

JEE Main 2026 Paper 1 (B.E./B.Tech) Apr 8 Shift 2

Question Paper with Solutions PDF



Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours

Maximum Marks: 300

General Instructions

- (i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2026, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 8 April 2026, Shift 2. The total duration is 3 hours.
- (ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.
- (iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.
- (iv) Marking scheme (multiple choice questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (v) Marking scheme (numerical value questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Consider the relation R on the set $\{-2, -1, 0, 1, 2\}$ defined by $(a, b) \in R$ if and only if $1 + ab > 0$. Then, among the statements :

I. The number of elements in R is 17

II. R is an equivalence relation

[4 marks]

(A) Only I is true

(B) Only II is true

(C) Both I and II are true

(D) Neither I nor II is true

Correct Answer: (A) Only I is true

Solution:

Concept:

- A relation is a set of ordered pairs, so count admissible second coordinates for each first coordinate separately.
- An equivalence relation must be reflexive, symmetric and transitive. Failure of even one property disproves equivalence.

Step 1: Count the ordered pairs Because the coordinates are integers, the strict inequality $1 + ab > 0$ is equivalent to $ab \geq 0$. For $a = -2$ or $a = -1$, the allowed values of b are $-2, -1, 0$, giving three pairs for each row. For $a = 0$, all five second coordinates are allowed. For $a = 1$ or $a = 2$, the allowed values are $0, 1, 2$, giving three pairs for each row. Thus

$$|R| = 3 + 3 + 5 + 3 + 3 = 17.$$

Ordered pairs are counted separately: for example, $(-2, -1)$ and $(-1, -2)$ are distinct. Hence statement I is true.

Step 2: Test equivalence using zero Every diagonal pair satisfies $1 + a^2 > 0$, so the relation is reflexive. It is symmetric because $ab = ba$. However, symmetry and reflexivity alone are insufficient. In particular, $(-1, 0) \in R$ and $(0, 1) \in R$, since both products are zero. Their transitive consequence would be $(-1, 1) \in R$, but $1 + (-1)(1) = 0$, which fails the strict inequality. Thus transitivity fails and statement II is false. Options B and C incorrectly assert equivalence, while option D incorrectly rejects the count 17.

Step 3: Conclusion Only statement I is true: $|R| = 17$, but R is not transitive and hence is not an equivalence relation. Hence, the correct option is (A).

Quick Tip to Solve: When zero relates to every element, test transitivity through zero using one positive and one negative element.

2. The number of values of $z \in \mathbb{C}$, satisfying the equations $|z - (4 + 8i)| = \sqrt{10}$ and $|z - (3 + 5i)| + |z - (5 + 11i)| = 4\sqrt{5}$, is : [4 marks]

- (A) 0
- (B) 2
- (C) 1
- (D) 4

Correct Answer: (B) 2

Solution:

Concept:

- The modulus $|z - z_0|$ is the Euclidean distance between the corresponding points in the Argand plane.
- An ellipse with distance sum $2a$ and focal half-distance c has semi-minor axis b , where $b^2 = a^2 - c^2$.

Step 1: Identify the common centre and the axes Let the two foci be $F_1 = (3, 5)$ and $F_2 = (5, 11)$. Their midpoint is $C = (4, 8)$, exactly the centre of the circle in the first equation. The distance between the foci is

$$F_1F_2 = \sqrt{(5-3)^2 + (11-5)^2} = 2\sqrt{10}.$$

Hence $c = \sqrt{10}$. The second equation fixes the sum of focal distances at $4\sqrt{5}$, so $a = 2\sqrt{5}$ and $a^2 = 20$. Consequently

$$b^2 = 20 - 10 = 10.$$

The given circle has radius $\sqrt{10} = b$.

Step 2: Count intersections in rotated coordinates Take coordinates u, v centred at C , with the u -axis along the line of foci and the v -axis perpendicular to it. A rotation preserves distances and does not change the number of points. The equations become

$$\frac{u^2}{20} + \frac{v^2}{10} = 1, \quad u^2 + v^2 = 10.$$

Multiplying the ellipse equation by 10 and comparing with the circle yields $u^2/2 + v^2 = 10$, and hence $u = 0$. Thus $v = \pm\sqrt{10}$. These are two distinct real plane points, each determining one complex number. There are no other solutions, excluding counts 0, 1 and 4.

Step 3: Conclusion Exactly two values of z satisfy both equations, corresponding to the two minor-axis endpoints of the ellipse. Hence, the correct option is (B).

Quick Tip to Solve: First compare the circle centre with the midpoint of the foci; a circle of radius equal to the ellipse semi-minor axis meets it at two points.

3. If the system of linear equations :

$$x + y + z = 6$$

$$x + 2y + 5z = 10$$

$$2x + 3y + \lambda z = \mu$$

has infinitely many solutions, then the value of $\lambda + \mu$ equals:

[4 marks]

(A) 12

(B) 16

(C) 22

(D) 28

Correct Answer: (C) 22

Solution:

Concept:

- A three-variable linear system has infinitely many solutions when it is consistent and its coefficient rank is less than three.
- If two equations are independent, the third must be a linear combination of them, including its constant term.

Step 1: Eliminate the first two variables Denote the first two equations by E_1 and E_2 . Their coefficient rows $(1, 1, 1)$ and $(1, 2, 5)$ are not proportional, so their rank is two. Adding them gives

$$E_1 + E_2 : \quad 2x + 3y + 6z = 16.$$

Subtract this equation from the third equation. The residual condition is

$$(\lambda - 6)z = \mu - 16.$$

If $\lambda \neq 6$, this determines z , after which the two independent equations determine x, y , producing a unique solution. It cannot give infinitely many solutions.

Step 2: Enforce consistency without a new constraint Therefore $\lambda = 6$ is necessary. The residual equation then reads $0 = \mu - 16$, so consistency requires $\mu = 16$. When both values hold, the third equation is precisely the sum of the first two and imposes no additional restriction. To verify the infinite family explicitly, set $z = t$. Subtracting E_1 from E_2 gives $y + 4z = 4$, so

$$z = t, \quad y = 4 - 4t, \quad x = 2 + 3t, \quad t \in \mathbb{R}.$$

Every real t solves all three equations. This confirms sufficiency, rather than relying only on a zero determinant. The requested sum is $6 + 16 = 22$, excluding 12, 16 and 28.

Step 3: Conclusion $\lambda = 6$ and $\mu = 16$, hence $\lambda + \mu = 22$. Hence, the correct option is (C).

Quick Tip to Solve: For infinite solutions, eliminate with the same row operation on coefficients and constants; a zero coefficient row must have zero constant too.

4. Let $A = \begin{bmatrix} \alpha & 1 & 2 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5\alpha & 0 \\ 0 & 4\alpha & -2\alpha \end{bmatrix} + \text{adj}(A)$. If $\det(B) = 66$, then $\det(\text{adj}(A))$ equals :

[4 marks]

(A) 289

(B) 361

(C) 441

(D) 529

Correct Answer: (C) 441

Solution:

Concept:

- The adjoint is the transpose of the cofactor matrix. Compute signed minors before adding the given matrix.
- For a square matrix of order three, $\det(\text{adj } A) = (\det A)^2$.

Step 1: Form the adjoint and simplify B The cofactor calculation gives

$$\text{adj } A = \begin{pmatrix} 15 & 3 & -6 \\ -10 & 5\alpha & 4 \\ 8 & -4\alpha & 3\alpha - 2 \end{pmatrix}.$$

For example, the entry in row 1, column 2 is the cofactor in row 2, column 1: $-[1 \cdot 5 - 2 \cdot 4] = 3$. Adding the matrix specified in the question cancels the second-column entries in rows 2 and 3. Thus

$$B = \begin{pmatrix} 16 & 3 & -6 \\ -10 & 0 & 4 \\ 8 & 0 & \alpha - 2 \end{pmatrix}.$$

The first entry is 16, because the added diagonal entry is 1; the lower-right entry is $\alpha - 2$, not $3\alpha - 2$.

Step 2: Use the sparse column to find alpha Expand along the second column. Its only nonzero entry has a negative cofactor sign, giving

$$\det B = -3 \begin{vmatrix} -10 & 4 \\ 8 & \alpha - 2 \end{vmatrix} = 30\alpha + 36.$$

Therefore $30\alpha + 36 = 66$, so $\alpha = 1$. Now expand A along its first row:

$$\det A = 15\alpha - 10 + 16 = 15\alpha + 6 = 21.$$

Applying the order-three adjoint identity yields $\det(\text{adj } A) = 21^2 = 441$. The other offered squares, $17^2, 19^2, 23^2$, do not arise from the value of α forced by $\det B = 66$.

Step 3: Conclusion $\det(\text{adj } A) = (21)^2 = 441$. Hence, the correct option is (C).

Quick Tip to Solve: Add the matrices before expanding: the cancellations in the second column make the determinant of B a single minor.

5. Let $\alpha = 3 + 4 + 8 + 9 + 13 + 14 + \dots$ upto 40 terms. If $(\tan \beta)^{\frac{\alpha}{1020}}$ is a root of the equation $x^2 + x - 2 = 0, \beta \in (0, \frac{\pi}{2})$, then $\sin^2 \beta + 3 \cos^2 \beta$ is equal to : [4 marks]

- (A) 2
(B) $\frac{7}{4}$
(C) $\frac{5}{2}$
(D) $\frac{3}{2}$

Correct Answer: (A) 2

Solution:

Concept:

- An interlaced series can be separated into its odd-position and even-position arithmetic progressions.
- For $0 < \beta < \pi/2$, $\tan \beta$ is positive. A positive real power cannot equal a negative root.

Step 1: Sum the two progressions The 40 terms consist of 20 pairs (3, 4), (8, 9), (13, 14), ... The first entries form an arithmetic progression of 20 terms, first term 3 and common difference 5; the second entries form another with first term 4 and the same difference. Their final entries are $3 + 19 \cdot 5 = 98$ and $4 + 19 \cdot 5 = 99$. Hence

$$\alpha = \frac{20}{2}(3 + 98) + \frac{20}{2}(4 + 99) = 1010 + 1030 = 2040.$$

Thus the exponent $\alpha/1020$ equals 2. Taking 40 terms of either progression separately would double the intended number of terms.

Step 2: Select the admissible root and evaluate The quadratic factorises as

$$x^2 + x - 2 = (x - 1)(x + 2) = 0,$$

so its roots are 1 and -2 . Since $\tan \beta > 0$, its square is positive and must be 1. Therefore $\tan \beta = 1$, with the negative square root excluded by the given first-quadrant interval. It follows that $\beta = \pi/4$, and $\sin^2 \beta = \cos^2 \beta = 1/2$. Thus

$$\sin^2 \beta + 3 \cos^2 \beta = \frac{1}{2} + \frac{3}{2} = 2.$$

In particular, the alternatives $7/4, 5/2, 3/2$ do not satisfy the equal sine-square and cosine-square values obtained from $\tan \beta = 1$.

Step 3: Conclusion The required value is 2. Hence, the correct option is (A).

Quick Tip to Solve: Before extracting trigonometric roots, use the interval for beta to discard negative polynomial roots and negative tangent values.

6. A candidate has to go to the examination centre to appear in an examination. The candidate uses only one means of transportation for the entire distance out of bus, scooter and car. The probabilities of the candidate going by bus, scooter and car, respectively, are $\frac{2}{5}$, $\frac{1}{5}$ and $\frac{2}{5}$. The probabilities that the candidate reaches late at the examination centre are $\frac{1}{5}$, $\frac{1}{3}$ and $\frac{1}{4}$ if the candidate uses bus, scooter and car, respectively. Given that the candidate reached late at the examination centre, the probability that the candidate travelled by bus is : [4 marks]

- (A) $\frac{11}{37}$
 (B) $\frac{12}{37}$
 (C) $\frac{13}{37}$
 (D) $\frac{14}{37}$

Correct Answer: (B) $\frac{12}{37}$

Solution:

Concept:

- For mutually exclusive and exhaustive travel modes, the total probability of lateness is the sum of the mode-wise joint probabilities.
- Bayes theorem reverses the conditioning: $P(B | L) = P(B)P(L | B)/P(L)$, where B means bus and L means late.

Step 1: Compute each contribution to lateness Let B, S, C denote travelling by bus, scooter and car, respectively. Exactly one mode is used, so the events form a partition. Their probabilities sum to $2/5 + 1/5 + 2/5 = 1$. Multiply each prior probability by its conditional lateness probability:

$$\begin{aligned} P(B \cap L) &= \frac{2}{5} \cdot \frac{1}{5} = \frac{2}{25} = \frac{12}{150}, \\ P(S \cap L) &= \frac{1}{5} \cdot \frac{1}{3} = \frac{1}{15} = \frac{10}{150}, \\ P(C \cap L) &= \frac{2}{5} \cdot \frac{1}{4} = \frac{1}{10} = \frac{15}{150}. \end{aligned}$$

These are joint probabilities.

Step 2: Condition on the observed lateness Add the three disjoint contributions to obtain

$$P(L) = \frac{12 + 10 + 15}{150} = \frac{37}{150}.$$

Conditioning on lateness restricts the sample space to L . Therefore

$$P(B | L) = \frac{P(B \cap L)}{P(L)} = \frac{12/150}{37/150} = \frac{12}{37}.$$

Using $1/5$ reverses the required condition. The numerators 11, 13 and 14 in the other options do not equal the bus joint contribution 12 when all weights are expressed over 150.

Step 3: Conclusion The required conditional probability is $\frac{12}{37}$. Hence, the correct option is (B).

Quick Tip to Solve: Put all joint probabilities over a common denominator; the posterior is the required mode weight divided by the sum of all weights.

7. A set of four observations has mean 1 and variance 13. Another set of six observations has mean 2 and variance 1. Then, the variance of all these 10 observations is equal to : [4 marks]

- (A) 5.96
- (B) 6.14
- (C) 6.04
- (D) 6.24

Correct Answer: (C) 6.04

Solution:

Concept:

- The population variance of a group is $\sigma^2 = \frac{\sum x_i^2}{n} - \mu^2$, where n is its size and μ is its mean.
- To combine groups, add their observation sums and square sums. Variances cannot simply be averaged when the group means differ.

Step 1: Recover the two square sums For the first group, the observation sum is $4(1) = 4$. Its square sum is $4(13 + 1^2) = 56$. For the second group, the observation sum is $6(2) = 12$, and its square sum is $6(1 + 2^2) = 30$. The extra mean-square term is essential because each supplied variance is measured about its own group mean.

Step 2: Calculate the combined mean and second moment There are ten observations altogether. Adding the two sums before dividing gives

$$\mu = \frac{4 + 12}{10} = \frac{8}{5} = 1.6.$$

Similarly, the mean of their squares is

$$\frac{\sum x_i^2}{10} = \frac{56 + 30}{10} = 8.6.$$

The weights are the group sizes, rather than equal weights for the two groups.

Step 3: Subtract the square of the combined mean Apply the variance identity to the complete collection:

$$\sigma^2 = 8.6 - (1.6)^2 = 8.6 - 2.56 = 6.04.$$

As a check, the within-group contribution is $(4 \cdot 13 + 6 \cdot 1)/10 = 5.8$; the between-group contribution is $[4(1 - 1.6)^2 + 6(2 - 1.6)^2]/10 = 0.24$. Their sum is again 6.04, excluding the other three decimal values.

Step 4: Conclusion The variance of all ten observations is 6.04. Hence, the correct option is (C).

Quick Tip to Solve: For pooled variance, recover each square sum using $n(\sigma^2 + \mu^2)$, then subtract the square of the pooled mean.

8. If $26 \left(\frac{2^3}{3} \binom{12}{2} + \frac{2^5}{5} \binom{12}{4} + \frac{2^7}{7} \binom{12}{6} + \cdots + \frac{2^{13}}{13} \binom{12}{12} \right) = 3^{13} - \alpha$, then α is equal to : [4 marks]

- (A) 45
- (B) 48
- (C) 51
- (D) 54

Correct Answer: (C) 51

Solution:

Concept:

- Adding $(1+t)^{12}$ and $(1-t)^{12}$ keeps only even powers: their half-sum is $\sum_{j=0}^6 \binom{12}{2j} t^{2j}$.
- Integrating a power t^k from zero to two produces $2^{k+1}/(k+1)$, matching the denominators in the given sum.

Step 1: Include the missing constant term Let S denote the parenthesised sum in the question. Its terms correspond to the even exponents $2, 4, \dots, 12$. The even-power binomial expansion also contains the constant term $\binom{12}{0} = 1$. Consequently,

$$S + 2 = \frac{1}{2} \int_0^2 ((1+t)^{12} + (1-t)^{12}) dt.$$

The added number is two, because the integral of the constant one over this interval is two.

Step 2: Evaluate the two elementary integrals Integrate each binomial power directly, remembering the negative derivative of $1-t$:

$$\begin{aligned} S + 2 &= \left[\frac{(1+t)^{13} - (1-t)^{13}}{26} \right]_0^2 \\ &= \frac{3^{13} - (-1)^{13}}{26} \\ &= \frac{3^{13} + 1}{26}. \end{aligned}$$

At the lower endpoint the two thirteenth powers cancel, so no additional constant remains.

Step 3: Match the prescribed expression Multiply the result by 26 and remove the added constant contribution:

$$26S = 3^{13} + 1 - 52 = 3^{13} - 51.$$

Comparing this with $26S = 3^{13} - \alpha$ gives a unique value. The other options arise neither from the endpoint sign nor from correctly removing the constant term; in particular, omitting that removal would give a completely different expression.

Step 4: Conclusion We obtain $\alpha = 51$. Hence, the correct option is (C).

Quick Tip to Solve: In a binomial sum with denominators one greater than the selected powers, integrate the even- or odd-power generating expression.

9. A person has three different bags and four different books. The number of ways, in which he can put these books in the bags so that no bag is empty, is : [4 marks]

- (A) 18
- (B) 36
- (C) 39
- (D) 72

Correct Answer: (B) 36

Solution:

Concept:

- Each different book is assigned to exactly one of the three different bags. The order of books inside a bag is irrelevant.
- If all three bags are nonempty and there are four books, their occupancy sizes must be 2, 1, 1.

Step 1: Identify the only possible occupancy pattern First put one book in every bag. Exactly one book remains, so one bag must ultimately contain two books and each of the other bags must contain one. There are no other positive integer occupancy patterns summing to four. This observation avoids counting empty bags and avoids introducing an order among books placed together.

Step 2: Choose the double bag and distribute the books Choose the bag that contains two books in 3 ways. Choose its unordered pair of books in $\binom{4}{2} = 6$ ways. The two remaining distinct books must occupy the two remaining distinct bags, one each; this can be done in $2! = 2$ ways. Therefore,

$$N = 3 \binom{4}{2} 2! = 3 \cdot 6 \cdot 2 = 36.$$

Each completed distribution has exactly one double bag, so this construction counts every allowed arrangement once.

Step 3: Check by excluding empty bags Without restrictions there are $3^4 = 81$ assignments. For each specified empty bag, all books have two choices, giving $2^4 = 16$ assignments. If two specified bags are empty, there is exactly one assignment to the remaining bag. Inclusion-exclusion yields

$$N = 3^4 - \binom{3}{1} 2^4 + \binom{3}{2} 1^4 = 81 - 48 + 3 = 36.$$

Thus neither 18, 72, nor 39 includes the full set of labelled-bag assignments.

Step 4: Conclusion There are 36 permitted distributions. Hence, the correct option is (B).

Quick Tip to Solve: For four distinct objects in three distinct nonempty containers, use the occupancy pattern 2, 1, 1 and choose the double container first.

10. If a straight line drawn through the point of intersection of the lines $4x + 3y - 1 = 0$ and $3x + 4y - 1 = 0$, meets the co-ordinate axes at the points P and Q, then the locus of the mid point of PQ is : [4 marks]

- (A) $x + y - 7 = 0$
(B) $x + y - 14xy = 0$
(C) $2x + y + 14xy = 0$
(D) $x + 2y - 14xy = 0$

Correct Answer: (B) $x + y - 14xy = 0$

Solution:

Concept:

- A line with nonzero finite intercepts a and b on the coordinate axes has equation $X/a + Y/b = 1$.
- The midpoint of $P(a, 0)$ and $Q(0, b)$ is $(a/2, b/2)$; thus intercepts can be replaced by twice the midpoint coordinates.

Step 1: Locate the fixed point of the moving line Subtract the two fixed-line equations:

$$(4X + 3Y - 1) - (3X + 4Y - 1) = 0 \implies X = Y.$$

Substitution into either equation gives $7X = 1$. Hence every line under consideration passes through the fixed point $R(1/7, 1/7)$. This point, rather than either original line separately, is the constraint on the moving line.

Step 2: Express the intercepts using the midpoint Let the midpoint be $M(x, y)$, and write $P(a, 0)$, $Q(0, b)$. The midpoint formula gives $a = 2x$, $b = 2y$. Since the moving line contains R , its intercept form implies

$$\frac{1/7}{a} + \frac{1/7}{b} = 1.$$

Replacing the intercepts now yields

$$\frac{1}{14x} + \frac{1}{14y} = 1.$$

This is the required relation before clearing denominators.

Step 3: Simplify the locus equation Multiplying by $14xy$, for nonzero intercepts, gives

$$x + y = 14xy \implies x + y - 14xy = 0.$$

This is the conic equation in option (B). The degenerate axis lines do not supply two distinct intercept endpoints; the derived curve is understood with those intercept restrictions. Option (A) is linear, while (C) and (D) have unequal linear coefficients incompatible with the symmetry between the two axes.

Step 4: Conclusion The locus has equation $x + y - 14xy = 0$. Hence, the correct option is (B).

Quick Tip to Solve: For a midpoint-of-intercepts locus, replace the line intercepts by $2x, 2y$ before imposing the fixed-point condition.

11. Let O be the vertex of the parabola $y^2 = 4x$ and its chords OP and OQ are perpendicular to each other. If the locus of the mid-point of the line segment PQ is a conic C , then the length of its latus rectum is :

[4 marks]

(A) 1

(B) 2

(C) 4

(D) 8

Correct Answer: (B) 2

Solution:

Concept:

- A point on $y^2 = 4x$ can be parametrised as $(t^2, 2t)$. The slope of its chord to the vertex is $2/t$, for a nondegenerate chord.
- Perpendicular finite slopes have product -1 . A parabola written $(y - k)^2 = 4a(x - h)$ has latus rectum length $4|a|$.

Step 1: Translate perpendicularity into parameters Write $P = (t_1^2, 2t_1)$ and $Q = (t_2^2, 2t_2)$. Their chords from $O = (0, 0)$ have slopes $2/t_1$ and $2/t_2$. Since the chords are perpendicular,

$$\frac{2}{t_1} \frac{2}{t_2} = -1 \implies t_1 t_2 = -4.$$

Neither endpoint is the vertex, since that would fail to define a nonzero chord.

Step 2: Eliminate the two point parameters Let the midpoint of PQ be (x, y) . Applying the midpoint formula gives

$$x = \frac{t_1^2 + t_2^2}{2}, \quad y = t_1 + t_2.$$

Square the second relation and use the product condition:

$$y^2 = t_1^2 + t_2^2 + 2t_1 t_2 = 2x - 8.$$

Therefore the midpoint locus is the right-opening parabola $y^2 = 2(x - 4)$. The parameter quadratic has discriminant $y^2 + 16 > 0$, so every real y on this parabola is obtainable.

Step 3: Read the latus rectum of the new parabola Compare $y^2 = 2(x - 4)$ with the translated standard form $y^2 = 4a(x - h)$. Thus $4a = 2$, so $a = 1/2$ and the latus rectum length is 2. Translation of the vertex to $(4, 0)$ does not alter that length. The value 4 belongs to the original parabola, while 1 and 8 do not match the coefficient of the new locus.

Step 4: Conclusion The length of the latus rectum of C is 2. Hence, the correct option is (B).

Quick Tip to Solve: For perpendicular vertex chords of $y^2 = 4x$, use $t_1 t_2 = -4$, then express the midpoint through the sum and sum of squares.

12. Let $\alpha = 3 \sin^{-1} \left(\frac{6}{11} \right)$ and $\beta = 3 \cos^{-1} \left(\frac{4}{9} \right)$, where inverse trigonometric functions take only the principal values.

Given below are two statements :

Statement I : $\cos(\alpha + \beta) > 0$.

Statement II : $\cos(\alpha) < 0$.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Both Statement I and Statement II are true
(B) Both Statement I and Statement II are false
(C) Statement I is true but Statement II is false
(D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- Principal values give $u = \sin^{-1}(6/11)$ and $v = \cos^{-1}(4/9)$ in the first quadrant, with $\alpha = 3u$ and $\beta = 3v$.
- The triple-angle identities are $\cos 3t = 4 \cos^3 t - 3 \cos t$ and $\sin 3t = 3 \sin t - 4 \sin^3 t$. The sign of $\cos(\alpha + \beta)$ follows from the addition formula.

Step 1: Determine both trigonometric signs for alpha Since $\sin u = 6/11$, first-quadrant positivity gives $\cos u = \sqrt{85}/11$. Hence

$$\begin{aligned}\cos \alpha &= \frac{\sqrt{85}}{11} \left(\frac{340}{121} - 3 \right) = -\frac{23\sqrt{85}}{1331} < 0, \\ \sin \alpha &= \frac{18}{11} - \frac{864}{1331} = \frac{1314}{1331} > 0.\end{aligned}$$

Thus Statement II is true. Specifying the principal values is essential for choosing the positive square root for $\cos u$.

Step 2: Determine both trigonometric signs for beta Here $\cos v = 4/9$ and $\sin v = \sqrt{65}/9$, again using the first quadrant. The triple-angle identities give

$$\begin{aligned}\cos \beta &= 4 \left(\frac{4}{9} \right)^3 - 3 \left(\frac{4}{9} \right) = -\frac{716}{729} < 0, \\ \sin \beta &= \frac{\sqrt{65}}{9} \left(\frac{64}{81} - 1 \right) = -\frac{17\sqrt{65}}{729} < 0.\end{aligned}$$

These signs also place β in the third quadrant.

Step 3: Apply the angle-addition formula Now

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta.$$

The first product is positive because both cosines are negative. The second product is negative because

$\sin \alpha > 0$ and $\sin \beta < 0$. Subtracting it makes another positive contribution, so the whole expression is strictly positive. Statement I is true as well; every option asserting a false statement is therefore excluded.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: Use triple-angle identities to find signs exactly; multiplying principal inverse angles by three can move them into a different quadrant.

13. For the function $f(x) = e^{\sin |x|} - |x|, x \in \mathbf{R}$, consider the following statements :

Statement I : f is differentiable for all $x \in \mathbf{R}$.

Statement II : f is increasing in $(-\pi, -\frac{\pi}{2})$.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Both Statement I and Statement II are true
- (B) Both Statement I and Statement II are false
- (C) Statement I is true but Statement II is false
- (D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- An absolute value can create a corner only at the point where its argument is zero. Away from zero the ordinary chain rule applies.
- Differentiability at zero requires equal finite left and right difference-quotient limits. A positive derivative throughout an interval proves that the function is strictly increasing there.

Step 1: Check the two derivatives at zero Here $f(0) = 1$. For $h > 0$, the difference quotient is $(e^{\sin h} - 1)/h - 1$, whose limit is $1 - 1 = 0$. For $h < 0$, write $h = -t$, where $t > 0$. Then the quotient is $-[(e^{\sin t} - 1)/t - 1]$, which also tends to zero. We used $(e^u - 1)/u \rightarrow 1$ and $\sin t/t \rightarrow 1$. Thus the apparent corner from $|x|$ cancels, giving $f'(0) = 0$.

Step 2: Check differentiability away from zero For positive x , $f(x) = e^{\sin x} - x$, a combination of differentiable functions. For negative x , $f(x) = e^{-\sin x} + x$, which is also differentiable. Combining these observations with the two-sided calculation at zero proves Statement I on the whole real line.

Step 3: Determine the derivative sign in the stated interval For $x < 0$, differentiation gives

$$f'(x) = 1 - e^{-\sin x} \cos x.$$

On $(-\pi, -\pi/2)$, the cosine is negative and the exponential is strictly positive. Hence the product being subtracted is negative, so $f'(x) > 1 > 0$. Statement II is therefore true as well; the options asserting that either statement is false contradict these direct checks.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: At the origin check cancellation in the complete difference quotient; the presence of an absolute value alone does not prove nondifferentiability.

14. Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 10\hat{i} + 2\hat{j} - \hat{k}$ and a vector $\vec{c}$ be such that $2(\vec{a} \times \vec{b}) + 3(\vec{b} \times \vec{c}) = \vec{0}$. If $\vec{a} \cdot \vec{c} = 15$, then $\vec{c} \cdot (\hat{i} + \hat{j} - 3\hat{k})$ is equal to : [4 marks]

- (A) -6
(B) -5
(C) -4
(D) -3

Correct Answer: (B) -5

Solution:

Concept:

- Use anticommutativity, $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$, to combine cross products with a common first vector.
- If a nonzero vector has zero cross product with another vector, the latter is its scalar multiple. The dot-product condition then determines the remaining scalar.

Step 1: Rewrite the cross-product condition The given equation becomes

$$\vec{b} \times (3\vec{c} - 2\vec{a}) = \vec{0}.$$

Since $\vec{b} = (10, 2, -1)$ is nonzero, $3\vec{c} - 2\vec{a}$ is parallel to $\vec{b}$. Consequently we may write

$$\vec{c} = \frac{2}{3}\vec{a} + t\vec{b}$$

for a real number t . This retains all possible solutions of the cross-product relation; setting the bracket itself equal to zero would unnecessarily discard the component parallel to $\vec{b}$.

Step 2: Apply the scalar-product condition Calculate the required scalar products directly from the coordinates:

$$\vec{a} \cdot \vec{a} = 16 + 1 + 9 = 26, \quad \vec{a} \cdot \vec{b} = 40 - 2 - 3 = 35.$$

Therefore

$$15 = \vec{a} \cdot \vec{c} = \frac{52}{3} + 35t, \quad t = -\frac{1}{15}.$$

The value is fixed uniquely because $\vec{a} \cdot \vec{b} \neq 0$.

Step 3: Evaluate only the requested projection Put $\vec{v} = (1, 1, -3)$. Instead of expanding all three coordinates of $\vec{c}$, use linearity:

$$\vec{a} \cdot \vec{v} = 4 - 1 - 9 = -6, \quad \vec{b} \cdot \vec{v} = 10 + 2 + 3 = 15.$$

Thus

$$\vec{c} \cdot \vec{v} = \frac{2}{3}(-6) - \frac{1}{15}(15) = -4 - 1 = -5.$$

The alternative numbers $-6, -4, -3$ do not satisfy this dot-product evaluation; in particular, -4 omits the contribution parallel to $\vec{b}$.

Step 4: Conclusion The required scalar product is -5 . Hence, the correct option is (B).

Quick Tip to Solve: Factor a common vector out of the cross products, then take dot products with the target vector instead of finding every coordinate.

15. Let the foot of perpendicular from the point $(\lambda, 2, 3)$ on the line $\frac{x-4}{1} = \frac{y-9}{2} = \frac{z-5}{1}$ be the point $(1, \mu, 2)$. Then the distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ and $\frac{x-\lambda}{2} = \frac{y-\mu}{3} = \frac{z+5}{6}$ is equal to : [4 marks]

- (A) $\frac{12}{7}$
 (B) $\frac{\sqrt{145}}{7}$
 (C) $\frac{\sqrt{146}}{7}$
 (D) $\frac{\sqrt{143}}{7}$

Correct Answer: (C) $\frac{\sqrt{146}}{7}$

Solution:

Concept:

- A point on $(x-4)/1 = (y-9)/2 = (z-5)/1 = t$ has coordinates $(4+t, 9+2t, 5+t)$. A perpendicular displacement has zero dot product with the line direction.
- For parallel lines with common direction $\vec{d}$ and points A, B , the perpendicular distance is $|(B - A) \times \vec{d}|/|\vec{d}|$.

Step 1: Find the foot and the unknown coordinates The foot has first coordinate 1, so $4+t=1$, giving $t=-3$. Its third coordinate is then $5-3=2$, as required, and $\mu=9-6=3$. If $P=(\lambda, 2, 3)$ and $H=(1, 3, 2)$, their displacement is $P-H=(\lambda-1, -1, 1)$. Perpendicularity to $(1, 2, 1)$ yields

$$(\lambda-1) - 2 + 1 = 0, \quad \lambda = 2.$$

Step 2: Identify points on the parallel lines The two lines have the same direction $\vec{d} = (2, 3, 6)$. Taking zero as their respective parameters gives $A = (1, 2, -4)$ and $B = (2, 3, -5)$, using the newly obtained values of λ and μ . Thus

$$B - A = (1, 1, -1), \quad |\vec{d}| = \sqrt{4+9+36} = 7.$$

The separation $|B - A|$ itself is not the shortest distance, because this displacement need not be perpendicular to the direction.

Step 3: Calculate the perpendicular separation The cross product is

$$(1, 1, -1) \times (2, 3, 6) = (9, -8, 1).$$

Hence

$$d = \frac{\sqrt{9^2 + (-8)^2 + 1^2}}{7} = \frac{\sqrt{146}}{7}.$$

As a check, the perpendicular-component formula gives $|B - A|^2 - [(B - A) \cdot \vec{d}]^2/|\vec{d}|^2 = 3 - 1/49 = 146/49$. This also excludes the other three numerators in the options.

Step 4: Conclusion The distance is $\sqrt{146}/7$ units. Hence, the correct option is (C).

Quick Tip to Solve: For parallel lines, pick their parameter-zero points and remove the displacement component along their common direction.

16. The value of the integral $\int_0^2 \frac{\sqrt{x(x^2+x+1)}}{(\sqrt{x+1})(\sqrt{x^4+x^2+1})} dx$ is equal to:

[4 marks]

- (A) $\frac{1}{3} \log_e(3 - 2\sqrt{2})$
- (B) $\frac{2}{3} \log_e(4 + \sqrt{2})$
- (C) $\frac{2}{3} \log_e(3 + 2\sqrt{2})$
- (D) $\frac{1}{3} \log_e(1 + 6\sqrt{2})$

Correct Answer: (C) $\frac{2}{3} \log_e(3 + 2\sqrt{2})$

Solution:

Concept:

- The polynomial identity $x^4 + x^2 + 1 = (x^2 + x + 1)(x^2 - x + 1)$ allows cancellation of positive square-root factors on the integration interval.
- After simplification, use $t = x^{3/2}$. The standard integral is $\int dt/\sqrt{1+t^2} = \log(t + \sqrt{1+t^2}) + C$.

Step 1: Simplify the square roots before integrating For $0 \leq x \leq 2$, all denominator factors are positive, so cancelling the common factor $\sqrt{x^2 + x + 1}$ is valid. Also, $(x+1)(x^2 - x + 1) = x^3 + 1$. The integral consequently reduces to

$$I = \int_0^2 \frac{\sqrt{x}}{\sqrt{(x+1)(x^2-x+1)}} dx = \int_0^2 \frac{\sqrt{x}}{\sqrt{1+x^3}} dx.$$

No sign is lost: the square roots denote their nonnegative real values throughout.

Step 2: Make the power substitution Choose $t = x^{3/2}$, giving $dt = \frac{3}{2}\sqrt{x} dx$. The limits become $t = 0$ and $t = 2\sqrt{2}$. Therefore

$$I = \frac{2}{3} \int_0^{2\sqrt{2}} \frac{dt}{\sqrt{1+t^2}} = \frac{2}{3} \left[\log(t + \sqrt{1+t^2}) \right]_0^{2\sqrt{2}}.$$

Differentiating the logarithmic expression gives $1/\sqrt{1+t^2}$, which verifies the antiderivative directly.

Step 3: Evaluate and compare the options At the upper limit, $\sqrt{1 + (2\sqrt{2})^2} = \sqrt{9} = 3$; at the lower limit the logarithm is $\log 1 = 0$. Thus

$$I = \frac{2}{3} \log(3 + 2\sqrt{2}).$$

The integrand is nonnegative, immediately excluding the negative logarithmic value in (A). Options (B) and (D) use different logarithm arguments or coefficients and do not match the evaluated endpoints.

Step 4: Conclusion The integral equals $\frac{2}{3} \log_e(3 + 2\sqrt{2})$. Hence, the correct option is (C).

Quick Tip to Solve: Factor the quartic under the radical first; the surviving cubic suggests the substitution $x^{3/2}$.

17. Let $y = y(x)$ be the solution of the differential equation $x\sqrt{1-x^2}dy + (y\sqrt{1-x^2} - x\cos^{-1}x)dx = 0$, $x \in (0, 1)$, $\lim_{x \rightarrow 1^-} y(x) = 1$. Then $y\left(\frac{1}{2}\right)$ equals : [4 marks]

- (A) $3 - \frac{\pi}{\sqrt{3}}$
 (B) $4 - \sqrt{3}\pi$
 (C) $4 - \frac{2\pi}{\sqrt{3}}$
 (D) $3 - \frac{\pi}{2\sqrt{3}}$

Correct Answer: (A) $3 - \frac{\pi}{\sqrt{3}}$

Solution:

Concept:

- After dividing by $x\sqrt{1-x^2}$, the equation is linear in y , with coefficient $1/x$. Its integrating factor is $e^{\int dx/x} = x$, since $x > 0$.
- The boundary condition is a one-sided limit at 1. Apply it after integrating on $(0, 1)$, where all divisions and inverse trigonometric derivatives are defined.

Step 1: Convert the equation to an exact derivative Division by the nonzero factor $x\sqrt{1-x^2}$ yields

$$y' + \frac{y}{x} = \frac{\cos^{-1}x}{\sqrt{1-x^2}}.$$

Multiply by the integrating factor x :

$$\frac{d}{dx}(xy) = \frac{x \cos^{-1}x}{\sqrt{1-x^2}}.$$

The right-hand side must still retain its factor x ; omitting it would produce a different differential equation.

Step 2: Integrate by parts Use $u = \cos^{-1}x$ and $dv = x dx/\sqrt{1-x^2}$. Then $du = -dx/\sqrt{1-x^2}$ and $v = -\sqrt{1-x^2}$, so

$$xy = -\sqrt{1-x^2} \cos^{-1}x - x + C.$$

The sign of the integral term is negative because $v du = dx$. Differentiating this expression reproduces $x \cos^{-1}x/\sqrt{1-x^2}$.

Step 3: Use the limit and evaluate the function As $x \rightarrow 1^-$, the radical times $\cos^{-1}x$ tends to zero and $xy \rightarrow 1$. Hence $1 = -1 + C$, so $C = 2$. At $x = 1/2$, $\cos^{-1}(1/2) = \pi/3$ and $\sqrt{1-x^2} = \sqrt{3}/2$. Consequently

$$\frac{1}{2}y\left(\frac{1}{2}\right) = \frac{3}{2} - \frac{\sqrt{3}\pi}{6}, \quad y\left(\frac{1}{2}\right) = 3 - \frac{\pi}{\sqrt{3}}.$$

This fixes both the constant term and the coefficient of π , excluding (B), (C) and (D).

Step 4: Conclusion $y(1/2) = 3 - \pi/\sqrt{3}$. Hence, the correct option is (A).

Quick Tip to Solve: For a term y/x , multiply by x first; then integrate the inverse-cosine term by parts and apply the endpoint limit.

18. Let $f : (1, \infty) \rightarrow \mathbf{R}$ be a function defined as $f(x) = \frac{x-1}{x+1}$. Let $f^{i+1}(x) = f(f^i(x))$, $i = 1, 2, \dots, 25$, where $f^1(x) = f(x)$. If $g(x) + f^{26}(x) = 0$, $x \in (1, \infty)$, then the area of the region bounded by the curves $y = g(x)$, $2y = 2x - 3$, $y = 0$ and $x = 4$ is : [4 marks]

- (A) $\frac{1}{8} + \log_e 2$
 (B) $\frac{1}{4} + \log_e 2$
 (C) $\frac{5}{6} + 3 \log_e 2$
 (D) $\frac{5}{6} + \log_e 2$

Correct Answer: No well-defined literal answer: the declared domain prevents iteration; rational extension gives (A).

Solution:

Concept:

- A composition requires the inner value to belong to the domain of the outer function. Here the declared domain is $(1, \infty)$, while $0 < f(x) < 1$ for every $x > 1$.
- For the stated rational iteration, simplify repeated compositions algebraically; a short cycle avoids computing all twenty-six expressions separately.
- Area bounded by several curves is found piecewise, identifying where the upper boundary changes and integrating upper minus lower on each interval.

Step 1: Check the domain of the iterates For $x > 1$, both $x - 1$ and $x + 1$ are positive and $x - 1 < x + 1$. Thus $0 < f(x) < 1$, which is outside the declared domain $(1, \infty)$. Consequently $f(f(x))$ and hence $f^{26}(x)$ are not defined for the stated function. Literally, g and the requested bounded region are not defined. The calculation below gives the conditional interpretation obtained by extending the same rational formula to the intermediate arguments; it does not remove the printed domain defect.

Step 2: Find the composition cycle Under this rational-extension interpretation, Substituting the rational expression into itself gives

$$f^2(x) = \frac{(x-1)/(x+1) - 1}{(x-1)/(x+1) + 1} = -\frac{1}{x}.$$

Continuing the same rational substitution,

$$f^3(x) = -\frac{x+1}{x-1}, \quad f^4(x) = x.$$

Thus the expressions repeat with period four, and $26 = 4 \cdot 6 + 2$ implies $f^{26}(x) = -1/x$. The relation defining g therefore gives $g(x) = 1/x$ on the interval used for the area.

Step 3: Locate the boundary intersections The straight line is $y = x - 3/2$, meeting the horizontal axis at $x = 3/2$. Its intersection with $y = 1/x$ satisfies

$$x - \frac{3}{2} = \frac{1}{x}, \quad 2x^2 - 3x - 2 = 0.$$

The roots are 2 and $-1/2$; only 2 lies in $x > 1$. From $3/2$ to 2, the enclosed region lies under the line and above the horizontal axis. From 2 to the right boundary 4, it lies under the hyperbola and above that axis.

Step 4: Add the two positive areas Under the rational extension, the area is therefore

$$\begin{aligned} A &= \int_{3/2}^2 \left(x - \frac{3}{2}\right) dx + \int_2^4 \frac{dx}{x} \\ &= \frac{1}{2} \left(2 - \frac{3}{2}\right)^2 + \log 4 - \log 2 \\ &= \frac{1}{8} + \log 2. \end{aligned}$$

The first part is also a triangle of base and height $1/2$, confirming $1/8$. The other options use incorrect triangular areas or an incorrect logarithmic contribution.

Step 5: Conclusion As printed, the iteration is undefined and there is no well-defined literal answer. If the rational formula is extended to the intermediate arguments, the resulting area is $\frac{1}{8} + \log_e 2$ square units, corresponding conditionally to option (A).

Quick Tip to Solve: Check that a function maps into its own domain before iterating it; a rational-expression cycle can still reveal the intended conditional answer.

19. Let $f(x) = \begin{cases} \frac{1}{3}, & x \leq \pi/2 \\ \frac{b(1-\sin x)}{(\pi-2x)^2}, & x > \pi/2 \end{cases}$. **If f is continuous at $x = \pi/2$, then the value of**

$\int_0^{3b-6} |x^2 + 2x - 3| dx$ **is :**

[4 marks]

- (A) 5
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (D) 4

Solution:

Concept:

- Continuity at the joining point requires the right-hand limit to equal the constant left-hand value, namely $1/3$.
- Use $1 - \cos h = 2 \sin^2(h/2)$ and $\lim_{z \rightarrow 0} \sin z/z = 1$. An absolute-value integral must be split wherever its polynomial changes sign.

Step 1: Find the parameter from continuity Put $h = x - \pi/2$, so that $\sin x = \cos h$ and $(\pi - 2x)^2 = 4h^2$. The right-hand limit is therefore

$$\lim_{h \rightarrow 0^+} \frac{b(1 - \cos h)}{4h^2} = \frac{b}{8}.$$

Equating this to the value at the joining point gives $b/8 = 1/3$, hence $b = 8/3$. Consequently, the upper limit of the required integral is $3b - 6 = 2$.

Step 2: Split the integral at the sign change The polynomial factors as $x^2 + 2x - 3 = (x - 1)(x + 3)$. On the interval from zero to two, its only zero is $x = 1$; it is negative before this point and positive

after it. With $F(x) = x^3/3 + x^2 - 3x$, we obtain

$$\begin{aligned} I &= -\int_0^1 (x^2 + 2x - 3) dx + \int_1^2 (x^2 + 2x - 3) dx \\ &= -[F(1) - F(0)] + [F(2) - F(1)] \\ &= \frac{5}{3} + \left(\frac{2}{3} + \frac{5}{3}\right) = 4. \end{aligned}$$

Integrating the polynomial without the absolute value would instead give only $2/3$, so the sign split is essential. The other proposed integers do not equal the sum of the two positive areas.

Step 3: Conclusion The required integral equals 4. Hence, the correct option is (D).

Quick Tip to Solve: First determine the parameter at the joining point; then factor the polynomial before removing its absolute value.

20. Let $\frac{x^2}{f(a^2+7a+3)} + \frac{y^2}{f(3a+15)} = 1$ represent an ellipse with major axis along y -axis, where f is a strictly decreasing positive function on $\mathbb{R}$. If the set of all possible values of a is $\mathbb{R} - [\alpha, \beta]$, then $\alpha^2 + \beta^2$ is equal to : [4 marks]

- (A) 28
- (B) 40
- (C) 61
- (D) 24

Correct Answer: (B) 40

Solution:

Concept:

- For $x^2/A + y^2/B = 1$ with $A, B > 0$, the major axis is along the y -axis exactly when $B > A$. Equal denominators describe a circle rather than an ellipse with a distinguished major axis.
- A strictly decreasing function reverses strict inequalities: $f(u) > f(v)$ is equivalent to $u < v$. Positivity guarantees that both squared semiaxes are admissible.

Step 1: Compare the squared semiaxes Here the squared semiaxis along x is $f(a^2 + 7a + 3)$, whereas that along y is $f(3a + 15)$. Thus the required orientation imposes

$$f(3a + 15) > f(a^2 + 7a + 3).$$

Because f is strictly decreasing, this is equivalent to

$$3a + 15 < a^2 + 7a + 3.$$

There is no additional denominator-positivity restriction on a , since the function is positive on the entire real line.

Step 2: Solve the strict quadratic inequality Rearranging and factoring gives

$$a^2 + 4a - 12 > 0 \iff (a + 6)(a - 2) > 0.$$

The product is positive outside its two roots, so

$$a \in (-\infty, -6) \cup (2, \infty) = \mathbf{R} - [-6, 2].$$

The endpoints are excluded because the denominators become equal there. Comparing with the interval in the question gives $\alpha = -6$, $\beta = 2$. Hence

$$\alpha^2 + \beta^2 = 36 + 4 = 40.$$

The values 28, 61 and 24 do not follow from these boundary roots; reversing the monotonicity inequality would incorrectly select the interval between them.

Step 3: Conclusion We obtain $\alpha^2 + \beta^2 = 40$. Hence, the correct option is (B).

Quick Tip to Solve: Compare the denominators first, then reverse the inequality once when applying a strictly decreasing function.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. The sum of squares of all the real solutions of the equation

$\log_{(x+1)}(2x^2 + 5x + 3) = 4 - \log_{(2x+3)}(x^2 + 2x + 1)$ is equal to ____ .

[4 marks]

Solution:

Concept:

- A real logarithm requires a positive argument and a positive base different from one. The identity $\log_a b = 1/\log_b a$ applies when both logarithms exist.
- Factor the arguments before introducing a logarithmic variable; this turns the equation into a quadratic while retaining the original domain restrictions.

Step 1: Establish the domain and simplify the logarithms Let $u = x + 1$. The first base requires $u > 0$, $u \neq 1$, so $x > -1$, $x \neq 0$. The second base is $2x + 3 = 2u + 1 > 1$, and both arguments are positive in this domain. Since

$$2x^2 + 5x + 3 = u(2u + 1), \quad x^2 + 2x + 1 = u^2,$$

put $t = \log_u(2u + 1)$. This is nonzero because $2u + 1 > 1$. The equation becomes

$$1 + t = 4 - \frac{2}{t}.$$

Step 2: Solve both branches and check admissibility Multiplication by nonzero t gives

$$t^2 - 3t + 2 = 0, \quad (t - 1)(t - 2) = 0.$$

For $t = 1$, we would need $2u + 1 = u$, giving $u = -1$, which violates positivity. For $t = 2$, we require $2u + 1 = u^2$, hence $u = 1 \pm \sqrt{2}$. Only $u = 1 + \sqrt{2}$ is positive; it is also different from one. Therefore $x = u - 1 = \sqrt{2}$ is the only real solution. Substitution gives the first logarithm equal to 3 and the second equal to 1, confirming the original equation. The requested sum contains just one square, rather than both roots of the auxiliary quadratic in u .

Step 3: Conclusion The only admissible real solution is $x = \sqrt{2}$, so the sum of squares is 2.

Quick Tip to Solve: Keep the positive-base restriction visible when solving the algebraic equations produced by logarithm substitutions.

22. If $\int_{\pi/6}^{\pi/4} (\cot(x - \frac{\pi}{3}) \cot(x + \frac{\pi}{3}) + 1) dx = \alpha \log_e(\sqrt{3} - 1)$, then $9\alpha^2$ is equal to ____ . [4 marks]

Solution:

Concept:

- The addition identity gives $\cot A \cot B + 1 = \cos(A - B)/(\sin A \sin B)$. Also, $\sin(x - c) \sin(x + c) = \sin^2 x - \sin^2 c$.
- After setting $t = \tan x$, use $dx = dt/(1 + t^2)$ and $\sin^2 x = t^2/(1 + t^2)$. Transform both endpoints along with the integrand.

Step 1: Reduce the integrand to a rational form Taking $A = x - \pi/3$, $B = x + \pi/3$, the numerator is $\cos(-2\pi/3) = -1/2$. Thus

$$\cot A \cot B + 1 = \frac{-1/2}{\sin^2 x - 3/4} = \frac{2}{3 - 4 \sin^2 x}.$$

With $t = \tan x$, the limits become $1/\sqrt{3}$ and 1, so

$$I = \int_{1/\sqrt{3}}^1 \frac{2 dt}{3 - t^2}.$$

The denominator is positive throughout this interval, so the integral is finite and positive.

Step 2: Evaluate and identify the logarithmic coefficient Partial fractions or the standard logarithmic antiderivative gives

$$\int \frac{2 dt}{3 - t^2} = \frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{3} + t}{\sqrt{3} - t} \right|.$$

The ratio inside the logarithm is 2 at the lower endpoint and $2 + \sqrt{3}$ at the upper endpoint. Therefore

$$I = \frac{1}{\sqrt{3}} \log \frac{2 + \sqrt{3}}{2}.$$

Since

$$(\sqrt{3} - 1)^2 = 4 - 2\sqrt{3}, \quad \frac{2 + \sqrt{3}}{2} = (\sqrt{3} - 1)^{-2},$$

it follows that $I = -2 \log(\sqrt{3} - 1)/\sqrt{3}$. The coefficient is negative because the logarithm in the question is negative, while the integral itself is positive.

Step 3: Conclusion Thus $\alpha = -2/\sqrt{3}$, and $9\alpha^2 = 9(4/3) = 12$.

Quick Tip to Solve: Use the cotangent identity before expanding; the tangent substitution then produces a simple difference of squares.

23. Let a line L_1 pass through the origin and be perpendicular to the lines

$L_2 : \vec{r} = (3 + t)\hat{i} + (2t - 1)\hat{j} + (2t + 4)\hat{k}$ and

$L_3 : \vec{r} = (3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (2 + s)\hat{k}, t, s \in \mathbf{R}$.

If $(a, b, c), a \in \mathbf{Z}$, is the point on L_3 at a distance of $\sqrt{17}$ from the point of intersection of L_1 and L_2 , then $(a + b + c)^2$ is equal to ____ . [4 marks]

Solution:

Concept:

- A line perpendicular to two given directions has a direction vector proportional to their cross product. A line through the origin therefore has the form $\mathbf{r} = \lambda \mathbf{d}$.
- For points P, Q , the squared distance is $|Q - P|^2$. The integer-coordinate condition must be applied after solving the resulting quadratic, because it selects the admissible point.

Step 1: Find the intersection with the perpendicular line The direction vectors of the two given lines are $(1, 2, 2)$ and $(2, 2, 1)$. Their cross product is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = (-2, 3, -2).$$

Hence L_1 is $\mathbf{r} = \lambda(-2, 3, -2)$. At its intersection with L_2 , compare the first and third coordinates: $3 + t = 2t + 4$, giving $t = -1$. The point is $P = (2, -3, 2)$, which equals $-1(-2, 3, -2)$, so all three coordinates satisfy L_1 .

Step 2: Use the distance and integer condition A point on L_3 is $Q = (3 + 2s, 3 + 2s, 2 + s)$. Thus

$$\begin{aligned} |Q - P|^2 &= (1 + 2s)^2 + (6 + 2s)^2 + s^2 \\ &= 9s^2 + 28s + 37. \end{aligned}$$

Setting this equal to 17 gives

$$9s^2 + 28s + 20 = 0 = (s + 2)(9s + 10).$$

For $s = -2$, the point is $(-1, -1, 0)$, whose first coordinate is an integer. For $s = -10/9$, the first coordinate is $7/9$, so that candidate is excluded. The retained point is indeed at squared distance $9 + 4 + 4 = 17$ from P .

Step 3: Conclusion We have $(a, b, c) = (-1, -1, 0)$. Therefore $(a + b + c)^2 = (-2)^2 = 4$.

Quick Tip to Solve: Use a cross product for the perpendicular direction, and postpone the integer-coordinate test until both distance roots are known.

24. Consider the circle $C : x^2 + y^2 - 6x - 8y - 11 = 0$. Let a variable chord AB of the circle C subtend a right angle at the origin. If the locus of the foot of the perpendicular drawn from the origin on the chord AB is the circle $x^2 + y^2 - \alpha x - \beta y - \gamma = 0$, then $\alpha + \beta + 2\gamma$ is equal to ____ . [4 marks]

Solution:

Concept:

- A chord whose endpoints subtend a right angle at a point has that point on the circle with the chord as diameter. Equivalently, for endpoint position vectors $\mathbf{a}, \mathbf{b}$, $\mathbf{a} \cdot \mathbf{b} = 0$.
- The foot of the perpendicular from the centre of a circle bisects a chord. Resolve the centre and the origin along a unit normal to the chord to relate its midpoint and half-length.

Step 1: Express the chord with a unit normal The circle is

$$(x - 3)^2 + (y - 4)^2 = 36,$$

with centre $C = (3, 4)$ and radius 6. Let the chord line be $\mathbf{n} \cdot \mathbf{r} = p$, where $|\mathbf{n}| = 1$. Its perpendicular foot from the origin is $H = p\mathbf{n} = (h, k)$, so $p^2 = h^2 + k^2$ and $p(\mathbf{n} \cdot C) = 3h + 4k$. Choose a perpendicular unit vector $\mathbf{u}$. The chord endpoints can be written as $p\mathbf{n} + (c \pm \ell)\mathbf{u}$, where $c = C \cdot \mathbf{u}$.

Step 2: Apply the right-angle condition and eliminate the normal The squared half-length is $\ell^2 = 36 - (C \cdot \mathbf{n} - p)^2$. The endpoints have zero dot product because the angle at the origin is right, so

$$0 = p^2 + c^2 - \ell^2.$$

Using $c^2 + (C \cdot \mathbf{n})^2 = |C|^2 = 25$, this simplifies to

$$2p^2 - 2p(C \cdot \mathbf{n}) - 11 = 0.$$

Hence the locus of $H = (h, k)$ is

$$h^2 + k^2 - 3h - 4k - \frac{11}{2} = 0.$$

Conversely, this equation gives the right-angle relation for the corresponding chord. Its radius is positive, and its points yield genuine intersections of the chord line with the original circle; the construction therefore supplies the stated circle locus.

Step 3: Conclusion Comparing coefficients gives $\alpha = 3$, $\beta = 4$, $\gamma = 11/2$. Thus $\alpha + \beta + 2\gamma = 3 + 4 + 11 = 18$.

Quick Tip to Solve: Use a unit normal to the chord: its distance from the origin is exactly the length of the perpendicular-foot vector.

25. Let f be a polynomial function such that

$$\log_2(f(x)) = \left(\log_2 \left(2 + \frac{2}{3} + \frac{2}{9} + \dots \infty \right) \right) \cdot \log_3 \left(1 + \frac{f(x)}{f(1/x)} \right), \quad x > 0,$$

and $f(6) = 37$. Then $\sum_{n=1}^{10} f(n)$ is equal to ____.

[4 marks]

Solution:

Concept:

- The infinite geometric series has sum $a/(1-r)$ when $|r| < 1$. Here its sum is $2/(1-1/3) = 3$.
- The change-of-base identity gives $\log_2 3 \log_3 A = \log_2 A$. Equality of logarithms with the same base gives equality of their positive arguments.
- A polynomial g for which $g(x)g(1/x) = 1$ for all positive x must be a monomial; its lowest and highest powers cannot be distinct.

Step 1: Remove the logarithms The geometric sum is 3. Consequently the given equation becomes $\log_2 f(x) = \log_2(1 + f(x)/f(1/x))$. Since the logarithms exist, $f(x) > 0$ and $f(1/x) > 0$. Thus

$$f(x) = 1 + \frac{f(x)}{f(1/x)}, \quad f(x)f(1/x) = f(x) + f(1/x).$$

Subtract the sum and add 1 to factor the identity:

$$(f(x) - 1)(f(1/x) - 1) = 1.$$

Step 2: Determine the polynomial Put $g(x) = f(x) - 1$. Let its smallest and largest nonzero powers be k and m , with coefficients a_k, a_m . If $m > k$, the product $g(x)g(1/x)$ has a unique term of highest power x^{m-k} , with nonzero coefficient $a_m a_k$; it cannot be the constant 1. Hence $m = k$, so $g(x) = ax^m$, and the product identity gives $a^2 = 1$. Since $g(6) = 36$, the sign is positive and $6^m = 36$, whence $m = 2$. Therefore $f(x) = 1 + x^2$, which also satisfies the original logarithmic equation for every positive x .

Step 3: Sum the requested values Use the sum of the first ten squares, adding the constant term once for each input:

$$\sum_{n=1}^{10} f(n) = \sum_{n=1}^{10} (1 + n^2) = 10 + \frac{10 \cdot 11 \cdot 21}{6} = 10 + 385 = 395.$$

Step 4: Conclusion The required numerical value is 395.

Quick Tip to Solve: After removing logarithms, subtract 1 from the polynomial: the reciprocal-product identity then forces a single power of x .

Physics – Section A: Multiple Choice Questions (80 marks)

26. A new unit (α) of length is chosen such that it is equal to the speed of light in vacuum. What is the distance between Venus and Earth in terms of α units if light takes 6 min. 40 s to cover this distance? [4 marks]

- (A) 200α
- (B) 400α
- (C) 300α
- (D) 500α

Correct Answer: (B) 400α

Solution:

Concept:

- A unit of length specifies a distance; the numerical measure of a distance is the physical distance divided by that unit.
- For uniform light propagation in vacuum, $d = ct$, where c is the speed of light and t is the travel time.

Step 1: Interpret the new length unit The statement uses the numerical value of the speed of light to define the new length unit. In SI terms the intended definition is $1\alpha = c \times 1\text{ s}$: one new length unit is the distance travelled by light in one second. This is how the length convention is used; a length is not dimensionally equated to a velocity. Taking $c = 3 \times 10^8 \text{ m s}^{-1}$, the unit has size $3 \times 10^8 \text{ m}$. The same value of c will appear in the physical distance and in the unit size, so it cancels.

Step 2: Convert the entire travel time to seconds The given time contains both minutes and seconds. Convert the minutes before combining them:

$$t = 6(60) + 40 = 400 \text{ s.}$$

Therefore the distance covered by light between the two planets is

$$d = c(400 \text{ s}) = 400 (c \times 1 \text{ s}) = 400\alpha.$$

Equivalently, in metres the distance is $1.2 \times 10^{11} \text{ m}$; dividing by $3 \times 10^8 \text{ m}$ per new unit again gives 400. The alternatives 200, 300 and 500 would correspond to travel times of 200, 300 and 500 seconds respectively, so none represents the stated time.

Step 3: Conclusion The Earth–Venus distance is 400α . Hence, the correct option is (B).

Quick Tip to Solve: When the length unit is the distance light travels in one second, the light-travel time in seconds directly gives the number of length units.

27. Consider the equation $H = \frac{x^p \epsilon^q E^r}{t^s}$

Where H = magnetic field; E = electric field, ϵ = permittivity, x = distance, t = time The values of p, q, r and s respectively are : [4 marks]

- (A) 1, 1, 1, 1
(B) -1, 1, 2, 1
(C) 1, -1, -2, 1
(D) -1, -2, -2, 1

Correct Answer: (A) 1, 1, 1, 1

Solution:

Concept:

- Magnetic field intensity H has dimensions $[H] = IL^{-1}$. The electric flux density is $D = \epsilon E$, with dimensions of charge per area.
- Use $[E] = MLT^{-3}I^{-1}$ and $[\epsilon] = M^{-1}L^{-3}T^4I^2$, then equate the exponents of independent fundamental dimensions.

Step 1: Write the dimensions of both sides The symbol H denotes magnetic field intensity, whose SI unit is ampere per metre. It is distinct from magnetic flux density B , measured in tesla. For the proposed expression, substitution of the dimensions gives

$$\left[\frac{x^p \epsilon^q E^r}{t^s} \right] = M^{-q+r} L^{p-3q+r} T^{4q-3r-s} I^{2q-r}.$$

This must be identical to $M^0 L^{-1} T^0 I^1$. Dimensional comparison determines the powers without requiring a numerical value for any field or length.

Step 2: Solve the exponent equations Comparison of mass and current exponents first gives

$$-q + r = 0, \quad 2q - r = 1.$$

Hence $r = q$, and then $q = r = 1$. Comparing the length exponent now yields $p - 3 + 1 = -1$, so $p = 1$. Finally the time exponent requires $4 - 3 - s = 0$, giving $s = 1$. Thus the combination is $x\epsilon E/t$. A direct units check is useful: ϵE has unit $C\,m^{-2}$, and multiplication by x/t changes this to $C\,m^{-1}s^{-1} = A\,m^{-1}$. The other triples fail the mass/current equations or the required length power; they cannot produce the unit of H .

Step 3: Conclusion The ordered values are $(p, q, r, s) = (1, 1, 1, 1)$. Hence, the correct option is (A).

Quick Tip to Solve: Identify H as magnetic field intensity before comparing dimensions; using the dimensions of B would test a different quantity.

28. A car moving with a speed of 54 km/h takes a turn of radius 20 m . A simple pendulum is suspended from the ceiling of the car. Determine the angle made by the string of the pendulum with the vertical during the turning. (Take $g = 10 \text{ m/s}^2$) [4 marks]

- (A) $\tan^{-1}(0.5)$
 (B) $\tan^{-1}(0.75)$
 (C) $\tan^{-1}(1.125)$
 (D) $\tan^{-1}(0.25)$

Correct Answer: (C) $\tan^{-1}(1.125)$

Solution:

Concept:

- For a car taking a circular turn, the horizontal centripetal acceleration is $a_c = v^2/R$, directed towards the centre of the turn.
- For a pendulum at rest relative to the car, tension supplies horizontal acceleration while its vertical component balances weight: $T \sin \theta = mv^2/R$, $T \cos \theta = mg$.

Step 1: Convert the speed and compute the acceleration The speed must be expressed in metres per second because the radius and gravitational acceleration are given in SI units. Thus

$$v = 54 \frac{1000}{3600} = 15 \text{ m s}^{-1}, \quad a_c = \frac{15^2}{20} = 11.25 \text{ m s}^{-2}.$$

During the steady turn the bob follows the same circular motion as the car. Its acceleration is horizontal; it has no vertical acceleration when the string has attained its steady inclination.

Step 2: Resolve the tension and find the inclination Let θ be the angle between the string and the vertical, and m the bob mass. The tension points along the string towards its support, with an inward horizontal component. The bob is displaced towards the outside of the turn. Resolving forces and dividing the two component equations eliminates both tension and mass:

$$\frac{T \sin \theta}{T \cos \theta} = \frac{mv^2/R}{mg}, \quad \tan \theta = \frac{v^2}{Rg} = \frac{225}{20 \cdot 10} = 1.125.$$

Consequently $\theta = \tan^{-1}(1.125)$, about 48.4° . The other arguments 0.5, 0.75 and 0.25 all give a horizontal-to-vertical force ratio smaller than the required 11.25/10; they do not satisfy the force components.

Step 3: Conclusion The string makes an angle $\tan^{-1}(1.125)$ with the vertical. Hence, the correct option is (C).

Quick Tip to Solve: For a pendulum in a steady horizontal acceleration, divide the horizontal and vertical tension equations: the mass and tension cancel immediately.

29. A gas balloon is going up with a constant velocity of 10 m/s . When this balloon reached a height of 75 m , a stone is dropped from it and balloon keeps moving up with the same velocity. The height of the balloon when the stone hits the ground is ____ m. (Take $g = 10 \text{ m/s}^2$) [4 marks]

- (A) 85
(B) 150
(C) 129
(D) 125

Correct Answer: (D) 125

Solution:

Concept:

- On release, an object retains the instantaneous velocity of its carrier. Here the stone starts with an upward velocity of 10 m s^{-1} , not zero.
- With upward positive, the stone obeys $y = y_0 + ut - \frac{1}{2}gt^2$; the balloon continues uniform motion $y_b = y_0 + v_b t$.

Step 1: Find when the stone reaches the ground Choose the ground as zero height and measure time from the instant of release. The initial height of the stone is 75 m , its upward initial velocity is 10 m s^{-1} , and its acceleration is -10 m s^{-2} . Thus

$$y_s(t) = 75 + 10t - 5t^2.$$

At impact $y_s = 0$, so

$$5t^2 - 10t - 75 = 0, \quad t^2 - 2t - 15 = (t - 5)(t + 3) = 0.$$

The root $t = -3 \text{ s}$ lies before release and is inadmissible. The physical time of flight is therefore 5 s . The positive initial velocity matters: the stone first rises before falling to the ground.

Step 2: Evaluate the balloon height at that instant The balloon is not in free fall. It continues upwards with its given constant velocity throughout these five seconds, so its additional rise is

$$\Delta y_b = 10 \times 5 = 50 \text{ m}.$$

Adding its initial height gives

$$y_b = 75 + 50 = 125 \text{ m}.$$

As a consistency check, the stone velocity at impact is $10 - 10(5) = -40 \text{ m s}^{-1}$; its displacement is $(10 - 40)5/2 = -75 \text{ m}$, exactly the drop from the release height to the ground. The proposed heights 85 , 150 and 129 m correspond to different flight times and fail the stone's impact equation.

Step 3: Conclusion The height of the balloon is $\boxed{125 \text{ m}}$. Hence, the correct option is (D).

Quick Tip to Solve: A stone dropped from a rising balloon inherits its upward velocity. Calculate its flight time first, then use uniform motion for the balloon.

30. A thin biconvex lens is prepared from the glass ($\mu = 1.5$) both curved surfaces of which have equal radii of 20 cm each. Left side surface of the lens is silvered from outside to make it reflecting. To have the position of image and object at the same place, the object should be placed, from the lens at a distance of ____ cm. [4 marks]

- (A) 10
(B) 12.5
(C) 13
(D) 13.5

Correct Answer: (A) 10

Solution:

Concept:

- A thin lens with one surface silvered acts as a reflecting optical system: the ray is refracted on entering, reflected at the silvered surface, and refracted again on leaving.
- For a spherical refracting surface, $n_2/v - n_1/u = (n_2 - n_1)/R$, using distances signed along the incident direction. Rays retrace their paths when they reach a spherical mirror normally.

Step 1: Apply the condition for retracing the rays Light must enter through the unsilvered right surface and reach the silvered left surface through the glass. For the final image to coincide with the object, the refracted rays must strike the reflecting surface normally and return along the same path. Thus the first refracting surface must form an image at the centre of curvature of the silvered surface. In the thin-lens approximation this centre lies 20 cm on the incident, right-hand side of the lens. It is a virtual image for the first refraction. Take the initial direction of propagation, from right to left, as positive; both this image and the object then have negative signed distances.

Step 2: Use refraction at the entrance surface Let the positive object distance be d . For the entrance surface, $n_1 = 1$, $n_2 = 1.5$, $u = -d$, $R = +20$ cm, and the required image distance is $v = -20$ cm. Therefore

$$\frac{1.5}{-20} - \frac{1}{-d} = \frac{1.5 - 1}{20}, \quad -0.075 + \frac{1}{d} = 0.025.$$

This yields $1/d = 0.100 \text{ cm}^{-1}$, so $d = 10$ cm. Reflection then reverses each ray, and the second refraction returns it to the original object point by reversibility. The other listed distances do not make the first refracted beam normal to the silvered surface, so they do not give coincident object and image.

Step 3: Conclusion The object should be placed 10 cm from the lens on its unsilvered side. Hence, the correct option is (A).

Quick Tip to Solve: For an object to coincide with its image in a silvered lens, require the first refracted rays to pass through the reflecting surface's centre of curvature.

31. Two identical bodies, projected with the same speed at two different angles cover the same horizontal range R . If the time of flight of these bodies are 5 s and 10 s, respectively, then the value of R is

_____ m. (Take $g = 10 \text{ m/s}^2$)

[4 marks]

(A) 250

(B) 25

(C) 500

(D) 125

Correct Answer: (A) 250

Solution:

Concept:

- For projection and landing at the same height, the time of flight is $T = 2u \sin \theta / g$, where u is the initial speed.
- The horizontal range is $R = u^2 \sin 2\theta / g$. Equal ranges at distinct angles for the same speed imply complementary projection angles.

Step 1: Relate the two projection angles Let the angles be θ_1 and θ_2 . Since the speeds and ranges are equal, $\sin 2\theta_1 = \sin 2\theta_2$. The two distinct physical launch angles therefore satisfy $\theta_1 + \theta_2 = 90^\circ$. Thus $\sin \theta_2 = \cos \theta_1$. This uses the usual projectile model with uniform gravity and no air resistance.

Step 2: Combine the two flight times The given flight times supply both components of the common initial speed:

$$T_1 = \frac{2u \sin \theta_1}{g}, \quad T_2 = \frac{2u \cos \theta_1}{g}.$$

Multiplying eliminates the separate angles:

$$T_1 T_2 = \frac{4u^2 \sin \theta_1 \cos \theta_1}{g^2} = \frac{2R}{g}.$$

Hence the range can be obtained without first finding either angle or the launch speed.

Step 3: Evaluate and compare the choices Substitute both times in seconds and the given gravitational acceleration:

$$R = \frac{g T_1 T_2}{2} = \frac{10 \times 5 \times 10}{2} = 250 \text{ m}.$$

The dimensions are $(\text{m s}^{-2})\text{s}^2 = \text{m}$, as required. The choices 25, 500 and 125 do not satisfy $T_1 T_2 = 2R/g = 50 \text{ s}^2$; in particular 500 results from forgetting the factor of two.

Step 4: Conclusion The common horizontal range is 250 m. Hence, the correct option is (A).

Quick Tip to Solve: For complementary launches with equal speed, use $R = g T_1 T_2 / 2$ directly; neither angle needs to be calculated.

32. A solid cylinder having radius R and length L is slipping on a rough horizontal plane. At time $t = 0$ the cylinder has a translational velocity $v_0 = 49 \text{ m/s}$, perpendicular to its axis and a rotational velocity $v_0/4R$ about the centre. The time taken by the cylinder to start rolling is ____ seconds. (coefficient of kinetic friction $\mu_K = 0.25$ and $g = 9.8 \text{ m/s}^2$) [4 marks]

- (A) 15
(B) 5
(C) 10
(D) 7.5

Correct Answer: (B) 5

Solution:

Concept:

- A slipping cylinder experiences kinetic friction $f = \mu_K mg$. This changes both its centre-of-mass speed and its angular speed.
- For a uniform solid cylinder, $I = \frac{1}{2}mR^2$. Pure rolling begins when the contact point has zero velocity relative to the ground, so $v = R\omega$.

Step 1: Determine the frictional accelerations Take the initial rotation in the rolling sense. Since $R\omega_0 = v_0/4 < v_0$, the bottom surface slips forward relative to the plane, and friction acts backwards. Thus

$$\frac{dv}{dt} = -\mu_K g.$$

The frictional torque increases rotation in the rolling sense:

$$\alpha = \frac{fR}{I} = \frac{\mu_K mgR}{mR^2/2} = \frac{2\mu_K g}{R}.$$

The mass, length and radius cancel from the eventual time.

Step 2: Impose the rolling condition While slipping persists, the accelerations are constant. Therefore

$$v = v_0 - \mu_K gt, \quad R\omega = \frac{v_0}{4} + 2\mu_K gt.$$

Set these expressions equal at the first instant of pure rolling:

$$v_0 - \mu_K gt = \frac{v_0}{4} + 2\mu_K gt.$$

This gives $3v_0/4 = 3\mu_K gt$, or

$$t = \frac{v_0}{4\mu_K g}.$$

It would be incorrect to keep the angular velocity fixed while friction slows translation.

Step 3: Substitute and verify Using the stated values,

$$t = \frac{49}{4(0.25)(9.8)} = 5 \text{ s}.$$

At this time $v = 49 - 2.45(5) = 36.75 \text{ m s}^{-1}$, and $R\omega = 12.25 + 4.9(5) = 36.75 \text{ m s}^{-1}$, confirming zero

slip. None of 15, 10 or 7.5 seconds is the first instant at which these velocities become equal.

Step 4: Conclusion The cylinder starts rolling after $\boxed{5\text{ s}}$. Hence, the correct option is (B).

Quick Tip to Solve: Track the relative slip speed $v - R\omega$; for a solid cylinder it decreases at $3\mu_K g$ while slipping.

33. A liquid of density 600 kg/m^3 flowing steadily in a tube of varying cross-section. The cross-section at a point A is 1.0 cm^2 and that at B is 20 mm^2 . Both the points A and B are in same horizontal plane, the speed of the liquid at A is 10 cm/s . The difference in pressures at A and B points is ____ Pa. [4 marks]

- (A) 18
- (B) 144
- (C) 36
- (D) 72

Correct Answer: (D) 72

Solution:

Concept:

- For steady incompressible flow, conservation of volume gives $A_A v_A = A_B v_B$. The narrower part therefore carries the faster flow.
- For two points at the same height, Bernoulli's equation gives $P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2$.

Step 1: Express the data in consistent units The area at A is $1.0\text{ cm}^2 = 100\text{ mm}^2$, whereas that at B is 20 mm^2 . Their ratio is therefore five, not fifty. Also

$$v_A = 10\text{ cm s}^{-1} = 0.10\text{ m s}^{-1}.$$

These conversions must precede substitution into the kinetic-energy term because the speed is squared.

Step 2: Find the speed in the narrower section Apply continuity to the same liquid at the two sections:

$$v_B = \frac{A_A}{A_B} v_A = \frac{100}{20}(0.10) = 0.50\text{ m s}^{-1}.$$

The speed rises by a factor of five. Consequently the static pressure at B must be lower than at A under the ideal Bernoulli description; the gravitational terms cancel since the two points are horizontal.

Step 3: Calculate the pressure difference Rearranging Bernoulli's equation,

$$P_A - P_B = \frac{1}{2}\rho(v_B^2 - v_A^2).$$

With $\rho = 600\text{ kg m}^{-3}$,

$$P_A - P_B = \frac{1}{2}(600)(0.50^2 - 0.10^2) = 300(0.25 - 0.01) = 72\text{ Pa}.$$

The values 18, 144 and 36 fail this direct substitution. In particular 144 doubles the required answer by omitting the factor one-half in the kinetic-energy density.

Step 4: Conclusion The pressure at A exceeds that at B by 72 Pa . Hence, the correct option is (D).

Quick Tip to Solve: Convert speeds to metres per second before squaring; use the area ratio only to find the second speed.

34. A spherical liquid drop of radius R acquires the terminal velocity v_1 when falls through a gas of viscosity η . Now the drop is broken into 64 identical droplets and each droplet acquires terminal velocity v_2 falling through the same gas. The ratio of terminal velocities v_1/v_2 is ____ . [4 marks]

- (A) 4
- (B) 0.25
- (C) 32
- (D) 16

Correct Answer: (D) 16

Solution:

Concept:

- For a small sphere falling in the Stokes regime, $v_t = 2(\rho_\ell - \rho_g)gr^2/(9\eta)$, with liquid and gas densities ρ_ℓ and ρ_g .
- Breaking a drop conserves liquid volume. Identical droplets share the original volume equally; their radii do not scale directly with their number.

Step 1: Find the radius of each droplet Let the radius of one of the 64 droplets be r . Equating the original liquid volume to the combined volume of the droplets gives

$$\frac{4\pi R^3}{3} = 64 \frac{4\pi r^3}{3}.$$

Cancelling the common spherical-volume factor gives $R^3 = 64r^3$, hence

$$r = \frac{R}{4}.$$

This cube-root dependence is essential: dividing the radius by 64 would violate conservation of volume.

Step 2: Compare the force balances At terminal speed, the effective weight (weight minus buoyancy) balances the viscous drag $6\pi\eta r v_t$. The effective weight varies as r^3 , while the drag coefficient varies as r . Their ratio therefore makes v_t proportional to r^2 . Both sizes fall through the same gas and have the same liquid density, so all other factors in Stokes' formula cancel in the ratio.

Step 3: Evaluate the terminal-speed ratio Thus

$$\frac{v_1}{v_2} = \frac{R^2}{r^2} = \left(\frac{R}{R/4}\right)^2 = 16.$$

The smaller droplets fall more slowly, which also checks that this ratio must exceed one. The choice 4 uses only the radius ratio, 0.25 reverses that radius ratio, and 32 does not follow either volume conservation or the squared-radius law. The stated viscosity has no numerical role because it is unchanged.

Step 4: Conclusion The ratio is $v_1/v_2 = 16$. Hence, the correct option is (D).

Quick Tip to Solve: For a drop divided into N identical parts, Stokes terminal speed gives $v_{\text{original}}/v_{\text{small}} = N^{2/3}$.

35. One mole of diatomic gas having rotational modes only is kept in a cylinder with a piston system. The cross-section area of the cylinder is 4 cm^2 . The gas is heated slowly to raise the temperature by 1.2°C during which the piston moves by 25 mm . The amount of heat supplied to the gas is ____ J. (Atmospheric pressure = 100 kPa , $R = 8.3 \text{ J/mol.K}$) (Neglect mass of the piston) [4 marks]

- (A) 24.8
- (B) 25
- (C) 15.04
- (D) 29.98

Correct Answer: The printed data are inconsistent, and none of the four options follows from them.

Solution:

Concept:

- For heat supplied, the first law is $Q = \Delta U + W$, where positive W is work done by the gas against its surroundings.
- A linear diatomic molecule has two rotational degrees of freedom, contributing $C_{V,\text{rot}} = R$. With translation also included and vibration inactive, its usual total molar heat capacity is $5R/2$.
- A massless piston heated slowly against fixed atmospheric pressure describes an isobaric process. For one mole of ideal gas, its measured changes must obey $P\Delta V = R\Delta T$.

Step 1: Calculate the stated piston work Convert the cross-sectional area and displacement before multiplying:

$$\Delta V = Ax = (4 \times 10^{-4})(25 \times 10^{-3}) = 10^{-5} \text{ m}^3.$$

The external pressure is $100 \text{ kPa} = 10^5 \text{ Pa}$, so the work specified by the piston displacement is

$$W = P\Delta V = (10^5)(10^{-5}) = 1.00 \text{ J}.$$

Step 2: Test the stated temperature change The temperature rise is 1.2 K , so

$$nR\Delta T = (1)(8.3)(1.2) = 9.96 \text{ J}.$$

This differs from $P\Delta V = 1.00 \text{ J}$. Thus the stated mole number, temperature rise, atmospheric pressure and piston displacement cannot simultaneously describe the specified slow ideal-gas process. The displacement would instead imply $\Delta T = 1/8.3 \text{ K}$, not 1.2 K .

Step 3: Check the heat-capacity interpretations Taking “rotational modes only” literally gives the formal rotational-energy increase $\Delta U_{\text{rot}} = nR\Delta T = 9.96 \text{ J}$. Adding the separately stated work gives 10.96 J , which is absent from the options. If the phrase were read as the usual diatomic model with translation and rotation active, the corresponding formal sum would instead be

$$Q = \frac{5}{2}(9.96) + 1 = 25.90 \text{ J},$$

also absent from the options. Neither calculation removes the isobaric inconsistency. In particular, option (B) is 25 J , not 10.96 J ; it cannot be obtained by relabelling the literal result.

Step 4: Conclusion As printed, the question has inconsistent physical data and no defensible unique option among (A)–(D). The formal rotational-only calculation gives 10.96 J , but it does not describe a self-consistent process with all the supplied data.

Quick Tip to Solve: For a slow massless-piston process, check $P\Delta V = nR\Delta T$ before selecting a heat-capacity formula or matching an option.

36. Initial pressure and volume of a monoatomic ideal gas are P and V . The change in internal energy of this gas in adiabatic expansion to volume $V_{\text{final}} = 27V$ is ____ J. [4 marks]

- (A) $-2PV(3\sqrt{3} - 1)$
- (B) $\frac{4}{3}PV$
- (C) $-\frac{4}{3}PV$
- (D) $\frac{3}{4}PV$

Correct Answer: (C) $-\frac{4}{3}PV$

Solution:

Concept:

- A monoatomic ideal gas has $C_V = 3R/2$, $C_P = 5R/2$, and heat-capacity ratio $\gamma = 5/3$.
- For the usual reversible adiabatic expansion, $TV^{\gamma-1}$ is constant. Internal energy is $U = \frac{3}{2}nRT$.

Step 1: Find the final temperature ratio Let the initial temperature be T_i . Apply the adiabatic relation between the initial volume V and final volume $27V$:

$$\frac{T_f}{T_i} = \left(\frac{V}{27V}\right)^{\gamma-1} = \left(\frac{1}{27}\right)^{2/3} = \frac{1}{9}.$$

The power is two-thirds because the gas is monoatomic. Expansion cools the gas, so a ratio smaller than one provides a useful physical check.

Step 2: Express both internal energies in terms of the initial state The initial ideal-gas equation gives $nRT_i = PV$, with P and V denoting the initial pressure and volume. Consequently

$$U_i = \frac{3}{2}PV, \quad U_f = \frac{3}{2}nRT_f = \frac{3}{2} \frac{PV}{9} = \frac{1}{6}PV.$$

The final internal energy remains positive even though the change will be negative. It would be

incorrect to use the initial pressure together with the final volume in the final-state ideal-gas equation.

Step 3: Subtract in the correct order The change requested is final minus initial:

$$\Delta U = U_f - U_i = \left(\frac{1}{6} - \frac{3}{2}\right) PV = -\frac{4}{3}PV.$$

A positive value would contradict cooling during adiabatic expansion, since no heat enters and the gas performs work. Thus both positive options are excluded by the sign, while the remaining negative expression has the wrong magnitude. The product PV has units of joules for pressure and volume in SI units.

Step 4: Conclusion The change in internal energy is $\Delta U = -\frac{4}{3}PV$. Hence, the correct option is (C).

Quick Tip to Solve: For a monoatomic reversible adiabat, use $T_f/T_i = (V_i/V_f)^{2/3}$ and then $\Delta U = \frac{3}{2}PV(T_f/T_i - 1)$.

37. The frequency of oscillation of a mass m suspended by a spring is ν_1 . If the length of the spring is cut to half, the same mass oscillates with frequency ν_2 . The value of ν_2/ν_1 is ____ . [4 marks]

- (A) 1
- (B) 2
- (C) $\sqrt{2}$
- (D) $\sqrt{3}$

Correct Answer: (C) $\sqrt{2}$

Solution:

Concept:

- For a mass attached to an ideal spring, the frequency is $\nu = (2\pi)^{-1}\sqrt{k/m}$, where k is the spring constant and m is the oscillating mass.
- For a uniform spring of fixed material and cross-section, stiffness is inversely proportional to its length: $k\ell = \text{constant}$. Cutting the spring changes its stiffness, even when the attached mass is unchanged.

Step 1: Relate the stiffnesses before and after cutting Let the original spring have length ℓ and stiffness k_1 . Its two equal halves, if rejoined, act as springs in series. Therefore each half must have stiffness twice that of the complete spring. Equivalently, inverse proportionality gives

$$\frac{k_2}{k_1} = \frac{\ell}{\ell/2} = 2.$$

This relation concerns the spring used to support the mass; it does not imply that the suspended mass is divided.

Step 2: Compare the frequencies for the same mass Apply the frequency formula separately to the original spring and to one half. Since the same mass is used, both the mass and the factor 2π cancel

in their ratio:

$$\frac{\nu_2}{\nu_1} = \frac{\sqrt{k_2/m}}{\sqrt{k_1/m}} = \sqrt{\frac{k_2}{k_1}} = \sqrt{2}.$$

The equilibrium extension changes when the stiffness changes, but this shift does not alter the small-oscillation frequency formula. A ratio of 1 would ignore the change in stiffness, while 2 would incorrectly make frequency directly proportional to stiffness. The value $\sqrt{3}$ would require three times the stiffness, which is not produced by halving the length.

Step 3: Conclusion The frequency increases by a factor $\sqrt{2}$, so $\boxed{\nu_2/\nu_1 = \sqrt{2}}$. Hence, the correct option is (C).

Quick Tip to Solve: First use the inverse relation between spring length and stiffness; then take the square root to obtain the frequency ratio.

38. A monochromatic source of light operating at 15 kW emits 2.5×10^{22} photons /s. The region of an electromagnetic spectrum to which the emitted electromagnetic radiation belongs to ____.

(Take $h = 6.6 \times 10^{-34}$ J.s and $c = 3 \times 10^8$ m/s).

[4 marks]

- (A) Microwave
- (B) Infrared
- (C) Visible
- (D) Ultraviolet

Correct Answer: (D) Ultraviolet

Solution:

Concept:

- The source power is energy emitted per second. For monochromatic light, $P = \dot{N}E$, where $\dot{N}$ is the number of photons emitted each second and E is the energy of one photon.
- Photon energy and wavelength are related by $E = h\nu = hc/\lambda$. Wavelengths of approximately 400–700 nm lie in the visible region.

Step 1: Calculate the energy of one photon Convert the power into SI units before dividing by the photon emission rate:

$$P = 15 \text{ kW} = 1.5 \times 10^4 \text{ J s}^{-1}.$$

All photons have the same energy because the source is monochromatic. Thus

$$E = \frac{P}{\dot{N}} = \frac{1.5 \times 10^4}{2.5 \times 10^{22}} = 6.0 \times 10^{-19} \text{ J}.$$

The unit is joule per photon, since the per-second factors in the numerator and denominator cancel. The total source power alone does not identify the spectral region; the energy per photon is the relevant quantity.

Step 2: Find the wavelength and classify the radiation Using the supplied values of Planck's constant

and the speed of light,

$$\lambda = \frac{hc}{E} = \frac{(6.6 \times 10^{-34})(3 \times 10^8)}{6.0 \times 10^{-19}} = 3.3 \times 10^{-7} \text{ m} = 330 \text{ nm}.$$

This wavelength is below the usual visible lower boundary near 400 nm, so it belongs to ultraviolet radiation. Infrared and microwave wavelengths are longer than visible wavelengths and are therefore excluded. Visible light is also excluded because 330 nm is too short; the high photon energy is consistent with this ultraviolet classification.

Step 3: Conclusion The emitted wavelength is 330 nm, in the ultraviolet region. Hence, the correct option is (D).

Quick Tip to Solve: Divide watts by photons per second to obtain energy per photon, then use $\lambda = hc/E$; compare the result in nanometres.

39. A current carrying circular loop of radius 2 cm with unit normal $\hat{n} = \frac{\hat{k} + \hat{i}}{\sqrt{2}}$ is placed in a magnetic field, $\vec{B} = B_o(3\hat{i} + 2\hat{k})$. If $B_o = 4 \times 10^{-3} \text{ T}$ and current $I = 100\sqrt{2} \text{ A}$, the torque experienced by the loop is ____ Wb.A. ($\pi = 3.14$) [4 marks]

- (A) $16 \times 10^{-5} \hat{k}$
(B) $5024 \times 10^{-7} \hat{k}$
(C) $5024 \times 10^{-7} \hat{i}$
(D) $5024 \times 10^{-7} \hat{j}$

Correct Answer: (D) $5024 \times 10^{-7} \hat{j}$

Solution:

Concept:

- The magnetic dipole moment of a single current loop is $\vec{\mu} = IA\hat{n}$, with $A = \pi r^2$ and $\hat{n}$ the specified unit normal.
- The torque in a uniform magnetic field is $\vec{\tau} = \vec{\mu} \times \vec{B}$. Its direction must be determined by the cross product rather than by multiplying magnitudes alone.

Step 1: Calculate the vector magnetic moment Express the radius in metres and use the given value of π :

$$A = 3.14(2 \times 10^{-2})^2 = 1.256 \times 10^{-3} \text{ m}^2.$$

The factor $\sqrt{2}$ in the current cancels the factor in the denominator of the unit normal. Therefore

$$\vec{\mu} = (100\sqrt{2})(1.256 \times 10^{-3}) \frac{\hat{i} + \hat{k}}{\sqrt{2}} = 0.1256(\hat{i} + \hat{k}) \text{ A m}^2.$$

Both components are positive, as required by the given normal. There is only one turn in the stated circular loop, so no additional turn factor is introduced.

Step 2: Evaluate the cross product with the field The magnetic field is $\vec{B} = 4 \times 10^{-3}(3\hat{i} + 2\hat{k}) \text{ T}$. Using

$$\hat{i} \times \hat{k} = -\hat{j} \text{ and } \hat{k} \times \hat{i} = \hat{j},$$

$$\begin{aligned}\vec{\tau} &= 0.1256(4 \times 10^{-3})[(\hat{i} + \hat{k}) \times (3\hat{i} + 2\hat{k})] \\ &= 0.1256(4 \times 10^{-3})(-2\hat{j} + 3\hat{j}) \\ &= 5.024 \times 10^{-4}\hat{j} \text{ N m.}\end{aligned}$$

The two vectors lie in the xz -plane, so the torque must be along the y -axis. This immediately excludes every option along $\hat{i}$ or $\hat{k}$. Also, $\text{T A m}^2 = \text{Wb A} = \text{N m}$, matching the requested unit.

Step 3: Conclusion $\vec{\tau} = 5024 \times 10^{-7}\hat{j} \text{ Wb A}$. Hence, the correct option is (D).

Quick Tip to Solve: Cancel the $\sqrt{2}$ factors in $I\hat{n}$ first, then evaluate the two nonzero cross-product terms to preserve the torque sign.

40. A 30 cm long solenoid has 10 turns per cm and area of 5 cm^2 . The current through the solenoid coil varies from 2 A to 4 A in 3.14 s. The e.m.f. induced in the coil is $\alpha \times 10^{-5} \text{ V}$. The value α is ____
[4 marks]

- (A) 60
- (B) 12
- (C) 120
- (D) 34

Correct Answer: (B) 12

Solution:

Concept:

- For a long air-core solenoid, the self-inductance is $L = \mu_0 n^2 A \ell$, where n is turns per metre, A is cross-sectional area and ℓ is length.
- The magnitude of the induced emf for the stated current change is $|\mathcal{E}| = L \Delta I / \Delta t$. Its polarity opposes the change in current according to Lenz's law.

Step 1: Convert the dimensions and determine the inductance The length is $\ell = 0.30 \text{ m}$, the area is $A = 5 \times 10^{-4} \text{ m}^2$, and the turn density is

$$n = 10 \text{ cm}^{-1} = 1000 \text{ m}^{-1}.$$

Thus the solenoid has $N = n\ell = 300$ turns. Using $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$,

$$\begin{aligned}L &= (4\pi \times 10^{-7})(1000)^2(5 \times 10^{-4})(0.30) \\ &= 6\pi \times 10^{-5} \text{ H.}\end{aligned}$$

The square on the turn density makes its unit conversion particularly important; using 10 in an SI formula would underestimate the result by a factor of 10^4 .

Step 2: Use the current-change rate and identify the coefficient The current increases by $\Delta I =$

$4 - 2 = 2$ A over 3.14 s. Hence

$$|\mathcal{E}| = (6\pi \times 10^{-5}) \frac{2}{3.14} \text{ V} \simeq 12 \times 10^{-5} \text{ V}.$$

Here $\pi \simeq 3.14$, consistent with the precision of the stated time. Comparing with $\alpha \times 10^{-5} \text{ V}$ gives $\alpha = 12$. The other listed coefficients do not follow from the SI dimensions and the change of current; using the final current instead of its change would also be incorrect. The positive coefficient describes emf magnitude, while its physical polarity opposes the increase.

Step 3: Conclusion The induced-emf magnitude is $12 \times 10^{-5} \text{ V}$, giving $\alpha = 12$. Hence, the correct option is (B).

Quick Tip to Solve: Convert turns per centimetre to turns per metre before squaring, and use $4 - 2$ A rather than the final current in the emf formula.

41. Two point charges $q_1 = 3\mu\text{C}$ **and** $q_2 = -4\mu\text{C}$ **are placed at points** $(2\hat{i} + 3\hat{j} + 3\hat{k})$ **and** $(\hat{i} + \hat{j} + \hat{k})$ **respectively. Force on charge** q_2 **is** ____ N. (Take $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ SI Units) [4 marks]

(A) $(12\hat{i} + 24\hat{j} + 24\hat{k}) \times 10^{-3}$

(B) $(4\hat{i} + 8\hat{j} + 8\hat{k}) \times 10^{-3}$

(C) $(3\hat{i} + 6\hat{j} + 6\hat{k}) \times 10^{-3}$

(D) $(-4\hat{i} - 8\hat{j} - 8\hat{k}) \times 10^{-3}$

Correct Answer: (B) $(4\hat{i} + 8\hat{j} + 8\hat{k}) \times 10^{-3}$

Solution:

Concept:

- Coulomb's law gives the magnitude $F = k|q_1q_2|/r^2$. The force between opposite charges is attractive, so the force on the negative charge points towards the positive charge.
- A vector force is its magnitude multiplied by the unit vector in its direction. The separation vector must therefore be divided by its length before multiplying by F .

Step 1: Find the direction and distance between the charges The force is requested on $q_2 = -4\mu\text{C}$, which lies at $\vec{r}_2 = \hat{i} + \hat{j} + \hat{k}$. It is attracted towards $q_1 = 3\mu\text{C}$, located at $\vec{r}_1 = 2\hat{i} + 3\hat{j} + 3\hat{k}$. The vector in the force direction is

$$\vec{r}_1 - \vec{r}_2 = \hat{i} + 2\hat{j} + 2\hat{k}.$$

Its length is $r = \sqrt{1^2 + 2^2 + 2^2} = 3$ m, giving the unit vector $(\hat{i} + 2\hat{j} + 2\hat{k})/3$. All components of the force are positive; choosing the opposite separation vector would give the wrong attractive-force direction.

Step 2: Calculate the magnitude and resolve its components Convert both microcoulomb values to coulombs. The product of their magnitudes is $12 \times 10^{-12} \text{ C}^2$, so

$$F = \frac{(9 \times 10^9)(12 \times 10^{-12})}{3^2} = 12 \times 10^{-3} \text{ N}.$$

Multiplying by the unit vector gives

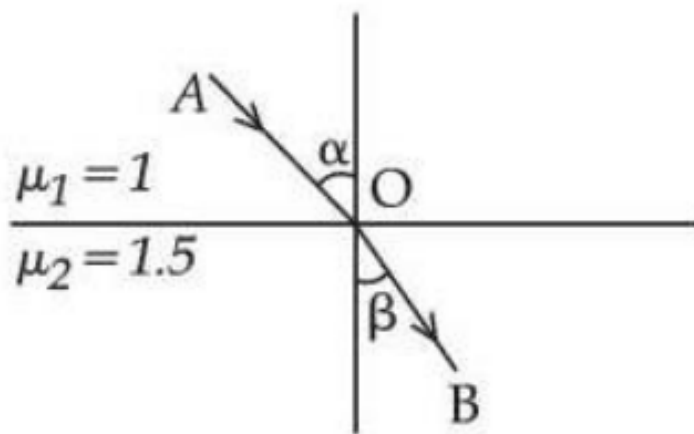
$$\vec{F}_2 = (12 \times 10^{-3}) \frac{\hat{i} + 2\hat{j} + 2\hat{k}}{3} = (4\hat{i} + 8\hat{j} + 8\hat{k}) \times 10^{-3} \text{ N}.$$

The first option mistakenly multiplies the force magnitude by a separation vector rather than a unit vector. The negative components in the last option point away from the attracting positive charge. The remaining positive option has the wrong magnitude.

Step 3: Conclusion $\vec{F}_2 = (4\hat{i} + 8\hat{j} + 8\hat{k}) \times 10^{-3} \text{ N}$. Hence, the correct option is (B).

Quick Tip to Solve: Decide the attractive-force direction first, then normalize the separation vector; a vector of length 3 is not a unit vector.

42. Light ray incident along a vector $\vec{AO}(\vec{AO} = 2\hat{i} - 3\hat{j})$ emerges out along vector $\vec{OB}(\vec{OB} = C\hat{i} - 4\hat{j})$ as shown in the figure below. The value of C is _____. [4 marks]



- (A) 1.6
- (B) 0.16
- (C) 11.6
- (D) 16

Correct Answer: (A) 1.6

Solution:

Concept:

- Snell's law is $\mu_1 \sin \alpha = \mu_2 \sin \beta$, with both angles measured from the normal. Here the horizontal interface has a vertical normal.
- For a ray direction $a\hat{i} + b\hat{j}$, the sine of its angle with the vertical normal is $|a|/\sqrt{a^2 + b^2}$. Direction vectors need not have equal lengths.

Step 1: Express the angles using the given ray components The figure shows the incident ray travelling through index $\mu_1 = 1$ and the refracted ray through index $\mu_2 = 1.5$. For $\vec{AO} = 2\hat{i} - 3\hat{j}$, the magnitude

is $\sqrt{13}$, so

$$\sin \alpha = \frac{2}{\sqrt{13}}.$$

For $\vec{OB} = C\hat{i} - 4\hat{j}$, the magnitude is $\sqrt{C^2 + 16}$, giving

$$\sin \beta = \frac{C}{\sqrt{C^2 + 16}}.$$

The diagram places the emergent ray to the right of the normal, so $C > 0$. The horizontal component is used because the angle is measured from the vertical, not from the interface.

Step 2: Apply Snell's law and solve for the positive coefficient Substituting the two sine expressions,

$$\frac{2}{\sqrt{13}} = \frac{3}{2} \frac{C}{\sqrt{C^2 + 16}}.$$

Squaring both sides and simplifying gives

$$\begin{aligned} 16(C^2 + 16) &= 117C^2, \\ 101C^2 &= 256, \\ C &= \frac{16}{\sqrt{101}} \simeq 1.592. \end{aligned}$$

Thus the matching listed value is 1.6. The ray bends towards the normal in the denser medium, as this value confirms: its horizontal-to-vertical ratio falls from $2/3$ to approximately $1.592/4$. The options 11.6 and 16 would bend the ray away from the normal; 0.16 would produce excessive bending and does not satisfy Snell's law.

Step 3: Conclusion $C = 16/\sqrt{101} \simeq 1.6$, to the precision of the options. Hence, the correct option is (A).

Quick Tip to Solve: For a horizontal boundary, use the horizontal component divided by the vector magnitude to obtain the sine of the angle with the normal.

43. K_1 and K_2 be the maximum kinetic energies of photoelectrons emitted from a surface of a given material for the light of wavelength λ_1 and λ_2 , respectively. If $\lambda_1 = 2\lambda_2$ then the work function of material is given by :

[4 marks]

- (A) $K_2 + 2K_1$
- (B) $2K_2 - K_1$
- (C) $K_1 - 2K_2$
- (D) $K_2 - 2K_1$

Correct Answer: (D) $K_2 - 2K_1$

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = hc/\lambda - \phi$, where ϕ is the work function of the metal.
- The work function depends on the emitting surface, so it has the same value for both incident wavelengths. Photon energy is inversely proportional to wavelength.

Step 1: Write the two energy balances Let $E = hc/\lambda_1$. Since $\lambda_1 = 2\lambda_2$, the second photon has energy $hc/\lambda_2 = 2E$. Consequently the maximum kinetic energies satisfy

$$K_1 = E - \phi, \quad K_2 = 2E - \phi.$$

These are maximum energies because no additional electron energy loss is included in Einstein's stopping-energy relation.

Step 2: Eliminate the photon energy Multiply the first equation by two and subtract it from the second. The unknown incident energy then cancels:

$$K_2 - 2K_1 = (2E - \phi) - 2(E - \phi) = \phi.$$

The plus sign in front of the remaining work function is essential. The expression $K_1 - 2K_2$ reverses the required coefficients, while $2K_2 - K_1$ does not eliminate E . Adding the energies as in option (A) also leaves an incident-energy term and therefore cannot give a material constant. The result is consistent with the fact that halving the wavelength increases the kinetic energy by the increase in photon energy.

Step 3: Conclusion The work function is $\phi = K_2 - 2K_1$. Hence, the correct option is (D).

Quick Tip to Solve: Use the wavelength ratio to relate photon energies first; then eliminate hc/λ between the two photoelectric equations.

44. Two radioactive substances A and B of mass numbers 200 and 212 respectively, shows spontaneous α -decay with same Q value of 1 MeV. The ratio of energies of α -rays produced by A and B is _____

[4 marks]

(A) $\frac{2548}{2650}$

(B) $\frac{2706}{2646}$

(C) $\frac{2597}{2600}$

(D) $\frac{2862}{2499}$

Correct Answer: (C) $\frac{2597}{2600}$

Solution:

Concept:

- In a two-body decay of a nucleus initially at rest, the alpha particle and daughter nucleus have equal and opposite momenta.
- For nonrelativistic recoil, $K = p^2/(2m)$. Hence the available energy Q is shared inversely in proportion to the two masses.

Step 1: Find the energy share of the alpha particle For a parent mass number M , the daughter has mass number $M - 4$. Using masses proportional to these mass numbers gives

$$\frac{K_\alpha}{K_d} = \frac{M - 4}{4}.$$

Conservation of energy gives $K_\alpha + K_d = Q$; therefore

$$K_\alpha = Q \frac{M - 4}{M}.$$

The alpha particle does not receive the full decay energy because the daughter must recoil to conserve momentum.

Step 2: Compare the two parent nuclei For A, substitute $M = 200$; for B, substitute $M = 212$. Their equal $Q = 1$ MeV cancels from the ratio:

$$\frac{K_{\alpha,A}}{K_{\alpha,B}} = \frac{196/200}{208/212} = \frac{196 \times 212}{200 \times 208} = \frac{2597}{2600}.$$

The ratio is slightly less than unity: the heavier daughter in decay B takes a smaller energy fraction, leaving its alpha particle slightly more energetic. Option (B), which exceeds unity, has the wrong trend. Options (A) and (D) give substantially larger deviations from unity than the small recoil correction allows.

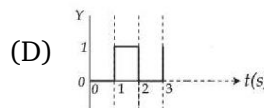
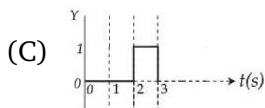
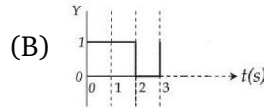
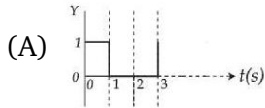
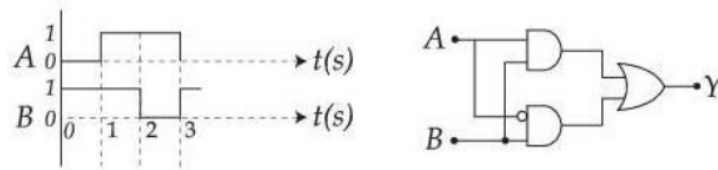
Step 3: Conclusion The required ratio of alpha-particle energies is $\frac{2597}{2600}$. Hence, the correct option is (C).

Quick Tip to Solve: For equal decay energies, compare the recoil factors $(M - 4)/M$, rather than assigning the entire Q value to the alpha particle.

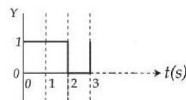
45.

The output Y for the given inputs A and B to the circuit is :

[4 marks]



Correct Answer: (B)



Solution:

Concept:

- An AND gate produces 1 only when both inputs are 1; an OR gate produces 1 when at least one input is 1.
- The small bubble on an input inverts that input. Complementary terms obey $A + \bar{A} = 1$, so Boolean expressions can be simplified before tracing a waveform.

Step 1: Read and simplify the circuit The upper AND gate receives A and B , so its output is AB . The lower AND gate has an inversion bubble on the branch carrying A ; its other input is B . Its output is therefore $\bar{A}B$. The final OR gate combines these outputs:

$$Y = AB + \bar{A}B = B(A + \bar{A}) = B.$$

The junctions on the wires are important: the inverted input is the A branch, not the B branch.

Step 2: Match the output with the input waveform The output must reproduce B at every instant, independently of A . In the given graph, B is high for $0 < t < 2$ s, low for $2 < t < 3$ s, and high again after $t = 3$ s. This is exactly the waveform shown in option (B). Option (A) falls at $t = 1$ s and therefore fails while B remains high. Option (C) is high in the interval where B is low. Option (D) is high only between 1 and 2 seconds, which corresponds to retaining only the upper AND term and ignoring the lower gate.

Step 3: Conclusion The circuit gives $\boxed{Y = B}$, so the output follows the B waveform. Hence, the correct option is (B).

Quick Tip to Solve: Simplify $AB + \bar{A}B$ before comparing graphs; complementary A branches make the output independent of A .

Physics – Section B: Numerical Value Questions (20 marks)

46. A parallel plate capacitor is having separation between plates 0.885 mm . It has a capacitance of $1\mu\text{ F}$ when the space between the plates is filled with an insulating material of resistivity $1 \times 10^{13}\Omega\text{ m}$ and resistance $17.7 \times 10^{14}\Omega$. Relative permittivity of the insulating material is $\alpha \times 10^7$. The value of α is ____ .

(Take permittivity of free space $= 8.85 \times 10^{-12}\text{ F/m}$)

[4 marks]

Solution:

Concept:

- A parallel-plate capacitor filled with a dielectric has capacitance $C = \epsilon_r \epsilon_0 A/d$, where A is the plate area and d the separation.
- The resistance across the same slab is $R = \rho d/A$. Multiplying these relations eliminates the geometric factor and gives $RC = \rho \epsilon_r \epsilon_0$.

Step 1: Eliminate the plate geometry The insulating material fills the whole space between the plates, so its electrical leakage path has length d and cross-sectional area A. Thus

$$\frac{A}{d} = \frac{\rho}{R}, \quad C = \epsilon_r \epsilon_0 \frac{\rho}{R}.$$

Rearranging gives

$$\epsilon_r = \frac{CR}{\rho \epsilon_0}.$$

No separate plate area is required. The stated separation cancels because precisely the same geometry appears in capacitance and resistance.

Step 2: Substitute the data and identify the coefficient Use $C = 1 \times 10^{-6}\text{ F}$, $R = 17.7 \times 10^{14}\Omega$, $\rho = 10^{13}\Omega\text{ m}$, and $\epsilon_0 = 8.85 \times 10^{-12}\text{ F/m}$. Then

$$\epsilon_r = \frac{(10^{-6})(17.7 \times 10^{14})}{(10^{13})(8.85 \times 10^{-12})} = \frac{17.7}{8.85} 10^7 = 2 \times 10^7.$$

Relative permittivity is dimensionless. Comparing this directly with the stated form $\alpha \times 10^7$ isolates the requested coefficient; the requested entry is not the full value of relative permittivity.

Step 3: Conclusion The relative permittivity is 2×10^7 ; hence $\boxed{\alpha = 2}$.

Quick Tip to Solve: When a dielectric slab serves as both a resistor and a capacitor, multiply R and C to cancel the area and thickness.

47. Some distant star is to be observed by some telescope of diameter of objective lens a , at an angular resolution of 3.0×10^{-7} radian. If the wavelength of light from the star reaching the telescope is 500 nm, the minimum diameter of the objective lens of the telescope is ____ cm. (nearest interger)

[4 marks]

Solution:

Concept:

- The Rayleigh criterion for a circular aperture gives the limiting angular separation $\theta_{\min} = 1.22\lambda/D$, with wavelength and diameter in the same length unit.
- To attain a prescribed resolution, the aperture diameter must be at least $D = 1.22\lambda/\theta_{\min}$. A larger aperture narrows the diffraction pattern.

Step 1: Apply the circular-aperture resolution criterion The telescope objective is a circular aperture. Its diffraction limit therefore includes the numerical factor 1.22, rather than the single-slit factor one. The required angle is 3.0×10^{-7} rad, and the wavelength is

$$\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{ m} = 5.00 \times 10^{-7} \text{ m}.$$

For the minimum acceptable objective diameter, set the limiting angle equal to the required angular resolution.

Step 2: Calculate and round in centimetres Substituting in the Rayleigh formula gives

$$D_{\min} = \frac{1.22(5.00 \times 10^{-7})}{3.0 \times 10^{-7}} \text{ m} = 2.0333 \dots \text{ m}.$$

Convert to the unit requested before rounding:

$$D_{\min} = 203.333 \dots \text{ cm} \simeq 203 \text{ cm}.$$

The question asks for a nearest-integer numerical entry, so ordinary rounding is used here. Keeping 2 as the entry would mistakenly report the value in metres, while omitting the circular-aperture factor would underestimate the diameter. The angular quantity is already in radians, requiring no degree conversion.

Step 3: Conclusion The minimum diameter is approximately 203.33 cm. The nearest-integer answer is 203.

Quick Tip to Solve: For a telescope use $1.22\lambda/D$, and convert the final diameter to centimetres before rounding the numerical entry.

48. A 5 mg particle carrying a charge of $5\pi \times 10^{-6} \text{ C}$ is moving with velocity of $(3\hat{i} + 2\hat{k}) \times 10^{-2} \text{ m/s}$ in a region having magnetic field $\vec{B} = 0.1\hat{k} \text{ Wb/m}^2$. It moves a distance of α meter along $\hat{k}$ when it completes 5 revolutions. The value of α is _____. [4 marks]

Solution:

Concept:

- A velocity component perpendicular to a uniform magnetic field produces circular motion, with period $T = 2\pi m / (|q|B)$.
- The parallel velocity component is unaffected by the magnetic force. Combining uniform parallel motion with circular transverse motion produces a helix.

Step 1: Separate the velocity components The field is along $+\hat{k}$, so $v_{\parallel} = 2 \times 10^{-2} \text{ m/s}$, while $v_{\perp} = 3 \times 10^{-2} \text{ m/s}$. Only the perpendicular component enters the circular motion; the axial displacement uses the parallel component. Convert the mass carefully:

$$m = 5 \text{ mg} = 5 \times 10^{-6} \text{ kg}.$$

The magnetic force is perpendicular to velocity and does no work, so these speed components remain constant throughout the motion.

Step 2: Find the elapsed time for five turns Using the charge magnitude $|q| = 5\pi \times 10^{-6} \text{ C}$ and field $B = 0.1 \text{ T}$,

$$T = \frac{2\pi(5 \times 10^{-6})}{(5\pi \times 10^{-6})(0.1)} = 20 \text{ s}.$$

Five revolutions therefore take $5T = 100 \text{ s}$. The distance moved parallel to the field is

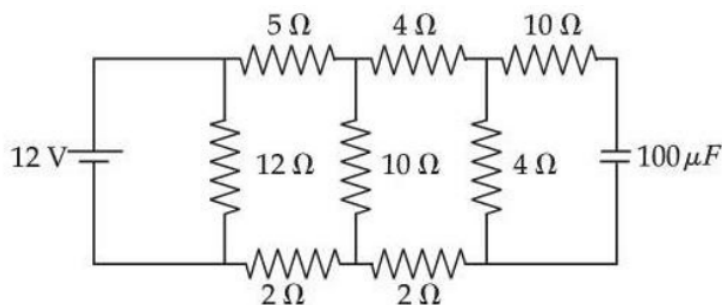
$$z = v_{\parallel}(5T) = (2 \times 10^{-2})(100) = 2 \text{ m}.$$

This is the axial distance asked for, rather than the longer distance travelled along the helical path. The perpendicular speed affects the radius but cancels from the period, so it is unnecessary to calculate that radius.

Step 3: Conclusion The particle advances 2 m along $+\hat{k}$ in five revolutions. Thus $\boxed{\alpha = 2}$.

Quick Tip to Solve: Helical pitch equals $v_{\parallel}T$; multiply it by the number of turns and keep the milligram-to-kilogram conversion explicit.

49. The stored charge in the capacitor in steady state of the following circuit is ____ μC . [4 marks]



Solution:

Concept:

- In the steady state of a direct-current circuit, the ideal capacitor carries no current. Its voltage is nevertheless set by the conducting resistor network connected across its terminals.
- Resistors in series carry the same current, whereas parallel branches share their terminal potential difference. The stored charge is $Q = CV$, where C is capacitance and V the voltage across the capacitor.

Step 1: Identify the conducting paths At steady state the rightmost capacitor branch is open. Consequently no current flows through the top-right 10Ω resistor, and its voltage drop is zero. The capacitor therefore has the same voltage as the adjacent vertical 4Ω resistor. The left vertical 12Ω resistor is directly across the ideal 12 V battery; it does not change the battery voltage applied to the other network.

Step 2: Reduce the middle network Between the two junctions carrying the vertical 10Ω resistor, the alternate path consists of the upper 4Ω , vertical 4Ω , and lower 2Ω resistors in series. Thus

$$R_{\text{path}} = 4 + 4 + 2 = 10\Omega, \quad R_{\text{middle}} = 10 \parallel 10 = 5\Omega.$$

This equivalent resistance is in series with the upper-left 5Ω and lower-left 2Ω resistors. The current entering this network is

$$I = \frac{12}{5 + 5 + 2} = 1\text{ A}.$$

Step 3: Find the capacitor voltage and charge The voltage across the middle parallel combination is $1 \times 5 = 5\text{ V}$. Its two equal-resistance branches each carry 0.5 A , giving a voltage of $0.5 \times 4 = 2\text{ V}$ across the vertical 4Ω resistor. This is also the capacitor voltage because the remaining series resistor has zero current. Therefore

$$Q = CV = (100\mu\text{F})(2\text{ V}) = 200\mu\text{C}.$$

Step 4: Conclusion The charge stored in the capacitor is $200\mu\text{C}$. The required numerical value is 200.

Quick Tip to Solve: Open the capacitor branch first, then find its terminal voltage; never include a resistor carrying zero steady-state current in a voltage-divider ratio.

50. Two masses of 3.4 kg and 2.5 kg are accelerated from an initial speed of 5 m/s and 12 m/s, respectively. The distances traversed by the masses in the 5th second are 104 m and 129 m, respectively. The ratio of their momenta after 10 s is $\frac{x}{8}$. The value of x is _____. [4 marks]

Solution:

Concept:

- For constant acceleration a and initial speed u , the displacement in the n th second is $s_n = u + \frac{1}{2}a(2n - 1)$, when times are measured in seconds.
- The velocity after time t is $v = u + at$, and momentum is $p = mv$. The ratio must include the different masses, not just the final speeds.

Step 1: Determine both accelerations The fifth-second distances refer to the intervals from $t = 4$ s to $t = 5$ s, rather than the complete first five seconds. Applying the n th-second relation separately gives

$$104 = 5 + \frac{9a_1}{2}, \quad a_1 = 22 \text{ m s}^{-2},$$

$$129 = 12 + \frac{9a_2}{2}, \quad a_2 = 26 \text{ m s}^{-2}.$$

The given distances and positive initial velocities yield positive accelerations in the direction of motion.

Step 2: Calculate the velocities at ten seconds Use the same starting instant for both bodies, with their respective initial speeds. At $t = 10$ s,

$$v_1 = 5 + 22(10) = 225 \text{ m s}^{-1},$$

$$v_2 = 12 + 26(10) = 272 \text{ m s}^{-1}.$$

The greater final speed of the second mass does not by itself decide which momentum is greater, because the first mass is heavier.

Step 3: Form and simplify the momentum ratio The momenta in the stated order are

$$p_1 = 3.4(225) = 765 \text{ kg m s}^{-1},$$

$$p_2 = 2.5(272) = 680 \text{ kg m s}^{-1}.$$

Their ratio is

$$\frac{p_1}{p_2} = \frac{765}{680} = \frac{9}{8} = \frac{x}{8}.$$

Matching the numerators gives the requested dimensionless value.

Step 4: Conclusion The momentum ratio is 9 : 8, so $x = 9$.

Quick Tip to Solve: For distance in the fifth second, subtract the displacement at four seconds from that at five seconds; then use each mass when comparing momenta.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51.

Match List - I with List - II.

List I: Mass of substance		List II: Number of atoms	
A.	1.8 mg water	I.	$2 \times 10^{-4} \times N_A$
B.	9.8 mg sulphuric acid	II.	$1.5 \times 10^{-4} \times N_A$
C.	1.8 mg carbon	III.	$3 \times 10^{-4} \times N_A$
D.	5.85 mg salt (NaCl)	IV.	$7 \times 10^{-4} \times N_A$

Choose the correct answer from the options given below :

[4 marks]

- (A) A-IV, B-III, C-I, D-II
 (B) A-III, B-II, C-IV, D-I
 (C) A-III, B-IV, C-II, D-I
 (D) A-III, B-IV, C-I, D-II

Correct Answer: (C) A-III, B-IV, C-II, D-I

Solution:

Concept:

- The number of formula units is nN_A , where $n = m/M$ is the amount in moles, m is mass in grams, and M is molar mass.
- To obtain the total number of atoms, multiply the number of molecules or formula units by the number of atoms in one unit. Count all elements, not only one selected element.

Step 1: Count atoms in water and sulphuric acid For $1.8 \text{ mg} = 1.8 \times 10^{-3} \text{ g}$ of water, the molar mass of H_2O is 18 g mol^{-1} . There are three atoms per molecule, so

$$N_{\text{water}} = \frac{1.8 \times 10^{-3}}{18} \times 3N_A = 3 \times 10^{-4} N_A.$$

Thus A matches III. For 9.8 mg of H_2SO_4 , the molar mass is 98 g mol^{-1} and each molecule contains seven atoms:

$$N_{\text{acid}} = \frac{9.8 \times 10^{-3}}{98} \times 7N_A = 7 \times 10^{-4} N_A.$$

Thus B matches IV.

Step 2: Count carbon atoms and atoms in sodium chloride Carbon is counted directly using its atomic molar mass, 12 g mol^{-1} :

$$N_C = \frac{1.8 \times 10^{-3}}{12} N_A = 1.5 \times 10^{-4} N_A.$$

Hence C matches II. Each formula unit of NaCl has two constituent atoms. With molar mass 58.5 g mol^{-1} ,

$$N_{\text{salt}} = \frac{5.85 \times 10^{-3}}{58.5} \times 2N_A = 2 \times 10^{-4} N_A.$$

Hence D matches I.

Step 3: Select the complete mapping The four assignments are A–III, B–IV, C–II and D–I. Only option (C) contains every one of these pairings. Option (A) mismatches water, option (B) mismatches sulphuric acid and carbon, and option (D) interchanges the correct carbon and sodium-chloride counts.

Step 4: Conclusion The correct mapping is A–III, B–IV, C–II, D–I. Hence, the correct option is (C).

Quick Tip to Solve: Convert milligrams to grams first, find moles, and multiply by the atoms per formula unit before matching the numerical coefficients.

52. Given below are two statements :

Given : Molar mass of C, H, O, Cl are 12, 1, 16 and 35.5 g mol^{-1} , respectively

Statement I : In 30% (w/w) solution of methanol in CCl_4 (at T K), the mole fraction of CCl_4 is equal to 0.33 .

Statement II : Mixture of methanol and CCl_4 shows positive deviation from Raoult's law. In the light of the above statements, choose the correct answer from the options given below : [4 marks]

- (A) Both Statement I and Statement II are true
- (B) Both Statement I and Statement II are false
- (C) Statement I is true but Statement II is false
- (D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- A 30% mass-by-mass solution contains 30 g of solute in 100 g of solution. Mole fraction is the amount of one component divided by the total amount of both components.
- A mixture shows positive deviation from Raoult's law when unlike intermolecular attractions are weaker than the attractions that dominate the pure liquids, increasing their escaping tendency and vapour pressure.

Step 1: Evaluate the stated mole fraction Take 100 g of solution. It contains 30 g of methanol and 70 g of carbon tetrachloride. From the given atomic molar masses,

$$M(\text{CH}_3\text{OH}) = 32 \text{ g mol}^{-1}, \quad M(\text{CCl}_4) = 154 \text{ g mol}^{-1}.$$

Their amounts are $30/32 = 0.9375 \text{ mol}$ and $70/154 = 0.4545 \text{ mol}$, respectively. Therefore

$$x_{\text{CCl}_4} = \frac{70/154}{30/32 + 70/154} = \frac{16}{49} \approx 0.3265 \approx 0.33.$$

Thus Statement I is true to the precision stated in the question.

Step 2: Assess the intermolecular interactions Pure methanol contains an extensive intermolecular

hydrogen-bonding network through its hydroxyl groups. Carbon tetrachloride is nonpolar and cannot replace those methanol–methanol hydrogen bonds with equally strong interactions. Mixing disrupts some of that network, and the unlike interactions are relatively weaker. The escaping tendency of molecules increases, so the total vapour pressure is above the ideal Raoult-law prediction. Hence the mixture shows positive deviation and Statement II is true.

Step 3: Compare the paired truth values Both statements are true: the numerical mole fraction follows from mole amounts, and the deviation follows from weakened interactions upon mixing. Options (B), (C) and (D) each label at least one of these true statements false and are therefore excluded. In particular, the mass fraction 0.70 of solvent must not be substituted for its mole fraction.

Step 4: Conclusion Statement I and Statement II are both true. Hence, the correct option is (A).

Quick Tip to Solve: For a mass-percent mixture, choose a 100 g basis; compute moles before fractions, and check whether mixing weakens the hydrogen bonds present in the pure liquid.

53. Bromine trifluoride autoionizes to form BrF_2^+ and BrF_4^- . The shapes of the cation and anion are respectively ____ and ____ . [4 marks]

- (A) bent, square planar
- (B) linear, square planar
- (C) bent, see-saw
- (D) linear, tetrahedral

Correct Answer: (A) bent, square planar

Solution:

Concept:

- In VSEPR theory, molecular shape depends on the number of bonding domains and lone pairs on the central atom. A single bond counts as one electron domain.
- For a central halogen bonded to monovalent fluorine atoms, the number of central electron pairs is $(7 + n - q)/2$, where n is the number of fluorine atoms and q is the algebraic positive charge.

Step 1: Determine the electron domains in the cation For BrF_2^+ , the positive charge removes one electron from the valence-electron count. The steric number is

$$\text{SN} = \frac{7 + 2 - 1}{2} = 4.$$

Two domains are Br–F bonds, leaving two lone pairs on bromine. This is an AX_2E_2 arrangement: the electron-domain geometry is tetrahedral, but the molecular shape, which describes only atom positions, is bent. Calling the cation linear would incorrectly ignore its two lone pairs.

Step 2: Determine the electron domains in the anion For BrF_4^- , the negative charge adds one electron. Its steric number is

$$\text{SN} = \frac{7 + 4 + 1}{2} = 6.$$

Four domains are Br–F bonds and two are lone pairs. The AX_4E_2 electron-domain arrangement is

octahedral. The two lone pairs occupy opposite positions to minimise repulsion, leaving the four fluorine atoms at the corners of a square in one plane. The molecular shape is therefore square planar.

Step 3: Eliminate the alternative shapes The required order is cation first and anion second, giving bent followed by square planar. Option (B) incorrectly makes the cation linear. Option (C) assigns a see-saw shape to the anion, whereas see-saw requires five electron domains and one lone pair. Option (D) is wrong for both ions: four bonds do not make the anion tetrahedral when two additional lone pairs are present.

Step 4: Conclusion The shapes are bent and square planar, respectively. Hence, the correct option is (A).

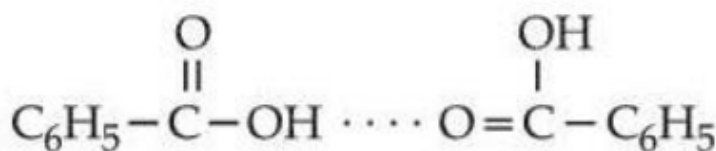
Quick Tip to Solve: Adjust the electron count for ionic charge before counting lone pairs; distinguish electron-domain geometry from the shape formed by the bonded atoms.

54. Which of the following statements are not correct?

- A. For water, magnitude of K_b is more than the magnitude of K_f .
- B. The elevation in boiling point of water when a non-volatile solute is added to it is larger in magnitude than its depression in freezing point.
- C. Osmotic pressure measurement is preferred over any other colligative property to determine molar mass of proteins and polymers.
- D. The dimerised form of benzoic acid in benzene is

Choose the correct answer from the options given below :

[4 marks]



- (A) A and B only
- (B) A and D only
- (C) A, B and D only
- (D) A, C and D only

Correct Answer: (C) A, B and D only

Solution:

Concept:

- For the same dilute solution, $\Delta T_b = iK_b m$ and $\Delta T_f = iK_f m$. For water, $K_b \approx 0.52$ and $K_f \approx 1.86$, in K kg mol^{-1} .
- Osmotic pressure obeys $\pi = cRT$ for dilute solutions and can be measured at room temperature. Carboxylic acids associate in nonpolar solvents through intermolecular hydrogen bonding.

Step 1: Test statements A and B For water, $K_b < K_f$, not the reverse. Statement A is therefore incorrect. In the same solution the factors i and m are common to the boiling-point elevation and freezing-point depression, so

$$\frac{\Delta T_b}{\Delta T_f} = \frac{K_b}{K_f} \approx \frac{0.52}{1.86} < 1.$$

The boiling-point elevation is smaller in magnitude, making Statement B incorrect as well.

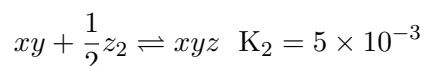
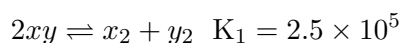
Step 2: Test the method for macromolecules Proteins and polymers have very large molar masses. Their dilute solutions give very small temperature changes, which are difficult to determine accurately. Osmotic pressure is appreciable even at low concentration and is measured at room temperature, helping avoid thermal damage to sensitive proteins. Consequently osmotic-pressure measurement is preferred for these substances, and Statement C is correct.

Step 3: Inspect the benzoic-acid dimer In benzene, benzoic-acid molecules form a cyclic dimer with two reciprocal $\text{O}-\text{H} \cdots \text{O}=\text{C}$ hydrogen bonds. Each molecule contributes an OH donor and a carbonyl-oxygen acceptor to the ring. The structure shown in Statement D contains only one intermolecular hydrogen bond and lacks the cyclic, doubly hydrogen-bonded arrangement; it is therefore not the correct representation of the dimer. Thus D is incorrect. The complete incorrect set is A, B and D. Options (A) and (B) each omit one of these, while (D) includes the correct Statement C and omits B.

Step 4: Conclusion The statements that are not correct are A, B and D only. Hence, the correct option is (C).

Quick Tip to Solve: Compare the two solvent constants using the same molality, and check that a carboxylic-acid dimer closes a ring with two hydrogen bonds.

55. Consider the following reactions in which all the reactants and products are present in gaseous state



The value of K_3 for the equilibrium $\frac{1}{2}x_2 + \frac{1}{2}y_2 + \frac{1}{2}z_2 \rightleftharpoons xyz$ is :

[4 marks]

(A) 2.5×10^{-3}

(B) 2.5×10^3

(C) 1.0×10^{-5}

(D) 5×10^{-3}

Correct Answer: (C) 1.0×10^{-5}

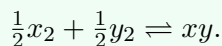
Solution:

Concept:

- When a balanced equilibrium is reversed, its equilibrium constant becomes its reciprocal. Multiplying every stoichiometric coefficient by a factor raises the constant to that factor.

- When two equilibria are added and an intermediate cancels, the constant of the resulting equilibrium is the product of the two constants. These rules apply to constants defined using the same convention.

Step 1: Reverse and halve the first equilibrium The target consumes half a mole each of x_2 and y_2 , so first reverse the dissociation equilibrium and divide all its coefficients by two:

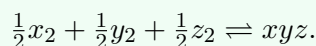


Its constant is

$$K_a = K_1^{-1/2} = \frac{1}{\sqrt{2.5 \times 10^5}} = \frac{1}{500} = 2.0 \times 10^{-3}.$$

The square root is essential because the coefficients were halved; reversing alone would give a different reaction.

Step 2: Add the second equilibrium Now add $xy + \frac{1}{2}z_2 \rightleftharpoons xyz$, whose constant remains $K_2 = 5 \times 10^{-3}$. The species xy cancels, leaving exactly



Therefore

$$K_3 = K_a K_2 = (2.0 \times 10^{-3})(5 \times 10^{-3}) = 1.0 \times 10^{-5}.$$

Option (D) would use the second reaction alone; (A) and (B) do not implement both reversal and halving correctly.

Step 3: Conclusion The required equilibrium constant is $K_3 = 1.0 \times 10^{-5}$. Hence, the correct option is (C).

Quick Tip to Solve: Write the target reaction as the second reaction minus half the first; immediately obtain $K_3 = K_2/\sqrt{K_1}$.

56. Given at 298 K :

$$E_{\text{Fe}^{2+}/\text{Fe}}^{\ominus} = X \text{ Volt}$$

$$E_{\text{Fe}^{3+}/\text{Fe}}^{\ominus} = Y \text{ Volt}$$

The $E_{\text{Fe}^{3+}/\text{Fe}^{2+}}^{\ominus}$ in Volt at 298 K is given by :

[4 marks]

(A) $2X - 3Y$

(B) $3Y - 2X$

(C) $3Y + 2X$

(D) $Y + X$

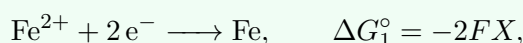
Correct Answer: (B) $3Y - 2X$

Solution:

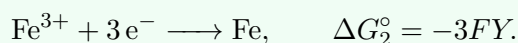
Concept:

- Standard electrode potentials are intensive quantities and cannot be added directly. First convert each reduction potential to standard Gibbs energy using $\Delta G^{\circ} = -nFE^{\circ}$, where n is the number of electrons and F is the Faraday constant.
- Gibbs energies are additive for reactions that add algebraically. All potentials here refer to reduction half-reactions at the same temperature.

Step 1: Write the three reduction half-reactions The given reductions are



and



Let the required potential for $\text{Fe}^{3+} + \text{e}^{-} \longrightarrow \text{Fe}^{2+}$ be E . The corresponding energy is $\Delta G_3^{\circ} = -FE$. Note that this last step transfers one electron, not two or three.

Step 2: Apply energy additivity and isolate the potential Adding the required one-electron reduction to the first, two-electron reduction cancels Fe^{2+} and gives the overall three-electron reduction. Hence

$$\Delta G_2^{\circ} = \Delta G_3^{\circ} + \Delta G_1^{\circ}.$$

Substituting the three expressions gives

$$-3FY = -FE - 2FX, \quad E = 3Y - 2X.$$

The minus sign on the $2X$ term comes from removing the Fe^{2+}/Fe step. Option (A) reverses this sign pattern, (C) adds rather than subtracts it, and (D) incorrectly adds the unweighted potentials.

Step 3: Conclusion The standard reduction potential is $E_{\text{Fe}^{3+}/\text{Fe}^{2+}}^{\circ} = (3Y - 2X) \text{ V}$. Hence, the correct option is (B).

Quick Tip to Solve: For successive reductions, use electron-weighted potentials: $3Y = E + 2X$, rather than adding the potentials themselves.

57. Given below are two statements :

$$R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1} \text{ and } 1\text{cal} = 4.2 \text{ J}$$

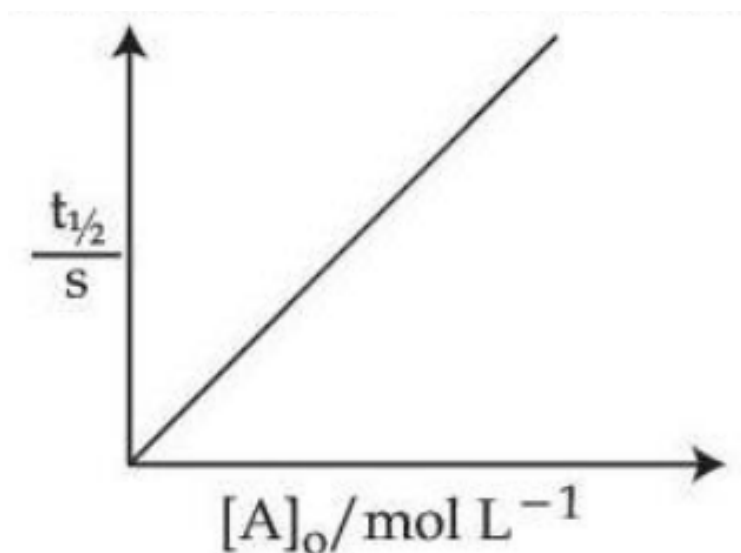
Statement I : When $E_a = 12.6\text{kcal/mol}$, the room temperature rate constant is doubled by a 10°C increase in temperature (298 K to 308 K)

Statement II : For a first order reactions $A \rightarrow B$,

Here $[A]_0$ is the initial concentration of A and $t_{1/2}$ is half life of reaction.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]



- (A) Both Statement I and Statement II are true
(B) Both Statement I and Statement II are false
(C) Statement I is true but Statement II is false
(D) Statement I is false but Statement II is true

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- The two-temperature Arrhenius equation is $\ln(k_2/k_1) = (E_a/R)(1/T_1 - 1/T_2)$. Activation energy and the gas constant must use consistent energy units.
- For a first-order reaction, $t_{1/2} = \ln 2/k$. At fixed temperature this half-life is independent of the initial reactant concentration.

Step 1: Test Statement I using the given constants Convert the activation energy using the conversion specified in the question:

$$E_a = 12.6 \times 10^3 \times 4.2 = 52920 \text{ J mol}^{-1}.$$

For $T_1 = 298 \text{ K}$ and $T_2 = 308 \text{ K}$,

$$\ln \frac{k_2}{k_1} = \frac{52920}{8.314} \frac{10}{298 \times 308} \approx 0.6935.$$

Therefore $k_2/k_1 \approx e^{0.6935} \approx 2.00$. Thus the stated doubling is correct to the precision of the data; Statement I is true.

Step 2: Test the graph in Statement II The supplied graph shows half-life increasing linearly from the origin as the initial concentration $[A]_0$ increases. But integration of the first-order rate law gives

$$\ln \frac{[A]_0}{[A]} = kt.$$

At half-life, $[A] = [A]_0/2$, so

$$t_{1/2} = \frac{\ln 2}{k}.$$

The initial concentration cancels. The correct half-life versus initial-concentration plot is a horizontal line, not the rising straight line shown. Statement II is false; this eliminates both options asserting it is true, while Statement I eliminates the both-false option.

Step 3: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: Convert kcal to joules before applying Arrhenius, and remember that concentration-independent half-life identifies first-order kinetics.

58. Match List-I with List-II.

List-I		List-II	
	Electronic configuration of neutral atom (where $n = 2$)		1 st Ionization Energy (kJ mol ⁻¹)
A.	ns ²	I.	2080
B.	ns ² np ¹	II.	899
C.	ns ² np ³	III.	800
D.	ns ² np ⁶	IV.	1402

Choose the correct answer from the options given below:

[4 marks]

(A) A-II, B-III, C-IV, D-I

(B) A-IV, B-III, C-II, D-I

(C) A-III, B-II, C-IV, D-I

(D) A-III, B-II, C-I, D-IV

Correct Answer: (A) A-II, B-III, C-IV, D-I

Solution:

Concept:

- First ionization energy is the energy required to remove the most loosely bound electron from an isolated gaseous neutral atom. Across a period it generally rises with effective nuclear charge, with subshell-related exceptions.
- For $n = 2$, the configurations in the question identify beryllium, boron, nitrogen and neon. A filled $2s$ subshell, half-filled $2p$ subshell and closed noble-gas shell explain their relative stability.

Step 1: Identify the four neutral atoms A has outer configuration $2s^2$, so it is Be. B has $2s^2 2p^1$, so it is B (boron). C has $2s^2 2p^3$, so it is N. D has $2s^2 2p^6$, so it is Ne. The core $1s^2$ electrons are understood in each configuration; these are neutral second-period atoms, not ions.

Step 2: Match energies using the subshell trend The first ionization energies are approximately

$$\begin{aligned}\text{Be} &: 899 \text{ kJ mol}^{-1} \quad (\text{II}), \\ \text{B} &: 800 \text{ kJ mol}^{-1} \quad (\text{III}), \\ \text{N} &: 1402 \text{ kJ mol}^{-1} \quad (\text{IV}), \\ \text{Ne} &: 2080 \text{ kJ mol}^{-1} \quad (\text{I}).\end{aligned}$$

Boron is lower than beryllium because its removed electron is in the higher-energy, less penetrating $2p$ subshell. Nitrogen has a stable half-filled $2p^3$ arrangement, while neon has the closed-shell configuration and the largest ionization energy. These identifications rule out the swapped Be/B assignments in (C) and (D), and the Be/N swap in (B).

Step 3: Conclusion The matching is A-II, B-III, C-IV, D-I. Hence, the correct option is (A).

Quick Tip to Solve: For the second period, remember the Be-to-B dip: removing the first $2p$ electron of boron is easier than breaking Be's filled $2s$ subshell.

59. Find the correct statements related to group 15 hydrides.

A. Reducing nature increases from NH_3 to BiH_3

B. Tendency to donate lone pair of electrons decreases from NH_3 to BiH_3

C. The stability of hydrides decreases from NH_3 to BiH_3

D. HEH bond angle decreases from NH_3 to SbH_3 (E = Elements of group 15)

Choose the correct answer from the options given below :

[4 marks]

(A) A and B only

(B) B and C only

(C) A, B, C and D

(D) A, C and D Only

Correct Answer: (C) A, B, C and D

Solution:

Concept:

- Group 15 hydrides have the formula EH_3 . Down the group the central atom becomes larger, the E–H bond weakens, and the hydrides become less thermally stable.
- Lone-pair availability controls their Lewis basicity, whereas bond strength controls how readily a hydride can act as a reducing agent. Bond angles also change with the central atom.

Step 1: Check reducing character and stability Statement A is correct: reducing character increases from NH_3 towards BiH_3 . The weaker E–H bonds down the group permit hydrogen to be released or transferred more readily. Statement C is also correct: thermal stability decreases in the same direction because the increasingly large central atom overlaps less effectively with the small hydrogen orbital. Thus the trend in reducing character is consistent with, rather than opposed to, the stability trend.

Step 2: Check lone-pair donation and bond angles Statement B is correct: the tendency to donate the lone pair decreases down the group; ammonia is the strongest base among these hydrides. The central atom grows and its lone pair becomes more diffuse and less effective in bonding to a proton or another electron-pair acceptor. Statement D is correct as well: the H–E–H angle decreases from about 107° in NH_3 to roughly 94° in PH_3 , 92° in AsH_3 , and 91° in SbH_3 . Bonding approaches the nearly orthogonal p-orbital limit down the group. Every listed statement therefore holds, so options that omit any of A, B, C or D are incomplete.

Step 3: Conclusion All four statements A, B, C and D are correct. Hence, the correct option is (C).

Quick Tip to Solve: Keep three linked downward trends together: weaker E–H bonds mean lower stability and stronger reducing behaviour; basicity and bond angle also decrease.

60. Given below are two statements :

Statement I : The number of pairs among $[\text{Ti}^{4+}, \text{V}^{2+}]$, $[\text{V}^{2+}, \text{Mn}^{2+}]$, $[\text{Mn}^{2+}, \text{Fe}^{3+}]$ and $[\text{V}^{2+}, \text{Cr}^{2+}]$ in which both ions are coloured is 3 .

Statement II : The number of pairs among $[\text{La}^{3+}, \text{Yb}^{2+}]$, $[\text{Lu}^{3+}, \text{Ce}^{4+}]$ and $[\text{Ac}^{3+}, \text{Lr}^{3+}]$ ions in which both are diamagnetic is 3 .

In the light of the above statements, choose the correct from the options given below : [4 marks]

- (A) Both Statement I and Statement II are correct
(B) Both Statement I and Statement II are incorrect
(C) Statement I is correct but Statement II is incorrect
(D) Statement I is incorrect but Statement II is correct

Correct Answer: (A) Both Statement I and Statement II are correct

Solution:

Concept:

- For the transition-metal ions listed here, partially occupied d subshells allow electronic transitions responsible for colour. A d^0 ion lacks such d–d transitions.
- Diamagnetic ions have no unpaired electrons. Empty and fully occupied f subshells, together with closed-shell cores, are diamagnetic in the ions listed. Remove the outermost electrons first when forming cations.

Step 1: Count the coloured d-block pairs The configurations are $\text{Ti}^{4+} : 3d^0$, $\text{V}^{2+} : 3d^3$, $\text{Mn}^{2+} : 3d^5$, $\text{Fe}^{3+} : 3d^5$, and $\text{Cr}^{2+} : 3d^4$. Thus Ti^{4+} is colourless, whereas each of the other ions is coloured. In particular, the weak colour of Mn^{2+} still counts as coloured; spin-forbidden transitions make it pale, not strictly colourless. The Ti/V pair fails, but the V/Mn, Mn/Fe and V/Cr pairs each have two coloured ions. The count is three, so Statement I is correct.

Step 2: Count the diamagnetic f-block pairs For the first pair, La^{3+} has the xenon core and Yb^{2+} has $4f^{14}$; both are closed-shell. For the second, Lu^{3+} has $4f^{14}$ and Ce^{4+} has the xenon core. For the third, Ac^{3+} has the radon core and Lr^{3+} has the filled $5f^{14}$ subshell. No unpaired electron remains in any of these six ions. Consequently all three pairs are diamagnetic, and Statement II is correct. Options declaring either statement incorrect are therefore excluded.

Step 3: Conclusion Statement I and Statement II are both correct. Hence, the correct option is (A).

Quick Tip to Solve: Write the ion configurations before counting pairs; remember that pale Mn^{2+} is coloured and filled f^{14} ions are diamagnetic.

61. Given below are two statements for catalytic properties of transition metals.

Statement I : First row transition metals which act as catalyst utilise their 3d electrons only for formation of bonds between reactant molecules and atoms on the surface of catalyst.

Statement II : There is increase in the concentration of reactants on the surface of catalyst which strengthens the bonds in reacting molecules.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Both Statement I and Statement II are correct
- (B) Both Statement I and Statement II are incorrect
- (C) Statement I is correct but Statement II is incorrect
- (D) Statement I is incorrect but Statement II is correct

Correct Answer: (B) Both Statement I and Statement II are incorrect

Solution:

Concept:

- Transition-metal surfaces adsorb reactants and provide an alternative reaction pathway of lower activation energy. Adsorption creates interactions between the metal surface and the reacting species.
- The availability of both partially filled d orbitals and outer s orbitals, together with variable oxidation states, helps transition metals bind and activate reactant molecules.

Step 1: Examine the orbital restriction in Statement I For first-row transition metals, the relevant valence orbitals include both 3d and 4s orbitals. Their catalytic behaviour cannot be attributed to participation of 3d electrons only. Electrons and available orbitals permit temporary bonding with species adsorbed on the metal surface. Thus the word “only” makes Statement I incorrect; catalytic interactions are not restricted in the manner claimed.

Step 2: Examine the effect of adsorption in Statement II Adsorption increases the local concentration of reactants at the catalyst surface, bringing reacting species into a favourable arrangement. However, interaction with the surface weakens the bonds that must break in the reaction. The activated species can then react along a pathway with a smaller activation barrier. Statement II incorrectly says that the bonds in the reacting molecules are strengthened, so it is also incorrect.

Step 3: Choose the matching pair The two statements must be judged separately: the first contains an incorrect orbital restriction and the second reverses the effect of surface activation on reactant bonds. Therefore neither statement is correct. Options (A), (C) and (D) each retain at least one of these false statements and are excluded.

Step 4: Conclusion Both Statement I and Statement II are incorrect. Hence, the correct option is (B).

Quick Tip to Solve: In adsorption catalysis, distinguish increased surface concentration from bond strength: reactant bonds are activated and weakened, not strengthened.

62. Given below are two statements :

Statement I : Vapours of the liquid with higher boiling point condense before vapours of the liquid with lower boiling points in fractional distillation.

Statement II : The vapours rising up in the fractionating column become richer in high boiling component of the mixture.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

- (A) Both Statement I and Statement II are true
- (B) Both Statement I and Statement II are false
- (C) Statement I is true but Statement II is false
- (D) Statement I is false but Statement II is true

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- Fractional distillation separates miscible liquids through repeated condensation and vaporisation in a fractionating column. The less volatile component has the higher boiling point.
- The vapour in equilibrium with a liquid mixture is richer in its more volatile, lower-boiling component. The column provides repeated stages of this enrichment.

Step 1: Follow condensation inside the column The fractionating column is hotter near the distillation flask and cooler towards its top. As mixed vapours rise into cooler regions, the component with the higher boiling point condenses more readily. Much of this higher-boiling material returns downwards as liquid reflux. The lower-boiling component can remain in the vapour phase and travel farther up the column. Therefore Statement I is true.

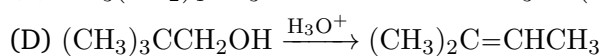
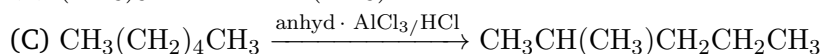
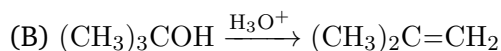
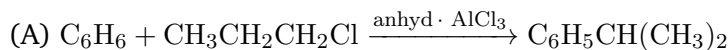
Step 2: Determine the composition of the rising vapour At successive stages, the descending liquid and ascending vapour exchange heat and material. Repeated condensation removes a greater proportion of the less volatile, higher-boiling component from the ascending stream. Repeated vaporisation favours the more volatile component. Consequently the vapour approaching the top is richer in the lower-boiling component, rather than the higher-boiling component asserted in Statement II. Statement II is false.

Step 3: Select the correct combination The combination is therefore true for Statement I and false for Statement II. Option (A) wrongly treats the enrichment claim as true, option (B) wrongly rejects preferential condensation, and option (D) reverses both evaluations. Only option (C) gives the required pair.

Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: In a fractionating column, high-boiling liquid preferentially returns downward while low-boiling vapour becomes enriched upward.

63. The major product of which of the following reactions is not obtained by rearrangement? [4 marks]



Correct Answer: (B) $(\text{CH}_3)_3\text{COH} \xrightarrow{\text{H}_3\text{O}^+} (\text{CH}_3)_2\text{C}=\text{CH}_2$

Solution:

Concept:

- A rearrangement changes the carbon skeleton or the positions of groups through migration, often by a hydride or alkyl shift that produces a more stable carbocation.
- Acid-catalysed dehydration can occur without a rearrangement when loss of water already gives a sufficiently stable carbocation and elimination directly yields the displayed alkene.

Step 1: Identify the reaction without migration In option (B), tert-butyl alcohol, $(\text{CH}_3)_3\text{COH}$, is protonated and loses water to give the tertiary tert-butyl carbocation. Removal of a beta hydrogen then forms $(\text{CH}_3)_2\text{C}=\text{CH}_2$, 2-methylpropene. The carbon skeleton is unchanged: the central carbon remains bonded to the same three carbon atoms. Thus this product is obtained by dehydration and elimination without a hydride or methyl shift.

Step 2: Check the other displayed transformations Option (A) gives isopropylbenzene from benzene and a straight-chain propyl halide under Friedel–Crafts conditions. Formation of the branched propyl substituent involves rearrangement to the more stable secondary electrophile. Option (C) converts straight-chain hexane into branched 2-methylpentane under anhydrous AlCl_3/HCl ; this is skeletal isomerisation and therefore rearrangement.

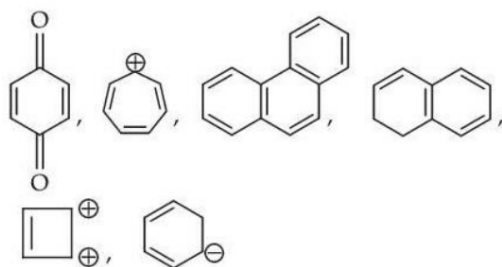
Step 3: Explain why option D rearranges In option (D), neopentyl alcohol, $(\text{CH}_3)_3\text{CCH}_2\text{OH}$, gives $(\text{CH}_3)_2\text{C}=\text{CHCH}_3$. The carbon adjacent to the alcohol-bearing carbon initially has no hydrogen available for ordinary elimination across that bond. A methyl migration changes the skeleton and provides a more favourable carbocation before alkene formation. This contrasts with the direct elimination possible for option (B).

Step 4: Conclusion The major product in option (B) is obtained without rearrangement. Hence, the correct option is (B).

Quick Tip to Solve: Compare carbon connectivity before and after reaction: dehydration of tert-butyl alcohol preserves the skeleton, whereas the other shown transformations require migration or skeletal isomerisation.

64. The total number of aromatic compounds/species from the following is:

[4 marks]



(A) 6

(B) 4

(C) 3

(D) 5

Correct Answer: (B) 4

Solution:

Concept:

- An aromatic species has a planar, cyclic, continuously conjugated pi-electron system and follows Huckel's rule: $4n + 2$ pi electrons, where n is a non-negative integer.
- Count electrons in the closed conjugated ring, including a participating lone pair or an empty p orbital where appropriate. Exocyclic double bonds and an interrupted ring cannot be treated as an ordinary benzene pi system.

Step 1: Classify the species in the first row The first species is para-benzoquinone. Its two exocyclic carbonyl groups produce a cross-conjugated system; it does not possess a benzene-like closed aromatic sextet around the ring, so it is non-aromatic. The second species is the tropylium cation: all seven carbons have p orbitals, and its three double bonds supply six pi electrons. It is aromatic. The fused three-ring species, phenanthrene, has an aromatic delocalised 14-pi-electron framework. The next fused two-ring species is dihydronaphthalene: its right-hand ring retains a benzene sextet, while two saturated carbons interrupt conjugation in the left-hand ring. It is counted as an aromatic compound because it contains the intact aromatic benzene ring. Both fused-ring species are therefore counted.

Step 2: Classify the charged species in the second row The four-membered ring with one double bond and two positively charged carbons is the cyclobutadienyl dication. The positive carbons supply empty p orbitals, so cyclic conjugation is possible with two pi electrons. This satisfies $4n + 2$ for $n = 0$ and is aromatic. The final six-membered anion has only two double bonds and one carbanionic centre; a saturated ring carbon interrupts continuous cyclic conjugation. Hence it is non-aromatic, irrespective of the electron count in the conjugated fragment.

Step 3: Count the aromatic members The aromatic members are the tropylium cation, phenanthrene, dihydronaphthalene and the cyclobutadienyl dication. The quinone and the final interrupted-ring anion are excluded. Thus the total is four; counting either excluded structure would incorrectly increase the answer to five or six, while overlooking the two-electron aromatic dication would give three.

Step 4: Conclusion There are 4 aromatic compounds/species. The four are tropylium, phenanthrene,

dihydronaphthalene with its intact benzene ring, and the two- π -electron cyclobutadienyl dication. Hence, the correct option is (B).

Quick Tip to Solve: Check continuous cyclic conjugation before applying $4n + 2$; a two-electron conjugated cyclic dication can be aromatic, while a ring containing an sp^3 interruption cannot.

65. *n*-Butane on monochlorination under photochemical condition gives an optically active compound "P". "P" on further chlorination gives dichloro compounds.

The number of dichloro compounds obtained (ignore stereoisomers) is :

[4 marks]

- (A) 3
- (B) 4
- (C) 5
- (D) 6

Correct Answer: (B) 4

Solution:

Concept:

- Photochemical chlorination replaces a hydrogen atom by chlorine through a free-radical substitution mechanism. The carbon skeleton of butane is retained.
- A carbon bonded to four different groups is a stereogenic centre. When stereoisomers are ignored, count distinct positional connectivities rather than separate enantiomers or diastereomers.

Step 1: Identify the optically active monochloro derivative Monochlorination of *n*-butane can give 1-chlorobutane or 2-chlorobutane. Only 2-chlorobutane, $\text{CH}_3\text{CHClCH}_2\text{CH}_3$, contains a stereogenic carbon: carbon 2 is bonded to hydrogen, chlorine, methyl and ethyl groups. Therefore the specified compound *P* is a 2-chlorobutane stereoisomer. The primary derivative has no carbon attached to four different groups and cannot be the stated optically active compound.

Step 2: Place the second chlorine at each distinct carbon Keep the original chlorine on carbon 2 of the four-carbon chain. Substitution of a hydrogen on carbon 1 gives 1,2-dichlorobutane; on carbon 2 it gives 2,2-dichlorobutane; on carbon 3 it gives 2,3-dichlorobutane; and on carbon 4 it gives 1,3-dichlorobutane after numbering the product from its nearer substituted end. These are four distinct constitutional structures. The two ends of 2-chlorobutane are not equivalent, because they lie at different distances from the original chlorine.

Step 3: Exclude stereochemical overcounting Several of these products can possess stereoisomers, especially the product with chlorine on both central carbons. The instruction explicitly says to ignore stereoisomers, so those forms are not counted separately. Likewise, further substitution is not continued beyond two chlorine atoms because only dichloro products are requested. The complete connectivity list therefore contains four members, excluding totals of three, five or six.

Step 4: Conclusion The number of dichloro compounds, ignoring stereoisomers, is 4. Hence, the correct option is (B).

Quick Tip to Solve: First fix *P* as 2-chlorobutane, then replace one remaining hydrogen at each of its four carbon positions; count structures before considering stereochemistry.

66. Given below are two statements:

Statement I: Due to increase in van der Waals forces, the order of boiling points is $\text{CH}_3\text{CH}_2\text{CH}_2\text{I} > \text{CH}_3\text{CH}_2\text{I} > \text{CH}_3\text{I}$.

Statement II: As *p*-dichlorobenzene is more symmetric, its melting point is higher than *o*-dichlorobenzene; however, its boiling point is lower than *o*-dichlorobenzene.

In the light of the above statements, choose the correct answer from the options given below:

[4 marks]

- (A) Both Statement I and Statement II are true
(B) Both Statement I and Statement II are false
(C) Statement I is true but Statement II is false
(D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- Within a homologous series of alkyl iodides, increasing molecular size and electron-cloud polarisability strengthen London dispersion forces and raise boiling points.
- Melting point depends strongly on crystal packing and symmetry. Boiling point concerns overcoming intermolecular attractions in the liquid and need not follow the same order as melting point.

Step 1: Evaluate Statement I using molecular size The three compounds all contain iodine, so the main change is increasing alkyl-chain length from methyl to ethyl to propyl. The larger molecules have more electrons and greater polarisability, which strengthens van der Waals dispersion attractions. More energy is required to separate the molecules on boiling. Consequently the order is $\text{CH}_3\text{CH}_2\text{CH}_2\text{I} > \text{CH}_3\text{CH}_2\text{I} > \text{CH}_3\text{I}$, as stated. Statement I is true.

Step 2: Separate melting behaviour from boiling behaviour Para-dichlorobenzene has a more symmetrical arrangement of its chlorine substituents than the ortho isomer. Its molecules pack more efficiently into a crystal lattice, accounting for its higher melting point. This symmetry argument refers to solid-state packing and does not require its boiling point to be higher as well. In the liquid, the ortho isomer has a greater net molecular dipole because its two carbon-chlorine bond dipoles do not cancel as they approximately do in the para isomer.

Step 3: Judge the full second statement and select The additional dipole-related attraction in ortho-dichlorobenzene is consistent with its higher boiling point relative to the para isomer. Thus para-dichlorobenzene can simultaneously have the higher melting point and the lower boiling point, exactly as Statement II says. Both statements are true. Options (B), (C) and (D) incorrectly reject one or both of these independently valid physical-property trends.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

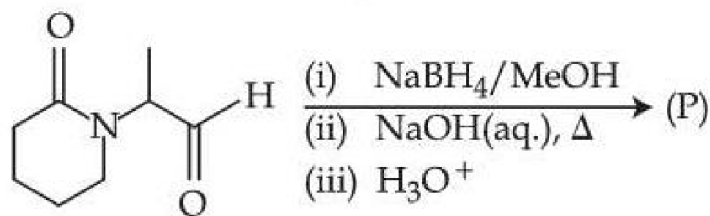
Quick Tip to Solve: Use symmetry and crystal packing for melting-point comparisons; use intermolecular attractions in the liquid for boiling-point comparisons.

67.

Consider the following reaction.

The major product (P) formed is :

[4 marks]

(A) *N*-[1-hydroxypropan-2-yl]piperidine

(B) 1-[1-oxopropan-2-yl]piperidin-2-ol

(C) HOOC(CH₂)₄NHCH(CH₃)CH₂OH(D) HO(CH₂)₅NHCH(CH₃)CH₂OH**Correct Answer:** (C) HOOC(CH₂)₄NHCH(CH₃)CH₂OH**Solution:****Concept:**

- NaBH₄ in methanol reduces an aldehyde to a primary alcohol under the stated conditions; it does not normally reduce an amide carbonyl.
- A lactam is a cyclic amide. Heating with aqueous alkali opens its ring by hydrolysis, forming a carboxylate and an amine; acid work-up gives the carboxylic acid.

Step 1: Identify the two carbonyl groups The starting compound contains a six-membered lactam ring and an aldehyde in the nitrogen substituent –CH(CH₃)CHO. These carbonyls have different reactivities. The lactam carbonyl is resonance-stabilised by the adjacent nitrogen, whereas the aldehyde carbonyl readily accepts hydride. Treating both groups as ordinary ketones would therefore predict the wrong product.

Step 2: Apply the reagents in their stated order The first reagent converts the side chain into –CH(CH₃)CH₂OH, retaining the ring amide. Next, hot aqueous NaOH hydrolyses the lactam carbonyl–nitrogen bond. Following the original ring from its carbonyl carbon to nitrogen leaves four methylene groups between them. The open-chain product is thus [–]OOC(CH₂)₄NHCH(CH₃)CH₂OH. Finally, H₃O⁺ protonates the carboxylate to the acid.

Step 3: Compare the four products Option (A) would require reduction of the lactam carbonyl without opening the ring, which these reagents do not accomplish. Option (B) incorrectly retains the aldehyde and reduces the amide to a hemiaminal. Option (D) changes the lactam carbonyl into an alcohol rather than hydrolysing it. Only option (C) preserves the required carbon skeleton and contains both the carboxylic acid and the newly formed primary alcohol.

Step 4: Conclusion The product is HOOC(CH₂)₄NHCH(CH₃)CH₂OH. Hence, the correct option is (C).

Quick Tip to Solve: Process a reagent sequence one reagent at a time: mild hydride reduces the aldehyde first, while hot aqueous alkali opens the lactam.

68. Which statements are True?

- A. In Hoffmann bromamide degradation, 4 moles of NaOH and 2 moles of Br₂ are consumed per mole of an amide
- B. Hoffmann bromamide reaction is not given by alkyl amides.
- C. Primary amines can be synthesized by Hoffmann bromamide degradation.
- D. Secondary amide on reaction with Br₂ and NaOH will give secondary amine.
- E. The by-products of Hoffmann degradation are Na₂CO₃, NaBr and H₂O.

Choose the correct answer from the options given below :

[4 marks]

- (A) A, C and E only
- (B) B, C and D only
- (C) C and E only
- (D) C, D and E only

Correct Answer: (C) C and E only

Solution:

Concept:

- Hofmann bromamide degradation converts a primary amide RCONH₂ into a primary amine RNH₂, with loss of its carbonyl carbon.
- The balanced overall equation is



Step 1: Check statements A and B The equation consumes four moles of NaOH, but only one mole of Br₂, per mole of primary amide. Statement A incorrectly specifies two moles of bromine, so it is false. Both aliphatic primary amides, such as ethanamide, and aromatic primary amides can undergo the reaction. For instance, ethanamide gives methylamine. Therefore the claim in B that alkyl amides do not give this reaction is also false.

Step 2: Check statements C, D and E The amine produced from RCONH₂ is RNH₂, so C is true. The usual degradation requires a primary amide: after N-bromination, the remaining nitrogen-bound hydrogen is needed in the rearrangement sequence. A secondary amide does not simply furnish a secondary amine by the stated Hofmann reaction. D is consequently false. The balanced equation explicitly contains sodium carbonate, sodium bromide and water as the by-products, making E true.

Step 3: Select the matching combination The true statements are C and E, with A, B and D false. Option (A) includes the incorrect bromine stoichiometry, option (B) includes the false B and D statements, and option (D) includes false statement D. The only combination containing exactly the true statements is option (C).

Step 4: Conclusion Only C and E are true. Hence, the correct option is (C).

Quick Tip to Solve: Write the balanced Hofmann equation before checking a statement set; it settles both reagent consumption and the by-products immediately.

69. The incorrect statement from the following with respect to carbohydrates is :

[4 marks]

- (A) All monosaccharides are reducing sugars.
- (B) The monosaccharide units obtained from hydrolysis of oligosaccharides are always the same.
- (C) Starch and cellulose are typical examples of polysaccharides, which are very high molecular weight compounds of more than ten monosaccharide units.
- (D) Open chain and cyclic structures co-exist at equilibrium that are responsible for certain properties as in the case of D – (+)-glucose.

Correct Answer: (B) The monosaccharide units obtained from hydrolysis of oligosaccharides are always the same.

Solution:

Concept:

- Oligosaccharides yield two to ten monosaccharide units on hydrolysis, but these units may be identical or different.
- A reducing sugar can supply a free carbonyl group through its solution equilibrium. Monosaccharides, including ketoses under alkaline test conditions, behave as reducing sugars.

Step 1: Test the absolute claim in option B The word "always" asserts that every oligosaccharide gives only identical monosaccharides. Sucrose is a counterexample: its hydrolysis yields one molecule each of glucose and fructose. Lactose similarly yields glucose and galactose. Maltose does give two glucose units, but that single example cannot establish a universal rule. Therefore the statement in (B) is incorrect.

Step 2: Verify the remaining statements Option (A) expresses the standard reducing behaviour of monosaccharides. Aldoses have an accessible aldehyde form, and ketoses can isomerise under the alkaline conditions used for reducing-sugar tests. Option (C) correctly identifies starch and cellulose as high-molecular-mass polysaccharides built from many monosaccharide units. They differ in their glycosidic linkage and resulting structure, but both belong to the stated class.

Step 3: Relate cyclic and open-chain forms In aqueous glucose, cyclic hemiacetal forms are in equilibrium with a small amount of the open-chain aldehyde. Ring opening and reclosing explains interconversion of anomers and mutarotation, while the open-chain form accounts for characteristic carbonyl reactions. Thus the coexistence described in (D) is correct. Since the question asks for the incorrect statement, these correct descriptions must not be selected.

Step 4: Conclusion Oligosaccharide hydrolysis need not give identical monosaccharides; sucrose gives glucose and fructose. Hence, the correct option is (B).

Quick Tip to Solve: For a statement containing "always", test sucrose and lactose as well as maltose; their hydrolysis products quickly expose this false generalisation.

70. Which of the following amino acid will give violet coloured complex with neutral ferric chloride solution? [4 marks]

- (A) Threonine
- (B) Serine
- (C) Tyrosine
- (D) Cysteine

Correct Answer: (C) Tyrosine

Solution:

Concept:

- Neutral ferric chloride gives characteristic coloured complexes with phenolic groups through coordination of the phenolic oxygen to iron(III).
- Tyrosine contains a phenolic hydroxyl group on an aromatic ring; an ordinary aliphatic alcohol group is chemically different.

Step 1: Identify the functional groups in the choices Tyrosine has the side chain $-\text{CH}_2-\text{C}_6\text{H}_4-\text{OH}$, with the hydroxyl group directly attached to its benzene ring. It therefore contains a phenol group in addition to the amino and carboxyl groups common to amino acids. Threonine and serine instead carry aliphatic hydroxyl groups. Cysteine carries a thiol group, $-\text{SH}$, rather than a phenolic hydroxyl group.

Step 2: Apply the ferric chloride test The phenolic oxygen of tyrosine can coordinate to Fe^{3+} , producing the violet coloured complex characteristic of this test. This is a functional-group identification: it does not follow merely from the presence of an oxygen atom or the general amino-acid structure. The aromatic attachment changes the acidity and coordination behaviour of the hydroxyl group and makes the phenol test appropriate.

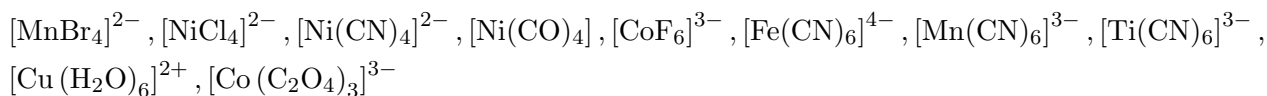
Step 3: Eliminate the other amino acids Threonine contains a secondary alcohol side chain and serine a primary alcohol side chain; neither possesses the phenolic group needed for the stated violet ferric chloride test. Cysteine is identified by sulfur-related chemistry, not by this phenolic colour reaction. The amino and carboxyl groups alone cannot distinguish the options because they are shared by all four amino acids. Hence tyrosine is uniquely selected by the side-chain functionality.

Step 4: Conclusion Tyrosine gives the violet coloured complex with neutral ferric chloride. Hence, the correct option is (C).

Quick Tip to Solve: Locate the hydroxyl group: an OH directly on a benzene ring signals the phenol test, whereas an OH on an aliphatic carbon does not.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. Number of paramagnetic complexes among the following is ____ .



[4 marks]

Solution:

Concept:

- A complex is paramagnetic if its metal-ion electron configuration contains one or more unpaired electrons. Determine oxidation state and d -electron count before applying crystal-field splitting.
- Weak-field tetrahedral and octahedral complexes generally favour high spin; strong-field cyanide and carbonyl ligands favour pairing. Square-planar nickel(II) cyanide is diamagnetic.

Step 1: Count the paramagnetic weak-field complexes In $[\text{MnBr}_4]^{2-}$, manganese is +2, hence d^5 ; its tetrahedral weak-field arrangement has five unpaired electrons. $[\text{NiCl}_4]^{2-}$ has nickel(II), d^8 , in a tetrahedral field and has two unpaired electrons. $[\text{CoF}_6]^{3-}$ contains cobalt(III), d^6 , and weak-field fluoride gives high-spin $t_{2g}^4 e_g^2$, with four unpaired electrons. These contribute three paramagnetic complexes.

Step 2: Inspect the remaining complexes with unpaired electrons $[\text{Mn}(\text{CN})_6]^{3-}$ contains manganese(III), d^4 ; even its low-spin t_{2g}^4 configuration has two unpaired electrons. $[\text{Ti}(\text{CN})_6]^{3-}$ contains titanium(III), d^1 , so its single electron is unpaired regardless of strong-field pairing preferences. $[\text{Cu}(\text{H}_2\text{O})_6]^{2+}$ contains copper(II), d^9 , with one unpaired electron. These add three more.

Step 3: Exclude the diamagnetic complexes $[\text{Ni}(\text{CN})_4]^{2-}$ is square-planar d^8 with all electrons paired. $[\text{Ni}(\text{CO})_4]$ has nickel in oxidation state zero, hence d^{10} , and is diamagnetic. $[\text{Fe}(\text{CN})_6]^{4-}$ is iron(II), low-spin d^6 , with t_{2g}^6 . Finally, $[\text{Co}(\text{C}_2\text{O}_4)_3]^{3-}$ is a low-spin cobalt(III), d^6 , complex and is also diamagnetic. These four do not enter the count.

Step 4: Conclusion The number of paramagnetic complexes is $3 + 3 = \boxed{6}$.

Quick Tip to Solve: Strong field does not guarantee diamagnetism: low-spin d^4 still has two unpaired electrons, while d^1 necessarily has one.

72. 'x' is the product which is obtained from benzene by reacting it with carbon monoxide and hydrogen chloride in the presence of cuprous chloride. 'y' is the major product obtained from the benzene by reacting it with ethanoyl chloride in the presence of anhydrous AlCl_3 . Product (major) obtained by heating x and y in the presence of alkali is z. Total number of π (pi) electrons in z is ____

[4 marks]

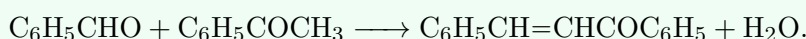
Solution:

Concept:

- Gattermann–Koch formylation introduces $-\text{CHO}$ into benzene, while Friedel–Crafts acylation with ethanoyl chloride introduces $-\text{COCH}_3$.
- Benzaldehyde has no alpha hydrogen, but acetophenone does. Their base-promoted crossed aldol condensation followed by dehydration forms an alpha,beta-unsaturated ketone.

Step 1: Identify x and y Carbon monoxide and hydrogen chloride in the stated cuprous-chloride-assisted formylation give benzaldehyde, $x = \text{C}_6\text{H}_5\text{CHO}$. Ethanoyl chloride with anhydrous AlCl_3 acylates benzene to give acetophenone, $y = \text{C}_6\text{H}_5\text{COCH}_3$. The former supplies the electrophilic aldehyde carbonyl; the latter supplies the enolate because its methyl group has alpha hydrogens.

Step 2: Form the crossed condensation product Alkali removes an alpha hydrogen from acetophenone. Its enolate adds to the carbonyl carbon of benzaldehyde, giving a beta-hydroxy ketone after protonation. Heating promotes loss of water and forms the conjugated product chalcone:



The carbonyl retained in the product belongs to acetophenone; the benzaldehyde carbonyl becomes the alkene end after dehydration. This identifies the full product before counting electrons.

Step 3: Count every pi-electron contribution The product contains two benzene rings, each with six pi electrons, contributing twelve in total. Its newly formed carbon–carbon double bond contributes two more pi electrons. The remaining carbonyl double bond also has one pi bond and contributes two pi electrons. Therefore

$$N_\pi = 2(6) + 2 + 2 = 16.$$

Counting only the aromatic rings would omit both non-aromatic pi bonds; counting only the conjugated alkene would omit the carbonyl contribution.

Step 4: Conclusion The major product is chalcone, $\text{C}_6\text{H}_5\text{CH}=\text{CHCOC}_6\text{H}_5$, containing 16 pi electrons.

Quick Tip to Solve: Draw the condensed chalcone skeleton before counting: two phenyl rings plus one $\text{C}=\text{C}$ and one $\text{C}=\text{O}$ contribute $12+2+2$ pi electrons.

73. Consider two radiations of wavelengths

$$\lambda_1 = 2000 \text{ \AA}$$

$$\lambda_2 = 6000 \text{ \AA}$$

The ratio of the energies of these two radiations $\left(\frac{E_1}{E_2}\right)$ is ____ (Nearest integer).

[4 marks]

Solution:

Concept:

- Electromagnetic radiation consists of photons whose energy is $E = h\nu$, where h is Planck's constant and ν is frequency.
- For radiation in vacuum, $c = \nu\lambda$, so $E = hc/\lambda$. Photon energy therefore varies inversely with wavelength, rather than directly with it.

Step 1: Express both energies using the same constants The wavelengths are $\lambda_1 = 2000 \text{ \AA}$ and $\lambda_2 = 6000 \text{ \AA}$. Since both radiations are electromagnetic waves, the constants h and c are identical in their energy expressions. Thus

$$E_1 = \frac{hc}{\lambda_1}, \quad E_2 = \frac{hc}{\lambda_2}.$$

There is no need to insert approximate numerical values for these constants: they cancel in the requested ratio.

Step 2: Evaluate the inverse wavelength ratio Divide the first expression by the second, retaining the order E_1/E_2 specified in the question:

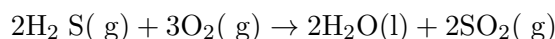
$$\frac{E_1}{E_2} = \frac{hc/\lambda_1}{hc/\lambda_2} = \frac{\lambda_2}{\lambda_1} = \frac{6000}{2000} = 3.$$

Both wavelengths use the same unit, so their unit cancels. Equivalently, converting both to metres gives $2.0 \times 10^{-7} \text{ m}$ and $6.0 \times 10^{-7} \text{ m}$, with the same ratio. The shorter wavelength must have the larger photon energy, which independently confirms that the ratio exceeds one.

Step 3: Conclusion The energy ratio is $E_1/E_2 = 3$. It is dimensionless and already an integer, so the numerical answer is $\boxed{3}$.

Quick Tip to Solve: For photon energy ratios, invert the wavelength ratio first. The shorter wavelength must correspond to the larger energy; this catches an accidental reciprocal.

74. Consider the reaction



The magnitude of enthalpy change for the reaction in kJ mol^{-1} is ____ . (Nearest integer)

$$\text{Given : } \Delta_f H^\ominus (\text{H}_2\text{S}) = -20.1 \text{ kJ mol}^{-1}$$

$$\Delta_f H^\ominus (\text{H}_2\text{O}) = -286.0 \text{ kJ mol}^{-1}$$

$$\Delta_f H^\ominus (\text{SO}_2) = -297.0 \text{ kJ mol}^{-1}$$

[4 marks]

Solution:

Concept:

- The standard reaction enthalpy follows Hess's law: $\Delta_r H^\ominus = \sum_{\text{products}} \nu \Delta_f H^\ominus - \sum_{\text{reactants}} \nu \Delta_f H^\ominus$. Each formation enthalpy is multiplied by its coefficient in the balanced reaction.
- The standard enthalpy of formation of an element in its reference state is zero. Consequently $\Delta_f H^\ominus (\text{O}_2(\text{g})) = 0$; the stated liquid state of water must be retained.

Step 1: Calculate the total product formation enthalpy The reaction produces two moles each of $\text{H}_2\text{O}(\text{l})$ and $\text{SO}_2(\text{g})$. Using the given formation enthalpies,

$$\begin{aligned} \sum_{\text{products}} \nu \Delta_f H^\ominus &= 2(-286.0) + 2(-297.0) \\ &= -572.0 - 594.0 \\ &= -1166.0 \text{ kJ mol}^{-1}. \end{aligned}$$

These coefficients refer to the reaction exactly as written, not to one mole of H_2S .

Step 2: Subtract the reactant enthalpy and take the magnitude The reactants contribute

$$\begin{aligned} \sum_{\text{reactants}} \nu \Delta_f H^\ominus &= 2(-20.1) + 3(0) \\ &= -40.2 \text{ kJ mol}^{-1}. \end{aligned}$$

Therefore

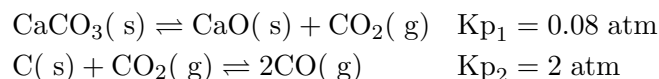
$$\Delta_r H^\ominus = -1166.0 - (-40.2) = -1125.8 \text{ kJ mol}^{-1}.$$

The negative sign indicates an exothermic reaction. The question asks for the magnitude, so use 1125.8, and then round to the nearest integer. Dividing by two would describe a different stoichiometric reaction and would not answer the expression printed here.

Step 3: Conclusion The magnitude of the reaction enthalpy is $1125.8 \text{ kJ mol}^{-1}$, which rounds to 1126 kJ mol^{-1} . The required numerical answer is 1126 .

Quick Tip to Solve: Write products minus reactants with coefficients before substituting negative formation enthalpies. Take the absolute value only after obtaining the signed reaction enthalpy.

75. Solid carbon, CaO and CaCO₃ are mixed and allowed to attain equilibrium at T K .



The partial pressure of CO is $__ \times 10^{-1}$ atm

[4 marks]

Solution:

Concept:

- Pure solids have unit activity and do not appear in the equilibrium-constant expression. The gaseous partial pressures alone enter the two heterogeneous equilibria.
- For $\text{CaCO}_3(\text{s}) \rightleftharpoons \text{CaO}(\text{s}) + \text{CO}_2(\text{g})$, $K_{p1} = p_{\text{CO}_2}$. For $\text{C}(\text{s}) + \text{CO}_2(\text{g}) \rightleftharpoons 2\text{CO}(\text{g})$, $K_{p2} = p_{\text{CO}}^2/p_{\text{CO}_2}$, using the pressure convention of the supplied constants.

Step 1: Use the carbonate equilibrium to fix the carbon dioxide pressure At the stated temperature, the calcium carbonate and calcium oxide solids coexist at equilibrium. Their activities cancel from the expression for the first reaction, leaving

$$p_{\text{CO}_2} = K_{p1} = 0.08 \text{ atm}.$$

This is the partial pressure of carbon dioxide, not the total gas pressure. The carbon present participates in the second equilibrium but does not add a concentration or pressure factor because it is also a pure solid.

Step 2: Substitute in the carbon monoxide equilibrium Apply the second equilibrium expression to the same equilibrium gas mixture:

$$\begin{aligned}K_{p2} &= \frac{p_{\text{CO}}^2}{p_{\text{CO}_2}}, \\ p_{\text{CO}}^2 &= (2 \text{ atm})(0.08 \text{ atm}) \\ &= 0.16 \text{ atm}^2.\end{aligned}$$

A partial pressure is nonnegative, so take the positive square root:

$$p_{\text{CO}} = \sqrt{0.16} \text{ atm} = 0.40 \text{ atm} = 4 \times 10^{-1} \text{ atm}.$$

Substitution back gives $0.40^2/0.08 = 2 \text{ atm}$, verifying the second equilibrium constant. The exponent two on the carbon monoxide pressure is essential because its stoichiometric coefficient is two.

Step 3: Conclusion The carbon monoxide partial pressure is $\boxed{0.40 \text{ atm}} = \boxed{4 \times 10^{-1} \text{ atm}}$. The coefficient required in the blank is $\boxed{4}$.

Quick Tip to Solve: In coupled heterogeneous equilibria, first use the reaction with a single gaseous species to fix its pressure; then substitute that pressure into the other equilibrium expression.