that SiF_4 , SnF_4 and PbF_4 are all ionic already fails for SiF_4 , regardless of the greater ionic character possible for heavier fluorides. Statement II is false.

Options (A) and (D) wrongly accept Statement II. Option (B) wrongly rejects the observed bond-angle order in Statement I. Only option (C) assigns the correct truth values to both statements.

Step 3: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: For a list claimed to be entirely ionic, look first for a clearly molecular member. In bond-angle comparisons, track central bonding density and terminal-atom repulsion as well as lone-pair count.

60. The correct order of first ($\Delta_i H_1$) and second ($\Delta_i H_2$) ionisation enthalpy values of Cr and Mn are :

- A. $\Delta_i H_1 : \text{Cr} > \text{Mn}$
- B. $\Delta_i H_2 : \text{Cr} > \text{Mn}$
- C. $\Delta_i H_1 : \text{Mn} > \text{Cr}$
- D. $\Delta_i H_2 : \text{Mn} > \text{Cr}$

Choose the correct answer from the options given below :

[4 marks]

- (A) A and B only
- (B) B and C only
- (C) A and D only
- (D) C and D only

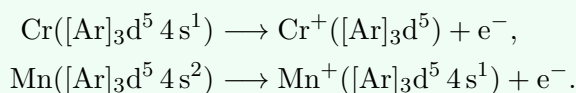
Correct Answer: (B) B and C only

Solution:

Concept:

- Chromium and manganese have ground-state configurations $\text{Cr} : [\text{Ar}]3d^5 4s^1$ and $\text{Mn} : [\text{Ar}]3d^5 4s^2$. During ionisation of these atoms, the $4s$ electrons are removed before the $3d$ electrons.
- A half-filled $3d^5$ subshell has extra stability associated with its symmetrical occupancy and exchange interactions. Compare the configurations before and after each separate electron removal.

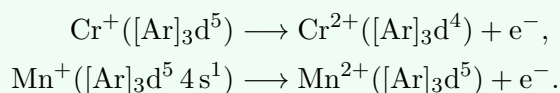
Step 1: Compare the first ionisation enthalpies The first removal processes are



Chromium readily loses its sole $4s$ electron to form the stable half-filled $3d^5$ cation. Manganese has greater nuclear charge and its first ionisation enthalpy is higher. Therefore $\Delta_i H_1(\text{Mn}) > \Delta_i H_1(\text{Cr})$: statement C is true and A is false.

Step 2: Compare the second ionisation enthalpies The second removal starts with the singly charged

ions, not with the neutral atoms:



Chromium must lose a $3d$ electron and disrupt its half-filled subshell, while manganese loses its remaining $4s$ electron and obtains a half-filled $3d^5$ ion. Thus $\Delta_i H_2(\text{Cr}) > \Delta_i H_2(\text{Mn})$: B is true and D is false. Options (A) and (C) include the false statement A, and option (D) includes the false statement D. Only B and C survive both comparisons.

Step 3: Conclusion The first ionisation enthalpy is higher for Mn, whereas the second is higher for Cr. Statements B and C are correct. Hence, the correct option is (B).

Quick Tip to Solve: Write the singly charged ion before comparing second ionisations. Removing an electron from stable $3d^5$ is costly; reaching $3d^5$ by losing a $4s$ electron is relatively favourable.

61. Which of the following sequences of hybridisation, geometry and magnetic nature are correct for the given coordination compounds?

- A. $[\text{NiCl}_4]^{2-}$ – sp^3 , tetrahedral, paramagnetic
B. $[\text{Ni}(\text{NH}_3)_6]^{2+}$ – $sp^3 d^2$, octahedral, paramagnetic
C. $[\text{Ni}(\text{CO})_4]$ – sp^3 , tetrahedral, paramagnetic
D. $[\text{Ni}(\text{CN})_4]^{2-}$ – dsp^2 , square planar, diamagnetic

Choose the correct answer from the options given below :

[4 marks]

- (A) A, B, C and D
(B) B, C and D only
(C) A, C and D only
(D) A, B and D only

Correct Answer: (D) A, B and D only

Solution:

Concept:

- Find the metal oxidation state before counting d electrons. Nickel has atomic number 28; neutral nickel forms a filled $3d^{10}$ shell in nickel tetracarbonyl, whereas Ni^{2+} is $3d^8$.
- Unpaired electrons cause paramagnetism. In the valence-bond description, four-coordinate nickel complexes can be tetrahedral sp^3 or square planar dsp^2 , depending on the ligand and electron pairing.

Step 1: Check A and B In $[\text{NiCl}_4]^{2-}$, four chloride ions contribute charge -4 , so nickel is $+2$. Chloride is weak field: the tetrahedral d^8 complex has two unpaired electrons. Thus its sp^3 , tetrahedral and paramagnetic description is correct. In $[\text{Ni}(\text{NH}_3)_6]^{2+}$, neutral ammonia leaves nickel as Ni^{2+} . Octahedral d^8 has configuration $t_{2g}^6 e_g^2$, with two unpaired electrons. Two empty inner $3d$ orbitals are unavailable; the stated outer-orbital $sp^3 d^2$, octahedral, paramagnetic description is correct.

Step 2: Check C and D In $\text{Ni}(\text{CO})_4$, carbon monoxide is neutral and nickel has oxidation state zero. Electron rearrangement gives a filled $3d^{10}$ shell, allowing sp^3 bonding with tetrahedral geometry. All electrons are paired, so this complex is diamagnetic. Statement C is false because it says paramagnetic. Cyanide is a strong-field ligand: $[\text{Ni}(\text{CN})_4]^{2-}$ is square planar, with paired electrons and dsp^2 hybridisation. Therefore D is correct.

Step 3: Select the combination The accepted statements are A, B and D. Each of options (A), (B) and (C) includes statement C, so each incorrectly classifies nickel tetracarbonyl as paramagnetic. Option (D) alone excludes C while retaining all three correct descriptions.

Step 4: Conclusion A, B and D only are correct. Hence, the correct option is (D).

Quick Tip to Solve: For nickel complexes, distinguish d^8 nickel(II) from d^{10} nickel(0) before using the ligand to infer geometry and magnetism.

62. Given below are two statements :

Statement I : A mixture of $\text{C}_{12}\text{H}_{22}\text{O}_{11}$ (sugar) and NaCl can be separated by dissolving sugar in alcohol, due to differential solubility.

Statement II : Rose essence from rose petals is separated by steam distillation due to its high volatility and insolubility in H_2O .

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Both Statement I and Statement II are true
- (B) Both Statement I and Statement II are false
- (C) Statement I is true but Statement II is false
- (D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- Differential solubility separates a mixture using a solvent that dissolves one component preferentially. The dissolved component is recovered after the insoluble material is filtered off.
- Steam distillation is suitable for a steam-volatile organic substance that is effectively immiscible with water. For two immiscible liquids the total vapour pressure is the sum of their separate vapour pressures.

Step 1: Evaluate Statement I Sugar and sodium chloride differ in their solubility in alcohol. Alcohol can selectively dissolve sugar relative to sodium chloride, which remains effectively insoluble for this separation. Shake the mixture with the solvent and filter; the salt remains in the residue, while the sugar is in the filtrate. Recovering or evaporating the alcohol yields the sugar. This is the differential-solubility principle described in Statement I, so Statement I is true. Water would be a poor choice here because it dissolves both components.

Step 2: Evaluate Statement II The volatile, water-insoluble components that constitute rose essence

can be carried out of the petals by steam. Their vapour pressure adds to that of water, so the combined vapour pressure reaches atmospheric pressure before the oil would need to boil alone. The vapours are condensed and the essential-oil fraction separates from water. This allows recovery of the fragrance without heating the organic material to its own high boiling temperature. Thus the steam-distillation description in Statement II is true.

Step 3: Compare the answer choices Statement I uses selective dissolution, whereas Statement II uses co-distillation with steam; both are valid separation principles for the mixtures specified. Option (B) wrongly rejects both statements. Options (C) and (D) each reject one valid statement. Therefore only the choice accepting both statements fits.

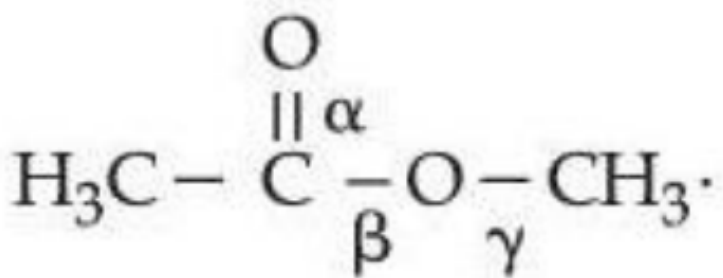
Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: Choose a selective solvent for two solids; choose steam distillation for a volatile essential oil that does not mix with water.

63. Shown below is the structure of methyl acetate with three different α , β and γ carbon - oxygen bonds.

The correct order of bond lengths of these bonds is :

[4 marks]



(A) $\alpha > \beta > \gamma$

(B) $\alpha < \beta < \gamma$

(C) $\alpha = \beta = \gamma$

(D) $\alpha < \beta = \gamma$

Correct Answer: (B) $\alpha < \beta < \gamma$

Solution:

Concept:

- Bond length generally decreases as bond order increases. A carbon-oxygen double bond is shorter than a carbon-oxygen single bond.
- In an ester, a lone pair on the singly bonded oxygen is conjugated with the carbonyl group. The acyl carbon-oxygen bond therefore has partial double-bond character, unlike the oxygen-alkyl bond.

Step 1: Identify the three bonds The labelled structure is methyl acetate, $\text{CH}_3\text{COOCH}_3$. The bond labelled α is the carbonyl $\text{C}=\text{O}$ bond. The bond labelled β joins the carbonyl carbon to the ester

oxygen. The bond labelled γ joins that oxygen to the methyl carbon. Although both β and γ are drawn as single bonds in the usual structural formula, they have different electronic environments.

Step 2: Apply ester resonance The non-carbonyl oxygen donates a lone pair into the carbonyl system. A charge-separated resonance contributor places a double bond between this oxygen and the acyl carbon, with negative charge on the original carbonyl oxygen and positive charge on the donating oxygen. Consequently β has partial double-bond character and is shorter than an ordinary oxygen-alkyl single bond. The γ bond leads to an sp^3 methyl carbon and does not acquire the same conjugative double-bond character. The predominantly double carbonyl bond α remains shortest.

Step 3: Order and eliminate the alternatives The carbonyl bond is shortest, the resonance-shortened acyl-oxygen bond is intermediate, and the oxygen-methyl single bond is longest. Thus $\alpha < \beta < \gamma$. Option (A) reverses the bond-order trend. Option (C) ignores the carbonyl double bond entirely. Option (D) recognises the shorter carbonyl bond but incorrectly treats the two formally single bonds as equivalent.

Step 4: Conclusion The bond-length order is $\alpha < \beta < \gamma$. Hence, the correct option is (B).

Quick Tip to Solve: In an ester, do not treat both C-O single bonds as equivalent: the bond adjoining the carbonyl is shortened by resonance.

64. 'x' is the product which is obtained by the hydrolysis of prop-1-yne in the presence of mercuric sulphate under dilute acidic medium at 333 K. 'y' is the product which is obtained by the reaction of ethane nitrile with methyl magnesium bromide in dry ether followed by hydrolysis. IUPAC name of product obtained from 'x' and 'y' in the presence of barium hydroxide followed by heating is : [4 marks]

- (A) 2 - Methylpent-4-en-3-one
- (B) 4-Methylpent-3-en-2-one
- (C) 4-Methylpent-1-ene
- (D) 2-Methylpent-3-one

Correct Answer: (B) 4-Methylpent-3-en-2-one

Solution:

Concept:

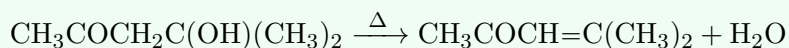
- Mercuric-ion-catalysed hydration of a terminal alkyne gives the Markovnikov enol, which tautomerises to a methyl ketone.
- A nitrile reacts with a Grignard reagent to give an imine magnesium salt; hydrolysis yields a ketone. Ketones having alpha hydrogen can undergo aldol addition followed by dehydration on heating.

Step 1: Identify product x Prop-1-yne is $\text{CH}_3\text{C}\equiv\text{CH}$. With HgSO_4 , dilute acid and water at 333 K, hydration forms $\text{CH}_3\text{C}(\text{OH})=\text{CH}_2$. Keto-enol tautomerism converts this enol into CH_3COCH_3 . Therefore x is propan-2-one (acetone), rather than an aldehyde. The carbonyl carbon is the internal carbon of the original triple bond.

Step 2: Identify product y Ethanenitrile is CH_3CN . Methylmagnesium bromide supplies a methyl

group to the nitrile carbon. The initial imine salt yields CH_3COCH_3 after hydrolysis. Thus y is also propan-2-one. Both starting routes have therefore led to the same three-carbon ketone; the subsequent step is acetone self-condensation.

Step 3: Condense and dehydrate acetone In barium hydroxide, an acetone enolate attacks the carbonyl carbon of another acetone molecule. Protonation gives diacetone alcohol, $\text{CH}_3\text{COCH}_2\text{C}(\text{OH})(\text{CH}_3)_2$. Heating removes water across the alpha and beta carbons to form the conjugated ketone:



The longest chain containing the carbonyl has five carbons. Numbering from the carbonyl end places the ketone at C2, the double bond at C3 and a methyl substituent at C4.

Step 4: Choose the IUPAC name The product is 4-methylpent-3-en-2-one, also called mesityl oxide. Option (A) places the carbonyl and double bond incorrectly. Option (C) lacks the retained ketone group. Option (D) lacks the double bond created by dehydration and gives the wrong carbonyl position.

Step 5: Conclusion The product is 4-methylpent-3-en-2-one. Hence, the correct option is (B).

Quick Tip to Solve: Identify each precursor first: alkyne hydration and nitrile-Grignard hydrolysis both give acetone here, so use acetone self-aldol condensation.

65. An optically active alkyl bromide $\text{C}_4\text{H}_9\text{Br}$, reacts with ethanolic KOH to form major compound [A] which reacts with bromine to give compound [B]. Compound [B] reacts with ethanolic KOH and sodamide to give compound [C]. One molecule of water adds to compound [C] on warming with mercuric sulphate and dilute sulphuric acid at 333 K to form compound [D]. The functional group in compound D will be confirmed by : [4 marks]

- (A) Haloform test
- (B) Lucas test
- (C) Silver mirror test
- (D) Benedict test

Correct Answer: (A) Haloform test

Solution:

Concept:

- An optically active alkyl bromide needs a stereogenic carbon bearing four different groups. Among the alkyl bromides with formula $\text{C}_4\text{H}_9\text{Br}$, this identifies 2-bromobutane.
- Alcoholic base promotes elimination; bromine adds across an alkene; successive dehydrohalogenations of a vicinal dibromide form an alkyne. Hydration of the alkyne gives a carbonyl compound after tautomerisation.

Step 1: Identify the alkene and dibromide The stereogenic carbon of 2-bromobutane is bonded to H, Br, methyl and ethyl groups. Ethanolic KOH eliminates HBr, giving the more substituted alkene but-2-ene as the major product A. Bromine adds across its double bond to give B, 2,3-dibromobutane,

$\text{CH}_3\text{CHBrCHBrCH}_3$. The carbon skeleton still has four carbon atoms.

Step 2: Form and hydrate the alkyne Treatment of the vicinal dibromide with ethanolic base and then the stronger base sodamide removes two molecules of HBr overall. Product C is but-2-yne, $\text{CH}_3\text{C}\equiv\text{CCH}_3$. Addition of one molecule of water in the presence of HgSO_4 and dilute H_2SO_4 forms an enol. Because the alkyne is symmetrical, either hydration orientation gives the same constitutional product. Tautomerisation gives D, butan-2-one, $\text{CH}_3\text{COCH}_2\text{CH}_3$.

Step 3: Identify the diagnostic test Butan-2-one is a methyl ketone: it contains the $\text{CH}_3\text{CO}-$ unit. It therefore gives a positive haloform test. For example, iodine in alkali converts it into yellow iodoform, CHI_3 , and propanoate. The relevant feature is the methyl group directly attached to the carbonyl carbon, not merely the presence of any ketone.

Step 4: Eliminate the remaining tests The Lucas test detects alcohols, so it is inappropriate for the ketone D. The silver mirror test detects aldehydes and certain reducing compounds, whereas this simple ketone is negative. Benedict reagent likewise does not give its characteristic reduction with butan-2-one. Thus neither (B), (C) nor (D) confirms the functional group present in the specified final product.

Step 5: Conclusion D is butan-2-one, a methyl ketone confirmed by the haloform test. Hence, the correct option is (A).

Quick Tip to Solve: For a reaction chain, retain the carbon skeleton and track the functional group: alkene, vicinal dibromide, alkyne, then methyl ketone.

66.

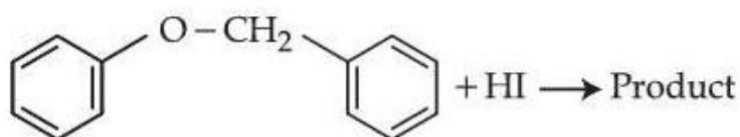
Consider the following reaction.

Statement I : In the above reaction, product formed will be a mixture of benzyl alcohol and iodobenzene.

Statement II : In the above reaction, the $-\text{O}-\text{CH}_2-$ bond is cleaved to give the product.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]



- (A) Both Statement I and Statement II are true
- (B) Both Statement I and Statement II are false
- (C) Statement I is true but Statement II is false
- (D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Concept:

- An ether first undergoes protonation at oxygen on treatment with hydrogen iodide. Iodide then cleaves an accessible carbon–oxygen bond.
- In an aryl–alkyl ether the aryl carbon–oxygen bond has partial double-bond character through resonance, and the aryl carbon is sp^2 -hybridised. Cleavage preferentially occurs on the alkyl side.

Step 1: Identify the two sides of the ether The reactant is benzyl phenyl ether, $C_6H_5OCH_2C_6H_5$. One oxygen bond leads directly to an aromatic carbon; the other leads to the benzylic CH_2 carbon. These are chemically different bonds even though the molecular structure contains a benzene ring on each side of oxygen. The benzylic carbon is an alkyl carbon, not part of its aromatic ring.

Step 2: Determine which bond breaks Protonation makes the ether oxygen a better leaving group. Iodide reacts at the benzylic carbon, and cleavage breaks the $O-CH_2$ bond. The phenyl group retains oxygen as phenol while the benzyl fragment forms benzyl iodide:



Attack by a usual backside substitution at an aromatic sp^2 carbon is not feasible. The aryl–oxygen bond is additionally strengthened by resonance; its cleavage to produce iodobenzene is therefore not the expected reaction.

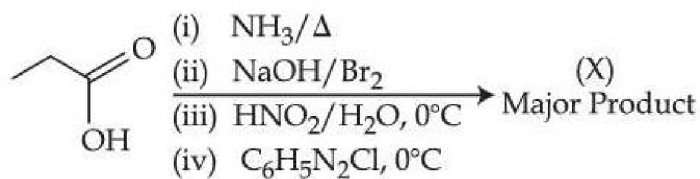
Step 3: Judge the two statements Statement I predicts benzyl alcohol and iodobenzene. That pair would require cleavage of the aryl–oxygen bond, contrary to the reaction above, so Statement I is false. Statement II explicitly identifies cleavage of the $O-CH_2$ bond and is true. Option (A) wrongly accepts Statement I, option (B) wrongly rejects Statement II, and option (C) reverses both truth values.

Step 4: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: In phenyl–O–alkyl ethers with HI, keep oxygen attached to phenyl: phenol and the alkyl iodide are the usual products.

67. Consider the following organic reaction sequence. Choose the final product (X) from the following (consider the major product in all intermediate reactions)

[4 marks]



- (A) Benzene
(B) Phenol
(C) Propanol
(D) Chlorobenzene

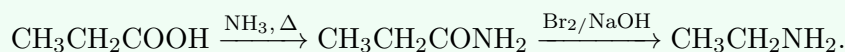
Correct Answer: (A) Benzene

Solution:

Concept:

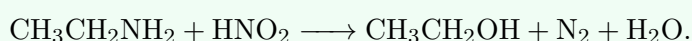
- A carboxylic acid reacts with ammonia on heating to give its primary amide. Hofmann bromamide degradation then converts RCONH_2 into RNH_2 , losing the carbonyl carbon.
- A primary aliphatic amine reacts with nitrous acid to give the corresponding alcohol. Ethanol reduces an aryl diazonium salt to the arene while itself being oxidised to ethanal.

Step 1: Convert the acid into the shorter amine The starting structure is propanoic acid, $\text{CH}_3\text{CH}_2\text{COOH}$, not ethanoic acid. Ammonia first gives the ammonium carboxylate, and heating removes water to form propanamide. The next reagent causes Hofmann degradation:

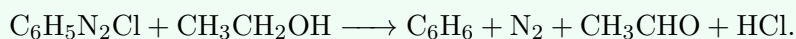


The product after these two operations is ethylamine, with two carbon atoms. Carbon counting prevents retaining three carbons incorrectly.

Step 2: Obtain the alcohol and use it in the final reaction At the third stage, primary aliphatic ethylamine undergoes deamination with nitrous acid. Its unstable aliphatic diazonium intermediate releases nitrogen and gives ethanol as the major alcohol:



In the last stage, this ethanol reacts with benzenediazonium chloride, reducing the diazonium group to hydrogen:



Thus the requested final product is the arene formed from the diazonium reagent, rather than the alcohol formed at the preceding stage.

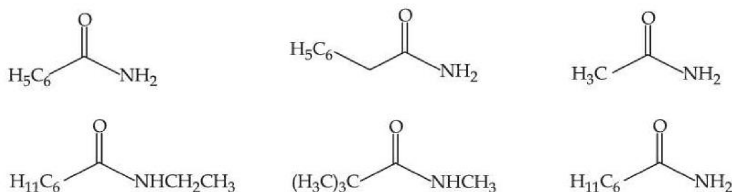
Step 3: Compare the listed final products Benzene is produced by reduction of the diazonium group. Phenol would result from hydrolysis of benzenediazonium salt under the appropriate aqueous warming conditions, whereas ethanol here acts as the reducing reagent. Propanol incorrectly preserves the carbon lost in Hofmann degradation. Chlorobenzene would require a chlorination route such as

the Sandmeyer reaction with cuprous chloride, which is not supplied.

Step 4: Conclusion The final product X is benzene, C_6H_6 . Hence, the correct option is (A).

Quick Tip to Solve: Track the carbon loss in Hofmann degradation, then recognise ethanol as a reducing agent for an aryl diazonium salt.

68. The number of compounds from the following which can undergo reaction with Br_2/KOH (alcoholic) to give respective products and these respective products can also be obtained separately by Gabriel phthalimide reaction is : [4 marks]



- (A) 5
(B) 4
(C) 3
(D) 6

Correct Answer: (C) 3

Solution:

Concept:

- Hofmann bromamide degradation requires a primary amide, $RCONH_2$, and gives the primary amine RNH_2 with one fewer carbon atom.
- Gabriel phthalimide synthesis produces primary aliphatic amines through substitution on an alkyl halide. Aniline is excluded because an aryl halide does not undergo the required ordinary nucleophilic substitution.

Step 1: Identify the primary amides in the six structures Read the six compounds from left to right across the upper row and then the lower row. Benzamide, phenylacetamide, acetamide and cyclohexanecarboxamide each contain the required $-CONH_2$ group. Their Hofmann products are, respectively, aniline, benzylamine, methylamine and cyclohexylamine. The two other structures contain $-CONHCH_2CH_3$ and $-CONHCH_3$. These are N-substituted secondary amides, not primary amides, so they fail the first requirement of the question.

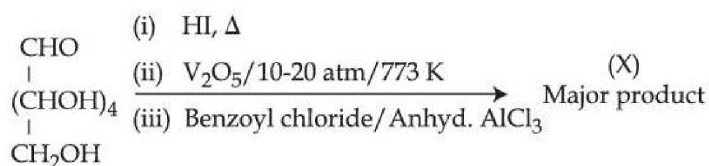
Step 2: Test which resulting amines are also accessible by Gabriel synthesis Benzamide gives $C_6H_5NH_2$, which cannot be obtained by the ordinary Gabriel route: the necessary aryl carbon does not permit the alkyl-halide substitution used in that method. Phenylacetamide gives $C_6H_5CH_2NH_2$, which is accessible using a benzyl halide; the nitrogen is attached to the side-chain carbon, not directly to the ring. Acetamide gives CH_3NH_2 , accessible using a methyl halide. Cyclohexanecarboxamide gives cyclohexylamine, a primary aliphatic amine accessible through the corresponding cycloalkyl halide route.

Step 3: Count only compounds satisfying both conditions The accepted starting amides are phenylacetamide, acetamide and cyclohexanecarboxamide. There are therefore three compounds. Counting four includes benzamide despite its inaccessible Gabriel product; counting five or six also includes one or both N-substituted amides despite their failure of the primary-amide requirement.

Step 4: Conclusion Exactly 3 compounds satisfy both requirements. Hence, the correct option is (C).

Quick Tip to Solve: Apply two filters separately: first keep only primary amides, then remove those whose Hofmann product is an aryl amine.

69. Consider the following reactions. Total number of electrons in the π bonds and lone pair of electrons in the product (X) is : [4 marks]



(A) 12

(B) 16

(C) 14

(D) 18

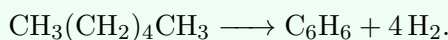
Correct Answer: (D) 18

Solution:

Concept:

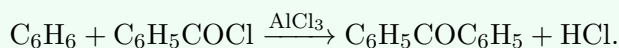
- Prolonged heating of glucose with hydrogen iodide removes oxygen and produces the straight-chain six-carbon alkane, n-hexane. Catalytic aromatisation of n-hexane gives benzene.
- Friedel–Crafts acylation with benzoyl chloride introduces $-\text{COC}_6\text{H}_5$ onto benzene. Each pi bond contains two electrons, and each lone pair also contains two electrons.

Step 1: Identify the carbon skeleton through the first two operations The displayed open-chain molecule has one aldehyde carbon, four CHOH carbons and one terminal CH_2OH carbon, for a total of six carbons. Treatment with hot HI gives n-hexane without changing that six-carbon skeleton. Passing this alkane over the stated vanadium oxide catalyst at high temperature and pressure causes dehydrocyclisation:



The intermediate that enters the last reaction is therefore benzene.

Step 2: Identify the final ketone Benzoyl chloride is $\text{C}_6\text{H}_5\text{COCl}$. In the presence of anhydrous AlCl_3 , it supplies the benzoyl electrophile for Friedel–Crafts acylation. Benzene forms benzophenone:



The product contains two phenyl rings joined through a carbonyl carbon. It contains one oxygen atom

and no chlorine atom; chloride leaves during the acylation.

Step 3: Count the requested electrons without including sigma bonds Each of the two benzene rings contributes three pi bonds, so the rings contribute six pi bonds altogether. The carbonyl group contributes one more. Hence there are seven pi bonds containing $7 \times 2 = 14$ electrons. The carbonyl oxygen has two lone pairs, containing four electrons. The requested combined number is

$$N = 14 + 4 = 18.$$

The value 14 omits the lone pairs, 12 counts only the aromatic pi electrons, and 16 fails to include all the required electrons.

Step 4: Conclusion The total number of electrons in pi bonds and lone pairs in benzophenone is 18. Hence, the correct option is (D).

Quick Tip to Solve: After identifying benzophenone, count two aromatic sextets, one carbonyl pi pair and the oxygen's two lone pairs separately.

70. Treatment of a gas 'X' with a freshly prepared ferrous sulphate solution gives a compound 'Y' as a brown ring. The compounds X and Y are. [4 marks]

- (A) NO and $[\text{Fe}(\text{NO})]\text{SO}_4$
- (B) NO_2 and $[\text{Fe}(\text{NO}_2)]\text{SO}_4$
- (C) N_2O and $[\text{Fe}(\text{N}_2\text{O})]\text{SO}_4$
- (D) N_2O_4 and $[\text{Fe}(\text{N}_2\text{O}_4)]\text{SO}_4$

Correct Answer: (A) NO and $[\text{Fe}(\text{NO})]\text{SO}_4$

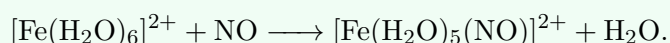
Solution:

Concept:

- The brown-ring test involves coordination of nitric oxide, NO, to iron in freshly prepared aqueous ferrous sulphate solution.
- The brown species is nitrosyl ferrous sulphate, conventionally represented as $[\text{Fe}(\text{H}_2\text{O})_5(\text{NO})]\text{SO}_4$. An abbreviated formula may suppress the coordinated water molecules.

Step 1: Recognise the gas that forms the brown complex Freshly prepared ferrous sulphate supplies iron(II) aquo ions. Nitric oxide binds as a nitrosyl ligand, replacing one water ligand of the aquo coordination sphere. The standard brown-ring species is thus a nitrosyl complex rather than a nitrito, nitrous oxide or dinitrogen tetroxide complex. In the familiar nitrate test, nitric oxide is first generated by reduction of nitrate; here it is supplied directly as the gas X. Therefore the identity required for X is NO.

Step 2: Write the coordination reaction and interpret the printed formula The formation of the coordinated cation can be represented by



Sulphate is the counter-ion, giving the salt $[\text{Fe}(\text{H}_2\text{O})_5(\text{NO})]\text{SO}_4$. The printed option writes this in the shortened form $[\text{Fe}(\text{NO})]\text{SO}_4$, omitting the water ligands. This abbreviation identifies the same nitrosyl ferrous sulphate responsible for the brown coloration. The coordination reaction, rather than a simple physical dissolution of the gas, accounts for formation of a characteristic coloured species.

Step 3: Eliminate the other gas–complex pairs Option (B) proposes nitrogen dioxide and its corresponding written complex; that is not the nitric-oxide nitrosyl species of the brown-ring test. Option (C) proposes nitrous oxide, which has different bonding and does not furnish the stated brown nitrosyl species. Option (D) proposes dinitrogen tetroxide, also not the ligand in this characteristic reaction. Only the first pair simultaneously identifies the correct gas and the correct abbreviated brown complex.

Step 4: Conclusion $X = \text{NO}$ and Y is nitrosyl ferrous sulphate, written in the options as $[\text{Fe}(\text{NO})]\text{SO}_4$. Hence, the correct option is (A).

Quick Tip to Solve: Associate the brown ring specifically with an iron–nitrosyl complex: the ligand is NO, and coordinated waters may be omitted in an abbreviated formula.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. An excess of AgNO_3 is added to 100 mL of a 0.05 M solution of tetraaquadichloridochromium (III) chloride. The number of moles of AgCl precipitated will be $___ \times 10^{-3}$.

(Nearest integer)

[4 marks]

Solution:

Concept:

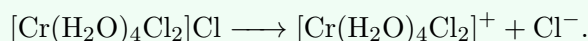
- In coordination nomenclature, ligands named before the metal belong inside the coordination sphere. The chloride named after the metal complex is an outside-sphere counter-ion.
- Excess silver nitrate precipitates free chloride quantitatively as AgCl . Chlorido ligands bound inside the coordination sphere are not counted as immediately ionisable chloride in this test.

Step 1: Deduce the coordination formula from the name Tetraaqua means four neutral water ligands and dichlorido means two coordinated chloride ligands. Chromium has oxidation state +3, so the complex ion has charge

$$+3 + 4(0) + 2(-1) = +1.$$

One external chloride balances that charge. The complete formula is therefore $[\text{Cr}(\text{H}_2\text{O})_4\text{Cl}_2]\text{Cl}$. It contains three chlorine atoms in all, but only one is the counter-ion outside the coordination brackets.

Step 2: Count the chloride ions available for precipitation On dissolution the coordination salt ionises as



Consequently each mole of the compound supplies one mole of freely ionisable chloride. The two chlorido ligands remain bound to chromium in the coordination complex for the standard silver-nitrate test. Counting all three chlorine atoms would confuse total chlorine content with the chloride detected by immediate precipitation.

Step 3: Calculate the precipitated amount Convert the solution volume to litres before using molarity:

$$V = 100 \text{ mL} = 0.100 \text{ L}.$$

The amount of coordination compound is

$$n = CV = (0.05 \text{ mol L}^{-1})(0.100 \text{ L}) = 0.005 \text{ mol}.$$

Since $\text{Ag}^+ + \text{Cl}^- \longrightarrow \text{AgCl(s)}$, and silver nitrate is in excess, the precipitation ratio is one mole of salt to one mole of silver chloride. Thus

$$n(\text{AgCl}) = 5 \times 10^{-3} \text{ mol}.$$

The coefficient multiplying the requested power of ten is already an integer.

Step 4: Conclusion The precipitated amount is $5 \times 10^{-3} \text{ mol}$ of AgCl ; the required nearest-integer entry is 5.

Quick Tip to Solve: Place the coordination brackets first. Only chloride outside the brackets contributes to the immediate silver chloride precipitate.

72. An alkane (Y) requires 8 moles of oxygen for complete combustion and on chlorination with $\text{Cl}_2/h\nu$, (Y) gives only one monochlorinated product (Z). The total number of primary carbon atoms in (Y) is ____ . [4 marks]

Solution:

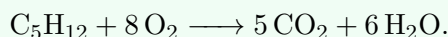
Concept:

- Complete combustion of one mole of an alkane obeys $\text{C}_n\text{H}_{2n+2} + ((3n+1)/2)\text{O}_2 \longrightarrow n\text{CO}_2 + (n+1)\text{H}_2\text{O}$, so oxygen consumption determines the molecular formula.
- Only one monochlorination product implies that all hydrogen positions are equivalent by molecular symmetry. A primary carbon is bonded to exactly one other carbon atom.

Step 1: Use oxygen consumption to determine the formula An alkane has formula $\text{C}_n\text{H}_{2n+2}$. Its n carbon atoms need n oxygen molecules for carbon dioxide, and its hydrogen atoms need another $(n+1)/2$ oxygen molecules for water. Therefore

$$n + \frac{n+1}{2} = 8, \quad 3n+1 = 16, \quad n = 5.$$

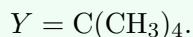
Thus Y is an isomer of pentane, C_5H_{12} . The balanced combustion check is



This stage fixes the molecular formula but does not yet distinguish its structural isomers.

Step 2: Apply the single-product chlorination condition The three structural isomers are n-pentane, 2-methylbutane and 2,2-dimethylpropane. In n-pentane, terminal and internal hydrogen positions are

different, producing more than one monochloro product. In 2-methylbutane, primary, secondary and tertiary environments are not all equivalent, again giving several products. In 2,2-dimethylpropane, or neopentane, a central carbon is bonded symmetrically to four methyl groups:



All twelve hydrogens are equivalent, so replacing any one hydrogen with chlorine gives the same constitutional product.

Step 3: Count primary carbons rather than primary hydrogens Each methyl carbon in neopentane is attached to the one central carbon, making each of these four carbons primary. The central carbon is attached to four other carbon atoms and is quaternary; it is not included. The monochlorinated product may be written $(CH_3)_3CCH_2Cl$, which confirms that every substitution starts from an equivalent methyl group. The requested count concerns carbon atoms, so neither the twelve equivalent hydrogens nor the total of five carbon atoms is the answer.

Step 4: Conclusion Y is neopentane, $C(CH_3)_4$, and it contains 4 primary carbon atoms.

Quick Tip to Solve: Use oxygen demand to find the carbon count, then use the number of monochloro products to test symmetry among its isomers.

73. 500 mL of $0.2M MnO_4^-$ solution in basic medium when mixed with 500 mL of 1.5 M KI solution, oxidises iodide ions to liberate molecular iodine. This liberated iodine is then titrated with a standard xM thiosulphate solution in presence of starch till the end point. If 300 mL of thiosulphate was consumed, then the value of x is _____. [4 marks]

Solution:

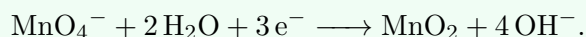
Concept:

- In basic medium, permanganate is reduced from manganese in oxidation state +7 to manganese dioxide, in which manganese is +4. Each mole of permanganate therefore accepts three moles of electrons.
- Iodide is oxidised according to $2I^- \longrightarrow I_2 + 2e^-$. In the subsequent titration, one mole of iodine consumes two moles of thiosulphate: $I_2 + 2S_2O_3^{2-} \longrightarrow 2I^- + S_4O_6^{2-}$.

Step 1: Find the amount of the limiting oxidant Convert the permanganate volume to litres before using molarity:

$$n(MnO_4^-) = 0.2 \times 0.500 = 0.100 \text{ mol.}$$

The basic-medium reduction half-reaction is



Consequently, complete reduction of the permanganate requires $3 \times 0.100 = 0.300$ mol of electrons. The available iodide is $1.5 \times 0.500 = 0.750$ mol. Since each iodide ion supplies one electron, iodide is in excess and the permanganate determines the iodine yield.

Step 2: Relate iodine to thiosulphate consumption Two moles of iodide produce one mole of iodine, so the liberated iodine is

$$n(\text{I}_2) = \frac{0.300}{2} = 0.150 \text{ mol.}$$

The iodine-thiosulphate equation then gives

$$n(\text{S}_2\text{O}_3^{2-}) = 2(0.150) = 0.300 \text{ mol.}$$

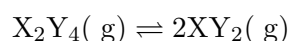
This amount is contained in the 300 mL of titrant delivered at the starch end point. Use that titrant volume, rather than the total volume of the original mixed solutions:

$$x = \frac{0.300}{0.300} = 1.00 \text{ mol L}^{-1}.$$

Step 3: Conclusion The thiosulphate concentration is 1.00 M; hence the requested numerical value is $x = 1$.

Quick Tip to Solve: For permanganate reduced to manganese dioxide, the electron count is three. In iodometry the iodine factor of two cancels, giving three moles of thiosulphate per mole of permanganate.

74. In a closed flask at 600 K , one mole of $\text{X}_2\text{Y}_4(\text{g})$ attains equilibrium as given below :



At equilibrium, 75% $\text{X}_2\text{Y}_4(\text{g})$ was dissociated and the total pressure is 1 atm . The magnitude of $\Delta_r G^\ominus$ (in kJ mol^{-1}) at this temperature is ____ . (Nearest Integer)

(Given : $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$; $\ln 10 = 2.3$, $\log 2 = 0.3$, $\log 3 = 0.48$, $\log 5 = 0.69$, $\log 7 = 0.84$) [4 marks]

Solution:

Concept:

- For $\text{X}_2\text{Y}_4(\text{g}) \rightleftharpoons 2\text{XY}_2(\text{g})$, the degree of dissociation determines the equilibrium mole fractions. An initial mole gives $1 - \alpha$ mole of reactant and 2α moles of product.
- Partial pressures equal mole fraction times total pressure. The dimensionless equilibrium constant is $K_p = (p_{\text{XY}_2}/p^\ominus)^2 / (p_{\text{X}_2\text{Y}_4}/p^\ominus)$, and $\Delta_r G^\ominus = -RT \ln K_p$.

Step 1: Calculate the equilibrium partial pressures Here $\alpha = 0.75$. Starting with one mole, the equilibrium amounts and their sum are

$$n(\text{X}_2\text{Y}_4) = 0.25, \quad n(\text{XY}_2) = 1.50, \quad n_{\text{total}} = 1.75.$$

Thus the reactant mole fraction is $1/7$ and the product mole fraction is $6/7$. At the stated total pressure of 1 atm,

$$p_{\text{X}_2\text{Y}_4} = \frac{1}{7} \text{ atm}, \quad p_{\text{XY}_2} = \frac{6}{7} \text{ atm.}$$

The total number of gas moles increases on dissociation, so the partial pressures must use 1.75 moles in the denominator, rather than the initial one mole.

Step 2: Evaluate the equilibrium constant and free energy Using the pressure convention implicit in the supplied data,

$$K_p = \frac{(6/7)^2}{1/7} = \frac{36}{7}.$$

Evaluate its logarithm with the numbers supplied in the question:

$$\begin{aligned}\log K_p &= 2 \log 2 + 2 \log 3 - \log 7 \\ &= 2(0.30) + 2(0.48) - 0.84 = 0.72, \\ \ln K_p &= 2.3(0.72) = 1.656.\end{aligned}$$

Since $K_p > 1$, the standard reaction Gibbs energy is negative. Its magnitude is

$$|\Delta_r G^\circ| = 8.3 \times 600 \times 1.656 = 8246.88 \text{ J mol}^{-1} = 8.24688 \text{ kJ mol}^{-1}.$$

Step 3: Conclusion The standard reaction Gibbs energy is approximately $-8.25 \text{ kJ mol}^{-1}$. Its magnitude, rounded to the nearest integer as requested, is $\boxed{8 \text{ kJ mol}^{-1}}$; enter $\boxed{8}$.

Quick Tip to Solve: For a one-to-two dissociation, total equilibrium moles are $1 + \alpha$. Calculate both partial pressures from this total before applying $-RT \ln K_p$.

75. Decomposition of a hydrocarbon follows the equation $k = (5.5 \times 10^{11} \text{ s}^{-1}) e^{\frac{-28000 \text{ K}}{T}}$. **The activation energy of reaction is _____ kJmol⁻¹. (Nearest Integer)**

Given : $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$

[4 marks]

Solution:

Concept:

- The Arrhenius equation is $k = A \exp[-E_a/(RT)]$, where A is the pre-exponential factor, E_a is the molar activation energy, R is the gas constant and T is absolute temperature.
- The exponential argument must be dimensionless. Therefore a coefficient expressed in kelvin in an exponent of the form $-B/T$ is $B = E_a/R$, rather than the activation energy itself.

Step 1: Compare the given exponential with the Arrhenius form The stated rate constant is

$$k = (5.5 \times 10^{11} \text{ s}^{-1}) \exp\left(-\frac{28000 \text{ K}}{T}\right).$$

Match its temperature-dependent exponent with $-E_a/(RT)$. Their coefficients of $1/T$ must agree, giving

$$\frac{E_a}{R} = 28000 \text{ K}, \quad E_a = (28000 \text{ K})R.$$

The factor $5.5 \times 10^{11} \text{ s}^{-1}$ is A . It determines the scale of the rate constant but does not enter the activation energy obtained by this comparison. The negative sign in the exponential is already part of the Arrhenius equation and does not make the activation energy negative.

Step 2: Substitute the gas constant and convert units With the supplied $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$,

$$E_a = 28000 \times 8.3 = 232400 \text{ J mol}^{-1}.$$

The kelvin in the coefficient cancels the inverse kelvin in the units of R , leaving energy per mole as required. Convert joules to kilojoules before rounding:

$$E_a = \frac{232400}{1000} = 232.4 \text{ kJ mol}^{-1}.$$

Since the fractional part is 0.4, the nearest integer is 232. Neither an experimental temperature nor the value of the rate constant is needed, because the activation energy has already been specified through the exponential coefficient.

Step 3: Conclusion The activation energy is $232.4 \text{ kJ mol}^{-1}$, so the requested nearest-integer answer is ; enter .

Quick Tip to Solve: In $k = Ae^{-B/T}$, multiply B by R to obtain E_a . Using R in joules gives joules per mole, so divide by 1000 before reporting kilojoules.